VOL. XIII. PART II.

1899.

TRANSACTIONS

OF THE

CANADIAN SOCIETY of CIVIL ENGINEERS

OCTOBER TO DECEMBER, 1899.

CONTENTS.

PROCEEDINGS.

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Canadian Society of Civil Engineers

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Montreal:
PRINTED FOR THE SOCIETY
BY JOHN LOVELL & SON, LIMITED,
1904.

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"The papers shall be the property of the Society, and no publication of any papers or discussion shall be made, except by the Society or with permission of Council."—By-Law No. 47.

INSTRUCTIONS FOR PREPARING PAPERS, ETC.

In writing papers, or discussions on papers, the use of the first person should be avoided.

They should be legibly written on foolscap paper, on one side only, with a margin on the left side.

Illustrations, when necessary, should be drawn on the dull side of tracing linen to as small a scale as is consistent with distinctness. They should not be more than ten inches in height and *in no case* should any one figure exceed this height. Black ink only should be used, and all lines, lettering, etc., must be clear and distinct.

When necessary to illustrate a paper for reading, diagrams must be furnished. These must be bold, distinct and clearly visible in detail for a distance of thirty feet.

Papers which have been read before other Societies, or have been published cannot be read at meetings of this Society.

All communications must be forwarded to the Secretary of the Society from whom any further information may be obtained.

The attention of Members is called to By-Laws 46 and 47.

CONTENTS.

PROCEEDINGS.

Thursday, 12th October.

H. Irwin, Member of Council, in the chair.

Presentations to the Library were reported as follows:—"Marine Steam Engine," "Engine-Room Practice," and "The Generation and Utilization of High Pressure Steam and Experience with Water Tube Boilers," from Mr. A. D. Watson, M. Can. Soc. C. E.

"The Canadian Canals," from Mr. G. Barnett Smith, Stud., Can. Soc. C. E.

A statement was received from the Treasurer as to the condition of the work on the building, 877 Dorchester Street.

Messrs. L. G. Papineau, E. S. Mattice, R. M. Hannaford, J. Ewing, G. Legrand and J. W. Heckman, having been appointed scrutineers of the ballot for the election of members, reported the following elected:—

MEMBERS.

G. L. Bourchier.
J. A. Mahood.
F. J. L. Tytler.
J. L. Weller.

ASSOCIATE MEMBERS.

J. Chalmers.
J. M. R. Fairbairn.
R. A. Galbraith.
F. J. Hetherington.
G. L. Law.
J. McLaren.
G. S. Maunsell.
W. H. Sullivan.

Transferred from the Class of Student to the Class of Associate Member.

W. A. Clement.
J. G. R. Wainwright.

STUDENT.

W. A. Hare.

Thursday, 26th October.

H. Irwin, Member of Council, in the chair.

On motion by Mr. W. McNab, seconded by Mr. E. A. Rhys-Roberts, it was resolved, "That the Council should issue a proposal for a change in the By-laws; that all Associate Members applying for transfer to the Class of Member should be required to present a paper with the application."

Paper No. 146.

THE KILAUEA PLANTATION RAILWAY.

By G. R. Ewart, Stud.Can.Soc.C.E.

The group known as the Hawaiian or Sandwich Islands consists of a number of islands, all of which are of comparatively limited area. Of these the Island of Kanai, upon which the railway referred to in this paper is situated, is about twenty-five miles in diameter, and has, like all the other islands, an exceedingly rough surface, rising in the centre to a height of six thousand feet. The Islands are volcanic in origin, and on one of them the craters are still active.

Only a narrow strip along the sea coast, varying in width from nothing up to about five miles, is capable of cultivation, and even this area is of a very rugged formation, consisting of a series of ridges and ravines, deeply cut by the heavy tropical rains.

The soil varies considerably in character, but for the most part is a loam of light red colour, resulting from the decomposition of the volcanic rock. It is thickly covered in many places with boulders of a hard grey rock, but when cleared proves to be exceedingly rich, and if properly irrigated yields very heavy crops.

The principal industry of the islanders is the cultivation of sugar cane, and this requires a very fertile soil, and a plentiful supply of water. In fact, except on the island of Hawaii, the area that is cultivatable is the area that is irrigable; for, although the rainfall is heavy, especially on the windward, that is, the north-east side of the islands, the land is so steep and the soil so porous that the water soon disappears, and, besides this, there are often long periods of dry weather, during which the cane would suffer severely were it not carefully watered.

The raising of a crop of sugar cane is a long and costly operation. The area to be planted is thoroughly ploughed, cultivated and harrowed by steam power. Contour lines for every two or three feet rise are then laid out with a level, and marked with stakes. A furrowing plough is run along the line of these stakes, and the intermediate space divided up by parallel furrows five or six feet apart. Ditches and leads are dug to connect with the main irrigation system, and the cane is planted in the furrows in the form of short pieces eight or ten inches long, laid on their sides and covered with about two inches of earth. Each of these pieces has several joints, at each of which is an eye, and these eyes sprout and draw their nourishment from the stick till they have thrown out enough roots to sustain themselves. The planting continues from the first of June until about the end of September, after which the cane fields must be regularly weeded and irrigated, the crop taking till about the first of November of the following year to mature. The cane is then cut and carried to the mill for grinding. A second crop, known as ratoon, is grown in the following year from the roots of the first crop.

The plantations vary greatly in size, that of Kilauea, of which the writer was at one time assistant head overseer, stretching about eight miles along the seacoast, and extending back to the crest of the mountains. About two thousand acres of this area are under cultivation at one time, the mountain slopes being held mainly for the water rights. The plantation is worked from one settlement, at which the mill is placed, and around which the labourers live. The working force on the plantation is about five hundred men, and this force, when working, is kept as close together as possible. With this system of organization it is necessary to convey all the labour, fertilizing material and seed cane to the fields, and to bring the harvest in to the mill, and it is for this traffic that the plantation railways are built.

As will be seen in the table attached, the tonnage handled in the grinding season, which lasts from about the middle of November to the end of April, rises as high as four hundred and seventy-five tons per day, and throughout the year from two to five hundred men are carried both ways daily. The tonnage taken off a field of plant cane will be about forty-five tons per acre, while a field of ratoon will not yield much over twenty tons per acre. On some plantations where the soil is richer, these figures are much larger. The maximum length of haul at Kilauea is about seven and a quarter miles.

The railroad system is designed to reach every part of the cultivatable land, and consists of portable, semi-portable and pemanent track.

