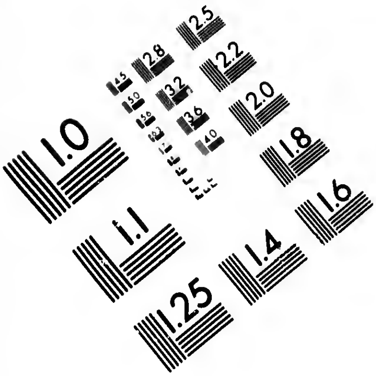

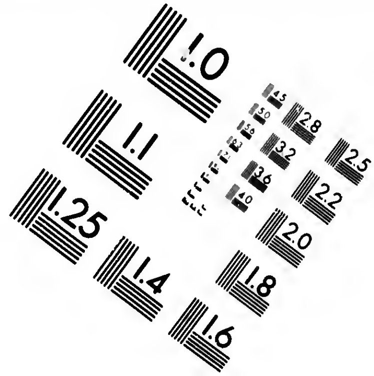

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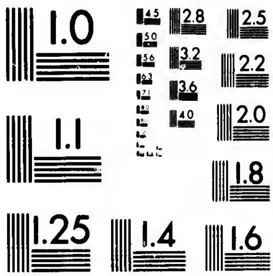

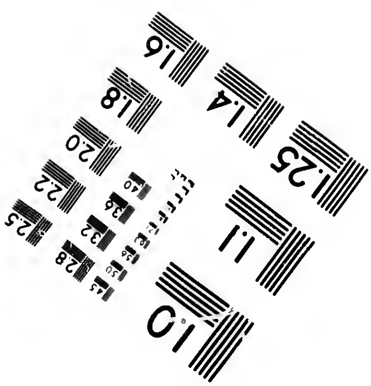

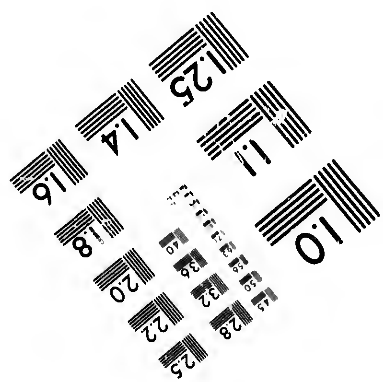

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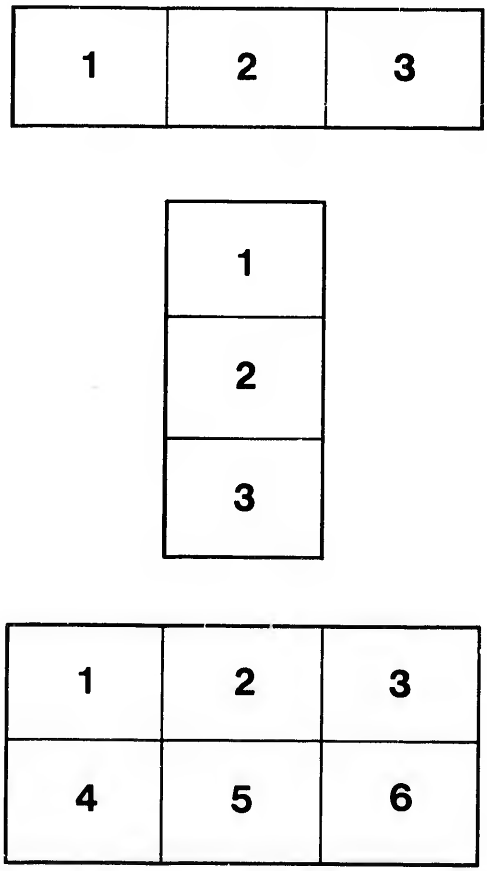

KEY

TO

BAILLAIRGE'S

STEREOMETRICAL TABLEAU.

NEW SYSTEM OF MEASURING

ALL

Bodies,---Segments, Frusta and Ungulæ of Such bodies

BY ONE AND THE SAME RULE.

FOR THE USE OF

ARCHITECTS, ENGINEERS, SURVEYORS, PROFESSORS OF MATHEMATICS, GEOMETRY, DESIGN, DIRECTORS OF UNIVERSITIES, COLLEGES, SEMINARIES, CONVENTS AND OTHER EDUCATIONAL INSTITUTIONS, SCHOOLS OF ARTS, TRADES AND INDUSTRIAL DESIGN, MACHINISTS MEASURERS, GUAGERS, CUSTOM HOUSE AND EXCISE OFFICERS, SHIP BUILDERS, CONTRACTORS, ARTISANS AND OTHERS.

By CHS. BAILLAIRGÉ,

ARCHITECT, ENGINEER, SURVEYOR,

HONORARY MEMBER

OF THE SOCIETY FOR THE GENERALIZATION OF EDUCATION IN FRANCE AND OF OTHER LITERARY AND SCIENTIFIC SOCIETIES.

Recipient of seven Medals in Gold, Silver and Bronze, awarded him in Europe for his discovery and invention.

QUEBEC:
C. DARVEAU, EDITOR AND PUBLISHER,
No. 8, Mountain Hill.

1874.

AL TABLEAU !

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BAILLAIRGE'S STEREOME

New system of determining the solid contents of a body

(Extract from the "Quebec Daily Mercur

Mr. Baillairgé's lecture on Wednesday evening last, before the Literary and Historical Society of Quebec, proved once more how very interesting, even in a popular sense, an otherwise dry and abstruse subject, may become, when ably handled.

The lecturer showed the relationship of geometry to all the industries of life. He traced its origin from remote antiquity, its gradual development up to the present time. He showed how it is the basis of all our public works, and how we are indebted to it for all the constructive arts: its relationship to mechanics, hydraulics, optics, and all the physical sciences. The fairer portion of mankind, said Mr. B., have the keenest, most appreciative perception of its advantages and beauties, as evidenced in the evervarying combinations so cunningly devised in their designs for needle tracery, laces and embroidery. He showed its relationship to chemistry in crystallization and polarization; to botany and zoology in the laws of morphology; to theology, and so on. In treating of the circle and other conic sections, he drew quite a poetical comparison between the engineer who traces out his curves among the woods and waters of the earth, and the astronomer who sweeps out his mighty circuits amidst the starry forests of the heavens. The parabola was fully illustrated in its application to the throwing of projectiles of war, also as evidenced in jets of water, the speaking trumpet, the mirror and the reflector, which, in light-houses, gathers the rays of light, as it were, into a bundle, and sends them forth together on their errand of humanity. In treating of the ellipse, this almost magic curve which is traced out in the heavens by every planet that revolves about the sun, by every satellite about its primary, he alluded to that most beautiful of all ovals —the face of lovely woman. He showed how the re-appearance of a comet may now be predicted even to the very day it heaves in sight, and though it has been absent for a century, and how in former ages, when these phenomena were unpredicted, they burst upon the world in unexpected moments, carrying terror everywhere and giving rise to the utmost anxiety and consternation, as if the end of all things were at hand; in a word, Mr. Baillairgé went over the whole field of geometry and mensuration, both plane and spherical; a difficult feat within the limits of a single lecture; and kept the audience, so to say, entranced with interest for two whole hours, which the president, Dr. Anderson, remarked: were to him as but one; and no doubt it must have been so to others, since Mr. Wilkie, in seconding the vote of thanks proposed by Capt. Ashe, alluded to the pleasure with which he had listened to the lecture as is, he said, it were like poetry to him, instead of the unpromising matter foreshadowed in the title. Mr. Baillairgé next explained in detail his stereometrical tableau, which we hope to see soon introduced into all the schools of this Dominion. He showed how conducive it will be in shortening the time heretofore devoted to the study of solids and

