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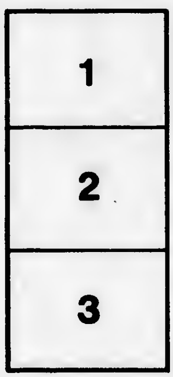

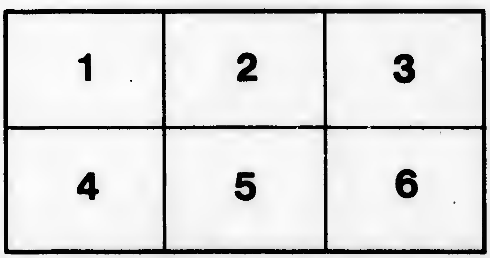

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THE ERRORS OF LEVELS AND LEVELLING.

PART II.—LEVELLING.

BY PROF. C. H. MCLEOD, MA. E., M. CAN. SOC. C. E.

To be read on Thursday, 18th December.

In considering the errors of levelling, reference will only be made to such operations as are ordinarily included under the name of spirit levelling, omitting any reference to trigonometrical levelling, in which the spirit level plays an equally important part. It will first be proper to remark that spirit levelling may be broadly divided into two classes: The ordinary levelling undertaken in connection with railways, canals, drainage, etc.; and "geodesic" or "precise" levelling, where the highest attainable accuracy is sought in fixing the elevations of inland points with reference to a common mean sea-level datum. Nearly all European countries are carrying out such systems of levelling in connection with trigonometrical surveys; and in the United States many thousands of miles have been run under the direction of the Coast and Geodetic Survey, the Lake Survey and the Mississippi River Commission. Some few hundreds of miles have, I believe, been levelled in Canada by precision methods, but I have not the details before me.

The instruments employed in spirit levelling may for convenience be divided into three classes,—the dumpy, the wye, and the precision level. The form of the dumpy and wye as employed for ordinary levelling need not be described here. The precision level appears under a variety of forms. Those which have been used in the best European work and in America are of the wye type. The level tube is detached and used as a striding level on the collars of the telescope. One wye is moveable in a vertical direction under the control of a micrometer screw, in order to obtain the small final adjustment necessary to bring the delicate level employed to a central position on the scale. There is a wide range of scale value in the levels employed in the two classes of work, and also amongst the individual instruments used in each class. This divergence is much more marked in the ordinary levelling instruments than in the geodesic instruments. In illustration of this point, the scale values of the levels (as they came from the hands of the maker) belonging to McGill College are given in the table below. The scale values employed for geodesic instruments are usually within the limits of 3″ and 6″ per division of 2^{mm}; equal respectively to 3.75″ and 7.5″ per div. of 0.1 in., the length of the division on the levels referred to below.

SCALE VALUES at 65° Fah.

Kind of Instrument.	Maker's Name.	Scale value per div'n of 0.1 in.	Optical powers.
12 in. Wye level.	Hammersley.	75″	18 and 26
14 in. Dumpy level "A."	Do	21″	18 " 26
14 in. do " "B."	Do	26″	20 " 25
14 in. do "	Stanley.	27″	20 " 25
14 in. do "	Troughton & Simms.	14″	18 " 30
18 in. Wye. "	Buff & Berger.	12″	37

The optical powers in this list will be referred to hereafter.

Before proceeding to a consideration of the errors of levelling, it will be necessary to review briefly the methods of adjustment in the several forms of instruments, and in this connection certain defects of construction will most conveniently be referred to.

