IMAGE EVALUATION TEST TARGET (MT-3)

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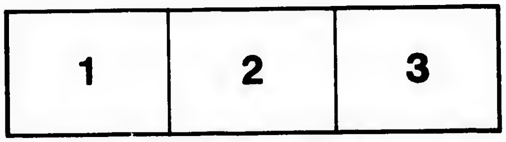
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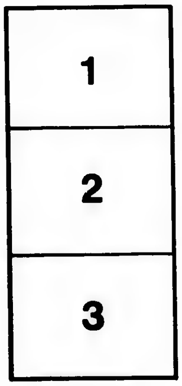
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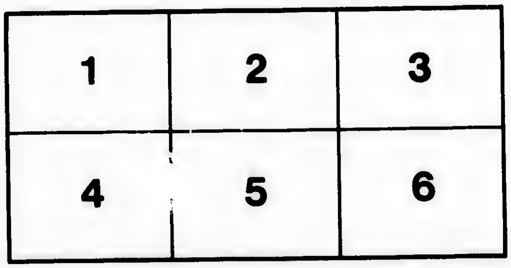
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12. On Hansen's Theory of the Physical Constitution of the Moon. By Simon Newcomb, of Washington.

The great reputation of the author has given extensive currency to the hypothesis put forth by Professor Hansen some years since, that the centre of gravity of the moon is considerably farther removed from us than the centre of figure. The consequences of this hypothesis are developed in an elaborate mathematical memoir to be found in the twenty-fourth volume of the Memoirs of the Royal Astronomical Society. But the reception of the doctrine seems to have been based rather on faith in its author, than on any critical examination of its logical foundation. Such an examination it is proposed to give it.

An indispensable preliminary to this examination is a clear understanding of what the basis of the doctrine is. Let us then consider these three propositions:

1. The moon revolves on her axis with a uniform motion equal to her mean motion around the earth.

2. Her motion around the earth is not uniform, but she is sometimes ahead of, and sometimes behind her mean place, owing both to the elliptic inequality of her motions and to perturbations.

3. Suppose her centre of gravity to be further removed from us than her centre of figure, and so placed that when the moon is in her mean position in her orbit, the line joining these centres passes through the centre of the earth.

Let us also conceive that these two centres are visible to an observer on the earth. Then a consideration of the geometrical arrangements of the problem will make it clear that when the moon is ahead of her mean place the observer will see the

Cambridge, 1869

two centres separated, the one nearest him being farther advanced in the orbit, while when the moon is behind her mean place the nearest centre will be behind the other. This apparent oscillation of the two centres is, indeed, an immediate effect of the moon's libration in longitude, as may be seen from the following figure in which the circle represents the moon, C and G her centres of figure and gravity, and E E′ E″ the

positions of an observer on the earth, relatively to the face of the moon, when she is behind, in, and ahead of her mean place.

Now the inequalities in the moon's motion computed from the theory of gravitation, are those of a supposed centre of gravity. But the inequalities given by observation are those of the centre of figure. Hence, in the case supposed, the inequalities of observation will be greater than those of theory. Also their ratio will be inversely as that of the distances of the centres which they represent.

Professor Hansen, in comparing his theory with observations, found that the theoretical inequalities would agree better with observation when multiplied by the constant factor 1.0001544. Supposing that this result could be accounted for on the hypothesis of a separation of the centres of gravity and figure, he thence inferred that the hypothesis was true. But the result cannot be entirely accounted for in this way, because the largest inequality of theory (evection) has a factor (eccentricity) which can only be determined from observation, and therefore, even the theoretical evection is that of the centre of figure and not of the centre of gravity. It must not be forgotten, that the eccentricity, which is not given by theory, is subject to be multiplied by the same factor that multiplies the other inequalities.