even to that of plane and convex superfic
gonometry, geometrical projection pers
the development of surfaces, shades and
like. Mr. Wilkie, so far as opportunity
him of proving the calculations, corrobora
tement in relation to the immense savin
many abstruse problems which general
days to solve, can now (if the rule be, as
serts, so generally applicable, and, as has
so many persons in testimonials over their
with the help of the new formula and table
in as many minutes: to say nothing of th
are in imparting at a glance a knowledge
clature or names, and an acquaintanceship
shapes and figures. He showed how, to
engineer, the builder and mechanic, the m
tive of the forms and relative proportio
roofs, domes, piers and quays, cisterns and
drons, vats, casks, tubs and other vessels
works of all kinds, comprising railroads a
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tent, the square or splayed opening of a
nich or loophole in a wall, the vault or ar
church or hall, the billiard or the cannon b
ger scale, the moon, earth, sun and pla
lairgé, we may add, has received an ord
from the Minister of Education of New
the view of introducing it into all the scho
vince; and Mr. Vannier, in writing to Mr
France, on the 10th of January last, to ad
granting of his letters-patent for that co
Messrs. Humbert & Noé, the President
the Society for the Generalization of Educ
have intimated their intention, at their nex
ing, of having some mark of distinction c
for the benefit which his invention and dis
to confer on education. Mr. Giard, in writ
lairgé, on the part of the Hon. Mr. Chauv
Public Instruction, says: "Il se fera un de
"mander l'adoption dans toutes les maison
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University, Mr. Maingui writes: "Plus o
"approfondit cette formule du cubage d
"est enchanté (the more one marvels) de
"sa clarté et surtout de sa grande général
McQuarries, B. A. "shall be delighted to
"tedious processes superseded by a formul
"so exact." Newton, of Yale College,
"considers the tableau a most useful
"showing the variety and extent of the ap
"formula." The College l'Assomption
"Baillairgé's system as part of their cours
Mr. Wilkie has written to the author that

EOMETRICAL TABLEAU!

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Daily Mercury" of 26th March, 1872.)

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rical projection perspective drawing,
urfaces, shades and shadows, and the
o far as opportunity had been afforded
alculations, corroborated Mr. B.'s sta-
the immense saving in time, where
ems which general required hours or
w (if the rule be, as Mr Baillairgé as-
plicable, and, as has been certified by
estimonials over their own signatures,)
new formula and tableau, be performed
to say nothing of the use the models
glance a knowledge of their nomen-
an acquaintanceship with their varied
He showed how, to the architect and
and mechanic, the models are sugges-
nd relative proportions of buildings,
nd quays, cisterns and reservoirs, caul-
bs and other vessels of capacity, eath-
omprising railroads and other cuttings
he shaft of the Greek and Roman col-
ancy timber, saw-logs, the camping
played opening of a door or window,
wall, the vault or arched ceiling of a
illiard or the cannon ball, or, on a lar-
, earth, sun and planets. Mr. Bail-
has received an order for a tableau
Education of New Brunswick, with
ng it into all the schools of that Pro-
nier, in writing to Mr Baillairgé, from
of January last, to advise him of the
rs-patent for that country, says that
Noé, the President and Secretary of
eneralization of Education in France,
ntention, at their next general meet-
mark of distinction conferred on him
his invention and discovery are likely
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BAILLAIRGE'S STEREO

HONORARY MEMBER OF THE SOCIETY FOR THE GENE

New system of measuring all Bodies, Segments, Frustums a

(Patented in Canada, in the United St

This is a Case 5 feet long, 3 feet wide and 5 inches deep, with a hin exhibiting and affording free access to some 200 well-finished Hardwo form, each of which being merely attached to the board, by means of a wi Student or Professor.

The use of the Tableau and accompanying Treatise, reduces the whole science and art of Mensuration from the study of a year to that of a day or two, and so simplifies the study and teaching of Solid Geometry, the Nomenclature of Geometrical and other forms, the development of surfaces, geometrical projection and perspective, plane and curved areas and Spherical Geometry, and Trigonometry, and mensuration of surfaces and solids, that the several branches hereinbefore mentioned may now be taught even in the most elementary schools, and in convents, where such study could not even have been dreamed of heretofore.

Each Tableau is accompanied by a Treatise explanatory of the mode of measurement by the "Prismoidal Formula," and an explanation of the solid, its nature, shape, opposite bases, and middle section.

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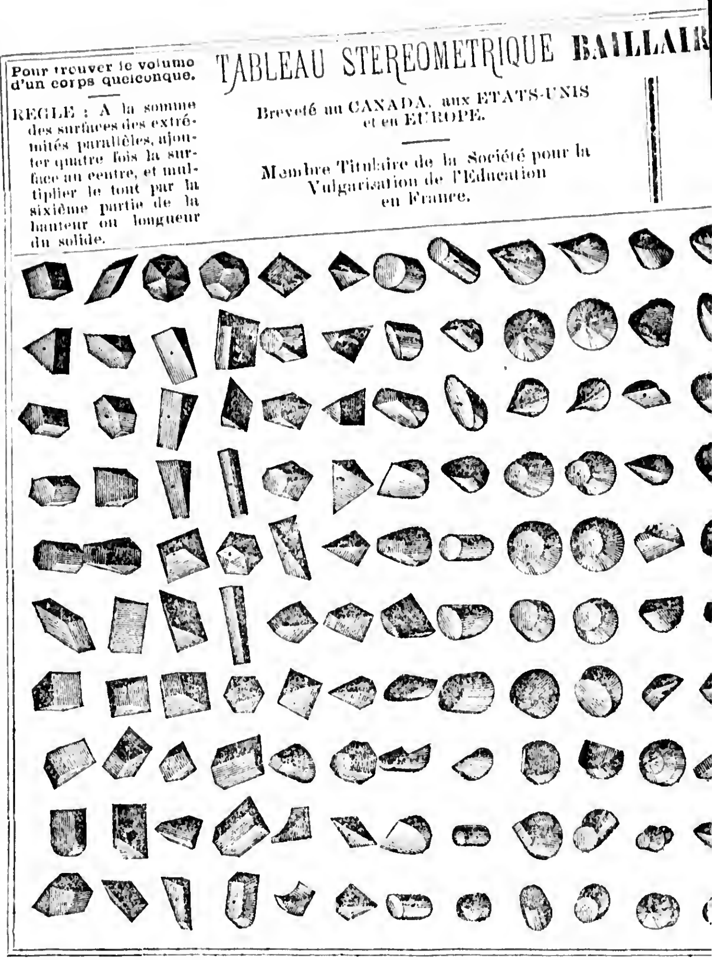
Agents wanted for the sale of the Tableau in Canada, the United States, &c.



