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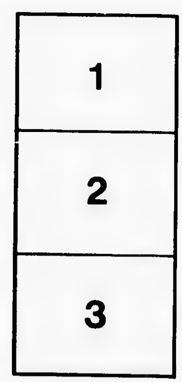

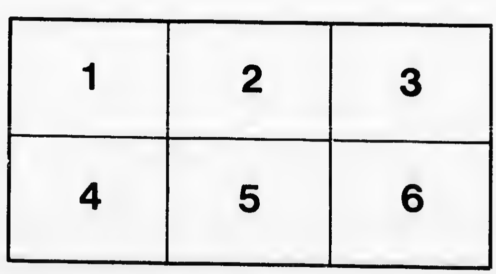

No. 2.

PRICE, 20 CENTS

Authorized by the Minister of Education.

THE HIGH SCHOOL DRAWING · COURSE

PRACTICAL GEOMETRY

BY

Arthur J. Reading.

Name,

Address,

Toronto; Grip Printing and Publishing Company.

INTRODUCTORY REMARKS.

All work in Practical Geometry requires to be done mechanically, except in the case of curves which cannot be drawn by means of compasses.

The following are the necessary instruments:—

PENCILS—either H or HH, sharpened to a wedge-shaped point, the flat side of which should rest against the rule in drawing lines. A piece of fine sand paper is about the best thing for keeping the point of the pencil sharp, and saves the blade of the pocket knife.

RULER—made of hard wood, at least six inches long, with a straight edge, and divided into inches, and halves, quarters, eighths, and sixteenths of an inch.

COMPASSES with steel, pencil, and pen points which fit into a socket in one of the legs. The stationary leg should have a needle point if possible, so that its length may be altered to correspond to whichever one of the moveable points is in use. The stationary leg should be a trifle longer than the other leg when the pencil or pen point is in use, and exactly the same length when the steel point is in use. The pencil used in the pencil point should be a little softer than that used with the ruler, as F or H, and should be sharpened in the same way. In drawing circles its edge should always be perpendicular to the radius. Properly constructed compasses have a hinge joint in each leg, so that when the pencil or pen point is in use, it can be kept perpendicular to the surface of the paper. If this is not attended to in the case of the pen point, the pen will not work properly. The joint of the compasses can be tightened or loosened by means of a little metal key which accompanies them. The joint should not be so loose that the legs will change their relative position when the compasses are being used, nor should it be so tight as to require any exertion to separate the legs. Practice will teach just how tight it should be. The compasses should be held loosely by the joint only, between the thumb and first finger, with the steel or needle point resting on the paper, without any pressure, and the other leg made to revolve around it. The student should practise until he can draw several concentric circles without puncturing the paper with the steel point. It is absolutely necessary that the steel point should be as sharp as it is possible to make it. India ink only should be used in the pens, as other inks corrode and spoil the points. The two steel points are used together when it is necessary to measure or to set off distances very accurately.

A DRAWING PEN for "inking in" straight lines. Its points should be exactly the same length and ground to a sharp rounded edge. In use it should be held nearly vertical, with the handle slightly inclined in the direction of the edge of the ruler, and drawn along the paper at a uniform rate of speed without any stoppages. It should be wiped out with a rag or piece of chamois skin every time it is filled, and before being put away.

PROTRACTOR, made of either metal, horn, ivory or wood; used for measuring angles. It is not absolutely necessary as the student can use problem xiv. for this purpose, but most boxes of mathematical instruments contain a protractor. Its form and instructions for constructing one are given in an exercise on problem xiii. In using it the centre of the semi-circle is placed over the point where the angle is to be constructed with the diameter coinciding with one line of the angle, and a pencil mark made at the circumference opposite the proper number. A line is then drawn through this point from the centre. In the form of the protractor shown in fig. 19, inside the semicircle, the point corresponding to the centre of the semicircle is in the middle of the lower edge.

A SET SQUARE, being a triangle of thin wood, will be found useful, though not necessary, for drawing parallel lines and erecting perpendiculars. The ruler is held in position and the set square slid along, with one edge firmly pressed against it. A square about five inches high, having angles of 30°, 60° and 90° will be most convenient.

When working the exercises it would be well to work them first on separate sheets of paper until the best form, and the amount of space required for each is ascertained; then to work them in pencil in the proper position, in the book, and afterwards "ink in" all the lines. Each step should be worked out by means of the methods given in the problems involved, but without referring to them unless necessary. When every problem is thoroughly understood mechanical methods may be adopted, that is, perpendiculars, parallel lines and angles may be drawn by means of the ruler and set square, and the protractor. One of the objects in view should be to commit the different methods to memory, and this becomes easy if the reasons why each particular construction is employed, are understood. The proofs of problems given will be helpful in this direction, besides satisfying the curious of the truth of the results.

Drawings on a given *scale* are sometimes asked for. When an object is represented as being ½ its natural size, it is evident that 6 inches in the drawing will represent 1 foot in the object. This scale would be indicated by the words: 6 inches to the foot, or by a fraction, ½. In the same way a scale of 1-12 would be one of 1 inch to the foot, or a scale of 1-48 one of ¼ inch to the foot, or a scale of 1-16 (3-48) one of ¾ inch to the foot. The fraction indicates the proportion which every measurement in the drawing bears to the corresponding measurement in the object drawn and that for every foot in the object, 1-2, 1-12, 1-16 or 1-48 of a foot must be taken in the drawing.

When measurements are asked for in the exercises, the distances between the proper points, or the length of the proper line should be taken by the compasses, the compasses applied to the proper scale, and the measurement carefully ascertained and written under the solution of the problem.

The sign ′ attached to a figure signifies *foot* or *feet*, and the sign ″, *inch* or *inches*: thus, *1′ 6″* reads 1 foot 6 inches, and *2′ 1″* reads, 2 feet 1 inch, and *Scale 4′ to 1″* reads, Scale 4 feet to the inch.

In the exercises, bisecting lines are supposed to be terminated by the points of intersection of the arcs employed.

Students are supposed to be familiar with Euclid's Elements.

Points are supposed to be joined by straight lines, and when the word *line* is used, a straight line is understood.

HIGH SCHOOL DRAWING COURSE.

PRACTICAL GEOMETRY.

PROBLEM I.

To Bisect a Given Line, A B.—*(Fig. 1.)*

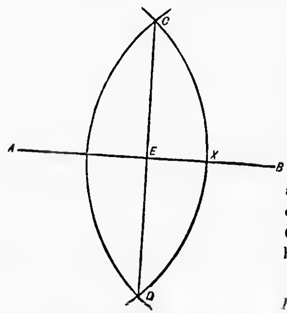

Fig. 1.

With A as a centre, using as a radius any distance greater than half the length of $A\ B$, as $A\ X$, draw an arc $C\ X\ D$. With B as a centre, using the same radius, draw another arc cutting the first one in C and D. Join $C\ D$. This line will bisect $A\ B$ in E.

Proof.—Join $A\ C$, $A\ D$, $B\ C$ and $B\ D$. Then in the $\triangle$s $A\ C\ D$ and $B\ C\ D$, $A\ C$, $A\ D$, $B\ C$ and $B\ D$ are all equal, because they are radii of equal circles, and the $< A\ C\ D =$ the $< B\ C\ D$. (*Euclid* i. 8.) Again in the $\triangle$s $A\ C\ E$ and $B\ C\ E$, $A\ C = B\ C$, $E\ C$ is common, and the $< A\ C\ E =$ the $< B\ C\ E$, $\therefore$ the base $A\ E =$ the base $E\ B$. (*Euclid* i. 4.)

EXERCISES.

1. Draw a horizontal line $1\frac{1}{2}$ inches long and bisect it.
2. (*a*) Draw an oblique line 2 inches long and bisect it.
   (*b*) Bisect each half of the bisecting line.

PROBLEM II.

To Bisect a Given Arc, A B.—*(Fig. 2.)*

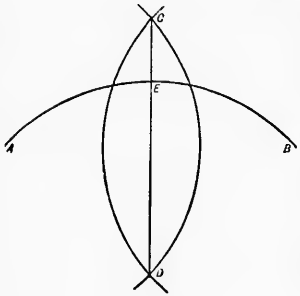

Fig. 2.

With *A* and *B* as centres, using any radius greater than half the length of *A B*, draw arcs to intersect in *C* and *D*. Join *C D*. This line will bisect the arc in *E*.

Proof.—If *A* and *B* be joined this problem will become similar to the preceding one. (*Euclid* iii. 30.)

PROBLEM III.

To Erect a Perpendicular to a Given Line, A B, at one of its extremities.—*(Fig. 3.)*

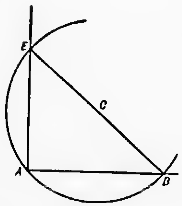

Fig. 3.

Select any point, as *C*. Using this as a centre and *C A*, the distance from it to the nearer extremity of the given line, as a radius, draw an arc cutting the given line or the given line produced in *D*. From *D* draw a line through *C*, cutting the arc in *E*. Join *A E*. This is the required perpendicular.

Proof.—The arc *E A D*, containing the < *E A D*, is a semicircle, ∴ this < is a ∟. (*Euclid* iii. 31.)

EXERCISES.

3. (*a*) Draw an arc of $1\frac{1}{2}$ inches radius and bisect it.
   (*b*) Bisect the bisecting line.
4. (*a*) On a horizontal line $1\frac{7}{8}$ inches long draw a semicircle of $\frac{3}{4}$ inch radius and bisect it.
   (*b*) Bisect each half of it.
5. (*a*) Draw a horizontal line $1\frac{5}{8}$ inches long and at its left hand extremity erect a perpendicular $1\frac{1}{2}$ inches long (by method shown in fig. 3).
   (*c*) Bisect the perpendicular.

PROBLEM IV.

To Erect a Perpendicular to a Given Line, A B, from a point, C, within it.—*(Fig. 4.)*

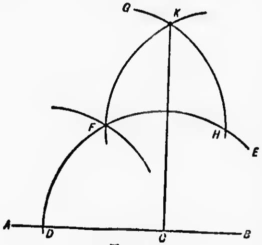

Fig. 4.

