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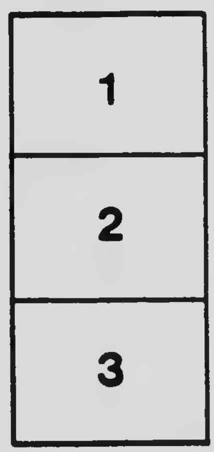
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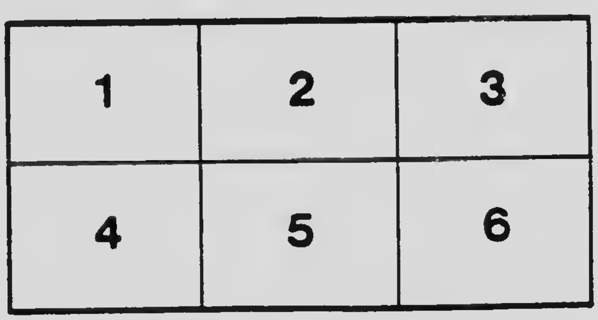
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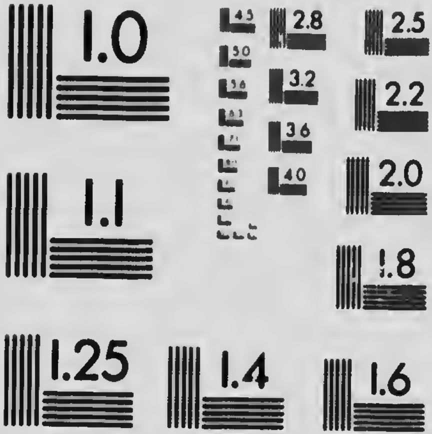

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PRACTICAL AND THEORETICAL

GEOMETRY

PART I

FOR CONTINUATION CLASSES IN PUBLIC SCHOOLS AND LOWER SCHOOL CLASSES IN SECONDARY SCHOOLS

BY

A. H. McDOUGALL, B.A.
Principal Ottawa Collegiate Institute.

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TO THE TEACHER.

In this manual I have followed the course in Geometry prescribed in the Regulations of the Department of Education of Ontario, for Form V of Public Schools, and for the Lower School of High Schools and Collegiate Institutes.

This course is defined as follows :—

"Definitions ; fundamental geometric conceptions ; use of instruments, compasses, protractor, graded rule, set-square ; measurement of lines and angles, and construction of lines and angles of given numerical magnitude ; accurate construction of figures ; some leading propositions in Euclidean plane geometry reached by induction as a result of the accurate construction of figures ; deduction also employed as principles are received and assured.

"NOTE.—The course should emphasize physical accuracy as well as accuracy of thought ; exactness in drawing lines of required length, in measuring lines that are drawn, in constructing angles of given magnitude, and in measuring angles that have been constructed."

A few definitions are given together in Chapter I, but generally they are brought in when required for the first time. An alphabetical index of the definitions is given at the end.

Practical methods are constantly used. I have found the use of tracing paper as a substitute for the mental superposition method of Euclid a satisfactory method with beginners. Other results are illustrated by paper folding. The test for the sum of the angles of a triangle is an elegant example of the use of this method.

About ten of the fundamental propositions of Euclid are investigated by experimental methods, and these afford the basis of much simple deductive reasoning. The usual proofs of these fundamental propositions are too abstract to be appreciated by the beginner, but if he is brought to accept the results as facts derived from experience, he readily applies them in deducing other simple relations, or in proving constructions.

Two extremes should be avoided in teaching Geometry to beginners; on the one hand purely abstract reasoning without a foundation of practical knowledge, and on the other hand a mass of disconnected experimental measurement or rule-work.

The answers that are given for many of the examples are intended to serve as checks on the accuracy of the drawing and on the correctness of the method. Results within one per cent. of the answers in the book may generally be taken as satisfactory.

The teacher should be provided with a pair of blackboard compasses, a straight-edge about three feet long, and a sixty-degree set-square with the shortest side about ten inches in length.

Each pupil should have:—

A supply of tracing paper.

A hard pencil.

A ruler graduated in inches and fractions of an inch, and also in centimetres and millimetres.

A pair of compasses with a hard pencil point.

A pair of dividers with two sharp steel points for making accurate measurements.

A protractor.

A set-square.

Too much stress cannot be placed on precision of language in the definitions and proofs. Such precision is essential to accuracy of thought, but it should be taught rather by questions and answers that require clearness of statement and description on the part of the pupils themselves than by causing set forms of words to be committed to memory.

CONTENTS.

PART I.

CHAPTER I.

Preliminary Definitions and Explanations.

1. A **point** has position but no size.

The position of a point on the blackboard, or on paper, is represented by a mark. This mark has some small size and therefore only roughly represents the idea of a point.

2. A **line** has length but neither breadth nor thickness.

Again, the mark that we use to represent a line has breadth and some small thickness, and consequently, only roughly represents the idea.

3. A **surface** has length and breadth but no thickness.

A sheet of tissue paper has length and breadth and very little thickness. It thus roughly represents the idea of a surface. In fact the sheet of paper has two well-defined surfaces separated by the substance of the paper.

4. A **solid** has length, br   th and thickness.

5. Lines may be either straight or curved.

6. Fold and crease a sheet of fairly stiff paper. The crease thus formed is a straight line.

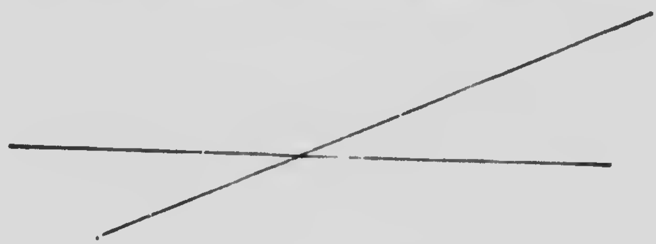
A ruler, or straight-edge, is used for drawing straight lines on paper or on the blackboard.

The edge of one ruler should exactly fit that of another. Place the edges of two rulers together and hold them up to the light. If you can see light between them they are not both straight.

Draw a line with your ruler. Turn the ruler over keeping the edge on the line you have drawn and see if it now fits the line. If it does the edge is straight.

The abbreviation st. is used for the word straight.

7. Draw two st. lines cutting each other.



The place where they cut is a point.

In drawing lines use a hard, sharp pencil. The best results are obtained with a flat, or chisel edge.

The intersection of two lines is a point.

The proper way to indicate a point on paper is by the intersection of two lines.

Points are indicated at A and B.

Do not use a dot for this purpose.

Two points fix the position of the st. line joining the points.

8. When we speak of a st. line we may mean one of indefinite length, that is, it may be thought of as extending without end in the two opposite directions and having the property of straightness throughout.

Or, we may mean only that part of a st. line from one fixed point to another. This latter may be distinguished from the other by calling it a **line-segment**.

9. To measure any line-segment, we take another line-segment of fixed length and find how many times the former segment contains the latter.

The segment used as a standard to measure the other is called the unit of measurement.

The line-segment AB contains the line-segment CD exactly three times.

The length of AB is thus expressed by the number 3 when CD is the unit.

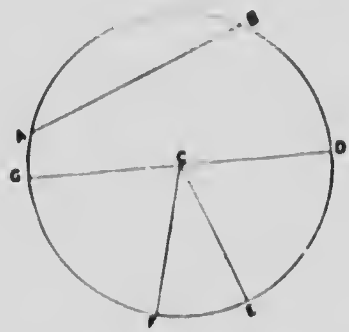
AB is also said to have to CD the ratio of 3 to 1.

Thus the length of a line-segment may be measured and expressed by a number.

There are two things required in expressing that measurement: the unit, and the number or ratio.

Certain fixed and established units of length have names, as: an inch, a mile, a metre, a centimetre, etc.

10. Draw a circle with your compasses.



11. The curved boundary is called the **circumference** of the circle.

12. The point C at which the steel point rested is called the **centre** of the circle.

13. All st. lines drawn from the centre to points on the circumference, as CD, CE, CF, CG, are equal in length, and are called **radii** of the circle.

Any one of them is called a **radius.**

14. A st. line drawn from one point on the circumference to another is called a **chord** of the circle.

AB is a chord of the circle.

15. If a chord passes through the centre, as GD, it is called a **diameter.**

The length of the diameter of a circle is twice that of a radius.

All diameters of the same circle are equal to each other.

16. A part of the circumference, as the curved line FED, is called an **arc** of the circle.

SYMMETRY.

17. Draw a circle on paper and draw one of its diameters. Cut out the circle and fold it so that the crease is along the diameter.

The part on one side of the diameter fits exactly on the part on the other side.

18. When a figure can be folded along a line so that the part on one side exactly fits the part on the other side, the figure is said to be **symmetrical** with respect to that line.

The line along which the figure is folded is called an **axis of symmetry** of the figure.

19. **Every diameter of a circle is an axis of symmetry of the circle.**

When the circle is folded along a diameter, press the point of a pin through both thicknesses of paper; withdraw the pin and spread the circle out.

The two marks made by the pin indicate corresponding points with respect to the axis of symmetry.

If a figure is symmetrical with respect to a st. line, for every point on one side of this axis of symmetry there is a corresponding point on the other side.

20. Draw a line-segment and fold the paper so that the ends of the segment are together. The crease cuts the segment at its middle point.

The ends of the segment are symmetrical with respect to its middle point.

The ends of any diameter of a circle are symmetrical with respect to the centre of the circle.

21. PROBLEM :—From the greater of two given line-segments to cut off a part equal to the less.

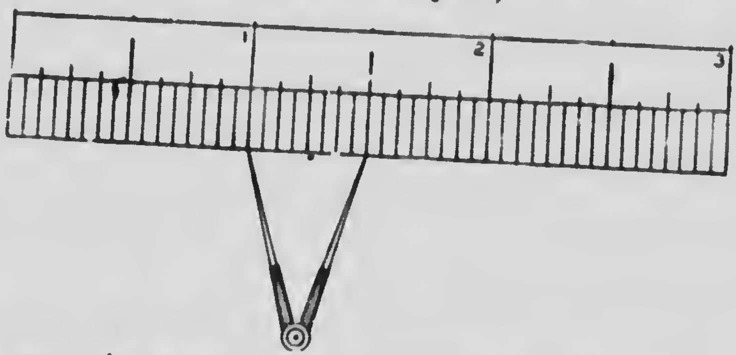
Draw two line-segments AB and CD, of which CD is the greater

Set the points of the compasses at the distance AB.

Place the steel point at C and draw a short arc cutting CD at E.

The part CE has been cut off equal in length to AB.

To measure off a given length from a st. line AB place the points of tne dividers on the scale so that they are the required number of units apart;



then carry the distance to the required position

To measure the length of a line-segment place the points of the dividers at the ends of the line-segment; and carry the distance to the scale. The length may then be read from the scale. If it does not give the exact number, an estimate may be made of the fraction.

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22. Exa[illegible]es.

1. It is required to cut off from a given st. line a part equal to twice a given line-segment.

2. From a given st. line cut off a part equal to five times a given length.

3. Make two line-segments that shall contain a given line-segment three and four times respectively.

These two line-segments are to each other in the ratio of 3 to 4 (ratio 3 : 4).

4. Make two line-segments that have to each other the ratio 5 : 6.

Take any line-segment of convenient length as unit.

5. From a given point draw a st. line in any direction 8 cm. in length.

6. From a given point draw a st. line in any direction equal to twice a given line-segment.

7. Draw a st. line BC of unlimited length and a line-segment D. Mark a point A not in BC. It is required to draw a line-segment, equal in length to D, having one end at A and the other in the line BC.

If the point A is not too far from BC, two such lines can be drawn.

8. Draw a st. line from a given point, equal to twice a given line-segment and terminated in a given st. line.

[illegible]aw a st. line from a given point to a given st. line and [illegible]o five times a given line-segment.

10. From a given point draw a line-segment that will be bisected at another given point.

11. Draw a line 2½ inches long and measure its length in millimetres.

12. Set the points of the dividers 6 cm. apart and measure the distance in inches. From your result find the number of inches in a metre.

If your rule is graduated to sixteenths of an inch, and the result is not an exact number of sixteenths, estimate the result to sixty-fourths. 1 m = 39.37 in. nearly.

13. Set the dividers at 2 inches and measure the distance in millimetres.

From your result find the number of centimetres in a yard. Test by calculation from value of metre in Ex. 12.

CHAPTER II.

ANGLES, TRIANGLES AND QUADRILATERALS.

23. From the same point, draw two st. lines of indefinite length.

The figure thus formed is called an **angle.**

The point from which the two lines are drawn is called the **vertex** of the angle.

The two lines are called the **arms** of the angle.

