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DLXXXII.*

NOTES ON THE THEORY OF SHAFT GOVERNORS.

BY ALBERT K. MANSFIELD, SALEM, OHIO.

(Member of the Society.)

AT our spring meeting of 1890, at Cincinnati, three papers on shaft governors were presented, which, together with the extended discussions thereon, make a valuable contribution to the literature of this subject.

It is the purpose of this paper to add something to that discussion, with the hope of getting a little nearer to a correct solution of the perplexing problems surrounding the subject.

The matter to be discussed will be divided into several topics, for the sake of clearness.

1. *The path of an unbalanced governor ball of a shaft governor, isochronously adjusted.*

This problem was proposed by Professor Sweet in one of the papers referred to, and in an elaborate mathematical analysis Prof. S. W. Robinson seems to prove that the centre of the approximately circular path is vertically above the centre of the shaft, and that the distance apart of these centres may be determined from the formula $h = \frac{9.78}{n^2}$; h being the distance sought, in inches, and n being the number of revolutions per *second.*

Confirmatory of this result, an experiment was made with an unbalanced weight, arranged according to the requirements of the problem. The figure illustrating the experimental apparatus is here reproduced (see Fig. 1).

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* Presented at the Montreal meeting (June, 1894) of the American Society of Mechanical Engineers, and forming part of Volume XV. of the *Transactions.*

The spring adjustment was such that this apparatus was thought to be isochronous at 555 revolutions per minute, and the experimental determination gave $h = 0.126$ inch (mean of several trials), while by the formula h becomes 0.114 inch—a fairly close agreement.

Following this a table of values of h was given, derived from the formula, revolutions per minute being taken at from 1,200 as a maximum down to 60; h in the former case being by calculation 0.02 inch, and in the latter 9.78 inches.

Suppose this table to be extended to cover slower speeds than sixty revolutions per minute, even down to one revolu-

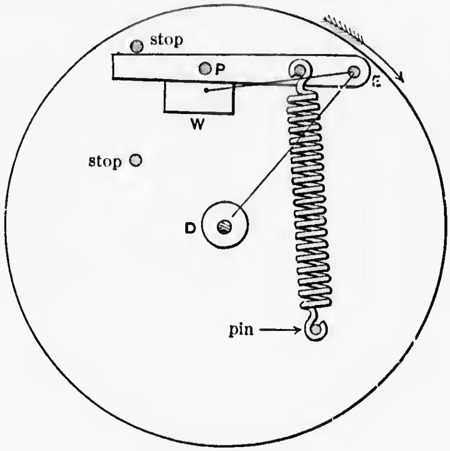


FIG. 1.

tion per minute, h would be found at this speed to be, by the formula, 35,200 inches, or 2,933 feet—more than half a mile above the centre of the shaft.

This extreme result is noted merely as a curious matter of interest.

The formula is doubtless correct, as deduced from the assumption on which it is based; but let us examine the assumption.

Referring to the figure of Professor Sweet's paper, here reproduced (Fig. 2), Professor Robinson says: "Suppose, to start with, that the weight B is at j, moving along a horizontal portion of arc. The action of gravity tends to deflect it downward instead of allowing it to move along a circular path concentric with the shaft, thus giving the mass B an accelerated motion relative to the wheel A, and along the radius AB, so that by the time the weight reaches a it will have a considerable velocity toward the centre C. From this point on gravity counteracts, and on reaching d will have destroyed the radial velocity toward C, when B will again be moving horizontally, or perpendicular to the radius, but will be at a point nearer the centre C than when at j. Now, from this point on a radial acceleration will occur, so that at g the weight will be moving outward with a radial velocity which, from g on to j, will again be destroyed by gravity, thus bringing the weight to rest on the radius at j, though at a greater radial distance from the centre C than at any point before in the revolution, and putting the weight in the position and condition supposed at the start, when it will go on in repetition of the curve as the next turn of the shaft is made, and so on, continuously, the curved path described being found to be nearly a circle, with its centre elevated above that of the shaft."

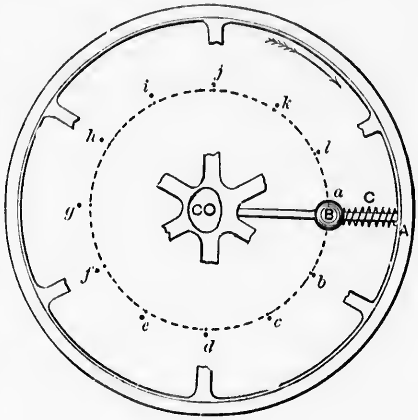


Fig. 2.

The questionable part of this reasoning lies in the first sentence: "Suppose, to start with, that the weight B is at j, moving along a horizontal portion of arc." Under this supposition what follows is justified; but the supposition is only one of an indefinite number that may be made with equal correctness, each leading to a different conclusion.

For example, suppose we start the analysis from the point a, the weight being assumed to have no radial movement at that point, and the motion to be right-handed. Then, during the entire first half-revolution, an accelerating force is drawing the weight away from the centre, always faster, until at g this force becomes zero, and the velocity outward is uniform. During the next half-revolution, a force of equal effect acts to draw the weight toward the centre, but this force must be entirely expended in overcoming the velocity of the weight outward, which it had when at the point g, therefore the movement of the weight is outward during the whole revolution. The same action occurs in succeeding revolutions, and the weight describes a spiral outward, finally reaching its outer stop.

