Technical and Bibliographic Notes / Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming are checked below.

- [x] Coloured covers / Couverture de couleur
- [ ] Covers damaged / Couverture endommagée
- [ ] Covers restored and/or laminated / Couverture restaurée et/ou pelliculée
- [ ] Cover title missing / Le titre de couverture manque
- [ ] Coloured maps / Cartes géographiques en couleur
- [ ] Coloured ink (i.e. other than blue or black) / Encre de couleur (i.e. autre que bleue ou noire)
- [ ] Coloured plates and/or illustrations / Planches et/ou illustrations en couleur
- [ ] Bound with other material / Relié avec d'autres documents
- [ ] Only edition available / Seule édition disponible
- [ ] Tight binding may cause shadows or distortion along interior margin / La reliure serrée peut causer de l'ombre ou de la distorsion le long de la marge intérieure.
- [ ] Blank leaves added during restorations may appear within the text. Whenever possible, these have been omitted from filming / Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.
- [ ] Additional comments / Commentaires supplémentaires:

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

- [ ] Coloured pages / Pages de couleur
- [ ] Pages damaged / Pages endommagées
- [ ] Pages restored and/or laminated / Pages restaurées et/ou pelliculées
- [x] Pages discoloured, stained or foxed / Pages décolorées, tachetées ou piquées
- [x] Pages detached / Pages détachées
- [x] Showthrough / Transparence
- [ ] Quality of print varies / Qualité inégale de l'impression
- [ ] Includes supplementary material / Comprend du matériel supplémentaire
- [ ] Pages wholly or partially obscured by errata slips, tissues, etc., have been refilmed to ensure the best possible image / Les pages totalement ou partiellement obscurcies par un feuillet d'errata, une pelure, etc., ont été filmées à nouveau de façon à obtenir la meilleure image possible.
- [ ] Opposing pages with varying colouration or discolourations are filmed twice to ensure the best possible image / Les pages s'opposant ayant des colorations variables ou des décolorations sont filmées deux fois afin d'obtenir la meilleure image possible.

This item is filmed at the reduction ratio checked below /
Ce document est filmé au taux de réduction indiqué ci-dessous.

10x	12x	14x	16x	18x	20x	22x	24x	26x	28x	30x	32x
							✓				

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)

1.0 2.8 2.5
3.2 2.2
3.6
4.0 2.0
1.1
1.8
1.25 1.4 1.6

DEPARTMENT OF THE INTERIOR, CANADA

HON. ARTHUR MEIGHEN, Minister; W. W. CORY, Deputy Minister

TOPOGRAPHICAL SURVEYS BRANCH

BULLETIN 41

TESTS OF SMALL TELESCOPES AT THE LABORATORY OF THE DOMINION LANDS SURVEYS

BY

E. DEVILLE, LL. D.,

SURVEYOR GENERAL OF DOMINION LANDS

OTTAWA

J. DE LABROQUERIE TACHÉ

[illegible] TO THE KING'S MOST EXCELLENT MAJESTY

1918

DEPARTMENT OF THE INTERIOR, CANADA

HON. ARTHUR MEIGHEN, Minister, W. W. CORY, Deputy Minister

TOPOGRAPHICAL SURVEYS BRANCH

BULLETIN 41

TESTS OF SMALL TELESCOPES AT THE LABORATORY OF THE DOMINION LANDS SURVEYS

BY

E. DEVILLE, LL. D.,

SURVEYOR GENERAL OF DOMINION LANDS

OTTAWA

J. DE LABROQUERIE TACHÉ

PRINTER TO THE KING'S MOST EXCELLENT MAJESTY

1918

30764—1

METHOD OF TESTING.

Introductory Note.

In surveying Dominion lands, astronomical observations are made on stars in daylight and the stadia is employed for measuring distances up to half a mile with rods divided to tenths of links. Both methods require telescopes of excellent quality in view of the limited dimensions compatible with ordinary survey instruments. The staff of the Surveys laboratory have thus been led to devote considerable attention to tests of small telescopes. It is proposed to describe their method of testing and incidentally, to give the experimental data gathered in working out the method.

The investigations were conducted by W. C. Way, who is in charge of the laboratory. All the data were observed by him and by his assistants.

The Essential Characteristics of a Surveyor's Telescope.

The telescope's features which are of interest to the surveyor are the brightness, the magnification, the field of view, the resolution or resolving power, and, to a lesser degree, the fall in resolution from the centre to the margin of the field. Distortion, which would affect the readings of a rod near the margin, is of no importance, rods being read always in the centre. The various aberrations do not interest the surveyor otherwise than as they affect the resolution.

The measurement of the magnification and of the field of view is a simple matter. Disregarding the loss of light through absorption and reflection, the brightness may, for the surveyor's purposes, be taken as proportional to the area of the exit pupil of the telescope, the diameter of which can be either measured or calculated from the objective's diameter and the magnification. All that remains to be ascertained by the tests is the resolution.

The Resolution.

In the system adopted by the Surveys Laboratory, the unit of resolution or resolving power is the resolution of unassisted vision. If a be the smallest detail of an object that can be seen with the naked eye, the resolution of a telescope is held to be R when the detail seen through it is:

$$\frac{a}{R}$$

In other words, the resolution is R when the details seen through the telescope are R times smaller than can be seen with the naked eye.