With *C* as a centre, using any radius, draw an arc *D E*. With *D* as a centre, and *D C* as radius, draw an arc to cut the first arc in *F*. With *F* as a centre, and *F D* as a radius, draw an arc, *G H*, to cut the first arc in *H*. With *H* as a centre and *H F* as radius, draw an arc to cut the last one in *K*. Join *K C*. This is the required perpendicular.

This problem may be used for erecting a perpendicular at the extremity of a line.

Proof.—The $\triangle$s *C F D*, *C F H* and *K F H* are equilateral, $\therefore$ the <s *D C F*, *C F H* and *K F H* are <s of 60°. Again, the $\triangle$ *C F K* is isosceles, having a vertical < of 120°, $\therefore$ *K C F* is an < of 30°. (*Euclid* i. 32.) But *D C F* is an > of 60°, $\therefore$ *D C K* is a ∟.

PROBLEM IV.—ANOTHER METHOD.—*(Fig 5.)*

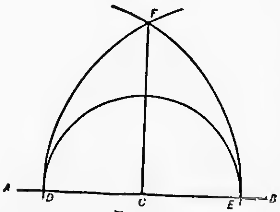

Fig. 5.

With *C* as a centre and with any radius, draw a semicircle, cutting *A B* in *D* and *E*. With *D* and *E* as centres, and with *D E* as radius, draw arcs to intersect in *F*. Join *C F*. This is the required perpendicular.

Proof.—Join *D F*, *E F*. Then in the $\triangle$s *D F C* and *E F C*, *D C* = *E C*, *F C* is common, and the base *D F* = the base *E F*, therefore the < *D C F* = the < *E C F* (*Euclid* i. 8) and these are adjacent <s.

EXERCISES.

6. (*a*) Draw two lines $1\frac{1}{4}$ inches long at right angles and with their point of intersection as a centre and a radius of $\frac{7}{8}$ inch, draw an arc to meet each of them.
   (*b*) Divide this arc or quadrant into four equal parts.
7. (*a*) Draw a vertical line $1\frac{1}{2}$ inches long and from a point in it $\frac{1}{2}$ inch from its upper extremity erect a perpendicular 1 inch long (by method shown in fig. 4).
   (*b*) With the point of intersection of the two perpendiculars as a centre, draw an arc of 1 inch radius to meet them, and bisect this arc.
8. A carpenter wishes to make a drawing board $1\frac{1}{2} \times 2$ feet with corners perfectly square. Show by a drawing how he would accomplish it without using a square. Make 1 inch in the drawing represent 1 foot of actual measurement in the drawing board.

PROBLEM V.

To Erect a Perpendicular to a Given Line, A B, from a Point, C, lying away from it.—*(Fig. 6.)*

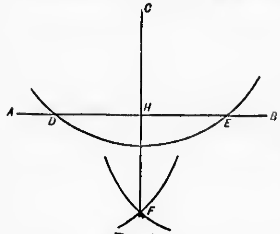

FIG. 6.

With C as a centre draw an arc to cut $A\,B$ in D and E. With D and E as centres, and with any radius, draw arcs to intersect in F. Join $C\,F$, This is the required perpendicular.

PROOF.—Join $D\,C$, $D\,F$, $E\,C$, and $E\,F$. Because $D\,C = E\,C$, and $D\,F = E\,F$ and $F\,C$ is common, the $< D\,C\,F =$ the $< E\,C\,F$ (*Euclid* i. 8.) In the $\triangle$s $D\,C\,H$ and $E\,C\,H$, $D\,C = E\,C$, $C\,H$ is common, and the $< D\,C\,H =$ the $< E\,C\,H$, $\therefore$ the $< D\,H\,C =$ the $< E\,H\,C$ (*Euclid* i. 4); and these are adjacent $<$s.

PROBLEM V.—ANOTHER METHOD.—*(Fig. 7.)*

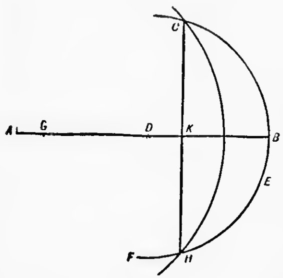

FIG. 7.

In $A\,B$ take any points D and G. With D as a centre and $D\,C$ as radius, draw an arc $C\,E\,F$. With G as a centre and $G\,C$ as radius, draw an arc to cut the first arc in C and H. Join $C\,H$. This is the required perpendicular.

PROOF.—Join $G\,C$, $G\,H$, $D\,C$ and $D\,H$. In the $\triangle$s $G\,C\,D$ and $G\,H\,D$, $G\,C = G\,H$, $D\,C = D\,H$ and $G\,D$ is common, $\therefore$ the $< C\,G\,D =$ the $< H\,G\,D$ (*Euclid* i. 8). Again in the $\triangle$s $C\,G\,K$ and $H\,G\,K$, $C\,G = H\,G$, $G\,K$ is common, and the $< C\,G\,K =$ the $< H\,G\,K$, $\therefore$ the $< G\,K\,C =$ the $< G\,K\,H$ (*Euclid* i. 4), and these are adjacent $<$s.

EXERCISES.

9. (*a*) Draw an oblique line 1½ inches long and from a point in it, ⅝ inch from either extremity, erect a perpendicular 1½ inches long (by method shown in fig. 5.)
   (*b*) Join the more distant extremities of these perpendiculars and divide this line into four equal parts.

10. Draw a vertical line 1$\frac{3}{16}$ inches long and select a point ¾ inch away from it and about opposite to its centre. From this point erect a perpendicular to the line.

11. (*a*) Draw a right angle formed by lines 1½ inches long.
    (*b*) With their point of contact as a centre and a radius of 1½ inches draw an arc to join their extremities and bisect it.
    (*c*) From the centre of this arc draw a line perpendicular to one of the original lines.

PROBLEM VI.

To Draw a Line Parallel to a Given Line, A B, at a Given Distance, C, from it.—*(Fig. 8.)*

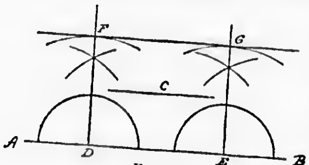

Fig. 8.

In $A\ B$ take any two points, D and E. From these points erect perpendiculars $D\ F$ and $E\ G$ (problem iv.). With D and E as centres, and with a radius equal to C, draw arcs to cut the perpendiculars in F and G. A line drawn through these points will be parallel to $A\ B$.

Proof.—$D\ F$ and $E\ G$ are equal and parallel, ∴ $F\ G$ is parallel to $D\ E$ (*Euclid* i. 33).

PROBLEM VII.

To Draw a Line Parallel to a Given Line, A B, to pass through a Given Point, C.—*(Fig. 9.)*

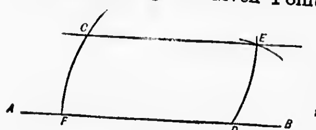

Fig. 9.

With C as a centre draw an arc to cut $A\ B$ in D. With D as a centre, and $D\ C$ as radius, draw an arc to cut $A\ B$ in F. With D as a centre and $F\ C$ as radius draw an arc to cut the arc from D, in E. A line drawn through C and E will be parallel to $A\ B$.

Proof.—Join $C\ D$, $F\ C$ and $E\ D$. Then in the △s $F\ C\ D$ and $E\ D\ C$, the sides $F\ D$, $D\ C$ = the sides $E\ C$, $C\ D$, and the bases $F\ C$ and $E\ D$ are equal, ∴ the < $F\ D\ C$ = the < $E\ C\ D$ (*Euclid* i. 8), and these are alternate <s. (*Euclid* i. 27.)

EXERCISES.

12. (*a*) Draw a vertical line $1\frac{1}{2}$ inches long and at one of its extremities erect a perpendicular equal to it in length.
    (*b*) Draw a straight line joining the extremities of the perpendiculars, and bisect it.
    (*c*) From the centre of this bisected line draw a line perpendicular to the vertical line, using method shown in fig. 7.
13. (*a*) Draw an oblique line 2 inches long and at a distance of $\frac{3}{4}$ inch from it draw a line $1\frac{1}{4}$ inches long parallel to it.
14. Draw two oblique lines $1\frac{1}{4}$ inches long forming a right angle and at a distance of 1 inch draw a line parallel to each of them.
15. Draw an oblique line $1\frac{5}{8}$ inches long, select a point $\frac{3}{4}$ inch distant from it, and draw a line parallel to it to pass through this point. Use method shown in fig. 9.

PROBLEM VIII.

To Divide a Given Line, A B, into any number of equal parts.—*(Fig. 10.)*

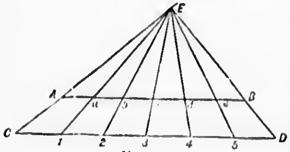

FIG. 10.

Draw CD parallel to AB at any distance from it (problem vi.). On CD set off any distance, as $C1$, as many times as the number of divisions required in AB. This may be done by means of the dividers, or, by means of a measure, ¼ or ½ an inch may be set off the required number of times. Through C and A draw a line of indefinite length, and through D and B draw a line to meet it in E. From *1*, *2*, *3*, *4* and *5* draw lines to E to cut AB in a, b, c, d and e. These points will divide AB into equal parts.

PROOF.—In the △s EAa and $EC1$, $EA : EC :: Aa : C1$ (*Euclid* vi. 4.) Similarly in the △s Eab and $E12$, $Ea : E1 :: ab : 12$, but $C1 = 12$, consequently $EA : EC :: Aa : 12$ and $Ea : E1 :: ab : C1$, ∴ $Aa : ab :: C1 : 12$. This proof may be applied to any of the divisions.

ANOTHER METHOD.—*(Fig. 11.)*

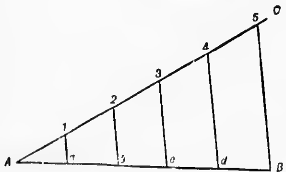

FIG. 11.