If we start the analysis from g, we find by similar reasoning that the weight describes a spiral inward.

The reasoning in the case of starting from g or a, which does not permit the ball to return to its starting point, is found to be rational, when compared with the case of any weight moved in a straight line, with no resistance except that of inertia, by an accelerating force, and stopped by an equal retarding force. The weight comes to rest, and no work is gained or lost, yet the weight is found in a new position.

Again, if we consider the weight to start, with no radial movement, from any other points intermediate between its positions, a, g, and j, d, it will be found to describe a spiral outward when the first position is taken at the right of the axis, and inward when at the left. These spirals are, as will be seen by consideration of the forces of inertia, in no case regular spirals, but are merely of spiral nature, not re-entering.

The conclusion to be derived from this analysis seems to be that the problem has no true solution, or if any expression based on correct reasoning could be found for the curve, it would be irrational.

Moreover, under the conditions of perfect isochronism assumed, one would be led to expect the motion of the ball to be erratic; as, for instance, it might first move in the outer spiral, when, reaching the outer stop, it may be compelled to move horizontally at the point j, which might start it in the eccentric circle. Slight disturbing influences, as of the atmosphere, would probably change it from this to other of its paths. This expectation seems to be to some extent verified by the experiment of Professor Robinson, for we find in his table of results the remarks "spiral inward," "spiral outward," "steady," etc.

The practical conclusion to be derived from this analysis is that an unbalanced weight in an isochronous shaft governor is not feasible.

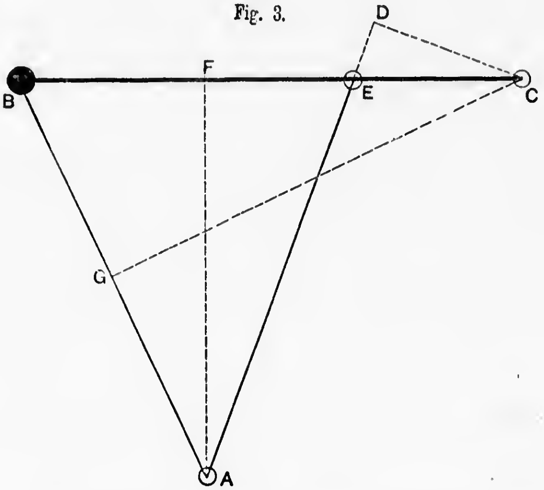
2. *Centrifugal moment of a governor weight.*

Theorem: The centrifugal moment about the weight pivot, like the simple weight moment, is the same wherever on the line through the centre of gravity of the weight to the centre of the pivot the weight be considered as concentrated.

Let A, Fig. 3, be the centre of the shaft, B the centre of gravity of the weight, and C the pivot centre or fulcrum.

In considering simple weight leverage about C, the effective weight may be assumed to act at any point on the line $B\,C$, as at

Fig. 3.



E, in which case the weight at E is to the actual weight at B as $B\,C$ to $E\,C$. The centrifugal moment of weight B for any given number of revolutions is proportional to the radius $B\,A$ times the line $G\,C$, drawn at right angles to $B\,A$ through C. The centrifugal force of the resultant weight E is similarly $A\,E$ times $D\,C$; $D\,C$ being drawn at right angles to $A\,E$ prolonged.

According to the theorem,

$$A\,B \times G\,C \times C\,E = A\,E \times D\,C \times B\,C,$$

the lines $C\,E$ and $B\,C$ being the relative weights in the two cases.

Proof: Draw the line $F\,A$ at right angles to $B\,C$. Then we have the similar triangles $A\,F\,B$ and $C\,G\,B$; also $A\,F\,E$ and $C\,D\,E$,

from which $AB : AF = CB : CG,$
and $CE : CD = AE : AF.$

Multiplying the equations together, and equating product of means to product of extremes, we have

$$AB \times GC \times CE = AE \times DC \times BC$$

as asserted.

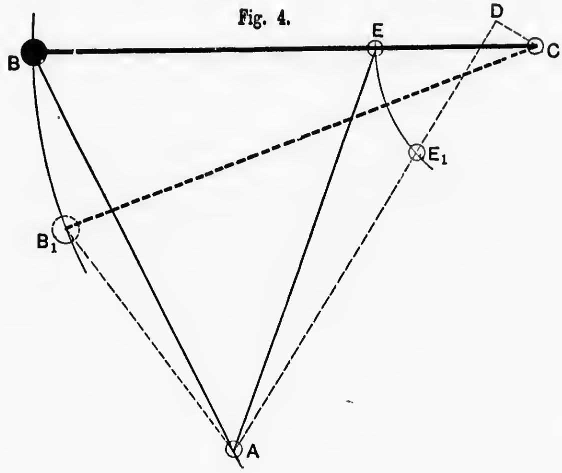
This goes to show that the attempt, which is evident in many shaft governors, to so design them that the centre of gravity of the weight shall move nearly in a radial line is unnecessary. Wherever the weight be concentrated on the line BC, provided its amount be sufficient, the result is the same whether the arc described approaches a radial line or not.

This demonstration leads up to another interesting detail, which seems to show that shaft governors are not always arranged for true isochronism.

3. *Position and tension of spring.*

In Fig. 4 let A be the centre of shaft, B the centre of gravity

Fig. 4.

