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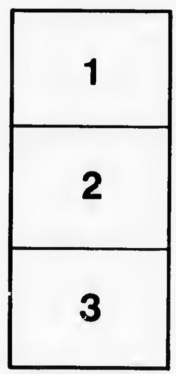

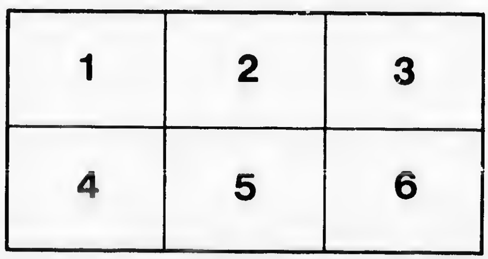

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"THE TRANSITION CURVE."

By Henry R. Lordly.

To be read on Friday, 28th October, 1892.

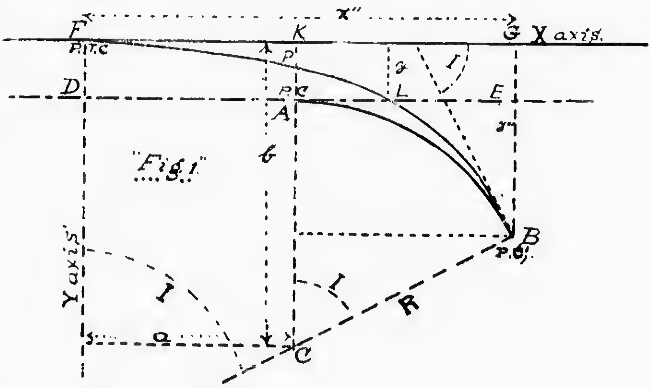
Although considerable has been written of late on the subject of "Transition Curves," much of which has been very interesting and of service to the profession, yet the question of deriving and laying out the curve practically has not yet received exhaustive treatment. The necessity of such a curve and the great benefit to be derived by its use has been so thoroughly discussed by different writers, that it will be treated as foreign to this paper. For reference to these points we recommend Mr. Wicksteed's paper, in "Transactions" of this Society, Vol. V, Part I.

The transition curve to which our attention will be given here, is the curve which has been worked up and developed by Prof. C. L. Crandall, C.E., of Cornell University. (Mr. Ellis Holbrook,[1] it is said, first proposed and used the principles in laying out curves for the Pittsburg, Cincinnati & St. Louis R.R., 1882, and a gentleman at Lehigh University worked up a solution of it a few years ago, but to Prof. Crandall, the honor is due for having put it in its present state of efficiency.)

This curve, as will be seen later, is strictly mathematically correct, and it has now been tested sufficiently in the field to show that besides this, its easy manipulation makes it invaluable to the engineer. In order to discuss it here, we will take it up under the following heading: (1.) Derivation of formulæ. (2.) Tables from the results of (1). (3.) Examples and general conclusions.

(1.) In order to counteract centrifugal force upon a circular curve the outer rail must be elevated, the change from the tangent being gradual to promote easy riding and to prevent twisting the trucks.

Therefore, taking the centrifugal force proportional to the elevation at every point, the curvature, in passing from the circular curve to the tangent, must increase directly with the distance.



[1] R. R. Gaz., Dec. 3, 1880.

Suppose in figure 1, AB to be a circular curve with centre C. Now if we begin at B to reduce the curvature directly with distance, continuing this reduction until curvature is zero, maintaining the same central angle I. as in circular curve, the new curve will pass outside AB, having a tangent of FG, parallel to DE, the tangent of AB, and at a certain distance, KA, from it. As is customary A is called the P.C.; F the P.T.C., and B the P.C[1]. Again let θ be the angle which the curve at any point L, makes with the initial tangent FG, s the length of arc FL. Then since by hypothesis the curvature at F is zero and increases with the distance from P; therefore the curvature of any point L, distant s from P, is equal to a constant multiplied by s. For convenience to avoid fractions later, this curvature is expressed by $2ks$, k being the constant depending on the rate of change of curvature.

The radius of the curvature equals $\frac{ds}{d\theta}$, θ being any angle, and and curvature varies inversely as the radius of curvature, we have $\frac{d\theta}{ds} = 2ks$, or, after integrating, $\theta = ks^2$.

If y is the ordinate, we have $dy = ds \sin\theta = ds \sin ks^2$ (a). but sin a (a being any angle) in series is equal to

$$* \quad a - \frac{a^3}{3!} + \frac{a^5}{5!} - \frac{a^7}{7!}\ rc.$$

$$\text{therefore in } (a)\ dy = ds\ (ks^2 - \frac{k^3s^6}{3!} + \frac{k^5s^{10}}{5!} \ \cdots$$

$$\text{therefore } y = \frac{ks^3}{3} - \frac{k^3s^7}{7.3.2} + \frac{k^5s^{11}}{11.5.4.3.2} \ \cdots$$

$$= \frac{ks^3}{3} - \frac{k^3s^7}{42} + \frac{k^5s^{11}}{1320} \quad \text{(b)}$$

* 3 ! = 1.2.3; 5 ! = 1.2.3.4.5. and so on.

