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THE

GRAND TRUNK RAILWAY

TRANSITION CURVE

BY

E. S. M. Lovelace, B.A.Sc.
A.M. Can. Soc. C.E.

PREFACE.

As most engineers who have given the subject the slightest consideration have acknowledged the advantages to be derived from the use of transition curves in the location of a line of Railway, the writer has no intention of discussing the question further than to say, that as sectionmen almost invariably ease off the ends of the circular curves as staked out (causing thereby either absolute kinks or else portions of track of a much sharper degree of curvature than the main curve), it would seem to be the duty of the engineer to avoid such sources of danger to a train becoming derailed by locating the curve at once in the position which it will be made to take finally.

The trouble hitherto has been that the transition curves proposed have either been of so complicated a nature as to render their location very troublesome, or else, mere approximations which engineers instinctively object to.

The transition curve which the writer has undertaken to describe is mathematically exact, and its location requires very little more work, either mental or otherwise, than does that of an ordinary circular curve.

In preparing the following, the writer received a good many suggestions from the papers on the subject read before the Canadian Society of Civil Engineers, and also from the remarks of those gentlemen who took part in the subsequent discussions on such papers.

E. S. M. L.

54 St. Matthew St.,
Montreal.

CHAPTER I.

FORMULÆ.

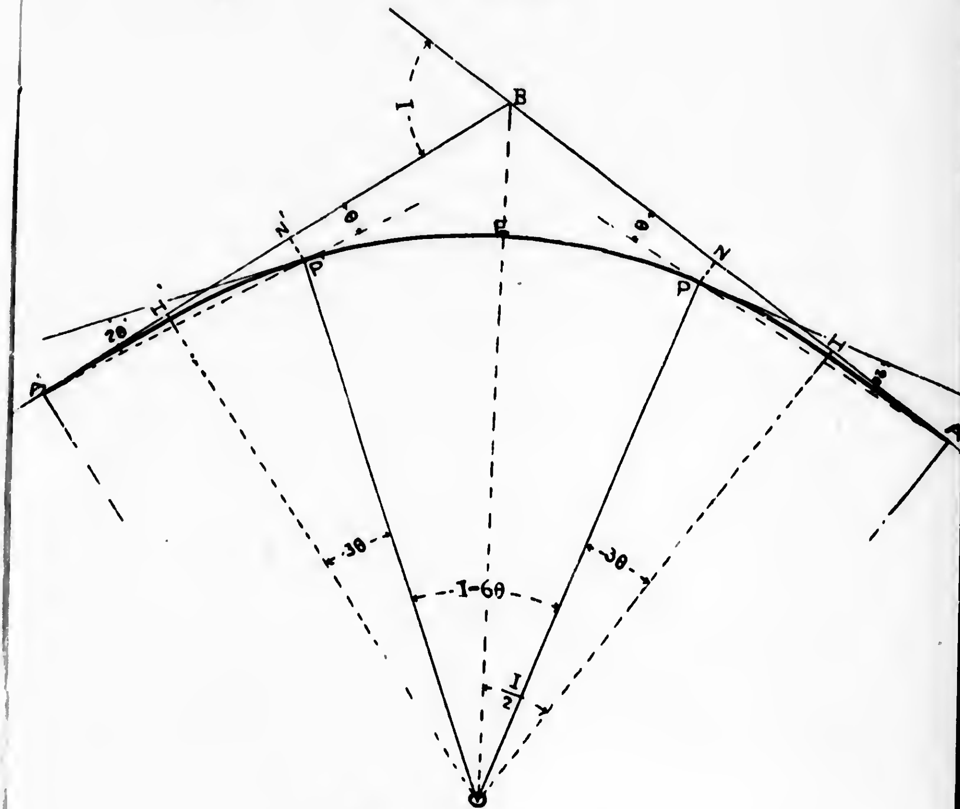

FIG. 1.

In Fig. 1 let AB and $A'B$ be two tangents, intersecting at B. It is proposed to connect them by the main curve PP', of radius OP or R and the transition curves AP, $A'P'$, the radii of which vary from R at the points P and P' to infinity at A and A'.

As most engineers are more concerned with practical results than with the theoretical transformations by which such results are obtained, all demonstrations, etc., have been purposely placed at the end of these notes, where all who choose may see for themselves that the following six equations (giving all the necessary information for the location of the proposed curves, and used in working up the table about to be described) are correct.

Assuming R, radius of main curve, and the constant number m, as known, then will AP, the chord of the transition curve, be equal to the constant number m divided by three times the radius of the main curve, or letting the chord $AP = c$.