The resolution is determined by trial of the telescope upon a test object.

Apparatus.

The test object is a transparency 40 x 45 centimetres presenting a series of sets of clear circular dots on an opaque ground. It is illuminated from behind by eight 40-watt tungsten lamps. Three suitably spaced ground glass plates between the lamps and the transparency diffuse the light and produce even illumination. The dots in each set are uniformly spaced, their diameter being one-tenth of their interval. This interval decreases in geometric progression from one set to the adjoining one, the ratio of the progression being:—

$$1{\cdot}05^{-1}$$

Each of the smaller sets contains 2,500 dots. To save space, the larger ones with coarse dots have not quite so many. The upper part of figure 8 represents portions of four adjoining sets.

Beyond the limit of resolution, a set appears as a surface uniformly illuminated. The test consists in ascertaining the finest set in which structure can be detected.

The apparatus is installed in the building of the comparator for measures of length. The transparency is against the wall at one end of the building and the telescope stand near the other end, 42·80 metres away. A small lamp behind the observer gives just enough light for manipulating the telescope: otherwise the room is in complete darkness.

Numbering of the Sets.

That set in which structure can just be detected from the telescope stand by unassisted vision is numbered one. The angle subtended by the dot interval of this set was found by the laboratory observers to be 0·000 405, or $\frac{1}{2470}$, about 1′23″. It is one inch at 206 feet, 8 inches at 25 chains, and 12·8 inches at half a mile.

Any other set is numbered *R* when the angle subtended by the dot interval is

$$\frac{0{\cdot}000\ 405}{R}$$

It follows that the resolution of a telescope is the serial number of the set in which structure can just be detected through the telescope.

The constant 0·000 405, from which the serial numbers are calculated, is a physiological factor which varies with individuals: it may even vary with the physical condition of the observer. When proceeding with resolution determinations, the observer must, from time to time, ascertain his personal equation or factor by observing with a telescope of known resolution. All the results of observation are multiplied by the personal factor found.

Observation of the Resolution.

The observation for resolution is first made by examining the image of the dots in the centre of the field and noting the smallest set in which structure can be detected. With a good telescope, structure is quite plain in one set and absent in the adjoining one. The correct number is interpolated by estimation. With a poor telescope, the determination is more uncertain: this uncertainty is a fairly good test of quality.

The falling off in definition away from the centre is measured by the decrease in resolution read upon an eccentric image. By adopting a constant angle for the direction of the eccentric image the results with different magnifications or telescopes are comparable.

Telescopes of equal resolution do not always disclose equal detail under all conditions. If the object viewed is feebly illuminated, the telescope having the largest exit pupil and the brightest image shows greater detail and the other telescopes appear to have less resolving power, but with the intense illumination provided behind the transparency dots, the changes in brightness due to difference in area of the exit pupil do not, within practical limits, affect the resolution of the telescope.

Aberrations.

The nature of the aberrations is disclosed by the changes which they cause in the aspect of the dots in and out of focus. So far, little attention has been devoted to the subject at the laboratory as it is not of material importance for the purpose in view.

Reading of Graduated Rods.

Among the problems which present themselves to the surveyor is to find the resolution necessary for reading graduated rods.

Let d be the width of one division of the rod and L the distance. The angle subtended by one division being $\frac{d}{L}$, the resolution required for detecting structure is:—

$$\frac{0{\cdot}000\ 405 \times L}{d}$$

This resolution is just enough to perceive that the rod is graduated. For counting and reading the divisions, the laboratory experiments indicate that about one-fourth more resolution is required.

$$R = \frac{5}{4} \times \frac{0{\cdot}000\ 405 \times L}{d}$$

or approximately:—

$$R = \frac{L}{2000d} \qquad (1)$$

A number of problems can be solved by means of the above relation between the distance of a rod, the width of the divisions, and the resolution needed to read them.

Example 1.—What resolution is needed for measuring half a mile with a stadia rod graduated to tenths of a link?

In equation (1), L and d are 40 and 0·001 chains respectively, hence:—

$$R = 20.$$

Example 2.—How far can a rod divided to hundredths of a foot be read with a resolution of 15?

Equation (1) gives:—

$$L = 300 \text{ feet}.$$

Example 3.—What is the smallest division which can be read on a rod at 440 metres with a resolution of 22?

Equation (1) gives:—

$$d = \cdot 01 \text{ metres}.$$

EXPERIMENTAL DATA.

Further Definitions.

In working out the method of testing just described, the laboratory staff had to make a number of experiments which throw some light upon the causes affecting the resolution and may be related as connected with the subject.

The definition of "resolution" may be extended to the case of an objective; the resolution may be called R when the dots of the set number R are separated in the focal plane image.

By definition, the resolution of the eye at full aperture is unity. The resolution becomes less than unity when the effective aperture of the pupil is sufficiently reduced, as when looking through a small hole or through the exit pupil of a telescope of high magnification.

Notation.

R – Resolution or resolving power of a telescope or of an objective. It has already been defined.

M—Magnification of a telescope. It is th ratio of the focal lengths of objective and ocular, or of the diameters of effective aperture and exit pupil.

D—Diameter of an objective or effective aperture of a telescope, in millimetres.

F—Focal length of objective in millimetres.

f—Equivalent focal length of ocular in millimetres.

e—Diameter of the exit pupil of a telescope or of a circular diaphragm placed in front of the eye, in millimetres.