In the same way, for the abscissa x, we have $dx = ds \cos\theta = ds \cos ks^2$

$$\text{and } \cos a \text{ in series} = 1 - \frac{a^2}{2!} + \frac{a^4}{4!} - \cdots$$

$$\text{therefore } dx = ds\left(1 - \frac{k^2s^4}{2!} + \frac{k^4s^8}{4!}\right) \ \cdots$$

$$\therefore \quad x = s - \frac{k^2s^5}{10} + \frac{k^4s^9}{216} - \cdots \text{(c)}$$

From these two equations (b) and (c) the length s of the arc FL is expressed in terms of the co-ordinates of the curve, but before using them the investigation must be carried further.

After passing this point two methods of procedure are open. First, reducing the curvature from the P. C.[1], in the same ratio as it was increased. Equation (b) will then give the radial ordinates for this part of the curve, besides the ordinates—from the tangent—for the first part. Also the curve and offset will bisect each other at P. If we should now place the second derivative for the first part at P equal to the second derivative of the second part at P, the K will be eliminated, leaving s to be ascertained for any given value of y or AK.

(The work of the above has been omitted here; it is simple application of the calculus, and, if interested, the reader may readily follow the steps through himself.)

Second—By continuing the curve to B, and placing the curve so that it shall be the circle of curvature, we will get for flat curves the same result as in the last case. When the offsets are larger, AK and the curve will not bisect each other, and the ordinates from the circle will differ slightly from the corresponding ones of the tangent. The author claims this to be theoretically the correct method, particularly if the curve is to be run with a transit. It also gives simpler formulæ.

Now from equation (1)

$$\text{(any arc) } S = \frac{ds}{d\theta} = \frac{1}{2ks} \text{ and } k = \frac{\theta}{s^2}$$

$$\therefore \quad = \frac{s}{2\theta}.$$

But at its limits S=R, and at the same time $2\theta = 2\,I_{(arc)}$; the s then being s'', the distance from the P. T. C. to the P. C.[1]

$$\text{Then } I^\circ = \frac{s''}{2\ R \sin 1^\circ} = 1.86 \sqrt{FD.}$$

The latter value is obtained by assuming the sine=arc and then putting $s''=\sqrt{63y''}=\sqrt{24\times5730\ F\div D}.\quad \theta=\frac{Ks^2}{\sin 1^\circ}.$

From equations (b) and (c), y and x may be found, and from these same equations, remembering that

$$R=\frac{5730}{D}\ \text{and}\ K=\frac{D}{11460s''},$$

we get, for the point B

$$y''=\frac{s''^2}{6\,R}-\frac{s''4}{336\ R^3}+\frac{s''4}{42240\ R^5}$$

$$\text{and}\ x=s''-\frac{s''3}{40\ R^2}+\frac{s''5}{3456\ R^4}.$$

(II.) We have now derived all the formulæ from which to make up our tables. In getting s'' for a given circular curve, the R would be assumed and the x'' and y'' then found. s'' may be more conveniently obtained from given values of F, the offset, for assuming $y''=4$ F, an approximate value of s'' results from using only the first term in the value of y''. This value, slightly increased, if substituted in the second and third terms, will give a value of s'' from the first term which will be sufficiently accurate. Having s'', we are now able to find I, K, F, etc. . . .

The values of F are now compared with the assumed one, and several trials may be necessary before the two values agree. If a few values of F in different parts of the table are ascertained, a certain relation is found to exist between F and y'', so that there is little trouble in getting subsequent values of F. When $x=a$ (fig. 1), the x' of the table is the length of the transition curve from P. C. to P. T. C. The $e=x'-a$, or the excess of transition curve over that of the tangent F K ; $e'=s''-x'-(100\ I\div D)$ or the excess of transition curve over that of circular curve from P. C. to P. C[1]; c is the chord length F B; x is tangent length to P. C.=F. K., and y is ordinate from tangent to the curve opposite P. C.=P. K.

Prof. Crandall has worked up a very complete set of tables, the curvature being up to 26°; 2° to 14° inclusive, and from 14° to 26°, taking 14°, 16°, 18°, etc. S'' ranges from 40 ft. to 800 ft.

For fractional values of F and D we may interpolate. F being very nearly proportional to y'', s''^2 will be proportional to F and therefore to $1\div D$.

S'' is half length of transition curve.

Below are given a few values from the tables, for illustration:

6° Curve.