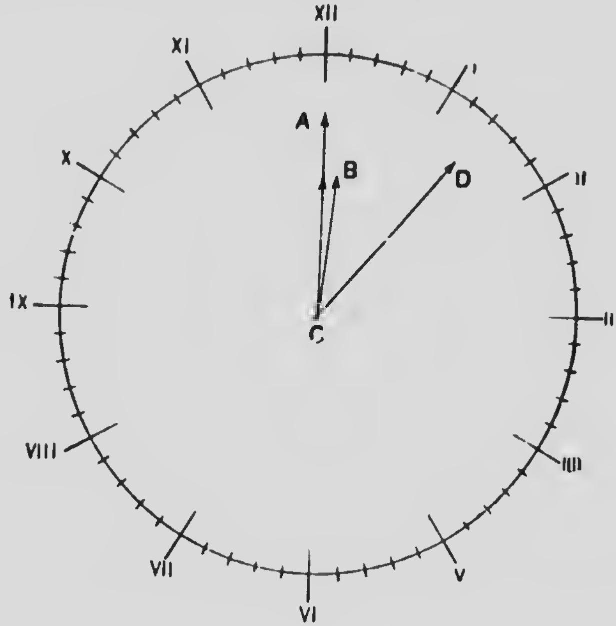
The angle in the figure may be called the angle BAC, or the angle CAB. The letter at the vertex must be the middle one in reading the angle.

The single letter at the vertex is sometimes used to denote the angle when there can be no doubt as to which angle is meant.

The symbol ∠ is used for the word angle. The plural is denoted by writing *s* after the symbol.

24. Draw a diagram of the dial of a clock, showing roughly the positions of the hour and minute marks.

Draw a line showing the position of the hands when they are together at twelve o'clock, and lines showing their positions at twelve minutes after twelve.



∠ ACD is made by the rotation of the minute hand from the position CA to the position CD, and this ∠ is twelve times that made by the rotation of the hour hand from CA to CB.

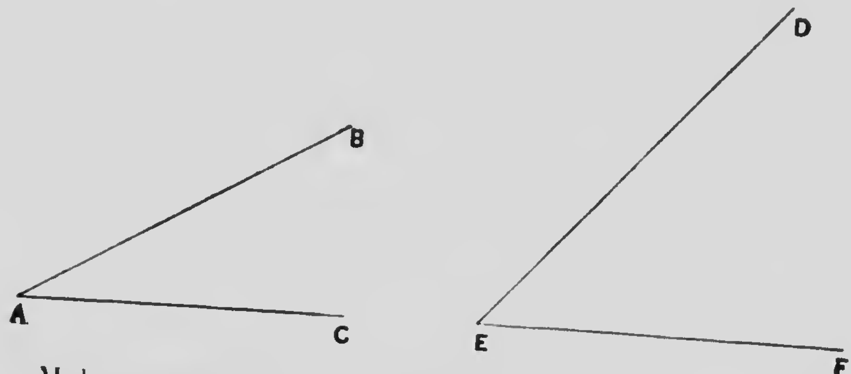
Draw a similar diagram showing the ∠s through which the hands rotate from 9 o'clock to 24 minutes after 9.

25. From § 24 we see that an ∠ may be considered to be made by a rotation, and that the amount of the rotation may be measured.

If the ∠ ACB were taken as the unit, the measure of the ∠ ACD would be 12.

The magnitude of an ∠ does not depend on the length to which its arms may happen to be produced.

26. Draw two ∠s BAC and DEF.



Make a tracing of ∠ BAC and mark the tracing with the same letters B, A, C.

Place the tracing on ∠ DEF so that the point A is exactly on the point E and the line AC falls along EF.

If AB falls between EF and ED, the ∠ BAC is less than ∠ DEF.

If AB falls exactly on ED, ∠ BAC is equal to ∠ DEF.

If AB falls beyond ED, ∠ BAC is greater than ∠ DEF.

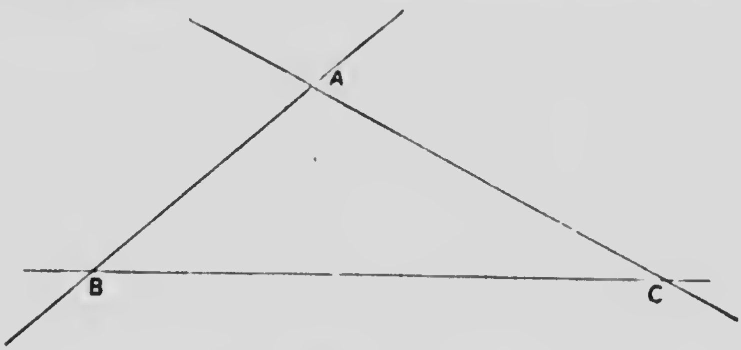
27. Draw three st. lines from a point and letter the diagram. Name three ∠s thus formed. Which ∠ is the sum of the other two? Which ∠s are respectively the differences of the other two?

Draw four st. lines from a point and name six ∠s thus formed. Which of these ∠s are the sums of other ∠s?

28. The figure formed by three st. lines which intersect each other is called a **triangle.**

ABC is a triangle.

The three points of intersection are called the **vertices** of the triangle.



The line-segments between the vertices of the triangle are called the **sides** of the triangle.

The symbol △ is used for the word triangle, and also sometimes to denote the area of the triangle.

Name the seven parts or elements of a △ which may be measured.

29. Draw a △ ABC.

The side BC is said to be opposite the ∠ BAC, or to subtend the ∠ BAC.

Which side subtends ∠ B?

Which side is opposite to ∠ C?

30. Draw an irregular △ and compare the lengths of its sides, using the dividers, if necessary. Which is the greatest and which the least? Write down your results.

Compare the ∠s of the △, using tracing paper. Note the greatest and the least.

In all the △s drawn by the class is it found that the greatest side subtends the greatest ∠ and the least side subtends the least ∠?

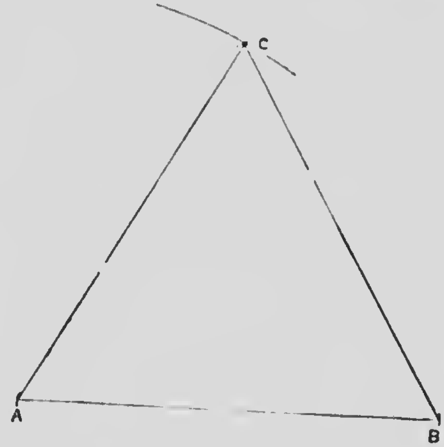
31. A △ having no two of its sides equal to one another is called a **scalene** △.

A △ having two sides equal to one another is called an **isosceles** △.

A △ having its sides all equal to one another is called an **equilateral** △.

All equilateral △s are therefore isosceles, but many isosceles △s are not equilateral.

32. Problem: To make an equilateral △ on a given line-segment.



Let AB be the given line-segment.

With centre A and radius AB draw an arc.

With centre B and the same radius draw another arc cutting the first at C.

Join AC and BC.

With the dividers compare the three sides. They are clearly equal in length, and the △ ABC is the required equilateral △.

33. Examples.

1. Make an equilateral △ having each side 6 cm.

2. Make an equilateral △ having each side double a given line-segment.

3. Make an isosceles △, having each of the equal sides double the base.

4. Make an isosceles △ having the base 3 cm. and each of the equal sides 9 cm.

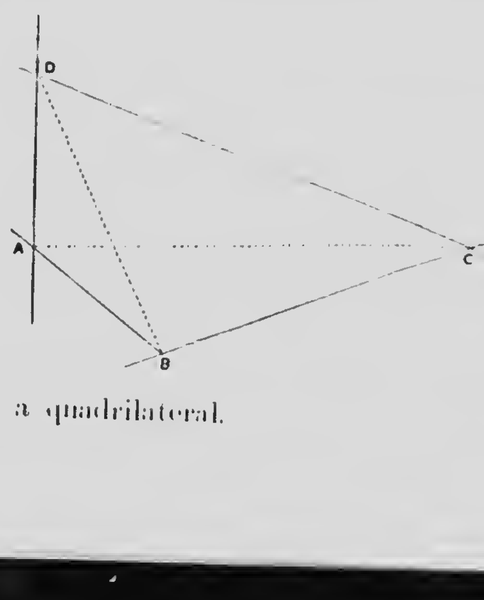
5. Draw a line-segment AB, 6 cm in length. Required to find a point 5 cm. from A. Can there be more than one such point? If so, how many? Where are they situated? Similarly find points 4 cm. from B. Is there a point 5 cm. from A and 4 cm. from B? How many such points are there? Join one of them to A and B. What figure is formed?

6. Make a △ having its sides respectively 3, 4 and 5 times a given line-segment.

7. Make a △, having its sides 47, 58 and 74 mm.

8. Make a △ having its sides 4, 6 and 8 cm. Describe an equilateral △ outwardly on each side. Draw a line from each vertex of the original △ to the remote vertex of the equilateral △ on the opposite side. Measure these lines in millimetres.

34. A figure formed by four st. lines is called a **quadrilateral.**



ABCD is a quadrilateral.

A quadrilateral is read by naming the vertices around the figure in circular order, as ABCD, or BADC.

A st. line joining two opposite vertices, as AC or BD, is called a **diagonal.**

35. A quadrilateral having its four sides equal is called a **rhombus.**

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36. Examples.

1. With a given line-segment AB as diagonal, make a rhombus having each side equal to AB.

2. Make a rhombus having each of the equal sides equal to twice one of the diagonals.

3. Make a rhombus having the sides and one diagonal each 4 cm. Measure the other diagonal.

4. Make a rhombus having one diagonal 6 cm. and each side 4 cm. Measure the other diagonal.

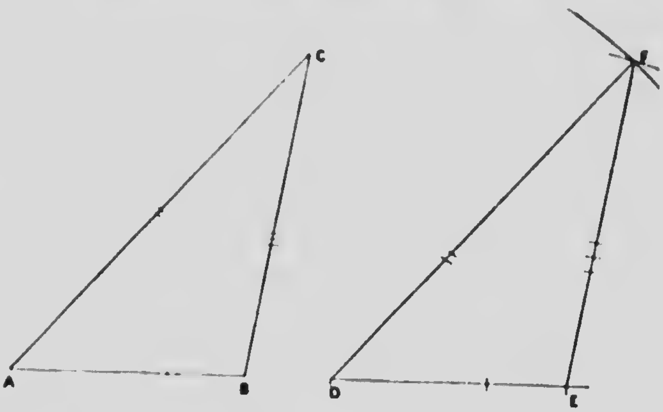
5. Make a rhombus having one diagonal 5 cm. and each side 4 cm. Measure the other diagonal.

6. Make a quadrilateral ABCD having the diagonal AC 7 cm., side AB 3 cm., BC 6 cm., AD 4 cm. and CD 5 cm. Find the length of BD.

7. Make a quadrilateral ABCD having AB 2 cm., BC 3 cm., CD 4 cm., AD 5 cm. and AC 25 mm. Measure BD.

CHAPTER III.

Equality of Triangles: Case I.



37. Draw any △ ABC.

Make a △ DEF having DE = AB, DF = AC and EF = BC.

Thus the three sides of one △ are respectively equal to the three sides of the other.

Make a tracing of the △ ABC and apply the tracing to the △ DEF, comparing the ∠s and the areas.

If the drawing and tracing are carefully made it will be found that the corresponding ∠s are equal; that is, ∠ D = ∠ A, ∠ E = ∠ B, and ∠ F = ∠ C, and as the tracing of △ ABC exactly fits △ DEF, the two △s must have the same area.

We have thus the important proposition:—

If two triangles have the three sides of one respectively equal to the three sides of the other, the two triangles are equal in all respects.

38 Figures that are equal in all respects, so that one may be made to fit the other exactly, are said to be **congruent.**

39. In using the proposition in § 37 we may always find the equal angles by noting that they are subtended by equal sides.

Thus the equal ∠s A and D are subtended by the equal sides BC and EF. By what sides are ∠s B and E subtended? Are they equal sides?

40.—Examples.

1. Draw a rhombus, and prove by the proposition in § 37 that the opposite ∠s of the rhombus must be equal.

2. Draw a kite-shaped figure by taking a line-segment AB, making an isosceles △ ACB on one side, and another isosceles △ ADB on the other side of AB. Prove that ∠ CAD = ∠ CBD, and also that the diagonal CD bisects ∠s ACB and ADB.

3. Draw two isosceles △s, CAB and DAB, on the same side of a line-segment AB, and prove ∠ CAD = ∠ CBD.

4. Make a quadrilateral, ABCD, having both pairs of opposite sides equal to each other, and prove that the opposite ∠s must be equal.

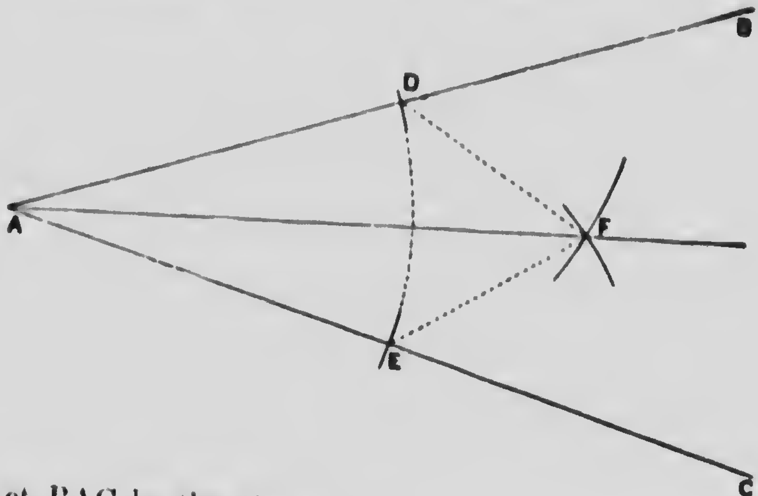
5. Prove that the diagonal of a rhombus bisects each of the ∠s through which it passes.