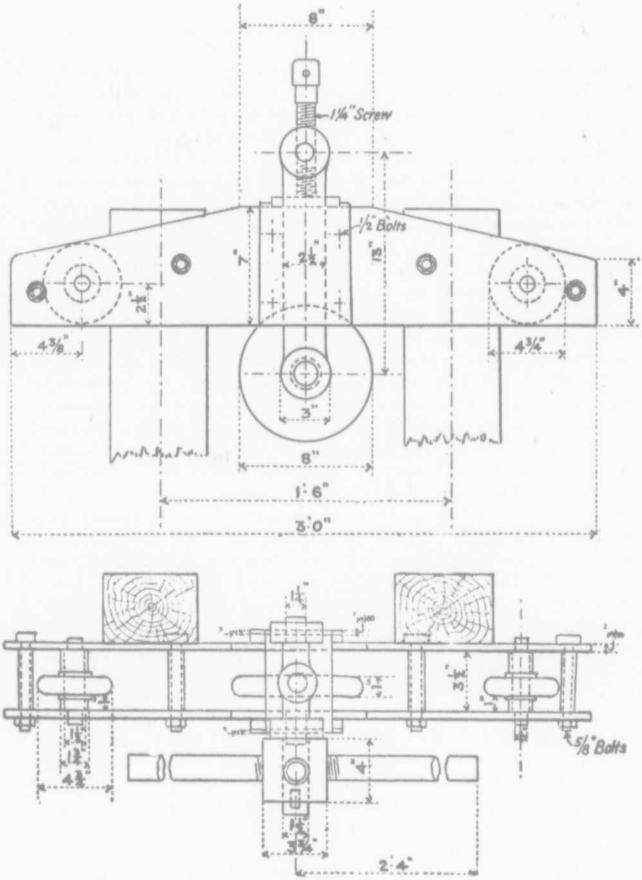
The permanent track is the main track of the plantation, and requires the most careful location and construction. The ruling principles in the location are that the grades coming from the fields must be kept as light as possible, and that the centre line must be made to correspond very closely to the country. This latter principle is dictated not only for the purpose of reducing first cost, but also because it is necessary in order to reach the fields that spurs of portable track can be thrown off at any point along the main line, and where the construction grade is not close to the surface, this would involve either heavy earthwork for temporary purposes, or heavy grades on the spurs. Again, it is essential that the irrigation system shall be interfered with as little as possible, and this necessitates a grade line, either in filling or close to the surface, so that the ditches may be carried under the track without altering their grades, or else in sufficiently heavy cutting that an overhead flume may be built.

As an illustration, an extension of the main line about 3.9 miles long, may be taken; it extends for about a mile through the roughest ground on the whole line, being along the face of a very steep side hill, cut up with numerous ravines and water courses, and covered with heavy boulders, the remainder being across the cane fields. The maximum grade from the field to the mill on this section is one per cent., the maximum in this direction on the whole line being one and an eighth per cent. The loads coming out from the mill being light, no particular effort is made to reduce the grades in this direction, the maximum being two and three-quarters per cent. The gauge is twenty-four inches, and on account of this and of the low speed at which the trains are operated, about ten miles per hour, it is possible to use very sharp curvature, the shortest radius on the main line being seventy feet, while curves of a hundred feet radius are frequent. The alignment was a series of short tangents connected by short curves of small radius; on the 3.9 miles, the maximum tangent was about five hundred feet long, and in case of reverse curvature, an effort was made to establish one hundred feet as the length of the minimum tangent, but in a few cases there was no straight track at all between reverse curves. These curves were nearly all run in by the ordinary field methods, with chords of twenty-five feet, the comparatively small central angle of the curves preventing the discrepancy between the arcs and the chords from materially affecting the result. The curves in many cases could have been traced directly, using the chain as a beam compass, had it not been for the great irregularity of the ground. Frequent use was made of the curve ranger, especially during the track laying. This little instrument is designed upon the principle of the sextant, and when set it is necessary to be exactly upon the curve in order to see the images of pickets set at the B. C. and E. C. in coincidence with each other.

The road-bed of the permanent way is made eight feet wide on the fills and ten feet wide in the cuts, the slopes being one and a half to one for the former and one to one for the latter. The grading was done by the plantation labourers, most of whom were Japanese coolies, who worked under a piecework system at the rate of fifty cents per day's labour. That is, that where the work to be done was simple casting, such as building a fill from side ditches, or making side hill cuttings, about four hundred cubic feet was laid out by the overseer as a day's work, and the labourer got no credit upon the time books until this was completed, a feat which was usually accomplished in eight hours. Where the material excavated had to be hauled, the coolies were worked in gangs of four or five, with an equipment of cars and rails, and the amount of work laid out per man varied from two to four hundred cubic feet, being dependent upon the nature of the soil in the cutting, and the length of the haul. No wheelbarrows or scrapers were used, as it proved more economical to lay down sections of the portable plantation track, which will be described later, and to furnish each gang with small four-wheeled trucks carrying boxes about four feet square and one foot deep, which contained about one cubic yard when properly loaded. The loaded cars can be moved very rapidly, and are easily handled by two men. When being loaded they are run as close as possible under the working face, and the earth is shovelled down into them, a method much superior in a labour-saving way to that generally followed in railroad cuttings operated with track. This method of handling material is also in very common use for the building of reservoir embankments and similar earth works, and is found to be very economical when the haul does not exceed five or six hundred feet. This limit was, however, much exceeded in places along the line, the haul being sometimes as great as a mile, and in these cases a temporary wooden trestle was built, and the fills made up with large cars run by locomotives from borrow pits. The excavation was carried on in these borrow pits, which had a face of eight to ten feet, by under-cutting the face for about two feet in, and then driving in a line of crow bars from the surface parallel to and about three feet away from the face. When these were driven into a depth of three feet, and used as a lever, the whole mass broke away down to the undercutting, and was broken up ready for the shovel by the fall. By this method the men could loosen up from five to six hundred feet per day.

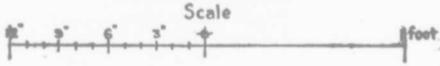
The hillside along which the line ran was so steep in places that it was necessary to terrace the slope in order to make a foothold for the filling material, and in some instances the slope of the fill was cut off by building up a dry retaining wall to grade, the masonry being similar to that used for open culverts, to be described later. In these cases the wall was rather light, but the bottom portions of the fill between the natural slope and the retaining wall was made of fragments of boulder stone, much of which was hand laid.