(1) $$c = \frac{m}{3R}$$

(2) $$\sin 2\theta = \frac{c^2}{m}$$

the co-ordinates of the point P are

(3) $$\begin{cases} AN = c \cos \theta \\ NP = c \sin \theta \end{cases}$$

the co-ordinates of the point O are

(4) $$\begin{cases} AH = \frac{c}{6}\left(\frac{2 + \cos 2\theta}{\cos \theta}\right) & \text{or } AH = AN - R \sin 3\theta \\ HO = \frac{c}{6}\left(\frac{2 - \cos 2\theta}{\sin \theta}\right) & \text{or } HO = NP + R \cos 3\theta \end{cases}$$

the distance AB is given by

(5) $$AB = HO \tan \frac{I}{2} + AH$$

Letting $\frac{L}{2}$ equal the length of transition curve from A to P

$$(6) \qquad \frac{L}{2} = c + \frac{c^5}{10m^2} + \frac{c^9}{24m^4}$$

The constant number m, which determines the length of AP (the chord of the transition curve) can of course be chosen at will.

To get on tangent at the point P, sight to A and turn off twice the deflection angle to P, that is turn off 2θ

The central angle POH is always 3θ or three times the deflection angle to the point P.

The above equations also apply to any other point on the transition curve between A and P.

CHAPTER II.

TABLES.

Assuming that the chord distance to the point P is sixty feet for each degree of curvature at that point (which assumption makes the value of the constant number m equal to 1031337), the following table gives the deflection angles and all necessary data for a transition curve that can be applied to any main curve (from an 0° to a 10° one), the degree of curvature of which is some multiple of 5 minutes.

When the degree of curvature of main curve is not a multiple of 5 minutes, all the necessary data (for the point where the transition curve meets the main curve) can be found by interpolation in the table with the exception of the distance HO, which varies so rapidly that it must be calculated from equation (4). The deflection angles for points five feet apart on transition curve can still be taken direct from the table.

To use the table :—

Look in 1st column for quantity corresponding to degree of curvature of main curve, and on the same line with it under the several headings will be found the deflection angle and distance to point where transition curve meets the main curve and also the quantities to be used in calculating the distance AB.

In laying out the curve by means of offsets from the tangent the necessary distances will be found in columns 6 and 7.

In laying out the curve by means of deflection angles from the point A, take from the 3rd column the chord distance from A to a desired point on curve, or take from column 4 the distance along curve itself to the same point, and on the same line in column (5) will be found the corresponding deflection angle.

For example:—

Suppose a case in which the chainage of the point B is $105 + 59.3$; the intersection angle $70° \ 0'$; and D, the degree of curvature of main curve $6° \ 35'$.

In the table, on the same line with $6° \ 35'$, the value of AH is found to be 197.3, and the value of HO 877.8,

also $\frac{L}{2}$ is 395.9, θ is $4° \ 21'.04$, and the chord length AP is 395.

By equation (5) $AB = HO \tan \frac{I}{2} + AH = 877.8 \tan 35°$

$$+ 197.3 = 813.9 \text{ feet}$$

also as $(I - 6\theta) = 70° \ 0' - 26° \ 6'.24 = 43° \ 53'.76$, therefore length of main curve $= 666.8$

Consequently the chainage of the point

$$A \text{ is } 105 + 59.3 - 8 + 13.9 = 97 + 45.4$$

$$P \text{ is } 97 + 45.4 + 3 + 95.9 = 101 + 41.3$$

$$P' \text{ is } 101 + 41.3 + 6 + 66.8 = 108 + 08.1$$

$$A' \text{ is } 108 + 08.1 + 3 + 95.9 = 112 + 04$$

Having established the points A and A' in the usual way by chaining along the tangents from B, suppose the instrument is at A, and also suppose that stakes are to be placed 50 feet apart. Referring to table, it will be seen that the deflection angle for a point

50 feet from A	or for	chainage	$97+95.4$	is	$0° 4'$
100	"	"	$98+45.4$	is	$0° 17'$
150	"	"	$98+95.4$	is	$0° 38'$
200	"	"	$99+45.4$	is	$1° 7'$
250	"	"	$99+95.4$	is	$1° 44'$
300	"	"	$100+45.4$	is	$2° 30'$
350	"	"	$100+95.4$	is	$3° 24'$
P 395.9	"	"	$101+41.3$	is	$4° 21'$

Now move the instrument to A' and set out the transition curve from A' to P' in precisely the same manner, using the above deflection angles.

Finally set up at P', sight to A', turn off 2θ or $8° 42'$ to get on tangent, and run in the circular curve in the usual way, checking on P.

Inspecting the table, it will be seen that, when the distance is greater than 220 feet, there is an appreciable difference between the chord length to a point and the length along curve itself; therefore in locating points on the curve (as it would be inconvenient to chain these fractional differences) the deflection angles are added corresponding to distances which are multiples of 30, 50, 60 and 100 feet (the lengths usually chosen in spacing stakes), these distances to be measured along the transition curve itself.

In bending the rails, if the chord distance from A to a point be divided by sixty, the result will be the degree of curvature (in degrees) of the transition curve at that point.

TABLE.

