LOCAL DEFLECTION OF THE PLUMB LINE

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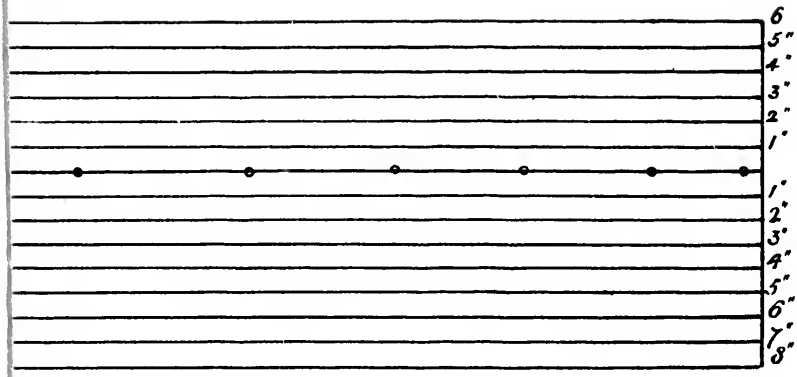
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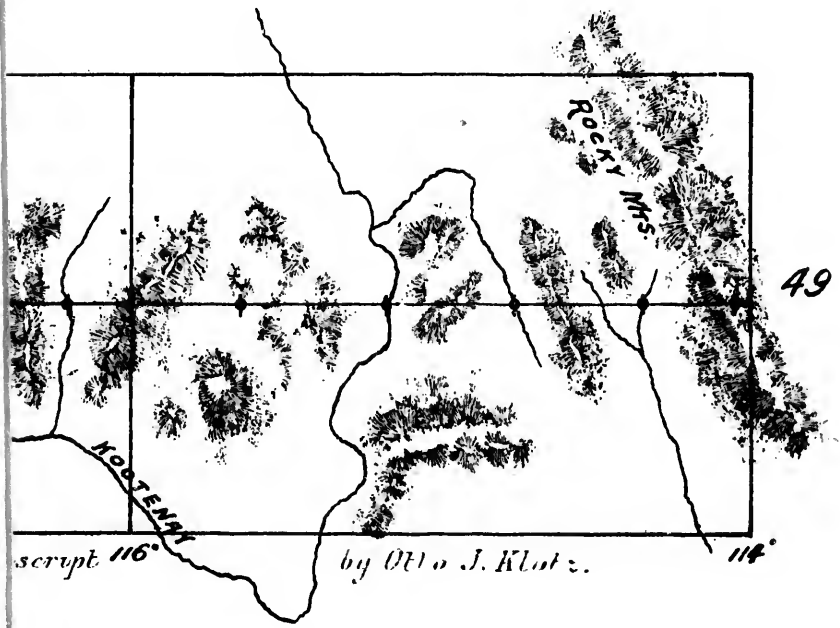
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by Otto J. Klotz.
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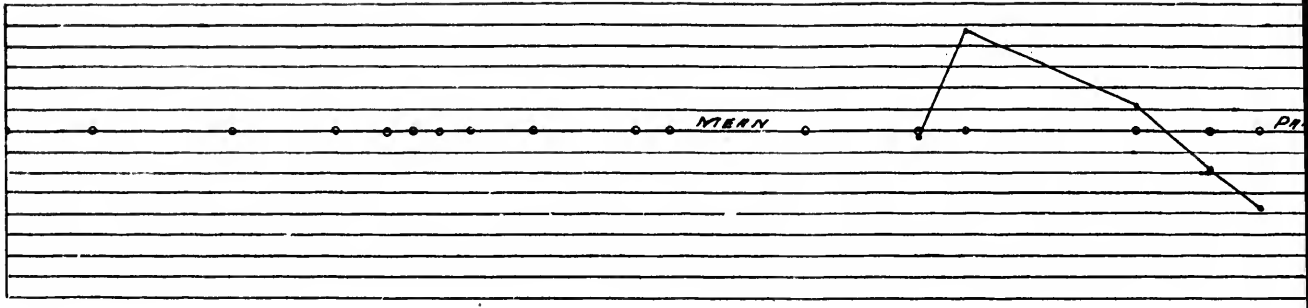
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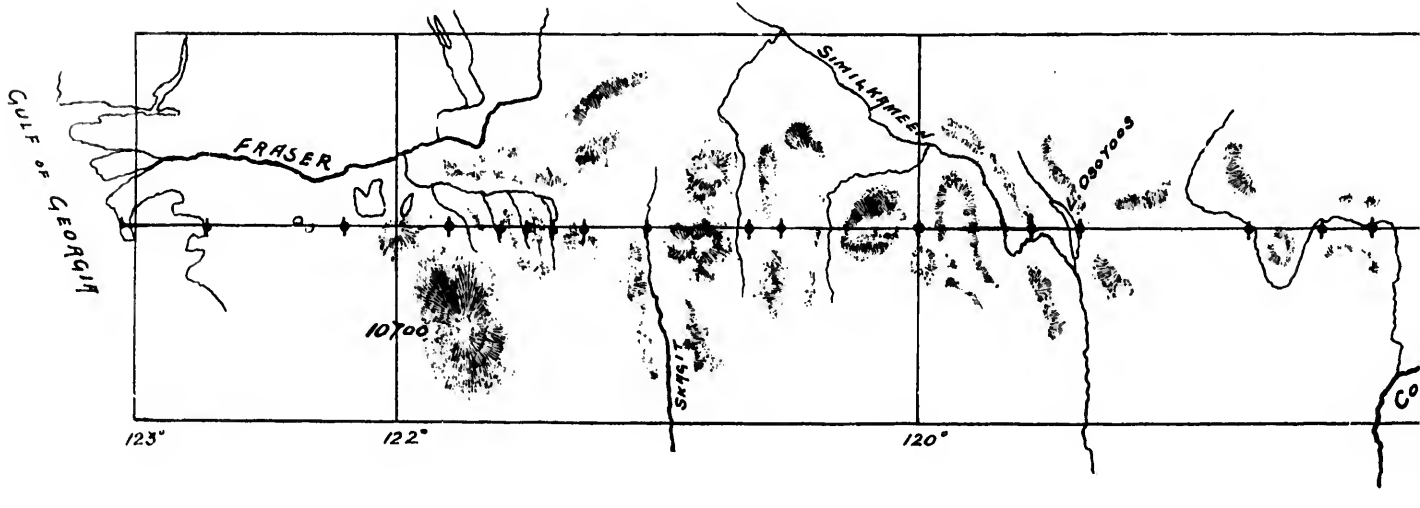
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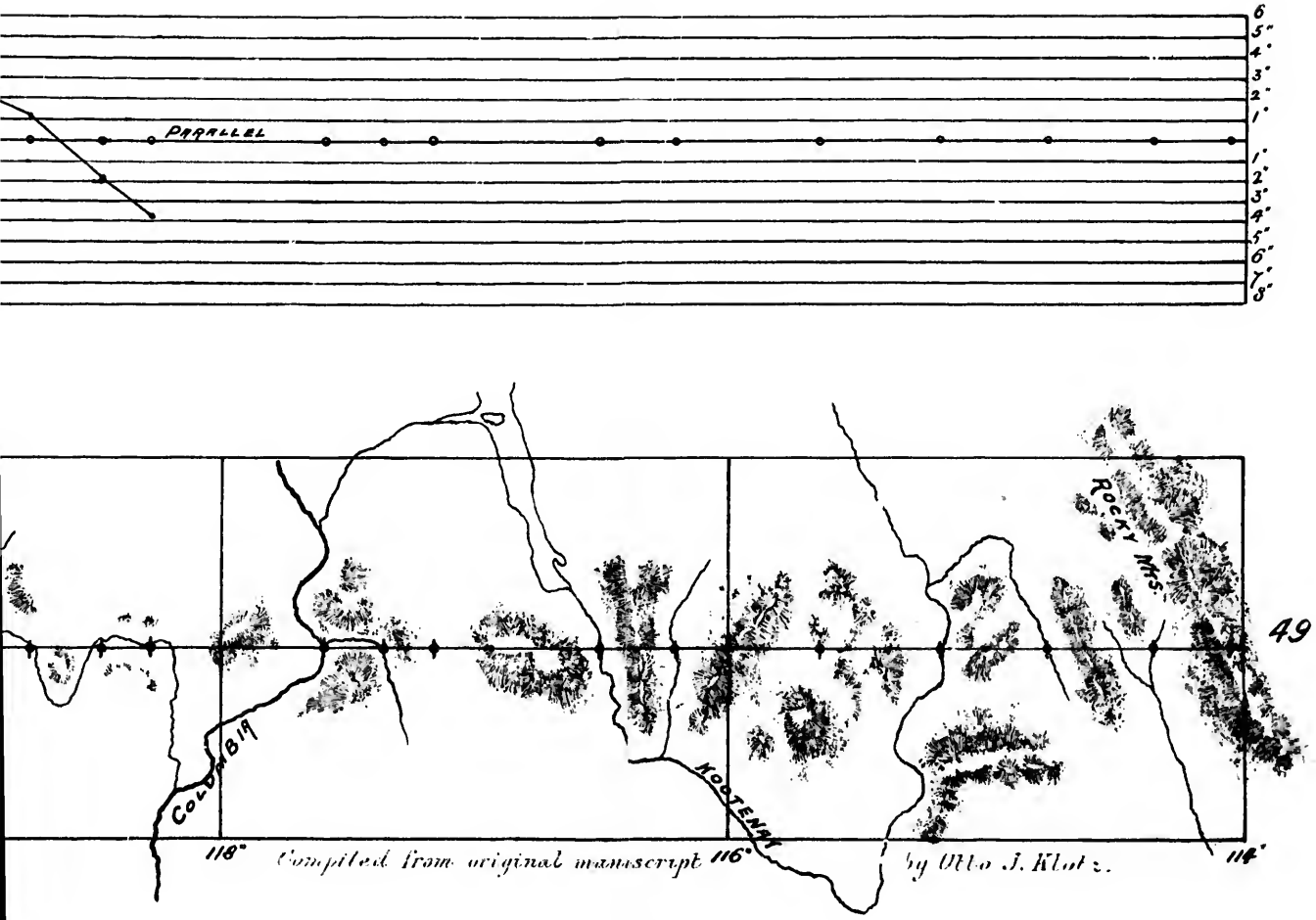
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49
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Compiled from original manuscript by Otto J. Klotz.