6. Draw a circle of radius 6 cm., and using the compasses and ruler draw a chord 5 cm. long in the circle.

7. Draw a circle of radius 4 cm., and in it place a chord 7 cm. long.

8. Draw a circle with any radius, place two equal chords in the circle. Prove that these equal chords must subtend equal ∠s at the centre.

41. Problem: To bisect a given angle.



Let BAC be the given ∠.

With the compasses cut off equal distances AD and AE from the arms of the ∠.

With centre D describe an arc.

With centre E and the same radius describe another arc cutting the first at F.

Join AF.

Then AF is the bisector of ∠ BAC.

Join DF and EF and prove by using the proposition in § 37 that the ∠ BAF = ∠ CAF.

Name an axis of symmetry of the figure.

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42. – Examples.

1. Divide a given ∠ into four equal parts.

2. Make a , and bisect the three ∠s. If your work is accurate the three bisectors will pass through one point.

3. Make a △ ABC having AB 36 mm., AC 42 mm., and BC 50 mm. Bisect ∠ A and produce the bisector to cut BC at D. Measure BD, CD and AD.

4. Make a △ having sides 42, 56 and 63 mm. Bisect the ∠ opposite the greatest side, produce the bisector to cut the side, and measure the segments.

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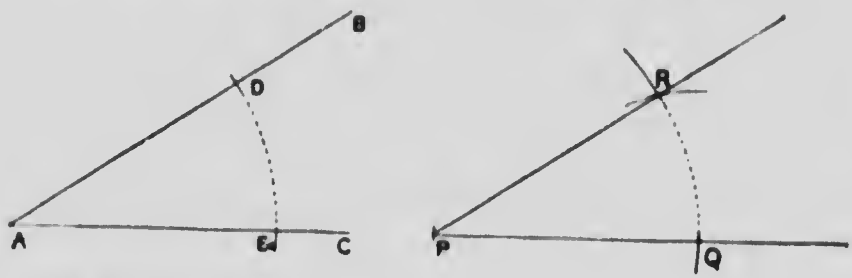
43. **Problem**: To make an ∠ equal to a given ∠.



Let BAC be the given ∠.

With the compasses cut off equal parts AE and AD from the arms of the ∠.

Draw a st. line and mark a point P in it.

Cut off PQ equal to AE or AD.

With centre P and radius PQ describe an arc.

Set the compasses at the distance DE, and with this radius and centre Q describe an arc. The two arcs cut at R.

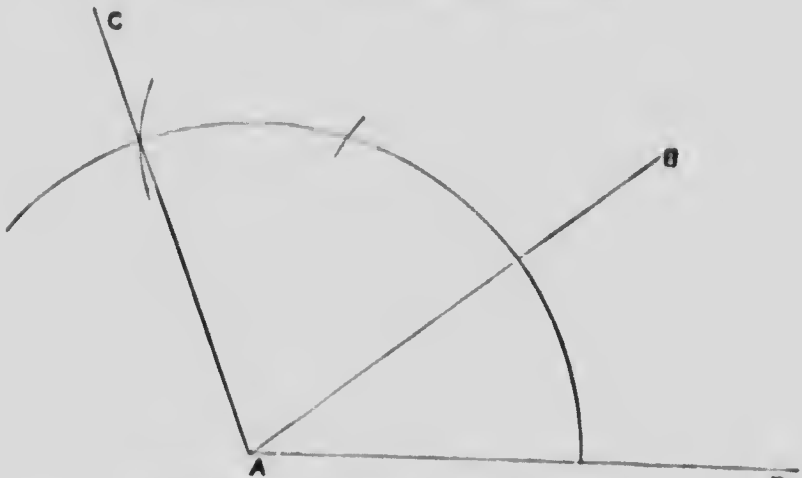
Join PR.

The ∠ RPQ is equal to the ∠ BAC.

Join DE and RQ, and prove that the △s are equal.

44. Examples.

1. Make an ∠ equal to three times a given ∠.



This diagram indicates a concise method.

2. Make an ∠ twice a given ∠.

3. Make an ∠ five times a given ∠.

4. Make an ∠ fourteen times a given ∠.

5. Make two ∠s that have to each other the ratio 3 : 4.

6. Make two ∠s that have to each other the ratio of 3 : 7.

7. Draw an equilateral △. Construct an isosceles △ having its vertical ∠ twice one of the ∠s of the equilateral △. How do the base ∠s of the isosceles △ compare with the vertical ∠?

---

45. In a △ ABC the small letters a, b, c are commonly used to represent the sides, a being the side opposite ∠ A, b the side opposite ∠ B, and c the side opposite ∠ C.

46. Examples.

1. Make a △ ABC, having a = 30, b = 35 and c = 45 mm. Make another △, having one ∠ equal to A, and the two sides which contain this ∠ respectively 7 and 9 cm. Measure the third side.

2. Make a △ ABC, having a = 5, b = 7 and c = 9 cm. Make another △, having one ∠ = ∠ A, and the two sides which contain the ∠ each 10 cm. Measure the third side.

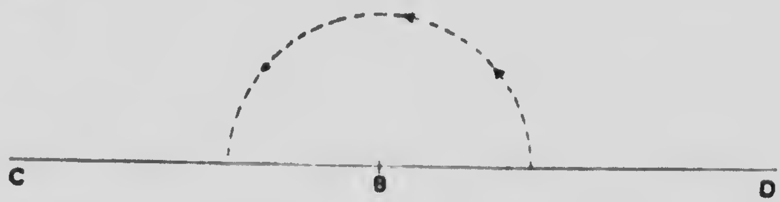
3. Make a △ ABC having a = 4, b = 5, c = 6 cm. Make another △ A'B'C' having B' = B, C' = C and a' = 9 cm. Measure b' and c'.

4. Make an equilateral △ ABC having each side 7 cm. Make a △ DEF having ∠ D one-half ∠ A, ∠ E twice ∠ B and side DE 8 cm. Measure DF.

Right Angles.

47. In one hour the minute hand of a clock makes a complete revolution about the centre of the dial. In half an hour it makes half a revolution, and the angle at the centre through which it rotates is called a **straight angle**. Thus in an hour it rotates through two straight angles.

48.

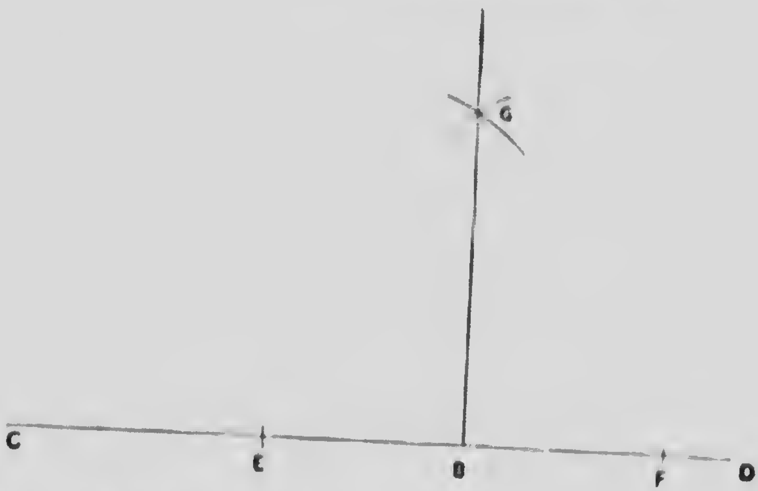


If any st. line CD rotates about a point B in itself until the line is exactly reversed, it is said to rotate through a straight ∠ at the vertex B.

The two arms of a st. ∠ are in the same st. line, and extend in opposite directions from the vertex.

49. Draw a st. line CD and mark a point B in the line.

Bisect the st. ∠ CBD, using the method of § 41.



50. Half a st. ∠ is called a **right** ∠.

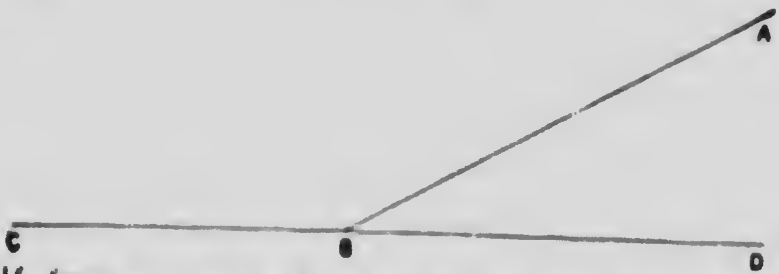
A right ∠ is thus one-quarter of a complete revolution. The minute hand of a clock rotates through four right ∠s in one hour.

Either arm of a right ∠ is said to be **perpendicular** to the other.

CBG and GBD, in the diagram of § 49, are right ∠s, and the st. lines BG and CD are perpendicular to each other.

51. An ∠ which is less than a right ∠ is called an **acute** ∠.

An ∠ which is greater than a right ∠ is called an **obtuse** ∠.



If, from any point B in the st. line CD, a line BA be drawn, the sum of the two adjacent ∠s ABC, ABD is a straight ∠; that is, ∠ ABC + ∠ ABD = 2 rt. ∠s.

Hence we have the proposition:—

The ∠s which one st. line makes with another, on the same side of that other, are together equal to two right ∠s.

53. If a right ∠ be divided into ninety equal parts, each of these parts is called a degree.

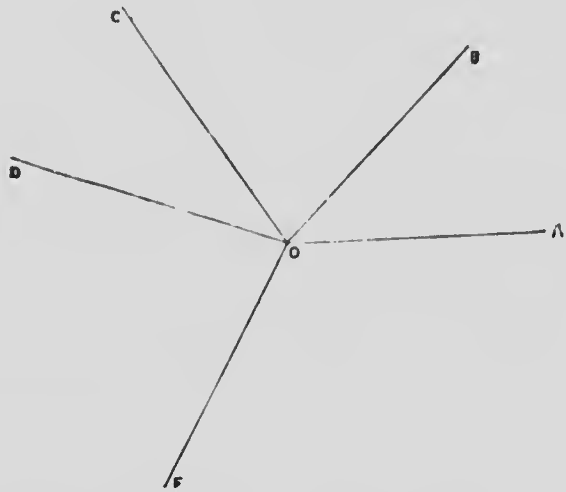
Thus 1 rt. ∠ = 90°,
1 st. ∠ = 180°,
1 revolution = 360°.

In order to get an idea of the size of a degree, draw a rt. ∠, bisect it and divide one half as carefully as you can into five equal parts. Each of these parts should contain 9°. Divide one of these parts into three equal ∠s of about 3° each, and one of these latter into three equal parts. A degree is found to be a small unit. Imagine this very small ∠ divided into sixty equal parts and you have some idea of the very small unit called a minute. Each minute contains sixty seconds.

Through how many degrees does the minute hand of a clock rotate in fifteen minutes? In one minute? In one hour?

Through how many degrees does the hour hand rotate in one hour? In four hours? In one minute?

54.



If any number of st. lines be drawn from a point, the sum of the ∠s about the point is equal to a complete revolution, or four right ∠s.

Thus ∠AOB + ∠BOC + ∠COD + ∠DOE + ∠EOA = 360°.

55. If one ∠ of a △ be a right ∠, the △ is called a **right-angled** △.

In a rt. ∠d △ the side opposite the rt. ∠ is called the **hypotenuse.**

If one ∠ of a △ be an obtuse ∠, the △ is called an **obtuse-angled** △.

If all three ∠s of a △ be acute ∠s, the △ is called an **acute-angled** △.

56. Examples.

1. Cut a piece of paper along a st. line. Fold over the paper so that one part of the edge falls on the other part, and crease the paper.

The crease is perpendicular to the edge.

2. Draw a st. line BC and mark a point A in the line. Using the set square draw from A a st. line perpendicular to BC.

3. Make an ∠ of 45°.

4. Make an ∠ of $22\frac{1}{2}$°.

5. Make a rt.-∠d △ having the sides that contain the rt. ∠ in the ratio 3 : 4.

6. Make a rt.-∠d △ having the hypotenuse twice one of the sides.

7. Make a rt.-∠d △ having one side 6 cm. and the hypotenuse 10 cm. Measure the other side.

8. A room is 40 ft. long and 25 ft. wide. Make a plan of the room and find the length of one of its diagonals. Use 2 millimetres to represent a foot.

9. The hypotenuse of a rt.-∠d △ is 8 cm. and one side is 7 cm. Find the third side in mm.

10. A man walks 5 miles north and then turns to the east. How far must he walk in this direction to be 13 miles in a direct line from the starting point?