At one extremity of AB, as A, draw a line AC, forming an angle with AB, and on it set off any convenient distance, as many times as the number of divisions required in AB. From the last of these points, as *5*, draw a line to B. From the other points, *4*, *3*, *2* and *1*, draw lines parallel to $5B$ (problem vii.). These lines will divide AB equally in the points a, b, c and d. (*Euclid* vi. 2.)

EXERCISES.

16. Draw a horizontal line 2 inches long and divide it into seven equal parts by method shown in fig. 10.
17. Draw a vertical line 1¼ inches long and divide it into ten equal parts by method shown in fig. 11.
18. A person is setting out cabbage plants at equal distances in rows 20 feet long. He puts 17 plants in each row. How far are they apart? Scale ⅛ inch to 1 foot.

PROBLEM IX.

To Divide a Straight Line, A B, into parts which will have a given ratio.—*(Fig. 12.)*

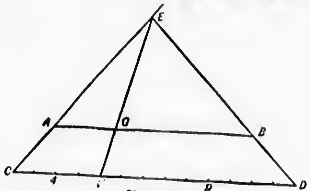

FIG. 12.

Let it be required to divide *A B* into two parts, which will bear to one another the ratio of 4 to 9. Draw *C D* parallel to *A B*, at any distance from it (problem vi.) and on *C D* set off any convenient distance, four times from *C* to *F*, and nine times from *F* to *D*. Through *A* draw *C A*, and through *B* draw *D B* to cut *C A* produced, in *E*. Join *E F*. Then *A G* is to *G B* as *C F* is to *F D*. (*Euclid* vi. 10.)

NOTE.—The method shown in fig. 11 may be used for working this problem.

PROBLEM

On a Given Line, A B, at a Given Point, B, to Construct an Angle equal to a Given Angle, C D E.—*(Fig. 13.)*

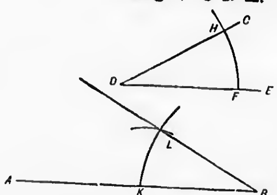

FIG. 13.

With *D* as a centre and with any radius, draw an arc to cut *D C* in *H* and *D E* in *F*. Using the same radius and *B* as a centre, draw an arc to cut *A B* in *K*. With *K* as a centre and *F H* as radius, draw an arc to cut the arc at *K* in *L*, and through *L* draw a line from *B*. The angle *A B L* is equal to the angle *E D C*.

PROOF.—Join *H F*, *L K*. Then *H D*, *D F*, *L B* and *B K*, are equal, as are also *H F* and *K L*, ∴ the $< H D F =$ the $< L B K$. (*Euclid* i. 8.)

EXERCISES.

19. Divide a vertical line $1\frac{1}{4}$ inches long into seven equal parts, by method shown in fig. 11.
20. Divide a vertical line 2 inches long into parts which will be to one another as 1, 4, 3 and 5.
21. Three boys invest in a stick of candy 3 inches long and wish to divide it so that the second shall have three times as much as the first, and the first shall have half as much as the third. By means of problem ix. show how the division would be effected.

PROBLEM XI.

To Bisect a Given Angle, A B C.—*(Fig. 14.)*

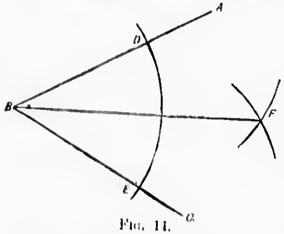

FIG. 14.

With *B* as a centre, draw an arc to cut *A B* in *D* and *C B* in *E*. With *D* and *E* as centres and with any radius, draw arcs to intersect in *F*. Join *B F*. This line will bisect the angle *A B C*.

PROOF.—Join *D F*, *E F*. In the △s *B D F* and *B E F*, the sides *B D*, *D F* = the sides *B E*, *E F* and *B F* is common, ∴ the < *D B F* = the < *E B F*. (*Euclid* i. 8.)

PROBLEM XII.

To Trisect a Right Angle.—*(Fig. 15.)*

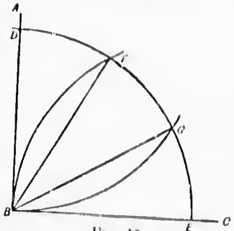

FIG. 15.

Construct the right angle *A B C* (problem iii.). With *B* as a centre draw an arc to cut *B A* in *D* and *B C* in *E*. With *D* and *E* as centres and with *D B* as radius, draw arcs to cut the arc *D E* in *F* and *G*. Join *B F*, *B G*. These lines trisect the right angle *A B C*.

PROOF.—Join *D G*, *F E*. Then *B F E* is an equilateral △ and the < *F B E* is an < of 60° (*Euclid* i. 32), ∴ *F B D* is an < of 30°. For a similar reason *D B G* is an < of 60°, and *G B E* an < of 30°, ∴ *F B G* is also an < of 30°.

EXERCISES.

22. At the extremities of a horizontal line $1\frac{3}{4}$ inches long, and on opposite sides of it construct equal angles which will be together less than a right angle.
23. Construct a right angle and bisect it.
24. (*a*) With lines one inch long construct an angle equal to half a right angle.
    (*b*) On the opposite side of one of them construct an angle equal to one-quarter of a right angle.
25. (*a*) Draw two lines 1 inch long forming an angle equal to two-thirds of a right angle.
    (*b*) At the other extremity of one of them on the opposite side of it, construct an angle equal to one-third of a right angle.

PROBLEM XIII.

To Construct an Angle of a Given Size,—41° and 19°.—(*Fig. 16.*)

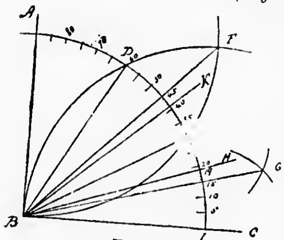

Fig. 16.

Construct a right angle and trisect it (problem xii.). By continuing the arcs passing through *D* and *E* till they meet in *F*, and joining *B F*, the right angle will be bisected. In this way angles of 30°, 45° and 60° will be obtained. Bisecting the arcs *A D* and *E L* (problem ii.) will give angles of 15° and 75°. By means of the dividers, divide each arc of 15° into 3 equal parts. These divisions of 5° will be small enough to be divided without mechanical aid into five parts, and thus the quadrant or arc of 90° will be divided into 90 parts, each one of which will represent 1°. A line drawn from *B* to division *41* will make *C B K* an angle of 41°, and a line to division *19* will make *C B H* an angle of 19°.

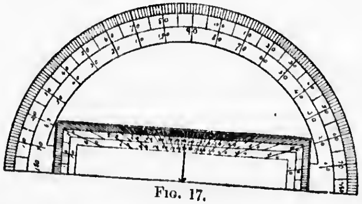

Fig. 17.

Note.—It will be a useful exercise to draw on a piece of thin but stiff cardboard a semicircle, and divide the circumference into 180 parts, as shown in *fig. 17*. It may be cut out around the circumference and along the diameter, and will then form a protractor. This instrument is usually made in one of the forms illustrated, of either metal, ivory, wood or horn. Its use has already been explained.

The measurement of angles can also be effected by means of a scale of chords, which is explained in the following problem.

EXERCISES.

26. (*a*) At one extremity of a line $1\frac{1}{2}$ inches long construct an angle of 52°.
    (*b*) At the other extremity, on the same side of it, construct an angle equal to half of the angle of 52°.
27. A gardener is instructed to plant seventeen trees so that sixteen of them will be equal distances apart and sixteen feet from the seventeenth tree. Show by a drawing on a scale of $\frac{1}{16}$ inch to 1 foot, the relative positions of the trees, and how their positions will be ascertained.
28. In the space below construct a semicircular protractor using a radius of 2 inches.

PROBLEM XIV.

To Construct a Scale of Chords. *(Fig. 18.)*

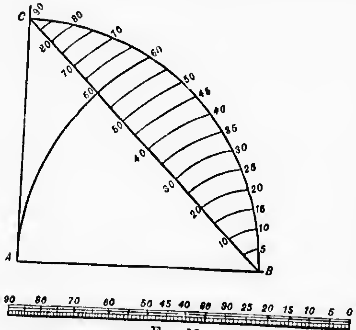

FIG. 18.

Construct a right angle CAB (problem iii.). With A as a centre draw an arc to cut AB in B and AC in C. Trisect the right angle, and further sub-divide it as explained in problem xiii. Join BC. With B as a centre, radii $B5$, $B10$, $B15$, $B20$, etc., respectively, draw arcs to cut BC in the points numbered *10*, *20*, *30*, *40*, *50*, etc. In a complete scale there should be ninety divisions in the line BC, which divisions could be transferred to another line of equal length, and arranged as shown in the lower part of *fig. 18.*

To use this scale of chords, take as a centre the point at which the required angle is to be constructed and draw an arc with the length of the chord of 60° as a radius. This chord is always equal to the radius of the arc by means of which the scale is constructed. Using as a centre the point where this arc cuts the straight line forming one arm of the angle, and a radius equal to the chord of the angle required, draw an arc to cut the first arc, and through this point draw a line from the centre of the first arc.

EXERCISES.

29. Construct a scale of chords with the chord of 60° $1\frac{1}{2}$ inches long, and use it in constructing the following angles.
30. At the left hand extremity of a horizontal line 1 inch long, construct an angle of 24°.
31. At the right hand extremity of a horizontal line $1\frac{1}{4}$ inches long, construct an angle of 75°

PROBLEM XV.

To Construct an Equilateral Triangle on a Given Line, A B.—(*Fig. 19.*)

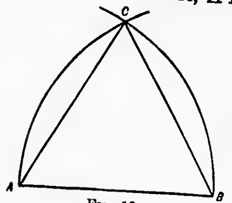

Fig. 19.

With A and B as centres and $A B$ as radius, draw arcs to intersect in C. Join $A C$, $B C$. The triangle $C A B$ is equilateral. (*Euclid* i. 1.)

PROBLEM XVI.

To Construct an Equilateral Triangle of a Given Altitude, A.—(*Fig. 20.*)

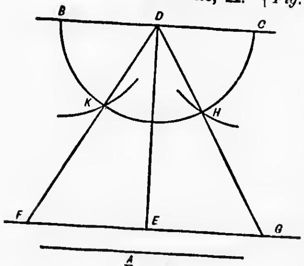

Fig. 20.