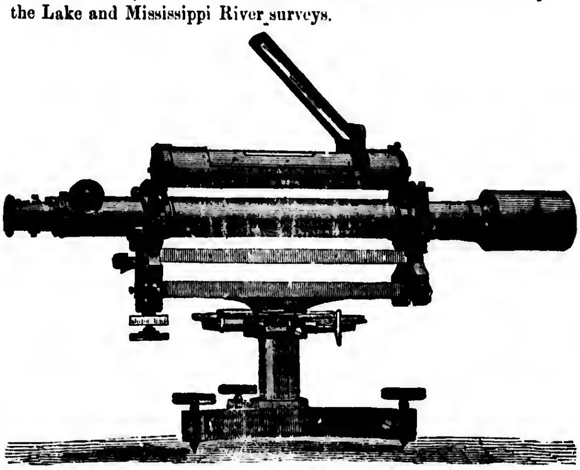
The object of all methods of adjustment is to bring the line of sight parallel to the line tangent to the inner surface of the level tube at its zero point,—this line will in this paper be referred to as the "bubble-axis." In the dumpy this condition can only be reached by the direct method of reading the rod on two "pegs," the difference in the level of which is known, one point being near the instrument while the other is several hundred feet distant, and shifting either the position of the cross hairs or of the level tube, until the readings give the true difference in level. The only error in this method is the slight one due to the curvature of the earth, amounting to about $\frac{1}{100}$th of a foot in 600 feet, and to one-quarter that amount in 300 feet.

The method usually employed to obtain the difference in level of the pegs is by setting the instrument at a point midway between them. The errors of the readings then being the same, the difference is the true one. A second method of determining this difference of level, which is not, I believe, so generally practised and perhaps not known to all Engineers, is as follows:—First set the instrument over one of the pegs, so that the eye end will just swing clear of the rod, held vertically on the peg. When the instrument is level, view the rod through the telescope, with the eye at the object end; the centre of the small circular portion of the rod thus seen can be accurately estimated and its position read by the aid of a pencil or knife edge held against the rod. Then read the rod as usual on the distant peg. Now move the instrument over the second peg, and obtain a reading of the rod as in the case of the first peg. Then read the rod on the first peg which is now the one distant from the instrument. If we call the difference of the rod readings in the first position of the instrument m, and the difference of those in the second position n, then the true difference in the elevation of the pegs is $\frac{m-n}{2}$. Results thus obtained, being from four separate readings instead of two, and being independent of the accuracy of any horizontal measurement, are susceptible of greater precision than the usual method. The instrument and rod are now in position for the necessary adjustment, and the distant rod reading will be corrected by the above amount.

It is customary to adjust the wye level by the indirect method. The line of sight being examined for coincidence with the axis of the collars by revolving the telescope in its wyes, the bubble-axis is then tested for parallelism with the bearings of the collars by turning the telescope end for end in its wyes, and the necessary corrections applied. This method is based on the assumption that the collars of the telescope are true cylinders, and have equal diameters—an assumption which is often wide of the mark. In the case of one level which I have used the error due to this cause amounts to somewhat over $\frac{1}{100}$th of a foot in 100 feet, a condition of adjustment which should be quite inadmissible for good railroad levelling. It is not an uncommon circumstance, in my experience, amongst users of wye levels, to find the fulfilment of the first portion of this test—the revolution of the telescope in its wyes—accepted as a guarantee of the perfection of the whole. Such an error of judgment is of course impossible with persons acquainted with the theory of the instrument; but alas, the holder of the position of "Engineer" in this free country of ours may readily be a touch above theoretical considerations.

For convenience only is it desirable that the bubble should remain central while the instrument rotates about its vertical axis. In the dumpy this adjustment must be made before, and in the wye after, the essential adjustments have been completed. It is commonly stated in defence of the indirect method, that there is a saving of time as compared with the direct or "peg" method. Such is however not my experience. The time-consuming portion of the work is the making of the necessary corrections to the level tube and reticule. In the essential adjustments by the indirect method both may require correction, whereas in the direct method only one of them—preferably the reticule—should be moved. We have then in comparing the relative convenience of the two, on the one hand the matter of setting up the instrument once and making two or three rod readings, and on the other the complete adjustment of the level tube. I had rather set up ten instruments and make as many rod readings, and could do it in *less* time, than adjust one bubble tube. This indirect method seems to me to have, for the purposes of ordinary levelling, no one point to commend it in preference to the direct method, and has the disadvantage that it does not guarantee correct adjustment, unless the pivots have, after careful examination, been ascertained to be equal; or the proper correction determined and applied. The wye is undoubtedly the most scientific instrument, and for precise work has some advantages,—such, for example, as the possibility of eliminating the error due to want of coincidence of the sliding tube with the line of sight. In a well constructed dumpy however this error must always be trifling. I have never been able to understand why the wye level should be used in ordinary levelling in preference to the dumpy. The wye form is more difficult of construction, more expensive, less rigid when constructed, and hence more liable to get out of repair. The most abominable instrument ever put in the hand of man is an old and shaky wye level. I submit that in the interest of good work, if not of economy, the construction of the wye form for ordinary levelling should be abandoned in favor of the more compact and rigid dumpy. The modification of the dumpy, such as Cushing's reversible level and other deviations from the type form, do not, for similar reasons, commend themselves to me. The adjustments for coincidence of the optical axes of the objective lens and the eye-piece with one another and with the axis of the tube, provided for in the wye but not in the dumpy, are really makers' adjustments, and except in the case in which the axis of the objective slide makes an angle with that of the tube are not essential to correct work. A very important point to be attended to in the construction of all levels, and in the examination of them from time to time by the Engineer, is that the object lens is not loose in its cell, and that the cell is not loose in the telescope tube. Both these defects are apt to occur through time. The tightening band at the back of the cell should screw into place in order to permit such a defect being rectified. An instrument having a loose objective is impossible of adjustment and fatal to good work.