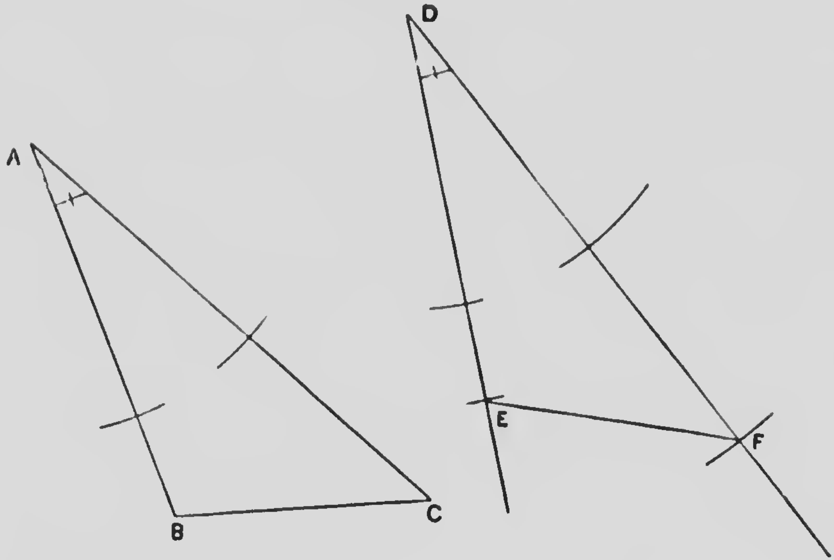
11. Two men start from the same point. One walks 3 miles north and turning to the east walks $5\frac{1}{2}$ miles farther. The other walks $6\frac{1}{2}$ miles west and then 4 miles south. Find how far apart the men now are.

12. A 51-foot ladder, whose foot is 24 ft. from the bottom of a wall just reaches a window. Find the height of the window from the ground.

13. A man goes from A 5 miles east to B, from B 6 miles north-east to C, from C 7 miles west to D and from D 4 miles north-west to E. Find the direct distance from A to E.

CHAPTER IV.

EQUALITY OF TRIANGLES: CASE II.



57. Draw any △ ABC.

Draw a line-segment DE equal to AB.

At D make an ∠ equal to ∠ A (§ 43), and cut off DF equal to AC.

Join EF.

With the dividers compare the lengths of BC and EF.

Make a tracing of △ ABC, and apply the tracing to the △ DEF.

If your work is correct you will find that

BC = EF,
∠ B = ∠ E,
∠ C = ∠ F,

and that area of △ ABC = area of △ DEF.

Hence we have the proposition :—

If two △s have two sides and the contained ∠ of one respectively equal to two sides and the contained ∠ of the other, the two △s are congruent.

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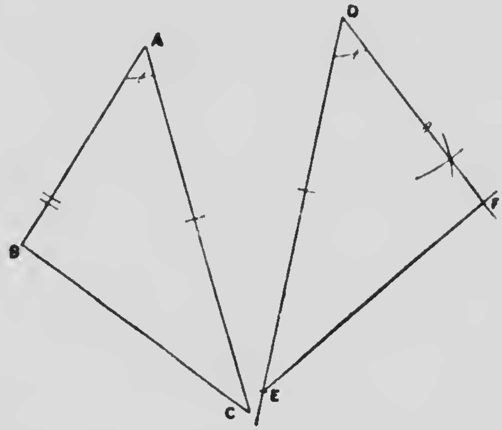
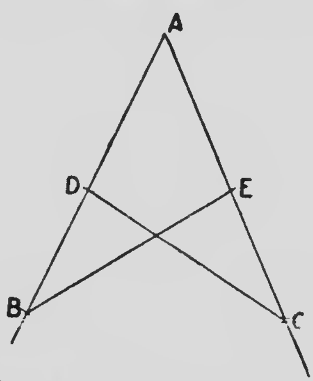
58. Examples.

1. Make two △s, as in the following figure, that have AB = DF, AC = DE, and ∠ A = ∠ D.



Name the other parts that must be equal by the proposition in § 57.

2. Draw any ∠ A. Mark two points D and B in one arm. From



the other arm cut off AC = AB and AE = AD. Join BE and CD. Show, by the proposition in § 57, that there are two △s in the figure that must be congruent, and name the parts that must be equal.

3. Draw any △ ABC. At A make, on the outside of the △, an ∠ CAD equal to the ∠ BAC (§ 43), and cut off AD = AB. Join DC.

Show that DC = BC and name the remaining ∠s in the two △s that must be equal.

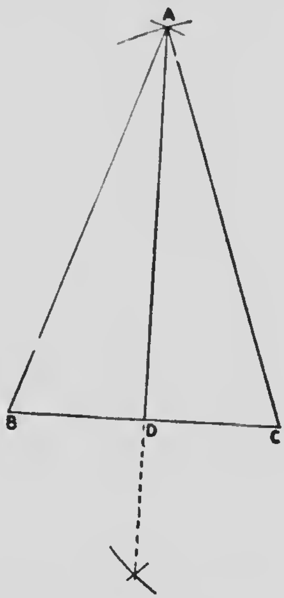
4. Make a rt.-∠d △ ABC having the rt. ∠ at C. Produce BC to D making CD = CB. Join DA. Show that DA = AB. Test with the dividers.

In the same diagram produce CA to E. Join EB and ED. Prove EB = ED.

5. Draw any line-segment AB. At A and B erect perpendiculars AD and BC equal to each other. Join AC and BD. Prove △ ABC congruent to △ ABD.

The Isosceles Triangle.

59. Draw an isosceles △ ABC, having AB = AC.



It is required to show that ∠ B = ∠ C.

Bisect ∠ A (§ 41), and let the bisector cut BC at D.

Prove by § 57 that ∠ B = ∠ C.

The ∠s at the base of an isosceles △ are equal.

60. Draw an isosceles △ ABC on good writing paper. Fold the paper so that one of the equal sides fits the other. The crease is the bisector of the vertical ∠, and the part of the △ on one side of the crease exactly fits the part on the other side.

Hence, the bisector of the vertical ∠ of an isosceles △ is the axis of symmetry of the △ (§ 18); and the bisector of the vertical ∠ bisects the base and cuts the base at right ∠s. This gives another proof of the proposition in § 59.

61. Draw an equilateral △ and show by the proposition in § 59 that the three ∠s of the △ are equal; that is, an equilateral △ is also equiangular.

Cut out the equilateral △ and show that it has three axes of symmetry.

62.—Examples.

1. Draw a rhombus and show that each diagonal bisects the other, and that the diagonals are perpendicular to each other.

Cut out the rhombus and show that it has two axes of symmetry.

2. Draw two unequal isosceles △s on opposite sides of the same base. Show that the st. line joining their vertices bisects the common base and cuts it at rt. ∠s.

Show that the line joining the vertices is an axis of symmetry of the kite-shaped figure.

3. Draw two isosceles △s on the same side of the same base and show that the line joining their vertices, when produced, bisects the common base and is perpendicular to it.

Show that the line joining the vertices, when produced, is an axis of symmetry of the arrow-head figure.

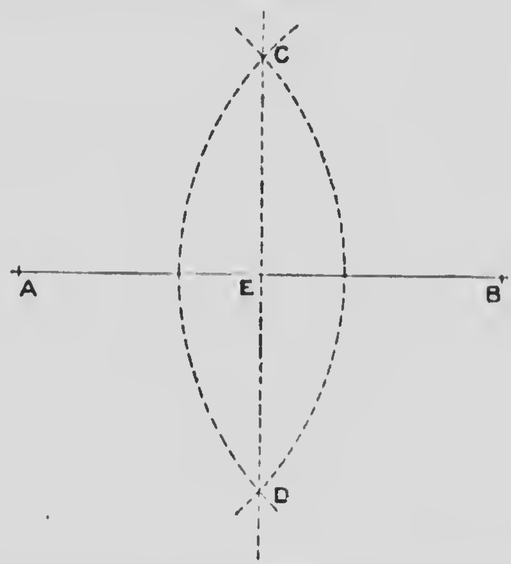
4. Make an equilateral △ ABC. Cut off equal parts AD, BE, CF, from AB, BC, CA respectively. Show that △ DEF must be equilateral.

5. Draw an isosceles △ ABC, having AB = AC. Produce AB and AC to D and E. Prove that the exterior ∠s thus formed are equal.

6. A given ∠ BAC is bisected; if CA be produced to D and the ∠ BAD be bisected, the two bisectors are at rt. ∠s.

Problem.

63. To bisect a given line segment.



Let AB be the given line-segment.

With centre A and any radius that is plainly greater than half of AB, draw two arcs, one on each side of AB.

With centre B and the same radius draw two arcs cutting the first two at C and D.

Join CD, cutting AB at E.

E is the middle point of AB.

Join AC, CB, BD, and DA, and prove that AE = EB. Test the result with the dividers.

Show that there are two axes of symmetry in the figure.

---

64. Examples.

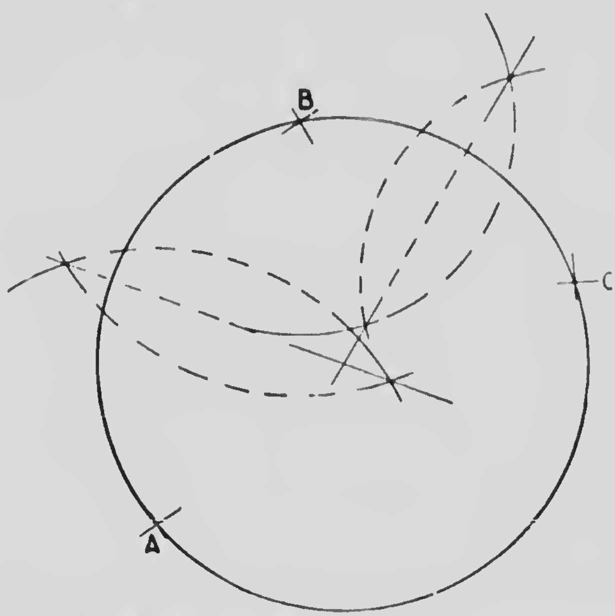
1. From a given line-segment cut off a part equal to one quarter of its length.

2. In the diagram of § 63 show that CD is perpendicular to AB.

3. Mark any point P in CD (diagram of § 63). Join PA and PB. Show that PB = PA. Hence if you draw a circle with centre P and radius PA, it will pass through B.

4. Describe a circle through two given points. Show that an infinite number of circles may be drawn through the two points. What line contains the centres of them all?

5. Draw a circle through three points, A, B, C, that are not in the same straight line.



What line contains the centres of all circles that could be drawn through A and B?

What line contains the centres of all circles that pass through B and C.

6. Draw any △ ABC. Draw a line bisecting AB and cutting it at rt ∠s. Draw a line bisecting AC and cutting it at rt. ∠s. Let these two lines cut at D; show that D is equally distant from A, B and C.

7. Draw a △ having sides 39, 42 and 45 mm. Describe a circle through the three vertices of the △ and find its diameter in millimetres.

8. Draw a △ ABC, having a = 55, b = 65, c = 86 mm. Bisect AB at D and join CD. Find the length of CD.

The line drawn from the vertex of a △ to the middle point of the opposite side is called a **median** of the △.

9. Draw an irregular △, and bisect the three sides. Join each vertex to the middle point of the opposite side. If your work is correct these three lines will pass through one point.

The point where the three medians meet is called the **centroid** of the △.

10. Draw a st. line and mark any two points B and C either in the line or outside of it. Find a point in the line that is equally distant from B and C.

11. Draw a circle with centre A. Draw any chord BC of the circle. Bisect BC at D. Join AD. Show that AD is perpendicular to BC.

12. Make a rhombus having one diagonal double the other.

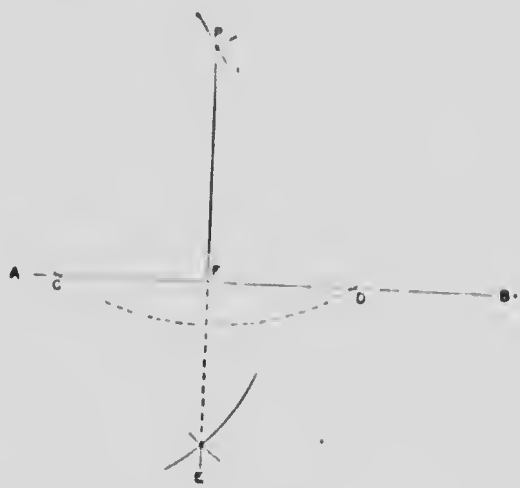
13. Make a rhombus having one diagonal 6 cm. and the other diagonal 52 mm. Find the side.

65. PROBLEM: To draw a perpendicular to a given st. line from a given point outside the line, using compasses and ruler.

Let P be the given point and AB the st. line.

Describe an arc with centre P to cut AB at C and D.

With centres C and D, and equal radii, describe two arcs to cut at E.



