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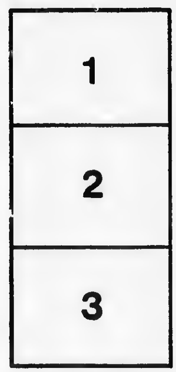

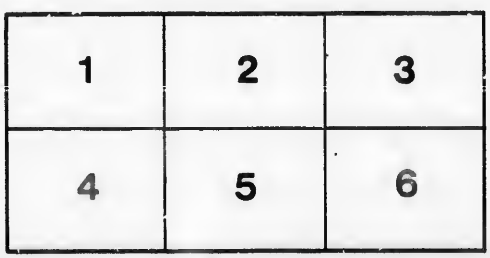

THE PRINCIPLES OF THE DOCTRINE OF LIFE-ANNUITIES;

EXPLAINED IN A FAMILIAR MANNER,

SO AS TO BE

Intelligible to Perſons not acquainted with the Doctrine of CHANCES;

AND ACCOMPANIED

With a VARIETY of NEW TABLES

Of the Values of ſuch ANNUITIES at ſeveral different Rates of Intereſt, both for Single Lives and for Two Joint Lives,

ACCURATELY COMPUTED FROM OBSERVATIONS.

By FRANCIS MASERES, Eſq; F.R.S.
CURSITOR-BARON OF HIS MAJESTY'S COURT OF EXCHEQUER.

LONDON:
Printed for B. WHITE, at HORACE'S-HEAD, FLEET-STREET.

M.DCC.LXXXIII.

PREFACE.

THE following work is grown to so much greater a size than was at first intended, that it seems to be necessary to give my readers some previous information of its contents, that they may be able to distinguish between the different parts of it, (which are by no means all equally interesting) and to select those which shall most excite their curiosity, and be thought most deserving of their perusal.

The principles of the whole doctrine are contained in the first 90 pages, which I would therefore recommend to the attentive perusal of every reader. Of these the two first pages contain an explanation of the *data*, or grounds, upon which the computations of the values of annuities for lives are built. These are, first, the decrease of the present value of a future sum of money arising from the mere distance of the time at which it is to be paid, and the consequent discount that is to be allowed to the purchaser of it for prompt payment, (the quantity of which discount, it is evident, will depend on the rate of the interest of money;) and, secondly, the chance which, when the payment of such future sum is not made certain, but is to depend on the continuance of the life of a person of a given age, the grantor of it has of escaping the necessity of paying it at all by means of the death of the said person before it becomes due; in order to determine which chance, it is necessary to have recourse to certain tables of the several probabilities of the duration of human life at every different year of age, which have been formed from observations of the numbers of persons who have died every year, in the course of a long series of years, at different ages, in divers cities and parishes, and other numerous bodies of men.

In

In pages 3, 4, 5, 6, an account is given of two tables of these probabilities of life that appear to me to be better grounded, and consequently fitter to be adopted, than any others; to wit, those of Monsieur Kersseboom and Monsieur de Parcieux: and the tables themselves are exhibited. And in pages 7, 8, 9, &c.——— 15, a comparison is made between these two tables, in order to discover which of them represents human life, at several different ages, as the more durable, or makes the probabilities of living greater than the other: and it is found, upon the said comparison, that till the age of 70 years, or for all persons under the age of 70 years, the probabilities of living are rather greater according to Monsieur de Parcieux's table than according to Monsieur Kersseboom's; but that after the age of 70 years, or for persons above the age of 70 years, the probabilities of living are greater according to Monsieur Kersseboom's table than according to Monsieur de Parcieux's.

In page 15 a preference is given to Monsieur de Parcieux's table above that of Monsieur Kersseboom; and the reason of the said preference is stated.

In page 16 mention is made of the *Breslaw* table of the probabilities of the duration of human life, which was formed and published by the celebrated Dr. Edmund Halley towards the end of the last century, (and which, I believe, was the first table of the kind ever published) and of the *London* table of those probabilities published by Mr. Smart and adopted by the learned Mr. Thomas Simpson, of Woolwich, and likewise of two other tables of those probabilities published by the Rev. Dr. Richard Price, of Newington-Green, which were derived from observations made at Norwich and Northampton. And in page 17 the reader is referred to the most celebrated writers on this subject for an explanation of the manner in which these tables of probabilities are formed from parish-registers, or other memorials, of the births and burials of mankind. And in pages 18 and 19, I have inserted some very useful remarks of the late very learned Mr. de Moivre on the several different merits of the four tables before-mentioned, of Dr. Halley, Monsieur Kersseboom, Monsieur de Parcieux, and Messieurs Smart and Simpson. And with these remarks I conclude the account of the *data*, or grounds, upon which the computations of the values of life-annuities are to be founded.

I then

I then proceed to lay down the fundamental maxim of the doctrine of life-annuities, or to give a definition of what is meant by the *fair price* or *value* of a life-annuity; which I have done with as much care and exactneſs as I was able, in pages 20 and 21, articles XXI and XXII.

And, having thus ſettled the grounds of the enſuing methods of computing the values of annuities either for terms of years, or for lives, I then proceed to deliver thoſe methods themſelves by the ſolution of four problems that contain them.

The firſt problem is, "To find the preſent value of a future ſum of money, which is certainly to be paid at the end of one or more years, according to any given rate of intereſt." The ſolution of this problem, with an illuſtration of it by an example, and a corollary to it, is contained in pages 21 and 22. And in page 23 an account is given of Mr. Smart's moſt valuable tables of intereſt, which are computed upon the principles explained in this problem.

The ſecond problem begins in page 24, and ſhews "how we may find the preſent value of a future ſum of money that is to be received at the end of a given number of years in caſe a perſon of a given age ſhall then be living, but not otherwiſe, according to any given rate of intereſt." Of this problem two ſolutions are given; the firſt a particular one, in which the given number of years, at the end of which the money is to be paid, and the number of years in the age of the perſon on whoſe life it depends, are both ſpecified; the other general, in which the ſaid numbers are repreſented, (as is uſual in Algebra,) by two letters of the alphabet. The former of theſe ſolutions, it is hoped, will make the ſubject familiar to the reader, and ſerve to facilitate to him the peruſal of the latter ſolution, which will be neceſſary for his more perfect ſatisfaction. Theſe two ſolutions take up pages 24, 25, 26, and 27. And in corollary 2 of this problem the ſolution is extended to the determination of the preſent value of a number of ſuch future payments of a given ſum of money, to be made at the ends of ſeveral ſucceſſive years, in caſe of the continuance of the life of a perſon of a given age, or, in other words, to the determination of the value

of

of a life-annuity. After which, in pages 28, 29, 30, 31, and 32, the subject is further illustrated by examples of the actual computation of the values of life-annuities by the method described in the said second corollary; and then, in pages 33 and 34, there is a 3d corollary to this problem, relating to the value of a remote life-annuity that is not to take place till the end of a given number of years, together with an example of the calculation of the value of such an annuity.

In this part of the work I have discovered an error in one of the arithmetical operations in page 30, which I must desire the reader to correct. In dividing £31.8212 by 118, I have made the quotient £.2612, whereas it ought to be £.2696. This error is attended with some others derived from it, which will all be particularly mentioned in the list of *Errata*.

In pages 34, 35, 36, 37, 38, 39, and 40, I have given a scholium containing an account of the substance of a bill that was patronized by Sir George Savile and the late Mr. Dowdeswell, (the member for Worcestershire) and other gentlemen of eminence and abilities, and which passed the House of Commons in the spring of the year 1773, but was thrown out of the House of Lords in consequence of a speech of Lord Camden. It was intended to operate as an encouragement to journeymen manufacturers, handicrafts-men, houshold servants, and others, to industry and frugality, by offering them a safe and convenient method of employing the money they could save out of their earnings, in the purchase of remote life-annuities that were to take place in the latter periods of their lives, when they should become less able to support themselves by their labour; which annuities were to have been secured upon the poor's rates of their respective parishes. As I still think such an establishment is very practicable, and might be attended with very useful consequences, I was willing to take this opportunity of again recommending it to the notice of the publick, and of removing, in the best manner I was able, the objections that had been made to it, and particularly that upon which the noble and learned Lord who opposed the bill, seemed to lay the greatest stress, which was the danger occasioned by it of bringing a new and heavy burthen upon the poor's rate.

After

After the scholium, in which this project for establishing life-annuities in parishes is set forth and defended, there follow a 4th and a 5th corollary to this 2nd problem, shewing how the value of an immediate, but imperfect, life-annuity, not reaching to the utmost extremity of life, and likewise that of a distant and imperfect life-annuity, may be determined upon the principles explained in the solution of the problem. These corollaries, with an example to each of them, are contained in pages 40 and 41, and conclude the whole doctrine of the computation of the values of life-annuities depend[illegible]g upon only one life.

Problem 3d relates to the computation of the present value of a future sum of one pound, sterling, that is to take place at the end of a certain number of years, provided two persons of given ages shall then be living, and upon the supposition of a given rate of the interest of money. This problem is solved in a double manner, as well as the former, to wit, 1st, in the case of a particular example, and, 2ndly, in general terms. These solutions are contained in pages 42, 43, 44, 45, 46, and 47. And from them are deduced, in pages 47 and 48, two corollaries which extend them to the determination of the present values of any future sums (that are greater or less than one pound, sterling,) a[illegible] r[illegible]mber of such sums to be paid at the end of every year dur[illegible] tinuance of the lives of both the persons of given ages, or[illegible]rds, of an annuity for their joint lives. And in pages [illegible]1, and 52, the method of computing the value of a[illegible] two joint lives, prescribed in corollary 2, is illustrated by a[illegible]e.

After the example to corollary 2, follow three more corollaries relating to annuities for two joint lives, to wit, coroll. 3, which relates to remote annuities for two joint lives; and coroll. 4, which relates to immediate, but imperfect, annuities for two joint lives; and coroll. 5, which relates to remote and imperfect annuities for two joint lives. These three corollaries are contained in pages 52 and 53.

After these five corollaries the principles of the solution of this problem are extended, in coroll. 6, to the case of a future payment depending on the joint continuance of three lives. This, as well as the former ſolutions, is done in a two-fold manner, to wit, firſt, in the caſe of a particular example, and afterwards in general terms. It takes up part of page 53, and all the 54th, 55th, and 56th pages, and part of page 57; it being rather a complex and difficult buſineſs, and much pains having been beſtowed upon it to make it as plain as poſſible. And in coroll. 7 the concluſions obtained in coroll. 6 are applied to the determination of the value of a number of ſucceſſive future payments of one pound each that are to be made at the end of every ſucceſſive year during the joint lives of three perſons of given ages, or, to the determination of the value of an annuity of one pound *per annum* for three joint lives. And this concludes the whole doctrine of the computation of the values of annuities for two, or more, *joint* lives.

The fourth problem relates to the value of a future ſum of money, the payment of which depends not upon the *joint* continuance of the lives of two perſons of given ages, but upon the continuance of the life of *either* of them; which value, it is evident, will be very different from the other, and will greatly exceed it. This problem is rather more difficult than the foregoing one, which relates to the joint continuance of the two lives. It is ſolved (as all the former ones,) in a two-fold manner, to wit, firſt, in the caſe of a particular example; and, ſecondly, in general terms. The former of theſe ſolutions is contained in pages 58 and 59; and the latter in pages 59, 60, and 61. From theſe ſolutions ſeveral corollaries are deduced.

In the 1ſt corollary (which is in page 62) it is ſhewn that the value of ſuch future payment of a given ſum of money, depending upon the longeſt of two given lives, has a remarkable relation to the value of the future payment of the ſame ſum of money depending upon the joint continuance of the ſame lives, and may be eaſily deduced from the ſaid latter value together with the values of the future payment of the ſame ſum of money depending upon the ſame two lives ſeparately; for that the value of ſuch future payment of a given ſum of money depending on the longeſt of two given lives, is equal to the exceſs of the ſum of the two values of the future payments of the ſame ſum of money depending upon the ſeparate continuance of the ſame two lives, above the value of the future payment

ment of the ſame ſum of money depending upon their joint continuance.

The 2nd corollary extends the ſolution of the problem, which related only to the value of a future payment of *one pound*, to the value of a future of *any other* ſum, greater, or leſs, than one pound.

The 3d corollary extends the ſolution of the problem to the determination of the value of a number of equal future payments of one pound each, to be received at the end of every year during the life of either of two perſons of given ages; or, in other words, to the determination of the value of an annuity of one pound a year during the longeſt of two given lives.

The 4th corollary ſhews that there is the ſame relation between the value of a life-annuity for the longeſt of two given lives, and that of the ſame annuity for the joint continuance of the ſame lives, together with thoſe of the ſame annuity for the two ſeparate lives, as there is between the value of a ſingle future payment depending on the continuance of either of the two lives, and the value of the ſame future payment in caſe of the continuance of both lives, together with the values of it in caſe of the continuance of each of the two lives ſeparately; or, that the value of an annuity of one pound a year for the longeſt of two given lives is equal to the exceſs of the ſum of the values of two ſeparate annuities of one pound a year for the ſame ſingle lives, above the value of an annuity of one pound a year for the joint continuance of both lives. This corollary is contained in pages 63, 64, 65, and 66, and is evidently of very great importance, inaſmuch as it enables us, when we have tables of the values of annuities for ſingle lives and for two joint lives ready computed to our hands, to derive from them the values of annuities for the longeſt of two given lives by the eaſy operations of addition and ſubtraction.

In pages 67 and 68 are three more corollaries, to wit, corollaries 5th, 6th, and 7th, which relate to the values of remote life-annuities for the longeſt of two given lives, and of immediate, but imperfect, life-annuities depending on the continuance of either of two given lives, and of remote and imperfect life-annuities depending likewiſe on the continuance of either of two given lives.

After

After these three corollaries (which are short and easy) comes the 8th corollary to this fourth problem, which is much longer and more difficult to understand than any preceding part of the book. It contains an extension of the solution of this problem to the case of a future sum of money depending on the continuance *of any one of three* lives of given ages; which, though it is determined by the same principles as the former case of a future sum of money depending on the continuance of *either of two given lives*, is greatly more complicated than that case, and consequently requires a much longer investigation. This investigation is carried on (like all the former solutions,) in two different manners, namely, first, in the case of a particular example, and afterwards in general terms. The particular investigation is contained in pages 68, 69, 70, 71, 72, 73, 74, and 75; and the general investigation is contained in pages 76, 77, 78, and part of page 79. And, after these two investigations, certain difficulties, that will probably arise in the attentive reader's mind after he has perused them, are examined and removed in pages 79 and 80, and the beginning of page 81.

These investigations are followed by three corollaries, namely, corollaries 9th, 10th, and 11th. The first of these corollaries is contained in pages 81 and 82, and is employed in shewing the relation of the value of a future payment of one pound depending on the longest of three lives of given ages to the value of a future payment of the same sum of money depending on the joint continuance of all the three lives, and to the three different values of a future payment of the same sum of money depending on the joint continuance of every two of the same three lives, and to the three different values of a future payment of the same sum of money depending on the continuance of each of the same three lives taken separately; the said first value being equal to the excess of the sum of the said second value (which relates to the joint continuance of all the three lives) and the said three last values (which relate to the three single lives) above the sum of the said third, fourth, and fifth values, which relate to the joint continuance of every two of the said lives. The 10th corollary extends the foregoing investigations to the case of a number of future payments of a sum of one pound that are to be made at the end of every year during the continuance of any one of three lives of given ages, or, in other words, to the case of a life-annuity depending

depending on the longest of three lives of given ages. This corollary is very short, and is contained in the first part of page 83. But the 11th corollary is a very long one, and takes up the remainder of page 83, and all the 84th, 85th, 86th, 87th, 88th, and 89th, pages, together with a part of page 90. But it is also very important and useful, and for that reason is so much enlarged upon. For it shews, that the value of an annuity of one pound a year for the longest of three lives of given ages, is equal to the excess of the sum of the value of a like annuity for the joint continuance of the same three lives and the three values of the like annuity for the same three lives taken separately, above the sum of the three values of the like annuity for the joint continuance of every two of the said lives; and consequently that, whenever we have tables of the values of annuities for single lives and for two and three joint lives ready calculated, we may easily deduce from them the values of annuities for the longest of three lives by mere addition and subtraction.

And here ends the fundamental part of the whole work, or the explanation of the principles of the doctrine of life-annuities. The remainder of the book, long as it is, is taken up in applications of these principles, and illustrations of them by numerous examples, in order to render these computations familiar to the reader, and in contrivances to abridge the labour of them, and in other such matters, which are much less curious and important than the explanation of the principles themselves of this useful species of computation, which are contained in the foregoing 90 pages. And therefore I expect that many of my readers will wholly pass over many large parts of this remainder, which they will esteem, and perhaps justly, not worth the trouble of perusing them; though others of my readers, (who may have more leisure and a greater liking to the subject,) will, I imagine, be inclined to go through every page of it. I shall therefore here continue the account of the contents of the book throughout this long and less interesting remainder of it, in the same manner as I have done already with respect to the first and more important 90 pages of it; to the end that the former set of readers may easily determine before-hand which parts of it they will pass over, and that even the latter class of readers may be the better able to judge which parts of it they will chuse to read first, and which they will read with the most attention.

Pages 90, 91, 92, 93, 94, and 95 relate to the computation of the values of annuities for terms of years certain, and contain short and convenient, algebräic, expressions of the said values. And here I have inserted a geometrical demonstration of the rule for finding the sum of the terms of a decreasing geometrical progression, consisting of a finite number of terms, as *A*, *B*, *C*, *D*, *E*, to wit, that it is equal to $\frac{AA - BE}{A - B}$, or to the quotient that arises by dividing the excess of the square of the first, or greatest, term above the rectangle, or product, contained under the second term and the last, or least, term, by the excess of the first, or greatest, term above the second term.

Page 96 contains a like short and convenient, algebräic, expression of the value of an immediate annuity of one pound a year for a given number of years depending on a single life of a given age; and page 97 contains a like expression for the value of such an immediate annuity during the whole life of a person of a given age, which is only a particular case of the former expression. And in the remainder of page 97, and in page 98 and part of page 99, are contained examples of the computation of the values of such life-annuities by means of the said algebräic expressions.

Then follows in pages 99 and 100 a like algebräic expression for the value of *a remote* annuity for a given number of years, depending on a life of a given age; and in pages 100, 101, and 102 is contained an example of the computation of the value of such a remote life-annuity by means of such algebräic expression.

All these examples of the computations of the values of life-annuities are the very same which were given in the former part of the work after the scholium of Prob. 2nd; only they are here performed by the help of the algebräic expressions here given of those values, in order to make the use of those expressions familiar to the reader. And it was in performing the computation of the last of these examples that I discovered the slip in an arithmetical operation which has been already mentioned in this preface to have been made in page 30, by making the quotient of the division of 31.8212 by 118 be .2612 instead of .2696. See the note in page 102, Art. XCII.

After

After these first examples, which relate to the values of annuities for the lives of very old persons, I have, in pages 103, 104, 105, 106, and 107, inserted the calculation of the value of an annuity of one pound a year for the life of a person only 10 years old; which exhibits a clear view of the labour that is necessary to be employed in making these computations exactly, when the lives are young, and, consequently, of the reason which, probably, induced Mr. De Moivre, and some other calculators of the values of life-annuities, to decline the foregoing accurate method of computing them, and resort to hypotheses concerning the probabilities of the duration of human life, which they knew to be not perfectly true, in order to facilitate the computation of them. The principal hypothesis used by Mr. De Moivre for this purpose is described in a subsequent part of the work.

After the above-mentioned long example of the computation of the value of a life-annuity of one pound a year for the whole life of a child of 10 years of age, there is in Art. XCVII, pages 107, 108, an example of the computation of the value of an annuity of one pound a year for the first 30 years of the said life; which is involved in, or makes only a part of, the foregoing computation.

Art. XCVIII, pages 108, 109, contains some remarks on the fluctuation of the price of the public funds, and of the interest of money in general; which makes it expedient to have the values of life-annuities calculated according to several different rates of interest.

In pages 109, 110, 111, and 112, an account is given of a very easy and convenient method of deducing the value of a life-annuity of one pound a year for a life of any given age from the value of the same annuity for a life that is older than the former by one year: by the help of which method a whole table of the values of a life-annuity of one pound a year for every age of human life, proceeding from the older ages to the younger by the constant difference of a year, may be computed with nearly the same labour as is necessary to obtain the value of the same annuity for the first, or youngest, life in the table. This method was first communicated to me by Dr. Price: but it was published in the year 1779 by Mr. William Morgan, the actuary to the Society for Equitable Assurances near

 Black-

Black-friars Bridge, in his Treatise on the Doctrine of Annuities and Assurances on Lives, pages 56, 57; and it had been published before by Dr. Price himself in his Treatise on Reversionary Payments, Note O of the Appendix, and likewise by Mr. Thomas Simpson in his book on Life-annuities, Prob. 1, Coroll. 7; which last book was published so long ago as the year 1742. But I should suspect that it was not known to Mr. De Moivre, when he calculated his tables of the values of life-annuities. For, if it had, I should imagine he would hardly have thought it necessary to have recourse to a certain inaccurate hypothesis concerning the probabilities of life, in order to diminish the labour of his computations, which would have been almost equally facilitated by the use of this excellent method. This method I have not only described, but explained and demonstrated in the fullest and clearest manner I was able, in the said pages 109, 110, 111, and 112.

In pages 112, 113, and 114 I have applied this method to the computation of the values of a life-annuity of one pound a year for lives of the ages of 8 years, 7 years, 6 years, 5 years, 4 years, and 3 years, from the value of a like annuity for a life of the age of 9 years (which had been computed by it in pages 110 111, from the value of a like annuity for a life of the age of 10 years,) and from each other. And afterwards, in pages 115, 116, 117, 118, 119, 120, &c. ——— 130, in order to make this method still more familiar to the curious and industrious reader (for all others, I suppose, will pass over this part of the book,) I have applied it to the computation of a compleat table of the values of a life-annuity of one pound a year for every age of life, from the age of 93 years up to the age of 3 years, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and upon a supposition that the interest of money is 3 ½ per cent. And here I have discovered another error in one of the arithmetical operations, which pervades all the subsequent computations, and renders them in a small degree erroneous. For at the bottom of page 123, in deducting the value of an annuity of one pound a year for a life of the age of 41 years from £15.580,939, or the value of the same annuity for a life of the age of 42 years, I have made the product of the multiplication of 16.580,939 by the fraction $\frac{643}{650}$ amount to

£16.448,528,

£16.448,528, whereas in truth it amounts to only £16.402,375; in consequence of which mistake the quantity $\frac{1}{1.035} \times \frac{643}{650} \times$ £16.580,939, (which is equal to the value of an annuity of one pound a year for a life of the age of 41 years,) appears to be equal to $\frac{1}{1.035} \times$ £16.448,528, or £15.892,297, or 15*l.* 17*s.* 10*d.* instead of $\frac{1}{1.035} \times$ £16.402,375, or £15.847,705, or 15*l.* 16*s.* 11½*d.* which is less than the former sum, 15*l.* 17*s.* 10*d.* by a difference of 10½*d.* This error necessarily affects all the following values of a life-annuity of one pound a year for lives of younger ages than 41 years, which are set down in pages 124, 125, 126, 127, 128, 129, and 130. But it is not of much importance with respect to the value of the annuity for a life of the age of 41 years, in which it occurs; the difference of 10½*d.* being but a trifling sum of money in comparison of 15*l.* 16*s.* 11½*d.* which is the true value of that annuity. And in all the following values of a life-annuity of one pound a year, for lives of younger ages than 41 years, the error, or difference of the false and true values, grows continually less and less. For the erroneous value of this annuity for a life of the age of 10 years, obtained in page 128, is only £20.752,981, or 20*l.* 15*s.* 0¾*d.* which exceeds the true value of it obtained before in Art. XCIV, page 106, to wit, £20.739,25, or 20*l.* 14*s.* 9¼*d.* by only 3½*d.* and the erroneous value of the same annuity for a life of the age of 3 years, obtained in page 130, is only £19.987,654, or 19*l.* 19*s.* 9*d.* which exceeds the true value of it, obtained before in Art. CV, page 114, to wit, £19.978,21, or 19*l.* 19*s.* 6¾*d*, by only 2¼. I therefore hope the reader will be of opinion that it was hardly worth while, on account of this mistake, to be at the trouble and expence of computing all these numbers over again, and of causing so many pages to be reprinted; more especially as the true numbers, which express the values of all these annuities, are all set down in two different places in the subsequent part of this work, namely, in Table XI, pages 214, 215, 216, 217, and in Table XV, page 224. See more concerning this mistake in Art. CC, pages 218, 219.

After

After this computation of a compleat ſet of the values of a life-annuity of one pound a year for every year of human life in pages 115, 116, &c. ——— 130, the ſaid values are all ſet down in regular order in Table III, pages 131, 132, 133, 134, 135, and are expreſſed there in two different manners, to wit, in decimal parts of a pound in the ſecond column, and in pounds, ſhillings, and pence in the fourth column. And the differences of the ſaid values are likewiſe ſet down in the ſame table, and are expreſſed alſo in two different manners, namely, in decimal parts of a pound in the third column, and in pounds, ſhillings, and pence in the fifth column.

In this table, it is evident, the numbers relating to ages younger than 42 years are in a ſmall degree greater than they ſhould be, being the ſame with thoſe obtained in the foregoing computation. But theſe numbers the reader may correct, if he thinks fit, by means of Table XI, pages 214, 215, &c. or Table XV. page 224.

In pages 136, 137, 138, 139, 140, 141, are ſome obſervations which will be of uſe to a calculator of ſuch a table in diſcovering any arithmetical errors he may happen to have fallen into in the courſe of his computations; though it does not anſwer that purpoſe ſo compleatly as another method given us for that purpoſe by the above-mentioned ingenious Mr. Morgan, in his *Doctrine of Annuities and Aſſurances on Lives*, and which I have afterwards inſerted in this work and explained in as ample a manner as I could, in pages 208, 209, 210, 211, 212, and 213.

Having in theſe firſt 142 pages of the book treated only of the true and exact methods of computing the values of life-annuities from tables of the probabilities of the duration of human life, I next proceed to give ſome account of what Mr. De Moivre and other writers on this ſubject have called *the decrements of human life*, and of the obſervations which they have made on the courſe, or variations, of the ſaid decrements, as the ages of life increaſe; by which obſervations Mr. De Moivre was induced to adopt the hypotheſis concerning the probabilities of human life which has been already alluded to, (though not perfectly agreeable to the truth,) for the ſake

ſake of facilitating his computations. Theſe *decrements of life* are the numbers of perſons who, out of a given original number of perſons, all of the ſame age, ſuppoſed to be living at a very early age, are repreſented, in tables of the probabilities of the duration of human life, as dying in the ſeveral ſucceeding years. In order to make the courſe of theſe decrements the more apparent, I have again inſerted the above-mentioned tables of the probabilities of life of Mr. Kerſſeboom and Monſieur de Parcieux, under the titles of Table IV and Table V, with an additional column in each, containing the decrements of life correſponding to every year of life, or the number of perſons who have died in the courſe of it. Theſe tables are contained in pages 144, 145, 146, 147, 148, and 149. And from them it appears that in ſeveral parts of human life, the decrements of life, or the numbers of perſons dying every year, are equal for ſeven or eight, and, in ſome parts, for twelve, or more, years together. Now, whenever this happens, the terms that expreſs the preſent values of the future payments of one pound each, which are to be received at the ends of thoſe years, will be a ſet of fractions, of which the numerators will form a decreaſing arithmetical progreſſion, and the denominators will form a decreaſing geometrical progreſſion; and therefore the ſum of all thoſe terms may be found at once by a ſingle ſhort expreſſion, without taking the pains to compute each of the terms ſeparately and then add them up into one ſum. The method of finding a fit expreſſion for this purpoſe is explained in the following 18 pages, to wit, pages 151, 152, 153, &c. —— 168. And then in pages 169, 170, an account is given of Mr. De Moivre's hypotheſis concerning the decrements of human life, and of the grounds upon which he conceived that the values of life-annuities computed from it would differ but in a ſmall degree from their true values.

Page 171 contains Table VI, which is a ſmall table of the values of a life-annuity of one pound a year for every fifth year of human life, computed by Dr. Halley from the Breſlaw table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 6 per cent. This was, probably, the firſt table of the values of life-annuities that ever was publiſhed. And with one of theſe values, but he does not ſay which, Mr. De Moivre tells us that he compared the value of the ſame annuity for a life of the

the ſame age computed from his own hypotheſis, and found the two values to be ſo very little different from each other that, for all uſeful and practical purpoſes, they might well be conſidered as the ſame. And from hence he concluded (but, I think, too haſtily,) that, in the buſineſs of computing the values of life annuities, he might ſafely neglect the tables of probabilities deduced from obſervations, and proceed upon the ground of his own hypotheſis.

Pages 172, 173, are employed in ſhewing how the conjectural probabilities of the duration of human life, reſulting from Mr. De Moivre's hypotheſis, may be compared with the real probabilities of the ſame, as exhibited in Monſieur de Parcieux's table of them. And then in pages 174, 175, a compariſon is made between the ſaid probabilities, by ſetting down in Table VII, in two columns adjoining to each other, the numbers that expreſs the probabilities of life according to Monſieur de Parcieux's table, and likewiſe thoſe which expreſs the ſame probabilities according to Mr. De Moivre's hypotheſis: and at the end of this table of compariſon between the ſaid probabilities, ſome obſervations are made, in pages 176, 177, and 178, concerning the differences between them and the manner in which the values of life-annuities, computed from them, would be affected by them.

Afterwards in page 178, Art. CLIX, it is ſhewn that the numbers exhibiting the probabilities of human life according to Mr. De Moivre's hypotheſis may be reduced to much ſmaller numbers, and even to ſuch as are only equal to thoſe which are neceſſary to compleat the numbers of years in the ſeveral ages of life to the number contained in the utmoſt ſuppoſed extent of life, and which, from this circumſtance, Mr. De Moivre calls the *complements of life*. And then in Table VIII, page 179, thoſe loweſt numbers expreſſing the probabilities of human life according to Mr. De Moivre's hypotheſis, and which are the complements of life to its greateſt poſſible extent, are ſet down in order, ſo as to enable the reader to compute the value of any life-annuity according to Mr. De Moivre's hypotheſis with the greateſt eaſe and readineſs.

After Table VIII, I have added the computations of the values of a life-annuity of one pound a year for the ſeveral ages of 1 year,

3 years,

3 years, 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35 years, 40 years, 45 years, 50 years, 55 years, 60 years, 65 years, and 70 years, from Table VIII, or Mr. De Moivre's hypothesis, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. in order to compare them with the true values of the same annuity for the same lives obtained before from Monsieur de Parcieux's table of probabilities. These computations are contained in pages 180, 181, 182, &c. —— 193. And the results of them, or the values of an annuity of one pound a year for lives of the said ages obtained by them, are set down in Table IX, page 194, together with the corresponding true values of the same annuity for the same lives according to Monsieur de Parcieux's table of probabilities, and with the differences between the said true values and the said conjectural values set down in an adjoining column.

By means of this table the reader is fully enabled to judge of the degree of exactness with which Mr. De Moivre's hypothesis enables us to find the values of life-annuities. And it appears that these conjectural values of life-annuities differ from their true values in many ages of life by more than a whole year's purchase, and after the age of 65 years by more than two years purchase, though about the age of 45 years they are very nearly equal to each other. These observations, with a few more of the same kind, are made in page 195, and a general conclusion is drawn from them against the expediency of ever making use of Mr. De Moivre's hypothesis in computing the values of life-annuities.

In pages 196, 197, 198, an account is given of another method of computing the values of life-annuities, which is different both from that above-explained in Prob. II, and its corollaries, and likewise from Mr. De Moivre's method by means of his hypothesis. This method was given by Mr. Weyman Lee, a Barrister at Law and Bencher of the Inner Temple, in a book he published on life-annuities in the year 1738. It is exceedingly erroneous, and gives the values of life-annuities, throughout the greatest part of human life, much greater than they should be. In the younger ages of life the difference of the erroneous value from the true one amounts to about 3 years purchase. Yet the principle, upon which Mr. Lee grounds this method, has something in it that is plausible at first sight, and is apt to mislead the understanding with an appearance of truth and simplicity, unless it be examined with a great degree of attention. And for this reason I have thought it worth mentioning and examining at considerable length. The method itself is described in page 197; and some remarks on the difference of the values of life-annuities, resulting from it, from the true values of the same annuities, are made in page 198. And in pages 199, 200, 201, I have inserted a whole table of the values of a life-annuity of one pound a year for every age of life, computed by this method, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. This table is called Table X; and in it are set down, besides these erroneous values of a life-annuity of one pound a year for the different ages of life, computed according to Mr. Lee's method, the true values of the same annuity for the same ages computed in the manner above-explained in Prob. II, and its corollaries, and likewise the differences between the said erroneous values and true values, in an adjoining column.

In page 202 some remarks are made on this table; and in the five following pages, to wit, pages 203, 204, 205, 206, and 207, the reasons alledged by Mr. Lee in support of his method of determining the values of life-annuities are stated in his own words, and examined, and the fallacy, or insufficiency, of them is pointed out.

From this account of what has been delivered concerning Mr. De Moivre's hypothesis and Mr. Weyman Lee's erroneous method of valuing life-annuities, the reader must see that all the last 66 pages, from page 142 to page 207, inclusively, are not necessary to the right understanding of the true method of computing the values of life-annuities, but only to the knowledge of the most remarkable methods and opinions that have been adopted for this purpose by writers of note. And therefore such of my readers as have no curiosity for this historical kind of information, will do well to pass over all these pages, and go at once from page 142 to page 208. And such of them as are only desirous of understanding the right method of computing the values of these annuities, without wishing to make these computations familiar to them, will do well to pass over in like manner the foregoing 28 pages from the end of page 114 to the end of Art. CXXIV in page 142.

In

In page 208 I return to the more useful part of the subject, to wit, the true method of computing the values of life-annuities according to the principles of Prob. II, and its corollaries, and have employed that and the five following pages, to wit, pages 209, 210, 211, 212, and 213, in explaining a method invented by the ingenious Mr. Morgan above-mentioned for proving the truth of the computations of the values of life-annuities, (when they are deduced one from the other in a regular succession proceeding from the older lives to the next younger,) as fast as they are made. This method is so satisfactory, and answers the purpose, for which it was invented, so compleatly, that nothing further need be wished for on the subject. And therefore I thought it would be proper, not only to explain it and demonstrate the truth of it, but also to illustrate it by a number of examples. This is done in Table XI, pages 214, 215, 216, and 217; in which table all the values of a life-annuity of one pound a year for every different age of life, are regularly computed one from the other by means of the expression given by Mr. Morgan for that purpose, from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 3½ per cent. and set down in the second column of the table over against the corresponding ages in the first column, and three other sets of numbers, which (according to this last-mentioned method of Mr. Morgan, that is explained in pages 208, 209, 210, 211, and 212,) are necessary to prove the truth of the former numbers, (or of the values of the annuities in the second column,) are set down in regular order in a third, a fourth, and a fifth column. These values may be depended upon, as exact: and it was by means of this table, (which carries its own proofs with it) that I found out the mistake I had made in page 123, where the value of an annuity of one pound a year for a life of the age of 41 years is made to be equal to £15.892,297, instead of £15.847,705; of which mistake and its consequences I have already said enough.

Pages 221, 222, 223, &c. —— 232, contain twelve tables of the values of a life-annuity of one pound a year for all the different ages of human life, from the age of 3 years to the age of 93 years, accurately computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, according to the twelve following rates of interest; to wit, 2 per cent. 2½ per cent.

3 per

3 per cent. 3½ per cent. 4 per cent. 4½ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. This is a greater variety of tables of the values of life-annuities than has ever yet been publiſhed. And I will add, for the ſatisfaction of my readers, and of all ſuch perſons as ſhall be inclined to make uſe of theſe tables, that theſe values have, all of them, been confirmed by the computation of the correſponding proof numbers, in the manner juſt now mentioned; tho' it has not been thought neceſſary to inſert thoſe numbers in the tables.

Theſe tables are all computed upon a ſuppoſition that the annuities are to be paid only once in every year, namely, at the end of every new compleat year after the purchaſe of them. But it more uſually happens that life-annuities are made payable every half-year, and ſometimes even every quarter of a year. In theſe caſes the values of them will be ſomewhat greater than when they are payable only at the end of every year. But to determine exactly *how much* greater the values of them will be in thoſe caſes than when they are paid at the end of every year, (as they are ſuppoſed to be in the foregoing tables,) is a difficult and intricate queſtion, and hardly worth the trouble of the ſolution. Yet, for the ſake of my more curious and induſtrious readers, I have taken the firſt of theſe caſes, to wit, that of annuities paid every half-year, into conſideration, and have given a ſolution of it by approximation, which I find to agree, in the reſult of it, with that given by Mr. Thomas Simpſon, in his doctrine of annuities; the reſult, or concluſion, of both inveſtigations being, "that a life-annuity payable half-yearly is worth more than the ſame annuity, when payable only at the end of every year, by ſomewhat leſs than a quarter of a year's purchaſe." The inveſtigation of this queſtion in the following work takes up no leſs than 27 pages, to wit, pages 233, 234, 235, &c.—259, and is extremely perplexing and unentertaining: Yet I do not know how to make it leſs ſo. I therefore adviſe the generality of my readers to paſs it over, and take mine and Mr. Simpſon's word for the truth of the concluſion juſt now mentioned, to wit, "that the difference of the values of a yearly and half-yearly life-annuity is leſs than a quarter of a year's purchaſe." I have followed this advice myſelf with reſpect to the values of life-annuities that are payable quarterly; not having taken the trouble of inveſtigating the ſaid values, but believing, upon the authority of the ſame Mr. Thomas Simpſon, that thoſe annuities are worth about three eighth parts of a year's purchaſe more than thoſe which are paid only once a year, or one eighth part of a year's purchaſe more than thoſe that are payable every half-year.

From

From the subject of the above-mentioned tedious and perplexing investigation, I was led to another inquiry, that bears some analogy to it, but is much easier and more entertaining, namely, concerning the limit of the sum of money to which the interest made of a given sum of money in a year, or in any other given time, may be made to increase, by increasing the number, and diminishing the lengths, of the terms for which the money is lent, so as to improve the money during the said given time, by means of the said repeated loans, at compound interest. This problem and its corollaries are contained in pages 260, 261, 262, &c.——271. And I imagine, the reader will be entertained by them. The conclusion from the whole is, that the advantages that may be made by lending a sum of money for very small portions of a year, (as, for example, for 52nd parts, or weeks,) and then receiving the interest due upon it, and immediately lending both principal and interest at the same rate of interest; I say, the advantages that may be made by this method of proceeding above the interest that will arise from it by lending it at once for the whole year, are so very small as not to be worth attending to. Thus, for example, the interest of £100 in a whole year, if thus lent for only a week at a time, at the interest of 5 per cent. *per annum*, will be less than £5. 2*s*. 6*d*. or will exceed the interest of it, when lent at once for a whole year, by less than half a crown; and the utmost possible increase of this interest, or, to speak more properly, the *limit* of the said increase, or the quantity to which the interest of the said sum of money approaches continually, and to which it may be made to come as near as we please, as the number of the short terms, for which the money is lent, is increased *ad infinitum*, is only £5. 2*s*. 6½*d*. together with the 158th part of a penny.

In pages 271, 272, a remarkable analogy is pointed out between the infinite series that had been found in pages 268, 269, 270, for the limit of the increase of a sum of money at compound interest in the course of a year, and a certain ordinate of a logarithmick curve. And in pages 272, 273, 274, &c.——278, the said infinite series, expressing the said limit of the increase of a sum of money at compound interest, is further considered and explained.

Pages 278, 279, contain a suggestion that it might be an useful publick measure for the government, when it establishes annuities for a term of 30 years, to enlarge the grant of such annuities to the extent of the lives of the purchasers of them, in consequence of such fuch an additional fum of money, to be paid by the faid purchafers for fuch enlargement of their intereft in the faid annuities, as fuch enlargement may be fairly worth. Now the value of fuch an enlargement of a purchafer's intereft in an annuity for 30 years certain is evidently the value of a remote life-annuity of the fame annual amount for the life of the faid purchafer, to commence at the expiration of the faid 30 years. And therefore in pages 280, 281, 282, I have fhewn how (without having recourfe to new computations upon the principles delivered in Prob. II. and itscorollaries) the value of fuch a remote life-annuity may eafily be derived from that of an immediate life-annuity for the life of the fame perfon, which, if the intereft of money is fuppofed to be either 2, 2½, 3, 3½, 4, 4½, 5, 6, 7, 8, 9, or 10 per cent. muft be contained in one of the preceeding twelve tables of the values of life-annuities in pages 221, 222, 223, &c. ——— 232. And then, in pages 283, 284, 285, 286, and 287, I have inferted four tables of the values of fuch remote life-annuities, computed from Monfieur de Parcieux's table of the probabilities of the duration of human life, at the four following rates of intereft, to wit, 3½ per cent. 4 per cent. 4½ per cent. and 5 per cent. which are the rates that are moft likely to be adopted in the eftablifhment of any government fecurities.

In this part of the work I have digreffed from the doctrine of life-annuities, (which is the proper fubject of it,) to make fome reflections on the National Debt, and the moft likely methods of paying off a part of it. Thefe reflections, together with various applications of the foregoing doctrine of life-annuities to different methods which may be taken for fhortening the duration of this enormous debt, by converting a part of the perpetual annuities of which it confifts, into annuities of a finite duration, fuch as life-annuities, or annuities for terms of years and likewife for life, and the like,) extend through no lefs than 102 pages, namely, from page 287 to page 389. And after this very long, but, I hope, not ufelefs, digreffion, I have reprinted a very valuable pamphlet on the fame fubject of the National Debt, that was written by Sir Nathaniel Gould, a merchant of eminence, and a Director of the Bank, in the reign of King George the 1ft. It was firft publifhed in the year 1726, and went through at leaft four editions: and Dr. Price has told us in his *Appeal to the Publick on the Subject of the National Debt*, that he fets fo high a value

value upon it that he wishes he could put it into every hand in the kingdom. This declaration of so eminent a writer induced me first to peruse it, and, afterwards, finding it answer the high character that he had given of it, to cause it to be reprinted. And, to the end that gentlemen may be able to procure it without purchasing this large volume upon a subject that may not happen to interest them, I have caused an additional number of copies of this pamphlet to be printed off, which may be bought separately. This pamphlet ends at page 447. And in the following pages I have added a few reflections connected with the subject of the said pamphlet, concerning the expediency of an equal assessment of the land-tax. These reflections are contained in pages 448, 449, 450, 451, 452, 453, and 454. And with them ends every thing in the book that relates to the values of life-annuities for *single lives*.

In pages 455, 456, a short and convenient, algebräic, expression is given of the value of an annuity of one pound a year for a certain number of years, depending on the *joint continuance of two lives* of given ages; and a like expression is given for the value of such an annuity for the *whole* of the joint continuance of the said lives; which is but a particular case of the former expression. And in pages 457. 458, 459, and 460, examples are given of the computation of the values of such annuities, depending on two joint lives, by means of the said expressions.

In pages 460, 461, a like short and general, algebräic, expression is given for the value of a remote annuity of one pound a year depending on the joint continuance of two lives of given ages; and in pages 462, 463, 464, an example is given of the computation of the value of such a remote annuity by means of the said expression.

Then, in pages 465, 466, 467, and 468, I have set forth and explained another very useful method, with which we have been favoured by the before-mentioned Mr. Morgan, in his *Doctrine of Annuities and Assurances on Lives*; by which the computations of the values of an annuity for two joint lives are as much facilitated as the computations of the values of annuities for single lives are by his former method, which is mentioned and explained in the foregoing

going part of this work, in pages 109, 110, 111, and 112. For by it we are enabled to deduce with great eaſe (by an expreſſion ſimilar to that obtained in Art. 109, 110, 111, and 112,) the value of an annuity of one pound *per annum* for the joint continuance of any two lives whatever, from the value of the ſame annuity for the joint continuance of two lives that are reſpectively older than the two former lives by one year; ſo that a whole table of the values of ſuch an annuity for two lives, that differ from each other by any given number of years, may be computed with almoſt as little labour as the two youngeſt lives poſſible that differ by the ſame number of years. Thus, for example, if the difference of the ages is 30 years, it will be almoſt as eaſy, by means of this expreſſion of Mr. Morgan, to compute a whole table of the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 93 years and 63 years, 92 years and 62 years, 91 years and 61 years, 90 years and 60 years, and ſo on up to the ages of 33 years and 3 years, as to compute the value of a ſingle annuity of 1 *l.* a year for the joint continuance of the ſaid laſt two lives, which are of the ages of 33 years and 3 years. The great uſefulneſs of ſuch a method in facilitating the buſineſs of computing compleat tables of the values of annuities for two joint lives, need not be pointed out. I have therefore not only ſtated it, and demonſtrated the truth of it, in the fulleſt and cleareſt manner I was able, in pages 464, 465, 466, and 467, but have afterwards illuſtrated it by a great number of examples in pages 468, 469, 470, 471, 472, 473, 474, and 475; ſo that I doubt not the attentive reader will have no difficulty in making himſelf compleatly maſter of it.

This method of Mr. Morgan for computing the values of annuities for two joint lives one from another, proceeding from the older lives to the younger, is evidently liable to the ſame inconvenience as his method of computing the values of annuities for ſingle lives one from another, proceeding likewiſe from the older lives to the younger; namely, that when an error has once crept into the calculation of the value of any one of the annuities, it will affect thoſe of all the remaining ones, that relate to younger lives. But Mr. Morgan has provided a ſimilar and a very compleat remedy for this inconvenience in both caſes, by ſhewing us how, by a certain counter-calculation accompanying that of the values of the annuities themſelves, we we may conſtantly verify thoſe values as faſt as we obtain them. This remedy in the caſe of annuities for ſingle lives I have deſcribed and demonſtrated in pages 209, 210, 211, 212, and 213, as has been already mentioned. And here, in pages 476, 477, 478, 479, 480, &c. —— 487, I have explained and illuſtrated by examples the method preſcribed by Mr. Morgan for effecting the ſame purpoſe in the caſe of annuities for two joint lives. And in pages 488, 489, I have inſerted a table, to wit, Table XXVIII, containing the values of an annuity of one pound a year for the joint lives of two perſons of the ages of 94 years and 84 years, 93 years and 83 years, 92 years and and 82 years, 91 years and 81 years, 90 years and 80 years, and the ten next younger ages, up to thoſe of 80 years and 70 years, computed one from the other by the above-mentioned expreſſion of Mr. Morgan, which is explained and illuſtrated in pages 464, 465, &c. —— 475; and containing likewiſe, in three adjoining columns, to wit, in columns 3^{d}, 4th, and 5th, the proof-numbers which the ſaid laſt-mentioned method of Mr. Morgan for confirming his calculations of the ſaid values, and which is explained and illuſtrated in pages 476, 477, &c. —— 487, directs us to compute: which proof-numbers manifeſtly confirm the values of the annuities ſet down in the foregoing, or third, column; ſo that no doubt can be entertained of the truth of the ſaid values.

And thus, by the help of Mr. Morgan's excellent methods, the computation of a table of the values of annuities for two joint lives, whoſe ages differ by the ſame number of years, is rendered as eaſily practicable, and as little liable to error, as that of a table of the values of annuities for ſingle lives.

But it is evident that the number of the differences that may be ſuppoſed to ſubſiſt between the ages of two perſons is ſo great that it is almoſt impoſſible to compute tables enough to exhibit the values of an annuity of one pound a year for two joint lives of all the different ages that may be taken. For this would require, at one rate of intereſt only, no fewer than 94 different tables, to wit, a table for two perſons whoſe ages differ by 93 years, another for two perſons whoſe ages differ by 92 years, a third for two perſons whoſe ages differ by 91 years, a fourth for two perſons whoſe ages differ by 90 years, and ſo on till we come to two perſons of the ſame age. And

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consequently, for the twelve different rates of interest above-mentioned, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. it would require twelve times 94, or near 1200, such tables: which it will, probably, never be thought worth while to cause to be computed. With respect therefore to the values of annuities for two joint lives, and still more with respect to those of annuities for three joint lives, (the varieties of which are still greater, and in a prodigious degree, than those of annuities for two joint lives,) we must be contented with a few tables accurately computed by the methods above-mentioned, for lives whose ages differ by only a moderate number of differences, far short of all the differences possible; and we must have recourse to methods of approximation for determining the values of such of these annuities as the tables we are possessed of do not exhibit. It is, however, desireable to have as many tables of the values of annuities for two joint lives accurately computed, as can be conveniently procured. And I think we have some reason to be surprized that so few tables of this kind have hitherto been published. To supply this defect in some degree I have procured ten different tables of the values of a life-annuity of one pound a year for two joint lives to be accurately computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. to wit, one table for two lives of the same age, another for two lives whose ages differ by five years, a third for two lives whose ages differ by 10 years, and a fourth, fifth, sixth, seventh, eighth, ninth, and tenth, for two lives whose ages differ by 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years. These tables are contained in pages 492, 493, 494, &c.——501; and in page 502 is given the value of an annuity of one pound a year for two joint lives of the ages of 93 years and 3 years; which is the only annuity that is possible, according to Monsieur de Parcieux's table of the probabilities of life, with a difference of 90 years between the ages.

After these tables, and a few remarks upon the difficulty of procuring compleat sets of the values of all possible annuities for two joint lives, which make the subject of Art. ccccx in pages 502, 503, I have shewn how, by a certain method of *interpolation* between the values

values of ſuch annuities for two joint lives as are contained in the foregoing tables, we may derive very uſeful approximations to the values of other annuities for two joint lives that are not contained in them, at the ſame intereſt of 3½ per cent. This method of interpolation is deſcribed and illuſtrated by examples in pages 503, 504, 505, 506, &c.——512. And the degree of exactneſs with which the numbers obtained by this method of interpolation, approach to, or exhibit, the true values of the annuities ſought by them, is inquired into and ſhewn in pages 512, 513, 514, &c.——518. And by thoſe means the reader is tolerably well enabled to find the value of one pound a year for the joint continuance of two lives of any ages, when the intereſt of money is 3½ per cent.

I have then added the like number of tables of the values of an annuity of one pound a year for two joint lives, computed likewiſe from Monſieur de Parcieux's table of the probabilities of life, when the intereſt of money is 4½ per cent. Theſe tables are contained in pages 519, 520, 521, 522, &c.——528; and they exhibit the values of the ſaid annuity of one pound a year for two joint lives, when the ages of the lives are equal, and when the differences of the ſaid ages are (as before) 5 years, 10 years, 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years. And in Art. CCCCXXXIII, page 529, is added the value of the ſaid annuity of one pound a year for two joint lives of the ages of 93 years and 3 years, which is the only annuity that can be ſuppoſed, according to Monſieur de Parcieux's table of the probabilities of life, for two lives whoſe ages differ from each other by 90 years.

By the help of this ſecond ſet of tables, and the method of interpolation before explained, the reader is tolerably well enabled to find the value of an annuity of one pound a year for the joint continuance of two lives of any ages whatſoever, when the intereſt of money is 4½ per cent.

After this ſecond ſet of tables of the values of annuities for two joint lives, which relates to the rate of 4½ per cent. I have endeavoured to extend the uſe of theſe two ſets of tables, which relate to the rates of 3½ and 4½ per cent. to the determination of the values of annuities for the ſame lives at the contiguous rates of intereſt of

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3 per cent. 4 per cent. and 5 per cent. This is done by a method of proceeding not very different from that of the foregoing method of interpolation, and which may be called *The method of interpolation and continuation.* This method is described and explained in pages 530, 531, 532. And in pages 532, 533, 534, &c. —— 539, an enquiry is made into the degree of exactness to which this method exhibits the values sought by it.

These methods of *interpolation* and *interpolation and continuation*, will only enable us to find near values of life-annuities for two joint lives at the same rates of interest as those which are computed in the two foregoing tables, to wit, the rates of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. and the three nearest rates of interest to them, and which differ from them by only $\frac{1}{2}$ per cent. to wit, 3 per cent. 4 per cent. and 5 per cent. In other rates of interest, that differ more considerably from those at which the tables are computed, it seems necessary to have recourse to some other method of approximating to the values of these annuities, that has no relation to those tables. This I have endeavoured to do in pages 540, 541, &c. —— 546; in which I have given an expression of the value of an annuity of one pound a year for two joint lives, which I conceive to be, in most cases, a tolerable approximation to the true value of such annuity, and which may be applied with an equal probability of success in all the different rates of interest. This expression of the value of such an annuity for two joint lives is derived, by conjectural, but probable, reasonings, from the value of the like annuity for the older of the two single lives; which latter value will, if the interest of money is either 2 per cent. or $2\frac{1}{2}$ per cent. or 3 per cent. or $3\frac{1}{2}$ per cent. or 4 per cent. or $4\frac{1}{2}$ per cent. or 5 per cent. or 6 per cent. 7 per cent. 8 per cent. 9 per cent. or 10 per cent. be contained in one of the twelve tables of the values of annuities for single lives in pages 221, 222, 223, 224, &c. —— 232. And by this means the said twelve tables of the values of annuities for single lives are rendered subservient to the purpose of finding the values of the like annuities for two joint lives: which seems to be a very convenient and useful principle to proceed upon in the valuation of these latter annuities, and of other annuities for more than two lives, which are still more numerous and complicated; provided the values of annuities for joint lives found by this method approach tolerably near to their

their true values. But, whether they do, or not, approach thus nearly to the ſaid true values, can only be determined by trying them in a few caſes, in which the true values have been actually computed, and comparing them with the ſaid true values. This inquiry is made, with reſpect to the above-mentioned expreſſion of the near value of an annuity of one pound a year for two joint lives (given in pages 540, 541, &c. —— 546,) in pages 546, 547, 548, &c. —— 556, by applying the ſaid expreſſion in a variety of examples to the inveſtigation of near values of ſome of the annuities which are contained in the two foregoing ſets of tables, and comparing the near values thereby obtained with the true values of the ſame annuities ſet down in the ſaid tables: and the reſults of this compariſon are ſet down in a table in pages 557 and 558. Theſe reſults ſuggeſt a correction of the aforeſaid expreſſion (given in pages 540, 541, &c. —— 546,) by multiplying it into the fraction $\frac{104}{100}$, which makes the ſaid expreſſion approach (in moſt caſes) conſiderably nearer than before to the true values of the annuities ſought by it. The reſults of the compariſon of the near values, ariſing from this corrected expreſſion, with the true values of the ſame annuities, are ſet down in another table in pages 558, 559, and 560. And in pages 560, 561, a remark is made on the ſaid laſt table, in which the ſaid corrected expreſſion, to wit, $\overset{£}{A} \times \frac{104}{100} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, is finally recommended as a tolerably exact method of obtaining a near value of an annuity of one pound a year for two joint lives, when the intereſt of money is either leſs than 3 per cent. or greater than 5 per cent. and the before-mentioned methods of *interpolation*, and *interpolation and continuation* conſequently cannot be applied.

The remaining part of the book relates to the values of annuities depending on three lives. In Art. CCCCLXXI, pages 561, 562, 563, a ſhort and convenient, algebräick, expreſſion is given of the value of an annuity of one pound a year for a given number of years, provided three lives of given ages ſhall all continue ſo long in being; which expreſſion, whenever the ſaid given number of years is equal to the greateſt poſſible number of years during which the oldeſt of the three lives can be extended, will become the expreſſion of the value of ſuch an annuity during the whole joint continuance of the three lives.

lives. And in Art. ccccLXXIII, pages 564, 565, 566, an example is given of the computation of the value of ſuch an annuity for the joint continuance of three lives of the ages of 75 years, 80 years, and 85 years, by means of the ſaid expreſſion. But, as the computation of theſe values of annuities for three joint lives occaſionally, or as we want them, by means of the aforeſaid expreſſion, in the manner exemplified in Art. ccccLXXIII, is in moſt caſes, and eſpecially when the lives are all young, extremely tedious; — and the immenſe variety of combinations with each other, which the ages of three different lives will admit of, makes it almoſt impoſſible to compute any competent numbers of theſe values; —— it is ſtill more neceſſary than in the caſe of annuities for two joint lives, to have recourſe to a ſhorter, though leſs accurate, method of computing them. With this view I have endeavoured to find a method of deriving the value of an annuity of one pound a year for three joint lives of given ages from the value of the ſame annuity for the joint continuance of the two older of the ſaid lives, which I have ſuppoſed to be either contained in ſome of the tables that have been computed for two joint lives, or to be derivable from thoſe tables by the above-mentioned methods of *Interpolation* and *Interpolation and Continuation*, or to be obtainable by means of the corrected expreſſion recommended for this purpoſe in pages 560, 561. And I have ſucceeded in ſome degree, (though not ſo well as I could wiſh,) in this endeavour, having found an expreſſion for the value of ſuch an annuity for three joint lives, which exhibits the ſaid value, for the moſt part, within leſs than a 20th part of the truth. The method in which this expreſſion is obtained, is explained in pages 568, 569, and 570. And the following twenty pages, to page 590, are employed in making a variety of trials of the degree of exactneſs with which this expreſſion exhibits the values of annuities for three joint lives, by comparing the values of ſuch annuities, obtained by means of the ſaid expreſſion, with the values of the ſame annuities accurately computed, as they are contained in certain tables of Mr. Morgan and Dr. Price. From all theſe trials, taken together, I have ventured to conclude in Art. ccccxcv, page 590, that it ſeems reaſonable to conjecture that the ſaid expreſſion given in pages 568, 569, and 570, to wit, $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times \dot{P}^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$, will always give

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us a tolerable approximation to the true value of an annuity of one pound a year for three joint lives of any ages whatsoever.

In pages 590, 591, 592, another method of approximating to the value of an annuity of one pound a year for the joint continuance of three lives of given ages, is delivered; which was published, and, I presume, invented, by the late very learned mathematician, Mr. Thomas Simpson, of Woolwich. And in pages 592, 593, an example is given of the computation of the value of such an annuity by this method in the case of three equal joint lives, all of the age of 60 years: and the near value of the said annuity, obtained by this computation, is found to be somewhat nearer than the near value of the same annuity resulting from the former expression, $\overset{£}{B} \times \frac{P^{1}}{P} \times \frac{g \times P^{1} - h \times P^{1}}{g \times P^{1} - h \times P^{11}}$, to the true value of that annuity. And in the following five pages, to wit, 594, 595, 596, 597, and 598, further trials are made of the exactness of this method of Mr. Simpson; from all which it is concluded in page 599, that the said method of Mr. Simpson is, for the most part, more exact, as well as always much shorter and easier to practice, than the other approximation, by means of the expression $\overset{£}{B} \times \frac{P^{1}}{P} \times \frac{g \times P^{1} - h \times P^{1}}{g \times P^{1} - h \times P^{11}}$, and upon the whole, deserves to be preferred to it. Yet, as there are now and then some instances in which the other method, by means of the expression $\overset{£}{B} \times \frac{P^{1}}{P} \times \frac{g \times P^{1} - h \times P^{1}}{g \times P^{1} - h \times P^{11}}$, comes nearer to the truth than this method of Mr. Simpson, I think it is convenient to be possessed of both methods, to the end that in doubtful cases we may resort to one of them as a kind of confirmation of the result obtained by the other, to a certain degree of exactness.

In Art. cccccx, page 599, it is observed, that, in all the trials of Mr. Simpson's approximation to the values of annuities for three joint lives, which have been made in the preceeding pages, the differences of the near values thereby obtained, from the true values of the same annuities, respectively, seldom exceed an eighth part of a

year's

year's annuity: which is a ſufficient degree of exactneſs for all practical purpoſes, and conſequently is a great recommendation of the ſaid method.

The five remaining pages of the book, to wit, pages 600, 601, 602, 603, 604, relate to the values of annuities for the longeſt of two, or three, lives of given ages. As it has been ſhewn in the former part of this work, in the 4th and 11th corollaries to Prob. IV, that the values of all annuities of this kind may be derived from thoſe of the ſame annuities for the ſame lives taken ſingly and jointly, it is ſhewn that, if there be three lives of different ages, and A be put for the value of an annuity of one pound a year for the youngeſt life, and B for the value of a like annuity for the middle life, and C for the value of a like annuity for the oldeſt life; and AB be put for the value of a like annuity for the joint continuance of the firſt, or youngeſt, life, and the middle life, and AC for the value of the like annuity for the joint continuance of the firſt, or youngeſt life, and the oldeſt life, and BC for the value of a like annuity for the joint continuance of the middle life and the youngeſt life; and ABC be put for the value of a like annuity for the joint continuance of all the three lives; I ſay, it is ſhewn that, if this notation be uſed, the value of a like annuity of one pound a year for the longeſt of the firſt and ſecond lives will be equal to $A + B - AB$, and the value of a like annuity for the longeſt of all the three lives will be equal to $A + B + C + ABC - AB - AC - BC$, or $A + B + C - AB - AC - BC + ABC$. Then, in Art. CCCCCXII, pages 601, 602, an example is given of the computation of the value of an annuity of one pound a year for the longeſt of two lives of the ages of 20 years and 30 years, by means of the expreſſion $A + B - AB$; and in Art. DXIII, pages 602, 603, 604, an example is given of the computation of a like annuity of one pound a year for the longeſt of three lives of the ages of 20 years, 30 years, and 40 years, by means of the expreſſion $A + B + C - AB - AC - BC + ABC$. After which example this treatiſe on life-annuities is concluded in page 604.

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The Appendix to this treatise on Life-annuities consists of the abovementioned *Bill for establishing Life-annuities in Parishes*, which the late Mr. Dowdeswell brought into the House of Commons in the year 1773, together with the two tables of the values of life-annuities, at the interest of 3 per cent. for the use of parishes in *London* and in the country, which Sir *George Savile* procured to be computed, under the inspection of Dr. *Price*, for the purposes of the said bill, and which were considered as a part of it. The reasons for reprinting these tables and the bill to which they refer, are stated in page 605.

A LIST of ERRATA, to be corrected in the following Work, and of some AMENDMENTS to be added to it.

In page 30, line 5th, instead of "£.2612" read "£.2696".

In page 31, line 5th, instead of "£3.7212, or £3. 14*s.* 5*d.*" read "£3.7296, or £3. 14*s.* 7*d.*"

In the same page, line 8th, instead of "£.2612" read "£.2696"; and in line 9th, instead of "£.7814" read "£.7898"; and in line 11th, instead of "£3.7212" read "£3.7296"; and in line 13th, instead of "£.7814" read "£.7898".

In page 32, dele the note at the end of Art. XXXIII, beginning thus, *N. B. This result is greater*, and containing six lines.

In page 34, line 4th, instead of "£.2612" read "£.2696"; and line 14th, instead of "£.8650" read "£.8734".

In page 48, Art. XLIX, line 3d, after the word "annuity" insert the words "of one pound a year."

In page 96, line 4th, dele the comma after the word "as".

In page 107, Art. XCVI, lines 8th and 9th, instead of the words "almost one half a year's payment, and consequently would be worth more than 21 years purchase," read these words, "about one half of one of those half-yearly payments, or a quarter of a year's annuity, and consequently would be worth very nearly 21 years purchase."

In page 123, the last line, instead of "$\frac{1}{1.035} \times$ £16.448,528 =) £15.892,297, or £15. 17s. 10d." read "$\frac{1}{1.035} \times$ £16.402,375 =) £15.847,705, or £15. 16s. $11\frac{1}{2}$d.

In page 136, line 2nd, after the words "one pound" insert the words "a year."

In page 137, line 7th, after the words "one pound" insert the words "a year."

——— line 18th, after the word "pence" place a semicolon; and add these words, "which does not differ greatly from 2 pence, 3 pence, farthing, and 3 pence, half-penny, which are the differences of the four next preceeding differences."

In page 159, line 5th, instead of "$\frac{1}{r-1} \times \frac{1}{r^n \times \overline{r-1}}$" read "$\frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}$."

In page 161, Art. CXLI, line 1st, instead of "n" read "a"; and in line 5th, or the line at the bottom of the page, instead of "equal to a" read "equal to o".

In page 168 the operation of the algebraic division of the series $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3}$ — &c. — $\frac{1}{nr^{n-1}}$ by $r - 1$ might have been set forth more fully by inserting the last term of it, to wit, $-\frac{1}{nr^{n-1}}$, after the &c. and at the same time inserting the corresponding terms in the quotient and in the series of successive dividends. If this had been done, (which, perhaps, it would have been better to do) the whole operation would have been as follows.

$r - 1$)

$$r-1)\ \frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \&c. - \frac{1}{nr^{n-1}} \left(\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \&c. + \frac{n-\overline{n-1}}{nr^{n-1}}, \text{ or } + \frac{1}{nr^{n-1}}.\right.$$

$$\frac{n-1}{n} - \left|\frac{n-1}{nr}\right.$$

$$* + \frac{n-2}{nr} - \frac{1}{nr^2}$$

$$+ \frac{n-2}{nr} - \left|\frac{n-2}{nr^2}\right.$$

$$* + \frac{n-3}{nr^2} - \frac{1}{nr^3}$$

$$+ \frac{n-3}{nr^2} - \left|\frac{n-3}{nr^3}\right.$$

$$* + \frac{n-4}{nr^3} - \&c.$$

$$+ \frac{n-\overline{n-1}}{nr^{n-2}} - \frac{1}{nr^{n-1}}.$$

$$+ \frac{n-\overline{n-1}}{nr^{n-2}} - \frac{1}{nr^{n-1}}$$

$$* \qquad *$$

In page 216, in the fifth column of numbers, the 22[nd] number, 4,697,003, instead of a comma place a full stop "." after the highest figure, 4.

In page 225, Table XVI, in the fourth column of numbers, in the first, the eighth, and the fifteenth numbers, which answer to the ages of 34 years, 41 years, and 48 years, place full stops, "." "." and "." after the two highest figures, 16, 14, and 13, of those numbers respectively.

In page 240, line 4[th], after the &c. insert these words, "and with $r^{\frac{1}{2}} \times \frac{£1}{2P} \times$."

And in line 5[th], instead of "$\frac{£1}{2P}$" read "$\frac{£1}{P}$."

And in line 7[th], instead of "$\frac{\Pi E-N+1}{r E-N+1}$" read "$r^{\frac{1}{2}} \times \frac{£1}{2P} \times \frac{\Pi E-N+1}{r E-N+1}$."

And

And in lines 8th and 9th, after "$+\frac{P\iota v}{r^5}+$ &c." read as follows,

"$+\frac{r^{\frac{1}{2}}}{4}\times\overset{\pounds}{V}+r^{\frac{1}{2}}+\frac{\overset{\pounds}{1}}{2P}\times$ the term $\frac{\Pi^{E-N+1}}{r^{E-N+1}}=$ (if we neglect this last quantity $r^{\frac{1}{2}}\times\frac{\overset{\pounds}{1}}{2P}\times\frac{\Pi^{E-N+1}}{r^{E-N+1}}$ on account of its extreme smallness) &c." as in the book.

In page 300, line 5th from the bottom, instead of "this third method" read "this fourth method".

In page 302, line 20th, instead of "$\frac{1000,0000}{554,338}$" read "$\frac{1000,000}{554,338}$".

In page 368, the last line, in the number £4,711,195,1609, place a full stop before the four last figures, thus, 4,711,195.1609.

In page 502, line 6th, after the words, "by 90 years," insert these words, "for the joint continuance of which an annuity will be of any value,"

——— In line 9th, at the end of Art. CCCCIX, add these words, "See below, page 529, Art. CCCCXXXIII."

In page 560, line 2nd from the bottom, instead of "to the instances" read "in the instances".

In page 581, Art. CCCCLXXXV, line 11th, instead of "livesin" read "lives in".

In page 592, Art. CCCCXCIX, line 7th, after the word "interest" insert the word "of".

In page 597, in the marginal abstract of Art. CCCCCV, line 10th of the said abstract, after the words "than the near values" insert the words, "of the same annuities".

THE

THE

PRINCIPLES, &c.

ARTICLE I.

THE doctrine of life-annuities is by no means of so abstruse and difficult a nature as many people are apt to imagine. A moderate share of common sense, or capacity to reason justly, and a knowledge of common arithmetick, are all the qualities that are necessary to a right understanding of the principles on which it is founded; even so far as to be able to compute the value of any proposed annuity for any given life or number of lives, if a person is disposed to undergo the labour of performing all the necessary arithmetical operations that arise in such a computation. To explain these principles in an easy and familiar manner, so as to make them intelligible to as many readers as possible, without having recourse to Algebra or the books written on the doctrine of chances, is the design of the following pages: which, as the subject of life-annuities is a matter of very general concern, will, I flatter myself, be considered by the publick as an useful and commendable undertaking. *Design of this tract.*

II. A life-annuity is a set of equal sums of money to be paid at certain future times, at the distance of a year one after the other, during the whole course of the life of the person for whose life the annuity is granted. Thus, if a man grants an annuity of 100*l.* a year to a young man of twenty years of age, he thereby undertakes to pay him one sum of 100*l.* at the end of one year from the time of making him the grant, and another sum of 100*l.* at the end of two years from the same time, and a third sum of 100*l.* at the end of three years from the same time, and a fourth sum of 100*l.* at the end of four years from the same time; and so on, paying a new sum of 100*l.* at the end of every following year throughout the whole life of the grantee. If therefore we can find the values of these several single payments of 100*l.* each, which are to be made to the grantee of the life-annuity at the end of the several future years of his life, but of which no part is to be paid to his executors, or other representatives, when he is dead, we may, by adding all these values together, find the value of the sum of all these payments, or of the whole life-annuity. We must there- *Definition of a life-annuity.*

fore

fore endeavour to find ſome method of determining the preſent value of a ſingle future payment of any given ſum of money to be made to a perſon of any given age at the diſtance of any given number of years, or when he ſhall have attained to any greater given age. Now this value will depend upon two circumſtances; which muſt therefore be previouſly agreed upon, before it can be known: namely, the rate of intereſt at which money may be improved, and the probability of the duration of human life. For, the higher the intereſt of money is, the leſs ought the granter to take for his grant of a given ſum of money, to be paid at a future time: becauſe, when the intereſt of money is high, he may increaſe ſuch leſſer ſum of money, in the interval of time between the grant and the future payment, ſo as to make it amount to as great a ſum at that time as would have been produced by a greater ſum of money received at the time of making the grant, and improved likewiſe at compound intereſt during the ſame interval, if the intereſt of money had been lower. And the greater is the frailty of human life, or the greater the probability that the grantee of the propoſed future payment will die before it becomes due, the leſs alſo ought to be the ſum of money paid to the granter for his grant of it, becauſe he will run a ſmaller riſque of being obliged to pay it. We muſt therefore in every queſtion of this ſort determine beforehand at what rate of intereſt the money paid to the granters for ſuch future payments may be improved, and with what degree of probability it may be expected that the perſons to whom they are granted will, or will not, live till they become due.

Of the preſent value of a future contingent payment of a given ſum of money.

Of the probability of the duration of human life at a given age.

III. Now, as to this latter circumſtance of the degree of probability that a perſon of a given age will, or will not, live to any other given age, or till the ſum of money granted to him becomes due, it is obviouſly in many caſes a matter of very great uncertainty, and will be often very different in different perſons of the ſame age. The chance which a man of 30 years of age, who is in good health and leads a temperate and quiet life in the country, has to live 20 years, or till he is 50 years of age, is evidently much greater than that of another man of the ſame age of 30 years, and of the ſame degree of health and vigour of body, who is going into a hot and unhealthy climate, to which he has never been accuſtomed, as, for example, to Senegal in Africa: and it is likewiſe greater than that of another man of 30 years of age and of the ſame degree of health and vigour, but who lives in a capital city and in ſcenes of pleaſure and debauchery; and ſtill more evidently it is greater than that of another man of 30 who is of a weakly and unhealthy conſtitution of body, or who by his daily occupation is expoſed to many dangers of his life, from which the generality of mankind are exempt, as is the caſe with ſoldiers and ſailors in time of war. But theſe are circumſtances out of the reach of calculation, and muſt be left to be conſidered by the perſons who grant and purchaſe life-annuities according to their own judgement and diſcretion

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in the particular cafes in which they occur. All that can be done by any general rules upon this fubject, is to eftimate the degree of probability with which it may reafonably be expected that a perfon of any given age will live to any other given age upon a fuppofition that he has neither a better nor a worfe chance of doing fo than the majority of other perfons of the fame age. And this medium, or average, chance of living is determined by tables that exhibit the numbers of perfons which, out of a certain pretty large number of children of one, or two, or three years of age, (which is ufually not lefs than 1000,) all living at the fame time, are found (by methods of reafoning that are grounded on long feriefes of obfervations,) to be living at the end of every fubfequent year of human life to its extreme period, which fome of the tables carry to 86, and others to more than 90, years. The inftances of the prolongation of human life to more than 100 years are fo unfrequent, that they are not thought to be worth attending to in forming any general rules upon this fubject.

Of tables of the faid probability.

IV. The moft exact tables of this kind that have hitherto been publifhed feem to be thofe of Mr. Kerffeboom, and Monfieur de Parcieux; which are inferted in the Appendix to Mr. De Moivre's Treatife on the Valuation of Annuities. The former was publifhed in an effay of the aforefaid Mr. Kerffeboom on the number of people in the provinces of Holland and Weft-Friefland, written in the Dutch language, about the year 1738, (of which an account is given in the 9th volume of the Abridgement of the Philofophical Tranfactions, page 326,) and is faid to have been formed from certain tables of affignable annuities for lives in Holland, which had been kept there for 125 years, and in which the ages of the feveral perfons dying in that period had been truly entered. And Mr. de Parcieux's table was made, as Mr. De Moivre informs us, by a like ufe of the lifts of the *French Tontines*, or *Long annuities*; and the numbers of it were verified by the *necrologies*, or mortuary regifters, of feveral religious houfes of both fexes. Thefe feem to be the moft folid and authentick grounds upon which it is poffible to form any tables of this kind: whereas there are fome circumftances of doubt and uncertainty in the methods of forming all the other tables of the probable duration of human life which prevent them from being intirely fatisfactory. And therefore I conceive thefe two tables to be more exact and fit to be adopted in computing the values of life-annuities than any other tables I have feen; and particularly in computing the values of any annuities for lives which the government of this kingdom may at any time think fit to grant, if that method of raifing money fhould hereafter be adopted, (as is the cafe at this time in Ireland,) or it fhould be thought expedient to difcharge a part of the national debt in that way, by converting a part of the perpetual three per cent. annuities, payable at the Bank, into annuities for the lives of their refpective proprietors, or for a term certain of 20 or 30 years and further for their lives. Thefe two tables are as follows.

Of the tables of probability of Mr. Kerffeboom and Monfieur de Parcieux.

V. TABLE I.

Representing the probabilities of the duration of human life at the several ages therein mentioned from the time of birth to the age of an hundred years: grounded on the registers of certain assignable annuities for lives granted by the government of Holland, which had been kept there for 125 years, and in which the ages of the several annuitants dying during that period had been truly entered.

By Mr. KERSSEBOOM.

Age.	*Persons living.*	*Age.*	*Persons living*	*Age.*	*Persons living.*	*Age.*	*Persons living.*
0	1400	26	760	52	482	78	130
1	1125	27	747	53	470	79	115
2	1075	28	735	54	458	80	100
3	1030	29	723	55	446	81	87
4	993	30	711	56	434	82	75
5	964	31	699	57	421	83	64
6	947	32	687	58	408	84	55
7	930	33	675	59	395	85	45
8	913	34	665	60	382	86	36
9	904	35	655	61	369	87	28
10	895	36	645	62	356	88	21
11	886	37	635	63	343	89	15
12	878	38	625	64	329	90	10
13	870	39	615	65	315	91	7
14	863	40	605	66	301	92	5
15	856	41	596	67	287	93	3
16	849	42	587	68	273	94	2
17	842	43	578	69	259	95	1
18	835	44	569	70	245	96	0.6
19	826	45	560	71	231	97	0.5
20	817	46	550	72	217	98	0.4
21	808	47	540	73	203	99	0.2
22	800	48	530	74	189	100	0.0
23	792	49	518	75	175		
24	783	50	507	76	160		
25	772	51	495	77	145		

VI. The

VI. The meaning of the foregoing table is this; That from the observations on which it is grounded there is reason to conclude that out of 1400 new-born children 1125 have lived to the end of one year, and 1075 to the end of two years, and 1030 to the end of three years; and so on throughout the table; the figures that are parallel and contiguous to every age expressing the number of persons out of the aforesaid original number of 1400 infants that are living at that age. As to the figures 0.6, 0.5, 0.4, 0.2, adjoining to the 96th, 97th, 98th, and 99th year of the hundred, which denote the decimal fractions $\frac{6}{10}$, $\frac{5}{10}$, $\frac{4}{10}$, and $\frac{2}{10}$, it is obvious that in a strict sense they are inapplicable to any human, or other living, creatures: and therefore they must be understood in the following manner. We must suppose the original number of new-born infants, who were the root, or basis, of the table, to have been increased tenfold, so as to become 14000 instead of 1400: in which case it is evident that all the following numbers in the table, representing the persons living at the several ages herein set down, will likewise be increased to ten times their former quantity; and therefore the number of persons living at the age of 95 years will be 10 instead of 1. Now the meaning of the figures 0.6, 0.5, 0.4, 0.2, and of the cyphers 0.0 adjoining to the 100th year, is that, out of these 10 persons who would in such case have been found to be living at the age of 95 years, only 6 would have been found to be living at the age of 96, and 5 at the age of 97, and 4 at the age of 98, and 2 at the age of 99, and none at all at the age of an hundred years.

VII. The second table above-mentioned, given us by Mr. De Parcieux, and inserted by Mr. De Moivre in the appendix to his treatise on the valuation of annuities, is as follows.

TABLE

TABLE II.

Representing the probabilities of the duration of human life at the several ages therein mentioned, from the age of 3 *years to the age of* 95: *grounded on lists of the French* Tontines, *or* Long Annuities, *and verified by a comparison thereof with the necrologies, or mortuary registers, of several religious houses of both sexes.*

By Monsieur DE PARCIEUX.

Age.	*Persons living.*	*Age.*	*Persons living.*	*Age.*	*Persons living.*	*Age.*	*Persons living.*	*Age.*	*Persons living.*
3 yrs.	1000	23	790	43	636	63	423	83	71
4	970	24	782	44	629	64	409	84	59
5	948	25	774	45	622	65	395	85	48
6	930	26	766	46	615	66	380	86	38
7	915	27	758	47	607	67	364	87	29
8	902	28	750	48	599	68	347	88	22
9	890	29	742	49	590	69	329	89	16
10	880	30	734	50	581	70	310	90	11
11	872	31	726	51	571	71	291	91	7
12	866	32	718	52	560	72	271	92	4
13	860	33	710	53	549	73	251	93	2
14	854	34	702	54	538	74	231	94	1
15	848	35	694	55	526	75	211	95	0
16	842	36	686	56	514	76	192		
17	835	37	678	57	502	77	173		
18	828	38	671	58	489	78	154		
19	821	39	664	59	476	79	136		
20	814	40	657	60	463	80	118		
21	806	41	650	61	450	81	101		
22	798	42	643	62	437	82	85		

VIII. Both

VIII. Both the foregoing tables represent human life as being more permanent, or the number of persons dying every year, out of a given number of persons alive at any given younger age, as being smaller, throughout both youth and middle age, and even till about the age of 70, than they appear to be by most other tables. And this is what we ought naturally to have expected, on account of the difference of the *data*, or grounds, on which these and those other tables have been constructed. For both these tables of Mr. Kersseboom and Mr. de Parcieux were formed, as we before observed, from the registers of the deaths of persons who had purchased life-annuities from the governments of Holland and France: whereas those other tables have been formed, for the most part, from the registers of the births and burials of all the inhabitants in general of the places for which they are computed. Now it is reasonable to believe that those persons who purchase annuities of a government are, for the most part, in easier circumstances, and less exposed to the dangers and hardships that destroy, or shorten, human life, than the rest of mankind, and, consequently, than the whole body of mankind (themselves included) upon an average: and therefore it is not likely that they should die off in the same average, or general, proportion, but at a slower rate.

Of the difference between the two foregoing tables of probabilities.

IX. But though both these tables of Mr. Kersseboom and Monsieur de Parcieux are formed from the registers of the deaths of government life-annuitants, and both of them represent the probability of the duration of human life as greater than it appears to be by any other tables, until towards the age of 70 years, yet they do not intirely agree with each other; but the table of Mr. de Parcieux represents that probability as still greater than that of Mr. Kersseboom, till towards the said advanced age of 70 years, and from that time somewhat less. The degree of difference between them in this respect may be collected from a comparison of them with each other at the several ages of 20, 30, 40, 50, 60, 70, and 80 years, which may be made in the following manner.

A comparison of them with each other with respect to persons of the age of 20 years.

X. Let us first take the persons living together at the age of 20 years in each of these tables, and see how many of those persons are represented in each of them to be living at the following ages of 30, 40, 50, 60, 70, 80, and 90, years, and then inquire in which of these two tables the proportions by which these numbers continually decrease are greatest.

Now

Now it appears by Mr. Kersseboom's table that out of 817 persons of 20 years of age, all living at the same time,

	711 will have lived	to the age of 30 years,
and	605	to the age of 40,
and	507	to the age of 50,
and	382	to the age of 60,
and	245	to the age of 70,
and	100	to the age of 80,
and	10	to the age of 90 years.

And by the table of Monsieur de Parcieux it appears that out of 814 persons of the same age of 20 years, all living at the same time,

	734 will have lived	to the age of 30 years,
and	657	to the age of 40,
and	581	to the age of 50,
and	463	to the age of 60,
and	310	to the age of 70,
and	118	to the age of 80,
and	11	to the age of 90 years.

It appears therefore that according to Monsieur de Parcieux's table there will be more persons found to have been living at the several ages of 30, 40, 50, 60, 70, 80, and 90, years, out of an original number of only 814 persons of the age of 20 years, all living at the same time, than are found to have been living at the same ages respectively, according to Mr. Kersseboom's table, out of the greater original number of 817 persons of the same age of 20 years all living at the same time; and that in the very considerable proportions of 734 to 711, 657 to 605, 581 to 507, 463 to 382, 310 to 245, 118 to 100, and 11 to 10. It is evident therefore that a life-annuity for the life of a person of 20 years of age is worth considerably more money according to Mr. de Parcieux's table than according to that of Mr. Kersseboom.

The like comparison of them with respect to the age of 30 *years.*

XI. In the next place we will compare these two tables together with respect to the age of 30 years.

Now

Now by Mr. Kerſſeboom's table it appears that out of 711 perſons of the age of 30 years, all living at the ſame time,

	605	will have lived to the age of 40 years,
and	507	to the age of 50,
and	382	to the age of 60,
and	245	to the age of 70,
and	100	to the age of 80,
and	10	to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 734 perſons of 30 years of age, all living at the ſame time,

	657	will have lived to the age of 40 years,
and	581	to the age of 50,
and	463	to the age of 60,
and	310	to the age of 70,
and	118	to the age of 80,
and	11	to the age of 90 years:

and conſequently out of 711 perſons of 30 years of age, all living at the ſame time, (which is the number of perſons living at that age in Mr. Kerſſeboom's table,)

($\frac{657 \times 711}{734}$, or) 636 will have lived to the age of 40 years,

and ($\frac{581 \times 711}{734}$, or) 563 to the age of 50,

and ($\frac{463 \times 711}{734}$, or) 448 to the age of 60,

and ($\frac{310 \times 711}{734}$, or) 300 to the age of 70,

and ($\frac{118 \times 711}{734}$, or) 114 to the age of 80,

and ($\frac{11 \times 711}{734}$, or) 10 to the age of 90 years.

It appears therefore that out of the ſame original number of 711 perſons of 30 years of age, all living at the ſame time, there will be more perſons found to be living at the ſeveral ſubſequent ages of 40, 50, 60, 70, and 80, years, according to Monſieur de Parcieux's table than according to

that

that of Mr. Kerſſeboom, in the ſeveral very conſiderable proportions of 636 to 605, 563 to 507, 448 to 382, 300 to 245, and 114 to 100, and the ſame number, to wit, 10 perſons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a perſon of 30 years of age would be worth a conſiderably greater ſum of money according to Monſieur de Parcieux's table than according to that of Mr. Kerſſeboom.

The like compariſon of them with reſpect to the age of 40 years.

XII. We will next compare theſe two tables together with reſpect to the age of 40 years.

Now by Mr. Kerſſeboom's table it appears that out of 605 perſons of 40 years of age, all living at the ſame time,

	507	will have lived	to the age of 50,
and	382		to the age of 60,
and	245		to the age of 70,
and	100		to the age of 80,
and	10		to the age of 90 years.

And by the table of Mr. de Parcieux it appears that out of 657 perſons of 40 years of age, all living at the ſame time,

	581	will have lived	to the age of 50 years,
and	463		to the age of 60,
and	310		to the age of 70,
and	118		to the age of 80,
and	11		to the age of 90 years:

and conſequently out of 605 perſons of 40 years of age, all living at the ſame time,

$\left(581 \times \frac{605}{657}, \text{ or}\right)$ 540 will have lived to the age of 50 years,

and $\left(463 \times \frac{605}{657}, \text{ or}\right)$ 426 to the age of 60,

and $\left(310 \times \frac{605}{657}, \text{ or}\right)$ 285 to the age of 70,

and $\left(118 \times \frac{605}{657}, \text{ or}\right)$ 108 to the age of 80,

and $\left(11 \times \frac{605}{657}, \text{ or}\right)$ 10 to the age of 90 years.

It

It appears therefore that out of the ſame original number of 605 perſons of 40 years of age, all living at the ſame time, there will be more perſons found to have been living at the ſeveral ſubſequent ages of 50, 60, 70, and 80 years according to Monſieur de Parcieux's table than according to that of Mr. Kerſſeboom, in the ſeveral conſiderable proportions of 540 to 507, 426 to 382, 285 to 245, and 108 to 100; and the ſame number, to wit, 10 perſons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a perſon of 40 years of age would be worth conſiderably more by Mr. de Parcieux's table than by that of Mr. Kerſſeboom.

The like compariſon of them with reſpect to the age of 50 years.

XIII. We will next compare theſe two tables with each other with reſpect to the age of 50 years.

Now by Mr. Kerſſeboom's table it appears that out of 507 perſons of 50 years of age, all living at the ſame time,

	382	will have lived	to the age of 60 years,
and	245		to the age of 70,
and	100		to the age of 80,
and	10		to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 581 perſons of the age of 50 years, all living at the ſame time,

	463	will have lived	to the age of 60 years,
and	310		to the age of 70,
and	118		to the age of 80,
and	11		to the age of 90 years:

and conſequently out of 507 perſons of 50 years of age, all living at the ſame time, (which is the number of perſons living at that age in Mr. Kerſſeboom's table)

$\left(463 \times \frac{507}{581}, \text{ or}\right)$ 404 will have lived to the age of 60 years,

and $\left(310 \times \frac{507}{581}, \text{ or}\right)$ 270 to the age of 70,

and $\left(118 \times \frac{507}{581}, \text{ or}\right)$ 103 to the age of 80,

and $\left(11 \times \frac{507}{581}, \text{ or}\right)$ 10 to the age of 90 years.

It

It appears therefore that out of the same original number of 507 persons of 50 years of age, all living at the same time, there will be more persons found to have been living at the several subsequent ages of 60, 70, and 80 years, according to Mr. de Parcieux's table than according to that of Mr. Kersseboom, in the proportions of 404 to 382, 270 to 245, and 103 to 100; and the same number, to wit, 10 persons, at the age of 90 years. It is evident therefore that a life-annuity for the life of a person of 50 years of age, as well as a life-annuity for the life of a person of 20, or 30, or 40, years of age, will be worth more according to the table of Monsieur de Parcieux than according to that of Mr. Kersseboom. But the difference of the values of such an annuity according to these two different tables will not be so great at the age of 50 as at those younger ages.

The like comparison of them with respect to the age of 60 years.

XIV. We will next examine the two tables with respect to the age of 60 years.

Now by Mr. Kersseboom's table it appears that out of 382 persons of 60 years of age, all living at the same time,

	245	will have lived	to the age of 70 years,
and	100		to the age of 80,
and	10		to the age of 90 years.

And by the table of Monsieur de Parcieux it appears that out of 463 persons of the age of 60 years, all living at the same time,

	310	will have lived	to the age of 70 years,
and	118		to the age of 80,
and	11		to the age of 90 years:

and consequently that out of 382 persons of 60 years of age, all living at the same time, (which is the number of persons living at that age in Mr. Kersseboom's table,)

$\left(310 \times \frac{382}{463}, \text{ or}\right)$ 256 will have lived to the age of 70 years,

and $\left(118 \times \frac{382}{463}, \text{ or}\right)$ 97 to the age of 80,

and $\left(11 \times \frac{382}{463}, \text{ or}\right)$ 9 to the age of 90 years.

It

It appears therefore that out of the ſame original number of 382 perſons of 60 years of age, all living at the ſame time, more perſons will be found to have been living at the age of 70 years by Mr. de Parcieux's table than by Mr. Kerſſeboom's in the proportion of 256 to 245; but that at the ſubſequent ages of 80 and 90 years fewer will be found to have been living by Mr. de Parcieux's table than by that of Mr. Kerſſeboom, in the proportions of 97 to 100, and 9 to 10. Now it is evident that the value of a life-annuity (which is, as we before obſerved, a ſet of equal future payments to be made at the end of every year during the continuance of the life for which it is granted,) depends more on the value of the payments that are to be made during the firſt years of its continuance than on thoſe which are to be made at more remote periods, both becauſe thoſe firſt payments are nearer at hand, and becauſe the probability of their becoming due by the continuance of the life for which they are granted is greater than the probability of the ſame event with reſpect to the latter payments. It ſeems reaſonable therefore to conclude, (though this cannot be affirmed with certainty without actually making the calculation,) that a life-annuity for the life of a perſon of 60 years of age will be worth ſomething more according to Mr. de Parcieux's table than according to that of Mr. Kerſſeboom, but not in ſo great a degree as at the younger ages before-mentioned.

The like compariſon with reſpect to the age of 70 years.

XV. We will next examine the two tables with reſpect to the age of 70 years.

Now by Mr. Kerſſeboom's table it appears that out of 245 perſons of 70 years of age, all living at the ſame time,

	231	will have lived	to the age of 71 years,
and	175		to the age of 75,
and	100		to the age of 80,
and	45		to the age of 85,
and	10		to the age of 90 years.

And by the table of Monſieur de Parcieux it appears that out of 310 perſons of 70 years of age, all living at the ſame time,

	291	will have lived	to the age of 71 years,
and	211		to the age of 75,
and	118		to the age of 80,
and	48		to the age of 85,
and	11		to the age of 90 years:

and

and consequently that out of 245 persons of 70 years of age, all living at the same time (which is the number of persons living at that age in Mr. Kersseboom's table,)

$$\left(\frac{291 \times 245}{310}, \text{ or}\right) 230 \text{ will have lived to the age of } 71 \text{ years,}$$

$$\text{and } \left(\frac{211 \times 245}{310}, \text{ or}\right) 167 \text{ to the age of } 75,$$

$$\text{and } \left(\frac{118 \times 245}{310}, \text{ or}\right) 93 \text{ to the age of } 80,$$

$$\text{and } \left(\frac{48 \times 245}{310}, \text{ or}\right) 38 \text{ to the age of } 85,$$

$$\text{and } \left(\frac{11 \times 245}{310}, \text{ or}\right) 9 \text{ to the age of } 90 \text{ years.}$$

It appears therefore that out of an original number of 245 persons of 70 years of age, all living at the same time, there will be fewer persons found to have been living at the several subsequent ages of 71, 75, 80, 85, and 90, years by Monsieur de Parcieux's table than by that of Mr. Kersseboom, in the several proportions of 230 to 231, 167 to 175, 93 to 100, 38 to 45, and 9 to 10. And consequently the value of a life-annuity for the life of a person of 70 years of age will be less according to Mr. de Parcieux's table than according to that of Mr. Kersseboom though it has been seen that in all younger ages from 20 years upwards the value of a life-annuity is greater by the former table than by the latter.

The like comparison with respect to the age of 80 years.

XVI. In the last place we will examine the two tables with respect to the age of 80 years.

Now it appears by Mr. Kersseboom's tables that out of 100 persons of the age of 80 years, all living at the same time,

	87	will have lived	to the age of 81 years,
and	45		to the age of 85,
and	10		to the age of 90 years.

And

And by the table of Monfieur de Parcieux it appears that out of 118 perfons of the age of 80 years, all living at the fame time,

	101	will have lived	to the age of 81 years,
and	48		to the age of 85,
and	11		to the age of 90 years:

and confequently that out of 100 perfons of the age of 80 years, all living at the fame time, (which is the number of perfons living at that age in Mr. Kerfieboom's table,)

$\left(101 \times \frac{100}{118}, \text{ or}\right)$ 85 will have lived to the age of 81 years,

and $\left(48 \times \frac{100}{118}, \text{ or}\right)$ 40 to the age of 85,

and $\left(11 \times \frac{100}{118}, \text{ or}\right)$ 9 to the age of 90 years.

It appears therefore that out of an original number of 100 perfons of the age of 80 years, all living at the fame time, there will be fewer perfons found to have been living at the feveral fubfequent ages of 81, 85, and 90 years according to Mr. de Parcieux's table than according to that of Mr. Kerffeboom, in the feveral proportions of 85 to 87, 40 to 45, and 9 to 10. And confequently the value of a life-annuity for the life of a perfon of 80 years of age, as well as that of an annuity for the life of a perfon of 70 years of age, will be lefs according to Mr. de Parcieux's table than according to that of Mr. Kerffeboom, though in the younger ages of 20, 30, and 40 years, the value of a life-annuity was confiderably greater by the former table than by the latter.

Of the preference due to Monfieur de Parcieux's table of probabilities above that of Mr. Kerffeboom, with refpect to tables of life-annuities to be calculated for the ufe of Englifhmen.

XVII. Thus much may be fufficient to fhew the difference between thefe two tables of Mr. Kerffeboom and Mr. de Parcieux. But which of them upon the whole deferves to be confidered as the more exact, I will not pretend to determine. Only thus much I will venture to obferve concerning them; That, as the foil and temperature of the air in England bear a greater refemblance, as I conceive, to the foil and temperature of the air in the northern parts of France than to thofe of Holland, which is fo full of moift vapours arifing from the waters amongft which it is fituated; and the Dutch are in general reckoned to be fhorter-lived than either

either the French or the Englifh; it feems reafonable to fuppofe that Mr. de Parcieux's table, which is formed from obfervations made in France, is more likely to afford a juft meafure of the duration of the lives of Englifhmen in the like fituation and circumftances of life, that is, proprietors of government life-annuities, than the table of Mr. Kerffeboom, which is formed from the like obfervations made in Holland. And therefore I conceive that, with refpect to the valuation of annuities on the lives of perfons living in England, and more efpecially of annuities to be granted at any time by the government, Mr. de Parcieux's table deferves to be preferred to the other. And accordingly I fhall have recourfe to it in the enfuing pages for the folution of the few queftions, or examples, upon this fubject which I fhall have occafion to confider.

Of the Breflaw and London tables of the probabilities of the duration of human life.

XVIII. Mr. De Moivre has given us, in his appendix above-mentioned to his treatife on the valuation of annuities, two other tables of the probable duration of human life, which he confiders as very ufeful for the purpofe of computing the values of life-annuities. Thefe are, 1ft, Dr. Halley's table, which he formed from obfervations on the births and burials of the inhabitants of the city of Breflaw, (which is the capital of the dutchy of Silefia in Germany,) during a feries of five years in the latter part of the laft century, to wit, the years 1687, 1688, 1689, 1690, and 1691; and 2dly, that of Mr. Smart, which he formed from fimilar obfervations on the births and burials in London. This laft table has been adopted by the late acute and very learned Mr. Thomas Simpfon, of Woolwich, in his treatife called "*The doctrine of annuities and reverfions*," and made the foundation of all his calculations of the values of annuities for lives, which he defigned principally for the ufe of the inhabitants of London. And Dr. Price has likewife given us thefe two tables of Dr. Halley and Mr. Smart, together with two other tables of the fame kind founded on obfervations of the births and burials at Norwich and Northampton, in his very ufeful treatife called "*Obfervations on reverfionary payments*," which contains the greateft variety of tables for the valuation of annuities, both for lives and terms of years, of any book that I have met with. And to this laft book, (which is defervedly in every body's hands,) I muft refer my readers for fuch of thefe tables as they may have occafion to confult, as I think it would be needlefs to infert them in this tract in which I propofe to make ufe only of Mr. de Parcieux's table for the folution of the few queftions and examples concerning future payments depending upon lives, that will occur in it.

Of

Of the manner of forming a table of the probabilities of the duration of human life.

XIX. The curious reader will probably be glad to be informed in what manner these tables of the probable duration of human life are formed from observations of the births and burials of mankind at the several different ages of human life in any given district. For satisfaction in this particular I must refer him to the two following discourses; to wit, first, to Dr. Halley's celebrated tract in the Philosophical Transactions, intitled, "*An Estimate of the degrees of the mortality of mankind, drawn from various tables of the births and funerals at the city of Breslaw; with an attempt to ascertain the price of annuities upon lives. By Edmund Halley, R. S. S*;" which is likewise published in the first volume of the *Miscellanea Curiosa*, page 280; and, secondly, to the before-mentioned treatise of Dr. Price upon reversionary payments, in which he will find, from page 235 to page 276, an ample and curious discourse upon this subject, intitled, "*Observations on the proper method of constructing tables for determining the rate of human mortality, the number of inhabitants, and the values of lives, in any town or district, from bills of mortality in which are given the numbers dying annually at all ages.*" The part of Dr. Halley's discourse above-mentioned which relates to this subject is contained in the first five pages of it, as it is printed in the *Miscellanea Curiosa*, which is the part preceding the table. The subsequent part of the discourse sets forth the several uses to which the table may be applied; of which the four first uses are explained in a very clear and easy manner, so as to be intelligible to a reader not acquainted with the mathematicks, but the following parts are much more difficult, and require much more knowledge and attention to understand them rightly. This I mention in order to save the unlearned reader, who shall look into that discourse, unnecessary trouble.

XX. As we have mentioned in the 18th article several different tables of the probability of the duration of human life, which different authors have formed from different sets of observations, and particularly those of Dr. Halley and Messieurs Smart and Simpson, it will, I doubt not, be a satisfaction to the reader to be informed of what so great a judge of these matters as Mr. De Moivre has delivered as his opinion concerning their respective merits and uses. I shall therefore now insert in this place the remarks of that able writer upon the four tables of Dr. Halley, Mr. Kersseboom, Mr. de Parcieux, and Messieurs Smart and Simpson, which he has inserted in the appendix to his learned treatise on the valuation of annuities. See his Doctrine of Chances, third edition, pages 345, 346, 347, 348. These remarks are as follows.

Mr. De Moivre's remarks on the four tables of the probability of the duration of human life published by Dr. Halley, Mr. Kersseboom, Mr. de Parcieux, and Messieurs Smart and Simpson, inserted in his treatise on the valuation of annuities.

" The first table is that of Dr. Halley, composed from the bills of " mortality of the city of Breslaw; the best, perhaps, as well as the first " of its kind; and which will always do honour to the judgement and " sagacity of its excellent author.

" Next follows a table of the ingenious Mr. Kersseboom, founded " chiefly upon registers of the Dutch annuitants, carefully examined and " compared for more than a century backward. And Monsieur de Parcieux " by a like use of the lists of the *French Tontines*, or *Long Annuities*, has " furnished us with Table III; whose numbers were likewise verified upon " the *necrologies*, or mortuary registers of several religious houses of both " sexes.

" To these is added the table of Messieurs Smart and Simpson, adapted " particularly to the city of London; whose inhabitants, for reasons too " well known, are shorter-lived than the rest of mankind.

" Each of these tables may have its particular use: The *Second* or " *Third* in valuing the *better* sort of lives, upon which one would chuse to " hold an annuity: The *Fourth* may serve for London, or for lives such " as those of its inhabitants are supposed to be: while Dr. Halley's num- " bers, falling between the two extremes, seem to approach nearer to the " general course of nature. And in cases of combined lives, two or more " of the tables may perhaps be usefully employed.

" Besides these, the celebrated Monsieur *de Buffon* has lately given us " a new table, from the actual observations of Monsieur *du Pré de Saint* " *Maur* of the French Academy. This gentleman, in order to strike a " just *mean*, takes three populous parishes in the city of Paris, and so many " country parishes as furnish him nearly an equal number of lives: and " his care and accuracy in that performance have been such as to merit the " high approbation of the learned Editor. It was therefore proposed to " add this table to the rest; after having purged its numbers of the " inequalities that necessarily happen in fortuitous things, as well as of " those arising from the careless manner in which ages are given in to the " parish-clerks; by which the years that are multiples of 10 are generally " over-loaded.

" But

"But this having been done with all due care, and the whole reduced "to Dr. Halley's denomination of 1000 infants of a year old; there "resulted only a mutual confirmation of the two tables; Mr. Du Pré's "table making the lives somewhat better as far as 39 years, and thence "a small matter worse than they are by Dr. Halley's.

"We may therefore retain this last as no bad standard for mankind in "general; till a better police, in this and other nations, shall furnish the "proper *data* for correcting it, and for expressing the decrements of life "more accurately and in larger numbers.

"For which purpose the parish registers ought to be kept in a better "manner, according to one or other of the forms that have been proposed "by authors. Or, if we suppose the numbers annually born to have "been nearly the same for an age past, the thing may be done at once, "by taking the numbers of the living, with their ages, throughout every "parish in the kingdom: as was in part ordered some time ago by the "right reverend the bishops: but their order was not universally obeyed; "for what reason we pretend not to guess. Certain it is that a *census* of "this kind once established, and repeated at proper intervals, would fur-"nish to our governours and to ourselves much important instruction, of "which we are now in a great measure destitute: Especially if the whole "was distributed into the proper classes of *married* and *unmarried*, *in-"dustrious* and *chargeable* poor, *Artificers* of every kind, *Manufacturers*, &c. "and if this was done in each county, city, and borough, separately; "that particular useful conclusions might thence be readily deduced, as "well as the general state of the nation discovered, and the rate according "to which human life is wasting from year to year. See, on this subject, "the judicious Observations of Mr. Corbyn Morris, addressed to Thomas "Potter, Esq; in the year 1751."

Thus far Mr. De Moivre. Since that time Dr. Price has published two more tables of the probability of the duration of human life formed from observations on the births and burials at Northampton and Norwich, which may be seen in his book above-mentioned, intitled, "*Observations on reversionary payments*," Tables IV and V, pages 317 and 318. And the grounds upon which they are constructed, as well as the method of forming them from those grounds, are set forth in the same book in the discourse intitled, "*Observations on the proper method of constructing tables, &c.*" from page 240 to page 281, third edition.

XXI. I now proceed to the main design of this tract, which is to shew how, by the help of any one such table of the probabilities of the duration of human life as those above-mentioned, the value of any future contin-

 gent

gent payment, depending upon the continuance of one or more lives, may be computed. Now the great fundamental maxim upon which all computations of this kind are grounded, may be explained as follows.

The fundamental maxim of the doctrine of life-annuities.

In every bargain between two perſons concerning a grant of a ſum of money to be paid by the one to the other at a given future time, in caſe the grantee, or purchaſer, ſhall be then alive, or in caſe the grantee and one or more other perſons of given ages ſhall be then alive, the fair price of ſuch future ſum of money, according to a given rate of the intereſt of money and a given table of the probabilities of the duration of human life, is to be aſcertained in the following manner. We muſt ſuppoſe, in the firſt place, that the grantor of the future ſum of money makes ſeveral hundred grants of the ſame kind, and upon exactly the ſame conditions, to as many different grantees, or purchaſers, all of the ſame age with the firſt grantee; and, in the ſecond place, that theſe ſeveral purchaſers and their companions, (or the perſons upon the continuance of whoſe lives, as well as their own, their right to the ſaid future ſums depends,) die off, in the interval between the time of making the grants and the time of payment, in the ſame proportions as perſons of the ſame ages reſpectively are repreſented to do in the table of the probabilities of the duration of human life by which the calculation is to be governed: and, in the third place, we muſt ſuppoſe that the ſeveral ſums of money paid by the ſeveral grantees of theſe future payments to the grantor of them as the price thereof, are improved by the ſaid grantor at compound intereſt, at the rate ſuppoſed in the queſtion, during the whole interval of time between the time of making the grants and the time at which the payments become due. And then we muſt inquire what ſum each of the ſaid grantees ought to pay to the grantor to the end that, upon theſe three ſuppoſitions, he may, at the end of the ſaid interval, or when the ſaid payments become due, be neither a gainer nor a loſer by the ſum total of all his bargains, but be poſſeſſed of juſt enough money, ariſing from the ſums formerly paid him by the ſaid grantees, to ſatisfy all the demands which will then be made upon him. And the ſum which ought thus to be paid him by each of the ſaid grantees, when he makes a great number of the ſaid grants to different perſons, is the fair price which a ſingle grantee ought to pay him for a grant of the ſame future ſum of money, ſubject to the ſame conditions and contingencies, when he makes only one ſuch grant.

This is a maxim which, I preſume, will be admitted as ſelf-evident; it being hardly poſſible to doubt of its truth. But if the reader ſhould not admit it upon its own evidence, I confeſs I am unable to demonſtrate it by means of any other propoſition more evident than itſelf. And therefore in in this caſe I muſt deſire him to conſider it as a definition of what is meant in the following pages by the expreſſions of the *fair price*, or *true value* of ſuch a future contingent payment; ſince it is only in that ſenſe that the fair price, or true value, of ſuch a future contingent payment can be collected from the tables of the probabilities of the duration of human life above-deſcribed.

Of the preſent values of future certain payments of money.

XXII. It appears from the foregoing article that, in order to find the true value of a future contingent payment, depending upon the continuance of one or more given lives, it is neceſſary that we ſhould be able to determine, what ſum of money, paid down at preſent to the grantor of the ſeveral future payments above-mentioned, will, if improved by him at compound intereſt at the rate of intereſt ſuppoſed in the queſtion, during the whole interval of time before the ſaid future payments become due, amount at that time to the whole ſum of money which he will then be obliged to pay to his ſeveral grantees. This preſent ſum of money which, being ſo improved at compound intereſt at a certain given rate of intereſt, will in a certain number of years amount to ſuch larger given ſum, is what the writers upon this ſubject uſually call the *preſent value* of ſuch larger ſum at the ſaid rate of intereſt. And, as this is a ſhort and convenient expreſſion to denote the ſaid leſſer ſum, I ſhall frequently make uſe of it in the courſe of the following pages.

XXIII. Now the method of finding the preſent value of any given ſum of money that is to be received at the end of a given number of years, according to any given rate of intereſt, is as follows.

P R O B L E M I.

To find the preſent value of any given ſum of money which is payable at the end of any given number of years, according to any given rate of intereſt.

S O L U T I O N.

Let S be put for the given ſum of money whoſe preſent value is to be determined; and n for the number of years at the end of which it is to be received. And let r be to 1 as the ſum that ariſes by adding together any ſum of money and its intereſt for one year according to the rate ſuppoſed in the queſtion, is to the ſaid principal ſum of money alone, or without its intereſt. And let x be put for the preſent value of the given ſum S, which is to be found. Then it is evident that x will be to x with its intereſt for

one

one year as 1 to r, and consequently that x with its interest for one year will be equal to $\frac{rx}{1}$, or rx; that is, the amount of x at the end of the first year will be rx. In like manner the amount of x at the end of the first year will be to the same amount together with the interest of it for one year, that is, to the amount of x at the end of the second year, as 1 to r; and consequently the amount of x at the end of the second year will be equal to r multiplied into the amount of x at the end of the first year, or to r multiplied into rx, or to rrx. And in like manner it will appear that the amount of x at the end of the third year will be r^3x, and at the end of the fourth year r^4x, and at the end of the fifth year r^5x, and at the end of the nth, or last, year r^nx. Therefore r^nx is equal to the given sum S whose present value x is to be found. Therefore x is equal to $\frac{S}{r^n}$; that is, the present value of the given sum S is equal to the quotient that arises by dividing that given sum by that power of r whose index is the number of years at the end of which the said sum becomes payable. Q.E.I.

An example of the foregoing solution.

Thus, for example, if the sum of 265*l.* is to be received at the end of three years, and the rate of interest is three per cent. we must proceed as follows.

Since the interest of money is 3 per cent. we shall have 100 : 103 : : 1 : r; and consequently r will be $= \frac{103\times 1}{100} = \frac{103}{100} = 1.03$. Therefore rr is $= 1.03 \times 1.03 = 1.0609$, and $r^3 = rr \times r = 1.0609 \times 1.03 = 1.092,727$. Therefore x, or the present value of S, or 265*l.* is $= \frac{£.265}{1.092,727} = £.242.51$, or 242*l.* 10*s.* 2*d.* ½. Q.E.I.

COROLL. It follows from the foregoing solution that the present value of one pound at the end of n years is $\frac{1}{r^n}$ of a pound. For in this case S is equal to £.1; and consequently x, or the present value of S, which is in all cases equal to $\frac{S}{r^n}$, is in this case equal to $\frac{1l.}{r^n}$, or $\frac{1}{r^n}$ of a pound. Q.E.D.

XXIV. If

XXIV. If the intereſt of money is

interest		r will be
2 per cent.		= 1.02,
$2\frac{1}{2}$ per cent.		= 1.025,
3 per cent.		= 1.03,
$3\frac{1}{2}$ per cent.		= 1.035,
4 per cent.		= 1.04,
$4\frac{1}{2}$ per cent.	r will be	= 1.045,
5 per cent		= 1.05,
6 per cent.		= 1.06,
7 per cent.		= 1.07,
8 per cent.		= 1.08,
9 per cent.		= 1.09,
10 per cent.		= 1.10.

Of Mr. Smart's tables of intereſt.

XXV. If one pound ſterling be denoted by 1, it is evident that r will be the amount of one pound ſterling together with its intereſt for one year, or the amount of one pound ſterling at the end of a year; and r^2 will be its amount at the end of two years; and r^3 its amount at the end of three years; and r^4 its amount at the end of four years; and, in general, r^n its amount at the end of n years. Now all theſe powers of r, from the firſt power to the hundredth, according to the twelve different values of it above-mentioned, to wit, 1.02, 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, and 1.10, (which correſpond to the ſeveral rates of intereſt above-mentioned, of 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent.) or all theſe amounts of one pound ſterling, at the end of one, two, three, and four, years, and of every following year as far as the hundredth, incluſively, according to thoſe ſeveral rates of intereſt; together with the ſeveral amounts of it at the end of all the intermediate half years from the beginning of the firſt year to the end of the hundredth year; are ready calculated to our hands, to no leſs than eight places of decimal fractions, by Mr. John Smart in his moſt uſeful tables of intereſt publiſhed in quarto in the year 1726. Theſe amounts, or powers of the different values of r, conſtitute what he calls his *firſt table of compound intereſt*, which takes up eight pages of his book, to wit, the 52d, 53d, 54th, 55th, 56th, 57th, 58th, and 59th pages. And in his *ſecond table of compound intereſt*, contained in pages 60, 61, 62, 63, 64, 65, 66, 67, he has given us the reciprocals of theſe amounts, or powers of r, or the *preſent values* of one pound payable at the end of any number of years or half years, in the ſpace of an hundred years, according to the ſaid rates of intereſt, computed to the ſame degree of exactneſs. By theſe moſt uſeful tables the greater part of the labour of computing

computing the value of a future contingent payment depending upon the continuance of a life or lives, or that of a life-annuity, (which is only the ſum total of ſeveral ſuch payments,) is taken away; as will be evident from the method of computing the values of ſuch payments, which we now proceed to explain.

PROBLEM II.

XXVI. To find the ſum of money which the purchaſer of a future payment of one pound ſterling, to be received at the end of any given number of years, provided the ſaid purchaſer ſhall be then living, ought to pay for it: the age of the ſaid purchaſer, and the rate of intereſt of money, and the probabilities of the duration of human life, being all given.

A ſolution of this problem in the caſe of a particular example.

Let the rate of intereſt of money be ſuppoſed to be 3 per cent. and the probabilities of the duration of human life ſuch as they are repreſented to be in Monſieur de Parcieux's table above-mentioned. And let the number of years at the end of which the ſaid ſum of one pound is to be paid to the grantee, or purchaſer of it, if he be then alive, be 30: and the age of the ſaid grantee, or purchaſer, 25 years.

Then, in the firſt place, we muſt look into Mr. de Parcieux's table to ſee how many perſons of 25 years of age are there ſuppoſed to be all living at the ſame time. This number we ſhall find to be 774. We muſt therefore ſuppoſe that the grantor of the one pound to the purchaſer propoſed in the queſtion does not confine himſelf to that ſingle grant, but makes 773 more ſuch grants, of one pound each, to as many different perſons of the ſame age of 25 years, to be paid to them at the end of 30 years, or when they ſhall be 55 years old, if they ſhall then be living, but not to be paid to their executors, or other repreſentatives, if they ſhall then be dead; that is, we muſt ſuppoſe that he makes 774 ſuch grants in all, including that to the purchaſer propoſed in the queſtion. And we muſt likewiſe ſuppoſe that all theſe 774 purchaſers have the ſame chance, one with the other, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. This done, we muſt inquire how many of theſe 774 purchaſers of one pound each will be alive at the end of 30 years, ſuppoſing them to die off in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 perſons of the age of 25 years, all living at the ſame time, 526 will be alive at the age of 55 years, or at the diſtance of 30 years. Therefore out

out of the said 774 purchasers of these future payments of one pound, to be received at the end of 30 years, 526 will live to be intitled to them. Therefore at the end of the said 30 years the grantor of these future payments will have 526 sums of one pound each to pay to the said surviving purchasers. And consequently, to the end that the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive at the time of making the said grants, 526 times the present value of one pound payable at the end of 30 years, when the interest of money is 3 per cent. or 526 times the sum which, being improved continually at compound interest during the said term of 30 years at the said rate of interest, will at the end of that time amount to one pound; because in that case, if he improves the said sum (of 526 times the present value of one pound) so received at compound interest at the said rate of 3 per cent. during the whole 30 years, it will in that time increase to just 526 pounds, which is the sum he will then be obliged to pay to the surviving purchasers. Now it appears by Mr. Smart's second table of compound interest, page 61, that the present value of one pound payable at the end of 30 years, without being liable to any contingency, when the interest of money is 3 per cent. is .411,986,76 of a pound. Therefore 526 times .411,986,76 of a pound, or £216.705,035,76, is the sum which the said grantor ought to receive at the time of making the said grants from all the 774 purchasers of them. Therefore the sum which each of them ought then to pay him is the 774th part of £216.705,035,76, or .279,980,66 of a pound, or, nearly, .28 of a pound, or 5*s.* 7*d.* $\frac{1}{4}$. And consequently, by Art. 21, when he makes only one such grant to a purchaser of 25 years of age, he ought to receive for it the same sum of .279,980,66 of a pound, or .28 of a pound, or 5*s.* 7*d.* $\frac{1}{4}$. Q.E.I.

XXVII. I have solved the foregoing problem in the case of a particular example for the sake of making the method of solution as clear and familiar as possible. But it is easy to see that the reasonings used in it extend to all other cases whatsoever, and consequently that the solution is really general. But now, that no doubt may be left on this head in the reader's mind, I shall repeat the solution, at the same length as before, in general terms and with a notation suited to them; by which means we shall obtain a short and general expression of the value of any such future and contingent payment as is hereby sought, which it will be easy to apply to any other instances.

A general solution of the foregoing problem.

XXVIII. Let 1 be put for one pound sterling, and r for the sum of one pound and its interest for one year according to the rate of interest given in the question, or the amount of one pound, improved at that rate of interest, at the end of a year. And let n denote the number of years

at

at the end of which the ſum of one pound is to be paid to the purchaſer, if he is then alive, but not to his executor, or other repreſentative, if he ſhall then be dead. And let N denote the number of years in the age of the purchaſer at the time of his making the purchaſe of this one pound; and conſequently $N+n$ the number of years contained in his age at the time he becomes intitled to receive it, if he lives ſo long.

Then, in the firſt place, we muſt look into the table of the probabilities of the duration of human life by which the calculation is to be governed, and which I ſhall ſuppoſe to be that of Mr. de Parcieux, to find how many perſons of N years of age are there repreſented as living at the ſame time. This number we will call P. We muſt then ſuppoſe that the grantor of the future payment of one pound to the purchaſer mentioned in the queſtion does not confine himſelf to that ſingle grant, but makes at the ſame time $P-1$ more ſuch grants to as many different purchaſers of them, all of the age of N years; ſo that, including the grant to the purchaſer mentioned in the queſtion, he makes in all P ſuch grants to P different purchaſers, all of the age of N years. And we muſt further ſuppoſe that all theſe P purchaſers have the ſame chance, one with the other, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. And we muſt likewiſe ſuppoſe them to die off, in the courſe of the n years which are to elapſe before the payments become due, in the proportion in which perſons of the ſame age are repreſented to die off in the ſame time in the table of the probabilities of the duration of human life adopted for the calculation. We muſt therefore look into that table to find how many perſons out of P perſons living at the age of N years are repreſented as living to the age of $N+n$ years. And this number we will call p. Then it is evident that out of all the P purchaſers of the future ſums of one pound each, payable at the end of n years, only p perſons will be alive at the end of the ſaid n years to claim their reſpective payments. Therefore at the end of the ſaid n years the grantor of theſe future payments will have p ſums of one pound each to pay to the ſaid ſurviving purchaſers. Therefore, to the end that the ſaid grantor, when theſe payments become due, may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive juſt p times the preſent value of one pound payable at the end of n years, when the intereſt of money is that which is denoted by the proportion of r to 1, or p times the ſum which, being improved continually at compound intereſt during the ſpace of n years at the ſaid rate of intereſt, will at the end of the ſaid term amount to one pound: becauſe in that caſe, if he ſo improves the money he then receives, it will amount at the end of the ſaid term to exactly p times one pound, ſo as to enable him to ſatisfy the juſt demands of the p ſurviving purchaſers. Now the preſent

value

value of one pound to be received at the end of n years, when the intereſt of money is that which is expreſſed by the proportion of r to 1, is juſt as much leſs than one pound as one pound is leſs than the amount of one pound improved at compound intereſt during n years at the ſame rate of intereſt. But the amount of one pound improved at compound intereſt for n years, at the rate of intereſt expreſſed by the proportion of r to 1, is r^n, or the nth power of r. Therefore the preſent value of one pound to be received at the end of n years, at the ſaid rate of intereſt, is to 1*l.* as 1*l.* is to r^n. And conſequently the ſaid preſent value of one pound is equal to $\frac{1}{r^n}$ of a pound. Therefore p times $\frac{1}{r^n}$ of a pound, or $\frac{p}{r^n}$ £ is the ſum which the ſaid grantor ought to receive at the time of making the grants aforeſaid from all the P purchaſers of them. Therefore the ſum which he ought to receive at that time from each of the ſaid purchaſers is the Pth part of $\frac{p}{r^n}$ £, or is equal to $\frac{p}{P \times r^n}$ £. And conſequently, by Art. 21, when the ſaid grantor makes only one ſuch grant to a purchaſer of N years of age, he ought likewiſe to receive for it the ſame ſum of $\frac{p}{P \times r^n}$ £. Q.E.I.

XXIX. This concluſion may be expreſſed in words in the manner following.

Find the preſent value of one pound certain, to be received at the end of the given number of years, according to the given rate of intereſt, by the help of Mr. Smart's ſecond table of compound intereſt, or otherwiſe. Then find in the given table of the probabilities of the duration of human life at the ſeveral different ages of it, the number of perſons living at the age of the purchaſer. Then add to the age of the purchaſer the number of years at the end of which the ſum of one pound is made payable to him, and find in the ſaid table the number of perſons living at the ſaid greater age. Then multiply the aforeſaid preſent value of one pound certain by this latter number of perſons living; and divide the product thence ariſing by the number of perſons living at the age of the purchaſer. And the quotient will be the value of this contingent ſum of one pound, which is payable to the purchaſer at the end of the given number of years, if he is then alive, expreſſed in decimal parts of a pound ſterling.

XXX. COROLL. 1. If the ſum of money to be received by the purchaſer at the end of the given number of years is greater or leſs than one pound, it is evident that the price he ought to pay for it to the grantor will be greater or leſs than the price of the future payment of one pound in

 the

the same proportion. Therefore if the sum to be received at the end of n years is denoted by the letter S, the price he ought to pay for it will be $\frac{p}{P} \times \frac{S}{r^n}$. For as £1 is to S£, so is $\frac{p}{P \times r^n}$, or the price of a future payment of £1, to $\frac{p \times S}{P \times r^n}$, or $\frac{p}{P} \times \frac{S}{r^n}$.

XXXI. COROLL. 2. By finding in the method described in the solution of the foregoing problem the several values of one pound sterling to be received by a purchaser of a given age at the end of every future year of the whole space of time through which it is possible that his life may be extended, provided he is living at the time when every such payment becomes due, and adding these values all together, we shall obtain the value of a set of equal payments of one pound each to be made to the purchaser at the end of every future year, or of an annuity of one pound a year for his life. And this is the manner in which tables of the values of life-annuities are, or ought to be, computed.

An example of the calculation of a life-annuity according to the method described in the foregoing articles.

XXXII. As an example of this method of computing the values of life-annuities, we will here compute at length the value of an annuity of one pound for the life of a purchaser of 80 years of age, upon a supposition that the interest of money is 3 per cent. and that the probabilities of the duration of human life are such as they are represented to be in Mr. de Parcieux's table.

Now it appears by Mr. de Parcieux's table that, out of 118 persons of the age of 80 years, all living at the same time, 101 will live to the age of 81 years. And by Mr. Smart's second table of compound interest, page 60, it appears that the present value of one pound certain, to be received at the end of one year, when the interest of money is 3 per cent. is .9708 of a pound. (For though he carries this value to eight places of figures, I omit the four last figures, as unnecessary in the present computation.) Therefore by Art. 28, or 29, $\frac{101 \times \overset{£}{9708}}{118}$, or $\frac{\overset{£}{98.0508}}{118}$, or .8309£, is the present value of the first year's payment of the annuity of one pound, which is to be made to the purchaser when he shall be 81 years old.

In

In like manner it appears by Mr. de Parcieux's table that of the original number of 118 persons of the age of 80 years, all living at the same time, there will be

	85 persons living	at the age of 82 years,
and	71	at the age of 83,
and	59	at the age of 84,
and	48	at the age of 85,
and	38	at the age of 86,
and	29	at the age of 87,
and	22	at the age of 88,
and	16	at the age of 89,
and	11	at the age of 90,
and	7	at the age of 91,
and	4	at the age of 92,
and	2	at the age of 93,
and	1	at the age of 94,
and none at all		at the age of 95.

And by Mr. Smart's second table of compound interest, page 60, it appears that, when the interest of money is 3 per cent. the present value of one pound certain, payable at the end of

			£
	2 years,	is	.9425;
and at the end of	3 years,		.9151;
and at the end of	4 years,		.8884;
and at the end of	5 years,		.8626;
and at the end of	6 years,		.8374;
and at the end of	7 years,		.8130;
and at the end of	8 years,		.7894;
and at the end of	9 years,		.7664;
and at the end of	10 years,		.7440;
and at the end of	11 years,		.7224;
and at the end of	12 years,		.7013;
and at the end of	13 years.		.6809;
and at the end of	14 years,		.6611.

Therefore, by Art. 28 or 29, the value of the second year's rent, or payment of one pound, which is to be received by the purchaser when he is 82 years old, is £ $\frac{85 \times .9425}{118}$, or £ $\frac{80.1125}{118}$, or £ .6789.

And

And the value of the third year's rent, which is to be paid him when he is 83 years old, is $\frac{71\times.9151}{118}$ £, or $\frac{65.0721}{118}$ £, or .5514.

And that of the fourth year's rent is $\frac{59\times.8884}{118} = \frac{52.4156}{118} = .4442$ £.

And that of the fifth year's rent is $\frac{48\times.8626}{118} = \frac{41.4048}{118} = .3508$ £.

And that of the sixth year's rent is $\frac{38\times.8374}{118} = \frac{31.8212}{118} = .2612$ £.

And that of the seventh year's rent is $\frac{29\times.8130}{118} = \frac{23.5770}{118} = .1998$ £.

And that of the eighth year's rent is $\frac{22\times.7894}{118} = \frac{17.3668}{118} = .1472$ £.

And that of the ninth year's rent is $\frac{16\times.7664}{118} = \frac{12.2624}{118} = .1039$ £.

And that of the tenth year's rent is $\frac{11\times.7440}{118} = \frac{8.1840}{118} = .0693$ £.

And that of the eleventh year's rent is $\frac{7\times.7224}{118} = \frac{5.0568}{118} = .0428$ £.

And

And that of the twelfth year's rent is $\frac{4 \times .7013}{118} = \frac{2.8052}{118} = .0237$ £.

And that of the thirteenth year's rent is $\frac{2 \times .6809}{118} = \frac{1.3618}{118} = .0115$ £.

And that of the fourteenth year's rent is $\frac{1 \times .6611}{118} = \frac{.6611}{118} = .0056$ £.

Add all theſe values of the ſeveral yearly rents together; and their ſum total, £3.7212, or 3*l.* 14*s.* 5*d.* will be the value of an annuity of one pound for the whole life of the perſon of 80 years of age who was ſuppoſed to be deſirous of purchaſing it. Q E I.

£	£	£	£
.8339	.2612	.0428	2.8562
.6759	.1998	.0237	.7814
.5514	.1472	.0115	.0836
.4442	.1039	.0056	3.7212
.3508	.0693	.0836	
2.8562	.7814		

XXXIII. The foregoing computation might have been ſomewhat ſhortened by firſt adding together all the ſeveral products £98.0508, £80.1125, £65.0721, £52.4156, &c. that ariſe from the multiplication of the numbers of perſons living at the ſeveral ages of 81, 82, 83, 84, &c. years into the correſponding preſent values of the ſum of one pound, and then dividing their ſum total by 118, or the number of perſons living at the age of 80 years, which is the age of the purchaſer of the annuity. For as all theſe products are to be divided by 118, it is evident that the concluſion will be the ſame whether we firſt divide each of them by 118, and then add the quotients together, as has been done in the foregoing computation, or whether we firſt add the products themſelves together, and then divide their ſum by 118. And by this latter method of proceeding we ſhall have only one operation of diviſion inſtead of ſeveral, which will conſiderably diminiſh the labour of the calculation. The addition of theſe products is as follows.

98.0508

£	£	£
98.0508	31.8212	5.0568
80.1125	23.5770	2.8052
65.0721	17.3668	1.3618
52.4156	12.2624	.6611
41.4048	8.1840	
337.0558	93.2114	9.8849
93.2114		
9.8849		
440.1521		

Therefore £440.1521 is the sum of all those products; which being divided by 118 gives us £3.7301, or 3*l.* 14*s.* 7*d.* $\frac{1}{2}$, for the value of an annuity of one pound for the life of a person of 80 years of age, when the interest of money is 3 per cent. Q E I.

N. B. This result is greater than the former by the small difference of the 100th part of a pound, or about 2*d.* $\frac{1}{2}$; which is owing, I imagine, to our not losing any part of the aforesaid products by this addition of them, whereas in dividing each of them by 118, according to the former method, we obtained quotients which were for the most part somewhat less than the truth. But this difference is not worth attending to.

XXXIV. The reader will observe that in adding together the fourteen products in the last article, and the quotients of the division of those products by 118 in Art. 32, I have made use of a round-about method of proceeding to obtain those sums; to wit, by first separating the said products and quotients, which were to be added together, into three different parcels, two of which consist of five terms and the third of four, and finding the sums of these several parcels of terms, and then adding these three sums together to obtain the sum total of the whole fourteen terms. My reason for doing this was because I find it easier to obtain the sum total of a great number of quantities in this manner than by adding them all up together in one operation; and I think there is less danger of making an error in the computation. But this is a matter in which every person, engaged in arithmetical calculations, must be governed by his own experience.

XXXV. The foregoing example of the computation of the value of a life-annuity for the life of a purchaser of fourscore years of age will do as well to illustrate the method of computing such annuities delivered in Coroll. 2d, Art. 31, as if we had taken a much younger life. And as this

this was the ſole intent of inſerting any example of this kind, an old life was pitched upon, in preference to a young one of only 20 or 30 years, for the ſake of avoiding the much greater number of multiplications and diviſions that would have been neceſſary in computing the value of an annuity for ſuch younger life, which might have appeared tedious to the reader.

XXXVI. COROLL. 3. The value of a remote life-annuity for the life of a purchaſer of a given age, that is, of an annuity that is to take place only at the end of a given number of years, and to continue from that time to the end of the purchaſer's life, according to any given rate of intereſt and any given table of the probabilities of human life, may likewiſe be computed by means of the foregoing problem. Of the value of a remote life-annuity.

For ſuch a remote life-annuity is a ſet of equal ſums of money which are to be paid to the purchaſer at the end of every year from the time when the annuity is to take place, and which are to continue from that time forwards to the end of the purchaſer's life. And conſequently, by finding the values of the ſaid equal contingent future payments from the time when the annuity is to commence to the end of the whole ſpace of time through which it is poſſible the annuitant's life may be prolonged, (that is, according to Mr. de Parcieux's table, to the end of that number of years which, being added to the purchaſer's preſent age, will make 94 years,) in the method deſcribed in the foregoing problem, and adding theſe values together, we ſhall obtain the value of the ſaid remote life-annuity which is to commence after a given number of years.

XXXVII. Thus, if a perſon of 80 years of age purchaſes a life-annuity of one pound, to commence when he ſhall be 85 years of age, or whereof he is to receive the firſt payment when he is 86 years old, the intereſt of money being 3 per cent. and the probabilities of the duration of human life ſuch as they are repreſented to be in Mr. de Parcieux's table, and it is required to find the value of ſuch annuity, we muſt find by the help of the foregoing problem the values of the ſeveral future payments of one pound each which are to be made to him at the end of his 86th, 87th, 88th, 89th, 90th, 91ſt, 92d, 93d, and 94th years, if he attains to thoſe reſpective ages, according to the ſaid rate of intereſt and table of probabilities. And the ſum of theſe values will be the value of the propoſed diſtant life-annuity, that was required to be found. Theſe values we have already found to be as follows. An example of the calculation of the value of ſuch an annuity.

F

The

The value of one pound a year to be received by a person of 80 years of age, according to the rate of interest and table of probabilities above-mentioned, when he shall be 86 years old, or

		£
at the end of 6 years,	is	.2612;
And the value of £1 to be received by him at the end of 7 years,	is	.1998;
And at the end of 8 years,	is	.1472;
And at the end of 9 years,	is	.1039;
And at the end of 10 years,	is	.0693;
And at the end of 11 years,	is	.0428;
And at the end of 12 years,	is	.0237;
And at the end of 13 years,	is	.0115;
And at the end of 14 years,	is	.0056.
And the sum of all these values is	- - -	.8650.

Therefore when the interest of money is 3 per cent. £.8650, or 17*s.* 3*d.* ½, is the value of an annuity of one pound for the life of a man of fourscore years of age, to commence when he shall be 85 years old, or whereof the first payment is to be made him when he shall be compleatly 86 years old. Q E I.

S C H O L I U M.

Concerning the bill for establishing certain remote life-annuities in parishes, which passed the House of Commons in the spring of the year 1773.

XXXVIII. In this manner may be computed all those tables of remote life-annuities which were prepared in the spring of the year 1773, and annexed to a bill at that time in the House of Commons, for enabling parishes to grant such annuities to their poor and industrious inhabitants. This bill was brought into the House of Commons by Mr. Dowdeswell, and seconded by Mr. George Rice, the member for Caermarthenshire, and supported by Sir George Savile, Sir Richard Sutton, Mr. Edmund Burke, Mr. Cornwall, Mr. Jackson, counsel to the Board of Trade, Mr. Thomas Townshend, junior, and many other members of parliament of eminent abilities. And it passed in that house upon a division, after a debate, by a majority of about two votes to one of all the members present. But it was thrown out by the House of Lords. As great pains had been taken in the framing this bill by Mr. Dowdeswell, (who brought it into the house,) Mr. Rice, and Sir George Savile, and many other gentlemen; who had often met together, for several hours at a time, at Sir George Savile's house in Leicester-Square to consider the several clauses of it; it may not be amiss to give my readers the following general account of it.

XXXIX. The

XXXIX. The deſign of this bill was to encourage the lower ranks of people to induſtry and frugality, by laying before them a ſafe and eaſy method of employing ſome part of the money they could ſave out of their wages, or daily earnings, in a manner that would be moſt ſtrikingly for their benefit. It was obſerved that their wanting opportunities of this kind was probably one very principal cauſe of their neglecting ſo obvious a piece of prudence.—That they knew, for the moſt part, but little of the public funds; and that, when it happened that they were acquainted with them, the ſmallneſs of the ſums they would be intitled to receive as the intereſt of the money they could afford to lay out in them, was no encouragement to them to diſpoſe of it in that way. For what inducement, for inſtance, can it be to a poor man who has ſaved ten pounds out of his year's wages to inveſt it in the 3 per cent. bank-annuities, to conſider that it will produce him about ſix or ſeven ſhillings a year? It is but the wages of three days labour. And, if they lend their money to tradeſmen of their acquaintance, as they ſometimes do, it happens not unfrequently that their creditor becomes a bankrupt, and the money they had truſted him with is loſt for ever; which diſcourages others of them from ſaving their money at all, and makes them reſolve to ſpend it in the enjoyment of preſent pleaſure.—But that, if they ſaw an eaſy method of employing the money they could ſpare in ſuch a manner as would procure them a conſiderable income in return for it in ſome future period of their lives, without any ſuch hazard of loſing it by another man's folly or misfortune, it was probable they would frequently embrace it: and thus a diminution of the poor's rate on the eſtates of the rich, an increaſe of preſent induſtry and ſobriety in the poor, and a more independant and comfortable ſupport of them in their old age than they can otherwiſe expect, would be the happy conſequences of ſuch an eſtabliſhment. To effect theſe uſeful purpoſes the bill provided as follows. The deſign of the ſaid bill.

1ſt, That in every pariſh in England or Wales, in which there were two churchwardens and two or more overſeers of the poor, or one churchwarden and three or more overſeers of the poor, that is, four, or more, pariſh-officers intruſted with the care of the poor, it ſhould be lawful for the body of the rateable inhabitants of ſuch pariſh, that is, of thoſe inhabitants who contributed to the poor's rate, to grant life-annuities, payable every quarter of a year, to ſuch of the inhabitants thereof as ſhould be willing to purchaſe them, at the prices ſet down in the tables annexed to the bill, which were computed upon a ſuppoſition that the intereſt of money was only 3 per cent. The principal proviſions of it.

2dly, That the money received from the purchaſers of theſe annuities ſhould be veſted in the 3 per cent. bank-annuities in the name of the pariſh which had granted it: and the dividends duely received by them every

 half-year,

half-year, and employed in the purchaſe of new ſtock, ſo as to be improved at compound intereſt, to the end that it may be able to anſwer the annuities bought with it when they ſhall become due.

3dly, That for the aforeſaid purpoſes of granting theſe life-annuities, and receiving the money paid for them, and holding the ſtock purchaſed with it in the bank-annuities, and the other purpoſes of this bill, the ſaid rateable inhabitants of every ſuch pariſh ſhould be made a body politick and corporate, and have a common ſeal.

4thly, That, if the pariſh-fund in the 3 per cent. bank-annuities ſhould, by the miſmanagement of it, or from any other cauſe, prove inſufficient to ſupply the life-annuities charged upon it, the poor's-rate ſhould be made a collateral ſecurity to the poor purchaſers of theſe annuities for the payment of them, and ſhould be increaſed to ſuch a degree as ſhould be ſufficient to make good the deficiencies.

5thly, That no ſuch annuity ſhould be granted to any one perſon of more than 20*l.* ſterling a year.

6thly, That no ſuch annuity ſhould be granted to any of the inhabitants of a pariſh but ſuch as were legally ſettled in it, or had a right to be relieved by it in caſe they became poor and helpleſs.

7thly, That no ſuch annuity granted to any man ſhould commence before he was compleatly 50 years of age; nor to any woman before ſhe was compleatly 35 years of age.

8thly, That no ſum leſs than five pounds ſhould be received by the managers of theſe annuities as the price of any ſuch annuity.

9thly, That the miniſters, and church-wardens and overſeers of the poor, ſhould be the managers of theſe annuities for the whole body politick and corporate of the rateable inhabitants of the pariſh, and ſhould receive the money from the purchaſers of them and veſt it in the 3 per cent. bank-annuities, and receive the dividends, and employ them in the purchaſe of freſh ſtock, and pay the annuities to the purchaſers when they became due: and that for the tranſacting of the ſaid buſineſs at the Bank they ſhould give a power of attorney to ſome perſon reſiding in London.

10thly, That nevertheleſs the ſaid managers ſhould not have the power of granting any of theſe annuities without the conſent of the rateable inhabitants of the pariſh, who ſhould be aſſembled in veſtry for that purpoſe after public notice of ſuch intended meeting given in the pariſh-church

church on two Sundays immediately after divine ſervice. And in theſe meetings of the pariſhioners it ſhould be neceſſary not only that the majority of them in number ſhould conſent to the granting the annuity propoſed, but that thoſe who ſo conſented ſhould have paid more than half the laſt poor's-rate paid by all the rateable inhabitants ſo aſſembled.

This reſtraint was intended to prevent the renters of ſmall tenements in the pariſh from involving the pariſh in the contingent burthen on the poor's-rate that might ariſe from theſe annuities, againſt the will of the more ſubſtantial inhabitants.

And it was further provided that no ſuch annuity ſhould be granted unleſs there were preſent at the meeting, in which it was granted, at leaſt twelve of the ſaid rateable inhabitants of the pariſh, except in pariſhes where the whole number of rateable inhabitants was leſs than nineteen; and that in that caſe it ſhould be neceſſary that at leaſt two third parts of the whole number of inhabitants ſhould be preſent at it.

And, in the 11th and laſt place, it was provided that the purchaſers of theſe annuities ſhould not be permitted to alienate them without firſt making an offer of them to the pariſh at the price they were worth at the time of ſuch offer according to the tables annexed to the bill, or at ſome lower price: and that ſuch of them as ſhould, at the time of purchaſing them, conſent to a clauſe that ſhould declare them to be abſolutely unalienable, ſhould, in conſequence of ſuch conſent, be incapable of alienating them at all.

The reaſon of this reſtraint upon the alienation of theſe annuities was to guard the poor owners of them againſt their own folly and weakneſs, by making it impoſſible for them to ſell their annuities for a ſmall part of their true value, over a pot of ale and without a proper degree of deliberation.

The reaſon of computing the values of theſe life-annuities upon the ſuppoſition of ſo low a rate of intereſt as 3 per cent. was to make the fund ariſing from the money paid for them be amply ſufficient to anſwer them when they ſhould become due; ſo that it ſhould be almoſt impoſſible, without great negligence in the management of this fund, that there ſhould ever be a neceſſity of having recourſe to an augmentation of the poor's-rate to make good its deficiencies. Yet even at this low rate of intereſt the purchaſers of theſe annuities would uſually get nine or ten per cent. for their money, if they purchaſed them only five years before the time of their commencement; and 30 or 40 per cent. if they would be content to wait for them 25 or 30 years; which men under 30 years of age might do, without any inconvenience. And the hope of this, it was preſumed, might

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be a ſufficient inducement to them to employ ſome part of their money in this way, and to be diligent in their callings, and frugal in their expences, with that view.

The principal objection that ſeemed likely to be made to the ſaid project.

XL. The only objection that ſeemed likely to be made to this project was the difficulty of carrying it into execution, ariſing from the inability of the miniſter and church-wardens and overſeers of the poor of the pariſh to manage the money received from the purchaſers of theſe annuities without an agent in London for that purpoſe; who would probably be, for the moſt part, either ſome ſtock-broker, or banker, or banker's clerk, or other man of buſineſs that dealt in money tranſactions, whom it might be difficult to engage in an employment of this kind without paying him for his trouble in a manner that the pariſh-fund could hardly afford. But this objection is not ſo ſtrong as it appears to be: becauſe the buſineſs of this kind to be done in London would not be ſo much as might at firſt be apprehended; and the price of brokerage upon buying and ſelling the pariſh-ſtock in the bank-annuities and receiving the dividends of it when they became due, is no great matter. But how far this difficulty was likely to hinder the execution of the bill could not be known with any tolerable degree of certainty without giving it a trial. And therefore the Houſe of Commons paſſed it. But that the experiment might be as little hazardous as poſſible, and pariſhes might not be involved by it againſt their wills in the danger of theſe remote incumbrances, the bill was made intirely optional, and the rateable inhabitants of every pariſh were left at liberty to grant or not grant any of theſe annuities, as they ſhould think fit, and even, after they had granted ſome ſuch, to deſiſt from granting any more. And this full liberty of proceeding herein according to their own judgements and inclinations was thought by the gentlemen who ſupported the bill in the Houſe of Commons to be a compleat anſwer to the objection above-mentioned ariſing from the ſuppoſed difficulty of carrying it into execution.

In order to remove the ſaid objection, the bill provided that every pariſh might chuſe whether it would, or would not, adopt the ſaid project.

An ingenious obſervation made in ſupport of the ſaid objection.

XLI. There was however a very ingenious and ſubtle obſervation made upon this anſwer, and in ſupport of the foregoing objection, by a noble lord, of diſtinguiſhed abilities, and who formerly filled the higheſt ſtation in the law with great reputation. This was, "that the option above-mentioned was not given to the right perſons, or to thoſe who were moſt likely to be affected by the burthens which the granting theſe annuities might hereafter bring upon the pariſhes.—For that the option was given to the rateable inhabitants of the pariſh, who were, for the moſt part, only renters of the lands they occupied; whereas the burthen upon the poor's-rate ariſing from the ſuppoſed deficiency of the annuity-fund was not likely to be felt till many years after the granting of the annuities, when

when the leaſes of the renters who had voted for the granting them, would be at an end, or, if they were renewed, would have been renewed at a lower rent than before, in conſideration of the approaching and probable increaſe of the poor's rate ariſing from the ſaid ſuppoſed deficiency; which would be an injury to the freeholders of the land, who were poſſeſſed of the permanent property of it:—and that therefore the conſent of the ſaid freeholders ought to be obtained to every act by which the lands of the pariſh might be expoſed to the danger of ſuch a future burthen."

An anſwer to the ſaid obſervation.

This obſervation ſeems to be ſomewhat refined; but will admit, as I conceive, of the following anſwer. The rateable inhabitants of pariſhes are of the three following ſorts; either owners of the houſes and lands which they occupy; or renters of them under long leaſes for 21 years, or for three lives, and often with a right of renewal; or renters of them under ſhort leaſes for one or two years, or merely at the will of the owners without any leaſes. If they are of the firſt ſort, they are the very perſons in whom the noble author of the obſervation thinks the option of granting, or refuſing to grant, theſe life-annuities ought to have been veſted. If they are of the ſecond ſort, that is, renters of the lands they occupy under long leaſes, they then are more likely to feel the burthen brought upon the pariſh by the ſuppoſed augmentation of the poor's rate than the freeholder or owner of the reverſion, and therefore are fitter than he is, according to the principle of the obſervation, to be truſted with the power of bringing this contingent burthen upon the pariſh. And laſtly, if they are renters of the lands they occupy under ſhort leaſes or at will, (which is the caſe ſuppoſed in the objection,) they are, in conſequence of the precariouſneſs of ſuch a tenure, ſo much under the influence of their landlord, that, if he ſhould but ſignify his pleaſure to them, by his ſteward or by a letter, that he does not chuſe that any of theſe annuities ſhould be granted in the pariſh, leſt his lands ſhould be expoſed to ſuch a future increaſe of the poor's rate, they will be ſure to give their votes againſt them. So that in all theſe caſes the intereſts of the perſons who are moſt likely to be affected by the apprehended burthen on the poor's rate, are ſufficiently protected by the proviſion that veſts this option in the rateable inhabitants of the pariſh. And beſides, experience ſhews that the inhabitants of pariſhes in general, as well thoſe who rent lands and houſes by the year, or at will, as thoſe who have more permanent intereſts in them, are wonderfully averſe to every thing that has even a remote tendency to increaſe the poor's rate. And conſequently there is no reaſon to apprehend that they would conſent to grant any of theſe pariſh-annuities whenever there was the ſmalleſt danger of their being ill managed, and producing, in conſequence thereof, an augmentation of the poor's rate to make good the deficiencies of their proper fund.

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Some other objections have also been made to this bill, which have been answered in a pamphlet, intitled, "*Considerations on the Bill now depending in the House of Commons for enabling parishes to grant life-annuities to poor persons, upon purchase, in certain circumstances, and under certain restrictions; being an appendix to the pamphlet intitled, "A proposal for establishing life-annuities in parishes for the benefit of the industrious poor." Sold by B. White, in Fleet-Street,* 1773;" to which I refer the reader.

I have this further to say in favour of the foregoing proposal, (which I hope will one day or other be again brought into parliament, and with better success;) that it has been carefully examined, and fully approved, by the learned and publick-spirited Dr. Price, of Newington-Green near Islington, the author of the "*Observations on reversionary payments*" abovementioned; and by the very acute and judicious Dr. Benjamin Franklyn, and by Mr. Wedderburn, who at the time of this bill's passing the House of Commons, was his Majesty's sollicitor-general, and now (in August, 1780,) is lord chief justice of the court of Common Pleas and baron Loughborough.

Of the value of an immediate, but imperfect, life-annuity.

XLII. COROLL. 4. The value of an immediate, but imperfect, life-annuity for the life of a purchaser of a given age; that is, of an annuity that is to commence immediately, (so that the first payment thereof shall be received at the distance of only one year from the time of purchasing it,) and to continue only during a certain number of years, which is less than that during which it is possible the purchaser's life may be prolonged, and then to cease, notwithstanding the purchaser should live on to a later period; may likewise be computed by the help of the foregoing problem.

For we need only in this case compute the values of the several future payments that are to be made to the purchaser at the ends of the several years during which the annuity is to continue, and add them together into one sum. And this sum will be the value of the proposed annuity. Q.E.I.

An example of the calculation of the value of such an annuity.

Thus, if a man of fourscore years of age wanted to purchase an annuity of one pound a year for the five following years of his life, and no longer, according to the rate of interest and table of probabilities mentioned in Coroll. 2, the price he ought to pay for it would be the sum total of the several sums following, to wit, £.8309, £.6789, £.5514, £.4442, and £.3508, which are the values of the several payments of one pound which would become due to him at the end of the 81st, 82d, 83d, 84th, and 85th years of his age; which sum is £2.8562, or 2*l.* 17*s.* 1*d.* $\frac{1}{4}$.

Note.

Note. An annuity of this kind might be uſeful to a man who had a reverſionary intereſt in a good eſtate that was to fall in to him at the end of a certain, not very long, term of years. Thus, for example, if a man was the ground-landlord of a number of houſes that paid him only a trifling ſum in ground-rents, but which were to fall into his poſſeſſion by the expiration of the leaſes in ſeven or eight years time, and then would afford him an ample income, an annuity of this kind, which ſhould continue only till the ſaid leaſes ſhould expire, would be of great convenience to him.

XLIII. COROLL. 5. And in like manner the value of a diſtant and imperfect life-annuity for the life of a purchaſer of a given age; or of an annuity, depending on the purchaſer's life, but that is not to commence till the end of a given number of years after the time of purchaſing it, and is to continue only during a certain period of time leſs than the whole ſpace through which it is poſſible the purchaſer's life may be prolonged, and then is to ceaſe, although the purchaſer ſhould live beyond it; may be computed by the help of the foregoing problem.

Of the value of a diſtant and imperfect life-annuity.

For we need only compute the values of the ſeveral future payments that are to be made to the purchaſer at the ends of the ſeveral years during which the annuity is to continue. And the ſum total of theſe values will be the value of the propoſed annuity. Q.E.I.

Thus, if a man of fourſcore years of age wants to purchaſe an annuity of one pound a year, that ſhall begin at the end of five years, and continue during the five following years, if he lives ſo long, and then ceaſe; ſo that he ſhall be intitled to receive the firſt payment of it at the end of the 86th year of his age, and to receive the other four payments at the ends of his 87th, 88th, 89th, and 90th years reſpectively, but nothing afterwards though he ſhould live to a greater age; according to the rate of intereſt and table of probabilities mentioned in Coroll. 2; the price he ought to pay for ſuch a diſtant and imperfect life-annuity would be the ſum total of the ſeveral following ſums, to wit, £.2612, £.1998, £.1472, £.1039, and £.0693, which are the values of the ſeveral payments of one pound which would be due to him at the ends of the 86th, 87th, 88th, 89th, and 90th years of his life reſpectively: which ſum is £.7814, or 15*s.* 6*d.* $\frac{1}{2}$.

An example of the calculation of the value of ſuch an annuity.

Thus much may ſuffice for the computation of the values of annuities depending upon one life. We now proceed to conſider thoſe which depend upon the joint continuance of two lives.

PROBLEM III.

XLIV. To find the ſum of money which the purchaſer of a future payment of one pound ſterling, to be received at the end of any given number of years, provided the ſaid purchaſer and another perſon, (who may be called *his companion*,) ſhall be then living, ought to pay for ſuch future ſum; the ages of the ſaid purchaſer and his companion, and the rate of intereſt of money, and the probabilities of the duration of human life, being all given.

A ſolution of this problem in the caſe of a particular example.

Let the rate of intereſt of money be ſuppoſed to be 3 per cent. and the probabilities of the duration of human life to be ſuch as they are repreſented to be in Monſieur de Parcieux's table above-mentioned. And let the number of years at the end of which the ſaid ſum of one pound is to be paid to the purchaſer of it, in caſe not only the ſaid purchaſer himſelf, but likewiſe his companion aforeſaid, ſhall be then alive, be 30: and the age of the ſaid purchaſer 25 years; and that of his ſaid companion 20 years.

Then, in the firſt place, we muſt look into Mr de Parcieux's table to ſee how many perſons of 25 years of age are there repreſented as all living at the ſame time. This number is 774. We muſt therefore ſuppoſe that the grantor of the one pound to the purchaſer propoſed in the queſtion makes at the ſame time 773 more ſuch grants of one pound to as many different perſons, all of the ſame age of 25 years, to be paid them at the end of 30 years, or when they ſhall be 55 years old, if, not only the grantees themſelves ſhall then be living, but certain other perſons, who may be called *their companions*, who are of the ſame age of 20 years with the companion of the purchaſer mentioned in the queſtion: that is, we muſt ſuppoſe that the grantor makes 774 ſuch grants in all, including that of the purchaſer propoſed in the queſtion. And we muſt likewiſe ſuppoſe that all theſe 774 purchaſers of theſe future payments of one pound have the ſame chance, one with another, of living any given number of years, or that there is no apparent reaſon for ſuppoſing that any one of them is more likely to live to any given future age than any other. This done, we muſt inquire how many of theſe 774 purchaſers of one pound each will be alive at the end of 30 years, ſuppoſing them to die off in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 perſons of the age of 25 years, all living at the ſame time, 526 will be alive at the age of 55 years, or at the end of 30 years. Therefore out of the ſaid 774 purchaſers of theſe future payments

payments of one pound each, only 526 will live to the end of the 30 years. And of theſe 526 ſurviving purchaſers only ſome part will be intitled to demand theſe payments, to wit, thoſe whoſe companions, who were of the age of 20 years at the time of making the grants, are likewiſe living at the end of the ſaid 30 years. For the other ſurviving purchaſers, whoſe companions are then dead, will, by the conditions of this problem, have no right to them. We muſt therefore, in the next place, inquire how many of the companions of the ſaid 526 ſurviving purchaſers will alſo be alive at the end of the ſaid 30 years. Now the companions of theſe 526 ſurviving purchaſers were at the time of making the grants juſt as many as thoſe purchaſers themſelves, that is, 526. We muſt therefore inquire by Mr. de Parcieux's table how many of theſe 526 companions of the ſaid 526 ſurviving purchaſers, who were all living and of the age of 20 years at the time of making the grants, will be alive at the end of the ſaid 30 years. Now it appears by Mr. de Parcieux's table that out of 814 perſons of the age of 20 years, all living at the ſame time, only 581 will be living at the age of 50 years; and conſequently out of 526 perſons of the age of 20 years, all living at the ſame time, only $526 \times \frac{581}{814}$, or 375 will be alive at the age of 50 years. Therefore of the 526 companions of the 526 ſurviving purchaſers only 375 will be living at the end of the ſaid 30 years. Therefore only 375 out of the ſaid 526 ſurviving purchaſers will be intitled to receive the ſaid payments of one pound each. Therefore at the end of the ſaid 30 years the grantor of the ſaid future payments will have only 375 ſums of one pound each, to pay to the ſurviving purchaſers; which will be due to thoſe 375 of them whoſe companions will be then alive. And conſequently, to the end that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive at the time of making the ſaid grants 375 times the preſent value of one pound payable at the end of 30 years, when the intereſt of money is 3 per cent. or 375 times the ſum which, being continually improved at compound intereſt during the ſaid term of 30 years at that rate of intereſt, will, at the end of that time, amount to one pound; becauſe in that caſe, if he improves the ſaid ſum of 375 times the preſent value of one pound, ſo received, at compound intereſt at the ſaid rate of 3 per cent. during the whole 30 years, it will in that time increaſe to juſt 375 pounds, which is the ſum which he will then be obliged to pay to the 375 ſurviving purchaſers who, by the continuance of the lives of their reſpective companions, will be intitled to their ſeveral payments of one pound a piece. Now, by Mr. Smart's ſecond table of compound intereſt, the preſent value of one pound payable at the end of 30 years, when the intereſt of money is 3 per cent. is .4119 of a pound. Therefore 375 tim. £.4119, or £154.4625, is the ſum which the ſaid grantor of theſe future payments ought to receive, at the time of making the ſaid grants, from all the 774 purchaſers of

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them. Therefore the sum which he ought then to receive from each of the said purchasers is the 774th part of £154.4625, that is, £.1995, or, nearly, £.2, or 4 shillings. And consequently, by Art. 21, when he makes only one such grant of one pound, payable at the end of 30 years, to a purchaser of 25 years of age, provided a companion of the purchaser, who is of the age of 20 years at the time of making the grant, shall also be living at the end of the said 30 years, he ought to receive for it the same sum of $\frac{2}{10}$ of a pound, or four shillings. Q E I.

XLV. I now proceed to give a solution of this problem in general terms, as I did before of problem 2d, in Art. 28.

A general solution of the foregoing problem.

Let 1 be to r as one pound sterling, or any other sum of money whatsoever supposed to be put out at interest at the rate supposed in the question, is to the amount of the same sum at the end of a year, or to the sum total of the said sum and its interest for one year. And let n denote the number of years at the end of which the sum of one pound is to be paid to the purchaser, if both he and his companion are then alive. And let N denote the number of years in the age of the purchaser at the time of his making the purchase of this future payment of one pound; and consequently $N+n$ the number of years contained in his age at the time he becomes intitled to receive it. And let M denote the number of years in the age of the companion of the purchaser at the time of making the purchase; and consequently $M+n$ the number of years contained in his age at the time when the payment becomes due.

Then, in the first place, we must look into the table of the probabilities of the duration of human life by which the calculation is to be governed, and which I shall suppose to be that of Mr. de Parcieux, to find how many persons of N years of age are there represented as all living at the same time. This number we will call P. We must then suppose that the grantor of the future payment of one pound to the purchaser mentioned in the question does not confine himself to that single grant, but makes at the same time $P-1$ more such grants of one pound each to as many different purchasers of them; so that, including that to the purchaser in the question, he makes in all P such grants to P different purchasers. And we must further suppose that all these P purchasers have the same chance, one with the other, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. And we must likewise suppose them to die off, in the course of the n years which are to elapse before

before the payments become due, in the ſame proportion in which perſons of the ſame age are repreſented to have died off in the ſame number of year. in the table of the probabilities of the duration of human life that is adopted for the calculation. We muſt therefore look into that table to find how many perſons out of P. perſons living at the age of N years are repreſented as living to the age of $N+n$ years. And this number we will call p. Then it is evident that out of all the P purchaſers of the future ſums of one pound each, payable at the end of n years, only p perſons will be alive at the end of the ſaid n years. And of theſe p ſurviving purchaſers only ſome part will be intitled to demand theſe payments, to wit, thoſe whoſe companions, who were of the age of M years at the time of making the grants, are likewiſe living at the end of the ſaid n years. For the other ſurviving purchaſers, whoſe companions are then dead, will, by the conditions of this problem, have no right to them. We muſt therefore, in the next place, inquire how many of the companions of the ſaid p ſurviving purchaſers will be alive at the end of the ſaid n years; without concerning ourſelves about the fate of the companions of the deceaſed purchaſers, becauſe it is immaterial to the preſent queſtion how many of them, or whether all, or any, of them, are ſtill alive. Now the companions of theſe p ſurviving purchaſers were, at the time of making the grants, juſt as many as thoſe purchaſers themſelves, that is, juſt p in number. We muſt therefore inquire how many of theſe p companions of the ſaid p ſurviving purchaſers will be alive at the end of the ſaid n years. Theſe companions of the ſaid purchaſers were of the age of M years at the time of making the ſaid grants. We muſt therefore look into the table of the probabilities of the duration of human life adopted in the calculation, to find how many perſons are there repreſented as living at the ſaid age of M years, and how many at the ſubſequent age of $M+n$ years. Let the former of theſe numbers be denoted by Q, and the latter by q. Then, ſince the p companions of the p ſurviving purchaſers of theſe future payments of one pound are ſuppoſed to die off, in the courſe of theſe n years, in the ſame proportion as the Q perſons of the ſame age of M years are repreſented to have done in the ſaid table of probabilities, the number of perſons out of the ſaid p companions of the ſaid ſurviving purchaſers that will be alive at the end of the ſaid n years will be to the number of thoſe companions who are alive at the beginning of the ſaid time in the ſame proportion as q is to Q; and conſequently will be equal to $\frac{p \times q}{Q}$, or $\frac{pq}{Q}$ Therefore only $\frac{pq}{Q}$ out of the whole number p of the ſaid ſurviving purchaſers will be intitled to theſe payments of one pound each at the end of the ſaid n years. Therefore at the end of the ſaid n years the grantor of the ſaid future payments will have $\frac{pq}{Q}$ ſums, of one pound each,

to

to pay to theſe $\frac{pq}{Q}$ more fortunate ſurviving purchaſers, whoſe companions will have lived, as well as themſelves, to the end of the ſaid n years. Therefore, to the end that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive, at the time of making the ſaid grants, $\frac{pq}{Q}$ times the preſent value of one pound payable at the end of n years, when the intereſt of money is that which is expreſſed by the proportion of r to 1, or $\frac{pq}{Q}$ times the ſum which, being continually improved at compound intereſt during the ſaid term of n years at that rate of intereſt, will, at the end of that time, amount to one pound: becauſe in that caſe, if he improves the ſaid ſum of $\frac{pq}{Q}$ times the preſent value of one pound, ſo received, at compound intereſt at the ſaid rate of intereſt during the whole term of n years, he will thereby have augmented it at the end of that time to juſt $\frac{pq}{Q}$ times one pound, or $\frac{pq}{Q}$ pounds, which is the ſum which he will then be obliged to pay to the $\frac{pq}{Q}$ fortunate ſurviving purchaſers of theſe future payments, whoſe companions will be then alive. Now the preſent value of one pound payable at the end of n years is $\frac{1}{r^n}$ of a pound. Therefore $\frac{pq}{Q}$ times $\frac{1}{r^n}$ of a pound, or $\frac{pq}{Qr^n}$ £, is the ſum of money which the ſaid grantor of theſe future payments ought to rece' at the time of making the ſaid grants, from all the P purchaſers t' 1. Therefore the ſum which he ought then to receive from each ſaid purchaſers is the Pth part of $\frac{pq}{Qr^n}$ £, or is equal to $\frac{pq}{PQr^n}$ £, or $\frac{pq}{PQ} \times \frac{1}{r^n}$ £. And conſequently, by Art. 21, when he makes only one ſuch grant of one pound, payable at the end of n years, to a purchaſer of N years of age, provided a companion of the purchaſer, who is of the age of M years at the time of making the grant, ſhall alſo be living at the end of the ſaid n years, he ought to receive for it from the ſaid ſingle purchaſer the ſame ſum of $\frac{pq}{PQr^n}$ £, or $\frac{pq}{PQ} \times \frac{1}{r^n}$ of a pound. Q.E.I.

XLVI. This

XLVI. This conclufion may be expreffed in words in the manner following.

The foregoing conclufion expreffed in words.

Find the prefent value of one pound certain, to be received at the end of the number of years (n) given in the problem, according to the rate of intereft therein alfo given, and expreffed by the proportion of r to 1; which prefent value will be $\frac{1}{r^n}$ of a pound. Then find in the given table of the probabilities of the duration of human life the number of perfons living at the age of the purchafer; and call it P. Then add to the age of the purchafer the number of years at the end of which the payment will become due; and find in the faid table the number of perfons living at the faid greater age; and call it p. Then find in the faid table of probabilities the number of perfons living at the age of the purchafer's companion; and call it Q. Then add to the age of the faid companion of the purchafer the number of years at the end of which the payment will become due; and find in the faid table the number of perfons that are there reprefented to be living at the faid greater age; and call it q. Then fay as the number Q is to the number q, fo is the number p to a fourth number; which will therefore be equal to $\frac{pq}{Q}$. Then multiply this fourth number into $\frac{1}{r^n}$ of a pound, or $\frac{1}{r^n}$£, or the prefent value of one pound certain, payable at the end of the given number of years. And laftly, divide the product, $\frac{pq}{Q} \times \frac{1}{r^n}$£, thence arifing, by P, the number of perfons reprefented in the given table of probabilities to be living at the age of the purchafer. And the quotient of this divifion, or the quantity $\frac{pq}{PQ} \times \frac{1}{r^n}$£, will be the value of the future contingent payment of one pound, which is payable to the purchafer at the end of the given number of years, provided both he and his companion are then alive, expreffed in decimal parts of a pound fterling.

XLVII. COROLL. 1. If the fum of money to be received by the purchafer at the end of the given number of years is greater or lefs than one pound, it is evident that the price he ought to pay for it to the grantor will be greater or lefs than the price of the future payment of one pound in the fame proportion. Therefore, if the fum to be received at the end of n years is denoted by S£, the price which the purchafer ought to pay for it, upon the conditions fuppofed in the foregoing problem, will be $\frac{pq}{PQ} \times \frac{S£}{r^n}$. For as as £1 is to S£, fo is $\frac{pq}{PQ} \times \frac{1}{r^n}$£, (which is the price of a future payment of one pound,) to $\frac{pq}{PQ} \times \frac{S}{r^n}$£; which is therefore the price of a future payment of S pounds. Q.E.D.

XLVIII. CO-

Of the value of an annuity for the joint lives of two perſons.

XLVIII. COROLL. 2. By finding in the method deſcribed in the ſolution of the foregoing problem the ſeveral values of one pound ſterling, to be received by a purchaſer of a given age at the end of every future year of the whole ſpace of time during which it is poſſible that his life and that of his companion may be prolonged, provided the ſaid purchaſer and his companion, whoſe age is likewiſe given, ſhall be both living at the time when every ſuch payment becomes due; and then adding theſe values all together into one ſum; we ſhall obtain the value of a ſet of equal payments of one pound each to be received by the purchaſer at the end of every future year during the continuance of the joint lives of the ſaid purchaſer and his companion, or of an annuity for their joint lives. And this is the way in which tables of the values of annuities for the joint lives of two perſons of given ages ought to be computed.

An example of the calculation of an annuity for two joint lives according to the method deſcribed in the foregoing articles.

XLIX. As an example of this method of computing the values of annuities for two joint lives, let it be required to compute the value of an annuity for the joint lives of a purchaſer aged 80 years and a companion aged 75 years, upon a ſuppoſition that the intereſt of money is 3 per cent. and the probabilities of the duration of human life ſuch as they are repreſented to be in Mr. de Parcieux's table.

Now it appears by Mr. de Parcieux's table that out of 118 perſons of the age of 80 years, all living at the ſame time, 101 will live to the end of one year, or to the age of 81 years; and out of 211 perſons of the age of 75 years, all living at the ſame time, 192 will live to the end of one year, or to the age of 76 years. And by Mr. Smart's ſecond table of compound intereſt, page 60, it appears that, when the intereſt of money is 3 per cent. or r is $= 1.03$, the preſent value of one pound certain, to be received at the end of a year, is .9708 of a pound, or £.9708. Therefore, in order to find the value of the firſt payment of this annuity for the joint lives of theſe two perſons, we have $P = 118$, $p = 101$, $Q = 211$, $q = 192$, and $\frac{1}{r^n} = \frac{1}{r^n} = £.9708$, and conſequently $\frac{pq}{PQ} \times \frac{1}{r^n}$, or $\frac{pq}{PQ} \times \frac{1}{r^1}$, $= \frac{101 \times 192}{118 \times 211} \times £.9708 = \frac{19392}{24898} \times £.9708 = \frac{£18825.7536}{24898} = £.7561$. Therefore £.7561 is the value of the firſt year's payment of the annuity of one pound, which is to be made to him at the end of one year from the time of granting it, provided both he and his companion are then alive; at which time the purchaſer himſelf will be 81 years old, and his companion 76.

In like manner it appears by Monſieur de Parcieux's table that of the original number of 118 perſons of the age of 80 years, all living at the ſame time, there will be

	85 perſons living	at the age of 82 years,
and	71	at the age of 83,
and	59	at the age of 84,
and	48	at the age of 85,
and	38	at the age of 86,
and	29	at the age of 87,
and	22	at the age of 88,
and	16	at the age of 89,
and	11	at the age of 90,
and	7	at the age of 91,
and	4	at the age of 92,
and	2	at the age of 93,
and	1	at the age of 94,
and none at all		at the age of 95.

And by the ſame table it appears that of the original number of 211 perſons of the age of 75 years, all living at the ſame time, there will be

	173 perſons living	at the age of 77 years,
and	154	at the age of 78,
and	136	at the age of 79,
and	118	at the age of 80,
and	101	at the age of 81,
and	85	at the age of 82,
and	71	at the age of 83,
and	59	at the age of 84,
and	48	at the age of 85,
and	38	at the age of 86,
and	29	at the age of 87,
and	22	at the age of 88,
and	16	at the age of 89 years.

It is needleſs to inquire how many perſons out of the aforeſaid original number of 211 perſons of the age of 75 years, all living at the ſame time, will live beyond the age of 89 years; becauſe, when the companion of the purchaſer is more than 89 years old, that is, when more than 14 years ſhall have elapſed after the time of granting the annuity, the purchaſer himſelf, who was 80 years old at the time of making the grant, will, according to Mr. de Parcieux's table, be dead, and conſequently the annuity will be at an end.

And

And by Mr. Smart's second table of compound interest, page 60, it appears that, when the interest of money is 3 per cent. the present value of one pound certain, payable at the end of

			£
	2 years,	is	.9425;
and at the end of	3 years,		.9151;
and at the end of	4 years,		.8884;
and at the end of	5 years,		.8626;
and at the end of	6 years,		.8374;
and at the end of	7 years,		.8130;
and at the end of	8 years,		.7894;
and at the end of	9 years,		.7664;
and at the end of	10 years,		.7440;
and at the end of	11 years,		.7224;
and at the end of	12 years,		.7013;
and at the end of	13 years,		.6809;
and at the end of	14 years,		.6611.

Therefore, by Art. 46, the value of the second year's rent, or payment of one pound, which is to be received by the purchaser when he is 82 years old, provided his companion is then alive, is $= \frac{85 \times 173}{118 \times 211} \times £.9425 =$

$$\frac{14705 \times .9425}{118 \times 211} = \frac{14705 \times .9425}{24898} = \frac{13859.4625}{24898} = £.5566.$$

And the value of the third year's rent, which is to be paid him when he is 83 years old, if his companion is then alive, is $\frac{71 \times 154}{118 \times 211} \times £.9151 =$

$$\frac{10934}{24898} \times £.9151 = \frac{10,005.7034}{24898} = £.4018.$$

And the value of the fourth year's rent, which is to be paid him when he is 84 years old, if his companion is then alive, is $\frac{59 \times 136}{118 \times 211} \times £.8884 =$

$$\frac{8024}{24898} \times £.8884 = \frac{7288.4336}{24898} = £.2927.$$

And

And the value of the fifth year's rent is $\frac{48 \times 118}{118 \times 211} \times$ £.8626 $= \frac{48}{211} \times$ £.8626 = £.1962.

And that of the fixth year's rent is $\frac{38 \times 101}{118 \times 211} \times$ £.8374 $= \frac{3838}{24898} \times$ £.8374 = £.1291.

And that of the feventh year's rent is $\frac{29 \times 85}{118 \times 211} \times$ £.8130 $= \frac{2465}{24898} \times$ £.8130 $= \frac{£\,2004.0450}{24898} =$ £.0805.

And that of the eighth year's rent is $\frac{22 \times 71}{118 \times 211} \times$ £.7894 $= \frac{1562}{24898} \times$ £.7894 $= \frac{£\,1233.0428}{24898} =$ £.0495.

And that of the ninth year's rent is $\frac{16 \times 59}{118 \times 211} \times$ £.7664 $= \frac{944}{24898} \times$ £.7664 $= \frac{£\,723.4816}{24898} =$ £.0290.

And that of the tenth year's rent is $\frac{11 \times 48}{118 \times 211} \times$ £.7440 $= \frac{528}{24898} \times$ £.7440 $= \frac{£\,392.8320}{24898} =$ £.0157.

And that of the eleventh year's rent is $\frac{7 \times 38}{118 \times 211} \times$ £.7224 $= \frac{266}{24898} \times$ £.7224 $= \frac{£\,192.1584}{24894} =$ £.0077.

And that of the twelfth year's rent is $\frac{4 \times 29}{118 \times 211} \times$ £.7013 $= \frac{116 \times £.7013}{24898} = \frac{£\,81.3508}{24898} =$ £.0032.

And

And that of the thirteenth year's rent is $\frac{2 \times 22}{118 \times 211} \times \pounds.6809 = \frac{44}{24898}$ $\times \pounds.6809 = \frac{\pounds\,29.9596}{24898} = \pounds.0012.$

And that of the fourteenth and last year's rent is $\frac{1 \times 16}{118 \times 211} \times \pounds.6611$ $= \frac{16 \times \pounds.6611}{24898} = \frac{\pounds\,10.5776}{24898} = \pounds.0004.$

The sum total of these values is £2.5197, or, nearly, £2.52, or 2*l.* 10*s.* 5*d.* which is therefore the value of the annuity proposed for the joint lives of the purchaser aged 80 years and his companion aged 75. Q E I.

£
.7561
.5566
.4018
.2927
.1962
.1291
.0805
.0495
.0290
.0157
.0077
.0032
.0012
.0004
2.5197

Of the value of a remote life-annuity for two joint lives.

L. COROLL. 3. The value of a remote life-annuity, that is to begin at the distance of a given number of years, and to continue during the joint lives of the purchaser and his companion, whose ages are given, may be easily computed by means of the foregoing problem.

For we need only compute the values of the several payments that are to be received at the end of every year after the said given number of years during the longest possible continuance of the joint lives of the said purchaser

purchaser and his companion, that is, during the longest possible continuance of the life of the older of them, according to the table of the probabilities of the duration of human life, that is, according to Mr. de Parcieux's table, till he is 94 years old. And the sum of these values will be the value of the proposed remote annuity. Q E I.

Of the value of an immediate, but imperfect, life-annuity for two joint lives.

LI. COROLL. 4. And in like manner the value of an immediate, but imperfect, life-annuity, that is to begin immediately, (or whereof the first payment is to be received at the end of a year,) but is to continue only during a certain number of years less than the utmost possible extent of the older of the two lives, provided both the purchaser and his companion continue to live during the said term, may be computed by means of the foregoing problem, by computing the values of the several payments that are to be received at the end of every year during the time that the said annuity is to continue, and adding up those values into one sum.

Of the value of a remote and imperfect life-annuity for two joint lives.

LII. COROLL. 5. And in like manner we may compute the value of a remote and imperfect life-annuity depending on the joint lives of the purchaser and his companion; that is, of an annuity depending on their joint lives, which is to commence at the end of a given number of years, and is to continue only during a certain space of time less than the utmost possible extent of the older of the two lives, and then to cease, though both the purchaser and his companion should be still alive.

For this value will be the sum of the values of the several payments that are to be received at the end of every year during the time that the annuity is to continue. Q E I.

LIII. COROLL. 6. It is easy to see that the method of reasoning used in the solution of the foregoing problem may be extended to the valuation of a future payment, to be received at the end of a given number of years, depending upon the continuance of three lives, or any greater number of lives whatsoever. In the case of three lives the additions to the two solutions in Art 44 and 45 will be as follows.

An extension of the particular solution of the foregoing problem to the valuation of a future payment of one pound depending on three joint lives.

LIV. In the particular example solved in Art. 44 let the right of the purchaser of 25 years of age to the future payment of one pound, at the end of 30 years, depend upon the continuance of two lives besides his own,

IMAGE EVALUATION TEST TARGET (MT-3)

6"


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2.5
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3.6
2.2
2.0
1.8
1.0

own, inſtead of one; to wit, upon the life of a companion of the age of 20 years, as is ſuppoſed in that example, and upon the life of a ſecond companion of the age of 10 years. And let 774 ſuch grants of one pound each, to be paid at the end of 30 years, be made to as many different purchaſers; as is likewiſe ſuppoſed in the ſaid example: and let each of theſe 774 purchaſers have two companions; the one of 20, the other of 10, years of age, upon the continuance of whoſe lives at the end of the ſaid 30 years his right to the ſaid future payment of one pound ſhall depend.

Then it is evident that the number of the ſurviving purchaſers that will be intitled, at the end of the ſaid 30 years, to theſe payments of one pound, will be leſs than it was before; becauſe ſome of thoſe whoſe firſt companions (who were aged 20 years at the time of making the grants) are ſtill alive, will have loſt their ſecond companions, (who were 10 years of age at the time of making the grants,) and with them their right to this future payment. We muſt therefore inquire how many of the ſaid ſurviving purchaſers there will be whoſe firſt and ſecond companions will be both alive at the end of the ſaid 30 years. Now we have ſeen that the whole number of purchaſers who will themſelves be living at the end of the ſaid 30 years is 526; and that of theſe 526 ſurviving purchaſers there will be 375, whoſe firſt companions (who were 20 years of age at the time of making the grants,) will be alſo living at the end of the ſaid term. But there will be only a part of theſe 375 purchaſers whoſe ſecond, as well as firſt, companions will be living at that time. We muſt therefore now inquire how many ſuch there will be, that is, how many of the ſecond companions of theſe 375 purchaſers will be alive at the end of the ſaid 30 years. Now theſe ſecond companions of theſe 375 purchaſers were alſo 375 in number at the time of making the grants; each of the ſaid purchaſers having one ſecond, as well as one firſt, companion. We muſt therefore inquire how many out of theſe 375 ſecond companions of the ſaid 375 purchaſers will be living at the end of the ſaid 30 years. Now theſe ſecond companions of the ſaid purchaſers were ſuppoſed to be 10 years of age at the time of making the ſaid grants. And it appears that out of 880 perſons of 10 years of age, all living at the ſame time, 657 will be living at the age of 40 years, or at the end of a term of 30 years. Therefore out of 375 perſons of the ſame age of 10 years, all living at the ſame time, there will be $375 \times \frac{657}{880}$, or 280, who will live to the age of 40, or to the end of 30 years. Therefore only 280 of the ſecond companions of the ſaid 375 ſurviving purchaſers will be alive at the end of the ſaid 30 years. Therefore out of the whole number of 526 purchaſers who will ſurvive to the end of the ſaid 30 years, there will be only 280 whoſe firſt and ſecond companions will both be living at that time; and who will conſequently

consequently be intitled to receive their respective payments of one pound each. Therefore 280 pounds is the sum which the grantor of these future payments will have then to pay to these surviving purchasers. And consequently 280 times the present value of one pound, payable at the end of 30 years, when the interest of money is 3 per cent. that is, 280 times .4119 of a pound, or £115.3320, is the sum which he ought to receive, at the time of making the grants, from all the 774 purchasers of these future payments. Therefore the sum he ought to receive from each of the said purchasers at the time of making the grants, is the 774th part of £115.3320, or £.1490, or, nearly, £.15, or 3 shillings. And consequently, by Art. 21, the same sum of 3 shillings is the price which he ought to receive for such a future payment of one pound, payable at the end of 30 years, provided the purchaser and both his said companions are then alive, when he makes only one such grant. Q.E.I.

An extension of the general solution of the same problem to the valuation of a future payment of one pound depending on three joint lives.

LV. In the general solution of the foregoing problem given in Art. 45 let the right of the purchaser to the future payment of one pound, to be received at the end of n years, depend upon the continuance of the lives of two companions instead of one, besides his own life. And let the age of the purchaser himself be N years, and that of his first companion M years, as before; and that of his second companion L years. Further, let it be supposed, as before, that the grantor makes P such grants of future sums of one pound, to be received at the end of n years, to P different purchasers, each of whom has two companions, the one of the age of M years, and the other of the age of L years, upon the continuance of both whose lives to the end of the said n years his right to the said future payment will depend.

Then it is evident that the number of the surviving purchasers that will be intitled, at the end of the said n years, to these payments of one pound each, will be less than it was before: because some of those whose first companions will be still living at the end of that term, will have lost their second companions before that time, and with them their right to this future payment. We must therefore inquire how many of the said surviving purchasers there will be whose first and second companions will be both living at the end of the said n years. Now the whole number of purchasers who will themselves be living at the end of the said n years, is p. And of these p purchasers it has been shewn in Art. 45 that there will be $\frac{p \times q}{Q}$, whose first companions (who were of the age of M years at the time

of

of making the grants,) will be living at the end of the ſaid term. But of theſe only a part will have their ſecond, as well as their firſt, companions living at the end of the ſaid term. We muſt therefore now inquire how great a part this will be of the ſaid $p \times \frac{q}{Q}$ purchaſers whoſe firſt companions will be then alive, that is, how many of the ſecond companions of theſe $p \times \frac{q}{Q}$ purchaſers will be alive at the end of the ſaid n years.

Now it is evident that theſe ſecond companions of the ſaid $p \times \frac{q}{Q}$ purchaſers were alſo $p \times \frac{q}{Q}$ in number at the time of making the ſaid grants; becauſe each of the ſaid purchaſers had a ſecond, as well as a firſt, companion. We muſt therefore inquire how many of theſe $p \times \frac{q}{Q}$ ſecond companions of the ſaid $p \times \frac{q}{Q}$ purchaſers will be living at the end of the ſaid n years, upon a ſuppoſition that they will die off in the courſe of the ſaid n years in the proportion repreſented in the table of the probabilities of the duration of human life by which the calculation is to be governed. Now theſe ſecond companions of the ſaid purchaſers were ſuppoſed to be L years of age at the time of making the grants. We muſt therefore look into the table of probabilities to ſee how many perſons are there repreſented to be living at the age of L years. Call this number T. We muſt then look again into the table to ſee how many perſons are there repreſented to be living at the age of $L+n$ years. Call this number t. Then, ſince out of T perſons of the age of L years, all living at the ſame time, only t perſons will live to the age of $L+n$ years, or to the end of n years, it follows that out of the $p \times \frac{q}{Q}$ ſecond companions of the aforeſaid $p \times \frac{q}{Q}$ purchaſers, who were all living at the time of making the ſaid grants, and were then L years old, only $p \times \frac{q}{Q} \times \frac{t}{T}$ will be alive at the end of the ſaid n years. Therefore only $p \times \frac{q}{Q} \times \frac{t}{T}$ of the ſaid ſurviving purchaſers will be intitled at the end of the ſaid n years to demand their reſpective payments of one pound. Therefore the ſum which the grantor of the ſaid payments will be obliged to pay to the ſaid purchaſers at the end of the ſaid n years, will be $p \times \frac{q}{Q} \times \frac{t}{T}$ pounds. Therefore the ſum

which

which he ought to receive, at the time of making the faid grants, from all the P purchafers of the faid future payments, is $p \times \frac{q}{Q} \times \frac{t}{T}$ times the prefent value of one pound certain, payable at the end of n years, that is, $p \times \frac{q}{Q} \times \frac{t}{T}$ times $\frac{1}{r^n}$ of a pound, or $p \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$£. Therefore the fum which he ought to receive from each of the faid purchafers, at the time of making the faid grants, as the price of his future payment of one pound, is the Pth part of $p \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$£, or is equal to $\frac{p}{P} \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$£. And confequently, by Art. 21, the price which he ought to receive from a fingle purchafer of fuch a future payment, when he makes only one fuch grant, is likewife equal to $\frac{p}{P} \times \frac{q}{Q} \times \frac{t}{T} \times \frac{1}{r^n}$£, or $\frac{pqt}{PQT} \times \frac{1}{r^n}$£. Q.E.I.

LVI. COROLL. 7. By finding, in the method defcribed in the laft corollary, the feveral values of one pound fterling, to be received by a purchafer of a given age at the end of every future year of the whole fpace of time during which it is poffible that his life and thofe of his two companions, whofe ages are likewife given, may be prolonged, provided that the faid purchafer and both his faid companions fhall be living at the time when every fuch payment becomes due; and then adding thefe values all together into one fum; we fhall obtain the value of a fet of equal payments of one pound each, to be received by the purchafer at the end of every year during the continuance of the joint lives of the faid purchafer and his two companions, or of an annuity for their joint lives. And this is the way in which tables of the values of annuities for the joint lives of three perfons of given ages ought to be computed.

Of the value of an annuity for three joint lives.

Of the value of future payments depending on the continuance of the longeft of two or more given lives.

LVII. We come now in the laft place to confider the value of a future fum of money depending upon the continuance of any one of two or more lives whofe ages are given.

PROBLEM IV.

Of the value of a future payment of one pound sterling, to be made at the end of a given number of years in case of the continuance of either of two given lives.

To find the sum of money which a purchaser ought to pay for a future sum of one pound sterling, to be received at the end of any given number of years, if either the said purchaser or another certain person (who may be called his companion,) shall be then living: the ages of the said purchaser and his companion, and the rate of interest of money, and the probabilities of the duration of human life, being all given.

A solution of this problem in the case of a particular example.

LVIII. Let the rate of interest of money be supposed to be 3 per cent. and the probabilities of the duration of human life to be such as they are represented to be in Mr. de Parcieux's table above-mentioned. And let the space of time at the end of which the said sum of one pound is to be paid to the purchaser of it, if he is then living, or to his companion, if the purchaser himself is then deceased, and his said companion is still alive, be 30 years: and the age of the said purchaser 25 years; and that of his said companion 20 years.

Then in the first place we must look into Mr. de Parcieux's table to see how many persons of 25 years of age are there represented to be all living at the same time. This number is 774. We must therefore suppose that the grantor of the future payment of one pound to the purchaser proposed in the question, makes at the same time 773 more such grants of one pound to as many different purchasers, all of the same age of 25 years, to be paid to them at the end of 30 years, or when they shall be 55 years old, if they shall then be living, and if they shall then be dead, but certain other persons, (who may be called their companions,) who are of the same age of 20 years with the companion of the purchaser mentioned in the question, shall be then alive, to be paid to their said companions respectively: that is, we must suppose that the grantor makes 774 such grants in all, including that to the purchaser proposed in the question. And we must likewise suppose that all these 774 purchasers of these future payments of one pound have the same chance, one with another, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. This done, we must inquire how many of these 774 purchasers of these remote payments will be alive at the end of 30 years, supposing them to die off in that interval of time in the proportion mentioned in Mr. de Parcieux's table. Now it appears by Mr. de Parcieux's table that out of 774 persons of the age of 25 years, all living at the same time, 526 will be alive at the age of 55 years, or at the end of 30 years. Therefore, out of the said 774 purchasers

purchasers of these future payments of one pound each, 526 will live to the end of the said 30 years; and consequently 248 will have died in the mean time. But by the conditions of this problem, (which differ widely from those of the last problem,) all these 526 surviving purchasers of these future payments will be intitled to receive them, and likewise all the surviving companions of the deceased 248 purchasers. We must therefore inquire, by the means of Mr. de Parcieux's table, how many of the companions of the said deceased 248 purchasers will be alive at the end of the said 30 years. Now the number of the companions of the said deceased 248 purchasers, at the time of making the grants, was 248, each of the said purchasers having had one companion. And their age, at the time of making the grants, was 20 years. Now it appears from Mr. de Parcieux's table that out of 814 persons of the age of 20 years, all living at the same time, 581 will be living at the age of 50 years, or at the end of 30 years. Therefore out of the 248 companions of the deceased purchasers, (who were all living at the time of making the said grants, and were then 20 years of age,) $248 \times \frac{581}{814}$, or 177, will be living at the end of the said 30 years. Therefore the grantor at the end of the said 30 years will have 526 pounds to pay to the 526 surviving purchasers, and 177 pounds to pay to the surviving 177 companions of the 248 deceased purchasers; that is, he will have, in all, 526 and 177 pounds, or 703 pounds, to pay to both. To the end therefore that the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive from all the purchasers of the said future payments, at the time of making the said grants, 703 times the present value of one pound, payable at the end of 30 years, when the interest of money is 3 per cent. that is, 703 times .4119 of a pound, or £289.5657. Therefore the sum which he ought to receive, at the time of making the said grants, from each of the said purchasers, who are 774 in number, is the 774th part of £289.5657, or .3741 of a pound. Therefore, by Art. 21, the sum which he ought to receive from a single purchaser, as the price of such a future payment of one pound, when he makes only one such grant, is likewise £.3741, or 7*s.* 5*d.* $\frac{3}{4}$. Q.E.I.

A general solution of the foregoing problem.

LIX. Let 1 be to r as the sum of one pound sterling, or any other sum of money whatsoever, that is put out to interest at the rate supposed in the question, is to the amount of the same sum at the end of a year, or to the sum total of the said sum and its interest for a year. And let n denote the number of years at the end of which the sum of one pound is to be paid to the purchaser, if he is then alive, or, if he is then dead but his companion is still alive, to his companion. And let N denote the number of years in the age of the purchaser, at the time of his purchasing this future

 payment

payment of one pound, and M the number of years in the age of his companion at the same time: whence it is evident that $N+n$ will be the number of years in the age of the purchaser, and $M+n$ the number of years in the age of his companion, at the end of the said n years, or when the payment becomes due.

Then, in the first place, we must look into the table of probabilities of the duration of human life by which the calculation is to be governed, to find how many persons of N years of age are there represented to be living at the same time. This number we will call P. We must then suppose that the grantor of the said future payment of one pound to the purchaser mentioned in the question does not confine himself to that single grant, but makes at the same time $P-1$ more such grants, of one pound each, to as many different purchasers of them, all of the same age of N years; so that, including the grant to the purchaser in the question, he makes, in all, P such grants to P different purchasers. And we must further suppose that all these P purchasers have the same chance, one with another, of living any given number of years, or that there is no apparent reason for supposing that any one of them is more likely to live to any given future age than any other. And we must likewise suppose them to die off, in the course of the said n years which are to elapse before the said payments become due, in the same proportion in which persons of the same age are represented to have died off in the same number of years in the table of the probabilities of the duration of human life that is adopted for the calculation. We must therefore look into that table to find how many persons out of P persons of the age of N years, all living at the same time, are represented as living to the age of $N+n$ years. And this number we will call p. Then it is evident that out of all the P purchasers of the future sums of one pound each, payable at the end of n years, only p persons will be alive at the end of the said n years. Therefore the number of these purchasers who will have died in the said n years is $P-p$. But by the conditions of this problem (which differ widely from those of the last problem,) all the said p surviving purchasers will be intitled to receive their respective payments of one pound, and likewise all the surviving companions of the $P-p$ deceased purchasers. We must therefore inquire, by means of the aforesaid table of probabilities, how many of the companions of the said deceased $P-p$ purchasers will be alive at the end of the said n years. Now the number of the companions of the said deceased $P-p$ purchasers, at the time of making the grants, was the same with that of the said deceased purchasers, that is, $P-p$; because each of these deceased purchasers was supposed to have one companion. And the age of these companions, at the time of making the grants, was M years. We must therefore look into the table of probabilities aforesaid, to find how many persons are there represented to be living at the age of M years, and how many at the subsequent age of $M+n$ years. Let the former of these numbers be denoted by Q, and the latter

latter by q. Then, ſince the $P-p$ companions of the $P-p$ deceaſed purchaſers of theſe future payments are ſuppoſed to die off, in the courſe of theſe n years, in the ſame proportion as the Q perſons of the ſame age of M years are repreſented to have done in the ſaid table of probabilities, the number of perſons out of the ſaid $P-p$ companions of the ſaid $P-p$ deceaſed purchaſers, that will be alive at the end of the ſaid n years, will be to the number of thoſe companions who were alive at the beginning of the ſaid time, in the ſame proportion as q is to Q; and conſequently will be equal to $\overline{P-p} \times \frac{q}{Q}$. Therefore the grantor of the ſaid future payments, will, at the end of the ſaid n years, be obliged to pay p ſums of one pound each to the p ſurviving purchaſers themſelves, and likewiſe $\overline{P-p} \times \frac{q}{Q}$ ſums of one pound each to the $\overline{P-p} \times \frac{q}{Q}$ ſurviving companions of the $P-p$ deceaſed purchaſers; that is, he will in all have $p + \overline{P-p} \times \frac{q}{Q}$ ſums of one pound to pay to them both. Therefore, to the end that he may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive from all the purchaſers of the ſaid future payments, at the time of making the ſaid grants, $p + \overline{P-p} \times \frac{q}{Q}$ times the preſent value of one pound, payable at the end of n years, when the intereſt of money is that which is expreſſed by the proportion of r to 1, that is, $p + \overline{P-p} \times \frac{q}{Q}$ times $\frac{1}{r^n}$ of a pound, or $p + \frac{Pq - pq}{Q}$ times $\frac{1}{r^n}$ £, or $\left|\frac{pQ + Pq - pq}{Q}\right| \times \frac{1}{r^n}$ £. Therefore the ſum which he ought to receive, at the time of making the ſaid grants, from each of the ſaid purchaſers, whoſe number is P, is the Pth part of $\left|\frac{pQ + Pq - pq}{Q}\right| \times \frac{1}{r^n}$ £, or is equal to $\left|\frac{pQ + Pq - pq}{PQ}\right| \times \frac{1}{r^n}$ £. And conſequently, by Art. 21, the ſum which he ought to receive from a ſingle purchaſer, as the price of ſuch a future payment of one pound, upon the conditions above-mentioned, when he makes only one ſuch grant, is likewiſe equal to this ſame quantity, $\left|\frac{pQ + Pq - pq}{PQ}\right| \times \frac{1}{r^n}$ £, (or $\left|\frac{pQ}{PQ} + \frac{Pq}{PQ} - \frac{pq}{PQ}\right| \times \frac{1}{r^n}$ £,) or $\left|\frac{p}{P} + \frac{q}{Q} - \frac{pq}{PQ}\right| \times \frac{1}{r^n}$ £. Q.E.I.

LX. CO-

Of the relation of the said value to the values of the same payment of one pound sterling to be made at the end of the same number of years in case of the continuance of the single lives of the same two persons and of the continuance of their joint lives.

LX. COROLL. 1. The value found by the foregoing problem, that is, the value of a future payment of one pound, to be received at the end of a given number of years denoted by n, provided that either of two persons of given ages shall be then alive, is equal to the excess of the value of the same payment, to be received at the same time, in case the first of the said persons shall be then alive, added to the value of it, to be received at the same time, in case the second of the said persons shall be then alive, above the value of it, to be received at the same time, in case both the said persons shall be then alive.

For the value found by the foregoing problem is $\overline{\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}}\Big| \times \frac{1}{r^n}$£, or $\frac{p}{P} \times \frac{1}{r^n}$£ $+ \frac{q}{Q} \times \frac{1}{r^n}$£ $- \frac{pq}{PQ} \times \frac{1}{r^n}$. And it has been shewn in Problem 2d, Art. 28, that $\frac{p}{P} \times \frac{1}{r^n}$£ is the value of a future payment of one pound, to be received at the end of n years in case a purchaser of the age of N years shall be then alive, and that $\frac{q}{Q} \times \frac{1}{r^n}$£ is the value of a future payment of one pound, to be received at the end of n years in case a purchaser of the age of M years shall be then alive: and it has been shewn in Problem 3d, Art. 45, that $\frac{pq}{PQ} \times \frac{1}{r^n}$£ is the value of a future payment of one pound, to be received at the end of n years in case a purchaser of the age of N years and a companion of the age of M years, shall be both living at that time. Therefore the value found in the foregoing problem is equal to the excess of the sum of the two former values above the third, or last, value; or the value of a future payment of one pound, to be received at the end of n years in case that either of two given lives shall be then in being, is equal to the excess of the sum of the two values of the same future payment, to be received at the same time, in case each of the same two lives shall be then in being, above the value of the same future payment, to be received at the same time, in case both the said lives shall be then in being. Q.E.D.

Of the value of a future payment of S pounds depending on the same contingencies.

LXI. COROLL. 2. If the sum of money to be received by the purchaser at the end of the given number of years is greater or less than one pound, it is evident that the price he ought to pay for it will be greater or less than the price of the future payment of one pound in the same proportion. Therefore for a future payment of S pounds, to be received at the end of n years in case either himself or his companion, whose ages are N and M years, shall be then alive, he ought to pay the sum of $\overline{\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}}\Big| \times \frac{S}{r^n}$£.

LXII. CO-

Of the value of an annuity of one pound sterling for the life of the longer liver of two persons of given ages.

LXII. COROLL. 3. By finding in the method described in the solution of the foregoing problem the several values of one pound sterling, to be received by a purchaser of a given age at the end of every future year of the whole space of time during which it is possible that either his life or that of his companion, whose age is also supposed to be given, may be prolonged, in case that either the said purchaser or his said companion shall be living at the time when every such payment becomes due; and then adding these values all together into one sum; we shall obtain the value of a set of equal payments of one pound each, to be received by the purchaser at the end of every future year during the lives of the said pu. chaser and his companion and the life of the longer liver of them, that is, of an annuity of one pound for the lives of the said purchaser and his companion and the life of the longer liver of them.