The accompanying cut taken from Fauth's catalogue, represents an instrument of the form used in geodesic work. The pattern employed by the United States coast and Geodetic Survey differs slightly from this instrument, which is the one prescribed by the International Geodetic Commission, held in Berlin in 1864, and used in this country on the Lake and Mississippi River surveys.



The following description of the instrument is taken from Johnson's theory and practice of surveying:—

"The bubble is enclosed in a wooden case (metal case in the cut), "and rests on top of the pivots or rings; it is carried in the hand "when the instrument is transported. A mirror is provided which "enables the observer to read the bubble without moving his eye from "the eye-piece. There is a thumb-screw with a very fine thread "under one wye, which is used for the final levelling of the telescope "when pointed on the rod. There are three levelling-screws and a "circular or box level for convenience in setting. The telescope "bubble is very delicate, one division on the scale corresponding to "about three seconds of arc. The bubble-tube is chambered also, "thus allowing the length of the bubble to be adjusted to different "temperatures. The magnifying power is about 45 diameters. "There are three horizontal wires provided, set at such a distance "apart that the wire interval is about one-hundredth of the distance "to the rod. The tripod legs are covered with white cloth to dim- "inish the disturbing effects of the sun upon them. The level itself is "always kept in the shade while at work.

"The levelling-rod is made in one piece, three metres long, of dry "pine, about four inches wide on the face, and strengthened by a "piece at the back, making a T-shaped cross-section. The rods are "self-reading, that is, they are without targets, and are graduated to "centimetres. An iron spur is provided at the bottom, which fits "into a socket in an iron foot-plate. The end of the spur should be "flat and the bottom of the socket turned out to a spherical form "convex upwards. A box-level is attached to the rod to enable the "rodman to hold it vertically, and this in turn is adjusted by "means of a plumb-line. Two handles are provided for holding the "rod, and a wooden tripod to be used in adjusting the rod-bubble. "The decimetres are marked on one side of the graduations and the "centimetres on the other; all figures are inverted since the telescope "is inverting."

In the use of such a level according to the methods of precise evelling, it is necessary to determine the instrumental constants:—

1. The angular value of one division of the level tube. This may be found in the field by sighting on a rod at a known distance, taking readings of the bubble along the whole length of the graduated part of the tube, but it is more satisfactorily determined on a solidly mounted level trier.

2. The inequality in the diameter of the telescope collars. This is found by reading the striding level on the collars in the two reversed positions, thus eliminating the error due to the unequal length of the striding level legs and obtaining the true inclination of the surface of the rings. The telescope is then reversed in its wyes and the levelling repeated. The difference in the two inclinations, divided by four, gives the angular value of the error or correction sought.* The following observations are given as an example:—

	Level readings. Eye end.		Object end.
Striding Level direct	7.4		9.0
" " reversed	8.0		8.4
	Means=7.7	and	8.7
			7.7
			2)1.0

Inclination of upper surface of collars.=0.50

Telescope reversed in Wyes.