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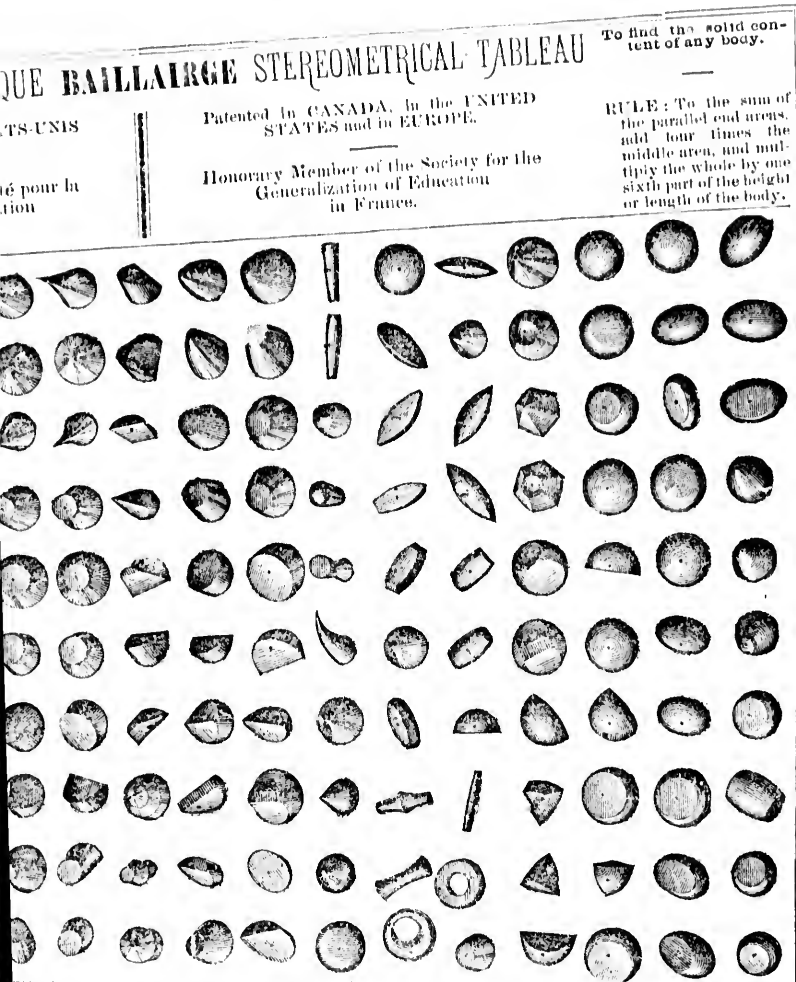
REOMETRICAL TABLEAU !

...OR THE GENERALIZATION OF EDUCATION IN FRANCE, &c., &c.

...s, Frustums and Ungulas of these bodies, by one and the same rule.

...the United States of America, and in Europe.)

...ep, with a hinged Glass Cover, under Lock and Key, so as to exclude dust while ...ished Hardwood Models of every conceivable Elementary, Geometrical or other ... means of a wire-peg or nail, can be removed and replaced at pleasure, by the



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QUEBEC,
CANADA.

Honorary Member of the Society for the Generalization of Education in France, &c., &c.

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EAL TABLEAU!

VTION IN FRANCE, &c., &c.

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exh: conceivable Elementary, Geometrical or other
fornbe removed and replaced at pleasure, by the
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TABLEAU

To find the solid content of any body.

RULE: To the sum of the parallel end areas, add four times the middle area, and multiply the whole by one sixth part of the height or length of the body.

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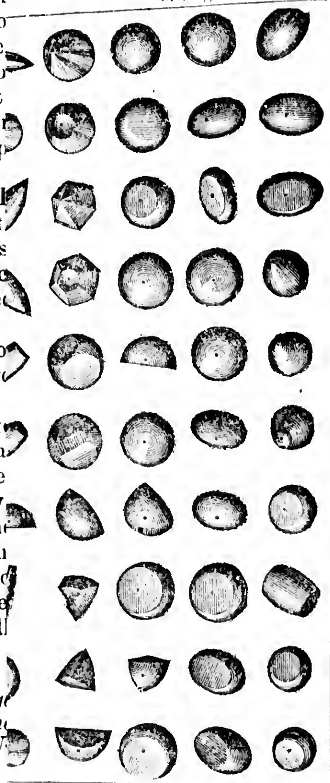

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QUEBEC:

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No. 8, Mountain Hill.

1874.

READ

THE PREFACE.

The question may be asked "If the system be so simple, why so voluminous a "KEY"? Now, it will be immediately seen that the present work is in reality, for the most part, a mere "Mensuration of Areas" which might perhaps have been omitted, since there are already many works which treat on that subject, and that the mode of measuring the surface or area of any solid is supposed to be known before its cubical contents can be arrived at. It is however more satisfactory for Teachers in general, Professors and Students to find thus brought together in a single volume, all that they require, than to have to seek it elsewhere. The mensuration of areas is not at all superfluous, even in the "Key"; since, in point of fact the whole difficulty and labor of computing the solid contents of any body, consists in determining the areas of certain of its component faces and sections.

That which also contributes largely to swell the dimensions of the "Key", is the great number of examples, fully worked out, of the author's system as applied to the computation of the most intricate solids, and the numerous tables of which the great utility will become apparent, when, having to compute the capacity of any boiler tub, vat or cask—the volume of a cylinder, sphere, Spheroid, conoid or of any segment, frustum or ungula of such bodies, the calculation will be found, so to say, fully worked out, since it will suffice to take out the requisite areas, add them and multiply their sum by the sixth part of the length or altitude of the body; after which a simple multiplication or division (as the case may be) of the units so obtained, will reduce them to inches, feet, mètres, gallons, litres, &c., or to any other units greater or less than the first.

At page XXIX, however; that is, after the testimonials, will be found an

ABRIDGED OR SYNOPTICAL KEY TO THE TABLEAU

and, to any one who understands the nomenclature of solid forms and the mensuration of areas, this Abridged Key contains all that is essential to the full and entire intelligence of the author's system.

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&c., &c., &c.

SYNOPTICAL OR ABRIDGED

KEY

TO THE AUTHOR'S NEW SYSTEM

OF MEASURING ANY SOLID,

SEGMENT, FRUSTUM OR UNGULA OF SUCH SOLID,

BY ONE AND THE SAME RULE.

(**1.**) *To the sum of the areas of the opposite and parallel ends or bases of the body to be measured, add four times the area of a section thereof parallel to these bases and equidistant from each of them, and multiply the whole by the sixth part of the height or length of the solid.*

(**2**) To be brief, we will call "*intermediate or half-way section*" the section in question in the formula; or again, and at will; "*centre section*" "*middle section,*" and we shall always designate this section by the letter M, initial letter of the word *middle* as we designate by B and B′ the opposite bases or ends of the solid, and by L or H its length or height.