Draw a straight line, $B C$. From any point, D, in it, erect a perpendicular $D E$ equal to A (problem iv.) Through E draw a line, $F G$, parallel to $B C$ (problem vii.). With D as a centre draw a semicircle cutting $B D C$ in B and C. With B and C as centres and $B D$ as radius, draw arcs to cut the semicircle in K and H. Through K and H draw lines from D to cut $F E G$ in F and G. Then $D F G$ is the triangle required.

Proof.—The $<$s $B D K$ and $C D H$ are $<$s of 60° (see proof of problem xii.), and $\because$ $B C$ is parallel to $F G$, the $<$s $D F G$ and $D G F$ are also $<$s of 60° (*Euclid* i. 29), $\therefore$ $F D G$ is an $<$ of 60°, and the $\triangle$ $D F G$ is equilateral, and its altitude $D E = A$.

EXERCISES.

32. (*a*) On a horizontal line $1\frac{1}{2}$ inches long construct an equilateral triangle.
    (*b*) From its vertical angle draw a line perpendicular to the opposite side.
33. (*a*) On a vertical line $1\frac{5}{8}$ inches long construct an equilateral triangle.
    (*b*) Bisect two of its angles.
34. (*a*) Construct an equilateral triangle the altitude of which will be $1\frac{1}{2}$ inches.
    (*b*) Divide one side so that its two parts will bear the same ratio as the side bears to the altitude

PROBLEM XVII.

To Construct a Triangle, its three sides A, B and C being given.—*(Fig. 21.)*

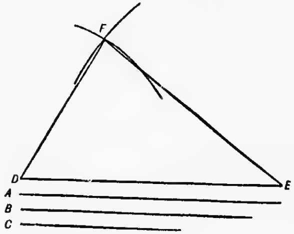

Fig. 21.

Make *D E* equal to *A*. With *E* as a centre and a radius equal to *B* draw an arc. With *D* as a centre and a radius equal to *C* draw another arc to intersect the first one in *F*. Join *D F*, *F E*. Then *F D E* is the triangle required. (*Euclid* i. 22.)

PROBLEM XVIII.

On a Given Line, A B, to Construct a Triangle Equiangular to a Given Triangle, CDE.—*(Fig. 22.)*

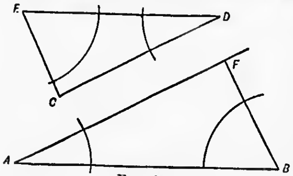

Fig. 22.

At *A* make the angle *F A B* equal to the angle *E D C*, and at *B* make the angle *F B A* equal to the angle *DEC* (problem x.). Produce the lines forming these angles till they meet in *F*. Then the triangle *F A B* is equiangular to the triangle *E C D*.

EXERCISES.

35. Construct a triangle the sides of which will be $1\frac{1}{2}$, $1\frac{3}{4}$, and 2 inches respectively.

36. Construct a triangle the sides of which will be as 5, 7 and 9.

37. A man is standing opposite a point 5 feet from the end of a wall 20 feet long, and 15 feet from its nearer extremity. Represent his position by a point and the wall by a line, and find out how far he is from the more distant end of the wall. Use a scale of 1 inch to 10 feet.

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PROBLEM XIX.

On a Given Line, A B, to Construct an Isosceles Triangle, having a Given Vertical Angle.—

(Figs. 23 and 24.)

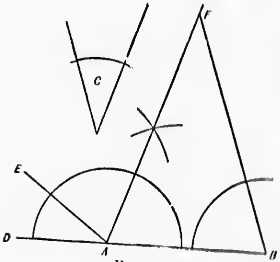

FIG. 23.

First.—Let C be the given vertical angle (*fig.* 23.)

Produce BA to D. At A make the angle DAE equal to C (problem x.). Bisect the angle EAB by AF (problem xi.). At B make the angle ABF equal to the angle BAF. Then the triangle FAB is the isosceles triangle required.

PROOF.—The $\angle$s EAF, BAF and ABF are equal. The $\angle DAF = \angle ABF + \angle BFA$ (*Euclid* I. 32); but $\angle EAF = \angle ABF$, $\therefore \angle DAE = \angle AFB$, and $\angle DAE = \angle C$.

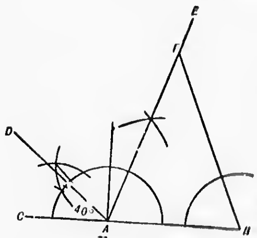

FIG. 24.

Second.—Let the vertical angle be an angle of 40° (*fig.* 24).

Produce BA to C. At A make the angle CAD an angle of 40°, and proceed as in first case.

EXERCISES.

38. A boy 3 feet 6 inches high stands 24 feet from the foot of a tree and finds that a line from his eye at an angle of 72° with a horizontal line exactly touches the top. What is the height of the tree? Scale 1 inch to 16 feet.

39. On a horizontal line 1 inch long construct an isosceles triangle having a vertical angle of 30°.

40. On a horizontal line 1½ inches long construct an isosceles triangle having a vertical angle equal to the angle which is opposite to the perpendicular of a right-angled triangle whose perpendicular is 1 inch long and base ½ inch long.

PROBLEM XX.

To Construct a Square on a Given Line, A B.—
(Fig. 25.)

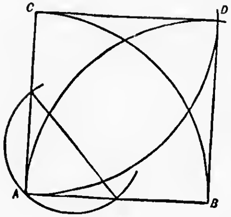

Fig. 25.

At one extremity, A, of the given line erect a perpendicular, $A\,C$ (problem iii.), and make it equal to $A\,B$ by means of an arc drawn with A as a centre and $A\,B$ as a radius. With the same radius, and B and C as centres, draw arcs to intersect in D. Join $B\,D$, $D\,C$. Then $A\,C\,D\,B$ is the square required.

Proof.—The sides $A\,B$ $A\,C$, $D\,B$ and $D\,C$ are all equal, and $C\,A\,B$ is a ∟, ∴ $A\,C\,D\,B$ is a square. (*Euclid* i. 34 *and cor.*)

PROBLEM XXI.

To Construct a Square on a Given Diagonal.—
(Fig. 26.)

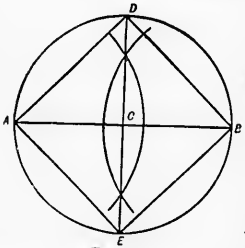

Fig. 26.

Bisect $A\,B$ in C by the line $D\,E$ perpendicular to it (problem i.). With C as a centre and $C\,A$ as radius draw a circle to cut $D\,C\,E$ in D and E. Join $A\,D$, $D\,B$, $B\,E$, $E\,A$. Then $A\,D\,B\,E$ is the square required.

Proof.—The lines $C\,D$, $C\,B$, $C\,E$ and $C\,A$ being equal and the <s about C being ∟s, the lines $A\,D$, $D\,B$, $B\,E$ and $E\,A$ are also equal. (*Euclid* i. 4.) The <s $D\,A\,E$, $D\,B\,E$, etc., being angles in semicircles are ∟s (*Euclid* iii. 31), ∴ $A\,D\,B\,E$ is a square.

EXERCISES.

41. On an oblique line $1\frac{1}{4}$ inches long construct a square.
42. (*a*) Construct a triangle, its sides being $\frac{3}{4}$, 1 and $1\frac{1}{2}$ inches respectively.
    (*b*) On each side construct a square.
43. Construct a square with a diagonal 2 inches long.
44. (*a*) Draw a circle of $\frac{1}{2}$ inch radius and make its diameter the diagonal of a square.
    (*b*) On each side construct an equilateral triangle.

PROBLEM XXII.

To Construct an Oblong when its sides, A and B, are given.—*(Fig. 27.)*

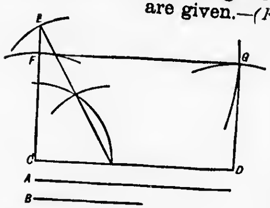

Fig. 27.

Make *C D* equal to *A*. At *C* erect a perpendicular, *C E* (problem iii.), and from it cut off *C F* equal to *B*. With *D* as a centre and a radius equal to *B*, draw an arc and cut it by another arc drawn with *F* as a centre and a radius equal to *A*. Join *F G*, *G D*. Then *C F G D* is the oblong required.

Proof.—*C F G D* is a parallelogram and *F C D* is a ∟, ∴ all its angles are ∟s. (*Euclid* i. 34 *and cor.*)

PROBLEM XXIII.

To Construct an Oblong when the Diagonal, A, and the length of one pair of sides, B, are given.—*(Fig. 28.)*

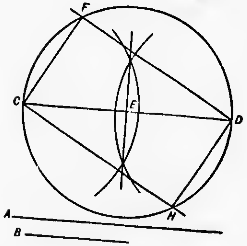

Fig. 28.

Make *C D* equal to *A*. Bisect *C D* in *E* (problem i.). With *E* as a centre and *E C* or *E D* as radius draw a circle. From *C* and *D* as centres, with a radius equal to *B*, draw arcs to cut the circumference of the circle in *F* and *H*. Join *C F*, *F D*, *D H* and *H C*. Then *C F D H* is the oblong required.

Proof.—In the semicircles *C F D* and *D H C* the arc *C F* = the arc *D H* (*Euclid* iii. 28), ∴ the arc *F D* = the arc *H C*, and the chords of these equal arcs are also equal. But the ∠s *C F D* and *D H C* are ∟s (*Euclid* iii. 31), ∴ *C F D H* is an oblong (*Euclid* i. 34).

EXERCISES.

45. Construct an oblong whose sides will be 1 inch and 1$\frac{3}{4}$ inches.
46. Make a line 2 inches long, one of the longer sides of an oblong whose sides will be as 4 to 7.
47. Construct an oblong having two sides 1 inch long, and its diagonal 2 inches long.
48. Construct an oblong on a diagonal 1$\frac{1}{2}$ inches long, making the shorter sides $\frac{1}{4}$ of the length of the diagonal

PROBLEM XXIV.