Degree of Curvature D	Radius of Curvature R	Chord Length C	Length of Transition Curve. $\frac{L}{2}$	Deflection Angle θ	Co-ordinates of Point P.		Co-ordinates of Point O.	
					AN or x	NP or y	AH	HO
0° 0′	infinite	0	0.0	0° 0′	0.0	0.00	0.0	infinite
5′	68755.8	5	5.0	0.04	5.0	0.00	2.5	68755.8
10′	34377.9	10	10.0	0.17	10.0	0.00	5.0	34377.9
15′	22918.6	15	15.0	0.38	15.0	0.00	7.5	22918.6
20′	17188.9	20	20.0	0.67	20.0	0.00	10.0	17189.0
25′	13751.2	25	25.0	1.04	25.0	0.01	12.5	13751.2
30′	11459.3	30	30.0	1.00	30.0	0.01	15.0	11459.3
35′	9822.3	35	35.0	2.54	35.0	0.02	17.5	9822.3
40′	8594.5	40	40.0	2.67	40.0	0 03	20.0	8594.5
45′	7639.5	45	45.0	3.38	45.0	0.04	22.5	7639.5
50′	6875.6	50	50.0	4.17	50.0	0.06	25.0	6875.6
55′	6250.5	55	55.0	5.04	55.0	0.08	27.5	6250.5
1° 0′	5729.6	60	60.0	6.00	60.0	0.11	30.0	5729.7
5′	5288.9	65	65.0	7.04	65.0	0.13	32.5	5288.9
10′	4911.1	70	70.0	8.17	70.0	0.17	35.0	4911.2
15′	4583.7	75	75.0	9.38	75.0	0.21	37.5	4583.8
20′	4297.2	80	80.0	10.67	80.0	0.25	40.0	4297.3
25′	4044.5	85	85.0	12.04	85.0	0.30	42.5	4044.5
30′	3819.8	90	90.0	13.50	90.0	0.35	45.0	3819.9
35′	3618.7	95	95.0	15.04	95.0	0.42	47.5	3618.8
40′	3437.8	100	100.0	16.67	100.0	0.49	50.0	3437.9
45′	3274.1	105	105.0	18.38	105.0	0.56	52.5	3274.2
50′	3125 3	110	110.0	20.17	110.0	0.65	55.0	3125.4
55′	2989.4	115	115.0	22.04	115.0	0.74	57.5	2989.6
2° 0′	2864.8	120	120.0	24.00	120.0	0.84	60.0	2865.0
5′	2750.2	125	125.0	26.04	125.0	0.95	62.5	2750.5
10′	2644.5	130	130.0	28.17	130.0	1.07	65.0	2644.7
15′	2546.5	135	135.0	30.38	135.0	1.19	67 5	2546.8
20′	2455.6	140	140.0	32.67	140.0	1.33	70.0	2455.9
25′	2370.9	145	145.0	35.04	145.0	1.48	72.5	2371.3
30′	2291.9	150	150.0	37.50	150.0	1.64	75.0	2292.3
35′	2217.9	155	155.0	40.05	155.0	1.81	77.5	2218.4
40′	2148.6	160	160.0	42.67	160.0	1.99	80.0	2149.1
45′	2083.5	165	165.0	45.38	165.0	2.18	82.5	2084.1
50′	2022.2	170	170.0	48.17	170.0	2.38	85.0	2022.8
55′	1964.5	175	175.0	51.05	175.0	2.62	87.5	1965.1
3° 0′	1909.9	180	180.0	54.01	180 0	2.83	90.0	1910.6
5′	1858.3	185	185.0	57.05	185.0	3.09	92.5	1859.1

TABLE.

Degree of Curvature D	Radius of Curvature	Chord Length C	Length of Transition Curve. L 2	Deflection Angle θ	Co-ordinates of Point P. AN or x	Co-ordinates of Point P. NP or y	Co-ordinates of Point O. AH	Co-ordinates of Point O. HO
3° 10′	1809.4	190	190.0	1° 0′.18	190.0	3.33	95.0	1810.2
15′	1763.0	195	195.0	3.39	195.0	3.60	97.5	1763.9
20′	1718.9	200	200.0	6.68	200.0	3.88	100.0	1719.9
25′	1677.0	205	205.0	10.06	205.0	4.18	102.5	1678.0
30′	1637.0	210	210.0	13.52	210.0	4.49	105.0	1638.2
35′	1599.0	215	215.0	17.07	214.9	4.82	107 5	1600.2
40′	1562.6	220	220.0	20.70	219.9	5.16	110.0	1563.9
45′	1527.9	225	225.1	24.41	224.9	5.52	112.5	1529.3
50′	1494.7	230	230.1	28.20	229.9	5.90	115.0	1496.2
55′	1462.9	235	235.1	32.08	234.9	6.29	117.5	1464.5
			240.0	35.97	239.8	6.70		
4° 0′	1432.4	240	240.1	36.05	239.9	6.71	120.0	1434.1
5′	1403.2	245	245.1	40.05	244.9	7.13	122.5	1405.0
			250.0	44.15	249.8	7.57		
10′	1375.1	250	250.1	44.23	249.9	7.58	125.0	1377.0
15′	1348.2	255	255.1	48.45	254.9	8.04	127.5	1350.2
20′	1322.2	260	260.1	52.75	259.9	8.53	130.0	1324.4
25′	1297.3	265	265.1	57·13	264.8	9.03	132.5	1299.5
			270.0	2° 1.51	269.7	9.54		
30′	1273.3	270	270.1	1.60	269.8	9.55	135.0	1275.6
35′	1250.1	275	275.1	6.15	274.8	10.09	137.5	1252.6
40′	1227.8	280	280.2	10.79	279.8	10.65	140.0	1230.4
45′	1206.2	285	285.2	15.51	284.8	11.23	142.5	1209.0
50′	1185.4	290	290.2	20.32	289.8	11.83	145.0	1188.4
55′	1165.4	295	295.2	25.21	294.7	12.46	147.5	1168.5
			300.0	29.99	299.5	13.07		
5° 0′	1145.9	300	300.2	30.19	299.7	13.10	150.0	1149.2
5′	1127.1	305	305.2	35.25	304.7	13.77	152.4	1130.6
10′	1109 0	310	310.3	40.40	309.7	14.46	154.9	1112.6
15′	1091.4	315	315.3	45.63	314.6	15.17	157.4	1095.2
20′	1074.3	320	320.3	50.95	319.6	15.91	159.9	1078.3
25′	1057.8	325	325.3	56.35	324.6	16.67	162.4	1061.9
			330.0	3° 1.40	329.1	17.38		
30′	1041.8	330	330.4	1.84	329.5	17.45	164.9	1046.1
35′	1026.2	335	335.4	7.41	334.5	18.25	167.4	1030.8
40′	1011.1	340	340.4	13.07	339.5	19.09	169.9	1015.9
45′	996.5	345	345.5	18.82	344.4	19.94	172.4	1001.4
			350.0	24.06	348.9	20.73		