(**3**) The *length* or *height* of the solid under consideration, shall always be the distance between its parallel bases or ends, that is the perpendicular drawn from one of these bases to the other or to the plane of this base, produced if necessary.

Then the formula will write:

$$\text{Volume} = (\text{area B} + 4 \text{ area M} + \text{area B}') \times \tfrac{1}{6} \text{ L or H.}$$

or:

$$\text{V.} = (\text{B} + 4\text{ M} + \text{B}')\, \tfrac{1}{6} \text{ L or H.}$$

or,

to dispose the areas so as to facilitate their addition:

$$\text{V} = \left\{ \begin{array}{l} + \quad \text{area B} \\ + 4 \text{ area M} \\ + \quad \text{area B} \end{array} \right\} \times \tfrac{1}{6} \text{ L or H,} \qquad \text{or} \quad \left\{ \begin{array}{l} + \quad \text{B} \\ + \ 4\text{M} \\ + \quad \text{B}' \end{array} \right\}$$

Sum of the areas

$\times \tfrac{1}{6}$ **L or H.**

Nature and value of the bases B, B'.

(4) Sometimes one of the ends or bases of the solid, as with the pyramid, cone, conoid, segment or ungula of a sphere, spheroid, or spindle, &c., will be but a point and its area, consequently, null or equal to zero (0). Sometimes, each of the bases will be null as to area or =0, as in the case of the sphere and spheroid; now, one of the bases will be a simple line, as for the wedge and certain prismoids and ungulæ, and its surface again null; at other times, each of the bases, as with certain prismoids, will be but a line and the surfaces null, as before; but in all cases, the author advises the pupil to maintain entire the formula and to write, as the case may be:

$$\text{V.} = \overset{\text{areas}}{\left\{\begin{matrix} + & B \\ + & 4\,M \\ + & B' \end{matrix}\right\}} \qquad \text{or, V.} = \overset{\text{areas}}{\left\{\begin{matrix} + & 0 \\ + & 4\,M \\ + & B' \end{matrix}\right\}}$$

$$\text{Sum} \times \tfrac{1}{6}\,\text{L or H.} \qquad \text{Sum} \times \tfrac{1}{6}\,\text{L or H.}$$

$$\text{or, V.} = \overset{\text{areas}}{\left\{\begin{matrix} + & B \\ + & 4\,M \\ + & 0 \end{matrix}\right\}} \qquad \text{or, V.} = \overset{\text{areas}}{\left\{\begin{matrix} + & 0 \\ + & 4\,M \\ + & 0 \end{matrix}\right\}}$$

$$\text{Sum} \times \tfrac{1}{6}\,\text{L or H.} \qquad \text{Sum} \times \tfrac{1}{6}\,\text{L or H.}$$

(5) **REM.** It is clear from what precedes that the respective surfaces in question are all plane surfaces, or must be considered as such, and that, with the author's system, every surface is null, to which a plane surface or a plane can touch but in one point, as in the sphere, spheroid and conoid; which does not prevent one from measuring in the same manner by the formula, and with the same accuracy, a spherical cone or pyramid, or any frustum of such a body comprised between paralled or concentric bases, one of which is consequently concave and the other convex.

(6) These enunciations would be quite sufficient to give a perfect understanding of the author's system, but some observations concerning more particularly, if not each of the solids of the tableau, at least every category or class thereof will perhaps not be useless.

(7) We say "class" or "category" and in fact it is proper to observe that the solids are disposed, on the tableau, by groups or families, each in one or several vertical rows. These rows are 20 in number and the horizontal rows 10 in number, forming 200 pieces.

The first row to the right (it would be indifferent to reverse the order and begin at the left) comprises the prism under some of its varied forms.

(**8**) The four following ranges offer the prismoid, under several diversified aspects (see introduction, page 6) including the regular or platonic solids, (dodecahedron, icosahedron, &c.,) and certain ungulæ of prisms.

(**9**) The sixth row, still going towards the left, is the pyramid and the frustum of that solid.

(**10**) Rows 7 and 8 show the right, inclined, truncated cylinder, and the numerous ungulæ, and frusta of ungulæ, of this solid, with also some cylindroids.

(**11**) 9 and 10 are the right and inclined cones, their frusta and ungulæ.

(**12**) 11 is the concave cone with its varieties and sections. 12 and 13 are the right and inclined parabolic and hyperbolic conoids, with their frusta, ungulæ and truncated ungulæ.

(**13**) 14, 15 and 16, the flattened and elongated spindles with their decomposed parts and varieties.

(**14**) 17 and 18 are the sphere and its segments, frusta, ungulæ, &c., spherical cone and pyramid and frusta of these bodies between parallel bases. These solids offer also to the appreciation the spherical, tri-rectangular, tri-acutangular, tri-obtusangular, &c., triangle, and facilitate to the pupil, the understanding of spherical geometry and trigonometry, and to the professor, the teaching of these sciences.

(**15**) 19 and 20, finally, are the flattened and elongated spheroid with the decomposed parts of these bodies.

See again on this subject "The Introduction" page 7.

Let us first consider the

PRISM OR CYLINDER,

Right, Inclined, Twisted. [1]

(**16**) The prism is a body whose breadth or size is every where equal or uniform; it is, in other terms, a solid which throughout its whole length or height is of invariable diameter or thickness, and the opposite and parallel bases or ends of which, as well as each

1. See the Introduction, page 11, last paragraph, letter of the Revd. M. Billion mathematician of the St. Sulpice Seminary, Montreal.

section parallel to these bases, are consequently, similar and equal plane figures; these figures may indifferently be rectilinear, curvilinear or mixtilinear.

We will then obtain the solidity or volume by making

$$V. = \frac{\left\{\begin{matrix} + \text{ area } B \\ + 4 \text{ area } M \\ + \text{ area } B' \end{matrix}\right\}}{\text{Sum of areas} \times \frac{1}{6} L \text{ or } H.} \quad \text{and, supposing the base } = 1, \quad = \frac{\left\{\begin{matrix} +1 \\ +4 \\ +1 \end{matrix}\right\}}{\text{Sum of areas} \times \frac{1}{6} L. \text{ or } H.}$$

$$= \left\{\begin{matrix} 6 \text{ a. } B \\ \text{or } 6 \text{ a. } M \\ \text{or } 6 \text{ a. } B' \end{matrix}\right\} \times \frac{1}{6} L. \text{ or } H = \left\{\begin{matrix} \text{a. } B \\ \text{or a. } M \\ \text{or a. } B' \end{matrix}\right\} \times L. \text{ or } H.$$

(**17**) That is: in the case of the prism, the general formula is reduced to the simplified expression: B or B′ or M × L; but we advise the pupil not to endeavour to remember this formula, simplified though it be, since he will always (see the introduction, page 9) return to it of himself; for one soon sees that it is the same thing to multiply any number by another number, or to multiply 6 times the first by the sixth part of the second.

PRISMOID

Right, Inclined, Twisted.