$\frac{R}{D}$—Resolution factor of an objective or telescope, or resolution per millimetre of aperture.

$\frac{M}{D}=\frac{1}{e}$—Magnification factor of a telescope or magnification per millimetre of aperture. It is equal to the reciprocal of the diameter of the exit pupil.

r—Resolution or resolving power of the eye when looking through a circular diaphragm or a telescope's exit pupil.

$\frac{r}{e}$—Resolution factor of the eye when the diameter of the pupil is e. It varies with the pupil aperture.

T, t—Angle subtended by a dot interval or by the radius of a diffraction ring.

λ—Wave length of light, in microns.

General Principles.

The angle t subtended by the dot interval of the set R is:—

$$t=\frac{0\cdot000\ 405}{R}$$

Seen through a telescope, the angle is magnified M times and becomes:—

$$T=0\cdot000\ 405\ \frac{M}{R}$$

To be able to resolve this angle, the resolution of the eye must be not less than:—

$$r\geq\frac{0\cdot000\ 405}{T}=\frac{R}{M}$$

or:—

$$R\leq Mr \qquad (2)$$

Hence:—

(1) *The resolution of a telescope cannot be greater than the product of the resolution of the eye corresponding to the size of the exit pupil, by the magnification.*

With high magnifications, the exit pupil is small and the resolution of the eye less than unity. Hence:—

(2) *With high magnifications, the resolution of a telescope is always less than the magnification.*

The aperture D of the telescope being equal to Me, equation 2 may be changed to:—

$$\frac{R}{D} \leq \frac{r}{e}$$

Hence:—

(3) *The resolution factor of a telescope cannot be greater than the resolution factor of the eye for a pupil aperture equal to the exit pupil of the telescope, nor can it be greater than the resolution factor of the objective. It must therefore be the lower of the two factors.*

According to these rules, the resolution of a telescope is limited either by the resolution of the objective, or by the resolution of the eye in conjunction with the ocular. Each must be investigated separately.

Diffracted Images.

It is shown by the mathematical theory of diffraction that the image of a luminous point in the focal plane of an objective is not a point, but a bright circular patch surrounded by alternately dark and bright rings. The angular radius t of the first dark ring is:—

$$t = 1220\frac{\lambda}{D}$$

in which λ is the wave length of light and D the diameter of the objective.

It is generally accepted that two luminous points are seen separated when the first dark ring of one image passes through the centre of the other image, that is, when the angular distance of the points is equal to the radius of the first dark diffraction ring. The illumination midway between the points is then 74 per cent of the maximum.

If this rule were applicable to the dot transparency, the resolution of the objective would be:—

$$R = \frac{0 \cdot 000\ 405}{t} = \frac{0 \cdot 322}{\lambda} D$$

Taking 0·56 microns for the wave length of the brightest part of the spectrum:—

$$R = 0 \cdot 59\ D$$

The rule is not strictly applicable to the dot transparency which must give slightly greater resolution. Moreover, the value of the coefficient depends upon the quality of the definition required in the image.[1]

The highest value derived from visual experiments by the laboratory staff is:—

$$R = 0 \cdot 63\ D$$

The illumination of the darkest spots of the image is then 59 per cent of the brightest spots.

Resolution of the Eye.

As already stated the angle subtended by the dot interval of transparency set number one, which can just be resolved by unassisted vision from the telescope stand, was found to be 0·000 405.

This coefficient can also be obtained by observation of the dots at closer range. No material change was noticed until the distance from the transparency became smaller than 7·50 metres, when the coefficient commenced to decrease. The observations could not be made at distances less than one metre because of the impracticability of producing by photography sets of dots sufficiently fine, the dots being enlarged by diffraction. The difficulty was overcome by observing with an ocular of long focal length the image in the focal plane of a good objective of large numerical aperture; the mean of a great number of observations gave 0·000 37. It would thus appear that the vision of the observers was ten per cent keener when looking at an image through an ocular than when looking at the object itself with the naked eye. According to this, a telescope of unit magnification, consisting of an objective and an ocular of equal focal lengths, would have a resolution of 1·10 and would show objects ten per cent better than unassisted vision. This deduction has not been verified experimentally.

The resolution of the eye at full aperture, which is unity when looking at objects from a distance exceeding 7·50 metres, becomes 1·10 at close range. Reverting to equation (2),

$$R \leqq Mr$$

the following additional rule may now be formulated:—

(4) *At low magnifications, the power of a telescope may be ten per cent greater than the magnification, but no more.*

This rule has been confirmed experimentally.

Whether the increase in the acuity of vision at close range is an actual fact or whether the effects observed are due to other causes and may be explained otherwise, is immaterial for the purposes of this investigation.

(1) The Resolving Power of Objectives, by P. G. Nutting—Bulletin of the Bureau of Standards. Vol. 6, No. 1.

Relation Between the Pupil Aperture and the Resolution of the Eye.

Several series of experiments were undertaken for ascertaining the relation between the effective pupil aperture and the resolution of the eye.

Circular holes were drilled through a thin brass plate and carefully measured with a micrometer microscope. The plate was fixed at various distances from the transparency and the dots examined through the holes. The resolution was obtained by multiplying the number of the observed set by:—

$$\frac{L}{42 \cdot 80}$$

L being the distance in metres between the perforated plate and the transparency.