...inates of ...nt O.
HO
1810.2
1763.9
1719.9
1678.0
1638.2
1600.2
1563.9
1529.3
1496.2
1464.5
1434.1
1405.0
1377.0
1350.2
1324.4
1299.5
1275.6
1252.6
1230.4
1209.0
1188.4
1168.5
1149.2
1130.6
1112.6
1095.2
1078.3
1061.9
1046.1
1030.8
1015.9
1001.4

TABLE.

Degree of Curvature D	Radius of Curvature	Chord Length C	Length of Transition Curve. $\frac{L}{2}$	Deflection Angle θ	Co-ordinates of Point P.		Co-ordinates of Point O.	
					AN or x	NP or y	AH	HO
5° 50′	982.2	350	350.5	3° 24′.65	349.4	20.82	174 9	987.4
55′	968.4	355	355.5	30.57	354.3	21.73	177.4	973.8
			360.0	35?85	358.7	22.55		
6° 0′	954.9	360	360.6	36.57	359.3	22.66	179.9	960.6
5′	941.9	365	365.6	42.66	364.2	23.62	182.4	947.8
10′	929.1	370	370.7	48.84	369.2	24.61	184.9	935.3
15′	916.7	375	375.7	55.11	374.1	25.63	187.4	923.1
20′	904.7	380	380.8	4° 1.46	379.1	26.67	189.8	911.3
25′	892.9	385	385.8	7.90	384.0	27.74	192.3	899.9
			390.0	13.24	388.0	28.64		
30′	881.5	390	390.9	14.43	388.9	28.84	194.8	888.7
35′	870.3	395	395.9	21.04	393.9	29.97	197.3	877.8
			400.0	26.40	397.8	30.89		
40′	859.4	400	401.0	27.75	398.8	31.12	199.8	867.2
45′	848.8	405	406.0	34.54	403 7	32.31	202.3	856.9
50′	838.5	410	411.1	41.42	408.6	33.53	204.8	846.9
55′	828.4	415	416.2	48.39	413.5	34.77	207.3	837.1
			420.0	53.75	417.3	35.74		
7° 0′	818.5	420	421.2	55.45	418.5	36.05	209.7	827.5
5′	808.9	425	426.3	5° 2.60	423.4	37.36	212.2	818.2
10′	799.5	430	431.4	9.84	428.3	38.70	214.7	809.1
15′	790.3	435	436.5	17.17	433.2	40.08	217.2	800.3
20′	781.3	440	441.6	24.59	438.0	41.48	219.7	791.7
25′	772.5	445	446.7	32.10	442.9	42.92	222.2	783.2
			450.0	36.96	446.0	43.86		
30′	764.0	450	451.8	39.70	447.8	44.39	224.6	775 0
35′	755.6	455	456.9	47.40	452.7	45.90	227.1	767.0
40′	747.3	460	462.0	55.19	457.5	47.44	229.6	759.2
45′	739.3	465	467.1	6° 3.06	462.4	49.02	232.1	751.5
50′	731.4	470	472.2	11.04	467.3	50.63	234.5	744.1
55′	723.7	475	477.3	19.10	472.1	52.28	237.0	736.8
			480.0	23.34	474.6	53.15		
8° 0′	716.2	480	482 4	27.26	477.0	53.96	239.5	729.7
5′	708.8	485	487.6	35.52	481.8	55.68	242.0	722.7
10′	701.6	490	492.7	43.87	486.6	57.43	244.4	715.9
15′	694.5	495	497.9	52.31	491.4	59.23	246.9	709.3
			500.0	55.89	493.5	59.99		
20′	687.6	500	503.0	7° 0.85	496.3	61.06	249.4	702.8

TABLE.

