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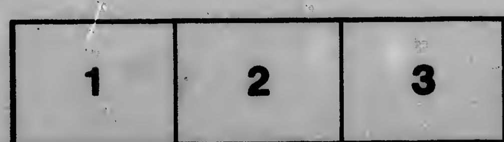

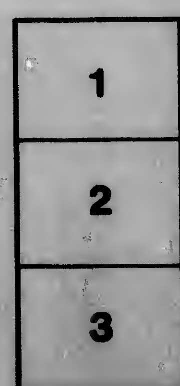

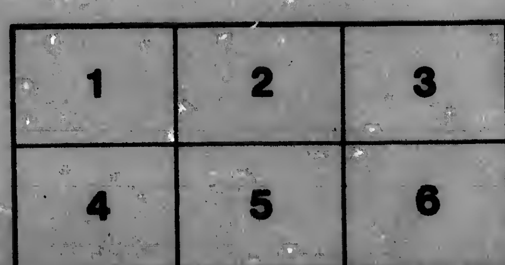

THE PLANET JUPITER.
As seen with the 26-inch telescope at Washington, 1875, June 24.

5, June 24.

AMERICAN SCIENCE SERIES V.3.

ASTRONOMY

FOR

SCHOOLS AND COLLEGES

BY

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PREFACE.

The following work is designed principally for the use of those who desire to pursue the study of Astronomy as a branch of liberal education. To facilitate its use by students of different grades, the subject-matter is divided into two classes, distinguished by the size of the type. The portions in large type form a complete course for the use of those who desire only such a general knowledge of the subject as can be acquired without the application of advanced mathematics. Sometimes, especially in the earlier chapters, a knowledge of elementary trigonometry and natural philosophy will be found necessary to the full understanding of this course, but it is believed that it can nearly all be mastered by one having at command only those geometrical ideas which are familiar to most intelligent students in our advanced schools.

The portions in small type comprise additions for the use of those students who either desire a more detailed and precise knowledge of the subject, or who intend to make astronomy a special study. In this, as in the elementary course, the rule has been never to use more advanced mathematical methods than are necessary to the development of the subject, but in some cases a knowledge of Analytic Geometry, in others of the Differential Calculus, and in others of elementary Mechanics, is necessarily presupposed. The object aimed at has been to lay a broad foundation for further study rather than to attempt the detailed presentation of any special branch.

As some students, especially in seminaries, may not desire so extended a knowledge of the subject as that embraced in the course in large type, the following hints are added for their benefit: Chapter I., on the relation of the earth to the heavens, Chapter III., on the motion of the earth, and the chapter on Chronology should, so far as possible, be mastered by all. The remaining parts of the course may be left to the selection of the teacher or student. Most persons will desire to know something of the telescope (Chapter II.), of the arrangement of the solar system (Chapter IV., §§ 1–2, and Part II., Chapter II.), of eclipses, of the phases of the moon, of the physical constitution of the sun (Part II., Chapter II.), and of the constellations (Part III., Chapter I.). It is to be expected that all will be interested in the subjects of the planets, comets, and meteors, treated in Part II., the study of which involves no difficulty.

An acknowledgment is due to the managers of the Clarendon Press, Oxford, who have allowed the use of a number of electrotypes from CHAMBERS's *Descriptive Astronomy*. Messrs. FAUTH & Co., instrument-makers, of Washington, have also lent electrotypes of instruments, and a few electrotypes have been kindly furnished by the editors of the *American Journal of Science* and of the *Popular Science Monthly*. The greater part of the illustrations have, however, been prepared expressly for the work.

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ASTRONOMY.

INTRODUCTION.

Astronomy (ἀστήρ—a star, and νόμος—a law) is the science which has to do with the heavenly bodies, their appearances, their nature, and the laws governing their real and their apparent motions.

In approaching the study of th , the most ancient of the sciences depending upon observation, it must be borne in mind that its progress is most intimately connected with that of the race, it having always been the basis of geography and navigation, and the soul of chronology. Some of the chief advances and discoveries in abstract mathematics have been made in its service, and the methods both of observation and analysis once peculiar to its practice now furnish the firm bases upon which rest that great group of exact sciences which we call physics.

It is more important to the student that he should become penetrated with the spirit of the methods of astronomy than that he should recollect its minutiæ, and it is most important that the knowledge which he may gain from this or other books should be referred by him to its true sources. For example, it will often be necessary to speak of certain planes or circles, the ecliptic, the equator, the meridian, etc., and of the relation of the apparent positions of stars and planets to them ; but his labor will be useless if it has not succeeded in giving him a precise notion of these circles and planes as they exist in the sky, and not merely in the figures of his text-book. Above all, the study of this science, in which not a single step could have been taken without careful and painstaking observation of the heavens, should lead its student himself to attentively regard the phenomena daily and hourly presented to him by the heavens.

Does the sun set daily in the same point of the horizon? Does a change of his own station affect this and other aspects of the sky? At what time does the full moon rise? Which way are the horns of the young moon pointed? These and a thousand other questions are already answered by the observant eyes of the ancients, who discovered not only the existence, but the motions, of the various planets, and gave special names to no less than fourscore stars. The modern pupil is more richly equipped for observation than the ancient philosopher. If one could have put a mere opera-glass in the hands of HIPPARCHUS the world need not have waited two thousand years to know the nature of that early mystery, the Milky Way, nor would it have required a GALILEO to discover the phases of *Venus* and the spots on the sun.

From the earliest times the science has steadily progressed by means of faithful observation and sound reasoning upon the data which observation gives. The advances in our special knowledge of this science have made it convenient to regard it as divided into certain portions, which it is often convenient to consider separately, although the boundaries cannot be precisely fixed.

Spherical and Practical Astronomy.—First in logical order we have the instruments and methods by which the positions of the heavenly bodies are determined from observation, and by which geographical positions are also fixed. The branch which treats of these is called spherical and practical astronomy. Spherical astronomy provides the mathematical theory, and practical astronomy (which is almost as much an art as a science) treats of the application of this theory.

Theoretical Astronomy deals with the laws of motion of the celestial bodies as determined by repeated observations of their positions, and by the laws according to which they ought to move under the influence of their mutual gravitation. The purely mathematical part of the science, by which the laws of the celestial motions are deduced from the theory of gravitation alone, is also called *Celestial Mechanics*, a term first applied by LA PLACE in the title of his great work *Mécanique Céleste*.

Cosmical Physics.—A third branch which has received its greatest developments in quite recent times may be called *Cosmical Physics.* Physical astronomy might be a better appellation, were it not sometimes applied to celestial mechanics. This branch treats of the physical constitution and aspects of the heavenly bodies as investigated with the telescope, the spectroscope, etc.

We thus have three great branches which run into each other by insensible gradations, but under which a large part of the astronomical research of the present day may be included. In a work like the present, however, it will not be advisable to follow strictly this order of subjects; we shall rather strive to present the whole subject in the order in which it can best be understood. This order will be somewhat like that in which the knowledge has been actually acquired by the astronomers of different ages.

Owing to the frequency with which we have to use terms expressing angular measure, or referring to circles on a sphere, it may be admissible, at the outset, to give an idea of these terms, and to recapitulate some properties of the sphere.

Angular Measures.—The unit of angular measure most used for considerable angles, is the degree, 360 of which extend round the circle. The reader knows that it is 90° from the horizon to the zenith, and that two objects 180° apart are diametrically opposite. An idea of distances of a few degrees may be obtained by looking at the two stars which form the pointers in the constellation *Ursa Major* (the Dipper), soon to be described. These stars are 5° apart. The angular diameters of the sun and moon are each a little more than half a degree, or 30′.

An object subtending an angle of only one minute appears as a point rather than a disk, but is still plainly visible to the ordinary eye. HELMHOLTZ finds that if two minute points are nearer together than about 1′ 12″, no eye can any longer distinguish them as two. If the objects are not plainly visible—if they are small stars, for instance, they may have to be separated 3′, 5′, or even 10′, to be seen as separate objects. Near the star *α Lyræ* are a pair of stars $3\frac{1}{2}'$ apart, which can be separated only by very good eyes.

If the object be not a point, but a long line, it may be seen by a good eye when its breadth subtends an angle of only a fraction of a minute; the limit probably ranges from 10″ to 15″.

If the object be much brighter than the background on which it is seen, there is no limit below which it is necessarily invisible. Its visibility then depends solely on the quantity of light which it sends to the eye. It is not likely that the brightest stars subtend an angle of $\frac{1}{100}$ of a second.

So long as the angle subtended by an object is small, we may regard it as varying directly as the linear magnitude of the body, and inversely as its distance from the observer. A line seen perpendicularly subtends an angle of 1° when it is a little less than 60 times its length distant from the observer (more exactly when it is 57·3 lengths distant); an angle of 1′ when it is 3438 lengths distant, and of 1″ when it is 206265 lengths distant. These numbers are obtained by dividing the number of degrees, minutes, and seconds, respectively, in the circumference, by 2 × 3·14159265, the ratio of the circumference of a circle to the radius.

Great Circles of the Sphere.—In Fig. 1 let the outline represent that of a sphere, around which are described the two great circles $A\,E\,B\,F$ and $C\,E\,D\,F$. These circles are the lines in which two planes passing through the centre O of the sphere intersect the latter. We may consider them as representing the planes.

The points P and P', each of which is 90° distant from every point of the circle $A\,E\,B\,F$, are called the

FIG. 1.—SECTIONS OF A SPHERE BY PLANES.

poles of that circle. The poles are the points in which a line passing through the centre O perpendicular to the plane of the circle meets the sphere. They may be considered as representing this line.

The angle $B\,D$, or $A\,C$, equal to the greatest distance of the two circles, is the same as the angle which the planes of the circles make with each other. The distance between the poles $P\,Q$ or $P'\,Q'$ is equal to the same angle. There are therefore three equivalent representatives for what may be considered the same element; namely: (1) the inclination of the planes of two circles; (2) the angle between their poles; and (3) the greatest angles, $A\,C$ or $B\,D$, between the circles on the celestial sphere.

SYMBOLS AND ABBREVIATIONS.

SIGNS OF THE PLANETS, ETC.

☉	The Sun.	♂	Mars.
☾	The Moon.	♃	Jupiter.
☿	Mercury.	♄	Saturn.
♀	Venus.	♅	Uranus.
⊕ or ♁	The Earth.	♆	Neptune.

The asteroids are distinguished by a circle inclosing a number, which number indicates the order of discovery, or by their names, or by both, as (100); *Hecate*

SIGNS OF THE ZODIAC.

Spring signs.	1.	♈	Aries.	Autumn signs.	7.	♎	Libra.
	2.	♉	Taurus.		8.	♏	Scorpius.
	3.	♊	Gemini.		9.	♐	Sagittarius.
Summer signs.	4.	♋	Cancer.	Winter signs.	10.	♑	Capricornus.
	5.	♌	Leo.		11.	♒	Aquarius.
	6.	♍	Virgo.		12.	♓	Pisces.

ASPECTS.

☌	Conjunction, or having the same longitude or right ascension.			
□	Quadrature, or differing 90° in	"	"	"
☍	Opposition, or differing 180° in	"	"	"

MISCELLANEOUS SYMBOLS.

☊ Ascending node.
☋ Descending node.
N. North. S. South.
E. East. W. West.
° Degrees.
′ Minutes of arc.
″ Seconds of arc.
ʰ Hours.
ᵐ Minutes of time.
ˢ Seconds of time.
L, Mean longitude of a body.
g, Mean anomaly.
f, True anomaly.
n, Mean sidereal motion in a unit of time.
r, Radius vector.
ϕ, Angle of eccentricity.
π, Longitude of perihelion (also parallax).

R.A. or α, Right ascension.
Dec. or δ, Declination.
ζ, True zenith distance.
ζ', Apparent zenith distance.
Δ Distance from the earth.
l, Heliocentric longitude.
b, Heliocentric latitude.
λ, Geocentric longitude.
β, Geocentric latitude.
θ or ☊, Longitude of ascending node.
i, Inclination of orbit to the ecliptic.
ω, Angular distance from perihelion to node.
u, Distance from node, or argument for latitude.
a, Altitude.
A, Azimuth.

ρ, Earth's Equatorial radius.

The Greek alphabet is here inserted to aid those who are not already familiar with it in reading the parts of the text in which its letters occur:

Letters.	Names.	Letters.	Names.
Α α	Alpha	Ν ν	Nu
Β β ϐ	Bēta	Ξ ξ	Xi
Γ γ ɼ	Gamma	Ο ο	Omicron
Δ δ	Delta	Π ϖ π	Pi
Ε ε	Epsilon	Ρ ρ ϱ	Rho
Ζ ζ ϛ	Zēta	Σ σ ς	Sigma
Η η	Eta	Τ τ ꚍ	Tau
Θ ϑ θ	Thēta	Υ υ	Upsilon
Ι ι	Iōta	Φ ϕ	Phi
Κ κ	Kappa	Χ χ	Chi
Λ λ	Lambda	Ψ ψ	Psi
Μ μ	Mu	Ω ω	Omega

THE METRIC SYSTEM.

The metric system of weights and measures being employed in this volume, the following relations between the units of this system most used and those of our ordinary one will be found convenient for reference:

MEASURES OF LENGTH.

1 kilometre	= 1000 metres	= 0·62137 mile.
1 metre	= the unit	= 39·37 inches.
1 millimetre	= $\frac{1}{1000}$ of a metre	= 0·03937 inch.

MEASURES OF WEIGHT.

1 millier or tonneau	= 1,000,000 grammes	= 2204·6 pounds.
1 kilogramme	= 1000 grammes	= 2·2046 pounds.
1 gramme	= the unit	= 15·432 grains.
1 milligramme	= $\frac{1}{1000}$ of a gramme	= 0·01543 grain.

The following rough approximations may be memorized:

The kilometre is a little more than $\frac{6}{10}$ of a mile, but less than $\frac{2}{3}$ of a mile.

The mile is $1\frac{6}{10}$ kilometres.

The kilogramme is $2\frac{1}{5}$ pounds.

The pound is less than half a kilogramme.

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CHAPTER I.

THE RELATION OF THE EARTH TO THE HEAVENS.

§ 1. THE EARTH.

In considering the relation of the earth to the heavens, we necessarily begin with the earth itself; not simply because we now know it to be one of the heavenly bodies, but because it is from its surface that all observations of the heavens have to be made.

A consideration of well-known facts will show that this earth upon which we live is, at least approximately, a globe whose dimensions are gigantic when compared to our ordinary and daily ideas of size. Its shape is in several ways known to be nearly that of a sphere.

FIG. 2.

Illustrating the fact that the portions of the earth visible from elevated positions, S, S', S'', etc., are bounded by circles.

I. It has been repeatedly circumnavigated in various directions.

II. Portions of its surface, visible from elevated positions in the midst of extensive plains or at sea, appear to be bounded by circles. This appearance at all points of the surface of a body is a geometrical attribute of a globular form only.

III. Further than this we know that careful measurements of portions of the globe by the various national geodetic surveys have agreed with this general conclusion.

More precise reasons will be apparent later, but these will be sufficient to base our general considerations upon. Of the *size* of the earth we may form a rough idea by the time required to travel completely around it, which is now about three months.

We find next that this globe is completely isolated in space. It neither rests on any thing else, nor is it in contact with any surrounding body. The most obvious proof of this which presents itself is, that mankind have visited nearly every part of its surface without finding any such connection, and that the heavenly bodies seem to perform complete circuits around it and under it without meeting with any obstacles. The sun which rose today is the same body as the setting sun of yesterday, but it has been seen to move (apparently) about the earth from east to west during the day, and it regularly reappears each morning. Moreover, if attentively watched, it will be found to rise and set at different parts of the horizon of any place at different times of the year, which negatives the ancient belief that its nocturnal journey was made through a huge subterranean tunnel.

§ 2. THE DIURNAL MOTION AND THE CELESTIAL SPHERE.

Passing now from the earth to the heavens, and viewing the sun by day, or the stars by night, the first phenomenon which claims our attention is that of the diurnal motion.

We must here caution the reader to carefully distinguish between *apparent* and *real* motions. For example, when the phenomena of the diurnal motion are set forth as real visible motions, he must be prepared to learn subsequently that this *appearance*, which is obvious to all, is yet a consequence of a *real* motion only to be detected by reason. We shall first describe the diurnal motion as it appears, and show that all the appearances to a spectator at any one place may be represented by supposing the earth to remain fixed in space, and the whole concave of the heavens to turn about it, and finally it will be shown that we have reason to believe that the solid earth itself is in constant rotation while the heavens remain immovable, presenting different portions in turn to the observer.

The motion in question is most obvious in the case of the sun, which appears to make a daily circuit in the heavens, rising in the east, passing over toward the south, setting in the west, and moving around under the earth until it reaches the eastern horizon again. Observations of the stars made through any one evening show that they also appear to perform a similar circuit. Whatever stars we see near the eastern horizon will be found constantly rising higher, and moving toward the south, while those in the west will be constantly setting. If we watch a star which is rising at the same point of the horizon where the sun rises, we shall find it to pursue nearly the same course in the heavens through the night that the sun follows through the day. Continued observations will show, however, that there are some stars which do not set at all—namely, those in the north. Instead of rising and setting, they appear to perform a daily revolution around a point in the heavens which in our latitudes is nearly half way between the zenith and the northern horizon. This central point is called the pole of the heavens. Near it is situated *Polaris*, or the pole star. It may be recognized by the *Pointers*, two stars in the constellation *Ursa Major*, familiarly known as *The Dipper*. These stars are shown in Fig. 3. If we watch any star between the pole and the north horizon, we shall find that instead of moving from east to west, as the stars generally appear to move, it really appears to move toward the east; but instead of continuing its motion and setting in the east, we shall find that it gradually curves its course upward. If we could follow it for twenty-four hours we should see it move upwards in the north-east, and then pass over toward the west between the zenith and the pole, then sink down in the north-west; and on the following night curve its course once more toward the east. The arc which it appears to describe is a perfect circle, having the pole in its centre. The farther from the pole we go, the larger the circle which each star seems to describe; and when we get to a distance equal to that between the pole and the horizon, each star in its apparent passage below the pole just grazes the horizon.

FIG. 3.—THE APPARENT DIURNAL MOTION.

As a result of this apparent motion, each individual constellation changes its configuration with respect to the horizon, that part which is highest when the constellation is above the pole being lowest when below it. This is shown in Figure 4, which represents a supposed constellation at five different times of the night.

Going farther still from the pole, the stars will dip be-

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low the horizon during a portion of their course, and the fraction of the circle which is below the pole will be continually increasing. Looking yet farther south we shall find one half of the circle to be above and one half below the horizon. Farther yet, we shall find the stars describing shorter arcs while above the horizon, and therefore longer ones below it. Near the south horizon, each star rises for only a short time a little to the east of south, and soon sets a little to the west of it.

FIG. 4.

If we carefully study this motion, we shall find that it does not arise from each star pursuing an independent course, for not only do all the stars perform this apparent revolution in the very same time, but they also preserve unchanged their relative distances from each other, with the exception of five, called *planets* or *wandering stars*. The thousands of others which are visible to the naked eye preserve their relative positions with such exactness that the ordinary observer could perceive no change even after the lapse of centuries. This fact naturally suggested to the ancients the idea that there must be some material connection between the stars. An apparent explanation, both of this and of the phenomena of the diurnal motion, was offered by the conception of the *celestial sphere.* The salient phenomena of the heavens, from whatever point of the earth's surface they might be viewed, were represented by supposing that the globe of the earth was situated centrally within an immensely larger hollow sphere of the heavens. The visible portion, or upper half of this hollow sphere, as seen from any point, constituted the celestial vault, and the whole sphere, with the stars which studded it, was called the firmament. The stars were set in its interior surface, or the firmament might be supposed to be of a perfectly transparent crystal, and the stars might be situated in any portion of its thickness. About one half of the sphere could be seen from any point of the earth's surface, the view of the other half being necessarily cut off by the earth itself. This sphere was conceived to make a diurnal revolution around an axis, necessarily a purely mathematical line, passing centrally through it and through the earth. The ends of this axis were the poles. The situation of the north end, or north pole, was visible in northern latitudes, while the south pole was invisible, being below the horizon. A navigator sailing south would so change his horizon, owing to the sphericity of the earth, that the location of the north pole would sink out of sight, while that of the south pole would come into view.

It was clearly seen, even by the ancients, that the diurnal motion could be as well represented by supposing the celestial sphere to be at rest, and the earth to revolve around this axis, as by supposing the sphere to revolve. This doctrine of the earth's rotation was maintained by several of the ancient astronomers, notably by ARISTARCHUS and TIMOCHARIS. The opposite view, however, was maintained by PTOLEMY, who could not conceive that the earth could be endowed with such a rapid rotation without disturbing the motion of bodies at its surface. We now know that PTOLEMY was wrong, and his opponents right. Still, so far as the apparent diurnal motion is concerned, it is indifferent whether we conceive the earth or the heavens to be in motion. Sometimes the one conception, and sometimes the other, will make the phenomena the more clear. As a matter of fact, astronomers speak of the sun rising and setting, just as others do, although it is in reality the earth which turns. This is a form of language which, being designed only to represent the appearances, need not lead us into error.

The celestial sphere which we have described has long ceased to figure in astronomy as a reality. We now know that the celestial spaces are practically perfectly void; that some of the heavenly bodies, which appear to be on the surface of the celestial sphere at equal distances from the earth as a centre, are thousands, or even millions of times farther from the earth than others; that there is no material connection between them, and that the celestial sphere itself is only a result of optical perspective. But the language and the conception are still retained, because they afford the most clear and definite method of representing the directions of the heavenly bodies from the observer, wherever he may be situated. In this respect it serves the same purpose that the geometric sphere does in spherical trigonometry. The student of this science knows that there is really no need of supposing a sphere or a spherical triangle, because every spherical arc is only the representative of an angle between two lines which emanate from the centre, one to each end of the arc, while the angles of the triangle are only those of the planes containing the three lines which are drawn to each angle from the centre. Spherical trigonometry is, therefore, in reality, only the trigonometry of solid angles; and the purpose of the sphere is only to afford a convenient method of conceiving of such angles. In the same way, although the celestial sphere has no real existence, yet by conceiving of it as a reality, and supposing certain lines of reference drawn upon it, we are enabled to form an idea of the relative directions of the heavenly bodies. We may conceive of it in two ways: firstly, as having an infinite radius, in which case the centre of the earth, or any point of its surface, may equally be supposed to be in the centre of the celestial sphere; or, secondly, we may suppose it to be finite, the observer carrying the cen-

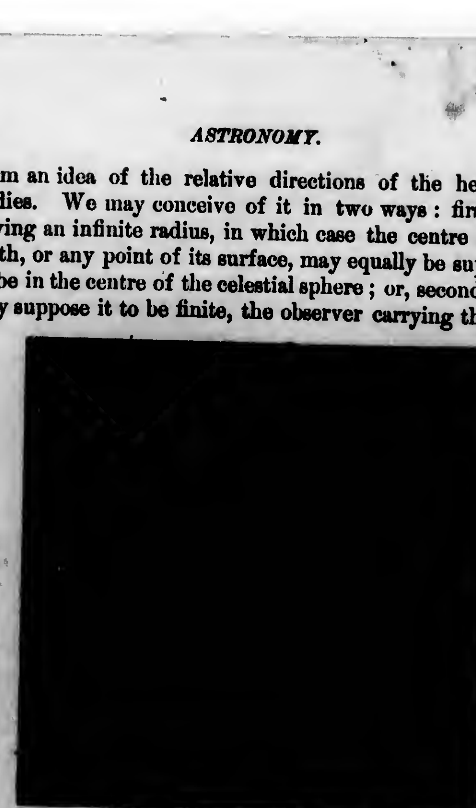

FIG. 5.—STARS SEEN ON THE CELESTIAL SPHERE.

tre with him wherever he goes. The first assumption will probably be the one which it is best to adopt. The object attained by each mode of representation is that of having the observer always in the centre of the supposed sphere. Fig. 5 will give the reader an idea of its application. He is supposed to be stationed in the centre, *O*, and to have around him the bodies *p q r s t*, etc. The sphere itself being supposed at an immense distance, outside of all these bodies, we may suppose lines to be drawn from each of them directly away from the centre until they reach the sphere. The points *P Q R S T*, etc., in which these lines intersect the sphere, will represent the apparent positions of the heavenly bodies as seen by the observer at O. If several of them, as those marked $t\ t\ t$, are in the same direction from the observer, they will appear to be projected on the same point of the sphere. Thus positions on the sphere represent simply the directions in which the bodies are seen, but have no direct relations to the distance.

It was seen by the ancients that the earth was only a point in comparison with the apparent sphere of the fixed stars. This was shown by the uniformity of the diurnal motion; if the earth had any sensible magnitude in comparison with the sphere of the heavens, the sun, or a star, would seem to be nearer to the observer when it passed the meridian, or any point near his zenith, than it would when it was below the horizon, or nearly under his feet, by a quantity equal to the diameter of the earth. Being nearer to him, it would seem to move more rapidly when above the horizon than when below, and its apparent angular dimensions would be greater in the zenith than in the horizon. As a matter of fact, however, the most refined observations do not show the slightest variation from perfect uniformity, no matter what the point at which the observer may stand. Therefore, observers all over the earth are apparently equally near the stars at every point of their apparent diurnal paths; whence their distance must be so great that in proportion to them the diameter of the earth entirely vanishes. This argument holds equally true whether we suppose the earth or the heavens to revolve, because the observer, carried around by the rotating earth, will be brought nearer to those stars which are over his head, and carried farther from them when he is on the opposite side of the circle in which he moves.

Suppose the earth to be at O, and the celestial sphere of the fixed stars to be represented in the figure by the circle $N\ Z\ Q\ S\ n$, etc. Suppose $N\ E\ S\ W$ to represent the plane of the *horizon* of some observer on the earth's surface. He will then see every thing *above* this plane, and nothing below it. If $N E S$ is his *eastern* horizon, stars will appear to rise at various points, g, E, d, a, etc., and will appear to describe circles until they attain their highest points at h, Q, e, b, etc., sinking into the western horizon at k, W, f, c, etc. These are facts of observation. The common *axis* of these circles is $P\ p$, and stars about P (the *pole*) never set. The apparent diurnal arc $l\ m$, for instance, represents the apparent orbit of a *circumpolar* star.

FIG. 6.

§ 3. CORRESPONDENCE OF THE TERRESTRIAL AND CELESTIAL SPHERES.

We have said that the direction of a heavenly body from an observer, or, which is the same thing, its apparent position, is defined by the point of the celestial sphere on which it seems to be. This point is that in which the straight line drawn from the observer to the body, and continued forward indefinitely, meets the celestial sphere. Its position is fixed by reference to certain fundamental circles supposed to be drawn on the sphere, on the same plan by which longitude and latitude on the earth are fixed. The system of thus defining terrestrial positions by reference to the earth's equator, and to some prime meridian from which we reckon the longitudes, is one with which the reader may be supposed familiar. We shall therefore commence with those circles of the celestial sphere which correspond to the meridians, parallels, etc., on the earth.

First, we remark that if we consider the earth to be at rest for a moment, every point on its surface is at the end of a radius which, if extended, would touch a corresponding point upon the celestial sphere. This point is called the *zenith* of the point on the earth. In other words, the zenith is defined by a line passing through the centre of the earth to the observer, and continuing directly upward until it meets the celestial sphere. To the observer this line necessarily appears vertical, because, wherever he may be, he understands by a vertical line one passing from where he stands toward the centre of the earth. As the earth revolves, the direction of this line in relation to any fixed diameter of the celestial sphere necessarily varies, and therefore the point in which it cuts the celestial sphere or the zenith of the observer varies also in space. Let us suppose first that the observer is on the earth's equator. Then he will see both the north and the south pole in the horizon directly opposite each other. Looking upward he will see his zenith half way between the poles. Then, as the earth revolves on its axis, his zenith will describe a great circle around the celestial sphere, every point of which will be equally distant from the two poles. If we imagine an infinitely long pencil reaching from any point of the earth's equator vertically up to the stars, we may conceive that its point marks out an equator among them. A complete revolution of the earth brings it back to the place from which it started, and thus completes the circle. The imaginary circle thus described in the heavens is called the *celestial equator*. The relation which it bears to the terrestrial equator is that every point of it is above a corresponding point of the latter. The two equators lie in the same plane, passing through the centre of the earth, which plane is called the *plane of the equator*, and belongs to both the celestial and terrestrial spheres.

Now suppose that the observer passes from the equator to 45° of north latitude. His horizon having changed by 45°, the north pole will now be 45° above the horizon, and 45° from the zenith. Then, by the revolution of the earth, his zenith will describe a circle on the celestial sphere which will be everywhere 45° distant from the celestial equator. This circle will thus correspond to the parallel of 45° north upon the earth. If he goes to latitude 60° north, he will see the pole at an elevation of 60°, and his zenith will in the same way describe a circle which will be everywhere 60° from the celestial equator, and 30° from the pole. If he passes to the pole, the latter will be directly over his head, and his zenith will not move at

FIG. 7.—TERRESTRIAL AND CELESTIAL SPHERES.

all. The celestial pole is simply the point in which the earth's axis of rotation, if continued out in a straight line of infinite length, would meet the celestial sphere. We thus have a series of circles on the celestial sphere corresponding to the parallels of latitude upon the earth. Unfortunately the celestial element corresponding to latitude on the earth is not called by that name, but by that of *declination*. The *declination* of a star is its distance north or south from the celestial equator, precisely as latitude on the earth is distance from the earth's equator.

Let L be a place on the earth, $P\,E\,p\,Q$, $P\,p$ being the earth's axis, and $E\,Q$ its equator. Z is the zenith and $H\,R$ the horizon of L. $L\,O\,Q$ is the *latitude* of L according to ordinary geographical definitions: *i.e.*, it is its angular distance from the equator.

FIG. 8.

Prolong $O\,P$ indefinitely to P', and draw $L\,P''$ parallel to it. To an observer at L the elevated pole of the heavens will be seen along the line $L\,P''$, because at an infinite distance the distance $P'\,P''$ will appear like a point. $H\,L\,Z = P\,O\,Q$ and $Z\,L\,P'' = Z\,O\,P'$, hence $P''\,L\,H = L\,O\,Q$—that is, *the elevation of the pole above the celestial horizon is equal to the latitude of the place.* Referring to Fig. 9, it can at once be seen that *the latitude of a place on the earth's surface is equal to the declination of the zenith of that place,* since the declination of the zenith is equal to the altitude of the elevated pole.

We have next to consider the correspondence between the celestial and terrestrial meridians. A terrestrial meridian is an imaginary line drawn along the earth's surface in a north and south direction from one pole to the other. These meridians diverge from one pole in every direction, and meet at the other pole. Sometimes they are called by the names of places they pass through, as the meridian of Greenwich, or the meridian of Washington. Each meridian may be considered as the intersection with the earth's surface of a plane passing through the axis of the earth, and therefore through both poles. Such a plane will cut the earth into two equal hemispheres, and will of course be vertical with the earth's surface along every part of its line of intersection. This plane is called the plane of the meridian; and by continuing it out to the celestial sphere, we should have a celestial meridian corresponding to each terrestrial one, precisely as we have circles of declination corresponding to parallels of latitude on the earth. But owing to the rotation of the earth, the circle in which the plane of the meridian of any place intersects the celestial sphere will be continually moving among the stars, so that there is no such permanent correspondence as in the case of the declinations. This does not prevent us from conceiving imaginary meridians passing from one pole of the heavens to the other precisely as the meridians on the earth do, only these meridians will be apparently in motion, owing to the rotation of the earth. We may, in fact, conceive of two sets of meridians—one really at rest among the stars, but apparently moving from east to west around the pole as the stars do, and the other the terrestrial meridians continued to the celestial sphere, apparently at rest, but really in motion from west to east. The relations of these meridians will be best understood when we explain the instruments and methods by which they are fixed, and by which the positions of the stars in the heavens are determined. At present we will confine ourselves to the consideration of the celestial meridians.

The reader will understand that these meridians pass from one pole of the celestial sphere to the other, precisely as on the globe terrestrial meridians pass from one pole to the other, and that being fixed among the stars, they appear to turn around the pole as the stars appear to do. As on the earth differences of longitude between different places are fixed by the differences between the meridians of the two places, so in the heavens what corresponds to longitude is fixed by the difference between the celestial meridians. This co-ordinate is, however, in the heavens not called longitude, but *right ascension*. Let the student very thoroughly impress upon his mind this term—right ascension—which is longitude on the celestial sphere, and also the term we have before spoken of—*declination*—which is latitude on the celestial sphere.

In order to fix the right ascension of a heavenly body, we must have a first meridian to count from, precisely as on the earth we count longitudes from the meridian of Greenwich or of Washington. It is indifferent what meridian we take as the first one; but it is customary to adopt the meridian of the vernal equinox. What the vernal equinox is will be described hereafter: for our present purposes, nothing more is necessary than to understand that a certain meridian is arbitrarily taken. If now we wish to fix the right ascension of a star, we have only to imagine a meridian passing through it, and to determine the angle which this meridian makes with the meridian of the vernal equinox, as measured from west to east on the equator. That angle will be the right ascension of a star. As already indicated, the declination of a star will be its angular distance from the equator measured on this meridian. Thus, the right ascension and declination of a star fix its apparent position on the celestial sphere, precisely as latitude and longitude fix the position of a point on the surface of the earth.

To give precision to the ideas, we present a brief condensation of this subject, with additional definitions.

Let $PZRN$ represent the celestial sphere of an observer in the northern hemisphere, O being the position of the earth. Pp is the *axis of the celestial sphere*, or the line about which the apparent diurnal orbits of the stars and the actual revolution of the earth are performed.

The *zenith*, Z, is the point immediately above, the *nadir* n, the point immediately below the observer. The direction Zn is defined in practice by the position freely assumed by the plumb line.

The *celestial horizon* is the plane perpendicular to the line joining the *zenith* and *nadir* $NESW$; or it is the terrestrial horizon continued till it meets the celestial sphere.

The celestial horizon intersects the earth in the *rational horizon*, which passes through the earth's centre, and which is so called in distinction to the *sensible horizon*, which is the plane tangent to the earth's surface at any point. But, since the earth itself is considered as but a point in comparison with the celestial sphere, the rational and sensible horizons are considered as one and the same circle on this sphere.

The *celestial poles* are the extremities of the *axis of the celestial sphere P p*, the *north pole* being that one which is above the horizon in the latitude of New York, in the northern hemisphere.

The circles apparently described by the stars in their diurnal orbits are called *parallels of declination*, *K N*;

FIG. 9.—CIRCLES OF THE SPHERE.

that one whose plane passes through the centre of the sphere being the *celestial equator*, or the *equinoctial*, *C W D*.

The *celestial equator* is then that parallel of declination which is a great circle of the celestial sphere.

The figure illustrates the phenomena which appear in the heavens to an observer upon the earth. The stars which lie in the equator have their diurnal paths bisected by the horizon, and are as long above the horizon as below it; those whose distances from the pole (*polar-distances*) are greater than 90° will be a shorter time above the horizon; those whose *polar-distances* are less than 90° a longer time.

The circle $N K$ drawn around the pole P as a centre so as to graze the horizon is called the *circle of perpetual apparition*, because stars situated within it never set. The corresponding circle $S R$ round the south pole is called the *circle of perpetual disappearance*, because stars within it never rise above our horizon.

The great circle passing through the *zenith* and the *pole* is the *celestial meridian*, $N P Z S$. The *meridian* intersects the *horizon* in the *meridian line*, and the points N and S are the *north* and *south points*.

The *prime vertical*, $E Z W$, is perpendicular to the *meridian line* and to the *horizon*: its extremities in the horizon are the *east* and *west points*.

The *meridian plane* is perpendicular to the *equator* and to the *horizon*, and therefore to their intersection. Hence this intersection is the *east* and *west line*, which is thus determined by the intersection of the planes of the *equator* and of the *horizon*.

The *altitude* of a heavenly body is its apparent distance above the horizon, expressed in degrees, minutes, and seconds of arc. In the zenith the altitude is 90°, which is the greatest possible altitude.

If A be any heavenly body, the angle $Z P A$ which the circle $P A$ drawn from the pole to the body makes with the meridian is called the *hour angle* of the body. The hour angle is the angle through which the earth has rotated on its axis since the body was on the meridian. It is so called because it measures the time which has elapsed since the passage of the body over the meridian.

That diameter of the earth which is coincident with the constant direction of the axis of the celestial sphere is its *axis*, and intersects the earth in its *north* and *south poles*.

§ 4. THE DIURNAL MOTION IN DIFFERENT LATITUDES.

As we have seen, the celestial horizon of an observer will change its place on the celestial sphere as the observer travels from place to place on the surface of the earth. If he moves directly toward the north his zenith will approach the north pole, but as the zenith is not a visible point, the motion will be naturally attributed to the pole, which will seem to approach the point overhead. The new apparent position of the pole will change the aspect of the observer's sky, as the higher the pole appears above the horizon the greater the circle of perpetual apparition, and therefore the greater the number of stars, which never set.

FIG. 10.—THE PARALLEL SPHERE.

If the observer is at the north pole his zenith and the pole itself will coincide: half of the stars only will be visible, and these will never rise or set, but appear to move around in circles parallel to the horizon. The horizon and equator will coincide. The meridian will be indeterminate since Z and P coincide; there will be no east and west line, and no direction but south. The sphere in this case is called a *parallel sphere.*

If instead of travelling to the north the observer should go toward the equator, the north pole would seem to approach his horizon. When he reached the equator both poles would be in the horizon, one north and the other south. All the stars in succession would then be visible, and each would be an equal time above and below the horizon.

FIG. 11.—THE RIGHT SPHERE.

The sphere in this case is called a *right sphere*, because the diurnal motion is at right angles to the horizon. If now the observer travels southward from the equator, the south pole will become elevated above his horizon, and in the southern hemisphere appearances will be reproduced which we have already described for the northern, except that the direction of the motion will, in one respect, be different. The heavenly bodies will still rise in the east and set in the west, but those near the equator will pass north of the zenith instead of south of it, as in our latitudes. The sun, instead of moving from left to right, there moves from right to left. The bounding line between the two directions of motion is the equator, where the sun culminates north of the zenith from March till September, and south of it from September till March.

If the observer travels west or east of his first station, his zenith will still remain at the same angular distance from the north pole as before, and as the phenomena caused by the earth's diurnal motion at any place depend only upon the altitude of the elevated pole at that place, these will not be changed except as to the times of their occurrence. A star which appears to pass through the zenith of his first station will also appear to pass through the zenith of the second (since each star remains at a constant angular distance from the pole), but later in time, since it has to pass through the zenith of every place between the two stations. The horizons of the two stations will intercept different portions of the celestial sphere at any one instant, but the earth's rotation will present the same portions successively, and in the same order, at both.

§ 5. RELATION OF TIME TO THE SPHERE.

As in daily life we measure time by the revolution of the hands of a clock, so, in astronomy, we measure it by the rotation of the earth, or the apparent revolution of the celestial sphere. Since the sphere seems to perform one revolution, or 360° in 24 hours, it follows that it moves through 15° in one hour, 1° in 4 minutes, 15′ in one minute of time, and 15″ in one second of time.

The hour angle of a heavenly body counted toward the west (see definition, p. 25) being the angle through which the sphere has revolved since the passage of the body over the meridian, it follows that the time which has elapsed since that passage may be found by dividing the hour angle, expressed in degrees, minutes, and seconds of arc, by 15, when the result will be the required interval expressed in hours, minutes, and seconds of time. If we know the time at which the body passed the meridian, and add this interval to it, we shall have the time corresponding to the hour angle. If we call it noon when the sun passes the meridian, the hour angle of the sun at any moment, divided by 15, gives the time since noon. *Mean solar time* is our ordinary time measured by the sun, after allowing for certain inequalities hereafter described.

Here, however, an important remark is to be made. Really the earth does not revolve on its axis in 24 of the hours used in ordinary life, but in about 4 minutes less than 24 hours (more exactly in 23 hours 56 minutes 4.09 seconds.) If we note the exact time at which a star crosses the meridian, or rises or sets, or disappears behind a chimney or other terrestrial object on one night, we shall find it to do the same thing 3 minutes 56 seconds earlier on the night following, an acceleration which, continued every day, amounts to a whole day in a year. The theory of this acceleration will be explained hereafter as arising from the annual revolution of the earth around the sun; at present we are concerned only with the fact. As a consequence of this fact, the starry sphere seems to revolve rather more than 15° in an hour, and the relation between the time and the arc through which the earth really turns, or the sphere seems to turn, becomes complex. To avoid this complexity, astronomers introduce a modified measure of time, known as *sidereal time*.

Sidereal Time.—The sidereal day is measured, not by the interval between two transits of the sun over the meridian, but by that between two transits of the same star. This day is supposed to commence at the moment of transit of the vernal equinox, or the meridian from which right ascensions are reckoned (a point among the stars to be hereafter defined), and is about 3 minutes 56 seconds shorter than the solar or common day. It is, however, divided into 24 *sidereal hours*, and the sidereal hour is subdivided into sidereal minutes and seconds exactly like the common hours. A simple calculation will show that the sidereal hour is nearly 10 seconds shorter than the solar hour, and, in general, each unit of sidereal time is $\frac{1}{366.25}$ part shorter than the corresponding unit of solar time. A sidereal clock is so constructed as to gain on the common clock at this rate—that is, it gains about one second in six minutes, ten seconds in an hour, 3 minutes 56 seconds in a day, two hours in a month, and 24 hours, or one day, in a year. The hours of the sidereal day are counted forward from 0 to 24, instead of being divided into two groups of 12 each, as in our civil reckoning of time. The face of the sidereal clock is divided into 24 hours, and the hour hand makes one revolution in this period instead of two. The minutes and seconds are each counted forward from 0 to 60, as in the common clock. The hands are set so as to mark $0^h\ 0^m\ 0^s$ at the moment when the vernal equinox passes the meridian of the observer. Thus, the sidereal time at any moment is simply the interval in hours, minutes, and seconds which has elapsed since the vernal equinox was on the meridian. By multiplying this time by 15, we have the number of degrees, minutes, and seconds through which the earth has turned since the transit of the vernal equinox.

The sidereal time of our common noon is given in the astronomical ephemeris for every day of the year. It can be found within ten or twelve minutes at any time by remembering that on March 22d it is sidereal 0 hours about noon, on April 22d it is about 2 hours sidereal time at noon, and so on through the year. Thus, by adding two hours for each month, and 4 minutes for each day after the 22d day last preceding, we have the sidereal time at the noon we require. Adding to it the number of hours since noon, and one minute more for ever fourth of a day on account of the constant gain of the clock, we have the sidereal time at any moment.

Example.—Find the sidereal time on July 4th, 1881, at 4 o'clock A.M. We have:

	h	m
June 22d, 3 months after March 22d; to be × 2,	6	0
July 3d, 12 days after June 22d; × 4,	0	48
4 A.M., 16 hours after noon, nearly $\frac{2}{3}$ of a day,	16	3
	22	51

This result is within a minute of the truth.

The reader now understands that a sidereal clock is one which keeps time, not by the apparent diurnal motion of the sun, but by that of the stars. Consequently, the astronomer, by looking at his clock, always knows the positions of the stars relatively to his meridian. We have now to show how he finds the right ascension of the stars by his sidereal clock. This is done by means of the meridian transit instrument, of which we shall here explain the first principles of construction, reserving a full description for the chapter on instruments. It consists essentially of a small telescope turning on an axis, which is fixed in an east and west line. With the axis thus fixed, the telescope can turn only in the plane of the meridian. When the observer looks into it, he will see the apparent diurnal motion of any star at which it may point, and this motion will be magnified in the ratio of the magnifying power of the telescope. With a high power it will therefore appear very rapid. When the star is exactly on the meridian it will appear in the middle of the field of view of the telescope, and, by means of apparatus to be hereafter described, the moment of crossing can be determined within a small fraction of a second.

Suppose now that the observer has his clock so set that it marks 0 hours 0 minutes 0 seconds at the moment that the vernal equinox crosses his meridian, and so regulated that when the equinox again reaches the meridian on the day following the hour hand will have made one revolution through the 24 hours, and come back to 0 hours again. Then, to find the right ascension of any star or other heavenly body, he watches when it is about to reach the meridian; then directs the transit instrument at the point where it is about to cross, and notes the exact time, in hours, minutes, and seconds, at which the star crosses the middle of the field of his transit. Multiplying this time by 15, he has the right ascension of the star in degrees, minutes, and seconds. In order to avoid the trouble of this multiplication, it is now customary to express the right ascensions of the heavenly bodies, not in degrees, but in time. The circle is divided into 24 hours, like the day, and these hours are divided into minutes and seconds in the usual way. Then the right ascension of a star is the same as the sidereal time at which it passes the meridian.

The relation of arc to time, as angular measures, can be readily remembered by noting that a minute or a second of *time* is fifteen times as great as the corresponding denomination in arc, while the hour is 15 times the degree. The minute and second of time are denoted by the initial letter of their names. So we have:

$1^h = 15^\circ$	$1^\circ = 4^m$
$1^m = 15'$	$1' = 4^s$
$1^s = 15''$	$1'' = 0^s.0666.$

Relation of Time and Longitude.—Considering our civil time as depending on the sun, it will be seen that it is noon at any and every place on the earth when the sun crosses the meridian of that place, or, to speak with more precision, when the meridian of the places passes under the sun. In the lapse of 24 hours, the rotation of the earth on its axis brings all its meridians under the sun in succession, or, which is the same thing, the sun appears to pass in succession all the meridians of the earth. Hence, *noon* continually travels westward at the rate of 15° in an hour, making the circuit of the earth in 24 hours. The difference between the time of day, or *local time* as it is called, at any two places, will be in proportion to the difference of longitude, amounting to one hour for every 15 degrees of longitude, four minutes for every degree, and so on. *Vice versa*, if at the same real moment of time we can determine the local times at two different places, the difference of these times, multiplied by 15, will give the difference of longitude.

The longitudes of places are determined astronomically on this principle. Astronomers are, however, in the habit of expressing the longitude of places on the earth like the right ascensions of the heavenly bodies, not in degrees, but in hours. For instance, instead of saying that Washington is 77° 3′ west of Greenwich, we commonly say that it is 5 hours 8 minutes 12 seconds west, meaning that when it is noon at Washington it is 5 hours 8 minutes 12 seconds after noon at Greenwich. This course is adopted to prevent the trouble and confusion which might arise from constantly having to change hours into degrees, and the reverse.

A question frequently asked in this connection is, Where does the day change? It is, we will suppose, Sunday noon at Washington. That noon travels all the way round the earth, and when it gets back to Washington again it is Monday. Where or when did it change from Sunday to Monday? We answer, wherever people choose to make the change. Navigators make the change occur in longitude 180° from Greenwich. As this meridian lies in the Pacific Ocean, and scarcely meets any land through its course, it is very convenient for this purpose. If its use were universal, the day in question would be Sunday to all the inhabitants east of this line, and Monday to every one west of it. But in practice there have been some deviations. As a general rule, on those islands of the Pacific which are settled by men travelling east, the day would at first be called Monday, even though they might cross the meridian of 180°. Indeed the Russian settlers carried their count into Alaska, so that when our people took possession of that territory they found that the inhabitants called the day Monday when they themselves called it Sunday. These deviations have, however, almost entirely disappeared, and with few exceptions the day is changed by common consent in longitude 180° from Greenwich.

§ 6. DETERMINATIONS OF TERRESTRIAL LONGITUDES.

We have remarked that, owing to the rotation of the earth, there is no such fixed correspondence between meridians on the earth and among the stars as there is between latitude on the earth and declination in the heavens. The observer can always determine his latitude by finding the declination of his zenith, but he cannot find his longitude from the right ascension of his zenith with the same facility, because that right ascension is constantly changing. To determine the longitude of a place, the element of time as measured by the diurnal motion of the earth necessarily comes in. Let us once more consider the plane of the meridian of a place extended out to the celestial sphere so as to mark out on the latter the celestial meridian of the place. Consider two such places, Washington and San Francisco for example; then there will be two such celestial meridians cutting the celestial sphere so as to make an angle of about forty-five degrees with each other in this case. Let the observer imagine himself at San Francisco. Then he may conceive the meridian of Washington to be visible on the celestial sphere, and to extend from the pole over toward his south-east horizon so as to pass at a distance of about forty-five degrees east of his own meridian. It would appear to him to be at rest, although really both his own meridian and that of Washington are moving in consequence of the earth's rotation. Apparently the stars in their course will first pass the meridian of Washington, and about three hours later will pass his own meridian. Now it is evident that if he can determine the interval which the star requires to pass from the meridian of Washington to that of his own place, he will at once have the difference of longitude of the two places by simply turning the interval in time into degrees at the rate of fifteen degrees to each hour.

Essentially the same idea may perhaps be more readily grasped by considering the star as apparently passing over the successive terrestrial meridians on the surface of the earth, the earth being now supposed for a moment to be at rest. If we imagine a straight line drawn from the centre of the earth to a star, this line will in the course of twenty-four sidereal hours apparently make a complete revolution, passing in succession the meridians of all the places on the earth at the rate of fifteen degrees in an hour of *sidereal* time. If, then, Washington and San Francisco are forty-five degrees apart, any one star, no matter what its declination, will require three sidereal hours to pass from the meridian of Washington to that of San Francisco, and the sun will require three *solar* hours for the same passage.

Whichever idea we adopt, the result will be the same: difference of longitude is measured by the time required for a star to apparently pass from the meridian of one place to that of another. There is yet another way of defining what is in effect the same thing. The sidereal time of any place at any instant being the same with the right ascension of its meridian at that instant, it follows that at any instant the sidereal times of the two places will differ by the amount of the difference of longitude. For instance: suppose that a star in 0 hours right ascension is crossing the meridian of Washington. Then it is 0 hours of local sidereal time at Washington. Three hours later the star will have reached the meridian of San Francisco. Then it will be 0 hours local sidereal time at San Francisco. Hence the difference of longitude of two places is measured by the difference of their sidereal times at the same instant of absolute time. Instead of sidereal times, we may equally well take mean times as measured by the sun. It being noon when the sun crosses the meridian of any place, and the sun requiring three hours to pass from the meridian of Washington to that of San Francisco, it follows that when it is noon at San Francisco it is three o'clock in the afternoon at Washington.*

* The difference of longitude thus depends upon the *angular distance of terrestrial meridians*, and not upon the motion of a celestial body,

The whole problem of the determination of terrestrial longitudes is thus reduced to one of these two: either to find the moment of Greenwich or Washington time corresponding to some moment of time at the place which is to be determined, or to find the time required for the sun or a star to move from the meridian of Greenwich or Washington to that of the place. If it were possible to fire a gun every day at Washington noon which could be heard in an instant all over the earth, then observers everywhere, with instruments to determine their local time by the sun or by the stars, would be able at once to fix their longitudes by noting the hour, minute, and second of local time at which the gun was heard. As a matter of fact, the time of Washington noon is daily sent by telegraph to many telegraph stations, and an observer at any such station who knows his local time can get a very close value of his longitude by observing the local time of the arrival of this signal. Human ingenuity has for several centuries been exercised in the effort to invent some practical way of accomplishing the equivalent of such a signal which could be used anywhere on the earth. The British Government long had a standing offer of a reward of ten thousand pounds to any person who would discover a practical method of determining the longitude at sea with the necessary accuracy. This reward was at length divided between a mathematician who constructed improved tables of the moon's motion and a mechanician who invented an improved chronometer. Before the invention of the telegraph the motion of the moon and the transportation of chronometers afforded almost the only practicable and widely extended methods of solving the problem in question. The invention of the telegraph offered a third, far more perfect in its appli-

and hence the longitude of a place is the same whether expressed as a difference of two sidereal times or of two solar times. The longitude of Washington west from Greenwich is $5^h\ 8^m$ or 77°, and this is, in fact, the ratio of the angular distance of the meridian of Washington from that of Greenwich to 360° or 24^h. It is thus plain that the longitude is the difference of the simultaneous local times, whether solar or sidereal.

cation, but necessarily limited to places in telegraphic communication with each other.

Longitude by Motion of the Moon.—When we describe the motion of the moon, we shall see that it moves eastward among the stars at the rate of about thirteen degrees per day, more or less. In other words, its right ascension is constantly increasing at the rate of a degree in something less than two hours. If, then, its right ascension can be predicted in advance for each hour of Greenwich or Washington time, an observer at any point of the earth, by noting the local time at his station, when the moon has any given right ascension, can thence determine the corresponding moment of Greenwich time ; and hence, from the difference of the local times, the longitude of his place. The moon will thus serve the purpose of a sort of clock running on Greenwich time, upon the face of which any observer with the proper appliances can read the Greenwich hour. This method of determining longitudes has its difficulties and drawbacks. The motion of the moon is so slow that a very small change in its right ascension will produce a comparatively large one in the Greenwich time deduced from it—about 27 times as great an error in the deduced longitudes as exists in the determination of the moon's right ascension. With such instruments as an observer can easily carry from place to place, it is hardly possible to determine the moon's right ascension within five seconds of arc ; and an error of this amount will produce an error of nine seconds in the Greenwich time, and hence of two miles or more in his deduced longitude. Besides, the mathematical processes of deducing from an observed right-ascension of the moon the corresponding Greenwich time are, under ordinary circumstances, too troublesome and laborious to make this method of value to the navigator.

Transportation of Chronometers.—The transportation of chronometers affords a simple and convenient method of obtaining the time of the standard meridian at any moment. The observer sets his chronometer as nearly as possible on Greenwich or Washington time, and determines its correction and *rate*. This he can do at any station of which the longitude is correctly known, and at which the local time can be determined. Then, wherever he travels, he can read the time of his standard meridian from the face of his chronometer at any moment, and compare it with the local time determined with his transit instrument or sextant. The principal error to which this method is subject arises from the necessary uncertainty in the rate of even the best chronometers. This is the method almost universally used at sea where the object is simply to get an approximate knowledge of the ship's position.

The accuracy can, however, be increased by carrying a large number of chronometers, or by repeating the determination a number of times, and this method is often employed for fixing the longitudes of seaports, etc. Between the years 1848 and 1855, great numbers of chronometers were transported on the Cunard steamers plying between Boston and Liverpool, to determine the difference of longitude between Greenwich and the Cambridge Observatory, Massachusetts. At Liverpool the chronometers were carefully compared with Greenwich time at a local observatory—that is, the astronomer at Liverpool found the error of the chronometer on its arrival in the ship, and then again when the ship was about to sail. When the chronometer reached Boston, in like manner its error on Cambridge time was determined, and the determination was repeated when the ship was about to return. Having a number of such determinations made alternately on the two sides of the Atlantic, the rates of the chronometers could be determined for each double voyage, and thus the error on Greenwich time could be calculated for the moment of each Cambridge comparison, and the moment of Cambridge time for each Greenwich comparison.

Longitude by the Electric Telegraph.—As soon as the electric telegraph was introduced it was seen by American astronomers that we here had a method of determining longitudes which for rapidity and convenience would supersede all others. The first application of this method was made in 1844 between Washington and Baltimore, under the direction of the late Admiral Charles Wilkes, U. S. N. During the next two years the method was introduced into the Coast Survey, and the difference of longitude between New York, Philadelphia, and Washington was thus determined, and since that time this method has had wide extension not only in the United States, but between America and Europe, in Europe itself, in the East and West Indies, and South America. The principle of the method is extremely simple. Each place, of which the difference of time (or longitude) is to be determined, is furnished with a transit instrument, a clock and a chronograph; instruments described in the next chapter. Each clock is placed in galvanic communication not only with its own chronograph, but if necessary is so connected with the telegraph wires that it can record its own beat upon a chronograph at the other station. The observer, looking into the telescope and noting the crossing of the stars over the meridian, can, by his signals, record the instant of transit both on his own chronograph and on that of the other station. The plan of making a determination between Philadelphia and Washington, for instance, was essentially this: When some previously selected star reached the meridian at Philadelphia, the observer pointed his transit upon it, and as it crossed the wires, recorded the signal of time not only on his own chronograph, but on that at Washington. About eight minutes afterward the star reached the meridian at Washington, and there the observer recorded its transit both on his own chronograph and on that at Philadelphia. The interval between the transit over the two places, as measured by either sidereal clock, at once gave the difference of longitude. If the record was instantaneous at the two stations, this interval ought to be the same, whether read off the Philadelphia or the Washington chronograph. It was found, however, that there was a difference of a small fraction of a second, arising from the fact that electricity required an interval of time, minute but yet appreciable, to pass between the two cities. The Philadelphia record was a little too late in being recorded at Washington, and the Washington one a little too late in being recorded at Philadelphia. We may illustrate this by an example as follows :

Suppose E to be a station one degree of longitude east of another station, W ; and that at each station there is a clock exactly regulated to the time of its own place, in which case the clock at E will be of course four minutes fast of the clock at W ; let us also suppose that a signal takes a quarter of a second to pass from one station to the other :

Then if the observer at E sends a signal to W at exactly noon by his clock	$12^h\ 0^m\ 0^s.00$
It will be received at W at	$11^h\ 56^m\ 0^s.25$
Showing an apparent difference of time of......	$3^m\ 59^s.75$
Then if the observer at W sends a signal at noon by his clock	$12^h\ 0^m\ 0^s.00$
It will be received at E at	$12^h\ 4^m\ 0^s.25$
Showing an apparent difference of time of......	$4^m\ 0^s.25$

One half the sum of these differences is four minutes, which is exactly the difference of time, or one degree of longitude; and one half their difference is twenty-five hundredths of a second, the time taken by the electric impulse to traverse the wire and telegraph instruments.

This is technically called the "wave and armature time."

We have seen that if a signal could be made at Washington noon, and observed by an observer anywhere situated who knew the local time of his station, his longitude would thus become known. This principle is often employed in methods of determining longitude other than those named. For example, the instant of the beginning and ending of an eclipse of the sun (by the moon) is a perfectly definite phenomenon. If this is observed by two observers, and these instants noted by each in the local time of his station, then the difference of these local times (subject to small corrections, due to parallax, etc.) will be the difference of longitude of the two stations.

The satellites of *Jupiter* suffer eclipses frequently, and the Greenwich and Washington times of these phenomena are computed and set down in the Nautical Almanac. Observations of these at any station will also give the difference of longitude between this station and Greenwich or Washington. As, however, they require a larger telescope and a higher magnifying power than can be used at sea, this method is not a practical one for navigators.

§ 7. MATHEMATICAL THEORY OF THE CELESTIAL SPHERE.

In this explanation of the mathematical theory of the relations of the heavenly bodies to circles on the sphere, an acquaintance with spherical trigonometry on the part of the reader is necessarily presupposed. The general method by which the position of a point on the sphere is referred to fixed points or circles is as follows:

A fundamental great circle $E\ V\ Q$, Fig. 12 is taken as a basis, and the first co-ordinate* of the body is its angular distance from this circle. When the earth's equator is taken as the fundamental circle, this distance is on the earth's surface called *Latitude;* on the celestial sphere the corresponding distance is called *Declination.* If the horizon is taken as the fundamental circle the distance is called *Altitude.* Altitude is therefore angular distance above the horizon. To distinguish between distances on opposite sides of the circle, distances on one side are regarded as algebraically positive quantities, and on the other side as negative. In the case of the equator the north side, and in that of the horizon the upper side, are considered positive. Hence, if a body is below the horizon its altitude is negative, and the latitude of a city south of the earth's equator is, in astronomical language, considered as negative.

Instead of the co-ordinate we have described, another called zenith or polar distance is frequently employed. The fundamental circle is

* The *co-ordinates* of a body are those *measures*, whether of angles or lines, which define its position. For instance, the geographical co-ordinates of a city are its latitude and longitude. To fix a position on a sphere or other surface, two co-ordinates are necessary, while in space three are required.

everywhere 90° from its positive pole, P. Hence, if A is the position of a star or other point on the sphere, and we put

FIG. 12.

δ, its declination or altitude, $= a A$.

p, its polar or zenith distance $= P A$, we shall have

$$\delta + p = 90°,$$

or,

$$p = 90° - \delta.$$

If the star is south of the fundamental circle, at B for example, δ being negative p will exceed 90°. This quantity p may range from zero at the one pole to 180° at the other, and will always be algebraically positive. It is on this account to be preferred to δ, though less frequently used.

II. The second co-ordinate required to fix a position on the celestial or terrestrial sphere is *longitude*, *right ascension*, or *azimuth*, according to the fundamental plane adopted. It is expressed by the position of the great circle or meridian $P\ A\ a\ P'$ which passes through the position from one pole to the other, at right angles to the fundamental circle. An arbitrary point, V for instance, is chosen on this latter circle, and the longitude is the angle Va, from this point to the intersection of the meridian or vertical circle passing through the object. We may also consider it as the angle $V\ P\ A$ which the circle passing through the object makes with the circle $P\ V$, because this angle is equal to $V a$. The angle is commonly counted from V toward the right, and from 0° round to 360°, so as to avoid using negative angles. If the observer is stationed in the centre of the sphere, with his head toward the positive pole P, the positive direction should be from right to left around the sphere. When the horizon is taken as the fundamental circle or plane, this secondary co-ordinate is called the *azimuth*, and should be counted from the south point toward east, or from the north point toward west, but is commonly counted the other way. It may be defined as the angular distance of the vertical circle passing through the object from the south point of the horizon.

FIG. 13.

The *hour angle* of a star is measured by the interval which has elapsed, or the angle through which the earth has revolved on its axis, since the star crossed the meridian. In Fig. 13 Z being the zenith and P the pole, the angle $Z\,P\,S$ is the hour angle of the star S. This angle is measured at the pole. If we put

τ, the sidereal time,
α, the right ascension of the object, we shall have

$$\text{Hour angle, } h = \tau - \alpha.$$

It will be negative before the object has passed the meridian, and positive afterward. It differs from right ascension only in the point from which it is reckoned, and the direction which is considered positive. The right ascension is measured toward the east from a point (the vernal equinox) which is fixed among the stars, while the hour angle is measured toward the west from the meridian of the observer, which meridian is constantly in motion, owing to the earth's rotation.

We have next to show the trigonometrical relations which subsist between the hour angle, declination, altitude, and azimuth. Let

FIG. 14.

Fig. 14 be a view of the celestial hemisphere which is above the horizon, as seen from the east. We then have:

$H\,E\,R\,W$, the horizon.
P, the pole.
Z, the zenith of the observer.
$H\,M\,Z\,P\,R$, the meridian of the observer.
$P\,R$, the latitude of the observer, which call ϕ.
$Z\,P, = 90° - \phi$, the co-latitude.
$P\,S$, the north polar distance of the star $= 90° -$ declination.
$T\,S$, its altitude, which call a.
$Z\,S$, its zenith distance $= 90° - a$.
$M\,Z\,S$, its azimuth, $= 180° -$ angle $S\,Z\,P$.
$Z\,P\,S$, its hour angle, which call h.

The spherical triangle $Z\,P\,S$, of which the angles are formed by the zenith, the pole, and the star, is the fundamental triangle of our problem. The latter, as commonly solved, may be put into two forms.

I. Given the latitude of the place, the declination or polar distance of the star, and its hour angle, to find its altitude and azimuth.

We have, by spherical trigonometry, considering the angles and sides of the triangle $Z\,P\,S$:

$$\begin{aligned}
\cos ZS &= \cos PZ \cos PS + \sin PZ \sin PS \cos P.\\
\sin ZS \cos Z &= \sin PZ \cos PS - \cos PZ \sin PS \cos P.\\
\sin ZS \sin Z &= \sin PS \sin P.
\end{aligned}$$

By the above definitions,

$ZS = 90° - a$, (a being the altitude of the star).
$PZ = 90° - \phi$, (ϕ being the latitude of the place).
$PS = 90° - \delta$, (δ being the declination of the star, + when north).
$P = h$, the hour angle.
$Z = 180° - z$, (z being the azimuth).

Making these substitutions, the equation becomes:

$$\begin{aligned}
\sin a &= \sin\phi \sin\delta + \cos\phi \cos\delta \cos h.\\
\cos a \cos z &= -\cos\phi \sin\delta + \sin\phi \cos\delta \cos h.\\
\cos a \sin z &= \cos\delta \sin h.
\end{aligned}$$

From these equations $\sin a$ and $\cos a$ may be obtained separately, and, if the computation is correct, they will give the same value of a. If the altitude only is wanted, it may be obtained from the first equation alone, which may be transformed in various [illegible] explained in works on trigonometry.

II. Given the latitude of the place, the declination of a star, and its altitude above the horizon, to find its hour angle and (if its right ascension is known) the sidereal time when it had the given altitude.

We find from the first of the above equations,

$$\cos h = \frac{\sin a - \sin\phi \sin\delta}{\cos\phi \cos\delta};$$

or we may use:

$$\sin^2 \tfrac{1}{2} h = \tfrac{1}{2}\,\frac{\cos(\phi - \delta) - \sin a}{\cos\phi \cos\delta}.$$

Having thus found h, we have

$$\text{Sidereal time} = h + \alpha,$$

α being the star's right ascension, and the hour angle h being changed into time by dividing by 15.

III. An interesting form of this last problem arises when we suppose $a = o$, which is the same thing as supposing the star to be in the horizon, and therefore to be rising or setting. The value of h will then be the hour angle at which it rises or sets; or being changed to time by dividing by 15, it will be the interval of sidereal time between its rising and its passage over the meridian, or between this passage and its setting. This interval is called the *semi-diurnal arc*, and by doubling it we have the time between the rising and setting of the star or other object. Putting $a = 0$ in the preceding expression for cos h we find for the semi-diurnal arc h,

$$\cos h = -\frac{\sin \phi \sin \delta}{\cos \phi \cos \delta}$$

$$= -\tan \phi \tan \delta,$$

and the arc during which the star is above the horizon is $2\,h$.

FIG. 15.—UPPER AND LOWER DIURNAL ARCS.

From this formula may be deduced at once many of the results given in the preceding sections.

(1). At the poles $\phi = 90°$, $\tan \phi =$ infinity, and therefore $\cos h =$ infinity. But the cosine of an angle can never be greater than unity; there is therefore no value of h which fulfils the condition. Hence, a star at the pole can neither rise nor set.

(2). At the earth's equator $\phi = 0°$, $\tan \phi = 0$, whence $\cos h = 0$, $h = 90°$, and $2\,h = 180°$, whatever be δ. This being a semicircumference all the heavenly bodies are half the time above the horizon to an observer on the equator.

(3). If $\delta = 0°$ (that is, if the star is on the celestial equator), then $\tan \delta = 0$, and $\cos h = 0, h = 90°$, $2\,h = 180°$, so that all stars on the equator are half the time above the horizon, whatever be the latitude of the observer. Here we except the pole, where, in this case, $\tan \phi \tan \delta = \infty \times 0$, an indeterminate quantity. In fact, a star on the celestial equator will, at the pole of the earth, seem to move round in the horizon.

(4). The above value of cos h may be expressed in the form:

$$\cos h = -\frac{\tan \delta}{\cot \phi} = -\frac{\tan \delta}{\tan (90° - \phi)}.$$

This shows that when δ lies outside the limits $+ (90° - \phi)$ and $- (90° - \phi)$, cos h will lie without the limits $- 1$ and $+ 1$, and there will be no value of h to correspond. Hence, in this case, the stars neither rise nor set. These limits correspond to those of perpetual apparition and perpetual disappearance.

(5). In the northern hemisphere ϕ and *tan.* ϕ are positive. Then, when δ is positive, cos h is negative, and $h > 90°$, $2\,h > 180°$. With negative δ, $\cos h$ is positive, $h < 90°$, $2h < 180°$. Hence, in northern latitudes, a northern star is more than half of the time above the horizon, and a southern star less. In the southern hemisphere, ϕ and $\tan\phi$ are negative, and the case is reversed.

(6). If, in the preceding case, the declination of a body is supposed constant and north, then the greater we make ϕ the greater the negative value of $\cos h$ and the greater h itself will be. Considering, in succession, the cases of north and south declination and north and south latitude, we readily see that the farther we go to the north on the earth, the longer bodies of north declination remain above the horizon, and the more quickly those of south declination set. In the southern hemisphere the reverse is true. Thus, in the month of June, when the sun is north of the equator, the days are shortest near the south pole, and continually increase in length as we go north.

Examples.

(1). On April 9, 1879, at Washington, the altitude of Rigel above the west horizon was observed to be 12° 25′. Its position was:

$$\text{Right ascension} = 5^h\ 8^m\ 44^s{\cdot}27 = \alpha.$$
$$\text{Declination} = -\ 8°\ 20'\ 36'' = \delta.$$

The latitude of Washington is $+\ 38°\ 53'\ 39'' = \phi$.

What was the hour angle of the star, and the sidereal time of observation?

$$
\begin{aligned}
\lg \sin a &= 9{\cdot}332478 \\
\lg \sin \phi &= 9{\cdot}797879 \\
\lg \sin \delta &= -\ 9{\cdot}161681 \\
\hline
-\ \lg \sin\phi \sin\delta &= 8{\cdot}959560 \\
-\ \sin\phi\sin\delta &= 0{\cdot}091109 \\
\sin a &= 0{\cdot}215020 \\
\hline
\sin a - \sin\phi\sin\delta &= 0{\cdot}306129 \\
\lg\cos\phi &= 9{\cdot}891151 \\
\lg\cos\delta &= 9{\cdot}995379 \\
\hline
\lg\cos\phi\cos\delta &= 9{\cdot}886530 \\
\lg(\sin a - \sin\phi\sin\delta, &= 9{\cdot}485905 \\
\hline
\lg\cos h &= 9{\cdot}599375 \\
h &= 66°\ 34'\ 33'' \\
h \div 15 &= 4^h\ 26^m\ 18^s.20 \\
\alpha &= 5^h\ \ 8^m\ 44^s.27 \\
\text{sidereal time} &= 9^h\ 35^m\ \ 2^s.47
\end{aligned}
$$

(2). Had the star been observed at the same altitude in the east, what would have been the sidereal time?

Ans. $\alpha - h = 0^h\ 42^m\ 26^s.07$.

(3). At what sidereal time does Rigel rise, and at what sidereal time does it set in the latitude of Washington?

$$\begin{aligned} -\operatorname{tg}\phi &= -\,9 \cdot 906728 \\ \operatorname{tg}\delta &= -\,9 \cdot 166301 \\ \hline \cos h &= -\,9 \cdot 073029 \\ h &= 83^\circ\, 12'\, 19'' \\ h \div 15 &= 5^h\, 32^m\, 49^s.27 \\ a &= 5^h\, 8^m\, 44^s.27 \\ \hline \text{rises } & 23^h\, 35^m\, 55^s.00 \\ \text{sets } & 10^h\, 41^m\, 33^s.54 \end{aligned}$$

(4). What is the greatest altitude of Rigel above the horizon of Washington, and what is its greatest depression below it? Ans. Altitude$=42^\circ\, 45'\, 45''$; depression$=59^\circ\, 26'\, 57''$.

(5). What is the greatest altitude of a star on the equator in the meridian of Washington? Ans. $51^\circ\, 6'\, 21''$.

(6). The declination of the pointer in the Great Bear which is nearest the pole is $62^\circ\, 30'$ N., at what altitude does it pass above the pole at Washington, and at what altitude does it pass below it? Ans. $66^\circ\, 23'\, 39''$ above the pole, and $11^\circ\, 23'\, 39'$ when below it.

(7). If the declination of a star is 50° N., what length of sidereal time is it above the horizon of Washington and what length below it during its apparent diurnal circuit? Ans. Above, $21^h\, 52^m$; below, $2^h\, 8^m$.

§ 8. DETERMINATION OF LATITUDES ON THE EARTH BY ASTRONOMICAL OBSERVATIONS.

Latitude from circumpolar stars.—In Fig. 16 let Z represent the zenith of the place of observation, P the pole, and $HPZR$ the meridian, the observer being at the centre of the sphere. Suppose S and S' to be the two points at which a circumpolar star crosses the meridian in the description of its apparent diurnal orbit. Then, since P is midway between S and S',

$$\frac{ZS + ZS'}{2} = ZP = 90^\circ - \phi,$$

or,

$$\frac{Z + Z'}{2} = 90^\circ - \phi.$$

If, then, we can measure the distances Z and Z', we have

FIG. 16.

$$\phi = 90^\circ - \frac{Z + Z'}{2}$$

which serves to determine ϕ. The distances Z and Z' can be measured by the meridian circle or the sextant—both of which instruments are described in the next chapter—and the latitude is then known. Z and Z' must be freed from the effects of refraction. In this method no previous knowledge of the star's declination is required, provided it remains constant between the upper and lower transit, which is the case for fixed stars.

Latitude by Circum-zenith Observations.—If two stars S and S', whose declinations δ and δ' are known, cross the meridian, one north and the other south of the zenith, at zenith distances ZS and ZS', which call Z and Z', and if we have measured Z and Z', we can from such measures find the latitude; for $\phi = \delta + Z$ and $\phi = \delta' - Z'$, whence

FIG. 17.

$$\phi = \tfrac{1}{2}[(\delta + \delta') + (Z - Z')].$$

It will be noted that in this method the latitude depends simply upon the mean of two declinations which can be determined beforehand, and only requires the *difference* of zenith distances to be accurately measured, while the absolute values of these are unknown. In this consists its capital advantage. This is the method invented by Capt. ANDREW TALCOTT, U.S.A., and now universally adopted in America in field astronomy, in the practice of the Coast Survey, etc.

Latitude by a Single Altitude of a Star.—In the triangle ZPS (Fig. 14) the sides are $ZP = 90° - \phi$; $PS = 90° - \delta$; $ZS = Z = 90° - a$; $ZPS = h =$ the hour angle. If we can measure at any known sidereal time θ the altitude a of the star S, and if we further know the right ascension, α, and the declination, δ, of the body (to be derived from the Nautical Almanac or a catalogue of stars), then we have from the triangle

$$\sin\phi = \sin a \sin\delta + \cos a \cos\delta \cos h;$$

or, since

$$t = \theta - \alpha;\ \sin\phi = \sin a \sin\delta + \cos a \cos\delta \cos(\theta - \alpha),$$

from which we can obtain ϕ. It is to be noted that in a place whose latitude (ϕ) is known, this observation will determine θ, the sidereal time, as explained in the last section; if the sun is observed, t is simply the solar time.

Latitude by a Meridian Altitude.—If the altitude of the body is observed on the meridian and south of the zenith, the equation above becomes, since $h = 0$ in this case,

$$\sin\phi = \sin a \sin\delta + \cos a \cos\delta,$$

or,

$$\sin\phi = \cos(a - \delta) \therefore \phi = 90° - a + \delta,$$

which is evidently the simplest method of obtaining ϕ from a measured altitude of a body of known declination. The last method is that commonly used at sea, the altitude being measured by the sextant. The student can deduce the formula for a northern altitude.

§ 9. PARALLAX AND SEMIDIAMETER.

An observation of the apparent position of a heavenly body can give only the *direction* in which it lies from the station occupied by the observer without any direct indication of the distance. It is evident that two observers stationed in different parts of the earth will not see such body in the same direction. In Fig. 18, let S' be a sta-

FIG. 18.—PARALLAX.

tion on the earth, P a planet, Z' the zenith of S', and the outer arc a part of the celestial sphere. An observation of the apparent right ascension and declination of P taken from the station S' will give us an apparent position P'. A similar observation at S'' will give an apparent position P'', while if seen from the centre of the earth the apparent position would be $P_{,}$. The angles $P' P P_{,}$ and $P'' P P_{,}$, which represent the differences of direction, are called *parallaxes.* It is clear that the parallax of a body depends upon its distance from the earth, being greater the nearer it is to the earth.

The word *parallax* having several distinct applications, we shall give them in order, commencing with the most general signification.

(1.) In its most general acceptation, parallax is the difference between the directions of a body as seen from two different standpoints. This difference is evidently equal to the angle made between two lines, one drawn from each point of observation to the body. Thus in Fig. 18 the difference between the direction of the body P as seen from C and from S' is equal to the angle $P' P P_{\prime\prime}$, and this again is equal to its opposite angle $S' P C$. This angle is, however, the angle between the two points C and S' as seen from P: we may therefore refer this most general definition of parallax to the body itself, and define parallax as the angle subtended by the line between two stations as seen from a heavenly body.

(2.) In a more restricted sense, one of the two stations is supposed to be some centre of position from which we imagine the body to be viewed, and the parallax is the difference between the direction of the body from this centre and its direction from some other point. Thus the parallax of which we have just spoken is the difference between the direction of the body as seen from the centre of the earth C and from a point on its surface as S'. If the observer at any station on the earth determines the exact direction of a body, the parallax of which we speak is the correction to be applied to that direction in order to reduce it to what it would have been had the observation been made at the centre of the earth. Observations made at different points on the earth's surface are compared by reducing them all to the centre of the earth.

We may also suppose the point C to be the sun and the circle $S' S''$ to be the earth's orbit around it. The parallax will then be the difference between the directions of the body as seen from the earth and from the sun. This is termed the *annual parallax*, because, owing to the annual revolution of the earth, it goes through its period in a year, always supposing the body observed to be at rest.

(3.) A yet more restricted parallax is the *horizontal*

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parallax of a heavenly body. The parallax first described in the last paragraph varies with the position of the observer on the surface of the earth, and has its greatest value when the body is seen in the horizon of the observer, as may be seen by an inspection of Fig. 19, in which the angle $C P S$ attains its maximum when the line $P S$ is tangent to the earth's surface, in which case P will appear in the horizon of the observer at S.

FIG. 19.—HORIZONTAL PARALLAX.

The horizontal parallax depends upon the distance of a body in the following manner: In the triangle $C P S$, right-angled at S, we have

$$CS = CP \sin CPS.$$

If, then, we put

ρ, the radius of the earth CS;

r, the distance of the body P from the centre of the earth;

π, the angle $S P C$, or the horizontal parallax,

we shall have,

$$\rho = r \sin \pi; \quad r = \frac{\rho}{\sin \pi}.$$

Since the earth is not perfectly spherical, the quantity ρ is not absolutely constant for all parts of the earth, and its greatest value is usually taken as that to which the horizontal value shall be referred. This greatest value is, as we shall hereafter see, the radius of the equator, and the corresponding value of the parallax is therefore called the *equatorial horizontal parallax.*

When the distance r of the body is known, the equatorial horizontal parallax can be found by the first of the above equations; when the parallax can be observed, the distance r is found from the second equation. How this is done will be described in treating the subject of celestial measurement.

It is easily seen that the equatorial horizontal parallax, or the angle $C\,P\,S$, is the same as the angular semi-diameter of the earth seen from the object P. In fact, if we draw the line $P\,S'$ tangent to the earth at S', the angle $S\,P\,S'$ will be the apparent angular diameter of the earth as seen from P, and will also be double the angle $C\,P\,S$. The apparent semi-diameter of a heavenly body is therefore given by the same formulæ as the parallax, its own radius being substituted for that of the earth. If we put,

ρ, the radius of the body in linear measure;

r, the distance of its centre from the observer, expressed in the same measure;

s, its angular semi-diameter, as seen by the observer; we shall have,

$$\sin s = \frac{\rho}{r}.$$

If we measure the semi-diameter s, and know the distance, r, the radius of the body will be

$$\rho = r \sin s.$$

Generally the angular semi-diameters of the heavenly bodies are so small that they may be considered the same as their sines. We may therefore say that the apparent angular diameter of a heavenly body varies inversely as its distance.

CHAPTER II.

ASTRONOMICAL INSTRUMENTS.

§ 1. THE REFRACTING TELESCOPE.

In explaining the theory and use of the refracting telescope, we shall assume that the reader is acquainted with the fundamental principles of the refraction and dispersion of light, so that the simple enumeration of them will recall them to his mind. These principles, so far as we have occasion to refer to them, are, that when a ray of light passing through a vacuum enters a transparent medium, it is refracted or bent from its course in a direction toward a line perpendicular to the surface at the point where the ray enters; that this bending follows a certain law known as the law of sines; that when a pencil of rays emanating from a luminous point falls nearly perpendicularly upon a convex lens, the rays, after passing through it, all converge toward a point on the other side called a focus; that light is compounded of rays of various degrees of refrangibility, so that, when thus refracted, the component rays pursue slightly different courses, and in passing through a lens come to slightly different foci; and finally, that the apparent angular magnitude subtended by an object when viewed from any point is inversely proportional to its distance.*

* More exactly, in the case of a globe, the sine of the angle is inversely as the distance of the object, as shown on the preceding page.

We shall first describe the telescope in its simplest form, showing the principles upon which its action depends, leaving out of consideration the defects of aberration which require special devices in order to avoid them. In the simplest form in which we can conceive of a telescope, it consists of two lenses of unequal focal lengths. The purpose of one of these lenses (called the *objective*, or *object glass*) is to bring the rays of light from a distant object at which the telescope is pointed, to a focus and there to form an image of the object. The purpose of the other lens (called the *eye-piece*) is to view this object, or, more precisely, to form another enlarged image of it on the retina of the eye.

FIG. 20.—ACTION OF OBJECTIVE IN FORMING AN IMAGE OF A DISTANT OBJECT.

The figure gives a representation of the course of one pencil of the rays which go to form the image $A\ I'$ of an object $I\ B$ after passing through the objective $O\ O'$. The pencil chosen is that composed of all the rays emanating from I which can possibly fall on the objective $O\ O'$. All these are, by the action of the objective, concentrated at the point I'. In the same way each point of the image out of the optical axis $A\ B$ emits an oblique pencil of diverging rays which are made to converge to some point of the image by the lens. The image of the point B of the object is the point A of the image. We must conceive the image of any object in the focus of any lens (or mirror) to be formed by separate bundles of rays as in the figure. The image thus formed becomes, in its turn, an object to be viewed by the eye-piece. After the rays meet to form the image of an object, as at I', they continue on their course, diverging from I' as if the latter were a material object reflecting the light. There is, however, this exception: that the rays, instead of diverging in every direction, only form a small cone having its vertex at I', and having its angle equal to $O\,I'\,O'$. The reason of this is that only those rays which pass through the objective can form the image, and they must continue on their course in straight lines after forming the image. This image can now be viewed by a lens, or even by the unassisted eye, if the observer places himself behind it in the direction A, so that the pencil of rays shall enter his eye. For the present we may consider the eye-piece as a simple lens of short focus like a common hand-magnifier, a more complete description being given later.

Magnifying Power.—To understand the manner in which the telescope magnifies, we remark that if an eye at the object-glass could view the image, it would appear of the same size as the actual object, the image and the object subtending the same angle, but lying in opposite directions. This angular magnitude being the same, whatever the focal distance at which the image is formed, it follows that the size of the image varies directly as the focal length of the object-glass. But when we view an object with a lens of small focal distance, its apparent magnitude is the same as if it were seen at that focal distance. Consequently the apparent angular magnitude will be inversely as the focal distance of the lens. Hence the focal image as seen with the eye-piece will appear larger than it would when viewed from the objective, in the ratio of the focal distance of the objective to that of the eye-piece. But we have said that, seen through the objective, the image and the real object subtend the same angle. Hence the angular magnifying power is equal to the focal distance of the objective, divided by that of the eye-piece. If we simply turn the telescope end for end, the objective becomes the eye-piece and the latter the objective. The ratio is inverted, and the object is diminished in size in the same ratio that it is increased when viewed in the ordinary way. If we should form a telescope of two lenses of equal focal length, by placing them at double their focal distance, it would not magnify at all.

The image formed by a convex lens, being upside down, and appearing in the same position when viewed with the eye-piece, it follows that the telescope, when constructed in the simplest manner, shows all objects inverted, or upside down, and right side left. This is the case with all refracting telescopes made for astronomical uses.

Light-gathering Power.—It is not merely by magnifying that the telescope assists the vision, but also by increasing the quantity of light which reaches the eye from the object at which we look. Indeed, should we view an object through an instrument which magnified, but did not increase the amount of light received by the eye, it is evident that the brilliancy would be diminished in proportion as the surface of the object was enlarged, since a constant amount of light would be spread over an increased surface; and thus, unless the light were faint, the object might become so darkened as to be less plainly seen than with the naked eye. How the telescope increases the quantity of light will be seen by considering that when the unaided eye looks at any object, the retina can only receive so many rays as fall upon the pupil of the eye. By the use of the telescope, it is evident that as many rays can be brought to the retina as fall on the entire object-glass. The pupil of the human eye, in its normal state, has a diameter of about one fifth of an inch; and by the use of the telescope it is virtually increased in surface in the ratio of the square of the diameter of the objective to the square of one fifth of an inch. Thus, with a two-inch aperture to our telescope, the number of rays collected is one hundred times as great as the number collected with the naked eye.

With a 5-inch object-glass, the ratio is	625 to 1
" 10 " " " " "	2,500 to 1
" 15 " " " " "	5,625 to 1
" 20 " " " " "	10,000 to 1
" 26 " " " " "	16,900 to 1

When a minute object, like a star, is viewed, it is necessary that a certain number of rays should fall on the retina in order that the star may be visible at all. It is therefore plain that the use of the telescope enables an observer to see much fainter stars than he could detect with the naked eye, and also to see faint objects much better than by unaided vision alone. Thus, with a 26-inch telescope we may see stars so minute that it would require many thousands to be visible to the unaided eye.

An important remark is, however, to be made here. Inspecting Fig. 20 we see that the cone of rays passing through the object-glass converges to a focus, then diverges at the same angle in order to pass through the eye-piece. After this passage the rays emerge from the eye-piece parallel, as shown in Fig. 22. It is evident that the diameter of this cylinder of parallel rays, or "emergent pencil," as it is called, is less than the diameter of the object-glass, in the same ratio that the focal length of the eye-piece is less than that of the object-glass. For the central ray $I I'$ is the common axis of two cones, $A I'$ and $O I' O'$, having the same angle, and equal in length to the respective focal distances of the glasses. But this ratio is also the magnifying power. Hence the diameter of the emergent pencil of rays is found by dividing the diameter of the object-glass by the magnifying power. Now it is clear that if the magnifying power is so small that this emergent pencil is larger than the pupil of the eye, all the light which falls on the object-glass cannot enter the pupil. This will be the case whenever the magnifying power is less than five for every inch of aperture of the glass. If, for example, the observer should look through a twelve-inch telescope with an eye-piece so large that the magnifying power was only 30, the emergent pencil would be two fifths of an inch in diameter, and only so much of the light could enter the pupil as fell on the central six inches of the object-glass. Practically, therefore, the observer would only be using a six-inch telescope, all the light which fell outside of the six-inch circle being lost. In order, therefore, that he may get the advantage of all his object-glass, he must use a magnifying power at least five times the diameter of his objective in inches.

When the magnifying power is carried beyond this limit, the action of a telescope will depend partly on the nature of the object one is looking at. Viewing a star, the increase of power will give no increase of light, and therefore no increase in the apparent brightness of the star. If one is looking at an object having a sensible surface, as the moon, or a planet, the light coming from a given portion of the surface will be spread over a larger portion of the retina, as the magnifying power is increased. All magnifying must then be gained at the expense of the apparent illumination of the surface. Whether this loss of illumination is important or not will depend entirely on how much light is to spare. In a general way we may say that the moon and all the planets nearer than *Saturn* are so brilliantly illuminated by the sun that the magnifying power can be carried many times above the limit without any loss in the distinctness of vision.

The Telescope in Measurement.—A telescope is generally thought of only as an instrument to assist the eye by its magnifying and light-gathering power in the manner we have described. But it has a very important additional function in astronomical measurements by enabling the astronomer to point at a celestial object with a certainty and accuracy otherwise unattainable. This function of the telescope was not recognized for more than half a century after its invention, and after a long and rather acrimonious contest between two schools of astronomers. Until the middle of the seventeenth century, when an astronomer wished to determine the altitude of a celestial object, or to measure the angular distance between two stars, he was obliged to point his quadrant or other measuring instrument at the object by means of "pinnules." These served the same purpose as the sights on a rifle. In using them, however, a difficulty arose. It was impossible for the observer to have distinct vision both of the object and of the pinnules at the same time, because when the eye was focused on either pinnule, or on the object, it was necessarily out of focus for the others. The only way to diminish this difficulty was to lengthen the arm on which the pinnules were fastened so that the latter should be as far apart as possible. Thus Tycho Brahe, before the year 1600, had measuring instruments very much larger than any in use at the present time. But this plan only diminished the difficulty and could not entirely obviate it, because to be manageable the instrument must not be very large.

About 1670 the English and French astronomers found that by simply inserting fine threads or wires exactly in the focus of the telescope, and then pointing it at the object, the image of that object formed in the focus could be made to coincide with the threads, so that the observer could see the two exactly superimposed upon each other. When thus brought into coincidence, it was known that the point of the object on which the wires were set was in a straight line passing through the wires, and through the centre of the object-glass. So exactly could such a pointing be made, that if the telescope did not magnify at all (the eye-piece and object-glass being of equal focal length), a very important advance would still be made in the accuracy of astronomical measurements. This line, passing centrally through the telescope, we call the *line of collimation* of the telescope, *A B* in Fig. 20. If we have any way of determining it we at once realize the idea expressed in the opening chapter of this book, of a pencil extended in a definite direction from the earth to the heavens. If the observer simply sets his telescope in a fixed position, looks through it and notices what stars pass along the threads in the eye-piece, he knows that those stars all lie in the line of collimation of his telescope at that instant. By the diurnal motion, a pencil-mark, as it were, is thus being made in the heavens, the direction of which can be determined with far greater precision than by any measurements with the unaided eye. The direction of this line of collimation can be determined by methods which we need not now describe in detail.

The Achromatic Telescope.—The simple form of telescope which we have described is rather a geometrical conception than an actual instrument. Only the earliest instruments of this class were made with so few as two lenses. GALILEO's telescope was not made in the form which we have described, for instead of two convex lenses having a common focus, the eye-piece was concave, and was placed at the proper distance inside of the focus of the objective. This form of instrument is still used in opera-glasses, but is objectionable in large instruments, owing to the smallness of the field of view. The use of two convex lenses was, we believe, first proposed by KEPLER. Although telescopes of this simple form were wonderful instruments in their day, yet they would not now be regarded as serving any of the purposes of such an instrument, owing to the aberrations with which a single lens is affected. We know that when ordinary light passes through a simple lens it is partially decomposed, the different rays coming to a focus at different distances. The focus for red rays is most distant from the object-glass, and that for violet rays the nearest to it. Thus arises the *chromatic aberration* of a lens. But this is not all. Even if the light is but of a single degree of refrangibility, if the surfaces of our lens are spherical, the rays which pass near the edge will come to a shorter focus than those which pass near the centre. Thus arises *spherical aberration.* This aberration might be avoided if lenses could be ground with a proper gradation of curvature from the centre to the circumference. Practically, however, this is impossible; the deviation from uniform sphericity, which an optician can produce, is too small to neutralize the defect.

Of these two defects, the chromatic aberration is much the more serious; and no way of avoiding it was known until the latter part of the last century. The fact had, indeed, been recognized by mathematicians and physicists, that if two glasses could be found having very different ratios of refractive to dispersive powers,* the defect could be cured by combining lenses made of these different kinds of glass. But this idea was not realized until the time of DOLLOND, an English optician who lived during the last century. This artist found that a concave lens of flint glass could be combined with a convex lens of crown of double the curvature in such a manner that the dispersive powers of the two lenses should neutralize each other, being equal and acting in opposite directions. But the crown glass having the greater refractive power, owing to its greater curvature, the rays would be brought to a focus without dispersion. Such is the construction of the achromatic objective. As now made, the outer or crown glass lens is double convex; the inner or flint one is generally nearly plano-concave. Fig. 21 shows the section of such an objective as made by ALVAN CLARK & SONS, the inner curves of the crown and flint being nearly equal.

FIG. 21.—SECTION OF OBJECT-GLASS.

* By the *refractive power* of a glass is meant its power of bending the rays out of their course, so as to bring them to a focus. By its *dispersive power* is meant its power of separating the colors so as to form a spectrum, or to produce chromatic aberration.

A great advantage of the achromatic objective is that it may be made to correct the spherical as well as the chromatic aberration. This is effected by giving the proper curvature to the various surfaces, and by making such slight deviations from perfect sphericity that rays passing through all parts of the glass shall come to the same focus.

The Secondary Spectrum.—It is now known that the chromatic aberration of an objective cannot be perfectly corrected with any combination of glasses yet discovered. In the best telescopes the brightest rays of the spectrum, which are the yellow and green ones, are all brought to the same focus, but the red and blue ones reach a focus a little farther from the objective, and the violet ones a focus still farther. Hence, if we look at a bright star through a large telescope, it will be seen surrounded by a blue or violet light. If we push the eye-piece in a little the enlarged image of the star will be yellow in the centre and purple around the border. This separation of colors by a pair of lenses is called a *secondary spectrum*.

Eye-Piece.—In the skeleton form of telescope before described the eye-piece as well as the objective was considered as consisting of but a single lens. But with such an eye-piece vision is imperfect, except in the centre of the field, from the fact that the image does not throw rays in every direction, but only in straight lines away from the objective. Hence, the rays from near the edges of the focal image fall on or near the edge of the eye-piece, whence arises distortion of the image formed on the retina, and loss of light. To remedy this difficulty a lens is inserted at or very near the place where the focal image is formed, for the purpose of throwing the different pencils of rays which emanate from the several parts of the image toward the axis of the telescope, so that they shall all pass nearly through the centre of the eye lens proper. These two lenses are together called the eye-piece.

There are some small differences of detail in the construction of eye-pieces, but the general principle is the same in all. The two recognized classes are the positive and negative, the former being those in which the image is formed before the light reaches the field lens; the negative those in which it is formed between the lenses.