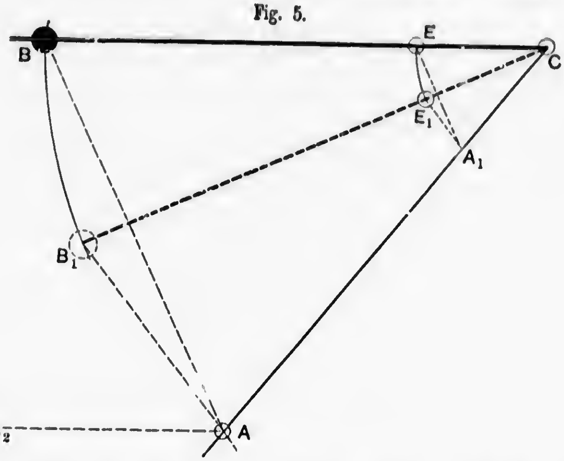


of weight, and C the weight pivot, as before. Since the line AB may be taken as a measure of centrifugal force of B, then a line from A to B at any other position of B in its arc will be, in an isochronous governor, assuming uniform velocity, a corresponding measure of the centrifugal force of B in this new position, as the line $A\,B_1$ for the position B_1. Suppose a pull spring pivoted so as to swing about the point A, and to be pivotally attached at its outer end to the centre of gravity of B; also, suppose the spring to be of such tension and strength as to exactly balance the centrifugal force of B. Suppose, also, that the line $A\,B$ represents the total extension of the spring—*i. e.*, that when the end B of the spring is at A the force of the spring is zero—then the spring, from the laws of spring tension, will just balance the centrifugal force of the weight B in other positions, as at B_1. The arrangement is therefore isochronous, for centrifugal and centripetal forces are exactly opposed to each other in direction and amount.

Take any point E, as before, on the line $B\,C$, and draw the line $A\,E$; consider $A\,E$ to represent a spring pivotally supported at A and pivotally connected at E, and of such force as to counterbalance the centrifugal force of B, or of its resultant weight at E. Then, if the zero of tension of this spring is at A, it will counterbalance correctly the centrifugal force of the weight at all other positions, as at E_1, for the spring lies in the line of action of centrifugal force, and its leverage $D\,C$ about C is the same as that of the centrifugal force. From which it is clear that a spring pivotally adjusted at A, and pivotally connected to the weight-arm *at any point* [illegible] the line $B\,C$, is correctly located to produce exact iso[illegible]

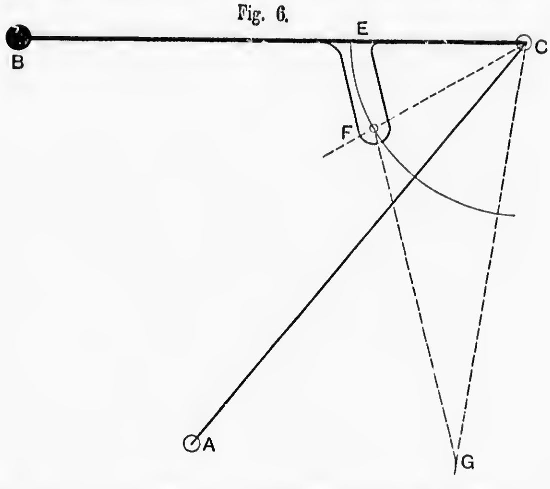
In Fig. 5 let A, B, B_1, and C [illegible] haft, weight, and pivot, as before. Draw lines $A\,C$ a[illegible] raw an arc from C through any point E on the lin[illegible] ing $B_1\,C$ at E_1. From the point E draw $E\,A_1$, parallel to [illegible], and connect A_1 to E_1. The figure $A_1\,E\,E_1$ is exactly similar to the figure $A\,B\,B_1$, and corresponding sides of the figures are parallel to each other. It will be clear from the foregoing and from inspection, that if a spring be pivotally connected from A_1 to E, and has its zero of tension at A_1, and is adjusted to balance the centrifugal moment of the weight, it will balance it in all other positions, as at E_1. But the arc $B\,B_1$ need not have been drawn through the centre of gravity B, for, from the previous demonstration (see Fig. 3), it could have been drawn through any other point of line $B\,C$; therefore, the line $B\,A$ might have had any direction between that of the direction of the line $A\,C$ to that of the direction of a line $A\ B_2$, parallel to $B\ C$. Such an indefinite number of constructions would bring the point A_1 at any position on the line $A\ C$, or its continuation through A.

Fig. 5.



It has therefore been shown that *a spring pivotally swung at any point on the line $C\ A$, or its continuation through A, and pivotally connected to any point on $C\ B$ or its continuation through B, and*

Fig. 6.



having its zero of extension at the first-named point, is correctly placed to produce exact isochronism.

Referring to Fig. 6, it will be clear without demonstration that the point of connection to weight-arm need not be on the line $B\,C$. It may be anywhere on the weight-arm, as at F, provided a new zero line $G\,C$ be drawn angularly the same distance and direction from $A\,C$ as the line $F\,C$ from $E\,C$.