	Level readings. Object end.		Eye end.
Striding level reversed	5.6		11.0
" " direct	5.0		11.6
	Means=5.3	and	11.3
			5.3
			2)6.0

Inclination of upper surface of collars=3.00

$$\frac{\text{Difference of readings}}{4}=\frac{3.00-0.50}{4}=0.625$$

The value of one division of the level was=5″. Hence, correction in seconds of arc = 5 × 0.625 = 3.12, and the eye end is the largest, requiring a negative correction to all rod readings.

3. The ratios of the portions of the rod intercepted between the three wires and the distances of the rod. This for the double purpose of obtaining a measure of the length of sights and furnishing a check on the readings.

The adjustments—making due allowance for the fact that the level tube is moveable—are the same as those for the ordinary wye. Since, however, it is impossible to do anything exactly, and in the best work the smallest errors should be provided for, the after-treatment of the adjustments is essentially different. They are reduced to as near zero

* See Chauventet's Astronomy, Vol. II, p. 153.

as possible, and the outstanding errors determined as follows:—1. The difference in the average of the three rod readings in the two reversed positions of the telescope tube gives twice the collimation error of the mean line of sight, at the known distance of the rod. 2. The inclination of the bubble-axis to the top surface of the rings is found by taking a series of readings of the bubble in the reversed positions, the average of the mean differences at the eye end and at the object end, for level direct and level reversed, gives twice the correction required for the inclination of the level tube. Thus, in the example already given, the difference is 0.6 div. and the correction $0.3 \times 5.0 = 1.''50$.

These determinations are made at the beginning and the end of each day's work, and their resulting values combined with the correction for inequality of collars, applied, in the reduction of the notes, to all rod readings.

Methods of work. The method of using these instruments differs from ordinary work only in that there are three rod readings for each setting of the rod, and the reading employed is the mean of these. In some cases the bubble is kept central by means of the micrometer screw and the reflecting mirror; and in other cases only approximately so, the ends being read and recorded by an assistant. A correction is, in the latter case, afterwards applied to each rod reading for the inclination of the line of sight. The three readings of the rod in each position should always be compared before the level is moved, in order, if necessary, that any doubtful reading may be checked. Self-reading rods usually graduated to centimeters and estimated to millimeters are employed on the Lakes and Mississippi River surveys, and in most of the European work. On the Coast and Geodetic Survey target rods are used, and the method of work there pursued is much more elaborate than the foregoing.

The methods of levelling are sometimes described as *single* or *double* levelling, according as single or double back and fore sight are taken. The latter has been the practice of the Coast Survey, and is a self-checking system—in so far at least as the readings, are concerned—really amounting to two lines of levels in the same direction. The single system is however the more generally adopted. There is a decided economy in time and in accuracy of result in the use of two rods alternating with one another on turning points.

In reviewing the construction of levelling instruments, the principal points to be held in mind as conducive to the best results in the various departments of work would seem to be. First,—stability of construction; second, properly constructed levels of sufficient delicacy; third, adequate optical power for the purpose in view. The first condition should be secured by a proper distribution of the metal, for the attainment of a maximum of strength from a minimum of material. The second and third are to a certain extent interdependent. For railroad and canal work from 10" to 20" per division of 0.1" would seem to be limiting values for the scale values of levels. Levels having a scale value of 12" are entirely satisfactory on ordinary work, while those beyond 20" I have considered as lacking in sensitiveness. It has always seemed to me better to have an over-sensitive than an under-sensitive level, admitting the difficulty of keeping the former central.

Increased optical power is of course secured at the sacrifice of light and definition. The powers demanded for our instrument are however much within the limit of good lenses, under the ordinary conditions of illumination. Referring to the list of instruments already given, the practice of the makers of dumpy levels seems to be keep in the neighborhood of 25 for the higher powers for 14 in. instruments. The object lenses of all these instruments would I think stand higher powers. With a good object glass of 1.5 in. aperture there should be no difficulty is using a power of 40 under the ordinary conditions of seeing. In the smaller apertures of the wye levels the matter of loss of light becomes serious. But the makers of wye levels seem altogether born to perversity,—having decided to use an object glass of small aperture, they must of necessity add to this a four lens eye-piece, as if it were any advantage to get an erect image. The four lens, inverting eye-piece has no advantage over the ordinary erecting eye-piece, and its use results in serious loss of light, giving a comparatively indistinct image —one of the most fruitful sources of error in levelling.