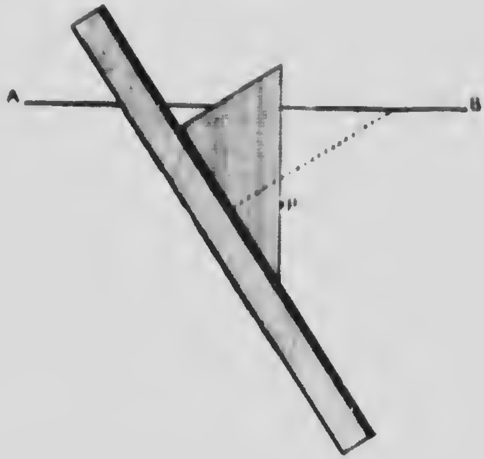
Join PE, cutting AB at F.

PF is the required perpendicular.

Join PC, CE, ED, DP, and prove that PF is perpendicular to AB. (§ 62, Ex. 2.)

Name an axis of symmetry in this diagram. Compare the rt. ∠ made by this method with the rt. ∠ of your set-square; and also with a rt. ∠ formed by paper folding. (§ 56, Ex. 1.)

66. From a point P outside a st. line AB draw a perpendicular to AB, using the set square.



The diagram shows an accurate and convenient method of using the set square to draw through P a st. line perpendicular to AB.

First place the set square in the position shown by the dotted line, with its longest side along AB.

Place a ruler along one of the other sides of the set-square and hold it firmly in that position.

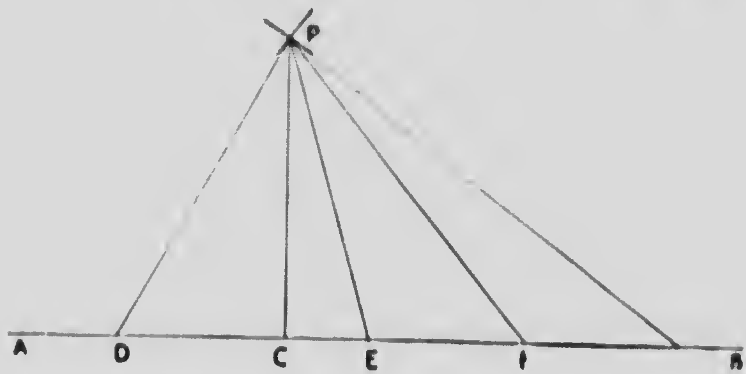
Rotate the set square through its right ∠, thus bringing the third side against the ruler, and slide the square along the ruler to the position shown by the shaded △.

A line drawn through P, along the hypotenuse of the set-square, will be perpendicular to AB.

Draw a st. line CD and mark any number of points on either side of CD. Draw perpendiculars from these points to CD and measure their lengths.

Find a point on each side of CD such that the perpendicular from each point to CD is 2 cm. in length.

67. Draw a st. line AB. Mark a point P outside of the line. Draw a perpendicular, PC, from P to AB. Draw any



number of other lines from P to points D, E, F, etc., in AB. With the dividers compare the lengths of PC, PD, PE, etc. What conclusion do you draw about the length of PC? Write down a general statement of the result.

The distance of a point from a line means the distance measured perpendicular to the line.

68. Examples.

1. Construct any quadrilateral and draw its two diagonals. Draw the four perpendiculars from the point of intersection of the diagonals to the sides of the quadrilateral.

2. Draw a △ ABC having a = 42, b = 45 and c = 39 mm. Draw the perpendicular from A to BC and find its length.

3. Draw a △ ABC having a = 3, b = 5 and c = 7 cm. Find the length of the perpendicular from A to BC.

4. Draw a △ ABC having a = 11, b = 4 and c = 9 cm. Draw AX perpendicular to BC. Bisect BC at D and join AD. Bisect the ∠ A and let the bisector cut BC at H. Measure AX, AD and AH.

5. Draw any irregular △ and draw the three perpendiculars from the vertices to the opposite sides. If the work is accurately done the three perpendiculars will pass through one point. This point is called the **orthocentre** of the △. If the △ be acute-∠d, the orthocentre is within the △; if it be obtuse-∠d the orthocentre is outside the △. Where is the orthocentre of a right-∠d △?

6. Draw a △ ABC having a = 75, b = 70 and c = 65 mm. Draw AX, BY and CZ perpendicular to BC, CA and AB respectively. Measure the two segments of each side and the three distances from the orthocentre O to the vertices.

7. Draw a △ having its sides 45, 50 and 85 mm. Find the distances from the orthocentre to the three vertices.

8. Two flagstaffs stand on level ground 100 feet apart. One is 60 ft. and the other 20 feet in height. Find the distance between their tops. Scale: 20 ft. to an inch.

9. Draw a diagram showing the four cardinal points of the compass. Show also north-east, north-west, south-east and south-west.

10. A man walks 3 miles to the north, turns to the east and walks 2 miles, then turns to the south-east and walks 3 miles farther. How far is he then from the starting point? Mark on the drawing the scale you have used.

11. A house stands some distance from a st. road. The distances from the house to two consecutive mile-stones on the road are known to be 1,400 and 1,200 yards. Find the perpendicular distance from the house to the road. Take one centimetre to represent 200 yards.

12. Draw a circle with centre O and any radius. Make two equal ∠s AOB and COD having A, B, C and D on the circumference. Join AB and CD. Prove AB = CD.

Give a general statement of the proposition just proved.

CHAPTER X

THE SUM OF THE ANGLES OF A TRIANGLE

69. Draw, on good writing paper, any rt. ∠d △ ABC having the rt. ∠ at C.

Cut out the △, fold and crease the paper so that the points A and B will fall at C.

The lines FD and FE show the positions of the creases, and FB and FA will fit along FC.

It will be found that the ∠s A and B together make up the ∠ C.

Thus the two acute ∠s of a rt. ∠d △ together make up a rt. ∠.

70. When the sum of two ∠s is a rt. ∠, each of the ∠s is said to be the complement of the other, and the two ∠s are said to be complementary.

Give the complements of the following ∠s:—40°, 30°, 75°, 48°, 27½°.

One of the acute ∠s of a rt.-∠d △ is 36°; What is the other one?

One of the acute ∠s of a rt. ∠d △ is 22½°; What is the other?

71. Draw any △. Divide it into two rt.-∠d △s. Thence show that the sum of the three ∠s of any △ is equal to two rt. ∠s.

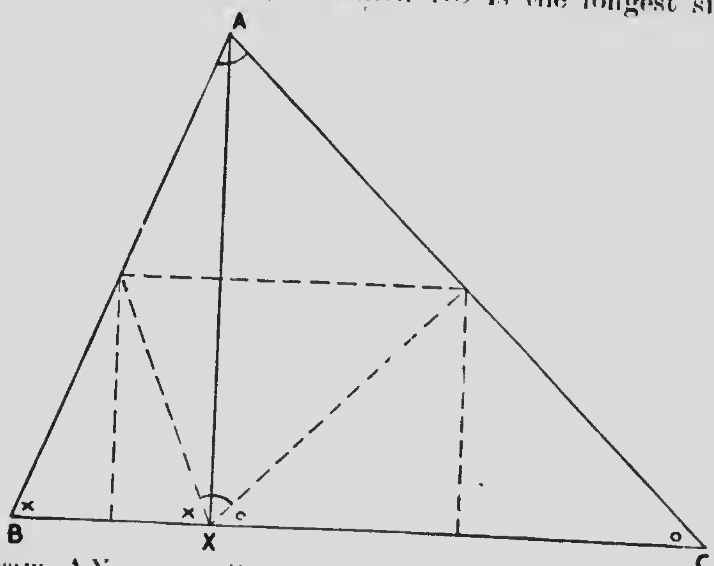
Thus it may be seen that:—

The sum of the three interior ∠s of any △ is equal to two right ∠s.

Ex.—Show that if two △s have two ∠s of the one respectively equal to two ∠s of the other, their third ∠s are equal.

72. Another method of showing that the three ∠s of a △ are together equal to two rt. ∠s.

Draw any △ ABC, of which BC is the longest side.



Draw AX perpendicular to BC.

Cut out the △ and fold so that A, B and C all meet at X.

The sum of the three ∠s coincides with the st. ∠ BXC, and is, therefore, equal to two rt. ∠s.

73. It follows from the proposition in § 71 that the sum of any two ∠s of a △ is less than two rt. ∠s.

74. Examples.

1. Two ∠s of a △ are 40° and 70°. Find the third ∠.

2. Two ∠s of a △ are 57° and 64°. Find the third ∠.

3. Two ∠s of a △ are 108° and 42°. Find the third ∠.

4. Show that each ∠ of an equilateral △ is 60°.

5. Make an ∠ of 60°.

6. Make the following ∠s: 30°, 15°, 75°, 105°, 150°, 37½°, 67½°, 187½°.

7. Trisect a rt. ∠.

8. Construct the △ ABC having b = 100 mm., c = 73 mm. and A = 60°. Measure a.

9. Construct △ ABC having a = 57 mm., B = 15° and C = 135°. Measure b and c.

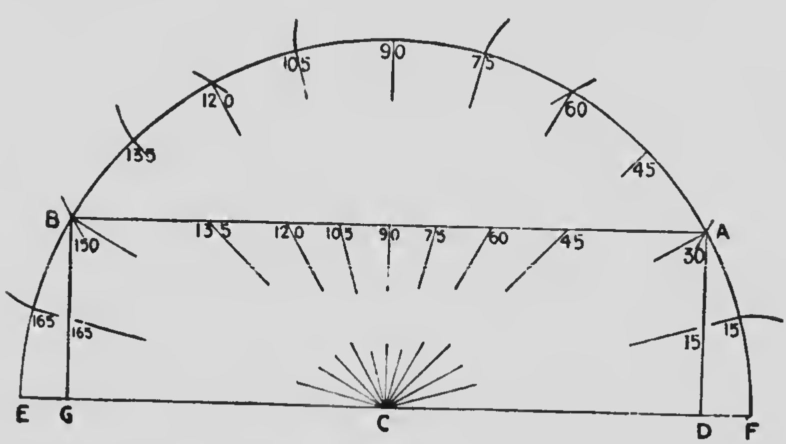
10. Construct △ ABC having a = 96 mm., A = 75° and B = 60°. Measure b and c.

11. Through what ∠ does the hour-hand of a clock rotate in one hour.

12. Draw a diagram of the dial of a clock showing accurately the position of each hour mark.

The Protractor and its Use.

75. A protractor is made by drawing a semi-circle and dividing the st. ∠ at the centre into equal parts.



Make, on stiff paper, a protractor with a unit of 15°.

76. To make a rectangular protractor, join AB and draw AD, BG perpendicular to ECF; mark the fifteen-degree points on DA, AB, BG, and cut out the rectangle.

77. A protractor is used for doing two things:

To make an ∠ containing a given number of degrees.

To measure a given ∠ in degrees.

78. Use the protractor you have made, in the following constructions:

1. Make ∠s of 15°, 75°, 150°, 165°, 195°, 240°, 285°.

2. Draw a diagram of the dial of a clock. Mark the centre C. Write the letters A, B, D, E, F at VII, VIII, IX, X, XI, respectively, and measure the ∠s ADF, BDE, CDE, CDF, ADB.

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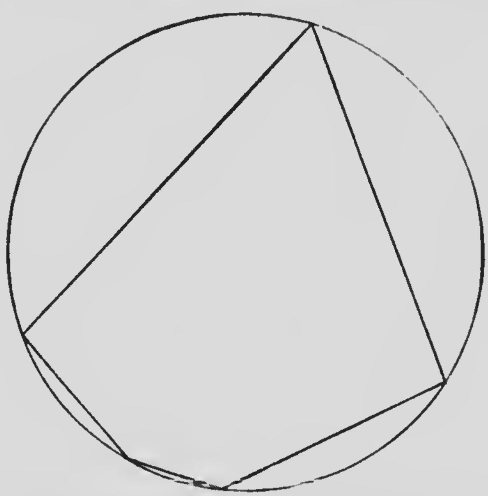
79. A figure is said to be **inscribed in a circle** when its vertices are on the circumference of the circle.



The circle is then said to be **circumscribed about the figure.**

80. A **polygon** is a closed figure bounded by any number of st. lines more than four. The number of ∠s in a polygon is the same as the number of sides.

No. of Sides.	Name.
5 . . .	pentagon.
6 . . .	hexagon.
7 . . .	heptagon.
8 . . .	octagon.
9 . . .	nonagon.
10 . . .	decagon.
12 . . .	dodecagon.
15 . . .	quindecagon.

81. A **regular polygon** has all its sides equal and all its ∠s equal.

82.—Examples.

1. Draw any quadrilateral and draw one of its diagonals. Show that the sum of the interior ∠s of the quadrilateral is four rt. ∠s.

2. Draw an irregular pentagon. Mark any point inside the pentagon, and join the point to the five vertices. Show that the sum of the interior ∠s of the pentagon is six rt. ∠s.

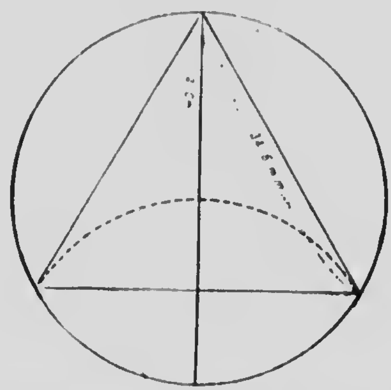
3. Find the number of degrees in each ∠ of a regular pentagon.