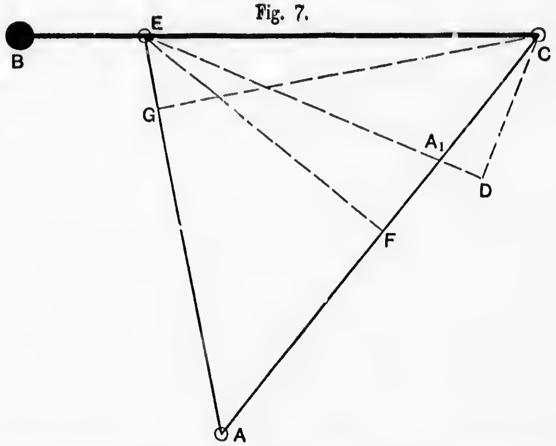
It follows that the point of connection of spring to weight-arm, and the direction of action of spring, *may be selected entirely at random*, or for convenience, provided only that the length between pivots and the tension of spring be fixed according to the principles laid down.

The conclusions arrived at by the preceding reasoning may be expressed in the form of a second

Theorem: The combined zero and fixed pivotal point of a spring, arranged to act isochronously on any point of the line of weight-arm from weight pivot through centre of gravity of weight, may be taken at any point on the line from weight pivot through centre of shaft. Moreover, the spring force required will be inversely as the distance of the fixed pivot from the weight-arm pivot.

In Fig. 7, letters $A\ A_1\ B\ C\ E$ represent the same parts as in Fig. 5. Drop perpendiculars $C\,G$ and $C\,D$ from C on lines $A\,E$

Fig. 7.



and $E\,A_1$ respectively; also from E drop the perpendicular $E\,F$ on line $A\,C$. Then similar triangles $A_1\,E\,F$ and $A_1\,C\,D$, as well as $A\,F\,E$ and $A\,C\,G$, are formed, from which proportions may be made as follows:

$$A\,E : E\,F = A\,C : C\,G,$$
$$A_1\,C : C\,D = A_1\,E : E\,F.$$

Multiplying together, equating products of extremes and means, and canceling EF, we have

$$AE \times CG \times A_1C = A_1E \times CD \times AC,$$

or

$$AE \times CG = A_1E \times CD \times \frac{AC}{A_1C}.$$

If we consider AC to be the unit of force of the spring when the fixed pivot is at A_1, and A_1C the unit of force when the pivot is at A, then this becomes intelligible.

It shows that the linear extensions of the springs AE and A_1E, multiplied by their leverages GC and CD and by the units of force of the springs, are equal. The points E and A_1 were taken at random, which makes the demonstration general.

A further consideration of Fig. 3 will show that when A is the fixed pivot the unit of spring force is inversely as the distance of the point of connection on line BC from C.

If, therefore, we have computed the centrifugal force of the weight B, we have merely to multiply this centrifugal force by the ratio $\frac{BC}{EC} \times \frac{AC}{A_1C}$ (see Fig. 5 for illustration) to find the corresponding balancing spring force; or to multiply the centrifugal force per inch of radius by this ratio to find the corresponding spring force per inch of extension. The linear extension of the spring was before shown to be EA_1.

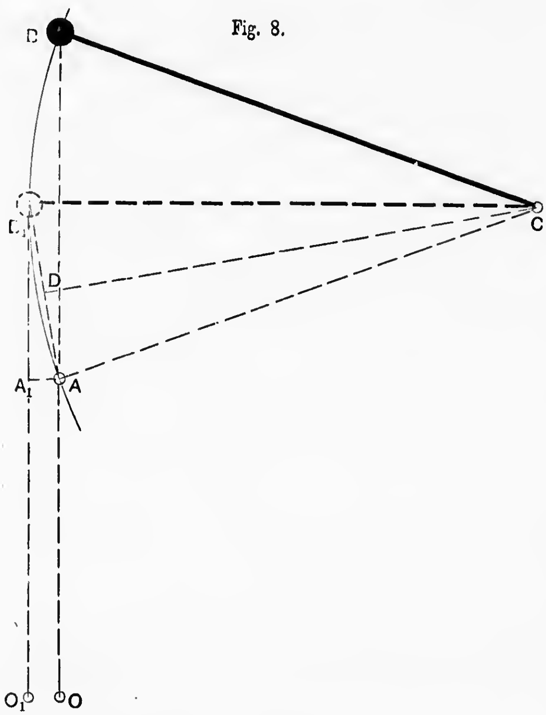
4. *Approximate isochronism.*

In Fig. 8 let ABC be the centres of shaft, of gravity of weight, and of pivot, as before. Draw a line from B through A to any point O. Let O be the fixed point of a spring pivotally attached to B, and having its zero of extension at A.

It is clear that the arrangement may be made isochronous for the two positions A and B. For the moment consider O to be infinitely removed from A, and investigate the mid-position of B at B_1. Draw B_1O_1 parallel to BO, and AA_1 parallel to B_1C; also AB_1, and DC at right angles to AB_1 through C.

Let AC be represented by R and the angle ACB_1 by a. Then the spring moment at B_1 is $R^2 \sin. a$, and the centrifugal moment is $2R^2 \sin. \frac{a}{2} \times \text{cosin.} \frac{a}{2}$, which expressions are equal to each other, by trigonometry. Therefore, with a spring so located and adjusted, a third point, B_1, is isochronous. If the point O is a finite distance from A, as shown, it will be found that there will still be a point B_1, near the middle of the arc $A\,B$, which will be isochronously balanced; other points, however, between A and B_1 will have their spring moment too small, and points between B_1 and B will have their spring moment too large.