To Find the Centre of a Given Circle.—(*Fig. 29.*)

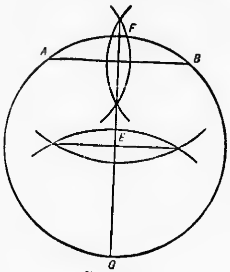

FIG. 29.

Draw any chord *A B*. Bisect it (problem i.) and produce the bisecting line to cut the circumference of the circle in two points *F* and *G*. Bisect *F G* in *E* (problem i.). Then *E* is the centre of the circle. (*Euclid* iii. 1.)

ANOTHER METHOD.—(*Fig. 30.*)

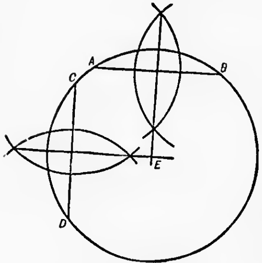

FIG. 30.

Draw any two chords as *A B* and *C D*. Bisect them (problem i.) and produce the bisecting lines to meet in *E*. Then *E* is the centre of the circle.

PROOF.—The lines bisecting the chords *A B* and *C D* both contain the centre (*Euclid* iii. 1), ∴ the point *E*, which is common to both of these lines, is the centre.

NOTE.—This method can be used for finding the centre of an arc.

EXERCISES.

49. Draw a circle 3 inches in diameter, and find its centre.
50. Lay a penny on the paper and trace a line around its circumference. Find the centre of this circle by means of the method shown in fig. 29. See note on page 35.
51. With a radius of 1 inch, draw an arc less than a semicircle and ascertain the position of its centre by means of the method shown in fig. 30.
52. (*a*) By means of some circular object about 2 inches in diameter, trace a circle and divide it into four equal parts.
    (*b*) Bisect one of the quadrants.
    (*c*) Trisect another of the quadrants.

PROBLEM XXV.

To Find the Centre of a Given Arc, A B.—*(Fig. 31.)*

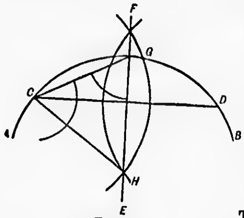

Fig. 31.

Draw any chord $C\,D$. Bisect it by the line $F\,E$ (problem i.) cutting the arc in G. Join $C\,G$. At C, make the angle $G\;C\;H$ equal to the angle $C\;G\;E$ (problem x.) and produce the line $C\,H$ to cut $G\,E$ in H. Then H is the centre of the arc.

Proof.—Join $G\,D$ and $D\,H$. The line $F\,E$ bisects the arc $C\,G\,D$ in G and the line $C\,D$ in X and the chord $C\,G$ = the chord $G\,D$. In the $\triangle$s $C\,G\,X$ and $D\,G\,X$, $C\,G = D\,G$, $G\,X$ is common and the base $C\,X$ = the base $D\,X$, $\therefore$ the $< C\,G\,X$ = the $< D\,G\,X$. (*Euclid* i. 8.) Again, in the $\triangle$s $C\,G\,H$ and $D\,G\,H$, $C\,G = D\,G$, $G\,H$ is common, and the $< C\,G\,H$ = the $< D\,G\,H$, $\therefore$ $C\,H = D\,H$. (*Euclid* i. 4.) But $C\,H = G\,H$, because the $\triangle$ $C\,H\,G$ is isosceles, $\therefore$ H is the centre of the arc. (*Euclid* iii. 9.)

PROBLEM XXVI.

To Draw a Circle of a Given Radius, A, whose Circumference will pass Through two Given Points, B and C.—*(Fig. 32.)*

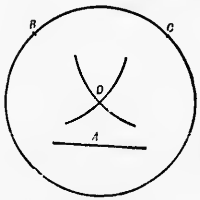

Fig. 32.

With B and C as centres and a radius equal to A, draw arcs to intersect in D. With D as a centre and the same radius as before, draw a circle.

EXERCISES.

53. With a radius of 2″ draw an arc whose chord is not greater than 2″, and find its centre.
54. By means of some circular object draw an arc and find its centre.
55. With a radius of 1″ draw a circle to pass through two points $\frac{3}{4}$″ apart.
56. A cow is tethered to a stake by a rope 24′ long. Where must the stake be placed so that the free end of the rope will not be nearer than 6′ to two trees planted 15′ apart?

PROBLEM XXVII.

To Draw a Circle whose Circumference will Pass through three Points, A, B and C, not in the same straight line.—*(Fig. 33.)*

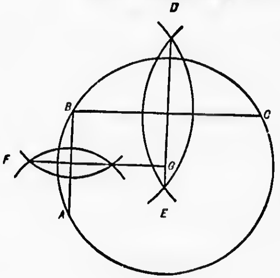

FIG. 33.

Join *A B* and *B C* and bisect these lines by the lines *F G* and *D E* (problem i.). Then the point of intersection of these bisecting lines is the centre of the circle re quired. With *G* as a centre and *G B* as radius draw a circle.

NOTE.—This problem is derived from cor. to *Euclid* iii. 1).

PROBLEM XXVIII.

To Draw a Tangent to a Circle from a point, B, in the Circumference.—*(Fig. 34.)*

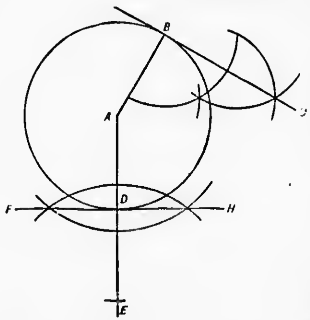

FIG. 34

Draw a radius from *B*. At *B* erect a perpendicular, *B C* (problem iii.). Then *B C* is the tangent required. (*Euclid* iii. 17-18.)

ANOTHER METHOD.

(Fig. 34.)

Draw any radius *A D*, and produce it, making *D E* equal to *D A*. Bisect *A E* (problem i.). Then *F H* is the tangent required. (*Euclid* iii. 18.)

EXERCISES.

57. Draw a triangle whose sides will be $1\frac{1}{4}''$, $1\frac{5}{8}''$ and $1\frac{3}{4}''$ long respectively, and draw the smallest circle that will contain it.

58. Draw a tangent to a circle $1\frac{3}{4}''$ in diameter.

59. Three upright posts are placed so that the distance from the first to the third is twice the distance of the second from the third, and the distance from the first to the second is two-thirds of the distance between the first and third. Show the position of a fourth post which will be equidistant from the other three.

60. A circle 2″ in diameter is touched by a line forming an angle of 24° with a horizontal line. Show the point of contact.

PROBLEM XXIX.

To Draw a Tangent to a given Circle from a point, A, without it.—*(Fig. 35.)*

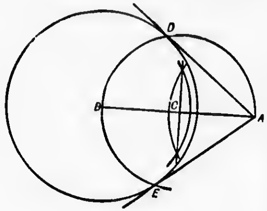

FIG. 35.

Join the given point, *A*, with the centre, *B*, of the circle. Bisect *A B* in *C* (problem i.). With *C* as a centre and *C A* as radius draw an arc to cut the circumference of the circle in *D* and *E*. Join *A D* and *A E*. Then either *A D* or *A E* will be the tangent required.

PROOF.—Join *B D*. Then the arc *A D B* is a semi-circle and the < *B D A* contained in it is a ∟. (*Euclid* iii. 31.) But *B D* is a radius of the circle, ∴ *D A* is a tangent.

ANOTHER METHOD.—*(Fig. 36.)*

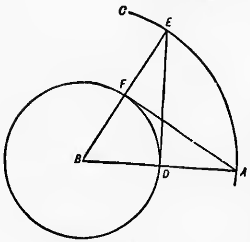

FIG. 36.

Draw a line from the given point, *A*, to the centre, *B*, of the circle, cutting the circumference in *D*. With *B* as a centre and *B A* as radius draw an arc, *A C* From *D* erect a perpendicular to cut this arc in *E* (problem iv.). Join *E B* cutting the circumference of the circle in *F*. Join *F A*. Then *F A* is the tangent required. (*Euclid* iii. 17.)

EXERCISES.

61. From a point 2″ from its centre draw a tangent to a circle $1\frac{1}{4}$″ in diameter.
62. A string is stretched tightly around a cylinder 1′ in diameter, and a rod $1\frac{1}{2}$′ distant from, and parallel to, its axis. Show the points where the string first touches the surface of the cylinder. Scale $\frac{1}{8}$.
63. From a point $2\frac{1}{2}$″ from the centre of a circle 2″ in diameter draw two lines which will be tangential to its circumference.

PROBLEM XXX.

On a Given line, A B, to Construct a Regular Pentagon—*Special Method.**—(*Fig. 37.*)

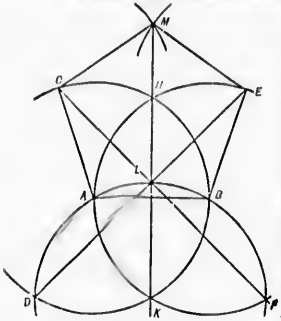

Fig. 37.

With *A* and *B* as centres and *A B* as radius draws arcs intersecting in *H* and *K*. With *K* as a centre and *K A* as radius draw an arc cutting the other two arcs in *D* and *F*. Join *H K*, cutting the last arc in *L*. From *D* and *F*, through *L*, draw lines to cut the first two arcs in *C* and *E*. Join *A C* and *B E*. With *C* and *E* as centres and *C A* as radius draw arcs to intersect in *M*. Join *C M* and *E M*.

This method cannot be proved.

ANOTHER METHOD—*General.*—(*Fig. 38.*)

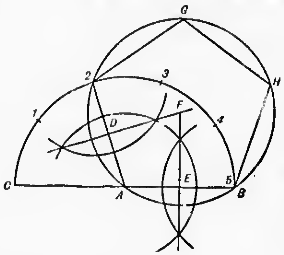

Fig. 38.

With *A* as a centre and *A B* as radius draw a semi-circle, cutting *A B* produced, in *C* and divide the circumference into five equal parts. From *A* through the *second* division draw a line *A 2*. Draw a circle to pass through *B*, *A* and *2* (problem xxvii.), and on its circumference set off the distance *A B* as many times as is possible. Join the points thus obtained.