The figure shows the positive eye-piece drawn accurately to scale. $O\ I$ is one of the converging pencils from the object-glass which forms one point (I) of the focal image $I\ a$. This image is viewed by the *field lens* F of the eye-piece as a real object, and the shaded pencil between F and E shows the course of these rays after deviation by F. If there were no *eye-lens* E an eye properly placed beyond F would see an image at $I'\ a'$. The eye-lens E receives the pencil of rays, and deviates it to the observer's eye placed at such a point that the whole incident pencil will pass through the pupil and fall on the retina, and thus be effective. As we saw in the

FIG. 22.—SECTION OF A POSITIVE EYE-PIECE.

figure of the refracting telescope, every point of the object produces a pencil similar to $O\ I$, and the whole surfaces of the lenses F and E are covered with rays. All of these pencils passing through the pupil go to make up the retinal image. This image is referred by the mind to the distance of distinct vision (about ten inches), and the image $A\ I''$ represents the dimension of the final image relative to the image $a\ I$ as formed by the objective and $\frac{A\ I''}{a\ I}$ is evidently the magnifying power of this particular eye-piece used in combination with this particular objective.

More Exact Theory of the Objective.—For the benefit of the reader who wishes a more precise knowledge of the optical principles on which the action of the objective or other system of lenses depends, we present the following geometrical theory of the subject. This theory is not rigidly exact, but is sufficiently so for all ordinary computations of the focal lengths and sizes of image in the usual combinations of lenses.

Centres of Convergence and Divergence.—Suppose $A\ B$, Fig. 23, to be a lens or combination of lenses on which the light falls from the left hand and passes through to the right. Suppose rays parallel to $R\ P$ to fall on every part of the first surface of the glass. After passing through it they are all supposed to converge nearly or exactly to the same point R'. Among all these rays there is one, and one only, the course of which, after emerging from the glass at Q, will be parallel to its original direction RP. Let $R\ P\ Q\ R'$ be this central ray, which will be completely determined by the direction from which it comes. Next, let us take a ray coming from another direction as $S\ P$. Among all the rays parallel to $S\ P$, let us take that one which, after emerging from the glass at T, moves in a line parallel to its original direction. Continuing the process, let us suppose isolated rays coming from all parts of a distant object subject to the single condition that the course of each, after passing through the glass or system of glasses, shall be parallel to its original course. These rays we may call *central rays*. They have this remarkable property, pointed out by GAUSS: that they all converge

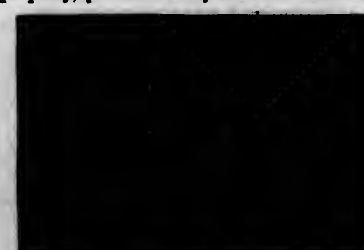

FIG. 23.

toward a single point, P, in coming to the glass, and diverge from another point, P', after passing through the last lens. These points were termed by GAUSS "Hauptpunkte," or principal points. But they will probably be better understood if we call the first one the centre of convergence, and the second the centre of divergence. It must not be understood that the central rays necessarily pass through these centres. If one of them lies outside the first or last refracting surface, then the central rays must actually pass through it. But if they lie between the surfaces, they will be fixed by the continuation of the straight line in which the rays move, the latter being refracted out of their course by passing through the surface, and thus avoiding the points in question. If the lens or system of lenses be turned around, or if the light passes through them in an opposite direction, the centre of convergence in the first case becomes the centre of divergence in the second, and *vice versa*. The necessity of this will be clearly seen by reflecting that a return ray of light will always keep on the course of the original ray in the opposite direction.

The figure represents a plano-convex lens with light falling on the convex side. In this case the centre of convergence will be on the convex surface, and that of divergence inside the glass about one third or two fifths of the way from the convex to the plano surface, the positions varying with the refractive index of the glass. In a double convex lens, both points will lie inside the glass, while if a glass is concave on one side and convex on the other, one of the points will be outside the glass on the concave side. It must be remembered that the positions of these centres of convergence and divergence depend solely on the form and size of the lenses and their refractive indices, and do not refer in any way to the distances of the objects whose images they form.

The principal properties of a lens or objective, by which the size of images are determined, are as follows: Since the angle $S' P' R'$ made by the diverging rays is equal to $R P S$, made by the converging ones, it follows, that if a lens form the image of an object, the size of the image will be to that of the object as their respective distances from the centres of convergence and divergence. In other words, the object seen from the centre of convergence P will be of the same angular magnitude as the image seen from the centre of divergence P'.

By *conjugate foci* of a lens or system of lenses we mean a pair of points such that if rays diverge from the one, they will converge to the other. Hence if an object is in one of a pair of such foci, the image will be formed in the other.

By the *refractive power* of a lens or combination of lenses, we mean its influence in refracting parallel rays to a focus which we may measure by the reciprocal of its focal distance or $1 \div f$. Thus, the power of a piece of plain glass is 0, because it cannot bring rays to a focus at all. The power of a convex lens is positive, while that of a concave lens is negative. In the latter case, it will be remembered by the student of optics that the virtual focus is on the same side of the lens from which the rays proceed. It is to be noted that when we speak of the focal distance of a lens, we mean the distance from the centre of divergence to the focus for parallel rays. In astronomical language this focus is called the stellar focus, being that for celestial objects, all of which we may regard as infinitely distant. If, now, we put

p, the power of the lens;

f, its stellar focal distance;

f', the distance of an object from the centre of convergence;

f'', the distance of its image from the centre of divergence; then the equation which determines f will be

$$\frac{1}{f'} + \frac{1}{f''} = \frac{1}{f} = p;$$

or,

$$f = \frac{f' f''}{f' + f''}; \quad f'' = \frac{f f'}{f' - f}.$$

From these equations may be found the focal length, having the distance at which the image of an object is formed, or *vice versa*.

§ 2. REFLECTING TELESCOPES.

As we have seen, the most essential part of a refracting telescope is the objective, which brings all the incident rays from an object to one focus, forming there an image of that object. In reflecting telescopes (reflectors) the objective is a mirror of speculum metal or silvered glass ground to the shape of a paraboloid. The figure shows the action of such a mirror on a bundle of parallel rays, which, after impinging on it, are brought by reflection to one focus F. The image formed at this focus may be viewed with an eye-piece, as in the case of the refracting telescope.

The eye-pieces used with such a mirror are of the kinds already described. In the figure the eye-piece would

FIG. 24.—CONCAVE MIRROR FORMING AN IMAGE.

have to be placed to the right of the point F, and the observer's head would thus interfere with the incident light. Various devices have been proposed to remedy this inconvenience, of which we will describe the two most common.

The Newtonian Telescope.—In this form the rays of light reflected from the mirror are made to fall on a small plane mirror placed diagonally just before they reach the principal focus. The rays are thus reflected out laterally through an opening in the telescope tube, and are there brought to a focus, and the image formed at the point marked by a heavy white line in Fig. 25, instead of at the point inside the telescope marked by a dotted line.

This focal image is then examined by means of an ordinary eye-piece, the head of the observer being outside of the telescope tube.

This device is the invention of Sir ISAAC NEWTON.

FIG. 25.
NEWTONIAN TELESCOPE.

FIG. 26.
CASSEGRAINIAN TELESCOPE.

The Cassegrainian Telescope.—In this form a secondary convex mirror is placed in the tube of the telescope about three fourths of the way from the large speculum to the focus. The rays, after being reflected from the large speculum, fall on this mirror before reaching the focus, and are reflected back again to the speculum; an opening is made in the centre of the latter to let the rays pass through. The position and curvature of the secondary mirror are adjusted so that the focus shall be formed just after passing through the opening in the speculum.

In this telescope the observer stands behind or under the speculum, and, with the eye-piece, looks through the opening in the centre, in the direction of the object. This form of reflector is much more convenient in use than the Newtonian, in using which the observer has to be near the top of the tube.

This form was devised by Cassegrain in 1672.

The advantages of reflectors are found in their cheapness, and in the fact that, supposing the mirrors perfect in figure, all the rays of the spectrum are brought to one focus. Thus the reflector is suitable for spectroscopic or photographic researches without any change from its ordinary form. This is not true of the refractor, since the rays by which we now photograph (the blue and violet rays) are, in that instrument, owing to the secondary spectrum, brought to a focus slightly different from that of the yellow and adjacent rays by means of which we see.

Reflectors have been made as large as six feet in aperture, the greatest being that of Lord Rosse, but those which have been most successful have hardly ever been larger than two or three feet. The smallest satellite of *Saturn* (*Mimas*) was discovered by Sir William Herschel with a four-foot speculum, but all the other satellites discovered by him were seen with mirrors of about eighteen inches in aperture. With these the vast majority of his faint nebulæ were also discovered.

The satellites of *Neptune* and *Uranus* were discovered by Lassell with a two-foot speculum, and much of the work of Lord Rosse has been done with his three-foot mirror, instead of his celebrated six-foot one.

From the time of Newton till quite recently it was usual to make the large mirror or objective out of speculum metal, a brilliant alloy liable to tarnish. When the mirror was once tarnished through exposure to the weather, it could be renewed only by a process of polishing almost equivalent to figuring and polishing the mirror anew. Consequently, in such a speculum, after the correct form and polish were attained, there was great difficulty in preserving them. In recent years this difficulty has been largely overcome in two ways: first, by improvements in the composition of the alloy, by which its liability to tarnish under exposure is greatly diminished, and, secondly, by a plan proposed by Foucault, which consists in making, once for all, a mirror of glass which will always retain its good figure, and depositing upon it a thin film of silver which may be removed and restored with little labor as often as it becomes tarnished.

In this way, one important defect in the reflector has been avoided. Another great defect has been less successfully treated. It is not a process of exceeding difficulty to give to the reflecting surface of either metal or glass the correct parabolic shape by which the incident rays are brought accurately to one focus. But to maintain this shape constantly when the mirror is mounted in a tube, and when this tube is directed in succession to various parts of the sky, is a mechanical problem of extreme difficulty. However the mirror may be supported, all the unsupported points tend by their weight to sag away from the proper position. When the mirror is pointed near the horizon, this effect of flexure is quite different from what it is when pointed near the zenith.

As long as the mirror is small (not greater than eight to twelve inches in diameter), it is found easy to support it so that these variations in the strains of flexure have little practical effect. As we increase its diameter up to 48 or

72 inches, the effect of flexure rapidly increases, and special devices have to be used to counterbalance the injury done to the shape of the mirror.

§ 3. CHRONOMETERS AND CLOCKS.

In Chapter I., § 5, we described how the right ascensions of the heavenly bodies are measured by the times of their transits over the meridian, this quantity increasing by a minute of arc in four seconds of time. In order to determine it with all required accuracy, it is necessary that the time-pieces with which it is measured shall go with the greatest possible precision. There is no great difficulty in making astronomical measures to a second of arc, and a star, by its diurnal motion, passes over this space in one fifteenth of a second of time. It is therefore desirable that the astronomical clock shall not vary from a uniform rate more than a few hundredths of a second in the course of a day. It is not, however, necessary that it should be perfectly correct; it may go too fast or too slow without detracting from its character for accuracy, if the intervals of time which it tells off—hours, minutes, or seconds—are always of exactly the same length, or, in other words, if it gains or loses exactly the same amount every hour and every day.

The time-pieces used in astronomical observation are the chronometer and the clock.

The *chronometer* is merely a very perfect time-piece with a balance-wheel so constructed that changes of temperature have the least possible effect upon the time of its oscillation. Such a balance is called a *compensation* balance.

The ordinary house clock goes faster in cold than in warm weather, because the pendulum rod shortens under the influence of cold. This effect is such that the clock will gain about one second a day for every fall of 3° Cent. (5°.4 Fahr.) in the temperature, supposing the pendulum rod to be of iron. Such changes of rate would be entirely inadmissible in a clock used for astronomical purposes. The astronomical clock is therefore provided with a compensation pendulum, by which the disturbing effects of changes of temperature are avoided.

There are two forms now in use, the *Harrison* (*gridiron*) and the *mercurial*. In the gridiron pendulum the rod is composed in part of a number of parallel bars of steel and brass, so connected together that while the expansion of the steel bars produced by an increase of temperature tends to depress the *bob* of the pendulum, the greater expansion of the brass bars tends to raise it. When the total lengths of the steel and brass bars have been properly adjusted a nearly perfect compensation occurs, and the centre of oscillation remains at a constant distance from the point of suspension. The rate of the clock, so far as it depends on the length of the pendulum, will therefore be constant.

FIG. 27.—GRIDIRON PENDULUM.

In the mercurial pendulum the weight which forms the bob is a cylindric glass vessel nearly filled with mercury. With an increase of temperature the steel suspension rod lengthens, thus throwing the centre of oscillation away from the point of suspension; at the same time the expanding mercury rises in the cylinder, and tends therefore to raise the centre of oscillation. When the length of the rod and the dimensions of the cylinder of mercury are properly proportioned, the centre of oscillation is kept at a constant distance from the point of suspension. Other methods of making this compensation have been used, but these are the two in most general use for astronomical clocks.

The *correction* of a chronometer (or clock) is the quantity of time (expressed in hours, minutes, seconds, and decimals of a second) which it is necessary to add algebraically to the indication of the hands, in order that the sum may be the correct time. Thus, if at sidereal 0^h, May 18, at New York, a sidereal clock or chronometer indicates $23^h\ 58^m\ 20^s\cdot 7$, its correction is $+\ 1^m\ 39^s\cdot 3$; if at 0^h (sidereal noon), of May 17, its correction was $+\ 1^m\ 38^s\cdot 3$, its daily *rate* or the change of its correction in a sidereal day is $+\ 1^s\cdot 0$: in other words, this clock is *losing* 1^s daily.

For clock *slow* the sign of the *correction* is +;
" " *fast* " " " " " is —;
" " *gaining* " " " " *rate* is —;
" " *losing* " " " " " is +.

A clock or chronometer may be well compensated for temperature, and yet its *rate* may be gaining or losing on the time it is intended to keep: it is not even necessary that the rate should be small (except that a small rate is practically convenient), provided only that it is constant. It is continually necessary to compute the clock correction at a given time from its known correction at some other time, and its known rate. If for some definite instant we denote the time as shown by the clock (technically "the clock-face") by T, the true time by T' and the clock correction by ΔT, we have

$$T' = T + \Delta T, \text{ and}$$
$$\Delta T = T' - T.$$

In all observatories and at sea observations are made daily to determine ΔT. At the instant of the observation the time T is noted by the clock; from the data of the observation the time T' is computed. If these agree, the clock is correct. If they differ, ΔT is found from the above equations.

If by observation we have found

ΔT_0 = the clock correction at a clock-time T_0,
ΔT = the clock correction at a clock-time T,
δT = the clock rate in a unit of time,

we have

$$\Delta T = \Delta T_0 + \delta T(T - T_0)$$

where $T - T_0$ must be expressed in days, hours, etc., according as δT is the rate in one day, one hour, etc.

When, therefore, the clock correction ΔT_0 and rate δT have been determined for a certain instant, T_0, we can deduce the true time from the clock-face T at any other instant by the equation $T' = T + \Delta T_0 + \delta T\,(T - T_0)$. If the clock correction has been determined at two different times, T_0 and T to be ΔT_0 and ΔT, the rate is inferred from the equation

$$\delta T = \frac{\Delta T - \Delta T_0}{T - T_0}.$$

These equations apply only so long as we can regard the rate as *constant*. As observations can be made only in clear weather, it is plain that during periods of overcast sky we must depend on these equations for our knowledge of T'—*i.e.*, the true time at a clock-time T.

The intervals between the determination of the clock correction should be small, since even with the best clocks and chronometers too much dependence must not be placed upon the rate. The following example from CHAUVENET'S Astronomy will illustrate the practical processes:

"*Example.*—At sidereal noon, May 5, the correction of a sidereal clock is $-16^m\ 47^s\cdot0$; at sidereal noon, May 12, it is $-16^m\ 13^s\cdot50$; what is the sidereal time on May 25, when the clock-face is $11^h\ 13^m\ 12^s\cdot6$, supposing the rate to be uniform?

$$
\begin{array}{rrl}
\text{May 5, correction} & = - & 16^m\ 47^s.30 \\
\text{" 12, \quad "} & = - & 16^m\ 13^s.50 \\
\hline
\text{7 days' rate} & = + & \quad\ 33^s\cdot50 \\
\delta T & = + & \quad\ \ 4^s\cdot829.
\end{array}
$$

Taking then as our starting-point $T_0 =$ May 12, 0^h, we have for the interval to $T =$ May 25, $11^h\ 13^m\ 12^s\cdot6$, $T - T_0 = 13^d\ 11^h\ 13^m\ 12^s\cdot6 = 13^d\cdot467$. Hence we have

$$
\begin{array}{rl}
\Delta T_0 = & -\quad 16^m\ 13^s\cdot50 \\
\delta T\,(T - T_0) = & +\quad\ \ 1^m\ \ 5^s\cdot03 \\
\hline
\Delta T = & -\quad 15^m\ \ 8^s\cdot47 \\
T = & 11^h\ 13^m\ 12^s\cdot60 \\
\hline
T' = & 10^h\ 58^m\ \ 4^s.13
\end{array}
$$

But in this example the rate is obtained for one true sidereal day, while the unit of the interval $13^d\cdot467$ is a sidereal day as shown by the clock. The proper interval with which to compute the rate in this case is $13^d\ 10^h\ 58^m\ 4^s\cdot13 = 13^d\cdot457$, with which we find

$$
\begin{array}{rl}
\Delta T_0 = & -\quad 16^m\ 13^s\cdot50 \\
\delta T \times 13\cdot457 = & +\quad\ \ 1^m\ \ 4^s\cdot98 \\
\hline
\Delta T = & -\quad 15^m\ \ 8^s.52 \\
T = & 11^h\ 13^m\ 12^s\cdot60 \\
\hline
T' = & 10^h\ 58^m\ \ 4^s\cdot08
\end{array}
$$

This repetition will be rendered unnecessary by always giving the rate in a *unit of the clock*. Thus, suppose that on June 3, at $4^h\ 11^m\ 12^s\cdot35$ by the clock, we have found the correction $+\ 2^m\ 10^s\cdot14$; and on June 4, at $14^h\ 17^m\ 49^s.82$ we have found the correction $+\ 2^m\ 19^s\cdot89$; the rate *in one hour of the clock* will be

$$\delta T = \frac{+\ 9^s\cdot75}{34^h\cdot1104} = +\ 0^s\cdot2858."$$

§ 4. THE TRANSIT INSTRUMENT.

The *meridian transit instrument*, or briefly the "transit," is used to observe the transits of the heavenly bodies,

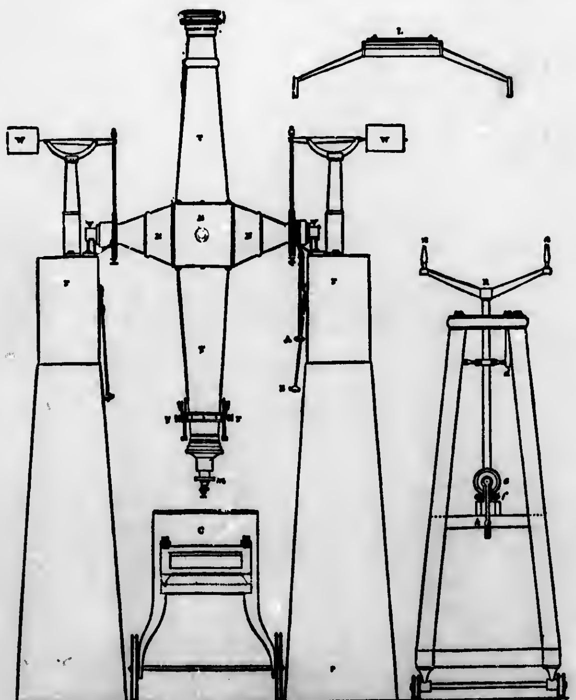

FIG. 28.—A TRANSIT INSTRUMENT.

and from the times of these transits as read from the clock to determine either the corrections of the clock or the right ascension of the observed body, as explained in Chapter I., § 5.

UMENT.

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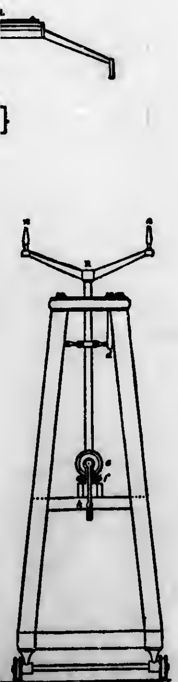
e heavenly bodies,



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It has two general forms, one (Fig. 28) for use in fixed observatories and one (Fig. 29) for use in the field.

It consists essentially of a telescope *T T* (Fig. 28) mounted on an axis *V V* at right angles to it.

FIG. 29.—PORTABLE TRANSIT INSTRUMENT.

The ends of this axis terminate in accurately cylindrical steel pivots which rest in metallic bearings *V V*, in shape like the letter Y, and hence called the Ys.

These are fastened to two pillars of stone, brick, or iron. Two counterpoises *W W* are connected with the axis as in the plate, so as to take a large portion of the weight of the axis and telescope from the Ys, and thus to diminish the friction upon these and to render the rotation about *V V* more easy and regular. In the ordinary use of the transit, the line *V V* is placed accurately level and perpendicular to the meridian, or in the east and west line. To effect this "adjustment," there are two sets of adjusting screws, by which the ends of *V V* in the Ys may be moved either up and down or north and south. The plate gives the form of transit used in permanent observatories, and shows the observing chair *C*, the reversing carriage *R*, and the level *L*. The arms of the latter have Y's, which can be placed over the pivots *V V*.

The *line of collimation* of the transit telescope is the line drawn through the centre of the objective perpendicular to the *rotation axis V V*.

The *reticle* is a network of fine spider lines placed in the focus of the objective.

In Fig. 30 the circle represents the field of view of a transit as seen through the eye-piece. The seven vertical lines, I, II, III, IV, V, VI, VII, are seven fine spider lines tightly stretched across a metal plate or diaphragm, and so adjusted as to be perpendicular to the direction of a star's apparent diurnal motion. This metal plate can be moved right and left by five screws. The horizontal wires, ***guide-wires***, *a* and *b*, mark the centre of the field. The field is illuminated at night by a lamp at the end of the axis which shines through the hollow interior of the latter, and causes the field to appear bright. The wires are dark against a bright ground. The *line of sight* is a line joining the centre of the objective and the central one, IV, of the seven vertical wires.

FIG. 30.

The whole transit is in adjustment when, first, the axis $V V$ is horizontal; second, when it lies east and west; and third, when the line of sight and the line of collimation coincide. When these conditions are fulfilled the line of sight intersects the celestial sphere in the meridian of the place, and when $T T$ is rotated about $V V$ the line of sight marks out the meridian on the sphere.

In practice the three adjustments are not exactly made, since it is impossible to effect them with mathematical precision. The errors of each of them are first made as small as is convenient, and are then determined and allowed for.

To find the *error of level*, we place on the pivots a fine level (shown in position in the figure of the portable transit), and determine how much higher one pivot is than the other in terms of the divisions marked on the level tube. Such a level is shown in Fig. 4 of plate 35, page 86. The value of one of these divisions in seconds of arc can be determined by knowing the length l of the whole level and the number n of divisions through which the bubble will run when one end is raised one hundredth of an inch.

If l is the length of the level in inches or the radius of the circle in which either end of the level moves when it is raised, then as the radius of any circle is equal to $57^\circ{\cdot}206$, $3437'{\cdot}75$ or $206,264''{\cdot}8$, we have in this particular circle one inch $= 206,264''{\cdot}8 \div l$; $0{\cdot}01$ inch $= 206,264{\cdot}8 \div 100\, l =$ a certain arc in seconds, say a''. That is, n divisions $= a''$, or one division $d = a'' \div n$.

The *error of collimation* can be found by pointing the telescope at a distant mark whose image is brought to the middle wire. The telescope (with the axis) is then lifted bodily from the Ys and replaced so that the axis $V V$ is reversed end for end. The telescope is again pointed to the distant mark. If this is still on the middle thread the line of sight and the line of collimation coincide. If not, the reticle must be moved bodily west or east until these conditions are fulfilled after repeated reversals.

To find the *error of azimuth* or the departure of the direction of $V V$ from an east and west line, we must observe the transits of two stars of different declinations δ and δ, and right ascensions α and α'. Suppose the clock to be running correctly—that is, with no rate—and the sidereal times of transit of the two stars over the middle thread to be θ and θ'. If $\theta - \theta' = \alpha - \alpha'$, then the middle wire is in the meridian and the azimuth is zero. For if the azimuth was not zero, but the west end of the axis was too far south, for example, the line of sight would fall east of the meridian for a south star, and further and further east the further south the star was. Hence if the two stars have widely different declinations δ and δ', then the star furthest south would come proportionately sooner to the middle wire than the other, and $\theta - \theta'$ would be different from $\alpha - \alpha'$. The amount of this difference gives a means of deducing the deviation of $A\ A$ from an east and west line. In a similar way the effect of a given error of level on the time of the transit of a star of declination δ is found.

Methods of Observing with the Transit Instrument.— We have so far assumed that the time of a star's transit over the middle thread was known, or could be noted. It is necessary to speak more in detail of how it is noted.

When the telescope is pointed to any star the earth's diurnal motion will carry the image of the star slowly across the field of view of the telescope (which is kept fixed), as before explained. As it crosses each of the threads, the time at which it is exactly on the thread is noted from the clock, which must be near the transit.

The mean of these times gives the time at which this star was on the middle thread, the threads being at equal intervals; or on the "*mean* thread," if, as is the case in practice, they are at unequal intervals.

If it were possible for an astronomer to note the *exact* instant of the transit of a star over a thread, it is plain that one thread would be sufficient; but, as all estimations of this time are, from the very nature of the case, but approximations, several threads are inserted in order that the accidental errors of estimations may be eliminated as far as possible. Five, or at most seven, threads are sufficient for this purpose. In the figure of the *reticle* of a transit instrument the star (the planet *Venus* in this case) may enter on the right hand in the figure, and may be supposed to cross each of the wires, the time of its transit over each of them, or over a sufficient number, being noted. The method of noting this time may be best understood by referring to the next figure. Suppose that the line in the middle of Fig. 32 is one of the transit-threads, and that the star is passing from the right hand of the figure toward the left; if it is on this wire at an

FIG. 31.

exact second by the clock (which is always near the observer, beating seconds audibly), this second must be written down as the time of the transit over this thread. As a rule, however, the transit cannot occur on the exact beat of the clock, but at the seventeenth second (for example) the star may be on the right of the wire, say at *a*; while at the eighteenth second it will have passed this wire and may be at *b*. If the distance of *a* from the wire is six tenths of the distance *a b*, then the time of transit is to be recorded as — *hours* — *minutes* (to be taken from the clock-face), and seventeen and six tenths *seconds;* and in this way the transit over each wire is observed. This is the method of "eye-and-ear" observation, the basis of such work as we have described, and it is so called from the part which both the eye and the ear play in the appreciation of intervals of time. The ear catches the beat of the clock, the eye fixes the place of the star at *a*; at the next beat of the clock, the eye fixes the star at *b*, and subdivides the space *a b* into tenths, at the same time appreciating the ratio which the distance from the thread to *a* bears to the distance *a b*. This is recorded as above. This method, which is still used in many observatories, was introduced by the celebrated Bradley, astronomer royal of England in 1750, and perfected by Maskelyne, his successor. A practiced observer can note the time within a tenth of a second in three cases out of four.

Fig. 32.

There is yet another method now in common use, which it is necessary to understand. This is called the American or chronographic method, and consists, in the present practice, in the use of a sheet of a paper wound about and fastened to a horizontal cylindrical barrel, which is caused to revolve by machinery once in one minute of time. A pen of glass which will make a continuous line is allowed to rest on the paper, and to this pen a continuous motion of translation in the direction of the length of the cylinder is given. Now, if the pen is allowed to mark, it is evident that it will trace on the paper an endless spiral line. An electric current is caused to run through the observing clock, through a key which is held in the observer's hand and through an electro-magnet connected with the pen.

A simple device enables the clock every second to give a slight lateral motion to the pen, which lasts about a thirtieth of a second. Thus every second is automatically marked by the clock on the chronograph paper. The observer also has the power to make a signal by his key (easily distinguished from the clock-signal by its different length), which is likewise permanently registered on the sheet. In this way, after the chronograph is in motion, the observer has merely to notice the instant at which the star is *on* the thread, and to press the key at that moment. At any subsequent time, he must mark some hour, minute, and second, taken from the clock, on the sheet at its appropriate place, and the translation of the spaces on the sheet into times may be done at leisure.

§ 5. GRADUATED CIRCLES.

Nearly every datum in practical astronomy depends either directly or indirectly upon the measure of an angle. To make the necessary measures, it is customary to employ what are called graduated or divided circles. These are made of metal, as light and yet as rigid as possible, and they have at their circumferences a narrow flat band of silver, gold, or platinum on which fine radial lines called "divisions" are cut by a "dividing engine" at regular and equal intervals. These intervals may be of 10′, 5′, or 2′, according to the size of the circle and the degree of accuracy desired. The narrow band is called the divided limb, and the circle is said to be divided to 10′, 5′, 2′. The separate divisions are numbered consecutively from 0° to 360° or from 0° to 90°, etc. The graduated circle has an axis at its centre, and to this may be attached the telescope by which to view the points whose angular distance is to be determined.

To this centre is also attached an arm which revolves with it, and by its motion past a certain number of divisions on the circle, determines the angle through which the centre has been rotated. This arm is called the index arm, and it usually carries a ***vernier*** on its extremity, by means of which the spaces on the graduated circle are subdivided. The *reading of the circle* when the index arm is in any position is the number of degrees, minutes, and seconds which correspond to that position; when the index arm is in another position there is a different reading, and the differences of the two readings $S^2—S^1$, $S^3—S^2$, $S^4—S^3$ are the angles through which the index arm has turned.


FIG. 33.

The process of measuring the angle between the objects by means of a divided circle consists then of pointing the telescope at the first object and reading the position of the index arm, and then turning the telescope (the index arm turning with it) until it points at the second object, and again reading the position of the index arm. The difference of these readings is the angle sought.

To facilitate the determination of the exact reading of the circle, we have to employ special devices, as the *vernier* and the *reading microscope*.

The Vernier.—In Fig. 34, MN is a portion of the divided limb of a graduated circle; CD is the index arm which revolves with the telescope about the centre of the circle. The end ab of CD is also a part of a circle concentric with MN, and it is divided into n parts or divisions. The length of these n parts is so chosen that it is the same as that of (n—1) parts on the divided limb $M\,N$ or the reverse.

The first stroke a is the *zero* of the vernier, and the reading is always determined by the position of this zero or pointer. If this has revolved past exactly twenty divisions of the circle, then the angle to be measured is $20 \times d$, d being the value of one division on the limb ($N\,M$) in arc.

FIG. 34.—THE VERNIER.

Call the angular value of one division on the vernier d';

$$(n-1)\,d = n \cdot d', \text{ or } d' = \frac{n-1}{n} \cdot d, \text{ and } d - d' = \frac{1}{n}\,d\,;$$

$d - d'$ is called the *least count* of the vernier which is one n^{th} part of a circle division.

If the zero a does not fall exactly on a division on the circle, but is at some other point (as in the figure), for example between two divisions whose numbers are P and $(P + 1)$, the whole reading of the circle in this position is $P \times d +$ the fraction of a division from P to a.

If the m^{th} division of the vernier is in the prolongation of a division on the limb, then this fraction $P\,a$ is m

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e figure), for ex-
bers are P and
n this position is
P to a.
the prolongation
action $P a$ is m

$(d - d') = \frac{m}{n} \cdot d$. In the figure $n = 10$, and as the 4th division is almost exactly in coincidence, $m = 4$, so that the whole reading of the circle is $P \times d + \frac{4}{10} \cdot d$. If d is 10′, for example, and if the division P is numbered 297° 40′, then this reading would be 297° 44′, the least count being 1′, and so in other cases. If the zero had started from the reading 280° 20′, it must have moved past 17° 24′, and this is the angle which has been measured.

§ 6. THE MERIDIAN CIRCLE.

The meridian circle is a combination of the transit instrument with a graduated circle fastened to its axis and moving with it. The meridian circle made by REPSOLD for the United States Naval Academy at Annapolis is shown in the figure. It has two circles, $c\ c$ and $c'\ c'$, finely divided on their sides. The graduation of each circle is viewed by four microscopes, two of which, $R\ R$, are shown in the cut. The microscopes are 90° apart. The cut shows also the hanging level $L\ L$, by which the error of level of the axis $A\ A$ is found.

The instrument can be used as a transit to determine right ascensions, as before described. It can be also used to measure declinations in the following way. If the telescope is pointed to the nadir, a certain division of the circles, as N, is under the first microscope. If it is pointed to the pole, the reading will change by the angular distance between the nadir and the pole, or by $90° + \phi$, ϕ being the latitude of the place (supposed to be known). The polar reading P is thus known when the nadir reading N is found. If the telescope is then pointed to various stars of unknown polar distances, p', p'', p''', etc., as they successively cross the meridian, and if the circle readings for these stars are P', P'', P''', etc., it follows that $p' = P' - P$; $p'' = P'' - P$; $p''' = P''' - P$, etc.

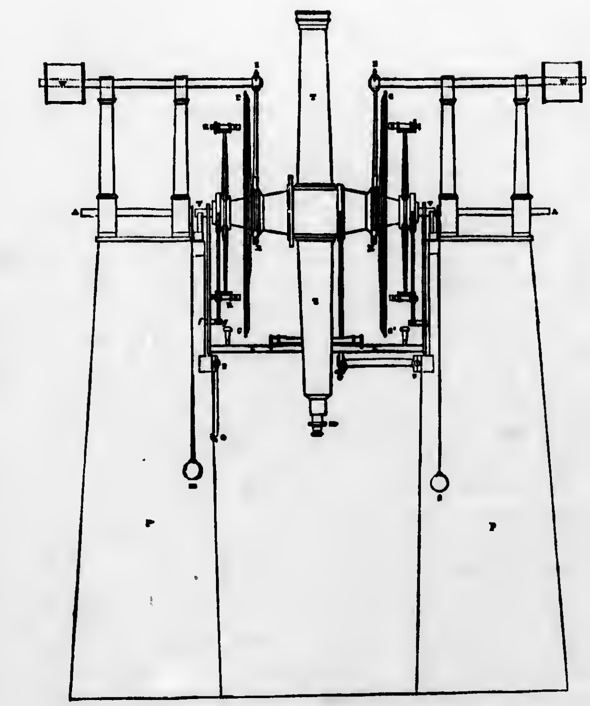

FIG. 35.—THE MERIDIAN CIRCLE.

To determine the readings P, P', P'', etc., we use the microscopes R, R, etc. The observer, after having set the telescope so that one of the stars shall cross the field of view exactly at its centre (which may be here marked by a single horizontal thread in the reticle), goes to each of the microscopes in succession and places his eye at A (see Fig. 1, page 86). He sees in the field of the microscope the image of the divisions of the graduated scale (Fig. 2) formed at D (Fig. 1), the common focus of the lenses A and C. Just at that focus is placed a notched scale (Fig. 2) and two crossed spider lines. These lines are fixed to a sliding frame a a, which can be moved by turning the graduated head F. This head is divided usually into sixty parts, each of which is 1″ of arc on the circle, one whole revolution of the head serving to move the sliding frame a a, and its crossed wires through 60″ or 1′ on the graduated circle. The notched scale is not movable, but serves to count the number of complete revolutions made by the screw, there being one notch for each revolution. The index i (Fig. 2) is fixed, and serves to count the number of parts of F which are carried past it by the revolution of this head.

If on setting the crossed threads at the centre of the motion of F, and looking into the microscope, a division on the circle coincides with the cross, the reading of the circle P is the exact number of degrees and minutes corresponding to that particular division on the divided circle.

Usually, however, the cross has been apparently carried *past* one of the exact divisions of the circle by a certain quantity, which is now to be measured and added to the reading corresponding to this adjacent division. This measure can be made by turning the screw back say four revolutions (measured on the notched scale) *plus* 37·3 parts (measured by the index i). If the division of the circle in question was 179° 50′, for example, the complete reading would be in this case 179° 50′ + 4′ 37″·3 or 179° 54′ 37″·3. Such a reading is made by each microscope, and the mean of the minutes and seconds from all four taken as the circle reading.

We now know how to obtain the readings of our circle when directed to any point. We require some zero of reference, as the nadir reading (N), the polar reading (P), the equator reading, (Q), or the zenith reading (Z). Any one of these being known, the circle readings for any stars as P', P'', P''', etc., can be turned into polar distances p', p'', p''', etc.

The nadir reading (N) is the zero commonly employed. It can be determined by pointing the telescope vertically downward at a basin of mercury placed immediately beneath the instrument, and turning the whole instrument about the axis until the middle wire of the reticle seen directly exactly coincides with the image of this wire seen by reflection from the surface of the quicksilver. When this is the case, the telescope is vertical, as can be easily seen, and the nadir reading may be found from the circles. The meridian circle thus serves to determine both the right ascension and declination of a given star at the same culmination. Zone observations are made with it by clamping the telescope in one

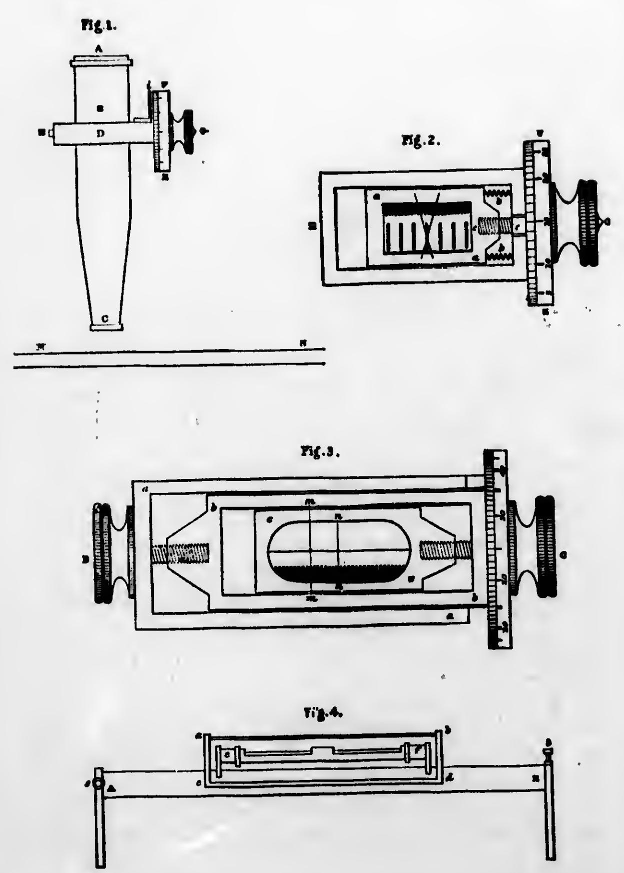

FIG. 36.—READING MICROSCOPE, MICROMETER AND LEVEL.

direction, and observing successively the stars which pass through its field of view. It is by this rapid method of observing that the largest catalogues of stars have been formed.

§ 7. THE EQUATORIAL.

To complete the enumeration and description of the principal instruments of astronomy, we require an account of the *equatorial.* This term, properly speaking, refers to a form of mounting, but it is commonly used to include both mounting and telescope. In this class of instruments the object to be attained is in general the easy finding and following of any celestial object whose apparent place in the heavens is known by its right ascension and declination. The equatorial mounting consists essentially of a pair of *axes* at right angles to each other. One of these $S\,N$ (the *polar axis*) is directed toward the elevated pole of the heavens, and it therefore makes an angle with the horizon equal to the latitude of the place (p. 21). This axis can be turned about its own axial line. On one extremity it carries another axis $L\,D$ (the *declination axis*), which is fixed at right angles to it, but which can again be rotated about *its* axial line.

To this last axis a telescope is attached, which may either be a reflector or a refractor. It is plain that such a telescope may be directed to any point of the heavens; for we can rotate the declination axis until the telescope points to any given polar distance or declination. Then, keeping the telescope fixed in respect to the declination axis, we can rotate the whole instrument as one mass about the polar axis until the telescope points to any portion of the parallel of declination defined by the given right ascension or hour-angle. Fig. 37 is an equatorial of six-inch aperture which can be moved from place to place.

If we point such a telescope to a star when it is rising (doing this by rotating the telescope first about its declination axis, and then about the polar axis), and fix the telescope in this position, we can, by simply rotating the

FIG. 37.—EQUATORIAL TELESCOPE POINTED TOWARD THE POLE.

whole apparatus on the polar axis, cause the telescope to trace out on the celestial sphere the apparent diurnal path which this star will appear to follow from rising to setting. In such telescopes a driving-clock is so arranged that it can turn the telescope round the polar axis at the same rate at which the earth itself turns about its own axis of rotation, but in a contrary direction. Hence such a telescope once pointed at a star will continue to point at it as long as the driving-clock is in operation, thus enabling the astronomer to observe it at his leisure.

FIG. 38.—MEASUREMENT OF POSITION-ANGLE.

Every equatorial telescope intended for making exact measures has a *filar micrometer*, which is precisely the same in principle as the reading microscope in Fig. 2, page 86, except that its two wires are parallel.

A figure of this instrument is given in Fig. 2, page 86. One of the wires is fixed and the other is movable by the screw. To measure the distance apart, of two objects A and B, wire 1 (the fixed wire) is placed on A and wire 2 (movable by the screw) is placed on B. The number of revolutions and parts of a revolution of the screw is noted, say $10^r \cdot 267$; then wires 1 and 2 are placed in coincidence, and this *zero-reading* noted, say $5^r \cdot 143$. The distance $A\,B$ is equal to $5^r \cdot 124$. Placing wires 1 and 2 a known number of revolutions apart, we may observe the transits of a star in the equator over them; and from the interval of time required for this star to move over say fifty revolutions, the value of one revolution is known, and can always be used to turn distances measured in revolutions to distances in time or arc.

By the filar micrometer we can determine the distance apart in seconds of arc of any two stars A and B. To completely fix the relative position of A and B, we require not only this distance, but also the angle which the line $A\,B$ makes with some fixed direction in space. We assume as the fixed direction that of the meridian passing through A. Suppose in Fig. 38 A and B to be two stars visible in the field of the equatorial. The clock-work is detached, and by the diurnal motion of the earth the two stars will cross the field slowly in the direction of the *parallel* of declination passing through A, or in the direction of the arrow in the figure from E. to W., east to west. The filar micrometer is constructed so that it can be rotated bodily about the axis of the telescope, and a graduated circle measures the amount of this rotation. The micrometer is then rotated until the star A will pass along one of its wires. This wire marks the direction of the parallel. The wire perpendicular to this is then in the meridian of the star.

The *position angle* of B with respect to A is then the angle which $A\,B$ makes with the meridian $A\,N$ passing through A toward the north. It is zero when B is north of A, 90° when B is east, 180 when B is south, and 270° when B is west of A. Knowing p, the position angle ($N\,A\,B$ in the figure), and s ($A\,B$) the distance of B, we can find the difference of right ascension ($\Delta\,\alpha$), and the difference of declination ($\Delta\,\delta$) of B from A by the formulæ,

$$\Delta\alpha = s \sin p\,; \quad \Delta\delta = s \cos p.$$

Conversely knowing $\Delta\alpha$ and $\Delta\delta$, we can deduce s and p from these formulæ. The angle p is measured while the clock-work keeps the star A in the centre of the field.

§ 8. THE ZENITH TELESCOPE.

The accompanying figure gives a view of the zenith telescope in the form in which it is used by the United States Coast Survey. It consists of a vertical pillar which supports two *Ys*. In these rests the horizontal axis of the instrument which carries the telescope at one end, and a counterpoise at the other. The whole instrument can revolve 180° in azimuth about this pillar. The telescope has a micrometer at its eye-end, and it also carries a divided circle provided with a fine level. A second level is provided, whose use is to make the rotation axis horizontal. The peculiar features of the zenith telescope are the divided circle and its attached level. The level is, as shown in the cut, in the plane of motion of the telescope (usually the plane of the meridian), and it can be independently rotated on the axis of the divided circle, and set by means of it to any angle with the optical axis of the telescope. The circle is divided from zero (0°) at its lowest point to 90° in each direction, and is firmly attached to the telescope tube, and moves with it.

By setting the vernier or index-arm of the circle to any degree and minute as a, and clamping it there (the level moving with it),

FIG. 89.—THE ZENITH TELESCOPE.

and then rotating the telescope and the whole system about the horizontal axis until the bubble of the level is in the centre of the level-tube, the axis of the telescopes will be directed to the zenith distance a. The filar micrometer is so adjusted that a motion of its screw measures differences of zenith distance. The use of the zenith telescope is for determining the latitude by TALCOTT's method. The theory of this operation has been already given on page 48. A description of the actual process of observation will illustrate the excellences of this method.

Two stars, A and B, are selected beforehand (from Star Catalogues), which culminate, A south of the zenith of the place of observation, B north of it. They are chosen at nearly equal zenith distances ζ^A and ζ^B, and so that $\zeta^A - \zeta^B$ is less than the breadth of the field of view. Their right ascensions are also chosen so as to be about the same. The circle is then set to the mean zenith distance of the two stars, and the telescope is pointed so that the bubble is nearly in the middle of the level. Suppose the right ascension of A is the smaller, it will then culminate first. The telescope is then turned to the south. As A passes near the centre of the field its distance from the centre is measured by the micrometer. The level and micrometer are read, the whole instrument is revolved 180°, and star B is observed in the same way.

By these operations we have determined the difference of the zenith distances of two stars whose declinations δ^A and δ^B are known. But ϕ being the latitude,

$\phi = \delta^A + \zeta^A$ and $\phi = \delta^B - \zeta^B$, whence

$$\phi = \tfrac{1}{2}(\delta^A + \delta^B) + \tfrac{1}{2}(\zeta^A - \zeta^B).$$

The first term of this is known; the second is measured; so that each pair of stars so observed gives a value of the latitude which depends on the measure of a very small arc with the micrometer, and as this arc can be measured with great precision, the exactness of the determination of the latitude is equally great.

§ 9. THE SEXTANT.

The sextant is a portable instrument by which the altitudes of celestial bodies or the angular distances between them may be measured. It is used chiefly by navigators for determining the latitude and the local time of the position of the ship. Knowing the local time, and comparing it with a chronometer regulated on Greenwich time, the longitude becomes known and the ship's place is fixed.

It consists of the arc of a divided circle usually 60° in extent, whence the name. This arc is in fact divided into 120 equal parts, each marked as a degree, and these are again divided into smaller spaces, so that by means of the vernier at the end of the index-arm $M\,S$ an arc of 10″ (usually) may be read.

The *index-arm* $M\,S$ carries the *index-glass* M, which is a silvered plane mirror set perpendicular to the plane of the divided arc. The *horizon-glass m* is also a plane mirror fixed perpendicular to the plane of the divided circle.

This last glass is fixed in position, while the first revolves with the index-arm. The horizon-glass is divided into two parts, of which the lower one is silvered, the upper half being transparent. *E* is a telescope of low power pointed toward the horizon-glass. By it any object to which it is directed can be seen through the unsilvered half of the horizon-glass. Any other object in the same plane can be brought into the same field by rotating the index-arm

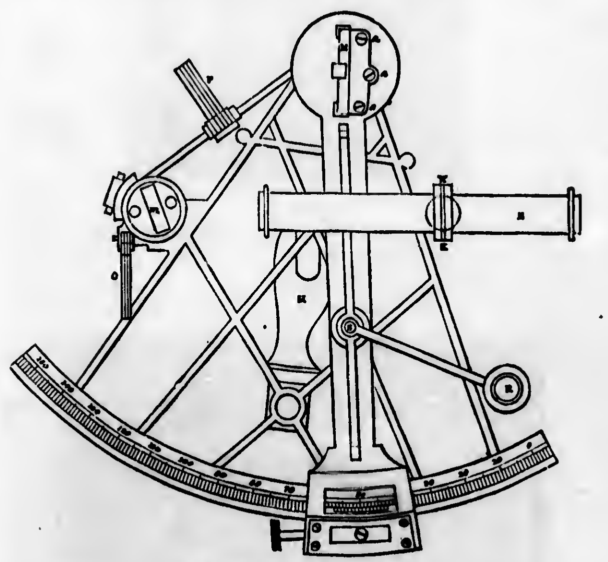

Fig. 40.—The Sextant.

(and the index-glass with it), so that a beam of light from this second object shall strike the index-glass at the proper angle, there to be reflected to the horizon-glass, and again reflected down the telescope *E*. Thus the images of any two objects in the plane of the sextant may be brought together in the telescope by viewing one directly, and the other by reflection.

The principle upon which the sextant depends is the following, which is proved in optical works. *The angle between the first and the last direction of a ray which has suffered two reflections in the same* *plane is equal to twice the angle which the two reflecting surfaces make with each other.*

In the figure $S\,A$ is the ray incident upon A, and this ray is by reflection brought to the direction $B\,E$. The theorem declares that the angle $B\,E\,S$ is equal to twice $D\,C\,B$, or twice the angle of

FIG. 41.

the mirrors, since $B\,C$ and $D\,C$ are perpendicular to B and A. To measure the altitude of a star (or the sun) at sea, the sextant is held in the hand, and the telescope is pointed to the sea-horizon, which appears like a definite line. The index-arm is then moved until the reflected image of the sun or of the star coincides with the

FIG. 42.—ARTIFICIAL HORIZON.

image of the sea-horizon seen directly. When this occurs the time is to be noted from a chronometer. If a star is observed, the reading of the divided limb gives the altitude directly; if it is the sun or moon which has been observed, the lower limb of these is brought to coincide with the horizon, and the altitude of the centre is found by applying the semi-diameter as found in the Nautical Almanac to the observed altitude of the limb.

The angular distance apart of a star and the moon can be measured by pointing the telescope at the star, revolving the whole sextant about the sight-line of the telescope until the plane of the divided arc passes through both star and moon, and then by moving the index-arm until the reflected moon is just in contact with the star's image seen directly.

On shore the horizon is broken up by buildings, trees, etc., and the observer is therefore obliged to have recourse to an *artificial horizon*, which consists usually of the reflecting surface of some liquid, as mercury, contained in a small vessel A, whose upper surface is necessarily parallel to the horizon $D\,A\,C$. A ray of light $S\,A$, from a star at S, incident on the mercury at A, will be reflected in the direction $A\,E$, making the angle $S\,A\,C = C\,A\,S'$ ($A\,S'$ being $E\,A$ produced), and the reflected image of the star will appear to an eye at E as far below the horizon as the real star is above it. With a sextant whose index and horizon-glasses are at I and H, the angle $S\,E\,S'$ may be measured; but $S\,E\,S' = S\,A\,S' - A\,S\,E$, and if $A\,E$ is exceedingly small as compared with $A\,S$, as it is for all celestial bodies, the angle $A\,S\,E$ may be neglected, and $S\,E\,S'$ will equal $S\,A\,S'$, or double the altitude of the object: hence one half the reading of the instrument will give the apparent altitude.

CHAPTER III.

MOTION OF THE EARTH.

§ 1. ANCIENT IDEAS OF THE PLANETS.

It was observed by the ancients that while the great mass of the stars maintained their positions relatively to each other not only during each diurnal revolution, but month after month and year after year, there were visible to them seven heavenly bodies which changed their positions relatively to the stars and to each other. These they called planets or wandering stars. Still calling the apparent crystalline vault in which the stars seem to be set the celestial sphere, and imagining it as at rest, it was found that the seven planets performed a very slow revolution around the sphere from west to east, in periods ranging from one month in the case of the moon, to thirty years in that of *Saturn*. It was evident that these bodies could not be considered as set in the same solid sphere with the stars, because they could not then change their positions among the stars. Various ways of accounting for their motions were therefore proposed. One of the earliest conceptions is associated with the name of Pythagoras. He is said to have taught that each of the seven planets had its own sphere inside of and concentric with that of the fixed stars, and that these seven hollow spheres each performed its own revolution, independently of the others. This idea of a number of concentric solid spheres was, however, apparently given up without any one having taken the trouble to refute it by argument. Although at first sight plausible enough, a close examination would show it to be entirely inconsistent with the observed facts. The idea of the fixed stars being set in a solid sphere was, indeed, in seemingly perfect accord with their diurnal revolution as observed by the naked eye. But it was not so with the planets. The latter, after continued observation, were found to move sometimes backward and sometimes forward; and it was quite evident that at certain periods they were nearer the earth than at other periods. These motions were entirely inconsistent with the theory that they were fixed in solid spheres. Still the old language continued in use—the word sphere meaning, not a solid body, but the space or region within which the planet moved.

These several conceptions, as well as those which followed, were all steps toward the truth. The planets were rightly considered as bodies nearer to us than the fixed stars. It was also rightly judged that those which moved most slowly were the most distant, and thus their order of distance from the earth was correctly given, except in the case of *Mercury* and *Venus*.

We now know that these seven planets, together with the earth, and a number of other bodies which the telescope has made known to us, form a family or system by themselves, the dimensions of which, although inconceivably greater than any which we have to deal with at the surface of the earth, are quite insignificant when compared with the distance which separates us from the fixed stars. The sun being the great central body of this system, it is called the *Solar System*. It is to the motions of its several bodies and the consequences which flow from them that the attention of the reader is directed in the following chapters. We premise that there are now known to be eight large planets, of which the earth is the third in the order of distance from the sun, and that these bodies all perform a regular revolution around the sun.

Mercury, the nearest, performs its revolution in three months; *Neptune*, the farthest, in 164 years.

First in importance to us, among the heavenly bodies which we see from the earth, stands the sun, the supporter of life and motion upon the earth. At first sight it might seem curious that the sun and seeming stars like *Mars* and *Saturn* should have been classified together as planets by the ancients, while the fixed stars were considered as forming another class. That the ancients were acute enough to do this tends to impress us with a favorable sense of the scientific character of their intellect. To any but the most careful theorists and observers, the star-like planets, if we may call them so, would never have seemed to belong in the same class with the sun, but rather in that of the stars; especially when it was found that they were never visible at the same time with the sun. But before the times of which we have any historic record, there were men who saw that, in a motion from west to east among the fixed stars, these several bodies showed a common character, which was more essential in a theory of the universe than were their immense differences of aspect and lustre, striking though these might be.

It must, however, be remembered that we no longer consider the sun as a planet. We have modified the ancient system by making the sun and the earth change places, so that the latter is now regarded as one of the eight large planets, while the former has taken the place of the earth as the central body of the system. In consequence of the revolution of the planets round the sun, each of them seems to perform a corresponding circuit in the heavens around the celestial sphere, when viewed from any other planet or from the earth.

§ 2. ANNUAL REVOLUTION OF THE EARTH.

To an observer on the earth, the sun seems to perform an annual revolution among the stars, a fact which has been known from the earliest ages. We now know that this is due to the annual revolution of the earth round the sun. It is to the nature and effects of this annual revolution of the earth that the attention of the reader is now directed. Our first lesson is to show the relations between it and the corresponding apparent revolution of the sun, which is its counterpart.

In Fig. 43, let S represent the sun, $A\ B\ C\ D$ the orbit of the earth around it, and $E\ F\ G\ H$ the sphere of the

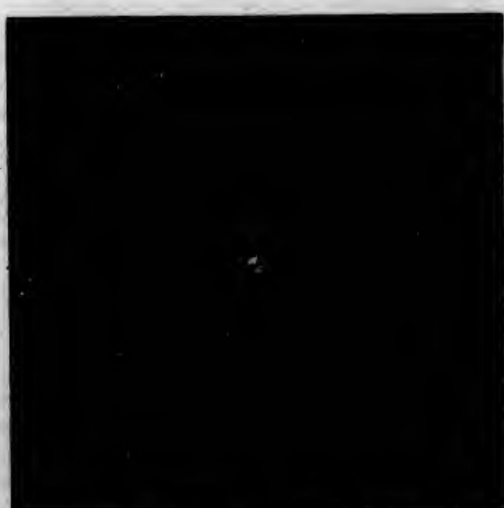

FIG. 43.—REVOLUTION OF THE EARTH.

fixed stars. This sphere, being supposed infinitely distant, must be considered as infinitely larger than the circle $A\ B\ C\ D$. Suppose now that 1, 2, 3, 4, 5, 6 are a number of consecutive positions of the earth. The line $1S$ drawn from the sun to the earth in the first position is called the radius vector of the earth. Suppose this line extended infinitely so as to meet the celestial sphere in the point $1'$. It is evident that to an observer on the earth at 1 the sun will appear projected on the sphere in the direction of 1′. When the earth reaches 2, it will appear in the direction of 2′, and so on. In other words, as the earth revolves around the sun, the latter will seem to perform a revolution among the fixed stars, which are immensely more distant than itself.

It is also evident that the point in which the earth would be projected, if viewed from the sun, is always exactly opposite that in which the sun appears as projected from the earth. Moreover, if the earth moves more rapidly in some points of its orbit than in others, it is evident that the sun will also appear to move more rapidly among the stars, and that the two motions must always accurately correspond to each other.

The radius vector of the earth in its annual course describes a plane, which in the figure may be represented by that of the paper. This plane continued to infinity in every direction will cut the celestial sphere in a great circle; and it is evident that the sun will always appear to move in this circle. The plane and the circle are indifferently termed the ecliptic. The plane of the ecliptic is generally taken as the fundamental one, to which the positions of all the bodies in the solar system are referred. By the fundamental principles of spherical trigonometry, it divides the celestial sphere into two equal parts. In thinking of the celestial motions, it is convenient to conceive of this plane as horizontal. Then if we draw a vertical line passing through the sun at right angles to it, or perpendicular to the plane of the paper on which the figure is represented, the point at which this line intersects the celestial sphere will be the pole of the ecliptic. This point is situated in the constellation *Draco*, and has an extremely slow motion of about half a second a year, owing to a change in the position of the ecliptic to be hereafter described.

Let us now study the apparent annual revolution of the sun produced in the way just mentioned. One result of this motion is probably familiar to every reader, in the different constellations which are seen at different times of the year. Let us take, for example, the bright star *Aldebaran*, which, on a winter evening, we may see northwest of *Orion*. Near the end of February this star crosses the meridian about six o'clock in the evening, and sets about midnight. If we watch it night after night through the months of March and April, we shall find that it is farther and farther toward the west on each successive evening at the same hour. By the end of April we shall barely be able to see it about the close of the evening twilight. At the end of May it will be so close to the sun as to be entirely invisible. This shows that during the months we have been watching it, the sun has been approaching the star from the west. If in July we watch the eastern horizon in the early morning, we shall see this star rising before the sun. The sun has therefore passed by the star, and is now east of it. At the end of November we will find it rising at sunset and setting at sunrise. The sun is therefore directly opposite the star. During the winter months it approaches it again from the west, and passes it about the end of May, as before. Any other star south of the zenith shows a similar change, since the relative positions of the stars do not vary.

§ 3. THE SUN'S APPARENT PATH.

It is evident that if the apparent path of the sun lay in the equator, it would, during the entire year, rise exactly in the east and set in the west, and would always cross the meridian at the same altitude. The days would always be twelve hours long, for the same reason that a star in the equator is always twelve hours above the horizon and twelve hours below it. But we know that this is not the case, the sun being sometimes north of the equator and sometimes south of it, and therefore having a motion in declination. To understand this motion, suppose that on March 19th, 1879, the sun had been observed with a meridian circle and a sidereal clock at the moment of transit over the meridian of Washington. Its position would have been found to be this :

Right Ascension, $23^h\ 55^m\ 23^s$; Declination, $0°\ 30'$ south.

Had the observation been repeated on the 20th and following days, the results would have been :

March 20,	R. Ascen.	$23^h\ 59^m\ 2^s$;	Dec.	$0°\ 6'$ South.
21,	"	$0^h\ 2^m\ 40^s$;	"	$0°\ 17'$ North.
22,	"	$0^h\ 6^m\ 19^s$;	"	$0°\ 41'$ North.

FIG. 44.—THE SUN CROSSING THE EQUATOR.

If we lay these positions down on a chart, we shall find them to be as in Fig. 44, the centre of the sun being south of the equator in the first two positions, and north of it in the last two. Joining the successive positions by a line, we shall have a small portion of the apparent path of the sun on the celestial sphere, or, in other words, a small part of the ecliptic.

It is clear from the observations and the figure that the sun crossed the equator between six and seven o'clock on the afternoon of March 20th, and therefore that the equator and ecliptic intersect at the point where the sun was at that hour. This point is called the *vernal equinox*, the first word indicating the season, while the second expresses the equality of the nights and days which occurs when the sun is on the equator. It will be remembered that this equinox is the point from which right ascensions are counted in the heavens in the same way that longitudes on the earth are counted from Greenwich or Washington. The s i d e r e a l clock is therefore so set that the hands shall read 0 hours 0 minutes 0 seconds at the moment when the vernal equinox crosses the meridian.

FIG. 45.—THE SUN'S APPARENT PATH IN SUMMER.

Continuing our observations of the sun's apparent course for six months from March 20th till September 23d, we should find it to be as in Fig. 45. It will be seen that Fig. 44 corresponds to the right-hand end of 45, but is on a much larger scale. The sun, moving along the great circle of the ecliptic, will reach its greatest northern declination about June 21st. This point is indicated on the figure as 90° from the vernal equinox, and is called the *summer solstice*. The sun's right ascension is then six hours, and its declination 23½° north.

The course of the sun now inclines toward the south, and it again crosses the equator about September 22d at a point diametrically opposite the vernal equinox. In virtue of the theorem of spherical trigonometry that all great circles intersect each other in two opposite points, the ecliptic and equator intersect at the two opposite equinoxes. The equinox which the sun crosses on September 22d is called the *autumnal equinox*.

During the six months from September to March the sun's course is a counterpart of that from March to September, except that it lies south of the equator. It attains its greatest south declination about December 22d, in right ascension 18 hours, and south declination 23½°. This point is called the *winter solstice*. It then begins to incline its course toward the north, reaching the vernal equinox again on March 20th, 1880.

The two equinoxes and the two solstices may be regarded as the four cardinal points of the sun's apparent annual circuit around the heavens. Its passage through these points is determined by measuring its altitude or declination from day to day with a meridian circle. Since in our latitude greater altitudes correspond to greater declinations, it follows that the summer solstice occurs on the day when the altitude of the sun is greatest, and the winter solstice on that when it is least. The mean of these altitudes is that of the equator, and may therefore be found by subtracting the latitude of the place from 90°. The time when the sun reaches this altitude going north marks the vernal equinox, and that when it reaches it going south marks the autumnal equinox.

These passages of the sun through the cardinal points have been the subjects of astronomical observation from the earliest ages on account of their relations to the change of the seasons. An ingenious method of finding the time when the sun reached the equinoxes was used by the astronomers of Alexandria about the beginning of our era. In the great Alexandrian Museum, a large ring or wheel was set up parallel to the plane of the equator—in other words, it was so fixed that a star at the pole would shine perpendicularly on the wheel. Evidently its plane if extended must have passed through the east and west points of the horizon, while its inclination to the vertical was equal to the latitude of the place, which was not far from 30°. When the sun reached the equator going north or south, and shone upon this wheel, its lower edge would be exactly covered by the shadow of the upper edge; whereas in any other position the sun would shine upon the lower inner edge. Thus the time at which the sun reached the equinox could be determined, at least to a fraction of a day. By the more exact methods of modern times, it can be determined within less than a minute.

It will be seen that this method of determining the annual apparent course of the sun by its declination or altitude is entirely independent of its relation to the fixed stars; and it could be equally well applied if no stars were ever visible. There are, therefore, two entirely distinct ways of finding when the sun or the earth has completed its apparent circuit around the celestial sphere; the one by the transit instrument and sidereal clock, which show when the sun returns to the same position among the stars, the other by the measurement of altitude, which shows when it returns to the same equinox. By the former method, already described, we conclude that it has completed an annual circuit when it returns to the same star; by the latter when it returns to the same equinox. These two methods will give slightly different results for the length of the year, for a reason to be hereafter described.

The Zodiac and its Divisions.—The zodiac is a belt in the heavens, commonly considered as extending some 8° on each side of the ecliptic, and therefore about 16° wide. The planets known to the ancients are always seen within this belt. At a very early age the zodiac was mapped out into twelve signs known as the *signs of the zodiac*, the names of which have been handed down to the present time. Each of these signs was supposed to be the seat of a constellation after which it was called. Commencing at the vernal equinox, the first thirty degrees through which the sun passed, or the region among the stars in which it was found during the month following, was called the sign *Aries*. The next thirty degrees was called *Taurus*. The names of all the twelve signs in their proper order, with the approximate time of the sun's entering upon each, are as follow:

Aries, the Ram,	March 20.
Taurus, the Bull,	April 20.
Gemini, the Twins,	May 20.
Cancer, the Crab,	June 21.
Leo, the Lion,	July 22.
Virgo, the Virgin,	August 22.
Libra, the Balance,	September 22.
Scorpius, the Scorpion,	October 23.
Sagittarius, the Archer,	November 23.
Capricornus, the Goat,	December 21.
Aquarius, the Water-bearer,	January 20.
Pisces, the Fishes,	February 19.

Each of these signs coincides roughly with a constellation in the heavens; and thus there are twelve constellations called by the names of these signs, but the signs and the constellations no longer correspond. Although the sun now crosses the equator and enters the *sign* Aries on the 20th of March, he does not reach the *constellation* Aries until nearly a month later. This arises from the precession of the equinoxes, to be explained hereafter.

§ 4. OBLIQUITY OF THE ECLIPTIC.

We have already stated that when the sun is at the summer solstice, it is about 23½° north of the equator, and when at the winter solstice, about 23½° south. This shows that the ecliptic and equator make an angle of about 23½° with each other. This angle is called the obliquity of the ecliptic, and its determination is very simple. It is only necessary to find by repeated observation the sun's greatest north declination at the summer solstice, and its greatest south declination at the winter solstice. Either of these declinations, which must be equal if the observations are accurately made, will give the obliquity of the ecliptic. It has been continually diminishing from the earliest ages at a rate of about half a second a year, or, more exactly, about forty-seven seconds in a century. This diminution is due to the gravitating forces of the planets, and will continue for several thousand years to come. It will not, however, go on indefinitely, but the obliquity will only oscillate between comparatively narrow limits.

The relation of the obliquity of the ecliptic to the seasons is quite obvious. When the sun is north of the equator, it culminates at a higher altitude in the northern hemisphere, and more than half of its apparent diurnal course is above the horizon, as explained in the chapter on the celestial sphere. Hence we have the heats of summer. In the southern hemisphere, of course, the case is reversed: when the sun is in north declination, less than half of his diurnal course is above the horizon in that hemisphere. Therefore this situation of the sun corresponds to summer in the northern hemisphere, and winter in the southern one. In exactly the same way, when the sun is far south of the equator, the days are shorter in the northern hemisphere and longer in the southern hemisphere. It is therefore winter in the former and summer in the latter. If the equator and the ecliptic coincided—that is, if the sun moved along the equator—there would be no such thing as a difference of seasons, because the sun would always rise exactly in the east and set exactly in the west, and always culminate at the same altitude. The days would always be twelve hours long the world over. This is the case with the planet *Jupiter*.

In the preceding paragraphs, we have explained the apparent annual circuit of the sun relative to the equator, and shown how the seasons depend upon this circuit. In order that the student may clearly grasp the entire subject, it is necessary to show the relation of these apparent movements to the actual movement of the earth around the sun.

To understand the relation of the equator to the ecliptic, we must remember that the celestial pole and the celestial equator have really no reference whatever to the heavens, but depend solely on the direction of the earth's axis of rotation. The pole of the heavens is nothing more than that point of the celestial sphere toward which the earth's axis points. If the direction of this axis changes, the position of the celestial pole among the stars will change also; though to an observer on the earth, unconscious of the change, it would seem as if the starry sphere moved while the pole remained at rest. Again, the celestial equator being merely the great circle in which the plane of the earth's equator, extended out to infinity in every direction, cuts the celestial sphere, any change in the direction of the pole of the earth necessarily changes the position of the equator among the stars. Now the positions of the celestial pole and the celestial equator among the stars seem to remain unchanged throughout the year. (There is, indeed, a minute change, but it does not affect our present reasoning.) This shows that, as the earth revolves around the sun, its axis is constantly directed toward nearly the same point of the celestial sphere.

§ 5. THE SEASONS.

The conclusions to which we are thus led respecting the real revolution of the earth are shown in Fig. 46. Here S represents the sun, with the orbit of the earth surrounding it, but viewed nearly edgeways so as to be much foreshortened. $A\ B\ C\ D$ are the four cardinal positions of the earth which correspond to the cardinal points of the apparent path of the sun already described. In each figure of the earth $N\,S$ is the axis, N being its north and S its south pole. Since this axis points in the

FIG. 46.—CAUSES OF THE SEASONS.

same direction relative to the stars during an entire year, it follows that the different lines $N\ S$ are all parallel. Again, since the equator does not coincide with the ecliptic, these lines are not perpendicular to the ecliptic, but are inclined from this perpendicular by $23\frac{1}{2}°$.

Now, consider the earth as at A; here it is seen that the sun shines more on the southern hemisphere than on the northern; a region of $23\frac{1}{2}°$ around the north pole is in darkness, while in the corresponding region around the south pole the sun shines all day. The five circles at right angles to the earth's axis are the parallels of latitude around which each region on the surface of the earth is carried by the diurnal rotation of the latter on its axis. It will be seen that in the northern hemisphere less than half of these are illuminated by the sun, and in the southern hemisphere more than half. This corresponds to our winter solstice.

When the earth reaches B, its axis $N\,S$ is at right angles to the line drawn to the sun, so that the latter shines perpendicularly on the equator, the plane of which passes through it. The diurnal circles on the earth are one half illuminated and one half in darkness. This position corresponds to the vernal equinox.

At C the case is exactly the reverse of that at A, the sun shining more on the northern hemisphere than on the southern one. North of the equator more than half the diurnal circles are in the illuminated hemisphere, and south of it less. Here then we have winter in the southern and summer in the northern hemisphere. The sun is above a region 23½° north of the equator, so that this position corresponds to our summer solstice.

At D the earth's axis is once more at right angles to a line drawn to the sun. The latter therefore shines upon the equator, and we have the autumnal equinox.

In whatever position we suppose the earth, the line $S N$, continued indefinitely, meets the celestial sphere at its north pole, while the middle or equatorial circle of the earth, continued indefinitely in every direction, marks out the celestial equator in the heavens. At first sight it might seem that, owing to the motion of the earth through so vast a circuit, the positions of the celestial pole and equator must change in consequence of this motion. We might say that, in reality, the pole of the earth describes a circle in the celestial sphere of the same size as the earth's orbit. But this sphere being infinitely distant, the circle thus described appears to us as a point, and thus the pole of the heavens seems to preserve its position among the stars through the whole course of the year. Again, we may suppose the equator to have a slight annual motion among the stars from the same cause. But for the same reason this motion is nothing when seen from the earth. On the other hand, the slightest change in the *direction* of the axis $S N$ will change the apparent position of the pole among the stars by an angle equal to that change of direction. We may thus consider the position of the celestial pole as independent of the position of the earth in its orbit, and dependent entirely on the direction in which the axis of the earth points.

If this axis were perpendicular to the plane of the ecliptic, it is evident that the sun would always lie in the plane of the equator, and there would be no change of seasons except such slight ones as might result from the small differences in the distance of the earth at different seasons.

§ 6. CELESTIAL LATITUDE AND LONGITUDE.

Besides the circles of reference described in the first chapter, still another system is used in which the ecliptic is taken as the fundamental plane. Since the motion of the earth around the sun takes place, by definition, in the plane of the ecliptic, and the motions of the planets very near that plane, it is frequently more convenient to refer the positions of the planets to the plane of the ecliptic than to that of the equator. The co-ordinates of a heavenly body thus referred are called its celestial *latitude* and *longitude*. To show the relation of these co-ordinates to right ascension and declination, we give a figure showing both co-ordinates at the same time, as marked on the celestial sphere. This figure is supposed to be the celestial sphere, having the solar system in its centre. The direction $P\ Q$ is that of the axis of the earth; $I\ J$ is the ecliptic, or the great circle in which the plane of the earth's orbit intersects the celestial sphere. The point in which these two circles cross is marked 0^h, and is the vernal equinox from which the right ascension and the longitude are both counted.

The horizontal and vertical circles show how right ascension and declination are counted in the manner described in Chapter I. As the right ascension is counted all the way around the equator from 0^h to 24^h, so longitude is counted along the ecliptic from the point 0^h, or the vernal equinox, toward J in degrees. The whole circuit measuring 360°, this distance will carry us all the way round. Thus if a body lies in the ecliptic, its longitude is simply the number of degrees from the vernal equinox to its position, measured in the direction from I toward J. If it does not lie in the ecliptic; if, for instance, it is at the point B, we let fall a perpendicular BJ from the body upon the ecliptic. The length of this perpendicular, measured in degrees, is called the *latitude* of the body, which may be north or south, while the distance of the foot of the perpendicular from the vernal equinox is called its *longitude*.

In astronomy it is usual to represent the positions of the bodies of the solar system, relatively to the sun, by their longitudes and latitudes, because in the ecliptic we have a

FIG. 47.—CIRCLES OF THE SPHERE.

plane more nearly fixed than that of the equator. On the other hand, it is more convenient to represent the position of all the heavenly bodies as seen from the earth by their right ascensions and declinations, because we cannot measure their longitudes and latitudes directly, but can only observe right ascension and declination. If we wish to determine the longitude and latitude of a body as seen from the centre of the earth, we have to first find its right ascension and declination by observation, and then change these quantities to longitude and latitude by trigonometrical formulæ.

CHAPTER IV.

THE PLANETARY MOTIONS.

§ 1. APPARENT AND REAL MOTIONS OF THE PLANETS.

Definitions.—The solar system, as we now know it, comprises so vast a number of bodies of various orders of magnitude and distance, and subjected to so many seemingly complex motions, that we must consider its parts separately. Our attention will therefore, in the present chapter, be particularly directed to the motions of the great planets, which we may consider as forming, in some sort, the fundamental bodies of the system. These bodies may, with respect to their apparent motions, be divided into three classes.

Speaking, for the present, of the sun as a planet, the first class comprises the *sun* and *moon*. We have seen that if, upon a star chart, we mark down the positions of the sun day by day, they will all fall into a regular circle which marks out the ecliptic. The monthly course of the moon is found to be of the same nature, although its motion is by no means uniform in a month, yet it is always toward the east, and always along or very near a certain great circle.

The second class comprises *Venus* and *Mercury*. The peculiarity exhibited by the apparent motion of these bodies is, that it is an oscillating one on each side of the sun. If we watch for the appearance of one of these planets after sunset from evening to evening, we shall find it to appear above the western horizon. Night after night it will be farther and farther from the sun until it attains a certain maximum distance; then it will appear to return to the sun again, and for a while to be lost in its rays. A few days later it will reappear to the west of the sun, and thereafter be visible in the eastern horizon before sunrise. In the case of *Mercury*, the time required for one complete oscillation back and forth is about four months; and in the case of *Venus* more than a year and a half.

The third class comprises *Mars*, *Jupiter*, and *Saturn* as well as a great number of planets not visible to the naked eye. The general or average motion of these planets is toward the east, a complete revolution in the celestial sphere being performed in times ranging from two years in the case of *Mars* to 164 years in that of *Neptune*. But, instead of moving uniformly forward, they seem to have a swinging motion; first, they move forward or toward the east through a pretty long arc, then backward or westward through a short one, then forward through a longer one, etc. It is only by the excess of the longer arcs over the shorter ones that the circuit of the heavens is made.

The general motion of the sun, moon, and planets among the stars being toward the east, the motion in this direction is called *direct;* whereas the occasional short motions toward the west are called *retrograde*. During the periods between direct and retrograde motion, the planets will for a short time appear stationary.

The planets *Venus* and *Mercury* are said to be at greatest *elongation* when at their greatest angular distance from the sun. The elongation which occurs with the planet east of the sun, and therefore visible in the western horizon after sunset, is called the eastern elongation, the other the western one.

A planet is said to be in *conjunction* with the sun when it is in the same direction, or when, as it seems to pass by the sun, it approaches nearest to it. It is said to be in *opposition* to the sun when exactly in the opposite direction—rising when the sun sets, and *vice versa*. If, when a planet is in conjunction, it is between the earth and the sun, the conjunction is said to be an *inferior* one; if beyond the sun, it is said to be *superior*.

Arrangements and Motions of the Planets.—We now know that the sun is the real centre of the solar system, and that the planets proper all revolve around it as the centre of motion. The order of the five innermost large planets, or the relative positions of their orbits, are shown in Fig. 48. These orbits are all nearly, but not exactly,

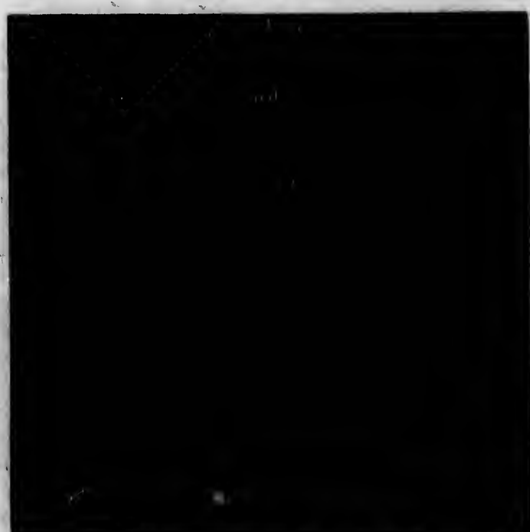

FIG. 48.—ORBITS OF THE PLANETS.

in the same plane. The planets *Mercury* and *Venus* which, as seen from the earth, never appear to recede very far from the sun, are in reality those which revolve inside the orbit of the earth. The planets of the third class, which perform their circuits at all distances from the sun, are what we now call the superior planets, and are more distant from the sun than the earth is. Of these, the orbits of *Mars*, *Jupiter*, and a swarm of telescopic planets are shown in the figure; next outside of *Jupiter* comes *Saturn*, the farthest planet readily visible to the naked eye, and then *Uranus* and *Neptune*, telescopic planets. On the scale of Fig. 48 the orbit of *Neptune* would be more than two feet in diameter. Finally, the moon is a small planet revolving around the earth as its centre, and carried with the latter as it moves around the sun.

Inferior planets are those whose orbits lie inside that of the earth, as *Mercury* and *Venus*.

Superior planets are those whose orbits lie outside that of the earth, as *Mars*, *Jupiter*, *Saturn*, etc.

The farther a planet is situated from the sun, the slower is its orbital motion. Therefore, as we go from the sun, the periods of revolution are longer, for the double reason that the planet has a larger orbit to describe and moves more slowly in its orbit. It is to this slower motion of the outer planets that the occasional apparent retrograde motion of the planets is due, as may be seen by studying Fig. 49. We first remark that the apparent direction of a planet, as seen from the earth, is determined by the line joining the earth and planet. Supposing this line to be continued onward to infinity, so as to intersect the celestial sphere, the apparent motion of the planet will be defined by the motion of the point in which the line intersects the sphere. If this motion is toward the east, it will be direct; if toward the west, retrograde.

Let us first take the case of an inferior planet. Suppose *H I K L M N* to be successive positions of the earth in its orbit, and *A B C D E F* to be corresponding positions of *Venus* or *Mercury*. It must be remembered that when we speak of east and west in this connection, we do not mean an absolute direction in space, but a direction around the sphere. In the figure we are supposed t[illegible]k down upon the planetary orbits from the north, and a direction west is, then, that in which the hands of a watch move, while east is in the opposite direction. When the earth is at H the planet is seen at A. The line HA being supposed tangent to the orbit of the planet, it is evident from geometrical considerations that this is the greatest angle which the planet can ever make with the sun as seen from the earth. This, therefore, corresponds to the greatest eastern elongation.

FIG. 49.

When the earth has reached I the planet is at B, and is therefore near the direction IB. The line has turned in a direction opposite that of the hands of a watch, and cuts the celestial sphere at a point farther east than the line HA did. Hence the motion of the planet during this period has been direct; but the direction of the sun having changed also in consequence of the advance of the earth, the angular distance between the sun and the planet is less than before.

While the earth is passing from I to K, the planet passes from B to C. The distance $B\,C$ exceeds $I\,K$, because the planet moves faster than the earth. The line joining the earth and planet, therefore, cuts the celestial sphere at a point farther west than it did before, and therefore the direction of the apparent motion is retrograde. At C the planet is in inferior conjunction. The retrograde motion still continues until the earth reaches L, and the planet D, when it becomes stationary. Afterward it is direct until the two bodies again come into the relative positions $I\,B$.

Let us next suppose that the inner orbit $A\,B\,C\,D\,E\,F$ represents that of the earth, and the outer one that of a superior planet, *Mars* for instance. We may consider $O\,Q\,P\,R$ to be the celestial sphere, only it should be infinitely distant. While the earth is moving from A to B the planet moves from H to I. This motion is direct, the direction $O\,Q\,P\,R$ being from west to east. While the earth is moving from B to D, the planet is moving from I to L; the former motion being the more rapid, the earth now passes by the planet as it were, and the line conjoining them turns in the same direction as the hands of a watch. Therefore, during this time the planet seems to describe the arc $P\,Q$ in the celestial sphere in the direction opposite to its actual orbital motion. The lines $L\,D$ and $M\,E$ are supposed to be parallel. The planet is then really stationary, even though as drawn it would seem to have a forward motion, owing to the distance of these two lines, yet, on the infinite sphere, this distance appears as a point. From the point M the motion is direct until the two bodies once more reach the relative positions $B\,I$. When the planet is at K and the earth at C, the former is in opposition. Hence the retrograde motion of the superior planets always takes place near opposition.

Theory of Epicycles.—The ancient astronomers represented this oscillating motion of the planets in a way which was in a certain sense correct. The only error they made was, in attributing the oscillation to a motion of the planet instead of a motion of the earth around the sun, which really causes it. But their theory was, notwithstanding, the means of leading COPERNICUS and others to the perception of the true nature of the motion. We allude to the celebrated theory of epicycles, by which the planetary motions were always represented before the time of COPERNICUS. Complicated though these motions were, it was seen by the ancient astronomers that they could be represented by a combination of two motions. First, a small circle or epicycle was supposed to move around the earth with a regular, though not uniform, forward motion, and then the planet was supposed to move around the circumference of this circle. The relation of this theory to the true one was this. The regular forward motion of the epicycle represents the real motion of the planet around the sun, while the motion of the planet around the circumference of the epicycle is an apparent one arising from the revolution of the earth around the sun. To explain this we must understand some of the laws of relative motion.

It is familiarly known that if an observer in unconscious motion looks upon an object at rest, the object will appear to him to move in a direction opposite that in which he moves. As a result of this law, if the observer is unconsciously describing a circle, an object at rest will appear to him to describe a circle of equal size. This is shown by the following figure. Let S represent the sun, and $A\ B\ C\ D\ E\ F$ the orbit of the earth. Let us suppose the observer on the earth carried around in this orbit, but imagining himself at rest at S, the centre of motion. Suppose he keeps observing the direction and distance of the planet P, which for the present we suppose to be at rest, since it is only the apparent motion that we shall have to consider. When the observer is at A he really sees the planet in a direction and distance $A\ P$, but imagining himself at S he thinks he sees the planet at the point a determined by drawing a line $S\,a$ parallel and equal to $A\,P$. As he passes from A to B the planet will seem to him to move in the opposite direction from A to b, the point b being determined by drawing $S\,b$ equal and parallel to $B\,P$. As he recedes from the planet through the arc $B\,C\,D$, the planet seems to recede from him through $b\,c\,d$; and while he moves from left to right through $D\,E$ the planet seems to move from right to left through $D\,E$. Finally, as he approaches the planet through the arc $E\,F\,A$ the planet seems to approach him through $E\,F\,A$, and when he returns to A the planet will appear at A, as in the beginning. Thus the planet, though really at rest, will seem to him to move over the circle $a\,b\,c\,d\,e\,f$ corresponding to that in which the observer himself is carried around the sun.

FIG. 50.

The planet being really in motion, it is evident that the combined effect of the real motion of the planet and the apparent motion around the circle $a\,b\,c\,d\,e\,f$ will be represented by carrying the centre of this circle P along the true orbit of the planet. The motion of the earth being more rapid than that of an outer planet, it follows that the apparent motion of the planet through $a\,b$ is more rapid than the real motion of P along the orbit. Hence in this part of the orbit the movement of the planet will be retrograde. In every other part it will be direct, because the progressive motion of P will at least overcome, sometimes be added to, the apparent motion around the circle.

In the ancient astronomy the apparent small circle $a\,b\,c\,d\,e\,f$ was called the *epicycle*.

In the case of the inner planets *Mercury* and *Venus* the relation of the epicycle to the true orbit is reversed. Here the epicyclic motion is that of the planet around its real orbit—that is, the true orbit of the planet around the sun was itself taken for the epicycle, while the forward motion was really due to the apparent revolution of the sun produced by the annual motion of the earth.

In the preceding descriptions of the planetary motions we have spoken of them all as circular. But it was found by HIPPARCHUS * that none of the planetary motions were really uniform. Studying the motion of the sun in order to determine the length of the year, he observed the times of its passage through the equinoxes and solstices with all the accuracy which his instruments permitted. He found that it was several days longer in passing through one half of its course than through the other. This was apparently incompatible with the favorite theory of the ancients that all the celestial motions were circular and uniform. It was, however, accounted for by supposing that the earth was not in the centre of the circle around which the sun moved, but a little to one side. Thus arose the celebrated theory of the *eccentric*. Careful observations of the planets showed that they also had similar inequalities of motion. The centre of the epicycle around which the real planet was carried was found to move more rapidly in one part of the orbit, and more slowly in the opposite part. Thus the circles in which the planets were supposed to move were not truly centred upon the earth. They were therefore called *eccentrics*.

This theory accounted in a rough way for the observed inequalities. It is evident that if the earth was supposed to be displaced toward one side of the orbit of the planet,

* HIPPARCHUS was one of the most celebrated astronomers of antiquity, being frequently spoken of as the father of the science. He is supposed to have made most of his observations at Rhodes, and flourished about one hundred and fifty years before the Christian era.

the latter would seem to move more rapidly when nearest the earth than when farther from it. It was not until the time of KEPLER that the eccentric was shown to be incapable of accounting for the real motion; and it is his discoveries which we are next to describe.

§ 2. KEPLER'S LAWS OF PLANETARY MOTION.

The direction of the sun, or its longitude, can be determined from day to day by direct observation. If we could also observe its distance on each day, we should, by laying down the distances and directions on a large piece of paper, through a whole year, be able to trace the curve which the earth describes in its annual course, this course being, as already shown, the counterpart of the apparent one of the sun. A rough determination of the relative distances of the sun at different times of the year may be made by measuring the sun's apparent angular diameter, because this diameter varies inversely as the distance of the object observed. Such measures would show that the diameter was at a maximum of 32′ 36″ on January 1st, and at a minimum of 31′ 32″ on July 1st of every year. The difference, 64″, is, in round numbers, $\frac{1}{30}$ the mean diameter—that is, the earth is nearer the sun on January 1st than on July 1st by about $\frac{1}{30}$. We may consider it as $\frac{1}{60}$ greater than the mean on the one date, and $\frac{1}{60}$ less on the other. This is therefore the actual displacement of the sun from the centre of the earth's orbit.

Again, observations of the apparent daily motion of the sun among the stars, corresponding to the real daily motion of the earth round the sun, show this motion to be least about July 1st, when it amounts to 57′ 12″ = 3432″, and greatest about January 1st, when it amounts to 61′ 11″ = 3671″. The difference, 239″, is, in round numbers, $\frac{1}{15}$ the mean motion, so that the range of variation is, in proportion to the mean, double what it is in the case of the distances. If the actual velocity of the earth in its orbit were uniform, the apparent angular motion round the sun would be inversely as its distance from the sun. Actually, however, the angular motion, as given above, is inversely as the square of the distance from the sun, because $(1 + \frac{1}{30})^2 = 1 + \frac{1}{15}$ very nearly. The actual velocity of the earth is therefore greater the nearer it is to the sun.

On the ancient theory of the eccentric circle, as propounded by HIPPARCHUS, the actual motion of the earth was supposed to be uniform, and it was necessary to suppose the displacement of the sun (or, on the ancient theory, of the earth) from the centre to be $\frac{1}{15}$ its mean distance, in order to account for the observed changes in the motion in longitude. We now know that, in round numbers, one half the inequality of the apparent motion of the sun in longitude arises from the variations in the distance of the earth from it, and one half from the earth's actually moving with a greater velocity as it comes nearer the sun. By attributing the whole inequality to a variation of distance, the ancient astronomers made the eccentricity of the orbit—that is, the distance of the sun from the geometrical centre of the orbit (or, as they supposed, the distance of the earth from the centre of the sun's orbit)—twice as great as it really was.

An immediate consequence of these facts of observation is KEPLER's second law of planetary motion, that the *radii vectores* drawn from the sun to a planet revolving round it, sweep over equal areas in equal times. Suppose, in Fig. 51, that S represents the position of the sun, and that the earth, or a planet, in a unit of time, say a day or a week, moves from P_3 to P_2. At another part of its orbit it moves from P to P_1 in the same time, and at a third part from P_4 to P_5. Then the areas SP_3P_2, SPP_1, SP_4P_5 will all be equal. A little geometrical consideration will, in fact, make it clear that the areas of the triangles are equal when the angles at S are inversely as the square of the radii vectores, SP, etc., since the expression for the area of a triangle in which the angle at S is very small is $\frac{1}{2}$ angle $S \times S P^2$.*

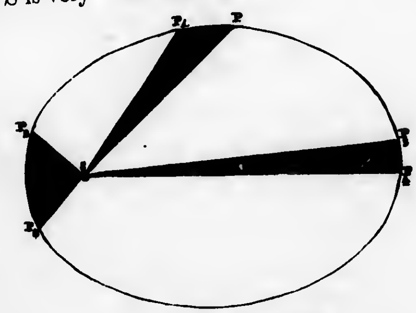

FIG. 51.—LAW OF AREAS.

In the time of KEPLER the means of measuring the sun's angular diameter were so imperfect that the preceding method of determining the path of the earth around the sun could not be applied. It was by a study of the motions of the planet *Mars*, as observed by TYCHO BRAHE, that KEPLER was led to his celebrated laws of planetary motion. He found that no possible motion of *Mars* in a truly circular orbit, however eccentric, would represent the observations. After long and laborious calculations, and the trial and rejection of a great number of hypotheses, he was led to the conclusion that the planet *Mars* moved in an ellipse, having the sun in one focus. As the analogies of nature led to the inference that all the planets, the earth included, moved in curves of the same class, and according to the same law, he was led to enunciate the first two of his celebrated laws of planetary motion, which were as follow:

* More exactly if we consider the arc PP_1 as a straight line, the area of the triangle PP_1S will be equal to $\frac{1}{2} SP \times SP_1 \times \sin$ angle S. But in considering only very small angles we may suppose $SP = SP_1$ and the sine of the angle S equal to the angle itself. This supposition will give the area mentioned above.

I. *Each planet moves around the sun in an ellipse, having the sun in one of its foci.*

II. *The radius vector joining each planet with the sun, moves over equal areas in equal times.*

To these he afterward added another showing the relation between the times of revolution of the separate planets.

III. *The square of the time of revolution of each planet is proportional to the cube of its mean distance from the sun.*

These three laws comprise a complete theory of planetary motion, so far as the main features of the motion are concerned. There are, indeed, small variations from these laws of KEPLER, but the laws are so nearly correct that they are always employed by astronomers as the basis of their theories.

Mathematical Theory of the Elliptic Motion.— The laws of KEPLER lead to problems of such mathematical elegance that we give a brief synopsis of the most important elements of the theory. A knowledge of the elements of analytic geometry is necessary to understand it.

Let us put:

a, the semi-major axis of the ellipse in which the planet moves. In the figure, if C is the centre of the ellipse, and S the focus in which the sun is situated, then $a = A\,C = C\,\pi$.

e, the eccentricity of the ellipse $= \dfrac{CS}{a}$.

π, the longitude of the perihelion, represented by the angle $\pi\,S\,E$, E being the direction of the vernal equinox from which longitudes are counted.

n, the mean angular motion of the planet round the sun in a unit of time. The actual motion being variable, the mean motion is found by dividing the circumference $= 360°$ by the time of revolution.

FIG. 58.

T, the time of revolution.

r, the distance of the planet from the sun, or its radius vector, a variable quantity.

I. The first remark we have to make is that the *ellipticities* of the planetary orbits—that is, the proportions in which the orbits are flattened—is much less than their *eccentricities*. By the properties of the ellipse we have :

$$SB = \text{semi-major axis} = a,$$
$$BC = \text{semi-minor axis} = a\sqrt{1-e^2},$$
$$\text{or, } BC = a(1 - \tfrac{1}{2}e^2) \text{ nearly, when } e \text{ is very small.}$$

The most eccentric of the orbits of the eight major planets is that of *Mercury*, for which $e = 0.2$. Hence for *Mercury*

$$BC = a(1 - \tfrac{1}{50})$$

very nearly, so that flattening of the orbit is only about $\frac{1}{50}$ or .02 of the major axis.

The next most eccentric orbit is that of *Mars* for which $e = .093$; $BC = a(1 - .0043)$, so that the flattening of the orbit is only about $\frac{1}{230}$.

We see from this that the hypothesis of the eccentric circle makes a very close approximation to the true form of the planetary orbits. It is only necessary to suppose the sun removed from the centre of the orbit by a quantity equal to the product of the eccentricity into the radius of the orbit to have a nearly true representation of the orbit and of the position of the sun.