The line is laid with steel rails, part of which weigh twenty-one pounds to the yard, and the remainder twenty-three pounds. Owing to the excessive curvature, it was necessary that the rails should be specially bent for the curves, and this is done upon the mounted rail-bender, drawings for which accompany the paper. Briefly described, the machine consists of three rollers, the rail being laid on its side and made to pass under the end rollers and above the centre one, this latter roller being capable of being raised or lowered at will by means of a screw. It was found that with these light



RAIL BENDER.

Scale



rail sections the setting of the centre roller to give any required ordinate in the centre after passing through the machine had to be varied, as some rails bent more readily than others, and the men handling the machine varied the setting to some extent by trial. After the roller was set the rail was drawn through the machine by turning the centre roller by hand levers, and the rail was thus curved throughout its entire length. The rail was T in section, and was laid directly upon the cross ties. These cross ties were 4 inches x 6 inches x 5 feet, and were put down at about eighteen inches centre to centre. On the 3.9 mile extension the ties were of Lihue wood, a native tree, which grows in great abundance in the mountains a few miles from the track. Lihue wood is a heavy dark red hardwood, which checks too readily to be used for lumber, but which proves to be an excellent cross tie and one from which it is very difficult to draw the spikes. These ties cost ten cents a piece delivered at the track. Formerly the cross tie in most general use was an imported California redwood tie, but this cost twenty-four cents delivered, and was quickly rotted out by the rust at the spike holes. The track when laid is lifted and surfaced with ordinary earth ballast, the elevation on the curves being calculated for a speed of ten miles per hour.

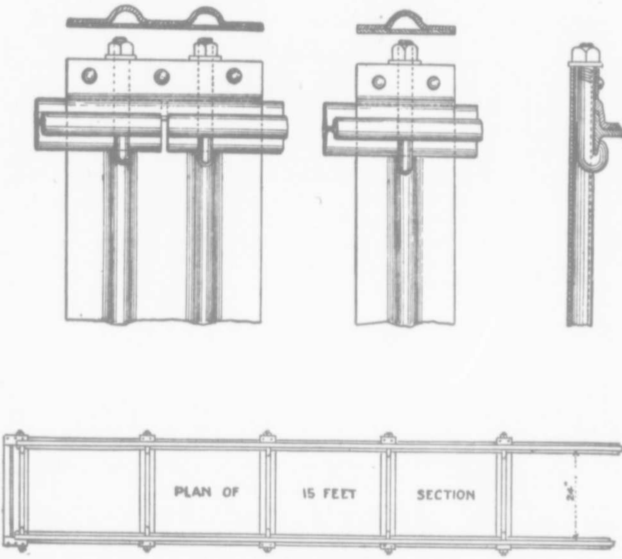
The drainage of the road bed has to be very carefully attended to, and provision made for carrying off large quantities of surface water during the heavy tropical rains, which frequently occur. In Kilauea on one occasion nine inches of rain fell in seven hours, and on another seven inches fell in three hours. Such rains turn every ravine into a rushing torrent, and unless ample provision is made for the passage of the water through the track, serious washouts would be the inevitable consequence. The accompanying sketch of an open culvert during the latter of the two rain storms referred to will give some idea of how the water rises in one of these storms. The dotted line represents the high water mark, and the heavy one the ordinary flow. The bed of the stream below the culvert was clear of all obstructions, and very steep, so that the water was not backed up in any way. At all the ravines of any size open culverts were put in from six to twelve feet wide at the bottom, and five to fifteen feet deep. The walls of the culverts were of dry stone masonry three feet wide on the top and vertical at the back, but with a batter of three inches to the foot on the side towards the opening. From the ends of these walls wing retaining walls were run back at an angle of about forty-five degrees till the foot of the slope was reached. The stone used was hard grey volcanic rock, which lay around in large boulders all along the track, and was cut into convenient sizes with plugs and feathers and roughly dressed with stone hammers.

OPEN CULVERT.

In the cuts, ditches a foot and a half to two feet wide and a foot deep are dug on both sides of the track, and carefully graded so that the water flows freely in them. Along the side hill similar ditches are dug on both sides and open stone drains are put across the track every four or five hundred feet, and an ample opening made for the water to escape down on the lower side.

The cost of grading, sleepers and track-laying on this section was $3,520 per mile. The reason that the cost of rails is not included is that they were bought at different times as required, and the price varied considerably.

At various places along the main line, branch lines are graded to some of the fields which lie at a distance from the main track. These vary in length from a few hundred yards to a mile and a half. The maximum grade with the load is three and a quarter per cent., and on one of them there is a grade against the load of two per cent. These lines are only used for a few weeks at a time, while the cane in the fields to which they lead is being cut. They are called semi-portable tracks for the reason that the rails are moved from place to place as needed. The rail used is eighteen pounds to the yard in twenty foot lengths, and is spiked to light redwood sleepers, so that when the fish plates are removed the sections can easily be lifted and piled on to flat cars and taken away to the next place required, and there be laid down again. On these semi-portable lines the curves are as far as possible of the same radius, so that the curved sections will readily fit any curve. At the end of the semi-portable track the locomotives stop, and beyond there the cars are hauled over portable track by bullocks or mules. The portable track is of light rail, fourteen pounds to the yard, in sections fifteen feet long, bolted to light steel ties, as shown in the accompanying drawings. The joints are made by means of a