Proof.—In any regular polygon each [illegible] the exterior angles is equal to 360° divided by the number of si[illegible] (*Euclid* i. 32, cor.) The $< CA2 = 2(180° \div 5)$ or 72°, which is [illegible]

* See note on Polygons on third page of cover.

EXERCISES.

64. On a vertical line $\frac{7}{8}''$ long construct a regular pentagon by a special method.
65. Construct a pentagon whose sides will be $1\frac{1}{4}''$ long.
66. Construct an isosceles triangle whose base will be $\frac{3}{4}''$ and vertical angle 72°. This triangle may be used as one of the triangles around the centre of a pentagon. The base will be the length of one side of the pentagon, and the vertex will be the centre of the circumscribing circle drawn with a radius equal to one of the equal sides of the triangle. The length of one side of the pentagon must be measured off around the circumference of the circle.

PROBLEM XXXI.

On a Given Line, A B, to Construct a Regular Hexagon—*Special Method—(Fig. 39).*

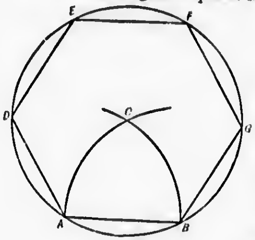

FIG. 39.

With *A B* as radius draw a circle whose circumference will pass through *A* and *B* (problem xxvi.). Around the circumference place chords *A D*, *DE*, *EF*, *F G* and *G B* equal to *A B*. Then *A D E F G B* will be the required hexagon. (*Euclid* iv. 15.)

PROBLEM XXXII.

Within a Given Circle to Inscribe any Regular Polygon.—*(Fig. 40.)*

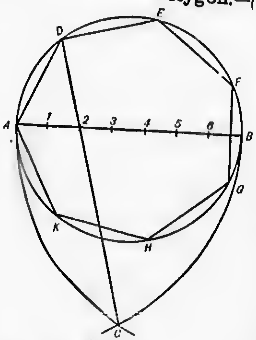

FIG. 40.

Let it be required to inscribe in the given circle a regular heptagon. Draw a diameter *A B* and divide it into as many equal parts, 7, as there are sides in the required polygon (problem viii.). With *A* and *B* as centres and *A B* as radius draw arcs to intersect in *C*. From *C* through the *second* division, *2*, draw a line to cut the circumference of the circle in *D*. Join *A D*. Around the circumference place chords equal to *A D*. Then *A D E F G H K* will be the heptagon required. This method cannot be proved.

EXERCISES.

67. On a line 1″ long construct a regular hexagon.
68. On a line $\frac{3}{4}$″ long construct a regular hexagon by means of method shown in Fig. 38.
69. Construct the greatest nonagon which will be contained by a circle $\frac{7}{8}$″ in diameter.
70. Place seven points so that each one will be $\frac{13}{16}$″ distant from three others.

PROBLEM XXXIII.

On a Given Line, A B, to Construct a Regular Octagon—*Special Method.—(Fig. 41.)*

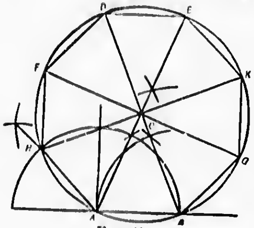

FIG. 41.

On $A\ B$ construct an isosceles triangle $A\ C\ B$ having a vertical angle of 45° (problem xix.). With C as a centre and $C\ A$ as radius draw a circle and produce $A\ C$ and $B\ C$ to meet its circumference in D and E. Through C draw diameters $F\ G$ and $H\ K$, perpendicular to $A\ E$ and $B\ D$ respectively, and join $A\ H$, $H\ F$, $F D$, $D E$, $E K$, $K G$ and $G B$.

PROOF.—The <s $A\ C B$ and $D\ C\ E$ are <s of 45° (*Euclid* i. 15), and the <s $D\ C\ K$ and $G\ C\ A$ are <s of 90°, ∴ the <s $E\ C\ K$ and $G\ C\ B$ are <s of 45° and for a similar reason the <s about C are all equal and are subtended by equal chords (*Euclid* iii. 26 and 29), ∴ the octagon constructed on $A\ B$ is equilateral. Again ∵ $C K$, $C\ G$, $C B$, $C A$, etc., are all equal and the <s contained by them are equal; the <s $C K G$, $C G K$, $C G B$, $C B G$, $C B A$, $C A B$, etc., are equal (*Euclid* i. 4), ∴ the <s $K G B$, $G\ B$ A, etc., are also equal. The octgaon on $A\ B$ is therefore regular, being both equilateral and equiangular.

PROBLEM XXXIV.

Within a Given Circle, C H G, to Inscribe a Regular Octagon—*Special Method.—(Fig. 42.)*

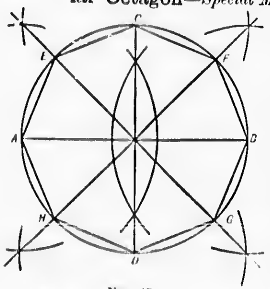

FIG. 42.

Draw any diameter, $A\ B$. Bisect it by the diameter $C\ D$ perpendicular to it (problem i.). Bisect the right angles about the centre of the circle, by the lines $E\ G$ and $H\ F$ (problem xi.). Join $A\ E$, $E\ C$, $C\ F$, etc. Then $A\ E\ C\ F\ B\ G\ D\ H$ will be the octagon required.

NOTE.—This problem requires no proof.

EXERCISES.

71. Construct a regular octagon whose sides will be $\frac{5}{8}''$ long.
72. Inscribe a regular octagon in a circle of 1″ radius.
73. Inscribe a regular octagon in a circle 3″ in diameter by means of method shown in fig. 40.

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PROBLEM XXXV.

To Draw an Ellipse when the Transverse Axis, A B, and Conjugate Axis, C D, are Given.—*(Fig. 44.)*

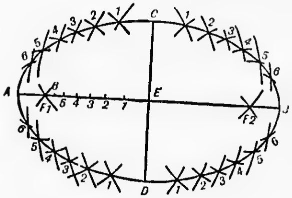

Fig. 43.

Draw *A B* and *C D*, at right angles, bisecting one another in *E*. With *C* and *D* as centres and *A E* as radius draw arcs to cut *A B* in *F 1* and *F 2*. These points will be the *foci* (plural of *focus*) of the ellipse. Between one of the foci, as *F 1*, and *E* mark off on *A B* any number of points, as *1, 2, 3, 4, 5, 6*, taking care to have their distance apart decrease, as they approach *F 1*. With *F 1* and *F 2* as centres and *1 A* as radius draw arcs *1, 1, 1, 1*, and cut them by other arcs drawn with the same centres and *1 B* as radius. With *F 1* and *F 2* as centres and *2 A* and *2 B*, *3 A* and *3 B*, *4 A* and *4 B*, etc., respectively, as radii draw arcs to intersect in *2, 2, 2, 2*; *3, 3, 3, 3*, etc., and through these points of intersection draw a curve which will be an ellipse.

Note.—The curve of an ellipse is such that the sum of the distance of any point in it, from each of the foci, is equal to the length of the transverse axis. The following practical method of drawing an ellipse is based upon this peculiarity: Place a pin in each of the foci and another in one of the extremities of the conjugate axis. Tie a piece of thread or string tightly around the three pins, remove the one at the end of the conjugate axis, and substitute a pencil or crayon. If the pencil be moved around the two remaining pins, keeping the string tightly stretched, it will trace a perfect ellipse.

EXERCISES.

74. By means of intersecting arcs draw an ellipse whose transverse axis is 4″ long and congugate axis 2″ long.
75. By the practical method described draw an ellipse whose axes will be 2″ and 3″ long respectively.

ANOTHER METHOD.—*(Fig. 44.)*

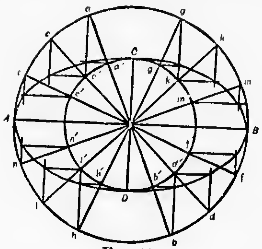

FIG. 44.

Draw $A B$ and $C D$ at right angles bisecting one another in E (problem i.). With E as a centre and radii $E A$ and $E C$ draw circles. Divide these circles into any number of equal parts, say 16, by the lines $a b$, $c d$, $e f$, etc. From the points a, c, e, n, l, h, etc., in the outer circle draw lines parallel to $C D$ and cut them by lines parallel to $A B$ drawn through the points a', c', e', n', l', h', etc., in the inner circle. A line drawn through the points of intersection of the perpendiculars will be the ellipse required.

See note on the ellipse on third page of cover.

PROBLEM XXXVI.

To Draw an Elliptical Curve when only the Transvers Axis, A B, is Given.—*(Fig. 45.)*

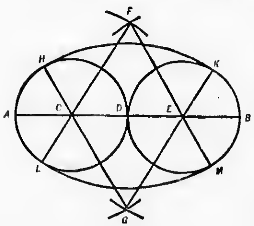

FIG. 45.

Divide $A B$ into 4 equal parts (problem viii.) in the points C, D and E. With C and E as centres and $C A$ as radius draw circles. With C and E as centres and $C E$ as radius draw arcs to intersect in F and G. From F and G draw lines through C and E to cut the circumferences of the circles in H, K, L and M. With F and G as centres and $F L$ as radius draw the arcs, $H K$ and $L M$ which will complete the curve.

EXERCISES.

76. By means of intersecting perpendiculars draw an ellipse whose axes will be as 2 to 3, the transverse axis being $3\frac{3}{4}''$ long.
77. Draw an elliptical curve whose greatest diameter will be $3''$ long.

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PROBLEM XXXVII.

An Ellipse being Given, to find the Transverse and Conjugate Axes.—*(Fig. 46.)*

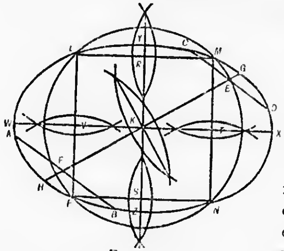

Fig. 46.