II. The least distance of the planet from the sun is

$$S\pi = a(1 - e),$$

and the greatest distance is

$$AS = a(1 + e).$$

III. The angular velocity of the planet around the sun at any point of the orbit, which we may call S, is, by the second law of KEPLER :

$$S = \frac{C}{r^2},$$

C being a constant to be determined. To determine it we remark that S is the angle through which the planet moves in a unit of time. If we suppose this unit to be very small, the quantity Sr^2 is double the area of the very small triangle swept over by the radius vector during such unit. This area is called the *areolar velocity* of the planet, and is a constant, by KEPLER's second law. Therefore, in the last equation, $C = Sr^2$ represents the double of the areolar velocity of the planet. When the planet completes an entire revolution, the radius vector has swept over the whole area of the ellipse which is $\pi a^2 \sqrt{1-e^2}$.* The time required to do this be-

* In this formula π represents the ratio of the circumference of the circle to its diameter.

ing called T, the area swept over with the areolar velocity $\frac{1}{2} C$ is also $\frac{1}{2} C\, T$. Therefore

$$\tfrac{1}{2}\, C\, T = \pi\, a^2 \sqrt{1 - e^2};$$

$$C = \frac{2\,\pi\, a^2 \sqrt{1 - e^2}}{T}.$$

The quantity $2\,\pi$ here represents 360°, or the whole circumference, which, being divided by T, the time of describing it will give the mean angular velocity of the planet around the sun which we have called n. Therefore

$$n = \frac{2\,\pi}{T},$$

and

$$C = a^2\, n \sqrt{1 - e^2}.$$

This value of C being substituted in the expression for S, we have

$$S = \frac{a^2\, n \sqrt{1 - e^2}}{r^2}.$$

IV. By KEPLER'S third law T^2 is proportioned to a^3; that is, $\frac{T^2}{a^3}$ is a constant for all the planets. The numerical value of this constant will depend upon the quantities which we adopt as the units of time and distance. If we take the year as the unit of time and the mean distance of the earth from the sun as that of distance, T and a for the earth will both be unity, and the ratio $\frac{T^2}{a^3}$ will therefore be unity for all the planets. Therefore

$$a^3 = T^2;\ a = T^{\frac{2}{3}}.$$

Therefore *if we square the period of revolution of any planet in years, and extract the cube root of the square, we shall have its mean distance from the sun in units of the earth's distance.*

It is thus that the mean distances of the planets are determined in practice, because, by a long series of observations, the times of revolution of the planets have been determined with very great precision.

V. To find the position of a planet we must know the epoch at which it passed its perihelion, or some equivalent quantity. To find its position at any other time let τ be the time which has elapsed since passing the perihelion. Then, by the law of areas, if P be the position of the planet at this time we shall have

$$\frac{\text{Area of sector } P\,S\,\pi}{\text{Area of whole ellipse}} = \frac{\tau}{T} \qquad (1).$$

The times τ and T being both given, the problem is reduced to that of cutting a given area of the ellipse by a line drawn from the focus to some point of its circumference to be found. This is known as KEPLER's problem, and may be solved by analytic geom-

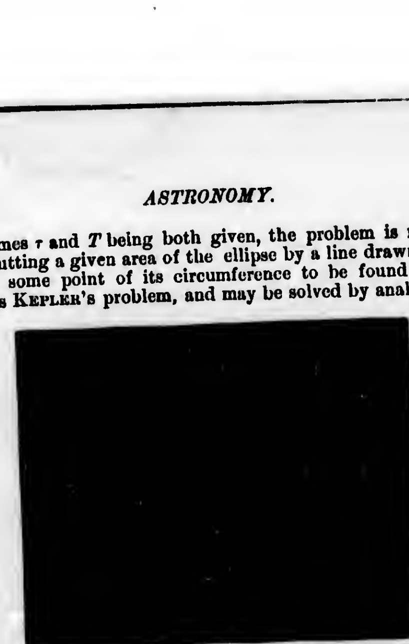

FIG. 53.

etry as follows: Let $A\,B$ be the major axis of the ellipse, P the position of the planet, and S that of the focus in which the sun is situated. On $A\,B$ as a diameter describe a circle, and through P draw the right line $P'\,P\,D$ perpendicular to $A\,B$.

The area of the elliptic sector $S\,P\,B$, over which the radius vector of the planet has swept since the planet passed the perihelion at B, is equal to the sector $C\,P\,B$ minus the triangle $C\,P\,S$. Since an ellipse is formed from a circle by shortening all the ordinates in the same ratio (namely, the ratio of the minor axis b to the major axis a), it follows that the elliptic sector $C\,P\,B$ may be formed from the circular sector $C\,P'\,B$ by shortening all the ordinates in the ratio of $D\,P$ to $D\,P'$, or of a to b. Hence,

$$\text{Area } C P B : \text{area } C P' B = b : a.$$

But area $C\,P'\,B$ = angle $P'\,C\,B \times \frac{1}{2}a^2$, taking the unit radius as the unit of angular measure. Hence, putting u for the angle $P'\,C\,B$ we have

$$\text{Area } C P B = \frac{b}{a} \text{ area } C P' B = \tfrac{1}{2}\,a\,b\,u \qquad (2).$$

Again, the area of the triangle $C\,P\,S$ is equal to $\frac{1}{2}$ base $C\,S \times$ altitude $P\,D$. Also $P\,D = \frac{b}{a}\,P'\,D$, and $P'\,D = C\,P' \sin u = a \sin u$. Wherefore,

$$P\,D = b \sin u \qquad (3).$$

By the first principles of conic sections, CS, the base of the triangle, is equal to ae. Hence

$$\text{Area } CPS = \tfrac{1}{2} a b e \sin u,$$

and, from (2) and (3),

$$\text{Area } SPB = \tfrac{1}{2} a b (u - e \sin u).$$

Substituting in equation (1) this value of the sector area, and $\pi a b$ for the area of the ellipse, we have

$$\frac{u - e \sin u}{2\pi} = \frac{\tau}{T},$$

or,

$$u - e \sin u = 2\pi \frac{\tau}{T}.$$

From this equation the unknown angle u is to be found. The equation being a transcendental one, this cannot be done directly, but it may be rapidly done by successive approximation, or the value of u may be developed in an infinite series.

Next we wish to express the position of the planet, which is given by its radius vector SP and the angle BSP which this radius vector makes with the major axis of the orbit. Let us put

r, the radius vector SP,
f, the angle BSP, called the *true anomaly*.

Then

$$r \sin f = PD = b \sin u \text{ (Equation 3)},$$

$$r \cos f = SD = CD - CS = CP' \cos u - ae = a(\cos u - e),$$

from which r and f can both be determined. By taking the square root of the sums of the squares, they give, by suitable reduction and putting $b^2 = a^2(1 - e^2)$,

$$r = a(1 - e \cos u),$$

and, by dividing the first by the second,

$$\tan f = \frac{b \sin u}{a(\cos u - e)} = \frac{\sqrt{1 - e^2} \sin u}{\cos u - e}.$$

Putting, as before, π for the longitude of the perihelion, the true longitude of the planet in its orbit will be $f + \pi$.

VI. To find the position of the planet relatively to the ecliptic, the inclination of the orbit to the ecliptic has to be taken into account. The orbits of the several large planets do not lie in the same plane, but are inclined to each other, and to the ecliptic, by various small angles. A table giving the values of these angles will be given hereafter, from which it will be seen that the orbit of *Mercury* has the greatest inclination, amounting to 7°, and that of *Uranus* the least, being only 46′. The reduction of the position of the planet to the ecliptic is a problem of spherical trigonometry, the solution of which need not be discussed here.

CHAPTER V.

UNIVERSAL GRAVITATION.

§ 1. NEWTON'S LAWS OF MOTION.

The establishment of the theory of universal gravitation furnishes one of the best examples of scientific method which is to be found. We shall describe its leading features, less for the purpose of making known to the reader the technical nature of the process than for illustrating the true theory of scientific investigation, and showing that such investigation has for its object the discovery of what we may call generalized facts. The real test of progress is found in our constantly increased ability to foresee either the course of nature or the effects of any accidental or artificial combination of causes. So long as prediction is not possible, the desires of the investigator remain unsatisfied. When certainty of prediction is once attained, and the laws on which the prediction is founded are stated in their simplest form, the work of science is complete.

The whole process of scientific generalization consists in grouping facts, new and old, under such general laws that they are seen to be the result of those laws, combined with those relations in space and time which we may suppose to exist among the material objects investigated. It is essential to such generalization that a single law shall suffice for grouping and predicting several distinct facts. A law invented simply to account for an isolated fact, however general, cannot be regarded in science as a law of nature. It may, indeed, be true, but its truth cannot be proved until it is shown that several distinct facts can be accounted for by it better than by any other law. The reader will call to mind the old fable which represented the earth as supported on the back of a tortoise, but totally forgot that the support of the tortoise needed to be accounted for as much as that of the earth.

To the pre-Newtonian astronomers, the phenomena of the geometrical laws of planetary motion, which we have just described, formed a group of facts having no connection with any thing on the earth. The epicycles of HIPPARCHUS and PTOLEMY were a truly scientific conception, in that they explained the seemingly erratic motions of the planets by a single simple law. In the heliocentric theory of COPERNICUS this law was still further simplified by dispensing in great part with the epicycle, and replacing the latter by a motion of the earth around the sun, of the same nature with the motions of the planets. But COPERNICUS had no way of accounting for, or even of describing with rigorous accuracy, the small deviations in the motions of the planets around the sun. In this respect he made no real advance upon the ideas of the ancients.

KEPLER, in his discoveries, made a great advance in representing the motions of all the planets by a single set of simple and easily understood geometrical laws. Had the planets followed his laws exactly, the theory of planetary motion would have been substantially complete. Still, further progress was desired for two reasons. In the first place, the laws of KEPLER did not perfectly represent all the planetary motions. When observations of the greatest accuracy were made, it was found that the planets deviated by small amounts from the ellipse of KEPLER. Some small emendations to the motions computed on the elliptic theory were therefore necessary. Had this requirement been fulfilled, still another step would have been desirable—namely, that of connecting the motions of the planets with motion upon the earth, and reducing them to the same laws.

Notwithstanding the great step which KEPLER made in describing the celestial motions, he unveiled none of the great mystery in which they were enshrouded. This mystery was then, to all appearance, impenetrable, because not the slightest likeness could be perceived between the celestial motions and motions on the surface of the earth. The difficulty was recognized by the older philosophers in the division of motions into "forced" and "natural." The latter, they conceived, went on perpetually from the very nature of things, while the former always tended to cease. So when KEPLER said that observation showed the law of planetary motion to be that around the circumference of an ellipse, as asserted in his law, he said all that it seemed possible to learn, supposing the statement perfectly exact. And it was all that could be learned from the mere study of the planetary motions. In order to connect these motions with those on the earth, the next step was to study the laws of force and motion here around us. Singular though it may appear, the ideas of the ancients on this subject were far more erroneous than their conceptions of the motions of the planets. We might almost say that before the time of GALILEO scarcely a single correct idea of the laws of motion was generally entertained by men of learning. There were, indeed, one or two who in this respect were far ahead of their age. LEONARDO DA VINCI, the celebrated painter, was noted in this respect. But the correct ideas entertained by him did not seem to make any headway in the world until the early part of the seventeenth century. Among those who, before the time of NEWTON, prepared the way for the theory in question, GALILEO, HUYGHENS, and HOOKE are entitled to especial mention. As, however, we cannot develop the history of this subject, we must pass at once to the general laws of motion laid down by NEWTON. These were three in number.

Law First: *Every body preserves its state of rest or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*

It was formerly supposed that a body acted on by no force tended to come to rest. Here lay one of the greatest difficulties which the predecessors of NEWTON found, in accounting for the motion of the planets. The idea that the sun in some way caused these motions was entertained from the earliest times. Even PTOLEMY had a vague idea of a force which was always directed toward the centre of the earth, or, which was to him the same thing, toward the centre of the universe, and which not only caused heavy bodies to fall, but bound the whole universe together. KEPLER, again, distinctly affirms the existence of a gravitating force by which the sun acts on the planets; but he supposed that the sun must also exercise an impulsive forward force to keep the planets in motion. The reason of this incorrect idea was, of course, that all bodies in motion on the surface of the earth had practically come to rest. But what was not clearly seen before the time of NEWTON, or at least before GALILEO, was, that this arose from the inevitable resisting forces which act upon all moving bodies around us.

Law Second: *The alteration of motion is ever proportional to the moving force impressed, and is made in the direction of the right line in which that force acts.*

The first law might be considered as a particular case of this second one arising when the force is supposed to vanish. The accuracy of both laws can be proved only by very carefully conducted experiments. They are now considered as mathematically proved.

Law Third: *To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and in opposite directions.*

That is, if a body A acts in any way upon a body B, B will exert a force exactly equal on A in the opposite direction.

These laws once established, it became possible to calculate the motion of any body or system of bodies when once the forces which act on them were known, and, *vice versa*, to define what forces were requisite to produce any given motion. The question which presented itself to the mind of NEWTON and his contemporaries was this: *Under what law of force will planets move round the sun in accordance with* KEPLER'S *laws?*

The laws of central forces had been discovered by HUYGHENS some time before NEWTON commenced his researches, and there was one result of them which, taken in connection with KEPLER'S third law of motion, was so obvious that no mathematician could have had much difficulty in perceiving it. Supposing a body to move around in a circle, and putting R the radius of the circle, T the period of revolution, HUYGHENS showed that the centrifugal force of the body, or, which is the same thing, the attractive force toward the centre which would keep it in the circle, was proportional to $\frac{R}{T^2}$. But by KEPLER'S third law T^2 is proportional to R^3. Therefore this centripetal force is proportional to $\frac{R}{R^3}$, that is, to $\frac{1}{R^2}$. Thus it followed immediately from KEPLER'S third law, that the central force which would keep the planets in their orbits was inversely as the square of the distance from the sun, supposing each orbit to be circular. The first law of motion once completely understood, it was evident that the planet needed no force impelling it forward to keep up its motion, but that, once started, it would keep on forever.

The next step was to solve the problem, what law of force will make a planet describe an ellipse around the sun, having the latter in one of its foci? Or, supposing a planet to move round the sun, the latter attracting it with a force inversely as the square of the distance; what will be the form of the orbit of the planet if it is not circular? A solution of either of these problems was beyond the power of mathematicians before the time of NEWTON; and it thus remained uncertain whether the planets moving under the influence of the sun's gravitation would or would not describe ellipses. Unable, at first, to reach a satisfactory solution, NEWTON attacked the problem in another direction, starting from the gravitation, not of the sun, but of the earth, as explained in the following section.

§ 2. GRAVITATION IN THE HEAVENS.

The reader is probably familiar with the story of NEWTON and the falling apple. Although it has no authoritative foundation, it is strikingly illustrative of the method by which NEWTON first reached a solution of the problem. The course of reasoning by which he ascended from gravitation on the earth to the celestial motions was as follows: We see that there is a force acting all over the earth by which all bodies are drawn toward its centre. This force is familiar to every one from his infancy, and is properly called gravitation. It extends without sensible diminution to the tops not only of the highest buildings, but of the highest mountains. How much higher does it extend? Why should it not extend to the moon? If it does, the moon would tend to drop toward the earth, just as a stone thrown from the hand drops. As the moon moves round the earth in her monthly course, there must be some force drawing her toward the earth; else, by the first law of motion, she would fly entirely away in a straight line. Why should not the force which makes the apple fall be the same force which keeps her in her orbit? To answer this question, it was not only necessary to calculate the intensity of the force which would keep the moon herself in her orbit, but to compare it with the intensity of gravity at the earth's surface. It had long been known that the distance of the moon was about sixty radii of the earth. If this force diminished as the inverse square of the distance, then, at the moon, it would be only $\frac{1}{3600}$ as great as at the surface of the earth. On the earth a body falls sixteen feet in a second. If, then, the theory of gravitation were correct, the moon ought to fall toward the earth $\frac{1}{3600}$ of this amount, or about $\frac{1}{19}$ of an inch in a second. The moon being in motion, if we imagine it moving in a straight line at the beginning of any second, it ought to be drawn away from that line $\frac{1}{19}$ of an inch at the end of the second. When the calculation was made with the correct distance of the moon, it was found to agree exactly with this result of theory. Thus it was shown that the force which holds the moon in her orbit is the same which makes the stone fall, only diminished as the inverse square of the distance from the centre of the earth.*

As it appeared that the central forces, both toward the sun and toward the earth, varied inversely as the squares of the distances, NEWTON proceeded to attack the mathematical problems involved in a more systematic way than any of his predecessors had done. KEPLER'S second law showed that the line drawn from the planet to the sun will describe equal areas in equal times. NEWTON showed that this could not be true, unless the force which held the planet was directed toward the sun. We have already stated that the third law showed that the force was inversely as the square of the distance, and thus agreed exactly with the theory of gravitation. It only remained to

* It is a remarkable fact in the history of science that NEWTON would have reached this result twenty years sooner than he did, had he not been misled by adopting an erroneous value of the earth's diameter. His first attempt to compute the earth's gravitation at the distance of the moon was made in 1665, when he was only twenty-three years of age. At that time he supposed that a degree on the earth's surface was sixty statute miles, and was in consequence led to erroneous results by supposing the earth to be smaller and the moon nearer than they really were. He therefore did not make public his ideas; but twenty years later he learned from the measures of PICARD in France what the true diameter of the earth was, when he repeated his calculation with entire success.

consider the results of the first law, that of the elliptic motion. After long and laborious efforts, NEWTON was enabled to demonstrate rigorously that this law also resulted from the law of the inverse square, and could result from no other. Thus all mystery disappeared from the celestial motions; and planets were shown to be simply heavy bodies moving according to the same laws that were acting here around us, only under very different circumstances. All three of KEPLER's laws were embraced in the single law of gravitation toward the sun. The sun attracts the planets as the earth attracts bodies here around us.

Mutual Action of the Planets.—It remained to extend and prove the theory by considering the attractions of the planets themselves. By NEWTON's third law of motion, each planet must attract the sun with a force equal to that which the sun exerts upon the planet. The moon also must attract the earth as much as the earth attracts the moon. Such being the case, it must be highly probable that the planets attract each other. If so, KEPLER's laws can only be an approximation to the truth. The sun, being immensely more massive than any of the planets, overpowers their attraction upon each other, and makes the law of elliptic motion very nearly true. But still the comparatively small attraction of the planets must cause some deviations. Now, deviations from the pure elliptic motion were known to exist in the case of several of the planets, notably in that of the moon, which, if gravitation were universal, must move under the influence of the combined attraction of the earth and of the sun. NEWTON, therefore, attacked the complicated problem of the determination of the motion of the moon under the combined action of these two forces. He showed in a general way that its deviations would be of the same nature as those shown by observation. But the complete solution of the problem, which required the answer to be expressed in numbers, was beyond his power.

Gravitation Resides in each Particle of Matter.—Still another question arose. Were these mutually attractive forces resident in the centres of the several bodies attracted, or in each particle of the matter composing them? NEWTON showed that the latter must be the case, because the smallest bodies, as well as the largest, tended to fall toward the earth, thus showing an equal gravitation in every separate part. The question then arose: what would be the action of the earth upon a body if the body was attracted—not toward the centre of the earth alone, but toward every particle of matter in the earth? It was shown by a quite simple mathematical demonstration that if a planet were on the surface of the earth or outside of it, it would be attracted with the same force as if the whole mass of the earth were concentrated in its centre. Putting together the various results thus arrived at, NEWTON was able to formulate his great law of universal gravitation in these comprehensive words: "*Every particle of matter in the universe attracts every other particle with a force directly as the masses of the two particles, and inversely as the square of the distance which separates them.*"

To show the nature of the attractive forces among these various particles, let us represent by m and m' the masses of two attracting bodies. We may conceive the body m to be composed of m particles, and the other body to be composed of m' particles. Let us conceive that each particle of the one body attracts each particle of the other with a force $\frac{1}{r^2}$. Then every particle of m will be attracted by each of the m' particles of the other, and therefore the total attractive force on each of these m particles will be $\frac{m'}{r^2}$. Each of the m particles being equally subject to this attraction, the total attractive force between the two bodies will be $\frac{m\,m'}{r^2}$. When a given force acts upon a body, it will produce less motion the larger the body is, the *accelerating* force being proportional to the total attracting force divided by the mass of the body moved. Therefore the accelerating force which acts on the body m', and which determines the amount of motion, will be $\frac{m}{r^2}$; and conversely the accelerating force acting on the body m will be represented by the fraction $\frac{m'}{r^2}$.

§ 3. PROBLEMS OF GRAVITATION.

The problem solved by NEWTON, considered in its greatest generality, was this: Two bodies of which the masses are given are projected into space, in certain directions, and with certain velocities. What will be their motion under the influence of their mutual gravitation? If their relative motion does not exceed a certain definite amount, they will each revolve around their common centre of gravity in an ellipse, as in the case of planetary motions. If, however, the relative velocity exceeds a certain limit, the two bodies will separate forever, each describing around the common centre of gravity a curve having infinite branches. These curves are found to be parabolas in the case where the velocity is exactly at the limit, and hyperbolas when the velocity exceeds it. Whatever curves may be described, the common centre of gravity of the two bodies will be in the focus of the curve. Thus, when restricted to two bodies, the problem admits of a perfectly rigorous mathematical solution.

Having succeeded in solving the problem of planetary motion for the case of two bodies, NEWTON and his contemporaries very naturally desired to effect a similar solution for the case of three bodies. The problem of motion in our solar system is that of the mutual action of a great number of bodies; and having succeeded in the case of two bodies, it was necessary next to try that of three.

Thus arose the celebrated problem of three bodies. It is found that no rigorous and general solution of this problem is possible. The curves described by the several bodies would, in general, be so complex as to defy mathematical definition. But in the special case of motions in the solar system, the problem admits of being solved by approximation with any required degree of accuracy. The principles involved in this system of approximation may be compared to those involved in extracting the square root of any number which is not an exact square ; 2 for instance. The square root of 2 cannot be exactly expressed either by a decimal or vulgar fraction ; but by increasing the number of figures it can be expressed to any required limit of approximation. Thus, the vulgar fractions $\frac{3}{2}$, $\frac{17}{12}$, $\frac{577}{408}$, etc., are fractions which approach more and more to the required quantity ; and by using larger numbers the errors of such fraction may be made as small as we please. So, in using decimals, we diminish the error by one tenth for every decimal we add, but never reduce it to zero. A process of the same nature, but immensely more complicated, has to be used in computing the motions of the planets from their mutual gravitation. The possibility of such an approximation arises from the fact that the planetary orbits are nearly circular, and that their masses are very small compared with that of the sun. The first approximation is that of motion in an ellipse. In this way the motion of a planet through several revolutions can nearly always be predicted within a small fraction of a degree, though it may wander widely in the course of centuries. Then suppose each planet to move in a known ellipse ; their mutual attraction at each point of their respective orbits can be expressed by algebraic formulæ. In constructing these formulæ, the orbits are first supposed to be circular ; and afterward account is taken by several successive steps of the eccentricity. Having thus found approximately their action on each other, the deviations from the pure elliptic motion produced by this action may be approximately calculated. This being done, the motions will be more exactly determined, and the mutual action can be more exactly calculated. Thus, the process can be carried on step by step to any degree of precision; but an enormous amount of calculation is necessary to satisfy the requirements of modern times with respect to precision.* As a general rule, every successive step in the approximation is much more laborious than all the preceding ones.

To understand the principle of astronomical investigation into the motion of the planets, the distinction between observed and theoretical motions must be borne in mind. When the astronomer with his meridian circle determines the position of a planet on the celestial sphere, that position is an observed one. When he calculates it, for the same instant, from theory, or from tables founded on the theory, the result will be a calculated or theoretical position. The two are to be regarded as separate, no matter if they should be exactly the same in reality, because they have an entirely different origin. But it must be remembered that no position can be calculated from theory alone independent of observation, because all sound theory requires some data to start with, which observation alone can furnish. In the case of planetary motions, these data are the elements of the planetary orbit already described, or, which amounts to the same thing, the velocity and direction of the motion of the planet as well as its mass at some given time. If these quantities were once given with mathematical precision, it would be possible, from the theory of gravitation alone, without recourse to observation, to predict the motions of the planets day by day and generation after generation with any required degree of precision, always supposing that they are subjected to no influence except their mutual gravitation according to the law of NEWTON. But it is impossible to determine the elements or the velocities without recourse to observation;

* In the works of the great mathematicians on this subject, algebraic formulæ extending through many pages are sometimes given.

and however correctly they may seemingly be determined for the time being, subsequent observations always show them to have been more or less in error. The reader must understand that no astronomical observation can be mathematically exact. Both the instruments and the observer are subjected to influences which prevent more than an approximation being attained from any one observation. The great art of the astronomer consists in so treating and combining his observations as to eliminate their errors, and get a result as near the truth as possible.

When, by thus combining his observations, the astronomer has obtained the elements of the planet's motion which he considers to be near the truth, he calculates from them a series of positions of the planet from day to day in the future, to be compared with subsequent observations. If he desires his work to be more permanent in its nature, he may construct tables by which the position can be determined at any future time. Having thus a series of theoretical or calculated places of the planet, he, or others, will compare his predictions with observation, and from the differences deduce corrections to his elements. We may say in a rough way that if a planet has been observed through a certain number of years, it is possible to calculate its place for an equal number of years in advance with some approach to precision. Accurate observations are commonly supposed to commence with BRADLEY, Astronomer Royal of England in 1750. A century and a quarter having elapsed since that time, it is now possible to construct tables of the planets, which we may expect to be tolerably accurate, until the year 2000. But this is a possibility rather than a reality. The amount of calculation required for such work is so immense as to be entirely beyond the power of any one person, and hence it is only when a mathematician is able to command the services of others, or when several mathematicians in some way combine for an object, that the best astronomical tables can hereafter be constructed.

§ 4. RESULTS OF GRAVITATION.

From what we have said, it will be seen that the problem of the motions of the planets under the influence of gravitation has called forth all the skill of the mathematicians who have attacked it. They actually find themselves able to reach a solution, which, so far as the mathematics of the subject are concerned, may be true for many centuries, but not a solution which shall be true for all time. Among those who have brought the solution so near to perfection, La Place is entitled to the first rank, although there are others, especially La Grange, who are fully worthy to be named along with him. It will be of interest to state the general results reached by these and other mathematicians.

We call to mind that but for the attraction of the planets upon each other, every planet would move around the sun in an invariable ellipse, according to Kepler's laws. The deviations from this elliptic motion produced by their mutual attraction are called *perturbations*. When they were investigated, it was found that they were of two classes, which were denominated respectively *periodic perturbations* and *secular variations*.

The *periodic* perturbations consist of oscillations dependent upon the mutual positions of the planets, and therefore of comparatively short period. Whenever, after a number of revolutions, two planets return to the same position in their orbits, the periodic perturbations are of the same amount so far as these two planets are concerned. They may therefore be algebraically expressed as dependent upon the longitude of the two planets, the disturbing one and the disturbed one. For instance, the perturbations of the earth produced by the action of *Mercury* depend on the longitude of the earth and on that of *Mercury*. Those produced by the attraction of *Venus* depend upon the longitude of the earth and on that of *Venus*, and so on.

The *secular perturbations*, or secular variations as they are commonly called, consist of slow changes in the forms and positions of the several orbits. It is found that the perihelia of all the orbits are slowly changing their apparent directions from the sun; that the eccentricities of some are increasing and of others diminishing; and that the positions of the orbits are also changing.

One of the first questions which arose in reference to these secular variations was, will they go on indefinitely? If they should, they would evidently end in the subversion of the solar system and the destruction of all life upon the earth. The orbits of the earth and planets would, in the course of ages, become so eccentric, that, approaching near the sun at one time and receding far away from it at another, the variations of temperature would be destructive to life. This problem was first solved by LA GRANGE. He showed that the changes could not go on forever, but that each eccentricity would always be confined between two quite narrow limits. His results may be expressed by a very simple geometrical construction. Let S represent the sun situated in the focus of the ellipse in which

FIG. 54.

the planet moves, and let C be the centre of the ellipse. Let a straight line $S B$ emanate from the sun to B, another line pass from B to D, and so on; the number of these lines being equal to that of the planets, and the last one terminating in C, the centre of the ellipse. Then the line $S B$ will be moving around the sun with a very slow motion; $B D$ will move around B with a slow motion somewhat different, and so each one will revolve in the same manner until we reach the line which carries on its end the centre of the ellipse. These motions are so slow that some of them require tens of thousands, and others hundreds of thousands of years to perform the revolution. By the combined motion of them all, the centre of the ellipse describes a somewhat irregular curve. It is evident, however, that the distance of the centre from the sun can never be greater than the sum of these revolving lines. Now this distance shows the eccentricity of the ellipse, which is equal to half the difference between the greatest and least distances of the planet from the sun. The perihelion being in the direction CS, on the opposite side of the sun from C, it is evident that the motion of C will carry the perihelion with it. It is found in this way that the eccentricity of the earth's orbit has been diminishing for about eighteen thousand years, and will continue to diminish for twenty-five thousand years to come, when it will be more nearly circular than any orbit of our system now is. But before becoming quite circular, the eccentricity will begin to increase again, and so go on oscillating indefinitely.

Secular Acceleration of the Moon.— Another remarkable result reached by mathematical research is that of the acceleration of the moon's motion. More than a century ago it was found, by comparing the ancient and modern observations of the moon, that the latter moved around the earth at a slightly greater rate than she did in ancient times. The existence of this acceleration was a source of great perplexity to La Grange and La Place, because they thought that they had demonstrated mathematically that the attraction could not have accelerated or retarded the mean motion of the moon. But on continuing his investigation, La Place found that there was one cause which he omitted to take account of—namely, the secular diminution in the eccentricity of the earth's orbit, of which we have just spoken. He found that this change in the eccentricity would slightly alter the action of the sun upon the moon, and that this alteration of action would be such that so long as the eccentricity grew smaller, the motion of the moon would continue to be accelerated. Computing the moon's acceleration, he found it to be equal to ten seconds into the square of the number of centuries, the law being the same as that for the motion of a falling body. That is, while in one century she would be ten seconds ahead of the place she would have occupied had her mean motion been uniform, she would, in two centuries, be forty seconds ahead, in three centuries ninety seconds, and so on; and during the two thousand years which have elapsed since the observations of HIPPARCHUS, the acceleration would be more than a degree. It has recently been found that LA PLACE's calculation was not complete, and that with the more exact methods of recent times the real acceleration computed from the theory of gravitation is only about six seconds. The observations of ancient eclipses, however, compared with our modern tables, show an acceleration greater than this; but owing to the rude and doubtful character of nearly all the ancient data, there is some doubt about the exact amount. From the most celebrated total eclipses of the sun, an acceleration of about twelve seconds is deduced, while the observations of PTOLEMY and the Arabian astronomers indicate only eight or nine seconds. There is thus an apparent discrepancy between theory and observation, the latter giving a larger value to the acceleration. This difference is now accounted for by supposing that the motion of the earth on its axis is retarded—that is, that the day is gradually growing longer. From the modern theory of friction, it is found that the motion of the ocean under the influence of the moon's attraction which causes the tides, must be accompanied with some friction, and that this friction must retard the earth's rotation. There is, however, no way of determining the amount of this retardation unless we assume that it causes the observed discrepancy between the theoretical and observed accelerations of the moon.

How this effect is produced will be seen by reflecting that if the day is continually growing longer without our knowing it, our observations of the moon, which we may suppose to be made at noon, for example, will be constantly made a little later, because the interval from one noon to another will be continually growing a little longer. The moon continually moving forward, the observation will place her further and further ahead than she would have been observed had there been no retardation of the time of noon. If in the course of ages our noon-dials get to be an hour too late, we should find the moon ahead of her calculated place by one hour's motion, or about a degree. The present theory of acceleration is, therefore, that the moon is really accelerated about six seconds in a century, and that the motion of the earth on its axis is gradually diminishing at such a rate as to produce an apparent additional acceleration which may range from two to six seconds.

§ 5. REMARKS ON THE THEORY OF GRAVITATION.

The real nature of the great discovery of NEWTON is so frequently misunderstood that a little attention may be given to its elucidation. Gravitation is frequently spoken of as if it were a theory of NEWTON'S, and very generally received by astronomers, but still liable to be ultimately rejected as a great many other theories have been. Not infrequently people of greater or less intelligence are found making great efforts to prove it erroneous. Every prominent scientific institution in the world frequently receives essays having this object in view. Now, the fact is that NEWTON did not discover any new force, but only showed that the motions of the heavens could be accounted for by a force which we all know to exist. Gravitation (Latin *gravitas*—weight, heaviness) is, properly speaking, the force which makes all bodies here at the surface of the earth tend to fall downward ; and if any one wishes to subvert the theory of gravitation, he must begin by proving that this force does not exist. This no one would think of doing. What NEWTON did was to show that this force, which, before his time, had been recognized only as acting on the surface of the earth, really extended to the heavens, and that it resided not only in the earth itself, but in the heavenly bodies also, and in each particle of matter, however situated. To put the matter in a terse form, what NEWTON discovered was not *gravitation*, but the *universality* of gravitation.

It may be inquired, is the induction which supposes gravitation universal so complete as to be entirely beyond doubt? We reply that within the solar system it certainly is. The laws of motion as established by observation and experiment at the surface of the earth must be considered as mathematically certain. Now, it is an observed fact that the planets in their motions deviate from straight lines in a certain way. By the first law of motion, such deviation can be produced only by a force; and the direction and intensity of this force admit of being calculated once that the motion is determined. When thus calculated, it is found to be exactly represented by one great force constantly directed toward the sun, and smaller subsidiary forces directed toward the several planets. Therefore, no fact in nature is more firmly established than is that of universal gravitation, as laid down by NEWTON, at least within the solar system.

We shall find, in describing double stars, that gravitation is also found to act between the components of a great number of such stars. It is certain, therefore, that at least some stars gravitate toward each other, as the bodies of the solar system do; but the distance which separates most of the stars from each other and from our sun is so immense that no evidence of gravitation between them has yet been given by observation. Still, that they do gravitate according to NEWTON'S law can hardly be seriously doubted by any one who understands the subject.

The reader may now be supposed to see the absurdity of supposing that the theory of gravitation can ever be subverted. It is not, however, absurd to suppose that it may yet be shown to be the result of some more general law. Attempts to do this are made from time to time by men of a philosophic spirit; but thus far no theory of the subject having the slightest probability in its favor has been propounded.

Perhaps one of the most celebrated of these theories is that of George Lewis Le Sage, a Swiss physicist of the last century. He supposed an infinite number of ultramundane corpuscles, of transcendent minuteness and velocity, traversing space in straight lines in all directions. A single body placed in the midst of such an ocean of moving corpuscles would remain at rest, since it would be equally impelled in every direction. But two bodies would advance toward each other, because each of them would screen the other from these corpuscles moving in the straight line joining their centres, and there would be a slight excess of corpuscles acting on that side of each body which was turned away from the other.*

One of the commonest conceptions to account for gravitation is that of a fluid, or ether, extending through all space, which is supposed to be animated by certain vibrations, and forms a vehicle, as it were, for the transmission of gravitation. This and all other theories of the kind are subject to the fatal objection of proposing complicated systems to account for the most simple and elementary facts. If, indeed, such systems were otherwise known to exist, and if it could be shown that they really would produce the effect of gravitation, they would be entitled to reception. But since they have been imagined only to account for gravitation itself, and since there is no proof of their existence except that of accounting for it, they

* Reference may be made to an article on the kinetic theories of gravitation by William B. Taylor, in the Smithsonian Report for 1876.

are not entitled to any weight whatever. In the present state of science, we are justified in regarding gravitation as an ultimate principle of matter, incapable of alteration by any transformation to which matter can be subjected. The most careful experiments show that no chemical process to which matter can be subjected either increases or diminishes its gravitating principles in the slightest degree. We cannot therefore see how this principle can ever be referred to any more general cause.

CHAPTER VI.

THE MOTIONS AND ATTRACTION OF THE MOON.

Each of the planets, except *Mercury* and *Venus*, is attended by one or more satellites, or *moons* as they are sometimes familiarly called. These objects revolve around their several planets in nearly circular orbits, accompanying them in their revolutions around the sun. Their distances from their planets are very small compared with the distances of the latter from each other and from the sun. Their magnitudes also are very small compared with those of the planets around which they revolve. Where there are several satellites revolving around a planet, the whole of these bodies forms a small system similar to the solar system in arrangement. Considering each system by itself, the satellites revolve around their central planets or "primaries," in nearly circular orbits, much as the planets revolve around the sun. But each system is carried around the sun without any serious derangement of the motion of its several bodies among themselves.

Our earth has a single satellite accompanying it in this way, the familiar moon. It revolves around the earth in a little less than a month. The nature, causes and consequences of this motion form the subject of the present chapter.

§ 1. THE MOON'S MOTIONS AND PHASES.

That the moon performs a monthly circuit in the heavens is a fact with which we are all familiar from childhood. At certain times we see her newly emerged from the sun's rays in the western twilight, and then we call her the new moon. On each succeeding evening, we see her further to the east, so that in two weeks she is opposite the sun, rising in the east as he sets in the west. Continuing her course two weeks more, she has approached the sun on the other side, or from the west, and is once more lost in his rays. At the end of twenty-nine or thirty days, we see her again emerging as new moon, and her circuit is complete. It is, however, to be remembered that the sun has been apparently moving toward the east among the stars during the whole month, so that during the interval from one new moon to the next the moon has to make a complete circuit relatively to the stars, and move forward some 30° further to overtake the sun. The revolution of the moon among the stars is performed in about $27\frac{1}{3}$ days,* so that if we observe when the moon is very near some star, we shall find her in the same position relative to the star at the end of this interval.

The motion of the moon in this circuit differs from the apparent motions of the planets in being always forward. We have seen that the planets, though, on the whole, moving directly, or toward the east, are affected with an apparent retrograde motion at certain intervals, owing to the motion of the earth around the sun. But the earth is the real centre of the moon's motion, and carries the moon along with it in its annual revolution around the sun. To form a correct idea of the real motion of these three bodies, we must imagine the earth performing its circuit around the sun in one year, and carrying with it the moon, which makes a revolution around it in 27 days, at a distance only about $\frac{1}{400}$ that of the sun.

In Fig. 55 suppose S to represent the sun, the large circle to represent the orbit of the earth around it, E to be some position of the earth, and the dotted circle to represent the orbit of the moon around the earth. We must

* More exactly, 27^d 32166.

imagine the latter to carry this circle with it in its annual course around the sun. Suppose that when the earth is at E the moon is at M. Then if the earth move to E_1 in $27\frac{1}{3}$ days, the moon will have made a complete revolution relative to the stars—that is, it will be at M_2, the line $E_1 M_2$ being parallel to $E M$. But new moon will not have arrived again because the sun is not in the same direction as before. The moon must move through the additional arc $M_1 E M_2$, and a little more, owing to the continual advance of the earth, before it will again be new moon.

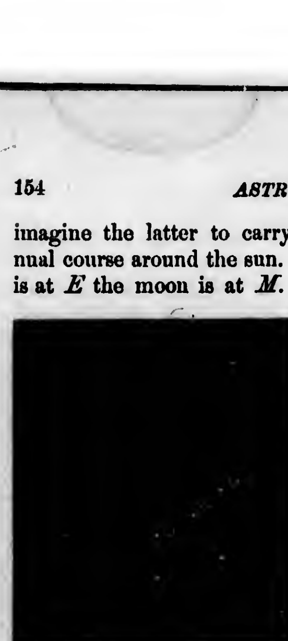

FIG. 55.

Phases of the Moon.—The moon being a non-luminous body shines only by reflecting the light falling on her from some other body. The principal source of light is the sun. Since the moon is spherical in shape, the sun can illuminate one half her surface. The appearance of the moon varies according to the amount of her illuminated hemisphere which is turned toward the earth, as can be seen by studying Fig. 56. Here the central globe is the earth; the circle around it represents the orbit of the moon. The rays of the sun fall on both earth and moon from the right, the distance of the sun being, on the scale of the figure, some 30 feet. Eight positions of the moon are shown around the orbit at A, E, C, etc., and the right-hand hemisphere of the moon is illuminated in each position. Outside these eight positions are eight others showing how the moon looks as seen from the earth in each position.

At A it is "new moon," the moon being nearly between the earth and the sun. Its dark hemisphere is then turned toward the earth, so that it is entirely invisible.

At E the observer on the earth sees about a fourth of the illuminated hemisphere, which looks like a crescent, as shown in the outside figure. In this position a great deal of light is reflected from the earth to the moon, rendering the dark part of the latter visible by a gray light.

Fig. 56.

This appearance is sometimes called the "old moon in the new moon's arms."

At C the moon is said to be in her "first quarter," and one half her illuminated hemisphere is visible.

At G three fourths of the illuminated hemisphere is visible, and at B the whole of it. The latter position, when the moon is opposite the sun, is called "full moon."

After this, at H, D, F, the same appearances are repeated in the reversed order, the position D being called the "last quarter."

The four principal phases of the moon are, "New moon," "First quarter," "Full moon," "Last quarter," which occur in regular and unending succession, at intervals of between 7 and 8 days.

§ 2. THE SUN'S DISTURBING FORCE.

The distances of the sun and planets being so immensely great compared with that of the moon, their attraction upon the earth and the moon is at all times very nearly equal. Now it is an elementary principle of mechanics that if two bodies are acted upon by equal and parallel forces, no matter how great these forces may be, the bodies will move relatively to each other as if those forces did not act at all, though of course the absolute motion of each will be different from what it otherwise would be. If we calculate the absolute attraction of the sun upon the moon, we shall find it to be about twice as great as that of the earth, because, although it is situated at 400 times the distance, its mass is about 330,000 times as great as that of the earth, and if we divide this mass by the square of the distance 400 we have 2 as the quotient.

To those unacquainted with mechanics, the difficulty often suggests itself that the sun ought to draw the moon away from the earth entirely. But we are to remember that the sun attracts the earth in the same way that it attracts the moon, so that the difference between the sun's attraction on the moon and on the earth is only a small fraction of the attraction between the earth and the moon.*

As a consequence of these forces, the moon moves around the earth nearly as if neither of them were attracted by

* In this comparison of the attractive forces of the sun upon the moon and upon the earth, the reader will remember that we are speaking not of the *absolute* force, but of what is called the *accelerating* force, which is properly the ratio of the absolute force to the mass of the body attracted. The earth having 80 times the mass of the moon, the sun must of course attract it with 80 times the absolute force in order to produce the same motion, or the same accelerating force.

the sun—that is, nearly in an ellipse, having the earth in its focus. But there is always a small difference between the attractive forces of the sun upon the moon and upon the earth, and this difference constitutes a disturbing force which makes the moon deviate from the elliptic orbit which it would otherwise describe, and, in fact, keeps the ellipse which it approximately describes in a state of constant change.

A more precise idea of the manner in which the sun disturbs the motion of the moon around the earth may be gathered from Fig. 57. Here *S* represents the sun, and the circle *F Q M N* represents the orbit of the moon. First suppose the moon at *N*, the position corresponding to new moon. Then the moon, being nearer to the sun than the earth is, will be attracted more powerfully by it than the earth is. It will therefore be drawn away from the earth, or the action of the sun will tend to separate the two bodies.

Fig. 57.

Next suppose the moon at *F* the position corresponding to full moon. Here the action of the sun upon the earth will be more powerful than upon the moon, and the earth will in consequence be drawn away from the moon. In this position also the effect of the disturbing force is to separate the two bodies. If, on the other hand, the moon is near the first quarter or near *Q*, the sun will exert a nearly equal attraction on both bodies; and ince the lines of attraction *E S* and *Q S* then converge toward *S*, it follows that there will be a tendency to bring the two bodies together. The same will evidently be true at the third quarter. Hence the influence of the disturbing force changes back and forth twice in the course of each lunar month.

The disturbing force in question may be constructed for any position of the moon in its orbit in the following way, which is believed to be due to Mr. R. A. Proctor: Let *M* be the position of the moon; let us represent the sun's attraction upon it by the line *M S*, and let us investigate what line will represent the sun's attraction upon the earth on the same scale. From *M* drop the perpen-

dicular MP upon the line ES joining the sun to the earth. This attraction being inversely as the square of the distance, we shall have,

$$\frac{\text{Attraction on earth}}{\text{Attraction on moon}} = \frac{SM^2}{SE^2}.$$

We have taken the line SM itself to represent the attraction on the moon, so that we have

$$\text{Attraction on moon} = SM.$$

Multiplying the two equations member by member, we find,

$$\text{Attraction on earth} = SM \times \frac{SM^2}{SE^2}.$$

The line SM is nearly equal to SP, so that we may take for an approximation to the required line,

$$SP \times \frac{SP^2}{SE^2} = SP \times \frac{SP^2}{(SP + PE)^2} = SP \times \frac{1}{\left(1 + \frac{PE}{SP}\right)^2}$$

$$= SP\left(1 - 2\frac{PE}{SP} + \text{etc.}\right),$$

the last equation being obtained by the binomial theorm. But the fraction $\frac{PE}{SP}$ is so small, being less than $\frac{1}{400}$, that its powers above the first will be small enough to be neglected. So we shall have for the required line,

$$SP - 2EP.$$

If, therefore, we take the point A so that PA shall be equal to 2 EP, the attraction of the sun upon the earth will on the same scale be represented by the line AS. The disturbing force which we seek is represented by the difference between the attraction of the sun upon the earth and that of the same body upon the moon. If then we suppose the force AS to be applied to the moon in the opposite direction, the resultant of the two forces MS and SA will represent the disturbing force required. By the law of the composition of forces, this resultant is represented by the line MA.

We are thus enabled to construct this force in a very simple manner, when the moon is in any given position. When the moon is at N, the line NA will be equal to $2\,EM$; the disturbing force will therefore be represented by twice the distance of the moon. On the other hand, when the moon is at Q the three points E N and A will all coincide. Hence the disturbing force which tends to bring the moon toward the earth will be represented by the line QE; hence the force which tends to draw the moon away from the earth at new and full moon is twice as great as that which draws the bodies together at the quarters. Consequently, upon the whole, the tendency of the sun's attraction is to diminish the attraction of the earth upon the moon.

§ 3. MOTION OF THE MOON'S NODES.

Among the changes which the sun's attraction produces in the moon's orbit, that which interests us most is the constant variation in the plane of the orbit. This plane is indicated by the path which the moon seems to describe in its circuit around the celestial sphere. Simple naked eye estimates of the moon's position, continued during a month, would show that her path was always quite near the ecliptic, because it would be evident to the eye that, like the sun, she was much farther north while passing from the vernal to the autumnal equinox than while describing the other half of her circuit from the autumnal to the vernal equinox. It would be seen that, like the sun, she was farthest north in about six hours of right ascension, and farthest south when in about eighteen hours of right ascension.

To map out the path with greater precision, we have to observe the position of the moon from night to night with a meridian circle. We thus lay down her course among the stars in the same manner that we have formerly shown it possible to lay down the sun's path, or the ecliptic. It is thus found that the path of the moon may be considered as a great circle, making an angle of 5° with the ecliptic, and crossing the ecliptic at this small angle at two opposite points of the heavens. These points are called the moon's *nodes*. The point at which she passes from the south to the north of the ecliptic is called the *ascending node;* that in which she passes from the north to the south is the *descending node*. To illustrate the motion of the moon near the node, the dotted line $a\ a$ may be taken as showing the path of the moon, while the circles show her position at successive intervals of one hour as she is approaching her ascending node. Position number 9 is exactly

FIG. 58.—MOTION OF THE MOON'S NODE.

at the node. If we continue following her course in this way for a week, we should find that she had moved about 90°, and attained her greatest north latitude at 5° from the ecliptic. At the end of another week, we should find that she had returned to the ecliptic and crossed it at her descending node. At the end of the third week very nearly, we should find that she had made three fourths the circuit of the heavens, and was now in her greatest south latitude, being 5° south of the ecliptic. At the end of six or seven days more, we should again find her crossing the ecliptic at her ascending node as before. We may thus conceive of four cardinal points of the moon's orbit, 90° apart, marked by the two nodes and the two points of greatest north and south latitude.

Motion of the Nodes.—A remarkable property of these points is that they are not fixed, but are constantly moving. The general motion is a little irregular, but, leaving out small irregularities, it is constantly toward the west. Thus returning to our watch of the course of the moon, we should find that, at her next return to the ascending node, she would not describe the line *a a* as before, but the line *b b* about one fourth of a diameter north of it. She would therefore reach the ecliptic more than $1\frac{1}{2}°$ west of the preceding point of crossing, and her other cardinal points would be found $1\frac{1}{2}°$ farther west as she went around. On her next return she would describe the line *c c*, then the line *d d*, etc., indefinitely, each line being farther toward the west. The figure shows the paths in five consecutive returns to the node.

A lapse of nine years will bring the descending node around to the place which was before occupied by the ascending node, and thus we shall have the moon crossing at a small inclination toward the south, as shown in the figure.

A complete revolution of the nodes takes place in 18·6 years. After the lapse of this period, the motion is repeated in the same manner.

One consequence of this motion is that the moon, after leaving a node, reaches the same node again sooner than she completes her true circuit in the heavens. How much sooner is readily computed from the fact that the retrograde motion of the node amounts to 1° 26′ 31″ during the period that the moon is returning to it. It takes the moon about two hours and a half (more exactly $0^d.10944$) to move through this distance; consequently, comparing with the sidereal period already given, we find that the return of the moon to her node takes place in $27^d.32166 - 0^d.10944 = 27^d.21222$. This time will be important to us in considering the recurrence of eclipses.

In Fig. 59 is illustrated the effect of these changes in the position of the moon's orbit upon her motion relative to the equator. E here represents the vernal and A the autumnal equinox, situated 180° apart. In March, 1876, the moon's ascending node corresponded with the vernal equinox, and her descending node with the autumnal one. Consequently she was 5° north of the ecliptic when in six hours of right ascension or near the middle of the figure. Since the ecliptic is 23½° north of the equator at this point, the moon attained a maximum declination of 28½°; she therefore passed nearer the zenith when in six hours of right ascension than at any other time during the eighteen years' period. In the language of the almanac, "the moon ran high." Of course when at her greatest distance south of the equator, in the other half of her orbit, she attained a corresponding south declination, and culminated at a lower altitude than she had for eighteen years. In 1885 the nodes will change places, and the orbit will deviate from the equator less than at any other time during the eighteen years. In 1880 the descending node will be in six hours of right ascension, and the greatest angular distance of the moon from the equator will be nearly equal to that of the sun.

FIG. 59.

§ 4. MOTION OF THE PERIGEE.

If the sun exerted no disturbing force on the moon, the latter would move round the earth in an ellipse according to KEPLER's laws. But the difference of the sun's attraction on the earth and on the moon, though only a small fraction of the earth's attractive force on the moon, is yet so great as to produce deviations from the elliptic motion very much greater than occur in the motions of the planets. It also produces rapid changes in the elliptic orbit. The most remarkable of these changes are the progressive motion of the nodes just described and a corresponding motion of the perigee. Referring to Fig. 52, which illustrated the elliptic orbit of a planet, let us suppose it to represent the orbit of the moon. S will then represent the earth instead of the sun, and π will be the lunar *perigee*, or the point of the orbit nearest the earth. But, instead of remaining nearly fixed, as do the orbits of the planets, the lunar orbit itself may be considered as making a revolution round the earth in about nine years, in the same direction as the moon itself. Hence if we note the longitude of the moon's perigee at any time, and again two or three years later, we shall find the two positions quite different. If we wait four years and a half, we shall find the perigee in directly the opposite point of the heavens.

The eccentricity of the moon's orbit is about 0.055, and in consequence the moon is about 6° ahead of its mean place when 90° past the perigee, and about the same distance behind when half way from apogee to perigee.

The disturbing action of the sun produces a great number of other inequalities, of which the largest are the *evection* and the *variation*. The former is more than a degree, and the latter not much less. The formulæ by which they are expressed belong to Celestial Mechanics, and the reader who desires to study them is referred to works on that subject.

§ 5. ROTATION OF THE MOON.

The moon rotates on her axis in the same time and in the same direction in which she revolves around the earth. In consequence she always presents very nearly the same face to the earth.* There is indeed a small oscillation called the *libration* of the moon, arising from the fact that her rotation on her axis is uniform, while her revolution around the earth is not uniform. In consequence of this we sometimes see a little of her farther hemisphere first on one side and then on the other, but the greater part of this hemisphere is forever hidden from human sight.

The axis of rotation of the moon is inclined to the ecliptic about 1° 29′. It is remarkable that this axis changes its direction in a way corresponding exactly to the motion of the nodes of the moon's orbit. Let us suppose a line passing through the centre of the earth perpendicular to the plane of the moon's orbit. In consequence of the inclination of the orbit to the ecliptic, this line will point 5° from the pole of the ecliptic. Then, suppose another line parallel to the moon's axis of rotation. This line will intersect the celestial sphere 1° 29′ from the pole of the ecliptic, and on the opposite side from the pole of the moon's orbit, so that it will be $6\frac{1}{2}°$ from the latter. As one pole revolves around the pole of the ecliptic in 18.6 years, the other will do the same, always keeping the same position relative to the first.

* This conclusion is often a *pons asinorum* to some who conceive that, if the same face of the moon is always presented to the earth, she cannot rotate at all. The difficulty arises from a misunderstanding of the difference between a relative and an absolute rotation. It is true that she does not rotate relatively to the line drawn from the earth to her centre, but she must rotate relative to a fixed line, or a line drawn to a fixed star.

§ 6. THE TIDES.

The ebb and flow of the tides are produced by the unequal attraction of the sun and moon on different parts of the earth, arising from the fact that, owing to the magnitude of the earth, some parts of it are nearer these attracting bodies than others, and are therefore more strongly attracted. To understand the nature of the tide-producing force, we must recall the principle of mechanics already cited, that if two neighboring bodies are acted on by equal and parallel accelerating forces, their motion relative to each other will not be altered, because both will move equally under the influence of the forces. When the forces are slightly different, either in magnitude or direction or both, the relative motion of the two bodies will depend on this difference alone. Since the sun and moon attract those parts of the earth which are nearest them more powerfully than those which are remote, there arises an inequality which produces a motion in the waters of the ocean. As the earth revolves on its axis, different parts of it are brought in in succession under the moon. Thus a motion is produced in the ocean which goes through its rise and fall according to the apparent position of the moon. This is called the *tidal wave*.

The tide-producing force of the sun and moon is so nearly like the disturbing force of the sun upon the motion of the moon around the earth that nearly the same explanation will apply to both. Let us then refer again to Fig. 57, and suppose E to represent the centre of the earth, the circle $F\ Q\ N$ its circumference, M a particle of water on the earth's surface, and S either the sun or the moon.

The entire earth being rigid, each part of it will move under the influence of the moon's attraction as if the whole were concentrated at its centre. But the attraction of the moon upon the particle M, being different from its mean attraction on the earth, will tend to make it move differently from the earth. The force which causes this difference of motion, as already explained, will be represented by the line MA. It is true that this same disturbing force is acting upon that portion of the solid earth at M as well as upon the water. But the earth cannot yield on account of its rigidity; the water therefore tends to flow along the earth's surface from M toward N. There is therefore a residual force tending to make the water higher at N than at M.

If we suppose the particle M to be near F, then the point A will be to the left of F. The water will therefore be drawn in an opposite direction or toward F. There will therefore also be a force tending to make the water accumulate around F. As the disturbing force of the sun tends to cause the earth and moon to separate both at new and full moon, so the tidal force of the sun and moon upon the earth tends to make the waters accumulate both at M and F. More exactly, the force in question tends to draw the earth out into the form of a prolate ellipsoid, having its longest axis in the direction of the attracting body. As the earth rotates on its axis, each particle of the ocean is, in the course of a day, brought in to the four positions $N\ Q\ F\ R$, or into some positions corresponding to these. Thus, the tide-producing force changes back and forth twice in the course of a lunar day. (By a lunar day we mean the interval between two successive passages of the moon across the meridian, which is, on the average, about $24^h\ 48^m$.) If the waters could yield immediately to this force, we should always have high tide at F and N and low tides at Q and R. But there are two causes which prevent this.

1. Owing to the inertia of the water, the force must act some time before the full amount of motion is produced, and this motion, once attained, will continue after the force has ceased to act. Again, the waters will continue to accumulate as long as there is any motion in the required direction. The result of this would be high tides at Q and R and low tides at F and N, if the ocean covered the earth and were perfectly free to move. That is, high tides would then be six hours after the moon crossed the meridian.

2. The principal cause, however, which interferes with the regularity of the motion is the obstruction of islands and continents to the free motion of the water. These deflect the tidal wave from its course in so many different ways, that it is hardly possible to trace the relation between the attraction of the moon and the motion of the tide; the time of high and low tide must therefore be found by observing at each point along the coast. By comparing these times through a series of years, a very accurate idea of the motion of the tidal wave can be obtained.

Such observations have been made over our Atlantic and Pacific coasts by the Coast Survey and over most of the coasts of Europe, by the countries occupying them. Unfortunately the tides cannot be observed away from the land, and hence little is known of the course of the tidal wave over the ocean.

We have remarked that both the sun and moon exert a tide-producing force. That of the sun is about $\frac{4}{10}$ of that of the moon. At new and full moon the two forces are united, and the actual force is equal to their sum. At first and last quarter, when the two bodies are 90° apart, they act in opposite directions, the sun tending to produce a high tide where the moon tends to produce a low one, and *vice versa*. The result of this is that near the time of new and full moon we have what are known as the spring tides, and near the quarters what are called neap tides. If the tides were always proportional to the force which produces them, the spring tides would be highest at full moon, but the tidal wave tends to go on for some time after the force which produces it ceases. Hence the highest spring tides are not reached until two or three days after new and full moon. Again, owing to the effect of friction, the neap tides continue to be less and less for two or three days after the first and last quarters, when the gradually increasing force again has time to make itself felt.

The theory of the tides offers very complicated problems, which have taxed the powers of mathematicians for several generations. These problems are in their elements less simple than those presented by the motions of the planets, owing to the number of disturbing circumstances which enter into them. The various depths of the ocean at different points, the friction of the water, its momentum when it is once in motion, the effect of the coast-lines, have all to be taken into account. These quantities are so far from being exactly known that the theory of the tides can be expressed only by some general principles which do not suffice to enable us to predict them for any given place. From observation, however, it is easy to construct tables showing exactly what tides correspond to given positions of the sun and moon at any port where the observations are made. With such tables the ebb and flow are predicted for the benefit of all who are interested, but the results may be a little uncertain on account of the effect of the winds upon the motion of the water.

CHAPTER VII.

ECLIPSES OF THE SUN AND MOON.

Eclipses are a class of phenomena arising from the shadow of one body being cast upon another, and thus wholly or partially obscuring it. In an eclipse of the sun, the shadow of the moon sweeps over the earth, and the sun is wholly or partially obscured to observers on that part of the earth where the shadow falls. In an eclipse of the moon, the latter enters the shadow of the earth, and is wholly or partially obscured in consequence of being deprived of some or all its borrowed light. The satellites of other planets are from time to time eclipsed in the same way by entering the shadows of their primaries; among these the satellites of *Jupiter* are objects whose eclipses may be observed with great regularity.

§ 1. THE EARTH'S SHADOW AND PENUMBRA.

In Fig. 60 let S represent the sun and E the earth. Draw straight lines, $D\,B\,V$ and $D'\,V'\,V$, each tangent to the sun and the earth. The two bodies being supposed spherical, these lines will be the intersections of a cone with the plane of the paper, and may be taken to represent that cone. It is evident that the cone $B\,V B'$ will be the outline of the shadow of the earth, and that within this cone no direct sunlight can penetrate. It is therefore called the earth's *shadow cone.*

Let us also draw the lines $D'\,B\,P$ and $D\,B'\,P'$ to represent the other cone tangent to the sun and earth. It is then evident that within the region VBP and $VB'P'$ the light of the sun will be partially but not entirely cut off.

FIG. 60.—FORM OF SHADOW.

Dimensions of Shadow.—Let us investigate the distance EV from the centre of the earth to the vertex of the shadow. The triangles VEB and VSD are similar, having a right angle at B and at D. Hence,

$$VE : EB = VS : SD = ES : (SD - EB).$$

So if we put

$l = VE$, the length of the shadow measured from the centre of the earth.
$r = ES$, the radius vector of the earth,
$R = SD$, the radius of the sun,
$\rho = EB$, the radius of the earth,
S, the angular semi-diameter of the sun as seen from the earth,
π, the horizontal parallax of the sun,

we have

$$l = VE = \frac{ES \times EB}{SD - EB} = \frac{r\rho}{R - \rho}.$$

But by the theory of parallaxes (Chapter I., § 7),

$$\rho = r \sin \pi$$

$$R = r \sin S$$

Hence,

$$l = \frac{\rho}{\sin S - \sin \pi}.$$

The mean value of the sun's angular semi-diameter, from which the real value never differs by more than the sixtieth part, is found by observations to be about $16' \, 0'' = 960''$, while the mean value of π

is about 8″·8. We find $\sin S - \sin \pi = 0\cdot00461$, and $\frac{1}{\sin S - \sin \pi} =$ $\frac{1}{\cdot00461} = 217$. We therefore conclude that the mean length of the earth's shadow is 217 times the earth's radius; in round numbers 1,380,000 kilometres, or 800,000 miles, the mean radius of the earth being 6370 kilometres. It will be seen from the figure that it varies directly as the distance of the earth from the sun; it is therefore about one sixtieth less than the mean in December, and one sixtieth greater in June.

The radius of the shadow diminishes uniformly with the distance as we go outward from the earth. At any distance z from the earth's centre it will be equal to $\left(1 - \frac{z}{l}\right)\rho$, for this formula gives the radius ρ when $z = 0$, and the diameter zero when $z = l$ as it should.*

§ 2. ECLIPSES OF THE MOON.

The mean distance of the moon from the earth is about 60 radii of the latter, while, as we have just seen, the length $E\ V$ of the earth's shadow is 217 radii of the earth. Hence when the moon passes through the shadow she does so at a point less than three tenths of the way from E to V. The radius of the shadow here will be $\frac{217-60}{217}$ of the radius $E\ B$ of the earth, a quantity which we readily find to be about 4600 kilometres. The radius of the moon being 1736 kilometres, it will be entirely enveloped by the shadow when it passes through it within 2864 kilometres of the axis $E\ V$ of the shadow. If its least distance from the axis exceed this amount, a portion of the lunar globe will be outside the limits $B\ V$ of the shadow cone, and will therefore receive a portion of the direct light of the sun. If the least distance of the centre of the moon from the axis of the shadow is greater than the sum of the radii of the moon and the shadow—that is, greater than 6336 kilometres—the moon will not enter the

* It will be noted that this expression is not, rigorously speaking, the semi-diameter of the shadow, but the shortest distance from a point on its central line to its conical surface. This distance is measured in a direction $E\ B$ perpendicular to $D\ B$, whereas the diameter would be perpendicular to the axis $S\ E$, and its half length would be a little greater than $E\ B$.

shadow at all, and there will be no eclipse proper, though the brilliancy of the moon must be diminished wherever she is within the penumbral region.

When an eclipse of the moon occurs, the phases are laid down in the almanac in the following manner: Supposing the moon to be moving around the earth from below upward, its advancing edge first meets the boundary $B' P'$ of the penumbra. The time of this occurrence is given in the almanac as that of "moon entering penumbra." A small portion of the sunlight is then cut off from the advancing edge of the moon, and this amount constantly increases until the edge reaches the boundary $B' V$ of the shadow. It is curious, however, that the eye can scarcely detect any diminution in the brilliancy of the moon until she has almost touched the boundary of the shadow. The observer must not therefore expect to detect the coming eclipse until very nearly the time given in the almanac as that of "moon entering shadow." As this happens, the advancing portion of the lunar disk will be entirely lost to view, as if it were cut off by a rather ill-defined line. It takes the moon about an hour to move over a distance equal to her own diameter, so that if the eclipse is nearly central the whole moon will be immersed in the shadow about an hour after she first strikes it. This is the time of beginning of total eclipse. So long as only a moderate portion of the moon's disk is in the shadow, that portion will be entirely invisible, but if the eclipse becomes total the whole disk of the moon will nearly always be plainly visible, shining with a red coppery light. This is owing to the refraction of the sun's rays by the lower strata of the earth's atmosphere. We shall see hereafter that if a ray of light $D B$ passes from the sun to the earth, so as just to graze the latter, it is bent by refraction more than a degree out of its course, so that at the distance of the moon the whole shadow is filled with this refracted light. An observer on the moon would, during a total eclipse of the latter, see the earth surrounded by a ring of light, and this ring would appear red, owing to the absorption of the blue and green rays by the earth's atmosphere, just as the sun seems red when setting.

The moon may remain enveloped in the shadow of the earth during a period ranging from a few minutes to nearly two hours, according to the distance at which she passes from the axis of the shadow and the velocity of her angular motion. When she leaves the shadow, the phases which we have described occur in reverse order.

It very often happens that the moon passes through the penumbra of the earth without touching the shadow at all. No notice is taken of these passages in our almanacs, because, as already stated, the diminution of light is scarcely perceptible unless the moon at least grazes the edge of the shadow.

§ 3. ECLIPSES OF THE SUN.

In Fig. 57 we may suppose $B\,E\,B'$ to represent the moon as well as the earth. The geometrical theory of the shadow will remain the same, though the length of the shadow will be much less. We may regard the mean semi-diameter of the sun as seen from the moon, and its mean parallax, as being the same for the moon as for the earth. Therefore in the formula which gives the length of the moon's shadow the denominator will retain the same value, while in the numerator we must substitute the radius of the moon for that of the earth. The radius of the moon is about 1736 kilometres, or 1080 miles. Multiplying this by 217, as before, we find the mean length of the moon's shadow to be 377,000 kilometres, or 235,000 miles. This is very nearly the same with the distance of the moon from the earth when she is in conjunction with the sun. We therefore conclude that when the moon passes between the earth and the sun, the former will be very near the vertex V of the shadow. As a matter of fact, an observer on the earth's surface will sometimes pass through the region $C\ V\ C$, and sometimes on the other side of V.

Now, in Fig. 60, still supposing $B\ E\ B'$ to be the moon, let us draw the lines $D\ B'\ P'$ and $D'\ B\ P$ tangent to both the moon and the sun, but crossing each other between these bodies at b. It is evident that outside the space $P\ B\ B'\ P'$ an observer will see the whole sun, no part of the moon being projected upon it; while within this space the sun will be more or less obscured. The whole obscured space may be divided into three regions, in each of which the character of the phenomenon is different from what it is in the others.

Firstly, we have the region $B\ V\ B'$ forming the shadow cone proper. Here the sunlight is entirely cut off by the moon, and darkness is therefore complete, except so far as light may enter by refraction or reflection. To an observer at V the moon would exactly cover the sun, the two bodies being apparently tangent to each other all around.

Secondly, we have the conical region to the right of V between the lines $B\ V$ and $B'\ V$ continued. In this region the moon is seen wholly projected upon the sun, the visible portion of the latter presenting the form of a ring of light around the moon. This ring of light will be wider in proportion to the apparent diameter of the sun, the farther out we go, because the moon will appear smaller than the sun, and its angular diameter will diminish in a more rapid ratio than that of the sun. This region is that of *annular eclipse*, because the sun will present the appearance of an annulus or ring of light around the moon.

Thirdly, we have the region $P\ B\ V$ and $P'\ B'\ V$, which we notice is connected, extending around the interior cone. An observer here would see the moon partly projected upon the sun, and therefore a certain part of the sun's light would be cut off. Along the inner boundary $B\ V$ and $B'\ V'$ the obscuration of the sun will be complete, but the amount of sunlight will gradually increase out to the outer boundary $B\ P\ B'\ P'$, where the whole sun is visible. This region of partial obscuration is called the *penumbra*.

To show more clearly the phenomena of solar eclipse, we present another figure representing the penumbra of

FIG. 61.—FIGURE OF SHADOW FOR ANNULAR ECLIPSE.

the moon thrown upon the earth.* The outer of the two circles S represents the limb of the sun. The exterior tangents which mark the boundary of the shadow cross each other at V before reaching the earth. The earth being a little beyond the vertex of the shadow, there can be no total eclipse. In this case an observer in the penumbral region, $C\ O$ or $D\ O$, will see the moon partly projected on the sun, while if he chance to be situated at O he will see an annular eclipse. To show how this is, we draw dotted lines from O tangent to the moon. The angle between these lines represents the apparent diameter of the moon as seen from the earth. Continuing them to the sun, they show the apparent diameter of the moon as projected upon the sun. It will be seen that in the case supposed, when

* It will be noted that all the figures of eclipses are necessarily drawn very much out of proportion. Really the sun is 400 times the distance of the moon, which again is 60 times the radius of the earth. But it would be entirely impossible to draw a figure of this proportion; we are therefore obliged to represent the earth as larger than the sun, and the moon as nearly half way between the earth and sun.

the vertex of the shadow is between the earth and moon, the latter will necessarily appear smaller than the sun, and the observer will see a portion of the solar disk on all sides of the moon, as shown in Fig. 62.

If the moon were a little nearer the earth than it is represented in the figure, its shadow would reach the earth

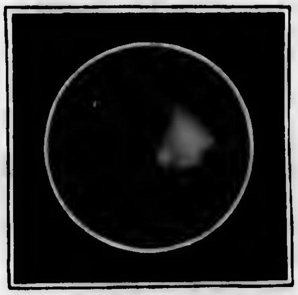

Fig. 62.—Dark body of moon projected on sun during an annular eclipse.

in the neighborhood of *O*. We should then have a total eclipse at each point of the earth on which it fell. It will be seen, however, that a total or annular eclipse of the sun is visible only on a very small portion of the earth's surface, because the distance of the moon changes so little that the earth can never be far from the vertex *V* of the shadow. As the moon moves around the earth from west to east, its shadow, whether the eclipse be total or annular, moves in the same direction. The diameter of the shadow at the surface of the earth ranges from zero to 150 miles. It therefore sweeps along a belt of the earth's surface of that breadth, in the same direction in which the earth is rotating. The velocity of the moon relative to the earth being 3400 kilometres per hour, the shadow would pass along with this velocity if the earth did not rotate, but owing to the earth's rotation the velocity relative to points on its surface may range from 2000 to 3400 kilometres (1200 to 2100 miles).

The reader will readily understand that in order to see a total eclipse an observer must station himself beforehand at some point of the earth's surface over which the shadow is to pass. These points are generally calculated some years in advance, in the astronomical ephemerides, with as much precision as the tables of the celestial motions admit of.

It will be seen that a partial eclipse of the sun may be visible from a much larger portion of the earth's surface than a total or annular one. The space CD (Fig. 61) over which the penumbra extends is generally of about one half the diameter of the earth. Roughly speaking, a partial eclipse of the sun may sweep over a portion of the earth's surface ranging from zero to perhaps one fifth or one sixth of the whole.

There are really more eclipses of the sun than of the moon. A year never passes without at least two of the former, and sometimes five or six, while there are rarely more than two eclipses of the moon, and in many years none at all. But at any one place more eclipses of the moon will be seen than of the sun. The reason of this is that an eclipse of the moon is visible over the entire hemisphere of the earth on which the moon is shining, and as it lasts several hours, observers who are not in this hemisphere at the beginning of the eclipse may, by the earth's rotation, be brought into it before it ends. Thus the eclipse will be seen over more than half the earth's surface. But, as we have just seen, each eclipse of the sun can be seen over only so small a fraction of the earth's surface as to more than compensate for the greater absolute frequency of solar eclipses.

It will be seen that in order to have either a total or annular eclipse visible upon the earth, the line joining the centres of the sun and moon, being continued, must strike the earth. To an observer on this line, the centres to 3400

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of the two bodies will seem to coincide. An eclipse in which this occurs is called a *central* one, whether it be total or annular. The accompanying figure will perhaps aid in giving a clear idea of the phenomena of eclipses of both sun and moon.

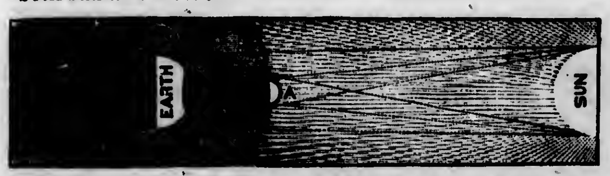

FIG. 63.—COMPARISON OF SHADOW AND PENUMBRA OF EARTH AND MOON. *A* IS THE POSITION OF THE MOON DURING A SOLAR, *B* DURING A LUNAR ECLIPSE.

§ 4. THE RECURRENCE OF ECLIPSES.

If the orbit of the moon around the earth were in or near the same plane with that of the latter around the sun—that is, in or near the plane of the ecliptic—it will be readily seen that there would be an eclipse of the sun at every new moon, and an eclipse of the moon at every full moon. But owing to the inclination of the moon's orbit, described in the last chapter, the shadow and penumbra of the moon commonly pass above or below the earth at the time of new moon, while the moon, at her full, commonly passes above or below the shadow of the earth. It is only when at the moment of new or full moon the moon is near its node that an eclipse can occur.

The question now arises, how near must the moon be to its node in order that an eclipse may occur? It is found by a trigonometrical computation that if, at the moment of new moon, the moon is more than 18°·6 from its node, no eclipse of the sun is possible, while if it is less than 13°·7 an eclipse is certain. Between these limits an eclipse may occur or fail according to the respective distances of the sun and moon from the earth. Half way between these limits, or say 16° from the node, it is an even chance that an eclipse will occur; toward the lower limit ($13° \cdot 7$) the chances increase to certainty; toward the upper one ($18° \cdot 6$) they diminish to zero. The corresponding limits for an eclipse of the moon are 9° and $12\frac{1}{2}°$—that is, if at the moment of full moon the distance of the moon from her node is greater than $12\frac{1}{2}°$ no eclipse can occur, while if the distance is less than 9° an eclipse is certain. We may put the mean limit at 11°. Since, in the long run, new and full moon will occur equally at all distances from the node, there will be, on the average, sixteen eclipses of the sun to eleven of the moon, or nearly fifty per cent more.

FIG. 64.—Illustrating lunar eclipse at different distances from the node. The dark circles are the earth's shadow, the centre of which is always in the ecliptic *A B*. The moon's orbit is represented by *C D*. At *G* the eclipse is central and total, at *F* it is partial, and at *E* there is barely an eclipse.

As an illustration of these computations, let us investigate the limits within which a central eclipse of the sun, total or annular, can occur. To allow of such an eclipse, it is evident, from an inspection of Fig. 61 or 63 that the actual distance of the moon from the plane of the ecliptic must be less than the earth's radius, because the line joining the centres of the sun and earth always lies in this plane. This distance must, therefore, be less than 6370 kilometres. The mean distance of the moon being 384,000 kilometres, the sine of the latitude at this limit is $\frac{6370}{384000}$, and the latitude itself is 57′. The formula for the latitude is, by spherical trigonometry,

$$\sin \text{latitude} = \sin i \ \sin u,$$

i being the inclination of the moon's orbit (5° 8′), and u the distance of the moon from the node. The value of $\sin i$ is not far from $\frac{1}{11}$, while, in a rough calculation, we may suppose the comparatively small angles u and the latitude to be the same as their sines. We may, therefore, suppose

$$u = 11 \ \text{latitude} = 10\tfrac{1}{2}°.$$

We therefore conclude that if, at the moment of new moon, the distance of the moon from the node is less than 10½° there will be a central eclipse of the sun, and if greater than this there will not be such an eclipse. The eclipse limit may range half a degree or more on each side of this mean value, owing to the varying distance of the moon from the earth. Inside of 10° a central eclipse may be regarded as certain, and outside of 11° as impossible.

If the direction of the moon's nodes from the centre of the earth were invariable, eclipses could occur only at the two opposite months of the year when the sun had nearly the same longitude as one node. For instance, if the longitudes of the two opposite nodes were respectively 54° and 234°, then, since the sun must be within 12° of the node to allow of an eclipse of the moon, its longitude would have to be either between 42° and 66°, or between 222° and 246°. But the sun is within the first of these regions only in the month of May, and within the second only during the month of November. Hence lunar eclipses could then occur only during the months of May and November, and the same would hold true of central eclipses of the sun. Small partial eclipses of the latter might be seen occasionally a day or two from the beginnings or ends of the above months, but they would be very small and quite rare. Now, the nodes of the moon's orbit were actually in the above directions in the year 1873. Hence during that year eclipses occurred only in May and November. We may call these months the seasons of eclipses for 1873.

But it was explained in the last chapter that there is a retrograde motion of the moon's nodes amounting to 19⅓° in a year. The nodes thus move back to meet the sun in its annual revolution, and this meeting occurs about 20 days earlier every year than it did the year before. The result is that the season of eclipses is constantly shifting, so that each season ranges throughout the whole year in 18·6 years. For instance, the season corresponding to that of November, 1873, had moved back to July and August in

1878, and will occur in May, 1882, while that of May, 1873, will be shifting back to November in 1882.

It may be interesting to illustrate this by giving the days in which the sun is in conjunction with the nodes of the moon's orbit during several years.

Ascending Node.	Descending Node.
1879. January 24.	1879. July 17.
1880. January 6.	1880. June 27.
1880. December 18.	1881. June 8.
1881. November 30.	1882. May 20.
1882. November 12.	1883. May 1.
1883. October 25.	1884. April 12.
1884. October 8.	1885. March 25.

During these years, eclipses of the moon can occur only within 11 or 12 days of these dates, and eclipses of the sun only within 15 or 16 days.

In consequence of the motion of the moon's node, three varying angles come into play in considering the occurrence of an eclipse, the longitude of the node, that of the sun, and that of the moon. We may, however, simplify the matter by referring the directions of the sun and moon, not to any fixed line, but to the node—that is, we may count the longitudes of these bodies from the node instead of from the vernal equinox. We have seen in the last chapter that one revolution of the moon relatively to the node is accomplished, on the average, in 27·21222 days. If we calculate the time required for the sun to return to the node, we shall find it to be 346·6201 days.

Now, let us suppose the sun and moon to start out together from a node. At the end of 346·6201 days the sun, having apparently performed nearly an entire revolution around the celestial sphere, will again be at the same node, which has moved back to meet it. But the moon will not be there. It will, during the interval, have passed the node 12 times, and the 13th passage will not occur for a week. The same thing will be true for

18 successive returns of the sun to the node; we shall not find the moon there at the same time with the sun; she will always have passed a little sooner or a little later. But at the 19th return of the sun and the 242d of the moon, the two bodies will be in conjunction within half a degree of the node. We find from the preceding periods that

242 returns of the moon to the node require					6585·357 days.	
19	"	" sun	"	"	"	6585·780 "

The two bodies will therefore pass the node within 10 hours of each other. This conjunction of the sun and moon will be the 223d new moon after that from which we started. Now, one lunation (that is, the interval between two consecutive new moons) is, in the mean, 29·530588 days; 223 lunations therefore require 6585·32 days. The new moon, therefore, occurs a little before the bodies reach the node, the distance from the latter being that over which the moon moves in $0^d·036$, or the sun in $0^d·459$. We readily find this distance to be 28′ of arc, somewhat less than the apparent semidiameter of either body. This would be the smallest distance from either node at which any new moon would occur during the whole period. The next nearest approaches would have occurred at the 35th and 47th lunations respectively. The 35th new moon would have occurred about 6° before the two bodies arrived at the node from which we started, and the 47th about $1\frac{1}{2}°$ past the opposite node. No other new moon would occur so near a node before the 223d one, which, as we have just seen, would occur 0° 28′ west of the node. This period of 223 new moons, or 18 years 11 days, was called the *Saros* by the ancient astronomers.

It will be seen that in the preceding calculations we have assumed the sun and moon to move uniformly, so that the successive new moon's occurred at equal intervals of 29·530588 days, and at equal angular distances around the ecliptic. In fact, however, the monthly inequalities in the motion of the moon cause deviations from her

mean motion which amount to six degrees in either direction, while the annual inequality in the motion of the sun in longitude is nearly two degrees. Consequently, our conclusions respecting the point at which new moon occurs may be astray by eight degrees, owing to these inequalities.

But there is a remarkable feature connected with the Saros which greatly reduces these inequalities. It is that this period of 6585¼ days corresponds very nearly to an integral number of revolutions both of the earth round the sun, and of the lunar perigee around the earth. Hence the inequalities both of the moon and of the sun will be nearly the same at the beginning and the end of a Saros. In fact, 6585¼ days is about 18 years and 11 days, in which time the earth will have made 18 revolutions, and about 11° on the 19th revolution. The longitude of the sun will therefore be about 11° greater than at the beginning of the period. Again, in the same period the moon's perigee will have made two revolutions, and will have advanced 13° 38′ on the third revolution. The sun and moon being 11° further advanced in longitude, the conjunction will fall at the same distance from the lunar perigee within two or three degrees. Without going through the details of the calculation, we may say as the result of this remarkable coincidence that the time of the 223d lunation will not generally be accelerated or retarded more than half an hour, though those of the intermediate lunations will sometimes deviate more than half a day. Also that the distance west of the node at which the new moon occurs will not generally differ from its mean value, 28′ by more than 20′.

In the preceding explanation, we have supposed the sun and moon to start out together from one of the nodes of the moon's orbit. It is evident, however, that we might have supposed them to start from any given distance east or west of the node, and should then at the end of the 223d lunation find them together again at nearly that distance from the node. For instance, on the 5th day of May, 1864, at seven o'clock in the evening, Washington time, new moon occurred with the sun and moon 2° 25′ west of the descending node of the moon's orbit. Counting forward 223 lunations, we arrive at the 16th day of May, 1882, when we find the new moon to occur 3° 20′ west of the same node. Since the character of the eclipse depends principally upon the relative position of the sun, the moon, and the node, the result to which we are led may be stated as follows:

Let us note the time of the middle of any eclipse, whether of the sun or of the moon. Then let us go forward 6585 days, 7 hours, 42 minutes, and we shall find another eclipse very similar to the first. Reduced to years, the interval will be 18 years and 10 or 11 days, according as a 29th day of February intervenes four or five times during the interval. This being true of every eclipse, it follows that if we record all the eclipses which occur during a period of 18 years, we shall find a new set to begin over again. If the period were an integral number of days, each eclipse of the new set would be visible in the same regions of the earth as the old one, but since there is a fraction of nearly 8 hours over the round number of days, the earth will be one third of a revolution further advanced before any eclipse of the new set begins. Each eclipse of the new set will therefore occur about one third of the way round the world, or 120° in longitude west of the region in which the old one occurred. The recurrence will not take place near the same region until the end of three periods, or 54 years; and then, since there is a slight deviation in the series, owing to each new or full moon occurring a little further west from the node, the fourth eclipse, though near the same region, will not necessarily be similar in all its particulars. For example, if it be a total eclipse of the sun, the path of the shadow may be a thousand miles distant from the path of 54 years previously.

As a recent example of the Saros, we may cite some total eclipses of the sun well known in recent times; for instance:

1842, July 8th, 1^h A.M., total eclipse observed in Europe;

1860, July 18th, 9^h A.M., total eclipse in America and Spain;

1878, July 29th, 4^h P.M., one visible in Texas, Colorado, and on the coast of Alaska.

A yet more remarkable series of total eclipses of the sun are those of the years 1850, 1868, 1886, etc., the dates and regions being:

1850, August 7th, 4h P.M., in the Pacific Ocean;

1868, August 17th, 12h P.M., in India;

1886, August 29th, 8h A.M., in the Central Atlantic Ocean and Southern Africa;

1904, September 9th, noon, in South America.

This series is remarkable for the long duration of totality, amounting to some six minutes.

Let us now consider a series of eclipses recurring at regular intervals of 18 years and 11 days. Since every successive recurrence of such an eclipse throws the conjunction 28′ further toward the west of the node, the conjunction must, in process of time, take place so far back from the node that no eclipse will occur, and the series will end. For the same reason there must be a commencement to the series, the first eclipse being east of the node. A new eclipse thus entering will at first be a very small one, but will be larger at every recurrence in each Saros. If it is an eclipse of the moon, it will be total from its 13th until its 36th recurrence. There will then be about 13 partial eclipses, each of which will be smaller than the last, when they will fail entirely, the conjunction taking place so far from the node that the moon does not touch the earth's shadow. The whole interval of time over which a series of lunar eclipses thus extend will be about 48 periods, or 865 years.

When a series of solar eclipses begins, the penumbra of the first will just graze the earth not far from one of the poles. There will then be, on the average, 11 or 12 partial eclipses of the sun, each larger than the preceding one, occurring at regular intervals of one Saros. Then the central line, whether it be that of a total or annular eclipse, will begin to touch the earth, and we shall have a series of 40 or 50 central eclipses. The central line will strike near one pole in the first part of the series; in the equatorial regions about the middle of the series, and will leave the earth by the other pole at the end. Ten or twelve partial eclipses will follow, and this particular series will cease. The whole number in the series will average between 60 and 70, occupying a few centuries over a thousand years.

§ 5. CHARACTERS OF ECLIPSES.

We have seen that the possibility of a total eclipse of the sun arises from the occasional very slight excess of the apparent angular diameter of the moon over that of the sun. This excess is so slight that such an eclipse can never last more than a few minutes. It may be of interest to point out the circumstances which favor a long duration of totality. These are:

(1) That the moon should be as near as possible to the earth, or, technically speaking, in perigee, because its angular diameter as seen from the earth will then be greatest.

(2) That the sun should be near its greatest distance from the earth, or in apogee, because then its angular diameter will be the least. It is now in this position about the end of June; hence the most favorable time for a total eclipse of very long duration is in the summer months. Since the moon must be in perigee and also between the earth and sun, it follows that the longitude of the perigee must be nearly that of the sun. The longitude of the sun at the end of June being 100°, this is the most favorable longitude of the moon's perigee.

(3) The moon must be very near the node in order that the centre of the shadow may fall near the equator. The reason of this condition is, that the duration of a total eclipse may be considerably increased by the rotation of the earth on its axis. We have seen that the shadow sweeps over the earth from west toward east with a velocity of about 3400 kilometres per hour. Since the earth rotates in the same direction, the velocity relative to the observer on the earth's surface will be diminished by a quantity depending on this velocity of rotation, and therefore greater, the greater the velocity. The velocity of rotation is greatest at the earth's equator, where it amounts to 1660 kilometres per hour, or nearly half the velocity of the moon's shadow. Hence the duration of a total eclipse may, within the tropics, be nearly doubled by the earth's rotation. When all the favorable circumstances combine in the way we have just described, the duration of a total eclipse within the tropics will be about seven minutes and a half. In our latitude the maximum duration will be somewhat less, or not far from six minutes, but it is only on very rare occasions, hardly once in many centuries, that all these favorable conditions can be expected to concur.

Of late years, solar eclipses have derived an increased interest from the fact that during the few minutes which they last they afford unique opportunities for investigating the matter which lies in the immediate neighborhood of the sun. Under ordinary circumstances, this matter is rendered entirely invisible by the effulgence of the solar rays which illuminate our atmosphere; but when a body so distant as the moon is interposed between the observer and the sun, the rays of the latter are cut off from a region a hundred miles or more in extent. Thus an amount of darkness in the air is secured which is impossible under any other circumstances when the sun is far above the horizon. Still this darkness is by no means complete, because the sunlight is reflected from the region on which the sun is shining. An idea of the amount of darkness may be gained by considering that the face of a watch can be read during an eclipse if the observer is careful to shade his eyes from the direct sunlight during the few minutes before the sun is entirely covered; that stars of the first magnitude can be seen if one knows where to look for them; and that all the prominent features of the landscape remain plainly visible. An account of the investigations made during solar eclipses belongs to the physical constitution of the sun, and will therefore be given in a subsequent chapter.

Occultation of Stars by the Moon.—A phenomenon which, geometrically considered, is analogous to an eclipse of the sun is the occultation of a star by the moon. Since all the bodies of the solar system are nearer than the fixed stars, it is evident that they must from time to time pass between us and the stars. The planets are, however, so small that such a passage is of very rare occurrence, and when it does happen the star is generally so faint that it is rendered invisible by the superior light of the planet before the latter touches it. There are not more than one or two instances recorded in astronomy of a well-authenticated observation of an actual occultation of a star by the opaque body of a planet, although there are several cases in which a planet has been known to pass over a star.

But the moon is so large and her angular motion so rapid, that she passes over some star visible to the naked eye every few days. Such phenomena are termed *occultations of stars by the moon.* It must not, however, be supposed that they can be observed by the naked eye. In general, the moon is so bright that only stars of the first magnitude can be seen in actual contact with her limb, and even then the contact must be with the unilluminated limb. But with the aid of a telescope, and the predictions given in the Ephemeris, two or three of these occultations can be observed during nearly every lunation.

CHAPTER VIII.

THE EARTH.

Our object in the present chapter is to trace the effects of terrestrial gravitation and to study the changes to which it is subject in various places. Since every part of the earth attracts every other part as well as every object upon its surface, it follows that the earth and all the objects that we consider terrestrial form a sort of system by themselves, the parts of which are firmly bound together by their mutual attraction. This attraction is so strong that it is found impossible to project any object from the surface of the earth into the celestial spaces. Every particle of matter now belonging to the earth must, so far as we can see, remain upon it forever.

§ 1. MASS AND DENSITY OF THE EARTH.

We begin by some definitions and some principles respecting attraction, masses, weight, etc.

The *mass* of a body may be defined as *the quantity of matter which it contains.*

There are two ways to measure this quantity of matter: (1) By the attraction or weight of the body—this weight being, in fact, the mutual force of attraction between the body and the earth; (2) By the inertia of the body, or the amount of force which we must apply to it in order to make it move with a definite velocity. Mathematically, there is no reason why these two methods should give the same result, but by experiment it is found that the attraction of all bodies is proportional to their inertia. In other words, all bodies, whatever their chemical constitution, fall exactly the same number of feet in one second under the influence of gravity, supposing them in a vacuum and at the same place on the earth's surface. Although the mass of a body is most conveniently determined by its weight, yet mass and weight must not be confounded.

The *weight* of a body is the apparent force with which it is attracted toward the centre of the earth. As we shall see hereafter, this force is not the same in all parts of the earth, nor at different heights above the earth's surface. It is therefore a variable quantity, depending upon the position of the body, while the mass of the body is regarded as something inherent in it, which remains constant wherever the body may be taken, even if it is carried through the celestial spaces, where its weight would be reduced to almost nothing.

The unit of mass which we may adopt is arbitrary; in fact, in different cases different units will be more convenient. Generally the most convenient unit is the weight of a body at some fixed place on the earth's surface—the city of Washington, for example. Suppose we take such a portion of the earth as will weigh one kilogram in Washington, we may then consider the mass of that particular lot of earth or rock as a kilogram, no matter to what part of the universe we take it. Suppose also that we could bring all the matter composing the earth to the city of Washington, one kilogram at a time, for the purpose of weighing it, returning each kilogram to its place in the earth immediately after weighing, so that there should be no disturbance of the earth itself. The sum total of the weights thus found would be the mass of the earth, and would be a perfectly definite quantity, admitting of being expressed in kilograms or pounds. We can readily calculate the mass of a volume of water equal to that of the earth because we know the magnitude of the earth in litres, and the mass of one litre of water. Dividing this into the mass of the earth, supposing ourselves able to determine this mass, and we shall have the specific gravity, or what is more properly called the *density* of the earth.

What we have supposed for the earth we may imagine for any heavenly body—namely, that it is brought to the city of Washington in small pieces, and there weighed one piece at a time. Thus the total mass of the earth or any heavenly body is a perfectly defined and determined quantity.

It may be remarked in this connection that our units of weight, the pound, the kilogram, etc., are practically units of mass rather than of weight. If we should weigh out a pound of tea in the latitude of Washington, and then take it to the equator, it would really be less heavy at the equator than in Washington; but if we take a pound weight with us, that also would be lighter at the equator, so that the two would still balance each other, and the tea would be still considered as weighing one pound. Since things are actually weighed in this way by weights which weigh one unit at some definite place, say Washington, and which are carried all over the world without being changed, it follows that a body which has any given weight in one place will, as measured in this way, have the same apparent weight in any other place, although its real weight will vary. But if a spring balance or any other instrument for determining actual weights were adopted, then we should find that the weight of the same body varied as we took it from one part of the earth to another. Since, however, we do not use this sort of an instrument in weighing, but pieces of metal which are carried about without change, it follows that what we call units of weight are properly units of mass.

Density of the Earth.—We see that all bodies around us tend to fall toward the centre of the earth. According to the law of gravitation, this tendency is not simply a single force directed toward the centre of the earth, but is the resultant of an infinity of separate forces arising from the attractions of all the separate parts which compose the earth. The question may arise, how do we know that each particle of the earth attracts a stone which falls, and that the whole attraction does not reside in the centre? The proofs of this are numerous, and consist rather in the exactitude with which the theory represents a great mass of disconnected phenomena than in any one principle admitting of demonstration. Perhaps, however, the most conclusive proof is found in the observed fact that masses of matter at the surface of the earth do really attract each other as required by the law of NEWTON. It is found, for example, that isolated mountains attract a plumb-line in their neighborhood. The celebrated experiment of CAVENDISH was devised for the purpose of measuring the attraction of globes of lead. The object of measuring this attraction, however, was not to prove that gravitation resided in the smallest masses of matter, because there was no doubt of that, but to determine the mean density of the earth, from which its total mass may be derived by simply multiplying the density by the volume.

It is noteworthy that though astronomy affords us the means of determining with great precision the *relative* masses of the earth, the moon, and all the planets, it does not enable us to determine the absolute mass of any heavenly body in units of the weights we use on the earth. We know, for instance, from astronomical research, that the sun has about 328,000 times the mass of the earth, and the moon only $\frac{1}{81}$ of this mass, but to know the absolute mass of either of them we must know how many kilograms of matter the earth contains. To determine this, we must know the mean density of the earth, and this is something about which direct observation can give us no information, because we cannot penetrate more than an insignificant distance into the earth's interior. The only way to determine the density of the earth is to find how much matter it must contain in order to attract bodies on its surface with a force equal to their observed weight—

that is, with such intensity that at the equator a body shall fall nearly ten metres in one second. To find this we must know the relation between the mass of a body and its attractive force. This relation can be found only by measuring the attraction of a body of known mass. An attempt to do this was made by MASKELYNE, Astronomer Royal of England, toward the close of the last century, the attracting object he selected being Mount Schehallien in Scotland. The specific gravity of the rocks composing this mountain was well enough known to give at least an approximate result. The density of the earth thus found was 4·71. That is, the earth has 4.71 times the mass of an equal volume of water. This result is, however, uncertain, owing to the necessary uncertainty respecting the density of the mountain and the rocks below it.