Degree of Curvature D	Radius of Curvature R	Chord Length C	Length of Transition Curve. $\frac{1}{2}$L	Deflection Angle θ	Co-ordinates of Point P. AN or x	Co-ordinates of Point P. NP or y	Co-ordinates of Point O. AH	Co-ordinates of Point O. HO
8° 25′	680.8	505	508.2	7° 9′.49	501.1	62.93	251.8	696.4
			510.0	12.63	502.8	63.61		
30′	674.1	510	513.3	18.23	505.9	64.84	254.3	690.2
35′	667.5	515	518.5	27.06	510.7	66 78	256.8	684.2
40′	661.1	520	523.7	35.99	515.4	68.77	259.2	678.2
45′	654.8	525	528.9	45.02	520.2	70.80	261.7	672.4
50′	648.6	530	534.1	54.15	525.0	72.87	264.2	666.8
55′	642.6	535	539.3	8° 3.38	529.7	74.98	266.6	661.2
			540.0	4.68	530.4	75.28		
9° 0′	636.6	540	544.5	12.71	534.5	77.13	269.1	655.8
5′	630.8	545	549.7	22.15	539.2	79.33	271.5	650.5
			550.0	22.72	539.5	79.46		
10′	625.1	550	554.9	31.68	543.9	81.56	274.0	645.3
15′	619.4	555	560.1	41.33	548.6	83.84	276.5	640.3
20′	613.9	560	565.4	51.07	553.3	86.17	278.9	635.3
			570.0	59.73	557.5	88.25		
25′	608.5	565	570.6	9° 0.92	558.0	88.54	281.4	630.5
30′	603.1	570	575.9	10.88	562.7	90.95	283.8	625.7
35′	597.9	575	581.2	20.94	567.4	93.41	286.2	621.1
40′	592.7	580	586.4	31.11	572.0	95.91	288.7	616.5
45′	587.7	585	591.7	41.39	576.7	98.46	291.1	612.1
50′	582.7	590	597.0	51.78	581.3	101.06	293.6	607.8
			600.0	57.65	583.9	102.54		
55′	577.8	595	602.4	10° 2.28	585.9	103.71	296.0	603.5
10° 0′	573.0	600	607.7	12.90	590.5	106.41	298.4	599.4

CHAPTER III.

PROBLEMS IN LOCATION OF TRANSITION CURVE.

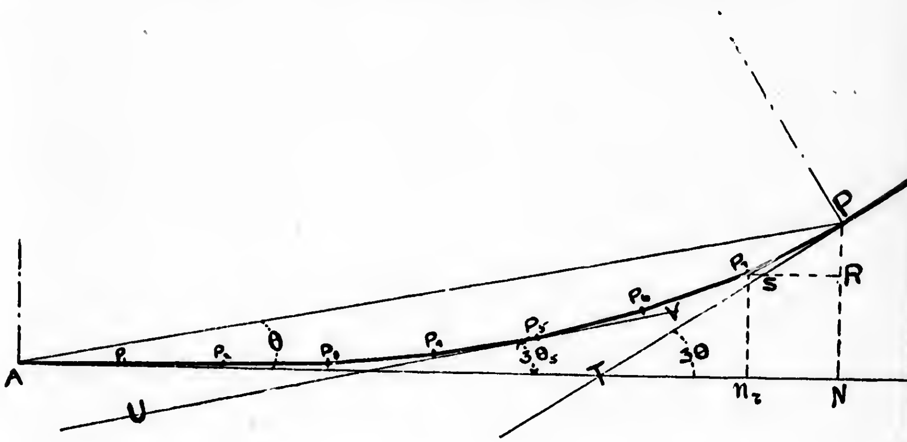

FIG. 2

Although theoretically both methods are equally good, practically it is better to run in the second piece of transition curve from A' before locating the main curve PP'; but should it be considered necessary to reverse this order by putting in the main curve first, and then the second piece of transition curve, proceed as follows :—

In Fig 2. Suppose that one piece of transition and the main curve have been located, that the transit is at P and sighting along PT the tangent to both transition and main curves.

Commencing at A call the points to be located P_1 P_2 P_3, etc., their rectangular co-ordinates $x_1\ y_1$, $x_2\ y_2$, $x_3\ y_3$, etc., and the deflection angles from A to these points $\theta^\circ_1\ \theta^\circ_2\ \theta^\circ_3$ etc.