Draw any chord, *A B*. Select any point, *C*, in the curve of the ellipse and through *C* draw a line, parallel to *A B* (problem vii.), cutting the curve in *D*. Bisect *A B* and *C D* (problem i.), and through *E* and *F* draw a line to cut the curve in *G* and *H*. Bisect *G H* (problem i.). With *K* as a centre draw a circle to cut the ellipse in four points, *L*, *M*, *N* and *P*, and join these points. Bisect *L M*, *M N*, *N P* and *P L* and through their centres *V*, *T* and *R*, *S* draw lines to cut the ellipse in *W*, *X*, *Y* and *Z*. Then *W X* will be the transverse axis, and *Y Z* the conjugate axis.

PROBLEM XXXVIII.

To Draw a Tangent or a Perpendicular to the Curve of an Ellipse, at a Given Point.—*(Fig. 47.)*

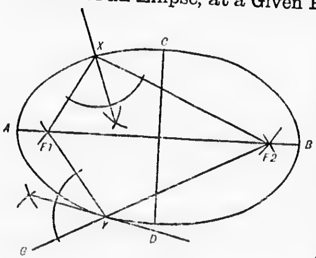

Fig. 47.

Let *X* and *Y* be two given points. Find the axes and the foci of the ellipse (problems xxxv. and xxxvii.). Draw lines from *X* and *Y* to each of the foci. Produce one of the lines, as $F_2 Y$, to *G* and bisect the angle $F_1 Y G$. The bisecting line will be the tangent required. Bisect the angle $F_1 X F_2$. The bisecting line will be the perpendicular required.

EXERCISES.

78. (*a*) Draw an ellipse by method shown in fig. 43 or fig. 44, its diameters being $1\frac{3}{4}''$ and $4\frac{1}{4}''$ long.
    (*b*) Erase everything but the curve, and show how its major and minor axes would be found.
79. (*a*) By method shown in fig 45 draw an elliptical curve whose transverse axis will be $3\frac{1}{2}''$ long.
    (*b*) Find its conjugate axis, and its foci.
    (*c*) Select any two points in the curve and from one draw a tangent and from the other draw a normal, *i.e.*, a line perpendicular to a tangent.

PROBLEM XXXIX.

To Draw an Oval of a Given Width, A B.—*(Fig. 48.)*

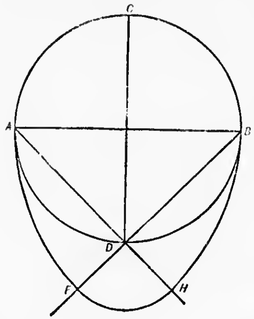

Fig. 48.

Draw a circle with a radius equal to one half of *A B*. Draw the diameter *C D* perpendicular to *A B* (problem iv.). From *A* and *B* draw lines through *D*. With *A* as a centre and *A B* as radius draw an arc from *B*, cutting *A D* produced, in *H*. With *B* as a centre and *B A* as radius draw an arc from *A*, cutting *B D* produced, in *F*. With *D* as a centre and *D F* as radius draw the arc *F H*.

PROBLEM XL.

To Draw the Involute of a Given Circle, A B C.—*(Fig. 49.)*

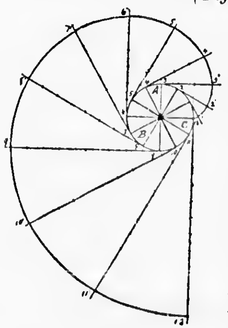

Fig. 49.

Divide the circle into any number of equal parts, say 12, in the points *1*, *2*, *3*, *4*, etc., and from these points draw tangents to the circle. On the tangent from *1* measure the length of one of the divisions of the circumference of the circle, as *1*, *1′*. On the tangent from *2* measure the length of two of the divisions, as *2*, *2′*. On each of the tangents measure the length of the number of divisions indicated by the number at the point from which the tangent is drawn, and through these points, *1′*, *2′*, *3′*, *4′*, etc., draw the curve.

EXERCISES.

80. The transverse axis of an ellipse is $2\frac{1}{4}''$ long. Its foci are $\frac{1}{4}''$ from each end. Find the length of the conjugate axis.

81. Draw the involute of a circle $\frac{3}{4}''$ in diameter.

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PROBLEM XLI.

To Draw a Circle of a Given Radius, A, to touch both lines of an Angle, B C D. *(Fig. 50.)*

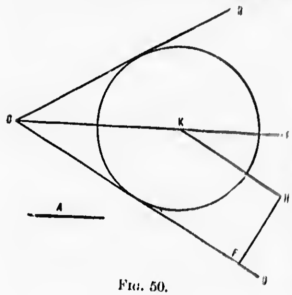

FIG. 50.

Bisect the given angle by *C E* (problem xi.). In one of the lines *B C* or *D C* take any point, *F*, and from it erect a perpendicular, *F H*, (problem iv.) equal in length to *A*. From *H* draw a line parallel to *D C* (problem vii.) to cut *C E* in *K*. With *K* as a centre and *A* or *H F* as a radius draw a circle.

PROBLEM XLII.

To Draw a Circle of a Given Radius, A, to Touch a Given Circle, B C D, and a Given Straight Line, E F. *(Fig. 51.)*

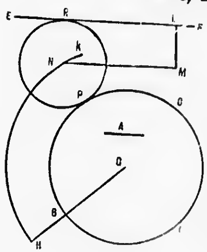

FIG. 51.

Draw any radius of the given circle, as *G B*, and produce it making *B H* equal to *A*. With *G* as a centre and *G H* as radius draw an arc *H K*. In *E F* select any point, *L*, and from it draw a line, *L M* perpendicular to *E F* (problem iv.) and equal to *A*. From *M* draw a line parallel to *E F* (problem vii.) to cut the arc *H K* in *N*. With *N* as a centre and *A*, or *L M* as radius draw a circle which will touch the circle *B C D* in *P*, and the straight line *E F* in *R*.

NOTE.—The radius of the required circle must not be less than one-half of the least distance between the given straight line and the given circle.

EXERCISES.

82. Draw an oval, the greatest width of which will be 2″.
83. Draw a circle of $\frac{1}{2}$″ radius to touch both lines of an angle of 45°.
84. A sphere $1\frac{1}{4}$″ in diameter is dropped into a hollow cone having a vertical angle of 50° and its axis 3″ long. How near could the centre of the ball approach to the vertex of the cone?
85. Two cylinders of $\frac{3}{8}$″ and $\frac{7}{8}$″ radius respectively are in contact with one another and with a horizontal surface. Show their relative positions.

PROBLEM XLIII.

Within a Given Circle, A B C, to Inscribe any Number, six, of Equal Circles, each one to Touch two Other Equal Circles and the Given Circle.—*(Fig. 52.)*

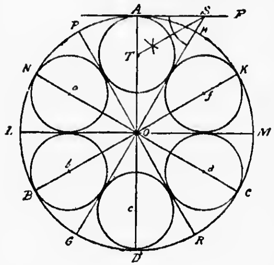

Fig. 52.

Draw any diameter *A D* and at *A* draw a tangent *A F* (problem xxviii.). Divide the circle into twice as many equal parts as the number of circles required, in this case twelve (problem xii.). Produce one of the diameters closest to *A D*, as *G H*, to meet the tangent in *S*. Bisect the angle *A S O* (problem xi.) and produce the bisecting line to meet *A D* in *T*. On the alternate radii *O N*, *O B*, *O D*, *O C*, and *O K* measure from *O* the distance *O T*. With the points *a*, *b*, *c*, *d*, *f* and *T* as centres and *T A* as radius draw circles which will fulfil the conditions required.

PROBLEM XLIV.

To draw a Circle Touching the Three Sides of a Given Triangle, A B C.—*(Fig. 53.)*

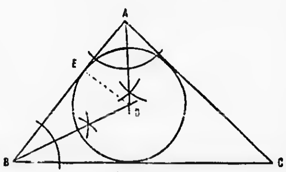

Fig. 53.

Bisect any two of the angles, as *C B A* and *B A C*, and produce the bisecting lines to meet in *D*. With *D* as a centre and its least distance, as *D E*, from any one of the sides, as a radius, draw a circle. (*Euclid* iv. 4.)

EXERCISES.

86. A tinsmith has a circular sheet of tin 24″ in diameter and wishes to cut out of it eight equal circles. Show by a drawing on a scale of $\frac{1}{3}$ how he would proceed to cut the large circle.

87. What will be the diameter of the largest circle which can be drawn in a right angled triangle the sides containing the right angle being $1\frac{3}{4}''$ and 2″ long respectively?

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PROBLEM XLV.

About a Given Circle, A B C, to Construct an Equilateral Triangle.—*(Fig. 54.)*

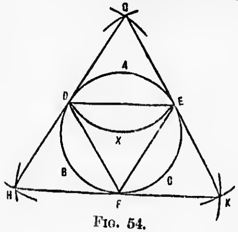

FIG. 54.

Find the centre, *X*, of the circle (problem xxiv.). Select any point, *A*, in the circumference and with *A* as a centre and *A X* as radius draw an arc cutting the circumference in *D* and *E*. Join *D E*. With *D* and *E* as centres and *D E* as radius draw arcs to intersect in *F* and join *D F* and *E F*. Then *D E F* will be an equilateral triangle inscribed in the circle. With *D*, *E* and *F* as centres and *D E* as radius, draw arcs to intersect in *G H* and *K*. Join *G H*, *G K* and *H K*. Then *G H K* will be the equilateral triangle required.

PROBLEM XLVI.

In a Given Circle, A B C, to Inscribe a Triangle Equiangular to a Given Triangle, D F G.—*(Fig. 55.)*

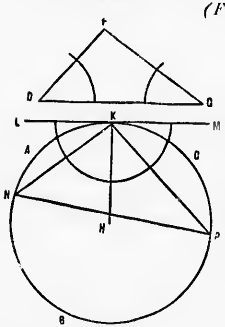

FIG. 55.

Draw any radius *H K* and at *K* draw a tangent, *L M* (problem xxviii.). At *K*, on *L M* make the angle *L K N* equal to the angle *F G D*, and make the angle *M K P* equal to the angle *F D G* (problem x.). Join *N P*. Then *K N P* will be the triangle required. (*Euclid* iv. 2.)