LOCAL DEFLECTION OF THE PLUMB LINE.

OTTO J. KLOTZ.

In the present short paper it is only intended to lay before the Society certain data of the deflection of the plumb line as found by latitude observations and azimuth surveys along the International Boundary, 49th parallel, from the Lake of the Woods to the Pacific Ocean. Those westward, as far as the Rocky Mountains, were published years ago in the United States Report of the Northern Boundary Survey, but those of British Columbia have, to my knowledge, never been published. The effect of deflection of the plumb line on longitude and azimuth observations is not considered in the present paper.

Deflection of the plumb line may be defined as the deviation of the vertical at any point from the normal at that point to the surface of an assumed figure of the earth. In dealing with the earth, we must assume its shape to be of some definite geometrical form and of certain dimensions, for only then can observations, at different points thereon, be correlated and adjusted. The best assumed form (Clarke's 1880 spheroid at present), however, differs at places widely from the actual form or geoid. To illustrate, if the continents were traversed by narrow canals, communicating with the ocean, their surface, although level, would be wavy or undulating, and would be in some places above, in other places below the surface of the spheroid or ellipsoid of revolution, the divergence of the two surfaces being probably confined to a few hundred feet.

The position assumed by the plumb line, is due to the law of gravitation, that is, it is the integrated result of the attraction of the individual particles, composing the mass of the earth, and hence the position is affected by the relative distribution of them. We may, therefore, say that the unsymmetrical distribution of the particles, whether on the surface, as mountains, etc., or in the thin crust, is the cause of the deflection of the plumb line

from its theoretical position. The ablest mathematicians have been engaged, ever since the era of precise measurement, upon this difficult question of the form and dimensions of the sea level surface.

In some instances we are quite prepared to find local deflections of the plumb line, for example, when observations are taken on the plains at a point near a more or less isolated upheaval, as the Three Buttes or Sweet Grass Hills in Montana, just south of the International Boundary. These hills, as we shall see later, pulled the 49th parallel out of its theoretical position about 800 feet. On the other hand, large deflections show themselves without any visible reason or cause as evolves from numerous observations and their geodetic connection. A remarkable instance is that of the comparatively plain area surrounding Moscow, which I visited recently, where, on the margins of an 18 mile east and west zone, large deflections of opposite signs were found. From this it must be concluded that there exists beneath the surface a cavity or at least matter of small density.

It is evident, therefore, that the observations alone, at any particular point, are not conclusive for fixing its geographical position upon the surface of the earth, but that numerous astronomically determined points must be connected linearly by triangulation, and from their inter-accordance, or discordance, the most probable values determined, based upon an assumed figure of the earth.

In our own city here, there appears, due to the topographical configuration, to be a deflection of the plumb line; for, the latitude determinations made by me at the observatory on the summit of the escarpment, compared with those made some years ago by Mr. Lindsay Russell on the opposite side of the river, show a discrepancy of about a second of arc, equal to about a hundred feet, a quantity greater than the error of observation. However, a more or less extended hypsometric survey would be necessary for a plausible *a priori* conclusion with reference to the probable discordance in latitude to be expected between two stations.