The CAVENDISH experiment for determining the attraction of a pair of massive balls affords a much more perfect method of determining this important element. The most careful experiments by this method were made by BAILY of England about the year 1845. The essential parts of the apparatus which he used are as follows:

A long narrow table *T* bears two massive spheres of lead *W W*, one at each end. This table admits of being turned around on a pivot in a horizontal direction. Above it is suspended a balance—that is, a very light deal rod *e* with a weight at each end suspended horizontally by a fine silver wire or fibre of silk *FE*. The weights to be attracted are attached to each end of the deal rod. The right-hand one is visible, while the other is hidden behind the left-hand weight *W*. In this position it will be seen that the attraction of the weights *W* tends to turn the balance in a direction opposite that of the hands of a watch. The fact is, the balance begins to turn in this direction, and being carried by its own momentum beyond the point of equilibrium, comes to rest by a twist of the thread. It is then carried part of the way back to its original position, and thus makes several vibrations which

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f known mass. An
SKELYNE, Astronomer
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require several minutes. At length it comes to rest in a position somewhat different from its original one. This position and the times of vibration are all carefully noted. Then the table *T* is turned nearly end for end, so that one weight *W* shall be between the observer and the right-hand ball, while the other weight is beyond the left-hand ball, and the observation is repeated. A series of observations made in this way include attractions in alternate di-

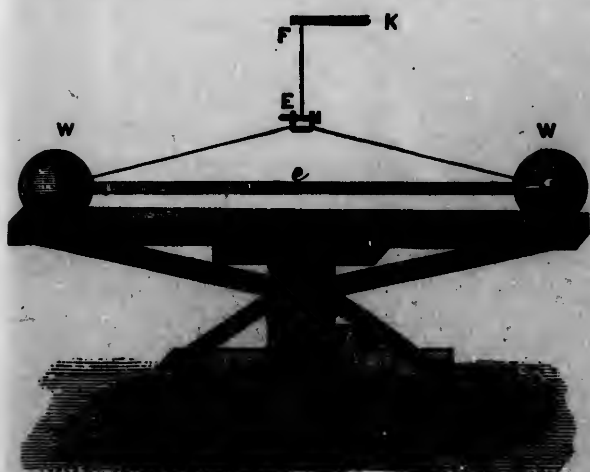

Fig. 65.

rections, giving a result from which accidental errors will be very nearly eliminated.

A third method of determining the density of the earth is founded on observations of the change in the intensity of gravity as we descend below the surface into deep mines. The principles on which this method rests will be explained presently. The most careful application of it was made by Professor AIRY in the Harton Colliery, England. The results of this and the other methods are as follows :

CAVENDISH and HUTTON, from the attraction of balls,	5·32
REICH, " " "	5·58
BAILY, " " "	5·66
MASKELYNE, from the attraction of Schehallien......	4·71
AIRY, from gravity in the Harton Colliery.........	6·56

Of these different results, that of BAILY is probably the best, and the most probable mean density of the earth is about 5⅔ times that of water. This is more than double the mean specific gravity of the materials which compose the surface of the earth ; it follows, therefore, that the inner portions of the earth are much more dense than its outer portions.

§ 2. LAWS OF TERRESTRIAL GRAVITATION.

The earth being very nearly spherical, certain theorems respecting the attraction of spheres may be applied to it. The fundamental theorems may be regarded as those which give the attraction of a spherical shell of matter. The demonstration of these theorems requires the use of the Integral Calculus, and will be omitted here, only the conditions and the results being stated. Let us then imagine a hollow shell of matter, of which the internal and external surfaces are both spheres, attracting any other masses of matter, a small particle we may suppose. This particle will be attracted by every particle of the shell with a force inversely as the square of its distance from it. The total attraction of the shell will be the resultant of this infinity of separate attractive forces. Determining this resultant by the Integral Calculus, it is found that :

Theorem I.—If the particle be outside the shell, it will be attracted as if the whole mass of the shell were concentrated in its centre.

Theorem II.—If the particle be inside the shell, the op-

posite attractions in every direction will neutralize each other, no matter whereabouts in the interior the particle may be, and the resultant attraction of the shell will therefore be zero.

To apply this to the attraction of a solid sphere, let us first suppose a body either outside the sphere or on its surface. If we conceive the sphere as made up of a great number of spherical shells, the attracted point will be external to all of them. Since each shell attracts as if its whole mass were in the centre, it follows that the whole sphere attracts a body upon the outside of its surface as if its entire mass were concentrated at its centre.

FIG. 66.

Let us now suppose the attracted particle inside the sphere, as at P, Fig. 66, and imagine a spherical surface $P\,Q$ concentric with the sphere and passing through the attracted particle. All that portion of the sphere lying outside this spherical surface will be a spherical shell having the particle inside of it, and will therefore exert no attraction whatever on the particle. That portion inside the surface will constitute a sphere with the particle on its surface, and will therefore attract as if all this portion were concentrated in the centre. To find what this attraction will be, let us first suppose the whole sphere of equal density. Let us put

a, the radius of the entire sphere.

r, the distance $P\,C$ of the particle from the centre.

The total volume of matter inside the sphere $P\,Q$ will then be, by geometry, $\frac{4}{3}\pi r^3$. Dividing by the square of the distance r, we see that the attraction will be represented by

$$\frac{4}{3}\pi r;$$

that is, inside the sphere the attraction will be directly as the distance of the particle from the centre. If the particle is at the surface we have $r = a$, and the attraction is

$$\frac{4}{3}\pi a.$$

Outside the surface the whole volume of the sphere $\frac{4}{3}\pi a^3$ will attract the particle, and the attraction will be

$$\frac{4}{3}\pi\frac{a^3}{r^2}.$$

If we put $r = a$ in this formula, we shall have the same result as before for the surface attraction.

Let us next suppose that the density of the sphere varies from its centre to its surface, but in such a way as to be equal at equal distances from the centre. We may then conceive of it as formed of an infinity of concentric spherical shells, each homogeneous in density, but not of the same density with the others. Theorems I. and II. will then still apply, but their result will not be the same as in the case of a homogeneous sphere for a particle inside the sphere. Referring to Fig. 66, let us put

D, the mean density of the shell outside the particle P.

D', the mean density of the portion $P\,Q$ inside of P.

We shall then have:

Volume of the shell, $\frac{4}{3}\pi(a^3 - r^3)$.

Volume of the inner sphere, $\frac{4}{3}\pi r^3$.

Mass of the shell $=$ vol. $\times D = \frac{4}{3}\pi D(a^3 - r^3)$.

Mass of the inner sphere $=$ vol. $\times D' = \frac{4}{3}\pi D' r^3$.

Mass of whole sphere $=$ sum of masses of shell and inner sphere $= \frac{4}{3}\pi\left(D a^3 + (D' - D) r^3\right)$.

Attraction of the whole sphere upon a point at its surface $= \frac{\text{Mass}}{a^2} = \frac{4}{3}\pi\left(Da + (D' - D)\frac{r^3}{a^2}\right)$.

Attraction of the inner sphere (the same as that of the whole shell) upon a point at $P = \frac{\text{Mass}}{r^2} = \frac{4}{3}\pi D' r$.

If, as in the case of the earth, the density continually increases toward the centre, the value of D' will increase also as r diminishes, so that gravity will diminish less rapidly than in the case of a homogeneous sphere, and may, in fact, actually increase. To show this, let us subtract the attraction at P from that at the surface. The difference will give:

$$\text{Diminution at } P = \frac{4}{3}\pi\left(Da + (D' - D)\frac{r^3}{a^2} - D'r\right).$$

Now, let us suppose r a very little less than a, and put

$$r = a - d,$$

d will then be the depth of the particle below the surface.

Cubing this value of r, neglecting the higher powers of d, and dividing by a^2, we find,

$$\frac{r^3}{a^2} = a - 3d.$$

Substituting in the above equation, the diminution of gravity at P becomes,

$$(3D - 2D')d.$$

We see that if $3D < 2D'$, that is, if the density at the surface is less than $\frac{2}{3}$ of the mean density of the whole inner mass, this quantity will become negative, showing that the force of gravity will be less at the surface than at a small depth in the interior. But it must ultimately diminish, because it is necessarily zero at the centre. It was on this principle that Professor Airy determined the density of the earth by comparing the vibrations of a pendulum at the bottom of the Harton Colliery, and at the surface of the earth in the neighborhood. At the bottom of the mine the pendulum gained about $2^s \cdot 5$ per day, showing the force of gravity to be greater than at the surface.

§ 3. FIGURE AND MAGNITUDE OF THE EARTH.

If the earth were fluid and did not rotate on its axis, it would assume the form of a perfect sphere. The opinion is entertained that the earth was once in a molten state, and that this is the origin of its present nearly spherical form. If we give such a sphere a rotation upon its axis, the centrifugal force at the equator acts in a direction opposed to gravity, and thus tends to enlarge the circle of the equator. It is found by mathematical analysis that the form of such a revolving fluid sphere, supposing it to be perfectly homogeneous, will be an oblate ellipsoid—that is, all the meridians will be equal and similar ellipses, having their major axes in the equator of the sphere and their minor axes coincident with the axis of rotation. Our earth, however, is not wholly fluid, and the solidity of its continents prevents its assuming the form it would take if the ocean covered its entire surface. When we speak of the figure of the earth, we mean, not the outline of the solid and liquid portions respectively, but the figure which it would assume if its entire surface were an ocean. Let us imagine canals dug down to the ocean level in every direction through the continents, and the water of the ocean to be admitted into them. Then the curved surface touching the water in all these canals, and coincident with the surface of the ocean, is that of the ideal earth considered by astronomers. By the figure of the earth is meant the figure of this liquid surface, without reference to the inequalities of the solid surface.

We cannot say that this ideal earth is a perfect ellipsoid, because we know that the interior is not homogeneous, but all the geodetic measures heretofore made are so nearly represented by the hypothesis of an ellipsoid that the latter is considered as a very close approximation to the true figure. The deviations hitherto noticed are of so irregular a character that they have not yet been reduced to any certain law. The largest which have been observed seem to be due to the attraction of mountains, or to inequalities of density beneath the surface.

Method of Triangulation.—Since it is practically impossible to measure around or through the earth, the magnitude as well as the form of our planet has to be found by combining measurements on its surface with astronomical observations. Even a measurement on the earth's surface made in the usual way of surveyors would be impracticable, owing to the intervention of mountains, rivers, forests, and other natural obstacles. The method of triangulation is therefore universally adopted for measurements extending over large areas. A triangulation is executed in the following way: Two points, *a* and *b*, a few

FIG. 67.—A PART OF THE FRENCH TRIANGULATION NEAR PARIS.

miles apart, are selected as the extremities of a base-line. They must be so chosen that their distance apart can be accurately measured by rods; the intervening ground should therefore be as level and free from obstruction as possible. One or more elevated points, *EF*, etc., must be visible from one or both ends of the base-line. By means of a theodolite and by observation of the pole-star, the directions of these points relative to the meridian are accurately observed from each end of the base, as is also the direction $a\,b$ of the base-line itself. Suppose F to be a point visible from each end of the base, then in the triangle $a\,b\,F$ we have the length $a\,b$ determined by actual measurement, and the angles at a and b determined by observations. With these data the lengths of the sides $a\,F$ and $b\,F$ are determined by a simple trigonometrical computation.

The observer then transports his instruments to F, and determines in succession the direction of the elevated points or hills $D\,E\,G\,H\,J$, etc. He next goes in succession to each of these hills, and determines the direction of all the others which are visible from it. Thus a network of triangles is formed, of which all the angles are observed with the theodolite, while the sides are successively calculated trigonometrically from the first base. For instance, we have just shown how the side $a\,F$ is calculated; this forms a base for the triangle $E\,F\,a$, the two remaining sides of which are computed. The side $E\,F$ forms the base of the triangle $G\,E\,F$, the sides of which are calculated, etc. In this operation more angles are observed than are theoretically necessary to calculate the triangles. This surplus of data serves to insure the detection of any errors in the measures, and to test their accuracy by the agreement of their results. Accumulating errors are further guarded against by measuring additional sides from time to time as opportunity offers.

Chains of triangles have thus been measured in Russia from the Danube to the Arctic Ocean, in England and France from the Hebrides to Algiers, in this country down nearly our entire Atlantic coast and along the great lakes, and through short distances in many other countries. An east and west line is now being run by the Coast Survey from the Atlantic to the Pacific Ocean. Indeed it may be expected that a network of triangles will be gradually extended over the surface of every civilized country, in order to construct perfect maps of it.

Suppose that we take two stations situated north and south of each other, determine the latitude of each, and measure the distance between them. It is evident that by dividing the distance in kilometres by the difference of latitude in degrees, we shall have the length of one degree of latitude. Then if the earth were a sphere, we should at once have its circumference by multiplying the length of one degree by 360. It is thus found, in a rough way, that the length of a degree is a little more than 111 kilometres, or between 69 and 70 English statute miles. Its circumference is therefore about 40,000 kilometres, and its diameter between 12,000 and 13,000.*

Owing to the ellipticity of the earth, the length of one degree varies with the latitude and the direction in which it is measured. The next step in the order of accuracy is to find the magnitude and the form of the earth from measures of long arcs of latitude (and sometimes of longitude) made in different regions, especially near the equator and in high latitudes. But we shall still find that different combinations of measures give slightly different results, both for the magnitude and the ellipticity, owing to the irregularities in the direction of attraction which we have already described. The problem is therefore to find what ellipsoid will satisfy the measures with the least sum total of error. New and more accurate solutions will be reached from time to time as geodetic measures are extended over a wider area. The following are among the most recent results hitherto reached: LISTING of Göttingen in 1878 found the earth's polar semidiameter, 6355·270 kilo-

* When the metric system was originally designed by the French, it was intended that the kilometre should be $\frac{1}{10000}$ of the distance from the pole of the earth to the equator. This would make a degree of the meridian equal, on the average, to $111\frac{1}{9}$ kilometres. But, owing to the practical difficulties of measuring a meridian of the earth, the correspondence with the metre actually adopted is not exact.

metres; earth's equatorial semidiameter, 6377·377 kilometres; earth's compression, $\frac{1}{288.5}$ of the equatorial diameter; earth's eccentricity of meridian, 0·08319. Another result is that of Captain CLARKE of England, who found: Polar semidiameter, 6356·456 * kilometres; equatorial semidiameter, 6378·191 kilometres.

It was once supposed that the measures were slightly better represented by supposing the earth to be an ellipsoid with three unequal axes, the equator itself being an ellipse of which the longest diameter was 500 metres, or about one third of a mile, longer than the shortest. This result was probably due to irregularities of gravity in those parts of the continents over which the geodetic measures have extended and is now abandoned.

Geographic and Geocentric Latitudes.—An obvious result of the ellipticity of the earth is that the plumb-line

FIG. 68.

does not point toward the earth's centre. Let Fig. 68 represent a meridional section of the earth, NS being the axis of rotation, EQ the plane of the equator, and O the position of the observer. The line HR, tangent to the

* Captain Clarke's results are given in feet, the polar radius being 20,854,895 feet. In changing to metres, the logarithm of the factor has been taken as 9.4840071.

earth at O, will then represent the horizon of the observer, while the line $Z N'$, perpendicular to $H R$, and therefore normal to the earth at Q, will be vertical as determined by the plumb-line. The angle $O N' Q$, or $Z O Q'$, which the observer's zenith makes with the equator, will then be his astronomical or geographical latitude. This is the latitude which in practice we nearly always have to use, because we are obliged to determine latitude by astronomical observation, and not by measurement from the equator. We cannot determine the direction of the true centre C of the earth by direct observation of any kind, but only that of the plumb-line, or of the perpendicular to a fluid surface. $Z O Q'$ is therefore the astronomical latitude. If, however, we conceive the line $C O z$ drawn from the centre of the earth through O, z will be the observer's *geocentric zenith*, while the angle $O C Q$ will be his *geocentric latitude*. It will be observed that it is the geocentric and not the geographic latitude which gives the true position of the observer relative to the earth's centre. The difference between the two latitudes is the angle $C O N'$ or $Z O z$; this is called the *angle of the vertical*. It is zero at the poles and at the equator, because here the normals pass through the centre of the ellipse, and it attains its maximum of $11' 30''$ at latitude 45°. It will be seen that the geocentric latitude is always less than the geographic. In north latitudes the geocentric zenith is south of the apparent zenith and in southern latitudes north of it, being nearer the equator in each case.

§ 4. CHANGE OF GRAVITY WITH THE LATITUDE.

If the earth were a perfect sphere, and did not rotate on its axis, the intensity of gravity would be the same over its entire surface. There is a slight variation from two causes, namely, (1) The elliptic form of our globe, and (2) the centrifugal force generated by its rotation on its axis. Strictly speaking, the latter is not a change in the real force of gravity, or of the earth's attraction, but only an apparent force of another kind acting in opposition to gravity.

The intensity of gravity is measured by the distance which a heavy body in a vacuum will fall in a unit of time, say one second. Either 10 metres or 32 feet may be regarded as a rough approximation to its value. There are, however, so many practical difficulties in the way of measuring with precision the distance a body falls in one second, that the force of gravity is, in practice, determined indirectly by finding the length of the second's pendulum. It is shown in mechanics that if a pendulum of length L vibrates in a time T, a heavy body will in this time T fall through the space $\pi^2 L$, π being the ratio of the circumference of a circle to its diameter. ($\pi = 3 \cdot 14159 \ldots \pi^2 = 9 \cdot 869604$.) Therefore, to find the force of gravity we have only to determine the length of the second's pendulum, and multiply it by this factor.

The determination of the mean attractive force of the earth is important in order that we may compute its action on the moon and other heavenly bodies, while the variations of this attraction afford us data for judging of the variations of density in the earth's interior. Scientific expeditions have therefore taken pains to determine the length of the second's pendulum at numerous points on the globe. To do this, it is not necessary that they should actually measure the length of the pendulum at all the places they visit. They have only to carry some one pendulum of a very solid construction to each point of observation, and observe how many vibrations it makes in a day. They know that the force of gravity is proportional to the square of the number of vibrations. Before and after the voyage, they count the vibrations at some standard point—London for instance. Thus, by simply squaring the number of vibrations and comparing the squares, they have the ratio which gravity at various points of the earth's surface bears to gravity at London. It is then only necessary to determine the absolute intensity of gravity at London to infer it at all the other points for which the ratio is known. From a great number of observations of this kind, it is found that the length of the second's pendulum in latitude ϕ may be nearly represented by the equation,

$$L = 0^{m} \cdot 99099 \, (1 + 0 \cdot 00520 \sin^2 \phi).$$

From this, the force of gravity is found by multiplying by $\pi^2 = 9 \cdot 8696$, giving the result:

$$g' = 9^{m} \cdot 7807 \, (1 + 0 \cdot 00520 \sin^2 \phi).$$

These formulæ show that the apparent force of gravity increases by a little more than $\frac{1}{193}$ of its whole amount from the equator to the poles. We can readily calculate how much of the diminution at the equator is due to the centrifugal force of the earth's rotation. By the formulæ of mechanics, the centrifugal force is given by the equation,

$$f = \frac{4 \pi^2 r}{T^2},$$

T being the time of one revolution, and r the radius of the circle of rotation. Supposing the earth a sphere, which will cause no important error in our present calculation, the distance of a point on the earth's surface in latitude ϕ from the axis of rotation of the earth is,

$$r = a \cos \phi,$$

a being the earth's radius. The centrifugal force in latitude ϕ is therefore

$$f = \frac{4\pi^2 a \cos\phi}{T^2}.$$

But this force does not act in the direction normal to the earth's surface, but perpendicular to the axis of the earth, which direction makes the angle ϕ with the normal. We may therefore resolve the force into two components, one, $f \sin \phi$, along the earth's surface toward the equator, the other, $f \cos \phi$, downward toward its centre. The first component makes the earth a prolate ellipsoid, as already shown, while the second acts in opposition to gravity. The centrifugal force, therefore, diminishes gravity by the amount,

$$f \cos \phi = \frac{4\pi^2 a \cos^2 \phi}{T^2}.$$

T, the sidereal day, is 86,164 seconds of mean time, while a, for the equator, is 6,377,377 metres. Substituting in this expression, the centrifugal force becomes

$$f \cos \phi = 0^m{\cdot}03391 \cos^2 \phi = 0^m{\cdot}03391\,(1 - \sin^2 \phi),$$

or at the equator a little more than $\frac{1}{289}$ the force of gravity. The expression for the apparent force of gravity given by observation, which we have already found, may be put in the form,

$$g' = 9^m{\cdot}7807 + 0^m{\cdot}05087 \sin^2 \phi.$$

This is the true force of gravity diminished by the centrifugal force; therefore, to find that true force we must add the centrifugal force to it, giving the result:

$$\begin{aligned} g &= 9^m{\cdot}8146 + 0^m{\cdot}01696 \sin^2 \phi \\ &= 9^m{\cdot}8146\,(1 + 0{\cdot}001728 \sin^2 \phi), \end{aligned}$$

for the real attraction of the spheroidal earth upon a body on its surface in latitude ϕ.

It will be interesting to compare this result with the attraction of a spheroid having the same ellipticity as the earth. It is found by integration that if e, supposed small, be the eccentricity of a homogeneous oblate ellipsoid, and g_0 its attraction upon a body on its equator, its attraction at latitude ϕ will be given by the equation,

$$g = g_0\left(1 + \frac{e^2}{10} \sin^2 \phi\right).$$

In the case of the earth, $e = 0 \cdot 0817$; $\frac{1}{10}e^2 = 0 \cdot 000667$; so that the expression for gravity would be,

$$g = g_0 (1 + 0 \cdot 000667 \sin^2 \phi).$$

We see that the factor of $\sin^2 \phi$, which expresses the ratio in which gravity at the poles exceeds that at the equator, has less than half the value ($\cdot 001780$), which we have found from observation. This difference arises from the fact that the earth is not homogeneous, but increases in density from the surface toward the centre. To see how this result follows, let us first inquire how the earth would attract bodies where its surface now is if its whole mass were concentrated in its centre. The distance of the equator from the centre is to that of the poles from the centre as 1 to $\sqrt{1 - e^2}$. Therefore, in the case supposed, attraction at the equator would be to attraction at the poles as $1 - e^2$ to 1. The ratio of increase of attraction at the poles is therefore in this extreme case about ten times what it is for the homogeneous ellipsoid. We conclude, therefore, that the more nearly the earth approaches this extreme case—that is, the more it increases in density toward the centre—the greater will be the difference of attraction at the poles and the equator.

§ 5. MOTION OF THE EARTH'S AXIS, OR PRECESSION OF THE EQUINOXES.

Sidereal and Equinoctial Year.—In describing the apparent motion of the sun, two ways were shown of finding the time of its apparent revolution around the sphere—in other words, of fixing the length of a year. One of these methods consists in finding the interval between successive passages through the equinoxes, or, which is the same thing, across the plane of the equator, and the other by finding when it returns to the same position among the stars. Two thousand years ago, HIPPARCHUS found, by comparing his own observations with those made two centuries before by TIMOCHARIS, that these two methods of fixing the length of the year did not give the same result. It had previously been considered that the length of a year was about 365¼ days, and in attempting to correct this period by comparing his observed times of the sun's passing the equinox with those of TIMOCHARIS, HIPPARCHUS found that it required a diminution of seven or eight minutes. He therefore concluded that the true length of the equinoctial year was 365 days, 5 hours, and about 53 minutes. When, however, he considered the return, not to the equinox, but to the same position relative to the bright star *Spica Virginis*, he found that it took some minutes more than 365¼ days to complete the revolution. Thus there are two years to be distinguished, the *tropical* or *equinoctial* year and the *sidereal* year. The first is measured by the time of the earth's return to the equinox; the second by its return to the same position relative to the stars. Although the sidereal year is the correct astronomical period of one revolution of the earth around the sun, yet the equinoctial year is the one to be used in civil life, because it is upon that year that the change of seasons depends. Modern determinations show the respective lengths of the two years to be:

Sidereal year, $365^d\ 6^h\ 9^m\ 9^s = 365^d.25636$.
Equinoctial year, $365^d\ 5^h\ 48^m\ 46^s = 365^d.24220$.

It is evident from this difference between the two years that the position of the equinox among the stars must be changing, and must move toward the west, because the equinoctial year is the shorter. This motion is called the *precession of the equinoxes*, and amounts to about 50″ per year. The equinox being simply the point in which the equator and the ecliptic intersect, it is evident that it can change only through a change in one or both of these circles. HIPPARCHUS found that the change was in the equator, and not in the ecliptic, because the declinations of the stars changed, while their latitudes did not.* Since

* To describe the theory of the ancient astronomers with perfect correctness, we ought to say that they considered the planes both of the equator and ecliptic to be invariable and the motion of precession to be due to a slow revolution of the whole celestial sphere around the pole of the ecliptic as an axis. This would produce a change in the position of the stars relative to the equator, but not relative to the ecliptic.

the equator is defined as a circle everywhere 90° distant from the pole, and since it is moving among the stars, it follows that the pole must also be moving among the stars. But the pole is nothing more than the point in which the earth's axis of rotation intersects the celestial sphere: it must be remembered too that the position of this pole in the celestial sphere depends solely upon the *direction* of the earth's axis, and is not changed by the motion of the earth around the sun, because the sphere is considered to be of infinite radius. Hence precession shows that the direction of the earth's axis is continually changing. Careful observations from the time of HIPPARCHUS until now show that the change in question consists in a slow revolution of the pole of the earth around the pole of the ecliptic as projected on the celestial sphere. The rate of motion is such that the revolution will be completed in between 25,000 and 26,000 years. At the end of this period the equinox and solstices will have made a complete revolution in the heavens.

The nature of this motion will be seen more clearly by referring to Fig. 46, p. 109. We have there represented the earth in four positions during its annual revolution. We have represented the axis as inclining to the right in each of these positions, and have described it as remaining parallel to itself during an entire revolution. The phenomena of precession show that this is not absolutely true, but that, in reality, the direction of the axis is slowly changing. This change is such that, after the lapse of some 6400 years, the north pole of the earth, as represented in the figure, will not incline to the right, but toward the observer, the amount of the inclination remaining nearly the same. The result will evidently be a shifting of the seasons. At *D* we shall have the winter solstice, because the north pole will be inclined toward the observer and therefore from the sun, while at *A* we shall have the vernal equinox instead of the winter solstice, and so on.

In 6400 years more the north pole will be inclined toward the left, and the seasons will be reversed. Another interval of the same length, and the north pole will be inclined from the observer, the seasons being shifted through another quadrant. Finally, at the end of about 25,800 years, the axis will have resumed its original direction.

Precession thus arises from a motion of the earth alone, and not of the heavenly bodies. Although the direction of the earth's axis changes, yet the position of this axis relative to the crust of the earth remains invariable. Some have supposed that precession would result in a change in the position of the north pole on the surface of the earth, so that the northern regions would be covered by the ocean as a result of the different direction in which the ocean would be carried by the centrifugal force of the earth's rotation. This, however, is a mistake. It has been shown by a mathematical investigation that the position of the poles, and therefore of the equator, on the surface of the earth, cannot change except from some variation in the arrangement of the earth's interior. Scientific investigation has yet shown nothing to indicate any probability of such a change.

The motion of precession is not uniform, but is subject to several inequalities which are called *Nutation*. These can best be understood in connection with the forces which produce precession.

Cause of Precession, etc.—Sir ISAAC NEWTON showed that precession was due to an inequality in the attraction of the sun and moon produced by the spheroidal figure of the earth. If the earth were a perfect homogeneous sphere, the direction of its axis would

FIG. 69.

never change in consequence of the attraction of another body. But the excess of matter around the equatorial regions of the earth is attracted by the sun and moon in such a way as to cause a turning force which tends to change the direction of the axis of rotation. To show the mode of action of this force, let us consider the earth as a sphere encircled by a large ring of matter extending around its equator, as in Fig. 69. Suppose a distant attracting body situated in the direction Cc, so that the lines in which the parts of the ring are attracted are Aa, Bb, Cc, etc., which will be nearly parallel. The attractive force will gradually diminish from A to B, owing to the greater distance of the latter from the attracting body. Let us put:

r, the distance of the centre C from the attracting body,

ρ, the radius $AC = BC$ of the equatorial ring, multiplied by the cosine of the angle ACc, so that the distance of A from the attracting centre is $r-\rho$, and that of B is $r+\rho$.

m, the mass of the attracting body;

The accelerative attraction exerted at the three points A, O, B will then be

$$\frac{m}{(r-\rho)^2};\ \frac{m}{r^2};\ \frac{m}{(r+\rho)^2}.$$

The radius ρ being very small compared with r, we may develop the denominators of the first and third fractions in powers of $\frac{\rho}{r}$, by the binomial theorem, and neglect all powers after the first. The attractions will then be approximately:

$$\frac{m}{r^2}+\frac{2m\rho}{r^3};\ \frac{m}{r^2};\ \frac{m}{r^2}-\frac{2m\rho}{r^3}.$$

The forces $\frac{2m\rho}{r^3}$ will be very small compared with $\frac{m}{r^2}$ on account of the smallness of ρ.

The principal force $\frac{m}{r^2}$ will cause all parts of the body to fall equally toward the attracting centre, and will therefore cause no rotation in the body and no change in the direction of the axis NS. Supposing the body to revolve around the centre in an orbit, we may conceive this attraction to be counterbalanced by the so-called centrifugal force.*

Subtracting this uniform principal force, there is left a force $\frac{2m\rho}{r^3}$ acting on A in the direction $A\,a$, and an equal force acting on B in the opposite direction $b\,B$. It is evident that these two forces tend to make the earth rotate around an axis passing through C in such a direction as to make the line $C\,A\,m$ coincide with $C\,e$, and that, if no cause modified the action of these forces, the earth would oscillate back and forth on that axis.

* We may here mention a very common misapprehension respecting what is sometimes called centrifugal force, and is supposed to be a force tending to make a body fly away from the centre. It is sometimes said that the body will fly from the centre when the centrifugal force exceeds the centripetal, and toward it in the opposite case. This is a mistake, such a force as this having no existence. The so-called centrifugal force is not properly a centrifugal force at all, but only the reaction of the whirling body against the centripetal force, which, by the third law of motion, is equal and opposite to that force. When a stone is whirled in a sling the tension on the string is simply the force necessary to make the stone constantly deviate from the straight line in which it tends to move, and is the same as the resistance which the stone offers to this deviation in consequence of its inertia. So, in the case of the planets, the centrifugal force is only the resistance offered by the inertia of the planet to the sun's attraction. If the sling should break, or if the sun should cease to attract the planet, the centripetal and centrifugal forces would both cease instantly, and the stone or planet would, in accordance with the first law of motion, fly forward in the straight line in which it was moving at the moment.

But a modifying cause is found in the rotation of the earth on its own axis, which prevents any change in the angle $m\,C\,e$, but causes a very slow revolution of the axis $N\,S$ around the perpendicular line $C\,E$, which motion is that of precession.*

Nutation.—It will be seen that, under the influence of the gravitation of the sun and moon, precession cannot be uniform. At the time of the equinoxes the equator $A\,B$ of the earth passes through the sun, and the latter lies in the line $B\,C\,A\,m$, so that the small precessional force tending to displace the equator must then vanish. This force increases on both sides of the equinox, and attains a maximum at the solstices when the angle $m\,C\,e$ is $23\frac{1}{2}°$. Hence the precession produced by the sun takes place by semi-annual steps. One of these steps, however, is a little longer than the other, because the earth is nearer the sun in December than in June.

Again, we have seen that the inclination of the moon's orbit to the equator ranges from $18\frac{1}{2}°$ to $28\frac{1}{2}°$ in a period of 18·6 years. Since the precessional force depends on this inclination, the amount of precession due to the action of the moon has a period equal to one revolution of the moon's node, or 18·6 years. These inequalities in the motion of precession are termed *nutation*.

Changes in the Right Ascensions and Declinations of the Stars.—Since the declination of a heavenly body is its angular distance from the celestial equator, it is evident that any change in the position of the equator must change the declinations of the fixed stars. Moreover, since right ascensions are counted from the position of the vernal equinox, the change in the position of this equinox produced by precession and nutation must change the right ascensions of the stars. The motion of the equator may be represented by supposing it to turn slowly around an axis lying in its plane, and pointing to 6^h and 18^h of right ascension. All that section of the equator lying within 6^h of the vernal equinox (see Fig. 45, page 103) is moving toward the south (downward in the figure), while the opposite section, from 6^h to 18^h right ascension, is moving north. The amount of this motion is $20''$ annually. It is evident that this motion will cause both equinoxes to shift toward the right, and the geometrical student will be able to see that the amount of the shift will be :

* The reason of this seeming paradox is that the rotative forces acting on A and B are as it were *distributed* by the diurnal rotation around $N\,S$. Suppose, for example, that A receives a downward and B an upward impulse, so that they begin to move in these directions. At the end of twelve hours A has moved around to B, so that its downward motion now tends to increase the angle $m\,C\,e$, and the upward motion of B has the same effect. If we suppose a series of impulses, a diminution of the inclination will be produced during the first 12 hours, but after that the effect of each impulse will be counterbalanced by that of 12 hours before, so that no further diminution will take place; but every impulse will produce a sudden permanent change in the direction of the axis $N\,S$, the end N moving toward and S from the observer.

This same law of rotation is exemplified in the gyroscope and the child's top, each of which are kept erect by the rotation, though gravity tends to make them fall.

On the equator, $20'' \cot \omega$;

On the ecliptic, $20'' \operatorname{cosec} \omega$;

ω being the obliquity of the ecliptic ($23° 27\frac{1}{2}'$). In consequence, the right ascensions of stars near the equator are constantly increasing by about $46''$ or arc, or $3^s.07$ of time annually. Away from the equator the increase will vary in amount, because, owing to the motion of the pole of the earth, the point in which the equator is intersected by the great circle passing through the pole and the star will vary as well as the equinox, it being remembered that the right ascension of the star is the distance of this point of intersection from the equinox.

The adept in spherical trigonometry will find it an improving exercise to work out the formulæ for the annual change in the right ascension and declination of the stars, arising from the motion of the equator, and consequently of the equinox. He will find the result to be as follows: Put

n, the annual angular motion of the equator ($20''\cdot06$),

ω, its obliquity ($23° 27'\cdot5$),

α δ, the right ascension and declination of the star;

Then we shall find:

Annual change in R. A. $= n \cot \omega + n \sin \alpha \tan \delta$.

Annual change in Dec. $= n \cos \alpha$.

CHAPTER IX.

CELESTIAL MEASUREMENTS OF MASS AND DISTANCE.

§ 1. THE CELESTIAL SCALE OF MEASUREMENT.

THE units of length and mass employed by astronomers are necessarily different from those used in daily life. For instance, the distances and magnitudes of the heavenly bodies are never reckoned in miles or other terrestrial measures for astronomical purposes; when so expressed it is only for the purpose of making the subject clearer to the general reader. The units of weight or mass are also, of necessity, astronomical and not terrestrial. The mass of a body may be expressed in terms of that of the sun or of the earth, but never in kilograms or tons, unless in popular language. There are two reasons for this course. One is that in most cases celestial distances have first to be determined in terms of some celestial unit—the earth's distance from the sun, for instance—and it is more convenient to retain this unit than to adopt a new one. The other is that the values of celestial distances in terms of ordinary terrestrial units are for the most part extremely uncertain, while the corresponding values in astronomical units are known with great accuracy.

An extreme instance of this is afforded by the dimensions of the solar system. By a long and continued series of astronomical observations, investigated by means of KEPLER's laws and the theory of gravitation, it is possible to determine the forms of the planetary orbits, their positions, and their dimensions in terms of the earth's mean distance from the sun as the unit of measure, with great precision. It will be remembered that KEPLER'S third law enables us to determine the mean distance of a planet from the sun when we know its period of revolution. Now, all the major planets, as far out as *Saturn*, have been observed through so many revolutions that their periodic times can be determined with great exactness—in fact within a fraction of a millionth part of their whole amount. The more recently discovered planets, *Uranus* and *Neptune*, will, in the course of time, have their periods determined with equal precision. Then, if we square the periods expressed in years and decimals of a year, and extract the cube root of this square, we have the mean distance of the planet with the same order of precision. This distance is to be corrected slightly in consequence of the attractions of the planets on each other, but these corrections also are known with great exactness. Again, the eccentricities of the orbits are exactly determined by careful observations of the positions of the planets during successive revolutions. Thus we are enabled to make a map of the planetary orbits which shall be so exact that the error would entirely elude the most careful scrutiny, though the map itself should be many yards in extent.

On the scale of this same map we could lay down the magnitudes of the planets with as much precision as our instruments can measure their angular semi-diameters. Thus we know that the mean diameter of the sun, as seen from the earth, is 32′, hence we deduce from formulæ given in connection with parallax (Chapter I., § 9), that the diameter of the sun is ·0093083 of the distance of the sun from the earth. We can therefore, on our supposed map of the solar system, lay down the sun in its true size, according to the scale of the map, from data given directly by observation. In the same way we can do this for each of the planets, the earth and moon excepted. There is no immediate and direct way of finding how large the earth or moon would look from a planet, hence the exception.

But without further special research into this subject, we shall know nothing about the *scale* of our map. It is clear that in order to fix the distances or the magnitudes of the planets according to any terrestrial standard, we must know this scale. Of course if we can learn either the distance or magnitude of any one of the planets laid down on the map, in miles or in semi-diameters of the earth, we shall be able at once to find the scale. But this process is so difficult that the general custom of astronomers is not to attempt to use an exact scale, but to employ the mean distance of the sun from the earth as the unit in celestial measurements. Thus, in astronomical language, we say that the distance of *Mercury* from the sun is 0·387, that of *Venus* 0·723, that of *Mars* 1·523, that of *Saturn* 9·539, and so on. But this gives us no information respecting the distances and magnitudes in terms of terrestrial measures. The unknown quantities of our map are the magnitude of the earth on the scale of the map, and its distance from the sun in terrestrial units of length. Could we only take up a point of observation from the sun or a planet, and determine exactly the angular magnitude of the earth as seen from that point, we should be able to lay down the earth of our map in its correct size. Then since we already know the size of the earth in terrestrial units, we should be able to find the scale of our map, and thence the dimensions of the whole system in terms of those units.

It will be seen that what the astronomer really wants is not so much the dimensions of the solar system in miles as to express the size of the earth in celestial measures. These, however, amount to the same thing, because having one, the other can be readily deduced from the known magnitude of the earth in terrestrial measures.

The magnitude of the earth is not the only unknown quantity on our map. From KEPLER's laws we can determine nothing respecting the distance of the moon from the earth, because unless a change is made in the units of time and space, they apply only to bodies moving around the sun. We must therefore determine the distance of the moon as well as that of the sun to be able to complete our map on a known scale of measurement.

§ 2. MEASURES OF THE SOLAR PARALLAX.

The problem of distances in the solar system is reduced by the preceding considerations to measuring the distances of the sun and moon in terms of the earth's radius. The most direct method of doing this is by determining their respective parallaxes, which we have shown to be the same as the earth's angular semi-diameter as seen from them. In the case of the sun, the required parallax can be determined as readily by measuring the parallaxes of any of the planets as by measuring that of the sun, because any one measured distance on the map will give us the scale of our map. Now, the planets *Venus* and *Mars* occasionally come much nearer the earth than the sun ever does, and their parallaxes also admit of more exact measurement. The parallax of the sun is therefore determined not by observations on the sun itself, but on these two planets. Three methods of finding the sun's parallax in this way have been applied. They are:

(1.) Observations of *Venus* in transit across the sun.

(2.) Observations of the declination of *Mars* from widely separated stations on the earth's surface.

(3.) Observations of the right ascension of *Mars*, near the times of its rising and setting, at a single station.

Solar Parallax from Transits of Venus.— The general principles of the method of determining the parallax of a planet by simultaneous observations at distant stations will be seen by referring to Fig. 18, p. 49. If two observers, situated at S' and S'', make a simultaneous observation of the direction of the body P, it is evident that the solution of a plane triangle will give the distance of P from each station. In practice, however, it would be impracticable to make simultaneous observations at distant stations, and as the planet is continually in motion, the problem is a much more complex one than that of simply solving a triangle. The actual solution is effected by a process which is algebraic rather than geometrical, but we may briefly describe the geometrical nature of the problem.

Considering the problem as a geometrical one, it is evident that, owing to the parallax of *Venus* being nearly four times as great as that of the sun, its path across the sun's disk will be different when viewed from different points of the earth's surface. The further south we go, the further north the planet will seem to be on the sun's disk. The change will be determined by the *difference* between the parallax of *Venus* and that of the sun, and this makes the geometrical explanation less simple than in the case of a determination into which only one parallax enters. It will be sufficient if the reader sees that when we know the relation between the two parallaxes—when, for instance, we know that the parallax of *Venus* is 3·78 times that of the sun—the observed displacement of *Venus* on the sun's disk will give us both parallaxes. The "relative parallax," as it is called, will be 2·78 times the sun's parallax, and it is on this alone that the displacement depends.

The algebraic process, which is that actually employed in the solution of astronomical problems of this class, is as follows:

Each observer is supposed to know his longitude and latitude, and to have made one or more observations of the angular distance of the centre of the planet from the centre of the sun.

To work up the observations, the investigator must have an *ephemeris* of *Venus* and of the sun—that is, a table giving the right ascension and declination of each body from hour to hour as calculated from the best astronomical data. The ephemeris can never be considered absolutely correct, but its error may be assumed as constant for an entire day or more. By means of it, the right ascension and declination of the planet and of the sun, as seen from the centre of the earth, may be computed at any time.

Each observer reduces the moments of his observations to Green-

wich mean time, or the mean time of any other meridian. Let those mean times for the observer S_1 be called T_1, T_2, T_3, etc. Suppose that at these mean times he has observed the distances of the centre of *Venus* from that of the sun to be D_1, D_2, D_3, etc. The corresponding geocentric distances are then computed from the ephemeris for these same times, T_1, T_2, T_3, etc. If the ephemeris and the observations were perfectly correct, and if there were no parallax, these calculated distances would come out the same as the observed ones. But this is never the case. It is therefore necessary to calculate what effect a change in the right ascension, declination, and parallax of the sun and *Venus* will have upon the calculated distance. In this operation these changes are considered as infinitely small, and the process used is that of differentiation.

Let us put:

α, δ, π, the right ascension, declination, and parallax of *Venus*.

α', δ', π', the same quantities for the sun.

$\Delta\alpha$, $\Delta\delta$, $\Delta\alpha'$, $\Delta\delta'$, the corrections necessary to the values of the quantities; α, δ, α', and δ in the ephemeris.

d_1, d_2, d_3, etc., the calculated geocentric distances of *Venus* from the sun's centre.

Then, the corrected calculated distances, which we shall call D'_1, D'_2, D'_3, etc., will be expressed in equations of the form:

$$d_1 + a_1 \Delta\alpha + a'_1 \Delta\alpha' + b_1 \Delta\delta + b'_1 \Delta\delta' + c_1 \pi + c'_1 \pi' = D'_1;$$
$$d_2 + a_2 \Delta\alpha + a'_2 \Delta\alpha' + b_2 \Delta\delta + b'_2 \Delta\delta' + c_2 \pi + c'_2 \pi' = D'_2.$$

In these equations d_1, d_2, etc., and the coefficients, a_1, a_1, a_2, etc., to c'_2, are all known quantities, being the direct results of calculation, while $\Delta\alpha$, $\Delta\alpha$, $\Delta\delta$, and $\Delta\delta$ are unknown corrections to the ephemeris, and π and π' are the parallaxes of *Venus* and the sun, also unknown. D'_1, D'_2, etc., are therefore also to be regarded as unknown.

But when all corrections are allowed for, these corrected calculated distances D'_1, D'_2, etc., ought to be the same as the observed distances D'_1, D'_2, etc., which are known quantities, being the direct result of observations. So if we put D_1 for D'_1, etc., and transpose d_1 to the other side of the equation, and perform the same process on the other equations, we shall have:

$$a_1 \Delta\alpha + a'_1 \Delta\alpha' + b_1 \Delta\delta + b'_1 \Delta\delta' + c_1 \pi + c'_1 \pi' = D_1 - d_1$$
$$a_2 \Delta\alpha + a'_2 \Delta\alpha' + b_2 \Delta\delta + b'_2 \Delta\delta' + c_2 \pi + c'_2 \pi' = D_2 - d_2, \text{ etc.}$$

These equations admit of being much simplified. If we suppose the right ascensions of the sun and *Venus* changed by the same amount—that is, if we suppose $\Delta a' = \Delta a$, it is evident that their distances will remain substantially unaltered. In order that this may be true in the equations, we must have

$$a_1 = -a'_1,$$

because the real change will be, in the case supposed,

$$a_1 \Delta\alpha + a'_1 \Delta\pi = (a_1 + a'_1) \Delta\alpha = 0.$$

In the same way, we must have very nearly,

$$b'_1 = -b_1;\ c'_1 = -c_1.$$

Then if we substitute these values of the accented coefficients, the first equation will be:

$$a_1(\Delta\alpha - \Delta\alpha') + b_1(\Delta\delta - \Delta\delta') + c_1(\pi - \pi') = D_1 - d_1.$$

If we put for brevity,

$$x = \Delta\alpha - \Delta\alpha';\ y = \Delta\delta - \Delta\delta',$$

the equations will become:

$$a_1 x + b_1 y + c_1(\pi - \pi') = D_1 - d_1$$
$$a_2 x + b_2 y + c_2(\pi - \pi') = D_2 - d_2.$$

The parallaxes of the sun and *Venus*, π' and π, are inversely as the distances of the respective bodies from the earth. During the transit of December, 1874, these distances were:

Distance of sun, 0·9847,
" " *Venus*, 0·2644.

So, if we put π_0 for the parallax at distance 1, we shall have:

$$\text{Actual parallax of the sun, } \pi' = \frac{\pi_0}{0{\cdot}9847} = 1{\cdot}0155\,\pi_0.$$

$$\text{Actual parallax of } Venus, \quad \pi = \frac{\pi_0}{0{\cdot}2644} = 3{\cdot}7822\,\pi_0;$$

whence

$$\pi - \pi' = 2{\cdot}7667\,\pi_0.$$

Substituting this value in our equations, they will become:

$$a_1 x + b_1 y + 2{\cdot}7667\,c_1\pi_0 = D_1 - d_1$$
$$a_2 x + b_2 y + 2{\cdot}7667\,c_2\pi_0 = D_2 - d_2, \text{ etc.}$$

All the corresponding equations being formed in this way, from the observations at the various stations, their solution will give the values of the three unknown quantities, x, y, and π_0. The value of π_0 will be the parallax corresponding to the astronomical unit—that is, the angular semi-diameter of the earth seen at the mean distance from the sun.

When many observations are made, we have more equations than there are unknown quantities to be determined. If all the equations were mathematically correct, we should not need them all, and could reject any of the surplus ones without affecting the result. But since each equation is necessarily affected with errors of observation, the problem presented to us is to obtain the most probable values of the unknown quantities from the combination of all the equations. These values are those which render the sum of the squares of the outstanding errors of observations (or, rather, of the outstanding differences between the observed quantities and the computed quantities) a minimum. For instance, suppose that we substitute in the equation

$$a_1 x + b_1 y + 2 \cdot 7767\, c_1\, \pi_0 = D_1 - d,$$

any assumed values of x, y, and π_0. In general the equation will not be satisfied, but there will remain a small difference between the two members, which we may call Δ_1. Let us call Δ_2 the difference obtained in the same way from the second equation, Δ_3 from the third, and so on, and let us put S for the sum of the squares of these quantities, so that

$$S = \Delta^2_1 + \Delta^2_2 + \Delta^2_3 + \text{etc.}$$

Then, for each system of values of x, y, and π_0, which we choose to assume, there will be a corresponding value of S, and the most probable system of values will be that which makes S the least.

The method by which this result is reached is called *the method of least squares*, and is developed in works on astronomical computations.

Measurements of the Parallax of Mars.—This parallax may be determined from observations in two ways. In that usually adopted there are two observers or sets of observers, one in the northern and the other in the southern hemisphere, each of whom determines the declination of the planet from day to day at the moment of transit over his meridian. These declinations will be different by the whole amount of parallactic difference between the two stations, or by the angle $S' P S''$ in Fig. 18, p. 49. The observations are continued through the period when *Mars* is nearest the earth, generally about a couple of months. Any opposition of the planet may be chosen for this purpose, but the most favorable ones are those when the planet is nearest its perihelion. Should the planet be exactly at its perihelion at the time of opposition, its distance from the earth would be only about 0·37, while at aphelion it would be 0·68. This great difference is owing to the considerable eccentricity of the orbit of *Mars*, as can be seen by studying Fig. 48, p. 115, which gives a plan of most of the orbits of the larger planets. The favorable oppositions occur at intervals of 15 or 17 years. One was that of 1862, which gave almost the first conclusive evidence that the old parallax of the sun found by ENCKE was too small. This parallax was 8″·577, and the corresponding distance of the sun was 95¼ millions of miles. The observations of 1862 seemed to show that this parallax must be increased by about one thirtieth part, and the distance diminished in about the same ratio. But the most recent results make it probable that the change should not be quite so great as this.

An extremely favorable opposition, in respect of distance, was that of September 5th, 1877, which occurred 15 days after *Mars* passed its perihelion. On September 3d its distance from the earth was only 0·377—less than it had been at any time since August, 1845.

Parallax of Mars in Right Ascension.—Another method of measuring the parallax of *Mars* is founded on principles entirely different from those we have hitherto considered. In the latter, observations have to be made by two observers in opposite hemispheres of the earth. But an observer at any point on the earth's surface is carried around on a circle of latitude every day by the diurnal motion of the earth. In consequence of this motion, there must be a corresponding apparent motion of each of the planets in an opposite direction. In other words, the parallax of the planet must be different at different times of the day. This diurnal change in the direction of the planet admits of being measured in the following way: The effect of parallax is always to make a heavenly body appear *nearer the horizon* than it would appear as seen from the centre of the earth. This will be obvious if we reflect that an observer moving rapidly from the centre of the earth to its circumference, and keeping his eye fixed upon a planet, would see the planet appear to move in an opposite direction—that is, downward relative to the point of the earth's surface which he aimed at. Hence a planet rising in the east will rise later in consequence of parallax, and will set earlier. Of course the rising and setting cannot be observed with sufficient accuracy for the purpose of parallax, but, since a fixed star has no parallax, the position of the planet relative to the stars in its neighborhood will change during the interval between the rising and setting of the planet. The observer therefore determines the positon of *Mars* relative to the stars surrounding him shortly after he rises and again shortly before he sets. The observations are repeated night after night as often as possible. Between each pair of east and west observations the planet will of course change its position among the stars in consequence of the orbital motions of the earth and planet, but these motions can be calculated and allowed for, and the changes still outstanding will then be due to parallax.

The most favorable regions for an observer to determine the parallax in this way are those near the earth's equator, because he is there carried around on the largest circle. If he is nearer the poles than the equator, the circle will be so small that the parallax will be hardly worth determining, while at the poles there will be no parallactic change at all of the kind just described.

Applications of this method have not been very numerous, although it was suggested by FLAMSTEED nearly two centuries ago. The latest and most successful trial of it was made by Mr. DAVID GILL of England during the opposition of *Mars* in 1877 above described. The point of observation chosen by him was the island of Ascension, west of Africa and near the equator. His measures indicate a considerable reduction in the recently received values of the solar parallax, and an increase in the distance of the sun, making the latter come somewhat nearer to the old value.

Accuracy of the Determinations of Solar Parallax.—The parallax of *Mars* at opposition is rarely more than

20″, and the relative parallax of *Venus* and the sun at the time of the transit is less than 24″. These quantities are so small as to almost elude very precise measurement; it is hardly possible by any one set of measures of parallax to determine the latter without an uncertainty of $\frac{1}{100}$ of its whole amount. In the distance of the sun this corresponds to an uncertainty of nearly half a million of miles. Astronomers have therefore sought for other methods of determining the sun's distance. Although some of these may be a little more certain than measures of parallax, there is none by which the distance of the sun can be determined with any approximation to the accuracy which characterizes other celestial measures.

Other Methods of Determining Solar Parallax.— A very interesting and probably the most accurate method of measuring the sun's distance is by using light as a messenger between the sun and the earth. We shall hereafter see, in the chapter on aberration, that the time required for light to pass from the sun to the earth is known with considerable exactness, being very nearly 498 seconds. If then we can determine experimentally how many miles or kilometres light moves in a second, we shall at once have the distance of the sun by multiplying that quantity by 498. But the velocity of light is about 300,000 kilometres per second. This distance would reach about eight times around the earth. It is rarely possible that two points on the earth's surface more than a hundred kilometres apart are visible from each other, and distinct vision at distances of more than twenty kilometres is rare. Hence to determine experimentally the time required for light to pass between two terrestrial stations requires the measurement of an interval of time, which even under the most favorable cases can be only a fraction of a thousandth of a second. Methods of doing it, however, have been devised and executed by the French physicists, FIZEAU, FOUCAULT, and CORNU, and quite recently by Ensign MICHELSON at the U. S. Naval Academy, Annapolis. From the experiments of the latter, which are probably the most accurate, the velocity of light would seem to be about 299,900 kilometres per second. Multiplying this by 498, we obtain 149,350,000 kilometres for the distance of the sun. The time required for light to pass from the sun to the earth is still uncertain by nearly a second, but this value of the sun's distance is probably the best yet obtained. The corresponding value of the sun's parallax is 8″·81.

Yet other methods of determining the sun's distance are given by the theory of gravitation. The best known of these depends upon the determination of the parallactic inequality of the moon. It is found by mathematical investigation that the motion of the moon is subjected to several inequalities, having the sun's horizontal parallax as a factor. In consequence of the largest of these inequalities, the moon is about two minutes behind its mean place near the first quarter, and as far in advance at the last quarter. If the position of the moon could be determined by observation with the same exactness that the position of a star or planet can, this would probably afford the most accurate method of determining the solar parallax. But an observation of the moon has to be made, not upon its centre, but upon its limb or circumference. Only the limb nearest the sun is visible, the other one being unilluminated, and thus the illuminated limb on which the observation is to be made is different at the first and third quarter. These conditions induce an uncertainty in the comparison of observations made at the two quarters which cannot be entirely overcome, and therefore leave a doubt respecting the correctness of the result.

Brief History of Determinations of the Solar Parallax.—The distance of the sun must at all times have been one of the most interesting scientific problems presented to the human mind. The first known attempt to effect a solution of the problem was made by ARISTARCHUS, who flourished in the third century before CHRIST. It was founded on the principle that the time of the moon's first quarter will vary with the ratio between the distance of the moon and sun, which may be shown as follows. In Fig. 70 let E represent the earth, M the moon, and S the sun. Since the sun always illuminates one half of the lunar globe, it is evident that when one half of the moon's disk appears illuminated, the triangle $E\,M\,S$ must be right-angled at M. The angle $M\,E\,S$ can be determined by measurement, being equal to the angular distance between the sun and the moon. Having two of the angles, the third can be determined, because the sum of the three must make two right angles. Thence we shall have the ratio between EM, the distance of the moon, and ES, the distance of the sun, by a trigonometrical computation.

FIG. 70.

Then knowing the distance of the moon, which can be determined with comparative ease, we have the distance of the sun by multiplying by this ratio. ARISTARCHUS concluded, from his supposed measures, that the angle $M\,E\,S$ was three degrees less than a right angle. We should then have $\frac{ES}{EM} = \sin 3° = \frac{1}{19}$ very nearly. It would follow from this that the sun was 19 times the distance of the moon. We now know that this result is entirely wrong, and that it is impossible to determine the time when the moon is exactly half illuminated with any approach to the accuracy necessary in the solution of the problem. In fact, the greatest angular distance of the earth and moon, as seen from the sun—that is, the angle ESM—is only about one quarter the angular diameter of the moon as seen from the earth.

The second attempt to determine the distance of the sun is mentioned by Ptolemy, though Hipparchus may be the real inventor of it. It is founded on a somewhat complex geometrical construction of a total eclipse of the moon. It is only necessary to state the result, which was, that the sun was situated at the distance of 1210 radii of the earth. This result, like the former, was due only to errors of observation. So far as all the methods known at the time could show, the real distance of the sun appeared to be infinite, nevertheless Ptolemy's result was received without question for fourteen centuries.

When the telescope was invented, and more accurate observations became possible, it was found that the sun's distance must be greater and its parallax smaller than Ptolemy had supposed, but it was still impossible to give any measure of the parallax. All that could be said was that it was less than the smallest quantity that could be decided on by measurement. The first approximation to the true value was made by Horrox of England, and afterward by Huyghens of Holland. It was not founded on any attempt to measure the parallax directly, but on an estimate of the probable magnitude of the earth on the scale of the solar system. The magnitude of the planets on this scale being known by measurement of their apparent angular diameters as seen from the earth, the solar parallax may be found when we know the ratio between the diameter of the earth and that of any planet whose angular diameter has been measured. Now, it was supposed by the two astronomers we have mentioned that the earth was probably of the same order of magnitude with the other planets.

Horrox had a theory, which we now know to be erroneous, that the diameters of the planets were proportional to their distances from the sun—in other words, that all the planets would appear of the same diameter when seen from the sun. This diameter he estimated at 28″, from which it followed that the solar parallax was 14″. HUYGHENS assumed that the actual magnitude of the earth was midway between those of the two planets *Venus* and *Mars* on each side of it; he thus obtained a result remarkably near the truth. It is true that in reality the earth is a little larger than either *Venus* or *Mars*, but the imperfect telescopes of that time showed the planets larger than they really were, so that the mean diameter of the enlarged planets, as seen in the telescope of HUYGHENS, was such as to correspond very nearly to the diameter of the earth.

The first really successful measure of the parallax of a planet was made upon *Mars* during the opposition of 1672, by the first of the two methods already described. An expedition was sent to the colony of Cayenne to observe the declination of the planet from night to night, while corresponding observations were made at the Paris Observatory. From a discussion of these observations, CASSINI obtained a solar parallax of 9″·5, which is within a second of the truth. The next steps forward were made by the transits of *Venus* in 1761 and 1769. The leading civilized nations caused observations on these transits to be made at various points on the globe. The method used was very simple, consisting in the determination of the times at which *Venus* entered upon the sun's disk and left it again. The absolute times of ingress and egress, as seen from different points of the globe, might differ by 20 minutes or more on account of parallax. The results, however, were found to be discordant. It was not until more than half a century had elapsed that the observations were all carefully calculated by ENCKE of Germany, who concluded that the parallax of the sun was 8″·857, and the distance 95 millions of miles.

In 1854 it began to be suspected that ENCKE's value of the parallax was much too small, and great labor was now devoted to a solution of the problem. HANSEN [illegible]e parallactic inequality of the moon, first found the parallax of the sun to be 8″·97, a quantity which he afterward reduced to 8″·916. This result seemed to be confirmed by other observations, especially those of *Mars* during the opposition of 1862. It was therefore concluded that the sun's parallax was probably between 8″·90 and 9″·00. Subsequent researches have, however, been diminishing this value. In 1867, from a discussion on all the data which were considered of value, it was concluded by one of the writers that the most probable parallax was 8″·848. The measures of the velocity of light made by MICHELSON in 1878 reduce this value to 8″·81, and it is now doubtful whether the true value is any larger than this.

The observations of the transit of *Venus* in 1874 have not been completely discussed at the time of writing these pages. When this is done some further light may be thrown upon the question. It is, however, to the determination of the velocity of light that we are to look for the best result. All we can say at present is that the solar parallax is probably between 8″·79 and 8″·83, or, if outside these limits, that it can be very little outside.

§ 3. RELATIVE MASSES OF THE SUN AND PLANETS.

In estimating celestial masses as well as distances, it is necessary to use what we may call celestial units—that is, to take the mass of some celestial body as a unit, instead of any multiple of the pound or kilogram. The reason of this is that the ratios among the masses of the planetary system, or, which is the same thing, the mass of each body in terms of that of some one body as the unit, can be determined independently of the mass of any one of them. To express a mass in kilograms or other terrestrial units, it is necessary to find the mass of the earth in such units, as already explained. This, however, is not necessary for astronomical purposes, where only the relative masses of the several planets are required. In estimating the masses of the individual planets, that of the sun is generally taken as a unit. The planetary masses will then all be very small fractions.

Masses of the Earth and Sun.—We shall first consider the mass of the earth because it is connected by a very curious relation with the parallax of the sun. Knowing the latter, we can determine the mass of the sun relative to the earth, which is the same thing as determining the astronomical mass of the earth, that of the sun being unity. This may be clearly seen by reflecting that when we know the radius of the earth's orbit we can determine how far the earth moves aside from a straight line in one second in consequence of the attraction of the sun. This motion measures the attractive force of the sun at the distance of the earth. Comparing it with the attractive force of the earth, and making allowance for the difference of distances from centres of the two bodies, we determine the ratio between their masses.

The calculation in question is made in the most simple and elementary manner as follows. Let us put:

π, the ratio of the circumference of a circle to its diameter ($\pi = 3{\cdot}14159\ldots$)

r, the mean radius of the earth, or the radius of a sphere having the same volume as the earth.

a, the mean distance of the earth from the sun.

g, the force of gravity on the earth's surface at a point where the radius is r—that is, the distance which a body will fall in one second.

g', the sun's attractive force at the distance a.

T, the number of seconds in a sidereal year.

M, the mass of the sun.

m, the mass of the earth.

P, the sun's mean horizontal parallax.

The force of gravity of the sun, g', may be considered as equal to the so-called centrifugal force of the earth, or to the distance which the earth falls toward the sun in one second. By the formula for centrifugal force given in Chapter VIII., p. 204, we have,

$$g' = \frac{4\pi^2 a}{T^2},$$

and by the law of gravitation,

$$g' = \frac{M}{a^2};$$

whence

$$\frac{M}{a^2} = \frac{4\pi^2 a}{T^2}$$

and

$$M = \frac{4\pi^2 a^3}{T^2}.$$

We have, in the same way, for the earth,

$$g = \frac{m}{r^2}$$

whence

$$m = g r^2.$$

Therefore, for the ratio of the masses of the earth and sun, we have:

$$\frac{M}{m} = \frac{4\pi^2}{gT^2} \cdot \frac{a^3}{r^2} = \frac{4\pi^2}{T^2} \cdot \frac{r}{g} \cdot \frac{a^3}{r^3} \qquad (a).$$

By the formulæ for parallax in Chapter I., § 8, we have:

$$r = a \sin P \;\therefore\; \frac{a^3}{r^3} = \frac{1}{\sin^3 P}.$$

Therefore

$$\frac{M}{m} = \frac{4\pi^2}{T^2} \cdot \frac{r}{g} \cdot \frac{1}{\sin^3 P} \qquad (b).$$

The quantities T, r and g may be regarded as all known with great exactness. We see that the mass of the earth, that of the sun being unity, is proportional to the cube of the solar parallax.

From data already given, we have:

$T =$ 365 days, 6 hours, $9^m\ 9^s$; in seconds, $T =$ 31 558 149,
Mean radius of the earth in metres,* . . $r =$ 6 370 008,
Force of gravity in metres, ... $g =$ 9·8202,

while $\log \pi^2 = 1 \cdot 59636$. Substituting these numbers in the formulæ, it may be put in the form,

$$\frac{m}{M} = [7 \cdot 58984] \sin^3 P,\dagger$$

where the quantity in brackets is the logarithm of the factor.

It will be convenient to make two changes in the parallax P. This angle is so exceedingly small that we may regard it as equal to its sine. To express it in seconds we must multiply it by the number of seconds in the unit radius—that is, by 206265". This will make P (in seconds) $= 206265'' \sin P$. Again, the standard to which parallaxes are referred is always the earth's equatorial radius, which is greater than r by about $\frac{1}{885}$ of its whole amount. So, if we put P'' for the *equatorial* horizontal parallax, expressed in seconds, we shall have,

$$P'' = (1 + \tfrac{1}{885})\, 206265'' \sin P = [5 \cdot 31492] \sin P,$$

whence, for $\sin P$ in terms of P'',

$$\sin P = \frac{P''}{[5 \cdot 31492]}.$$

* The mean radius of the earth is not the mean of the polar and equatorial radii, but one third the sum of the polar radius and twice the equatorial one, because we can draw three such radii, each making a right angle with the other two.

† A number enclosed in brackets is frequently used to signify the logarithm of a coefficient or divisor to be used.

If we substitute this value in the expression for the quotient of the masses, it may be put into either of the forms:

$$\frac{M}{m} = \frac{[8\cdot35493]}{P''^3},$$

$$P'' = [2\cdot78498]\left(\frac{m}{M}\right)^{\frac{1}{3}}.$$

The first formula gives the ratio of the masses when the solar parallax is known; the second, the parallax when the ratio of the masses is known. The following table shows, for different values of the solar parallax, the corresponding ratio of the masses, and distance of the sun in terrestrial measures:

Solar Parallax. P''	$\frac{M}{m}$	Distance of the Sun. In equatorial radii of the earth.	Distance of the Sun. In millions of miles.	Distance of the Sun. In millions of kilometres.
8"·75	337992	23578	93·421	150·343
8"·76	336835	23546	93·314	150·172
8"·77	335684	23519	93·208	150·001
8"·78	334538	23492	93·102	149·830
8"·79	333396	23466	92·996	149·660
8"·80	332262	23439	92·890	148·490
8"·81	331132	23413	92·785	149·320
8"·82	330007	23386	92·680	149·151
8"·83	328887	23360	92·575	148·983
8"·84	327778	23333	92·470	148·814
8"·85	326664	23307	92·366	148·646

We have said that the solar parallax is probably contained between the limits 8".79 and 8".83. It is certainly hardly more than one or two hundredths of a second without them. So, if we wish to express the constants relating to the sun in round numbers, we may say that—

Its *mass* is 330,000 times that of the earth.

Its *distance* in miles is 93 millions, or perhaps a little less.

Its distance in kilometres is probably between 149 and 150 millions.

Density of the Sun.—A remarkable result of the preceding investigation is that the density of the sun, relative to that of the earth, can be determined independently of the mass or distance of the sun by measuring its apparent angular diameter, and the force of gravity at the earth's surface. Let us put

D, the density of the sun.

d, that of the earth.

s, the sun's angular semi-diameter, as seen from the earth. Then, continuing the notation already given, we shall have:

Linear radius of the sun $= a \sin s$.

Volume of the sun $\quad = \frac{4\pi}{3} a^3 \sin^3 s$

(from the formula for the volume of a sphere).

Mass of the sun, $M = \frac{4\pi}{3} a^3 D \sin^3 s$.

Mass of the earth, $m = \frac{4\pi}{3} r^3 d$.

Substituting these values of M and m in the equation (a), and dividing out the common factors, it will become

$$\frac{D}{d} \sin^3 s = \frac{4\pi^2 r}{T^2 g},$$

from which we find, for the ratio of the density of the earth to that of the sun,

$$\frac{d}{D} = \frac{g\,T^2}{4\pi^2 r} \sin^3 s.$$

This equation solves the problem. But the solution may be transformed in expression. We know from the law of falling bodies that a heavy body will, in the time t, fall through the distance $\frac{1}{2} g t^2$. Hence the factor $g\,T^2$ is double the distance which a body would fall in a sidereal year, if the force of gravity could act upon it continuously with the same intensity as at the surface of the earth. Hence $\frac{g\,T^2}{2r}$ will be the number of radii of the earth through which the body will fall in a sidereal year. If we put F for this number, the preceding equation will become,

$$\frac{d}{D} = \frac{F \sin^3 s}{2\pi^2}.$$

We therefore have this rule for finding the density of the earth relative to that of the sun:

Find how many radii of the earth a heavy body would fall through in a sidereal year in virtue of the force of gravity at the earth's surface. Multiply this number by the cube of the sine of the sun's angular semi-diameter, as seen from the earth, and divide by the numerical factor $2\pi^2 = 19 \cdot 7392$. The quotient will be the ratio of the density of the earth to that of the sun.

From the numerical data already given, we find:

Density of earth, that of sun being unity,

$$\frac{d}{D} = 3 \cdot 9206.$$

Density of the sun, that of the earth being unity,

$$\frac{D}{d} = 0 \cdot 25505.$$

These relations do not give us the actual density of either body. We have said that the mean density of the earth is about $5\frac{1}{2}$, that of water being unity. The sun is therefore about 40 or 50 per cent denser than water.

Masses of the Planets.—If we knew how far a body would fall in one second at the surface of any other planet than the earth, we could determine its mass in much the same way as we have determined that of the earth. Now if the planet has a satellite revolving around it, we can make this determination—not indeed directly on the surface of the planet, but at the distance of the satellite, which will equally give us the required datum. Indeed by observing the periodic time of a satellite, and the angle subtended by the major axis of its orbit around the planet, we have a more direct datum for determining the mass of the planet than we actually have for determining that of the earth. (Of course we here refer to the masses of the planets relative to that of the sun as unity.) In fact could an astronomer only station himself on the planet *Venus* and make a series of observations of the angular distance of the moon from the earth, he could determine the mass of the earth, and thence the solar parallax, with far greater precision than we are likely to know it for centuries to come. Let us again consider the equation for M found on page 228:

$$M = \frac{4\pi^2 a^3}{T^2}.$$

Here a and T may mean the mean distance and periodic time of any planet, the quotient $\frac{a^3}{T^2}$ being a constant by KEPLER's third law. In the same equation we may suppose a the mean distance of a satellite from its primary, and T its time of revolution, and M will then represent the mass of the planet. We shall have therefore for the mass of the planet,

$$m = \frac{4\pi^2 a'^3}{T'^2},$$

a' being the mean distance of the satellite from the planet, and T' its time of revolution. Therefore, for the mass of the planet relative to that of the sun we have:

$$\frac{m}{M} = \frac{a'^3\,T^2}{a^3\,T'^2}.$$

Let us suppose a to be the mean distance of the planet from the sun, in which case T must represent its time of revolution. Then, if we put s for the angle subtended by the radius of the orbit of the satellite, as seen from the sun, we shall have, assuming the orbit to be seen edgewise,

$$\sin s = \frac{a'}{a}.$$

If the orbit is seen in a direction perpendicular to its plane, we should have to put tang s for sin s in this formula, but the angle s is so small that the sine and tangent are almost the same. If we put τ for the ratio of the time of revolution of the planet to that of the satellite, it will be equivalent to supposing

$$\tau = \frac{T}{T'}.$$

The equation for the mass of the planet will then become

$$\frac{m}{M} = \tau^2 \sin^3 s,$$

which is the simplest form of the usual formula for deducing the mass of a planet from the motion of its satellite. It is true that we cannot observe s directly, since we cannot place ourselves on the sun, but if we observe the angle s from the earth we can always reduce it to the sun, because we know the proportion between the distances of the planet from the earth and from the sun.

All the large planets outside the earth have satellites; we can therefore determine their masses in this simple way. The earth having also a satellite, its mass could be determined in the same way but for the circumstance already mentioned that we cannot determine the distance of the moon in planetary units, as we can the distance of the satellites of the other planets from their primaries.

The planets *Mercury* and *Venus* have no satellites. It is therefore necessary to determine their masses by their influence in altering the elliptic motions of the other planets round the sun. The alterations thus produced are for the most part so small that their determination is a practical problem of some difficulty. Thus the action of *Mercury* on the neighboring planet *Venus* rarely changes the position of the latter by more than one or two seconds of arc, unless we compare observations more than a century apart. But regular and accurate observations of *Venus* were rarely made until after the beginning of this century. The mass of *Venus* is best determined by the influence of the planet in changing the position of the plane of the earth's orbit. Altogether, the determination of the masses of *Mercury* and *Venus* presents one of the most complicated problems with which the mathematical astronomer has to deal.

CHAPTER X.

THE REFRACTION AND ABERRATION OF LIGHT.

§ 1. ATMOSPHERIC REFRACTION.

When we refer to the *place* of a planet or star, we usually mean its *true* place—*i.e.*, its direction from an observer situated at the centre of the earth, considered as a geometrical point. We have shown in the section on parallax how observations which are necessarily taken at the surface of the earth are reduced to what they would have been if the observer were situated at the earth's centre. In this, however, we have supposed the star to appear to be projected on the celestial sphere in the prolongation of the line joining the observer and the star. The ray from the star is considered as if it suffered no deflection in passing through the stellar spaces and through the earth's atmosphere. But from the principles of physics, we know that such a luminous ray passing from an empty space (as the stellar spaces are), and through an atmosphere, must suffer a refraction, as every ray of light is known to do in passing from a rare into a denser medium. As we see the star in the direction which its light beam has when it enters the eye—that is, as we project the star on the celestial sphere by prolonging this light beam backward into space—there must be an apparent displacement of the star from refraction, and it is this which we are to consider.

We may recall a few definitions from physics. The ray which leaves the star and impinges on the outer surface of the earth's atmosphere is called the *incident ray;* after its deflection by the atmosphere it is called the *refracted ray.* The difference between these directions is called the *astronomical refraction.* If a normal is drawn (perpendicular) to the surface of the refracting medium at the point where the incident ray meets it, the acute angle between the incident ray and the normal is called the angle of incidence, and the acute angle between the normal and the refracted ray is called the angle of refraction. The refraction itself is the difference of these angles. The normal and both incident and refracted rays are in the same vertical plane. In Fig. 69 SA is the ray incident upon the surface BA of the refracting medium $B'BAN$, AC is the refracted ray, MN the normal, SAM and CAN the angles of incidence and refraction respectively. Produce CA backward in the direction AS' : SAS' is the refraction. An observer at C will see the star S as if it were at S'. AS' is the apparent direction of the ray from the star S, and S' is the *apparent place* of the star as affected by refraction.

FIG. 71.—REFRACTION.

This supposes the space above BB' in the figure to be entirely empty spaces, and the earth's atmosphere, equally dense throughout, to fill the space below BB'. In fact, however, the earth's atmosphere is most dense at the surface of the earth, and gradually diminishes in density to its exterior boundary. Therefore, if we wish to represent the facts as they are, we must suppose the atmosphere to be divided into a great number of parallel layers of air, and by assuming an infinite number of these we may also assume that throughout each of them the air is equally dense. Hence the preceding figures will only represent the refraction at a single one of these layers. It follows from this that the path of a ray of light through the atmosphere is not a straight line like $A\ C$, but a curve. We may suppose this curve to be represented in Fig. 70, where the number of layers has been taken very small to avoid confusing the drawing.

Let C be the centre and A a point of the surface of the earth; let S be a star, and $S\ e$ a ray from the star which is refracted at the various layers into which we suppose the atmosphere to be divided, and which finally

FIG. 72.—REFRACTION OF LAYERS OF AIR.

enters the eye of an observer at A in the apparent direction $A\ S'$. He will then see the star in the direction S' instead of that of $S\ S$, and $S\ A\ S'$, the refraction, will throw the star nearer to the zenith Z.

The angle $S'\ A\ Z$ is the apparent zenith distance of S; the true zenith distance of S is $Z\ A\ S$, and this may be assumed to coincide with $S e$, as for all heavenly bodies except the moon it practically does. The line $S\ e$ prolonged will meet the line $A\ Z$ in a point above A, suppose at b'.

Law of Refraction.—A consideration of the physical conditions involved has led to the following form for the refraction in zenith distance $(\Delta \zeta)$,

$$(\Delta \zeta) = A \tan(\zeta' - \zeta(\Delta \zeta)),$$

in which ζ' is the apparent zenith distance of the star, and A is a constant to be determined by observation. A is found to be about 57″, so that we may write $(\Delta \zeta) = 57'' \tan \zeta'$ approximately.

This expression gives what is called the mean refraction—that is, the refraction corresponding to a mean state of the barometer and thermometer. It is clear that changes in the temperature and pressure will affect the density of the air, and hence its refractive power. The tables of the mean refraction made by Bessel, based on a more accurate formula than the one above, are now usually used, and these are accompanied by auxiliary tables giving the small corrections for the state of the meteorological instruments.

Let us consider some of the consequences of refraction, and for our purpose we may take the formula $(\Delta \zeta) = 57'' \tan \zeta'$, as it very nearly represents the facts. At $\zeta' = 0$ $(\Delta \zeta) = 0$, or at the apparent zenith there is no refraction. This we should have anticipated as the incident ray in itself normal to the refracting surface.

The following extract from a refraction table gives the amount of refraction at various zenith distances:

ζ'	$(\Delta\zeta)$	ζ'	$(\Delta\zeta)$
0°	0′ 0″	70°	2′ 39″
10°	0′ 10″	80°	5′ 20″
20°	0′ 33″	85°	10′ 0″
45°	0′ 58″	88°	18′ 0″
50°	1′ 09″	89°	24′ 25″
60°	1′ 40″	90°	34′ 30″

Quantity and Effects of Refraction.—At 45° the refraction is about 1′, and at 90° it is 34′ 30″—that is, bodies at the zenith distances of 45° and 90° appear elevated above their true places by 1′ and 34½′ respectively. If the sun has just risen—that is, if its lower limb is just in apparent contact with the horizon, it is, in fact, entirely below the true horizon, for the refraction (35′) has elevated its centre by more than its whole apparent diameter (32′).

The moon is full when it is exactly opposite the sun, and therefore were there no atmosphere, moon-rise of a full moon and sunset would be simultaneous. In fact, both bodies being elevated by refraction, we see the full moon risen before the sun has set. On April 20th, 1837, the full moon rose eclipsed before the sun had set.

We see from the table that the refraction varies comparatively little between 0° and 60° of zenith distance, but that beyond 80° or 85° its variation is quite rapid.

The refraction on the two limbs of the sun or moon will then be different, and of course greater on the lower limb. This will apparently be lifted up toward the upper limb more than the upper limb is lifted away from it, and hence the sun and moon appear oval in shape when near the horizon. For example, if the zenith distance of the sun's lower limb is 85°, that of the upper will be about 84° 28′, and the refractions from the tables for these two zenith distances differ by 1′; therefore, the sun will appear oval in shape, with axes of 32′ and 31′ approximately.

Determination of Refraction.—If we know the law according to which refraction varies—that is, if we have an accurate formula which will give ($\Delta \zeta$) in terms of ζ, we can determine the absolute refraction for any one point, and from the law deduce it for any other points. Thus knowing the horizontal refraction, or the refraction in the horizon, we can determine the refraction at other known zenith distances.

We know the time of (theoretical or true) sunrise and sunset by the formula of § 7, p. 44, and we may observe the time of apparent rising and setting of the sun (or a star). The difference of these times gives a means of determining the effect of refraction.

Or, in the observations for latitude by the method of § 8, p. 47, we can measure the apparent polar distances of a circumpolar star at its upper and lower culmination. Its polar distances above and below pole should be equal; if there were no refraction they would be so, but they really differ by a quantity which it is easy to see is the difference of the refractions at lower and upper culminations. By choosing suitable circumpolar stars at various polar distances, this difference may be determined for all polar distances, and therefore at all zenith distances.

§ 2. ABERRATION AND THE MOTION OF LIGHT.

Besides refraction, there is another cause which prevents our seeing the celestial bodies exactly in the true direction in which they lie from us—namely, the progressive motion of light. We now know that we see objects only by the light which emanates from them and reaches our eyes, and we also know that this light requires time to pass over the space which separates us from the object. After the ray of light once leaves the object, the latter may move away, or even be blotted out of existence, but the ray of light will continue on its course. Consequently when we look at a star, we do not see the star that now is, but the star that was several years ago. If it should be annihilated, we should still see it during the years which would be required for the last ray of light emitted by it to reach us. The velocity of light is so great that in all observations of terrestrial objects, our vision may be regarded as instantaneous. But in celestial observations the time required for the light to reach us is quite appreciable and measurable.

The discovery of the propagation of light is among the most remarkable of those made by modern science. The fact that light requires time to travel was first learned by the observations of the satellites of *Jupiter*. Owing to the great magnitude of this planet, it casts a much longer and larger shadow than our earth does, and its inner satellite is therefore eclipsed at every revolution. These eclipses can be observed from the earth, the satellite vanishing from view as it enters the shadow, and suddenly reappearing when it leaves it again. The accuracy with which the times of this disappearance and reappearance could be observed, and the consequent value of such observations for the determination of longitudes, led the astronomers of the seventeenth century to make a careful study of the motions of these bodies. It was, however, necessary to make tables by which the times of the eclipses could be predicted. It was found by ROEMER that these times depended on the distance of *Jupiter* from the earth. If he made his tables agree with observations when the earth was nearest *Jupiter*, it was found that as the earth receded from *Jupiter* in its annual course around the sun, the eclipses were constantly seen later, until, when at its greatest distance, the times appeared to be 22 minutes late. ROEMER saw that it was in the highest degree improbable that the actual motions of the satellites should be affected with any such inequality; he therefore propounded the bold theory that it took *time* for light to come from *Jupiter* to the earth. The extreme differences in the times of the eclipse being 22 minutes, he assigned this as the time required for light to cross the orbit of the earth, and so concluded that it came from the sun to the earth in 11 minutes. We now know that this estimate was too great, and that the true time for this passage is about 8 minutes and 18 seconds.

Discovery of Aberration.—At first this theory of ROEMER was not fully accepted by his contemporaries. But in the year 1729 the celebrated BRADLEY, afterward Astronomer Royal of England, discovered a phenomenon of an entirely different character, which confirmed the theory. He was then engaged in making observations on the star γ *Draconis* in order to determine its parallax. The effect of parallax would have been to make the declination greatest in June and least in December, while in March and September the star would occupy an intermediate or mean position. But the result was entirely different. The declinations of June and December were the same, showing no effect of parallax; but instead of remaining constant the rest of the year, the declination was some 40 seconds greater in September than in March, when the effect of parallax would be the same. This showed that the direction of the star appeared different, not according to the position of the earth, but according to the direction of its motion around the sun, the star being apparently displaced in this direction.

It has been said that the explanation of this singular anomaly was first suggested to BRADLEY while sailing on the Thames. He noticed that when his boat moved rapidly at right angles to the true direction of the wind, the apparent direction of the wind changed toward the point whither the boat was going. When the boat sailed in an opposite direction, the apparent direction of the wind suddenly changed in a corresponding way. Here was a phenomenon very analogous to that which he had observed in the stars, the direction from which the wind appeared to come corresponding to the direction in which the light reached the eye. This direction changed with the motion of the observer according to the same law in the two cases. He now saw that the apparent displacement of the star was due to the motion of the rays of light combined with that of the earth in its orbit, the apparent direction of the star depending, not upon the absolute direction from which the ray comes, but upon the relation of this direction to the motion of the observer.

To show how this is, let $A\,B$ be the optical axis of a telescope, and S a star from which emanates a ray moving in the true direction $S\,A\,B'$. Perhaps the reader will have a clearer conception of the subject if he imagines $A\,B$ to be a rod which an observer at B seeks to point at the star S. It is evident that he will point this rod in such a way that the ray of light shall run accurately along its length. Suppose now that the observer is moving from B toward B' with such a velocity that he moves from B to B' during the time required for a ray of light to move from A to B'. Suppose also that the ray of light $S\,A$ reaches A at the same time that the end of his rod does. Then it is clear that while the rod is moving from the position $A\,B$ to the position $A'\,B'$, the ray of light will move from A to B', and will therefore run accurately along the length of the rod. For instance, if b is one third of the way from B to B', then the light, at the instant of the rod tak-

FIG. 73.

ing the position $b\ a$, will be one third of the way from A to B', and will therefore be accurately on the rod. Consequently, to the observer, the rod will appear to be pointed at the star. In reality, however, the pointing will not be in the true direction of the star, but will deviate from it by an angle of which the tangent is the ratio of the velocity with which the observer is carried along to the velocity of light. This presupposes that the motion of the observer is at right angles to that of a ray of light. If this is not his direction, we must resolve his velocity into two components, one at right angles to the ray and one parallel to it. The latter will not affect the apparent direction of the star, which will therefore depend entirely upon the former.

Effects of Aberration.—The apparent displacement of the heavenly bodies thus produced is called the *aberration of light*. Its effect is to cause each of the fixed stars to ascribe an apparent annual oscillation in a very small orbit. The nature of the displacement may be conceived of in the following way: Suppose the earth at any moment, in the course of its annual revolution, to be moving toward a point of the celestial sphere, which we may call P. Then a star lying in the direction P or in the opposite direction will suffer no displacement whatever. A star lying in any other direction will be displaced in the direction of the point P by an angle proportional to the sine of its angular distance from P. At 90° from P the displacement will be a maximum, and its angular amount will be such that its tangent will be equal to the ratio of the velocity of the earth to that of light. If A be the "aberration" of the star, and $P\,S$ its angular distance from the point P, we shall have,

$$\tan A = \frac{v}{v'} \sin P\,S,$$

v' and v being the respective velocities of light and of the earth.

Now, if the star lies near the pole of the ecliptic, its direction will always be nearly at right angles to the direction in which the earth is moving. A little consideration will show that it will seem to describe a circle in consequence of aberration. If, however, it lies in the plane of the earth's orbit, then the various points toward which the earth moves in the course of the year all lying in the ecliptic, and the star being in this same plane, the apparent motion will be an oscillation back and forth in this plane, and in all other positions the apparent motion will be in an ellipse more and more flattened as we approach the ecliptic.

Velocity of Light.—The amount of aberration can be determined in two ways. If we know the time which light requires to come from the sun to the earth, a simple calculation will enable us to determine the ratio between this velocity and that of the earth in its orbit. For instance, suppose the time to be 498 seconds; then light will cross the orbit of the earth in 996 seconds. The circumference of the earth being found by multiplying its diameter by 3·1416, we thus find that, on the supposition we have made, light would move around the circumference of the earth's orbit in 52 minutes and 8 seconds. But the earth makes this same circuit in 365¼ days, and the ratio of these two quantities is 10090. The maximum displacement of the star by aberration will therefore be the angle of which the tangent is $\frac{1}{10090}$, and this angle we find by trigonometrical calculation to be 20″·44.

This calculation presupposes that we know how long light requires to come from the sun. This is not known with great accuracy owing to the unavoidable errors with which the observations of *Jupiter's* satellites are affected. It is therefore more usual to reverse the process and determine the displacement of the stars by direct observation, and then, by a calculation the reverse of that we have just made, to determine the time required by light to reach us from the sun. Many painstaking determinations of this quantity have been made since the time of BRADLEY, and as the result of them we may say that the value of the "*constant of aberration*," as it is called, is certainly between 20″.4 and 20″.5; the chances are that it does not deviate from 20″.44 by more than two or three hundredths of a second.

It will be noticed that by determining the constant of aberration, or by observing the eclipses of the satellites of *Jupiter*, we may infer the time required for light to pass from the sun to the earth. But we cannot thus determine the velocity of light unless we know how far the sun is. The connection between this velocity and the distance of the sun is such that knowing one we can infer the other. Let us assume, for instance, that the time required for light to reach us from the sun is 498 seconds, a time which is probably accurate within a single second. Then knowing the distance of the sun, we may obtain the velocity of light by dividing it by 498. But, on the other hand, if we can determine how many miles light moves in a second, we can thence infer the distance of the sun by multiplying it by the same factor. During the last century the distance of the sun was found to be certainly between 90 and 100 millions of miles. It was therefore correctly concluded that the velocity of light was something less than 200,000 miles per second, and probably between 180,000 and 200,000. This velocity has since been determined more exactly by the direct measurements at the surface of the earth already mentioned.

CHAPTER XI.

CHRONOLOGY.

§ 1. ASTRONOMICAL MEASURERS OF TIME.

THE most intimate relation of astronomy to the daily life of mankind has always arisen from its affording the only reliable and accurate measure of long intervals of time. The fundamental units of time in all ages have been the day, the month, and the year, the first being measured by the revolution of the earth on its axis, the second, primitively, by that of the moon around the earth, and the third by that of the earth round the sun. Had the natural month consisted of an exact entire number of days, and the year of an exact entire number of months, there would have been no history of the calendar to write. There being no such exact relations, innumerable devices have been tried for smoothing off the difficulties thus arising, the mere description of which would fill a volume. We shall endeavor to give the reader an idea of the general character of these devices, including those from which our own calendar originated, without wearying him by the introduction of tedious details.

Of the three units of time just mentioned, the most natural and striking is the shortest—namely, the day. Marking as it does the regular alternations of wakefulness and rest for both man and animals, no astronomical observations were necessary to its recognition. It is so nearly uniform in length that the most refined astronomical observations of modern times have never certainly indicated any change. This uniformity, and its entire freedom from all ambiguity of meaning, have always made the day a common fundamental unit of astronomers. Except for the inconvenience of keeping count of the great number of days between remote epochs, no greater unit would ever have been necessary, and we might all date our letters by the number of days after CHRIST, or after a supposed epoch of creation.

The difficulty of remembering great numbers is such that a longer unit is absolutely necessary, even in keeping the reckoning of time for a single generation. Such a unit is the year. The regular changes of seasons in all extra-tropical latitudes renders this unit second only to the day in the prominence with which it must have struck the minds of primitive man. These changes are, however, so slow and ill-marked in their progress, that it would have been scarcely possible to make an accurate determination of the length of the year from the observation of the seasons. Here astronomical observations came to the aid of our progenitors, and, before the beginning of extant history, it was known that the alternation of seasons was due to the varying declination of the sun, as the latter seemed to perform its annual course among the stars in the "oblique circle" or ecliptic. The common people, who did not understand the theory of the sun's motion, knew that certain seasons were marked by the position of certain bright stars relatively to the sun—that is, by those stars rising or setting in the morning or evening twilight. Thus arose two methods of measuring the length of the year—the one by the time when the sun crossed the equinoxes or solstices, the other when it seemed to pass a certain point among the stars. As we have already explained, these years were slightly different, owing to the precession of the equinoxes, the first or equinoctial year being a little less and the second or sidereal year a little greater than 365¼ days.

The number of days in a year is too great to admit of their being easily remembered without any break; an intermediate period is therefore necessary. Such a period is measured by the revolution of the moon around the earth, or, more exactly, by the recurrence of new moon, which takes place, on the average, at the end of nearly 29½ days. The nearest round number to this is 30 days, and 12 periods of 30 days each only lack 5¼ days of being a year. It has therefore been common to consider a year as made up of 12 months, the lack of exact correspondence being filled by various alterations of the length of the month or of the year, or by adding surplus days to each year.

The true lengths of the day, the month, and the year having no common divisor, a difficulty arises in attempting to make months or days into years, or days into months, owing to the fractions which will always be left over. At the same time, some rule bearing on the subject is necessary in order that people may be able to remember the year, month, and day. Such rules are found by choosing some *cycle* or period which is very nearly an exact number of two units, of months and of days for example, and by dividing this cycle up as evenly as possible. The principle on which this is done can be seen at once by an example, for which we shall choose the lunar month. The true length of this month is 29·5305884 days. We see that two of these months is only a little over 59 days; so, if we take a cycle of 59 days, and divide it into two months, the one of 30 and the other of 29 days, we shall have a first approximation to a true average month. But our cycle will be too short by $0^d \cdot 061$, the excess of two months over 59 days, and this error will be added at the end of every cycle, and thus go on increasing as long as the cycle is used without change. At the end of 16 cycles, or of 32 lunar months, the accumulated error will amount to one day. At the end of this time, if not sooner, we should have to add a day to one of the months.

Seeing that we shall ultimately be wrong if we have a two-month cycle, we seek for a more exact one. Each month of 30 days is nearly $0^d.47$ too long, and each month of 29 days is rather more than $0^d.53$ too short. So in the long run the months of 30 days ought to be more numerous than those of 29 days in the ratio that 53 bears to 47, or, more exactly, in the ratio that ·5305884 bears to ·4694116. A close approximation will be had by having the long months one eighth more numerous than the short ones, the numbers in question being nearly in the ratio of 9 : 8. So, if we take a cycle of 17 months, 9 long and 8 short ones, we find that $9 \times 30 + 8 \times 29 = 502$ days for the assumed length of our cycle, whereas the true length of 17 months is very near $502^d.0200$. The error will therefore be ·02 of a day for every cycle, and will not amount to a day till the end of 50 cycles, or nearly 70 years.

A still nearer approach will be found by taking a cycle of 49 months, 26 to be long and 23 short ones. These 49 months will be composed of $26 \times 30 + 23 \times 29 =$ 1447 days, whereas 49 true lunar months will comprise 1446·998832 days. Each cycle will therefore be too long by only ·001168 of a day, and the error would not amount to a day till the end of 84 cycles, or more than 3000 years.

Although these cycles are so near the truth, they could not be used with convenience because they would begin at different times of the year. The problem is therefore to find a cycle which shall comprise an entire number of years. We shall see hereafter what solutions of this problem were actually found.

§ 2. FORMATION OF CALENDARS.

The months now or heretofore in use among the peoples of the globe may for the most part be divided into two classes:

(1.) The lunar month pure and simple, or the mean interval between successive new moons.

(2.) An approximation to the twelfth part of a year, without respect to the motion of the moon.