Of the relation of the said value to the values of the like annuities of one pound for the single lives of the same two persons and for their joint lives.

LXIII. COROLL. 4. The value of an annuity of one pound for the lives of two persons of given ages and the life of the longer liver of them, is equal to the excess of the sum of the values of two separate annuities of one pound each, for the single lives of the same persons, above the value of an annuity of one pound for their joint lives.

In the foregoing articles P denoted the number of persons represented, in the table of the probabilities of the duration of human life adopted for the calculation, to be living at the age of the purcha ... ie age of N years; and Q denoted the number of persons there ... ed to be living at the age of the purchaser's companion, or at ... years. Now let P^{I} denote the number of persons therein repr ... ving at the age of $N+1$, and P^{II} those living at the age o ... ^{III} those living at the age of $N+3$, and P^{IV} those living at th ... $+4$; and so on for the following years; the Roman numeral figures p... over every new letter P denoting the number of years by which every new age, corresponding to it, exceeds the age of N years, or the age of the purchaser. And, in like manner, let Q^{I} denote the number of persons represented in the said table to be living at the age of $M+1$ years; and Q^{II} the number of persons living at the age of $M+2$ years; and Q^{III} those living at the age of $M+3$ years; and Q^{IV} those living at the age of $M+4$ years; and so on for the following years; the Roman numeral figures placed over every new letter Q denoting the number of years by which every new age corresponding to it, exceeds the age of M years, or the age of the purchaser's companion.

Then it is evident from the foregoing problem that the value of a sum of one pound, to be received at the end of one year, in case either

either the purchaser or his companion shall be then alive, will be equal to $\overline{\frac{P'}{P}+\frac{Q'}{Q}-\frac{P'}{P}\frac{Q'}{Q}}\Big| \times \frac{1}{r}£$; and the value of the same sum, to be received at the end of two years, subject to the same contingency, will be equal to $\overline{\frac{P''}{P}+\frac{Q''}{Q}-\frac{P''}{P}\frac{Q''}{Q}}\Big| \times \frac{1}{r^2}£$; and the value of the same sum, to be received at the end of three years, subject to the same contingency, will be $\overline{\frac{P'''}{P}+\frac{Q'''}{Q}-\frac{P'''}{P}\frac{Q'''}{Q}}\Big| \times \frac{1}{r^3}£$; and the value of the same sum, to be received at the end of four years, subject to the same contingency, will be $\overline{\frac{P^{iv}}{P}+\frac{Q^{iv}}{Q}-\frac{P^{iv}}{P}\frac{Q^{iv}}{Q}}\Big| \times \frac{1}{r^4}£$; and so on for the following years.

Therefore the sum of these values continued to the utmost possible extent of the younger of these two lives, that is, the value of a life-annuity of one pound for the lives of the purchaser and his companion and the life of the longer liver of them, is equal to the following set of values; to wit,

$$\overline{\frac{P'}{P}+\frac{Q'}{Q}-\frac{P'}{P}\frac{Q'}{Q}} \times \frac{1}{r}£$$

$$+\overline{\frac{P''}{P}+\frac{Q''}{Q}-\frac{P''}{P}\frac{Q''}{Q}}\Big| \times \frac{1}{r^2}£,$$

$$+\overline{\frac{P'''}{P}+\frac{Q'''}{Q}-\frac{P'''}{P}\frac{Q'''}{Q}}\Big| \times \frac{1}{r^3}£,$$

$$+\overline{\frac{P^{iv}}{P}+\frac{Q^{iv}}{Q}-\frac{P^{iv}}{P}\frac{Q^{iv}}{Q}}\Big| \times \frac{1}{r^4}£,$$

$$+\overline{\frac{P^{v}}{P}+\frac{Q^{v}}{Q}-\frac{P^{v}}{P}\frac{Q^{v}}{Q}}\Big| \times \frac{1}{r^5}£,$$

+&c. continued to the utmost extent of human life;

And

And consequently to the following set of quantities; to wit,

$$\frac{P'}{P} \times \frac{1}{r}\pounds \quad + \frac{Q'}{Q} \times \frac{1}{r}\pounds \quad - \frac{P' \; Q'}{P \; Q} \times \frac{1}{r}\pounds$$

$$+ \frac{P''}{P} \times \frac{1}{r^2}\pounds \quad + \frac{Q''}{Q} \times \frac{1}{r^2}\pounds \quad - \frac{P'' \; Q''}{P \; Q} \times \frac{1}{r^2}\pounds$$

$$+ \frac{P'''}{P} \times \frac{1}{r^3}\pounds \quad + \frac{Q'''}{Q} \times \frac{1}{r^3}\pounds \quad - \frac{P''' \; Q'''}{P \; Q} \times \frac{1}{r^3}\pounds$$

$$+ \frac{P^{iv}}{P} \times \frac{1}{r^4}\pounds \quad + \frac{Q^{iv}}{Q} \times \frac{1}{r^4}\pounds \quad - \frac{P^{iv} \; Q^{iv}}{P \; Q} \times \frac{1}{r^4}\pounds$$

$$+ \frac{P^{v}}{P} \times \frac{1}{r^5}\pounds \quad + \frac{Q^{v}}{Q} + \frac{1}{r^5}\pounds \quad - \frac{P^{v} \; Q^{v}}{P \; Q} \times \frac{1}{r^5}\pounds$$

+&c. continued to the utmost extent of human life:

And consequently to the three following sets of quantities, which are the same with the quantities last mentioned, but only are placed in a different order, (those which before were placed in perpendicular columns, one under the other, being now placed in separate horizontal lines,) to wit,

1st, $\frac{P'}{P} \times \frac{1}{r}\pounds + \frac{P''}{P} \times \frac{1}{r^2}\pounds + \frac{P'''}{P} \times \frac{1}{r^3}\pounds + \frac{P^{iv}}{P} \times \frac{1}{r^4}\pounds + \frac{P^{v}}{P} \times \frac{1}{r^5}\pounds$ +&c. continued to the utmost extent of human life;

And 2dly, $\frac{Q'}{Q} \times \frac{1}{r}\pounds + \frac{Q''}{Q} \times \frac{1}{r^2}\pounds + \frac{Q'''}{Q} \times \frac{1}{r^3}\pounds + \frac{Q^{iv}}{Q} \times \frac{1}{r^4}\pounds + \frac{Q^{v}}{Q} \times \frac{1}{r^5}\pounds$ +&c. continued to the utmost extent of human life;

And 3dly, $- \frac{P' \; Q'}{P \; Q} \times \frac{1}{r}\pounds - \frac{P'' \; Q''}{P \; Q} \times \frac{1}{r^2}\pounds - \frac{P''' \; Q'''}{P \; Q} \times \frac{1}{r^3}\pounds - \frac{P^{iv} \; Q^{iv}}{P \; Q} \times \frac{1}{r^4}\pounds - \frac{P^{v} \; Q^{v}}{P \; Q} \times \frac{1}{r^5}\pounds$ —&c. continued to the utmost extent of human life;

that is, the value of the life-annuity of one pound for the lives of the said purchaser and his companion and the life of the longer liver of them, is equal to the excess of the sum of the first and second of the three last sets of quantities above the third set.

K

Now

Now it appears from Problem 2d, Coroll. 2, Art. 31, that the first of these three sets of quantities, to wit, $\frac{P^{I}}{P} \times \frac{1}{r}£ + \frac{P^{II}}{P} \times \frac{1}{r^2}£ + \frac{P^{III}}{P} \times \frac{1}{r^3}£ + \frac{P^{IV}}{P} \times \frac{1}{r^4}£ + \frac{P^{V}}{P} \times \frac{1}{r^5}£ +$ &c. continued to the utmost extent of human life, is the value of an annuity of one pound for the life of the purchaser, who was supposed to be of the age of N years, which corresponds to the number P in the table.

And it appears, in like manner, that the 2d of these 3 sets of quantities, to wit, $\frac{Q^{I}}{Q} \times \frac{1}{r}£ + \frac{Q^{II}}{Q} \times \frac{1}{r^2}£ + \frac{Q^{III}}{Q} \times \frac{1}{r^3}£ + \frac{Q^{IV}}{Q} \times \frac{1}{r^4}£ + \frac{Q^{V}}{Q} \times \frac{1}{r^5}£$ + &c. continued to the utmost extent of human life, is the value of an annuity of one pound for the life of the said purchaser's companion, who was supposed to be of the age of M years, which corresponds to the number Q in the table.

And it appears from Prob. 3, Coroll. 2, Art. 48, that the last of the said 3 sets of quantities, to wit, $\frac{P^{I}}{P}\frac{Q^{I}}{Q} \times \frac{1}{r}£ + \frac{P^{II}}{P}\frac{Q^{II}}{Q} \times \frac{1}{r^2}£ + \frac{P^{III}}{P}\frac{Q^{III}}{Q} \times \frac{1}{r^3}£ + \frac{P^{IV}}{P}\frac{Q^{IV}}{Q} \times \frac{1}{r^4}£ + \frac{P^{V}}{P}\frac{Q^{V}}{Q} \times \frac{1}{r^5}£ +$ &c. continued to the utmost extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser and his companion, whose ages N and M correspond to the numbers P and Q in the said table.

Therefore the value of a life-annuity of one pound for the lives of the said purchaser and his companion, and the life of the longer liver of them, is equal to the excess of the sum of the values of two separate life-annuities of one pound each, for the single lives of the said purchaser and his companion, above the value of an annuity of one pound for their joint lives. Q E D.

LXIV. Therefore when tables of the values of life-annuities for single lives and for two joint lives have been computed and prepared for use, it seems to be unnecessary, and hardly worth the labour of it, to compute a third table of the values of life-annuities for the lives of two persons of given ages, and the life of the longer liver: because the values of these latter annuities may be so easily derived, by means of the foregoing corollary, from the values of the corresponding annuities of the two former kinds, which are supposed to be set down in the tables.

LXV.

LXV. COROLL. 5. The value of a remote life-annuity of one pound, that is to begin at the diftance of a given number of years, and to continue during the lives of the purchafer and his companion and the life of the longer liver of them, whofe ages are given, is equal to the excefs of the fum of two feparate remote annuities of one pound each, for the fingle lives of the fame two perfons, that are to begin at the fame future period, above the value of a third remote annuity of one pound, that is alfo to begin at the fame future period and to continue during the joint lives of both the faid perfons.

Of the value of a remote annuity of one pound for the life of the longer liver of two perfons of given ages.

For all the reafoning ufed in the foregoing corollary applies equally to thefe remote life-annuities and to thofe compleat and immediate life-annuities that are the fubject of that corollary; of which compleat annuities the former remote annuities are only the latter parts. This I take to be fo evident, that a repetition of that reafoning to apply it to this cafe would probably be tedious to the reader. And therefore I omit it.

LXVI. COROLL. 6. And in like manner the value of an immediate, but imperfect, life-annuity of one pound, that is to begin immediately (or whereof the firft payment is to be received at the end of a year,) but is to continue only during a certain number of years, lefs than the utmoft extent of human life, if either the purchafer or his companion, whofe ages are given, continues to live through that period, is equal to the excefs of the fum of the values of two feparate, immediate, and imperfect life-annuities of one pound each, for the fingle lives of the purchafer and his companion, to continue during the fame limited period, if each of them continues to live fo long, above the value of a like immediate and imperfect annuity of one pound for the joint lives of the faid purchafer and his companion during the fame term of years, if they both continue to live fo long.

Of the value of an immediate, but imperfect, life-annuity of one pound for a certain number of years, if the life of either of two perfons of given ages fhall fo long continue.

This alfo may be eafily collected from the reafoning ufed in Coroll. 4, thefe annuities being only the firft parts of the compleat life-annuities therein mentioned.

LXVII. COROLL. 7. And in like manner the value of a remote and imperfect life-annuity of one pound, for the lives of the purchafer and his companion, whofe ages are given, and the life of the longer liver of them; that is, of an annuity that is to begin at the end of a given number of years, and is to continue only during a certain fpace of time, lefs than the utmoft poffible extent of the life of the younger of them, if either of them lives to the end of the faid period, and then to ceafe, though one, or both, of them fhould be ftill alive; is equal to the excefs of the fum of the values

Of the value of a remote and imperfect life-annuity of one pound for the life of the longer liver of two perfons whofe ages are given.

of two ſeparate annuities, of one pound each, for the ſingle lives of the ſaid purchaſer and his companion, that ſhould commence at the end of the ſame given number of years as the former, and continue only during the ſame time, in caſe each of them ſhould continue to live ſo long, and then ceaſe, though they ſhould be ſtill alive, above the value of a like remote and imperfect life-annuity for the joint lives of the ſame perſons, that ſhould commence at the end of the ſame number of years, and continue during the ſame ſpace of time, if they both ſhall continue to live ſo long.

This alſo may be eaſily collected from the reaſoning uſed in Coroll. 4; theſe remote and imperfect life-annuities for the lives of the purchaſer and his companion and the life of the ſurvivor of them, being only the middle parts of the compleat life-annuities of the ſame kind therein mentioned. And to this corollary I therefore refer the reader.

Of the value of a future payment depending on the continuance of any one of three given lives.

LXVIII. COROLL. 8. The reaſoning uſed in the ſolution of the foregoing problem may be extended to the valuation of a future payment, to be received at the end of a given number of years in caſe any one of three perſons, or more, whoſe ages are given, ſhall be then alive. In the caſe of three lives the additions neceſſary to be made to the two ſolutions in Art. 58 and 59, will be as follows.

An inveſtigation of the ſaid value in the caſe of a particular example.

LXIX. In the particular example ſolved in Art. 58 let the right of the purchaſer, of 25 years of age, to the future payment of one pound, at the end of 30 years, be extended to two other perſons beſides himſelf, inſtead of one; ſo that, if either the purchaſer himſelf or either of his ſaid companions ſhall be then alive, the ſaid future ſum ſhall be payable by the grantor of it to one of the ſaid three perſons. And to make the caſe more clear and definite, let it be ſuppoſed that the older of theſe two perſons is called his *firſt companion* and the younger his *ſecond companion*; and that, if the purchaſer himſelf is alive at the end of the ſaid 30 years, the ſaid ſum of one pound ſhall be payable to him alone, though either or both his ſaid companions ſhould be alſo living at the ſame time; and that, if he is then dead, but his companions are both alive, it ſhall be paid to the elder of the two, or his firſt companion; and that, if only one of them is then alive, it ſhall be paid to the ſaid only ſurviving companion. And let the age of the ſaid purchaſer's older, or firſt, companion, be 20 years, as as was ſuppoſed in the foregoing example; and that of his younger, or ſecond, companion 10 years; at the time of making the grant. And further, let it be ſuppoſed that the grantor of the ſaid future payment of one pound makes 774 ſuch grants of one pound each, to be received at the end of 30 years, to as many different purchaſers, all of the ſame age of 25 years as the purchaſer propoſed in the queſtion; and that the grant to the purchaſer propoſed in the queſtion is one of thoſe 774 grants; and that each of theſe purchaſers has two companions, to wit, an older, or firſt, companion of 20 years of age, and a younger, or ſecond, companion of 10 years of age; and that each of the ſums ſo granted is to be paid by the ſaid grantor, at the end of the ſaid ſpace of 30 years, provided the purchaſer himſelf, or either of his two companions, is then alive; namely, to the purchaſer himſelf, if he is then alive; or otherwiſe to the older of his two companions, if they are then both alive; or, if only one of them is then alive, to the ſaid only ſurviving companion.

Theſe things being ſuppoſed, it is evident that the number of perſons that will be intitled to receive theſe payments of one pound each at the end of the ſaid 30 years, will be greater than in the caſe ſuppoſed in the ſolution of the foregoing problem, Art. 58: becauſe, not only all the 526 ſurviving purchaſers themſelves, together with the 177 ſurviving firſt companions of the 248 deceaſed purchaſers, making in all 703 perſons, will be intitled to theſe payments, as they were in that ſolution, but there will alſo be ſome ſurviving ſecond companions of the deceaſed purchaſers who will alſo have a right to them What the number of theſe will be, we muſt now proceed to inquire.

Now it is evident, in the firſt place, from the conditions of this queſtion, that the ſurviving ſecond companions of the 526 ſurviving purchaſers can have no claim to theſe payments of one pound; becauſe they are to be made to the ſaid ſurviving purchaſers themſelves. And, for the like reaſon, it is evident, in the ſecond place, that the ſecond companions of thoſe deceaſed purchaſers whoſe firſt companions are alive at the end of the ſaid 30 years, can have no claim to theſe payments; becauſe it is provided that when both the companions of a deceaſed purchaſer are alive at the time his payment becomes due, it ſhall be made to the ſaid purchaſer's firſt companion, and nor to his ſecond companion. Now it has been ſhewn in Art. 58 that out of the 248 purchaſers, who will have died in the courſe of the ſaid 30 years, there will be 177, whoſe firſt companions (who were 20 years old at the time of making the grants,) will be alive at the end of the ſaid 30 years. Therefore the number of the ſaid deceaſed purchaſers, whoſe firſt companions will alſo be dead before the end of the ſaid 30 years, is the exceſs of 248 above 177 perſons, that is, 71 perſons. There are therefore 71 deceaſed purchaſers, whoſe ſecond companions will have a right

to

to receive these payments of one pound each, if they live to the end of the said 30 years. We must therefore inquire how many of the second companions of these 71 deceased purchasers will live to the end of the said 30 years, supposing them to die off in the proportion set forth in Mr. de Parcieux's table of the probabilities of the duration of human life. Now these companions are evidently 71 in number, because each of the said 71 deceased purchasers had one second as well as one first companion. And the age of these second companions, at the time of making the grants, is supposed to be 10 years. Now it appears from Mr. de Parcieux's table that out of 880 persons of the age of 10 years, all living at the same time, 657 will live to the age of 40 years, or to the end of a term of 30 years. Therefore out of the said 71 second companions of the said deceased purchasers there will be $71 \times \frac{657}{880}$, or 53, who will live to the end of the said 30 years. Therefore the whole number of persons intitled to receive the said payments of one pound each, at the end of the said 30 years, will be, first, the 526 surviving purchasers, and, secondly, the 177 surviving first companions of the 248 deceased purchasers, and, thirdly, the said 53 surviving second companions of those 71 of the said 248 deceased purchasers whose first companions will have died before the end of the said 30 years; that is, in all, 756 persons. Therefore at the end of the said 30 years the aforesaid grantor will be obliged to pay to the said surviving purchasers and the said first and second companions of the purchasers that are deceased, the sum of 756 pounds. Therefore, to the end that, when the said payments become due, the said grantor may be neither a gainer nor a loser by the sum total of all his bargains, it is necessary that he should receive, at the time of making the said grants, 756 times the present value of one pound certain, payable at the end of 30 years, when the interest of money is 3 per cent. that is, 756 times .4119 of a pound, or £311.3964, from all the 774 purchasers of these future payments. Therefore the sum which each of the said purchasers ought then to pay him, as the price of the said future payment of one pound to be received at the end of the said 30 years, is the 774th part of £311.3964, or £.4023, or 8*s.* ½*d.* And consequently, by Art. 21, this sum of 8*s.* ½*d.* is likewise the price which a single purchaser ought to pay for a grant of such a future payment of one pound, to be received at the end of 30 years, if either himself or either of his two companions aforesaid shall be then alive, when the grantor of it makes only one such grant. Q E I.

LXX. In the foregoing article it was supposed that, where both the companions of the deceased purchasers happened to live to the end of the said 30 years, the said sums of one pound were to be paid to their first, or elder, companions, who were 20 years of age at the time of making the grants

grants of them, in preference to their ſecond, or younger, companions, who were only 10 years old at that time. But the values of theſe future ſums of one pound will be exactly the ſame, if we ſhould ſuppoſe that they are to be paid to the ſaid ſecond, or younger, companions of the ſaid deceaſed purchaſers in preference to the firſt, or older. This is what we might reaſonably expect and ſuppoſe to be true without a particular demonſtration. But, to remove all doubt from the reader's mind, we ſhall prove it diſtinctly by applying the reaſoning uſed in the foregoing article to this new ſuppoſition, in the manner following.

Upon this new ſuppoſition it is evident that the perſons who will be intitled to receive the ſaid payments of one pound at the end of the ſaid 30 years, will be; in the firſt place, thoſe of the 774 purchaſers themſelves who will then be alive, whoſe number has been ſhewn to be 526; and, in the ſecond place, thoſe of the ſecond companions of the 248 deceaſed purchaſers who will then be living; and, in the third place, after ſubtracting from the ſaid 248 deceaſed purchaſers thoſe of them whoſe ſecond companions will have lived to the end of the ſaid 30 years, thoſe of the firſt companions of the remaining deceaſed purchaſers (that is, of thoſe deceaſed purchaſers whoſe ſecond companions have died in the courſe of the ſaid 30 years,) who will alſo then be living. We muſt therefore inquire by the means of Mr. de Parcieux's table, in the firſt place, how many of the ſecond companions of the ſaid 248 deceaſed purchaſers will be alive at the end of the ſaid 30 years: and then, ſubtracting this number from that of all the deceaſed purchaſers, that is, from 248, in order to obtain the number of the ſaid deceaſed purchaſers whoſe ſecond companions will have died in the courſe of the ſaid 30 years, we muſt inquire, in the next place, by means of the ſaid table, how many of the firſt companions of the ſaid remaining number of the ſaid deceaſed purchaſers will be alive at the end of the ſaid 30 years.

Now it is evident that the ſecond companions of the 248 deceaſed purchaſers are alſo 248 in number, each of the ſaid purchaſers having had one ſecond as well as one firſt companion. And theſe ſecond companions are ſuppoſed to have been 10 years old at the time of making the grants. Now it appears by Mr. de Parcieux's table above-mentioned that out of 880 perſons of 10 years of age, all living at the ſame time, 657 will be alive at the age of 40 years, or at the end of 30 years. Therefore out of the aforeſaid 248 ſecond companions of the 248 deceaſed purchaſers there will be $248 \times \frac{657}{880}$, or 185, perſons alive at the end of the ſaid 30 years. Therefore the number of the ſaid ſecond companions of the ſaid 248 deceaſed purchaſers who will have died in the courſe of the ſaid 30 years, will be the exceſs of 248 above 185, that is, 63 perſons. Therefore there are

are 63 perſons out of the ſaid 248 deceaſed purchaſers whoſe ſecond companions will have died in the courſe of the ſaid 30 years. But, though the ſecond companions of theſe 63 deceaſed purchaſers will be dead at the end of the ſaid 30 years, yet ſome of their firſt companions, who were of the age of 20 years at the time of making the grants, will probably be ſtill alive at that time; and, if they are ſo, will be intitled to theſe payments of one pound each. We muſt therefore inquire, by Mr. de Parcieux's table, how many of the ſaid firſt companions of theſe remaining 63 deceaſed purchaſers will be alive at the end of the ſaid 30 years. Now theſe firſt companions of the ſaid 63 deceaſed purchaſers are likewiſe 63 in number at the time of making the ſaid grants; becauſe each of the ſaid purchaſers is ſuppoſed to have had at that time two companions. And the age of theſe firſt companions at the time of making the ſaid grants, was ſuppoſed to be 20 years. Now it appears by Mr. de Parcieux's table that out of 814 perſons of the age of 20 years, all living at the ſame time, 581 will be alive at the age of 50 years, or at the end of 30 years. Therefore of the ſaid 63 firſt companions of the ſaid 63 deceaſed purchaſers, there will be $63 \times \frac{581}{814}$, or 45, perſons living at the end of the ſaid 30 years, and will be intitled to receive their reſpective payments of one pound each. Therefore the whole number of perſons intitled to receive the ſaid payments, at the end of the ſaid 30 years, will be, firſt, the aforeſaid 526 ſurviving purchaſers, and, ſecondly, the 185 ſurviving ſecond companions of the 248 deceaſed purchaſers, and thirdly, the 45 ſurviving firſt companions of the 63 deceaſed purchaſers whoſe ſecond companions will have died in the courſe of the ſaid 30 years; that is, in all, 756 perſons, as before. Therefore at the end of the ſaid 30 years the aforeſaid grantor will be obliged to pay to the ſaid ſurviving purchaſers, and the ſaid firſt and ſecond companions of the purchaſers that are deceaſed, the ſum of 756 pounds. Therefore, to the end that, at the expiration of the ſaid 30 years, the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive, at the time of making the ſaid grants, 756 times the value of one pound certain, payable at the end of 30 years, when the intereſt of money is 3 per cent. that is, 756 times .4119 of a pound, or £311.3964, from all the 774 purchaſers of the ſaid future payments. Therefore the ſum which he ought then to receive from each of the ſaid purchaſers as the price of his ſaid future ſum of one pound, is the 774th part of £311.3964, or 4023*l.* or 8*s.* $\frac{1}{2}$*d.* And conſequently, by Art. 21, this ſum of £.4023, or 8*s.* $\frac{1}{2}$*d.* is likewiſe the price which a purchaſer ought to pay the grantor for ſuch a future payment, when the latter makes only one ſuch grant; which is the ſame price which was to be paid for it on the former ſuppoſition of the firſt, or older, companion's being intitled to receive the ſaid payment in preference to the younger. Q. E. D.

LXXI. In

LXXI. In the two foregoing articles it is ſhewn that the price to be paid for the future ſum of one pound, to be received at the end of 30 years, if the purchaſer or either of his two companions ſhall be then alive, is exactly the ſame, whether, in caſe of the purchaſer's death and the ſurvivorſhip of both his companions at the end of the ſaid time, the purchaſer's firſt, or older, companion be intitled to receive it in preference to his ſecond, or younger, companion, or his ſecond, or younger, companion, be intitled to receive it in preference to the firſt: for that in both caſes, if we ſuppoſe 774 ſuch grants of future ſums of one pound to be made to as many different purchaſers upon the conditions above-mentioned, the number of perſons living at the end of the ſaid 30 years who will be intitled to receive the ſaid ſums, will be the ſame, to wit, 756 perſons. Now that this is not accidental, or peculiar to the ages and numbers that occur in this particular example, but would have been true, if any other ages than thoſe of 25, 20, and 10 years, had been pitched upon, and any other number of years than 30 had been ſuppoſed to intervene before the ſaid future payments became due, will appear by ſetting down the ſeveral factors and diviſors by whoſe multiplication and diviſion of each other the ſaid reſulting number of 756 claimants of theſe ſums of one pound is obtained in both caſes: which may be done in the manner following.

In Art. 69 the number of the perſons intitled to receive the ſaid payments of one pound each at the end of the ſaid 30 years was compoſed of the whole number of purchaſers who were then alive, which was 526, and of the 177 firſt, or older, companions of the deceaſed 248 purchaſers, and of the 53 ſurviving ſecond companions of thoſe (248—177, or) 71 deceaſed purchaſers whoſe firſt companions had died in the courſe of the ſaid 30 years.

Now the ſaid number, 177, of the ſurviving firſt companions of the deceaſed 248 purchaſers, is $= 248 \times \frac{581}{814} = \overline{774-526} \times \frac{581}{814} = \frac{774 \times 581}{814} - \frac{526 \times 581}{814}$.

And 53, the number of the ſurviving ſecond companions of the ſaid 71 deceaſed purchaſers, whoſe firſt companions will have died in the courſe of the ſaid 30 years, is $= 71 \times \frac{657}{880} = \overline{248-177} \times \frac{657}{880} = \frac{248 \times 657}{880} - \frac{177 \times 657}{880}$

$$= \frac{\overline{774-526} \times 657}{880} - \frac{177 \times 657}{880} = \frac{774 \times 657}{880} - \frac{526 \times 657}{880} - \frac{177 \times 657}{880}$$

$$= \frac{774 \times 657}{880} - \frac{526 \times 657}{880} - \frac{774 \times 581}{814} \times \frac{657}{880} + \frac{526 \times 581}{814} \times \frac{657}{880} = \frac{774 \times 657}{880}$$

$$- \frac{526 \times 657}{880} - \frac{774 \times 581 \times 657}{814 \times 880} + \frac{526 \times 581 \times 657}{814 \times 880}.$$

Therefore 526+177+53, or the whole number of perſons intitled to receive theſe payments at the end of the ſaid 30 years, is $= 526 + \frac{774\times581}{814} - \frac{526\times581}{814} + \frac{774\times657}{880} - \frac{526\times657}{880} - \frac{774\times581\times657}{814\times880} + \frac{526\times581\times657}{814\times880}$.

And in Art. 70 the number of perſons intitled to receive the ſaid payments of one pound each at the end of the ſaid 30 years, is compoſed of the ſaid 526 ſurviving purchaſers, and of the 185 ſurviving ſecond, or younger, companions of the 248 deceaſed purchaſers, and of the 45 ſurviving firſt, or older, companions of thoſe (248—185, or) 63 deceaſed purchaſers whoſe ſecond companions had died in the courſe of the ſaid 30 years.

Now the ſaid number, 185, of the ſecond companions of the 248 deceaſed purchaſers, is $= 248\times\frac{657}{880} = \overline{774-526}\times\frac{657}{880} = \frac{774\times657}{880} - \frac{526\times657}{880}$.

And 45, the number of the ſurviving firſt companions of thoſe 63 purchaſers whoſe ſecond companions have died in the courſe of the ſaid 30 years, is $= 63\times\frac{581}{814} = \overline{248-185}\times\frac{581}{814} = \frac{248\times581}{814} - \frac{185\times581}{814}$ $= \overline{774-526}\times\frac{581}{814} - \frac{185\times581}{814} = \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{185\times581}{814}$ $= \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{774\times657}{880}\times\frac{581}{814} + \frac{526\times657}{880}\times\frac{581}{814} = \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{774\times657\times581}{880\times814} + \frac{526\times657\times581}{880\times814}$.

Therefore 526+185+45, or the whole number of perſons intitled to receive theſe payments at the end of the ſaid 30 years is $= 526 + \frac{774\times657}{880} - \frac{526\times657}{880} + \frac{774\times581}{814} - \frac{526\times581}{814} - \frac{774\times657\times581}{880\times814} + \frac{526\times657\times581}{880\times814}$.

Now

Now it is easy to observe that this expression consists of the same number of terms, to wit, seven, as the expression found above for the number of claimants of these sums of one pound upon the former supposition of preferring the surviving first companions of the purchasers to their surviving second companions; and that the terms of both the expressions are the same, being composed of the same factors and divisors, though placed in a somewhat different order. For in both expressions the first term is 526; and the second term of the first expression is $\frac{774 \times 581}{814}$, which makes the fourth term in the second expression; and the third term of the first expression is $\frac{526 \times 581}{814}$, with the mark of subtraction prefixed to it, which makes the fifth term in the second expression, where it has the same mark of subtraction; and the fourth term of the first expression is $\frac{774 \times 657}{880}$, which makes the second term of the second expression; and the fifth term of the first expression is $\frac{526 \times 657}{880}$, with the mark of subtraction prefixed to it, which makes the third term in the second expression, where it has the same mark of subtraction prefixed to it; and the sixth term of the first expression is $\frac{774 \times 581 \times 657}{814 \times 880}$, with the mark of subtraction prefixed to it; and $\frac{774 \times 657 \times 581}{880 \times 814}$ with the mark of subtraction prefixed to it, is the sixth term of the second expression, which is equal to the sixth term of the former expression, because it is composed of the same factors both in the numerator and denominator, though placed in a different order; and the seventh term in the first expression is $\frac{526 \times 581 \times 657}{814 \times 880}$, which is composed of the same factors with, and therefore is equal to, the seventh term of the second expression, or $\frac{526 \times 657 \times 581}{880 \times 814}$. Therefore the whole first expression is equal to the whole second expression, or the number of persons intitled to receive these payments at the end of the said 30 years is the same upon both the said suppositions. Q E D.

A general

A general investigation of the value of a future payment of one pound depending on the continuance of any one of three given lives.

LXXII. We proceed now to extend the general ſolution of the foregoing problem given in Art. 59 to the caſe of a grant of a future payment of one pound made to a purchaſer, to be received at the end of a given number of years, in caſe either the purchaſer himſelf, or either of two other perſons named by him, and who may be called his *companions*, ſhall be then alive; the ages of the purchaſer and his two companions, and the rate of intereſt of money, and the table of the probabilities of the duration of human life, being all given.

Let the rate of intereſt of money be expreſſed, as before, by the proportion of r to 1. And let n denote the number of years at the end of which the ſaid ſum of one pound is to be received; and N the number of years in the age of the purchaſer himſelf; and M the number of years in the age of the purchaſer's firſt, or older, companion; and L the number of years in the age of his ſecond, or younger, companion. And let the ſaid ſum of one pound be payable, at the end of the ſaid n years, to the ſaid purchaſer himſelf, if he is then alive; and, if he is then dead, but both his companions are alive, to his firſt, or older, companion; and, if he alſo is then dead, but his ſecond, or younger, companion is then alive, to his ſaid ſecond, or younger, companion. And let P denote the number of perſons repreſented in the given table of probabilities to be living at the age of N years, or the age of the purchaſer; and Q the number of perſons therein repreſented to be living at the age of M years, or the age of the purchaſer's firſt, or older, companion; and T the number of perſons therein repreſented to be living at the age of L years, or the age of the purchaſer's ſecond, or younger, companion; and p, q, and t the numbers of perſons therein repreſented to be living at the ages of $N+n$, $M+n$, and $L+n$ reſpectively. And let it be ſuppoſed that the grantor of the ſaid future ſum of one pound, to be received at the end of n years, makes not only one ſuch grant, but P ſuch grants to P different purchaſers, each of whom has two companions, an older, or firſt, companion of the age of M years, and a younger, or ſecond, companion of the age of L years; of which P grants that to the purchaſer in the queſtion is one: and that each of theſe grants is ſubject to the ſame conditions as the grant to the purchaſer in the queſtion.

Then it is evident that the number of perſons that will be intitled to receive theſe payments of one pound each at the end of the ſaid n years, will be greater than in the caſe ſuppoſed in the ſolution of the foregoing problem, Art. 59; becauſe not only all the p purchaſers who will be living at the end of the ſaid 30 years, together with the $\frac{\overline{P-p} \times q}{Q}$ ſurviving firſt

companions

companions of the $P-p$ deceafed purchafers, making in all $p+\overline{P-p}\times\frac{q}{Q}$ perfons, will be intitled to thefe payments, as they were in that folution, but there will alfo be fome furviving fecond companions of the faid deceafed purchafers who will have a right to them. What the number of thefe will be, we muft now proceed to inquire.

Now it is evident, in the firft place, from the conditions of this queftion that the furviving fecond companions of the p purchafers, who will be alive at the end of the faid n years, can have no claim to the faid fums of one pound; becaufe they are to be paid to the faid furviving purchafers themfelves. And, for the like reafon, it is evident, in the fecond place, that the fecond companions of thofe deceafed purchafers whofe firft companions are alive at the end of the faid n years, can have no claim to thefe payments; becaufe it is provided that, when both the companions of a deceafed purchafer are alive at the time the payment becomes due, it fhall be received by the faid purchafer's firft, or older, companion in preference to the fecond. Now it has been fhewn in Art. 58 that out of the $P-p$ purchafers who will have died in the courfe of the faid n years, there will be $\overline{P-p}\times\frac{q}{Q}$ perfons whofe firft, or older, companions, who were M years of age at the time of making the grants, will be alive at the end of the faid n years. Therefore the number of the faid deceafed purchafers whofe firft, or older, companions will alfo be dead at the end of the faid n years, is the excefs of $P-p$ above $\overline{P-p}\times\frac{q}{Q}$, or $\frac{Pq-pq}{Q}$, perfons, or is equal to $P-p-\frac{Pq-pq}{Q}=\frac{PQ-pQ-Pq+pq}{Q}$. There are therefore $\frac{PQ-pQ-Pq+pq}{Q}$ deceafed purchafers whofe fecond companions will have a right to receive thefe payments of one pound each, if they live to the end of the faid n years. We muft therefore inquire how many of the fecond companions of thefe $\frac{PQ-pQ-Pq+pq}{Q}$ deceafed purchafers will live to the end of the faid n years, fuppofing them to die off in the proportion fet forth in the given table of the probabilities of the duration of human life. Now thefe fecond companions of the faid deceafed purchafers are evidently juft as many as the faid deceafed purchafers themfelves, that is, $\frac{PQ-pQ-Pq+pq}{Q}$; becaufe each of the faid deceafed purchafers had one fecond as well as one firft companion. And the age of thefe fecond companions at the time of making the grants, is fuppofed to be L years. Now the numbers of perfons reprefented in the given table of probabilities to be living at the ages of L years and $L+n$ years are fuppofed to be T and t. Therefore, fince out of T perfons of L years of age, all living at the fame time, t perfons perſons will be alive at the end of n years, it is evident that of the aforeſaid $\frac{PQ-pQ-Pq+pq}{Q}$ ſecond companions of the ſaid deceaſed purchaſers, who are all living, and of the age of L years, at the time of making the grants, there will be $\frac{t}{T} \times \frac{PQ-pQ-Pq+pq}{Q}$, or $\frac{PQt-pQt-Pqt+pqt}{QT}$, alive at the end of the ſaid n years. Therefore the whole number of perſons intitled to receive the ſaid payments of one pound each at the end of the ſaid n years, will be, firſt, the p ſurviving purchaſers themſelves, and, ſecondly, the $\overline{P-p} \times \frac{q}{Q}$, or $\frac{Pq-pq}{Q}$, ſurviving firſt companions of the $P-p$ deceaſed purchaſers, and, thirdly, the ſaid $\frac{PQt-pQt-Pqt+pqt}{QT}$ ſurviving ſecond companions of the ſaid $\frac{PQ-pQ-Pq+pq}{Q}$ deceaſed purchaſers, whoſe firſt companions will have died in the courſe of the ſaid n years: that is, the number of the perſons intitled to receive theſe payments will be, in all, $p+\frac{Pq-pq}{Q}+\frac{PQt-pQt-Pqt+pqt}{QT}$, or $p+\frac{Pq}{Q}-\frac{pq}{Q}+\frac{Pt}{T}-\frac{pt}{T}-\frac{Pqt}{QT}+\frac{pqt}{QT}$. Therefore at the end of the ſaid n years the aforeſaid grantor will be obliged to pay to the ſaid ſurviving purchaſers and the ſaid firſt and ſecond companions of the purchaſers that are deceaſed, the ſum of $p+\frac{Pq}{Q}-\frac{pq}{Q}+\frac{Pt}{T}-\frac{pt}{T}-\frac{Pqt}{QT}+\frac{pqt}{QT}$ pounds. Therefore, to the end that the ſaid grantor may be neither a gainer nor a loſer by the ſum total of all his bargains, it is neceſſary that he ſhould receive, at the time of making the ſaid grants, $p+\frac{Pq}{Q}-\frac{pq}{Q}+\frac{Pt}{T}-\frac{pt}{T}-\frac{Pqt}{QT}+\frac{pqt}{QT}$ times the preſent value of one pound certain, payable at the end of n years, when the intereſt of money is expreſſed by the proportion of r to 1; that is, $p+\frac{Pq}{Q}-\frac{pq}{Q}+\frac{Pt}{T}-\frac{pt}{T}-\frac{Pqt}{QT}+\frac{pqt}{QT}$ times $\frac{1}{r^n}$ of a pound, or $\frac{1}{r^n}$ £, from all the P purchaſers of theſe future payments. Therefore the ſum which he ought then to receive from each of the ſaid purchaſers is the Pth part of $p+\frac{Pq}{Q}-\frac{pq}{Q}+\frac{Pt}{T}-\frac{pt}{T}-\frac{Pqt}{QT}+\frac{pqt}{QT}$ times $\frac{1}{r^n}$ £, or is equal to $\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}+\frac{t}{T}-\frac{pt}{PT}-\frac{qt}{QT}+\frac{pqt}{PQT}$ times $\frac{1}{r^n}$ £. And conſequently, by Art. 21, the price which a purchaſer ought to pay for ſuch a grant of a future fum of one pound, to be received at the end of n years, upon the conditions above-mentioned, when the grantor of it makes only one fuch grant, is likewife equal to $\frac{p}{P} + \frac{q}{\mathcal{Q}} - \frac{pq}{P\mathcal{Q}} + \frac{t}{T} - \frac{pt}{PT} - \frac{qt}{\mathcal{Q}T} + \frac{pqt}{P\mathcal{Q}T}$ times $\frac{1}{r^n}$ £. QEI.

LXXIII. In the foregoing article it was fuppofed that when both the companions of the deceafed purchafers were alive at the end of the faid n years, the faid fums of one pound were to be paid to their firft, or older, companions, who were of the age of M years at the time of making the grants, in preference of their fecond, or younger, companions, who were at that time of the age of L years. But the values of thefe future fums of one pound will be exactly the fame, if we fhould fuppofe that they are to be paid to the faid fecond, or younger, companions of the faid deceafed purchafers in preference to the firft or older. This may be fhewn in the manner following.

Upon this new fuppofition it is evident that the perfons who will be intitled to receive the faid fums of one pound at the end of the faid n years, will be compofed, in the firft place, of the p furviving purchafers themfelves, and, in the fecond place, of the furviving fecond, or younger, companions of the $P-p$ deceafed purchafers, and, in the third place, of the furviving firft, or older, companions of thofe of the faid $P-p$ deceafed purchafers whofe fecond companions will have died in the courfe of the faid n years. Now, fince of T perfons of the age of L years, or of the age of the fecond companions of the faid purchafers at the time of making the grants, t perfons are found in the given table of probabilities to be living at the end of n years, it follows, that of the $P-p$ fecond companions of the $P-p$ deceafed purchafers, who were living at the time of making the grants, there will be $\overline{P-p} \times \frac{t}{T}$, or $\frac{Pt-pt}{T}$, perfons alive at the end of the faid n years. Therefore the number of deceafed purchafers whofe fecond companions will die in the courfe of the faid n years is $P-p-\overline{\frac{Pt-pt}{T}}$, or $\frac{PT-pT-Pt+pt}{T}$; and confequently the number of the firft companions of thefe deceafed purchafers is likewife equal to $\frac{PT-pT-Pt+pt}{T}$. But the age of thefe firft companions is that of M years. And it appears by the table that of $\mathcal{Q}$ perfons of the age of M years, all living at the fame time,

time, there will be q persons living at the end of n years. Therefore of the said $\frac{PT-pT-Pt+pt}{T}$ first companions of the said deceased purchasers, who were living at the time of making the grants, there will be living at the end of the said n years only $\overline{\frac{PT-pT-Pt+pt}{T}} \times \frac{q}{Q}$, or $\frac{PTq-pTq-Ptq+ptq}{TQ}$, persons. Therefore the whole number of persons intitled to receive the said payments of one pound each at the end of the said n years is $p+\frac{Pt-pt}{T}+\frac{PTq-pTq-Ptq+ptq}{TQ}$, or $p+\frac{Pt}{T}-\frac{pt}{T}+\frac{Pq}{Q}-\frac{pq}{Q}-\frac{Ptq}{TQ}+\frac{ptq}{TQ}$. Therefore this will be the number of pounds which the grantor of the said future payment will have to pay at the end of the said n years. Therefore at the time of making the grants he ought to receive from all the P purchasers of these future payments a sum equal to $p+\frac{Pt}{T}-\frac{pt}{T}+\frac{Pq}{Q}-\frac{pq}{Q}-\frac{Ptq}{TQ}+\frac{ptq}{TQ}$ times the present value of one pound payable at the end of n years, or $p+\frac{Pt}{T}-\frac{pt}{T}+\frac{Pq}{Q}-\frac{pq}{Q}-\frac{Ptq}{TQ}+\frac{ptq}{TQ}$ times $\frac{1}{r^n}$ of a pound; and consequently from each of the said purchasers the Pth part of the former sum, or $\frac{p}{P}+\frac{t}{T}-\frac{pt}{PT}+\frac{q}{Q}-\frac{pq}{PQ}-\frac{tq}{TQ}+\frac{ptq}{PTQ}$ times $\frac{1}{r^n}$ of a pound.

Therefore, by Art. 21, this is likewise the sum which a purchaser ought to pay to the grantor for a grant of such a future sum of one pound, when he makes only one such grant. Now this sum of $\frac{p}{P}+\frac{t}{T}-\frac{pt}{PT}+\frac{q}{Q}-\frac{pq}{PQ}-\frac{tq}{TQ}+\frac{ptq}{PTQ}$ times $\frac{1}{r^n}$ of a pound, is evidently equal to the sum which was found in the last article to be the price of such a grant of a future payment of one pound to be received at the end of n years, in case the purchaser himself who was aged N years, or either of his two companions, who were aged M years and L years at the time of making the grant, should be then alive, upon a supposition that his first, or older, companion should be intitled to the said payment in preference to his second, or younger, companion, namely, the sum of $\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}+\frac{t}{T}-\frac{pt}{PT}-\frac{qt}{QT}+\frac{pqt}{PQT}$ times $\frac{1}{r^n}$ of a pound; because these two expressions consist of the same terms connected together by addition and subtraction in the same manner, and differing from each other only in the order in which they are placed, which cannot

cannot affect their magnitude. It is therefore true, as was before affirmed, that whether the future ſum of one pound be made payable to the firſt, or older, companion of the purchaſer in preference to his ſecond, or younger, companion, or to the ſecond companion in preference to the firſt, or older, the price to be paid to the grantor for ſuch future ſum, will in both caſes be the ſame. Q E D.

LXXIV. COROLL. 9. It follows from the laſt corollary that the value of a future payment of one pound, to be received at the end of n years if either the purchaſer himſelf or either of his two companions, whoſe ages are given, ſhall be then alive, is equal to the exceſs of the ſum of the three values of the like future payment of one pound, to be received at the end of the ſaid n years in caſe each of the ſaid three perſons, to wit, the purchaſer and his two companions, ſhall be then alive, and the value of the like future payment to be received at the end of the ſaid time in caſe all the ſaid three perſons ſhall be then alive, above the ſum of the three values of the like future payment of one pound in caſe the purchaſer himſelf and his firſt companion ſhall be then alive, and in caſe the purchaſer himſelf and his ſecond companion ſhall be then alive, and in caſe his firſt and ſecond companion ſhall both be then alive.

Of the relation of the value of a future payment of one pound depending on the continuance of the life of any one of three perſons of given ages to the values of the ſame payment to be made at the ſame future time in caſe of the continuance of the ſingle lives of each of the ſame three perſons, and in caſe of the joint continuance of the lives of every two of them, and in caſe of the joint continuance of the lives of all the three.

For it has been ſhewn that the ſaid value of a future payment of one pound, to be received at the end of n years if either the purchaſer himſelf or either of his two companions ſhall be then alive, is equal to $\frac{p}{P}+\frac{q}{Q}-\frac{pq}{PQ}+\frac{t}{T}-\frac{pt}{PT}-\frac{qt}{QT}+\frac{pqt}{PQT}$ times $\frac{1}{r^n}$ of a pound. And conſequently it is equal to $\frac{p}{P}\times\frac{1}{r^n}£+\frac{q}{Q}\times\frac{1}{r^n}£+\frac{t}{T}\times\frac{1}{r^n}£+\frac{pqt}{PQT}\times\frac{1}{r^n}£-\frac{pq}{PQ}\times\frac{1}{r^n}£-\frac{pt}{PT}\times\frac{1}{r^n}£-\frac{qt}{QT}\times\frac{1}{r^n}£$. But, by Prob. 2, Art. 28, $\frac{p}{P}\times\frac{1}{r^n}£$ is the value of a future payment of one pound, to be received at the end of n years in caſe a purchaſer of the age of N years, which correſponds to the number P in the table of the probabilities of the duration of human life, ſhall be then alive; and $\frac{q}{Q}\times\frac{1}{r^n}£$ is the value of a like future payment of one pound, to be received at the ſame time, in caſe a perſon of the age of M years, which correſponds to the number Q in the table, or of the age of the aforeſaid purchaſer's firſt, or older, companion, ſhall be then alive; and $\frac{t}{T}\times\frac{1}{r^n}£$ is the value of a like future payment

M of

of one pound, to be received at the same time in case a person of the age of L years, which corresponds to the number T in the said table, or of the age of the aforesaid purchaser's second, or younger, companion, shall be then alive. And, by Prob. 3, Coroll. 6, Art. 55, $\frac{pqt}{PQT} \times \frac{1}{r^n}$ £ is the value of a future payment of one pound, to be received at the end of n years in case a purchaser of the age of N years, which corresponds to the number P in the table, and two companions of the said purchaser, one of the age of M years, which corresponds to the number Q in the table, and the other of the age of L years, which corresponds to the number T in the table, shall all three be living at the end of the said time. And, by Prob. 3, Art. 45, $\frac{pq}{PQ} \times \frac{1}{r^n}$ £ is the value of a future payment of one pound, to be received at the end of n years if both a purchaser of the age of N years, which corresponds to the number P in the table, and a companion of the age of M years, which corresponds to the number Q in the table, shall be living at that time: and $\frac{pt}{PT} \times \frac{1}{r^n}$ £ is the value of one pound, to be received at the end of n years if two persons of the ages of N and L years, which correspond to the numbers P and T in the table, shall be then alive: and $\frac{qt}{QT} \times \frac{1}{r^n}$ £ is the value of a future payment of one pound, to be received at the end of n years, if two persons of the ages of M and L years, which correspond to the numbers Q and T in the table, shall be then alive. Therefore the value of a future payment of one pound, to be received at the end of n years if either the purchaser aforesaid, aged N years, or either of his two companions, aged M years and L years, shall be then alive, is equal to the excess of the sum of the four following values, to wit, the values of the same payment, to be received at the same time, depending upon the single lives of the said purchaser and his two companions, and the value of it, to be received at the same time in case of the joint continuance of the lives of all the three, above the sum of the three following values, to wit, the values of the same payment, to be received at the same time, in case of the joint continuance of the two lives of the purchaser and his first, or older, companion, and in case of the joint continuance of the lives of the purchaser and his second, or younger, companion, and in case of the joint continuance of the lives of his two companions. Q.E.D.

LXXV. CO-

LXXV. COROLL. 10. By finding in the method defcribed in the two laft corollaries the feveral values of one pound fterling, to be received at the end of every year of the whole fpace of time during which it is poffible that either a purchafer of a given age, or either of his two companions, whofe ages are likewife given, may be prolonged, in cafe that either the faid purchafer or either of his faid companions fhall be living at the time when every fuch payment becomes due; and then adding thefe values all together into one fum; we fhall obtain the value of a fet of equal payments of one pound each, to be received by the purchafer or his companions at the end of every future year during the lives of either of them, that is, the value of an annuity of one pound for the lives of the faid purchafer and his two companions and the life of the longeft liver of them.

Of the value of an annuity of one pound fterling for the life of the longeft liver of three perfons of given ages.

LXXVI. COROLL. 11. The value of an annuity of one pound fterling for the lives of three perfons of given ages and the life of the longeft liver of them, is equal to the excefs of the fum of the four following values, to wit, the value of an annuity of one pound for the life of the firft of the faid three perfons, the value of a like annuity for the life of the fecond of them, and the value of a like annuity for the life of the third of them, and the value of a like annuity for the joint lives of all the three, above the fum of the three following values, to wit, the value of a like annuity of one pound during the joint lives of the firft and fecond perfons, and the value of a like annuity during the joint lives of the firft and third perfons, and the value of a like annuity during the joint lives of the fecond and third perfons.

Of the relation of the faid value to the values of the like annuities of one pound for the fingle lives of the fame three perfons, and for the joint lives of every two of them, and for the joint lives of all the three.

Let the firft of thefe three perfons be fuppofed, as before, to be the purchafer of the faid annuity of one pound, and the other two be called his companions. And let N be the number of years in the age of the faid purchafer; and M the number of years in the age of the faid purchafer's firft, or older, companion; and L the number of years in the age of his fecond, or younger, companion. And let P denote, as before, the number of perfons reprefented in the table of the probabilities of the duration of human life adopted for the calculation, to be living at the age of the purchafer, or at the age of N years; and Q denote the number of perfons therein reprefented to be living at the age of the purchafer's firft, or older, companion, or the age of M years; and T the number of perfons therein reprefented to be living at the age of the purchafer's fecond, or younger, companion, or the age of L years. And further, let P^{I} denote the number of perfons reprefented in the faid table to be living at the age of $N+1$ years; and P^{II} the number of thofe living at the age of $N+2$ years; and P^{III} the number of thofe living at the age of $N+3$ years; and P^{IV} the number of thofe living at the age of $N+4$ years; and fo on for the following

years;

years; the Roman numeral figures placed over every new letter P denoting the number of years by which every new age, corresponding to it, exceeds the age of N years, or the age of the purchaser. And, in like manner, let Q^{I} denote the number of persons represented in the said table to be living at the age of $M+1$ years; and Q^{II} the number of persons living at the age of $M+2$ years; and Q^{III} the number of persons living at the end of $M+3$ years; and Q^{IV} the number of persons living at the end of $M+4$ years; and so on, for the following years; the Roman numeral figures placed over every new letter Q denoting the number of years by which every new age corresponding to it, exceeds the age of M years, or of the purchaser's first, or older, companion. And, lastly, let T^{I} denote the number of persons represented in the said table to be living at the age of $L+1$ years; and T^{II} the number of persons living at the age of $L+2$ years; and T^{III} the number of persons living at the age of $L+3$ years; and T^{IV} the number of persons living at the age of $L+4$ years; and so on for the following years; the Roman numeral figures placed over every new letter T denoting the number of years by which every new age, corresponding to it, exceeds the age of L years, or of the purchaser's second, or younger, companion.

Then it is evident from Coroll. 8 that the value of a sum of one pound to be received at the end of one year, in case either the purchaser or one of his two companions shall be then alive, will be equal to

$$\left|\frac{P^{I}}{P}+\frac{Q^{I}}{Q}+\frac{T^{I}}{T}-\frac{P^{I}Q^{I}}{PQ}-\frac{P^{I}T^{I}}{PT}-\frac{Q^{I}T^{I}}{QT}+\frac{P^{I}Q^{I}T^{I}}{PQT}\right|\times\frac{1}{r}\text{ of a}$$

pound; because in this case n is $=1$, and p, q, t are respectively equal to P^{I}, Q^{I}, and T^{I}.

And in like manner it is evident that the value of the same sum of one pound, to be received at the end of two years, subject to the same contingency, is equal to

$$\left|\frac{P^{II}}{P}+\frac{Q^{II}}{Q}+\frac{T^{II}}{T}-\frac{P^{II}Q^{II}}{PQ}-\frac{P^{II}T^{II}}{PT}-\frac{Q^{II}T^{II}}{QT}+\frac{P^{II}Q^{II}T^{II}}{PQT}\right|$$

$\times\frac{1}{r^2}$ of a pound; because in this case n is equal to 2, and p, q, t are respectively equal to P^{II}, Q^{II}, and T^{II}.

And in like manner the value of the same sum, to be received at the end of three years, subject to the same contingency, is equal to

$$\left|\frac{P^{III}}{P}+\frac{Q^{III}}{Q}+\frac{T^{III}}{T}-\frac{P^{III}Q^{III}}{PQ}-\frac{P^{III}T^{III}}{PT}-\frac{Q^{III}T^{III}}{QT}+\frac{P^{III}Q^{III}T^{III}}{PQT}\right|$$

$\times\frac{1}{r^3}$ of a pound; because in this case n is $=3$, and p, q, t, are equal to P^{III}, Q^{III}, T^{III}, respectively.

And

And in like manner the value of the ſame ſum, to be received at the end of four years, ſubject to the ſame contingency, is equal to

$$\overline{\frac{P^{iv}}{P}+\frac{Q^{iv}}{Q}+\frac{T^{iv}}{T}-\frac{P^{iv}Q^{iv}}{PQ}-\frac{P^{iv}T^{iv}}{PT}-\frac{Q^{iv}T^{iv}}{QT}+\frac{P^{iv}Q^{iv}T^{iv}}{PQT}}\Big|\times\frac{1}{r^4}$$

of a pound; becauſe in this caſe n is $=4$, and p, q, t, are equal to P^{iv}, Q^{iv}, T^{iv}, reſpectively.

And in the ſame manner we may find the values of one pound, to be received at the end of the fifth, ſixth, ſeventh, and every following year of the whole ſpace of time during which it is poſſible that the lives of the aforeſaid purchaſer, or either of his two companions, may be prolonged.

Therefore the ſum of theſe values, continued to the utmoſt poſſible extent of the youngeſt of thoſe three lives, that is, the value of a life-annuity of one pound for the lives of the purchaſer and his two companions, and the life of the longeſt liver of them, is equal to the following ſet of values, to wit,

$$\overline{\frac{P^{i}}{P}+\frac{Q^{i}}{Q}+\frac{T^{i}}{T}-\frac{P^{i}Q^{i}}{PQ}-\frac{P^{i}T^{i}}{PT}-\frac{Q^{i}T^{i}}{QT}+\frac{P^{i}Q^{i}T^{i}}{PQT}}\Big|\times\frac{1}{r}\text{£},$$

$$+\overline{\frac{P^{ii}}{P}+\frac{Q^{ii}}{Q}+\frac{T^{ii}}{T}-\frac{P^{ii}Q^{ii}}{PQ}-\frac{P^{ii}T^{ii}}{PT}-\frac{Q^{ii}T^{ii}}{QT}+\frac{P^{ii}Q^{ii}T^{ii}}{PQT}}\Big|\times\frac{1}{r^2}\text{£},$$

$$+\overline{\frac{P^{iii}}{P}+\frac{Q^{iii}}{Q}+\frac{T^{iii}}{T}-\frac{P^{iii}Q^{iii}}{PQ}-\frac{P^{iii}T^{iii}}{PT}-\frac{Q^{iii}T^{iii}}{QT}+\frac{P^{iii}Q^{iii}T^{iii}}{PQT}}\Big|$$
$$\times\frac{1}{r^3}\text{£},$$

$$+\overline{\frac{P^{iv}}{P}+\frac{Q^{iv}}{Q}+\frac{T^{iv}}{T}-\frac{P^{iv}Q^{iv}}{PQ}-\frac{P^{iv}T^{iv}}{PT}-\frac{Q^{iv}T^{iv}}{QT}-\frac{P^{iv}Q^{iv}T^{iv}}{PQT}}\Big|\times\frac{1}{r^4}\text{£},$$

$$+\overline{\frac{P^{v}}{P}+\frac{Q^{v}}{Q}+\frac{T^{v}}{T}-\frac{P^{v}Q^{v}}{PQ}-\frac{P^{v}T^{v}}{PT}-\frac{Q^{v}T^{v}}{QT}+\frac{P^{v}Q^{v}T^{v}}{PQT}}\Big|\times\frac{1}{r^5}\text{£},$$

+&c. continued to the utmoſt extent of human life;

And consequently to the following set of quantities, to wit,

$$\frac{P^{i}}{P}\times\frac{1}{r}\pounds\ +\frac{Q^{i}}{Q}\times\frac{1}{r}\pounds\ +\frac{T^{i}}{T}\times\frac{1}{r}\pounds\ -\frac{P^{i}Q^{i}}{P\ Q}\times\frac{1}{r}\pounds\ -\frac{P^{i}T^{i}}{P\ T}\times\frac{1}{r}\pounds$$
$$-\frac{Q^{i}T^{i}}{Q\ T}\times\frac{1}{r}\pounds\ +\frac{P^{i}Q^{i}T^{i}}{P\ Q\ T}\times\frac{1}{r}\pounds.$$

$$+\frac{P^{ii}}{P}\times\frac{1}{r^{2}}\pounds\ +\frac{Q^{ii}}{Q}\times\frac{1}{r^{2}}\pounds\ +\frac{T^{ii}}{T}\times\frac{1}{r^{2}}\pounds\ -\frac{P^{ii}Q^{ii}}{P\ Q}\times\frac{1}{r^{2}}\pounds$$
$$-\frac{P^{ii}T^{ii}}{P\ T}\times\frac{1}{r^{2}}\pounds\ -\frac{Q^{ii}T^{ii}}{Q\ T}\times\frac{1}{r^{2}}\pounds\ +\frac{P^{ii}Q^{ii}T^{ii}}{P\ Q\ T}\times\frac{1}{r^{2}}\pounds,$$

$$+\frac{P^{iii}}{P}\times\frac{1}{r^{3}}\pounds\ +\frac{Q^{iii}}{Q}\times\frac{1}{r^{3}}\pounds\ +\frac{T^{iii}}{T}\times\frac{1}{r^{3}}\pounds\ -\frac{P^{iii}Q^{iii}}{P\ Q}\times\frac{1}{r^{3}}\pounds$$
$$-\frac{P^{iii}T^{iii}}{P\ T}\times\frac{1}{r^{3}}\pounds\ -\frac{Q^{iii}T^{iii}}{Q\ T}\times\frac{1}{r^{3}}\pounds\ +\frac{P^{iii}Q^{iii}T^{iii}}{P\ Q\ T}\times\frac{1}{r^{3}}\pounds,$$

$$+\frac{P^{iv}}{P}\times\frac{1}{r^{4}}\pounds\ +\frac{Q^{iv}}{Q}\times\frac{1}{r^{4}}\pounds\ +\frac{T^{iv}}{T}\times\frac{1}{r^{4}}\pounds\ -\frac{P^{iv}Q^{iv}}{P\ Q}\times\frac{1}{r^{4}}\pounds$$
$$-\frac{P^{iv}T^{iv}}{P\ T}\times\frac{1}{r^{4}}\pounds\ -\frac{Q^{iv}T^{iv}}{Q\ T}\times\frac{1}{r^{4}}\pounds\ +\frac{P^{iv}Q^{iv}T^{iv}}{P\ Q\ T}\times\frac{1}{r^{4}}\pounds,$$

$$+\frac{P^{v}}{P}\times\frac{1}{r^{5}}\pounds\ +\frac{Q^{v}}{Q}\times\frac{1}{r^{5}}\pounds\ +\frac{T^{v}}{T}\times\frac{1}{r^{5}}\pounds\ -\frac{P^{v}Q^{v}}{P\ Q}\times\frac{1}{r^{5}}\pounds$$
$$-\frac{P^{v}T^{v}}{P\ T}\times\frac{1}{r^{5}}\pounds\ -\frac{Q^{v}T^{v}}{Q\ T}\times\frac{1}{r^{5}}\pounds\ +\frac{P^{v}Q^{v}T^{v}}{P\ Q\ T}\times\frac{1}{r^{5}}\pounds,$$

+ &c. continued to the utmost extent of human life.

And consequently to the seven following sets of quantities, which are the same with the quantities last-mentioned, but only are placed in a different order, (those which before were placed in perpendicular columns, one under the other, being now placed in separate horizontal lines;) to wit,

1st, $\frac{P^{i}}{P}\times\frac{1}{r}\pounds\ +\frac{P^{ii}}{P}\times\frac{1}{r^{2}}\pounds\ +\frac{P^{iii}}{P}\times\frac{1}{r^{3}}\pounds\ +\frac{P^{iv}}{P}\times\frac{1}{r^{4}}\pounds$ $+\frac{P^{v}}{P}\times\frac{1}{r^{5}}\pounds$ + &c. continued to the utmost extent of human life:

And

And 2dly, $\frac{Q}{Q} \times \frac{1}{r}\text{£} + \frac{Q^{\text{II}}}{Q} \times \frac{1}{r^2}\text{£} + \frac{Q^{\text{III}}}{Q} \times \frac{1}{r^3}\text{£} + \frac{Q^{\text{IV}}}{Q} \times \frac{1}{r^4}\text{£}$ $+ \frac{Q^{\text{V}}}{Q} \times \frac{1}{r^5}\text{£}$ +&c. continued to the utmost extent of human life:

And 3dly, $\frac{T}{T} \times \frac{1}{r}\text{£} + \frac{T^{\text{II}}}{T} \times \frac{1}{r^2}\text{£} + \frac{T^{\text{III}}}{T} \times \frac{1}{r^3}\text{£} + \frac{T^{\text{IV}}}{T} \times \frac{1}{r^4}\text{£}$ $+ \frac{T^{\text{V}}}{T} \times \frac{1}{r^5}\text{£}$ +&c. continued to the utmost extent of human life:

And 4thly, $-\frac{P^{\text{I}}Q^{\text{I}}}{P\ \ Q} \times \frac{1}{r}\text{£} - \frac{P^{\text{II}}Q^{\text{II}}}{P\ \ Q} \times \frac{1}{r^2}\text{£} - \frac{P^{\text{III}}Q^{\text{III}}}{P\ \ Q} \times \frac{1}{r^3}\text{£}$ $-\frac{P^{\text{IV}}Q^{\text{IV}}}{P\ \ Q} \times \frac{1}{r^4}\text{£} - \frac{P^{\text{V}}Q^{\text{V}}}{P\ \ Q} \times \frac{1}{r^5}\text{£}$ −&c. continued to the utmost extent of human life:

And 5thly, $-\frac{P^{\text{I}}T^{\text{I}}}{P\ \ T} \times \frac{1}{r}\text{£} - \frac{P^{\text{II}}T^{\text{II}}}{P\ \ T} \times \frac{1}{r^2}\text{£} - \frac{P^{\text{III}}T^{\text{III}}}{P\ \ T} \times \frac{1}{r^3}\text{£}$ $-\frac{P^{\text{IV}}T^{\text{IV}}}{P\ \ T} \times \frac{1}{r^4}\text{£} - \frac{P^{\text{V}}T^{\text{V}}}{P\ \ T} \times \frac{1}{r^5}\text{£}$ −&c. continued to the utmost extent of human life:

And 6thly, $-\frac{Q^{\text{I}}T^{\text{I}}}{Q\ \ T} \times \frac{1}{r}\text{£} - \frac{Q^{\text{II}}T^{\text{II}}}{Q\ \ T} \times \frac{1}{r^2}\text{£} - \frac{Q^{\text{III}}T^{\text{III}}}{Q\ \ T} \times \frac{1}{r^3}\text{£}$ $-\frac{Q^{\text{IV}}T^{\text{IV}}}{Q\ \ T} \times \frac{1}{r^4}\text{£} - \frac{Q^{\text{V}}T^{\text{V}}}{Q\ \ T} \times \frac{1}{r^5}\text{£}$ −&c. continued to the utmost extent of human life:

And 7thly, $\frac{P^{\text{I}}Q^{\text{I}}T^{\text{I}}}{P\ \ Q\ \ T} \times \frac{1}{r}\text{£} + \frac{P^{\text{II}}Q^{\text{II}}T^{\text{II}}}{P\ \ Q\ \ T} \times \frac{1}{r^2}\text{£} + \frac{P^{\text{III}}Q^{\text{III}}T^{\text{III}}}{P\ \ Q\ \ T} \times \frac{1}{r^3}\text{£}$ $+\frac{P^{\text{IV}}Q^{\text{IV}}T^{\text{IV}}}{P\ \ Q\ \ T} \times \frac{1}{r^4}\text{£} + \frac{P^{\text{V}}Q^{\text{V}}T^{\text{V}}}{P\ \ Q\ \ T} \times \frac{1}{r^5}\text{£}$ +&c. continued to the utmost extent of human life;

that

that is, the value of a life-annuity of one pound for the lives of the ſaid purchaſer and his two companions, and the life of the longeſt liver of them, is equal to the exceſs of the ſum of the firſt, ſecond, third, and ſeventh, of the ſeven laſt ſets of quantities above the ſum of the fourth, fifth, and ſixth ſets.

Now it appears from Prob. 2, Coroll. 2, Art. 31, that the firſt of theſe ſeven ſets of quantities, to wit, $\frac{P'}{P} \times \frac{1}{r}£ + \frac{P''}{P} \times \frac{1}{r^2}£ + \frac{P'''}{P} \times \frac{1}{r^3}£ + \frac{P^{iv}}{P} \times \frac{1}{r^4}£ + \frac{P^{v}}{P} \times \frac{1}{r^5}£$ + &c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the purchaſer, who was ſuppoſed to be of the age of N years, which correſponds to the number P in the table.

And, in like manner, the ſecond of theſe ſeven ſets of quantities, to wit,

$$\frac{Q'}{Q} \times \frac{1}{r}£ + \frac{Q''}{Q} \times \frac{1}{r^2}£ + \frac{Q'''}{Q} \times \frac{1}{r^3}£ + \frac{Q^{iv}}{Q} \times \frac{1}{r^4}£ + \frac{Q^{v}}{Q} \times \frac{1}{r^5}£$$

+ &c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the ſaid purchaſer's firſt, or older, companion, who was ſuppoſed to be of the age of M years, which correſponds to the number Q in the table.

And, in like manner, the third of theſe ſeven ſets of quantities, to wit,

$$\frac{T'}{T} \times \frac{1}{r}£ + \frac{T''}{T} \times \frac{1}{r^2}£ + \frac{T'''}{T} \times \frac{1}{r^3}£ + \frac{T^{iv}}{T} \times \frac{1}{r^4}£ + \frac{T^{v}}{T} \times \frac{1}{r^5}£$$

+ &c. continued to the utmoſt extent of human life, is the value of an annuity of one pound for the life of the ſaid purchaſer's ſecond, or younger, companion, who was ſuppoſed to be of the age of L years, which correſponds to the number T in the table.

And

And it appears from Prob. 3, Coroll. 2, Art. 48, that the fourth of the said seven sets of quantities, to wit, $\frac{P'Q'}{PQ} \times \frac{1}{r}£ + \frac{P''Q''}{PQ} \times \frac{1}{r^2}£ + \frac{P'''Q'''}{PQ} \times \frac{1}{r^3}£ + \frac{P^{iv}Q^{iv}}{PQ} \times \frac{1}{r^4}£ + \frac{P^{v}Q^{v}}{PQ} \times \frac{1}{r^5}£$ + &c. continued to the utmost extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser and his said first, or older, companion, whose ages N and M correspond to the letters, or numbers, P and Q in the said table.

And in like manner the fifth of the said seven sets of quantities, to wit, $\frac{P'T'}{PT} \times \frac{1}{r}£ + \frac{P''T''}{PT} \times \frac{1}{r^2}£ + \frac{P'''T'''}{PT} \times \frac{1}{r^3}£ + \frac{P^{iv}T^{iv}}{PT} \times \frac{1}{r^4}£ + \frac{P^{v}T^{v}}{PT} \times \frac{1}{r^5}£$ + &c. continued to the utmost extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser and his said second, or younger, companion, whose ages N and L correspond to the numbers P and T in the said table.

And in like manner the sixth of the said seven sets of quantities, to wit, $\frac{Q'T'}{QT} \times \frac{1}{r}£ + \frac{Q''T''}{QT} \times \frac{1}{r^2}£ + \frac{Q'''T'''}{QT} \times \frac{1}{r^3}£ + \frac{Q^{iv}T^{iv}}{QT} \times \frac{1}{r^4}£ + \frac{Q^{v}T^{v}}{QT} \times \frac{1}{r^5}£$ + &c. continued to the utmost extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser's two companions, whose ages M and L correspond to the numbers Q and T in the said table.

And, lastly, it appears from Prob. 3d, Coroll. 7, Art. 56, that the seventh, or last, of the said seven sets of quantities, to wit, $\frac{P'Q'T'}{PQT} \times \frac{1}{r}£ + \frac{P''Q''T''}{PQT} \times \frac{1}{r^2}£ + \frac{P'''Q'''T'''}{PQT} \times \frac{1}{r^3}£ + \frac{P^{iv}Q^{iv}T^{iv}}{PQT} \times \frac{1}{r^4}£ + \frac{P^{v}Q^{v}T^{v}}{PQT} \times \frac{1}{r^5}£$ + &c. continued to the utmost extent of human life, is the value of an annuity of one pound for the joint lives of the said purchaser and his two companions, whose ages N, M, and L correspond to the numbers P, Q, and T.

Therefore

Therefore the value of an annuity of one pound for the three lives of the said purchaser and his two companions, and the life of the longest liver of them, is equal to the excess of the sum of the values of three separate life-annuities of one pound each for the single lives of the said purchaser and his two companions, together with the value of an annuity of the same sum for the joint lives of all the three, above the sum of the three values of a like annuity for the joint lives of the said purchaser and his first, or older, companion, and for the joint lives of the said purchaser and his second, or younger, companion, and for the joint lives of both his said companions. Q.E.D.

Short and general Expressions for the Values of the Annuities that are the Subject of the foregoing Problems.

LXXVII. Having now explained, in as full and clear a manner as I was able, the grounds and reasons of the methods above delivered for the calculation of the values of annuities, so far as relates to annuities for given numbers of years, not depending upon the continuance of any life, or upon any other contingent event, and to annuities for given numbers of years depending upon the continuance of a single life of a given age, or upon the joint continuance of two, or of three, lives of given ages, or upon the continuance of any one of two, or of three, lives of given ages, the interest of money, in all these cases, being likewise given; it will now be convenient to recapitulate the substance of the conclusions that have been obtained in the foregoing articles, by exhibiting short and general expressions of the values of these several kinds of annuities, to which the reader, whenever he shall have occasion to compute any of these annuities, may readily have recourse. This may be done in the manner following.

A short

A short expression of the value of an annuity of one pound per annu for a given number of years, not depending on the continuance of any life or on any other uncertain event, the interest of money being also given.

LXXVIII. Let £ r denote the value of one pound sterling together with its interest for one year at any given rate of interest: and let n be the number of years during which an annuity of one pound per annum is to continue.

Then by Art. 2, pages 21 and 22, it is evident that the present value of this annuity will be $= £1 \times$ the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. continued to the term $\frac{1}{r^n}$, or to n terms.

A method of obtaining another and still shorter expression of the same value.

LXXIX. As this series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. is a geometrical progression, the sum of its terms may be found in a single short expression without taking the trouble of computing them all separately, and then adding them into one sum total. This may be done by means of the following Lemma.

A LEMMA.

LXXX. The sum of the terms of every decreasing geometrical progression is equal to the quotient that arises by dividing the excess of the square of its first, or greatest, term above the product, or rectangle under its second term and its least term, by the excess of its first, or greatest, term above its second term. Of the sum of the terms of a decreasing geometrical progression.

For, if A, B, C, D, E, are a series of terms in geometrical proportion, we shall have $A : B :: B : C$, and $A : B :: C : D$, and $A : B :: D : E$.

Therefore, by El. 5, 12, the sum of all the antecedents A, B, C, D, will be to the sum of all the consequents B, C, D, E, as the first antecedent A is to the first consequent B. But the sum of all the antecedents A, B, C, D, is the sum of all the terms of the series except the last term E;

and

and the ſum of all the conſequents B, C, D, E, is the ſum of all the terms of the ſeries except the firſt term A. Therefore, if the ſum of all the terms of the ſeries be called S, we ſhall have $S-E : S-A :: A : B$. Therefore $B \times \overline{S-E}$ will be $= A \times \overline{S-A}$, or $BS - BE$ will be $= AS - AA$. Therefore, adding AA to both ſides, we ſhall have $AA - BE + BS = AS$; and, ſubtracting BS (which is leſs than AS, becauſe B is leſs than A, and which therefore is alſo leſs than the other ſide of the equation, to wit, $AA - BE + BS$) from both ſides, we ſhall have $AA - BE = AS - BS = S \times \overline{A-B}$. Therefore S, or the ſum of all the terms, A, B, C, D, E, will be equal to $\frac{AA-BE}{A-B}$, that is, to the quotient that ariſes by dividing the exceſs of AA, the ſquare of the firſt, or greateſt, term A, above BE, the product, or rectangle under the ſecond term B and the leaſt term E, by the exceſs of the firſt, or greateſt, term A, above the ſecond term B. And it is eaſy to ſee that the ſame reaſonings will take place, and conſequently that the ſame concluſion will follow from them, if the ſeries A, B, C, D, &c. ſhould conſiſt of any other number of terms whatever, as well as when it conſiſts of five terms. Q E D.

LXXXI. Now in the ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^n}$, $\frac{1}{r}$ is $= A$, and $\frac{1}{rr}$ is $= B$, and $\frac{1}{r^n}$ is $= E$. Therefore AA is $= \frac{1}{rr}$, and BE is $= \frac{1}{rr} \times \frac{1}{r^n} = \frac{1}{r^{n+2}}$, and conſequently $AA - BE$ is $= \frac{1}{rr} - \frac{1}{r^{n+2}} = \frac{r^{n+2} - r^2}{r^{n+4}} = \frac{r^n - 1}{r^{n+2}}$. And $A - B$ is $= \frac{1}{r} - \frac{1}{r^2} = \frac{r^2 - r}{r^3} = \frac{r-1}{rr}$. And conſequently $\frac{AA-BE}{A-B}$ is $= \frac{\frac{r^n-1}{r^{n+2}}}{\frac{r-1}{r^2}} = \frac{r^n-1}{r^{n+2}} \times \frac{r^2}{r-1} =$

$$\frac{r^n - 1}{r^n \times \overline{r-1}} = \frac{r^n}{r^n \times \overline{r-1}} - \frac{1}{r^n \times \overline{r-1}} = \frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}.$$

A ſecond, and very ſhort, expreſſion of the value of an annuity of one pound a year for a given number of years certain.

Therefore the ſum of all the terms of the ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. continued to n terms, is $= \frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}$, and conſequently the value of an annuity of one pound per annum for n years is $= £1 \times \overline{\frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}}$. Q E I.

LXXXII. I ſhall

LXXXII. I shall now proceed to give an example of the computation of the value of an annuity of one pound for a given number of years by means of both these expressions, that is, the expression $\frac{£}{1}$ × the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^n}$ and the expression $\frac{£}{1} \times \overline{\frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}}$.

An example of the computation of the value of an annuity of one pound for a given number of years by means of the expression $\frac{£}{1}$ × *the series* $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^n}$.

LXXXIII. Let n, or the given number of years during which the annuity is to continue, be 30 years, and let the rate of the interest of money be $3\frac{1}{2}$ per cent.

Then will r be = 1.035, and $\frac{1}{r}$ will be = $\frac{1}{1.035}$ = .966,184. And

$\frac{1}{r^2}$ will be = .933,511, and $\frac{1}{r^3}$ = .901,943,

and $\frac{1}{r^4}$ = .871,442, and $\frac{1}{r^5}$ = .841,973,

and $\frac{1}{r^6}$ = .813,501, and $\frac{1}{r^7}$ = .785,991,

and $\frac{1}{r^8}$ = .759,412, and $\frac{1}{r^9}$ = .733,731,

and $\frac{1}{r^{10}}$ = .708,919, and $\frac{1}{r^{11}}$ = .[illegible],946,

and $\frac{1}{r^{12}}$ = .661,783, and $\frac{1}{r^{13}}$ = .639,404,

and $\frac{1}{r^{14}}$ = .617,782, and $\frac{1}{r^{15}}$ = .596,891,

and

and $\frac{1}{r^{16}}$ will be = .576,706, and $\frac{1}{r^{17}}$ = .557,204,

and $\frac{1}{r^{18}}$ = .538,361, and $\frac{1}{r^{19}}$ = .520,156,

and $\frac{1}{r^{20}}$ = .502,566, and $\frac{1}{r^{21}}$ = .485,571,

and $\frac{1}{r^{22}}$ = .469,151, and $\frac{1}{r^{23}}$ = .453,286,

and $\frac{1}{r^{24}}$ = .437,957, and $\frac{1}{r^{25}}$ = .423,147,

and $\frac{1}{r^{26}}$ = .408,838, and $\frac{1}{r^{27}}$ = .395,012,

and $\frac{1}{r^{28}}$ = .381,654, and $\frac{1}{r^{29}}$ = .368,748,

and $\frac{1}{r^{30}}$ = .356,278. Therefore the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \frac{1}{r^8} + \frac{1}{r^9} + \frac{1}{r^{10}} + \frac{1}{r^{11}} + \frac{1}{r^{12}} + \frac{1}{r^{13}} + \frac{1}{r^{14}} + \frac{1}{r^{15}} + \frac{1}{r^{16}} + \frac{1}{r^{17}} + \frac{1}{r^{18}} + \frac{1}{r^{19}} + \frac{1}{r^{20}} + \frac{1}{r^{21}} + \frac{1}{r^{22}} + \frac{1}{r^{23}} + \frac{1}{r^{24}} + \frac{1}{r^{25}} + \frac{1}{r^{26}} + \frac{1}{r^{27}} + \frac{1}{r^{28}} + \frac{1}{r^{29}} + \frac{1}{r^{30}}$ is =

.966,184,	+ .933,511,	+ .901,943,	+ .871,442,
+ .841,973,	+ .813,501,	+ .785,991,	+ .759,412,
+ .733,731,	+ .708,919,	+ .684,946,	+ .661,783,
+ .639,404,	+ .617,782,	+ .596,891,	+ .576,706,
+ .557,204,	+ .538,361,	+ .520,156,	+ .502,566,
+ .485,571,	+ .469,151,	+ .453,286,	+ .437,957,
+ .423,147,	+ .408,838,	+ .395,012,	+ .381,654,
+ .368,748,	+ .356,278	=	
+ 5.015,962,	+ 4.846,341,	+ 4.338,225,	+ 4.191,520,

= 18.392,048; and consequently the value of an annuity of one pound a year for 30 years certain, when the interest of money is $3\frac{1}{2}$ per cent. is = £1 × 18.392,048, or £18.392,048, or 18*l.* 7*s.* 10*d.*$\frac{1}{2}$. Q.E.I.

A com-

A computation of the same annuity for 30 *years by means of the expression* $£1 \times \overline{\frac{1}{r-1} - \frac{1}{r^n \times r-1}}$.

LXXXIV. Here r is $= 1.035$; and consequently $\frac{1}{r}$ is $= \frac{1}{1.035} =$ .966,184; and $\frac{1}{r^n}$ is $= \frac{1}{r^{30}} =$ .356,278; and $r-1$ is $= .035 = \frac{35}{1000}$; and $\frac{1}{r-1}$ is $= \frac{1}{\frac{35}{1000}} = 1 \times \frac{1000}{35} =$ 28.571,428; and $\frac{1}{r^n \times r-1}$, or $\frac{1}{r^n} \times \frac{1}{r-1}$, is $=$ .356,278 $\times \frac{1000}{35} = \frac{356.278}{35} =$ 10.179,37. Therefore $\frac{1}{r-1} - \frac{1}{r^n \times r-1}$ is $=$ 28.571,42 $-$ 10.179,37 $=$ 18.392,05. Therefore the value of the proposed annuity of one pound per annum for 30 years, when the interest of money is $3\frac{1}{2}$ per cent. is £1 × 18.392,05, or £18.392,05, or 18*l.* 7*s.* 10*d.* $\frac{1}{2}$, as before. Q E I.

LXXXV. Note. A table of the values of annuities of this kind, shewing the present value of an annuity of one pound a year for any number of years certain, not exceeding 90 years, when the interest of money is at 5, 4, and 3, per cent. has been given us by the late learned Mr. Thomas Simpson, of Woolwich, in the 114th and 115th pages of his comprehensive little treatise, called, "*The doctrine of annuities and reversions, &c.*" in small octavo, which was published in the year 1742. And another table of the same kind had been published before by Mr. John Smart in the year 1726 in his very useful work above-mentioned in Art. 25, intitled "*Tables of Interest, &c.*" This latter table is computed to eight places of decimal fractions; and for the twelve different rates of interest mentioned above in Art. 24, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. and for every number of years, and even half-years, in the space of an hundred years. So that nothing can be expected, or need be desired, more compleatly convenient for the purpose of estimating the values of annuities for terms of years certain, than this table. It is Mr. Smart's fourth table of compound interest, pages 76, 77, 78, 79, 80, 81, 82, 83, of his valuable book above-mentioned.

A short

A short expression of the value of an annuity of one pound per annum for a given number of years, depending on the continuance of one life of a given age, the rate of the interest of money being also given.

LXXXVI. Let $\overset{\pounds}{r}$ be, as before, the sum of one pound together with its interest for one year according to the given rate of interest. And let N be the number of years in the age of the person on whose life the annuity is to depend, and E be the greatest number of years through which it is supposed to be possible for human life to be extended, according to the table of the probabilities of the duration of human life adopted for the calculation; which number is in Monsieur de Parcieux's table 94 years. Then will $E-N$ be the greatest number of years through which, according to such supposition, it is possible that the life of the annuitant of the given age of N years can be extended. Let n be any number of years not greater than $E-N$ years. And let the annuity of one pound per annum be granted for the term of n years, provided the said annuitant of the age of N years shall so long live, but otherwise to cease at his death. Let P be the number of persons of the said age of N years represented to be all living at the same time in Monsieur de Parcieux's table of the probabilities of the duration of human life, or in such other table of these probabilities as is thought proper by the calculator to be adopted as the ground of his calculation. And let P^{I} be the number of persons represented in the said table to be living at the age of $N+1$ years; and P^{II} the number living at the age of $N+2$ years; and P^{III} the number living at the age of $N+3$ years; and P^{IV}, P^{V}, P^{VI}, P^{VII}, P^{VIII}, P^{IX}, P^{X}, &c. the numbers living at the several following ages of $N+4$ years, $N+5$ years, $N+6$ years, $N+7$ years, $N+8$ years, $N+9$ years, $N+10$ years, &c. respectively.

Then will the present value of an annuity of one pound a year for the space of n years, in case the said life of N years shall so long continue, be equal to the following expression, to wit, $\overset{\pounds}{1} \times$ the series $\frac{P^{I}}{Pr} + \frac{P^{II}}{Pr^2} + \frac{P^{III}}{Pr^3} + \frac{P^{IV}}{Pr^4} + \frac{P^{V}}{Pr^5} + \frac{P^{VI}}{Pr^6} + \frac{P^{VII}}{Pr^7} +$ &c. continued to n terms, or to the term $\frac{P^{n}}{Pr^{n}}$, or to the expression $\frac{\overset{\pounds}{1}}{P} \times$ the series $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} +$ &c. continued to n terms, or to the term $\frac{P^{n}}{r^{n}}$. This is evident from Problem 2, and its second corollary, Art. 26, 27, 28, 29, 31, pages 24, 25, 26, 27, 28.

LXXXVII. If

LXXXVII. If n years is the greateſt number of years through which it is poſſible (according to the table of probabilities of the duration of human life adopted in the calculation,) for the given life of N years to be extended, the ſaid expreſſion $\frac{1}{P}$ × the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} +$ &c. continued to n terms, or to the term $\frac{P^n}{r^n}$, will be the value of an annuity of one pound *per annum* for the whole life of a perſon of N years of age: but, if n is leſs than the ſaid complement of N years to the utmoſt duration of human life, the ſaid expreſſion will be leſs than the value of an annuity of one pound *per annum* for the whole life of a perſon of the age of N years, and will be the value of an immediate, but imperfect, life-annuity during n years of the life of a perſon of that age. This is evident from Art. 31, 32, 33, pages 28, 29, 30, 31, 32.

An example of the computation of the value of an immediate and compleat life-annuity of one pound per annum for the whole life of a perſon of a given age, by means of the foregoing expreſſion.

LXXXVIII. Let it be required to find the value of an annuity of one pound *per annum* for the life of a man of fourſcore years of age, according to Monſieur de Parcieux's table of the probabilities of the duration of human life, and upon a ſuppoſition that the intereſt of money is 3 per cent.

Here n, or the number of years through which the annuity is to continue, in caſe the life of fourſcore years of age ſhall laſt ſo long, is the greateſt poſſible number of years through which, according to Monſieur de Parcieux's table, a life of fourſcore years of age can be extended, that is, 94—80 years, or 14 years. Therefore the ſeries $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} +$ &c. in the foregoing expreſſion muſt be continued to 14 terms; which may be computed as follows.

P is $= 118$, $P^{I} = 101$, $P^{II} = 85$, $P^{III} = 71$, $P^{IV} = 59$, $P^{V} = 48$, $P^{VI} = 38$, $P^{VII} = 29$, $P^{VIII} = 22$, $P^{IX} = 16$, $P^{X} = 11$, $P^{XI} = 7$, $P^{XII} = 4$, $P^{XIII} = 2$, and $P^{XIV} = 1$. And r is $= 1.03$, and $\frac{1}{r} = \frac{1}{1.03} = .9708$,

and

and $\frac{1}{r^2} = .9425$, $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, $\frac{1}{r^9} = .7664$, $\frac{1}{r^{10}} = .7440$, $\frac{1}{r^{11}} = .7224$, $\frac{1}{r^{12}} = .7013$, $\frac{1}{r^{13}} = .6809$, and $\frac{1}{r^{14}} = .6611$. Therefore the expression

$$\text{£}\frac{1}{P} \times \text{ the series } \frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} + \frac{P^{\text{VIII}}}{r^8} + \frac{P^{\text{IX}}}{r^9} + \frac{P^{\text{X}}}{r^{10}} + \frac{P^{\text{XI}}}{r^{11}} + \frac{P^{\text{XII}}}{r^{12}} + \frac{P^{\text{XIII}}}{r^{13}} + \frac{P^{\text{XIV}}}{r^{14}}$$

will be equal to

$$\text{£}\frac{1}{118} \times \text{ the series } 101 \times .9708 + 85 \times .9425 + 71 \times .9151 + 59 \times .8884 + 48 \times .8626 + 38 \times .8374 + 29 \times .8130 + 22 \times .7894 + 16 \times .7664 + 11 \times .7440 + 7 \times .7224 + 4 \times .7013 + 2 \times .6809 + 1 \times .6611$$

$$= \text{£}\frac{1}{118} \times \text{ the series } 98.0508 + 80.1125 + 65.0721 + 52.4156 + 41.4048 + 31.8212 + 23.5770 + 17.3668 + 12.2624 + 8.1840 + 5.0568 + 2.8052 + 1.3618 + .6611$$

$$= \text{£}\frac{1}{118} \times 440.1521 = \text{£}1 \times \frac{440.1521}{118} = \text{£}1 \times 3.7301 = \text{£}3.7301 = \text{£}3,\ 14s.,\ 7\tfrac{1}{2}d.$$

Consequently the value of an annuity of one pound per annum for the whole life of a man of fourscore years of age, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and when the interest of money is 3 per cent. is 3*l.* 14*s.* 7*d.*½. Q. E. I.

An example of the computation of the value of an immediate, but imperfect, life-annuity, depending on the life of a person of a given age, by means of the same expression.

LXXXIX. Let it be required to find the value of an annuity of one pound *per annum* for the first five years of the life of a man of fourscore years of age, according to the same table of the probabilities of the duration of human life and the same rate of interest as before.

For

For this purpose we need only take the first five terms of the foregoing series $\frac{P^{I}}{r} + \frac{P^{II}}{r^{2}} + \frac{P^{III}}{r^{3}} + \frac{P^{IV}}{r^{4}} +$ &c. and multiply their sum into the fraction $\frac{£1}{P}$; and the product will be the value of the proposed annuity. These terms are $101 \times .9708 + 85 \times .9425 + 71 \times .9151 + 59 \times .8884 + 48 \times .8626$, which are equal to $98.0508 + 80.1125 + 65.0721 + 52.4156 + 41.4048 = 337.0558$; which, being multiplied into the fraction $\frac{£1}{P}$, or $\frac{£1}{118}$, is $= £1 \times \frac{337.0558}{118} = £1 \times 2.8564 = £2.8564 = 2l.\ 17s.\ 1d.\frac{1}{2}$. Therefore the value of an annuity of one pound *per annum* for the first five years of the life of a man of fourscore years of age, according to the foregoing suppositions of the duration of human life and the interest of money, is $2l.\ 17s.\ 1d.\frac{1}{2}$. Q E I.

XC. And, if the annuity to be purchased is to be a remote one, or to be paid at the distance of more than one year, a short expression of its value, similar to the foregoing expressions, may be found as follows.

A short and general expression of the value of a remote annuity of one pound per annum *for a given number of years, depending on a life of a given age, when the interest of money is also given.*

Let r be, as before, the value of one pound together with its interest for a year at the given rate of the interest of money. And let m be the number of years at the end of which the annuity is to commence, so that the first payment of it shall be made at the end of $m+1$ years. And let N be the number of years in the age of the purchaser of the annuity at the time of purchasing it; and let E, as before, be the whole number of years through which it is possible, according to the table of probabilities adopted in the calculation, for human life to be extended; which in Monsieur de Parcieux's table is 94 years. Then will $N+m$ be the number of years in the age of the said purchaser at the time of the commencement of the said annuity, and $N+m+1$ the number of years in his age at the time when the first payment of the said annuity will become due; and $E-\overline{N+m}$, or $E-N-m$, will be the greatest possible number of years through which the life of the said purchaser can be extended after he shall have attained the age of $N+m$ years and the annuity shall have commenced. Let n be any number of years not greater than $E-N-m$; and let P^{m+1}

denote

denote the number of persons of the age of $N+m+1$ years represented, in the table of probabilities of life adopted in the calculation, to be all living at the same time; and P^{m+II} the number of persons represented in the said table to be living at the age of $N+m+2$ years; and P^{m+III} the number of persons represented to be living at the age of $N+m+3$ years; and P^{m+IV} the number of persons living at the age of $N+m+4$ years; and P^{m+V} the number of persons living at the age of $N+m+5$ years; and so on for all the following ages in the table.

Then will the value of an annuity of one pound per annum to commence at the distance of m years, (so that the first payment of it shall be made at the end of $m+1$ years,) and to continue during n years, provided a person aged N years at the time of the purchase shall so long live, but to cease as soon as such person shall be dead, be equal to the following expression, to wit, £ $\frac{1}{P} \times$ the series $\frac{P^{m+I}}{r^{m+1}} + \frac{P^{m+II}}{r^{m+2}} + \frac{P^{m+III}}{r^{m+3}} + \frac{P^{m+IV}}{r^{m+4}} + \frac{P^{m+V}}{r^{m+5}} + \frac{P^{m+VI}}{r^{m+6}} + \frac{P^{m+VII}}{r^{m+7}} +$ &c. continued to n terms, or to the term $\frac{P^{m+n}}{r^{m+n}}$. This is evident from Art. 36 and 37, pages 33, 34.

An example of the computation of the value of a remote annuity, depending upon a life of a given age, by means of the foregoing expression.

XCI. Let it be required to find the value of an annuity of one pound *per annum* for the life of a person of fourscore years of age, but which shall not commence till five years after the purchase of it, so that the first payment of it shall be made to the said person of fourscore years of age at the end of six years, or when he shall be fourscore and six years of age, if he shall be then living, and which shall continue during the whole remainder of the life of the said purchaser; the interest of money being 3 per cent. (as in the last example,) and the probabilities of the duration of human life such as they are represented to be in Monsieur de Parcieux's table.

Here N, or the number of years in the age of the purchaser of the annuity, is 80; and m, the number of years before the annuity is to commence, is 5; and consequently $N+m$, or the number of years in the age of the annuitant at the time the annuity is to commence, is 85. E, the greatest number of years through which human life can be extended, is, according to Monsieur de Parcieux's table of probabilities, 94 years; and consequently $E-N-m$, or the greatest number of years through which it is is possible that the life of the proposed annuitant can be extended after the annuity shall have commenced, is 94 — 80 — 5, or 94 — 85, or 9, years. And, n, or the number of years during which it is possible the annuity may continue, will consequently be equal to 9 years, because the annuity, when once it has taken place, is supposed to continue during the whole remaining part of the annuitant's life; and consequently the series $\frac{P_{m+\mathrm{I}}}{r^{m+1}} + \frac{P_{m+\mathrm{II}}}{r^{m+2}} + \frac{P_{m+\mathrm{III}}}{r^{m+3}} + \frac{P_{m+\mathrm{IV}}}{r^{m+4}} + \frac{P_{m+\mathrm{V}}}{r^{m+5}}$ + &c. will consist of nine terms. These terms may be computed as follows.

Since m is $= 5$, we shall have $m + \mathrm{I} = \mathrm{VI}$, and $m + \mathrm{II} = \mathrm{VII}$, and $m + \mathrm{III} = \mathrm{VIII}$, and $m + \mathrm{IV} = \mathrm{IX}$, and $m + \mathrm{V} = \mathrm{X}$, and $m + \mathrm{VI} = \mathrm{XI}$, and $m + \mathrm{VII} = \mathrm{XII}$, and $m + \mathrm{VIII} = \mathrm{XIII}$, and $m + \mathrm{IX} = \mathrm{XIV}$; and, in like manner, $m + 1 = 6$, and $m + 2 = 7$, and $m + 3 = 8$, and $m + 4 = 9$, and $m + 5 = 10$, and $m + 6 = 11$, and $m + 7 = 12$, and $m + 8 = 13$, and $m + 9 = 14$. Therefore the series $\frac{P_{m+\mathrm{I}}}{r^{m+1}} + \frac{P_{m+\mathrm{II}}}{r^{m+2}} + \frac{P_{m+\mathrm{III}}}{r^{m+3}} + \frac{P_{m+\mathrm{IV}}}{r^{m+4}} + \frac{P_{m+\mathrm{V}}}{r^{m+5}} + \frac{P_{m+\mathrm{VI}}}{r^{m+6}} + \frac{P_{m+\mathrm{VII}}}{r^{m+7}} + \frac{P_{m+\mathrm{VIII}}}{r^{m+8}} + \frac{P_{m+\mathrm{IX}}}{r^{m+9}}$ is $= \frac{P_{\mathrm{VI}}}{r^6} + \frac{P_{\mathrm{VII}}}{r^7} + \frac{P_{\mathrm{VIII}}}{r^8} + \frac{P_{\mathrm{IX}}}{r^9} + \frac{P_{\mathrm{X}}}{r^{10}} + \frac{P_{\mathrm{XI}}}{r^{11}} + \frac{P_{\mathrm{XII}}}{r^{12}} + \frac{P_{\mathrm{XIII}}}{r^{13}} + \frac{P_{\mathrm{XIV}}}{r^{14}}$.

But r is, as before, $= 1.03$; and consequently $\frac{1}{r}$ is $= \frac{1}{1.03} = .9708$, and $\frac{1}{r^6}$ is $= .8374$, and $\frac{1}{r^7}$ is $= .8130$, and $\frac{1}{r^8}$ is $= .7894$, and $\frac{1}{r^9}$ is $= .7664$, and $\frac{1}{r^{10}}$ is $= .7440$, and $\frac{1}{r^{11}}$ is $= .7224$, and $\frac{1}{r^{12}}$ is $= .7013$, and $\frac{1}{r^{13}}$ is $= .6809$, and $\frac{1}{r^{14}}$ is $= .6611$. [illegible] P is $= 118$, and $P_{\mathrm{VI}} = 38$, and $P_{\mathrm{VII}} = 29$, and $P_{\mathrm{VIII}} = 22$, and $P_{\mathrm{IX}} = 16$, and $P_{\mathrm{X}} = 11$, and $P_{\mathrm{XI}} = 7$, and $P_{\mathrm{XII}} = 4$, and $P_{\mathrm{XIII}} = 2$, and $P_{\mathrm{XIV}} = 1$. Therefore the expression $\frac{\pounds 1}{P} \times$ the series $\frac{P_{\mathrm{VI}}}{r^6} + \frac{P_{\mathrm{VII}}}{r^7} + \frac{P_{\mathrm{VIII}}}{r^8} + \frac{P_{\mathrm{IX}}}{r^9} + \frac{P_{\mathrm{X}}}{r^{10}} + \frac{P_{\mathrm{XI}}}{r^{11}} + \frac{P_{\mathrm{XII}}}{r^{12}} + \frac{P_{\mathrm{XIII}}}{r^{13}} + \frac{P_{\mathrm{XIV}}}{r^{14}}$ is equal to $\frac{\pounds 1}{118} \times$ the series $38 \times .8374$

$+ 29 \times .8130$

$+ 29 \times .8130 + 22 \times .7894 + 16 \times .7664 + 11 \times .7440 + 7 \times .7224 + 4 \times .7013 + 2 \times .6809 + 1 \times .6611 = \frac{\pounds 1}{118} \times$ the series $31.8212 + 23.5770 + 17.3668 + 12.2624 + 8.1840 + 5.0568 + 2.8052 + 1.3618 + .6611 = \frac{\pounds 1}{118} \times 103.0963 = \pounds 1 \times \frac{103.0963}{118} = \pounds 1 \times .8736$ $= \pounds .8736 = 17s.\ 5d.\frac{1}{2}$. Therefore $17s.\ 5d.\frac{1}{2}$ is the value of an annuity of one pound *per annum* for the life of a man of fourscore years of age, to commence at the distance of five years, or when he shall be fourscore and five years old, so that the first payment of it shall be made to him when he shall be 86 years old, according to Monsieur de Parcieux's table of the probabilities of life and when the interest of money is 3 per cent. Q E I.

XCII. N. B. This value of this annuity is a little greater than that which is found for it above in Art. 37, page 34, which is $17s.\ 3d.\frac{1}{2}$. The reason of the difference is a small mistake which I have discovered to have been made in one of the arithmetical operations in Art. 32, page 30. The quotient of the division of 31.8212 by 118 is there made to be .2612, whereas it ought to be .2696, which is greater than .2612 by .0084. This difference .0084, being added to the number .8650 in Art. 37, page 34, will make it equal to .8734, which agrees in its three highest figures with the number .8736, just now found for the value of this annuity. The discovery of this mistake, if I had made it sooner, would have prevented the insertion of the note at the end of Art. 33, page 32, which now appears to be ill-grounded.

XCIII. The examples that have been given, in the several foregoing articles, of the computation of the values of life-annuities, both immediate and remote, are the same which were given above in the several articles that come after the solution of Problem 11. The reason of repeating them in this place was that they might serve to illustrate the short and general expressions of those values which have been set forth in this latter part of the present tract, and might thereby enable the reader, if he chose it, to become more familiarly acquainted with the practice of computing the values of life-annuities by means of those expressions. With the same view I shall here subjoin another example of the computation of the values of life-annuities by means of those general expressions, and shall chuse for that purpose a younger life than those in the foregoing examples. This will occasion a long and tedious calculation, which would, I thought, have too much interrupted the chain of the reasonings used in Problem 11 and its corollaries,

corollaries, which are intended to explain the theory, or the grounds and reasons, of these computations; but in this part of the work, which is intended to illustrate the practice of these computations, I conceive that the exhibition of such a calculation may be of use.

A calculation of the value of an annuity of one pound sterling for the life of a person of 10 years of age; upon a supposition that the interest of money is 3 and an half per cent. and that the probabilities of the duration of human life are such as they are represented to be in Monsieur de Parcieux's table.

XCIV. The number of persons of the age of 10 years, represented in Monsieur de Parcieux's table to be all living at the same time, is 880; that is, according to the notation of Art. 86, P is $= 880$. And it appears by the same table that the number living at 11 years of age is 872, and the number living at 12 years of age is 866, and those living at 13, 14, 15, and all the following years of age up to 94 years, are 860, 854, 848, 842, 835, 828, 821, 814, 806, 798, 790, 782, 774, 766, 758, 750, 742, 734, 726, 718, 710, 702, 694, 686, 678, 671, 664, 657, 650, 643, 636, 629, 622, 615, 607, 599, 590, 581, 571, 560, 549, 538, 526, 514, 502, 489, 476, 463, 450, 437, 423, 409, 395, 380, 364, 347, 329, 310, 291, 271, 251, 231, 211, 192, 173, 154, 136, 118, 101, 85, 71, 59, 48, 38, 29, 22, 16, 11, 7, 4, 2, and 1; that is, according to the notation of Art. 86, P^{I} is $= 872$, and P^{II} is $= 866$, and P^{III}, P^{IV}, P^{V}, &c. are equal to 860, 854, 848, and the other numbers just now mentioned, respectively.

And it appears by Mr Smart's second table of Compound Interest, pages 60—67, that, when the interest of money is $3\frac{1}{2}$ per cent. r is $= 1.035$, and $\frac{1}{r} = \frac{1}{1.035} = .96618$, and $\frac{1}{r^2} = .93351$, and $\frac{1}{r^3} = .90194$, and $\frac{1}{r^4} = .87144$, and $\frac{1}{r^5} = .84197$, and $\frac{1}{r^6} = .81350$, and $\frac{1}{r^7} = .78599$, and $\frac{1}{r^8} = .75941$, and $\frac{1}{r^9} = .73373$, and $\frac{1}{r^{10}} = .70891$, and $\frac{1}{r^{11}} = .68494$, and $\frac{1}{r^{12}} = .66178$, and $\frac{1}{r^{13}} = .63940$, and $\frac{1}{r^{14}} = .61778$, and $\frac{1}{r^{15}} = .59689$, and $\frac{1}{r^{16}} = .57670$, and $\frac{1}{r^{17}} = .55720$, and $\frac{1}{r^{18}} = .53836$,

$= .53836$, and $\frac{1}{r^{19}} = .52015$, and $\frac{1}{r^{20}} = .50256$, and $\frac{1}{r^{21}} = .48557$, and $\frac{1}{r^{22}} = .46915$, and $\frac{1}{r^{23}} = .45328$, and $\frac{1}{r^{24}} = .43795$, and $\frac{1}{r^{25}} = .42314$, and $\frac{1}{r^{26}} = .40883$, and $\frac{1}{r^{27}} = .39501$, and $\frac{1}{r^{28}} = .38165$, and $\frac{1}{r^{29}} = .36874$, and $\frac{1}{r^{30}} = .35627$, and $\frac{1}{r^{31}} = .34423$, and $\frac{1}{r^{32}} = .33258$, and $\frac{1}{r^{33}} = .32134$, and $\frac{1}{r^{34}} = .31047$, and $\frac{1}{r^{35}} = .29997$, and $\frac{1}{r^{36}} = .28983$, and $\frac{1}{r^{37}} = .28003$, and $\frac{1}{r^{38}} = .27056$, and $\frac{1}{r^{39}} = .26141$, and $\frac{1}{r^{40}} = .25257$, and $\frac{1}{r^{41}} = .24403$, and $\frac{1}{r^{42}} = .23577$, and $\frac{1}{r^{43}} = 22780$, and $\frac{1}{r^{44}} = .22010$, and $\frac{1}{r^{45}} = .21265$, and $\frac{1}{r^{46}} = .20546$, and $\frac{1}{r^{47}} = .19851$, and $\frac{1}{r^{48}} = .19180$, and $\frac{1}{r^{49}} = .18532$, and $\frac{1}{r^{50}} = .17905$, and $\frac{1}{r^{51}} = .17299$, and $\frac{1}{r^{52}} = .16714$, and $\frac{1}{r^{53}} = .16149$, and $\frac{1}{r^{54}} = .15603$, and $\frac{1}{r^{55}} = .15075$, and $\frac{1}{r^{56}} = .14566$, and $\frac{1}{r^{57}} = .14073$, and $\frac{1}{r^{58}} = .13597$, and $\frac{1}{r^{59}} = .13137$, and $\frac{1}{r^{60}} = .12693$, and $\frac{1}{r^{61}} = .12264$, and $\frac{1}{r^{62}} = .11849$, and $\frac{1}{r^{63}} = .11448$, and $\frac{1}{r^{64}} = 11061$, and $\frac{1}{r^{65}} = .10687$, and $\frac{1}{r^{66}} = .10326$, and $\frac{1}{r^{67}} = .09976$, and $\frac{1}{r^{68}} = .09639$, and $\frac{1}{r^{69}} = .09313$, and $\frac{1}{r^{70}} = .08998$, and $\frac{1}{r^{71}} = .08694$, and $\frac{1}{r^{72}} = .08400$, and $\frac{1}{r^{73}} = .08116$, and $\frac{1}{r^{74}} = .07841$, and $\frac{1}{r^{75}} = .07576$, and $\frac{1}{r^{76}} = .07320$, and $\frac{1}{r^{77}} = .07072$, and $\frac{1}{r^{78}} = .06833$. and $\frac{1}{r^{79}} = .06602$, and $\frac{1}{r^{80}} = .06379$, and $\frac{1}{r^{81}} = .06163$,

= .06163, and $\frac{1}{r^{82}}$ = .05955, and $\frac{1}{r^{83}}$ = .05753, and $\frac{1}{r^{84}}$ = .05559.

Therefore the expression $\frac{£1}{P}$ × the series $\frac{P^{I}}{r} + \frac{P^{II}}{r^{2}} + \frac{P^{III}}{r^{3}} + \frac{P^{IV}}{r^{4}} + \frac{P^{V}}{r^{5}} + \frac{P^{VI}}{r^{6}} + \frac{P^{VII}}{r^{7}}$ + &c. to $\frac{P^{LXXXIV}}{r^{84}}$ is in this case equal to $\frac{£1}{880}$ × the series 872 × .96618 + 866 × .93351 + 860 × .90194
+ 854 × .87144 + 848 × .84197 + 842 × .81350 + 835 × .78599
+ 828 × .75941 + 821 × .73373 + 814 × .70891 + 806 × .68494
+ 798 × .66178 + 790 × .63940 + 782 × .61778 + 774 × .59689
+ 766 × .57670 + 758 × .55720 + 750 × .53836 + 742 × .52015
+ 734 × .50256 + 726 × .48557 + 718 × .46915 + 710 × .45328
+ 702 × .43795 + 694 × .42314 + 686 × .40883 + 678 × .39501
+ 671 × .38165 + 664 × .36874 + 657 × .25627 + 650 × .34423
+ 643 × .33258 + 636 × .32134 + 629 × .31047 + 622 × .29997
+ 615 × .28983 + 607 × .28003 + 599 × .27056 + 590 × .26141
+ 581 × .25257 + 571 × .24403 + 560 × .23577 + 549 × .22780
+ 538 × .22010 + 526 × .21265 + 514 × .20546 + 502 × .19851
+ 489 × .19180 + 476 × .18532 + 463 × .17905 + 450 × .17299
+ 437 × .16714 + 423 × .16149 + 409 × .15603 + 395 × .15075
+ 380 × .14566 + 364 × .14073 + 347 × .13597 + 329 × .13137
+ 310 × .12693 + 291 × .12264 + 271 × .11849 + 251 × .11448
+ 231 × .11061 + 211 × .10687 + 192 × .10326 + 173 × .09976
+ 154 × .09639 + 136 × .09313 + 118 × .08998 + 101 × .08694
+ 85 × .08400 + 71 × .08116 + 59 × .07841 + 48 × .07576
+ 38 × .07320 + 29 × 07072 + 22 × .06833 + 16 × .06602
+ 11 × .06379 + 7 × .06163 + 4 × .05955 + 2 × .05753
+ 1 × .05559 = $\frac{£1}{880}$ × the series 842.50896 + 808.41966
+ 775.66840 + 744.20976 + 713 99056 + 684.96700
+ 656.30165 + 628.79148 + 602.39[illegible]33 + 577.05274
+ 552.06164 + 528.10044 + 505.12600 + 483.10396
+ 461.99286 + 441.75220 + 422.35760 + 403.77000
+ 385.95130 + 368.87904 + 352.52382 + 336.84970
+ 321.82880

+ 321.82880	+ 307.44090	+ 293.65916	+ 280.45738
+ 267.81678	+ 256.08715	+ 244.84336	+ 234.06939
+ 223.74950	+ 213.84894	+ 204.37224	+ 195.28563
+ 186.58134	+ 178.24545	+ 169.97821	+ 162.06544
+ 154.23190	+ 146.74317	+ 139.34113	+ 132.03120
+ 125.06220	+ 118.41380	+ 111.85390	+ 105.60644
+ 99.65202	+ 93.79020	+ 88.21232	+ 82.90015
+ 77.84550	+ 73.04018	+ 68.31027	+ 63.81627
+ 59.54625	+ 55.35080	+ 51.22572	+ 47.18159
+ 43.22073	+ 39.34830	+ 35.68824	+ 32.11073
+ 28.73448	+ 25.55091	+ 22.54957	+ 19.82592
+ 17.25848	+ 14.84406	+ 12.66568	+ 10.61764
+ 8.78094	+ 7.14000	+ 5.76236	+ 4.62619
+ 3.63628	+ 2.78160	+ 2.05088	+ 1.50326
+ 1.05632	+ .70169	+ .43141	+ .23820

$+ .11506 \quad + .05559 = \frac{£1}{880} \times 18250.54036 = £1 \times \frac{18250.54036}{880} = £1 \times 20.73925 = £20.73925$, or $20l.\ 14s.\ 9d.\frac{1}{4}$. Therefore the value of an annuity of one pound *per annum* for the life of a perſon of the age of 10 years, when the intereſt of money is $3\frac{1}{2}$ per cent. is, according to Monſieur de Parcieux's table of probabilities, $20l.\ 14s.\ 9d.\frac{1}{4}$. Q E I.

XCV. The addition of the terms of the foregoing ſeries may be performed as follows.

842.50896	552.06164	352.52382	223.74950	139.34113
808.41966	528.10044	326.84970	213.84894	132.03120
775.66840	505.12600	321.82880	204.37224	125.06220
744.20976	483.10396	307.44090	195.28563	118.41380
713.99056	461.99286	293.65916	186.58134	111.85390
684.96700	441.75220	280.45738	178.24545	105.60644
656.30165	422.35760	267.81678	169.97821	99.65202
628.79148	403.77000	256.08715	162.06544	93.79020
602.39233	385.95130	244.84336	154.23190	88.21232
577.05274	368.87904	234.06939	146.74317	82.90015
7034.30254	4553.09504	2895.57644	1835.10182	1096.86336

77.84550	35.68824	8.78094	7034.30254
73.04018	32.11079	7.14000	4553.09504
68.31077	28.73448	5.76236	2895.57644
63.81627	25.55091	4.62619	1835.10182
59.54625	22.54957	3.63628	1096.86336
53.35080	19.82592	2.78160	576.88561
51.22572	17.25848	2.05088	219.84577
47.18159	14.84406	1.50326	38.86978
43.22073	12.66568	1.05632	18250.54036
39.34830	10.61764	.70169	
576.88561	219.84577	.43141	
		.23820	
		.11506	
		.05559	
		38.86978	

XCVI. It appears from the foregoing calculation that, when the interest of money is $3\frac{1}{2}$ per cent. an annuity for the life of a person of 10 years of age is worth near 21 years purchase: and, if the annuity, or rather in this case we should say, the *pension*, was to be paid every half-year, instead of every year, so that the first payment of it should be made at the end of half a year after the time of granting it, or when the grantee is ten years and a half old, the value of it would be greater than before by almost one half a year's payment, and consequently would be worth more than 21 years purchase. And, if we were to make the like calculation upon a supposition that the interest of money was only 3 per cent. we should find that such an annuity would be worth about 23 years purchase. But these are greater prices than, I believe, any life-annuities have ever been sold for. The real values of annuities for lives are greater than they are generally thought to be.

XCVII. It appears from the operations in the foregoing calculation, that, when the interest of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for 30 years depending on the life of a person of 10 years of age, or that shall continue for 30 years in case the said person shall so long live, but shall cease upon his death, is, according to Monsieur de Parcieux's table of probabilities of the duration of human life, equal to Of the value of an annuity for 30 years depending on the life of a person of the age of 10 years.

$$\frac{£1}{880} \times \text{the series } 7034.30254 + 4553.09504 + 2895.57644 = \frac{£1}{880} \times 14482.97402 = £1 \times \frac{14482.97402}{880} = £1 \times 16.45792 =$$

£16.45792,

Of the value of a remote annuity for the life of a person of the age of 10 years, to commence at the end of 30 years.

£16.45792, or 16*l.* 9*s.* 2*d.* And it appears likewise that the value of a remote annuity of one pound a year for the life of a person of ten years of age, to commence at the end of 30 years, (or whereof the first payment shall be made at the end of 31 years, or when the said person shall be compleatly 41 years of age,) and to continue during the whole remainder of his life, will, (upon the same suppositions of the interest of money and the probabilities of the duration of human life,) be equal to

$$\pounds\frac{1}{880} \times \text{the series } 1835.10182 + 1096.86336 + 576.88561 + 219.84577 + 38.86978 = \frac{1}{880} \times 3768.46634 = \pounds 1 \times \frac{3768.46634}{880} = \pounds 1 \times 4.28234$$

$= \pounds 4.28234$, or 4*l.* 5*s.* 7*d.* $\frac{1}{4}$. Therefore any life-annuity for the life of a person of 10 years of age, to commence at the distance of 30 years, is (upon the suppositions here made,) worth about four years and a quarter's purchase.

Of the different interests that might be made of money by vesting it in the publick funds in the years 1773 and 1780.

XCVIII. The interest that might be made of money by vesting it in the publick funds in January, 1773, was something less than $3\frac{1}{2}$ per cent. For 100 pounds stock in the consolidated 3 per cent. annuities might then be bought for 87 pounds; and, when it is worth 85*l.* 14*s.* 3*d.* $\frac{1}{2}$, or £85.7142, or, when £85.7142 will purchase a perpetual annuity of 3 pounds *per annum*, £100 will purchase a perpetual annuity of $\pounds\frac{3 \times 100}{85.7142}$, or $3\frac{1}{2}$ pounds, *per annum*. Therefore, if the Government had then wanted to borrow a moderate sum of money of the people, (as, for instance, a million or two of pounds sterling,) they could, probably, have borrowed it at the rate of $3\frac{1}{2}$ per cent. This therefore was at that time the most useful rate of interest, and the fittest to be attended to and made the ground of calculations of the values of annuities either for terms of years or for lives. But now, in October, 1780, £100 stock in the 3 per cent. consolidated annuities, or a perpetual annuity of 3 pounds *per annum*, is worth only about 60 pounds; and consequently £100, employed in the purchase of those annuities, would purchase a perpetual annuity of 5 pounds a year. Therefore 5 per cent. or, perhaps, $5\frac{1}{2}$, or even 6, per cent. is now the rate of interest which most deserves to be considered in the estimation of the value of annuities, and to be made the foundation of our calculations concerning them. In order therefore to make proper estimates of the values of annuities in different periods of national wealth and prosperity, it will be most convenient to have tables of the values of annuities adapted to

Expedience of having several different tables of life annuities adapted to different rates of interest.

to ſeveral different rates of intereſt; as we have in Mr. Smart's excellent book above-mentioned for the values of annuities for terms of years at the twelve different rates of intereſt mentioned above in Art. 24, page 23, to wit, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, 8, 9, and 10, per cent. And accordingly I ſhall inſert in the following part of this work the like tables for annuities for ſingle lives of all ages from 3 years to 93 years, at the ſame twelve different rates of intereſt and according to Monſieur de Parcieux's table of the probabilities of the duration of human life; which have all been faithfully calculated, under the inſpection and direction of the learned and worthy Dr. Price above-mentioned, upon the principles that have been above explained.

XCIX. In peruſing the foregoing calculation of the value of an annuity for the life of a perſon of 10 years of age, in Art. 94, the reader will, I doubt not, have entertained a high idea of the difficulty of compoſing a whole table of the values of life-annuities for every year of human life as far as 93 years, if the ſame method is to be taken for the computation of every one of thoſe values as is there purſued for that of an annuity for a life of the age of ten years, He will therefore, probably, be glad to be informed that the repetition of this laborious proceſs for every year of human life is not neceſſary, but that the value of an annuity for a life of any age may be deduced from the value of an annuity for a life that is one year older by a very ſhort and eaſy proceſs, inſomuch that a whole table of the values of life-annuities for every year of human life may be computed with very little more labour than has been employed above, in Art. 94, in computing the value of only that one annuity for the life of a perſon of ten years of age. The method of doing this was communicated to me by the learned Dr. Price above-mentioned, and may be explained as follows.

A ſhort and eaſy method of deducing from the value of an annuity of one pound for any given life the value of a like annuity for a life one year younger than the former.

C. We have ſeen in Art. 94 that, if the intereſt of money be $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life ſuch as they are repreſented in Monſieur de Parcieux's table, the value of an annuity of one pound for the life of a perſon of 10 years of age is £20.73925, or 20*l.* 14*s.* 9*d.*$\frac{1}{4}$. Now from this value we may derive that of a like annuity of one pound for the life of a perſon of the age of 9 years in the following manner.

An explanation of the ſaid method in the caſe of a particular example, or age of human life.

CI. In

CI. In Monſieur de Parcieux's table of probabilities there are 890 perſons repreſented to be living at the age of 9 years. Now let us ſuppoſe that a man were to make 890 grants of annuities, of one pound a year each, to all theſe 890 perſons of the age of 9 years, for their reſpective lives. And let the payments which the ſaid grantor would thereby oblige himſelf to make to the ſaid grantees be divided into the two following parts, or claſſes, to wit, firſt, the payments he is to make at the end of the firſt year to ſuch of the 890 grantees as ſhall be then alive, and, ſecondly, all the other payments which he will be obliged to make to ſuch of them as ſhall ſurvive beyond the ſaid firſt year, and which will become due at the ends of the ſecond, third, fourth, fifth, and other following years reſpectively, till all the ſaid grantees ſhall be dead. And let us inveſtigate ſeparately the prices he ought to receive from the ſaid grantees for theſe two ſets, or claſſes, of payments which he thus binds himſelf to make.

Now it appears from Monſieur de Parcieux's table that of theſe 890 grantees, all of the age of 9 years, 880 will live to the end of one year, or till they are 10 years old. Therefore at the end of the ſaid firſt year the ſaid grantor will have 880 payments, of one pound each, to make to the ſaid 880 ſurviving grantees. He ought therefore to receive, as the fair price of thoſe payments, 880 times the preſent value of one pound to be received at the end of a year, or 880 times $\frac{1}{r}$ of a pound, or 880 × £ $\frac{1}{r}$, or $\frac{1}{r} \times 880 \times$ £1.

Secondly, it is evident that the ſubſequent payments which the ſaid grantor will have to make, at the ends of the ſecond, third, fourth, fifth, and other following years, to ſuch of the ſaid 880 grantees (who are living at the end of the firſt year,) as ſhall further ſurvive to the ends of the ſaid ſecond, third, fourth, fifth, and other following years, will be preciſely the ſame as he would have had to make to them at the ſame times reſpectively if he had poſt-poned making any of theſe grants for the ſpace of a year, or till the ſaid 880 perſons were 10 years old, and had then made them 880 grants, of one pound a year each, for their reſpective lives. Thus, for example, the payment he will have to make at the end of the ſecond year, or when the grantees are 11 years old, will be in both caſes £872, becauſe there will be 872 of the ſaid grantees then alive; and the payment he will have to make at the end of the third year, or when the grantees are 12 years old, will be in both caſes £866, becauſe there will be 866 of the ſaid grantees alive at that time; and the ſame is true of the payments at the ends of all the following years, to wit, that they will be the ſame in both caſes. If therefore the prices of theſe ſeveral payments were to be paid when the ſaid grantees were 10 years old, they muſt be preciſely the ſame as if the grants had been made when the grantees were 10 years old; and

and confequently the fum total of all thefe prices would be precifely equal to the value of 880 life-annuities of one pound a year granted to 880 perfons of the age of 10 years, that is, (as appears by Art. 94) to 880 times £20.73925. But the prices of thefe feveral payments of £872, £866, &c. are fuppofed in the prefent cafe to be paid to the grantor when the grantees are only 9 years old, or a year fooner than on the laft fuppofition. Therefore (by Problem I, Art. XXIII, page 21,) they muft be lefs than thofe other prices of the fame payments, refpectively, in the proportion of $\frac{1}{r}$ to 1. Therefore the fum total of all the faid prices muft be lefs than the fum total of all thofe other prices of the fame payments in the fame propor : of $\frac{1}{r}$ to 1, and confequently muft be equal to $\frac{1}{r} \times 880 \times £20.73925$; that is, the price to be paid to the grantor by all the 890 grantees of the age of 9 years, on account of all the faid future payments of the fecond clafs, which are to be made at the ends of the fecond, third, fourth, fifth, and other following years from the time of making the grants, is $\frac{1}{r} \times 880 \times £20.73925$.

But it was before fhewn that the price to be paid to the faid grantor by the faid 890 grantees on account of the payments to be made to them at the end of the firft year, is $\frac{1}{r} \times 880 \times £1$.

Therefore the price to be paid to the faid grantor by the faid 890 grantees on both the faid accounts, or for the whole life-annuities of one pound that are thus granted to them at the age of 9 years, is $\frac{1}{r} \times 880 \times £1 + \frac{1}{r} \times 880 \times £20.73925$. And confequently the price that ought to be paid by each of the faid 890 grantees for his annuity is the 890th part of the faid fum, or $\frac{1}{r} \times \frac{880}{890} \times £1 + \frac{1}{r} \times \frac{880}{890} \times £20.73925$, or $\frac{1}{r} \times \frac{880}{890} \times \overline{£1 + £20.73925}$. Therefore by Art. 21, if only one fuch grant is made to a perfon of the age of 9 years, the price of the faid fingle annuity ought alfo to be $\frac{1}{r} \times \frac{880}{890} \times \overline{£1 + £20.73925}$; that is, (becaufe $\frac{1}{r}$ is in this cafe $= \frac{1}{1.035}$,) the price of the faid annuity will be $\frac{1}{1.035} \times \frac{880}{890} \times \overline{1£ + 20.73925£}$ $(= \frac{1}{1.035} \times \frac{880}{890} £21.73925 = \frac{1}{1.035} \times £21.494988) = £20.76810$, or 20*l.* 15*s.* 4*d.* $\frac{1}{4}$. Q.E.I.

CII. Thus

CII. Thus it appears that from the value of an annuity of one pound for a life of the age of 10 years we may, by very easy arithmetical operations, deduce that of a like annuity for a life of the age of 9 years, to wit, by only adding £1 to the former value, and then multiplying the sum thence arising, first, into the fraction $\frac{880}{890}$, (of which the numerator expresses the number of persons represented in Monsieur de Parcieux's table of probabilities to be living at the age of 10 years, and the denominator expresses the number of persons that are therein represented to be living at the age of 9 years,) and, secondly, into the fraction $\frac{1}{r}$, which bears the same proportion to 1 as the present value of one pound to be received at the end of a year bears to one pound. And, as the reasonings employed in making this deduction are not peculiar to the ages of 9 and 10 years, but may be applied to any other two ages of human life that differ from each other only by a year, we may safely conclude in general terms, that, if $\overset{£}{V}$ denotes the value of an annuity of one pound a year for the life of a person of the age of N years, and P is the number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life, (or in such other table of those probabilities as is adopted as the ground of the calculation,) as living at the age of N years, and $P+d$ is the number of persons represented there as living at the age of $N-1$ years, the value of an annuity of one pound a year for the life of a person of the age of $N-1$ years will be equal to $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$. Q.E.I.

An extension of the said method to all other ages of human life, and a general expression of the value of an annuity of one pound for the younger of the two lives.

Other examples of the foregoing method of deducing the value of an annuity for any proposed life from that of a like annuity for a life that is one year older.

CIII. Let it be supposed that the value of an annuity of one pound a year for the life of a person of 9 years of age is already known, and that it is (as we have just now found it to be by the computation of it made above in Art. 101,) equal to £20.76810; the interest of money being $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life such as they are represented in Monsieur de Parcieux's table. And let it be required to deduce from this value of an annuity of one pound for a life of a person of the age of 9 years the value of a like annuity for the life of a person of the age of 8 years, in the method above explained, or by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$.

Here

Here P is $= 890$, and $P+d$ is $= 902$, and $\overset{£}{V}$ is $=$ £20.76810, and consequently $\overline{1+V}|£$ is $=$ £21.76810; and $\frac{1}{r}$ is (as before) $= \frac{1}{1.035}$. Therefore $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$ is $= \frac{1}{1.035} \times \frac{890}{902} \times £21.76810 = \frac{1}{1.035} \times £21.47850 = £20.75217$. Consequently the value of an annuity of one pound for the life of a person of the age of 8 years is £20.75217, or 20*l.* 15*s.* ½*d.* QEI.

CIV. In like manner by putting $\overset{£}{V} = £20.75217$, and $P = 902$, and $P+d = 915$, and computing the expression $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$, or $\frac{1}{1.035} \times \frac{902}{915} \times £21.75217$, we shall find the value of an annuity of one pound for a life of 7 years to be equal to $\left(\frac{1}{1.035} \times £21.44312\text{, or}\right)$ £20.71799, or 20*l.* 14*s.* 4*d.* ¼.

CV. And in like manner an annuity of one pound for a life of the age of 6 years will be found to be worth $\frac{1}{1.035} \times \frac{915}{930} \times £21.71799$ $\left(= \frac{1}{1.035} \times £21.367,699,8\right) = £20.64512$, or 20*l.* 12*s.* 10*d.* ¼.

And an annuity of one pound for a life of the age of 5 years will be worth $\frac{1}{1.035} \times \frac{930}{948} \times £21.64512$ $\left(= \frac{1}{1.035} \times £21.23413\right) = £20.51606$, or 20*l.* 10*s.* 3*d.* ¼.

And an annuity of one pound for a life of the age of 4 years will be worth $\frac{1}{1.035} \times \frac{948}{970} \times £21.51606$ $\left(= \frac{1}{1.035} \times £21.028,06\right) =$ £20.31696, or 20*l.* 6*s.* 4*d.*

 And

And an annuity of one pound for a life of the age of 3 years will be worth $\frac{1}{1.035} \times \frac{970}{1000} \times$ £21.31696 $\left(= \frac{1}{1.035} \times \text{£}20.67745\right) =$ £19.97821, or 19*l.* 19*s.* 6*d.* $\frac{3}{4}$.

CVI. Therefore, if no miſtakes have been made in the foregoing calculations, we may conclude that the values of an annuity of one pound for the lives of children of 3, 4, 5, 6. 7, 8, 9, and 10 years of age, when the intereſt of money is $3\frac{1}{2}$ per cent. are (according to Monſieur de Parcieux's table of the probabilities of the duration of human life,) reſpectively, equal to the following ſums of money, to wit,

	£ 19.97821,	or	£ 19.	19*s.*	6*d.* $\frac{3}{4}$.
And	£ 20.31696,	or	£ 20.	6*s.*	4*d.*
And	£ 20.51606,	or	£ 20.	10*s.*	3*d.* $\frac{3}{4}$.
And	£ 20.64512,	or	£ 20.	12*s.*	10*d.* $\frac{3}{4}$.
And	£ 20.71799,	or	£ 20.	14*s.*	4*d.* $\frac{3}{4}$.
And	£ 20.75217,	or	£ 20.	15*s.*	$\frac{1}{2}$*d.*
And	£ 20.76810,	or	£ 20.	15*s.*	4*d.* $\frac{1}{2}$.
And	£ 20.73925,	or	£ 20.	14*s.*	9*d.* $\frac{1}{4}$.

CVII. All the other values of a compleat table of life-annuities (grounded upon the foregoing ſuppoſitions that the intereſt of money is $3\frac{1}{2}$ per cent. and that the life of man decays in the manner repreſented in Monſieur de Parcieux's table of probabilities,) may be computed in the ſame manner, one from another, by beginning with the oldeſt life in the table and proceeding downwards to the next younger life till we have obtained the value of an annuity of one pound for a life of 10 years of age, which we have already computed in Art. 94, and found to be £20.73925. And the trouble of computing the values of the annuities of one pound belonging to all theſe different ages will not be very much greater than that of computing, by the method uſed in Art. 94, the value of only that one annuity for the life of a perſon of the age of 10 years, which is computed in that article. The operations neceſſary for this purpoſe are as follows.

A com-

A computation of the values of an annuity of one pound for every different year of human life from the age of 93 years to the age of 10 years, inclusively; upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. and that the probabilities of the duration of human life are such as they are represented to be in Monsieur de Parcieux's table.

CVIII. It appears by Monsieur de Parcieux's table that of two persons aged 93 years, living at the same time, only one will live another year, or to the age of 94, and that the said survivor at 94 years of age will die before he is 95. If therefore a man were to grant to 2 old persons of the age of 93 years annuities of one pound a year each for their lives, he would, at the end of the first year, have only one payment of one of the said annuities to make, to wit, that which would be then due to the survivor of the said two grantees; and he would never after be obliged to make another payment of either of the annuities, because both the said grantees would be dead before the end of the second year, or before the second payment would become due. To the end therefore that the grantor of the said annuities might be neither a gainer nor a loser by granting them, it would be necessary that he should receive from both the said grantees, at the time of making the said grants, the present value of the said only payment of one pound which he would have to make to the survivor of the said two grantees at the end of the first year; which present value is the sum of $\frac{1}{r}$ of a pound, or £$\frac{1}{r}$. And consequently he ought to receive from each of the said two grantees one half of that sum, or $\frac{1}{2} \times$ £$\frac{1}{r}$, or (because $\frac{1}{r}$ is in this case $= \frac{1}{1.035}$, the interest of money being $3\frac{1}{2}$ per cent.) $\frac{1}{2} \times$ £$\frac{1}{1.035}$, or $\frac{1}{2} \times$ £.96618, or £.48309. Therefore, by Art. 21, if the grantor were to make only one such grant of an annuity of one pound a year to a person of the age of 93 years for his life, he ought to receive, as the price of such single grant, the same sum of £.48309; or, in other words, the value of an annuity of one pound a year for a life of 93 years is £.48309, or 9*s*. 8*d*. Q. E I.

CIX. Having

CIX. Having thus found the value of an annuity of one pound a year for a life of 93 years, the values of the like annuities for all younger lives may be found by the continual application of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|$ £, or $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|$ £, in the manner following.

CX. To find the value of an annuity of one pound for a life of 92 years, we must put $\overset{£}{V}$ = £.48309, and $P = 2$, and $P+d = 4$; because there are 2 persons living, according to Monf. de Parcieux's table, at the age of 93 years, and 4 persons living at the age of 92 years. And we shall have $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|$ £ $= \frac{1}{1.035} \times \frac{2}{4} \times \overline{1+.48309}|$ £ $= \frac{1}{1.035} \times \frac{2}{4} \times$ £1.48309 $= \frac{1}{1.035} \times$ £.741,545 = £.716,468. Therefore the value of an annuity of one pound for a life of 92 years is £.716,468, or 14*s.* 4*d.* Q E I.

CXI. To find the value of an annuity of one pound for a life of 91 years, we must put $\overset{£}{V}$ = £.716,468, and consequently $\overline{1+V}|$ £ = £1.716,468. And, by the table of probabilities, the numbers of persons living at the ages of 91 and 92 years are 7 and 4; that is, P is $= 4$, and $P+d$ is $= 7$. Therefore $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|$ £ is $= \frac{1}{1.035} \times \frac{4}{7} \times$ £1.716,468 $= \frac{1}{1.035} \times$ £ 980,838 = .947,669. Consequently the value of an annuity of one pound for the life of a person of the age of 91 years is .947,669, or 18*s.* 11*d.* $\frac{1}{2}$.

CXII. Again, put $\overset{£}{V}$ = £.947,669, and consequently $\overline{1+V}|$ £ = £1.947,669; and let P be $= 7$, and $P+d = 11$, which are the numbers of persons represented in the table as living at the ages of 91 and 90 years. And we shall have $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|$ £ $= \frac{1}{1.035} \times \frac{7}{11} \times$ £1.947,669 $= \frac{1}{1.035} \times$ £1.239,425 = £1.197,512. Therefore the value of an annuity of one pound for a life of the age of 90 years is = £1.197,512, or 1*l.* 3*s.* 11*d.* $\frac{1}{2}$.

CXIII. And

CXIII. And in the ſame manner we ſhall find that the value of an annuity of one pound for a life of the age of 89 years is $\left(= \frac{1}{1.035} \times \frac{11}{16} \times \overline{1+1.197512}|£ = \frac{1}{1.035} \times \frac{11}{16} \times £2.197,512 = \frac{1}{1.035} \times £1.510,789 = \right)$ £1.459,699, or 1*l.* 9*s.* 2*d.* $\frac{1}{4}$.

And that the value of the like annuity for a life of the age of 88 years is $\left(= \frac{1}{1.035} \times \frac{16}{22} \times £2.459,699 = \frac{1}{1.035} \times £1.788,872 = \right)$ £1.728,378, or 1*l.* 14*s.* 6*d.* $\frac{3}{4}$.

And that the value of the like annuity for a life of 87 years is $\left(= \frac{1}{1.035} \times \frac{22}{29} \times £2.728,378 = \frac{1}{1.035} \times £2.069,804 = \right)$ £1.999,810, or 2*l.* 0*s.* 0*d.*

And that the value of the like annuity for a life of 86 years is $\left(= \frac{1}{1.035} \times \frac{29}{38} \times £2.999,810 = \frac{1}{1.035} \times £2.289,328 = \right)$ £2.211,911, or 2*l.* 4*s.* 2*d.* $\frac{3}{4}$.

And that the value of the like annuity for a life of 85 years is $\left(= \frac{1}{1.035} \times \frac{38}{48} \times £3.211,911 = \frac{1}{1.035} \times £2.542,762 = \right)$ £2.456,774, or 2*l.* 9*s.* 1*d.* $\frac{1}{2}$.

And that the value of the like annuity for a life of 84 years is $\left(= \frac{1}{1.035} \times \frac{48}{59} \times £3.456,774 = \frac{1}{1.035} \times £2.812,290 = \right)$ £2.717,188, or 2*l.* 14*s.* 4*d.*

And that the value of the like annuity for a life of 83 years is $\left(= \frac{1}{1.035} \times \frac{59}{71} \times £3.717,188 = \frac{1}{1.035} \times £3.088,930 = \right)$ £2.984,473, or 2*l.* 19*s.* 8*d.* $\frac{1}{4}$.

And

And that the value of the like annuity for a life of 82 years is $\left(= \frac{1}{1.035} \times \frac{71}{85} \times £3.984,473 = \frac{1}{1.035} \times £3.328,206 = \right) £3.215,658$, or 3*l.* 4*s.* 3*d.* $\frac{1}{4}$.

And that for a life of 81 years it is $\left(= \frac{1}{1.035} \times \frac{85}{101} \times £4.215,658 = \frac{1}{1.035} \times £3.547,831 = \right) £3.427,856$, or 3*l.* 8*s.* 6*d.* $\frac{1}{4}$.

And that for a life of 80 years it is $\left(= \frac{1}{1.035} \times \frac{101}{118} \times £4.427,856 = \frac{1}{1.035} \times £3.789,944 = \right) £3.661,781$, or 3*l.* 13*s.* 2*d.* $\frac{1}{4}$.

CXIV. And for a life of 79 years it is $\left(= \frac{1}{1.035} \times \frac{118}{136} \times £4.661,781 = \frac{1}{1.035} \times £4.044,780 = \right) £3.908,000$, or 3*l.* 18*s.* 2*d.*

And for a life of 78 years it is $\left(= \frac{1}{1.035} \times \frac{136}{154} \times £4.908,000 = \frac{1}{1.035} \times £4.334,337 = \right) £4.187,758$, or 4*l.* 3*s.* 9*d.*

And for a life of 77 years it is $\left(= \frac{1}{1.035} \times \frac{154}{173} \times £5.187,758 = \frac{1}{1.035} \times £4.618,004 = \right) £4.461,839$, or 4*l.* 9*s.* 2*d.* $\frac{1}{4}$.

And for a life of 76 years it is $\left(= \frac{1}{1.035} \times \frac{173}{192} \times £5.461,839 = \frac{1}{1.035} \times £4.921,344 = \right) £4.754,921$, or 4*l.* 15*s.* 1*d.* $\frac{1}{4}$.

And

And for a life of 75 years it is $\left(= \frac{1}{1.035} \times \frac{192}{211} \times £5.754{,}921 = \frac{1}{1.035} \times £5.236{,}705 = \right)$ £5.059,623, or 5*l.* 1*s.* 2*d.* ¼.

And for a life of 74 years it is $\left(= \frac{1}{1.035} \times \frac{211}{231} \times £6.059{,}623 = \frac{1}{1.035} \times £5.534{,}980 = \right)$ £5.347,806, or 5*l.* 6*s.* 11*d.* ½.

And for a life of 73 years it is $\left(= \frac{1}{1.035} \times \frac{231}{251} \times £6.347{,}806 = \frac{1}{1.035} \times £5.842{,}004 = \right)$ £5.644,448, or 5*l.* 12*s.* 10*d.* ½.

And for a life of 72 years it is $\left(= \frac{1}{1.035} \times \frac{251}{271} \times £6.644{,}448 = \frac{1}{1.035} \times £6.154{,}082 = \right)$ £5.945,972, or 5*l.* 18*s.* 11*d.*

And for a life of 71 years it is $\left(= \frac{1}{1.035} \times \frac{271}{291} \times £6.945{,}972 = \frac{1}{1.035} \times £6.468{,}585 = \right)$ £6.249,840, or 6*l.* 5*s.* 0*d.*

And for a life of 70 years it is $\left(= \frac{1}{1.035} \times \frac{291}{310} \times £7.249{,}840 = \frac{1}{1.035} \times £6.805{,}495 = \right)$ £6.575,357, or 6*l.* 11*s.* 6*d.*

And for a life of 69 years it is $\left(= \frac{1}{1.035} \times \frac{310}{329} \times £7.575{,}357 = \frac{1}{1.035} \times £7.137{,}874 = \right)$ £6.896,496, or 6*l.* 17*s.* 11*d.* ¼.

And

And for a life of 68 years it is $\left(= \frac{1}{1.035} \times \frac{329}{347} \times £7.896,496 = \frac{1}{1.035} \times £7.486,879 = \right)$ £7.233,699, or 7*l.* 4*s.* 8*d.*

And for a life of 67 years it is $\left(= \frac{1}{1.035} \times \frac{347}{364} \times £8.233,699 = \frac{1}{1.035} \times £7.849,158 = \right)$ £7.583,727, or 7*l.* 11*s.* 8*d.*

And for a life of 66 years it is $\left(= \frac{1}{1.035} \times \frac{364}{380} \times £8.583,727 = \frac{1}{1.035} \times £8.222,307 = \right)$ £7.944,258, or 7*l.* 18*s.* 10*d.* ½.

And for a life of 65 years it is $\left(= \frac{1}{1.035} \times \frac{380}{395} \times £8.944,258 = \frac{1}{1.035} \times £8.604,\text{[illegible]} = \right)$ £8.313,625, or 8*l.* 6*s.* 3*d.* ¼.

And for a life of 64 years it is $\left(= \frac{1}{1.035} \times \frac{395}{409} \times £9.313,625 = \frac{1}{1.035} \times £8.994,821 = \right)$ £8.690,648, or 8*l.* 13*s.* 9*d.* ¾.

And for a life of 63 years it is $\left(= \frac{1}{1.035} \times \frac{409}{423} \times £9.690,648 = \frac{1}{1.035} \times £9.369,917 = \right)$ £9.053,059, or 9*l.* 1*s.* 0*d.* ¾.

And for a life of 62 years it is $\left(= \frac{1}{1.035} \times \frac{423}{437} \times £10.053,059 = \frac{1}{1.035} \times £9.730,993 = \right)$ £9.401,925, or 9*l.* 8*s.* 0*d.* ½.

And

And for a life of 61 years it is $\left(= \frac{1}{1.035} \times \frac{437}{450} \times £10.401,925\right.$ $= \frac{1}{1.035} \times £10.101,424 =\Big)$ £9.759,829, or 9*l.* 15*s.* 2*d.* $\frac{1}{4}$.

And for a life of 60 years it is $\left(= \frac{1}{1.035} \times \frac{450}{463} \times £10.759,829\right.$ $= \frac{1}{1.035} \times £10.457,717 =\Big)$ £10.104,074, or 10*l.* 2*s.* 1*d.*

And for a life of 59 years it is $\left(= \frac{1}{1.035} \times \frac{463}{476} \times £11.104,074\right.$ $= \frac{1}{1.035} \times £10.800,811 =\Big)$ £10.435,566, or 10*l.* 8*s.* 8*d.* $\frac{1}{2}$.

And for a life of 58 years it is $\left(= \frac{1}{1.035} \times \frac{476}{489} \times £11.435,566\right.$ $= \frac{1}{1.035} \times £11.131,553 =\Big)$ £10.755,123, or 10*l.* 15*s.* 1*d.* $\frac{1}{4}$.

And for a life of 57 years it is $\left(= \frac{1}{1.035} \times \frac{489}{502} \times £11.755,123\right.$ $= \frac{1}{1.035} \times £11.450,707 =\Big)$ £11.063,485, or 11*l.* 1*s.* 3*d.* $\frac{1}{4}$.

And for a life of 56 years it is $\left(= \frac{1}{1.035} \times \frac{502}{514} \times £12.063,485\right.$ $= \frac{1}{1.035} \times £11.781,847 =\Big)$ £11.383,427, or 11*l.* 7*s.* 8*d.*

And for a life of 55 years it is $\left(= \frac{1}{1.035} \times \frac{514}{526} \times £12.383,427\right.$ $= \frac{1}{1.035} \times £12.100,915 =\Big)$ £11.691,801, or 11*l.* 13*s.* 10*d.*

And for a life of 54 years it is $\left(= \frac{1}{1.035} \times \frac{526}{538} \times \pounds 12.691,801 = \frac{1}{1.035} \times \pounds 12.408,712 = \right)$ £11.989,093, or 11*l.* 19*s.* 9*d.* $\frac{1}{4}$.

And for a life of 53 years it is $\left(= \frac{1}{1.035} \times \frac{538}{549} \times \pounds 12.989,093 = \frac{1}{1.035} \times \pounds 12.728,838 = \right)$ £12.298,386, or 12*l.* 5*s.* 11*d.* $\frac{1}{2}$.

And for a life of 52 years it is $\left(= \frac{1}{1.035} \times \frac{549}{560} \times \pounds 13.298,386 = \frac{1}{1.035} \times \pounds 13.037,167 = \right)$ £12.596,296, or 12*l.* 11*s.* 11*d.*

And for a life of 51 years it is $\left(= \frac{1}{1.035} \times \frac{560}{571} \times \pounds 13.596,296 = \frac{1}{1.035} \times \pounds 13.334,370 = \right)$ £12.883,449, or 12*l.* 17*s.* 8*d.*

And for a life of 50 years it is $\left(= \frac{1}{1.035} \times \frac{571}{581} \times \pounds 13.883,449 = \frac{1}{1.035} \times \pounds 13.644,491 = \right)$ £13.183,083, or 13*l.* 3*s.* 8*d.*

And for a life of 49 years it is $\left(= \frac{1}{1.035} \times \frac{581}{590} \times \pounds 14.183,083 = \frac{1}{1.035} \times \pounds 13.966,730 = \right)$ £13.494,425, or 13*l.* 9*s.* 10*d.* $\frac{1}{2}$.

And for a life of 48 years it is $\left(= \frac{1}{1.035} \times \frac{590}{599} \times \pounds 14.494,425 = \frac{1}{1.035} \times \pounds 14.276,645 = \right)$ £13.793,859, or 13*l.* 15*s.* 10*d.* $\frac{1}{2}$.

And

And for a life of 47 years it is $\left(= \frac{1}{1.035} \times \frac{599}{607} \times £14.793,859 = \frac{1}{1.035} \times £14,598,882 = \right)$ £14.105,200, or 14*l.* 2*s.* 1*d.* $\frac{1}{4}$.

And for a life of 46 years it is $\left(= \frac{1}{1.035} \times \frac{607}{615} \times £15.105,200 = \frac{1}{1.035} \times £14.908,709 = \right)$ £14.404,549, or 14*l.* 8*s.* 1*d.*

And for a life of 45 years it is $\left(= \frac{1}{1.035} \times \frac{615}{622} \times £15.404,549 = \frac{1}{1.035} \times £15.231,185 = \right)$ £14.716,120, or 14*l.* 14*s.* 3*d.* $\frac{3}{4}$.

And for a life of 44 years it is $\left(= \frac{1}{1.035} \times \frac{622}{629} \times £15.716,120 = \frac{1}{1.035} \times £15.541,218 = \right)$ £15.015,669, or 15*l.* 0*s.* 3*d.* $\frac{3}{4}$.

And for a life of 43 years it is $\left(= \frac{1}{1.035} \times \frac{629}{636} \times £16.015,669 = \frac{1}{1.035} \times £15.839,395 = \right)$ £15.303,763, or 15*l.* 6*s.* 1*d.*

And for a life of 42 years it is $\left(= \frac{1}{1.035} \times \frac{636}{643} \times £16.303,763 = \frac{1}{1.035} \times £16.126,272 = \right)$ £15.580,939, or 15*l.* 11*s.* 7*d.* $\frac{1}{2}$.

And for a life of 41 years it is $\left(= \frac{1}{1.035} \times \frac{643}{650} \times £16.580,939 = \frac{1}{1.035} \times £16.448,528 = \right)$ £15.892,297, or 15*l.* 17*s.* 10*d.*

And

And for a life of 40 years it is $\left(= \frac{1}{1.035} \times \frac{650}{657} \times £16.892,297 = \frac{1}{1.035} \times £16.712,318 = \right)$ £16.147,167, or 16*l.* 2*s.* 11*d.* $\frac{1}{4}$.

And for a life of 39 years it is $\left(= \frac{1}{1.035} \times \frac{657}{664} \times £17.147,167 = \frac{1}{1.035} \times £16.966,398 = \right)$ £16.392,655, or 16*l.* 7*s.* 10*d.* $\frac{1}{4}$.

And for a life of 38 years it is $\left(= \frac{1}{1.035} \times \frac{664}{671} \times £17.392,655 = \frac{1}{1.035} \times £17.211,211 = \right)$ £16.629,189, or 16*l.* 12*s.* 7*d.*

And for a life of 37 years it is $\left(= \frac{1}{1.035} \times \frac{671}{678} \times £17\ 629,189 = \frac{1}{1.035} \times £17.447,176 = \right)$ £16.857,174, or 16*l.* 17*s.* 1*d.* $\frac{3}{4}$.

And for a life of 36 years it is $\left(= \frac{1}{1.035} \times \frac{678}{686} \times £17.857,174 = \frac{1}{1.035} \times £17.648,927 = \right)$ £17.052,103, or 17*l.* 1*s.* $\frac{1}{2}$*d.*

And for a life of 35 years it is $\left(= \frac{1}{1.035} \times \frac{686}{694} \times £18.052,103 = \frac{1}{1.035} \times £17.844,009 = \right)$ £17.240,588, or 17*l.* 4*s.* 9*d.* $\frac{3}{4}$.

And for a life of 34 years it is $\left(\frac{1}{1.035} \times \frac{694}{702} \times £18.240,588 = \frac{1}{1.035} \times £18.032,718 = \right)$ £17.422,915, or 17*l.* 8*s.* 5*d.* $\frac{1}{2}$.

And

And for a life of 33 years it is $\left(= \frac{1}{1.035} \times \frac{702}{710} \times £18.422,915 = \frac{1}{1.035} \times £18.215,332 = \right)$ £17.599,354, or 17*l.* 11*s.* 11*d*$\frac{3}{4}$.

And for a life of 32 years it is $\left(= \frac{1}{1.035} \times \frac{710}{718} \times £18.599,354 = \frac{1}{1.035} \times £18.392,118 = \right)$ £17.770,162, or 17*l.* 15*s.* 2*d.* $\frac{3}{4}$.

And for a life of 31 years it is $\left(= \frac{1}{1.035} \times \frac{718}{726} \times £18.770,162 = \frac{1}{1.035} \times £18.563,328 = \right)$ £17.935,582, or 17*l.* 18*s.* 8*d.* $\frac{1}{2}$.

And for a life of 30 years it is $\left(= \frac{1}{1.035} \times \frac{726}{734} \times £18.935,582 = \frac{1}{1.035} \times £18.729,199 = \right)$ £18.095,844, or 18*l.* 1*s.* 11*d.*

And for a life of 29 years it is $\left(= \frac{1}{1.035} \times \frac{734}{742} \times £19.095,844 = \frac{1}{1.035} \times £18.889,958 = \right)$ £18.251,167, or 18*l.* 5*s.* o*d.* $\frac{1}{4}$.

And for a life of 28 years it is $\left(= \frac{1}{1.035} \times \frac{742}{750} \times £19.251,167 = \frac{1}{1.035} \times £19.045,821 = \right)$ £18.401,759, or 18*l.* 8*s.* $\frac{1}{2}$*d.*

And for a life of 27 years it is $\left(= \frac{1}{1.035} \times \frac{750}{758} \times £19.401,759 = \frac{1}{1.035} \times £19.196,991 = \right)$ £18.547,817, or 18*l.* 10*s.* 11*d.* $\frac{1}{2}$.

And

And for a life of 26 years it is $\left(= \frac{1}{1.035} \times \frac{758}{766} \times £19.547,817 = \frac{1}{1.035} \times £19.343,662 = \right)$ £18.689,528, or 18*l.* 13*s.* 9*d.* $\frac{1}{2}$.

And for a life of 25 years it is $\left(= \frac{1}{1.035} \times \frac{766}{774} \times £19.689,528 = \frac{1}{1.035} \times £19.486,018 = \right)$ £18.827,070, or 18*l.* 16*s.* 6*d.* $\frac{1}{2}$.

And for a life of 24 years it is $\left(= \frac{1}{1.035} \times \frac{774}{782} \times £19.827,070 = \frac{1}{1.035} \times £19.624,235 = \right)$ £18.960,613, or 18*l.* 19*s.* 2*d.* $\frac{1}{2}$.

And for a life of 23 years it is $\left(= \frac{1}{1.035} \times \frac{782}{790} \times £19.960,613 = \frac{1}{1.035} \times £19.758,480 = \right)$ £19.090,318, or 19*l.* 1*s.* 9*d.* $\frac{1}{2}$.

And for a life of 22 years it is $\left(= \frac{1}{1.035} \times \frac{790}{798} \times £20,090,318 = \frac{1}{1.035} \times £19.888,911 = \right)$ £19.216,339, or 19*l.* 4*s.* 4*d.*

And for a life of 21 years it is $\left(= \frac{1}{1.035} \times \frac{798}{806} \times £20.216,339 = \frac{1}{1.035} \times £20.015,680 = \right)$ £19.338,821, or 19*l.* 6*s.* 9*d.* $\frac{1}{4}$.

And for a life of 20 years it is $\left(= \frac{1}{1.035} \times \frac{806}{814} \times £20.338,821 = \frac{1}{1.035} \times £20.138,930 = \right)$ £19.457,903, or 19*l.* 9*s.* 1*d.* $\frac{3}{4}$.

And

And for a life of 19 years it is $\left(= \frac{1}{1.035} \times \frac{814}{821} \times £20.457,903\right.$ $= \frac{1}{1.035} \times £20.283,475 = \left.\right)$ £19.597,560, or 19*l.* 11*s.* 11*d.* ½.

And for a life of 18 years it is $\left(= \frac{1}{1.035} \times \frac{821}{828} \times £20.597,560\right.$ $= \frac{1}{1.035} \times £20.423,426 = \left.\right)$ £19.732,778, or 19*l.* 14*s.* 7*d.* ¾.

And for a life of 17 years it is $\left(= \frac{1}{1.035} \times \frac{828}{835} \times £20.732,778\right.$ $= \frac{1}{1.035} \times £20.558,970 = \left.\right)$ £19.863,739, or 19*l.* 17*s.* 3*d.* ¼.

And for a life of 16 years it is $\left(= \frac{1}{1.035} \times \frac{835}{842} \times £20.863,739\right.$ $= \frac{1}{1.035} \times £20.690,287 = \left.\right)$ £19.990,615, or 19*l.* 19*s.* 9*d.* ¾.

And for a life of 15 years it is $\left(= \frac{1}{1.035} \times \frac{842}{848} \times £20.990,615\right.$ $= \frac{1}{1.035} \times £20.842,096 = \left.\right)$ £20.137,194, or 20*l.* 2*s.* 9*d.*

And for a life of 14 years it is $\left(= \frac{1}{1.035} \times \frac{848}{854} \times £21.137,194\right.$ $= \frac{1}{1.035} \times £20.988,677 = \left.\right)$ £20.278,914, or 20*l.* 5*s.* 7*d.*

And for a life of 13 years it is $\left(= \frac{1}{1.035} \times \frac{854}{860} \times £21.278,914\right.$ $= \frac{1}{1.035} \times £21.130,456 = \left.\right)$ £20.415,899, or 20*l.* 8*s.* 3*d.* ¾.

And

And for a life of 12 years it is $(=\frac{1}{1.035}\times\frac{860}{866}\times$ £21.415,899 $=\frac{1}{1.035}\times$ £21.267,520 =) £20.548,338, or 20*l.* 10*s.* 11*d.* ¼.

And for a life of 11 years it is $(=\frac{1}{1.035}\times\frac{866}{872}\times$ £21.548,338 $=\frac{1}{1.035}\times$ £21.400,069 =) £20.676,395, or 20*l.* 13*s.* 6*d.* ¼.

And for a life of 10 years it is $(=\frac{1}{1.035}\times\frac{872}{880}\times$ £21.676,395 $=\frac{1}{1.035}\times$ £21.479,336 =) £20.752,981, or 20*l.* 15*s.* 0*d.* ¾.

CXV. This value of an annuity of one pound for a life of 10 years is somewhat greater than that which was found for it in Art 94, which was £20.73925. But the difference between them is but small, being only .013,731 of a pound, or (240 × .013,731, or) 3.295,440 of a penny, or about three pence, farthing, upon the sum of twenty pounds, fifteen shillings. They may therefore be considered as being equal to each other, and consequently as affording a proof of the agreement of the two different methods of calculation by which they have been obtained, and a confirmation of their truth. Which of these two values is the more exact, I do not know; but I am inclined to think it is this last, which we have just now obtained by means of the repeated application of the expression $\frac{1}{1.035}\times\frac{P}{P+d}\times\overline{1+V|}$ £, to wit, £20.752,981. And, if this number does exhibit the value of the said annuity more exactly than the other number £20.73925, it is evident that the values of the remaining annuities for lives of the ages of 9, 8, 7, 6, 5, 4, and 3 years, will be somewhat greater than they are found to be in Art. 103, 104, 105, and 106. In order, therefore, to make the table of these values (which we have here computed,) uniform and regular throughout, I shall compute the values of an annuity of one pound for lives of 9, 8, 7, 6, 5, 4, and 3 years of age a second a ſecond time by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V|}£$, upon a ſuppoſition that the firſt value of $V£$, or the value of an annuity of one pound for a life of the age of 10 years, is $= £20.752,981$. This may be done in the manner following.

CXVI. If $\overset{£}{V}$ is $= £20.752,981$, we ſhall have $\overline{1+V|}£ = £21.752,981$. And in this caſe P is $= 880$, and $P+d$ is $= 890$. Therefore the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V|}£$ will be $= \frac{1}{1.035} \times \frac{880}{890} \times £21.752,981$ $(= \frac{1}{1.035} \times £21.508,565) = £20.781,222$; or the value of an annuity of one pound for a life of 9 years is £20.781,222, or 20*l.* 15*s.* 7*d.*$\frac{1}{4}$.

CXVII. And in like manner the value of an annuity of one pound for a life of 8 years will be $(= \frac{1}{1.035} \times \frac{890}{902} \times £21.781,222 = \frac{1}{1.035} \times £21.491,449 =)$ £20.764,685, or 20*l.* 15*s.* 3*d.* $\frac{1}{2}$.

And for a life of 7 years it is $(= \frac{1}{1.035} \times \frac{902}{915} \times £21.764,685 = \frac{1}{1.035} \times £21.455,459 =)$ £20.729,912, or 20*l.* 14*s.* 7*d.*

And for a life of 6 years it is $(= \frac{1}{1.035} \times \frac{915}{930} \times £21.729,912 = \frac{1}{1.035} \times £21.379,322 =)$ £20.656,349, or 20*l.* 13*s.* 1*d.* $\frac{1}{2}$.

And for a life of 5 years it is $(= \frac{1}{1.035} \times \frac{930}{948} \times £21.656,349 = \frac{1}{1.035} \times £21.245,152 =)$ £20.526,716, or 20*l.* 10*s.* 6*d.* $\frac{1}{2}$.

S And

And for a life of 4 years it is $\left(= \frac{1}{1.035} \times \frac{948}{970} \times £21.526,72[illegible] = \frac{1}{1.035} \times £21.038,481 = \right) £20.327,034$, or 20*l.* 6*s.* 6*d.* ½.

And for a life of 3 years it is $\left(= \frac{1}{1.035} \times \frac{970}{1000} \times £21.327,034 = \frac{1}{1.035} \times £20.687,222 = \right) £19.987,654$, or 19*l.* 19*s.* 9*d.*

CXVIII. Having now compleated the computation of the values of an annuity of one pound a year for every year of human life from the age of 93 years down to the age of 3 years, inclusively, I shall proceed to range them in a regular series or table, and shall express them (as I have hitherto done) both in decimal parts of a pound sterling, and in pounds, shillings, and pence. And, that the rate, or degree, at which they increase, or decrease, in the different periods of human life, may appear the more readily, I shall subjoin to them a table of their differences, expressed likewise in a double manner, to wit, in decimal parts of a pound sterling and in pounds, shillings, and pence. This table is as follows.

TABLE

TABLE III.

Containing the values of an annuity of one pound sterling a year for the lives of persons of all ages from the age of 3 years to the age of 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 3½ per cent.

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the said values.*	*Values of the same annuity, in pounds, shillings, and pence.*	*Differences of the said values.*
	£	£	£ s. d.	£ s. d.
3	19.987,654		19 19 9	
		0.339,380		0 6 9½
4	20.327,034		20 6 6½	
		0.199,682		0 4 0
5	20.526,716		20 10 6½	
		0.129,633		0 2 7
6	20.656,349		20 13 1½	
		0.073,563		0 1 5½
7	20.729,912		20 14 7	
		0.034,773		0 0 8½
8	20.764,685		20 15 3½	
		0.016,537		0 0 4
9	20.781,222		20 15 7½	
		0.028,241		0 0 6¾
10	20.752,981		20 15 0¾	
		0.076,586		0 1 6½
11	20.676,395		20 13 6¼	
		0.128,057		0 2 6¾
12	20.548,338		20 10 11½	
		0.132,439		0 2 7¾
13	20.415,899		20 8 3¾	
		0.136,985		0 2 8¾
14	20.278,914		20 5 7	
		0.141,720		0 2 10
15	20.137,194		20 2 9	
		0.146,579		0 2 11¼

Years of Age.	Values of an annuity of one pound, in decimal parts of a pound.	Differences of the said values.	Values of the same annuity, in pounds, shillings, and pence.			Differences of the said values.		
	£	£	£	s.	d.	£	s.	d.
16	19.990,615	0.126,876	19	19	9¾	0	2	6½
17	19.863,739	0.130,961	19	17	3¼	0	2	7½
18	19.732,778	0.135,218	19	14	7¾	0	2	8¼
19	19.597,560	0.138,657	19	11	11½	0	2	9¼
20	19.457,903	0.119,082	19	9	1¾	0	2	4½
21	19.338,821	0.122,482	19	6	9¼	0	2	5¼
22	19.216,339	0.126,021	19	4	4	0	2	6½
23	19.090,318	0.129,705	19	1	9½	0	2	7
24	18.960,613	0.133,543	18	19	2½	0	2	8
25	18.827,070	0.137,542	18	16	6½	0	2	9
26	18.689,528	0.141,711	18	13	9½	0	2	10
27	18.547,817	0.146,058	18	10	11½	0	2	11
28	18.401,759	0.150,592	18	8	0½	0	3	0¼
29	18.251,167	0.155,323	18	5	0¼	0	3	1¼
30	18.095,844	0.160,262	18	1	11	0	3	2½
31	17.935,582	0.165,420	17	18	8½	0	3	5¾
32	17.770,162	0.170,808	17	15	2¾	0	3	3
33	17.599,354	0.176,439	17	11	11¾	0	3	6¼
34	17.422,915	0.182,327	17	8	5½	0	3	7¾
35	17.240,588	0.188,485	17	4	9¾	0	3	9¼

Years of Age.	Values of an annuity of one pound, in decimal parts of a pound.	Differences of the said values.	Values of the same annuity, in pounds, shillings, and pence.	Differences of the said values.
	£	£	£ s. d.	£ s. d.
36	17.052,103		17 1 0½	
		0.194,929		0 3 10¾
37	16.857,174		16 17 1¾	
		0.227,985		0 4 6¼
38	16.629,189		16 12 7	
		0.236,534		0 4 8¾
39	16.392,655		16 7 10¼	
		0.245,488		0 4 11
40	16.147,167		16 2 11¼	
		0.254,870		0 5 1¼
41	15.892,297		15 17 10	
		0.311,358		0 6 2½
42	15.580,939		15 11 7½	
		0.277,176		0 5 6½
43	15.303,763		15 6 1	
		0.288,094		0 5 9¼
44	15.015,669		15 0 3¾	
		0.299,549		0 6 0
45	14.716,120		14 14 3¾	
		0.311,571		0 6 2¾
46	14.404,549		14 8 1	
		0.299,349		0 5 11¾
47	14.105,200		14 2 1¼	
		0.311,341		0 6 2¾
48	13.793,859		13 15 10½	
		0.299,434		0 6 0
49	13.494,425		13 9 10½	
		0.311,342		0 6 2½
50	13.183,083		13 3 8	
		0.299,634		0 6 0
51	12.883,449		12 17 8	
		0.287,153		0 5 9
52	12.596,296		12 11 11	
		0.297,910		0 5 11¼
53	12.298,386		12 5 11½	
		0.309,293		0 6 2¼
54	11.989,093		11 19 9¼	
		0.297,292		0 5 11¼
55	11.691,801		11 13 10	
		0.308,374		0 6 2

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the said values.*	*Values of the same annuity, in pounds, shillings, and pence.*			*Differences of the said values.*		
	£	£	£	s.	d.	£	s.	d.
56	11.383,427		11	7	8			
		0.319,942				c	6	4¾
57	11.063,485		11	1	3¼			
		0.308,362				0	6	2
58	10.755,123		10	15	1¼			
		0.319,557				0	6	4¾
59	10.435,566		10	8	8½			
		0.331,492				0	6	7½
60	10.104,074		10	2	1			
		0.344,245				0	6	10¾
61	9.759,829		9	15	2¼			
		0.357,904				0	7	1¾
62	9.401,925		9	8	0½			
		0.348,866				0	6	11¾
63	9.053,059		9	1	0¾			
		0.362,411				0	7	3
64	8.690,648		8	13	9¾			
		0.377,023				0	7	6½
65	8.313,625		8	6	3¼			
		0.369,367				0	7	4¾
66	7.944,258		7	18	10½			
		0.360,531				0	7	2½
67	7 583,727		7	11	8			
		0.350,028				0	7	0
68	7.233,699		7	4	8			
		0.337,203				0	6	8¾
69	6.896,496		6	17	11¼			
		0.321,139				0	6	5¼
70	6.575.357		6	11	6			
		0.325,517				0	6	6
71	6.249,840		6	5	0			
		0.303,868				0	6	1
72	5.945,972		5	18	11			
		0.301,524				0	6	1½
73	5.644,448		5	12	10½			
		0.296,642				0	5	11
74	5.347,806		5	6	11½			
		0.288,183				0	5	9¼
75	5.059,623		5	1	2¼			
		0.304,702				0	6	1

Years of Age.	*Values of an annuity of one pound, in decimal parts of a pound.*	*Differences of the said values.*	*Values of the same annuity, in pounds, shillings, and pence.*			*Differences of the said values.*		
	£	£	£	s.	d.	£	s.	d.
76	4.754,921		4	15	1¼			
		0.293,082				0	5	10½
77	4.461,839		4	9	2¾			
		0.274,081				0	5	5¾
78	4.187,758		4	3	9			
		0.279,758				0	5	7
79	[illegible]08,000		3	18	2			
		0.246,219				0	4	11¼
80	[illegible],781		3	13	2¾			
		0.233,925				0	4	8
81	[illegible]6		3	8	6¾			
		0.212,198				0	4	3
82	[illegible]15,658		3	4	3¾			
		0.231,185				0	4	7½
83	2.984,473		2	19	8¼			
		0.267,285				0	5	4¼
84	2.717,188		2	14	4			
		0.260,414				0	5	2½
85	2.456,774		2	9	1½			
		0.244,[illegible]03				0	4	10¾
86	2.211,911		2	4	2¾			
		0.212,101				0	4	2¾
87	1.999,810		2	0	0			
		0.271,432				0	5	5¼
88	1.7[illegible],378		1	14	6¾			
		0.268,679				0	5	4½
89	1.459,699		1	9	2¼			
		0.262,187				0	5	2¾
90	1.197,512		1	3	11½			
		0.249,843				0	5	0
91	0.947.669		0	18	11½			
		0.231,201				0	4	7½
92	0.716,468		0	14	4			
		0.233,378				0	4	8
93	0.48309		0	9	8			

CXIX. In

CXIX. In the foregoing table it may be obſerved that the differences of the values of an annuity of one pound for the ſeveral ſucceſſive years of age in human life, (which are ſet down in pounds, ſhillings, and pence in the fifth column of the ſaid table,) vary gradually, by increaſing or decreaſing, within the limits of 4 pence on the one hand, and 7 ſhillings, and 6 pence, half-penny, on the other; the leaſt of them, (which is the difference of the ſaid values for the ages of 8 and 9 years,) being 4 pence, and the greateſt of them (which is the difference of the ſaid values for the ages of 64 and 65 years,) being 7*s.* 6*d.* ½. And, between theſe limits, they vary, in ſome places by increaſing, and in others by decreaſing, (while the years of life increaſe,) by the ſmall differences of a penny, two pence, or three pence, for the moſt part, and in a few places by ſix pence, or ſeven pence, or, at moſt, eleven pence, excepting only the beginning of the table, between the ages of 3 and 12 years, in which they decreaſe (while the years of life increaſe,) by differences of 2*s.* 9*d.* ½, ſeventeen pence, and 13 pence, halfpenny.

CXX. This obſervation may be of uſe in computing other tables of this kind according to other rates of intereſt, by enabling the computer to detect in good time any error he may happen to have fallen into in the courſe of his arithmetical operations. Thus, for example, if in computing the value of an annuity for a life of 60 years by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V|}\,£$, or $\frac{1}{1.035} \times \frac{450}{463} \times \overline{£1 + £9.759,829}$, or $\frac{1}{1.035} \times \frac{450}{463} \times £10.759,829$, I had found the ſaid value to be £10.154,074, inſtead of £10.104,074, that is, 10*l.* 3*s.* 1*d.* inſtead of 10*l.* 2*s.* 1*d.* I might have ſoon diſcovered that I had made a miſtake in the calculation, by ſubtracting from this value the value of the like annuity of one pound for a life of 61 years, which I had before found to be £9.759,829, or 9*l.* 15*s.* 2*d.* ¼, and then comparing the remainder, or difference, with the foregoing differences between the values of the ſame annuity for lives of 61, 62, 63 and 64 years, all which I had before computed. For I ſhould then have found that this laſt difference, (or the exceſs of 10*l.* 3*s.* 1*d.* above 9*l.* 15*s.* 2*d.* ¼,) was 7*s.* 10*d.* ¾; which exceeds the difference next preceeding it, or that between the values of two annuities of one pound each for lives of 61 and 62 years, (which is 7*s.* 1*d.* ¾,) by no leſs a ſum than 9 pence: whereas that difference (7*s.* 1*d.* ¾,) exceeds the next preceeding difference (which is 6*s.* 11*d.* ¾) by only 2 pence; and that difference (6*s.* 11*d.* ¾) is exceeded by the next preceeding difference, to wit, 7*s.* 3*d.* by only 3*d.* ¼; and this laſt difference, (7*s.* 3*d.*) is exceeded by the next preceeding difference (which is 7*s.* 6*d.* ½,) by only 3*d.* ½. Now each of theſe three differences of the differences

differences ſet down in the table, to wit, 2*d.* 3*d.* $\frac{1}{4}$, and 3*d.* $\frac{1}{2}$, is much leſs than 9 pence, which is the exceſs of the new difference, 7*s.* 10*d.* $\frac{3}{4}$, above the next preceeding difference, 7*s.* 1*d* $\frac{3}{4}$. I ſhould therefore, upon obſerving the uncommon magnitude of this laſt-mentioned exceſs, have been led to ſuſpect that the laſt-found difference, 7*s.* 10*d.* $\frac{3}{4}$, was greater than the truth, aud conſequently that the number found for the value of an annuity of one pound for a life of 60 years, to wit, 10*l.* 3*s.* 1*d.* or £10.154,074, was too great. And upon this ſuſpicion, ſo ariſing from the compariſon of the ſaid differences in the fifth column of the foregoing table with each other, I ſhould have reviſed the calculation of the value of the ſaid annuity by means of the expreſſion $\frac{1}{1.035} \times \frac{450}{463} \times$ £10.759,829, and ſhould have diſcovered my miſtake, and have found that the true value of the ſaid annuity for a life of 60 years was not £10.154,074, but £10.104,074, or 10*l.* 2*s.* 1*d.* And this number there is no reaſon to ſuſpect of being erroneous, becauſe it exceeds the value of the like annuity for a life of 61 years (which is 9*l.* 15*s.* 2*d.* $\frac{1}{4}$) by only 6*s.* 10*d.* $\frac{3}{4}$, which is exceeded by the next preceeding difference, to wit, 7*s.* 1*d.* $\frac{3}{4}$, by only 3 pence. And in fact I did in this manner diſcover and ſet right ſome miſtakes I had fallen into in computing the foregoing table.

CXXI. There is, however, another and more accurate method of diſcovering the errors that may have been made in computing ſuch tables of the values of life-annuities by means of the expreſſion above-mentioned, as ſoon as they ariſe, and conſequently of confirming the truth of the computations, when they have been rightly performed. But it requires conſiderably more labour than is neceſſary to the making of ſuch a table of differences as is above ſet down. Yet its uſefulneſs in aſcertaining the truth of theſe computations is ſo great, that, I think, all perſons who undertake to compute tables of the values of life-annuities by means of the expreſſion above-mentioned, would do well to undergo that labour, and to examine and correct their calculations by it. It is given, and explained, and illuſtrated by examples, by Mr. William Morgan, actuary to the Society of Equitable Aſſurances on Lives and Survivorſhips, in the 58th, 59th, 60th, &c.—73d, and the 212th and 213th, pages of his uſeful and learned treatiſe, intitled, *The Doctrine of Annuities and Aſſurances on Lives and Survivorſhips, ſtated and explained*; which was publiſhed in octavo in the year 1779, and printed for T. Cadell in the Strand. And I propoſe to make farther mention of it in the ſubſequent part of this diſcourſe.

Of certain irregularities in the increase and decrease of the differences in the foregoing table.

CXXII. But, though the observation made in Art. 119 concerning the differences set down in the fifth column of the foregoing table, to wit, that they increase, or decrease, in different parts of the table, by small and almost equal increments or decrements, be true in general, yet it is liable to several exceptions. For it may also be observed, that, in several places in the said table, the said differences, after having increased or decreased almost regularly, or by nearly equal increments or decrements, for several years together, do, on a sudden, increase, or decrease, by a much larger increment, or decrement, than before; after which they again increase, or decrease, as before, in a nearly regular manner, or by nearly equal increments, or decrements, for several more years together; and then again receive an uncommonly large increment, or decrement. Of this it will be proper to give some examples: and in examining these differences with this view, it will be most convenient to proceed in the order in which the values of the annuities set down in the foregoing table were computed, that is, from the older lives to the younger.

Now, if we begin this examination from the age of 41 years and continue it upwards to the age of 19 years, we shall find the variations of these differences to be as follows.

From the age of 41 years to the age of 37 years these differences are 5*s*. 1*d*. $\frac{1}{4}$, 4*s*. 11*d*. 4*s*. 8*d* $\frac{3}{4}$, and 4*s*. 6*d*. $\frac{3}{4}$; which differ from each other by only 2*d*. $\frac{1}{4}$, 2*d*. $\frac{1}{4}$ and 2*d*: and then, from the age of 37 years to the age of 36 years, the difference is 3*s*. 10*d*. $\frac{3}{4}$; which is less than the next preceeding difference, 4*s*. 6*d* $\frac{3}{4}$, by the much larger sum of 8 pence, instead of 2*d*. or 2*d*. $\frac{1}{4}$. After this, from the age of 36 years to the age of 32 years, the differences are 3*s*. 9*d*. $\frac{1}{4}$, 3*s*. 7*d*. $\frac{3}{4}$, 3*s*. 6*d*. $\frac{1}{4}$, and 3*s*. 3*d*. which differ from the next preceeding difference, 3*s*. 10*d*. $\frac{3}{4}$, and from each other by the small sums of 1*d*. $\frac{1}{2}$, 1*d*. $\frac{1}{2}$, 1*d* $\frac{1}{2}$, and 3*d*. $\frac{1}{4}$. And thus far, that is, from the age of 41 years to the age of 32 years, the said differences have continually decreased from 5*s*. 1*d*. $\frac{1}{4}$, to 3*s*. 3*d* But here they increase for one year, to wit, from the age of 32 years to the age of 31 years, and then decrease again for the next 11 years, or from the age of 31 years to the age of 20 years, and then the said difference increases again from the age of 20 years to the age of 19 years. From the age of 32 years to the age of 31 years the said difference increases from 3*s*. 3*d*. to 3*s*. 5*d*. $\frac{3}{4}$, and therefore differs from the value it would have had if it had gone on decreasing as before, (which would have been about 3*s*.) by about 5*d*. $\frac{3}{4}$. And from the age of 31 years to the age of 20 years, the said differences are 3*s*. 2*d* $\frac{1}{2}$, 3*s*. 1*d*. $\frac{1}{4}$, 3*s*. 0*d*. $\frac{1}{4}$, 2*s*. 11*d*. 2*s*. 10*d*. 2*s*. 9*d*. 2*s*. 8*d*. 2*s*. 7*d*. 2*s* 6*d*. $\frac{1}{2}$, 2*s*. 5*d* $\frac{1}{4}$, and 2*s*. 4*d*. $\frac{1}{2}$; which decrease continually, and differ from the next preceeding difference, 3*s*. 5*d*. $\frac{3}{4}$, and from each other, by the moderate decrements of 3*d*. $\frac{1}{4}$, 1*d*. $\frac{1}{4}$, 1*d*.$\frac{1}{4}$, 1*d*. 1*d*.$\frac{1}{4}$, 1*d*. 1*d*. 1*d*. 1*d*. o*d*.$\frac{1}{2}$, 1*d*.$\frac{1}{4}$, and o*d*.$\frac{3}{4}$. And from the age of 20 years to the age of 19 years, the faid difference (inftead of decreafing, as it had done for the preceeding 11 years;) increafes from 2*s*. 4*d*$\frac{1}{2}$ to 2*s*. 9*d*.$\frac{3}{4}$, or by an increment of 5*d*.$\frac{1}{4}$, and therefore is greater than it would have been if it had gone on decreafing as before, by about 6 pence, or 7 pence.

And the like irregularities, or inequalities, in the variation of thefe differences may be obferved in other parts of the faid table.

CXXIII. This irregularity feems, at firft fight, ftrange and unaccountable, and (agreeably to what has been obferved in Art. 119, 120) may give room to a fufpicion that the values which increafe, or decreafe, by thefe uncommonly large increments, or decrements, have been erroneoufly computed. And yet I am perfuaded, from the care I took in the computation of them, that they are not erroneous. But, I believe, it will be generally found that this irregularity is owing to the variations in the decreafe of the numbers of perfons who are reprefented, in Monfieur de Parcieux's table above-mentioned, as living at the ends of the feveral fucceffive years of human life. For it is remarkable that, in feveral parts of that table, the number of the perfons who die in the fpace of a year continues the fame for feveral years together, and then changes to fome greater, or leffer, number. Thus, for example, from the age of 11 years to the age of 16 years 6 perfons are reprefented in the faid table as dying every year; the numbers of perfons reprefented there as living at the ages of 11, 12, 13, 14, 15, and 16, years being 872, 866, 860, 854, 848, and 842, refpectively; which numbers differ from each other by 6. And, in like manner, from the age of 16 years to the age of 20 years the number of thofe who die every year is 7; the numbers of perfons reprefented to be living at the feveral ages of 16, 17, 18, 19, and 20, years being 842, 835, 828, 821, and 814, refpectively; which numbers differ from each other by 7. And afterwards, for 17 years together, from the age of 20 years to the age of 37 years, the number of thofe who die every year is 8; the numbers of perfons reprefented as living at the age of 20 years and the feveral fubfequent ages of 21, 22, 23, 24, 25, &c. years, up to 37 years, being 814, 806, 798, 790, 782, 774, 766, 758, 750, 742, 734, 726, 718, 710, 702, 694, 686, and 678; which all differ from each other by 8. And for the next 9 years, or from the age of 37 years to the age of 46 years, the number of thofe who die every year is again only 7; the numbers of perfons reprefented in the faid table as living at the ages of 37, 38, 39, 40, 41, 42, 43, 44, 45, and 46, years being 678, 671, 664, 657, 650, 643, 636, 629, 622, and 615, refpectively; which differ from each other by 7. And the fame thing may be obferved in other parts

Of the caufe of many of thofe irregularities.

parts of Monsieur de Parcieux's said table, and likewise in Mr. Kersseboom's table above-mentioned and in Dr. Halley's Breslaw table, and in all other tables of the probabilities of the duration of human life that have ever Been published.

Now it will be found that the extraordinary decrements and increments above-mentioned in the differences which are set down in the fifth column of the foregoing table, happen, for the most part, in those years of human life in which the number of persons who die in the said years increases or decreases, or varies from the number of those who died in the year next preceeding. Thus, for example, we have observed that, from the age of 41 years to the age of 37 years, the said differences decrease from 5*s*. 1*d*. $\frac{1}{4}$ to 4*s*. 6*d*. $\frac{3}{4}$ by the moderate decrements of 2*d*. $\frac{1}{4}$, 2*d*. $\frac{1}{4}$, and 2*d*.; and during the same period we find in Monsieur de Parcieux's table of probabilities, that the number of those who die every year is always the same, to wit, 7. But in the next year, or from the age of 37 years to the age of 36 years, the corresponding difference in the fifth column of the aforesaid table is 3*s*. 10*d*. $\frac{3}{4}$; which is less than the next preceeding difference, 4*s*. 6*d*. $\frac{1}{4}$, by the much greater sum of 8 pence: and in this same year the number of persons that die in the course of a year (which from the age of 46 years to the age of 37 years had been constantly 7 in a year,) is increased to 8; which occasions an uncommon increase in the value of d in the denominator of the fraction $\frac{P}{P+d}$ in the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|$ £, which expresses the value of the next younger life. By this uncommon increase of d the whole denominator $P+d$ receives also a greater increase than usual, and consequently the whole fraction $\frac{P}{P+d}$, and therefore also the whole expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|$ £, becomes less than it would have been if d, the number of persons dying in the year, had not increased to the number 8, but had continued to be 7, as it had been for the foregoing 9 years, or from the age of 46 years to the age of 37 years. Therefore the value of the annuity of one pound for a life of 36 years (which is equal to the said expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}|$ £) will be less than it would have been if d had continued to be equal to 7 instead of becoming equal to 8. And hence it comes to pass that the excess of this value above the value of the like annuity for a life of 37 years, is less than it would have been if d had continued to be equal to 7, or that the difference 3*s*. 10*d*. $\frac{1}{4}$ is less than it would have been, if d had continued to be equal to 7, and consequently that it falls short of the preceeding difference, 4*s*. 6*d*. $\frac{3}{4}$, by the uncommonly large decrement of 8 pence.

And

And in the ſame manner we have ſeen that from the age of 31 years to the age of 20 years the ſaid differences continually decreaſe from 3*s.* 5*d.*$\frac{3}{4}$ to 2*s.* 4*d* $\frac{1}{2}$ by the moderate decrements of 3*d.*$\frac{1}{4}$, 1*d.*$\frac{1}{4}$, 1*d.* 1*d.*$\frac{1}{4}$, 1*d.* 1*d.* 1*d.* 1*d.* 0*d.*$\frac{1}{2}$, 1*d.*$\frac{1}{4}$, and 0*d.*$\frac{3}{4}$: and during the ſame period of time we find in Monſieur de Parcieux's table of probabilities, that the number of thoſe who die every year (and which is denoted by the letter *d* in the denominator of the fraction $\frac{P}{P+d}$ in the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V|}$ £) is always the ſame, to wit, 8. But in the next year, or from the age of 20 years to the age of 19 years, the correſponding difference in the fifth column of the aforeſaid table is 2*s.* 9*d.*$\frac{3}{4}$; which is not only not leſs than the preceeding difference, 2*s.* 4*d.*$\frac{1}{2}$, by about a penny, (as might have been expected from a view of the aforeſaid decrements 3*d.*$\frac{1}{4}$, 1*d.*$\frac{1}{4}$, 1*d.* 1*d* $\frac{1}{4}$, 1*d.* 1*d.* 1*d.* 1*d.* 0*d.*$\frac{1}{2}$, 1*d.*$\frac{1}{4}$, and 0*d.*$\frac{3}{4}$ of the next preceeding differences,) but exceeds it by 5*d.*$\frac{1}{4}$, and conſequently is greater than might have been expected by about 6 pence: and in this ſame year, or from the age of 20 years to the age of 19 years, the number of perſons who die in the courſe of a year (and which from the age of 37 years to the age of 20 years had conſtantly been 8 in a year) is diminiſhed to 7; which occaſions a leſs diminution than uſual in the denominator of the fraction $\frac{P}{P+d}$ in the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V|}$ £, by which every new value is derived from that of the annuity for the next preceeding life which is one year older. Hence it comes to paſs that the ſaid expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V|}$ £, (which is equal to the value of an annuity for a life of 19 years,) is greater than it would have been if *d* had not decreaſed to 7, but had continued to be equal to 8, as it had been for ſeveral years before; and conſequently the ſaid value of an annuity for a life of 19 years exceeds the value of an annuity for a life of 20 years by a greater difference than it would have done if *d* had continued to be equal to 8, and the ſaid difference, to wit, 2*s.* 9*d.*$\frac{3}{4}$, inſtead of being leſs than the preceeding difference 2*s.* 4*d.*$\frac{1}{2}$, by about a penny, exceeds it by 5*d.*$\frac{1}{4}$.

CXXIV. We have ſeen in the foregoing article that the uncommonly great increments and decrements of the aforeſaid differences, (which are ſet down in the fifth column of the foregoing table,) are often occaſioned by correſpondent changes in the number of perſons who are repreſented in Monſieur de Parcieux's table of probabilities as dying in a year, or in the number

number d in the denominator of the fraction $\frac{P}{P+d}$ in the expression $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{|1+V}$ £. But they also sometimes happen without any such change in the value of d. Thus, for example, from the age of 36 years to the age of 32 years, the differences in the said fifth column are 3*s*. 9*d*.$\frac{1}{4}$, 3*s*. 7*d*.$\frac{3}{4}$, 3*s*. 6*d*.$\frac{1}{4}$ and 3*s*. 3*d*. which all decrease by the moderate decrements of 1*d*.$\frac{1}{2}$, 1*d*.$\frac{1}{2}$, and 3*d*.$\frac{1}{4}$. But in the next year, or from the age of 32 years to the age of 31 years, the difference is 3*s*. 5*d*.$\frac{3}{4}$; which is not only not less than the next preceeding difference, 3*s*. 3*d*. by about 3*d*. (as might have been expected from the course of the foregoing decrements,) but exceeds it by the sum of 2*d*.$\frac{3}{4}$, and therefore is greater than it might have been naturally expected to be by about 5*d*.$\frac{3}{4}$, or 6 pence. And yet the number of persons who die between the ages of 32 and 31 years is the same with that of those who die between the ages of 33 and 32 years, and between the ages of 34 and 33 years, and of 35 and 34 years, and of 36 and 35 years, and of 37 and 36 years, to wit, 8. How to account for this, and the like uncommonly great variations in the said differences, when there is no correspondent change in the number of persons represented in the table of probabilities as dying in the space of a year, I do not know. But thus much, I think, we may safely conclude, to wit, that, as it *often* happens that an uncommonly large decrement, or increment, of the differences in the said fifth column of the foregoing table, is occasioned by a change in the number of persons dying in the year, corresponding to such difference, there will always be reason, whenever such uncommonly large decrement, or increment, of the said differences is observed, and there is no change in the number of persons dying in the correlponding year of human life, to occasion it; —I say, there will always be reason in these cases to suspect that some error has crept into the calculation, and it will therefore be prudent to re-examine the operations of it with great care.

Of the decrements of human life.

CXXV. The numbers of persons represented, in a table of the probabilities of the duration of human life, as dying every year, are called by many writers on this subject *the decrements of human life*. And, because these decrements continue the same for several years together, in several different periods of human life, it is said by these writers that human life wastes, or *decreases*, *uniformly*, for several years together, in several of its stages, or periods; the numbers of the persons living at the ends of the several years in each of the said stages, or periods, (if reckoned from the younger ages to the older,) forming a regular, decreasing, arithmetical progression. Thus, for example, the numbers of persons living at the several

ſeveral ages of 20 years, 21 years, 22 years, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, and 37, years according to Monſieur de Parcieux's table of probabilities, form the following regular, decreaſing, arithmetical progreſſion, in which the common difference of the terms is 8, to wit, 814 + 806 + 798 + 790 + 782 + 774 + 766 + 758 + 750 + 742 + 734 + 726 + 718 + 710 + 702 + 694 + 686 + 678, or 814 + 814 — 8, + 814 — 2 x 8, + 814 — 3 x 8, + 814 — 4 x 8 + 814 — 5 x 8, + 814 — 6 x 8, + 814 — 7 x 8, + 814 — 8 x 8, + 814 — 9 x 8, + 814 — 10 x 8, + 814 — 11 x 8, + 814 — 12 x 8, + 814 — 13 x 8, + 814 — 14 x 8, + 814 — 15 x 8, + 814 — 16 x 8, + 814 — 17 x 8.

CXXVI. To make the courſe of theſe decrements in the different periods of human life more apparent, it will be convenient to preſent again to the reader's view the above-mentioned tables of probabilities of Mr. Kerſſeboom and Monſieur de Parcieux, and to ſet down in a ſeparate column, adjoining to that which exhibits the numbers of the perſons living at the ends of the ſeveral ſucceſſive years of human life in each table, the numbers of the perſons who have died in the next preceeding year. Theſe tables will then be as follows.

TABLE

TABLE IV.

Being the foregoing table of probabilities of Mr. Kerſſeboom, above ſet down in page 4, together with the numbers of the perſons dying in every year, or the decrements of human life, ſet down in an adjoining column.

Years of Age.	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*	*Years of Age.*	*Perſons living.*	*Perſons dying in a year, or the decrements of human life.*
0	1400	275	15	856	7
1	1125	50	16	849	7
2	1075	45	17	842	7
3	1030	37	18	835	9
4	993	29	19	826	9
5	964	17	20	817	9
6	947	17	21	808	8
7	930	17	22	800	8
8	913	9	23	792	9
9	904	9	24	783	11
10	895	9	25	772	12
11	886	8	26	760	13
12	878	8	27	747	12
13	870	7	28	735	12
14	863	7	29	723	12

Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.	Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.
30	711	12	50	507	12
31	699	12	51	495	13
32	687	12	52	482	12
33	675	10	53	470	12
34	665	10	54	458	12
35	655	10	55	446	12
36	645	10	56	434	13
37	635	10	57	421	13
38	625	10	58	408	13
39	615	10	59	395	13
40	605	9	60	382	13
41	596	9	61	369	13
42	587	9	62	356	13
43	578	9	63	343	14
44	569	9	64	329	14
45	560	10	65	315	14
46	550	10	66	301	14
47	540	10	67	287	14
48	530	8	68	273	14
49	518	11	69	259	14

Years of Age.	*Persons living.*	*Persons dying in a year, or the decrements of human life.*	*Years of Age.*	*Persons living.*	*Persons dying in a year, or the decrements of human life.*
70	245	14	86	36	8
71	231	14	87	28	7
72	217	14	88	21	6
73	203	14	89	15	5
74	189	14	90	10	3
75	175	15	91	7	2
76	160	15	92	5	2
77	145	15	93	3	1
78	130	15	94	2	1
79	115	15	95	1	0.4
80	100	13	96	0.6	0.1
81	87	12	97	0.5	0.1
82	75	11	98	0.4	0.2
83	64	9	99	0.2	0.2
84	55	10	100	0.0	
85	45	9			

TABLE

TABLE V.

Being the foregoing ta[illegible]f probabilities of Monsieur de Parcieux, above set down in page [illegible], together with the numbers of the persons dying in every year, or the decrements of human life, set down in an adjoining column.

Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.	Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.
3	1000	30	18	828	7
4	970	22	19	821	7
5	948	18	20	814	8
6	930	15	21	806	8
7	915	13	22	798	8
8	902	12	23	790	8
9	890	10	24	782	8
10	880	8	25	774	8
11	872	6	26	766	8
12	866	6	27	758	8
13	860	6	28	750	8
14	854	6	29	742	8
15	848	6	30	734	8
16	842	7	31	726	8
17	835	7	32	718	8

Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.	Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.
33	710	8	52	560	11
34	702	8	53	549	11
35	694	8	54	538	12
36	686	8	55	526	12
37	678	7	56	514	12
38	671	7	57	502	13
39	664	7	58	489	13
40	657	7	59	476	13
41	650	7	60	463	13
42	643	7	61	450	13
43	636	7	62	437	14
44	629	7	63	423	14
45	622	7	64	409	14
46	615	8	65	395	15
47	607	8	66	380	16
48	599	9	67	364	17
49	590	9	68	347	18
50	581	10	69	329	19
51	571	11	70	310	19

Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.	Years of Age.	Persons living.	Persons dying in a year, or the decrements of human life.
71	291	20	84	59	11
72	271	20	85	48	10
73	251	20	86	38	9
74	231	20	87	29	7
75	211	19	88	22	6
76	192	19	89	16	5
77	173	19	90	11	4
78	154	18	91	7	3
79	136	18	92	4	2
80	118	17	93	2	1
81	101	16	94	1	1
82	85	14	95	0	
83	71	12			

A remark

A remark on the decrements of human life in the foregoing tables of Mr. Kerſſeboom and Monſieur de Parcieux.

CXXVII. It appears from the foregoing table of Mr. Kerſſeboom that in ſeveral different periods of human life, from the age of 12 years to the age of 80 years, the decrements of life, or the numbers of perſons dying in a year, continue the ſame for five, ſix, or ſeven, years together, and in one period for twelve years together, namely, from the age of 63 years to the age of 75 years, in each of which years the number of perſons dying is 14. And it appears in like manner from the foregoing table of Monſieur de Parcieux, that in ſeveral parts of human life, from the age of 11 years to the age of 80 years, the decrements of life, or the numbers of perſons dying in a year, continue the ſame for four, five, or eight, years together, and in one period for no leſs than ſeventeen years together, namely, from the age of 20 years to the age of 37 years, in each of which years the number of perſons dying is 8. Now by the help of this obſervation we may in ſome degree abridge the labour of computing the values of life-annuities, when thoſe values are not derived one from the other by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{|1+V|}$ £, but are computed ſeparately in the manner deſcribed in Art. 86 and exemplified in Art. 94 by the computation of the value of an annuity of one pound for a life of the age of 10 years. For it is there ſhewn that the value of an annuity of one pound a year during n years, in caſe a perſon aged N years ſhall ſo long live, is equal to $\frac{1}{P}$ £ $\times$ the ſeries $\frac{P^{i}}{r} + \frac{P^{ii}}{r^{2}} + \frac{P^{iii}}{r^{3}} + \frac{P^{iv}}{r^{4}} + \frac{P^{v}}{r^{5}} + \frac{P^{vi}}{r^{6}} + \frac{P^{vii}}{r^{7}} +$ &c. continued to n terms, or to the term $\frac{P^{n}}{r^{n}}$.