4. Find the number of degrees in each ∠ of a regular hexagon.

5. Find the number of degrees in each ∠ of a regular octagon.

6. Find the sum of the interior ∠s of any heptagon.

7. Draw a circle of radius 5 cm. Inscribe an equilateral △ in the circle. Find the side of the △ in millimetres. How many degrees are there in the ∠ at the centre subtended by a side?



8. Draw a diagram showing the space around a point filled in by six equilateral △s.

9. Draw a circle of any radius and inscribe a regular hexagon in the circle. Show that each side must be equal to the radius of the circle, and that each side must subtend an ∠ of 60° at the centre.

10. Draw a line-segment AB 6 cm. in length. Describe a regular hexagon ABCDEF. Measure the diagonals AC and AD.

11. Draw a figure showing the space around a point filled up by three regular hexagons.

12. Draw a circle of radius 7 cm. Inscribe a regular octagon in the circle. Measure a side of the octagon.

13. Draw a line-segment 5 cm. in length. Describe a regular octagon on the line-segment. Circumscribe a circle about the octagon. Find the radius of the circle.

14. Draw any pentagon ABCDE. Produce the sides, AB to F, BC to G, etc. Measure the exterior ∠s FBC, GCD, etc., thus formed, and find their sum. Show that the sum would be the same for a polygon with any number of sides.

83.

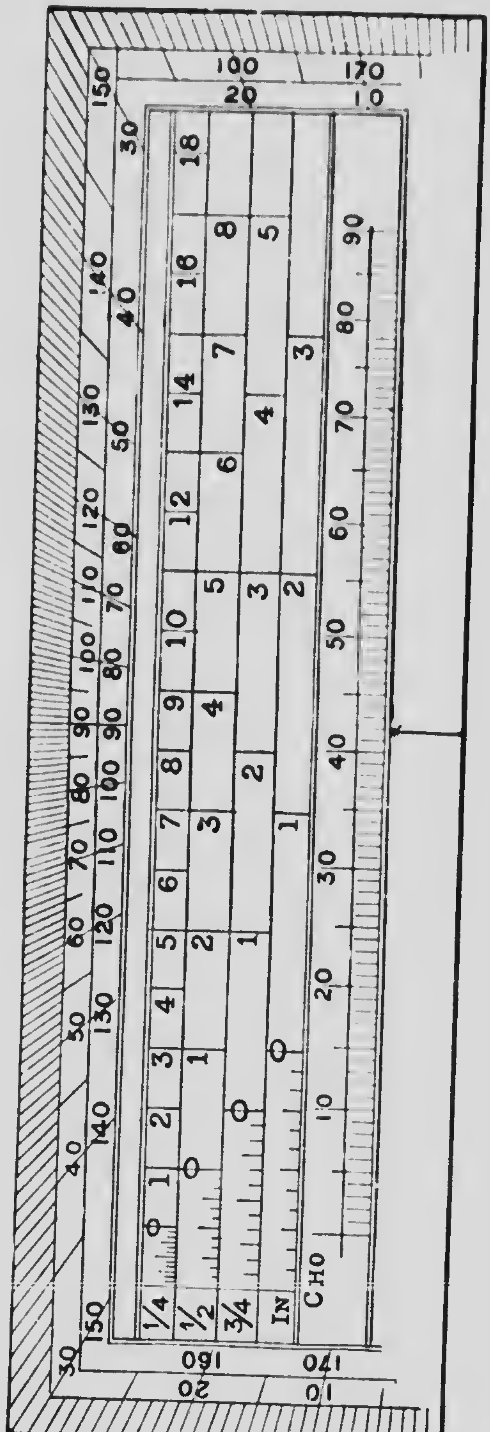


Diagram of rectangular protractor graduated in degrees.

84. Examples.

1. Draw an ∠ of 79°.

2. Make successive ∠s BAC, CAD, DAE, EAF, of 17°, 42°, 13°, 18° respectively. Is the ∠ BAF exactly a rt. ∠?

3. Construct the △, b = 3 cm., c = 5 cm. and A = 23°. Measure a.

4. Construct the △ a = 13 cm., b = 7 cm. and C = 60°. Measure A and B.

5. Construct the △ b = 70 mm., c = 55 mm., A = 60°. Measure B and C.

6. Construct a = 84 mm., b = 42 mm. and C = 120°, and measure c, A and B.

7. The sides of a △ are 56, 65 and 33 mm. Measure the greatest ∠.

8. If a = 9, b = 10, c = 11, find the measure of B.

9. The sides of a △ are 2, 3, 4; measure the greatest ∠.

10. The sides of a △ are 32, 40, 66; measure the greatest ∠.

11. If A = 30°, b = 8 cm. and a = 6 cm., show that c has two different values. Draw both △s and find both values of c.

12. If a = 9, b = 12 and A = 30°, find the length of c.

13. The base of a △ is 64 mm.; the base ∠s are 22½° and 112½°. Find the vertical height of the △.

14. At a point on level ground 50 ft. from the base of a flagstaff the height of the flagstaff subtends an ∠ of 40°. Find the height of the flagstaff. (Take a millimetre to represent a foot.)

15. A church spire at a distance of 500 feet has an elevation of 15°. Find its height. (Take 1 cm. to represent 50 ft.)

16. A man finds that a church spire has an elevation of 45°. On walking 100 ft. towards the spire he finds it has an elevation of 60°. Find its height.

17. A person standing on the bank of a river, observes that the ∠ subtended by a tree at the same level as himself on the opposite bank is 28°; when he retires horizontally 40 ft. from the bank he

finds the ∠ to be 20°. Find the height of the tree and the breadth of the river.

18. Find the height of a chimney when it is found that, on walking towards it 100 ft. in a horizontal line through its base, the ∠ of elevation of the top of the chimney changes from 30° to 45°.

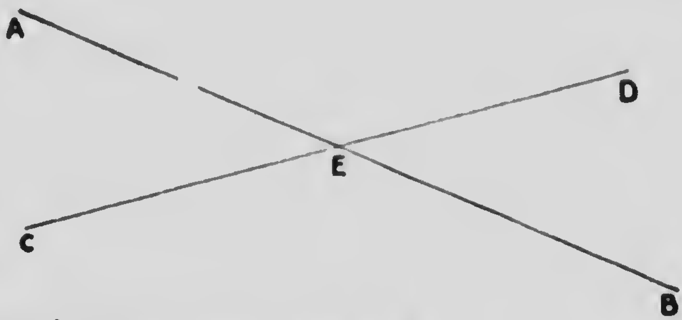
19. A man measures a distance AB along the bank of a river. C is a mark on the opposite shore. He finds c = 200 yds., A = 30° and B = 60°. Find the breadth of the river.

20. A, B, C and D are four towns. A traveller goes from A to B 4 miles due east, turns to his left through an ∠ of 60° and goes 5 miles to C. From C he goes 3 miles due north to D. Find the direct distance from A to D and the ∠ AD makes with the east and west line.

CHAPTER VI.

Vertically Opposite Angles.

85. **If** two st. lines cut each other, the vertically opposite ∠s are equal.



AB and CD cut each other at E.

It is required to show that ∠ AED = ∠ BEC.

Because AEB is a st. line,

∠ AED + ∠ DEB = a st. ∠.

Because CED is a st. line,

∠ CEB + ∠ DEB = a st. ∠.

All st. ∠s are equal to each other.

Therefore ∠ AED + ∠ DEB = ∠ CEB + ∠ DEB.

From each of these equal sums take away the common ∠ DEB and the remainders must be equal.

Therefore ∠ AED = ∠ CEB.

Prove in the same way that ∠ AEC = the vertically opposite ∠ DEB.

86. Examples.

1. Draw two st. lines AB and CD, cutting at E. Make CE = ED and AE = EB.

Join AD and BC. Prove AD = BC.

Join AC and BD. Prove AC = BD.

2. Draw two st. lines AB and CD cutting at E. Make CE = AE and ED = EB.

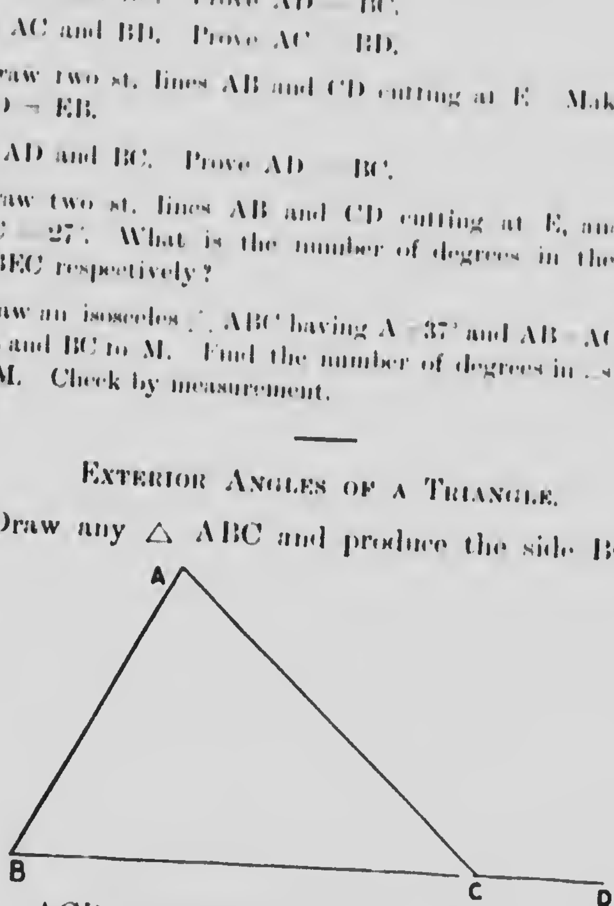
Join AD and BC. Prove AD = BC.

3. Draw two st. lines AB and CD cutting at E, and such that ∠ AEC = 27°. What is the number of degrees in the ∠ s AED, BED, BEC respectively?

4. Draw an isosceles △ ABC having A = 37° and AB = AC. Produce AC to L and BC to M. Find the number of degrees in ∠ s ACB, BCL and ACM. Check by measurement.

Exterior Angles of a Triangle.

87. Draw any △ ABC and produce the side BC to D.



The ∠ ACD contained by the side CA and the produced part CD is called an exterior ∠ of the △ ABC.

ACB is the adjacent interior ∠ and ABC and BAC are the two interior and opposite ∠ s.

88. Draw a △ and produce each side both ways. Read the six exterior ∠ s, and with each one name the adjacent interior ∠ and the two interior and opposite ∠ s.

Which of the exterior ∠s are equal?

Ex. Make a △ having two ∠s 15° and 45° respectively. Find the number of degrees in each exterior ∠.

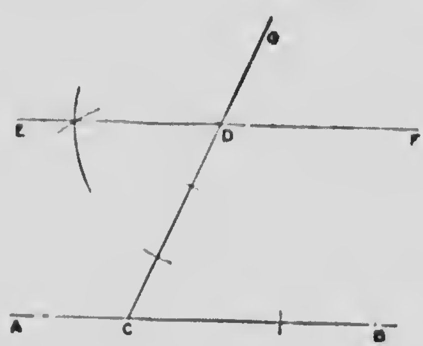
89. Show that in any △ each exterior ∠ is equal to the sum of the two interior and opposite ∠s. (§ 71).

90. Show that in any △ each exterior ∠ is greater than either of the interior and opposite ∠s.

Ex. Draw any △ ABC and bisect the ∠ A. Produce the bisector to cut BC at D. Show that ∠ ADC is greater than ∠ DAC, and that ∠ ADB is greater than ∠ BAD.

PARALLEL STRAIGHT LINES.

91. Draw a st. line AB and mark a point C in the line. At C make any ∠ BCD and mark a point D in CD.



At D, and on the opposite side of CD from ∠ BCD, make ∠ CDE = ∠ BCD (§ 43).

Produce ED to F.

No matter how far EF and AB be produced towards F and B they cannot meet, for, if they did, they would form, with DC, a △ and the exterior ∠ EDC of the △

would be equal to an interior and opposite ∠ DCB, which is known to be impossible from § 90.

FE and BA cannot meet when produced towards E and A for, if they did, they would form, with BC, a △, having the exterior ∠ BCD equal to the interior and opposite ∠ CDE, which is impossible.

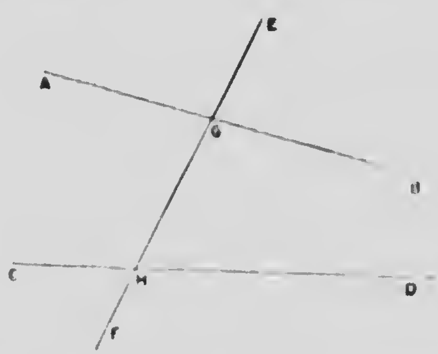
Thus, EF and AB would not meet if produced to any distance in either direction.

92. St. lines which are in the same plane and which do not meet when produced to any finite distance in either direction, are said to be **parallel** st. lines.