(**18**) The prismoid of which we treat at length, from page 161 to 167 of this work, has for its opposite and paralled bases or ends, any plane figures, equal or unequal, similar or dissimilar, rectilineal, curvilineal, or mixtilineal, and one of which, as in the case of the pyramid or the wedge, may be a simple point or a line, or each of the bases a mere line as already stated (**4**).

We must then write, according to the case:

$$V = \frac{\left\{\begin{matrix} + \text{ a. } B \\ +4 \text{ a. } M \\ + \text{ a. } B' \end{matrix}\right\}}{\text{Sum of the a.} \times \frac{1}{6} L \text{ or } H.} \quad \text{or} \quad \frac{\left\{\begin{matrix} + O \\ +4 M \\ + B' \end{matrix}\right\}}{\text{Sum of the a.} \times \frac{1}{6} L \text{ or } H.} \quad \text{or} \quad \frac{\left\{\begin{matrix} + B \\ +4 M \\ + O \end{matrix}\right\}}{\text{Sum of the a.} \times \frac{1}{6} L \text{ or } H.} \quad \text{or} \quad \frac{\left\{\begin{matrix} + O \\ + 4M \\ + O \end{matrix}\right\}}{\text{Sum of the a.} \times \frac{1}{6} L \text{ or } H.}$$

PYRAMID, CONE

Regular, Irregular. Right, Inclined.

(**19**) In the pyramid, the base, or one of the ends is any plane figure and the intermediate section a figure similar to the base and equal in area to the fourth part of the base (**95, T.**).

The section of the cone, as of the pyramid, by a plane passing through its axis and apex, is a triangle, and the breadth of this triangle, taken at the half of its altitude is (page 85, rem.) half that of the base. Now, this same half-way breadth of the triangle furnishes the corresponding diameter of the pyramid or of the cone; that is, the diameter of the half way section of the solid by a plane parallel to the plane of its base. The cone, if right, has for base a circle; if inclined, an ellipse, and for its middle section parallel to the base, a circle or ellipse similar to this base and equal in surface to the fourth part of it; the other base or end, of the cone or pyramid, is a mere point, and its area in consequence is null or = 0.

Which gives us:

$$V. = \left\{\begin{matrix} + & a.\ B' \\ +4 & a.\ M \\ + & a.\ B \end{matrix}\right\} \qquad = \left\{\begin{matrix} + & 0 \\ + & 4M \\ + & B \end{matrix}\right\}$$

Sum of the areas $\times \frac{1}{6}$ L or H. Sum of the areas $\times \frac{1}{6}$ L or H.

And suppos. the base =1,

$$= \left\{\begin{matrix} + & 0 \\ + & 4 \times \frac{1}{4} \\ + & 1 \end{matrix}\right\} \qquad = \left\{\begin{matrix} +0 \\ +1 \\ +1 \end{matrix}\right\} \qquad = \left\{\begin{matrix} 1 \times \frac{1}{3}L,\ H \\ \text{or} \\ B \times \frac{1}{3}L,\ H \end{matrix}\right\}$$

S. of the a. $\times \frac{1}{6}$ L or H. S. of the a. $\times \frac{1}{6}$ L or H.

That is: for the pyramid, the cone, the formula reduces to multiplying the surface or area of the base by the $\frac{1}{3}$ of the height.

PARABOLIC, HYPERBOLIC CONOID

Right, Inclined.

(20) Here the base is a circle or an ellipse, according as the solid is right or inclined, and the half-way section between the base and the apex or the opposite ends, is, as any other section parallel to the base, a figure similar to such base and in the paraboloid, equal (**7**) in area to the half of it; or, which is the same thing, the diameter of this section is equal to the square root (see the tables) of half the square of the corresponding diameter of the base. The other base or end of the solid is but a point, since we have agreed to consider as such every curved surface which a plane surface or a plane can touch, at a time, but on an infinitely small extent; that is, a point.

Whence:

$$V. = \left\{\begin{matrix} + & \text{area } B' \\ + & 4 \text{ area } M \\ + & \text{area } B \end{matrix}\right\} \qquad = \left\{\begin{matrix} + & 0 \\ + & 4\ M \\ + & B \end{matrix}\right\}$$

Sum of the areas $\times \frac{1}{6}$ L or H. Sum of the areas $\times \frac{1}{6}$ L or H.

And suppos. the base $=1$, $= \left\{\begin{matrix} + & 0 \\ +4 \times \frac{1}{2} \\ + & 1 \end{matrix}\right\}$ $= \left\{\begin{matrix} +0 \\ +2 \\ +1 \end{matrix}\right\}$ $= \left\{\begin{matrix} 3 \times \frac{1}{6} \text{ H or L} \\ \text{or} \\ 1 \times \frac{1}{2} \text{ H or L} \end{matrix}\right\}$

$\times \frac{1}{6}$ L or H $\times \frac{1}{6}$ L or H

(**21**) So that for the paraboloid, the general formula is reduced to that of multiplying the area of the base by half the height; but as this expression, simplified though it be, differs from the general formula and may confuse the memory, (Introduction page 9) the pupil will do well not to endeavour to retain it; but instead of that, and to remove all doubt concerning the simplified formula, to resort immediately—although, it is true, with a few additional figures—to the sole and universal formula of the author; for, it cannot be denied that a longer process under a less tension of the mind, is less toilsome, and causes less anxiety as to the accuracy of the result, than a shorter but more arduous operation.

SPHERE, SPHEROID

Flattened, Elongated.

(**22**) In the sphere and spheroid, the only area to be computed is that of the central or half-way section, each of the two other areas being, as that for the top of the conoid, null or $= 0$. The central section is either a circle or an ellipsis.

Thence:

$V = \left\{\begin{matrix} + & \text{area B}' \\ +4 & \text{area M} \\ + & \text{area B} \end{matrix}\right\} = \left\{\begin{matrix} + & 0 \\ +4\text{M} \\ + & 0 \end{matrix}\right\} = \left\{\begin{matrix} \text{four great circles} \\ \text{or ellipses as the} \\ \text{case may be.} \end{matrix}\right\} \times \frac{1}{3} \text{ R.}$

Sum of the areas $\times \frac{1}{6}$ diam. or H. or L. — Sum of the areas. $\times \frac{1}{6}$ D or $\frac{1}{3}$ R.

(**23**) **REM.** As for the spheroid or ellipsoid, it is indifferent under which aspect it be considered, respecting its half-way section and its height, length or diameter: but as it is more simple to find, either by calculation or from the tables at the end of this treatise, the area of a circle than that of an ellipse, matters can be managed so that its central section be a circle, which will be done by performing the imaginary section of the solid by a plane perpendicular to the fixed axis. The solid would equally be measured in an inclined position (**171, R.**) being attentive however, as has been said (**3**) to take for the height or length a perpendicular to the plane of section and terminated on both sides by planes parallel to such a section and both of them on opposite sides tangential to the solid under consideration.