Application of the ſaid remark to the purpoſe of abridging the computations of life-annuities, when performed in the method deſcribed in Art. 86.

Now this ſeries may be divided into ſeveral different ſerieſes, in ſome of which the numerators of the terms will (as appears from the foregoing obſervation) decreaſe by equal differences, or form arithmetical progreſſions. And where this happens, the ſaid leſſer ſerieſes may be ſummed in a more eaſy and expeditious manner than by, firſt, computing the values of every ſingle term in them, and then adding them all together into one ſum, as was done in Art. 94. For the denominators of the terms of theſe ſerieſes are the terms of a geometrical progreſſion, and decreaſe continually in the proportion of $\frac{1}{r}$ to 1, or of 1 to r: and where the terms of a ſeries conſiſt of fractions whoſe numerators form an arithmetical, and their denominators a geometrical, progreſſion, the ſum of the terms of ſuch a ſeries may be found in a compendious manner, without an actual computation and addition of them all together, by a method not unlike that explained above in Art. 80 and 81 for finding the ſum of the terms of a ſimple

ſimple geometrical progreſſion, (ſuch as $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n},$) and which is derived from that method. The manner of deriving ſuch a method of ſumming theſe compound ſerieſes, conſiſting of terms whoſe numerators form an arithmetical, and denominators a geometrical, progreſſion, from the aforeſaid method of ſumming a ſimple geometrical progreſſion, I will now endeavour to explain.

Of the ſummation of a ſeries of fractions, conſiſting of a given number of terms, whoſe numerators form a decreaſing arithmetical progreſſion, and denominators an increaſing geometrical progreſſion.

CXXVIII. Let $a, + a - d, + a - 2d, + a - 3d, + a - 4d, + a - 5d, + a - 6d, + \&c.$ be a decreaſing arithmetical progreſſion, conſiſting of any number of terms denoted by the letter n. Then, it is evident, its laſt term will be $a - \overline{n - 1} \times d$. or $a - \overline{nd - d}$, or $a - nd + d$.

In the next place let r be any quantity greater than 1, and let $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ be a geometrical progreſſion conſiſting likewiſe of n terms, which, it is evident, will decreaſe in the common ratio of 1 to r. The laſt term of this ſeries will evidently be $\frac{1}{r^n}$.

Laſtly, let the terms of this latter, or geometrical, progreſſion, $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$, be multiplied into the correſpondent terms of the arithmetical progreſſion, $a, + a - d, + a - 2d, + a - 3d, + a - 4d, + a - 5d, + a - 6d, + \&c. + a - nd + d$; and we ſhall thereby form a third ſeries, which will be as follows, to wit, $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} + \&c. + \frac{a-nd+d}{r^n}$. Now the ſum of the terms of a compound ſeries of this form may be obtained without actually computing every ſeparate term in it and then adding them all up together, by a method derived from that given in Art. 80 for finding the ſum of the terms of the geometrical progreſſion A + B + C + D + E. This method may be explained as follows.

CXXIX. The

CXXIX. The foregoing compound ſeries may be divided into the following ſets of quantities, to wit,

$$\begin{array}{lllllllll}
\frac{a}{r} & +\frac{a}{r^2} & +\frac{a}{r^3} & +\frac{a}{r^4} & +\frac{a}{r^5} & +\frac{a}{r^6} & +\frac{a}{r^7} & +\&c. & +\frac{a}{r^n} \\
 & -\frac{d}{r^2} & -\frac{d}{r^3} & -\frac{d}{r^4} & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & -\frac{d}{r^3} & -\frac{d}{r^4} & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & -\frac{d}{r^4} & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & -\frac{d}{r^5} & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & & -\frac{d}{r^6} & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & & & -\frac{d}{r^7} & -\&c. & -\frac{d}{r^n} \\
 & & & & & & & -\&c. & -\frac{d}{r^n} \\
 & & & & & & & & -\frac{d}{r^n}.
\end{array}$$

In this ſet of quantities it is evident in the 1ſt place that the upper line $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \&c.$ will conſiſt of n terms, and the ſecond line $-\frac{d}{r^2} - \frac{d}{r^3} - \frac{d}{r^4} - \&c.$ of one term leſs than the upper line, or of $n-1$ terms, and the third line of $n-2$ terms, and the fourth line of $n-3$ terms, and ſo on, every new line conſiſting of one term leſs than that which immediately preceeds it; and conſequently that the loweſt line will conſiſt of only one term, to wit, $-\frac{d}{r^n}$.

Secondly,

Secondly, the number of these horizontal lines of terms will be the same as the number of terms in the upper line $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4}$ + &c. that is, n.

Thirdly, the terms in all the lines, except the first, or upper, line, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4}$ + &c. have the sign — prefixed to them, and are to be subtracted from the upper line.

And, fourthly, all the lines of terms except the two last, (which consist of only two terms and one term,) are geometrical progressions whose terms decrease in the common ratio of 1 to r. Consequently they may be summed by the application of the expression $\frac{AA - BE}{A - B}$ given in Art. 80 for the sum of the terms of the geometrical progression $A + B + C + D + E$.

CXXX. Now, if we apply this expression to the first, or upper, series, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7}$ + &c. $+ \frac{a}{r^n}$, we shall have $A = \frac{a}{r}$, and $B = \frac{a}{r^2}$, and $E = \frac{a}{r^n}$. Therefore AA will be $= \frac{aa}{r^2}$; and BE will be $\frac{a}{r^2} \times \frac{a}{r^n} = \frac{aa}{r^{n+2}}$; and $AA - BE$ will be $= \frac{aa}{r^2} - \frac{aa}{r^{n+2}}$ $\left(= \frac{r^{n+2}aa - r^2 aa}{r^{n+4}}\right) = \frac{r^n aa - aa}{r^{n+2}}$; and A-B will be $= \frac{a}{r} - \frac{a}{r^2}$ $\left(= \frac{r^2 a - ra}{r^3}\right) = \frac{ra - a}{r^2}$; and consequently $\frac{AA - BE}{A - B}$ will be $= \frac{\frac{r^n aa - aa}{r^{n+2}}}{\frac{ra - a}{r^2}}$ $\left(= \frac{r^n aa - aa}{r^{n+2}} \times \frac{r^2}{ra - a} = \frac{r^n a - a}{r^n \times \overline{r - 1}} = \frac{a}{r - 1} \times \frac{r^n - 1}{r^n}\right) = \frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}}$. Therefore the sum of all the terms of

 the

the series $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} + \&c. + \frac{a}{r^n}$ is $= \frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}}$. QEI.

CXXXI. And, in like manner, if the same expression $\frac{AA - BE}{A - B}$ be applied to the second series $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$, we shall have $A = \frac{d}{r^2}$, and $B = \frac{d}{r^3}$, and $E = \frac{d}{r^n}$; and consequently $AA = \frac{dd}{r^4}$, and $BE = \frac{dd}{r^{n+3}}$ and $AA - BE = \frac{dd}{r^4} - \frac{dd}{r^{n+3}} \left(= \frac{r^{n+3}dd - r^4 dd}{r^{n+7}}\right) = \frac{r^{n-1}dd - dd}{r^{n+3}}$; and $A - B = \frac{d}{r^2} - \frac{d}{r^3} \left(= \frac{r^3 d - r^2 d}{r^5}\right) = \frac{rd - d}{r^3}$, and $\frac{AA - BE}{A - B} = \dfrac{\frac{r^{n-1}dd - dd}{r^{n+3}}}{\frac{rd - d}{r^3}} \left(= \frac{r^{n-1}dd - dd}{r^{n+3}} \times \frac{r^3}{rd - d} = \frac{r^{n-1}d - d}{r^n \times \overline{r-1}} = \frac{d}{r-1} \times \frac{r^{n-1} - 1}{r^n}\right) = \frac{d}{r-1} \times \overline{\frac{1}{r} - \frac{1}{r^n}}$. Therefore the sum of all the terms of the said second series, $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\frac{1}{r} - \frac{1}{r^n}}$. QEI.

CXXXII. The third series $\frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$ is equal to the second series $\frac{d}{r^2} + \frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$ without the first term $\frac{d}{r^2}$, and therefore is =

$\frac{d}{r-1}$

$$\frac{d}{r-1} \times \overline{\left|\frac{1}{r} - \frac{1}{r^n}\right.} - \frac{d}{r^2} \left(= \frac{r^{n-1}d - d}{r^n \times \overline{r-1}} - \frac{d}{r^2} = \frac{r^{n-1}d - d}{r^n \times \overline{r-1}} - \frac{r^{n-2} \times \overline{r-1} \times d}{r^n \times \overline{r-1}} = \frac{r^{n-1}d - d}{r^n \times \overline{r-1}} - \frac{r^{n-2} \times \overline{rd - d}}{r^n \times \overline{r-1}} = \frac{r^{n-1}d - d}{r^n \times \overline{r-1}} - \frac{r^{n-1}d + r^{n-2}d}{r^n \times \overline{r-1}} = \frac{r^{n-2}d - d}{r^n \times \overline{r-1}} = \frac{d}{r-1} \times \frac{r^{n-2} - 1}{r^n}\right) = \frac{d}{r-1} \times \overline{\left|\frac{1}{rr} - \frac{1}{r^n}\right.}.$$ Q.E.I.

CXXXIII. And the fourth series, $\frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$, is, in like manner, equal to the third series $\frac{d}{r^3} + \frac{d}{r^4} + \frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$ without its first term $\frac{d}{r^3}$, and therefore is $= \frac{d}{r-1} \times \overline{\left|\frac{1}{rr} - \frac{1}{r^n}\right.} - \frac{d}{r^3} \left(= \frac{r^{n-2}d - d}{r^n \times \overline{r-1}} - \frac{d}{r^3} = \frac{r^{n-2}d - d}{r^n \times \overline{r-1}} - \frac{r^{n-3} \times \overline{r-1} \times d}{r^n \times \overline{r-1}} = \frac{r^{n-2}d - d}{r^n \times \overline{r-1}} - \frac{r^{n-3} \times \overline{rd - d}}{r^n \times \overline{r-1}} = \frac{r^{n-2}d - d}{r^n \times \overline{r-1}} - \frac{r^{n-2}d + r^{n-3}d}{r^n \times \overline{r-1}} = \frac{r^{n-3}d - d}{r^n \times \overline{r-1}} = \frac{d}{r-1} \times \frac{r^{n-3} - 1}{r^n} = \frac{d}{r-1} \times \frac{r^{n-3} - 1}{r^n}\right) = \frac{d}{r-1} \times \overline{\left|\frac{1}{r^3} - \frac{1}{r^n}\right.}$. Q.E.I.

CXXXIV. And by proceeding in the same manner we shall find that the fifth series, $\frac{d}{r^5} + \frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\left|\frac{1}{r^4} - \frac{1}{r^n}\right.}$, and that the sixth series, $\frac{d}{r^6} + \frac{d}{r^7} + \&c. + \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\left|\frac{1}{r^5} - \frac{1}{r^n}\right.}$, and that the seventh series, $\frac{d}{r^7} + \frac{d}{r^8} + \&c.$

 $+ \frac{d}{r^n}$,

$+\frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\frac{1}{r^5} - \frac{1}{r^n}}$, and that, in general, the mth series, $\frac{d}{r^m} + \frac{d}{r^{m+1}} +$ &c. $+ \frac{d}{r^n}$, is $= \frac{d}{r-1} \times \overline{\frac{1}{r^{m-1}} - \frac{1}{r^n}}$.

CXXXV. Consequently the value of all the serieses in Art. 129, except the first, that is, of all the serieses whose terms are marked with the sign — and are therefore to be subtracted from the first, or upper, series $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} +$ &c. $+ \frac{a}{r^n}$, will be $= \frac{d}{r-1} \times \overline{\frac{1}{r} - \frac{1}{r^n}} + \frac{d}{r-1} \times \overline{\frac{1}{r^2} - \frac{1}{r^n}} + \frac{d}{r-1} \times \overline{\frac{1}{r^3} - \frac{1}{r^n}} + \frac{d}{r-1} \times \overline{\frac{1}{r^4} - \frac{1}{r^n}} + \frac{d}{r-1} \times \overline{\frac{1}{r^5} - \frac{1}{r^n}} + \frac{d}{r-1} \times \overline{\frac{1}{r^6} - \frac{1}{r^n}}$ + &c. continued to $n-1$ terms, and consequently is $= \frac{d}{r-1} \times$ the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} +$ &c. continued to $n-1$ terms, or to the term $\frac{1}{r^{n-1}}$, $- \frac{d}{r-1} \times \overline{n-1} \times \frac{1}{r^n}$ $= \frac{d}{r-1} \times$ the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6}$ + &c. $+ \frac{1}{r^{n-1}}$, $- \frac{d}{r-1} \times \overline{\frac{n-1}{r^n}}$. Now it has been shewn above in Art. 81, page 92, that the sum of all the terms of the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} +$ &c. continued to the term $\frac{1}{r^n}$, is $= \frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}$. Therefore the sum of all the terms of the same geometrical series continued to the term $\frac{1}{r^{n-1}}$, or to one term less than before, will be $= \frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}} - \frac{1}{r^n}$ $\left(= \frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}\right.$ $- \frac{1 \times \overline{r-1}}{r^n \times \overline{r-1}}$

$-\frac{1 \times \overline{r-1}}{r^n \times \overline{r-1}} = \frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}} - \frac{r+1}{r^n \times \overline{r-1}} = \frac{1}{r-1} - \frac{r}{r^n \times \overline{r-1}})$ $= \frac{1}{r-1} - \frac{1}{r^{n-1} \times \overline{r-1}}$. Therefore $\frac{d}{r-1} \times$ the geometrical series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} +$ &c. $+ \frac{1}{r^{n-1}}$ is $= \frac{d}{r-1} \times \frac{1}{r-1} - \frac{d}{r-1} \times \frac{1}{r^{n-1} \times \overline{r-1}}$. Consequently the sum of all the serieses in Art. 129, except the first series, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4}$ $+$ &c. $+ \frac{a}{r^n}$ is $= \frac{d}{r-1} \times \frac{1}{r-1} - \frac{d}{r-1} \times \frac{1}{r^{n-1} \times \overline{r-1}} - \frac{d}{r-1} \times \frac{n-1}{r^n}$ $= \frac{d}{\overline{r-1}|^2} - \frac{d}{r^{n-1} \times \overline{r-1}|^2} - \frac{nd+d}{r^n \times \overline{r-1}}$. But it has been shewn above, in Art. 130, that the sum of all the terms of the first, or highest, series, $\frac{a}{r} + \frac{a}{r^2} + \frac{a}{r^3} + \frac{a}{r^4} + \frac{a}{r^5} + \frac{a}{r^6} + \frac{a}{r^7} +$ &c. $+ \frac{a}{r^n}$, is $= \frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}}$. Therefore the excess of this first series above the sum of all the rest is $= \frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}} - \frac{d}{\overline{r-1}|^2} +$ [illegible] $\overline{\ldots 1}|^2$ $+ \frac{nd-d}{r^n \times \overline{r-1}}$. Therefore the sum of all the terms of the [illegible] $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6}$ $+$ &c. $+ \frac{a-nd+d}{r^n}$, (which is equal to the said excess of the said first series in Art. 129 above the sum of all the rest,) is also equal to $\frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}} - \frac{d}{\overline{r-1}|^2} + \frac{d}{r^{n-1} \times \overline{r-1}|^2} + \frac{nd-d}{r^n \times \overline{r-1}}$. Q.E.I.

CXXXVI. The

CXXXVI. The foregoing expreſſion may be made ſomewhat more ſimple by means of the following ſubſtitutions.

A reduction of the value of the aforeſaid compound ſeries found in the laſt article to a ſhorter and ſimpler expreſſion.

Let G be equal to $\frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}$, or to the ſum of the terms of the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. continued to n terms, or to the term $\frac{1}{r^n}$; and let H be equal to $G - \frac{1}{r^n}$, or to the ſame geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. continued to the term $\frac{1}{r^{n-1}}$ inſtead of the term $\frac{1}{r^n}$. Then will the foregoing expreſſion $\frac{a}{r-1} \times \overline{1 - \frac{1}{r^n}} - \frac{d}{\overline{r-1}|^2} + \frac{d}{r^{n-1} \times \overline{r-1}|^2} + \frac{nd-d}{r^n \times \overline{r-1}}$ be $= aG - \frac{dH}{r-1} + \frac{nd-d}{r^n \times \overline{r-1}} = aG - \frac{d}{r-1} \times \overline{G - \frac{1}{r^n}} + \frac{nd-d}{r^n \times \overline{r-1}} = aG - \frac{dG}{r-1} + \frac{d}{r^n \times \overline{r-1}} + \frac{nd-d}{r^n \times \overline{r-1}} = aG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}}$. Therefore the compound ſeries $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} +$ &c. $+ \frac{a-nd+d}{r^n}$ is $= aG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}}$. Q.E.I.

CXXXVII. I will now proceed to apply this expreſſion to the computation of a part of the ſeries $\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} +$ &c. in the caſe of an annuity for a life of the age of 10 years according to Monſieur de Parcieux's table of the probabilities of the duration of human life; of the value of which annuity we have already exhibited the full computation above in Art. 94.

An

An example of the summation of a compound series of this form, to wit, $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7}$ + *&c. continued to n terms, or to the term* $\frac{a-nd+d}{r^n}$, *by means of the expression* $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}}$; *in which the capital letter G is put for* $\frac{1}{r-1} \times \frac{1}{r^n \times \overline{r-1}}$, *or the sum of the terms of the geometrical progression* $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + *&c. continued to n terms, or to the term* $\frac{1}{r^n}$.

CXXXVIII. The tenth, eleventh, twelfth, and other following terms of the series $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7}$ + &c. in Art. 94, as far as the 27th term, or $\frac{P^{XXVII}}{r^{27}}$, are as follows, to wit, $\frac{814}{r^{10}} + \frac{806}{r^{11}} + \frac{798}{r^{12}} + \frac{790}{r^{13}} + \frac{782}{r^{14}} + \frac{774}{r^{15}} + \frac{766}{r^{16}} + \frac{758}{r^{17}} + \frac{750}{r^{18}} + \frac{742}{r^{19}} + \frac{734}{r^{20}} + \frac{726}{r^{21}} + \frac{718}{r^{22}} + \frac{710}{r^{23}} + \frac{702}{r^{24}} + \frac{694}{r^{25}} + \frac{686}{r^{26}} + \frac{678}{r^{27}}$, in which the numerators 814, 806, 798, 790, &c. decrease continually by the same common difference, 8. This series is equal to the product of $\frac{1}{r^9}$ × the series $\frac{814}{r} + \frac{806}{r^2} + \frac{798}{r^3} + \frac{790}{r^4} + \frac{782}{r^5} + \frac{774}{r^6} + \frac{766}{r^7} + \frac{758}{r^8} + \frac{750}{r^9} + \frac{742}{r^{10}} + \frac{734}{r^{11}} + \frac{726}{r^{12}} + \frac{718}{r^{13}} + \frac{710}{r^{14}} + \frac{702}{r^{15}} + \frac{694}{r^{16}} + \frac{686}{r^{17}} + \frac{678}{r^{18}}$; and this

this last series is of the same form with the series $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7}$ + &c. $+ \frac{a-nd+d}{r^n}$, which is the subject of the foregoing articles; and consequently it may be summed by means of the expression $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$. This may be done in the following manner.

CXXXIX. Here r is $= 1.035$; and n, (or the number of terms in the series,) is 18; and a, (the numerator of the first term,) is 814; and d, (the common difference of the numerators,) is 8. Therefore the expression $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times r-1}$ is $= 814G - \frac{8G}{.035} + \frac{18 \times 8}{1.035|^{18} \times .035}$ $= 814G - 228.571,428\,G + \frac{144}{1.035|^{18} \times .035} = 585.428,572\,G + \frac{4114.285,714}{1.035|^{18}}$. Now it appears from Mr. Smart's first table of compound interest, page 52, that the 18th power of 1.035 is 1.857,489; and it appears by his fourth table of compound interest, page 76, that G, or the value of the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7}$ + &c. $+ \frac{1}{r^{18}}$, when r is $= 1.035$, is 13.189.681. Therefore $585.428,572\,G + \frac{4114.285,714}{1.035|^{18}}$ is $= 585.428,572 \times 13.189,681 + \frac{4114.285,714}{1.857,489} = 7721.616,112 + 2214.971,778 = 9936.587,890$. Therefore the series $\frac{814}{r} + \frac{806}{r^2} + \frac{798}{r^3} + \frac{790}{r^4} + \frac{782}{r^5} + \frac{774}{r^6} + \frac{766}{r^7} + \frac{758}{r^8} + \frac{750}{r^9} + \frac{742}{r^{10}} + \frac{734}{r^{11}} + \frac{726}{r^{12}} + \frac{718}{r^{13}} + \frac{710}{r^{14}} + \frac{702}{r^{15}} + \frac{694}{r^{16}} + \frac{686}{r^{17}} + \frac{678}{r^{18}}$ is equal to 9936.587,890. Consequently

quently the series $\frac{814}{r^{10}} + \frac{806}{r^{11}} + \frac{798}{r^{12}} + \frac{790}{r^{13}} + \frac{782}{r^{14}} + \frac{774}{r^{15}} + \frac{766}{r^{16}} + \frac{758}{r^{17}} + \frac{750}{r^{18}} + \frac{742}{r^{19}} + \frac{734}{r^{20}} + \frac{726}{r^{21}} + \frac{718}{r^{22}} + \frac{710}{r^{23}} + \frac{702}{r^{24}} + \frac{694}{r^{25}} + \frac{686}{r^{26}} + \frac{678}{r^{27}}$ is $= 9936.587,890 \times \frac{1}{r^9} = \frac{9936.587,890}{\overline{1.035}|^{9}}$ = (by Mr. Smart's first table of compound interest, page 52,) $\frac{9936.587,890}{1.362,897} = 7290.784,182$. Q E I.

CXL. If the terms of this last series be computed separately, (as they have been above in Art. 94) they will be equal to the following numbers, to wit, 577.05274 + 552.06164 + 528.10044 + 505.12600 + 483.10396 + 461.99286 + 441.75220 + 422.35760 + 403.77000 + 385.95130 + 368.87904 + 352.52382 + 336.84970 + 321.82880 + 307.44090 + 293.65916 + 280.45738 + 267.81678; the sum of which is 7290.72432, which agrees with the foregoing value of them found by means of the expression $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}}$ together with the multiplication by $\frac{1}{r^9}$, to wit, 7290.784,182, to five places of figures, and thereby confirms the truth of the said expression. See these numbers, 577.05274 + 552.06164 + &c. above in Art. 94, pages 105, 106.

A particular case of the foregoing compound series.

CXLI. When a, (or the numerator of the first and greatest term of the series $\frac{a}{r} + \frac{a-d}{r^2} + \frac{a-2d}{r^3} + \frac{a-3d}{r^4} + \frac{a-4d}{r^5} + \frac{a-5d}{r^6} + \frac{a-6d}{r^7} +$ &c. $+ \frac{a-nd+d}{r^n}$) is an exact multiple of d (the common difference of the numerators of the several terms,) and the series is continued so far as to make $a - nd + d$, or $a - \overline{n-1}| \times d$, (the numerator of its last term) equal to a, or $\overline{n-1}| \times d$ equal to a, the series will

be

be as follows, to wit, $\frac{nd-d}{r} + \frac{nd-d-d}{r^2} + \frac{nd-d-2d}{r^3} + \frac{nd-d-3d}{r^4} + \frac{nd-d-4d}{r^5} + \frac{nd-d-5d}{r^6} + \frac{nd-d-6d}{r^7} + \&c. + \frac{nd-d-nd+d}{r^n}$, or $\frac{nd-d}{r} + \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} + \frac{nd-7d}{r^7} + \&c. + \frac{nd-nd}{r^n}$, or $+\frac{0}{r^n}$; the laſt term $\frac{nd-nd}{r^n}$, being equal to 0, and being here ſet down only for the ſake of analogy and uniformity, and to make the ſeries appear to conſiſt of n terms, as it did in the former caſe, though in reality it contains in this caſe only $n-1$ terms. For ſuch a preſervation of the general forms of expreſſions in caſes that do not ſtrictly come under them, is often found to be of uſe in theſe ſciences.

The expreſſion of its ſum in that caſe.

CXLII. Now, when a is thus $= nd-d$, the expreſſion $aG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}}$, obtained in Art. 136, will be $= \overline{nd-d} \times G - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}} = ndG - dG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}} = nd \times \overline{\frac{1}{r-1} - \frac{1}{r^n \times \overline{r-1}}} - dG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}} = \frac{nd}{r-1} - \frac{nd}{r^n \times \overline{r-1}} - dG - \frac{dG}{r-1} + \frac{nd}{r^n \times \overline{r-1}} = \frac{nd}{r-1} - dG - \frac{dG}{r-1} = \frac{nd}{r-1} - \frac{dG \times \overline{r-1}}{r-1} - \frac{dG}{r-1} = \frac{nd}{r-1} - \frac{rdG+dG}{r-1} - \frac{dG}{r-1} = \frac{nd-rdG}{r-1}$. Therefore the ſeries $\frac{nd-d}{r} + \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} + \frac{nd-7d}{r^7} + \&c. + \frac{nd-nd}{r^n}$, or $+\frac{0}{r^n}$, is equal to $\frac{nd-rdG}{r-1}$, or to $d \times \overline{\frac{n-rG}{r-1}}$; which is a very ſhort and ſimple expreſſion.

CXLIII. And

CXLIII. And, if d, the common difference of the numerators of the terms in the last-mentioned series $\frac{nd-d}{r} + \frac{nd-2d}{r^2} + \frac{nd-3d}{r^3} + \frac{nd-4d}{r^4} + \frac{nd-5d}{r^5} + \frac{nd-6d}{r^6} + \frac{nd-7d}{r^7}$ + &c. $+ \frac{nd-nd}{r^n}$, is $= 1$, both the said series itself and the expression $\frac{nd-rdG}{r-1}$ for the sum of all its terms, will be thereby still further simplified. For the series will then be $= \frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7}$ + &c. $+ \frac{n-n}{r^n}$; and the expression $\frac{nd-rdG}{r-1}$ will be $= \frac{n-rG}{r-1}$. And consequently the series $\frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7}$ + &c. $+ \frac{n-n}{r^n}$ is $= \frac{n-rG}{r-1}$.

A reduction of the series obtained in Art. 141 to a more simple form.

The expression of the sum of the said reduced series.

CXLIV. Therefore, if we divide both the last series $\frac{n-1}{r} + \frac{n-2}{r^2} + \frac{n-3}{r^3} + \frac{n-4}{r^4} + \frac{n-5}{r^5} + \frac{n-6}{r^6} + \frac{n-7}{r^7}$ + &c. $+ \frac{n-n}{r^n}$ and the expression $\frac{n-rG}{r-1}$, (to which it is equal) by n, the quotients thence arising will be equal; that is, the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7}$ + &c. $+ \frac{n-n}{nr^n}$ is $\left(= \frac{\frac{n}{n} - \frac{r}{n}G}{r-1}\right) = \frac{1 - \frac{r}{n}G}{r-1}$.

An expression for the sum of a series derived from the series in the last article by dividing its terms by n.

 CXLV. If

Mr. De Moivre's expression for the sum of the terms of the last series.

CXLV. If the letter P be used, instead of the letter G, to denote the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c. + \frac{1}{r^n}$, the expression $\frac{1 - \frac{rP}{n}}{r-1}$ will be equal to the sum of all the terms of the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} + \&c. + \frac{n-n}{nr^n}$.

CXLVI. This last expression $\frac{1 - \frac{rP}{n}}{r-1}$ is that which Mr. De Moivre has given us for the purpose of summing the last series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \&c. + \frac{n-n}{nr^n}$ in his treatise of Annuities on Lives, Part 2d, pages 310, 311, of the third edition published in quarto, at the end of his treatise on the doctrine of Chances, in the year 1756. And in the same place he has given us a synthetick demonstration of the truth of this expression, or of its equality with the said series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \&c. + \frac{n-n}{nr^n}$. But he has not informed us by what steps, or in what manner, he discovered that this expression was equal to the said series, but has alledged as an excuse for this omission, that *the reasonings that led him to this expression require something more than an ordinary skill in the doctrine of serieses.* He probably therefore had deduced it from some more abstruse propositions in that doctrine than that which we have had recourse to for the same purpose in the foregoing articles, which is only the well-known theorem that was demonstrated in Art. 80, that the sum of the terms of a decreasing geometrical progression, $A + B + C + D + E$, is equal to the fraction $\frac{AA - BE}{A - B}$. His synthetick demonstration of the truth of this expression is for the most part very clear and satisfactory. But in the last step

He has given us a synthetick demonstration of the truth of the said expression.

Observations on the said synthetick demonstration.

ſtep of it, I think there is ſome obſcurity. For, inſtead of dividing the ſeries $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} - \frac{1}{nr^7} - \&c. - \frac{1}{nr^{n-1}}$ (which conſiſts of only a finite number of terms, to wit, n terms,) by $r-1$, as the courſe of the demonſtration requires, he multiplies it by the infinite ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ *ad infinitum*, which is equal to $\frac{1}{r-1}$. This multiplication is certainly equivalent to a diviſion by $r-1$; and, I imagine, Mr. De Moivre conſidered it as a ſomewhat ſimpler and eaſier operation. But in this he ſeems on the preſent occaſion to have been miſtaken. For it is hardly, if at all, eaſier than the diviſion by that very ſhort and convenient diviſor, $r-1$; and it is certainly much leſs ſimple, becauſe it introduces to the reader's notice a ſeries conſiſting of an infinite number of terms in a matter relating to a ſeries conſiſting of only a finite number of terms. And it is natural to ſuppoſe that the product ariſing by the multiplication of a finite ſeries by an infinite one muſt be an infinite ſeries: and ſo in truth it muſt, unleſs, after a certain number of terms in the firſt part of it, the ſeveral numbers, that compoſe each of the ſubſequent terms, are equal to each other and marked with contrary ſigns + and —, ſo as thereby to deſtroy one another and make the whole term equal to o; in which caſe the ultimate product (when properly collected by the addition of all the ſeparate products, or horizontal lines of terms, of which it is compoſed,) may poſſibly be a ſeries conſiſting of a finite number of terms, notwithſtanding the number of the ſeveral products, of which it is compoſed, and conſequently the number of terms in thoſe products, may be infinite. And this is in truth the caſe with the ſeries that is the product of the multiplication of the finite ſeries $\frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \frac{1}{nr^4} - \frac{1}{nr^5} - \frac{1}{nr^6} - \&c. - \frac{1}{nr^{n-1}}$ by the infinite ſeries $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} + \&c.$ *ad infinitum*. Though the number of the ſeparate products of which it is compoſed, and conſequently the number of terms in thoſe products, is infinite, it is nevertheleſs a finite ſeries, or conſiſts of only a finite number of terms; as will appear upon a cloſe and attentive examination of its nature and conſtitution. But Mr. De Moivre does not ſhew that it will be ſo, nor even aſſert that the number of its terms will not be infinite, though the number of thoſe of one of its factors is ſo: inſomuch

much that a reader who does not consider the subject with more than ordinary care, when he has gone through the whole demonstration, and is come to the last series $\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}+\frac{n-5}{nr^5}+\frac{n-6}{nr^6}+\frac{n-7}{nr^7}+$ &c. may very well be at a loss to know, whether the number of its terms be finite or infinite. But, if Mr. De Moivre had added the necessary observations to explain this matter and to shew that the number of terms in the said ultimate series will only be finite, it would still have been an impropriety to introduce an infinite series in a demonstration relating to a finite one, when it might be so easily avoided. I shall therefore, in presenting my reader with Mr. De Moivre's synthetick demonstration of this proposition, take the liberty of changing that step in it, and dividing the series $\frac{n-1}{n}-\frac{1}{nr}-\frac{1}{nr^2}-\frac{1}{nr^3}-\frac{1}{nr^4}-\frac{1}{nr^5}-\frac{1}{nr^6}-$ &c. $-\frac{1}{nr^{n-1}}$ by $r-1$, instead of multiplying it by the infinite series $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+\frac{1}{r^6}+\frac{1}{r^7}+$ &c. *ad infinitum.* With this single change that demonstration is as follows.

CXLVII. *A synthetick demonstration that the expression* $\dfrac{1-\frac{rP}{n}}{r-1}$ (*in which* P *denotes the sum of the terms of the geometrical progression* $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+\frac{1}{r^6}+\frac{1}{r^7}+$ &c. $+\frac{1}{r^n}$) *is equal to the sum of the terms of the series* $\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}+\frac{n-5}{nr^5}+\frac{n-6}{nr^6}+\frac{n-7}{nr^7}+$ &c. $+\frac{n-n}{nr^n}$.

Since P is $=\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+\frac{1}{r^6}+\frac{1}{r^7}+$ &c. $+\frac{1}{r^n}$, we shall have $rP=\frac{r}{r}+\frac{r}{r^2}+\frac{r}{r^3}+\frac{r}{r^4}+\frac{r}{r^5}+\frac{r}{r^6}+\frac{r}{r^7}$

$+\frac{r}{r^7}+\&c.+\frac{r}{r^n}=1+\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+\frac{1}{r^6}+\&c.+\frac{1}{r^{n-1}}$, and $\frac{r}{n}P=\frac{1}{n}+\frac{1}{nr}+\frac{1}{nr^2}+\frac{1}{nr^3}+\frac{1}{nr^4}+\frac{1}{nr^5}+\frac{1}{nr^6}+\&c.+\frac{1}{nr^{n-1}}$.

Therefore $1-\frac{rP}{n}$ is $=1-\frac{1}{n}-\frac{1}{nr}-\frac{1}{nr^2}-\frac{1}{nr^3}-\frac{1}{nr^4}-\frac{1}{nr^5}-\frac{1}{nr^6}-\&c.-\frac{1}{nr^{n-1}}=\frac{n-1}{n}-\frac{1}{nr}-\frac{1}{nr^2}-\frac{1}{nr^3}-\frac{1}{nr^4}-\frac{1}{nr^5}-\frac{1}{nr^6}-\&c.-\frac{1}{nr^{n-1}}$; and, dividing both sides by $r-1$,

$$\frac{1-\frac{rP}{n}}{r-1}=\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}+\frac{n-5}{nr^5}+\frac{n-6}{nr^6}+\frac{n-7}{nr^7}+\&c.+\frac{n-n}{nr^n}.$$ Q.E.D.

CXLVIII. The division of the series $\frac{n-1}{n}-\frac{1}{nr}-\frac{1}{nr^2}-\frac{1}{nr^3}-\frac{1}{nr^4}-\frac{1}{nr^5}-\frac{1}{nr^6}-\&c.-\frac{1}{nr^{n-1}}$ by $r-1$ may be performed as follows.

$r-1)$

The operation of division mentioned in the foregoing article.

$$
\begin{array}{l}
r-1\,\Big)\ \frac{n-1}{n} - \frac{1}{nr} - \frac{1}{nr^2} - \frac{1}{nr^3} - \&c. \ \Big(\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \&c. \\
\qquad \frac{n-1}{n} - \frac{n-1}{nr} \\
\qquad * + \frac{n-2}{nr} - \frac{1}{nr^2} \\
\qquad\quad + \frac{n-2}{nr} - \frac{n-2}{nr^2} \\
\qquad\qquad * + \frac{n-3}{nr^2} - \frac{1}{nr^3} \\
\qquad\qquad\quad + \frac{n-3}{nr^2} - \frac{n-3}{nr^3} \\
\qquad\qquad\qquad * + \frac{n-4}{nr^3}
\end{array}
$$

Of the use made of the foregoing expression by Mr. De Moivre in the computation of life-annuities.

CXLIX. Mr. De Moivre, in his treatise of Life-annuities, makes great use of this expression $\frac{1-\frac{rP}{n}}{r-1}$. For he employs it in the solution of his first and most important problem, or the computation of the value of an annuity upon a life of any given age. And with this view he adopts an hypothesis, concerning the probabilities of the duration of human life, which is fitted to make that expression applicable to his purpose, and which he conceives to be so little different from the real estimate of those probabilities, as deduced from observations by Dr. Halley, Monsieur de Parcieux, and others, that it will make the values of life-annuities computed by it very nearly the same as they would be, if computed strictly from tables founded on observations in the manner that has been above explained. This hypothesis is as follows.

Of Mr. De Moivre's Hypothesis concerning the decrements of human life.

CL. We have seen above, in Art. 126, that, both in Mr. Kerseboom's and Monsieur de Parcieux's tables of the probabilities of the duration of human life, the numbers of persons dying in a year, or the decrements of human life, (as they are called,) continue the same for several years together in several different stages, or periods, of life from the age of 11 or 12 years to the age of 75 or 80 years. And in this interval (from the age of 11 or 12 years to the age of 75 or 80 years,) the number of persons dying in a year during one of these stages, or periods, is not either constantly greater, or constantly less, than the number of those who die in a year during the next older period of life, but in some parts of human life is greater than the said next number, and in other parts of it is less. Thus, for example, in Monsieur de Parcieux's table of probabilities, it appears that from the age of 11 years to the age of 16 years the number of persons who die in every year is 6; and from the age of 16 years to the age of 20 years the number of persons who die in every year is 7; and from the age of 20 years to the age of 37 years, the number of persons who die in every year is 8: and thus far the number of persons dying every year in these three successive stages, or periods, of human life, is found to increase as the period of human life advances, or becomes older. But in the next period of life, namely, from the age of 37 years to the age of 46 years, the number of persons dying in a year is only 7; after which, in the interval between the age of 46 years and the age of 75 years, it again increases to 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20 persons in a year; and then it decreases again to 19 and 18 persons in the next five years, or from the age of 75 years to the age of 80 years. And the same variation, from a greater number to a lesser, and from a lesser number to a greater, may be observed in the decrements of human life, or the number of persons dying in a year, in different stages, or periods, of it, in Mr. Kerseboom's table above-mentioned, and in Dr. Halley's table of probabilities deduced from the observations made at Breslaw, and in several other tables of the same nature.

The decrements of human life in some periods of life become greater, and in others become less, as the age of life advances.

CLI. From this observation on the course of the decrements of human life Mr. De Moivre was led to conjecture that, if the said decrements were supposed to be equal in every year, throughout the whole extent of life, as well as during particular periods of it, and the values of life-annuities were computed upon that supposition, they would be very nearly equal to the

Mr. De Moivre's conjectural supposition derived from the foregoing observation.

the true values of them when computed in ſtrict conformity to Dr. Halley's ſaid Breſlaw table, or any other table of the probabilities of the duration of human life that was grounded on actual obſervations. For, as, by ſuch a reduction of the irregular variation of the decrements of life to an abſolute equality of the ſaid decrements throughout life, the values of ſome of the future payments of an annuity for any given age, in ſome particular years of life, would be made greater than they ought to be, ſo, it is evident, the values of ſome others of the future payments of it, in other years of human life, would be made leſs than they ought to be; and theſe contrary variations in the values of different future payments of the annuity in different parts of human life, Mr. De Moivre ſuppoſed, would pretty nearly counter-balance each other, and make the ſum of all the ſaid future payments, or the value of the whole annuity throughout the whole extent of life, pretty nearly the ſame as it would have been if it had been regularly computed in ſtrict conformity to Dr. Halley's, or ſome other, table of the probabilities of human life derived from actual obſervations, in the manner above deſcribed and exemplified in Art. 94. And he tells us that, upon trial, he found this conjecture to be well-founded: for that, having computed the value of an annuity for a life of a certain age upon the ground of this hypotheſis, and compared it with the value of the ſame annuity for the ſame life, as computed by Dr. Halley in ſtrict conformity to the Breſlaw table of probabilities, he found theſe values to be ſo very little different from each other that, for all uſeful and practical purpoſes, they might well be conſidered as the ſame, and conſequently that, in the buſineſs of computing the values of life-annuities, he might ſafely neglect the tables of probabilities deduced from obſervations, and proceed upon the ground of his own Hypotheſis.

CLII. Mr. De Moivre does not inform us what was the age of the life for which he calculated an annuity upon the ground of his Hypotheſis, in order to compare it with Dr Halley's value of the ſame annuity computed ſtrictly from the Breſlaw table of probabilities. But it ſeems not unlikely to have been an annuity for a life of the age of 10 years, that being one of thoſe which Dr. Halley had computed and ſet down in his tract above-mentioned, and being in ſome degree fitter than the others there ſet down for the purpoſe of the ſaid compariſon. The ſeveral annuities there ſet down as having been accurately computed from the Breſlaw table of probabilities, are for lives of the ages of one year, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70, years. They were computed upon a ſuppoſition that the intereſt of money was 6 per cent. as at that time (in the year, 1692,) it generally was. This little table of the values of life-annuities is as follows.

Of Dr. Halley's table of the values of life-annuities for lives of certain ages, computed from the Breſlaw table of probabilities.

TABLE

TABLE VI.

Containing the values of an annuity of one pound a year for the lives of persons of the ages of 1 year, 5 years, 10 years, 15 years, 20 years, and every following age of human life that exceeds the next preceeding age by 5 years, as far as the age of 70 years: computed by Dr. Halley from the Breslaw table of the probabilities of the duration of human life, upon a supposition that the interest of money is 6 per cent.

Years of Age.	*Values of an annuity of one pound.*	*Years of Age.*	*Values of an annuity of one pound.*
	£		£
1	10.28	40	10.57
5	13.40	45	9.91
10	13.44	50	9.21
15	13.33	55	8.51
20	12.78	60	7.60
25	12.27	65	6.54
30	11.72	70	5.32
35	11.12		

This was, I believe, the first table of the values of life-annuities that ever was published.

 CLIII. That

Of the manner of comparing the conjectural probabilities of the duration of human life, resulting from Mr. De Moivre's Hypothesis, with the real probabilities of the same as exhibited in Monsieur de Parcieux's table.

CLIII. That we may be the better able to judge of the degree in which Mr. De Moivre's Hypothesis above-mentioned, concerning the decrements of human life, approaches to, or varies from, the real decrements of life, as represented in Monsieur de Parcieux's table of probabilities, it will be proper to form a new table of the numbers of persons that would be living at the ends of the several years of human life after the age of 3 years, (which is the youngest age in Monsieur de Parcieux's table,) out of the same original number of persons living at the said age of 3 years as are represented in Monsieur de Parcieux's table to be living at that age, that is, out of 1000 persons of the age of 3 years, if the whole number were to die in the course of the same number of years in which they are represented to be all dead in Monsieur de Parcieux's table, that is, in the course of 92 years, but were to die in equal numbers in every year of that period, instead of dying in the numbers represented as dying in the several years of the same period in Monsieur de Parcieux's table. Now for this purpose it is necessary, in the first place, to divide the whole original number of persons represented in Monsieur de Parcieux's table as living at the age of 3 years, that is, 1000, by 92, or the number of years in which all the said 1000 lives are there represented to become extinct; because, it is evident, the quotient of this division will give us the number of persons who would die in every year of the said period of 92 years, upon Mr. De Moivre's supposition that that number were the same in every year. This quotient is 10.8697, which consists of the whole number 10 and the decimal fraction .8697; which shews that it is not possible that the number of persons dying every year in the said period of 92 years, (from the age of 3 years to the age of 95 years,) out of the said original number of 1000 persons living at the age of 3 years, should be always exactly the same. But, if we somewhat increase this original number, and make it 1012 instead of 1000, we shall thereby render it capable of an exact division by 92, and the quotient will be 11. And this variation, I apprehend, will not be of much importance, notwithstanding we do not suppose the other numbers in Monsieur de Parcieux's table of probabilities to be increased at the same time in the same proportion of 1012 to 1000, but to continue as before. For, if a table of the values of life-annuities were to be calculated from Monsieur de Parcieux's table of probabilities so altered in its first number only, all the said values would be precisely the same as before, except the value of an annuity for a life of the age of 3 years, which would be less than before in the proportion of $\frac{1}{1012}$ to $\frac{1}{1000}$, or 1000 to 1012. This variation, therefore, in the first number of Monsieur de Parcieux's said table may well enough be made on the present occasion for the sake of avoiding the inconvenience and obscurity that might arise from

from reprefenting the number of perfons who are fuppofed to die every year according to Mr. De Moivre's Hypothefis by a number that involves a fraction, fuch as the quotient 10.8697 above found.

CLIV. Let us therefore fuppofe this variation to be made in the firft number of Monfieur de Parcieux's table of probabilities, but that all the other numbers in it continue as before. And from the firft number of this table fo increafed, or from the number 1012, (which contains the number 11 exactly 92 times,) let us continually fubtract the number 11, till the faid number 1012 be reduced to nothing. The table of numbers thence arifing will reprefent the numbers of perfons fuppofed to be living at the ages of 4 years, 5 years, 6 years, and every following age of life that exceeds the age next preceeding by one year, as far as the age of 95 years, according to Mr. De Moivre's Hypothefis. This table, together with Monfieur de Parcieux's table, with its firft number increafed from 1000 to 1012, as aforefaid, will be as follows.

TABLE

TABLE VII.

Containing Monsieur de Parcieux's table of the probabilities of the duration of human life, from the age of 3 *years to the age of* 95 *years, with the first number of it increased from* 1000 *to* 1012; *and likewise an artificial table fitted to the same original number* 1012, *in which the decrements of human life, or persons dying in a year, are supposed to be always the same throughout the said period of* 92 *years, or from the age of* 3 *years to the age of* 95 *years, agreeably to Mr. De Moivre's Hypothesis.*

Years of Age.	*Persons living according to Monsieur de Parcieux's table, with an increase of the first number from* 1000 *to* 1012.	*Persons living according to Mr. De Moivre's Hypothesis.*	*Years of Age.*	*Persons living according to Monsieur de Parcieux's table.*	*Persons living according to Mr. De Moivre's Hypothesis.*
3	1012	1012	26	766	759
4	970	1001	27	758	748
5	948	990	28	750	737
6	930	979	29	742	726
7	915	968	30	734	715
8	902	957	31	726	704
9	890	946	32	718	693
10	880	935	33	710	682
11	872	924	34	702	671
12	866	913	35	694	660
13	860	902	36	686	649
14	854	891	37	678	638
15	848	880	38	671	627
16	842	869	39	664	616
17	835	858	40	657	605
18	828	847	41	650	594
19	821	836	42	643	583
20	814	825	43	636	572
21	806	814	44	629	561
22	798	803	45	622	550
23	790	792	46	615	539
24	782	781	47	607	528
25	774	770	48	599	517

Years of Age.	Persons living according to Monsieur de Parcieux's table.	Persons living according to Mr. De Moivre's Hypothesis.	Years of Age.	Persons living according to Monsieur de Parcieux's table.	Persons living according to Mr. De Moivre's Hypothesis.
49	590	506	73	251	242
50	581	495	74	231	231
51	571	484	75	211	220
52	560	473	76	192	209
53	549	462	77	173	198
54	538	451	78	154	187
55	526	440	79	136	176
56	514	429	80	118	165
57	502	418	81	101	154
58	489	407	82	85	143
59	476	396	83	71	132
60	463	385	84	59	121
61	450	374	85	48	110
62	437	363	86	38	99
63	423	352	87	29	88
64	409	341	88	22	77
65	395	330	89	16	66
66	380	319	90	11	55
67	364	308	91	7	44
68	347	297	92	4	33
69	329	286	93	2	22
70	310	275	94	1	11
71	291	264	95	0	0
72	271	253			

An observation on the foregoing table of the real and conjectural probabilities of the duration of human life.

CLV. It appears from the foregoing table, that, from the age of 3 years to the age of 24 years, the numbers of persons represented as living at the end of every year are greater in the arithmetical progression 1012, 1001, 990, 979, &c. formed upon Mr. De Moivre's Hypothesis, than in Monsieur de Parcieux's series of numbers 1012, 970, 948, 930, 915, &c. and that at the age of 24 years they are nearly the same in both serieses of numbers, being 781 and 782; and that, from the age of 24 years to the age of 74 years, the numbers in Mr. De Moivre's series are less than those in Monsieur de Parcieux's; and that at the age of 74 years they are equal in both serieses, being 231 in both serieses; and that then, after the age of 74 years, Mr. De Moivre's numbers become a second time greater than Monsieur de Parcieux's, and continue so to the end of the table.

A conclusion from the foregoing observation concerning the value of an annuity for a life of the age of 74 years.

CLVI. It follows from the foregoing observation that, if the value of an annuity for a life of 74 years were to be computed according to both these serieses of numbers, the value of it derived from Mr. De Moivre's numbers would be greater than the value of it derived from Monsieur de Parcieux's numbers. For the number P, or the number of persons living at the given age, would be the same in both cases, to wit, 231; and the numbers P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. would be greater in the calculation grounded on Mr. De Moivre's series than in that which was built on Monsieur de Parcieux's series: and consequently $\frac{\pounds 1}{P} \times$ the series

$$\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} + \frac{P^{VII}}{r^7} + \&c.$$

(which, by Art. 87, is equal to the value of the annuity for the given life,) would be greater in the former case than in the latter.

Conclusions from the same observation concerning the values of annuities for lives of other ages.

CLVII. But, if we were to compute the value of an annuity for 50 years, and no longer, dependent upon a life of the age of 24 years, by means of these two serieses of numbers, the value of such an annuity derived from Mr. De Moivre's numbers would be less than the other value of it derived from Monsieur de Parcieux's numbers. For in this case the value of P in the former calculation would be 781, and in the latter it would be 782, which is almost equal to 781; and the values of P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. in the former calculation would be 770, 759, 748, 737, 726, 715, 704, &c. and in the latter calculation they

they would be 774, 766, 758, 750, 742, 734, 726, &c. which are respectively greater than the other numbers: and consequently the expression £

$$\frac{1}{P} \times \text{the series } \frac{P^{I}}{r} + \frac{P^{II}}{r^{2}} + \frac{P^{III}}{r^{3}} + \frac{P^{IV}}{r^{4}} + \frac{P^{V}}{r^{5}} + \frac{P^{VI}}{r^{6}} + \frac{P^{VII}}{r^{7}} + \&c.$$

when derived from Mr. De Moivre's numbers, will be less than the same expression, when derived from Monsieur de Parcieux's numbers, or the value of an annuity for 50 years, dependent upon a life of the age of 24, will be less according to Mr. De Moivre's numbers than according to Monsieur de Parcieux's numbers.

CLVIII. But, if we continue the last annuity for a life of the age of 24 years to the end of life, or to 70 years from its commencement, instead of restraining it to 50 years, the values of the future contingent payments of it to be made in the last 20 years of its duration, or from the age of 74 to the age of 94 years, will be greater according to Mr. De Moivre's numbers than according to Monsieur de Parcieux's. And consequently the value of a compleat life-annuity for a person of the age of 24 years, computed from Mr. De Moivre's numbers, will differ less from the value of it, computed from Monsieur de Parcieux's numbers, than the values of a limited annuity for only 50 years, depending upon the same life, computed according to the same two sets of numbers, differ from each other; because the excess of the values of the future payments to be made after the age of 74 years according to Mr. De Moivre's numbers, above the values of those payments according to Monsieur de Parcieux's numbers, will, in some degree, make amends for the deficiencies of the values of the preceeding future payments, from the age of 24 years to the age of 74 years, according to Mr. De Moivre's numbers, below the values of the same payments according to Monsieur de Parcieux's numbers. But it will only do this *in some degree*, and not by any means compleatly; because the values of the 20 last future payments of the annuity from the age of 74 years to the age of 94 years, are so much less (on account of the distance of time at which those payments are to be made,) than the values of the 50 preceeding future payments of it from the age of 24 years to the age of 74 years, that it does not greatly signify towards obtaining the true value of the whole annuity for a life of the age of 24 years, by which of the two serieses of numbers they are computed. And therefore, upon the whole, the value of a compleat life-annuity for a person of the age of 24 years, computed according to Mr. De Moivre's numbers, will still be considerably less than its value, computed according to Monsieur de Parcieux's numbers, or than its true value. And, in general, it will be found that the value of an annuity for a life of any age under 45 years, computed according to Mr. De Moivre's numbers, will be less than its value computed from Monsieur de Parcieux's numbers, or its true value. But, that we may be the better able to judge of this matter, I will now proceed to compute a small table of the values of life-annuities (such as that which was computed exactly from the Breslaw table of probabilities by Dr. Halley, and which is inserted above in Art. 152,) according to Mr. De Moivre's numbers, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. which values we may afterwards compare with the values of the same annuities, as computed above (in Art. 108, 109, &c.—114) from the numbers of Monsieur de Parcieux's table.

The numbers of persons living according to Mr. De Moivre's Hypothesis in Table VII. may be reduced to smaller numbers.

CLIX. As the numbers in Mr. De Moivre's series above-mentioned are, all of them, exact multiples of 11, they will all be divisible by 11 without any remainders, and consequently may be reduced to smaller numbers without altering their proportions to one another. Let them all be so divided by 11; and the numbers thence arising will be as follows, to wit, 92, 91, 90, 89, 88, 87, 86, 85, 84, 83, 82, 81, 80, 79, 78, 77, 76, 75, 74, 73, 72, 71, 70, 69, 68, 67, 66, 65, 64, 63, 62, 61, 60, 59, 58, 57, 56, 55, 54, 53, 52, 51, 50, 49, 48, 47, 46, 45, 44, 43, 42, 41, 40, 39, 38, 37, 36, 35, 34, 33, 32, 31, 30, 29, 28, 27, 26, 25, 24, 23, 22, 21, 20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, which is an arithmetical progression of the simplest nature possible, in which the terms continually decrease by the same common difference 1. Yet, as these numbers are to each other in exactly the same proportions as the former numbers 1012, 1001, 990, 979, 968, 957, 946, &c. (of which they are exact 11th parts,) it is evident that the values of any life-annuities of one pound deduced from the general expression $\frac{\pounds 1}{P} \times$ the series $\frac{P^{\mathrm{I}}}{r} + \frac{P^{\mathrm{II}}}{r^2} + \frac{P^{\mathrm{III}}}{r^3} + \frac{P^{\mathrm{IV}}}{r^4} + \frac{P^{\mathrm{V}}}{r^5} + \frac{P^{\mathrm{VI}}}{r^6} + \frac{P^{\mathrm{VII}}}{r^7} +$ &c. by substituting in its terms, instead of P, P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. any numbers taken from the series 92, 91, 90, 89, 88, 87, 86, &c. will be the same with the values deduced from the same general expression by substituting in its terms, instead of P, P^{I}. P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, P^{VII}, &c. the corresponding numbers of the series 1012, 1001, 990, 979, 968, 957, 946, &c.

CLX. If these new numbers are set down in a regular series near the ages to which they respectively belong, and are supposed to begin from the age of one year, instead of beginning from the age of 3 years (as the numbers in the last table do,) they will be as follows.

TABLE

TABLE VIII.

Containing an artificial estimate of the probabilities of the duration of human life, similar to that contained in the second column of Table VII. but expressed in the smallest numbers possible; derived from Mr. De Moivre's Hypothesis that the numbers of persons dying every year, out of any given original number, are always the same throughout the whole extent of human life, and formed upon a supposition that the utmost possible duration of human life is less than 95 years.

Years of Age.	*Persons living.*	*Years of Age.*	*Persons living.*	*Years of Age.*	*Persons living.*	*Years of Age.*	*Persons living.*
1	94	25	70	49	46	73	22
2	93	26	69	50	45	74	21
3	92	27	68	51	44	75	20
4	91	28	67	52	43	76	19
5	90	29	66	53	42	77	18
6	89	30	65	54	41	78	17
7	88	31	64	55	40	79	16
8	87	32	63	56	39	80	15
9	86	33	62	57	38	81	14
10	85	34	61	58	37	82	13
11	84	35	60	59	36	83	12
12	83	36	59	60	35	84	11
13	82	37	58	61	34	85	10
14	81	38	57	62	33	86	9
15	80	39	56	63	32	87	8
16	79	40	55	64	31	88	7
17	78	41	54	65	30	89	6
18	77	42	53	66	29	90	5
19	76	43	52	67	28	91	4
20	75	44	51	68	27	92	3
21	74	45	50	69	26	93	2
22	73	46	49	70	25	94	1
23	72	47	48	71	24	95	0
24	71	48	47	72	23		

Of the complement of Life.

CLXI. In the foregoing table the number of persons represented as living at any given age is always the same with the number of years which are necessary to be added to the number of years in the given age in order to make it equal to 95 years, or the period in the course of which it is supposed that all the persons set down in the table as living at the ends of the several foregoing years, will be dead. Thus, the number of persons living at the age of one year is 94; which, being added to one year, the age of the said persons, makes 95: and the number living at the age of 10 years is 85; which being added to 10, the number of years in the age of the said persons, is 95, as before. And the same observation is true throughout the table, to wit, that the number in the second column, which expresses the persons living at any given age, is always the complement of the number of years in the said age to 95. This complement Mr. De Moivre calls *the complement of life*: so that, according to his Hypothesis, the complement of life at any given age is always equal to the number of persons living at the same age.

A computation of the values of an annuity of one pound a year for the lives of persons of several different ages, according to the foregoing artificial table of the probabilities of the duration of human life, grounded on Mr. De Moivre's Hypothesis, and upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

For the life of a child of the age of one year.

CLXII. Let it now be required to find, by means of the foregoing artificial table of probabilities, the value of an annuity of one pound a year for the life of a child of the age of one year, when the interest of money is $3\frac{1}{2}$ per cent.

By Art. 87, the general expression for the value of an annuity for a life of any given age is £ $\frac{1}{P}\times$ the series $\frac{P^{\text{I}}}{r}+\frac{P^{\text{II}}}{r^2}+\frac{P^{\text{III}}}{r^3}+\frac{P^{\text{IV}}}{r^4}+\frac{P^{\text{V}}}{r^5}+\frac{P^{\text{VI}}}{r^6}+\frac{P^{\text{VII}}}{r^7}+$ &c. in which P signifies the number of persons living at the given age, P^{I} the number of persons living at the end of the next year, P^{II} the number living at the end of the second year, P^{III} the number living at the end of the third year, and so on to the utmost extent of human life.

These numbers P, P^{I}, P^{II}, P^{III}, &c. are in the present case, and according to Mr. De Moivre's Hypothesis, 94, 93, 92, 91, &c. Therefore the value of the annuity sought is = £ $\frac{1}{94}\times$ the series $\frac{93}{r}+\frac{92}{r^2}+\frac{91}{r^3}+\frac{90}{r^4}+\frac{89}{r^5}+\frac{88}{r^6}+\frac{87}{r^7}+$ &c. continued to 93 terms, or to the term

$\frac{1}{r^{93}}$

$\frac{1}{r^{93}} = \frac{£}{1} \times$ the series $\frac{93}{94r} + \frac{92}{94r^2} + \frac{91}{94r^3} + \frac{90}{94r^4} + \frac{89}{94r^5} + \frac{88}{94r^6} + \frac{87}{94r^7} +$ &c. $+ \frac{1}{94r^{93}} = \frac{£}{1} \times$ the series $\frac{94-1}{94r} + \frac{94-2}{94r^2} + \frac{94-3}{94r^3} + \frac{94-4}{94r^4} + \frac{94-5}{94r^5} + \frac{94-6}{94r^6} + \frac{94-7}{94r^7} +$ &c. $+ \frac{94-93}{94r^{93}}$. Let one term more be added to this series, to wit, the term $\frac{94-94}{94r^{94}}$, in order to bring the series to the same form as the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} +$ &c. $+ \frac{n-n}{nr^n}$, in Art. 144. Such addition of the term $\frac{94-94}{94r^{94}}$ will make no alteration in the value of the series to which it is added, because the said term is $= \left(\frac{0}{94r^{94}} \text{ or}\right) 0$. Therefore the value of the annuity sought will be $= \frac{£}{1} \times$ the series $\frac{94-1}{94r} + \frac{94-2}{94r^2} + \frac{94-3}{94r^3} + \frac{94-4}{94r^4} + \frac{94-5}{94r^5} + \frac{94-6}{94r^6} + \frac{94-7}{94r^7} +$ &c. $+ \frac{94-94}{94r^{94}}$. But it has been shewn in Art. 144, that, if n be any whole number whatsoever, the series $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4} + \frac{n-5}{nr^5} + \frac{n-6}{nr^6} + \frac{n-7}{nr^7} +$ &c. $+ \frac{n-n}{nr^n}$ will be equal to the fraction $\frac{1 - \frac{rG}{n}}{r-1}$, in which G denotes the sum of the terms of the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} + \frac{1}{r^7} +$ &c. $+ \frac{1}{r^n}$. Therefore the series $\frac{94-1}{94r} + \frac{94-2}{94r^2} + \frac{94-3}{94r^3} + \frac{94-4}{94r^4} + \frac{94-5}{94r^5} + \frac{94-6}{94r^6} + \frac{94-7}{94r^7} +$ &c. $+ \frac{94-94}{94r^{94}}$ will be $= \frac{1 - \frac{rG}{94}}{r-1}$; that is, (because the interest of money is supposed to be $3\frac{1}{2}$ per cent.

cent. and conſequently r is $= 1.035$,) the ſaid ſeries will be $= \dfrac{1-\frac{1.035}{94}G}{.035}$, in which G will denote the ſum of the terms of the geometrical progreſſion $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}+\frac{1}{r^4}+\frac{1}{r^5}+\frac{1}{r^6}+\frac{1}{r^7}$ + &c. $+\frac{1}{r^{94}}$, or $\frac{1}{1.035}+\frac{1}{\overline{1.035}|^2}+\frac{1}{\overline{1.035}|^3}+\frac{1}{\overline{1.035}|^4}+\frac{1}{\overline{1.035}|^5}+\frac{1}{\overline{1.035}|^6}+\frac{1}{\overline{1.035}|^7}$ + &c. $+\frac{1}{\overline{1.035}|^{94}}$, or continued to 94 terms. Now it appears from Mr. Smart's fourth table of compound intereſt, page 82, that G, or the ſum of the terms of the ſaid geometrical progreſſion, is 27.445,426. Therefore $\frac{1.035}{94}G$ is $= \frac{1.035}{94} \times 27.445,426 = \frac{28.406,015,910}{94} = .302,191,658$; and $1-\frac{1.035}{94}G$ is $= 1 - .302,191,658 = .697,808,342$; and $\dfrac{1-\frac{1.035}{94}G}{1.035}$ is $= \frac{.697,808,342}{.035} = 19.908,809$. Therefore the value of an annuity of one pound a year for the life of a child of the age of one year, is, according to Mr. De Moivre's Hypotheſis, $=$ £ $1 \times 19.908,809$, $=$ £ $19.908,809$, or 19*l.* 18*s.* 2*d.* Q.E.I.

CLXIII. By the like reaſonings as thoſe in the preceeding article it may be ſhewn, that, if n be the complement of life to any other given age whatſoever, the value of an annuity of one pound for a life of the ſaid age, according to Mr. De Moivre's Hypotheſis, will be $=$ £ $1 \times$ the fraction $\dfrac{1-\frac{r}{n}G}{r-1}$; as, I preſume, the reader muſt eaſily perceive. I will now therefore proceed to calculate, by means of this expreſſion, £ $1 \times \dfrac{\overline{1-\frac{r}{n}G}}{r-1}$, the values of an annuity of one pound for lives of the ſeveral

ſeveral ages of 3 years, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70 years, according to Mr. De Moivre's Hypotheſis, to the end that we may be able to compare them with the values of the ſame annuity for lives of the ſame ages, as computed above in Art. 108, 109, &c.—114, from Monſieur de Parcieux's table.

CLXIV. When the given life is of the age of 3 years, we ſhall have n, or the complement of life, $(= 95 - 3) = 92$, and $\frac{r}{n} = \frac{r}{92} = \frac{1.035}{92}$, and G = the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} +$ &c. continued to $\frac{1}{r^{92}}$, or to 92 terms, $= \frac{1}{1.035} + \frac{1}{\overline{1.035}|^2} + \frac{1}{\overline{1.035}|^3} + \frac{1}{\overline{1.035}|^4} +$ &c. continued to $\frac{1}{\overline{1.035}|^{92}}$, or to 92 terms; which (by Mr. Smart's fourth table of compound intereſt, page 82,) is = 27.365,227. Therefore $\frac{r}{n}G$ will be $= \frac{1.035}{92} \times 27.365,227 = \frac{28.323,009,945}{92} = .307,858,803$; and $1 - \frac{r}{n}G$ will be $= 1.000,000,000 - .307,858,803 = .692,141,197$; and $\frac{1 - \frac{r}{n}G}{r-1}$ will be $= \frac{.692,141,197}{r-1} = \frac{.692,141,197}{.035} = 19.775,462$. Conſequently $\frac{£}{1} \times \overline{\left|\frac{1 - \frac{r}{n}G}{r-1}\right.}$, or the value of an annuity of one pound a year for a life of the age of 3 years, according to Mr. De Moivre's Hypotheſis, will be $= \frac{£}{1} \times 19.775,462$, $= £19.775,462$, or 19*l.* 15*s.* 6*d.* For the life of a child of the age of 3 years.

N. B. This value of the ſaid annuity is almoſt a quarter of a year's purchaſe leſs than its true value, as computed above from Monſieur de Parcieux's table of probabilities, that value being £19.987,654, or 19*l.* 19*s.* 9*d.* See Table III. Art. 118.

CLXV. When

For a life of the age of 5 years.

CLXV. When the given life is of the age of 5 years, we shall have n, or the complement of it to 95 years, $= 90$, and $\frac{r}{n} = \frac{r}{90} = \frac{1.035}{90}$, and $G =$ the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$ + &c. continued to $\frac{1}{r^{90}}$, or to 90 terms, and consequently (by Mr. Smart's tables, page 82) $= 27.279,315$. Therefore $\frac{r}{n}G$ will be $= \frac{1.035}{90} \times 27.279,315 = \frac{28.234,091,025}{90} = .313,712,122$; and $1 - \frac{r}{n}G$ will be $= 1.000,000,000 - .313,712,122 = .686,287,878$; and $\frac{1 - \frac{r}{n}G}{r-1}$ will be $= \frac{.686,287,878}{r-1} = \frac{.686,287,878}{.035} = 19.608,225$. Therefore $\overset{£}{1} \times \frac{1 - \frac{r}{n}G}{r-1}$, or the value of an annuity of one pound for a life of the age of 5 years, according to Mr. De Moivre's Hypothesis, will be $= \overset{£}{1} \times 19.608,225 =$ £19.608,225, or 19*l.* 12*s.* 2*d.* Q.E.I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £20.526,716, or 20*l.* 10*s.* 6*d.*$\frac{1}{2}$; which is greater than the value just now found for it by almost a year's purchase.

For a life of the age of 10 years.

CLXVI. When the given life is of the age of 10 years, we shall have n $(= 95 - 10) = 85$, and $\frac{r}{n} = \frac{1.035}{85}$, and $G =$ the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4}$ + &c. continued to $\frac{1}{r^{85}}$, or to 85 terms, which (by Mr. Smart's tables, page 82,) is $= 27.036,803$. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{85} \times 27.036,803 = \frac{27.983,091,105}{85} =$ .329,212,836;

.329,212,836; and $1-\frac{r}{n}G$ is = 1.000,000,000 − .329,212,836 =

.670,787,164; and $\dfrac{1-\frac{r}{n}G}{r-1}$ is = $\dfrac{.670,787,164}{.035}$ = 19.165,347.

Therefore $\overset{£}{1} \times \overline{\dfrac{1-\frac{r}{n}G}{r-1}}$, or the value of an annuity of one pound a year for a life of the age of 10 years, according to Mr. De Moivre's Hypothesis, is = $\overset{£}{1}$ × 19.165,347 = £19.165,347, or 19*l.* 3*s.* 3*d.*¼. Q. E. I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £20.752,981, or 20*l.* 15*s.* 7*d.*½; which is greater than the value just now found for it by more than a year and a half's purchase.

CLXVII. When the given life is of the age of 15 years, we shall have n (= 95 − 15) = 80, and $\frac{r}{n} = \frac{1.035}{80}$, and G = the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$ + &c. continued to $\frac{1}{r^{80}}$, or to 80 terms, which (by Mr. Smart's tables, page 82) is = 26.748,775. *For a life of the age of 15 years.*

Therefore $\frac{r}{n}G$ is = $\dfrac{1.035}{80}$ × 26.748,775 = $\dfrac{27.684,982,125}{80}$ =

.346,062,276; and $1-\frac{r}{n}G$ is = 1.000,000,000 − .346,062,276 =

.653,937,724; and $\dfrac{1-\frac{r}{n}G}{r-1}$ is = $\dfrac{.653,937,724}{.035}$ = 18.683,934.

Therefore $\overset{£}{1} \times \overline{\dfrac{1-\frac{r}{n}G}{r-1}}$, or the value of an annuity of one pound a year for a life of the age of 15 years, according to Mr. De Moivre's Hypothesis, is = $\overset{£}{1}$ × 18.683,934 = £18.683,934, or 18*l.* 13*s.* 8*d.* Q. E. I.

N. B. The

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £20.137,194, or 20*l.* 2*s.* 9*d.* which is greater than the value juſt now found for it by almoſt a year and a half's purchaſe.

For a life of the age of 20 years.

CLXVIII. When the given life is of the age of 20 years, we ſhall have n $(= 95 - 20) = 75$; and $\frac{r}{n} = \frac{1.035}{75}$; and G = the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$ + &c. continued to the term $\frac{1}{r^{75}}$, or to 75 terms, which (by Mr. Smart's tables, page 80,) is = 26.406,688. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{75} \times 26.406{,}688 = \frac{27.330{,}922{,}080}{75}$ $= .364{,}412{,}294$; and $1 - \frac{r}{n}G$ is $= 1.000{,}000{,}000, - .364{,}412{,}294 =$ $.635{,}587{,}706$; and $\frac{1 - \frac{r}{n}G}{r - 1}$ is $= \frac{.635{,}587{,}706}{.035} = 18.159{,}648.$

Therefore £1 $\times \frac{1 - \frac{r}{n}G}{r - 1}$, or the value of an annuity of one pound a year for a life of the age of 20 years, according to Mr. De Moivre's Hypotheſis, is = £1 × 18.159,648 = £18.159,648, or 18*l.* 3*s.* 2*d.* ¼. Q E I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £19.457,903, or 19*l.* 9*s.* 1*d.* ¾; which is greater than the value juſt now found for it by about a year and a quarter's purchaſe.

For a life of the age of 25 years.

CLXIX. When the given life is of the age of 25 years, we ſhall have n $(= 95 - 25) = 70$, and $\frac{r}{n} = \frac{1.035}{70}$, and G = the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4}$ + &c. continued to the term $\frac{1}{r^{70}}$, or to 70 terms, which (by Mr. Smart's table, page 80,) is = 26.000,396.

Therefore

Therefore $\frac{r}{n}G$ is $= \frac{1.035}{70} \times 26.000,396 = \frac{26.910,411,060}{70} =$ $.384,434,443$; and $1-\frac{r}{n}G$ is $= 1.000,000,000 - .384,434,443 =$ $.615,565,557$; and $\frac{1-\frac{r}{n}G}{r-1}$ is $= \frac{.615,565,557}{.035} = 17.587,587$.

Therefore $\overset{£}{1} \times \frac{1-\frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 25 years, according to Mr De Moivre's Hypothesis, is $= \overset{£}{1} \times 17.587,587 =$ £17.587,587, or 17*l.* 11*s.* 9*d.* Q.E.I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £18.827,070, or 18*l.* 16*s.* 6*d.* ½; which is greater than the value just now found for it by about a year and a quarter's purchase.

CLXX. When the given life is of the age of 30 years, we shall have n $(= 95 - 30) = 65$, and $\frac{r}{n} = \frac{1.035}{65}$, and $G =$ the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. continued to the term $\frac{1}{r^{65}}$, or to 65 terms, which (by Mr. Smart's tables, page 80,) is $= 25.517,849$. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{65} \times 25.517,849 = \frac{26.410,974,715}{65}$ $= .406,322,672$; and $1-\frac{r}{n}G$ is $= 1.000,000,000 - .406,322,672 =$ $.593,677,328$; and $\frac{1-\frac{r}{n}G}{r-1}$ is $= \frac{.593,677,328}{.035} = 16.962,209$. For a life of the age of 30 years.

Therefore $\overset{£}{1} \times \frac{1-\frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 30 years, according to Mr. De Moivre's Hypothesis, is $= \overset{£}{1} \times 16.962,209 =$ £16.962,209, or 16*l.* 19*s.* 3*d.*

 N. B. The

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £18.095,844, or 18*l.* 1*s.* 11*d.* which is greater than the value just now found for it by about a year and half a quarter's purchase.

For a life of the age of 35 years.

CLXXI. When the given life is of the age of 35 years, we shall have n (= 95 — 35) = 60, and $\frac{r}{n} = \frac{1.035}{60}$, and G = the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{60}}$ = (by Mr. Smart's tables, page 80,) 24.944,734. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{60} \times 24.944,734 = \frac{25.817,799,690}{60} = .430,296,661$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000 - .430,296,661 = .569,703,339$; and $\frac{1 - \frac{r}{n}G}{r - 1}$ is $= \frac{.569,703,339}{.035} = 16.277,238$. Therefore $\frac{£}{1} \times \frac{1 - \frac{r}{n}G}{r - 1}$, or the value of an annuity of one pound a year for a life of the age of 35 years, according to Mr. De Moivre's Hypothesis, is $= \frac{£}{1} \times 16.277,238 =$ £16.277,238, or 16*l.* 7*s.* 6*d.* $\frac{1}{2}$. Q E I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £17.240,588, or 17*l.* 4*s.* 9*d.* $\frac{3}{4}$; which is greater than the value just now found for it by somewhat less than a year's purchase.

For a life of the age of 40 years.

CLXXII. When the given life is of the age of 40 years, we shall have n (= 95 — 40) = 55, and consequently $\frac{r}{n} = \frac{1.035}{55}$, and G = the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{55}}$ = (by Mr. Smart's tables, page 80,) 24.264,053. Therefore $\frac{r}{n}G$ is =

$\frac{1.035}{55}$

$\frac{1.035}{55} \times 24.264,053 = \frac{25.113,294,855}{55} = .456,605,361$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000 - .456,605,361 = .543,394,639$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.543,394,639}{.035} = 15.525,561$. Therefore $\overset{\pounds}{1} \times \overline{\frac{1 - \frac{r}{n}G}{r-1}}$, or the value of an annuity of one pound a year for a life of the age of 40 years, according to Mr. De Moivre's Hypothefis, is $= \overset{\pounds}{1} \times 15.525,561 =$ £15.525,561, or 15*l.* 10*s.* 6*d.* Q.E.I.

N. B. The value of this annuity, as computed above from Monfieur de Parcieux's table, is £16.147,167, or 16*l.* 2*s.* 11*d.* $\frac{1}{4}$; which is greater than the value just now found for it by fomething more than half a year's purchafe.

CLXXIII. When the given life is of the age of 45 years, we fhall have n ($= 95 - 45$) $= 50$, and confequently $\frac{r}{n} = \frac{1.035}{50}$, and $G =$ the geometrical progreffion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{50}} =$ (by Mr. Smart's tables, page 78,) 23.455,617. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{50} \times 23.455,617 = \frac{24.276,563,595}{50} = .485,531,271$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000, - .485,531,271 = .514,468,729$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.514,468,729}{.035} = 14.699,106$. Therefore $\overset{\pounds}{1} \times \overline{\frac{1 - \frac{r}{n}G}{r-1}}$, or the value of an annuity of one pound a year for a life of the age of 45 years, according to Mr. De Moivre's Hypothefis, is $= \overset{\pounds}{1} \times 14.699,106$ $=$ £14.699,106, or 14*l.* 14*s.* 0*d.* Q.E.D.

For a life of the age of 45 years.

N. B. The

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £14.716,120, or 14*l.* 14*s.* 3*d.* $\frac{3}{4}$; which is greater than the value just now found for it by only the trifling sum of 3*d.* $\frac{3}{4}$.

For a life of the age of 50 years.

CLXXIV. When the given life is of the age of 50 years, we shall have n $(= 95 - 50) = 45$, and consequently $\frac{r}{n} = \frac{1.035}{45}$, and $G =$ the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{45}}$ = (by Mr. Smart's tables, page 78,) 22.495,450. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{45} \times 22.493,450 = \frac{23.282,790,750}{45} = .517,395,350$; and $1 - \frac{r}{n}G$ is $= 1.000,000,000, - .517,395,350, = .482,604,650$; and $\frac{1 - \frac{r}{n}G}{r-1}$ is $= \frac{.482,604.650}{.035} = 13.788,704$. Therefore $\overset{£}{1} \times \frac{\overline{1 - \frac{r}{n}G}}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 50 years, according to Mr. De Moivre's Hypothesis, is $= \overset{£}{1} \times 13.788,704 =$ £13.788,704, or 13*l.* 15*s.* 9*d.* $\frac{1}{4}$. Q E I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £13.183,083, or 13*l.* 3*s.* 8*d.* which, instead of being greater than the value just now found for it, (as has been the case in the preceeding instances,) is less than that value by more than half a year's purchase.

For a life of the age of 55 years.

CLXXV. When the given life is of the age of 55 years, we shall have n $(= 95 - 55) = 40$, and consequently $\frac{r}{n} = \frac{1.035}{40}$, and $G =$ the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \&c. + \frac{1}{r^{40}}$ = (by Mr. Smart's tables, page 78,) 21.355,072. Therefore $\frac{r}{n}G$ is =

$\frac{1.035}{40}$

$\frac{1.035}{40} \times 21.355,072 = \frac{22.102.499}{40} = .552,562,487$; and $\frac{1-\frac{r}{n}G}{r-1}$ is $= 1.000,000,000, - .552,562,487, = .447,437,513$; and $\frac{1-\frac{r}{n}G}{r-1}$ is $= \frac{.447,437,513}{.035} = 12.783.928$. Therefore £1 × $\overline{\frac{1-\frac{r}{n}G}{r-1}}$, or the value of an annuity of one pound a year for a life of the age of 55 years, is = £1 × 12.783,928 = £12.783,928, or 12*l.* 15*s.* 8*d.* Q.E.I.

N. B. The value of this annuity, as computed above from Monſieur de Parcieux's table, is £11.691,801, or 11*l.* 13*s.* 10*d.* which is leſs than the value juſt now found for it by more than a year's purchaſe.

CLXXVI. When the given life is of the age of 60 years, we ſhall have n (= 95 — 60) = 35, and conſequently $\frac{r}{n} = \frac{1.035}{35}$, and G = the geometrical progreſſion $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} +$ &c. $+ \frac{1}{r^{35}}$ = (by Mr. Smart's tables, page 78,) 20.000,661. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{35} \times 20.000,661 = \frac{20.700,684,135}{35} = .591,448,118$; and $1-\frac{r}{n}G$ is $= 1.000,000,000, - .591,448,118 = 408,551,882$; and $\frac{1-\frac{r}{n}G}{r-1}$ is $= \frac{.408,551,882}{.035} = 11.672,910$. Therefore £1 × $\overline{\frac{1-\frac{r}{n}G}{r-1}}$, or the value of an annuity of one pound a year for a life of the age of 60 years, according to Mr. De Moivre's Hypotheſis, is = £1 × 11.672,910 = £11.672,910, or 11*l.* 13*s.* 5*d.* $\frac{1}{2}$. Q.E.I.

For a life of the age of 60 years.

N. B. The

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £10.104,074, or 10*l.* 2*s.* 1*d.* which is less than the value just now found for it by more than a year and a half's purchase.

For a life of the age of 65 years.

CLXXVII. When the given life is of the age of 65 years, we shall have n (= 95 — 65) = 30, and consequently $\frac{r}{n} = \frac{1.035}{30}$, and G = the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$ + &c. $+ \frac{1}{r^{30}}$, = (by Mr. Smart's tables, page 78,) 18.392,045. Therefore $\frac{r}{n}G$ is $= \frac{1.035}{30} \times 18.392,045 = \frac{19.035,766,575}{30} = .634,525,552$; and $1 - \frac{r}{n}G$ is = 1.000,000,000, — .634,525,552 = .365,474,448; and $\frac{1 - \frac{r}{n}G}{r - 1}$ is $= \frac{.365,474,448}{.035}$ = 10.442,127. Therefore £1 × $\frac{\overline{1 - \frac{r}{n}G}}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 65 years, according to Mr. De Moivre's Hypothesis, is = £1 × 10.442,127 = £10.442,127, or 10*l.* 8*s.* 10*d.* Q E I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £8.813,625, or 8*l.* 6*s.* 3*d.* $\frac{1}{4}$; which is less than the value just now found for it by more than two years purchase.

For a life of the age of 70 years.

CLXXVIII. When the given life is of the age of 70 years, we shall have n (= 95 — 70) = 25, and consequently $\frac{r}{n} = \frac{1.035}{25}$, and G = the geometrical progression $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5}$ + &c. $+ \frac{1}{r^{25}}$

$+\frac{1}{r^{25}}$ = (by Mr. Smart's tables, page 76,) 16.481,514. Therefore

$\frac{r}{n}G$ is $= \frac{1.035}{25} \times 16.481,514 = \frac{17.058,366,990}{25} = .682,334,679$;

and $1-\frac{r}{n}G$ is $= 1.000,000,000 - .682,334,679 = .317,665,321$;

and $\dfrac{1-\frac{r}{n}G}{r-1}$ is $= \dfrac{.317,665,321}{.035} = 9.076,152$. Therefore

£1 × $\dfrac{1-\frac{r}{n}G}{r-1}$, or the value of an annuity of one pound a year for a life of the age of 70 years, according to Mr. De Moivre's Hypothesis, is = £1 × 9.076,152 = £9.076,152, or 9*l.* 1*s.* 6*d.*¼ Q.E.I.

N. B. The value of this annuity, as computed above from Monsieur de Parcieux's table, is £6.575,357, or 6*l.* 11*s.* 6*d.* which is less than the value just now found for it by about two years and a half's purchase.

CLXXIX. If the foregoing values of an annuity of one pound for the lives of persons of the several ages of 1 year, 3 years, 5 years, 10 years, 15 years, 20 years, 25, 30, 35, 40, 45, 50, 55, 60, 65, and 70, years, which are derived from Mr. De Moivre's Hypothesis, are set down in a regular series, and the values of the same annuity for all the same lives, except the first, as computed above from Monsieur de Parcieux's table, are set down in an adjoining column, and the differences between the said values are likewise set down in a third column, they will together form the following table.

TABLE IX.

Containing the conjectural values of an annuity of one pound a year for the lives of persons of several different ages, derived from Mr. De Moivre's Hypothesis, "That the decrements of human life, or the numbers of "persons dying every year out of any given original number of persons "living at the beginning of life, continue the same throughout the whole "extent of human life;" and containing likewise the true values of the same annuity of one pound a year for all the same lives, except the first, (which is a life of the age of 1 year) computed according to Monsieur de Parcieux's table of the probabilities of the duration of human life; upon a supposition, in both ways of computing the values of this annuity, that the interest of money is $3\frac{1}{2}$ per cent. and that the utmost extent of human life is less than 95 years; and containing likewise, in an adjoining column, the differences of the said values of the said annuity from each other.

Years of Age.	*Values of a life-annuity of one pound a year, according to Mr. De Moivre's Hypothesis.*	*Values of the same life-annuity according to Monsieur de Parcieux's table of probabilities.*	*Differences of the said values.*
	£		
1	19.908,909	£	
3	19.775,462	19.987,654	0.212,192
5	19.608,225	20.526,716	0.918,491
10	19.165,347	20.752,981	1.587,634
15	18.683,934	20.137,194	1.453,260
20	18.159,648	19.457,903	1.298,255
25	17.587,587	18.827,070	1.239,483
30	16.962,[illegible]09	18.095,844	1.133,635
35	16.277,238	17.240,588	0.963,350
40	15.525,561	16.147,167	0.621,606
45	14.699,106	14.716,120	0.017,014
50	13.788,704	13.183,083	0.605,621
55	12.783,928	11.691,801	1.092,127
60	11.672,910	10.104,074	1.568,836
65	10.442,127	8.313,625	2.128,5[illegible]2
70	9.076,152	6.575,357	2.500,795

CLXXX. It

CLXXX. It appears from the foregoing table, in the 1ſt place, That, from the age of 3 years to the age of 45 years, the values of life-annuities, according to Mr. De Moivre's Hypotheſis, are leſs than their true values, as computed above from Monſieur de Parcieux's table of probabilities; *Remarks on the foregoing table.*

And, 2dly, That during the ſaid interval from the age of 3 years to the age of 45 years, the exceſs of the true value of a life-annuity of one pound, above its conjectural value, derived from Mr. De Moivre's Hypotheſis, increaſes frou £0.212,192, or leſs than a quarter of a year's purchaſe, to £1.587,634, or more than a year and a half's purchaſe, (which is its magnitude at the age of 10 years,) and then decreaſes again to £0.017,014, or leſs than one week's purchaſe, or, we may ſay, to 0;

And, 3dly, That, from the age of 45 years to the age of 70 years, the values of life-annuities, according to Mr. De Moivre's Hypotheſis, are greater than their true values, as computed above from Monſieur de Parcieux's table of probabilities;

And, 4thly, That, during the ſaid interval from the age of 45 years to the age of 70 years, the exceſs of the conjectural value of a life-annuity of one pound, (according to Mr. De Moivre's Hypotheſis,) above its true value increaſes continually, as the age advances, or grows older, from 0.017,014, or 0, to £2.500,795, or more than two years and a half's purchaſe, which is its magnitude at the age of 70 years.

CLXXXI. Upon the whole matter, the differences of theſe values ſeem to me to be ſo conſiderable that I cannot but wonder that Mr. De Moivre ſhould have thought fit to diſregard them, and to compute his tables of life-annuities from his own Hypotheſis, inſtead of following Dr. Halley's, or Monſieur de Parcieux's, or ſome other table of the probabilities of life deduced from obſervations. And this appears the more ſurprizing when we conſider, that the labour of deriving the value of a life-annuity for a life of any given age from that of a lik nuity for a life only one year older, in ſtrict conformity to a table of the probabilities of life, by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|$£, is ſcarcely, if at all, greater than that of computing the ſame annuity, according to Mr. De Moivre's Hypothef, by means of the expreſſion $\text{£}1 \times \frac{\overline{1 - \frac{r}{n}G}}{r\ \ 1}$. *A general concluſion concerning Mr. De Moivre's Hypotheſis.*

 I would

I would therefore recommend it to all future calculators of the values o life-annuities, or at least to those who undertake only to calculate annuities for single lives, to lay aside all consideration of Mr. De Moivre's Hypothesis, and to compute those values from Monsieur de Parcieux's table of probabilities, or some other table of the same kind which they may esteem to be of still better authority, by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}$ £, in the manner that has been above explained and exemplified in Art. 100, 101, 102, &c.—117.

End of the explanation of Mr. De Moivre's Hypothesis.

An account of Mr. Weyman Lee's method of computing the values of Life-annuities.

CLXXXII. Some gentlemen who have undertaken to write upon the subject of life-annuities, but who appear to have been unacquainted with the Mathematicks and the doctrine of chances, have considered the method of computing the value of a life-annuity above-delivered in Problem 11, Coroll. 2, Art. 31, page 28, and exemplified in many instances in the subsequent part of this tract, and which is that which all mathematical writers, (from Dr. Halley in the year 1692, down to Dr. Price and Mr. Morgan in their late treatises,) have universally adopted;——I say, some gentlemen have considered this method as erroneous. One of these gentlemen was the late Weyman Lee, Esq; a barrister at law and a bencher of the Society of the Inner Temple. This gentleman published a book in one volume octavo in the year 1738, upon the doctrine of life-annuities, intitled, "*An Essay on the method of ascertaining the value of annuities, and of Leases reduced to annuities certain, for one or more lives*;" and in the year 1751 he published a second pamphlet on the same subject, intitled, "*A valuation of annuities and leases certain for a single life.*" This latter pamphlet I have seen. And in it, I observe, he argues against the method of computing the values of life-annuities above-explained, in a style of great confidence, and with a high degree of contempt of his numerous mathematical adversaries, Dr. Halley, a Mr. H. B, Mr. Richards, Mr. Hodgson, Mr. Hayes, and Mr. De Moivre: but at the same time he expresses himself with great obscurity, as might naturally be expected from a person who endeavoured to invalidate so clear and certain a proposition. And, in lieu of the said method of computing the values of life-annuities (which he conceives himself to have shewn to be erroneous,) he gives us another method of estimating them, which is as follows.

CLXXXIII. When

CLXXXIII. When the value of an annuity for the life of a person of any given age is to be determined, we must look into Dr. Halley's, or Monsieur de Parcieux's, or some other, table of the probabilities of the duration of human life, to find what number of persons are therein represented as living at the said age. And then we must divide this number by 2, and look again into the said table, to find at what later age of life the number of persons represented therein as living will be reduced to this latter number, or to one half of the former number. And an annuity of one pound a year for the life of a person of the first, or given, age will be equal in value, says Mr. Lee, to a like annuity of one pound a year for a term of years certain, equal to the difference of the said two ages, or to the space of time in which the number of persons represented in the table as living at the given age will be reduced to half.

Mr. Weyman Lee's method of valuing life-annuities.

Thus, for example, if the value of an annuity for the life of a person of the age of 10 years is to be determined, we shall find in Monsieur de Parcieux's table above-mentioned, that 880 persons are there represented as living at the age of 10 years. The half of this number is 440. We must therefore, in the second place, look out in the same table the age at which the number of persons living is reduced to 440. Now we shall find, upon this inspection of the said table, that at the age of 61 years there will be 450 persons living, and at the age of 62 years 437 persons living, out of the said 880 persons who were living at the age of 10 years. Therefore at some intermediate age between 61 and 62, as, for instance, about the age of 61 years and nine months, we may suppose that the number of persons living, (out of the said original 880 persons, who were living at the age of 10 years,) will be 440; or, in other words, the original 880 persons, living at the age of 10 years, will be reduced to 440, or to one half, in the course of 51 years and 9 months. Therefore (according to Mr. Weyman Lee's method of valuing annuities,) an annuity of one pound a year for the life of a person of the age of 10 years is of the same value with an annuity of one pound a year for the space of 51 years and 9 months certain, or, in round numbers, for the space of 52 years.

An example of the said method.

CLXXXIV. According to this method of estimating life-annuities, an annuity of one pound a year for the life of a person of the age of 10 years, when the interest of money is $3\frac{1}{2}$ per cent. is £23.795,764, or 23*l.* 15*s.* 11*d.* that being the value of an annuity of one pound a year for a term certain of 52 years, when the interest of money is $3\frac{1}{2}$ per cent. as appears from Mr. Smart's tables, page 80.

CLXXXV. But

Of the difference of the values of life-annuities computed by this method from their true values.

CLXXXV. But we have seen above, in Art. 94, that the true value of an annuity of one pound a year for the life of a person of the age of 10 years, when the interest of money is $3\frac{1}{2}$ per cent. is only £20.73925, or 20*l.* 14*s.* 9*d.* $\frac{1}{4}$: and by the other calculation of it, in Art. 114, it appeared to be only £20.752,981, or 20*l.* 15*s* 0*d.* $\frac{1}{4}$. Therefore the foregoing value of it, obtained by the method of Mr. Weyman Lee, to wit, 23*l.* 15*s.* 11*d.* is greater than its true value by no less than three years purchase. And in like manner it will be found upon trial, that in all other ages of life, except from the age of 74 years to the age of 83 years, the values of life-annuities obtained by this method of Mr. Lee, will be greater, and for the most part *very much* greater, than their true values.

CLXXXVI. But that the difference of these values of life-annuities, found by Mr. Lee's method, from their true values, as computed according to the directions of Problem 11, Coroll. 2, may be the more apparent, I shall here present the reader with a table containing both sets of values of an annuity of one pound a year for the lives of persons of all ages, differing from each other by one year, from the age of 3 years to the age of 93 years, inclusively, computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. and containing likewise the differences of the said values. This table is as follows.

TABLE

TABLE X.

Containing the values of an a[illegible]ity of one pound a year for the lives of perſons of the ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years incluſively, upon a ſuppoſition that the probabilities of the duration of human life are ſuch as they are repreſented to be in Monſieur de Parcieux's table of them, and that the intereſt of money is 3½ per cent. computed according to two different methods, to wit, Mr. Weyman Lee's method juſt now mentioned in Art. 183, and the true method that has bee[illegible] adopted by Dr. Halley and all other mathematicians, and which has been explained above in Problem 11 and its corollaries; and containing likewiſe, in an adjoining column, the differences of the ſaid values.

Years of Age.	*Values of an annuity of one pound by Mr. Weyman Lee's method.*	*Values of the ſame annuity by Dr. Halley's method.*	*Differences of the ſaid values.*
	£	£	£
3	24.113,295	19.987,654	4.125,641
4	24.113,295	20.327,034	3.786,261
5	24.113,295	20.526,716	3.586,579
6	24.113,295	20.656,349	3.456,946
7	24.035,948	20.729,912	3.306,036
8	23.957,260	20.764,685	3.192,575
9	23.877,206	20.781,222	3.095,984
10	23.795,764	20.752,981	3.042,783
11	23.628,616	20.676,395	2.952,221
12	23.455,617	20.548,338	2.9[illegible]7,[illegible]79
13	23.366,801	20.4[illegible]5,899	2.950,962
14	23.276,564	20.278,914	2.9[illegible]7,650
15	23.091,244	20.137,194	2.954,050
16	22.890,437	19.990,615	2.9[illegible]8,822
17	22.700,918	19.803,739	2.837,179
18	22.599,067	19.732,778	2.866,289
19	22.495,450	19.597,560	2.897,8[illegible]0
20	22.282,791	19.4[illegible]7,[illegible]03	2.824,8[illegible]8
21	22.173,686	19.338,[illegible]21	2.[illegible],8[illegible]5
22	21.949,7[illegible]5	19.[illegible]16,339	2.733,426
23	21.834,882	19.[illegible]90,318	2.744,564
24	21.599,103	18.960,613	2.638,490

Years of Age.	*Values of an annuity of one pound by Mr. Weyman Lee's method.*	*Values of the same annuity by Dr. Halley's method.*	*Differences of the said values.*
	£	£	£
25	21.478,137	18.827,070	2.651,067
26	21.355,072	18.689,528	2.665,54[illegible]
27	21.102,499	18.547,817	2.[illegible]54,682
28	20.972,917	18.401,759	2.571,158
29	20.706,969	18.251,167	2.455,802
30	2[illegible].570,525	18.095,8[illegible]4	2.[illegible]74,681
31	20.290,493	17.935,582	2.354,911
32	20.146,823	17.770,162	2.376,661
33	19.851,962	17.599,354	2.[illegible]52,608
34	19.700,684	17.422,915	2.277,769
35	19.390,208	17.240,588	2.149,620
36	19.068,815	17.052,103	2.016,762
37	18.904,000	16.857,174	2.046,826
38	18.565,640	16.629,189	1.936,451
39	18.392,045	16.392,655	1.999,390
40	18.035,[illegible]67	16.147,167	1.888,60[illegible]
41	17.667,018	15.892,297	1.774,[illegible]21
42	17.285,364	15.580,939	1.704,425
43	17.089,556	15.303,763	1.[illegible]85,793
44	16.890,552	1[illegible].015,669	1.874,083
45	16.481,514	14.716,120	1.765,394
46	16.058,367	14.404,549	1.653,8[illegible]8
47	15.841,272	14.105,200	1.736,072
48	15.395,716	13.793,859	1.601,857
49	15.167,124	13.494,425	1.672,699
50	14.697,974	13.183,083	1.514,891
51	14.212,403	12.883,449	1.328,954
52	13.963,281	12.596,206	1.366,985
53	13.709,837	12.298,386	1.411,451
54	13.189,681	11.989,093	1.200,588
55	12.922,816	11.691,801	1.231,015
56	12.651,320	11.383,427	1.267,893
57	12.094,116	1[illegible].063,485	1.030,631
58	11.[illegible]08,243	10.755,123	1.053,120
59	11.517,410	10.435,566	1.081,844
60	10.920,520	10.104,074	0.816,446
61	10.302,758	9.759,829	0.542,9[illegible]9
62	9.985,785	9.401,925	0.583,860
63	9.663,334	9.053,059	0.610,275
64	0.235,288	8.690,648	0.644,640

Years of Age.	*Values of an annuity of one pound by Mr. Weyman Lee's method.*	*Values of the same annuity by Dr. Halley's method.*	*Differences of the said values.*
	£	£	£
65	8.662,023	8.313,625	0.348,398
66	8.316,605	7.944,258	0.372,347
67	7.965,194	7.583,727	0.381,467
68	7.607,686	7.233,699	0.373,987
69	7.243,976	6.896,496	0.347,480
70	6.873,955	6.575,357	0.298,598
71	6.497,515	6.249,840	0.247,675
72	6.114,543	5.945,972	0.168,571
73	5.724,9[illegible]	5.644,448	0.080,480
74	5.328,[illegible]	5.347,806	0.019,253
75	4.925,3[illegible]	5.059,623	0.134,323
76	4.515,052	4.754,921	0.239,809
77	4.515,052	4.461,839	0.053,213
78	4.097,686	4.187,758	0.[illegible]9 ,070
79	3.673,079	3.908,000	0 234,921
80	3.673,079	3.661,781	0,011,298
81	3.241,105	3.427,856	0.186,751
82	3.241,105	3.215,658	0.025,447
83	2.801,636	2.984,473	0.182,837
84	2.801,636	2.717,188	0.084,448
85	2.801,636	2.456,774	0 344,862
86	2,354,544	2.211,911	0.142,633
87	2.354,544	1.999,810	0.354,734
88	1.899,694	1.728,378	0.171,316
89	1.899,694	1.459,699	0.439,995
90	1.436,953	1.197,512	0.239,441
91	1.436,9[illegible]3	0.947,669	0.489,284
92	0.966,183	0.716.468	0.249,715
93	0.966,183	0.483[illegible]9.	0.4[illegible]3,093
94	0.000,000		

Obfervations on the differences of the two fets of values of life-annuities fet down in the foregoing table.

CLXXXVII. From the foregoing table it appears, that at the age of 3 years the value of a life-annuity obtained by Mr. Lee's method is greater than its true value by more than 4 years purchafe; and that, from the age of 3 years to the age of 10 years, inclufively, it exceeds the true value by more than three years purchafe; and that, from the age of 10 years to the age of 37 years, inclufively, it exceeds the true value by more than two years purchafe; and that, from the age of 37 years to the age of 59 years, inclufively, it exceeds the true value by more than one year's purchafe; and that from the age of 59 years to the age of 74 years, inclufively, it exceeds the true value by lefs than one year's purchafe, and that in that laft interval, from the age of 59 years to the age of 74 years, its excefs above the true value becomes gradually lefs and lefs, till, at the faid latter age of 74 years, it is almoft equal to 0. But, after the faid age of 74 years, the value of a life-annuity obtained by the faid method of Mr. Lee, is, for about 9 years, or till the age of 83 years, nearly equal to, but for the moft part fomewhat lefs than, its true value; after which it again becomes greater than the faid true value at the age of 84 years, and continues to exceed the faid true value throughout the whole remainder of the table, or in every following year of human life.

CLXXXVIII. The differences of the foregoing values of life-annuities, obtained by Mr. Lee's method of computation, from their true values, are fo confiderable before the ages of 39 or 40 years, that it would be by no means advifeable to have recourfe to thofe values in fettling the prices of life-annuities that are to be granted for the lives of perfons under thofe ages. And even to the age of 64 years, inclufively, I fhould think thefe differences too great to be neglected. But from the age of 65 years to the age of 85, or 86, years, thofe values might do tolerably well as a guide in the purchafe of life-annuities, if a table containing the true values of the annuities for lives of thofe ages were not at hand. But, when tables of the true values of life-annuities, (computed ftrictly from a table of the probabilities of human life, and not from Mr. De Moivre's Hypothefis, or any other inaccurate fuppofition adopted by calculators for the fake of abridging their trouble,) are ready-computed to our hands, it feems to be imprudent and unfatisfactory to have recourfe to any other values of them.

CLXXXIX. We have feen in the foregoing table, that Mr. Lee's method of eftimating life-annuities is doubly erroneous. For it makes the values of them much greater than they ought to be throughout the greater part of human life; and, for a few years between the ages of 74 years and

and 84 years, it makes them rather lefs than they ought to be. Yet Mr. Lee is very confident that it is the only right method of valuing life-annuities, and feems to wonder that any body can doubt of its being fo. And, to remove thefe doubts, and fully eftablifh the juftnefs of his faid method he has given us what he calls two diftinct proofs of it. Thefe proofs are expreffed in very vague and loofe terms, infomuch that I muft confefs myfelf unable to underftand them. The words *chance* and *chances* occur in them very frequently, and fometimes the expreffion of *the chance of, or to, a chance:* but they feem to be ufed without any diftinct meanings annexed to them. However, as this is a matter which relates to the very foundation of the doctrine of life-annuities, I fhould be glad that my readers would exercife their own judgements concerning it; and therefore, to enable them to do fo, (as Mr. Lee's tracts are not now eafily to be met with) I fhall here prefent them with a tranfcript of the paffage, in Mr. Lee's fecond pamphlet above-mentioned, which contains the faid two fuppofed proofs of the juftnefs of his method of valuing life-annuities. In this paffage Mr. Lee has introduced fome numbers into his argument, by way of example and illuftration, which are grounded upon Dr. Halley's Breflaw table of the probabilities of the duration of human life, and on a fuppofition that the intereft of money is 6 per cent. Thefe numbers I have taken the liberty to alter fo as to make them fuit with Monfieur de Parcieux's table of probabilities and a fuppofition that the intereft of money is $3\frac{1}{2}$ per cent. in order to avoid the unneceffary trouble and perplexity that might have arifen from the confideration of a different table of probabilities and a different rate of intereft from thofe which are become familiar to us by our frequent ufe of them in the courfe of the foregoing articles. Thefe alterations in Mr. Lee's numbers, it is evident, cannot in the leaft affect the force of his reafonings, (if there is any force in them,) and therefore may be made without injuring him. And, with thefe alterations in the numbers, the paffage containing Mr. Lee's two fuppofed proofs of the juftnefs of his method of valuing life-annuities, in pages 8, 9, and 10 of his fecond pamphlet above-mentioned, intitled, *A valuation of annuities for lives*, is in the words following.

Of Mr. Weyman Lee's attempts to prove the juftnefs of his method of valuing life-annuities.

CXC. " Since the author [Mr. H. B.] is fo full of, and fo learned in, " the doctrine of chances, as he calls it, I will take the liberty of offering " to his confideration an argument, or two, in behalf of my rule, according to his way of thinking. The value of the chances in this cafe " depends on, or is one and the fame thing as, the value of the annuities " dependant on thofe chances. All the chances which an annuitant has on " the life of A [a perfon of the age of 10 years] are agreed to be 880: " and of thofe he has an even chance to enjoy a moiety, that is, 440.

Extracts from Mr. Lee's fecond tract, publifhed in the year 1751.

" The

" The annuities which attend thoſe chances for the whole life are annuities " for the term of 85 years, it being ſuppoſed poſſible that he may live for " ſo many years. The total value of them for 85 years, as I compute it, " is 27.036,803; and the value of thoſe in poſſeſſion for 52 years is " 23.795,764, and of thoſe in reverſion 3.241,039. Since then the an- " nuitant on the life of A has an even chance to 440 chances on his life, " and the annuities will attend thoſe chances;——and, ſince theſe 440 " chances will, and, from the nature of the life of man, of neceſſity muſt, " ariſe in the firſt part of thoſe 85 years, if they ariſe at all; (and 'tis " ſuppoſed in the caſe to be an even chance that ſo many chances will " ariſe in his life:) he muſt then of neceſſity have the annuities attending " thoſe very chances, that is, he muſt have the annuities for the firſt 52 " years of his life. This author [Mr. H. B.], or whoever will undertake " to anſwer this argument, muſt ſhew that 'tis poſſible for an annuitant on " the life of A to have the benefit of all theſe 440 chances (to which " number of chances he has without diſpute a right, or an even chance to " enjoy,) unleſs he does enjoy thoſe which ariſe in the firſt part of life and " in immediate poſſeſſion, that is, thoſe of the greateſt value. And this, " I am very ſure, he cannot ſhew.

" Again; theſe poſitions are not conteſted:——That an annuity for a " term of 52 years is in value £23.795,764;——and that an annuitant " for the life of A has an even chance that A lives for 52 years.—'Tis a " maxim made uſe of by this author, and is certainly a true one, That, " in eſtimating the value of annuities for a life, all the poſſible chances of " life muſt be computed. From hence I argue thus. To eſtimate the " value of an annuity for the life of A, we muſt compute all the poſſible " chances on the life of A. The chance that A lives for 52 years is an " even chance, and conſequently is one of the poſſible chances on the life " of A. Therefore, to eſtimate the value of an annuity for his life, we " muſt compute this even chance. The even chance on this life is to a term " for 52 years. A term for 52 years is in value £23.795,764. Therefore " the value of an annuity for this life is £23.795,764."

CXCI. Theſe are the words of Mr. Weyman Lee's proofs of the truth of his method of valuing life-annuities. But what meaning is to be found in them, or what train of reaſoning Mr. Lee might purſue obſcurely in his own mind, when he uſed them, I will not pretend to determine. And yet I am inclined to conjecture that his meaning (though certainly it is not expreſſed

expreſſed in the foregoing words,) might be as follows; to wit, "that, if a perſon were to make 880 different grants, of one pound a year each, to as many different grantees, all of the age of 10 years, for their reſpective lives, half the ſaid grantees would be dead in the ſpace of 52 years, and the other half would live beyond that term; and conſequently 440 of the ſaid 880 annuities would become extinct in the courſe of the ſaid 52 years, and the other 440 of them would continue to be payable after the ſaid 52 years. And hence, I preſume, Mr. Lee was inclined to conclude, that therefore it would be the ſame thing, in point of advantage, to the grantor of theſe 880 annuities, whether he granted them to the ſaid 880 perſons for the life of each of them reſpectively, or whether he granted them to the ſaid perſons and their executors, or other repreſentatives, for a term certain of 52 years; becauſe that, if he had, firſt, granted them for a term certain of 52 years, and afterwards was, by the conſent of all parties, to change thoſe grants into grants of the ſame annuities of one pound each to the ſame 880 perſons of the age of 10 years for the lives of the ſeveral grantees reſpectively, *the burthen* that would fall on the ſaid grantor in conſequence of this change in the nature of the grants, by means of the payments he would thereby be obliged to make after the expiration of the ſaid term of 52 years, to the 440 annuitants who would live beyond that term, would be counter-balanced by *the profit*, or ſaving, that would accrue to him before the end of the ſaid term of 52 years by the extinction of the annuities that had been granted to the other 440 annuitants who would have died in the courſe of the ſaid term." This is the only argument that I can imagine to have been intended to be expreſſed in the foregoing paſſage of Mr. Lee's pamphlet. But, if this was the argument Mr. Lee relied on, he was greatly miſtaken in thinking it a juſt one, though it may, at firſt ſight, appear plauſible. For the truth is, that the profit accruing to the grantor of the 880 annuities of one pound by the aforeſaid change in his bargain, by means of the extinction of the annuities that had been granted to the 440 annuitants who had died before the end of the 52 years, will *much more than counter-balance* the burthen that would fall on him, in conſequence of ſuch a change of his firſt bargain, by means of the payments he would thereby be obliged to make to the other 440 annuitants who would live beyond the ſaid term of 52 years. And one very obvious reaſon why we ſhould expect that the ſaid *profit* ſhould *more than counter-balance* the ſaid *burthen*, in ſo young an age as this of 10 years, is, that the payments which conſtitute the ſaid profit, namely, the payments of the annuities ſaved by the deaths of the 440 annuitants who die before the end of the ſaid 52 years, are *much leſs remote*, and conſequently *much more valuable*, than the payments which conſtitute the ſaid burthen, or the payments incurred by the obligation of continuing the annuities, during their reſpective lives, to the other 440 annuitants who will live beyond the ſaid term of 52 years. In what preciſe proportion, or degree, the ſaid profit will

A conjecture concerning the meaning of the foregoing extracts from Mr. Lee's pamphlet.

Mr. Lee's miſtake in the ſaid argument ſuppoſed to be contained in the foregoing extracts.

will exceed the said burthen, is a nice question, and can only be determined by observing, in Monsieur de Parcieux's table of the probabilities of human life, how many of the 880 persons who are there represented as living at the age of 10 years, (and to whom we suppose annuities of one pound to have been granted for their respective lives,) will die off in every individual year of the whole term of 85 years through which it is possible that their lives may be extended, and computing, first, the values of the several payments that will be saved in each of the first 52 years of the said term of 85 years, by the deaths of those 440 persons who will die in the course of the said 52 years, and, secondly, the values of the payments that will be incurred in each of the remaining 33 years of the said term of 85 years after the 52d year, by the continuance of the lives of the other 440 annuitants, who will live beyond the said 52 years, and then comparing the sum of the values of the former payments with the sum of the values of the latter payments. This would be a nice and tedious inquiry, and would be tantamount to the computation of the value of an annuity of one pound for the life of a person of the age of 10 years in the manner explained above in Problem 11 and its corollaries, and exemplified in Art. 94. But it is sufficient to invalidate the foregoing argument, (which I have supposed to be that which was meant to be advanced by Mr. Lee;) that the *exact equality* of the said profit to the said burthen (which in the said argument is taken for granted, as a thing self-evident,) is by no means apparent; but that there is, on the contrary, good reason to suppose (without going into the nice inquiry just now mentioned,) that the said profit in this case of an annuity for a life of the age of 10 years, is *much greater* than the said burthen. For then it will not be the same thing (as it is supposed to be in the said argument) in point of advantage, to the grantor of the said 880 annuities of one pound to as many grantees, all of the age of 10 years, whether he grant them the said annuities for the term of 52 years certain, or whether he grant them for the lives of each several grantee; but the latter bargain will be less burthensome to the grantor than the former; and therefore he ought to receive a less price from the said 880 grantees in the latter case than in the former. And consequently, it will follow from Art. 21, that, when the grantor makes only one such grant of an annuity of one pound to a person of the age of 10 years, he ought to receive a less price for it than he ought to receive for the grant of the same annuity for a term certain of 52 years; or, in other words, there is reason to suppose (even without going into the nice inquiry above-mentioned,) that the value of an annuity of one pound a year for the life of a person of the age of 10 years is less than the value of the same annuity for a term certain of 52 years; contrary to what Mr. Lee has asserted.

CXCII. The

CXCII. The only way of determining truly the value of any propofed life-annuity is that which is explained above in Art 21, and made the foundation of this whole doctrine, namely, to fuppofe the grantor of the annuity to make many more fuch grants to many other grantees, all of the fame age as the firft, or propofed, grantee, and then to inquire what price ought to be paid by each of the faid grantees to the grantor of all the faid annuities, in order to make him, at the clofe of the whole tranfaction, or when all the faid annuitants fhall be dead, be neither a gainer nor a lofer by the fum total of all his bargains; upon a fuppofition that he improves the money he receives from the faid grantees, as the price of their annuities, at compound intereft according to a certain given rate, and that the grantees of the faid annuities die off every year in the proportions reprefented in fome particular table of the probabilities of the duration of human life that is adopted as the ground of the calculation. That price, and no other, is the *true, or fair, value* of fuch a life-annuity: and, whenever we attempt to eftimate a life-annuity without thus fuppofing a great number of the like annuities to be granted at the fame time to other perfons of the fame age as the propofed annuitant, we fhall find ourfelves bewildered and confufed, and without any folid foundation to build upon. And I have no doubt that it is only to the want of fuch *a criterion*, or *meafure*, whereby to judge of the true value of a life-annuity, or, perhaps I ought to fay, of fuch *a definition* of fuch true value, that we ought to afcribe the obfcurity and confufion with which Mr. Weyman Lee treats of this fubject, both in the paffage above-cited from him, in which he endeavours to prove the rectitude of his own erroneous method of eftimating life-annuities, and in other parts of the fame tract, in which he endeavours to prove the falfhood of the method which has been ufed by Dr. Halley, and all other mathematicians, for that purpofe, and which has been above explained and illuftrated in Prob. 11 and its corollaries, and in feveral of the following articles. All this perplexity would have been avoided, if he had previoufly fettled clearly and diftinctly in his own mind what he meant by *the fair price, or value*, of a life-annuity.

Of the neceffity of keeping conftantly in view the idea of *the fair price of a life-annuity* given above in Art. XXI.

CXCIII. I have been induced to enlarge the more on the falfhood of this erroneous method of valuing life-annuities, becaufe I have obferved, that not only Mr. Weyman Lee in the year 1751, but a later and much abler writer on the fame fubject, Mr. Dale, in his ufeful book on life-annuities, intitled, *Calculations deduced from firft principles, &c.* publifhed in the year 1772, feems likewife to be perfuaded of its truth. See the faid book of Mr. Dale, article 2d of the Addenda.

End of the examination of Mr. Weyman Lee's erroneous method of valuing Life-annuities.

CXCIV. From

CXCIV. From these digressions, concerning Mr. De Moivre's Hypothesis for abridging the computations of life-annuities, and Mr. Weyman Lee's erroneous method of valuing those annuities, I now return to the only true and accurate method of computing the values of the said annuities, which is that which has been above explained in Problem 11 and its corollaries. From this method, (as we have shewn above in Art. 100, 101, and 102,) the expression $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|$£ may be derived, by which the value of an annuity of one pound for any proposed life may be computed from the value of the same annuity for a life that is one year older. Nothing can well be expected more concise and convenient for the purpose of computing a table of life-annuities, than this expression. Nevertheless it is liable to this one inconvenience, that, as the values of life-annuities for different ages are deduced by it one from another, in regular succession from the older ages to the younger, an error that should happen to be made in computing the value of one of these life-annuities would affect the values of all the following life-annuities, or of the annuities for all the ages that were younger than that in which the error was originally made. This inconvenience we have in some measure endeavoured to guard against in Art. 120, by shewing how an uncommonly large increase, or decrease, in the differences of the values of these successive life-annuities will, for the most part, afford a just ground of suspicion that some mistake has been made in the computation of the last value by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|$£, and thereby will induce the calculator to revise his last computation; in consequence of which revision such mistake, if any has been made, will be discovered. But Mr. Morgan in his useful and learned tract upon life-annuities, intitled, *The doctrine of annuities and assurances on lives and survivorships*, (pages 59, 60, 61, &c.——68,) has given us a much better and more satisfactory method of removing this inconvenience, and proving the truth of the several operations, as they arise. This method may be explained as follows.

Of Mr. Morgan's method of proving the truth of the computations of the values of Life-annuities, as fast as they are made.

CXCV. Let A denote the number of persons represented in any table of the probabilities of the duration of human life as living at the youngest age that is set down in the said table; which age in Monsieur de Parcieux's table is the age of 3 years. Secondly, Let P be the number of persons represented in the said table as living at any subsequent, or older, age, consisting of N years; and let n be the number of years by which the said faid age of N years exceeds the youngeft age in the table. And, thirdly, let £ r be the value of one pound together with its intereft for one year.

An expreffion of the value of a fingle future payment of one pound, to be received by a perfon of the youngeft age in the table, after a given number of years, if he fhall be then alive.

Then, it is evident, that the value of a fingle payment of one pound to be received at the end of n years by a perfon of the youngeft age in the table, if the faid perfon fhall fo long live, that is, if he fhall live to the age of N years, will be £ $\frac{1}{r^n} \times \frac{P}{A}$, or £1 $\times \frac{P}{Ar^n}$. In Monfieur de Parcieux's table of probabilities the number of perfons reprefented as living at the age of 3 years (which is the youngeft age in the table,) is 1000; and confequently A is $= 1000$, and £1 $\times \frac{P}{Ar^n}$ is $=$ £1 $\times \frac{P}{1000r^n}$.

An expreffion of the value of a remote life-annuity of one pound a year, for the life of a perfon of the youngeft age in the table, to commence at a given future age.

CXCVI. Further, if P^{I} denote the number of perfons reprefented in the table of probabilities as living at the age of $N+1$ years, and P^{II} the number living at the age of $N+2$ years, and P^{III} the number living at the age of $N+3$ years, and fo on throughout the remainder of the table, the number annexed, (in Roman figures,) to the top of the letter P being every where the fame with the number of years added to N, and the faid letter P, with fuch number fo annexed, denoting the number of perfons living at the age denoted by N with the fame number added to it; it is evident that the value of a remote annuity of one pound a year for the life of a perfon of the youngeft age in the table of probabilities, to commence at the end of n years, or whereof the firft payment is to be received at the end of $n+1$ years, will be equal to £ $\frac{1}{A} \times$ the feries

$$\frac{P^{\mathrm{I}}}{r^{n+1}} + \frac{P^{\mathrm{II}}}{r^{n+2}} + \frac{P^{\mathrm{III}}}{r^{n+3}} + \frac{P^{\mathrm{IV}}}{r^{n+4}} + \frac{P^{\mathrm{V}}}{r^{n+5}} + \frac{P^{\mathrm{VI}}}{r^{n+6}} + \frac{P^{\mathrm{VII}}}{r^{n+7}} + \&c.$$

continued to the end of the table, or to the utmoft poffible extent of human life, which in Monfieur de Parcieux's table is 95 years; or it will be equal to £1 $\times$ the feries $\frac{P^{\mathrm{I}}}{Ar^{n+1}} + \frac{P^{\mathrm{II}}}{Ar^{n+2}} + \frac{P^{\mathrm{III}}}{Ar^{n+3}} + \frac{P^{\mathrm{IV}}}{Ar^{n+4}} + \frac{P^{\mathrm{V}}}{Ar^{n+5}} + \frac{P^{\mathrm{VI}}}{Ar^{n+6}} + \frac{P^{\mathrm{VII}}}{Ar^{n+7}} + \&c.$ continued to the end of the table, or to the utmoft poffible extent of human life.

 CXCVII. Now

The Propofition that is the ground of Mr. Morgan's method above-mentioned.

CXCVII. Now this remote annuity of one pound a year for the life of a perfon of the youngeft age in the table is equal to the product that arifes by multiplying the value of a like annuity of one pound a year for the life of a perfon of the age of N years, into $£1 \times \frac{P}{Ar^n}$, or the prefent value of the above-mentioned fingle payment of one pound to be made to a perfon of the youngeft age in the table at the end of n years, or when he fhall have attained the age of N years, if he fhall fo long live; or, if we put $\overset{£}{V}$ for the value of an annuity of one pound a year for the life of a perfon of the age of N years, and $\overset{£}{R}$ for the value of a remote annuity of the fame fum of one pound a year for the life of a perfon of the youngeft age in the table to commence at the diftance of n years, or whereof the firft payment is to be received at the end of $n + 1$ years, we fhall have $\overset{£}{R} = \overset{£}{V} \times \overset{£}{1} \times \frac{P}{Ar^n}$.

DEMONSTRATION.

For $\overset{£}{V}$, or the value of an annuity of one pound a ye.. for the life of a perfon of the age of N years, is (by Art. 86 and 87) $= £1 \times$ the feries $\frac{P^{I}}{Pr} + \frac{P^{II}}{Pr^2} + \frac{P^{III}}{Pr^3} + \frac{P^{IV}}{Pr^4} + \frac{P^{V}}{Pr^5} + \frac{P^{VI}}{Pr^6} + \frac{P^{VII}}{Pr^7} + \&c.$ continued to the end of the table of probabilities, or to the utmoft extent of human life. Therefore $£1 \times \frac{P}{Ar^n} \times \overset{£}{V}$ is $= £1 \times \frac{P}{Ar^n} \times £1 \times$ the feries $\frac{P^{I}}{Pr} + \frac{P^{II}}{Pr^2} + \frac{P^{III}}{Pr^3} + \frac{P^{IV}}{Pr^4} + \frac{P^{V}}{Pr^5} + \frac{P^{VI}}{Pr^6} + \frac{P^{VII}}{Pr^7} + \&c.$ continued to the end of the table of probabilities, and confequently is $= £1 \times$ the feries $\frac{P^{I}}{Ar^{n+1}} + \frac{P^{II}}{Ar^{n+2}} + \frac{P^{III}}{Ar^{n+3}} + \frac{P^{IV}}{Ar^{n+4}} + \frac{P^{V}}{Ar^{n+5}} + \frac{P^{VI}}{Ar^{n+6}} + \frac{P^{VII}}{Ar^{n+7}} + \&c.$ continued to the end of the table of probabilities. But, by Art. 196, $\overset{£}{R}$, or the value of a remote annuity

annuity of one pound a year for the life of a perſon of the youngeſt age in the table of probabilities, to commence at the end of n years, or whereof the firſt payment is to be received at the end of $n+1$ years, is $= £1 \times$ the ſeries $\frac{P^{I}}{Ar^{n+1}} + \frac{P^{II}}{Ar^{n+2}} + \frac{P^{III}}{Ar^{n+3}} + \frac{P^{IV}}{Ar^{n+4}} + \frac{P^{V}}{Ar^{n+5}} + \frac{P^{VI}}{Ar^{n+6}} + \frac{P^{VII}}{Ar^{n+7}}$ + &c. continued to the end of the table of probabilities. Therefore $\overset{\pounds}{1} \times \frac{P}{Ar^{n}} \times \overset{\pounds}{V}$, or $\overset{\pounds}{V} \times \overset{\pounds}{1} \times \frac{P}{Ar^{n}}$, is $= \overset{\pounds}{R}$. Q.E.D.

Mr. Morgan's rule for proving the truth of the computation of the value of a life-annuity for any given age.

CXCVIII. From the foregoing propoſition Mr. Morgan's rule for proving the truth of a computation of a life-annuity for a life of any given age is an eaſy conſequence. This rule is as follows. Let $\overset{\pounds}{V}$ be put for the value of an annuity of one pound a year for the life of a perſon of the given age of N years, which number of years exceeds the number of years in the youngeſt age ſet down in the table of probabilities by n years. Then, in order to try whether $\overset{\pounds}{V}$ has been rightly computed, compute the quantity $£1 \times \frac{P}{Ar^{n}}$, or the value of a ſingle future payment of one pound to be made to a perſon of the youngeſt age ſet down in the table of probabilities at the end of n years, if ſuch perſon ſhall then be living; and likewiſe compute the ſeveral terms of the ſeries $\frac{P^{I}}{Ar^{n+1}} + \frac{P^{II}}{Ar^{n+2}} + \frac{P^{III}}{Ar^{n+3}} + \frac{P^{IV}}{Ar^{n+4}} + \frac{P^{V}}{Ar^{n+5}} + \frac{P^{VI}}{Ar^{n+6}} + \frac{P^{VII}}{Ar^{n+7}}$ + &c. continued to the end of the table of probabilities, or to the utmoſt extent of human life; and then add the terms of this ſeries up together into one ſum total, which call S: and, laſtly, multiply $\overset{\pounds}{V}$ into $\overset{\pounds}{1} \times \frac{P}{Ar^{n}}$. And, if the product of this multiplication is equal to $£1 \times S$, or if $V \times \frac{P}{Ar^{n}}$ is equal to S, we may conclude that $\overset{\pounds}{V}$, or the value of the ſaid annuity of one pound a year for a life of the age of N years, has been rightly computed.

For

Proof of the ſaid rule.

For it is evident from Art. 196 that $\overset{\pounds}{1} \times S$ is the value of a remote annuity of one pound a year for the life of a perſon of the youngeſt age in the table, to commence at the diſtance of n years, ſo that the firſt payment thereof ſhall be received at the diſtance of $n+1$ years, which value is called $\overset{\pounds}{R}$ in Art. 197; that is, $\overset{\pounds}{1} \times S$ is $= \overset{\pounds}{R}$. Therefore when $\overset{\pounds}{V} \times \overset{\pounds}{1} \times \frac{P}{Ar^n}$ is equal to $\overset{\pounds}{1} \times S$, it will be equal likewiſe to $\overset{\pounds}{R}$, and conſequently (by Art. 197,) the value denoted by $\overset{\pounds}{V}$ muſt have been rightly computed. Q.E.D.

An account of the conſtruction of the table of numbers next enſuing, or Table XI.

CXCIX. To make the manner of applying this rule of Mr. Morgan more apparent, it will be proper to ſet down again in regular order, in a new table, the values of the life-annuities which we computed above in Art. 108, ——— 115, from Monſieur de Parcieux's table by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V}\,\pounds$. And, as, in obtaining the ſaid values one from another by means of that expreſſion, we proceeded upwards, or from the older lives to the younger, it will be convenient to ſet down the ſaid values in the ſame order in the following table in a column adjoining to another which contains the numbers of years in the correſponding ages. After theſe two columns of the ages of the lives and the values of the annuities, I ſhall ſet down, in a third column, the preſent values of a ſingle payment of one pound to be received by a perſon of the age of 3 years (which is the youngeſt age in Monſieur de Parcieux's table of probabilities,) at the ends of 91 years, 90 years, 89 years, 88 years, and every following number of years reſpectively (reckoning in this backward order) to the end of one year, or at the ages of 94 years, 93 years, 92 years, 91 years, and every following younger age (proceeding by a difference of one year,) to the age of 4 years; which ſeries, it is evident, will contain in it all the terms of the ſeries $\frac{P^{\text{I}}}{Ar^{n+1}} + \frac{P^{\text{II}}}{Ar^{n+2}} + \frac{P^{\text{III}}}{Ar^{n+3}} + \frac{P^{\text{IV}}}{Ar^{n+4}} + \frac{P^{\text{V}}}{Ar^{n+5}} + \frac{P^{\text{VI}}}{Ar^{n+6}} + \frac{P^{\text{VII}}}{Ar^{n+7}} +$ &c. in whatever part of the table of probabilities the number P^{I} be taken, or whatever be the number of years denoted by $N+1$, or by N. And then, in a fourth column, I ſhall ſet down the ſums of the terms in the foregoing ſeries of values contained in the third column, as they ariſe; ſo that every term in this fourth

fourth column that is even with any given age, ſhall be equal to the ſum of all the terms in the third column that correſpond to the ages that are older than the ſaid given age. Thus, for example, the term in the fourth column that is even with the age of 50 years is equal to the ſum of all the terms in the third column that correſpond to the ages that are older than 50 years. Theſe ſums will be equal to the values of remote annuities of one pound a year for the life of a perſon of the age of 3 years, to commence at the diſtances of 90 years, 89 years, 88 years, 87 years, &c. or ſo that the firſt payments of them ſhall become due at the diſtances of 91 years, 90 years, 89 years, 88 years, &c. and will comprize all the different values of the quantity which in Art. 197 is called *R*, and in Art. 198 is called £ 1 × *S*. And, laſtly, in a fifth column, I ſhall ſet down the products that ariſe by multiplying the terms of the ſecond column, or the values of the life-annuities for the ſeveral ages ſet down in the firſt column, by the correſponding terms, or terms that are placed even with them, in the third column. And ſo far as we find theſe products, ſet down in the fifth column, to coincide with the numbers in the fourth column that are placed even with them, we may conclude that the numbers in the ſecond column, or the values of the life-annuities correſponding to the ages ſet down in the firſt column, have been rightly computed. This table will be as follows.

TABLE

TABLE XI.

Consisting of five columns of numbers; in the first of which the numbers of years in the several ages of human life, that differ from each other by a year, from the age of 94 years to the age of 3 years, inclusively, are set down in regular order; and in the second column are set down the several values of an annuity of one pound a year for the lives of persons of the several ages set down in the first column, respectively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 3½ per cent. and in the third column are set down the present values of a single payment of one pound to be received by a child of the age of 3 years at the ends of 91 years, 90 years, 89 years, 88 years, and of every following lesser number of years down to one year, respectively, if the child shall live to the ends of the said years; and in the fourth column are set down the numbers that arise by the continual addition of the numbers set down in the third column, so that each number in the said fourth column is equal to the sum of all the numbers in the third column that are placed above it, or that correspond to the preceeding, or older, ages; and in the fifth and last column are set down the products that arise by multiplying the terms of the second column (or the values of the life-annuities for the several ages set down in the first column,) by the corresponding terms of the third column, respectively.

Years of Age.	*Values of a life-annuity of one pound a year.*	*Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.*	*Sums of the values in the third column.*	*Products of the multiplication of the numbers in the second and third columns.*
	£	£		
94	0.000,000	0.000,043	£	£
93	0.483,091	0.000,090	0.000,043	0.000,043
92	0.716,468	0.000,187	0.000,133	0.000,133
91	0.947,669	0.000,339	0.000,320	0.000,321
90	1.197,512	0.000,551	0.000.659	0.000,659
89	1.4[illegible]9,699	0.000,830	0.001,210	0.001,211

Years of Age.	Values of a life-annuity of one pound a year.	Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.	Sums of the values in the third column.	Products of the multiplication of the numbers in the second and third columns.
	£	£	£	£
88	1.728,378	0.001,181	0.002,040	0.002,041
87	1.999,810	0.001,612	0.003,221	0.003,223
86	2.211,911	0.002,186	0.004,833	0.004,835
85	2.456,774	0.002,858	0.007,019	0.007,021
84	2.717,188	0.003,636	0.009,877	0.009,879
83	2.984,473	0.004,529	0.013,513	0.013,516
82	3.215,658	0.005,612	0.018,042	0.018,046
81	3.427,856	0.006,901	0.023,654	0.023,655
80	3.661,781	0.008,345	0.030,555	0.030,557
79	3.908,000	0.009,955	0.038,900	0.038,904
78	4.187,75[illegible]	0.011,667	0.048,855	0.048,858
77	4.461,839	0.013,566	0.060,52[illegible]	0.060,529
76	4.754,921	0.015,583	0.074,088	0.074,095
75	5.059,623	0.017,724	0.089,671	0.089,676
74	5.347,806	0.020,083	0.107,395	0.107,399
73	5.644,448	0.02[illegible]86	0.127,478	0.127,485
72	5.945,972	0.025,239	0.150,064	0.150,070
71	6.249,840	0.028,050	0.175,303	0.175,308
70	6.575,357	0.030,928	0.203,353	0.203,362
69	6.896,496	0.033,972	0.23[illegible],281	0.234,287
68	7.233,699	0.037,085	0.268,253	0.268,261
67	7.583,727	0.040,203	0.305,338	0,305,343
66	7.944,258	0.043,50[illegible]	0.345,601	0.345,614
65	8.313,625	0.046,805	0.389,106	0.389,119
64	8.690,648	0.050,160	0.435,911	0.435,922
63	9.053,059	0.053,693	0.486,071	0.486,085
62	9.401,925	0.057,41[illegible]	0.539,764	0.539,773
61	9.759,829	0.061,188	0.597,175	0.597,184
60	10.104,074	0.065,159	0.658,363	0.658,3[illegible]1
59	10.435,566	0.069,334	0.723,522	0.723,539
58	10.755,123	0.073,720	0.792,856	0.792,867
57	11.[illegible]63,485	0.078,329	0.866,576	0.866,591
56	11.383,427	0.083,008	0.944,905	0.944,915
55	11.691,801	0.087,919	1.027,913	1.027,931
54	11.989,093	0.093,072	1.115,832	1.115,848

Years of Age.	*Values of a life-annuity of one pound a year.*	*Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.*	*Sums of the values in the third column.*	*Products of the multiplication of the numbers in the second and third columns.*
	£	£	£	£
53	12.298,386	0.098,300	1.208,904	1.208,931
52	12.596,296	0.103,779	1.307,204	1.307,231
51	12.883,449	0.109,521	1.410,983	1.411,008
50	13.183,083	0.115,339	1.520,504	1.520,523
49	13.494,425	0.121,225	1.635,843	1.635,861
48	13.793,8[illegible]9	0.127,382	1.757,068	1.757,089
47	14.105,200	0.133,601	1.884,450	1.884,468
46	14.404,5[illegible]9	0.140,100	2.018,051	2.018,077
45	14.716,120	0.146,6[illegible]4	2.158,151	2.158,177
44	15.015,669	0.153,495	2.304,805	2.304,830
43	15.303,763	0.160,635	2.458,300	2.458,320
42	15.580,939	0.168,087	2.618,935	2.618,953
41	15.847,705	0.175,864	2.787,022	2.787,040
40	16.104,542	0.183,980	2.962,886	2.962,904
39	16.351,906	0.192,448	3.146,866	3.146,891
38	16.590,228	0.201,283	3.339,314	3.339,330
37	16.819,920	0.210,502	3.540,597	3.540,626
36	17.016,528	0.220,440	3.751.099	3.751,123
35	17.206,612	0.230,816	3.971,539	3.971,561
34	17.390,462	0.241,649	4.202,355	4.202,387
33	17.568,352	0.252,957	4.444,004	4.444,037
32	17.740,543	0.264,761	4.696,961	4.697,003
31	17.907,280	0.277,080	4.961,722	4.961,748
30	18.068,798	0.289,938	5.238,802	5.238,831
29	18.225,318	0.303,357	5.528,740	5.528,777
28	18.377,051	0.317,360	5.832,097	5.832,140
27	18.524,196	0.331,971	6.149,457	6.149,495
26	18.666,944	0.347,216	6.481,428	6.481,461
25	18.805,476	0.363,122	6.828,644	6.828,682
24	18.939,963	0.379,7[illegible]5	7.191,766	7.191,788
23	19.070.569	0.397,026	7.571,481	7.571,511
22	19.197,449	0.415,083	7.968,507	7.968,534
21	19.320,751	0.433,918	8.383,590	8.383,621
20	19.440,616	0.453,563	8.817,508	8.817,544

Years of Age.	*Values of a life-annuity of one pound a year.*	*Present values of a single payment of one pound, to be received by a child of the age of 3 years, at the ends of 91, 90, 89, 88, &c. years, if he shall be living at the ends of those years respectively.*	*Sums of the values in the third column.*	*Products of the multiplication of the numbers in the second and third columns.*
	£	£	£	£
19	19.581,000	0.473:474	9.271,071	9.271,094
18	19.716,914	0.494,224	9.744,545	9.744,572
17	19.848,540	0.515,847	10.238,769	10.238,809
16	19.976,052	0.538,378	10.754,616	10.754,666
15	20.123,320	0.561,191	11.292,994	11.293,026
14	20.265,616	0.584,943	11.854,185	11.854,230
13	20.403,141	0.609,669	12.439,128	12.439,162
12	20.536,088	0.635,410	13.048,797	13.048,835
11	20.664,641	0.662,206	13.684,207	13.684,249
10	20.741,729	0.691,671	14.346,413	14.346,452
9	20.770,473	0.724,015	15.038,084	15.038,134
8	20.754,438	0.759,459	15.762,099	15.762,144
7	20.720,153	0.797,369	16.521,558	16.521,607
6	20.647,176	0.838,806	17.318,927	17.318,975
5	20.518,022	0.884,967	18.157,733	18.157,772
4	20.318,825	0.937,197	19.042,700	19.042,741
3	19.979,961		19.979,897	

Of the degree of exactness of the values of life-annuities given in the foregoing table.

In this table the numbers in the fifth column agree with those in the fourth column in either the five or the six first places of figures; and those of the last year, or that correspond to the age of 4 years, to wit, £19.042,741 and £19.042,700, agree with each other in the six first places of figures, their difference being only £0.000,041, or 41 millionth parts of a pound. And therefore we may safely conclude that all the values of an annuity of one pound a year set down in this table have been rightly computed to the same degree of exactness, or to at least five places of figures.

An account of an error in the computation of Table XI, which makes the values of an annuity of one pound for lives under the age of 41 years a little too great.

CC. The reader will obferve that the numbers fet down in the foregoing table as the values of annuities of one pound a year for the lives of perfons under the age of 42 years differ a little from the values of them exhibited in Table III, page 131, et feq. This is owing to an error in the calculation of the value of an annuity for a life of 41 years of age in that table, which error runs through all the reft of the table, or all the values of it that are derived from that value, that is, all the values of it for lives of younger ages than that of 41 years. The value of an annuity of one pound a year for a life of 41 years is equal to $\frac{1}{1.035} \times \frac{643}{650}$ $\times$ £16.580,939, which in page 123 (where the values afterwards fet down in Table III, are computed,) is made to be equal to $\frac{1}{1.035}$ $\times$ £16.448,528, or £15.892,297, whereas it is in truth equal to $\frac{1}{1.035} \times$ £16.402,375, or £15.847,705. But the difference between thefe two values, £15.892,297 and £15.847,705, is only £0.044,592, or about $10d.\frac{3}{4}$, which is but a trifling difference upon a fum of near $15l.\ 17s.$ And the differences between the fubfequent values in the two tables are ftill lefs than $10d.\frac{3}{4}$. Thus, for example, the difference between £17.770,162, the value of an annuity of one pound a year for a life of the age of 32 years in Table III, page 132, and £17.740,543, the value of the fame annuity given in the laft table, is £0.029,619; which is lefs than $7d.\frac{1}{4}$; and the difference between £20.752,981, the value of an annuity of one pound a year for a life of the age of 10 years, in Table III, page 131, and £20.741,729, the value of the fame annuity given in the laft table, is £0.011,252; which is lefs than $2d.\frac{3}{4}$; and the difference between £19.987,654, the value of an annuity of one pound a year given in Table III, page 131, for a life of only 3 years of age (which is the youngeft age in the table,) and £19.979,961, the value of the fame annuity given in the laft table, is £0.007,693; which is lefs than $2d.$ We may therefore confider the values given in Table III as being exact to all purpofes of practice, or as co-inciding with thofe in the laft table, which has been computed without any miftake. And therefore I did not think it worth while to caufe the errors of Table III, and of fome articles that follow it and refer to it, to be corrected, when I had difcovered them, which was not till after the fheets in which that table and thofe fubfequent articles are contained had been printed off. I difcovered thefe errors by means of Mr. Morgan's method, above-mentioned, of proving the truth of thefe computations as we go on. For when, in computing the 5th column in the laft table (which confifts of the products of the numbers in the

the ſecond and third columns,) I came to the product belonging to the age of 41 years, I found that the ſaid product was greater than it ought to be, or exceeded by a greater difference than it ought to have done the correſponding number in the fourth column; and thereupon I computed anew the value of an annuity of one pound a year for a life of the age of 41 years from that of the like annuity for a life of the age of 42 years, by means of the expreſſion $\frac{1}{1.035} \times \frac{643}{650} \times £16.580,939$, and found that the ſaid expreſſion was not equal to $\frac{1}{1.035} \times £16.448,528$, or $£15.892,297$, (as it is ſuppoſed to be in page 123,) but to $\frac{1}{1.035} \times £16.402,375$, or $£15.847,705$. And, having thus found out and corrected this error belonging to the age of 41 years, I computed the values of an annuity of one pound a year for lives of all ages younger than 41 years over again by means of the expreſſion $\frac{1}{1.035} \times \frac{P}{P+d} \times \overline{1+V|}\,£$, and proved them, as I went on, by Mr. Morgan's method above-mentioned, and therefore I am confident that this laſt table is correct.

CCI. Having now gone through every thing I propoſed to offer concerning the *manner* of computing the values of annuities for ſingle lives, I ſhall proceed to exhibit ſeveral different tables of thoſe values ſuited to the ſeveral following rates of the intereſt of money, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. all computed fairly and ſtrictly from Monſieur de Parcieux's table of the probabilities of the duration of human life above-mentioned by means of the expreſſion $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V|}\,£$. One of them, to wit, the fourth, or that which relates to the intereſt of $3\frac{1}{2}$ per cent. is the ſame with the firſt and ſecond columns of Table XI, which is given above in Art. 199. I have inſerted this part of the ſaid table a ſecond time in its proper place among the other tables, becauſe I thought it would be more convenient for the readers who ſhall want to conſult and make uſe of it, to look for it there than in the former parts of this tract. But I have there ſet down only the ſaid two firſt columns of Table XI, or the numbers of years in the ſeveral ages of human life and the

An account of the following twelve tables of the values of a life-annuity of one pound.

the numbers that exhibit the values of the annuities that correſpond to them, without the additional numbers that are contained in the third, fourth, and fifth columns of Table XI. and which are only uſeful as proofs of the exactneſs of the former numbers. This fourth of the following ſet of tables, or table relating to the intereſt of $3\frac{1}{2}$ per cent. I have computed myſelf. The others have been computed by Mr. Denſham, an able arithmetician, whom Dr. Price recommended for that purpoſe. And they were proved by him, as he proceeded in the work, by Mr. Morgan's method above-mentioned, though I have not thought it neceſſary to cauſe all thoſe proof-numbers to be printed. But I have looked over thoſe proof-numbers myſelf, and found that they confirm the truth of the computations.

And, as theſe tables are not inſerted in this place to illuſtrate the manner in which they have been computed, (as was the caſe with Table XI,) but are intended to be reſorted to and made uſe of, as exhibiting the true values of life-annuities for the different ages of human life, according to the different rates of the intereſt of money, without any reference to the method of computing them, I have ſet down the values of the annuities in each table in their natural order, beginning with the youngeſt age, which is that of 3 years. Theſe tables are as follows.

TABLE

TABLE XII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 2 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	27.382,830	34	21.782,266	65	9.238,319
4	27.794,317	35	21.474,025	66	8.795,049
5	28.008,120	36	21.158,940	67	8.365,278
6	28.121,217	37	20.836,776	68	7.950,607
7	28.153,865	38	20.475,233	69	7.553,307
8	28.130,823	39	20.104,909	70	7.176,577
9	28. 80,318	40	19.725,499	71	6.798,054
10	27.96 ,401	41	19.336,686	72	6.445,751
11	27.788,463	42	18.938,139	73	6.098,544
12	27.540,613	43	18.529,510	74	5.759,088
13	27.287,412	44	18.110,436	75	5.431,073
14	27.028,710	45	17.680,537	76	5.087,894
15	26.764,350	46	17.239,415	77	4.759,614
16	26.494,172	47	16.815,956	78	4.453,776
17	26.250,605	48	16.381,354	79	4.144,112
18	26.001,981	49	15.963,864	80	3.871,790
19	25.748,153	50	15.535,376	81	3.613,948
20	25.488,966	51	15.123,598	82	3.380,105
21	25.256,792	52	14.729,083	83	3.127,536
22	25.020,193	53	14.324,686	84	2.838,919
23	24.779,034	54	13.909,922	85	2.559,295
24	24.533,179	55	13.511,804	86	2.297,450
25	24.282,487	56	13.103,799	87	2.070,661
26	24.026,812	57	12.685,379	88	1.784,299
27	23.766,002	58	12.283,071	89	1.502,199
28	23.499,897	59	11.870,902	90	1.228,718
29	23.228,331	60	11.448,295	91	0.969,461
30	22.951,131	61	11.014,604	92	0.730,488
31	22.668,117	62	10.569,115	93	0.490,196
32	22.379,101	63	10.137,299	94	0.000,000
33	22.083,886	64	9.693,983		

TABLE XIII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $2\frac{1}{2}$ per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	24.462,676	34	20.136,887	65	8.911,789
4	24.849,735	35	19.878,238	66	8.495,160
5	25.062,078	36	19.612,791	67	8.090,289
6	25.185,830	37	19.340,316	68	7.698,809
7	25.238,680	38	19.030,630	69	7.323,022
8	25.242,492	39	18.712,036	70	6.966,149
9	25.222,411	40	18.384,189	71	6.606,509
10	25.146,756	41	18.046,727	72	6.271,426
11	25.011,897	42	17.699,272	73	5.940,421
12	24.814,820	43	17.341,428	74	5.616,112
13	24.612,646	44	16.972,778	75	5.302,157
14	24.405,208	45	16.592,885	76	4.972,521
15	24.192,333	46	16.201,291	77	4.656,602
16	23.973,844	47	15.825,188	78	4.361,896
17	23.779,193	48	15.437,457	79	4.062,687
18	23.579,731	49	15.064,768	80	3.799,480
19	23.375,296	50	14.680,583	81	3.549,972
20	23.165,719	51	14.311,129	82	3.323,657
21	22.980,544	52	13.957,047	83	3.078,502
22	22.791,199	53	13.592,614	84	2.797,255
23	22.597,546	54	13.217,294	85	2.524,251
24	22.399,442	55	12.856,800	86	2.268,242
25	22.196,735	56	12.485,883	87	2.046,484
26	21.989,269	57	12.103,960	88	1.765,079
27	21.776,880	58	11.736,386	89	1.487,784
28	21.559,396	59	11.358,341	90	1.218,152
29	21.336,639	60	10.969,190	91	0.962,095
30	21.108,421	61	10.568,230	92	0.725,758
31	20.874,547	62	10.154,683	93	0.487,804
32	20.634,811	63	9.753,041	94	0.000,000
33	20.388,999	64	9.339,034		

TABLE XIV.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 3 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	22.02[illegible],85	34	18.681,86	65	8.603,99
4	22.3[illegible],40	35	18.464,13	66	8.211,93
5	22 597,31	36	18.239,84	67	7.830,08
6	22.725,72	37	18.008,72	68	7.460,10
7	22.791,23	38	17.742,48	69	7.104,30
8	22.8[illegible]3,30	39	17 467,42	70	6.765,92
9	22.[illegible]14,53	40	17.183,14	71	6.423,92
10	22.766,00	41	16.889,24	72	6.104,99
11	22.664,17	42	16.585,30	73	5.789,19
12	22.505,84	43	16.270,90	74	5.479,13
13	22.342,75	44	15.945,54	75	5.178,44
14	22.174,71	45	15.608,74	76	4.861,62
15	22.001,56	46	15.260,00	77	4.557,43
16	21.823,10	47	14.924,96	78	4.273,29
17	21.666,23	48	14.578,02	79	3.984,05
18	21.504,89	49	14.244,41	80	3.729,54
19	21.338,90	50	13.899,02	81	3.488,01
20	21.168,08	51	13.566,71	82	3.268,91
21	21.019,54	52	13.248,20	83	3.030,89
22	20.867,17	53	12.919,06	84	2.756,77
23	20 710,84	54	12.578,70	85	2.490,19
24	20.550,40	55	12.251,64	86	2.239,87
25	20.385,70	56	11.913,80	87	2.023,06
26	20.210,57	57	11.564,55	88	1.746,59
27	20.042,84	58	11.228,16	89	1.473,61
28	19.864,33	59	10.880,86	90	1.207,74
29	19.680,86	60	10.521,96	91	0.954,82
30	19.492,23	61	10.150,71	92	0.721,08
31	19.298,24	62	9.766,26	93	0.485,43
32	19.098,66	63	9.392,18	94	0.000,00
33	18.893,29	64	9.005,09		

TABLE XV.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	19.979,961	34	17.390,462	65	8.313,625
4	20.318,825	35	17.206,612	66	7.944,258
5	20.518,022	36	17.016,528	67	7.583,727
6	20.647,176	37	16.819,920	68	7.233,699
7	20.720,153	38	16.590,228	69	6.896,496
8	20.754,438	39	16.351,906	70	6.575,357
9	20.770,473	40	16.104,542	71	6.249,840
10	20.741,729	41	15.847,705	72	5.945,972
11	20.664,641	42	15.580,939	73	5.644,448
12	20.536,088	43	15.303,763	74	5.347,806
13	20.403,141	44	15.015,669	75	5.059,623
14	20.265,616	45	14.716,120	76	4.754,921
15	20.123,320	46	14.404,549	77	4.461,839
16	19.976,052	47	14.105,200	78	4.187,758
17	19.848,540	48	13.793,859	79	3.908,000
18	19.716,914	49	13.494,425	80	3.661,781
19	19.581,000	50	13.183,083	81	3.427,856
20	19.440,616	51	12.883,449	82	3.215,658
21	19.320,751	52	12.596,296	83	2.984,473
22	19.197,449	53	12.298,386	84	2.717,188
23	19.070,569	54	11.989,093	85	2.456,774
24	18.939,963	55	11.691,801	86	2.211,911
25	18.805,476	56	11.383,427	87	1.999,810
26	18.666,944	57	11.063,485	88	1.728,378
27	18.524,195	58	10.755,123	89	1.459,699
28	18.377,051	59	10.435,566	90	1.197,512
29	18.225,318	60	10.104,074	91	0.947,669
30	18.068,798	61	9.759,829	92	0.716,468
31	17.907,280	62	9.401,925	93	0.483,091
32	17.740,543	63	9.053,059	94	0.000,000
33	17.568,352	64	8.690,648		

TABLE XVI.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	18.242,464	34	16 239,735	65	8.039,349
4	18.558,931	35	16.084,014	66	7.690,960
5	18.749,209	36	15.922,447	67	7.350,186
6	18.876,581	37	15.754,736	68	7.018,692
7	18.953,475	38	15.555,857	69	6.698,802
8	18.995,706	39	15.348,644	70	6.393,749
9	19.021,902	40	15.132,664	71	6.083,658
10	19.007,582	41	14 907,457	72	5.793,941
11	18.949,243	42	14.672,537	73	5.505,835
12	18.843,753	43	14.427,388	74	5.221,832
13	18.734,230	44	14.171,466	75	4.945,465
14	18.620,487	45	13.904,190	76	4.652,254
15	18.502,326	46	13.624,948	77	4.369,724
16	18.379,539	47	13.356,700	78	4.105,200
17	18,274,964	48	13 076,491	79	3.834,477
18	18.166,641	49	12.807,002	80	3.596,174
19	18 054,395	50	12.525,605	81	3.369,530
20	17.938,041	51	12.254,766	82	3.163,947
21	17.840,730	52	11.995,305	83	2.939,337
22	17.740,368	53	11.725,075	84	2.678,656
23	17.63[illegible],818	54	11.443,400	85	2.424,216
24	17.529,936	55	11.172,645	86	2.184,654
25	17 419,599	56	10.890,825	87	1.977,157
26	17.305,557	57	10.597,210	88	1.710,503
27	17.187,730	58	10.314,093	89	1.446,020
28	17.065,909	59	10.019,612	90	1.187,434
29	16.939,905	60	9.712,978	91	0.940,608
30	16.809,518	61	9.393,319	92	0.711,907
31	16.674,537	62	9.059,665	93	0.48[illegible],769
32	16.534,739	63	8.733,893	94	0.000,000
33	16.389,888	64	8.394,166		

TABLE XVII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4½ per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	16.756,318	34	15.210,67	65	7.779,99
4	17.051,91	35	15.078,39	66	7.451,02
5	17.232,78	36	14.940,68	67	7.128,57
6	17.356,81	37	14.797,23	68	6.814,32
7	17.435,21	38	14.624,42	69	6.510,57
8	17.482,39	39	14.443,64	70	6.220,54
9	17.515,42	40	14.254,42	71	5.924,89
10	17.511,62	41	14.056,29	72	5.648,45
11	17.467,53	42	13.848,74	73	5.372,97
12	17.380,04	43	13.631,21	74	5.100,89
13	17.288,86	44	13.403,15	75	4.835,69
14	17.193,80	45	13 163,92	76	4.553,37
15	17.094,65	46	12.912,88	77	4.280,86
16	16.991,21	47	12.671,81	78	4.025,43
17	16.904,67	48	12.418,90	79	3.763,33
18	16.814,73	49	12.175,72	80	3.532,59
19	16.721,21	50	11.920,73	81	3.312,91
20	16.623,93	51	11.675,33	82	3.113,67
21	16.544,44	52	11.440,38	83	2.895,38
22	16 462,26	53	11.194,74	84	2.641,07
23	16.377,28	54	10.937,70	85	2.392,40
24	16.289,34	55	10.690,66	86	2.157,98
25	16.198,31	56	10.432,56	87	1.954,95
26	16.104,02	57	10.162,64	88	1.692,95
27	16.006,32	58	9.902,29	89	1.432,57
28	15.905,03	59	9.630,51	90	1.177,51
29	15.799,96	60	9.346,46	91	0.933,64
30	15.690,92	61	9.049,21	92	0.707,40
31	15.577,70	62	8.737,74	93	0.478,468
32	15.460,07	63	8.433,15	94	0.000,000
33	15.337,82	64	8.114,30		

TABLE XVIII.

Containing the values of an annuity of one pound a year for the lives of persons of the ſeveral ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years, incluſively; computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, upon a ſuppoſition that the intereſt of money is 5 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
	£		£		£
3	15.475,38	34	14.287,24	65	7.534,57
4	15.751,70	35	14.174,54	66	7.223,59
5	15.923,11	36	14.056,84	67	6.918,17
6	16.042,87	37	13.933,83	68	6.619,96
7	16.121,17	38	13.783,16	69	6.331,26
8	16.171,20	39	13.624,89	70	6.055,28
9	16.208,71	40	13.458,56	71	5.773,18
10	16.212,55	41	13.283,68	72	5.505,20
11	16.179,35	42	13.099,71	73	5.245,60
12	16.106,03	43	12.906,09	74	4.984,76
13	16.029,32	44	12.702,21	75	4.730,12
14	15.949,04	45	12.487,42	76	4.458,12
15	15.864,99	46	12.261,04	77	4.195,13
16	15.776,95	47	12.043,77	78	3.948,35
17	15.704,68	48	11.814,85	79	3.694,48
18	15.629,33	49	11.594,84	80	3.470,95
19	15.550,71	50	11.363,18	81	3.257,93
20	15.468,67	51	11.140,30	82	3.064,75
21	15.403,32	52	10.927,09	83	2.852,53
22	15.335,63	53	10.703,34	84	2.604,35
23	15.265,47	54	10.468,30	85	2.361,25
24	15.192,73	55	10.242,48	86	2.131,77
25	15.117,25	56	10.005,69	87	1.933,03
26	15.038,89	57	9.757,12	88	1.675,04
27	14.957,50	58	9.517,33	89	1.418,35
28	14.872,90	59	9.266,13	90	1.166,21
29	14.784,92	60	9.002,61	91	0.924,25
30	14.693,37	61	8.725,83	92	0.702,94
31	14.598,05	62	8.434,68	93	0.476,19
32	14.498,74	63	8.149,54	94	0.000,00
33	14.395,22	64	7.849,93		

TABLE XIX.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 6 per cent.

Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.	Years of Age.	Values of an annuity of one pound a year.
	£		£		£
3	13.390,78	34	12.704,49	65	7.081,91
4	13.633,23	35	12.622,00	66	6.803,15
5	13.786,59	36	12.535,35	67	6.528,33
6	13.896,64	37	12.444,26	68	6.259,06
7	13.971,93	38	12.328,53	69	5.997,22
8	14.023,69	39	12.206,01	70	5.746,68
9	14.065,55	40	12.076,23	71	5.489,21
10	14.078,91	41	11.938,66	72	5.247,98
11	14.060,56	42	11.792,75	73	5.006,12
12	14.007,46	43	11.637,89	74	4.765,93
13	13.951,50	44	11.473,46	75	4.530,73
14	13.892,50	45	11.298,74	76	4.277,82
15	13.830,25	46	11.112,99	77	4.032,52
16	13.764,54	47	10.935,03	78	3.801,85
17	13.712,73	48	10.745,94	79	3.563,34
18	13.658,38	49	10.564,46	80	3.353,32
19	13.001,33	50	10.371,80	81	3.152,81
20	13.541,40	51	10.186,65	82	2.971,06
21	13.496,36	52	10.009,95	83	2.770,32
22	13.449,57	53	9.823,15	84	2.533,81
23	13.400,92	54	9.625,44	85	2.301,35
24	13.350,30	55	9.435,74	86	2.081,39
25	13.297,59	56	9.235,40	87	1.890,99
26	13.242,66	57	9.023,54	88	1.642,24
27	13.185,37	58	8.819,24	89	1.393,57
28	13.125,58	59	8.603,71	90	1.148,64
29	13.063,13	60	8.376,00	91	0.913,30
30	12.997,84	61	8.135,06	92	0.694,19
31	12.929,54	62	7.879,69	93	0.471,69
32	12.858,02	63	7.628,91	94	0.000,00
33	12.783,08	64	7.363,46		

TABLE XX.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 7 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	11.777,44	34	11.404,51	65	6.673,74
4	11.9[illegible]62	35	11.343,49	66	6.422,78
5	12.120,80	36	11.279,09	67	6.174,46
6	12.229,00	37	11.211,03	68	5.930,35
7	12.[illegible]99,56	38	11.120,95	69	5.692,65
8	12.350,20	39	11.024,87	70	5.464,47
9	12.392,90	40	10.922,30	71	5.228,74
10	12.41[illegible],09	41	10.812,72	72	5.007,66
11	12.401,71	42	10.695,57	73	4.785,15
12	12.361,77	43	10.570,[illegible]	74	4.563,42
13	12.319,38	44	10.436,[illegible]1	75	4.345,69
14	12.274,34	45	10.292,20	76	4.110,04
15	12.226,48	46	10.138,01	77	3.880,74
16	12.175,56	47	9.990,64	78	3.664,71
17	12.137,07	48	9.832,76	79	3.440,23
18	12.096,46	49	9.681,55	80	3.242,57
19	12.053,56	50	9.519,73	81	3.053,54
20	12.008,23	51	9.364,51	82	2.882,31
21	11.976,34	52	9.216,85	83	2.692,20
22	11.943,15	53	9.059,63	84	2.466,56
23	11.908,59	54	8.892,00	85	2.244,04
24	11.872,55	55	3.731,51	86	2.033,01
25	11.834,93	56	8.560,84	87	1.850,43
26	11.795,64	57	8.379,07	88	1.609,95
27	11.7[illegible]4,55	58	8.203,96	89	1.368,64
28	11.711,53	59	8.017,98	90	1.130,11
29	11.666,45	60	7.820,13	91	0.900,21
30	11.619,16	61	7.609,27	92	0.685,649
31	11.569,50	62	7.384,13	93	[illegible]4[illegible]7,289
32	11.517,30	63	7.162,51	94	0.000,000
33	11.462,37	64	6.926,23		

TABLE XXI.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 8 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	10.498,90	34	10.323,80	65	6.305,04
4	10.689,50	35	10.278,24	66	6.078,24
5	10.812,58	36	10.229,96	67	5.853,05
6	10.903,61	37	10.176,73	68	5.630,99
7	10.968,95	38	10.107,71	69	5.414,20
8	11.017,21	39	10.031,41	70	5.205,72
9	11.059,02	40	9.949,36	71	4.989,28
10	11.079,47	41	9.861,03	72	4.786,10
11	11.075,61	42	9.765,86	73	4.580,86
12	11.044,54	43	9.663,21	74	4.375,67
13	11.011,33	44	9.552,42	75	4.173,66
14	10.975,79	45	9.432,72	76	3.953,62
15	10.937,73	46	9.303,30	77	3.738,86
16	10.896,93	47	9.179,99	78	3.536,16
17	10.867,35	48	9.046,31	79	3.324,52
18	10.835,97	49	8.910,[illegible]9	80	3.138,19
19	10.802,63	50	8.782,39	81	2.959,71
20	10.767.17	51	8.651,10	82	2.798,19
21	10.743,97	52	8.526,72	83	2.6[illegible],95
22	10.719,82	53	8.393,38	84	2.402,45
23	10.694,65	54	8.250,20	85	2.189,26
24	10.668,38	55	8.113,50	86	1.986,62
25	10.640,95	56	7.967,16	87	1.811,41
26	10.612,25	57	7.810,22	88	1.578, 0
27	10.582 20	58	7.659,28	89	1.344,52
28	10.550,69	59	7.497,95	90	1.112,13
29	10.517,60	60	7.325,16	91	0.887,458
30	10.482,32	61	7.139,72	92	0.[illegible]77,297
31	10.446,20	62	6.940,29	93	0.462 962,9
32	10.407,60	63	6.743,60	94	0.000,000
33	10.366,85	64	6.532,39		

TABLE XXII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 93 years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 9 per cent.

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	9.464,49	34	9.415,24	65	5.970,66
4	9.635,36	35	9.380,92	66	5.764,92
5	9.746,28	36	9.344,45	67	5.559,98
6	9.829,07	37	9.305,64	68	5.357,29
7	9.889,33	38	9.248,97	69	5.158,93
8	9.934,73	39	9.187,66	70	4.967,89
9	9.974,87	40	9.121,25	71	4.768,56
10	9.99[illegible]	41	9.049,24	72	4.581,32
11	9.[illegible]	42	8.971,06	73	4.391,54
12	[illegible]	43	8.886,08	74	4.201,23
13	[illegible]	44	8.793,62	75	4.013,41
14	9.915,24	45	8.692,92	76	3.807,52
15	9.804,08	46	8.583,14	77	3.606,01
16	9.850,43	47	8.478,93	78	3.415,49
17	9.826,98	48	8.365,47	79	3.215,61
18	9.801,97	49	8.257,46	80	3.039,69
19	9.775,24	50	8.140,06	81	2.870,94
20	9.746,65	51	8.028,06	82	2.718,38
21	9.729,30	52	7.922,48	83	2.547,30
22	9.711,26	53	7.808,53	84	2.341,29
23	9.692,47	54	7.685,33	85	2.136,85
24	9.672,88	55	7.568,13	86	1.942,11
25	9.652,42	56	7.441,86	87	1.773,87
26	9.631,02	57	7.305,54	88	1.548,74
27	9.608,60	58	7.174,74	89	1.321,18
28	9.5[illegible]5,10	59	7.034,06	90	1.094,68
29	9.560,41	60	6.882,41	91	0.8[illegible],038
30	9.534,43	61	6.718,55	92	0.669,135
31	9.507,05	62	6.541,08	93	0.458,715
32	9.478,15	63	6.365,76	94	0.000,000
33	9.447,60	64	6.170,19		

TABLE XXIII.

Containing the values of an annuity of one pound a year for the lives of persons of the several ages of 3 *years,* 4 *years,* 5 *years, and every following number of years up to* 93 *years inclusively; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is* 10 *per cent.*

Years of Age.	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*	*Years of Age.*	*Values of an annuity of one pound a year.*
	£		£		£
3	8.612,41	34	8.643,42	65	5.666,42
4	8.766,66	35	8.617,37	66	5.479,11
5	8.867,12	36	8.589,66	67	5.291,94
6	8.942,62	37	8.560,12	68	5.106,33
7	8.998,15	38	8.514,37	69	4.924,28
8	9.040,62	39	8.464,55	70	4.748,71
9	9.078,77	40	8.410,21	71	4.564,65
10	9.100,13	41	8.350,86	72	4.391,68
11	9.101,99	42	8.285,95	73	4.215,77
12	9.081,56	43	8.214,86	74	4.038,86
13	9.059,42	44	8.136,92	75	3.863,86
14	9.035,38	45	8.051,35	76	3.670,85
15	9.009,25	46	7.957,30	77	3.481,41
16	8.980,80	47	7.868,40	78	3.302,03
17	8.961,70	48	7.770,84	79	3.112,98
18	8.941,21	49	7.678,32	80	2.946,03
19	8.919,19	50	7.576,99	81	2.786,85
20	8.895,48	51	7.480,66	82	2.642,58
21	8.882,16	52	7.390,37	83	2.48 ,03
22	8.868,33	53	7.292,30	84	2.282,89
23	8.853,95	54	7.185,54	85	2.086,67
24	8.838,99	55	7.084,42	86	1.899,38
25	8.823,39	56	6.974,80	87	1.737,73
26	8.807,10	57	6.855,69	88	1.519,72
27	8.790,06	58	6.741,74	89	1.298,58
28	8.772,21	59	6.618,46	90	1.077,73
29	8.753,47	60	6.484,73	91	0.802,938
30	8.733,77	61	6.339,28	92	0.661,157
31	8.713,02	62	6.180,65	93	0.454,545
32	8.691,12	63	6.023,74	94	0.000,000
33	8.667,96	64	5.852,93		

Concerning life-annuities that are payable every half-year or every quarter of a year.

CC. The foregoing tables exhibit the values of life-annuities, according to the above rates of intereſt, upon a ſuppoſition that thoſe annuities are to be paid only once a year at the ends of the ſeveral years of the annuitant's life that ſhall follow the time of granting them. But it is often ſtipulated between the grantors and grantees of life-annuities that they ſhall be paid every half-year, and ſometimes that they ſhall be paid every quarter of a year, during the life of the annuitant. In theſe caſes they will be ſomewhat more valuable than if they are to be paid only once a year; and conſequently a ſmall addition ought to be made to the values of them ſet down in the foregoing tables. Mr. Simpſon (in his doctrine of annuities, pages 78, 79, 80, 81,) ſays that this addition ought, in the caſe of half-yearly payments, to be about a quarter of a year's annuity, and, in the caſe of quarterly payments, to be about three eighth parts of a year's annuity. But theſe are only approximations to the values of theſe additions: and it is impoſſible to determine them with perfect preciſion without a table of the probabilities of the duration of human life that ſhould exhibit the numbers of perſons who die off in every half-year and quarter of a year, as well as in every whole year, of the whole poſſible extent of human life, out of a given original number of perſons of the ſame age all living at the ſame time: and no ſuch table has yet been publiſhed. But we may determine the value of the firſt of theſe additions, or that which is to be made in the caſe of half-yearly payments, to a ſufficient degree of exactneſs for all uſeful purpoſes in the manner following.

Obſervations on the value of a life-annuity that is payable every half-year.

CCI. It has been ſhewn above in Art. 86, 87, that the value of an annuity of one pound a year for the life of a perſon of the age of N years is $= \frac{£\,1}{P} \times$ the ſeries $\frac{P^{\text{I}}}{r} + \frac{P^{\text{II}}}{r^2} + \frac{P^{\text{III}}}{r^3} + \frac{P^{\text{IV}}}{r^4} + \frac{P^{\text{V}}}{r^5} + \frac{P^{\text{VI}}}{r^6} + \frac{P^{\text{VII}}}{r^7} +$ &c. continued to $E-N$ terms, or, (according to Monſieur de Parcieux's table of probabilities,) to $94-N$ terms, or to the utmoſt poſſible extent of human life.

Now let us ſuppoſe this grant of an annuity of one pound a year for the life of the perſon aged N years to be changed into a grant of an annuity, or rather *penſion*, of one pound a year for the life of the ſame perſon, to be paid by half-yearly payments of 10 ſhillings each during the ſaid perſon's life. Then it is evident that the ſaid perſon will gain by this change the three following advantages. In the firſt place, he will have a chance of receiving one half-year's payment after he has attained the age of 94 years; becauſe it is ſuppoſed, in Monſieur de Parcieux's table of the

 probabilities

probabilities of the duration of human life, that one perfon, out of the original 1000 perfons who are living at the age of 3 years, will live beyond the age of 94 years and die before he attains the age of 95 years: and therefore, if the annuitant fhould be that perfon, and fhould live to the age of 94 years and a half, he may, upon this fecond fuppofition of the annuity's being paid by half-yearly payments, receive a payment of 10 fhillings in that remote period of his life, which it is abfolutely impoffible he fhould receive upon the former fuppofition of its being paid only at the end of every year. This advantage is, however, fo exceeding fmall, that it is hardly worth attending to. And, in the fecond place, the annuitant will, upon this fecond fuppofition of the annuity's being paid every half-year, receive one half of all the reft of the money which will be paid him by the grantor of the annuity, half a year fooner than on the former fuppofition, namely, all thofe payments of 10 fhillings each which he will receive at the ends of half a year, a year and a half, two years and a half, three years and a half, and every following intermediate half-year during his life; from which circumftance thofe half-yearly payments will be more valuable to him, and more burthenfome to the grantor of the annuity, than they were before, when they were to be made only at the ends of a whole year, two years, three years, four years, and every following whole number of years during the fame perfon's life. And, in the laft place, the annuitant's chances of receiving thofe payments on the fecond fuppofition of their being made at the ends of half a year, a year and a half, two years and a half, three years and a half, &c. will be fomewhat better than they were before, when he was to receive them half a year later, or at the ends of a whole year, two years, three years, four years, &c. becaufe fewer perfons out of the whole number P of perfons reprefented in Monfieur de Parcieux's table as livin[illegible] at the age of the annuitant, at the time of granting the annuity, will [illegible]ad at the ends of half a year, a year and a half, two years and a [illegible]ree years and a half, &c. than at the ends of a whole year, two y[illegible]hree years, four years, &c. And for thefe reafons, and chiefly for the two latter, (as the firft is not worth attending to,) the value of an annuity of one pound a year payable every half-year by payments of 10 fhillings each, during the life of a perfon of a given age, will be fomewhat greater than that of a like annuity of one pound a year that is payable only at the end of every year.

CCII. In order to determine the value of one of thefe half-yearly annuities with exactnefs it would be neceffary to have a table of the probabilities of the duration of human life that fhould fet forth the numbers of perfons who may be fuppofed to die in every half-year, as well as year, of human life, out of a given original number of very young perfons of the fame age all living at the fame time. But no fuch tables have yet been

been publiſhed. We muſt therefore content ourſelves with determining the values of theſe annuities by an approximation; which may be done to a conſiderable and ſufficient degree of exactneſs in the following manner.

An inveſtigation of the ſaid value by approximation.

CCIII. Let $\overset{\pounds}{V}$ denote (as in the foregoing articles,) the value of an annuity of one pound a year payable at the end of every year, during the life of a perſon of the age of N years; and $\overset{\pounds}{W}$ the value of a like annuity of one pound a year payable at the end of every year, during the life of a perſon one year younger than the former, or of the age of $N-1$ years. And let P be the number of perſons repreſented in Monſieur de Parcieux's table of probabilities as living at the age of N years, and $P+d$ be the number of perſons repreſented therein as living at the age of $N-1$ years. And let $\overset{\pounds}{r}$ be the value of one pound together with its intereſt for one year, as in the foregoing articles.

Then will $\overline{r-1|} \times \pounds 1$ be the intereſt of one pound for a year, and conſequently $\frac{r-1}{2} \times \pounds 1$ will be the intereſt of one pound for half a year: for, when money is lent for leſs than a whole year at any particular rate of intereſt denominated from the whole year, the parties to ſuch ſhort loans generally mean that the intereſt of the ſum lent for the time the loan is to laſt ſhall bear the ſame proportion to its intereſt for a whole year as the time for which it is lent bears to a whole year. Thus, if the intereſt of money is 4 per cent. per annum, and 100 pounds are lent for half a year, it is generally underſtood that the borrower, when he repays the 100 pounds at the end of the half-year, is to pay with it an additional ſum of 40 ſhillings by way of intereſt for it. And, in like manner in general, if the intereſt upon one pound for a whole year is $\overline{r-1|} \times \pounds 1$, its intereſt for half a year will be $\frac{r-1}{2} \times \pounds 1$, and its intereſt for the m^{th} part of a year will be $\frac{r-1}{m} \times \pounds 1$.

Therefore the value of one pound together with its intereſt for half a year will be $\pounds 1 + \frac{r-1}{2} \times \pounds 1$, (or $\frac{2}{2} \times \pounds 1 + \frac{r-1}{2} \times \pounds 1$, or $\frac{2+r-1}{2} \times \pounds 1$,) or $\frac{1+r}{2} \times \pounds 1$. Let this value be denoted by $\overset{\pounds}{b}$.

CCIV. Now this quantity b, or $\frac{1+r}{2}$, will be greater than $r^{\frac{1}{2}}$. For, ſince 1 is to $r^{\frac{1}{2}}$ as $r^{\frac{1}{2}}$ is to r, it follows from Euclid's Elements, book 5, prop. 25, that the ſum of the two extreme terms 1 and r will be greater than twice the middle term $r^{\frac{1}{2}}$, and conſequently that half that ſum, or $\frac{1+r}{2}$, will be greater than the ſaid middle term $r^{\frac{1}{2}}$. Therefore b^2, b^3, b^4, b^5, b^6, b^7, b^8, b^9, &c. will be greater than $(\overline{r^{\frac{1}{2}}}|^2$, $\overline{r^{\frac{1}{2}}}|^3$, $\overline{r^{\frac{1}{2}}}|^4$, $\overline{r^{\frac{1}{2}}}|^5$, $\overline{r^{\frac{1}{2}}}|^6$, $\overline{r^{\frac{1}{2}}}|^7$, $\overline{r^{\frac{1}{2}}}|^8$, $\overline{r^{\frac{1}{2}}}|^9$, &c. or $r^{\frac{2}{2}}$, $r^{\frac{3}{2}}$, $r^{\frac{4}{2}}$, $r^{\frac{5}{2}}$, $r^{\frac{6}{2}}$, $r^{\frac{7}{2}}$, $r^{\frac{8}{2}}$, $r^{\frac{9}{2}}$, &c. or) r, $r^{\frac{3}{2}}$, r^2, $r^{\frac{5}{2}}$, r^3, $r^{\frac{7}{2}}$, r^4, $r^{\frac{9}{2}}$, &c. reſpectively.

CCV. Further, let us ſuppoſe that a table of the probabilities of the duration of human life had been prepared from obſervations on births and burials to every half-year of human life, and that in ſuch table the number of perſons repreſented as living at the age of N years and a half had been Π', and thoſe living at the age of $N+1$ years and a half had been Π'', and thoſe living at the age of $N+2$ years and a half had been Π''', and in like manner that thoſe living at the ends of $N+3$ years and a half, $N+4$ years and a half, $N+5$ years and a half, and ſo on, had been denoted by Π^{iv}, Π^{v}, Π^{vi}, &c. out of the number P who are living at the age of N years.

Then it is evident that the numbers Π', Π'', Π''', Π^{iv}, Π^{v}, &c. will be reſpectively greater than the numbers P', P'', P''', P^{iv}, P^{v}, &c. which repreſent the numbers of perſons living at the ages of $N+1$ years, $N+2$ years, $N+3$ years, $N+4$ years, $N+5$ years, &c. but leſs than the numbers P, P', P'', P''', P^{iv}, &c. which repreſent the numbers of perſons living at the ages of N years, $N+1$ years, $N+2$ years, $N+3$ years, $N+4$ years, &c.

CCVI. Theſe things being premiſed, we may now find a quantity which ſhall always be ſomewhat, though very little, greater than the true value of an annuity of one pound a year to be paid half-yearly by payments of 10 ſhillings each, during the life of a perſon of the age of N years, by proceeding in the manner following.

CCVII. We have already seen in Art. 78,——87, that, when the interest of money is supposed to be paid once a year, and then added to the principal, and the whole lent out again immediately at the same rate of interest, the present value of an annuity, or yearly payment, of one pound for a certain number of years denoted by the letter n will be equal to $\overset{£}{1} \times$ the series $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} + \frac{1}{r^5} + \frac{1}{r^6} +$ &c. $+ \frac{1}{r^n}$; and that the value of the same annuity for the same term of years, but depending on the life of a person of the age of N years, so as to cease whenever he shall die, is $= \frac{\overset{£}{1}}{P} \times$ the series $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} +$ &c. $+ \frac{P^{n}}{r^n}$; and that the value of the same annuity for the whole life of the said person of the age of N years is $= \frac{\overset{£}{1}}{P} \times$ the series $\frac{P^{I}}{r} + \frac{P^{II}}{r^2} + \frac{P^{III}}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \frac{P^{VI}}{r^6} +$ &c. $+ \frac{P^{E-N}}{r^{E-N}}$, the letter E denoting 94, or, in general, the greatest number of years to which it is supposed, in the table of probabilities which is made the ground of the calculation, that human life can be extended.

CCVIII. Now, by reasonings similar to those by which these conclusions were obtained, it will follow that, if we suppose the interest of money to be paid every half-year, instead of every year, and then to be added to the principal, and the sum of both to be lent out again at the same rate of interest, the present value of a half-yearly set of payments of one pound each, for the life of a person of the age of N years, will be equal to $\frac{\overset{£}{1}}{P} \times$ the series $\frac{\Pi^{I}}{h} + \frac{P^{I}}{h^2} + \frac{\Pi^{II}}{h^3} + \frac{P^{II}}{h^4} + \frac{\Pi^{III}}{h^5} + \frac{P^{III}}{h^6} + \frac{\Pi^{IV}}{h^7} + \frac{P^{IV}}{h^8} + \frac{\Pi^{V}}{h^9} +$ &c. continued to the utmost extent of human life, or to the term $\frac{\Pi^{E-N+1}}{h^{2E-2N+1}}$, and therefore (since h has been shewn to be greater than $r^{\frac{1}{2}}$,) will be less than $\frac{\overset{£}{1}}{P} \times$ the series $\frac{\Pi^{I}}{r^{\frac{1}{2}}} + \frac{P^{I}}{r} + \frac{\Pi^{II}}{r^{\frac{3}{2}}}$

$+ \frac{P^{II}}{r^2}$

$+\frac{P''}{r^2} + \frac{\Pi'''}{r^{\frac{5}{2}}} + \frac{P'''}{r^3} + \frac{\Pi^{IV}}{r^{\frac{7}{2}}} + \frac{P^{IV}}{r^4} + \frac{\Pi^{V}}{r^{\frac{9}{2}}} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$, or than $\frac{\pounds 1}{P} \times$ the feries $\frac{\Pi'}{r^{\frac{1}{2}}} + \frac{\Pi''}{r^{\frac{3}{2}}} + \frac{\Pi'''}{r^{\frac{5}{2}}} + \frac{\Pi^{IV}}{r^{\frac{7}{2}}} + \frac{\Pi^{V}}{r^{\frac{9}{2}}} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$ together with $\frac{\pounds 1}{P} \times$ the feries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$. Therefore the prefent value of a fet of half-yearly payments of only 10 fhillings, or half a pound, each, for the life of the fame perfon will be lefs than $\frac{\pounds 1}{2P} \times$ the feries $\frac{\Pi'}{r^{\frac{1}{2}}} + \frac{\Pi''}{r^{\frac{3}{2}}} + \frac{\Pi'''}{r^{\frac{5}{2}}} + \frac{\Pi^{IV}}{r^{\frac{7}{2}}} + \frac{\Pi^{V}}{r^{\frac{9}{2}}} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$ together with $\frac{\pounds 1}{2P} \times$ the feries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$.

CCIX. Now $\frac{\pounds 1}{P} \times$ the feries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$ is equal to $\pounds V$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a perfon of the age of N years. Therefore $\frac{\pounds 1}{2P} \times$ the feries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{IV}}{r^4} + \frac{P^{V}}{r^5} + \&c. + \frac{P^{E-N}}{r^{E-N}}$ is $= \frac{\pounds V}{2}$. Therefore the prefent value of a fet of half-yearly payments of 10 fhillings each for the life of a perfon of the age of N years is lefs than $\frac{\pounds 1}{2P} \times$ the feries $\frac{\Pi'}{r^{\frac{1}{2}}} + \frac{\Pi''}{r^{\frac{3}{2}}} + \frac{\Pi'''}{r^{\frac{5}{2}}} + \frac{\Pi^{IV}}{r^{\frac{7}{2}}} + \frac{\Pi^{V}}{r^{\frac{9}{2}}} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+\frac{1}{2}}}$ together with $\frac{\pounds V}{2}$.

Therefore

Therefore it is lefs than $r^{\frac{1}{2}} \times \frac{\pounds 1}{2P} \times$ the feries $\frac{\Pi'}{r} + \frac{\Pi''}{r^2} + \frac{\Pi'''}{r^3} + \frac{\Pi^{iv}}{r^4} + \frac{\Pi^{v}}{r^5} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ together with $\frac{\pounds V}{2}$.

CCX. Now it is probable that the number of perfons who die in the firft half of every year of human life is fomewhat lefs than the number of thofe who die in the fecond half of the fame year; and confequently the number of perfons living at the end of the firft half-year will be fomewhat greater than an arithmetical mean proportional between the numbers of perfons living at the beginning of the firft half-year and at the end of the fecond half-year. Therefore Π' will be fomewhat greater than an arithmetical mean between P and P', and Π'' than an arithmetical mean between P' and P'', and Π''' than an arithmetical mean between P'' and P''', and Π^{iv}, Π^{v}, &c. than arithmetical means between P''' and P^{iv}, and between P^{iv} and P^{v}, &c. Confequently Π', Π'', Π''', Π^{iv}, Π^{v}, &c. will be fomewhat greater than $\frac{P+P'}{2}$, $\frac{P'+P''}{2}$, $\frac{P''+P'''}{2}$, $\frac{P'''+P^{iv}}{2}$, and $\frac{P^{iv}+P^{v}}{2}$, &c. But the difference will probably be fo extremely fmall that we may very well confider them as equal. And upon this fuppofition the feries $\frac{\Pi'}{r} + \frac{\Pi''}{r^2} + \frac{\Pi'''}{r^3} + \frac{\Pi^{iv}}{r^4} + \frac{\Pi^{v}}{r^5} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ will be equal to the feries $\frac{P+P'}{2r} + \frac{P'+P''}{2r^2} + \frac{P''+P'''}{2r^3} + \frac{P'''+P^{iv}}{2r^4} + \frac{P^{iv}+P^{v}}{2r^5} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$, and confequently to one half of the feries $\frac{P+P'}{r} + \frac{P'+P''}{r^2} + \frac{P''+P'''}{r^3} + \frac{P'''+P^{iv}}{r^4} + \frac{P^{iv}+P^{v}}{r^5} + \&c.$ together with the term $\frac{\Pi^{E-N+1}}{r^{E-N+1}}$, and confequently to one half of the feries $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4} + \frac{P^{iv}}{r^5} + \&c.$ together with one half of the feries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{iv}}{r^4} + \frac{P^{v}}{r^5} + \&c.$ and with the term $\frac{\Pi^{E-N+1}}{r^{E-N+1}}$.

Therefore

Therefore $r^{\frac{1}{2}} \times \frac{\overset{\pounds}{1}}{2P} \times$ the ſeries $\frac{\Pi'}{r} + \frac{\Pi''}{r^2} + \frac{\Pi'''}{r^3} + \frac{\Pi^{iv}}{r^4} + \frac{\Pi^{v}}{r^5} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ is equal $r^{\frac{1}{2}} \times \frac{\overset{\pounds}{1}}{2P} \times \frac{1}{2} \times$ the ſeries $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4} + \frac{P^{iv}}{r^5} + \&c.$ together with $r^{\frac{1}{2}} \times \frac{\overset{\pounds}{1}}{2P} \times \frac{1}{2} \times$ the ſeries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{iv}}{r^4} + \frac{P^{v}}{r^5} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ $= \frac{r^{\frac{1}{2}}}{4} \times \frac{\overset{\pounds}{1}}{2P}$ the ſeries $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4} + \frac{P^{iv}}{r^5} + \&c.$ $+ \frac{r^{\frac{1}{2}}}{4} \times \frac{\overset{\pounds}{1}}{P} \times$ the ſeries $\frac{P'}{r} + \frac{P''}{r^2} + \frac{P'''}{r^3} + \frac{P^{iv}}{r^4} + \frac{P^{v}}{r^5} + \&c.$ $+ \frac{\Pi^{E-N+1}}{r^{E-N+1}} = \frac{r^{\frac{1}{2}}}{4} \times \frac{\overset{\pounds}{1}}{P} \times$ the ſeries $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4} + \frac{P^{iv}}{r^5} + \&c.$ $+ \frac{r^{\frac{1}{2}}}{4} \times \overset{\pounds}{V} + \frac{r^{\frac{1}{2}}}{4} \times \frac{\overset{\pounds}{1}}{P} \times$ the term $\frac{\Pi^{E-N+1}}{r^{E-N+1}} =$ (if we neglect this laſt quantity $\frac{r^{\frac{1}{2}}}{4} \times \frac{\overset{\pounds}{1}}{P} \times \frac{\Pi^{E-N+1}}{r^{E-N+1}}$, on account of its extreme ſmallneſs,) $\frac{r^{\frac{1}{2}}}{4} \times \frac{\overset{\pounds}{1}}{P} \times$ the ſeries $\frac{P}{r} + \frac{P'}{r^2} + \frac{P''}{r^3} + \frac{P'''}{r^4} + \frac{P^{iv}}{r^5} + \&c. + \frac{r^{\frac{1}{2}} \times \overset{\pounds}{V}}{4}$.

CCXI. Therefore $r^{\frac{1}{2}} \times \frac{\overset{\pounds}{1}}{2P} \times$ the ſeries $\frac{\Pi'}{r} + \frac{\Pi''}{r^2} + \frac{\Pi'''}{r^3} + \frac{\Pi^{iv}}{r^4} + \frac{\Pi^{v}}{r^5} + \&c. + \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ together with $\frac{\overset{\pounds}{V}}{2}$ is $= \frac{r^{\frac{1}{2}}}{4}$

$\times \frac{\overset{\pounds}{1}}{P}$

$\times \overset{\pounds}{\frac{1}{P}} \times$ the series $\frac{P}{r} + \frac{P^{\prime}}{r^2} + \frac{P^{\prime\prime}}{r^3} + \frac{P^{\prime\prime\prime}}{r^4} + \frac{P^{iv}}{r^5} +$ &c. $+ \frac{r^{\frac{1}{2}} \times \overset{\pounds}{V}}{4} + \frac{\overset{\pounds}{V}}{2} = \frac{r^{\frac{1}{2}}}{4} \times \overset{\pounds}{\frac{1}{P}} \times$ the series $\frac{P}{r} + \frac{P^{\prime}}{r^2} + \frac{P^{\prime\prime}}{r^3} + \frac{P^{\prime\prime\prime}}{r^4} + \frac{P^{iv}}{r^5} +$ &c. $+ \frac{r^{\frac{1}{2}} + 2}{4} \times \overset{\pounds}{V}$.

CCXII. But it has been shewn, in Art. 209, that the present value of a set of half-yearly payments of 10 shillings each, during the life of a person of the age of N years, is somewhat less than $r^{\frac{1}{2}} \times \overset{\pounds}{\frac{1}{2P}} \times$ the series $\frac{\Pi^{\prime}}{r} + \frac{\Pi^{\prime\prime}}{r^2} + \frac{\Pi^{\prime\prime\prime}}{r^3} + \frac{\Pi^{iv}}{r^4} + \frac{\Pi^{v}}{r^5} +$ &c. $+ \frac{\Pi^{E-N+1}}{r^{E-N+1}}$ together with $\frac{\overset{\pounds}{V}}{2}$. Therefore the said value of the said half-yearly payments is somewhat less than $\frac{r^{\frac{1}{2}}}{4} \times \overset{\pounds}{\frac{1}{P}} \times$ the series $\frac{P}{r} + \frac{P^{\prime}}{r^2} + \frac{P^{\prime\prime}}{r^3} + \frac{P^{\prime\prime\prime}}{r^4} + \frac{P^{iv}}{r^5} +$ &c. $+ \frac{r^{\frac{1}{2}}+2}{4} \times \overset{\pounds}{V}$.

CCXIII. Now $\overset{\pounds}{W}$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a person of the age of $N-1$ years, is $= \overset{\pounds}{\frac{1}{P+d}} \times$ the series $\frac{P}{r} + \frac{P^{\prime}}{r^2} + \frac{P^{\prime\prime}}{r^3} + \frac{P^{\prime\prime\prime}}{r^4} + \frac{P^{iv}}{r^5} +$ &c. Therefore $\frac{P+d}{P} \times \overset{\pounds}{W}$ is $= \overset{\pounds}{\frac{1}{P}} \times$ the series $\frac{P}{r} + \frac{P^{\prime}}{r^2} + \frac{P^{\prime\prime}}{r^3} + \frac{P^{\prime\prime\prime}}{r^4} + \frac{P^{iv}}{r^5} +$ &c. and $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \overset{\pounds}{W}$ is $= \frac{r^{\frac{1}{2}}}{4} \times \overset{\pounds}{\frac{1}{P}} \times$ the series $\frac{P}{r} + \frac{P^{\prime}}{r^2} + \frac{P^{\prime\prime}}{r^3} + \frac{P^{\prime\prime\prime}}{r^4} + \frac{P^{iv}}{r^5} +$ &c.

 Therefore

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Therefore $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \frac{£}{W} + \frac{r^{\frac{1}{2}}+2}{4} \times \frac{£}{V}$ is $= \frac{r^{\frac{1}{2}}}{4} \times \frac{£}{\frac{1}{P}}$ $\times$ the series $\frac{P}{r} + \frac{P^{I}}{r^{2}} + \frac{P^{II}}{r^{3}} + \frac{P^{III}}{r^{4}} + \frac{P^{IV}}{r^{5}}$ + &c. $+ \frac{r^{\frac{1}{2}}+2}{4} \times \frac{£}{V}$; and consequently the present value of a set of half-yearly payments of 10 shillings each, for the life of a person of the age of *N* years, is less than $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \frac{£}{W} + \frac{r^{\frac{1}{2}}+2}{4} \times \frac{£}{V}$.

CCXIV. But $\frac{£}{W}$ is $= \frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£$. Therefore $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \frac{£}{W}$ is $= \frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|£ = \frac{r^{\frac{1}{2}}}{4r} \times \overline{1+V}|£ = \frac{1}{4r^{\frac{1}{2}}} \times \overline{1+V}|£ = \frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{\frac{£}{V}}{4r^{\frac{1}{2}}}$. Therefore $\frac{r^{\frac{1}{2}}}{4} \times \frac{P+d}{P} \times \frac{£}{W} + \frac{r^{\frac{1}{2}}+2}{4} \times \frac{£}{V}$ is $= \frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{\frac{£}{V}}{4r^{\frac{1}{2}}} + \frac{r^{\frac{1}{2}}+2}{4} \times \frac{£}{V} = \frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{\frac{£}{V}}{4r^{\frac{1}{2}}} + \frac{r+2r^{\frac{1}{2}}}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$. Therefore the present value of a set of half-yearly payments of 10 shillings each, or of an annuity of one pound a year payable half-yearly, for the life of a person of the age of *N* years, is somewhat less than $\frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$, or $\frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$ is a quantity somewhat greater than, but very nearly equal to, the value of the said half-yearly annuity. Q E I.

CCXV. Since

The exceſs of the quantity $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$, or the ſecond member of the foregoing near value of an annuity of one pound a year, paid half-yearly, above $\frac{£}{V}$, or the value of the ſame annuity, when paid at the end of every year, is always extremely ſmall.

CCXV. Since 1 is to $r^{\frac{1}{2}}$ as $r^{\frac{1}{2}}$ is to r, $1+r$ will be greater than $2r^{\frac{1}{2}}$, and conſequently $1+r+2r^{\frac{1}{2}}$ will be greater than $2r^{\frac{1}{2}}+2r^{\frac{1}{2}}$, or $1+2r^{\frac{1}{2}}+r$ will be greater than $4r^{\frac{1}{2}}$, and therefore $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$ will be greater than $\frac{£}{V}$. But the difference between them will be extremely ſmall. Thus, for example, if the intereſt of money is 4 per cent. we ſhall have $r = 1.04$, and $r^{\frac{1}{2}} = \sqrt{1.04} = 1.0198$, and conſequently $2r^{\frac{1}{2}} = 2 \times 1.0198 = 2.0396$, and $1+2r^{\frac{1}{2}}+r = 1+2.0396+1.04 = 4.0796$, and $4r^{\frac{1}{2}} = 4.0792$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.0796}{4.0792} \times \frac{£}{V} = 1.000,09 \times \frac{£}{V}$, or leſs than $\frac{£}{V} + .0001 \times \frac{£}{V}$, or $\frac{£}{V} + \frac{1}{10000}$ part of $\frac{£}{V}$. And, when the intereſt of money is 10 per cent. (which is the higheſt intereſt ſuppoſed in the foregoing tables,) and r is conſequently equal to 1.1, we ſhall have $r^{\frac{1}{2}} = 1.0488$, and $2r^{\frac{1}{2}} = 2.0976$, and $1+2r^{\frac{1}{2}}+r = 1+2.0976+1.1 = 4.1976$, and $4r^{\frac{1}{2}} = 4 \times 1.0488 = 4.1952$, and conſequently $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.1976}{4.1952} \times \frac{£}{V} = 1.00057 \times \frac{£}{V}$, or $\frac{£}{V} + \frac{57}{100,000} \times \frac{£}{V}$, or $\frac{£}{V} + \frac{1}{1754}$ part of $\frac{£}{V}$. Therefore, if the intereſt of money be either 10 per cent. or any lower intereſt, the exceſs of $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$ above $\frac{£}{V}$ will be either $\frac{1}{1754}$ part, or ſome leſſer , of $\frac{£}{V}$; and conſequently the exceſs of $\frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$ above $\frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{£}{V}$ will be only the 1754th, or ſome leſſer, part of $\frac{£}{V}$. And therefore, if we ſuppoſe the ſaid quantity $\frac{\frac{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$,

 which

A conjectural conclusion concerning the magnitude of the true value of the said half-yearly annuity.

which has been shewn to be always somewhat, and but a very little, greater than the true value of an annuity of one pound a year paid half-yearly for the life of a person of the age of N years, to be greater than the said true value by the same excess by which it exceeds the quantity £ $\frac{1}{4r^{\frac{1}{2}}}$ + £ $\frac{}{V}$, the said true value will be equal to £ $\frac{1}{4r^{\frac{1}{2}}}$ + £ $\frac{}{V}$. Q E I.

Agreement of the said conjectural conclusion with the value of the said half-yearly annuity assigned by Mr. Thomas Simpson.

CCXVI. If this last supposition be a true one, the value of an annuity of one pound a year for the life of a person of the age of N years, when paid half-yearly, will be obtained by adding to £ V, or the value of the like annuity, when paid at the end of every year, (which is given in the foregoing tables,) the sum of £ $\frac{1}{4r^{\frac{1}{2}}}$, or somewhat less than £ $\frac{1}{4}$, or a quarter of a year's annuity; which agrees with the direction given by Mr. Thomas Simpson for this purpose in his *Doctrine of Annuities, &c.* page 81.

A computation of the several different values of the expression £ $\frac{1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times$ £ V, *(found in Art.* 203—214,*) according to the twelve different values of* r, *or the twelve different rates of interest, above-mentioned.*

CCXVII. But, if this last supposition should not be true, or if the truth of it should not be thought sufficiently evident or probable, it will be necessary to have recourse to the full expression £ $\frac{1}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}}$ $\times$ £ V, which has been shewn in the foregoing articles to be always somewhat greater than, but very nearly equal to, the true value of an annuity of one pound a year that is paid half-yearly, during the life of a person of the

the age of N years. Now, if we compute the different values of this expreſſion according to the twelve different rates of intereſt above-mentioned, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. thoſe values will be as follows.