The exterior angle $PSR = SPP_7 + RP_7P$

$$\text{therefore} \quad SPP_7 = PSR - RP_7P$$

but tangent of angle RP_7P is equal to

$$\frac{PR}{RP_7} = \frac{NP - NR}{AN - AN_7} = \frac{y - y_7}{x - x_7}$$

also the angle PSR is equal to the angle PTN or 3θ;

therefore the angle TPP_7, the deflection angle from the tangent PT, to P_7 the first point to be located, is given by

$$\text{angle } TPP_7 = 3\theta - \tan^{-1}\frac{y - y_7}{x - x_7}$$

$$\text{Similarly angle } TPP_6 = 3\theta - \tan^{-1}\frac{y - y_6}{x - x_6}$$

$$\text{" \quad " } TPP_5 = 3\theta - \tan^{-1}\frac{y - y_5}{x - x_5}$$

$$\text{" \quad " } TPP_4 = 3\theta - \tan^{-1}\frac{y - y_4}{x - x_4}$$

$$\text{" \quad " } TPP_3 = 3\theta - \tan^{-1}\frac{y - y_3}{x - x^3}$$

$$\text{" \quad " } TPP_2 = 3\theta - \tan^{-1}\frac{y - y_2}{x - x_2}$$

$$\text{" \quad " } TPP_1 = 3\theta - \tan^{-1}\frac{y - y_1}{x - x_1}$$

$$\text{" \quad " } TPA = 3\theta - \theta = 2\theta.$$

As the distances from A to points $P_1\ P_2\ P_3$ etc., will have been already decided on when running in the first piece of transition curve, it is only necessary to take from the table the values of $x_1\ y_1$, $x_2\ y_2$, $x_3\ y_3$, etc., corresponding to these distances, and insert them in the above equations :—

For instance: in the example given in the last chapter the point P_4 is 200 feet from A,

$$\text{therefore from table } x_4 = 200.0 \text{ and } y_4 = 3.88$$

and P is 395.9 feet from A,

$$\text{therefore from table } x = 393.9 \text{ and } y = 29.97$$

$$\theta \text{ is also } 4^\circ\ 21'.04, \quad \text{therefore } 3\theta \text{ is } 13^\circ\ 3'.$$

Consequently the angle

$$TPP_4 = 13^\circ\ 3' - \tan^{-1} \frac{29.97 - 3.88}{393.9 - 200} = 13^\circ\ 3' - 7^\circ\ 40' = 5^\circ\ 23'$$

A similar problem is that in which it becomes necessary to put in an intermediate hub at any point, say P_5, for then the transit being moved to P_5 and sighted along the tangent UP_5

$$\text{the deflection angle } P_4P_5U = 3\theta_5 - \tan^{-1} \frac{y_5 - y_4}{x_5 - x_4}$$

$$\text{“ \quad “ \quad “ } P_3P_5U = 3\theta_5 - \tan^{-1} \frac{y_5 - y_3}{x_5 - x_3}$$

$$\text{“ \quad “ \quad “ } P_2P_5U = 3\theta_5 - \tan^{-1} \frac{y_5 - y_2}{x_5 - x_2}$$

$$\text{“ \quad “ \quad “ } P_1P_5U = 3\theta_5 - \tan^{-1} \frac{y_5 - y_1}{x_5 - x_1}$$

$$\text{“ \quad “ \quad “ } A\ P_5U = 3\theta_5 - \theta_5 = 2\theta_5$$

or if running the curve forward to P

the deflection angle $P_6 P_5 V = \tan^{-1} \dfrac{y_6 - y_5}{x_6 - x_5} - 3\theta_5$

" " " $P_7 P_5 V = \tan^{-1} \dfrac{y_7 - y_5}{x_7 - x_5} - 3\theta_5$

" " " $P\, P_5 V = \tan^{-1} \dfrac{y - y_5}{x - x_5} - 3\theta_5$

If there be more than one intermediate hub, proceed in an exactly similar way.

Referring again to Fig. 1 it will be seen that $OH - R$ is the offset distance which a circular curve of radius OH would have to be moved towards the centre O to make room for the transition curves, so that if an external distance corresponding to a radius equal to OH be taken from a volume on circular curves, this offset will have to be added to it to give the external distance EB in Fig. 1.

also since $EB = OH \sec \dfrac{I}{2} - R$

a value of R may be chosen which will (with the corresponding value of OH taken from the table) make EB equal to any required distance.

For a given intersection angle I, EB is as small as possible when $3\theta = \dfrac{I}{2}$, or when in the table $\theta = \dfrac{I}{6}$, at the same time the main curve PP' reduces to zero, the points P and P' come together at E, and R is the minimum radius which can be used so long as $m = 1031337$, the value chosen in the construction of the table.

Conversely, for a given minimum radius R, EB is as small as possible when m has a value equal to $9R^2 \sin \dfrac{I}{3}$ found by combining equations (1) and (2); theoretically this would be the correct curve to adopt in every case, but to use it one would have to be content to do without the aid of tables, as such would require to be infinite in extent.

In special cases when the values given in the table are not suitable, and it becomes necessary to depend entirely on the formulæ given in the 1st Chapter, proceed as follows :—

Assume a convenient length for the chord distance AP or c; then, as R is supposed given; by equation (1) $m = 3R, c$.

Knowing m and c, θ is given by equation (2).

By equation (4) AH and HO can now be easily calculated, and their values substituted in equation (5) will give the tangent distance AB.