SEGMENT

of Sphere, Spheroid. [1]

(**24**) The segment having but one compatible base, the formula to measure it does not differ in any way from that of the cone or conoid, except however that the relation between the area of its base and that of its intermediate section varies with the height of the segment. The radius of this section in the segment of a sphere "small circle of the sphere" is equal (**374, G.**) to the square root (see the table) of the product of the half-versed sine (height) of the segment, by the remainder of the diameter of the sphere of which the segment is a part, and when necessary this diameter is obtained by dividing the square (see the tables) of the radius of the base of the segment, by its height, to get the remainder of the diameter.

$$V = \left\{ \begin{array}{l} + \ \text{area } B' \\ + \ 4 \text{ area } M \\ + \ \text{area } B \end{array} \right\} = \left\{ \begin{array}{l} + 0 \\ + 4M \\ + B \end{array} \right\}$$

$$\text{Sum of the areas} \times \tfrac{1}{6} H. \qquad \text{Sum of the areas} \times \tfrac{1}{6} H.$$

FRUSTUM

of Pyramid, Cone, Conoid, Sphere, Spheroid.

(**25**) In all these solids with two parallel bases, the bases and half-way section are similar figures: circles, if the frustum be that of a right cone or conoid, sphere or spheroid cut by planes perpendicular to its fixed axis; similar regular polygons, if the frustum is a part of a regular pyramid of the same name; and, similar rectilineal, mixtilineal or curvilineal figures, if the pyramid is irregular.

(**26**) In each of these cases, the vertical section of the solid by a plane parallel to its axis, presents a trapezium. Now, the mean breadth of the trapezium is obtained by taking the half-sum of its parallel sides, that is, their arithmetical mean; and this mean is precisely the diameter of the frustum at half-height between its two bases; whence it is easy to arrive at the factors of the half-way section of the solid, and consequently at the area of such a section (see the tables.)

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1. We do not add: "segment of pyramid, cone and conoid" simply because all such segments, that is, all such parts cut off from the apices of these solids by a plane parallel or not to the base, is still a pyramid, a cone, a conoid and its volume subject to the formula already given.

$$V = \left\{ \begin{array}{l} + \quad \text{area } B' \\ + \; 4 \text{ area } M \\ + \quad \text{area } B \end{array} \right\}$$

Sum of the areas $+ \frac{1}{6}$ H.

UNGULA

of Sphere, Spheroid, comprised between planes of section passing in any direction through the centre of the solid.

(27) In each of these solids, the opposite bases or ends are null as to area or $=0$; the central section alone has any value and this section, in the sphere, is a sector of a circle (a part of a circle comprised between an arc and two radii) whilst in the spheroid, the same section is circular, if the planes of section have their common intersection in the fixed axis of the solid, in the other case it is elliptical.

Whence, the cubic content is:

$$V = \left\{ \begin{array}{l} + \quad \text{area } B' \\ + 4 \text{ area } M \\ + \quad \text{area } B \end{array} \right\} \qquad = \left\{ \begin{array}{r} 0 \\ + 4 \text{ area } M \\ 0 \end{array} \right\}$$

Sum of areas $\times \frac{1}{6}$ H. or L. Sum of areas $\times \frac{1}{6}$ H. or L.

(28) **NOTE.** In practice, the length of the arc of the sector may be directly measured, by means of a metallic ribbon or the like, or of a thin rod that can be fitted to the curve of the solid, to determine its circular or elliptical circumference, or any part of such circumference.

UNGULA

of any Prism, Prismoid, Cylinder, Cylindroid, Pyramid, Cone, Conoid, comprised between planes of section having their common intersection in the axis of the solid.

(29) It is clear that the ungula of a prism or prismoid, cylinder or cylindroid is nothing else itself but a solid of the same name; that the ungula of a pyramid or of a cone is simply a pyramid having for base, in the case of the pyramid, any plane figure, and in the case of the cone, a circular or elliptical sector, according as the cone of which the ungula forms part, is right or oblique. As for the ungula of a conoid, it will be considered, with respect to its measurement, as the segment of an ungula of a sphere or spheroid (see the following paragraph). It is clear that the apex or one of the bases of the ungula is but a line or point, as the case may be, and that

In all such cases the formula is:

$$V. = \frac{\left\{\begin{array}{r} \text{area } B' \\ +\ 4 \text{ area } M \\ +\ \text{area } B \end{array}\right\}}{\text{Sum of areas}} \times \tfrac{1}{6}\text{ H. or L.} \quad \text{that is, as the case may be.} \quad V = \frac{\left\{\begin{array}{r} O \\ +\ 4\ M \\ +\ B \end{array}\right\}}{\text{Sum of areas}} \times \tfrac{1}{6}\text{ H. or L.} \quad \text{or } V. = \frac{\left\{\begin{array}{r} O \\ +\ 4\ M \\ +\ O \end{array}\right\}}{\text{Sum of areas}} \times \tfrac{1}{6}\text{ L. or H.}$$

SEGMENT, FRUSTUM OF AN UNGULA

in the conditions of the enunciation, par. 127, of the treatise; that is, in the conditions enumerated in the two last paragraphs (27 and 29).

(30) It is plain that if the segment in question be that of an ungula of a sphere or spheroid, this segment will have but one base of any value, the other base being a mere point. The base will be a circular or elliptical sector and the section at half-height and parallel to the base, will be a sector similar to the base. We will then have for the expression of the volume of the proposed segment:

$$V. = \frac{\left\{\begin{array}{r} +\ \ \text{area } B' \\ +\ 4 \text{ area } M \\ +\ \ \text{area } B \end{array}\right\}}{\text{Sum of the areas}} \times \tfrac{1}{6}\text{ H. or L.} \qquad = \frac{\left\{\begin{array}{r} +\ \ O \\ +\ 4\ M \\ +\ \ B \end{array}\right\}}{\text{Sum of the areas}} \times \tfrac{1}{6}\text{ H. or L.}$$

(31) If it be a **frustum of an ungula, sphere or spheroid between parallel bases,** the expression will be:

$$V = \frac{\left\{\begin{array}{r} +\ \ \text{area } B' \\ +\ 4 \text{ area } M \\ +\ \ \text{area } B \end{array}\right\}}{\text{Sum of the areas}} + \tfrac{1}{6}\text{ H. or L.}$$

(32) Finally if it be a **frustum of an ungula of a prism or prismoid, pyramid, cone or conoid** (for the segment of an ungula of a pyramid, cone or conoid, is evidently a solid of the same name as that of which the ungula forms part) the formula will be, as always:

$$V = \left\{ \begin{array}{l} + \quad \text{area } B' \\ +4 \ \text{area } M \\ + \quad \text{area } B \end{array} \right\}$$

Sum of areas $\times \frac{1}{6}$ H. or L.

SPHERICAL CONE OR SECTOR, SPHERICAL PYRAMID.

(33) To arrive at the volume of these bodies, we must do precisely as for the ordinary cone and pyramid, save that the base and middle section will be convex or concave surfaces which will be measured according to the rules found (**165, 167**), the volume being always:

$$V. = \left\{ \begin{array}{l} \text{area } B' \\ 4 \ \text{area } M \\ \text{area } B \end{array} \right\} \qquad = \left\{ \begin{array}{l} 0 \\ +4M \\ + \ B \end{array} \right\}$$

Sum of the areas $\times \frac{1}{6}$ H. Sum of the areas $\times \frac{1}{6}$ H.