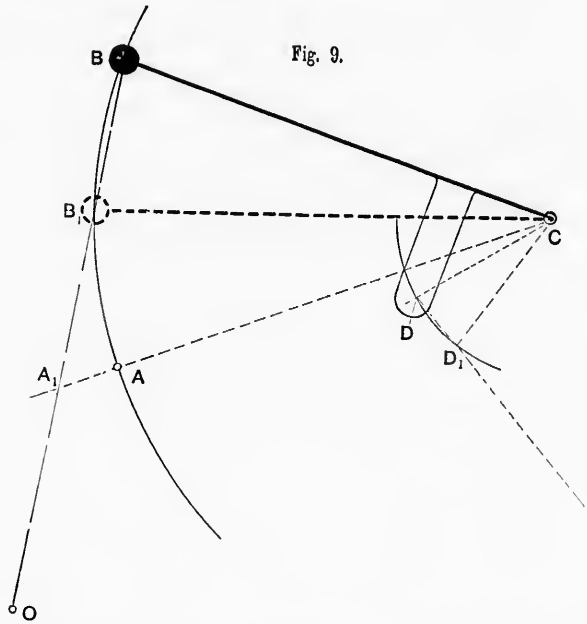
Fig. 8.



Suppose the spring to be arranged as in Fig. 9, the point B_1 being the inner or initial position of the weight B; then clearly, from previous demonstrations—O being the pivotal point of the spring—to produce isochronism at points B_1 and B, A_1 must be the zero point of the spring. It will be found as before that a point nearly midway between B and B_1 is also isochronous; also, if the angle $B\,C\,B_1$ is not large, the approach to complete isochronism is very close.

This corresponds to the arrangement commonly used in practice. Clearly, the arc $B\,B_1$ may be drawn in any other place from C as a centre, as at $D\,D_1$, the line of the spring being made to pass through these two points, and their angular distance apart being the same as that of B and B_1.

Fig. 9.



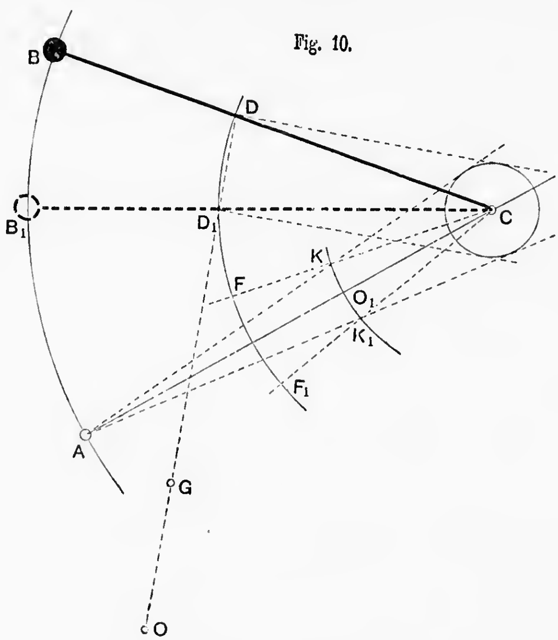
5. *Influence of the weight of the spring.*

Let the spring be applied, as in Fig. 10, and let G be its centre of gravity. Determine its moment m of centrifugal force about its pivot O, and divide by the length of the spring $D_1\,O$, which we will call l. Owing to the weight G being constant, and the direction of l practically always the same, $\frac{m}{l}$ is very nearly constant—m being the moment of centrifugal force and l the length of the spring—for any degree of extension between D_1 and D, and the quantity $\frac{m}{l}$ is the tangential force at D, due to the centrifugal force of the spring. Extend the arc $D\,D_1$ across the line $A\,C$, and lay off each side of the line to F and F_1 one-half the arc $D\,D_1$. Draw radial lines from C to F and F_1. Draw a circle through C whose diameter is the chord $D\,D_1$. From A

draw lines tangent to this circle, crossing the radial lines CF and CF_1 at K and K_1. Let d represent the distance AK.

Then, since $\frac{m}{l}$ is the force, $\frac{m}{ld}$ is the weight, which, concentrated at K or K_1—according to the position of the weight-arm BC—will produce a centrifugal moment about C almost exactly equivalent, at the three points K, K_1, and O_1, to the centrifugal moment due to the spring weight. Between the extreme positions K and K_1 and the central position O the action of such a

Fig. 10.



weight is not exactly equivalent to that of the spring, but with a moderate arc the error is extremely small.

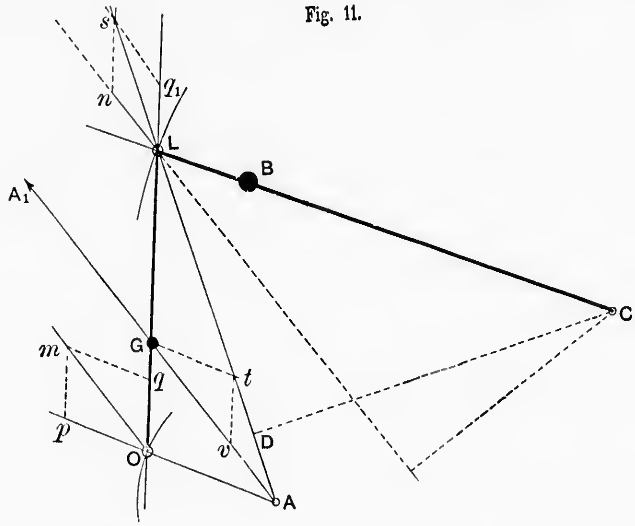
The object of determining the location and amount of the equivalent weight K is to find the influence of the spring on the location of a correct centre of gravity line BC.