In the firſt place, when the intereſt of money is 2 per cent. we ſhall have $r = 1.02$, and $r^{\frac{1}{2}} = \sqrt{1.02} = 1.009,950$, and $2r^{\frac{1}{2}} = 2.019,900$, and $4r^{\frac{1}{2}} = 4.039,800$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.019,900 + 1.02)$ $= 4.039,900$; and conſequently $\frac{\pounds 1}{4r^{\frac{1}{2}}} = \frac{\pounds 1}{4.039,800} = .2475\,\pounds$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\pounds}{V} = \frac{4.039,900}{4.039,800} \times \frac{\pounds}{V} = 1.000,024 \times \frac{\pounds}{V}$. Therefore, when the intereſt of money is 2 per cent. the value of an annuity of one pound a year, for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2475\,\pounds + 1.000,024 \times \frac{\pounds}{V}$.

Secondly, when the intereſt of money is $2\frac{1}{2}$ per cent. we ſhall have $r = 1.025$, and $r^{\frac{1}{2}} = \sqrt{1.025} = 1.012,422$, and $2r^{\frac{1}{2}} = 2.024,844$, and $4r^{\frac{1}{2}} = 4.049,688$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.024,844 + 1.025)$ $= 4.049,844$; and conſequently $\frac{\pounds 1}{4r^{\frac{1}{2}}} = \frac{\pounds 1}{4.049,688} = .2469\,\pounds$, and $\frac{1 + 2r^{\frac{1}{2}} + r}{4r^{\frac{1}{2}}} \times \frac{\pounds}{V} = \frac{4.049,844}{4.049,688} \frac{\pounds}{V} = 1.000,038 \times \frac{\pounds}{V}$. Therefore, when the intereſt of money is $2\frac{1}{2}$ per cent. the value of an annuity of one pound a year, for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2469\,\pounds + 1.000,038 \times \frac{\pounds}{V}$.

Thirdly, when the intereſt of money is 3 per cent. we ſhall have $r = 1.03$, and $r^{\frac{1}{2}} = \sqrt{1.03} = 1.014,889$, and $2r^{\frac{1}{2}} = 2.029,778$, and $4r^{\frac{1}{2}} = 4.059,556$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.029,778 + 1.03)$ $= 4.059,778$;

$= 4.059,778$; and conſequently $\frac{\overset{£}{1}}{4r^{\frac{1}{2}}} = \frac{\overset{£}{1}}{4.059,556} = .2463£$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.059,778}{4.059,556} \times \frac{£}{V} = 1.000,054 \times \frac{£}{V}$. Therefore, when the intereſt of money is 3 per cent. the value of an annuity of one pound a year, for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2463£ + 1.000,054 \times \frac{£}{V}$.

Fourthly, when the intereſt of money is $3\frac{1}{2}$ per cent. we ſhall have $r = 1.035$, and $r^{\frac{1}{2}} = \sqrt{1.035} = 1.017,349$, and $2r^{\frac{1}{2}} = 2.034,698$, and $4r^{\frac{1}{2}} = 4.069,396$, and $1+2r^{\frac{1}{2}}+r$ $(= 1+2.034,698 + 1.035)$ $= 4.069,698$; and conſequently $\frac{\overset{£}{1}}{4r^{\frac{1}{2}}} = \frac{\overset{£}{1}}{4.069,396} = .2457£$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.069,698}{4.069,396} \times \frac{£}{V} = 1.000,074 \times \frac{£}{V}$. Therefore, when the intereſt of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2457£ + 1.000,074 \times \frac{£}{V}$.

Fifthly, when the intereſt of money is 4 per cent. we ſhall have $r = 1.04$, and $r^{\frac{1}{2}} = \sqrt{1.04} = 1.019,804$, and $2r^{\frac{1}{2}} = 2.039,608$, and $4r^{\frac{1}{2}} = 4.079,216$, and $1+2r^{\frac{1}{2}}+r$ $(= 1+2.039,608+1.04)$ $= 4.079,608$; and conſequently $\frac{\overset{£}{1}}{4r^{\frac{1}{2}}} = \frac{\overset{£}{1}}{4.079,216} = .2451£$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.079,608}{4.079,216} \times \frac{£}{V} = 1.000,096 \times \frac{£}{V}$. Therefore, when the intereſt of money is 4 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2451£ + 1.000,096 \times \frac{£}{V}$.

Sixthly,

Sixthly, when the intereſt of money is $4\frac{1}{2}$ per cent. we ſhall have $r = 1.045$, and $r^{\frac{1}{2}} = \surd 1.045 = 1.022,252$, and $2r^{\frac{1}{2}} = 2.044,504$, and $4r^{\frac{1}{2}} = 4.089,008$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.044,504 + 1.045)$ $= 4.089,504$; and conſequently $\frac{\pounds 1}{4r^{\frac{1}{2}}} = \frac{\pounds 1}{4.089,008} = .2445\,\pounds$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\pounds}{V} = \frac{4.089,504}{4.089,008} \times \frac{\pounds}{V} = 1.000,121 \times \frac{\pounds}{V}$. Therefore, when the intereſt of money is $4\frac{1}{2}$ per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2445\,\pounds + 1.000,121 \times \frac{\pounds}{V}$.

Seventhly, when the intereſt of money is 5 per cent. we ſhall have $r = 1.05$, and $r^{\frac{1}{2}} = \surd 1.05 = 1.024,695$, and $2r^{\frac{1}{2}} = 2.049,390$, and $4r^{\frac{1}{2}} = 4.098,780$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.049,390 + 1.05)$ $= 4.099,390$; and conſequently $\frac{\pounds 1}{4r^{\frac{1}{2}}} = \frac{\pounds 1}{4.098,780} = .2439\,\pounds$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\pounds}{V} = \frac{4.099,390}{4.098,780} \times \frac{\pounds}{V} = 1.000,148 \times \frac{\pounds}{V}$. Therefore, when the intereſt of money is 5 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2439\,\pounds + 1.000,148 \times \frac{\pounds}{V}$.

Eighthly, when the intereſt of money is 6 per cent. we ſhall have $r = 1.06$, and $r^{\frac{1}{2}} = \surd 1.06 = 1.029,563$, and $2r^{\frac{1}{2}} = 2.059,126$, and $4r^{\frac{1}{2}} = 4.118,252$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.059,126 + 1.06)$ $= 4.119,126$; and conſequently $\frac{\pounds 1}{4r^{\frac{1}{2}}} = \frac{\pounds 1}{4.118,252} = .2428\,\pounds$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{\pounds}{V} = \frac{4.119,126}{4.118,252} \times \frac{\pounds}{V} = 1.000,212 \times \frac{\pounds}{V}$. Therefore,

when

when the intereſt of money is 6 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of *N* years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2428\,£ + 1.000{,}212 \times \frac{£}{V}$.

Ninthly, when the intereſt of money is 7 per cent. we ſhall have $r = 1.07$, and $r^{\frac{1}{2}} = \sqrt{1.07} = 1.034{,}408$, and $2r^{\frac{1}{2}} = 2.068{,}816$, and $4r^{\frac{1}{2}} = 4.137{,}632$, and $1 + 2r^{\frac{1}{2}} + r$ $(1 + 2.068{,}816 + 1.07)$ $= 4.138{,}816$; and conſequently $\frac{£\,1}{4r^{\frac{1}{2}}} = \frac{£\,1}{4.137{,}632} = .2416\,£$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.138{,}816}{4.137{,}632} \times \frac{£}{V} = 1.000{,}286 \times \frac{£}{V}$. Therefore, when the intereſt of money is 7 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of *N* years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2416\,£ + 1.000{,}286 \times \frac{£}{V}$.

Tenthly, when the intereſt of money is 8 per cent. we ſhall have $r = 1.08$, and $r^{\frac{1}{2}} = \sqrt{1.08} = 1.039{,}230$, and $2r^{\frac{1}{2}} = 2.078{,}460$, and $4r^{\frac{1}{2}} = 4.156{,}920$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.078{,}460 + 1.08)$ $= 4.158{,}460$; and conſequently $\frac{£\,1}{4r^{\frac{1}{2}}} = \frac{£\,1}{4.156{,}920} = .2405\,£$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V} = \frac{4.158{,}460}{4.156{,}920} \times \frac{£}{V} = 1.000{,}370 \times \frac{£}{V}$. Therefore, when the intereſt of money is 8 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of *N* years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2405\,£ + 1.000{,}370 \times \frac{£}{V}$.

Eleventhly,

Eleventhly, when the intereſt of money is 9 per cent. we ſhall have $r = 1.09$, and $r^{\frac{1}{2}} = \sqrt{1.09} = 1.044,031$, and $2r^{\frac{1}{2}} = 2.088,062$, and $4r^{\frac{1}{2}} = 4.176,124$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.088,062 + 1.09)$ $= 4.178,062$; and conſequently $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}} = \frac{\overset{\pounds}{1}}{4.176,124} = .2394\,\pounds$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V} = \frac{4.178,062}{4.176,124} \times \overset{\pounds}{V} = 1.000,464 \times \overset{\pounds}{V}$. Therefore, when the intereſt of money is 9 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2394\,\pounds + 1.000,464 \times \overset{\pounds}{V}$.

And, laſtly, when the intereſt of money is 10 per cent. we ſhall have $r = 1.1$, and $r^{\frac{1}{2}} = \sqrt{1.1} = 1.048,809$, and $2r^{\frac{1}{2}} = 2.097,618$, and $4r^{\frac{1}{2}} = 4.195,236$, and $1 + 2r^{\frac{1}{2}} + r$ $(= 1 + 2.097,618 + 1.1)$ $= 4.197,618$; and conſequently $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}} = \frac{\overset{\pounds}{1}}{4.195,236} = .2383\,\pounds$, and $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V} = \frac{4.197,618}{4.195,236} \times \overset{\pounds}{V} = 1.000,567 \times \overset{\pounds}{V}$. Therefore, when the intereſt of money is 10 per cent. the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid half-yearly, is ſomewhat leſs than, but very nearly equal to, $.2383\,\pounds + 1.000,567 \times \overset{\pounds}{V}$.

CCXVIII. The ſubſtance of the concluſions obtained in the laſt article may be expreſſed more conciſely in the manner following.

The concluſions obtained in the laſt article, expreſs'd more conciſely.

Let $\overset{\pounds}{V}$ denote, as before, the value of an annuity of one pound a year for the life of a perſon of the age of N years, to be paid at the end of every year; and let $\overset{\pounds}{H}$ denote the value of the ſame annuity for the

ſame

ſame life, when it is to be paid at the end of every half-year by payments of 10 ſhillings each. Then, if the intereſt of money be

2 per cent. $\overset{£}{H}$ will be (nearly) $= .2475£ + 1.000,024 \times \overset{£}{V}$;

If $2\frac{1}{2}$ per cent. $\overset{£}{H}$ will be (nearly) $= .2469£ + 1.000,038 \times \overset{£}{V}$;

If 3 per cent. $\overset{£}{H}$ will be (nearly) $= .2463£ + 1.000,054 \times \overset{£}{V}$;

If $3\frac{1}{2}$ per cent. $\overset{£}{H}$ will be (nearly) $= .2457£ + 1.000,074 \times \overset{£}{V}$;

If 4 per cent. $\overset{£}{H}$ will be (nearly) $= .2451£ + 1.000,096 \times \overset{£}{V}$;

If $4\frac{1}{2}$ per cent. $\overset{£}{H}$ will be (nearly) $= .2445£ + 1.000,121 \times \overset{£}{V}$;

If 5 per cent. $\overset{£}{H}$ will be (nearly) $= .2439£ + 1.000,148 \times \overset{£}{V}$;

If 6 per cent. $\overset{£}{H}$ will be (nearly) $= .2428£ + 1.000,212 \times \overset{£}{V}$;

If 7 per cent. $\overset{£}{H}$ will be (nearly) $= .2416£ + 1.000,286 \times \overset{£}{V}$;

If 8 per cent. $\overset{£}{H}$ will be (nearly) $= .2405£ + 1.000,370 \times \overset{£}{V}$;

If 9 per cent. $\overset{£}{H}$ will be (nearly) $= .2394£ + 1.000,464 \times \overset{£}{V}$;

And, if 10 per cent. $\overset{£}{H}$ will be (nearly) $= .2383£ + 1.000,567 \times \overset{£}{V}$.

Expreſſions of the differences of the values of an annuity of one pound a year, when paid yearly and when paid half-yearly, according to the aforeſaid rates of intereſt.

CCXIX. Therefore the values of $\overset{£}{H} - \overset{£}{V}$, or the differences of the values of the ſaid yearly and half-yearly annuities of one pound a year for the life of a perſon of the age of N years, according to the aforeſaid different rates of the intereſt of money, will be as follows.

If the intereſt of money is

2 per cent. $\overset{£}{H} - \overset{£}{V}$ will be (nearly) $= .2475£ + 0.000,024 \times \overset{£}{V}$;

If $2\frac{1}{2}$ per cent. $\overset{£}{H} - \overset{£}{V}$ will be (nearly) $= .2469£ + 0.000,038 \times \overset{£}{V}$;

If 3 per cent. $\overset{£}{H} - \overset{£}{V}$ will be (nearly) $= .2463£ + 0.000,054 \times \overset{£}{V}$;

If

If $3\frac{1}{2}$ per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2457\pounds + 0.000,074 \times \overset{\pounds}{V}$;

If 4 per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2451\pounds + 0.000,096 \times \overset{\pounds}{V}$;

If $4\frac{1}{2}$ per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2445\pounds + 0.000,121 \times \overset{\pounds}{V}$;

If 5 per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2439\pounds + 0.000,148 \times \overset{\pounds}{V}$;

If 6 per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2428\pounds + 0.$[illegible]$0,212 \times \overset{\pounds}{V}$;

If 7 per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2416\pounds + 0.000,286 \times \overset{\pounds}{V}$;

If 8 per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2405\pounds + 0.000,370 \times \overset{\pounds}{V}$;

If 9 per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2394\pounds + 0.000,464 \times \overset{\pounds}{V}$;

And, if 10 per cent. $\overset{\pounds}{H}-\overset{\pounds}{V}$ will be (nearly) $= .2383\pounds + 0.000,567 \times \overset{\pounds}{V}$.

CCXX. The second terms of the foregoing values of $\overset{\pounds}{H}-\overset{\pounds}{V}$, to wit, $0.000,024 \times \overset{\pounds}{V}$, $0.000,038 \times \overset{\pounds}{V}$, 0.000,[illegible] &c. are always very small in comparison of the first terms, [illegible] 9£, .2463£, &c. as will be evident from the following co[illegible] their magnitudes when they are greatest, or when the values o[illegible] est.

Of the smallness of the second terms of the foregoing expressions in comparison of their first terms.

CCXXI. When the interest of money is 2 per cent. it appears from the foregoing tables, (Table XII, page 221,) that $\overset{\pounds}{V}$, or the value of an annuity of one pound a year, paid at the end of every year, for the life of a person of the age of N years, is greatest when N is $= 7$, or the annuitant is 7 years of age: and then the said annuity is worth $\overset{\pounds}{28}.153,865$. Therefore the greatest magnitude of $.000,024 \times \overset{\pounds}{V}$, or of the second term of the expression $.2475\pounds + 0.000,024 \times \overset{\pounds}{V}$, (which, at this rate of

The magnitudes of the said second terms when they are greatest.

of intereft, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000,024 \times \overset{\pounds}{28.153,865} = \overset{\pounds}{0.000,675}$; which is lefs than the 366th part of .2475£, or the firft term of the faid expreffion.

When the intereft of money is $2\frac{1}{2}$ per cent. it appears from the foregoing tables (Table XIII, page 222,) that $\overset{\pounds}{V}$, or the value of a life-annuity of one pound a year, paid yearly, is greateft when the annuitant is of the age of 8 years, and that it is then $= \overset{\pounds}{25.242,492}$. Therefore the greateft magnitude of $0.000,038 \times \overset{\pounds}{V}$, or of the fecond term of the expreffion $.2469\pounds + .000,038 \times \overset{\pounds}{V}$, (which, at this rate of intereft, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000,038 \times \overset{\pounds}{25.242,492} = \overset{\pounds}{0.000,959}$; which is lefs than $\overset{\pounds}{0.001}$, or $\frac{1}{1000}$ part of a pound, and confequently than the 246th part of .2469£, or of the firft term of the faid expreffion.

When the intereft of money is 3 per cent. it appears from Table XIV, page 223, that the greateft magnitude of $\overset{\pounds}{V}$ is at the age of 9 years, and that it then is $= \overset{\pounds}{22.814,53}$. Therefore the greateft magnitude of $0.000,054 \times \overset{\pounds}{V}$, or of the fecond term of the expreffion $.2463\pounds + 0.000,054 \times \overset{\pounds}{V}$, (which, at this rate of intereft, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000,054 \times \overset{\pounds}{22.814,53} = \overset{\pounds}{0.001,221}$; which is lefs than the 201ft part of 2463£, or of the firft term of the faid expreffion.

When the intereft of money is $3\frac{1}{2}$ per cent. it appears from Table XV, page 224, that the greateft magnitude of $\overset{\pounds}{V}$ is at the age of 9 years, and that it is then $= \overset{\pounds}{20.770,473}$. Therefore the greateft magnitude of of 0.000,074 × $\overset{\pounds}{V}$, or of the second term of the expression .2457£ + 0.000,074 × $\overset{\pounds}{V}$ (which, at this rate of interest, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is = 0.000,074 × $\overset{\pounds}{20.770,473}$ = $\overset{\pounds}{0.001,537}$; which is less than the 159th part of .2457£, or of the first term of the said expression.

When the interest of money is 4 per cent. it appears from Table XVI, page 225, that the greatest magnitude of $\overset{\pounds}{V}$ is at the age of 9 years, and that it is then = $\overset{\pounds}{19.021,902}$. Therefore the greatest magnitude of 0.000,096 × $\overset{\pounds}{V}$, or of the second term of the expression .2451£ + 0.000,096 × $\overset{\pounds}{V}$, (which, at this rate of interest, is equal to $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is = 0.000,096 × $\overset{\pounds}{19.021,902}$ = $\overset{\pounds}{0.001,826}$; which is less than the 134th part of .2451£, or of the first term of the said expression.

When the interest of money is 4½ per cent. it appears from Table XVII, page 226, that the greatest magnitude of $\overset{\pounds}{V}$ is at the age of 9 years, and that it is then = $\overset{\pounds}{17.51542}$. Therefore the greatest magnitude of 0.000,121 × $\overset{\pounds}{V}$, or of the second term of the expression .2445£ + 0.000,121 × $\overset{\pounds}{V}$, (which, at this rate of interest, is = $\overset{\pounds}{H} - \overset{\pounds}{V}$,) is = 0.000,121 × 17.51542 = 0.002,129; which is less than the 114th part of .2445£, or of the first term of the said expression.

When the interest of money is 5 per cent. it appears from Table XVIII, page 227, that the greatest magnitude of $\overset{\pounds}{V}$ is at the age of 10 years, and that it is then = $\overset{\pounds}{16.212,55}$. Therefore the greatest magnitude of 0.000,148 × $\overset{\pounds}{V}$, or of the second term of the expression .2439£ + 0.000,148 × $\overset{\pounds}{V}$,

$+\ 0.000{,}148 \times \overset{\pounds}{V}$, (which, at this rate of intereſt, is $= \overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000{,}148 \times 16.212{,}55 = 0.002{,}499$; which is leſs than the 97[th] part of $.2439\,\pounds$, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 6 per cent. it appears from Table XIX, page 228, that the greateſt magnitude of $\overset{\pounds}{V}$ is at the age of 10 years, and that it is then $= \overset{\pounds}{14.078{,}91}$. Therefore the greateſt magnitude of $0.000{,}212 \times \overset{\pounds}{V}$, or of the ſecond term of the expreſſion $.2428\,\pounds + 0.000{,}212 \times \overset{\pounds}{V}$, (which, at this rate of intereſt, is $= \overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000{,}212 \times \overset{\pounds}{14.07891} = \overset{\pounds}{0.002{,}984}$; which is leſs than the 81[ſt] part of $.2428\,\pounds$, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 7 per cent. it appears from Table XX, page 229, that the greateſt magnitude of $\overset{\pounds}{V}$ is at the age of 10 years, and that it is then $= \overset{\pounds}{12.411{,}09}$. Therefore the greateſt magnitude of $0.000{,}286 \times \overset{\pounds}{V}$, or of the ſecond term of the expreſſion $.2416\,\pounds + 0.000{,}286 \times \overset{\pounds}{V}$, (which, at this rate of intereſt, is $= \overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000{,}286 \times \overset{\pounds}{12.411{,}09} = \overset{\pounds}{0.003{,}549}$; which is leſs than the 68[th] part of $.2416\,\pounds$, or of the firſt term of the ſaid expreſſion.

When the intereſt of money is 8 per cent. it appears from Table XXI, page 230, that the greateſt magnitude of $\overset{\pounds}{V}$ is at the age of 10 years, and that it is then $= \overset{\pounds}{11.079{,}47}$. Therefore the greateſt magnitude of $\overset{\pounds}{0.000{,}370} \times \overset{\pounds}{V}$, or of the ſecond term of the expreſſion $.2405\,\pounds + 0.000{,}370 \times \overset{\pounds}{V}$, (which, at this rate of intereſt, is $= \overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000{,}370 \times \overset{\pounds}{11.079{,}47} = \overset{\pounds}{0.004{,}099}$; which is leſs than the 58[th] part of $.2405\,\pounds$, or of the firſt term of the ſaid expreſſion.

When

When the intereſt of money is 9 per cent. it appears from Table XXII, page 231, that the greateſt magnitude of $\overset{\pounds}{V}$ is at the age of 10 years, and that it is then $= \overset{\pounds}{9.996,17}$. Therefore the greateſt magnitude of $0.000,464 \times \overset{\pounds}{V}$, or of the ſecond term of the expreſſion $.2394\pounds + 0.000,464 \times \overset{\pounds}{V}$, (which, at this rate of intereſt, is $= \overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000,464 \times \overset{\pounds}{9.996,17} = \overset{\pounds}{0.004,638}$; which is leſs than the 51ſt part of $.2394\pounds$, or of the firſt term of the ſaid expreſſion.

And, when the intereſt of money is 10 per cent. it appears from Table XXIII, page 232, that the greateſt magnitude of $\overset{\pounds}{V}$ is at the age of 11 years, and that it is then $= \overset{\pounds}{9.101,99}$. Therefore the greateſt magnitude of $0.000,567 \times \overset{\pounds}{V}$, or of the ſecond term of the expreſſion $.2383\pounds + 0.000,567 \times \overset{\pounds}{V}$, (which, at this rate of intereſt, is $= \overset{\pounds}{H} - \overset{\pounds}{V}$,) is $= 0.000,567 \times \overset{\pounds}{9.101,99} = \overset{\pounds}{0.005,160}$; which is leſs than the 46th part of $.2383\pounds$, or of the firſt term of the ſaid expreſſion.

The magnitudes of the whole expreſſions mentioned in Art. 219, when they are greateſt.

CCXXII. It follows from the foregoing article that the greateſt poſſible magnitudes of the ſeveral expreſſions $.2475\pounds + 0.000,024 \times \overset{\pounds}{V}$, $.2469\pounds + 0.000,038 \times \overset{\pounds}{V}$, $.2463\pounds + 0.000,054 \times \overset{\pounds}{V}$, $.2457\pounds + 0.000,074 \times \overset{\pounds}{V}$, $.2451\pounds + 0.000,096 \times \overset{\pounds}{V}$, $.2445\pounds + 0.000,121 \times \overset{\pounds}{V}$, $.2439\pounds + 0.000,148 \times \overset{\pounds}{V}$, $.2428\pounds + 0.000,212 \times \overset{\pounds}{V}$, $.2416\pounds + 0.000,286 \times \overset{\pounds}{V}$, $.2405\pounds + 0.000,370 \times \overset{\pounds}{V}$, $.2394\pounds + 0.000,464 \times \overset{\pounds}{V}$, and $.2383\pounds + 0.000,567 \times \overset{\pounds}{V}$, (to which the general expreſſion

$\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}}$

$\frac{\overset{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \frac{£}{V}$ becomes equal when r is equal to 1.02, 1.025, 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, 1.07, 1.08, 1.09, and 1.10, or when the intereſt of money is 2 per cent. 2½ per cent. 3 per cent. 3½ per cent. 4 per cent. 4½ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent.) are as follows.

When the intereſt of money is 2 per cent. the greateſt poſſible magnitude of the expreſſion $.2475£ + 0.000,024 \times \frac{£}{V}$ is (= .2475£ + 0.000,675£) = .248,175£.

When the intereſt of money is 2½ per cent. the greateſt poſſible magnitude of the expreſſion $.2469£ + 0.000,038 \times \frac{£}{V}$ is (= .2469£ + 0.000,959£) = .247,859£.

When the intereſt of money is 3 per cent. the greateſt poſſible magnitude of the expreſſion $.2463£ + 0.000,054 \times \frac{£}{V}$ is (= .2463£ + 0.001,221) = .247,521£.

When the intereſt of money is 3½ per cent. the greateſt poſſible magnitude of the expreſſion $.2457£ + 0.000,074 \times \frac{£}{V}$ is (= .2457£ + 0.001,537£) = .247,237£.

When the intereſt of money is 4 per cent. the greateſt poſſible magnitude of the expreſſion $.2451£ + 0.000,096 \times \frac{£}{V}$ is (= .2451£ + 0.001,826£) = .246,926£.

When the intereſt of money is 4½ per cent. the greateſt poſſible magnitude of the expreſſion $.2445£ + 0.000,121 \times \frac{£}{V}$ is (= .2445£ + 0.002,129£) = .246,629£.

When

When the intereſt of money is 5 per cent. the greateſt poſſible magnitude of the expreſſion $.2439\pounds + 0.000{,}148 \times \frac{\pounds}{V}$ is $(= .2439\pounds + 0.002{,}499\pounds) = .246{,}399\pounds$.

When the intereſt of money is 6 per cent. the greateſt poſſible magnitude of the expreſſion $.2428\pounds + 0.000{,}212 \times \frac{\pounds}{V}$ is $(= .2428\pounds + 0.002{,}984\pounds) = .245{,}784\pounds$.

When the intereſt of money is 7 per cent. the greateſt poſſible magnitude of the expreſſion $.2416\pounds + 0.000{,}286 \times \frac{\pounds}{V}$ is $(= .2416\pounds + 0.003{,}549\pounds) = .245{,}149\pounds$.

When the intereſt of money is 8 per cent. the greateſt poſſible magnitude of the expreſſion $.2405\pounds + 0.000{,}370 \times \frac{\pounds}{V}$ is $(= .2405\pounds + 0.004{,}099\pounds) = .244{,}599\pounds$.

When the intereſt of money is 9 per cent. the greateſt poſſible magnitude of the expreſſion $.2394\pounds + 0.000{,}464 \times \frac{\pounds}{V}$ is $(= .2394\pounds + 0.004{,}638\pounds) = .244{,}038\pounds$.

And, when the intereſt of money is 10 per cent. the greateſt poſſible magnitude of the expreſſion $.2383\pounds + 0.000{,}567 \times \frac{\pounds}{V}$ is $(= .2383\pounds + 0.005{,}160\pounds) = .243{,}460\pounds$.

The said greatest magnitudes of the whole expressions mentioned in Art. 217 are, all of them, less than a quarter of a year's annuity.

CCXXIII. These several quantities, .248,175£, .247,859£, .247,521£, .247,237£, .246,926£, .246,629£, .246,399£, .245,784£, .245,149£, .244,599£, .244,038£, and .243,460£, or (neglecting the two last figures, as inconsiderable,) .2482£, .2478£, .2475£, .2472£, .2469£, .2466£, .2464£, .2458£, .2451£, .2446£, .2440£, and .2434£, (which are the greatest possible magnitudes of the several values of the expression $\frac{\overset{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{£}{V}$, or $\overset{£}{H} - \overset{£}{V}$, according to the twelve different rates of interest above-mentioned,) are all of them less than .2500£, or $\frac{1}{4}$ of a pound, or a quarter of a year's annuity. Therefore the greatest possible magnitude of the excess of the value of an annuity of one pound a year, paid half-yearly by payments of 10 shillings each, above the value of a like annuity of one pound a year, paid at the end of every year, when the interest of money is at any of the rates before-mentioned, is less than $\frac{1}{4}$ part of a pound, or a quarter of a year's annuity.

The second terms of the said expressions, mentioned in Art 217, are so small in comparison of the first terms, that they may justly be neglected.

CCXXIV. Since the several values of $\frac{\overset{£}{1}}{4r^{\frac{1}{2}}}$, or of the first term of the expression $\frac{\overset{£}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{£}{V}$, at the several rates of interest before-mentioned, have been shewn, in Art. 217, to be .2475£, and .2469£, and .2463£, and .2457£, and .2451£, and .2445£, and .2439£, and .2428£, and .2416£, and .2405£, and .2394£, and .2383£; and (by Art. 221) the greatest possible magnitudes of the second term, $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{£}{V}$, of the same expression, at the same rates of interest, are only equal to 0.000,675£, and 0.000,959£, and 0.001,221£, and 0.001,537£, and 0.001,926£, and 0.002,129£, and 0.002,499£, and 0.002,984£, and 0.003,549£, and 0.004,099£, and 0.004,638£, and 0.005,160£; of which latter quantities the last and greatest, to wit, 0.005,160£, is only about 5 thousandth parts, or one two hundredth part, of a pound, or one two hundredth part of a year's year's annuity, or leſs than two days annuity; we may ſafely neglect the ſecond term, $\frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$, of the expreſſion $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}} + \frac{1+2r^{\frac{1}{2}}+r}{4r^{\frac{1}{2}}} \times \overset{\pounds}{V}$, as being inconſiderable in comparison of the firſt term, $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}}$, and may therefore eſteem the firſt term $\frac{\overset{\pounds}{1}}{4r^{\frac{1}{2}}}$ to be equal to the whole expreſſion, and conſequently to $\overset{\pounds}{H} - \overset{\pounds}{V}$, or to the difference of the values of an annuity of one pound a year paid yearly and paid half-yearly. And, upon this ſuppoſition, we ſhall have $\overset{\pounds}{H} - \overset{\pounds}{V}$,

The values of the ſaid expreſſions, when their ſecond terms are neglected.

When the intereſt of money is 2 per cent. = .2475 £;
And, when it is $2\frac{1}{2}$ per cent. = .2469 £;
And, when it is 3 per cent. = .2463 £;
And, when it is $3\frac{1}{2}$ per cent. = .2457 £;
And, when it is 4 per cent. = .2451 £;
And, when it is $4\frac{1}{2}$ per cent. = .2445 £;
And, when it is 5 per cent. = .2439 £;
And, when it is 6 per cent. = .2428 £;
And, when it is 7 per cent. = .2416 £;
And, when it is 8 per cent. = .2405 £;
And, when it is 9 per cent. = .2394 £;
And, when it is 10 per cent. = .2383 £.

Agreement of theſe values with Mr. Thomas Simpſon's determination.

CCXXV. And, if theſe laſt-mentioned quantities, .2475 £, .2469 £, .2463 £, &c. (which, it is evident, differ very little from each other,) be conſidered as equal to each other, and alſo as equal to .25 £, (from which they differ by only about the 100th part of a pound, or about four days annuity,) we ſhall then have $\overset{\pounds}{H} - \overset{\pounds}{V}$, or the difference of the values of an annuity of one pound a year, paid half-yearly, and paid yearly, in all the different rates of intereſt above-mentioned, equal to .25 £, or $\frac{1}{4}$ part of a pound, or a quarter of a year's annuity; agreeably to Mr. Thomas Simpſon's determination above-mentioned.

 CCXXVI. When

Of the value of a life-annuity that is paid at the end of every quarter of a year.

CCXXVI. When the annuity is to be paid at the end of every quarter of a year, it will be worth somewhat more than when it is to be paid at the end of every half-year; and Mr. Simpson says that it will be worth an eighth part of a year's annuity more than in the other case, or $\frac{3}{8}$ of a year's annuity more than when it is to be paid at the end of every year. And this, I think, seems very probable. But after the foregoing very tedious investigation of the difference of the values of a yearly and a half-yearly annuity, (which yet I knew not how to make shorter or easier,) I shall not trouble either myself or my reader with any attempt to determine it exactly. Indeed I thought the difference of the values of a yearly and a half-yearly annuity hardly worth attending to, and should not therefore have gone into the foregoing investigation of it, (especially when I found into what length it led me) if I had not observed that several other writers upon this subject, and particularly Mr. De Moivre, Mr. Simpson, and Dr. Price, have thought fit to mention it as an object of some importance. But, having now found that the said difference is worth only about .24£, or less than a quarter of a year's annuity, we may be the better satisfied that the further addition that ought to be made to $\overset{£}{V}$, or to the value of the annuity, when paid yearly, in order to make it equal to the value of the same annuity paid quarterly, can be but a very trifling quantity. And therefore we may safely decline any further consideration of it.

[*End of the consideration of the values of life-annuities paid half-yearly and paid quarterly.*]

Of the limit of the increase of a sum of money at compound interest.

CCXXVII. Nevertheless, as it is evident that the advantage arising from a given sum of money put out at compound interest continually increases in *some* degree when the term for which the money is lent is diminished, it will, I doubt not, be entertaining to the reader to see a determination of *the limit* of this increase, or the sum to which the given sum of money may, (by increasing the number, and diminishing the lengths, of the terms for which the money is lent,) be made to approach as near as we please, in the space of a year, or in any other given time, but which it can never exceed, or even absolutely become equal to, into whatever number of parts the year be divided. Now this limit may be determined without much difficulty by the solution of the following Problem.

A Problem

A Problem in the Doctrine of Compound Interest.

CCXXVIII. Let a be any given sum of money, and b its interest for one year. Let m be any whole number whatsoever; and let it be supposed that the money a is lent out for the m^{th} part of a year at an interest that bears the same proportion to the interest b, (which would be due for a whole year,) as the term for which it is lent, to wit, the m^{th} part of a year, bears to a whole year; and that, when the interest and principal are both repaid at the end of the m^{th} part of a year, they are added together and immediately lent out again at the same rate of interest for another m^{th} part of a year; and so on continually, at the end of every m^{th} part of a year, till the whole year is exhausted. It is required to find the sum to which the original sum a will have increased, by these repeated loans of principal and interest, at the end of the whole year.

S O L U T I O N.

Since b is the interest of the sum a for a whole year, it is evident that $\frac{b}{m}$ will be the interest of the same sum a for the m^{th} part of a year. Therefore the sum to be lent at the end of the first, and the beginning of the second, m^{th} part of a year will be $a+\frac{b}{m}$. This sum is to be lent for the m^{th} part of a year, as the sum a was at first; and it is to be lent at the same interest. Therefore the said sum $a+\frac{b}{m}$ together with its interest for the m^{th} part of a year will bear the same proportion to the said sum $a+\frac{b}{m}$ alone as the said sum $a+\frac{b}{m}$ (which is equal to the original sum a together with its interest for the m^{th} part of a year,) bears to the original sum a. Therefore the said sum $a+\frac{b}{m}$, together with its interest for the m^{th} part of a year, is equal to $\frac{\left(a+\frac{b}{m}\right)^2}{a}$; or the quantity to which the original sum a will have increased by these loans at the end of the second fecond m^{th} part of a year will be $\frac{\overline{a+\frac{b}{m}}\big|^{2}}{a}$. In the fame manner it may be fhewn that the quantity to which the original fum a will have increafed in the next, or third, m^{th} part of the year will be $= \frac{\overline{a+\frac{b}{m}}\big|^{2}}{a} \times \frac{a+\frac{b}{m}}{a}$, or $\frac{\overline{a+\frac{b}{m}}\big|^{3}}{aa}$; and the quantity to which it will have increafed in the fourth m^{th} part of the year will be $= \frac{\overline{a+\frac{b}{m}}\big|^{3}}{aa} \times \frac{a+\frac{b}{m}}{a}$, or $\frac{\overline{a+\frac{b}{m}}\big|^{4}}{a^{3}}$; and confequently that the quantity to which it will have increafed in the laft m^{th} part of the year, or in the courfe of the whole year, will be $= \frac{\overline{a+\frac{b}{m}}\big|^{m}}{a^{m-1}}$. QEI.

Examples of the foregoing Solution.

Cafe of intereft paid every half-year.

CCXXIX. COROLL. 1. Let m be $= 2$; or let the fum a be lent for only half a year, and then paid in with the intereft due upon it, and immediately lent out again, together with the intereft, at the fame rate of intereft, for the other half-year. Then will $\frac{\overline{a+\frac{b}{m}}\big|^{m}}{a^{m-1}}$ be $= \frac{\overline{a+\frac{b}{2}}\big|^{2}}{a^{2-1}} = \frac{\overline{a+\frac{b}{2}}\big|^{2}}{a} = \frac{aa+2a\times\frac{b}{2}+\frac{bb}{4}}{a} = a+b+\frac{bb}{4a}$. Therefore by lending the fum a in this manner for two fucceffive half-years, inftead of lending it at once for a whole year, the faid fum will, at the end of the year, be increafed to $a+b+\frac{bb}{4a}$, inftead of being increafed only to $a+b$.

Thus,

Thus, for example, if we ſuppoſe the ſum a to be £100, and b, the intereſt of it for a year, to be 5 pounds, we ſhall have $bb = \overset{£}{25}$, and $\frac{bb}{4a}\left(= \overset{£}{\frac{25}{4\times100}} = \overset{£}{\frac{25}{4\times4\times25}} = \overset{£}{\frac{1}{16}} = \overset{S}{\frac{20}{16}} = \overset{S}{\frac{5}{4}}\right) = \overset{S}{1}.\ \overset{D}{3}.$ Therefore, if a man were to lend a hundred pounds at the intereſt of 5 per cent. per annum, for two ſucceſſive half-years, ſo that the principal together with its intereſt for the firſt half-year, ſhould be made a new principal and lent out at the ſame intereſt for the ſecond half-year, he would thereby increaſe it in the courſe of the whole year to only 1*s.* 3*d.* more than it would have been increaſed to in the ſame time by lending it at once for a whole year at the ſame intereſt.

Caſe of intereſt paid every quarter of a year.

CCXXX. COROLL. 2. If m is $= 4$, we ſhall have $\overline{a+\frac{b}{m}}\Big|^{m} = \overline{a+\frac{b}{4}}\Big|^{4}\left(a^4 + \frac{4a^3b}{4} + \frac{6aabb}{16} + \frac{4a\times b^3}{64} + \frac{b^4}{256}\right) = a^4 + a^3b + \frac{3aabb}{8} + \frac{ab^3}{16} + \frac{b^4}{256}$, and $a^{m-1} = a^{4-1} = a^3$; and conſequently $\frac{\overline{a+\frac{b}{m}}\big|^{m}}{a^{m-1}} = a + b + \frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$. Therefore, if the ſum a be lent out for only a quarter of a year, or three months, at the intereſt of b per annum, or $\frac{b}{4}$ for the ſaid three months, and it be then paid in with the intereſt due upon it; and then the ſaid principal and intereſt be added together, ſo as to make a new principal, and the ſaid new principal be lent out again immediately, at the ſame rate of intereſt as before, for three months more; and ſo on for the two remaining quarters of the year; the quantity to which the original ſum a will have increaſed at the end of the year by means of theſe four ſucceſſive loans, will be $a + b + \frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$, which is greater than $a+b$, (or the quantity to which the ſaid ſum a would have increaſed in the ſame time by means of a ſingle loan at the ſame intereſt for the whole year,) by $\frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$.

If

If a is 100 pounds, and b is 5 pounds, we shall have $\frac{3bb}{8a}$ $\left(= \overset{£}{\frac{3\times25}{8\times100}} = \overset{£}{\frac{3\times1}{8\times4}} = \overset{£}{\frac{3\times1}{32}} = \overset{S}{\frac{3\times20}{32}} = \overset{S}{\frac{60}{32}} = \overset{D}{\frac{720}{32}} = \overset{D}{22.5}\right) = \overset{S}{1},\ \overset{D}{10\tfrac{1}{2}}$; and $\frac{b^3}{16aa}$ $\left(= \overset{£}{\frac{125}{16\times10{,}000}} = \overset{£}{\frac{25}{16\times2000}} = \overset{£}{\frac{5}{16\times400}} = \overset{£}{\frac{1}{16\times80}} = \overset{£}{\frac{1}{1280}} = \overset{D}{\frac{240}{1280}} = \overset{D}{\frac{24}{128}} = \overset{D}{\frac{3}{16}}\right) = \overset{D}{0.1875}$; and $\frac{b^4}{256a^3}$ $\left(= \overset{£}{\frac{625}{256\times1000{,}000}} = \overset{£}{\frac{125}{256\times200{,}000}} = \overset{£}{\frac{25}{256\times40{,}000}} = \overset{£}{\frac{5}{256\times8000}} = \overset{£}{\frac{1}{256\times1600}} = \overset{D}{\frac{240}{256\times1600}} = \overset{D}{\frac{24}{256\times160}} = \overset{D}{\frac{3}{256\times20}} = \overset{D}{\frac{3}{5120}}\right)$ $= 0.000{,}581$. Therefore $\frac{3bb}{8a} + \frac{b^3}{16aa} + \frac{b^4}{256a^3}$ will be $= \overset{S}{1},\ \overset{D}{10.5} + \overset{D}{0.18750} + \overset{D}{0.000{,}58} = \overset{S}{1},\ \overset{D}{10.\ 68808}$, or less than $\overset{S}{1},\ \overset{D}{10\tfrac{3}{4}}$; that is, the excess of the increase of the original sum of a hundred pounds in the space of a year by means of these four quarterly loans above the increase of it in the same time by means of a single loan of it for the whole year at the interest of 5 pounds, is a little more than $\overset{S}{1},\ \overset{D}{10\tfrac{1}{2}}$, but less than $\overset{S}{1},\ \overset{D}{10\tfrac{3}{4}}$.

Case of interest paid every month.

CCXXXI. COROLL. 3. If m is $= 12$, or the sum a is lent out for only the 12[th] part of a year, or a calendar month, and then is paid in, and the principal and interest added together so as to form a new principal, and the said new principal lent out again immediately for another calendar month at the same interest as before; and the like loans of the principal with its monthly interest, be repeated for the remaining ten months of the year, we shall have $\left.\overline{a+\frac{b}{m}}\right|^{m} = \left.\overline{a+\frac{b}{12}}\right|^{12}$, and $a^{m-1} = a^{11}$, and consequently $\frac{\left.\overline{a+\frac{b}{m}}\right|^{m}}{a^{m-1}} = \frac{\left.\overline{a+\frac{b}{12}}\right|^{12}}{a^{11}}$.

Now

Now, by Sir Isaac Newton's binomial theorem, $\overline{a+\frac{b}{12}}\Big|^{12}$ is $\Big(= a^{12} + \frac{12}{1} \times a^{12-1}\frac{b}{12} + \frac{12}{1} \times \frac{12-1}{2} \times \frac{a^{12-2} \times bb}{12 \times 12} + \frac{12}{1} \times \frac{12-1}{2} \times \frac{12-2}{3} \times a^{12-3} \times \frac{b^3}{12 \times 12 \times 12}$ + &c. continued to the term $\frac{b^{12}}{\overline{12}|^{12}} = a^{12} + \frac{12a^{11}b}{12} + 12 \times \frac{11}{2} \times \frac{a^{10}bb}{12 \times 12} + 12 \times \frac{11}{2} \times \frac{10}{3} \times \frac{a^9 b^3}{12 \times 12 \times 12}$ + &c. continued to the term $\frac{b^{12}}{\overline{12}|^{12}}\Big) = a^{12} + a^{11}b + \frac{11a^{10}bb}{24} + \frac{55a^9b^3}{432}$ + &c. continued to the term $\frac{b^{12}}{\overline{12}|^{12}}$. Therefore $\dfrac{\overline{a+\frac{b}{12}}\Big|^{12}}{a^{11}}$ is $=$ $a + b + \frac{11bb}{24a} + \frac{55b^3}{432aa}$ + &c. continued to the term $\frac{b^{12}}{\overline{12}|^{12}a^{11}}$. Therefore the sum a will, in the course of the whole year, have increased, by means of these twelve repeated monthly loans, to $a + b + \frac{11bb}{24a} + \frac{55b^3}{432aa}$ + &c. continued to the term $\frac{b^{12}}{\overline{12}|^{12}a^{11}}$; the excess of which above $a + b$, (or above the quantity to which it would have increased in the same time by a single loan for the interest b,) is $\frac{11bb}{24a} + \frac{55b^3}{432aa}$ + &c. continued to the term $\frac{b^{12}}{\overline{12}|^{12}a^{11}}$; in which series all the terms after $\frac{55b^3}{432aa}$ are omitted on account of their extreme smallness in comparison of $\frac{11bb}{24a}$ and $\frac{55b^3}{432aa}$.

 If

If a is $\overset{£}{100}$, and b is $\overset{£}{5}$, we shall have $bb = \overset{£}{25}$, and $\frac{11bb}{24a}$

$$\left(= \overset{£}{\frac{11\times25}{24\times100}} = \overset{£}{\frac{11\times25}{24\times4\times25}} = \overset{£}{\frac{11\times1}{24\times4}} = \overset{£}{\frac{11}{96}} = \overset{D}{\frac{11\times240}{96}} = \overset{D}{\frac{2640}{96}}\right.$$

$= \left.\overset{D}{27.5}\right) = \overset{S}{2},\ \overset{D}{3.5}$, or $\overset{S}{2},\ \overset{D}{3\tfrac{1}{2}}$; and $\frac{55b^3}{432aa}$ $\left(= \overset{£}{\frac{55\times125}{432\times100\times100}} =\right.$

$$\overset{£}{\frac{55\times25\times5}{432\times4\times25\times5\times20}} = \overset{£}{\frac{55}{432\times4\times20}} = \overset{£}{\frac{55}{432\times4\times4\times5}} = \overset{£}{\frac{11}{432\times4\times4}}$$

$= \overset{£}{\frac{11}{432\times16}} = \overset{£}{\frac{11}{6912}} = \overset{D}{\frac{11\times240}{6912}} = \left.\overset{D}{\frac{2640}{6912}}\right) = \overset{D}{0.38}$, or less than one half-penny. Therefore $\frac{11bb}{24a} + \frac{55b^3}{432aa}$ will be equal to $\overset{S}{2},\ \overset{D}{3.5} + \overset{D}{0.38}$, or $\overset{S}{2},\ \overset{D}{3.88}$, or less than $\overset{S}{2},\ \overset{D}{4}$. So that the excess of the interest of 100 pounds for a year, arising from these 12 monthly loans, at this rate of interest, above 5 pounds, or the interest of the same sum of money for the same time, arising from a single loan of it for the whole year, is only $\overset{S}{2},\ \overset{D}{3.88}$, or less than $\overset{S}{2},\ \overset{D}{4}$.

Case of interest paid every week.

CCXXXII. COROLL. 4. If m is $= 52$, or the sum a is lent out for only the 52d part of a year, or for one week, and then paid in, and, with its interest, immediately lent out again for another week, and so on for every week throughout the whole year, we shall have $\overline{a+\frac{b}{m}}\Big|^{m} = \overline{a+\frac{b}{52}}\Big|^{52}$ and $a^{m-1} = a^{51}$, and consequently $\frac{\overline{a+\frac{b}{m}}\Big|^{m}}{a^{m-1}}$ $= \frac{\overline{a+\frac{b}{52}}\Big|^{52}}{a^{51}}$. Now, by Sir Isaac Newton's binomial theorem, $\overline{a+\frac{b}{52}}\Big|^{52}$ is $\left(= a^{52} + \frac{52}{1} \times a^{51} \times \frac{b}{52} + \frac{52}{1} \times \frac{52-1}{2} \times \frac{a^{50}\times bb}{52\times52}\right.$

$+ \frac{52}{1}$

$$+\frac{52}{1}\times\frac{52-1}{2}\times\frac{52-2}{3}\times\frac{a^{49}\times b^3}{52\times52\times52}+\&c. = a^{52}+a^{51}b+\frac{51a^{50}bb}{104}+\frac{51}{2}\times\frac{50}{3}\times\frac{a^{49}b^3}{52\times52}+\&c. = a^{52}+a^{51}b+\frac{51a^{50}bb}{104}+\frac{17\times25\times a^{49}b^3}{52\times52}+\&c.) = a^{52}+a^{51}b+\frac{51a^{50}bb}{104}+\frac{425a^{49}b^3}{2704}+\&c.$$

Therefore $\dfrac{\overline{a+\frac{b}{52}}\Big|^{52}}{a^{51}}$ is $= a+b+\dfrac{51bb}{104a}+\dfrac{425b^3}{2704aa}+\&c.$

Therefore the ſum a will, in the courſe of the whole year, have increaſed, by means of theſe 52 repeated weekly loans, to $a+b+\frac{51bb}{104a}+\frac{425b^3}{2704aa}+\&c.$ which exceeds $a+b$ (or the quantity to which the ſaid ſum a would have increaſed in the ſame time by a ſingle loan for the intereſt b,) by the difference $\frac{51bb}{104a}+\frac{425b^3}{2704aa}+\&c.$ in which difference all the terms after the ſecond term, $\frac{425b^3}{2704aa}$, are omitted on account of their extreme ſmallneſs in compariſon of $\frac{51bb}{104a}$ and $\frac{425b^3}{2704aa}$.

Let a be $= \overset{£}{100}$, as before, and $b = \overset{£}{5}$. Then will bb be $= \overset{£}{25}$, and $\frac{51bb}{104a}$ will be $\left(= \frac{\overset{£}{51\times25}}{104\times100} = \frac{\overset{£}{51\times25}}{104\times4\times25} = \frac{\overset{£}{51\times1}}{104\times4} = \frac{\overset{D}{51\times240}}{104\times4} = \frac{\overset{D}{51\times60}}{104} = \frac{\overset{D}{51\times15}}{26} = \frac{\overset{D}{766}}{26}\right) = \overset{D}{29.46}$, or $\overset{S}{2}, \overset{D}{5.46}$; and $\frac{425b^3}{2704aa}$ will be $\left(= \frac{\overset{£}{425\times5\times5\times5}}{2704\times100\times100} = \frac{\overset{£}{425\times5\times5\times5}}{2704\times5\times5\times4\times5\times20} = \frac{\overset{£}{425\times1}}{2704\times4\times20} = \frac{\overset{£}{425\times1}}{2704\times4\times4\times5} = \frac{\overset{£}{85\times1}}{2704\times4\times4} = \frac{\overset{D}{85\times240}}{2704\times4\times4} = \frac{\overset{D}{85\times60}}{2704\times4} = \frac{\overset{L}{85\times15}}{2704}\right.$

$= \overset{D}{\dfrac{85 \times 15}{2704}} = \overset{D}{\dfrac{1275}{2704}}) = \overset{D}{0.47}$; and consequently $\dfrac{51\,bb}{104\,a} + \dfrac{425\,b^3}{2704\,aa}$ will be $= \overset{S}{2},\ \overset{D}{5.46} + \overset{D}{0.47} = \overset{S}{2},\ \overset{D}{5.93}$, or lefs than $\overset{S}{2},\ \overset{D}{6}$. Confequently the excefs of the compound intereft of 100 pounds for a year arifing from thefe 52 weekly loans of it, at this rate of intereft, above 5 pounds, or the fimple intereft of the fame fum of money for the fame time, or the intereft arifing from a fingle loan of it for the whole year, is only $\overset{S}{2},\ \overset{D}{5.93}$, or lefs than $\overset{S}{2},\ \overset{D}{6}$.

Recapitulation of the conclufions obtained in the four preceeding articles.

CCXXXIII. Thus it appears that, when the intereft of money is 5 per cent. per annum, the excefs of the compound intereft of 100 pounds for a year, arifing from two fucceffive half-yearly loans of it at that intereft, above the fimple intereft of it, is only $\overset{S}{1},\ \overset{D}{3}$; and the excefs of the compound intereft above the fimple intereft when the loans of it are made for a quarter of a year at a time, is about $\overset{S}{1},\ \overset{D}{10\frac{1}{2}}$; and, when the faid loans are made for a month at a time, it is $\overset{S}{2},\ \overset{D}{3\frac{1}{2}}$; and, when they are made for only a week at a time, it is about $\overset{S}{2},\ \overset{D}{6}$. So that very little advantage arifes to the lender of a fum of money by lending it for terms lefs than a year, or by receiving the intereft of it at the end of every half-year, or quarter of year, or month, or week, inftead of lending it for a whole year at once, or receiving his intereft only at the end of the year.

Cafe of intereft paid at the end of every minute, or fecond, of time, or rather, of every infinitely fmall portion of time.

CCXXXIV. COROLL. 5. The *limit* of the magnitude to which the fum a will increafe in the courfe of a year by fuppofing the terms for which it is lent to be continually diminifhed, and the number of them to be increafed in the fame proportion, may be found by fuppofing the number m (which denotes the number of thofe terms contained in the whole year,) to become infinite.

Now, when m is infinite, the feveral fractions $\dfrac{m-1}{2}$, $\dfrac{m-2}{3}$, $\dfrac{m-3}{4}$, $\dfrac{m-4}{5}$, $\dfrac{m-5}{6}$, &c. will become equal to $\dfrac{m}{2}$, $\dfrac{m}{3}$, $\dfrac{m}{4}$, $\dfrac{m}{5}$, &c. be-

caufe

cause the subtracted numbers 1, 2, 3, 4, 5, &c. in the several numerators of those fractions will be infinitely smaller than the number m, from which they are subtracted. Therefore $\overline{a+\frac{b}{m}}\big|^{m}$, (which, by Sir Isaac Newton's binomial theorem, is in all cases $= a^{m} + \frac{m}{1}\, a^{m-1}\frac{b}{m} + \frac{m}{1} \times \frac{m-1}{2} \times a^{m-2} \times \frac{bb}{mm} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times a^{m-3} \times \frac{b^{3}}{m^{3}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times a^{m-4} \times \frac{b^{4}}{m^{4}} + \frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4} \times \frac{m-4}{5} \times a^{m-5} \times \frac{b^{5}}{m^{5}}$ + &c. continued to the $m+1^{\text{th}}$ term,) will in this case be $= a^{m} + \frac{m}{1} \times a^{m-1} \times \frac{b}{m} + \frac{m}{1} \times \frac{m}{2} \times a^{m-2} \times \frac{bb}{mm} + \frac{m}{1} \times \frac{m}{2} \times \frac{m}{3} \times a^{m-3} \times \frac{b^{3}}{m^{3}} + \frac{m}{1} \times \frac{m}{2} \times \frac{m}{3} \times \frac{m}{4} \times a^{m-4} \times \frac{b^{4}}{m^{4}} + \frac{m}{1} \times \frac{m}{2} \times \frac{m}{3} \times \frac{m}{4} \times \frac{m}{5} \times a^{m-5} \times \frac{b^{5}}{m^{5}}$ + &c. *ad infinitum* $= a^{m} + a^{m-1}b + \frac{a^{m-2}bb}{1.2} + \frac{a^{m-3}b^{3}}{1.2.3} + \frac{a^{m-4}b^{4}}{1.2.3.4.} + \frac{a^{m-5}b^{5}}{1.2.3.4.5.}$ + &c. *ad infinitum*; and consequently $\dfrac{\overline{a+\frac{b}{m}}\big|^{m}}{a^{m-1}}$ will in this case be $= a + b + \frac{bb}{1.2.a} + \frac{b^{3}}{1.2.3.aa} + \frac{b^{4}}{1.2.3.4.a^{3}} + \frac{b^{5}}{1.2.3.4.5.a^{4}}$ + &c. *ad infinitum*. Therefore the limit of the quantity to which the original sum a may be made to increase in the space of a year by continually diminishing the magnitude, and increasing the number, of the parts into which the year is divided, or the periods for which the money is to be lent, is the said series $a + b + \frac{bb}{1.2.a} + \frac{b^{3}}{1.2.3.aa} + \frac{b^{4}}{1.2.3.4.a^{3}} + \frac{b^{5}}{1.2.3.4.5.a^{4}}$ + &c. *ad infinitum*, or $a + b + \frac{bb}{2a} + \frac{b^{3}}{2.3.aa} + \frac{b^{4}}{2.3.4.a^{3}} + \frac{b^{5}}{2.3.4.5.a^{4}}$ + &c. *ad infinitum*;

and

and the exceſs of the ſaid limit above $a+b$, or the quantity to which the ſaid ſum a would have increaſed in the ſame time by a ſingle loan of it at the intereſt b for the whole year, is = the ſeries

$$\frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5a^4} + \&c. \textit{ ad infinitum.}\quad \text{Q.E.I.}$$

Now let a (as before) be $= \overset{\pounds}{100}$, and b be $= \overset{\pounds}{5}$. Then will $\frac{bb}{2a}$ be $\left(= \frac{b\times b}{2a} = \overset{\pounds}{5} \times \frac{5}{2\times 100} = \overset{\pounds}{5} \times \frac{1}{2} \times \frac{1}{20} = \overset{\pounds}{5} \times \frac{1}{40} = \frac{\overset{S}{100}}{40} = \frac{\overset{D}{1200}}{40}\right) = \overset{D}{30} = \overset{S}{2},\ \overset{D}{6}$; and $\frac{b^3}{2.3.aa}$ will be $\left(= \frac{bb}{2a} \times \frac{b}{3a} = \frac{bb}{2a} \times \frac{1}{3\times 20} = \frac{bb}{2a} \times \frac{1}{60} = \overset{D}{30} \times \frac{1}{60}\right) = \overset{D}{\frac{1}{2}}$; and $\frac{b^4}{2.3.4.a^3}$ will be $\left(= \frac{b^3}{2.3.aa} \times \frac{b}{4a} = \frac{b^3}{2.3.aa} \times \frac{1}{4\times 20} = \frac{b^3}{2.3.aa} \times \frac{1}{80} = \overset{D}{\frac{1}{2}} \times \frac{1}{80}\right) = \overset{D}{\frac{1}{160}}$; and $\frac{b^5}{2.3.4.5.a^4}$ will be $\left(= \frac{b^4}{2.3.4.a^3} \times \frac{b}{5a} = \frac{b^4}{2.3.4.a^3} \times \frac{1}{5\times 20} = \frac{b^4}{2.3.4.a^3} \times \frac{1}{100} = \overset{D}{\frac{1}{160}} \times \frac{1}{100}\right) = \overset{D}{\frac{1}{16000}}$; and conſequently the ſeries $\frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c.$ will be $\left(= \overset{D}{30} + \overset{D}{\frac{1}{2}} + \overset{D}{\frac{1}{160}} + \overset{D}{\frac{1}{16000}} + \&c. = \overset{D}{30} + \overset{D}{0.500,000} + \overset{D}{0.006,250} + 0.000,062 + \&c. = \overset{D}{30.506,312}\right) = \overset{S}{2},\ \overset{D}{6.506,312}$, or $\overset{S}{2},\ \overset{D}{6\tfrac{1}{2}} + \overset{D}{0.006,312}$, or $\overset{S}{2},\ \overset{D}{6\tfrac{1}{2}}$ together with $\frac{6312}{1000,000}$ parts, or one 158th part, of a penny. Therefore the limit of the exceſs of the ſum to which it is poſſible to increaſe the ſum of 100 pounds in the ſpace of a year by lending it at the intereſt of 5 per cent. per annum for very ſmall portions of a year, above 105 pounds, or the ſum to which it will increaſe in in the ſame time by a ſingle loan of it for the whole year at the ſame rate of intereſt, is only two ſhillings, and ſixpence, half-penny, together with the 158^{th} part of a penny. Q.E.I.

From this and the preceeding articles we may conclude that the advantages that may be made by lending money for very ſmall portions of a year, inſtead of lending it for a year, or half a year, at a time, are ſo very ſmall as not to be worth attending to.

A remarkable analogy between the foregoing limit of the increaſe of a ſum of money at compound intereſt, in the courſe of a year, and a certain ordinate of a logarithmick curve.

CCXXXV. Let the ſum a be repreſented by the ſubtangent of any logarithmick curve; and let an ordinate to the axis of the ſaid curve be drawn that ſhall be equal to the ſubtangent of it; which, from the nature of the ſaid curve, (which admits of ordinates of all poſſible magnitudes,) is always poſſible: and from the bottom of the ſaid ordinate, (or the point in which it touches the axis of the curve,) and on that ſide of the curve on which the ordinates increaſe, let a portion of the ſaid axis be taken that ſhall bear the ſame proportion to the ſaid ordinate, (or to the ſubtangent of the curve,) as b, (the intereſt of the ſum a for a year, when lent at once for the whole year,) bears to the ſaid ſum a itſelf; and at the extremity of the ſaid portion, or abſciſs, of the axis, let another ordinate be erected; which, it is evident, will be greater than the former ordinate, or than the ſubtangent. Then will the ſaid ſecond, or greater, ordinate repreſent, or be proportional to, the ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c.$ *ad infinitum*; or the ſaid firſt, or leſſer ordinate, (which is equal to the ſubtangent,) the ſaid abſciſs of the axis intercepted between the two ordinates, and the ſaid greater ordinate, will bear to each other the ſame proportions, reſpectively, as the original ſum a, the ſum b (which is the intereſt of a for one year, when lent out at once for the whole year,) and the ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c.$ *ad infinitum*, or the *limit* of the quantity to which the ſum a may be made to increaſe in the ſame ſpace of one year by lending it out at compound intereſt for very ſhort terms.

For

For it is ſhewn in my Elements of Plane Trigonometry, Art. 354, page 470, that, if a be the ſubtangent of a logarithmick curve, and a and $a + x$ be two ordinates to the axis of the ſaid curve, and z the abſciſs of the axis intercepted between the ſaid ordinates, the difference, x, of the ſaid ordinates will be equal to the ſeries $z + \frac{zz}{2a} + \frac{z^3}{2.3.aa} + \frac{z^4}{2.3.4.a^3} + \frac{z^5}{2.3.4.5.a^4} +$ &c. *ad infinitum.* Conſequently $a + x$, or the greater of the ſaid two ordinates, will be equal to the ſeries $a + z + \frac{zz}{2a} + \frac{z^3}{2.3.aa} + \frac{z^4}{2.3.4.a^3} + \frac{z^5}{2.3.4.5.a^4} +$ &c. *ad infinitum.* Therefore, if we denote the ſaid abſciſs of the axis by the letter b inſtead of the letter z, we ſhall have $a + x =$ the ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum.* Q.E.D.

N. B. This analogy between the foregoing limit of the increaſe of the ſum a in the courſe of a year, by being lent out for ſhort periods at compound intereſt, and the ordinate of a logarithmick curve, has ſomething in it that is pleaſing and amuſing to the imagination: and therefore I thought the reader might be glad to ſee it ſtated in this place, though I do not know of any practical uſe in the doctrine of annuities to which it can be applied. See upon this ſubject Dr. Keil's tract on Logarithms, at the end of Commandine's edition of Euclid's Elements, printed at Oxford in the year 1747, page 71.

Of the foregoing infinite ſeries, obtained for the limit of the increaſe of the ſum a *at compound intereſt, when* b *denotes the ſimple intereſt of it for a greater, or leſſer, term than one year.*

CCXXXVI. In Art. 227 and all the following articles we have ſuppoſed b to be the intereſt of the ſum a for a ſingle year, becauſe that is the moſt common way of eſtimating, or denominating, the intereſt of money. But the reaſonings uſed in thoſe articles will be equally juſt, and the concluſions obtained in them will be equally true, if b be made to denote the intereſt of a for any other given time, either greater or leſs than a year; as, for example, if b be made to denote the intereſt of a for

for fifteen or twenty years. And therefore it will always be true, (whatever be the length of the term for which the money is lent, or whatever be the magnitude of the interest b,) that the limit of the quantity to which the original sum a may be made to increase, (in the same time in which the interest b becomes due,) by lending it at compound interest for a great number of very short terms, is equal to the foregoing series $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*.

The foregoing series is true in all cases, whatever be the proportion of the interest b to the original sum a.

One of these cases seems more curious than the rest, namely, that in which the interest b, which is to be paid for the original sum a at the end of the term for which it is lent, is equal to the said sum a itself; as, for example, if the sum a were to be lent for a term of 20 years, and were then to be repaid with an interest equal to itself; which, it is evident, would be by no means a hard or unreasonable agreement. Now in this case the foregoing series $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum* will become equal to $a + a + \frac{aa}{2a} + \frac{a^3}{2.3.aa} + \frac{a^4}{2.3.4.a^3} + \frac{a^5}{2.3.4.5.a^4} +$ &c. *ad infinitum* $= a + a + \frac{a}{2} + \frac{a}{2.3} + \frac{a}{2.3.4} + \frac{a}{2.3.4.5} +$ &c. *ad infinitum* $= a \times$ the series $1 + 1 + \frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5} +$ &c. *ad infinitum* $= a \times 2.718,281,828,459,036$, &c.

Case of the equality of b to a.

Therefore, if the interest due for the sum a for any given time were equal to the said sum itself, instead of being only a 20th, or 25th, part of it, (as is most commonly the case, when money is lent only for a year,) the limit of the quantity to which the said sum might be made to increase in the same space of time by improving it at compound interest by means of very frequent payments of the interest and immediate renewals of the loan of both principal and interest, would be equal to 2.718, 281, 828, 459, 036, &c. times, or less than three times, the original sum a.

The value of the said series in that case.

CCXXXVII. The computation of the terms of the series $1 + 1 + \frac{1}{2} + \frac{1}{2.3} \frac{1}{2.3.4} + \frac{1}{2.3.4.5} +$ &c. so far as to obtain the foregoing number 2.718,281,828,459,036, may be performed as follows.

A computation of the value of the said series in that case to sixteen places of figures.

Let the capital letters A, B, C, D, E, F, G, H, I, K, L, M, &c. be put for the ſeveral terms of the ſaid ſeries, as they ariſe; ſo that A ſhall be $= 1$, and B ſhall be $= 1$, and C ſhall be $= \frac{1}{2}$, and D ſhall be $= \frac{1}{2.3}$, and E ſhall be $= \frac{1}{2.3.4}$, and F ſhall be $= \frac{1}{2.3.4.5}$, and ſo on of the following terms. Then, from the law of the continuation of the terms of this ſeries, it is evident that D will be $= \frac{C}{3}$, and E will be $= \frac{D}{4}$, and F will be $= \frac{E}{5}$, and G will be $= \frac{F}{6}$, and H will be $= \frac{G}{7}$, and I will be $= \frac{H}{8}$, and K will be $= \frac{I}{9}$, and L will be $= \frac{K}{10}$, and M will be $= \frac{L}{11}$, and N will be $= \frac{M}{12}$, and O will be $= \frac{N}{13}$, and P will be $= \frac{O}{14}$, and Q will be $= \frac{P}{15}$, and R will be $= \frac{Q}{16}$, and S will be $= \frac{R}{17}$. We ſhall therefore have

$A = 1 = 1.000,000,000,000,000$;

and $B = 1 = 1.000,000,000,000,000$;

and $C = \frac{1}{2} = .500,000,000,000,000$;

and $D = \frac{C}{3} = .166,666,666,666,666$;

and $E = \frac{D}{4} = .041,666,666,666,666$;

and $F = \frac{E}{5} = .008,333,333,333,333$;

and $G = \frac{F}{6} = .001,388,888,888,888$;

and $H = \frac{G}{7} = .000,198,412,698,412$;

and

$$\text{and } I = \frac{H}{8} = .000,024,801,587,301;$$

$$\text{and } K = \frac{I}{9} = .000,002,755,731,922;$$

$$\text{and } L = \frac{K}{10} = .000,000,275,573,192;$$

$$\text{and } M = \frac{L}{11} = .000,000,025,052,108;$$

$$\text{and } N = \frac{M}{12} = .000,000,002,087,675;$$

$$\text{and } O = \frac{N}{13} = .000,000,000,160,590;$$

$$\text{and } P = \frac{O}{14} = .000,000,000,011,470;$$

$$\text{and } Q = \frac{P}{15} = .000,000,000,000,764;$$

$$\text{and } R = \frac{Q}{16} = .000,000,000,000,047;$$

$$\text{and } S = \frac{R}{17} = .000,000,000,000,002.$$

2.718,281,828,459,036.

N. B. The proportion of this number, 2.718,281,828,459,036, &c. to 1, is what Mr. Cotes calls the *ratio modularis*, being the ratio, or proportion, of two ordinates to the axis of a logarithmick curve drawn through the extremities of an abſciſs of the axis equal to the *ſubtangent* of the curve, or that ratio of which the ſubtangent of a logarithmick curve, and, in general, in any other ſyſtem of the meaſures of ratios, the *modulus* of the ſyſtem, is the meaſure.

Of the value of the foregoing ſeries when the intereſt b is greater than the original ſum a.

CCXXXVIII. If the ſum a be ſuppoſed to be lent for ſo long a term of years before its intereſt b is to be paid, that the ſaid intereſt ſhall be greater than the principal, it is evident that the literal parts of the terms of the foregoing ſeries $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4}$

$+ \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*, to wit, a, b, $\frac{bb}{a}$, $\frac{b^3}{aa}$, $\frac{b^4}{a^3}$, $\frac{b^5}{a^4}$, &c. will continually increase. And hence it may seem, perhaps, at first sight, that the terms of the series will in these cases continually diverge, and that consequently their sum, or the series itself, will become infinitely great, and therefore will not exhibit truely the value of the aforesaid limit. But, if we consider the matter with a little attention, we shall perceive that the series must in all cases be a converging one, whatever be the proportion in which the interest b may exceed the original sum a, though the convergency will not, in these cases, begin from the first term a, as it does when the interest b is less than a. For, as, upon this supposition, the literal parts of the terms, to wit, a, b, $\frac{bb}{a}$, $\frac{b^3}{aa}$, $\frac{b^4}{a^3}$, $\frac{b^5}{a^4}$, &c. will diverge, or increase, continually in the same given proportion of b to a, and the numeral parts of the third, fourth, fifth, sixth, and other following terms, to wit, $\frac{1}{2}$, $\frac{1}{2.3}$, $\frac{1}{2.3.4}$, $\frac{1}{2.3.4.5}$, &c. do, in all cases, decrease in proportions which are continually and indefinitely increasing, to wit, in the proportions of 2 to 1, 3 to 1, 4 to 1, and so on, it is evident that the proportion of the numeral parts of two contiguous terms in the series (in which the latter of the said numeral parts will be less than the former of them,) must always, in some part or other of the series, become greater than the proportion of the literal parts of the same terms, (in which the latter of the said literal parts will be greater than the former of them,) and consequently that the latter of the said two contiguous terms will, upon the whole, be less than the former of them. And after the terms in which this happens all the following terms of the series will continually decrease in a greater and greater proportion. Thus, for example, if b is $= 10a$, the first ten terms of the series $a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} +$ &c. *ad infinitum*, or (if we denote the numeral parts of these terms, to wit, 1, 1, $\frac{1}{2}$, $\frac{1}{2.3}$, $\frac{1}{2.3.4}$, $\frac{1}{2.3.4.5}$, &c. by the capital letters A, B, C, D, E, F, &c. as in Art. 237,) of the series $Aa + Bb + \frac{Cbb}{a} + \frac{Db^3}{aa} + \frac{Eb^4}{a^3} + \frac{Fb^5}{a^4}$ + &c.

The said series will in this case diverge in its first terms, but afterwards converge in all the remaining terms.

$+$ &c. *ad infinitum*, or of the series $Aa + Bb + \frac{Bbb}{2a} + \frac{Cb^3}{3aa} + \frac{Db^4}{4a^3} + \frac{Eb^5}{5a^4} + \frac{Fb^6}{6a^5} + \frac{Gb^7}{7a^6} + \frac{Hb^8}{8a^7} + \frac{Ib^9}{9a^8} + \frac{Kb^{10}}{10a^9} + \frac{Lb^{11}}{11a^{10}} + \frac{Mb^{12}}{12a^{11}} + \frac{Nb^{13}}{13a^{12}} +$ &c. will diverge, or increase; because the proportion of b to a, or of 10 to 1, (in which the literal part of every term exceeds the literal part of the term that immediately preceeds it,) is greater than the proportions of 2 to 1, 3 to 1, 4 to 1, 5 to 1, 6 to 1, 7 to 1, 8 to 1, and 9 to 1, in which the numeral parts of third, fourth, fifth, sixth, seventh, eighth, ninth, and tenth terms are exceeded by the numeral parts of the terms that immediately preceed them: but the eleventh term $\frac{Kb^{10}}{10a^9}$ will not be greater than the tenth term $\frac{Ib^9}{9a^8}$, but exactly equal to it, being equal to $\left(K \times \frac{b^{10}}{10a^9} = \frac{I}{9} \times \frac{b^{10}}{10a^9} = \frac{I}{9} \times \frac{b^9}{a^8} \times \frac{b}{10a} = \frac{Ib^9}{9a^8} \times \frac{b}{10a} = \frac{Ib^9}{9a^8} \times \frac{1}{10} \times \frac{b}{a} = \frac{Ib^9}{9a^8} \times \frac{1}{10} \times \frac{10}{1} =\right) \frac{Ib^9}{9a^8} \times \frac{10}{10}$, which is $= \frac{Ib^9}{9a^8}$: and the twelfth term $\frac{Lb^{11}}{11a^{10}}$ will be less than the eleventh term $\frac{Kb^{10}}{10a^9}$, being $\left(= L \times \frac{b^{11}}{11a^{10}} = \frac{K}{10} \times \frac{b^{11}}{11a^{10}} = \frac{K}{10} \times \frac{b^{10}}{a^9} \times \frac{b}{11a} = \frac{Kb^{10}}{10a^9} \times \frac{b}{11a} = \frac{Kb^{10}}{10a^9} \times \frac{1}{11} \times \frac{b}{a} = \frac{Kb^{10}}{10a^9} \times \frac{1}{11} \times \frac{10}{1}\right) = \frac{Kb^{10}}{10a^9} \times \frac{10}{11}$, which is less than $\frac{Kb^{10}}{10a^9}$ in the proportion of 10 to 11. And in like manner it will be found, that the thirteenth term $\frac{Mb^{12}}{12a^{11}}$ is less than the twelfth term $\frac{Lb^{11}}{11a^{10}}$ in the proportion of 10 to 12, and that the fourteenth term $\frac{Nb^{13}}{13a^{12}}$ is less than the thirteenth term $\frac{Mb^{12}}{12a^{11}}$ in the proportion of 10 to 13, and that every following term is less than the term that imme-

diately

diately preceeds it in the increafing proportions of 10 to 14, 10 to 15, 10 to 16, and 10 to every following higher number whatfoever. So that from the eleventh term $\frac{Kb^{10}}{10a^9}$ the foregoing feries will be a converging feries. And the fame convergency muft take place, (though farther from the beginning of the feries,) if we fhould fuppofe the intereft b to exceed the original fum a in the proportion of ten thoufand to one, or any other and greater proportion, how great foever, inftead of exceeding it only in the proportion of 10 to 1. We may therefore conclude that the faid feries

$$a + b + \frac{bb}{2a} + \frac{b^3}{2.3.aa} + \frac{b^4}{2.3.4.a^3} + \frac{b^5}{2.3.4.5.a^4} + \&c. \textit{ ad infinitum}$$

will always exhibit the true value of the aforefaid limit of the quantity to which the original fum a may be made to increafe by compound intereft, in the fame time as at fimple intereft it increafes to $a + b$, whatever be the proportion of the intereft b to the faid original fum a.

But it is now time to return from this digreffion, concerning the increafe of money at compound intereft, to the confideration of the values of annuities depending upon lives, which is the principal fubject of this tract.

[*End of the inquiry concerning the increafe of a fum of money by means of compound intereft.*]

Concerning remote life-annuities, that are to commence at the diftance of 30 years.

CCXXXIX. Now the next thing I propofe to enter upon, concerning this fubject of Life-annuities, is the computation of the values of certain remote life-annuities, which feem to deferve particular confideration: I mean life annuities that are to commence at the diftance of 30 years, or whereof the firft payments are to be made at the end of 31 years;—I fay, at the end of 31 years, and not at the end of 30 years and a half, becaufe I fhall now return to the fuppofition that has been made throughout all the former part of this book, as far as Art. 200, "that the payments of annuities are to be made only at the end of every year," and fhall not trouble either my reader or myfelf any more with the perplexing, but not very important, confideration of the difference between the values of annuities paid yearly, and annuities paid half-yearly. Now thefe particular remote life-annuities, which are to take place at the diftance of 30 years, feem to me to be more interefting than

than any others (as, for inſtance, than life-annuities that are to commence at the diſtance of 20, or 25, or 35, or 40, years,) becauſe I conceive that it would be a very uſeful and convenient meaſure, both for the Publick, and the individuals whom it would concern, if the Parliament were to eſtabliſh ſuch annuities, which the people ſhould be at liberty to purchaſe at their full and proper values according to the ſeveral ages of the purchaſers. For, as the Parliament has, within theſe few years paſt, thought fit to eſtabliſh annuities for a term of 30 years certain, it ſeems reaſonable to ſuppoſe that it would be a great ſatiſfaction to the younger part of the proprietors of thoſe annuities to be able, for a moderate ſum of money, (ſuch as about two years annuity,) to purchaſe an additional intereſt in them for their own lives, and thereby to rid themſelves of the uneaſy apprehenſion, that, without living to an uncommonly great old age, they might poſſibly outlive the income that ſupport them. If this objection to theſe annuities for 30 years were removed, by eſtabliſhing ſuch additional life-annuities as have been juſt now mentioned, it would probably induce more people to become purchaſers of them, and conſequently would tend to enhance their value, and enable the Government to get a better price for them. And thus both the Government and the individuals who ſhould purchaſe theſe 30 years annuities, or any other annuities for the ſame term that may be hereafter created, would find their account in the eſtabliſhment of the ſaid additional remote life-annuities.

It would probably be an uſeful publick meaſure for the Parliament to eſtabliſh ſuch remote life-annuities.

Nor would the eſtabliſhment of ſuch remote life-annuities retard, in any *conſiderable* degree, the diminution of the national debt ariſing from the expiration of the annuities for 30 years; becauſe it is almoſt certain that much the greater part of the proprietors of thoſe annuities will be dead before the expiration of the ſaid term of 30 years; and of the few that will live beyond it, and who conſequently might, if they had become purchaſers of theſe additional life-annuities, continue to enjoy their reſpective annuities beyond it, much the greater part will die in the courſe of the next ten or fifteen years: and it muſt alſo be remembered that they will have paid to the Government a full and fair price for the ſaid prolongation of their ſaid annuities during the time they do enjoy them.

For theſe reaſons I am inclined to think that the meaſure of eſtabliſhing ſuch a ſet of remote life-annuities is not liable to any objection but that of the difficulty (as it may be ſuppoſed) of diſtinctly aſcertaining their ſeveral values according to the ſeveral ages of the perſons who ſhall purchaſe them. But this difficulty is a ſmall one, and may be eaſily removed by computing a few tables of thoſe values according to the moſt received and cuſtomary rates of intereſt at which money is at this time

time (July 1781,) borrowed by the Government, and will, probably, be borrowed by them when the blessing of peace shall be restored to us. And with this view I have procured the four following tables of the values of such remote life-annuities to be computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon suppositions that the interest of money is 5 per cent. $4\frac{1}{2}$ per cent. 4 per cent. and $3\frac{1}{2}$ per cent. It does not seem likely that Government should be able to borrow money at a lower interest than $3\frac{1}{2}$ per cent. for many years to come, even though peace should be speedily re-established.

Of the manner of computing the values of these remote annuities.

CCXL. The values of remote life-annuities may be either computed in the manner explained above in Problem II, Coroll. 3, Art. 36, page 33, without any reference to the values of immediate life-annuities for the lives of persons of the ages at which the remote annuities are to take place, or they may be derived from the values of those immediate life-annuities by an easy operation or two of multiplication and division. And, as the values of those immediate life-annuities have been already computed, and are contained in the foregoing tables, I conceive the latter method of obtaining the values of the said remote annuities to be upon the whole the most convenient. This method may be explained in the manner following.

A method of deriving them from the values of immediate life-annuities contained in the foregoing tables in Art. 201, pages 221, 222, &c.—232.

CCXLI. Let $\overset{\pounds}{R}$ be the value sought of an annuity of one pound a year, that is to commence at the end of m years, (or so that the first payment of it shall become due at the end of $m+1$ years,) in case a person who is now of the age of N years shall be then alive, and which is to continue during the whole remainder of the said person's life. And let $\overset{\pounds}{V}$ be the value of an immediate annuity of one pound a year for the life of a person of the age of $N+m$ years. Let P be the number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life, as being alive at the age of N years, and F the number of persons represented there as living at the age of $N+m$ years, and F^{I} the number living at the age of $N+m+1$ years, and F^{II} the number living at the age of $N+m+2$ years, and F^{III}, F^{IV}, F^{V}, F^{VI}, F^{VII}, &c. the numbers living at the ages of $N+m+3$ years, $N+m+4$ years, $N+m+5$ years, $N+m+6$ years,

years, $N+m+7$ years, &c. respectively. Then, by Art. 86, we shall have $\overset{\pounds}{V} = \frac{\overset{\pounds}{1}}{F} \times$ the series $\frac{F^{I}}{r} + \frac{F^{II}}{r^2} + \frac{F^{III}}{r^3} + \frac{F^{IV}}{r^4} + \frac{F^{V}}{r^5} + \frac{F^{VI}}{r^6} + \frac{F^{VII}}{r^7}$ + &c. continued to the end of the table of probabilities. And, because $F, F^{I}, F^{II}, F^{III}, F^{IV}, F^{V}, F^{VI}, F^{VII}$, &c. in this notation are respectively equal to $P^{m}, P^{m+I}, P^{m+II}, P^{m+III}, P^{m+IV}, P^{m+V}, P^{m+VI}, P^{m+VII}$, &c. in the notation used in Art. 90, page 99, we shall have $\overset{\pounds}{R}$ (which, by that article, is $= \frac{\overset{\pounds}{1}}{P} \times$ the series $\frac{P^{m+I}}{r^{m+1}} + \frac{P^{m+II}}{r^{m+2}} + \frac{P^{m+III}}{r^{m+3}} + \frac{P^{m+IV}}{r^{m+4}} + \frac{P^{m+V}}{r^{m+5}} + \frac{P^{m+VI}}{r^{m+6}} + \frac{P^{m+VII}}{r^{m+7}}$ + &c. continued to the end of the table of probabilities,) $= \frac{\overset{\pounds}{1}}{P} \times$ the series $\frac{F^{I}}{r^{m+1}} + \frac{F^{II}}{r^{m+2}} + \frac{F^{III}}{r^{m+3}} + \frac{F^{IV}}{r^{m+4}} + \frac{F^{V}}{r^{m+5}} + \frac{F^{VI}}{r^{m+6}} + \frac{F^{VII}}{r^{m+7}}$ + &c. continued to the end of the table of probabilities, $= \frac{\overset{\pounds}{1}}{P} \times \frac{1}{r^m} \times$ the series $\frac{F^{I}}{r} + \frac{F^{II}}{r^2} + \frac{F^{III}}{r^3} + \frac{F^{IV}}{r^4} + \frac{F^{V}}{r^5} + \frac{F^{VI}}{r^6} + \frac{F^{VII}}{r^7}$ + &c. continued to the end of the table of probabilities, $= \frac{\overset{\pounds}{1}}{P} \times \frac{1}{r^m} \times F \times \frac{\overset{\pounds}{1}}{F} \times$ the series $\frac{F^{I}}{r} + \frac{F^{II}}{r^2} + \frac{F^{III}}{r^3} + \frac{F^{IV}}{r^4} + \frac{F^{V}}{r^5} + \frac{F^{VI}}{r^6} + \frac{F^{VII}}{r^7}$ + &c. continued to the end of the table of probabilities, $= \frac{\overset{\pounds}{1}}{P} \times \frac{1}{r^m} \times F \times \overset{\pounds}{V} = \frac{F}{P} \times \frac{1}{r^m} \times \overset{\pounds}{V}$; that is, the value of the remote annuity of one pound, that is to take place at the end of m years, in case a person that is now of the age of N years shall be then alive, and which is to continue during the whole remainder of the said person's life, is equal

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to the product that arises by multiplying $\overset{\pounds}{V}$, (or the value of an immediate annuity of one pound a year for the life of a person of the age of $N+m$ years,) first into the fraction $\frac{1}{r^m}$, and then into the fraction $\frac{F}{P}$. Q. E. F.

CCXLII. Thus, for example, if it is required to find the value of a remote annuity of one pound a year for the life of a person of the age of 10 years, to commence at the distance of 30 years, or when the said annuitant shall be 40 years of age, or so that the first payment of it shall become due at the end of 31 years, or when the said annuitant shall be 41 years old, the interest of money being $3\frac{1}{2}$ per cent. we must proceed in the manner following.

In the first place we must look out in the foregoing tables, (Table XV, page 224,) the value, $\overset{\pounds}{V}$, of an immediate annuity of one pound a year for the life of a person of the age of ($N+m$, or $10+30$, or) 40 years; which we shall find to be $\overset{\pounds}{16.104,542}$.

Secondly, since the interest of money is supposed to be $3\frac{1}{2}$ per cent. we shall have $r = 1.035$, and $\frac{1}{r} = \frac{1}{1.035} = 0.966,183$, and $\frac{1}{r^m} = \frac{1}{r^{30}} = 0.356,278$; as appears by Mr. Smart's second table of compound interest, pages 60 and 61.

And, lastly, it appears from Monsieur de Parcieux's table of the probabilities of the duration of human life, that P, or the number of persons living at the age of (N, or) 10 years, out of an original number of 1000 living at the age of 3 years, is 880, and that F, or the number of persons living at the age of ($N+m$, or $10+30$, or) 40 years, out of the same original number, is 657.

Therefore $\frac{F}{P} \times \frac{1}{r^m} \times \overset{\pounds}{V}$, or $\frac{F}{P} \times \frac{1}{r^{30}} \times \overset{\pounds}{V}$, is $= \frac{657}{880} \times 0.356,278 \times \overset{\pounds}{16.104,542}$ $\left(= \frac{234.074,646}{880} \times \overset{\pounds}{16.104,542} = 0.265,993 \times \overset{\pounds}{16.104,542}\right)$

× (£16.104,542) = £4.283,695; or the present value of a remote annuity of one pound a year for the life of a person of the age of 10 years, that is to commence at the end of 30 years, or when the annuitant shall be 40 years of age, or so that the first payment of it shall become due at the end of 31 years, or when the said annuitant shall be 41 years old, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent. is £4.283,695, or 4*l.* 5*s.* 8*d.* or somewhat more than four years and a quarter's annuity. Q. E. I.

And in this manner all the values of remote life-annuities contained in the four following tables have been computed.

TABLE XXIV.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years inclusively, that is to commence only at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

Years of Age.	*Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.*	*Years of Age.*	*Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.*
	£		£
3	4.444,037	16	3.748,438
4	4.483,991	17	3.653,162
5	4.487,811	18	3.555,256
6	4.471,977	19	3.455,031
7	4.440,391	20	3.352,405
8	4.397,007	21	3.251,937
9	4.346,451	22	3.149,313
10	4.283,695	23	3.044,957
11	4.208,738	24	2.938,658
12	4.121,688	25	2.830,830
13	4.032,235	26	2.721,424
14	3.940,261	27	2.610,440
15	3.845,704	28	2.498,339

Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
	£		£
29	2.385,100	47	0.453,064
30	2.270,749	48	0.383,586
31	2.155,438	49	0.320,944
32	2.038,874	50	0.264,962
33	1.921,611	51	0.216,020
34	1.803,961	52	0.173,893
35	1.685,836	53	0.137,512
36	1.567,839	54	0.106,163
37	1.450,584	55	0.079,874
38	1.332,772	56	0.058,249
39	1.217,431	57	0.041,158
40	1.105,337	58	0.027,702
41	0.996,861	59	0.017,798
42	0.892,829	60	0.010,135
43	0.793,643	61	0.005,251
44	0.699,717	62	0.002,352
45	0.611,500	63	0.000,813
46	0.528,105	64	0.000,000

TABLE XXV.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years inclusively, that is to commence only at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4 per cent.

Years of Age.	Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
	£		£		£
3	3.587,85	24	2.427,34	45	0.517,24
4	3.623,63	25	2.341,00	46	0.447,80
5	3.630,32	26	2.253,22	47	0.383,98
6	3.621,19	27	2.163,84	48	0.325,40
7	3.599,31	28	2.073,38	49	0.272,51
8	3.567,87	29	1.981,77	50	0.225,18
9	3.530,60	30	1.889,02	51	0.183,76
10	3.483,36	31	1.795,12	52	0.148,06
11	3.426,10	32	1,700,08	53	0.117,20
12	3.358,91	33	1.604,31	54	0.090,57
13	3.289,62	34	1.507,87	55	0.068,20
14	3.218,16	35	1.410,77	56	0.049,79
15	3.144,42	36	1.313,53	57	0.035,21
16	3.068,30	37	1.216,66	58	0.023,72
17	2.993,65	38	1.119,08	59	0.014,98
18	2.916,67	39	1.023,35	60	0.008,69
19	2.837,63	40	0.930,14	61	0.004,51
20	2.750,45	41	0.839,74	62	0.002,009
21	2.676,12	42	0.752,89	63	0.000,700
22	2.595,36	43	0.669,94	64	0.000,000
23	2.512,24	44	0.591,26		

TABLE XXVI.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years, inclusively, that is to commence only at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4½ per cent.

Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
	£		£		£
3	2.907,57	24	2.009,15	45	0.437,99
4	2.939,17	25	1.939,82	46	0.379,55
5	2.947,25	26	1.869,12	47	0.325,76
6	2.942,54	27	1.797,02	48	0.276,32
7	2.927,52	28	1.723,83	49	0.231,62
8	2.904,73	29	1.649,54	50	0.191,56
9	2.877,17	30	1.574,14	51	0.156,46
10	2.841,47	31	1.497,61	52	0.126,19
11	2.797,56	32	1.419,93	53	0.099,98
12	2.745,46	33	1.341,48	54	0.077,33
13	2.691,56	34	1.262,26	55	0.058,29
14	2.635,79	35	1.182,30	56	0.042,60
15	2.578,05	36	1.102,01	57	0.030,15
16	2.518,24	37	1.021,84	58	0.020,34
17	2.459,53	38	0.940,89	59	0.012,86
18	2.398,78	39	0.861,31	60	0.007,47
19	2.336,23	40	0.783,67	61	0.003,88
20	2.271,78	41	0.708,23	62	0.001,73
21	2.208,42	42	0.635,62	63	0.000,604
22	2.143,56	43	0.566,16	64	0.000,000
23	2.077,16	44	0.500,17		

TABLE XXVII.

Containing the values of a remote annuity of one pound a year for the lives of persons of the several ages of 3 years, 4 years, 5 years, and every following number of years up to 63 years, inclusively, that is to commence at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, after the time of purchasing it: computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is 5 per cent.

Years of Age.	Values of a remote life annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.	Years of Age.	Values of a remote life-annuity of one pound a year, that is to commence at the end of 30 years.
	£		£		£
3	2.364,81	24	1.666,37	45	0.371,26
4	2.393,50	25	1.610,16	46	0.322,03
5	2.400,38	26	1.553,46	47	0.276,64
6	2.399,10	27	1.495,12	48	0.234,87
7	2.388,90	28	1.435,76	49	0.197,04
8	2.372,38	29	1.375,37	50	0.163,10
9	2.351,96	30	1.313,93	51	0.133,33
10	2.324,88	31	1.251,42	52	0.107,63
11	2.291,05	32	1.187,80	53	0.085,35
12	2.250,48	33	1.123,40	54	0.066,08
13	2.208,88	34	1.058,21	55	0.049,85
14	2.164,67	35	0.992,23	56	0.036,46
15	2.119,27	36	0.925,83	57	0.025,83
16	2.072,10	37	0.859,37	58	0.017,43
17	2.025,74	38	0.792,10	59	0.011,03
18	1.977,62	39	0.725,83	60	0.006,410
19	1.927,94	40	0.661,07	61	0.003,326
20	1.876,60	41	0.598,01	62	0.001,488
21	1.826,07	42	0.536,84	63	0.000.520
22	1.774,23	43	0.478,99	64	0.000,000
23	1.721,01	44	0.423,57		

OBSER-

OBSERVATIONS

ON THE

NATIONAL DEBT,

AND

The most likely Methods of paying off a Part of it.

ART. CCXLIII.

WHEN the blessing of peace shall be restored to us (which, in our present friendless and declining condition, we ought surely to wish for upon almost any terms;) it is to be hoped that our rulers will set themselves seriously to work to reduce to a more moderate quantity the immense load of debt under which the nation now labours, and which will then be at least two hundred and twenty millions of pounds sterling, even if a peace should be made as soon as possible. For at this present time, November, 1781, it amounts to more than a hundred and ninety-eight millions, two hundred thousand pounds; or at least it will do so on the first of next January, 1782; as will appear from an accurate statement of it which has been printed in the Publick Advertiser of the second of October last, and which the reader may see in the note below.* And to

* An account of the taxes laid on since the beginning of the American War.

Computed produce per annum.

1776.	£	
Stamps on deeds,	30,000	
Ditto on news-papers,	18,000	
Ditto on cards,	6,000	
Additional duty on coaches, &c.	19,000	
		73,000
1777.		
Tax on servants,	105,000	
Stamps,	55,000	
Additional duty on glass,	45,000	
Duty on sales by auction,	37,000	
		242,000

Tax

to this ſum we muſt add at leaſt 25 millions more for the expence attending the concluſion of the war, before every thing ſhall be compleatly ſettled on a peace eſtabliſhment: ſo that at the final re-eſtabliſhment of peace the whole debt will be at leaſt two hundred and twenty millions. Now, when the important buſineſs of reducing this enormous debt to a leſs ſum ſhall be undertaken, it ſeems probable that one, or more, of the following methods will be adopted for that purpoſe.

The

1778.		
Tax upon houſe rents,	£.264,000	
Additional duty on wines,	72,000	
		336,000
1779.		
A tax upon taxes, viz. an additional ſurcharge of 5 per cent. on cuſtoms and exciſe,	314,000	
A tax upon poſt horſes,	164,000	
		478,000
1780.		
An additional tax upon malt,	310,000	
Additional duty on Britiſh low wines,	20,617	
Ditto on Britiſh ſpirits,	34,557	
Ditto on brandy,	35,310	
Ditto on rum,	70,958	
Second additional duty on wines,	72,000	
Additional duty on coals exported,	12,899	
Additional 5 per cent. on the above laid taxes,	46,193	
Additional duty upon ſalt,	69,000	
Additional ſtamp duties,	21,000	
Duty on licences to ſell tea,	9,082	
		701,616
1781.		
Five per cent. on exciſe, except malt, ſoap, candles, and hides,	150,000	
Diſcount of the cuſtoms,	167,000	
Tobacco one penny three farthings per pound,	61,000	
Sugar halfpenny per pound,	326,000	
Since laid,		704,000
Duty on paper,	100,000	
Ditto on almanacks,	10,000	
		110,000
		2,644,616

The exact national debt up to July 5, 1781, is £.177,206,000.
The annual intereſt raiſed on the publick is £.6,889,000.

So far the funded debt, and the taxes laid, in order to diſcharge the intereſt to the publick creditors. The debt unfunded may be computed as here under:

Navy

The first method of employing a given sum of money every year in the reduction of the national debt.

The firſt method is to employ the annual Surplus of the Publick Revenue, which can be ſpared for this good work, in repaying to ſome of the national creditors a part of the capital which is due to them, and alſo to employ the annual ſavings of the intereſt upon the capital ſo diſcharged (which, it is evident, will increaſe every year,) in the ſame manner, that is, in diſcharging, or repaying, ſome more of the ſaid capital due to the publick creditors. Thus, for example, if the ſum that can be ſpared out of the publick revenue for this purpoſe ſhould be a million of pounds ſterling a year, and ſome of the publick funds, as, for example, the 4 per cent. annuities, ſhould riſe to par, or above par, ſo that 100 pounds in thoſe annuities ſhall ſell for a hundred pounds, or more, in hard money, the ſaid ſurplus million of the publick revenue may be employed

Navy debt on the 1ſt of January, 1782, about —	£.9,000,000	
Army extraordinaries, — — —	3,000,000	
Vote of credit of laſt ſeſſion, — — —	1,000,000	
Ordnance debt, — — —	1,000,000	
Money to be voted for navy extras, — —	1,000,000	
Exchequer bills in circulation, about — —	4,000,000	
Borrowed from the Bank of England, — — —	2,000,000	
	———	21,000,000
Suppoſe, when this ſum comes to be funded, that the loan or bargain with the publick may, as it has for the two or three laſt years, be negociated at 5 and a half per cent. the annual intereſt to be paid on 21 millions will be	1,155,000	

RECAPITULATION.

Principal funded on the 5th of July, 1781, — —	177,206,000	
Principal which will remain unfunded on the 1ſt of January, 1782, — — —	21,000,000	
Total of the national debt on the ſaid laſt mentioned day,	———	198,206,000
Intereſt paid, for which proviſions have been made by taxes, 5th July, 1781, — — —	6,889,000	
Intereſt to be paid for the debts not yet funded, which will ſtand due on the 1ſt of January, 1782, — —	1,155,000	
	———	8,044,000

So that on the 1ſt of January, 1782, the national debt, funded and unfunded, will amount to *one hundred ninety-eight millions*, and a conſiderable fraction, and the intereſt to *eight millions*, which is nearly double to what was paid by the people, in taxes, previous to the breaking out of the preſent war, the annual intereſt, on the 1ſt of January, 1776, being in or about *four millions three hundred thouſand pounds*.

in

in paying off a million of the capital of the ſaid annuities: and thus the national debt will be diminiſhed by the ſum of one million by this firſt payment. But by this firſt diminution of the debt it is evident the publick will have leſs intereſt to pay to the publick creditors at the beginning of the next year than it had before by the intereſt of the million that has been ſo paid off, that is, (if the annuities are 4 per cents,) by 40,000 pounds. Therefore this ſum of 40,000 pounds, which, before the diſcharge of the ſaid million, uſed to be paid away every year to the publick creditors as the intereſt of it, may then be employed, together with the annual million of this year originally deſtined to this purpoſe, in diſcharging, or repaying, ſome more of the capital of the ſaid 4 per cent. annuities. And in like manner there will be in every following year a greater and greater ſum of money, by means of theſe ſavings of the intereſt of the diſcharged debt, that may be ſo applied. And in the courſe of fifty, or ſixty, years the amount of theſe annual payments to the publick creditors, or the portion of the national debt extinguiſhed by them, will be exceeding great, being evidently equal to the amount of an annual ſum of one million of pounds for the ſame period of time, improved at compound intereſt. In fifty years time it would be more than a hundred and fifty-two millions; and in ſixty years it would be nearly two hundred and thirty-eight millions: as appears from Mr. Smart's third table of compound intereſt, pages 70 and 72. And conſequently, as the annual intereſt of a hundred and fifty-two millions, at 4 per cent. is 152 times 40,000 pounds, or 6,080,000 pounds, and the annual intereſt of two hundred and thirty-eight millions, at the ſame rate, is 238 times 40,000 pounds, or 9,520,000 pounds; the quantity of the annual intereſt of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed by the operation of this annual ſurplus million in the courſe of fifty years would be £.6,080,000 *per annum*; and the quantity of thoſe perpetual annuities that would be redeemed by the operation of this annual million in the courſe of ſixty years would be £.9,520,00 *per annum*; which is more than the whole intereſt of the preſent national debt, though increaſed to ſo alarming a quantity. So great would be the effect of the faithful application of only the moderate ſum of one million of pounds *per annum* for the ſpace of only 60 years to this important purpoſe!

CCLXV. This firſt method of employing the ſuppoſed ſurplus part of the publick revenue is only fit to be applied to ſuch of the public funds as ſhall riſe to *par* or above *par*, as I have ſuppoſed may be the caſe with the 4 per cent. annuities. But the greateſt part of the national debt conſiſts of 3 per cent. annuities; which will not, in all probability, riſe at the peace to any thing like their *par*, or nominal value, they being at this time, November, 1781, at the very low price of 56 per cent. But, **A remark on the ſaid method.**

perhaps, they may rise, upon a peace, to £75 per cent. Now, if they should then rise to this value, or to any other value less than a hundred pounds per cent. it is not to be expected that the Government will redeem them at the full price of 100 pounds per cent. or pay the publick creditors a real hundred pounds for every nominal hundred pounds of their stock in these annuities; because the nation is not under any obligation to redeem them at that, or any other, price, provided it continues to pay the interest of them with punctuality. The Government will therefore chuse to continue to pay the interest due upon these funds, unless the owners of them will consent to be paid off at a less price than the *par*, or nominal value, of their stocks, and will accept a price equal to, or a little exceeding, the price at which those stocks shall then be found to sell at the publick market. And many of the owners of these stocks will, doubtless, be glad to accept of such a price for them. This brings me to the consideration of the second method in which it is probable that a given annual sum of money, that can be spared from the publick revenue for that purpose, may be employed in diminishing the national debt, and redeeming the perpetual annuities that are now due to the publick creditors as the interest of it.

A second method of employing a given sum of money every year in the reduction of the national debt.

CCXLVI. This second method of employing a given annual sum of money in the reduction, or diminution, of the national debt consists in buying up with it, from the owners of some of the publick stocks that shall be sold at the publick market for less than their *par*, or nominal value, a part of their said stocks, with the full and free consent of the said owners, at the market-price, or a price a little exceeding the market-price, of the said stocks; and in employing the interest that would thereby be every year redeemed to the nation, in the same manner.

Thus, for example, if we suppose, as before, that the annual surplus of the publick revenue that can be spared for this purpose, is a million of pounds sterling, and that the value of the 3 per cent. annuities should rise, after the conclusion of a peace, to 74 or 75 pounds per cent. the said million of pounds sterling *per annum* may be employed in buying up a part of the capital of the said 3 per cent. annuities, with the consent of the respective owners of it, at the price of £75 per cent. and the interest that would thereby be redeemed to the nation at the end of every year may also be employed in the same manner.

CCXLVII. If

CCXLVII. If a million of pounds ſterling were to be ſo employed, it is evident that 750,000 pounds of it would buy up, or extinguiſh, a million of the ſaid capital of the 3 per cent. annuities, and conſequently that the whole million ſo employed would extinguiſh a million and the third part of a million, or £1,333,333, of the ſaid capital; of which capital of £1,333,333, the annual intereſt is 40,000 pounds.

Of the effect of this ſecond method of diminiſhing the national debt.

This would be the effect of employing a ſingle million of pounds ſterling in this manner, or of the firſt year's execution of the ſuppoſed project of employing a million of pounds in this way for ſeveral years together. But at the end of the ſaid firſt year it is evident that the Government would have the 40,000 pounds, which uſed before to pay the intereſt of the capital of £1,333,333 ſo extinguiſhed, to employ in the ſame manner, over and above the annual million belonging to this year, which was originally deſtined to this purpoſe; ſo that the quantity of the ſaid capital of the 3 per cent. annuities that would be extinguiſhed by the ſecond operation of this method would be ſo much as could be purchaſed at the price of £75 per cent. by the ſum of 1,040,000 pounds, that is,

$$\left(\frac{100}{75} \times £1.040,000, \text{ or } \frac{4}{3} \times £1,040,000, \text{ or } \frac{£\,4,160,000}{3}, \text{ or}\right)$$

£1.386,666; of which ſum the annual intereſt is 41,600 pounds.

Therefore at the beginning of the third year the Government would have the three following ſums to employ in this manner; to wit, firſt, the original million belonging to the ſaid third year; 2dly, the 40,000 pounds of the intereſt of the national debt which was redeemed by the firſt operation; and, 3dly, the 41,600 pounds of the intereſt of the ſame debt which was redeemed by the ſecond operation; which ſums together amount to 1,081,600 pounds. And this ſum would be ſufficient to extinguiſh

$\left(\frac{4}{3} \times £1,081,600, \text{ or}\right)$ £1,442,133 of the ſaid capital of the 3 per cent. annuities; the intereſt of which capital is 43,264 pounds *per annum*.

Therefore the ſums that the Government will have to employ in this manner at the beginnings of the firſt, ſecond, third, and fourth years during the time that this method ſhall be purſued, will be as follows; to wit,

1ſt, £1,000,000,
2dly, £1,000,000 + £40,000, or £1,040,000,
3dly, £1,000,000 + £40,000 + £41,600, or £1,081,600, and
4thly, £1,000,000 + £40,000 + £41,600 + £43,264, or £1,124,864.

Theſe

These ſums, £1,000,000, £1,040,000, £1,081,600, and £1,124,864, are in continual geometrical proportion, the common ratio of the terms being that of 100 to 104; and the ſecond, third and fourth terms are the values of a million of pounds at the ends of one, two, and three years, when improved at compound intereſt at the rate of 4 per cent. And in like manner it will be found that the ſums which the Government will have to employ in the ſame manner at the beginnings of the fifth, ſixth, ſeventh, and other following years during which this method ſhall be purſued, will the values of a million of pounds at the ends of four, five, ſix, and every following number of years, when the ſaid million is improved during thoſe years at compound intereſt at the rate of 4 per cent. Therefore the total amount of all the annual ſums which the Government would have to employ in this manner during any given number of years, in conſequence of having ſet apart an original annual ſum of a million of pounds for this purpoſe, would be equal to the total amount of an annual ſum of a million of pounds at the end of the ſame number of years, if the ſaid annual ſum were, as faſt as it was received by the perſon to whom it was due, to be immediately lent out and improved at compound intereſt at the rate of 4 per cent. But we have already obſerved that it appears from Mr. Smart's tables of compound intereſt, pages 70 and 72, that the total amount of an annual payment of a million of pounds, ſo improved at compound intereſt at the rate of 4 per cent. for a period of fifty years, is more than a hundred and fifty-two millions; and that the amount of it for a period of ſixty years is very nearly two hundred and thirty-eight millions. Therefore the total amount of all the ſums which the Government would have to employ in thus buying up the ſtock of the 3 per cent. annuities at the price of £75 per cent. if an annual ſum of a million of pounds ſterling *per annum*, together with the intereſt that would be continually redeemed by it, were to be faithfully applied to this purpoſe for a period of fifty years, would be more than a hundred and fifty-two millions; and the amount of all the ſums which the Government would have to employ in this manner in the courſe of ſixty years would be very nearly two hundred and thirty-eight millions. And, as every million of this money would be ſufficient to extinguiſh £1,333,333 of the capital of the ſaid 3 per cent. annuities, it follows that the amount of all the ſaid annual ſums that would, according to this ſecond method of employing the ſaid annual ſurplus million, be applied to the extinguiſhment of the capital of the ſaid 3 per cent. annuities, in the courſe of fifty years, would be ſufficient to have extinguiſhed in that time more than 152 times £1,333,333, or than £202,666,666 of that capital, and conſequently to have redeemed for the nation more than 152 times 40,000 pounds *per annum*, or 6,080,000 pounds *per annum* of the perpetual annuities which pay the intereſt of that capital: and the amount of the ſaid annual ſums, that might be ſo employed, in the courſe of ſixty years would be ſufficient to have extinguiſhed, very

very nearly, 238 times £1,333,333, or, very nearly, £317,33,254, of the faid capital of the 3 per cent. annuities, and confequently to have redeemed for the nation very nearly 238 times 40,000 pounds *per annum*, or very nearly 9,520,000 pounds *per annum* of the perpetual annuities which pay the intereft of the faid capital: which latter fum, £9,520,000, is greater than the whole intereft of the prefent national debt, though increafed of late to fo alarming a quantity.

The two foregoing methods of diminifhing the national debt would be equally efficacious and beneficial to the nation.

CCXLVIII. It is evident that the two foregoing methods of applying an annual fum of a million of pounds, together with the intereft continually redeemed by it, for a given number of years, to the diminution of the national debt, to wit, by paying off at *par* thofe ftocks which carry an intereft of 4 per cent. when the price of thofe ftocks fhall rife to *par*, and by buying up at the price of £75 per cent. (with the confent of the owners of them,) thofe ftocks which fhall carry an intereft of 3 per cent. would be equally beneficial to the publick, or that the quantity of the intereft of the national debt that would be redeemed by either of them in a given time would be exactly the fame as the quantity of the faid intereft that would be redeemed by the other in the fame time.

Expediency of appropriating by act of parliament, in the ftricteft manner poffible, the fum of one million of pounds fterling per annum *out of the finking fund, during a term of fifty, or fixty, years, to the difcharge of the national debt.*

CCXLIX. The foregoing very great operation of only one million of pounds fterling a year, with the intereft continually redeemed by it, in the courfe of fo moderate a period of time as fifty, or fixty, years, when ftrictly applied to this purpofe of diminifhing the national debt without any interruption, in either of the two foregoing methods, ought, one would think, to induce the Parliament to appropriate that fum out of the finking fund to this important purpofe in the ftricteft manner that can be devifed, for the fpace of fifty, or fixty, years, and to forbear to interrupt its operation during that period upon any account, or occafion, whatfoever, however urgent. And it feems the more reafonable to expect that fuch a meafure will foon be adopted, becaufe the finking fund has of late years produced no lefs a fum than three millions of pounds fterling *per annum*: and our minifters of ftate, as well as the owners of property in the publick funds, ought to recollect that the whole of the faid fund, as its name imports, was once appropriated by Parliament to this very purpofe, of *finking*, or diminifhing, the national debt, in the manner now recommended for one third of it.

A third

A third method of employing a given annual sum of money in the reduction of the national debt.

CCL. A third method of employing the said annual surplus million, or other part of the publick revenue that can be spared for this purpose, to the diminution of the national debt, would be to convert some of the *perpetual* annuities now due to the publick creditors as the interest of the said debt, into *temporary* annuities, or annuities that should continue only for a limited number of years, as, for example, for 100 years, or 80 years, or 60 years, or 30 years, but which should be greater than the perpetual annuities in the room of which they should be substituted, in a certain just and reasonable proportion that was adapted to the change made in their duration: the said change being made with the consent of the proprietors of the perpetual annuities that should be so converted. For without this consent a measure of this kind would be a breach of the national faith: but with such consent of the proprietors concerned it would be a perfectly just measure, and might, I imagine, be sometimes an expedient one. And, if it were adopted, the said annual surplus million, or other part of the publick revenue that could be spared for this purpose, might be employed in paying the additions that would be necessary to be made to the said perpetual annuities in compensation for the abridgement of their duration.

A remark on the said third method of diminishing the national debt.

CCLI. By this third method of proceeding the national debt would not be diminished gradually, or every year a little, as in the two former methods; but, on the contrary, the interest annually paid to the publick creditors would be somewhat increased during the continuance of the new temporary annuities that had been substituted in the room of some of the former perpetual ones. But at the end of that time the whole of the said temporary annuities would cease at once, and consequently the perpetual annuities, in lieu of which they had been substituted, would be redeemed to the nation.

The said third method would be equally efficacious with either of the two former in diminishing the national debt.

CCLII. This third method of diminishing the national debt will have precisely the same effect at the end of the term during which the new temporary annuities which are substituted in lieu of the former perpetual annuities, are to continue, as either of the two former methods; that is, the quantity of the interest of the national debt which will be redeemed by this third method at the end of the said limited number of years, by employing a million of pounds sterling, or any other given sum of money, every year in this manner, will be the same as the quantity of the

the ſaid intereſt that would be redeemed in the ſame courſe of time by employing the ſame million of pounds, or other annual ſum, together with the ſavings of intereſt annually produced by it, in either of the two former methods.

An example of its effect in a period of ſixty years.

CCLIII. Thus, for example, if the intereſt of money is ſuppoſed, as before, to be 4 per cent. and the term during which the new temporary annuities, (which are ſubſtituted in lieu of the former perpetual annuities,) are to continue, is ſixty years, we ſhall find that the quantity of the intereſt of the national debt, or of the perpetual annuities, that would be redeemed by the application of a million of pounds every year during the ſaid ſixty years according to this third method, or by employing the ſaid million in paying the additions which would have been made to the perpetual annuities which had been converted into temporary ones, would be exactly the ſame as the quantity of the intereſt of the national debt, or of the perpetual annuities, that would be redeemed in the ſame period of ſixty years by employing the ſame ſum of a million every year, together with the annual ſavings of intereſt produced by it, in either of the two former methods; which ſum we have ſeen above to be £9,520,000 *per annum.* This may be ſhewn in the manner following.

When the intereſt of money is 4 per cent. or a perpetual annuity of four pounds a year is worth a hundred pounds, and conſequently a perpetual annuity of one pound a year is worth twenty-five pounds, a temporary annuity of one pound a year, that is to continue only for ſixty years, will be worth only £22.6234; as appears from Mr. Smart's fourth table of compound intereſt, page 80. Therefore a temporary annuity for 60 years that ſhall be worth 25 pounds, or ſhall be equal in value to a perpetual annuity of one pound a year, muſt be greater than a like temporary annuity of one pound a year in the proportion of £25 to £22.6234, and therefore will be $= £1 \times \frac{25}{22.6234}$, or £1.1050, or ſomething leſs than 1*l.* 2*s.* 1*d.*$\frac{1}{4}$ a year. And conſequently, if the Government were to convert ſome of the perpetual annuities, now due to the publick creditors, into temporary annuities for 60 years, (with the conſent of the proprietors of them,) and were to allow to each proprietor the ſum of 1*l.* 2*s.* 1*d.*$\frac{1}{4}$ *per annum* during the ſaid term of 60 years, inſtead of every ſum of 1*l.* *per annum* that had formerly been due to him for ever, ſuch a bargain ought to be conſidered as fair and equal on both ſides.

Now, ſince for every perpetual annuity of one pound a year that ſhould be thus changed into a temporary annuity for 60 years, it would be neceſſary ceffary to allow the proprietor of it an annuity of 1*l.* 2*s.* 1*d.*$\frac{1}{4}$, or £1.1050, a year during the faid term of 60 years: it follows that for every million of pounds *per annum* of the faid perpetual annuities that fhould be thus changed into temporary annuities for a term of 60 years, it would be neceffary to allow to the proprietors of the faid million of perpetual annuities a million times £1.1050 *per annum*, or £1,105,000, or a million and a hundred and five thoufand pounds *per annum*, to be continued during the faid term of 60 years; or, in other words, an annual additional fum of £105,000 would be required to enable the Government to convert £1000,000 *per annum* of the perpetual annuities into temporary annuities for 60 years. Therefore an annual additional fum of £1000,000 would be fufficient to enable the Government to convert into annuities for 60 years a quantity of the perpetual annuities that is as much greater than £1,000,000 a year as £1,000,000 is greater than £105,000; which greater quantity is (£1,000,000 $\times \frac{1,000,000}{105,000}$, or £1000,000 $\times \frac{1000}{105}$, or £1,000,000 × 9.52. or) £9,520,000. Therefore the quantity of the intereft of the national debt, or of the perpetual annuities, that would be redeemed to the nation in the courfe of 60 years by employing the fum of a million of pounds *per annum* during that time in this third manner would be £9,520,000 *per annum*. Q.E.D.

In one refpect this third method of applying a given fum of money every year to the diminution of the national debt feems to be preferable to either of the two former methods of applying it to the fame purpofe.

CCLIV. It appears therefore that thefe three different methods of applying a given annual fum of money to the diminution of the national debt, or to the redemption of the publick revenue that is mortgaged for the payment of the intereft of it, are equally efficacious, if they are purfued with equal fteadinefs. But in this laft refpect, I mean the probability of their being purfued with fteadinefs for a confiderable length of time, they do not feem to be quite equal; but the third method feems rather to have the advantage of the former two. For, if either the firft method of applying a given annual fum of money to this purpofe, to wit, "that of employing it every year in repaying, at their *par*, or nominal value, fome of thofe ftocks, or branches of the national debt, which fhall have rifen at the publick market either to, or above, their faid *par*, or nominal value;" or the fecond method of applying it to the fame purpofe, to wit, "by buying up at the market price, or at a price very little exceeding the market price, fome of the publick ftocks that were under their *par*, or nominal value, with the confent of the refpective owners of them;" I fay, if either of thefe methods were to be adopted, there would be reafon to apprehend that, whenever any particular emergence fhould arife that required a greater expenditure of publick money than ordinary, the minifters of ftate might be tempted to propofe to the Parliament, and the

Parliament

Parliament to adopt the propoſal, that the operation of diminiſhing the national debt ſhould be ſuſpended for a year, and the ſaid annual ſum that had been allotted to this purpoſe, ſhould be applied to the diſcharge of the extra-ordinary expence that was made neceſſary by the ſuppoſed emergency: becauſe by ſo doing they would avoid the odium and difficulty of raiſing the ſaid extra-ordinary money by laying freſh taxes upon the people. The experience of the nation affords ſufficient grounds for ſuch an apprehenſion. For we have had but too many inſtances of ſuch a diſpoſition both in our miniſters of ſtate and in our parliaments ever ſince the year 1733, when Sir Robert Walpole firſt began to divert the ſinking fund from its original deſtination of *ſinking*, or diminiſhing, the national debt, and employed it in defraying the current ſervices of the year: without which change in the application of it, the continual operation of that large and uſeful fund from the year 1733 to the preſent year, 1781, would have extinguiſhed almoſt all our publick debts ſoon after they had been contracted, though they have now increaſed to ſuch an enormous quantity. But, if the aforeſaid annual ſurplus million were to be applied to the diminution of the national debt in the third method above-mentioned, that is, "by paying with it the additions that ſhould have been made to ſome of the perpetual annuities now due to the publick creditors, upon a converſion of the ſaid annuities (with the conſent of the creditors to whom they were due,) into greater annuities for a limited number of years," it could never afterwards be diverted from this deſtination, and applied to any other purpoſe, without an abſolute breach of the national faith; which, I preſume, we may conſider as a kind of moral impoſſibility. For, as ſuch a meaſure would not only be exorbitantly unjuſt and cruel towards the numerous unhappy individuals who would be the immediate ſufferers by it, but likewiſe moſt ruinous to the general credit and commerce of the kingdom, and likely to produce the moſt dreadful ſcenes of internal miſery and confuſion; we may reaſonably hope that our miniſters of ſtate, and our parliaments, will always look upon it with the utmoſt horror and deteſtation, and will conſequently think themſelves bound in duty to prevent it by the moſt vigorous and extenſive exertions of their right of impoſing new taxes that may become neceſſary for that purpoſe, however diſagreeable and unpopular ſuch exertions may appear. And therefore, I think, we may conſider ſuch an event as morally impoſſible. And, if we are right in ſo conſidering it, we may conclude that the aforeſaid third method of applying any given annual ſum to the diminution of the national debt will deſerve to be preferred to both the former methods of applying it to the ſame purpoſe, becauſe of the poſſibility, and, perhaps, even probability, that thoſe former methods of applying it may occaſionally be ſuſpended and interrupted.

A fourth method of employing a given ſum of money every year in the reduction of the national debt.

CCLV. A fourth method of applying a given annual ſum of money to the diminution of the national debt, or the redemption of the intereſt of it, might be as follows. Some of the perpetual annuities now due to the publick creditors might, with the conſent of the owners of them, be converted into life-annuities for the lives of their reſpective owners, or for the lives of other perſons whom they ſhould name, and who may be called their *nominees*: which life-annuities, it is evident, muſt be greater than the correſponding perpetual annuities, in the room of which they would have been ſubſtituted, in various proportions ſuited to the ſeveral ages of the perſons for whoſe lives they would be to continue. And the ſaid annual ſum of money, that could be ſpared out of the publick revenue for the purpoſe of diminiſhing the national debt, might be employed in the payment of the additions neceſſary to be made to the ſaid perpetual annuities upon their being ſo converted into life-annuities.

This fourth method of diminiſhing the national debt, if properly purſued, would be equally effectual for that purpoſe with either of the three preceeding methods.

CCLVI. In this method of diminiſhing the national debt, it is evident, there will be continual ſavings of intereſt ariſing to the nation every year by the deaths of ſome of the life-annuitants. Now, if theſe ſavings are continually employed, for the ſame purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-mentioned, that is to ſay, either in paying off at *par* ſome part of thoſe ſtocks which ſhall have riſen to *par*, or above *par*, or in buying up at the market price, or at a price very little greater than the market price, (with the conſent of the reſpective owners of it) ſome part of thoſe ſtocks which ſhall continue under *par*; the quantity of annual intereſt that would be redeemed to the nation by applying a given ſum of money every year in this fourth method during the life of the longeſt liver of all the ſaid life-annuitants, would be the very ſame that would be redeemed by applying the ſame ſum of money to the diminution of the national debt during the ſame length of time in either of the three preceeding methods. Of which it may not be amiſs to give an example: which may be done as follows.

An example of this third method of applying a given ſum of money every year to the diminution of the national debt.

CCLVII. It has been ſhewn above that a million of pounds ſterling *per annum*, applied conſtantly to the diminution of the national debt, or the redemption of the intereſt of it, for the ſpace of 60 years, in either of the

the three foregoing methods, will redeem an annual intereſt of 9,520,000 pounds. We will therefore now inquire what quantity of annual intereſt the ſame annual ſum of a million of pounds ſterling will redeem in the ſame courſe of time, if applied in this fourth method; making the ſame ſuppoſitions as before, to wit, that the ſtocks which carry an intereſt of 4 per cent. will riſe to *par*, and that thoſe which carry an intereſt of 3 per cent. will riſe to £75 per cent. and uſing the table of the values of life-annuities when the intereſt of money is 4 per cent. which is given above in page 225, for the valuation of the life-annuities which we are now going to conſider, and Monſieur de Parcieux's table of the probabilities of the duration of human life (upon which the ſaid table in page 225 is founded,) for the purpoſe of aſcertaining the numbers of the life-annuitants that will die off in every year of the whole term, and the conſequent ſavings of intereſt that will accrue every year to the publick by their deaths.

CCLVIII. Now, that this calculation (which will unavoidably run into conſiderable length,) may not be too tedious and intricate, we will ſuppoſe that all the perſons for whoſe lives the new annuities, that are to be ſubſtituted inſtead of the former perpetual annuities, ſhall be granted, are of the ſame age. And further,—as it is ſuppoſed that the whole operation of this fourth method is to continue during the ſpace of 60 years;——and in Monſieur de Parcieux's table of the probabilities of the duration of human life, only one perſon, out of the whole original number of 1000 perſons living at the age of 3 years, is ſuppoſed to live to the age of 94 years, and that one perſon is ſuppoſed to die before he attains the age of 95 years;——we will ſuppoſe the ſeveral perſons, for whoſe lives the ſaid new annuities ſhall be granted, to be all of the age of 35 years: for then it may be ſuppoſed that the longeſt liver of them will live almoſt to the very end of the ſaid term of 60 years during which this operation is to continue.

CCLIX. Now it appears above by Table XVI, page 225, that, when the intereſt of money is 4 per cent. or the value of a perpetual annuity of four pounds a year is a hundred pounds, and conſequently the value of a perpetual annuity of one pound a year is twenty-five pounds, the value of an annuity of one pound a year for the life of a perſon of the age of 35 years is £16.084,014. Therefore an annuity for the life of a perſon of the age of 35 years that ſhall be worth 25 pounds, or ſhall be equal in value to a perpetual annuity of one pound a year, will be greater than an annuity of one pound a year for the life of the ſame perſon in the proportion of £25 to £16.084,014, and therefore will be equal

A computation of the quantity of the intereſt of the national debt that would be redeemed to the publick by this method of life-annuities at the end of 60 years without the help of the annual ſavings

that would accrue to the publick by the deaths of ſome of the life-annuitants in the courſe of the ſaid term.

equal to £1 × $\frac{25.000,000}{16.084,014}$, or £1.554,338. This therefore is the life-annuity which each of the persons aged 35 years for whose lives we suppose the said life-annuities to be granted, ought to receive in lieu of the perpetual annuity of one pound a year in the room of which it is substituted. Therefore for every million of pounds *per annum* of the perpetual annuities, now due to the publick creditors, which should be thus converted into life-annuities for the lives of persons of the age of 35 years, it would be necessary that Government should establish life-annuities to the amount of a million times £1.554,338 *per annum*, or to the amount of £1,554,338, *per annum*; which would require an additional annual interest of £554,338: so that an additional annual interest of 554,338 pounds would be sufficient to enable the Government to convert a million of pounds *per annum* of the perpetual annuities into life-annuities for the lives of persons of the age of 35 years. Therefore an additional annual interest of £1000,000 will be sufficient to enable the Government to convert into life-annuities for the lives of persons of the age of 35 years a quantity of the perpetual annuities that is greater than £1000,000 *per annum* in the proportion of £1000,000 to £554,338, that is, a quantity of the said perpetual annuities that is equal to (£1000,000 × $\frac{1000,0000}{554,338}$, or) £1,803,953 *per annum*. Therefore at the end of the said term of 60 years, when all the said life-annuitants (who were of the age of 35 years at the beginning of the said term,) will be dead, the said sum of £1,803,953 *per annum* of the perpetual annuities will be intirely redeemed to the nation by means of this conversion of them into life-annuities.

Of the savings that would accrue every year to the publick by the deaths of some of the life-annuitants.

CCLX. This would be the quantity of the perpetual annuities that would be redeemed by this operation in the course of 60 years, if all the aforesaid life-annuitants had lived to within a day of the end of the whole 60 years, or if, when they had died off in the different years of the said term according to the course of nature, the savings which would have accrued every year to the publick by their deaths had not been applied to the same purpose of diminishing the national debt, but had been spent upon some other service. But it was supposed above that these savings were to be employed continually to the same purpose of diminishing the national debt in either the first or the second of the methods above-described. We must now therefore inquire what these savings would amount to in every year of this whole term, and what quantity of the perpetual annuities they would respectively be sufficient to redeem in the course of the said term, if they were applied to that use in either the first or the second method above-mentioned. These inquiries may be made in the manner following.

CCLXI. The

CCLXI. The number of perſons repreſented in Monſieur de Parcieux's table as living at the age of 35 years is 694. The money to be paid annually by the Government in life-annuities, ſo long as the life-annuitants ſhall be all alive, in lieu of the £1,803,953 *per annum* of perpetual annuities which are to be redeemed, is the ſaid ſum of £1.803,953 *per annum* together with the annual million of pounds which is deſtined to this purpoſe, and conſequently is £2,803,953 *per annum*. Therefore, if we ſuppoſe theſe life-annuities to be diſtributed equally amongſt 694 perſons, all of the age of 35 years, each of the ſaid perſons will be poſſeſſed of a life-annuity of $\left(\frac{\pounds\,2,803,953}{694},\ \text{or}\right)$ £4040.27 *per annum*. And conſequently, if we ſuppoſe them to be equally diſtributed amongſt 6940 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of a life-annuity of £404.027 *per annum:* and, if we ſuppoſe them to be equally diſtributed amongſt 69400 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of a life-annuity of £40.4027, or 40*l.* 8*s.* ½*d.* *per annum.* Let us make this laſt ſuppoſition.

Of the number of perſons ſuppoſed to be life-annuitants, and the value of their annuities.

CCLXII. Then, ſince it appears by Monſieur de Parcieux's table of the probabilities of the duration of human life, that out of 694 perſons of the age of 35 years, all living at the ſame time, only 686 perſons will be living at the end of a year, or at the age of 36 years, it follows that out of the ſaid 69400 perſons above-mentioned, of the age of 35 years, all living at the ſame time, for whoſe lives theſe annuities of £40.4027, or 40*l.* 8*s.* ½*d.* each, are ſuppoſed to be granted, only 68600 will be living at the end of a year, or at the age of 36 years. And conſequently the annuities that would have been payable to the other 800 life-annuitants at the end of the year, if the ſaid annuitants had been then living, will be ſaved to the publick, and will be ready in the publick treaſury to be employed, in any manner that may be thought fit, towards the diminution of the national debt. The amount of theſe annuities is 800 times £40.4027, or £32,322.1600. Now, if this ſum £32,322.1600 is immediately laid out either in paying off ſome of the capital of the 4 per cent. annuities at *par*, according to the firſt method above-deſcribed, or in buying up ſome of the capital of the 3 per cent. annuities at the price of £75 per cent. according to the ſecond method above-deſcribed; and the intereſt of the capital thereby extinguiſhed, or the portion of the perpetual annuities that will be redeemed at the end of the next, or ſecond, year, by thus employing the ſaid ſum of £32,322.160, be likewiſe employed in the ſame manner; and the intereſt of the capital extinguiſhed by this ſecond operation, or the portion of the perpetual annuities that will thereby be redeemed at

Of the ſaving made at the end of the firſt year by the deaths of ſome of the life-annuitants.

Of the manner in which the ſaid ſaving is to be employ'd during the remaining 59 years of the ſaid term of 60 years.

at the end of the third year, be likewiſe employed in the ſame manner; and all the following portions of the perpetual annuities that will be thus redeemed by means of this firſt ſum of £32,322.1600, that is ſaved to the nation at the end of the firſt year by the death of the aforeſaid 800 life-annuitants, be likewiſe employed in the ſame manner during the whole remainder of the ſaid term of 60 years;—I ſay, if theſe ſucceſſive ſums of money be ſo employed without interruption during the remainder of the ſaid term of 60 years, the quantity of the capital of the national debt which will be thereby extinguiſhed at the end of the ſaid term of 60 years, or at the end of 59 years from the time when this firſt ſaving of £32,322.1600 will have accrued, will be equal in value to the ſum to which the ſaid firſt ſaving of £32,322.1600 would have increaſed in the ſame period of 59 years, if it had been improved all the time at compound intereſt at the rate of 4 per cent. and therefore it will, (according to Mr. Smart's firſt table of compound intereſt, page 56,) be equal in value to the ſum of £32,322.1600 × 10.115,026, or £326,939.4887; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £326{,}939.4887, \text{ or}\right)$ £13077.5795 *per annum.* Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities now due to the publick creditors, which would be redeemed at the end of the whole term of 60 years by means of this firſt ſaving of £32,322.1600, which will accrue to the publick at the end of the firſt year of the ſaid term by the deaths of the aforeſaid 800 life-annuitants, would be £13,077.5795 *per annum.*

Of the quantity of the capital of the national debt that will have been extinguiſhed by means of the ſaid ſaving at the end of the ſaid term of 60 years.

Of the ſaving made at the end of the ſecond year by the deaths of life-annuitants.

CCLXIII. In the courſe of the ſecond year of this term the 68600 life-annuitants of the age of 36 years, who are living at the end of the firſt year, will be reduced to 67800. Therefore 800 more of the ſaid life-annuitants will have died in the courſe of this ſecond year. And conſequently at the end of this ſecond year there will be a ſaving to the Government of twice 800 payments of £40.4027 each, which would have then become due to the 1600 life-annuitants who will have died in the two preceeding years, if they had lived till that time. There will therefore be in the hands of the Government at the end of this ſecond year the ſum of 1600 × £40.4027, or £64,644.3200, to be employed during the remainder of the ſaid term of 60 years, that is, during 58 years, in diminiſhing the national debt; which will be ſufficient to extinguiſh as much of the capital of the ſaid debt as is equal in value to the ſum to which this ſecond ſaving of £64,644.3200 will increaſe in the courſe of 58 years, if conſtantly improved during that time at compound intereſt at the rate of 4 per cent. Now the amount of £64,644.3200 at the end of 58 years, when improved during that time at compound intereſt at 4 per cent. appears

appears by Mr. Smart's firſt table of compound intereſt, page 56, to be = £64,644.3200 × 9.725,986, or £628,729.7513; the intereſt of which at 4 per cent. is ($\frac{4}{100}$ × £628,729 7513, or $\frac{£2,514,919.0052}{100}$, or) £25,149 1900. Therefore the quantity of the capital of the national debt that would be extinguiſhed at the end of the ſaid term of 60 years by means of this ſecond ſaving of £64,644.3200 (which would have accrued to the publick at the end of the ſecond year of the ſaid term by the deaths of the ſaid 1600 life-annuitants who would have died in the two preceeding years,) will be equal in value to £628,729.7513; and the quantity of the intereſt of the ſaid debt, or of the perpetual annuities now due to the publick creditors, that would then be redeemed to the nation by means of this ſecond ſaving, would be the ſum of £25,149.1900 *per annum*.

Of the quantity of the capital of the national debt that will have been extinguiſhed by means of this ſecond ſaving at the end of the ſaid term of 60 years.

CCLXIV. In the ſame manner we muſt proceed to inveſtigate the ſavings that will accrue to the publick at the ends of the third, fourth, fifth, ſixth, and every following year of the whole term of 60 years, and the quantities of the capital of the national debt that will be extinguiſhed by each of the ſaid ſavings at the end of the ſaid term of 60 years, if they are conſtantly employed for that purpoſe in either the firſt or the ſecond method above-mentioned, during the ſeveral remaining years of the ſaid term after the times when they will have reſpectively accrued. And, in order to theſe inveſtigations, it will be neceſſary in the firſt place to aſcertain, by the help of Monſieur de Parcieux's table of the probabilities of the duration of human life, the numbers of life-annuitants who will be alive at the ends of the ſeveral remaining years of this term, except the laſt year of it, (at the end of which they will be all dead,) and thence to determine the numbers of thoſe who will have died in each of the ſaid years, and from whoſe deaths the ſeveral ſavings to the publick will ariſe.

Of the ſavings that will be made at the ends of the third, and fourth, and other following years of the ſaid term of 60 years, by the deaths of life-annuitants.

CCLXV. Now of the aforeſaid original number of 69400 life-annuitants, all of the age of 35 years, who were ſuppoſed to be living at the beginning of the ſaid term of 60 years, and for whoſe lives the ſaid annuities, of £4[illegible].4027 to each, were ſuppoſed to be granted, it appears from Monſieur de Parcieux's table of probabilities that the following numbers will be living at the ſubſequent ages of 38 years, 39 years, 40 years, 41 years, &c. to the age of 94 years, incluſive, or at the ends of the third, fourth, fifth, and ſixth, and every following, year of the ſaid term of 60 years; to wit,

The numbers of life-annuitants who will be living at the ends of all the ſaid years, except the laſt year.

67100,	59900,	48900,	34700,	15400,	2200,
66400,	59000,	47600,	32900,	13600,	1600,
65700,	58100,	46300,	31000,	11800,	1100,
65000,	57100,	45000,	29100,	10100,	700,
64300,	56000,	43700,	27100,	8500,	400,
63600,	54900,	42300,	25100,	7100,	200,
62900,	53800,	40900,	23100,	5900,	100.
62200,	52600,	39500,	21100,	4800,	
61500,	51400,	38000,	19200,	3800,	
60700,	50200,	36400,	17300,	2900,	

The numbers of life-annuitants who will have died in each of the said years, and in the last year.

Therefore the numbers who will have died in each of those years, and in the last year of the said period of 60 years, or from the age of 94 years to the age of 95 years, will be as follows; to wit,

700,	800,	1300,	1700,	1900,	700,
700,	900,	1300,	1800,	1800,	600,
700,	900,	1300,	1900,	1800,	500,
700,	1000,	1300,	1900,	1700,	400,
700,	1100,	1300,	2000,	1600,	300,
700,	1100,	1400,	2000,	1400,	200,
700,	1100,	1400,	2000,	1200,	100,
700,	1200,	1400,	2000,	1100,	100.
700,	1200,	1500,	1900,	1000,	
800,	1200,	1600,	1900,	900,	

CCLXVI. Consequently the savings of interest that will be made by the publick at the ends of those years in consequence of the deaths of life-annuitants that will have happened in the preceeding years of the said term, will be as follows; to wit,

At the end of the third year

$$\overline{1600 + 700} \times £40.4027,$$
$$\text{or } 2300 \times £40.4027 = £92,926.2100.$$

At the end of the fourth year

$$\overline{2300 + 700} \times £40.4027,$$
$$\text{or } 3000 \times £40.4027, = £121,208.1000.$$

At

At the end of the fifth year,

$\overline{3000 + 700}|$ × £40.4027,

or 3700 × £40.4027, = £149,489.9900.

At the end of the sixth year,

$\overline{3700 + 700}|$ × £40.4027,

or 4400 × £40.4027, = £177,771.8800.

At the end of the seventh year,

$\overline{4400 + 700}|$ × £40.4027,

or 5100 × £40.4027, = £206,053.7700.

At the end of the eighth year,

$\overline{5100 + 700}|$ × £40.4027,

or 5800 × £40.4027, = £234,335.6600.

At the end of the ninth year,

$\overline{5800 + 700}|$ × £40.4027,

or 6500 × £40.4027, = £262,617.5500.

At the end of the tenth year,

$\overline{6500 + 700}|$ × £40.4027,

or 7200 × £40.4027, = £290,899.4400.

At the end of the eleventh year,

$\overline{7200 + 700}|$ × £40.4027,

or 7900 × £40.4027, = £319,181.3300.

At the end of the twelfth year,

$\overline{7900 + 800} \times £40.4027$,

or $8700 \times £40.4027, = £351{,}503.4900$.

At the end of the thirteenth year,

$\overline{8700 + 800} \times £40.4027$,

or $9500 \times £40.4027, = £383{,}825.6500$.

At the end of the fourteenth year,

$\overline{9500 + 900} \times £40.4027$,

or $10{,}400 \times £40.4027, = £420{,}188.0800$.

At the end of the fifteenth year,

$\overline{10{,}400 + 900} \times £40.4027$,

or $11{,}300 \times £40.4027, = £456{,}550.5100$.

At the end of the sixteenth year,

$\overline{11{,}300 + 1000} \times £40.4027$,

or $12{,}300 \times £40.4027, = £496{,}953.2100$.

At the end of the seventeenth year,

$\overline{12{,}300 + 1100} \times £40.4027$,

or $13{,}400 \times £40.4027, = £541{,}396.1800$.

At the end of the eighteenth year,

$\overline{13{,}400 + 1100} \times £40.4027$,

or $14{,}500 \times £40.4027, = £585{,}839.1500$.

At

At the end of the nineteenth year,

$\overline{14{,}500 + 1100} \times$ £40.4027,

or 15,600 × £40.4027, = £630,282.1200.

At the end of the twentieth year,

$\overline{15{,}600 + 1200} \times$ 40.4207,

or 16,800 × £40.4027, = £678,765.3600.

At the end of the twenty-first year,

$\overline{16{,}800 + 1200} \times$ £40.4027,

or 18,000 × £40.4027, = £727,248.6000.

At the end of the twenty-second year,

$\overline{18{,}000 + 1200} \times$ £40.4027,

or 19,200 × £40.4027, = £775,731.8400.

At the end of the twenty-third year,

$\overline{19{,}200 + 1300} \times$ £40.4027,

or 20,500 × £40.4027, = £828,255.3500.

At the end of the twenty-fourth year,

$\overline{20{,}500 + 1300} \times$ £40.4027,

or 21,800 × £40.4027, = £880,778.8600.

At the end of the twenty-fifth year,

$\overline{21{,}800 + 1300} \times$ £40.4027,

or 23,100 × £40.4027, = £933,302.3700.

At the end of the twenty-sixth year,

$\overline{23,100 + 1300} \times$ £40.4027,

or 24,400 × £40.4027, = £985,825.8800.

At the end of the twenty-seventh year,

$\overline{24,400 + 1300} \times$ £40.4027,

or 25,700 × £40.4027, = £1,038,349.3900.

At the end of the twenty-eighth year,

$\overline{25,700 + 1400} \times$ £40.4027,

or 27,100 × £40.4027, = £1,094,913.1700.

At the end of the twenty-ninth year,

$\overline{27,100 + 1400} \times$ £40.4027,

or 28,500 × £40.4027, = £1,151,476.9500.

At the end of the thirtieth year,

$\overline{28,500 + 1400} \times$ £40.4027,

or 29,900 × £40.4027, = £1,208,040.7300.

At the end of the thirty-first year,

$\overline{29,900 + 1500} \times$ £40.4027,

or 31,400 × £40.4027, = £1,268,644.7800.

At the end of the thirty-second year,

$\overline{31,400 + 1600} \times$ £40.4027,

or 33,000 × £40.4027, = £1,333,289.1000.

At

At the end of the thirty-third year,

$\overline{33{,}000 + 1700}l \times £40.4027,$

or $34{,}700 \times £40.4027, = £1{,}401{,}973.6900.$

At the end of the thirty-fourth year,

$\overline{34{,}700 + 1800}l \times £40.4027,$

or $36{,}500 \times £40.4027, = £1{,}474{,}698.5500.$

At the end of the thirty-fifth year,

$\overline{36{,}500 + 1900}l \times £40.4027,$

or $38{,}400 \times £40.4027, = £1{,}551{,}463.6800.$

At the end of the thirty-ſixth year,

$\overline{38{,}400 + 1900}l \times £40.4027,$

or $40{,}300 \times £40.4027. = £1{,}628{,}228.8100.$

At the end of the thirty-ſeventh year,

$\overline{40{,}300 + 2000}l \times £40.4027,$

or $42{,}300 \times £40.4027, = £1{,}709{,}034.2100.$

At the end of the thirty-eighth year,

$\overline{42{,}300 + 2000}l \times £40.4027,$

or $44{,}300 \times £40.4027, = £1{,}789{,}839.6100.$

At the end of the thirty-ninth year,

$\overline{44{,}300 + 2000}l \times £40.4027,$

or $46{,}300 \times £40.4027, = £1{,}870{,}645.0100.$

At

At the end of the fortieth year,

$\overline{46,300 + 2000|} \times £40.4027,$

or $48,300 \times £40.4027, = £1,951,450.4100.$

At the end of the forty-first year,

$\overline{48,300 + 1900|} \times £40.4027,$

or $50,200 \times £40.4027, = £2,028,215.5400.$

At the end of the forty-second year,

$\overline{50,200 + 1900|} \times £40.4027,$

or $52,100 \times £40.4027, = £2,104,980.6700.$

At the end of the forty-third year,

$\overline{52,100 + 1900|} \times £40.4027,$

or $54,000 \times £40.4027, = £2,181,745.8000.$

At the end of the forty-fourth year,

$\overline{54,000 + 1800|} \times £40.4027,$

or $55,800 \times £40.4027, = £2,254,470.6600.$

At the end of the forty-fifth year,

$\overline{55,800 + 1800|} \times £40.4027,$

or $57,600 \times £40.4027, = £2,327,195.5200.$

At the end of the forty-sixth year,

$\overline{57,600 + 1700|} \times £40.4027,$

or $59,300 \times £40.4027, = £2,395,880.1100.$

At

At the end of the forty-seventh year,

$\overline{59,300 + 1600l} \times £40.4027,$
or $60,900 \times £40.4027, = £2,460,524.4300.$

At the end of the forty-eighth year,

$\overline{60,900 + 1400l} \times £40.4027,$
or $62,300 \times £40.4027, = £2,517,088.2100.$

At the end of the forty-ninth year,

$\overline{62,300 + 1200l} \times £40.4027,$
or $63,500 \times £40.4027, = £2,565,571.4500.$

At the end of the fiftieth year,

$\overline{63,500 + 1100l} \times £40.4027,$
or $64,600 \times £40.4027. = £2,610,014.4200.$

At the end of the fifty-first year,

$\overline{64,600 + 1000l} \times £40.4027,$
or $65,600 \times £40.4027, = £2,650,417.1200.$

At the end of the fifty-second year,

$\overline{65,600 + 900l} \times £40.4027,$
or $66,500 \times £40.4027, = £2,686,779.5500.$

At the end of the fifty-third year,

$\overline{66,500 + 700l} \times £40.4027,$
or $67,200 \times £40.4027, = £2,715,061.4400.$

At the end of the fifty-fourth year,

$\overline{67,200 + 600} \times £40.4027,$

or $67,800 \times £40.4027, = £2,739,303.0600.$

At the end of the fifty-fifth year,

$\overline{67,800 + 500} \times £40.4027,$

or $68,300 \times £40.4027, = £2,759,504.4100.$

At the end of the fifty-sixth year,

$\overline{68,300 + 400} \times £40.4027,$

or $68,700 \times £40.4027, = £2,775,665.4900.$

At the end of the fifty-seventh year,

$\overline{68,700 + 300} \times £40.4027,$

or $69,000 \times £40.4027, = £2,787,786.3000.$

At the end of the fifty-eighth year,

$\overline{69,000 + 200} \times £40.4027,$

or $69,200 \times £40.4027, = £2,795,866.8400.$

At the end of the fifty-ninth year,

$\overline{69,200 + 100} \times £40.4027,$

or $69,300 \times £40.4027, = £2,799,907.1100.$

And at the end of the fixtieth and laſt year,

$\overline{69,300 + 100} \times £40.4027,$

[illegible]

Note,

Note. This laſt ſaving £2,803,947.3800, ought to be equal to the whole ſum allotted to the payment of the life-annuities; ſince at the time of its accruing to the publick all the life-annuitants are ſuppoſed to be dead. It ought therefore to be equal to £1000,000 + £1,803,953, or £2,803,953, which was the ſum allotted to that purpoſe. And ſo we find it very nearly is, ſince it differs from it by leſs than £6, which, upon ſo large a ſum as £2,803,953, is not worth attending to. And it would have been more nearly equal to £2,803,953, if in dividing £2,803,953 by 694 (in Art. CCLXI) we had carried the quotient to more than ſix places of figures. For, if we had carried it only to one figure more, we ſhould have found it to be 4040.278. And, in conſequence of this increaſe of this quotient, the annuity which would have belonged to each of the aforeſaid 69400 life-annuitants for his life, would have been £40.40278, inſtead of £40.4027; and 69400 × £40.40278 is = £2,803,952.93200, which differs from £2,803,953 by leſs than $\frac{1}{10}$ of a pound, or than two ſhillings. But the numbers £40.4027 and £2,803,947.3800 are near enough to the truth to anſwer the purpoſe of this computation.

A remark on the laſt ſaving, £ 2,803,947.3000.

CCLXVII. Now all theſe ſavings are to be employed, as faſt as they ariſe, for the purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-mentioned. By this means the laſt ſaving, £2,803,947.3800, will either extinguiſh £2,803,947.3800 of the capital of the 4 per cent. annuities, (which are ſuppoſed to ſell at their *par*, or nominal value,) or, if employed in the ſecond method above-deſcribed, it will extinguiſh a proportionally greater capital of the 3 per cent. annuities, which are ſuppoſed to ſell at the price of £75 per cent. and in either caſe it will redeem to the publick the intereſt of 4 per cent. upon it, or the annual ſum of $\left(\frac{4}{100} \times £2,803,947.3800, \text{ or } \frac{£ \, 11,215,789.5200}{100}, \text{ or}\right)$ £112,157.8952. But all the other ſavings will have time to perform more than one operation of this kind towards the diminution of the national debt, to wit, a new operation at the end of every year of the term that is remaining after the time when it accrued: and, in conſequence of theſe repeated operations, the quantity of the national debt which each of theſe ſavings will have extinguiſhed at the end of the ſaid term of 60 years, will be equal in value to the amount of ſuch ſaving at the end of the ſaid term, if improved in the mean time at compound intereſt at the rate of four per cent.

Of the effect of the ſaid ſavings in diminiſhing the national debt.

We muſt therefore now compute the amounts of theſe ſavings at the end of the ſaid term, if ſo improved at compound intereſt in the mean time, or during the remaining years of the ſaid term after the times at which they will have reſpectively accrued, that is, during 57 years, 56 years, 55 years, 54 years, &c. to the laſt year of the ſaid term. Now theſe amounts will be as follows.

A computation of the amounts of all the foregoing ſavings, except the laſt, at the end of the ſaid term of 60 years, if improved in the mean time at compound intereſt at the rate of 4 per cent.

CCLXVIII. The amount of £92,926.21, improved at compound intereſt at 4 per cent. during 57 years, is = £92,926.21 × 9.351,910 = £869,037.5525.

The amount of £121,208.1000, improved in the ſame manner during 56 years, is = £121,208.1000 × 8.992,221 = £1,089,930.0221.

The amount of £149,489.9900, improved in the ſame manner during 55 years, is = £149,489.9900 × 8.640,367 = £1,292,545.3163.

And the amounts of all the other ſavings, improved in the ſame manner during the remaining years of the ſaid term of 60 years after they ſhall have reſpectively accrued to the publick, will be as follows.

£177,771.8800	×	8.313,814	=	£1,477,962.3447.
£206,053.7700	×	7.994,052	=	£1,647,204.5521.
£234,335.6600	×	7.686,588	=	£1,801,241.6721.
£262,617.5500	×	7.390,950	=	£1,940,993.1811.
£290,899.4400	×	7.106,683	=	£2,067,330.1049.
£319,181.3300	×	6.833,349	=	£2,181,077.4221.
£351,503.4900	×	6.570,528	=	£2,309,563.5231.
£383,825.6500	×	6.317,815	=	£2,424,939.4489.
£420,188.0800	×	6.074,822	=	£2,552,567.7925.
£456,550.5100	×	5.841,175	=	£2,666,791.4252.
£496,953.2100	×	5.616,515	=	£2,791,145.1576.
£541,396.1800	×	5.400,495	=	£2,923,807.3631.
£585,839.1500	×	5.192,783	=	£3,042,135.5788.
£630,282.1200	×	4.993,061	=	£3,147,037.0723.
£678,765.3600	×	4.801,020	=	£3,258,766.0686.
£727,248.6000	×	4.616,366	=	£3,357,245.7105.

£775,731.8400

£775,731.8400	×	4.438,813	=	£3,443,328.5759.
£828,255.3500	×	4.268,089	=	£3,535,067.5285.
£880,778.8600	×	4.103,932	=	£3,614,656.5484.
£933,302.3700	×	3.946,089	=	£3,682,894.2159.
£985,825.8800	×	3.794,316	=	£3,740,534.9097.
£1,038,349.3900	×	3.648,381	=	£3,788,294.1858.
£1,094,913.1700	×	3.5[illegible]8,058	=	£3,841,018.9[illegible]53.
£1,151,476.9500	×	3.373,133	=	£3,884,084.8987.
£1,208,040.7300	×	3.243,397	=	£3,918,155.6795.
£1,268,644.7800	×	3.118,651	=	£3,956,460.3117.
£1,333,289.1000	×	2.998,703	=	£3,998,138.0240.
£1,401,97[illegible].6900	×	2.883,368	=	£4,042,406.0745.
£1,[illegible]74,698.5500	×	2.772,469	=	£4,088,556.0142.
£1,551,463.6800	×	2.665,836	=	£4,135,947.7308.
£1,628,228.8100	×	2.563,304	=	£4,173,645.4215.
£1,709,034.2100	×	2.464,7[illegible]5	=	£4,212,282.2529.
£1,789,839.6100	×	2.369,918	=	£4,241,773.0758.
£1,870,645.0100	×	2.278,768	=	£4,262,765.9881.
£1,951,450.4100	×	2.191,123	=	£4,275,867.8767.
£2,028,215.5400	×	2.106,849	=	£4,273,143.8822.
£2,104,980.6700	×	2.025,816	=	£4,264,203.5209.
£2,181,745.8000	×	1.947,900	=	£4,249,822.6438.
£2,254,470.6600	×	1.872,981	=	£4,222,580.7112.
£2,327,195.5200	×	1.80[illegible],943	=	£4,191,146.4813.
£2,395,880.1100	×	1.731,676	=	£4,148,[illegible]88.0853.
£2,460,524.4300	×	1.665,073	=	£4,096,952.7942.
£2,517,088.2100	×	1.601,032	=	£4,029,938.7710.
£2,565,571.4500	×	1.539,454	=	£3,94[illegible],579.2309.
£2,610,014.4200	×	1.480,244	=	£3,863,458.1851.
£2,650,417.1200	×	1.423,311	=	£3,772,3[illegible]7.8414.

£2,686,779.5500

£2,686,779.5500	×	1.368,569	=	£3,677,043.2019.
£2,715,061.4400	×	1.315,931	=	£3,572,853.5158.
£2,739,303.0600	×	1.265,319	=	£3,466,092.2085.
£2,759,504.4100	×	1.216,652	=	£3,357,356.5594.
£2,775,665.4900	×	1.169,858	=	£3,247,134.4788.
£2,787,786.3000	×	1.124,864	=	£3,135,880.4485.
£2,795,866.8400	×	1.081,600	=	£3,024,009.5741.
£2,799,907.1100	×	1.040,000	=	£2,911,903.3944.

The addition of the amounts of all the yearly savings at the end of the said term of 60 years, into one sum.

CCLXIX. Having thus found the amounts of the several yearly savings at the end of the said term of 60 years, we must now add these several amounts together; which may be done as follows.

The amounts of the savings made at the ends of the first ten years, the second ten years, and the third ten years, of the said term, are as follows.

£	£	£
326,939.4887	2,181,077.4221	3,357,245.7105
628,729.7513	2,309,563.5231	3,443,328.5759
869,037.5525	2,424,939.4489	3,535,067.5285
1,089,930.0221	2,552,567.7925	3,614,656.5484
1,292,545.1668	2,666,791.4252	3,682,894.2159
1,477,962.3447	2,791,145.1576	3,740,534.9097
1,647,204.5521	2,923,807.3631	3,788,294.1858
1,801,241.6721	3,042,135.5788	3,841,018.9053
1,940,993.1811	3,147,037.0723	3,884,084.8987
2,067,330.1049	3,258,766.0686	3,918,155.6795
13,141,913.8363	27,297,830.8522	36,805,281.1582

And

And the amounts of the ſavings made at the ends of the fourth ten years, the fifth ten years, and the ſixth ten years, are as follows.

£	£	£
3,956,460.3117	4,273,143.8822	3,772,367.8414
3,998,138.0240	4,264,203.5209	3,677,043.2019
4,042,406.0745	4,249,822.6438	3,572,833.5158
4,088,556.0142	4,222,580.7112	3,466,092.2085
4,135,947.7308	4,191,146.4813	3,357,356.5594
4,173,645.4215	4,148,888.0853	3,247,134.4788
4,212,282.2529	4,096,952.7942	3,135,880.4485
4,241,772.0758	4,029,938.7710	3,024,009.5741
4,262,765.9881	3,949,579.2309	2,911,903.3944
4,275,867.8767	3,863,458.1851	2,803,947.3800
41,387,842.7702	41,289,714.3059	32,968,568.6028

And the ſum total of theſe ſix ſeveral ſums, or the ſum total of the amounts that will ariſe from all the ſavings made at the ends of all the 60 years of the ſaid term, at the end of the ſaid term, is £192,891,151.5256.

£
13,141,913.8363
27,297,830.8522
36,805,281.1582
41,387,842.7702
41,289,714.3059
32,968,568.6028
192,891,151.5256

CCLXX. Therefore the quantity of the capital of the national debt that would have been extinguiſhed by means of all theſe ſavings at the end of the ſaid term of 60 years, will be equal in value to the ſaid ſum of £192,891,151.5256; of which the annual intereſt at 4 per cent. is $\left(\frac{4}{100} \times £192,891,151.5256, \text{ or } \frac{£771,564,606.1024}{100}, \text{ or}\right)$ £7,715,646.0610. The quantity of the national debt that will be extinguiſhed at the end of the ſaid term of 60 years by means of all the ſaid yearly ſavings.

Therefore

The quantity of the interest of the said debt that will thereby be redeemed to the publick at the end of the said term of 60 years.

Therefore the quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of the said interest, that would be redeemed to the publick at the end of the said term of 60 years by means of all these savings, (so that at the end of the next, or 61st, year it would be free to be disposed of by the Parliament in any other way,) is £7,715,646.0610 *per annum*. Q E I.

The whole quantity of the interest of the national debt that will be redeemed to the publick at the end of the said term of 60 years by the foregoing scheme of life-annuities.

CCLXXI. This sum of £7,715,646.0610 *per annum* must therefore be added to £1,803,953 *per annum*, which was the quantity of the perpetual annuities that was supposed to be converted into life-annuities for the lives of 69400 persons, all of the age of 35 years, and which will therefore be now wholly redeemed to the publick by the deaths of all the said life-annuitants: and the sum of both, or the sum of £9,519,599.0610, or (neglecting the fraction .0610,) £9,519,599 *per annum* will be the whole quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the said term of 60 years (so as to be free at the end of the next, or 61st, year to be disposed of by the Parliament in any other way,) by the faithful application of a million of pounds sterling a year to that salutary purpose during that period according to the fourth method above-described. Q E I.

Agreement between the effect of the foregoing scheme of life-annuities, and the effects of the three preceeding methods.

CCLXXII. This sum of £9,519,599 *per annum*, which is the quantity of the interest of the national debt that would be redeemed to the publick in the course of 60 years by the application of a million of pounds sterling *per annum* in this fourth method, or in the way of life-annuities, is the same with the quantity of the interest of the said debt that would have been redeemed to the publick in the same time by applying a million of pounds *per annum* to the same purpose in either of the three former methods. For that was shewn above, (in Art. CCXLIV, CCXLVII, CCLIII,) to be £9,520,000 *per annum*; which differs from £9,519,599 *per annum*, by only £401 *per annum*, which is a mere trifle upon so large a sum. And, if we had computed the said former quantity more accurately, we should have found it to approach still nearer to £9,519,599. For it is not exactly equal to £9,520,000 *per annum*, or 238 × £40,000 *per annum*, (as it is there, for the sake of round numbers, supposed to be,) but more nearly to 237.9906 × £40,000 *per annum*, that is, to £9,519,624.0000, or £9,519,624, *per annum*; which differs from £9,519,599 *per annum* by only £25 *per annum*. They may therefore be considered as equal; agreeably to what is asserted in Art. CCLVI.

CCLXXIII. We

General conclusion concerning the efficacy of this fourth method of applying a given sum of money every year to the diminution of the national debt.

CCLXXIII. We may therefore conclude that this fourth method of applying a million of pounds sterling, or any other given sum of money, every year, during a given number of years, to the purpose of diminishing the national debt, by converting some of the perpetual annuities, now due to the publick creditors, into life-annuities, would be just as efficacious in redeeming the interest of the said debt, and consequently just as beneficial to the nation, as either of the three former methods of applying the same sum of money to the same purpose during the same period of time; to prove which was the principal object of the foregoing computation. But this, it must be remembered, is true only upon a supposition that the savings of the interest of the national debt, that will accrue to the publick every year by the deaths of some of the life-annuitants, are immediately employed in discharging a part of the said debt in either the first or second method above-described, or in some other method that is equivalent to them. For, if those savings are not so employed, but are diverted to other uses, the conversion of the perpetual annuities, now due to the publick creditors, into life-annuities would be evidently a *much less* efficacious method of diminishing the national debt than either of the three former methods above-mentioned. Thus, for example, we have seen that, if the said savings are employed in the manner above-described, the sum of one million of pounds, sterling, a year will be sufficient to redeem to the publick, in the course of 60 years, the sum of £9,519,599 *per annum* of the interest of the national debt; but without the help of the said savings, it will redeem in that time only £1,803,953 *per annum*. Therefore, whenever any proposal is made to pay off any part of the national debt by means of life-annuities, great care must be taken that the savings of the interest of the said debt, that will continually accrue to the publick by the deaths of some of the life-annuitants, shall be constantly applied to the same purpose; or otherwise the nation must (as Dr. Price has justly observed in his Essay on Publick Credit and the National Debt in the third chapter of his Treatise on Reversionary Payments, first edition, page 149,) lose greatly by all schemes of this kind.

A comparison between this fourth method and the third method, with respect to the advantage of securing to the publick the faithful application of the said annual sum of money to the purposes of its destination.

CCLXXIV. This fourth method of employing a given sum of money every year in the diminution of the national debt, "by converting some of the perpetual annuities, that now pay the interest of it, into life-annuities, and allotting the said annual sum to the payment of the additions thereupon made to the perpetual annuities so converted," partakes in some degree of the advantage mentioned in Art CCLIV as belonging to the third method, to wit, that of securing to the publick the faithful application of such annual sum to the purposes of its destination, without suspension or interruption. For it would then be impossible to with-hold any

part

part of the life-annuities fo created from the perfons to whom they would be due, without an abfolute breach of the publick faith; which, (as we before obferved,) may be confidered as a moral impoffibility. But this advantage will relate only to fo much of the publick revenue as will be due at any time to the life-annuitants that are ftill alive; and not to that other part of it which was allotted to the payment of the annuities of the other life-annuitants who will have died fince the eftablifhment of them, and which will be every year increafing by the deaths of more of the faid life-annuitants. For this part of the revenue, which will be thus faved every year to the publick, will be as liable to be diverted, by the minifters of ftate and the Parliament, from its original deftination of diminifhing the national debt, in order to defray the expence of fome temporary meafure of Government, as the money that fhould be allotted to the diminution of the national debt in either the firft or the fecond method above-defcribed: whereas in the third method of applying a given fum of money every year to the diminution of the national debt, "by converting fome of the perpetual annuities, that now pay the intereft of it, into greater temporary annuities that fhould continue for a certain limited number of years, and allotting the faid annual fum to the payment of the additions thereupon made to the perpetual annuities fo converted," the whole of the faid annual fum is effectually fecured from being diverted from its original and proper deftination, and applied to any other purpofe whatfoever, during the whole of the faid term.

A fifth method of employing a given fum of money every year in the reduction of the national debt.

CCLXXV. A fifth method of applying a given fum of money every year to the diminution of the national debt, would be to combine the foregoing third and fourth methods together, by converting fome of the perpetual annuities now due to the publick creditors, into greater temporary annuities of a mixt nature, or that fhould continue both for a certain moderate number of years at all events, or whether the proprietors of them lived to the end of the faid term, or not, and fhould, in cafe of their dying before the end of the faid term, be paid to their executors or other perfonal reprefentatives, and fhould likewife continue during the lives of fuch of the faid proprietors, or of other perfons to be named by the faid proprietors, as fhould live beyond the end of the faid term, and by employing the faid annual fum of money in paying the additions that it would be neceffary to make to the perpetual annuities fo converted, in compenfation for the abridgement of their duration: fuch converfion being made (as in the fecond, third, and fourth methods above-mentioned,) with the confent of the proprietors of the annuities fo converted.

CCLXXVI. In

Of the savings that would accrue every year to the publick, after the expiration of a certain term, by the deaths of life-annuitants.

CCLXXVI. In this fifth method of employing a given sum of money every year for the purpose of diminishing the national debt, it is evident there would be no savings accruing to the publick till after the expiration of the limited term of years during which the new annuities, into which the perpetual annuities had been converted, were to continue at all events, whether the proprietors of the said perpetual annuities, or their *nominees*, were living or dead. But after the expiration of the said term there would accrue to the publick very large savings every year by the deaths of some of the persons for whose lives the said annuities should have been granted. Now these savings ought to be employed, as fast as they arose, in diminishing the national debt in either the first or the second method above-described, or in some other method equivalent to them.

CCLXXVII. Thus, for example, it might be expedient to convert a million, or two millions, or, perhaps, three millions, of pounds, sterling, *per annum* of the perpetual annuities, now due to the publick creditors, (with the consent of the proprietors of such annuities,) into annuities which should continue for 30 years at all events, and likewise, after the expiration of the said 30 years, during the lives of their respective proprietors, or of other persons that should be named by the said proprietors, and who may be called their *nominees*. And then the million of pounds, sterling, a year, or other given annual sum that could be spared out of the publick revenue for this purpose, or a part of the said sum, might be employed in paying the additions which it would be necessary to make to the perpetual annuities so converted, in order to make amends to the proprietors of them for this abridgement of the time of their continuance. And, as there would accrue to the publick at the end of the said term of 30 years, during which the said annuities were to continue at all events, a great saving of interest by the deaths of the many life-annuitants who would have died in the course of the said term;—and other and still greater savings of interest would accrue in like manner to the publick at the end of every following year after the expiration of the said term by the deaths of life-annuitants, until all the said life-annuitants were dead;—these several savings should, as fast as they accrued, be employed, for the same purpose of diminishing the national debt, in either the first or the second method above-mentioned.

This fifth method of diminishing the national debt, if properly pursued, would be equally effectual for that purpose with either of the four preceding methods.

CCLXXVIII. This fifth method of paying off a part of the national debt, by converting some of the perpetual annuities, now due to the publick creditors, into these compound, temporary, annuities, (which are to continue at all events during a certain given number of years, and further for the lives of certain life-annuitants,) would have exactly the same effect towards diminishing the national debt, in any given period of time, as either of the four preceeding methods; that is, if a given sum of money was every year to be faithfully applied to the diminution of the national debt in this fifth method during a given number of years;—and the savings of interest accruing to the publick, by the deaths of the said life-annuitants, at the expiration of the said term certain (during which the annuities were to continue at all events,) and at the ends of all the following years, after the expiration of the said term, during the lives of any of the said life-annuitants, were, as soon as they accrued, to be faithfully employed, for the same purpose of diminishing the national debt, in either the first or the second method above-described;—the quantity of the interest of the national debt, or of the perpetual annuities now allotted to the payment of it, that would be redeemed to the publick at the end of any supposed number of years, by employing the said given sum of money in this fifth method, would be the same as the quantity of the said interest that would have been redeemed to the publick by the same annual sum of money, if it had been applied to the same useful purpose during the same number of years in either of the four preceeding methods. Of this it may not be amiss to give the following example.

An example of the said fifth method of applying a given sum of money every year to the diminution of the national debt.

Preliminary suppositions.

CCLXXIX. Let us suppose (as in the foregoing articles,) that the sum of money that is to be applied every year to the diminution of the national debt in this fifth method, is a million of pounds, sterling, a year; and that the period during which it is to be so applied, is 60 years. And let us suppose further, that the persons upon whose lives the new annuities, (which are, according to this fifth method, to be established in lieu of some of the perpetual annuities now due to the publick creditors,) are to depend, are all, at the time of establishing these annuities, of the age of 35 years; and that they will die off in the course of the following years in the proportions expressed in Monsieur de Parcieux's table of the probabilities of the duration of human life; and consequently that the longest liver of them will live almost through the whole of the said period of 60 years, and die a few days before the end of it. And let the number of years during which the said annuities are to continue at all events, or whether the said life-annuitants are alive or dead, (and during which they are to be paid to the said life-annuitants, if they are alive, and, if they are dead, to their executors or other personal representatives,) be 30 years. And let the interest of money be supposed (as in the foregoing articles) to be 4 per cent. and the price at which the 3 per cent. annuities sell at the publick market, to be £75 per cent.

CCLXXX. These

CCLXXX. Thefe things being premifed, we muft inquire, in the next place, what quantity of the perpetual annuities, which now are employed in paying the intereft of the national debt, might be converted into thefe compound, temporary, annuities (which are at the fame life-annuities and annuities for a term certain of 30 years,) by the help of the faid fum of a million of pounds, fterling, a year. Now this quantity may be determined in the manner following.

Of the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into thefe compound life annuities by the help of a million of pounds *per annum.*

CCLXXXI. It appears from Mr. Smart's fourth table of compound intereft, page 78, that the value of an annuity of one pound a year for a term certain of 30 years, is £17.292,03. And it appears above in Table XXV, page 285, that the value of a remote life-annuity of one pound a year for the life of a perfon of the age of 35 years, that is to commence at the diftance of 30 years, or whereof the firft payment is to be received at the end of 31 years, is £1.410,77. It follows therefore that the value of a compound, temporary, annuity of one pound a year, that is to continue for 30 years certain, and alfo for the life of a perfon of the age of 35 years, muft be £17.292,03 + £1.410,77, or £18.702,80. Now, when the intereft of money is 4 per cent. (as it is here fuppofed to be,) the value of a perpetual annuity of one pound a year is £25. Therefore, in order to make an annuity of one pound a year of the compound and temporary kind here defcribed, or that is to continue only for 30 years certain and during the life of a perfon of the age of 35 years, equal in value to a perpetual annuity of one pound a year, we muft increafe it in the proportion of £25 to £18.702,80; by which means it will become equal to $\left(\frac{25.00000}{18.70280} \times £1, \text{ or}\right)$ £1.336,698. Therefore a compound, temporary, annuity of £1.336,698 a year, that is to continue for 30 years certain and alfo for the life of a perfon of the age of 35 years, will be equal in value to a perpetual annuity of one pound a year: and confequently, if the Government were to convert fome of the perpetual annuities, now due to the publick creditors as the intereft of their debts, into compound, temporary, annuities of the kind here defcribed, or that fhould continue for 30 years certain and further for the lives of perfons of the age of 35 years, they ought to allow to the faid publick creditors, whofe perpetual annuities fhould be fo converted into thefe temporary ones, an annuity of £1.336,698 a year of the kind here defcribed, for every perpetual annuity of £1 a year of which they had been before poffeffed. Therefore for every million of pounds *per annum* of the perpetual annuities that fhould be fo converted, it would be neceffary to allow to the proprietors of them the fum of a million times £1.336,698 *per annum*, or of £1,336,698 *per annum* of thefe compound and temporary annuities: and confequently

An inveftigation of the faid quantity.

consequently an additional annual sum of £336,698 would be required to enable the Government to convert £1,000,000 *per annum* of the perpetual annuities, now due to the publick creditors as the interest of their debts, into these compound, temporary, annuities for 30 years certain and for the lives of persons of the age of 35 years. Therefore an additional annual sum of £1,000,000 would be sufficient to enable the Government to convert into these compound, temporary, annuities for 30 years certain and for the lives of persons of the age of 35 years, a quantity of the perpetual annuities that is greater than £1,000,000 *per annum* in the same proportion as £1,000,000 is greater than £336,698, that is, a quantity of the said perpetual annuities that is equal to

$$\left(£1{,}000{,}000 \times \frac{1{,}000{,}000}{336{,}698}, \text{ or } \frac{\overset{£}{1{,}000{,}000{,}000{,}000}}{336{,}698}, \text{ or}\right) £2{,}970{,}020 \textit{ per annum}.$$ Q.E.I.

CCLXXXII. It is evident therefore that at the end of the said term of 60 years. when all the said life-annuitants (who were of the age of 35 years at the beginning of the said term,) will be dead, the said sum of £2,970,020 *per annum* of the perpetual annuities, now due to the publick creditors, will be intirely redeemed to the publick by means of this conversion of them into these compound, temporary, annuities.

Of the savings that would accrue to the publick at the end of every year, after the first 30 years, by the deaths of life-annuitants.

CCLXXXIII. This would be the quantity of the said perpetual annuities that would be redeemed to the nation by this operation in the course of 60 years, if all the aforesaid life-annuitants had lived to within a day of the end of the whole 60 years, or if, when they had died off in the different years of the said term of 60 years according to the course of nature, the savings which would have accrued every year to the publick by their deaths during the last 30 years of the said term, had not been applied to the same purpose of diminishing the national debt, but had been spent upon some other service. But it was supposed above that these savings were to be employed continually, as fast as they arose, for the same purpose of diminishing the national debt, in either the first, or the second, of the methods above-described. We must therefore now proceed to inquire, what these savings would amount to in every year of the said term of 60 years after the first 30 years of it, and what quantity of the perpetual annuities, now due to the publick creditors, they would be sufficient to redeem in the course of the latter half of the said term, if they were constantly employed for that purpose, until the end of the said term, in either the first or the second method above-mentioned. These inquiries may be made in the manner following.

CCLXXXIV. The

Of the number of perſons ſuppoſed to be life-annuitants, and the value of their annuities.

CCLXXXIV. The number of perſons repreſented in Monſieur de Parcieux's table of probabilities as living at the age of 35 years, is 694. The money that is to be paid annually by the Government in the aforeſaid compound annuities (for the lives of ſeveral perſons of the age of 35 years, and, in caſe of their dying within the ſpace of 30 years, to the executors of ſuch deceaſed life-annuitants for the remainder of the ſaid term of 30 years,) in lieu of the £2,970,020 *per annum* of perpetual annuities which are to be thereby redeemed, is the ſaid ſum of £2,970,020 *per annum* together with the annual million of pounds which is deſtined to this purpoſe, and conſequently is £3,970,020 *per annum.* Therefore, if we ſuppoſe theſe compound annuities to be diſtributed equally amongſt 694 perſons, all of the age of 35 years, each of the ſaid perſons will be poſſeſſed of an annuity, for 30 years certain and alſo for his life, of

$$\left(\frac{£3,970,020}{694}, \text{ or } £5720.4899, \text{ or, very nearly,}\right) £5720.4900 \textit{ per annum.}$$

And conſequently, if we ſuppoſe them to be equally diſtributed amongſt 6940 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of an annuity, of the ſame kind, or £572.0490 *per annum:* and, if we ſuppoſe them to be equally diſtributed amongſt 69400 perſons, all of the ſame age of 35 years, each of the ſaid perſons will be poſſeſſed of an annuity, of the ſame kind, of £57.2049, or 57*l.* 4*s.* 1*d.*$\frac{1}{4}$, *per annum.* Let us make this laſt ſuppoſition.

Of the ſaving made at the end of the 31ſt year by the deaths of ſome of the life-annuitants.

CCLXXXV. Then, ſince it appears by Monſieur de Parcieux's table of the probabilities of the duration of human life, that out of 694 perſons of the age of 35 years, all living at the ſame time, only 380 perſons will be living at the end of 31 years, or at the age of 66 years, it follows that out of the ſaid 69400 perſons above-mentioned, of the age of 35 years, all living at the ſame time, (for whoſe lives, and likewiſe for a term certain of 30 years, theſe annuities of £57.2049, or 57*l.* 4*s.* 1*d.*$\frac{1}{4}$ each, are ſuppoſed to be granted,) only 38000 will be living at the end of 31 years, or at the age of 66 years. And conſequently the annuities of £57.2049, or 57*l.* 4*s.* 1*d.*$\frac{1}{4}$ each, which would have been payable to the other (69400 — 38000, or) 31400 life-annuitants at the end of the ſaid 31 years, if the ſaid annuitants had been then living, will be ſaved to the publick, and will be ready in the publick treaſury to be employed, in any manner that may be thought fit, towards the diminution of the national debt. The amount of theſe annuities is 31,400 times £57.2049, or £1,796,233.8600. Now, if this ſum of £1,796,233.8600 is immediately laid out either in paying off ſome of the capital of the 4 per cent. annuities at *par*, according to the firſt method above-deſcribed, or in buying up ſome of the capital of the 3 per cent. annuities at the price of £75 per cent.

Of the manner in which the ſaid ſaving is to be employ'd during the remaining 29 years of ſaid term 60 years.

cent. according to the fecond method above-defcribed; and the intereft of the capital thereby extinguifhed, or the portion of the perpetual annuities that will be redeemed at the end of the next, or 32d, year, by thus employing the faid fum of £1,796,233.8600, be likewife employed in the fame manner; and the intereft of the capital extinguifhed by this fecond operation, or the portion of the perpetual annuities that will thereby be redeemed at the end of the 33d year, be likewife employed in the fame manner; and all the following portions of the perpetual annuities that will be thus redeemed by means of this firft fum of £1,796,233.8600, that is faved to the nation at the end of the 31ft year by the deaths of the aforefaid 31400 life-annuitants, be likewife employed in the fame manner during the whole remainder of the faid term of 60 years;——I fay, if thefe fucceffive fums of money be fo employed without interruption during the remainder of the faid term of 60 years, the quantity of the capital of the national debt that will be thereby extinguifhed at the end of the faid term of 60 years, or at the end of 29 years from the time when this firft faving of £1,796,233.8600 will have accrued, will be equal in value to the fum to which the faid firft faving of £1,796,233.8600 would have increafed in the fame period of 29 years, if it had been improved all the time at compound intereft at the rate of 4 per cent. and therefore it will (according to Mr. Smart's firft table of compound intereft, page 54,) be equal in value to the fum of £1,796,233.8600 × 3.118,651, or £5,601,826.5237; the intereft of which at 4 per cent. is $\left(\frac{4}{100}\times\right.$ £5,601,826.5237, or $\frac{£22,407,306.0948}{100}$, or$\left.\right)$ £224,073.0609 *per annum*.

The quantity of the capital of the national debt that will have been extinguifhed by means of the faid faving at the end of the faid term of 60 years.

The quantity of the intereft of the faid debt that will have been thereby redeemed at the end of the faid term of 60 years.

Therefore the quantity of the intereft of the national debt, or of the perp[illegible]nuities now due to the publick creditors, which would be redeeme[illegible] end of the whole term of 60 years by means of this firft faving of [illegible]96,233.8600, which would have accrued to the publick at the end of the 31ft year of the faid term by the deaths of the aforefaid 31,400 life-annuitants, would be £224,073.0609 *per annum*.

Of the faving made at the end of the thirty-fecond year by the deaths of life-annuitants.

CCLXXXVI. In the courfe of the thirty-fecond year of this term the 38,000 life-annuitants of the age of 66 years who were living at the end of the thirty-firft year, or at the age of 66 years, will be reduced to 36,400. Therefore 1600 more of the faid life-annuitants will have died in the courfe of this thirty-fecond year. And confequently at the end of this thirty-fecond year there will be a faving to the Government of (31,400 + 1,600, or) 33,000 payments of £57.2049 each, which would have then become due to the 33,000 life annuitants who will have died in the thirty-

two

two preceeding years, if they had lived till that time. There will therefore be in the hands of Government at the end of this thirty-ſecond year the ſum of 33,000 × £57.2049, or £1,887,761.7000, to be employed during the remainder of the ſaid term of 60 years, that is, during 28 years, in diminiſhing the national debt; which will be ſufficient to extinguiſh as much of the capital of the ſaid debt as is equal in value to the ſum to which this ſecond ſaving of £1,887,761.7000 will increaſe in the courſe of 28 years, if it be conſtantly improved during that time at compound intereſt at the rate of 4 per cent. Now the amount of £1,887,761.7000 at the end of 28 years, when improved during that time at compound intereſt at the rate of 4 per cent. appears by Mr. Smart's firſt table of compound intereſt, page 54, to be £1,887,761.7000 × 2.998,703, or £5,660,836.6730; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times \right.$ £5,660,836.6730, or $\frac{£22,643,346.6920}{100}$, or$\left.\right)$ £226,433.4669 *per annum*.

Therefore the quantity of the capital of the national debt which would be extinguiſhed at the end of the whole term of 60 years by means of this ſecond ſaving of £1,887,761.7000, (which would have accrued to the publick at the end of the thirty-ſecond year of the ſaid term by the deaths of the aforeſaid 33,000 life-annuitants who would have died in the preceeding 32 years,) would be equal in value to the ſum of £5,660,836.6730; and the quantity of the intereſt of the ſaid debt, or of the perpetual annuities now due to the publick creditors, which would be redeemed at the end of the ſaid term of 60 years by means of the ſaid ſecond ſaving, would be the ſum of £226,433.4669 *per annum*. Q.E.I.

Of the ſavings that will be made at the ends of the 33d, 34th, 35th, and other following years of the ſaid term of 60 years, by the deaths of life-annuitants.

CCLXXXVII. In the ſame manner we muſt proceed to inveſtigate the ſavings that will accrue to the publick at the ends of the thirty-third, the thirty-fourth, the thirty-fifth, the thirty-ſixth, and every following year of the whole term of 60 years, and the quantities of the capital of the national debt that will be extinguiſhed by each of the ſaid ſavings at the end of the ſaid term of 60 years, if they are conſtantly employed for that purpoſe, in either the firſt or the ſecond method above deſcribed, during the ſeveral remaining years of the ſaid term after the times when they will have reſpectively accrued. And, in order to theſe inveſtigations, it will be neceſſary, in the firſt place, to aſcertain, by the help of Monſieur de Parcieux's table of the probabilities of the duration of human life, the numbers of life-annuitants who will be alive at the ends of the ſeveral remaining years of this term, and thence to determine the numbers of thoſe who will have died in each of the ſaid years, and from whoſe deaths the ſeveral new and ſucceſſive ſavings to the publick will ariſe.

The numbers of life-annuitants who will be living at the ends of all the faid years, except the laft year.

CCLXXXVIII. Now of the aforefaid original number of 69,400 life-annuitants, all of the age of 35 years, who were fuppofed to be living at the beginning of the faid term of 60 years, and for whofe lives (as well as for a term certain of 30 years,) the faid annuities, of £57.2049 each, were fuppofed to be granted, it appears from Monfieur de Parcieux's table of probabilities that the following numbers will be living at the fubfequent ages of 68 years, 69 years, 70 years, 71 years, &c. as far as the age of 94 years inclufive, or at the ends of the thirty-third, the thirty-fourth, the thirty-fifth, the thirty-fixth, and every following year of the faid term of 60 years, except the laft; to wit,

34,700,	21,100,	8,500,	1,600,
32,900,	19,200,	7,100,	1,100,
31,000,	17,300,	5,900,	700,
29,100,	15,400,	4,800,	400,
27,100,	13,600,	3,800,	200,
25,100,	11,800,	2,900,	100.
23,100,	10,100,	2,200,	

The numbers of life-annuitants who will have died in each of the faid years, and in the laft year.

Therefore the numbers of the faid life-annuitants who will have died in each of thofe years, to wit, the 33d, the 34th, the 35th, the 36th, &c. and in the laft year of the faid period of 60 years, or from the age of 94 years to the age of 95 years, are as follows; to wit,

1700,	2000,	1600,		600,
1800,	1900,	1400,		500,
1900,	1900,	1200,		400,
1900,	1900,	1100,		300,
2000,	1800,	1000,		200,
2000,	1800,	900,		100,
2000,	1700,	700,	and	100.

The favings that will be made at the ends of the faid years in confequence of the deaths of the faid life-annuitants.

CCLXXXIX. Therefore the favings of intereft that will be made by the publick at the ends of thofe years in confequence of the deaths of life-annuitants that will have happened in the courfe of them refpectively, will be as follows; to wit,

At

At the end of the thirty-third year,

$\overline{33{,}000 + 1{,}700} \times £57.2049,$

or $34{,}700 \times £57.2049, = £1{,}985{,}010.0300.$

At the end of the thirty-fourth year,

$\overline{34{,}700 + 1800} \times £57.2049,$

or $36{,}500 \times £57.2049, = £2{,}087{,}978.8500.$

At the end of the thirty-fifth year,

$\overline{36{,}500 + 1900} \times £57.2049,$

or $38{,}400 \times £57.2049, = £2{,}196{,}668.1600.$

At the end of the thirty-sixth year,

$\overline{38{,}400 + 1900} \times £57.2049,$

or $40{,}300 \times £57.2049, = £2{,}305{,}357.4700.$

At the end of the thirty-seventh year,

$\overline{40{,}300 + 2000} \times £57.2049,$

or $42{,}300 \times £57.2049, = £2{,}419{,}767.2700.$

At the end of the thirty-eighth year,

$\overline{42{,}300 + 2000} \times £57.2049,$

or $44{,}300 \times £57.2049, = £2{,}534{,}177.0700.$

At the end of the thirty-ninth year,

$\overline{44{,}300 + 2000} \times £57.2049,$

or $46{,}300 \times £57.2049, = £2{,}648{,}586.8700.$

At the end of the fortieth year,

$\overline{46,300 + 2000} \times \pounds 57.2049,$

or $48,300 \times \pounds 57.2049, = \pounds 2,762,996.6700.$

At the end of the forty-first year,

$\overline{48,300 + 1900} \times \pounds 57.2049,$

or $50,200 \times \pounds 57.2049, = \pounds 2,871,685.9800.$

At the end of the forty-second year,

$\overline{50,200 + 1900} \times \pounds 57.2049,$

or $52,100 \times \pounds 57.2049, = \pounds 2,980,375.2900.$

At the end of the forty-third year,

$\overline{52,100 + 1900} \times \pounds 57.2049,$

or $54,000 \times \pounds 57.2049, = \pounds 3,089,064.6000.$

At the end of the forty-fourth year,

$\overline{54,000 + 1800} \times \pounds 57.2049,$

or $55,800 \times \pounds 57.2049, = \pounds 3,192,033.4200.$

At the end of the forty-fifth year,

$\overline{55,800 + 1800} \times \pounds 57.2049,$

or $57,600 \times \pounds 57.2049, = \pounds 3,295,002.2400.$

At the end of the forty-sixth year,

$\overline{57,600 + 1700} \times \pounds 57.2049,$

or $59,300 \times \pounds 57.2049, = \pounds 3,392,250.5700.$

At

At the end of the forty-seventh year,

$$\overline{59{,}300 + 1600}| \times £57.2049,$$
$$\text{or } 60{,}900 \times £57.2049, = £3{,}483{,}778.4100.$$

At the end of the forty-eighth year,

$$\overline{60{,}900 + 1400}| \times £57.2049,$$
$$\text{or } 62{,}300 \times £57.2049, = £3{,}563{,}865.2700.$$

At the end of the forty-ninth year,

$$\overline{62{,}300 + 1200}| \times £57.2049,$$
$$\text{or } 63.500 \times £57.2049, = £3{,}632{,}511.1500.$$

At the end of the fiftieth year,

$$\overline{63{,}500 + 1100}| \times £57.2049,$$
$$\text{or } 64{,}600 \times £57.2049, = £3{,}695{,}436.5400.$$

At the end of the fifty-first year,

$$\overline{64{,}600 + 1000}| \times £57.2049,$$
$$\text{or } 65{,}600 \times £57.2049, = £3{,}752{,}641.4400.$$

At the end of the fifty-second year,

$$\overline{65{,}600 + 900}| \times £57.2049,$$
$$\text{or } 66{,}500 \times £57.2049, = £3{,}804{,}125.8500.$$

At the end of the fifty-third year,

$$\overline{66{,}500 + 700}| \times £57.2049,$$
$$\text{or } 67{,}200 \times £57.2049, = £3{,}844{,}169.2800.$$

At the end of the fifty-fourth year,

$\overline{67{,}200 + 600} \times$ £57.2049,

or 67,800 × £57.2049, = £3,878,492.2200.

At the end of the fifty-fifth year,

$\overline{67{,}800 + 500} \times$ £57.2049,

or 68,300 × £57.2049, = £3,907,094.6700.

At the end of the fifty-sixth year,

$\overline{68{,}300 + 400} \times$ £57.2049,

or 68,700 × £57.2049, = £3,929,976.6300.

At the end of the fifty-seventh year,

$\overline{68{,}700 + 300} \times$ £57.2049,

or 69,000 × £57.2049, = £3,947,138.1000.

At the end of the fifty-eighth year,

$\overline{69{,}000 + 200} \times$ £57.2049,

or 69,200 × £57.2049, = £3,958,579.0800.

At the end of the fifty-ninth year,

$\overline{69{,}200 + 100} \times$ £57.2049,

or 69,300 × £57.2049, = £3,964,299.5700.

And at the end of the sixtieth and last year,

$\overline{69{,}300 + 100} \times$ £57.2049,

or 69,400 × £57.2049, = £3,970,020.0600.

CCXC. All

CCXC. All these savings are to be laid out in diminishing the national debt in either the first or the second method above-described; by which means the portion of the national debt that will have been extinguished at the end of the said term of 60 years in consequence of each of these savings, except the last, to [illegible] £3,970,020.0600, or (neglecting the fraction .0600,) £3,970,020, will [illegible] qual in value to a sum that is greater than such saving, namely, to the sum which is the amount to which such saving will have increased at the end of the said term, if we suppose it to be improved in the mean time at compound interest at the rate of 4 per cent. We must therefore, in the next place, compute the amounts of these savings at the end of the said term, if so improved at compound interest in the mean time, or during the remaining years of the said term after the times at which they will have respectively accrued, that is, during 27 years, 26 years, 25 years, 24 years, &c. to the last year of the said term. Now these amounts will be as follows.

The said savings are to be employed in diminishing the national debt in either the first or the second method above-described.

CCXCI. The amount of £1,985,010.0300, improved at compound interest at the rate of 4 per cent. during 27 years, is = £1,985,010.0300 × 2.883,368 = £5,723,514.4001.

A computation of the amounts of all the said savings, except the last, at the end of the said term of 60 years, if improved in the mean time at compound interest at the rate of 4 per cent.

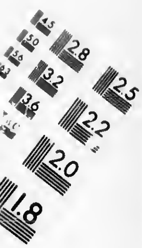
The amount of £2,087,978.8500, improved in the same manner during 26 years, is £2,087,978.8500 × 2.772,469 = £5,788,856.6342.

The amount of £2,196,668.1600, improved in the same manner during 25 years, is £2,196,668.1600 × 2.665,836 = £5,855,957.0609.

And the amounts of all the following savings, improved in the same manner during the remaining years of the said term of 60 years after the times at which they will have respectively accrued to the publick, will be as follows.

£2,305,357.4700	×	2.563,304	=	£5,909,332.0242.
£2,419,767.27	×	2.464,715	=	£5,964,036.6868.
£2,534,177.0700	×	2.369,918	=	£6,005,791.8533.
£2,648,586.8700	×	2.278,768	=	£6,035,515.0045.
£2,762,996.6700	×	2.191,123	=	£6,054,065.5525.
£2,871,685.9800	×	2.106,849	=	£6,050,208.7352.
£2,980,375.2900	×	2.025,816	=	£6,037,691.9484.
£3,089,064.6000	×	1.947,900	=	£6,017,188.9343.

£3,192,033. [illegible]



	£3,192,033.4200	× 1.872,981	=	£5,978,617.9647.
	£3,295,002.2400	× 1.800,943	=	£5,934,111.2191.
	£3,392,250.5700	× 1.731,676	=	£5,874,278.8980.
	£3,483,778.4100	× 1.665,073	=	£5,800,745.3684.
	£3,563,865.2700	× 1.601,032	=	£5,705,862.3409.
	£3,632,511.1500	× 1.539,454	=	£5,592,083.8199.
	£3,695,436.5400	× 1.480,244	=	£5,476,147.7657.
	£3,752,641.4400	× 1.423,311	=	£5,341,175.8406.
	£3,804,125.8500	× 1.368,569	=	£5,206,208.7104.
	£3,844,169.2800	× 1.315,931	=	£5,058,661.5248.
	£3,878,492.2200	× 1.265,319	=	£4,907,529.8973.
	£3,907,094.6700	× 1.216,652	=	£4,753,574.5444.
	£3,929,976.6300	× 1.169,858	=	£4,597,514.6004.
	£3,947,138.1000	× 1.124,864	=	£4,439,993.5517.
	£3,958,579.0800	× 1.081,600	=	£4,281,599.1329.
And	£3,964,299.5700	× 1.040,000	=	£4,122,871.5528.

The addition of the amounts of all the yearly savings at the end of the said term of 60 years, into one sum.

CCXCII. Having thus found the amounts to which the several yearly savings made at the ends of the thirty-first year and every following year of the said term of 60 years, except the last, will have increased at the end of the said term by being improved in the mean time at compound interest at the rate of 4 per cent. we must now add all these amounts, together with the saving of £3,970,020.0600, which will accrue at the end of the last year, into one sum: which may be done as follows.

The amounts of the savings made in the first period of six years after the expiration of the term certain of 30 years, and in the second period of six years after the expiration of the said term, and in the third period of six years after the expiration of the said term, will be as follows.

£	£	£
5,601,826.5237	5,964,036.6868	6,017,188.9343
5,660,836.6730	6,005,791.8533	5,978,617.9647
5,723,514.4001	6,035,515.0045	5,934,111.2191
5,788,856.6342	6,054,065.5525	5,874,278.8980
5,855,957.0609	6,050,208.7352	5,800,745.3684
5,909,332.0242	6,037,691.9484	5,705,862.3409
34,540,323.3161	36,147,309.7807	35,310,804.7254

And

And the amounts of the favings made in the fourth period of fix years and the fifth period of fix years after the expiration of the faid term certain of 30 years, will be as follows.

£	£
5,592,083.8199	4,753,574.5444
5,476,147.7657	4,597,514.6004
5,341,175.8406	4,439,993.5517
5,206,208.7104	4,281,599.1329
5,058,661.5248	4,122,871.5528
4,907,529.8973	3,970,020.0600
31,581,807.5587	26,165,573.4422

Therefore the fum total of the amounts of all thefe favings at the end of the faid term of 60 years, is £163,745,818 8231.

£
34,540,323.3161
36,147,309.7807
35,310,804.7254
31,581,807.5587
26,165,573.4422
163,745,818.8231

CCXCIII. Therefore the quantity of the capital of the national debt that would have been extinguifhed, by means of all thefe favings, at the end of the faid term of 60 years, will be equal in value to the faid fum of £163,745,818.8231; the annual intereft of which at 4 per cent.

$$\left(\frac{4}{100} \times £163,745,818.8231, \text{ or } \frac{£\,654,983,275.2924}{100}, \text{ or}\right) £6,549,832.7529.$$

Therefore the quantity of the intereft of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed to the publick at the end of the faid term of 60 years by means of all thefe favings, (fo that at the end of the next, or 61ft, year it would be free to be difpofed of by the Parliament in any other way,) would be £6,549,832.7529 *per annum*, or (neglecting the fractional part .7529,) £6,549,832 *per annum*. Q. E. I.

The quantity of the national debt that will have been extinguifhed by means of all the foregoing favings, at the end of the faid term of 60 years.

The quantity of the intereft of the national debt that will be then redeemed to the publick by means of the faid favings.

CCXCIV. This

The whole quantity of the interest of the national debt that will be redeemed to the publick at the end of the said term of 60 years by the foregoing operation.

CCXCIV. This sum of £6,549,832 *per annum* must therefore be added to £2,970,020 *per annum*; which was the quantity of the perpetual annuities, now due to the publick creditors, which was supposed to be converted into these compound, temporary, annuities for 30 years certain and for the lives of 69400 persons, all of the age of 35 years; and which will therefore be now wholly redeemed to the publick by the expiration of the said term of 30 years, and the deaths of all the said life-annuitants: and the sum of both, or the sum of £9,519,852 *per annum*, will be the whole quantity of the interest of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the said term of 60 years, (so as to be free at the end of the next, or 61st, year to be disposed of by the Parliament in any other way,) by the faithful application of a million of pounds, sterling, a year to that salutary purpose during that period according to this fifth method. Q E I.

Agreement between the effect of the foregoing scheme, " of life-annuities with a term certain annexed to them," and the effects of the four preceeding methods.

CCXCV. This sum of £9,519,852 *per annum* may be considered as equal to the quantity of the perpetual annuities, now due to the publick creditors, which would have been redeemed to the publick in the same space of 60 years by employing the same annual sum of a million of pounds, sterling, for the same purpose in either of the four preceeding methods; which quantity we have seen in the foregoing pages to be £9,520,000 *per annum*, or, more accurately, $\left(\frac{4}{100} \times \text{£}237{,}990{,}685.24\right.$, or $\frac{\text{£}951{,}962{,}740.96}{100}$, or$\left.\right)$ £9,519,627.4096, or (neglecting the fraction .4096,) £9,519,627 *per annum*. For the difference between them is only £225 *per annum*, which, upon so large a sum as £9,519,627, is a mere trifle.

General conclusion concerning the efficacy of this fifth method of applying a given sum of money every year to the diminution of the national debt.