The Lunar Month.—The mean interval between consecutive new moons being nearly $29\frac{1}{2}$ days, it was common in the use of the pure lunar month to have months of 29 and 30 days alternately. This supposed period, however, as just shown, will fall short by a day in about $2\frac{1}{2}$ years. This defect was remedied by introducing cycles containing rather more months of 30 than of 29 days, the small excess of long months being spread uniformly through the cycle. Thus the Greeks had a cycle of 235 months (to be soon described more fully), of which 125 were full or long months, and 110 were short or deficient ones. We see that the length of this cycle was 6940 days ($125 \times 30 + 110 \times 29$), whereas the length of 235 true lunar months is $235 \times 29 \cdot 53088 = 6939 \cdot 688$ days. The cycle was therefore too long by less than one third of a day, and the error of count would amount to only one day in more than 70 years. The Mohammedans, again, took a cycle of 360 months, which they divided into 169 short and 191 long ones. The length of this cycle was 10631 days, while the true length of 360 lunar months is $10631 \cdot 012$ days. The count would therefore not be a day in error until the end of about 80 cycles, or nearly 23 centuries. This month therefore follows the moon closely enough for all practical purposes.

Months other than Lunar.—The complications of the system just described, and the consequent difficulty of making the calendar month represent the course of the moon, are so great that the pure lunar month was generally abandoned, except among people whose religion required important ceremonies at the time of new moon. In cases of such abandonment, the year has been usually divided into 12 months of slightly different lengths. The ancient Egyptians, however, had 12 months of 30 days each, to which they added 5 supplementary days at the close of each year.

Kinds of Year.—As we find two different systems of months to have been used, so we may divide the calendar years into three classes—namely:

(1.) The lunar year, of 12 lunar months.

(2.) The solar year.

(3.) The combined luni-solar year.

The Lunar Year.—We have already called attention to the fact that the time of recurrence of the year is not well marked except by astronomical phenomena which the casual observer would hardly remark. But the time of new moon, or of beginning of the month, is always well marked. Consequently, it was very natural for people to begin by considering the year as made up of twelve lunations, the error of eleven days being unnoticeable in a single year, unless careful astronomical observations were made. Even when this error was fully recognized, it might be considered better to use the regular year of 12 lunar months than to use one of an irregular or varying number of months. Such a year is the religious one of the Mohammedans to this day. The excess of 11 days will amount to a whole year in 33 years, 32 solar years being nearly equal to 33 lunar years. In this period therefore each season will have coursed through all times of the year. The lunar year has therefore been called the "wandering year."

The Solar Year.—In forming this year, the attempt to measure the year by revolutions of the moon is entirely abandoned, and its length is made to depend entirely on the change of the seasons. The solar year thus indicated is that most used in both ancient and modern times. Its length has been known to be nearly 365¼ days from the times of the earliest astronomers, and the system adopted in our calendar of having three years of 365 days each, followed by one of 366 days, has been employed in China from the remotest historic times. This year of 365¼ days is now called by us the *Julian Year*, after JULIUS CÆSAR, from whom we obtained it.

The Luni-Solar Year.—If the lunar months must, in some way, be made up into solar years of the proper average length, then these years must be of unequal length, some having twelve months and others thirteen. Thus, a period or cycle of eight years might be made up of 99 lunar months, 5 of the years having 12 months each, and 3 of them 13 months each. Such a period would comprise 2923½ days, so that the average length of the year would be 365 days 10½ hours. This is too great by about 4 hours 42 minutes. This very plan was proposed in ancient Greece, but it was superseded by the discovery of the *Metonic Cycle*, which figures in our church calendar to this day. A luni-solar year of this general character was also used by the Jews.

The Metonic Cycle.—The preliminary considerations we have set forth will now enable us to understand the origin of our own calendar. We begin with the Metonic Cycle of the ancient Greeks, which still regulates some religious festivals, although it has disappeared from our civil reckoning of time. The necessity of employing lunar months caused the Greeks great difficulty in regulating their calendar so as to accord with their rules for religious feasts, until a solution of the problem was found by METON, about 433 B.C. The great discovery of METON was that a period or cycle of 6940 days could be divided up into 235 lunar months, and also into 19 solar years. Of these months, 125 were to be of 30 days each, and 110 of 29 days each, which would, in all, make up the required 6940 days. To see how nearly this rule represents the actual motions of the sun and moon, we remark that:

	Days.	Hours.	Min.
235 lunations require	6939	16	31
19 Julian years "	6939	18	0
19 true solar years require	6939	14	27

We see that though the cycle of 6940 days is a few hours too long, yet, if we take 235 true lunar months, we find their whole duration to be a little less than 19 Julian years of 365¼ days each, and a little more than 19 true solar years.

The problem now was to take these 235 months and divide them up into 19 years, of which 12 should have 12 months each, and 7 should have 13 months each. The long years, or those of 13 months, were probably those corresponding to the numbers 3, 5, 8, 11, 13, 16, and 19, while the first, second, fourth, sixth, etc., were short years. In general, the months had 29 and 30 days alternately, but it was necessary to substitute a long month for a short one every two or three years, so that in the cycle there should be 125 long and 110 short months.

Golden Number.—This is simply the number of the year in the Metonic Cycle, and is said to owe its appellation to the enthusiasm of the Greeks over METON'S discovery, the authorities having ordered the division and numbering of the years in the new calendar to be inscribed on public monuments in letters of gold. The rule for finding the golden number is to divide the number of the year by 19, and add 1 to the remainder. From 1881 to 1899 it may be found by simply subtracting 1880 from the year. It is employed in our church calendar for finding the time of Easter Sunday.

Period of Callypus.—We have seen that the cycle of 6940 days is a few hours too long either for 235 lunar months or for 19 solar years. CALLYPUS therefore sought to improve it by taking one day off of every fourth cycle, so that the four cycles should have 27759 days, which were to be divided into 940 months and into 76 years. These years would then be Julian years, while the recurrence of new moon would only be six hours in error at the end of the 76 years. Had he taken a day from every third cycle, and from some year and month of that cycle, he would have been yet nearer the truth.

The Mohammedan Calendar.—Among the most remarkable calendars which have remained in use to the present time is that of the Mohammedans. The year is composed of 12 lunar months, and therefore, as already mentioned, does not correspond to the course of the seasons. As with other systems, the problem is to find such a cycle that an entire number of these lunar years shall correspond to an integral number of days. Multiplying the length of the lunar month by 12, we find the true length of the lunar year to be 354·36706 days. The fraction of a day being not far from one third, a three-year cycle, comprising two years of 354 and one of 355 days, would be a first approximation to three lunar years, but would still be one tenth of a day too short. In ten such cycles or thirty years, this deficiency would amount to an entire day, and by adding the day at the end of each tenth three-year cycle, a very near approach to the true motion of the moon will be obtained. This thirty-year cycle will consist of 10631 days, while the true length of 360 lunar months is 10631·0116 days. The error will not amount to a day until the end of 87 cycles, or 2610 years, so that this system is accurate enough for all practical purposes. The common Mohammedan year of 354 days is composed of months containing alternately 30 and 29 days, the first having 30 and the last 29. In the years of 355 days the alternation is the same, except that one day is added to the last month of the year.

The old custom was to take for the first day of the month that following the evening on which the new moon could first be seen in the west. It is said that before the exact arrangement of the Mohammedan calendar had been completed, the rule was that the visibility of the crescent moon should be certified by the testimony of two witnesses. The time of new moon given in our modern almanacs is that when the moon passes nearly between us and the sun, and is therefore entirely invisible. The moon is generally one or two days old before it can be seen in the evening, and, in consequence, the lunar month of the Mohammedans and of others commences about two days after the actual almanac time of new moon.

The civil calendar now in use throughout Christendom had its origin among the Romans, and its foundation was laid by JULIUS CÆSAR. Before his time, Rome can hardly be said to have had a chronological system, the length of the year not being prescribed by any invariable rule, and being therefore changed from time to time to suit the caprice or to compass the ends of the rulers. Instances of this tampering disposition are familiar to the historical student. It is said, for instance, that the Gauls having to pay a certain monthly tribute to the Romans, one of the governors ordered the year to be divided into 14 months, in order that the pay days might recur more rapidly. To remedy this, CÆSAR called in the aid of SOSIGENES, an astronomer of the Alexandrian school, and by them it was arranged that the year should consist of 365 days, with the addition of one day to every fourth year. The old Roman months were afterward adjusted to the Julian year in such a way as to give rise to the somewhat irregular arrangement of months which we now have.

Old and New Styles.—The mean length of the Julian year is $365\frac{1}{4}$ days, about $11\frac{1}{4}$ minutes greater than that of the true equinoctial year, which measures the recurrence of the seasons. This difference is of little practical importance, as it only amounts to a week in a thousand years, and a change of this amount in that period is productive of no inconvenience. But, desirous to have the year as correct as possible, two changes were introduced into the calendar by Pope GREGORY XIII. with this object. They were as follows:

1. The day following October 4, 1582, was called the 15th instead of the 5th, thus advancing the count 10 days.

2. The closing year of each century, 1600, 1700, etc., instead of being always a leap year, as in the Julian calendar, is such only when the number of the century is divisible by 4. Thus while 1600 remained a leap year, as before, 1700, 1800, and 1900 were to be common years.

This change in the calendar was speedily adopted by all

Catholic countries, and more slowly by Protestant ones, England holding out until 1752. In Russia it has never been adopted at all, the Julian calendar being still continued without change. The Russian reckoning is therefore 12 days behind ours, the ten days dropped in 1582 being increased by the days dropped from the years 1700 and 1800 in the new reckoning. This modified calendar is called the *Gregorian Calendar*, or *New Style*, while the old system is called the *Julian Calendar*, or *Old Style*.

It is to be remarked that the practice of commencing the year on January 1st was not universal until comparatively recent times. During the first sixteen centuries of the Julian calendar there was such an absence of definite rules on this subject, and such a variety of practice on the part of different powers, that the simple enumeration of the times chosen by various governments and pontiffs for the commencement of the year would make a tedious chapter. The most common times of commencing were, perhaps, March 1st and March 22d, the latter being the time of the vernal equinox. But January 1st gradually made its way, and became universal after its adoption by England in 1752.

Solar Cycle and Dominical Letter.—In our church calendars January 1st is marked by the letter A, January 2d by B, and so on to G, when the seven letters begin over again, and are repeated through the year in the same order. Each letter there indicates the same day of the week throughout each separate year, A indicating the day on which January 1st falls, B the day following, and so on. An exception occurs in leap years, when February 29th and March 1st are marked by the same letter, so that a change occurs at the beginning of March. The letter corresponding to Sunday on this scheme is called the *Dominical* or Sunday letter, and, when we once know what letter it is, all the Sundays of the year are indicated by that letter, and hence all the other days of the week by their letters. In leap years there will be two Dominical letters, that for the last ten months of the year being the one next preceding the letter for January and February. In the Julian calendar the Dominical letter must always recur at the end of 28 years (besides three recurrences at unequal intervals in the mean time). This period is called the *solar cycle*, and determines the days of the week on which the days of the month fall during each year.

Since any day of the year occurs one day earlier in the week than it did the year before, or two days earlier when a 29th of February has intervened, the Dominical letters recur in the order G, F, E, D, C, B, A, G, etc. A similar fact may be expressed by saying that any day of the year occurs one day later in the week for every year that has elapsed, and, in addition, one day later for every 29th of February that has intervened. This fact will make it easy to calculate the day of the week on which any historical event happened from the day corresponding in any past or future year. Let us take the following example:

On what day of the week was WASHINGTON born, the date being 1732, February 22d, knowing that February 22d, 1879, fell on Saturday. The interval is 147 years: dividing by 4 we have a quotient of 36 and a remainder of 3, showing that, had every fourth year in the interval been a leap year, there were either 36 or 37 leap years. As a February 29th followed only a week after the date, the number must be 37;* but as 1800 was dropped from the list of leap years, the number was really only 36. Then $147 + 36 = 183$ days advanced in the week. Dividing by 7, because the same day of the week recurs after seven days, we find a remainder of 1. So February 22d, 1879, is one day further advanced than was February 22d, 1732; so the former being Saturday, WASHINGTON was born on Friday.

* Perhaps the most convenient way of deciding whether the remainder does or does not indicate an additional leap year is to subtract it from the last date, and see whether a February 29th then intervenes. Subtracting 3 years from February 22d, 1879, we have February 22d, 1876, and a 29th occurs between the two dates, only a week after the first.

§ 3. DIVISION OF THE DAY.

The division of the day into hours was, in ancient and mediæval times, effected in a way very different from that which we practice. Artificial time-keepers not being in general use, the two fundamental moments were sunrise and sunset, which marked the day as distinct from the night. The first subdivision of this interval was marked by the instant of noon, when the sun was on the meridian. The day was thus subdivided into two parts. The night was similarly divided by the times of rising and culmination of the various constellations. EURIPIDES (480–407 B.C.) makes the chorus in *Rhesus* ask :

> "CHORUS.—Whose is the guard? Who takes my turn? *The first signs are setting, and the seven Pleiades are in the sky, and the Eagle glides midway through heaven.* Awake! Why do you delay? Awake from your beds to watch! See ye not the brilliancy of the moon? Morn, morn indeed is approaching, and *hither is one of the forerunning stars.*" —The Tragedies of Euripides. Literally Translated by T. A. Buckley. London: H. G. Bohn. 1854. Vol. 2, p. 322.

The interval between sunrise and sunset was divided into twelve equal parts called hours, and as this interval varied with the season, the length of the hour varied also. The night, whether long or short, was divided into hours of the same character, only, when the night hours were long, those of the day were short, and *vice versa.* These variable hours were called *temporary hours.* At the time of the equinoxes, both the day and the night hours were of the same length with those we use—namely, the twenty-fourth part of the day; these were therefore called *equinoctial hours.*

The use of these temporary hours was intimately associated with the time of beginning of the day. Instead of commencing the civil day at midnight, as we do, it was customary to commence it at sunset. The Jewish Sabbath, for instance, commenced as soon as the sun set on Friday, and ended when it set on Saturday. This made a more distinctive division of the astronomical day than that which we employ, and led naturally to considering the *day* and the *night* as two distinct periods, each to be divided into 12 hours.

So long as temporary hours were used, the beginning of the day and the beginning of the night, or, as we should call it, six o'clock in the morning and six o'clock in the evening, were marked by the rising and setting of the sun; but when equinoctial hours were introduced, neither sunrise nor sunset could be taken to count from, because both varied too much in the course of the year. It therefore became customary to count from noon, or the time at which the sun passed the meridian. The old custom of dividing the day and the night each into 12 parts was continued, the first 12 being reckoned from midnight to noon, and the second from noon to midnight. The day was made to commence at midnight rather than at noon for obvious reasons of convenience, although noon was of course the point at which the time had to be determined.

Equation of Time.—To any one who studied the annual motion of the sun, it must have been quite evident that the intervals between its successive passages over the meridian, or between one noon and the next, could not be the same throughout the year, because the apparent motion of the sun in right ascension is not constant. It will be remembered that the apparent revolution of the starry sphere, or, which is the same thing, the diurnal revolution of the earth upon its axis, may be regarded as absolutely constant for all practical purposes. This revolution is measured around in right ascension as explained in the opening chapter of this work. If the sun increased its right ascension by the same amount every day, it would pass the meridian 3′ 56″ later every day, as measured by sidereal time, and hence the intervals between successive passages would be equal. But the motion of the sun in right ascension is unequal from two causes: (1) the unequal motion of the earth in its annual revolution around it, arising from the eccentricity of the orbit, and (2) the obliquity of the ecliptic. How the first cause produces an inequality is obvious, and its approximate amount is readily computed. We have seen that the angular velocity of a planet around the sun is inversely as the square of its radius vector. Taking the distance of the earth from the sun as unity, and putting e for the eccentricity of its orbit, its greatest distance about the end of June is $1 + e = 1 \cdot 0168$, and its least distance about the end of December is $1 - 0 \cdot 0168$. The squares of these quantities are $1 \cdot 034$ and $1 - \cdot 034$ very nearly; therefore the motion is about one thirtieth greater than the mean in December and one thirtieth less in June. The mean motion is $3^m\ 56^s$; the actual motion therefore varies from $3^m\ 48^s$ to $4^m\ 4^s$.

The effect of the obliquity of the ecliptic is still greater. When the sun is near the equinox, its motion along the ecliptic makes an angle of $23\frac{1}{2}°$ with the parallels of declination. Since its motion in right ascension is reckoned along the parallel of declination, we see that it is equal to the motion in longitude multiplied by the cosine of $23\frac{1}{2}°$. This cosine is less than unity by about $\cdot 07$; therefore at the times of the equinox the mean motion is diminished by this fraction, or by 20 seconds. Therefore the days are then 20 seconds shorter than they would be were there no obliquity. At the solstices the opposite effect is produced. Here the different meridians of right ascension are nearer together than they are at the equator in the proportion of the cosine of $23\frac{1}{2}°$ to unity; therefore, when the sun moves through one degree along the ecliptic, it changes its right ascension by $1 \cdot 08°$; here, therefore, the days are about 19 seconds longer than they would be if the obliquity of the ecliptic was zero.

We thus have to recognize two slightly different kinds of days: *solar* days and *mean* days. A solar day is the interval of time between two successive transits of the sun over the same meridian, while a mean day is the mean of all the solar days in a year. If we had two clocks, the one going with perfect uniformity, but regulated so as to keep as near the sun as possible, and the other changing its rate so as to always follow the sun, the latter would gain or lose on the former by amounts sometimes rising to 22 seconds in a day. The accumulation of these variations through a period of several months would lead to such deviations that the sun-clock would be 14 minutes slower than the other during the first half of February, and 16 minutes faster during the first week in November. The time-keepers formerly used were so imperfect that these inequalities in the solar day were nearly lost in the necessary irregularities of the rate of the clock. All clocks were therefore set by the sun as often as was found necessary or convenient. But during the last century it was found by astronomers that the use of units of time varying in this way led to much inconvenience; they therefore substituted *mean* time for solar or *apparent* time.

Mean time is so measured that the hours and days shall always be of the same length, and shall, on the average, be as much behind the sun as ahead of it. We may imagine a fictitious or mean sun moving along the equator at the rate of 3^m 56^s in right ascension every day. Mean time will then be measured by the passage of this fictitious sun across the meridian. Apparent time was used in ordinary life after it was given up by astronomers, because it was very easy to set a clock from time to time as the sun passed a noon-mark. But when the clock was so far improved that it kept much better time than the sun did, it was found troublesome to keep putting it backward and forward, so as to agree with the sun. Thus mean time was gradually introduced for all the purposes of ordinary life except in very remote country districts, where the farmers may find it more troublesome to allow for an equation of time than to set their clocks by the sun every few days.

The common household almanac should give the equation of time, or the mean time at which the sun passes the meridian, on each day of the year. Then, if any one wishes to set his clock, he knows the moment of the sun passing the meridian, or being at some noon-mark, and sets his time-piece accordingly. For all purposes where accurate time is required, recourse must be had to astronomical observation. It is now customary to send time-signals every day at noon, or some other hour agreed upon, from observatories along the principal lines of telegraph. Thus at the present time the moment of Washington noon is signalled to New York, and over the principal lines of railway to the South and West. Each person within reach of a telegraph-office can then determine his local time by correcting these signals for the difference of longitude.

§ 4. REMARKS ON IMPROVING THE CALENDAR.

It is an interesting question whether our calendar, this product of the growth of ages, which we have so rapidly described, would admit of decided improvement if we were free to make a new one with the improved materials of modern science. This question is not to be hastily answered in the affirmative. Two small improvements are undoubtedly practicable: (1) a more regular division of the 365 days among the months, giving February 30 days, and so having months of 30 and 31 days only; (2) putting the additional day of leap year at the end of the year instead of at the end of February. The smallest change from our present system would be made by taking the two additional days for February, the one from the end of July, and the other from the end of December, leaving the last with 30 days in common years and 31 in leap years. When we consider more radical changes than this, we find advantages set off by disadvantages. For instance, it would on some accounts be very convenient to divide the year into 13 months of 4 weeks each, the last month having one or two extra days. The months would then begin on the same day of the week through each year, and would admit of a much more convenient subdivision into halves and quarters than they do now. But the year would not admit of such a subdivision without dividing the months also, and it is possible that this inconvenience would balance the conveniences of the plan.

An actual attempt in modern times to form an entirely new calendar is of sufficient historic interest to be mentioned in this connection. We refer to the so-called Republican Calendar of revolutionary France. The year sometimes had 365 and sometimes 366 days, but instead of having the leap years at defined intervals, one was inserted whenever it might be necessary to make the autumnal equinox fall on the first day of the year. The division of the year was effected after the plan of the ancient Egyptians, there being 12 months of 30 days each, followed by 5 or 6 supplementary days to complete the year, which were kept as feast-days.* The sixth day of course occurred only in the leap years, or *Franciads* as they were called. It was called the Day of the Revolution, and was set apart for a quadrennial oath to remain free or die.

No attempt was made to fit the new calendar to the old one, or to render the change natural or convenient. The year began with the autumnal equinox, or September 22d of the Gregorian calendar; entirely new names were given to the months; the week was abolished, and in lieu of it the month was divided into three decades, the last or tenth day of each decade being a holiday set apart for the adoration of some sentiment. Even the division of the day into 24 hours was done away with, and a division into ten hours was substituted.

The Republican Calendar was formed in 1793, the year 1 commencing on September 22d, 1792, and it was abolished on January 1st, 1806, after 13 years of confusion.

* They received the nickname of *sans-culottides*, from the opponents of the new state of things.

§ 5. THE ASTRONOMICAL EPHEMERIS, OR NAUTICAL ALMANAC.

The *Astronomical Ephemeris*, or, as it is more commonly called, the *Nautical Almanac*, is a work in which celestial phenomena and the positions of the heavenly bodies are computed in advance. The need of such a work must have been felt by navigators and astronomers from the time that astronomical predictions became sufficiently accurate to enable them to determine their position on the surface of the earth. At first works of this class were prepared and published by individual astronomers who had the taste and leisure for this kind of labor. MANFREDI, of Bonn, published *Ephemerides* in two volumes, which gave the principal aspects of the heavens, the positions of the stars, planets, etc., from 1715 until 1725. This work included maps of the civilized world, showing the paths of the principal eclipses during this interval.

The usefulness of such a work, especially to the navigator, depends upon its regular appearance on a uniform plan and upon the fulness and accuracy of its data; it was therefore necessary that its issue should be taken up as a government work. Of works of this class still issued the earliest was the *Connaissance des Temps* of France, the first volume of which was published by PICARD in 1679, and which has been continued without interruption until the present time. The publication of the British *Nautical Almanac* was commenced in the year 1767 on the representations of the Astronomer Royal showing that such a work would enable the navigator to determine his longitude within one degree by observations of the moon. An astronomical or nautical almanac is now published annually by each of the governments of Germany, Spain, Portugal, France, Great Britain, and the United States. They have gradually increased in size and extent with the advancing wants of the astronomer until those of Great Britain and this country have become octavo volumes of between 500 and 600 pages. These two are published three years or more beforehand, in order that navigators going on long voyages may supply themselves in advance. The *American Ephemeris and Nautical Almanac* has been regularly published since 1855, the first volume being for that year. It is designed for the use of navigators the world over, and the greater part of it is especially arranged for the use of astronomers in the United States.

The immediate object of publications of this class is to enable the wayfarer and traveller upon land and the voyager upon the ocean to determine their positions by observations of the heavenly bodies. Astronomical instruments and methods of calculation have been brought to such a degree of perfection that an astronomer, armed with a nautical almanac, a chronometer regulated to Greenwich or Washington time, a catalogue of stars, and the necessary instruments of observation, can determine his position at any point on the earth's surface within a hundred yards by a single night's observations. If his chronometer is not so regulated, he can still determine his latitude, but not his longitude. He could, however, obtain a rough idea of the latter by observations upon the planets, and come within a very few miles of it by a single observation on the moon.

The Ephemeris furnishes the fundamental data from which all our household almanacs are calculated.

The principal quantities given in the American Ephemeris for each year are as follows:

The positions of the sun and the principal large planets for Greenwich noon of every day in each year.

The right ascension and declination of the moon's centre for every hour in the year.

The distance of the moon from certain bright stars and planets for every third hour of the year.

The right ascensions and declinations of upward of two hundred of the brighter fixed stars, corrected for precession, nutation, and aberration, for every ten days.

The positions of the principal planets at every visible transit over the meridian of Washington.

Complete elements of all the eclipses of the sun and moon, with maps showing the passage of the moon's shadow or penumbra over those regions of the earth where the eclipses will be visible, and tables whereby the phases of the eclipses can be accurately computed for any place.

Tables for predicting the occultations of stars by the moon.

Eclipses of *Jupiter's* satellites and miscellaneous phenomena.

To give the reader a still further idea of the *Ephemeris*, we present a small portion of one of its pages for the year 1882:

FEBRUARY, 1882—AT GREENWICH MEAN NOON.

Day of the week.	Day of the month.	The Sun's Apparent right ascension.			The Sun's Diff. for 1 hour.	The Sun's Apparent declination.			The Sun's Diff. for 1 hour.	Equation of time to be subtracted from mean time.		Diff. for 1 hr.	Sidereal time or right ascension of mean sun.		
		H.	M.	S.	S.	°	′	″	″	M.	S.	S.	H.	M.	S.
Wed.	1	21	0	12·04	10·175	S 17	2	23·4	+42·88	13	51·34	0·318	20	46	21·70
Thur.	2	21	4	16·84	10·141	16	45	5·4	43·57	13	58·58	0·284	20	50	18·26
Frid.	3	21	8	19·82	10·107	16	27	30·9	44·30	14	5·01	0·250	20	54	14·81
Sat.	4	21	12	21·96	10·073	16	9	39·2	+44·99	14	10·61	0·216	20	58	11·37
Sun.	5	21	16	23·38	10·040	15	51	30·8	45·69	14	15·41	0·183	21	2	7·92
Mon.	6	21	20	23·88	10·007	15	33	6·1	46·38	14	19·40	0·150	21	6	4·48
Tues.	7	21	24	23·62	9·974	15	14	26·4	+47·03	14	22·60	0·117	21	10	1·03
Wed.	8	21	28	22·60	9·941	14	55	39·1	47·66	14	25·01	0·084	21	13	57·58
Thur.	9	21	32	20·79	9·909	14	36	17·7	48·28	14	26·65	0·052	21	17	54·14
Frid.	10	21	36	18·21	9·877	14	16	51·6	48·88	14	27·51	0·020	21	21	50·70
Sat.	11	21	40	14·86	9·846	13	57	11·2	49·47	14	27·63	0·011	21	25	47·25
Sun.	12	21	44	10·80	9·815	13	37	16·9	50·03	14	26·99	0·042	21	29	43·81
Mon.	13	21	48	5·98	9·784	13	17	9·1	+50·59	14	25·62	0·073	21	33	40·36
Tues.	14	21	52	0·42	9·753	12	56	48·3	51·12	14	23·53	0·104	21	37	36·91
Wed.	15	21	55	54·16	9·723	12	36	14·9	51·63	14	20·70	0·134	21	41	33·46
Thur.	16	21	59	47·17	9·693	12	15	28·8	+52·14	14	17·15	0·164	21	45	30·02
Frid.	17	22	3	39·47	9·664	11	54	30·1	52·62	14	12·90	0·193	21	49	26·57
Sat.	18	22	7	31·07	9·635	11	33	20·6	53·07	14	7·94	0·222	21	53	23·12

Of the same general nature with the Ephemeris are catalogues of the fixed stars. The object of such a catalogue is to give the right ascension and declination of a number of stars for some epoch, the beginning of the year 1875 for instance, with the data by which the position of a star can be found at any other epoch. Such catalogues are, however, imperfect owing to the constant small changes in the positions of the stars and the errors and imperfections of the older observations. In consequence of these imperfections, a considerable part of the work of the astronomer engaged in accurate determinations of geographical positions consist in finding the most accurate positions of the stars which he makes use of.

PART II.

THE SOLAR SYSTEM IN DETAIL.

CHAPTER I.

STRUCTURE OF THE SOLAR SYSTEM.

THE solar system, as it is known to us through the discoveries of COPERNICUS, KEPLER, NEWTON and their successors, consists of the sun as a central body, around which revolve the major and minor planets, with their satellites, a few periodic comets, and an unknown number of meteor swarms. These are permanent members of the system. At times other comets appear, and move usually in parabolas through the system, around the sun, and away from it into space again, thus visiting the system without being permanent members of it.

The bodies of the system may be classified as follows:

1. The central body—the Sun.
2. The four inner planets—*Mercury*, *Venus*, the *Earth*, *Mars*.
3. A group of small planets, sometimes called *Asteroids*, revolving outside of the orbit of *Mars*.
4. A group of four outer planets—*Jupiter*, *Saturn*, *Uranus*, and *Neptune*.
5. The satellites, or secondary bodies, revolving about the planets, their primaries.
6. A number of comets and meteor swarms revolving in very eccentric orbits about the Sun.

The eight planets of Groups 2 and 4 are sometimes classed together as the *major planets*, to distinguish them from the two hundred or more *minor planets* of Group 3. The formal definitions of the various classes, laid down by Sir WILLIAM HERSCHEL in 1802, are worthy of repetition :

Planets are celestial bodies of a certain very considerable size.

They move in not very eccentric ellipses about the sun.

The planes of their orbits do not deviate many degrees from the plane of the earth's orbit.

Their motion about the sun is direct.

They may have satellites or rings.

They have atmospheres of considerable extent, which, however, bears hardly any sensible proportion to their diameters.

Their orbits are at certain considerable distances from each other.

Asteroids, now more generally known as *small* or *minor planets*, are celestial bodies which move about the sun in orbits, either of little or of considerable eccentricity, the planes of which orbits may be inclined to the ecliptic in any angle whatsoever. They may or may not have considerable atmospheres.

Comets are celestial bodies, generally of a very small mass, though how far this may be limited is yet unknown.

They move in very eccentric ellipses or in parabolic arcs about the sun.

The planes of their motion admit of the greatest variety in their situation.

The direction of their motion is also totally undetermined.

They have atmospheres of very great extent, which show themselves in various forms as tails, coma, haziness, etc.

Relative Sizes of the Planets.—The comparative sizes of the major planets, as they would appear to an observer situated at an equal distance from all of them, is given in the following figure.

The relative apparent magnitudes of the sun, as seen from the various planets, is shown in the next figure.

Flora and *Mnemosyne* are two of the asteroids.

A curious relation between the distances of the planets, known as Bode's law, deserves mention. If to the numbers,

0, 3, 6, 12, 24, 48, 96, 192, 384,

each of which (the second excepted) is twice the preceding, we add 4, we obtain the series,

4, 7, 10, 16, 28, 52, 100, 196, 388.

These last numbers represent approximately the dis-

FIG. 75.—APPARENT MAGNITUDES OF THE SUN AS SEEN FROM DIFFERENT PLANETS.

tances of the planets from the sun (except for *Neptune*, which was not discovered when the so-called law was announced).

This is shown in the following table:

is twice the preced-

, 196, 388.

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HE SUN AS SEEN FROM DIF-
ETS.

un (except for *Neptune*,
ne so-called law was an-

table:

PLANETS.	Actual Distance.	Bode's Law.
Mercury	3·9	4·0
Venus	7·2	7·0
Earth	10·0	10·0
Mars	15·2	16·0
[Ceres]	27·7	28·0
Jupiter	52·0	52·0
Saturn	95·4	100.0
Uranus	191·8	196·0
Neptune	300·4	388·0

It will be observed that *Neptune* does not fall within this ingenious scheme. *Ceres* is one of the minor planets.

The relative brightness of the sun and the various planets has been measured by ZÖLLNER, and the results are given below. The column *per cent* shows the percentage of error indicated in the separate results :

SUN AND	Ratio : 1 to	Percent. of Error.
Moon	618,000	1·6
Mars	6,994,000,000	5·8
Jupiter	5,472,000,000	5·7
Saturn (ball alone)	130,980,000,000	5·0
Uranus	8,486,000,000,000	6·0
Neptune	[illegible]9,680,000,000,000	5·5

The differences in the density, size, mass and distance of the several planets, and in the amount of solar light and heat which they receive, are immense. The distance of *Neptune* is eighty times that of *Mercury*, and it receives only [illegible] as much light and heat from the sun. The density of the earth is about six times that of water, while *Saturn's* mean density is less than that of water.

The mass of the sun is far greater than that of any single planet in the system, or indeed than the combined mass of all of them. In general, it is a remarkable fact that the mass of any given planet exceeds the sum of the masses of all the planets of less mass than itself. This is shown in the following table, where the masses of the planets are taken as fractions of the sun's mass, which we here express as 1,000,000,000:

Mercury.	Mars.	Venus.	Earth.	Uranus.	Neptune.	Saturn.	Jupiter.	Sun.	PLANETS.
200	324	2,353	3,060	44,250	51,600	285,580,	954,305	1,000,000,000	Masses.

The mass of Mercury is less than the mass of Mars:	200 <	324
The sum of masses of Mercury and Mars is less than the mass of Venus:	524 <	2,353
Mercury + Mars + Venus < Earth:	2,877 <	3,060
Mercury + Mars + Venus + Earth < Uranus:	5,937 <	44,250
Mercury + Mars + Venus + Earth + Uranus < Neptune:	50,187 <	51,600
Mercury + Mars + Venus + Earth + Uranus + Neptune < Saturn:	101,787 <	285,580
Mercury + Mars + Venus + Earth + Uranus + Neptune + Saturn < Jupiter:	387,367 <	954,305
Combined mass of all the planets is less than that of the Sun:	1,341,672 <	1,000,000,000

The total mass of the small planets, like their number, is unknown, but it is probably less than one thousandth that of our earth, and would hardly increase the sum total of the above masses of the solar system by more than one or two units. The sun's mass is thus over 700 times that of all the other bodies, and hence the fact of its central position in the solar system is explained. In fact, the *centre of gravity* of the whole solar system is very little outside the body of the sun, and will be inside of it when *Jupiter* and *Saturn* are in opposite directions from it.

Planetary Aspects.—The motions of the planets about the sun have been explained in Chapter IV. From what is there said it appears that the best time to see one of the outer planets will be when it is in opposition—that is, when its geocentric longitude or its right ascension differs 180° or 12^h from that of the sun. At such a time the planet will rise at sunset and culminate at midnight. During the three months following opposition, the planet will rise from three to six minutes earlier every day, so that, knowing when a planet is in opposition, it is easy to find it at any other time. For example, a month after opposition the

FIG. 76.

planet will be two to three hours high about sunset, and will culminate about nine or ten o'clock. Of course the inner planets never come into opposition, and hence are best seen about the times of their greatest elongations.

The above figure gives a rough plan of part of the solar system as it would appear to a spectator immediately above or below the plane of the ecliptic.

It is drawn approximately to scale, the mean distance of the earth (= 1) being half an inch. The mean distance of *Saturn* would be 4·77 inches, of *Uranus* 9·59 inches, of *Neptune* 15·03 inches. On the same scale the distance of the nearest fixed star would be 103,133 inches, or over one and one half miles.

The arrangement of the planets and satellites is then—

The Inner Group.	Asteroids.	The Outer Group.
Mercury.	200 minor planets, and probably many more.	Jupiter and 4 moons.
Venus.		Saturn and 8 moons.
Earth and Moon.		Uranus and 4 moons.
Mars and 2 moons.		Neptune and 1 moon.

To avoid repetitions, the elements of the major planets and other data are collected into the two following tables, to which reference may be made by the student. The units in terms of which the various quantities are given are those familiar to us, as miles, days, etc., yet some of the distances, etc., are so immensely greater than any known to our daily experience that we must have recourse to illustrations to obtain any idea of them at all. For example, the distance of the sun is said to be 92½ *million* miles. It is of importance that some idea should be had of this distance, as it is the unit, in terms of which not only the distances in the solar system are expressed, but which serves as a basis for measures in the stellar universe. Thus when we say that the distance of the stars is over 200,000 times the mean distance of the sun, it becomes necessary to see if some conception can be obtained of one factor in this. Of the abstract number, 92,500,000, we have no conception. It is far too great for us to have counted. We have never taken in at one view, even a million similar discrete objects. To count from 1 to 200 requires, with very rapid counting, 60 seconds. Suppose this kept up for a day without intermission; at the end we should have counted 288,000, which is about $\frac{1}{321}$ of 92,500,000. Hence over 10 months' uninterrupted counting by night and day would be required simply to enumerate the *number*, and long before the expiration of the task all *idea* of it would have vanished. We may take other and perhaps more striking examples. We know, for instance, that the time of the fastest express-trains between New York and Chicago, which average 40 miles per hour, is about a day. Suppose such a train to start for the sun and to continue running at this rapid rate. It would take 363 years for the journey. Three hundred and sixty-three years ago there was not a European settlement in America.

A cannon-ball moving continuously across the intervening space at its highest speed would require about nine years to reach the sun. The report of the cannon, if it could be conveyed to the sun with the velocity of sound in air, would arrive there five years after the projectile. Such a distance is entirely inconceivable, and yet it is only a small fraction of those with which astronomy has to deal, even in our own system. The distance of *Neptune* is 30 times as great.

If we examine the dimensions of the various orbs, we meet almost equally inconceivable numbers. The diameter of the sun is 860,000 miles; its radius is but 430,000, and yet this is nearly twice the mean distance of the moon from the earth. Try to conceive, in looking at the moon in a clear sky, that if the centre of the sun could be placed at the centre of the earth, the moon would be far within the sun's surface. Or again, conceive of the force of gravity at the surface of the various bodies of the system. At the sun it is nearly 28 times that known to us. A pendulum beating seconds here would, if transported to the sun, vibrate with a motion more rapid than that of a watch-balance. The muscles of the strongest man would not support him erect on the surface of the sun: even lying down he would crush himself to death under his own weight of two tons. We may by these illustrations get some rough idea of the meaning of the numbers in these tables, and of the incapability of our limited ideas to comprehend the true dimensions of even the solar system.

ELEMENTS OF THE ORBITS OF THE EIGHT MAJOR PLANETS FOR 1850.

NAME.	Mean Motion in 365¼ Days.	Mean Distance from Sun. Astronomical Units.	Mean Distance from Sun. Millions of Miles.	Eccentricity of Orbit.	Longitude of Perihelion. ° ′ ″	Inclination to Ecliptic. ° ′ ″	Longitude of the Node. ° ′ ″	Mean Longitude of Planet, 1849. Dec. 9, 21·6. ° ′ ″	Authority.
Mercury...	5381016·2925	0·3870988	35½	·20560478	75 7 13·8	7 0 7·71	46 33 8·6	323 11 23·53	Leverrier.
Venus..	2106641·3960	0·7233332	66½	·00684331	129 27 14·4	3 23 34·83	75 19 52·2	243 57 44·34	Leverrier.
	2106641·3040	0·723	.....	·00684311	129 27 42·9	3 23 35·01	75 19 53·1	243 57 23·82	G. W. Hill.
Earth...	1295977·4200	1.0	92½	·01677110	100 21 21·4	....	....	99 48 18·66	Leverrier.
	1295977·4312	1·0	.....	·01677120	100 21 41·0	....	....	99 48 17·71	Hansen.
Mars.....	689050·8012	1·5236914	141	·09326113	333 17 53·5	1 51 2·28	48 23 53·0	83 9 16·92	Leverrier.
Jupiter...	109256·6197	5·202800	481	·0482519	11 54 58·2	1 18 41·37	98 56 16·9	159 56 12·94	Leverrier.
Saturn..	43996·0586	9·538852	882	·0559428	90 6 56·5	2 29 39·80	112 20 52·9	14 50 28·49	Leverrier.
	43996·209	9·5388	.....	·0560470	90 8 59·8	2 29 39·20	112 20 0·0	14 49 43·50	G. W. Hill.
Uranus...	15424·797	19·18338	1,774	·0463592	170 38 48·7	0 46 20·29	73 14 37·6	29 13 43·73	Newcomb.
Neptune...	7865·862	30·05437	2,780	·0089903	49 9 13·1	1 46 58·75	130 7 18·3	334 30 5·75	Newcomb.

Name.	Masses.	Mean Angular Semidiameters.		Angular Diameters at Distance Unity.				Mean Diameter in Miles.	Density.		Axial Rotation.	Gravity at Surface ⊕ = 1.	Periodic Time.	Orbital Velocity in Miles per Second.
				Polar.		Equatorial.			Water = 1.	Earth = 1.				
		"	At Dist.	'	"	'	"						Days.	
Sun......	Unity.	960·0	1·00	32	0·00	32	0·00	860,000	1·444	0·2553	25ᵈ 5ᵐ 38ˢ	27·71	......	......
Mercury...	[illegible]	3·34	1·00	0	6·68	0	6·68	2,992	6·85	1·21	24ʰ 5ᵐ	0·46	87·97	29·55
Venus.....	[illegible]	8·55	1·00	0	17·10	0	17·10	7,660	4·81	0·850	23ʰ 21ᵐ	0·82	224·70	21·61
Earth.....	[illegible]	8·84	1·00	0	17·64	0	17·70	7,918	5·66	1·000	23ʰ 56ᵐ 4·09ˢ	1·00	365·26	18·38
Mars......	[illegible]	4·69	1·00	0	9·36	0	9·42	4,211	4·17	0·737	24ʰ 37ᵐ 22.7ˢ	0·39	686·98	14·99
													Years.	
Jupiter....	[illegible]	18·26	5·20	0	184·2	0	195·8	86,000	1·378	0·2435	9ʰ 55ᵐ 20·0ˢ	2·64	11·86	8·06
Saturn....	[illegible]	8·10	9·54	0	146·3	0	163·8	70,500	0·750	0·1325	10ʰ 14ᵐ 23·8ˢ	1·18	29·46	5·95
Uranus....	[illegible]	1·84	19·2	0	70·7	0	70·7	31,700	1·28	0·226	Unknown.	0·90	84·02	4·20
Neptune...	[illegible]	1·28	30·0	0	77·0	0	77·0	34,500	1·15	0·204	Unknown.	0·89	164·78	3·36

CHAPTER II.

THE SUN.

§ 1. GENERAL SUMMARY.

To the student of the present time, armed with the powerful means of research devised by modern science, the sun presents phenomena of a very varied and complex character. To enable the nature of these phenomena to be clearly understood, we preface our account of the physical constitution of the sun by a brief summary of the main features seen in connection with that body.

Photosphere.—To the simple vision the sun presents the aspect of a brilliant sphere. The visible shining *surface* of this sphere is called the *photosphere*, to distinguish it from the body of the sun as a whole. The apparently flat surface presented by a view of the photosphere is called the sun's *disk*.

Spots.—When the photosphere is examined with a telescope, small dark patches of varied and irregular outline are frequently found upon it. These are called the *solar spots*.

Rotation.—When the spots are observed from day to day, they are found to move over the sun's disk in such a way as to show that the sun rotates on its axis in a period of 25 or 26 days. The sun, therefore, has *axis*, *poles*, and *equator*, like the earth, the axis being the line around which it rotates.

Faculæ.—Groups of minute specks brighter than the general surface of the sun are often seen in the neighborhood of spots or elsewhere. They are called *faculæ*.

Chromosphere, or Sierra.—The solar photosphere is covered by a layer of glowing vapors and gases of very irregular depth. At the bottom lie the vapors of many metals, iron, etc., volatilized by the fervent heat which reigns there, while the upper portions are composed principally of hydrogen gas. This vaporous atmosphere is commonly called the *chromosphere*, sometimes the *sierra*. It is entirely invisible to direct vision, whether with the telescope or naked eye, except for a few seconds about the beginning or end of a total eclipse, but it may be seen on any clear day through the spectroscope.

Prominences, Protuberances, or Red Flames.—The gases of the chromosphere are frequently thrown up in irregular masses to vast heights above the photosphere, it may be 500,000, 100,000, or even 200,000 kilometres. Like the chromosphere, these masses have to be studied with the spectroscope, and can never be directly seen except when the sunlight is cut off by the intervention of the moon during a total eclipse. They are then seen as rose-colored flames, or piles of bright red clouds of irregular and fantastic shapes. They are now usually called "prominences" by the English, and "protuberances" by French writers.

Corona.—During total eclipses the sun is seen to be enveloped by a mass of soft white light, much fainter than the chromosphere, and extending out on all sides far beyond the highest prominences. It is brightest around the edge of the sun, and fades off toward its outer boundary, by insensible gradations. This halo of light is called the *corona*, and is a very striking object during a total eclipse.

§ 2. THE PHOTOSPHERE.

Aspect and Structure of the Photosphere.—The disk of the sun is circular in shape, no matter what side of the sun's globe is turned toward us, whence it follows that the sun itself is a sphere. The aspect of the disk, when viewed with the naked eye, or with a telescope of low power, is that of a uniform bright, shining surface, hence called the *photosphere*. With a telescope of higher power the photosphere is seen to be diversified with groups of spots, and under good conditions the whole mass has a mottled or curdled appearance. This mottling is caused by the presence of cloud-like forms, whose outlines though faint are yet distinguishable. The background is also covered with small white dots or forms still smaller than the clouds. These are the "rice-grains," so called. The clouds themselves are composed of small, intensely bright bodies, irregularly distributed, of tolerably definite shapes, which seem to be suspended in or superposed on a darker medium or background. The spaces between the bright dots vary in diameter from 2″ to 4″ (about 1400 to 2800 kilometres). The rice-grains themselves have been seen to be composed of smaller granules, sometimes not more than 0″·3 (135 miles) in diameter, clustered together. Thus there have been seen at least three orders of aggregation in the brighter parts of the photosphere: the larger cloud-like forms; the rice grains; and, smallest of all, the granules. These forms have been studied with the telescope by SECCHI, HUGGINS, and LANGLEY, and their relations tolerably well made out.

In the *Annuaire* of the Bureau of Longitudes for 1878 (p. 689), M. JANSSEN gives an account of his recent discovery of the reticulated arrangement of the solar photosphere. The paper is accompanied by a photograph of the appearances described, which is enlarged threefold. Photographs less than four inches in diameter cannot satisfactorily show such details. As the granulations of the solar surface are, in general, not greatly larger than 1″ or 2″, the photographic irradiation, which is sometimes 20″ or more, may completely obscure their characteristics. This difficulty M. JANSSEN has overcome by enlarging the image and shortening the time of exposure. In this way the irradiation is diminished, because as the diameters increase, the linear dimensions of the details are increased, and "the imperfections of the sensitive plate have less relative importance."

Again, M. JANSSEN has noted that in short exposures the photographic spectrum is almost monochromatic.

In this way it differs greatly from the visible spectrum, and to the advantage of the former for this special purpose. The diameter of the solar photograms have since 1874 been successively increased to 12, 15, 20, and 30 centimetres. The exposure is made equal all over the surface. In summer this exposure for the largest photograms is less than 0^{s}·0005. The development of such pictures is very slow.

FIG. 77.—RETICULATED ARRANGEMENT OF THE PHOTOSPHERE.

These photograms, on examination, show that the solar surface is covered with a fine granulation. The forms and the dimensions of the elementary surfaces are very various. They vary in size from 0″·3 or 0″·4 to 3″ or 4″ (200 to 3000 kilometres). Their forms are generally circles or ellipses, but these curves are sometimes greatly altered. This granulation is apparently spread equally all over the disk. The brilliancy of the points is very variable, and they appear to be situated at different depths below the photosphere: the most luminous particles, those to which the solar light is chiefly due, occupy only a small fraction of the solar surface.

The most remarkable feature, however, is "the reticulated arrangement of the parts of the photosphere." "The photograms show that the constitution of the photosphere is not uniform throughout, but that it is divided in a series of regions more or less distant from each other, and having each a special constitution. These regions have, in general, rounded contours, but these are often almost rectilinear, thus forming polygons. The dimensions of these figures are very variable; some are even 1′ in diameter (over 25,000 miles)." "Between these figures the grains are sharply defined, but in their interior they are almost effaced and run together as if by some force." These phenomena can be best understood by a reference to the figure of M. Janssen (p. 281).

Light and Heat from the Photosphere.—The *photosphere* is not equally bright all over the apparent disk. This is at once evident to the eye in observing the sun with a telescope. The centre of the disk is most brilliant, and the edges or *limbs* are shaded off so as to forcibly suggest the idea of an absorptive atmosphere, which, in fact, is the cause of this appearance.

Such absorption occurs not only for the rays by which we see the sun, the so-called *visual rays*, but for those which have the most powerful effect in decomposing the salts of silver, the so-called *chemical rays*, by which the ordinary photograph is taken.

The amount of heat received from different portions of the sun's disk is also variable, according to the part of the apparent disk examined. This is what we should expect. That is, if the intensity of any one of these radiations (as felt at the earth) varies from centre to circumference, that of every other should also vary, since they are all modifications of the same primitive motion of the sun's constituent particles. But the constitution of the sun's atmosphere is such that the law of variation for the three classes is different. The intensity of the radiation in the sun itself and inside of the absorptive atmosphere is probably nearly constant. The ray which leaves the centre of the sun's disk in passing to the earth, passes through the smallest possible thickness of the solar atmosphere, while the rays from points of the sun's body which appear to us near the limbs pass, on the contrary, through the maximum thickness of atmosphere, and are thus longest subjected to its absorptive action.

This is plainly a rational explanation, since the part of the sun which is seen by us as the limb varies with the position of the earth in its orbit and with the position of the sun's surface in its rotation, and has itself no physical peculiarity. The various absorptions of different classes of rays correspond to this supposition, the more refrangible rays suffering most absorption, as they must do, being composed of waves of shorter wave length.

The following table gives the observed ratios of the amount of heat, light, and chemical action at the centre of the sun and at various distances from the centre toward the limb. The first column of the table gives the apparent distances from the centre of the disk, the sun's radius being 1·00. The second column gives the percentage of heat-rays received by an observer on the earth from points at these various distances. That is, for every 100 heat-rays reaching the earth from the sun's centre, 95 reach us from a point half way from the centre to the limb, and so on.

Analogous data are given for the light-rays and the chemical rays. The data in regard to heat are due to Professor LANGLEY; those in regard to light and chemical action to Professor PICKERING and Dr. VOGEL respectively.

DISTANCE FROM CENTRE.	Heat Rays.	Light Rays.	Chemical Rays.
0·00..........	100	100	100
0·25..........	99	97	96
0·50..........	95	91	90
0·75..........	86	79	66
0·85..........	....	69	45
0·95..........	....	55	25
0·96..........	62	....	23
0·98..........	50	....	18
1·00..........	....	37	13

For two equal apparent surfaces, *A* near the sun's centre and *B* near the limb, we may say that the rays from the two surfaces when received at the earth have approximately the following relative effects:

A has twice as much effect on a thermometer as *B* (heat);

A has three times as much illuminating effect as *B* (light);

A has seven times as much effect in decomposing the photographic salts of silver as *B* (actinic effect).

It is to be carefully borne in mind that the above numbers refer to variations of the sun's rays received from different equal surfaces *A* and *B*, *in their effect upon certain arbitrary terrestrial standards of measure.* If, for example, the decomposition of other salts than those employed for ordinary photographic work be taken as standards, then the numbers will be altered, and so on. We are simply measuring the power of solar rays selected from different parts of the sun's apparent disk, and hence exposed to different conditions of absorption in his atmosphere, to do work of a certain selected kind, as to raise the temperature of a thermometer, to affect the human retina, or to decompose certain salts of silver.

In this the absorption of the earth's atmosphere is rendered constant for each kind of experiment. This atmosphere has, however, a very strong absorptive effect. We know that we can look at the setting or rising sun, which sends its light rays through great depths of the earth's atmosphere, but not upon the sun at noonday. The temperature is lower at sunrise or at sunset than at noon, and the absorption of chemical rays is so marked that a photograph of the solar spectrum which can be taken in three seconds at noon requires six hundred seconds about sunset—that is, two hundred times as long (DRAPER).

Amount of Heat Emitted by the Sun.—Owing to the absorption of the solar atmosphere, it follows that we receive only a portion—perhaps a very small portion—of the rays emitted by the sun's photosphere.

If the sun had no absorptive atmosphere, it would seem to us hotter, brighter, and more blue in color.

Exact notions as to how great this absorption is are hard to gain, but it may be said roughly that the best authorities agree that although it is quite possible that the sun's atmosphere absorbs half the emitted rays, it probably does not absorb four fifths of them.

It is a curious, and as yet we believe unexplained fact, that the absorption of the solar atmosphere does not affect the darkness of the Fraunhofer lines. They seem equally black at the centre and edge of the sun.* The amount

* Prof. YOUNG has spoken of a slight observable difference.

of this absorption is a practical question to us on the earth. So long as the central body of the sun continues to emit the same quantity of rays, it is plain that the thickness of the solar atmosphere determines the number of such rays reaching the earth. If in former times this atmosphere was much thicker, then less heat would have reached the earth. Professor LANGLEY suggests that the glacial epoch may be explained in this way. If the central body of the sun has likewise had different emissive powers at different times, this again would produce a variation in the temperature of the earth.

Amount of Heat Radiated.—There is at present no way of determining accurately either the absolute amount of heat emitted from the central body or the amount of this heat stopped by the solar atmosphere itself. All that can be done is to measure (and that only roughly) the amount of heat really received by the earth, without attempting to define accurately the circumstances which this radiation has undergone before reaching the earth.

The difficulties in the way of determining how much heat reaches the earth in any definite time, as a year, are twofold. First, we must be able to distinguish between the heat as received by a thermometric apparatus from the sun itself and that from external objects, as our own atmosphere, adjacent buildings, etc.; and, second, we must be able to allow for the absorption of the earth's atmosphere.

POUILLET has experimented upon this question, making allowance for the time that the sun is below the horizon of any place, and for the fact that the solar rays do not in general strike perpendicularly but obliquely upon any given part of the earth's surface. His conclusions may be stated as follows: if our own atmosphere were removed, the solar rays would have energy enough to melt a layer of ice 9 centimetres thick over the whole earth daily, or a layer of about 30 metres thick in a year.

Of the total amount of heat radiated by the sun, the earth receives but an insignificant share. The sun is capable of heating the entire surface of a sphere whose radius is the earth's mean distance to the same degree that the earth is now heated. The surface of such a sphere is 2,170,000,000 times greater than the angular dimensions of the earth as seen from the sun, and hence the earth receives less than one two billionth part of the solar radiation. The rest of the solar rays are, so far as we know, lost in space.

It is found, from direct measures, that a sun-spot gives less heat, area for area, than the unspotted photosphere, and it is an interesting question how much the climate of the earth can be affected by this difference.

Professor LANGLEY, of Pittsburgh, has made measurements of the direct effect of sun-spots on terrestrial temperature. The observations consisted in measuring the relative amounts of umbral, penumbral, and photospheric radiation. The relative umbral, penumbral, and photospheric areas were deduced from the Kew observations of spots; and from a consideration of these data, and confining the question strictly to changes of terrestrial temperature due to this cause alone, LANGLEY deduces the result that "sun-spots do exercise a direct effect on terrestrial temperature by decreasing the mean temperature of the earth at their maximum." This change is, however, very small, as "it is represented by a change in the mean temperature of our globe in eleven years not *greater* than 0·3° C., and not *less* than 0·5° C." It is not intended to show that the earth is, on the whole, cooler in maximum sun-spot years, but that, as far as this cause goes, it tends to make the earth cooler by this minute amount. What other causes may co-exist with the maximum spot-frequency are not considered.

Solar Temperature.—From the amount of heat actually radiated by the sun, attempts have been made to determine the actual temperature of the solar surface. The estimates reached by various authorities differ widely, as the laws which govern the absorption within the solar envelope are almost unknown. Some such law of absorption has to be supposed in any such investigation, and the estimates have differed widely according to the adapted law.

Secchi estimates this temperature as about 6,100,000° C. Other estimates are far lower, but, according to all sound philosophy, the temperature must far exceed any terrestrial temperature. There can be no doubt that if the temperature of the earth's surface were suddenly raised to that of the sun, no single chemical element would remain in its present condition. The most refractory materials would be at once volatilized.

We may concentrate the heat received upon several square feet (the surface of a huge burning-lens or mirror, for instance), examine its effects at the focus, and, making allowance for the condensation by the lens, see what is the minimum possible temperature of the sun. The temperature at the focus of the lens cannot be higher than that of the source of heat in the sun; we can only concentrate the heat received on the surface of the lens to one point and examine its effects. If a lens three feet in diameter be used, the most refractory materials, as fire-clay, platinum, the diamond, are at once melted or volatilized. The effect of the lens is plainly the same as if the earth were brought closer to the sun, in the ratio of the diameter of the focal image to that of the lens. In the case of the lens of three feet, allowing for the absorption, etc., this distance is yet greater than that of the moon from the earth, so that it appears that any comet or planet so close as this to the sun, if composed of materials similar to those in the earth, must be vaporized.

If we calculate at what rate the temperature of the sun would be lowered annually by the radiation from its surface, we shall find it to be 1¼° Centigrade yearly if its specific heat is that of water, and between 3° and 6° per annum if its specific heat is the same as that of the various constituents of the earth itself. It would therefore cool down in a few thousand years by an appreciable amount.

§ 3. SUN-SPOTS AND FACULÆ.

A very cursory examination of the sun's disk with a small telescope will generally show one or more dark spots upon the photosphere. These are of various sizes, from minute black dots 1″ or 2″ in diameter (1000 kilometres or less) to large spots several minutes of arc in extent.

Solar spots generally have a dark central *nucleus* or *umbra*, surrounded by a border or *penumbra* of grayish tint, intermediate in shade between the central blackness and the bright photosphere. By increasing the power of the telescope, the spots are seen to be of very complex forms. The umbra is often extremely irregular in shape, and is sometimes crossed by bridges or ligaments of shining matter. The *penumbra* is composed of filaments of brighter and darker light, which are arranged in striæ. The appearances of the separate filaments are as if they were directed downward toward the interior of the spot in an oblique direction. The general aspect of a spot under considerable magnifying power is shown in Fig. 78.

The first printed account of solar spots was given by FABRITIUS in 1611, and GALILEO in the same year (May, 1611) also described them. They were also attentively

FIG. 78.—UMBRA AND PENUMBRA OF SUN-SPOT.

studied by the Jesuit SCHEINER, who supposed them to be small planets projected against the solar disk. This idea was disproved by GALILEO, whose observations showed them to belong to the sun itself, and to move uniformly across the solar disk from east to west. A spot just visible at the east limb of the sun on any one day travelled slowly across the disk for 12 or 1[illegible] days, when it reached the west limb, behind which it disappeared. After about the same period, it reappeared at the eastern limb, unless, as is often the case, it had in the mean time vanished.

The spots are not permanent in their nature, but are formed somewhere on the sun, and disappear after lasting a few days, weeks, or months. But so long as they last they move regularly from east to west on the sun's apparent disk, making one complete rotation in about 25 days. This period of 25 days is therefore approximately the rotation period of the sun itself.

Spotted Region.—It is found that the spots are chiefly confined to two zones, one in each hemisphere, extending from about 10° to 35° or 40° of heliographic latitude. In the polar regions, spots are scarcely ever seen, and on the solar equator they are much

Fig. 79.—Photograph of the Sun.

more rare than in latitudes 10° north or south. Connected with the spots, but lying on or above the solar surface, are *faculæ*, mottlings of light brighter than the general surface of the sun. The formation of a sun-spot is said to be often presaged by the appearance of faculæ near the point where the spot is to form.

Solar Rotation.—To obtain the exact period of rotation, the spots must be carefully fixed in position by micrometric measures from day to day, the times of the measures being noted. Better still, daily photographs may be made and afterward measured. This has been done by several observers, and the remarkable result reached that the spots do not all rotate exactly in the same period, but that this time, as determined from any spot, depends upon the heliographic latitude of the spot, or its angular distance from the solar equator. A series of observations made by Mr. CARRINGTON of England (by the eye) give the following values of the rotation times T, for spots in different heliographic latitudes L:

$L =$ 0°	5°	10°	15°	20°
$T =$ 25·187 days	25·222	25·327	25·500	25·739
$L =$ 25°	30°	35°	40°	45°
$T =$ 26·040	26·398	26·804	27·252	27·730

The period of rotation seems also to vary somewhat in different years even for spots in the same heliographic latitude, so that we really cannot assign any one definite rotation time to the sun, as we can to the earth or the moon.

"The probability is that the sun, not being solid, has really no one period of rotation, but different portions of its surface and of its internal mass move at different rates, and to some extent independently of each other, though approximately in one plane inclined about 7° to the ecliptic, and around a common axis. The individual spots drift in latitude as well as in longitude, and, on the whole, it appears that spots within 15° or 20° of the solar equator on either side move toward the equator, while beyond this limit they move away from it." (YOUNG.)

Solar Axis and Equator.—The spots must revolve with the surface of the sun about his axis, and the directions of their motions must be approximately parallel to his equator. Fig. [illegible] shows the appearances as actually observed, the dotted lines representing the apparent paths of the spots across the sun's disk at different times of the year. In June and December these paths, to an observer on the earth, seem to be right lines, and hence at these times the observer must be in the plane of the solar equator. At other times the paths are ellipses, and in March and September the planes of these ellipses are most oblique, showing the spectator to be then farthest from the plane of the solar equator. The inclination of the solar equator to the ecliptic is, as already stated, about 7° 9′, and the axis of rotation is of course perpendicular to it.

Nature of the Spots.—The sun-spots are really depressions in the photosphere, as was first pointed out by ANDREW WILSON of Glasgow. When a spot is seen at the edge of the disk, it appears as a notch in the limb, and is elliptical in shape. As the rotation carries it further and further on to the disk, it becomes more and more nearly circular in shape, and after passing the centre of the disk the appearances take place in reverse order.

These observations were explained by WILSON, and more fully by Sir WILLIAM HERSCHEL, by supposing the sun to consist of an interior dark cool mass, surrounded by two layers of clouds. The outer layer, which forms the visible photosphere, was supposed extremely brilliant. The inner layer, which could not be seen except when a cavity existed in the photosphere, was supposed to be dark. The appearance of the edges of a spot, which has been described as the penumbra, was supposed to arise from those dark clouds. The spots themselves are, according to this view, nothing but openings through both of the atmospheres, the

FIG. 59.—APPARENT PATH OF SOLAR SPOT AT DIFFERENT SEASONS.

nucleus of the spot being simply the black surface of the inner sphere of the sun itself.

This theory, which the figure on the next page exemplifies, accounts for the facts as they were known to HERSCHEL. But when it is confronted with the questions of the cause of the sun's heat and of the method by which this heat has been maintained constant in amount for centuries, it breaks down completely. The conclusions of WILSON and HERSCHEL, that the spots are depressions in the sun's surface, are undoubted. But the existence of a cool central and solid nucleus to the sun is now known to be impossible. The apparently black centres of the spots are so mostly by contrast. If they were seen against a perfectly black background, they would appear very bright, as has been proved by the photometric measures of Professor LANGLEY. And a cool solid nucleus beneath such an atmosphere as HERSCHEL supposed would soon become gaseous by the conduction and radiation of the heat of the photosphere. The supply of solar heat, which has been very nearly constant during the historic period, would in a sun so constituted have sensibly diminished in a few hundred years. For these and other reasons, the hypothesis of HERSCHEL must be modified, save as to the fact that the spots are really cavities in the photosphere.

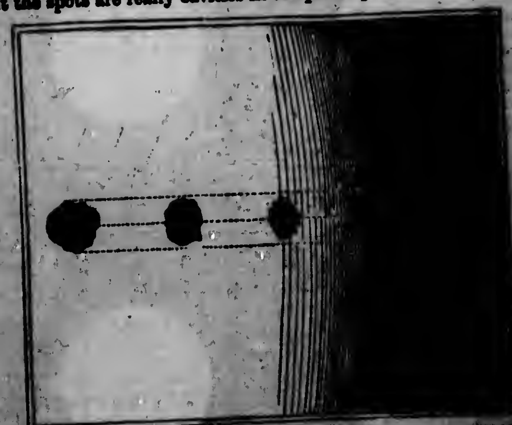

FIG. 81.—APPEARANCE OF A SPOT NEAR THE LIMB AND NEAR THE CENTRE OF THE SUN.

Number and Periodicity of Solar Spots.—The number of solar spots which come into view varies from year to year. Although at first sight this might seem to be what we call a purely accidental circumstance, like the occurrence of cloudy and clear years on the earth, yet the series of observations of sun-spots by Hofrath SCHWABE of Dessau (see the table), continued by him for forty years, established the fact that this number varied *periodically*. This had indeed been previously suspected by HORREBOW, but it was independently suggested and completely proved by SCHWABE.

TABLE OF SCHWABE'S RESULTS.

YEAR.	Days of Observation.	Days of no Spots.	New Groups.	Mean Diurnal Variation in Declination of the Magnetic Needle.
1826	277	22	118	9·75
1827	273	2	161	11·33
1828	282	0	225	11·28
1829	244	0	199	14·74
1830	217	1	190	12·13
1831	239	3	149	12·22
1832	270	49	84	.....
1833	247	139	33	.....
1834	273	120	51	.....
1835	244	18	173	9·57
1836	200	0	272	12·34
1837	168	0	333	12·27
1838	202	0	282	12·74
1839	205	0	162	11·03
1840	263	3	152	9·91
1841	283	15	102	7·82
1842	307	64	68	7·08
1843	312	149	34	7·15
1844	321	111	52	6·61
1845	332	29	114	8·13
1846	314	1	157	8·81
1847	276	0	257	9·55
1848	278	0	330	11·15
1849	285	0	238	10·64
1850	308	2	186	10·44
1851	308	0	151	8·32
1852	337	2	125	8·09
1853	299	3	91	7·09
1854	334	65	67	6·81
1855	313	146	79	6·41
1856	321	193	34	5·98
1857	324	52	98	6·95
1858	335	0	188	7·41
1859	343	0	205	10·37
1860	332	0	211	10·05
1861	322	0	204	9·17
1862	317	3	160	8·59
1863	330	2	124	8·84
1864	325	4	130	8·03
1865	307	25	93	8·14
1866	349	76	45	7·65
1867	316	195	25	7·09
1868	301	23	101	8·15

The periodicity of the spots is evident from the table. It will appear in a more striking way from the following summary:

From 1828 to 1831,	sun without	spots	on only	....	1	day.
In 1833,	"	"	"	....	139	days.
From 1836 to 1840,	"	"	"	....	3	"
In 1843,	"	"	"	....	147	"
From 1847 to 1851,	"	"	"	....	2	"
In 1856,	"	"	"	....	193	"
From 1858 to 1861,	"	"	"	....	no	day.
In 1867,	"	"	"	....	195	days.

Every 11 years there is a minimum number of spots, and about 5 years after each minimum there is a maximum. If instead of merely counting the number of spots, measurements are made on solar photograms, as they are called, of the extent of *spotted area*, the period comes out with greater distinctness. This periodicity of the area of the solar spots appears to be connected with magnetic phenomena on the earth's surface, and with the number of auroras visible. It has been supposed to be connected also with variations of temperature, of rainfall, and with other meteorological phenomena such as the monsoons of the Indian Ocean, etc. The cause of this periodicity is as yet unknown. CARRINGTON, DE LA RUE, LOEWY, and STEWART have given reasons which go to show that there is a connection between the spotted area and the configurations of the planets, particularly of *Jupiter*, *Venus*, and *Mercury*. ZÖLLNER says that the cause lies within the sun itself, and assimilates it to the periodic action of a geyser, which seems to be *à priori* probable. Since, however, the periodic variations of the spots correspond to the magnetic variation, as exhibited in the last column of the table of SCHWABE's results, it appears that there may be some connection of an unknown nature between the sun and the earth at least. But at present we can only state our limited knowledge and wait for further information.

Dr. Wolf (Director of the Zurich Observatory) has collected all the available observations of the solar spots, and it is found that since 1610 we have a tolerably complete record of these appearances. The number and character of the spots are now noted every day by observers in many quarters of the civilized world. This long series of observations has served as a basis to determine each epoch of maximum and minimum which has occurred since 1610, and from thence to determine the length of each single period.

The following table gives Dr. Wolf's results:

Table giving the Times of Maximum and Minimum Sun-Spot Frequency, according to Wolf.

First Series.				Second Series.			
Minima.	Diff.	Maxima.	Diff.	Minima.	Diff.	Maxima.	Diff.
A.D. 1610·8		1615·5		1745·0		1750·3	
	8·2		10·5		10·2		11·2
1619·0		1626·0		1755·2		1761·5	
	15·0		13·5		11·3		8·2
1634·0		1639·5		1766·5		1769·7	
	11·0		9·5		9·0		8·7
1645·0		1649·0		1775·5		1778·4	
	10·0		11·0		9·2		9·7
1655·0		1660·0		1784·7		1788·1	
	11·0		15·0		13·6		16·1
1666·0		1675·0		1798·3		1804·2	
	13·5		10·0		12·3		12·2
1679·5		1685·0		1810·6		1816·4	
	10·0		8·0		12·7		13·5
1689·5		1693·0		1823·3		1829·9	
	8·5		12·5		10·6		7·3
1698·0		1705·5		1833·9		1837·2	
	14·0		12·7		9·6		10·9
1712·0		1718·2		1843·5		1848·1	
	11·5		9·3		12·5		12·0
1723·5		1727·5		1856·0		1860·1	
	10·5		11·2		11·2		10·5
1734·0		1738·7		1867·2		1870·1	
11·20 ± 2·11 years. ±0·64		11·20 ± 2·06 ys. ±0·63		11·11 ± 1·54 ys. ±0·47		10·94 ± 2·52 ys. ±0·76	

From the first series of earlier observations, the period comes out from observed *minima*, 11·20 years, with a variation of two years; from observed *maxima* the period is 11·20 years, with variation of three years—that is, this series shows the period to vary between 13·3 and 9·1 years. If we suppose these errors to arise only from errors of observation, and not to be real changes of the period itself, the *mean* period is 11·20 ± 0·64.

The results from the second series are also given at the foot of the table. From a combination of the two, it follows that the *mean* period is 11·111 ± 0·307 years, with an oscillation of ± 2·030 years.

These results are formulated by Dr. WOLF as follows: The frequency of solar spots has continued to change periodically since their discovery in 1610; the mean length of the period is 11⅑ years, and the separate periods may differ from this mean period by as much as 2·03 years.

A general relation between the frequency of the spots and the variation of the magnetic needle is shown by the numbers which have been given in the table of SCHWABE's results. This relation has been most closely studied by WOLF. He denotes by g the number of groups of spots seen on any day on the sun, counting each isolated spot as a group; by f is denoted the number of spots in each group (fg is then proportional to the spotted area); by k a coefficient depending upon the size of the telescope used for observation, and by r the daily *relative number* so called; then he supposes

$$r = k\,(f + 10 \cdot g).$$

From the daily relative numbers are formed the mean monthly and the mean annual relative numbers r. Then, according to WOLF, if v is the mean annual variation of the magnetic needle at any place, two constants for that place, α and β, can be found, so that the following formula is true for all years:

$$v = \alpha + \beta \cdot r.$$

Thus for Munich the formula becomes,

$$v = 6'\cdot 27 + 0'\cdot 051\ r;$$

and for Prague,

$$v = 5'\cdot 80 + 0'\cdot 045\ r, \text{ and so on.}$$

YEAR.	MUNICH.			PRAGUE.		
	Observed.	Computed.	Δ	Observed.	Computed.	Δ
	′	′		′	′	
1870......	12·27	12·77	− 0·50	11·41	12·10	− 0·69
1871......	11·70	11·56	+ 0·14	11·60	10·89	+ 0·71
1872......	10·96	11·13	− 0·17	10·70	10·46	+ 0·24
1873......	9·12	9·54	− 0·42	9·05	8·87	+ 0·18

The above comparison bears out the conclusion that the magnetic variations are subjected to the same perturbations as the development of the solar spots, and it may be said that the changes in the frequency of solar spots and the like changes of magnetic variations show that these two phenomena are dependent the one on the other, or rather upon the same cosmical cause. What this cause is remains as yet unknown.

§ 4. THE SUN'S CHROMOSPHERE AND CORONA.

Phenomena of Total Eclipses.—The beginning of a total solar eclipse is an insignificant phenomenon. It is marked simply by the small black notch made in the luminous disk of the sun by the advancing edge or limb of the moon. This always occurs on the western half of the sun, as the moon moves from west to east in its orbit. An hour or more must elapse before the moon has advanced sufficiently far in its orbit to cover the sun's disk. During this time the disk of the sun is gradually hidden until it becomes a thin crescent. To the general spectator there is little to notice during the first two thirds of this period from the beginning of the eclipse, unless it be perhaps the altered shapes of the images formed by small holes or apertures. Under ordinary circumstances, the image of the sun, made by the solar rays which pass through a small hole—in a card, for example—are circular in shape, like the shape of the sun itself. When the sun is crescent, the image of the sun formed by such rays is also crescent, and, under favorable circumstances, as in a thick forest where the interstices of the leaves allow such images to be formed, the effect is quite striking. The reason for this phenomenon is obvious.

The actual amount of the sun's light may be diminished to two thirds or three fourths of its ordinary amount without its being strikingly perceptible to the eye. What is first noticed is the change which takes place in the color of the surrounding landscape, which begins to wear a ruddy aspect. This grows more and more pronounced, and gives to the adjacent country that weird effect which lends so much to the impressiveness of a total eclipse. The reason for the change of color is simple. We have already said that the sun's atmosphere absorbs a large proportion of the bluer rays, and as this absorption is dependent on the thickness of the solar atmosphere through which the rays must pass, it is plain that just before the sun is totally covered the rays by which we see it will be redder than ordinary sunlight, as they are those which come from points near the sun's limb, where they have to pass through the greatest thickness of the sun's atmosphere.

The color of the light becomes more and more lurid up to the moment when the sun has nearly disappeared. If the spectator is upon the top of a high mountain, he can then begin to see the moon's shadow rushing toward him at the rate of a mile in about two seconds. Just as the shadow reaches him there is a sudden increase of darkness—the brighter stars begin to shine in the dark lurid sky, the thin crescent of the sun breaks up into small points or dots of light, which suddenly disappear, and the moon itself, an intensely black ball, appears to hang isolated in the heavens.

An instant afterward, the corona is seen surrounding the black disk of the moon with a soft effulgence quite different from any other light known to us. Near the moon's limb it is intensely bright, and to the naked eye uniform in structure; 5′ or 10′ from the limb this inner corona has a boundary more or less defined, and from this extend streamers and wings of fainter and more nebulous light. These are of various shapes, sizes, and brilliancy. No two solar eclipses yet observed have been alike in this respect.

These wings seem to vary from time to time, though at nearly every eclipse the same phenomena are described by observers situated at different points along the line of totality. That is, these appearances, though changeable, do not change in the time the moon's shadow requires to pass from Vancouver's Island to Texas, for example, which is some fifty minutes.

Superposed upon these wings may be seen (sometimes with the naked eye) the red flames or protuberances which were first discovered during a solar eclipse. These need not be more closely described here, as they can now be studied at any time by aid of the spectroscope.

The total phase lasts for a few minutes (never more than six or seven), and during this time, as the eye becomes more and more accustomed to the faint light, the outer corona is seen to stretch further and further away from the sun's limb. At the eclipse of 1878, July 29th, it was seen by Professor LANGLEY, and by one of the writers, to extend more than 6° (about 9,000,000 miles) from the sun's limb. Just before the end of the total phase there is a sudden increase of the brightness of the sky, due to the increased illumination of the earth's atmosphere near the observer, and in a moment more the sun's rays are again visible, seemingly as bright as ever. From the end of totality till the last contact the phenomena of the first half of the eclipse are repeated in inverse order.

Telescopic Aspect of the Corona.—Such are the appearances to the naked eye. The corona, as seen through a telescope, is, however, of a very complicated structure. The inner corona is usually composed of bright striæ or filaments separated by darker bands, and some of these latter are sometimes seen to be almost totally black. The appearances are extremely irregular, but they are often as if the inner corona were made up of brushes of light on a darker background. The direction of these brushes is often radial to the sun, especially about the poles, but where the outer corona joins on to the inner these brushes are sometimes bent over so as to join, as it were, the boundaries of the outer light.

The great difficulties in the way of studying the corona have been due to the short time at the disposal of the observer, and to the great differences which even the best draughtsmen will make in their rapid sketches of so complicated a phenomenon. The figure of the inner corona on the next page is a copy of one of the best drawings made of the eclipse of 1869, and is inserted chiefly to show the nature of the only drawings possible in the limited time. The numbers refer to the red prominences around the limb. The radial structure of the corona and its different extension and nature at different points are also indicated in the drawing.

The figure on page 302, is acopy of a crayon drawing made in 1878. The best evidence which we can gain of the details of the corona comes, however, from a series of photographs taken during the whole of totality. A photograph with a short exposure gives the details of the inner corona well, but is not affected by the fainter outlying parts. One of longer exposure shows details further away from the sun's limb, while those near it are lost in a glare of light, being over-exposed, and so on. In this way a series of photographs gives us the means of building up, as it were, the whole corona from its brightest parts near the sun's limb out to the faintest portions which will impress themselves on a photographic plate.

The corona and red prominences are solar appendages. It was formerly doubtful whether the corona was an atmosphere belonging to the sun or to the moon. At the eclipse of 1860 it was proved by measurements that the red prominences belonged to the sun and not to the moon, since the moon gradually covered them by its motion, they remaining attached to the sun. The corona has also since been shown to be a solar appendage.

The eclipse of 1851 was total in Sweden and neighboring parts, and was very carefully observed. Similar prominences were seen about the sun's limb, and one of so bizarre a form as to show that it could by no possibility

FIG. 53.—DRAWING OF THE CORONA MADE DURING THE ECLIPSE OF AUGUST 7, 1869.

by a mountain or solid mass, since if such had been the case it would inevitably have overturned. It was therefore a gaseous or cloud-like appendage belonging to the

FIG. [illegible].—SUN'S CORONA DURING THE ECLIPSE OF JULY 29, 1878.

sun. There were others of various and perhaps varying shapes, and the bases of these were connected by a low band of serrated rose-colored light. One of these protuberances was shown to be entirely above the sun, as if floating within its atmosphere. Around the whole disk of the sun a ring of similar nature to the prominences exists, which is brighter than the corona, and seems to form a base for the protuberances themselves; this is the sierra. Some of the red flames were of enormous height; one of at least 80,000 miles.

FIG. [illegible].—SOME OF THE SOLAR PROMINENCES AS SEEN WITH THE SPECTROSCOPE.

Gaseous Nature of the Prominences.—The next eclipse (1868, July) was total in India, and was observed by many skilled astronomers. A discovery of M. JANSSEN's* will make this eclipse forever memorable. He was provided with a spectroscope, and by it observed the prominences. One prominence in particular was of vast size, and when the spectroscope was turned upon it, its spectrum was discontinuous, showing the bright lines of hydrogen gas.

* Now Director of the Solar Observatory of Meudon, near Paris.

The brightness of the spectrum was so marked that Janssen determined to keep his spectroscope fixed upon it even after the reappearance of sunlight, to see how long it could be followed. It was found that its spectrum could still be seen after the return of complete sunlight; and not only on that day, but on subsequent days, similar phenomena could be observed.

One great difficulty was conquered in an instant. The red flames which formerly were only to be seen for a few moments during the comparatively rare occurrences of total eclipses, and whose observation demanded long and expensive journeys to distant parts of the world, could now be regularly observed with all the facilities offered by a fixed observatory.

This great step in advance was independently made by Mr. Lockyer,* and his discovery was derived from pure theory, unaided by the eclipse itself. By this method the prominences have been carefully mapped day by day all around the sun, and it has been proved that around this body there is a vast atmosphere of hydrogen gas—the *chromosphere* or *sierra*. From out of this the prominences are projected sometimes to heights of 100,000 kilometres or more.

It will be necessary to recall the main facts of observation which are fundamental in the use of the spectroscope. When a brilliant point is examined with the spectroscope, it is spread out by the prism into a band—the spectrum. Using two prisms, the spectrum becomes longer, but the light of the surface, being spread over a greater area, is enfeebled. Three, four, or more prisms spread out the spectrum proportionally more. If the spectrum is of an incandescent solid or liquid, it is always continuous, and it can be enfeebled to any degree; so that any part of it can be made as feeble as desired.

This method is precisely similar in principle to the use of the telescope in viewing stars in the daytime. The telescope lessens the brilliancy of the sky, while the disk of the star is kept of the same intensity, as it is a point in itself. It thus becomes visible. If it is a glowing gas, its spectrum will consist of a definite number of lines, say three—A, B, C, for example. Now suppose the spectrum of this gas to be superposed on the continuous spectrum of the sun; by using only one prism, the

* Mr. J. Norman Lockyer, F.R.S., of London, now attached to the Science and Art Department of the South Kensington Museum.

solar spectrum is short and brilliant, and every part of it may be more brilliant than the line spectrum of the gas. By increasing the dispersion (the number of prisms), the solar spectrum is proportionately enfeebled. If the ratio of the light of the bodies themselves, the sun and the gas, is not too great, the continuous spectrum may be so enfeebled that the line spectrum will be visible when superposed upon it, and the spectrum of the gas may then be seen even in the presence of true sunlight. Such was the process imagined and successfully carried out by Mr. LOCKYER, and such is in essence the method of viewing the prominences to-day adopted.

The Coronal Spectrum.—In 1869 (August 7th) a total solar eclipse was visible in the United States. It was probably observed by more astronomers than any preceding eclipse. Two American astronomers, Professor YOUNG, of Dartmouth College, and Professor HARKNESS, of the Naval Observatory, especially observed the spectrum of the corona. This spectrum was found to consist of one faint greenish line crossing a faint continuous spectrum. The place of this line in the maps of the solar spectrum published by KIRCHHOFF was occupied by a line which he had attributed to the *iron* spectrum, and which had been numbered 1474 in his list, so that it is now spoken of as 1474 K. This line is probably due to some gas which must be present in large and possibly variable quantities in the corona, and which is not known to us on the earth, in this form at least. It is probably a gas even lighter than hydrogen, as the existence of this line has been traced 10′ or 20′ from the sun's limb nearly all around the disk.

In the eclipse of July 29th, 1878, which was total in Colorado and Texas, the continuous spectrum of the corona was found to be crossed by the dark lines of the solar spectrum, showing that the coronal light was composed in part of reflected sunlight.

§ 5. SOURCES OF THE SUN'S HEAT.

Theories of the Sun's Constitution.—No considerable fraction of the heat radiated from the sun returns to it from the celestial spaces, since if it did the earth would intercept some of the returning rays, and the temperature of night would be more like that of noonday. But we know the sun is daily radiating into space 2,170,000,000 times as much heat as is daily received by the earth, and it follows that unless the supply of heat is infinite (which we cannot believe), this enormous daily radiation must in time exhaust the supply. When the supply is exhausted, or even seriously trenched upon, the result to the inhabitants of the earth will be fatal. A slow diminution of the daily supply of heat would produce a slow change of climates from hotter toward colder. The serious results of a fall of 50° in the mean annual temperature of the earth will be evident when we remember that such a fall would change the climate of France to that of Spitzbergen. The temperature of the sun cannot be kept up by the mere combustion of its materials. If the sun were solid carbon, and if a constant and adequate supply of oxygen were also present, it has been shown that, at the present rate of radiation, the heat arising from the combustion of the mass would not last more than 5000 years.

An explanation of the solar heat and light has been suggested, which depends upon the fact that great amounts of heat and light are produced by the collision of two rapidly moving heavy bodies, or even by the passage of a heavy body like a meteorite through the earth's atmosphere. In fact, if we had a certain mass available with which to produce heat in the sun, and if this mass were of the best possible materials to produce heat by burning, it can be shown that, by burning it at the surface of the sun, we should produce vastly less heat than if we simply allowed it to fall into the sun. In the last case, if it fell from the earth's distance, it would give 6000 times more heat than by its burning.

The *least* velocity with which a body from space could fall upon the sun's surface is in the neighborhood of 280 miles in a second of time, and the velocity may be as great as 350 miles. From these facts, the meteoric theory of solar heat originated. It is in effect that the heat of the sun is kept up by the impact of meteors upon its surface.

No doubt immense numbers of meteorites fall into the sun daily and hourly, and to each one of them a certain considerable portion of heat is due. It is found that, to account for the present amount of radiation, meteorites equal in mass to the whole earth would have to fall into the sun every century. It is extremely improbable that a mass one tenth as large as this is added to the sun in this way per century, if for no other reason because the earth itself and every planet would receive far more than its present share of meteorites, and would itself become quite hot from this cause alone.

There is still another way of accounting for the sun's constant supply of energy, and this has the advantage of appealing to no cause outside of the sun itself in the explanation. It is by supposing the heat, light, etc., to be generated by a constant and gradual contraction of the dimensions of the solar sphere. As the globe cools by radiation into space, it must contract. In so contracting its ultimate constituent parts are drawn nearer together by their mutual attraction, whereby a form of energy is developed which can be transformed into heat, light, electricity, or other physical forces. ✓

This theory is in complete agreement with the known laws of force. It also admits of precise comparison with facts, since the laws of heat enable us, from the known amount of heat radiated, to infer the exact amount of contraction in inches which the linear dimensions of the sun must undergo in order that this supply of heat may be kept unchanged, as it is practically found to be. With the present size of the sun, it is found that it is only necessary to suppose that its diameter is diminishing at the rate of about 220 feet per year, or 4 miles per century, in order that the supply of heat radiated shall be constant. It is plain that such a change as this may be taking place, since we possess no instruments sufficiently delicate to have detected a change of even ten times this amount since the invention of the telescope.

It may seem a paradoxical conclusion that the cooling of a body may cause it to become hotter. This indeed is true only when we suppose the interior to be gaseous, and not solid or liquid. It is, however, proved by theory that this law holds for gaseous masses.

If a spherical mass of gas be condensed to one half the primitive diameter, the central attraction upon any part of its mass will be increased fourfold, while the surface subjected to this attraction will be reduced to one fourth. Hence the pressure per unit of surface will be augmented sixteen times, while the density will be increased but eight times. If the elastic and the gravitating forces were in equilibrium in the original condition of the mass, the temperature must be doubled in order that they may still be in equilibrium when the diameter is reduced to one half.

If, however, the primitive body is originally solid or liquid, or if, in the course of time, it becomes so, then this law ceases to hold, and radiation of heat produces a lowering of the temperature of the body, which progressively continues until it is finally reduced to the temperature of surrounding space.

We cannot say whether the sun has yet begun to liquefy in his interior parts, and hence it is impossible to predict at present the duration of his constant radiation. Theory shows us that after about 5,000,000 years, the sun radiating heat as at present, and still remaining gaseous, will be reduced to one half of its present volume. It seems probable that somewhere about this time the solidification will have begun, and it is roughly estimated, from this line of argument, that the present conditions of heat radiation cannot last greatly over 10,000,000 years.

The future of the sun (and hence of the earth) cannot, as we see, be traced with great exactitude. The past can be more closely followed if we assume (which is tolerably safe) that the sun up to the present has been a gaseous, and not a solid or liquid mass. Four hundred years ago, then, the sun was about 100 miles greater in diameter than now; and if we suppose this process of contraction to have regularly gone on at the same rate (an uncertain supposition), we can fix a date when the sun filled any given space, out even to the orbit of *Neptune*—that is, to the time when the solar system consisted of but one body, and that a gaseous or nebulous one. It will subsequently be seen that the ideas here reached *à posteriori* have a striking analogy to the *à priori* ideas of Kant and La Place.

It is not to be taken for granted, however, that the amount of heat to be derived from the contraction of the

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sun's dimensions is infinite, no matter how large the primitive dimensions may have been. A body falling from any distance to the sun can only have a certain finite velocity depending on this distance and the mass of the sun itself, which, even if the fall be from an infinite distance, cannot exceed, for the sun, 350 miles per second. In the same way the amount of heat generated by the contraction of the sun's volume from any size to any other is finite, and not infinite.

It has been shown that if the sun has always been radiating heat at its present rate, and if it had originally filled all space, it has required 18,000,000 years to contract to its present volume. In other words, assuming the present rate of radiation, and taking the most favorable case, the age of the sun does not exceed 18,000,000 years. The earth, is of course, less aged. The supposition lying at the base of this estimate is that the radiation of the sun has been constant throughout the whole period. This is quite unlikely, and any changes in this datum affect greatly the final number of years which we have assigned. While this number may be greatly in error, yet the method of obtaining it seems, in the present state of science, to be satisfactory, and the main c[illegible]clusion remains that the past of the sun is finite, and that [illegible] probability its future is a limited one. The exact num[illegible]turies that it is to last are of no moment even were [illegible]ta at hand to obtain them : the essential point is, that, so far as we can see, the sun, and incidentally the solar system, has a finite past and a limited future, and that, like other natural objects, it passes through its regular stages of birth, vigor, decay, and death, in one order of progress.

1 If the sun, as believed by many, corresponds to the Lord, why should it not be correspondingly infinite?

2 What would we or rather the future habitants of the earth, do [illegible]

CHAPTER III.

THE INFERIOR PLANETS.

§ 1. MOTIONS AND ASPECTS.

THE inferior planets are those whose orbits lie between the sun and the orbit of the earth. Commencing with the more distant ones, they comprise *Venus*, *Mercury*, and, in the opinion of some astronomers, a planet called *Vulcan*, or a group of planets, inside the orbit of *Mercury*. The planets *Mercury* and *Venus* have so much in common that a large part of what we have to say of one can be applied to the other with but little modification.

The real and apparent motions of these planets have already been briefly described in Part I., Chapter IV. It will be remembered th[illegible]ccordance with KEPLER's third law, their peri[illegible]ution around the sun are less than that of [illegible]nsequently they overtake the latter betw[illegible]ferior conjunctions.

The interval between th[illegible] conjunctions is about four months in the case of *Mercury*, and between nineteen and twenty months in that of *Venus*. At the end of this period each repeats the same series of motions relative to the sun. What these motions are can be readily seen by studying Fig. 84. In the first place, suppose the earth, at any point, *E*, of its orbit, and if we draw a line, *E L* or *E M*, from *E*, tangent to the orbit of either of these planets, it is evident that the angle which this line makes with that drawn to the sun is the greatest elongation of the planet from the sun. The orbits being eccentric, this elongation varies with the position of the earth. In the case of *Mercury* it ranges from 16° to 29°, while in the case of *Venus*, the orbit of which is nearly circular, it varies very little from 45°. These planets, therefore, seem to have an oscillating motion, first swinging toward the east of the sun, and then toward the west of it, as already explained in Part I., Chapter IV. Since, owing to the annual revolution of the earth, the sun has a constant eastward motion among the stars, these planets must have, on the whole, a corresponding though intermittent motion in the same direction. Therefore the ancient astronomers supposed their period of revolution to be one year, the same as that of the sun.

FIG. 84.

If, again, we draw a line ESC from the earth through the sun, it is evident that the first point I, in which this line cuts the orbit of the planet, or the point of inferior conjunction, will (leaving eccentricity out of the question) be the least distance of the planet from the earth, while the second point C, or the point of superior conjunction, on the opposite side of the sun, will be the greatest distance. Owing to the difference of these distances, the apparent magnitude of these planets, as seen from the earth, is subject to great variations.

FIG. 85.—APPARENT MAGNITUDES OF THE DISK OF MERCURY.

Fig. 85 shows these variations in the case of *Mercury*, A representing its apparent magnitude when at its greatest distance, B when at its mean distance, and C when at its least distance. In the case of *Venus* (Fig. 86) the variations are much greater than in that of *Mercury*, the greatest distance, 1·72, being more than six times the least distance, which is only 0·28. The variations of apparent magnitude are therefore great in the same proportion.

In thus representing the apparent angular magnitude of these planets, we suppose their whole disks to be visible, as they would be if they shone by their own light. But since they can be seen only by the reflected light of the sun, only those portions of the disk can be seen which are at the same time visible from the sun and from the earth. A very little consideration will show that the proportion of the disk which can be seen constantly diminishes as the planet approaches the earth, and looks larger.

FIG. 86.—APPARENT MAGNITUDES OF DISK OF VENUS.

When the planet is at its greatest distance, or in superior conjunction (C, Fig. 84), its whole illuminated hemisphere can be seen from the earth. As it moves around and approaches the earth, the illuminated hemisphere is gradually turned from us. At the point of greatest elongation, M or L, one half the hemisphere is visible, and the planet has the form of the moon at first or second quarter. As it approaches inferior conjunction, the apparent visible disk assumes the form of a crescent, which becomes thinner and thinner as the planet approaches the sun.

Fig. 87 shows the apparent disk of *Mercury* at various times during its synodic revolution. The planet will appear brightest when this disk has the greatest surface.

This occurs about half way between greatest elongation and inferior conjunction.

In consequence of the changes in the brilliancy of these planets produced by the variations of distance, and those produced by the variations in the proportion of illuminated disk visible from the earth, partially compensating each other, their actual brilliancy is not subject to such great variations as might have been expected. As a general rule, *Mercury* shines with a light exceeding that of a star of the first magnitude. But owing to its proximity to the sun, it can never be seen by the naked eye except in the west a short time after sunset, and in the east a little before sunrise. It is then of necessity near the horizon, and

FIG. 87.—APPEARANCE OF MERCURY AT DIFFERENT POINTS OF ITS ORBIT.

therefore does not seem so bright as if it were at a greater altitude. In our latitudes we might almost say that it is never visible except in the morning or evening twilight. In higher latitudes, or in regions where the air is less transparent, it is scarcely ever visible without a telescope. It is said that COPERNICUS died without ever obtaining a view of the planet *Mercury*.

On the other hand, the planet *Venus* is, next to the sun and moon, the most brilliant object in the heavens. It is so much brighter than any fixed star that there can seldom be any difficulty in identifying it. The unpractised observer might under some circumstances find a difficulty in distinguishing between *Venus* and *Jupiter*, but the different motions of the two planets will enable him to distinguish them if they are watched from night to night during several weeks.

§ 2. ASPECT AND ROTATION OF MERCURY.