6. *Influence of the weight of the link.*

Theorem: If a governor link be constrained to move at one end in a circular path about the centre of the driving shaft, and at the other end in a circular path about the centre of the weight pivot, then is the centrifugal effect of the link the same as if that portion of the weight of the link were concentrated at its weight-arm pivot, which would rest on its support if the link were placed in a horizontal position on two end supports.

In Fig. 11 let A be the shaft centre, C the weight-arm pivot, $L\,O$ the link, and G the centre of gravity of the link. $G\,A = G\,A_1$ may be taken to represent the centrifugal force of the link. Clearly, this force may be resolved into two forces, $O\,m$ and $L\,n$, the sum of which is equal to $G\,A$, while their ratio is as $G\,L$ to $G\,O$, and their direction of action parallel to $G\,A$.

Fig. 11.



Resolve $O\,m$ into the components $O\,p$ and $O\,q$, $O\,p$ having a radial direction from A, and $O\,q$ lying in the line of the link. Lay off $O\,q$ from L to q_1, in the direction of the link, and combine $L\,q_1$ with $L\,n$ by the parallelogram of forces, which gives the force $L\,s$ as the total resultant force of the link tending to rotate the weight-arm about its pivot.

Draw $L\,A$, also $G\,t$ parallel to $O\,A$, and $t\,v$ parallel to $O\,L$. By geometry $G\,A$ is divided at v, and $L\,A$ at t, in the same ratio as $L\,O$ at G; therefore, since $O\,m$ was made equal to $G\,v$ in amount and direction, the triangles $m\,O\,q$ and $G\,v\,t$ are equal. $L\,n$ was made parallel to and equal to $v\,A$, and $n\,s$ is parallel to and equal to $v\,t$ by construction; therefore the triangle $n\,s\,L$ is equal to the triangle $v\,t\,A$, and $L\,s$ is equal to $A\,t$ in amount and direction.

$$\text{But} \qquad \frac{A\,t}{A\,L} = \frac{O\,G}{O\,L}, \quad \text{or,} \quad A\,t = A\,L \times \frac{O\,G}{O\,L}.$$

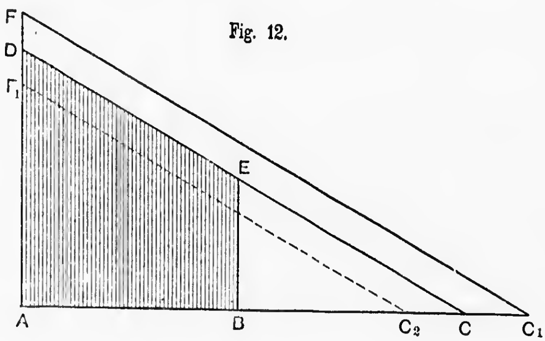
In other words, when the centrifugal force acting at G is represented by the radius $A\,G$, that acting at L may be represented by the radius $A\,L$, multiplied by a fraction which is the ratio of the weight G, which would be supported at L, provided the link were to rest in a horizontal position on two supports at L and O; which was to be proved.

If the centre of gravity of the link were at its centre, as is common, then it would be exactly right to consider one-half the link concentrated at L.

7. *Frictional effect of valve.*

In Fig. 12 let $A\,C$ represent the maximum tension of spring and $B\,C$ the tension to inner position of weight-arm. Let $A\,D$

Fig. 12.



and $B\,E$ represent the spring force corresponding to positions A and B of weight-arm. Assuming perfect isochronism between weight and spring, then $A\,D$ and $B\,E$ also represent the balanced centrifugal force, and this force for any intermediate position of weight-arm is the corresponding height from the line $A\,B$ to the line $D\,E$. Up to this time we have neglected the effect of valve-gear friction.

Supposing this effect to be a constant force acting in the same direction as the centrifugal force of the weight, then it may be represented by a line parallel to and above $D\,C$, as $F\,C_1$. If the constant friction of the valve-gear acts against centrifugal force, or with the spring, then $F_1\,C_2$, parallel to $D\,C$, may represent its

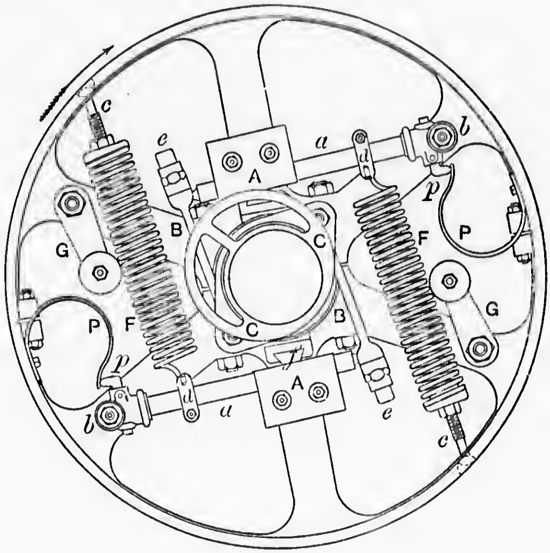
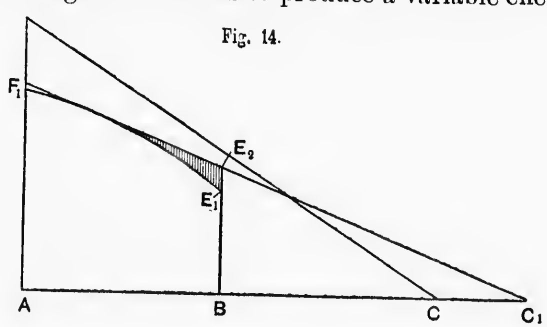


Fig. 13.

effect. In the former of these cases the maximum spring force becomes $A\,F$, and the maximum spring tension $A\,C_1$, while in the latter case these quantities become $A\,F_1$ and $A\,C_2$.