The use of the mirror on level tubes to view the bubble is not in such favor with engineers as it should be. The usual process of reading the rod while standing in a different position from that in which the instrument was levelled must necessarily introduce errors of a serious nature. When the mirror is used it should always be placed at the same inclination, and the observer should by trial, assisted by another person, obtain that position of the eye in which the bubble may be viewed without parallax, and from which he will always afterwards observe it.

It is usual to classify sources of error somewhat as follows:—1, instrumental errors; 2, errors from unstable supports; 3, errors of observation; 4, personal errors; 5, atmospheric errors. The chief source of error from *instrumental* causes is no doubt due to either a permanent or temporary lack of parallelism between the line of sight and the bubble-axis. No adjustment can be perfect, and even if perfect under stated conditions is liable to change under other conditions. The causes which might produce a change in adjustment due to the influence of varying temperature on the glass of the level has been discussed in the first part of this paper. A temporary change may result from the unequal heating of the metal of the instrument, which would probably take place in bright weather when levelling, in a direction towards or away from the sun. The direct action of the sun should be avoided by the use of a shade—a heavy canvas umbrella is used on geodetic work. In order to obviate the effect of any possible change in the scale value of the level, readings should never be taken when the bubble is at any considerable distance from central. If the bubble is sluggish there is a possible source of error in its being read before final settlement. This latter difficulty can be overcome, with a chambered level, by avoiding the use of short bubbles. Errors arising from defects of adjustment are of course completely eliminated by equality of sights, except in such a case as that mentioned above, where, on account of the direct heating action of the sun, a change may take place between sights. Such changes are much more likely to affect the results of ordinary levelling where there is an interval of time between centering the bubble and reading the rod. Errors of this nature are, though small for any given sight, of serious consequence through always being in the same direction, or cumulative in character. If there be, of necessity, a difference in the lengths of sights, he possible error so introduced should be neutralized by making, on the first opportunity, a similar difference in the opposite direction. In the Indian Survey sights are made equal by chaining. Where, as for example in crossing a wide river, a long foresight is unavoidable, the method of reciprocal levelling, explained in connection with the peg adjustment, should be employed. This also eliminates the error due to curvature of the earth. Where there are two vertical wires and the rod is read anywhere between them, there may be a slight error introduced through the lack of horizontality of the wire. This should be provided for in making the adjustment by swinging the telescope, when correctly levelled, around its vertical axis, and rocking the reticule ring until the wire is observed to continuously bisect a fixed point. Where no rod level is used, as is usually the case in ordinary levelling, it is however more important that the vertical wire should be truly vertical; and where both cannot, in this case, be secured, the rod should always be read in the same position with horizontal wire. Wye levels should always have a means of preventing the telescope from rotating in the wyes.

Errors due to the rod will also fall under this class, and we should first look to the accuracy of its length and uniformity of its graduation. Mr. I. W. Wright in his work on the adjustment of observations remarks that:—

"An important source of error in spirit levelling, and one very "commonly overlooked, is the change in length of the levelling road "from variations of temperature. From experiments made by the "Prussian Land Survey, in which the rods were compared daily with "a steel standard, the following fluctuations in length were found for "four rods made of seasoned fir:—

Rod 13, from	May	19	to Aug.	18	0.51	*mm.*	per	metre.
14	"	" 20	"	15	0.46	"	"	"
9	"	" 24	Sept.	6	0.37	"	"	"
10	"	" 24	"	6	0.43	"	"	"

" It is quite possible that errors from this source may largely exceed " the errors arising from the levelling itself. Each field party should " therefore be provided with the means of making a daily comparison " of the rods used, with a standard of length. A steel metre and a " micrometer microscope mounted on a stand would be all that would " be necessary."