FRUSTUM

of a spherical cone or pyramid between parallel bases.

(34) Will be expressed as the frustum of the ordinary cone and pyramid by:

$$V. = \left\{ \begin{array}{l} \quad \text{area } B' \\ +4 \ \text{area } M \\ + \quad \text{area } B \end{array} \right\}$$

Sum of the areas $\times \frac{1}{6}$ H.

FRUSTUM OF A TRIANGULAR PRISM

that is, having its opposite bases or ends not parallel to one another.

(35) The frustum of a triangular prism, considering any of its lateral faces as one of its bases, and the edge or opposed side as the other base, is nothing else but a prismoid; such is the wedge when the edge of that solid is of unequal breadth with the head. Under this view, the edge or side in question being but a mere line and consequently null as to area, we will have as an expression of the volume:

$$V = \left\{ \begin{array}{l} \quad \text{area } B' \\ +4 \ \text{area } M \\ + \quad \text{area } B \end{array} \right\} \qquad \text{that is } V. = \left\{ \begin{array}{l} \ 0 \\ + \ 4M \\ + \ B \end{array} \right\}$$

Sum of the areas $\times \frac{1}{6}$ H. Sum of the areas $\times \frac{1}{6}$ H.

If the **frustum of the triangular prism** (of the last paragraph) is itself **truncated** by a plane parallel to one of its lateral faces, we will still have a prismoid whose volume will be:

$$V.= \frac{\left\{\begin{array}{r} \text{area B} \\ +\ 4\ \text{area M} \\ +\ \ \text{area B} \end{array}\right\}}{\text{Sum of areas} \times \frac{1}{6}\ \text{H.}}$$

SPHEROID WITH THREE AXES.

(**36**) This solid, as also any segment, frustum, or ungula thereof, segment or frustum of such ungula, is exactly measured by the formula, whatever the direction of the planes of section may be. Therefore, as the case may be:

$$V.= \frac{\left\{\begin{array}{r} +\ \ \ \text{O} \\ 4\ \text{a. M} \\ +\ \ \text{a. O} \end{array}\right\}}{\begin{array}{c}\text{Sum of the a.} \\ \times \frac{1}{6}\ \text{H or L.}\end{array}} \quad \text{or} \quad \frac{\left\{\begin{array}{r} \text{O} \\ +4\ \text{M} \\ +\ \ \ \text{B} \end{array}\right\}}{\begin{array}{c}\text{Sum of the a.} \\ \times \frac{1}{6}\ \text{L or H.}\end{array}} \quad \text{or} \quad \frac{\left\{\begin{array}{r} \text{B} \\ +4\,\text{M} \\ +\ \ \ \text{B} \end{array}\right\}}{\begin{array}{c}\text{Sum of the a.} \\ \times \frac{1}{6}\ \text{L or H.}\end{array}}$$

COMPOUND BODIES.

(**37**) The tableau presents a certain number of these bodies; for instance a cylinder terminated at one end by a segment of a sphere or spheroid (such would be a mortar); a frustum of a cone ending in the same way (a gun for instance); a cylinder or frustum of a cone crowned with a cone (a hay-stack or circular tower with a conical roof); a cone ending at its base by a segment of a sphere or spheroid, like certain kinds of bueys. It is plain that to measure these compound bodies or any other forms that can be decomposed into elements of the kind already treated on, the composing parts thereof must be separately computed, in order to make up afterwards the sum of such parts, according to the rules which have just been given

APPROXIMATELY.

(See the general expression, par. 127).

(**38**) And very nearly, say generally at .005 or at about ($\frac{1}{2}$) one half per cent, more or less, often (see the detailed problems of the treatise) with perfect accuracy or very near an exact result; is the volume obtained of

ANY FRUSTUM

of a Prism or Prismoid, Cylinder or Cylindroid, Pyramid, Cone or Conoid, Sphere or Spheroid, comprised between non parallel bases.

(39) By decomposing it, by an imaginary section parallel to one of its bases and passing through the nearest point of the other base into a frustum with parallel bases (the exact volume of which is obtained by the rules already given) and an ungula.

ANY UNGULA

of a Prism or Prismoid, Cylinder or Cylindroid, Pyramid, Cone or Conoid, Sphere or Spheroid.

(40) In this solid, as in the regular ungula of paragraphs (**27** and **29**) the apex or one of the bases or ends, is but a mere line or point, and its volume is very nearly.

(See the detailed ungulæ of the treatise).

$$V. = \left\{\begin{array}{r} \text{area B}' \\ +4\text{ area M} \\ +\text{ area B} \end{array}\right\} \qquad \text{That is } V. = \left\{\begin{array}{r} 0 \\ 4\text{ M} \\ +\text{ B} \end{array}\right\}$$

Sum of the areas $\times \frac{1}{6}$ H. Sum of the areas $\times \frac{1}{6}$ H.

REM. As will be seen (**120**) if the base of the cylindrical ungula be not truncated, that is, if this base is a circle or an ellipsis, the formula gives the exact volume of the solid, and in the same manner under the same conditions, the exact volume of an ungula of a prism will be arrived at.

FRUSTUM OF AN UNGULA.

If the ungula of the last paragraph is cut off by a plane parallel to its base, of which the tableau offers examples, the volume will not the less be, as usual:

$$V. = \left\{\begin{array}{r} \text{area B}' \\ +\ 4\text{ area M} \\ +\text{ area B} \end{array}\right\}$$

Sum of the areas $\times \frac{1}{6}$ H.

ELONGATED SPINDLE, FLATTENED SPINDLE.

(41) The spindle considered, as a whole, is not a usual solid; it has little importance, and to be convinced that it cannot be measured at once, as with the elongated or flattened spheroid, it is sufficient to compare it in one's mind to an exact spheroid having the same axes or diameters. It is then seen how much its volume is less than that of the corresponding spheroid which is more swollen towards the ends of its axis in the elongated spheroid, and in the opposite direction, if it be a flattened spheroid.

(**42**) But if it be impossible to arrive at once at the volume of the spindle, one succeeds almost immediately, by measuring the half of this solid, and afterwards doubling the result, since then, by taking its section half-way between the centre of the spindle and its apex or end, the very element which contributes especially to make its volume vary, is considered, and this process applied to the flattened spindle, will give the exact volume if the perimeter of a section of the half-spindle by a plane passing through its fixed axis, is an arc of a conic section, as will generally be the case, the flattened spindle being then considered as two equal segments of a sphere or spheroid, united by their bases or planes perpendicular to the fixed axis of the solid, of which the composing elements of the spindle form part.

It will be seen (prob. LI) that it is sufficient to divide the half-spindle into two parts which will be measured separately and the sum of which will be afterwards taken, to arrive at a result which shall differ from the truth but by the 9th part to the quarter of one per cent.

CENTRAL FRUSTUM OF ELONGATED SPINDLE.

(Cask).