In some constructions the connection between the governor and the valve-gear is such as to produce a variable effect of fric-

Fig. 14.



tion. This is the case with the "Buckeye" governor, shown in Fig. 13, in which, moreover, this friction has a centripetal effect.

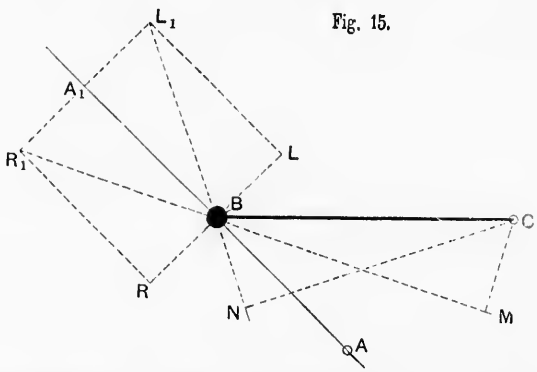
In Fig. 14 this variable resistance is illustrated by the curved line $F_1 E_1$.

By reference to Fig. 13 an auxiliary spring P will be seen, which is designed to act through a little more than half the range of the weight-arm, and to produce an effect illustrated by the shaded portion of Fig. 14. The result is that a line $F_1 C_1$ approximately straight, illustrates the centripetal action of the main spring, $B\,C_1$ being its initial tension, and $B\,E_2$ its total force at first stop, or initial position.

8. *Inertia in a shaft governor.*

In Fig. 15 let A and C be the centres of shaft and weight pivot, respectively, and consider the total effective weight of the governor weight and arm centred at B. Inertia acts on the weight B at right angles to the line $A\,B$.

Fig. 15.



From analytical mechanics (see Weisbach) the *force* of inertia may be represented by the expression $P = M\,R\dfrac{d\,\omega}{d\,t}$, while centrifugal force is $F = \omega^2\,M\,R$; in both expressions ω is the angular velocity, M the mass, R the radius $A\,B$, and $d\,t$ the small interval of time in which a change of velocity occurs.

Substituting for ω its equivalent value $2\,\pi\,T$, in which T is the number of revolutions per second, and differentiating the equation for centrifugal force, since it is only the difference of force due to change of speed which is effective, we have

$$P = 2\,\pi\,M\,R\frac{d\,T}{d\,t},$$

and $$d\,F = 8\,\pi^2\,M\,R\,T\,d\,T.$$

If $B\,L$ or $B\,R$, according to the direction of motion, represents the force P, and $B\,A_1$ the force $d\,F$, then $B\,L_1$ or $B\,R_1$ is the resultant of these forces, and the tangent of the angle $A_1\,B\,L_1$, or $A_1\,B\,R_1$, which angle we designate by α, is

$$\text{tangent } \alpha = \frac{P}{d\,F},$$

or, substituting above values,

$$\text{tangent } \alpha = \frac{1}{4\,\pi\,T\,d\,t}.$$

We see from this that the effect of inertia to increase or decrease (according to the direction of motion) the moment of

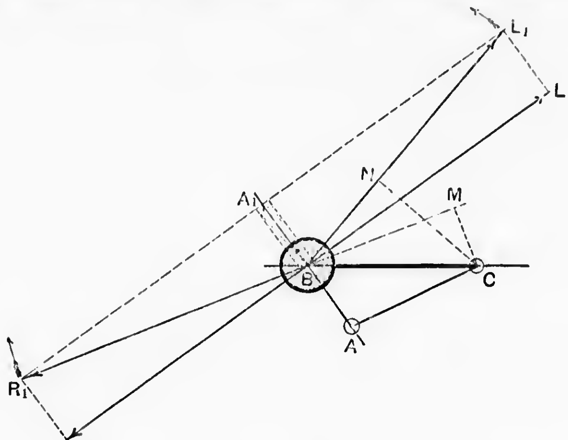


FIG. 16.

force about the weight pivot is less the greater the number of revolutions per unit of time, and is greater the less the interval of time in which the change of speed takes place.

Let us assume that the weight B is no longer concentrated in a point, but is spread out into a disk of considerable size, as in Fig. 16, whose radius we call r; then the force of inertia relative to the axis A is greater than before.