Finally find the length of the transition curve from equation (6). To locate the curve :—

Call the chord distances from A to the points to be located a, $2a$, $3a$, $4a$, etc., and the deflection angles to these points, θ_1°, θ_2°, θ_3°, θ_4°, etc.,

then by equation (2) $\sin 2\theta_1 = \dfrac{a^2}{m}$

" " " $\sin 2\theta_2 = 4\dfrac{a^2}{m} = 4 \sin 2\theta_1$

" " " $\sin 2\theta_3 = 9\dfrac{a^2}{m} = 9 \sin 2\theta_1$

thus it is only necessary to calculate the value of $\sin 2\theta_1$, for this, multiplied by 4, 9, 16, 25, etc., will at once give the values of $\sin 2\theta_2$, $\sin 2\theta_3$, $\sin 2\theta_4$, $\sin 2\theta_5$, etc.

θ_1°, θ_2°, θ_3°, etc., can now be found from a table of natural sines by inspection.

When θ is not greater than say 6° it would be correct enough to assume that $\sin 2\theta_1 = 2\theta_1$, or $\theta_1{}^\circ = \frac{a^2}{2m}\frac{180}{\pi}$, therefore if $\theta_1{}^\circ$ be multiplied by 4, 9, 16, 25, etc., $\theta_2{}^\circ$ $\theta_3{}^\circ$ $\theta_4{}^\circ$, etc., will at once be given.

CHAPTER IV.

DERIVATION OF FORMULÆ AND GENERAL CONCLUSIONS RELATIVE TO CURVE.

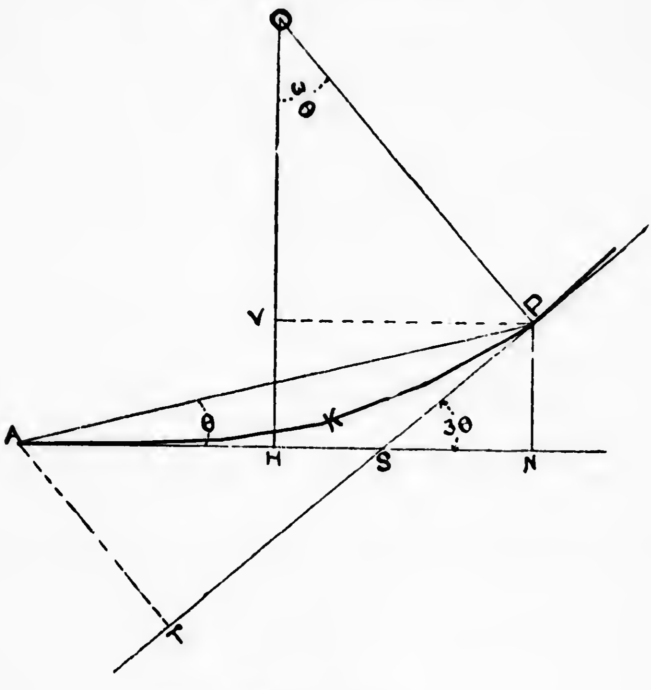

FIG. (3)

Let AKP be the proposed transition curve, and assume that the radius of curvature at any point P of this curve varies inversely as the chord distance from A to that point, that is, Let $r = \frac{m}{3AP}$

and therefore $AP = \frac{m}{3r}$ = equation (1)

Assume a system of polar co-ordinates in which AP is the radius vector, and the angle which it makes with the tangent AN the vectorial angle.

PST is tangent to the transition curve at P and AT is the polar distance

Let $AP = c$

" $AT = p$

The formula for the radius of curvature at any point P is

$$r = \frac{c\,dc}{dp}$$

$$\therefore dp = \frac{c\,dc}{r}$$

by (1) $dp = \frac{3c^2\,dc}{m}$

integrating $p = \frac{c^3}{m}$ (a)

there is no constant of integration as p and c vanish together.

The formula for the Polar distance is

$$p = \frac{c^2\,d\theta}{\sqrt{(dc)^2 + c^2\,(d\theta)^2}}$$

Squaring both sides, transposing and simplifying

$$d\theta = \frac{p\,dc}{c\,\sqrt{c^2 - p^2}}$$

$$\text{by } (a) \quad d\theta = \frac{c\,dc}{m\sqrt{1-\frac{c^4}{m^2}}} \quad (b)$$

$$\text{Integrating } \theta = \frac{1}{2}\sin^{-1}\frac{c^2}{m}$$

$$\therefore \sin 2\theta = \frac{c^2}{m} = \text{equation } (2)$$

there is no constant of integration as c and θ vanish together.