(**43**) This solid which gives its form to the thousand and one varieties and dimensions of casks, throughout the whole world, is, respecting the measurement of its capacity or volume, of great importance, on account of the generally high price of the contents. Well! as will be seen (prob. LII), it is sufficient to measure at once the half-cask to arrive at the exact volume, within the quarter to the fortieth part of one per cent; maximum error of one quart on a hundred gallons or of one litre on 400 litres and which does not exceed generally $\frac{1}{10}$ to $\frac{1}{20}$ of a gallon or litre for every 100 gallons or 100 litres, and can, besides, by that itself, be rectified, that the error is known to be always in excess and that consequently the result may be diminished by so much, if required.

$$V. = \left\{ \begin{array}{r} \text{area B} \\ + \ 4 \text{ area M} \\ + \ \text{area B}' \end{array} \right\}$$

Sum of the areas × $\frac{1}{6}$ L or H.

CONCAVE CONE.

(44) The concave cone is analogous, as to its volume, to the elongated half-spindle, which may also be called a convex cone; and in the same way as we very nearly arrive at the volume of the half-spindle, by measuring it in two slices; so, if the hollow or convex cone is decomposed into two parts, by a plane parallel to that of its base, to measure separately each of these parts and add them together afterwards, the volume will be obtained at less than one half per cent loss. [1]

FRUSTUM OF CONCAVE CONE

between parallel bases.

(45) A great many vessels of capacity assume the form of this solid and as the hollow or concave cone is analogous to the half-spindle or convex cone, so the frustum of the concave cone may be considered as analogous to the central half-frustum of the elongated spindle or half-cask. Then by measuring it at once, provided its curve be uniform throughout its height and especially if this curve is not considerable, the volume or desired content will be very nearly arrived at, and if this curvature of the lateral face of the solid or vessel of capacity in question, is considerable or of unequal radii in various parts of the height of the frustum, a nearly perfect accuracy can be secured, by decomposing it mentally with a view to its measurement, into two or at most into three parts or slices by planes of section parallel to the bases.

The volume of each of the component slices will be:

$$V. = \left\{ \begin{array}{r} \text{area B} \\ + \ 4\,\text{area M} \\ + \ \ \text{area B}' \end{array} \right\}$$

Sum of the areas $\times \frac{1}{6}$ H or L.

COMPOUND BODIES.

(46) These bodies may assume many varied forms. The tableau presents some of them: for instance, a central or eccentric frustum of a sphere or spheroid surmounted by a concave cone (kind of

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1 For forms with concave sides the volume is less; as for convex bodies the volume is more.

dome or minaret); segment of a sphere or spheroid surmounted by a segment of an elongated spindle or convex cone, or of a hollow or concave cone; two frusta of right cones united by their broader bases; two others, by their smaller bases; two frusta of concave cones and two others in the same conditions. And it is clear that other forms may be conceived, in almost infinite variety, but of which the rules already given are sufficient to determine the respective and composing volumes.

SUNDRY.

(**47**) Besides the solids which have just been enumerated, it is proper to say a word of certain forms which the tableau presents and of which the origin or the whole solid of which the body under consideration forms a part, might not at once suggest themselves to the mind. Thus, the **eccentric solid ring** may be considered as the central frustum of a very elongated spindle bent on itself. Then will it be measured by adding to the sum of the areas of both its less and greater sections, 4 times the area of the half-way section between these first, to multiply the whole by the length of the half-circumference used as the imaginary axis of the ring.

(**48**) The **bent cone or half-spindle** in form of the horn of an ox is measured like the inclined cone, considering as its height, the perpendicular drawn from its apex to the plane of its base.

(**49**) There is the **eccentric frustum of an elongated spindle**, which may represent the shaft of the roman column, swollen as it is, about the third part of its height, and the volume of which may be had by taking separately that of each of its composing half-frusta.

(**50**) The **regular polyhedrons**, as it is seen, may be decomposed into as many regular and equal pyramids as the solid has faces and be easily measured in this manner, each pyramid having for base one of the faces of the polyhedron and for height the half-height of the polyhedron, that is the half-diameter or radius of the inscribed sphere.

(**51**) The **decomposed parts of the flattened and elongated spindles and of certain other solids** furnish the idea and in consequence the manner of measuring, or gauging any sailing vessel, steamer or other, by decomposing it, if necessary, into elements of the kind already treated of.

(52) **REM.** The regular polyhedrons could equally be at once measured, by taking the trouble of finding the area of the central section of each of them. All these solids have two parallel bases, one of the bases being, for the tetrahedon, a point—for the tetrahedron is nothing but a pyramid.—The octahedron may be considered as a double pyramid or a compound of two pyramids, base to base, and be measured in this manner. As to the dodecahedron, it will be seen that while each of its parallel bases is a regular pentagon, its half-way section between these bases, is a regular decagon or a ten sided regular polygon, and each side of which is equal in length to the half-diagonal of the pentagon. As to the middle section of the exahedron, if it be taken paralled to two opposite and paralled sides of the solid, it will be a twelve sided regular polygon, the perimeter of which it would be too long to determine here. If on the contrary the half-way section is supposed parallel to or equidistant from two opposite vortices of the solid, that is, perpendicular to the axis or diameter uniting two opposite points or extremities of the solid, this section will be a regular decagon each side of which will be equal to the half-side of the triangle forming the face of the polyhedron. Finally, if any two opposite sides or edges be taken for the parallel bases of the icosahedron, the half-way section parallel to these edges and perpendicular to the plane which unites them will be a six sided symmetrical polygon, two opposite and paralled sides of which, each equal to the side of the triangle forming the face of the polyhedron and being one of these sides, and the other sides of the exagon, parallel, two and two, and respectively equal to the height or right radius of the said component face.

REDUCED TABLEAU.

(53) **REM.** It is hardly necessary to say that, in this treatise and in the abridged key, every thing which relates to the so called tableau of 200 models, is equalled applicable to the reduced tableau of 105 models which the author is preparing for elementary schools and with a view to reduce the price of it in order to place it within the means of persons less capable of ordering it.

In the reduced tableau the models will be 105 in number, disposed in 15 vertical rows and on 7 horizontal rows ($15 \times 7 = 105$). Then beginning for instance by the left, the

1st vertical row would be the prism, its frustums and ungulas.
2nd, 3rd, 4th row, the prismoid and various frusta and ungulæ.
5th row, the pyramid, its frusta and ungulæ.
6th row, the cylinder, its frusta and ungulæ.
7th row, the cone, its frusta and ungulæ.
8th row, the conoid, its frusta and ungulæ.
9th row, the flattened spindle, its segments, frusta and ungulæ.
10th row, the elongated spindle, its segments, frusta and ungulæ.
11th, 12th and 13th row, the sphere, its segments, frusta and ungulæ.
14th row, the flattened spheroid, its segments, frusta and ungulæ.
15th row, the elongated spheroid, segments, frusta and ungulæ.

And if in the tableau any segment, frustum or ungula is wanting to complete the number of those included in the nomenclature of the solids to which the formula relates, it can easily be mentally supplied in the same way, as, if required, any compound solid may equally be decomposed by imaginary planes of section, into elementary forms, to submit its volume to the required computation.

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