CCXCVI. We may therefore conclude that this fifth method of employing a million of pounds, sterling, or any other given sum of money, every year to the purpose of diminishing the national debt, " by converting some of the perpetual annuities, now due to the publick creditors, into these compound, temporary, annuities, which are to continue at all events during a certain term of years, and are also to be extended contingently beyond the said term and during the lives of certain life-annuitants," would be just as efficacious in redeeming the perpetual annuities that are now due to the publick creditors as the interest of the said debt, and therefore would be just as beneficial to the nation as either of the four preceeding methods of applying the same sum of money to the same

purpose

purpoſe during the ſame period of time: which was the principal object of the foregoing computation. But this, it muſt be remembered, is true only upon a ſuppoſition that the annual ſavings which will accrue to the publick at the end of every year after the expiration of the ſaid term certain (during which the ſaid annuities are to continue at all events,) by the deaths of the ſaid life-annuitants, are immediately employed in diſcharging a part of that debt in either the firſt or the ſecond method above-deſcribed, or in ſome other method that is equivalent to them. For, if thoſe ſavings are not ſo employed, but are diverted to other uſes, the converſion of the perpetual annuities, now due to the publick creditors, into compound, temporary, annuities of the kind deſcribed in this fifth method, would be evidently a much *leſs* efficacious method of diminiſhing the national debt than either of the four preceeding methods. Thus, for example, we have ſeen that, by employing the ſaid ſavings in the manner above-deſcribed, the ſum of one million of pounds, ſterling, a year, employed in this fifth method, will be ſufficient to redeem to the publick, in the courſe of 60 years, the ſum of £9,519,852 *per annum* of the intereſt of the national debt; but without the help of the ſaid ſavings, it will redeem in that time only £2,970,020 *per annum* of the ſaid intereſt. Therefore, whenever any propoſal is made to pay off any part of the national debt by means of theſe compound, temporary, annuities, (which are to continue for a certain number of years at all events, and afterwards during the lives of certain life-annuitants,) great care muſt be taken that the ſavings of the publick revenue, which will accrue every year to the publick, (after the expiration of the term certain, during which ſuch annuities are to continue at all events,) by the deaths of the ſaid life-annuitants, ſhall be conſtantly applied to the ſame purpoſe; or otherwiſe the nation muſt (as Dr. Price has obſerved concerning the converſion of the perpetual annuities, now due to the publick creditors, into life-annuities,) loſe greatly by all ſchemes of this kind.

A ſixth method of employing a given ſum of money every year in the reduction of the national debt, by means of life-annuities with benefit of ſurvivorſhip during a certain term of years.

CCXCVII. A ſixth method of applying a given ſum of money every year to the diminution of the national debt, might be grafted on the fourth method, in which ſome of the perpetual annuities, now due to the publick creditors, are ſuppoſed to be converted into life-annuities. For, inſtead of ſuppoſing (as we did in the ſaid fourth method,) that, upon the deaths of the ſaid life-annuitants, their annuities ſhall revert to the publick, and be immediately employed, for the purpoſe of diminiſhing the

the national debt, in either the firſt, or the ſecond, method above-mentioned, we may ſuppoſe them to accrue to the ſurviving life annuitants, and to be equally divided amongſt them, untill their annuities (which, it is evident, will, upon this plan, be continually increaſing,) ſhall have increaſed to ten times their original quantity, or to ſome other very great ſum, or during a certain number of years leſs than the whole number of years during which it is probable that ſome few of the ſaid life-annuitants may live: and then the annuities of thoſe life-annuitants who ſhall die after the expiration of the ſaid term of years, or after the annuities of the ſurviving life-annuitants ſhall have been increaſed to their greateſt allowed magnitude, may be ſuppoſed to accrue to the publick, (as in the aforeſaid fourth method,) and to be employed, for the purpoſe of diminiſhing the national debt, in either the firſt, or the ſecond, method above-mentioned. And, if this plan were adopted, the annual ſum of money, which could be ſpared from the publick revenue for this purpoſe, might be employed in paying to the proprietors of the perpetual annuities ſo converted into temporary ones, the additions which it would be neceſſary to make to the ſaid perpetual annuities in compenſation for the ſaid abridgement of their duration.

In this method, as well as in the four preceeding methods, it is ſuppoſed that the proprietors of the perpetual annuities that are ſo to be converted into temporary ones, have previouſly conſented to ſuch converſion.

CCXCVIII. Thus, for example, if a million of pounds, ſterling, *per annum* could be ſpared from the publick revenue for this purpoſe, the Government might convert a proportionable quantity of the perpetual annuities, now due to the publick creditors, with the conſent of the proprietors of ſuch annuities, into annuities for the lives of the ſaid proprietors, or of ſuch perſons as they ſhould name, with the benefit of ſucceeding to each other's annuities by ſurvivorſhip during the ſpace of 30 years; after which the annuities of the then ſurviving life-annuitants ſhould no longer accrue, upon the deaths of the ſaid life-annuitants, to their ſurviving companions, but ſhould belong to the publick, and be employed every year in the diminution of the national debt in either the firſt, or the ſecond, method above-mentioned. And the ſaid million of pounds *per annum* would, in ſuch caſe, be employed in paying the additions which it would be neceſſary to make to the perpetual annuities that ſhould be ſo converted, as a compenſation to the proprietors of them for the abridgement of the time of their continuance.

CCXCIX. If a given ſum of money were to be employed every year for the purpoſe of diminiſhing the national debt in this ſixth method, its effect would be exactly the ſame in a given period of time as it would be, if employed for the ſame purpoſe during the ſame period in either of the five preceeding methods; that is, the quantity of the intereſt of the national debt, or of the perpetual annuities now due to the publick creditors, that would be redeemed to the publick by the application of a given annual ſum of money to the diminution of the national debt during a given number of years in this ſixth method would be exactly the ſame as that which would be redeemed to the publick by applying the ſame ſum of money every year to the ſame purpoſe during the ſame number of years in either of the five preceeding methods. Of this I will now proceed to give an example.

This ſixth method of diminiſhing the national debt, if properly purſued, would be equally effectual for that purpoſe with either of the five preceeding methods.

An example of the ſaid ſixth method of employing a given ſum of money every year in the reduction of the national debt.

CCC. Let the ſum of money to be employed in the reduction of the national debt according to this ſixth method be (as in the former examples,) a million of pounds, ſterling, a year; and the period during which it is to be ſo employed, be alſo the ſame as was choſen in the ſaid former examples, to wit, a period of 60 years. And let the proprietors of the perpetual annuities which are to be converted into life-annuities with benefit of ſurvivorſhip according to this ſixth method, or the *nominees* of the ſaid proprietors, during whoſe lives the new life-annuities are to continue, be all of the age of 35 years; ſo that it may reaſonably be expected, (according to Monſieur de Parcieux's table of the probabilities of the duration of human life,) that the laſt ſurvivor of them will live almoſt through the whole period of 60 years, and die a few days before the end of it. And let the number of the life-annuitants for whoſe lives the new annuities are to continue, be ſuppoſed to be the ſame as in the examples above given of the foregoing fourth and fifth methods, to wit, 69,400. And let the ſaid 69,400 life-annuitants, (who are all of the age of 35 years at the time of granting the ſaid life-annuities,) be ſuppoſed to die off every year in the proportions ſet forth in Monſieur de Parcieux's table of the probabilities of the duration of human life. And, laſtly, let the intereſt of money be ſuppoſed to be 4 per cent. and the price at which the 3 per cent. annuities ſell at the publick market, to be £75 per cent.

Preliminary ſuppoſitions.

CCCI. Theſe

An inveſtigation of the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into theſe compound life-annuities, with benefit of ſurvivorſhip, by the help of a million of pounds, ſterling, per annum.

CCCI. Theſe things being premiſed, we muſt, in the next place, inquire what quantity of the perpetual annuities, now due to the publick creditors, might, by the help of a million of pounds, ſterling, a year, be converted into temporary annuities of the kind above-deſcribed. Now this may be determined in the manner following.

Let us firſt ſuppoſe that theſe 69,400 life-annuitants (who are all of the age of 35 years,) receive from the Government, for an adequate conſideration, grants of annuities of £10 a piece, of the kind juſt now deſcribed, or that ſhall increaſe by ſurvivorſhip during the ſpace of 30 years, and then ſhall continue at their laſt magnitude, (to which they will have increaſed in the courſe of the ſaid 30 years,) during the lives of each of the then ſurviving life-annuitants reſpectively. The ſum of all theſe annuities will be £694,000 *per annum*; which, it is evident, the Government will be obliged to pay for 30 years together to ſuch of the ſaid life-annuitants as ſhall be living to receive it. But at the end of the ſaid term of 30 years the Government will have leſs than £694,000 to pay every year to the ſaid ſurviving life-annuitants; becauſe then the right of ſucceeding to each other's annuities by ſurvivorſhip is ſuppoſed to ceaſe, and the annuities of the life-annuitants who ſhall die after the end of the ſaid 30 years are ſuppoſed to accrue to the publick. The Government will therefore have a ſaving at the end of every year after the expiration of the ſaid 30 years, which may be employed in diminiſhing the national debt in either the firſt or the ſecond method above-mentioned: and theſe ſavings will be greater and greater every year untill all the ſaid life-annuitants are dead, or till the end of the whole term of 60 years. The number of life-annuitants will be reduced in the courſe of 30 years from 69,400 to 39,500; ſo that at the end of the 30th year the ſum of £694,000 will be to be divided between 39,500 perſons, inſtead of 69,400 perſons, and conſequently each of the ſaid 39,500 perſons will receive, for his ſhare of the ſaid £694,000, the ſum of $\left(\frac{£694,000}{39,500}, \text{ or}\right)$ £17.569,620. And this ſum of £17.569,620, which he will be intitled to receive at the end of the ſaid 30th year, he will continue to receive at the end of every following year of his whole life.

An original life-annuity of £10 a year for the life of a perſon of the age of 35 years, will increaſe in the courſe of 30 years to £17.569,620 per annum.

Diviſion of a compound life-annuity of £10 a year, for the life of a perſon of the age of 35 years, with benefit of ſurvivorſhip during 30 years, into two parts.

We may therefore divide the whole annuity which each of the ſaid 69,400 life-annuitants will be intitled to according to the foregoing ſuppoſitions, into the two following parts; to wit, 1ſt, an annuity of £10 a year for the firſt 30 years of his life, and no longer, but accompanied with the benefit of ſucceeding by ſurvivorſhip to the annuities of ſuch of his fellow life-annuitants as ſhould die in the courſe of the ſaid 30 years; by which means the ſaid annuity of £10 a year will gradually increaſe from £10 a year to £17.569,620 a year; and ſecondly, a remote life-annuity of £17.569,620, that is to commence at the end of 30 years, (or whereof the firſt payment is is to be received at the end of 31 years,) and that is to continue during the remainder of his life. Now the values of these two annuities may be found in the manner following.

A computation of the value of the first part of such compound life-annuity.

CCCII. As the several annuities of those life-annuitants who shall die every year during a term of 30 years, are supposed to be divided amongst their surviving companions, and not to accrue to the publick, it is evident that the price which ought to be paid to the publick by the whole body of the said 69,400 life-annuitants for the enjoyment of the said life-annuities during the said term of 30 years, in case they shall live so long, accompanied with the aforesaid benefit of survivorship during the said term, will be the same that ought to be paid by them for the same annuities, if the said annuities were to be granted to them for a term certain of 30 years, so as to become payable to their executors, or other personal representatives, in case the said life-annuitants themselves should happen to die in the course of the said term: because it would be a matter of indifference to the publick, whether the annuities of the life-annuitants that died in the course of the said term of 30 years, were made payable, during the several remaining years of the said term, to the executors, or other personal representatives, of the said deceased life-annuitants, or to their surviving companions, who would, in such case, supply the place of their executors. And consequently, as all the said 69,400 life-annuitants are supposed to be of the same age of 35 years, and there is no reason that any one of them should pay more for his annuity than any other; the price which each of them ought to pay for the enjoyment of his life-annuity during the said term of 30 years, or such part of the said term as he shall happen to live through, when accompanied with this benefit of succeeding, by survivorship, to his proportional share of the annuities of those of his companions who shall die in the course of the said term, will be the same as he ought to pay for the purchase of the same annuity for the same term of 30 years, when it is to be enjoyed at all events during the whole of the said term either by himself or his executor, or other personal representative, but without any chance of augmentation. Now it appears by Mr. Smart's fourth table of compound interest, page 78, that when the interest of money is 4 per cent. the value of an annuity of one pound a year for a term of 30 years certain (or not depending on the continuance of any life, or any other contingency,) is £17.292,033. Therefore the value of an annuity of £10 a year for the same term of 30 years is £172.920,33. And consequently the value of a life-annuity of £10 a year for the life of one of the aforesaid 69,400 life-annuitants of the age of 35 years, during the same term of 30 years, if he shall so long live, but not to continue after the said term, accompanied with the above-mentioned benefit of succeeding, by survivorship, to the annuities of such of his fellow life-annuitants as shall die in the course of the said term, will also be £172.920,33. Q.E.I.

CCCIII. The

A computation of the value of the second part of such compound life-annuity.

CCCIII. The other annuity, of which we were to determine the value, is a remote life-annuity of £17.569,620 *per annum*, for the life of a person of the age of 35 years, that is to commence at the end of 30 years, or whereof the first payment is to be received at the end of 31 years, the interest of money being 4 per cent. Now the value of a life-annuity of only one pound a year of this remote kind for the life of a person of the age of 35 years, when the interest of money is 4 per cent. appears above (in Table xxv, page 285,) to be £1.410,77. Therefore the value of a life-annuity of £17.569,620 *per annum* of this remote kind for the life of a person of the age of 35 years, is = 17.569,620 × £1.410,77, or £24.786,692. Q E I.

The value of the whole of the said compound life-annuity of 10*l.* *per annum.*

CCCIV. Therefore the value of both the foregoing annuities, or of the whole annuity which is supposed to be granted by the Government to each of the aforesaid 69,400 life-annuitants of the age of 35 years, is £172.920,33 + £24.786,69, or £197.707,02. Therefore £197.707,02 is the price which each of the said 69,400 life-annuitants ought to pay to the Government for a grant of an annuity of £10 a year of the kind above-described. Q E I.

The said compound life-annuity of 10*l.* *per annum* is equal in value to a perpetual annuity of £7.908,28 *per annum.*

CCCV. Now, when the interest of money is 4 per cent. the interest of £197.707,02 will be $\left(\frac{4}{100} \times £197.707,02\text{, or } \frac{790.828,08}{100}\text{, or}\right)$ £7.908,28 *per annum*; and consequently a perpetual annuity of £7.908,28 *per annum* is worth the sum of £197.707,02. Therefore a perpetual annuity of £7.908,28 *per annum* is equal in value to a life-annuity of the kind above-described of £10 *per annum* for the life of a person of the age of 35 years. Therefore the 69,400 life-annuities of £10 each which are above supposed to be granted by the Government to 69,400 persons, all of the age of 35 years, will be equal in value to 69,400 perpetual annuities of £7.908,28 *per annum*, or to a perpetual annuity of 69,400 × £7.908,28, or £548,834.63200 *per annum*. Therefore, if the yearly sum of £548,834.63200 of the perpetual annuities, now due to the publick creditors, were the property of the said 69,400 persons of the age of 35 years, to whom we supposed above that the Government had granted life-annuities of £10 each of the kind above-described, and the said 69,400 life-annuitants were to resign to the Government their right to the said annual sum of £548,834.63200 of the perpetual annuities, in exchange for, and as the price of, their said life-annuities of £10 each, or the annual sum of £694,000 amongst them, such a bargain would be fair and equal

on

on both sides. Therefore the difference between £694,000 *per annum* and £548,834.63200 *per annum*, that is, the sum of £145,165.36800 *per annum*, would be the addition which it would be necessary for the Government to make to the said sum of £548,834.63200 of the perpetual annuities, upon their being converted into life-annuities of the kind above-described, as a compensation to the proprietors of them for the abridgement of the time of their continuance. And consequently, since the additional sum of £145,165.36800 *per annum* would be sufficient to enable the Government to convert the sum of £548,834.63200 *per annum* of the perpetual annuities into life-annuities of the kind above-described, the additional sum of a million of pounds *per annum* would be sufficient to enable the Government to convert into life-annuities of the same kind a quantity of the perpetual annuities that is greater than £548,834.63200 *per annum* in the same proportion in which £1000,000 *per annum* is greater than £145,165.36800 *per annum*, or a quantity of the perpetual annuities that is equal to $\left(\frac{1000,000}{145,165.36800} \times £548,834.63200\right.$, or $\frac{548,834,632,000}{145,165.368,00}$, or) £3,780,754 *per annum*. Therefore the quantity which we proposed in Art. CCCI as necessary to be determined, to wit, the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into life-annuities of the kind above-described by the help of a fund of a million of pounds *per annum*, is the sum of £3,780,754 *per annum*. Q E I.

The quantity of the perpetual annuities mentioned in Art. 301 as necessary to be determined, is the sum of £3,780,754 *per annum*.

CCCVI. Let us now therefore suppose that the [illegible] ere to employ the said fund of a million of pounds *per an*[illegible]ting £3,780,754 *per annum* of the perpetual annuities into lif[illegible] the kind above-described; and that the number of persons of [illegible] 35 years, to whom, or for whose lives, the said life-annuities were to be granted, was 69,400. Then it is evident that the whole sum of money which would be paid every year to these 69,400 life-annuitants, or the survivors of them, for the space of 30 years, would be £3,780,754 *per annum* together with the said additional sum of £1,000,000 *per annum*, which would be necessary to enable the Government to make this conversion; that is, it would be £4,780,754 *per annum*. And consequently the life-annuity which would be due to each of the said 69,400 life-annuitants at first, or before any of his companions had died, would be equal to $\frac{£4,780,754}{69,400}$, or £68.886,94 *per annum*.

The original quantity of each man's life annuity, upon a supposition that the whole sum of £3,780,754 *per annum* of the perpetual annuities is converted into these life-annuities, and that the number of the life-annuitants continues (as before) to be 69,400, is £68.88694 *per annum*.

CCCVII. The

Another manner of determining the quantity of the ſaid original life-annuity.

CCCVII. The quantity of each of theſe life-annuities may likewiſe be determined in the following manner.

Since, when each of the 69,400 perſons of the age of 35 years was ſuppoſed to have a life-annuity, of the kind above-deſcribed, of £10 a year granted to him by Government, the additional annual ſum of money which was required to enable the Government to convert a portion of the perpetual annuities due to the publick creditors, that was equivalent to the annual ſum of £694,000 (which was thus ſuppoſed to be granted, in life-annuities of £10 a piece, to the ſaid 69,400 life-annuitants,) into the ſaid life-annuities, was £145,165.36800 *per annum*, it follows that, when an additional ſum of £1000,000 *per annum* is employed for the ſame purpoſe, and the number of the life-annuitants, who are to receive theſe life-annuities from Government, ſtill continues the ſame as before, to wit, 69,400, (as is the caſe on the preſent ſuppoſition,) the life-annuity which each of the ſaid perſons will be intitled to upon this ſecond ſuppoſition will be greater than the life-annuity of £10, (to which he was intitled upon the former ſuppoſition,) in the proportion £1000,000 to £145,165.36800, and therefore will be equal to £10 $\times \frac{1,000,000}{145,165.36800}$, or $\frac{£10,000,000}{145,165.36800}$, or £68.88695, which agrees with the number found before, to wit, £68.88694, to ſix places of figures, and differs from it only by an unit in the ſeventh figure. We may therefore be confident that the ſaid firſt ſix figures are true, or that the quantity of each of the original life-annuities belonging to the ſaid 69,400 life-annuitants, or that which he would be intitled to at the end of the firſt year, if none of his companions had died in the courſe of it, is £68.8869 *per annum.*

Of the increaſe of the ſaid original life-annuity of £68.88694 *per annum*, by the benefit of ſurvivorſhip, in the courſe of 30 years.

CCCVIII. We muſt now proceed to inquire to what quantity this original life-annuity, belonging to each of the ſaid 69,400 life-annuitants, will increaſe by the deaths of ſome of the ſaid life-annuitants, in the courſe of the ſaid term of 30 years. Now, ſince, when each of the original life-annuities was ſuppoſed to be £10 a year, it was found (in Art. CCCI,) to increaſe, in the courſe of the ſaid 30 years, to £17.569,620 a year, it follows that the original life-annuity of £68.8869, or, more accurately, £68.88694 *per annum*, to which each of the ſaid 69,400 life-annuitants of the age of 35 years will be intitled upon this ſecond ſuppoſition, will increaſe in the courſe of the ſaid 30 years to a quantity that will exceed £68.886,94 *per annum* in the ſame proportion as £17.569,620 exceeds £10, that

that is, to the quantity $\left(\frac{17,569,620}{10} \times £68.88694\right.$, or $\frac{£1210,317,368,761,8}{10}$, or) £121.031,736,876,28, or (neglecting the seven last figures,) £121.0317. Therefore at the end of the 30th year of the said term of 60 years, each of the survivors, out of the original 69,400 life-annuitants, will be intitled to receive from the Government the sum of £121.0317; and he will also be intitled to receive the same sum at the end of every following year during his whole life; but without any further increase, because it is supposed that from this time the annuities of the several life-annuitants, who shall die in the following years of the said term of 60 years, are to accrue to the publick.

Its last and greatest magnitude will be £121.0317 *per annum*.

CCCIX. That this sum of £121.0317 is the true quantity of the increased annuity, to which each of the surviving life-annuitants will be intitled at the end of the 30th year of the said term of 60 years, will likewise appear from the following consideration. The number of the said life-annuitants will, in the course of the said 30 years, be reduced from 69,400 to 39,500: and consequently the sum total of all the money paid at the end of every year, during the said 30 years, to all the surviving life-annuitants together, to wit, the sum £4,780,754, will be divided at the end of the 30th year amongst 39,500 persons; and consequently the annuity belonging to each of the said 39,500 persons will be $\frac{£4,780,754}{39,500}$, or £121.0317 *per annum*. Q.E.I.

Another way of determining the said greatest magnitude.

CCCX. We must therefore, in the next place, inquire into the savings which will now accrue to the publick at the end of every following year after the 30th, by the deaths of the remaining life-annuitants: which savings are to be employed, for the purpose of diminishing the national debt, in either the first, or the second, method above-mentioned.

Of the savings that will accrue to the publick after the expiration of the said term of 30 years.

CCCXI. Now it appears from Monsieur de Parcieux's table of the probabilities of the duration of human life, that, out of the 39,500 life-annuitants who will be living at the end of the 30th year of the said term of 60 years, and who will be then of the age of 65 years, only 38,000 will be living at the age of 66 years, or at the end of the 31st year of the said term, and that the numbers living at the subsequent ages of 67 years, 68 years, 69 years, 70 years, &c. or at the ends of the thirty second year, the thirty-third year, the thirty-fourth year, the thirty-fifth year, and every following year of the said term of 60 years, will be as follows; to wit,

The numbers of the said 69,400 life-annuitants that will be living at the ends of the 31st year, and every following year, of the said period of 60 years.

 36,400,

36,400,	21,100,	7,100,	700,
34,700,	19,200,	5,900,	400,
32,900,	17,300,	4,800,	200,
31,000,	15,400,	3,800,	100,
29,100,	13,600,	2,900,	000.
27,100,	11,800,	2,200,	
25,100,	10,100,	1,600,	
23,100,	8,500,	1,100,	

The numbers of the said life-annuitants that will have died in each of the said years.

CCCXII. Therefore the numbers of the said life-annuitants who will have died in the course of the 31st year, the 32d year, the 33d year, the 34th year, and every following year of the whole period of 60 years, in which they will all have died, will be as follows; to wit,

1,500,	2,000,	1,600,	500,
1,600,	2,000,	1,400,	400,
1,700,	1,900,	1,200,	300,
1,800,	1,900,	1,100,	200,
1,900,	1,900,	1000,	100,
1,900,	1,800,	900,	and 100.
2,000,	1,800,	700,	
2,000,	1,700,	600,	

The savings that will accrue to the publick at the ends of each of those years by the deaths of the said life-annuitants.

CCCXIII. Therefore at the end of the 31st year there will accrue to the Government a saving of the annuities of the 1500 life-annuitants who will have died in the course of the said year, or which would have become due to them at the end of the said year, if they had been then alive; that is, a saving of 1500 times £121.0317, or of the sum of £181,547.5500. And at the end of the thirty-second year there will accrue to the Government a still greater saving, to wit, a saving of the annuities of the 1500 life-annuitants who will have died in the course of the 31st year, and likewise of the annuities of the 1600 life-annuitants who will have died in the course of the 32d year, that is, a saving of 1500 + 1600 times £121.0317, or 3100 times £121.0317, or of the sum of £375,198.2700. And in like manner we shall find the savings at the ends of the 33d year, the 34th year, the 35th year, the 36th year, and every following year, of the whole term of 60 years, to be as follows; to wit,

At

At the end of the thirty-third year,

$\overline{3,100 + 1700} \times £121.0317,$

or $4,800 \times £121.0317, = £580,952.1600.$

At the end of the thirty-fourth year,

$\overline{4,800 + 1800} \times £121.0317,$

or $6,600 \times £121.0317, = £798,809.2200.$

At the end of the thirty-fifth year,

$\overline{6,600 + 1900} \times £121.0317,$

or $8,500 \times £121.0317, = £1,028,769.4500.$

At the end of the thirty-fixth year,

$\overline{8,500 + 1900} \times £121.0317,$

or $10,400 \times £121.0317, = £1,258,729.6800.$

At the end of the thirty-feventh year,

$\overline{10,400 + 2000} \times £121.0317,$

or $12,400 \times £121.0317, = £1,500,793.0800.$

At the end of the thirty-eighth year,

$\overline{12,400 + 2000} \times £121.0317,$

or $14,400 \times £121.0317, = £1,742,856.4800.$

At the end of the thirty-ninth year,

$\overline{14,400 + 2000} \times £121.0317,$

or $16,400 \times £121.0317, = £1,984,919.8800.$

At

At the end of the fortieth year,

$\overline{16,400 + 2000} \times$ £121.0317,

or 18,400 × £121.0317, = £2,226,983.2800.

At the end of the forty-firſt year,

$\overline{18,400 + 1900} \times$ £121.0317,

or 20,300 × £121.0317, = £2,456,943.5100.

At the end of the forty-ſecond year,

$\overline{20,300 + 1900} \times$ £121.0317,

or 22,200 × £121.0317, = £2,686,903.7400.

At the end of the forty-third year,

$\overline{22,200 + 1900} \times$ £121.0317,

or 24,100 × £121.0317, = £2,916,863.9700.

At the end of the forty-fourth year,

$\overline{24,100 + 1800} \times$ £121.0317,

or 25,900 × £121.0317, = £3,134,721.0300.

At the end of the forty-fifth year,

$\overline{25,900 + 1800} \times$ £121.0317,

or 27,700 × £121.0317, = £3,352,578.0900.

At the end of the forty-ſixth year,

$\overline{27,700 + 1700} \times$ £121.0317,

or 29,400 × £121.0317, = £3,558,331.9800.

At

At the end of the forty-seventh year,

$\overline{29,400 + 1600} \times$ £121.0317,

or 31,000 × £121.0317, = £3,751,982.7000.

At the end of the forty-eighth year,

$\overline{31,000 + 1400} \times$ £121.0317,

or 32,400 × £121.0317, = £3,921,427.0800.

At the end of the forty-ninth year,

$\overline{32,400 + 1200} \times$ £121.0317,

or 33,600 × £121.0317, = £4,066,665.1200.

At the end of the fiftieth year,

$\overline{33,600 + 1100} \times$ £121.0317,

or 34,700 × £121.0317, = £4,199,799.9900.

At the end of the fifty-first year,

$\overline{34,700 + 1000} \times$ £121.0317,

or 35,700 × £121.0317, = £4,320,831.6900.

At the end of the fifty-second year,

$\overline{35,700 + 900} \times$ £121.0317,

or 36,600 × £121.0317, = £4,429,760.2200.

At the end of the fifty-third year,

$\overline{36,600 + 700} \times$ £121.0317,

or 37,300 × £121.0317, = £4,514,482.4100.

At

At the end of the fifty-fourth year,

$\overline{37{,}300 + 600} \times £121.0317,$

or $37{,}900 \times £121.0317, = £4{,}587{,}101.4300.$

At the end of the fifty-fifth year,

$\overline{37{,}900 + 500} \times £121.0317,$

or $38{,}400 \times £121.0317, = £4{,}647{,}617.2800.$

At the end of the fifty-sixth year,

$\overline{38{,}400 + 400} \times £121.0317,$

or $38{,}800 \times £121.0317, = £4{,}696{,}029.9600.$

At the end of the fifty-seventh year,

$\overline{38{,}800 + 300} \times £121.0317,$

or $39{,}100 \times £121.0317, = £4{,}732{,}339.4700.$

At the end of the fifty-eighth year,

$\overline{39{,}100 + 200} \times £121.0317,$

or $39{,}300 \times £121.0317, = £4{,}756{,}545.8100.$

At the end of the fifty-ninth year,

$\overline{39{,}300 + 100} \times £121.0317,$

or $39{,}400 \times £121.0317, = £4{,}768{,}648.9800.$

And at the end of the sixtieth and last year,

$\overline{39{,}400 + 100} \times £121.0317,$

or $39{,}500 \times £121.0317, = £4{,}780{,}752.1500.$

CCCXIV. Now

The faid favings are to be employed in diminifhing the national debt, in either the firft or the fecond method above-defcribed.

CCCXIV. Now all thefe favings are to be employed, as faft as they arife, for the purpofe of diminifhing the national debt, in either the firft or the fecond method above-mentioned. By this means the laft faving, £4,780,752.1500, (which accrues to the publick at the end of the whole period of 60 years,) will either extinguifh £4,780,752.1500 of the capital of the 4 per cent. annuities (which are fuppofed to fell at their *par*, or nominal value,) or, if employed in the fecond method above-defcribed, it will extinguifh a proportionably greater capital of the 3 per cent. annuities, which are fuppofed to fell at the price of £75 per cent. and in either cafe it will redeem to the publick the intereft of 4 per cent. upon it, or the annual fum of $\left(\frac{4}{100} \times £4,780,752.1500, \text{ or } \frac{£19,123,008.6000}{100}, \text{ or}\right)$ £191,230.0860. But all the other favings will have time to perform more than one operation of this kind towards the diminution of the national debt, to wit, a new operation at the end of every year of the term that is remaining after the time when it accrued: and, in confequence of thefe repeated operations, the quantity of the national debt which each of thefe favings will have extinguifhed at the end of the faid term of 60 years, will be equal in value to the amount of fuch faving at the end of the faid term, if improved in the mean time at compound intereft at the rate of 4 per cent.

We muft therefore now compute the amounts of thefe favings at the end of the faid term, if fo improved at compound intereft in the mean time, or during the remaining years of the faid term after the times at which they will have refpectively accrued, that is, during 29 years, 28 years, 27 years, 26 years, &c. to the laft year of the faid term. Now thefe amounts will be as follows.

A computation of the amounts of all the faid favings, except the laft, at the end of the faid term of 60 years, if improved in the mean time at compound intereft at the rate of 4 per cent.

CCCXV. The amount of £181,547.5500, improved at compound intereft at 4 per cent. during 29 years, is = £181,547.5500 × 3.118,651 = £566,183.4483.

The amount of £375,198.2700, improved in the fame manner during 28 years, is = £375,198,2700 × 2.998,703 = £1,125,108.1778.

And the amounts of all the other favings, improved in the fame manner during the remaining years of the faid term of 60 years after they fhall have refpectively accrued to the publick, will be as follows.

£580,952.1600 × 2.883,368 will be = £1,675,098.8676.
£798,809.2200 × 2.772,469 = £2,214,673.7993.
£1,028,769.4500 × 2.665,836 = £2,742,530.6375.
£1,258,729.6800 × 2.563,304 = £3,226,506.8236.
£1,500,793.0800 × 2.464,715 = £3,699,027.2161.
£1,742,856.4800 × 2.369,918 = £4,130,426.9433.
£1,984,919.8800 × 2.278,768 = £4,523,171.9051.
£2,226,983.2800 × 2.191,123 = £4,879,594.2854.
£2,456,943.5100 × 2.106,849 = £5,176,408.9871.
£2,686,903.7400 × 2.025,816 = £5,443,172.5869.
£2,916,863.9700 × 1.947,900 = £5,681,759.3271.
£3,134,721.0300 × 1.872,981 = £5,871,272.9294.
£3,352,578.0900 × 1.800,943 = £6,037,802.0431.
£3,558,331.9800 × 1.731,676 = £6,161,878.0897.
£3,751,982.7000 × 1.665,073 = £6,247,325.0902.
£3,921,427.0800 × 1.601,032 = £6,278,330.2407.
£4,066,665.1200 × 1.539,454 = £6,260,443.8856.
£4,199,799.9900 × 1.480,244 = £6,216,728.7363.
£4,320,831.6900 × 1.423,311 = £6,149,887.2735.
£4,429,760.2200 × 1.368,569 = £6,062,432.5145.
£4,514,482.4100 × 1.315,931 = £5,940,747.3522.
£4,587,101.4300 × 1.265,319 = £5,804,146.5943.
£4,647,617.2800 × 1.216,652 = £5,654,532.8589.
£4,696,029.9600 × 1.169,858 = £5,493,688.2149.
£4,732,339.4700 × 1.124,864 = £5,323,238.3055.
£4,756,545.8100 × 1.081,600 = £5,144,679.9480.
£4,768,648.9800 × 1.040,000 = £4,959,394.9392.

CCCXVI. Having

CCCXVI. Having thus found the amounts to which the feveral yearly favings made at the ends of the thirty-firft year and every following year of the faid term of 6c years, except the laft, will have increafed at the end of the faid term by being improved in the mean time at compound intereft at the rate of 4 per cent. we muft now add up all thefe amounts, together with the faving of £4,780,752.1500, (which will accrue at the end of the 60th, or laft, year,) into one fum: which may be done as follows.

The addition of the amounts of all the aforefaid yearly favings, at the end of the faid term of 60 years, into one fum.

The amounts of the favings made in the firft period of fix years after the expiration of the above-mentioned term of 30 years, and in the fecond period of fix years after the expiration of the faid term, and in the third period of fix years after the expiration of the faid term, will be as follows.

£	£	£
566,183.4483,	3,699,027.2161,	5,681,759.3271,
1,125,108.1778,	4,130,426.9433,	5,871,272.9294,
1,675,098.8676,	4,523,171.9051,	6,037,802.0431,
2,214,673.7993,	4,879,594.2854,	6,161,878.0897,
2,742,530.6375,	5,176,408.9871,	6,247,325.0902,
3,226,506.8236.	5,443,172.5869.	6,278,330.2407,
11,550,101.7541	27,851,801.9239	36,278,367.7202

And the amounts of the favings made in the fourth period of fix years after the expiration of the faid term of 30 years, and in the fifth and laft period of fix years after the expiration of the faid term, will be as follows.

£	£
6,260,443.8856,	5,654,532.8589,
6,216,728.7363,	5,493,688.2149,
6,149,887.2735,	5,323,238.3055,
6,062,432.5145,	5,144,679.9480,
5,940,747.3522,	4,959,394.9392,
5,804,146.5943.	4,780,752.1500.
36,434,386.3564	31,356,286.4165

And

And the ſum total of theſe five ſeveral ſums, or the ſum total of the amounts that will ariſe from all the ſavings made at the ends of all the years of the ſaid term of 60 years after the 30th year, at the end of the ſaid term of 60 years, is £143,470,944.1711.

£
11,550,101.7541,
27,851,801.9239,
36,278,367.7202,
36,434,386.3564,
31,356,286.4165.
143,470,944.1711.

The quantity of the capital of the national debt that will have been extinguiſhed, by means of all the foregoing ſavings, at the end of the ſaid term of 60 years.

CCCXVII. Therefore the quantity of the capital of the national debt that would have been extinguiſhed, by means of all theſe ſavings, at the end of the ſaid term of 60 years, will be equal in value to the ſaid ſum of £143,470,944.1711; of which the annual intereſt at 4 per cent. is $\left(\frac{4}{100} \times £143,470,944.1711, \text{ or } \frac{£573,883,776.6844}{100}, \text{ or}\right)$ £5,738,837.7668.

The quantity of the intereſt of the national debt that will be then redeemed to the publick by means of the ſaid ſavings.

Therefore the quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of the ſaid intereſt, that would be redeemed to the publick at the end of the ſaid term of 60 years by means of all theſe ſavings, (ſo that at the end of the next, or 61ſt, year it would be free to be diſpoſed of by the Parliament in any other way,) is £5,738,837.7668 *per annum*. Q E I.

The whole quantity of the intereſt of the national debt that will be redeemed to the publick, at the end of the ſaid term of 60 years, by the foregoing operation.

CCCXVIII. This ſum of £5,738,837.7668 *per annum* muſt therefore be added to £3,780,753 *per annum*, which was the quantity of the perpetual annuities, now due to the publick creditors, which was ſuppoſed to be converted into life-annuities of the kind above-deſcribed for the lives of 69400 perſons, all of the age of 35 years, and which will therefore be now wholly redeemed to the publick by the deaths of all the ſaid life-annuitants: and the ſum of both, or the ſum of £9,519,590.7668, or (neglecting the fraction .7668) £9,519,590 *per annum* will be the whole quantity of the intereſt of the national debt, or of the perpetual annuities employed in the payment of it, which will be redeemed to the publick at the end of the ſaid term of 60 years, (ſo as to be free at the end of the next, or 61ſt, year to be diſpoſed of by the Parliament in any other

other way,) by the faithful application of a million of pounds, ſterling, *per annum* to that ſalutary purpoſe, during that period, according to the ſixth method above-deſcribed. Q E I.

Agreement between the effect of the foregoing ſcheme, "of life-annuities with the benefit of ſurvivorſhip during a certain number of years," in redeeming to the publick the intereſt of the national debt, and the effects of the five preceeding methods.

CCCXIX. This ſum of £9,519,590 *per annum*, which is the quantity of the intereſt of the national debt that would be redeemed to the publick in the courſe of 60 years by the application of a million of pounds, ſterling, *per annum* to that purpoſe in this ſixth method, or in the way of life-annuities with a right of ſucceſſion by ſurvivorſhip to the annuities of the deceaſed life-annuitants for a term of 30 years, is the ſame with the quantity of the intereſt of the ſaid debt that would have been redeemed to the publick in the ſame time by applying the ſame ſum of a million of pounds *per annum* to the ſame purpoſe in either of the five former methods. For the ſum that would have been ſo redeemed in either the firſt or the ſecond method is £9,519,624 *per annum*; which differs from the preſent ſum of £9,519,590 *per annum* by only £34 *per annum*, which, upon ſo large a ſum as £9,519,624, is perfectly inconſiderable. And the quantities of the ſaid intereſt that would be redeemed by the application of the ſame ſum of a million of pounds *per annum* to the ſame purpoſe during the ſame period of ſixty years, in either the third, or the fourth, or the fifth, method above-mentioned, have been before ſhewn to be very nearly equal to the ſame ſum of £9,519,624 *per annum*. Therefore the quantity of the ſaid intereſt which would be redeemed by this ſixth method may be conſidered as equal to that which would be redeemed in the ſame time by the ſame annual ſum of money in either of the five preceeding methods; agreeably to what is aſſerted above in Art. CCXCIX. Q E D.

A compariſon between this ſixth method and the third and fourth methods above-mentioned, with reſpect to the advantage of ſecuring to the publick the faithful application of the ſaid annual ſum of money to the purpoſes of its deſtination.

CCCXX. This ſixth method of employing a given ſum of money every year in the diminution of the national debt, "by means of the above-mentioned life-annuities with the benefit of ſurvivorſhip during a certain number of years," partakes in a greater degree than the fourth method, (which conſiſted in eſtabliſhing life-annuities without the benefit of ſurvivorſhip,) of the advantage mentioned in Art. CCLIV as belonging to the third method, to wit, that of ſecuring to the publick the faithful application of ſuch annual ſum to the purpoſes of its deſtination, without ſuſpenſion or interruption. For it would then be impoſſible to with-hold not only any part of the original life-annuities ſo created, but alſo any part of the additions that would accrue to the ſurviving life-annuitants every year by the deaths of ſome of their companions in the courſe of the given term of years during which the ſaid benefit of ſurvivorſhip had been granted

granted to them, without an abſolute breach of the publick faith; which (as we obſerved above in Art. CCLIV) may be conſidered as a moral impoſſibility. And conſequently during the whole of the ſaid term throughout which ſuch benefit of ſurvivorſhip was to continue, the annual ſum of money allotted to the purpoſe of diminiſhing the national debt would be tied down to the purpoſe of its deſtination. But, when the ſaid term was at an end, the ſubſequent ſavings that would ariſe from the deaths of thoſe life-annuitants who would die after the ſaid term, would be as liable to be diverted, by the miniſters of ſtate and the parliament, from its original deſtination of diminiſhing the national debt, and to be employed in defraying the expence of ſome temporary meaſure of Government, as the money that ſhould be allotted to the ſame purpoſe, of diminiſhing the national debt, in either the firſt or the ſecond method above-deſcribed: whereas in the *third* method of applying a given ſum of money every year to the diminution of the national debt, "by converting ſome of the perpetual annuities that now pay the intereſt of it, into greater temporary annuities that ſhould continue for a certain limited number of years, and allotting the ſaid annual ſum to the payment of the additions thereupon made to the perpetual annuities ſo converted," *the whole* of the ſaid annual ſum is effectually ſecured from being diverted from its original and proper deſtination, and applied to any other purpoſe whatſoever, during the *whole* of the ſaid term.

A ſeventh method of employing a given ſum of money every year in the reduction of the national debt, by means of life-annuities, which ſhould continue at all events for a certain term of years, and, in caſe of the deaths of the life-annuitants, ſhould become payable to their executors, or other perſonal repreſentatives, during the remainder of the ſaid term, and which ſhould afterwards be accompanied with the benefit of ſurvivorſhip during a ſecond term of years.

CCCXXI. A ſeventh method of applying a given ſum of money every year to the diminution of the national debt would be to combine the foregoing fifth and ſixth methods together in the following manner.

A portion of the perpetual annuities, now due to the publick creditors, might (with the conſent of the proprietors of ſuch annuities) be converted into equivalent life-annuities of the following kind. The ſaid annuities ſhould continue for a certain moderate term of years at all events, ſo that, in caſe the proprietors themſelves ſhould die before the expiration of the ſaid term, the annuities ſhould be payable to the executors, or other perſonal

ſonal repreſentatives, of the ſaid proprietors during the remaining years of it. At the end of the ſaid term of years the annuities payable to the executors of the deceaſed proprietors ſhould ceaſe; but thoſe that were payable to the ſurviving proprietors ſhould continue during their lives, (as in the fifth method above-deſcribed,) and ſhould alſo, during a ſecond term of years, increaſe continually (as in the laſt, or ſixth, method above-deſcribed) by a diviſion of the annuities of the life-annuitants that died in every year after the end of the firſt term, amongſt their ſurviving companions. And after the end of this ſecond term of years there ſhould be no further increaſe of the ſaid life-annuities by ſurvivorſhip; but the then ſurviving life-annuitants ſhould receive during the remainder of their lives the ſame annuities as they had received at the end of the laſt year of the ſaid ſecond term. And the ſavings that would accrue to the publick after the end of the ſaid ſecond term by the deaths of the few remaining life-annuitants, ſhould be applied, as faſt as they aroſe, to the diminution of the national debt in either the firſt or the ſecond method above-deſcribed.

Thus, for example, if the life-annuitants for whoſe lives theſe annuities were to be granted, were all of the age of 35 years. and conſequently a few of the longeſt livers of them would (according to Monſieur de Parcieux's table of the probabilities of the duration of human life,) live to almoſt the end of a period of 60 years, the ſaid period of 60 years might be divided into three parts, to wit, 1ſt, a term of 30 years, 2dly, a term of 20 years, and, 3dly, a term of 10 years. And during the firſt of theſe terms the annuities might be made payable not only to the ſaid life-annuitants themſelves, if they were living, but, in caſe of their deceaſe, to their executors, or other perſonal repreſentatives; but at the end of the ſaid firſt term of 30 years the payments of theſe annuities to the repreſentatives of the ſaid deceaſed life-annuitants ſhould intirely ceaſe, and only thoſe due to the ſurviving life-annuitants ſhould be continued. And during the ſecond period of 20 years the ſurviving life-annuitants ſhould not only continue to receive their original life-annuities, but ſhould ſucceed, by ſurvivorſhip, to the life annuities of ſuch of their companions as ſhould die in the courſe of the ſaid 20 years; by which means the annuities of the few perſons who ſhould live to the end of the ſaid 20 years, or to the age of 85 years, would be increaſed in the proportion of the number of the ſaid life-annuitants who would be living at the ſaid age of 85 years to the number of them that had been living at the beginning of the ſaid term of 20 years, or at the age of 65 years, that is (according to Monſieur de Parcieux's table,) in the proportion of 48 to 395, or ſomething more than in the proportion of 1 to 8. At the end of the ſaid ſecond term of 20 years this benefit of ſurvivorſhip ſhould ceaſe, and the few life-annuitants that ſhould live beyond that term ſhould continue to receive the ſame annuities during the remaining years of their lives as they had received at the

the end of the last year of the said term of 20 years, or when they were of the age of 85 years: and the savings that would then arise every year by the deaths of some of the said life-annuitants, should accrue to the publick, and be employed in the diminution of the national debt in either the first or the second method above-described.

And, if this method of diminishing the national debt were adopted, the annual sum of money that could be spared out of the publick revenue for this purpose, or a part of that sum, must be employed in paying the additions which it would be necessary for the Government to make to the perpetual annuities, now due to the publick creditors, which should be converted into life-annuities of the kind just now described, in compensation of the abridgement of the time of their continuance.

This seventh method of diminishing the national debt, if properly pursued, would be equally effectual for that purpose with either of the six preceeding methods.

CCCXXII. This seventh method of applying a given sum of money every year to the diminution of the national debt would have exactly the same effect in the course of any given period of time as either of the six preceeding methods; or the quantity of the interest of the national debt, or of the perpetual annuities now employed in paying it, which would be redeemed to the publick in any given number of years by employing a given sum of money every year for that purpose according to this seventh method, would be the very same that would be redeemed to the publick in the same period of time by employing the same sum of money every year for the same purpose according to either of the preceeding methods. Of this I shall now proceed to give an example.

An example of the said seventh method of employing a given sum of money every year in the reduction of the national debt.

Preliminary suppositions.

CCCXXIII. Let the sum of money that is to be employed every year for the purpose of diminishing the national debt according to this seventh method, be, (as in the foregoing examples,) a million of pounds, sterling, *per annum*; and the period during which it is to be so employed, be also (as before,) a term of 60 years. Let the life-annuitants for whose lives the new life-annuities, of the compound nature above-described, are to be granted, in lieu of certain portions of the perpetual annuities, now due to the publick creditors, (which are thereupon, with the consent of the proprietors of them, to cease,) be all of the age of 35 years: and let them be supposed to die off every year in the proportions set forth in Monsieur de Parcieux's table of the probabilities of the duration of human life; in consequence of which it must be supposed that some few of them will

will live almost throughout the whole of the said period of 60 years, and die a few days before the end of it. Also let the number of life-annuitants for whose lives these life-annuities are granted, be (as before) 69,400.

Further, let the first term, during which the life-annuities of the deceased life-annuitants are to be paid to their executors, or other personal representatives, be a term of 30 years; and the second term, during which the life-annuitants, who shall outlive the first term, shall be intitled to succeed by survivorship to the annuities of such of their companions as shall die in the course of the said second term, be 20 years. And, lastly, let the interest of money be supposed to be 4 per cent. and the perpetual annuities of 4 per cent. now due to the publick creditors, be supposed to sell at their *par*, or nominal value; and the perpetual annuities of 3 per cent. now due to the publick creditors, be supposed to sell at the price of £75 per cent.

These things being premised, we must now proceed to inquire what quantity of the perpetual annuities, now due to the publick creditors, the Government would be able to convert into life-annuities of the compound kind just now described, by means of the said sum of a million of pounds, sterling, *per annum*, allotted to that purpose during the space of 60 years. Now this quantity may be determined by proceeding in the manner following.

An investigation of the quantity of the perpetual annuities, now due to the publick creditors, which might be converted into th[illegible] [illegible] life-annuities, with a term of 30 years certain, and the benefit of survivorship during a second term of 20 years, by the help of a million of pounds, sterling, *per annum* allotted to that purpose during a period of 60 years.

CCCXXIV. The number of life-annuitants of the age of 35 years, to whom these life-annuities are supposed to be granted, is 69,400. If therefore we suppose the annuities granted to them to be £10 a year a piece, the annual expence to the Government arising from these grants would be £694,000. And this expence would continue without any diminution during the whole of the said first term of 30 years, notwithstanding the numbers of the said life-annuitants who would be continually dying in every year of the said term; because during the whole of the said term the annuities of the deceased life-annuitants are to be paid to their respective executors, or other personal representatives.

CCCXXV. But at the end of the said term of 30 years a considerable saving would accrue every year to the publick from the said annuities of the life-annuitants who had died in the course of the said term; because then the said annuities are no longer to be paid to the representatives of the deceased life-annuitants. This saving will be equal to £10 multiplied into the number of life-annuitants who will have died in the course of the said

 30 years.

30 years. Now it appears from Monsieur de Parcieux's table of the probabilities of the duration of human life, that the said original 69,400 life-annuitants, of the age of 35 years, for whose lives the said annuities of £10 each were supposed to be granted, will, in the course of 30 years, be reduced to 39,500. Therefore the number of the said life-annuitants who will have died in the course of the said 30 years is (69,400 — 39,500, or) 29,900: and consequently the saving which will accrue every year to the publick, after the expiration of the said term of 30 years, in consequence of the deaths of the said life-annuitants, who will have died in the course of the said term, will be (29,900 × £10, or) £299,000. And therefore the sum which the Government will be obliged to pay, in consequence of the grants of these annuities, after the expiration of the said term of 30 years, will not, as before, be £694,000 a year, but only (£694,000 — £299,000, or) £395,000 a year. And this expence of £395,000 *per annum* will continue the same during the whole second term of 20 years, notwithstanding the deaths of several of the said life-annuitants in every year of the said term; because it is supposed that the annuities of the several life-annuitants who shall die in the course of the said second term are to be divided amongst their surviving companions.

CCCXXVI. But after the expiration of this second term of 20 years, the annual expence of the Government, in consequence of the grants of these annuities, will grow less and less every year; because then the annuities of the life-annuitants who shall die in the remaining 10 years of the said period of 60 years are no longer to be enjoyed, by right of survivorship, by their few surviving companions, but are to accrue to the publick.

An original life-annuity of 10 pounds a year, for the life of a person of the age of 35 years, will increase in the course of the aforesaid second term, (which consists of 20 years,) to £82.291,666 *per annum*.

CCCXXVII. In the course of the said second term of 20 years the annuities of the life-annuitants who shall have lived to the end of it, will have increased from £10 *per annum* to £82.291,666 *per annum*. For the number of the said life-annuitants will have decreased in the said term from 39,500 to 4,800: and consequently the sum of £395,000, which is every year to be divided among them, will, at the end of the last year of the said term, be divided amongst only 4,800 persons; and therefore the annuity then paid to each of them will be $= \frac{£395,000}{4,800} = \frac{£3950}{48} =$ £82.291,666. And this annuity, of £82.291,666 *per annum*, each of the said 4,800 surviving life-annuitants will be intitled to receive during the remainder of his life, but without any further increase of it by survivorship.

CCCXXVIII. In

Diviſion of a compound life-annuity of 10 pounds a year, for the life of a perſon of the age of 35 years, with a term certain of 30 years and the benefit of ſurvivorſhip during another term conſiſting of 20 years, into three parts.

CCCXXVIII. In order to diſcover the value of one of theſe compound life-annuities of £10 *per annum*, it will be convenient to divide it into the three following parts, to wit, 1ſt, an annuity of £10 a year for a term of 30 years, to be certainly paid either to the life-annuitant, or, in caſe of his deceaſe, to his perſonal repreſentative; and, 2dly, A remote annuity of £10 a year, depending upon the life of the life-annuitant, ſo as not to take place unleſs the ſaid life-annuitant ſhall be then alive, and afterwards to ceaſe immediately in caſe of his death; and which, even in the other event, of his living, ſhall continue only during 20 years; but which during the ſaid 20 years ſhall be accompanied with the advantage of a right in the life-annuitant to ſucceed, by ſurvivorſhip, to a ſhare of the annuities of ſuch of his fellow life-annuitants as ſhall die in the courſe of the ſaid 20 years; and 3dly, A remote life-annuity of £82.291,666 *per annum*, that is to commence at the diſtance of 50 years, or whereof the firſt payment is to be received at the end of 51 years. The values of theſe three annuities may be determined in the manner following.

A computation of the value of the firſt part of the ſaid compound life-annuity of 10l. per annum.

CCCXXIX. In the firſt place, it appears from Mr. Smart's fourth table of compound intereſt, page 78, that the value of an annuity of one pound a year for 30 years certain, when the intereſt of money is 4 per cent. is £17.292,033,30. And conſequently the value of an annuity of £10 a year for 30 years certain is £172.920,333. Q. E. I.

A computation of the value of the ſecond part of the ſaid compound life-annuity of 10l. per annum.

CCCXXX. Secondly, ſince during the whole ſecond term of 20 years the Government is every year to pay to the ſurviving life-annuitants the ſum of £395,000 *per annum*, it is evident that the price which the Government ought to receive from all the 69,400 original life-annuitants together for this annual payment of £395,000 during the ſaid ſecond term of 20 years, muſt be the ſame as if the ſame annual payment were to be made to them during the ſame term of 20 years in the more ſimple form of a remote annuity for a term certain of 20 years, that was to be paid either to them or their executors, or other perſonal repreſentatives, during the ſaid term, and was to commence at the end of 30 years, or whereof the firſt payment was to be received at the end of 31 years. Now the value of a remote annuity of £395,000 *per annum* for a term of 20 years, that is to commence at the end of 30 years, is equal to the exceſs of the value of an immediate annuity of the ſame magnitude, or £395,000 *per annum*, for a term of 50 years above the value of an immediate annuity of the ſame magnitude for a term of 30 years. Now it appears from Mr. Smart's fourth table of compound intereſt, page 78, that the value of an immediate annuity of £395,000 *per annum* for a term of 50 years, when the intereſt

 of

of money is 4 per cent. is £395,000 × 21.482,184,62, or £8,485,462.9249, and that the value of an immediate annuity of £395,000 *per annum* for a term of 30 years is £395,000 × 17.292,033,30, or £6,830,353.1535. Therefore the value of a remote annuity of £395,000 *per annum* for a term of 20 years, that is to commence at the end of 30 years, is equal to £8,485,462.9249 — £6,830,353.1535, or £1,655,109.7714. Therefore the sum which all the 69,400 life-annuitants, taken together, ought to pay to the Government for the said second annuity of £395,000 *per annum*, which they are to receive amongst them during the said second term of 20 years, which is to commence at the end of the first term of 30 years, is £1,655,109.7714; and consequently the sum which each of the said 69,400 life-annuitants ought to pay for his contingent share of the said annuity of £395,000 *per annum* during the said second term of 20 years, accompanied with the benefit of survivorship during the said term, in the manner above-described, is $\frac{£1,655,109.7714}{69,400}$, or £23.848,843. Q.E.I.

A computation of the value of the third part of the said compound life annuity of 10*l*. *per annum*.

CCCXXXI. Lastly, the value of a remote life-annuity of £82.291,666 *per annum*, for the life of a person of the age of 35 years, that is to commence at the end of 50 years, or whereof the first payment is to be received at the end of 51 years, may be determined by the help of Art. CCXLI in the manner following.

The number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life as living at the age of 35 years, is 694; and the number of persons represented in the same table as living at the age of 85 years, or at the end of 50 years, is 48. Therefore, if $\overset{£}{V}$ be put for the value of an immediate annuity of one pound a year for the life of a person of the age of 85 years, it follows from Art. CCXLI that the value of a remote annuity of one pound a year for the life of a person of the age of 35 years, that is to commence at the end of 50 years, or when the said person shall be 85 years old, will be equal to $\frac{48}{694} \times \frac{1}{r^{50}} \times \overset{£}{V}$. Now $\overset{£}{V}$, or the value of an immediate annuity of one pound a year for the life of a person of the age of 85 years, when the interest of money is 4 per cent. appears above in Table XVI, page 225, to be = £2.424,216. And it appears by Mr. Smart's second table of compound interest, pages 60 and 62, that, when the interest of money is 4 per cent. or r is = 1.04, the fraction $\frac{1}{r^{50}}$,

or

or $\frac{1}{1.04^{50}}$, is = .140,712,62. Therefore $\frac{48}{694} \times \frac{1}{r^{50}} \times \frac{£}{V}$ is = $\frac{48}{694} \times .140,712,62 \times £2.424,216 = \frac{48}{694} \times .341,117,784,805,92,£$ = $\frac{£16.373,653,670,684,16}{694}$ = £0.023,593,160; that is, the value of a remote life-annuity of one pound a year for the life of a perſon of the age of 35 years, that is to commence at the end of 50 years, or when the ſaid perſon ſhall have attained the age of 85 years, or ſo that he ſhall receive the firſt payment thereof at the age of 86 years, is = £0.023,593,160. Therefore the value of a remote life-annuity of £82.291,666 *per annum* for the life of a perſon of the age of 35 years, that is to commence at the end of 50 years, or when the ſaid perſon ſhall have attained the age of 85 years, is = 82.291,666 × £0.023,593,160 = £1.941,520,442. Q.E.I.

CCCXXXII. Therefore the ſum of the values of theſe three annuities, or the value of the whole life-annuity of £10 a year ſuppoſed to be granted to each of the aforeſaid 69,400 life-annuitants of the age of 35 years, is = £172.920,333 + £23.848,843 + 1.941,520 = £198.710,696. Q.E.I.

The value of the whole of the ſaid compound life-annuity of 10l. per annum.

CCCXXXIII. The annual intereſt of £198.710,696, at 4 per cent. is $\left(\frac{4}{100} \times £198.710,696, \text{ or } \frac{£794.842,784}{100}, \text{ or}\right)$ £7.948,427,84. Therefore a perpetual annuity of £7.948,427,84 *per annum* is equal in value to a life-annuity of £10 *per annum*, of the compound kind above-deſcribed, for the life of a perſon of the age of 35 years. Therefore, if each of the above-mentioned 69,400 life-annuitants of the age of 35 years, to whom we have ſuppoſed life-annuities of £10 each, of the compound kind above-mentioned, to have been granted, had been poſſeſſed of a perpetual annuity in the publick funds, of £7.948,427,84 *per annum*, and were to reſign, or relinquiſh, to the publick his right to ſuch perpetual annuity, in exchange for, and as the price of, the ſaid life-annuity of £10 a year, of the compound kind above-deſcribed, ſuch a bargain would be fair and equal on both ſides. Therefore the ſum of all the ſaid 69,400 life-annuities of £10 each, added together, would be equal in value to 69,400 perpetual annuities of £7.948,427,84 *per annum*, or to a portion of the perpetual annuities, now due to the publick creditors, of 69,400 × £7.948,427,84, or) £551,620.892,096 *per annum*. In order therefore to convert a portion of the perpetual annuities, now due to the publick creditors,

The ſaid compound life-annuity of 10l. per annum for the life of a perſon of the age of 35 years, is equal in value to a perpetual annuity of £7.948,427,84 per annum.

creditors, amounting to £551,620.892,096 *per annum*, into life-annuities of £10 each, of the compound kind above-described, it would be necessary for the Government to apply to this purpose an additional sum of money every year, during the space of 30 years, that is equal to the difference between £551,620.892,096, and £694,000, or that is equal to £142,379.107,904. And consequently, since an additional sum of £142,379.107,904 *per annum* would be sufficient to enable the Government to convert into life-annuities, of the compound kind above-mentioned, for the lives of persons of the age of 35 years, a portion of the perpetual annuities, now due to the publick creditors, amounting to £551,620.892,096 *per annum*; we may conclude that an additional sum of a million of pounds every year would be sufficient to enable the Government to convert into life-annuities of the same kind, for the lives of persons of the same age of 35 years, a portion of the said perpetual annuities that is greater than £551,620.892,096 *per annum* in the same proportion in which £1,000,000 is greater than £142,379.107,904, that is, a portion of the said perpetual annuities

The quantity of the perpetual annuities mentioned in Art. 323 as necessary to be determined, is the sum of £3,874,310.6360 *per annum*.

amounting to $\left(\frac{1,000,000}{142,379.107,904} \times \text{£}551,620.892,096, \text{ or to } \frac{551,620,892,096.000,000}{142,379.107,904}, \text{ or }\right)$ £3,874,310.635,995, or £3,874,310.6360 *per annum*. This therefore is the quantity which was proposed in Art. CCCXXIII as necessary to be determined in order to ascertain the effect of the application of a million of pounds, sterling, *per annum* to the diminution of the national debt, during a period of 60 years, according to this seventh method.

Of the original quantity of each man's life-annuity, upon a supposition that the whole sum of £3,874,310 *per annum* of the perpetual annuities is converted into these compound life-annuities, and that the number of the life-annuitants continues, as before, to be 69,400.

CCCXXXIV. Let us therefore now suppose that the whole of the said sum of a million of pounds *per annum* is employed in the diminution of the national debt according to this seventh method, and consequently that the sum of £3,874,310 *per annum* of the perpetual annuities, now due to the publick creditors, is converted into life-annuities, of the compound kind above-described, for the lives of persons of the age of 35 years. And let us suppose the number of life-annuitants between whom the said sum of £3,874,310 of the perpetual annuities, (so converted into life-annuities,) together with the sum of £1,000,000 *per annum*, (which is to be added to it, on account of the said conversion,) is to be divided, to be (as on the former supposition made in Art. CCCXXIII,) 69,400. And let the life-annuities granted to these 69,400 life-annuitants be supposed to be all equal to each other.

CCCXXXV. Then,

CCCXXXV. Then, ſince the ſum that is to be paid every year by the Government to the ſaid 69,400 life-annuitants, or their ſeveral executors, or other perſonal repreſentatives, during a term of 30 years, is £3,874,310 + £1,000,000, or £4,874,310; and this ſum is to be equally divided between all the ſaid 69,400 life-annuitants, or their repreſentatives; it follows that the annuity which each of the ſaid life-annuitants, or his repreſentative, will be intitled to receive at the end of every year during the ſaid term of 30 years, will be $\frac{\pounds 4{,}874{,}310}{69{,}400}$, or £70.235,014 *per annum*.

The original quantity of ſuch compound life-annuity is £70.235,014 *per annum*.

CCCXXXVI. At the end of the ſaid term of 30 years the ſaid 69,400 life-annuitants will be reduced to 39,500; and in the courſe of 20 years more the ſaid 39,500 life-annuitants will be further reduced to 4,800. But it is ſuppoſed that during this latter term of 20 years the benefit of ſurvivorſhip is to take place between the ſaid life-annuitants. Therefore the ſame ſum of money which was divided, at the end of the laſt year of the firſt term of 30 years, amongſt the 39,500 life annuitants who were then living, will, at the end of the laſt year of the ſaid ſecond term of 20 years, be divided amongſt only 4,800 perſons. That ſum is 39,500 times £70.235,014, or £2,774,283.053,000. Therefore $\frac{\pounds 2{,}774{,}283.053{,}000}{4{,}800}$, or £577.975,636, will be the ſum which each of the ſaid 4,800 ſurviving life-annuitants will be intitled to receive at the end of the ſaid ſecond term of 20 years, or when he ſhall have attained the age of 85 years. And this ſum of £577.975,636 each of the ſaid 4,800 ſurviving life-annuitants will be intitled to receive likewiſe at the end of every following year during his life; but without any further augmentation.

The ſaid original life-annuity of £70.235,014 *per annum*, will increaſe by the benefit of ſurvivorſhip, in the courſe of the aforeſaid ſecond term of years, conſiſting of 20 years, to £577.975,636 *per annum*.

CCCXXXVII. We muſt now inquire what are the ſavings which will accrue to the publick by the deaths of any of the ſaid life-annuitants in any part of the ſaid period of 60 years.

Of the ſavings that will accrue to the publick in the courſe of the whole period of 60 years.

Now it is evident that during the whole of the firſt term of 30 years the deaths of the ſeveral life-annuitants who will die in the courſe of the ſaid term, will produce no ſaving to the publick; becauſe the executors, or other perſonal repreſentatives, of the deceaſed life-annuitants will be intitled to their annuities. But at the end of the ſaid term of 30 years a very great ſaving will accrue to the publick all at once. For then the annuities of the deceaſed life-annuitants are no longer to be paid to their repreſentatives,

The firſt and greateſt ſaving will be at the end of the firſt term above-mentioned, conſiſting of 30 years.

repreſentatives, but are to become the property of the publick and to be employed in the reduction of the national debt in either the firſt or the ſecond method above-deſcribed. The number of life-annuitants that will have died in the courſe of the ſaid term of 30 years is the difference between 69,400, the original number of the ſaid life-annuitants, and 39,500, the number of them who will be living at the end of the ſaid term of 30 years; which difference is 29,900. Therefore the ſaving that will accrue to the publick after the expiration of the ſaid term of 30 years will be 29,900 annuities of £70.235,014, or the ſum of £2,100,026.918,600. This ſaving will accrue to the publick at the end of the 31ſt year; and the like ſaving will accrue to the publick at the ends of the 32d year, the 33d year, the 34th year, the 35th year, and of every following year during the whole period of 60 years, to the laſt year of it incluſively; ſo that there will be thirty ſuch ſavings of £2,100,026.918,600, in the ſaid time. Now all theſe ſavings are to be employed, as faſt as they ariſe, for the purpoſe of diminiſhing the national debt, in either the firſt, or the ſecond, method above-mentioned: and conſequently the quantity of the capital of the national debt that will have been extinguiſhed by them in the courſe of the ſaid latter thirty years of the ſaid period of 60 years, will be equal in value to the ſum that would be produced by means of an annual revenue of £2,100,026.918,600 that ſhould be received for 30 years together, and ſhould, at the end of every year, or as ſoon as it was received, be immediately lent out at intereſt at the rate of 4 per cent. together with the intereſt that had been already produced by it; which ſum, according to Mr. Smart's third table of compound intereſt, page 70, is 56.084,937,76 × £2,100,026.918,600, or £117,779,879.024,005,586,336, or (neglecting all the decimal fraction .024,005,586,336, except the four firſt places of figures,) £117,779,879.0240. Therefore the quantity of the capital of the national debt which will be extinguiſhed at the end of the ſaid period of 60 years by means of theſe 30 annual ſavings of £2,100,026.918,600 each, which will accrue to the publick at the ends of the latter 30 years of the ſaid period, will be equal in value to the ſum of £117,779,879.0240; and conſequently the quantity of the intereſt of the ſaid debt, or of the perpetual annuities that are now employed in paying it, which will be redeemed to the publick at the end of the ſaid period of 60 years by means of the ſaid ſavings, will be $\left(\frac{4}{100} \times £117,779,879.0240, \text{ or } \frac{£471,119,516.0960}{100}, \text{ or}\right)$ £4,711,195.1609.

This ſaving will be £2,100,026.918,600 *per annum.*

The quantity of the capital of the national debt which will be extinguiſhed by means of this ſaving at the end of the whole period of 60 years, will be equal in value to £117,779,879.0240. And the quantity of the intereſt of the ſaid debt that will then be redeemed to the publick by means of the ſaid ſaving, will be £4,711,195.1609.

CCCXXXVIII.

CCCXXXVIII. Besides this very large saving of £2,100,026.918,600 *per annum*, (which will accrue to the publick at the end of the first 30 years in consequence of the deaths of the 29,900 life-annuitants who will have died in the course of the said 30 years,) there will be other savings that will accrue to the publick, after the expiration of the said second term of 20 years, by the deaths of the 4,800 then surviving life-annuitants in the course of the remaining 10 years of the said period of 60 years. We must therefore now inquire what these savings will amount to in each of the said 10 years.

Of the savings that will accrue to the publick after the expiration of the aforesaid second term, consisting of 20 years, by the deaths of life-annuitants.

CCCXXXIX. Now, in order to discover what these savings will amount to in each of the said 10 years, it will be necessary to determine, by the help of Monsieur de Parcieux's table of the probabilities of the duration of human life, what numbers of the said 4,800 life-annuitants (who are supposed to be living at the end of the said second term of 20 years,) will die in each of the said 10 years. Now it appears by Monsieur de Parcieux's table that, out of 4,800 persons of the age of 85 years, all living at the same time, the numbers living at the subsequent ages of 86 years, 87 years, 88 years, 89 years, &c. will be as follows; to wit,

The numbers of life-annuitants who will be living at the ends of the last ten years of the aforesaid period of 60 years.

3,800,		700,
2,900,		400,
2,200,		200,
1,600,		100,
1,100,	and	000.

Therefore the numbers of the said 4,800 life-annuitants (who will have lived to the end of the said second term of 20 years, or to the age of 85 years,) that will die in the 10 succeeding years, will be as follows; to wit,

The numbers of the said life-annuitants who will die in each of the said ten years.

1,000,		400,
900,		300,
700,		200,
600,		100,
500,	and	100.

CCCXL. It

The favings accruing to the publick at the ends of each of the faid ten years, by the deaths of the faid life-annuitants.

CCCXL. It has been fhewn above in Art. CCCXXXVI, that each of the faid 4,800 life-annuitants (who will be living at the end of the faid fecond term of 20 years, or at the age of 85 years,) will be intitled to receive at the end of every following year of his life the fum of £577.975,636. Therefore the faving that will accrue to the publick at the end of the firft of the faid 10 remaining years, by the deaths of the 1000 life-annuitants who will have died in the courfe of the faid firft year, will be 1000 payments of £577.975,636 each, or the fum of £577,975.6360; and the faving that will accrue to the publick at the end of the fecond of the faid remaining 10 years by the deaths of the faid 1000 perfons who will have died in the faid firft year and likewife of the 900 perfons who will have died in the faid fecond year, will be 1000 + 900, or 1900, payments of £577.975,636 each, or the fum of (1900 × £577.975,636, or) 1,098,153.708,400. And in like manner it may be fhewn that the favings that will accrue to the publick at the ends of the 3d, 4th, 5th, 6th, and other following years of the faid remaining 10 years, will be as follows; to wit,

At the end of the third year,

$\overline{1900 + 700} \times$ £577.975,636,

or 2,600 × £577.975,636, = £1,502,736.6536.

At the end of the fourth year,

$\overline{2,600 + 600} \times$ £577.975,636,

or 3,200 × £577.975,636, = £1,849,522.0352.

At the end of the fifth year,

$\overline{3,200 + 500} \times$ £577.975,636,

or 3,700 × £577.975,636, = £2,138,509.8532.

At the end of the fixth year,

$\overline{3,700 + 400} \times$ £577.975,636,

or 4,100 × £577.975,636, = £2,369,700.1076.

At

At the end of the seventh year,

$$\overline{4{,}100 + 300} \times £577{\cdot}975{,}636,$$

or $4{,}400 \times £577{\cdot}975{,}636, = £2{,}543{,}092.7984.$

At the end of the eighth year,

$$\overline{4{,}400 + 200} \times £577{\cdot}975{,}636,$$

or $4{,}600 \times £577{\cdot}975{,}636, = £2{,}658{,}687.9256.$

At the end of the ninth year,

$$\overline{4{,}600 + 100} \times £577{\cdot}975{,}636,$$

or $4{,}700 \times £577{\cdot}975{,}636, = £2{,}716{,}485.4892.$

And at the end of the tenth year,

$$\overline{4{,}700 + 100} \times £577{\cdot}975{,}636,$$

or $4{,}800 \times £577{\cdot}975{,}636, = £2{,}774{,}283.0528.$

Of the quantity of the national debt that will have been extinguished at the end of the whole aforesaid period of 60 years, by means of the said savings.

CCCXLI. These savings are to be employed in diminishing the national debt in either the first, or the second, method above-mentioned: by which means the portion of the national debt that will have been extinguished at the end of the said period of 60 years, in consequence of each of these savings, except the last, to wit, £2,774,283.0528, (which will accrue at the end of the last year of the said period,) will be equal in value to a sum that is greater than such saving, namely, to the sum which is the amount to which such saving will have increased at the end of the said period of 60 years, if we suppose it to have been improved in the mean time at compound interest at the rate of 4 per cent. We must therefore, in the next place, compute the amounts of these savings at the end of the said period of 60 years, if so improved at compound interest in the mean time, or during the remaining years of the said term after the times at which they will have respectively accrued, that is, during 9 years, 8 years, 7 years, 6 years, and so on, to the last year of the said period. Now these amounts will be as follows.

The amounts of all the ſaid ſavings, except the laſt, at the end of the ſaid period of 60 years, if improved in the mean time by compound intereſt at the rate of 4 per cent.

CCCXLII. The amount of the firſt of the foregoing ſavings, (which will accrue to the publick at the end of the 51ſt year of the ſaid period of 60 years,) to wit, £577,975.6360, improved at compound intereſt at the rate of 4 per cent. during 9 years, is = £577,975.6360 × 1.423,311,81 = £822,639.5486.

The amount of the ſecond ſaving, £1,098,153.7084, improved in the ſame manner during 8 years, is = £1,098,153.7084 × 1.368,569,05 = £1,502,899.1774.

The amount of the third ſaving, £1,502,736.6536, improved in the ſame manner during 7 years, is = £1,502,736.6536 × 1.315,931,78 = £1,977,498.9194.

And the amounts of the ſix following ſavings, improved in the ſame manner during the remaining years of the ſaid period of 60 years after the times at which they will have reſpectively accrued to the publick, will be as follows.

£1,849,522.0352	×	1.265,319,02	=	£2,340,235.4090.
£2,138,509.8532	×	1.216,652,90	=	£2,601,824.2145.
£2,369,700.1076	×	1.169,858,56	=	£2,772,213.9555.
£2,543,092.7984	×	1.124,864,00	=	£2,860,633.5375.
£2,658,687.9256	×	1.081,600,00	=	£2,875,636.8603.
£2,716,485.4892	×	1.040,000,00	=	£2,825,144.9087.

The addition of the ſaid nine amounts, together with the ſaving that will accrue at the end of the laſt, or 60th, year of the ſaid period, into one ſum

CCCXLIII. Having thus found the amounts to which the ſeveral yearly ſavings made at the ends of the fifty-firſt year and every following year of the ſaid period of 60 years, except the laſt, will have increaſed at the end of the ſaid term by being improved in the mean time at compound intereſt at the rate of 4 per cent. we muſt now add up all theſe amounts, together with the ſaving of £2,774,283.0528, (which will accrue at the end of the laſt year,) into one ſum: which may be done as follows.

£822,639 5486,

£
822,639.5486,
1,502,899.1774,
1,977,498.9194,
2,3[illegible]0,235.4090,
2,601,824.2145,
2,772,213.9555,
2,860,633.5375,
2,875,636.8603,
2,825,144.9087,
2,774,283.0528.
23,353,009.5837.

Therefore the ſum of the amounts of all theſe ten ſavings at the end of the ſaid period of 60 years, is £23,353,009.5837.

CCCXLIV. This ſum of the amounts of theſe laſt ten ſavings made in the laſt ten years of the ſaid period of 60 years, muſt now be added to the amount of the former great ſavings of £2,100,026.918,600 *per annum*, which began to accrue to the publick at the end of the firſt term of 30 years; which amount was ſhewn above (in Art. CCCXXXVII,) to be £117,779,879.0240. Therefore the ſum total of the amounts of all the ſavings at the end of the whole period of 60 years will be £117,779,879.0240 + £23,353,009.5837, or £141,132,888.6077; the intereſt of which at 4 per cent. is $\left(\frac{4}{100} \times £141,132,888.6077, \text{ or } \frac{£564,531,554.4308}{100}, \text{ or}\right)$ £5,645,315.5443 *per annum*. Therefore the quantity of the capital of the national debt which will have been extinguiſhed, at the end of the ſaid period of 60 years, by means of all the ſaid ſavings, will be equal in value to the ſum of £141,132,888.6077; and the quantity of the intereſt of the ſaid debt, or of the perpetual annuities now employed in paying the ſaid intereſt, which would be thereby redeemed to the publick at the end of the ſaid period of 60 years, (ſo as to be free, at the end of the next, or 61ſt, year, to be diſpoſed of in any way the Parliament ſhould think fit,) would be the ſum of £5,645,315.5443 *per annum*, or (neglecting the fraction .5443) the ſum of £5,645,315 *per annum*. Q E I.

The addition of the laſt ſum total to the amount of the former great ſavings which began to accrue to the publick at the end of the firſt term above-mentioned, conſiſting of 30 years.

The quantity of the capital of the national debt that will have been extinguiſhed at the end of the whole period of 60 years by means of all the ſavings above-mentioned.

The quantity of the intereſt of the ſaid debt that will have been then redeemed to the publick by means of all the ſaid ſavings.

CCCXLV. To

The whole quantity of the intereft of the national debt that will have been redeemed to the publick at the end of the faid whole period of 60 years by the faithful application of a million of pounds, fterling, *per annum* according to this feventh method.

CCCXLV. To this fum of £5,645,315 *per annum*, which will thus have been redeemed to the publick at the end of the faid period of 60 years by means of all the foregoing favings, we muft add the fum of £3,874,310, *per annum*, which was the quantity of the perpetual annuities, due to the publick creditors, which were fuppofed to be converted into life-annuities of the compound kind above-defcribed; and the fum thence arifing, to wit, the fum of £9,519,625 *per annum*, will be the whole quantity of the intereft of the national debt, or of the perpetual annuities employed in the payment of the faid intereft, which will be redeemed to the publick at the end of the faid term of 60 years, (fo as to be free, at the end of the next, or 61ft, year, to be difpofed of by the Parliament in any other way,) by the faithful application of a million of pounds, fterling, a year to that ufeful purpofe during that period according to this feventh method.

This fum is the fame with the quantity of the intereft of the national debt that would be redeemed to the publick in the fame time by the application of the fame annual fum of a million according to either of the fix foregoing methods.

CCCXLVI. This fum of £9,519,625 *per annum* of the perpetual annuities, (now due to the publick creditors,) which will be thus redeemed to the publick in the courfe of 60 years by means of an annual fum of a million of pounds, fterling, employed for this purpofe according to this feventh method, is equal to the quantity of the faid perpetual annuities which would have been redeemed to the publick in the fame fpace of 60 years by employing the fame annual fum of a million of pounds, fterling, for the fame purpofe in either of the fix preceeding methods. For we have feen above that the faid quantity is £9,520,000 *per annum*, or, more accurately, £9,519,627 *per annum* in the firft, fecond, and third methods, and £9,519,599 *per annum* in the fourth method, and £9,519,852 *per annum* in the fifth method, and £9,519,590 *per annum* in the fixth method; all which quantities differ fo very little from each other, that they may be confidered as equal.

A comparifon of this feventh method of applying a given annual fum of money to the diminution of the national debt, with the former methods of doing the fame thing, with refpect to the advantage mentioned in Art. 254.

CCCXLVII. This feventh method of employing a given fum of money every year in the diminution of the national debt, "by converting fome of the perpetual annuities that now pay the intereft of it, into life-annuities of the foregoing compound kind, or that fhall continue at all events for a certain number of years (fo as to be payable during the faid term either to the life-annuitants themfelves, or their executors, or other perfonal reprefentatives,) and afterwards fhall be accompanied, during a fecond term of years, with the benefit of furvivorfhip, and then, after the expiration of the faid fecond term, fhall continue at their laft, or greateft, magnitude during the remaining years of the lives of the faid life-annuitants," partakes, in a confiderable degree, of the advantage mentioned in Art. CCLIV as belonging to the third method, to wit, the advantage of ſecuring to the publick the faithful application of ſuch annual ſum to the purpoſes of its deſtination, without ſuſpenſion or interruption. For it would then be impoſſible to with-hold any part of the life-annuities ſo created from the perſons to whom they would be due, without an abſolute breach of the publick faith; which (as we before obſerved,) may be conſidered as a moral impoſſibility. And this advantage will relate not only to ſo much of the publick revenue as will at any time be due to the life-annuitants themſelves who will be ſtill alive, (as was the caſe in the fourth method above-mentioned, by which ſome of the perpetual annuities were converted into ſimple life-annuities, without a concomitant term of years, or any benefit of ſurvivorſhip;) but it will relate alſo, during the aforeſaid firſt term of years, to that part of the publick revenue which was allotted to the payment of the annuities of the deceaſed life-annuitants; becauſe the annuities of ſuch deceaſed life-annuitants will be due, during the ſaid firſt term of years, to their reſpective executors, or other perſonal repreſentatives: and it will relate alſo, during the ſecond term of years above-mentioned, to the annuities of all the life-annuitants who were living at the beginning of the ſaid ſecond term, or at the end of the aforeſaid firſt term; becauſe, during the whole of the ſaid ſecond term, the annuities of the life-annuitants who ſhall die in the courſe of the ſaid ſecond term, are to accrue to their ſurviving companions. But it will not relate to the great annual ſaving which will accrue to the publick at the end of the ſaid firſt term, nor to the latter annual ſavings which will accrue to the publick at the end of the ſaid ſecond term by the deaths of the life-annuitants who ſhall die after the end of the ſaid ſecond term. For this part of the publick revenue, which will be thus ſaved every year to the publick, will be as liable to be diverted, by the miniſters of ſtate and the Parliament, from its original deſtination of diminiſhing the national debt, as the money that ſhould be allotted to the diminution of the national debt in either the firſt or the ſecond method above-deſcribed: whereas in the third method of applying a given ſum of money every year to the diminution of the national debt, to wit, "by converting ſome of the perpetual annuities, that now pay the intereſt of it, into greater temporary annuities, that ſhould continue for a limited number of years, and allotting the ſaid annual ſum to the payment of the additions thereupon made to the perpetual annuities ſo converted," the whole of the ſaid annual ſum would be effectually ſecured from being diverted from its original and proper deſtination, and applied to any other purpoſe whatſoever, during the whole of the ſaid term.

CCCXLVIII. Theſe

A remark on the seven foregoing methods of applying a given annual sum of money to the diminution of the national debt.

CCCXLVIII. These are the methods that, I imagine, the Government would be most likely to adopt, if they were earnestly to set about this good work of reducing the national debt to a more moderate quantity by setting apart and employing a given sum of money every year, out of the publick revenue, for that purpose. And I have therefore thought it might be useful to set them forth in a very full and plain manner, and illustrate the effects of each of them by an example. And, in order the better to compare these effects with one another, I have thought it best, in the said examples, to suppose the same annual sum of money, to wit, a million of pounds, sterling, *per annum*, to be employed for the same number of years, to wit, a period of 60 years, in each of the said methods. And the result is, that all these methods are equally efficacious towards the great purpose of diminishing the national debt, if they are pursued with equal steadiness and fidelity.

Some of the foregoing methods could not, probably, be carried into execution to any great extent.

CCCXLIX. But, though all these methods would be equally efficacious towards the reduction of the national debt, they are not all equally fit to be adopted, or capable of being carried into execution to any great extent. For it is probable that some of them would be thought convenient by only a small number of the present publick creditors, and consequently could be adopted only to a small extent; the consent of the said creditors being necessary to all the foregoing methods, except the first. Thus, for example, the fourth method, "by which some of the perpetual annuities, now due to the publick creditors, would, with the consent of the said creditors, be converted into mere life-annuities, without a term of years certain annexed to them, or any benefit of survivorship," would probably suit only a small number of the present owners of the perpetual annuities; as, for instance, the owners of £180,393.5 *per annum* of the said perpetual annuities, instead of the owners of £1,803,935, *per annum* of the said annuities, as was supposed above in Art. CCLIX; and consequently an annual sum of only £100,000 *per annum*, and not of £1000,000 *per annum*, cou'd be employed in this method. And, indeed it does not seem probable that any one of the five latter methods of employing a given sum of money in the diminution of the national debt could be carried into execution to the extent supposed in the foregoing examples, or so as to employ so large a fund as a million of pounds, sterling, *per annum*; because it is not likely that any one of the said methods should suit the convenience of a sufficient number of the publick creditors, or proprietors of the perpetual annuities now due from the Government, to make such a measure practicable. But, perhaps, if the Government were to adopt *all the five* methods, and to propose to the people, in the first place, that a part of the said annual sum of a million of pounds, sterling, as, for instance, half of it,

But it seems likely that each of the five latter methods might be found practicable and convenient to a certain moderate extent.

or

or £500,000 *per annum*, ſhould be employed in converting ſome of the perpetual annuities, now due to the publick creditors, into greater temporary annuities, that ſhould continue only during a certain limited number of years, as, for example, 60 years, according to the third method above-deſcribed;——and, in the ſecond place, that another part of the ſaid annual million, as, for inſtance, £100,000 *per annum*, ſhould be employed in converting a ſecond portion of the ſaid perpetual annuities into ſimple life-annuities, according to the fourth method above-deſcribed; ——and, thirdly, that another part of the ſaid annual million, as, for inſtance, £200,000 *per annum*, ſhould be employed in converting a third portion of the ſaid perpetual annuities into mixt annuities, or life-annuities, with a term of years certain annexed to them, or that ſhould continue during the lives of certain perſons of known ages, and likewiſe, in caſe thoſe life-annuitants ſhould die within a certain moderate number of years, (as, for inſtance, 30 years,) ſhould continue during the remainder of the ſaid term of years, and be payable to the executors, or other perſonal repreſentatives, of the ſaid deceaſed life-annuitants, according to the fifth method above-mentioned;——and, fourthly, that another part of the ſaid annual million, as, for inſtance, £100,000 *per annum*, ſhould be employed in converting a fourth portion of the ſaid perpetual annuities into life-annuities, with the benefit of ſurvivorſhip during a certain moderate number of years, as, for example, 30 years, according to the ſixth method above-mentioned;——and, fifthly and laſtly, that the remaining part of the ſaid annual million ſhould be employed in converting a fifth portion of the ſaid perpetual annuities into life-annuities accompanied with both the foregoing advantages, that is, with a permanent intereſt in the ſaid annuities to be enjoyed at all events by either the ſaid life-annuitants or their repreſentatives, during a certain moderate term of years, (as, for inſtance, 30 years,) and likewiſe, after the expiration of the ſaid term, with the benefit of ſucceeding by ſurvivorſhip to each other's annuities during a ſecond term of years, as, for example, 20 years;——; I ſay, if the Government were to make theſe propoſals to the people, it ſeems not unlikely that they might meet with the concurrence of a ſufficient number of the publick creditors to enable the Government to carry them into execution to a ſufficient extent to exhauſt the whole of the ſaid ſuppoſed fund of a million of pounds *per annum*. But, if they did not meet with that concurrence to ſo great an extent, the remaining part of the ſaid annual million, which could not be employed in either of thoſe five methods, might be employed, for the ſame good purpoſe of diminiſhing the national debt, in either the firſt or the ſecond method above-deſcribed; and with equal advantage to the publick, if employed in either of thoſe methods with equal ſteadineſs and fidelity.

Of the disadvantage that may arise to the public by granting life-annuities to persons of different ages for the same price.

CCCL. When life-annuities have, on former occasions, been established in England by the Government, it has, I believe, been customary to take the same price for annuities of the same yearly amount from persons of all ages, or, at least, of several different ages within certain, pretty distant, limits. Now this seems to be a very injudicious practice, and would be likely to prove disadvantageous to the nation, even though the price taken for the several annuities so granted should be a just medium between the true values of the same annuities for the oldest and the youngest lives for which they were made grantable: because it is probable that the greater part of the persons who would purchase such annuities would be nearer to the youngest age for which they would be grantable than to the oldest. Thus, for example, if a thousand annuities of 10 pounds a year each, were to be offered to sale by the Government for the lives of any persons between the ages of 20 years and 40 years at the price which is the true value of a like annuity of £10 a year for the life of a person of the age of 30 years, it is probable that the greater part of these annuities would be purchased for the lives of persons under 30 years of age, and consequently for less than their true value: and therefore the Government, or Nation, would lose more by the sale of these annuities to persons under the age of 30 years, than they would gain by the few annuities, of the same price, or class, which would be sold to persons who were above the age of 30 years, but under that of 40; and consequently they would, upon the whole, be losers by such a bargain. It would therefore, I apprehend, be more prudent in the Government, when annuities are to be granted to persons of different ages, to sell them at their true and proper prices according to the several ages of the persons for whose lives they are granted, having recourse, for the determination of the said prices, to the tables above-computed and set forth in pages 221, 222, &c.—232, and considering every such life-annuitant as being of the age to which he had attained on his last preceeding birth-day.

This disadvantage would be still greater in the case of life-annuities with the benefit of survivorship.

CCCLI. The disadvantage to the publick, mentioned in the foregoing article as resulting from the practice of granting life-annuities to persons of different ages at the same price, would, it is evident, be considerably greater in the case of life-annuities accompanied with the benefit of survivorship, than in the case of simple life-annuities: because in the case of survivorship those life-annuitants whose ages would have but little exceeded the youngest age of the class, and who would consequently have paid too little for their annuities, would, by means of this benefit of survivorship, if they happened to live a good number of years, enjoy much larger annuities than in the other case of simple life-annuities; and consequently the loss of the publick would be proportionably greater in the former case than in the latter. It would therefore be very highly expedient,

expedient, if the Government were to reſolve to eſtabliſh any life-annuities with the benefit of ſurvivorſhip, to take care that there ſhould be as many diſtinct claſſes of life-annuitants, intitled to the ſaid benefit of ſurvivorſhip with reſpect to each other, as there were different ages, differing from each other by a year, amongſt the ſaid life-annuitants, ſo that thoſe perſons only who were of the ſame age, or within a year of the ſame age, ſhould belong to the ſame claſs, and pay the ſame price for their annuities. This might be done, as I imagine, without occaſioning much difficulty or confuſion, by dividing the whole annual ſum which it was propoſed to apply to the converſion of ſome of the perpetual annuities into life-annuities with the benefit of ſurvivorſhip, into as many parts as there were different years in the ages of the life-annuitants for whoſe lives the ſaid annuities were intended to be granted. Thus, for example, if it were the intention of Government that perſons of any age from the age of 10 years to the age of 59 years, incluſively, ſhould be admitted to become purchaſers of theſe life-annuities with benefit of ſurvivorſhip, and the whole annual ſum intended to be employed in converting ſome of the perpetual annuities into theſe life-annuities with benefit of ſurvivorſhip, was £100,000 *per annum*; it would be expedient to divide the ſaid ſum of £100,000 *per annum* into fifty equal parts of £2000 *per annum* each, and to apply one of theſe parts, or the annual ſum of £2000, to the converſion of ſome of the perpetual annuities into theſe life-annuities with benefit of ſurvivorſhip, for the lives of perſons of the age of 10 years; and to apply another ſum of £2000 *per annum* to the converſion of ſome of the perpetual annuities into theſe life-annuities, with benefit of ſurvivorſhip, for the lives of perſons of the age of 11 years; and to employ the like annual ſums of £2000 each, in converting ſome of the perpetual annuities into theſe life-annuities, with the benefit of ſurvivorſhip, for the lives of perſons of the ſeveral ages of 12 years, 13 years, 14 years, 15 years, and every following number of years to 59 years incluſively. And the only difficulty that would attend this diviſion of the ſaid life-annuitants into the ſaid fifty claſſes according to their ages, would be to determine what would be the proper prices of the life-annuities that ſhould be granted to each claſs of life-annuitants, or what would be the quantity of the perpetual annuities, now due to the publick creditors, which the additional ſum of £2000 *per annum* would enable the Government to convert into life-annuities of the kind intended, accompanied with the benefit of ſurvivorſhip, for the lives of perſons of each of the ſaid fifty ages: which might be done by making a computation for each of the ſaid ages ſimilar to the computations entered into above in the examples of the fifth and ſeventh methods above-mentioned. And theſe computations the Government might eaſily procure to be made for the purpoſe by perſons that were but moderately converſant with this doctrine of Life-annuities, if they ſhould be of opinion that theſe methods of diminiſhing the national debt by eſtabliſhing life-annuities with benefit of ſurvivorſhip, were worth adopting.

Of the manner in which ſuch life annuities might be eſtabliſhed, ſo as to avoid that diſadvantage.

 CCCLII. It

Of the frequent alienations of the Sinking Fund, that have been made for many years paſt.

CCCLII. It has been obſerved above, in Art. CCLIV, that our miniſters of ſtate and our parliaments have, ever ſince the year 1733, (and we might have ſaid, ever ſince the year 1727;) gone into a moſt pernicious practice, of diverting the Sinking Fund (which had been appropriated in the year 1717 to the gradual diſcharge of the national debt,) from the original purpoſes of its deſtination, and applying it to the current ſervices of the year. The conſequence of this practice has been the increaſe of the national debt to a much greater quantity than it would otherwiſe have arrived at, notwithſtanding the three wars of 1739, 1755, and 1775, in which we have been unfortunately, and, perhaps, unneceſſarily, engaged.

A miſtaken opinion that has ſometimes been advanced in juſtification of the ſaid alienations.

Yet it has been often alledged in juſtification of this practice, "That it is "a matter of indifference to the nation, and to the increaſe or diminution "of its debts, whether, when any given ſum of money is wanted for any "particular ſervice, the ſaid ſum be borrowed at intereſt, and a new tax "be laid for the payment of the ſaid intereſt, or whether the ſaid ſum "be taken at once out of the ſinking fund and no new tax be laid to "ſupply its place." "Thus, for example, ſay theſe gentlemen, ſuppoſe "it ſhould be neceſſary at any time to apply the ſum of a million of "pounds, ſterling, to ſome particular purpoſe not included in the con-"ſtant and ordinary expences of the ſtate, as, for inſtance, to the building "ſome new ſhips of war for the Royal Navy, or purchaſing naval ſtores; "and ſuppoſe that the ſinking fund produces an overplus of ſomething "more than a million of pounds, one year with another, above the "neceſſary expences of the ordinary government in time of peace; why "ſhould we not take the ſaid million, (which is wanted for the ſaid "purpoſe,) out of the ſinking fund, and thereby only ſuſpend, or rather "leſſen, for one year the operation of the ſaid fund in diminiſhing the "national debt, inſtead of borrowing the ſaid ſum of a million at intereſt "of new creditors, and leaving the whole of the ſinking fund to be "applied to the purpoſes of its deſtination? By the former method of "proceeding the national debt will not, it is true, have been diminiſhed "in the courſe of the year by the ſum of £1,000,000, as it would other-"wiſe have been; but then, on the other hand, it will not have been "increaſed by a new loan of £1,000,000, as it will be in the latter "caſe: and therefore, upon the whole, the quantity of the ſaid debt "at the end of the year will be the ſame in both caſes." This, I believe, is the manner in which the ſaid remark is uſually ſtated. But it is, as Dr. Price has obſerved, a very great and dangerous ſophiſm.

Remarks on the ſaid miſtaken opinion.

For it is tantamount to ſaying, that it comes to the ſame thing, with reſpect to the diminution of the national debt, whether the Government does, or does not, raiſe new taxes towards defraying the extraordinary expences of the ſtate; which, it is evident, cannot be true. If, indeed, the aforeſaid million were to be borrowed at intereſt, and no new tax were to be laid to pay the ſaid intereſt, but the ſaid intereſt were to be taken every

every year out of the ſinking fund, the above-mentioned reaſoning would be juſt, and ſuch a proceeding would have exactly the ſame effect in retarding the diſcharge of the national debt as if the Government had taken the whole million itſelf out of the ſinking fund at once. For a million of pounds, ſterling, paid at once, is equal in value to the intereſt of a million of pounds, ſterling, to be paid every year for ever. But, when a new tax is provided to pay the intereſt of the new loan, the ſaid two methods of procuring the ſaid million, or other ſum that is wanted, can be no longer equivalent to each other, but the former method, to wit, "that of borrowing the money, and at the ſame time laying a new tax to pay the intereſt of it," muſt tend leſs than the other method, to wit, "that of taking the whole ſum at once out of the ſinking fund," to retard the diſcharge of the national debt, and therefore will be preferable to it with reſpect to that important operation. This is ſo evident that, I confeſs, I am ſurprized it can ever have been doubted of. But, ſince it has been doubted of, and ſince it is a truth of very conſiderable importance, I will endeavour to illuſtrate it by an example: which may be done as follows.

An example, to illuſtrate the falſhood of the opinion ſtated and confuted in the preceeding article.

CCCLIII. Let it be ſuppoſed (as in the ſeveral examples above-mentioned,) that the intereſt of money is 4 per cent. and that the 4 per cent.. annuities, now due to the publick creditors, ſell at the publick market at their *par*, or nominal value. And let it further be ſuppoſed that the Parliament had ſet apart the ſum of a million of pounds, ſterling, a year out of the ſinking fund, for the diminution of the national debt, and that the ſaid £1,000,000 *per annum* were to be faithfully applied to that purpoſe during a period of 20 years, in the firſt of the ſeven methods above-deſcribed, that is, in paying off ſome of the 4 per cent. annuities, now due to the publick creditors, at their *par*, or nominal value; by which means it is evident that a very conſiderable portion of the national debt will be extinguiſhed at the end of the ſaid 20 years. And let it be ſuppoſed that during every one of the ſaid 20 years the Parliament ſhall find it neceſſary to employ the ſame ſum of £1,000,000 in ſome other important ſervices, and that, being unwilling to ſuſpend, or impede, the operation of the ſinking fund, they ſhall forbear taking this million out of it, and ſhall, inſtead thereof, borrow a million every year of new creditors at 4 per cent. and at the ſame time impoſe every year a new perpetual tax of £40,000 *per annum* to pay the intereſt of the million ſo borrowed. And then let us inquire what will be the effect of this method of proceeding at the end of the ſaid 20 years, with reſpect to the increaſe or diminution of the national debt. This may be done in the manner following.

CCCLIV. The

CCCLIV. The quantity of the capital of the 4 per cent. annuities which will be extinguiſhed at the end of the ſaid 20 years by employing the ſaid £1,000,000 every year for that purpoſe in the firſt of the ſeven methods above-deſcribed, will be equal to the ſum to which an annual revenue of £1,000,000 will amount at the end of 20 years, if every payment of the ſaid revenue, together with the intereſt ariſing from the ſaid payment, ſhould, immediately after it is received, be lent out at intereſt at the rate of 4 per cent. Now it appears from Mr. Smart's third table of compound intereſt, page 68, that this ſum is = £1,000,000 × 29.778,078,58, or £29,778,078.58. Therefore the ſum of £29,778,078.58 of the capital of the 4 per cent. annuities, now due to the publick creditors, would be extinguiſhed by means of the ſaid £1,000,000 *per annum* in the courſe of the ſaid 20 years; or the national debt would be diminiſhed, by means of the ſaid £1,000,000 *per annum*, at the end of the ſaid 20 years, by the ſum of £29,778,078.58. But on the other hand it will be increaſed in the courſe of the ſaid 20 years by means of the aforeſaid loans of a million each, (which we have ſuppoſed to be made every year with new funds to pay the intereſt of them,) by only the ſum of £20,000,000. Therefore it will have been more diminiſhed than increaſed, and conſequently will, upon the whole, have been diminiſhed, in the courſe of the ſaid 20 years, by the exceſs of £29,778,078.58 above £20,000,000, that is, by the ſum of £9,778,078.58; which is not very much leſs than ten millions. Such will be the good effect, with reſpect to the diminution of the national debt, of the ſaid firſt method of proceeding, "by borrowing the money that is wanted every year, and laying new taxes to pay the intereſt of it."

CCCLV. But, if the other way of proceeding be adopted, or the million of pounds, that is wanted every year for the ſuppoſed ſervices, be taken out of the ſinking fund, it is evident, the operation of the ſaid million, in diminiſhing the national debt, will be thereby totally ſuſpended, and the ſaid debt will be exactly the ſame at the end of the ſaid 20 years as at the beginning of them.

CCCLVI. Therefore the difference between the quantity of the national debt at the end of 20 years, if the former method be taken, and the quantity of it at the ſame time, if the latter method be adopted, is no leſs a ſum than £9,778,078.58, or not much leſs than ten millions of pounds. Q.E.I.

CCCLVII. If

CCCLVII. If we had taken a longer period than 20 years, the difference of the two quantities of the national debt at the end of it, arising from these different methods of raising the said annual sum of a million of pounds, sterling, would have much greater than £9,778,078.58. In a period of 30 years it would have been (£56,084,937.76 — £30,000,000, or) £26,084,937.76; in a period of 40 years it would have been (£95,025,515.72 — £40,000,000, or) £55,025,515.72; in a period of 50 years it would have been (£152,667,083.68 — £50,000,000, or) £102,667,083.68; and in a period of 60 years it would have been (£237,990,685.24 — £60,000,000, or) £177,990,685.24; as will appear from Mr. Smart's third table of compound interest, pages 68, 70, and 72.

Other examples for the same purpose.

CCCLVIII. The difference between these two methods of raising the money that may from time to time become necessary to defray the extra-ordinary expences of the state, is a matter of such importance that it can hardly be too much insisted on. And it seems to have been well understood, and properly attended to, by our ministers of state in the reign of king George the 1st, when the sinking fund was first established. For at that time it was customary for the Parliament to borrow money to supply the occasional exigencies of the state, and lay new taxes to pay the interest of the money so borrowed, instead of taking the said money out of the sinking fund and thereby interrupting the operation of that useful establishment. Of this salutary practice Dr. Price in his excellent *Appeal to the Publick on the Subject of the National Debt*, 2d edition, published in the year 1772, page 29, note *b*, has given us several examples. He there informs us, that

The falshood of the said mistaken opinion seems to have been well understood in the reign of king George the 1st.

The Sinking Fund was faithfully applied to the discharge of the national debt to the end of that reign.

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In 1718 was borrowed towards the supplies — — —	505,995;
In 1719 — — — — — —	312,737;
In 1720 — — — — —	500,000;
In 1721 — — — — — —	1,000,000;
In 1725 — — — — — —	500,000;
In 1726 — — — — — —	370,000;
And In 1727 — — — — —	1,750,000.

In all these instances (if I understand the doctor rightly,) the Parliament not only borrowed these several sums, rather than take them out of the sinking fund, (as they might have done,) but laid new taxes to pay the interest of them. And in the year 1726, (the nation being then under the apprehension of a war,) the land-tax was raised from two shillings in the pound to four shillings in the pound, rather than take any money from the sinking fund. And in the following year, 1727, (in which the nation still continued

continued under the apprehension of a war,) king George the 1st in his speech at the opening of the parliament, (on the 17th of January, 172$\frac{6}{7}$,) after congratulating the gentlemen of the House of Commons on the great addition that would be made that year to the *Sinking Fund*, warns them against being led by the NECESSITIES OF THE NATION to a diversion of it. "Let all that wish well (are the words of the speech,) "to the peace and quiet of my government, have the satisfaction to see "that our PRESENT NECESSITIES shall make no interruption in the "progress of that desireable work of gradually discharging the national "debt. I hope therefore you will make a provision for the immediate "application of the produce of the Sinking Fund to the uses for which "it was so wisely contrived, and to which it stands now appropriated." To which the House of Commons in their address to the king in answer to his said speech, (which was carried, upon a debate and division, by 251 voices against 81, and was presented to the king on the 19th of January, 172$\frac{6}{7}$,) reply in the words following, to wit, "And that all who "wish well to the peace and quiet of your Majesty's government, may "have the satisfaction to see, that our PRESENT NECESSITIES shall "make no interruption in the progress of that desireable work of gradually discharging the national debt, we will consider of the most proper "methods for immediately applying the produce of the Sinking Fund "to the uses for which it was so wisely contrived, and to which it stands "now appropriated." By this speech of the King and address of the House of Commons it seems clear that the advantages attending a faithful application of the Sinking Fund to the purposes of its destination, were at that time well understood and generally allowed.

A change of conduct in the ministers of state and the Parliament, with respect to the Sinking Fund, took place in the year 1728.

CCCLIX. But soon after the death of king George the 1st a different opinion seems to have prevailed with many people, and, most unhappily for the kingdom, a different practice began to be adopted by the Government. Dr. Price informs us, in the passage above-mentioned, that in the years 1728 and 1729 the Sinking Fund was charged with the interest of the money borrowed in those years; which was just as pernicious a measure, with respect to the interruption of the great object of diminishing the national debt, as if the Parliament had taken the whole sums themselves at once from the Sinking Fund, instead of borrowing them: as we have already observed above in Art. CCCLII, page 380, line 4 from the bottom, &c. It was, however, a less open and manifest violation of the Sinking Fund than the other measure of taking the gross sums themselves out of it; and therefore probably it gave less offence to that part of the nation which had most approved of the institution of the Sinking Fund, and it contributed less to expose Sir Robert Walpole (who was chancellor

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of the Exchequer at the institution of the Sinking Fund in the month of March, 17$\frac{16}{17}$,* and was the principal adviser and promoter of that useful measure, and who was likewise in the same great office at the time of the aforesaid misapplications of the said fund in the years 1728 and 1729,) to the charge of *inconsistency* and of becoming the destroyer of his own favourite and justly-applauded measure. And accordingly we find that, while this appearance of delicacy with respect to the violation of the Sinking Fund continued, the King and Parliament, both, spoke of it with respect, and pretended to be unwilling to break into it. For in the year 1729, when (the nation being still threatened with a war,) extra-ordinary supplies were wanted, and the Sinking Fund would have afforded all that was necessary, king George the second, in his speech to the Parliament at the opening of the session, on the 21st of January, 172$\frac{8}{9}$, expressed himself to the members of the House of Commons in these words; "And, as the produce of the Sinking Fund has exceeded our expecta-" tions, I must recommend it to your care to make a farther application "of it to its proper uses." To which the Commons, in their address to the King upon this speech, make the following answer, to wit, "That "they will not fail to make the proper disposition of the growing produce "of the Sinking Fund." And accordingly a million of the South-Sea annuities was ordered to be paid off that year out of the Sinking Fund; but at the same time the interest of the new money that was borrowed this year, was charged upon the said fund, and became a perpetual incumbrance on it. And the same thing had been done in the preceeding year, 1728, with respect to the interest of the money borrowed in that year.

But yet an appearance of an unwillingness to break into it was kept up by the king and Parliament for about five years longer.

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* The Sinking Fund was established by Stat. 3 George I, Cap. vii, Sect. 37, which is in these words. "And be it enacted and declared by the authority afore-" said, That all the monies to arise from time to time, as well of, or for, the said "excess, or surplus, by virtue of the said act made for redeeming the Funds of the "Governour and Company of the Bank of England; and of, or for, the said excess, "or surplus, by virtue of the said act made for redeeming the Funds of the said "Governour and Company of Merchants of Great-Britain trading to the South-"Seas, and other parts of America, and for encouraging the Fishery; as also of and "for the said excess, or surplus, of the said duties and revenues by this act appro-"priated, as aforesaid; and the said overplus monies of the said general yearly Fund "by this act established, or intended to be established, as aforesaid; *shall be appro-" priated, reserved, and employed, to and for the discharging the principal and interest of " such national debts and incumbrances as were incurred before the five and twentieth day " of December, one thousand, seven hundred, and sixteen*, and are declared to be national "debts, and are provided for by act of Parliament, in such manner and form as shall "be directed or appointed by any future act or acts of Parliament to be discharged "therewith or out of the same, *and to, or for, none other use or purpose whatsoever*."

The words of the clause in the statute of the 3d of Geo. I, by which the Sinking Fund was established.

In the year 1733 the Parliament took a groſs ſum of half a million of pounds ſterling, out of the Sinking Fund, in order to keep the land-tax at only one ſhilling in the Pound.

CCCLX. But in a few years time this appearance of delicacy with reſpect to the miſapplication of the Sinking Fund was laid aſide, and the miniſters of ſtate and parliament openly took out of it whatever groſs ſums of money they thought neceſſary for the publick exigencies. The manner in which this unfortunate change of conduct happened, is related by Dr. Price in his tract above-mentioned, intitled *An Appeal to the Publick on the Subject of the National Debt*, in theſe words. "In the year 1732 the "land-tax had been reduced to one ſhilling in the pound; and, in order "to ſupply the deficiency ariſing from hence, half a million had been "procured for the current ſervice by the revival of the ſalt-duties, which, "but two years before, had been repealed, becauſe reckoned too burthen-"ſome to the poor.—In the year 1733, in order to keep the land-tax as "low as it had been the year before, it was neceſſary either to borrow "another half-million, or to take it from the *Sinking Fund*. The latter "method was choſen; and propoſed by Sir Robert Walpole to the Houſe "of Commons.—Long and warm debates enſued.—A propoſal to alienate, "in a time of profound peace, a fund which the law had made ſacred, "and the alienation of which no poſſible exigence of publick affairs could "juſtify, only for the ſake of keeping the land-tax for one year at one "ſhilling in the pound, juſtly kindled the indignation of the patriotick "party. They urged, in oppoſition to it, the prohibition of the law, "the faith of Parliament, and the ſecurity of the kingdom. The pro-"poſer of the alienation was reminded of his inconſiſtency and treachery "in endeavouring to beat down that very monument of glory which he "had boaſted of having erected for himſelf; and Sir John Barnard warned "him that he was drawing upon himſelf the curſes of poſterity.—But all "arguments were vain.—The miniſtry pleaded that the landed intereſt "wanted eaſe; that there was no occaſion for being in a hurry to pay the "national debt; and that the circumſtances of the kingdom had altered "ſo much ſince the eſtabliſhment of the *Sinking Fund*, that the competition "then among the publick creditors was, not who ſhould be *firſt*, but who "ſhould be *laſt*, paid. Thus argued, among others, Sir Walpole. His "reaſons prevailed; and the Houſe of Commons, not uſed to refuſe him "any thing, conſented.

Dr. Price's account of the debates upon this occaſion.

And the ſame pernicious practice has continued ever ſince.

"The practice of alienating the Sinking Fund having been thus begun, "it went on of courſe. In the next year, or 1734, £[illegible],20[illegible],000 was taken "from it. In 1735 it was even anticipated and mortgaged.

"Thus then expired, after an exiſtence of about elever years, the "*Sinking Fund*,—that ſacred bleſſing—once the nation's only [illegible]e; pre-"maturely and cruelly deſtroyed by its own parent.——Could [illegible] have "eſcaped the hands of violence, it would have made us the envy and the "[illegible]ur of the world, by leaving us at this time [in the year 1772] "not

"not only TAX-FREE, but in poſſeſſion of a treaſure greater, perhaps, "than was ever enjoyed by any kingdom.—But let me not dwell on a "recollection ſo grievous!"

CCCLXI. Dr. Price informs us further in the excellent tract above-mentioned, that about the year 1726 an opinion had been propagated, that, notwithſtanding the eſtabliſhment of the Sinking Fund, the publick debts had been for ſome years increaſing rather than decreaſing. "And "this, ſays he, occaſioned the publication of a very curious and important "pamphlet in defence of the Sinking Fund and the miniſtry, intitled, "*An Eſſay on the Publick Debts of this Kingdom.* I have now by me the "fourth edition of this pamphlet; and I wiſh I could put it into every "hand in the kingdom. It contains an excellent account of the import-"ance of diſcharging the publick debts, and of the proviſions made for "that purpoſe by the inſtitution of the Sinking Fund. It proves particu-"larly, in oppoſition to the opinion I have mentioned, that the publick "debts *had decreaſed*; and that, of the 52 millions then due, 50 millions "would in 28 years be extinguiſhed by the *Sinking Fund.* The ſame "explanation is given of the nature of this fund with that which I have "given: the ſame repreſentation is made of its powers; and the ſame "arguments are uſed to demonſtrate the evil of alienating it, in order to "avoid making new loans charged on new funds.—And he concludes "with this general inference from all his obſervations, "'That the pro-"'viſion which had been made of the *Sinking Fund*, was an expedient "'from which the full and effectual payment of the principal of the "'national debt, in a few years, might, with great aſſurance, be "'expected.'"

Dr Price's account of an excellent pamphlet on the publick debts of this kingdom, that was publiſhed in the year 1726.

CCCLXII. This pamphlet, Dr. Price further informs us, was anſwered in a pamphlet intitled, *A State of the National Debt*; and this produced another pamphlet by the author of the firſt, (or the *Eſſay on the Publick Debts,*) which was intitled, *A Defence of an Eſſay on the Publick Debts of this Kingdom, in anſwer to a pamphlet, intitled, A State of the National Debt.* And in this laſt pamphlet, intitled, *A Defence of an Eſſay, &c.* there are theſe words. "The ſucceſs of my *Eſſay on the Publick Debts,* "and the ſatisfaction of hearing from the throne, that my zeal for the pre-"ſervation of the Sinking Fund cannot have been diſagreeable to his Ma-"jeſty, gives me a pleaſure that, alone, is an ample reward for my attempts "to ſerve my country." From theſe words it ſeems probable that this pamphlet (of which I have now in my hands the ſecond edition, which was publiſhed in the year 1726;) had been very much read and approved of, and had tended very much to confirm the miniſters of ſtate in their reſolu-

Of the good reception the ſaid pamphlet ſeems to have met with from the publick.

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tion to continue, for that year at leaſt, to apply the Sinking Fund to the purpoſe of its deſtination, and to induce them to adviſe the king to make uſe of the words above mentioned in his ſpeech to the Parliament on the 17th of January, 172$\frac{6}{7}$, to wit, "Let all that wiſh well to the peace and "quiet of my government have the ſatisfaction to ſee, that our preſent "neceſſities ſhall make no interruption in the progreſs of that deſireable "work of gradually diſcharging the national debt. I hope, therefore, you "will make a proviſion for the immediate application of the produce of "*the Sinking Fund* to the uſes for which it was ſo wiſely contrived, and to "which it ſtands now appropriated."

A conjecture concerning the author of the ſaid pamphlet.

CCCLXIII. The aforeſaid excellent pamphlet, intitled, *An Eſſay on the Publick Debts of the Kingdom*, was publiſhed without a name. But it ſeems probable that it was written by Sir Nathaniel Gould, an eminent merchant, who was one of the directors of the Bank in the year 1726, when the ſecond edition (and, perhaps, alſo the firſt edition of it,) was publiſhed. And the other pamphlet above-mentioned, which was written in anſwer to it and was intitled, *A State of the National Debt*, ſeems to have been written by the famous Mr. William Pulteney, who was afterwards created Earl of Bath. This may be conjectured with a conſiderable degree of probability from the following extract from Chandler's Collection of the Proceedings of the Houſe of Commons, vol. vii. page 23. "On "the 23d of February, 172$\frac{7}{8}$, the Commons being in a committee of "ſupply, Mr. William Pulteney [who was then in oppoſition to the mi- "niſtry,] obſerved, "'That, notwithſtanding the great merit that ſome "'perſons had built upon the eſtabliſhment of the *Sinking Fund*, it appeared "'that the national debt had been increaſed ſince the ſetting up of that "'*Pompous Project*.'" Upon which Sir Nathaniel Gould, an eminent "merchant, ſaid, "'That he apprehended that gentleman had his notions "'out of a treatiſe, intitled, *A State of the National Debt*, *&c.* ſuppoſed "'to be written by that very gentleman: but that, if he [Sir Nathaniel "'Gould] underſtood any thing, it was *Numbers*; and he durſt pawn his "'credit and reputation to prove that author's calculations and inferences "'to be falſe and erroneous.'" To this Mr. Pulteney replied, "'That he "'took them to be right; and he would likewiſe pawn his credit and "'reputation to make good his aſſertion.'" Upon this Sir Robert Walpole "took up the cudgels, and ſaid, "'He would maintain what Sir "'Nathaniel Gould had advanced.'" Several warm expreſſions having "paſſed on both ſides, Mr. Hungerford interpoſed, in a jocular ſpeech, "that put the Houſe in good humour; and ſo the diſpute ended."

CCCLXIV. As

CCCLXIV. As this pamphlet, intitled, *An Essay on the Publick Debts of the Kingdom*, (which, for the reason just now given, I conjecture to have been written by Sir Nathaniel Gould,) is, in Dr. Price's opinion, of such importance as to deserve to be put into every hand in the kingdom; and, as it tends strongly to illustrate and enforce the utility of the measure I have ventured to recommend above in Art. CCXLIX and CCLIV, (pages 29[illegible] and 298, 299,) "of appropriating, in the strictest manner possible, some part, at least, of the Sinking Fund to the purposes of its original destination, or the gradual discharge of the national debt;" I am persuaded my readers will not be sorry to have an opportunity of perusing it; and I therefore have determined to reprint it, word for word, as a part of the present Treatise. It is as follows.

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AN
ESSAY
ON THE
PUBLICK DEBTS
OF
THIS KINGDOM.

WHEREIN

The Importance of discharging them is considered; the Provisions for that Purpose by the SINKING FUND, and the Progress therein hitherto made, are stated and explained; the Sufficiency of those Provisions is demonstrated; some general Mistakes about the Nature and Efficacy of this Expedient examined and removed; and the Progress of the SINKING FUND described and computed from *Midsummer*, 1727.

To which is subjoined,

An Enquiry into the General Convenience of reducing farther the Interest of our Publick Debts below 4 *per Cent. per Annum.*

In a Letter to a Member of the House of Commons.

Reprinted from the Second Edition, which was published by J. PEELE, in Pater-Noster-Row, in the Year 1726.

N. B. This Pamphlet is supposed to have been written by Sir NATHANIEL GOULD, an eminent Merchant and a Director of the Bank.

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PREFACE.

THERE may perhaps appear ſomething too aſſuming in the attempts of an author, to inform the publick, or to direct their ſentiments about matters of general importance to admit of a better reception or entertainment from the town than what performances of this kind have of late years generally met with. I have thought therefore, that it may not be improper to mention my reaſon for the publication of the following ſheets, as an apology for it with the reader; to whom I can with great truth and ſincerity repreſent, that I ſhould never have thought any knowledge which I had, or any diſcoveries in my power, of our circumſtances with regard to our preſent debts, worth the publick notice, if I had not frequently met with ſome miſtakes on this ſubject, which appeared to me very generally to prevail, and firmly to be inſiſted on and believed much to the diſadvantage of our publick credit, and which at the ſame time I have flattered myſelf, might be confuted and removed, from ſuch informations only as I ſhould be able, on this occaſion, to collect. I have ſo often heard it affirmed, that our publick debts have increaſed upon us ſince the proviſions made for the diſcharge of them, that it has ſometimes ſeemed to me to be the more common opinion even of thoſe perſons who are moſt intereſted to be rightly informed in this particular; and have almoſt as often heard it from hence inferred, that thoſe proviſions are therefore inſufficient to anſwer the expectations we are ſuppoſed to have from them. And from the bad influence that the belief of this aſſertion, and the inference from it, muſt have on our publick credit, eſpecially when it falls in with any general apprehenſions for the publick peace or welfare on any other account, I have been induced to think, that as this fact is not true, nor the inference from it rightly made, it would be of general convenience that they were publickly contradicted, and proved to be otherwiſe; and that this were better done from that leſs exact and partial information which I have been able to come at upon this ſubject, than not done at all, or perpetually put off in expectation of its being ſome time done by ſuch perſons who have the exacteſt knowledge of our circumſtances in this reſpect, or the beſt capacity for improving it for this purpoſe.

Nor ſhould I have been diverted from communicating the few or partial diſcoveries that the following ſheets may be thought to contain, by being told, that ſuch miſrepreſentations of our circumſtances were made with deſign only, and by perſons who better than myſelf knew the ſtate of our affairs in this reſpect; becauſe in this caſe in particular, it appears to be the publick intereſt, that the truth ſhould be as generally known as may be, and that every perſon who is or may be an adventurer in our publick funds, ſhould, as diſtinctly as can be, underſtand the proviſions that have been made for ſupporting the credit of them.

What I have farther added beyond the general deſign, by which I was at firſt engaged to write upon this ſubject, the reader will judge of on the peruſal of it. I am not inſenſible that there are ſeveral parts of this performance open to exceptions; but I have more hopes of the reader's indulgence to theſe faults as they ſhall occur to him, than I have that he will forgive the recital of them here, or my detaining him by endeavouring in this place to explain away or obviate any exceptions of this kind. I ſhall therefore mention but two particulars, in which I may be thought more than once to have offended. One is, that I have not every where uſed the utmoſt exactneſs in ſuppoſing, ſtating, or deſcribing the publick debts, or the variations in them. To this fault, as often as I have been guilty of it from any other cauſe than my want of materials for that purpoſe, I have been chiefly induced by the views of being thereby more intelligible; having preſumed that it would be better to omit any ſuch degrees of exactneſs in this reſpect, as were more than ſufficient to anſwer the general deſign of this eſſay, which would at the ſame time render it more tedious and perplexing. I have alſo, for much the ſame reaſon, been induced to content myſelf with the uſe of ſome words in what has ſeemed to me to have been their more ordinary acceptation, when applied to this ſubject; which in a longer or more elaborate enquiry, I ſhould have thought myſelf obliged to define and explain diſtinctly before I ventured upon the uſe of them.

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ESSAY

ON THE

PUBLICK DEBTS

OF

THIS KINGDOM.

In a LETTER to a Member of the House of Commons.

SIR,

UPON recollecting the conversation that was the occasion of your desiring my thoughts in writing on the subject of our Publick Debts; I have concluded, that I should best answer your expectations from me in this affair, by confining my thoughts,

1. To the consideration of what advantage to the publick may be reasonably expected from the discharge of those debts, and the redemption of the duties provided for the payment of their interest.

2. To an enquiry into the reasons we have at present to expect or hope that these debts, or any considerable part of them, will within any reasonable compass of time be discharged and paid off. And,

3. To

3. To ſuch reflections as have occurred to me upon thoſe meaſures that may for the future be entered upon, for the more ſpeedy and effectual diſcharge of our preſent debts, from the income of the Sinking Fund already provided for that purpoſe; or for ſtill farther increaſing the annual income of that fund by ſuch reductions as may yet be made in the intereſt or annuities payable for the principal ſums of which the preſent debt conſiſts.

Of the advantages that will arise to the publick from the diſcharge of the national debts.

As to the firſt of theſe, or the advantage ariſing to the publick by the diſcharge of the preſent debts; there ſeems to be but little room to enlarge, after the conſideration of that great annual revenue at preſent levied and applied to the payment of our debts, which, after the total diſcharge of them, will, without any loſs or injury to private perſons, be redeemed to, and become the property of, the publick. The preſent yearly expence to the Government, on account of our publick debts, computing the annual income of the Sinking Fund and the yearly intereſt of thoſe debts together, will be found to amount to little leſs than, if not to exceed, the ſum of £3,000,000. A revenue exceeding the whole farther annual expence of our civil and military government in a time of peace; and which, together with the ordinary ſupplies which our Government requires in a time of peace, may perhaps be a fund ſufficient to anſwer our utmoſt probable expences during the moſt expenſive war.

I do not think myſelf at liberty to ſuppoſe, or promiſe it as one advantage ariſing to the publick from the diſcharge of the preſent debts, that the ſeveral duties appropriated to the payment of them will, as ſoon as they are redeemed, be immediately removed or determined; for reaſons, which in the following ſheets I ſhall have a further occaſion to mention; when I ſhall recommend it to be conſidered, whether the revenues ariſing from thoſe duties, or the greateſt part of them, are not raiſed with more eaſe, greater equality, and more to the common benefit of the ſubjects of Great-Britain, than ſome part of the ſupplies that are annually voted for the current ſervice of the year; and conſequently, how far it may be reaſonable to ſubſtitute a great part of the revenues ariſing from thoſe duties, after the redemption of them, in the place of our annual taxes. But it will, I preſume, appear no ſmall convenience to the publick, ariſing from the redemption of the aforeſaid duties, that, when they ſhall be no longer appropriated to the payment of our debts, the principal difficulty will be removed, which has at any time obſtructed the removal or leſſening any of theſe duties, though the convenience of the publick may, upon other accounts, have perſuaded to it; either as ſuch duties may have appeared to give too great perplexity to perſons employed in trade, or to prevent or obſtruct any profitable branch of our commerce with foreign countries; as they may have been thought to require too ſtrict an enquiry, or or too great ſeverity or expence in the collecting them; as by being laid on any commodities univerſally neceſſary, they may have ſeemed too great a burthen on the pooreſt of our inhabitants; or as by bearing too great a proportion to the bulk of the commodities on which they have been laid, they may have made the gain ariſing from defrauding the publick, or the temptation to attempt it, bear too great a proportion to the hazard of being diſcovered; or, as in any other reſpect they may be found to be attended with general inconvenience, or unreaſonable hardſhip on particular perſons, employments or conditions of life amongſt us.

And, however it ſhall be determined, after the diſcharge of our preſent debts, as to the continuance or removal of the whole or any part of the duties appropriated for the payment of them; the revenues ariſing from them, being redeemed, will become the property of the publick, and, if not from thenceforth removed, will be employed in the room of, and take away the occaſion for, ſuch other taxes as ſhall then appear a greater burthen to, or to be more unequally levied upon, the ſubjects of this kingdom.

Having mentioned the quantity of annual expence to the Government, occaſioned by our publick debts, it ſeems unneceſſary to proceed further in proving the importance of diſcharging them, or to deſcend to or enumerate any further inconveniencies, that upon this account we labour under. An uncomfortable employment! and which, I hope, I ſhall be excuſed from, for this further reaſon; that the inconvenience of our preſent debts, and the importance of diſcharging them, are ſo univerſally believed and felt, and ſo unanimouſly agreed to, that I know none of my fellow-ſubjects who want to be convinced of them. I ſhall proceed therefore to what I propoſed in the

Of the probability that the ſaid debts will be diſcharged in a few years by means of the Sinking Fund.

Second place, to make out the probability, and repreſent the reaſons we have to hope, that the preſent publick debts will, within the compaſs of a few years, be effectually and honourably diſcharged.

What I have chiefly propoſed under this head, is to deſcribe and explain, as far as my materials for that purpoſe will carry me, thoſe meaſures which have been already taken for the diſcharge of our publick debts by the proviſion of the Sinking Fund. To which attempt, though this proviſion has already been made as publick as our acts of parliament, and though the operation and progreſs of it, in the diſcharge of our debts, is without any difficulty to be computed, I find myſelf induced, from that general ſuſpicion of the inefficacy of this proviſion to anſwer the ends propoſed by it; and which ſeems to have prevailed amongſt ſome people, who have either not had leiſure for that purpoſe, or who have declined the trouble of collecting the materials for, or making thoſe computations from

from them which are requisite, in order to their satisfaction about the use and efficacy of the Sinking Fund.

Of the establishment of the Sinking Fund in the month of March, 1716/7.

The first material provision that was made for discharging the principal of our present debts, was enacted in the third year of his present Majesty's reign, by three several acts of parliament at that time made; the first of which (in the order that they should have been printed amongst the statutes published for that sessions) is intitled, *An act for redeeming several funds of the Governour and Company of the Bank of England, pursuant to former provisoes of redemption; and for securing to them several new funds and allowances redeemable by Parliament; and for obliging them to advance further sums, not exceeding £2,500,000 at five per cent. as shall be found necessary to be employed in lessening the national debts and incumbrances; and for continuing certain provisions formerly made for the expences of his Majesty's civil government, and for the payment of annuities formerly purchased at the rate of five per cent. and for other purposes in this act mentioned*, page 331. The second, intitled, *An act for redeeming the yearly fund of the South-Sea Company (being after the rate of six pound per cent. per annum, and settling on the said Company a yearly fund after the rate of 5 per cent. per annum, and to raise for an annuity or annuities, at 5 per cent. per annum, any sum not exceeding £2,000,000, to be employed in lessening the national debts and incumbrances, and for making the said new yearly fund and annuities to be hereafter redeemable in the time and manner thereby prescribed*, page 375. And the third, intitled, *An act for redeeming the duties and revenues which were settled to pay off principal and interest on the orders made forth on four lottery acts, passed in the 9th and 10th years of her late Majesty's reign; and for redeeming certain annuities payable on orders, according to a former act in that behalf; and for establishing a general yearly fund, &c.* page 291.

Of the several sums of which the Sinking Fund is composed.

The *Sinking Fund* of late years, (called so from its being understood to be appropriated to the sinking and discharging, as far as it will go, the principal sums of the present publick debt,) is made up of money arising yearly into the Exchequer, as the surplus of the produce of three several funds established by the three aforesaid acts of parliament, by the names of the Aggregate Fund, the South-Sea Fund, and the General Fund; the surplusses of which three funds, or what they annually produce more than the yearly sums to the payment of which they are first appropriated, are by the last of the aforesaid acts of parliament reserved for, and made applicable only to, the discharge of the principal and interest of such debts as had been before the year 1716 contracted and provided for by Parliament. The yearly sums to the payment of which those funds are first appropriated (except the sum of £700,000 *per annum* to his Majesty for the expence of his civil government) are generally the interest, or annuities, payable for several principal sums, of which our publick debts consist. As

As often therefore as any of thoſe principal ſums are paid off, or the proprietors of any part of the publick debt are induced to accept of a leſs intereſt or annuity for the ſame principal ſums, the Sinking Fund is underſtood to increaſe by the yearly addition of the intereſt of ſuch ſums as are paid off, or the abatement of the annuity for ſuch part of the publick debt as is agreed to be continued at a lower rate of intereſt. But the duties, the ſurplus of which the Sinking Fund conſiſts of, not bringing in every year an equal ſum of money, the ſurplus likewiſe is not every year alike; and therefore, in order to compute in what time the preſent publick debt may be diſcharged by the Sinking Fund, it is neceſſary to enquire from what different annual ſums have of late years been produced by it, what yearly ſum it is reaſonable to ſuppoſe may for the future be produced by it at a medium, or one year with another. To which yearly ſum we are from time to time to add the yearly intereſt or annuity of ſuch principal ſums, part of the preſent publick debt, as may be paid off by it; and all ſuch abatements of intereſt of all or any part of the publick debts, as are already agreed hereafter to take place, or may for the future be agreed to by any of the creditors to the publick.

There are a great many particulars which you will ſee, Sir, I muſt want the knowledge of, in order to make this ſuppoſition with any great exactneſs. I could wiſh here to be able to ſtate the produce of the ſeveral particular duties, the exceſſes of which conſtitute and ſupply the Sinking Fund; the different ſums produced by them in different years, from the times they were ſeverally granted; to aſſign the moſt probable cauſes of their variation, and from thence infer the probability of their producing more or leſs for the future: but however unprovided I am with materials for an inquiry of this kind, it may be yet worth while to proceed in deſcribing the proportions in which any determined yearly ſum (though by miſtake) ſuppoſed to be the preſent yearly produce at a medium of the Sinking Fund, will increaſe, when applied to the payment of the publick debts; as thoſe proportions will be the ſame with thoſe in which any other ſum, with more truth or probability ſuppoſed to be produced one year with another by the Sinking Fund, will increaſe when applied to the ſame purpoſe.

Of the annual amount of the Sinking Fund.

The beſt account I have been able to get of the produce of the Sinking Fund for ſome years laſt paſt lies now before me, and ſtates the produce of the ſurpluſſes of the ſeveral funds, commonly called the Aggregate Fund, the General Fund, and the South-Sea Fund, (the ſum of which ſurpluſſes our acts of parliament call the *Sinking Fund)* to be from the 31ſt of December, 1722, to the ſame time in the year 1723, £619,000 and upward; and the produce of the ſame ſurpluſſes from thence to the 31ſt of December, 1724, to amount to upwards of £653,000. This amount

amount of the produce of the Sinking Fund for the two years above-mentioned, exceeds the produce of the ſame fund for ſome years before, by a greater ſum than can be accounted for by the diſcharge or reduction of the intereſt of any part of the publick debt before that time; and which therefore I am inclined to attribute to ſeveral proviſions about that time made by the Legiſlature, for preventing frauds in the payment, and for the more fully and effectually collecting of ſeveral duties which in part ſupply the revenues appropriated to the payment of our publick debts; and of which proviſions I would hope we may long enjoy the benefit in the increaſe of the Sinking Fund. And from hence, I ſhould think, we might venture to expect an annual produce from the Sinking Fund for the future, equal to the produce of the ſame fund at a medium for the two years above-mentioned, ending in December 1724; and increaſing by the yearly addition of the intereſt of ſuch principal ſums as may be henceforth paid off, and of the abatements of the intereſt, or annuities, of any of the publick debts when the ſame ſhall take place, that already are or may hereafter be agreed for.

Of the ſavings of intereſt that will accrue to the Sinking Fund in the year 1727.

The abatements of intereſt in the year 1727 are ſo conſiderable, and the time when they are to take place ſo near, that I believe it will be thought reaſonable to ſtep forwards to the time when the Sinking Fund will be increaſed by the addition of thoſe abatements; and from that time to conſider the progreſs that may be made in diſcharging the preſent publick debt by the Sinking Fund.

From Midſummer in the year 1727, it is already provided, that the Sinking Fund be increaſed by the reduction of the intereſt from 5 to 4 *per cent.* or an abatement of 1 *per cent. per annum* on the principal ſums following.

	l.	*s.*	*d.*
On £13,061,878, being the amount of the publick debt to the South-Sea Company, excluding £3,839,363, part of it, for which an annuity at 4 *per cent.* only is at preſent payable,	130,618	15	7
On 16,901,241*l.* 17*s.* of the South-Sea annuities, - -	169,012	8	4
On 3,775,027*l.* 17*s.* 10½, part of the debt to the Bank of England, - - - -	37,750	5	6¾
On £4,000,000, farther part of the debt to the Bank of England, purchaſed by them of the South-Sea Company, - - - - - - -	40,000	0	0

To

	l.	s.	d.
To this, if the annual income of the Sinking Fund on the 31st of December, 1724, be added, supposed to be	600,000	0	0
And the increase of it by the discharge of £600,000 *per annum* of such Exchequer bills as remained uncancelled on the 31st of December, 1724, and are made payable out of the Sinking Fund from the said 31st of December, to the 24th of June, 1727, viz. the interest and charge of circulating £1,500,000 Exchequer bills at 3 *per cent.* - - - - - - - -	45,000	0	0
	1,022,381	9	5

The amount of the said several annual sums will be upward of £1,022,000, the produce of the Sinking Fund from the 24th of June, 1727.

The produce of the *Sinking Fund* from the 24th of June, 1727, will be upwards of £1,022,000 *per annum*; and the whole of the national debts at that time will be less than 50 millions.

The publick debts on the 31st of December, 1724, are stated to amount to £52,363,471, or thereabouts: from which, if it be allowed me to deduct £1,500,000 Exchequer bills above supposed to be paid off by the Sinking Fund on the 24th of June, 1727, and such further principal sums as provision is made for the discharge of otherwise than by the Sinking Fund, the remainder to be paid off on the 24th of June, 1727, will be considerably less than 50 millions. Which sum however, (that I may not be thought to strain matters in favour of this scheme of discharging the publick debts by a Sinking Fund) I will suppose to be the principal debt to be paid off on the 24th of June, 1727, and the annual produce of the Sinking Fund to be from the same time one million only. I will likewise suppose, (as is most generally true) that the above-mentioned principal sum of 50 millions, will from the same time carry interest after the rate of 4 per cent. And, because there are some persons so sanguine as to imagine, that by force of our Sinking Fund, or some schemes formed upon it, the same debt may be still further reduced to a lower rate of interest, and the Sinking Fund increased further by such reduction; I will likewise suppose such a scheme to have taken effect, and the above-mentioned principal sum to carry 3 per cent. interest only, and the annual produce of the Sinking Fund to be increased, by an abatement of 1 per cent. interest on 50 millions, to £1,500,000. Upon both which suppositions, I shall subjoin a computation, describing in what number of years, from Midsummer 1727, the above-mentioned principal sum of 50 millions, or any particular part of it, may be discharged and paid off; in which, when I had not time to correct them, I discovered a small mistake or two; which I hope the reader will excuse, when I have assured him, that they no where misrepresent the time in which the aforesaid debt, or any part of it, may be paid off, by so much as two days.

Computations of the numbers of years in which the aforesaid national debt of 50 millions may be paid off by means of the Sinking Fund, upon suppositions of its bearing the two different rates of interest of 4 per cent. and 3 per cent.

Computation at 4l. per Cent.

	Payments made at Midsummer every year.			Total of all the payments from the beginning in every year.		
	£.	S.	D.	£.	S.	D.
1728	1,000,000	0	0	1,000,000	0	0
	40,000	0	0			
29	1,040,000	0	0	2,040,000	0	0
	41,600	0	0			
30	1,081,600	0	0	3,121,600	0	0
	43,264	0	0			
31	1,124,864	0	0	4,246,464	0	0
	44,994	11	$2\frac{3}{8}$			
32	1,169,858	11	$2\frac{3}{8}$	5,416,32[illegible]	11	$2\frac{3}{8}$
	46,794	6	$10\frac{1}{8}$			
33	1,216,652	18	$0\frac{1}{2}$	6,632,975	9	$0\frac{7}{8}$
	48,666	2	$3\frac{3}{4}$			
34	1,265,319	0	$4\frac{1}{4}$	7,898,294	9	$5\frac{1}{8}$
	50,612	15	$2\frac{1}{2}$			
35	1,315,93[illegible]	15	$6\frac{3}{4}$	9,214,226	4	$11\frac{7}{8}$
	52,637	5	5			
36	1,368,569	0	$11\frac{3}{4}$	10,582,795	5	$10\frac{5}{8}$
	54,742	15	$2\frac{3}{8}$			
37	1,423,311	16	$2\frac{1}{8}$	12,006,107	2	$0\frac{3}{4}$
	56,932	9	5			
38	1,480,244	5	$7\frac{1}{2}$	13,486,351	7	$8\frac{1}{4}$
	59,209	15	$2\frac{1}{2}$			
39	1,539,454	0	10	15,025,805	8	$6\frac{1}{4}$
	61,578	3	$2\frac{3}{4}$			
40	1,601,032	4	$0\frac{3}{4}$	16,626,837	12	7
	64,041	5	$9\frac{1}{4}$			
41	1,665,073	9	10	18,291,911	2	5
	66,602	18	$9\frac{1}{2}$			
42	1,731,676	8	$7\frac{1}{2}$	20,023,587	11	$0\frac{1}{2}$
	69,267	1	$1\frac{3}{4}$			

Computation at 3l. per Cent.

	Payments made at Midsummer every Year.			*Total of a[illegible] the payments from the beginning in every year.*		
	£.	S.	D.	£.	S.	D.
1728	1,500,000	0	0	1,500,000	0	0
	45,000	0	0			
29	1,545,000	0	0	3,045,000	0	0
	46,350	0	0			
30	1,591,350	0	0	4,636,350	0	0
	47,740	10	0			
31	1,639,090	10	0	6,275,440	10	0
	49,272	14	0			
32	1,688,363	4	0	7,963,803	14	0
	50,650	17	4½			
33	1,739,014	1	4½	9,702,817	15	4½
	52,170	8	5¼			
34	1,791,184	9	9¾	11,494,002	5	1¼
	53,735	10	8¼			
35	1,844,920	0	6	13,338,922	5	7¼
	55,347	12	0			
36	1,900,[illegible]7	12	6	15,239,189	18	1¼
	57,008	0	6¾			
37	1,957,275	13	0¾	17,196,465	11	2
	58,718	0	6½			
38	2,015,993	13	6¼	19,212,459	4	8¼
	60,47[illegible]	16	2½			
39	2,076,473	9	8¾	21,289,032	14	5
	62,544	4	1			
40	2,139,017	13	9¾	23,428,050	8	2¾
	64,170	10	7¾			
41	2,203,188	4	5½	25,631,238	12	7¼
	66,095	12	11¼			
42	2,269,283	17	4¾	27,900,522	10	0
	68,078	10	3¾			

Computation at 4l. per Cent.

	Payments made at Midsummer every year. £.	s.	d.	Total of all the payments from the beginning in every year. £.	s.	d.
1743	1,800,943	9	9¼	21,824,531	0	9¾
	72,037	14	9½			
44	1,872,981	4	6¾	23,697,512	5	3½
	74,919	4	11¾			
45	1,947,900	9	6½	25,645,412	14	10
	77,916	0	4½			
46	2,025,816	9	11	27,671,229	4	9
	81,032	13	2¼			
47	2,106,849	3	1¼	29,778,078	7	10¼
	84,273	19	3¾			
48	2,191,123	2	5	31,969,201	10	3¼
	87,644	18	6			
49	2,278,768	0	11	34,247,969	11	2¼
	91,150	14	1			
50	2,369,918	15	0	36,617,988	6	2¼
	94,796	15	0			
51	2,464,715	10	0	39,082,703	16	2¼
	98,588	12	4¾			
52	2,563,304	2	4¾	41,646,007	18	7
	102,532	3	3½			
53	2,665,836	5	8¼	44,311,844	4	3¼
	106,633	9	0½			
54	2,772,469	14	8¾	47,094,313	19	0
	110,898	15	9½			
55	2,883,368	10	6¼	49,977,682	9	6¼
	115,334	14	9¾	22,317	10	5¾
56	2,998,703	5	4	50,000,000	0	0
	1296	14	8			
	3,000,000	0	0			

Computation at 3l. per Cent.

	Payments made at Midsummer every year.			*Total of all the payments from the beginning in every year.*		
	£.	s.	d.	£.	s.	d.
1743	2,337,362	7	8½	30,237,884	17	8½
	70,120	17	5			
44	2,407,483	5	1½	32,645,368	2	10
	72,224	9	11½			
45	2,479,707	15	1	35,125,075	17	11
	74,391	4	7¾			
46	2,554,098	19	8¾	37,679,174	17	7¾
	76,622	19	4¼			
47	2,630,721	19	1	40,309,896	16	8¾
	78,921	13	2			
48	2,709,643	12	3	43,019,540	8	11¼
	81,289	6	2			
49	2,790,932	16	5	45,810,473	7	4¾
	83,727	19	9			
50	2,874,660	18	2	48,685,134	5	6¾
	86,239	16	8½	1,314,865	14	5¼
51½	2,960,900	14	10½	50,000,000	0	0
	39,099	5	1½			
	3,000,000	0	0			

Obſervations on the preceeding computations.

You will be pleaſed to obſerve, Sir, that the annual income of the Sinking Fund, in this manner applied to the diſcharge of the principal of the publick debts, increaſes yearly in the ſame manner and proportion as a principal ſum put out and continued at compound intereſt, or intereſt upon intereſt, at ſuch a rate of intereſt as the principal ſum to be paid off is ſuppoſed to carry: that the increaſe of it in every year, is by the intereſt of that principal ſum which was paid off in the year next before it; and that the whole of the increaſe of it in any one year, from the beginning to apply it in diſcharge of the principal debt, is the ſum of the intereſt of all the principal ſums that have been in the year before paid off by it: and that the whole of the debt propoſed to be paid off by a Sinking Fund in this manner applied, will be compleatly diſcharged the year before the Sinking Fund itſelf is increaſed, by the addition of the whole intereſt of the debt to be paid off.

From which obſervations, it will be eaſy to compute the progreſs of any other annual ſum, greater or leſs, than what I have ſuppoſed to be the produce of the Sinking Fund in the year 1727, in the payment of a principal ſum of 50 millions, at 4 per cent. or any other rate of intereſt, or any other principal ſum which you may think it more reaſonable (as our affairs now ſtand) to provide for the payment of, by the common rules for calculating the increaſe of principal ſums continued at compound intereſt.

In the uſe of which rules, you will find, Sir, if you ſhould think it more reaſonable to ſet the income of the Sinking Fund, from the year 1727, at £800,000, or (as ſome perſons have repreſented it) at £1200,000 *per annum*, that a debt of 50 millions, carrying 4 per cent. intereſt, would in the firſt of theſe caſes be paid off in about 32 years, and in the other in 25 years and one month; or if in either caſe the Sinking Fund ſhould be ſuppoſed to be increaſed by £500,000 *per annum* added to it, from the intereſt of the ſame debt, reduced to 3 per cent. from the ſame time, it would appear that it might be fully diſcharged and paid off, by a Sinking Fund of £1,300,000 *per annum* in about 25 years, and by a Sinking Fund of £1,700,000 in 21 years and 8 months, or thereabouts.

A miſtaken opinion has of late been propagated, that the publick debts have increaſed ſince the eſtabliſhment of the Sinking Fund.

But to whatever may in this manner be obſerved or proved, relating to the efficacy or progreſs of the Sinking Fund increaſing annually by addition of the intereſt of ſuch debts as are diſcharged by it, I have heard it objected and ſtrongly inſiſted on to be true, that our publick debts have been far from decreaſing or made leſs ſince the contrivance and application of this expedient for that purpoſe; but, on the contrary, have been growing upon us, and are now conſiderably greater than they were about the time when the ſurpluſſes of ſeveral funds were firſt appropriated to the diſcharge

discharge of those debts. And this melancholy circumstance the same persons aggravate, with observing, that the increase of our debts has been in a time of almost uninterrupted peace; and infer, that our debts must increase still faster upon us, in case of any publick troubles.

I have often wondered how so uncomfortable a mistake could so generally prevail, against the testimony that the memory of every person at all acquainted with publick transactions of this kind must bear, that our publick loans of late years (except such as have been made on funds provided to discharge the monies advanced upon them within the year,) have not been equal to the sums that have within the same time been paid off; till upon further enquiry upon this subject, I have had put into my hands copies of accounts, supposed to be made up at the Exchequer, stating the totals of the publick debts for different years to be greater considerably from the year 1720, than in that year, and in that year to be more than in any year before it. From which accounts I cannot but think this mistake must arise and prevail with persons who satisfied themselves with observing the totals only, and have not attended to the particular articles of which they were made up; but in examining the particular articles of which those totals are made up, they will find that the great increase of figures in the description of our present incumbrances, is not owing to any real increase of their true quantity.

An account of the circumstances that have given rise to the said mistaken opinion.

In an account now before me, of the amount of the publick debts on the 31st of December in several years, beginning in 1717, and ending in the year 1724, the amount of the publick debts in the first of those years is described to be £47,894,950, and in the last to be £52,363,471. Of which great increase in the description of our debts, the chief reasons are; first, the subscription of several irredeemable annuities for different terms of years into the South Sea Company's stock, in the years 1719 and 1720; by which those annuities were converted into a redeemable debt from the Government, and purchased back from the proprietors at higher rates, or a greater number of years purchase, than were paid by the proprietors for the same annuities when they were first purchased from the Government. Before these subscriptions made, this part of our publick incumbrances is described in the aforesaid account, by the principal sums originally advanced by the proprietors on the purchase of them; and afterwards by the quantity of redeemable debt, for which by virtue of the aforesaid subscriptions they were exchanged; which generally exceeds by four years and one half's purchase the sum originally contributed by the proprietors of those annuities, and which upon the whole of the said annuities at those different times subscribed, amounts to about £3,155,858. This in the present view must, I think, be admitted to be no real increase of the publick incumbrances, or at least not properly brought into the account

1st, The purchase of certain irredeemable annuities for different terms of years.

account of thoſe years in which the aforeſaid ſubſcriptions were made; thoſe ſubſcriptions being well enough known and underſtood to have been of great advantage to the publick, and very much to have facilitated the diſcharge of the whole of our preſent debts; and it being very obvious, that whatever real incumbrance has been growing upon us on account of thoſe annuities, it is to be attributed only to the increaſing value of thoſe annuities, and to be computed from the times of their being valued at higher prices, and not from the times of the ſubſcriptions above-mentioned, by which the further increaſe of their value was moſt fortunately prevented; and about which, all that we have to wiſh is, that it had been done ſooner.

2dly, Army-debentures.

Another article increaſing in the aforeſaid accounts of the publick debts from the year 1717 to the year 1724, is of army-debentures, or annuities charged and made payable out of the fund commonly called the General Fund, after the rate of 4 per cent. for ſuch principal ſums, as in purſuance of ſeveral acts of parliament for appointing commiſſioners to ſtate the debt due to the army, have been certified to have been due for ſervices in the late war, and before the year 1717. This, Sir, from 40,157*l.* 8*s.* 5*d.* which on the 31ſt of December, 1717, is only ſtated to be due from the publick under this article, is on the 31ſt of December, 1724, by the aforeſaid accounts deſcribed to amount to upwards of £2,140,157. But as this debt was due before the year 1717, in the preſent inquiry, whether the publick debts are ſince that time increaſed or no, this ſum is (now that the quantity of it is determined,) to be reckoned in the amount of the publick debt, as well in the year 1717 as in the year 1724; or, in other words, to be conſidered as due from the publick from the time it was contracted, and not from the time only when it was certified to be due.

3dly, Exchequer notes.

Another article increaſing the total amount of the publick debts in the year 1724 beyond that of the year 1717, in the aforeſaid accounts, is, that of £1,000,000 of Exchequer notes made out and lent to the South-Sea Company in the year 1720, and in that year added to the amount of the publick debts. This ſum, on the re-payment of it by the South-Sea Company, would have been deducted from the amount of the publick debts in that year in which it was repaid, if it had not been provided by a ſubſequent act of parliament, that the aforeſaid Exchequer notes ſhould be cancelled and paid out of the Sinking Fund; and that the ſum of £1,000,000, due from the South-Sea Company, ſhould be applied, when paid, to the diſcharge of a farther million of Exchequer notes made forth in the year 1722, and upon which money was raiſed for the diſcharge of a like ſum in arrear to the navy; which ſaid ſum of £1,000,000 being in this manner ultimately ſupplied out of the Sinking Fund, it is neceſſary to

ſuppoſe

ſuppoſe it to have been due from the publick before the year 1716, the Sinking Fund being, as I have above obſerved, about that time appropriated to the diſcharge of ſuch debts only as were due before that year; and conſequently this ſum of £1,000,000 being in the year 1717 owing, and in arrear from the Government, ſhould alſo in our preſent inquiry about the increaſe of the publick debts, be in that year added to the amount of them.

As ſhould alſo, for much the ſame kind of reaſons, the following leſs material articles, viz. — Other leſs material articles.

	l.	s.	d.
Navy-annuities, a debt, though before due, not brought into the publick accounts till the year 1718,	110,312	0	0
A further proviſion for the ſufferers at Nevis and St. Chriſtophers, about — — — —	41,000	0	0
The increaſe of a deficiency on the Eaſt-India Company's fund ſtated in the publick accounts, to be from the year 1717 to the year 1720, about — —	67,500	0	0
A ſum in the year 1723, raiſed for immediate ſervice on the credit of Exchequer notes, the payment of which was at the ſame time provided for by a tax on the eſtates of Roman Catholicks, — —	100,000	0	0
To theſe articles are to be added the three firſt above-mentioned, viz.			
The increaſe computed on the ſubſcription of irredeemables, — — — — —	3,155,858	0	0
Of army debentures, — — —	2,100,000	0	0
And the ſum raiſed for diſcharging arrears to the navy,	1,000,000	0	0
The amount of which ſums together is, — —	6,574,670	0	0

And this ſum, Sir, muſt be added to the above-mentioned total of our publick debts in the year 1717, before the comparing it with the total of the ſame debts in the year 1724 will truly determine how far our debts are increaſed or grown leſs from one time to another. Let this then be done.

	l.	s.	d.
Let this then be done, — — —	47,894,950	0	0
	6,574,670	0	0
	54,469,620	0	0
And the aforeſaid amount of our debts in 1724,	52,363,471	0	0
deducted from it, — — — —	2,106,149	0	0

And

The national debts have really been diminished by the sum of £2,106,149 from the year 1717 to the year 1724.

And it will appear, that our debts are not in reality increased from the year 1717, to the year 1724; but, on the contrary, are diminished by the sum of £2,106,149, or thereabouts.

Another method of estimating the diminution of the national debts in the same period.

The same thing will appear from enumerating the particulars of the real increase or decrease of our debts from one time to the other; of which, Sir, the following is very nearly a true account, viz.

	l.	*s.*	*d.*
Money at different times borrowed on the duty on coals for building churches, more than in the mean time has been paid off by the particular provision made for that purpose, — — —	92,778	2	0
Money borrowed for the service of the year 1719, more than paid off by the provision made for that purpose on the 31st of December, 1724, — —	439,300	0	0
Money borrowed on the plate-act for the service of the year 1720, — — — —	312,000	0	0
Total,	844,078	2	0

And this sum of 844,078 *l.* 2*s.* is the whole sum that our debts can, with any propriety, be said to be increased by from the year 1717. Such other sums as have been since that time borrowed having been employed in aid of the Sinking Fund, and applied in the discharge of some other debts at a higher interest; of which the following (except what of this kind has been already mentioned) is likewise a true account, viz.

	l.	*s.*	*d.*
Borrowed in the year 1719 by lottery, — —	500,000	0	0
Advanced in the same year by the South-Sea Company on the increase of their stock and funds, about	544,142	0	0
Advanced in the year 1723, towards the discharge of the lottery annuities unsubscribed to the South-Sea Company, about — — — —	1,000,000	0	0
Total,	2,044,142	0	0

By which sum, together with the Sinking Fund, have been paid off from 1717 to 1724, viz.

	l.	*s.*	*d.*
Of Exchequer notes, — — —	2,924,612	0	0
Of lottery annuities unsubscribed, — —	1,204,786	0	0

Bank

	l.	s.	d.
Bank annuities unſubſcribed, — — —	235,297	0	0
Deficiency of the Eaſt-India Company's fund, —	191,028	0	0
Beſides, there has been in the ſame time paid in part of a principal debt contracted by two lotteries in the years 1713 and 1714, by proviſion for that purpoſe at the ſame time made, about — — —	429,490	0	0
Total,	4,985,213	0	0
From whence the total of the laſt above-mentioned loans being deducted, viz. — — —	2,044,142	0	0
The remainder — — — —	2,941,071	0	0

will be the ſum of what has been paid off from the year 1717 to the year 1724 by the Sinking Fund, or otherwiſe without the aſſiſtance of thoſe loans.

And from thence — — — —	2,941,071	0	0
Let us farther deduct the total of the aforeſaid articles by which our debts have really, in the mean time, been increaſed, viz. — — —	844,078	0	0
And that remainder — — —	2,096,993	0	0

will be the ſum by which our debts, within the aforeſaid ſeven years, appear by this computation really to have been diminiſhed; differing indeed from that ſum which I have from the firſt computation ſtated to be the decreaſe of the publick debts in the ſame time by near £10,000. But which difference, if I pretended to the utmoſt exactneſs, might be removed, by either adding to the laſt remainder, or reckoning amongſt the particulars by which our debts have decreaſed within the time aforeſaid, the value of ſuch annuities for lives as within that time have reverted to the Crown.

The ſaid debts have, according to this ſecond eſtimation, been diminiſhed in the ſame period by the ſum of £2,096,993.

This ſum, perhaps, eſpecially if it be farther reduced by the deduction of one million at two different times borrowed, to ſupply the deficiencies of the proviſion for the expence of his Majeſty's civil government, conſidered as part of, and an addition to, our publick debts, may be thought too inconſiderable a diminution of our debts to be boaſted of as the effect of this expedient for ſo great a length of time. But, as it is no real objection to the truth of thoſe computations which I have made, of the progreſs of the Sinking Fund from the year 1727, I preſume it will likewiſe be no diſcouragement to our dependance on this proviſion for the payment of our debts; eſpecially after we have conſidered the great addition that will be made to the Sinking Fund in the year 1727, and

 have

have farther obſerved the much greater diſpatch which a yearly ſum applied to the payment of any determined debt at intereſt, and increaſing annually in the manner above-deſcribed, will make in the diſcharge of ſuch a debt in a few years after the firſt application of it to that purpoſe, than it will do when it firſt begins in that manner to be applied.

Another miſtaken opinion which has lately been advanced concerning the Sinking Fund.

The little progreſs, however, hitherto made in the diminution of our debts, leads me to the examination of another opinion, which I think I have obſerved to prevail with the ſame perſons, who affirm our publick debts to have increaſed upon us; which is, that upon the ſuppoſition that ſuch debts are really increaſing upon us by new loans equal to or exceeding the diſcharges made in the ſame time by the Sinking Fund, the Sinking Fund is in ſuch caſe making no effectual progreſs at all in the diminution of our debts. It is perhaps the more material to conſider here, how far this opinion is true, for this reaſon, that though this ſuppoſition on which it is founded has not been true hitherto, it muſt be, however, admitted to be not improbable, that ſome future exigencies of the Government may make ſuch new loans neceſſary, as may exceed any ſums in the ſame compaſs of time produced by or applied to the diſcharge of our debts from the Sinking Fund. And in this caſe, upon the ſuppoſition that ſuch new loans are made upon further funds found out for payment of the intereſt of the money ſo to be advanced upon them, this opinion, that the Sinking Fund, applied as aforeſaid, would be making no effectual advance to the compleat diſcharge of the whole of our publick debts, would not be true. This will be beſt explained, if during the time that the above ſuppoſed Sinking Funds are employed in the diſcharge of the aforeſaid debt of 50 millions, the whole of our debts ſhould be ſuppoſed, by new loans upon further funds borrowed at 4 or 3 per cent. intereſt, to be increaſed by a further ſum of 50 millions, and that ſum to be diſcharged in the ſame manner, and by the ſame Sinking Funds, after the diſcharge of the firſt 50 millions; or if the account of the progreſs of the above ſuppoſed Sinking Funds be carried on, till inſtead of 50 millions they ſhall have diſcharged a principal debt of 100 millions.

Computations of the times in which another debt of 50 millions of pounds, (contracted upon new funds during the time of diſcharging the firſt debt of 50 millions by means of the Sinking Fund.) might be paid off by means of the ſaid fund, if the intereſt of money is 4 and 3 per cent.

Computation

Computation at 4l. per Cent.

	Payments made at Midsummer every year.			*Total of all the payments from the beginning in every year.*		
	£.	S.	D.	£.	S.	D.
1755				49,977,682	9	6¼
56	2,998,703	5	4	52,976,385	14	10¼
	119,948	2	7¼			
57	3,118,651	7	11¼	56,095,037	2	9½
	124,746	1	1¼			
58	3,243,397	9	0½	59,338,434	11	10
	129,735	17	11½			
59	3,373,113	7	0	62,711,547	18	10
	134,924	10	8			
60	3,508,037	17	8½	66,219,585	16	6
	140,321	10	3½			
61	3,648,359	7	11½	69,867,945	4	5½
	145,934	7	6			
62	3,794,293	15	5½	73,662,238	19	11
	151,771	15	0			
63	3,946,065	10	5½	77,608,304	9	4½
	157,842	12	5¼			
64	4,103,908	2	10¾	81,712,212	12	3¼
	164,156	6	6			
65	4,268,064	9	4¾	85,980,277	1	8
	170,722	11	6½			
66	4,438,787	0	11¼	90,419,064	2	7¼
	177,551	9	6¾			
67	4,616,338	10	6	95,035,402	13	1¼
	184,653	10	9¾			
68	4,800,992	1	3¾	99,836,394	14	5
	192,039	13	7¾	163,605	5	7
69	4,993,031	14	11½	100,000,000	0	0
	6,968	5	0½			
	5,000,000	0	0			

Computation at 3l. per Cent.

	Payments made at Midsummer every Year.			*Total of all the payments from the beginning in every year.*		
	£.	S.	D.	£.	S.	D.
1750				48,685,134	5	6¼
51	2,960,900	14	10½	51,646,034	0	5¼
	88,827	0	5½			
52	3,049,727	15	4	54,695,761	15	9¼
	91,491	16	1¾			
53	3,141,219	11	5¼	57,836,981	7	3
	94,236	11	7¾			
54	3,235,456	3	1½	61,072,437	10	4½
	97,063	13	8¼			
55	3,332,519	16	9¾	64,404,957	7	2¼
	99,975	11	10¾			
56	3,432,495	8	8½	67,837,452	15	10¾
	102,974	17	3			
57	3,535,470	5	11½	71,372,923	1	10¼
	106,064	2	2			
58	3,641,534	8	1½	75,014,457	9	11¾
	109,246	0	7¾			
59	3,750,780	8	9¼	78,765,237	18	9
	112,523	8	3¼			
60	3,863,303	17	0½	82,628,541	15	9½
	115,899	2	3¾			
61	3,979,202	19	4¼	86,607,744	15	1¾
	119,376	1	10			
62	4,098,579	1	2¼	90,706,323	16	4
	122,9[illegible]7	7	4½			
63	4,221,536	8	6¼	94,927,860	4	10¾
	126,646	1	9¼			
64	4,348,182	10	4	99,276,042	15	2¾
	130,745	9	6	723,957	4	9¼
65	4,478,927	19	10	100,000,000	0	0
	21,072	0	2			
	4,500,000	0	0			

From

From hence, Sir, it presently appears that the above ſuppoſed Sinking Funds, in this manner increaſing by the addition of the intereſt of the principal ſums in every year paid off, and conſequently by additions in every year greater than thoſe made to it in the year before, will be ſufficient not only to diſcharge our preſent debts, but any probable addition in the mean time to be made to them by further loans on new-invented funds, in a few years after the preſent debts ſhall be diſcharged: and that the time required for the diſcharge of our debts, increaſed by any addition in this manner made, will by no means be lengthened out, or the payment of the whole of our debts by the Sinking Fund retarded or delayed in proportion to the addition to or increaſe of the debt itſelf: the total payment of our publick debts becoming by no means deſperate from any Sinking Fund, however leſs than thoſe above ſuppoſed, upon account of any determined increaſe of or additions made to them; unleſs thoſe additions are ſuppoſed to be continued increaſing in every year in the ſame or a greater proportion to one another than that in which the additions yearly made to the Sinking Fund increaſe. This is ſo true, that ſuppoſitions about the increaſe of the publick debt might be carried to the utmoſt extravagance, and ſtill appear to be provided for by the above-mentioned Sinking Fund of £1,000,000, increaſing at the rate of 4 per cent. compound intereſt; which, if it were worth while, might be ſhewed to be ſufficient, in about 105 years, to pay off a debt of 1575 millions, allowing for the increaſe of the preſent debt of 50 millions, by an addition of 15 millions in every year in which that Sinking Fund ſhould be ſo applied. Nor will this at all ſurprize perſons who have been accuſtomed to attend to the increaſe of money put out at compound intereſt, or quantities continued in geometrical progreſſion; an enquiry into which will remove all doubts about the truth of what I have here advanced. It would however be true, that if at any time, on the diſcharge of any part of the principal of the preſent debt, the intereſt were not added to, and applied in the further diſcharge of, the remaining debt, but another equal or greater principal ſum ſhould be borrowed on the ſame annuity; the progreſs of the Sinking Fund would by ſuch meaſures, if the ſame ſum were borrowed, be ſtopped; and, if a greater, be put backwards: but as long as theſe meaſures are not taken, or the Sinking Fund diverted or applied to any other purpoſe than the diſcharge of our debts; the full and effectual payment of all our debts by this expedient, is by no means to be deſpaired of from the increaſe of them by new loans on further duties.

Obſervations on the great efficacy of the Sinking Fund in diminiſhing the national debts, notwithſtanding new ſums of money ſhould in the mean time be borrowed by the publick upon new funds to pay the intereſt of them.

But this is upon a ſuppoſition that the Sinking Fund is never charged with the intereſt of the new loans.

And that the Sinking Fund will, from time to time, be applied to the diſcharge of the publick debts, and not be diverted or applied to any other purpoſe whatſoever, is what, I think, we may ſecurely promiſe ourſelves; from conſidering that the aforeſaid fund has been appropriated to that purpoſe by the Legiſlature, and our publick faith in the ſame manner engaged

The publick faith is engaged to the proprietors of the national debt, that the Sinking Fund ſhall never be applied to any other purpoſe but that of diſcharging the ſaid national debt.

engaged to the creditors of the Government, that the ſurplus of the aforeſaid duties ſhould be applied to the diſcharge of the principal of their debts, as the funds themſelves to the payment of the intereſt or annuities contracted for: which faith of the publick in this manner engaged, I think we have all the reaſon in the world to believe will be as inviolably obſerved in this as in any other part of their contract with the proprietors of the publick debts.

The clauſe of the act of parliament, (3 Geo. 1, cap. 7, ſect. 37.) by which the Sinking Fund is ſo appropriated.

This appropriation of the Sinking Fund to the purpoſe aforeſaid, you will find, Sir, to have been made by the aforeſaid acts of parliament. In the laſt of which, taking them in that order in which I have referred to them, page 3[illegible]0, after reciting that by the two other acts of parliaments, the ſurpluſſes of the Aggregate and South-Sea Funds are provided to be reſerved to the diſpoſition of Parliament only; it is enacted, That the ſurpluſſes of the General Fund thereby created, ſhould in like manner be accounted for and reſerved for the diſpoſition of Parliament. And then it is further enacted in the words following, "That all the monies to ariſe "from time to time, as well of or for the ſaid exceſs or ſurplus, by virtue "of the ſaid act made for redeeming the funds of the Governour and "Company of the Bank of England [viz. the Aggregate Fund] and of "or for the ſaid exceſs or ſurplus, by virtue of the ſaid act for redeeming "the funds of the ſaid Governour and Company of Merchants trading to "the South-Seas, &c. and of or for the ſaid exceſs or ſurplus of the ſaid "duties and revenues by this act appropriated as aforeſaid, [viz. the "General Fund] and the ſaid overplus monies of the ſaid general yearly "fund by this act eſtabliſhed or intended to be eſtabliſhed as aforeſaid, "ſhall be appropriated, reſerved, and employed to and for the diſcharging "the principal and intereſt of ſuch national debts and incumbrances as "were incurred before the 25th of December, 1716, and are declared to "be national debts, and are provided for by act of parliament in ſuch "manner and form as ſhall be directed and appointed by any future act "or acts of parliament to be diſcharged therewith or out of the ſame, and "to and for none other uſe, intent or purpoſe whatſoever."

The ſaid clauſe ought to be conſidered as a ſolemn contract entered into by the Government with the publick creditors.

By theſe words, I think, the ſurpluſſes therein mentioned, of which the annual income of the Sinking Fund is made up, ſufficiently appear to have been appropriated by the legiſlative power to the payment of our publick debts, till they ſhall be intirely diſcharged and paid off. Nor can this proviſion well be underſtood as made by the Government for what then appeared for publick convenience only, and conſequently to be altered by ſubſequent acts whenever it ſhall appear, or be pretended to be otherwiſe; but muſt, I think, be conſidered as a contract by the Government with the publick creditors, if the occaſion of theſe acts of parliament be attended to. In which caſe it will appear, that the ſeveral proviſions by by these acts made, were enacted and proposed to the creditors aforesaid, as inducements to them to accept of an interest, or annuity, for their debts by one sixth part less than that which till that time they had received; of which the most obvious inducement was, that what was thus deducted from the yearly interest of their debts, should be applied for the better securing and gradual discharge of the principal of the said debts. To which security, amongst the other benefits by the same acts of parliament proposed to them, they must, I think, be considered to have intitled themselves by their subscriptions afterwards made, subsequent to, and in consideration of, such proposals made to them by the Legislature. And whoever will be at the trouble of turning over the several subsequent acts of parliament relating to the publick debts, will find this provision for the application of the Sinking Fund frequently repeated and confirmed: and in cases where by act of parliament application of monies in the Sinking Fund to the discharge of debts that were less obviously, or less generally, known to have been within the description of the debts intended by the provision above-recited, such debts have been, by the recitals, declared and explained to have been debts incurred before the 25th of December, 1716, and provided for by Parliament in a manner that has plainly intimated it to be understood by the Legislature, that the above-recited provision was an engagement, or contract, of the Government with the publick creditors, about the punctual observation of which from time to time they were intitled to have all possible satisfaction; or at least, that the punctual application of the above-mentioned surplusses to the discharge of our present debts, was regarded by them as a matter of the highest consequence to the publick welfare. And as long as the publick welfare shall be in the least regarded, and this continues to be the only expedient for removing such heavy incumbrances on our affairs, and redeeming so considerable a revenue to the use of the publick, I think we may confidently expect, that no persons whatsoever, whose hands the administration of our affairs may at any time for the future be committed to, can ever be induced to approve of, or recommend, the application of the produce of the Sinking Fund, in any possible exigence of our affairs, to any other uses than those to which it stands now appropriated, though there were no other considerations to enforce it.

Therefore it cannot be supposed that any ministers of state whatsoever will ever presume to divert the produce of the Sinking Fund from the purposes of its original destination.

For let us inquire a little, what publick exigencies can be supposed to happen, that can make it at any time adviseable to divert or apply the produce of the Sinking Fund to any other purpose till after the entire payment of our publick debts. Let the expence that the circumstances of our affairs may at any time make necessary, be, or be supposed to be, ever so much more than what can be conveniently raised within the year; it must, I think, always appear more eligible in regard to the publick interest, as well as more easy to those persons in the administration, to whom the care

In any supposed exigencies of state, that are likely to arise, it will be more expedient to raise money by new loans, with new funds, or taxes, to pay the interest of them, than to break in upon the Sinking Fund.

care of providing the necessary supplies shall at any time be allotted, to raise what shall be further wanted by increasing the publick debt with further loans upon interest provided for by new duties, than to supply the same sums in any way from the produce of the Sinking Fund.

The computation that I have last made was to shew, that the time in which the above supposed Sinking Fund of £1,000,000 will be sufficient to compleat the discharge of the publick debts, will by no means increase equally to the increase of the principal sum of those debts by further loans on new funds: but it may be of further use to shew, how much less the increase of the publick debts, by borrowing further sums at interest provided for by new funds, will retard the discharge of the whole of the publick debts, than the supplying the same sums in any way from the produce of the Sinking Fund would do.

An illustration of this proposition by an example.

Let us suppose, for instance, that the Government were obliged for 25 years together to increase the present debt, by a million borrowed in every year at an interest of 4 per cent. provided for by further funds, the above-made computation will shew that that additional debt of 25 millions would be paid off by a Sinking Fund of one million, applied as is therein supposed, in little more than 7 years after the discharge of the present 50 millions. But if the same sum were to be supplied out of the produce of the Sinking Fund, it is obvious that the payment of the publick debts must stand still for 25 years, and be by more than two thirds of that time retarded beyond the time in which they would otherwise be discharged, though increased as aforesaid; and the greater the sum is supposed to be, that in these different ways is to be supplied, the greater will be the proportion in which the payment of our debts will be delayed, by supplying such expences from the Sinking Fund, more than by the other way: or if the sums in these different ways supplied should be supposed less, the difference of the delay in these two cases will be indeed less: but on supposition of the smallest sum to be these two different ways supplied, the delay arising to the discharge of the publick debt by this misapplication of the Sinking Fund, will be at least three times as great as that which will be occasioned by increasing the publick debt in the other method.

The borrowing money, and charging the interest of it on the Sinking Fund instead of laying new taxes for that purpose, would be as pernicious an alienation of the Sinking Fund as the taking greater sums out of it that were equal to the money so borrowed.

The borrowing money on the income of the Sinking Fund in any form, if no more were in any one year borrowed than what had been by the Sinking Fund the year before paid off; and if that money be supposed to be borrowed at the same rate of interest that was payable for the debt before paid off; will have the same effect in delaying the payment of the publick debts, as the misapplication of the revenue of the Sinking Fund the year before would have had: but if greater sums be at any time borrowed on that fund, the payment of the publick debts will not only be stopped, but put backwards; and that in a manner that obviously leads

not

not only to delay the payment of the publick debts, but the taking away intirely the only ſecurity yet provided, that they ſhall ever be paid off. For which reaſon I ſhall not trouble you, Sir, with any computation of the different degrees in which different ſteps in purſuing theſe meaſures will affect us; but at once ſuppoſe it impoſſible that any perſons can propoſe to borrow money, (or much leſs to ſucceed in it) on the credit of ſchemes that themſelves deſtroy all probability of the re-payment of it; which, ſuch meaſures as theſe, muſt evidently appear to do, to thoſe that conſider, that we have already had the greateſt advantage from the reduction of intereſt that can with reaſon be hoped for in the proviſion of the preſent Sinking Fund; which if we once part with in exchange for an increaſed principal debt at a lower rate of intereſt only, it will be madneſs to expect that either ſuch a lower rate of intereſt, or any alteration in our circumſtances for the better, will admit of the ſame kind of proviſion to be made again for the payment of our debts increaſed by ſuch meaſures as theſe are.

I cannot therefore, Sir, amongſt the ordinary viciſſitudes of the affairs of any nation, not even amongſt any long and expenſive wars, that it may be neceſſary for the defence and ſafety of theſe kingdoms to carry on with our neighbours, find out that exigence of our affairs that can make the miſapplication of the Sinking Fund appear neceſſary, or probable to be put in practice; while it is ſo certain, that the lands, eſtates, expence, or commerce of Great-Britain, will yet eaſily admit of farther duties ſufficient to furniſh new funds to anſwer the intereſt of ſuch ſums as any publick occaſions that I can repreſent to myſelf can call for. Nor can I fear, that ſuch duties will not be chearfully voted and ſubmitted to, when they ſhall appear neceſſary to prevent the miſapplication of an annual ſum employed in ſo uſeful and neceſſary a ſervice to the publick, as the reduction of our debts; while that appears to be retarded ſo much more by diſcontinuing the payment of thoſe debts, than by the increaſe of them.

Examination of another reaſon which has lately been advanced for ſuſpecting that the miniſters of ſtate will not long continue to apply the whole of the Sinking Fund to the diſcharge of the national debt.

There is another objection to the probability of the payment of our publick debts, which, if I did not frequently meet with it, I ſhould chuſe not to mention, from my apprehenſions, that in ſtating of it as I have met with it, I ſhould be obliged to mention my ſuperiours with leſs decency, than that grateful ſenſe of the happineſs we enjoy under the preſent reign would on all other occaſions lead me to, or than you, Sir, from the ſame motives would expect from me. But as you are pleaſed to admit you have often met with it from others, you will give me leave to mention it, in my way to anſwer it. The objection I mean is, That the continuance of our publick debts is, and always muſt be, the intereſt of perſons in the adminiſtration; that the greateſt profit of their employments ariſes from hence; and that the neceſſary power and influence to ſupport themſelves in thoſe employments,

employments, depend greatly on their having reſerved to themſelves the diſpoſition of the various offices and employments in collecting and applying the revenues appropriated to the payment of the publick debts; which, when thoſe debts ſhall be diſcharged, can ſubſiſt no longer.

It muſt be obſerved, in anſwer to this reaſon, that both the king and the miniſters of ſtate have hitherto been anxiouſly careful to apply the Sinking Fund to its proper uſe, the diſcharge of the national debt.

Whatever truth we ſhould admit to be in this objection, we have the pleaſure of obſerving, that it appears to be equally true, from the frequent and earneſt recommendations from his Majeſty of the neceſſary meaſures for diſcharging the publick debts to the care and endeavours of the legiſlature; the ſeveral ſteps that have been taken by them; and the great and effectual proviſion that is already made for this purpoſe; that nothing can have been, or will be, more ſincerely intended and endeavoured by his Majeſty, or the perſons who have had, or ſhall have, the honour to be employed by him.

It may further be obſerved, in anſwer to the ſaid reaſon, that it is not neceſſary, or probable that, when the national debt ſhall be all paid off, the taxes which now pay the intereſt of it will be aboliſhed.

But from the ſenſe I have juſt now profeſſed to have of the bleſſings we enjoy under the preſent government, I muſt confeſs, I ſhould with no pleaſure look forwards on that period of time, when his preſent Majeſty or his ſucceſſors ſhould be deprived of the means of ſupporting it, or even of rewarding and encouraging the fidelity and ſervices of their beſt ſubjects. The chief uſe therefore that I have propoſed to make of this objection, is to take an occaſion from it, of conſidering how far it is probable that ſuch a reform as is above ſuppoſed, of the various employments in collecting and receiving the preſent revenues, will take place on the diſcharge of the publick debts; or how far it is reaſonable that it ſhould do ſo. And this ſuppoſition being founded on a preſumption, that the particular duties now appropriated to the payment of the publick debts will, after the payment of them, be immediately removed; the reaſonableneſs of that preſumption will be the matter in queſtion.

The Government is now ſupported by other taxes, that are granted anew every year, to wit, the land-tax and the malt-tax.

For the purpoſe of this inquiry, Sir, I ſhould propoſe it to be conſidered, that the ſupport of our government neceſſarily requires a conſiderable annual expence, that is at preſent ordinarily ſupplied by other taxes than thoſe which have been provided to anſwer the payment of the publick debts; that the preſent ordinary proviſion for that annual expence has been hitherto determined, rather by the neceſſities of the publick, than by choice; and that it yet remains to be debated, how far the duties at preſent appropriated to the payment of our debts, or part of them, may, after the diſcharge of thoſe debts, be continued and made to anſwer the ordinary annual expence of our government, more to the advantage of the publick, with leſs burden and expence to the particular eſtates of his Majeſty's ſubjects in this kingdom, and conſiſtently with a more equal and reaſonable proportion of the burthen or expence by every ſubject ſubmitted to,

Theſe taxes might, when the national debt was all paid off, be dropt, and their place ſupplied by ſome of the duties which now pay the intereſt of the ſaid debt.

to, to the benefit he receives from the ſupport of our government, than is now done by the preſent proviſion made for the aforeſaid ordinary annual expence.

It is in vain to ſuppoſe, that the neceſſary expences of a government are to be ſupplied by any taxes that are no ways burthenſome to the whole or ſome part of the community, and conſequently to which ſome objections may not be dreſſed up by perſons intereſted in avoiding them; which objections, however, when ſuch taxes appear neceſſary, it is unreaſonable to propoſe or aggravate. I ſhall not therefore point out any inequality or hardſhip that I may apprehend to be in the ordinary annual proviſion made amongſt us by a land-tax; but content myſelf with making ſome obſervations, tending to recommend the greateſt part of the duties now appropriated to the payment of our debts, as the moſt convenient and reaſonable taxes to ſupply the ordinary expence of our government, when redeemed by the payment of thoſe debts.

This would be found to be very equitable and convenient.

Upon enumerating the ſeveral duties which at different times have been provided to anſwer the demands of the publick creditors, it will appear that the greateſt part of them (whether collected by cuſtom or exciſe) have been laid upon commodities in general uſe and conſumption amongſt that part of the inhabitants of this country, whoſe circumſtances will admit of the expence.

The latter duties (which now pay the intereſt of the national debt,) are, for the moſt part, duties on the conſumption of commodities.

About theſe duties it will appear upon reflection to be generally true that they have been added to the price which thoſe commodities had before the impoſition of ſuch duties, and from thenceforth to be ultimately paid in the laſt price of ſuch commodities by the conſumer.

Upon which ſuppoſition, if the aforeſaid duties are either, by way of cuſtom or exciſe, generally collected throughout the country where ſuch commodities are conſumed; it is plain that the ſaid duties will generally be paid by every perſon reſiding in ſuch a country, nearly in proportion to his ordinary annual expence.

And this, Sir, is the ſhare or proportion which, of all others, I think moſt eligible to be taken from every perſon reſiding in a country where great part of the inhabitants ſubſiſt by commerce, towards the publick expences of the government of that country, when it can in this manner be done, without enquiring exactly into the expence of every particular inhabitant.

Theſe are the moſt convenient taxes that can be laid in a commercial country.

For,

An advantage belonging to these taxes.

For, first, in this way the publick expence is least sensibly felt by those who really contribute towards it; every person being voluntary in his expence, and gratifying himself while he is contributing from his estate to the expence of the government.

A second advantage belonging to them.

2. Contributions in this manner generally made by the inhabitants of a country in proportion to their expence, will be likewise made in a near proportion to the real value of the property of the same inhabitants; perhaps, a nearer than it would be done by a law made, directing the publick expences to be levied in that proportion, from the great difficulty of finding out, and plain inconvenience of exactly inquiring into the real value of every man's property for a purpose of this kind, in a country so much engaged in traffick as our's is. Nor will taxes upon our expences vary much from taxes proportioned to the value of our property, (if long continued) from what may at first sight appear a reason for that conclusion; I mean, the different choice of the thrifty and extravagant in the proportion of their expences; the first of which, by contributing little himself to the publick expence, is providing for larger contributions by his successors; and the other, by contributing too largely in haste, is incapacitating himself for contributing at all.

Advantages of these taxes above direct taxes on property.

I think also, that in those particulars in which a tax proportioned to our expences, either does, or may be contrived to, vary from one intended to levy the same sum in proportion to the value of property in Great-Britain, such a tax on our expences appears the more eligible.

1. A tax proportioned to the expences of persons residing in Great-Britain, will collect a proportion of the income of the various profitable professions and emp[illegible]nts amongst us, and of the annual gains of foreign and inland [illegible]ce; all which being received and enjoyed by virtue of the laws, [illegible]der the protection of this government, should, together with the annu[illegible]ncome of our property, contribute towards it.

2. It will likewise collect and take in a proportion of the annual income of such estates or employments as supply the expence of foreigners on different accounts residing in Great-Britain, as well as of such of his Majesty's subjects who chuse to reside here and support their expences by the income of estates in Ireland, or any of our colonies or plantations in America or elsewhere; from whom, in return for the protection their estates receive from the arms or influence of Great-Britain, supported at our expence, no contributions in common with the inhabitants of this kingdom can be thought unreasonable.

Contributions

Contributions thus made by perſons reſiding in Great-Britain, in proportion to their expences, will likewiſe include a proportion of the annual income of ſuch eſtates as may be brought hither by foreigners chuſing to ſettle amongſt us, or by any of our own countrymen returning with their gains from other countries.

In ſhort, it will include a proportion of all eſtates whatſoever, whether within or without the kingdom of Great-Britain, and whether diſcovered or not diſcovered, that any way ſupply the expences of our inhabitants, in a manner (as is above obſerved,) not grievous to, or liable to be complained of by, the contributors themſelves, and with the further good œconomy of ſparing on ordinary occaſions, and increaſing, that publick ſtock, that unmoveable part of our property within this kingdom, to which in times of extraordinary danger and expence we muſt neceſſarily have recourſe.

Duties of this kind may alſo be made inſtrumental to the judicious regulation of our trade.

It may likewiſe be conſidered, in recommendation of this manner of ſupplying the ordinary expences of our government by duties in the manner above ſuppoſed, levied in proportion to our expences, what farther conveniencies to the publick may be procured by ſuch duties, over and above ſuch a ſupply to its ordinary expences; ſuch as diſcouraging the conſumption of ſuch foreign commodities as may, in a manner plainly inconvenient to the publick, interfere with, or hinder the conſumption of, the produce or manufactures of our own country; abating the extraordinary price of foreign commodities, or the exorbitant gains of foreigners by the importation of them; the diminiſhing a trade carried on with any of our neighbours, the balance of which is too evidently in their favour; the encouraging any other more profitable branch of the Britiſh commerce; or the preventing the increaſe of any particular article of expence, that may too plainly tend to debauch the manners, or abate the induſtry, of his Majeſty's ſubjects. Of this kind many are the conveniencies that may be procured to a country, by the ſame meaſures that ſupply the ordinary expences of its government. And when it ſhall be conſidered to how many publick uſes of this ſort ſeveral of the duties appropriated to the payment of our debts are ſubſervient, beſides the annual income produced by them; I believe it will appear by no means eligible, and much leſs neceſſary, that the whole of thoſe duties ſhould, immediately after the payment of the publick debts, be removed and determined; when the ſame conveniencies may be ſtill preſerved to us by the continuance of them, and the income of thoſe duties be made to ſupply ſuch of our expences as are now provided for by leſs equal, or leſs beneficial, taxes.

We may therefore conclude that it will not be expedient, when the national debt ſhall be totally diſcharged, to aboliſh all the duties that now are employed in paying the intereſt of it.

Such conſiderations as theſe, I think, are ſufficient to remove the above-mentioned ſuppoſition, that the payment of our publick debts is inconſiſtent with the intereſt of a Britiſh miniſtry; in which, however, I could

It may further be observed, in answer to the reason advanced above in page 419, that the views mentioned in that reason are too remote to influence the ministers of state for many years to come.

could still advance farther, by remarking how remote the views of any interest of this kind are placed, by the length of time that will be necessarily required for the discharge of our present debts from a Sinking Fund; and by observing, that the removal of any part of the present duties, which are any ways inconvenient to the publick, and are continued now only because appropriated to the payment of some part of our debts, will by no means imply or even admit of a reduction of officers employed in the collection of those kind of duties, either by way of custom or excise, in the several ports or districts in Great-Britain, in proportion to the income of such abolished duties; and from several other reflections that have occurred to me on this subject, if I did not think it unnecessary any farther to follow so groundless and indecent a jealousy of the integrity and publick spirit of such of my countrymen, who shall for the future deserve and attain to the favour and confidence of his Majesty or his successors.

Thus far I have been endeavouring to make out, that the provision already made of the present Sinking Fund is an expedient, from which we may with great confidence expect the full and effectual payment of the principal of our present debts within a few years. Upon which, Sir, if I have dwelt longer than you may have thought necessary, I hope you will be pleased to consider in excuse of it, how far I must have been led to do so, by attending to the happy influence that a general confidence in the efficacy of this expedient would have on the credit of our publick funds, especially in case that the measures lately taken by some neighbouring princes should make a rupture with them necessary to us; and how far such a general opinion of the efficacy of this scheme has a tendency to forward and increase the success of it.

An inquiry whether the Government ought, in prudence, to endeavour to procure a further reduction of the interest of the national debt.

I am now brought, Sir, to the last task that, in obedience to your commands, I have assigned myself; and am to inquire what measures it may be most for the interest of the publick to take in the application and use of the Sinking Fund from the year 1727. About which the only question that can, as I think, be put is, Whether it shall be from thenceforth adviseable for us to endeavour after a greater increase of the Sinking Fund, by a farther reduction of the interest of the publick debts? Or if it may not be then on the whole more for the publick interest, to endeavour only after such an increase of the aforesaid fund, as will be produced by the application of it from time to time to the discharge of the publick debts, and the addition of the yearly interest of such of the said debts as shall be from time to time paid off.

Before I proceed to any other consideration which it may be thought material to attend to in determining this question, I shall take leave to state the greater effect the first of these different measures would have in accele-

rating

rating the payment of the publick debts than the other of them. And this I chuſe firſt to do, becauſe in an affair of this publick concern, and where we are not to be ſuppoſed to give ourſelves the trouble of the ſame exactneſs in computation that we ſhould uſe in our own private affairs, I am a little apprehenſive that people, when they turn their thoughts to this ſubject, are apt, upon any increaſe of the Sinking Fund, to promiſe themſelves a farther degree of diſpatch in the payment of the publick debts in proportion to ſuch increaſe. For an inſtance, to explain my meaning: I fear, that upon ſtating from the above-mentioned ſuppoſition, that the Sinking Fund of £1,000,000 was increaſed to £1500,000 *per annum*, by an abatement of 1 per cent. intereſt on 50 millions, the debt ſuppoſed to be paid off by it; on ſtating ſuch a caſe, I ſay, I fear it would be in haſte inferred, from the Sinking Fund's being increaſed to half as much again as it was before, that the publick debts would be likewiſe paid off by the Sinking Fund ſo increaſed half as ſoon again, or that the publick debts would be paid off by a Sinking Fund of one million and a half *per annum* in two third parts of the time that would be taken up in diſcharging it by a Sinking Fund of one million *per annum* only. But this inference would not be true, by whatever means the Sinking Fund were ſuppoſed to be ſo increaſed; and leaſt true, when the increaſe of the Sinking Fund is made by a reduction of the intereſt of the debt to be paid off by it.

Of the effect which an increaſe of the Sinking Fund from £1,000,000 *per annum* to £1500,000 *per annum*, by a reduction of the intereſt upon a debt of 50 millions from 4 per cent. to 3 per cent. would have in accelerating the time in which the ſaid debt would be compleatly diſcharged.

If the aforeſaid fund of £1,000,000 *per annum* were increaſed to £1500,000 by an addition made to it of £500,000 *per annum* provided by a new tax, or any otherwiſe than by an abatement of the intereſt of the 50 millions to be paid off, which ſhould continue to carry 4 per cent. intereſt, it would be true, that while the ſaid increaſed Sinking Fund is ſuppoſed to be applied to the diſcharge of that debt, it would pay off in every year half as much again as the Sinking Fund of one million only, beginning at the ſame time to be applied to the ſame purpoſe, would do in the ſame year; and at the end of any number of years, in which both funds are ſuppoſed to continue ſo applied, will have paid off a principal ſum exceeding the principal ſum paid off by the Sinking Fund of one million only, by one half part of the latter; or in other words, the principal ſum paid off by the aforeſaid greater fund will be to that paid off by the leſſer, either in an equal number of years from the time they begin to be applied, or in any one year equally diſtant from that time, in the proportion of three to two. And in this ſenſe the aforeſaid greater fund may be ſaid to pay off the publick debt half as faſt again, as in the ſame time it will pay off half as much again. But from hence it is not to be inferred, that the leſs fund will be half as long again as the greater in diſcharging the ſame principal ſum; or that the ſame principal ſum would be paid off by the greater fund in two thirds of the time that would be taken up in diſcharging it by the ſmaller fund: and of this the plain reaſon will ſoon appear on inſpecting the above-made computations; from which it may be obſerved, That the Sinking Fund applied, as we have all along ſuppoſed it, is increaſing by an addition in every year made to it of the intereſt of that principal ſum which was paid off by it in the year before; from whence both the income of the fund itſelf, and the principal ſums annually paid off by it, are in every year greater than in the year before, and increaſing in every year by an addition greater than the addition made to it in the year before: from whence it neceſſarily follows, that in a ſeries of payments made by the Sinking Fund for any number of years carried on, the payments towards the latter end of ſuch ſeries muſt be conſiderably greater than thoſe before; and that the amount of the payments for any number of years ſeparated at the latter end from the reſt of the ſeries, muſt greatly exceed the amount of the payments for any equal number of years in any other part of the ſame ſeries. And from hence it muſt appear, that the exceſs of the payments made by the greater Sinking Fund above thoſe made by the leſs in the ſame number of years, will not be a rule for determining the time in which they muſt ſeverally be employed in diſcharging the ſame principal ſums.

The time of ſuch compleat diſcharge would not be thereby diminiſhed to two thirds of what it was before.

And it will be further from the truth, in the caſe of the Sinking Fund increaſed from an abatement of the intereſt of the debt to be paid off, by an addition of an annual income equal to one half part of its income before ſuch increaſe, to ſuppoſe, that from thenceforth the debt will be diſcharged in two third parts of the time which would have been otherwiſe required; becauſe the additions from time to time made to a Sinking Fund employed in the payment of a debt carrying 3 per cent. intereſt only, do not increaſe in the ſame or ſo great a proportion, as thoſe made annually to a Sinking Fund in the diſcharge of a debt at 4 per cent. From which circumſtance the leſs Sinking Fund increaſing by this greater ratio or proportion, would in a longer ſeries than I hope we have any thing to do with in the preſent caſe, have ſo conſiderable an advantage, as to overtake the greater Sinking Fund in its payments, and from thence to be every year diſcharging a greater debt.

But it would only be leſs than before by about one ſixth part of its former quantity.

But in the caſe we have ſuppoſed, of a debt of fifty millions, the time in which we have before computed that that debt, carrying 3 per cent. intereſt, may be paid off by a Sinking Fund of £1500,000, is 23 years and one half nearly; and by the Sinking Fund of £1,000,000, the debt continuing at 4 per cent. intereſt, it may be paid off in about 28 years; ſo that the time ſaved in the diſcharge of our debts by the reduction of them to 3 per cent. intereſt, appears, on the aforeſaid ſuppoſitions, to be 4 years and a half, or thereabouts; which is ſomething leſs than one ſixth part of the time in which the ſame debt might be diſcharged without any further reduction of the intereſt.

Another

Another way of estimating the advantage to the publick which would arise from such a diminution of the time in which the national debt would be discharged.

Another way of stating the advantage to the publick in this contraction of the time which our debts may take up in the discharge of them, from 28 to 23 years and a half, would be to find out and assign that annual sum, which, added to the above-supposed Sinking Fund of £1,000,000 at the publick expence, and without any further reduction of the interest of the debt to be paid off, would answer the same purpose as the addition of £500,000 to that fund taken from the income of the publick creditors, and contract the time in which the payment of 50 millions would be compleated, from 28 years to 23 and a half. And this, Sir, will be found to be almost £322,000; which yearly expence to the Government for 23 years and a half, would answer the same purpose as the above-supposed deduction of £500,000 *per annum* from the income of the publick debts. And this advantage I chuse to state distinctly as it is, before I proceed farther, because I think in all the discourse I have met with on this publick affair, I have seldom heard any distinction made about the convenience of the several reductions of interest from 6 to 5 per cent. and from thence to the rate of interest, at 4 per cent. which is shortly to take place; or relating to the further reduction to 3 per cent. which we seem to intend and be providing for; but on the contrary, they seem all to be considered and expected alike, as of equal advantage in dispatching the discharge of the publick debts; though it be at the same time true, that by the first of these reductions we came only to have any Sinking Fund at all; and to the second of these reductions, together with the provisions at the same time made about the unredeemable annuities, we owe it, that the total payment of our debts by this expedient begins to appear practicable. But in those circumstances in which we now are, and with those views which we at present have of the payment of our debts within no great length of time, from the provisions already made for that purpose, by the reduction of interest hitherto effected or contracted for; I think we are at liberty, before any further steps of this kind, to consider of some probable consequences that may follow upon them; which to have produced as objections to any former reductions of publick interest, while they appeared so necessary, might have been thought impertinent or untimely.

It is by no means a just conclusion, that, because the former reductions of the interest of the publick debts from 6 per cent. to 5 per cent. and from 5 per cent to 4 per cent. have been beneficial to the nation, therefore a further reduction of the fund interest from 4 per cent. to 3 per cent. will also be beneficial to it.

It seems to me to have been an opinion of late years pretty generally agreed to, (perhaps as long since as the celebrated Mr. Locke's performance on that subject) That all attempts to reduce interest by compulsive methods, or by force of any laws made for that purpose, are not only unlikely to succeed, but on other accounts inconvenient to the publick: but I know not if the interest of the publick in the reduction of it by any other means effected, has been much considered; or if such a reduction of interest is not usually expected by us with general satisfaction, arising from our regarding it as the effect of the common and natural causes of a lower interest in every country, and such alterations in our circumstances as are truly enumerated amongst the instances of publick prosperity.

A reduction of the interest of money, if it is brought about without compulsion, is commonly esteemed a sign of general prosperity.

Of the causes of the variations of the rate of interest for money.