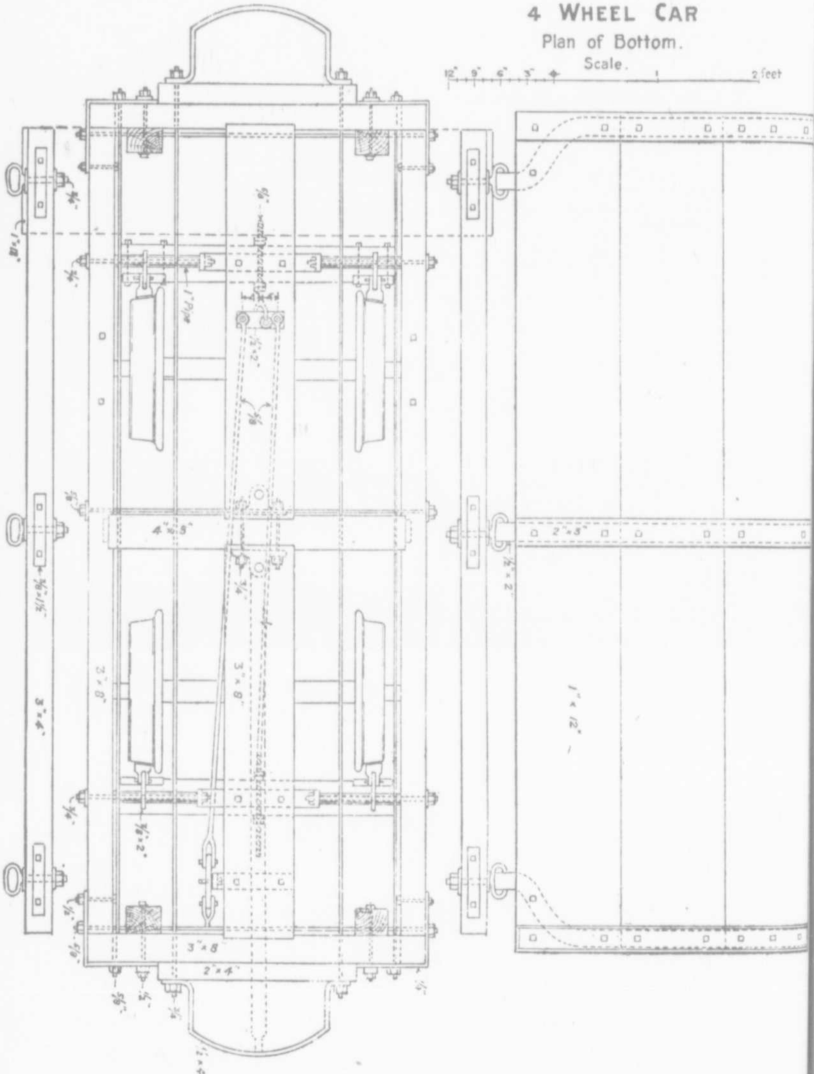


PORTABLE TRACK.

double tie to which the ends of both sections are bolted. A section weighs about one hundred and seventy-five pounds, and can therefore be readily carried by two men. In the fields the cane leaves are removed, the ground roughly levelled with hoes, and the track laid down. For handling the rails small trucks are used, each capable of carrying three hundred feet of track, and these are moved along as fast as the track is laid, so that it never has to be carried far by hand. A main line is run down the top of a ridge, with branches to both sides about two hundred feet apart. Split switches are used, and the points are thrown over by hand in a manner similar to ordinary street railway switching. In laying these tracks the best available grades are used, without resorting to cutting and filling, and the men working on them acquire great skill in picking out the best location for the lines. The grades, of course, are at times excessive; in fact, the tracks are often laid to places which seem at first sight to be completely inaccessible, such expedients as switchbacking down the side of a gulch to reach

4 WHEEL CAR

Plan of Bottom.



the cane in the bottom of it being by no means uncommon. There is practically no limit to the grades used on these switches, and runaways occasionally occur, the best expedient for stopping which is to cover the track ahead of the cars with the cane leaves, which are lying about everywhere to the thickness of perhaps a foot. In case the runaways are empty, a very successful expedient is to drop the shutters on both sides, and these dragging along the ground will nearly always bring the cars to rest. When the grade is against the load it is not a serious matter, as the brakes are usually sufficient to let the empties down safely, and hauling the full cars up is merely a question of more bullocks or mules, but when the grade is with the load it is often very difficult to keep the cars under control. Various means are employed to overcome this difficulty. One is to keep the rails on the steep grades well sanded, which greatly increases the friction when the brakes are applied with sufficient force to skid the wheels, but where the cars are hauled by bullocks by far the safest method is to take a team of ten or twelve yokes and hitch it on behind, not in the usual way, but so that they travel in the same direction as the cars, and the pull tends to draw the yokes over their heads. As soon as they feel the yokes against their horns they pull back, and the cars can be safely lowered down almost any grade. The haul by portable track varies from a few hundred feet to a maximum of about a mile. At all places where branches of portable or semi-portable track leave the main line, split rail switches are put in. These are made just the length of a rail, and are riveted to ties made of boiler plate, so that they can be put down or taken up in a very short time.

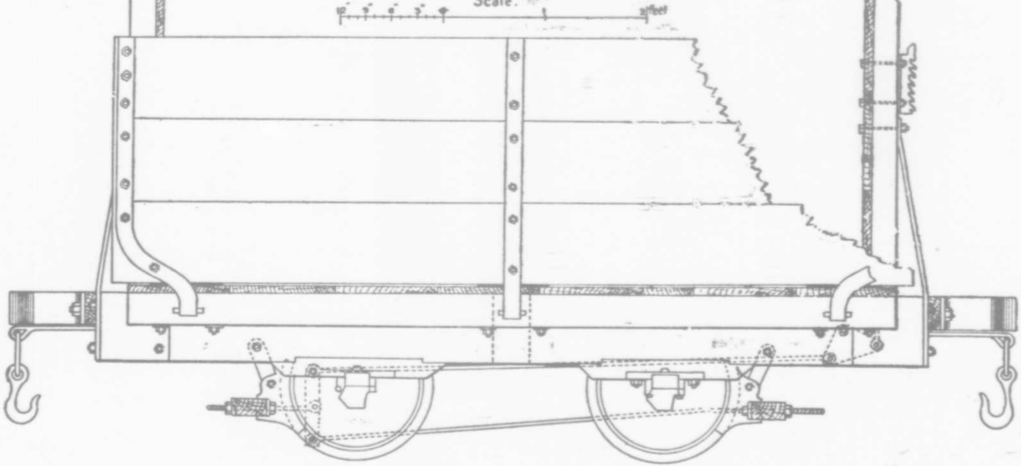
The rolling stock consists of three locomotives and one hundred and thirty cars. The three locomotives are of different designs and weights, the heaviest being a Baldwin double-ender, weighing thirty-one thousand pounds, with inside wheels and a rigid wheel base of three feet six, and capable of hauling trains of from twenty to twenty-five loaded cars, the others being a Fowler engine, weighing fifteen thousand pounds, with a wheel base of three feet six, and having a forward truck only, and a German engine weighing fourteen thousand pounds, with a wheel base of three feet four, and having, like the Fowler, a forward truck only. The two latter haul from ten to twelve loaded cars.