All observations for the determinations of positions upon the earth depend upon the direction of the vertical. Latitude and

longitude observations, the surveyors' and engineers' operations, all have their zero of reckoning in the centre of the level bubble, and any displacement of the latter, which is equivalent to the displacement of the plumb line, affects the results, and will show discordances when widely separated observations are geodetically connected.

It may be stated that a delicate level used for latitude work, reading to a second of arc, has usually a radius of about 1,700 feet, or nearly a third of a mile, for the curve ground on its inner upper surface.

To digress for a moment.

Boundary lines may be divided into three classes:—those representing a social unity, those representing a physical unity, and those representing a political unity. Those of the first find the largest number of representatives in the older settled countries, for the primal concept of boundary was to conserve the social unity. It was not to define territorial extent as much as to define or assert the domain of a like people; like by language, race, or religion or other affiliation. Such boundaries are, as a rule, very irregular and difficult to describe. When the social organism reached that development that written treaties became necessary between adjoining peoples, the description of the separating boundary was made from the boundary *de facto*, and the boundary not laid down from the description.

The second class we may consider an expansion of the first, resulting from conquest, whereby a physical as well as a social unity was to be preserved. Of the physical boundaries,—mountains, rivers and lakes,—to serve the purpose of barriers, by mountains, that end is undoubtedly best attained. Europe furnishes a number of examples of this.

The third class, which we may call the astronomic boundary, is a development of recent times, and applies invariably to areas practically unsettled, unsurveyed and little known. Such can be laid down on paper, or described in treaties without scarcely any knowledge of the country whatsoever.

While Europe furnishes the most examples of the first, America does so of the last. Many of the State boundaries of the United States are astronomic lines, either meridians, parallels,

or lines of definite azimuth. Similarly with the subdivision of Australia and recent boundaries in Africa. Astronomic boundaries may generally be taken as an index of the ignorance existing of the country or area involved and its resources.

Boundaries under the first division are difficult for definition or restoration when such is necessary. Those of the second, always dependent upon water, are generally self evident, while those of the third are comparatively easy of determination.

There are instances of a fourth class of boundaries—where the position of it is dependent upon the position of a physical feature,—e. g. that it be at or within a given distance from a river or the sea.

A notable case of the last is that of south-eastern Alaska, as described in Article 4 of the Convention of February 28th, 1825. Such boundaries are exceedingly difficult to lay down on the ground, requiring, too, beforehand laborious mathematical calculations. The Railway Belt of British Columbia, extending twenty miles on each side of the Canadian Pacific Railway gave the writer an example of such computation.

A river, and least of all a large river, a commercial artery, forms an undesirable international boundary. The very nature thereof makes it a route of travel, and hence of settlement on its banks, which, if in possession of two countries, is almost sure to lead to trouble. Hence we find few or no large rivers forming such boundaries, although our own St. Lawrence does for a short distance separate us from our southern neighbor. A summit or watershed boundary is pretty satisfactory, if restricted to mountains, but when applied to plains or undulating country, is fraught with difficulties. The difficulty consists in establishing the line of watershed, as was presented in the Maine-New Brunswick controversy early in the century.

Of the several classes of boundaries spoken of, none is as immutable as the astronomic one. Of the first, the original monuments and records may disappear, and personal evidence be wanting. Of the second or physical boundary, time may bring about changes quite marked and cause the line to move therewith. As a well-known instance, the Mississippi may be cited. In a recent report of the "Idaho and Montana Boundary Line"

we have another illustration. It is stated that: "from a geological standpoint, but hardly from a practical one, however, there is another reason why monuments should not be placed on the summit of the Bitter Root range, as marking the boundary line between Idaho and Montana. There is abundant evidence that the summit is what is known as a retreating or migrating divide; in other words, the waters tributary to the Bitter Root River in Montana are continually capturing by erosion those of the Clearwater river in Idaho, so that the divide is slowly being shifted to the westward, thus adding to the territory of Montana and diminishing that of Idaho. The existing divide is uniformly from six to eight miles from the irregular line representing the original divide, if the latter may be accepted as having passed through the highest points of the range, which seems probable."

When a boundary is defined by a parallel of latitude, the question invariably arises, in the demarcation of it, whether the astronomic or mean parallel is to be adopted. The astronomic parallel is that line on the surface of the earth on which direct observations for latitude give the same elevation of the pole; geometrically, for the spheroid or ellipsoid of revolution, it is the intersection of the cone, having its apex in the minor axis of the earth and making an angle therewith equal to the complement of the latitude, with the surface of the earth. Principally owing to the local deflection of the plumb line, points astronomically determined in latitude will not "close," that is, the line projected or run from one station as a parallel will not meet the next point or astronomic station.

That line with reference to which the sum of the discrepancies north is equal to the sum of those south is the mean parallel.

However, as the latter can only be determined *after* the location and connection of the astronomic points, entailing revision of the whole work, and besides the difficulty of re-establishing points on the mean parallel in case of loss or disappearance of monuments and marks, it has generally been decided to adhere to the simpler and more readily established astronomic parallel. All such parallels traced upon the earth are irregular curves.

The International Boundary Line, between the Lake of the Woods and the Rocky (Stony) Mountains is defined in the second

article of the Convention of 1818 as being the parallel of 49 degrees north latitude. The immediate cause of the delimitation of the boundary was the discovery that the fort of the Hudson's Bay Company at Pembina was nearly a mile within United States territory.

The field work was begun in September 1872 and finished in the same month two years later, while the proceedings of the joint commission were brought to a conclusion May 29th, 1876.

On this line of 860 miles, 40 astronomic stations were established, and 388 monuments erected.

After due consideration, the commissioners agreed upon the astronomic parallel. The recommendation for this, by the chief astronomers of the commission, was based on the following grounds: "1st That the portion of the parallel of 49° included within the operations of the commission, being only about one twentieth of the entire circle of latitude, was not sufficient to fix, with any mathematical accuracy, the true position of the mean line of 49°, and that, therefore, if such a parallel were described, depending on the mean of the astronomic stations, no known point of the boundary would be in latitude 49°; 2nd That as the amplitude of the arcs, included between the mean and the astronomical parallels, would in many cases be very considerable, grave errors and complications might arise in the subsequent re-survey of any lost portion of the boundary; 3rd That the definition of a mean line would involve a re-adjustment of the whole boundary, after the first careful survey should have been completed, and consequently a very considerable increase of expense, without any practical benefit accruing; 4th That for every purpose except that of geodetic computation, a parallel of points determined astronomically (instrumental errors aside), is a true parallel of latitude, and therefore, fulfils the stipulations of the treaty under which the joint commission was organized."