Mr. Locke, in his aforesaid treatise on this subject, mentions, as the natural causes of the variations of the rate of interest in any country, the variations of the proportion that the quantity of money bears in such a country to the demand for it, arising either from the quantity of debts contracted amongst the inhabitants, or of the trade carried on by them. To which occasions of a demand for money, I think should be generally added all other circumstances in the affairs of such a country, as may be there supposed ordinarily to contribute to, or be the occasion of, a greater or less demand of that kind. And as a further natural and ordinary cause of a variation any where in the rate of interest, I should chuse to add such alterations in the circumstances or situation of the affairs of the country where such a variation happens, as may make it more or less dangerous or secure to advance money upon loans in all or any of the different scenes of business, where negotiations of this kind are usually carried on; by which last cause I am apt to think, that the more sudden and sensible variations in the rate of interest have been chiefly and most frequently every where occasioned. And when a lower rate of interest is supposed to be produced amongst us, by such causes as these are, it is perhaps most reasonable that it should be regarded with general satisfaction; as it is a proof of such a situation of our affairs as is of itself, and independently on this consequence from it, an instance and part of the description of our general welfare and prosperity; and as the monied man himself has in this case an equivalent for what he may be supposed to lose by the abatement of his income, in the greater safety, with which, on such an occasion, he lends his money, or the less hazard which he runs of the repayment of it, as well as in the greater frequency of opportunities which such a situation in our affairs produces, of putting out and improving money with greater safety.

When the interest of money is lowered in a country by the causes here mentioned, it may be considered as a sign of general prosperity.

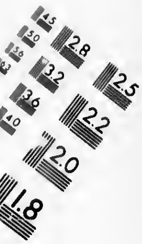
But when the interest of money is lowered by the operation of other causes than those above-mention'd, it is not certain that it ought to be so considered.

But as far as a lower rate of interest may be produced amongst us, either without such compulsive methods, or the concurrence of such natural and ordinary causes for it as are above mentioned, I apprehend that it yet remains to be inquired into, if it be the interest of the publick that it should be so? and, as far as the success of any measures entered upon for this purpose may be uncertain, if it be, with regard to the publick, adviseable that such a reduction of our interest should be attempted or endeavoured after? And in this case it will be allowed me, that a lower rate of interest thus produced, or supposed to be so, is no longer to be considered as a proof of, or attended with the above-mentioned instances of, our publick happiness; such a rate of interest having no tendency in itself to increase our money, or the lenders security for the repayment of it, nor, consequently, being of any effect to produce a real increase of our negotiations in advancing money; which, while no provision is made for increasing either, our capacity or disposition to lend money can by no means become greater, or more frequent, from the greater application to borrow money only.

I shall

Of the effects of ſtock-jobbing on the prices of the publick ſtocks.

I ſhall therefore endeavour, Sir, to deſcribe ſuch tranſactions amongſt us with reſpect to our publick debts, as I apprehend may have been ſuppoſed to have had a great ſhare and influence in producing amongſt us lately very great and general variations in the rate of intereſt, and from which a ſtill further reduction of rate of intereſt may be yet expected; I mean thoſe great adventures in .. ublick funds, of late years ſo apparently undertaken with a view to ſuch gains, as might be quickly made by the different prices of them, and which have ſo much contributed to the late great and ſudden variations in the market prices of theſe ſecurities. In the infancy of theſe adventures, the chief or only motives to them probably were thoſe pieces of intelligence about the ſituation of our publick affairs, from the publication of which the adventurer might reaſonably infer the general ſatisfaction or diffidence of the proprietors of the publick debts in their ſeveral ſecurities. And as far as intelligence of this kind was true, and the general ſenſe of the proprietors upon the publication of it rightly conjectured or inferred, the riſe or fall of ſtocks produced by theſe adventures might be regarded as an event, which in a longer time or in a leſs proportion would have happened, if theſe adventures had not been made; and in this view may not improperly have been called the growth or declenſion of our publick credit. But as this practice grew upon us, it is not to be wondered at, if from the general induſtry of great numbers to be firſt acquainted with every material occurrence to the publick, and to be earlieſt in the improvement of their information in adventures of this nature, ſeveral variations in the prices of our funds have been produced by tranſactions in them, undertaken upon falſe or uncertain intelligence, and groundleſs inferences and conjectures from it; which variations have not been afterwards to be accounted for from any real alteration in the poſture of our affairs, or the general ſentiments of the proprietors of the publick debts; and from which therefore the real ſtate of publick credit at ſuch a time would be uncertainly, if not falſely, inferred or determined. The later variations in the prices of our ſtocks would be ſtill more improperly deſcribed to be the growth or declenſion of our publick credit; which credit, ſince the reſtoration of our tranquillity, and during the abſence of our apprehenſions for the publick ſafety, can only with propriety be ſaid not to have been diſputed or called in queſtion, and which cannot, I think, be ſuppoſed to have been of late at all attended to by the purchaſers of our publick ſecurities, at premiums and advanced prices far beyond thoſe ſums for the re-payment of which the credit of the Government is any ways depended on. In ſhort, by whatever names we have been accuſtomed, or may chuſe, to deſcribe the riſe or fall of our ſtocks, I ſubmit it to ſuch perſons who have made any obſervations on the late tranſactions in Exchange-Alley, if they have not (and eſpecially the riſe of them) been generally occaſioned by ſuch adventures made in them, as perſons have been induced to from the hopes of gain, from a further



further ſpeedy variation in the price of them, without any regard to the continuance of it; and if theſe variations are not of late come to be expected from any the moſt inconſiderable occaſions, or perhaps for no reaſon at all, but what is to be inquired for in the market, and amongſt the accounts and contracts depending there.

While this diſpoſition continues amongſt numbers to be conſtantly adventuring in the publick funds, and conſequently upon expectations that muſt be generally ſupported by the moſt inconſiderable reaſons, it is hardly to be doubted but that at any time in the abſence of our apprehenſions of any general danger, the intelligence being ſpread amongſt them that any ſcheme or propoſals were to be ſet on foot, by which the riſe of ſtocks was either intended or ſuppoſed, would generally determine theſe adventurers to expect and provide for ſuch a riſe of ſtocks, and by their contracts founded on theſe expectations in a great meaſure to produce it: to effect which purpoſe, I hardly think it material, that any further reaſonable proviſion ſhould be made in the propoſals or ſchemes themſelves, or that any thing would be further neceſſary for this purpoſe, than declaring the riſe of ſtocks to be intended by them. Such a riſe of ſtocks I am almoſt inclined to believe might be the firſt effect of any intelligence communicated in Exchange-Alley at ſuch a time as I have above ſuppoſed, that ſome propoſals were ſhortly to be made to all or great part of the publick creditors, to agree to the further reduction of their intereſt or annuities, as diſagreeable as this muſt at firſt appear to the greateſt part of the creditors themſelves. But how far ſuch a riſe of ſtocks may be in this caſe expected, and how far it may proceed in forwarding any propoſals of this nature, I ſubmit to be conjectured from the following conſiderations.

If the government were to propoſe to reduce the intereſt of the publick debts (with the conſent of the owners of them,) from 4 per cent. to 3, or 3½, per cent. ſuch a propoſal would, probably, occaſion a riſe in the price of ſtocks.

Reaſons in ſupport of this opinion.

Firſt, Such a propoſal muſt ſuppoſe and lead our expectations to a riſe of ſtocks in general; without which, or at leaſt if the contrary ſhould happen, ſuch a propoſal could by no means be executed or complied with, it being neceſſary to the ſucceſs of this propoſal, that the market price ſhould, at the time of making it, offer the proprietor as much, or more, as, if he declined to comply with it, would be payable to him by the Government. And as the greateſt part of the proprietors of the publick debts have been at different times incorporated for the purpoſe of carrying on certain trades, from the profits of which (as I would willingly hope) 1 per cent, or more, has been annually divided over and above the income of their intereſt in the publick debts; if their annuity from the Government, when reduced and diminiſhed, continues to be valued as before, the price of that part which is not liable to any diminution from theſe propoſals, may well enough be expected to riſe in ſome proportion to ſuch a reduction of their annuity. Thus, if to the proprietors of South-Sea ſtock, for inſtance, it were propoſed that their annuities in the year 1727, ſhould be be from thence reduced to 3 per cent. upon the ſuppoſition that their ſhares in the publick debt ſhould, after ſuch a reduction, continue to be valued, as before, at par, it might be as reaſonable to expect that the 1 per cent. continuing to be divided on every hundred pound ſtock, ſhould be from thenceforth valued in the price of it at 33*l.* 6*s.* 8*d.* as it was before to expect it ſhould be ever valued at 25*l.* And from the riſe of that part of our publick ſecurities which fall under this conſideration, ſome advance in our other ſecurities may likewiſe be expected, as the money received on the ſale of thoſe ſtocks which ſhall firſt, and in the greateſt proportion, riſe on this occaſion, is generally obſerved to be applied to the purchaſe of that part of our publick debts which is conceived to be leſs liable to variation in the prices of them.

Secondly, Such perſons as are obſerved to be conſtantly adventuring in the ſtocks from expectations of gain, either from the riſe or fall of them, muſt be generally ſuppoſed to be determined to theſe adventures by the loweſt degree of probability, that they ſhall ſucceed in them; and it is hardly therefore to be doubted, but that the ordinary adventures in our ſtocks would be made upon expectations of the riſe of them, upon the publication of any propoſals from authority that ſuppoſed the riſe of ſtocks, or implied that it was expected by our ſuperiours.

Thirdly, The riſe of ſtocks upon this occaſion would be further favoured, by the diſpoſition of thoſe proprietors who are not ordinarily engaged in adventures of this kind, to wait for the utmoſt advantage to be made of the riſe of ſtocks, whatever might be their ſentiments about continuing proprietors of the publick debts when reduced to a lower intereſt.

Fourthly, A riſe of ſtocks on this, as well as former occaſions, may be ſtill further advanced, by the ſpreading of falſe computations of the value of our ſtocks, and idle opinions about credit and circulation, and by the force of a general example, aſſiſted by the confidence of the proprietors of our publick debts in the authority by which theſe propoſals may be recommended.

And when the ſtocks ſhall be ſufficiently advanced to colour any propoſals of this nature, it is perhaps not impoſſible that the concurrence of the proprietors to ſuch propoſals ſhould be obtained; though at the ſame time they may be generally diſſatisfied with the lower rate of intereſt propoſed to them, and ſeverally determined on that account to quit their intereſt in the publick debts on the next convenient opportunity. For it is to be conſidered,

That

That the reason for their objecting publickly or declaring their sentiments against any proposals of this kind, is removed by the price of their securities at market, where they are offered for the present as much, or more, for them, as, if they thought proper to decline these proposals, would be payable to them by the Government; and that the general dissatisfaction of the proprietors should not determine them to take advantage of the then market price for the sale of their securities, and by that means occasion the fall of stocks, and prevent the success of these proposals, may in a great measure be accounted for from a general inclination to have the utmost possible advantage from the rise of stocks, from the difficulty of finding on such an occasion any immediate employment for their money, and their impatience of its lying by them unemployed, joined with that dependance which men generally have on their own foresight and skill in the choice of the fittest opportunity for this purpose; from which motives, while the bulk of our publick securities may be supposed to be kept from market, that part of them which shall be brought there, by the more wary or determined of the proprietors, will be found for a time provided for, by those considerable sums which the estates and credit that persons engaged in such adventures in the stocks as I have above described, will for a time supply the market with.

From the foregoing reasons it seems probable that the consent of the publick creditors might be obtained to such a reduction of the interest of their debts.

I think, Sir, from hence it appears possible, that a proposal for reducing the interest of our publick debts, though without any reasonable foundation, may, as our affairs now stand, succeed even so far as to obtain the concurrence of the proprietors. And the inference which I would from thence make is, that it belongs to them, by whose influence or advice such a proposal shall at any time be made, first to consider if there be a solid foundation for it, or if it be likely still farther to succeed; and not to depend on the consent of the proprietors, in this manner obtained, as a sufficient proof that such a proposal was reasonable, or as a security for the still further success of it.

Nevertheless such a measure might not, upon the whole, be adviseable.

Whenever therefore the further reduction of the interest of our publick debts shall be attempted, it should be first enquired if the real proportion of our ordinary necessities for money to our capacity and disposition to supply them, have been so far altered as to admit of it; and if such an alteration has proceeded from those reasonable and general causes of it, which are likely long to continue and support it. Nor will such an alteration be safely inferred from the market-prices of our stocks, any further than those prices are determined and produced by such purchases only as are made with a view to the improvement of the money laid out in these securities from the interest or income of them; by which purchases of late years the prices of our stocks have so seldom been determined, that perhaps it may be more reasonable in this case to conclude from an enquiry into the

rate

rate of intereſt ordinarily reſerved on private loans, or into thoſe other tranſactions in which we are uſually directed by computations upon the cuſtomary rate of intereſt amongſt us. Theſe tranſactions, however, muſt, as well as the prices of ſtocks, in this caſe be conſidered and attended to on this account, that perſons in the diſpoſition of their money will, as often as they think themſelves equally ſecure, be determined by the greater intereſt they are offered for it; from whence it muſt be expected, that when the income of our publick ſecurities, compared with the prices they are ſold for, offer a leſs improvement for our money than may with equal ſecurity be made of it by private loans or otherwiſe, the general induſtry of mankind to make the beſt improvement of their eſtates, will quickly reduce either the price of our publick ſecurities, or the rate of intereſt in ſuch private tranſactions as aforeſaid.

I queſtion therefore, if any attempts to reduce the intereſt of our publick debts below 4 per cent. at preſent will be of any laſting convenience to the publick, or ever can be ſo till ſuch a lower rate of intereſt ſhall be preceeded by its being cuſtomarily accepted of upon private loans on unqueſtionable ſecurities. For let it be conſidered how ſuch a lower rate of intereſt can otherwiſe appear to be founded on any real variation in the proportion of our neceſſities for money to our capacity or diſpoſition to ſupply them; or if the contrary does not appear, from a higher rate of intereſt ordinarily paid at the ſame time upon private loans. And while this continues to be the caſe, how reaſonable is it to apprehend, that when the money and credit of thoſe adventurers, who firſt advanced the price of ſtock, ſhall be withdrawn, the ſame ordinary neceſſities for money, without any increaſe of the proviſion for ſupplying them, will bring the proprietors of the reduced ſecurities to expect and look out for the former annual income for their money, and thereby occaſion a declenſion in the price of theſe ſecurities proportioned to the diminution of their intereſt?

The reduction of the intereſt of the national debts, from 4 per cent. to $3\frac{1}{2}$ or 3 per cent ought not to be attempted till a proportional reduction of the intereſt of money lent upon private mortgages ſhall have previouſly taken place.

How far the continuance of thoſe adventures, by which the price of ſtocks is ſuppoſed firſt on ſuch an occaſion to be advanced, may be depended on for the ſupport of it, may be collected from the motives by which the adventurers were firſt engaged in them; and is from thence to be expected but till the utmoſt probable riſe of ſtocks from ſuch propoſals has been effected: after which that the former ſupplies from their credit and eſtates ſhould be withdrawn from market, is not all that is in this caſe to be apprehended, it being further probable that they will be from thence employed in depreciating thoſe ſecurities which were at firſt advanced by them, with a view to the ſame kind of profits from the fall, as they before expected from the riſe of ſtocks. From which, together with the freſh neceſſities which the more inconſiderate of theſe adventures will naturally produce, it would not be at all ſtrange if the price of ſtocks ſhould be carried lower,

 beyond

beyond the declenſion of them in proportion to the diminution of their income, and the rate of intereſt for a time become higher than it would have been, if ſuch an attempt for the reducing it had not been made. And this conſequence of a riſe of ſtock from adventures of this kind muſt ſome time or other be expected, as far as that rate of intereſt which our real neceſſities would produce is varied from or miſrepreſented by ſuch adventures. It may not probably immediately ſucceed a riſe of ſtocks by theſe means effected; theſe adventures in a time of general tranquillity, may for a conſiderable time be protracted by further views, or the market ſupplied by a ſucceſſion of them; and this has often been the caſe, till upon the arrival of ſome intelligence about the ſituation of our affairs, which we call bad news, theſe views have been given over, and the declenſion of our ſtocks on that occaſion attributed to, and accounted for only from, that intelligence: but if the real occaſion of the great variation in our ſtocks at ſuch a time were further enquired for, it would be found to be the precipitate ſale of great quantities of ſtock, which, with ſuch views as aforeſaid, had been before bought up; and that this declenſion would as certainly, if not ſo ſuddenly, have happened from the ſame occaſion, without the intervention of ſuch intelligence, when theſe views ſhould have been on any other account given over; or when (as, I think, I have heard ſome perſons acquainted with theſe tranſactions expreſs themſelves) *the game had been played out.*

It is very much to be wiſhed that the market-price of the publick debts may be always either equal to, or greater than, their *par*, or original, or nominal, value.

It is true, indeed, that the Government by thoſe terms on which they borrow, I mean, by engaging only for the payment of the intereſt or annuities contracted for till the repayment of the principal, avoid all inconvenience from the intereſt of money advancing after their contract for the reduction of it, and leave the entire diſappointment upon the proprietors. But I ſubmit it, how far the publick can be conſidered to be unconcerned in a diſappointment of the publick creditors, obviouſly owing to their concurrence with propoſals recommended to them by authority; or in that general mutiny and diſcontent, which will be the neceſſary conſequence of ſuch a diſappointment: which from a remarkable inſtance of this nature, after the execution of the late South-Sea ſcheme in the year 1720, we muſt have obſerved to have been once regarded by the Legiſlature, as of ſufficient moment to induce them to releaſe the moſt conſiderable advantage that the publick had agreed for from that ſcheme, though ſet on foot upon the propoſals of the creditors themſelves. And if ever the publick creditors ſhould be generally diſappointed by a conſiderable diſcount upon their ſecurities, obviouſly owing to their concurrence to an abatement of intereſt recommended by publick authority, and propoſed for the convenience of the publick; I doubt if their expectations of relief from the Government could be thought leſs reaſonable. I cannot, for my own part, but think, that the general ſubmiſſion of the publick creditors in their contracts with the

the Government, to wait for the re-payment of their principal till the publick convenience will admit of it, and waving any such agreement about a determined time for the re-payment of it, as in private contracts is ordinarily provided, would be far from removing their expectations of redress under a disappointment of this nature; and rather apprehend, that this submission would be urged on such an occasion, as a meritorious instance of their confidence in the care and protection of our Government, and as a reason for their expecting in return for it, that what may be then called Publick Credit should be kept up, and their securities by all possible means preserved at *par*, till the time when they could be discharged.

Of the causes of the success of the first of the two late attempts to reduce the interest of the publick debt; by which it was reduced from 6 per cent. to 5 per cent.

The success attending the reduction of so great a part of our debt from 6 to 5 per cent. and from thence afterwards to 4 per cent. cannot certainly be looked on as a foundation for expecting the same event of our endeavours to reduce interest still further. As to the first of these reductions, we shall find it, on looking back, to have been attempted quickly after such an alteration in the circumstances of our affairs, as furnished the best foundation for our hopes of succeeding in it: at the end of a long and expensive war, that threatened us with the loss of every thing valuable, but more particularly of that part of our property which had been advanced for the services of the publick; at a time when those necessities of the Government were removed, which had obliged us for several years before to be continually increasing the publick debts, and at the same time admitted of no provisions for the discharging of them; at a time when the lasting prosperity of Great-Britain was lately secured to us by his present Majesty's accession to the throne, and, soon after, by the entire defeat of the last attempt that was likely to be made to disturb or prevent the present happy establishment. From such a foundation as was then laid for the growing wealth of these kingdoms, from the increase of our people, our commerce and manufacture, and for the particular security and greater confidence of the publick creditors, it was most reasonable to expect, that the abatement of publick interest then proposed should take place; especially, when these proposals were attended with the provisions that were then first made, for securing and rendering practicable the discharging the principal of the publick debts. Nor do I think it unreasonable to have expected, that by degrees, and from the fruits and sensible effects of this happy alteration in our circumstances, the further reduction of publick interest to 4 per cent. which has since been agreed for, and which in the year 1727 is generally to take place, might likewise be effected. And, though it may be doubted, whether the effect of this last reduction of publick interest has been yet fully tried, upon recollecting how little the interest of the proprietors of the publick debts in this reduction was attended to by themselves, at the time when it was agreed to; and how possible it is, that a far greater number of the proprietors of the present funds may have proposed to quit their interest in them, when the

Of the further reduction of the said interest from 5 per cent. to 4 per cent. which is to take place in the year 1727.

reduction is actually to take place, than will be able to find customers for it, unless at a considerable discount: yet when, on the other hand, it shall be considered, how far this reduction has been precedented by considerable loans amongst us at the same rate of interest, as well before the exigencies of our government during the late war with France, as since his Majesty's accession to the throne; I hope we may chearfully conclude, that this reduction of the publick interest may well enough be supported by the regular application of the annual income of the Sinking Fund to the discharging of our present debts. But no difficulty of finding employment for money at 4 per cent. interest, nor any private loans at a lower rate of interest amongst us, can yet, I think, lead us to expect that a further reduction of publick interest to 3 per cent. will be for any length of time submitted to.

There is no reason to think that a further reduction of the said interest to 3 per cent. will, for many years to come, be practicable.

And while this continues to be the case, and from 4 to 5 per cent. interest is every day offered upon unexceptionable securities, I should think it a more reasonable use made of recollecting the late reductions of our publick interest, to place them to the account of our present happy circumstances; and, before we proceed to expect from the late alteration in our affairs a further reduction of interest, to consider how far we are indebted to it upon that account already.

I have indeed sometimes heard it said, that the last reduction of our annuities to 4 per cent. still wants to be taken care of; and that the price of our publick securities proportioned to that rate of interest, is only to be supported by such adventures as will be encouraged by keeping in view the prospect of a still further reduction of those annuities to be attempted. From these persons I very much differ; and cannot but think that this last reduction (if no new troubles presently fall out) would be effectually supported by the future regular application of the Sinking Fund; from which, in a number of years, I should rather expect that a further reduction of interest may be naturally and reasonably produced, if the effect of this provision be not before-hand too far presumed upon and anticipated. But whoever really thinks that the further reduction of publick interest must be kept in view, in order to support the reductions already made, evidently supposes us to be proceeding in measures with regard to our publick debts, in which we must somewhere stop, and whenever we do so, repent of every step we have taken in advancing thither.

Examination of an argument in favour of a farther reduction of the said interest, drawn from the low rate of interest in some of the towns of Holland.

For want of examples amongst ourselves, as I suppose, to support our expectations of the further reduction of publick interest, I have sometimes heard the present low rate of interest in some of the trading towns of our neighbours the Hollanders, quoted to prove the probability, that the same, or something near the same, low rate of interest may be made to take place amongst us too. But I see not why the low rate of interest in that country should

ſhould be more regarded as the ſtandard for the rate of intereſt amongſt us, than the higher rate of intereſt in other neighbouring countries, unleſs on account of our greater commerce and negotiation with the Hollanders. And after I have admitted that the rate of intereſt amongſt them is on this account moſt likely to have ſome influence upon our's; I muſt expect it ſhould be allowed me, that this lower rate of intereſt than our's having for ſeveral years prevailed amongſt the Hollanders, has already had its effect with us in the reduction of our intereſt to that rate which we now conſider it to be brought to; and that the further effect or influence of their example in the reduction of our intereſt, is only to be expected from the further reduction of the rate of intereſt below what it is at preſent ſuppoſed to be amongſt them.

That the circumſtances of our affairs are the ſame in all thoſe particulars that lead to a low intereſt in any country with thoſe of our before-named neighbours, is by no means, in the preſent inquiry, to be preſumed; if they were ſo, our rate of intereſt muſt now be pretty nearly the ſame with theirs. But if our rate of intereſt conſiderably exceeds theirs, and has (which, as I have been informed, is true) for a long ſucceſſion of years conſtantly done ſo; it muſt be inferred, that our circumſtances, in ſome particulars that influence the rate of intereſt, differ much from theirs. And the conſtancy with which our rate of intereſt has been obſerved for a long time to have exceeded theirs, is enough to ſatisfy us that the occaſion of it is to be enquired for in ſome difference in our circumſtances which has continued with equal conſtancy, and for the ſame length of time; and not amongſt any projects or contrivances at different times ſet on foot by either of us, to anſwer any purpoſe of this nature.

The true reaſon of the difference between the rates of intereſt given for money in England and in Holland.

The true and general reaſon of this difference between our rate of intereſt and theirs, has, perhaps, been long ſince aſſigned by the abovementioned Mr. Locke, and ſeems moſt probably to be the very different proportion which the lands or property of any other kind producing a certain annual income amongſt the Hollanders, taxed as that kind of property has been with them, bear to the great ſtocks and other perſonal eſtates of the inhabitants of that country, from that which the value of lands and other property of the ſame kind here bear to the perſonal eſtates in this kingdom. To this difference it ſeems owing, that while the Hollander can find little other employment for the money he can ſpare from his own adventures within his own country, than in ſupplying the neceſſities which their commerce from time to time produces, the monied inhabitant of this country, beſides the opportunities offered him from the ordinary neceſſities of perſons engaged in trade, is hardly ever without propoſals for the employment of his money in ſupplying the wants of the proprietors of our lands, by either purchaſing or advancing money upon their eſtates;

and

and from hence is in a condition to demand and obtain a greater reward for the uſe of money than the Hollander can do, where the demand for it in his own country is ſo much leſs. This difference between us, as far as it will be allowed to have been one cauſe why our intereſt has hitherto exceeded theirs, will be allowed alſo as a reaſon why it ſhould continue to do ſo, till the inhabitants, wealth and commerce of Great-Britain, ſhall have increaſed in the ſame proportion to the extent and value of our lands, as it may be obſerved they have done in the Seven Provinces.

The expectation of a further reduction of the intereſt of the publick debt has contributed to increaſe the pernicious practice of *ſtock jobbing.*

I cannot forbear thinking, that upon this occaſion it deſerves moſt ſeriouſly to be conſidered, how far our late expectations of continued attempts to reduce the rate of intereſt, has contributed to promote and increaſe the aforeſaid traffick in Exchange-Alley; a practice that, in the midſt of thoſe reproaches which it lies under by the name of *ſtock-jobbing*, and the moſt ſerious complaints of its ill conſequences to the kingdom upon every declenſion of our ſtocks, ſeems to me to be ſtill growing upon us. Whenever it ſhall be ſeriouſly intended to prevent or reſtrain this practice, I believe it will appear, that whatever contrivances may be provided for prohibiting the contracts in Exchange-Alley in the manner they are now made, or altering the manner of conveying our intereſt in ſtocks from one perſon to another, while they increaſe the difficulty of the moſt innocent and neceſſary tranſactions in the publick funds, will have little further effect on this practice, than to force it into ſome other channel, and perhaps increaſe the profit and employment of the banker only, by making his credit or aſſiſtance further neceſſary; and that the moſt reaſonable method of preventing it, will be removing the encouragement and temptation to it. And though our complaints of theſe adventures are then only generally made, when they ſeem to contribute to the declenſion of our ſtocks, a little enquiry will convince us, that the foundation for ſuch a *fall* of ſtocks was really laid by thoſe adventures which ſeemed to contribute to, and attended, the *riſe* of them. If during a time of general tranquillity, from unlimited expectations of the perpetual advance of publick credit, countenanced amongſt us beyond any ſufficient foundation for it, perſons are induced to ſpread their eſtates upon the utmoſt price of our publick funds, in ſuch a manner that a variation of 2 or 3 per cent. in the price of them threaten them with the loſs of the greateſt part of their eſtates; what can be expected, but from the earlieſt appearance of publick troubles, an idle rumour, (though improbable to be true,) or the apprehenſion of any ill accident, (though moſt unlikely to fall out,) ſhould determine them, in this ſituation of their affairs, to conſult their ſafety with the utmoſt precipitation, and crowd the market with the ſtock of which before they continued proprietors upon ſuch deſperate terms. A fall of ſtocks by this means occaſioned, with perſons leſs exactly acquainted with the reaſon of it, ſerves as a confirmation of every falſe report at the ſame time publiſhed

to

to the prejudice of our affairs: from hence ſtill further quantities of ſtock are brought to market, and a further declenſion in the price of them occaſioned; from whence, to greater apprehenſions of publick danger, and from thence to the further fall of ſtocks, by turns producing and increaſing each other, we may have often been obſerved to proceed without any poſſibility of putting a ſtop to either of them. Upon ſuch an occaſion as this it has often, and perhaps conſtantly, happened, that ſeveral perſons propoſing to themſelves gain from the calamities of the publick, have, on a preſumption of the fall of ſtocks, contracted for the delivery of ſtock which they had not, and could propoſe to furniſh only by the purchaſe of what the growing apprehenſions of others ſhould afterwards bring to market; and of theſe adventures it has been uſual, on the fall of ſtocks, principally to complain. This is a practice, which has doubtleſs often contributed to the misfortunes of the publick on an occaſion of this nature; but which, I doubt not, would in a great meaſure be prevented for the future, if the exceſſive adventures in the purchaſe of ſtocks, in expectations of gain from the riſe of them, were firſt prevented; to which the contrary practice is chiefly owing. For it will, on enquiry, be found, that ſtock-jobbing begins from, and people have been uſually initiated into this practice by, general expectations of the riſe of ſtocks; in which when they are once habituated, and the expectations of gain from the variations in the price of ſtock are become the only end of their tranſactions, their deſpair of advantage by the riſe of ſtock is quickly changed for hopes of profit from the fall of it.

Beſides, the unreaſonable prices to which t[illegible]ant adventures for the riſe of ſtock have carried them, muſt firſt [illegible] the more wary and leſs credulous adventurers to expect th[illegible] and have been the occaſion of that ſucceſs that has encourag[illegible]ance of theſe adventures for the fall of ſtocks: nor would [illegible]ance of their contracts, for the delivery of ſtock, which they had n[illegible]rtain times, be generally practicable, but from the contracts for the ſame time ordinarily made for purchaſing ſtock without providing the money to be paid for it.

And if it be to theſe ſanguine expectations of the riſe of ſtocks, and the adventures founded on them, that the frequent and exceſſive variations in the price of them are really and ultimately owing; how dearly do the publick pay, in every inſtance of perplexity in our affairs, for any convenience to be reaped from, or uſe to be made of, this prevailing humour in the abſence of our apprehenſions of publick danger?

The

Ill consequences that result from the variations in the price of the publick funds occasioned by stock-jobbing contracts.

The rise of our stocks, produced by the assistance of such inconsiderate adventures in the purchase of them as have been above described, is at best, of itself, and without attending to any consequence from it, a matter of absolute indifference to the publick in the absence of general danger; but the consequence of it in the declension of our stocks, upon the approach of publick troubles, is by no means so; then it is that the general diffidence in our securities and wreck of publick credit is of the utmost disservice to us, by rendering difficult, if not impracticable, the raising such supplies as an occasion of this kind may necessarily call for; and as the variation in our publick funds at such a time may be regarded by our neighbours as the measure of our apprehensions from their attempts upon us, and encourage them in their presumption on the unsettled circumstances of our affairs: all which difficulties in our affairs on such an occasion, attended with false and groundless reports and apprehensions of our danger, general mutiny and discontent, seditious exceptions to the conduct of our superiours, and great distress and interruption to our commerce, I cannot but think we in a great measure owe to such inconsiderate purchases of our publick funds during the general tranquillity; and that they might in a great degree for the future be prevented, if, by removing all encouragement to the extraordinary rise of stocks, the publick funds were suffered to fall generally into the hands of such persons, who, satisfied with their income, shall purchase them as a supply for their ordinary expences, with money which they are not soon likely to have any other occasion for.

The proprietors of the publick debts ought to be treated with lenity and tenderness, as persons who have deserved well of the publick.

The proprietors of our debts have, as such, not deserved severity from the publick; but, as subjects of this kingdom, are intitled to have their interest regarded by the Government, as far as the publick convenience will admit of it. And in this view there may be some room to consider the unequal hardship to the publick creditors, by the loss of a fourth part of the annual income of their estates, implied in the success of an attempt to reduce their annuities to 3 per cent. And while the convenience to the publick, to be obtained by such a reduction, is supposed to be the earlier discharge of the publick debts; the hardship appears greater from this circumstance, that what shall be thus annually deducted and taken from their income, will not go so far in answering this purpose, as two thirds of the same yearly sum any other way supplied, and for this general convenience, more equally levied upon the subjects of this kingdom: £322,000 *per annum*, or thereabouts, raised at the general expence, and added to the Sinking Fund of £1,000,000, being, as I have before observed, sufficient, on the above-made suppositions, to effect the total discharge of the publick debts, as soon as the addition of £500, 00 *per annum* deducted from the interest of those debts when reduced to 3 per cent. For it should be attended to, that though the gain or convenience to the publick is to be computed upon such of our debts only as from time to time remain unsatisfied,

unsatisfied, and as long as they remain so; yet supposing the continuance and general success of this reduction of our interest, the loss to the publick creditors is from the time of such a reduction to be computed on the whole of the annuities that shall be reduced. If this, as a hardship on the publick creditors, should not be proper in this case to be considered; it may be so, however, for the purpose of collecting what their sentiments on this affair must some time or other be. The loss and inconvenience to them by this reduction will be so sensibly felt, that no misrepresentations can possibly long mislead them: the continuance of their submission to former abatements of their interest has been already accounted for, by the then late alteration in our circumstances for the better, making the purchaser of our reduced annuities a large amends in his greater security, and being convenient to the monied men in general in the frequency of opportunities of improving their estates with safety. But will the present happy situation of our affairs admit of a further equal alteration in our circumstances for the better, or that shall in the same proportion increase our security in advancing money upon the publick credit? The former reductions of our annuities may have been recommended to the publick creditors, as the only means that could render the discharge of our debts practicable to the Government; but, as far as they are interested in it, is not that end sufficiently obtained? or is the prospect that the payment of the publick debts may be thereby effected sooner, by 4 years and a half in 28, than it would be otherwise, of consequence enough to the proprietor of any part of them, to induce him for that purpose only to part with for the future one fourth part of the annual income of his estate? However the reduction of publick interest hitherto effected may have contributed to the security of the proprietors, from the next reduction it is perhaps not unreasonable to apprehend a contrary effect; and next to the great difficulty in the discharging of our debts, the most reasonable foundation of our apprehensions may be, its becoming, in the opinion of some persons, a matter of too much indifference to the publick whether they are ever discharged or no. When the publick debts, by the further reduction of their interest, shall sit so easy upon us, as to require but one moiety of the annual provision at first made for the payment of it, and leave the other at the service of the publick, the danger seems to me by no means inconsiderable, that it may soon after be determined to employ the annual income of the Sinking Fund in the room of, and to ease the publick of some other taxes by which our ordinary expences are supplied; and that it may be thought as reasonable to rest contented with the recovery of half the annual income of the publick funds without any expence to us, as to redeem the whole of them with the trouble and expence of really discharging so considerable a debt.

The further reduction of the interest of the publick debts would be a great hardship on the publick creditors, and would raise great discontents amongst them.

And if this reduction of interest be successfully carried on, the loss and inconvenience aforesaid cannot be confined to the proprietors of our debts only,

And it would also be highly inconvenient to many other persons besides the publick creditors.

only, or to their property in the publick funds; for if it were ſo, it is plain, the price of them, after ſuch a reduction, muſt be abated in proportion to it: it muſt therefore, if it ſucceeds, take place in the intereſt of all priv e loans, in the profits of all the different employments of our money, and by degrees muſt affect the profits of our commerce, and ſpread itſelf throughout the kingdom: an effect, which when not produced by, or attended with the increaſe of our wealth, the revival of commerce amongſt us, the ſucceſſion of general tranquillity to a dangerous or unſettled ſituation of our affairs, or other like inſtances of general proſperity, I know not how to regard otherwiſe, than as an uncomfortable and general inconvenience in a country where the perſonal eſtates are ſo conſiderable as here they are; which, if it ſhould be thought not material to attend to, as a hardſhip or inconvenience merely to particular perſons, ſhould at leaſt put us in mind of the oppoſition that muſt ſooner or later be expected amongſt us to meaſures from which a great reduction of intereſt is apprehended.

An examination of the advantages which ſome people imagine would ariſe from ſuch a reduction.

Againſt this great and general inconvenience to the proprietors of perſonal eſtates from a lower intereſt, I would willingly place any further publick or private advantage that may ariſe from it, beſides haſtening the payment of our publick debts. The chief, if not the only, advantages of this kind that I have met with by any perſons propoſed from a lower intereſt, have been the increaſe of our foreign commerce, and the advance of the value of our lands and irredeemable annuities of any kind.

The increaſe of our foreign commerce.

As to the firſt of theſe, it muſt be admitted, that caſes may be put about the particular circumſtances of any country in which a lower rate of intereſt would have a tendency to increaſe their commerce; as it might be an inducement to ſuch perſons who could no longer ſupport themſelves, or were not contented with the income of their eſtates at ſuch a lower rate of intereſt, to engage in trade; and as it might be the means of furniſhing others with money for the purpoſe of undertaking any particular branch of trade, at ſuch intereſt and upon ſuch terms as the profit of ſuch a trade would only anſwer. But all the advantage of this kind that in our circumſtances, and in the preſent caſe, we have to expect, is to be collected only by an inquiry into the preſent ſtate of our commerce; from whence, if it cannot be made to appear that there is at preſent any profitable branch of foreign commerce neglected by us, the profit of which will, over and above the hazard and other expences of adventuring, exactly bear 3 per cent. intereſt for the money employed in it, but will not anſwer four; I ſhould think we have more reaſon to apprehend ſome ill conſequences from a ſudden reduction of intereſt amongſt us with relation to our foreign commerce, which are by no means inconſiderable: ſuch as

It is rather to be apprehended that ſuch a reduction will be hurtful to our foreign commerce, and produce the following ill conſequences.

The

The raſhly engaging unexperienced perſons in unprofitable adventures, to their own and the nation's prejudice.

1ſt ill conſequence.

The increaſing our adventures in the ſeveral branches of our preſent commerce beyond the demand for our commodities, or the poſſibility of vending them with advantage at foreign markets; and thereby rendering the whole of our foreign commerce for the future leſs profitable; and by this means,

2d ill conſequence.

The furniſhing a temptation to the more ſkilful and experienced perſons, at preſent employed in our foreign commerce, to remove with their effects and ſettle in other countries, from whence the commerce they are beſt acquainted with may be carried on with more advantage. An inconvenience which we have the greateſt reaſon to guard againſt at this particular juncture, when our neighbours in the different parts of Europe are ſo generally attempting to rival us in our foreign commerce.

3d ill conſequence.

And if amongſt theſe conſequences of the ſudden reduction of our intereſt, the money of foreigners, which either our Government or private perſons amongſt us at preſent have the uſe of, (to whom moſt certainly a higher rate of intereſt than they can have at home for it, muſt have been the general inducement for their truſting it here) ſhould be called from us, and applied to other uſes; a higher rate of intereſt than before may not only be apprehended, but an abſolute impoſſibility of ſupplying the ordinary demands of our commerce for ſome time at any rate at all.

4th ill conſequence.

Of the increaſe of the value of lands, or the price they will ſell for, by ſuch a reduction.

As to the proprietors of our lands and irredeemable annuities, I am content to admit that they may reaſonably expect a higher price to be offered for their eſtates, in ſome meaſure proportioned to, and regulated by, a lower rate of intereſt produced by, and in proportion to, any ſolid and reaſonable cauſes for it. But I think it has been with truth obſerved by Mr. Locke on this occaſion, that in this higher price of their eſtates, thoſe proprietors are only intereſted who have contracted, or want to contract, debts upon their eſtates; it being of no conſequence to the perſon who neither ſells nor mortgages his eſtate, or intends to do ſo, what price he may procure for it; and it being as plain, that the perſon who on this occaſion receives a higher price on the ſale of his eſtate, from thenceforth ſtands in the place of the monied man, poſſeſſed of a greater ſum of money indeed than he could have had before, but which will produce no greater annual income, nor, generally ſpeaking, go farther in any proviſion he has intended for himſelf or family, nor in any other uſe that he can apply it to (except the diſcharge of ſuch debts as he may have contracted) than a leſs ſum would have done when the rate of intereſt was higher. The principal, if not the only, general advantage of a lower intereſt to

This is no real publick advantage.

the proprietors of land, is therefore so far as they have contracted debts; which advantage to them, and to all other persons who have contracted debts, is exactly balanced in the publick accounts by an equal loss and inconvenience to their creditors.

Such a reduction would likewise, probably, be attended with a diminution of the general expences of the people, and, consequentially, of the publick revenues arising from taxes on the consumption of commodities.

I will desire your attention, Sir, but to one consequence more, which I think will naturally and necessarily follow a further reduction of our interest, if it can by any means be effected, or for any length of time prevail amongst us, without the concurrence of what I have hitherto supposed to be the natural and only reasonable causes of it, viz. a considerable diminution of our expences, which the publick, as our affairs now stand, and the proprietors of land in particular, seem to me not a little interested to prevent. A fourth part of the income and usual profits of the personal estates in this kingdom, withdrawn and deducted from the whole of our ordinary annual expence, must occasion a very considerable diminution in it, when not supplied by the increase of those personal estates, or the growing wealth of our inhabitants, and must from thence occasion a considerable diminution in the price and consumption of our commodities. And this I apprehend will be the sooner and more sensibly felt, as the interest of money and the profits of personal estates are more generally the funds for, and supply the expence of, the inhabitants of this metropolis of the kingdom, than of any other part of it; and as a variation in our fashionable expences here is most likely to spread itself, by the force of our example, throughout the other parts of this kingdom, where perhaps there may not be the same occasion for it; from hence it deserves well to be considered, if the publick may not lose as much, or more, in their revenues arising from different commodities consumed amongst us, as may be saved by the reduction of our interest; or if the proprietors of land may not at last find themselves obliged to furnish from their own revenues those supplies for the service of the Government, which have been hitherto furnished by our expences. And if it be possible that this diminution of our expences should proceed further, in reducing the price of labour, and from thence of our necessary provisions and the produce of our lands, the proprietors of those estates must in their turn suffer from the reduction of their annual revenues.

And it might be necessary to increase the land-tax to make good the deficiencies of these taxes.

It might be of great publick benefit to determine immediately, by an act of parliament, the order in which the publick debts should be successively discharged by the application of the Sinking Fund.

From such reflections as these, Sir, it has seemed to me not unreasonable, that we should at least for some time rest contented with such reductions of publick interest as have been hitherto made: from whence I have been further induced to think, that it would be of considerable convenience to the publick, if the application of the present Sinking Fund, which stands now appropriated to the discharge of the publick debts in general, were by act of parliament determined as to the course and order in which those debts should be for the future discharged by it. These measures

meaſures with regard to the publick debts may poſſibly have not been hitherto propoſed, on account of that advantage which the publick may have been ſuppoſed for the future to be in a condition to make in the further reduction of publick intereſt, while they reſerved to themſelves the preference of one creditor to another in the order of diſcharging them; but I ſubmit it, how far this advantage would be prudently exchanged for the following conveniencies to the publick, from determining the order in which the Sinking Fund ſhould be applied in the diſcharge of our preſent debts.

Advantages that would ariſe from ſuch a meaſure.

Firſt, The annual income of the Sinking Fund will, by this means, be more fully appropriated to the payment of the publick debts, and the application of it to that purpoſe more effectually ſecured, by entitling every particular creditor to expect the application of it in the order that ſhall be ſo determined.

Secondly, It will be of conſiderable uſe in fixing the credit of the publick funds, and the confidence of the proprietors on ſuch foundations, as will ſupport them in any time of publick difficulty, by removing all grounds for thoſe apprehenſions, which, on ſuch occaſions, are obſerved (greatly to the diſadvantage of the Government) to prevail amongſt us, that the income of the Sinking Fund will be applied to ſome other purpoſes than the diſcharge of our debts; and by giving every particular creditor an opportunity of computing and ſatisfying himſelf in the value of his intereſt in the publick funds, from the knowledge of that time when his principal will be punctually paid off.

Thirdly, It will in a great meaſure prevent ſtock-jobbing, by removing the temptation to it from the great variations in the market-prices of our debts, from ſuch extravagant premiums paid for them in a time of peace, as if the income of them was conceived to be an irredeemable annuity; and ſuch diſcounts on the other hand allowed upon them, in a time of the leaſt general apprehenſion, as if they were regarded as debts almoſt deſperate.

Fourthly, It will lay a further foundation for a greater equality in the prices of our publick debts, by giving an opportunity to the proprietors to ſuit their own convenience in the purchaſe of ſuch part of thoſe debts as are determined to be payable, as near the time as poſſible when they expect any occaſion for their money; and prevent in a great meaſure the neceſſities of the proprietors being brought to market, eſpecially in the manner in which, when any declenſion in the price of ſtocks is apprehended, it may be obſerved often to be done long before they have any real occaſion for their money.

Fifthly,

Fifthly, It will tend to the increaſe of our credit, and the facilitating both of publick and private loans at the preſent, or as far as is reaſonable to wiſh for it, at a lower rate of intereſt, by capacitating ſuch of the creditors, whoſe debts ſhall be in a leſs remote order of payment, to lend out ſuch ſums as they may have by them reſerved for diſtant uſes, in expectation of being ſupplied for ſuch diſtant occaſions by the payment of their ſhare in the publick debts in the order and at the time appointed for it.

And laſtly, ſuch a determination of the order in which the Sinking Fund ſhould be applied in the diſcharge of our publick debts, and the notice the creditors would thereby have when they ſhould be paid off, would give them an opportunity of looking out for, and providing, the moſt apt and convenient employment for their money againſt the time of receiving it; a convenience to the creditors themſelves, which, as the publick is always intereſted in the innocent improvement of our eſtates, may, I think, be eſteemed a general advantage.

A recapitulation of the ſeveral propoſitions that have been advanced and recommended in the courſe of this Eſſay.

I ſhall conclude, by putting together what I have been endeavouring to repreſent about the reduction of our publick intereſt, viz. That the general and uſual rate of intereſt in every country is determined by the proportion that the ordinary neceſſities or demands for money amongſt the inhabitants bear to their capacity and diſpoſition to ſupply them; That any other rate of intereſt produced without a variation in the proportion aforeſaid, or a foundation laid for it, is not likely to continue; That we ſeem here to have had the effect of the late happy alteration in our publick circumſtances, in ſuch reductions of our intereſt as have been made already; That the proſpects of a ſtill further reduction of publick intereſt are a continued encouragement to adventures, which, though they may be made to contribute to the producing ſuch a reduction for a time, are not to be depended on for the ſupport of it; That theſe adventures are themſelves at all times a general inconvenience, and particularly prejudicial to the publick on the approach of troubles. I have likewiſe endeavoured to repreſent, that the further reduction of publick intereſt is neither equally neceſſary, nor of equal advantage to the publick, as either of thoſe that have been already made; nor does it want to be explained, that the ſame addition to the Sinking Fund, to be now made by the next reduction of the publick intereſt below 4 per cent. will diminiſh the remaining income of the creditors in a greater proportion than thoſe before made, and be a greater inconvenience to them. I have recommended it to be conſidered, how far a reduction beginning with the publick intereſt muſt, if it ſucceeds, neceſſarily ſpread itſelf, and affect the reſt of our perſonal eſtates; and from thence the oppoſition that meaſures for reducing intereſt will ſome time meet with, where ſome real alteration in our circumſtances does not perſuade to it. I have propoſed it to be inquired, if there be any other general advantage to be obtained by a lower rate of intereſt amongſt us, than in regard

regard to our publick debts only. And from ſuch conſiderations, would ſubmit it, if it might not be convenient, that not only our meaſures for further reducing intereſt, but our expectations of it were at leaſt for ſome time ſuſpended; till after the regular application of the Sinking Fund now provided for a few years, and the intermiſſion of ſuch extraordinary adventures as aforeſaid in our publick ſtocks, we might with more certainty collect what lower of intereſt our real circumſtances will admit of.

In what little I have ſaid about the conſequences of a lower intereſt on our commerce and expences, I have referred myſelf, Sir, to ſentiments in which I have had the honour to agree with you, and muſt not pretend to have made out any thing to general ſatisfaction: if I had attempted to do ſo, I ſhould have been carried too far beyond the deſign of this eſſay; and ſhould have been obliged to examine ſome prevailing opinions on this ſubject, which ſeem to me ſo far from being reaſonable or true, that I have ſometimes thought that part of them which the private intereſt of particular perſons have not introduced amongſt us, to have been taken up merely on account of their reſemblance to paradoxes, and for that reaſon affording the greateſt amuſement in converſation.

I would not have it, from any thing I have ſaid, inferred, that I am in general againſt any expedient for the much ſpeedier diſcharge of our preſent debts; I ſhould be glad if any reaſonable method for this purpoſe could be thought of; nor would any new burthen, or variation in the preſent burthen, on the ſubjects of this kingdom, implied in any propoſals for this purpoſe, be with me an objection to them, if the means were but found out of proportioning ſuch a new burthen, either to the property or expences of our inhabitants, in ſuch a manner as would be generally ſubmitted and agreed to: and that ſuch an expedient were found out, I wiſh for this general reaſon, that whatever in publick affairs is thought of great and general importance to be done at all, ſhould be done, if poſſible, as ſoon as it appears to be ſo; that the moſt eligible methods for effecting it are ſuch as may be carried on, and finiſhed under the direction of the ſame perſons who were firſt engaged in them; and that the ſucceſs of ſuch meaſures ſhould be as little as poſſible hazarded by the different ſentiments of their ſucceſſors. But this conſideration will not go far in recommending the further increaſe of the Sinking Fund, by reducing the intereſt of the publick debts; which, upon the ſuppoſitions on which my calculations have been made, would not, if the Sinking Fund were increaſed by reducing their intereſt to 2 per cent. be paid off in leſs than 20 years and a half, or thereabouts; if to 1 per cent. in leſs than 18 years and 4 months; or if the creditors would be ſatisfied without any intereſt at all till the payment of their principal, in leſs than ſixteen years and eight months.

I am, SIR, &c.

[*End of Sir Nathaniel Gould's Eſſay on the Publick Debts of this Kingdom.*]

Article CCCLXV. In the foregoing excellent pamphlet of Sir Nathaniel Gould, pages 420 and 421, the judicious author, having occasion to make mention of the land-tax, takes notice of the hardship arising from its inequality; but adds that, in his opinion, there was no great occasion to dwell upon that circumstance, because there was *then* a prospect that, by the operation of the Sinking Fund, the national debt would be wholly discharged in the course of a moderate number of years, after which he advises, and likewise supposes, that the taxes which had been before appropriated to the payment of the said debt, would be employed in defraying the ordinary expences of Government, so as to make the annual grants of the land-tax and malt-tax become no longer necessary. But now (in April, 1782,) that pleasing prospect is at an end, and, instead of it, there is reason to apprehend, that it will be necessary, not only to continue the land-tax upon its present footing of four shillings in the pound, or to raise by it the sum of two millions of pounds, sterling, *per annum*, for a great number of years to come, (perhaps, for a hundred years,) but even to increase the said tax to double its present quantity, or to raise by it the sum of four millions of pounds, sterling, *per annum*, in order to make good the payment of the interest of the prodigious sum of money to which the national debt will, probably, have increased before the blessing of a general peace shall be compleatly restored to us;——more especially, if our ministers of state should resolve to apply a million, or twelve hundred thousand, pounds a year out of the Sinking Fund to the gradual diminution of the capital of the said debt in the manner that has been above recommended. The land-tax must, therefore, in our present unfortunate circumstances, be considered as a *permanent* part of the publick revenue; and consequently the very great inequality with which it is assessed on the inhabitants of different counties in the kingdom, and even on those of different districts of the same county, is an object well worthy the attention of the Legislature; and, if it be really a grievance and a measure of great injustice (as, I must confess, it has always appeared to me to be, and as Sir Nathaniel Gould in the passages above referred to, in pages 420, 421, seems to have considered it,) it is highly fit to be speedily corrected and redressed. I therefore hope I shall be excused, if, before I return from this political digression concerning the Sinking Fund and the national debt, to the Doctrine of Life-annuities, (which is the proper subject of this book,) I lay before the reader some reflections on this subject of the inequality of the land-tax, and on the expediency of assessing it for the future in a more equal manner, which were first drawn up and published in the Publick Advertiser in the month of January, 1780, together with a proposal to make the interest of all future sums of money that should be borrowed by the publick, liable to pay land-tax in the same proportion with the rents of lands, to the end that all the subjects of the Crown, those who are possessed of personal

perſonal property as well as thoſe who have landed eſtates, may both contribute, and be ſeen and known to contribute, in the ſame degree, in proportion to their reſpective annual incomes, to the publick expences which the exigencies of the ſtate make neceſſary. Theſe reflections are as follow:

On the expediency of an EQUAL ASSESSMENT of the LAND-TAX, and of making the annual intereſt of all new publick loans of money ſubject to the ſame tax.

CCCLXVI. It ſeems to be an indiſputable maxim, founded both on equity and good policy, that, when taxes are neceſſary to the exigencies of the ſtate, perſons poſſeſſed of equal incomes ſhould contribute equally towards them. And, whenever this maxim is departed from, even though the ground of ſuch departure may be juſt by virtue of ſome original compact in favour of the perſons who pay leſs than their ſhare of the publick taxes, it is ſure to create ſome jealouſy and uneaſineſs in the other members of the ſtate, and thereby to render the property ſo exempted ſomewhat leſs ſecure than it otherwiſe would be.

We have, indeed, in our government, two ſtriking inſtances of a departure from this prudent and equitable maxim, which are frequently the ſubjects of very great complaints: I mean, the ſhamefully unequal aſſeſſment of the land-tax on the lands and houſes of the kingdom, and the total exemption of all the intereſt of the publick funds from that and every other tax. By the unequal aſſeſſment of the land-tax, ſome people pay more than four ſhillings in the pound upon the rents of their lands, while others pay only eight-pence or nine-pence, or in ſome places, (as the two northern counties and the new buildings at Marybone) not more than four-pence upon theirs. And this is done by acts of parliament renewed every year, and not by any permanent and original act of parliament that could be conſidered as a plighting of the national faith to the purchaſers of land, that their lands ſhould always be taxed according to the rule of aſſeſſment then obſerved. On the contrary, the perſons who have been lightly taxed have always feared, and thoſe who have been heavily taxed have always hoped, that the Parliament would, one day or other, have a ſufficient regard to juſtice to correct this groſs inequality, and to impoſe the land-tax according to a new and equal aſſeſſment; or rather, indeed, according to a certain proportion of the rents received by every man; or, if the land is kept in the owner's hands, of the rents which were received for it when it was laſt lett, or which it might eaſily be lett for, in the judgment of the commiſſioners of the land-tax; as is done, if I miſtake not, in the caſe of

Of the very unequal aſſeſſment of the land tax, the late houfe-tax. And they all have hoped, at times, (though now, I believe, that hope is at an end) that the land-tax would be reduced to two fhillings in the pound, and fometimes even that it would be intirely taken off, or (to fpeak more correctly) permitted to expire without being re-impofed; as I am fully perfuaded it might have been, if prudent and œconomical meafures had been purfued by our feveral minifters of ftate for thefe laft forty years, and both this and the two laft wars had been avoided. There is, therefore, as I apprehend, no weight in the reafoning of thofe who fay that fuch a correction of the inequality of the land-tax would be unjuft with refpect to thofe purchafers of land who have bought their land at a greater price than they otherwife would have done, upon an expectation that the land-tax would continue to be raifed according to the then prefent mode of affeffment. The nation is not bound to continue in the practice of impofing this tax unequally, becaufe thefe gentlemen have flattered themfelves that they would do fo.

I am the more confirmed in this opinion of the injuftice of continuing the land-tax on its prefent unequal footing, becaufe it was that of Dr. Benjamin Franklin, whom I confider as one of the moft judicious and wifeft men now living upon earth, and of whofe talents we now feel the force, fince, by our attack upon one of the charters of the Americans, and our other alarming acts of authority againft them, we have driven him to employ them againft us. This truly great man ufed always to quote the continuance of the land-tax, upon its prefent very unequal footing, as a proof of the little regard that was had to juftice and common fenfe in our national deliberations. And I remember once in particular, that when it was faid that fome people thought the correction of this inequality would be unjuft with refpect to thofe who had purchafed land upon a fuppofition of its continuance, he replied with fome quicknefs, "Unjuft! yes, it would be unjuft; for it would be doing but half of what ftrict juftice would require; which would be to create a counter-inequality in the affeffment of the land-tax, whereby the lands which had been heavily taxed fhould hereafter be taxed lightly, and thofe which had been lightly taxed fhould hereafter be taxed heavily, for the fpace of about fourfcore years, or for a time that fhould be equal to the time during which the prefent unequal affeffment had been permitted to continue." This counter-inequality he, perhaps, would not ferioufly have wifhed to fee eftablifhed; but, I think, *that* manner of expreffing himfelf fhewed ftrongly his opinion of the propriety of an exact equality for the future, and his contempt of the arguments derived from its fuppofed injuftice with refpect to purchafers.

This firft deviation, therefore, from the rule of equal taxation in our government ought, as I conceive, to be corrected in the next land-tax act which fhall be paffed, there being no valid nor juft objection to be made to fuch correction.

CCCLXVII. But

CCCLXVII. But the caſe is different with reſpect to the other inſtance of a deviation from the ſame rule, to wit, the exemption of the intereſt of the publick debts from the land-tax and all other taxes. For this intereſt cannot be made ſubject to the land-tax without a direct breach of the national faith to the proprietors of it, there being expreſs clauſes in the ſeveral acts of parliament by which the loans that conſtitute theſe debts have been eſtabliſhed, which provide that the ſeveral annuities granted to the perſons who have advanced their money to the Government, ſhall be *free from all taxes, charges, and impoſitions whatſoever.** And the money ſo advanced to Government has been advanced at a lower rate of intereſt in conſequence of theſe clauſes. And much of it, I believe, has been lent to Government in the war of 1741, at the moderate intereſt of about four per cent. which, if it had been left ſubject to the land-tax (as all other perſonal eſtate is, according to the ſtrict letter of the land-tax acts, though, from the difficulty of coming at it, the tax is ſeldom actually paid upon it;) could not have been obtained for leſs than five per cent. The owners of this debt may, therefore, be ſaid to have paid the land-tax upon it in the very act of lending it on the terms propoſed to them; ſince, in conſideration of their exemption from that and other taxes, they conſented to take four, inſtead of five, per cent. for their money. Nevertheleſs, in proceſs of time, theſe original compacts grow to be in a manner forgot by the generality of mankind, who are apt to conſider this exemption of the ſtock-holders from paying the land-tax, as an unjuſt diſtinction in their favour: and the land-holders in general are apt to hold this language, partly, perhaps, from ignorance of the aforeſaid original clauſe of exemption, and partly from the bias of ſelf-intereſt, which makes them wiſh to ſee the ſtock-holders bear a ſhare of the burthen which they labour under, whether they have, or have not, been ſo exempted. I have known men of very good underſtandings and education talk in this manner, and, when they have been told of the ſaid clauſe of exemption in the ſeveral acts of parliament for borrowing the ſaid money, either refuſe to believe that there was ſuch a clauſe, or, if convinced of the exiſtence of it, deny its efficacy and validity, in point of juſtice and good policy, to entitle the ſtock-holders to be ſo exempted. This opinion and inclination in the land-owners of the kingdom, and perhaps in other claſſes of men that are not themſelves ſtock-holders, certainly contributes to make the publick funds leſs ſecure than they would be, if they had not been ſo exempted in their firſt eſtabliſh-

Of the exemption of the publick funds from the payment of taxes.

The ground of it.

The ſaid exemption is odious to the land-holders, and thereby leſſens the ſecurity of the ſaid funds.

* Thus, for example, in the ſtatute of the 21ſt of George the Second, chap. ii, ſect. 15, it is enacted, "That all the ſeveral and reſpective annuities, payable in purſuance of this act, after the rate of four pounds *per centum per annum*, on all and every the principal ſums for which the ſame are payable, *ſhall be free from all taxes, charges, and impoſitions whatſoever.*"

ment, but had been left liable (like all other property, both real and perſonal,) to pay their proportion of the land-tax: for then, as both the claſſes would conſtantly and viſibly have contributed at the ſame time to the relief of the exigencies of the ſtate, the land-holders would have had no pretence to grudge the ſtock-holders the enjoyment of the intereſt of their money, which would, in every view, have been as much their rightful property as the rents of freehold land are the property of their reſpective owners.

It would tend to the ſecurity of the publick funds, if the proprietors of them would conſent to make the intereſt of them liable to the land-tax.

This diminution of the ſecurity of the publick funds, ariſing from their being thus exempted from taxes, appears to me to be a matter of ſo much importance, (as their ſecurity evidently depends, in a conſiderable degree, on the opinion the nation at large entertains of the juſtice of continuing them) that I ſhould be ready, with reſpect to my little property in them, to give up my right to the ſaid exemption, and make it liable to pay the land-tax at that which may be ſuppoſed to be the medium rate at which it is levied upon the lands of the kingdom, which I have heard people eſtimate at about eighteen pence in the pound, or, at moſt, two ſhillings, when the land-tax is called four ſhillings in the pound, or (to ſpeak more correctly) when the ſum raiſed by the land-tax in England and Wales, (excluſive of Scotland) is about two millions: for it is ſuppoſed that an equal aſſeſſment of the land-tax in England and Wales at eighteen-pence or two ſhillings in the pound, would raiſe the ſaid ſum of two millions. This, therefore, would be the rate at which property in the publick funds ought to be charged to the land-tax, if it could, conſiſtently with juſtice and the preſervation of the national faith, be charged to it at all. And at this rate, I ſay, I would freely conſent to have my property there charged, notwithſtanding my aforeſaid right to an exemption, if all the other proprietors of ſtock, or a majority of them, were willing to do the ſame: and this, not ſo much from a motive of publick ſpirit (though I hope that motive is not without its influence) as from a deſire of rendering the remaining part of the intereſt due to me, the eighteen ſhillings and ſixpence in the pound, more ſecure: for then nothing but the moſt ſhameleſs and bare-faced injuſtice could ever prompt any land-holder to wiſh for, or encourage, any attempt to get rid of the publick debt by any other method than that of fairly paying it off.

But this is hardly to be expected.

But ſuch a general conſent of the ſtock-holders to let their ſtock be made liable to pay the land-tax, is what we can hardly expect to ſee; more eſpecially as many of them are foreigners, reſident in Holland and Switzerland, who will probably conceive themſelves to be no way obliged to pay taxes for the ſupport of our government.

Nor can ſuch a meaſure be taken, conſiſtently with juſtice, without their conſent.

And to ſubject their ſtock to the land-tax by act of parliament, without their conſent, or even with the conſent of ſome, but not all of them, would undoubtedly be a breach of the national faith, and an act of bankruptcy, *pro tanto*. With reſpect, therefore, to the immenſe publick debt already ſubſiſting, I do not expect, or wiſh, to ſee any endeavours uſed by

publick

But it would be a useful measure in the government, to make the intereſt of all *future* publick loans of money liable to the land-tax.

publick authority to carry this meaſure, of making it liable to the land-tax, into execution. But with reſpect to the loans of the preſent year, 1780, and of the following years, (if more ſuch are to be expected,) I muſt own I ſhould like to ſee it tried. It would certainly have the good effect of increaſing the ſecurity of the money ſo lent, for the reaſon already ſuggeſted and it would be no immediate diminution of the intereſt received and enjoyed for the ſaid money; becauſe the intereſt which would be given by Government for the money, when liable to this tax, would be proportionably greater than if lent under a clauſe of exemption. And yet Government would be no loſer by it, ſince it would receive back, in the ſhape of a tax on the ſaid intereſt, the additional intereſt it would be obliged to give on this account. This, it may be ſaid, is mere trifling and doing nothing, ſince you give with one hand what you take back with the other. But the advantage reſulting from it is this: the land-holder, and the ſtock-holder, whoſe ſtock is originally made liable to this tax, will ever after run the ſame fortune, and experience the ſame increaſe or diminution of their reſpective incomes, as the affairs of the ſtate are proſperous or unfortunate. If, contrary to all preſent appearance, the land-tax ſhould ever again be leſs than four ſhillings in the pound, the ſtock-holder upon this new eſtabliſhment would enjoy a proportional diminution of the tax upon his property: and, on the other hand, if it ſhould become neceſſary, in the courſe of this moſt ruinous and unhappy war, to make the land-tax double or treble of what it now is, (which ſeems to be a much more likely event than the former) the ſtock-holder will pay a double or treble tax as well as the owner of land, and viſibly bear his proportion of the common burthen, and thereby eſcape the dangerous envy of being conſidered as a kind of foreigner, or neutral perſon unconcerned in the welfare or calamities of his country.

An ingenious and publick-ſpirited writer of ſome letters that have appeared in the Whitehall Evening Poſt, dated from Windſor, and which have been lately collected and publiſhed in a pamphlet printed for Dodſley, has recommended a meaſure of this kind, with reſpect to the publick debt already exiſting, and has given very powerful reaſons in ſupport of it, which, together with the many other important particulars contained in thoſe letters, are well worth the moſt ſerious attention of the publick. But, as ſuch a ſtep would be attended with conſiderable difficulties, I cannot but doubt whether it be, upon the whole, adviſeable.

But the meaſure which I have here ventured to recommend, and which relates only to the ſtock hereafter to be created, would be attended with the ſame advantages, as far as it went, and would not be liable to any of the ſame objections.

And,

It would, probably, be expedient to admit the owners of such stock in the publick funds as should be made liable to pay the land-tax, to a right of voting for members of parliament.

And, perhaps, if this measure were adopted, either with respect to the interest of future loans of money, or to that of the money already due to the publick creditors, it might be adviseable at the same time to admit the stock-holders who would thus become contributors to the land-tax, to a right of voting for members of the House of Commons, by whom the said tax is granted. Every proprietor of such stock, who was of the male sex, and a native of Great Britain, or Ireland, or any of the British dominions, and had been in possession of an annuity of 10 pounds a year, standing in his own name, in any of the publick funds for more than a whole year, and had resided for more than a year together in any particular county in England, Wales, or Scotland, might, as I imagine, without any inconvenience, be permitted to vote at the election of the knights of the shire, or commissioners of the shire, in which he had so resided. Perhaps the offer of such a privilege might induce some of the present proprietors of the publick funds to consent that the interest of their shares of the national debt should for the future be made liable to the land-tax: more especially, if they shall apprehend themselves to be under a kind of necessity (from the enormity of the present load upon the publick revenue,) of making some sacrifice of this nature, or of giving up a part of their annual income, arising from the funds, in order to preserve the remainder; which the *Earl of Stair*, in his very able and most interesting pamphlet on the State of the Publick Revenue, published about January, 1782, and intitled, *Facts, and Consequences*, &c. positively declares it will be absolutely necessary that they should do.

When the stock-holders should have thus become liable to the land-tax, and, in consideration of their thereby contributing, like the owners of land, to the common burthens of the nation, should have been admitted to a share in the election of the national representative, the security of their property in the publick funds would be rendered as compleat as any laws, or publick regulations, can make it: though they would still have reason to wish, at least as heartily as any other class of men in the kingdom, that the blessing of peace may speedily be restored to us, and that then the Government may adopt such measures of vigour for increasing the publick revenue, and such measures of œconomy in the management of it, as may enable them gradually to discharge some part of this enormous debt, and thereby render the payment of the remainder of it less precarious.

[*End of the reflections on the Land-tax and the Interest of the National Debt.*]

CCLXVIII. I now

CCCLXVIII. I now return from theſe political digreſſions to the Doctrine of Life-Annuities, which is the proper ſubject of this treatiſe.

And, as I have gone through every thing that has appeared to be neceſſary to the illuſtration of the doctrine of annuities for ſingle lives, I ſhall now proceed to ſubjoin the like ſhort and convenient expreſſions for the values of annuities dependent on more than one life as were given above, in Art. 86 *et ſeq.* for the values of annuities depending on a ſingle life.

A ſhort expreſſion of the value of an annuity of one pound per annum *for a given number of years, depending on the joint continuance of two lives of given ages, according to a given table of the probabilities of the duration of human life, and a given rate of the intereſt of money.*

CCCLXIX. Let r be, as before, the ſum of one pound together with its intereſt for one year according to the given rate of intereſt. And let N be the number of years in the age of the younger of the two perſons on the joint continuance of whoſe lives the annuity is to depend; and $N+a$ the number of years in the age of the older of the ſaid perſons; and E the greateſt number of years through which it is ſuppoſed to be poſſible for human life to be extended, according to the table of probabilities of the duration of human life adopted for the calculation; which number is in Monſieur de Parcieux's table 94 years. Let n be any number of years not greater than $E - \overline{N+a}$, or $E-N-a$, or than the greateſt number of years during which it is poſſible that the older of the two lives may be prolonged. And let the annuity of one pound *per annum* be granted for the term of n years, provided both the ſaid perſons of the ages of N years and $N+a$ years ſhall ſo long live, but otherwiſe to ceaſe upon the death of either of them. Let P be the number of perſons repreſented in Monſieur de Parcieux's table of the probabilities of the duration of human life, (or in ſuch other table of thoſe probabilities as is thought proper by the calculator to be adopted as the ground of his calculation,) as being all living together at the ſaid age of N years; and $P^{\prime}$ the number of perſons repreſented in the ſaid table to be living at the age of $N+1$ years; and $P^{\prime\prime}$ the number living at the age of $N+2$ years; and $P^{\prime\prime\prime}$ the number living at the age of $N+3$ years; and P^{iv}, P^{v}, P^{vi}, P^{vii}, P^{viii}, P^{ix}, P^{x}, &c. the numbers living at the ſeveral following ages of $N+4$ years, $N+5$ years, $N+6$ years, $N+7$ years,

$N+8$

$N+8$ years, $N+9$ years, $N+10$ years, &c. respectively. And let $\mathcal{Q}$ be the number of persons represented in the said table as living at the age of $N+a$ years; and $\mathcal{Q}^{I}$ the number of persons represented there as living at the age of $N+a+1$ years; and $\mathcal{Q}^{II}$ the number living at the age of $N+a+2$ years; and $\mathcal{Q}^{III}$ the number living at the age of $N+a+3$ years; and $\mathcal{Q}^{IV}$, $\mathcal{Q}^{V}$, $\mathcal{Q}^{VI}$, $\mathcal{Q}^{VII}$, $\mathcal{Q}^{VIII}$, $\mathcal{Q}^{IX}$, $\mathcal{Q}^{X}$, &c. the numbers living at the several following ages of $N+a+4$ years, $N+a+5$ years, $N+a+6$ years, $N+a+7$ years, $N+a+8$ years, $N+a+9$ years, $N+a+10$ years, &c. respectively.

Then will the present value of an annuity of one pound a year, to be enjoyed during the space of n years, in case both the said lives, of the ages of N years and $N+a$ years, shall so long continue, be equal to the expression £1 × the series $\frac{P^{I}\times \mathcal{Q}^{I}}{P\mathcal{Q}\times r}+\frac{P^{II}\times \mathcal{Q}^{II}}{P\mathcal{Q}\times r^{2}}+\frac{P^{III}\times \mathcal{Q}^{III}}{P\mathcal{Q}\times r^{3}}+\frac{P^{IV}\times \mathcal{Q}^{IV}}{P\mathcal{Q}\times r^{4}}+\frac{P^{V}\times \mathcal{Q}^{V}}{P\mathcal{Q}\times r^{5}}+\frac{P^{VI}\times \mathcal{Q}^{VI}}{P\mathcal{Q}\times r^{6}}+\frac{P^{VII}\times \mathcal{Q}^{VII}}{P\mathcal{Q}\times r^{7}}+$ &c. continued to n terms, or to the term $\frac{P^{n}\times \mathcal{Q}^{n}}{P\mathcal{Q}\times r^{n}}$, or equal to the expression $\frac{£1}{P\mathcal{Q}}\times$ the series $\frac{P^{I}\times \mathcal{Q}^{I}}{r}+\frac{P^{II}\times \mathcal{Q}^{II}}{r^{2}}+\frac{P^{III}\times \mathcal{Q}^{III}}{r^{3}}+\frac{P^{IV}\times \mathcal{Q}^{IV}}{r^{4}}+\frac{P^{V}\times \mathcal{Q}^{V}}{r^{5}}+\frac{P^{VI}\times \mathcal{Q}^{VI}}{r^{6}}+\frac{P^{VII}\times \mathcal{Q}^{VII}}{r^{7}}+$ &c. continued to n terms, or to the term $\frac{P^{n}\times \mathcal{Q}^{n}}{r^{n}}$. This is evident from Problem III, and its second Corollary, Art. XLIV, XLV, XLVI, and XLVIII, pages 42, 43, 44, 45, 46, 47, 48.

The expression of the value of an annuity of one pound a year for the whole joint continuance of two lives of given ages.

CCCLXX. If n years is the greatest number of years through which it is possible (according to the table of the probabilities of the duration of human life adopted in the calculation,) for the older of the two given lives, or the life of the age of $N+a$ years, to be extended, or, in other words, if n is equal to $E-N-a$, the said expression $\frac{£1}{P\mathcal{Q}}\times$ the series $\frac{P^{I}\times \mathcal{Q}^{I}}{r}+\frac{P^{II}\times \mathcal{Q}^{II}}{r^{2}}+\frac{P^{III}\times \mathcal{Q}^{III}}{r^{3}}+\frac{P^{IV}\times \mathcal{Q}^{IV}}{r^{4}}+\frac{P^{V}\times \mathcal{Q}^{V}}{r^{5}}+\frac{P^{VI}\times \mathcal{Q}^{VI}}{r^{6}}+\frac{P^{VII}\times \mathcal{Q}^{VII}}{r^{7}}+$ &c. continued to n terms, or to

the

the term $\frac{P^n \times Q^n}{r^n}$, (which term will in this caſe be $\frac{P^{E-N-a} \times Q^{E-N-a}}{r^n}$,) will be the value of an annuity of one pound *per annum* for the whole joint continuance of the two given lives of N years and $N+a$ years: but, if n is leſs than the ſaid complement of $N+a$ years to E, or to the utmoſt duration of human life, the ſaid expreſſion will be leſs than the value of an annuity of one pound *per annum* for the whole joint continuance of the two given lives of the ages of N years and $N+a$ years, and will be the value of an immediate, but imperfect, life-annuity of one pound *per annum* during n years of the joint continuance of the lives of two perſons of the ſaid ages. This is evident from Art. XLVIII, XLIX, LI, pages 48, 49, 50, 51, 52, 53.

An example of the computation of the value of an immediate and compleat life-annuity of one pound per annum *for the whole joint continuance of the lives of two perſons of given ages, by means of the foregoing expreſſion.*

CCCLXXI. Let it be required to find the value of an annuity of one pound *per annum* for the whole joint continuance of the lives of two perſons of the ages of 75 years and 80 years, according to Monſieur de Parcieux's table of the probabilities of the duration of human life, and upon a ſuppoſition that the intereſt of money is 3 per cent.

Here n, or the number of years through which the annuity is to continue, in caſe both the lives (of which the older is of the age of fourſcore years,) ſhall laſt ſo long, is the greateſt poſſible number of years through which, according to Monſieur de Parcieux's table, a life of fourſcore years of age can be extended, that is, (94 — 80 years, or) 14 years. Therefore the ſeries

$$\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5} + \frac{P^{VI} \times Q^{VI}}{r^6} + \frac{P^{VII} \times Q^{VII}}{r^7} + \text{\&c.}$$

in the foregoing expreſſion muſt be continued to 14 terms; which terms may be computed as follows.

Here P is $= 211$, $P^{I} = 192$, $P^{II} = 173$, $P^{III} = 154$, $P^{IV} = 136$, $P^{V} = 118$, $P^{VI} = 101$, $P^{VII} = 85$, $P^{VIII} = 71$, $P^{IX} = 59$, $P^{X} = 48$, $P^{XI} = 38$, $P^{XII} = 29$, $P^{XIII} = 22$, and $P^{XIV} = 16$; and Q is $= 118$, $Q^{I} = 101$, $Q^{II} = 85$, $Q^{III} = 71$, $Q^{IV} = 59$, $Q^{V} = 48$,

$Q^{v} = 48$, $Q^{vi} = 38$, $Q^{vii} = 29$, $Q^{viii} = 22$, $Q^{ix} = 16$, $Q^{x} = 11$, $Q^{xi} = 7$, $Q^{xii} = 4$, $Q^{xiii} = 2$, and $Q^{xiv} = 1$. And r is $= 1.03$, and $\frac{1}{r} = \frac{1}{1.03} = .9708$, and $\frac{1}{r^2} = .9425$, $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, $\frac{1}{r^9} = .7664$, $\frac{1}{r^{10}} = .7440$, $\frac{1}{r^{11}} = .7224$, $\frac{1}{r^{12}} = .7013$, $\frac{1}{r^{13}} = .6809$, and $\frac{1}{r^{14}} = .6611$. Therefore the expression $\frac{£1}{P \times Q}$ × the series

$$\frac{P^{i} \times Q^{i}}{r} + \frac{P^{ii} \times Q^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} + \frac{P^{v} \times Q^{v}}{r^5}$$
$$+ \frac{P^{vi} \times Q^{vi}}{r^6} + \frac{P^{vii} \times Q^{vii}}{r^7} + \frac{P^{viii} \times Q^{viii}}{r^8} + \frac{P^{ix} \times Q^{ix}}{r^9}$$
$$+ \frac{P^{x} \times Q^{x}}{r^{10}} + \frac{P^{xi} \times Q^{xi}}{r^{11}} + \frac{P^{xii} \times Q^{xii}}{r^{12}} + \frac{P^{xiii} \times Q^{xiii}}{r^{13}}$$
$$+ \frac{P^{xiv} \times Q^{xiv}}{r^{14}}$$

will be equal to $\frac{£1}{211 \times 118}$ × the series

$$192 \times 101 \times .9708 + 173 \times 85 \times .9425 + 154 \times 71 \times .9151$$
$$+ 136 \times 59 \times .8884 + 118 \times 48 \times .8626 + 101 \times 38 \times .8374$$
$$+ 85 \times 29 \times .8130 + 71 \times 22 \times .7894 + 59 \times 16 \times .7664$$
$$+ 48 \times 11 \times .7440 + 38 \times 7 \times .7224 + 29 \times 4 \times .7013$$
$$+ 22 \times 2 \times .6809 + 16 \times 1 \times .6611 = \frac{£1}{24{,}898} \times \text{the series}$$

$$19{,}392 \times .9708 + 14{,}705 \times .9425 + 10{,}934 \times .9151$$
$$+ 8024 \times .8884 + 5664 \times .8626 + 3838 \times .8374$$
$$+ 2465 \times .8130 + 1562 \times .7894 + 944 \times .7664$$
$$+ 528 \times .7440 + 266 \times .7224 + 116 \times .7013$$
$$+ 44 \times .6809 + 16 \times .6611 = \frac{£1}{24{,}898} \times \text{the series}$$

$$18{,}825.7536 + 13{,}859.4625 + 10{,}005.7034 + 7{,}288.4336$$
$$+ 4{,}885.7664 + 3{,}213.9412 + 2004.0450 + 1233.0428$$
$$+ 723.4816 + 392.8320 + 192.1584 + 81.3508$$
$$+ 29.9596$$

$+ 29.9596 + 10.5776 = \frac{\pounds 1}{24,898} \times 62,746.5085 = \pounds 1 \times \frac{62,746.5085}{24,898} = \pounds 1 \times 2.5201 = \pounds 2.5201$. Therefore the value of an annuity of one pound a year for the whole joint continuance of two lives of the ages of 75 years and 80 years, when the intereſt of money is 3 per cent. is £2.5201, or 2*l.* 10*s.* 5*d.* QEI.

Note. This value 2.5201 exceeds the value found for the ſame annuity in Art. XLIX, to wit, £2.5197, by £.0004. But this very ſmall difference (which begins only in the fourth and laſt place of decimal fractions,) is rather to be conſidered as a proof of the agreement of the two calculations with each other than of an error in either of them, and probably has ariſen from the different order in which the arithmetical operations have been performed in them, the terms of the ſeries having been ſeparately divided by 24898 in Art. XLIX, and the ſeveral quotients thence ariſing added together into one ſum, whereas in this latter calculation the ſame terms have been all added together into one ſum without ſuch diviſion, and then the ſaid ſum has been divided by 24898.

An example of the computation of the value of an immediate, but imperfect, life-annuity, depending on the joint continuance of the lives of two perſons of given ages, by means of the ſame expreſſion.

CCCLXXII. Let it be required to find the value of an annuity of one pound *per annum* for the firſt five years of the joint continuance of the lives of two perſons of the ages of 75 years and 80 years, according to the ſame table of the probabilities of the duration of human life, and the ſame rate of intereſt, as in the laſt example.

For this purpoſe we need only take the firſt five terms of the foregoing ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. and multiply their ſum into the fraction $\frac{\pounds 1}{P \times Q}$; and the product will be the value of the propoſed annuity. Theſe terms are $192 \times 101 \times .9708 + 173 \times 85 \times .9425 + 154 \times 71 \times .9151 + 136 \times 59 \times .8884 + 118 \times 48 \times .8626$; which are equal to

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19392 × .9708 + 14,705 × .9425 + 10,934 × .9151 + 8024 × .8834 + 5664 × .8626 = 18,825.7536 + 13,859.4625 + 10,005.7034 + 7,288.4336 + 4,885.7664 = 54,865.1195; which being multiplied into the fraction $\frac{\pounds 1}{P \times Q}$, or $\frac{\pounds 1}{211 \times 118}$, or $\frac{\pounds 1}{24,898}$, is $= \pounds 1 \times \frac{54,865.1195}{24,898} = \pounds 1 \times 2.2035 = \pounds 2.2035$. Therefore the value of an annuity of one pound *per annum* for the first five years of the joint continuance of the lives of two persons of the ages of 75 years and 80 years, according to the foregoing suppositions concerning the duration of human life and the interest of money, is £2.2035, or 2*l*. 4*s*. 0*d*. $\frac{3}{4}$. Q E I.

CCCLXXIII. And, if the annuity to be purchased is a remote one, or is to be paid at the distance of more than one year, a short expression of its value, similar to the foregoing expressions, may be found as follows.

A short and general expression of the value of a remote annuity of one pound per annum *for a given number of years, depending on the joint continuance of two lives of given ages, when the rate of the interest of money is also given.*

Let *r* be, as before, the value of one pound together with its interest for a year at the given rate of the interest of money. And let *m* be the number of years at the end of which the annuity is to commence, so that the first payment of it shall be made at the end of $m + 1$ years. And let *N* be the number of years in the age of the younger of the two persons upon the joint continuance of whose lives the annuity is to depend; and let $N + a$ be the number of years in the age of the older of the said persons; at the time of purchasing the said annuity. And let *E* be, as before, the whole number of years through which it is possible, according to the table of probabilities adopted in the calculation, for human life to be extended; which in Monsieur de Parcieux's table is 94 years. Then will $N + a + m$ be the number of years in the age of the older of the said two persons at the time of the commencement of the said annuity, and $N + a + m + 1$ the number of years in his age at the time when the first

first payment of the said annuity will become due; and $E-\overline{N+a+m}$, or $E-N-a-m$, will be the greatest possible number of years through which the life of the said older person can be extended after he shall have attained the age of $N+a+m$ years, and the annuity shall have commenced. Let n be any number of years not greater than $E-N-a-m$; and let P be the number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life, (or in such other table of those probabilities as shall be adopted as the ground of the calculation) as being alive at the age of N years, or of the younger of the said two persons, and Q be the number represented there as living at the age of $N+a$ years, or of the older of the said persons; and out of the P persons represented in the said table as living at the age of N years, let $P^{m+\mathrm{I}}$, $P^{m+\mathrm{II}}$, $P^{m+\mathrm{III}}$, $P^{m+\mathrm{IV}}$, $P^{m+\mathrm{V}}$, &c. be the numbers of persons represented therein as living at the subsequent ages of $N+m+1$ years, $N+m+2$ years, $N+m+3$ years, $N+m+4$ years, $N+m+5$ years, &c. or at the ends of $m+1$ years, $m+2$ years, $m+3$ years, $m+4$ years, $m+5$ years, &c. from the time of purchasing the annuity; and out of the Q persons represented therein as living at the age of $N+a$ years, let $Q^{m+\mathrm{I}}$, $Q^{m+\mathrm{II}}$, $Q^{m+\mathrm{III}}$, $Q^{m+\mathrm{IV}}$, $Q^{m+\mathrm{V}}$, &c. be the numbers of persons represented therein as living at the subsequent ages of $N+a+m+1$ years, $N+a+m+2$ years, $N+a+m+3$ years, $N+a+m+4$ years, $N+a+m+5$ years, &c. or at the ends of $m+1$ years, $m+2$ years, $m+3$ years, $m+4$ years, $m+5$ years, &c. from the time of purchasing the annuity; and so on for all the following ages in the table.

Then will the value of an annuity of one pound *per annum*, to commence at the distance of m years, (so that the first payment of it shall be made at the end of $m+1$ years,) and to continue during n years, provided two persons, who are of the ages of N years and $N+a$ years at the time of purchasing the annuity, shall so long live, but which shall cease as soon as either of the said persons shall die; — I say, the value of such a remote annuity will be equal to the following expression, to wit,

$$\frac{\pounds 1}{P\times Q}\times \text{the series } \frac{P^{m+\mathrm{I}}\times Q^{m+\mathrm{I}}}{r^{m+1}}+\frac{P^{m+\mathrm{II}}\times Q^{m+\mathrm{II}}}{r^{m+2}}+\frac{P^{m+\mathrm{III}}\times Q^{m+\mathrm{III}}}{r^{m+3}}+\frac{P^{m+\mathrm{IV}}\times Q^{m+\mathrm{IV}}}{r^{m+4}}+\frac{P^{m+\mathrm{V}}\times Q^{m+\mathrm{V}}}{r^{m+\mathrm{V}}}+\text{\&c. continued to } n \text{ terms,}$$

or to the term $\frac{P^{m+n}\times Q^{m+n}}{r^{m+n}}$. This is evident from Art. LII, page 53.

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An example of the computation of the value of a remote life-annuity, depending upon the joint continuance of two lives of given ages, by means of the foregoing expreſſion.

CCCLXXIV. Let it be required to find the value of an annuity of one pound *per annum* during the joint lives of two perſons of the ages of 75 and 80 years, but which ſhall not commence till five years after the purchaſe of it, ſo that the firſt payment of it ſhall not become due until the end of ſix years, or till the younger of the ſaid two perſons (if he ſhall be then living,) ſhall have attained the age of fourſcore and one years, and the older of the ſaid perſons (if he ſhall be then living,) ſhall have attained the age of fourſcore and ſix years; and which ſhall then continue during the whole remainder of the joint continuance of the lives of the ſaid two perſons; the intereſt of money being 3 per cent. (as in the laſt example,) and the probabilities of the duration of human life being ſuch as they are repreſented to be in Monſieur de Parcieux's table of them.

Here N, or the number of years in the age of the younger of the ſaid two perſons, is 75; and $N+a$, or the number of years in the age of the older of the ſaid two perſons, is 80; and m, or the number of years before the propoſed annuity is to commence, is 5; and conſequently $N+a+m$, or the number of years in the age of the older of the ſaid two perſons, at the time when the annuity is to commence, is 85. E, or the greateſt number of years through which human life can be extended, is, according to Monſieur de Parcieux's table of probabilities, 94 years; and conſequently $E - |\overline{N+a+m}$, or the greateſt number of years through which it is poſſible that the life of the older of the ſaid two perſons can be extended after the annuity ſhall have commenced, is $94 - 85$, or 9, years. Therefore 9 years is likewiſe the greateſt number of years through which it is poſſible that the lives of both the ſaid perſons ſhould continue together in being. Therefore n, or the greateſt number of years through which it is poſſible the annuity may continue, will be equal to 9 years; becauſe, the annuity, when once it has taken place, is ſuppoſed to continue during the whole remainder of the joint continuance of the lives of the ſaid two perſons. And conſequently the ſeries $\frac{Pm+\text{I} \times \mathcal{Q}m+\text{I}}{r^{m+1}} + \frac{Pm+\text{II} \times \mathcal{Q}m+\text{II}}{r^{m+2}}$

$$+ \frac{Pm+\text{III} \times \mathcal{Q}m+\text{III}}{r^{m+3}} + \frac{Pm+\text{IV} \times \mathcal{Q}m+\text{IV}}{r^{m+4}} + \frac{Pm+\text{V} \times \mathcal{Q}m+\text{V}}{r^{m+5}} + \&c.$$

will conſiſt of nine terms. Theſe terms may be computed as follows.

Since

Since m is = v, we ſhall have $m + \text{i} = \text{vi}$, and $m + \text{ii} = \text{vii}$, $m + \text{iii} = \text{viii}$, $m + \text{iv} = \text{ix}$, $m + \text{v} = \text{x}$, $m + \text{vi} = \text{xi}$, $m + \text{vii} = \text{xii}$, $m + \text{viii} = \text{xiii}$, and $m + \text{ix} = \text{xiv}$; and in like manner $m + 1 = 6$, $m + 2 = 7$, $m + 3 = 8$, $m + 4 = 9$, $m + 5 = 10$, $m + 6 = 11$, $m + 7 = 12$, $m + 8 = 13$, and $m + 9 = 14$. Therefore the ſeries $\frac{P^{m+\text{i}} \times Q^{m+\text{i}}}{r^{m+1}} + \frac{P^{m+\text{ii}} \times Q^{m+\text{ii}}}{r^{m+2}} + \frac{P^{m+\text{iii}} \times Q^{m+\text{iii}}}{r^{m+3}}$ $+ \frac{P^{m+\text{iv}} \times Q^{m+\text{iv}}}{r^{m+4}} + \frac{P^{m+\text{v}} \times Q^{m+\text{v}}}{r^{m+5}} + \frac{P^{m+\text{vi}} \times Q^{m+\text{vi}}}{r^{m+6}}$ $+ \frac{P^{m+\text{vii}} \times Q^{m+\text{vii}}}{r^{m+7}} + \frac{P^{m+\text{viii}} \times Q^{m+\text{viii}}}{r^{m+8}} + \frac{P^{m+\text{ix}} \times Q^{m+\text{ix}}}{r^{m+9}}$ is equal to $\frac{P^{\text{vi}} \times Q^{\text{vi}}}{r^6} + \frac{P^{\text{vii}} \times Q^{\text{vii}}}{r^7} + \frac{P^{\text{viii}} \times Q^{\text{viii}}}{r^8} + \frac{P^{\text{ix}} \times Q^{\text{ix}}}{r^9}$ $+ \frac{P^{\text{x}} \times Q^{\text{x}}}{r^{10}} + \frac{P^{\text{xi}} \times Q^{\text{xi}}}{r^{11}} + \frac{P^{\text{xii}} \times Q^{\text{xii}}}{r^{12}} + \frac{P^{\text{xiii}} \times Q^{\text{xiii}}}{r^{13}}$ $+ \frac{P^{\text{xiv}} \times Q^{\text{xiv}}}{r^{14}}$.

But r is, as before, = 1.03; and conſequently $\frac{1}{r}$ is $= \frac{1}{1.03} = .9708$, and $\frac{1}{r^6}$ is = .8374, and $\frac{1}{r^7} = .8130$, and $\frac{1}{r^8} = .7894$, and $\frac{1}{r^9} = .7664$, and $\frac{1}{r^{10}} = .7440$, and $\frac{1}{r^{11}} = .7224$, and $\frac{1}{r^{12}} = .7013$, and $\frac{1}{r^{13}} = .6809$, and $\frac{1}{r^{14}} = .6611$. And P is = 211, and Q is = 118, and P^{vi} is = 101, $P^{\text{vii}} = 85$, $P^{\text{viii}} = 71$, $P^{\text{ix}} = 59$, $P^{\text{x}} = 48$, $P^{\text{xi}} = 38$, $P^{\text{xii}} = 29$, $P^{\text{xiii}} = 22$, and $P^{\text{xiv}} = 16$; and Q^{vi} is = 38, and $Q^{\text{vii}} = 29$, and $Q^{\text{viii}} = 22$, and $Q^{\text{ix}} = 16$, and $Q^{\text{x}} = 11$, and $Q^{\text{xi}} = 7$, and $Q^{\text{xii}} = 4$, and $Q^{\text{xiii}} = 2$, and $Q^{\text{xiv}} = 1$. Therefore the expreſſion £ $\frac{1}{P \times Q}$ × the ſeries

$\frac{P^{\text{vi}} \times Q^{\text{vi}}}{r^6}$

$$\frac{P^{VI}\times Q^{VI}}{r^{6}} + \frac{P^{VII}\times Q^{VII}}{r^{7}} + \frac{P^{VIII}\times Q^{VIII}}{r^{8}} + \frac{P^{IX}\times Q^{IX}}{r^{9}} + \frac{P^{X}\times Q^{X}}{r^{10}} + \frac{P^{XI}\times Q^{XI}}{r^{11}} + \frac{P^{XII}\times Q^{XII}}{r^{12}} + \frac{P^{XIII}\times Q^{XIII}}{r^{13}} + \frac{P^{XIV}\times Q^{XIV}}{r^{14}}$$

is equal to $\frac{\pounds 1}{211\times 118}$ × the series $101\times 38\times .8374 + 85\times 29\times .8130 + 71\times 22\times .7894 + 59\times 16\times .7664 + 48\times 11\times .7440 + 38\times 7\times .7224 + 29\times 4\times .7013 + 22\times 2\times .6809 + 16\times 1\times .6611 = \frac{\pounds 1}{211\times 118}$ × the series $3838\times .8374 + 2465\times .8130 + 1562\times .7894 + 944\times .7664 + 528\times .7440 + 266\times .7224 + 116\times .7013 + 44\times .6809 + 16\times .6611 = \frac{\pounds 1}{211\times 118}$ × the series $3213.9412 + 2004.0450 + 1233.0428 + 723.4816 + 392.8320 + 192.1584 + 81.3508 + 29.9596 + 10.5776 = \frac{\pounds 1}{211\times 118}\times 7881.3890 = \frac{\pounds 1}{24{,}898}\times 7881.3890 = \pounds 1\times\frac{7881.3890}{24{,}898} = \pounds 1\times .3165 = \pounds 0.3165 =$ *6s. 4d.* Therefore *6s. 4d.* is the value of a remote annuity of one pound *per annum*, that is to commence at the distance of five years, (or whereof the first payment is to be made at the end of six years,) and that is to continue during the joint lives of two persons who, at the time of purchasing it, are of the ages of 75 and 80 years; according to Monsieur de Parcieux's table of the probabilities of life, and when the interest of money is 3 per cent. Q.E.I.

CCCLXXV. The foregoing examples are, I presume, sufficient to illustrate the manner of computing the values of annuities, whether immediate or remote, that are to depend on the joint continuance of two lives of given ages. But, when a whole table of the values of such annuities is to be computed, it is not necessary to make a new calculation, similar to that given above in Art. CCCLXXI, for every different year of human

human life; but the values of these annuities for different pairs of lives whose ages differ from each other by the same number of years, may be deduced one from another by an easy arithmetical process similar to that of Mr. Morgan, above explained in Art. c, ci, cii, pages 109, 110, 111, 112, by which the value of an annuity for a single life of any given age is deduced from that of the like annuity for a life that is one year older. This method, (as well as that explained above in Art. c, ci, cii,) has been given by Mr. Morgan in his learned treatise on the Doctrine of Assurances and Annuities for Lives, page 73, and may be explained as follows.

A short and easy method of deducing from the value of an annuity of one pound a year during the joint continuance of any two given lives, the value of a like annuity for the joint continuance of two other lives that are one year younger than the former lives.

CCCLXXVI. Let the number of years in the age of the younger of the two given lives be called N, and the number of years in the age of the older of the said lives be called $N+a$. Then will the numbers of years in the ages of two lives that are younger than the given lives by a year, be $N-1$ years and $N+a-1$ years.

Let the value of an annuity of one pound a year for the joint continuance of the two given lives of the ages of N years and $N+a$ years be £ V; and the value of the like annuity of one pound a year for the joint continuance of two other lives that are one year younger than the two given lives, and which consequently are of the ages of $N-1$ years and $N+a-1$ years, be £ Y.

Let P be the number of persons who are supposed, in the table of the probabilities of the duration of human life which is adopted as the ground of the calculation, to be living at the age of the younger of the two given lives, that is, at the age of N years, and $P+d$ the number of persons who are therein represented as living at the age which is younger than the former age by one year, that is, at the age of $N-1$ years; and let Q be the number of persons who are therein represented as living at the age of the older of the two given lives, or at the age of $N+a$ years, and $Q+e$ the number of persons represented as living at the age which is younger than the said older age by one year, or at the age of $N+a-1$ years.

 And

And let r be the value of one pound together with its intereſt for one year, according to the rate of intereſt ſuppoſed in the calculation.

Then will $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives that are of the ages of $N-1$ years and $N+a-1$ years, or are reſpectively younger than the two given lives by one year, be equal to $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overset{\pounds}{\overline{1+V}|}$, or to the quantity which ariſes by adding one pound to $\overset{\pounds}{V}$, the value of an annuity of one pound a year for the joint continuance of the two given lives of the ages of N years and $N+a$ years, and then multiplying the ſum $\overline{1+V}|\pounds$, firſt, into the fraction $\frac{1}{r}$, and, ſecondly, into the fraction $\frac{P \times Q}{\overline{P+d} \times \overline{Q+e}}$. This may be demonſtrated in the manner following.

DEMONSTRATION.

CCCLXXVII. It is evident from Art. CCCLXX, that $\overset{\pounds}{V}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the given ages of N years and $N+a$ years, is (if we make uſe of the notation uſed in Art. CCCLXIX, CCCLXX,) equal to the expreſſion $\frac{\overset{\pounds}{1}}{P \times Q}$ × the ſeries $\frac{P' \times Q'}{r} + \frac{P'' \times Q''}{r^2} + \frac{P''' \times Q'''}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} +$ &c. continued to $E-N-a$ terms, and that $\overset{\pounds}{Y}$, or the value of a like annuity of one pound a year for the joint continuance of two lives of the ages of $N-1$ years and $N+a-1$ years, is equal to the expreſſion $\frac{\overset{\pounds}{1}}{\overline{P+d} \times \overline{Q+e}}$ × the ſeries $\frac{P \times Q}{r}$ $\frac{P' \times Q'}{r^2} + \frac{P'' \times Q''}{r^3} + \frac{P''' \times Q'''}{r^4} + \frac{P^{iv} \times Q^{iv}}{r^5} +$ &c. continued to $E-N-a+1$ terms.

Now

Now this laſt expreſſion (which is $= \overset{\pounds}{Y}$,) may be derived from the former (which is $= \overset{\pounds}{V}$,) by adding $\overset{\pounds}{1}$ to it, and then multiplying the ſum, firſt, into the fraction $\frac{1}{r}$, and, ſecondly, into the fraction $\frac{P \times Q}{\overline{P+d} \times \overline{Q+e}}$.

For, if we add $\overset{\pounds}{1}$ to the expreſſion $\frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E-N-a$ terms, it will become equal to $\overset{\pounds}{1} + \frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E-N-a$ terms, $= \frac{\overset{\pounds}{1}}{P \times Q} \times P \times Q + \frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E-N-a$ terms, $= \frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $P \times Q + \frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E-N-a+1$ terms. And this quantity, being multiplied by $\frac{1}{r}$, will become equal to $\frac{\overset{\pounds}{1}}{P \times Q} \times$ the ſeries $\frac{P \times Q}{r} + \frac{P^{I} \times Q^{I}}{r^2} + \frac{P^{II} \times Q^{II}}{r^3} + \frac{P^{III} \times Q^{III}}{r^4} + \frac{P^{IV} \times Q^{IV}}{r^5}$ + &c. continued to $E-N-a+1$ terms; and,

being

being further multiplied by the fraction $\frac{P \times Q}{P+d \times \overline{Q+e}}$, it will become equal to $\frac{\pounds 1}{\overline{P+d} \times \overline{Q+e}}$ × the series $\frac{P \times Q}{r} + \frac{P^{I} \times Q^{I}}{r^2} + \frac{P^{II} \times Q^{II}}{r^3} + \frac{P^{III} \times Q^{III}}{r^4} + \frac{P^{IV} \times Q^{IV}}{r^5}$ + &c. continued to $E - N - a + 1$ terms, that is, to $\pounds Y$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \frac{\pounds}{\overline{1+V}}$ is $= \pounds Y$. Q.E.D.

CCCLXXVIII. I will now proceed to give a few examples of this method of deducing the values of annuities for joint lives from those of annuities for joint lives that are one year older, by which it will become familiar to the reader.

Examples of the foregoing method.

CCCLXXIX. Let us therefore suppose that the interest of money is $3\frac{1}{2}$ per cent. and that the two lives for whose joint continuance an annuity of one pound a year is to be granted are, successively, of the ages of 83 years and 93 years, of 82 years and 92 years, of 81 years and 91 years, of 80 years and 90 years, of 79 years and 89 years, of 78 years and 88 years, of 77 years and 87 years, of 76 years and 86 years, of 75 years and 85 years, of 74 years and 84 years, of 73 years and 83 years, of 72 years and 82 years, of 71 years and 81 years, and of 70 years and 80 years; in all which pairs of lives the difference of the two ages is constantly 10 years. And let the probabilities of the duration of human life be supposed to be such as they are represented to be in Monsieur de Parcieux's table of them.

CCCLXXX. Then, in the first place, we must compute the value of an annuity of one pound a year for the joint continuance of the two oldest lives, to wit, those of the ages of 83 years and 93 years, by means of the expression $\frac{\pounds 1}{P \times Q}$ × the series $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4}$ + &c. continued to $E - N - a$ terms, which is given for that purpose in Art. CCCLXX.

Here

Here r is $= 1.035$; and E is 94 years; N, or the number of years in the age of the younger of the two lives, is 83; a, or the difference of the ages of the two lives, is 10 years; $N+a$, or the number of years in the age of the older of the two lives, is 93; $E-N-a$ is $94-93$ years, or 1 year; P, or the number of persons living at the age of the younger life, or the age of 83 years, is 71; P', or the number of persons living at the age of 84 years, is 59; Q, or the number of persons living at the age of the older of the two lives, or at the age of 93 years, is 2; and Q', or the number of persons living at the age of 94 years, is 1; and Q'', Q''', Q^{iv}, &c. or the numbers of persons living at the ages of 95 years, 96 years, 97 years, &c. are equal to 0. Therefore all the terms of the series $\frac{P' \times Q'}{r} + \frac{P'' \times Q''}{r^2} + \frac{P''' \times Q'''}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4}$ + &c. after the first term $\frac{P' \times Q'}{r}$, are equal to 0; and consequently the expression $\frac{\pounds 1}{P \times Q}$ × the series $\frac{P' \times Q'}{r} + \frac{P'' \times Q''}{r^2} + \frac{P''' \times Q'''}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4}$ + &c. is $= \frac{\pounds 1}{P \times Q} \times \frac{P' \times Q'}{r}$

$$= \frac{\pounds 1}{71 \times 2} \times \frac{59 \times 1}{1.035} = \frac{\pounds 1}{142} \times \frac{59}{1.035} = \pounds 1 \times \frac{59}{142 \times 1.035} = \pounds 1 \times \frac{59}{146.970} = \pounds 1 \times .401,442 = \pounds 0.401,442.$$

Therefore, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 83 years and 93 years, when the interest of money is $3\frac{1}{2}$ per cent. is £0.401,442, or 8*s.* $\frac{1}{4}$*d.* Q E I.

CCCLXXXI. Having thus found the value of an annuity of one pound a year for the joint continuance of the oldest pair of lives by means of the expression $\frac{\pounds 1}{P \times Q}$ × the series $\frac{P' \times Q'}{r} + \frac{P'' \times Q''}{r^2} + \frac{P''' \times Q'''}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4}$ + &c. given for that purpose in

Art.

Art. CCCLXX, we may now proce[ed] to find the values of a like annuity for the joint continuance of all the younger pairs of lives above-mentioned, by means of the expression $\frac{1}{r} \times \frac{PQ}{\overline{P+d}| \times \overline{Q+e}|} \times \overline{1+V}|$ £. This may be done in the manner following.

CCCLXXXII. To find the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 82 years and 92 years, we must proceed as follows.

The value of an annuity of one pound a year for the joint continuance of two lives of the ages of 83 years and 93 years has been found to be £0.401,442. Therefore $\overset{£}{V}$ is = £0.401,442. The numbers of persons living at the ages of 83 years and 82 years are 71 and 85; and the numbers of persons living at the ages of 93 years and 92 years are 2 and 4. Therefore P is = 71, and $P+d$ is = 85, and Q is = 2, and $Q+e$ is = 4. And r is = 1.035. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{Q+e}|} \times \overline{1+V}|$ £ is $= \frac{1}{1.035} \times \frac{71 \times 2}{85 \times 4} \times$ £1.401,442 $= \frac{1}{1.035} \times \frac{71}{85 \times 2} \times$ £1.401,442 $= \frac{1}{1.035} \times \frac{71}{170} \times$ £1.401,442 $= \frac{1}{1.035 \times 170} \times 71 \times$ £1.401,442 $= \frac{1}{175.950} \times$ £99.502,382 $= \frac{99.502,382}{175.950} =$ £0.565,515. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 82 years and 92 years, is = £0.565,515, or 11*s.* 3*d.* ¾. Q.E.I.

CCCLXXXIII. When the two lives are of the ages of 81 years and 91 years, we shall have $\overset{£}{V}$ = £0.565,515, and consequently $\overset{£}{1+V}$ = £1.565,515. And P will be = 85, and $P+d$ = 101, and Q = 4, and $Q+e$ = 7. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{Q+e}|} \times \overline{1+V}|$ £ will

be

be $= \frac{1}{1.035} \times \frac{85 \times 4}{101 \times 7} \times £1.565,515 = \frac{1}{1.035} \times \frac{340}{707} \times £1.565,515 = \frac{340 \times \overset{£}{1.565,515}}{1.035 \times 707} = \frac{340 \times \overset{£}{1.565,515}}{731.745} = \frac{\overset{£}{532.275,100}}{731.745}$ $= £0.727,405$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 81 years and 91 years, is $= £0.727,405$, or 14*s.* 6*d.*$\frac{1}{2}$ Q E I.

CCCLXXXIV. When the two lives are of the ages of 80 years and 90 years, we shall have $\overset{£}{V} = \overset{£}{0.727,405}$, and consequently $\overset{£}{1} + \overset{£}{V}$ $= £1.727,405$. And P will be $= 101$, $P + d = 118$, $Q = 7$, and $Q + e = 11$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V}\,£$ will be $= \frac{1}{1.035} \times \frac{101 \times 7}{118 \times 11} \times £1.727,405 = \frac{707}{1.035 \times 118 \times 11}$ $\times £1.727,405 = \frac{\overset{£}{1221.275,335}}{1.035 \times 118 \times 11} = \frac{\overset{£}{1221.275,335}}{1.035 \times 1298} = \frac{\overset{£}{1221.275,335}}{1343.430}$ $= \overset{£}{0.909,072}$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 80 years and 90 years, is $= £0.909,072$, or 18*s.* 2*d.*$\frac{1}{4}$. Q E I.

CCCLXXXV. When the two lives are of the ages of 79 years and 89 years, we shall have $\overset{£}{V} = \overset{£}{0.909,072}$, and consequently $\overset{£}{1} + \overset{£}{V}$ $= £1.909,072$. And P will be $= 118$, and $P + d = 136$, and $Q = 11$, and $Q + e = 16$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V}\,£$

will

will be $= \frac{1}{1.035} \times \frac{118 \times 11}{136 \times 16} \times \pounds 1.909{,}072 = \frac{1298 \times \overset{\pounds}{1}.909{,}072}{1.035 \times 136 \times 16}$

$= \frac{1298 \times \overset{\pounds}{1}.909{,}072}{1.035 \times 2176} = \frac{2477.975{,}456}{2252.160} = \pounds 1.100{,}266.$ Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 79 years and 89 years, is £1.100,266, or 1*l.* 2*s.* 0*d.* Q E I.

CCCLXXXVI. When the two lives are of the ages of 78 years and 88 years, we ſhall have $\overset{\pounds}{V} = \overset{\pounds}{1}.100{,}266$, and conſequently $\overset{\pounds}{1} + \overset{\pounds}{V} = \pounds 2.100{,}266$. And P will be $= 136$, and $P + d = 154$, and $Q = 16$, and $Q + e = 22$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V} \, \pounds$ will be $= \frac{1}{1.035} \times \frac{136 \times 16}{154 \times 22} \times \pounds 2.100{,}266 = \frac{1}{1.035} \times \frac{2176}{3388} \times \pounds 2.100{,}266 = \frac{\overset{\pounds}{4570}.178{,}816}{3506.580} = \pounds 1.303{,}315.$ Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 78 years and 88 years, is £1.303.315, or 1*l.* 6*s.* 0*d.* $\frac{3}{4}$. Q E I.

CCCLXXXVII. When the two lives are of the ages of 77 and 87 years, we ſhall have $\overset{\pounds}{V} = \overset{\pounds}{1}.303{,}315$, and conſequently $\overset{\pounds}{1} + \overset{\pounds}{V} = \pounds 2.303{,}315$. And P will be $= 154$, and $P + d = 173$, and $Q = 22$, and $Q + e = 29$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V} \, \pounds$ will be $= \frac{1}{1.035} \times \frac{154 \times 22}{173 \times 29} \times \pounds 2.303{,}315, = \frac{1}{1.035} \times \frac{3388}{173 \times 29} \times \pounds 2.303{,}315, = \frac{1}{1.035} \times \frac{3388}{5017} \times \pounds 2.303{,}315, = \frac{3388}{5192.595} \times$

£2 303,315

$\pounds 2.303,315 = \frac{\overset{\pounds}{7803.631,220}}{5192.595} = \pounds 1.502,838$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 77 years and 87 years, is £1.502,838, or 1*l.* 10*s.* ½*d.* QEI.

CCCLXXXVIII. When the two lives are of the ages of 76 and 86 years, we ſhall have $\overset{\pounds}{V} = \overset{\pounds}{1}.502,838$, and conſequently $\overset{\pounds}{1} + \overset{\pounds}{V} = \pounds 2.502,838$. And P will be $= 173$, $P + d = 192$, and $\mathcal{Q} = 29$, and $\mathcal{Q} + e = 38$. Therefore $\frac{1}{r} \times \frac{P \times \mathcal{Q}}{\overline{P+d}| \times |\overline{\mathcal{Q}+e}} \times \overline{1+V}|\pounds$ will be $= \frac{1}{1.035} \times \frac{173 \times 29}{192 \times 38} \times \pounds 2.502,838 = \frac{1}{1.035} \times \frac{5017}{7296} \times \pounds 2.502,838 = \frac{\overset{\pounds}{12556.738,246}}{7551.360} = \pounds 1.662,844$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 76 years and 86 years, is £1.662,844, or 1*l.* 13*s.* 3*d.* QEI.

CCCLXXXIX. When the two lives are of the ages of 75 years and 85 years, we ſhall have $\overset{\pounds}{V} = \overset{\pounds}{1}.662,844$, and conſequently $\overline{1+V}|\pounds = \pounds 2.662,844$. And P will be $= 192$, and $P + d = 211$, and $\mathcal{Q} = 38$, and $\mathcal{Q} + e = 48$. Therefore $\frac{1}{r} \times \frac{P \times \mathcal{Q}|}{\overline{P+d}| \times |\overline{\mathcal{Q}+e}} \times \overline{1+V}|\pounds$ will be $= \frac{1}{1.035} \times \frac{192 \times 38}{211 \times 48} \times \pounds 2.662,844 = \frac{1}{1.035} \times \frac{7296}{211 \times 48} \times \pounds 2.662,844 = \frac{1}{1.035} \times \frac{7296}{10128} \times \pounds 2.662,844 = \frac{7296 \times \overset{\pounds}{2}.662,844}{10482.480} = \frac{\overset{\pounds}{19,428.109,824}}{10482.480} = \pounds 1.853,388$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 75 years and 85 years, is £1.853,388, or 1*l.* 17*s.* 0*d.* ¾. QEI.

CCCXC. When the two lives are of the ages of 74 years and 84 years, we shall have $\overset{\pounds}{V} = \overset{\pounds}{1.853{,}388}$, and consequently $\overline{1+V}|\,\pounds = \pounds 2.853{,}388$. And P will be $= 211$, and $P+d = 231$, and $Q = 48$, and $Q+e = 59$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{|Q+e}} \times \overline{1+V}|\,\pounds$ will be $= \frac{1}{1.035} \times \frac{211 \times 48}{231 \times 59} \times \pounds 2.853{,}388 = \frac{1}{1.035} \times \frac{10128}{13629} \times \pounds 2.853{,}388 = \frac{\overset{\pounds}{28{,}899.113{,}664}}{14{,}106.015} = \pounds 2.048{,}708$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 74 years and 84 years, is £2.048,708, or 2*l.* 0*s.* 11*d.* $\frac{3}{4}$. Q E I.

CCCXCI. When the two lives are of the ages of 73 years and 83 years, we shall have $\overset{\pounds}{V} = \overset{\pounds}{2.048{,}708}$, and consequently $\overline{1+V}|\,\pounds = \pounds 3.048{,}708$. And P will be $= 231$, and $P+d = 251$, and $Q = 59$, and $Q+e = 71$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{|Q+e}} \times \overline{1+V}|\,\pounds$ will be $= \frac{1}{1.035} \times \frac{231 \times 59}{251 \times 71} \times \pounds 3.048{,}708 = \frac{1}{1.035} \times \frac{13629}{17821} \times \pounds 3.048{,}708 = \frac{\overset{\pounds}{41{,}550.841{,}332}}{18{,}444.735} = \pounds 2.252{,}720$. Therefore $\overset{\pounds}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 73 years and 83 years, is £2.252,720, or 2*l.* 5*s.* $\frac{1}{2}$*d.* Q E I.

CCCXCII. When the two lives are of the ages of 72 years and 82 years, we shall have $\overset{\pounds}{V} = \overset{\pounds}{2.252{,}720}$, and consequently $\overline{1+V}|\,\pounds = \pounds 3.252{,}720$. And P will be $= 251$, and $P+d = 271$, and $Q = 71$, and $Q+e = 85$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{|Q+e}} \times \overline{1+V}|\,\pounds$ will

be

be $= \frac{1}{1.035} \times \frac{251 \times 71}{271 \times 85} \times £3.252,720 = \frac{1}{1.035} \times \frac{17821}{23035} \times$ $£3.252,720 = \frac{\overset{£}{57,966.723,120}}{23,841.225} = £2.431,365$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 72 years and 82 years, is $= £2.431,365$, or 2*l.* 8*s.* 7*d.* $\frac{1}{2}$ Q E I.

CCCXCIII. When the two lives are of the ages of 71 years and 81 years, we shall have $\overset{£}{V} = \overset{£}{2.431,365}$, and consequently $\overline{1+V}|£$ $= £3.431,365$. And P will be $= 271$, and $P+d = 291$, and $Q = 85$, and $Q+e = 101$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{|Q+e}} \times \overline{1+V}|£$ will be $= \frac{1}{1.035} \times \frac{271 \times 85}{291 \times 101} \times £3.431,365 = \frac{1}{1.035} \times \frac{23035}{29391}$ $\times £3.431,365 = \frac{\overset{£}{79,041.492,775}}{30,419.685} = £2.598,366$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 71 years and 81 years, is $= £2.598,366$, or 2*l.* 11*s.* 11*d.* $\frac{1}{2}$. Q E I.

CCCXCIV. When the two lives are of the ages of 70 years and 80 years, we shall have $\overset{£}{V} = \overset{£}{2.598,366}$, and consequently $\overline{1+V}|£$ $= £3.598,366$. And P will be $= 291$, and $P+d = 310$, and $Q = 101$, and $Q+e = 118$. Therefore $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{|Q+e}} \times \overline{1+V}|£$ will be $= \frac{1}{1.035} \times \frac{291 \times 101}{310 \times 118} \times £3.598,366 = \frac{1}{1.035} \times \frac{29,391}{36,580}$ $\times £3.598,366 = \frac{\overset{£}{105,759.605,106}}{37,860.300} = £2.793,416$. Therefore $\overset{£}{Y}$, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 80 years, is $= £2.793,416$, or 2*l.* 15*s.* 10*d.* $\frac{1}{2}$. Q E I.

End of the examples of the foregoing method.

 CCCXCV. The

CCCXCV. The foregoing examples are, I presume, sufficient to illustrate, and render familiar to the reader, the method given in Art. CCCLXXVI of deriving the values of annuities for two joint lives from those of equal annuities for two joint lives one year older than the former by means of the expression $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V}|\pounds$.

A remark on the foregoing method of computing a table of the values of annuities for two joint lives.

And this is undoubtedly the best method that can be taken for the purpose of computing a table of these annuities. Yet it is liable to the inconvenience mentioned above in Art. CXCIV, page 208, as belonging to the other method above explained in Art. C, CI, CII, by which the values of annuities for single lives are successively deduced from the values of annuities for the next older lives by means of the expression $\frac{1}{r} \times \frac{P}{P+d} \times \overline{1+V}|\pounds$, to wit, that, as the values of these annuities are deduced by it one from another in regular succession from the older ages to the younger, any error that should happen to be made in computing the value of any one of them would affect the values of all the following life-annuities, which would belong to younger ages than those in which the error arose. But this inconvenience Mr. Morgan has enabled us to remove by giving us a method of examining and proving the truth of our calculations, as fast as we make them, which is similar to the method given by him for the like purpose in the case of annuities for single lives, which has been stated and explained above in Art. CXCV, CXCVI, CXCVII, and CXCVIII. This method is as follows.

Of Mr. Morgan's method of proving the truth of the computations of the values of annuities for two joint lives, that are made by means of the foregoing expression, $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d} \times \overline{Q+e}} \times \overline{1+V}|\pounds$, *as fast as they are made.*

CCCXCVI. Let N be (as before, in Art. CCCLXXVI,) the number of years in the age of the younger of any two given lives, upon the joint continuance of which an annuity of one pound *per annum* is to depend; and $N+a$ be the number of years in the older of the said lives. And let P, P^{I}, P^{II}, P^{III}, P^{IV}, P^{V}, P^{VI}, &c be the numbers of persons represented in the table of the probabilities of life that is adopted as the ground of the calculation, as living at the ages of N years, $N+1$ years,

years, $N+2$ years, $N+3$ years, $N+4$ years, $N+5$ years, $N+6$ years, &c. respectively; and Q, Q^{I}, Q^{II}, Q^{III}, Q^{IV}, Q^{V}, Q^{VI}, &c. be the numbers of persons represented therein as living at the ages of $N+a$ years, $N+a+1$ years, $N+a+2$ years, $N+a+3$ years, $N+a+4$ years, $N+a+5$ years, $N+a+6$ years, &c. respectively. Also let A be the number of persons represented in the said table as living at the youngest age in the said table; (which in Monsieur de Parcieux's table of these probabilities is the age of 3 years:) and let K be the number of persons represented therein as living at the age which is greater than the said youngest age by a years. And let the number of years by which the age of N years, or of the younger of the two given lives, exceeds the youngest age in the table, be m years. Then will the age of $N-m$ years be the youngest age in the table, at which the number of persons represented in the table as living is A; and $N-m+a$ will be the number of years in the age which exceeds the said youngest age by a years, and at which the number of persons represented in the table as living is K.

CCCXCVII. These things being premised, it is evident, in the first place, That the present value of a single future payment of one pound, to be received at the end of m years in case two persons, who are now of the ages of $N-m$ years (or the youngest age in the table,) and $N-m+a$ years, shall both be then alive, is £ $\frac{1}{r^m} \times \frac{P \times Q}{A \times K}$. This is evident from Problem III, Art. XLV, pages 44, 45, 46.

Preliminary propositions necessary to the demonstration of the principal proposition on which Mr. Morgan's method is founded.

In the second place, it follows from Art. CCCLXXIII, that the value of a remote annuity of one pound a year, that is to commence at the distance of m years, (or whereof the first payment is to be received at the end of $m+1$ years,) and that is to continue during the joint lives of two persons of the ages of $N-m$ years (or the youngest age in the table,) and $N-m+a$ years, will be £ $\frac{1}{A \times K}$ × the series $\frac{P^{\text{I}} \times Q^{\text{I}}}{r^{m+1}} + \frac{P^{\text{II}} \times Q^{\text{II}}}{r^{m+2}} + \frac{P^{\text{III}} \times Q^{\text{III}}}{r^{m+3}} + \frac{P^{\text{IV}} \times Q^{\text{IV}}}{r^{m+4}} + \frac{P^{\text{V}} \times Q^{\text{V}}}{r^{m+5}} +$ &c. continued to the end of the table, and consequently will be = £1 the series $\frac{P^{\text{I}} \times Q^{\text{I}}}{A \times K \times r^{m+1}} + \frac{P^{\text{II}} \times Q^{\text{II}}}{A \times K \times r^{m+2}} + \frac{P^{\text{III}} \times Q^{\text{III}}}{A \times K \times r^{m+3}}$

$$+ \frac{P^{\text{IV}} \times Q^{\text{IV}}}{A \times K \times r^{m+4}}$$

$+ \frac{P^{iv} \times Q^{iv}}{A \times K \times r^{m+4}} + \frac{P^{v} \times Q^{v}}{A \times K \times r^{m+5}} +$ &c. continued to the end of the table. Let the value of this remote annuity be called $\overset{£}{R}$.

And, in the third place, it is evident that the value of an immediate life-annuity of one pound a year (or one of which the firſt payment is to be received at the end of a year,) that is to continue during the joint lives of two perſons of the ages of N years and $N+a$ years, is $= \frac{\overset{£}{1}}{P \times Q} \times$ the ſeries $\frac{P^{i} \times Q^{i}}{r} + \frac{P^{ii} \times Q^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} + \frac{P^{v} \times Q^{v}}{r^5} +$ &c. continued to the end of the table. Let this value be (as before,) called $\overset{£}{V}$.

The ſaid princi-pal propoſition itſelf.

CCCXCVIII. Now, if the laſt of theſe three quantities (which is $= \overset{£}{V}$) be multiplied into the firſt of them, to wit, $\frac{\overset{£}{1}}{r^m} \times \frac{P \times Q}{A \times K}$, or the preſent value of a future ſingle payment of one pound, to be received at the end of m years in caſe of the joint continuance of two lives of the ages of $N-m$ years and $N-m+a$ years, the product thence ariſing will be equal to the ſecond of them, (to wit, £1 $\times$ the ſeries $\frac{P^{i} \times Q^{i}}{A \times K \times r^{m+1}} + \frac{P^{ii} \times Q^{ii}}{A \times K \times r^{m+2}} + \frac{P^{iii} \times Q^{iii}}{A \times K \times r^{m+3}} + \frac{P^{iv} \times Q^{iv}}{A \times K \times r^{m+4}} + \frac{P^{v} \times Q^{v}}{A \times K \times r^{m+5}} +$ &c. continued to the end of the table,) or to $\overset{£}{R}$, or to the value of a remote annuity of one pound a year, that is to commence at the diſtance of m years, or ſo that the firſt payment of it ſhall be received at the end of $m+1$ years, and that ſhall continue during the joint lives of two perſons of the ages of $N-m$ years (or the youngeſt age in the table,) and $N-m+a$ years.

DEMONSTRATION.

For $\frac{\overset{£}{1}}{r^m} \times \frac{P \times Q}{A \times K} \times \frac{\overset{£}{1}}{P \times Q} \times$ the ſeries $\frac{P^{i} \times Q^{i}}{r} + \frac{P^{ii} \times Q^{ii}}{r^2} + \frac{P^{iii} \times Q^{iii}}{r^3} + \frac{P^{iv} \times Q^{iv}}{r^4} + \frac{P^{v} \times Q^{v}}{r^5} +$ &c. continued to

the

the end of the table, is = $\overset{£}{\frac{1}{r^m}} \times \frac{1}{A \times K} \times$ the ſame ſeries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5}$ + &c. continued to the end of the table, = £1 × the ſeries $\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}} + \frac{P^{V} \times Q^{V}}{A \times K \times r^{m+5}}$ + &c. continued to the end of the table, = $\overset{£}{R}$. QED.

Mr. Morgan's method itſelf, derived from the foregoing propoſition.

CCCXCIX. If, therefore, in computing a table of the ſeveral values of an annuity of one pound a year for the joint continuance of ſeveral ſucceſſive pairs of lives whoſe ages differ from each other by the ſame number of years, or a years, we at the ſame time compute the correſponding values of the expreſſion $\overset{£}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$, and of the expreſſion £1 × the ſeries $\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}} + \frac{P^{V} \times Q^{V}}{A \times K \times r^{m+5}}$ + &c. continued to the end of the table, or of $\overset{£}{R}$, and then multiply the expreſſion $\overset{£}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ into $\overset{£}{V}$, or the value of the annuity for two lives which we have before computed, and it ſhall appear that the product thereby obtained is equal to $\overset{£}{R}$, or to £1 × the ſeries $\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}} + \frac{P^{V} \times Q^{V}}{A \times K \times r^{m+5}}$ + &c. continued to the end of the table, we may ſafely conclude that the value of $\overset{£}{V}$ has been accurately computed.

CCCC. To

An explanation of the manner of applying the foregoing rule, or method, of Mr. Morgan to the proof of the truth of the computations of a table of the values of annuities for two joint lives.

CCCC. To make the manner of applying this rule of Mr. Morgan more apparent, it will be proper to ſet down again in regular order, in a new table, the values of the life-annuities for the joint continuance of two given lives, which we have computed above in Art. CCCLXXX, CCCLXXXI, CCCLXXXII, &c.—CCCXCIV, from Monſieur de Parcieux's table by means of the expreſſion $\frac{1}{1.035} \times \frac{P \times Q}{\overline{P+d}| \times |\overline{Q+e}} \times \overline{1+V}|$ £. And, as, in obtaining the ſaid values one from another by means of that expreſſion, we proceeded upwards, or from the older lives to the younger, it will be convenient to ſet down the ſaid values in the ſame order in the ſaid new table in a column adjoining to two other columns that contain the numbers of years in the two ages correſponding to the ſaid values. After theſe three columns, (containing the years in the ages of the two given lives, and the values of the correſponding annuities,) I ſhall ſet down, in a fourth column, the ſeveral ſucceſſive values of the expreſſion £ $\frac{1}{r^m} \times \frac{P \times Q}{A \times K}$, or the preſent values of a ſingle payment of one pound, to be received at the ends of 81 years, 80 years, 79 years, 78 years, and every following leſſer number of years down to 70 years, in caſe two perſons of the ages of 3 years (which is the youngeſt age in the table,) and 13 years ſhall be then living. And then, in a fifth column, I ſhall ſet down the ſums of the terms in the foregoing ſeries of values contained in the fourth column, as they ariſe; ſo that every term in this fifth column that is even with any two given ages in the firſt and ſecond columns, ſhall be equal to the ſum of all the terms in the fourth column that correſpond to the ages that are older than the ſaid two given ages. Thus, for example, the term in the fifth column that is even with the ages of 75 years and 85 years in the firſt and ſecond columns, is equal to the ſum of all the terms in the fourth column that correſpond to the ages that are older than the ages of 75 and 85 years. Theſe ſums, contained in this fifth column, will be equal to the values of remote annuities of one pound a year for the joint continuance of the lives of two perſons of the ages of 3 years and 13 years, that are to commence at the diſtances of 80 years, 79 years, 78 years, 77 years, &c. down to 70 years, or ſo that the firſt payments of them ſhall become due at the ends of 81 years, 80 years, 80 years, 79 years, 78 years, &c. down to 71 years; and they will comprize all the different values of the quantity which in Art. CCCXCVII is called $\overset{£}{R}$, that relate to remote periods that are greater than 70 years. For they are the sums of the successive values of the expression $\frac{\overset{£}{1}}{r^m} \times \frac{P \times Q}{A \times K}$, which, it is evident, are the same with the terms of the series

$$\frac{P^{I} \times Q^{I}}{A \times K \times r^{m+1}} + \frac{P^{II} \times Q^{II}}{A \times K \times r^{m+2}} + \frac{P^{III} \times Q^{III}}{A \times K \times r^{m+3}} + \frac{P^{IV} \times Q^{IV}}{A \times K \times r^{m+4}} + \frac{P^{V} \times Q^{V}}{A \times K \times r^{m+5}} + \text{\&c.}$$

continued to the end of the table, which series is $= \overset{£}{R}$.

And, lastly, in a sixth column, I shall set down the products that arise by multiplying the terms of the third column, or the values of the life-annuities for the joint continuance of two lives of the several ages set down in the first and second columns, by the corresponding terms, or terms that are placed even with them, in the fourth column, or by the values of single future payments of one pound, depending on the joint continuance of two lives of the ages of 3 years and 13 years.

And, when we have thus obtained the numbers that are to be placed in these several columns, we must compare those in the sixth column with those that stand even with them in the fifth column: and so far as we find them to be equal to the said numbers in the fifth column, we may conclude that the numbers in the third column, or the values of an annuity of one pound for two joint lives of the ages set down in the first and second columns, have been rightly computed.

Of the computation of the successive values of the expression $\frac{\overset{£}{1}}{r^m} \times \frac{P \times Q}{A \times K}$.

CCCCI. The only difficulty that can occur in forming a table of this kind is in the computation of the successive values of the expression $\frac{\overset{£}{1}}{r^m} \times \frac{P \times Q}{A \times K}$. But this will be found to be a work of no great labour, and may be performed in the manner following.

 CCCCII. A is

CCCCII. A is the number of persons represented in Monsieur de Parcieux's table of probabilities as living at the age of 3 years, which is 1000; and K is the number of persons therein represented as living at the age of 13 years, which is 860. Therefore $A \times K$ is $= 1000 \times 860 = 860{,}000$. And r is $= 1.035$; and therefore (by Mr. Smart's second table of compound interest, page 60 *et seq.*) $\frac{1}{r}$ is $= .966{,}183{,}57$;

and $\frac{1}{r^{81}}$ is $= .061{,}635{,}61$; $\frac{1}{r^{80}}$ is $= .063{,}792{,}85$;

$\frac{1}{r^{79}}$ is $= .066{,}025{,}60$; $\frac{1}{r^{78}}$ is $= .068{,}336{,}50$;

$\frac{1}{r^{77}}$ is $= .070{,}728{,}27$; $\frac{1}{r^{76}}$ is $= .073{,}203{,}76$;

$\frac{1}{r^{75}}$ is $= .075{,}765{,}90$; $\frac{1}{r^{74}}$ is $= .078{,}417{,}70$;

$\frac{1}{r^{73}}$ is $= .081{,}162{,}32$; $\frac{1}{r^{72}}$ is $= .084{,}003{,}00$;

$\frac{1}{r^{71}}$ is $= .086{,}943{,}11$; $\frac{1}{r^{70}}$ is $= .089{,}986{,}12$;

$\frac{1}{r^{69}}$ is $= .093{,}135{,}63$; $\frac{1}{r^{68}}$ is $= .096{,}395{,}38$;

and $\frac{1}{r^{67}}$ is $= .099{,}769{,}22$. And the values of P and Q when m is $= 81$, or the numbers of persons represented in Monsieur de Parcieux's table of probabilities as living at the ages of $(81 + 3$ years, or) 84 years and $(81 + 13$ years, or) 94 years, are 59 and 1; and the following values of P and Q at the following lesser ages of 83 years and 93 years, 82 years and 92 years, 81 years and 91 years, 80 years and 90 years, 79 years and 80 years, 78 years and 88 years, 77 years and 87 years, 76 years and 86 years, 75 years and 85 years, 74 years and 84 years, 73 years and 83 years, 72 years and 82 years, 71 years and 81 years, and 70 years and 80 years, are 71 and 2, 85 and 4, 101 and 7, 118 and 11, 136 and 16, 154 and 22, 173 and 29, 192 and 38, 211 and 48, 231 and 59, 251 and 71, 271 and 85, 291 and 101, and, lastly, 310 and 118.

CCCCIII. Therefore the first value of the expression $\frac{\overset{\pounds}{1}}{r^m} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 81$, is $= \pounds 0.061,635,61 \times \frac{59 \times 1}{860,000}$ $= \frac{59 \times \overset{\pounds}{0.061,635,61}}{860,000} = \frac{3.636,400}{860,000} = \pounds 0.000,004,22$.

The second value of the expression $\frac{\overset{\pounds}{1}}{r^m} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 80$, is $= \pounds 0.063,792,85 \times \frac{71 \times 2}{860,000} =$ $\frac{142 \times \overset{\pounds}{0.063,792,85}}{860,000} = \frac{\overset{\pounds}{9.058,584}}{860,000} = \pounds 0.000,010,53$.

The third value of the expression $\frac{\overset{\pounds}{1}}{r^m} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 79$, is $= \pounds 0.066,025,60 \times \frac{85 \times 4}{860,000} =$ $\frac{340 \times \overset{\pounds}{0.066,025,60}}{860,000} = \frac{34 \times \overset{\pounds}{0.066,025,60}}{86,000} = \frac{\overset{\pounds}{2.244,870}}{86,000} = \pounds 0.000,026,10$.

The fourth value of the expression $\frac{\overset{\pounds}{1}}{r^m} \times \frac{P \times Q}{A \times K}$, or that which it has when m is $= 78$, is $= \pounds 0.068,336,50 \times \frac{101 \times 7}{860,000} =$ $\frac{707 \times \overset{\pounds}{0.068,336,50}}{860,000} = \frac{\overset{\pounds}{48.313,905}}{860,000} = \pounds 0.000,056,17$.

 The

The fifth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.070,728,27 \times$

$$\frac{118 \times 11}{860,000} = \frac{1298 \times \overset{\pounds}{0.070,728,27}}{860,000} = \frac{\overset{\pounds}{91.805,094}}{860,000} = \pounds 0.000,106,75.$$

The sixth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.073,203,76 \times$

$$\frac{136 \times 16}{860,000} = \frac{2176 \times \overset{\pounds}{0.073,203,76}}{860,000} = \frac{\overset{\pounds}{159.291,381}}{860,000} \pounds 0.000,185,22.$$

The seventh value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.075,765,90 \times$

$$\frac{154 \times 22}{860,000} = \frac{3388 \times \overset{\pounds}{0.075,765,90}}{860,000} = \frac{\overset{\pounds}{256.694,869}}{860,000} = \pounds 0.000,298,48.$$

The eighth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.078,417,70 \times$

$$\frac{173 \times 29}{860,000} = \frac{5017 \times \overset{\pounds}{0.078,417,70}}{860,000} = \frac{\overset{\pounds}{393.421,600}}{860,000} = \pounds 0.000,457,46.$$

The ninth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.081,$[illegible]$,32 \times$.

$$\frac{192 \times 38}{860,000} = \frac{7296 \times \overset{\pounds}{0.081,162,32}}{860,000} = \frac{\overset{\pounds}{592.160,286}}{860,000} = \pounds 0.000,688,55.$$

The

The tenth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.084,003,05 \times$

$$\frac{211 \times 48}{860,000} = \frac{10,128 \times \overset{\pounds}{0.084,003}}{860,000} = \frac{\overset{\pounds}{850\ 782,384}}{860,000} = \pounds 0.000.989,28.$$

The eleventh value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.086,943,11 \times$

$$\frac{231 \times 59}{860,000} = \frac{13629 \times \overset{\pounds}{0.086,943,11}}{860,000} = \frac{\overset{\pounds}{1184.947,646}}{860,000} = \pounds 0.001,377,84.$$

The twelfth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.089,986,12 \times$

$$\frac{251 \times 71}{860,000} = \frac{17821 \times \overset{\pounds}{0.089,986,12}}{860,000} = \frac{\overset{\pounds}{1603.642,644}}{860,000} = \pounds 0.001,864,70.$$

The thirteenth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.093,135,63 \times$

$$\frac{271 \times 85}{860,000} = \frac{23035 \times \overset{\pounds}{0.093,135,63}}{860,000} = \frac{\overset{\pounds}{2145.379,237}}{860,000} = \pounds 0.002,494,62.$$

The fourteenth value of $\overset{\pounds}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.096,395,38 \times$

$$\frac{291 \times 101}{860,000} = \frac{29391 \times \overset{\pounds}{0.096,395;38}}{860,000} = \frac{\overset{\pounds}{2833.156,513}}{860,000} = \pounds 0.003,294,36.$$

And

And the fifteenth value of $\frac{\pounds 1}{r^m} \times \frac{P \times Q}{A \times K}$ is $= \pounds 0.099,769,22 \times$

$$\frac{310 \times 118}{860,000} = \frac{36580 \times \pounds 0.099,769,22}{860,000} = \frac{\pounds 3649.558,067}{860,000} = \pounds 0.004,243,67.$$

CCCCIV. These several successive values of the expression $\frac{\pounds 1}{r^m} \times \frac{P \times Q}{A \times K}$, if ranged in order, will be as follows.

£
0.000,004,22,
0.000,010,53,
0.000,026,10,
0.000,056,17,
0.000,106,75,
0.000,185,22,
0.000,298,48,
0.000,457,46,
0.000,688,55,
0.000,989,28,
0.001,377,84,
0.001,864,70,
0.002,494,62,
0.003,294,36,
0.004,243,67.

These numbers therefore will constitute the fourth column of the ensuing table.

CCCCV. The

CCCCV. The ſums that ariſe by the continual addition of theſe numbers will be as follows.

The ſums of the ſaid ſucceſſive values of the expreſſion £ $\frac{1}{r^m} \times \frac{P \times Q}{A \times K}$.

£
0.000,004,22,
0.000,014,75,
0.000,040,85,
0.000,097,02,
0.000,203,77,
0.000,388,99,
0.000,687,47,
0.001,144,93,
0.001,833,48,
0.002,822,76,
0.004,200,60,
0.006,065,30,
0.008,559,92,
0.011,854,28,
0.016,097,95.

Theſe numbers therefore will conſtitute the fifth column of the enſuing table.

CCCCVI. This

CCCCVI. This table is as follows.

T A B L E XXVIII.

Consisting of six columns of numbers; in the first of which the numbers of years in the several ages of human life, that differ from each other by a year, from the age of 84 *years to the age of* 70 *years, inclusively, are set down in regular order; and in the second column are set down the numbers of years in the several ages of human life, that differ from each other by a year, from the age of* 94 *years to the age of* 80 *years, inclusively; and in the third column are set down the several values of an annuity of one pound a year for the joint continuance of the lives of two persons of the ages set down in the first and second columns even with the said values; computed from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is* $3\frac{1}{2}$ *per cent. and in the fourth column are set down the present values of a single payment of one pound, to be received at the ends of* 81 *years,* 80 *years,* 79 *years,* 78 *years, and every following lesser number of years down to* 67 *years, inclusively, if two persons of the ages of* 3 *years and* 13 *years shall both be living at the ends of the said years; and in the fifth column are set down the numbers that arise by the continual addition of the numbers set down in the fourth column; so that each number in the said fifth column is equal to the sum of all the numbers in the said fourth column that are placed above it, or that correspond to the preceeding, or older, ages; and in the sixth and last column are set down the products that arise by multiplying the terms of the third column, (or the values of a life-annuity of one pound a year for the joint continuance of two lives of the ages set down in the first and second columns,) by the corresponding terms of the fourth column, respectively.*

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Present values of a single payment of one pound, to be received at the ends of 81 years, 80 years, 79 years, 78, 77, 76, &c. years, in case two persons of the ages of 3 years and 13 years shall both be living at the ends of those years respectively.*	*Sums of the values in the fourth column.*	*Products of the multiplication of the numbers in the third and fourth columns.*
		£	£		
84	94	0.000,000	0.000,004,22	£	£
83	93	0.401,442	0.000,010,53	0.000,004,22	0,000,004,22
82	92	0.565,515	0.000,026,10	0.000,014,75	0.000,014,76
81	91	0.727,405	0.000.056,17	0.000,040,85	0.000,040,85
80	90	0.909,072	0.000,106,75	0.000,097,02	0.000,097,04
79	89	1.100,266	0.000,185,22	0.000,203,77	0.000,203,79
78	88	1.303,315	0.000,298,48	0.000,388,99	0.000,389,01
77	87	1.502,838	0.000,457,46	0.000,687,47	0.000,687,48
76	86	1.662,844	0.000,688,55	0.001,144,93	0.001,144,95
75	85	1.853,388	0.000,989,28	0.001,833,48	0.001,833,51
74	84	2.048,708	0.001,377,84	0.002,822,76	0.002,822,89
73	83	2.252,720	0.001,864,70	0.004,200,60	0.004,200,64
72	82	2.431,365	0.002,494,62	0.006,065,30	0.006,065,33
71	81	2.598,366	0.003,294,36	0.008,559,92	0.008,559,95
70	80	2.793,416	0.004,243,67	0.011,854,28	0.011,854,33

S C H O L I U M.

CCCCVII. In the foregoing table I have computed only the firſt fourteen values (reckoning from the oldeſt ages to the younger,) of an annuity of one pound a year for the joint continuance of the lives of two perſons whoſe ages differ from each other by 10 years. It would not have been difficult (as the reader muſt perceive,) to compleat the table by computing the values of an annuity of one pound a year for the joint continuance of all the younger lives whoſe ages differ from each other by the ſame difference of 10 years, down to the value of a like annuity for the joint continuance of two lives of the ages of 3 years and 13 years. But this was not neceſſary to the deſign with which the foregoing computations were undertaken; which was only to ſhew how ſuch a table might be formed by means of the expreſſion $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d}| \times \overline{Q+e}|} \times \overline{1+V}|$ £, and how the ſeveral numbers thereby obtained might be verified, as faſt as they were computed, by multiplying them into the correſponding values of the expreſſion $\frac{\pounds\, 1}{r^m} \times \frac{P \times Q}{A \times K}$, or the correſponding numbers that are ſet down in the fourth column of the ſaid table, and comparing the products thence ariſing with the numbers in the fifth column. This, I apprehend, is made ſufficiently manifeſt by the computation of the fourteen values ſet down in the foregoing table: and therefore I have declined the trouble of continuing theſe computations any further.

CCCCVIII. Neverthelefs, as it will be of great convenience to ſuch perſons as have occaſion to deal in the purchaſe of annuities for joint lives, to have a *compleat* table of the values of an annuity of one pound for the joint continuance of two lives whoſe ages differ from each other by 10 years, (of which the foregoing table contains only the firſt fourteen numbers,) and likewiſe to have other tables of the values of the like annuities for the joint continuance of two lives whoſe ages differ from each other by more, or leſs, than 10 years, I have cauſed the foregoing table to be compleated by another hand under the inſpection of the learned Mr. Morgan, above-mentioned, (the preſent actuary of the Society for Equitable Aſſurances on Lives and Survivorſhips,) and alſo nine other tables of the ſame kind to be computed from Monſieur de Parcieux's table

table of the probabilities of the duration of human life upon a supposition that the intereſt of money is $3\frac{1}{2}$ per cent. (as it is ſuppoſed to be in the foregoing table,) to wit, a table of the values of an annuity of one pound a year for the lives of two perſons of equal ages, another of the values of the like annuity for the lives of two perſons whoſe ages differ from each other by five years, a third for two lives whoſe ages differ by 20 years, a fourth for two lives whoſe ages differ by 30 years, and a fifth, ſixth, ſeventh, eighth, and ninth, table, for two lives whoſe ages differ by 40 years, 50 years, 60 years, 70 years, and 80 years. Theſe tables have all been computed by means of the expreſſion $\frac{1}{r} \times \frac{P \times Q}{\overline{P+d|} \times \overline{|Q+e}} \times \overline{1+V|}\,£$, in the ſame manner as the fourteen numbers above computed in Table XXVIII; and the computations have alſo been verified in the ſame manner, to wit, by computing the ſucceſſive values of the expreſſion $\overset{£}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$, and by finding the ſums of thoſe ſucceſſive values, and, laſtly, by multiplying the values of the ſeveral annuities into the correſponding values of the expreſſion $\overset{£}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$, and obſerving that the products thereby obtained were equal to the correſponding ſums of the ſucceſſive values of $\overset{£}{\frac{1}{r^m}} \times \frac{P \times Q}{A \times K}$. But I have not thought it neceſſary to cauſe all theſe latter numbers (which ſerve only to prove the truth of the computations) to be printed; and therefore I ſhall preſent the reader with only the values of the annuities themſelves, as was done in the tables of the values of an annuity of one pound a year for a ſingle life given above in Art. ccI, Tables XII, XIII, XIV, XV,——XXIII, pages 221, 222, 223,—232. And, as it is moſt uſual, in exhibiting tables of the values of life-annuities, to begin with thoſe of the younger ages and proceed on to thoſe of older ages, I ſhall obſerve the ſame order in ſetting down the values contained in the following tables, notwithſtanding they were computed one from another by proceeding in a contrary order, or from the older ages to the younger. Theſe tables are as follows.

TABLE XXIX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons of the same age, when the interest of money is 3½ per cent.------Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the first life.	*Years in the age of the second life*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the first life.*	*Years in the age of the second life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the first life.*	*Years in the age of the second life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	3	15.785,24	34	34	14.172,18	65	65	5.699,541
4	4	16.363,93	35	35	14.008,32	66	66	5.373,930
5	5	16.731,89	36	36	13.838,74	67	67	5.061,732
6	6	16.994,35	37	37	13.603,10	68	68	4.764,788
7	7	17.170,57	38	38	13.437,90	69	69	4.485,940
8	8	17.287,49	39	39	13.203,01	70	70	4.229,522
9	9	17.378,30	40	40	12.957,85	71	71	3.967,857
10	10	17.397,65	41	41	12.701,78	72	72	3.735,261
11	11	17.338,47	42	42	12.434,14	73	73	3.506,636
12	12	17.194,84	43	43	12.154,19	74	74	3.285,035
13	13	17.045,84	44	44	11.861,14	75	75	3.075,111
14	14	16.891,22	45	45	11.554,15	76	76	2.843,825
15	15	16.730,68	46	46	11.232,32	77	77	2.625,379
16	16	16.563,92	47	47	10.933,91	78	78	2.429,124
17	17	16.432,30	48	48	10.620,90	79	79	2.223,694
18	18	16.296,21	49	49	10.330,55	80	80	2.057,238
19	19	16.155,41	50	50	10.025,94	81	81	1.906,337
20	20	16.009,67	51	51	9.743,490	82	82	1.785,767
21	21	15.900,58	52	52	9.484,577	83	83	1.649,026
22	22	15.788,72	53	53	9.213,854	84	84	1.471,612
23	23	15.673,96	54	54	8.930,288	85	85	1.301,203
24	24	15.556,18	55	55	8.669,387	86	86	1.148,822
25	25	15.435,19	56	56	8.396,670	87	87	1.041,569
26	26	15.310,85	57	57	8.111,004	88	88	0.873,177
27	27	15.183,00	58	58	7.847,174	89	89	0.708,630
28	28	15.051,43	59	59	7.571,510	90	90	0.551,722
29	29	14.915,97	60	60	7.282,755	91	91	0.410,10[illegible]
30	30	14.776,38	61	61	6.979,449	92	92	0.299,890
31	31	14.632,46	62	62	6.659,908	93	93	0.241,546
32	32	14.483 96	63	63	6.356,826	94	94	0.000,000
33	33	14.330,63	64	64	6.037,443			

TABLE XXX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 5 years; when the interest of money is 3½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of theduration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	8	16.510,17	32	37	14.035,08	61	66	6.065,430
4	9	16.854,05	33	38	13.843,23	62	67	5.748,630
5	10	17.051,58	34	39	13.643,78	63	68	5.447,890
6	11	17.155,02	35	40	13.436,29	64	69	5.150,625
7	12	17.171,55	36	41	13.220,25	65	70	4.858,151
8	13	17.154,48	37	42	12.995,13	66	71	4.567,925
9	14	17.120,70	38	43	12.739,85	67	72	4.299,865
10	15	17.048,09	39	44	12.473,04	68	73	4.040,373
11	16	16.933,54	40	45	12.193,97	69	74	3.792,444
12	17	16.79[illegible],58	41	46	11.901,[illegible]8	70	75	3.560,612
13	18	16.652,69	42	47	11.616,67	71	76	3.314,346
14	19	16.504,62	43	48	11.317,93	72	77	3.088,057
15	20	16.351,08	44	49	11.025,10	73	78	2.876,561
16	21	16.213,13	45	50	10.718,15	74	79	2.663,173
17	22	16.090,90	46	51	10.416,04	75	80	2.477,972
18	23	15.964,95	47	52	10.137,24	76	81	2.292,901
19	24	15.835,03	48	53	9.845,200	77	82	2.129,559
20	25	15.701,10	49	54	9.556,736	78	83	1.964,259
21	26	15.583,33	50	55	9.273,595	79	84	1.770,303
22	27	15.462,37	51	50	8.994,268	80	85	1.595,707
23	28	15.338,05	52	57	8.718,822	81	86	1.437,314
24	29	15.210,20	53	58	8.449,496	82	87	1.316,220
25	30	15.078,62	54	59	8.167,754	83	88	1.149,833
26	31	14.943,13	55	60	7.889,259	84	89	0.969,173
27	32	14.803,51	56	61	7.597,410	85	90	0.793,411
28	33	14.659,54	57	62	7.290,800	86	91	0.630,011
29	34	14.510,98	58	63	7.002,970	87	92	0.495,243
30	35	14.357,56	59	64	6.700,90	88	93	0.351,339
31	36	14.199,03	60	65	6.382,877	89	94	0.000.000

TABLE XXXI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 10 years; when the interest of money is $3\frac{1}{2}$ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	13	16.366,17	31	41	13.500,30	59	69	5.611,286
4	14	16.585,56	32	42	13.282,31	60	70	5.336,698
5	15	16.688,70	33	43	13.055,10	61	71	5.054,106
6	16	16.732,59	34	44	12.818,10	62	72	4.784,148
7	17	16.749,69	35	45	12.570,68	63	73	4.523,082
8	18	16.734,46	36	46	12.312,20	64	74	4.260,822
9	19	16.703,36	37	47	12.063,41	65	75	3.999,072
10	20	16.634,79	38	48	11.784,38	66	76	3.728,183
11	21	16.547,42	39	49	11.513,43	67	77	3.470,693
12	22	16.418,12	40	50	11.229,92	68	78	3.233,054
13	23	16.284,59	41	51	10.953,88	69	79	2.996,397
14	24	16.146,60	42	52	10.685,81	70	80	2.793,419
15	25	16.003,93	43	53	10.405,59	71	81	2.598,368
16	26	15.856,33	44	54	10.112,29	72	82	2.431,367
17	27	15.723,54	45	55	9.825,467	73	83	2.252,722
18	28	15.586,50	46	56	9.525,226	74	84	2.048,709
19	29	15.444,98	47	57	9.227,309	75	85	1.853,389
20	30	15.298,75	48	58	8.935,094	76	86	1.662,845
21	31	15.167,59	49	59	8.645,309	77	87	1.502,839
22	32	15.032,49	50	60	8.341,630	78	88	1.303,316
23	33	14.893,27	51	61	8.038,570	79	89	1.100,267
24	34	14.749,68	52	62	7.735,712	80	90	0.909,073
25	35	14.601,50	53	63	7.437,175	81	91	0.727,405
26	36	14.448,46	54	64	7.123,728	82	92	0.565,515
27	37	14.290,29	55	65	6.808,550	83	93	0.4[illegible]1,442
28	38	14.104,15	56	66	6.496,025	84	94	0.000,000
29	39	13.910,74	57	67	6.186,700			
30	40	13.709,61	58	68	5.894,503			

TABLE XXXII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 20 years; when the interest of money is 3½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	23	15.560,29	27	47	12.402,10	51	71	5.515,096
4	24	15.772,[illegible]4	28	48	12.1[illegible]6,36	52	72	5.249,786
5	25	15.876,39	29	49	11.900,87	53	73	4.984,023
6	26	15.925,[illegible]3	30	50	11.644,53	54	74	4.719,681
7	27	15.929,42	31	51	11.398,29	55	75	4.469,896
8	28	15.902,97	32	52	11.102,98	56	76	4.202,853
9	29	15.861,35	33	53	10.917,97	57	77	3.943,096
10	30	15.784,[illegible]0	34	54	10.662,54	58	78	3.706,498
11	31	15.667,99	35	55	10.417,61	59	79	3.462,599
12	32	15.510,67	36	56	10.162,63	60	80	3.246,444
13	33	15.347,68	37	57	9.896,823	61	81	3.039,033
14	34	15.178,75	38	58	9.625,224	62	82	2.848,658
15	35	15.003,54	39	59	9.342,070	63	83	2.646,551
16	36	14.821,70	40	60	9.046.436	64	84	2.409,133
17	37	14.65[illegible],5[illegible]	41	61	8.737.294	65	85	2.173,497
18	38	14.452,13	42	62	8.413,488	66	86	1.953,728
19	39	14.244,52	43	63	8.095,180	67	87	1.766,127
20	40	14.028,29	44	64	7.751,737	68	88	1.527,607
21	41	13.821,31	45	65	7.411,735	69	89	1.292,917
22	42	13.6[illegible]5,76	46	66	7.064,716	70	90	1.065,724
23	43	13.381,12	4[illegible]	67	6.733,989	71	91	0.84[illegible],497
24	44	13.146,84	48	68	6.408,777	72	92	0.646,370
25	45	12.902,34	49	69	6.102,706	73	93	0.444,598
26	46	12.646,97	50	70	5 807,267	74	94	0.000,000

TABLE XXXIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 30 years; when the interest of money is 3½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	33	14.605,10	24	54	10.805,23	45	75	4.717,841
4	34	14.761,38	25	55	10.556,78	46	76	4.427,255
5	35	14.812,79	26	56	10.298,13	47	77	4.152,480
6	36	14.810,22	27	57	10.028,53	48	78	3.892,549
7	37	14.763,70	28	58	9.769,124	49	79	3.631,597
8	38	14.662,37	29	59	9.499,177	50	80	3.399,171
9	39	14.542,30	30	60	9.217,864	51	81	3.182,288
10	40	14.384,49	31	61	8.924,272	52	82	2.990,528
11	41	14.186,34	32	62	8.617,370	53	83	2.779,762
12	42	13.945,55	33	63	8.317,989	54	84	2.533,005
13	43	13.694,31	34	64	8.005,274	55	85	2.295,974
14	44	13.432,04	35	65	7.678,014	56	86	2.071,761
15	45	13.158,08	36	66	7.356,765	57	87	1.876,901
16	46	12.871,77	37	67	7.042,733	58	88	1.628,765
17	47	12.611,02	38	68	6.726,106	59	89	1.381,241
18	48	12.338,55	39	69	6.419,796	60	90	1.137,781
19	49	12.075,74	40	70	6.126,861	61	91	0.903,579
20	50	11.801,15	41	71	5.828,088	62	92	686,040
21	51	11.551,46	42	72	5.547,756	63	[illegible]	0.467,103
22	52	11.312,81	43	73	5.267,684	64	94	0.000,000
23	53	11.064,30	44	74	4.990,020			

TABLE XXXIV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 40 years; when the interest of money is 3½ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	43	12.995,63	21	61	8.987,406	39	79	3.742,057
4	44	13.020,79	22	62	8.674,708	40	80	3.511,390
5	45	12.944,45	23	63	8.369,404	41	81	3.291,726
6	46	12.812,26	24	64	8.050,492	42	82	3.092,312
7	47	12.655,71	25	65	7.716,753	43	83	2.873,803
8	48	12.404,90	26	66	7.388,815	44	84	2.619,179
9	49	12.274,57	27	67	7.067,833	45	85	2.369,586
10	50	12.047,58	28	68	6.755,442	46	86	2.133,182
11	51	11.804,02	29	69	6.453,925	47	87	1.931,165
12	52	11.543,45	30	70	6.166,483	48	88	1.669,911
13	53	11.271,87	31	71	5.873,947	49	89	1.412,744
14	54	10.988,55	32	72	5.600,948	50	90	1.159,767
15	55	10.714,92	33	73	5.329,415	51	91	0.919,313
16	56	10.429,73	34	74	5.061,816	52	92	0.697,812
17	57	10.145,47	35	75	4.801,680	53	93	0.473,412
18	58	9.870,862	36	76	4.525,229	54	94	0.000,000
19	59	9.584,837	37	77	4.239,330			
20	60	9.286,549	38	78	4.003,965			

TABLE XXXV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 50 years; when the interest of money is 3½ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	53	10.680,23	24	74	5.075,965
4	54	10.628,92	25	75	4.811,044
5	55	10.513,03	26	76	4.529,340
6	56	10.350,53	27	77	4.257,630
7	57	10.148,69	28	78	4.003,127
8	58	9.938,555	29	79	3.742,189
9	59	9.709,814	30	80	3.512,640
10	60	9.449,236	31	81	3.294,317
11	61	9.154,806	32	82	3.096,568
12	62	8.[illegible]24,696	33	83	2.880,142
13	63	8.501,682	34	84	2.628,12[illegible]
14	64	8.164,376	35	85	2.382,003
15	65	7.811,53[illegible]	36	86	2.150,472
16	66	7.463,965	37	87	1.950,898
17	67	7.132,382	38	88	1.689,411
18	68	6.809,136	39	89	1.429,589
19	69	6.496,404	40	90	1.175,112
20	70	6.197,247	41	91	0.931,818
21	71	5.900,765	42	92	0.706,129
22	72	5.623,758	43	93	0.477,775
23	73	5.348,020	44	94	0.000 000

TABLE XXXVI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 60 years; when the interest of money is $3\frac{1}{2}$ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	63	8.049,257	20	80	3.526,172
4	64	7.882,630	21	81	3.306,196
5	65	7.643,727	22	82	3.106,801
6	66	7.382,707	23	83	2.888,572
7	67	7.107,740	24	84	2.634,545
8	68	6.828,138	25	85	2.386,279
9	69	6.554,273	26	86	2.152,327
10	70	6.281,256	27	87	1.949,808
11	71	5.989,106	28	88	1.688,533
12	72	5.702,314	29	89	1.428,905
13	73	5.416,621	30	90	1.174,597
14	74	5.134,386	31	91	0.931,450
15	75	4.858,956	32	92	0.705,886
16	76	4.566,065	33	93	0.477,648
17	77	4.288,874	34	94	0.000,000
18	78	4.028,814			
19	79	[illegible]61,967			

TABLE XXXVII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 70 years; when the interest of money is 3½ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	73	5.137,907	14	84	2.656,044
4	74	4.956,842	15	85	2.402,893
5	75	4.746,962	16	86	2.163,852
6	76	4.503,802	17	87	1.959,233
7	77	4.258,192	18	88	1.695,616
8	78	4.022,334	19	89	1.433,647
9	79	3.777,077	20	90	1.176,850
10	80	3.557,527	21	91	0.933,060
11	81	3.341,255	22	92	0.706,947
12	82	3.137,624	23	93	0.478,200
13	83	2.914,904	24	94	0.000,000

TABLE XXXVIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 80 years; when the interest of money is 3½ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£
3	83	2.783,857	9	89	1.432,098
4	84	2.574,553	10	90	1.180,457
5	85	2.351,322	11	91	0.937,542
6	86	2.133,541	12	92	0.709,889
7	87	1.940,957	13	93	0.479,721
8	88	1.686,249	14	94	0.000,000

CCCCIX. According

CCCCIX. According to Monſieur de Parcieux's table of the probabilities of the duration of human life, (which begins with the age of 3 years, and ends with the age of 94 years, and ſuppoſes the utmoſt poſſible extent of human life to be ſomewhat leſs than 95 years,) it is evident that only one pair of lives can be found whoſe ages will differ from each other by 90 years, to wit, two lives of the ages of 3 and 93 years. And the value of an annuity of one pound a year for the joint continuance of two lives of theſe ages, upon a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. is £0.468,599.

Remarks on the great number of tables (of the ſame kind as thoſe above-computed,) that would be neceſſary in order to exhibit the values of annuities for two joint lives in all their poſſible varieties.

CCCCX. It would evidently be a work of great uſe and convenience to thoſe perſons who have occaſion to ſell or purchaſe annuities for two joint lives, to compute as many more tables, of the ſame kind as the ten foregoing ones, as there may be taken different numbers of years for the difference of the ages of the two lives upon which the annuity is to depend; ſo that, for example, if the difference of the ages of any two lives was 23 years, or 27 years, inſtead of 20 years, or 30 years, the value of an annuity of one pound a year during the joint continuance of both the lives ſhould be accurately exhibited in ſome one or other of the tables, in the ſame manner as the values of the like annuities are exhibited in the foregoing tables when the difference of the ages is either 0, or 5 years, or 10 years, or 20, 30, 40, 50, 60, 70, or 80, years. But this would require an immenſe quantity of calculation. For, in order to have the exact values of an annuity of one pound a year for two joint lives in all the poſſible varieties in which the ages of the two lives may be combined together, when the intereſt of money is $3\frac{1}{2}$ per cent. (as it is ſuppoſed to be in the ten foregoing tables,) it would be neceſſary to compute no fewer than 93 different tables, or 83 more than the ten that are above computed. For, beſides the ſaid ten tables, which exhibit thoſe values when the ages of the two lives are equal, and when they differ from each other by 5 years, or 10 years, or 20 years, or 30 years, or 40 years, or 50 years, or 60 years, or 70 years, or 80 years, it would be neceſſary to compute an eleventh table of the ſame kind that ſhould exhibit the values of a like annuity for the joint continuance of two lives whoſe ages differed from each other by one year; and a twelfth table of the ſame kind when the difference of the ages is two years; and a thirteenth table of the ſame kind when the difference of the ages is three years; and, in general, a new table of the ſame

ſame kind for every new difference that can be taken between the ſaid two ages: which differences (if we reckon 0 for one of them, or include the caſe of two lives of equal ages,) will, all together, (or including the ten differences in the tables above-computed,) amount to the number of 93, if the youngeſt life ſet down in the table of probabilities is of the age of one year; or, if we make uſe of Monſieur de Parcieux's table of probabilities, (in which the youngeſt age is that of 3 years,) will amount to the number of 91. And, if we were deſirous of having the values of the like annuities for two joint lives in all their poſſible varieties when the intereſt of money is either 2 per cent. or $2\frac{1}{2}$ per cent. or 3 per cent. or 4 per cent. or $4\frac{1}{2}$ per cent. or 5 per cent. or 6 per cent. or 7 per cent. or 8 per cent. or 9 per cent. or 10 per cent. as well as when it is $3\frac{1}{2}$ per cent. (as we have the values of the like annuities for ſingle lives of all ages for all thoſe different rates of intereſt in the tables exhibited above in pages 221, 222, — — — 232,) it would be neceſſary to compute the ſame number of tables of this kind, to wit, 93, or 91, tables, for each of theſe rates of intereſt; which would make, in all, 12 times 93, or 12 times 91, ſuch tables, or more than eleven hundred ſuch tables. Now the computation of ſuch a prodigious number of tables would be a buſineſs of ſo much length and labour that it, probably, will never be undertaken; though, perhaps, it might be worth the while of the Government, or of ſome of the ſocieties for making inſurances upon lives, or of the dean and chapter of ſome rich cathedral church, whoſe lands are leaſed out upon lives, or of ſome other wealthy body of men, to whom ſuch tables might be peculiarly uſeful, to cauſe *two, or three,* ſets of theſe tables to be computed for *two, or three,* of the moſt common and uſeful rates of intereſt, as, for example, for $3\frac{1}{2}$ per cent. 4 per cent. and $4\frac{1}{2}$ per cent. If this were to be undertaken and carefully performed, under ſuch encouragement, it would, I doubt not, be allowed on all hands to be a work of great merit and of general advantage to the publick.

A method of finding by Interpolation *the values of ſuch annuities for two joint lives, as are not contained in any of the foregoing tables.*

CCCCXI. But in the mean while, and until ſuch tables ſhall be publiſhed, it will be deſireable to find out, if poſſible, ſome tolerably eaſy method of deriving the values of ſuch annuities for two joint lives as are *not* ſet down in the foregoing ten tables, from the values of thoſe which *are* therein exhibited. Now this may be done to a moderate degree of exactneſs, ſufficient for common purpoſes, by a kind of *Interpolation*, which may be explained in the following manner.

CCCCXII. The

The principles of the said method of Interpolation.

CCCCXII. The principles upon which this method of interpolation is founded are as follows.

In the first place it is evident beyond a doubt, that the value of an annuity for the joint continuance of any two given lives is greater than the value of the like annuity for two other lives whereof the younger is of the same age with the younger of the two former lives, and the older is older than the older of the two former lives: or, in other words, the value of an annuity for two joint lives of N years and $N+n$ years is greater than the value of the same annuity for two joint lives of the ages of N years and $N+n+e$ years. Thus, for example, the value of an annuity of one pound a year for two joint lives of the ages of 23 years and 30 years is greater than the value of the like annuity for two joint lives of the ages of 23 years and 40 years.

In the second place it seems highly probable, that, if we take a moderate number of lives that are successively older the one than the other by one year, and combine them, one after another, with another life that is younger than any of them, the values of an annuity for these successive pairs of joint lives, (in all of which the youngest life is of the same age,) will be nearly in arithmetical proportion, as well as the ages of the older lives in these successive pairs of lives, which are supposed to increase by the equal difference of one year. Thus, if the younger life in each of these pairs of lives is of the age of N years, and the several older lives, with which this life is to be successively combined, are of the ages of $N+n$ years, $N+n+1$ years, $N+n+2$ years, $N+n+3$ years, $N+n+4$ years, $N-+n+5$ years, &c. it is probable that the values of an annuity

for two joint lives of the ages of N years and $N+n$ years,
and for two joint lives of the ages of N years and $N+n+1$ years,
and for two joint lives of the ages of N years and $N+n+2$ years,
and for two joint lives of the ages of N years and $N+n+3$ years,
and for two joint lives of the ages of N years and $N+n+4$ years,
and for two joint lives of the ages of N years and $N+n+5$ years, &c.

will form, pretty nearly, an arithmetical progression, as well as the older ages, $N+n$ years, $N+n+1$ years, $N+n+2$ years, $N+n+3$ years, $N+n+4$ years, $N+n+5$ years, &c. themselves: only the series of those values will be a decreasing progression, whereas the series of the older ages is an increasing one. This, I say, seems probable, (though it is not absolutely evident,) and will be found to be sufficiently near the truth, when the number of the terms in these progressions is not greater than 11, to be the foundation of a very useful method of approximating to the values of these joint annuities.

CCCCXIII. These

The said method itself.

CCCCXIII. These things being premised, let it be proposed to find the value of an annuity of one pound a year for the joint continuance of two lives of the ages of N years and $N+a$ years, in which a, the difference of the two ages, is not either 5 years, or 10 years, or 20 years, or 30 years, or 40 years, or 50 years, or 60 years, or 70 years, or 80 years, (which are the differences of the two ages in the foregoing tables,) but some intermediate number of years between some two of these differences, that are contiguous to each other.

Let the greatest difference (amongst the differences set down in the foregoing tables, which are either 5 years, or 10 years, or some multiple of 10 years,) than which the given difference a is greater, be $10 \times m$, or $10\,m$. Then, it is evident, the next greater difference, or the least difference (amongst the differences set down in the foregoing tables,) that is greater than the difference a, will be $10 \times \overline{m+1}$, or $10\,m+10$. Therefore (by the first principle above-mentioned,) the value of the proposed annuity of one pound a year for the joint continuance of two lives of the ages of N years and $N+a$ years will be of an intermediate magnitude between the value of a like annuity for two joint lives of the ages of N years and $N+10\,m$ years, and the value of a like annuity for two joint lives of the ages of N years and $N+10\,m+10$ years; both which values may be found in some of the foregoing ten tables. And thus we may obtain, by means of the first principle above-mentioned, two limits of the value of the proposed annuity, between which we may be sure it is of an intermediate magnitude.

And, by the second principle above-mentioned, we may make a nearer approximation to its true value by reasoning as follows.

The eleven following values of an annuity of one pound a year for two joint lives, (of which eleven values we can find the first and the last in the ten foregoing tables,) will form, pretty nearly, a decreasing arithmetical progression; to wit, the values of the said annuity for two joint lives of the ages of

N years and $N+10\,m$ years,
N years and $N+10\,m+1$ years,
N years and $N+10\,m+2$ years,
N years and $N+10\,m+3$ years,
N years and $N+10\,m+4$ years,
N years and $N+10\,m+5$ years,

N years and $N + 10m + 6$ years,
N years and $N + 10m + 7$ years,
N years and $N + 10m + 8$ years,
N years and $N + 10m + 9$ years,
and N years and $N + 10m + 10$ years.

Subtract therefore the last of these values from the first, and divide the difference by 10; and the quotient thence arising will be the quantity which must be continually added to the last value, or subtracted from the first value, in order to form an arithmetical progression of terms between the first value and the last. Let these continual additions, or subtractions, be made. And, amongst the terms thereby obtained, that which corresponds to the ages of N years and $N + a$ years (which latter age must be equal to one of the intermediate ages between $N + 10m$ years and $N + 10m + 10$ years,) will be a near value of the proposed annuity of one pound a year for two joint lives of the ages of N years and $N + a$ years. Q E I.

Examples of the foregoing method of discovering the values of the above-mentioned intermediate, or omitted, annuities for two joint lives by Interpolation.

First example. CCCCXIV. As an example of this method of interpolation, let the two lives for whose joint continuance an annuity of one pound a year is to be granted, be of the ages of 70 years and 77 years.

Then it is evident, in the first place, that the value of this annuity is not contained in any of the foregoing ten tables; because 7 years is not the difference of the two ages in any of them.

But, in the second place, we may observe that the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 77 years must be less than the value of a like annuity for the joint continuance of two lives of the ages of 70 years and 75 years, but greater than the value of the like annuity for the joint continuance of two lives of the ages of 70 years and 80 years.

In the third place we may observe that the values of the two latter annuities, (between which the value of the proposed annuity lies,) to wit, the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 75 years and of a like annuity for the joint continuance of two lives of the ages of 70 years and 80 years, are both contained in the foregoing tables; the former of these values being contained in Table XXX, in which the difference of the ages is 5 years, and the latter of them being contained in Table XXXI, in which the

the difference of the ages is 10 years. The former of these values appears in Table XXX to be = £3.560,612; and the latter of them appears in Table XXXI to be = £2.793,419. Therefore the value of the proposed annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 77 years is of an intermediate magnitude between £3.560,612 and £2.793,419.

In the fourth place we may reasonably suppose that the values of an annuity of one pound a year for the joint continuance of the six following pairs of lives, to wit,

two lives of the ages of 70 years and 75 years,
two lives of the ages of 70 years and 76 years,
two lives of the ages of 70 years and 77 years,
two lives of the ages of 70 years and 78 years,
two lives of the ages of 70 years and 79 years,
and two lives of the ages of 70 years and 80 years,

will form, pretty nearly, an arithmetical progression, or will decrease by nearly equal differences. Therefore, if we subtract the last of them, which is = £2.793,419, from the first, or £3.560,612, and divide the remainder, to wit, £0.767,193, by 5, the quotient, £0.153,438, will be the common difference by which these terms will decrease; and consequently, if this quotient be either continually added to the last term, £2.793,419, or continually subtracted from the first term, £3.560,612, we shall thereby obtain the values of the intermediate terms of the progression. If we proceed by addition, these intermediate terms will be as follows, to wit,

	£		£		£
	2.793,419	+	0.153,438,	or	2.946,857,
	2.946,857	+	0.153,438,	or	3.100,295,
	3.100,295	+	0.153,438,	or	3.253,732,
and	3.253,732	+	0.153,438,	or	3.407,171;

and, if we proceed by subtraction, they will be as follows, to wit,

	£		£		£
	3.560,612	—	0.153,438,	or	3.407,174,
	3.407,174	—	0.153,438,	or	3.253,736,
	3.253,736	—	0.153,438,	or	3.100,298,
and	3.100,298	—	0.154,438,	or	2.946,860.

 Therefore

Therefore the values of an annuity of one pound a year for the joint continuance of two lives of the ages of

70 years and 76 years,
70 years and 77 years,
70 years and 78 years,
and 70 years and 79 years,

Will be nearly equal to

£
3.407,174,
3.253,736,
3.100,298,
and 2.946,860;

of which values the fecond, to wit, £3.253,736, is that we were in fearch of. Q E I.

Note. This value, £3.253,736, (which is the third term of the arithmetical progreffion, confifting of fix terms, whofe firft and laft terms are £3.560,612 and £2.793,419) might have been found feparately, or without finding the other intermediate terms of the faid progreffion, by dividing the difference of the extreme terms, to wit, £0.767,193, by 5, fo as to find the common difference of the terms, or £0.153,438, and then fubtracting twice the faid difference, or £0.306,876, from the firft, or greateft, term, £3.560,612. For £3.560,612 — £0.306,876, is = £3.253,736.

Second example.

CCCCXV. As another example of this method of Interpolation, let us fuppofe the ages of the two lives, for the joint continuance of which an annuity of one pound a year is to be granted, to be 59 years and 70 years.

Here we muft obferve in the firft place, that the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years muft be fomewhat lefs than the value of a like annuity for two joint lives of the ages of 59 and 69 years; which appears by Table XXXI to be = £5.611,286.

In

In the ſecond place we muſt obſerve that the value of the ſaid annuity of one pound a year for the joint continuance of two lives of the ages of 59 years and 70 years will be greater than the value of a like annuity for the joint continuance of two lives of the ages of 59 years and 79 years; which appears by Table XXXII to be = £3.462,599. Therefore the value of the ſaid annuity of one pound a year for two joint lives of the ages of 59 years and 70 years is greater than £3.462,599, but leſs than £5.611,286. And it will evidently be much nearer to the greater of theſe values, or £5.611,286, than to the leſſer value, £3.462,599.

In the third place, in order to make a nearer approach to the value of this annuity, we muſt ſuppoſe that the values of an annuity of one pound a year for the eleven following pairs of lives, to wit,

for two lives of the ages of 59 years and 69 years,
two lives of the ages of 59 years and 70 years,
two lives of the ages of 59 years and 71 years,
two lives of the ages of 59 years and 72 years,
two lives of the ages of 59 years and 73 years,
two lives of the ages of 59 years and 74 years,
two lives of the ages of 59 years and 75 years,
two lives of the ages of 59 years and 76 years,
two lives of the ages of 59 years and 77 years,
two lives of the ages of 59 years and 78 years,
and two lives of the ages of 59 years and 79 years,

will form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences. And, if they do ſo decreaſe, the difference between the firſt and ſecond of thoſe values will be nearly the tenth part of the difference between the firſt and the laſt values. Now the firſt of theſe values has been ſhewn to be = £5.611,286, and the laſt of them has been ſhewn to be = £3.462,599: and the difference of theſe values is £2.148,687; of which the tenth part is £0.214,868. Therefore the ſecond of the foregoing eleven values will be = £5.611,286 — £0.214,868, or £5.396,418. Therefore the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years, will be, nearly, equal to £5.396,418. Q E I.

CCCCXVI. There is another way of obtaining a near value of this annuity by means of the foregoing tables, beſides this of Interpolation; but which differs very little from it, and is founded on exactly the ſame principles. It is as follows.

It

It appears by Table XXX, (in which the difference of the ages of the two lives is 5 years,) that the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 59 years and 64 years is £6.700,900. And we have feen that the value of a like annuity for the joint continuance of two lives of the ages of 59 and 69 years is £5.611,286; which is lefs than £6.700,900 by the difference £1.089,614.

Now we may reafonably fuppofe that the values of an annuity of one pound a year for the joint continuance of the feven following pairs of lives, to wit,

two lives of the ages of 59 years and 64 years,
two lives of the ages of 59 years and 65 years,
two lives of the ages of 59 years and 66 years,
two lives of the ages of 59 years and 67 years,
two lives of the ages of 59 years and 68 years,
two lives of the ages of 59 years and 69 years,
and two lives of the ages of 59 years and 70 years,

will form, pretty nearly, an arithmetical progreffion, or will decreafe by equal differences. And, if they do fo decreafe, the difference between the firft of them (which is = £6.700,900,) and the laft, (which is the value fought,) will be equal to fix times their common difference; and confequently, if we can find the common difference of thefe values, the laft of them may be derived from the firft by fubtracting from it fix times the faid common difference. And it may alfo be derived from the fixth value (which we have feen to be = £5.611,286,) by fubtracting from it the faid common difference itfelf. We muft therefore inquire what is the faid common difference.

Now, fince the firft of thefe values (which are fuppofed to form an arithmetical progreffion,) is = £6.700,900, and the fixth of them is = £5.611,286, it follows that the common difference of thefe values muft be a fifth part of the difference of £6.700,900 and £5.611,286, that is, a fifth part of £1.089,614, and therefore will be = £0.217,922. Therefore fix times the faid common difference will be = (fix times £0.217,922, or) £1.307,532; which, being fubtracted from £6.700,900, or the firft of the foregoing feven values, leaves £5.393,368 for the laft of the faid values, or the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years. Q. E. I.

Or, if we fubtract the common difference, £0.217,922, itfelf, from £5.611,286, or the fixth of the foregoing values, the remaining quantity, £5.393,364,

£5.393,364, will b the laſt of the ſaid ſeven values, or the value of an annuity of one pound a year for two joint lives of the ages of 59 years and 70 years. Q E I.

Note. Theſe values £5.393,368 and £5.393,364 are ſo nearly equal to the value found for this annuity in the laſt article, to wit, £5.396,418, that the difference is not worth attending to. But, if it were, we ought to conſider the value laſt obtained, to wit, £5.393,368, as being nearer to the exact value of the propoſed annuity than £5.396,418, becauſe it is obtained by means of an arithmetical progreſſion conſiſting of fewer terms than the other, in Art. ccccxv, by means of which the former value, £5.396,418, had been found.

CCCCXVII. As a third example of this method of Interpolation, let the ages of the two lives, upon the joint continuance of which an annuity of one pound a year is to depend, be 26 years and 53 years; the difference of which is 27 years. Third example.

Here we muſt obſerve, in the firſt place, that the value of this annuity is leſs than the value of a like annuity for two joint lives of the ages of 26 years and 46 years, and greater than the value of a like annuity for two joint lives of the ages of 26 years and 56 years; whence it follows, by Tables XXXII and XXXIII, that it is leſs than £12.646,97, and greater than £10.298,13.

In the ſecond place we may reaſonably ſuppoſe that the values of an annuity of one pound a year for the joint continuance of the following eleven pairs of lives, to wit,

two lives of the ages of 26 years and 46 years,
two lives of the ages of 26 years and 47 years,
two lives of the ages of 26 years and 48 years,
two lives of the ages of 26 years and 49 years,
two lives of the ages of 26 years and 50 years,
two lives of the ages of 26 years and 51 years,
two lives of the ages of 26 years and 52 years,
two lives of the ages of 26 years and 53 years,
two lives of the ages of 26 years and 54 years,
two lives of the ages of 26 years and 55 years,
and two lives of the ages of 26 years and 56 years,

will

will form, pretty nearly, an arithmetical progreffion, or will decreafe by nearly equal differences. And, if they do fo decreafe, their common difference will be nearly a tenth part of the difference of the extreme terms. Now the extreme terms are £12.646,97 and £10.298,13, the difference of which is £2.348,84; and the tenth part of this difference is £0.234,884. Therefore £0.234,884, is the common difference of the terms; and confequently £10.298,13 + 3 × £0.234,884, (or £10.298,13 + £0.704,652,) or £11.002,78 will be the value of the laft term but three, or of the eighth term, or will be the value of the propofed annuity of one pound a year for two joint lives of the ages of 26 years and 53 years. Q E I.

CCCCXVIII. Thefe three examples will, I prefume, be fufficient to illuftrate this method of deriving the values of fuch annuities for two joint lives as *are not* contained in the foregoing ten tables, from the values of the annuities that *are* contained in thofe tables, by Interpolation. And therefore I fhall not add any more examples of this method with *that* defign.

Other examples of the foregoing method of Interpolation, *which ferve to fhew to what degree of exactnefs the near values of thefe annuities for two joint lives, that are obtained by means of it, may be fuppofed to co-incide with their true values.*

CCCCXIX. But it will be neceffary to give a few more inftances of this method of Interpolation with *another* view, namely, in order to fhew that the values of annuities for two joint lives which are obtained by means of it, are pretty nearly equal to their true values, and may confequently be ufed, on all common occafions, inftead of the faid true values, without any fenfible inconvenience. Now this will beft appear by computing, by means of this method of Interpolation, a few of the near values of an annuity of one pound a year for the joint continuance of two lives whofe ages differ from each other by 5 years, and then comparing the faid near values of thefe annuities, thereby obtained, with the exact values of the fame annuities exhibited above in Table XXX, page 493.

First example. CCCCXX. Let it therefore be required, in the firft place, to find, by this method of Interpolation, a near value of an annuity of one pound a year for the joint continuance of two lives of the ages of 10 years and 15 years; the exact value of which annuity appears in Table XXX to be £17.048,09.

Now

Now it appears by Table XX X that the value of an annuity of one pound a year for two joint lives that are both of the same age of 10 years, is £17.397,65. And it appears from Table XXXI that the value of a like annuity of one pound a year for two joint lives of the ages of 10 years and 20 years is £16.634,79. Therefore, according to this method of Interpolation, the value of an annuity of one pound a year for two joint lives of the ages of 10 years and 15 years will be nearly equal to an arithmetical mean proportional between £17.397,65 and £16.634,79, and consequently will be nearly equal to $\left(\frac{\pounds 17.397,65 + \pounds 16.634,79}{2}\right.$, or $\frac{\pounds 34.032,44}{2}$, or$\left.\right)$ £17.016,22. Q E I.

The difference between this near value, £17.016,22, of the proposed annuity, and £17.048,09, its exact value, is £0.031,87, which is less than one 534th part of the said true value.

CCCCXXI. Let the two lives be of the ages of 20 years and 25 years. *Second example.*

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £15.701,10.

The near value of the same annuity will, according to the foregoing method of Interpolation, be an arithmetical mean proportional between £16.009,67, which appears by Table XXIX to be the value of a like annuity for two joint lives that are both of the age of 20 years, and £15.298,75, which appears by Table XXXI to be the value of a like annuity for two joint lives of the ages of 20 years and 30 years; and consequently the said near value will be $\left(= \frac{\pounds 16.009,67 + \pounds 15.298,75}{2}\right.$, $= \left.\frac{\pounds 31.308,42}{2}\right)$ = £15.654,21. Q E I.

The difference between this near value, £15.6[illegible],21, of the proposed annuity, and its more exact value, £15.701,10, is £0.046,89, which is less than the 334th part of the said exact value.

Third example. CCCCXXII. Let the two lives be of the ages of 30 years and 35 years.

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £14.357,56.

And the near value of the same annuity will, according to the foregoing method of Interpolation, be an arithmetical mean proportional between the value of a like annuity for two joint lives that are both of the same age of 30 years, which appears by Table XXIX to be £14.776,38, and the value of a like annuity for two joint lives of the ages of 30 years and 40 years, which appears by Table XXXI to be £13.709,61; and consequently it will be $= \left(\frac{\overset{£}{14.776,38} + \overset{£}{13.709,61}}{2} = \frac{\overset{£}{28.485,99}}{2} =\right)$ £14.242,99. Q E I.

The difference between this near value, £14.242,99, of the proposed annuity, and its more exact value, £14.357,56, is £0.114,57; which is less than the 125th part of the said exact value.

Fourth example. CCCCXXIII. Let the two lives be of the ages of 40 years and 45 years.

Then will the exact value of an annuity of one pound a year for these two joint lives, according to Table XXX, be £12.193,97.

The value of an annuity of one pound a year for two joint lives that are both of the age of 40 years, appears by Table XXIX to be £12.957,85; and the value of a like annuity for two joint lives of the ages of 40 years and 50 years appears by Table XXXI to be £11.229,92. Therefore the value of the like annuity for two joint lives of the ages of 40 years and 45 years is nearly equal to an arithmetical mean between £12.957,85 and £11.229,92, and consequently is nearly equal to $\left(\frac{\overset{£}{12.957,85} + \overset{£}{11.229,92}}{2}\right.$; or $\frac{\overset{£}{24.187,77}}{2}$, or$\left.\right)$ £12.093,88. Q E I.

The

The difference between this near value, £12.093,88, of the proposed annuity, and its more exact value, £12.193,97, is £0.100,09; which is less than the 121st part of the said exact value.

Fifth example.

CCCCXXIV. In like manner, if the two lives are of the ages of 50 years and 55 years, the exact value of an annuity of one pound during their joint continuance, given in Table XXX, is £9.273,595; and the near value of the same annuity, obtained by the foregoing method of Interpolation, will be an arithmetical mean between £10.025,94 and £8.341,632, and consequently will be equal to $\frac{\pounds 18.367,572}{2}$, or £9.183,786. Q E I.

The difference between this near value, £9.183,786, of the proposed annuity, and its more exact value, £9.273,595, is £0.089,809; which is less than the 103d part of the said exact value.

Sixth example.

CCCCXXV. If the two lives are of the ages of 60 years and 65 years, the exact value of the annuity will be £6.382,877; and the near value of it will be $= \left(\frac{\pounds 7.282,755 + \pounds 5.336,698}{2} = \frac{\pounds 12.619,453}{2} =\right)$ £6.309,726; which differs from the exact value, £6.382,877, by £0.073,151, which is less than the 87th part of the said exact value.

Seventh example.

CCCCXXVI. If the two lives are of the ages of 70 years and 75 years, the exact value of the annuity will be £3.560,612; and the near value of it will be $= \left(\frac{\pounds 4.229,522 + \pounds 2.793,419}{2} = \frac{\pounds 7.022,941}{2} =\right)$ £3.511,470; which differs from the exact value, £3.560,612, by £0.049,142; which is less than the 72d part of the said exact value.

Eighth example.

CCCCXXVII. And, if the two lives are of the ages of 80 years and 85 years, the exact value of the annuity will be £1.595,707; and the near value of it will be $= \left(\frac{\pounds 2.057,238 + \pounds 0.909,073}{2} = \frac{\pounds 2.966,311}{2} =\right)$

 £1.483,155;

£1.483,155; which differs from the exact value, £1.595,707, by £0.112,552, which is less than the 14th part of the said exact value.

Conclusions from the foregoing examples.

First conclusion. CCCCXXVIII. From these examples we may conclude that, when the difference of the ages of two lives is 5 years, and the age of the older of them is not greater than 75 years, the near value of an annuity of one pound a year for the joint continuance of both lives, that is derived, by the foregoing method of Interpolation, from the values, (given in Tables XXIX and XXXI,) of a like annuity for two joint lives both of the same age with the younger of the two proposed lives, and of a like annuity for two joint lives whose ages differ by 10 years, and of which the younger is of the same age with the younger of the two proposed lives; —— I say, we may conclude that the said near value of the said annuity, so obtained, will differ from the true value of the same annuity by less than the 72d part of the said true value; and that, while the age of the older life is not greater than 65 years, the difference of the said near and true values will be less than an 103d part of the said true value. For, I presume, it cannot be doubted that, what we have found to be true concerning the differences of these values in the foregoing examples of two lives of the ages, of 10 years and 15 years, of 20 years and 25 years, of 30 years and 35 years, of 40 years and 45 years, of 50 years and 55 years, of 60 years and 65 years, of 70 years and 75 years, and of 80 years and 85 years, will be true with respect to all other lives of ages that lie between the ages herein specified, and which differ from each other by the same difference of 5 years; as, for example, of two lives of the ages of 13 years and [illegible] years, 23 years and 28 years, 33 years and 38 years, 43 years a[illegible] years, &c. But, if any person should doubt of this conclusion, [illegible] easily satisfy himself of the truth of it by applying this method of interpolation to the discovery of the near values of as many of these latter annuities as he shall think fit, by means of Tables XXIX and XXXI, in the manner above exemplified in the preceeding articles, and then comparing the near values, thereby obtained, with the true values of the same annuities exhibited in Table XXX.

Second conclusion. CCCCXXIX. And, secondly, we may conclude with a good degree of probability, that, since the near values of the annuities contained in Table XXX, for the joint continuance of two lives whose ages differ from each other by 5 years, which are obtained by this method of Interpolation, differ from the true values of the same annuities by such small quantities as a 72d, or a 103d, part, of the said true values, the near values of other annuities for two joint lives, where the difference of the ages is greater or

less

leſs than 5 years, obtained by the ſame method of Interpolation, will differ from their true values by almoſt as ſmall quantities, or by quantities not very different from the 72d part, or the 103d part, of the ſaid true values. For in both caſes the near values of the annuities ſought are obtained by ſuppoſing that the principle laid down in Art. CCCCXII is nearly true, or that the values of eleven annuities of one pound a year for the joint continuance of eleven pairs of lives of the following ages, to wit,

N years and N years,
N years and $N+1$ years,
N years and $N+2$ years,
N years and $N+3$ years,
N years and $N+4$ years,
N years and $N+5$ years,
N years and $N+6$ years,
N years and $N+7$ years,
N years and $N+8$ years,
N years and $N+9$ years,
and N years and $N+10$ years,

form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences. If therefore we find upon trial (as we have done in the foregoing examples,) that the near value of an annuity for two joint lives of the ages of N and $N+5$ years obtained in this manner, differs but by a ſmall quantity from its true value, we may conclude with a high degree of probability, that the near values of the other intermediate annuities, (as, for example, of annuities for two joint lives of the ages of N years and $N+3$ years or of the ages of N years and $N+7$ years,) that are obtained in the ſame manner, will differ from their true values either by as ſmall quantities, or by quantities that will be very little greater than the ſaid difference between the near value and the true value of the ſaid annuity for two joint lives of the ages of N years and $N+5$ years, which is the middlemoſt annuity of the whole eleven.

CCCCXXX. And, if this concluſion be juſt, (which, I think, it is hardly poſſible to doubt of,) the near values of annuities for two joint lives, obtained by this method of Interpolation, may be conſidered as differing from their true values by only about the 72d part of the ſaid true values, when the age of the older life is not greater than 75 years, and by only about the 103d part of the ſaid true values when the age of the older life is not greater than 65 years, and by a much ſmaller part of the ſaid true values when the age of the older life is only 30 or 20 years. Third concluſion.

CCCCXXXI. Now

The values of annuities for two joint lives, obtained by the foregoing method of Interpolation, are sufficiently exact for ordinary purposes.

CCCCXXXI. Now either the 103d part of the true value of an annuity, or the 72d part of it, is too small a difference to be of much importance in the bargains that are made for the purchase of life-annuities. And consequently this method of finding the values of annuities for two joint lives by Interpolation between the values of other contiguous annuities that have been already computed, may justly be considered as a very useful and a sufficient supplement to the want of such compleat tables of the values of these joint annuities, adapted to all the possible differences of ages in the two lives, as are mentioned in Art. ccccx.

End of the explanation and illustration of the foregoing method of finding the values of annuities for two joint lives by Interpolation.

CCCCXXXII. I shall now present the reader with another set of tables of the values of annuities for two joint lives, of the same kind as those above exhibited in Art. ccccviii, and which I have procured to be computed, (like the former,) under the inspection of the learned Mr. Morgan, from Monsieur de Parcieux's table of the probabilities of the duration of human life, upon a supposition that the interest of money is $4\frac{1}{2}$ per cent. These tables are as follows.