The cars are of two general types, there being seventy-two eight-wheeled cars and forty-eight four-wheeled ones. The eight-wheeled cars are imported from England, weigh about twenty-five hundred pounds, and carry an average load of 3.7 tons, as given in the statistics of 1898-99, the proportion of dead load to total load being one to four. The four-wheeled cars, which are built upon the

4 WHEEL CAR

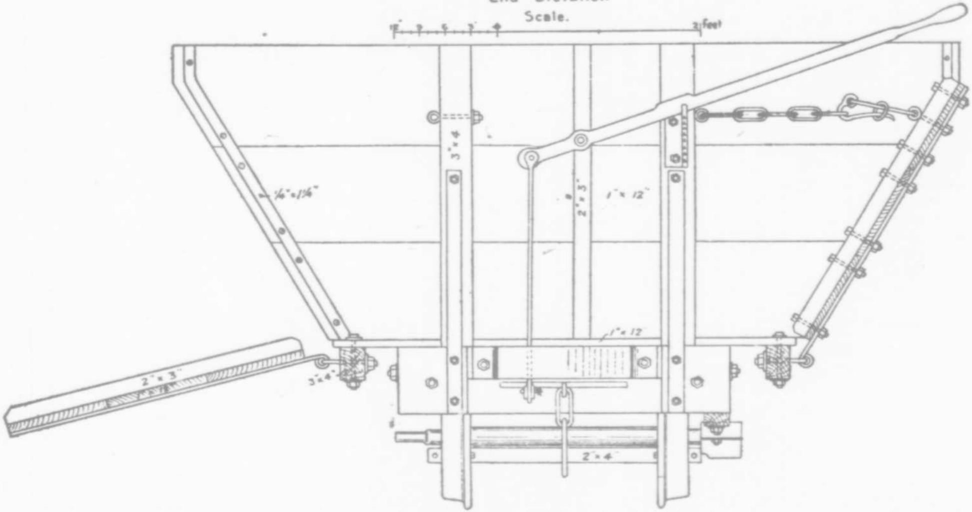
Side Elevation.

Scale.



4 WHEEL CAR

End Elevation



plantation, weigh about sixteen hundred pounds, and are preferred in operation because of their light weight, which makes them easy to handle on the steep grades, and easy to replace on the somewhat irregular track, on which derailments are by no means unknown. They are of simpler construction than the eight wheelers, and are more cheaply kept in working order. They carry, according to the 1898-99 figures, a load of 2.36 tons, the proportion of dead weight to total load being, as in the case of the eight wheelers, one to four.

The method of loading the cars with cane is rather peculiar. Boards two by twelve inches and about eight feet long, with cleats nailed across them and having iron hooks at one end, are hung from the shutters. The men gather the cane in armfuls, and walk up these boards, and throw it into the bed of the car until it is full; after this the cane is laid alternately lengthwise and across the car, and the whole is pinned together with long straight sticks of cane. In this way the load is built up to a height of six or eight feet above the bottom of the car. When the cars arrive at the mill they are weighed, and then run on to a track alongside the cane carrier, which consists of two endless chains with slats between them about five feet long; this revolves and feeds the cane to the rollers, which crush it. The track beside the cane carrier is about six feet above it, and when the cars arrive there the shutters on the side towards the carrier are dropped, and men standing on a platform on the opposite side pull the load down by means of long poles with hooks on the ends.

The accompanying table shows the costs and quantities for the 1898-99 crop. The columns of cost include under the head of "loading" the actual cost of the labour engaged in loading the cars; under "track," all expenses of laying the spurs of portable and semi-portable track, and of lifting them after the cane was loaded, and under "hauling," all expenses of the trains and of the bullock teams used on the switches. The latter item amounts to about ten or twelve cents per ton of cane on each of the fields, the variation in this column being due to the varying length of haul on the main track. In the track column the difference of cost per ton of cane on ratoon and plant fields is particularly noticeable.

TABLE SHOWING COSTS AND QUANTITIES OF THE 1898-99 CROP.

Crop 1898-9.		Haul. in Miles by Locomotive.	Acres	Tons of Cane.	Cost of			Cost per Ton Cane.			Total.	Tons of Cane per Acre.	Average Tons Cane per Car.	Tons. Cane per Day.	No. of Days.	Acres per Day.
Fields.	Cane.				Loading.	Track.	Hauling.	Loading.	P Track.	Hauling.						
Lower Waiakalua.	Plant.	4.3	306	13,502.7	$2,358.77	$ 786.73	$1,801.23	c. 17.5	c. 5.8	c. 13.3	c. 36.6	44.1	3.04	456.6	29	10.55
Pilaa	"	6.0	305	13,748.4	2,613.84	1,038.69	2,467.13	19.0	7.6	17.9	44.5	45.1	2.87	416.6	33	9.24
Lepeuli	"	7.2	206	8,037.3	1,534 07	679.30	1,799.60	19.1	8.4	22.4	49.9	39.0	2.72	446.5	18	11.4
Upper Waiakalua	Rattoon.	4.3	282	6,157.2	1,108.24	578.42	942.39	18.0	9.4	15.3	42.7	21.8	2.99	397 2	15½	18.2
Store....	"	0.0	196	3,684.9	701.78	385.84	366.53	19.0	10.5	10.0	39.5	18.8	2.80	350.9	10½	18.7
Namahana.......	"	0.5	310	5,389.8	970.94	528,79	503.69	18.0	9.8	9 4	37.2	17.4	2.90	359.3	15	20.7

Thursday, 9th November.

P. W. St. George, Vice-President, in the chair.

Arrangements for the Annual Meeting were fully discussed.

On motion by Mr. C. de B. Leprohon, seconded by Mr. L. Skaife, it was resolved "that the Council be asked to extend the time for another list of officers and members of Council to be presented, by one month, that is to 15th December.

On motion by Mr. J. G. Kerry, seconded by Mr. G. H. Duggan, it was resolved, "that in the opinion of the meeting it is desirable that the Council should send out a notice to members, asking for opinions as to the most suitable time of the year for a Convention or Annual Meeting."

On motion by Mr. W. J. Sproule, seconded by Mr. C. de B. Leprohon, it was resolved, "that the Council be asked to have the Annual Meeting held the last Tuesday in January."

Thursday, 23rd November.

P. W. St. George, Vice-President, in the chair.

Messrs. H. Irwin and W. C. Thomson, having been appointed scrutineers, reported that His Excellency the Earl of Minto, Governor-General of Canada, had been unanimously elected an Honorary Member of the Society.

Professor Owens delivered a lecture upon "The Transmission of Electrical Power."

Thursday, 7th December.

P. W. St. George, Vice-President, in the chair.

Presentations to the Library were reported as follows:—

"The Graphical Solution of Hydraulic Problems," from Mr. F. C. Coffin, M. Can. Soc. C. E.

"Graphical Analysis of Roof Trusses," from Mr. A. Crumpton, A. M. Can. Soc. C. E.

Paper No. 147.