For data at very close range, the plate was placed at the exit pupil of good telescopes of large numerical aperture and low magnification. Two of the series are given below, one at a distance of 7·50 metres from the transparency and the other with oculars of 32 mm. focal length and smaller. The figures are the mean of a large number of observations.

RELATION BETWEEN THE PUPIL APERTURE AND THE RESOLUTION OF THE EYE.

Pupil Aperture, *e*, in Millimetres.	Resolution, *r*, at a Distance of	
	7·50 metres.	0·032 metres and less.
0·25	......	0·157
0·37	......	0·230
0·53	......	0·336
0·65	0·380	0·404
0·77	0·454	0·485
1·02	0·588	0·627
1·18	0·661	......
1·19	......	0·726
1·34	0·738	0·805
1·56	0·829	0·890
1·84	0·916	0·956
2·02	0·962	1·041
2·50	1·000	1·102
2·75	1·000	1·104

The observations are plotted in Fig. 1. At 7·50 metres from the transparency, the resolution of the eye for apertures between 0 and 0·9 mm. is proportional to the aperture, the equation being:—

$$r = 0{\cdot}59\ e$$

It is the same equation as for a good objective with a somewhat smaller resolution factor.

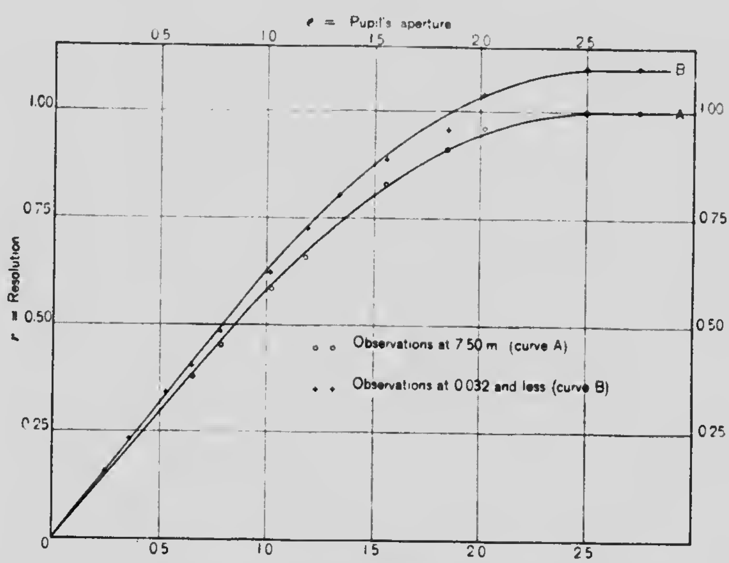


FIG. 1—Relation between the pupil's aperture and the resolution of the eye.

Above 0·9 mm. the resolution increases more slowly than the aperture until it reaches unity for an aperture of 2·5 mm. Beyond 2·5 mm. there is no increase in the resolution; it remains constant at unity, which agrees with the definition of resolution.

With oculars at close range, the resolution for apertures between 0 and 0·9 mm. is also proportional to the aperture, but the equation is:—

$$r = 0{\cdot}63\ e$$

This is precisely the equation for an objective of good quality.

Above 0·9 mm, the resolution increases more slowly than the aperture until it reaches a value of 1·10 for an aperture of 2·5 mm. Beyond 2·5 mm. the resolution remains constant at 1·10. The resolution of the eye at full aperture, which, by definition, is 1·00 for distant objects, becomes 1·10 at close range, thus showing keener vision at close range than at a distance.

Taking 15 mm. for the equivalent in air of the focal length of the eye, the pupil's diameters of 0·9 mm. and 2·5 mm. correspond to numerical apertures of $F/17$ and $F/6$. Up to $F/17$, its resolution is, like that of a lens, proportional to the diameter of the pupil, but it falls short of a good lens at greater apertures.

For application to telescope tests, it is more convenient to plot the resolution and magnification factors. The data are as follows:

RELATION BETWEEN THE RESOLUTION AND MAGNIFICATION FACTORS OF THE EYE.

Magnification Factor. $\frac{1}{e}$	Resolution Factor, $\frac{r}{e}$, at a Distance of	
	7·50 metres.	0·032 metres and less.
0·364	0·364	0·402
0·400	0·400	0·441
0·495	0·476	0·516
0·544	0·498	0·520
0·641	0·532	0·570
0·746	0·551	0·601
0·84	.....	0·610
0·85	0·560	.....
0·98	0·577	0·615
1·30	0·590	0·630
1·54	0·585	0·622
1·89	.....	0·634
2·70	.....	0·622
4·00	.....	0·628

The data are plotted in Fig. 2. The observations at $7^m.50$ are marked with a small circle: those at short range with a cross. The smooth curves *A* and *B* represent very nearly the two sets of observations.