Accordingly, astronomic positions were determined at approximate intervals of twenty miles. These stations were connected by tracing upon the ground tangents to the prime vertical circles at each successive point. From these tangents, checked and corrected for errors of azimuth, the calculated offsets to the small circle of latitude were measured at convenient intervals,

varying from one to three miles. From the last mentioned offset the relative station error (deflection of plumb line) was found and distributed between the two stations in the ratio of the distances where offsets were taken. From this method it results that the boundary line, as actually traced, is an irregular curve, affected at each astronomical point by instrumental errors and by the local deflection of the plumb line, making the closest probable approximation, at every point, to a true astronomical parallel.

Of the forty astronomical stations on the 49th parallel, four were observed jointly, seventeen by the United States astronomer and nineteen by the British. The mean of the probable errors of the British stations was, ±".088 and of the United States ±".059. The average of the probable errors is then a little over seven feet.

The greatest difference of station errors is 13."89 or 1,407 feet, being in a distance of 97 7/10 miles, between the Cypress Hills to the north of the boundary, and the Three Buttes or Sweet Grass Hills near, and to the south of the 49th parallel. The station error of the former is + 5."94, of the latter – 7."95, that is the Three Buttes pulled the 49th parallel 805 feet south, and the Cypress Hills 602 feet north of the mean parallel. The greatest discrepancy between adjacent stations, about twenty miles apart, is 7."28 or 738 feet, near the Three Buttes.

Looking at the accompanying diagram, in which the upper figure is a representation (much exaggerated) of the relative position of the astronomic and mean parallels, while the lower figure shows the main features of the topography for a distance of about thirty-five miles on each side of the boundary line, the large deflections appear obvious from the topography. From the Lake of the Woods, westward, into the valley of the Red River, the station errors increase, and for a reason, which from our lack of knowledge of the underlying strata, must be conjectural. The escarpment of the Pembina Mountains, (elevation would be a more appropriate term, height 1,695 feet), naturally draws the vertical southward, continuing to do so until the Turtle Mountains (of moderate elevation, 2,550 feet) are reached, which too deflect to the south. After entering the Côteau of the Missouri we pass along the southern base of the high ridge, separating the waters flowing into the Gulf of Mexico, from those flowing into Hudson's Bay, and find, naturally, a deflection to the north, increasing to a

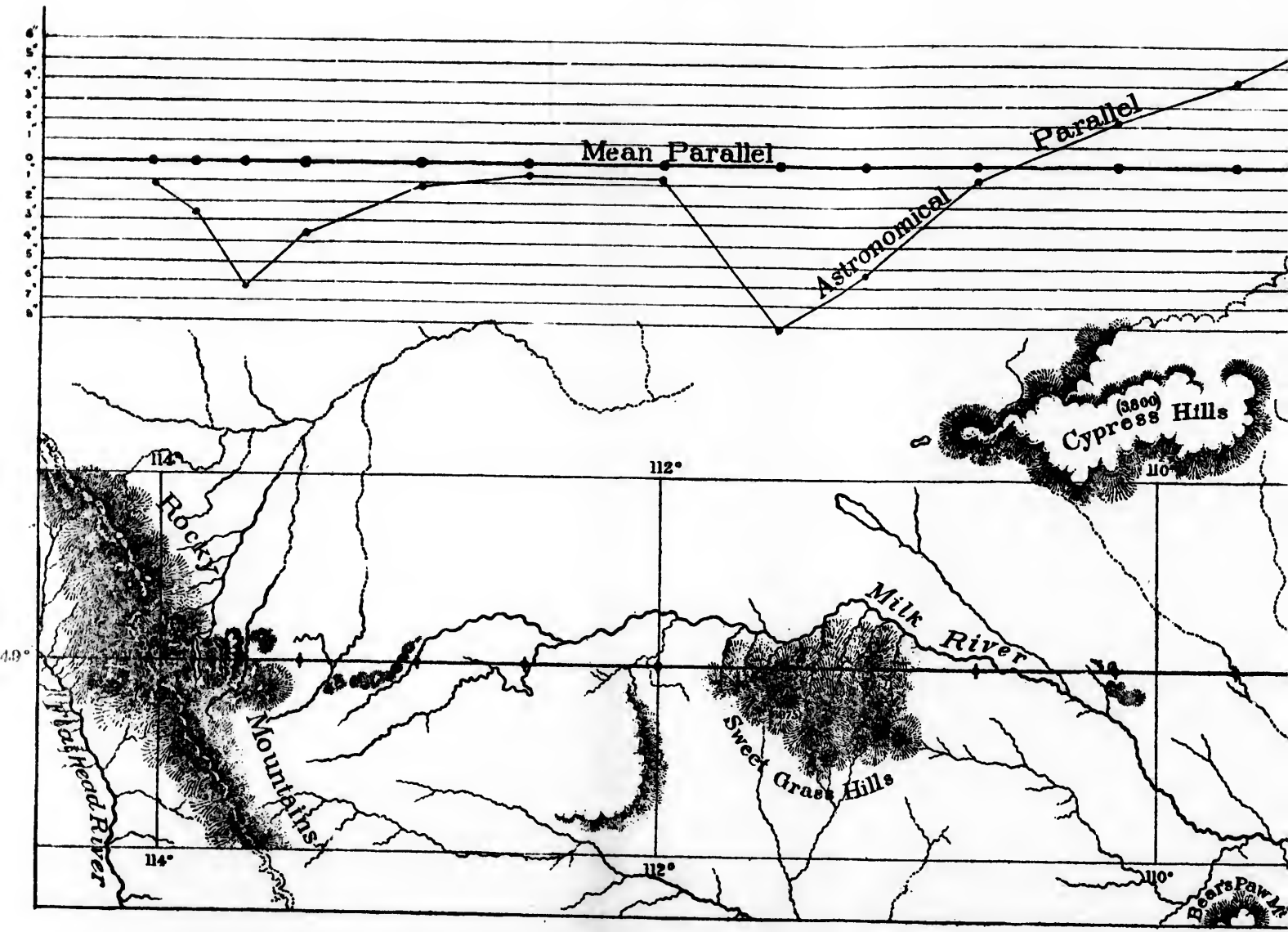
maximum, south of the Cypress Hills (3,800 feet). Here the enormous intrusive masses of the Three Buttes, produce a violent disturbing effect, drawing the astronomical parallel to the south, at an average rate of 14 feet to a mile, for a distance of about one hundred miles. When we actually enter the tumultuous Rocky Mountains, with all their varied conditions of compositions, of faults and dykes, and our lack of hypsometric maps, we are unable to even make a plausible estimate in which direction the local deflection is to be expected. Even the relative deflection between adjacent stations remains unknown in most cases on account of the great difficulty in connecting them geodetically.