S''	I°	x	y	c	e	e'	F	x'	y
60	1.80	60	0.63	60		—.01	0.16	30	0.08
100	3.00	100	1.74	100		.01	0.44	50	0.22
200	6.00	199.8	6.98	199.9	.01	—.01	1.75	100	0.87
300	9.00	299.3	15.68	299.7	.02	.04	3.92	149.9	1.96
400	12.00	398.2	27.83	399.2	.05	.18	6.96	199.7	3.48
600	18.00	594.1	62.38	597.3	.20	.80	15.67	299.0	7.79

S'' is taken for every 20 ft. In computing S''. I. F. and x' only, are really required.

(III.) The curve may be laid out by deflection angles or by offsets. The distance s'' is divided into 20 equal parts, y and x being found from the formula, for each point, then $y\div x$ gives the tangent for the respective deflection angles, the transit being at the P. T. C. These angles are tabulated in parts of I, and are almost proportional to I as D and F vary. The greatest error is really very small. Also from the values of θ in the formula, the central angles, beginning at the P. T. C. are proportional to I, the same being true of the central angles subtended by the short chords. Below we give the notes for a transition curve, by deflections, just as it appears in the transit book. From this we believe the reader will see the method of operation without much further explanation.

STATION.	POINT.	BEARING.	VERNIER.	CURVE DATA.
52+40.5	P. T. C.[1]		3° 00′=$\frac{1}{3}$ I	
+80.5			3° 43′	
51+20.5			4° 41′	
+60.5			5° 53′	
50+00.5			7° 19′	
● +40.5	P. C.[1] ⊙		4° 44′	
49			3° 31′	
48			0° 31′	Vertex=48+67
* +82.7	P. C[1] ⊙	6° Left.	3° 00′=$\frac{1}{3}$ I	△=27° 28′
47+22.7			1° 55′.2	D= 6°
+62.7			1° 04′.5	I= 9°
46+02.7			0° 28′.8	T=234.44
45+42.7			0° 07′.2	S″=300
44+82.7	Offset 3.92 P. T. C. ⊙	N. 20 W.		F=0° 03′.92

Here we have taken S″=300. ∴ F=3.92, and s or x = 149.9. T[1]=234.44. We divide 300 by 5, which is 60 ft. for chord length, which is reasonable length. Then as central angles are as square of distance:

$(\frac{1}{5})^2 \times 9° = 21°\ 06'$ ∴ 07′.2 =deflection.
$(\frac{2}{5})^2 \times 9° = 1°\ 26.4'$ ∴ 0° 28′.8= "
$(\frac{3}{5})^2 \times 9° = 3°\ 14.4'$ ∴ 1° 04′.8= "
$(\frac{4}{5})^2 \times 9° = 5°\ 45.6$ ∴ 1° 55′.2= "
$(\frac{5}{5})^2 \times 9° = 9°\ 00'$ ∴ 3° 00′ = " $=\frac{1}{3}$ I.

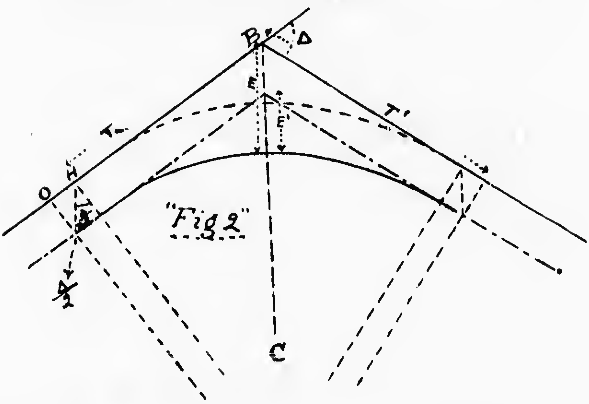
Since △=27° 28′ and 18° is used up for T. curve ∴ 9° 28′ remains for simple curve, which is (9° 28÷6°) long=1+57.8. (The deflections from P. C.[1] to P. T. C.[1] are taken from Table No. 2, a description of which has been omitted for want of space. It is constructed from the tangent, in series and an equation, $d=\frac{1}{3}\cdot\frac{1}{s'^4}(s^2+s'^2+ss)-$ . . . . This table should be very valuable to the engineer. Without it we begin from the P. T. C.[1] and run to the P. C.[1] with the deflections first found.