The maximum expansion above quoted would amount to .007 of a foot in the length of a 14 foot rod. It will be noted that the variation of temperature is not given. Experiments conducted by Prof. Van de Sande-Bakhuyzen on the staves used in the Netherlands, give results of much less magnitude then the above. He found the rate of expansion for seasoned fir rods to be 4.4 micra per metre, per degree centigrade, and that other changes amounted in all to not more then .05 mm. per metre.

For ordinary work the self-reading telescope rod is almost universally adopted, on account of its portability and convenience in use. The rods of this form, though sold as "standard" by the makers, are liable to be most inaccurate. Of the seven 14 ft. self-reading rods in use at McGill College, four are within .002 of a foot of being correct, one is .006 too short, and two are .014 too long, at 62° Fah. There are then two of these rods, and surprising as it may seem, both from the same makers—a London firm of high repute—which differ between themselves to the extent of .020 of a foot. Two 12 ft. target rods by different American makers are within .002 of the correct length. Amongst a lot of five telescope rods, recently measuredin the warehouse of a dealer, one was found to be .020 too long at 14 feet and .015 too long at 17 feet. In most of the above cases the total error was roughly distributed throughout the length of the rod. In that last mentioned, and in one of the College rods, the graduation was somewhat irregular.

Where rods are properly constructed they should not be influenced to any great extent by moisture. The utmost precaution should however be exercised to keep them dry. In the Swiss levelling the errors due to temperature and moisture are stated as being "small, slow in action, and somewhat irregular." The reports published from time to time of extreme changes from moisture are no doubt due to improper protection. Where rods are used under proper direction they are of course subject to careful comparison with the national standard, and any errors in their lengths allowed for.

Errors arising from *unstable supports* may occur through the instrument or the rod, and are usually of a cumulative character. The instrument may settle slightly between the reading of the back and fore sights, or the converse may occur, depending upon the character of the ground. Similarly, the turning points may settle or spring up between sights. In soft ground, settlement is likely to occur; while in stiff clay both instrument and rod—if the latter is supported on a peg driven in the ground—may spring back slightly. Pegs are preferred as turning points for soft ground and for hard soil or roads, plates having a projecting knob or spherical hole for the rod, as already explained. The rod should never be removed from the point until all the readings have been made, and the weight of the rodman should never be allowed to rest upon it. When both instrument and rod move in the same direction the effects are additive, and the character of the soil continuing the same over a considerable stretch of country, a large error may be accumulated. To obviate such a result, Colonel Walker adopted, in India, the plan of alternating the order of observations at successive stations of the instrument, by reading the back staff first on one station and the forward staff first on the next. The error may in part at least be eliminated by levelling between bench-marks in opposite directions. Duplicate levels in opposite directions between benches or along the whole length of line should completely eliminate it. This is indeed the sovereign cure for all errors of a cumulative character,

Mr. Hirsch, one of the Directors of the Swiss precision levelling.

has shown that the error due to settlement, other things being equal, is proportional to the length of line run. In ordinary levelling operation, the character of the ground affects the work in a different way, the feet of the observer compress the ground near the tripod legs, and displace the line of sight in the interval of time between levelling and reading the rod. To obviate this, Colonel Goulier has recommended that two of the legs be always placed parallel to the line of sight. The use of the mirror, already noticed, would also remove this source of error.

The largest source of *observational errors* is believed to be due to the want of careful centering or reading the bubble. Every leveller should know what rod reading is covered by a range of one division of his bubble at a given distance of, say 100 feet, in order that he may fully appreciate the effect of errors of this kind.

When the illumination of the two ends of the bubble is different, an error in centering is almost sure to follow, there being a tendency to bring the bubble too much towards the light. Error is also introduced through parallax, the bubble being viewed obliquely to its length. It has recently been urged by a German observer—Dr. Reinhertz—that the bubbles should be viewed in profile. Clearness of the glass and distinctness of the graduations have much to do with the accuracy of bubble readings. Errors of rod reading are more common with a target than with a speaking rod. The best check on the former is for both rodman and leveller to make independent readings. Where three wires are used, errors with self-reading rods are of very rare occurrence. The mean of the three readings is also without doubt more accurate than a single reading on a target rod. It does not by any means follow that because a target rod reads to .001 of a foot that the reading is accurate within that limit. A difference in the illumination of the rod will also affect the relative accuracy of the readings, and a line running east and west will probably show different results, according as it is levelled in the forenoon or afternoon.