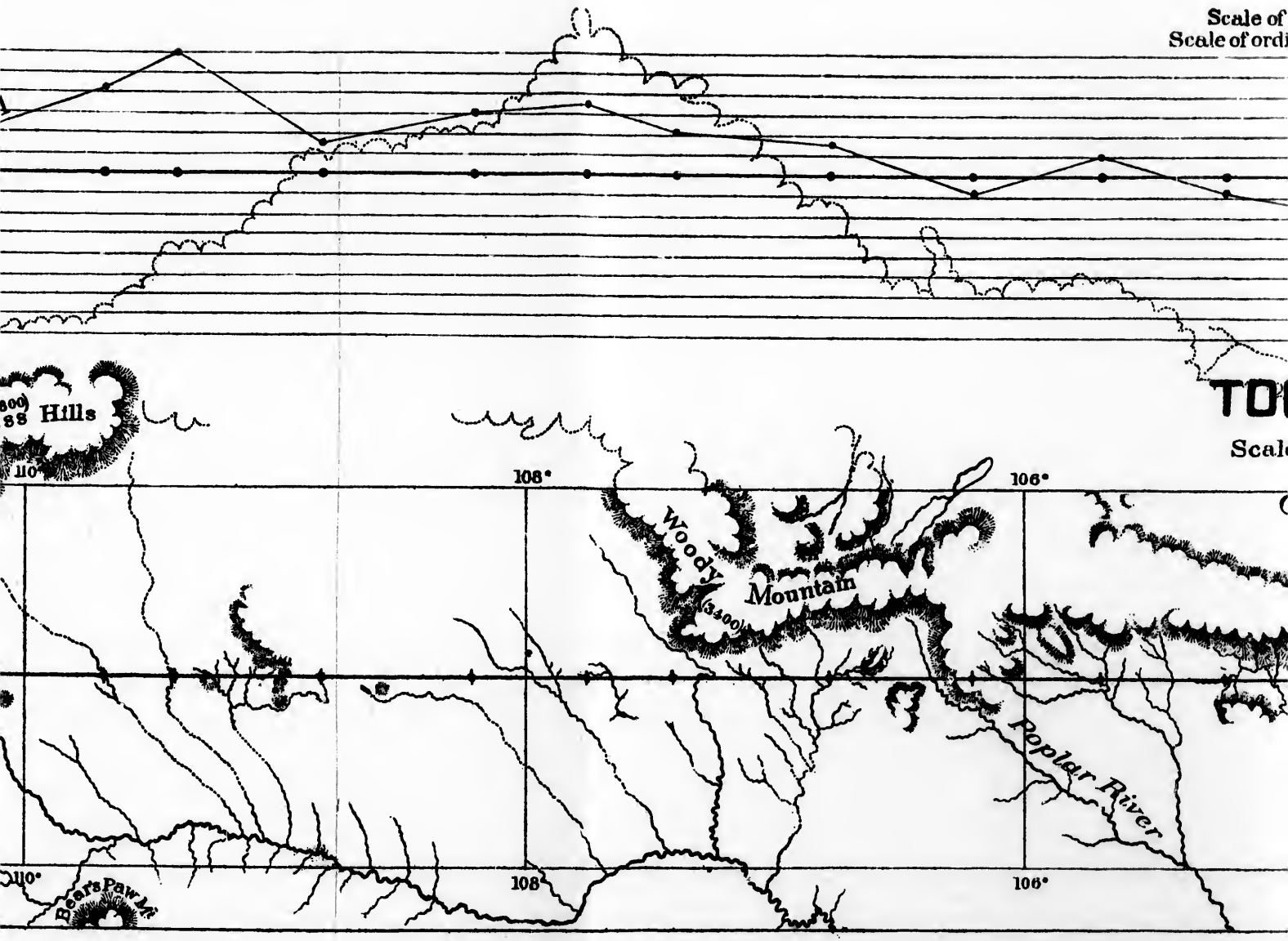
As a very remarkable example of the deflection of the plumb line may be mentioned, the one on the arc of the meridian between Andrate and Mondivi, in northern Italy, where in a distance of a little over seventy-seven miles, a difference of nearly forty-one seconds was found, that is to say the difference in the distance between those two terminal points determined by direct astronomic observation, and also linearly by triangulation was found to be about four tenths of a mile. How much of this quantity is attributable to each place for local deflection, and again how much is due to relief or topography, and how much to the unequal distribution of masses beneath the surface of the earth, is not known.

It is evident that observations at two places which are also geodetically connected, can only give the relative deflection of the plumb line.

For the boundary between the Lake of the Woods to the summit of the Rocky Mountains, the Commissioners agreed that the line joining any two adjacent monuments shall be an arc of the parallel. This was to apply, too, in the case of restoring any monument whose position was lost. This agreement differs from that of the boundary commissioners, who had charge, some 17 years previously of defining the boundary from the Gulf of Georgia to the summit of the Rocky Mountains. They agreed that the connecting line between monuments shall be a straight or direct line, i.e. an arc of a great circle.

The international boundary commission appointed to define the boundary under the first article of the Treaty of June 15th, 1846, (the present southern boundary of British Columbia) was


Mean Parallel
Astronomical Parallel
(3800)
Cypress Hills
Rocky Mountains
Milk River
Sweet Grass Hills
Flat Head River
Bear's Paw M
114°
112°
110°
49°