The various phases of *Mercury*, as dependent upon its various positions relative to the sun, have already been shown. If the planet were an opaque sphere, without inequalities and without an atmosphere, the apparent disk would always be bounded by a circle on one side and an ellipse on the other, as represented in the figure. Whether any variation from this simple and perfect form has ever been detected is an open question, the balance of evidence being very strongly in the negative. Since no spots are visible upon it, it would follow that unless variations of form due to inequalities on its surface, such as mountains, can be detected, it is impossible to determine whether the planet rotates on its axis. The only evidence in favor of such rotation is that of SCHRÖTER, the celebrated astronomer of Lilienthal, who made the telescopic study of the moon and planets his principal work. About the beginning of the present century he noticed that at certain times the south horn of the crescent of *Mercury* seemed to be blunted. Attributing this appearance to the shadow of a lofty mountain, he concluded that the planet *Mercury* revolved on its axis in a little more than 24 hours. But this planet has since been studied with instruments much more powerful than those of SCHRÖTER, and no confirmation of his results has been obtained. We must therefore conclude that the period of rotation of *Mercury* on its axis is entirely unknown.

Respecting an atmosphere of *Mercury*, the evidence is also conflicting. The spectrum of this planet has been studied by Dr. VOGEL, now astronomer at the Physical Observatory of Potsdam, who finds that its principal lines coincide with those of the sun. Of course we should expect this because the planet shines by reflected solar light. But he also finds that certain lines are seen in the spectrum of *Mercury* which we know to be due to the absorption of the earth's atmosphere, and which appear more dense than they should from the simple passage through our atmosphere. This would seem to show that *Mercury* has an envelope of gaseous matter somewhat like our own. On the other hand, Dr. ZÖLLNER, of Leipsic, by measuring the amount of light reflected by the planet at various times, concludes that *Mercury*, like our moon, is devoid of any atmosphere sufficient to reflect the light of the sun. We may therefore regard it as doubtful whether any evidence of an atmosphere of *Mercury* can be obtained, and it is certain that we know nothing definite respecting its physical constitution.

§ 3. THE ASPECT AND SUPPOSED ROTATION OF VENUS.

As *Venus* sometimes comes nearer the earth than any other primary planet, astronomers have examined its surface with great interest ever since the invention of the telescope. But no conclusive evidence respecting the rotation of the planet and no proof of any changes or any inequalities on its surface have ever been obtained. The observations are either very discordant, or so difficult and unreliable that we may readily suppose the observers to have been misled as to what they saw. In 1767 CASSINI thought he saw a bright spot on *Venus* during several successive evenings, and concluded, from his supposed observation that the planet revolved on its axis in a little more than 23 hours. The subject was next taken up by BIANCHINI, an Italian astronomer, who supposed that he saw a number of dark regions on the planet. These he considered to be seas or oceans, and he went so far as to give them names. Watching them from night to night, he concluded that the time of rotation of *Venus* was more than 24 days. Again, SCHRÖTER thought that, when *Venus* was a crescent, one of its sharp points was blunted at certain intervals, as in the case of *Mercury*. He formed the same theory of the cause of this appearance—namely, that it was due to the shadow of a high mountain. He concluded that the time of rotation found by CASSINI was nearly correct. Finally, in 1842, DE VICO, of Rome, thought he could see the same dark regions or oceans on the planet which had been seen by BLANCHINI. He concluded that the true time of rotation was $23^h\ 21^m\ 22^s$. This result has gone into many of our text-books as conclusive, but it is contradicted by the investigation of many excellent observers with much better instruments. HERSCHEL was never able to see any permanent markings on *Venus*. If he ever caught a glimpse of spots, they were so transient that he could gather no evidence respecting the rotation of the planet. He therefore concluded that if they really existed, they were due entirely to clouds floating in an atmosphere, and that no time of rotation could be deduced by observing them. This view of HERSCHEL, so far as concerns the aspect of the planet, is confirmed by a study with the most powerful telescopes in recent times. With the great Washington telescope, no permanent dark spots and no regular blunting of either horn has ever been observed.

It may seem curious that skilled observers could have been deceived as to what they saw; but we must remember that there are many celestial phenomena which are extremely difficult to make out. By looking at a drawing of a planet or nebula, and seeing how plain every thing seems in the picture, we may be entirely deceived as to the actual aspect with a telescope. Under the circumstances, if the observer has any preconceived theory, it is very easy for him to think he sees every thing in accordance with that theory. Now, there are at all times great differences in the brilliancy of the different parts of the disk of *Venus*. It is brightest near the round edge which is turned toward the sun. Over a small space the brightness is such that some recent observers have formed a theory that the sun's light is reflected as from a mirror. On the other hand, near the boundary between light and darkness, the surface is much darker. Moreover, owing to the undulations of our atmosphere, the aspect of any planet so small and bright as *Venus* is constantly changing. The only way to reach any certain conclusion respecting its appearance is to take an average, as it were, of the appearances as modified by the undulations. In taking this average, it is very easy to imagine variations of light and darkness which have no real existence; it is not, therefore, surprising that one astronomer should follow in the footsteps of another in seeing imaginary markings.

Atmosphere of Venus.—The evidence of an atmosphere of *Venus* is perhaps more conclusive than in the case of any other planet. When *Venus* is observed very near its inferior conjunction, and when it therefore presents the view of a very thin crescent, it is found that this crescent extends over more than 180°. This would be evidently impossible unless the sun illuminated more than one half the planet. One of the most fortunate observers of this phenomenon was Professor C. S. LYMAN, of Yale College, who observed *Venus* in December, 1866. The inferior conjunction of the planet occurred near the ascending node, so that its angular distance from the sun was less than it had been at any former time during the present century. Professor LYMAN saw the disk, not as a thin crescent, but as an entire and extremely fine circle of light. We therefore conclude that *Venus* has an atmosphere which exercises so powerful a refraction upon the light of the sun that the latter illuminates several degrees more than one half the globe. A phenomenon which must be attributed to the same cause has several times been observed during transits of *Venus*. During the transit of December 8th, 1874, most of the observers who enjoyed a fine steady atmosphere saw that when *Venus* was partially projected on the sun, the outline of that part of its disk outside the sun could be distinguished by a delicate line of light. A similar appearance was noticed by DAVID RITTENHOUSE, of Philadelphia, on June 3d, 1769. From these several observations, it would seem that the refractive power of the atmosphere of *Venus* is greater than that of the earth. Attempts have been made to determine its exact amount, but they are too uncertain to be worthy of quotation.

§ 4. TRANSITS OF MERCURY AND VENUS.

When *Mercury* or *Venus* passes between the earth and sun, so as to appear projected on the sun's disk, the phenomenon is called a *transit*. If these planets moved around the sun in the plane of the ecliptic, it is evident that there would be a transit at every inferior conjunction. But since their orbits are in reality inclined to the ecliptic, transits can occur only when the inferior conjunction takes place near the node. In order that there may be a transit, the latitude of the planet, as seen from the earth, must be less than the angular semi-diameter of the sun—that is, less than 16′.*

The longitude of the descending node of *Mercury* at the present time is 227°, and therefore that of the ascending node 47°. The earth has these longitudes on May 7th and November 9th. Since a transit can occur only within a few degrees of a node, *Mercury* can transit only within a few days of these epochs.

The longitude of the descending node of *Venus* is now

* The mathematical student, knowing that the inclination of the orbit of *Mercury* is 7° 0′ and that of *Venus* 3° 24′, will find it an interesting problem to calculate the limits of distance from the node within which inferior conjunction must take place in order that a transit may occur. From the geocentric latitude 16′ the heliocentric latitude may be found by multiplying by the distance from the earth and dividing by that from the sun. He will find these limits to be a little greater for *Mercury* than for *Venus*, notwithstanding its greater inclination, and to be only a few degrees in either case.

about 256°, and therefore that of the ascending node is 76°. The earth has these longitudes on June 6th and December 7th of each year. Transits of *Venus* can therefore occur only within two or three days of these times.

Recurrence of Transits of Mercury.—The transits of *Mercury* and *Venus* recur in cycles which resemble the eighteen-year cycle of eclipses, but in which the precision of the recurrence is less striking. From the mean motions of *Mercury* and the earth already given, we find that the mean synodic period of *Mercury* is, in decimals of a Julian year, $0^y\cdot317256$. Three synodic periods are therefore some eighteen days less than a year. If, then, we suppose an inferior conjunction of *Mercury* to occur exactly at a node, the third conjunction following will take place about eighteen days before the earth again reaches the node, and therefore about 18° from the node, since the earth moves nearly 1° in a day. This is far outside the limit of a transit; we must, therefore, wait until another conjunction occurs near the same place. To find when this will be, the successive vulgar fractions which converge toward the value of the above period may be found by the method of continued fractions. The first five of these fractions are:

$$\frac{1}{3} \qquad \frac{6}{19} \qquad \frac{7}{22} \qquad \frac{13}{41} \qquad \frac{46}{145}$$

Here the denominators are numbers of synodic periods, while the numerators are the approximate corresponding number of years. By actual multiplication we find:

3 Periods	$= 0^y\cdot951768 =$	$1^y - \cdot048232.$	Error	$= -17°$
19 "	$= 6\cdot037864 =$	$6 + \cdot037864.$	"	$= +10°$
22 "	$= 6\cdot979632 =$	$7 - \cdot020368.$	"	$-7°$
41 "	$= 13\cdot007496 =$	$13 + \cdot007496.$	"	$= +2°\cdot7$
145 "	$= 46\cdot002120 =$	$46 + \cdot002120.$	"	$= +0°\cdot76$

In this table the errors show the number of degrees from the node at which the inferior conjunction will occur at the end of one year, six years, seven years, etc. They are found by multiplying the fraction by which the intervals exceed or fall short of an entire number of years by 360°. It will be seen that the 19th, 22d, 41st, and 145th conjunctions occur nearer and nearer the node, or, supposing that we do not start from a node, nearer and nearer the point of the orbits from which we do start. It follows that the recurrence of a transit of *Mercury* at the same node is possible at the end of 7 years, probable at the end of 13 years, and almost certain at the end of 46 years. The latter is the cycle which it would be most convenient to take as that in which all the transits would recur, but it would still not be so exact as the eclipse cycle of 18 years 11 days.

The following table shows the dates of occurrence of transits of *Mercury* during the present century. They are separated into May transits, which occur near the descending node, and November ones, which occur near the ascending node. November transits are the most numerous, because *Mercury* is then nearer the sun, and the transit limits are wider.

	1799, May 6.		1802, Nov. 9.	
33	1832, May 5.	13	1815, Nov. 11.	
13	1845, May 8.	7	1822, Nov. 5.	
33	1878, May 6.	13	1835, Nov. 7.	
13	1891, May 9.	13	1848, Nov. 10.	
33	1924	13	1861, Nov. 12.	
		7	1868, Nov. 5.	
		13	1881, Nov. 7.	
		13	1894, Nov. 10.	1914
		13	1907 Nov. 12.	

It will be seen that in a cycle of 46 years there are two May transits and four November ones, so that the latter are twice as numerous as the former. These numbers may, however, change slightly at some future time through the failure of a recurrence, or the entrance of a new transit into the series. Thus, in the May series, it is doubtful whether there will be an actual transit 46 years after 1891—that is, in 1937—or whether *Mercury* will only pass very near the limb of the sun. On the other hand, *Mercury* passed within a few minutes of the sun's limb on May 3d, 1865, and it will probably graze the limb 46 years later—that is, on May 4th or 5th, 1911.

Recurrence of Transits of Venus.—For many centuries past and to come, transits of *Venus* occur in a cycle more exact than those of *Mercury*. It happens that eight times the mean motion of *Venus* is very nearly the same as thirteen times the mean motion of the earth; in other words, *Venus* makes 13 revolutions around the sun in nearly the same time that the earth makes 8 revolutions—that is, in eight years. During this period there will be 5 inferior conjunctions of *Venus*, because the latter has made 5 revolutions more than the earth. Consequently, if we wait eight years from an inferior conjunction of *Venus*, we shall, at the end of that time, have another inferior conjunction, the fifth in regular order, at nearly the same point of the two orbits. It will, therefore, occur at the same time of the year, and in nearly the same position relative to the node of *Venus*. In Fig. 33 let *S* represent the sun, and the circle drawn around it the orbit of the earth.

FIG. 33.—CONJUNCTIONS OF VENUS.

Suppose also that at the moment of the inferior conjunction of *Venus*, we draw a straight line *S* 1 through *Venus* to the earth at 1. We shall then have to wait about $1\frac{3}{5}$ years for another inferior conjunction, during which time the earth will have made one revolution and $\frac{3}{5}$ of another, and *Venus* $2\frac{3}{5}$ revolutions. The straight line drawn through the point of inferior conjunction will then be *S* 2. The third conjunction will in the same way take place in the position *S* 3, which is $1\frac{3}{5}$ revolutions further advanced; the fourth in the position *S* 4, and the fifth in the position *S* 5. If the correspondence of the motions were exact, the sixth conjunction, at the end of 8 years ($5 \times 1\frac{3}{5} = 8$), would again take place in the original position *S* 1, and all subsequent ones would follow in the same order. All inferior conjunctions would then take place at one of these five points, and no transit would ever be possible unless one of these points should chance to be very near the line of nodes.

In fact, however, the correspondence is not perfectly exact, but, at the end of 8 years, the sixth conjunction will take place not exactly along the line *S* 1, but a little before the two bodies reach this line. The actual angle between the line *S* 1 and that of the sixth conjunction will be about 2° 22′, the point shifting back toward the direction *S* 4. Of course, each following conjunction will take place at the same distance back from that of eight years before, leaving out small changes due to the eccentricities of the orbits and the variations of their elements. It follows then that if we suppose the five lines of conjunction to have a retrograde motion in a direction the opposite of that of the arrow, amounting to 2° 22′ in eight years, all the inferior conjunctions will take place along these five lines. The distance apart of the lines being 72° and the motion about 18′ per year, the intervals between the passages of the several conjunction lines over the line of nodes will be about 240 years. Really, the exact time is 243 years.

Suppose, now, that a conjunction should take place exactly at a node, then the fifth following conjunction would take place 2° 22′ before reaching the node. The limits within which a transit can occur are, however, only 1° 46′ on each side of the node; consequently, there would be no further transit at that node until the next following conjunction point reached it, which would happen at the end of 243 years. If, however, the conjunction should take place between 0° 36′ and 1° 46′ *after* reaching the node, there would be a transit, and the fifth following conjunction would also occur within the limit on the other side of the node, so that we should have two transits eight years apart. We may, therefore, have either one transit or two according to the distance from the node at which the first transit occurs. We thus have at any one node either a single transit, or a pair of transits eight years apart, in a cycle of 243 years. At the middle of this cycle the node will be half way between two of the conjunction points—the points 1 and 3, for instance; but it is evident that in this case the opposite node will coincide with the conjunction point 2, since there is an odd number of such points. It follows, therefore, that about the middle of the interval between two consecutive sets of transits at one node we shall have a transit or a pair of transits at the opposite node.

The earth passes through the line of the descending node of the orbit of *Venus* early in June of each year, and through the ascending node early in December. It follows, therefore, that the series will be a transit or a pair of transits in June; then an interval of about 120 years, to be followed by a transit or a pair of transits in December, and so on. Owing to the eccentricity of the orbits, the intervals will not be exactly equal, the motions of the several conjunction points not being uniform, nor their distance exactly 72°. The dates and intervals of the transits for three cycles nearest to the present time are as follows:

			Intervals.
1518, June 2.	1761, June 5.	2004, June 8.	8 years.
1526, June 1.	1769, June 3.	2012, June 6.	105½ "
1631, Dec. 7.	1874, Dec. 9.	2117, Dec. 11.	8 "
1639, Dec. 4.	1882, Dec. 6.	2125, Dec. 8.	121½ "

The 243-year cycle is so exact that the actual deviations from it are due almost entirely to the secular variation of the orbits of *Venus* and the *Earth*. Moreover, the conjunction of December 8th, 1874, took place 1° 25′ past the ascending node, so that the conjunction of 1882 takes place about 1° 4′ before reaching the node. Owing to the near approach of the period to exactness, several pairs of transits near this node have taken place in the past, at equal intervals of 243 years, and will be repeated for three or four cycles in the future.

Nearly the same remark applies to those which take place at the descending node, where pairs of transits eight years apart will occur for about three cycles in the future. Owing, however, to secular variations of the orbit, the conjunction point for the second June transit of each pair and the first December transit will, after perhaps a thousand years, take place so far from the node that the planet will not quite touch the sun, and then during a period of many centuries there will only be one transit at each node in every 243 years, instead of two, as at present.

§ 5. SUPPOSED INTRAMERCURIAL PLANETS.

Some astronomers are of opinion that there is a small planet or a group of planets revolving around the sun inside the orbit of *Mercury*. To this supposed planet the name *Vulcan* has been given; but astronomers generally discredit the existence of such a planet of considerable size, because the evidence in its favor is not regarded as conclusive.

The evidence in favor of the existence of such planets may be divided into three classes, as follows, which will be considered in their order:

(1) A motion of the perihelion of the orbit of *Mercury*, supposed to be due to the attraction of such a planet or group of planets.

(2) Transits of dark bodies across the disk of the sun which have been supposed to be seen by various observers during the past century.

(3) The observation of certain unidentified objects by Professor WATSON and Mr. LEWIS SWIFT during the total eclipse of the sun, July 29th, 1878.

(1) In 1858, LE VERRIER made a careful collection of all the observations on the transits of *Mercury* which had been recorded since the invention of the telescope. The result of that investigation was that the observed times of transit could not be reconciled with the calculated motion of the planet, as due to the gravitation of the other bodies of the solar system. He found, however, that if, in addition to the changes of the orbit due to the attraction of the other planets, he supposed a motion of the perihelion amounting to 36″ in a century, the observations could all be satisfied. Such a motion might be produced by the attraction of an unknown planet inside the orbit of *Mercury*. Since, however, a single planet, in order to produce this effect, would have to be of considerable size, and since no such object had ever been observed during a total eclipse of the sun, he concluded that there was probably a group of planets much too small to be separately distinguished. So far as the discrepancy between theory and observation is concerned, these results of LE VERRIER's have been completely confirmed by the mathematical researches of Mr. G. W. HILL, and by observations of transits since LE VERRIER's calculations were completed. Indeed, the result of these researches and observations is that the motion of the perihelion is even greater than that found by LE VERRIER, the surplus motion being more than 40″ in a century. There is no known way of accounting for this motion in accordance with well-established laws, except by supposing matter of some sort to be revolving around the sun in the supposed position. At the same time it is always possible that the effect may be produced by some unknown cause.*

(2) Astronomical records contain upward of twenty instances in which dark bodies have been supposed to be seen in transit across the disk of the sun. If we suppose these observations to be all perfectly correct, the existence of a great number of considerable planets within the orbit of *Mercury* would be placed beyond doubt. But a critical analysis shows that these observations, considered as a class, are not entitled to the slightest credence. In the first place,

* An electro-dynamic theory of attraction has been within the past twenty years suggested by several German physicians, which involves a small variation from the ordinary theory of gravitation. It has been shown that, by supposing this theory true, the motion of the perihelion of *Mercury* could be accounted for by the attraction of the sun.

scarcely any of them were made by experienced observers with powerful instruments. It is very easy for an unpractised observer to mistake a round solar spot for a planet in transit. It may therefore be supposed that in many cases the observer saw nothing but a spot on the sun. In fact, the very last instance of the kind on record was an observation by WEBER at Peckeloh, on April 4th, 1876. He published an account of his observation, which he supposed was that of a planet, but when the publication reached other observers, who had been examining the sun at the same time, it was shown conclusively that what he saw was nothing more than an unusually round solar spot. Again, in most of the cases referred to, the object seen was described as of such magnitude that it could not fail to have been noticed during total eclipses if it had any real existence. It is also to be noted that if such planets existed they would frequently pass over the disk of the sun. During the past fifty years the sun has been observed almost every day with the greatest assiduity by eminent observers, armed with powerful instruments, who have made the study of the sun's surface and spots the principal work of their lives. None of these observers has ever recorded the transit of an unknown planet. This evidence, though negative in form, is, under the circumstances, conclusive against the existence of such a planet of such magnitude as to be visible in transit with ordinary instruments.

(3) The observations of Professor WATSON during the total eclipse above mentioned seem to afford the strongest evidence yet obtained in favor of the real existence of the planet. His mode of proceeding was briefly this: Sweeping to the west of the sun during the eclipse, he saw two objects in positions where, supposing the pointing of his telescope accurately known, no fixed star existed. There is, however, a pair of known stars, one of which is about a degree distant from one of the unknown objects, and the other about the same distance and direction from the second. It is considered by some that Professor WATSON's supposed planets may have been this pair of stars. Still, if Professor WATSON's planets were capable of producing the motion of the perihelion of *Mercury* already referred to, we should regard their existence as placed beyond reasonable doubt. But his observations and the theoretical results of LE VERRIER do not in any manner strengthen each other, because, if we suppose the observed perturbations in the orbit of *Mercury* to be due to planets so small as those seen by WATSON, the number of these planets must be many thousands. Now, it is very certain that there are not thousands of planets there brighter than the sixth magnitude, because they would have been seen by other telescopes engaged in the same search. The smaller we suppose the individual planets, the more numerous they must be, and, finally, if we consider them as individually invisible, they will probably be numbered by tens of thousands. The smaller and more numerous they are, supposing their combined mass the same, the greater the sum total of light they would reflect. At a certain point the amount of light would become so considerable that the group would appear as a cloud-like mass. Now, there is

a phenomenon known as the zodiacal light, which is probably caused by matter either in a gaseous state or composed of small particles revolving around the sun at various distances from it. This light can be seen rising like a pillar from the western horizon on any very clear night in the winter or spring. Of its nature scarcely any thing is yet known. The spectroscopic observations of Professor WRIGHT, of Yale College, seem to indicate that it is seen by reflected sunlight. Very different views, however, have obtained respecting its constitution, and even its position, some having held that it is a ring surrounding the earth. We can therefore merely suggest the possibility that the observed motion of the perihelion of *Mercury* is produced by the attraction of this mass.

CHAPTER IV.

THE MOON.

In Chapter VII. of the preceding part we have described the motions of the moon and its relation to the earth. We shall now explain its physical constitution as revealed by the telescope.

When it became clearly understood that the earth and moon were to be regarded as bodies of one class, and that the old notion of an impassable gulf between the character of bodies celestial and bodies terrestrial was unfounded, the question whether the moon was like the earth in all its details became one of great interest. The point of most especial interest was whether the moon could, like the earth, be peopled by intelligent inhabitants. Accordingly, when the telescope was invented by Galileo, one of the first objects examined was the moon. With every improvement of the instrument, the examination became more thorough, so that the moon has been an object of careful study by the physical astronomer.

The immediate successors of Galileo thought that they perceived the surface of the moon, like that of our globe, to be diversified with land and water. Certain regions appeared dark and, for the most part, smooth, while others were bright and evidently broken up into hills and valleys. The former regions were supposed to be oceans, and received names to correspond with this idea. These names continue to the present day, although we now know that there are no oceans there.

With every improvement in the means of research, it has become more and more evident that the surface of the moon is totally unlike that of our earth. There are no oceans, seas, rivers, air, clouds, or vapor. We can hardly suppose that animal or vegetable life exists under such circumstances, the fundamental conditions of such existence on our earth being entirely wanting. We might almost as well suppose a piece of granite or lava to be the abode of life as the surface of the moon to be such.

Before proceeding with a description of the lunar surface, as made known to us by the telescopes of the present time, it will be well to give some estimates of the visibility of objects on the moon by means of our instruments. Speaking in a rough way, we may say that the length of one mile on the moon would, as seen from the earth, subtend an angle of 1″ of arc. More exactly, the angle subtended would range between 0″·8 and 0″·9, according to the varying distance of the moon. In order that an object may be plainly visible to the naked eye, it must subtend an angle of nearly 1′. Consequently, a magnifying power of 60 is required to render a round object one mile in diameter on the surface of the moon plainly visible. Starting from this fact, we may readily form the following table, showing the diameters of the smallest objects that can be seen with different magnifying powers, always assuming that vision with these powers is perfect:

Power 60; diameter of object 1 mile.
Power 150; diameter 2000 feet.
Power 500; diameter 600 feet.
Power 1000; diameter 300 feet.
Power 2000; diameter 150 feet.

If telescopic power could be increased indefinitely, there would of course be no limit to the minuteness of an object visible on the moon's surface. But the necessary imperfections of all telescopes are such that only in extraordinary cases can any thing be gained by increasing the magnifying power beyond 1000. The influence of warm and cold currents in our atmosphere is such as will forever prevent the advantageous use of high magnifying powers. After a certain limit we see nothing more by increasing the power, vision becoming indistinct in proportion as the power is increased. It may be doubted whether the moon was ever seen through a telescope to so good advantage as she would be seen with a magnifying power of 500, unaccompanied by any drawback from atmospheric vibrations or imperfection of the telescope. In other words, it is hardly likely that an object less than 600 feet in extent could ever be seen on the moon by any telescope whatever, unless it were possible to mount the instrument above the atmosphere of the earth. It is therefore only the great features on the surface of the moon, and not the minute ones, which can be made out with the telescope.

Character of the Moon's Surface.—The most striking point of difference between the earth and moon is seen in the total absence from the latter of any thing that looks like an undulating surface. No formations similar to our valleys and mountain-chains have been detected. The lowest surface of the moon which can be seen with the telescope appears to be nearly smooth and flat, or, to speak more exactly, spherical (because the moon is a sphere). This surface has different shades of color in different regions. Some portions are of a bright, silvery tint, while others have a dark gray appearance. These differences of tint seem to arise from differences of material.

Upon this surface as a foundation are built numerous formations of various sizes, but all of a very simple character. Their general form can be made out by the aid of Fig. 89, and their dimensions by the scale of miles at the bottom of it. The largest and most prominent features are known as craters. They have a typical form consisting of a round or oval ragged wall rising from the plane in the manner of a circular fortification. These walls are frequently from three to six thousand metres in height, very rough and broken. In their interior we see

FIG. [illegible].—ASPECT OF THE MOON'S SURFACE.

the plane surface of the moon already described. It is, however, generally covered with fragments or broken up by small inequalities so as not to be easily made out. In the centre of the craters we frequently find a conical formation rising up to a considerable height, and much larger than the inequalities just described. In the craters we have a vague resemblance to volcanic formations upon the earth, the principal difference being that their magnitude is very much greater than any thing known here. The diameter of the larger ones ranges from 50 to 200 kilometres, while the smallest are so minute as to be hardly visible with the telescope.

When the moon is only a few days old, the sun's rays strike very obliquely upon the lunar mountains, and they cast long shadows. From the known position of the sun, moon, and earth, and from the measured length of these shadows, the heights of the mountains can be calculated. It is thus found that some of the mountains near the south pole rise to a height of 8000 or 9000 metres (from 25,000 to 30,000 feet) above the general surface of the moon. Heights of from 3000 to 7000 metres are very common over almost the whole lunar surface.

Next to the so-called craters visible on the lunar disk, the most curious features are certain long bright streaks, which the Germans call *rills* or *furrows.* These extend in long radiations over certain of the craters, and have the appearance of cracks in the lunar surface which have been subsequently filled by a brilliant white material. NASMYTH and CARPENTER have described some experiments designed to produce this appearance artificially. They took hollow glass globes, filled them with water, and heated them until the surface was cracked. The cracks generated at the weakest point of the surface radiate from the point in a manner strikingly similar in appearance to the rills on the moon. It would, however, be premature to conclude that the latter were actually produced in this way.

The question of the origin of the lunar features has a bearing on theories of terrestrial geology as well as upon various questions respecting the past history of the moon itself. It has been considered in this aspect by various geologists.

Lunar Atmosphere.—The question whether the moon has an atmosphere has been much discussed. The only conclusion which has yet been reached is that no positive evidence of an atmosphere has ever been obtained, and that if one exists it is certainly several hundred times rarer than the atmosphere of our earth. The most delicate method of detecting such an appendage would be by its refracting the light of a star seen through it. As the moon advances in her monthly course around the earth, she frequently appears to pass over bright stars. These phenomena are called *occultations*. Just before the limb of the moon appears to reach the star, the latter will be seen through the moon's atmosphere, if there is one, and will be displaced in a direction from the moon's centre. But the most careful observations have failed to show the slightest evidence of any such displacement. Hence the most delicate test for a lunar atmosphere gives no evidence whatever that it exists.

The spectra of stars when about to be occulted have also been examined in order to see whether any absorption lines which might be produced by the lunar atmosphere became visible. The evidence in this direction has also been negative. Moreover, the spectrum of the moon itself does not seem to differ in the slightest from that of the sun. We conclude therefore that if there is a lunar atmosphere, it is too rare to exert any sensible absorption upon the rays of light.

Light and Heat of the Moon.—Many attempts have been made to measure the ratio of the light of the full moon and that of the sun. The results have been very discordant, but all have agreed in showing that the sun emits several hundred thousand times as much light as the full moon. The last and most careful determination is that of Zöllner, who finds the sun to be 618,000 times as bright as the full moon.

The moon must reflect the heat as well as the light of the sun, and must also radiate a small amount of its own heat. But the quantities thus reflected and radiated are so minute that they have defied detection except with the most delicate instruments of research now known. By collecting the moon's rays in the focus of one of his large reflecting telescopes, Lord Rosse was able to show that a certain amount of heat is actually received from the moon, and that this amount varies with the moon's phase, as it should do. He also sought to learn how much of the moon's heat was reflected and how much radiated. This he did by ascertaining its capacity for passing through glass. It is well known to students of physics that a very much larger portion of the heat radiated by the sun or other extremely hot bodies will pass through glass than of heat radiated by a cooler body. Experiments show that about 86 per cent of the sun's heat will pass through ordinary optical glass. If the heat of the moon were entirely reflected sun heat, it would possess the same property, and the same proportion would pass through glass. But the experiments of Lord Rosse have shown that instead of 86 per cent, only 12 per cent passed through the glass. As a general result of all his researches, it may be supposed that about six sevenths of the heat given out by the moon is radiated and one seventh reflected.

Is there any change on the surface of the Moon?—When the surface of the moon was first found to be covered by craters having the appearance of volcanoes at the surface of the earth, it was very naturally thought that these supposed volcanoes might be still in activity, and exhibit themselves to our telescopes by their flames. Sir William Herschel supposed that he saw several such volcanoes, and, on his authority, they were long believed to exist. Subsequent observations have shown that this was a mistaken opinion, though a very natural one under the circumstances. If we look at the moon with a telescope when she is three or four days old, we shall see the darker portion of her surface, which is not reached by the sun's rays, to be faintly illuminated by light reflected from the earth. This appearance may always be seen at the right time with the naked eye. If the telescope has an aperture of five inches or upward, and the magnifying power does not exceed ten to the inch, we shall generally see one or more spots on this dark hemisphere of the moon so much brighter than the rest of the surface that they may well suggest the idea of being self-luminous. It is, however, known that these are only spots possessing the power of reflecting back an unusually large portion of the earth's light. Not the slightest sound evidence of any incandescent eruption at the moon's surface has ever been found.

Several instances of supposed changes on the moon's surface have been described in recent times. A few years ago a spot known as Linnæus, near the centre of the moon's visible disk, was found to present an appearance entirely different from its representation on the map of BEER and MAEDLER, made forty years before. More recently KLEIN, of Cologne, supposed himself to have discovered a yet more decided change in another feature of the moon's surface.

The question whether these changes are proven is one on which the opinions of astronomers differ. The difficulty of reaching a certain conclusion arises from the fact that each feature necessarily varies in appearance, owing to the different ways in which the sun's light falls upon it. Sometimes the changes are very difficult to account for, even when it is certain that they do not arise from any change on the moon itself. Hence while some regard the apparent changes as real, others regard them as due only to differences in the mode of illumination.

CHAPTER V.

THE PLANET MARS.

§ 1. DESCRIPTION OF THE PLANET.

Mars is the next planet beyond the earth in the order of distance from the sun, being about half as far again as the earth. It has a decided red color, by which it may be readily distinguished from all the other planets. Owing to the considerable eccentricity of its orbit, its distance, both from the sun and from the earth, varies in a larger proportion than does that of the other outer planets.

At the most favorable oppositions, its distance from the earth is about 0·38 of the astronomical unit, or, in round numbers, 57,000,000 kilometres (35,000,000 of miles). This is greater than the least distance of *Venus*, but we can nevertheless obtain a better view of *Mars* under these circumstances than of *Venus*, because when the latter is nearest to us its dark hemisphere is turned toward us, while in the case of *Mars* and of the outer planets the hemisphere turned toward us at opposition is fully illuminated by the sun.

The period of revolution of *Mars* around the sun is a little less than two years, or, more exactly, 687 days. The successive oppositions occur at intervals of two years and one or two months, the earth having made during this interval a little more than two revolutions around the sun, and the planet *Mars* a little more than one. The dates of several past and future oppositions are shown in the following table:

1871	March 20th.
1873	April 27th.
1875	June 20th.
1877	September 5th.
1879	November 12th.
1881	December 26th.
1884	January 31st.
1886	March 6th.

Owing to the unequal motion of the planet, arising from the eccentricity of its orbit, the intervals between successive oppositions vary from two years and one month to two years and two and a half months.

About August 26th of each year the earth is in the same direction from the sun as the perihelion of the orbit of *Mars*. Hence if an opposition occurs about that time, *Mars* will be very near its perihelion, and at the least possible distance from the earth. At the opposite season of the year, near the end of February, the earth is on the line drawn from the sun to the aphelion of the orbit *Mars*. The least favorable oppositions are therefore those which occur in February. The distance of *Mars* is then about 0·65 of the astronomical unit.

The favorable oppositions occur at intervals of 15 or 17 years, the period being that required for the successive increments of one or two months between the times of the year at which successive oppositions occur to make up an entire year. This will be readily seen from the preceding table of the times of opposition, which shows how the oppositions ranged through the entire year between 1871 and 1886. The opposition of 1877 was remarkably favorable. The next most favorable opposition will occur in 1892.

Mars necessarily exhibits phases, but they are not so well marked as in the case of *Venus*, because the hemisphere which it presents to the observer on the earth is always more than half illuminated. The greatest phase occurs when its direction is 90° from that of the sun, and even then six sevenths of its disk is illuminated, like that of the moon, three days before or after full moon. The phases of *Mars* were observed by GALILEO in 1610, who, however, could not describe them with entire certainty.

Rotation of Mars.—The early telescopic observers noticed that the disk of *Mars* did not appear uniform in color and brightness, but had a variegated aspect. In 1666 the celebrated Dr. ROBERT HOOKE found that the markings on *Mars* were permanent and moved around in such a way as to show that the planet revolved on its axis. The markings given in his drawing can be traced at the present day, and are made use of to determine the exact period of rotation of the planet. Drawings made by HUYGHENS abont the same time have been used in the same way. So well is the rotation fixed by them that the astronomer can now determine the exact number of times the planet has rotated on its axis since these old drawings were made. The period has been found by Mr. PROCTOR to be $24^h\ 37^m\ 22^s \cdot 7$, a result which appears certain to one or two tenths of a second. It is therefore less than an hour greater than the period of rotation of the earth.

Surface of Mars.—The most interesting result of these markings on *Mars* is the probability that its surface is diversified by land and water, covered by an atmosphere, and altogether very similar to the surface of the earth. Some portions of the surface are of a decided red color, and thus give rise to the well-known fiery aspect of the planet. Other parts are of a greenish hue, and are therefore supposed to be seas. The most striking features are two brilliant white regions, one lying around each pole of the planet. It has been supposed that this appearance is due to immense masses of snow and ice surrounding the poles. If this were so, it would indicate that the processes of evaporation, cloud formation, and condensation of vapor into rain and snow go on at the surface of *Mars* as at the surface of the earth. A certain amount of color is given to

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this theory by supposed changes in the magnitude of these ice-caps. But the problem of establishing such changes is one of extreme difficulty. The only way in which an adequate idea of this difficulty can be formed is by the reader himself looking at *Mars* through a telescope.

If he will then note how hard it is to make out the different shades of light and darkness on the planet, and

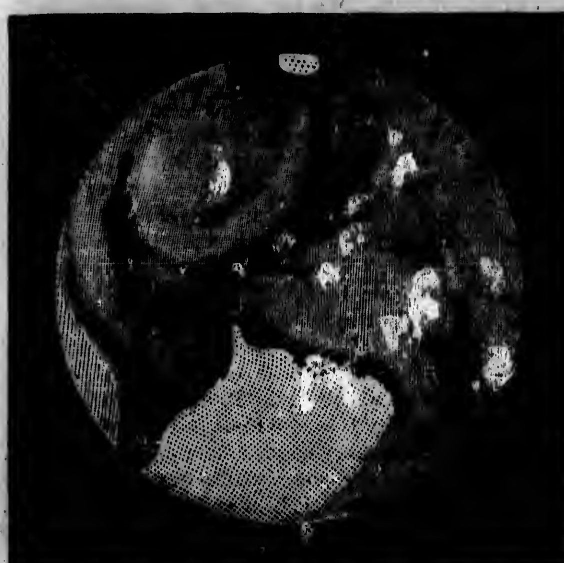

FIG. 90.—TELESCOPIC VIEW OF MARS.

how they must vary in aspect under different conditions of clearness in our own atmosphere, he will readily perceive that much evidence is necessary to establish great changes. All we can say, therefore, is that the formation of the ice-caps in winter and their melting in summer has some evidence in its favor, but is not yet completely proven.

§ 2. SATELLITES OF MARS.

Until the year 1877, *Mars* was supposed to have no satellites, none having ever been seen in the most powerful telescopes. But in August of that year, Professor Hall, of the Naval Observatory, instituted a systematic search with the great equatorial, which resulted in the discovery of two such objects. We have already described the opposition of 1877 as an extremely favorable one; otherwise it would have been hardly possible to detect these bodies. They had never before been seen, partly on account of their extreme minuteness, which rendered them invisible except with powerful instruments and at the most favorable times, and partly on account of the fact, already alluded to, that the favorable oppositions occur only at intervals of 15 or 17 years. There are only a few weeks during each of these intervals when it is practicable to distinguish them.

These satellites are by far the smallest celestial bodies known. It is of course impossible to measure their diameters, as they appear in the telescope only as points of light. A very careful estimate of the amount of light which they reflect was made by Professor E. C. Pickering, Director of the Harvard College Observatory, who calculated how large they ought to be to reflect this light. He thus found that the outer satellite was probably about six miles and the inner one about seven miles in diameter, supposing them to reflect the solar rays precisely as *Mars* does. The outer one was seen with the telescope at a distance from the earth of 7,000,000 times this diameter. The proportion would be that of a ball two inches in diameter viewed at a distance equal to that between the cities of Boston and New York. Such a feat of telescopic seeing is well fitted to give an idea of the power of modern optical instruments.

Professor Hall found that the outer satellite, which he called *Deimos*, revolves around the planet in $30^h\ 18^m$, and the inner one, called *Phŏbos*, in $7^h\ 38^m$. The latter is only 5800 miles from the centre of *Mars*, and less than 4000 miles from its surface. It would therefore be almost possible with one of our telescopes on the surface of *Mars* to see an object the size of a large animal on the satellite.

This short distance and rapid revolution make the inner satellite of *Mars* one of the most interesting bodies with which we are acquainted. It performs a revolution in its orbit in less than half the time that *Mars* revolves on its axis. In consequence, to the inhabitants of *Mars*, it would seem to rise in the west and set in the east. It will be remembered that the revolution of the moon around the earth and of the earth on its axis are both from west to east; but the latter revolution being the more rapid, the apparent diurnal motion of the moon is from east to west. In the case of the inner satellite of *Mars*, however, this is reversed, and it therefore appears to move in the actual direction of its orbital motion. The rapidity of its phases is also equally remarkable. It is less than two hours from new moon to first quarter, and so on. Thus the inhabitants of *Mars* may see their inner moon pass through all its phases in a single night.

CHAPTER VI.

THE MINOR PLANETS.

When the solar system was first mapped out in its true proportions by Copernicus and Kepler, only six primary planets were known — namely, *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn*. These succeeded each other according to a nearly regular law, as we have shown in Chapter I., except that between *Mars* and *Jupiter* a gap was left, where an additional planet might be inserted, and the order of distance be thus made complete. It was therefore supposed by the astronomers of the seventeenth and eighteenth centuries that a planet might be found in this region. A search for this object was instituted toward the end of the last century, but before it had made much progress a planet in the place of the one so long expected was found by Piazzi, of Palermo. The discovery was made on the first day of the present century, 1801, January 1st.

In the course of the following seven years the astronomical world was surprised by the discovery of three other planets, all in the same region, though not revolving in the same orbits. Seeing four small planets where one large one ought to be, Olbers was led to his celebrated hypothesis that these bodies were the fragments of a large planet which had been broken to pieces by the action of some unknown force.

A generation of astronomers now passed away without the discovery of more than these four. But in December 1845, Hencke, of Dreisen, being engaged in mapping down the stars near the ecliptic, found a fifth planet of the group. In 1847 three more were discovered, and discoveries have since been made at a rate which thus far shows no signs of diminution. The number has now reached 200, and the discovery of additional ones seems to be going on as fast as ever. The frequent announcements of the discovery of planets which appear in the public prints all refer to bodies of this group.

The minor planets are distinguished from the major ones by many characteristics. Among these we may mention their great number, which exceeds that of all the other known bodies of the solar system; their small size; their positions, all being situated between the orbits of *Mars* and *Jupiter;* the great eccentricities and inclinations of their orbits.

Number of Small Planets.—It would be interesting to know how many of these planets there are in all, but it is as yet impossible even to guess at the number. As already stated, fully 200 are now known, and the number of new ones found every year ranges from 7 or 8 to 10 or 12. If ten additional ones are found every year during the remainder of the century, 400 will then have been discovered.

The discovery of these bodies is a very difficult work, requiring great practice and skill on the part of the astronomer. The difficulty is that of distinguishing them amongst the hundreds of thousands of telescopic stars which are scattered in the heavens. A minor planet presents no sensible disk, and therefore looks exactly like a small star. It can be detected only by its motion among the surrounding stars, which is so slow that hours or even days must elapse before it can be noticed.

Magnitudes.—In consequence of the minor planets having no visible disks in the most powerful telescopes, it is impossible to make any precise measurement of their diameters. These can, however, be estimated by the amount of light which the planet reflects. Supposing the proportion of light reflected about the same as in the case of the larger planets, it is estimated that the diameters of the three or four largest, which are those first discovered, range between 300 and 600 kilometres, while the smallest are probably from 20 to 50 kilometres in diameter. The average diameter of all that are known is perhaps less than 150 kilometres—that is, scarcely more than one hundredth that of the earth. The volumes of solid bodies vary as the cubes of their diameters; it might therefore take a million of these planets to make one of the size of the earth.

Form of Orbits.—The orbits of the minor planets are much more eccentric than those of the larger ones; their distance from the sun therefore varies very widely. The most eccentric orbit yet known is that of *Aethra*, which was discovered by Professor WATSON in 1873. Its least distance from the sun is 1·61, a very little further than *Mars*, while at aphelion it is 3·59, or more than twice as far. Two or three others are twice as far from the sun at aphelion as at perihelion, while nearly all are so eccentric that if the orbits were drawn to a scale, the eye would readily perceive that the sun was not in their centres. The largest inclination of all is that of *Pallas*, which is one of the original four, having been discovered by OLBERS in 1802. The inclination to the ecliptic is 34°, or more than one third of a right angle. Five or six others have inclinations exceeding 20°; they therefore range entirely outside the zodiac, and in fact sometimes culminate to the north of our zenith.

Origin of the Minor Planets.—The question of the origin of these bodies was long one of great interest. The features which we have described associate themselves very naturally with the celebrated hypothesis of OLBERS, that we here have the fragments of a single large planet which in the beginning revolved in its proper place between the orbits of *Mars* and *Jupiter*. OLBERS himself suggested a test of his theory. If these bodies were really formed by an explosion of the large one, the separate orbits of the fragments would all pass through the point where the explosion occurred. A common point of intersection was therefore long looked for; but although two or three of the first four did pass pretty near each other, the required point could not be found for all four.

It was then suggested that the secular changes in the orbits produced by the action of the other planets would in time change the positions of all the orbits in such a way that they would no longer have any common intersection. The secular variations of their orbits were therefore computed, to see if there was any sign of the required intersection in past ages, but none could be found. No support has been given to OLBERS' hypothesis by subsequent investigations, and it is no longer considered by astronomers to have any foundation. So far as can be judged, these bodies have been revolving around the sun as separate planets ever since the solar system itself was formed.

CHAPTER VII.

JUPITER AND HIS SATELLITES.

§ 1. THE PLANET JUPITER.

Jupiter is much the largest planet in the system. His mean distance is nearly 800,000,000 kilometres (480,000,000 miles). His diameter is 140,000 kilometres, corresponding to a mean apparent diameter, as seen from the sun of 36″·5. His linear diameter is about $\frac{1}{10}$, his surface is $\frac{1}{100}$, and his volume $\frac{1}{1000}$ that of the sun. His mass is $\frac{1}{1048}$, and his density is thus nearly the same as the sun's—viz., 0·24 of the earth's. He rotates on his axis in $9^h\ 55^m\ 20^s$.

He is attended by four satellites, which were discovered by GALILEO on January 7th, 1610. He named them in honor of the MEDICIS, the *Medicean stars*. These satellites were independently discovered on January 16th, 1610, by HARRIOT, of England, who observed them through several subsequent years. SIMON MARIUS also appears to have early observed them, and the honor of their discovery is claimed for him. They are now known as Satellites I, II, III, and IV, I being the nearest.

The surface of *Jupiter* has been carefully studied with the telescope, particularly within the past 20 years. Although further from us than *Mars*, the details of his disk are much easier to recognize. The most characteristic features are given in the drawings appended. These features are, *firstly*, the dark bands of the equatorial regions, and, *secondly*, the cloud-like forms spread over nearly the whole surface. At the limb all these details become indistinct, and finally vanish, thus indicating a highly absorptive atmosphere. The light from the centre of the disk is twice as bright as that from the poles (ARAGO). The bands can be seen with instruments no more powerful than those used by GALILEO, yet he makes no mention of them, although they were seen by ZUCCHI, FONTANA, and others before 1633. HUYGHENS (1659) describes the bands as brighter than the rest of the disk—a unique observation, on which we must look with some distrust, as since 1660 they have constantly been seen darker than the rest of the planet.

The color of the bands is frequently described as a brick-red, but one of the authors has made careful studies in

FIG. 91.—TELESCOPIC VIEW OF JUPITER AND HIS SATELLITES.

color of this planet, and finds the prevailing tint to be a salmon color, exactly similar to the color of *Mars*. The position of the bands varies in latitude, and the shapes of the limiting curves also change from day to day; but in the main they remain as permanent features of the region to which they belong. Two such bands are usually visible, but often more are seen. For example, CASSINI (1690, December 16th) saw six parallel bands extending completely around the planet. HERSCHEL, in the year 1793, attributed the aspects of the bands to zones of the planet's atmosphere more tranquil and less filled with clouds than the remaining portions, so as to permit the true surface of the planet to be seen through these zones, while the prevailing clouds in the other regions give a brighter tint to these latter. The color of the bands seems to vary from time to time, and their bordering lines sometimes alter with such rapidity as to show that these borders are formed of something like clouds.

The clouds themselves can easily be seen at times, and they have every variety of shape, sometimes appearing as

FIG. [illegible].—TELESCOPIC VIEW OF JUPITER, WITH A SATELLITE AND ITS SHADOW SEEN ON IT.

brilliant circular white masses, but oftener they are similar in form to a series of white cumulous clouds such as are frequently seen piled up near the horizon on a summer's day. The bands themselves seem frequently to be veiled over with something like the thin *cirrus* clouds of our atmosphere. On one occasion an annulus of white cloud was seen on one of the dark bands for many days, retaining its shape through the whole period.

Such clouds can be tolerably accurately observed, and may be used to determine the rotation time of the planet. These observations show that the clouds have often a motion of their own, which is also evident from other considerations.

The following results of observation, of spots situated in various regions of the planet will illustrate this :

				h.	m.	s.
CASSINI............	1665,	rotation time	=	9	56	00
HERSCHEL..........	1778,	" "	=	9	55	40
HERSCHEL..........	1779,	" "	=	9	50	48
SCHROETER.........	1785,	" "	=	9	56	56
BEER & MÄDLER....	1835,	" "	=	9	55	26
AIRY................	1835,	" "	=	9	55	21
SCHMIDT............	1862,	" "	=	9	55	29

§ 2. THE SATELLITES OF JUPITER.

Motions of the Satellites.— The four satellites move about *Jupiter* from west to east in nearly circular orbits. When one of these satellites passes between the sun and *Jupiter*, it casts a shadow upon *Jupiter's* disk (see Fig. 92) precisely as the shadow of our moon is thrown upon the earth in a solar eclipse. If the satellite passes through *Jupiter's* own shadow in its revolution, an eclipse of this satellite takes place. If it passes between the earth and *Jupiter*, it is projected upon *Jupiter's* disk, and we have a transit ; if *Jupiter* is between the earth and the satellite, an occultation of the latter occurs. All these phenomena can be seen from the earth with a common telescope, and the times of observation are all found predicted in the *Nautical Almanac.* In this way we are sure that the black spots which we see moving across the disk of *Jupiter* are really the shadows of the satellites themselves, and not phenomena to be otherwise explained. These shadows being seen black upon *Jupiter's* surface, show that this planet shines by reflecting the light of the sun.

Telescopic Appearance of the Satellites.—Under ordinary circumstances, the satellites of *Jupiter* are seen to have disks—that is, not to be mere points of light. Under very favorable conditions, markings have beeen seen on these disks, and it is very curious that the anomalous appearances given in Fig. 98 (by Dr. HASTINGS) have been seen at various times by other good observers, as SECCHI, DAWES, and RUTHERFURD. Satellite III, which is much the largest, has decided markings on its face; IV sometimes appears, as in the figure, to have its circular outline

FIG. 98.—TELESCOPIC APPEARANCE OF JUPITER'S SATELLITES.

cut off by right lines, and satellite I sometimes appears gibbous. The opportunities for observing these appearances are so rare that nothing is known beyond the bare fact of their existence, and no plausible explanation of the figure shown in IV has been given.

Phenomena of the Satellites.—The phenomena of the satellites are illustrated in Fig. 94. Here *S* represents the sun, *A T* the orbit of the earth (the earth itself being at *T*), the outer circle the orbit of *Jupiter*, and the four small circles upon the latter four different positions of [illegible] of a satellite. In the centre of each of the satellite orbits [illegible] small white circle designed to represent the planet *Jupiter* itself. The dotted lines drawn from each edge of the sun to the corresponding edges of the planet and continued until they meet in a point show the outlines of the shadow of *Jupiter*.

Let us first consider the position of *Jupiter* marked *J* to the left of the figure, it being then in opposition to the sun. The observer on the earth at *T* could not then see an object anywhere in the shadow of *Jupiter* because the latter is entirely behind the planet. Hence, as the satellite moves around, he will see it disappear behind the right-hand limb of the planet and reappear from the left-hand limb. Such a phenomenon is called an occultation, and is designated as disappearance or reappearance, according to the phase.

It may be remarked, however, that the inclination of the outer satellite to the orbit of *Jupiter* is so great that it sometimes passes entirely above or below the planet, and therefore is not occulted at all.

Let us next consider *Jupiter* in the position J'' near the bottom of the figure, the shadow, as before, pointing from the planet directly away from the sun. If the shadow were a visible object, the observer on the earth at T could see it projected out on the right of the planet, because he is not in the line between *Jupiter* and the sun. Hence as a satellite moves around and enters the shadow, he will see it disappear from sight, owing to the sunlight being cut off; this

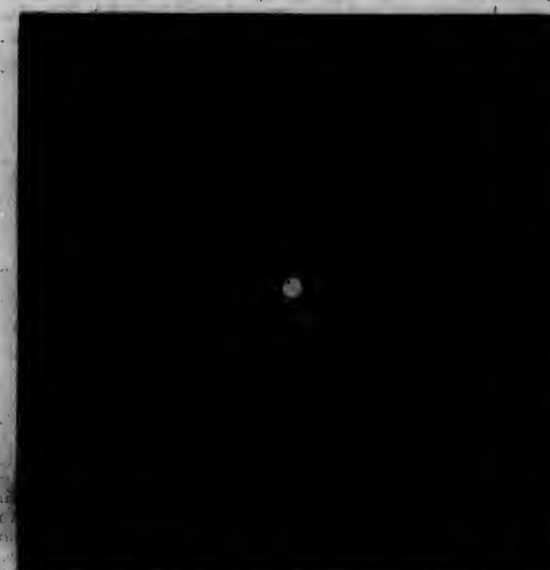

FIG. 94.—PHENOMENA OF JUPITER'S SATELLITES.

is called an *eclipse disappearance*. If the satellite is one of the two outer ones, he will be able to see it reappear again after it comes out of the shadow before it is occulted behind the planet.

Soon afterward the occultation will occur, and it will afterward reappear on the left. In the case of the inner or first satellite, however, the point of emergence from the shadow is hidden behind the planet, consequently the observer, after it once disappears in the shadow, will not see it reappear until it emerges from behind the planet.

If the planet is in the position J', the satellite will be occulted behind the planet where it reaches the first dotted line. If it is the inner satellite, it will not be seen to reappear on the other side of the planet, because when it reaches the second dotted line it has entered the shadow. After a while, however, it will reappear from the shadow some little distance to the left of the planet; this phenomenon is called an *eclipse reappearance.* In the case of the outer satellites, it may sometimes happen that they are visible for a short time after they emerge from behind the disk and before they enter the shadow.

These different appearances are, for convenience, represented in the figure as corresponding to different positions of *Jupiter* in his orbit, the earth having the same position in all; but since *Jupiter* revolves around the sun only once in twelve years, the changes of relative position really correspond to different positions of the earth in its orbit during the course of the year.

The satellites completely disappear from telescopic view when they enter the shadow of the planet. This seems to show that neither planet nor satellite is self-luminous to any great extent. If the satellite were self-luminous, it would be seen by its own light, and if the planet were luminous the satellite might be seen by the reflected light of the planet.

The motions of these objects are connected by two curious and important relations discovered by La Place, and expressed as follows:

I. *The mean motion of the first satellite added to twice the mean motion of the third is exactly equal to three times the mean motion of the second.*

II. *If to the mean longitude of the first satellite we add twice the mean longitude of the third, and subtract three times the mean longitude of the second, the difference is always* 180°.

The first of these relations is shown in the following table of the mean daily motions of the satellites:

Satellite I in one day moves	203°·4890
" II " "	101°·3748
" III " "	50°·3177
" IV " "	21°·5711
Motion of Satellite I	203°·4890
Twice that of Satellite III	100°·6354
Sum	304°·1244
Three times motion of Satellite II	304°·1244

Observations showed that this condition was fulfilled as exactly as possible, but the discovery of La Place consisted in showing that if the approximate coincidence of the mean motions was once established, they could never deviate from exact coincidence with the law. The case is analogous to that of the moon, which always presents the same face to us and which always will since the relation being once approximately true, it will become exact and ever remain so.

The discovery of the gradual propagation of light by means of these satellites has already been described, and it has also been explained that they are of use in the rough determination of longitudes. To facilitate their observation, the Nautical Almanac gives complete ephemerides of their phenomena. A specimen of a portion of such an ephemeris for 1865, March 7th, 8th, and 9th, is added. The times are Washington mean times. The letter *W* indicates that the phenomenon is visible in Washington.

1865—March.

				d.	h.	m.	s.
I.	Eclipse	Disapp.		7	18	27	38·5
I.	Occult.	Reapp.		7	21	56	
III.	Shadow	Ingress		8	7	27	
III.	Shadow	Egress		8	9	58	
III.	Transit	Ingress		8	12	31	
II.	Eclipse	Disapp.		8	13	1	22·7
III.	Transit	Egress	W.	8	15	6	
II.	Eclipse	Reapp.	W.	8	15	24	11·1
II.	Occult.	Disapp.	W.	8	15	27	
I.	Shadow	Ingress	W.	8	15	43	
I.	Transit	Ingress	W.	8	16	58	
I.	Shadow	Egress		8	17	57	
II.	Occult.	Reapp.		8	17	59	
I.	Transit	Egress		8	19	13	
I.	Eclipse	Disapp.		9	12	55	59·4
I.	Occult.	Reapp.	W.	9	16	25	

Suppose an observer near New York City to have determined his local time accurately. This is about 13^m faster than Washington time. On 1865, March 8th, he would look for the reappearance of II at about $15^h\ 34^m$ of his local time. Suppose he observed it at $15^h\ 36^m\ 22^s{\cdot}7$ of his time: then his meridian is $12^m\ 11^s{\cdot}6$ east of Washington. The difficulty of observing these eclipses with accuracy, and the fact that the aperture of the telescope employed has an important effect on the appearances seen, have kept this method from a wide utility, which it at first seemed to promise.

The apparent diameters of these satellites have been measured by Struve, Secchi, and others, and the best results are:

I, 1″·0; II, 0″·9; III, 1″·5; IV, 1″·3.

Their masses (*Jupiter*=1) are:

I, 0·000017; II, 0·000023; III, 0·000088; IV, 0·000043.

The third satellite is thus the largest, and it has about the density of the planet. The true diameters vary from 2200 to 3700 miles. The volume of II is about that of our moon; III approaches our earth in size.

Variations in the light of these bodies have constantly been noticed which have been supposed to be due to the fact that they turned on their axes once in a revolution, and thus presented various faces to us. The recent accurate photometric measures of Engelmann show that this hypothesis will not account for all the changes observed, some of which appear to be quite sudden.

ELEMENTS OF THE SATELLITES OF JUPITER.

SATELLITE.	Mass. (Jupiter = 1.)	Mean Daily Tropical Motion.	Synodic Period, or Interval between Eclipses.	Paris Mean Time of First Superior Mean Conjunction in 1880.	Mean Distance from Jupiter. In Arc at Distance = 5·20278.	Mean Distance from Jupiter. In Miles.
		°	d. h. m. s. d.	h. m. s.	"	
I......	·000016877	203·488993385	1 18 28 35·945375 = 1·7698605	Jan. 1, 6 16 45·1	111·83	266,000
II......	·000023227	101·374762063	3 13 17 53·735363 = 3·5540942	Jan. 2, 17 3 2·7	177·81	414,000
III......	·000088437	50·317646432	7 3 59 35·854197 = 7·1663872	Jan. 3, 20 27 5·3	283·63	661,000
IV......	·000042475	21·571109430	16 18 5 6·928880 = 16·7535524	Jan. 0, 15 6 37·8	498·85	1,169,000

CHAPTER VIII.

SATURN AND ITS SYSTEM.

§ 1. GENERAL DESCRIPTION.

Saturn is the most distant of the major planets known to the ancients. It revolves around the sun in 29½ years, at a mean distance of nearly 1,500,000,000 kilometres (890,000,000 miles). The angular diameter of the ball of the planet is about 16″·2, corresponding to a true diameter of about 110,000 kilometres (70,500 miles). Its diameter is therefore nearly nine times and its volume about 700 times that of the earth. It is remarkable for its small density, which, so far as known, is less than that of any other heavenly body, and even less than that of water. Consequently, it cannot be composed of rocks, like those which form our earth. It revolves on its axis, according to the recent observations of Professor HALL, in $10^h\ 14^m\ 24^s$, or less than half a day.

Saturn is perhaps the most remarkable planet in the solar system, being itself the centre of a system of its own, altogether unlike any thing else in the heavens. Its most noteworthy feature is seen in a pair of rings which surround it at a considerable distance from the planet itself. Outside of these rings revolve no less than eight satellites, or twice the greatest number known to surround any other planet. The planet, rings, and satellites are altogether called the *Saturnian system*. The general appearance of this system, as seen in a small telescope, is shown in Fig. 95.

To the naked eye, *Saturn* is of a dull yellowish color, shining with about the brilliancy of a star of the first magnitude. It varies in brightness, however, with the way in which its ring is seen, being brighter the wider the ring appears. It comes into opposition at intervals of one year and from twelve to fourteen days. The following are the times of some of these oppositions, by studying which one will be enabled to recognize the planet:

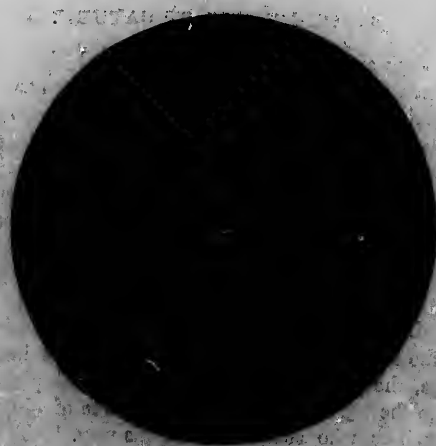

FIG. 95.—TELESCOPIC VIEW OF THE SATURNIAN SYSTEM.

1879..................... October 5th.
1880..................... October 18th.
1881..................... October 31st.
1882..................... November 14th.
1883..................... November 28th.
1884..................... December 11th.

During these years it will be best seen in the autumn and winter.

When viewed with a telescope, the physical appearance of the ball of *Saturn* is quite similar to that of *Jupiter*, having light and dark belts parallel to the direction of its rotation. But these cloud-like belts are very difficult to see, and so indistinct that it is not easy to determine the time of rotation from them. This has been done by observing the revolution of bright or dark spots which appear on the planet on very rare occasions.

§ 2. THE RINGS OF SATURN.

The rings are the most remarkable and characteristic feature of the Saturnian system. Fig. 96 gives two views of the ball and rings. The upper one shows one of their aspects as actually presented in the telescope, and the lower one shows what the appearance would be if the planet were viewed from a direction at right angles to the plane of the ring (which it never can be from the earth).

The first telescopic observers of *Saturn* were unable to see the rings in their true form, and were greatly perplexed to account for the appearance which the planet presented. GALILEO described the planet as "tri-corporate," the two ends of the ring having, in his imperfect telescope, the appearance of a pair of small planets attached to the central one. "On each side of old *Saturn* were servitors who aided him on his way." This supposed discovery was announced to his friend KEPLER in the following logogriph:

smaismrmilmepoelalevmibonenugttaviras, which, being transposed, becomes—

"Altissimam planetam tergeminam obsevavi" (I have observed the most distant planet to be triform).

The phenomenon constantly remained a mystery to its first observer. In 1610 he had seen the planet accompanied, as he supposed, by two lateral stars; in 1612 the latter had vanished, and the central body alone remained. After that GALILEO ceased to observe *Saturn*.

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FIG. 96.—RINGS OF SATURN.

The appearances of the ring were also incomprehensible to HEVELIUS, GASSENDI, and others. It was not until 1655 (after seven years of observation) that the celebrated HUYGHENS discovered the true explanation of the remarkable and recurring series of phenomena present by the tricorporate planet.

He announced his conclusions in the following logograph :—

"aaaaaa ccccc d eeeee g h iiiiiii llll mm nnnnnnnnn oooo pp q rr s ttttt uuuuu," which, when arranged, read—

"Annulo cingitur, tenui, plano, nusquam cohærente, ad eclipticam inclinato" (it is girdled by a thin plane ring, nowhere touching, inclined to the ecliptic).

This description is complete and accurate.

In 1665 it was found by BALL, of England, that what HUYGHENS had seen as a single ring was really two. A division extended all the way around near the outer edge. This division is shown in the figures.

In 1850 the Messrs. BOND, of Cambridge, found that there was a third ring, of a dusky and nebulous aspect, inside the other two, or rather attached to the inner edge of the inner ring. It is therefore known as *Bond's dusky ring.* It had not been before fully described owing to its darkness of color, which made it a difficult object to see except with a good telescope. It is not separated from the bright ring, but seems as if attached to it. The latter shades off toward its inner edge, which merges gradually into the dusky ring so as to make it difficult to decide precisely where it ends and the dusky ring begins. The latter extends about one half way from the inner edge of the bright ring to the ball of the planet.

Aspect of the Rings.—As *Saturn* revolves around the sun, the plane of the rings remains parallel to itself. That is, if we consider a straight line passing through the centre of the planet, perpendicular to the plane of the ring, as the axis of the latter, this axis will always point in the same direction. In this respect, the motion is similar to that of the earth around the sun. The ring of *Saturn* is inclined about 27° to the plane of its orbit. Consequently, as the planet revolves around the sun, there is a change in the direction in which the sun shines upon it similar to that which produces the change of seasons upon the earth, as shown in Fig. 46, page 109.

The corresponding changes for *Saturn* are shown in Fig. 97. During each revolution of *Saturn* the plane

FIG. 97.—DIFFERENT ASPECTS OF THE RING OF SATURN AS SEEN FROM THE EARTH.

of the ring passes through the sun twice. This occurred in the years 1862 and 1878, at two opposite points of the orbit, as shown in the figure. At two other points, midway between these, the sun shines upon the plane of the ring at its greatest inclination, about 27°. Since the earth is little more than one tenth as far from the sun as *Saturn* is, an observer always sees *Saturn* nearly, but not quite, as if he were upon the sun. Hence at certain times the rings of *Saturn* are seen edgeways, while at other times they are at an inclination of 27°, the aspect depending upon the position of the planet in its orbit. The following are the times of some of the phases :

1878, February 7th.—The edge of the ring was turned toward the sun. It could then be seen only as a thin line of light.

1885.—The planet having moved forward 90°, the south side of the rings may be seen at an inclination of 27°.

1891, December.—The planet having moved 90° further, the edge of the ring is again turned toward the sun.

1899.—The north side of the ring is inclined toward the sun, and is seen at its greatest inclination.

The rings are extremely thin in proportion to their extent. Their form is much the same as if they were cut out of large sheets of thin paper. Consequently, when their edges are turned toward the earth, they appear as a thin line of light, which can be seen only with powerful telescopes. With such telescopes, the planet appears as if it were pierced through by a piece of very fine wire, the ends of which project on each side more than the diameter of the planet. It has frequently been remarked that this appearance is seen on one side of the planet, when no trace of the ring can be seen on the other.

There is sometimes a period of a few weeks during which the plane of the ring, extended outward, passes between the sun and the earth. That is, the sun shines on one side of the ring, while the other or dark side is turned toward the earth. In this case, it seems to be established that only the edge of the ring is visible. If this be so, the substance of the rings cannot be transparent to the sun's rays, else it would be seen by the light which passes through it.

Possible Changes in the Rings.—In 1851 OTTO STRUVE propounded a noteworthy theory of changes going on in the rings of *Saturn*. From all the descriptions, figures, and measures given by the older astronomers, it appeared that two hundred years ago the space between the planet and the inner ring was at least equal to the combined breadth of the two rings. At present this distance is less than one half of this breadth. Hence STRUVE concluded that the inner ring was widening on the inside, so that its edge had been approaching the planet at the rate of about 1″·3 in a century. The space between the planet and the inner edge of the bright ring is now about 4″, so that if STRUVE's theory were true, the inner edge of the ring would actually reach the planet about the year 2200. Notwithstanding the amount of evidence which STRUVE cited in favor of his theory, astronomers generally are incredulous respecting the reality of so extraordinary a change. The measures necessary to settle the question are so difficult and the change is so slow that some time must elapse before the theory can be established, even if it is true. The measures of KAISER render this doubtful.

Shadow of Planet and Ring.—With any good telescope it is easy to observe both the shadow of the ring upon the ball of *Saturn* and that of the ball upon the ring. The form which the shadows present often appears different from that which the shadow ought to have according to the geometrical conditions. These differences probably arise from irradiation and other optical illusions.

Constitution of the Rings of Saturn.—The nature of these objects has been a subject both of wonder and of investigation by mathematicians and astronomers ever since they were discovered. They were at first supposed to be solid bodies; indeed, from their appearance it was difficult to conceive of them as anything else. The question then arose: What keeps them from falling on the planet? It was shown by LA PLACE that a homogeneous and solid ring surrounding the planet could not remain in a state of equilibrium, but must be precipitated upon the central ball by the smallest disturbing force. HERSCHEL having thought that he saw certain irregularities in the figure of the ring, LA PLACE concluded that the object could be kept in equilibrium by them. He simply assumed this, but did not attempt to prove it.

About 1850 the investigation was again begun by Professors BOND and PEIRCE, of Cambridge. The former supposed that the rings could not be solid at all, because they had sometimes shown signs of being temporarily broken up into a large number of concentric rings. Although this was probably an optical illusion, he concluded that the rings must be liquid. Professor PEIRCE took up the problem where LA PLACE had left it, and showed that even an irregular solid ring would not be in equilibrium about *Saturn*. He therefore adopted the view of BOND, that the rings were fluid; but finding that even a fluid ring would be unstable without a support, he supposed that such a support might be furnished by the satellites. This view has also been abandoned.

It is now established beyond reasonable doubt that the rings do not form a continuous mass, but are really a countless multitude of small separate particles, each of which revolves on its own account. These satellites are individually far too small to be seen in any telescope, but so numerous that when viewed from the distance of the earth they appear as a continuous mass, like particles of dust floating in a sunbeam. This theory was first propounded by CASSINI, of Paris, in 1715. It had been forgotten for a century or more, when it was revived by Professor CLERK MAXWELL in 1856. The latter published a profound mathematical discussion of the whole question, in which he shows that this hypothesis and this alone would account for the appearances presented by the rings.

KAISER's measures of the dimensions of the Saturnian system are:

BALL OF SATURN.

Equatorial diameter	17''·274
Polar "	15''·392

RINGS.

Major axis of outer ring	39''·471
" " " the great division	34''·227
" " " the inner edge of ring	27''·859
Width of the ring	5''·806
Dark space between ball and ring	5''·292

§ 3. SATELLITES OF SATURN.

Outside the rings of *Saturn* revolve its eight satellites, the order and discovery of which are shown in the following table:

No.	NAME.	Distance from Planet.	Discoverer.	Date of Discovery.
1	Mimas.	3·3	Herschel.	1789, September 17
2	Enceladus.	4·3	Herschel.	1789, August 28.
3	Tethys.	5·3	Cassini.	1684, March.
4	Dione.	6·8	Cassini.	1684, March.
5	Rhea.	9·5	Cassini.	1672, December 23.
6	Titan.	20·7	Huyghens.	1655, March 25.
7	Hyperion.	26·8	Bond.	1848, September 16
8	Japetus.	64·4	Cassini.	1671, October.

The distances from the planet are given in radii of th latter. The satellites *Mimas* and *Hyperion* are visibl only in the most powerful telescopes. The brightest o all is *Titan*, which can be seen in a telescope of the small est ordinary size. *Japetus* has the remarkable peculiarit of appearing nearly as bright as *Titan* when seen west of the planet, and so faint as to be visible only in large telescopes when on the other side. This appearance is explained by supposing that, like our moon, it always presents the same face to the planet, and that one side of it is black and the other side white. When west of the planet, the bright side is turned toward the earth and the satellite is visible. On the other side of the planet, the dark side is turned toward us, and it is nearly invisible. Most of the remaining five satellites can be ordinarily seen with telescopes of moderate power.

The elements of all the satellites are shown in the following table :

SATELLITE.	Mean Daily Motion.	Mean Distance from Saturn.	Longitude of Peri-Sat.	Eccentricity.	Inclination to Ecliptic.		Longitude of Node	
	°		° ′	° ′	°	′	°	′
Mimas.....	381·958	......	?	?	28	00	168	00
Enceladus.	262·731	......	?	?	28	00	168	00
Tethys....	190·69773	43·70	?	?	28	10	167	38
Diane.....	131·534930	54·60	?	?	28	10	167	38
Rhea......	79·690216	76·12	?	?	28	11	166	34
Titan......	22·577033	176·75	257.16	·0286	27	34	167	56
Hyperion..	16·914	214·22	40·00	·125	28	00	168	00
Japetus...	4·538036	514·64	351·25	·0289	18	44	142	53

CHAPTER IX.

THE PLANET URANUS.

Uranus was discovered on March 13th, 1781, by Sir WILLIAM HERSCHEL (then an amateur observer) with a ten-foot reflector made by himself. He was examining a portion of the sky near H *Geminorum*, when one of the stars in the field of view attracted his notice by its peculiar appearance. On further scrutiny, it proved to have a planetary disk, and a motion of over 2″ per hour. HERSCHEL at first supposed it to be a comet in a distant part of its orbit, and under this impression parabolic orbits were computed for it by various mathematicians. None of these, however, satisfied subsequent observations, and it was finally announced by LEXELL and LA PLACE that the new body was a planet revolving in a nearly circular orbit. We can scarcely comprehend now the enthusiasm with which this discovery was received. No new body (save comets) had been added to the solar system since the discovery of the third satellite of *Saturn* in 1684, and all the major planets of the heavens had been known for thousands of years.

HERSCHEL suggested, as a name for the planet, *Georgium Sidus*, and even after 1800 it was known in the English *Nautical Almanac* as the Georgian Planet. LALANDE suggested *Herschel* as its designation, but this was judged too personal, and finally the name *Uranus* was adopted. Its symbol was for a time written ♅, in recognition of the name proposed by LALANDE.

Uranus revolves about the sun in 84 years. Its apparent diameter as seen from the earth varies little, being about 3″·9. Its true diameter is about 50,000 kilometres, and its figure is, so far as we yet know, exactly spherical.

In physical appearance it is a small greenish disk without markings. It is possible that the centre of the disk is slightly brighter than the edges. At its nearest approach to the earth, it shines as a star of the sixth magnitude, and is just visible to an acute eye when the attention is directed to its place. In small telescopes with low powers, its appearance is not markedly different from that of stars of about its own brilliancy.

It is customary to speak of HERSCHEL's discovery of *Uranus* as an accident; but this is not entirely just, as all conditions for the detection of such an object, if it existed, were fulfilled. At the same time the early identification of it as a planet was more easy than it would have been eleven days earlier, when, as ARAGO points out, the planet was stationary.

Sir WILLIAM HERSCHEL suspected that *Uranus* was accompanied by six satellites.

Of the existence of two of these satellites there has never been any doubt, as they were steadily observed by HERSCHEL from 1787 until 1810, and by Sir JOHN HERSCHEL during the years 1828 to 1832, as well as by other later observers. None of the other four satellites described by HERSCHEL have ever been seen by other observers, and he was undoubtedly mistaken in supposing them to exist. Two additional ones were discovered by LASSELL in 1847, and are, with the satellites of *Mars*, the faintest objects in the solar system. Neither of them is identical with any of the missing ones of HERSCHEL. As Sir WILLIAM HERSCHEL had suspected six satellites, the following names for the true satellites are generally adopted to avoid confusion:

			DAYS.
I, *Ariel*	Period	=	2·520383
II, *Umbriel*	"	=	4·144181
III, *Titania*, HERSCHEL's (II.)	"	=	8·705897
IV, *Oberon*, HERSCHEL's (IV.)	"	=	13·463269

It is an interesting question whether the observatio which HERSCHEL assigned to his supposititious satellite may not be composed of observations sometimes of *Ari* sometimes of *Umbriel*. In fact, out of nine suppose observations of I, one case alone was noted by HERSCH in which his positions were entirely trustworthy, and this night *Umbriel* was in the position of his suppose satellite I.

It is likely that *Ariel* varies in brightness on differe sides of the planet, and the same phenomenon has al been suspected for *Titania*.

The most remarkable feature of the satellites of *Uranus* is th their orbits are nearly perpendicular to the ecliptic instead having a small inclination to that plane, like those of all the orbi of both planets and satellites previously known. To form a corre idea of the position of the orbits, we must imagine them tipped ov until their north pole is nearly 8° below the ecliptic, instead of 9 above it. The pole of the orbit which should be considered as t north one is that from which, if an observer look down upon a r volving body, the latter would seem to turn in a direction opposi that of the hands of a watch. When the orbit is tipped over mo than a right angle, the motion from a point in the direction of t north pole of the ecliptic will seem to be the reverse of this; it therefore sometimes considered to be *retrograde*. This term is f quently applied to the motion of the satellites of *Uranus*, but rather misleading, since the motion, being nearly perpendicular the ecliptic, is not exactly expressed by the term.

The four satellites move in the same plane, so far as the most fined observations have ever shown. This fact renders it high probable that the planet *Uranus* revolves on its axis in the sa plane with the orbits of the satellites, and is therefore an obl spheroid like the earth. This conclusion is founded on the cons eration that if the planes of the satellites were not kept together some cause, they would gradually deviate from each other owing the attractive force of the sun upon the planet. The different sat lites would deviate by different amounts, and it would be extrem improbable that all the orbits would at any time be found in same plane. Since we see them in the same plane, we conclude t some force keeps them there, and the oblateness of the planet wo cause such a force.

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CHAPTER X.

THE PLANET NEPTUNE.

AFTER the planet *Uranus* had been observed for some thirty years, tables of its motion were prepared by BOUVARD. He had as data available for this purpose not only the observations since 1781, but also observations made by LE MONNIER, FLAMSTEED, and others, extending back as far as 1695, in which the planet was observed for a fixed star and so recorded in their books. As one of the chief difficulties in the way of obtaining a theory of the planet's motion was the short period of time during which it had been regularly observed, it was to be supposed that these ancient observations would materially aid in obtaining exact accordance between the theory and observation. But it was found that, after allowing for all perturbations produced by the known planets, the ancient and modern observations, though undoubtedly referring to the same object, were yet not to be reconciled with each other, but differed systematically. BOUVARD was forced to omit the older observations in his tables, which were published in 1820, and to found his theory upon the modern observations alone. By so doing, he obtained a good agreement between theory and the observations of the few years immediately succeeding 1820.

BOUVARD seems to have formulated the idea that a possible cause for the discrepancies noted might be the existence of an unknown planet, but the meagre data at his disposal forced him to leave the subject untouched. In 1830 it was found that the tables which represented the motion of the planet well in 1820–25 were 20″ in error, in 1840 the error was 90″, and in 1845 it was over 120″.

These progressive and systematic changes attracted the attention of astronomers to the subject of the theory of the motion of *Uranus*. The actual discrepancy (120″) in 1845 was not a quantity large in itself. Two stars of the magnitude of *Uranus*, and separated by only 120″, would be seen as one to the unaided eye. It was on account of its systematic and progressive increase that suspicion was excited. Several astronomers attacked the problem in various ways. The elder STRUVE, at Pulkova, prosecuted a search for a new planet along with his double star observations; BESSEL, at Koenigsberg, set a student of his own, FLEMING, at a new comparison of observation with theory, in order to furnish data for a new determination; ARAGO, then Director of the Observatory at Paris, suggested this subject in 1845 as an interesting field of research to LE VERRIER, then a rising mathematician and astronomer. Mr. J. C. ADAMS, a student in Cambridge University, England, had become aware of the problems presented by the anomalies in the motion of *Uranus*, and had attacked this question as early as 1843. In October, 1845, ADAMS communicated to the Astronomer Royal of England elements of a new planet so situated as to produce the perturbations of the motion of *Uranus* which had actually been observed. Such a prediction from an entirely unknown student, as ADAMS then was, did not carry entire conviction with it. A series of accidents prevented the unknown planet being looked for by one of the largest telescopes in England, and so the matter apparently dropped. It may be noted, however, that we now know ADAMS' elements of the new planet to have been so near the truth that if it had been really looked for by the powerful telescope which afterward discovered its satellite, it could scarcely have failed of detection.

BESSEL's pupil FLEMING died before his work was done and BESSEL's researches were temporarily brought t an end. STRUVE's search was unsuccessful. Only LE VERRIER continued his investigations, and in the most thorough manner. He first computed anew the perturbations of *Uranus* produced by the action of *Jupiter* and *Saturn*. Then he examined the nature of the irregularities observed. These showed that if they were caused by an unknown planet, it could not be between *Saturn* and *Uranus*, or else *Saturn* would have been more affected than was the case.

The new planet was outside of *Uranus* if it existed at all, and as a rough guide BODE's law was invoked, which indicated a distance about twice that of *Uranus*. In the summer of 1846, LE VERRIER obtained complete elements of a new planet, which would account for the observed irregularities in the motion of *Uranus*, and these were published in France. They were very similar to those of ADAMS, which had been communicated to Professor CHALLIS, the Director of the Observatory of Cambridge.

A search was immediately begun by CHALLIS for such an object, and as no star-maps were at hand for this region of the sky, he began mapping the surrounding stars. In so doing the new planet was actually observed, both on August 4th and 12th, 1846, but the observations remaining unreduced, and so the planetary nature of the object was not recognized.

In September of the same year, LE VERRIER wrote to Dr. GALLE, then Assistant at the Observatory of Berlin, asking him to search for the new planet, and directing him to the place where it should be found. By the aid of an excellent star chart of this region, which had just been completed by Dr. BREMIKER, the planet was found September 23d, 1846.

The strict rights of discovery lay with LE VERRIER, but the common consent of mankind has always credited ADAMS with an equal share in the honor attached to this most brilliant achievement. Indeed, it was only by the most unfortunate succession of accidents that the discovery

did not attach to ADAMS' researches. One thing must i
fairness be said, and that is that the results of LE VE
RIER, which were reached after a most thorough invest
gation of the whole ground, were announced with an en
tire confidence, which, perhaps, was lacking in the othe
case.

This brilliant discovery created more enthusiasm tha
even the discovery of *Uranus*, as it was by an exercise o
far higher qualities that it was achieved. It appeared t
savor of the marvellous that a mathematician could sa

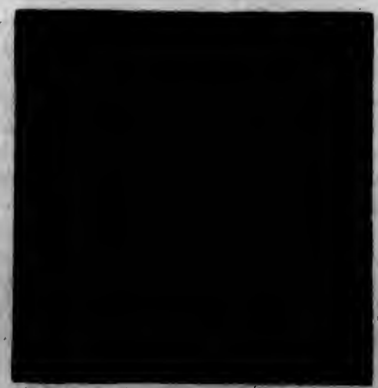

FIG. 98.

to a working astronomer that by pointing his telescope
a certain small area, within it should be found a ne
major planet. Yet so it was.

The general nature of the disturbing force which r
vealed the new planet may be seen by Fig. 98, whi
shows the orbits of the two planets, and their respecti
motions between 1781 and 1840. The inner orbit is th
of *Uranus*, the outer one that of *Neptune*. The arro
passing from the former to the latter show the directio
of the attractive force of *Neptune*. It will be seen th

the two planets were in conjunction in the year 1822. Since that time *Uranus* has, by its more rapid motion, passed more than 90° beyond *Neptune*, and will continue to increase its distance from the latter until the beginning of the next century.

Our knowledge regarding *Neptune* is mostly confined to a few numbers representing the elements of its motion. Its mean distance is more than 4,000,000,000 kilometres (2,775,000,000 miles); its periodic time is 164·78 years; its apparent diameter is 2″·6 seconds, corresponding to a true diameter of 55,000 kilometres. Gravity at its surface is about nine tenths of the corresponding terrestrial surface gravity. Of its rotation and physical condition nothing is known. Its color is a pale greenish blue. It is attended by one satellite, the elements of whose orbit are given herewith. It was discovered by Mr. Lassell, of England, in 1847. It is abont as faint as the two outer satellites of *Uranus*, and requires a telescope of twelve inches aperture or upward to be well seen.

Elements of the Satellite of Neptune, from Washington Observations.

Mean Daily Motion	61°·25679
Periodic Time	5^d·87690
Distance (log. Δ = 1·47814)	16″·275
Inclination of Orbit to Ecliptic	145° 7′
Longitude of Node (1850)	184° 30′
Increase in 100 Years	1° 24′

The great inclination of the orbit shows that it is turned nearly upside down; the direction of motion is therefore retrograde.

CHAPTER XI.

THE PHYSICAL CONSTITUTION OF THE PLANETS.

It is remarkable that the eight large planets of the solar system, considered with respect to their physical constitution as revealed by the telescope and the spectroscope, may be divided into four pairs, the planets of each pair having a great similarity, and being quite different from the adjoining pair. Among the most complete and systematic studies of the spectra of all the planets are those made by Mr. Huggins, of London, and Dr. Vogel, of Berlin. In what we have to say of the results of spectroscopy, we shall depend entirely upon the reports of these observers.

Mercury and Venus.—Passing outward from the sun, the first pair we encounter will be *Mercury* and *Venus*. The most remarkable feature of these two planets is a negative rather than a positive one, being the entire absence of any certain evidence of change on their surfaces. We have already shown that *Venus* has a considerable atmosphere, while there is no evidence of any such atmosphere around *Mercury*. They have therefore not been proved alike in this respect, yet, on the other hand, they have not been proved different. In every other respect than this, the similarity appears perfect. No permanent markings have ever been certainly seen on the disk of either. If, as is possible, the atmosphere of both planets is filled with clouds and vapor, no change, no openings, and no formations among these cloud masses are visible from the earth. Whenever either of these planets is in a certain position relative to the earth and the sun, it seemingly presents the same appearance, and not the slightest change occurs in that appearance from the rotation of the planet on its axis, which every analogy of the solar system leads us to believe must take place.

When studied with the spectroscope, the spectra of *Mercury* and *Venus* do not differ strikingly from that of the sun. This would seem to indicate that the atmospheres of these planets do not exert any decided absorption upon the rays of light which pass through them; or, at least, they absorb only the same rays which are absorbed by the atmosphere of the sun and by that of the earth. The one point of difference which Dr. VOGEL brings out is, that the lines of the spectrum produced by the absorption of our own atmosphere appear darker in the spectrum of *Venus*. If this were so, it would indicate that the atmosphere of *Venus* is similar in constitution to that of our earth, because it absorbs the same rays. But the means of measuring the darkness of the lines are as yet so imperfect that it is impossible to speak with certainty on a point like this. Dr. VOGEL thinks that the light from *Venus* is for the most part reflected from clouds in the higher region of the planet's atmosphere, and therefore reaches us without passing through a great depth of that atmosphere.

The Earth and Mars.—These planets are distinguished from all the others in that their visible surfaces are marked by permanent features, which show them to be solid, and which can be seen from the other heavenly bodies. It is true that we cannot study the earth from any other body, but we can form a very correct idea how it would look if seen in this way (from the moon, for instance). Wherever the atmosphere was clear, the outlines of the continents and oceans would be visible, while they would be invisible where the air was cloudy.

Now, so far as we can judge from observations made at so great a distance, never much less than forty millions of miles, the planet *Mars* presents to our telescopes very much the same general appearance that the earth would if observed from an equally great distance. The only exception is that the visible surface of *Mars* is seemingly much less obscured by clouds than that of the earth would be. In other words, that planet has a more sunny sky than ours. It is, of course, impossible to say what conditions we might find could we take a much closer view of *Mars*: all we can assert is, that so far as we can judge from this distance, its surface is like that of the earth.

This supposed similarity is strengthened by the spectroscopic observations. The lines of the spectrum due to aqueous vapor in our atmosphere are found by Dr. VOGEL to be so much stronger in *Mars* as to indicate an absorption by such vapor in its atmosphere. Dr. HUGGINS had previously made a more decisive observation, having found a well-marked line to which there is no corresponding strong line in the solar spectrum. This would indicate that the atmosphere of *Mars* contains some element not found in our own, but the observations are too difficult to allow of any well-established theory being yet built upon them.

Jupiter and Saturn.—The next pair of planets are *Jupiter* and *Saturn*. Their peculiarity is that no solid crust or surface is visible from without. In this respect they differ from the earth and *Mars*, and resemble *Mercury* and *Venus*. But they differ from the latter in the very important point that constant changes can be seen going on at their surfaces. The nature of these changes has been discussed so fully in treating of these planets individually, that we need not go into it more fully at present. It is sufficient to say that the preponderance of evidence is in favor of the view that these planets have no solid crusts whatever, but consist of masses of molten matter, surrounded by envelopes of vapor constantly rising from the interior.

The view that the greater part of the apparent volume of these planets is made of a seething mass of vapor is further strengthened by their very small specific gravity. This can be accounted for by supposing that the liquid interior is nothing more than a comparatively small central core, and that the greater part of the bulk of each planet is composed of vapor of small density.

That the visible surfaces of *Jupiter* and *Saturn* are covered by some kind of an atmosphere follows not only from the motion of the cloud forms seen there, but from the spectroscopic observations of HUGGINS in 1864. He found visible absorption-bands near the red end of the spectrum of each of these planets. VOGEL found a complete similarity between the spectra of the two planets, the most marked feature being a dark band in the red. What is worthy of remark, though not at all surprising, is that this band is not found in the spectrum of *Saturn's* rings. This is what we should expect, as it is hardly possible that these rings should have any atmosphere, owing to their very small mass. An atmosphere on bodies of so slight an attractive power would expand away by its own elasticity and be all attracted around the planet.

Uranus and Neptune.—These planets have a strikingly similar aspect when seen through a telescope. They differ from *Jupiter* and *Saturn* in that no changes or variations of color or aspect can be made out upon their surfaces; and from the earth and *Mars* in the absence of any permanent features. Telescopically, therefore, we might classify them with *Mercury* and *Venus*, but the spectroscope reveals a constitution entirely different from that of any other planets. The most marked features of their spectra are very dark bands, evidently produced by the absorption of dense atmospheres. Owing to the extreme faintness of the light which reaches us from these distant bodies, the regular lines of the solar spectrum are entirely invisible in their spectra, yet these dark bands which are peculiar to them have been seen by HUGGINS, SECCHI, VOGEL, and perhaps others.

FIG. 99.—SPECTRUM OF URANUS.

This classification of the eight planets into pairs is rendered yet more striking by the fact that it applies to what we have been able to discover respecting the rotations of these bodies. The rotation of the inner pair, *Mercury* and *Venus*, has eluded detection, notwithstanding their comparative proximity to us. The next pair, the earth and *Mars*, have perfectly definite times of rotation, because their outer surfaces consist of solid crusts, every part of which must rotate in the same time. The next pair, *Jupiter* and *Saturn*, have well-established times of rotation, but these times are not perfectly definite, because the surfaces of these planets are not solid, and different portions of their mass may rotate in slightly different times. *Jupiter* and *Saturn* have also in common a very rapid rate of rotation. Finally, the outer pair, *Uranus* and *Neptune*, seem to be surrounded by atmospheres of such density that no evidence of rotation can be gathered. Thus it seems that of the eight planets, only the central four have yet certainly indicated a rotation on their axes.

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CHAPTER XII.

METEORS.

§ 1. PHENOMENA AND CAUSES OF METEORS.

DURING the present century, evidence has been collected that countless masses of matter, far too small to be seen with the most powerful telescopes, are moving through the planetary spaces. This evidence is afforded by the phenomena of "aerolites," "meteors," and "shooting stars." Although these several phenomena have been observed and noted from time to time since the earliest historic era, it is only recently that a complete explanation has been reached.

Aerolites.—Reports of the falling of large masses of stone or iron to the earth have been familiar to antiquarian students for many centuries. ARAGO has collected several hundred of these reports. In one instance a monk was killed by the fall of one of these bodies. One or two other cases of death from this cause are supposed to have occurred. Notwithstanding the number of instances on record, aerolites fall at such wide intervals as to be observed by very few people, consequently doubt was frequently cast upon the correctness of the narratives. The problem where such a body could come from, or how it could get into the atmosphere to fall down again, formerly seemed so nearly incapable of solution that it required some credulity to admit the facts. When the evidence became so strong as to be indisputable, theories of their origin began to be propounded. One theory quite fashionable in the early part of this century was that they were thrown from volcanoes in the moon. This theory, though the subject of mathematical investigation by LA PLACE and others, is now no longer thought of.

The proof that aerolites did really fall to the ground first became conclusive by the fall being connected with other more familiar phenomena. Nearly every one who is at all observant of the heavens is familiar with *bolides*, or fire-balls—brilliant objects having the appearance of rockets, which are occasionally seen moving with great velocity through the upper regions of the atmosphere. Scarcely a year passes in which such a body of extraordinary brilliancy is not seen. Generally these bodies, bright though they may be, vanish without leaving any trace, or making themselves evident to any sense but that of sight. But on rare occasions their appearance is followed at an interval of several minutes by loud explosions like the discharge of a battery of artillery. On still rarer occasions, masses of matter fall to the ground. It is now fully understood that the fall of these aerolites is always accompanied by light and sound, though the light may be invisible in the daytime.

When chemical analysis was applied to aerolites, they were proved to be of extramundane origin, because they contained chemical combinations not found in terrestrial substances. It is true that they contained no new chemical elements, but only combination of the elements which are found on the earth. These combinations are now so familiar to mineralogists that they can distinguish an aerolite from a mineral of terrestrial origin by a careful examination. One of the largest components of these bodies is iron. Specimens having very much the appearance of great masses of iron are found in the National Museum at Washington.

Meteors.—Although the meteors we have described are of dazzling brilliancy, yet they run by insensible gradations into phenomena, which any one can see on any clear night. The most brilliant meteors of all are likely to be seen by one person only two or three times in his life. Meteors having the appearance and brightness of a distant rocket may be seen several times a year by any one in the habit of walking out during the evening and watching the sky. Smaller ones occur more frequently; and if a careful watch be kept, it will be found that several of the faintest class of all, familiarly known as *shooting stars*, can be seen on every clear night. We can draw no distinction between the most brilliant meteor illuminating the whole sky, and perhaps making a noise like thunder, and the faintest shooting star, except one of degree. There seems to be every gradation between these extremes, so that all should be traced to some common cause.

Cause of Meteors.—There is now no doubt that all these phenomena have a common origin, being due to the earth encountering innumerable small bodies in its annual course around the sun. The great difficulty in connecting meteors with these invisible bodies arises from the brilliancy and rapid disappearance of the meteors. The question may be asked why do they burn with so great an evolution of light on reaching our atmosphere? To answer this question, we must have recourse to the mechanical theory of heat. It is now known that heat is really a vibratory motion in the particles of solid bodies and a progressive motion in those of gases. By making this motion more rapid, we make the body warmer. By simply blowing air against any combustible body with sufficient velocity, it can be set on fire, and, if incombustible, the body will be made red-hot and finally melted. Experiments to determine the degree of temperature thus produced have been made by Sir William Thompson, who finds that a velocity of about 50 metres per second corresponds to a rise of temperature of one degree Centigrade. From this the temperature due to any velocity can be readily calculated on the principle that the increase of temperature is proportional to the "energy" of the particles, which again is proportional to the square of the velocity. Hence a velocity of 500 metres per second would correspond to a rise of 100° above the actual temperature of the air, so that if the latter was at the freezing-point the body would be raised to the temperature of boiling water. A velocity of 1500 metres per second would produce a red heat. This velocity is, however, much higher than any that we can produce artificially.