The sine of APT, the angle which the tangent PST makes with the chord AP is $\frac{AT}{AP} = \frac{p}{c}$

$$\text{but by } (a) \quad \frac{p}{c} = \frac{c^2}{m}$$

$$\text{and by } (2) \quad \frac{c^2}{m} = \sin 2\theta$$

$$\therefore \sin APT = \sin 2\theta$$

$$\therefore APT = 2\theta$$

Also the central angle $POH = PSN = SAP + APS = \theta + 2\theta = 3\theta$

The rectangular co-ordinates of P are

$$\left.\begin{aligned} AN &= c\cos\theta \\ \text{and } NP &= c\sin\theta \end{aligned}\right\} = \text{equation } (3)$$

The rectangular co-ordinates of O are

$$\left.\begin{aligned} AH &= AN - HN = AN - VP = AN - r\sin 3\theta \\ HO &= HV + VO = NP + VO = NP + r\cos 3\theta \end{aligned}\right\} = \text{equation } (4)$$

Substituting in the above $c\cos\theta$ for AN, $c\sin\theta$ for NP,

$\frac{c}{3 \sin 2\theta}$ (found by combining (1) and (2) for r, simplifying and reducing,

$$\left.\begin{aligned} AH \text{ becomes} &= \frac{c}{6}\,\frac{(2+\cos 2\theta)}{\cos\theta} \\ \text{and } HO \text{ becomes} &= \frac{c}{6}\,\frac{(2-\cos 2\theta)}{\sin\theta} \end{aligned}\right\} = \text{equation (4)}$$

Referring to Fig. (1)

$$AB = AH + HB = AH + OH \tan\frac{I}{2} = \text{equation (5)}$$

To find the length of transition curve itself:—
the formula for a differential of the curve is

$$dL = \sqrt{(dc)^2 + c^2\,(d\theta)^2}$$

$$\text{but by } (b) \quad (d\theta)^2 = \frac{c^2\,dc^2}{m^2 - c^4}$$

$$\therefore\ dL = \frac{m\,dc}{\sqrt{m^2 - c^4}} = m\,(m^2 - c^4)^{-\frac{1}{2}}\,dc = \left(1 - \frac{c^4}{m^2}\right)^{-\frac{1}{2}} dc$$

expanding by the Binomial Theorem

$$dL = \left(1 + \frac{c^4}{2m^2} + \frac{3\,c^8}{8\,m^4} + \right) dc$$

integrating, $L = c + \frac{c^5}{10m^2} + \frac{c^9}{24m^4}$ = equation (6)

when $m = 1031337$, (the number chosen in making up the table); $\frac{c^5}{10m^2}$ the second term in the above series $= \frac{1}{10}$ of a foot when

$c = 254.3$ feet, and $\frac{c^9}{24m^4}$ the third term in the above series $= \frac{1}{10}$ of a foot when $c = 518.6$ feet, therefore it is only necessary to use the second term when $D > 4° \text{-} 10'$, and the third when $D > 8° \; 35'$.

In constructing the table, a value for the constant number m was chosen which would give a reasonable length of transition curve for at least the great majority of cases (viz., those in which the degree of curvature of main curve varies from say 3° to 7°); and which would also give to the table a convenient form for comparison with such tables on circular curves as may be found in the works of Searle, Shunk, etc.; it was found as follows:—

Let the chord distance in feet to any point on transition curve be numerically equal to the number of minutes contained in the degree of curvature at that point, or if D = degree of curvature at the point, then $60D$ will equal the number of minutes in the degree of curvature at the same point

that is, Let $c = 60D$

giving about 60 feet of transition curve for each degree of curvature of main curve

$$\text{by (1)} \quad 60D = \frac{m}{3r}$$

$$\text{but when } D = 1° \quad r = 5729.65$$

$$\text{and} \therefore m = 3,\ 5729.65,\ 60 = 1031337$$

In bending the rails it is also a convenience to know that if the chord length to a point on the transition curve be divided by sixty, the degree of curvature at that point is at once given.

When necessary, other values of m may be found similarly.

The curve is symmetrical with respect to a line, making an angle of 45° with the initial line AN, for since

$$\sin(90 - 2\phi) = \sin(90 + 2\phi)$$

$$\text{and } \therefore \sin 2(45 - \phi) = \sin 2(45 + \phi)$$

c has the same value in equation (2) when $\theta = \underline{45 - \phi}$, as it has when $\theta = \underline{45 + \phi}$.

When ϕ is 0, $\theta = 45°$, $2\theta = 90°$, and $\sin 2\theta = 1$

$$\therefore c = \sqrt{m} = a \text{ maximum.}$$

If a few points be plotted, it will be seen that the curve takes the form of a loop, the point of the loop being at the origin A.

If the negative values of c be also taken, a second loop is obtained, so that the complete curve is in the form of the figure eight, and is, in fact, identical with a well-known curve called the Lemniscata.

If it be assumed that $\sin 2\theta = 2\theta$

$$\text{equation (2) becomes } 2\theta = \frac{c^2}{m} \quad \text{or } \theta = k\,c^2, \text{ where } k = \frac{1}{2m}$$

This is the formula that a member of the Canadian Society of Civil Engineers proposed using; but looking at Figure 1 it will be seen that the length of the main curve depends upon the value of 6θ, and therefore if 6θ is greater than say 6° or θ greater then 1°, a correct length for the main curve will not be obtained if such a formula has been used to calculate the value of θ.

As has been already pointed out, however, the assumption may be used to advantage in the mere location of points on the transition curve.

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