Scale of
Scale of ordi
TO
Scal
Hills
110°
108°
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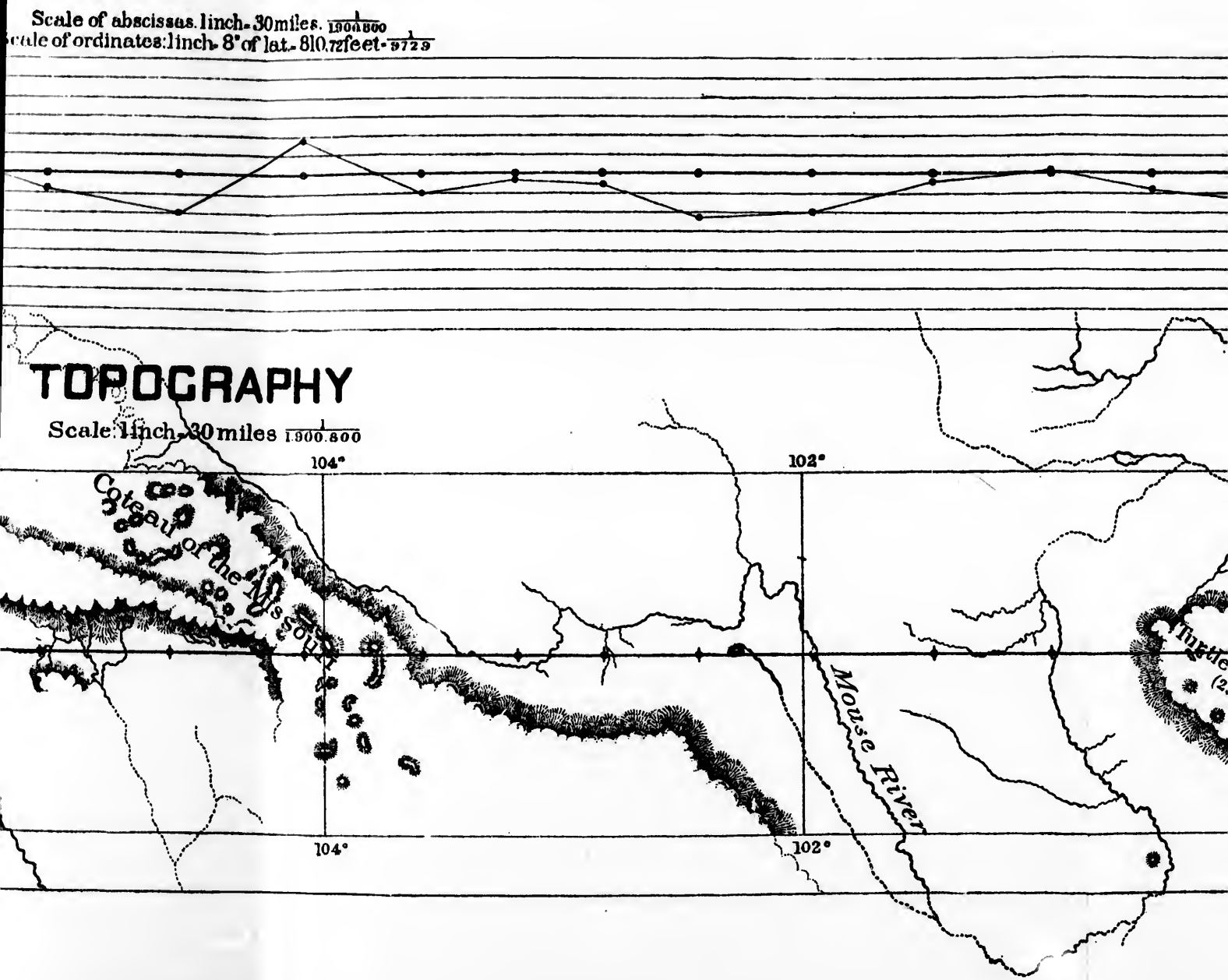
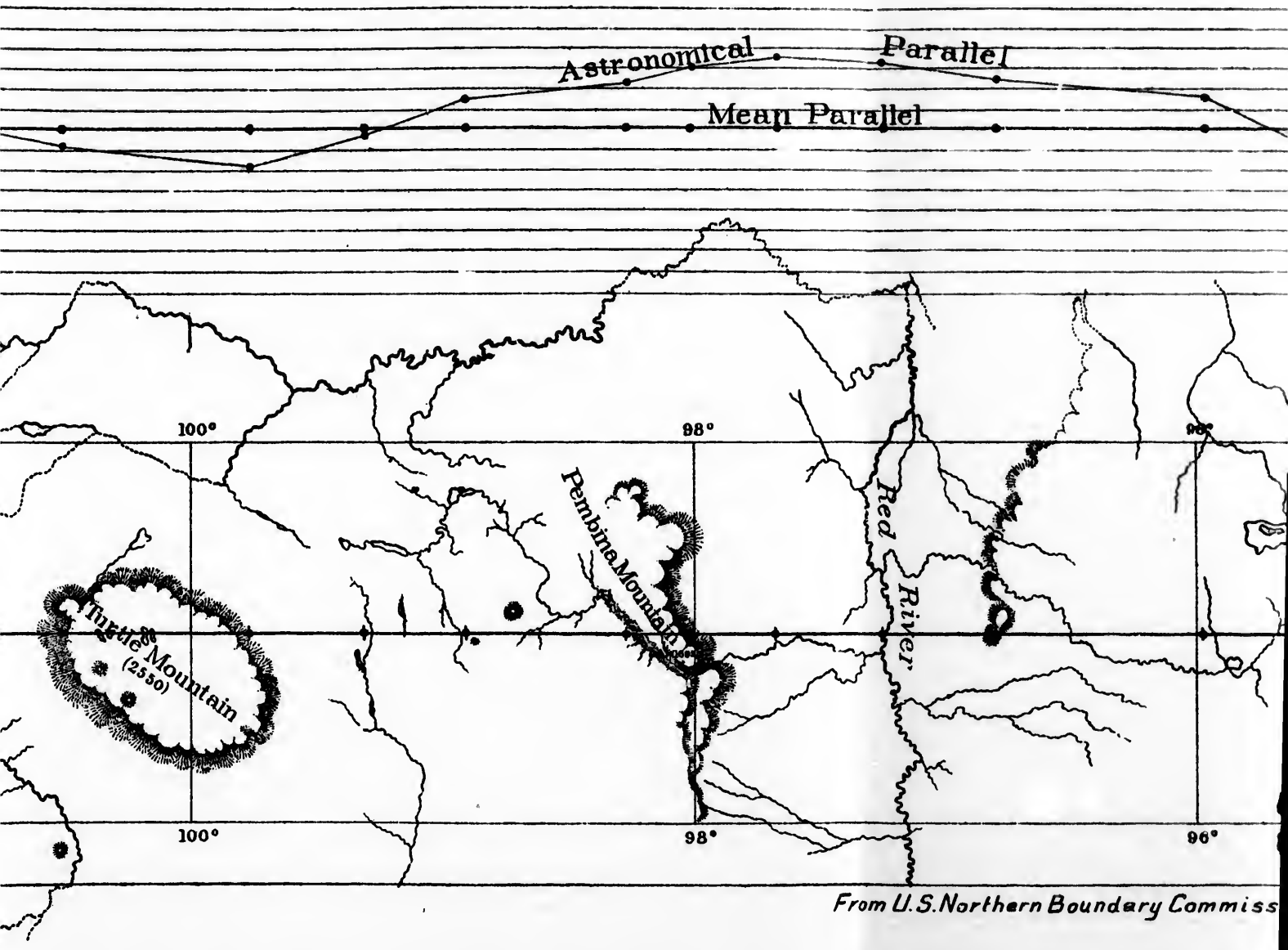
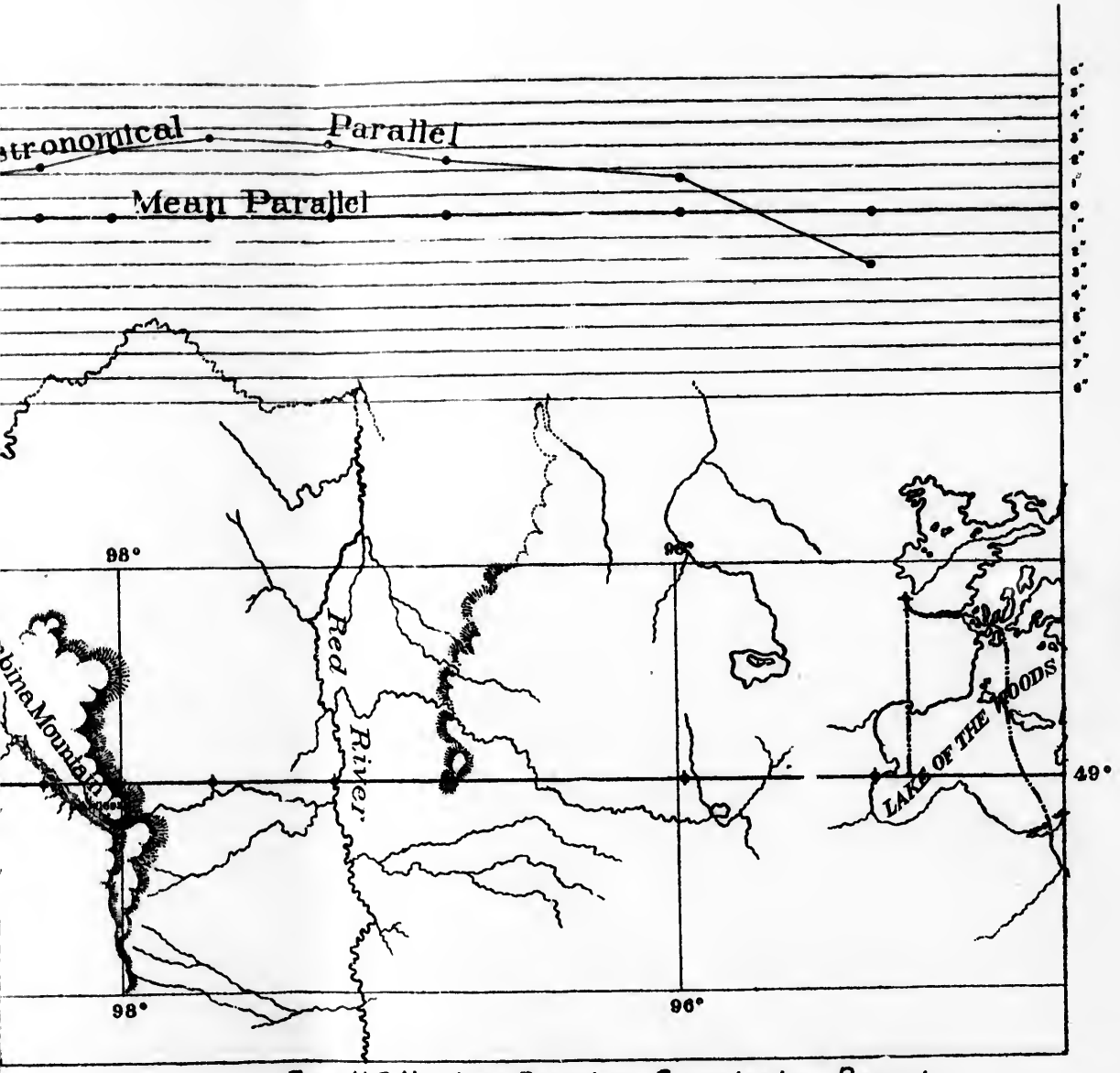
Woody
Mountain
(3400)
Poplar River
Bears Paw Mt
110°
108°
106°