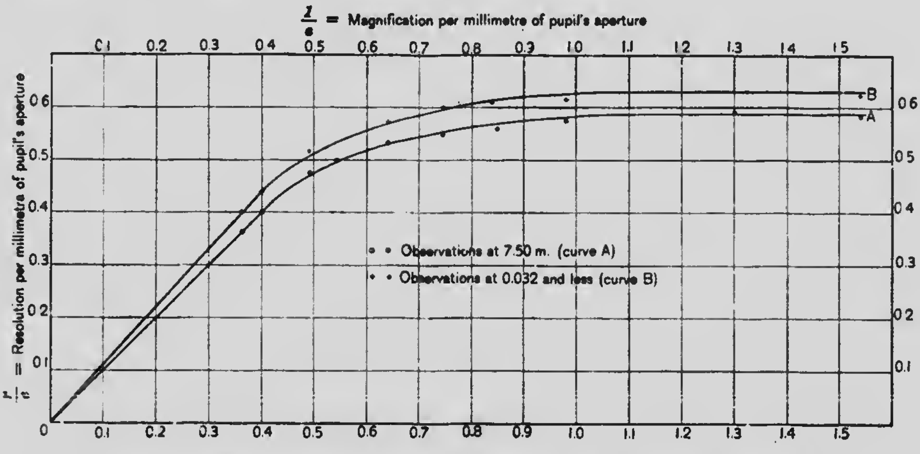


FIG. 2—Relation between the resolution and magnification factors of the eye.

At 7^{m} 50, the magnification and resolution factors are equal so long as they are below 0·4 (pupil 2·5 mm or more). Above 0·4 the resolution factor increases more slowly than the magnification factor until it reaches a maximum and constant value of 0·59 for a magnification factor of 1·10.

In the test at close range, the magnification and resolution factors are proportional so long as they are below 0·4 but they are not equal.

The relation is:—

$$\frac{r}{e} = \frac{1}{e} \times 1{\cdot}10$$

which means that the resolution is constant and equal to 1·10. Above 0·4, the resolution factor increases more slowly than the magnification factor until it reaches a maximum value of 0·63 for a magnification factor of 1·10, beyond which it remains constant. The end of the curve *B* is not shown in the figure: it merely extends to the right as a parallel to the axis of abscissae, as far as magnification factor 4·00.

This maximum value of 0·63 compared with 0·59 at 7^{m} 50, illustrates again but in a different manner, keener vision at close range.

The values of the resolution factor at close range, measured from curve *B* of Fig. 2, are given below: they are the values to be used in telescope tests.

MAGNIFICATION AND RESOLUTION FACTORS OF THE EYE AT CLOSE RANGE.

Magnification Factor.	Resolution Factor.	Magnification Factor.	Resolution Factor.
0·2	0·220	0·7	0·587
0·3	0·330	0·8	0·607
0·4	0·440	0·9	0·619
0·5	0·511	1·0	0·627
0·6	0·556	1·1	0·630

Resolution of a Telescope.

It is now possible to understand how a telescope should work. In Fig. 3, *O D F* is the curve of the eye's resolution factor (curve *B* of Fig. 2). The resolution factor of the objective being a constant, it is represented upon this diagram by a parallel to the axis of abscissae, which may occupy one of three positions.

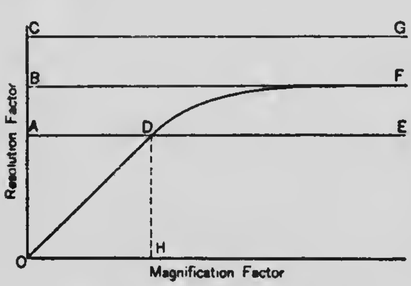


Fig. 3—Resolution of a telescope

If the factor is less than 0·63, the line *A E* cuts the curve at *D*. For a magnification factor comprised between *O* and *H*, the telescope's resolution factor is that of the eye: for a greater magnification, it is that of the objective. The resolution is limited by the objective: the eye could see more than the objective can resolve.

An objective with a resolution factor of 0·63 is represented by the line *B F* tangent to the curve. The resolution factor of the telescope is that of the eye, whatever the magnification.

C G represents an objective with a resolution factor greater than 0·63. The telescope's resolution factor must also in this case, be that of the eye, whatever the magnification. The resolution is limited by the eye: the objective can resolve more than the eye can see. If such an objective exists, the eye is not perfect enough as an optical instrument to make use of its full resolving power.

It would appear from the figure that a telescope of low magnification does not require a good objective; provided the objective's resolution factor is a little greater than the magnification factor, objects should be seen as well as if the objective were of better quality.

Conversely, high magnification does not improve the resolution when the objective is poor.

Experiments with Telescopes.

In order to ascertain whether these theoretical deductions are confirmed by observation, experiments were made with three telescopes which, tested by ordinary methods, had proved to be of excellent quality. Five oculars were used and the telescope's aperture was varied by diaphragms in front of the objective. This procedure is open to the objection that a high grade objective is corrected to work at its best with one particular ocular. If the ocular is of short focus, the objective appears undercorrected for colour at lower magnifications and the definition is impaired.[1]

This was evidently the case, more or less, with two of the three telescopes experimented with. In the other telescope, which had a comparatively small numerical aperture, the effect is not so apparent.

The results of the tests are tabulated below and are plotted in Figs. 4, 5, and 6. On each figure, the curve represents the resolution factor of the eye, transferred from Fig. 2 (curve *B*).

RESOLUTION EXPERIMENTS WITH TELESCOPE NO. 797.