If we wish to simply put in the offsets and run the curve later, we place the stakes as follows:

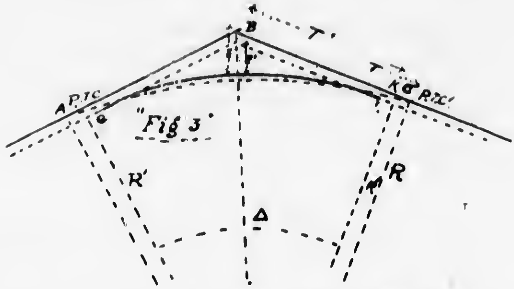
	P. I. (point of intersection)=	48+67
	T[1]=	2+34.4
∴ P. C.		=46+32.6
		+ 4+57.8
	P. T.	50+90.4

Then at Sts. 46+32.6 and 50+90.4 offsets are placed. (4+57.8)= $\frac{\triangle}{D}=\frac{27°\ 28'}{6°}$.

As will be seen, it is better to work forward instead of from the P. T. C. and P. T. C.[1] to the circular curve.



* Set up transit and set to 6° for backsight.
● " " " 4° 44′ + I = 13° 44′ for backsight, etc.



If the front tangent has not been located, in beginning the circular curve proceed as follows. Set up transit at offset distance inside the tangent, or at the P. C.; backsight to a point similarly offset; then run the curve as usual. At the P. T. the operation will have to be reversed. If, on the contrary, the front tangent has been fixed, T^1 and not T, must be measured from the vertex to locate a point from which to lay off F from the P. C., $T^1 = (R+F)\tan\frac{\triangle}{2}$, and is found, from fig. 2, as follows:

$$T^1 = HB + HO$$
$$= R\tan\frac{\triangle}{2} + F\tan\frac{\triangle}{2}$$
$$= (R+F)\tan\frac{\triangle}{2}$$
$$\text{or } = T + F\tan\frac{\triangle}{2}.$$

Here we have △, D and F given.

As the figure indicates, the circular curve is moved parallel to itself to a distance F, from its former position, in order to make room for the transition curve. The new curve then has an external distance, with reference to the old tangent, equal to or slightly less than the old, the offset being small. Thus $E^1 \leqq E - \frac{F}{\cos\frac{\triangle}{2}}$. From Searles' Table VI we may take the E for a 1° curve; divide this value by E^1 for D^1, and then change the latter value enough to avoid fractional minutes, before finding the length of the curve and T^1.

In case the new curve should fit the roadbed better by extending as far outside the old curve at centre as inside at the P. C. we would have:

$$E^1 = E - \frac{F}{\cos\frac{\triangle}{2}} - \frac{F}{2}.$$

Another important case arises where a transition curve is to be put in on old track, the new track being same length as the old. This is to prevent cutting the rails. In fig. 3 let

$$BC = T = R\tan\frac{\triangle}{2}$$

$$\text{and } BK = T^1 = (R^1+F)\tan\frac{\triangle}{2}.$$

The arc AC = length of old track
= R△° arc 1°
and arc GL = R^1△° arc 1°.

Now, the length of new track from A to C, the transition curve being put in, is equal to $GL + 2\,(BC - BK) + 2\,(e+e^1)$, therefore by substitution we get $R\triangle^\circ \text{ arc } 1^\circ = R^1\triangle^\circ \text{ arc } 1^\circ + 2\,R\tan\frac{\triangle^\circ}{2} - 2\,(R^1+F)\tan\frac{\triangle}{2} + 2\,(e+e^1)$, therefore

$$R^1 = \frac{R\triangle^\circ \text{ arc } 1^\circ - 2\,(R-F)\tan\frac{\triangle}{2} - 2\,(e+e^1)}{\triangle^\circ \text{ arc } 1^\circ - 2\tan\frac{\triangle}{2}}.$$

The following will show the use of the above equation. Find the data for a transition curve where the track is already laid on a 6° curve, 800 ft. long.

Taking 25 ft. of transition curve per degree we have $c = 2\times150 = 300$. Then from the tables we get F=3.92 ft.; $(e+e^1)=$

$(.02+.04)=0.06$; $\triangle=48°$; $R=\frac{5730}{6}=955$. Substituting in the equation

$$R^1=\frac{799.90-846.69-0.12}{0.83760-0.89046}$$

$$=\frac{-46.91}{-0.05286}=887.43.$$

therefore from Searles' Table VI we get

$$D^1=6°28'=6°.466.$$

$$L^1=48\div6.466=742.35 \text{ ft.}$$

$$T^1=(887.43+3.92 \tan\frac{\triangle}{2}=396.85 \text{ ft.},$$

which is the data required. Other problems might be taken, but we believe enough has been given to show the working of the curve.

Some engineers, it might be remarked, seem to think that using what they call "elaborate transition curves" is a waste of time. No reason is offered, however, to show why it should take more time to do it right than wrong. At any rate, present railroad practice demands the best, and a properly qualified engineer is one able to respond to these demands. Surely, if a thing is worth doing at all, it is worth doing well.

In conclusion the writer wishes to extend thanks to Prof. Crandall for his kindness in allowing the use of his notes and tables.