TRANSITION CURVES.

By E. S. M. Lovelace, B.A.Sc., A. M. Can. Soc. C. E.

Some years ago the writer published a pamphlet on the "Lemniscata as a Transition Curve," a copy of which is in the possession of the Society. The pamphlet was rather favorably reviewed in the "Railroad Gazette," June 30th, 1893, and in "Engineering News," November 2nd of the same year, and as a result a good many copies were sold to engineers in different parts of the United States.

As was indicated, however, in the articles in the above-mentioned journals, and as the writer himself realized shortly after the book was printed, the tables given in it were not sufficiently extensive, while the general formulae giving the elements of the proposed curve were perhaps a little too complicated to lend themselves readily to work in the field, and consequently the pamphlet was not as successful as had been confidently hoped it would be. Before submitting to the Society the modified set of formulae which the writer now proposes to use, it may be of interest to look at the simpler way in which those given in the pamphlet might have been written, but which unfortunately was not discovered until too late.

Referring to the accompanying diagram and calling

the distance measured along chord A P	= C
the distance measured along curve A S P	= S
the offset distance H V	= K
the radius of main curve O P	= R
the deflection angle P A N	= θ

the formulae given in pamphlet are as follows:—

(1)″ $\text{Sin } 2\theta = \dfrac{C}{3R}$

$$(2'')\quad AH = \frac{C}{6}\left(\frac{2+\cos 2\theta}{\cos\theta}\right)$$

$$(3'')\quad OH = \frac{C}{6}\left(\frac{2-\cos 2\theta}{\sin\theta}\right)$$

$$(4'')\quad AB = OH \tan\frac{1}{2} + AH$$

$$(5'')\quad S = C + \frac{C^3}{90\,R^2} + \frac{C^5}{1944\,R^4}$$

In place of the above write:—

$$(1')\quad \sin 2\theta = \frac{C}{3\,R}$$

$$(2')\quad AH = \frac{R}{2}\,(3\sin\theta + \sin 3\,\theta)$$

$$(3')\quad OH = \frac{R}{2}\,(3\cos\theta - \cos 3\,\theta)$$

$$(4')\quad AB = \frac{R}{2}\sec\frac{I}{2}\left[\,3\sin\left(\frac{I}{2}+\theta\right) - \sin\left(\frac{I}{2}-3\theta\right)\right]$$

$$(5')\quad S = \frac{C}{1-\frac{2}{5}\theta^2} = C + \frac{20}{3}\frac{K^2}{C}$$

$(6')$ deflection to point distant l along curve from $A = \frac{\theta l^2}{S^2}$

Equation (4) is the first expression giving the value of A B in one simple formula, which the writer has seen. It can be used to special advantage when the curves of an existing track are to be eased off in such a manner that the total length of the track remain the same and, so, no rails will have to be cut. To insure this, looking at the Fig. on page 218 it will be necessary that:

$$R'\,\frac{I}{2} + \left(AB - R'\tan\frac{I}{2}\right) = S + R\left(\frac{I}{2} - 3\theta\right)$$

where R' = radius of existing track

$$\text{that is } R'\left(\frac{I}{2} - \tan\frac{I}{2}\right) = S - AB + R\left(\frac{I}{2} - 3\,\theta\right).$$

substituting for S and A B, their values given by (5)′ and (4)′.

$$R = \frac{R^1\left(\frac{I}{2} - \tan\frac{I}{2}\right)}{\frac{3\sin 2\theta}{1-\frac{2}{5}\theta^2} - \frac{1}{2}\sec\frac{I}{2}\left[\,3\sin\left(\frac{I}{2}+\theta\right) - \sin\left(\frac{I}{2}-3\theta\right)\right] + \left(\frac{I}{2}-3\theta\right)}$$

finally, assume a convenient value for θ and solve for R.

While this is rather a formidable looking equation to work out,

it can easily be seen that the 1st and 2nd quantities in the denominator multiplied by R at once give S and A B, so that nearly everything for the subsequent laying out of the curves is found by the one calculation.

In the spring of 1894 the writer in a letter to the "Editor Engineering News" suggested that the above might be an improvement on those given in the pamphlet, and received a reply from the late Mr. A. M. Wellington which is interesting as showing the strong practical grasp he had of all engineering subjects. He states in his letter: "I thought at first you had this simplified into an acceptable form, and, you will see, intended to publish it, but I fear it is still too complex to make it suitable to lay before engineers as a practicable and practical method. A suggestion! Forget the lemniscata or any other particular curve—Assume a curve in which A S P and H V bisect each other N A P = T P A = 2θ and V P P′V′ is the original main curve of any length whatever, and you have something that can be called general and proper. Then assume your curve V V′ already run, and H V to be assumed at discretion, and work out all the other elements for laying out the curve either by homologous offsets from tangents and main curve, or by angles from P or A. All this is simple. I have done it and used it, but I have no time to put it in shape. Do it, and you will do a good work. Less won't do—too complex for no gain."

The writer has tried to work out a curve using the assumptions suggested by Mr. Wellington, but has not on the whole succeeded in obtaining as simple a set of formulae as may be derived from those numbered (1)′ to (6)′, and which may therefore fairly be called formulae for *modified Lemnicasta.*

Looking at equation (1)′ and remembering that 2θ is always a little greater than Sin 2θ and S a little greater than C, within the limits over which θ will likely range, the curve represented by (1)′ will be very slightly altered by writing:

$$2\theta = \frac{S}{3R} \quad \text{or} \quad \theta = \frac{S}{6R} \qquad \text{(a)}$$

By equation (3)′ OH = R + K = $\frac{R}{2}$ $(3\cos\theta - \cos 3\theta)$

expanding $\cos\theta$ and $\cos 3\theta$ into a series it will be found that

$$K = \frac{3}{2} R\,(\theta^2 - \theta^4),$$

as K for modified will always be slightly greater than K for true Lemniscata θ^4 can be discarded and therefore

$$\theta^2 = \frac{2K}{3R} \quad \text{or} \quad \theta = \sqrt{\frac{2K}{3R}} \qquad \text{(b)}$$

combine (a) and (b) $\therefore K = \frac{S^2}{24\ R}$ (c)

combine (a) and (c) $\therefore \theta = \frac{4\ K}{s}$ (d)

By (5)′ $C = S - \frac{2}{5} S \theta^2 = S - \frac{2}{5}\ S \frac{16\ K^2}{S^2} \ = S - \frac{32}{5}\ \frac{K^2}{S}$ (e)