To be more explicit: —

Let e be the true eccentricity of the orbit described by the moon's centre of gravity; then the true evection in the same orbit will be

$$e \times A;$$

A being a factor depending principally on the mean motions of the sun and moon; and on Hansen's hypothesis the apparent evection, or that of the centre of figure, will be

$$e \times A \times 1.0001544.$$

On the same hypothesis, the eccentricity derived from observation, being half the coefficient of the principal term of the equation of the centre, will be

$$e \times 1.0001544,$$

and the theoretical evection computed with this eccentricity will be

$$e \times 1.0001544 \times A,$$

which is the same with that derived from observation. Hence:

The theoretical evection will agree with that of observation notwithstanding a separation of the centres of gravity and figure of the moon.

That Hansen overlooked this point is to be attributed to his method of determining the lunar perturbations, by numerical computation from the various elements of the moon's motion, so that the manner in which the inequality depends on the elements does not appear. It is only when we determine the perturbations in algebraic form that this dependence appears.

Passing now from the evection, the next great perturbation of the moon's motion is the variation. But the value of this perturbation has not been accurately determined from observation, because, attaining its maxima and minima in the moon's octants, it is complicated with the moon's semi-diameter, and parallactic inequality. Even if the semi-diameter is known, the two inequalities in question cannot be determined separately with precision, because their coefficients have the same sign in that part of the moon's orbit where nearly all the meridian observations are made. From this cause Airy's value of the parallactic inequality from all the Greenwich observations from 1750 to 1830 was 3″ in error. And when in his last investigation Airy rejected the observations previous to 1811, owing to some uncertainty as to what semi-diameter should be employed, the result was still a second too small. It is therefore interesting to find what value of the variation will result if we substitute the known value of the parallactic inequality in Airy's equations for the determination of that element. Neglecting those unknowns which have small coefficients, these equations are from 1806 to 1851,

1806 — 15	10.66 W	+	28.14 V	= +	17.2
16 — 24	9.45	+	30.92	+	24.9
25 — 33	9.43	+	29.26	+	42.1
34 — 42	9.29	+	27.28	+	10.8
43 — 51	9.05	+	23.36	+	7.9
Sum, . .	47.88 W	+	138.96 V	= +	102.9

In these equations $W \times 0''.73$ represents the correction to the coefficient of variation, and $V \times 3''.77$ that to the coefficient of parallactic inequality. We now know from recent special investigations that the latter coefficient is very near 125″.50. Airy's provisional one was 122″.10, whence

$$V = \frac{125''.50 - 122''.10}{3''.77} = 0.90$$

The sum of the preceding equations gives

$$W = 2.15 - 2.90\, V = -0.46.$$

The resulting correction to the provision variation (2370″.3) is therefore $-0.46 \times 0''.73 = -0''.34$, making the variation derived from observation . . . . . . 2369″.96, while Hansen's theoretical value is . . . . 2369 .86, and Delaunay's . . . . 2369 .74. The differences are too minute to found any theory upon. Leaving the evection and variation, the other inequalities are so minute that their product by Hansen's coefficient is altogether insensible.

Summing up the results of our inquiry, it appears that in the case of the evection, the supposed discordance between theory and observation would not follow from Hansen's hypothesis, and, therefore, even if it exists, cannot be attributed to that hypothesis as a cause. In the case of the variation no such discordance has been proved. In the case of the other inequalities the discordance would be insensible. The hypothesis is therefore without logical foundation.

The question whether the evection given by observation is really greater than that deduced from theory, although it does not affect our conclusion, is yet interesting and important. It appears from the commencement of Hansen's computations in his "*Darlegung*,"* that his theoretical perturbations were computed with an eccentricity equal to .05490079, and that, on comparing with observation, he found that this eccentricity should be increased to .05490807, which is the value adopted in the *Tables de La Lune*, and is greater than that employed in his theory in the ratio 1 : 1.0001326. If, now, he had employed this corrected eccentricity to recompute the evection, the latter would have been increased in the same ratio, and the outstanding discrepency between theory and observation would have been reduced .0001544 — .0001326 = .0000218 of it value, or 0″.10, a quantity no larger than may be attributed to the errors of theory and observation.

It does not, therefore, appear that there is any sensible discordance between the values of the two great perturbations of the moon which result from Hansen's theory, and those which result from all the Greenwich observations from 1811 to 1851.

*Erste Abhandlung, § 4, pp. 173-175.