TABLE

TABLE XXXIX.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons of the same age, when the interest of money is 4½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the first life.	Years in the age of the second life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	3	13.570,765	34	34	12.659,86	65	65	5.423,067
4	4	14.072,22	35	35	12.536,31	66	66	5.123,340
5	5	14 395,92	36	36	12.407,73	67	67	4.834,907
6	6	14.031,71	37	37	12.273,92	68	68	4.559,660
7	7	14.795,55	38	38	12.095,25	69	69	4.300,488
8	8	14.910,24	39	39	11.907,44	70	70	4.061,770
9	9	15.004,20	40	40	11.709,84	71	71	3.816,915
10	10	15.037,76	41	41	11.501,77	72	72	3.599,136
11	11	15.004,12	42	42	11.282,46	73	73	3.384,355
12	12	14.897,33	43	43	11.051,13	74	74	3.175,568
13	13	14.785,69	44	44	10.806,91	75	75	2.977,377
14	14	14.668,91	45	45	10.548,84	76	76	2.757,619
15	15	14.546,70	46	46	10.275,91	77	77	2.549,448
16	16	14.418,73	47	47	10.023,25	78	78	2.362,120
17	17	14.321,26	48	48	9.755,94[illegible]	79	79	2.165,060
18	18	14.219,84	49	49	9.508,360	80	80	2.005,385
19	19	14.114,20	50	50	9.246,458	81	81	1.860,452
20	20	14.004,10	51	51	9.003,958	82	82	1.744,983
21	21	13.926,24	52	52	8.782,414	83	83	1.613,537
22	22	13.846,16	53	53	8.549,082	84	84	1.441,788
23	23	13.763,77	54	54	8.302,844	85	85	1.276,351
24	24	13.678,93	55	55	8.076,876	86	86	1.128,148
25	25	13.591,50	56	56	7.839,037	87	87	1.024,200
26	26	13.501,34	57	57	7.588,114	88	88	0.859,738
27	27	13.408,28	58	58	7.356,793	89	89	0.698,585
28	28	13.312,16	59	59	7.113,508	90	90	0.544,507
29	29	13.212,80	60	60	6.856,915	91	91	0.405,106
30	30	13.109,99	61	61	6.585,462	92	92	2.296,467
31	31	13.003,53	62	62	6.297,341	93	93	0.239,234
32	32	12.893,19	63	63	6.023,53[illegible]	94	94	0.000,000
33	33	12.778,72	64	64	5.732,893			

TABLE XL.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 5 years; when the interest of money is 4½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	8	14.219,29	32	37	12.556,49	61	66	5.756,635
4	9	14.525,27	33	38	12.407,81	62	67	5.466,931
5	10	14.707,65	34	39	12.252,17	63	68	5.191,171
6	11	14.810,70	35	40	12.089,10	64	69	4.917,419
7	12	14.839,89	36	41	11.918,07	65	70	4.646,949
8	13	14.840,94	37	42	11.738,52	66	71	4.377,326
9	14	14.828,32	38	43	11.531,14	67	72	4.127,800
10	15	14.782,57	39	44	11.312,59	68	73	3.885,426
11	16	14.700,60	40	45	11.082,06	69	74	3.653,183
12	17	14.598,23	41	46	10.838,70	70	75	3.435,590
13	18	14.491,45	42	47	10.600,65	71	76	3.203,078
14	19	14.379,98	43	48	10.349,18	72	77	2.988,987
15	20	14.263,55	44	49	10.102,06	73	78	2.788,448
16	21	14.160,62	45	50	9.840,830	74	79	2.585,274
17	22	14.071,49	46	51	9.582,860	75	80	2.408,860
18	23	13.979,20	47	52	9.345,367	76	81	2.231,987
19	24	13.883,53	48	53	9.094,623	77	82	2.07[illegible],853
20	25	13.784,30	49	54	8.846,134	78	83	1.917,4[illegible]0
21	26	13.699,50	50	55	8.601,572	79	84	1.730,370
22	27	13.612,1[illegible]	51	56	8.359,590	80	85	1.561,668
23	28	13.521,96	52	57	8.120,294	81	86	1.408,369
24	29	13.428,91	53	58	7.885,844	82	87	1.291,505
25	30	13.332,79	54	59	7.638,860	83	88	1.129,847
26	31	13.233,41	55	60	7.393,970	84	89	0.953,637
27	32	13.130,57	56	61	7.135,517	85	90	0.781,712
28	33	13.024,06	57	62	6.861,983	86	91	0.621,495
29	34	12.913,66	58	63	6.605,[illegible]43	87	92	0.489,284
30	35	12.799,11	59	64	6.333,493	88	93	0.347,9[illegible]8
31	36	12.680,15	60	65	6.045,498	89	94	0.000,000

TABLE XLI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 10 years; when the interest of money is $4\frac{1}{2}$ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	13	13.534,84	31	41	12.132,94	59	69	5.339,366
4	14	14.344,57	32	42	11.959,77	60	70	5.087,882
5	15	14.446,47	33	43	11.777,89	61	71	4.827,611
6	16	14.498,41	34	44	11.586,69	62	72	4.578,320
7	17	14.528,31	35	45	11.385,50	63	73	4.336,530
8	18	14.531,10	36	46	11.173,61	64	74	4.092,576
9	19	14.520,95	37	47	10.969,90	65	75	3.848,068
10	20	14.478,80	38	48	10.737,84	66	76	3.593,607
11	21	14.420,71	39	49	10.512,31	67	77	3.350,952
12	22	14.326,17	40	50	10.274,39	68	78	3.126,500
13	23	14.227,96	41	51	10.042,43	69	79	2.902,025
14	24	14.125,85	42	52	9.816,964	70	80	2.709,443
15	25	14.019,61	43	53	9.579,45	71	81	2.523,916
16	26	13.908,99	44	54	9.328,882	72	82	2.365,251
17	27	13.811,42	45	55	9.083,299	73	83	2.194,845
18	28	13.710,22	46	56	8.824,214	74	84	1.999,081
19	29	13.605,12	47	57	8.566,170	75	85	1.811,170
20	30	13.495 90	48	58	8.312,358	76	86	1.627,327
21	31	13.400,16	49	59	8.059,770	77	87	1.473,044
22	32	13.301,14	50	60	7.793,079	78	88	1.279,463
23	33	13.198,65	51	61	7.525,77[illegible]	79	89	1.081,750
24	34	13.092,47	52	62	7.257,462	80	90	0.895,079
25	35	12.982,38	53	63	6.992,041	81	91	0.717,24[illegible]
26	36	12.868,14	54	64	6.711,295	82	92	0.558,568
27	37	12.749,48	55	65	6.427,546	83	93	0.397,601
28	38	12.605,79	56	66	6.144,923	84	94	0.000,000
29	39	12.455.44	57	67	5.863,955			
30	40	12.297,99	58	68	5.597,977			

TABLE XLII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 20 *years; when the interest of money is* 4½ *per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.*

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£			£
3	23	13.566,18	27	47	11.235,80	51	71	5.249,784
4	24	13.764,63	28	48	11.025,14	52	72	5.006,611
5	25	13.869,97	29	49	10.823,13	53	73	4.761,978
6	26	13.928,96	30	50	10.610,56	54	74	4.517,666
7	27	13.950,52	31	51	10.406,55	55	75	4.286,359
8	28	13.946,14	32	52	10.212,00	56	76	4.037,427
9	29	13.929,47	33	53	10.008,01	57	77	3.794,414
10	30	13.882,15	34	54	9.793,822	58	78	3.572,788
11	31	13.801,26	35	55	9.588,700	59	79	3.343,173
12	32	13.684,05	36	56	9.373,704	60	80	3.139,598
13	33	13.561,85	37	57	9.148,020	61	81	2.943,841
14	34	13.434,35	38	58	8.916,202	62	82	2.764,131
15	35	13.301,20	39	59	8.672,806	63	83	2.572,536
16	36	13.162,05	40	60	8.416,828	64	84	2.345,809
17	37	13.033,30	41	61	8.147,138	65	85	2.119,938
18	38	12.878,23	42	62	7.862,472	66	86	1.908,778
19	39	12.715,57	43	63	7.581,639	67	87	1.728,597
20	40	12.544,83	44	64	7.285,200	68	88	1.497,798
21	41	12.382,05	45	65	6.971,573	69	89	1.269,895
22	42	12.211,23	46	66	6.659,066	70	90	1.048,546
23	43	12.031,83	47	67	6.360,345	71	91	0.834,287
24	44	11.843,25	48	68	6.065,302	72	92	0.638,300
25	45	11.644,84	49	69	5.786,990	73	93	0.440,344
26	46	11.435,90	50	70	5.517,470	74	94	0.000,000

TABLE XLIII.

Containing the va[illegible] of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 30 years; when the interest of money is 4½ per cent.----Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	33	12.892,77	24	54	9.910,650	45	75	4.518,077
4	34	13.047,91	25	55	9.702,397	46	76	4.247,671
5	35	13.112,32	26	56	9.484,074	47	77	3.991,243
6	36	13.130,47	27	57	9.254,869	48	78	3.748,011
7	37	13.110,84	28	58	9.034,348	49	79	3.502,708
8	38	13.043,28	29	59	8.803,302	50	80	3.284,035
9	39	12.959,63	30	60	8.560,832	51	81	3.079,666
10	40	12.842,64	31	61	8.305,940	52	82	2.899,153
11	41	12.689,53	32	62	8.037,502	53	83	2.699,676
12	42	12.497,79	33	63	7.774,945	54	84	2.464,370
13	43	12.296,06	34	64	7.498,687	55	85	2.237,648
14	44	12.083,66	35	65	7.207,393	56	86	2.022,653
15	45	11.859,89	36	66	6.920,335	57	87	1.835,846
16	46	11.623,98	37	67	6.638,708	58	88	1.596,108
17	47	11.410,32	38	68	6.353,241	59	89	1.356,043
18	48	11.185,18	39	69	6.076,191	60	90	1.119,059
19	49	10.967,99	40	70	5.810,588	61	91	0.890,743
20	50	10.739,19	41	71	5.538,183	62	92	0.677,405
21	51	10.532,33	42	72	5.282,170	63	93	0.462,633
22	52	10.334,98	43	73	5.025,29[illegible]	64	94	0.000,000
23	53	10.128,01	44	74	4.769,599			

IMAGE EVALUATION
TEST TARGET (MT-3)

6"

2.8
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3.2
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TABLE XLIV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 40 years; when the interest of money is 4½ per cent.---Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£			£
3	43	11.664,55	21	61	8.361,570	39	79	3.606,689
4	44	11.706,30	22	62	8.087,980	40	80	3.390,203
5	45	11.657,84	23	63	7.820,090	41	81	3.183,644
6	46	11.559,58	24	64	7.538,184	42	82	2.996,187
7	47	11.439,70	25	65	7.240,908	43	83	2.789,655
8	48	11.288,63	26	66	6.947,583	44	84	2.547,150
9	49	11.138,05	27	67	6.659,348	45	85	2.308,580
10	50	10.953,87	28	68	6.377,813	46	86	2.082,011
11	51	10.754,12	29	69	6.105,244	47	87	1.888,493
12	52	10.538,20	30	70	5.844,810	48	88	1.636,142
13	53	10.311,44	31	71	5.578,317	49	89	1.386,792
14	54	10.073,02	32	72	5.329,295	50	90	1.140,577
15	55	9.842,622	33	73	5.080,618	51	91	0.905,794
16	56	9.600,677	34	74	4.834,663	52	92	0.689,009
17	57	9.358,650	35	75	4.594,865	53	93	0.468,882
18	58	9.124,664	36	76	4.338,334	54	94	0.000,000
19	59	8.879,210	37	77	4.090,833			
20	60	8.621,334	38	78	3.852,445			

TABLE XLV.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 50 years; when the interest of money is 4½ per cent.---- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£
3	53	9.769,927	24	74	4.848,025
4	54	9.740,534	25	75	4.603,719
5	55	9.652,684	26	76	4.342,180
6	56	9.522,340	27	77	4.089,075
7	57	9.355,740	28	78	3.851,485
8	58	9.181,316	29	79	3.606,634
9	59	8.989,406	30	80	3.391,199
10	60	8.767,436	31	81	3.185,9[illegible]7
11	61	8.5[illegible]3,132	32	82	3.000,036
12	62	8.224,342	33	83	2.795,513
13	63	7.940,830	34	84	2.555,537
14	64	7.642,510	35	85	2.320,372
15	65	7.327,995	36	86	2.098,610
16	66	7.016,758	37	87	1.907,556
17	67	6.718,992	38	88	1.655,071
18	68	6.427,600	39	89	1.403,200
19	69	6.144,730	40	90	1.155,588
20	70	5.873,408	41	91	0.918,076
21	71	5.603,355	42	92	0.697,208
22	72	5.350,681	43	93	0.473,205
23	73	5.098,132	44	94	0.000,000

TABLE XLVI.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 60 years; when the interest of money is 4½ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.	Years in the age of the younger life.	Years in the age of the older life.	Values of an annuity of one pound a year for the joint continuance of both lives.
		£			£
3	63	7.519,867	20	80	3.404,081
4	64	7.378,607	21	81	3.197,263
5	65	7.169,216	22	82	3.009,860
6	66	6.938,290	23	83	2.803,637
7	67	6.693,300	24	84	2.561,759
8	68	6.442,915	25	85	2.324,537
9	69	6.196,954	26	86	2.100,434
10	70	5.950,820	27	87	1.906,501
11	71	5.685,410	28	88	1.654,218
12	72	5.423,923	29	89	1.402,532
13	73	5.162,329	30	90	1.155,084
14	74	4.902,882	31	91	0.917,714
15	75	4.648,840	32	92	0.696,969
16	76	4.376,827	33	93	0.473,078
17	77	4.118,650	34	94	0.000,000
18	78	3.875,876			
19	79	3.625,462			

TABLE XLVII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 70 years; when the interest of money is 4½ per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	73	4.898,531	14	84	2.582,455
4	74	4.734,189	15	85	2.340,580
5	75	4.541,851	16	86	2.111,582
6	76	4.316,868	17	87	1.915,651
7	77	4.088,644	18	88	1.661,118
8	78	3.868,953	19	89	1.407,168
9	79	3.639,892	20	90	1.157,288
10	80	3.433,726	21	91	0.919,296
11	81	3.230,664	22	92	0.698,015
12	82	3.039,329	23	93	0.473,624
13	83	2.828,899	24	94	0.000,000

TABLE XLVIII.

Containing the values of an annuity of one pound a year for the joint continuance of the lives of two persons whose ages differ from each other by 80 *years; when the interest of money is* $4\frac{1}{2}$ *per cent.--- Computed from Monsieur de Parcieux's table of the probabilities of the duration of human life.*

Years in the age of the younger life.	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*	*Years in the age of the younger life.*	*Years in the age of the older life.*	*Values of an annuity of one pound a year for the joint continuance of both lives.*
		£			£
3	83	2.702,580	9	89	1.405,610
4	84	2.503,721	10	90	1.160,806
5	85	2.290,611	11	91	0.923,697
6	86	2.082,129	12	92	0.700,914
7	87	1.897,819	13	93	0.475,131
8	88	1.651,923	14	94	0.000,000

CCCCXXXIII. According

CCCCXXXIII. According to Monſieur de Parcieux's table of the probabilities of the duration of human life (which begins with the age of 3 years, and ends with the age of 94 years, and ſuppoſes the utmoſt poſſible extent of human life to be ſomewhat leſs than 95 years,) it is evident that only two pairs of lives can be found whoſe ages will differ from each other by 90 years, to wit, two lives of the ages of 3 years and 93 years, and two lives of the ages of 4 years and 94 years. And an annuity for the joint continuance of this latter pair of lives cannot, according to this table of probabilities, be of any value, becauſe it is ſuppoſed to be certain, according to this table, that the older life, to wit, the life of 94 years, will be extinct before the end of the year, or before the payment of the annuity will become due. Therefore the only two lives, whoſe ages differ from each other by 90 years, for the joint continuance of which an annuity can be of any value, are two lives of the ages of 3 years and 93 years. And the value of an annuity of one pound a year for the joint continuance of two lives of theſe ages, when the intereſt of money is $4\frac{1}{2}$ per cent. is £0.464,115.

The value of an annuity of one pound a year for two joint lives of the ages of 3 years and 93 years.

CCCCXXXIV. When the intereſt of money is $4\frac{1}{2}$ per cent. and the difference of the ages of the two lives is not either 0, or 5 years, or 10 years, or 20 years, or 30 years, or 40, 50, 60, 70, 80, or 90, years, but ſome intermediate number of years lying between ſome two of theſe differences that are contiguous to each other, we muſt have recourſe to the method of interpolation above explained in Art. CCCCXIII, in order to obtain a near value of an annuity of one pound a year for their joint continuance; as we did above when the intereſt of money was ſuppoſed to be $3\frac{1}{2}$ per cent. For this method of interpolation will be equally applicable to the diſcovery of theſe near values of annuities for two joint lives at one rate of the intereſt of money as at another.

When the value of an annuity for two joint lives, (upon a ſuppoſition that the intereſt of money is $4\frac{1}{2}$ per cent) is not contained in any of the foregoing tables, recourſe muſt be had to the method of *interpolation* above-deſcribed.

CCCCXXXV. We may therefore, by the help of the two foregoing ſets of tables, together with the method of interpolation above-explained, diſcover either the true values, or tolerably near values, of all annuities for the joint continuance of two lives of any ages whatſoever, when the intereſt of money is either $3\frac{1}{2}$ per cent. or $4\frac{1}{2}$ per cent.

End of the directions for finding the values of annuities for two joint lives when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.

Of the values of annuities for two joint lives, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent.

Theſe values may alſo be found to a tolerable degree of exactneſs by means of the two foregoing ſets of tables.

The conjectural ſuppoſition by means of which the ſaid values may be deduced from the values of the ſame annuities when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. which are given in the two foregoing ſets of tables.

CCCCXXXVI. And from the values of theſe annuities for two joint lives when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. we may find tolerably near values of the ſame annuities when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. by ſuppoſing that the values of an annuity for the joint continuance of any two given lives at the five following different rates of intereſt, to wit, 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent. (which differ from each other by only $\frac{1}{2}$ per cent.) form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences. For, if this ſuppoſition be true, the third value of the annuity, or that which it has when the intereſt of money is 4 per cent. will be nearly equal to an arithmetical mean proportional between the ſecond value of it, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. and the fourth value of it, or that which it has when the intereſt of money is $4\frac{1}{2}$ per cent. which two extreme values may be found in the two foregoing ſets of tables. We, therefore, need only ſubtract the fourth value of the annuity, or that which it has when the intereſt of money is $4\frac{1}{2}$ per cent. from the ſecond value of it, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. and divide the remainder by 2; and, if the quotient, thence ariſing, be ſubtracted from the ſecond value of the annuity, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. the remainder will be nearly equal to the third value of it, or that which it has when the intereſt of money is 4 per cent. And, in like manner, if we ſubtract the ſaid quotient from the fourth value of the annuity, or that which it has when the intereſt of money is $4\frac{1}{2}$ per cent. the remainder will be the fifth value of the annuity, or that which it has when the intereſt of money is 5 per cent. and, if we add the ſaid quotient to the ſecond value of the annuity, or that which it has when the intereſt of money is $3\frac{1}{2}$ per cent. the ſum will be equal to the firſt value of it, or that which it has when the intereſt of money is 3 per cent.

An example of the method of deducing the former values from the latter.

Thus, for example, if it were required to aſſign the values of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. by means of the two foregoing ſets of tables, which exhibit the values of this annuity only when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. we muſt proceed in the following manner.

The value of an annuity of one pound a year for the joint continuance of two lives of the ages of 25 years and 35 years, when the intereſt of money is $3\frac{1}{2}$ per cent. appears by Table XXXI, page 494, to be = £14.601,50;

£14.601,50; and the value of the ſame annuity, when the intereſt of money is 4½ per cent. appears by Table XLI to be = £12.982,38. The difference between £14.601,50 and £12.982,38 is £1.619,12; and half this difference is £0.809,56. Therefore the value of this annuity when the intereſt of money is 4 per cent. will be nearly equal to £14.601,50 — £0.809,56, or £13.791,94; and the value of it when the intereſt of money is 5 per cent. will be nearly equal to £12.982,38 — £0.809,56, or £12.172,82; and the value of it when the intereſt of money is 3 per cent. will be nearly equal to £14.601,50 + £0.809,56, or £15.411,06. Therefore the values of an annuity of one pound a year for the joint continuance of two lives of the ages of 25 years and 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. are nearly equal to £15.411,06, £13.791,94, and £12.172,82. Q. E. I.

Of the ſimilarity of this method of deducing the ſaid former values from the latter, to the method of interpolation deſcribed above in Art. 413.

CCCCXXXVII. In the foregoing article we have ſhewn how to derive a near value of any given annuity upon a ſuppoſition that the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. from the two values of the ſame annuity when the intereſt of money is 3½ per cent. and 4½ per cent. which two values are either given exactly in ſome of the tables of the two foregoing ſets of tables, or may be derived, by the aforeſaid method of Interpolation, from the values which are there ſet down. And the method of doing this (and which we have deſcribed in the laſt article,) is very ſimilar to the aforeſaid method of Interpolation (deſcribed above in Art. CCCCXIII,) and may itſelf likewiſe be called with propriety *a method of Interpolation*, as it proceeds (like the former method,) on a ſuppoſition that the unknown value of the propoſed annuity, when the intereſt of money is 4 per cent. is nearly equal to an arithmetical mean between the two known values of the ſame annuity when the intereſt of money is 3½ per cent. and 4½ per cent. or conſiſts in the *interpolation*, or *interpoſition*, of an arithmetical mean between thoſe two known values. Indeed, as to the manner of determining the other two unknown values of the propoſed annuity, or thoſe which it has when the intereſt of money is 3 per cent. and 5 per cent. the word *Interpolation* is not quite ſo proper for it; becauſe they are not found by *interpolating*, or *interpoſing*, any new terms between the two known values of the ſaid annuity (which it has when the intereſt of money is 3½ per cent. and 4½ per cent.) but by adding new terms, at both ends, to the arithmetical progreſſion, (conſiſting of three terms) whereof the two known values of the annuity (which it has when the intereſt of money is 3½ per cent. and 4½ per cent.) are the firſt and laſt terms, and the arithmetical mean between thoſe two known values is the middle term. With reſpect, therefore, to theſe values of the propoſed annuity, which it has when the intereſt of money is 3 per cent. and 5 per cent. and which are thus determined to a moderate degree of exactneſs by *continuing* the aforeſaid arithmetical progreſſion (conſiſting of the aforeſaid three terms,) at both its ends, we ought rather to call this method of proceeding *the method of Continuation* than *the method of Interpolation*. And therefore, perhaps, it may be proper to call this whole method of finding all the ſaid three unknown values of the propoſed annuity, (which it has when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent.) taken together, *the method of Interpolation and Continuation*. But, by whatever name we call them, the two parts of this method are both founded on the ſame principle, or ſuppoſition, to wit, "that, when three rates of the intereſt of money are taken that are ſucceſſively greater the one than the other by only one half per cent. (as either the rates of 3 per cent. $3\frac{1}{2}$ per cent. and 4 per cent. or of $3\frac{1}{2}$ per cent. 4 per cent. and $4\frac{1}{2}$ per cent. or of 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent.) the values of an annuity for two joint lives of any given ages at theſe three rates of intereſt, will form, pretty nearly, an arithmetical progreſſion, or will decreaſe by nearly equal differences;" which ſuppoſition (though I do not know any method of demonſtrating it,) has ſo great an appearance of probability that, I imagine, it is hardly poſſible to doubt the truth of it.

This method may be called, with propriety, *the method of interpolation and continuation.*

Of the degree of exactneſs *to which the near values of annuities for two joint lives, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. that are deduced, by the foregoing* method of Interpolation and Continuation, *from the values of the ſame annuities when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. may, upon reaſonable grounds, be ſuppoſed to be true.*

CCCCXXXVIII. But, though it can hardly be doubted that the near values of an annuity for two joint lives of any given ages when the intereſt of money is 3 per cent. and 4 per cent. and 5 per cent. that are obtained by the foregoing method of *Interpolation and Continuation* from the values of the ſame annuity when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. are not very different from their true values, yet it would be deſireable (if it can be done,) to diſcover to *what degree of exactneſs* we may conſider theſe near values of the propoſed annuity as agreeing, or coinciding, with its correſpondent true values.

CCCCXXXIX. Now, to do this in the moſt ſatisfactory manner, it would be neceſſary to compute the exact values of an annuity of one pound a year for ſeven or eight pairs of joint lives of different ages when the

the intereſt of money is 3 per cent. and 4 per cent. and 5 per cent. and then to compare the ſaid exact values with the near values of the ſame annuity, at the ſame rates of the intereſt of money, that are derived, by means of the foregoing method of Interpolation and Continuation, from the values of the ſame annuity when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. which are given in the two foregoing ſets of tables: upon which compariſon I have no doubt that the differences between the ſaid near values and exact values would be found to be very inconſiderable. But this compariſon I am not at preſent able to make, not being poſſeſſed of the exact values of any annuities for two joint lives at the intereſt of 3 per cent. 4 per cent or 5 per cent. computed from Monſieur de Parcieux's table of the probabilities of the duration of human life. And I do not think it an object of ſufficient importance to make it worth while, on this account only, to procure any of theſe exact values to be computed.

CCCCXL. But we may form a very probable conjecture concerning the degree of exactneſs to which the aforeſaid near values of annuities for two joint lives at 3 per cent. 4 per cent. and 5 per cent. (which are obtained, by the foregoing method of *interpolation and continuation*, from the values of the ſame annuities at the intereſts of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) agree, or co-incide. with their exact values, by ſuppoſing what, I imagine, it is hardly po[illegible]ubt of, to wit, that there is ſuch an analogy between the valu[illegible]ties for two joint lives and the values of annuities for ſingle li[illegible] ages with the older of the two joint lives, that, if the near v[illegible]ties for ſingle lives at the intereſts of 3 per cent. 4 per cent. a[illegible] (that are obtained from the exact values of the ſame annuitie[illegible]reſts of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method o[illegible]*polation and continuation*,) are found to agree, or co-incide, with their true values to a certain degree of exactneſs, ſo as (for example,) to differ from the ſaid true values by only a 200th, or an 100th, part of the ſaid true values, it may be concluded that the near values of annuities for two joint lives (of which the older is of the ſame age with the ſingle life with which it is compared,) at the ſame intereſts of 3 per cent. 4 per cent. and 5 per cent. (that are obtained, by the ſaid method of interpolation and continuation, from the exact values of the ſame annuities at the intereſts of $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) will likewiſe agree, or co-incide, with their true values to the ſame, or very nearly the ſame, degree of exactneſs as the near values and the true values of the correſponding annuities for ſingle lives were found to do, or ſo as to differ from the ſaid true values by only about a 200th, or a 100th, part of the ſaid true values. Thus, for example, if we ſhould derive the value of an annuity of one pound a year for a ſingle life of the age of 35 years, when the intereſt of money is 4 per cent. from the two values of the

A probable method of diſcovering the ſaid degree of exactneſs.

the ſame annuity for the ſame life when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of interpolation, or by taking an arithmetical mean between thoſe two values; and, upon comparing this near value of the ſaid annuity with its exact value, (which is contained above in Table XVI, page 225,) we ſhould find the difference between them to be only a 200th, or a 100th, part of the ſaid exact value; we may conclude that the near value of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years, when the intereſt of money is 4 per cent. that is derived from the exact values of the ſame annuity for the ſame two joint lives, when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the foregoing method of interpolation, or by taking an arithmetical mean between the ſaid two exact values, will alſo differ from the true value of the ſame annuity by only about a 200th, or a 100th, part of the ſaid true value. If this concluſion be allowed to be juſt, (and it certainly appears ſo highly probable, that it ſeems difficult to doubt of the truth of it,) we may then try the degree of exactneſs to which this method of *interpolation and continuation* exhibits the values of annuities for two joint lives when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. by applying it to the determination of the values of the like annuities for ſingle lives (of the ſame ages as the older lives in the correſponding joint lives,) when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. and comparing the near values of the ſaid annuities for ſingle lives, thereby obtained, with the exact values of the ſame annuities for ſingle lives at the ſame rates of intereſt, which are exhibited above in Tables XIV, XVI, and XVIII. Of this I ſhall now proceed to give a few inſtances.

An example of the ſaid method.

CCCCXLI. In the firſt place therefore, let us ſuppoſe that the two lives for whoſe joint continuance an annuity of one pound a year is to be granted, are of the ages of 25 years and 35 years.

We have ſeen in Art. CCCCXXXVI, that the two exact values of this annuity, when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. are £14.601,50 and £12.982,38, and that the near values of the ſame annuity, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. that are derived from the ſaid two exact values by the foregoing method of interpolation and continuation, are £15.411,06, £13.791,94, and £12.172,82. We are therefore to endeavour to determine, (by a reaſonable conjecture founded on the analogy juſt now ſuppoſed to take place between the values of annuities for joint lives and the values of annuities for ſingle lives,) *to what degree of exactneſs* theſe three near values, £15.411,06, £13.791,94, and £12.172,82, of the aforeſaid annuity when the

the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. agree, or co-incide, with the true values of the ſame annuity at the ſame rates of intereſt. Now, in order to lay a ground for a conjecture of this kind, we may proceed as follows.

The value of an annuity of one pound a year for a ſingle life of the age of 35 years, when the intereſt of money is 3 per cent. appears by Table XIV, page 223, to be £18.464,13; and the values of the ſame annuity, when the intereſt of money is 3½ per cent. 4 per cent. 4½ per cent. and 5 per cent. appear by Tables XV, XVI, XVII, and XVIII, to be £17.206,612, £16.084,014, £15.078,39, and £14.174,54. Theſe are the true, or exact, values of this annuity at theſe five different rates of intereſt.

Now let the firſt, third, and fifth values of this annuity be derived from the ſecond and fourth values of it, to wit, £17.206,612 and £15.078,39, by the aforeſaid method of *interpolation and continuation.* This may be done in the manner following. The exceſs of the ſecond term, £17.206,612, of this progreſſion above the fourth term, £15.078,39, is = £2.128,222; and half this difference = £1.064,111. Therefore the firſt of the ſaid five terms (which are ſuppoſed, for the preſent purpoſe, to conſtitute a decreaſing arithmetical progreſſion,) will be = £17.206,612 + £1.064,111, or £18.270,723; and the third term will be = £17.206,612 — £1.064,111, or £16.142,501; and the fifth term will be £15.078,39 — £1.064,111, or £14.014,279; and conſequently the values of the propoſed annuity of one pound a year for the life of a perſon of the age of 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. will be nearly equal to £18.270,723, £16.142,501, and £14.014,279. Now, if we compare theſe three near values of this annuity, at theſe three different rates of intereſt, with the three exact values of the ſame annuity at the ſame rates of intereſt, (which we have ſeen to be £18.464,13, £16.084,014, and £14.174,54,) we ſhall find, that the firſt near value, to wit, £18.270,723, is leſs than the correſponding true value, £18.464,13, by the difference £0.193,407, which is leſs than the 95th part of the ſaid true value; and that the ſecond near value, to wit, £16.142,501, is greater than the correſponding true value, £16.084,014, by the difference £0.058,487, which is leſs than the 275th part of the ſaid true value; and that the third near value, £14.014,279, is leſs than the correſponding true value, £14.174,54, by the difference, £0.160,261, which is leſs than the 88th part of the ſaid true value. Thus it appears that the near values of an annuity of one pound a year for a ſingle life of the age of 35 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by this method of *interpolation and continuation*, differ from the correſponding true values of

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it by lefs than the 95th, the 275th, and the 88th, parts of the faid true values refpectively. And from hence we may conclude (by means of the analogy above-mentioned between the values of annuities for fingle lives and the values of annuities for joint lives,) that the near values of an annuity of one pound a year for two joint lives of the ages of 25 years and 35 years, when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained in Art. CCCCXXXV by the fame method of *interpolation and continuation*, to wit, £15.411,06, £13.791,94, and £12.172,82, will likewife differ from the correfponding true values of the fame annuity at thofe three rates of intereft, by only about the 95th part, the 275th part, and the 88th part, of the faid true values refpectively.

Another example of the faid method.

CCCCXLII. In the fecond place, Let us fuppofe the two lives to be of the ages of 25 years and 45 years.

The value of an annuity of one pound a year for two joint lives of thefe ages, when the intereft of money is $3\frac{1}{2}$ per cent. appears by Table XXXII, page 495, to be £12.902,34; and the value of the fame annuity for the fame joint lives, when the intereft of money is $4\frac{1}{2}$ per cent. appears by Table XLII to be £11.644,84. The difference of thefe values is £1.257,50; and half this difference is £0.628,75. Therefore, according to the foregoing method of *interpolation and continuation*, the value of the fame annuity for the fame joint lives, when the intereft of money is 3 per cent. will be nearly equal to (£12.902,34 + £0.628,75, or) £13.531,09; and the value of it, when the intereft of money is 4 per cent. will be nearly equal to (£12.902,34 — £0.628,75, or) £12.273,59; and the value of it, when the intereft of money is 5 per cent. will be nearly equal to (£11.644,84 — £0.628,75, or) £11.016,09.

Now, in order to form a probable conjecture concerning the degree of exactnefs to which thefe near values of the aforefaid annuity for two joint lives agree with its correfponding true values, let us derive the values of a like annuity of one pound a year for a fingle life of the age of 45 years when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. from the values of it when the intereft of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the fame method of *interpolation and continuation*; and then compare the near values, thereby obtained, with the correfpondent true values of the fame annuity at the fame rates of intereft, (to wit, 3 per cent. 4 per cent. and 5 per cent.) as exhibited above in Tables XIV, XVI, and XVIII. This may be done as follows.

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The value of an annuity of one pound a year for a life of the age of 45 years, when the intereft of money is $3\frac{1}{2}$ per cent. appears by Table XV to be £14.716,120; and the value of the fame annuity for the fame life, when the intereft of money is $4\frac{1}{2}$ per cent. appears by Table XVII to be £13.163,92. The difference of thefe values is £1.552,200; and half the faid difference is £0.776,100. Therefore the value of an annuity of one pound a year for a life of the age of 45 years, when the intereft of money is 3 per cent. will be nearly equal to (£14.716,120 + £0.776,100, or) £15.492,220; and the value of the fame annuity for the fame life, when the intereft of money is 4 per cent. will be nearly equal to (£14.716,120 — £0.776,100, or) £13.940,020; and the value of the fame annuity for the fame life, when the intereft of money is 5 per cent. will be nearly equal to (£13.163,92 — £0.776,100, or) £12.387,820.

Now the true values of this annuity, when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. appear by Tables XIV, XVI, and XVIII, to be £15.608,74, £13.904,190, and £12.487,42. Therefore the firft of the foregoing near values of the faid annuity, to wit, £15.492,220, falls fhort of the correfponding true value of it, £15.608,74, by the difference, £0.116,520, which is lefs than the 133d part of the faid true value; and the fecond near value of the faid annuity, to wit, £13.940,020, exceeds the correfponding true value of it, to wit, £13.904,190, by the difference, £0.035,830, which is lefs than the 388th part of the faid true value; and the third near value of the faid annuity, to wit, £12.387,820, is lefs than the correfponding true value of it, to wit, £12.487,42, by the difference, £0.099,600, which is lefs than the 125th part of the faid true value. We may therefore conclude (by means of the analogy above-mentioned between the values of annuities for fingle lives and the values of annuities for joint lives,) that the three near values of an annuity of one pound a year for two joint lives of the ages of 25 years and 45 years, when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained above by the faid method of *interpolation and continuation*, to wit, £13.531,09, £12.273,59, and £11.016,09, will differ from the correfponding true values of the fame annuity by only about the 133d part, the 388th part, and the 125th part, of the faid true values refpectively.

CCCCXLIII. As a third example of this method of determining by analogy the degree of exactnefs of the values of annuities for two joint lives, when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by the foregoing method of *interpolation and continuation*, let us fuppofe the two lives to be of the ages of 20 years and 70 years.

A third example of the fame method.

The value of an annuity of one pound a year for two joint lives of these ages, when the intereſt of money is $3\frac{1}{2}$ per cent. appears by Table XXXV, page 495, to be £6.197,247; and the value of the ſame annuity for the ſame joint lives, when the intereſt of money is $4\frac{1}{2}$ per cent. appears by Table XLV to be £5.873,408 The difference of theſe values is £0.323,839; and half this difference is £0.161,919. Therefore, according to the foregoing method of *interpolation and continuation*, the values of the ſame annuity for the ſame joint lives, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. will be nearly equal to (£6.197,247 + £0.161,919, or) £6.359,166, (£6.197,247 — £0.161,919, or) £6.035,328, and (£5.873,408 — £0.161,919, or) £5.711,489, reſpectively.

Now, in order to form a conjecture concerning the degree of exactneſs of theſe near values of an annuity of one pound a year for theſe two joint lives of the ages of 20 years and 70 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. we muſt derive the values of a like annuity for a ſingle life of the age of 70 years, at the ſame rates of the intereſt of money, from the values of it, when the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. by the ſame method of *interpolation and continuation*, and then compare the near values, thereby obtained, with the correſponding true values of the ſame annuity for a ſingle life of the ſame age of 70 years, at the ſame rates of intereſt, as they are exhibited above in Tables XIV, XVI, and XVIII. This may be done as follows.

The value of an annuity of one pound a year for a ſingle life of the age of 70 years, when the intereſt of money is $3\frac{1}{2}$ per cent. appears by Table XV to be £6.575,357; and the value of the ſame annuity for the ſame life, when the intereſt of money is $4\frac{1}{2}$ per cent. appears by Table XVII to be £6.220,54. The difference of theſe values is £0.354,817; and half the ſaid difference is £0.177,408. The ſum of £6.575,357 and £0.177,408 is £6.752,765; and the difference of £6.575,357 and £0.177,408 is £6.397,949; and the difference of £6.220,54 and £0.177,408 is £6.043,132. Therefore £6.752,765 will be a near value of an annuity of one pound a year for a ſingle life of the age of 70 years, when the intereſt of money is 3 per cent. and £6.397,949 will be a near value of the ſame annuity for the ſame life, when the intereſt of money is 4 per cent. and £6.043,132 will be a near value of the ſame annuity for the ſame life, when the intereſt of money is 5 per cent.

Now the true values of an annuity of one pound a year for a ſingle life of the age of 70 years, when the intereſt of money is 3 per cent. 4 per cent. and 5 per cent. appear by Tables XIV, XVI, and XVIII, to be £6.765,920, £6.393,749, and £6.055,280; which differ from the foregoing near values of the ſame annuity, obtained by the method of

interpolation

interpolation and continuation, to wit, £6.752,765, £6.397,949, and £6.043,132, by only £0.013,155, £0.004,200, and £0.012,148, which are lefs than the 514th part, the 1522d part, and the 498th part, of the faid true values £6.765,920, £6.393,749, and £6.055,280, refpectively. We may therefore conclude (by means of the analogy above-mentioned between the values of annuities for fingle lives and the values of annuities for joint lives,) that the three near values of an annuity of one pound a year for two joint lives of the ages of 20 years and 70 years, when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. which were obtained above by the faid method of *interpolation and continuation*, to wit, £6.359,166, £6.035,328, and £5.711,489, will differ from the correfponding true values of the fame annuity by only about the 514th, the 1522d, and 498th, part of the faid true values.

Conclufion drawn from the three foregoing examples, concerning the degree of exactnefs of the near values of annuities for two joint lives obtained by means of the foregoing method of Interpolation and Continuation.

CCCCXLIV. It appears from the three foregoing examples, that the near values of an annuity of one pound a year for a fingle life of either 35 years, or 45 years, or 70 years, of age, when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained by the foregoing method of *interpolation and continuation*, differ but little from the correfponding true values of the fame annuity; and further, that the differences of the near values of the faid annuity, when the intereft of money is 4 per cent. (which are obtained by *interpolation*, or by interpofing arithmetical means between the values of the fame annuity, when the intereft of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) from its correfpondent true values, are much fmaller than the differences of the near values of the faid annuity from its true values, when the intereft of money is 3 per cent. and 5 per cent. in which cafes the faid near values are not obtained by *interpolation*, but by *continuation*. Therefore, if the foregoing analogy between the values of annuities for fingle lives and the values of annuities for joint lives be allowed to fubfift, it will follow, that the near values of annuities for two joint lives, when the intereft of money is 3 per cent. 4 per cent. and 5 per cent. which are obtained, by the foregoing method of *interpolation and continuation*, from the values of the like annuities when the intereft of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. will likewife differ but little from the true values of the fame annuities; and that the differences of the near values of fuch annuities from their true values, will be ftill lefs, when the intereft of money is 4 per cent. than when it is 3 per cent, or 5 per cent. But in all the three rates of intereft the faid near values of thefe annuities will be *near enough* to their true values for moft of the common purpofes of bufinefs.

End of the explanation and illuftration of the foregoing method of Interpolation and Continuation.

Of

Of the values of annuities for two joint lives, when the intereſt of money is either lower than 3 *per cent. or higher than* 5 *per cent.*

CCCCXLV. We have ſeen in the foregoing articles how, by the help of the two ſets of tables of the values of annuities for two joint lives, given above in Art. CCCCVIII and CCCCXXXII, (which are founded on a ſuppoſition that the intereſt of money is $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent.) we may find tolerably near values of the ſame annuities when the intereſt of money is 3 per cent. or 4 per cent. or 5 per cent. by means of the foregoing method of *Interpolation and Continuation.* But what ſhall we do, may the reader aſk, when the intereſt of money is not 3, 4, or 5, per cent. but 2 per cent. or 6 per cent. or 7 per cent. or 8, or 9, or 10, per cent? Will the ſame method of Interpolation and Continuation enable us to find the values of annuities for two joint lives at theſe ſeveral rates of intereſt to a tolerable degree of exactneſs? or, in other words, may it be ſuppoſed that the values of an annuity of one pound a year for two joint lives of given ages at the ſeveral following rates of intereſt of money, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. $5\frac{1}{2}$ per cent. 6 per cent. $6\frac{1}{2}$ per cent. 7 per cent. $7\frac{1}{2}$ per cent. 8 per cent. $8\frac{1}{2}$ per cent. 9 per cent. $9\frac{1}{2}$ per cent. and 10 per cent. (as well as at the intereſts of 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. and 5 per cent.) will form, pretty nearly, an arithmetical progreſſion, or decreaſe by nearly equal differences? For, if this ſuppoſition is true, it is evident that, when any two of the terms of this progreſſion are known, all the other terms may be derived from them by additions, or ſubtractions, of the common difference, by which the terms decreaſe; and conſequently the values of the propoſed annuity at all the other rates of intereſt may be deduced in this manner from its values when the intereſt of money is $3\frac{1}{2}$ per cent. and 4 per cent. which are exhibited in the two foregoing ſets of tables. In anſwer to this queſtion I muſt obſerve, that it does not ſeem probable that this ſuppoſition will be nearly true in ſuch a variety of different rates of intereſt. For we have, in the laſt article, ſeen reaſon to conclude, that the near values of an annuity for two joint lives of given ages when the intereſt of money is 3 per cent. and 5 per cent. which are obtained by means of this ſuppoſition, differ much more from its correſponding true values than the near value of it, (obtained by means of the ſame ſuppoſition,) when the intereſt of money is 4 per cent. differs from its true value. And therefore we have reaſon to conclude, that the near values of the ſame annuity when the intereſt of money is $2\frac{1}{2}$ per cent. or 2 per cent. or $5\frac{1}{2}$ per cent. or 6 per cent, or 7, 8, 9, or 10 per cent. that would be obtained by means of this ſuppoſition, would differ ſtill more from its correſponding true values, and, probably, would differ from them too much to make it adviſeable to neglect the differences and conſider the ſaid near values, in practice, as equal to the correſponding true values: at leaſt we may well ſuppoſe this to be the caſe at the very

very

very high interests of 8, 9, and 10 per cent. It seems desirable, therefore, to discover, if possible, some other method of finding a tolerably near value of an annuity for two joint lives of given ages, when the interest of money is either lower than 3 per cent or higher than 5 per cent, that may exempt us from the necessity of computing its exact value by means of the expression £ $\frac{1}{P \times Q} \times$ the series $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5} + \frac{P^{VI} \times Q^{VI}}{r^6} + \frac{P^{VII} \times Q^{VII}}{r^7} +$ &c. continued to the end of the table of the probabilities of the duration of human life; the computation of which, when the two lives are young, is very tedious. Now one method of doing this to a moderate degree of exactness (though not to so great a degree of exactness as might be desired) is as follows.

A method of approximating to the value of an annuity of one pound a year for the joint continuance of two lives of given ages, without computing the expression $\frac{1}{P \times Q} \times$ the series $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5} +$ &c. (continued to the end of the table of the probabilities,) which is accurately equal to it.

CCCCXLVI. It is evident that the series $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5} +$ &c. continued to the end of the table of probabilities, will consist of the same number of terms as the series $\frac{Q^{I}}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^{V}}{r^5} +$ &c. continued likewise to the end of the table of probabilities: because, when all the persons who are represented in the table of probabilities as living at the age of the older of the two lives, and whose number is originally Q, shall be dead, the annuity (which is to depend on the joint continuance of both the lives,) must necessarily cease, notwithstanding several of the persons who are there represented as living at the age of the younger of the two lives, and whose number is originally P, may be still alive.

CCCCXLVII. Since

CCCCXLVII. Since therefore the two feriefes $\frac{Q^{I}}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^{V}}{r^5} +$ &c. and $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5} +$ &c. confift of the fame number of terms;—and, fince they alfo have the fame quantities for the denominators of their feveral terms, to wit, the quantities r, r^2, r^3, r^4, r^5, &c.—and the numerators of the terms of the former feries, to wit, Q^{I}, Q^{II}, Q^{III}, Q^{IV}, Q^{V}, &c. are involved in, or are factors of, the numerators of the terms of the latter feries, to wit, $P^{I} \times Q^{I}$, $P^{II} \times Q^{II}$, $P^{III} \times Q^{III}$, $P^{IV} \times Q^{IV}$, $P^{V} \times Q^{V}$, &c.—it feems reafonable to fuppofe that the value of the whole latter feries may, in fome way or other, be deduced from the value of the whole former feries, to a tolerable degree of exactnefs, fo as to make it unneceffary to take the trouble of computing all the terms of the latter feries and then adding them up into one fum. And, if this fhall appear to be practicable, it will follow, that the value of an annuity of one pound a year for the joint continuance of two lives of any given ages, may be deduced from the value of an annuity of the like amount for the older of the fame two lives fingly; which value we fhall always be able to find in one of the twelve tables above exhibited in Art. CCI, pages 221, 222, 223, &c. —— 232. And thus the faid tables of the values of annuities for fingle lives may be made fubfervient to the difcovery of the values of the like annuities for two joint lives. For let $\overset{\pounds}{A}$ be the value of an annuity of one pound a year for the older of the two lives fingly; and let $\overset{\pounds}{B}$ be the value of an annuity of one pound a year for the joint continuance of both lives. Then will $\overset{\pounds}{A}$ be equal to $\frac{\overset{\pounds}{1}}{Q} \times$ the feries $\frac{Q^{I}}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^{V}}{r^5} +$ &c. and $\overset{\pounds}{B}$ will be equal to $\frac{\overset{\pounds}{1}}{P \times Q} \times$ the feries $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5} +$ &c. and confequently the feries $\frac{Q^{I}}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^{V}}{r^5} +$ &c. will be $= Q \times \overset{\pounds}{A}$, and the feries series $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5}$ + &c. will be $= P \times Q \times \overset{£}{B}$. If therefore we can derive the latter series from the former, we shall thereby obtain the value of $P \times Q \times \overset{£}{B}$; which, being divided by $P \times Q$, will give us $\overset{£}{B}$, or the value of an annuity of one pound a year for the two proposed joint lives.

We must therefore endeavour to find a method of deriving the value of the series $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5}$ + &c. from the value of the series $\frac{Q^{I}}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^{V}}{r^5}$ + &c. which is equal to $Q \times \overset{£}{A}$. Now this may be done to a certain moderate degree of exactness, by means of a conjectural supposition that has an appearance of great probability, in the manner following.

CCCCXLVIII. Put S for the value of the simple series $\frac{Q^{I}}{r} + \frac{Q^{II}}{r^2} + \frac{Q^{III}}{r^3} + \frac{Q^{IV}}{r^4} + \frac{Q^{V}}{r^5}$ + &c. and the Greek capital Σ for the value of the compound series $\frac{P^{I} \times Q^{I}}{r} + \frac{P^{II} \times Q^{II}}{r^2} + \frac{P^{III} \times Q^{III}}{r^3} + \frac{P^{IV} \times Q^{IV}}{r^4} + \frac{P^{V} \times Q^{V}}{r^5}$ + &c. and put $g = \frac{Q^{I}}{r}$, and $h = \frac{Q^{II}}{r^2}$, and $i = \frac{P^{I} \times Q^{I}}{r}$, and $k = \frac{P^{II} \times Q^{II}}{r^2}$. And let G denote the value of an infinite series of terms in geometrical proportion, of which $\frac{Q^{I}}{r}$ and $\frac{Q^{II}}{r}$, or g and h, are the two first terms, and the Greek capital Γ denote the value of an infinite series of terms in geometrical proportion, of which $\frac{P^{I} \times Q^{I}}{r}$ and $\frac{P^{II} \times Q^{II}}{r^2}$, or i and k, are the two first terms.

Then

Then will the infinite geometrical ſeries G, or $g + h + \frac{hh}{g} + \frac{h^3}{gg} + \frac{h^4}{g^3} + \frac{h^5}{g^4} + \frac{h^6}{g^5} +$ &c. be $= \frac{gg}{g-h}$, and the infinite geometrical ſeries Γ, or $i + \frac{kk}{i} + \frac{k^3}{ii} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5} +$ &c. be $= \frac{ii}{i-k}$; as will be evident from Art. LXXX, pages 91, 92.

CCCCXLIX. Now it ſeems reaſonable to ſuppoſe that the geometrical ſeries G, or $g + h + \frac{hh}{g} + \frac{h^3}{gg} + \frac{h^4}{g^3} + \frac{h^5}{g^4} + \frac{h^6}{g^5} +$ &c. *ad infinitum*, will bear, pretty nearly, the ſame proportion to the ſimple ſeries S, or $\frac{Q^{\text{I}}}{r} + \frac{Q^{\text{II}}}{r^2} + \frac{Q^{\text{III}}}{r^3} + \frac{Q^{\text{IV}}}{r^4} + \frac{Q^{\text{V}}}{r^5} + \frac{Q^{\text{VI}}}{r^6} + \frac{Q^{\text{VII}}}{r^7} +$ &c. continued to the end of the table of probabilities, (of which ſeries the two firſt terms, $\frac{Q^{\text{I}}}{r}$ and $\frac{Q^{\text{II}}}{r^2}$, are the ſame with the two firſt terms, g and h, of the ſaid geometrical ſeries,) as the geometrical ſeries Γ, or $i + k + \frac{k^2}{i} + \frac{k^3}{ii} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5} +$ &c. *ad infinitum*, bears to the compound ſeries Σ, or $\frac{P^{\text{I}} \times Q^{\text{I}}}{r} + \frac{P^{\text{II}} \times Q^{\text{II}}}{r^2} + \frac{P^{\text{III}} \times Q^{\text{III}}}{r^3} + \frac{P^{\text{IV}} \times Q^{\text{IV}}}{r^4} + \frac{P^{\text{V}} \times Q^{\text{V}}}{r^5} + \frac{P^{\text{VI}} \times Q^{\text{VI}}}{r^6} + \frac{P^{\text{VII}} \times Q^{\text{VII}}}{r^7} +$ &c. continued to the end of the table of probabilities, of which ſeries the two firſt terms $\frac{P^{\text{I}} \times Q^{\text{I}}}{r}$ and $\frac{P^{\text{II}} \times Q^{\text{II}}}{r^2}$, are the ſame with the two firſt terms, i and k, of the ſaid geometrical ſeries. And, if this ſuppoſition is true, the ſeries Σ, or $\frac{P^{\text{I}} \times Q^{\text{I}}}{r} + \frac{P^{\text{II}} \times Q^{\text{II}}}{r^2} + \frac{P^{\text{III}} \times Q^{\text{III}}}{r^3} + \frac{P^{\text{IV}} \times Q^{\text{IV}}}{r^4} + \frac{P^{\text{V}} \times Q^{\text{V}}}{r^5} + \frac{P^{\text{VI}} \times Q^{\text{VI}}}{r^6}$

$+\frac{P^{VI}\times Q^{VI}}{r^6}+\frac{P^{VII}\times Q^{VII}}{r^7}+$ &c. will be nearly equal to $\frac{\Gamma\times S}{G}$.

But we have seen in Art. XLVII, that the series Σ, or $\frac{P^{I}\times Q^{I}}{r}+\frac{P^{II}\times Q^{II}}{r^2}+\frac{P^{III}\times Q^{III}}{r^3}+\frac{P^{IV}\times Q^{IV}}{r^4}+\frac{P^{V}\times Q^{V}}{r^5}+\frac{P^{VI}\times Q^{VI}}{r^6}+\frac{P^{VII}\times Q^{VII}}{r^7}+$ &c. is $=P\times Q\times\overset{\pounds}{B}$, and the series S, or $\frac{Q^{I}}{r}+\frac{Q^{II}}{r^2}+\frac{Q^{III}}{r^3}+\frac{Q^{IV}}{r^4}+\frac{Q^{V}}{r^5}+\frac{Q^{VI}}{r^6}+\frac{Q^{VII}}{r^7}+$ &c. is $=Q\times\overset{\pounds}{A}$. Therefore $P\times Q\times\overset{\pounds}{B}$ will be nearly equal to $\frac{\Gamma\times Q\times\overset{\pounds}{A}}{G}$.

But the series G is $=\frac{gg}{g-h}$, and the series Γ is $=\frac{ii}{i-k}$; as we have seen in Art. CCCCXLVIII. Therefore $P\times Q\times\overset{\pounds}{B}$ will be nearly equal to $\frac{Q\times\overset{\pounds}{A}\times\frac{ii}{i-k}}{\frac{gg}{g-h}}$, or $Q\times\overset{\pounds}{A}\times\frac{ii}{i-k}\times\frac{g-h}{gg}$, or $Q\times\overset{\pounds}{A}\times\frac{ii}{gg}\times\frac{g-h}{i-k}$; and $\overset{\pounds}{B}$, or the value of an annuity of one pound a year for the two joint lives whose ages correspond to the numbers P and Q, will be nearly equal to $\frac{Q\times\overset{\pounds}{A}}{P\times Q}\times\frac{ii}{gg}\times\frac{g-h}{i-k}$, or $\frac{\overset{\pounds}{A}}{P}\times\frac{ii}{gg}\times\frac{g-h}{i-k}$.

But g is $=\frac{Q^{I}}{r}$, and h is $=\frac{Q^{II}}{r^2}$, and i is $=\frac{P^{I}\times Q^{I}}{r}$, and k is $=\frac{P^{II}\times Q^{II}}{r^2}$. Therefore $i=g\times P^{I}$, and k is $=h\times P^{II}$. Consequently ii is $=gg\times P^{I}\times P^{I}$, and $i-k$ is $=g\times P^{I}-h\times P^{II}$, and $\frac{ii}{gg}\times\frac{g-h}{i-k}$

 is =

is $= \frac{gg \times P^{I} \times P^{I}}{gg} \times \frac{g-h}{g \times P^{I} - h \times P^{II}} = P^{I} \times P^{I} \times \frac{g-h}{g \times P^{I} - h \times P^{II}}$.

Therefore $\frac{\overset{£}{A}}{P} \times \frac{ii}{gg} \times \frac{\overline{g-h}}{i-k}$ is $= \frac{\overset{£}{A}}{P} \times P^{I} \times P^{I} \times \frac{g-h}{g \times P^{I} - h \times P^{II}} =$ $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{P^{I} \times \overline{g-h}}{g \times P^{I} - h \times P^{II}} = \overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{II} - h \times P^{II}}$. Therefore $\overset{£}{B}$, or the value of an annuity of one pound a year for the two joint lives whose ages correspond to the numbers P and Q, will be nearly equal to $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, or may be derived from $\overset{£}{A}$, or the value of the like annuity for a single life of the age corresponding to Q, by multiplying it into the fraction $\frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$. Q.E.I.

Examples of the foregoing method of approximating to the values of annuities for two joint lives.

CCCCL. I shall now proceed to try the truth of this expression, $\overset{£}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, by applying it to the computation of the values of an annuity of one pound a year for the joint continuance of two lives of some of the ages set down in the tables above exhibited in Art. CCCCVIII and CCCCXXXII. And, if we shall find that the values of those annuities which we shall obtain by means of this expression, differ but little from the values of them contained in those tables, (which have been accurately computed,) we shall have reason to conclude that the same expression will give us the like approximations to the true values of annuities for two joint lives in other instances.

Examples of the said method, upon a supposition that the interest of money is $3\frac{1}{2}$ per cent.

First example; in which the difference of the ages of the two lives is 10 years.

CCCCLI. Let us therefore suppose that the two lives for the joint continuance of which an annuity of one pound a year is to be purchased, are of the ages of 10 years and 20 years; and that the interest of money is $3\frac{1}{2}$ per cent. and that the table of probabilities of the duration of human life, by which the calculation is to be governed, is that of Monsieur de Parcieux.

Then

Then we ſhall have (by Table XV.) $\overset{\pounds}{A}$, or the value of an annuity of one pound a year for a ſingle life of the age of 20 years, $=\overset{\pounds}{19.440,616}$, and $P = 880$, $P^{I} = 872$, $P^{II} = 866$, $Q = 814$, $Q^{I} = 806$, $Q^{II} = 798$, $r = 1.035$, $\frac{1}{r}\ \left(=\frac{1}{1.035}\right) = 0.966,\ 183$, $\frac{1}{r^2} = 0.933,510$; and conſequently $g\ \left(=\frac{Q^{I}}{r} = \text{[illegible]} \times .966,183\right) = 778.743,498$, and $b\ \left(=\frac{Q^{II}}{r^2} = 798 \times .933,510\right) = 744.940,980$. Therefore $g \times P^{I}$ will be $(= 778.743,498 \times 872) = 679,064.330,256$, and $b \times P^{I}$ will be $= 744.940,980 \times 872) = 649,588.534,560$, and $b \times P^{II}$ will be $(= 744.940,980 \times 866) = 643,629.006,720$; and conſequently $g \times P^{I} - b \times P^{I}$ will be $(= 679,064.330,256 - 649,588.534,560) = 29,475.795,696$, and $g \times P^{I} - b \times P^{II}$ will be $(= 679,064.330,256 - 643,629.006,720) = 35,435.323,536$. Therefore $\frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$ will be $= \frac{29,475.795,696}{35,435.323,536} =$ (nearly) $\frac{29,475.795,696}{35,435} = .8318$; and $\frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$ will be = (nearly) $\frac{P^{I}}{P} \times .8318\ \left(= \frac{872}{880} \times .8318 = \frac{109}{110} \times .8318 = \frac{90.6662}{110}\right) = .8242$. Therefore $\overset{\pounds}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$ will be $(= \overset{\pounds}{A} \times .8242 = \overset{\pounds}{19.440,616} \times .8242) = \overset{\pounds}{16.022,955}$. Q E I.

The true value of this annuity for two joint lives appears by Table XXXI, page 494, to be $\overset{\pounds}{16.634,79}$; which is greater than the near value of it juſt now found, to wit. $\overset{\pounds}{16.022,955}$, by the difference $\overset{\pounds}{0.611,835}$, which is ſomewhat leſs than a 27th part of the ſaid true value.

Difference between the foregoing near value and the true value of this annuity.

CCCCLII. Now let the two lives be of the ages of 20 years and 30 years.

In this caſe $\overset{\pounds}{A}$ will be the value of an annuity of one pound a year for a ſingle life of the age of 30 years, and therefore (by Table XV.) will be

Second example; in which the difference of the two lives is likewiſe 10 years.

be = 18.068,798. And P will be = 814, P^{1} = 806, P^{11} = 798, Q = 734, Q^{1} = 726, Q^{11} = 718, r = 1.035, $\frac{1}{r}\left(=\frac{1}{1.035}\right) = 0.966,183$, and $\frac{1}{r^2} = 0.933,510$. Therefore $\frac{Q^1}{r}$ will be $(= 726 \times .966,183) = 701.448,858$, and $\frac{Q^{11}}{r^2}$ will be $= (=718 \times .933,510) = 670.260,180$, that is, g will be = 701.448,858, and h will be = 670.260,180. Therefore $g \times P^1$ will be $(= 701.448,858 \times 806) = 565,367.779,548$, and $h \times P^1$ will be $(=670.260,180 \times 806) = 540,229.705,080$, and $h \times P^{11}$ will be $(= 670.260,180 \times 798) = 534,867.623,640$; and consequently $g \times P^1 - h \times P^1$ will be $(= 565,367.779,548 - 540,229.705,080) = 25,138.074,468$, and $g \times P^1 - h \times P^{11}$ will be $(= 565,367.779,548 - 534,867.623,640) = 30,500.155,808$. Therefore $\frac{g \times P^1 - h \times P^1}{g \times P^1 - h \times P^{11}}$ will be $= \frac{25,138.074,468}{30,500.155,908} =$ (nearly) $\frac{25,138.074,468}{30,500} = .8241$; and $\frac{P^1}{P} \times \frac{g \times P^1 - h \times P^1}{g \times P^1 - h \times P^{11}}$ will be $\left(= \frac{P^1}{P} \times .8241 = \frac{806}{814} \times .8241 = \frac{403}{407} \times .8241 = \frac{332.1123}{407}\right) = .8160$. Therefore $\overset{£}{A} \times \frac{P^1}{P} \times \frac{g \times P^1 - h \times P^1}{g \times P^1 - h \times P^{11}}$ will be $= \overset{£}{A} \times .8160 = \overset{£}{18.068,798} \times .8160 = \overset{£}{14.7441}$. Q.E.I.

Difference between the foregoing near value and the true value of this annuity.

The true value of this annuity for the said joint lives of the ages of 20 years and 30 years appears by Table XXXI, page 494, to be £15.298,75; which is greater than the value just now found for it, to wit, £14.7441, by the difference £0.554,65, which is less than the 27th part of the said true value, £15.298,75.

Five more examples of the same method, in all which the difference of the ages of the two lives is the same as in the two former examples, to wit, 10 years.

CCCCLIII. If we derive in the same manner the values of an annuity of one pound a year for the joint continuance of the five following pairs of lives, to wit, two lives of the ages of 30 years and 40 years, two lives of the ages of 40 years and 50 years, two lives of the ages of 50 years and 60 years, two lives of the ages of 60 years and 70 years, and two lives of the ages of 70 years and 80 years, from the values of the same annuity for single lives of the same ages as the elder lives in each of these pairs of lives respectively, that is, for single lives of the ages of 40 years, 50 years, 60 years,

years, 70 years, and 80 years, (which values appear by Table XV to be £16.104,542, £13.183,083, £10.104,074, £6.575,357, and £3.661,781,); —I say, if we derive near values of the said annuity for the said pairs of joint lives from these values of the same annuity for the said single lives by means of the aforesaid expression $\overset{£}{A} \times \frac{p_1}{p} \times \frac{g \times p_1 - h \times p_1}{g \times p_1 - h \times p_{11}}$, those near values will be found to be as follows; to wit,

for two joint lives of the ages of 30 years and 40 years, £12.865,18;
for two joint lives of the ages of 40 years and 50 years, £10.917,67;
for two joint lives of the ages of 50 years and 60 years, £7.680,967;
for two joint lives of the ages of 60 years and 70 years, £5.074,649;
and for two joint lives of the ages of 70 years and 80 years, £2.645,883.

CCCCLIV. Now it appears from Table XXXI, that the true values of these last five annuities for joint lives are as follows, to wit,

The true values of the last five annuities.

for two joint lives of the ages of 30 years and 40 years, £13.709,61;
for two joint lives of the ages of 40 years and 50 years, £11.229,92;
for two joint lives of the ages of 50 years and 60 years, £8.341,630;
for two joint lives of the ages of 60 years and 70 years, £5.336,698;
and for two joint lives of the ages of 70 years and 80 years, £2.703,419.

CCCCLV. The difference between the near value, £12.865,18, of the first of these annuities, and its true value, £13.709,61, is £0.844,43; which is less than a 16th part of the said true value.

The differences between their foregoing near values and their true values.

The difference between the near value, £10.917,67, of the second of these annuities, and its true value, £11.229,92, is £0.312,25; which is less than a 35th part of the said true value.

The difference between the near value, £7.680,967, of the third of these annuities, and its true value, £8.341,630, is £0.660,663; which is less than a 12th part of the said true value.

The

The difference between the near value, £5.074,649, of the fourth of the foregoing annuities, and its true value, £5.336,698, is £0.262,049; which is less than the 20th part of the said true value.

And the difference between the near value, £2.645,883, of the fifth and last of the foregoing annuities, and its true value, £2.793,419, is £0.147,536; which is less than the 19th part of the said true value.

CCCCLVI. In all these examples it appears that the near values of annuities for two joint lives, obtained by means of the expression $\overset{£}{A} \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$, are less than their respective true values, and that the differences of the said near values and true values are in some cases about a 35th part of the said true values respectively, and in other cases a 27th, a 20th, a 19th, and a 16th part of the said true values, and in some cases almost a 12th part of them.

Five other examples of the foregoing method of finding near values of annuities for two joint lives; in which the difference of the ages of the two lives is 30 years.

CCCCLVII. Let us now examine the near values of some annuities for two joint lives, that are obtained by means of the said expression, $\overset{£}{A} \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$, when the difference of the ages of the two lives is considerably greater than 10 years. Let this difference, therefore, be 30 years; and let us derive, by means of the said expression, $\overset{£}{A} \times \frac{P_1}{P} \times \frac{g \times P_1 - h \times P_1}{g \times P_1 - h \times P_{11}}$, near values of an annuity of one pound a year for the five following pairs of joint lives, upon a supposition that the interest of money is (as it was supposed to be in the foregoing examples,) $3\frac{1}{2}$ per cent. to wit,

two lives of the ages of 10 years and 40 years,
two lives of the ages of 20 years and 50 years,
two lives of the ages of 30 years and 60 years,
two lives of the ages of 40 years and 70 years,
and two lives of the ages of 50 years and 80 years,

from

from the values of a like annuity of one pound a year for single lives of the ages of 40 years, 50 years, 60 years, 70 years, and 80 years, respectively, which are given above in Table XV, to wit, £16.104,542, £13.183,083, £10.104,074, £6.575,357, and £3,661,781. These near values will be found to be as follows; to wit,

The near values of the said annuities.

for two joint lives of the ages of 10 years and 40 years, £13.892,33;
for two joint lives of the ages of 20 years and 50 years, £11.068,05;
for two joint lives of the ages of 30 years and 60 years, £8.560,176;
for two joint lives of the ages of 40 years and 70 years, £5.931,793;
and for two joint lives of the ages of 50 years and 80 years, £3.320,420.

Their true values.

CCCCLVIII. Now it appears from Table XXXIII, that the true values of these last five annuities for two joint lives are as follows; to wit,

for two joint lives of the ages of 10 years and 40 years, £14.384,49;
for two joint lives of the ages of 20 years and 50 years, £11.801,15;
for two joint lives of the ages of 30 years and 60 years, £9.217,864;
for two joint lives of the ages of 40 years and 70 years, £6.126,861;
and for two joint lives of the ages of 50 years and 80 years, £3.399,171.

The differences between their said near values and true values.

CCCCLIX. The difference between the near value, £13.892,33, of the first of these annuities and its true value, £14.384,49, is £0.492,16; which is less than the 29th part of the said true value.

The difference between the near value, £11.068,05, of the second annuity, and its true value, £11.801,15, is £0.733,10; which is less than the 16th part of the said true value.

The difference between the near value, £8.560,176, of the third annuity, and its true value, £9.217,864, is £0.657,688; which is less than the 14th part of the said true value.

The difference between the near value, £5.931,793, of the fourth annuity, and its true value, £6.126,861, is £0.195,068; which is less than the 31st part of the said true value.

And

And the difference between the near value, £3.320,420, and the true value, £3.399,171, of the fifth and laft annuity, is £0.078,751; which is lefs than the 43d part of the faid true value.

CCCCLX. In thefe five examples it appears that the near values of annuities for two joint lives, whofe ages differ from each other by 30 years, obtained by means of the expreffion $\overset{\pounds}{A} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$, are lefs than their refpective true values, (as was the cafe with the near values of the annuities mentioned in Art. CCCCLI, &c. —— CCCCLVI) and that the differences of the faid near values and true values are in fome cafes lefs than the 43d part of the faid true values refpectively, and in other cafes a 31ft, a 29th, and a 16th part of the faid true values, and in fome cafes about a 14th part of them.

Thefe differences are rather fmaller than the differences of the near values and true values of the annuities for two joint lives whofe ages differ from each other by only 10 years, which are mentioned in Art. CCCCLVI.

Examples of the faid method, upon a fuppofition that the intereft of money is $4\frac{1}{2}$ per cent.

CCCCLXI. We will next examine the near values of annuities for two joint lives, which may be obtained by means of the expreffion $\overset{\pounds}{A} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$, when the intereft of money is $4\frac{1}{2}$ per cent.

Examples in which the differences of the ages of the two lives is 10 years.

Let there be feven pairs of lives whofe ages differ from each other by 10 years, to wit,

two lives of the ages of 10 years and 20 years,
two lives of the ages of 20 years and 30 years,
two lives of the ages of 30 years and 40 years,
two lives of the ages of 40 years and 50 years,
two lives of the ages of 50 years and 60 years,
two lives of the ages of 60 years and 70 years,
and two lives of the ages of 70 years and 80 years.

When

When the intereſt of money is $4\frac{1}{2}$ per cent. the near values of an annuity of one pound a year for the joint continuance of theſe ſeveral pairs of lives, that may be derived by means of the expreſſion $\frac{£}{A} \times \frac{P1}{P} \times \frac{g \times P1 - b \times P1}{g \times P1 - b \times P11}$, from the values of the like annuity of one pound a year for ſingle lives of the ages of 20 years, 30 years, 40 years, 50 years, 60 years, 70 years, and 80 years, (which values appear by Table XVII to be £16.623,93, £15.690,92, £14.254,42, £11.920,73, £9.346,46, £6.220,54, and £3.532,59,) will be found to be as follows; to wit,

The near values of an annuity of one pound a year for the ſaid ſeven pairs of joint lives.

The near value of an annuity of one pound a year for two joint lives of the ages of 10 years and 20 years, will be £14.655,20;

for two joint lives of the ages of 20 years and 30 years, £13.359,42;

for two joint lives of the ages of 30 years and 40 years, £11.594,96;

for two joint lives of the ages of 40 years and 50 years, £10.126,50;

for two joint lives of the ages of 50 years and 60 years, £7.329,817;

for two joint lives of the ages of 60 years and 70 years, £4.889,327;

and for two joint lives of the ages of 70 years and 80 years, £2.580,00.

CCCCLXII. Now it appears from Table XLI that the true values of theſe ſeven annuities of one pound a year for two joint lives are as follows; to wit,

The true values of the ſame annuity for the ſame pairs of joint lives.

for two joint lives of the ages of 10 years and 20 years, £14.478,80;

for two joint lives of the ages of 20 years and 30 years, £13 495,90;

for two joint lives of the ages of 30 years and 40 years, £12.297,99;

for two joint lives of the ages of 40 years and 50 years, £10.274,39;

for two joint lives of the ages of 50 years and 60 years, £7.793,079;

for two joint lives of the ages of 60 years and 70 years, £5.087,882;

and for two joint lives of the ages of 70 years and 80 years, £2.709,443.

The differences between the ſaid near values and true values.

CCCCLXIII. The difference between the near value, £14.655,20, of the firſt of theſe annuities, and its true value, £14.478,80, (which, it is worth obſerving, is, leſs than the near value, though in all the former examples the true values of the annuities have been greater than their near values,) is £0.176,40; which is leſs than the 82d part of the ſaid true value.

The difference between the near value, £13.359,42, of the ſecond annuity, and its true value, £13.495,90, is £0.136,48; which is leſs than the 98th part of the ſaid true value.

The difference between the near value, £11.594,96, of the third annuity, and its true value, £12.297,99, is £0.703,03; which is leſs than the 17th part of the ſaid true value.

The difference between the near value, £10.126,50, of the fourth annuity and its true value, £10.274,39, is £0.147,89; which is leſs than the 69th part of the ſaid true value.

The difference between the near value, £7.329,817, of the fifth annuity, and its true value, £7.793,079, is £0.463,262; which is leſs than the 17th part of the ſaid true value.

The difference between the near value, £4.889,327, of the ſixth annuity, and its true value, £5.087,882, is £0.198,555; which is leſs than the 25th part of the ſaid true value.

And the difference between the near value, £2.580,00, of the ſeventh and laſt annuity, and its true value, £2.709,443, is £0.129,443; which is leſs than the 20th part of the ſaid true value.

A remark on the ſaid differences.

Theſe differences are ſmaller than thoſe of the near values and the true values of the annuities mentioned in Art. CCCCLVII and CCCCLVIII, for two joint lives whoſe ages differ from each other by 30 years, when the intereſt of money is $3\frac{1}{2}$ per cent. and are ſmaller in a ſtill greater degree than the differences of the near values and the true values of the annuities for two joint lives, mentioned in Art. CCCCLVI, whoſe ages differ by only 10 years, when the intereſt of money is $3\frac{1}{2}$ per cent.

CCCCLXIV. We

CCCCLXIV. We will now examine the near values of ſome annuities for two joint lives, that are obtained by means of the expreſſion $\overset{\pounds}{A} \times \frac{P1}{P} \times \frac{g \times P1 - h \times P1}{g \times P1 - h \times P11}$, when the difference of the ages of the two lives is 30 years, and the intereſt of money (as in the two preceeding articles,) is $4\frac{1}{2}$ per cent.

Five other examples of the foregoing method of finding the values of annuities for two joint lives; in which the difference of the ages of the two lives is 30 years.

Now, when the intereſt of money is $4\frac{1}{2}$ per cent. the near values of an annuity of one pound a year for the five following pairs of joint lives, to wit,

two joint lives of the ages of 10 years and 40 years,
two joint lives of the ages of 20 years and 50 years,
two joint lives of the ages of 30 years and 60 years,
two joint lives of the ages of 40 years and 70 years,
and two joint lives of the ages of 50 years and 80 years,

which may be obtained by means of the expreſſion $\overset{\pounds}{A} \times \frac{P1}{P} \times \frac{g \times P1 - h \times P1}{g \times P1 - h \times P11}$, will be found to be as follows; to wit,

The near values of the ſaid five annuities.

for two joint lives of the ages of 10 years and 40 years, £12.446,40;
for two joint lives of the ages of 20 years and 50 years, £10.248,45;
for two joint lives of the ages of 30 years and 60 years, £8.075,146;
for two joint lives of the ages of 40 years and 70 years, £5.655,547;
and for two joint lives of the ages of 50 years and 80 years, £3.214,679.

CCCCLXV. Now it appears from Table XLIII, that the true values of theſe laſt five annuities for two joint lives are as follows; to wit,

Their true values.

for two joint lives of the ages of 10 years and 40 years, £12 842,64;
for two joint lives of the ages of 20 years and 50 years, £10.739,19;
for two joint lives of the ages of 30 years and 60 years, £8.560,832;
for two joint lives of the ages of 40 years and 70 years, £5.810,588;
and for two joint lives of the ages of 50 years and 80 years, £3.284,035.

 CCCCLXVI. All

CCCCLXVI. All theſe true values are greater than the near values of the ſame annuities obtained in Art. CCCCLXIV by means of the expreſſion $\overset{£}{A} \times \frac{P_{I}}{P} \times \frac{g \times P_{I} - h \times P_{I}}{g \times P_{I} - h \times P_{II}}$, as was the caſe with reſpect to the true values of the annuities mentioned in all the preceeding examples, except the firſt example in Art. CCCCLXII, in which it appeared that the true value of an annuity of one pound a year for two joint lives of the ages of 10 years and 20 years, when the intereſt of money is $4\frac{1}{2}$ per cent, to wit, £14.478,80, was leſs than its near value, £14.655,20, obtained by means of the expreſſion $\overset{£}{A} \times \frac{P_{I}}{P} \times \frac{g \times P_{I} - h \times P_{I}}{g \times P_{I} - h \times P_{II}}$. This remarkable exception I do not know how to account for.

The differences between the ſaid near values and true values.

CCCCLXVII. The difference between the near value, £12.446,40, of the firſt of the five laſt-mentioned annuities for two joint lives, and its true value, £12.842,64, is £0.396,24; which is leſs than the 32d part of the ſaid true value.

The difference between the near value, £10.248,45, of the ſecond of the ſaid five annuities, and its true value, £10.739,19, is £0.490,74; which is leſs than the 21ſt part of the ſaid true value.

The difference between the near value, £8.075,146, of the third annuity, and its true value, £8.560,832, is £0.485,686; which is leſs than the 17th part of the ſaid true value.

The difference between the near value, £5.655,547, of the fourth annuity, and its true value, £5.810,588, is £0.155,041; which is leſs than the 37th part of the ſaid true value.

And the difference between the near value, £3.214,679, of the fifth and laſt annuity, and its true value, £3.284,035, is £0.069,356; which is leſs than the 47th part of the ſaid true value.

A table

A table of the foregoing near values of an annuity of one pound a year for different pairs of joint lives, and of the corresponding true values of the same annuity, and of the differences of the said near values and true values, and of the fractions that express the proportions of the said differences to the said true values.

CCCCLXVIII. If the near values and true values of all the annuities mentioned in the foregoing articles, from Art. CCCCLI to the last article, inclusively, be ranged in regular order in two contiguous columns, and their differences be set down in a third column adjoining to the second column, and the fractions that express the proportions of the said differences to the said true values, be set down in a fourth column adjoining to the said third column, the said several numbers will be as follows.

The near values of an annuity of one pound a year for two joint lives of several different ages, obtained by means of the expression £ $A \times \frac{P1}{P} \times \frac{g \times P1 - h \times P1}{g \times P1 - h \times P11}$, *when the interest of money is* $3\frac{1}{2}$ *per cent.*	*The true values of the same annuity for the same joint lives, when the interest of money is* $3\frac{1}{2}$ *per cent.*	*The differences of the said near and true values.*	*The proportions of the said differences to the said true values.*
£	£	£	
16.022,955	16.634,79	0.611,835	$\frac{1}{27}$
14.7441	15.298,75	0.554,65	$\frac{1}{27}$
12.865,18	13.709,61	0.844,43	$\frac{1}{16}$
10.917,67	11.229,92	0.312,25	$\frac{1}{35}$
7.680,967	8.341,630	0.660,663	$\frac{1}{12}$
5.074,649	5.336,698	0.262,049	$\frac{1}{20}$
2.645,883	2.793,419	0.147,536	$\frac{1}{19}$
£	£	£	
13.892,33	14 384,49	0.492,16	$\frac{1}{29}$
11.068,05	11.801,15	0.733,10	$\frac{1}{16}$
8.560,176	9.217,864	0.657,688	$\frac{1}{14}$
5.931,793	6.126,861	0.195,068	$\frac{1}{31}$
3.320,420	3.399,171	0.078,751	$\frac{1}{43}$

The near values of the same annuity for the same joint lives, obtained by means of the same expression $\overset{£}{A} \times \frac{P1}{P} \times \frac{g \times P1 - h \times P1}{g \times P1 - h \times P11}$, *when the interest of money is* $4\frac{1}{2}$ *per cent.*	*The true values of the same annuity for the same joint lives, when the interest of money is* $4\frac{1}{2}$ *per cent.*	*The differences of the said near and true values.*	*The proportions of the said differences to the said true values.*
£	£	£	
14.655,20	14.478,80	0.176,40	$\frac{1}{82}$
13.359,42	13.495,90	0.136,48	$\frac{1}{98}$
11.594,96	12.297,99	0.703,03	$\frac{1}{17}$
10.126,50	10.274,39	0.147,89	$\frac{1}{69}$
7.329,817	7.793,079	0.463,262	$\frac{1}{16}$
4.889,327	5.087,882	0.198,555	$\frac{1}{25}$
2.580,00	2.709,443	0.129,443	$\frac{1}{20}$
£	£	£	
12.446,40	12.842,64	0.396,24	$\frac{1}{32}$
10.248,45	10.739,19	0.490,74	$\frac{1}{21}$
8.075,146	8.560,832	0.485,686	$\frac{1}{17}$
5.655,547	5.810,588	0.155,041	$\frac{1}{37}$
3.214,679	3.284,035	0.069,356	$\frac{1}{47}$

A second table of the same kind as the last, in which the near values of the annuities for two joint lives are derived from the former near values of them, exhibited in the last table, by multiplying the said former near values into the fraction $\frac{104}{100}$.

CCCCLXIX. If the foregoing near values of an annuity of one pound a year for two joint lives (which were obtained by means of the expression $\overset{£}{A} \times \frac{P1}{P} \times \frac{g \times P1 - h \times P1}{g \times P1 - h \times P11}$) be increased in the proportion of 104 to 100, or be multiplied into the fraction $\frac{104}{100}$, we shall thereby obtain a second

cond fet of near values of the faid annuity for the fame joint lives, that will, for the moft part, differ lefs than the foregoing near values of it from its correfponding true values. This will appear upon tryal, by computing thefe fecond near values, (which will be equal to £$A \times \frac{104}{100} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$) and placing them one under another in a column (as the former near values were placed in the laft article,) and then fetting down the correfponding true values of the fame annuity in a fecond column adjoining to the former, and the differences between the faid near values and true values in a third column adjoining to the fecond, and the fractions that exprefs the proportions of the faid differences to the faid true values in a fourth column adjoining to the third. This may be done in the manner following.

The near values of an annuity of one pound a year for two joint lives of feveral different ages, obtained by means of the expreffion £$A \times \frac{104}{100} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$, *when the intereft of money is* $3\frac{1}{2}$ *per cent.*	*The true values of the fame annuity for the fame joint lives.*	*The differences of the faid near and true values.*	*The proportions of the faid differences to the faid true values.*
£	£	£	
16.663,86	16.634,79	0.029,07	$\frac{1}{572}$
15.333,86	15.298,75	0.035,11	$\frac{1}{435}$
13.379,78	13.709,61	0.329,83	$\frac{1}{41}$
11.354,37	11.229,92	0.124,45	$\frac{1}{90}$
7.988,205	8.341,630	0.353,425	$\frac{1}{23}$
5.277,634	5.336,698	0.059,064	$\frac{1}{90}$
2.751,718	2.793,419	0.041,701	$\frac{1}{66}$
£	£	£	
14.448,02	14.384,49	0.063,53	$\frac{1}{226}$
11.510,77	11.801,15	0.290,38	$\frac{1}{46}$
8.902,583	9.217,864	0.315,281	$\frac{1}{29}$
6.169,064	6.126,861	0.04[illegible],203	$\frac{1}{145}$
3.453,236	3.399,171	0.054,065	$\frac{1}{62}$

The near values of the ſame annuity for the ſame joint lives, obtained by means of the ſame expreſſion $\overset{\pounds}{A} \times \frac{104}{100} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$*, when the intereſt of money is* $4\frac{1}{2}$ *per cent.*	*The true values of the ſame annuity for the ſame joint lives, when the intereſt of money is* $4\frac{1}{2}$ *per cent.*	*The differences of the ſaid near values and true values.*	*The proportions of the ſaid differences to the ſaid true values.*
£	£	£	
15.241,40	14.478,80	0.762,60	$\frac{1}{18}$
13.893,89	13.495,90	0.397,99	$\frac{1}{33}$
12.057,85	12.297,99	0.240,14	$\frac{1}{51}$
10.531,56	10.274,39	0.257,17	$\frac{1}{39}$
7.623,009	7.793,079	0.170,070	$\frac{1}{45}$
5.084,900	5.087,882	0.002,982	$\frac{1}{1705}$
2.683,200	2.709,443	0.026,243	$\frac{1}{103}$
£	£	£	
12.944,256	12.842,64	0.101,61	$\frac{1}{126}$
10.658,38	10.739,19	0.080,81	$\frac{1}{132}$
8.398,151	8.560,832	0.162,681	$\frac{1}{52}$
5.881,768	5.810,588	0.071,180	$\frac{1}{81}$
3.343,266	3.284,035	0.059,231	$\frac{1}{55}$

A remark on the near values contained in the laſt table.

CCCCLXX. By theſe inſtances it appears that the expreſſion $\overset{\pounds}{A} \times \frac{104}{100} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$ almoſt always gives us the value of an annuity of one pound a year for two joint lives to a conſiderably greater degree of exactneſs than the expreſſion $\overset{\pounds}{A} \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - h \times P^{\text{I}}}{g \times P^{\text{I}} - h \times P^{\text{II}}}$, and, in moſt caſes, to a ſufficient degree of exactneſs for practical purpoſes. And from its anſwering ſo well to the inſtances here given, it ſeems reaſonable to ſuppoſe that it will give the values of annuities for two joint lives to nearly the ſame

ſame degree of exactneſs in other inſtances, or where the ages of the two joint lives are different from thoſe above-ſuppoſed: and this it ſeems likely to do as well at one rate of the intereſt of money as at another; though for want of tables of the true values of annuities for two joint lives at any other rates of intereſt than $3\frac{1}{2}$ per cent. and $4\frac{1}{2}$ per cent. to try it by, the foregoing examples are confined to annuities at only thoſe two rates of intereſt. And therefore I think upon the whole, I may venture to recommend the ſaid expreſſion $\overset{\pounds}{A} \times \frac{104}{100} \times \frac{p_1}{P} \times \frac{g \times P_1 - b \times p_1}{g \times P_1 - b \times P_{11}}$, as a tolerably convenient approximation to the value of an annuity of one pound a year for two joint lives, when the intereſt of money is either leſs than 3 per cent. or greater than 5 per cent. and conſequently the value of ſuch annuity cannot be found in either of the two foregoing ſets of tables for two joint lives in Art. CCCCVIII and CCCCXXXII, nor be derived from the values exhibited in thoſe tables by either the method of *Interpolation* explained in Art. CCCCXII, CCCCXIII, &c. —— CCCCXXXI, or the method of *Interpolation and Continuation* explained in Art. CCCCXXXVI, CCCCXXXVII, &c.

I now proceed to conſider the values of annuities that depend on the joint continuance of *three* lives.

Of the values of annuities depending on the joint continuance of three lives.

CCCCLXXI. Let r be, as before, the ſum of one pound, together with its intereſt for a year, according to any given rate of intereſt. And let N be the number of years in the age of the younger of the three perſons on the joint continuance of whoſe lives the annuity is to depend; and $N + a$ the number of years in the age of the next older of the ſaid three perſons; and $N + a + b$ the number of years in the age of the oldeſt of the ſaid three perſons; and E the greateſt number of years through which it is ſuppoſed to be poſſible for human life to be extended, according to the table of probabilities of the duration of human life adopted for the calculation; which number is in Monſieur de Parcieux's table 94 years. Let n be any number of years not greater than $E - \overline{N + a + b}$, or $E - N - a - b$, or than the greateſt number of years during which it is poſſible that the oldeſt of the three lives may be prolonged. And let an annuity of one pound *per annum* be granted for the term of n years, provided all the three perſons aforeſaid, of the ages of N years, $N + a$ years, and $N + a + b$ years, ſhall ſo long live, but otherwiſe to ceaſe upon the death

A ſhort expreſſion of the value of an annuity of one pound a year depending on the joint continuance of three lives of given ages.

death of either of them. Let P be the number of persons represented in Monsieur de Parcieux's table of the probabilities of the duration of human life, (or in such other table of those probabilities as is thought proper by the calculator to be adopted as the ground of his calculation,) as being all living together at the said first, or youngest, age of N years; and P^{I} the number of persons represented in the said table to be living at the age of $N+1$ years; and P^{II} the number living at the age of $N+2$ years; and P^{III} the number living at the age of $N+3$ years; and P^{IV}, P^{V}, P^{VI}, P^{VII}, P^{VIII}, P^{IX}, P^{X}, &c. the numbers living at the several following ages of $N+4$ years, $N+5$ years, $N+6$ years, $N+7$ years, $N+8$ years, $N+9$ years, $N+10$ years, &c. respectively. And let Q be the number of persons represented in the said table as living at the second, or next older, age of $N+a$ years; and Q^{I} the number of persons represented there as living at the age of $N+a+1$ years; and Q^{II} the number living at the age of $N+a+2$ years; and Q^{III} the number living at the age of $N+a+3$ years; and Q^{IV}, Q^{V}, Q^{VI}, Q^{VII}, Q^{VIII}, Q^{IX}, Q^{X}, &c. the numbers living at the several following ages of $N+a+4$ years, $N+a+5$ years, $N+a+6$ years, $N+a+7$ years, $N+a+8$ years, $N+a+9$ years, $N+a+10$ years, &c. respectively. And let R be the number of persons represented in the said table as living at the third, or oldest, age of $N+a+b$ years; and R^{I} the number of persons represented there as living at the age of $N+a+b+1$ years; and R^{II} the number living at the age of $N+a+b+2$ years; and R^{III} the number of persons living at the age of $N+a+b+3$ years; and R^{IV}, R^{V}, R^{VI}, R^{VII}, R^{VIII}, R^{IX}, R^{X}, &c. the numbers living at the several following ages of $N+a+b+4$ years, $N+a+b+5$ years, $N+a+b+6$ years, $N+a+b+7$ years, $N+a+b+8$ years, $N+a+b+9$ years, $N+a+b+10$ years, &c. respectively.

These things being supposed, the present value of an annuity of one pound a year, to be enjoyed during the space of n years, in case all the said three lives, of the ages of N years, $N+a$ years, and $N+a+b$ years, shall so long continue, will be equal to the expression, £1 × the series

$$\frac{P^{I}\times Q^{I}\times R^{I}}{P\times Q\times R\times r}+\frac{P^{II}\times Q^{II}\times R^{II}}{P\times Q\times R\times r^{2}}+\frac{P^{III}\times Q^{III}\times R^{III}}{P\times Q\times R\times r^{3}}+\frac{P^{IV}\times Q^{IV}\times R^{IV}}{P\times Q\times R\times r^{4}}+\frac{P^{V}\times Q^{V}\times R^{V}}{P\times Q\times R\times r^{5}}+\frac{P^{VI}\times Q^{VI}\times R^{VI}}{P\times Q\times R\times r^{6}}+\frac{P^{VII}\times Q^{VII}\times R^{VII}}{P\times Q\times R\times r^{7}}+\text{\&c.}$$

continued to n terms, or to the term $\frac{P^{n}\times Q^{n}\times R^{n}}{P\times Q\times R\times r^{n}}$, or equal to the expression, $\frac{£1}{P\times Q\times R}$ + the series

$\frac{P^{I} \times Q^{I} \times R^{I}}{r} + \frac{P^{II} \times Q^{II} \times R^{II}}{r^2} + \frac{P^{III} \times Q^{III} \times R^{III}}{r^3} + \frac{P^{IV} \times Q^{IV} \times R^{IV}}{r^4} + \frac{P^{V} \times Q^{V} \times R^{V}}{r^5} + \frac{P^{VI} \times Q^{VI} \times R^{VI}}{r^6} + \frac{P^{VII} \times Q^{VII} \times {}^{VII}}{r^7}$ + &c. continued to n terms, or to the term $\frac{P^{n} \times Q^{n} \times R^{n}}{r^n}$, or (dropping, for the sake of brevity, the marks $\times$ of the several multiplications,) equal to the expression $\frac{£1}{PQR} \times$ the series $\frac{P^{I} Q^{I} R^{I}}{r} + \frac{P^{II} Q^{II} R^{II}}{r^2} + \frac{P^{III} Q^{III} R^{III}}{r^3} + \frac{P^{IV} Q^{IV} R^{IV}}{r^4} + \frac{P^{V} Q^{V} R^{V}}{r^5} + \frac{P^{VI} Q^{VI} R^{VI}}{r^6} + \frac{P^{VII} Q^{VII} R^{VII}}{r^7} +$ &c. continued to n terms, or to the term $\frac{P^{n} Q^{n} R^{n}}{r^n}$. This is evident from Prob. III, and its 6th and 7th Corollaries, Art. LIII, LIV, LV, LVI, pages 53, 54, 55, 56, 57.

CCCCLXXII. If n years is the greatest number of years through which it is possible (according to the table of the probabilities of the duration of human life adopted in the calculation,) for the oldest of the three given lives, or the life of the age of $N + a + b$ years, to be extended; or, in other words, if n is equal to $E - N - a - b$, the said expression $\frac{£1}{PQR} \times$ the series $\frac{P^{I} Q^{I} R^{I}}{r} + \frac{P^{II} Q^{II} R^{II}}{r^2} + \frac{P^{III} Q^{III} R^{III}}{r^3} + \frac{P^{IV} Q^{IV} R^{IV}}{r^4} + \frac{P^{V} Q^{V} R^{V}}{r^5} + \frac{P^{VI} Q^{VI} R^{VI}}{r^6} + \frac{P^{VII} Q^{VII} R^{VII}}{r^7} +$ &c. continued to n terms, or to the term $\frac{P^{n} Q^{n} R^{n}}{r^n}$, (which term in this case will be $\frac{P^{E-N-a-b} \times Q^{E-N-a-b} \times R^{E-N-a-b}}{r^n}$) will be the value of an annuity of one pound *per annum* for the whole joint continuance of the three given lives of the ages of N years, $N + a$ years, and $N + a + b$ years. But, if n is less than $E - N - a - b$, or the complement of $N + a + b$ (the number of years in the age of the oldest life) to E, or to the utmost possible duration of human life, the said expression will be less than the value of an annuity of one pound a year for the whole joint continuance of the said three lives, of the ages of N years, $N + a$ years, and $N + a + b$ years, and will be the value of an immediate, but

 imperfect,

imperfect, life-annuity of one pound *per annum* during n years of the joint continuance of the said three lives

An example of the computation of the value of an immediate and compleat life-annuity of one pound per annum *for the whole joint continuance of the lives of three persons of given ages, by means of the foregoing expression.*

CCCCLXXIII. Let it be required to find the value of an annuity of one pound *per annum* for the whole joint continuance of the lives of three persons of the ages of 75 years, 80 years, and 85 years, according to Monsieur de Parcieux's table of the probabilities of the duration of human life, and upon a supposition that the interest of money is 3 per cent.

Here n, or the number of years through which the annuity is to continue, in case all the three lives (of which the oldest is of the age of 85 years,) shall last so long, is the greatest possible number of years through which, according to Monsieur de Parcieux's table, a life of the age of 85 years can be extended, that is (94 — 85 yeays, or) 9 years. Therefore the series $\frac{P^{I}\,Q^{I}\,R^{I}}{r} + \frac{P^{II}\,Q^{II}\,R^{II}}{r^2} + \frac{P^{III}\,Q^{III}\,R^{III}}{r^3} + \frac{P^{IV}\,Q^{IV}\,R^{IV}}{r^4}$ + &c. in the foregoing expression, must be continued to 9 terms; which terms may be computed as follows.

Here P is $= 211$, P^{I} is $= 192$, $P^{II} = 173$, $P^{III} = 154$, $P^{IV} = 136$, $P^{V} = 118$, $P^{VI} = 101$, $P^{VII} = 85$, $P^{VIII} = 71$, and $P^{IX} = 59$; and Q is $= 118$, $Q^{I} = 101$, $Q^{II} = 85$, $Q^{III} = 71$, $Q^{IV} = 59$, $Q^{V} = 48$, $Q^{VI} = 38$, $Q^{VII} = 29$, $Q^{VIII} = 22$, and $Q^{IX} = 16$; and R is $= 48$, $R^{I} = 38$, $R^{II} = 29$, $R^{III} = 22$, $R^{IV} = 16$, $R^{V} = 11$, $R^{VI} = 7$, $R^{VII} = 4$, $R^{VIII} = 2$, and $R^{IX} = 1$. And r is $= 1.03$, and $\frac{1}{r} = \frac{1}{1.03} = .9708$, and $\frac{1}{r^2} = .9425$, and $\frac{1}{r^3} = .9151$, $\frac{1}{r^4} = .8884$, $\frac{1}{r^5} = .8626$, $\frac{1}{r^6} = .8374$, $\frac{1}{r^7} = .8130$, $\frac{1}{r^8} = .7894$, and $\frac{1}{r^9} = .7664$. Therefore the expression

$\frac{\pounds 1}{PQR} \times$ the series $\frac{P^{I}Q^{I}R^{I}}{r} + \frac{P^{II}Q^{II}R^{II}}{r^2} + \frac{P^{III}Q^{III}R^{III}}{r^3} + \frac{P^{IV}Q^{IV}R^{IV}}{r^4} + \frac{P^{V}Q^{V}R^{V}}{r^5} + \frac{P^{VI}Q^{VI}R^{VI}}{r^6} + \frac{P^{VII}Q^{VII}R^{VII}}{r^7} + \frac{P^{VIII}Q^{VIII}R^{VIII}}{r^8} + \frac{P^{IX}Q^{IX}R^{IX}}{r^9}$ will be equal to $\frac{\pounds 1}{211 \times 118 \times 48} \times$ the series

$$
\begin{aligned}
& 192 \times 101 \times 38 \times .9708 \\
+\ & 173 \times 85 \times 29 \times .9425 \\
+\ & 154 \times 71 \times 22 \times .9151 \\
+\ & 136 \times 59 \times 16 \times .8884 \\
+\ & 118 \times 48 \times 11 \times .8626 \\
+\ & 101 \times 38 \times 7 \times .8374 \\
+\ & 85 \times 29 \times 4 \times .8130 \\
+\ & 71 \times 22 \times 2 \times .7894 \\
+\ & 59 \times 16 \times 1 \times .7664
\end{aligned}
$$

$= \frac{\pounds 1}{24{,}898 \times 48} \times$ the series

$$
\begin{aligned}
& 19392 \times 38 \times .9708 \\
+\ & 14705 \times 29 \times .9425 \\
+\ & 10934 \times 22 \times .9151 \\
+\ & 8024 \times 16 \times .8884 \\
+\ & 5664 \times 11 \times .8626 \\
+\ & 3838 \times 7 \times .8374 \\
+\ & 2465 \times 4 \times .8130 \\
+\ & 1562 \times 2 \times .7894 \\
+\ & 944 \times 1 \times .7664
\end{aligned}
$$

$= \frac{\pounds 1}{24{,}898 \times 48} \times$ the series

$$
\begin{aligned}
& 38 \times 18{,}825.7536 \\
+\ & 29 \times 13{,}859.4625 \\
+\ & 22 \times 10{,}005.7034 \\
+\ & 16 \times 7{,}288.4336 \\
+\ & 11 \times 4{,}885.7664 \\
+\ & 7 \times 3{,}213.9412
\end{aligned}
$$

$$
\begin{array}{rl}
+ & 4 \times 2004.0450 \\
+ & 2 \times 1233.0428 \\
+ & 1 \times 723.4816 \\
= & \dfrac{£1}{1,195,104} \times \text{the series} \\
 & 715,378.6368 \\
+ & 401,924.4125 \\
+ & 220,125.4748 \\
+ & 116,614.9376 \\
+ & 53,743.4304 \\
+ & 22,497.5884 \\
+ & 8,016.1800 \\
+ & 2,466.0856 \\
+ & 723.4816
\end{array}
$$

$= \dfrac{£1}{1,195,104} \times 1,541,490.2277 = \dfrac{£1,541,490.2277}{1,195,104} = £1.2898$. Therefore the value of an annuity of one pound a year for the whole joint continuance of three lives of the ages of 75 years, 80 years, and 85 years, when the intereſt of money is 3 per cent. is 1.2898, or £.1 5*s*. $9\frac{1}{2}$*d*. Q E I.

S C H O L I U M.

CCCCLXXIV. When three, or more, lives are combined together, the differences that may be taken between their ſeveral ages are ſo immenſely numerous that it is totally impracticable to form tables that ſhall exhibit the values of all, or even any conſiderable part of, the ſeveral annuities that may be ſuppoſed to depend on them. And conſequently, whenever the values of ſuch annuities are wanted, it will be neceſſary to take the trouble of computing them.

And, if the accurate value of ſuch an annuity is wanted, I believe there is no other way of finding it but that which has been above ſet forth by computing the expreſſion, $\frac{£1}{PQR} \times$ the ſeries $\frac{P^{I} Q^{I} R^{I}}{r} + \frac{P^{II} Q^{II} R^{II}}{r^2}$

$\frac{P^{III}\,Q^{III}\,R^{III}}{r^3} + \frac{P^{IV}\,Q^{IV}\,R^{IV}}{r^4} + \frac{P^{V}\,Q^{V}\,R^{V}}{r^5} + \frac{P^{VI}\,Q^{VI}\,R^{VI}}{r^6} + \frac{P^{VII}\,Q^{VII}\,R^{VII}}{r^7} +$ &c. continued to the end of the table of probabilities of the duration of human life; as was done in the foregoing example. But this method of obtaining it is in all cafes rather tedious, even when the oldeft life is a very old one: but when the three lives are all young, the computation is fo very long and troublefome that few perfons will, probably, care to undertake it. It feems therefore to be highly expedient to endeavour to find out fome lefs difficult method of obtaining the value of an annuity of this kind by a tolerable approximation. Now this may be done to a moderate degree of exactnefs by a method analogous to that explained in Art. CCCCXLVI, CCCCXLVII, CCCCXLVIII, CCCCXLIX, whereby the value of an annuity of one pound a year for two joint lives of given ages was derived from the value of the fame annuity for the older of the two fingle lives by means of the expreffion $\overset{\pounds}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$. For we fhall find, upon examination, that the value of an annuity of one pound a year for three joint lives of given ages may be derived in the like manner from the value of the fame annuity for the joint continuance of the two older lives, by means of an expreffion exactly fimilar to the aforefaid expreffion $\overset{\pounds}{A} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$. For if $\overset{\pounds}{B}$ is the value of an annuity of one pound a year for the joint continuance of the two older of the three lives, and g is $= \frac{Q^{I}\,R^{I}}{r}$, and $h = \frac{Q^{II}\,R^{II}}{r^2}$; the value of an annuity of one pound a year for the joint continuance of all the three lives will be, nearly, equal to $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$; as may be fhewn in the manner following.

A method

A method of deriving the value of an annuity of one pound a year for the joint continuance of three lives of any given ages, from the value of the same annuity for the joint continuance of the two older of the said lives, by approximation.

CCCCLXXV. Let S be put for the series $\frac{Q^{I} R^{I}}{r} + \frac{Q^{II} R^{II}}{r^2} + \frac{Q^{III} R^{III}}{r^3} + \frac{Q^{IV} R^{IV}}{r^4} + \frac{Q^{V} R^{V}}{r^5} + \frac{Q^{VI} R^{VI}}{r^6} + \frac{Q^{VII} R^{VII}}{r^7} +$ &c. continued to the end of the table of the probabilities of the duration of human life. And let the Greek capital letter Σ be put for the series $\frac{P^{I} Q^{I} R^{I}}{r} + \frac{P^{II} Q^{II} R^{II}}{r^2} + \frac{P^{III} Q^{III} R^{III}}{r^3} + \frac{P^{IV} Q^{IV} R^{IV}}{r^4} + \frac{P^{V} Q^{V} R^{V}}{r^5} + \frac{P^{VI} Q^{VI} R^{VI}}{r^6} + \frac{P^{VII} Q^{VII} R^{VII}}{r^7} +$ &c. continued likewise to the end of the same table of probabilities.

Then it is evident that $\overset{£}{B}$, or the value of an annuity of one pound a year for the joint continuance of the two older lives (which answer to the letters Q and R,) will be $= \frac{£1}{QR} \times S$; and that $\overset{£}{C}$, or the value of the same annuity for the joint continuance of all the three lives, will be $= \frac{£1}{PQR} \times \Sigma$.

Let i be put $= \frac{P^{I} Q^{I} R^{I}}{r}$, and k be put $= \frac{P^{II} Q^{II} R^{II}}{r^2}$.

Then, since g was taken $= \frac{Q^{I} R^{I}}{r}$, and h was taken $= \frac{Q^{II} R^{II}}{r^2}$, we shall have $i = g \times P^{I}$, and $k = h \times P^{II}$.

Let G be put for the infinite geometrical progression $g + h + \frac{hh}{g} +$

$\frac{h^3}{gg} + \frac{h^4}{g^3} + \frac{h^5}{g^4} + \frac{h^6}{g^5} +$ &c. of which the two firſt terms, g and h are reſpectively equal to, $\frac{Q^I R^I}{r}$ and $\frac{Q^{II} R^{II}}{r^2}$, the two firſt terms of the ſeries S. And let the Greek capital letter Γ be put for the infinite geometrical progreſſion $i + k + \frac{kk}{i} + \frac{k^3}{ii} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5} +$ &c. of which the two firſt terms, i and k, are reſpectively equal to, $\frac{P^I Q^I R^I}{r}$ and $\frac{P^{II} Q^{II} R^{II}}{r^2}$, the two firſt terms of the ſeries Σ.

Then will G be $= \frac{gg}{g-h}$; and Γ will be $= \frac{ii}{i-k}$.

CCCCLXXVI. Theſe things being premiſed, it ſeems reaſonable to conjecture that the geometrical progreſſion $g + h + \frac{hh}{g} + \frac{h^3}{gg} + \frac{h^4}{g^3} + \frac{h^5}{g^4} + \frac{h^6}{g^5} +$ &c. *ad infinitum*, or G, will bear pretty nearly the ſame proportion to its kindred ſeries S, (of which the two firſt terms, $\frac{Q^I R^I}{r}$ and $\frac{Q^{II} R^{II}}{r^2}$, are reſpectively equal to its two firſt terms g and h,) as the geometrical progreſſion $i + k + \frac{kk}{i} + \frac{k^3}{ii} + \frac{k^4}{i^3} + \frac{k^5}{i^4} + \frac{k^6}{i^5} +$ &c. *ad infinitum* bears to its kindred ſeries Σ, of which the two firſt terms, $\frac{P^I Q^I R^I}{r}$ and $\frac{P^{II} Q^{II} R^{II}}{r^2}$, are reſpectively equal to its two firſt terms, i and k. And, if this conjecture is well-founded, we ſhall have Σ nearly $\frac{\Gamma \times S}{G}$, and conſequently nearly $= \frac{\frac{ii}{i-k} \times S}{\frac{gg}{g-h}} = \frac{ii}{i-k} \times S \times \frac{g-h}{gg} =$

A conjectural ſuppoſition upon which this method of approximation is founded.

$\frac{g\times P^{I}\times g\times P^{I}}{g\times P^{I}-h\times P^{II}}\times S\times\frac{g-h}{gg} = \frac{gg\times P^{I}\times P^{I}}{g\times P^{I}-h\times P^{II}}\times S\times\frac{g-h}{gg} = \frac{P^{I}\times P^{I}}{g\times P^{I}-h\times P^{II}}\times S\times\overline{g-h} = \frac{P^{I}}{g\times P^{I}-h\times P^{II}}\times S\times\overline{g-h}$ $\times P^{I} = \frac{P^{I}}{g\times P^{I}-h\times P^{II}}\times S\times\overline{g\times P^{I}-h\times P^{I}} = P^{I}\times S$ $\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}}$.

But $\overset{£}{B}$ is $= \frac{£1}{QR}\times S$; and consequently S is $= \overset{£}{B}\times QR$. Therefore $P^{I}\times S\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}}$ is $= P^{I}\times\overset{£}{B}\times QR\times$ $\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}}$; and consequently Σ is, nearly, $= P^{I}\times\overset{£}{B}\times QR$ $\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}=h\times P^{II}}$.

Therefore $\overset{£}{C}$, or $\frac{£1}{PQR}\times\Sigma$, is nearly $= \frac{£1}{PQR}\times P^{I}\times\overset{£}{B}\times QR$ $\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}} = \frac{1}{P}\times P^{I}\times\overset{£}{B}\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}} = \overset{£}{B}\times\frac{P^{I}}{P}$ $\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}}$; that is, the value of an annuity of one pound a year for the joint continuance of all the three lives will be, nearly, equal to $\overset{£}{B}\times\frac{P^{I}}{P}\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}}$, or may be derived from $\overset{£}{B}$, the value of the same annuity for the joint continuance of the two older lives, by multiplying it into the fraction $\frac{P^{I}}{P}\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}}$. Q E I.

CCCCLXXVII. This expression $\overset{£}{B}\times\frac{P^{I}}{P}\times\frac{g\times P^{I}-h\times P^{I}}{g\times P^{I}-h\times P^{II}}$ will, I believe, be almost always less than the true value of $\overset{£}{C}$. But in what *de-*

gree,

gree, or within what *limits*, it will differ from it, I do not know. But, that we may form some tolerable conjecture upon the subject, I will now proceed to apply this expression to the computation of the values of a few annuities for three joint lives, of which the learned Mr. Morgan has given us the true values to five places of figures, computed strictly by means of the above-mentioned expression, $\frac{\pounds 1}{PQR} \times$ the series $\frac{P^{I}Q^{I}R^{I}}{r} + \frac{P^{II}Q^{II}R^{II}}{r^{2}} + \frac{P^{III}Q^{III}R^{III}}{r^{3}} + \frac{P^{IV}Q^{IV}R^{IV}}{r^{4}} + \frac{P^{V}Q^{V}R^{V}}{r^{5}} + \frac{P^{VI}Q^{VI}R^{VI}}{r^{6}} + \frac{P^{VII}Q^{VII}R^{VII}}{r^{7}} +$ &c. continued to the end of the table of the probabilities of the duration of human life. These true values are contained in the seventh table of Mr. Morgan's treatise on the doctrine of annuities, page 273. They are the values of an annuity of one pound a year for three joint lives of equal ages, from the age of 60 years to the age of 91 years, inclusive of both. And they are computed from the Northampton table of the probabilities of the duration of human life, upon a supposition that the interest of money is 4 per cent. This table is as follows.

TABLE

T A B L E XLIX.

Containing the true values of an annuity of one pound a year for the joint continuance of three lives of equal ages, from the age of 60 *years to the age of* 91 *years, both included; when the interest of money is* 4 *per cent.*

Computed from the Northampton table of the probabilities of the duration of human life.

Years in the age of the first life.	*Years in the age of the second life.*	*Years in the age of the third life.*	*Values of an annuity of 1 l. a year for the joint continuance of all the three lives.*	*Years in the age of the first life.*	*Years in the age of the second life.*	*Years in the age of the third life.*	*Values of an annuity of 1 l. a year for the joint continuance of all the three lives.*
			£				£
60	60	60	4.7826	76	76	76	1.9089
61	61	61	4.6115	77	77	77	1.7846
62	62	62	4.4382	78	78	78	1.5843
63	63	63	4.2626	79	79	79	1.3906
64	64	64	4.0849	80	80	80	1.2121
65	65	65	3.9050	81	81	81	1.0685
66	66	66	3.7230	82	82	82	1.0117
67	67	67	3.5390	83	83	83	0.9617
68	68	68	3.3533	84	84	84	0.8981
69	69	69	3.1662	85	85	85	0.7906
70	70	70	2.9780	86	86	86	0.7690
71	71	71	2.7895	87	87	87	0.7568
72	72	72	2.6015	88	88	88	0.5368
73	73	73	2.4160	89	89	89	0.3233
74	74	74	2.2352	90	90	90	0.1346
75	75	75	2.0636	91	91	91	0.1202

CCCCLXXVIII. Mr.

CCCCLXXVIII. Mr. Morgan has alſo given us, in his valuable treatiſe on annuities before-mentioned, pages 74, 75, and 76, a compleat table of the true values of an annuity of one pound a year for the joint continuance of two lives of equal ages, when the intereſt of money is 4 per cent. computed ſtrictly from the aforeſaid Northampton table of the probabilities of the duration of human life, for every age of life from the age of one year to that of 91 years, incluſively. This table is as follows.

TABLE L.

Containing the true values of an annuity of one pound a year for the joint continuance of two lives of equal ages, from the age of one year to the age of 91 years, incluſively; when the intereſt of money is 4 per cent.

Computed from the Northampton table of the probabilities of the duration of human life.

Years in the age of the firſt life.	Years in the age of the ſecond life.	Values of an annuity of 1*l.* a year for the joint continuance of both lives.	Years in the age of the firſt life.	Years in the age of the ſecond life.	Values of an annuity of 1*l.* a year for the joint continuance of both lives.
		£			£
1	1	10.[illegible]432	14	14	13.5638
2	2	11.3227	15	15	13.3673
3	3	12.5931	16	16	13.1616
4	4	13.1723	17	17	12.9461
5	5	13.63[illegible]2	18	18	12.7201
6	6	13.9355	19	19	12.5346
7	7	14.1[illegible]20	20	20	12.3928
8	8	14.2031	21	21	12.2985
9	9	14.2356	22	22	12.2[illegible]38
10	10	[illegible]	23	23	12.1089
11	11	[illegible]	24	24	12.0139
12	12	[illegible]	25	25	[illegible]
13	13	[illegible]	26	26	11.8242

Years in the age of the first life.	*Years in the age of the second life.*	*Values of an annuity of 1l. a year for the joint continuance of both lives.*	*Years in the age of the first life.*	*Years in the age of the second life.*	*Values of an annuity of 1l. a year for the joint continuance of both lives.*
		£			£
27	27	11.7292	60	60	6.2468
28	28	11.6351	61	61	6.0407
29	29	11.5422	62	62	5.8313
30	30	11.4500	63	63	5.6188
31	31	11.3007	64	64	5.4031
32	32	11.1462	65	65	5.1843
33	33	10.9880	66	66	4.9626
34	34	10.8240	67	67	4.7381
35	35	10.6538	68	68	4.5111
36	36	10.4782	69	69	4.2821
37	37	10.2962	70	70	4.0516
38	38	10.1073	71	71	3.8203
39	39	9.9110	72	72	3.5893
40	40	9.7065	73	73	3.3605
41	41	9.5532	74	74	3.1364
42	42	9.3960	75	75	2.9192
43	43	9.2358	76	76	2.7182
44	44	9.0723	77	77	2.5427
45	45	8.9054	78	78	2.2976
46	46	8.7350	79	79	2.0694
47	47	8.5608	80	80	1.8570
48	48	8.3829	81	81	1.6868
49	49	8.2008	82	82	1.5998
50	50	8.0779	83	83	1.5144
51	51	7.9599	84	84	1.4143
52	52	7.7818	85	85	1.244,62
53	53	7.6006	86	86	1.213,51
54	54	7.4165	87	87	1.132,87
55	55	7.2294	88	88	0.840,92
56	56	7.0392	89	89	0.554,77
57	57	6.8459	90	90	0.298,169
58	58	6.6494	91	91	0.240,383
59	59	6.4497			

CCCCLXXIX. And

CCCCLXXIX. And the Northampton table of the probabilities of the duration of human life, from which the two foregoing tables of the values of an annuity of one pound for two and three joint lives were computed, is given us likewife in Mr. Morgan's aforefaid treatife on annuities, page 267, and is as follows.

TABLE LI.

Reprefenting the probabilities of the duration of human life at the feveral ages therein mentioned, from the age of one year to the age of 92 years, inclufively; as deduced by Dr. Price from obfervations on the bills of mortality at Northampton.

Age.	*Perfons living.*	*Age.*	*Perfons living.*	*Age.*	*Perfons living.*	*Age.*	*Perfons living.*
1 year.	849	25	475	49	293	73	99
2 years.	722	26	467	50	284	74	91
3	672	27	459	51	275	75	83
4	646	28	451	52	267	76	75
5	625	29	443	53	259	77	67
6	609	30	435	54	251	78	60
7	596	31	428	55	243	79	53
8	586	32	421	56	235	80	46
9	577	33	414	57	227	81	39
10	570	34	407	58	219	82	32
11	564	35	400	59	211	83	26
12	558	36	393	60	203	84	21
13	553	37	386	61	195	85	17
14	548	38	379	62	187	86	13
15	543	39	372	63	179	87	10
16	538	40	365	64	171	88	8
17	533	41	357	65	163	89	6
18	528	42	349	66	155	90	4
19	522	43	341	67	147	91	2
20	515	44	333	68	139	92	1
21	507	45	325	69	131	93	0
22	499	46	317	70	123		
23	491	47	309	71	115		
24	483	48	301	72	107		

CCCCLXXX. By the help of theſe two laſt tables we may apply the expreſſion $\overset{\pounds}{B} \times \frac{P^{\mathrm{I}}}{P} \times \frac{g \times P^{\mathrm{I}} - h \times P^{\mathrm{I}}}{g \times P^{\mathrm{I}} - h \times P^{\mathrm{II}}}$ to the diſcovery of near values of the ſeveral annuities for three joint lives of equal ages, of which we have ſeen the true values above in Table XLIX, Art. CCCCLXXVII. This may be done in the manner following.

CCCCLXXXI. When the three lives are all of the ſame age, Q and R will be equal to P, and Q^{I} and R^{I} will be equal to P^{I}, and Q^{II} and R^{II} will be equal to P^{II}. Therefore in this caſe g, or $\frac{Q^{\mathrm{I}}\,R^{\mathrm{I}}}{r}$, will be $= \frac{P^{\mathrm{I}}\,P^{\mathrm{I}}}{r}$, and h, or $\frac{Q^{\mathrm{II}}\,R^{\mathrm{II}}}{r^2}$, will be $= \frac{P^{\mathrm{II}}\,P^{\mathrm{II}}}{r^2}$; and conſequently $g \times P^{\mathrm{I}}$ will be $= \frac{P^{\mathrm{I}}\,P^{\mathrm{I}}}{r} \times P^{\mathrm{I}} = \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}}}{r}$; and $h \times P^{\mathrm{I}}$ will be $= \frac{P^{\mathrm{II}}\,P^{\mathrm{II}}}{r^2} \times P^{\mathrm{I}} = \frac{P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{I}}}{r^2}$; and $h \times P^{\mathrm{II}}$ will be $= \frac{P^{\mathrm{II}}\,P^{\mathrm{II}}}{r^2} \times P^{\mathrm{II}} = \frac{P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{II}}}{r^2}$. Therefore $g \times P^{\mathrm{I}} - h \times P^{\mathrm{I}}$ will be $= \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}}}{r} - \frac{P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{I}}}{r^2} = \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r}{r^2} - \frac{P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{I}}}{r^2} = \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r - P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{I}}}{r^2}$; and $g \times P^{\mathrm{I}} - h \times P^{\mathrm{II}}$ will be $= \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}}}{r} - \frac{P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{II}}}{r^2} = \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r}{r^2} - \frac{P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{II}}}{r^2} = \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r - P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{II}}}{r^2}$; and conſequently $\frac{g \times P^{\mathrm{I}} - h \times P^{\mathrm{I}}}{g \times P^{\mathrm{I}} - h \times P^{\mathrm{II}}}$ will be $= \dfrac{\frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r - P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{I}}}{r^2}}{\frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r - P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{II}}}{r^2}}$

$= \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$. Therefore $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P}{g \times P^{I} - h \times P^{II}}$, or the near value of an annuity of one pound a year for the three joint lives, will, in this case of an equality of the three ages, be $= \overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$; which I take to be the most convenient form to which this expression can be reduced.

An example of the computation of the near value of an annuity of one pound a year for the joint continuance of three lives of equal ages, by means of the foregoing expression.

CCCCLXXXII. Let it now be required, by means of this expression, $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, to find a near value of an annuity of one pound a year for three joint lives all of the age of 60 years, when the interest of money is 4 per cent. according to the foregoing Northampton table of the probabilities of the duration of human life; of which annuity we have seen above, in Table XLIX, that the true value is £4.7826.

Now it appears from Table L, Art. CCCCLXXVIII, that $\overset{£}{B}$, or the value of an annuity of one pound a year for the joint continuance of two lives that are both of the same age of 60 years, when the interest of money is 4 per cent. and according to the Northampton table of the probabilities of the duration of human life, is £6.2468. And it appears by the said table of probabilities, that the number of persons therein supposed to be living at the several ages of 60 years, 61 years, and 62 years, are 203, 195, and 187. Therefore P is $= 203$, $P^{I} = 195$, and P^{II} is $= 187$.

Also, since the interest of money is supposed to be 4 per cent. r will be $= 1.04$.

Therefore $P^{I} \times P^{I} \times P^{I} \times r$ will be $(= 195 \times 195 \times 195 \times 1.04 = 7,414,875 \times 1.04) = 7,711,470.00$; and $P^{II} \times P^{II} \times P^{I}$ will be $(= 187 \times 187 \times 195 = 34,969 \times 195) = 6,818,955$; and $P^{II} \times P^{II} \times P^{II}$ will be $(= 187 \times 187 \times 187 = 34,969 \times 187) = 6,539,203$. Therefore

 $P^{I} \times$

$P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}$ will be $(= 7{,}711{,}470 - 6{,}818{,}955) = 892{,}515$; and $P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}$ will be $(= 7{,}711{,}470 - 6{,}539{,}203) = 1{,}172{,}267$; and consequently $\frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ will be $= \frac{892{,}515}{1{,}172{,}267} = 0.761{,}35$. Therefore $\frac{P^{I}}{P} \times$ the fraction $\frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ will be $(= \frac{P^{I}}{P} \times 0.761{,}35 = \frac{195}{203} \times 0.761{,}35 = \frac{148.46325}{203}) = 0.731{,}34$; and $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ will be $(= B \times 0.731{,}34 = 6.2468 \times 0.731{,}34) = 4.5685$. Therefore 4.5685 is a near value of the proposed annuity of one pound a year for the joint continuance of three lives, all of the age of 60 years, when the interest of money is 4 per cent. according to the Northampton table of the probabilities of the duration of human life. Q E I.

This value of the proposed annuity is less than its true value £4.7826, (given above in Table XLIX,) by the difference £0.2141, which is about a 22d part of the said true value, £4.7826.

Near values of an annuity of one pound a year for three equal joint lives of the ages of 61 years, 62 years, 63 years, 64 years, &c. to the age of 91 years, inclusively, computed from the same expression.

CCCCLXXXIII. If the values of an annuity of one pound a year for the joint continuance of three equal lives of the ages of 61 years, 62 years, 63 years, 64 years, &c. to the age of 91 years, inclusively, are computed in the same manner by means of the said expression, $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, or $\overset{\pounds}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$ they will be found to be as follows; to wit,

for three equal joint lives of the age of 61 years, = 4.3992,
of the age of 62 years, = 4.2281,
of the age of 63 years, = 4.0555,
of the age of 64 years, = 3.8814,
of the age of 65 years, = 3.7059,

of the

of the age of 66 years, = 3.5293,
of the age of 67 years, = 3.3516,
of the age of 68 years, = 3.1730,
of the age of 69 years, = 2.9942,
of the age of 70 years, = 2.8153,
of the age of 71 years, = 2.6370,
of the age of 72 years, = 2.4600.
of the age of 73 years, = 2.2857,
of the age of 74 years, = 2.1156,
of the age of 75 years, = 1.9513,
of the age of 76 years, = 1.7961,
of the age of 77 years, = 1.6861,
of the age of 78 years, = 1.5067,
of the age of 79 years, = 1.3398,
of the age of 80 years, = 1.1842,
of the age of 81 years, = 1.0439,
of the age of 82 years, = 0.98201,
of the age of 83 years, = 0.92350,
of the age of 84 years, = 0.87917,
of the age of 85 years, = 0.72954,
of the age of 86 years, = 0.72759,
of the age of 87 years, = 0.69995,
of the age of 88 years, = 0.50505,
of the age of 89 years, = 0.31932,
of the age of 90 years, = 0.12872,
and of the age of 91 years, = 0.12019.

Theſe near values, we may obſerve, are all leſs than the true values of the ſame annuities, given above in Table XLIX. And this will be found to be the caſe in moſt inſtances, except when ſome of the lives are very young; and then it ſometimes happens that the near values of an annuity for three joint lives, obtained by the foregoing expreſſion, is greater than its true value.

 CCCCLXXXIV. The

The differences of the foregoing near values of an annuity of one pound a year for three joint lives and the corresponding true values of the same annuity.

CCCCLXXXIV. The differences of theſe near values of the aforeſaid annuities for three equal joint lives from their true values contained above in Table XLIX, are as follows; to wit,

£.		£.		£.
4.6115	—	4.3992	=	0.2123;
4.4382	—	4.2281	=	0.2101;
4.2626	—	4.0555	=	0.2071;
4.0849	—	3.8814	=	0.2035;
3.9050	—	3.7059	=	0.1991;
3.7230	—	3.5293	=	0.1937;
3.5390	—	3.3516	=	0.1874;
3.3533	—	3.1730	=	0.1803;
3.1662	—	2.9942	=	0.1720;
2.9780	—	2.8153	=	0.1627;
2.7895	—	2.6370	=	0.1525;
2.6015	—	2.4600	=	0.1415;
2.4160	—	2.2857	=	0.1303;
2.2352	—	2.1156	=	0.1196;
2.0636	—	1.9513	=	0.1123;
1.9089	—	1.7961	=	0.1128;
1.7846	—	1.6861	=	0.0985;
1.5843	—	1.5067	=	0.0776;
1.3906	—	1.3398	=	0.0508;
1.2121	—	1.1842	=	0.0279;
1.0685	—	1.0439	=	0.0246;
1.0117	—	0.982,01	=	0.029,69;
0.9617	—	0.923,50	=	0.038,20;
0.8981	—	0.879,17	=	0.078,93;
0.7606	—	0.729,54	=	0.031,06;
0.7690	—	0.727,59	=	0.041,41;
0.7568	—	0.699,95	=	0.056,85;
0.5368	—	0.505,05	=	0.031,75;

0.3233 — 0.319,32 = 0.003,98 ;
0.1346 — 0.128,72 = 0.005,88 ;
0.1202 — 0.120,19 = 0.000,01.

The proportions of the foregoing differences to the said several true values, respectively.

CCCCLXXXV. And the proportions of these differences to the said true values themselves are expressed by the following fractions, to wit,

$\frac{1}{21}$, $\frac{1}{21}$, $\frac{1}{20}$, $\frac{1}{20}$, $\frac{1}{19}$, $\frac{1}{19}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{18}$, $\frac{1}{20}$, $\frac{1}{27}$, $\frac{1}{43}$, $\frac{1}{43}$, $\frac{1}{34}$, $\frac{1}{25}$, $\frac{1}{11}$, $\frac{1}{24}$, $\frac{1}{18}$, $\frac{1}{13}$, $\frac{1}{16}$, $\frac{1}{81}$, $\frac{1}{22}$, and $\frac{1}{12020}$.

A conclusion from the foregoing examples in favour of the usefulness of the foregoing expression, when the ages of the three lives are equal to each other and are not less than 60 years.

These fractions (though they are not so small as might have been wished,) are sufficiently small to make the foregoing near values of an annuity of one pound a year for three equal joint lives of the ages of 60 years, 61 years, 62 years, &c. to the end of life, of very considerable use in practice. And therefore we may conclude from them that, in estimating annuities for three equal joint livesin the latter period of human life, after the age of 60 years, the expression

$$\text{£}B \times \frac{P^{\text{I}}}{P} \times \frac{P^{\text{I}} \times P^{\text{I}} \times P^{\text{I}} \times r - P^{\text{II}} \times P^{\text{II}} \times P^{\text{I}}}{P^{\text{I}} \times P^{\text{I}} \times P^{\text{I}} \times r - P^{\text{II}} \times P^{\text{II}} \times P^{\text{II}}}, \text{ or } \text{£}B \times \frac{P^{\text{I}}}{P} \times \frac{g \times P^{\text{I}} - b \times P^{\text{I}}}{g \times P^{\text{I}} - b \times P^{\text{II}}},$$

may safely be adopted, as a means of determining the values of such annuities to a tolerable degree of exactness.

An inquiry into the usefulness of the said expression in other cases.

CCCCLXXXVI. But it may be doubted, perhaps, whether this expression will be equally useful in determining the values of annuities for three joint lives, when the ages of the lives are younger than 60 years, and when the three lives are not all of equal ages. Now, in order to form some judgement upon this matter, it will be necessary to try this approximation in some other instances, in which the three lives shall be much younger than 60 years of age, and likewise in some instances in which the three lives shall not be all of equal ages. And with this view I shall now present the reader

reader with four more tables of values of annuities for three joint lives, of which the two former contain the accurate value of an annuity of one pound a year for three equal joint lives of the ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35, 40, 45, 50, 55, 60, 65, and 70 years, and for three lives of unequal ages that differ from each other by 10 years and 20 years, and of which the youngest lives are of the ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35, 40, 45, 50, 55, 60, 65, 70, and 75 years, and the two latter contain the near values of the annuities mentioned in the two former tables, which refult from the expreffion £ $B \times \frac{P\text{I}}{P} \times \frac{g \times P\text{I} - h \times P\text{I}}{g \times P\text{I} - h \times P\text{II}}$. The two firft tables have been communicated to me by Dr. Price, and the two latter have been computed by an able arithmetician, who was employed for that purpofe by Mr. Morgan, of the Society for equitable Affurances near Black-friars Bridge. In all the four tables the intereft of money is fuppofed to be 4 per cent. and the table of the probabilities of the duration of human life, upon which the calculations are founded, is not that of Monfieur de Parcieux, (fo often mentioned in the courfe of this work,) nor yet the Northampton table of thofe probabilities given above in Art. CCCCLXXIX, but a new table derived from the faid Northampton table by Dr. Price, and called by him "*The new Northampton Table of thofe probabilities*," being an improvement on the faid former Northampton table, and (as I am informed,) differing but little from it. Thefe four tables are as follows.

TABLE

TABLE LII.

Containing the true values of an annuity of one pound a year for the joint continuance of the lives of three persons all of the same age, at the several ages of 5 years, 10 years, 15 years, 20 years, 25 years, 30 years, 35 years, 40 years, 45 years, 50 years, 55 years, 60 years, 65 years, and 70 years; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life.

Years in the age of the first life.	*Years in the age of the second life.*	*Years in the age of the third life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	5	5	11.1704
10	10	10	12.2006
15	15	15	11.2746
20	20	20	10.3429
25	25	25	9.796,42
30	30	30	9.221,10
35	35	35	8.585,22
40	40	40	7.865,05
45	45	45	7.126,40
50	50	50	6.317,17
55	55	55	5.550,60
60	60	60	4.755,03
65	65	65	3.914,00
70	70	70	2.995,84

TABLE

TABLE LIII.

Containing the true values of an annuity of one pound a year for the joint continuance of the lives of three persons of unequal ages, that differ from each other by 10 *years and* 20 *years, when the age of the youngest life is either* 5 *years, or* 10 *years, or* 15 *years, or* 20 *years, or* 25 *years, or* 30 *years, or* 35, *or* 40, 45 50, 55, 60, 65, 70, *or* 75 *years; upon a supposition that the interest of money is* 4 *per cent.*

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life.

Years in the age of the first, or youngest, life.	*Years in the age of the second life.*	*years in the age of the third, or oldest, life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	15	25	10.6551
10	20	30	10.4379
15	25	35	9.738,56
20	30	40	8.986,72
25	35	45	8.313,10
30	40	50	7.570,83
35	45	55	6.816,07
40	50	60	5.994,15
45	55	65	5.145,62
50	60	70	4.219,37
55	65	75	3.297,98
60	70	80	2.408,48
65	75	85	1.623,48
70	80	90	1.122,51
75	85	95	0.169,378

TABLE

TABLE LIV.

Containing approximations to the values of the annuities for three equal joint lives mentioned above in Table LII; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by means of the expression $\frac{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, or $\frac{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, *given above in Art.* CCCCLXXVI *and* CCCCLXXXI.

Years in the age of the first life.	*Years in the age of the second life.*	*Years in the age of the third life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	5	5	10.4912
10	10	10	12.3942
15	15	15	11.4322
20	20	20	10.2377
25	25	25	9.667,62
30	30	30	9.073,29
35	35	35	8.426,20
40	40	40	7.691,08
45	45	45	6.941,47
50	50	50	6.118,36
55	55	55	5.342,53
60	60	60	4.543,77
65	65	65	3.718,59
70	70	70	2.839,99

TABLE LV.

Containing approximations to the values of the annuities for three joint lives of different ages, mentioned above in Table LIII; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by means of the expression $\frac{£}{B} \times \frac{P^1}{P} \times \frac{g \times P^1 - h \times P^1}{g \times P^1 - h \times P^{11}}$, *given above in Art.* CCCCLXXVI.

Years in the age of the first, or youngest, life.	*Years in the age of the second life.*	*years in the age of the third, or oldest, life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	15	25	9.1325
10	20	30	10 5080
15	25	35	9.858,45
20	30	40	8.756,27
25	35	45	8.068,36
30	40	50	7.343,65
35	45	55	6.594,61
40	50	60	5.786,88
45	55	65	4.947,33
50	60	70	4.044,10
55	65	75	3.181,57
60	70	80	2.332,36
65	75	85	1.604,14
70	80	90	1.093,25
75	85	95	0.169,378

CCCCLXXXVII. The

CCCCLXXXVII. The differences between the true values of the annuities for three equal joint lives, mentioned in Table LII, and the near values of the same annuities which are contained in Table LIV, are as follows; to wit,

The differences of the true values of the annuities for three equal joint lives in Table LII and the near values of the same annuities in Table LIV.

£.		£.		£.
11.1704	−	10.4912	=	0.6792;
− 12.2006	+	12.3940	=	0.1936;
− 11.2746	+	11.4322	=	0.1576;
10.3429	−	10.2377	=	0.1052;
9.796,42	−	9.667,62	=	0.128,80;
9.221,10	−	9.073,29	=	0.147,81;
8.585,22	−	8.426,20	=	0.159,02;
7.865,05	−	7.691,08	=	0.173,97;
7.126,40	−	6.941,47	=	0.184,93;
6.317,17	−	6.118,36	=	0.198,81;
5.550,60	−	5.342,53	=	0.208,07;
4.755,03	−	4.543,77	=	0.211,26;
3.914,00	−	3.718,59	=	0.195,41;
2.995,84	−	2.839,99	=	0.155,85.

CCCCLXXXVIII. It may be here observed that all the foregoing near values of annuities for three equal joint lives, contained in Table LIV, and which were obtained by means of the expression $\frac{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, or $\frac{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - b \times P^{I}}{g \times P^{I} - b \times P^{II}}$, are not less than the true values of the same annuities respectively, (as was the case with the near values of annuities for three equal joint lives of the ages of 60 years, 61 years, &c. to 91 years, obtained above in Art. CCCCLXXXII and CCCCLXXXIII by means of the same expression;) but two of the said near values, to wit, £12.3940 and £11.4322, (which relate to the ages of 10 years and 15 years,) are greater than the corresponding true values, £12.2006 and 11.2746.

A comparison of the said near values with the corresponding true values.

A remark on the magnitude of the faid differences with refpect to one year's annuity.

CCCCLXXXIX. It may alfo be obferved that all the foregoing differences between the true values and the near values of thofe annuities, except the firft difference £0.6792, (which relates to the age of 5 years, and is much greater than any of the others,) are lefs than £0.25, or a quarter of a year's annuity; which, with a view to practical purpofes, can hardly be confidered as a very important error.

The proportions of the faid differences to the feveral correfponding true values of the faid annuities.

CCCCXC. The proportions of the foregoing differences to their feveral correfponding true values are expreffed by the following fractions; to wit, $\frac{1}{16}$, $\frac{1}{63}$, $\frac{1}{71}$, $\frac{1}{98}$, $\frac{1}{76}$, $\frac{1}{62}$, $\frac{1}{53}$, $\frac{1}{45}$, $\frac{1}{38}$, $\frac{1}{31}$, $\frac{1}{26}$, $\frac{1}{22}$, $\frac{1}{20}$, and $\frac{1}{19}$. Thefe fractions are, for the moft part, confiderably lefs than the fractions in Art. CCCCLXXXV, which exprefs the proportions of the differences of the near values and true values of the former fet of annuities for three equal joint lives, (where the ages were 60 years and upwards,) to their correfponding true values. We may therefore conclude that the expreffion $\overset{£}{B} \times \frac{P^{\mathrm{I}}}{P} \times \frac{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r - P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{I}}}{P^{\mathrm{I}} \times P^{\mathrm{I}} \times P^{\mathrm{I}} \times r - P^{\mathrm{II}} \times P^{\mathrm{II}} \times P^{\mathrm{II}}}$, or $\overset{£}{B} \times \frac{P^{\mathrm{I}}}{P} \times \frac{g \times P^{\mathrm{I}} - b \times P^{\mathrm{I}}}{g \times P^{\mathrm{I}} - b \times P^{\mathrm{II}}}$, will be as ufeful, or rather more fo, in computing the values of annuities for three equal joint lives, when the lives are under the age of 60 years as when they go beyond it.

The differences between the true values of the annuities for three joint lives of unequal ages, mentioned in Table LIII, and the near values of the fame annuities in Table LV.

CCCCXCI. The differences between the true values of the annuities for three joint lives of unequal ages which differ from each other by 10 years and 20 years, given above in Table LIII, and the near values of the fame annuities which are contained in Table LV, are as follows; to wit,

	£		£		£
	10.6551	−	9.1325	=	1.5226,
−	10.4379	+	10.5080	=	0.0701;
−	9.738,56	+	9.858,45	=	0.119,89;

8.986,72

£.		£.		£.
8.986,72	—	8.756,27	=	0.230,45 ;
8.313,10	—	8.068,36	=	0.244,74 ;
7.570,83	—	7.343,65	=	0.227,18 ;
6.816,07	—	6.594,61	=	0.221,46 ;
5.994,15	—	5.786,88	=	0.207,27 ;
5.145,62	—	4.947,33	=	0.198,29 ;
4.219,37	—	4.044,10	=	0.175,27 ;
3.297,98	—	3.181,57	=	0.116,41 ;
2.408,48	—	2.332,36	=	0.076,12 ;
1.623,48	—	1.604,14	=	0.019,34 ;
1.122,51	—	1.093,25	=	0.029,26 ;
0.169,378	—	0.169,378	=	0.000,000.

A compariſon of the ſaid near values with the correſponding true values.

CCCCXCII. In looking over the near values and true values contained in the foregoing article we may obſerve, that the near values are, for the moſt part, leſs than the correſponding true values; but not conſtantly ſo: for the near values £10.5080 and £9.858,45, (the former of which relates to three lives of the ages of 10, 20, and 30 years, and the latter to three lives of the ages of 15, 25, and 35 years,) are greater than the correſponding true values, £10.4379 and £9.738,56: which agrees pretty much with what was obſerved in Art. CCCCLXXXVIII concerning the near values and the true values contained in Art. CCCCLXXXVII.

A remark on the magnitude of the ſaid differences with reſpect to one year's annuity.

CCCCXCIII. It may alſo be obſerved that all the foregoing differences between the near values in Table LV and the correſponding true values in Table LIII, contained in Art. CCCCXCI, except the firſt difference, £1.5226, (which relates to three lives of the ages of 5 years, 15 years, and 25 years, and which is vaſtly greater than any of the others,) are leſs than £0.25, or a quarter of a year's annuity; which (as was before obſerved) is no very important variation from the truth.

The proportions of the ſaid differences to the ſeveral correſponding true values of the ſaid annuities.

CCCCXCIV. And the proportions of the foregoing differences (contained in Art. CCCCXCI) to their ſeveral correſponding true values are expreſſed

preſſed by the following fractions; to wit, $\frac{1}{7}$, $\frac{1}{148}$, $\frac{1}{81}$, $\frac{1}{38}$, $\frac{1}{23}$, $\frac{1}{33}$, $\frac{1}{30}$, $\frac{1}{28}$, $\frac{1}{25}$, $\frac{1}{24}$, $\frac{1}{28}$, $\frac{1}{31}$, $\frac{1}{83}$, and $\frac{1}{38}$.

Theſe fractions are for the moſt part ſomewhat greater than the fractions in Art. ccccxc, but ſomewhat leſs than thoſe in Art. ccccLxxxv, which expreſs the proportions of the differences of the near values and true values of the former ſet of annuities for three equal joint lives (where the ages were 60 years and upwards,) to their correſponding true values. We may therefore conclude that the expreſſion $\frac{£}{B} \times \frac{P'}{P} \times \frac{g \times P' - h \times P'}{g \times P' - h \times P''}$ will be as uſeful, or rather more ſo, in computing the values of annuities for three joint lives whoſe ages differ from each other by 10 years and 20 years, throughout all the periods of life, or, at leaſt, when the youngeſt life is not younger than 10 years, as in computing the values of annuities for three equal joint lives of the age of 60 years, or upwards.

A general concluſion, from all the foregoing trials, in favour of the uſefulneſs of the expreſſion $\frac{£}{B} \times \frac{P'}{P} \times \frac{g \times P' - h \times P'}{g \times P' - h \times P''}$.

CCCCXCV. And from all theſe trials, taken together, it ſeems reaſonable to conjecture, that the expreſſion $\frac{£}{B} \times \frac{P'}{P} \times \frac{g \times P' - h \times P'}{g \times P' - h \times P''}$ will always give us a tolerable approximation to the true value of an annuity for three joint lives of any ages whatſoever.

Another method of approximating to the value of an annuity for the joint continuance of three lives of given ages.

Of Mr. Thomas Simpſon's method of finding a near value of an annuity for three joint lives.

CCCCXCVI. The late very learned mathematician, Mr. Thomas Simpſon, of Woolwich Academy, in his book entitled, *Select Exerciſes in the Mathematicks*, page 279, has given us another method of finding a near value of an annuity for the joint continuance of three lives of any given ages, which is exceedingly ſhort and ſimple, and which alſo (as he informs us in the ſame book, page 312,) gives the quantity ſought to a very conſiderable degree of exactneſs, ſo as ſeldom to differ from the true value of the annuity by by more than an eighth part of a years annuity. This method may be defcribed as follows.

CCCCXCVII. Let the propofed annuity, of which we are to find the value, be an annuity of one pound a year, as in all the foregoing inftances. And let it be fuppofed that we have tables of the values of an annuity of one pound a year for fingle lives of all ages, already computed to our hands; and likewife that we have tables of the values of an annuity of one pound a year for the joint continuance of two lives, computed to our hands, which contain a fufficient variety of different ages of the two lives to enable us to find, (by the help of the method of Interpolation above explained in Art CCCCXI, &c. —— CCCCXXXI,) the value of fuch an annuity for any two propofed lives of any given ages whatfoever to a confiderable degree of exactnefs. [illegible] faid me[illegible]

Thefe things being fuppofed, we muft, in the firft place, find (by the tables, or by the rules given for finding the values of annuities for two joint lives,) the value of the propofed annuity of one pound a year for the joint continuance of the two oldeft of the three given lives. This value, it is evident, will be equal to the value of the fame annuity of one pound a year for a fingle life that is ftill older than either of the faid two oldeft of the three given lives. Let this fourth life be found by means of a table of the values of an annuity of one pound a year for fingle lives, by looking along the column containing the values of fuch an annuity till we meet with one that is equal, or nearly equal, to the faid value of the fame annuity of one pound a year for the joint continuance of the faid two oldeft of the three given lives. For the age correfponding to this value will be the age of this fourth, or imaginary, life.

And, laftly, find the value of a like annuity of one pound a year for the joint continuance of this fourth, or imaginary, life, and the firft, or youngeft of the three given lives.

This laft value will be nearly equal (as Mr. Simpfon affures us) to the value of the propofed annuity of one pound a year during the joint continuance of the three given lives. Q E I.

CCCCXCVIII. This may be expreffed more concifely in the following manner. Call the youngeft life A, the next B, and the third, or oldeft, C. And let D be the fourth, or imaginary, life, an annuity for which is equal

A more concife manner of expreffing the foregoing defcription.

equal in value to the fame annuity for the joint continuance of the lives *B* and *C*.

Then, in the firft place, we muft find the value of an annuity of one pound a year for the joint continuance of the lives *B* and *C*. And, fecondly, we muft find the fingle life *D*, an annuity for which is equal in value to the fame annuity for the two joint lives *B* and *C*. And, thirdly, we muft find the value of an annuity of one pound a year for the joint continuance of the two lives *D* and *A*. This laft value will be nearly equal to the value fought, or the value of an annuity of one pound a year for the joint continuance of all the three given lives. Q. E. I.

An example of the foregoing method of approximating to the value of an annuity for three joint lives.

CCCCXCIX Let us fuppofe the three given lives to be all of the fame age of 60 years, as in the example given above in Art. CCCCLXXXII; and let the annuity be, as before, an annuity of one pound a year. And let it be required to find the value of this annuity for the joint continuance of thefe three lives of the age of 60 years, according to the Northampton table of the probabilities of the duration of human life given above in Art. CCCCLXXIX, Table XLI, and upon a fuppofition that the intereft money is 4 per cent. To find this value by the foregoing method, we muft proceed as follows.

The value of an annuity of one pound a year during the joint continuance of two of thefe lives, upon the fuppofitions here made, appears from Table L, Art. CCCLXXVIII, to be £6.2468. We muft therefore look into Mr. Morgan's table of the values of an annuity of one pound a year for fingle lives in his treatife on the doctrine of annuities, pages 64, 65, 66, in order to find a value equal, or nearly equal, to the faid value £6.2468, that table of Mr. Morgan having been formed from the aforefaid Northampton table of the probabilities of the duration of human life, and upon a fuppofition that the intereft of money is 4 per cent. And we fhall there find that the value that comes neareft to £.6.2468 is £.6.263,15, which anfwers to the age of 70 years, or is the value of an annuity of one pound a year for a fingle life of the age of 70 years. We muft therefore, in the laft place, feek for the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 70 years and 60 years. Now the value of fuch an annuity appears by Mr. Morgan's fixth table (in the Appendix to his

his aforesaid Treatise on Annuities, pages 271, 272) to be, nearly, £.4.858. Therefore £.4.858 will (according to Mr. Simpson's assertion,) be nearly equal to the value of an annuity of one pound a year for the joint continuance of the three given lives, all of the age of 60 years. Q. E. I.

The difference between the foregoing near value of an annuity for three equal joint lives (obtained by Mr. Simpson's method,) and the true value of the same annuity.

CCCCC. This near value, £4.858, of the said annuity of one pound a year for the said three equal joint lives of the age of 60 years, is somewhat greater than its true value, which we have seen above, in Table XLIX, to be £.4.7826. But the difference between them is but trifling, being only £.0.0754, which is less than the 63d part of the said true value £.4.7826, and less also than £.0.0833, or than the 12th part of a pound, or than the 12th part of a year's annuity, or than a month's annuity.

This near value (obtained by Mr. Simpson's method,) is more exact than the former near value of the same annuity, obtained in Art. CCCCLXXXII.

CCCCCI. The former near value of this annuity for three equal joint lives of the age of 60 years, which was obtained in Art. CCCCLXXXII, by means of the expression $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, or $\overset{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, was £4.585; which is less than the true value £4.7826, and differs from it by the quantity £0.1976, which is greater than the difference £0.0754. Therefore in the present instance Mr. Simpson's method of approximating to the values of these annuities comes nearer to the truth than the other method. But whether or no it does so in general, can only be known by trying it in a variety of instances, and comparing the values resulting from it with the true values of the same annuities. With this view I shall present the reader with the two following tables of near values of the two sets of annuities, whose true values have been set down in Tables LII and LIII, computed according to the foregoing method of Mr. Simpson; which we may afterwards compare with the said true values. These tables of near values are as follows.

Two tables of near values of annuities for three joint lives, computed by Mr. Simpson's method of approximation.

TABLE LVI.

Containing approximations to the values of the annuities for three equal joint lives mentioned above in Table LII; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by the foregoing method of Mr. Thomas Simpson, of Woolwich.

Years in the age of the first life.	*Years in the age of the second life.*	*Years in the age of the third life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	5	5	11.2119
10	10	10	12.2447
15	15	15	11.3543
20	20	20	10.4653
25	25	25	9.902,20
30	30	30	9.321,74
35	35	35	8.718,71
40	40	40	7.939,79
45	45	45	7.274,62
50	50	50	6.403,16
55	55	55	5.656,00
60	60	60	4.900,62
65	65	65	3.969,56
70	70	70	3.099,22

TABLE

TABLE LVII.

Containing approximations to the values of the annuities for three joint lives of different ages, mentioned above in Table LIII; when the interest of money is 4 per cent.

Computed from Dr. Price's new Northampton table of the probabilities of the duration of human life, by the foregoing method of Mr. Thomas Simpson, of Woolwich.

Years in the age of the first or youngest, life.	*Years in the age of the second life.*	*years in the age of the third, or oldest, life.*	*Values of an annuity of one pound a year for the joint continuance of the three lives.*
			£.
5	15	25	10.5974
10	20	30	10.4853
15	25	35	9.872,64
20	30	40	9.037,79
25	35	45	8.426,81
30	40	50	7.622,29
35	45	55	6.944,67
40	50	60	6.003,39
45	55	65	5.243,74
50	60	70	4.285,36
55	65	75	3.397,35
60	70	80	2.362,16
65	75	85	1.575,23
70	80	90	1.043,73

 CCCCCII. The

The differences of the true values of the annuities for three equal joint lives, in Table LII, and the near values of the same annuities (obtained by Mr. Simpson's method,) in Table LVI.

CCCCCII. The differences between the near values of annuities for three equal joint lives in Table LVI, and the true values of the same annuities in Table LII, are as follows; to wit,

—	11.1704	+	11,2119	=	0.0415;
—	12.2006	+	12.2447	=	0.0441;
—	11.2746	+	11.3543	=	0.0797;
—	10.3429	+	10.4653	=	0.1224;
—	9.796,42	+	9.902,20	=	0.105,78;
—	9.221,10	+	9.321,74	=	0.100,64;
—	8.585,22	+	8.718,71	=	0.133,49;
—	7.865,05	+	7.939,79	=	0.074,74;
—	7.126,40	+	7.274,62	=	0.148,22;
—	6.317,17	+	6.403,16	=	0.085,99;
—	5.550,60	+	5.656,00	=	0.105,40;
—	4.755,03	+	4.900,62	=	0.145,59;
—	3.914,00	+	3.969.56	=	0.055,56;
—	2.995,84	+	3.099,22	=	0.103,38.

A comparison between the said near values and the corresponding true values.

CCCCCIII. It is remarkable that all the near values in the foregoing article (which were obtained by Mr. Simpson's method,) are greater than the true values of the same annuities respectively; whereas the near values of them obtained above in Art. CCCCLXXXVI, Table LIV, by means of the expression $\frac{£}{B} \times \frac{P^{I}}{P} \times \frac{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{I}}{P^{I} \times P^{I} \times P^{I} \times r - P^{II} \times P^{II} \times P^{II}}$, or $\frac{£}{B} \times \frac{P^{I}}{P} \times \frac{g \times P^{I} - h \times P^{I}}{g \times P^{I} - h \times P^{II}}$, were, for the most part, less than the said true values.

The proportions of the foregoing differences of the true and near values of the said annuities to the said true values, respectively.

CCCCCIV. The proportions of the foregoing differences in Art. CCCCCII to their corresponding true values are expressed by the following fractions;

fractions; to wit, $\frac{1}{269}$, $\frac{1}{290}$, $\frac{1}{141}$, $\frac{1}{84}$, $\frac{1}{92}$, $\frac{1}{91}$, $\frac{1}{64}$, $\frac{1}{105}$, $\frac{1}{48}$, $\frac{1}{73}$, $\frac{1}{52}$, $\frac{1}{32}$, $\frac{1}{70}$, and $\frac{1}{28}$.

CCCCCV. All these fractions, except the fourth, $\frac{1}{84}$, are considerably less than the fractions in Art. CCCCXC, which express the proportions of the differences of the former near values of the same annuities (which were obtained by means of the expression $\frac{£}{B} \times \frac{P'}{P} \times \frac{P' \times P' \times P' \times r - P'' \times P'' \times P'}{P' \times P' \times P' \times r - P'' \times P'' \times P''}$, or $\frac{£}{B} \times \frac{P'}{P} \times \frac{g \times P' - h \times P'}{g \times P' - h \times P''}$,) from their several true values respectively, to the said true values. Therefore in these instances, as well as in the example given in Art. CCCCXCIX, Mr. Simpson's method of approximation seems to be preferable to the former method by means of the said expression.

All the foregoing near values of annuities for three equal joint lives (obtained by Mr. Simpson's method,) except one, are more exact than the near values obtained above in Art. CCCCLXXXVI, Table LIV.

CCCCCVI. The differences between the near values of annuities for three joint lives of different ages, contained above in Table LVII, and the true values of the same annuities in Table LIII, are as follows; to wit,

The differences between the true values of annuities for three joint lives of different ages, in Table LIII, and the near values of the same annuities (obtained by Mr. Simpson's method,) in Table LVII.

	£.		£.		£.
	10.6551	−	10.5974	=	0.0577;
−	10.4379	+	10.4853	=	0.0474;
−	9.738,56	+	9.872,64	=	0.134,08;
−	8.986,72	+	9.037,79	=	0.051,05;
−	8.313,10	+	8.426,81	=	0.113,71;
−	7.570,83	+	7.622,29	=	0.051,46;
−	6.816,07	+	6.994,67	=	0.128,60;
−	5.994,15	+	6.003,39	=	0.009,24;
−	5.145,62	+	5.243,74	=	0.098,12;
−	4.219,37	+	4.285,36	=	0.065,99;
−	3.297,98	+	3.397,35	=	0.099,37;
				+	2.408,48

$$+ \quad 2.408{,}48 \;-\; 2.362{,}16 \;=\; 0.046{,}32\,;$$
$$+ \quad 1.623{,}48 \;-\; 1.575{,}23 \;=\; 0.048{,}25\,;$$
$$+ \quad 1.122{,}51 \;-\; 1.043{,}73 \;=\; 0.078{,}78.$$

All the foregoing near values of annuities for three joint lives, of different ages, (which were obtained by Mr. Simpſon's method,) except the firſt and the three laſt, are greater than the true values of the ſame annuities, reſpectively.

CCCCCVII. It is remarkable that all the near values ſet down in Table LVII and in the foregoing article, (and which were obtained by Mr. Simpſon's method of approximation,) except the firſt and the three laſt, are greater than the ſeveral true values of the ſame annuities, reſpectively; as was obſerved in Art. CCCCCIII concerning all the near values, without any exception, that are ſet down in Table LVI and in Art. CCCCCII, and which were likewiſe obtained by Mr. Simpſon's method of approximation; whereas the near values of theſe annuities obtained above by means of the expreſſion $\frac{£}{B} \times \frac{P1}{P} \times \frac{g \times P^1 - b \times P^1}{g \times P^1 - b \times P^{11}}$, and ſet down in Table LV and Art. CCCCXCI, are, for the moſt part, leſs than the ſaid true values, reſpectively.

The proportions of the foregoing differences to the correſponding true values.

CCCCCVIII. The proportions of the foregoing differences in Art. CCCCCVI to their correſponding true values are expreſſed by the following fractions; to wit, $\frac{1}{184}$, $\frac{1}{220}$, $\frac{1}{72}$, $\frac{1}{176}$, $\frac{1}{73}$, $\frac{1}{147}$, $\frac{1}{53}$, $\frac{1}{648}$, $\frac{1}{52}$, $\frac{1}{63}$, $\frac{1}{33}$, $\frac{1}{51}$, $\frac{1}{33}$, and $\frac{1}{14}$.

All the foregoing near values of annuities for three joint lives of different ages, (obtained by Mr. Simpſon's method, except the firſt and the two laſt, are more exact than the near values of the ſame annuities obtained above in Art. CCCCLXXXVI, Table LV.

CCCCCIX. All theſe fractions, except the third, to wit, $\frac{1}{72}$, and the two laſt, to wit, $\frac{1}{33}$, and $\frac{1}{14}$, are conſiderably leſs than the fractions in Art. CCCCXCIV, which expreſs the proportions of the differences of the former near values of the ſame annuities (which were obtained by means of the expreſſion $\frac{£}{B} \times \frac{P1}{P} \times \frac{g \times P1 - b \times P1}{g \times P1 - b \times P11}$) from their ſeveral true values reſpectively to their ſaid true values. Therefore in the greater part of theſe inſtances, as well as in thoſe of the annuities for three equal joint lives contained

tained in Table LVI, and in the example given in Art. CCCCXCIX, this method of Mr. Simpson seems to be preferable, in point of exactness, to the former method of approximation by means of the expression $\frac{£}{B} \times \frac{p_1}{P} \times \frac{g \times P_1 - b \times p_1}{g \times P_1 - b \times P_{11}}$. And it is certainly much shorter and easier in practice than that other method, which (as we have seen above in Art. CCCCLXXXII) requires a good deal of calculation. And therefore I think it must be considered, upon the whole, as the better method of the two. Yet, as there are now and then some instances in which the other method comes nearer to the truth than this, I think it is convenient to be possessed of both methods, to the end that in doubtful cases we may resort to one of them as a kind of confirmation of the result obtained by the other to a moderate degree of exactness.

Mr. Simpson's method of approximation seems, therefore, upon the whole, to be preferable to the former method of approximation by means of the expression $\frac{£}{B} \times \frac{P^1}{P} \times \frac{g \times P^1 - b \times P^1}{g \times P^1 - b \times P^{11}}$.

CCCCCX. The only thing that seems wanting to make Mr. Simpson's method satisfactory, is a demonstration of its truth, or an investigation of it in some way or other. But this is what Mr. Simpson has not given us. For he only says (in his *Select Exercises*, page 312,) "That the "reasonableness of this method of proceeding is evident from the nature "of the subject, without calling in the assistance of any kind of computa- "tion; and that, in a number of examples respecting lives of different "ages, he scarce ever found the error to exceed an eighth part of a year's "purchase." And this account of the degree of exactness to which this method of Mr. Simpson gives the values of these annuities is confirmed by the foregoing trials; since of all the fourteen differences in Art. CCCCCII (which relate to annuities for three equal joint lives) only the seventh difference, to wit £0.133,49, the ninth, to wit, £.0.148,22, and the twelfth, to wit, £.0.145,59, are greater than £0.125,00, or an eighth part of a year's annuity; and of all the fourteen differences in Art. CCCCCVI, (which relate to annuities for three joint lives of different ages, which differ from each other by 10 years and 20 years,) only the third difference, to wit, £.0.134,08, and the seventh difference, to wit, £.0.128,60, are greater than £.0.125,00, or an eighth part of a year's annuity. It seems probable, therefore, that the differences of the near values of annuities for three joint lives, that would be obtained by this method of Mr. Simpson in any other instances, (in which the ages of the lives were different from those above-supposed,) from the true values of the same annuities, respectively, would seldom be greater than £.0.125,00, or one eighth part of a year's annuity; and consequently, that this method of Mr. Simpson will, in all those

Of the degree of exactness to which it may reasonably be conjectured that Mr. Simpson's method of approximation will give, in most cases, the value of an annuity for three joint lives.

thoſe caſes be a very uſeful method of approximating to the values of ſuch annuities. And with this I ſhall conclude what I had to offer concerning the valuation of annuities for the joint continuance of three lives of given ages.

[*End of the doctrine of the valuation of annuities for the joint continuance of three lives.*]

Of the values of annuities that depend on the continuance of the longeſt of two, or more, lives of given ages.

CCCCCXI. Having now gone through the doctrine of the valuation of annuities depending on the *joint* continuance of two, or more, lives of given ages, it remains that I ſhew how to eſtimate the values of annuities that depend on the continuance of *any one* out of two, or more, lives of given ages.

The principles upon which the determination of theſe values is founded, are explained above in Prob. IV. and its corollaries, Art. LVIII, LIX, LX, &c. —— —— LXXVI, pages 58, 59, 60, &c. —— —— 90. It is there ſhewn in Coroll. IV, pages 63, 64, 65, 66, " That the value of " an annuity of one pound a year for the lives of two perſons of given ages " and the life of the longer liver of them, is equal to the exceſs of the ſum " of two ſeparate annuities of one pound a year each, for the ſingle lives " of the ſame perſons, above the value of a like annuity of one pound a " year for their joint lives." And it is ſhewn in Coroll. XI. Art. LXXVI, pages 83, 84, 85, &c. —— —— 90, " That the value of an annuity of " one pound a year for the lives of three perſons of given ages, and the " life of the longeſt liver of them, is equal to the exceſs of the ſum of the " four following values, to wit, 1ſt. the value of an annuity of one pound " a year for the life of the firſt of the ſaid three perſons; 2dly, the value " of a like annuity for the life of the ſecond of them; 3dly, the value of a " like annuity for the life of the third of them; and, 4thly, the value of a " like annuity for the joint lives of all the three, above the ſum of the three " following values, to wit, 1ſt. the value of a like annuity of one pound a " year during the joint lives of the firſt and ſecond perſons; 2dly, the va-

" lue

"lue of the like annuity during the joint lives of the first and third persons; "and, 3dly, the value of a like annuity during the joint lives of the se-"cond and third persons." Therefore, if *A* be put for the value of an annuity of one pound a year for the life of the younger of two persons of given ages, and *B* for the value of a like annuity of one pound a year for the life of the older of the said two persons, and *AB* be put for the value of a like annuity of one pound a year for the joint continuance of both lives; I say, if these are supposed to be the values of *A*, *B*, and *AB*, the value of a like annuity of one pound a year for the lives of both the said persons and the life of the longer liver of them, will be equal to $A + B - AB$. And, if *C* be put for the value of a like annuity of one pound a year for the life of a third person that is older than either of the two former persons, and *AC* be put for the value of an annuity of one pound a year for the joint continuance of the first and third lives, and *BC* be put for the value of a like annuity of one pound a year for the joint continuance of the second and third lives, and *ABC* be put for the value of a like annuity of one pound a year for the joint continuance of all the three lives; I say, if these are supposed to be the values of *A*, *B*, *C*, *AB*, *AC*, *BC*, and *ABC*, the value of an annuity of one pound a year for the life of the longer liver of the said three persons will be equal to $A + B + C + ABC - AB - AC - BC$, or $A + B + C - AB - AC - BC + ABC$. And, in like manner the value of an annuity for the longest of four, or more, lives may be deduced from the values of the like annuities for the same single lives and for their joint continuance, by the principles laid down above in Prob. IV. and its corollaries, pages 58, 59, 60, &c.———90, whatever the number of lives may be. But it is seldom thought necessary, in treating of this subject of life-annuities, to suppose the lives to be more than three.

An example of the computation of the value of an annuity of one pound a year for the longest of two lives of given ages, by means of the expression $A + B - AB$.

CCCCCXII. Let the younger life be supposed to be of the age of 20 years, and the older of the age of 30 years. And let the interest of money be supposed to be $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life to be such as they are represented to be in Monsieur de Parcieux's table.

Then will *A*, or the value of an annuity of one pound a year for the first, or younger life, be = £19.440,616; and *B*, or the value of a like annuity of one pound a year for the second, or older, life, will be = £18.068,798; and *AB*, or the value of a like annuity of one pound a year for the joint continuance of both lives, will be = £15.298,75; as appears by Tables XV and XXXI, in pages 224 and 494. Therefore

$A + B - AB$, or the value of an annuity of one pound a year for the longer of the two given lives of the ages of 20 years and 30 years, will be $= £19.440,616 + £18.068,798 - £15.298,750$ $(= £37.509,414 - £15.298,750) = £22.210,664$. Q. E. I.

An example of the computation of the value of an annuity of one pound a year for the longest of three lives of given ages, by means of the expression $A + B + C - AB - AC - BC + ABC$.

CCCCCXIII. Let the youngest life be supposed (as in the last example) to be of the age of 20 years, and the second life to be of the age of 30 years; and let the third, or oldest, life be of the age of 40 years. And let the interest of money be supposed (as in the last example) to be $3\frac{1}{2}$ per cent. and the probabilities of the duration of human life to be such as they are represented to be in Monsieur de Parcieux's table of them.

Then, in the first place, it appears from Table XV, page 224, that A, or the value of an annuity of one pound a year for the first, or youngest, life (which is of the age of 20 years) is $= £19.440,616$; and that B, or the value of an annuity of one pound a year for the second life (which is of the age of 30 years) is $= £18.068,798$; and that C, or the value of an annuity of one pound a year for the third, or oldest, life (which is of the age of 40 years) is $= £16.104,542$. Therefore $A + B + C$ will be $(= £19.440,616 + £18.068,798 + £16.104,542) = £53.613,956$.

In the second place, it appears from Table XXXI, page 494, that, when the interest of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for two joint lives of the ages of 20 years and 30 years is £15.298,75; and the value of a like annuity for two joint lives of the ages of 30 years and 40 years is £13.709,61. And it appears from Table XXXII, page 495, that the value of an annuity of one pound a year for two joint lives of the ages of 20 years and 40 years (when the interest of money is at the same rate of $3\frac{1}{2}$ per cent.) is £14.028,29. Therefore AB is $= £15.298,75$, and BC is $= £13.709,61$, and AC is $= £14.028,29$; and consequently $AB + BC + AC$ is $(= £15.298,75 + £13.709,61 + £14.028,29) = £43.036,65$. Therefore $A + B + C - AB - BC - AC$, or $A + B + C - AB - AC - BC$, is $(= A + B + C - £43.036,65 = £53.613,956 - 43.036,65) = £10.577,306$.

In the last place we must find the value of ABC, or of an annuity of one pound a year for the joint continuance of the three given lives of the ages of 20 years, 30 years, and 40 years. Now this may be done to a moderate degree of exactness by means of Mr. Simpson's method of approximation, (above explained in Art. CCCCXCVII, CCCCXCVIII, pages 591, 592) in the manner following.

The

The value of an annuity of one pound a year for the joint continuance of the two older lives (which are of the ages of 30 years and 40 years) has been ſeen to be £13.709,61. We muſt therefore look into Table XV, page 224, in order to find the age of the ſingle life, for which an annuity of one pound a year will be worth the ſame ſum, or nearly the ſame ſum, as is the value of the ſame annuity of one pound a year for the joint continuance of the ſaid two older lives, (of the ages of 30 years and 40 years) to wit, the ſum of £13.709,61. Now it appears from Table XV, page 224, that the value of an annuity of one pound a year for a ſingle life of the age of 48 years (when the intereſt of money is $3\frac{1}{2}$ per cent.) is £13.793,859, which is but little greater than £13.709,61. Therefore 48 years is, pretty nearly, the age ſought. We muſt therefore now ſeek the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 48 years. This may be done in the manner following.

It appears from Table XXXII, page 495, that, when the intereſt of money is $3\frac{1}{2}$ per cent. the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 40 years is £14 28,29; and it appears from Table XXXIII, page 496, that, at the ſame rate of intereſt, the value of a like annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 50 years is £11.801,15. The difference of theſe two values, (or £14.028,29 — £11.801,15) is £2.227,14; and the fifth part of this difference is £0.445,428. Therefore, if we ſuppoſe the values of an annuity of one pound a year for the ſix following pairs of joint lives, to wit,

two joint lives of the ages of 20 years and 40 years,
two joint lives of the ages of 20 years and 42 years,
two joint lives of the ages of 20 years and 44 years,
two joint lives of the ages of 20 years and 46 years,
two joint lives of the ages of 20 years and 48 years,
and two joint lives of the ages of 20 years and 50 years,

to form, pretty nearly, an arithmetical progreſſion, or to decreaſe by, nearly, equal differences, the difference of the laſt value but one from the laſt value will be equal to one fifth part of the difference of the firſt value from the laſt, that is, to £0.445,428. Therefore by adding £0.445,428 to the laſt value, which is £11.801,15, we ſhall obtain, pretty nearly, the laſt value but one, or the value of an annuity of one pound a year for the joint continuance of two lives of the ages of 20 years and 48 years; which value will therefore be, nearly = £12.246,578. This therefore is, pretty nearly, equal to the value of ABC, or of an annuity of one pound a year for the joint continuance of the three lives originally propoſed, which are of the ages of 20 years, 30 years, and 40 years. Therefore $A + B + C - AB - AC - BC + ABC$ will be nearly equal to ($A + B + C - AB - AC$

$- AC - BC +$ £12.246,578, or to £10.577,306 + £12.246,578, or) £22.823,884; that is, the value of an annuity of one pound a year for the longeſt of three lives of the ages of 20 years, 30 years, and 40 years, when the intereſt of money is $3\frac{1}{2}$ per cent. will be, nearly, equal to £22.823,884.

C O N C L U S I O N.

CCCCCCXIV. I have now gone through every thing that ſeemed to me to be neceſſary to compleat the object of this treatiſe, which was to explain in a clear and familiar manner *the principles* of the doctrine of life-annuities. Much more, indeed, might be added to it concerning *the application* of this doctrine to a variety of uſeful queſtions concerning life-annuities, that may often occur in the courſe of men's dealings with each other; ſuch as the methods of finding the values of an annuity for one or more lives of given ages, that ſhall take place in caſe of the failure of one or more other lives of given ages, or in caſe of the failure, of one, or more, other lives of given ages, before a third ſet of lives of given ages, or that ſhall depend on a variety of other contingencies. But for the ſolution of queſtions of this kind I ſhall refer my reader to Mr. Thomas Simpſon's *Doctrine of Life-Annuities*, and to his *Select Exerciſes in the Mathematicks*, and to Dr. Price's *Obſervations on Reverſionary Payments*, and Mr. Morgan's *Doctrine of Annuities and Aſſurances on Lives and Survivorſhips*, and to various other learned and uſeful tracts on this ſubject. I am contented with having explained the fundamental principles of the doctrine, and with having preſented the reader with a compleat ſet of tables of the values of life-annuities for ſingle lives at the ſeveral different rates of the intereſt of money at which Mr. Smart has given us tables of the values of annuities for terms of years, to wit, 2 per cent. $2\frac{1}{2}$ per cent. 3 per cent. $3\frac{1}{2}$ per cent. 4 per cent. $4\frac{1}{2}$ per cent. 5 per cent. 6 per cent. 7 per cent. 8 per cent. 9 per cent. and 10 per cent. all fairly computed from Monſieur de Parcieux's table of the probabilities of the duration of human life, without having recourſe to Mr. de Moivre's Hypotheſis, or any other inaccurate ſuppoſition, in order to facilitate the computation; and likewiſe with a conſiderable number of tables of the values of annuities for two joint lives at the two different rates of intereſt of $3\frac{1}{2}$ per cent and $4\frac{1}{2}$ per cent all fairly computed likewiſe from the ſaid table of Monſieur de Parcieux; and with having furniſhed the reader with rules for finding tolerably near values of other annuities, for two joint lives, and likewiſe of annuities for *three* joint lives, of any ages whatſoever, at the ſame, or any other, rates of intereſt; by all which, I flatter myſelf, the foregoing treatiſe will be juſtly intitled to be conſidered as a *uſeful appendix*, or *ſupplement*, to Mr. John Smart's very valuable *tables of intereſt*.

F I N I S.

APPENDIX.

AS the tables of the values of remote life-annuities that were computed for the bill mentioned above in the ſcholium, page 34, (which was brought into the Houſe of Commons by the late Mr. Dowdeſwell in the year 1773,) have never been publiſhed, I preſume they will be thought to make no improper addition to thoſe which have been inſerted in the preceeding work. Nor can I ſuppoſe that a copy of that bill itſelf, to which the ſaid tables of the values of remote life-annuities were annexed, will be unwelcome to ſuch of my readers as ſhall approve the ſcope and view of it, which was "to encourage the poor to induſtry and frugality "by accommodating them with a ſafe and convenient method of "laying out what little money they could ſave out of the earnings "of their labour." I ſhall therefore now proceed to add to the foregoing ſheets an exact copy of this bill, in its laſt form, as it paſſed the Houſe of Commons, after a variety of amendments and improvements made in it, with great care and pains, by the gentlemen who were concerned for its ſucceſs, and alſo a copy of the tables of the values of remote life-annuities which were annexed to the ſaid bill, and were conſidered as a part of it. This bill was as follows.

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BILL,

INTITULED

An ACT for the better Support of Poor Persons in certain Circumstances, by enabling Parishes to grant them Annuities for Life, upon Purchase, and under certain Restrictions.

The preamble.

Whereas it often happens that persons engaged as journeymen in manufactures and handicraft trades, and likewise household servants, labourers, and divers other persons, get more money, as the wages of their labour and service, than is sufficient for their present maintenance, and might easily, if they were so minded, lay by, out of their said gettings, a sufficient sum to provide for their support in their old age:

And whereas it would be highly useful, both to the said persons themselves and to the nation in general, that they should endeavour to make such provision for their support, in the latter periods of their lives, as they would thereby become more sober and virtuous in their ordinary course of life, and more industrious in the prosecution of their several callings and employments,

employments, which would tend to the increase of the riches and manufactures of this kingdom:

And whereas it is probable that many of the said persons might be induced to lay up some part of their earnings in their youth and middle age, in order to make such provision for their old age, if a convenient opportunity were offered them of employing the money they should so lay up in a safe and advantageous manner:

And whereas the most safe and advantageous way, in which the said frugal and industrious persons can employ the several sums of money, which they may be able to save out of their wages, for the support of their old age, seems to be to purchase therewith annuities for their lives, which should commence at some remote period, when their strength and ability to work will be considerably impaired; and the poor's rates of the several parishes in England and Wales seem likely both to be, and to be thought by the said industrious and frugal persons, a sufficient and convenient fund to secure, at all events, the payment of such life-annuities as aforesaid, to the several persons who shall have purchased them, in case any deficiency should happen in the fund created for the payment of the said annuities, by the monies that shall have been paid for the purchase of them; be it therefore enacted by the king's most excellent majesty, by and with the advice and consent of the lords spiritual and temporal, and commons, in this present parliament assembled, and by the authority of the same, That, from and after the fifth day of July, one thousand seven hundred and seventy-three, the plan and method herein-after prescribed, for granting annuities to such industrious and frugal persons for their lives, shall take place and be carried into execution, in all parishes and townships which maintain their own poor, within England and Wales, and the town of Berwick upon Tweed, where there shall be two churchwardens, and two or more overseers of the poor, or one churchwarden, and three or more overseers of the poor, and where the same shall have been approved by a majority of the inhabitants of the said parishes or townships, who are liable to be charged to the poor's rates of the same, assembled at two different parish meetings, held in the churches of the said parishes or townships, after due notice given thereof, and at the distance of, at least, twenty-one days one from the other, subject to the rules and regulations herein-after prescribed; such majority in number being also charged, in the last rate made for the relief of the poor of the said parish or township, in a sum greater than that which was assessed in the said rate upon the rest of the inhabitants assembled at the said meeting; and such notice shall express the times of both the said meetings, and shall be published in the respective parish churches, on a Sunday immediately after divine service, and affixed upon the door of the said parish church,

After July 5, 1773, the following plan shall take place in all parishes and townships in England, in which the majority of the inhabitants, both in number and value, shall adopt it in two different parish-meetings held for the purpose after due publick notice.

church, by the clerk of the ſaid pariſh, or the perſon acting as ſuch, who is and are hereby required to publiſh and affix the ſame, upon the requeſt of any perſon deſirous of purchaſing any ſuch annuity; and the rector, vicar, or perpetual curate of any ſuch pariſh or townſhip, together with the churchwardens and overſeers of the poor in the ſame, ſhall be, and they are hereby, authoriſed and required to receive, on the behalf of all the ſaid inhabitants of ſuch pariſh or townſhip ſo liable, as is aforeſaid, to be charged to the poor's rate of the ſame, from any perſon to whom the ſaid inhabitants ſhall have agreed to grant a like annuity in manner herein-after mentioned, any ſum of money that ſuch perſon ſhall have agreed to pay them for the purchaſe of the ſame; and to grant to ſuch perſon, in the name and on the part of the ſaid rateable inhabitants of the ſame pariſh or townſhip, an annuity for the life of the grantee, equivalent to the purchaſe-money, to commence at ſuch future period of ſuch grantee's life, as ſhall have been ſettled between the ſaid inhabitants and the ſaid purchaſer, ſubject to the reſtrictions herein-after mentioned; and the poor's rate of ſuch pariſh or townſhip ſhall be, and they are hereby declared to be, ſubject by ſuch grant to the payment of the annuity thereby granted, from time to time, as the ſame ſhall become due and payable; and, if it ſhall happen that the rector, vicar, or perpetual curate of any ſuch pariſh or townſhip, ſhall be abſent from ſuch pariſh or townſhip, when the grant of any ſuch annuity ſhall be demanded, and ſhall continue to be abſent from the ſame for the ſpace of thirty days from the ſame time, it ſhall and may be lawful for the churchwardens and overſeers of the poor of the ſaid pariſh or townſhip, and they are hereby impowered and required, after the expiration of the ſaid thirty days from ſuch demand being made, notwithſtanding the abſence of ſuch rector, vicar, or perpetual curate, to make grants of ſuch annuities, without the concurrence of ſuch rector, vicar, or perpetual curate; and ſuch grants ſhall be, and are hereby declared to be, as good, valid, and effectual, as if ſuch rector, vicar, or perpetual curate, had been preſent, and concurred in the making thereof.

The rector, or vicar, or perpetual curate, of the pariſh, together with the church wardens and overſeers thereof, ſhall have power to make grants of life annuities to any inhabitants thereof for certain reaſonable prices to be paid by the ſaid grantees, under certain reſtrictions.

And the poors rates of ſuch pariſhes ſhall become liable, in conſequence of theſe grants, to the payment of ſuch annuities, when they become due.

And in caſe of the abſence of the rector, vicar, or perpetual curate, for the ſpace of 30 days together, the ſaid grants may be made by the church wardens and overſeers of the poor without them.

And for obviating all doubts that may ariſe touching the value of ſuch life-annuities, and the money to be paid for the purchaſe thereof, be it further enacted by the authority aforeſaid, That in all pariſhes and townſhips within the cities of London and Weſtminſter, or of Southwark, the ſaid annuities ſhall be granted according to the reſpective rates herein-after mentioned and expreſſed in the ſet of tables hereunto annexed, No. I. and in all other pariſhes and townſhips within England, Wales, and the town of Berwick upon Tweed, the ſaid annuities ſhall be granted according to the reſpective prices herein-after mentioned and expreſſed in the ſet of tables hereunto annexed, No. II; and if any ſuch life-annuity ſhall be granted to any

The prices to be paid for ſuch annuities ſhall be theſe which are contained in the tables hereunto annexed; of which the firſt ſet, called No. I, ſhall relate to annuities granted in the cities of London, Weſtminſter, and the borough of Southwark; and the ſecond ſet, called No. II, ſhall relate to thoſe granted in all other parts of England and Wales.

any perſon under the age of fifteen years, the ſame price ſhall be paid for ſuch annuity as if the grantee thereof was fifteen years of age.

No ſum leſs than 5l. ſhall be employed in the purchaſe of any of theſe annuities. And no annuity ſhall be greater than 20l. *per annum.*

Annuities granted to men ſhall not take place till they ſhall be 50 years old, nor to women till they ſhall be 35 years old.

Provided always, That no ſmaller ſum than five pounds of lawful money of Great Britain ſhall be laid out in the purchaſe of any of the ſaid annuities, nor ſhall any perſon be allowed to purchaſe any annuity or annuities from any one pariſh or townſhip, to a greater amount than twenty pounds per annum; nor ſhall any of the ſaid annuities commence after the grantee thereof ſhall have attained his age of ſeventy-five years; nor ſhall it commence for the life of any man, before he ſhall have attained the age of fifty years, nor for the life of any woman before ſhe ſhall have attained the age of thirty-five years; nor ſhall any ſuch annuity be granted to any perſon for any other life than that of the grantee.

Theſe annuities ſhall not be granted without the conſent of ſuch a majority of the pariſhioners preſent at the meetings in which they are granted, as pays more to the poor's rate than the other pariſhioners there preſent.

Nor ſhall they be granted to any perſons but ſuch as are ſuppoſed to have a legal ſettlement in the pariſh.

And be it further enacted, That no ſuch annuity ſhall be granted, in any pariſh or townſhip, where the number of inhabitants aſſeſſed towards the relief of the poor ſhall exceed the number of eighteen, unleſs twelve, at the leaſt, of ſuch inhabitants ſhall be preſent in a veſtry, or other publick meeting to be held for that purpoſe; and in pariſhes or townſhips where the whole number of inhabitants, liable to be aſſeſſed as aforeſaid, ſhall be leſs than nineteen, no ſuch annuity ſhall be granted, unleſs at a veſtry or other publick meeting, where two-thirds of ſuch inhabitants ſhall be preſent; nor ſhall any ſuch annuity be granted, at any of the ſaid meetings, unleſs the major part of the inhabitants ſo aſſembled, being alſo charged in the laſt poor's rate, as aforeſaid, in a ſum greater than what is aſſeſſed in the ſame rate upon the reſt of the inhabitants there aſſembled, ſhall agree to grant the ſame; and no ſuch meeting ſhall be held for granting ſuch annuities, unleſs previous notice of holding a meeting for that purpoſe ſhall be publiſhed in the pariſh church upon two Sundays immediately after divine ſervice, and a written copy of ſuch notice affixed upon the door of ſuch church previous to the firſt publication thereof, as aforeſaid; and the clerk of the ſaid pariſh, or the perſon acting as ſuch, is and are hereby required to publiſh and affix the ſame, at the requeſt of any perſon deſirous of purchaſing ſuch annuity; and no ſuch annuity ſhall be granted to any perſon who ſhall not (in the opinion of the majority of ſuch inhabitants ſo aſſembled) appear to have a legal ſettlement within the pariſh or townſhip upon which the ſame ſhall be charged.

Yet, if, through miſtake, they are granted in the manner directed in this act, to inhabitants of the pariſh who have not a legal ſettlement in it, they ſhall nevertheleſs be valid and binding.

Provided nevertheleſs, That if any annuity ſhall have been granted in the manner directed by this act, ſuch grant ſhall be valid and binding upon the pariſh, or townſhip, granting the ſame; and the annuity thereby granted ſhall be regularly paid, according to the purport and true meaning of ſuch grant,

grant, notwithſtanding it ſhall afterwards be diſcovered that, at the time of making ſuch grant, the grantee thereof was not legally ſettled in ſuch pariſh or townſhip.

And be it further enacted, That the ſaid annuities ſhall be granted by deeds of grant, fairly written or printed on parchment, and ſigned and ſealed by the churchwardens and overſeers of the poor, and by the rector, vicar, or perpetual curate (unleſs he ſhall be abſent as aforeſaid) of the pariſh, or townſhip, in behalf of which ſuch annuities ſhall be granted, and may be made in the form, or to the effect, following:

The deeds of grant ſhall be made out on parchment.

"AT a public meeting of the inhabitants of the pariſh or townſhip "of in the county of "holden in the veſtry of the church of the ſaid pariſh or townſhip, on the " day of in the year of the reign "of our ſovereign lord King George the Third, and in the year of our "Lord Chriſt after due "notice of the ſaid meeting being firſt given.

The form of the deeds by which theſe annuities ſhall be granted.

"*A. B.* rector (or vicar, or perpetual curate, as it may happen to be) "of the pariſh or townſhip of aforeſaid, in the county "aforeſaid, *C. D.* and *E. F.* churchwardens of the ſaid pariſh or townſhip, "*G. H.* *I. K.* and *L. M.* overſeers of the poor of the ſaid pariſh or town-"ſhip; To all to whom this preſent writing ſhall come, ſend greeting: "Whereas *N. O.* of the ſaid pariſh or townſhip aforeſaid, Bricklayer, (or "houſhold ſervant to P. Q. Eſquire, of the ſaid pariſh or townſhip, or day-"labourer or otherwiſe, *according to his proper addition or employment*) "appearing to us to be a perſon lawfully ſettled in the ſaid pariſh or town-"ſhip, and intitled to be relieved by the poors rate raiſed in the ſame, in "caſe he ſhould become poor and helpleſs, and now of the the age of " years, hath paid unto the hands of us the rector, or vicar, or "perpetual curate, churchwardens, and overſeers of the poor of the pariſh "or townſhip, aforeſaid, the ſum of pounds of lawful "money of Great Britain, as the price of an annuity for his life, that ſhall "begin when he is years of age, that is, at the end "of years from this preſent time, and in the year of our "our Lord Chriſt to be paid "to him, or his certain attorney, by the rector, churchwardens, and over-"ſeers of the poor of the ſaid pariſh or townſhip for the time being, by "equal quarterly payments to be made at the four following feaſt days; "to wit, the feaſt day of the annunciation of the Bleſſed Virgin Mary, the "feaſt day of Saint John the Baptiſt, the feaſt day of Saint Michael the

" Archangel, and the feaſt day of the Nativity of our Lord Chriſt, or
" within ſeven days after each of the ſaid feaſt days, according to the ſtatute
" of the thirteenth year of the reign of King George the Third, in that
" behalf made and provided: NOW know ye, that we the rector, (vicar, or
" perpetual curate,) churchwardens, and overſeers of the poor of the ſaid
" pariſh or townſhip, in conſideration of the ſaid ſum of pounds,
" to us in hand paid, (the receipt whereof is hereby acknowledged,) and
" in purſuance of the ſtatute aforeſaid, do by this preſent writing, in the
" behalf of the inhabitants of the ſaid pariſh or townſhip of
" give and grant unto the ſaid *N. O.* an annuity or yearly penſion of
" of of lawful money of Great Gritain, which ſhall
" commence on the feaſt day of that ſhall be in
" the year of our Lord Chriſt
" and continue to be paid to the ſaid *N. O.* from thence forwards during
" his life, by equal quarterly payments, at the four feaſt days above men-
" tioned, or within ſeven days after each of them reſpectively: and fur-
" ther, by virtue of the ſtatute aforeſaid, we do bind and engage the rates
" of the ſaid pariſh or townſhip, that ſhall hereafter be raiſed therein for
" the relief of the poor thereof, for the full and due payment of the ſaid
" annuity of to the ſaid *N. O.* by equal quarterly
" payments, on the four feaſt days above-mentioned, or within ſeven days
" after each of them reſpectively, from the ſaid feaſt day of
" to the feaſt day or day of payment that ſhall happen
" next before the death of the ſaid *N. O.* including both the ſaid days.
" In witneſs whereof, we have hereunto ſet our hands, and have fixed our
" common ſeal, this day of in the
" year of the reign of our ſovereign lord George the Third, and in the
" year of our Lord Chriſt.

A. B. *Rector, or Vicar, or perpetual Curate, of the ſaid Pariſh or Townſhip.*

The Common Seal of the Pariſh of in the County of

C. D.
E. F. } *Churchwardens.*

G. H.
I. K.
L. M. } *Overſeers of the Poor.*

And in caſe of the abſence of the rector, vicar, or perpetual curate as aforeſaid, ſuch grant ſhall be made in the names of the churchwardens and overſeers of the poor only; and ſuch abſence of the rector, vicar, or perpetual curate, ſhall be ſpecified in the ſame, and the charges and expences attending

ing the making out every such grant shall be defrayed by the person to whom such annuity shall be granted, who shall pay to the said parish officers the sum of two shillings and sixpence for the same, and no more.

Such deed of grant shall be made out by the said grantors for the sum of 2s. 6d. to be paid them by the grantee.

And be it further enacted, That, in every such deed of grant, the pounds, shillings, and pence, thereby granted, and the date of the year of our Lord Christ in which such annuity is to commence, shall be written in words at length, and not in figures; and such deeds of grant, immediately after the same shall be signed and sealed as aforesaid, shall be delivered to the grantee of the annuity thereby granted, to be kept by him or her, as the proof of his or her right to such annuity.

The amount of these annuities, and the dates of their commencement, shall be written in these deeds in words at length.

And be it further enacted by the authority aforesaid, That the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor of every such parish or township, shall, and they are hereby required to cause a copy of every such deed of grant to be entered in a book, to be kept in the parish chest for that purpose, and to seal and sign the said copy in the same manner as the original deed is hereby directed to be signed and sealed, (both which shall be executed at a public meeting of the inhabitants of the parish or township, to be holden for that purpose;) and the grantee of the said annuity shall at the same time sign and seal an acknowledgement in writing, which shall be put at the bottom of the said copy, in the words or to the effect following:

A copy of every such deed of grant shall be entered by the grantors of it in a book to be kept in the parish for that purpose; and an acknowledgement of its being a true copy, shall be signed and sealed by the grantee of the annuity.

I N. O. *of the parish or township of*   *in the county of*   *do acknowledge, that the above is a true copy of the deed of grant of a certain life-annuity, which has been this day granted to me by the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor of the said parish or township, or by the churchwardens and overseers of the poor*, in case of the absence of the rector, vicar, or perpetual curate, as aforesaid.

N. O.

Which said copy shall be made at the expence of the inhabitants of the said parish or township, and be paid for out of the money received as the price of the said annuity, from the said grantee thereof, without any new expence to the said grantee: and if it shall at any time happen, that any original deed of grant, delivered as aforesaid, shall be lost or destroyed, the copy thereof so entered as aforesaid shall be deemed sufficient evidence that such grant had been made, and shall intitle such grantee to receive his annuity according to the purport of such copy. And, if such grantee shall desire to have a copy of such deed of grant made from the said parish copy, instead of the original deed so lost or destroyed, the rector, vicar, or perpetual curate,

And, if the grantee of the annuity shall lose his deed of grant of it, such copy of the said deed in the parish book shall be sufficient evidence of it.

And the parish-officers shall give the grantee a new copy of the lost deed of grant of his annuity from such parish-copy of it, for the sum of 2s. 6d.

curate, churchwardens, and overſeers of the poor of the ſaid pariſh or townſhip for the time being, ſhall cauſe ſuch copy to be made out upon parchment, and ſhall ſign the ſame, in atteſtation of its being a true copy from the ſaid copy in the pariſh book, and ſhall deliver ſuch atteſted copy to ſuch grantee, without requiring any new conſideration-money for the ſame, or any fee or reward, except the neceſſary expence of making out ſuch new copy; for which they ſhall take the ſum of two ſhillings and ſixpence, and no more: and the ſaid new copy, ſo atteſted, ſhall be ſufficient evidence that ſuch grant had been made, and ſhall intitle the grantee to receive his annuity from the ſaid pariſh or townſhip, according to the purport of ſuch atteſted copy, notwithſtanding the copy in the pariſh book ſhould afterwards happen to be loſt or deſtroyed.

And the ſaid new copy of the ſaid deed of grant ſhall be ſufficient evidence of its having been made, although the copy in the pariſh-book ſhould afterwards be loſt.

The overſeers of the poor ſhall enter all proceedings relating to the execution of this act, in proper books to be provided for that purpoſe.

And be it further enacted by the authority aforeſaid, That, previous to the holding of any meeting in purſuance of this act, the overſeers of the poor for the pariſh or townſhip where ſuch meeting ſhall be held, ſhall cauſe proper books to be provided and kept in the public cheſt, belonging to ſuch pariſh or townſhip; in which books all orders, proceedings, and accounts, relating to the execution of this act, ſhall be fairly entered at length, by ſome perſon to be appointed by the reſpective churchwardens and overſeers of the poor; and the names of all the perſons who ſhall give a vote for, or againſt, the making of any order, or the coming to any reſolution relating to the execution of this act, ſhall be entered in one of the ſaid books, and the perſons agreeing to, or concurring in, ſuch order or reſolution ſhall ſign the ſame; and an account of all monies received or payed in purſuance of this act ſhall alſo be entered in one of the ſaid books, expreſſing particularly the times of all ſuch receipts and payments, the perſon to, or from, whom the ſame were paid or received, and for what purpoſes; which accounts ſhall be balanced and cloſed once in every year, as near as conveniently may be to the time for holding the petty ſeſſions for appointing overſeers of the poor for ſuch reſpective pariſh or townſhip, and (being ſigned by the rector, vicar, or perpetual curate, and the churchwardens and overſeers of the poor, for each ſuch pariſh or townſhip,) ſhall be laid before the juſtices of the peace at ſuch petty ſeſſions, who ſhall examine ſuch accounts with the vouchers thereto; and, if the ſame ſhall appear to ſuch juſtices to be juſt and true accounts, they ſhall certify the ſame in writing upon ſuch accounts; and a duplicate of every ſuch account ſhall be tranſmitted by the reſpective overſeers of the poor to the clerk of the peace for the county, riding, or diviſion, wherein ſuch pariſh or townſhip is ſituated, (whether the ſame be a town-corporate, having excluſive juriſdiction or not,) before the next general quarter ſeſſions of the peace; and each and every clerk of the peace is hereby required to cauſe every ſuch account to be filed amongſt the records of his office.

An account of all the monies received and paid by virtue of this act ſhall be entered in one of theſe books. And the accounts ſhall be balanced once in every year, and laid before the juſtices of the peace at the petty ſeſſions, to be examined by them; and ſhall be certified by the ſaid juſtices to be juſt and true, if found to be ſo.

And a duplicate of every ſuch account ſhall be tranſmitted to the clerk of the peace for the county, before the next general quarter ſeſſions of the peace.

And

And be it further enacted, That no deed of grant, or any of the copies thereof herein before directed to be made, nor any power of attorney for accepting and transferring ſtock, and for the receiving the dividends due thereon, ſhall be charged or chargeable with any ſtamp-duty whatſoever, but ſhall be good, valid, and effectual, to all intents and purpoſes, without any ſtamp being impreſſed thereon.

No ſtamp-duty ſhall be paid for the ſaid deeds of grant, or the copies of them.

And be it further enacted, That if any grantee of any ſuch life-annuity ſhall conſent that the ſame may be made unalienable, and thereupon a clauſe for that purpoſe, expreſſing his conſent that it ſhould be made ſo, ſhall be inſerted in the deed of grant delivered to him, and in the copy thereof kept in the pariſh regiſter of the ſaid grants of life-annuities, and which he ſhall have acknowledged to be a true copy of his ſaid grant in the manner above directed, in ſuch caſe the ſaid annuity ſhall be payable to the ſaid grantee alone, or his certain attorney, during his life, without any power in him to alienate, or aſſign, it to any perſon, or in any manner, whatſoever; and every aſſignment of ſuch annuity that ſhall afterwards be made by the ſaid grantee to any other perſon, ſhall be totally void in law and equity, and of no effect or operation whatſoever.

The ſaid annuities may, with the conſent of the grantees thereof expreſſed in the deeds of grant, be made unalienable.

And be it further enacted, That if any grantee of any of the ſaid annuities that ſhall not have conſented as above to make his ſaid annuity unalienable, ſhall be deſirous of ſelling or diſpoſing of any ſuch annuity to any other perſon, he ſhall, in the firſt place, make an offer to ſell the ſame to the rector, vicar, or perpetual curate, churchwardens and overſeers of the poor, and other inhabitants chargeable to the poors rates of the ſaid pariſh or townſhip where the ſame was bought, at the price which ſuch annuity ſhall be then worth, according to the rates herein-after mentioned, or at any lower price; which offer ſhall be made at a veſtry or other publick meeting of the inhabitants of the ſaid pariſh or townſhip, notice of which meeting ſhall be given on two different Sundays, in the ſame manner as herein-before directed, concerning meetings for granting the ſaid annuities; and, if, upon ſuch offer being made, the major part of the rateable inhabitants ſo aſſembled, being alſo charged in the laſt poors rate as aforeſaid, in a ſum greater than what is aſſeſſed in the ſame rate upon the reſt of the inhabitants there aſſembled, ſhall think proper to purchaſe the ſame, the rector, vicar, or perpetual curate, and the churchwardens and overſeers of the poor of ſuch pariſh or townſhip, or the churchwardens and overſeers of the poor only, in the abſence of ſuch rector, vicar, or perpetual curate as aforeſaid, ſhall be, and are hereby, authoriſed and required to buy up the ſaid annuity, at the ſaid price, or at any lower price as ſhall be agreed for, and to defray the expence of ſuch purchaſe out of the fund created by the monies received from the grantees of the ſaid life-annuities; and in caſe the major part of the inhabitants

And, when they are not made unalienable, they ſhall not be alienated before an offer has been made of them to the pariſh by which they have been granted, at the values ſet down in the tables hereunto annexed.

But, if the pariſh refuſes to buy them in, the grantees ſhall be at liberty during the ſpace of ſix months to ſell them to any other perſons.

inhabitants, so assembled as aforesaid, shall not agree with the said grantee for the purchase of the said annuity, the grantee thereof shall, for the space of six months from the time of such refusal or non-agreement, be at liberty to sell and assign his said annuity to any person whatsoever, by a deed duly sealed and delivered in the presence of two credible witnesses, and signed by the said witnesses, in attestation of their having seen the same so signed and delivered: and after the expiration of the said six months, the said grantee shall not have power to sell or assign such annuity to any person whatsoever, unless he shall again make an offer thereof, in the manner before prescribed, to the rector, vicar, or perpetual curate, and churchwardens and overseers of the poor, and other rateable inhabitants of the said parish or township; but in case of their refusal or non-agreement to purchase the same, such grantee shall again be at liberty, during the space of six months from such refusal or non-agreement, to assign such annuity to any person whatsoever: and so, in like manner, as often as he shall be desirous of selling the said annuity, he shall acquire a right so to do, for the space of six months after making such offer, and the refusal of the same, or non-agreement, as aforesaid. And all assignments made of such annuities by the grantees thereof, otherwise than according to the rules herein prescribed, shall be utterly void and of no effect. But, when any of the said annuities shall have been assigned by the grantee thereof to any other person, the said person, to whom it shall have been so assigned, shall be at liberty to assign it to whom he shall think fit, and in whatever time and manner he shall think fit, agreeable to the usual rules of law, without being obliged to make a previous offer of it to the parish officers and rateable inhabitants of the parish, or township, in which it was granted, as is required of the grantee thereof; and every subsequent assignee thereof shall have the same liberty.

After any such annuity shall have been sold, or assigned once, it may be assigned again without making any previous offer of it to the parish.

If any person shall make any fraudulent alteration in any of the said parish deeds of grant, or, knowing any such fraudulent alteration to have been made, shall receive any money upon the same, he shall thereby become guilty of felony without benefit of clergy. And, upon his conviction the annuity shall cease.