Looking at diagram,— $AH = C \cos \theta - R \sin 3\ \theta$

$$= C\left(1 - \frac{\theta^2}{2}\right) - R\left(3\ \theta - \frac{9}{2}\ \theta^3\right)$$

$$= C\left(1 - \frac{8K^2}{s^2}\ \right) - \frac{S}{2} + 12\ \frac{K^2}{s} \text{ by (a) and (d)}$$

$$= C - \frac{8K^2}{S} - \frac{S}{2} + 12\ \frac{K^2}{S} \text{ very nearly.}$$

$$S - \frac{32}{5}\ \frac{K^2}{S} - \frac{8K^2}{S} - \frac{S}{2} + \frac{12\ K^2}{S} \text{ by (e}$$

$$\therefore AH = \frac{S}{2} - \frac{5}{2}\ \frac{K^2}{S} \quad \text{(f)}$$

Equation (a) may be written

$$\frac{\theta}{60{,}57.30} = \frac{S}{6\ \frac{5730}{D}}$$

where θ is the number of minutes contained in the angle, and $D = \frac{5730}{R}$

That is θ in minutes $= \frac{S\ D}{10}$ a very important formula, and the one that would likely generally be used in calculating θ.

The complete set of formulae for modified lemniscata are therefore:

(1) $K = \frac{S^2}{24\ R}$

(2) $\theta = \frac{S}{6\ R} = \frac{4\ K}{S} = \sqrt{\frac{2\ K}{3\ R}}$

(3) θ in minutes $= \frac{S\ D}{10}$ where $D = \frac{5730}{R}$

(4) $AH = \frac{S}{2} - \frac{5}{2}\ \frac{K^2}{S}$

(5) $C = S - \frac{32}{5}\frac{K^2}{S}$

(6) deflection to point distant l from $A = \theta \frac{l^2}{S^2} = \theta'$

do do $2l$ do $= 4\theta \frac{l^2}{S^2} = 4\theta'$

do do nl do $n^2 \theta \frac{l^2}{S^2} = n^2 \theta'$

Each of the above equations contains three unknown quantities, any two of which may be assumed at will, so that the curve is extremely flexible and can be made to fulfil almost any conditions.

The correction $\frac{5}{2}\left(\frac{K^2}{S}\right)$ for the distance A H given in equation (4) need only be used when K is fairly large. It is the want of such a correction that has caused trouble (in the case of high degrees of curve) when the otherwise admirable method (first made known to the Society by Mr. Wicksteed) is used.

The accuracy of the above formulae can best be illustrated by an example taken at random.

Let $I = 102° \, 0'$

" $R = 410.65$ corresponding to about a 14° curve

" $K = 27$

by (1) $S^2 = 24 \, K \, R$

$\therefore 2 \log S = \log 24 + \log K + \log R \quad \therefore S = 515.85$

$\therefore \log S = 2.7125244$

by equation (3) $\theta' = \frac{SD}{10} = \frac{515.85}{10} \times \frac{5729.58}{410.65}$

$\therefore \log \theta' = \log s + \log 5729.58 - \log 410.65$

$\therefore \log \theta' = 2.857\,1733$

$\therefore \quad \theta = 719.736$ minutes $= 11° \, 59' \, 44''$

$$\frac{K^2}{S} = \frac{27^2}{515.85} = 1.413$$

$\therefore$ by (4) $AH = \frac{S}{2} - \frac{5}{2} \frac{K^2}{S} = 257.925 - 3.533 = 254.39$

by (5) $AP = S - \frac{32}{5} \frac{K^2}{S} = 515.85 - 9.04 = 506.81$

$A B = (R + K) \tan \frac{I}{2} + A H$

$= 437.65 \tan 51° + 254.39 = 794.85$

now $AA' = 2\ AB \cos \frac{I}{2} = 2,794.85 \cos 51^\circ = 1000.43$

but AA' should also $= 2\left[AP\left(\cos 51^\circ - \theta\right) + R \sin\left(\frac{I}{2} - 3\theta\right)\right]$

$$= 2\ (506.81 \cos 39^\circ\ 0'\ 16'' + 410.65 \sin 15^\circ\ 0'\ 48'')$$

$$= 2\ (393.84 + 106.38)$$

$$= 1000.44$$

proving that all the equations given above are almost exactly right, and that if chained correctly the curves would close precisely.

In conclusion the writer would draw attention to a rather peculiar coincidence in connection with the curve represented by the above formulae.

As already mentioned, a short review of the writer's pamphlet was given in "The Railroad Gazette," June 30th, 1893, while in the same journal's issue of August 4th, 1893, there appeared an article entitled: "A New Transition Curve,—The Lemniscata," by Charles H. Tutton, Dept. Public Works, Bureau of Engineering, Buffalo, N.Y.

Mr. Tutton was unaware that anything had been contributed on this curve prior to his paper on the subject, but, hearing in November of the same year that such was not the case, at once wrote, enclosing his own article and asking for the writer's in exchange.

In his letter acknowledging receipt of this, Mr. Tutton gives some very interesting information in connection with his own work in the matter.

He states: "I investigated the curve several years ago, and wrote quite a letter to Prof. Jamieson over two years ago on it, immediately following his series of articles on the Cubic Parabola in the 'American Engineer and Railroad Journal.'

"Its investigation started from Prof. Airy's article in vol. 1 of Van Nostrand's Magazine for 1867, when a set of tables for the Cubic Parabola were given."

"The close approximation of the deflections to an arithmetical progression at once struck me, and for a time I worked on the idea of developing a curve whose equation could be written $2\ K\ \ \theta = S$ where $K =$ some constant, or in plain English one whose deflection angles from the main tangent were in direct arithmetical progression for equal distances chained along the curve. This curve comes *very close* to the lemniscata, but I have never been able to throw it into what you would term usable shape. Another curve that promised very fair results could be written $2\ K \tan \theta = s'$ and can be plotted by laying off the tangents of the angles in arithmetical progression for equal area."

"I think, altogether, the lemniscata is the preferable curve for field use to any that I know of. I have seen handbooks on transition curves alone that were larger than the entire field book (Henck for instance) usually carried by the engineer, and my experience is that while in the multitude of counsellors there may be safety, yet in the multitude of tables there is a decided chance of error, increasing in rather more than arithmetical progression."

After making some further comparisons Mr. Tutton concludes: "I am busier now on sewers and pavements than on track problems, but if you have leisure I have an idea that the development of the curve $2\,K\theta = S^2$ might prove interesting, as it is flatter at the beginning and sharper at the centre than the lemniscata."