EXERCISES.

88. What will be the length of the sides of the smallest equilateral triangle which will contain a circle 1″ in diameter?

89. A carpenter wishes to make a box whose section will be an equilateral triangle, to contain a cylinder 6″ in diameter and 3′ long. What will be the width of the sides of the box, inside measurement? Scale $\frac{1}{4}$.

90. In a circle $2\frac{1}{2}$″ in diameter construct the largest possible triangle whose sides will be as 4, 5 and 6.

PROBLEM XLVII.

About a given Circle, A B C, to Construct a Triangle Equiangular to a Given Triangle, D G F.—*(Fig. 56.)*

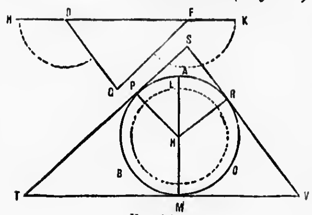

Fig. 56.

Produce one side $D\ F$ of the given triangle, indefinitely to H and K. Draw any diameter, $L\ M$, of the given circle. At N on $L\ M$ make the angle $M\ N\ P$ equal to the angle $K\ F\ G$, and make the angle $M\ N\ R$ equal to the angle $H\ D\ G$ (problem x.). At M, P and R draw tangents to the circle (problem xxviii.) to intersect in S, T and V. Then $S\ T\ V$ will be the triangle required. (*Euclid* iv. 3.)

PROBLEM XLVIII.

Within a Given Square, A B C D, to Construct a Regular Octagon.—*(Fig. 57.)*

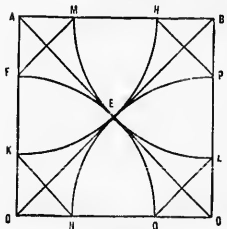

Fig. 57.

Draw the diagonals, $A\ C$ and $B\ D$, of the square, intersecting in E. With A, B, C and D as centres and $A\ E$ as radius draw arcs to cut the sides of the square in F, G, H, K, L, M and N, P. Join $F\ M$, $H\ P$, $L\ G$ and $N\ K$. Then $N\ K\ F\ M\ N\ P\ L\ G$ will be the octagon required.

EXERCISES.

91. About a circle 1″ in diameter construct a triangle equiangular to the one mentioned in exercise 90.
92. A carpenter has a piece of timber 12″ square and wishes to make it octagonal in form. How would the change be effected? Scale $\frac{1}{6}$.
93. What will be length of the sides of an octagon whose diameter is $2\frac{1}{4}$″?

PROBLEM XLIX.

To Find a Third Proportional to Two Given Lines, A and B.—*(Figs. 59 and 60.)*

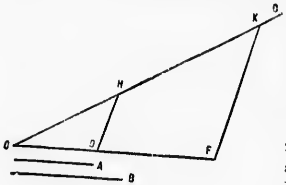

FIG. 58.

FIRST.—Let it be required to find a third proportional which will be greater than *A* or *B*. —*(Fig. 59.)*

Draw any line *C F* making *C D* equal to *A* and *D F* equal to *B*. From *C* draw a line, *C G*, of indefinite length forming any angle with *C F*, and on it measure *C H* equal to *B*. Join *H D*. From *F* draw a line parallel to *D H* (problem vii.), to meet *C G* in *K*. Then *H K* will be the proportional required, that is *C D* : *C H* :: *C H* : *H K*. (*Euclid* vi. 11.)

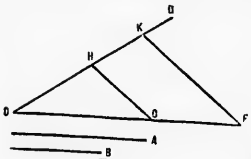

FIG. 59.

SECOND.—Let it be required to find a third proportional which will be less than *A* or *B*.—*(Fig. 60.)*

Draw any line, *C F*, making *C D* equal to *A* and *D F* equal to *B*. From *C* draw a line, *C G*, of indefinite length, forming any angle with *C F*, and on it measure *C H* equal to *B*. Join *H D*. From *F* draw a line parallel to *H D* (problem vii.) to meet *C G* in *K*. Then *H K* will be the proportional required, that is, *C D* : *C H* :: *C H* : *H K*. (*Euclid* vi. 11.)

EXERCISES.

94. Draw two lines ½″ and ⅞″ long respectively and find a third proportional which will be greater than either,
95. Draw two lines bearing the ratio of 4 and 5 and find a third preportional less than either.
96. Find a third proportional (greater) to two lines bearing the ratio of 3 and 5.
97. Find a third proportional (less) to two lines 1¼″ a. d 1″ long respectively.

PROBLEM L.

To Find a Mean Proportional Between two Given Lines, A and B.—(*Fig. 58.*)

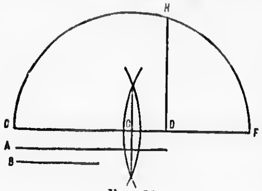

Fig. 60.

Draw any line, *C F*, making *C D* equal to *A* and *D F* equal to *B*. Bisect *C F* in *G* (problem i.). With *G* as a centre and *G C* as radius draw a semicircle. From *D* erect a perpendicular (problem iv.) to cut the circumference of the semicircle in *H*. Then *D H* will be the mean proportional required, that is, *C D* : *D H* :: *D H* : *D F*. (*Euclid* vi. 13.)

Note.—This problem is used when it is required to construct a square equal in area to a given oblong. For instance: Let it be required to construct a square which will be equal in area to an oblong whose sides are equal to *A* and *B* respectively. (*Fig. 61.*)

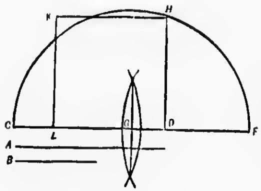

Fig. 61.

Draw any line *C F* equal in length to the sum of *A* and *B*. Bisect this line in *C*. On it draw a semicircle, and from *D* erect a perpendicular to cut its circumference in *H*. On *D H* construct a square *D L K H* which will be the square required.

EXERCISES.

98. Find the mean proportional between two lines 1″ and 1¼″ long, respectively.

99. What will be the length of the sides of a square equal in area to an oblong ¾″ x 1″?

100. A farmer has a field 100′ x 500′ and exchanges it for a square one of the same area. What will be the length of its sides? Scale 200′ = 1″

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GENERAL NOTES.

INKING IN.—The teacher should encourage pupils to do as much work with ink as is possible. For use in school hours, the most convenient ink is the liquid India ink sold by stationers, but if this is not obtainable the India ink in sticks should be used, avoiding the cheap qualities, as they are not perfectly black. The ink should be rubbed with a few drops of water in a saucer or on a porcelain or glass slab, until it is thick enough to make a perfectly black mark. It should be kept free from dust, and as the water evaporates more should be added. The pens may be filled by means of a strip of hard finished paper or cardboard, or a small brush, and care should be taken not to smear the ink on the outside of the pen. The thickness of the lines can be regulated by means of the thumb-screw at the side of the drawing pen. Do not leave the stick of India ink standing in water, but after rubbing as much as is needed set it on end to dry.

This work of "inking in" may be made more interesting if ink of different colors is used. For instance, make all *given* lines blue; all *construction* lines red; and the *result, i.e.* the circle, square, polygon, etc., required, black. Water colors will supply the necessary tints, and should be treated in the same way as the India ink.

CIRCLES.—In the exercises will be found some questions requiring circles to be drawn by other means than the compasses. It would be well for the teacher to provide for the use of the pupils, or the pupils to provide for their own use, a few circles of different sizes, cut out of thin cardboard or tin. They can be laid on the paper and a line traced around their circumference.

POLYGONS.—Polygons are named according to the number of sides. Those most used are the Pentagon, 5 sides; Hexagon, 6 sides; Heptagon, 7 sides; Octagon, 8 sides; Nonagon, 9 sides; Decagon, 10 sides; Undecagon, 11 sides; Duodecagon, 12 sides. They may be regular or irregular—*regular* when the sides and angles are all equal, and *irregular* when either the sides or angles, or both, are unequal. The methods for constructing polygons may be classed as special and general. The *special* methods are those that can be used for the construction of only one particular polygon, and the *general* methods are those that can be used in constructing any or all of the polygons. For instance, the methods illustrated in figs 37, 39, 41 and 42 are special, because they are of no use for any polygons except those specified in the particular problems; and those illustrated in figs. 38 and 40 are general, because any of the polygons can be constructed by them. In problem xxx., fig. 38, if a duodecagon be required, the circumference of the semicircle will be divided into twelve parts, and the line corresponding to $A\,2$, drawn through the second division. If an octagon be required the circumference of the semicircle will be divided into eight parts, etc. In the method described in problem xxxii. the diameter of the semicircle is divided into the same number of parts as there are sides in the polygon required, as explained for an octagon, into eight parts; for a nonagon, into nine parts, etc., and the line corresponding to CD, fig. 40, drawn through the second division from either end of the diameter.

This problem supplies a useful method for dividing a circle into any number of equal parts. Suppose it is required to divide a circle into nineteen equal parts. Draw a diameter and divide it into nineteen equal parts. With its extremities as centres and a radius equal to its length draw arcs to intersect on each side of it in points X and Y corresponding to C, fig. 40. From X and Y draw lines through the alternate points of division, commencing with 2, as 2, 4, 6, 8, 10, 12, 14, 16 and 18, to cut the circumference.

If the work is carefully done the result seems to be exact, yet as far as the author can learn, the method admits of no mathematical proof.

ELLIPSE.—In the methods for drawing an ellipse, illustrated in figs. 43 and 44, the perfection of the curve depends upon the number of divisions in the line $F\,I\,E$ in the one case, and of the circumference of the circles in the other. In drawing large ellipses a greater number of divisions should be taken.

No part of the curve of an ellipse can be considered to be a portion of the circumference of a circle, hence the result of problem xxxvi. is an elliptical curve and not a true ellipse.

INVOLUTE OF CIRCLE.—In problem xl. where the length of the divisions of the circle is referred to, the length of the curve is meant, and not the length of the chord. A slight allowance must be made for the difference between the two. It is apparent that the longer the arc is, the greater will be the difference in length between it and its chord.