The earth moves around the sun with a velocity of about 30,000 metres per second; consequently if it met a body at rest the concussion between the latter and the atmosphere would correspond to a temperature of more than 300,000°. This would instantly dissolve any known substance.

As the theory of this dissipation of a body by moving with planetary velocity through the upper regions of our air is frequently misunderstood, it is necessary to explain two or three points in connection with it.

(1.) It must be remembered that when we speak of these enormous temperatures, we are to consider them as *potential*, not *actual*, temperatures. We do not mean that the body is actually raised to a temperature of 300,000°, but only that the air acts upon it as if it were put into a furnace heated to this temperature—that is, it is rapidly destroyed by the intensity of the heat.

(2.) This potential temperature is independent of the density of the medium, being the same in the rarest as in the densest atmosphere. But the actual effect on the body is not so great in a rare as in a dense atmosphere. Every one knows that he can hold his hand for some time in air at the temperature of boiling water. The rarer the air the higher the temperature the hand would bear without injury. In an atmosphere as rare as ours at the height of 50 miles, it is probable that the hand could be held for an indefinite period, though its temperature should be that of red-hot iron; hence the meteor is not consumed so rapidly as if it struck a dense atmosphere with planetary velocity. In the latter case it would probably disappear like a flash of lightning.

(3.) The amount of heat evolved is measured not by that which would result from the combustion of the body, but by the *vis viva* (energy of motion) which the body loses in the atmosphere. The student of physics knows that motion, when lost, is changed into a definite amount of heat. If we calculate the amount of heat which is equivalent to the energy of motion of a pebble having a velocity of 20 miles a second, we shall find it sufficient to raise about 1300 times the pebble's weight of water from the freezing to the boiling point. This is many times as much heat as could result from burning even the most combustible body.

(4.) The detonation which sometimes accompanies the passage of very brilliant meteors is not caused by an explosion of the meteor, but by the concussion produced by its rapid motion through the atmosphere. This concussion is of much the same nature as that produced by a flash of lightning. The air is suddenly condensed in front of the meteor, while a vacuum is left behind it.

The invisible bodies which produce meteors in the way just described have been called *meteoroids*. Meteoric phenomena depend very largely upon the nature of the meteoroids, and the direction and velocity with which they are moving relatively to the earth. With very rare exceptions, they are so small and fusible as to be entirely dissipated in the upper regions of the atmosphere. Even of those so hard and solid as to produce a brilliant light and the loudest detonation, only a small proportion reach the earth. It has sometimes happened that the meteoroid only grazes the atmosphere, passing horizontally through its higher strata for a great distance and continuing its course after leaving it. On rare occasions the body is so hard and massive as to reach the earth without being entirely consumed. The potential heat produced by its passage through the atmosphere is then all expended in melting and destroying its outer layers, the inner nucleus remaining unchanged. When such a body first strikes the denser portion of the atmosphere, the resistance becomes so great that the body is generally broken to pieces. Hence we very often find not simply a single aerolite, but a small shower of them.

Heights of Meteors.—Many observations have been made to determine the height at which meteors are seen. This is effected by two observers stationing themselves several miles apart and mapping out the courses of such meteors as they can observe. In order to be sure that the same meteor is seen from both stations, the time of each observation must be noted. In the case of very brilliant meteors, the path is often determined with considerable precision by the direction in which it is seen by accidental observers in various regions of the country over which it passes.

The general result from numerous observations and investigations of this kind is that the meteors and shooting stars commonly commence to be visible at a height of about 160 kilometres, or 100 statute miles. The separate results of course vary widely, but this is a rough mean of them. They are generally dissipated at about half this height, and therefore above the highest atmosphere which reflects the rays of the sun. From this it may be inferred that the earth's atmosphere rises to a height of at least 160 kilometres. This is a much greater height than it was formerly supposed to have.

§ 2. METEORIC SHOWERS.

As already stated, the phenomena of shooting stars may be seen by a careful observer on almost any clear night. In general, not more than three or four of them will be seen in an hour, and these will be so minute as hardly to attract notice. But they sometimes fall in such numbers as to present the appearance of a meteoric shower. On rare occasions the shower has been so striking as to fill the beholders with terror. The ancient and mediæval records contain many accounts of these phenomena which have been brought to light through the researches of antiquarians. The following is quoted by Professor NEWTON from an Arabic record :

> "In the year 599, on the last day of Moharrem, stars shot hither and thither, and flew against each other like a swarm of locusts ; this phenomena lasted until daybreak ; people were thrown into consternation, and made supplication to the Most High : there was never the like seen except on the coming of the messenger of God, on whom be benediction and peace."

It has long been known that some showers of this class occur at an interval of about a third of a century. One was observed by HUMBOLDT, on the Andes, on the night of November 12th, 1799, lasting from two o'clock until daylight. A great shower was seen in this country in 1833, and is well known to have struck the negroes of the Southern States with terror. The theory that the showers occur at intervals of 34 years was now propounded by OLBERS, who predicted a return of the shower in 1867. This prediction was completely fulfilled, but instead of appearing in the year 1867 only, it was first noticed in 1866. On the night of November 13th of that year a remarkable shower was seen in Europe, while on the corresponding night of the year following it was again seen in this country, and, in fact, was repeated for two or three years, gradually dying away.

The occurrence of a shower of meteors evidently shows that the earth encounters a swarm of meteoroids. The recurrence at the same time of the year, when the earth is in the same point of its orbit, shows that the earth meets the swarm at the same point in successive years. All the meteoroids of the swarm must of course be moving in the same direction, else they would soon be widely scattered. This motion is connected with the *radiant point*, a well-marked feature of a meteoric shower.

Radiant Point.—Suppose that, during a meteoric shower, we mark the path of each meteor on a star map, as in the figure. If we continue the paths backward in a straight line, we shall find that they all meet near one and the same point of the celestial sphere—that is, they move as if they all radiated from this point. The

FIG. 100.—RADIANT POINT OF METEORIC SHOWER.

latter is, therefore, called the *radiant point*. In the figure the lines do not all pass accurately through the same point. This is owing to the unavoidable errors made in marking out the path.

It is found that the radiant point is always in the same position among the stars, wherever the observer may be situated, and that it does not partake of the diurnal motion of the earth—that is, as the stars apparently move toward the west, the radiant point moves with them.

The radiant point is due to the fact that the meteoroids which strike the earth during a shower are all moving in the same direction. If we suppose the earth to be at rest, and the actual motion of the meteoroids to be compounded with an imaginary motion equal and opposite to that of the earth, the motion of these imaginary bodies will be the same as the actual relative motion of the meteoroids seen from the earth. These relative motions will all be parallel; hence when the bodies strike our atmosphere the paths described by them in their passage will all be parallel straight lines. Now, by the principles of spherical trigonometry, a straight line seen by an observer at any point is projected as a great circle of the celestial sphere, of which the observer supposes himself to be the centre. If we draw a line from the observer parallel to the paths of the meteors, the direction of that line will represent a point of the sphere through which all the paths will seem to pass; this will, therefore, be the radiant point in a meteoric shower.

A slightly different conception of the problem may be formed by conceiving the plane passing through the observer and containing the path of the meteor. It is evident that the different planes formed by the parallel meteor paths will all intersect each other in a line drawn from the observer parallel to this path. This line will then intersect the celestial sphere in the radiant point.

Orbits of Meteoric Showers.—From what has just been said, it will be seen that the position of the radiant point indicates the direction in which the meteoroids move relatively to the earth. If we also knew the velocity with which they are really moving in space, we could make allowance for the motion of the earth, and thus determine the direction of their actual motion in space. It will be remembered that, as just explained, the apparent or relative motion is made up of two components—the one the actual motion of the body, the other the motion of the earth taken in an opposite direction. We know the second of these components already; and if we know the velocity relative to the earth and the direction as given by the radiant point, we have given the resultant and one component in magnitude and direction. The computation of the other component is one of the simplest problems in kinematics. Thus we shall have the actual direction and velocity of the meteoric swarm in space. Having this direction and velocity, the orbit of the swarm around the sun admits of being calculated.

Relations of Meteors and Comets.—The velocity of the meteoroids does not admit of being determined from observation. One element necessary for determining the orbits of these bodies is, therefore, wanting. In the case of the showers of 1799, 1833, and 1866, commonly called the November showers, this element is given by the time of revolution around the sun. Since the showers occur at intervals of about a third of a century, it is highly probable this is the periodic time of the swarm around the sun. The periodic time being known, the velocity at any distance from the sun admits of calculation from the theory of gravitation. Thus we have all the data for determining the real orbits of the group of meteors around the sun.

The calculations necessary for this purpose were made by LE VERRIER and other astronomers shortly after the great shower of 1866. The following was the orbit as given by LE VERRIER:

Period of revolution	33·25 years.
Eccentricity of orbit	0·9044.
Least distance from the sun	0·9890.
Inclination of orbit	165° 19′.
Longitude of the node	51° 18′.
Position of the perihelion	(near the node).

The publication of this orbit brought to the attention of the world an extraordinary coincidence which had never before been suspected. In December, 1865, a faint telescopic comet was discovered by TEMPEL at Marseilles, and afterward by H. P. TUTTLE at the Naval Observatory, Washington. Its orbit was calculated by Dr. OPPOLZER, of Vienna, and his results were finally published on January 28th, 1867, in the *Astronomische Nachrichten*; they were as follows:

Period of revolution	33·18 years.
Eccentricity of orbit	0·9054.
Least distance from the sun	0·9765.
Inclination of orbit	162° 42′.
Longitude of the node	51° 26′.
Longitude of the perihelion	42° 24′.

The publication of the cometary orbit and that of the orbit of the meteoric group were made independently within a few days of each other by two astronomers, neither of whom had any knowledge of the work of the other. Comparing them, the result is evident. *The swarms of meteoroids which cause the November showers move in the same orbit with* TEMPEL'*s comet.*

TEMPEL'S comet passed its perihelion in January, 1866. The most striking meteoric shower commenced in the following November, and was repeated during several years. It seems, therefore, that the meteoroids which produce these showers follow after TEMPEL'S comet, moving in the same orbit with it. This shows a curious relation between comets and meteors, of which we shall speak more fully in the next chapter. When this fact was brought out, the question naturally arose whether the same thing might not be true of other meteoric showers.

Other Showers of Meteors.—Although the November showers are the only ones so brilliant as to strike the ordinary eye, it has long been known that there are other nights of the year in which more shooting stars than usual are seen, and in which the large majority radiate from one point of the heavens. This shows conclusively that they arise from swarms of meteoroids moving together around the sun.

August Meteors.—The best marked of these minor showers occurs about August 9th or 10th of each year. The radiant point is in the constellation *Perseus.* By watching the eastern heavens toward midnight on the 9th or 10th of August of any year, it will be seen that numerous meteors move from north-east toward south-west, having often the distinctive characteristic of leaving a trail behind, which, however, vanishes in a few moments. Assuming their orbits to be parabolic, the elements were calculated by SCHIAPARELLI, of Milan, and, on comparing with the orbits of observed comets, it was found that these meteoroids moved in nearly the same orbit as the second comet of 1862. The exact period of this comet is not known, although the orbit is certainly elliptic. According to the best calculation, it is 124 years, but for reasons given in the next chapter, it may be uncertain by ten years or more.

There is one remarkable difference between the August and the November meteors. The latter, as we have seen, appear for two or three consecutive years, and then are not seen again until about thirty years have elapsed. But the August meteors are seen every year. This shows that the stream of August meteoroids is endless, every part of the orbit being occupied by them, while in the case of the November ones they are gathered into a group.

We may conclude from this that the November meteoroids have not been permanent members of our system. It is beyond all probability that a group comprising countless millions of such bodies should all have the same time of revolution. Even if they had the same time in the beginning, the different actions of the planets on different parts of the group would make the times different. The result would be that, in the course of ages, those which had the most rapid motion would go further and further ahead of the others until they got half a revolution ahead of them, and would finally overtake those having the slowest motion. The swiftest and slowest one would then be in the position of two race-horses running around a circular track for so long a time that the swiftest horse has made a complete run more than the slowest one and has overtaken him from behind. When this happens, the meteoroids will be scattered all around the orbit, and we shall have a shower in November of every year. The fact that has not yet happened shows that they have been revolving for only a limited length of time, probably only a very few thousand years.

Although the total mass of these bodies is very small, yet their number is beyond all estimation. Professor NEWTON has estimated that, taking the whole earth, about seven million shooting stars are encountered every twenty-four hours. This would make between two and three thousand million meteoroids which are thus, as it were, destroyed every year. But the number which the earth can encounter in a year is only an insignificant fraction of the total number, even in the solar system. It may be interesting to calculate the ratio of the space swept over by the earth in the course of a year to the volume of the sphere surrounding the sun and extending out to the orbit of *Neptune*. We shall find this ratio to be only as one to about three millions of millions. If we measure by the number of meteoroids in a cubic mile, we might consider them very thinly scattered. There is, in fact, only a single meteor to several million cubic kilometres of space in the heavens. Yet the total number is immensely great, because a globe including the orbit of *Neptune* would contain millions of millions of millions of millions of cubic kilometres.* If we reflect, in addition, that the meteoroids probably

* The computations leading to this result may be made in the following manner:

I. *To find the cubical space swept through by the earth in the course of a year.* If we put π for the ratio of the circumference of a circle to its diameter, and ρ for the radius of the earth, the surface of a plane section of the earth passing through its centre will be $\pi\rho^2$. Multiplying this by the circumference of the earth's orbit, we shall have the space required, which we readily find to be more than 80,000 millions of millions of kilometres. Since, in sweeping through this space, the earth encounters about 2500 millions of meteoroids, it follows that weigh but a few grains each, we shall see how it is that they are entirely invisible to vision, even with powerful telescopes.

The Zodiacal Light.—If we observe the western sky during the winter or spring months, about the end of the evening twilight, we shall see a stream of faint light, a little like the Milky Way, rising obliquely from the west, and directed along the ecliptic toward a point south-west from the zenith. This is called the *zodiacal light.* It may also be seen in the east before daylight in the morning during the autumn months, and has sometimes been traced all the way across the heavens. Its origin is still involved in obscurity, but it seems probable that it arises from an extremely thin cloud either of meteoroids or of semi-gaseous matter like that composing the tail of a comet, spread all around the sun inside the earth's orbit. The researches of Professor A. W. WRIGHT show that its spectrum is probably that of reflected sunlight, a result which gives color to the theory that it arises from a cloud of meteoroids revolving round the sun.

there is only one meteoroid to more than ten millions of cubic kilometres.

II. *To find the ratio of the sphere of space within the orbit of Neptune to the space swept through by the earth in a year.* Let us put r for the distance of the earth from the sun. Then the distance of Neptune may be taken as $30\ r$, and this will be the radius of the sphere. The circumference of the earth's orbit will than be $2\ \pi\ r$, and the space swept over will be $2\ \pi^2\ r\ \rho^2$. The sphere of *Neptune* will be

$$\tfrac{4}{3}\ \pi\ 30^3\ r^3 = 36{,}000\ \pi\ r^3, \text{ nearly.}$$

The ratio of the two spaces will be

$$\frac{18{,}000\ r^2}{\pi\ \rho^2} = 6{,}000\ \frac{r^2}{\rho^2}, \text{ nearly.}$$

The ratio $\frac{r}{\rho}$ is more than 23,000, showing the required ratio to be about three millions of millions. The total number of scattered meteoroids is therefore to be reckoned by millions of millions of millions.

CHAPTER XIII.

COMETS.

§ 1. ASPECT OF COMETS.

COMETS are distinguished from the planets both by their aspects and their motions. They come into view without anything to herald their approach, continue in sight for a few weeks or months, and then gradually vanish in the distance. They are commonly considered as composed of three parts, the *nucleus*, the *coma* (or hair), and the *tail*.

The nucleus of a comet is, to the naked eye, a point of light resembling a star or planet. Viewed in a telescope, it generally has a small disk, but shades off so gradually that it is difficult to estimate its magnitude. In large comets, it is sometimes several hundred miles in diameter, but never approaches the size of one of the larger planets.

The nucleus is always surrounded by a mass of foggy light, which is called the *coma*. To the naked eye, the nucleus and coma together look like a star seen through a mass of thin fog, which surrounds it with a sort of halo. The coma is brightest near the nucleus, so that it is hardly possible to tell where the nucleus ends and where the coma begins. It shades off in every direction so gradually that no definite boundaries can be fixed to it. The nucleus and coma together are generally called the *head* of the comet.

The *tail* of the comet is simply a continuation of the coma extending out to a great distance, and always directed away from the sun. It has the appearance of a stream of milky light, which grows fainter and broader as it recedes from the head. Like the coma, it shades off so gradually that it is impossible to fix any boundaries to it. The length of the tail varies from 2° or 3° to 90° or more. Generally the more brilliant the head of the comet, the longer and brighter is the tail. It is also often brighter and more sharply defined at one edge than at the other.

The above description applies to comets which can be plainly seen by the naked eye. After astronomers began to sweep the heavens carefully with telescopes, it was found that many comets came into sight which would entirely escape the unaided vision. These are called *telescopic comets.* Sometimes six or more of such comets are discovered in a single year, while one of the brighter class may not be seen for ten years or more.

FIG. 101.—TELESCOPIC COMET WITHOUT A NUCLEUS.

FIG. 102.—TELESCOPIC COMET WITH A NUCLEUS.

When comets are studied with a telescope, it is found that they are subject to extraordinary changes of structure. To understand these changes, we must begin by saying that comets do not, like the planets, revolve around the sun in nearly circular orbits, but always in orbits so elongated that the comet is visible in only a very small part of its course. When one of these objects is first seen, it is generally approaching the sun from the celestial spaces. At this time it is nearly always devoid of a tail, and sometimes of a nucleus, presenting the aspect of a thin patch of cloudy light, which may or may not have a nucleus in its centre. As it approaches the sun, it is generally seen to grow brighter at some one point, and there a nucleus gradually forms, being, at first, so faint that it can scarcely be distinguished from the surrounding nebulosity. The latter is generally more extended in the direction of the sun, thus sometimes giving rise to the erroneous impression of a tail turned toward the sun. Continuing the watch, the true tail, if formed at all, is found to begin very gradually. At first so small and faint as to be almost invisible, it grows longer and brighter every day, as long as the comet continues to approach the sun.

§ 2. THE VAPOROUS ENVELOPES.

If a comet is very small, it may undergo no changes of aspect, except those just described. If it is an unusually bright one, the next object noticed by telescopic examination will be a bow surrounding the nucleus on the side toward the sun. This bow will gradually rise up and spread out on all sides, finally assuming the form of a semicircle having the nucleus in its centre, or, to speak with more precision, the form of a parabola having the nucleus near its focus. The two ends of this parabola will extend out further and further so as to form a part of the tail, and finally be lost in it. Continuing the watch, other bows will be found to form around the nucleus, all slowly rising from it like clouds of vapor. These distinct vaporous masses are called the *envelopes:* they shade off gradually into the coma so as to be with difficulty distinguished from it, and indeed may be considered as part of it. The inner envelope is sometimes connected with the nucleus by one or more fan-shaped appendages, the centre of the fan being in the nucleus, and the envelope forming its round edge. This appearance is apparently caused by masses of vapor streaming up from that side of the nucleus nearest the sun, and gradually spreading around the comet on each side. The form of a bow is not the real form of the envelopes, but only the apparent one in which we see them projected against the background of the sky. Their true form is similar to that of a paraboloid of revolution, surrounding the nucleus on all sides, except that turned from the sun. It is, therefore, a surface and not a line. Perhaps its form can be best imagined by supposing the sun to be directly above the comet, and a fountain, throwing a liquid horizontally on all sides, to be built upon that part of the comet which is uppermost. Such a fountain would throw its water in the form of a sheet, falling on all sides of the cometic nucleus, but not touching it. Two or three vapor surfaces of this kind are sometimes seen around the comet, the outer one enclosing each of the inner ones, but no two touching each other.

FIG. 108.—FORMATION OF ENVELOPES.

To give a clear conception of the formation and motion of the envelopes, we present two figures. The first of these shows the appearance of the envelopes in four successive stages of their course, and may be regarded as sections of the real umbrella-shaped surfaces which they form. In all these figures, the sun is supposed to be above the comet in the figure, and the tail of the comet to be directed dowaward. In *a* the sheet of vapor has just begun to rise. In *b* it is risen and expanded yet further. In *c* it has begun to move away and pass around the comet on all sides. Finally, in *d* this last motion has gone so far that the higher portions have nearly disappeared, the larger part of the matter having moved away toward the tail. Before the stage *c* is reached, a second envelope will commonly begin to rise as at *a*, so that two or three may be visible at the same time, enclosed within each other.

In the next figure the actual motion of the matter composing the envelopes is shown by the courses of the several dotted lines. This motion, it will be seen, is not very unlike that of water thrown up from a fountain on the part of the nucleus nearest the sun and then falling down on all sides. The point in which the motion of the cometic matter differs from that of the fountain is that, instead of being thrown in continuous streams, the action is intermittent, the fountain throwing up successive sheets of matter instead of continuous streams.

From the gradual expansion of these envelopes around the head of the comet and the continual formation of new ones in the immediate neighborhood of the nucleus, they would seem to be due to a process of evaporation going on from the surface of the latter. Each layer of vapor thus formed rises up and spreads out continually until the part next the sun attains a certain maximum height. Then it gradually moves away from the sun, keeping its distance from the comet, at least until it passes the latter on every side, and continues onward to form the tail.

FIG. 104.—FORMATION OF COMET'S TAIL.

These phenomena were fully observed in the great comet of 1858, the observations of which were carefully collected by the late Professor BOND, of Cambridge. The envelopes of this comet were first noticed on September [illegible]th, when the outer one was 16′ above the nucleus and the inner one 8″. The outer one disappeared on September 30th at a height of about 1′. In the mean while, however, a third had appeared, the second having gradually expanded so as to take the place of the first. Seven successive envelopes in all were seen to rise from this comet, the last one commencing on October 20th, when all the others had been dissipated. The rate at which the envelopes ascended was generally from 50 to 60 kilometres per hour, the ordinary speed of a railway-train.

The first one rose to a height of about 30,000 kilometres, but it was finally dissipated. But the successive ones disappeared at a lower and lower elevation, the sixth being lost sight of at a height of about 10,000 kilometres.

In the great comet of 1861, eleven envelopes were seen between July 2d, when portions of three were in sight, and the 19th of the same month, a new one rising at regular intervals of every second day. Their evolution and dissipation were accomplished with much greater rapidity than in the case of the great comet of 1858, an envelope requiring but two or three days instead of two or three weeks to pass through all its phases.

§ 3. THE PHYSICAL CONSTITUTION OF COMETS.

To tell exactly what a comet is, we should be able to show how all the phenomena it presents would follow from the properties of matter, as we learn them at the surface of the earth. This, however, no one has been able to do, many of the phenomena being such as we should not expect from the known constitution of matter. All we can do, therefore, is to present the principal characteristics of comets, as shown by observation, and to explain what is wanting to reconcile these characteristics with the known properties of matter.

In the first place, all comets which have been examined with the spectroscope show a spectrum composed, in part at least, of bright lines or bands. These lines have been supposed to be identified with those of carbon; but although the similarity of aspect is very striking, the identity cannot be regarded as proven.

FIG. 105.—SPECTRA OF OLEFIANT GAS AND OF A COMET.

In the annexed figure the upper spectrum, A, is that of carbon taken in olefiant gas, and the lower one, B, that of a comet. These spectra interpreted in the usual way would indicate, firstly, that the comet is gaseous; secondly, that the gases which compose it are so hot as to shine by their own light. But we cannot admit

these interpretations without bringing in some additional theory. A mass of gas surrounding so minute a body as the nucleus of a telescopic comet would expand into space by virtue of its own elasticity unless it were exceedingly rare. Moreover, if it were incandescent, it would speedily cool off so as to be no longer self-luminous. We must, therefore, propose some theory to account for the continuation of the luminosity through many centuries, such as electric activity or phosphorescence. But without further proof of action of these causes we cannot accept their reality. We are, therefore, unable to say with certainty how the light in the spectrum of comets which produces the bright lines has its origin.

In the last chapter it was shown that swarms of minute particles called meteoroids follow certain comets in their orbits. This is no doubt true of all comets. We can only regard these meteoroids as fragments or *débris* of the comet. The latter has therefore been considered by Professor NEWTON as made up entirely of meteoroids or small detached masses of matter. These masses are so small and so numerous that they look like a cloud, and the light which they reflect to our eyes has the milky appearance peculiar to a comet. On this theory a telescopic comet which has no nucleus is simply a cloud of these minute bodies. The nucleus of the brighter comets may either be a more condensed mass of such bodies or it may be a solid or liquid body itself.

If the reader has any difficulty in reconciling this theory of detached particles with the view already presented, that the envelopes from which the tail of the comet is formed consist of layers of vapor, he must remember that vaporous masses, such as clouds, fog, and smoke, are really composed of minute separate particles of water or carbon.

Formation of the Comet's Tail.—The tail of the comet is not a permanent appendage, but is composed of the masses of vapor which we have already described as ascending from the nucleus, and afterward moving away from the sun. The tail which we see on one evening is not absolutely the same we saw the evening before, a portion of the latter having been dissipated, while new matter has taken its place, as with the stream of smoke from a steamship. The motion of the vaporous matter which forms the tail being always away from the sun, there seems to be a repulsive force exerted by the sun upon it. The form of the comet's tail, on the supposition that it is composed of matter thus driven away from the sun with a uniformly accelerated velocity, has been several times investigated, and found to represent the observed form of the tail so nearly as to leave little doubt of its correctness. We may, therefore, regard it as an observed fact that the vapor which rises from the nucleus of the comet is repelled by the sun instead of being attracted toward it, as larger masses of matter are.

No adequate explanation of this repulsive force has ever been given. It has, indeed, been suggested that it is electrical in its character, but no one has yet proven experimentally that the attraction exerted by the sun upon terrestrial bodies is influenced by their electrical state. If this were done, we should have a key to one of the most difficult problems connected with the constitution of comets. As the case now stands, the repulsion of the sun upon the comet's tail is to be regarded as a well-ascertained and entirely isolated fact which has no known counterpart in any other observed fact of nature.

In view of the difficulties we find in explaining the phenomena of comets by principles based upon our terrestrial chemistry and physics, the question will arise whether the matter which composes these bodies may not be of a constitution entirely different from that of any matter we are acquainted with at the earth's surface. If this were so, it would be impossible to give a complete explanation of comets until we know what forms matter might possibly assume different from those we find it to have assumed in our laboratories. This is a question which we merely suggest without attempting to speculate upon it. It can be answered only by experimental researches in chemistry and physics.

§ 4. MOTIONS OF COMETS.

Previous to the time of NEWTON, no certain knowledge respecting the actual motions of comets in the heavens had been acquired, except that they did not move around the sun like the planets. When NEWTON investigated the mathematical results of the theory of gravitation, he found that a body moving under the attraction of the sun might describe either of the three conic sections, the ellipse, parabola, or hyperbola. Bodies moving in an ellipse, as the planets, would complete their orbits at regular intervals of time, according to laws already laid down. But if the body moved in a parabola or a hyperbola, it would never return to the sun after once passing it, but would move off

FIG. 108.—ELLIPTIC AND PARABOLIC ORBITS.

to infinity. It was, therefore, very natural to conclude that comets might be bodies which resemble the planets in moving under the sun's attraction, but which, instead of describing an ellipse in regular periods, like the planets, move in parabolic or hyperbolic orbits, and therefore only approach the sun a single time during their whole existence.

This theory is now known to be essentially true for most of the observed comets. A few are indeed found to be revolving around the sun in elliptic orbits, which differ from those of the planets only in being much more eccentric. But the greater number which have been observed have receded from the sun in orbits which we are unable to distinguish from parabolas, though it is possible they may be extremely elongated ellipses. Comets are therefore divided with respect to their motions into two classes: (1) *periodic comets*, which are known to move in elliptic orbits, and to return to the sun at fixed intervals; and (2) *parabolic comets*, apparently moving in parabolas, never to return.

The first discovery of the periodicity of a comet was made by HALLEY in connection with the great comet of 1682. Examining the records of observations, he found that a comet moving in nearly the same orbit with that of 1682 had been seen in 1607, and still another in 1531. He was therefore led to the conclusion that these three comets were really one and the same object, returning to the sun at intervals of about 75 or 76 years. He therefore predicted that it would appear again about the year 1758. But such a prediction might be a year or more in error, owing to the effect of the attraction of the planets upon the comet. In the mean time the methods of calculating the attraction of the planets were so far improved that it became possible to make a more accurate prediction. As the year 1759 approached, the necessary computations were made by the great French geometer CLAIRAUT, who assigned April 13th, 1759, as the day on which the comet would pass its perihelion. This prediction was remarkably correct. The comet was first seen on Christmas-day, 1758, and passed its perihelion March 12th, 1759, only one month before the predicted time. The comet returned again in 1835, within three days of the moment predicted by DE PONTÉCOULANT, the most successful calculator. The next return will probably take place in 1911 or 1912, the exact time being still unknown, because the necessary computations have not yet been made.

We give a figure showing the position of the orbit of HALLEY's comet relative to the orbits of the four outer planets. It attained its greatest distance from the sun, far beyond the orbit of *Neptune*, about the year 1873, and then commenced its return journey. The figure shows the probable position of the comet in 1874. It was then far beyond the reach of the most powerful telescope, but its distance and direction admit of being calculated with so much precision that a telescope could be pointed at it at any required moment.

FIG. 107.—ORBIT OF HALLEY'S COMET.

We have already stated that great numbers of comets, too faint to be seen by the naked eye, are discovered by telescopes. A considerable number of these telescopic comets have been found to be periodic. In most cases, the period is many centuries in length, so that the comets have only been noticed at a single visit. Eight or nine, however, have been found to be of a period so short that they have been observed at two or more returns.

We present a table of such of the periodic comets as have been actually observed at two or more returns. A number of others are known to be periodic, but have been observed only on a single visit to our system.

PERIODIC COMETS SEEN AT MORE THAN ONE RETURN.

Designation of Comet.	Last Return to Perihelion Observed.	Least Distance from Sun.	Greatest Distance from Sun.	Inclination.		Longitude of Node.		Distance from Node to Perihelion.		Periodic Time.
				°	′	°	′	°	′	Years.
Encke's..........	1878, July 26.	0·342	4·10	13	7	334	39	176	19	3·3035
Biela's..........	1852, Sept. 23.	0·860	6·19	12	33	245	52	223	13	6·620
Faye's..........	1873, July 18.	1·686	5·93	11	22	209	42	200	15	7·413
Brorsen's..........	1879, March 30.	0·591	5·66	29	59	102	17	14	29	5·561
D'Arrest's..........	1857, Nov. 28.	1·17	5·79	13	56	148	26	174	33	6·39
Winnecke's..........	1875, March 12.	0·78	5·50	11	17	111	33	165	10	5·55
Tuttle's..........	1871, Dec. 2.	1·03	10·51	54	17	269	17	206	47	13·76
Tempel's..........	1879, May 7.	1·771	4·81	9	47	78	46	159	27	5·98
Halley's..........	1835, Nov. 15.	0·586	35·3	162	15	57	18	112	48	76·00

Theory of Cometary Orbits.—There is a property of all orbits of bodies around the sun, an understanding of which will enable us to form a clear idea of some causes which affect the motion of comets. It may be expressed in the following theorem:

The mean distance of a body from the sun, or the major axis of the ellipse in which it revolves, depends only upon the velocity of the body at a given distance from the sun, and may be found by the formula,

$$a = \frac{\mu r}{2\mu - r v^2},$$

in which r is the distance from the sun, v the velocity with which the body is moving, and μ a constant proportional to the mass of the sun and depending on the units of time and length we adopt.

To understand this formula, let us imagine ourselves in the celestial spaces, with no planets in our neighborhood. Suppose we have a great number of balls and shoot them out with the same velocity, but in different directions, so that they will describe orbits around the sun. Then the bodies will all describe different orbits, owing to the different directions in which we threw them, but these orbits will all possess the remarkable property of having equal major axes, and therefore equal mean distances from the sun. Since, by KEPLER'S third law, the periodic time depends only upon the mean distance, it follows that the bodies will have the same time of revolution around the sun. Consequently, if we wait patiently at the point of projection, they will all make a revolution in the same time, and will all come back again at the same moment, each one coming from a direction the opposite of that in which it was thrown.

In the above formula the major axis is given by a fraction, having the expression $2\mu - r v^2$ for its denominator; it follows that if the square of the velocity is almost equal to $\frac{2\mu}{r}$, the value of a will become very great, because the denominator of the fraction will be very small. If the velocity is such that $2\mu - r v^2$ is zero, the mean distance will become infinite. Hence, in this case the body will fly off to an infinite distance from the sun and never return. Much less will it return if the velocity is still greater. Such a velocity will make the value of a algebraically negative and will correspond to the hyperbola.

If we take one kilometre per second as the unit of velocity, and the mean distance of the earth from the sun as the unit of distance, the value of μ will be represented by the number 875, so that the formula for a will be $a = \frac{875 r}{1750 - r v^2}$. From this equation, we may calculate what velocity a body moving around the sun must have at any given distance r, in order that it may move in a parabolic orbit—that is, that the denominator of the fraction shall vanish. This condition will give $v^2 = \frac{1750}{r}$. At the distance of the earth from the sun we have $r = 1$, so that, at that distance, v will be the square root of 1750, or nearly 42 kilometres per second. The further we get out from the sun, the less it will be; and we may remark, as an interesting theorem, that whenever the comet is at the distance of one of the planetary orbits, its velocity must be equal to that of the planet multiplied by the square root of 2, or 1·414, etc. Hence, if the velocity of any planet were suddenly increased by a little more than $\frac{4}{10}$ of its amount, its orbit would be changed into a parabola, and it would fly away from the sun, never to return.

It follows from all this that if the astronomer, by observing the course of a comet along its orbit, can determine its exact velocity from point to point, he can thence calculate its mean distance from the sun and its periodic time. But it is found that the velocity of a large majority of comets is so nearly equal to that required for motion in a parabola, that the difference eludes observation. It is hence concluded that most comets move nearly in parabolas, and will either never return at all or, at best, not until after the lapse of many centuries.

§ 5. ORIGIN OF COMETS.

All that we know of comets seems to indicate that they did not originally belong to our system, but became members of it through the disturbing forces of the planets. From what was said in the last section, it will be seen that if a comet is moving in a parabolic orbit, and its velocity is diminished at any point by ever so small an amount, its orbit will be changed into an ellipse; for in order that the orbit may be parabolic, the quantity $2\mu - r v^2$ must remain exactly zero. But if we then diminish v by the smallest amount, this expression will become finite and positive, and a will no longer be infinite. Now, the attraction of a planet may have either of two opposite effects; it may either increase or diminish the velocity of the comet. Hence if the latter be moving in a parabolic orbit, the attraction of a planet might either throw it out into a hyperbolic orbit, so that it would never again return to the sun, but wander forever through the celestial spaces, or it might change its orbit into a more or less elongated ellipse.

Suppose $C D$ to represent a small portion of the orbit of the planet and $A B$ a small portion of the orbit of a comet passing near it. Suppose also that the comet passes a little in front of the planet, and that the simultaneous positions of the two bodies are represented by the corresponding letters of the alphabet, a, b, c, d, etc.; the shortest distance of the two bodies will be the line $c\ c$, and it is then that the attraction will be the most powerful. Between $c\ c$ and $d\ d$ the planet will attract the comet almost directly backward. It follows then that if a comet pass the planet in the way here represented, its velocity will be retarded by the attraction of the latter. If therefore it be a parabolic comet, the orbit will be changed into an ellipse. The nearer it passes to the planet, the greater will be the change, so long as it passes in front of it. If it passes behind, the reverse effect will follow, and the motion will be accelerated. The orbit will then be changed into a hyperbola. The orbit finally described after the comet leaves our system will depend upon whether its velocity is accelerated or retarded by the combined attraction of all the planets.

FIG. 108.—ATTRACTION OF PLANET ON COMET.

All the studies which have been made of comets seem to show that they originally moved in parabolic orbits, and were brought into elliptic orbits in this way by the attraction of some planet. The planet which has thus brought in the greatest number is no doubt *Jupiter*. In fact, the orbits of several of the periodic comets pass very near to that planet. It might seem that these orbits ought almost to intersect that of the planet which changed them. This would be true at first, but owing to the constant change in the position of the cometary orbit, produced by the attraction of the planets, the orbits would gradually move away from each other, so that in time there might be no approach whatever of the planet to the comet.

A remarkable case of this sort was afforded by a comet discovered in June, 1770. It was observed in all nearly four months, and was for some time visible to the naked eye. On calculating its orbit from all the observations, the astronomers were astonished to find it to be an ellipse with a period of only five or six years. It ought therefore to have appeared again in 1776 or 1777, and should have returned to its perihelion twenty times before now, and should also have been visible at returns previous to that at which it was first seen. But not only was it never seen before, but it has never been seen since! The reason of its disappearance from view was brought to light on calculating its motions after its first discovery. At its return in 1776, the earth was not in the right part of its orbit for seeing it. On passing out to its aphelion again, about the beginning of 1779, it encountered the planet *Jupiter*, and approached so near it that it was impossible to determine on which side it passed. This approach, it will be remembered, could not be observed, because the comet was entirely out of sight, but it was calculated with absolute certainty from the theory of the comet's motion. The attraction of *Jupiter*, therefore, threw it into some orbit so entirely different that it has never been seen since.

It is also highly probable that the comet had just been brought in by the attraction of *Jupiter* on the very revolution in which it was first observed. Its history is this: Approaching the sun from the stellar spaces, probably for the first time, it passed so near *Jupiter* in 1767 that its orbit was changed to an ellipse of short period. It made two complete revolutions around the sun, and in 1779 again met the planet near the same place it had met him before. The orbit was again altered so much that no telescope has found the comet since. No other case so remarkable as this has ever been noticed.

Not only are new comets occasionally brought in from the stellar spaces, but old ones may, as it were, fade away and die. A case of this sort is afforded by BIELA's comet, which has not been seen since 1852, and seems to have entirely disappeared from the heavens. Its history is so instructive that we present a brief synopsis of it. It was first observed in 1772, again in 1805, and then a third time in 1826. It was not until this third apparition that its periodicity was recognized and its previous appearances identified as those of the same body. The period of revolution was found to be between six and seven years. It was so small as to be visible in ordinary telescopes only when the earth was near it, which would occur only at one return out of three or four. So it was not seen again until near the end of 1845. Nothing remarkable was noticed in its appearance until January, 1846, when all were astonished to find it separated into two complete comets, one a little brighter than the other. The computation of Professor HUBBARD makes the distance of the two bodies to have been 200,000 miles.

The next observed return was that of 1852, when the two comets were again viewed, but far more widely separated, their distance having increased to about a million and a half of miles. Their brightness was so nearly equal that it was not possible to decide which should be considered the principal comet, nor to determine with certainty which one should be considered as identical with the comet seen during the previous apparition.

Though carefully looked for at every subsequent return, neither comet has been seen since. In 1872, Mr. POGSON, of Madras, thought that he got a momentary view of the comet through an opening between the clouds on a stormy evening, but the position in which he supposed himself to observe it was so far from the calculated one that his observation has not been accepted.

Instead of the comet, however, we had a meteoric shower. The orbit of this comet almost intersects that of the earth. It was therefore to be expected that the latter, on passing the orbit of the comet, would intersect the fragmentary meteoroids supposed to follow it, as explained in the last chapter. According to the calculated orbit of the comet, it crossed the point of intersection in September, 1872, while the earth passes the same point on November 27th of each year. It was therefore predicted that a meteoric shower would be seen on the night of November 27th, the radiant point of which would be in the constellation *Andromeda*. This prediction was completely verified, but the meteors were so faint that though they succeeded each other quite rapidly, they might not have been noticed by a casual observer. They all radiated from the predicted point with such exactness that the eye could detect no deviation whatever.

We thus have a third case in which meteoric showers are associated with the orbit of a comet. In this case, however, the comet has been completely dissipated, and probably has disappeared forever from telescopic vision, though it may be expected that from time to time its invisible fragments will form meteors in the earth's atmosphere.

§ 6. REMARKABLE COMETS.

It is familiarly known that bright comets were in former years objects of great terror, being supposed to presage the fall of empires, the death of monarchs, the approach of earthquakes, wars, pestilence, and every other calamity which could afflict mankind. In showing the entire groundlessness of such fears, science has rendered one of its greatest benefits to mankind.

In 1456, the comet known as HALLEY'S, appearing when the Turks were making war on Christendom, caused such terror that Pope CALIXTUS ordered prayers to be offered in the churches for protection against it. This is supposed to be the origin of the popular myth that the Pope once issued a bull against the comet.

The number of comets visible to the naked eye, so far as recorded, has generally ranged from 20 to 40 in a century. Only a small portion of these, however, have been so bright as to excite universal notice.

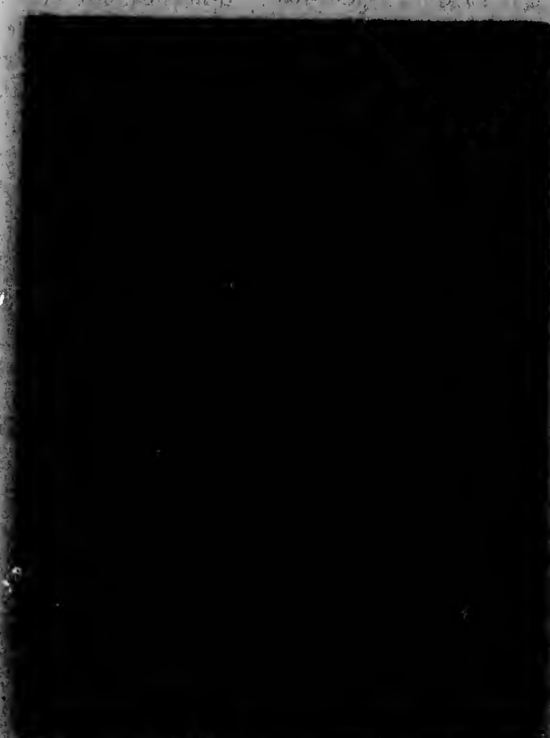
Comet of 1680.—One of the most remarkable of these brilliant comets is that of 1680. It inspired such terror that a medal, of which we present a figure, was struck upon the Continent of Europe to quiet apprehension. A free translation of the inscription is: "The star threatens evil things; trust only! God will turn them to good." What makes this comet especially remarkable in history is that NEWTON calculated its orbit, and showed that it moved around the sun in a conic section, in obedience to the law of gravitation.

FIG. 109.—MEDAL OF THE GREAT COMET OF 1680.

Great Comet of 1811.—Fig. 110 shows its general appearance. It has a period of over 3000 years, and its aphelion distance is about 40,000,000,000 miles.

Great Comet of 1843.—One of the most brilliant comets which have appeared during the present century was that of February, 1843. It was visible in full daylight close to the sun. Considerable terror was caused in some quarters, lest it should presage the end of the world, which had been predicted for that year by MILLER. At perihelion it passed nearer the sun than any other body has ever been known to pass, the least distance being only about one fifth of the sun's semi-diameter. With a very slight change of its original motion, it would have actually fallen into the sun.

Great Comet of 1858.—Another remarkable comet for the length of time it remained visible was that of 1858. It is frequently called after the name of DONATI, its first discoverer. No comet visiting our neighborhood in



recent [illegible] favorable an opportunity for studying its physical constitution. Some of the results of the observations made upon it have already been presented.

FIG. 111.—DONATI'S COMET OF 1858.

Its greatest brilliancy occurred about the beginning of October, when its tail was 40° in length and 10° in breadth at its outer end.

DONATI's comet had not long been observed when it was found that its orbit was decidedly elliptical. After it disappeared, the observations were all carefully investigated by two mathematicians, Dr. VON ASTEN, of Germany, and Mr. G. W. HILL, of this country. The latter found a period of 1950 years, which is probably within a half a century of the truth. It is probable, therefore, that this comet appeared about the first century before the Christian era, and will return again about the year 3800.

Encke's Comet and the Resisting Medium.—Of telescopic comets, that which has been most investigated by astronomers is known as ENCKE's comet. Its period is between three and four years. Viewed with a telescope, it is not different in any respect from other telescopic comets, appearing simply as a mass of foggy light, somewhat brighter near one side. Under the most favorable circumstances, it is just visible to the naked eye. The circumstance which has lent most interest to this comet is that the observations which have been made upon it seem to indicate that it is gradually approaching the sun. ENCKE attributed this change in its orbit to the existence in space of a resisting medium, so rare as to have no appreciable effect upon the motion of the planets, and to be felt only by bodies of extreme tenuity, like the telescopic comets. The approach of the comet to the sun is shown, not by direct observation, but only by a gradual diminution of the period of revolution. It will be many centuries before this period would be so far diminished that the comet would actually touch the sun.

If the change in the period of this comet were actually due to the cause which ENCKE supposed, then other faint comets of the same kind ought to be subject to a similar influence. But the investigations which have been made in recent times on these bodies show no deviation of the kind. It might, therefore, be concluded that the change in the period of ENCKE's comet must be due to some other cause. There is, however, one circumstance which leaves us in doubt. ENCKE's comet passes nearer the sun than any other comet of short period which has been observed with sufficient care to decide the question. It may, therefore, be supposed that the resisting medium, whatever it may be, is densest near the sun, and does not extend out far enough for the other comets to meet it. The question is one very difficult to settle. The fact is that all comets exhibit slight anomalies in their motions which prevent us from deducing conclusions from them with the same certainty that we should from those of the planets.

PART III.

THE UNIVERSE AT LARGE.

INTRODUCTION.

In our studies of the heavenly bodies, we have hitherto been occupied almost entirely with those of the solar system. Although this system comprises the bodies which are most important to us, yet they form only an insignificant part of creation. Besides the earth on which we dwell, only seven of the bodies of the solar system are plainly visible to the naked eye, whereas it is well known that 2000 stars or more can be seen on any clear night. We now have to describe the visible universe in its largest extent, and in doing so shall, in imagination, step over the bounds in which we have hitherto confined ourselves and fly through the immensity of space.

The material universe, as revealed by modern telescopic investigation, consists principally of shining bodies, many millions in number, a few of the nearest and brightest of which are visible to the naked eye as stars. They extend out as far as the most powerful telescope can penetrate, and no one knows how much farther. Our sun is simply one of these stars, and does not, so far as we know, differ from its fellows in any essential characteristic. From the most careful estimates, it is rather less bright than the average of the nearer stars, and overpowers them by its brilliancy only because it is so much nearer to us.

The distance of the stars from each other, and therefore from the sun, is immensely greater than any of the distances which we have hitherto had to consider in the solar system. Suppose, for instance, that a walker through the celestial spaces could start out from the sun, taking steps 3000 miles long, or equal to the distance from Liverpool to New York, and making 120 steps a minute. This speed would carry him around the earth in about four seconds; he would walk from the sun to the earth in four hours, and in five days he would reach the orbit of *Neptune*. Yet if he should start for the nearest star, he would not reach it in a hundred years. Long before he got there, the whole orbit of *Neptune*, supposing it a visible object, would have been reduced to a point, and finally vanish from sight altogether. In fact, the nearest known star is about seven thousand times as far as the planet *Neptune*. If we suppose the orbit of this planet to be represented by a child's hoop, the nearest star would be three or four miles away. We have no reason to suppose that contiguous stars are, on the average, nearer than this, except in special cases where they are collected together in clusters.

The total number of the stars is estimated by millions, and they are probably separated by these wide intervals. It follows that, in going from the sun to the nearest star, we would be simply taking one step in the universe. The most distant stars visible in great telescopes are probably several thousand times more distant than the nearest one, and we do not know what may lie beyond.

The point we wish principally to impress on the reader in this connection is that, although the stars and planets present to the naked eye so great a similarity in appearance, there is the greatest possible diversity in their distances and characters. The planets, though many millions of miles away, are comparatively near us, and form a little family by themselves, which is called the solar system. The fixed stars are at distances incomparably greater—the nearest star, as just stated, being thousands of times more distant than the farthest planet. The planets are, so far as we can see, worlds somewhat like this on which we live, while the stars are suns, generally larger and brighter than our own. Each star may, for aught we know, have planets revolving around it, but their distance is so immense that the largest planets will remain invisible with the most powerful telescopes man can ever hope to construct.

The classification of the heavenly bodies thus leads us to this curious conclusion. Our sun is one of the family of stars, the other members of which stud the heavens at night, or, in other words, the stars are suns like that which makes the day. The planets, though they look like stars, are not such, but bodies more like the earth on which we live.

The great universe of stars, including the creation in its largest extent, is called the *stellar system*, or *stellar universe*. We have first to consider how it looks to the naked eye.

CHAPTER I.

THE CONSTELLATIONS.

§ 1. GENERAL ASPECT OF THE HEAVENS.

When we view the heavens with the unassisted eye, the stars appear to be scattered nearly at random over the surface of the celestial vault. The only deviation from an entirely random distribution which can be noticed is a certain grouping of the brighter ones into constellations. We notice also that a few are comparatively much brighter than the rest, and that there is every gradation of brilliancy, from that of the brightest to those which are barely visible. We also notice at a glance that the fainter stars outnumber the bright ones; so that if we divide the stars into classes according to their brilliancy, the fainter classes will be far the more numerous.

The total number one can see will depend very largely upon the clearness of the atmosphere and the keenness of the eye. From the most careful estimates which have been made, it would appear that there are in the whole celestial sphere about 6000 stars visible to an ordinarily good eye. Of these, however, we can never see more than a fraction at any one time, because one half of the sphere is always of necessity below the horizon. If we could see a star in the horizon as well as in the zenith, one half of the whole number, or 3000, would be visible on any clear night. But stars near the horizon are seen through so great a thickness of atmosphere as greatly to obscure their light; consequently only the brightest ones can there be seen. As a result of this obscuration, it is not likely that more than 2000 stars can ever be taken in at a single view by any ordinary eye. About 2000 other stars are so near the South Pole that they never rise in our latitudes. Hence out of the 6000 supposed to be visible, only 4000 ever come within the range of our vision, unless we make a journey toward the equator.

The Galaxy.—Another feature of the heavens, which is less striking than the stars, but has been noticed from the earliest times, is the *Galaxy*, or *Milky Way*. This object consists of a magnificent stream or belt of white milky light 10° or 15° in breadth, extending obliquely around the celestial sphere. During the spring months, it nearly coincides with our horizon in the early evening, but it can readily be seen at all other times of the year spanning the heavens like an arch. It is for a portion of its length split longitudinally into two parts, which remain separate through many degrees, and are finally united again. The student will obtain a better idea of it by actual examination than from any description. He will see that its irregularities of form and lustre are such that in some places it looks like a mass of brilliant clouds. In the southern hemisphere there are vacant spaces in it which the navigators call coal-sacks. In one of these, 5° by 18°, there is scarcely a single star visible to the naked eye (see Figs. 121 and 132).

Lucid and Telescopic Stars. — When we view the heavens with a telescope, we find that there are innumerable stars too small to be seen by the naked eye. We may therefore divide the stars, with respect to brightness, into two great classes.

Lucid Stars are those which are visible without a telescope.

Telescopic Stars are those which are not so visible.

When GALILEO first directed his telescope to the heavens, about the year 1610, he perceived that the Milky Way was composed of stars too faint to be individually seen by the unaided eye. We thus have the interesting fact that although telescopic stars cannot be seen one by one, yet in the region of the Milky Way they are so numerous that they shine in masses like brilliant clouds. HUYGHENS in 1656 resolved a large portion of the Galaxy into stars, and concluded that it was composed entirely of them. KEPLER considered it to be a vast ring of stars surrounding the solar system, and remarked that the sun must be situated near the centre of the ring. This view agrees very well with the one now received, only that the stars which form the Milky Way, instead of lying around the solar system, are at a distance so vast as to elude all our powers of calculation.

Such are in brief the more salient phenomena which are presented to an observer of the starry heavens. We shall now consider how these phenomena have been classified by an arrangement of the stars according to their brilliancy and their situation.

§ 2. MAGNITUDES OF THE STARS.

In ancient times, the stars were arbitrarily classified into six orders of magnitude. The fourteen brightest visible in our latitude were designated as of the first magnitude, while those which were barely visible to the naked eye were said to be of the sixth magnitude. This classification, it will be noticed, is entirely arbitrary, since there are no two stars which are absolutely of the same brilliancy, while if all the stars were arranged in the order of their actual brilliancy, we should find a regular gradation from the brightest to the faintest, no two being precisely the same. Therefore the brightest star of any one magnitude is about of the same brilliancy with the faintest one of the next higher magnitude. It depends upon the judgment of the observer to what magnitude a given star shall be assigned; so that we cannot expect an agreement on this point. The most recent and careful division into magnitudes has been made by Heis, of Germany, whose results with respect to numbers are as follows. Between the North Pole and 35° south declination, there are:

14	stars	of the	first	magnitude.
48	"	"	second	"
152	"	"	third	"
313	"	"	fourth	"
854	"	"	fifth	"
3974	"	"	sixth	"
5355	of the first six magnitudes.			

Of these, however, nearly 2000 of the sixth magnitude are so faint that they can be seen only by an eye of extraordinary keenness.

In order to secure a more accurate classification and expression of brightness, Heis and others have divided each magnitude into three orders or sub-magnitudes, making eighteen orders in all visible to the naked eye. When a star was considered as falling between two magnitudes, both figures were written, putting the magnitude to which the star most nearly approached first. For instance, the faintest stars of the fourth magnitude were called 4·5. The next order below this would be the brightest of the fifth magnitude; these were called 5·4. The stars of the average fifth magnitude were called 5 simply. The fainter ones were called 5·6, and so on. This notation is still used by some astronomers, but those who aim at greater order and precision express the magnitudes in tenths. For instance, the faintest stars of the fifth magnitude they would call 4·6, those one tenth fainter 4·7, and so on until they reached the average of the fifth magnitude, which would be 5·0. The division into tenths of magnitudes is as minute a one as the ordinary eye is able to make.

This method of designating the brilliancy of a star on a scale of magnitudes is not at all accurate. Several attempts have been made in recent times to obtain more accurate determinations, by measuring the light of the stars. An instrument with which this can be done is called a *photometer*. The results obtained with the photometer have been used to correct the scale of magnitudes and make it give a more accurate expression for the light of the stars. The study of such measures shows that, for the most part, the brightness of the stars increases in geometrical progression as the magnitudes vary in arithmetical progression. The stars of one magnitude are generally about $2\frac{1}{4}$ times as bright as those of the magnitude next below it. Therefore if we take the light of a star of the sixth magnitude, which is just visible to the naked eye, as unity, we shall have the following scale:

Magnitude	6th,	brightness	1
"	5th,	"	$2\frac{1}{2}$
"	4th,	"	$6\frac{1}{4}$
"	3d,	"	16 nearly
"	2d,	"	40
"	1st,	"	100

Therefore, according to these estimates, an average star of the first magnitude is about 100 times as bright as one of the sixth. There is, however, a deviation from this scale in the case of the brighter magnitudes, an average star of the second magnitude being perhaps three times as bright as one of the third, and most of the stars of the first magnitude brighter than those of the second in a yet larger ratio. Indeed, the first magnitude stars differ so greatly in brightness that we cannot say how bright a standard star of that magnitude really is. *Sirius*, for instance, is probably 500 times as bright as a sixth magnitude star.

The logarithm of $2\frac{1}{2}$ being very nearly 0·40, we can readily find how many stars of any one magnitude are necessary to make one of the higher magnitude by multiplying the difference of the magnitude by 0·40, and taking the number corresponding to this logarithm.

This scale will enable us to calculate in a rough way the magnitude of the smallest stars which can be seen with a telescope of given aperture. The quantity of light which a telescope admits is directly as the square of its aperture. The amount of light emitted by the faintest star visible in it is therefore inversely as this square. If we increase the aperture 50 per cent, we increase the seeing power of our telescope about one magnitude. More exactly, the ratio of increase of aperture is $\sqrt{2\frac{1}{2}}$, or 1·58. The pupil of the eye is probably equivalent to a telescope of about $\frac{1}{4}$ of an inch in aperture; that is, in a telescope of this size the faintest visible star would be about of the sixth magnitude. To find the exact magnitude of the faintest star visible with a larger telescope, we recall that the quantity of light received by the objective is proportional to the square of the aperture. As just shown, every time we multiply the square of the aperture by $2\frac{1}{2}$, or the aperture itself by the square root of this quantity, we add one magnitude to the power of our telescope. Therefore, if we call a_0 the aperture of a telescope which would just show a star one magnitude brighter than the first (or mag. 0), the aperture necessary to show a star of magnitude m will be found by mltiplying a_0 by 1·58 m times—that is, it will be $1.58^m\, a_0$. So, calling a this aperture, we have:

$$a = 1{\cdot}58^m\, a_0 = a_0 \sqrt{2{\cdot}5^m}.$$

Taking the logarithms of both sides of the equation, and using approximate round numbers which are exact enough for this purpose:

$$\log.\ a = m \log.\ 1{\cdot}58 + \log.\ a_0 = \frac{m}{2} \log.\ 2{\cdot}5 + \log.\ a_0 = \frac{m}{5} + \log.\ a_0.$$

Now, as just found, when $m = 6$, $a = 0^{in}{\cdot}25 = 6{\cdot}4$ millimetres. With these values of a and m we find:

$$\log. a_0 = -1{\cdot}802 \text{ in fractions of an inch.}$$
$$= -0{\cdot}397 \text{ in fractions of a millimetre.}$$

Hence, when the magnitude is given, and we wish to find the aperture:

$$\log. a = \frac{m}{5} - 1{\cdot}802 \text{ [will give aperture in inches.]}$$

$$\log. a = \frac{m}{5} - 0{\cdot}397 \text{ [will give aperture in millimetres.]}$$

If the aperture is given, and we require the limiting magnitude.

$$m = 5 \log. a + 9{\cdot}0 \text{ [if } a \text{ is in inches.]}$$
$$m = 5 \log. a + 2{\cdot}0 \text{ [if } a \text{ is in millimetres.]}$$

The magnitudes for different apertures is shown in the following table:

Aperture.	*Minimum Visible.*	Aperture.	*Minimum Visible.*
Inches.	Magnitude.	Inches.	Magnitude.
1·0	9·0	6·5	13·1
1·5	9·9	7·0	13·3
2·0	10·5	8·0	13·6
2·5	11·0	9·0	13·8
3·0	11·4	10·0	14·0
3·5	11·7	11·0	14·2
4·0	12·0	12·0	14·4
4·5	12·3	15·0	14·9
5·0	12·5	18·0	15·3
5·5	12·7	26·0	16·1
6·0	12·9	34·0	16·6

§ 3. THE CONSTELLATIONS AND NAMES OF THE STARS.

The earliest astronomers divided the stars into groups, called constellations, and gave special proper names both to these groups and to many of the more conspicuous stars. We have no record of the process by which this was done, or of the considerations which led to it. It was long before the commencement of history, as we may infer from different allusions to the stars and constellations in the book of *Job*, which is supposed to be among the most ancient writings now extant. We have evidence that more than 3000 years before the commencement of the Christian chronology the star *Sirius*, the brightest in the heavens, was known to the Egyptians under the name of *Sothis*. *Arcturus* is mentioned by JOB himself. The seven stars of the *Great Bear*, so conspicuous in our northern sky, were known under that name to HOMER and HESIOD, as well as the group of the *Pleiades*, or Seven Stars, and the constellation of *Orion*. Indeed, it would seem that all the earlier civilized nations, Egyptians, Chinese, Greeks, and Hindoos, had some arbitrary division of the surface of the heavens into irregular, and often fantastic shapes, which were distinguished by names.

In early times, the names of heroes and animals were given to the constellations, and these designations have come down to the present day. Each object was supposed to be painted on the surface of the heavens, and the stars were designated by their position upon some portion of the object. The ancient and mediæval astronomers would speak of "the bright star in the left foot of *Orion*," "the eye of the *Bull*," "the heart of the *Lion*," "the head of *Perseus*," etc. These figures are still retained upon some star-charts, and are useful where it is desired to compare the older descriptions of the constellations with our modern maps. Otherwise they have ceased to serve any purpose, and are not generally found on maps designed for astronomical uses.

The Arabians, who used this clumsy way of designating stars, gave special names to a large number of the brighter ones. Some of these names are in common use at the present time, as *Aldebaran*, *Fomalhaut*, etc. A few other names of bright stars have come down from prehistoric times, that of *Arcturus* for instance: they are, however, gradually falling out of use, a system of nomenclature introduced in modern times having been substituted.

In 1654, BAYER, of Germany, mapped down the constellations upon charts, designating the brighter stars of each constellation by the letters of the Greek alphabet. When this alphabet was exhausted, he introduced the letters of the Roman alphabet. In general, the brightest star was designated by the first letter of the alphabet α, the next by the following letter β, etc. Although this is sometimes supposed to have been his rule, the Greek letter affords only an imperfect clue to the average magnitude of a star. In a great many of the constellations there are deviations from the order, the brightest star being β; but where stars differ by an entire magnitude or more, the fainter ones nearly always follow the brighter ones in alphabetical order.

On this system, a star is designated by a certain Greek letter, followed by the genitive of the Latin name of the constellation to which it belongs. For example, *α Canis Majoris*, or, in English, α of the Great Dog, is the designation of *Sirius*, the brightest star in the heavens. The seven stars of the *Great Bear* are called *α Ursæ Majoris*, *β Ursæ Majoris*, etc. *Arcturus* is *α Boötis*. The reader will here see a resemblance to our way of designating individuals by a Christian name followed by the family name. The Greek letters furnish the Christian names of the separate stars, while the name of the constellation is that of the family. As there are only fifty letters in the two alphabets used by Bayer, it will be seen that only the fifty brightest stars in each constellation could be designated by this method. In most of the constellations the number thus chosen is much less than fifty.

When by the aid of the telescope many more stars than these were laid down, some other method of denoting them became necessary. Flamsteed, who observed before and after 1700, prepared an extensive catalogue of stars, in which those of each constellation were designated by numbers in the order of right ascension. These numbers were entirely independent of the designations of Bayer—that is, he did not omit the Bayer stars from his system of numbers, but numbered them as if they had no Greek letter. Hence those stars to which Bayer applied letters have two designations, the letter and the number.

FLAMSTEED's numbers do not go much above 100 for any one constellation—*Taurus*, the richest, having 139. When we consider the more numerous minute stars, no systematic method of naming them is possible. The star can be designated only by its position in the heavens, or the number which it bears in some well-known catalogue.

§ 4. DESCRIPTION OF THE CONSTELLATIONS.

The aspect of the starry heavens is so pleasing that nearly every intelligent person desires to possess some knowledge of the names and forms of the principal constellations. We therefore present a brief description of the more striking ones, illustrated by figures, so that the reader may be able to recognize them when he sees them on a clear night.

We begin with the constellations near the pole, because they can be seen on any clear night, while the southern ones can, for the most part, only be seen during certain seasons, or at certain hours of the night. The accompanying figure shows all the stars within 50° of the pole down to the fourth magnitude inclusive. The Roman numerals around the margin show the meridians of right ascension, one for every hour. In order to have the map represent the northern constellations exactly as they are, it must be held so that the hour of sidereal time at which the observer is looking at the heavens shall be at the top of the map. Supposing the observer to look at nine o'clock in the evening, the months around the margin of the map show the regions near the zenith. He has therefore only to hold the map with the month upward and face the north, when he will have the northern heavens as they appear, except that the stars near the bottom of the map will be cut off by the horizon.

The first constellation to be looked for is *Ursa Major*, the *Great Bear*, familiarly known as "the Dipper." The two extreme stars in this constellation point toward the pole-star, as already explained in the opening chapter.

Ursa Minor, sometimes called "the Little Dipper," is the constellation to which the pole-star belongs. About

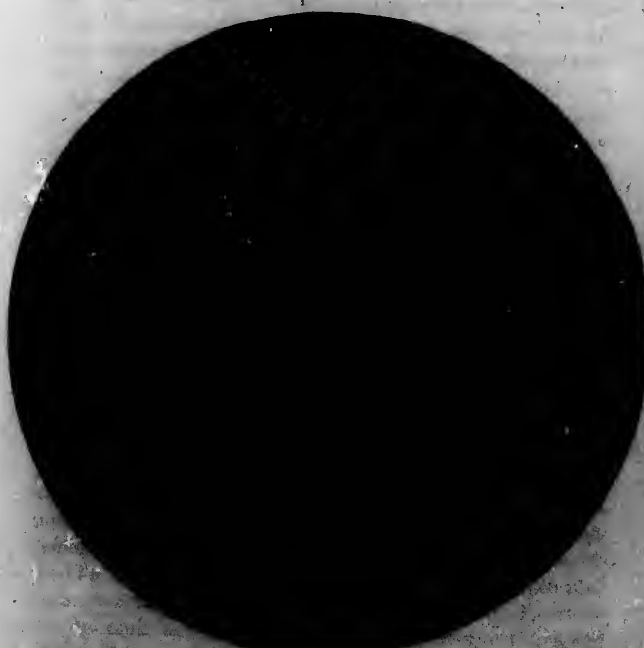


FIG. 112.—MAP OF THE NORTHERN CONSTELLATIONS.

15° from the pole, in right ascension XV. hours, is a star of the second magnitude, β *Ursæ Minoris*, about as bright as the pole-star. A curved row of three small stars lies between these two bright ones, and forms the handle of the supposed dipper.

Cassiopeia, or "the Lady in the Chair," is near hour I of right ascension, on the opposite side of the pole-star from *Ursa Major*, and at nearly the same distance. The six brighter stars are supposed to bear a rude resemblance to a chair. In mythology, *Cassiopeia* was the queen of *Cepheus*, and in the mythological representation of the constellation she is seated in the chair from which she is issuing her edicts.

In hour III of right ascension is situated the constellation *Perseus*, about 10° further from the pole than *Cassiopeia*. The Milky Way passes through these two constellations.

Draco, the Dragon, is formed principally of a long row of stars lying between *Ursa Major* and *Ursa Minor*. The head of the monster is formed of the northernmost three of four bright stars arranged at the corners of a lozenge between XVII and XVIII hours of right ascension.

Cepheus is on the opposite side of *Cassiopeia* from *Perseus*, lying in the Milky Way, about XXII hours of right ascension. It is not a brilliant constellation.

Other constellations near the pole are *Camelopardalis*, *Lynx*, and *Lacerta* (the Lizard), but they contain only small stars.

In describing the southern constellations, we shall take four separate positions of the starry sphere corresponding respectively to VI hours, XII hours, XVIII hours, and 0 hours of sidereal time or right ascension. These hours of course occur every day, but not always at convenient times, because they vary with the time of the year, as explained in Chapter I., Part I.

We shall first suppose the observer to view the heavens at VI hours of sidereal time, which occurs on December 21st about midnight, January 1st about 11.30 P.M., February 1st about 9.30 P.M., March 1st about 7.30 P.M., and so on through the year, two hours earlier every month. In this position of the sphere, the Milky Way spans the heavens like an arch, resting on the horizon between north and north-west on one side, and between south and south-east on the other. We shall first describe the constellations which lie in its course, beginning at the north. *Cepheus* is near the north-west horizon, and above it is *Cassiopeia*, distinctly visible at an altitude nearly equal to that of the pole. Next is *Perseus*, just north-west of the zenith. Above *Perseus* lies *Auriga*, the Charioteer, which may be recognized by a bright star of the first magnitude called *Capella* (the Goat), now quite near the zenith. *Auriga* is represented as holding a goat in his arms, in the body of which the star is situated. About 10° east of *Capella* is the star β *Aurigæ* of the second magnitude.

Going farther south, the Milky Way next passes between *Taurus* and *Gemini*.

Taurus, the Bull, may be recognized by the *Pleiades*, or "Seven Stars." Really there are only six stars in the group clearly visible to ordinary eyes, and any eye strong enough to see seven will probably see four others, or eleven in all. This group forms an interesting object of study with a small telescope, as sixty or eighty stars can then readily be seen. We therefore present a telescopic view of it, the six larger stars being those visible to any ordinary eye, the five next in size those which can be seen by a remarkably good eye, and the others those which require a telescope. East of the *Pleiades* is the bright red star *Aldebaran*, or "the Eye of the Bull." It lies in a group called the *Hyades*, arranged in the form of the letter V, and forming the face

FIG. 112.—TELESCOPIC VIEW OF THE PLEIADES.

of the Bull. In the middle of one of the legs of the V will be seen a beautiful pair of stars of the fourth magnitude very close together. They are called *θ Tauri*.

Gemini, the Twins, lie east of the Milky Way, and may be recognized by the bright stars *Castor* and *Pollux*, which lie 20° or 30° south-east or south of the zenith.

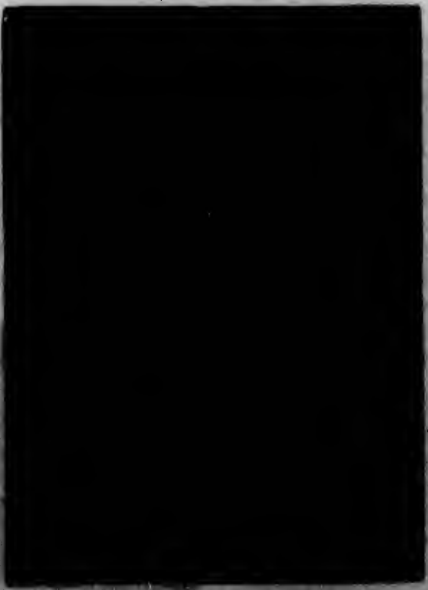


FIG. 114.—THE CONSTELLATION ORION.

They are about 5° apart, and *Pollux*, the southernmost one, is a little brighter than *Castor*.

Orion, the most brilliant constellation in the heavens, is very near the meridian, lying south-east of *Taurus* and south-west of *Gemini*. It may be readily recognized by the figure which we give. Four of its bright stars form the corners of a rectangle about 15° long from north to south, and 5° wide. In the middle of it is a row of three bright stars of the second magnitude, which no one can fail to recognize. Below this is another row of three smaller ones. The middle star of this last row is called *θ Orionis*, and is situated in the midst of the great nebula of *Orion*, one of the most remarkable telescopic objects in the heavens. Indeed, to the naked eye this star has a nebulous hazy appearance. The two stars of the first magnitude are *α Orionis*, or *Betelguese*, which is the highest, and may be recognized by its red color, and *Rigel*, or *β Orionis*, a sparkling white star lower down and a little to the west. The former is in the shoulder of the figure, the latter in the foot. A little north-west of *Betelguese* are three small stars, which form the head. The row of stars on the west form his arm and club, the latter being raised as if to strike at *Taurus*, the Bull, on the west.

Canis Minor, the Little Dog, lies across the Milky Way from *Orion*, and may be recognized by the bright star *Procyon* of the first magnitude. The three stars *Pollux*, *Procyon*, and *Betelguese* form a right-angled triangle, the right angle being at *Procyon*.

Canis Major, the Great Dog, lies south-east of *Orion*, and is easily recognized by *Sirius*, the brightest fixed star in the heavens. A number of bright stars south and south-east of *Sirius* belong to this constellation, making it one of great brilliancy.

Argo Navis, the ship Argo, lies near the south horizon, partly above it and partly below it. Its brightest star is *Canopus*, which, next to *Sirius*, is the brightest star in the heavens. Being in 53° of south declination, it never rises to an observer within 53° of the North Pole—that is, north of 37° of north latitude. In our country it is visible only in the Southern States, and even there only between six and seven hours of sidereal time.

We next trace out the zodiacal constellations, which are of interest because it is through them that the sun passes in its apparent annual course. We shall commence in the west and go toward the east, in the order of right ascension.

Aries, the Ram, is in the west, about one third of the way from the horizon to the zenith. It may be recognized by three stars of the second, third, and fourth magnitudes, respectively, forming an obtuse-angled triangle. The brightest star is the highest. Next toward the east is *Taurus*, the Bull, which brings us nearly to the meridian, and east of the meridian lies *Gemini*, the Twins, both of which constellations have just been described.

FIG. 115.—THE CONSTELLATION LEO, THE LION.

Cancer, the Crab, lies east of *Gemini*, but contains no bright star. The most noteworthy object in this constellation is *Præsepe*, a group of telescopic stars, which appears to the naked eye like a spot of milky light. To see it well, the night must be clear and the moon not in the neighborhood.

Leo, the Lion, is from one to two hours above the eastern horizon. Its brightest star is *Regulus*, one third of the way from the eastern horizon to the zenith, and between the first and second magnitudes. Five or six stars north of it in a curved line are in the form of a sickle, of which *Regulus* is the handle. As the Lion was drawn among the old constellations, *Regulus* formed his heart, and was therefore called *Cor Leonis*. The sickle forms his head, and his body and tail extend toward the horizon. The tail ends near the star *Denebola*, which is quite near the horizon.

Leo Minor lies to the north of *Leo*, and *Sextans*, the Sextant, south of it, but neither contains any bright stars.

Eridanus, the River Po, south-west of *Orion; Lepus*, the Hare, south of *Orion* and west of *Canis Major; Columba*, the Dove, south of *Lepus*, are constellations in the south and south-west, which, however, have no striking features.

The constellations we have described are those seen at six hours of sidereal time. If the sky is observed at some other hour near this, we may find the sidereal time by the rule given in Chapter I., § 5, p. 30, and allow for the diurnal motion during the interval.

Appearance of the Constellations, at 12 Hours Sidereal Time.—This hour occurs on April 1st at 11.30 P.M., on May 1st at 9.30 P.M., and on June 1st at 7.30 P.M.

At this hour, *Ursa Major* is near the zenith, and *Cassiopeia* near or below the north horizon. The Milky Way is too near the horizon to be visible. *Orion* has set in the west, and there is no very conspicuous constellation in the south. *Castor* and *Pollux* are high up in the north-west, and *Procyon* is about an hour and a half above the horizon, a little to the south of west. All the constellations in the west and north-west have been previously described, *Leo* being a little west of the meridian. Three zodiacal constellations have, however, risen, which we shall describe.

Virgo, the Virgin, has a single bright star, *Spica*, about as bright as *Regulus*, now about one hour east of the meridian, and but little more than half way from the zenith to the horizon.

Libra, the Balance, is south-east from *Virgo*, but has no conspicuous stars.

Scorpius, the Scorpion, is just rising in the south-east, but is not yet high enough to be well seen.

Hydra is a very long constellation extending from *Canis Minor* in a south-east direction to the south horizon. Its brightest star is *α Hydra*, of the second magnitude, 25° below *Regulus*.

Corvus, the Crow, is south of *Virgo*, and may be recognized by four or five stars of the second or third magnitude, 15° south-west from *Spica*.

Next, looking north of the zodiacal constellations, we see:

Coma Berenices, the Hair of Berenice, now exactly on the meridian, and about 10° south of the zenith. It is a close irregular cluster of very small stars, unlike any thing else in the heavens. In ancient mythology, Berenice had vowed her hair to Venus, but Jupiter carried it away from the temple in which it was deposited, and made it into a constellation.

Bootes, the Bear-Keeper, is a large constellation east of *Coma Berenices*. It is marked by *Arcturus*, a bright but somewhat red star of the first magnitude, about 20° east of the zenith. *Bootes* is represented as holding two dogs in a leash. These dogs are called *Canes Venatici*, and are at the time supposed exactly in our zenith chasing *Ursa Major* around the pole.

FIG. 116.—CORONA BOREALIS.

Corona Borealis, the Northern Crown, lies next east of *Bootes* in the north-east. It is a small but extremely beautiful constellation. Its principal stars are arranged in the form of a semicircular chaplet or crown.

Appearance of the Constellations at 18 Hours of Sidereal Time.—This hour occurs on July 1st at 11.30 P.M., on August 1st at 9.30 P.M., and on September 1st at 7.30 P.M.

In this position, the Milky Way seems once more to span the heavens like an arch, resting on the horizon in the north-west and south-east. But we do not see the same parts of it which were visible in the first position at six hours of right ascension. *Cassiopeia* is now in the north-east and *Ursa Major* has passed over to the west.

Arcturus is two or three hours above the western horizon. We shall commence, as in the first position of the sphere, by describing the constellations which lie along on the Milky Way, starting from *Cassiopeia*. Above *Cassiopeia* we have *Cepheus*, and then *Lacerta*, neither of which contains any striking stars.

Cygnus, the Swan, may be recognized by four or five stars forming a cross directly in the centre of the Milky Way, and a short distance north-east from the zenith. The brightest of these stars, α *Cygni*, forms the northern end of the cross, and is nearly of the first magnitude.

Lyra, the Harp, is a beautiful constellation south-west of *Cygnus*, and nearly in the zenith. It contains the brilliant star *Vega*, or α *Lyræ*, of the first magnitude, and of a bluish white color. South of *Vega* are four stars of the fourth magnitude, forming an oblique parallelogram, by which the constellation can be readily recognized. East of *Vega*, and about as far from it as the nearest star of the parallelogram, is ϵ *Lyræ*, a very interesting object, because it is really composed of two stars of the fourth magnitude, which can be seen separately by a very keen eye. The power to see this star double is one of the best tests of the acuteness of one's vision (see Fig. 122).

FIG. 117.—LYRA, THE HARP.

Aquila, the Eagle, is the next striking constellation in the Milky Way. It is two hours east of the meridian, and about midway between the zenith and horizon. It is readily recognized by the bright star *Altair* or *α Aquilæ*, situated between two smaller ones, the one of the third and the other of the fourth magnitude. The row of three stars lies in the centre of the Milky Way.

FIG. 118.—AQUILA, DELPHINUS, AND SAGITTA.

Sagitta, the Arrow, is a very small constellation, formed of three stars immediately north of *Aquila*.

Delphinus, the Dolphin, is a striking little constellation north-east of *Aquila*, recognized by four stars in the form of a lozenge. It is familiarly called "Job's Coffin."

In this position of the celestial sphere three new zodiacal constellations have arisen.

Scorpius, the Scorpion, already mentioned, now two hours west of the meridian, and about 30° above the horizon, is quite a brilliant constellation. It contains *Antares*, or *α Scorpii*, a reddish star of nearly the first magnitude, and a long row of curved stars west of it.

FIG. 119.—SCORPIUS, THE SCORPION.

Sagittarius, the Archer, comprises a large collection of second magnitude stars in and near the Milky Way, and now very near the meridian. The westernmost stars form the arrow of the archer.

Capricornus, the Goat, is now in the south-east, but contains no bright stars. *Aquarius*, the Water-bearer, which has just risen, and *Pisces*, the Fishes, which have partly risen, contain no striking objects.

Ophiuchus, the Serpent-bearer, is a very large constellation north of *Scorpius* and west of the Milky Way. *Ophiuchus* holds in his hands an immense serpent, lying with its tail in an opening of the Milky Way, south-west of *Aquila*, while its head and body are formed of a collection of stars of the third and fourth magnitudes, extending north of *Scorpius* nearly to *Bootes*.

FIG. 120.—SAGITTARIUS, THE ARCHER.

Hercules is a very large constellation between *Corona Borealis* and *Lyra*. It is now in the zenith, but contains no bright stars. It has, however, a number of interesting telescopic objects, among them the great cluster of *Hercules*, barely visible to the naked eye, but containing an almost countless mass of stars. The head of *Draco*, already described, is just north of *Hercules*.

Constellations Visible at 0 Hours of Sidereal Time.—This time will occur on October 1st at 11.30 P.M., on November 1st at 9.30 P.M., on December 1st at 7.30 P.M., and on January 1st at 5.30 P.M.

In this position, the Milky Way appears resting in the east and west horizons, but in the zenith it is inclined over toward the north. All the constellations, either in or north of its course, are among those already described. We shall therefore consider only those in the south.

Pegasus, the Flying Horse, is distinguished by four stars of the second magnitude, which form a large square about 15° on each side, called the square of *Pegasus*. The eastern side of this square is almost exactly on the meridian.

Andromeda is distinguished by a row of three or four bright stars, extending from the north-east corner of *Pegasus*, in the direction of *Perseus*.

Cetus, the Whale, is a large constellation in the south and south-east. Its brightest star is β *Ceti*, standing alone, 30° above the horizon, and a little east of the meridian.

Piscis Australis, the Southern Fish, lies further west than *Cetus*. It has the bright star *Fomalhaut*, about 15° above the horizon, and an hour west of the meridian.

§ 5. NUMBERING AND CATALOGUING THE STARS.

As telescopic power is increased, we still find stars of fainter and fainter light. But the number cannot go on increasing forever in the same ratio as with the brighter magnitudes, because, if it did, the whole sky would be a blaze of starlight.

If telescopes with powers far exceeding our present ones were made, they would no doubt show new stars of the 20th and 21st magnitudes. But it is highly probable that the *number* of such successive orders of stars would not increase in the same ratio as is observed in the 8th, 9th, and 10th magnitudes, for example. The enormous labor of estimating the number of stars of such classes will long prevent the accumulation of statistics on this question; but this much is certain, that in special regions of the sky, which have been searchingly examined by various telescopes of successively increasing apertures, the number of new stars found is by no means in proportion to the increased instrumental power. Thus, in the central portions of the nebula of *Orion*, only some half dozen stars have been found with the Washington 26-inch refractor which were not seen with the Cambridge 15-inch, although the visible magnitude has been extended from $15^m \cdot 1$ to $16^m \cdot 3$. If this is found to be true elsewhere, the conclusion may be that, after all, the stellar system can be experimentally shown to be of finite extent, and to contain only a finite number of stars.

We have already stated that in the whole sky an eye of average power will see about 6000 stars. With a telescope this number is greatly increased, and the most powerful telescopes of modern times will probably show more than 20,000,000 stars. As no trustworthy estimate has ever been made, there is great uncertainty upon this point, and the actual number may range anywhere between 15,000,000 and 40,000,000. Of this number, not one out of twenty has ever been catalogued at all.

The gradual increase in the number of stars laid down in various of the older catalogues is exhibited in the following table from CHAMBERS's *Descriptive Astronomy*:

CONSTELLATION.	Ptolemy. B.C. 130.	Tycho Brahe. A.D. 1570.	Hevelius. A.D. 1660.	Flamsteed. A.D. 1690.	Bode. A.D. 1800.
Aries........	18	21	27	66	148
Ursa Major..	35	56	73	87	338
Boötes.... ..	23	28	52	54	319
Leo.........	35	40	50	95	337
Virgo.......	32	39	50	110	411
Taurus......	44	43	51	141	394
Orion........	38	62	62	78	304

The most famous and extensive series of star observations are noticed below.

The uranometries of BAYER, FLAMSTEED, ARGELANDER, HEIS, and GOULD give the lucid stars of one or both hemispheres laid down on maps. They are supplemented by the star catalogues of other observers, of which a great number has been published. These last were undertaken mainly for the determination of star positions, but they usually give as an auxiliary datum the magnitude of the star observed. When they are carried so far as to cover the heavens, they will afford valuable data as to the distribution of stars throughout the sky.

The most complete catalogue of stars yet constructed is the *Durchmusterung des Nördlichen Gestirnten Himmels*, the joint work of ARGELANDER and his assistants, KRUEGER and SCHÖNFELD. It embraces all the stars of the first nine magnitudes from the North

Pole to 2° of south declination. This work was begun in 1852, and at its completion a catalogue of the approximate places of no less than 314,926 stars, with a series of star-maps, giving the aspect of the northern heavens for 1855, was published for the use of astronomers. ARGELANDER's original plan was to carry this *Durchmusterung* as far as 23° south, so that every star visible in a small comet-seeker of 2¼ inches aperture should be registered. His original plan was abandoned, but his former assistant and present successor at the observatory of Bonn, Dr. SCHÖNFELD, is now engaged in executing this important work.

The Catalogue of Stars of the British Association for the Advancement of Science contains 8377 stars in both hemispheres, and gives all the stars visible to the eye. It is well adapted to learn the unequal distribution of the lucid stars over the celestial sphere. The table on the opposite page is formed from its data.

From this table it follows that the southern sky has many more stars of the first seven magnitudes than the northern, and that the zones immediately north and south of the Equator, although greater in surface than any others of the same width in declination, are absolutely poorer in such stars.

The meaning of the table will be much better understood by consulting the graphical representation of it on page 438, by PROCTOR. On this chart are laid down all the stars of the British Association Catalogue (a dot for each star), and beside these the Milky Way is represented. The relative richness of the various zones can be at once seen, and perhaps the scale of the map will allow the student to trace also the zone of brighter stars (1st–3d magnitude), which is inclined to that of the Milky Way by a few degrees, and is approximately a great circle of the sphere.

The distribution and number of the brighter stars (1st–7th magnitude) can be well understood from this chart.

In ARGELANDER's *Durchmusterung* of the stars of the northern heavens, there are recorded as belonging to the northern hemisphere:

10	stars	between the	1·0	magnitude	and the	1·9	magnitude.
37	"	"	2·0	"	"	2·9	"
128	"	"	3·0	"	"	3·9	"
310	"	"	4·0	"	"	4·9	"
1,016	"	"	5·0	"	"	5·9	"
[illegible],328	"	"	6·0	"	"	6·9	"
13,593	"	"	7·0	"	"	7·9	"
57,960	"	"	8·0	"	"	8·9	"
237,544	"	"	9·0	"	"	9·5	"

In all 314,926 stars from the first to the 9·5 magnitudes are enumerated in the northern sky, so that there are about 600,000 in the whole heavens.

We may readily compute the amount of light received by the earth on a clear but moonless night from these stars. Let us assume

DISTRIBUTION OF THE LUCID STARS, ACCORDING TO THE BRITISH ASSOCIATION CATALOGUE.

R. A.	Declination.												Sum.
	+90° to +75°	+75° to +60°	+60° to +45°	+45° to +30°	+30° to +15°	+15° to +0°	+0° to −15°	−15° to −30°	−30° to −45°	−45° to −60°	−60° to −75°	−75° to −90°	
0h–1	11	19	35	22	35	49	34	11	39	30	27	10	322
1–2	14	29	25	27	44	48	24	15	39	37	21	7	330
2–3	6	11	37	28	45	37	26	18	44	22	38	7	319
3–4	7	16	29	22	68	23	23	21	44	18	24	7	302
4–5	8	18	27	24	77	39	24	17	41	23	15	5	318
5–6	5	11	30	36	70	45	46	22	55	31	18	7	376
6–7	7	14	26	21	69	28	15	34	74	55	28	4	370
7–8	6	13	27	16	62	28	18	39	66	70	19	8	387
8–9	5	22	18	27	72	40	17	26	56	80	24	7	394
9–10	4	18	23	31	37	45	30	14	38	59	44	6	344
10–11	7	9	15	39	29	55	27	24	24	62	48	11	344
11–12	8	10	14	17	24	54	25	17	29	38	40	9	260
12–13	14	11	20	15	46	34	37	17	30	52	28	5	309
13–14	6	7	16	32	25	20	58	37	30	38	29	7	305
14–15	4	9	24	27	20	20	41	48	25	26	24	10	290
15–16	11	13	21	27	31	21	26	106	37	53	29	5	390
16–17	6	17	22	28	29	43	18	103	75	44	33	3	417
17–18	3	11	22	24	25	26	16	99	72	37	28	3	366
18–19	9	17	30	32	23	17	24	125	71	24	29	5	406
19–20	3	16	40	34	49	30	24	88	29	23	27	4	363
20–21	14	19	46	50	80	27	42	108	28	27	37	6	429
21–22	8	27	44	27	27	23	48	68	23	48	26	6	370
22–23	12	25	36	32	19	25	68	28	28	42	24	10	347
23–24	6	25	31	22	24	37	41	31	30	38	31	9	325
Sum	179	377	658	660	980	814	750	1,108	1,047	974	679	156	8,377

The Northern Milky Way, from Heis; the Southern, from Sir J. Herschel.—Drawn by Richard A. Proctor.

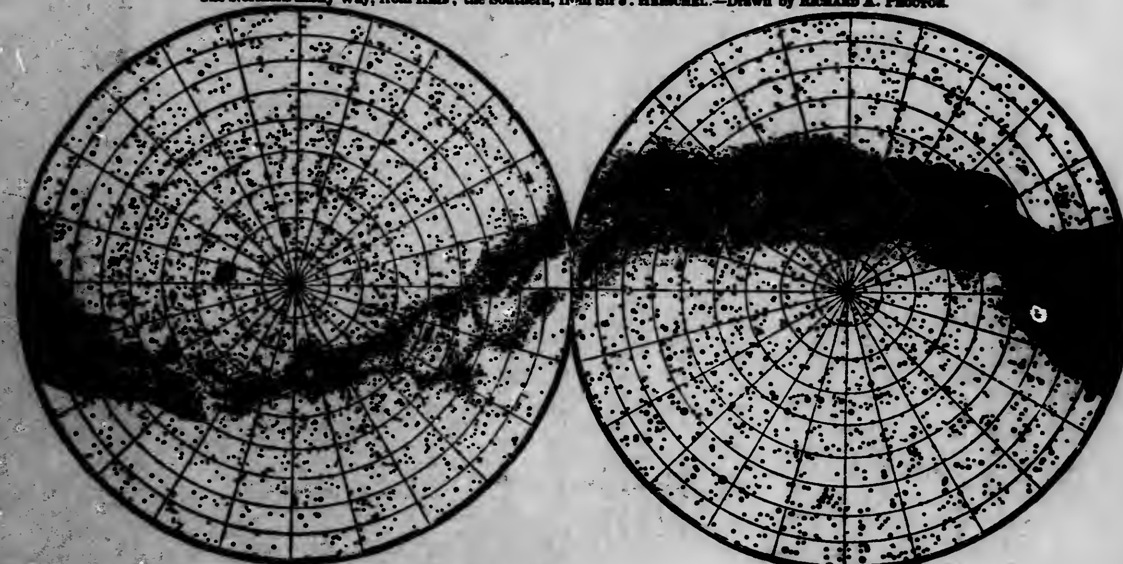


Fig. 121.—The stars to sixth magnitude in the B. A. Catalogue on an equal surface projection.

FIG. 121.—THE STARS TO SIXTH MAGNITUDE IN THE B. A. CATALOGUE ON AN EQUAL SURFACE PROJECTION.

that the brightness of an avetage star of the first magnitude is about 0·5 of that of *α Lyræ*. A star of the 2d magnitude will shine with a light expressed by $0 \cdot 5 \times 0 \cdot 4 = 0 \cdot 20$, and so on.

The total brightness of		10	1st	magnitude	stars is	5·0
"	"	37	2d	"	"	7·4
"	"	128	3d	"	"	10·1
"	"	310	4th	"	"	9·9
"	"	1,016	5th	"	"	13·0
"	"	4,322	6th	"	"	22·1
"	"	13,593	7th	"	"	27·8
"	"	57,960	8th	"	"	47·4
					Sum =	142·7

It thus appears that from the stars to the 8th magnitude, inclusive, we receive 143 times as much light as from *α Lyræ*. *α Lyræ* has been determined by ZÖLLNER to be about 44,000,000,000 times fainter than the sun, so that the proportion of starlight to sunlight can be computed. It also appears that the stars of magnitudes too high to allow them to be individually visible to the naked eye are yet so numerous as to affect the general brightness of the sky more than the so-called lucid stars (1st–6th magnitude).

CHAPTER II.

VARIABLE AND TEMPORARY STARS.

§ 1. STARS REGULARLY VARIABLE.

ALL stars do not shine with a constant light. Since the middle of the seventeenth century, stars variable in brilliancy have been known, and there are also stars which periodically change in color. The period of a variable star means the interval of time in which it goes through all its changes, and returns to the same brilliancy.

The most noted variable stars are *Mira Ceti* (*o Ceti*) and *Algol* (*β Persei*). *Mira* appears about twelve times in eleven years, and remains at its greatest brightness (sometimes as high as the 2d magnitude, sometimes not above the 4th) for some time, then gradually decreases for about 74 days, until it becomes invisible to the naked eye, and so remains for about five or six months. From the time of its reappearance as a lucid star till the time of its maximum is about 43 days (HEIS). The *mean* period, or the interval from minimum to minimum, is about 333 days (ARGELANDER), but this period, as does the maximum light, varies greatly.

Algol has been known as a variable star since 1667. Its period is about $2^d\ 20^h\ 49^m$, and is supposed to be from time to time subject to slight fluctuations. This star is commonly of the 2d magnitude; after remaining so about 2½ days, it falls to 4^m in the short time of 4½ hours, and remains of 4^m for 20 minutes. It then commences to increase in brilliancy, and in another 3½ hours it is again of the 2d magnitude, at which point it remains for the remainder of its period, about $2^d\ 12^h$.

These two examples of the class of variable stars give a rough idea of the extraordinary nature of the phenomena they present. A closer examination of others discloses minor variations of great complexity and apparently without law.

The following are some of the more prominent variable stars visible to the naked eye:

NAME.	R. A. 1870.			Declination, 1870.		Period.	Changes of Magnitude.	
	h.	*m.*	*s.*	°	′	*d.*	*from*	*to*
β Persei......	2	59	43	+ 40	27·2	2·86	2½	4
δ Cephei......	22	24	21	+ 57	45·0	5·36	3·7	4·8
η Aquilæ......	19	45	51	+ 0	40·4	7·17	3·6	4·7
β Lyræ......	18	45	17	+ 33	12·7	12·91	3½	4½
α Herculis....	17	8	43	+ 14	32·4	88·5	3·1	3·9
o Ceti........	2	12	47	− 3	34·1	330·0	2	10
υ Hydræ.....	13	22	37	− 22	36·4	436·0	4	10
η Argus.....	10	40	2	− 59	0·1	70 years.	1	6

Abont 90 variable stars are well known, and as many more are suspected to vary. In nearly all cases the mean period can be fairly well determined, though anomalies of various kinds frequently appear. The principal anomalies are:

First. The period is seldom constant. For some stars the changes of the period seem to follow a regular law; for others no law can be fixed.