Under *personal errors* we have merely to note that each observer has his own peculiarities, which will largely affect the resulting difference of level over a great length of line. This is in fact the personal equation of the observer. To quote from the report of the Chief Engineer, U. S. A., for 1884: "These discrepancies vary with different observers, and are not even constant for the same observer, are "nearly proportional to the distance, and seem to be independent of "the nature of the ground, the direction in which the work is done, "the season or the manner of supporting the rod." The results of some recent levels on the Mississippi survey go to show that this personal equation may be somewhat evanescent, particularly with young observers, and that every line of levels should be duplicated in opposite directions by the same observer within the shortest possible limits of time, in order to reduce the probability of change in the personal equation. With skilled observers of long practice, this habit is probably a constant from year to year.

For the effects of *atmosphere errors* I cannot do better than quote from Professor J. B. Johnson, who was for some time engaged on work in connection with the Lake survey. (Van Nostrands Mag. for Oct. 1883.)

"Errors from this source may be classified as coming from: 1, "Wind; 2, Tremulousness; 3, Variable Refraction.

"1. Wind generally shakes the instrument, and makes the holding "of the rod difficult or impossible. For two seasons I have used a "tent on windy days to protect the instrument, and with great success. "Good work can be done in this way so long as the rod can be held. "We also have large square canvas umbrellas that can be set on the "ground to the windward of the instrument, and these effectually "shield them in ordinary windy weather.

"The tents used were wall-tents, 5 × 6 feet, and one 8-foot centre "pole. A square iron frame, 3 × $3\frac{1}{2}$ feet, sewed into the canvas near "the top, formed the lateral support there. It was held down by six "or eight steel pins, 18 inches long and $\frac{1}{2}$ inch diameter, with flat "heads. These passed through iron rings sewed into the bottom. "There were openings for the line of sight and a flap for the observer "to enter and pass out with the instrument. These tents were made "to be used on Gulf coast at a very windy season, when one half the "time would have been lost from high winds without them. The rod-"men supported their rods by sticks held in the hand and braced "against the rod at an angle, resting on the ground. Care had to be "exercised that the rods were not thereby lifted from their sockets in "the foot plates.

"2. Tremulousness is caused by a difference of temperature between "air and ground, and always occurs in clear weather after the sun is "a few hours high. This causes the target, or figures on a speaking "rod, to appear to move up and down, giving rise to what is known as "dancing" or "boiling." This simply causes an uncertainty in "the reading, depending directly on the degree of unsteadiness. It "is a compensating error, and the observer must be his own judge as "to when he must stop work in order to obtain the required degree of "precision. The only remedy is to shorten the length of sight; but as "there are some errors that multiply directly with the number of sights "taken in a given distance, there is also a limit to which this remedy "may be profitably carried. I do not think it advisable to use sights "less than 100 feet if the highest accuracy is sought, and perhaps "never more than 400 feet, even when the atmosphere is perfectly "clear and steady. In clear weather not more than 3 or 4 hours a "day can be utilized for the best work.

"3. Variable refraction occurs when the sunshine suddenly comes "up or leaves the line; this happens along the edge of timber or under "the brow of a hill, as when the line rapidly emerges from or comes "into the shade from the sun's movement, or on partially cloudy days, "when the sun is alternately covered and clear. When from the first "source, it occurs about 8 a.m. and 4 p.m. It is a peculiar phenome-"non, and is more common in winter than in summer. The atmosphere "is apparently steady and the sight well taken; but upon checking it, "the reading has changed, and may be observed to change gradually or "suddenly, and sometimes to recover a part or all of its original move-"ment, when the instruments were known to be stable. I have seen "these changes of reading amount to 5 millimeters, or 1-5 of an inch "in a distance of 100 meters, or 328 feet. If the atmosphere is found "to be in this condition, the work should be stopped for a while, as "this state of affairs is not likely to continue long."

Errors due to carelessness—and their name is legion—need not be discussed. We can make no provision for the acts of the rodman, who, being sent to hunt up a turning point, triumphantly brings it to you in his hand; or yet for the leveller, who fails persistently to distinguish between a 6 and a 9.