Scale of abscissas. 1 inch = 30 miles. 1/1900800
Scale of ordinates: 1 inch = 8' of lat. = 810.72 feet = 1/9729
TOPOGRAPHY
Scale: 1 inch = 30 miles 1/1900800
104°
102°
Coteau of the Missouri
Mouse River
Turtle
104°
102°


Astronomical Parallel
Mean Parallel
100°
98°
96°
Pembina Mountain
Red River
Turtle Mountain
(2550)
100°
98°
96°
From U.S. Northern Boundary Commiss



From U.S. Northern Boundary Commission Report.

organized in 1858 (first meeting August 13th), and in the summer of the same year began the field work at the western terminus of the boundary. The observations and surveys were carried eastward under great difficulties, owing to the heavy forest and mountainous character of much of the country. Early in 1862 the field work was completed and the preparation of the maps begun, which were, however, not completed and jointly signed by the commissioners till 7th May, 1869. A report was never published.

Between the extreme east and west points, upon the watershed of the Rocky Mountains, and the eastern shore of the channel which separates the continent of North America from Vancouver Island in west longitude 114° 03′ 34″ and 123° 3′ 53″ respectively, the exact length of the boundary line upon the 49th parallel of north latitude is 409 4/10 miles. The position of the parallel was determined by 28 astronomical stations, 11 of which were established by the British Commission, 14 by the American Commission, and 3 were observed by both. Another station was fixed by the British Commission at Schweltza Lake, but it was at the time rejected on account of the apparently large deflection of the plumb line, though the after experience of the most accurate instrumental observations in that mountainous country, led to the conclusion that the result at Schweltza was quite as trustworthy as any of the others. It is, however, not included in the final determinations.

At the first meeting of the Commissioners at Semiahmoo, Aug. 13, 1858, it was concluded, after discussing plans for determining and marking the line as far as the Cascade Mountains, to be inexpedient at that time, in consequence of the great expense, consumption of time, and the impracticable nature of the country, to mark the whole boundary by cutting a track through the dense forest. It was therefore agreed to ascertain points on the line by the determination of astronomical points at convenient intervals on or near the boundary ; and to mark such astronomical stations or points fixed on the parallel forming the boundary, by cutting a track of not less than 20 feet in width on each side for the distance of half a mile or more, according to circumstance. Further, that the boundary be determined and similarly marked where it crosses streams of any size, permanent

trails, or any striking natural features of the country. In the vicinity of settlements, the line was to be cut a greater distance. Bessel's value of the figure of the earth was adopted.

From the two points on the parallel, dependent, respectively, on the Sumass and Schweltza astronomical stations, cuttings were made to connect the points. When the cuttings met, there was found to be a discrepancy of 8″, say about 810 feet; they were, however, connected, though the line thus defined is obviously not strictly *the* boundary of the treaty. The distance is about 9¼ miles. This relative deflection of the plumb line, 8″, in so short a distance, is the largest on the whole 49th parallel, from the Lake of the Woods to the Pacific. When the cuttings on the parallel from Sumass and the British station at Semiahmoo met, there was a discrepancy of 114 feet in the twenty miles, and between the U. S. astronomical station at the east shore of Semiahmoo Bay, and the British one five miles east thereof, a discrepancy on the parallel of nearly nine feet was found, an error quite within the error of observation.