The symbol ∥ is used for the word parallel, or for the words 'is parallel to.'

Give examples of st. lines which do not meet when produced and yet are not ∥.

93. Draw a st. line EF cutting two other st. lines AB and CD at G and H.



Eight ∠s are thus formed, four of which, AGH, BGH, CHG, DHG, being between AB and CD, are called **interior** ∠s. The other four are called **exterior** ∠s.

The st. line EF which cuts the other two is sometimes called a **transversal.**

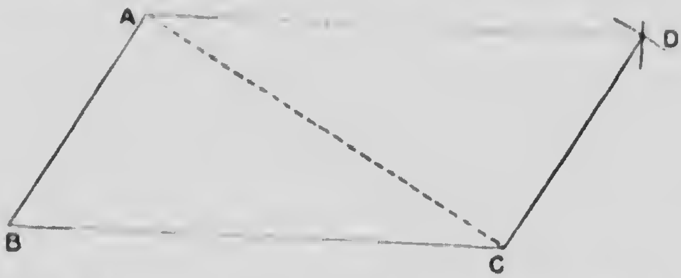
The interior ∠s AGH and GHD, on opposite sides of the transversal, are called **alternate** ∠s. Thus also, BGH and GHC are alternate ∠s.

Name four pairs of equal angles in the diagram.

94. The statement of the proposition in § 92 may now be given as follows:

If a st. line cuts two other st. lines making the alternate ∠s equal, the two st. lines are ∥.

95. Make a quadrilateral ABCD, having AB equal to the opposite side CD, and AD equal to the opposite side



BC. Join AC. Show that AB ∥ CD and that AD ∥ BC.

Thus, **if both pairs of opposite sides of a quadrilateral be equal, the opposite sides are ∥.**

96. A quadrilateral having both pairs of opposite sides ∥ is called a **parallelogram.**

The symbol ∥gm. is used for the word parallelogram.

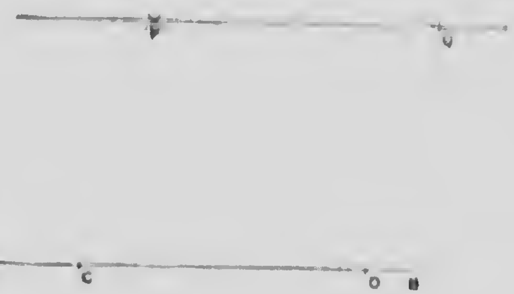
97. PROBLEM: To draw from a given point a st. line ∥ to a given st. line, using compasses and ruler.

P is the given point, and AB the given st. line.

It is required to draw from P a st. line ∥ AB.

Mark any two points C and D in AB.

Set the compasses at distance CD, and with this radius and centre P describe an arc.



Set the compasses at distance PC, and with this radius and centre D describe an arc.

The two arcs cut at Q.

Draw PQ.

PQ is the required line.

Join CP, DQ and show that PQ is ∥ AB. (See § 95.)

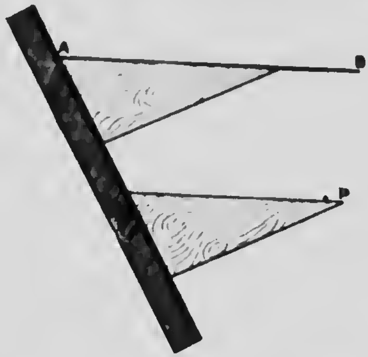
98. Draw a st. line and mark any two points in it. Erect perpendiculars to the line at the two points, and show that they are ∥ to each other.

Hence, **st. lines that are perpendicular to the same st. line are ∥.**

99. Draw a st. line AB and mark in it any two points C and D. From C draw a st. line CE, and on the same side of AB draw from D a st. line DF making the ∠ ADF equal to the ∠ ACE. Prove that DF ∥ CE.

Hence, **if a st. line cuts two other st. lines so as to make an exterior ∠ equal to the interior and opposite ∠ on the same side of the cutting line, the two lines are ∥.**

100. To draw a st. line through a given point P ∥ to a given st. line AB, using set-square and ruler.



Place the set-square with the hypotenuse along the st. line AB.

Place a ruler against another side of the set-square as in the diagram.

Hold the ruler firmly in position and slide the set-square along it until the hypotenuse comes to the point P.

A line may then be drawn through P that will be ∥ AB.

101.—Examples.

1. Show that a rhombus is a ∥gm.

2. Draw any △ ABC, and through A draw a st. line ∥ BC.

3. Draw a △ ABC with sides 43, 53 and 62 mm. Through A, B and C draw st. lines ∥ BC, CA and AB respectively, meeting at P, Q and R. Measure the sides of the △ PQR.

4. Make a rhombus with one ∠ 45° and each of the equal sides 6 cm. Measure the two diagonals.

5. Make a ∥gm. having one ∠ 60°, one pair of equal sides 75 mm. each, and the other pair each 35 mm. Measure the longer diagonal.

6. Make a ∥gm. having sides 35 mm. and 50 mm. and one diagonal 40 mm. Measure one of the small ∠s.

7. Draw two st. lines AOB and COD. Cut off AO = OB and CO = OD. Join AC, CB, BD and DA. Prove that ACBD is a ||gm.

Thus, if the diagonals of a quadrilateral bisect each other, the quadrilateral is a ||gm.

8. Make a ||gm. having one diagonal 7 cm., the other diagonal 6 cm., and an ∠ between the diagonals 60°. Measure the sides of the ||gm.

9. Make a ||gm. having one diagonal 8 cm., the other 10 cm. and one side 6 cm. Measure the ∠s between the diagonals.

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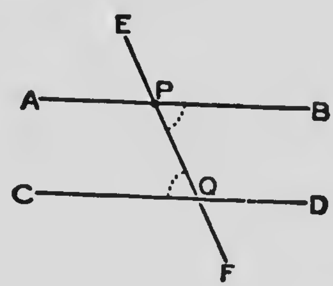
102. Draw, with set-square and ruler, two || st. lines AB and CD. Draw any transversal EPQ, cutting AB at P and CD at Q.

Measure the alternate ∠s BPQ and PQC. They will be found to be equal to each other.



Show (1) ∠ APQ = ∠ PQD.
(2) ∠ EPA = ∠ PQC.
(3) ∠ EPB = ∠ PQD.
(4) ∠ BPQ + ∠ PQD = 2 rt. ∠s.
(5) ∠ APQ + ∠ PQC = 2 rt. ∠s.

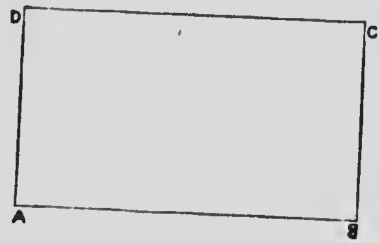
From (2), or (3), it follows that: **if a transversal cuts two || st. lines, the exterior ∠ is equal to the interior and opposite ∠ on the same side of the transversal.**

Give a general statement of the proposition that follows from (4), or (5).

From the same figure, show that the eight ∠s made by the transversal with the two ∥ st. lines are divided into two sets, of four each, such that the ∠s of each set are equal to each other.

Rectangle and Square.

103. Draw any line-segment AB. Draw AD and BC perpendicular to AB.



Thus AD is ∥ BC. (§ 98.)

Mark any point D in AD, and draw DC perpendicular to DA.

Then, because DC and AB are both perpendicular to DA, DC is ∥ AB. (§ 98.)

Thus ABCD is a ∥gm. having its three ∠s B, A and D rt. ∠s, and as the ∠s of a quadrilateral are together equal to four rt. ∠s, ∠ C must also be a rt. ∠.

104. A ∥gm. that has its ∠s rt. ∠s is called a **rectangle.**

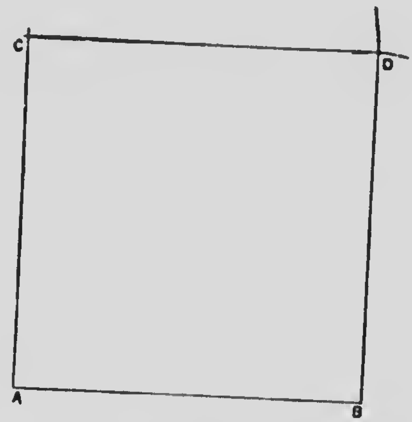
105. If you compare the opposite sides of a rectangle with the dividers they will be found to be equal.

106. Prove that the diagonals of a rectangle are equal to each other.

Ex. Make a rectangle 5 cm. by 12 cm. and measure the diagonals.

107. A rectangle that has all its sides equal is called a **square.**

108. It is required to draw a square on a given line-segment:



Let AB be the given line-segment.

Make a rt. ∠ at A, and with the compasses, make AC = AB.

With centre C and radius AB describe an arc.

With centre B and radius AB describe another arc.

D is the point where the arcs cut.

Join CD and BD.

ABDC is the required square.

Because all the sides have been made equal ABDC is a rhombus, and hence, by § 95, it is a ∥gm.

Because AC cuts the ∥ lines AB and CD ∠ A + ∠ C = two rt. ∠s (§ 102); but ∠ A is a rt. ∠, and hence ∠ C is also a rt. ∠.

In the same way it may be shown that ∠ B is a rt. ∠.

Because A, B and C are rt. ∠s, and the four ∠s of any quadrilateral are together equal to four rt. ∠s, ∠ D must also be a rt. ∠.

Construct a square on a line-segment AB, using the compasses only once (to make AC = AB), making rt. ∠s at A, B and C with the set-square.

109.—Examples.

1. Make a square having each side 54 mm. Measure its diagonal.

2. Make a square having each side 2.5 inches. Measure its diagonal.

3. Make a square having one diagonal 9 cm. Measure the side of the square.

4. Make a square having one diagonal 2 inches. Measure the side.

5. Draw a circle with a radius of 5 cm. Inscribe a square in the circle. Measure the side of the square.

6. Make a square with a side 75 mm. Describe a circle through the vertices of the square. Find the radius of the circle.

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110. The following properties hold for all ||gms:

1. The opposite sides of a ||gm are equal.
2. The opposite ∠s of a ||gm are equal.
3. The diagonals of a ||gm bisect each other.
4. Either diagonal of a ||gm bisects the area of the ||gm.

111. Properties of the rhombus.

A rhombus is a ||gm, and therefore every rhombus has the four properties given in § 110.

The following additional properties have been shown to be true for the rhombus, but they do not hold for all ||gms.

5. The diagonals of a rhombus cut at rt. ∠s. (§ 62, Ex. 1.)

6. Either diagonal of a rhombus bisects each of the ∠s through which it passes. (§ 40, Ex. 5.)

112. A rectangle is a ||gm., and therefore has the four properties of the ||gm.

The following additional property has been shown to be true for all rectangles, but does not hold for the rhombus or for all ||gms.

The diagonals of a rectangle are equal to each other. (§ 106.)

113. As a square is a ||gm., a rectangle and a rhombus, it has all the seven properties of these figures, viz.: the opposite sides and ∠s are equal, and the diagonals are equal, bisect the ∠s, bisect each other and cut at right ∠s, and each diagonal bisects the area.

114. Draw a rectangle and cut it out.

Fold the paper so as to show that the rectangle has two axes of symmetry, and that all the ∠s may be thus made to fit on each other.

Draw a square and cut it out.

Fold the paper so as to show that the square has four axes of symmetry.

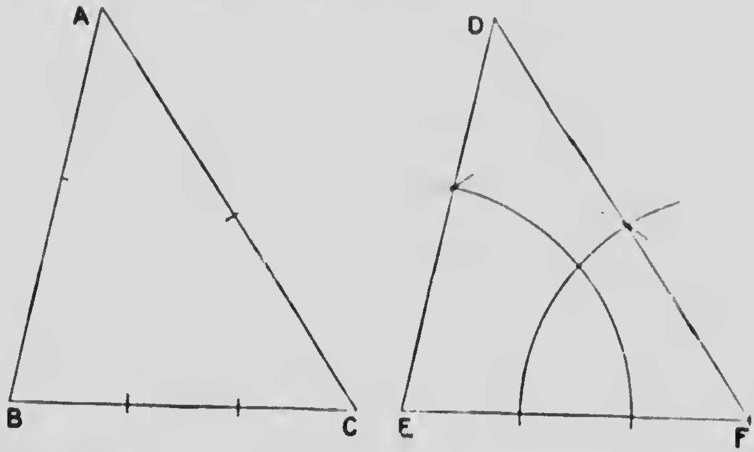
By folding along two of the axes of symmetry all four ∠s may be made to fit one another; and by folding along the other two all four sides may be made to fit one another.

115. Draw any ||gm., and find the pt. where the diagonals intersect. Through this pt. draw a number of line-segments terminated by the sides of the ||gm. Using the dividers, find whether these lines are bisected at the pt. of intersection.

A ||gm. is symmetrical with respect to the pt. of intersection of its diagonals.

CHAPTER VII.

Equality of Triangles: Case III.



116. Draw any △ ABC.

Draw a line-segment EF = BC.

Make ∠ E = ∠ B and ∠ F = ∠ C, and produce the arms of these ∠s to meet at D.