The coincidence the writer spoke of consists in the fact that the curve represented by the above equation is identical with that dealt with in this paper, and called for want of a better name the "modified Lemniscata." Thus, Mr. Tutton's foresight in the matter has been, in the writer's opinion, amply justified.

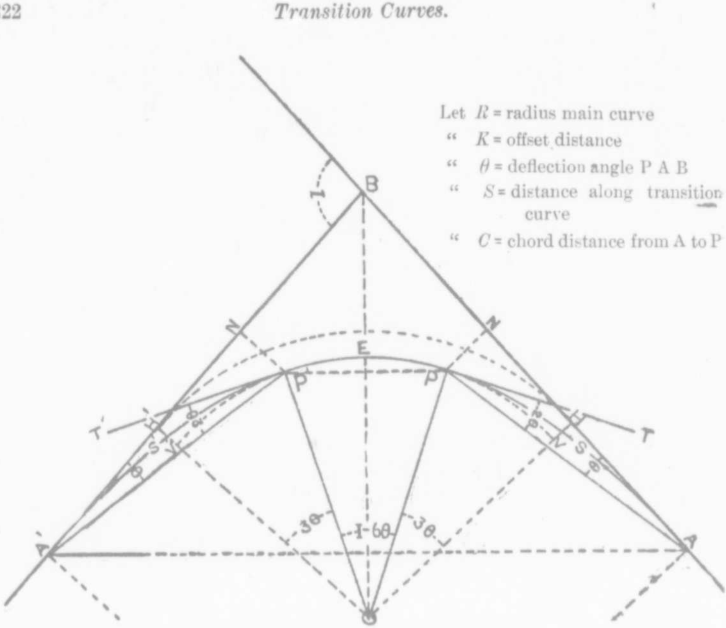
That the two curves are one and the same can easily be seen for calling the deflection angle to a point on curve , and the deflection angle to the point where transition joins main curve θ the corresponding distances along curve being S and S'.

by (6) $\theta = \dfrac{\theta' S^2}{S'^2}$

$$\therefore S^2 = \frac{S'^2}{\theta'} \quad \theta = 2\,K\,\theta \text{ where } 2\,K = \frac{S'^2}{\theta'}$$

The writer has not touched upon the question of the actual laying out of the curves in the field, for the reason that this has been very fully entered into by Mr. Wicksteed and others, whose methods and conclusions apply also to the curve which the writer has attempted to describe.

The only contribution, therefore, that the writer has to offer to the Society is a complete set of properly tabulated general formulae, which will ensure the transition and main curves closing under any and all conditions.



Let R = radius main curve
" K = offset distance
" θ = deflection angle P A B
" S = distance along transition curve
" C = chord distance from A to P

Formulae for modified Lemniscata.

(1) $K = \frac{S^2}{24\,R}$

(2) $\theta = \frac{S}{6\,R} = \frac{4\,K}{S} = \sqrt{\frac{2\,K}{3\,R}}$

(3) θ in minutes $= \frac{S\,D}{10}$ where $D = \frac{5730}{R}$

(4) $A\,H = \frac{S}{2} - \frac{5}{2}\;\frac{K^2}{S}$

(5) $C = S - \frac{32}{5}\;\frac{K^2}{S}$

(6) Deflection to point distant l from A = $\theta \frac{l^2}{S^2} = \theta'$

" " " " $2l$ " " $= 4\,\theta \frac{l^2}{S^2} = 4\,\theta'$

" " " " $3l$ " " $= 9\,\theta \frac{l^2}{S^2} = 9\,\theta'$

DISCUSSION.

MR. J. H. KENNEDY said that the Author was to be congratulated on getting on the right track in the development of the curve $2 K\theta = S^2$, and a set of tables calculated for this curve would be a valuable acquisition to railway engineers. Mr. Kennedy.

He was, however, of the opinion that the suggestion given the Author by Mr. A. M. Wellington, was the most practical idea that he had yet seen, and he had on many occasions made the same assumptions, and figured out the intermediate points and staked them on the ground with considerable success.

Some years ago he was placed in charge of construction of some railway work in Dakota, which had already been located in this way, viz.:

The tangent being run up to the point H, a hub was then placed back at A for the $B\ C$ of the transition. Then an offset $H\ V$ was measured from H and the curve started from V instead of from H. At the distance $V\ P$ on the curve equal to $A\ H$ a hub was placed for the $B\ C\ C$ of the main curve, and at the end of the main curve the operation was reversed and the offset measured from the $E\ C$ to the next tangent. The hubs at A and P and also the offset HV, were recorded in the transit notes, and no attempt was made to run in the transitions. As a matter of fact the transitions were never run in until wanted for track centres after the grading was completed. A blue print giving a table of offsets and a somewhat formidable looking formula for calculating the intermediate points on the transition curves was furnished, but it was seldom on hand when wanted, and it was on this work that he had made use of the method suggested to the Author by Mr. Wellington.

The offset HV was bisected for the central point, and generally only one other point in each direction from this central point being necessary, these were run in by transit from the points A and P respectively, by assuming $APT = 2\ HAP$ also $APT + HAP = P\ T\ N =$ angle subtended by the arc $V\ P$ which was known.

This, of course, could be only an approximation to a mathematically correct curve, but the result seemed to give satisfaction, and he believed that something definite, useful and practical might yet be developed along this line.

The very practical way in which the provision was made for the transitions in the original location in the case here related must impress engineers very favourably, as all would agree that, to be satisfactory, provision for easing off curves must be made in some way in the original location.

Thursday, 21st December.

P. W. St. George, Vice-President, in the chair.

Professor Owens delivered the second of a series of lectures upon "The Transmission of Electrical Power."

Messrs. Stuart Howard, H. A. Budden, J. S. Costigan and A. R. Archer, having been appointed scrutineers of the ballot for the election of members reported the following elected:—

MEMBER.

C. N. Coburn.

ASSOCIATE MEMBERS.

A. Campbell.
H. T. Hughes.
T. L. Simmons.
C. Thomson.
C. H. Topp.

Transferred from the Class of Associate Member to the Class of Member:—

T. W. Lesage.
J. S. Vindin.

Transferred from the Class of Student to the Class of Associate Member:—

F. A. Creighton.
A. T. LeFevre.
M. C. Macfarlane.
Angus Smith.

STUDENTS.

N. McL. Campbell.
F. de C. Davies.
J. A. Fraser.
J. D. Mackerras.
V. D. Tilley.

INDEX TO THE TRANSACTIONS.