The only other cutting on the whole boundary line west of the Rocky Mountains, connecting adjacent astronomic stations is between the Similkameen and the Columbia rivers, a distance of 96 miles. The stations there in order eastward are: Similkameen (U.S.); Lake Osoyoos (Br.); 1st Crossing Newhoilapitkw (U.S.); 2nd Crossing Inshwointum (Br.); 3rd Crossing Statapoosten (U.S.); and on the Columbia (Br. and U.S.) From the point on the parallel at Lake Osoyoos, a line was run east and west 30 1/5 miles, connecting with similar points at Similkameen and at the 1st Crossing. The line was found to strike 509 feet north of the former point and north of the latter 364 feet, showing a marked deflection of the plumb line. When, similarly, an east and west line was run from a point on the parallel at Inshwointum, it was found to be south 300 feet of the point on the parallel at the 1st crossing, and 180 feet north of the point at Statapoosten.

This shows, therefore, a discrepancy between the latitude of Lake Osoyoos (Br.) and Statapoosten (U.S.) of 844 feet, due to local attraction or difference of local attraction. After verifying the accuracy of the latitude observations, it was decided to adopt the mean parallel, based on the differences found, between Similkameen and Statapoosten—a distance of 71 miles This is the

only part of the whole boundry line between the Lake of the Woods and the gulf of Georgia, where a mean parallel has been adopted for the boundary, instead of the astronomic parallel. These seventy-one miles were re-cut on the mean parallel. From the extremity of the mean parallel at Statapoosten, an east line was run to the Columbia, where a difference of 212 feet was found between the mean of the British and United States latitude determinations there and the mean parallel. The line (for final boundary) was thereupon deflected from Statapoosten so as to strike the above mean Columbia position of the 49th parallel.

The actual definition of the boundary is as follows: Its western extremity is marked by a substantial granite obelisk in longitude 123° 03′ 53″, west, standing upon a steep cliff on the western face of the promintory of Point Roberts, about 160 feet above the sea. For 44.8 miles eastward there are 42 iron pillars placed at suitable points on the boundary. One pillar stands on the eastern face of Point Roberts, 2 miles 704 yards from the obelisk, and there are two intermediate pillars in the interval at average distances apart of somewhat more than ¾ mile. A pillar on the west shore of Semiahmoo Bay is 12 miles 1,177 yds. from that on Point Roberts on the opposite side of the bay; and thence in 29¾ miles to the eastermost pillar the average distance apart is about 1380 yds., varying between 1 mile 1245 yds. and 198 yds. on the opposite bank of the Sumass River. These pillars all stand in a continuous cutting through the forest or in intervening patches of swamp and prairie. From the eastern-most iron pillar, to the right or west bank of the Similkameen river is 107.9 miles, the boundary is defined in the vicinity of 9 astronomical stations by 19 cairns or pyramids built of dry stones, and one bench mark cut on the face of a rock at Ensakwatch; and at several stations short vistas were also cut in the forest, between the cairns. This wide interval comprises the rugged and inhospitable region of the Cascade Mountains. One of the widest unmarked intervals on the boundary occurs in these mountains, between Pasayten and Naisnulch, the distance between the marked points being 23.7 miles. From a cairn at the foot of the mountains on the west side of the Similkameen river to the east or left bank of the Columbia, the boundary for 95 miles is well and continuously marked by 69 stone cairns and one mound of earth, and by forest cutting in all necessary cases.

This was the most favorable portion of the work, part of the line passing over rolling prairie country interspersed with wood ; but very considerable portions were also mountainous, rugged and heavily timbered, though more accessible from the valley of the Newhoialpitkw (Kettle) river than were the Cascade Mountains. Two cairns stand within 129 yards of each other on the east bank of the Columbia (one having been placed by each Commission) and the average distance apart of the remainder is 1 mile 679 yds. From the hill tops the line of boundary defined by cairns and cuttings can be traced for many miles. For the remaining 161.8 miles between the eastern cairn on the left bank of the Columbia river and the terminal point on the watershed of the Rocky Mountains in west longitude 114° 03′ 28″, the boundary passes over successive mountain ranges intersected only by the valley of the Kootenay River at two points 75¾ miles apart and by the adjacent valleys of the Flathead river and its tributary Kishenehu creek. This portion of the line is marked in the vicinity of 9 astronomical stations, by 26 cairns and one bench mark cut in the face of the rock at the Kootenay Mountain Station, and by a cairn fixed by survey on the trail between Kootenay west and Mooyie station ; and the usual forest vistas were cut at the usual defined points, besides longer cuttings of 7 and 10 miles at the eastern crossing of the Kootenay, and between the Flathead and Kishenehu rivers. On the summit of the Rocky Mountains the monument consists of a pyramid of dry stones, situate on a narrow saddle with precipitous sides connecting two lofty mountains, serving to identify the locality between the Columbia and the Rocky Mountains, exclusive of the Mooyie trail cairn, and the intervals between the Kootenay mountain and Kootenay west stations, and Mooyie and Yahk stations, the distance between the consecutively marked points at the several astronomical stations averages about 13¼ miles ; but between the stations named they extend to 25 and 24 miles owing to the inaccessible nature of the intervening country which is quite as bad as the Cascade Mountains.

As already stated, the Boundary Commissioners had agreed to understand the boundary laid out by them, to consist of a series of straight lines between the successively marked points, without regard to the distances between those points or the curve

of the parallel in the longer intervals. That they did upon the consideration that it was of the greatest importance that nothing should be left for future discussion of settlement, and that the operations should be final and conclusive. It may be stated that opposite the centre of a chord of 25 miles in length, the departure from the 49th parallel would be about 40 yards, and of 12 miles, 9 yards. Both these departures are probably far smaller than the deflection of the plumb line, at the governing astronomical stations.

We have, therefore, in the actual boundary line of British Coulmbia, a deviation from the 49th parallel, as given in the treaty of 15th June, 1846, in so far, that the straight lines replace the curve of the parallel between all the stations, and furthermore, that between Similkameen and Statapoosten, the mean parallel was adopted instead of the astronomically determined points.

We have followed now the 49th parallel for 1,270 miles, about one thirteenth of its circumference, and it has disclosed to us some of its vagaries as manifested in the latitude component of the deflection of the plumb line. This boundary line is the longest astronomic one on the earth, the nearest approach to it being the meridian separating West Australia from North and South Australia.

The same law or force which causes the deflection of the plumb line, determines the length of the seconds pendulum, preserves the planets in their orbits, and maintains the stability of the universe—is the law of gravitation. Our earth furnishes us with many interesting problems, and the very discordances observed—apparent though they are—tend to lead us on to unravelling the mysteries and intricacies of nature, and to unfolding the unity and harmony of the cosmos.