$F = 949$mm.

f	*M*	*R* for Objective Apertures of					
		76·0mm	69·0mm	55·3mm	46·4 mm	37·4mm	30·7mm
32·00mm	29·66	32·9	32·0	28·9	26·6	22·7	19·5
20·95	45·30	41·0	38·2	32·9	28·9	23·6	19·5
15·75	60·26	44·7	41·8	34·9	29·3	23·8	19·5
9·06	104·7	47·5	44·0	34·9	29·8	24·1	19·5
4·10	231·5	47·7	44·7	35·6	29·5	23·9	19·5

(1) On the Adjustment and Testing of Telescopic Objectives. By H. Dennis Taylor.

RESOLUTION EXPERIMENTS WITH TELESCOPE NO. 21.

$F = 410 \cdot 4$mm

f	*M*	*R* for Objective Apertures of						
		50·6mm	38·9mm	33·5mm	30·0mm	24·7mm	20·2mm	16·3mm
32·00mm	12·82	12·9	12·6	12·5	12·4	12·1	11·2	9·9
20·95	19·59	19·4	18·4	17·2	17·0	14·4	12·1	10·4
15·75	26·06	24·4	21·1	19·2	18·0	14·8	12·2	10·4
9·06	45·30	30·2	24·3	21·0	18·9	15·4	12·3	10·4
4·10	100·10	32·3	25·1	21·5	19·2	15·4	1.	10·1

RESOLUTION EXPERIMENTS WITH TELESCOPE NO. 13167.

$F = 255.8$mm

f	*M*	*R* for Objective Apertures of					
		38·0mm	30·9mm	23·9mm	21·3mm	19·2mm	15·4mm
32·00mm	7·99	8·53	8·53	8·53	8·39	8·17	7·79
20·95	12·21	12·1	11·8	10·9	10·9	10·4	9·00
15·75	16·24	13·8	14·5	12·7	12·0	12·1	9·44
9·06	28·23	21·7	18·7	14·6	13·2	12·0	9·72
4·10	62·39	24·0	19·6	14·7	13·2	12·1	9·74

For telescope No. 797, Fig. 4, the experimental points lie very close to the curve: the agreement is almost perfect. The observations at magnification

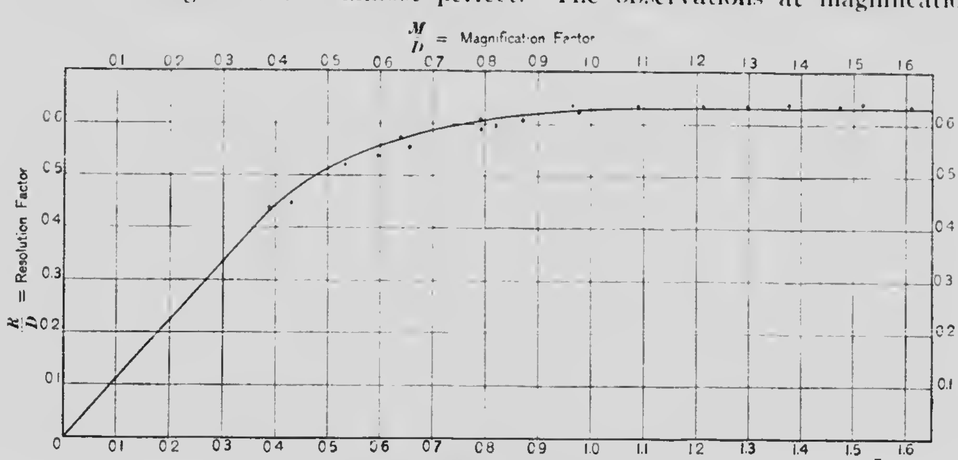


Fig. 4—Experiments with telescope No. 797.

factors greater than 1·65 have not been plotted because the figure would extend too far to the right; the resolution factor for all these observations is very nearly 0·63 so that they all lie on the extension of the curve parallel to the axis of abscissae. These observations have also been omitted from the other figures for the same reason.

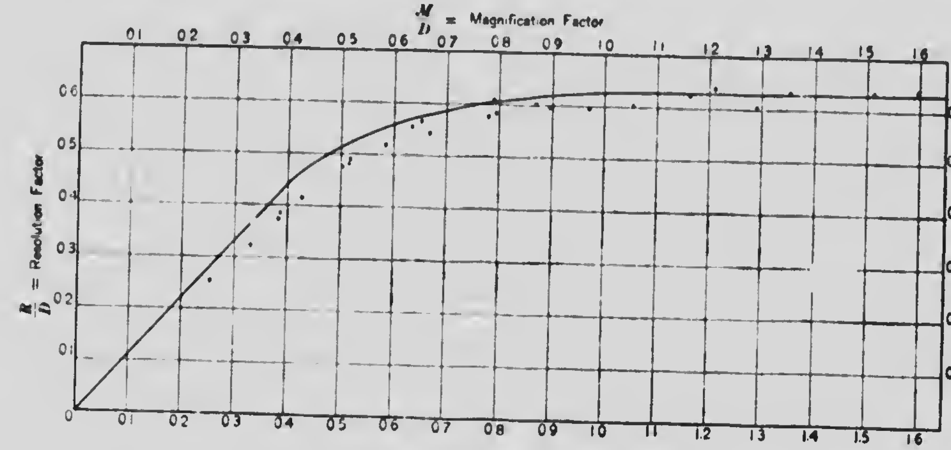


FIG. 5—Experiments with telescope No. 21.

With telescope No. 21, Fig. 5, the agreement is good at high magnifications but there is a slight falling off at medium magnifications.