Looking at the unavoidable errors of levelling in a more comprehensive manner, we may regard them as composed of three classes,—compensating errors, cumulative errors, and accidental errors. The first classes should be so manipulated as to eliminate themselves during the progress of the work. The second should be removed by the same observer repeating the work under as nearly as possible the same conditions, and in an opposite direction. Levels checked only in the same direction give fallacious results. The third are the legitimate errors inseparable from all observations, and are proportional to the square root of the distance. The errors of a properly conducted system of levels are usually considered to be of this character, and their precision tested accordingly.

The limit of error allowed n the.......... Ft.

U. S. Coast and Geodetic Survey is...... $0.029\sqrt{\text{Dist. in miles.}}$

Lake Survey.................................... $.041\sqrt{\text{Dist. in miles.}}$

Mississippi River Commission.................. $.021\sqrt{\text{Dist. in miles.}}$

between duplicate lines.

The following interesting table of the results of levelling in Great Britain, India and Switzerland has been compiled by Mr. Wilfrid Airy, M. Inst. C.E.

Average differences in a single mile of the results obtained by two observers, on ground of different degrees of inclination.

Character of Ground.	Great Britain.	India.	Switzerland.
	Foot.	Foot.	Foot.
Nearly level, very favorable circumstances of weather.	**0230**	**.0142**	.0125
Slightly undulating, gradients not exceeding 1 in 100.	.0238	.0168	.0148
Gradients between 1 in 100 and 1 in 20.	.0379	**.0208**	.0183
Gradients between 1 in 20 and 1 in 10.	**.0566**	**.0350**	.0308
Gradients steeper than 1 in 20.			.0416

Note.—The quantities in bold face type are estimated from analogy afforded by Swiss levelling, as no direct data could be furnished.

In illustration of the high degree of accuracy attained over long lines, the following is taken from the report of the levelling operations in India for 1866, by Colonel Walker :—

Section.	Length in Miles.	Maximum Divergence of two Observers.	Terminal Divergence.
		Foot.	Foot.
Calcutta to Tilliagarhi.	242	0.20	0.15
Tilliagarhi to Patka Gerouli.	346	0.40	0.38
Agra to Patka Gerouli.	342	0.15	0.05

Some excellent results over duplicated lines have in recent years been obtained with the Wye levels used in the engineering branch U. S. A. The methods adopted were practically those of precision levelling. As an example of these I extract the following :—

Section.	Length in Miles.	Max. Divergence of two observers in feet.	Terminal Divergence in Feet.
Sioux City to Fort Randal	179	.082	.060
Fort Randall to Pierre, Dak.	190	.156	.154

The best levelling has however undoubtedly been done in Switzerland. The field rules there adopted are as follows :—

1. The levelling to be executed by equal rights whenever possible ; the difference between the length of back and fore sights never to exceed ten metres.
2. The length of sight is as a rule to be limited as under :—
   (*a*) Upon railroads with gradients 1 in 100, to 100 metres.
   (*b*) " " " steep gradients 50, to 100 metres.
   (*c*) " highroads in the plains 30,to 60 metres.
   (*d*) " mountain roads 10, to 25 metres.
3. The spirit level to be always shaded from the sun.
4. The three instrumental errors, viz.: Collimation of optical axis, inequality of pivots, and bubble error to be determined at least once each day.
5. The field work to be carried on continuously except on wet or windy days. Three kilometres at least should be the length of line levelled per day along railways and two along highways.
6. Bench marks to be made at every kilometre, and to be clearly described in the field book.

In preparing this paper I have endeavoured to touch upon all classes of engineering levelling, naturally however the subject being one which bears more particularly on geodesic work, I have given greater attention to that department. In deprecation of a possible criticism to the effect that the major portion of the methods herein detailed are of no consequence to "practical" Engineers, I would beg to remind any so disposed that possibly their particular line of work has not embraced the whole sphere of labours of the profession. I would also wish to express the hope that the members of the Canadian Society of Civil Engineers may at some day not far distant be called to do geodesic levelling within the boundaries of their own country.