Second. The time from a minimum to the next maximum is usually shorter than from this maximum to the next minimum.

Third. Some stars (as β *Lyræ*) have not only one maximum between two consecutive principal minima, but two such maxima. For β *Lyræ*, according to ARGELANDER, $3^d\ 2^h$ after the principal minimum comes the first maximum; then, $3^d\ 7^h$ after this, a secondary minimum in which the star is by no means so faint as in the principal minimum, and finally $3^d\ 3^h$ afterward comes the principal maximum, the whole period being $12^d\ 21^h\ 47^m$. The course of one period is illustrated below, supposing the period to begin at $0^d\ 0^h$, and opposite each phase is given the intensity of light in terms of γ *Lyræ* $= 1$, according to photometric measures by KLEIN.

Phase.			Relative Intensity.
Principal Minimum	0^d	0^h	0·40
First Maximum	3^d	2^h	0·83
Second Minimum	6^d	9^h	0·58
Principal Maximum	9^d	12^h	0·89
Principal Minimum	12^d	22^m	0·40

The periods of 94 well-determined variable stars being tabulated, it appears that they are as follows :

Period between		No. of Stars.	Period between		No. of Stars.
1 d. and	20 d.	13	350 d. and	400 d.	13
20	50	1	400	450	8
50	100	4	450	500	8
100	150	4	500	550	0
150	200	5	550	600	0
200	250	9	600	650	1
250	300	14	650	700	0
300	350	13	700	750	1
					$\Sigma = 94$

It is natural that there should be few known variables of periods of 500 days and over, but it is not a little remarkable that the periods of over half of these variables should fall between 250 and 450 days.

The color of over 80 per cent of the variable stars is red or orange. Red stars (of which 600 to 700 are known) are now receiving close attention, as there is a strong likelihood of finding among them many new variables.

The spectra of variable stars show changes which appear to be connected with the variations in their light.

Another class of variations occurs among the fixed stars—namely, variations in color, either with or without corresponding changes of magnitude.

In the *Uranometry*, composed in the middle of the tenth century by the Persian astronomer Al Sûfi, it is stated that at the time of his observations the star *Algol* was reddish—a term which he applies also to the stars *Antares*, *Aldebaran*, and some others. Most of these still exhibit a reddish aspect. But *Algol* now appears as a white star, without any sign of color. Dr. Klein, of Cologne, discovered that *α Ursæ Majoris* periodically changes color from an intense fiery red to a yellow or yellowish-red every five weeks. Weber, of Peckeloh, has observed this star lately, and finds this period to be well established.

§ 2. TEMPORARY OR NEW STARS.

There are a few cases known of apparently *new* stars which have suddenly appeared, attained more or less brightness, and slowly decreased in magnitude, either disappearing totally, or finally remaining as comparatively faint objects.

The most famous one was that of 1572, which attained a brightness greater than that of *Sirius* or *Jupiter* and approached to *Venus*, being even visible to the eye in daylight. Tycho Brahe first observed this star in November, 1572, and watched its gradual increase in light until its maximum in December. It then began to diminish in brightness, and in January, 1573, it was fainter than *Jupiter*. In February and March it was of the 1st magnitude, in April and May of the 2d, in July and August of the 3d, and in October and November of the 4th. It continued to diminish until March, 1574, when it became invisible, as the telescope was not then in use. Its color, at first intense white, decreased through yellow and red. When it arrived at the 5th magnitude its color again became white, and so remained till its disappearance. Tycho measured its distance carefully from nine stars near it, and near its place there is now a star of the 10th or 11th magnitude, which is possibly the same star.

The history of temporary stars is in general similar to that of the star of 1572, except that none have attained so great a degree of brilliancy. More than a score of such objects are known to have appeared, many of them before the making of accurate observations, and the conclusion is probable that many have appeared without recognition. Among telescopic stars, there is but a small chance of detecting a new or temporary star.

Several supposed cases of the disappearance of stars exist, but here there are so many possible sources of error that great caution is necessary in admitting them.

Two temporary stars have appeared since the invention of the spectroscope (1859), and the conclusions drawn from a study of their spectra are most important as throwing light upon the phenomena of variable stars in general.

The first of these stars is that of 1866, called *T Coronæ*. It was first seen on the 12th of May, 1866, and was then of the 2d magnitude. Its changes were followed by various observers, and its magnitude found to diminish as follows:

1866.	m.	1866.	m.
May 12	2·0	May 18	5·5
13	2·2	19	6·0
14	3·0	20	6·5
15	3·5	21	7·0
16	4·0	22	7·5
17	4·5	23	8·0

By June 7th it had fallen to $9^m \cdot 0$, and July 7th it was $9^m \cdot 5$. SCHMIDT's observations of this star (*T Coronæ*), continued up to 1877, show that, after falling from the second to the seventh magnitude in nine days, its light diminished very gradually year after year down to nearly the tenth magnitude, at which it has remained pretty constant for some years. But during the whole period there have been fluctuations of brightness at tolerably regular intervals of ninety-four days, though of successively decreasing extent. After the first sudden fall, there seems to have been an increase of brilliancy, which brought the star above the seventh magnitude again, in October, 1866, an increase of a full magnitude; but since that time the changes have been much smaller, and are now but little more than a tenth of a magnitude. The color of the star has been pale yellow throughout the whole course of observations.

The spectroscopic observations of this star by Huggins and Miller showed it to have a spectrum then absolutely unique. The report of their observations says, "the spectrum of this object is twofold, showing that the light by which it shines has emanated from two distinct sources. The principal spectrum is analogous to that of the sun, and is formed of light which was emitted by an incandescent solid or liquid photosphere, and which has suffered a partial absorption by passing through an atmosphere of vapors at a lower temperature than the photosphere. Superposed over this spectrum is a second spectrum consisting of a few *bright* lines which is due to light which has emanated from intensely heated matter in the state of gas."

In November, 1876, Dr. Schmidt discovered a new star in *Cygnus*, whose telescopic history is similar to that given for *T Coronæ*. When discovered it was of the 3d magnitude, and it fell rapidly below visibility to the naked eye.

This new star in *Cygnus* was observed by Cornu, Copeland, and Vogel, by means of the spectroscope; and from all the observations it is plain that the hydrogen lines, at first prominent, have gradually faded. With the decrease in their brilliancy, a line corresponding in position with the brightest of the lines of a nebula has strengthened. On December 8th, 1876, this last line was much fainter than F (hydrogen line in the solar spectrum), while on March 2d, 1877, F was very much the fainter of the two.

At first it exhibited a continuous spectrum with numerous bright lines, but in the latter part of 1877 it emitted only monochromatic light, the spectrum consisting of a single bright line, corresponding in position to the characteristic line of gaseous nebulæ. The intermediate stages were characterized by a gradual fading out, not only of the continuous spectrum, but also of the bright lines which crossed it. From this fact, it is inferred that this star, which has now fallen to 10·5 magnitude, has actually become a planetary nebula, affording an instance of a remarkable reversal of the process imagined by La Place in his nebular theory.

§ 3. THEORIES OF VARIABLE STARS.

The theory of variable stars now generally accepted by investigators is founded on the following general conclusions:

(1) That the only distinction which can be made between the various classes of stars we have just described is one of degree. Between stars as regular as *Algol*, which goes through its period in less than three days, and the sudden blazing out of the star de-

scribed by TYCHO BRAHE, there is every gradation of irregularity. The only distinction that can be drawn between them is in the length of the period and the extent and regularity of the changes. All such stars must, therefore, for the present, be included in the single class of variables.

It was at one time supposed that newly created stars appeared from time to time, and that old ones sometimes disappeared from view. But it is now considered that there is no well-established case either of the disappearance of an old star or the creation of a new one. The supposed cases of disappearance arose from cataloguers accidentally recording stars in positions where none existed. Subsequent astronomers finding no stars in the place concluded that the star had vanished when in reality it had never existed. The view that temporary stars are new creations is disproved by the rapidity with which they always fade away again.

(3) That all stars may be to a greater or less extent variable; only in a vast majority of cases the variations are so slight as to be imperceptible to the eye. If our sun could be viewed from the distance of a star, or if we could actually measure the amount of light which it transmits to our eyes, there is little doubt that we should find it to vary with the presence or absence of spots on its surface. We are therefore led to the result that variability of light may be a common characteristic of stars, and if so we are to look for its cause in something common to all such objects.

The spots on the sun may give us a hint of the probable cause of the variations in the light of the stars. The general analogies of the universe, and the observations with the spectroscope, all lead us to the conclusion that the physical constitution of the sun and stars is of the same general nature. As we see spots on the sun which vary in form, size and number from day to day, so if we could take a sufficiently close view of the faces of the stars we should probably see spots on a great number of them. In our sun the spots never cover more than a very small fraction of the surface; but we have no reason to suppose that this would be the case with the stars. If the spots covered a large portion of the surface of the star, then their variations in number and extent would cause the star to vary in light.

This view does not, however, account for those cases in which the light of a star is suddenly increased in amount hundreds of times. But the spectroscopic observations of *T Coronæ* show another analogy with operations going on in our sun. Mr. HUGGINS's observations, which we have already cited, seem to show that there was a sudden and extraordinary outburst of glowing hydrogen from the star, which by its own light, as well as by heating up the whole surface of the star, caused an increase in its brilliancy.

Now, we have on a very small scale something of this same kind going on in our sun. The red flames which are seen during a total eclipse are caused by eruptions of hydrogen from the interior of the sun, and these eruptions are generally connected with the faculæ or portions of the sun's disk more brilliant than the rest of the photosphere.

The general theory of variable stars which has now the most evidence in its favor is this: These bodies are, from some general cause not fully understood, subject to eruptions of glowing hydrogen gas from their interior, and to the formation of dark spots on their surfaces. These eruptions and formations have in most cases a greater or less tendency to a regular period.

In the case of our sun, the period is 11 years, but in the case of many of the stars it is much shorter. Ordinarily, as in the case of the sun and of a large majority of the stars, the variations are too slight to affect the total quantity of light to any visible extent. But in the case of the variable stars this spot-producing power and the liability to eruptions are very much greater than in the case of our sun, and thus we have changes of light which can be readily perceived by the eye. Some additional strength is given to this theory by the fact just mentioned, that so large a proportion of the variable stars are red. It is well known that glowing bodies emit a larger proportion of red rays and a smaller proportion of blue ones the cooler they become. It is therefore probable that the red stars have the least heat. This being the case, it is more easy to produce spots on their surface; and if their outside surface is so cool as to become solid, the glowing hydrogen from the interior when it did burst through would do so with more power than if the surrounding shell were liquid or gaseous.

There is, however, one star of which the variations may be due to an entirely different cause—namely, *Algol*. The extreme regularity with which the light of this object fades away and disappears suggests the possibility that a dark body may be revolving around it, and partially eclipsing it at every revolution. The law of variation of its light is so different from that of the light of other variable stars as to suggest a different cause. Most others are near their maximum for only a small part of their period, while *Algol* is at its maximum for nine tenths of it. Others are subject to nearly continuous changes, while the light of *Algol* remains constant during nine tenths of its period.

CHAPTER III.

MULTIPLE STARS.

§ 1. CHARACTER OF DOUBLE AND MULTIPLE STARS.

When we examine the heavens with telescopes, we find many cases in which two or more stars are extremely close together, so as to form a pair, a triplet, or a group. It is evident that there are two ways to account for this appearance.

1. We may suppose that the stars happen to lie nearly in the same straight line from us, but have no connection with each other. It is evident that in this case a pair of stars might appear double, although the one was hundreds or thousands of times farther off than the other. It is, moreover, impossiblo, from mere inspection, to determine which is the farther.

2. We may suppose that the stars are really as near together as they appear, and are to be considered as forming a connected pair or group.

A couple of stars in the first case are said to be *optically double*, and are not generally classed by astronomers as double stars.

Stars which are considered as really double are those which are so near together that we are justified in considering them as physically connected. Such stars are said to be *physically double*, and are generally designated as *double stars* simply.

Though it is impossible by mere inspection to decide to which class a pair of stars should be considered as belonging, yet the calculus of probabilities will enable us to decide in a rough way whether it is likely that two stars not physically connected should appear so very close together as most of the double stars do. This question was first considered by the Rev. JOHN MICHELL, F.R.S., of England, who in 1777 published a paper on the subject in the *Philosophical Transactions.* He showed that if the lucid stars were equally distributed over the celestial sphere, the chances were 80 to 1 against any two being within three minutes of each other, and that the chances were 500,000 to 1 against the six visible stars of the *Pleiades* being accidentally associated as we see them. When the millions of telescopic stars are considered, there is a greater probability of such accidental juxtaposition. But the probability of many such cases occurring is so extremely small that astronomers regard all the closest pairs as physically connected. It is now known that of the 600,000 stars of the first ten magnitudes, at least 10,000, or one out of every 60, has a companion within a distance of 30″ of arc. This proportion is many times greater than could possibly be the result of chance.

There are several cases of stars which appear double to the naked eye. Two of these we have already described—namely, *θ Tauri* and *ε Lyræ.* The latter is a most curious and interesting object, from the fact that each of the two stars which compose it is itself double. No more striking idea of the power of the telescope can be formed than by pointing a powerful instrument upon this object. It will then be seen that this minute pair of points, capable of being distinguished only by the most perfect eye, is really composed of two pairs of stars wide apart, with a group of smaller stars between and around them. The figure shows the appearance in a telescope of considerable power.

FIG. 12[illegible].—THE QUADRUPLE STAR *ε* LYRÆ.

Revolutions of Double Stars—Binary Systems.— The most interesting question suggested by double stars is that of their relative motion. It is evident that if these bodies are endowed with the property of mutual gravitation, they must be revolving around each other, as the earth and planets revolve around the sun, else they would be drawn together as a single star. With a view of detecting this revolution, astronomers measure the *position-angle*, and *distance* of these objects. The *distance* of the

FIG. 136.—MEASUREMENT OF POSITION-ANGLE.

components of the double star is simply the apparent angle which separates them, as seen by the observer. It is always expressed in seconds or fractions of a second of arc.

The *angle of position*, or "position-angle" as it is often called for brevity, is the angle which the line joining the two stars makes with the line drawn from the brightest star to the north pole. If the fainter star is directly north of the brighter one, this angle is zero; if east, it is 90°; if south, it is 180° ; if west, it is 270°. This is illustrated by the figure, which is supposed to represent the field of view of an inverting telescope pointed toward the south. The arrow shows the direction of the apparent diurnal motion. The telescope is supposed to be so pointed that the brighter star may be in the centre of the field. The numbers around the surrounding circle then show the angle of position, supposing the smaller star to be in the direction of the number.

The letters *s n*, *s f*, *n p*, and *n f* show the methods of dividing the four quadrants, *s* meaning south, *n* north, *f* following, and *p* preceding. The two latter words refer to the direction of the diurnal motion. Fig. 124 is an example of a pair of stars in which the position-angle is about 44°.

FIG. 124.—POSITION-ANGLE OF A DOUBLE STAR.

If, by measures of this sort extending through a series of years, the distance or position-angle of a pair of stars is found to change, it shows that one star is revolving around the other. Such a pair is called a *binary star* or *binary system*. The only distinction which we can make between binary systems and ordinary double stars is founded on the presence or absence of observed motion. It is probable that nearly all the double stars are really binary systems, but that many thousands of years are required to perform a revolution, so that the motion has not yet been detected.

The discovery of binary systems is one of great scientific interest, because from them we learn that the law of gravitation includes the stars as well as the solar system in its scope, and may therefore be regarded as a universal property of matter.

Colors of Double Stars.—There are a few noteworthy statistics in regard to the colors of the components of double stars which may be given. Among 596 of the brighter double stars, there are 375 pairs where each component has the same color and intensity; 101 pairs where the components have same color, but different intensity; 120 pairs of different colors. Among those of the same color, the vast majority were both white. Of the 476 stars of the same color, there were 295 pairs whose components were both white; 118 pairs whose components were both yellow or both red; 63 pairs whose components were both bluish. When the components are of different colors, the brighter generally appears to have a tinge of red or yellow; the other of blue or green.

These data indicate in part real physical laws. They also are partly due to the physiological fact that the fainter a star is, the more blue it will appear to the eye.

Measures of Double Stars.—The first systematic measures of the relative positions of the components of double stars were made by CHRISTIAN MAYER, Director of the Ducal Observatory of Mannheim, 1778, but it is to SIR WILLIAM HERSCHEL that we owe the basis of our knowledge of this branch of sidereal astronomy. In 1780 HERSCHEL measured the relative situation of more than 400 double stars, and after repeating his measures some score of years later, he found in about 50 of the pairs evidence of relative motion of the components. In this first survey he found 97 stars whose distance was under 4", 168 between 4" and 8", 114 between 8" and 16", and 132 between 16" and 32".

Since HERSCHEL's observations, the discoveries of Sir JOHN HERSCHEL, Sir JAMES SOUTH, DAWES, and many others in England, of W. STRUVE, OTTO STRUVE, MADLER, SECCHI, DEMBOWSKI, DUNER, in Europe, and of G. P. BOND, ALVAN CLARK, and S. W. BURNHAM, in the United States, have increased the number of known double stars to about 10,000.

Besides the double stars, there are also triple, quadruple, etc., stars. These are generically called *multiple stars*. The most remarkable multiple star is the *Trapezium*, in the centre of the nebula of *Orion*, commonly called θ *Orionis*, whose four stars are, without doubt, physically connected.

The next combination beyond a multiple star is a cluster of stars; and beginning with clusters of 1' in diameter, such objects may be found up to 30' or more in diameter, every intermediate size being represented. These we shall consider shortly.

§ 2. ORBITS OF BINARY STARS.

When it was established that many of the double stars were really revolving around each other, it became of great interest to determine the orbit and ascertain whether it was an ellipse, with the centre of gravity of the two objects in one of the foci; if so, it would be shown that gravitation among the stars followed the same law as in the solar system. As an illustration of how this may be done, we present the following measures of the position-angle and distance of the binary star ξ *Ursæ Majoris*, which was the first one of which the orbit was investigated. The following notation is used: p, the angle of position; s, the distance; A, the brighter star; B, the fainter one.

ξ URSÆ MAJORIS = Σ 1523.*

EPOCH.	p	s.	Observer.
	°	"	
1782·0	143·8	....	W. Herschel.
1802·1	97·5	....	"
1820·1	276·4	....	W. Struve.
1821·8	264·7	1·92	"
1831·3	201·1	1·90	J. Herschel.
1840·3	150·9	2·45	Dawes.
1851·6	132·6	2·99	Mädler.
1863·2	96·7	2·56	Dembowski.
1872·5	16·5	0·91	Dunér.

If these measures be plotted on a sheet of squared paper, the several positions of B will be found to lie in an ellipse. This ellipse is the projection of the real orbit on the plane perpendicular to the line of sight, or line joining the earth with the star A. It is a question of analysis to determine the true orbit from the times and from the values of p and s.

If the real orbit happened to lie in a plane perpendicular to the line of sight, the star A would lie in the focus of the ellipse. If this coincidence does not take place, then the plane of the true orbit is seen obliquely.

The first two of KEPLER's laws can be employed in determining such orbits, but the third law is inapplicable.

Masses of Binary Systems.—When the parallax or distance, the semi-major axis of the orbit, and the time of revolution of a binary system are known, we can determine the combined mass of the pair of stars in terms of the mass of the sun. Let us put:

a'', the mean distance of the two components as measured in seconds;
a, their mean distance from each other in astronomical units;
T, the time of revolution in years;
M, M_c, the masses of the two component stars;
P, their annual parallax;
D, their distance in astronomical units.

* Σ 1523 signifies that this star is No. 1523 of W. Struve's Dorpat Catalogue.

From the generalization of KEPLER's third law, given by the theory of gravitation, we have

$$M_0 + M = \frac{a^3}{T^2}.$$

From the formulæ explained in treating of parallax we have

$$D = 1 + \sin. P.$$

If a'' is the major axis in seconds, a being the same quantity in astronomical units, then

$$a = D \cdot \sin. a''.$$

From these two equations,

$$a = \frac{\sin. a''}{\sin. P} = \frac{a''}{P}.$$

because a'' and P are so small that the arcs may be taken for their sines.

Putting this value of a in the equation for $M + M_0$,

we have $$M + M_0 = \frac{a''^3}{+T^2 P^3}.$$

α Centauri and *p Ophiuchi* are two binary stars whose parallaxes have been determined ($0''\cdot 98$ and $0''\cdot 16$) from direct measures. For *α Centauri*

$$T = 77\cdot 0 \text{ years};\ a'' = 15''\cdot 5;\ P = 0''\cdot 98;$$

for *p Ophiuchi*,

$$T = 94\cdot 4 \text{ years};\ a'' = 4''\cdot 70;\ P = 0''\cdot 16.$$

If we substitute in the last equation these values for T, P, and a'', we have

$$M_0 + M = 0\cdot 67 \text{ for } \alpha \textit{ Centauri},$$
$$M_0 + M = 2\cdot 84 \text{ for } p \textit{ Ophiuchi}.$$

The last number is quite uncertain, owing to the difficulty of measuring so small a parallax. We can only conclude that the mass of these two systems is not many times greater or less than the mass of our sun. From the agreement in these two cases, it is probable that in other systems, if the mass could be determined, it would not be greatly different from the mass of our sun. We may on this supposition, which amounts to supposing $M_0 + M = 1$, apply the formula

$$P = a'' \div T^{\frac{2}{3}}$$

to other binaries, and deduce a value for P in each case which is called the hypothetical parallax (Gyldén), and which is probably not far from the truth.

There are, beside binary systems, multiple ones as *ζ Cancri*, where the distance of A and B is $0''\cdot 8$; and from the middle point between A and B to C is $5''\cdot 5$. The period of revolution of $\frac{A + B}{2}$ about C is supposed to be about 730 years. If in the last formula we put $T = 730$ years and $a'' = 5''\cdot 5$, we have the hypothetical parallax

$$p = 0''\cdot 062.$$

Following are given the elements of several of the more important binary stars. Eight of these have moved through an entire revolution—360°—since the first observation, and about 150 are known which have certainly moved through an arc of over 10° since they were first observed.

In the tables the semi-major axis, or mean distance, must be given in seconds, since we have usually no data by which its value in linear measures of any kind can be fixed.

Periods of revolution exceeding 120 years must be regarded as quite uncertain.

ELEMENTS OF BINARY STARS.

STAR'S NAME.	Period (Years.)	Time of Periastron.	Semi-Axis Major.	Eccentricity.	Calculator.
42 Comæ Ber.	25·7	1869·9	0″·65	0·48	Dubiago.
ζ Herculis	34·6	1864·9	1·36	0·41	Flammarion.
Σ 3121 *	37·03	1842·8	[0·71]	0·26	Doberck.
η Coronæ Bor.	40·2	1849·9	0·99	0·29	Flammarion.
ξ Libræ	95.90	1859·6	1·26	0·08	Doberck.
γ Coronæ Aus.	55·5	1882·7	2·40	0·69	Schiaparelli.
ξ Ursæ Maj.	60·6	1875.6	2·58	0·38	Hind.
	60·6	1875·5	2·54	0·37	Flammarion.
ζ Cancri	62·4	1869·8	0·90	0·00	O. Struve.
	60·5	1869.9	0·91	0·37	Flammarion.
α Centauri	85·0	1874·9	21·80	0·67	Hind.
70 Ophiuchi	92·8	1807·9	4·88	0·39	Flammarion.
γ Coronæ Bor.	95·5	1843·7	0·70	0·35	Doberck.
3062 Σ	104·4	1834·9	1·27	0·46	Doberck.
ω Leonis	114·6	1841·6	0·85	0·55	Doberck.
λ Ophiuchi	233·9	1808·9	1·19	0·49	Doberck.
ρ Eridani	117·5	1817·5	3·82	0·38	Doberck.
1768 Σ	124·5	1863·0	....	0·66	Doberck.
ξ Boötis	127·4	1770·7	4·86	0·71	Doberck.
γ Virginis	175·0	1836·5	3·89	0·87	Flammarion.
τ Ophiuchi	217·9	1821·9	1·40	0·61	Doberck.
η Cassiopeæ	222·4	1909·2	9·88	0·57	Doberck.
44 Boötis	261·1	1783·0	3·09	0·71	Doberck.
1938 Σ, μ^2 Boötis	280·3	1863·5	1·47	0·60	Doberck.
36 Andromedæ	349·1	1798·8	1·54	0·65	Doberck.
γ Leonis	402·6	1741·1	2·00	0·74	Doberck.
δ Cygni	415·1	1904·1	2·31	0·28	Behrmann.
61 Cygni	452·0	......	15·4	....	....
σ Coronæ Bor.	845·9	1826·9	5·89	0·75	Doberck.
α Geminorum	1001·2	1749·8	7·43	0·88	Doberck.
ζ Aquarii	1578·3	1924·2	7·64	0·65	Doberck.

* 3121 Σ signifies No. 3121 of W. STRUVE'S Dorpat Catalogue.

The first computation of the orbit of a binary star was made by SAVARY (Astronomer at the Paris Observatory) about 1826, and his results were the first which demonstrated that the laws of gravitation, which we knew to be operative over the extent of the solar system, and even over the vast space covered by the orbit of HALLEY's comet, extended even further, to the fixed stars. It might have been before 1825 a hazardous extension of our views to suppose even the nearest fixed stars to be subject to the laws of NEWTON; but as many of the known binaries have no measurable parallax, it is by no means an unsafe conclusion that every fixed star which our best telescopes will show is subjected to the same laws as those which govern the fall of bodies upon the earth.

CHAPTER IV.

NEBULÆ AND CLUSTERS.

§ 1. DISCOVERY OF NEBULÆ.

In the star-catalogues of Ptolemy, Hevelius and the earlier writers, there was included a class of nebulous or cloudy stars, which were in reality star-clusters. They appeared to the naked eye as masses of soft diffused light of greater or less extent. In this respect, they were quite analogous to the Milky Way. When Galileo first directed his telescope to the sky, the nebulous appearance of these spots vanished, and they were seen to consist of clusters of stars.

As the telescope was improved, great numbers of such patches of light were found, some of which could be resolved into stars, while others could not. The latter were called *nebulæ* and the former *star-clusters*.

About 1650, Huyghens described the great nebula of *Orion*, one of the most remarkable and brilliant of these objects. During the last century, Messier, of Paris, made a list of 103 northern nebulæ, and Lacaille noted a few of those of the southern sky. The careful sweeps of the heavens by Sir William Herschel with his great telescopes first gave proof of the enormous number of these masses. In 1786, he published a catalogue of one thousand new nebulæ and clusters. This was followed in 1789 by a catalogue of a second thousand, and in 1802 by a third catalogue of five hundred new objects of this class. A similar series of sweeps, carried on by Sir JOHN HERSCHEL in both hemispheres, added about two thousand more nebulæ. The general catalogue of nebulæ and clusters of stars of the latter astronomer, published in 1864, contains 5079 nebulæ: 6251 are known in 1879. Over two thirds of these were first discovered by the HERSCHELS.

The mere enumeration of over 4000 nebulæ is, however, but a small part of the labor done by these two distinguished astronomers. The son has left a great number of studies, drawings, and measures of nebulæ, and the memoirs of the father on the Construction of the Heavens owe their suggestiveness and much of their value to his long-continued observations on this class of objects, which gave him the clue to his theories.

§ 2. CLASSIFICATION OF NEBULÆ AND CLUSTERS.

In studying these objects, the first question we meet is this: Are all these bodies clusters of stars which look diffused only because they are so distant that our telescopes cannot distinguish them separately? or are some of them in reality what they seem to be—namely, diffused masses of matter?

In his early memoirs of 1784 and 1785, Sir WILLIAM HERSCHEL took the first view. He considered the Milky Way as nothing but a congeries of stars, and all nebulæ naturally seemed to him to be but stellar clusters, so distant as to cause the individual stars to disappear in a general milkiness or nebulosity.

In 1791, however, his views underwent a change. He had discovered a *nebulous star* (properly so called), or a star which was undoubtedly similar to the surrounding stars, and which was encompassed by a halo of nebulous light.*

* This was the 69th nebula of his *fourth class* of planetary nebulæ. (H. iv. 69.)

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He says: "Nebulæ can be selected so that an insensible gradation shall take place from a coarse cluster like the *Pleiades* down to a milky nebulosity like that in *Orion*, every intermediate step being represented. This tends to confirm the hypothesis that all are composed of stars more or less remote.

"A comparison of the two *extremes* of the series, as a coarse cluster and a nebulous star, indicates, however, that *the nebulosity about the star is not of a starry nature.*

"Considering H, iv. 69, as a typical nebulous star, and supposing the nucleus and chevelure to be connected, we may, first, suppose the whole to be of stars, in which case either the nucleus is enormously larger than other stars of its stellar magnitude, or the envelope is composed of stars indefinitely small; or, second, we must admit that the star is *involved in a shining fluid of a nature totally unknown to us.*

"The shining fluid might exist independently of stars. The light of this fluid is no kind of reflection from the star in the centre. If this matter is self-luminous, it seems more fit to produce a star by its condensation than to depend on the star for its existence.

"Both diffused nebulosities and planetary nebulæ are better accounted for by the hypothesis of a shining fluid than by supposing them to be distant stars."

This was the first exact statement of the idea that, beside stars and star-clusters, we have in the universe a totally distinct series of objects, probably much more simple in their constitution. The observations of HUGGINS and SECCHI on the spectra of these bodies have, as we shall see, entirely confirmed the conclusions of HERSCHEL.

Nebulæ and clusters were divided by HERSCHEL into classes. Of his names, only a few are now in general use. He applied the name *planetary nebulæ* to certain circular or elliptic nebulæ which in his telescope presented disks like the planets. *Spiral nebulæ* are those whose convolutions have a spiral shape. This class is quite numerous.

The different kinds of nebulæ and clusters will be better understood from the cuts and descriptions which follow than by formal definitions. It must be remembered that there is an almost infinite variety of such shapes.

The figure by Sir JOHN HERSCHEL on the next page gives a good idea of a spiral or ring nebula. It has a central nucleus and a small and bright companion nebula near it. In a larger telescope than HERSCHEL's its aspect is even more complicated. See also Fig. 128.

The *Omega* or *horseshoe* nebula, so called from the resemblance of the brightest end of it to a Greek Ω, or to a horse's iron shoe, is one of the most complex and remarkable of the nebulæ visible in the northern hemisphere. It is particularly worthy of note, as there is some reason to believe that it has a proper motion. Certain it is that the bright star which in the figure is at the left-hand upper corner of one of the squares, and on the left-hand (west) edge of the streak of nebulosity, was in the older drawings placed on the other side of this streak, or within the dark bay, thus making it at least probable that either the star or the nebula has moved.

FIG. 126.—SPIRAL NEBULA.

The *trifid* nebula, so called on account of its three branches which meet near a central dark space, is a striking object, and was suspected by Sir JOHN HERSCHEL to have a proper motion. Later observations seem to confirm this, and in particular the three bright stars on the left-hand edge of the right-hand (east) mass are now more deeply immersed in the nebula than they were observed to be by HERSCHEL (1833) and MASON, of Yale College (1837). In 1784, Sir WILLIAM HERSCHEL described them as "in the middle of the [dark] triangle." This description does not apply to their present situation. (Fig. 127).

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FIG. 126.—THE OMEGA OR HORSESHOE NEBULA.

§ 3. STAR CLUSTERS.

The most noted of all the clusters is the *Pleiades*, which have already been briefly described in connection with the constellation *Taurus*. The average naked eye can easily distinguish six stars within it, but under favorable conditions ten, eleven, twelve, or

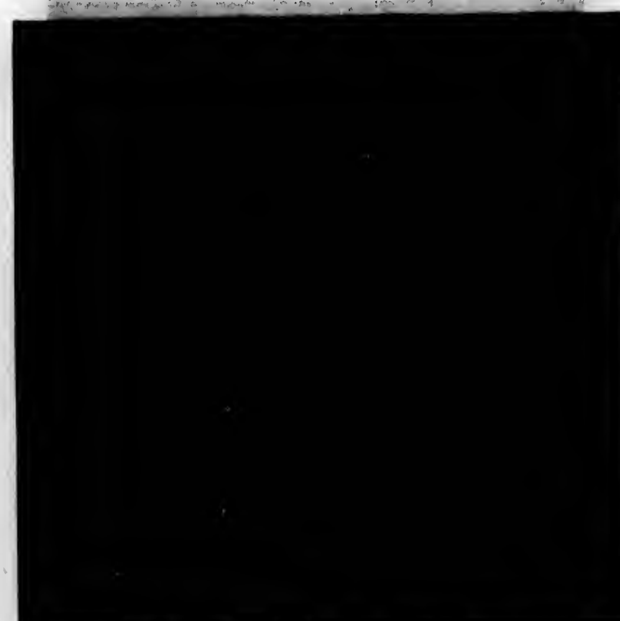

FIG. 127.—THE TRIFID NEBULA.

more stars can be counted. With the telescope, over a hundred stars are seen. A view of these is given in the map accompanying the description of the *Pleiades*, Fig. 118, p. 425. This group contains TEMPEL's variable nebula, so called because it has been supposed to be subject to variations of light. This is probably not a variable nebula.

The clusters represented in Figs. 129 and 130 are good examples of their classes. The first is globular and contains several thousand small stars. The central regions are densely packed with stars, and from those radiate curved hairy-looking branches of a spiral form. The second is a cluster of about 200 stars, of magnitudes varying from the ninth to the thirteenth and fourteenth, in which the brighter stars are scattered in a somewhat unusual manner

FIG. 128.—THE RING NEBULA IN LYRA.

over the telescopic field. This cluster is an excellent example of the "compressed" form so frequently exhibited. In clusters of this class the spectroscope shows that each of the individual stars is a true sun, shining by its native brightness. If we admit that a cluster is real—that is, that we have to do with a collection of stars physically connected—the globular clusters become important. It is a fact of observation that in general the stars composing such clusters are about of equal magnitude, and are more condensed at the centre than at the edges. They are probably subject to central powers or forces. This was seen by Sir WILLIAM HERSCHEL in 1789. He says :

"Not only were *round* nebulæ and clusters formed by central powers, but likewise every cluster of stars or nebula that shows a gradual condensation or increasing brightness toward a centre. This theory of central power is fully established on grounds of observation which cannot be overturned.

"Clusters can be found of 10 diameter with a certain degree of compression and stars of a certain magnitude, and smaller clusters of 4', 3' or 2' in diameter, with smaller stars and greater compression, and so on through resolvable nebulæ by imperceptible steps, to the smallest and faintest [and most distant] nebulæ. Other clusters

FIG. 129.—GLOBULAR CLUSTER. FIG. 130.—COMPRESSED CLUSTER.

there are, which lead to the belief that either they are more compressed or are composed of larger stars. Spherical clusters are probably not more different in size among themselves than different individuals of plants of the same species. As it has been shown that the spherical figure of a cluster of stars is owing to central powers, it follows that those clusters which, *cæteris paribus*, are the most complete in this figure must have been the longest exposed to the action of these causes.

"The maturity of a sidereal system may thus be judged from the disposition of the component parts.

"Though we cannot see any individual nebula pass through all its stages of life, we can select particular ones in each peculiar stage," and thus obtain a single view of their entire course of development.

§ 4. SPECTRA OF NEBULÆ AND CLUSTERS.

In 1864, five years after the invention of the spectroscope, Dr. Huggins, of London, commenced the examination of the spectra of the nebulæ, and was led to the discovery that while the spectra of stars were invariably continuous and crossed with dark lines similar to those of the solar spectrum, those of many nebulæ were *discontinuous*, showing these bodies to be composed of glowing gas. The figure shows the spectrum of one of the most famous planetary nebulæ. (H. iv. 37.) The gaseous nebulæ include nearly all the planetary nebulæ, and very frequently have stellar-like condensations in the centre.

Singular enough, the most milky looking of any of the nebulæ (that in *Andromeda*) gives a continuous spectrum, while the nebula of *Orion*, which fairly glistens with small stars, has a discontinuous

Fig. 131.—Spectrum of a Planetary Nebula.

spectrum, showing it to be a true gas. Most of these stars are too faint to be separately examined with the spectroscope, so that we cannot say whether they have the same spectrum as the nebulæ.

The spectrum of most clusters is continuous, indicating that the individual stars are truly stellar in their nature. In a few cases, however, clusters are composed of a mixture of nebulosity (usually near their centre) and of stars, and the spectrum in such cases is compound in its nature, so as to indicate radiation both by gaseous and solid matter.

§ 5. DISTRIBUTION OF NEBULÆ AND CLUSTERS ON THE SURFACE OF THE CELESTIAL SPHERE.

The following map (Fig. 132) by Mr. R. A. Proctor, gives at a glance the distribution of the nebulæ on the celestial sphere with reference to the Milky Way, whose boundaries only are indicated.

Isographic Projection.—*R. A. Proctor.*

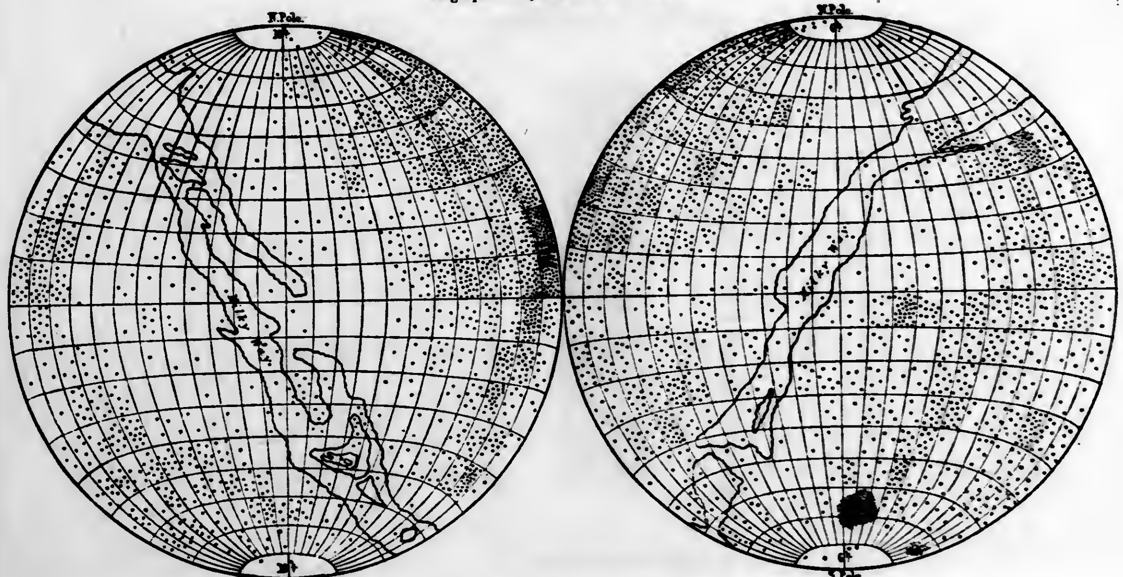

FIG. 132.—DISTRIBUTION OF THE NEBULÆ. THE ZONE OF FEW NEBULÆ.

FIG. 192.—DISTRIBUTION OF THE NEBULÆ. THE ZONE OF FEW NEBULÆ.

The position of each nebula is marked by a dot; where the dots are thickest there is a region rich in nebulæ. A casual examination shows that such rich regions are distant from the Galaxy, and it would appear that it is a general law that the nebulæ are distributed in greatest number around the two poles of the galactic circle, and that in a general way their number at any point of the sphere increases with their distance from this circle. This was noticed by the elder HERSCHEL, who constructed a map similar to the one given. It is precisely the reverse of the law of apparent distribution of the true star-clusters, which in general lie in or near the Milky Way.

CHAPTER V.

SPECTRA OF FIXED STARS.

1. CHARACTERS OF STELLAR SPECTRA.

Soon after the discovery of the spectroscope, Dr. Huggins and Professor W. A. Miller applied this instrument to the examination of stellar spectra, which were found to be, in the main, similar to the solar spectrum—*i.e.*, composed of a continuous band of the prismatic colors, across which dark lines or bands were laid, the latter being fixed in position. These results showed the fixed stars to resemble our own sun in general constitution, and to be composed of an incandescent nucleus surrounded by a gaseous and absorptive atmosphere of lower temperature. This atmosphere around many stars is different in constitution from that of the sun, as is shown by the different position and intensity of the various black lines and bands.

The various stellar spectra have been classified by Secchi into four *types*, distinguished from one another by marked differences in the position, character, and number of the dark lines.

Type I is composed of the white stars, of which *Sirius* and *Vega* are examples (the upper spectrum in the plate Fig. 133). The spectrum of these stars is continuous, and is crossed by four dark lines, due to the presence of large quantities of hydrogen in the envelope. Sodium and magnesium lines are also seen, and others yet fainter.

Type II is composed mainly of the yellow stars, like our own sun, *Arcturus*, *Capella*, *Aldebaran*, and *Pollux*. The spectrum of the sun is shown in the second place in the plate. The vast majority of the stars visible to the naked eye belong to this class.

Type III (see the third and fourth spectra in the plate) is composed of the brighter reddish stars like α *Orionis*, *Antares*, α *Herculis*, etc. These spectra are much contracted toward the violet end, and are crossed by eight or more dark bands, these bands being themselves resolvable into separate lines.

These three types comprise nearly all the lucid stars, and it is not a little remarkable that the essential differences between the three classes were recognized by Sir William Herschel as early as 1798, and published in 1814. Of course his observations were made without a slit to his spectroscopic apparatus.

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FIG. 183.—TYPES OF STELLAR SPECTRA. AFTER SECCHI.

Type IV comprises the red stars, which are mostly telescopic. The characteristic spectrum is shown in the last figure of the plate. It is curiously banded with three bright spaces separated by darker ones.

It is probable that the hotter a star is the more simple a spectrum it has; for the brightest, and therefore probably the hottest stars, such as *Sirius*, give spectra showing only very thick hydrogen lines and a few very thin metallic lines, while the cooler stars, such as our sun, are shown by their spectra to contain a much larger number of metallic elements than stars of the type of *Sirius*, but no non-metallic elements (oxygen possibly excepted). The coolest stars give band-spectra characteristic of compounds of metallic with non-metallic elements, and of the non-metallic elements uncombined.

§ 2. MOTION OF STARS IN THE LINE OF SIGHT.

Spectroscopic observations of stars not only give information in regard to their chemical and physical constitution, but have been applied so as to determine approximately the velocity in kilometres per second with which the stars are approaching to or receding from the earth along the line joining earth and star. The theory of such a determination is briefly as follows:

In the solar spectrum we find a group of dark lines, as a, b, c, which always maintain their relative position. From laboratory experiments, we can show that the three bright lines of incandescent hydrogen (for example) have always the same relative position as the solar dark lines a, b, c. From this it is inferred that the solar dark lines are due to the presence of hydrogen in its absorptive atmosphere.

Now, suppose that in a stellar spectrum we find three dark lines a', b', c', whose relative position is exactly the same as that of the solar lines a, b, c. Not only is their relative position the same, but the characters of the lines themselves, so far as the fainter spectrum of the star will allow us to determine them, are also similar—that is, a' and a, b' and b, c' and c are alike as to thickness, blackness, nebulosity of edges, etc., etc. From this it is inferred that the star really contains in its atmosphere the substance whose existence has been shown in the sun.

If we contrive an apparatus by which the stellar spectrum is seen in the lower half (say) of the eye-piece of the spectroscope, while the spectrum of hydrogen is seen just above it, we find in some cases this remarkable phenomenon. The three dark stellar lines, a', b', c', instead of being exactly coincident with the three hydrogen lines a, b, c, are seen to be all thrown to one side or the other by a like amount—that is, the whole group a', b', c', while preserving its relative distances the same as those of the comparison group a, b, c, is shifted toward either the violet or red end of the spectrum by a small yet measurable amount. Repeated experiments by different instruments and observers show always a shifting in the same direction and of like amount. The figure shows the shifting of the *F* line in the spectrum of *Sirius*, compared with one fixed line of hydrogen.

FIG. 134.—F-LINE IN SPECTRUM OF SIRIUS.

This displacement of the spectral lines is now accounted for by a motion of the star toward or from the earth. It is shown in Physics that if the source of the light which gives the spectrum *a*′, *b*′, *c*′ is moving away from the earth, this group will be shifted toward the red end of the spectrum; if toward the earth, then the whole group will be shifted toward the blue end. The amount of this shifting is a function of the velocity of recession or approach, and this velocity in miles per second can be calculated from the measured displacement. This has been done for many stars by Dr. HUGGINS, Dr. VOGEL, and Mr. CHRISTIE. Their results agree well, when the difficult nature of the research is considered. The rates of motion vary from insensible amounts to 100 kilometres per second; and in some cases agree remarkably with the velocities computed from the proper motions and probable parallaxes.

CHAPTER VI.

MOTIONS AND DISTANCES OF THE STARS.

§ 1. PROPER MOTIONS.

WE have already stated that, to the unaided vision, the fixed stars appear to preserve the same relative position in the heavens through many centuries, so that if the ancient astronomers once more saw them, they could hardly detect the slightest change in their arrangement. But the refined methods of modern astronomy, in which the power of the telescope is applied to celestial measurement, have shown that there are slow changes in the positions of the brighter stars, consisting in a motion forward in a straight line and with uniform velocity. These motions are, for the most part, so slow that it would require thousands of years for the change of position to be perceptible to the unaided eye. They are called *proper motions.*

As a general rule, the fainter the stars the smaller the proper motions. For the most part, the proper motions of the telescopic stars are so minute that they have not been detected except in a very few cases. This arises partly from the actual slowness of the motion, and partly from the fact that the positions of these stars have not generally been well determined. It will be readily seen that, in order to detect the proper motion of a star, its position must be determined at periods separated by considerable intervals of time. Since the exact determinations of star positions have only been made since the year 1750, it follows that no proper motion can be detected unless it is large enough to become perceptible at the end of a century and a quarter. With very few exceptions, no accurate determination of the positions of telescopic stars was made until about the beginning of the present century. Consequently, we cannot yet pronounce upon the proper motions of these stars, and can only say that, in general, they are too small to be detected by the observations hitherto made.

To this rule, that the smaller stars have no sensible proper motions, there are a few very notable exceptions. The star *Groombridge* 1830, is remarkable for having the greatest proper motion of any in the heavens, amounting to about 7" in a year. It is only of the seventh magnitude. Next in the order of proper motion comes the double star 61 *Cygni*, which is about of the fifth magnitude. There are in all seven small stars, all of which have a larger proper motion than any of the first magnitude. But leaving out these exceptional cases, the remaining stars show, on an average, a diminution of proper motion with brightness. In general, the proper motions even of the brightest stars are only a fraction of a second in a year, so that thousands of years would be required for them to change their place in any striking degree, and hundreds of thousands to make a complete revolution around the heavens.

§ 2. PROPER MOTION OF THE SUN.

A very interesting result of the proper motions of the stars is that our sun, considered as a star, has a considerable proper motion of its own. By observations on a star, we really determine, not the proper motion of the star itself, but the relative proper motion of the observer and the star—that is, the difference of their motions. Since the earth with the observer on it is carried along with the sun in space, his proper motion is the same as that of the sun, so that what observation gives us is the difference between the proper motion of the star and that of the sun. There is no way to determine absolutely how much of the apparent proper motion is due to the real motion of the star and how much to the real motion of the sun. If, however, we find that, on the average, there is a large preponderance of proper motions in one direction, we may conclude that there is a real motion of the sun in an opposite direction. The reason of this is that it is more likely that the average of a great mass of stars is at rest than that the sun, which is only a single one, should be at rest. Now, observation shows that this is really the case, and that the great mass of stars appear to be moving from the direction of the constellation *Hercules* and toward that of the constellation *Argus.** A number of astronomers have investigated this motion with a view of determining the exact point in the heavens toward which the sun is moving. Their results are shown in the following table:

	Right Ascension.		Declination.		
Argelander	257°	49′	28°	50′	N.
O. Struve	261°	22′	37°	36′	N.
Lundahl	252°	24′	14°	26′	N.
Galloway	260°	1′	34°	23′	N.
Mädler	261°	38′	39°	54′	N.
Airy and Dunkin	262°	29′	28°	58′	N.

It will be perceived that there is some discordance arising from the diverse characters of the motions to be investigated. Yet, if we lay these different points down on a map of the stars, we shall find that they all fall in the constellation *Hercules.* The amount of the motion is such that if the sun were viewed at right angles to the direction of motion from an average star of the first magnitude, it would appear to move about one third of a second per year.

§ 3. DISTANCES OF THE FIXED STARS.

The problem of the distance of the stars has always been one of the greatest interest on account of its involving the question of the extent of the visible universe. The ancient astronomers supposed all the fixed stars to be situated at a short distance outside of the orbit of the planet *Saturn,* then the outermost known planet. The idea was prevalent that Nature would not waste space by leaving a great region beyond *Saturn* entirely empty.

When Copernicus announced the theory that the sun was at rest and the earth in motion around it, the problem of the distance of the stars acquired a new interest.

* This was discovered by Sir William Herschel in 1783.

It was evident that if the earth described an annual orbit, then the stars would appear in the course of a year to oscillate back and forth in corresponding orbits, unless they were so immensely distant that these oscillations were too small to be seen. Now, the apparent oscillation of *Saturn* produced in this way was described in Part I., and shown to amount to some 6° on each side of the mean position. These oscillations were, in fact, those which the ancients represented by the motion of the planet around a small epicycle. But no such oscillation had ever been detected in a fixed star. This fact seemed to present an almost insuperable difficulty in the reception of the Copernican system. This was probably the reason why Tycho Brahe was led to reject the system. Very naturally, therefore, as the instruments of observation were from time to time improved, this apparent annual oscillation of the stars was ardently sought for. When, about the year 1704, Roemer thought he had detected it, he published his observations in a dissertation entitled "*Copernicus Triumphans.*" A similar attempt, made by Hooke of England, was entitled "*An Attempt to Prove the Motion of the Earth.*"

This problem is identical with that of the annual parallax of the fixed stars, which has been already described in the concluding section of our opening chapter. This parallax of a heavenly body is the angle which the mean distance of the earth from the sun subtends when seen from the body. The distance of the body from the sun is inversely as the parallax (nearly). Thus the mean distance of *Saturn* being 9·5, its annual parallax exceeds 6°, while that of *Neptune*, which is three times as far, is about 2°. It was very evident, without telescopic observation, that the stars could not have a parallax of one half a degree. They must therefore be at least twelve times as far as *Saturn* if the Copernican system were true.

When the telescope was applied to measurement, a continually increasing accuracy began to be gained by the improvement of the instruments. Yet for several generations the parallax of the fixed stars eluded measurement. Very often indeed did observers think they had detected a parallax in some of the brighter stars, but their successors, on repeating their measures with better instruments, and investigating their methods anew, found their conclusions erroneous. Early in the present century it became certain that even the brighter stars had not, in general, a parallax as great as 1″, and thus it became certain that they must lie at a greater distance than 200,000 times that which separates the earth from the sun.

Success in actually measuring the parallax of the stars was at length obtained almost simultaneously by two astronomers, BESSEL of Königsberg, and STRUVE of Dorpat. BESSEL selected for his star to be observed 61 *Cygni*, and commenced his observations on it in August, 1837. The result of two or three years of observation was that this star had a parallax of 0″·35, or about one third of a second. This would make its distance from the sun nearly 600,000 astronomical units. The reality of this parallax has been well established by subsequent investigators, only it has been shown to be a little larger, and therefore the star a little nearer than BESSEL supposed. The most probable parallax is now found to be 0″·51, corresponding to a distance of 400,000 radii of the earth's orbit.

The star selected by STRUVE for the measure of parallax was the bright one, *α Lyræ*. His observations were made between November, 1835, and August, 1838. He first deduced a parallax of 0″·25. Subsequent observers have reduced this parallax to 0″·20, corresponding to a distance of about 1,000,000 astronomical units.

Shortly after this, it was found by HENDERSON, of England, Astronomer Royal for the Cape of Good Hope, that the star *α Centauri* had a still larger parallax of about 1″. This is the largest parallax now known in the case of any fixed star, so that *α Centauri* is, beyond all reasonable doubt, the nearest fixed star. Yet its distance is more than 200,000 astronomical units, or thirty millions of millions of kilometres. Light, which passes from the sun to the earth in 8 minutes, would require 3¼ years to reach us from *α Centauri*.

Two methods of determining parallax have been applied in astronomy. The parallax found by one of these methods is known as *absolute*, that by the other as *relative parallax*. In determining the absolute parallax, the observer finds the polar distance of the star as often as possible through a period of one or more years with a meridian circle, and then, by a discussion of all his observations, concludes what is the magnitude of the oscillation due to parallax. The difficulty in applying this method is that the refraction of the air and the state of the instrument are subject to changes arising from varying temperature, so that the observations are always uncertain by an amount which is important in such delicate work.

In determining the *relative parallax*, the astronomer selects two stars in the same field of view of his telescope, one of which is many times more distant than the other. It is possible to judge with a high degree of probability which star is the more distant, from the magnitudes and proper motions of the two objects. It is assumed that a star which is either very bright or has a large proper motion is many times nearer to us than the extremely faint stars which may be nearly always seen around it. The effect of parallax will then be to change the apparent position of the bright star among the small stars around it in the course of a year. This change admits of being measured with great precision by the micrometer of the equatorial, and thus the relative parallax may be determined.

It is true that this relative parallax is really not the absolute parallax of either body, but the difference of their parallaxes. So we must necessarily suppose that the parallax of the smaller and more distant object is zero. It is by this method of relative parallax that the great majority of determinations have been made.

The distances of the stars are sometimes expressed by the time required for light to pass from them to our system. The velocity of light is, it will be remembered, about 300,000 kilometres per second, or such as to pass from the sun to the earth in 8 minutes 18 seconds.

The time required for light to reach the earth from some of the stars, of which the parallax has been measured, is as follows:

Star.	Years.	Star.	Years.
α *Centauri*	3·5	70 *Ophiuchi*	19·1
61 *Cygni*	6·7	ι *Ursæ Majoris*	24·8
21,185 Lalande	6·3	*Arcturus*	25·4
β *Centauri*	6·9	γ *Draconis*	35·1
μ *Cassiopeia*	9·4	1830 Groombridge	35·9
34 Groombridge	10·5	*Polaris*	42·4
21,258 Lalande	11·9	3077 Bradley	46·1
17,415 Oeltzen	13·1	85 *Pegasi*	64·5
Sirius	16·7	α *Aurigæ*	70·1
α *Lyræ*	17·9	σ *Draconis*	129·1

CHAPTER VII.

CONSTRUCTION OF THE HEAVENS.

THE visible universe, as revealed to us by the telescope, is a collection of many millions of stars and of several thousand nebulæ. It is sometimes called the stellar or sidereal system, and sometimes, as already remarked, the stellar universe. The most far-reaching question with which astronomy has to deal is that of the form and magnitude of this system, and the arrangement of the stars which compose it.

It was once supposed that the stars were arranged on the same general plan as the bodies of the solar system, being divided up into great numbers of groups or clusters, while all the stars of each group revolved in regular orbits round the centre of the group. All the groups were supposed to revolve around some great common centre, which was therefore the centre of the visible universe.

But there is no proof that this view is correct. The only astronomer of the present century who held any such doctrine was MAEDLER. He thought that the centre of motion of all the stars was in the *Pleiades*, but no other astronomer shared his views. We have already seen that a great many stars are collected into clusters, but there is no evidence that the stars of these clusters revolve in regular orbits, or that the clusters themselves have any regular motion around a common centre. Besides, the large majority of stars visible with the telescope do not appear to be grouped into clusters at all.

The first astronomer to make a careful study of the arrangement of the stars with a view to learn the structure of the heavens was Sir WILLIAM HERSCHEL. He published in the *Philosophical Transactions* several memoirs on the construction of the heavens and the arrangement of the stars, which have become justly celebrated. We shall therefore begin with an account of HERSCHEL's methods and results.

HERSCHEL's method of study was founded on a mode of observation which he called *star-gauging*. It consisted in pointing a powerful telescope toward various parts of the heavens and ascertaining by actual count how thick the stars were in each region. His 20-foot reflector was provided with such an eye-piece that, in looking into it, he would see a portion of the heavens about 15′ in diameter. A circle of this size on the celestial sphere has about one quarter the apparent surface of the sun, or of the full moon. On pointing the telescope in any direction, a greater or less number of stars were nearly always visible. These were counted, and the direction in which the telescope pointed was noted. Gauges of this kind were made in all parts of the sky at which he could point his instrument, and the results were tabulated in the order of right ascension.

The following is an extract from the gauges, and gives the average number of stars in each field at the points noted in right ascension and north polar distance:

R. A.		N. P. D. 92° to 94° No. of Stars.	R. A.		N. P. D. 78° to 80° No. of Stars.
h.	m.		h.	m.	
15	10	9·4	11	6	3·1
15	22	10·6	12	31	3·4
15	47	10·6	12	44	4·6
16	8	12·1	12	49	3·9
16	25	13·6	13	5	3·3
16	37	18·6	14	30	3·6

In this small table, it is plain that a different law of clustering or of distribution obtains in the two regions. Such differences are still more marked if we compare the extreme cases found by HERSCHEL, as R. A. = 19^h 41^m, N. P. D. = 74° 33′, number of stars per field; 588, and R. A. = 16^h 10^m, N. P. D., 113° 4′, number of stars = 1·1.

The number of these stars in certain portions is very great. For example, in the Milky Way, near *Orion*, six fields of view promiscuously taken gave 110, 60, 70, 90, 70, and 74 stars each, or a mean of 79 stars per field. The most vacant space in this neighborhood gave 63 stars. So that as HERSCHEL'S sweeps were two degrees wide in declination, in one hour (15°) there would pass through the field of his telescope 40,000 or more stars. In some of the sweeps this number was as great as 116,000 stars in a quarter of an hour.

On applying this telescope to the Milky Way, HERSCHEL supposed at the time that it completely resolved the whole whitish appearance into small stars. This conclusion he subsequently modified. He says:

"It is very probable that the great stratum called the Milky Way is that in which the sun is placed, though perhaps not in the very centre of its thickness.

"We gather this from the appearance of the Galaxy, which seems to encompass the whole heavens, as it certainly must do if the sun is within it. For, suppose a number of stars arranged between two parallel planes, indefinitely extended every way, but at a given considerable distance from each other, and calling this a sidereal stratum, an eye placed somewhere within it will see all the stars in the direction of the planes of the stratum projected into a great circle, which will appear lucid on account of the accumulation of the stars, while the rest of the heavens, at the sides, will only seem to be scattered over with constellations, more or less crowded, according to the distance of the planes, or number of stars contained in the thickness or sides of the stratum."

Thus in HERSCHEL'S figure an eye at *S* within the stratum *ab* will see the stars in the direction of its length *ab*, or height *cd*, with all those in the intermediate situations, projected into the lucid circle *A C B D*, while those in the sides *mv*, *nw*, will be seen scattered over the remaining part of the heavens *M V N W*.

"If the eye were placed somewhere without the stratum, at no very great distance, the appearance of the stars within it would assume the form of one of the smaller circles of the sphere, which

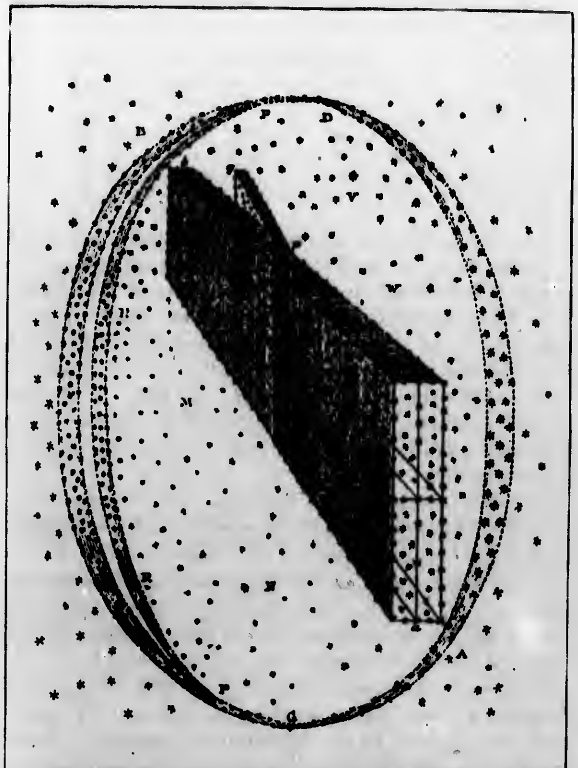

FIG. 135.—HERSCHEL'S THEORY OF THE STELLAR SYSTEM.

would be more or less contracted according to the distance of the eye; and if this distance were exceedingly increased, the whole stratum might at last be drawn together into a lucid spot of any shape, according to the length, breadth, and height of the stratum.

"Suppose that a smaller stratum $p\ q$ should branch out from the former in a certain direction, and that it also is contained between two parallel planes, so that the eye is contained within the great stratum somewhere before the separation, and not far from the place where the strata are still united. Then this second stratum will not be projected into a bright circle like the former, but it will be seen as a lucid branch proceeding from the first, and returning into it again at a distance less than a semicircle.

"In the figure the stars in the small stratum $p\ q$ will be projected into a bright arc $P\ R\ R\ P$, which, after its separation from the circle $C\ B\ D$, unites with it again at P.

"If the bounding surfaces are not parallel planes, but irregularly curved surfaces, analogous appearances must result."

The Milky Way, as we see it, presents the aspect which has been just accounted for, in its general appearance of a girdle around the heavens and in its bifurcation at a certain point, and HERSCHEL's explanation of this appearance, as just given, has never been seriously questioned. One doubtful point remains: are the stars in Fig. 135 scattered all through the space $S - a\,b\,p\,d$? or are they near its bounding planes, or clustered in any way within this space so as to produce the same result to the eye as if uniformly distributed?

HERSCHEL assumed that they were nearly equably arranged all through the space in question. He only examined one other arrangement—viz., that of a ring of stars surrounding the sun, and he pronounced against such an arrangement, for the reason that there is absolutely nothing in the size or brilliancy of the sun to cause us to suppose it to be the centre of such a gigantic system. No reason except its importance to us personally can be alleged for such a supposition. By the assumptions of Fig. 135, each star will have its own appearance of a galaxy or milky way, which will vary according to the situation of the star.

Such an explanation will account for the general appearances of the Milky Way and of the rest of the sky, supposing the stars equally or nearly equally distributed in space. On this supposition, the system must be deeper where the stars appear more numerous. The same evidence can be strikingly presented in another way so as to include the results of the southern gauges of Sir JOHN HERSCHEL. The Galaxy, or Milky Way, being nearly a great circle of the sphere, we may compute the position of its north or south pole; and as the position of our own polar points can evidently have no relation to the stellar universe, we express the position of the gauges in *galactic* polar distance, north or south. By subtracting these polar distances from 90°, we shall have the distance of each gauge from the central plane of the Galaxy itself, the stars near 90° of polar distance being within the Galaxy. The average number of stars per field of 15′ for each zone of 15° of galactic polar distance has been tabulated by STRUVE and HERSCHEL as follows:

Zones of Galactic North Polar Distance.	Average Number of Stars per Field of 15′.	Zones of Galactic South Polar Distance.	Average Number of Stars per Field of 15′.
0° to 15°	4·32	0° to 15°	6·05
15° to 30°	5·42	15° to 30°	6·62
30° to 45°	8·21	30° to 45°	9·08
45° to 60°	13·61	45° to 60°	13·49
60° to 75°	24·09	60° to 75°	26·29
75° to 90°	53·43	75° to 90°	59·06

This table clearly shows that the *superficial* distribution of stars from the first to the fifteenth magnitudes over the apparent celestial sphere is such that the vast majority of them are in that zone of 30° wide, which includes the Milky Way. Other independent researches have shown that the fainter lucid stars, considered alone, are also distributed in greater number in this zone.

HERSCHEL endeavored, in his early memoirs, to find the physical explanation of this inequality of distribution in the theory of the universe exemplified in Fig. 136, which was based on the fundamental assumption that, on the whole, the stars were nearly equably distributed in space.

If they were so distributed, then the number of stars visible in any gauge would show the thickness of the stellar system in the direction in which the telescope was pointed. At each pointing, the field of view of the instrument includes all the visible stars situated within a cone, having its vertex at the observer's eye, and its base at the very limits of the system, the angle of the cone (at the eye) being 15′ 4″. Then the cubes of the perpendiculars let fall from the eye on the plane of the bases of the various visual cones are proportional to the solid contents of the cones themselves, or, as the stars are supposed equally scattered within all the cones, the cube roots of the numbers of stars in each of the fields express the relative lengths of the perpendiculars. A *section* of the sidereal system along any great circle can thus be constructed as in the figure, which is copied from HERSCHEL.

The solar system is supposed to be at the dot within the mass of stars. From this point lines are drawn along the directions in which the gauging telescope was pointed. On these lines are laid off lengths proportional to the cube roots of the number of stars in each gauge.

FIG. 136.—ARRANGEMENT OF THE STARS ON THE HYPOTHESIS OF EQUABLE DISTRIBUTION.

The irregular line joining the terminal points is approximately the bounding curve of the stellar system in the great circle chosen. Within this line the space is nearly uniformly filled with stars. Without it is empty space. A similar section can be constructed in any other great circle, and a combination of all such would give a representation of the shape of our stellar system. The more numerous and careful the observations, the more elaborate the representation, and the 863 gauges of HERSCHEL are sufficient to mark out with great precision the main features of the Milky Way, and even to indicate some of its chief irregularities. This figure may be compared with Fig. 135.

On the fundamental assumption of HERSCHEL (equable distribution), no other conclusions can be drawn from his statistics but that drawn by him.

This assumption he subsequently modified in some degree, and was led to regard his gauges as indicating not so much the depth of the system in any direction as the clustering power or tendency of the stars in those special regions. It is clear that if in any given part of the sky, where, on the average, there are 10 stars (say) to a field, we should find a certain small portion of 100 or more to a field, then, on HERSCHEL's first hypothesis, rigorously interpreted, it would be necessary to suppose a spike-shaped protuberance directed from the earth in order to explain the increased number of stars. If many such places could be found, then the probability is great that this explanation is wrong. We should more rationally suppose some real inequality of star distribution here. It is, in fact, in just such details that the system of HERSCHEL breaks down, and the careful examination which his system has received leads to the belief that it must be greatly modified to cover all the known facts, while it undoubtedly has, in the main, a strong basis.

The stars are certainly not uniformly distributed, and any general theory of the sidereal system must take into account the varied tendency to aggregation in various parts of the sky.

The curious convolutions of the Milky Way, observed at various parts of its course, seem inconsistent with the idea of very great depth of this stratum, and Mr. PROCTOR has pointed out that the circular forms of the two "coal-sacks" of the Southern Milky Way indicate that they are really globular, instead of being cylindric tunnels of great length, looking into space, with their axes directed toward the earth. If they are globular, then the depth of the Milky Way in their neighborhood cannot be greatly different from their diameters, which would indicate a much smaller depth than that assigned by HERSCHEL.

In 1817, HERSCHEL published an important memoir on the same subject, in which his first method was largely modified, though not abandoned entirely. Its fundamental principle was stated by him as follows:

"It is evident that we cannot mean to affirm that the stars of the fifth, sixth, and seventh magnitudes are really smaller than those of the first, second, or third, and that we must ascribe the cause of the difference in the apparent magnitudes of the stars to a difference in their relative distances from us. On account of the great number of stars in each class, we must also allow that the stars of each succeeding magnitude, beginning with the first, are, one with another, further from us than those of the magnitude immediately preceding. The relative magnitudes give only relative distances, and can afford no information as to the real distances at which the stars are placed.

"A standard of reference for the arrangement of the stars may be had by comparing their distribution to a certain properly modified equality of scattering. The equality which I propose does not require that the stars should be at equal distances from each other, nor is it necessary that all those of the same nominal magnitude should be equally distant from us."

It consists of allotting a certain equal portion of space to every star, so that, on the whole, each equal portion of space within the stellar system contains an equal number of stars.

The space about each star can be considered spherical. Suppose such a sphere to surround our own sun, its radius will not differ greatly from the distance of the nearest fixed star, and this is taken as the unit of distance.

FIG. 137.—ORDERS OF DISTANCE OF STARS.

Suppose a series of larger spheres, all drawn around our sun as a centre, and having the radii 3, 5, 7, 9, etc. The contents of the spheres being as the cubes of their diameters, the first sphere will have $3 \times 3 \times 3 = 27$ times the volume of the unit sphere, and will therefore be large enough to contain 27 stars; the second will have 125 times the volume, and will therefore contain 125 stars, and so with the successive spheres. The figure shows a section of portions of these spheres up to that with radius 11. Above the centre are given the various orders of stars which are situated between the several spheres, while in the correspondin: spaces below the centre are given the number of stars which the region is large enough to contain; for instance, the sphere of radius 7 has room for 343 stars, but of this space 125 parts belong to the spheres inside of it: there is, therefore, room for 218 stars between the spheres of radii 5 and 7.

HERSCHEL designates the several distances of these layers of stars as orders; the stars between spheres 1 and 3 are of the first order of distance, those between 3 and 5 of the second order, and so on. Comparing the room for stars between the several spheres with the number of stars of the several magnitudes, he found the result to be as follows:

Order of Distance.	Number of Stars there is Room for.	Magnitude.	Number of Stars of that Magnitude.
1	26	1	17
2	98	2	57
3	218	3	206
4	386	4	454
5	602	5	1,161
6	866	6	6,103
7	1,178	7	6,146
8	1,538		

The result of this comparison is, that, if the order of magnitudes could indicate the distance of the stars, it would denote at first a gradual and afterward a very abrupt condensation of them.

If, on the ordinary scale of magnitudes, we assume the brightness of any star to be inversely proportional to the square of its distance, it leads to a scale of distance different from that adopted by HERSCHEL, so that a sixth-magnitude star on the common scale would be about of the eighth order of distance according to this scheme—that is, we must remove a star of the first magnitude to eight times its actual distance to make it shine like a star of the sixth magnitude.

On the scheme here laid down, HERSCHEL subsequently assigned the *order* of distance of various objects, mostly star-clusters, and his estimates of these distances are still quoted. They rest on the fundamental hypothesis which has been explained, and the error in the assumption of equal brilliancy for all stars, affects these estimates. It is perhaps most probable that the hypothesis, of equal brilliancy for all stars is still more erroneous than the hypothesis of equal distribution, and it may well be that there is a very large range indeed in the actual dimensions and in the intrinsic brilliancy of stars at the same order of distance from us, so that the tenth-magnitude stars, for example, may be scattered throughout the spheres, which HERSCHEL would assign to the seventh, eighth, ninth, tenth, eleventh, twelfth, and thirteenth magnitudes.

Since the time of HERSCHEL, one of the most eminent of the astronomers who have investigated this subject is STRUVE the elder, formerly director of the Pulkowa Observatory. His researches were founded mainly on the numbers of stars of the several magnitudes found by BESSEL in a zone thirty degrees wide extending all around the heavens, 15° on each side of the equator. With these he combined the gauges of Sir WILLIAM HERSCHEL. The hypothesis on which he based his theory was similar to that employed by HERSCHEL in his later researches, in so far that he supposed the magnitude of the stars to furnish, on the average, a measure of their relative distances. Supposing, after HERSCHEL, a number of concentric spheres to be drawn around the sun as a centre, the successive spaces between which corresponded to stars of the several magnitudes, he found that the further out he went, the more the stars were condensed in and near the Milky Way. This conclusion may be drawn at once from the fact we have already mentioned, that the smaller the stars, the more they are condensed in the region of the Galaxy. STRUVE found that if we take only the stars plainly visible to the naked eye—that is, those down to the fifth magnitude—they are no thicker in the Milky Way than in other parts of the heavens. But those of the sixth magnitude are a little thicker in that region, those of the seventh yet thicker, and so on, the inequality of distribution becoming constantly greater as the telescopic power is increased.

From all this, STRUVE concluded that the stellar system might be considered as composed of layers of stars of various densities, all parallel to the plane of the Milky Way. The stars are thickest in and near the central layer, which he conceives to be spread out as a wide, thin sheet of stars. Our sun is situated near the middle of this layer. As we pass out of this layer, on either side we find the stars constantly growing thinner and thinner, but we do not reach any distinct boundary. As, if we could rise in the atmosphere, we should find the air constantly growing thinner, but at so gradual a rate of progress that we could hardly say where it terminated; so, on STRUVE's view, would it be with the stellar system, if we could mount up in a direction perpendicular to the Milky Way. STRUVE gives the following table of the thickness of the stars on each side of the principal plane, the unit of distance being that of the extreme distance to which HERSCHEL's telescope could penetrate:

Distance from Principal Plane.	Density.	Mean Distance between Neighboring Stars.
In the principal plane............	1·0000	1·000
0·05 from principal plane........	0·48568	1·272
0·10 " "	0·33288	1·458
0·20 " "	0·23895	1·611
0·30 " "	0·17980	1·772
0·40 " "	0·13021	1·973
0·50 " "	0·08646	2·261
0·60 " "	0·05510	2·628
0·70 " "	0·03079	3·190
0·80 " "	0·01414	4·131
0·866 " "	0·00532	5·729

This condensation of the stars near the central plane and the gradual thinning-out on each side of it are only designed to be the expression of the general or average distribution of those bodies. The probability is that even in the central plane the stars are many times as thick in some regions as in others, and that, as we leave the plane, the thinning-out would be found to proceed at very different rates in different regions. That there may be a gradual thinning-out cannot be denied; but STRUVE's attempt to form a table of it is open to the serious objection that, like HERSCHEL, he supposed the differences between the magnitudes of the stars to arise entirely from their different distances from us. Although where the scattering of the stars is nearly uniform, this supposition may not lead us into serious error, the case will be entirely different where we have to deal with irregular masses of stars, and especially where our telescopes penetrate to the boundary of the stellar system. In the latter case we cannot possibly distinguish between small stars lying within the boundary and larger ones scattered outside of it, and STRUVE's gradual thinning-out of the stars may be entirely accounted for by great diversities in the absolute brightness of the stars.

Distribution of Stars.—The brightness B of any star, as seen from the earth, depends upon its surface S, the intensity of its light per unit of surface, i, and its distance D, so that its brightness can be expressed thus:

$$B = \frac{S \times i}{D^2};$$

for another star:

$$B' = \frac{S' \times i'}{D'^2},$$

and

$$\frac{B}{B'} = \frac{S \cdot i}{S' \cdot i'} \cdot \frac{D'^2}{D^2}.$$

Now this ratio of the brightness $B \div B'$ is the only fact we usually know with regard to any two stars. D has been determined for only a few stars, and for these it varies between 200,000 and 2,000,000 times the major axis of the earth's orbit. S and i are not known for any star. There is, however, a probability that i does not vary greatly from star to star, as the great majority of stars are white in color (only some 700 red stars, for instance, are known out of the 300,000 which have been carefully examined). Among 476 double stars of STRUVE's list 295 were white, 63 being bluish, only one fourth, or 118, being yellow *or* red.

If B is of the nth mag. its light in terms of a first magnitude star is δ^{n-1} where $\delta = 0 \cdot 397$, and if B' is of the mth mag., its light is δ^{m-1}, both expressed in terms of the light of a first magnitude star as unity ($\delta^0 = 1$).

Therefore we may put $B = \delta^{n-1}$, $B' = \delta^{m-1}$, and we have

$$\frac{\delta^{n-1}}{\delta^{m-1}} = \delta^{n-m} = \frac{S \cdot i \cdot D'^2}{S' \cdot i \cdot D^2}$$

In this general expression we seek the ratio $\frac{D}{D'}$, and we have it expressed in terms of four unknown quantities. We must therefore make some supposition in regard to these.

1. *If all stars are of equal intrinsic brilliancy and of equal size,* then

$$Si,\ S'i',\ \text{and } \delta^{n-m} = \text{a constant} = \frac{D'^2}{D^2},$$

whence the relative distance of any two stars would be known on this hypothesis.

II. Or, ***suppose the stars to be uniformly distributed in space,*** or the star-density to be equal in all directions. From this we can also obtain some notions of the relative distances of stars.

Call D_1, D_2, D_3 ..... D_n the average distances of stars of the 1, 2, 3, ... nth magnitudes.

If K stars are situated within the sphere of radius 1, then the number of stars (Q_n), situated within the sphere of radius D_n, is

$$Q_n = K \cdot (D_n)^3,$$

since the cubic contents of spheres are as the cubes of their radii. Also

$$Q_{n-1} = K\,(D_{n-1})^3,$$

whence

$$\frac{D_n}{D_{n-1}} = \sqrt[3]{\frac{Q_n}{Q_{n-1}}}.$$

If we knew Q_n and Q_{n-1}, the number of stars contained in the spheres of radii D_n and D_{n-1}, then the ratio of D_n and D_{n-1} would be known. We cannot know Q_n, Q_{n-1}, etc., directly, but we may suppose these quantities to be proportional to the numbers of stars of the nth and $(n-1)$th magnitudes found in an enumeration of all the stars in the heavens of these magnitudes, or, failing in these data, we may confine this enumeration to the northern hemisphere, where LITTROW has counted the number of stars of each class in ARGELANDER's *Durchmusterung*. As we have seen (p. 436)

$$Q_7 = 19{,}699 \text{ and } Q_8 = 77{,}794,$$

whence

$$\frac{D_8}{D_7} = \sqrt[3]{\frac{Q_8}{Q_7}} = 1 \cdot 58,$$

and this would lead us to infer that the stars of the 8th magnitude were distributed inside of a sphere whose radius was about 1·6 times that of the corresponding sphere for the 7th magnitude stars provided that, 1st, the stars in general are equally or about equally distributed, and, 2d, that on the whole the stars of the 8 .... n magnitudes are further away from us than those of the 7..... (n — 1) magnitudes.

We may have a kind of test of the truth of this hypothesis, and of the first employed, as follows, we had:

$$\frac{D_n}{D_{n-1}} = \sqrt[3]{\frac{Q_n}{Q_{n-1}}}.$$

Also from the first hypothesis the brightness B_n of a star of the nth magnitude in terms of a first magnitude star = 1 was

$$B_n = \delta^{n-1}.$$

If here, again, we suppose the distance of a first magnitude star to be = 1 and of an nth magnitude star D_n, then

$$B_n = \frac{1}{D_n^2}$$

or
$$D_n = \frac{1}{\sqrt{B_n}} = \left(\frac{1}{\sqrt{\delta}}\right)^{n-1}.$$

Also
$$D_{n-1} = \left(\frac{1}{\sqrt{\delta}}\right)^{n-2}.$$

whence
$$\frac{D_n}{D_{n-1}} = \frac{1}{\sqrt{\delta}}.$$

Comparing the expression for $\frac{D_n}{D_{n-1}}$, in the two cases, we have

$$\sqrt[3]{\frac{Q_n}{Q_{n-1}}} = \frac{1}{\sqrt{\delta}} \text{ or } \delta = \left(\frac{Q_{n-1}}{Q_n}\right)^{\frac{2}{3}}.$$

If the value of δ in this last expression comes near to the value which has been deduced for it from direct photometric measures of the relative intensity of various classes of stars, *viz.*, $\delta = 0{\cdot}40$, then this will be so far an argument to show that a certain amount of credence may be given to both hypotheses I. and II. Taking the values of Q_7 and Q_8, we have

$$\delta_{(7,\,8)} = \left(\frac{19{,}699}{77{,}794}\right)^{\frac{2}{3}} = 0{\cdot}40.$$

From the values of Q_6 and Q_7, there results $\delta_{(6,\,7)} = 0{\cdot}45$. These, then, agree tolerably well with the independent photometric values for δ, and show that the equation

$$D_n = \left(\frac{1}{\sqrt{\delta}}\right)^{n-1}$$

gives the average distance of the stars of the nth magnitude with a certain approach to accuracy. For the stars from 1st to 8th magnitude these distances are:

1 to 1·9 magnitude	1·00
2 to 2·9 "	1·54
3 to 3·9 "	2·36
4 to 4·9 "	3·64
5 to 5·9 "	5·59
6 to 6·9 "	8·61
7 to 7·9 "	13·23
8 to 8·9 "	20·35

This presentation of the subject is essentially that of Prof. HUGO GYLDÉN.

CHAPTER VIII.

COSMOGONY.

A THEORY of the operations by which the universe received its present form and arrangement is called *Cosmogony*. This subject does not treat of the origin of matter, but only with its transformations.

Three systems of Cosmogony have prevailed among thinking men at different times.

(1.) That the universe had no origin, but existed from eternity in the form in which we now see it.

(2.) That it was created in its present shape in a moment, out of nothing.

(3.) That it came into its present form through an arrangement of materials which were before "without form and void."

The last seems to be the idea which has most prevailed among thinking men, and it receives many striking confirmations from the scientific discoveries of modern times. The latter seem to show beyond all reasonable doubt that the universe could not always have existed in its present form and under its present conditions; that there was a time when the materials composing it were masses of glowing vapor, and that there will be a time when the present state of things will cease. The explanation of the processes through which this occurs is sometimes called the *nebular hypothesis*. It was first propounded by the philosophers SWEDENBORG, KANT, and LAPLACE, and although since greatly modified in detail, the views of these men have in the main been retained until the present time.

We shall begin its consideration by a statement of the various facts which appear to show that the earth and planets, as well as the sun, were once a fiery mass.

The first of these facts is the gradual but uniform increase of temperature as we descend into the interior of the earth. Wherever mines have been dug or wells sunk to a great depth, it is found that the temperature increases as we go downward at the rate of about one degree centigrade to every 30 metres, or one degree Fahrenheit to every 50 feet. The rate differs in different places, but the general average is near this. The conclusion which we draw from this may not at first sight be obvious, because it may seem that the earth might always have shown this same increase of temperature. But there are several results which a little thought will make clear, although their complete establishment requires the use of the higher mathematics.

The first result is that the increase of temperature cannot be merely superficial, but must extend to a great depth, probably even to the centre of the earth. If it did not so extend, the heat would have all been lost long ages ago by conduction to the interior and by radiation from the surface. It is certain that the earth has not received any great supply of heat from outside since the earliest geological ages, because such an accession of heat at the earth's surface would have destroyed all life, and even melted all the rocks. Therefore, whatever heat there is in the interior of the earth must have been there from before the commencement of life on the globe, and remained through all geological ages.

The interior of the earth being hotter than its surface, and hotter than the space around it, must be losing heat. We know by the most familiar observation that if any object is hot inside, the heat will work its way through to the surface by the process of conduction. Therefore, since the earth is a great deal hotter at the depth of 30 metres than it is at the surface, heat must be continually coming to the surface. On reaching the surface, it must be radiated off into space, else the surface would have long ago become as hot as the interior. Moreover, this loss of heat must have been going on since the beginning, or, at least, since a time when the surface was as hot as the interior. Thus, if we reckon backward in time, we find that there must have been more and more heat in the earth the further back we go, so that we must finally reach back to a time when it was so hot as to be molten, and then again to a time when it was so hot as to be a mass of fiery vapor.

The second fact is that we find the sun to be cooling off like the earth, only at an incomparably more rapid rate. The sun is constantly radiating heat into space, and, so far as we can ascertain, receiving none back again. A small portion of this heat reaches the earth, and on this portion depends the existence of life and motion on the earth's surface. The quantity of heat which strikes the earth is only about $\frac{1}{\text{[illegible]}}$ of that which the sun radiates. This fraction expresses the ratio of the apparent surface of the earth, as seen from the sun, to that of the whole celestial sphere.

Since the sun is losing heat at this rate, it must have had more heat yesterday than it has to-day; more two days ago than it had yesterday, and so on. Thus calculating backward, we find that the further we go back into time the hotter the sun must have been. Since we know that heat expands all bodies, it follows that the sun must have been larger in past ages than it is now, and we can trace back this increase in size without limit. Thus we are led to the conclusion that there must have been a time when the sun filled up the space now occupied by the planets, and must have been a very rare mass of glowing vapor. The planets could not then have existed separately, but must have formed a part of this mass of vapor. The latter was therefore the material out of which the solar system was formed.

The same process may be continued into the future.

Since the sun by its radiation is constantly losing heat, it must grow cooler and cooler as ages advance, and must finally radiate so little heat that life and motion can no longer exist on our globe.

The third fact is that the revolutions of all the planets around the sun take place in the same direction and in nearly the same plane. We have here a similarity amongst the different bodies of the solar system, which must have had an adequate cause, and the only cause which has ever been assigned is found in the nebular hypothesis. This hypothesis supposes that the sun and planets were once a great mass of vapor, as large as the present solar system, revolving on its axis in the same plane in which the planets now revolve.

The fourth fact is seen in the existence of nebulæ. We have already stated that the spectroscope shows these bodies to be masses of glowing vapor. We thus actually see matter in the celestial spaces under the very form in which the nebular hypothesis supposes the matter of our solar system to have once existed. Since these masses of vapor are so hot as to radiate light and heat through the immense distance which separates us from them, they must be gradually cooling off. This cooling must at length reach a point when they will cease to be vaporous and condense into objects like stars and planets. We know that every star in the heavens radiates heat as our sun does. In the case of the brighter stars the heat radiated has been made sensible in the foci of our telescopes by means of the thermo-multiplier. The general relation which we know to exist between light and radiated heat shows that all the stars must, like the sun, be radiating heat into space.

A fifth fact is afforded by the physical constitution of the planets *Jupiter* and *Saturn*. The telescopic examination of these planets shows that changes on their surfaces are constantly going on with a rapidity and violence to which nothing on the surface of our earth can compare. Such operations can be kept up only through the agency of heat or some equivalent form of energy. But at the distance of *Jupiter* and *Saturn* the rays of the sun are entirely insufficient to produce changes so violent. We are therefore led to infer that *Jupiter* and *Saturn* must be hot bodies, and must therefore be cooling off like the sun, stars and earth.

We are thus led to the general conclusion that, so far as our knowledge extends, nearly all the bodies of the universe are hot, and are cooling off by radiating their heat into space. Before the discovery of the "conservation of energy," it was not known that this radiation involved the waste of a something which is necessarily limited in supply. But it is now known that heat, motion, and other forms of force are to a certain extent convertible into each other, and admit of being expressed as quantities of a general something which is called *energy*. We may define the unit of energy in two or more ways: as the quantity which is required to raise a certain weight through a certain height at the surface of the earth, or to heat a given quantity of water to a certain temperature. However we express it, we know by the laws of matter that a given mass of matter can contain only a certain definite number of units of energy. When a mass of matter either gives off heat, or causes motion in other bodies, we know that its energy is being expended. Since the total quantity of energy which it contains is finite, the process of radiating heat must at length come to an end.

It is sometimes supposed that this cooling off may be merely a temporary process, and that in time something may happen by which all the bodies of the universe will receive back again the heat which they have lost. This is founded upon the general idea of a compensating process in nature. As a special example of its application, some have supposed that the planets may ultimately fall into the sun, and thus generate so much heat as to reduce the sun once more to vapor. All these theories are in direct opposition to the well-established laws of heat, and can be justified only by some generalization which shall be far wider than any that science has yet reached. Until we have such a generalization, every such theory founded upon or consistent with the laws of nature is a necessary failure. All the heat that could be generated by a fall of all the planets into the sun would not produce any change in its constitution, and would only last a few years. The idea that the heat radiated by the sun and stars may in some way be collected and returned to them by the mere operation of natural laws is equally untenable. It is a fundamental principle of the laws of heat that the latter can never pass from a cooler to a warmer body, and that a body can never grow warm or acquire heat in a space that is cooler than the body is itself. All differences of temperature tend to equalize themselves, and the only state of things to which the universe can tend, under its present laws, is one in which all space and all the bodies contained in space are at a uniform temperature, and then all motion and change of temperature, and hence the conditions of vitality, must cease. And then all such life as ours must cease also unless sustained by entirely new methods.

The general result drawn from all these laws and facts is, that there was once a time when all the bodies of the universe formed either a single mass or a number of masses of fiery vapor, having slight motions in various parts, and different degrees of density in different regions. A gradual condensation around the centres of greatest density then went on in consequence of the cooling and the mutual attraction of the parts, and thus arose a great number of nebulous masses. One of these masses formed the material out of which the sun and planets are supposed to have been formed. It was probably at first nearly globular, of nearly equal density throughout, and endowed with a very slow rotation in the direction in which the planets now move. As it cooled off, it grew smaller and smaller, and its velocity of rotation increased in rapidity by virtue of a well-established law of mechanics, known as that of *the conservation of areas.* According to this law, whenever a system of particles of any kind whatever, which is rotating around an axis, changes its form or arrangement by virtue of the mutual attractions of its parts among themselves, the sum of all the areas described by each particle around the centre of rotation in any unit of time remains constant. This sum is called the *areolar velocity*.

If the diameter of the mass is reduced to one half, supposing it to remain spherical, the area of any plane passing through its centre will be reduced to one fourth, because areas are in proportion to the square of the diameters. In order that the areolar velocity may then be the same as before, the mass must rotate four times as fast. The rotating mass we have described must have had an axis around which it rotated, and therefore an equator defined as being everywhere 90° from this axis. In consequence of the increase in the velocity of rotation, the centrifugal force would also be increased as the mass grew smaller. This force varies as the radius of the circle described by the particle multiplied by the square of the angular velocity. Hence when the masses, being reduced to half the radius, rotate four times as fast, the centrifugal force at the equator would be increased $\frac{1}{2} \times 4^2$, or eight times. The gravitation of the mass at the surface, being inversely as the square of the distance from the centre, or of the radius, would be increased four times. Therefore as the masses continue to contract, the centrifugal force increases at a more rapid rate than the central attraction. A time would therefore come when they would balance each other at the equator of the mass. The mass would then cease to contract at the equator, but at the poles there would be no centrifugal force, and the gravitation of the mass would grow stronger and stronger. In consequence the mass would at length assume the form of a lens or disk very thin in proportion to its extent. The denser portions of this lens would gradually be drawn toward the centre, and there more or less solidified by the process of cooling. A point would at length be reached, when solid particles would begin to be formed throughout the whole disk. These would gradually condense around each other and form a single planet, or they might break up into small masses and form a group of planets. As the motion of rotation would not be altered by these processes of condensation, these planets would all be rotating around the central part of the mass, which is supposed to have condensed into the sun.

It is supposed that at first these planetary masses, being very hot, were composed of a central mass of those substances which condensed at a very high temperature, surrounded by the vapors of those substances which were more volatile. We know, for instance, that it takes a much higher temperature to reduce lime and platinum to vapor than it does to reduce iron, zinc, or magnesium. Therefore, in the original planets, the limes and earths would condense first, while many other metals would still be in a state of vapor. The planetary masses would each be affected by a rotation increasing in rapidity as they grew smaller, and would at length form masses of melted metals and vapors in the same way as the larger mass out of which the sun and planets were formed. These masses would then condense into a planet, with satellites revolving around it, just as the original mass condensed into sun and planets.

At first the planets would be so hot as to be in a molten condition, each of them probably shining like the sun. They would, however, slowly cool off by the radiation of heat from their surfaces. So long as they remained liquid, the surface, as fast as it grew cool, would sink into the interior on account of its greater specific gravity, and its place would be taken by hotter material rising from the interior to the surface, there to cool off in its turn. There would, in fact, be a motion something like that which occurs when a pot of cold water is set upon the fire to boil. Whenever a mass of water at the bottom of the pot is heated, it rises to the surface, and the cool water moves down to take its place. Thus, on the whole, so long as the planet remained liquid, it would cool off equally throughout its whole mass, owing to the constant motion from the centre to the circumference and back again. A time would at length arrive when many of the earths and metals would begin to solidify. At first the solid particles would be carried up and down with the liquid. A time would finally arrive when they would become so large and numerous, and the liquid part of the general mass become so viscid, that the motion would be obstructed. The planet would then begin to solidify. Two views have been entertained respecting the process of solidification.

According to one view, the whole surface of the planet would solidify into a continuous crust, as ice forms over a pond in cold weather, while the interior was still in a molten state. The interior liquid could then no longer come to the surface to cool off, and could lose no heat except what was conducted through this crust. Hence the subsequent cooling would be much slower, and the globe would long remain a mass of lava, covered over by a comparatively thin solid crust like that on which we live.

The other view is that, when the cooling attained a certain stage, the central portion of the globe would be solidified by the enormous pressure of the superincumbent portions, while the exterior was still fluid, and that thus the solidification would take place from the centre outward.

It is still an unsettled question whether the earth is now solid to its centre, or whether it is a great globe of molten matter with a comparatively thin crust. Astronomers and physicists incline to the former view; geologists to the latter one. Whichever view may be correct, it appears certain that there are great lakes of lava in the interior from which volcanoes are fed.

It must be understood that the nebular hypothesis, as we have explained it, is not a perfectly established scientific theory, but only a philosophical conclusion founded on the widest study of nature, and pointed to by many otherwise disconnected facts. The widest generalization associated with it is that, so far as we can see, the universe is not self-sustaining, but is a kind of organism which, like all other organisms we know of, must come to an end in consequence of those very laws of action which keep it going. It must have had a beginning within a certain number of years which we cannot yet calculate with certainty, but which cannot much exceed 20,000,000, and it must end in a chaos of cold, dead globes at a calculable time in the future, when the sun and stars shall have radiated away all their heat, unless it is re-created by the action of forces of which we at present know nothing.

FINIS.

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☞ This index is intended to point out the subjects treated in the work, and further, to give references to the pages where technical terms are defined or explained.