And be it further enacted, That, if any person shall at any time make, or cause to be made, any alteration in any deed of grant of any such annuity, or any copy thereof which shall be made out as aforesaid, so as to make the annual sum thereby granted appear to be greater than it really was, or the time of the commencement of such annuity appear to be earlier than that which was appointed by the said deed, or shall know that any such alteration has been made in such deed or copy, and having made, or caused to be made, such alteration, or knowing the same to have been made therein, shall demand and receive of the parish officers of the parish or township in which such annuity was granted, any part of such annuity, according to the purport of such fraudulent alteration, every person so offending, and being convicted thereof, shall be deemed and adjudged guilty of felony, and shall suffer death as a felon, without benefit of clergy; and, upon such conviction, such annuity shall cease and determine.

And

And be it further enacted, That if any perſon ſhall forge or frame, or cauſe to be forged or framed, any deed of grant of any ſuch pariſh annuity as is above mentioned, with an intent to defraud the inhabitants of the pariſh or townſhip, in which ſuch deed ſhall purport ſuch annuity to have been ted, and ſhall for that purpoſe counterfeit, or cauſe to be counterfeited, the names of the pariſh officers which are herein required to be ſubſcribed to every ſuch deed of grant, and, by means of ſuch forged deed, ſhall obtain from the ſaid officers of the pariſh, or townſhip, in which ſuch forged deed ſhall purport the ſaid annuity to have been granted, any ſum of money whatſoever, as part of ſuch pretended annuity, every perſon ſo offending, and being convicted thereof, ſhall be deemed and adjudged guilty of felony, and ſhall ſuffer death as a felon, without benefit of clergy.

If any perſon ſhall forge, or cauſe to be forged, any ſuch deed of grant, and ſhall, by means of ſuch forged deed, obtain from the officers of any pariſh any ſum of money, he ſhall thereby become guilty of felony without benefit of clergy.

And be it further enacted, That if any perſon ſhall, by any means whatſoever, get poſſeſſion of any real deed of grant of any ſuch annuity, or any copy thereof, given by the pariſh, or townſhip, wherein ſuch annuity is granted, as is above mentioned, without having a right to the annuity thereby granted, either as the grantee thereof, or the lawful aſſignee thereof, by one or more aſſignment or aſſignments thereof, in the manner above preſcribed, and ſhall falſely pretend to be the perſon lawfully intitled to ſuch annuity, and under ſuch pretence ſhall produce any ſuch deed of grant to the pariſh officers of the pariſh or townſhip in which ſuch annuity was granted, and in conſequence thereof ſhall demand and obtain, from the ſaid officers, any ſum of money whatſoever, as a part of ſuch annuity, every perſon ſo offending, and being convicted thereof, ſhall be deemed and adjudged to be guilty of felony.

And, if any perſon ſhall, by any means, get poſſeſſion of any *real* deed of grant of ſuch an annuity, belonging to another perſon, and ſhall, by falſely pretending that he is the perſon to whom the ſaid annuity belongs, obtain payment of any of the money due upon it, he ſhall thereby become guilty of felony.

And be it further enacted, That the ſeveral perſons intitled to the ſaid annuities ſhall apply to the ſaid officers of the pariſhes, or townſhips, in which the ſame have been reſpectively granted, for the payment thereof, as ſoon as they conveniently can, after the ſeveral quarterly feaſt days in which they are made payable; and if any ſuch annuitant ſhall neglect to apply (either by himſelf or his lawful attorney) for the payment of his annuity, to the ſaid officers of the pariſh or townſhip in which the ſame was granted, for more than the ſpace of one year and a quarter, ſo that five or more quarterly payments thereof ſhall be in arrear, he ſhall not be entitled to receive more than the four laſt quarterly payments of ſuch annuity, and ſhall forfeit his right to all the former payments ſo in arrear.

Any grantee of any one of theſe annuities who ſhall neglect to apply for payment of it for more than five quarters of a year, ſhall forfeit all the arrears due to him, except thoſe of the laſt four quarters of a year.

Provided always, That nothing in this clauſe ſhall extend, or be conſtrued to extend, to any grantee, who, from being beyond the ſeas, or any unavoidable incapacity, ſhall have been diſabled from making his demand within the

A proviſoe in favour of perſons beyond the ſeas, or who ſhall have been under any unavoidable incapacity of making application for the payment of their ſaid annuities in due time.

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the time so limited, and shall make such demand within six months after his return from abroad, or the removal of such incapacity.

The money paid for the purchase of these parish-annuities shall be the property of the inhabitants of the parish that are chargeable to the poors rates; and shall be invested in the three per cent. bank annuities in their name.

And be it further enacted, That the money to be received by the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor of every parish or township, in which any such annuities shall be granted, shall be the property of all the inhabitants of such parish or township, chargeable to the poors rates. And the respective rectors, vicars, or perpetual curates, churchwardens, and overseers of the poor shall, and they are hereby required as soon as conveniently may be, after the receipt of any such purchase-money, to lay out the same in the purchase of some of the public annuities, established by authority of parliament, and payable at the bank of England, after the rate of three *per centum per annum:* which money so laid out, shall stand in the books of the governor and company of the said bank, in the name of the rector, vicar, or perpetual curate, churchwardens, overseers of the poor, and inhabitants of such parish or township, chargeable to the poors rates of the same; who shall be, and are hereby declared to be, a body politick and corporate, having perpetual succession for the several purposes of this act, and shall be known by the name of *The Rector, Vicar, or perpetual Curate, Churchwardens, Overseers of the Poor, and Inhabitants of such Parish or Township, chargeable to the Poors Rates of the same*, and shall have and use a common seal, on which seal shall be engraven the name of such parish or township, and the county, city, or town in which the same is situated, and such seal shall be carefully kept in the chest of such parish or township. And the rector, vicar, or perpetual curate of such parish or township, and the churchwardens and overseers of the poor thereof, or a majority of them, shall be, and are hereby constituted and declared to be, *the acting members* of such body corporate, and shall have power to purchase the said bank annuities, in the name of the whole body, and to receive the dividends of interest that shall become due thereupon, and to sell and transfer the said annuities whensoever they shall think fit.

And the said inhabitants shall be a body politick and corporate for the several purposes of this act, and shall have a corporate name and a common seal.

And the rector, vicar, or perpetual curate, of the parish, and the churchwardens and overseers of the poor, or a majority of them, shall be *the acting members* of every such body corporate.

The said corporations may appoint agents, or attornies, to transact their business at the Bank of England.

And, for the more easy transacting the business relating to the said bank annuities, be it further enacted, That the rector, vicar, or perpetual curate, and the churchwardens and overseers of the poor of every such parish or township, or the majority of them, shall, and they are hereby impowered, with the concurrence of a majority of the rateable inhabitants assembled at a parish meeting, (such majority in number being also charged in the last rate made for the relief of the poor of the said parish or township, in a sum greater than that which was assessed in the same rate upon the rest of the inhabitants assembled at the said meeting,) upon notice given as aforesaid, to constitute and appoint any person or persons that they shall think proper to be be their agent or attorney, agents or attornies, to purchase, sell, or transfer, in their stead and place, and in the behalf of the said body politick and corporate, the said bank annuities, or to receive the dividends of interest due thereupon; which appointment shall be by a letter, or power, of attorney for that purpose, in writing, sealed and delivered by a majority of them, the said rector, vicar, or perpetual curate, churchwardens and overseers of the poor in manner herein-after directed, and likewise signed by them in presence of two credible witnesses, who shall likewise sign their names thereto, in attestation of their having seen the said power so signed, sealed, and delivered; and shall continue in force until it shall be expresly revoked by another instrument in writing, made and executed in the same manner, by such rector, vicar, or perpetual curate, churchwardens and overseers of the poor, or the major part of them, as aforesaid, and attested by the same number of subscribing witnesses, as the said letter, or power, of attorney is hereby directed to be attested by. And every such letter, or power, of attorney shall (during the time it shall continue in force) be sufficient to impower the person to whom it was given, (if it shall purport so to impower him) to purchase and accept any stock in the said bank-annuities, for, and in behalf of, the said body politick and corporate; and likewise to receive, in the same behalf, all the dividends of interest that shall become due on the stock of such body; and likewise to sell and transfer all, or any part of, the stock which shall belong to such body corporate, either at the time of executing such letter of attorney, or at any time after, while such letter of attorney shall continue in force.

The aforesaid money so invested in the three per cent. bank-annuities shall be kept and used as a fund for the payment of the said parish life-annuities, and shall be applied to no other use or purpose whatsoever.

And be it further enacted, That all the money which shall be paid for the purchase of any of the said life-annuities, and which is herein before directed to be invested in the three *per centum* bank annuities, shall be kept and used as a fund, for the payment of the said life-annuities, as they shall become due, and shall not be applied to any other use, or purpose, whatsoever; and the respective rectors, vicars, or perpetual curates, and churchwardens and overseers of the poor of every parish or township, in which such life-annuities shall be granted, shall, and they are hereby required, either by themselves or their lawful attorney, impowered in the manner before-mentioned, to receive the dividends of interest that shall become due upon the said fund or stock of the said bodies politick and corporate respectively, in the said bank annuities, as often and as soon as they shall become due and payable, and to invest the money, arising by such dividends, immediately in the purchase of new stock in the said bank-annuities, so as thereby to encrease such parish fund continually, and enable it to furnish the payments of the several life-annuities charged thereupon, and to pay out of the said dividends such annuities as shall be then due to the persons entitled to receive the same, by equal quarterly payments in every year, at the four feast-days before-mentioned, or or within seven days after each of the said feast days respectively; and in case such dividends shall not be sufficient for that purpose, to pay the same out of the fund so vested in bank-annuities; and for that purpose to sell and transfer, from time to time, whenever it shall be necessary, so much of the said fund or stock of the said parishes or townships respectively, in the said bank annuities, as shall be sufficient to enable them to make the said payments.

Excepting only the necessary charges of managing this fund; which may be defrayed out of it.

Provided always, and be it hereby further enacted, That it shall and may be lawful for the said rector, vicar, or perpetual curate, and churchwardens and overseers of the poor in every such parish or township, to defray the necessary charges and expences of investing the money received from the purchasers of the said life-annuities in the said bank-annuities, and of receiving the dividends thereof, and investing them in the purchase of fresh stock in the same, and of selling and transferring the same, and of preparing the said letters of attorney for the transaction of this business, and of procuring the aforesaid books for entering and registering copies of the grants of the said annuities, or any other books necessary for carrying this act into execution, and of every other thing necessary to be done by any other person than themselves in the execution of this act, out of the monies received by them from time to time from the purchasers of such life-annuities, or out of the dividends payable out of the said stock or fund, they keeping an exact account of the particulars of the said charges and expences; any thing to the contrary hereof above-mentioned in any-wise notwithstanding.

Sums of money given by charitable persons in aid of the funds to be established by this act shall be employed in the purchase of three per cent bank-annuities in the same manner as the money paid for the purchase of the said life-annuities.

And be it further enacted, That if any sum or sums of money shall be given by charitable persons, or otherwise, for the purpose of enlarging the said original fund of any parish or township, and enabling it with more certainty to furnish the several payments of the life-annuities charged thereupon, without any more particular directions from the donor or donors, as to the manner of applying the same, the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor of such parish or township, shall, and they are hereby authorised and required to, invest the money arising by such gifts, or otherwise, in the said bank-annuities, in the same manner as the money contributed by the purchasers of the said life-annuities is herein-before directed to be invested, and to employ and use such money as an additional fund for the payment of the said life-annuities, in the same manner as they are herein-before authorised and required to employ the aforesaid original fund, contributed by the purchasers of the said life-annuities.

If these funds should prove insufficient for the payment of the life-annuities that shall have been granted by virtue of this act, the deficiencies shall be made good out of the poors rate.

And, for securing the payment of the said life-annuities, **be it further enacted,** That if it shall at any time happen in any parish or township, where such

ſuch life-annuities ſhall have been granted, that the aforeſaid original fund ariſing from the contributions of the purchaſers of the ſaid life-annuities, together with the ſaid additional fund, ſhall not be ſufficient to furniſh the quarterly payments of the ſaid life-annuities, the churchwardens and overſeers of the poor ſhall, and they are hereby required to, make the ſaid payments out of any other money in their hands, raiſed for the relief of the poor of ſuch pariſh or townſhip; and if ſuch money in their hands be not ſufficient for that purpoſe, they ſhall, and are hereby required to, make ſuch an addition to the rates, which they are impowered to make, by a ſtatute made in the forty-third year of the reign of Queen Elizabeth, intituled, *An Act for the Relief of the Poor*, as ſhall be ſufficient to ſupply the deficiencies of the ſaid funds, and to enable them to compleat the ſaid quarterly payments of the ſaid life-annuities, and to defray the neceſſary charges and expences of executing this act; which ſaid additional rate ſhall be made, levied, and recovered, under the ſame regulations and reſtrictions, and by the ſame ways and means, as the rates made for the relief of the poor are, by the ſaid laſt-mentioned ſtatute, or any ſubſequent act or acts of parliament, directed to be made, levied, and recovered.

And be it further enacted by the authority aforeſaid, That, if any of the capital ſtock of any pariſh or townſhip, in the ſaid bank annuities, ſhall remain, after all the annuities ſhall become extinct, the intereſt of ſuch ſtock ſhall be applied in aid of the rate to be raiſed and levied for the relief of the poor of the ſaid pariſh or townſhip, from time to time, as ſuch intereſt ſhall become due; and it ſhall and may be lawful for the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor of ſuch pariſh or townſhip, with the advice and conſent of the inhabitants of ſuch pariſh or townſhip, rateable towards the relief of the poor, or the major part of them, aſſembled in veſtry, or other publick meeting for that purpoſe, (due notice of the ſaid meeting being given in manner herein-before directed,) to ſell and diſpoſe of ſuch capital ſtock, and to apply the money ariſing by ſuch ſale, either in repairing, amending, or new-building any alms-houſes, in ſuch pariſh or townſhip, or furniſhing them in a more convenient manner, or in building or furniſhing new alms-houſes, or in any other manner for the benefit of the poor of ſuch pariſh or townſhip, as the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor, and inhabitants ſo aſſembled, ſhall order and direct.

And, on the other hand, if after the extinction of all the life-annuities that have been granted in any pariſh, there ſhall remain any part of the ſaid pariſh-fund in the ſaid three per cent. bank-annuities, ſuch fund ſhall be employed in aid of the poors rate in the ſaid pariſh.

And be it further enacted by the authority aforeſaid, That every ſuch rector, vicar, or perpetual curate, concerned in the execution of this act, who ſhall remove from any ſuch pariſh, and every churchwarden, or overſeer of the poor, at the time of going out of his office, and the executors or adminiſtrators

The acting members of theſe corporations who ſhall have any of the money belonging to the funds in their hands at the time of their removal from the pariſh or the expiration of their offices in it, ſhall pay over the ſame to their ſucceſſors within one month after ſuch removal or expiration of their offices. And their executors or adminiſtrators ſhall do the ſame after their deaths.

miniſtrators of every ſuch rector, vicar, or perpetual curate, churchwarden, or overſeer of the poor, who ſhall happen to die, during the time he ſhall be concerned in the execution of this act, ſhall deliver in a true and perfect account of, and ſhall pay, or cauſe to be paid, all monies remaining in his or their hands, for the purpoſes of this act, to the rector, vicar, or perpetual curate, churchwardens, or overſeers of the poor, for ſuch pariſh or townſhip, within one month after any ſuch removal, going out of office, or death; and in caſe of neglect or refuſal ſo to do, it ſhall and may be lawful for any two juſtices of the peace for the county or place where the offender ſhall reſide, to make inquiry concerning the ſame, in a ſummary way, either by confeſſion of the party, or by the teſtimony of any credible witneſs or witneſſes upon oath, (which oath ſuch juſtices are hereby impowered to adminiſter;) and to cauſe the money remaining in the hands of ſuch rector, vicar, or perpetual curate, churchwarden, overſeer of the poor, executor, or adminiſtrator reſpectively, to be recovered by diſtreſs and ſale of his goods and chattels, rendering the overplus to the owner of ſuch goods and chattels, after deducting the charges attending ſuch diſtreſs and ſale; and ſuch juſtices are hereby impowered to cauſe the books and accounts beforementioned to be brought, and ſuch witneſſes to be ſummoned to appear before them as they ſhall think neceſſary for their information: and if any perſon ſhall refuſe to appear, or to give evidence, or to produce ſuch books and accounts, as aforeſaid, to ſuch juſtices, it ſhall and may be lawful for ſuch juſtices to impoſe any fine or fines upon ſuch perſon and perſons ſo offending as they ſhall think fit, (ſo as no ſuch fine ſhall exceed the ſum of ten pounds upon any one perſon for one offence,) and to levy ſuch fines by diſtreſs and ſale of the offenders goods and chattels; and, if no ſuch diſtreſs can be found, it ſhall and may be lawful for ſuch juſtices to commit the offender to the common gaol of the county or place, for any time not exceeding ſix months, unleſs ſuch fine ſhall be ſooner paid.

And, in caſe of neglect or refuſal ſo to do, any two juſtices of the peace for the county in which the offender ſhall reſide, may inquire into the matter in a ſummary way, and compel him to do juſtice.

And be it further enacted by the authority aforeſaid, That, if any ſuch annuity ſhall be behind and unpaid for the ſpace of ſeven days after the ſame ſhall become due, and have been demanded, it ſhall and may be lawful for any one juſtice of the peace, for the county or place where the pariſh or townſhip from which ſuch annuity ſhall be ſo due and unpaid ſhall be ſituate, to make enquiry concerning the ſame, in a ſummary way, either by confeſſion of the party, or by the teſtimony of any credible witneſs or witneſſes upon oath, (which oath ſuch juſtice is hereby impowered to adminiſter,) and to cauſe the money ſo due to be levied by warrant under his hand and ſeal, by diſtreſs and ſale of the goods and chattels of any one of the churchwardens or overſeers of the poor of ſuch pariſh, rendering the overplus to the owner of ſuch goods and chattels, after deducting the charges attending ſuch diſtreſs and ſale.

And, in caſe the churchwardens and overſeers of the poor ſhall neglect, or refuſe, to pay any annuity for the ſpace of ſeven days after it ſhall have become due and been demanded, any one juſtice of the peace of the county in which the pariſh is ſituated, may inquire into the matter in a ſummary way, and compel them to do juſtice.

And

And whereas, by sundry acts of parliament, several parishes and townships have been united into several corporations, and the management, maintenance, and regulation of the poor of such parishes and townships is thereby vested in the governors, guardians, and acting members of such corporations respectively; be it enacted by the authority aforesaid, That in all places where such corporations exist, and where the plan and method herein-before prescribed for granting annuities to industrious and frugal persons, shall have been approved by the majority of a general court, to be called and held for that purpose, the power of granting life-annuities in pursuance of this act to persons appearing to be legally settled in any parish or township, so incorporated as aforesaid, shall be vested in the governors, guardians, and acting members of the corporation, to which such parish or township does respectively belong, and not in the rector, vicar, or perpetual curate, churchwardens, and overseers of the poor, and rateable inhabitants, of such parish or township; and the governors, guardians, and acting members of such corporation, assembled in a general court, or the major part of them so assembled, are hereby impowered to agree, according to the respective prices expressed in the set of tables hereunto annexed, No. 2, for the sale of any annuity or annuities for life, to any person or persons, appearing to them to have a lawful settlement in any parish or township, within the limits of the jurisdiction of the said corporation, the purchase-money not being less, nor the annuity or annuities greater, than the sums before limited, nor the commencement of any such annuity at any other age than as before limited for other parishes and townships; and upon such agreement made, the said governors, guardians, and acting members, are hereby impowered and required to receive the purchase-money for, and to do every act for granting and securing such life-annuity or annuities by deeds, to be executed by them in the same form and manner as other deeds are executed by them, for vesting the purchase-money received by them in the purchase of three *per cent.* bank-annuities, in the name of the said corporation, for receiving the dividends, and transferring the said stock, so purchased by them, for paying, or causing to be paid, the life-annuities so granted by them, and for re-purchasing the same in the manner before directed, which the several rectors, vicars, or perpetual curates, churchwardens, and overseers of the poor of other parishes and townships, are by this act impowered or required to do; for all and every the purposes before mentioned, and in the same form and manner, as near as the institution of such corporation, and the several regulations touching its manner of acting and proceeding, will admit. And in case at any time the fund or funds for securing such life-annuities shall prove deficient, the governors, guardians, and acting members of every such corporation, are hereby respectively impowered and required to pay the same out of any money in their hands, applicable to the maintenance

The foregoing regulations shall extend to such parishes and townships as have been united together, by acts of parliament, into several corporations for the purposes of maintaining and managing their poor.

And they shall be carried into execution by the governors, guardians, and acting members of such corporations.

nance of the poor of the pariſh or townſhip in which ſuch annuitant appeared to have been lawfully ſettled at the time of granting ſuch annuity, or to make, or cauſe to be made, a ſufficient rate upon ſuch inhabitants within the limits of the juriſdiction of the ſaid corporation, as would have been liable to contribute to the relief of the poor of ſuch pariſh or townſhip; and the ſaid governors, guardians, and acting members of the ſaid corporations, are hereby further impowered and required to compel the ſeveral perſons concerned in the receipt and payment of money, upon account of ſuch life-annuities, to produce and paſs their accounts of all monies received and paid by them, in ſuch manner, and by ſuch means, as the ſeveral accountants are compellable to account for other monies received and paid by them, by the ſaid ſeveral acts of parliament; and no conſent of the rateable inhabitants of the ſaid ſeveral pariſhes and townſhips ſhall be requiſite to make valid, or confirm any of the ſaid proceedings of the governors, guardians, and acting members of ſuch corportions.

And the aforeſaid penal clauſes againſt perſons guilty of frauds with reſpect to annuities granted by ſingle pariſhes ſhall extend to perſons guilty of the like frauds with reſpect to annuities granted by the ſaid corporations conſiſting of ſeveral pariſhes united together.

And be it further enacted by the authority aforeſaid, That every clauſe, proviſion, and regulation herein contained, as well with reſpect to perſons who ſhall alter, forge, frame, in manner before mentioned, or, not being the owner thereof, ſhall get poſſeſſion of, and demand and obtain money for, any grant, or copy of any grant, of any annuity, to be granted by virtue of this act, by rectors, vicars, or perpetual curates, churchwardens, and overſeers of the poor, as with reſpect to every other matter or thing directed to be done by them in the execution hereof, ſhall, in every caſe where, and as far as it can be made applicable, extend, and take place, alſo with reſpect to ſuch grants, or copies of grants, of annuities, which, in purſuance hereof, ſhall be granted by the ſaid corporations.

And be it further enacted by the authority aforeſaid, That in all pariſhes and townſhips, which have not been united into ſeveral corporations, for the management, maintenance, and regulation of their poor, the following form ſhall be obſerved in all powers of attorney to be executed by the rector, vicar, or perpetual curate, churchwardens, and overſeers of the poor of ſuch pariſhes or townſhips, for the acceptance of ſtock, and the receipt of dividends due thereupon; that is to ſay,

The Pariſh of *in the County of*

The form of the powers of attorney which ſhall be granted by ſingle pariſhes for the acceptance of ſtock in the three per cent. bank-annuities and the receipt of dividends due thereupon.

" KNOW all men by theſe preſents, That we, the rector, (*vicar, or*
" *perpetual curate, as the caſe may happen,*) churchwardens and over-
" ſeers of the poor, and rateable inhabitants of the pariſh of
" in the county of in veſtry aſſembled, by virtue
" of

" of an act of parliament of the thirteenth year of the reign of his Majesty
" King George the Third, (intituled, *An Act for the better Support of*
" *Poor Persons in certain Circumstances, by enabling Parishes to grant them*
" *Annuities for Life upon Purchase, and under certain Restrictions,*) do make,
" constitute, and appoint, *A. B.* of the parish of
" in the county of Gentleman, our true and
" lawful attorney, for us, in our names, and on our behalf, to accept all
" transfers that are, or may, at any time or times, be made unto us, the
" said rector, (*vicar, or perpetual curate, as the case may happen,*) church-
" wardens, overseers of the poor, and rateable inhabitants of the parish
" of in the county of and
" our successors, of any interest or shares in the capital, or joint, stock of
" three *per cent.* reduced, (*or consolidated,*) bank annuities, (*as the case may*
" *happen*); and also, on our behalf, to receive, and give receipts for, all divi-
" dends and interest that shall grow due and payable, on our interest or
" share in the above-said capital, or joint-stock, or on any part thereof, for
" the time being, and to do all lawful acts requisite for effecting the pre-
" mises, hereby ratifying and confirming all that our said attorney shall do
" therein by virtue hereof. In witness whereof, we, the acting members
" of the said body corporate, have hereunto set our hands, and affixed
" our common seal, the day of
" in the year of our Lord

Signed, sealed, and deliver-
ed in the presence of
N. O. of Gentleman,
R. S. of Yeoman.

D. E. Rector of the
Parish of in
the County of

J. R.
S. M. Churchwardens of the said Parish.

T. N.
E. A.
F. M. Overseers of the Poor of the said Parish.

The Common Seal of the Parish of in the County of

And in all such parishes or townships the following form shall be observed in all powers of attorney, for the transfer of all such stock as shall have been purchased by them; that is to say, *The Parish of in the County of*

The form of the powers of attorney which shall be granted by single parishes for the selling and transferring of stock in the said 3 per cent. bank-annuities.

" KNOW all men by these presents, That we, the rector, *(vicar, or perpetual curate, as the case may happen,)* churchwardens, overseers of the poor, and rateable inhabitants of the parish of in the county of in vestry assembled, by virtue of an act of parliament, of the thirteenth year of the reign of his Majesty King George the Third, (intituled, *An Act for the better Support of Poor Persons in certain Circumstances, by enabling Parishes to grant them Annuities for Life, upon Purchase, and under certain Restrictions,)* do make, constitute, and appoint *A. B.* of the parish of in the county of Gentleman, our true and lawful attorney, for us, in our names, and on our behalf, to sell, assign, and transfer or any part thereof, being part of [*or* all] the interest or share in the capital, or joint stock, of three *per cent.* reduced *(or consolidated)* bank-annuities, *(as the case may happen)* standing in the names of us the rector, *(vicar, or perpetual curate, as the case may happen,)* churchwardens, overseers of the poor, and rateable inhabitants of the said parish of in the county of and to receive the consideration-money, and to give discharges and receipts for the same, and to do all lawful acts requisite for effecting the premises, hereby ratifying and confirming all that our said attorney shall do herein by virtue hereof. In witness whereof, we, the acting members of the said body corporate, have hereunto set our hands, and affixed our common seal, the day of in the year of our Lord

Signed, sealed, and delivered in the presence of
N. O. of Gentleman,
P. S. of Yeoman.

D. E. Rector of the Parish of in the County of

J. R., *S. M.* — Churchwardens of the said Parish.

T. N., *E. A.*, *F. M.* — Overseers of the Poor of the said Parish.

The Common Seal of the Parish of in the County of

TABLES

TABLES

SHEWING THE VALUES,

IN A SINGLE PRESENT PAYMENT,

OF AN

ANNUITY OF ONE POUND,

PAYABLE QUARTERLY

For the Lives of Perfons of all Ages, from . . .

Such Annuity being fuppofed to commence at any Age, not younger than 35 nor older than 75 Years.

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DIRECTIONS

FOR USING THE FOLLOWING

TABLES.

Drawn up by the Rev. Dr. Richard Price, of Newington-Green, near Islington, in the County of Middlesex.

THESE tables give the present payments that ought to be made (reckoning the interest at three per cent.) by persons of all ages, from 15 to 73, for a life-annuity of one pound, payable quarterly, (that is, five shillings each quarter,) to commence at any given age not less than 35, nor exceeding 75.

For instance. Table the first, in the second set of tables, shews, that the payment due from a person whose age is between $14\frac{3}{4}$ and $15\frac{1}{4}$ is £4 0*s.* 1*d.* for a life-annuity of one pound, payable quarterly, to commence when he attains to 43 years of age, or (more exactly) at the end of 28 years and a quarter from the day on which the payment is made. In like manner the same table shews, that the present payment due from a person of the same age, for the same annuity, is *one pound*, supposing the first quarterly payment of the annuity to be made at the end of forty-five years and a quarter, or not till he attains to 60 years of age; and *six shillings*, if he chuses to wait for the first quarterly payment 55 years and a quarter, or till he attains to 70 years of age.

A life-annuity therefore of £2. to commence at these different periods, must be worth double these sums; and a life-annuity of £10. must be worth ten times these sums.

Universally

Universally therefore. In order to find from the following tables the present money that ought to be given for any annuity payable quarterly to purchasers of giv... ages, after attaining any other given ages----*Multiply the value in the table for the given age, by the number of pounds in the proposed annuity, and the product will be the answer.*

Example. From table 7th, in the second set of tables, (calculated for the country,) it will appear by inspection, that the value to a purchaser aged 21, of an annuity of £1. for life, payable quarterly, to commence at 50 years of age, is £3. 1*s.* 2*d.* If therefore the annuity is £5. its value will be this sum multiplyed by 5, or £15. 5*s.* 10*d.* If the annuity is £20. its value will be the same multiplyed by 20, or £61. 3*s.* 4*d.*

Second Example. Suppose a person whose age is 25, to apply for an annuity (payable quarterly) of £10. for his life, after attaining his 55th year. From table 11, in the second set of tables, it appears, that, if the annuity had been one pound, its present value would have been £2. 9*s.* therefore the annuity being £10. its value is £2. 9*s.* multiplied by 10, or £24. 10*s.*

The value of the same annuity, according to table 11, in the first set of tables, calculated for London, is £1. 14*s.* 3*d.* multiplied by 10, or £17. 2*s.* 6*d.*

Again. Suppose a person of a given age to desire to be informed "what annuity, as a provision for old age, he can purchase with a given sum of money, of which he is in possession."— In order to discover the answer, look over the table for the given age, and find there the given sum; or, if it be not found exactly, find the sum nearest to it; and the correspondent age will shew, that with the given sum he may purchase an annuity of one pound for life, to commence at that age.——In like manner, the ages corresponding to half, a third, a fourth, &c. of the sum will shew, that it will purchase for him an annuity of £2. £3. £4. &c. to commence at those ages respectively.

Example. A poor person, aged 22, has saved by his industry the the sum of £5. and with this in his hands, he applies for such an annuity, to commence in some future year of his life, as it can purchase for him.

By looking over the table for the age of 22, or table 8 in the second set of tables, it will appear, that £4. 19*s.* 11*d.* or, very nearly the sum he offers, will purchase for him an annuity of one pound for his life, to to commence at the age of 44 years; and it appears alſo from the ſame table, that £2. 10*s.* 3*d.* (or about half his money,) will purchaſe the ſame annuity, to commence when he is 53; and that £1. 4*s.* 2*d.* (or about a quarter of his money,) will purchaſe the ſame annuity, to commence when he is 61; and that 19*s.* 7*d.* (or about a fifth part of his money,) will purchaſe the ſame annuity, to commence when he is 63. If, therefore, he thinks an annuity of one pound, for his life, to commence at 44, too little, he may be offered for £5. 0*s.* 6*d.* £4. 16*s.* 8*d.* £4 17*s.* 11*d.* (that is for ſums nearly equal to £5.) an annuity of £2. for life, to commence at 53, or of £4. for life, to commence at 61, or of £5. for life, to commence at 63. From table 8, in the firſt ſet of tables, it will in the ſame way appear, that in London £5. would intitle ſuch a perſon to a life-annuity of either £2. to commence at 49, or of £4. to commence at 57, or of £5. to commence at 59, or four years earlier than by the ſecond ſet of tables, which are calculated for the country.

Obſervation 1ſt. The payments of perſons who happen to die before the age agreed on for the commencement of their annuities, are in theſe tables ſuppoſed to be an advantage ſhared amongſt ſurvivors, without which the money advanced would be inſufficient to bear the expence of the annuities.

Theſe tables alſo ſuppoſe, that annuitants will be intitled to nothing for any part of that quarter of the year in which they ſhall happen to die.

Obſervation 2d. It ſhould be remembered, that the firſt of the three columns in theſe tables is intended to be an explanation of the column next to it, and to expreſs with more preciſion the time at which the calculations ſuppoſe the annuity to commence. — Thus, if it were only expreſſed in table 1ſt, that a perſon, whoſe age is exactly 14$\frac{3}{4}$, would be entitled, for a preſent payment of £2. 7*s.* 11*d.* to an annuity, payable quarterly, of £1. for life, to commence at the age of 50, it would only apper that the annuity was to commence ſome time or other after he had attained to that age, or entered his 51ſt year. But the firſt column removes this uncertainty, by ſpecifying, that the firſt quarterly payment is to be made at the end of 35 years and a quarter after purchaſing, or exactly upon his attaining the age of 50. In like manner; ſuppoſing his age 15, 15 and a quarter, or 15 and a half, the ſame column, by ſpecifying that the annuity was to commence at the end of 35 years and $\frac{1}{4}$, would ſhew, that the firſt payment was to be made when he came to be a quarter of a year, half a year, or three quarters of a year, turned of 50.

Obſervation

Observation 3d. When there is any uncertainty with respect to the precise age of a purchaser, a younger age should always be taken, rather than an older, in order to guard against the losses to parishes, that would arise from intitling persons to higher annuities than are adequate to their payments. Much, however, will not depend on determining the age of any purchaser to greater exactness than half a year, or a year.

Observation 4th. A considerable difference will be found at all ages under 45 or 50 years between the values in the following tables for London, and for the country. The reason is, that the inhabitants of London, and of great towns in general, are much more short-lived than the inhabitants of small towns, and country parishes and villages. This appears from undeniable observations, and has created the necessity of calculating distinct tables for London and the country.

It may be proper to add, for the satisfaction of those who may wish to examine the following tables, that they have been calculated in the method explained and demonstrated, by Mr. Maseres, in a pamphlet intitled, *A Proposal for establishing Life-Annuities in Parishes, for the Benefit of the industrious Poor*, or by a rule in Dr. Price's Treatise on Annuities, Quest. 6th, page 17th.——In calculating the second set of tables, the probabilities of life at Northampton, as given in Table 4th, page 323, of the said Treatise of Dr. Price, have been combined with the values of lives in Table 6th, page 325. And in calculating the first set of tables, the probabilities and values of lives have been taken from Mr. Simpson's tables given in pages 332, and 334, of the same Treatise, or in page 254, and 260, of Mr. Simpson's Select Exercises, with no other than the following difference.——The tables which have been mentioned give the values of lives in yearly payments only. An annuity, payable quarterly, is worth three eighths of a years purchase, or 7s. 6d. more than an annuity payable yearly. Three eighths, therefore, or, in decimals, .375, have been always added to the values of lives taken from these tables, in order to obtain from thence the values in the following tables.

FIRST

FIRST SET OF TABLES.

Intended for the Use of LONDON.

TABLE I.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 14 3/4 to 15 1/2.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d
20 1/4,	or at Age 35	5	10	8	41 1/4,	or at Age 56	1	0	0
21 1/4,	— — 36	5	3	5	42 1/4,	— — 57	0	18	2
22 1/4,	— — 37	4	16	6	43 1/4,	— — 58	0	16	4
23 1/4,	— — 38	4	9	8	44 1/4,	— — 59	0	14	11
24 1/4,	— — 39	4	3	5	45 1/4,	— — 60	0	13	5
25 1/4,	— — 40	3	17	3	46 1/4,	— — 61	0	12	1
26 1/4,	— — 41	3	11	7	47 1/4,	— — 62	0	10	10
27 1/4,	— — 42	3	6	0	48 1/4,	— — 63	0	9	9
28 1/4,	— — 43	3	1	2	49 1/4,	— — 64	0	8	8
29 1/4,	— — 44	2	16	7	50 1/4,	— — 65	0	7	9
30 1/4,	— — 45	2	12	3	51 1/4,	— — 66	0	6	9
31 1/4,	— — 46	2	8	0	52 1/4,	— — 67	0	6	0
32 1/4,	— — 47	2	4	5	53 1/4,	— — 68	0	5	2
33 1/4,	— — 48	2	0	10	54 1/4,	— — 69	0	4	7
34 1/4,	— — 49	1	17	8	55 1/4,	— — 70	0	4	0
35 1/4,	— — 50	1	14	7	56 1/4,	— — 71	0	3	6
36 1/4,	— — 51	1	11	10	57 1/4,	— — 72	0	3	0
37 1/4,	— — 52	1	9	0	58 1/4,	— — 73	0	2	7
38 1/4,	— — 53	1	6	6	59 1/4,	— — 74	0	2	2
39 1/4,	— — 54	1	4	0	60 1/4,	— — 75	0	1	11
40 1/4,	— — 55	1	2	0					

TABLE II.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 15 ¾ to 16 ½.

To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d.	To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d.
19 ¼,	or at Age 35	5	15	3	40 ¼,	or at Age 56	1	0	11
20 ¼,	— — 36	5	7	11	41 ¼,	— — 57	0	18	11
21 ¼,	— — 37	5	0	6	42 ¼,	— — 58	0	17	2
22 ¼,	— — 38	4	13	8	43 ¼,	— — 59	0	15	6
23 ¼,	— — 39	4	6	11	44 ¼,	— — 60	0	14	0
24 ¼,	— — 40	4	0	9	45 ¼,	— — 61	0	12	7
25 ¼,	— — 41	3	14	7	46 ¼,	— — 62	0	11	4
26 ¼,	— — 42	3	9	2	47 ¼,	— — 63	0	10	2
27 ¼,	— — 43	3	3	10	48 ¼,	— — 64	0	9	1
28 ¼,	— — 44	2	19	1	49 ¼,	— — 65	0	8	0
29 ¼,	— — 45	2	14	5	50 ¼,	— — 66	0	7	2
30 ¼,	— — 46	2	10	4	51 ¼,	— — 67	0	6	3
31 ¼,	— — 47	2	6	3	52 ¼,	— — 68	0	5	6
32 ¼,	— — 48	2	2	9	53 ¼,	— — 69	0	4	9
33 ¼,	— — 49	1	19	3	54 ¼,	— — 70	0	4	2
34 ¼,	— — 50	1	16	2	55 ¼,	— — 71	0	3	8
35 ¼,	— — 51	1	13	2	56 ¼,	— — 72	0	3	2
36 ¼,	— — 52	1	10	5	57 ¼,	— — 73	0	2	8
37 ¼,	— — 53	1	7	8	58 ¼,	— — 74	0	2	3
38 ¼,	— — 54	1	5	3	59 ¼,	— — 75	0	1	11
39 ¼,	— — 55	1	2	11					

TABLE III.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 16 ¼ to 17 ¼.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
18 ¼, or at Age	35	6	0	7	39 ¼, or at Age	56	1	1	9
19 ¼, — —	36	5	12	7	40 ¼, — —	57	0	19	9
20 ¼, — —	37	5	5	1	41 ¼, — —	58	0	17	9
21 ¼, — —	38	4	17	7	42 ¼, — —	59	0	16	2
22 ¼, — —	39	4	10	11	43 ¼, — —	60	0	14	7
23 ¼, — —	40	4	4	2	44 ¼, — —	61	0	13	2
24 ¼, — —	41	3	18	0	45 ¼, — —	62	0	11	10
25 ¼, — —	42	3	11	11	46 ¼, — —	63	0	10	8
26 ¼, — —	43	3	6	9	47 ¼, — —	64	0	9	6
27 ¼, — —	44	3	1	7	48 ¼, — —	65	0	8	5
28 ¼, — —	45	2	17	0	49 ¼, — —	66	0	7	4
29 ¼, — —	46	2	12	4	50 ¼, — —	67	0	6	6
30 ¼, — —	47	2	8	5	51 ¼, — —	68	0	5	8
31 ¼, — —	48	2	4	6	52 ¼, — —	69	0	5	0
32 ¼, — —	49	2	1	1	53 ¼, — —	70	0	4	4
33 ¼, — —	50	1	17	9	54 ¼, — —	71	0	3	10
34 ¼, — —	51	1	14	9	55 ¼, — —	72	0	3	4
35 ¼, — —	52	1	11	9	56 ¼, — —	73	0	2	11
36 ¼, — —	53	1	9	0	57 ¼, — —	74	0	2	5
37 ¼, — —	54	1	6	3	58 ¼, — —	75	0	2	0
38 ¼, — —	55	1	4	0					

TABLE IV.

For the Ufe of LONDON.

Shewing the Payment due, (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 17¾ to 18½.

To commence at the end of Years after purchafing.	Value of an annuity of £1. in one prefent payment. £.	.	d.
17¼, or at Age 35	6	5	8
18¼, — — 36	5	17	7
19¼, — — 37	5	9	6
20¼, — — 38	5	2	1
21¼, — — 39	4	14	8
22¼, — — 40	4	8	0
23¼, — — 41	4	1	3
24¼, — — 42	3	15	5
25¼, — — 43	3	9	7
26¼, — — 44	3	4	5
27¼, — — 45	2	19	3
28¼, — — 46	2	14	10
29¼, — — 47	2	10	5
30¼, — — 48	2	6	7
31¼, — — 49	2	2	10
32¼, — — 50	1	19	6
33¼, — — 51	1	16	2
34¼, — — 52	1	13	2
35¼, — — 53	1	10	2
36¼, — — 54	1	7	7
37¼, — — 55	1	5	0

To commence at the end of Years after purchafing.	Value of an annuity of £1. in one prefent payment. £.	s.	d.
38¼, or at Age 56	1	2	9
39¼, — — 57	1	0	6
40¼, — — 58	0	18	8
41¼, — — 59	0	16	10
42¼, — — 60	0	15	3
43¼, — — 61	0	13	9
44¼, — — 62	0	12	4
45¼, — — 63	0	11	1
46¼, — — 64	0	9	11
47¼, — — 65	0	8	9
48¼, — — 66	0	7	9
49¼, — — 67	0	6	9
50¼, — — 68	0	6	0
51¼, — — 69	0	5	3
52¼, — — 70	0	4	7
53¼, — — 71	0	4	0
54¼, — — 72	0	3	6
55¼, — — 73	0	3	0
56¼, — — 74	0	2	7
57¼, — — 75	0	2	2

TABLE V.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 18$\frac{3}{4}$ to 19$\frac{1}{2}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.
16$\frac{1}{4}$,	or at Age 35	6	11	6	37$\frac{1}{4}$,	or at Age 56	1	3	8
17$\frac{1}{4}$,	36	6	2	9	38$\frac{1}{4}$,	57	1	1	6
18$\frac{1}{4}$,	37	5	14	7	39$\frac{1}{4}$,	58	0	19	4
19$\frac{1}{4}$,	38	5	6	5	40$\frac{1}{4}$,	59	0	17	8
20$\frac{1}{4}$,	39	4	19	1	41$\frac{1}{4}$,	60	0	16	0
21$\frac{1}{4}$,	40	4	11	9	42$\frac{1}{4}$,	61	0	14	5
22$\frac{1}{4}$,	41	4	5	1	43$\frac{1}{4}$,	62	0	12	11
23$\frac{1}{4}$,	42	3	18	5	44$\frac{1}{4}$,	63	0	11	7
24$\frac{1}{4}$,	43	3	12	9	45$\frac{1}{4}$,	64	0	10	4
25$\frac{1}{4}$,	44	3	7	2	46$\frac{1}{4}$,	65	0	9	2
26$\frac{1}{4}$,	45	3	2	1	47$\frac{1}{4}$,	66	0	8	1
27$\frac{1}{4}$,	46	2	17	0	48$\frac{1}{4}$,	67	0	7	1
28$\frac{1}{4}$,	47	2	12	9	49$\frac{1}{4}$,	68	0	6	2
29$\frac{1}{4}$,	48	2	8	6	50$\frac{1}{4}$,	69	0	5	6
30$\frac{1}{4}$,	49	2	4	10	51$\frac{1}{4}$,	70	0	4	9
31$\frac{1}{4}$,	50	2	1	1	52$\frac{1}{4}$,	71	0	4	2
32$\frac{1}{4}$,	51	1	17	10	53$\frac{1}{4}$,	72	0	3	7
33$\frac{1}{4}$,	52	1	14	7	54$\frac{1}{4}$,	73	0	3	1
34$\frac{1}{4}$,	53	1	11	7	55$\frac{1}{4}$,	74	0	2	7
35$\frac{1}{4}$,	54	1	8	7	56$\frac{1}{4}$,	75	0	2	3
36$\frac{1}{4}$,	55	1	6	2					

TABLE VI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 19¾ to 20½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
15¼,	or at Age 35	6	17	1	36¼,	or at Age 56	1	4	10
16¼,	— — 36	6	8	3	37¼,	— — 57	1	2	5
17¼,	— — 37	5	19	5	38¼,	— — 58	1	0	5
18¼,	— — 38	5	11	4	39¼,	— — 59	0	18	5
19¼,	— — 39	5	3	3	40¼,	— — 60	0	16	8
20¼,	— — 40	4	15	11	41¼,	— — 61	0	15	0
21¼,	— — 41	4	8	8	42¼,	— — 62	0	13	6
22¼,	— — 42	4	2	3	43¼,	— — 63	0	12	1
23¼,	— — 43	3	15	10	44¼,	— — 64	0	10	10
24¼,	— — 44	3	10	3	45¼,	— — 65	0	9	7
25¼,	— — 45	3	4	8	46¼,	— — 66	0	8	6
26¼,	— — 46	2	19	10	47¼,	— — 67	0	7	5
27¼,	— — 47	2	14	11	48¼,	— — 68	0	6	7
28¼,	— — 48	2	10	10	49¼,	— — 69	0	5	8
29¼,	— — 49	2	6	8	50¼,	— — 70	0	5	0
30¼,	— — 50	2	3	0	51¼,	— — 71	0	4	4
31¼,	— — 51	1	19	5	52¼,	— — 72	0	3	10
32¼,	— — 52	1	16	2	53¼,	— — 73	0	3	3
33¼,	— — 53	1	12	11	54¼,	— — 74	0	2	10
34¼,	— — 54	1	10	0	55¼,	— — 75	0	2	5
35¼,	— — 55	1	7	2					

TABLE VII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 20$\frac{3}{4}$ to 21$\frac{3}{4}$.

To commence at the end of Years after purchaſing.	Value of an annuity of £1 in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.	Value of an annuity of £1. in one preſent payment. £.	s.	d.
14$\frac{1}{4}$, or at Age 35	7	3	5	35$\frac{1}{4}$, or at Age 56	1	5	10
15$\frac{1}{4}$, — — 36	6	13	11	36$\frac{1}{4}$, — — 57	1	3	6
16$\frac{1}{4}$, — — 37	6	5	0	37$\frac{1}{4}$, — — 58	1	1	1
17$\frac{1}{4}$, — — 38	5	16	1	38$\frac{1}{4}$, — — 59	0	19	3
18$\frac{1}{4}$, — — 39	5	8	1	39$\frac{1}{4}$, — — 60	0	17	5
19$\frac{1}{4}$, — — 40	5	0	1	40$\frac{1}{4}$, — — 61	0	15	8
20$\frac{1}{4}$, — — 41	4	12	10	41$\frac{1}{4}$, — — 62	0	14	0
21$\frac{1}{4}$, — — 42	4	5	7	42$\frac{1}{4}$, — — 63	0	12	7
22$\frac{1}{4}$, — — 43	3	19	5	43$\frac{1}{4}$, — — 64	0	11	3
23$\frac{1}{4}$, — — 44	3	13	3	44$\frac{1}{4}$, — — 65	0	10	0
24$\frac{1}{4}$, — — 45	3	7	9	45$\frac{1}{4}$, — — 66	0	8	10
25$\frac{1}{4}$, — — 46	3	2	2	46$\frac{1}{4}$, — — 67	0	7	10
26$\frac{1}{4}$, — — 47	2	17	7	47$\frac{1}{4}$, — — 68	0	6	10
27$\frac{1}{4}$, — — 48	2	12	11	48$\frac{1}{4}$, — — 69	0	6	0
28$\frac{1}{4}$, — — 49	2	8	10	49$\frac{1}{4}$, — — 70	0	5	2
29$\frac{1}{4}$, — — 50	2	4	10	50$\frac{1}{4}$, — — 71	0	4	7
30$\frac{1}{4}$, — — 51	2	1	3	51$\frac{1}{4}$, — — 72	0	3	11
31$\frac{1}{4}$, — — 52	1	17	9	52$\frac{1}{4}$, — — 73	0	3	5
32$\frac{1}{4}$, — — 53	1	14	6	53$\frac{1}{4}$, — — 74	0	2	11
33$\frac{1}{4}$, — — 54	1	11	2	54$\frac{1}{4}$, — — 75	0	2	7
34$\frac{1}{4}$, — — 55	1	8	6				

TABLE VIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 21 ¼ to 22 ¼.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.
13 ¼,	or at Age 35	7	9	6	34 ¼,	or at Age 56	1	7	1
14 ¼,	— — 36	6	19	11	35 ¼,	— — 57	1	4	5
15 ¼,	— — 37	6	10	4	36 ¼,	— — 58	1	2	3
16 ¼,	— — 38	6	1	6	37 ¼,	— — 59	1	0	0
17 ¼,	— — 39	5	12	7	38 ¼,	— — 60	0	18	2
18 ¼,	— — 40	5	4	8	39 ¼,	— — 61	0	16	4
19 ¼,	— — 41	4	16	8	40 ¼,	— — 62	0	14	9
20 ¼,	— — 42	4	9	9	41 ¼,	— — 63	0	13	1
21 ¼,	— — 43	4	2	9	42 ¼,	— — 64	0	11	9
22 ¼,	— — 44	3	16	8	43 ¼,	— — 65	0	10	5
23 ¼,	— — 45	3	10	7	44 ¼,	— — 66	0	9	3
24 ¼,	— — 46	3	5	3	45 ¼,	— — 67	0	8	1
25 ¼,	— — 47	3	0	0	46 ¼,	— — 68	0	7	2
26 ¼,	— — 48	2	15	5	47 ¼,	— — 69	0	6	3
27 ¼,	— — 49	2	11	0	48 ¼,	— — 70	0	5	6
28 ¼,	— — 50	2	7	0	49 ¼,	— — 71	0	4	9
29 ¼,	— — 51	2	3	0	50 ¼,	— — 72	0	4	2
30 ¼,	— — 52	1	19	5	51 ¼,	— — 73	0	3	6
31 ¼,	— — 53	1	15	11	52 ¼,	— — 74	0	3	0
32 ¼,	— — 54	1	12	10	53 ¼,	— — 75	0	2	7
33 ¼,	— — 55	1	9	8					

TABLE IX.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 22¼. to 23½.

To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d.	To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d
12¼,	or at Age 35	7	16	6	33¼,	or at Age 56	1	8	2
13¼,	— — 36	7	6	2	34¼,	— — 57	1	5	8
14¼,	— — 37	6	16	5	35¼,	— — 58	1	3	1
15¼,	— — 38	6	6	9	36¼,	— — 59	1	1	0
16¼,	— — 39	5	18	0	37¼,	— — 60	0	19	0
17¼,	— — 40	5	9	2	38¼,	— — 61	0	17	2
18¼,	— — 41	5	1	3	39¼,	— — 62	0	15	4
19¼,	— — 42	4	13	4	40¼,	— — 63	0	13	10
20¼,	— — 43	4	6	8	41¼,	— — 64	0	12	3
21¼,	— — 44	4	0	0	42¼,	— — 65	0	10	11
22¼,	— — 45	3	13	11	43¼,	— — 66	0	9	7
23¼,	— — 46	3	7	11	44¼,	— — 67	0	8	6
24¼,	— — 47	3	2	10	45¼,	— — 68	0	7	5
25¼,	— — 48	2	17	9	46¼,	— — 69	0	6	6
26¼,	— — 49	2	13	4	47¼,	— — 70	0	5	8
27¼,	— — 50	2	9	0	48¼,	— — 71	0	5	0
28¼,	— — 51	2	5	1	49¼,	— — 72	0	4	4
29¼,	— — 52	2	1	2	50¼,	— — 73	0	3	9
30¼,	— — 53	1	17	8	51¼,	— — 74	0	3	2
31¼,	— — 54	1	14	1	52¼,	— — 75	0	2	8
32¼,	— — 55	1	11	2					

TABLE X.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 23 ¼ to 24 ¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
11 ¼,	or at Age 35	8	3	9	32 ¼,	or at Age 56	1	9	8
12 ¼,	— — 36	7	13	3	33 ¼,	— — 57	1	6	9
13 ¼,	— — 37	7	2	8	34 ¼,	— — 58	1	4	4
14 ¼,	— — 38	6	13	0	35 ¼,	— — 59	1	2	0
15 ¼,	— — 39	6	3	5	36 ¼,	— — 60	0	19	11
16 ¼,	— — 40	5	14	8	37 ¼,	— — 61	0	17	11
17 ¼,	— — 41		5	11	38 ¼,	— — 62	0	16	2
18 ¼,	— — 42	4	18	3	39 ¼,	— — 63	0	14	5
19 ¼,	— — 43	4	10	7	40 ¼,	— — 64	0	12	11
20 ¼,	— — 44	4	4	0	41 ¼,	— — 65	0	11	4
21 ¼,	— — 45	3	17	4	42 ¼,	— — 66	0	10	1
22 ¼,	— — 46	3	11	6	43 ¼,	— — 67	0	8	10
23 ¼,	— — 47	3	5	8	44 ¼,	— — 68	0	7	10
24 ¼,	— — 48	3	0	9	45 ¼,	— — 69	0	6	10
25 ¼,	— — 49	2	15	9	46 ¼,	— — 70	0	6	0
26 ¼,	— — 50	2	11	5	47 ¼,	— — 71	0	5	2
27 ¼,	— — 51	2	7	1	48 ¼,	— — 72	0	4	6
28 ¼,	— — 52	2	3	2	49 ¼,	— — 73	0	3	11
29 ¼,	— — 53	1	19	4	50 ¼,	— — 74	0	3	4
30 ¼,	— — 54	1	16	0	51 ¼,	— — 75	0	2	10
31 ¼,	— — 55	1	[illegible]	[illegible]					

TABLE

TABLE XI.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 24 ¼ to 25 ¼.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
10 ¼,	or at Age 35	8	12	1	31 ¼,	or at Age 56	1	11	0
11 ¼,	— — 36	8	0	8	32 ¼,	— — 57	1	8	2
12 ¼,	— — 37	7	10	0	33 ¼,	— — 58	1	5	4
13 ¼,	— — 38	6	19	3	34 ¼,	— — 59	1	3	1
14 ¼,	— — 39	6	9	8	35 ¼,	— — 60	1	0	11
15 ¼,	— — 40	6	0	1	36 ¼,	— — 61	0	18	10
16 ¼,	— — 41	5	11	4	37 ¼,	— — 62	0	16	10
17 ¼,	— — 42	5	2	8	38 ¼,	— — 63	0	15	2
18 ¼,	— — 43	4	15	3	39 ¼,	— — 64	0	13	6
19 ¼,	— — 44	4	7	11	40 ¼,	— — 65	0	12	0
20 ¼,	— — 45	4	1	3	41 ¼,	— — 66	0	10	6
21 ¼,	— — 46	3	14	8	42 ¼,	— — 67	0	9	4
22 ¼,	— — 47	3	9	1	43 ¼,	— — 68	0	8	2
23 ¼,	— — 48	3	3	6	44 ¼,	— — 69	0	7	2
24 ¼,	— — 49	2	18	8	45 ¼,	— — 70	0	6	3
25 ¼,	— — 50	2	13	10	46 ¼,	— — 71	0	5	6
26 ¼,	— — 51	2	9	6	47 ¼,	— — 72	0	4	9
27 ¼,	— — 52	2	5	3	48 ¼,	— — 73	0	4	1
28 ¼,	— — 53	2	1	4	49 ¼,	— — 74	0	3	6
29 ¼,	— — 54	1	17	6	50 ¼,	— — 75	0	3	0
30 ¼,	— — 55	1	14	3					

TABLE XII.

For the Ufe of LONDON.

Shewing the Payment due, (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 25¾ to 26½.

To commence at the end of Years after purchafing.	or at Age	Value of an annuity of £1. in one prefent payment. £.	s.	d.	To commence at the end of Years after purchafing.	or at Age	Value of an annuity of £1. in one prefent payment. £.	s.	d.
9¼,	or at Age 35	9	0	2	30¼,	or at Age 56	1	12	7
10¼,	— — 36	8	8	7	31¼,	— — 57	1	9	5
11¼,	— — 37	7	17	0	32¼,	— — 58	1	6	9
12¼,	— — 38	7	6	4	33¼,	— — 59	1	4	2
13¼,	— — 39	6	15	8	34¼,	— — 60	1	1	11
14¼,	— — 40	6	6	1	35¼,	— — 61	0	19	8
15¼,	— — 41	5	16	6	36¼,	— — 62	0	17	9
16¼,	— — 42	5	8	1	37¼,	— — 63	0	15	10
17¼,	— — 43	4	19	8	38¼,	— — 64	0	14	2
18¼,	— — 44	4	12	4	39¼,	— — 65	0	12	6
19¼,	— — 45	4	5	0	40¼,	— — 66	0	11	1
20¼,	— — 46	3	18	7	41¼,	— — 67	0	9	9
21¼,	— — 47	3	12	3	42¼,	— — 68	0	8	7
22¼,	— — 48	3	6	9	43¼,	— — 69	0	7	6
23¼,	— — 49	3	1	4	44¼,	— — 70	0	6	7
24¼,	— — 50	2	16	7	45¼,	— — 71	0	5	8
25¼,	— — 51	2	11	10	46¼,	— — 72	0	5	0
26¼,	— — 52	2	7	6	47¼,	— — 73	0	4	3
27¼,	— — 53	2	3	3	48¼,	— — 74	0	3	8
28¼,	— — 54	1	19	6	49¼,	— — 75	0	3	1
29¼,	— — 55	1	15	10					

TABLE

TABLE XIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 26¾ to 27½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
8¼, or at Age 35	9	9	4	29¼, or at Age 56	1	14	2
9¼, — — 36	8	16	10	30¼, — — 57	1	11	0
10¼, — — 37	8	5	1	31¼, — — 58	1	7	11
11¼, — — 38	7	13	3	32¼, — — 59	1	5	5
12¼, — — 39	7	2	8	33¼, — — 60	1	3	0
13¼, — — 40	6	12	1	34¼, — — 61	1	0	9
14¼, — — 41	6	2	7	35¼, — — 62	0	18	6
15¼, — — 42	5	13	0	36¼, — — 63	0	16	8
16¼, — — 43	5	4	11	37¼, — — 64	0	14	11
17¼, — — 44	4	16	9	38¼, — — 65	0	13	3
18¼, — — 45	4	9	6	39¼, — — 66	0	11	7
19¼, — — 46	4	2	2	40¼, — — 67	0	10	3
20¼, — — 47	3	16	0	41¼, — — 68	0	9	0
21¼, — — 48	3	9	11	42¼, — — 69	0	7	11
22¼, — — 49	3	4	6	43¼, — — 70	0	6	11
23¼, — — 50	2	19	2	44¼, — — 71	0	6	0
24¼, — — 51	2	14	6	45¼, — — 72	0	5	2
25¼, — — 52	2	9	11	46¼, — — 73	0	4	6
26¼, — — 53	2	5	7	47¼, — — 74	0	3	10
27¼, — — 54	2	1	3	48¼, — — 75	0	3	3
28¼, — — 55	1	17	8				

TABLE XIV.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 27 ¾ to 28 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
7 ¼,	or at Age 35	9	18	9	28 ¼,	or at Age 56	1	16	0
8 ¼,	— — 36	9	5	11	29 ¼,	— — 57	1	12	6
9 ¼,	— — 37	8	13	2	30 ¼,	— — 58	1	9	7
10 ¼,	— — 38	8	1	5	31 ¼,	— — 59	1	6	8
11 ¼,	— — 39	7	9	8	32 ¼,	— — 60	1	4	2
12 ¼,	— — 40	6	19	1	33 ¼,	— — 61	1	1	9
13 ¼,	— — 41	6	8	6	34 ¼,	— — 62	0	19	7
14 ¼,	— — 42	5	19	3	35 ¼,	— — 63	0	17	6
15 ¼,	— — 43	5	10	0	36 ¼,	— — 64	0	15	8
16 ¼,	— — 44	5	1	11	37 ¼,	— — 65	0	13	10
17 ¼,	— — 45	4	13	9	38 ¼,	— — 66	0	12	3
18 ¼,	— — 46	4	6	9	39 ¼,	— — 67	0	10	9
19 ¼,	— — 47	3	19	8	40 ¼,	— — 68	0	9	6
20 ¼,	— — 48	3	13	8	41 ¼,	— — 69	0	8	3
21 ¼,	— — 49	3	7	8	42 ¼,	— — 70	0	7	3
22 ¼,	— — 50	3	2	5	43 ¼,	— — 71	0	6	4
23 ¼,	— — 51	2	17	2	44 ¼,	— — 72	0	5	6
24 ¼,	— — 52	2	12	5	45 ¼,	— — 73	0	4	8
25 ¼,	— — 53	2	7	9	46 ¼,	— — 74	0	4	0
26 ¼,	— — 54	2	3	7	47 ¼,	— — 75	0	3	5
27 ¼,	— — 55	1	19	6					

TABLE

TABLE XV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 28 ¾ to 29 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
6 ¼,	or at Age 35	10	9	5	27 ¼,	or at Age 56	1	17	9
7 ¼,	— — 36	9	15	7	28 ¼,	— — 57	1	14	3
8 ¼,	— — 37	9	2	6	29 ¼,	— — 58	1	10	11
9 ¼,	— — 38	8	9	6	30 ¼,	— — 59	1	8	2
10 ¼,	— — 39	7	17	9	31 ¼,	— — 60	1	5	5
11 ¼,	— — 40	7	6	1	32 ¼,	— — 61	1	2	11
12 ¼,	— — 41	6	15	6	33 ¼,	— — 62	1	0	6
13 ¼,	— — 42	6	4	11	34 ¼,	— — 63	0	18	6
14 ¼,	— — 43	5	15	11	35 ¼,	— — 64	0	16	6
15 ¼,	— — 44	5	7	0	36 ¼,	— — 65	0	14	8
16 ¼,	— — 45	4	18	11	37 ¼,	— — 66	0	12	10
17 ¼,	— — 46	4	10	10	38 ¼,	— — 67	0	11	4
18 ¼,	— — 47	4	4	0	39 ¼,	— — 68	0	9	11
19 ¼,	— — 48	3	17	3	40 ¼,	— — 69	0	8	9
20 ¼,	— — 49	3	11	4	41 ¼,	— — 70	0	7	7
21 ¼,	— — 50	3	5	6	42 ¼,	— — 71	0	6	8
22 ¼,	— — 51	3	0	4	43 ¼,	— — 72	0	5	10
23 ¼,	— — 52	2	15	2	44 ¼,	— — 73	0	5	0
24 ¼,	— — 53	2	10	4	45 ¼,	— — 74	0	4	2
25 ¼,	— — 54	2	5	7	46 ¼,	— — 75	0	3	7
26 ¼,	— — 55	2	1	8					

TABLE

TABLE XVI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 29 $\frac{1}{4}$ to 30 $\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
5 $\frac{1}{4}$, or at Age	35	11	0	3	26 $\frac{1}{4}$, or at Age	56	1	19	11
6 $\frac{1}{4}$, — —	36	10	6	1	27 $\frac{1}{4}$, — —	57	1	16	0
7 $\frac{1}{4}$, — —	37	9	11	11	28 $\frac{1}{4}$, — —	58	1	12	9
8 $\frac{1}{4}$, — —	38	8	18	11	29 $\frac{1}{4}$, — —	59	1	9	7
9 $\frac{1}{4}$, — —	39	8	5	11	30 $\frac{1}{4}$, — —	60	1	6	10
10 $\frac{1}{4}$, — —	40	7	14	2	31 $\frac{1}{4}$, — —	61	1	4	1
11 $\frac{1}{4}$, — —	41	7	2	5	32 $\frac{1}{4}$, — —	62	1	1	9
12 $\frac{1}{4}$, — —	42	6	12	2	33 $\frac{1}{4}$, — —	63	0	19	4
13 $\frac{1}{4}$, — —	43	6	1	11	34 $\frac{1}{4}$, — —	64	0	17	4
14 $\frac{1}{4}$, — —	44	5	12	11	35 $\frac{1}{4}$, — —	65	0	15	4
15 $\frac{1}{4}$, — —	45	5	4	0	36 $\frac{1}{4}$, — —	66	0	13	8
16 $\frac{1}{4}$, — —	46	4	16	2	37 $\frac{1}{4}$, — —	67	0	11	11
17 $\frac{1}{4}$, — —	47	4	8	4	38 $\frac{1}{4}$, — —	68	0	10	7
18 $\frac{1}{4}$, — —	48	4	1	8	39 $\frac{1}{4}$, — —	69	0	9	2
19 $\frac{1}{4}$, — —	49	3	15	0	40 $\frac{1}{4}$, — —	70	0	8	1
20 $\frac{1}{4}$, — —	50	3	9	2	41 $\frac{1}{4}$, — —	71	0	7	0
21 $\frac{1}{4}$, — —	51	3	3	4	42 $\frac{1}{4}$, — —	72	0	6	1
22 $\frac{1}{4}$, — —	52	2	18	2	43 $\frac{1}{4}$, — —	73	0	5	3
23 $\frac{1}{4}$, — —	53	2	12	11	44 $\frac{1}{4}$, — —	74	0	4	6
24 $\frac{1}{4}$, — —	54	2	8	4	45 $\frac{1}{4}$, — —	75	0	3	9
25 $\frac{1}{4}$, — —	55	2	3	10					

TABLE XVII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 30¾ to 31½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d
4¼,	or at Age 35	11	12	9	25¼,	or at Age 56	2	2	0
5¼,	— — 36	10	17	4	26¼,	— — 57	1	18	1
6¼,	— — 37	10	2	10	27¼,	— — 58	1	14	3
7¼,	— — 38	9	8	4	28¼,	— — 59	1	11	3
8¼,	— — 39	8	15	4	29¼,	— — 60	1	8	3
9¼,	— — 40	8	2	4	30¼,	— — 61	1	5	6
10¼,	— — 41	7	10	7	31¼,	— — 62	1	2	10
11¼,	— — 42	6	18	10	32¼,	— — 63	1	0	7
12¼,	— — 43	6	8	10	33¼,	— — 64	0	18	3
13¼,	— — 44	5	18	11	34¼,	— — 65	0	16	3
14¼,	— — 45	5	9	11	35¼,	— — 66	0	14	3
15¼,	— — 46	5	1	0	36¼,	— — 67	0	12	8
16¼,	— — 47	4	13	5	37¼,	— — 68	0	11	0
17¼,	— — 48	4	5	11	38¼,	— — 69	0	9	9
18¼,	— — 49	3	19	4	39¼,	— — 70	0	8	6
19¼,	— — 50	3	12	10	40¼,	— — 71	0	7	5
20¼,	— — 51	3	7	0	41¼,	— — 72	0	6	5
21¼,	— — 52	3	1	3	42¼,	— — 73	0	5	7
22¼,	— — 53	2	16	0	43¼,	— — 74	0	4	8
23¼,	— — 54	2	10	8	44¼,	— — 75	0	4	0
24¼,	— — 55	2	6	4					

TABLE XVIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 31 $\frac{1}{4}$ to 32 $\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
3 $\frac{1}{4}$,	or at Age 35	12	5	1	24 $\frac{1}{4}$,	or at Age 56	2	4	5
4 $\frac{1}{4}$,	— — 36	11	9	4	25 $\frac{1}{4}$,	— — 57	2	0	1
5 $\frac{1}{4}$,	— — 37	10	13	7	26 $\frac{1}{4}$,	— — 58	1	16	6
6 $\frac{1}{4}$,	— — 38	9	19	1	27 $\frac{1}{4}$,	— — 59	1	12	11
7 $\frac{1}{4}$,	— — 39	9	4	7	28 $\frac{1}{4}$,	— — 60	1	9	10
8 $\frac{1}{4}$,	— — 40	8	11	7	29 $\frac{1}{4}$,	— — 61	1	6	10
9 $\frac{1}{4}$,	— — 41	7	18	6	30 $\frac{1}{4}$,	— — 62	1	4	2
10 $\frac{1}{4}$,	— — 42	7	7	1	31 $\frac{1}{4}$,	— — 63	1	1	7
11 $\frac{1}{4}$,	— — 43	6	15	8	32 $\frac{1}{4}$,	— — 64	0	19	4
12 $\frac{1}{4}$,	— — 44	6	5	8	33 $\frac{1}{4}$,	— — 65	0	17	1
13 $\frac{1}{4}$,	— — 45	5	15	8	34 $\frac{1}{4}$,	— — 66	0	15	2
14 $\frac{1}{4}$,	— — 46	5	7	0	35 $\frac{1}{4}$,	— — 67	0	13	3
15 $\frac{1}{4}$,	— — 47	4	18	4	36 $\frac{1}{4}$,	— — 68	0	11	9
16 $\frac{1}{4}$,	— — 48	4	10	11	37 $\frac{1}{4}$,	— — 69	0	10	2
17 $\frac{1}{4}$,	— — 49	4	3	6	38 $\frac{1}{4}$,	— — 70	0	9	0
18 $\frac{1}{4}$,	— — 50	3	17	0	39 $\frac{1}{4}$,	— — 71	0	7	10
19 $\frac{1}{4}$,	— — 51	3	10	6	40 $\frac{1}{4}$,	— — 72	0	6	10
20 $\frac{1}{4}$,	— — 52	3	4	8	41 $\frac{1}{4}$,	— — 73	0	5	10
21 $\frac{1}{4}$,	— — 53	2	18	10	42 $\frac{1}{4}$,	— — 74	0	5	0
22 $\frac{1}{4}$,	— — 54	2	13	9	43 $\frac{1}{4}$,	— — 75	0	4	3
23 $\frac{1}{4}$,	— — 55	2	8	8					

TABLE

TABLE XIX.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 32 ¾ to 33 ¼.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.
2 ¼,	or at Age 35	12	19	3	23 ¼,	or at Age 56	2	6	9
3 ¼,	— — 36	12	2	1	24 ¼,	— — 57	2	2	6
4 ¼,	— — 37	11	5	11	25 ¼,	— — 58	1	18	2
5 ¼,	— — 38	10	9	10	26 ¼,	— — 59	1	14	10
6 ¼,	— — 39	9	15	4	27 ¼,	— — 60	1	11	6
7 ¼,	— — 40	9	0	11	28 ¼,	— — 61	1	8	5
8 ¼,	— — 41	8	7	9	29 ¼,	— — 62	1	5	4
9 ¼,	— — 42	7	14	8	30 ¼,	— — 63	1	2	10
10 ¼,	— — 43	7	3	7	31 ¼,	— — 64	1	0	4
11 ¼,	— — 44	6	12	6	32 ¼,	— — 65	0	18	2
12 ¼,	— — 45	6	2	6	33 ¼,	— — 66	0	15	11
13 ¼,	— — 46	5	12	6	34 ¼,	— — 67	0	14	1
14 ¼,	— — 47	5	4	1	35 ¼,	— — 68	0	12	3
15 ¼,	— — 48	4	15	8	36 ¼,	— — 69	0	10	10
16 ¼,	— — 49	4	8	4	37 ¼,	— — 70	0	9	5
17 ¼,	— — 50	4	1	1	38 ¼,	— — 71	0	8	3
18 ¼,	— — 51	3	14	8	39 ¼,	— — 72	0	7	2
19 ¼,	— — 52	3	8	3	40 ¼,	— — 73	0	6	2
20 ¼,	— — 53	3	2	4	41 ¼,	— — 74	0	5	2
21 ¼,	— — 54	2	16	6	42 ¼,	— — 75	0	4	6
22 ¼,	— — 55	2	11	7					

TABLE XX.

For the Use of LONDON.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 33 ¾ to 34 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
1 ¼,	or at Age 35	13	13	4	22 ¼,	or at Age 56	2	9	6
2 ¼,	— — 36	12	15	10	23 ¼,	— — 57	2	4	8
3 ¼,	— — 37	11	18	2	24 ¼,	— — 58	2	0	8
4 ¼,	— — 38	11	2	0	25 ¼,	— — 59	1	16	8
5 ¼,	— — 39	10	5	11	26 ½,	— — 60	1	13	3
6 ¼,	— — 40	9	11	4	27 ¼,	— — 61	1	9	11
7 ¼,	— — 41	8	16	9	28 ¼,	— — 62	1	7	0
8 ¼,	— — 42	8	4	0	29 ¼,	— — 63	1	4	0
9 ¼,	— — 43	7	11	3	30 ¼,	— — 64	1	1	6
10 ¼,	— — 44	7	0	2	31 ¼,	— — 65	0	19	0
11 ¼,	— — 45	6	9	0	32 ¼,	— — 66	0	17	0
12 ¼,	— — 46	5	19	4	33 ¼,	— — 67	0	14	10
13 ¼,	— — 47	5	9	8	34 ¼,	— — 68	0	13	1
14 ¼,	— — 48	5	1	4	35 ¼,	— — 69	0	11	4
15 ¼,	— — 49	4	13	1	36 ¼,	— — 70	0	10	0
16 ¼,	— — 50	4	5	10	37 ¼,	— — 71	0	8	8
17 ¼,	— — 51	3	18	7	38 ¼,	— — 72	0	7	7
18 ¼,	— — 52	3	12	2	39 ¼,	— — 73	0	6	6
19 ¼,	— — 53	3	5	8	40 ¼,	— — 74	0	5	7
20 ¼,	— — 54	3	0	0	41 ¼,	— — 75	0	4	9
21 ¼,	— — 55	2	14	4					

TABLE

TABLE XXI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $34\frac{3}{4}$ to $35\frac{1}{4}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 35	14	9	6	$21\frac{1}{4}$,	or at Age 56	2	12	2
$1\frac{1}{4}$,	— — 36	13	10	4	$22\frac{1}{4}$,	— — 57	2	7	5
$2\frac{1}{4}$,	— — 37	12	12	4	$23\frac{1}{4}$,	— — 58	2	2	8
$3\frac{1}{4}$,	— — 38	11	14	3	$24\frac{1}{4}$,	— — 59	1	18	11
$4\frac{1}{4}$,	— — 39	10	18	2	$25\frac{1}{4}$,	— — 60	1	15	2
$5\frac{1}{4}$,	— — 40	10	2	0	$26\frac{1}{4}$,	— — 61	1	11	9
$6\frac{1}{4}$,	— — 41	9	7	4	$27\frac{1}{4}$,	— — 62	1	8	4
$7\frac{1}{4}$,	— — 42	8	12	8	$28\frac{1}{4}$,	— — 63	1	5	7
$8\frac{1}{4}$,	— — 43	8	0	3	$29\frac{1}{4}$,	— — 64	1	2	9
$9\frac{1}{4}$,	— — 44	7	7	11	$30\frac{1}{4}$,	— — 65	1	0	3
$10\frac{1}{4}$,	— — 45	6	16	9	$31\frac{1}{4}$,	— — 66	0	17	9
$11\frac{1}{4}$,	— — 46	6	5	7	$32\frac{1}{4}$,	— — 67	0	15	9
$12\frac{1}{4}$,	— — 47	5	16	2	$33\frac{1}{4}$,	— — 68	0	13	8
$13\frac{1}{4}$,	— — 48	5	6	10	$34\frac{1}{4}$,	— — 69	0	12	1
$14\frac{1}{4}$,	— — 49	4	18	8	$35\frac{1}{4}$,	— — 70	0	10	6
$15\frac{1}{4}$,	— — 50	4	10	7	$36\frac{1}{4}$,	— — 71	0	9	3
$16\frac{1}{4}$,	— — 51	4	3	4	$37\frac{1}{4}$,	— — 72	0	8	0
$17\frac{1}{4}$,	— — 52	3	16	2	$38\frac{1}{4}$,	— — 73	0	6	11
$18\frac{1}{4}$,	— — 53	3	9	8	$39\frac{1}{4}$,	— — 74	0	5	10
$19\frac{1}{4}$,	— — 54	3	3	1	$40\frac{1}{4}$,	— — 75	0	5	0
$20\frac{1}{4}$,	— — 55	2	17	8					

TABLE XXII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 35 ½ to 36 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 36	14	5	6	20 ¼,	or at Age 56	2	15	4
1 ¼,	— — 37	13	6	4	21 ¼,	— — 57	2	10	0
2 ¼,	— — 38	12	8	3	22 ¼,	— — 58	2	5	6
3 ¼,	— — 39	11	10	2	23 ¼,	— — 59	2	1	0
4 ¼,	— — 40	10	14	0	24 ¼,	— — 60	1	17	2
5 ¼,	— — 41	9	17	8	25 ¼,	— — 61	1	13	5
6 ¼,	— — 42	9	3	5	26 ¼,	— — 62	1	10	2
7 ¼,	— — 43	8	9	2	27 ¼,	— — 63	1	6	11
8 ¼,	— — 44	7	16	8	28 ¼,	— — 64	1	4	1
9 ¼,	— — 45	7	4	3	29 ¼,	— — 65	1	1	4
10 ¼,	— — 46	6	13	5	30 ¼,	— — 66	0	18	11
11 ¼,	— — 47	6	2	7	31 ¼,	— — 67	0	16	6
12 ¼,	— — 48	5	13	4	32 ¼,	— — 68	0	14	8
13 ¼,	— — 49	5	4	1	33 ¼,	— — 69	0	12	9
14 ¼,	— — 50	4	16	0	34 ¼,	— — 70	0	11	2
15 ¼,	— — 51	4	8	0	35 ¼,	— — 71	0	9	8
16 ¼,	— — 52	4	0	8	36 ¼,	— — 72	0	8	6
17 ¼,	— — 53	3	13	5	37 ¼,	— — 73	0	7	3
18 ¼,	— — 54	3	7	1	38 ¼,	— — 74	0	6	3
19 ¼,	— — 55	3	0	9	39 ¼,	— — 75	0	5	3

TABLE XXIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 36 ¾ to [illegible] ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 37	14	1	6	20¼,	or at Age 57	2	13	1
1¼,	— — 38	13	2	5	21¼,	— — 58	2	7	9
2¼,	— — 39	12	4	3	22¼,	— — 59	2	3	7
3¼,	— — 40	11	6	2	23¼,	— — 60	1	19	4
4¼,	— — 41	10	9	9	24¼,	— — 61	1	15	6
5¼,	— — 42	9	13	4	25¼,	— — 62	1	11	8
6¼,	— — 43	8	19	6	26¼,	— — 63	1	8	7
7¼,	— — 44	8	5	8	27¼,	— — 64	1	5	6
8¼,	— — 45	7	13	2	28¼,	— — 65	1	2	8
9¼,	— — 46	7	0	8	29¼,	— — 66	0	19	11
10¼,	— — 47	6	10	1	30¼,	— — 67	0	17	7
11¼,	— — 48	5	19	7	31¼,	— — 68	0	15	4
12¼,	— — 49	5	10	6	32¼,	— — 69	0	13	7
13¼,	— — 50	5	1	4	33¼,	— — 70	0	11	10
14¼,	— — 51	4	13	4	34¼,	— — 71	0	10	4
15¼,	— — 52	4	5	4	35¼,	— — 72	0	8	11
16¼,	— — 53	3	18	0	36¼,	— — 73	0	7	9
17¼,	— — 54	3	10	8	37¼,	— — 74	0	6	6
18¼,	— — 55	3	4	6	38¼,	— — 75	0	5	7
19¼,	— — 56	2	18	5					

TABLE XXIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 37 ½ to 38 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age	38	13	17	6	19 ¼, or at Age	57	2	16	2
1 ¼, — —	39	12	18	9	20 ¼, — —	58	2	11	1
2 ¼, — —	40	12	0	6	21 ¼, — —	59	2	6	1
3 ¼, — —	41	11	2	2	22 ¼, — —	60	2	1	10
4 ¼, — —	42	10	6	1	23 ¼, — —	61	1	17	7
5 ¼, — —	43	9	10	1	24 ¼, — —	62	1	13	10
6 ¼, — —	44	8	16	1	25 ¼, — —	63	1	10	2
7 ¼, — —	45	8	2	2	26 ¼, — —	64	1	7	1
8 ¼, — —	46	7	10	0	27 ¼, — —	65	1	4	0
9 ¼, — —	47	6	17	9	28 ¼, — —	66	1	1	3
10 ¼, — —	48	6	7	4	29 ¼, — —	67	0	18	7
11 ¼, — —	49	5	17	0	30 ¼, — —	68	0	16	5
12 ¼, — —	50	5	7	11	31 ¼, — —	69	0	14	3
13 ¼, — —	51	4	18	9	32 ¼, — —	70	0	12	7
14 ¼, — —	52	4	10	8	33 ¼, — —	71	0	11	0
15 ¼, — —	53	4	2	6	34 ¼, — —	72	0	9	6
16 ¼, — —	54	3	15	5	35 ¼, — —	73	0	8	1
17 ¼, — —	55	3	8	3	36 ¼, — —	74	0	7	0
18 ¼, — —	56	3	2	2	37 ¼, — —	75	0	5	11

TABLE

TABLE XXV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 38 ¼ to 39 ¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 39	13	13	6	19 ¼,	or at Age 58	2	13	11
1 ¼,	— — 40	12	15	2	20 ¼,	— — 59	2	9	2
2 ¼,	— — 41	11	16	8	21 ¼,	— — 60	2	4	5
3 ¼,	— — 42	10	18	2	22 ¼,	— — 61	2	0	1
4 ¼,	— — 43	10	2	6	23 ¼,	— — 62	1	15	10
5 ¼,	— — 44	9	6	11	24 ¼,	— — 63	1	12	3
6 ¼,	— — 45	8	12	9	25 ¼,	— — 64	1	8	9
7 ¼,	— — 46	7	18	8	26 ¼,	— — 65	1	5	7
8 ¼,	— — 47	7	6	10	27 ¼,	— — 66	1	2	5
9 ¼,	— — 48	6	14	11	28 ¼,	— — 67	0	19	10
10 ¼,	— — 49	6	4	8	29 ¼,	— — 68	0	17	3
11 ¼,	— — 50	5	14	4	30 ¼,	— — 69	0	15	3
12 ¼,	— — 51	5	5	4	31 ¼,	— — 70	0	13	3
13 ¼,	— — 52	4	16	3	32 ¼,	— — 71	0	11	8
14 ¼,	— — 53	4	8	0	33 ¼,	— — 72	0	10	1
15 ¼,	— — 54	3	19	8	34 ¼,	— — 73	0	8	8
16 ¼,	— — 55	3	12	10	35 ¼,	— — 74	0	7	4
17 ¼,	— — 56	3	5	11	36 ¼,	— — 75	0	6	3
18 ¼,	— — 57	2	19	11					

TABLE XXVI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 39 ¾ to 40 ½.

To commence at the end of — Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of — Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 40	13	11	6	18 ¼,	or at Age 58	2	17	9
1 ¼,	— — 41	12	11	1	19 ¼,	— — 59	2	12	1
2 ¼,	— — 42	11	13	0	20 ¼,	— — 60	2	7	3
3 ¼,	— — 43	10	14	10	21 ¼,	— — 61	2	2	6
4 ¼,	— — 44	9	19	1	22 ¼,	— — 62	1	18	4
5 ¼,	— — 45	9	3	3	23 ¼,	— — 63	1	14	2
6 ¼,	— — 46	8	9	6	24 ¼,	— — 64	1	10	7
7 ¼,	— — 47	7	15	9	25 ¼,	— — 65	1	7	1
8 ¼,	— — 48	7	4	0	26 ¼,	— — 66	1	4	0
9 ¼,	— — 49	6	12	3	27 ¼,	— — 67	1	1	0
10 ¼,	— — 50	6	2	0	28 ¼,	— — 68	0	18	7
11 ¼,	— — 51	5	11	8	29 ¼,	— — 69	0	16	2
12 ¼,	— — 52	5	2	6	30 ¼,	— — 70	0	14	3
13 ¼,	— — 53	4	13	3	31 ¼,	— — 71	0	12	4
14 ¼,	— — 54	4	5	3	32 ¼,	— — 72	0	10	9
15 ¼,	— — 55	3	17	2	33 ¼,	— — 73	0	9	2
16 ¼,	— — 56	3	10	4	34 ¼,	— — 74	0	7	10
17 ¼,	— — 57	3	3	5	35 ¼,	— — 75	0	6	8

TABLE

TABLE XXVII.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 40 ¾ to 41 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 41	13	7	6	18 ¼,	or at Age 59	2	15	8
1 ¼,	— — 42	12	7	1	19 ¼,	— — 60	2	10	3
2 ¼,	— — 43	11	9	4	20 ¼,	— — 61	2	5	5
3 ¼,	— — 44	10	11	7	21 ¼,	— — 62	2	0	7
4 ¼,	— — 45	9	15	8	22 ¼,	— — 63	1	16	7
5 ¼,	— — 46	8	19	8	23 ¼,	— — 64	1	12	7
6 ¼,	— — 47	8	6	3	24 ¼,	— — 65	1	9	0
7 ¼,	— — 48	7	12	10	25 ¼,	— — 66	1	5	5
8 ¼,	— — 49	7	1	2	26 ¼,	— — 67	1	2	6
9 ¼,	— — 50	6	9	7	27 ¼,	— — 68	0	19	7
10 ¼,	— — 51	5	19	4	28 ¼,	— — 69	0	17	4
11 ¼,	— — 52	5	9	1	29 ¼,	— — 70	0	15	1
12 ¼,	— — 53	4	19	8	30 ¼,	— — 71	0	13	3
13 ¼,	— — 54	4	10	3	31 ¼,	— — 72	0	11	5
14 ¼,	— — 55	4	2	6	32 ¼,	— — 73	0	9	11
15 ¼,	— — 56	3	14	8	33 ¼,	— — 74	0	8	4
16 ¼,	— — 57	3	7	10	34 ¼,	— — 75	0	7	1
17 ¼,	— — 58	3	1	0					

TABLE XXVIII.

For the Use of LONDON.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 41 ¾ to 42 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 42	13	3	6	17¼,	or at Age 59	2	19	3
1¼,	— — 43	12	4	6	18¼,	— — 60	2	13	9
2¼,	— — 44	11	6	6	19¼,	— — 61	2	8	3
3¼,	— — 45	10	8	6	20¼,	— — 62	2	3	7
4¼,	— — 46	9	12	10	21¼,	— — 63	1	18	10
5¼,	— — 47	8	17	2	22¼,	— — 64	1	14	10
6¼,	— — 48	8	3	10	23¼,	— — 65	1	10	9
7¼,	— — 49	7	10	5	24¼,	— — 66	1	7	4
8¼,	— — 50	6	18	9	25¼,	— — 67	1	3	11
9¼,	— — 51	6	7	1	26¼,	— — 68	1	1	2
10¼,	— — 52	5	16	7	27¼,	— — 69	0	18	4
11¼,	— — 53	5	6	1	28¼,	— — 70	0	16	3
12¼,	— — 54	4	17	0	29¼,	— — 71	0	14	1
13¼,	— — 55	4	7	9	30¼,	— — 72	0	12	3
14¼,	— — 56	4	0	0	31¼,	— — 73	0	10	5
15¼,	— — 57	3	12	3	32¼,	— — 74	0	9	0
16¼,	— — 58	3	5	9	33¼,	— — 75	0	7	7

TABLE XXIX.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $42\frac{3}{4}$ to $43\frac{1}{2}$.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$, or at Age 43	12	19	6
$1\frac{1}{4}$, — — 44	12	2	0
$2\frac{1}{4}$, — — 45	11	3	8
$3\frac{1}{4}$, — — 46	10	5	5
$4\frac{1}{4}$, — — 47	9	10	1
$5\frac{1}{4}$, — — 48	8	14	8
$6\frac{1}{4}$, — — 49	8	1	5
$7\frac{1}{4}$, — — 50	7	8	1
$8\frac{1}{4}$, — — 51	6	16	4
$9\frac{1}{4}$, — — 52	6	4	8
$10\frac{1}{4}$, — — 53	5	13	11
$11\frac{1}{4}$, — — 54	5	3	2
$12\frac{1}{4}$, — — 55	4	14	3
$13\frac{1}{4}$, — — 56	4	5	4
$14\frac{1}{4}$, — — 57	3	17	7
$15\frac{1}{4}$, — — 58	3	9	10
$16\frac{1}{4}$, — — 59	3	3	8
$17\frac{1}{4}$, or at Age 60	2	17	6
$18\frac{1}{4}$, — — 61	2	11	11
$19\frac{1}{4}$, — — 62	2	6	4
$20\frac{1}{4}$, — — 63	2	1	9
$21\frac{1}{4}$, — — 64	1	17	2
$22\frac{1}{4}$, — — 65	1	13	1
$23\frac{1}{4}$, — — 66	1	9	0
$24\frac{1}{4}$, — — 67	1	5	9
$25\frac{1}{4}$, — — 68	1	2	5
$26\frac{1}{4}$, — — 69	0	19	10
$27\frac{1}{4}$, — — 70	0	17	2
$28\frac{1}{4}$, — — 71	0	15	2
$29\frac{1}{4}$, — — 72	0	13	1
$30\frac{1}{4}$, — — 73	0	11	3
$31\frac{1}{4}$, — — 74	0	9	6
$32\frac{1}{4}$, — — 75	0	8	2

TABLE

TABLE XXX.

For the Ufe of LONDON.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 43 ¾ to 44 ½.

To commence at the end of Years after purchafing.	Value of an annuity of £1. in one prefent payment. £.	s.	d.	To commence at the end of Years after purchafing.	Value of an annuity of £1. in one prefent payment. £.	s.	d.
¼, or at Age 44	12	17	0	16¼, or at Age 60	3	1	4
1¼, — — 45	11	18	0	17¼, — — 61	2	15	1
2¼, — — 46	11	0	0	18¼, — — 62	2	9	9
3¼, — — 47	10	2	2	19¼, — — 63	2	4	4
4¼, — — 48	9	6	11	20¼, — — 64	1	19	8
5¼, — — 49	8	11	7	21¼, — — 65	1	15	1
6¼, — — 50	7	18	4	22¼, — — 66	1	11	2
7¼, — — 51	7	5	0	23¼, — — 67	1	7	3
8¼, — — 52	6	13	0	24¼, — — 68	1	4	2
9¼, — — 53	6	1	1	25¼, — — 69	1	1	0
10¼, — — 54	5	10	7	26¼, — — 70	0	18	6
11¼, — — 55	5	0	2	27¼, — — 71	0	16	0
12¼, — — 56	4	11	3	28¼, — — 72	0	14	0
13¼, — — 57	4	2	4	29¼, — — 73	0	12	0
14¼, — — 58	3	15	0	30¼, — — 74	0	10	3
15¼, — — 59	3	7	8	31¼, — — 75	0	8	8

TABLE XXXI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 44 ¾ to 45 ¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 45	12	13	6	16 ¼,	or at Age 61	2	19	1
1 ¼,	— — 46	11	13	11	17 ¼,	— — 62	2	12	9
2 ¼,	— — 47	10	16	5	18 ¼,	— — 63	2	7	7
3 ¼,	— — 48	9	18	11	19 ¼,	— — 64	2	2	4
4 ¼,	— — 49	9	3	9	20 ¼,	— — 65	1	17	8
5 ¼,	— — 50	8	8	7	21 ¼,	— — 66	1	13	0
6 ¼,	— — 51	7	15	3	22 ¼,	— — 67	1	9	3
7 ¼,	— — 52	7	1	11	23 ¼,	— — 68	1	5	6
8 ¼,	— — 53	6	9	8	24 ¼,	— — 69	1	2	7
9 ¼,	— — 54	5	17	6	25 ¼,	— — 70	0	19	7
10 ¼,	— — 55	5	7	4	26 ¼,	— — 71	0	17	3
11 ¼,	— — 56	4	17	2	27 ¼,	— — 72	0	14	11
12 ¼,	— — 57	4	8	3	28 ¼,	— — 73	0	12	11
13 ¼,	— — 58	3	19	5	29 ¼,	— — 74	0	10	11
14 ¼,	— — 59	3	12	5	30 ¼,	— — 75	0	9	3
15 ¼,	— — 60	3	5	6					

TABLE XXXII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 45¾ to 46¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 46	12	9	6	15¼,	or at Age 61	3	2	11
1¼,	— — 47	11	10	9	16¼,	— — 62	2	16	9
2¼,	— — 48	10	13	4	17¼,	— — 63	2	10	7
3¼,	— — 49	9	15	11	18¼,	— — 64	2	5	4
4¼,	— — 50	9	0	8	19¼,	— — 65	2	0	1
5¼,	— — 51	8	5	6	20¼,	— — 66	1	15	7
6¼,	— — 52	7	11	10	21¼,	— — 67	1	11	1
7¼,	— — 53	6	18	2	22¼,	— — 68	1	7	6
8¼,	— — 54	6	6	3	23¼,	— — 69	1	4	0
9¼,	— — 55	5	14	4	24¼,	— — 70	1	1	2
10¼,	— — 56	5	4	2	25¼,	— — 71	0	18	3
11¼,	— — 57	4	14	0	26¼,	— — 72	0	16	0
12¼,	— — 58	4	5	7	27¼,	— — 73	0	13	8
13¼,	— — 59	3	17	2	28¼,	— — 74	0	11	9
14¼,	— — 60	3	10	0	29¼,	— — 75	0	9	11

TABLE XXXIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 46¾ to 47¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 47	12	5	6	15 ¼,	or at Age 62	3	0	4
1 ¼,	— — 48	11	7	7	16 ¼,	— — 63	2	14	5
2 ¼,	— — 49	10	10	3	17 ¼,	— — 64	2	8	6
3 ¼,	— — 50	9	12	11	18 ¼,	— — 65	2	3	2
4 ¼,	— — 51	8	17	8	19 ¼,	— — 66	1	17	10
5 ¼,	— — 52	8	2	5	20 ¼,	— — 67	1	13	6
6 ¼,	— — 53	7	8	5	21 ¼,	— — 68	1	9	2
7 ¼,	— — 54	6	14	5	22 ¼,	— — 69	1	5	10
8 ¼,	— — 55	6	2	10	23 ¼,	— — 70	1	2	5
9 ¼,	— — 56	5	11	2	24 ¼,	— — 71	0	19	9
10 ¼,	— — 57	5	1	0	25 ¼,	— — 72	0	17	0
11 ¼,	— — 58	4	10	11	26 ¼,	— — 73	0	14	8
12 ¼,	— — 59	4	2	11	27 ¼,	— — 74	0	12	5
13 ¼,	— — 60	3	14	11	28 ¼,	— — 75	0	10	7
14 ¼,	— — 61	3	7	8					

TABLE XXXIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 47¾ to 48¼.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 48	12	3	6
1 ¼,	—— 49	11	4	2
2 ¼,	—— 50	10	6	9
3 ¼,	—— 51	9	9	5
4 ¼,	—— 52	8	13	9
5 ¼,	—— 53	7	18	1
6 ¼,	—— 54	7	4	6
7 ¼,	—— 55	6	10	10
8 ¼,	—— 56	5	19	3
9 ¼,	—— 57	5	7	7
10 ¼,	—— 58	4	18	0
11 ¼,	—— 59	4	8	4
12 ¼,	—— 60	4	0	2
13 ¼,	—— 61	3	12	0
14 ¼,	—— 62	3	5	0
15 ¼,	—— 63	2	17	11
16 ¼,	—— 64	2	11	10
17 ¼,	—— 65	2	5	10
18 ¼,	—— 66	2	0	9
19 ¼,	—— 67	1	15	7
20 ¼,	—— 68	1	11	6
21 ¼,	—— 69	1	7	5
22 ¼,	—— 70	1	4	2
23 ¼,	—— 71	1	1	0
24 ¼,	—— 72	0	18	3
25 ¼,	—— 73	0	15	7
26 ¼,	—— 74	0	13	5
27 ¼,	—— 75	0	11	4

TABLE XXXV.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchaſer from 48¾ to 49¼.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 49	11	19	6
1 ¼,	—— 50	11	0	10
2 ¼,	—— 51	10	3	4
3 ¼,	—— 52	9	5	11
4 ¼,	—— 53	8	9	10
5 ¼,	—— 54	7	13	10
6 ¼,	—— 55	7	0	7
7 ¼,	—— 56	6	7	3
8 ¼,	—— 57	5	15	8
9 ¼,	—— 58	5	4	0
10 ¼,	—— 59	4	14	11
11 ¼,	—— 60	4	5	9
12 ¼,	—— 61	3	17	5
13 ¼,	—— 62	3	9	1
14 ¼,	—— 63	3	2	3
15 ¼,	—— 64	2	15	6
16 ¼,	—— 65	2	9	4
17 ¼,	—— 66	2	3	3
18 ¼,	—— 67	1	18	4
19 ¼,	—— 68	1	13	5
20 ¼,	—— 69	1	9	6
21 ¼,	—— 70	1	5	8
22 ¼,	—— 71	1	2	7
23 ¼,	—— 72	0	19	6
24 ¼,	—— 73	0	16	10
25 ¼,	—— 74	0	14	2
26 ¼,	—— 75	0	12	2

TABLE XXXVI.

For the Use of London.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 49½ to 50½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age	50	11	15	6
1 ¼, ——	51	10	16	9
2 ¼, ——	52	9	18	10
3 ¼, ——	53	9	1	0
4 ¼, ——	54	8	5	4
5 ¼, ——	55	7	9	8
6 ¼, ——	56	6	16	5
7 ¼, ——	57	6	3	1
8 ¼, ——	58	5	12	1
9 ¼, ——	59	5	1	0
10 ¼, ——	60	4	11	9
11 ¼, ——	61	4	2	4
12 ¼, ——	62	3	14	4
13 ¼, ——	63	3	6	3
14 ¼, ——	64	2	19	4
15 ¼, ——	65	2	12	6
16 ¼, ——	66	2	6	7
17 ¼, ——	67	2	0	9
18 ¼, ——	68	1	16	1
19 ¼, ——	69	1	11	4
20 ¼, ——	70	1	7	8
21 ¼, ——	71	1	4	0
22 ¼, ——	72	1	0	11
23 ¼, ——	73	0	17	10
24 ¼, ——	74	0	15	4
25 ¼, ——	75	0	13	0

TABLE XXXVII.

For the Use of London.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 50½ to 51½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age	51	11	11	6
1 ¼, ——	52	10	12	8
2 ¼, ——	53	9	14	4
3 ¼, ——	54	8	16	0
4 ¼, ——	55	8	0	10
5 ¼, ——	56	7	5	7
6 ¼, ——	57	6	12	4
7 ¼, ——	58	5	19	0
8 ¼, ——	59	5	8	7
9 ¼, ——	60	4	18	1
10 ¼, ——	61	4	8	7
11 ¼, ——	62	3	19	0
12 ¼, ——	63	3	11	3
13 ¼, ——	64	3	3	6
14 ¼, ——	65	2	16	6
15 ¼, ——	66	2	9	6
16 ¼, ——	67	2	3	11
17 ¼, ——	68	1	18	3
18 ¼, ——	69	1	13	10
19 ¼, ——	70	1	9	4
20 ¼, ——	71	1	5	10
21 ¼, ——	72	1	2	3
20 ¼, ——	73	0	19	3
23 ¼, ——	74	0	16	3
24 ¼, ——	75	0	13	11

TABLE XXXVIII.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 51½ to 52½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 52	11	7	6
1¼,	—— 53	10	7	8
2¼,	—— 54	9	9	9
3¼,	—— 55	8	11	9
4¼,	—— 56	7	16	7
5¼,	—— 57	7	1	3
6¼,	—— 58	6	8	8
7¼,	—— 59	5	16	0
8¼,	—— 60	5	5	3
9¼,	—— 61	4	14	6
10¼,	—— 62	4	5	3
11¼,	—— 63	3	16	0
12¼,	—— 64	3	8	1
13¼,	—— 65	3	0	3
14¼,	—— 66	2	13	6
15¼,	—— 67	2	6	9
16¼,	—— 68	2	1	5
17¼,	—— 69	1	16	0
18¼,	—— 70	1	11	9
19¼,	—— 71	1	7	6
20¼,	—— 72	1	4	0
21¼,	—— 73	1	0	6
22¼,	—— 74	0	17	7
23¼,	—— 75	0	14	5

TABLE XXXIX.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 52½ to 53½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 53	11	1	6
1¼,	—— 54	10	2	8
2¼,	—— 55	9	5	2
3¼,	—— 56	8	7	7
4¼,	—— 57	7	12	4
5¼,	—— 58	6	17	0
6¼,	—— 59	6	5	0
7¼,	—— 60	5	12	11
8¼,	—— 61	5	2	0
9¼,	—— 62	4	11	0
10¼,	—— 63	4	2	0
11¼,	—— 64	3	13	1
12¼,	—— 65	3	5	0
13¼,	—— 66	2	17	0
14¼,	—— 67	2	10	6
15¼,	—— 68	2	4	0
16¼,	—— 69	1	18	11
17¼,	—— 70	1	13	10
18¼,	—— 71	1	9	9
19¼,	—— 72	1	5	8
20¼,	—— 73	1	2	2
21¼,	—— 74	0	18	9
22¼,	—— 75	0	16	0

TABLE XL.

For the Use of London.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 53¾ to 54¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 54	10	17	6
1 ¼,	— 55	9	18	6
2 ¼,	— 56	9	0	11
3 ¼,	— 57	8	3	3
4 ¼,	— 58	7	8	7
5 ¼,	— 59	6	14	0
6 ¼,	— 60	6	1	7
7 ¼,	— 61	5	9	2
8 ¼,	— 62	4	18	7
9 ¼,	— 63	4	7	10
10 ¼,	— 64	3	18	8
11 ¼,	— 65	3	9	7
12 ¼,	— 66	3	1	9
13 ¼,	— 67	2	14	0
14 ¼,	— 68	2	7	9
15 ¼,	— 69	2	1	7
16 ¼,	— 70	1	16	8
17 ¼,	— 71	1	11	9
18 ¼,	— 72	1	7	9
19 ¼,	— 73	1	3	8
20 ¼,	— 74	1	0	4
21 ¼,	— 75	0	17	3

TABLE XLI.

For the Use of London.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 54¾ to 55¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 55	10	13	6
1 ¼,	— 56	9	14	5
2 ¼,	— 57	8	16	8
3 ¼,	— 58	7	18	11
4 ¼,	— 59	7	4	11
5 ¼,	— 60	6	10	11
6 ¼,	— 61	5	18	2
7 ¼,	— 62	5	5	6
8 ¼,	— 63	4	15	2
9 ¼,	— 64	4	4	9
10 ¼,	— 65	3	15	5
11 ¼,	— 66	3	6	1
12 ¼,	— 67	2	18	7
13 ¼,	— 68	2	11	0
14 ¼,	— 69	2	5	1
15 ¼,	— 70	1	19	2
16 ¼,	— 71	1	14	6
17 ¼,	— 72	1	9	9
18 ¼,	— 73	1	5	9
19 ¼,	— 74	1	1	8
20 ¼,	— 75	0	18	7

TABLE XLII.

For the Use of London.

Shewing the Payment due (reckoning Intereſt at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 55¼ to 56¼.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	56	10	9	6
1 ¼,	——	57	9	10	2
2 ¼,	——	58	8	13	1
3 ¼,	——	59	7	16	0
4 ¼,	——	60	7	1	7
5 ¼,	——	61	6	7	2
6 ¼,	——	62	5	14	8
7 ¼,	——	63	5	2	4
8 ¼,	——	64	4	11	8
9 ¼,	——	65	4	1	0
10 ¼,	——	66	3	12	0
11 ¼,	——	67	3	2	10
12 ¼,	——	68	2	15	8
13 ¼,	——	69	2	8	5
14 ¼,	——	70	2	2	8
15 ¼,	——	71	1	17	0
16 ¼,	——	72	1	12	3
17 ¼,	——	73	1	7	7
18 ¼,	——	74	1	3	8
19 ¼,	——	75	1	0	1

TABLE XLIII.

For the Uſe of London.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75,

Age of the Purchaſer from 56¼ to 57¼.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d
¼,	or at Age	57	10	5	6
1 ¼,	——	58	9	5	11
2 ¼,	——	59	8	9	6
3 ¼,	——	60	7	13	2
4 ¼,	——	61	6	18	3
5 ¼,	——	62	6	3	5
6 ¼,	——	63	5	11	3
7 ¼,	——	64	4	19	2
8 ¼,	——	65	4	8	3
9 ¼,	——	66	3	17	4
10 ¼,	——	67	3	8	6
11 ¼,	——	68	2	19	8
12 ¼,	——	69	2	12	10
13 ¼,	——	70	2	5	11
14 ¼,	——	71	2	0	4
15 ¼,	——	72	1	14	10
16 ¼,	——	73	1	10	1
17 ¼,	——	74	1	5	5
18 ¼,	——	75	1	1	8

TABLE

TABLE XLIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 57½ to 58½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
½,	or at Age 58	9	19	6
1½,	—— 59	9	1	9
2½,	—— 60	8	5	0
3½,	—— 61	7	8	2
4½,	—— 62	6	13	8
5½,	—— 63	5	19	2
6½,	—— 64	5	6	9
7½,	—— 65	4	14	5
8½,	—— 66	4	3	10
9½,	—— 67	3	13	3
10½,	—— 68	3	4	10
11½,	—— 69	2	16	5
12½,	—— 70	2	9	9
13½,	—— 71	2	3	1
14½,	—— 72	1	17	7
15½,	—— 73	1	12	2
16½,	—— 74	1	7	7
17½,	—— 75	1	3	5

TABLE XLV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 58½ to 59½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
½,	or at Age 59	9	15	6
1½,	—— 60	8	17	8
2½,	—— 61	8	0	5
3½,	—— 62	7	3	2
4½,	—— 63	6	9	1
5½,	—— 64	5	15	0
6½,	—— 65	5	2	4
7½,	—— 66	4	9	8
8½,	—— 67	3	19	6
9½,	—— 68	3	9	3
10½,	—— 69	3	1	2
11½,	—— 70	2	13	2
12½,	—— 71	2	6	9
13½,	—— 72	2	0	4
14½,	—— 73	1	14	11
15½,	—— 74	1	9	6
16½,	—— 75	1	5	2

TABLE

TABLE XLVI.

For the Ufe of LONDON.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 59¾ to 60¾.

To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d.
¼,	or at Age 60	9	11	6
1 ¼,	—— 61	8	12	7
2 ¼,	—— 62	7	15	9
3 ¼,	—— 63	6	18	10
4 ¼,	—— 64	6	4	5
5 ¼,	—— 65	5	10	0
6 ¼,	—— 66	4	17	8
7 ¼,	—— 67	4	5	4
8 ¼,	—— 68	3	15	7
9 ¼,	—— 69	3	5	9
10 ¼,	—— 70	2	18	0
11 ¼,	—— 71	2	10	2
12 ¼,	—— 72	2	3	10
13 ¼,	—— 73	1	17	5
14 ¼,	—— 74	1	12	1
15 ¼,	—— 75	1	7	0

TABLE XLVII.

For the Ufe of LONDON.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 60¾ to 61¾.

To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d.
¼,	or at Age 61	9	5	6
1 ¼,	—— 62	8	7	7
2 ¼,	—— 63	7	11	1
3 ¼,	—— 64	6	14	7
4 ¼,	—— 65	5	19	10
5 ¼,	—— 66	5	5	0
6 ¼,	—— 67	4	13	0
7 ¼,	—— 68	4	1	1
8 ¼,	—— 69	3	11	8
9 ¼,	—— 70	3	2	3
10 ¼,	—— 71	2	14	9
11 ¼,	—— 72	2	7	3
12 ¼,	—— 73	2	0	11
13 ¼,	—— 74	1	14	6
14 ¼,	—— 75	1	9	3

TABLE

TABLE XLVIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 61¼ to 62¼.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	62	9	1	6
1 ¼,	—	63	8	3	4
2 ¼,	—	64	7	6	4
3 ¼,	—	65	6	9	4
4 ¼,	—	66	5	14	11
5 ¼,	—	67	5	0	5
6 ¼,	—	68	4	8	11
7 ¼,	—	69	3	17	4
8 ¼,	—	70	3	8	3
9 ¼,	—	71	2	19	1
10 ¼,	—	72	2	11	6
11 ¼,	—	73	2	4	0
12 ¼,	—	74	1	17	10
13 ¼,	—	75	1	12	0

TABLE XLIX.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 62¼ to 63¼.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	63	8	17	6
1 ¼,	—	64	7	19	2
2 ¼,	—	65	7	1	8
3 ¼,	—	66	6	4	2
4 ¼,	—	67	5	10	0
5 ¼,	—	68	4	15	11
6 ¼,	—	69	4	4	10
7 ¼,	—	70	3	13	8
8 ¼,	—	71	3	4	10
9 ¼,	—	72	2	15	11
10 ¼,	—	73	2	8	4
11 ¼,	—	74	2	0	10
12 ¼,	—	75	1	14	10

TABLE L.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 63¾ to 64¾.

To commence at the end of Years after purchasing			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	64	8	13	6
1 ¼,	——	65	7	14	0
2 ¼,	——	66	6	16	9
3 ¼,	——	67	5	19	6
4 ¼,	——	68	5	5	9
5 ¼,	——	69	4	12	1
6 ¼,	——	70	4	1	3
7 ¼,	——	71	3	10	4
8 ¼,	——	72	3	1	4
9 ¼,	——	73	2	12	5
10 ¼,	——	74	2	5	0
11 ¼,	——	75	1	18	0

TABLE LI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75,

Age of the Purchaser from 64¾ to 65¾.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	65	8	7	6
1 ¼,	——	66	7	8	11
2 ¼,	——	67	6	11	11
3 ¼,	——	68	5	14	11
4 ¼,	——	69	5	1	7
5 ¼,	——	70	4	8	4
6 ¼,	——	71	3	17	8
7 ¼,	——	72	3	7	0
8 ¼,	——	73	2	17	11
9 ¼,	——	74	2	8	11
10 ¼,	——	75	2	1	9

TABLE LII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 65½ to 66½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
½,	or at Age 66	8	3	6
1½,	—— 67	7	4	6
2½,	—— 68	6	7	11
3½,	—— 69	5	11	3
4½,	—— 70	4	18	1
5½,	—— 71	4	5	0
6½,	—— 72	3	14	2
7½,	—— 73	3	3	3
8½,	—— 74	2	14	5
9½,	—— 75	2	6	0

TABLE LIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 66½ to 67½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
½,	or at Age 67	7	19	6
1½,	—— 68	7	0	2
2½,	—— 69	6	3	11
3½,	—— 70	5	7	8
4½,	—— 71	4	14	8
5½,	—— 72	4	1	8
6½,	—— 73	3	10	8
7½,	—— 74	2	19	7
8½,	—— 75	2	10	11

TABLE LIV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 67¾ to 68½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	68	7	15	6
1 ¼,	——	69	6	16	1
2 ¼,	——	70	6	0	0
3 ¼,	——	71	5	3	11
4 ¼,	——	72	4	10	7
5 ¼,	——	73	3	17	4
6 ¼,	——	74	3	6	6
7 ¼,	——	75	2	16	0

TABLE LV.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 68¾ to 69½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	69	7	9	6
1 ¼,	——	70	6	12	0
2 ¼,	——	71	5	16	0
3 ¼,	——	72	5	0	2
4 ¼,	——	73	4	6	7
5 ¼,	——	74	3	13	1
6 ¼,	——	75	3	2	4

TABLE

TABLE LVI.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 69¾ to 70½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	70	7	5	6
1 $\frac{1}{4}$,	——	71	6	7	7
2 $\frac{1}{4}$,	——	72	5	11	3
3 $\frac{1}{4}$,	——	73	4	15	0
4 $\frac{1}{4}$,	——	74	4	1	8
5 $\frac{1}{4}$,	——	75	3	9	0

TABLE LVII.

For the Use of LONDON.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 70¾ to 71½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	71	7	1	6
1 $\frac{1}{4}$,	——	72	6	3	3
2 $\frac{1}{4}$,	——	73	5	6	7
3 $\frac{1}{4}$,	——	74	4	9	11
4 $\frac{1}{4}$,	——	75	3	16	9

TABLE LVIII.

For the Use of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $71\frac{3}{4}$ to $72\frac{3}{4}$.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age	72	6	17	3
$1\frac{1}{4}$, ——	73	5	18	1
$2\frac{1}{4}$, ——	74	5	1	9
$3\frac{1}{4}$, ——	75	4	6	6

TABLE LIX.

For the Uſe of LONDON.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from $72\frac{3}{4}$ to $73\frac{3}{4}$.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
$\frac{1}{4}$, or at Age	73	6	11	6
$1\frac{1}{4}$, ——	74	5	13	7
$2\frac{1}{4}$, ——	75	4	16	11

SECOND

SECOND SET OF TABLES.

Intended for the Use of COUNTRY PARISHES.

TABLE I.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $14\frac{3}{4}$ to $15\frac{1}{4}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$20\frac{1}{4}$,	or at Age 35	6	12	8
$21\frac{1}{4}$,	— — 36	6	4	10
$22\frac{1}{4}$,	— — 37	5	17	8
$23\frac{1}{4}$,	— — 38	5	10	7
$24\frac{1}{4}$,	— — 39	5	4	1
$25\frac{1}{4}$,	— — 40	4	17	7
$26\frac{1}{4}$,	— — 41	4	11	7
$27\frac{1}{4}$,	— — 42	4	5	7
$28\frac{1}{4}$,	— — 43	4	0	1
$29\frac{1}{4}$,	— — 44	3	14	7
$30\frac{1}{4}$,	— — 45	3	9	8
$31\frac{1}{4}$,	— — 46	3	4	9
$32\frac{1}{4}$,	— — 47	3	0	4
$33\frac{1}{4}$,	— — 48	2	15	11
$34\frac{1}{4}$,	— — 49	2	11	11
$35\frac{1}{4}$,	— — 50	2	7	11
$36\frac{1}{4}$,	— — 51	2	4	4
$37\frac{1}{4}$,	— — 52	2	0	8
$38\frac{1}{4}$,	— — 53	1	17	7
$39\frac{1}{4}$,	— — 54	1	14	6
$40\frac{1}{4}$,	— — 55	1	11	9
$41\frac{1}{4}$,	or at Age 56	1	9	0
$42\frac{1}{4}$,	— — 57	1	6	7
$43\frac{1}{4}$,	— — 58	1	4	2
$44\frac{1}{4}$,	— — 59	1	2	1
$45\frac{1}{4}$,	— — 60	1	0	0
$46\frac{1}{4}$,	— — 61	0	18	1
$47\frac{1}{4}$,	— — 62	0	16	2
$48\frac{1}{4}$,	— — 63	0	14	7
$49\frac{1}{4}$,	— — 64	0	13	0
$50\frac{1}{4}$,	— — 65	0	11	8
$51\frac{1}{4}$,	— — 66	0	10	3
$52\frac{1}{4}$,	— — 67	0	9	2
$53\frac{1}{4}$,	— — 68	0	8	0
$54\frac{1}{4}$,	— — 69	0	7	0
$55\frac{1}{4}$,	— — 70	0	6	0
$56\frac{1}{4}$,	— — 71	0	5	2
$57\frac{1}{4}$,	— — 72	0	4	5
$58\frac{1}{4}$,	— — 73	0	3	9
$59\frac{1}{4}$,	— — 74	0	3	1
$60\frac{1}{4}$,	— — 75	0	2	6

TABLE II.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 15¾ to 16½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
19¼,	or at Age 35	6	17	5	40¼,	or at Age 56	1	10	3
20¼,	— — 36	6	9	10	41¼,	— — 57	1	7	7
21¼,	— — 37	6	2	2	42¼,	— — 58	1	5	3
22¼,	— — 38	5	15	1	43¼,	— — 59	1	2	11
23¼,	— — 39	5	8	1	44¼,	— — 60	1	0	9
24¼,	— — 40	5	1	6	45¼,	— — 61	0	18	9
25¼,	— — 41	4	15	0	46¼,	— — 62	0	16	11
26¼,	— — 42	4	9	0	47¼,	— — 63	0	15	2
27¼,	— — 43	4	3	1	48¼,	— — 64	0	13	8
28¼,	— — 44	3	17	8	49¼,	— — 65	0	12	1
29¼,	— — 45	3	12	3	50¼,	— — 66	0	10	10
30¼,	— — 46	3	7	6	51¼,	— — 67	0	9	4
31¼,	— — 47	3	2	8	52¼,	— — 68	0	8	4
32¼,	— — 48	2	18	3	53¼,	— — 69	0	7	3
33¼,	— — 49	2	13	10	54¼,	— — 70	0	6	3
34¼,	— — 50	2	9	11	55¼,	— — 71	0	5	5
35¼,	— — 51	2	6	0	56¼,	— — 72	0	4	7
36¼,	— — 52	2	2	5	57¼,	— — 73	0	3	10
37¼,	— — 53	1	19	0	58¼,	— — 74	0	3	3
38¼,	— — 54	1	10	0	59¼,	— — 75	0	2	8
39¼,	— — 55	1	12	11					

TABLE

TABLE III.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 16 ¾ to 17 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
18 ¼,	or at Age 35	7	3	4	39 ¼,	or at Age 56	1	11	4
19 ¼,	— — 36	6	14	10	40 ¼,	— — 57	1	8	9
20 ¼,	— — 37	6	7	1	41 ¼,	— — 58	1	6	2
21 ¼,	— — 38	5	19	5	42 ¼,	— — 59	1	3	11
22 ¼,	— — 39	5	12	6	43 ¼,	— — 60	1	1	7
23 ¼,	— — 40	5	5	7	44 ¼,	— — 61	0	19	7
24 ¼,	— — 41	4	19	0	45 ¼,	— — 62	0	17	7
25 ¼,	— — 42	4	12	5	46 ¼,	— — 63	0	15	10
26 ¼,	— — 43	4	6	6	47 ¼,	— — 64	0	14	1
27 ¼,	— — 44	4	0	7	48 ¼,	— — 65	0	12	8
28 ¼,	— — 45	3	15	4	49 ¼,	— — 66	0	11	2
29 ¼,	— — 46	3	10	0	50 ¼,	— — 67	0	9	11
30 ¼,	— — 47	3	5	3	51 ¼,	— — 68	0	8	7
31 ¼,	— — 48	3	0	6	52 ¼,	— — 69	0	7	7
32 ¼,	— — 49	2	16	1	53 ¼,	— — 70	0	6	6
33 ¼,	— — 50	2	11	9	54 ¼,	— — 71	0	5	7
34 ¼,	— — 51	2	7	11	55 ¼,	— — 72	0	4	9
35 ¼,	— — 52	2	4	0	56 ¼,	— — 73	0	4	0
36 ¼,	— — 53	2	0	8	57 ¼,	— — 74	0	3	3
37 ¼,	— — 54	1	17	4	58 ¼,	— — 75	0	2	8
38 ¼,	— — 55	1	14	4					

TABLE IV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 17¾ to 18½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
17¼,	or at Age 35	7	8	11	38¼,	or at Age 56	1	12	9
18¼,	— — 36	7	0	6	39¼,	— — 57	1	9	10
19¼,	— — 37	6	12	1	40¼,	— — 58	1	7	4
20¼,	— — 38	6	4	6	41¼,	— — 59	1	4	9
21¼,	— — 39	5	16	10	42¼,	— — 60	1	2	7
22¼,	— — 40	5	9	10	43¼,	— — 61	1	0	3
23¼,	— — 41	5	2	9	44¼,	— — 62	0	18	4
24¼,	— — 42	4	16	4	45¼,	— — 63	0	16	4
25¼,	— — 43	4	9	10	46¼,	— — 64	0	14	8
26¼,	— — 44	4	4	1	47¼,	— — 65	0	13	0
27¼,	— — 45	3	18	2	48¼,	— — 66	0	11	8
28¼,	— — 46	3	12	11	49¼,	— — 67	0	10	3
29¼,	— — 47	3	7	8	50¼,	— — 68	0	9	0
30¼,	— — 48	3	3	0	51¼,	— — 69	0	7	9
31¼,	— — 49	2	18	3	52¼,	— — 70	0	6	10
32¼,	— — 50	2	13	11	53¼,	— — 71	0	5	9
33¼,	— — 51	2	9	8	54¼,	— — 72	0	4	11
34¼,	— — 52	2	6	0	55¼,	— — 73	0	4	2
35¼,	— — 53	2	2	2	56¼,	— — 74	0	3	5
36¼,	— — 54	1	18	11	57¼,	— — 75	0	2	9
37¼,	— — 55	1	15	8					

TABLE

TABLE V.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 18 ¼ to 19 ¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
16 ¼,	or at Age 35	7	15	3	37 ¼,	or at Age 56	1	14	0
17 ¼,	— — 36	7	6	1	38 ¼,	— — 57	1	11	2
18 ¼,	— — 37	6	17	9	39 ¼,	— — 58	1	8	4
19 ¼,	— — 38	6	9	5	40 ¼,	— — 59	1	5	10
20 ¼,	— — 39	6	1	11	41 ¼,	— — 60	1	3	4
21 ¼,	— — 40	5	14	4	42 ¼,	— — 61	1	1	2
22 ¼,	— — 41	5	7	2	43 ¼,	— — 62	0	19	0
23 ¼,	— — 42	5	0	0	44 ¼,	— — 63	0	17	1
24 ¼,	— — 43	4	13	8	45 ¼,	— — 64	0	15	2
25 ¼,	— — 44	4	7	4	46 ¼,	— — 65	0	13	7
26 ¼,	— — 45	4	1	7	47 ¼,	— — 66	0	12	0
27 ¼,	— — 46	3	15	10	48 ¼,	— — 67	0	10	8
28 ¼,	— — 47	3	10	7	49 ¼,	— — 68	0	9	4
29 ¼,	— — 48	3	5	5	50 ¼,	— — 69	0	8	2
30 ¼,	— — 49	3	0	9	51 ¼,	— — 70	0	7	0
31 ¼,	— — 50	2	16	1	52 ¼,	— — 71	0	6	1
32 ¼,	— — 51	2	11	10	53 ¼,	— — 72	0	5	1
33 ¼,	— — 52	2	7	7	54 ¼,	— — 73	0	4	4
34 ¼,	— — 53	2	4	0	55 ¼,	— — 74	0	3	7
35 ¼,	— — 54	2	0	5	56 ¼,	— — 75	0	2	11
36 ¼,	— — 55	1	17	2					

TABLE VI.

For the Use of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 19¾ to 20¾.

To commence at the end of Years after pur-chaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after pur-chaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
15¼,	or at Age 35	8	2	0	36¼,	or at Age 56	1	15	7
16¼,	— — 36	7	12	10	37¼,	— — 57	1	12	6
17¼,	— — 37	7		9	38¼,	— — 58	1	9	9
18¼,	— — 38	6	15	5	39¼,	— — 59	1	6	11
19¼,	— — 39	6	7	2	40¼,	— — 60	1	4	6
20¼,	— — 40	5	19	6	41¼,	— — 61	1	2	1
21¼,	— — 41	5	11	10	42¼,	— — 62	0	19	11
22¼,	— — 4?	5	4	9	43¼,	— — 63	0	17	10
23¼,	— — 43	4	17	8	44¼,	— — 64	0	16	0
24¼,	— — 44	4	11	4	45¼,	— — 65	0	14	2
25¼,	— — 45	4	5	1	46¼,	— — 66	0	12	7
26¼,	— — 46	3	19	4	47¼,	— — 67	0	11	1
27¼,	— — 47	3	13	8	48¼,	— — 68	0	9	9
28¼,	— — 48	3	8	5	49¼,	— — 69	0	8	5
29¼,	— — 49	3	3	3	50¼,	— — 70	0	7	4
30¼,	— — 50	2	18	8	51¼,	— — 71	0	6	
31¼,	— — 51	2	14	0	52¼,	— — 72	0	5	
32¼,	— — 52	2	9	11	53¼,	— — 73	0	4	[illegible]
33¼,	— — 53	2	5	10	54¼,	— — 74	0	3	9
34¼,	— — 54	2	2	4	55¼,	— — 75	0	3	0
35¼,	— — 55	1	18	9					

TABLE

TABLE VII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 20¾ to 21½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
14¼, or at Age 35	8	9	7	35¼, or at Age 56	1	17	2
15¼, — — 36	7	19	7	36¼, — — 57	1	14	0
16¼, — — 37	7	10	6	37¼, — — 58	1	11	0
17¼, — — 38	7	1	5	38¼, — — 59	1	8	3
18¼, — — 39	6	13	2	39¼, — — 60	1	5	7
19¼, — — 40	6	4	11	40¼, — — 61	1	3	2
20¼, — — 41	5	17	2	41¼, — — 62	1	0	10
21¼, — — 42	5	9	4	42¼, — — 63	0	18	9
22¼, — — 43	5	2	4	43¼, — — 64	0	16	9
23¼, — — 44	4	15	5	44¼, — — 65	0	15	0
24¼, — — 45	4	9	1	45¼, — — 66	0	13	2
25¼, — — 46	4	2	10	46¼, — — 67	0	11	8
26¼, — — 47	3	17	2	47¼, — — 68	0	10	2
27¼, — — 48	3	11	7	48¼, — — 69	0	8	11
28¼, — — 49	3	6	4	49¼, — — 70	0	7	7
29¼, — — 50	3	1	2	50¼, — — 71	0	6	7
30¼, — — 51	2	16	7	51¼, — — 72	0	5	7
31¼, — — 52	2	12	0	52¼, — — 73	0	4	9
32¼, — — 53	2	8	1	53¼, — — 74	0	3	11
33¼, — — 54	2	4	2	54¼, — — 75	0	3	2
34¼, — — 55	2	0	8				

TABLE

TABLE VIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 21¾ to 22½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
13¼, or at Age 35	8	17	2	34¼, or at Age 56	1	18	11
14¼, — — 36	8	7	2	35¼, — — 57	1	15	6
15¼, — — 37	7	17	2	36¼, — — 58	1	12	6
16¼, — — 38	7	8	2	37¼, — — 59	1	9	6
17¼, — — 39	6	19	1	38¼, — — 60	1	6	9
18¼, — — 40	6	10	9	39¼, — — 61	1	4	2
19¼, — — 41	6	2	4	40¼, — — 62	1	1	10
20¼, — — 42	5	14	7	41¼, — — 63	0	19	7
21¼, — — 43	5	6	10	42¼, — — 64	0	17	7
22¼, — — 44	4	19	11	43¼, — — 65	0	15	7
23¼, — — 45	4	13	0	44¼, — — 66	0	13	11
24¼, — — 46	4	6	10	45¼, — — 67	0	12	2
25¼, — — 47	4	0	7	46¼, — — 68	0	10	9
26¼, — — 48	3	14	11	47¼, — — 69	0	9	3
27¼, — — 49	3	9	4	48¼, — — 70	0	8	1
28¼, — — 50	3	4	2	49¼, — — 71	0	6	11
29¼, — — 51	2	19	1	50¼, — — 72	0	5	10
30¼, — — 52	2	14	8	51¼, — — 73	0	4	11
31¼, — — 53	2	10	3	52¼, — — 74	0	4	1
32¼, — — 54	2	6	3	53¼, — — 75	0	3	4
33¼, — — 55	2	2	5				

TABLE

TABLE IX.

For the Use of Country Parishes

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 22¾ to 23½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
12¼,	or at Age 35	9	5	10	33¼,	or at Age 56	2	0	8
13¼,	— — 36	8	14	10	34¼,	— — 57	1	17	3
14¼,	— — 37	8	4	10	35¼,	— — 58	1	13	11
15¼,	— — 38	7	14	10	36¼,	— — 59	1	11	0
16¼,	— — 39	7	5	10	37¼,	— — 60	1	8	0
17¼,	— — 40	6	16	10	38¼,	— — 61	1	5	5
18¼,	— — 41	6	8	4	39¼,	— — 62	1	2	10
19¼,	— — 42	5	19	10	40¼,	— — 63	1	0	7
20¼,	— — 43	5	12	1	41¼,	— — 64	0	18	4
21¼,	— — 44	5	4	5	42¼,	— — 65	0	16	5
22¼,	— — 45	4	17	7	43¼,	— — 66	0	14	5
23¼,	— — 46	4	10	8	44¼,	— — 67	0	12	10
24¼,	— — 47	4	4	7	45¼,	— — 68	0	11	2
25¼,	— — 48	3	18	5	46¼,	— — 69	0	9	10
26¼,	— — 49	3	12	9	47¼,	— — 70	0	8	5
27¼,	— — 50	3	7	1	48¼,	— — 71	0	7	3
28¼,	— — 51	3	2	0	49¼,	— — 72	0	6	2
29¼,	— — 52	2	17	0	50¼,	— — 73	0	5	2
30¼,	— — 53	2	12	8	51¼,	— — 74	0	4	3
31¼,	— — 54	2	8	5	52¼,	— — 75	0	3	6
32¼,	— — 55	2	4	6					

TABLE X.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 23¾ to 24½.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.
11 ¼,	or at Age 35	9	14	3	32 ¼,	or at Age 56	2	2	10
12 ¼,	— — 36	9	3	3	33 ¼,	— — 57	1	18	11
13 ¼,	— — 37	8	12	4	34 ¼,	— — 58	1	15	8
14 ¼,	— — 38	8	2	4	35 ¼,	— — 59	1	12	6
15 ¼,	— — 39	7	12	5	36 ¼,	— — 60	1	9	6
16 ¼,	— — 40	7	3	3	37 ¼,	— — 61	1	6	6
17 ¼,	— — 41	6	14	1	38 ¼,	— — 62	1	4	0
18 ¼,	— — 42	6	5	8	39 ¼,	— — 63	1	1	7
19 ¼,	— — 43	5	17	2	40 ¼,	— — 64	0	19	4
20 ¼,	— — 44	5	9	6	41 ¼,	— — 65	0	17	1
21 ¼,	— — 45	5	1	11	42 ¼,	— — 66	0	15	3
22 ¼,	— — 46	4	15	3	43 ¼,	— — 67	0	13	6
23 ¼,	— — 47	4	8	7	44 ¼,	— — 68	0	11	10
24 ¼,	— — 48	4	2	3	45 ¼,	— — 69	0	10	2
25 ¼,	— — 49	3	16	0	46 ¼,	— — 70	0	8	11
26 ¼,	— — 50	3	10	7	47 ¼,	— — 71	0	7	8
27 ¼,	— — 51	3	5	2	48 ¼,	— — 72	0	6	8
28 ¼,	— — 52	3	0	1	49 ¼,	— — 73	0	5	7
29 ¼,	— — 53	2	15	0	50 ¼,	— — 74	0	4	6
30 ¼,	— — 54	2	10	10	51 ¼,	— — 75	0	3	8
31 ¼,	— — 55	2	6	9					

TABLE XI.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 24$\frac{3}{4}$ to 25$\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
10$\frac{1}{4}$, or at Age	35	10	3	9	31$\frac{1}{4}$, or at Age	56	2	5	1
11$\frac{1}{4}$, — —	36	9	11	9	32$\frac{1}{4}$, — —	57	2	1	2
12$\frac{1}{4}$, — —	37	9	0	9	33$\frac{1}{4}$, — —	58	1	17	2
13$\frac{1}{4}$, — —	38	8	9	10	34$\frac{1}{4}$, — —	59	1	14	1
14$\frac{1}{4}$, — —	39	7	19	11	35$\frac{1}{4}$, — —	60	1	11	1
15$\frac{1}{4}$, — —	40	7	10	0	36$\frac{1}{4}$, — —	61	1	8	0
16$\frac{1}{4}$, — —	41	7	0	8	37$\frac{1}{4}$, — —	62	1	5	0
17$\frac{1}{4}$, — —	42	6	11	5	38$\frac{1}{4}$, — —	63	1	2	8
18$\frac{1}{4}$, — —	43	6	3	0	39$\frac{1}{4}$, — —	64	1	0	5
19$\frac{1}{4}$, — —	44	5	14	7	40$\frac{1}{4}$, — —	65	0	18	2
20$\frac{1}{4}$, — —	45	5	7	0	41$\frac{1}{4}$, — —	66	0	15	11
21$\frac{1}{4}$, — —	46	4	19	5	42$\frac{1}{4}$, — —	67	0	14	2
22$\frac{1}{4}$, — —	47	4	12	11	43$\frac{1}{4}$, — —	68	0	12	6
23$\frac{1}{4}$, — —	48	4	6	6	44$\frac{1}{4}$, — —	69	0	10	10
24$\frac{1}{4}$, — —	49	4	0	0	45$\frac{1}{4}$, — —	70	0	9	2
25$\frac{1}{4}$, — —	50	3	13	7	46$\frac{1}{4}$, — —	71	0	8	1
26$\frac{1}{4}$, — —	51	3	8	5	47$\frac{1}{4}$, — —	72	0	7	0
27$\frac{1}{4}$, — —	52	3	3	3	48$\frac{1}{4}$, — —	73	0	6	0
28$\frac{1}{4}$, — —	53	2	18	2	49$\frac{1}{4}$, — —	74	0	4	11
29$\frac{1}{4}$, — —	54	2	13	0	50$\frac{1}{4}$, — —	75	0	3	10
30$\frac{1}{4}$, — —	55	2	9	0					

TABLE XII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 25¾ to 26½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
9¼, or at Age 35	10	13	1	30¼, or at Age 56	2	7	1
10¼, — — 36	10	1	1	31¼, — — 57	2	3	2
11¼, — — 37	9	9	1	32¼, — — 58	1	19	3
12¼, — — 38	8	18	2	33¼, — — 59	1	15	4
13¼, — — 39	8	7	3	34¼, — — 60	1	12	4
14¼, — — 40	7	17	2	35¼, — — 61	1	9	5
15¼, — — 41	7	7	1	36¼, — — 62	1	6	5
16¼, — — 42	6	17	10	37¼, — — 63	1	3	6
17¼, — — 43	6	8	7	38¼, — — 64	1	1	3
18¼, — — 44	6	0	3	39¼, — — 65	0	19	0
19¼, — — 45	5	11	10	40¼, — — 66	0	16	10
20¼, — — 46	5	4	4	41¼, — — 67	0	14	8
21¼, — — 47	4	16	10	42¼, — — 68	0	13	0
22¼, — — 48	4	10	5	43¼, — — 69	0	11	5
23¼, — — 49	4	4	0	44¼, — — 70	0	9	10
24¼, — — 50	3	17	7	45¼, — — 71	0	8	3
25¼, — — 51	3	11	2	46¼, — — 72	0	7	2
26¼, — — 52	3	6	1	47¼, — — 73	0	6	1
27¼, — — 53	3	1	0	48¼, — — 74	0	5	0
28¼, — — 54	2	16	0	49¼, — — 75	0	4	0
29¼, — — 55	2	11	0				

TABLE

TABLE XIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 26 ½ to 27 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
8 ¼,	or at Age 35	11	3	7	29 ¼,	or at Age 56	2	9	0
9 ¼,	— — 36	10	10	5	30 ¼,	— — 57	2	5	2
10 ¼,	— — 37	9	18	6	31 ¼,	— — 58	2	1	4
11 ¼,	— — 38	9	6	6	32 ¼,	— — 59	1	17	5
12 ¼,	— — 39	8	15	8	33 ¼,	— — 60	1	13	7
13 ¼,	— — 40	8	4	9	34 ¼,	— — 61	1	10	8
14 ¼,	— — 41	7	14	6	35 ¼,	— — 62	1	7	10
15 ¼,	— — 42	7	4	3	36 ¼,	— — 63	1	4	11
16 ¼,	— — 43	6	15	0	37 ¼,	— — 64	1	2	0
17 ¼,	— — 44	6	5	10	38 ¼,	— — 65	0	19	10
18 ¼,	— — 45	5	17	6	39 ¼,	— — 66	0	17	8
19 ¼,	— — 46	5	9	2	40 ¼,	— — 67	0	15	7
20 ¼,	— — 47	5	1	9	41 ¼,	— — 68	0	13	5
21 ¼,	— — 48	4	14	4	42 ¼,	— — 69	0	11	11
22 ¼,	— — 49	4	7	11	43 ¼,	— — 70	0	10	5
23 ¼,	— — 50	4	1	6	44 ¼,	— — 71	0	8	11
24 ¼,	— — 51	3	15	1	45 ¼,	— — 72	0	7	5
25 ¼,	— — 52	3	8	8	46 ¼,	— — 73	0	6	4
26 ¼,	— — 53	3	3	9	47 ¼,	— — 74	0	5	3
27 ¼,	— — 54	2	18	10	48 ¼,	— — 75	0	4	2
28 ¼,	— — 55	2	13	11					

TABLE XIV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 27 ¼ to 28 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
7 ¼,	or at Age 35	11	14	1	28 ¼,	or at Age 56	2	11	9
8 ¼,	— — 36	11	0	10	29 ¼,	— — 57	2	7	2
9 ¼,	— — 37	10	7	8	30 ¼,	— — 58	2	3	2
10 ¼,	— — 38	9	15	9	31 ¼,	— — 59	1	19	2
11 ¼,	— — 39	9	3	10	32 ¼,	— — 60	1	15	7
12 ¼,	— — 40	8	12	9	33 ¼,	— — 61	1	12	1
13 ¼,	— — 41	8	1	8	34 ¼,	— — 62	1	9	0
14 ¼,	— — 42	7	11	5	35 ¼,	— — 63	1	6	0
15 ¼,	— — 43	7	1	2	36 ¼,	— — 64	1	3	4
16 ¼,	— — 44	6	12	0	37 ¼,	— — 65	1	0	9
17 ¼,	— — 45	6	2	11	38 ¼,	— — 66	0	18	6
18 ¼,	— — 46	5	14	8	39 ¼,	— — 67	0	16	3
19 ¼,	— — 47	5	6	5	40 ¼,	— — 68	0	14	4
20 ¼,	— — 48	4	19	0	41 ¼,	— — 69	0	12	7
21 ¼,	— — 49	4	11	7	42 ¼,	— — 70	0	10	10
22 ¼,	— — 50	4	5	0	43 ¼,	— — 71	0	9	3
23 ¼,	— — 51	3	18	6	44 ¼,	— — 72	0	7	11
24 ¼,	— — 52	3	12	5	45 ¼,	— — 73	0	6	8
25 ¼,	— — 53	3	6	4	46 ¼,	— — 74	0	5	6
26 ¼,	— — 54	3	1	4	47 ¼,	— — 75	0	4	5
27 ¼,	— — 55	2	16	4					

TABLE

TABLE XV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 28 ¾ to 29 ¼.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
6 ¼, or at Age 35	12	5	11	27 ¼, or at Age 56	2	13	10
7 ¼, — — 36	11	11	5	28 ¼, — — 57	2	9	7
8 ¼, — — 37	10	18	2	29 ¼, — — 58	2	5	5
9 ¼, — — 38	10	5	0	30 ¼, — — 59	2	1	2
10 ¼, — — 39	9	13	1	31 ¼, — — 60	1	17	0
11 ¼, — — 40	9	1	2	32 ¼, — — 61	1	13	10
12 ¼, — — 41	8	9	10	33 ¼, — — 62	1	10	7
13 ¼, — — 42	7	18	7	34 ¼, — — 63	1	7	5
14 ¼, — — 43	7	8	4	35 ¼, — — 64	1	4	2
15 ¼, — — 44	6	18	2	36 ¼, — — 65	1	1	10
16 ¼, — — 45	6	9	1	37 ¼, — — 66	0	19	6
17 ¼, — — 46	6	0	0	38 ¼, — — 67	0	17	2
18 ¼, — — 47	5	11	11	39 ¼, — — 68	0	14	10
19 ¼, — — 48	5	3	9	40 ¼, — — 69	0	13	1
20 ¼, — — 49	4	16	3	41 ¼, — — 70	0	11	6
21 ¼, — — 50	4	8	10	42 ¼, — — 71	0	9	9
22 ¼, — — 51	4	2	2	43 ¼, — — 72	0	8	1
23 ¼, — — 52	3	15	6	44 ¼, — — 73	0	6	11
24 ¼, — — 53	3	9	9	45 ¼, — — 74	0	5	10
25 ¼, — — 54	3	4	0	46 ¼, — — 75	0	4	8
26 ¼, — — 55	2	18	11				

TABLE

TABLE XVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 29 $\frac{3}{4}$ to 30 $\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
5 $\frac{1}{4}$,	or at Age 35	12	17	2	26 $\frac{1}{4}$,	or at Age 56	2	16	6
6 $\frac{1}{4}$,	— — 36	12	2	9	27 $\frac{1}{4}$,	— — 57	2	11	6
7 $\frac{1}{4}$,	— — 37	11	8	3	28 $\frac{1}{4}$,	— — 58	2	7	3
8 $\frac{1}{4}$,	— — 38	10	15	1	29 $\frac{1}{4}$,	— — 59	2	3	0
9 $\frac{1}{4}$,	— — 39	10	1	11	30 $\frac{1}{4}$,	— — 60	1	19	1
10 $\frac{1}{4}$,	— — 40	9	9	9	31 $\frac{1}{4}$,	— — 61	1	15	3
11 $\frac{1}{4}$,	— — 41	8	17	7	32 $\frac{1}{4}$,	— — 62	1	11	11
12 $\frac{1}{4}$,	— — 42	8	6	4	33 $\frac{1}{4}$,	— — 63	1	8	7
13 $\frac{1}{4}$,	— — 43	7	15	2	34 $\frac{1}{4}$,	— — 64	1	5	8
14 $\frac{1}{4}$,	— — 44	7	5	1	35 $\frac{1}{4}$,	— — 65	1	2	9
15 $\frac{1}{4}$,	— — 45	6	1[illegible]	0	36 $\frac{1}{4}$,	— — 66	1	0	4
16 $\frac{1}{4}$,	— — 46	6	5	11	37 $\frac{1}{4}$,	— — 67	0	17	10
17 $\frac{1}{4}$,	— — 47	5	16	11	38 $\frac{1}{4}$,	— — 68	0	15	9
18 $\frac{1}{4}$,	— — 48	5	8	9	39 $\frac{1}{4}$,	— — 69	0	13	8
19 $\frac{1}{4}$,	— — 49	5	0	7	40 $\frac{1}{4}$,	— — 70	0	11	10
20 $\frac{1}{4}$,	— — 50	4	13	3	41 $\frac{1}{4}$,	— — 71	0	10	2
21 $\frac{1}{4}$,	— — 51	4	5	10	42 $\frac{1}{4}$,	— — 72	0	8	8
22 $\frac{1}{4}$,	— — 52	3	1[illegible]	4	43 $\frac{1}{4}$,	— — 73	0	7	2
23 $\frac{1}{4}$,	— — 53	3	12	9	44 $\frac{1}{4}$,	— — 74	0	5	11
24 $\frac{1}{4}$,	— — 54	3	7	2	45 $\frac{1}{4}$,	— — 75	0	4	10
25 $\frac{1}{4}$,	— — 55	3	1	7					

TABLE XVII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 30¾ to 31½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
4¼,	or at Age 35	13	10	0	25¼,	or at Age 56	2	19	2
5¼,	— — 36	12	14	0	26¼,	— — 57	2	14	2
6¼,	— — 37	11	19	7	27¼,	— — 58	2	9	2
7¼,	— — 38	11	5	2	28¼,	— — 59	2	4	11
8¼,	— — 39	10	12	0	29¼,	— — 60	2	0	7
9¼,	— — 40	9	18	10	30¼,	— — 61	1	17	1
10¼,	— — 41	9	6	5	31¼,	— — 62	1	13	7
11¼,	— — 42	8	14	0	32¼,	— — 63	1	10	1
12¼,	— — 43	8	2	11	33¼,	— — 64	1	6	7
13¼,	— — 44	7	11	10	34¼,	— — 65	1	4	0
14¼,	— — 45	7	1	10	35¼,	— — 66	1	1	5
15¼,	— — 46	6	11	10	36¼,	— — 67	0	18	10
16¼,	— — 47	6	2	10	37¼,	— — 68	0	16	2
17¼,	— — 48	5	13	10	38¼,	— — 69	0	14	4
18¼,	— — 49	5	5	8	39¼,	— — 70	0	12	6
19¼,	— — 50	4	17	6	40¼,	— — 71	0	10	8
20¼,	— — 51	4	10	3	41¼,	— — 72	0	8	10
21¼,	— — 52	4	2	11	42¼,	— — 73	0	7	7
22¼,	— — 53	3	16	7	43¼,	— — 74	0	6	3
23¼,	— — 54	3	10	2	44¼,	— — 75	0	5	0
24¼,	— — 55	3	4	8					

TABLE

TABLE XVIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $31\frac{3}{4}$ to $32\frac{1}{4}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.
$3\frac{1}{4}$,	or at Age 35	14	2	3	$24\frac{1}{4}$,	or at Age 56	3	2	0
$4\frac{1}{4}$,	— — 36	13	6	4	$25\frac{1}{4}$,	— — 57	2	16	7
$5\frac{1}{4}$,	— — 37	12	10	5	$26\frac{1}{4}$,	— — 58	2	11	8
$6\frac{1}{4}$,	— — 38	11	16	0	$27\frac{1}{4}$,	— — 59	2	6	10
$7\frac{1}{4}$,	— — 39	11	1	7	$28\frac{1}{4}$,	— — 60	2	2	8
$8\frac{1}{4}$,	— — 40	10	8	3	$29\frac{1}{4}$,	— — 61	1	18	5
$9\frac{1}{4}$,	— — 41	9	14	11	$30\frac{1}{4}$,	— — 62	1	14	11
$10\frac{1}{4}$,	— — 42	9	2	7	$31\frac{1}{4}$,	— — 63	1	11	4
$11\frac{1}{4}$,	— — 43	8	10	3	$32\frac{1}{4}$,	— — 64	1	8	2
$12\frac{1}{4}$,	— — 44	7	19	3	$33\frac{1}{4}$,	— — 65	1	5	0
$13\frac{1}{4}$,	— — 45	7	8	2	$34\frac{1}{4}$,	— — 66	1	2	4
$14\frac{1}{4}$,	— — 46	6	18	3	$35\frac{1}{4}$,	— — 67	0	19	7
$15\frac{1}{4}$,	— — 47	6	8	4	$36\frac{1}{4}$,	— — 68	0	17	4
$16\frac{1}{4}$,	— — 48	5	19	4	$37\frac{1}{4}$,	— — 69	0	15	0
$17\frac{1}{4}$,	— — 49	5	10	5	$38\frac{1}{4}$,	— — 70	0	13	1
$18\frac{1}{4}$,	— — 50	5	2	3	$39\frac{1}{4}$,	— — 71	0	11	2
$19\frac{1}{4}$,	— — 51	4	14	2	$40\frac{1}{4}$,	— — 72	0	9	6
$20\frac{1}{4}$,	— — 52	4	7	1	$41\frac{1}{4}$,	— — 73	0	7	11
$21\frac{1}{4}$,	— — 53	3	19	11	$42\frac{1}{4}$,	— — 74	0	6	7
$22\frac{1}{4}$,	— — 54	3	13	9	$43\frac{1}{4}$,	— — 75	0	5	3
$23\frac{1}{4}$,	— — 55	3	7	6					

TABLE

TABLE XIX.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 32 ¼ to 33 ¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	.	d.
2 ¼,	or at Age 35	14	16	1
3 ¼,	— — 36	13	18	7
4 ¼,	— — 37	13	2	8
5 ¼,	— — 38	12	6	10
6 ¼,	— — 39	11	12	5
7 ¼,	— — 40	10	18	0
8 ¼,	— — 41	10	4	6
9 ¼,	— — 42	9	11	0
10 ¼,	— — 43	8	13	10
11 ¼,	— — 44	8	6	7
12 ¼,	— — 45	7	15	7
13 ¼,	— — 46	7	4	7
14 ¼,	— — 47	6	14	9
15 ¼,	— — 48	6	4	10
16 ¼,	— — 49	5	15	11
17 ¼,	— — 50	5	7	0
18 ¼,	— — 51	4	18	11
19 ¼,	— — 52	4	10	10
20 ¼,	— — 53	4	3	11
21 ¼,	— — 54	3	17	0
22 ¼,	— — 55	3	10	11

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
23 ¼,	or at Age 56	3	4	10
24 ¼,	— — 57	2	19	5
25 ¼,	— — 58	2	14	0
26 ¼,	— — 59	2	9	3
27 ¼,	— — 60	2	4	7
28 ¼,	— — 61	2	0	6
29 ¼,	— — 62	1	16	4
30 ¼,	— — 63	1	12	9
31 ¼,	— — 64	1	9	2
32 ¼,	— — 65	1	6	4
33 ¼,	— — 66	1	3	6
34 ¼,	— — 67	1	0	8
35 ¼,	— — 68	0	17	10
36 ¼,	— — 69	0	15	10
37 ¼,	— — 70	0	13	10
38 ¼,	— — 71	0	11	10
39 ¼,	— — 72	0	9	10
40 ¼,	— — 73	0	8	5
41 ¼,	— — 74	0	7	0
42 ¼,	— — 75	0	5	7

TABLE XX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 33 ¾ to 34 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
1 ¼,	or at Age 35	15	9	9	22 ¼,	or at Age 56	3	8	1
2 ¼,	— — 36	14	12	3	23 ¼,	— — 57	3	2	1
3 ¼,	— — 37	13	14	9	24 ¼,	— — 58	2	16	9
4 ¼,	— — 38	12	18	11	25 ¼,	— — 59	2	11	6
5 ¼,	— — 39	12	3	1	26 ¼,	— — 60	2	6	10
6 ¼,	— — 40	11	8	5	27 ¼,	— — 61	2	2	2
7 ¼,	— — 41	10	13	9	28 ¼,	— — 62	1	18	2
8 ¼,	— — 42	10	0	4	29 ¼,	— — 63	1	14	2
9 ¼,	— — 43	9	6	11	30 ¼,	— — 64	1	10	8
10 ¼,	— — 44	8	14	10	31 ¼,	— — 65	1	7	3
11 ¼,	— — 45	8	2	8	32 ¼,	— — 66	1	4	4
12 ¼,	— — 46	7	11	9	33 ¼,	— — 67	1	1	6
13 ¼,	— — 47	7	0	10	34 ¼,	— — 68	0	19	0
14 ¼,	— — 48	6	11	0	35 ¼,	— — 69	0	16	6
15 ¼,	— — 49	6	1	1	36 ¼,	— — 70	0	14	5
16 ¼,	— — 50	5	12	3	37 ¼,	— — 71	0	12	2
17 ¼,	— — 51	5	3	5	38 ¼,	— — 72	0	10	6
18 ¼,	— — 52	4	15	7	39 ¼,	— — 73	0	8	9
19 ¼,	— — 53	4	7	9	40 ¼,	— — 74	0	7	3
20 ¼,	— — 54	4	0	11	41 ¼,	— — 75	0	5	10
21 ¼,	— — 55	3	14	2					

TABLE

TABLE XXI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 34 $\frac{3}{4}$ to 35 $\frac{1}{4}$.

To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	or at Age	Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 35	16	5	0	21 $\frac{1}{4}$,	or at Age 56	3	11	4
1 $\frac{1}{4}$,	— — 36	15	6	0	22 $\frac{1}{4}$,	— — 57	3	5	4
2 $\frac{1}{4}$,	— — 37	14	8	0	23 $\frac{1}{4}$,	— — 58	2	19	4
3 $\frac{1}{4}$,	— — 38	13	11	0	24 $\frac{1}{4}$,	— — 59	2	14	2
4 $\frac{1}{4}$,	— — 39	12	15	2	25 $\frac{1}{4}$,	— — 60	2	9	0
5 $\frac{1}{4}$,	— — 40	11	19	5	26 $\frac{1}{4}$,	— — 61	2	4	5
6 $\frac{1}{4}$,	— — 41	11	4	6	27 $\frac{1}{4}$,	— — 62	1	19	10
7 $\frac{1}{4}$,	— — 42	10	9	7	28 $\frac{1}{4}$,	— — 63	1	15	11
8 $\frac{1}{4}$,	— — 43	9	16	2	29 $\frac{1}{4}$,	— — 64	1	12	0
9 $\frac{1}{4}$,	— — 44	9	2	10	30 $\frac{1}{4}$,	— — 65	1	8	8
10 $\frac{1}{4}$,	— — 45	8	10	10	31 $\frac{1}{4}$,	— — 66	1	5	4
11 $\frac{1}{4}$,	— — 46	7	18	9	32 $\frac{1}{4}$,	— — 67	1	2	5
12 $\frac{1}{4}$,	— — 47	7	8	0	33 $\frac{1}{4}$,	— — 68	0	19	7
13 $\frac{1}{4}$,	— — 48	6	17	2	34 $\frac{1}{4}$,	— — 69	0	17	5
14 $\frac{1}{4}$,	— — 49	6	7	4	35 $\frac{1}{4}$,	— — 70	0	15	2
15 $\frac{1}{4}$,	— — 50	5	17	5	36 $\frac{1}{4}$,	— — 71	0	13	0
16 $\frac{1}{4}$,	— — 51	5	8	7	37 $\frac{1}{4}$,	— — 72	0	10	9
17 $\frac{1}{4}$,	— — 52	4	19	10	38 $\frac{1}{4}$,	— — 73	0	9	2
18 $\frac{1}{4}$,	— — 53	4	12	3	39 $\frac{1}{4}$,	— — 74	0	7	8
19 $\frac{1}{4}$,	— — 54	4	4	8	40 $\frac{1}{4}$,	— — 75	0	6	2
20 $\frac{1}{4}$,	— — 55	3	18	0					

TABLE XXII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 35¾ to 36½.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age 36	16	0	10	20¼, or at Age 56	3	14	10
1¼, — — 37	15	2	0	21¼, — — 57	3	8	3
2¼, — — 38	14	4	6	22¼, — — 58	3	2	5
3¼, — — 39	13	7	2	23¼, — — 59	2	16	7
4¼, — — 40	12	11	0	24¼, — — 60	2	11	6
5¼, — — 41	11	15	0	25¼, — — 61	2	6	5
6¼, — — 42	11	0	2	26¼, — — 62	2	1	11
7¼, — — 43	10	5	3	27¼, — — 63	1	17	6
8¼, — — 44	9	12	0	28¼, — — 64	1	13	8
9¼, — — 45	8	18	8	29¼, — — 65	1	9	11
10¼, — — 46	8	6	9	30¼, — — 66	1	6	7
11¼, — — 47	7	14	9	31¼, — — 67	1	3	4
12¼, — — 48	7	3	11	32¼, — — 68	1	0	7
13¼, — — 49	6	13	1	33¼, — — 69	0	17	10
14¼, — — 50	6	3	4	34¼, — — 70	0	15	8
15¼, — — 51	5	13	7	35¼, — — 71	0	13	5
16¼, — — 52	5	5	0	36¼, — — 72	0	11	6
17¼, — — 53	4	16	5	37¼, — — 73	0	9	7
18¼, — — 54	4	8	11	38¼, — — 74	0	7	11
19¼, — — 55	4	1	5	39¼, — — 75	0	6	6

TABLE XXIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $36\frac{3}{4}$ to $37\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 37	15	16	9	$20\frac{1}{4}$,	or at Age 57	3	11	8
$1\frac{1}{4}$,	— — 38	14	18	0	$21\frac{1}{4}$,	— — 58	3	5	2
$2\frac{1}{4}$,	— — 39	14	0	6	$22\frac{1}{4}$,	— — 59	2	19	6
$3\frac{1}{4}$,	— — 40	13	3	4	$23\frac{1}{4}$,	— — 60	2	13	10
$4\frac{1}{4}$,	— — 41	12	6	11	$24\frac{1}{4}$,	— — 61	2	8	10
$5\frac{1}{4}$,	— — 42	11	10	7	$25\frac{1}{4}$,	— — 62	2	3	10
$6\frac{1}{4}$,	— — 43	10	15	10	$26\frac{1}{4}$,	— — 63	1	19	6
$7\frac{1}{4}$,	— — 44	10	1	0	$27\frac{1}{4}$,	— — 64	1	15	2
$8\frac{1}{4}$,	— — 45	9	7	10	$28\frac{1}{4}$,	— — 65	1	11	6
$9\frac{1}{4}$,	— — 46	8	14	7	$29\frac{1}{4}$,	— — 66	1	7	10
$10\frac{1}{4}$,	— — 47	8	2	8	$30\frac{1}{4}$,	— — 67	1	4	7
$11\frac{1}{4}$,	— — 48	7	10	10	$31\frac{1}{4}$,	— — 68	1	1	5
$12\frac{1}{4}$,	— — 49	6	19	11	$32\frac{1}{4}$,	— — 69	0	18	10
$13\frac{1}{4}$,	— — 50	6	9	0	$33\frac{1}{4}$,	— — 70	0	16	2
$14\frac{1}{4}$,	— — 51	5	19	5	$34\frac{1}{4}$,	— — 71	0	14	0
$15\frac{1}{4}$,	— — 52	5	9	10	$35\frac{1}{4}$,	— — 72	0	11	10
$16\frac{1}{4}$,	— — 53	5	1	5	$36\frac{1}{4}$,	— — 73	0	10	1
$17\frac{1}{4}$,	— — 54	4	13	0	$37\frac{1}{4}$,	— — 74	0	8	5
$18\frac{1}{4}$,	— — 55	4	5	7	$38\frac{1}{4}$,	— — 75	0	6	9
$19\frac{1}{4}$,	— — 56	3	18	2					

TABLE XXIV.

For the Ufe of Country Parifhes.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 37 ¼ to 38 ½.

To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d.	To commence at the end of Years after purchafing.		Value of an annuity of £1. in one prefent payment. £.	s.	d.
¼,	or at Age 38	15	12	6	19 ¼,	or at Age 57	3	15	0
1 ¼,	— — 39	14	13	11	20 ¼,	— — 58	3	8	7
2 ¼,	— — 40	13	16	0	21 ¼,	— — 59	3	2	0
3 ¼,	— — 41	12	18	7	22 ¼,	— — 60	2	16	7
4 ¼,	— — 42	12	2	3	23 ¼,	— — 61	2	11	0
5 ¼,	— — 43	11	5	11	24 ¼,	— — 62	2	6	2
6 ¼,	— — 44	10	11	3	25 ¼,	— — 63	2	1	4
7 ¼,	— — 45	9	16	6	26 ¼,	— — 64	1	17	1
8 ¼,	— — 46	9	3	5	27 ¼,	— — 65	1	12	10
9 ¼,	— — 47	8	10	3	28 ¼,	— — 66	1	9	3
10 ¼,	— — 48	7	18	4	29 ¼,	— — 67	1	5	8
11 ¼,	— — 49	7	6	5	30 ¼,	— — 68	1	2	7
12 ¼,	— — 50	6	15	8	31 ¼,	— — 69	0	19	7
13 ¼,	— — 51	6	4	11	32 ¼,	— — 70	0	17	1
14 ¼,	— — 52	5	15	6	33 ¼,	— — 71	0	14	7
15 ¼,	— — 53	5	6	1	34 ¼,	— — 72	0	12	6
16 ¼,	— — 54	4	17	10	35 ¼,	— — 73	0	10	5
17 ¼,	— — 55	4	9	7	36 ¼,	— — 74	0	8	[illegible]
18 ¼,	— — 56	4	2	3	37 ¼,	— — 75	0	7	1

TABLE

TABLE XXV.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 38 ¾ to 39 ½.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age 39	15	8	6
1 ¼,	— — 40	14	9	10
2 ¼,	— — 41	13	11	10
3 ¼,	— — 42	12	13	10
4 ¼,	— — 43	11	17	7
5 ¼,	— — 44	11	1	4
6 ¼,	— — 45	10	6	8
7 ¼,	— — 46	9	12	0
8 ¼,	— — 47	8	19	0
9 ¼,	— — 48	8	6	0
10 ¼,	— — 49	7	14	0
11 ¼,	— — 50	7	2	1
12 ¼,	— — 51	6	11	6
13 ¼,	— — 52	6	0	10
14 ¼,	— — 53	5	11	7
15 ¼,	— — 54	5	2	5
16 ¼,	— — 55	4	14	3
17 ¼,	— — 56	4	6	2
18 ¼,	— — 57	3	19	0
19 ¼,	or at Age 58	3	11	10
20 ¼,	— — 59	3	5	6
21 ¼,	— — 60	2	19	2
22 ¼,	— — 61	2	13	8
23 ¼,	— — 62	2	8	2
24 ¼,	— — 63	2	3	6
25 ¼,	— — 64	1	18	10
26 ¼,	— — 65	1	14	8
27 ¼,	— — 66	1	10	7
28 ¼,	— — 67	1	7	1
29 ¼,	— — 68	1	3	7
30 ¼,	— — 69	1	0	8
31 ¼,	— — 70	0	17	10
32 ¼,	— — 71	0	15	5
33 ¼,	— — 72	0	13	0
34 ¼,	— — 73	0	11	0
35 ¼,	— — 74	0	9	0
36 ¼,	— — 75	0	7	5

TABLE XXVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 39 ¾ to 40 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 40	15	4	4	18 ¼,	or at Age 58	3	15	8
1 ¼,	— — 41	14	5	2	19 ¼,	— — 59	3	8	7
2 ¼,	— — 42	13	7	2	20 ¼,	— — 60	3	2	5
3 ¼,	— — 43	12	9	2	21 ¼,	— — 61	2	16	3
4 ¼,	— — 44	11	13	0	22 ¼,	— — 62	2	10	10
5 ¼,	— — 45	10	16	10	23 ¼,	— — 63	2	5	6
6 ¼,	— — 46	10	2	4	24 ¼,	— — 64	2	0	11
7 ¼,	— — 47	9	7	9	25 ¼,	— — 65	1	16	4
8 ¼,	— — 48	[illegible]	14	8	26 ¼,	— — 66	1	12	4
9 ¼,	— — 49	[illegible]	1	6	27 ¼,	— — 67	1	8	4
10 ¼,	— — 50	[illegible]	9	8	28 ¼,	— — 68	1	5	0
11 ¼,	— — 51	[illegible]	17	10	29 ¼,	— — 69	1	1	8
12 ¼,	— — 52	6	7	5	30 ¼,	— — 70	0	18	10
13 ¼,	— — 53	5	17	0	31 ¼,	— — 71	0	16	1
14 ¼,	— — 54	5	7	10	32 ¼,	— — 72	0	13	9
15 ¼,	— — 55	4	18	9	33 ¼,	— — 73	0	11	6
16 ¼,	— — 56	4	10	9	3[illegible] ¼,	— — 74	0	9	7
17 ¼,	— — 57	4	2	9	35 ¼,	— — 75	0	7	9

TABLE XXVII.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 40 $\frac{3}{4}$ to 41 $\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 41	15	0	0	18 $\frac{1}{4}$,	or at Age 59	3	12	5
1 $\frac{1}{4}$,	— — 42	14	0	7	19 $\frac{1}{4}$,	— — 60	3	5	5
2 $\frac{1}{4}$,	— — 43	13	2	7	20 $\frac{1}{4}$,	— — 61	2	19	4
3 $\frac{1}{4}$,	— — 44	12	4	7	21 $\frac{1}{4}$,	— — 62	2	13	4
4 $\frac{1}{4}$,	— — 45	11	8	6	22 $\frac{1}{4}$,	— — 63	2	8	1
5 $\frac{1}{4}$,	— — 46	10	12	5	23 $\frac{1}{4}$,	— — 64	2	2	10
6 $\frac{1}{4}$,	— — 47	9	18	0	24 $\frac{1}{4}$,	— — 65	1	18	4
7 $\frac{1}{4}$,	— — 48	9	3	7	25 $\frac{1}{4}$,	— — 66	1	13	10
8 $\frac{1}{4}$,	— — 49	8	10	4	26 $\frac{1}{4}$,	— — 67	1	10	0
9 $\frac{1}{4}$,	— — 50	7	17	1	27 $\frac{1}{4}$,	— — 68	1	6	2
10 $\frac{1}{4}$,	— — 51	7	5	4	28 $\frac{1}{4}$,	— — 69	1	2	11
11 $\frac{1}{4}$,	— — 52	6	13	7	29 $\frac{1}{4}$,	— — 70	0	19	9
12 $\frac{1}{4}$,	— — 53	6	3	5	30 $\frac{1}{4}$,	— — 71	0	17	1
13 $\frac{1}{4}$,	— — 54	5	13	2	31 $\frac{1}{4}$,	— — 72	0	14	5
14 $\frac{1}{4}$,	— — 55	5	4	2	32 $\frac{1}{4}$,	— — 73	0	12	2
15 $\frac{1}{4}$,	— — 56	4	15	2	33 $\frac{1}{4}$,	— — 74	0	10	0
16 $\frac{1}{4}$,	— — 57	4	7	3	34 $\frac{1}{4}$,	— — 75	0	8	2
17 $\frac{1}{4}$,	— — 58	3	19	5					

TABLE XXVIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 41 ¾ to 42 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 42	14	15	6	17 ¼,	or at Age 59	3	16	1
1 ¼,	— — 43	13	16	2	18 ¼,	— — 60	3	9	2
2 ¼,	— — 44	12	18	2	19 ¼,	— — 61	3	2	4
3 ¼,	— — 45	12	0	3	20 ¼,	— — 62	2	16	4
4 ¼,	— — 46	11	4	2	21 ¼,	— — 63	2	10	5
5 ¼,	— — 47	10	8	1	22 ¼,	— — 64	2	5	4
6 ¼,	— — 48	9	13	7	23 ¼,	— — 65	2	0	2
7 ¼,	— — 49	8	19	0	24 ¼,	— — 66	1	15	10
8 ¼,	— — 50	8	5	11	25 ¼,	— — 67	1	11	5
9 ¼,	— — 51	7	12	9	26 ¼,	— — 68	1	7	9
10 ¼,	— — 52	7	1	2	27 ¼,	— — 69	1	4	0
11 ¼,	— — 53	6	9	8	28 ¼,	— — 70	1	0	11
12 ¼,	— — 54	5	19	7	29 ¼,	— — 71	0	17	10
13 ¼,	— — 55	5	9	6	30 ¼,	— — 72	0	15	4
14 ¼,	— — 56	5	0	7	31 ¼,	— — 73	0	12	9
15 ¼,	— — 57	4	11	8	32 ¼,	— — 74	0	10	7
16 ¼,	— — 58	4	3	10	33 ¼,	— — 75	0	8	7

TABLE XXIX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 42 ¾ to 43 ¾.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 43	14	11	0	17 ¼,	or at Age 60	3	12	9
1 ¼,	— — 44	13	11	9	18 ¼,	— — 61	3	6	0
2 ¼,	— — 45	12	13	10	19 ¼,	— — 62	2	19	3
3 ¼,	— — 46	11	15	11	20 ¼,	— — 63	2	13	5
4 ¼,	— — 47	10	19	10	21 ¼,	— — 64	2	7	7
5 ¼,	— — 48	10	3	10	22 ¼,	— — 65	2	2	7
6 ¼,	— — 49	9	9	2	23 ¼,	— — 66	1	17	7
7 ¼,	— — 50	8	14	6	24 ¼,	— — 67	1	13	4
8 ¼,	— — 51	8	1	6	25 ¼,	— — 68	1	9	1
9 ¼,	— — 52	7	8	5	26 ¼,	— — 69	1	5	6
10 ¼,	— — 53	6	17	1	27 ¼,	— — 70	1	1	11
11 ¼,	— — 54	6	5	10	28 ¼,	— — 71	0	18	11
12 ¼,	— — 55	5	15	10	29 ¼,	— — 72	0	16	0
13 ¼,	— — 56	5	5	10	30 ¼,	— — 73	0	13	7
14 ¼,	— — 57	4	17	0	31 ¼,	— — 74	0	11	2
15 ¼,	— — 58	4	8	2	32 ¼,	— — 75	0	9	1
16 ¼,	— — 59	4	0	6					

TABLE XXX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 43 ¾ to 44 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 44	14	6	9	16 ¼,	or at Age 60	3	17	0
1 ¼,	— — 45	13	7	2	17 ¼,	— — 61	3	9	4
2 ¼,	— — 46	12	9	3	18 ¼,	— — 62	3	2	9
3 ¼,	— — 47	11	11	4	19 ¼,	— — 63	2	16	1
4 ¼,	— — 48	10	15	2	20 ¼,	— — 64	2	10	5
5 ¼,	— — 49	9	19	0	2[illegible] ¼,	— — 65	2	4	9
6 ¼,	— — 50	9	4	5	22 ¼,	— — 66	1	19	10
7 ¼,	— — 51	8	9	10	23 ¼,	— — 67	1	14	11
8 ¼,	— — 52	[illegible]	17	0	24 ¼,	— — 68	1	10	10
9 ¼,	— — 53	7	4	2	25 ¼,	— — 69	1	6	9
10 ¼,	— — 54	6	13	0	26 ¼,	— — 70	1	3	3
11 ¼,	— — 55	6	1	10	27 ¼,	— — 71	0	19	10
12 ¼,	— — 56	5	11	11	28 ¼,	— — 72	0	17	0
13 ¼,	— — 57	5	2	0	29 ¼,	— — 73	0	14	2
14 ¼,	— — 58	4	13	3	30 ¼,	— — 74	0	11	10
15 ¼,	— — 59	4	4	7	31 ¼,	— — 75	0	9	7

TABLE

TABLE XXXI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 44 ¾ to 45 ¼.

To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.	Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age 45	14	2	0	16 ¼, or at Age 61	3	13	6
1 ¼, — — 46	13	2	7	17 ¼, — — 62	3	6	0
2 ¼, — — 47	12	4	8	18 ¼, — — 63	2	19	6
3 ¼, — — 48	11	6	10	19 ¼, — — 64	2	13	0
4 ¼, — — 49	10	10	6	20 ¼, — — 65	2	7	5
5 ¼, — — 50	9	14	2	21 ¼, — — 66	2	1	11
6 ¼, — — 51	8	19	8	22 ¼, — — 67	1	17	1
7 ¼, — — 52	8	5	2	23 ¼, — — 68	1	12	4
8 ¼, — — 53	7	12	7	24 ¼, — — 69	1	8	5
9 ¼, — — 54	7	0	0	25 ¼, — — 70	1	4	5
10 ¼, — — 55	6	8	11	26 ¼, — — 71	1	1	1
11 ¼, — — 56	5	17	10	27 ¼, — — 72	0	17	10
12 ¼, — — 57	5	8	0	28 ¼, — — 73	0	15	1
13 ¼, — — 58	4	18	2	29 ¼, — — 74	0	12	5
14 ¼, — — 59	4	9	7	30 ¼, — — 75	0	10	2
15 ¼, — — 60	4	1	0				

TABLE

TABLE XXXII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 45 ¾ to 46 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 46	13	17	6	15 ¼,	or at Age 61	3	17	3
1 ¼,	— — 47	12	17	10	16 ¼,	— — 62	3	9	11
2 ¼,	— — 48	11	19	9	17 ¼,	— — 63	3	2	7
3 ¼,	— — 49	11	1	8	18 ¼,	— — 64	2	16	2
4 ¼,	— — 50	10	5	6	19 ¼,	— — 65	2	9	10
5 ¼,	— — 51	9	9	3	20 ¼,	— — 66	2	4	5
6 ¼,	— — 52	8	15	0	21 ¼,	— — 67	1	19	0
7 ¼,	— — 53	8	0	8	22 ¼,	— — 68	1	14	4
8 ¼,	— — 54	7	8	2	23 ¼,	— — 69	1	9	9
9 ¼,	— — 55	6	15	8	24 ¼,	— — 70	1	5	11
10 ¼,	— — 56	6	4	8	25 ¼,	— — 71	1	2	2
11 ¼,	— — 57	5	13	8	26 ¼,	— — 72	0	18	11
12 ¼,	— — 58	5	4	0	27 ¼,	— — 73	0	15	10
13 ¼,	— — 59	4	14	3	28 ¼,	— — 74	0	13	2
14 ¼,	— — 60	4	5	9	29 ¼,	— — 75	0	10	9

TABLE XXXIII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

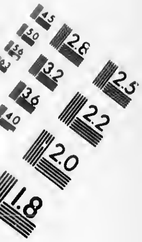
Age of the Purchaſer from 46 3/4 to 47 1/2.

To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.	To commence at the end of Years after purchaſing.		Value of an annuity of £1. in one preſent payment. £.	s.	d.
1/4,	or at Age 47	13	12	6	15 1/4,	or at Age 62	3	13	7
1 1/4,	— — 48	12	13	2	16 1/4,	— — 63	3	6	5
2 1/4,	— — 49	11	14	11	17 1/4,	— — 64	2	19	2
3 1/4,	— — 50	10	16	7	18 1/4,	— — 65	2	12	11
4 1/4,	— — 51	10	0	6	19 1/4,	— — 66	2	6	9
5 1/4,	— — 52	9	4	4	20 1/4,	— — 67	2	1	5
6 1/4,	— — 53	8	10	4	21 1/4,	— — 68	1	16	2
7 1/4,	— — 54	7	16	3	22 1/4,	— — 69	1	11	8
8 1/4,	— — 55	7	3	10	23 1/4,	— — 70	1	7	2
9 1/4,	— — 56	6	11	5	24 1/4,	— — 71	1	3	0
10 1/4,	— — 57	6	0	6	25 1/4,	— — 72	0	19	10
11 1/4,	— — 58	5	9	7	26 1/4,	— — 73	0	16	10
12 1/4,	— — 59	5	0	0	27 1/4,	— — 74	0	13	10
13 1/4,	— — 60	4	10	4	28 1/4,	— — 75	0	11	4
14 1/4,	— — 61	4	2	0					

TABLE

IMAGE EVALUATION TEST TARGET (MT-3)

6"


2.8
2.5
3.2
2.2
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1.0

TABLE XXXIV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 47 ¾ to 48 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 48	13	7	9	14 ¼,	or at Age 62	3	18	1
1 ¼,	— — 49	12	7	10	15 ¼,	— — 63	3	9	10
2 ¼,	— — 50	11	9	7	16 ¼,	— — 64	3	2	10
3 ¼,	— — 51	10	11	5	17 ¼,	— — 65	2	15	8
4 ¼,	— — 52	9	15	6	18 ¼,	— — 66	2	9	7
5 ¼,	— — 53	8	19	7	19 ¼,	— — 67	2	3	7
6 ¼,	— — 54	8	5	7	20 ¼,	— — 68	1	18	5
7 ¼,	— — 55	7	11	8	21 ¼,	— — 69	1	13	3
8 ¼,	— — 56	6	19	4	22 ¼,	— — 70	1	8	11
9 ¼,	— — 57	6	7	0	23 ¼,	— — 71	1	4	8
10 ¼,	— — 58	5	16	2	24 ¼,	— — 72	1	1	2
11 ¼,	— — 59	5	5	4	25 ¼,	— — 73	0	17	8
12 ¼,	— — 60	4	15	10	26 ¼,	— — 74	0	14	9
13 ¼,	— — 61	4	6	4	27 ¼,	— — 75	0	12	0

TABLE

TABLE XXXV.

For the Use of Country Parishes.

Shewing the Payment due, (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 48 $\frac{3}{4}$ to 49 $\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 49	13	2	6
1 $\frac{1}{4}$,	— — 50	12	2	6
2 $\frac{1}{4}$,	— — 51	11	4	5
3 $\frac{1}{4}$,	— — 52	10	6	3
4 $\frac{1}{4}$,	— — 53	9	10	6
5 $\frac{1}{4}$,	— — 54	8	14	10
6 $\frac{1}{4}$,	— — 55	8	0	11
7 $\frac{1}{4}$,	— — 56	7	7	1
8 $\frac{1}{4}$,	— — 57	6	14	10
9 $\frac{1}{4}$,	— — 58	6	2	7
10 $\frac{1}{4}$,	— — 59	5	11	10
11 $\frac{1}{4}$,	— — 60	5	1	1
12 $\frac{1}{4}$,	— — 61	4	11	9
13 $\frac{1}{4}$,	— — 62	4	2	5
14 $\frac{1}{4}$,	or at Age 63	3	14	3
15 $\frac{1}{4}$,	— — 64	3	6	2
16 $\frac{1}{4}$,	— — 65	2	19	3
17 $\frac{1}{4}$,	— — 65	2	12	3
18 $\frac{1}{4}$,	— — 67	2	6	4
19 $\frac{1}{4}$,	— — 68	2	0	5
20 $\frac{1}{4}$,	— — 69	1	15	5
21 $\frac{1}{4}$,	— — 70	1	10	5
22 $\frac{1}{4}$,	— — 71	1	6	3
23 $\frac{1}{4}$,	— — 72	1	2	2
24 $\frac{1}{4}$,	— — 73	0	18	10
25 $\frac{1}{4}$,	— — 74	0	15	6
26 $\frac{1}{4}$,	— — 75	0	12	8

TABLE XXXVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 49 ¾ to 50 ½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age	50	12	17	8	13 ¼, or at Age	63	3	18	7
1 ¼, — —	51	11	17	10	14 ¼, — —	64	3	10	7
2 ¼, — —	52	10	19	10	15 ¼, — —	65	3	2	7
3 ¼, — —	53	10	1	11	16 ¼, — —	66	2	15	10
4 ¼, — —	54	9	6	2	17 ¼, — —	67	2	8	11
5 ¼, — —	55	8	10	6	18 ¼, — —	68	2	3	2
6 ¼, — —	56	7	16	7	19 ¼, — —	69	1	17	5
7 ¼, — —	57	7	2	10	20 ¼, — —	70	1	12	7
8 ¼, — —	58	6	10	7	21 ¼, — —	71	1	7	9
9 ¼, — —	59	5	18	5	22 ¼, — —	72	1	3	9
10 ¼, — —	60	5	7	9	23 ¼, — —	73	0	19	10
11 ¼, — —	61	4	17	1	24 ¼, — —	74	0	16	7
12 ¼, — —	62	4	7	10	25 ¼, — —	75	0	13	6

TABLE XXXVII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 50 ¼ to 51 ¼.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.	To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age	51	12	12	6	13 ¼, or at Age	64	3	14	10
1 ¼, — —	52	11	13	2	14 ¼, — —	65	3	6	11
2 ¼, — —	53	10	15	4	15 ¼, — —	66	2	19	1
3 ¼, — —	54	9	17	7	16 ¼, — —	67	2	12	5
4 ¼, — —	55	9	1	11	17 ¼, — —	68	2	5	8
5 ¼, — —	56	8	6	2	18 ¼, — —	69	2	0	0
6 ¼, — —	57	7	12	4	19 ¼, — —	70	1	14	5
7 ¼, — —	58	6	18	7	20 ¼, — —	71	1	9	9
8 ¼, — —	59	6	6	5	21 ¼, — —	72	1	5	1
9 ¼, — —	60	5	14	3	22 ¼, — —	73	1	1	4
10 ¼, — —	61	5	3	9	23 ¼, — —	74	0	17	6
11 ¼, — —	62	4	13	2	24 ¼, — —	75	0	14	4
12 ¼, — —	63	4	4	0					

TABLE XXXVIII.

For the Ufe of Country Parifhes

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 51 $\frac{1}{4}$ to 52 $\frac{1}{4}$.

To commence at the end of Years after purchafing.			Value of an annuity of £1. in one prefent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	52	12	7	6
1 $\frac{1}{4}$,	——	53	11	7	10
2 $\frac{1}{4}$,	——	54	10	10	1
3 $\frac{1}{4}$,	——	55	9	12	4
4 $\frac{1}{4}$,	——	56	8	16	9
5 $\frac{1}{4}$,	——	57	8	1	1
6 $\frac{1}{4}$,	——	58	7	7	4
7 $\frac{1}{4}$,	——	59	6	13	8
8 $\frac{1}{4}$,	——	60	6	1	7
9 $\frac{1}{4}$,	——	61	5	9	7
10 $\frac{1}{4}$,	——	62	4	19	2
11 $\frac{1}{4}$,	——	63	4	8	8
12 $\frac{1}{4}$,	——	64	3	19	8
13 $\frac{1}{4}$,	——	65	3	10	8
14 $\frac{1}{4}$,	——	66	3	2	11
15 $\frac{1}{4}$,	——	67	2	15	3
16 $\frac{1}{4}$,	——	68	[illegible]	8	9
17 $\frac{1}{4}$,	——	69	[illegible]	2	3
18 $\frac{1}{4}$,	——	70	1	16	9
19 $\frac{1}{4}$,	——	71	1	11	4
20 $\frac{1}{4}$,	——	72	1	6	10
21 $\frac{1}{4}$,	——	73	1	2	5
22 $\frac{1}{4}$,	——	74	0	18	9
23 $\frac{1}{4}$,	——	75	0	15	3

TABLE XXXIX.

For the Ufe of Country Parifhes.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchafer from 52 $\frac{1}{4}$ to 53 $\frac{1}{4}$.

To commence at the end of Years after purchafing.			Value of an annuity of £1. in one prefent payment. £.	s.	d.
$\frac{1}{4}$,	or at Age	53	12	2	0
1 $\frac{1}{4}$,	——	54	11	2	7
2 $\frac{1}{4}$,	——	55	10	4	11
3 $\frac{1}{4}$,	——	56	9	7	2
4 $\frac{1}{4}$,	——	57	8	11	8
5 $\frac{1}{4}$,	——	58	7	16	1
6 $\frac{1}{4}$,	——	59	7	2	5
7 $\frac{1}{4}$,	——	60	6	8	9
8 $\frac{1}{4}$,	——	61	5	16	10
9 $\frac{1}{4}$,	——	62	5	4	11
10 $\frac{1}{4}$,	——	63	4	14	7
11 $\frac{1}{4}$,	——	64	4	4	3
12 $\frac{1}{4}$,	——	65	3	15	5
13 $\frac{1}{4}$,	——	66	3	6	7
14 $\frac{1}{4}$,	——	67	2	19	0
15 $\frac{1}{4}$,	——	68	2	11	6
16 $\frac{1}{4}$,	——	69	2	5	2
17 $\frac{1}{4}$,	——	70	1	18	10
18 $\frac{1}{4}$,	——	71	1	13	7
19 $\frac{1}{4}$,	——	72	1	8	3
20 $\frac{1}{4}$,	——	73	1	4	0
21 $\frac{1}{4}$,	——	74	0	19	9
22 $\frac{1}{4}$,	——	75	0	16	2

TABLE XL.

For the Use of Country Parishes.

Shewing the Payment due (reckoning interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 53¼ to 54½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	54	11	16	8
1 ¼,	——	55	10	17	2
2 ¼,	——	56	9	19	6
3 ¼,	——	57	9	1	10
4 ¼,	——	58	8	6	4
5 ¼,	——	59	7	10	10
6 ¼,	——	60	6	17	3
7 ¼,	——	61	6	3	8
8 ¼,	——	62	5	11	11
9 ¼,	——	63	5	0	1
10 ¼,	——	64	4	9	11
11 ¼,	——	65	3	19	9
12 ¼,	——	66	3	11	1
13 ¼,	——	67	3	2	4
14 ¼,	——	68	2	15	0
15 ¼,	——	69	2	7	8
16 ¼,	——	70	2	1	6
17 ¼,	——	71	1	15	5
18 ¼,	——	72	1	10	4
19 ¼,	——	73	1	5	3
20 ¼,	——	74	1	1	1
21 ¼,	——	75	0	17	2

TABLE XLI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 54¼ to 55½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	55	11	11	3
1 ¼,	——	56	10	11	9
2 ¼,	——	57	9	14	1
3 ¼,	——	58	8	16	6
4 ¼,	——	59	8	1	0
5 ¼,	——	60	7	5	7
6 ¼,	——	61	6	12	1
7 ¼,	——	62	5	18	7
8 ¼,	——	63	5	7	0
9 ¼,	——	64	4	15	4
10 ¼,	——	65	4	5	4
11 ¼,	——	66	3	15	3
12 ¼,	——	67	3	6	9
13 ¼,	——	68	2	18	2
14 ¼,	——	69	2	11	0
15 ¼,	——	70	2	3	11
16 ¼,	——	71	1	17	11
17 ¼,	——	72	1	12	0
18 ¼,	——	73	1	7	2
19 ¼,	——	74	1	2	4
20 ¼,	——	75	0	18	2

TABLE XLII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 55½ to 56½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age	56	11	5	6
1 ¼, —	57	10	5	10
2 ¼, —	58	9	8	6
3 ¼, —	59	8	10	11
4 ¼, —	60	7	15	6
5 ¼, —	61	7	0	2
6 ¼, —	62	6	6	9
7 ¼, —	63	5	13	4
8 ¼, —	64	5	1	11
9 ¼, —	65	4	10	5
10 ¼, —	66	4	0	7
11 ¼, —	67	3	10	8
12 ¼, —	68	3	2	4
13 ¼, —	69	2	14	0
14 ¼, —	70	2	7	0
15 ¼, —	71	2	0	1
16 ¼, —	72	1	14	4
17 ¼, —	73	1	8	8
18 ¼, —	74	1	3	11
19 ¼, —	75	0	19	6

TABLE XLIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 56½ to 57½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼, or at Age	57	11	0	0
1 ¼, —	58	10	0	5
2 ¼, —	59	9	2	11
3 ¼, —	60	8	5	4
4 ¼, —	61	7	10	0
5 ¼, —	62	6	14	9
6 ¼, —	63	6	1	6
7 ¼, —	64	5	8	2
8 ¼, —	65	4	16	10
9 ¼, —	66	4	5	6
10 ¼, —	67	3	15	10
11 ¼, —	68	3	6	1
12 ¼, —	69	2	18	0
13 ¼, —	70	2	9	10
14 ¼, —	71	2	3	1
15 ¼, —	72	1	16	4
16 ¼, —	73	1	10	10
17 ¼, —	74	1	5	4
18 ¼, —	75	1	0	9

TABLE XLIV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $57\frac{1}{2}$ to $58\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 58	10	14	0
$1\frac{1}{4}$,	—— 59	9	14	7
$2\frac{1}{4}$,	—— 60	9	17	1
$3\frac{1}{4}$,	—— 61	7	19	7
$4\frac{1}{4}$,	—— 62	7	4	4
$5\frac{1}{4}$,	—— 63	6	9	2
$6\frac{1}{4}$,	—— 64	5	16	0
$7\frac{1}{4}$,	—— 65	5	2	10
$8\frac{1}{4}$,	—— 66	4	11	8
$9\frac{1}{4}$,	—— 67	4	0	6
$10\frac{1}{4}$,	—— 68	3	11	0
$11\frac{1}{4}$,	—— 69	3	1	5
$12\frac{1}{4}$,	—— 70	2	13	7
$13\frac{1}{4}$,	—— 71	2	5	8
$14\frac{1}{4}$,	—— 72	1	19	1
$15\frac{1}{4}$,	—— 73	1	12	7
$16\frac{1}{4}$,	—— 74	1	7	3
$17\frac{1}{4}$,	—— 75	1	2	2

TABLE XLV.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $58\frac{1}{2}$ to $59\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 59	10	8	0
$1\frac{1}{4}$,	—— 60	9	8	9
$2\frac{1}{4}$,	—— 61	8	11	3
$3\frac{1}{4}$,	—— 62	7	13	10
$4\frac{1}{4}$,	—— 63	6	18	8
$5\frac{1}{4}$,	—— 64	6	3	7
$6\frac{1}{4}$,	—— 65	5	10	7
$7\frac{1}{4}$,	—— 66	4	17	7
$8\frac{1}{4}$,	—— 67	4	6	6
$9\frac{1}{4}$,	—— 68	3	15	6
$10\frac{1}{4}$,	—— 69	3	6	2
$11\frac{1}{4}$,	—— 70	2	16	10
$12\frac{1}{4}$,	—— 71	2	9	2
$13\frac{1}{4}$,	—— 72	2	1	6
$14\frac{1}{4}$,	—— 73	1	15	2
$15\frac{1}{4}$,	—— 74	1	8	11
$16\frac{1}{4}$,	—— 75	1	3	8

TABLE

TABLE XLVI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 59½ to 60½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 60	10	2	0
1¼,	—— 61	9	2	8
2¼,	—— 62	8	5	2
3¼,	—— 63	7	7	10
4¼,	—— 64	6	12	9
5¼,	—— 65	5	17	9
6¼,	—— 66	5	4	11
7¼,	—— 67	4	12	1
8¼,	—— 68	4	1	2
9¼,	—— 69	3	10	4
10¼,	—— 70	3	1	3
11¼,	—— 71	2	12	2
12¼,	—— 72	2	4	9
13¼,	—— 73	1	17	5
14¼,	—— 74	1	11	2
15¼,	—— 75	1	5	5

TABLE XLVII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 60½ to 61½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age 61	9	16	0
1¼,	—— 62	8	16	7
2¼,	—— 63	7	19	2
3¼,	—— 64	7	1	10
4¼,	—— 65	6	6	11
5¼,	—— 66	5	12	0
6¼,	—— 67	4	19	4
7¼,	—— 68	4	6	7
8¼,	—— 69	3	15	11
9¼,	—— 70	3	5	3
10¼,	—— 71	2	16	5
11¼,	—— 72	2	7	7
12¼,	—— 73	2	0	5
13¼,	—— 74	1	13	4
14¼,	—— 75	1	7	2

TABLE XLVIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 61½ to 62½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
½,	or at Age 62	9	9	8
1½,	—— 63	8	10	3
2½,	—— 64	7	12	11
3½,	—— 65	6	15	8
4½,	—— 66	6	0	9
5½,	—— 67	5	6	0
6½,	—— 68	4	13	6
7½,	—— 69	4	1	0
8½,	—— 70	3	10	7
9½,	—— 71	3	0	1
10½,	—— 72	2	11	6
11½,	—— 73	2	3	0
12½,	—— 74	1	15	8
13½,	—— 75	1	9	3

TABLE XLIX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 62½ to 63½.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
½,	or at Age 63	9	3	3
1½,	—— 64	8	3	11
2½,	—— 65	7	6	8
3½,	—— 66	6	9	6
4½,	—— 67	5	14	9
5½,	—— 68	5	0	0
6½,	—— 69	4	7	9
	—— 70	3	15	6
	—— 71	3	5	3
	— 72	2	15	0
	73	2	6	8
	74	1	18	5
	75	1	11	5

TABLE L.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 63¾ to 64¼.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	64	8	16	8
1 ¼,	——	65	7	17	4
2 ¼,	——	66	7	0	2
3 ¼,	——	67	6	3	0
4 ¼,	——	68	5	8	6
5 ¼,	——	69	4	13	11
6 ¼,	——	70	4	1	10
7 ¼,	——	71	3	9	9
8 ¼,	——	72	2	19	10
9 ¼,	——	73	2	9	10
10 ¼,	——	74	2	1	7
11 ¼,	——	75	1	14	0

TABLE LI.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75,

Age of the Purchaser from 64¾ to 65¼.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	65	8	10	0
1 ¼,	——	66	7	10	10
2 ¼,	——	67	6	13	8
3 ¼,	——	68	5	16	7
4 ¼,	——	69	5	2	3
5 ¼,	——	70	4	7	11
6 ¼,	——	71	3	16	0
7 ¼,	——	72	3	4	1
8 ¼,	——	73	2	14	5
9 ¼,	——	74	2	4	9
10 ¼,	——	75	1	16	7

TABLE LII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 65½ to 66½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	66	8	3	4
1 ¼,	——	67	7	4	0
2 ¼,	——	68	6	6	11
3 ¼,	——	69	5	10	0
4 ¼,	——	70	4	15	10
5 ¼,	——	71	4	1	8
6 ¼,	——	72	3	10	0
7 ¼,	——	73	2	18	4
8 ¼,	——	74	2	8	8
9 ¼,	——	75	1	19	9

TABLE LIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from 66½ to 67½.

To commence at the end of Years after purchasing.			Value of an annuity of £1. in one present payment. £.	s.	d.
¼,	or at Age	67	7	16	6
1 ¼,	——	68	6	17	2
2 ¼,	——	69	6	0	3
3 ¼,	——	70	5	3	5
4 ¼,	——	71	[illegible]	9	5
5 ¼,	——	72	3	15	5
6 ¼,	——	73	3	4	0
7 ¼,	——	74	2	12	7
8 ¼,	——	75	2	3	0

TABLE LIV.

For the Ufe of Country Parifhes

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchafer from 67¾ to 68¾.

To commence at the end of Years after purchafing.			Value of an annuity of £1. in one prefent payment. £.	s.	d.
¼,	or at Age	68	7	9	6
1 ¼,	——	69	6	10	1
2 ¼,	——	70	5	13	4
3 ¼,	——	71	4	16	7
4 ¼,	——	72	4	2	10
5 ¼,	——	73	3	9	0
6 ¼,	——	74	2	17	6
7 ¼,	——	75	2	7	1

TABLE LV.

For the Ufe of Country Parifhes.

Shewing the Payment due (reckoning Intereft at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any age from 35 to 75.

Age of the Purchafer from 68¾ to 69¾.

To commence at the end of Years after purchafing.			Value of an annuity of £1. in one prefent payment. £.	s.	d.
¼,	or at Age	69	7	2	4
1 ¼,	——	70	6	3	1
2 ¼,	——	71	5	6	5
3 ¼,	——	72	4	9	9
4 ¼,	——	73	3	16	3
5 ¼,	——	74	3	2	8
6 ¼,	——	75	2	11	2

TABLE

TABLE LVI.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 69¾ to 70¼.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	70	6	15	1
1¼,	——	71	5	15	9
2¼,	——	72	4	19	3
3¼,	——	73	4	2	9
4¼,	——	74	3	9	0
5¼,	——	75	2	16	0

TABLE LVII.

For the Uſe of Country Pariſhes.

Shewing the Payment due (reckoning Intereſt at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaſer from 70¾ to 71¼.

To commence at the end of Years after purchaſing.			Value of an annuity of £1. in one preſent payment. £.	s.	d.
¼,	or at Age	71	6	7	9
1¼,	——	72	5	8	6
2¼,	——	73	4	12	1
3¼,	——	74	3	15	9
4¼,	——	75	3	1	11

TABLE

TABLE LVIII.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $71\frac{1}{2}$ to $72\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 72	6	0	0
$1\frac{1}{4}$,	—— 73	5	0	11
$2\frac{1}{4}$,	—— 74	4	4	2
$3\frac{1}{4}$,	—— 75	3	8	6

TABLE LIX.

For the Use of Country Parishes.

Shewing the Payment due (reckoning Interest at 3 per Cent.) for a Life-Annuity of One Pound, payable Quarterly. To commence at any Age from 35 to 75.

Age of the Purchaser from $72\frac{1}{2}$ to $73\frac{1}{2}$.

To commence at the end of Years after purchasing.		Value of an annuity of £1. in one present payment. £.	s.	d.
$\frac{1}{4}$,	or at Age 73	5	12	6
$1\frac{1}{4}$,	—— 74	4	13	4
$2\frac{1}{4}$,	—— 75	3	16	3

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