The falling off at medium magnifications is a little more marked with telescope No. 13167, Fig. 6, but the agreement is good at high and low magnifications.

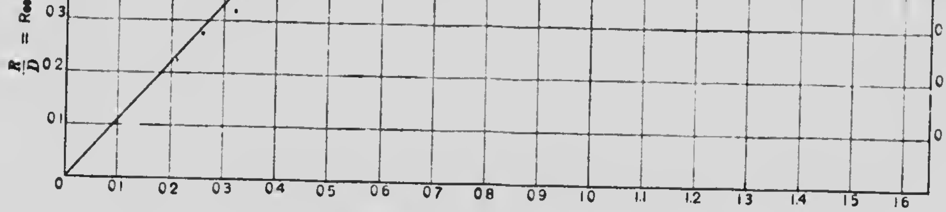


FIG. 6—Experiments with telescope No. 13167.

On the whole, the agreement is about as good as could be expected in observing with objectives and oculars which were not corrected to work together.

Experiments with an objective of poor quality are given below: they are plotted in Fig. 7.

RESOLUTION EXPERIMENTS WITH TELESCOPE NO. 1074.

$F = 380$ mm. $D = 43 \cdot 5$ mm.

Magnification	11·9	18·1	21·8	41·8	92·
Resolution	12·8	17·0	20·0	21·5	21

At the low magnification of 11·9 (exit pupil = 3·66 mm) which is little more than one-quarter of the objective's aperture in millimetres, the resolution of the telescope is as good as that of a perfect instrument. It is limited by what the

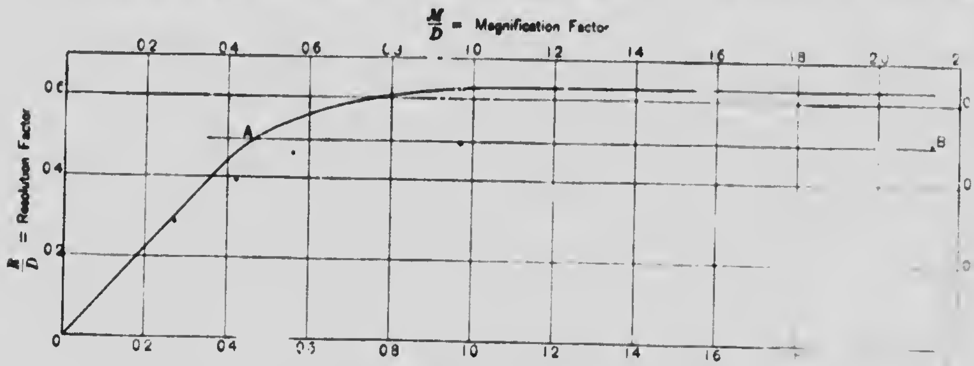


FIG. 7—Experiments with telescope No. 1074.

eye is able to see through the ocular, which is less than is resolved by the objective. The quality of the objective does not come into play.

The defective performance of the objective is brought out at the high magnifications of 41·8 and 92·6 (Exit pupils = 1·04 and 0·47 mm). If the objective were a good one, the resolution should be 27 and 27·4 while it is only 21·5 and 21·8. The eye could see through the ocular more detail than is resolved by the objective.

On the figure, the resolution factor of the objective, 0·50, is represented by the line *A B* parallel to the axis of abscissae and intersecting the curve of the eye's resolution factor at *A*. The curve for the resolution factor of the telescope should therefore be the line *O A B*. The observed data agree fairly [illegible] this conclusion but the sharp angle at *A* is rounded off. The effect of the [illegible] quality of the objective is felt before the limit of its resolution is reached.

Resolution of a Perfect Objective.

The maximum resolution factor of telescopes 797, 21, and 13167 found by observation is 0·63; it could not be more because that is the limit for the eye. What the observation shows is that the factor for the objectives is no less than 0·63, but it may be greater. The test fixed only a minimum limit for the objective's factor; the actual value cannot be determined by visual observation.

In order to elucidate what the actual factor may be, four consecutive sets of the dot transparency near the limit of resolution were photographed on a bromide dry plate.

The diameter of the objective was 0·9 mm. With a resolution factor of 0·63, the resolution of the lens would have been:—

$$R \text{ (visual)} = 0\cdot567$$

Resolution being inversely proportional to the wave length of light, this number must be multiplied by the ratio of the wave lengths of the light acting on the eye and on the bromide plate, about 0·56 and 0·45 microns respectively —

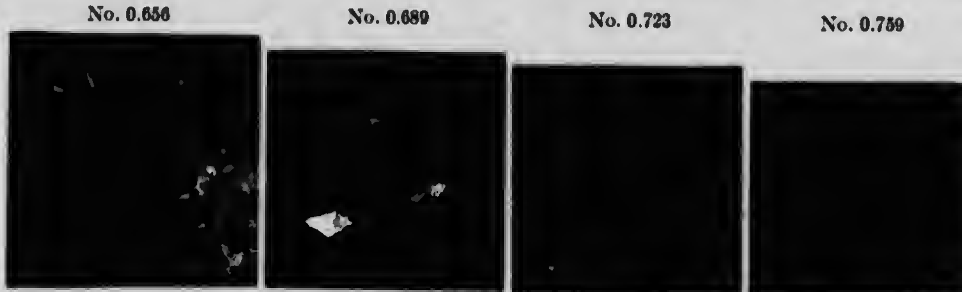
$$R \text{ (actinic)} = 0\cdot706$$

In other words, if 0·567 is the resolution for visual rays, it is 0·706 for actinic rays.