By a well-known law of inertia, the radius of gyration of the weight is $R + \frac{r}{\sqrt{2}}$, therefore the force of inertia acting at B is

$$P = \frac{\left(R + \frac{r}{\sqrt{2}}\right)^2}{R} M \frac{d\,\omega}{d\,t}.$$

Substituting $2\,\pi\,T$ for ω, and dividing by $d\,F$ as before, we have

$$\text{tangent } \alpha = \frac{\left(1 + \dfrac{r}{R\,\sqrt{2}}\right)^2}{4\,\pi\,T\,d\,T}$$

Suppose r to be $\frac{1}{3}$ of R, T to be three revolutions per second, and $d\,t$ to be one second; then tangent α becomes 0.0406, and α is less than $2\frac{1}{2}^\circ$. If $d\,t$ is [illegible] of one second, then α becomes about 22°, and if $d\,t$ is $\frac{1}{100}$ of one second, α is about 76°. The extremes of these three cases are shown graphically in Fig. 16 for both right and left hand motion. This illustrates to how great an extent, when changes of speed are sudden, inertia force may be useful to assist centrifugal force; also to what a slight extent inertia acts when changes are not sudden.

It also shows that if the direction of motion be badly chosen,

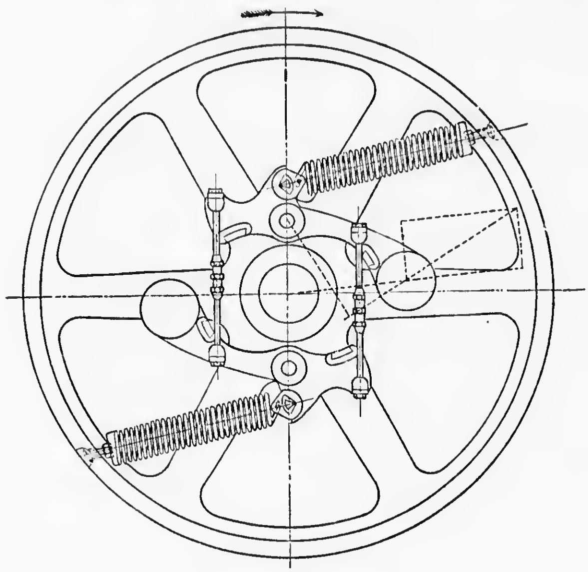


Fig. 17.

the combined forces may produce an instantaneous moment about the weight pivot in the wrong direction, thus interfering with sensitive governing.

As to the actual value of $d\,t$ in practice, it may often be a very small quantity, for in an engine having dead points the velocity changes a number of times, to a greater or less extent, during each revolution. These changes are less the heavier the fly-wheel, therefore with a light fly-wheel an inertia governor should be specially efficient.

In one of the papers referred to at the beginning of this article, Mr. Armstrong advocates the use of inertia in the way which would reduce the effective moment about C, for the sake of "stability."

The fact seems to be, however, that stability and sensitiveness are best arrived at by using the force of inertia to aid centrifugal force, as in the left-hand motion of Fig. 16.

Fig. 17 illustrates a governor for a single-valve engine—designed by Mr. J. W. Thompson—which is said to have performed so perfectly that no perceptible variation of speed in the range of the governor could be detected by careful test, and there was no trouble from racing.

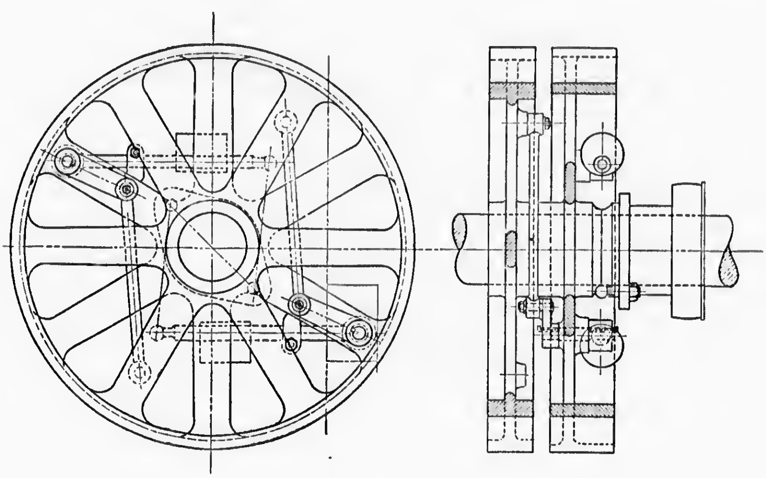


FIG. 18.

It will be noted that the arc through which the weight-arm moves is so small that isochronism could be practically perfect, while inertia was utilized to a great degree.

In Fig. 18 is represented a very ingenious method of combining a separate inertia weight with a shaft governor. This was applied by Messrs Bancroft & Lewis, of Philadelphia, to a Buckeye engine in the works of Wm. Sellers & Co.

The inertia weight consists of a wheel, which, being centred on the shaft, has its centrifugal force completely balanced, while its inertia force acts to *aid* the centrifugal force of the governor. Instead of causing racing this is said to have overcome all tendency to race, thus enabling the governor to be adjusted for practically perfect isochronism.

It is well known that an increase in the amount of the balanced forces—centrifugal and centripetal—of a governor tends to increase the effectiveness of the governor to overcome disturbing influences; yet an increase in these forces may produce an increase of friction in the pivots, which may defeat the desired object. The friction of pivots is not increased, however, by so designing the governor as to utilize inertia to aid centrifugal force.

In this respect the shaft governor may have a decided advantage over the old ball governor, which is purely centrifugal.

Referring back to Fig. 18, it is not essential that the inertia weight be centred on the shaft. It may be centred at the weight-arm pivot, thus forming a part of the weight-arm. If its centre of gravity is coincident with the centre of pivot it will not affect the centrifugal adjustment of the governor weight, but will aid the governing moment by its inertia.