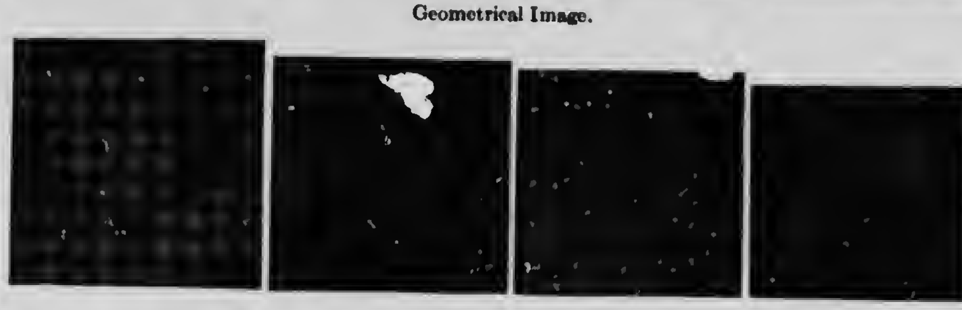
The sets photographed were Nos. 0·656, 0·689, 0·723, and 0·759. The first two only should have been resolved.

The photograph, enlarged three and a half times, is reproduced below (Fig. 8). The upper part of the figure is the geometrical image of the dots and the lower part the actual photograph. Structure is quite distinct in Nos. 0·656 and 0·689; it is fairly plain in No. 0·723 but is nearly obliterated in No. 0·759. In

No. 0.656 No. 0.689 No. 0.723 No. 0.759



Geometrical Image.



Actual Photograph.

FIG. 8—Transparency dots and their photograph near the limit of resolution

the visual tests, an image like this would be read as 0·75; it corresponds for visual rays to a resolution of 0·60 and a resolution factor of 0·67. The objective appears to resolve 6 per cent more than can be seen by the eye. However, further experiments are required before a definite conclusion can be reached.

General Considerations.

As long as the diameter of the exit pupil is not less than 2·5 mm, the resolution of an ordinary telescope of fair quality should not be less than the theoretical value. Lower values are however found in prismatic binoculars, probably due to the kind of glass used for the prisms.

Little is to be gained by reducing the exit pupil below 1·32 mm (magnification factor = 0·76). The resolution, if the telescope is good, is only five per cent below the limit and any increase in the magnification is more than counterbalanced by the loss of brightness of the image.

If a telescope has an exit pupil of 2·5 mm, the substitution of an objective of larger aperture without change in the magnification has no other effect than increasing the brightness of the image. It does not increase the resolution.

Resolution of a definite value can be obtained in a number of ways and with telescopes of various sizes; it can be had with a large objective aperture and low magnification, or with a small objective aperture and high magnification. A resolution of 14, for instance, is obtained with a good telescope of one and a quarter inch aperture (31·7 mm) and a magnification of 12·7, also with an aperture of seven-eighths of an inch (22·2 mm) and a magnification of 22·3. In good light, a graduated rod can be read as far with one telescope as with the other but there is a vast difference in the aspect of the images. In the larger telescope, the image is very bright and sharp, the field is large and the divisions of the rod, although very minute, can easily be read. In the smaller telescope, the image is coarse and dark and the field very restricted; the divisions of the rod, although nearly twice as large as in the smaller telescope, cannot be read any better or more accurately. To the superficial observer, it looks as if the larger telescope were of better optical quality. On the other hand, the volume and weight of the larger telescope, which are proportional to the cube of the linear dimensions, are nearly three times those of the smaller telescope, but unless the size and weight are inconvenient, the larger one is the more pleasant to work with.

The Proper Size of Telescope for a Land Surveyor's Theodolite.

The best size for a surveyor's telescope is the smallest that will just meet his requirements. From a mechanical point of view, there is an advantage in reducing the bulk and weight of a theodolite's telescope; the instrument is less subject to strains and the measurements are more accurate.

For general purposes, a magnification of 12 to 14 is convenient and sufficient. With an objective of $1\frac{1}{8}$ inches aperture (28·6 mm) the resolution is 13·1 to 14·7 and the exit pupil 2·38 to 2·04 mm, which gives a fairly bright field.

For Dominion land surveyors who have to observe the pole star in day light and to read at half a mile stadia rods divided to tenths of links, a resolution of not less than 20 is needed. This is obtained with an objective aperture of $1\frac{3}{8}$ inches (35 mm) and a magnification of 22·7. The exit pupil is 1·54 mm, which still leaves the field fairly bright. The magnification may be increased to 25 for which the resolution is 20·6, but the field is not so bright.

Magnifications of 22·7 and 25 are inconvenient fo. general purposes; the field is too restricted and time is lost in finding the pickets or other marks of the survey. A low power eye-piece should be provided for use when the high power is not needed.

The above remarks apply to the ordinary engineering and surveying instruments intended to measure angles to half a minute or thereabouts. The resolution necessary for resolving half a minute is only 2·8; the power of the telescopes specified is therefore more than ample for the measurement of angles.

The size of the telescope of a level instrument depends upon the graduation of the rod and the distance at which it is to be read. The necessary resolution is given by equation (1).