Then ∠ A = ∠ D. (§ 71, *Ex.*)

Make a tracing of △ ABC and fit it on △ DEF.

It will be found that:

AB = DE
AC = DF

and area of △ ABC = area of △ DEF.

Thus: **if two △s have the ∠s and a side of one respectively equal to the ∠s and the corresponding side of the other, the △s are congruent.**

117.—Examples.

1. Draw any ∠ and bisect it. Mark any point P in the bisector and draw a perpendicular from P to each arm of the ∠. Prove that these perpendiculars are equal to each other.

2. Draw a △ ABC having a = 46 mm., b = 55 mm. and c = 61 mm. Find the point P in AB from which perpendiculars drawn to CA and CB are equal. Measure these perpendiculars.

3. Draw a △ ABC having a = 4 cm., b = 8 cm. and c = 5 cm. Find the point P in BC from which perpendiculars drawn to AB and AC are equal and measure these perpendiculars.

Find also the point Q, in CB produced, from which perpendiculars drawn to AB and AC are equal to each other and measure these perpendiculars.

4. Draw any △ ABC. Bisect BC at D. Draw the median AD and produce it. Draw perpendiculars from B and C to AD. Prove that these perpendiculars are equal to each other.

5. A farmer has two barns, A, 160 yds. east of his house and B, 240 yds. north-east of the house. He wishes to build a straight road from his house that will run between the barns and equally distant from them. Find what ∠ the centre line of the road must make with the east and west line and how far it will be from each barn. (Scale: 80 yds. to the inch.)

6. Draw any ∠ A and bisect it. Mark any point P in the bisector, and at P draw a st. line perpendicular to AP and meeting the arms of ∠ A at B and C. Show that ABC is an isosceles ∠.

7. Make an equilateral ∠ having an altitude of 7 cm. Measure the side of the △.

8. Make an isosceles △ having the vertical ∠ = 74° and the perpendicular from the vertex to the base = 6 cm. Measure the equal sides.

9. Draw two st. lines AB and AC and mark any point P. Draw a line from P that shall make with AB and AC an isosceles △. Show that there are generally two solutions.

10. Draw any △ and find a point within the △ that is equally distant from the three sides.

11. Draw a △ ABC having a = 78 mm., b = 84 mm., and c = 90 mm. Find the point, within the △, that is equally distant from the three sides. Measure its distance from each side.

Produce the sides of the △ and find three points without the △, each of which is equally distant from the three sides. Measure the distance in each case.

AREAS OF TRIANGLES AND PARALLELOGRAMS.

118. Examples.

1. Draw a square inch.

2. Draw a square decimetre.

3. Draw a rectangle 3 inches by 2 inches and divide it into square inches, showing that there are 2 rows of 3 sq. inches each: or, 3 rows of 2 sq. inches each.

The area of the rectangle is thus 2 × 3, or 3 × 2, square inches.

4. Draw a rectangle 5 cm. by 3 cm., and divide it into 15 sq. centimetres.

5. Draw a rectangle $2\frac{1}{2}$ inches by $3\frac{1}{2}$ inches and divide it up so as to show the number of square inches.

6. Draw a diagram to show that $30\frac{1}{4}$ sq. yds. make one sq. rod. (Scale: $\frac{1}{2}$ inch to the yard.)

In each case, if the figure is rectangular, the number of square units of area may be found by multiplying the number of units of length by the number of units of breadth.

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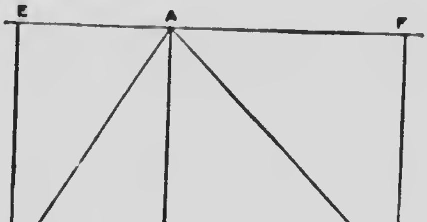
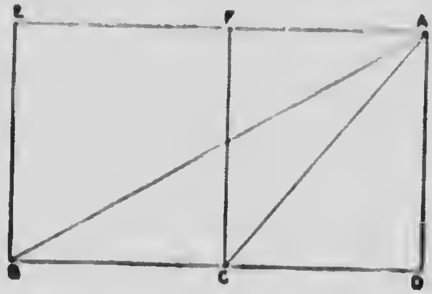
119. Draw a △ ABC.



Through A draw EF ∥ BC. From A, B and C draw lines AD, BE and CF perpendicular to the two ∥ st. lines.

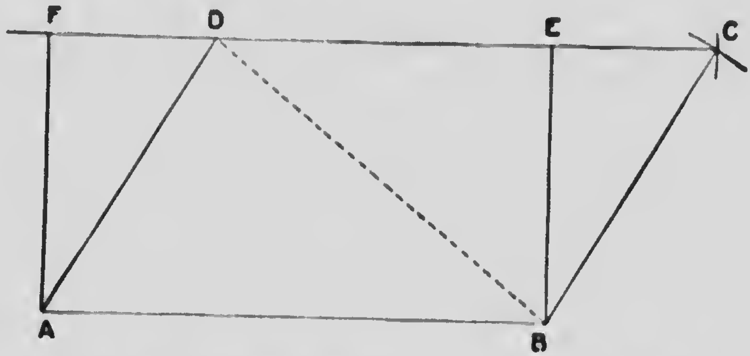
Show that the area of the rectangle BEFC is twice that of the △ ABC.



Prove the same thing from the second diagram.

The area of a △ is half that of a rectangle of the same base and altitude as the △.

120. Draw any ||gm. ABCD.



Draw AF and BE perpendicular to the two ‖ lines AB and DC.

Join BD.

Show that the area of the rectangle ABEF = the area of the ||gm. ABCD.

The area of a ||gm. is equal to that of a rectangle on the same base and between the same ||s as the ||gm.

121.—Examples.

1. Draw a △, having its sides 65 mm., 70 mm. and 75 mm. Make a rectangle 70 mm. long, that has twice the area of the △. Make a rectangle 65 mm. long, that has twice the area of the △. From each of these rectangles find the area of the △. Take the average of your two results as your answer.

2. Make △ ABC, having b = 6 cm., c = 8 cm., and ∠ A = 72°. Find its area.

3. Draw a △, having its sides 73 mm., 57 mm. and 48 mm. Find its area.

4. Find the area of the △: a = 10 cm., ∠ B = 42°, ∠ C = [illegible]8°.

5. Draw a ▱gm. having two adjacent sides 6·4 cm. and 7·3 cm. and the contained ∠ 30°. Find its area.

6. Draw a ▱gm. having the two diagonals 4·8 cm. and 6·8 cm. and an ∠ between the diagonals 75°. Find its area.

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122. In a plan, if one inch represents 3 yards, what area does 1 sq. inch represent?

If 2 cm. represents 15 m., what does one sq. cm. represent?

123. In Surveyor's Measure, 1 chain = 100 links = 66 feet.

Find the number of chains in a mile.

Find the number of square chains in an acre.

124.—Examples.

1. The sides of a triangular field are 36 chains, 25 chains and 29 chains. Draw a diagram and find the number of acres in the field. (Scale: 1 mm. to the chain.)

2. Two sides of a triangular field are 41 and 38 chains and the contained ∠ is 70°. Find its area in acres.

3. Draw a diagram to show that if the side of one square is 3 times the side of another, the area of the first square is 9 times the area of the second.

4. Draw a rt. ∠d △, having the sides that contain the right ∠ 56 mm. and 72 mm. Find the area of the △.

5. The area of a ||gm. is 50 sq. cm., one side is 10 cm. and one ∠ is 60°. Construct the ||gm., and measure the other side.

6. Draw a rectangle of base 7 cm. and height 4 cm. On the same base construct a ||gm. having the same area as the rectangle and two of its sides each 65 mm. Measure one of the smaller ∠s of the ||gm.

7. Make a ||gm. having sides 10 and 7 cm. and one ∠ 60°. Make a rhombus equal in area to the ||gm. and having each side 10 cm. Measure the shorter diagonal of the rhombus.

8. Make a rectangle 8 cm. by 5 cm. Construct a ||gm. equal in area to the rectangle and having two sides 7 cm. and 8 cm. Construct a rhombus equal in area to the ||gm. and having each side 7 cm. Measure the shorter diagonal of the rhombus.

9. Make a rhombus having each side 8 cm. and its area 50 sq. cm. Measure the shorter diagonal.

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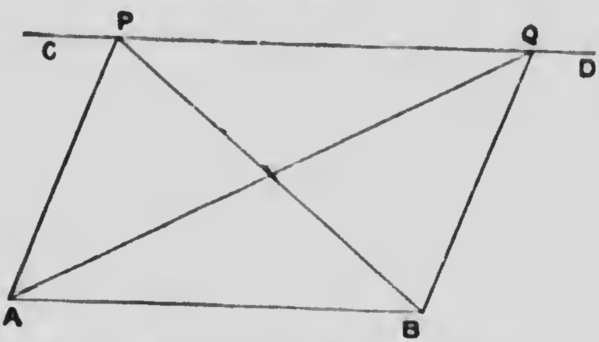
125. Draw a line-segment AB.



Draw a st. line CD || AB, and in CD mark any two points P and Q.

Join PA, PB, QA, QB.

Show that △ PAB = △ QAB.

Thus: **△s on the same base and between the same ∥s are equal in area.**

---

126. Examples.

1. Draw any △ ABC and also any st. line DE not ∥ BC. Make a △ PBC having its vertex P in DE and equal in area to △ ABC.

2. Draw any △ ABC. Through A, B, C draw st. lines EF, FD, DE ∥ respectively to BC, CA, AB and meeting in D, E, F. Show that the three ∥gms. thus formed are equal in area.

3. Draw two ∥gms. on the same base and between the same ∥s. Show that they are equal in area. (See § 120.)

4. Draw two ∥gms on equal bases and between the same ∥s. Show that they are equal in area. (Assume that rectangles of the same length and breadth are equal.)

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127. Draw a rt.-∠d △, having the two sides that contain the rt. ∠ 6 cm and 8 cm. Describe squares on the three sides. Measure the hypotenuse and find the areas of the three squares. Compare the sum of the squares on the two sides with the area of the square on the hypotenuse.

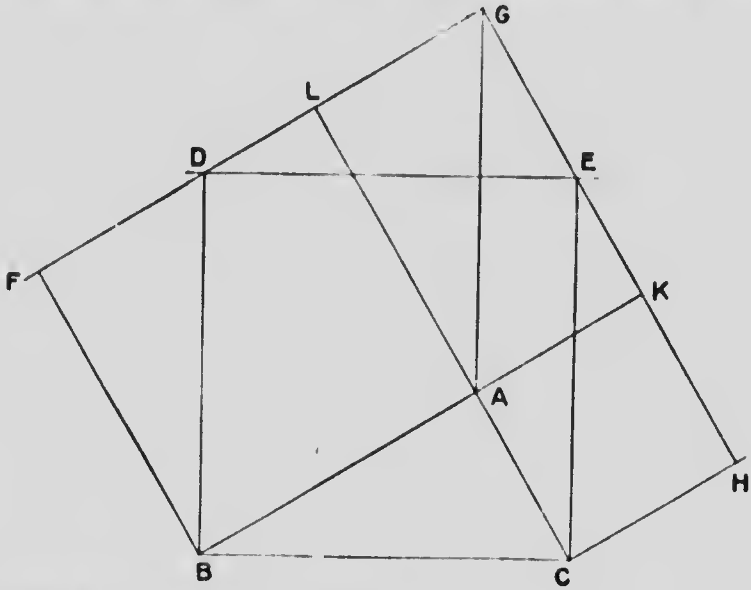
Do the same with a rt.-∠d △ [illegible] the sides are 60 mm. and 25 mm.

Draw any rt.-∠d △ and describe squares on the three sides. Measure the three sides in millimetres and find the areas of the squares. Compare the sum of the areas of the squares on the two sides with the area of the square on the hypotenuse.

It will be found that:—

The area of the square on the hypotenuse of a rt.-∠d △ is equal to the sum of the areas of the squares on the two sides.

128. Further illustration of the proposition in § 127.



Draw any rt.-∠d △ BAC.

Describe a square BDEC on BC. Through D draw FDG ∥ BA. Through E draw GEH ∥ CA. Through B draw BF ∥ CA. Through C draw CH ∥ BA.

Produce CA to L and BA to K. Join AG.

Make an exact tracing of this figure and cut out both figure and tracing.

The two pieces of paper are then exactly equal.

From one piece cut away the three △s BFD, DGE and CEH. The remainder is the square BDEC.

From the other piece cut away the three △s BAC, ALG, AGK. The remainder will be the two figures BFLA and AKHC.

Fit the six △s on each other and they will be found to be all equal, and as three have been taken from each of the equal pieces of paper, the remainders must be equal.

Test the figure BFLA by folding it along the two pairs of axes of symmetry, and it will be found to be a square. (See § 114.) Thus BFLA is the square on BA.

In the same way CAKH will be found to be the square on CA.

Hence, the square on BC is equal to the sum of the squares on BA and AC.

129.—Examples.

1. Draw two line-segments 5 cm. and 6 cm. in length. Describe squares on both, and make a square equal in area to the two squares. Measure the side of this last square and check your result by calculation.

2. Draw three squares having sides 1 in., 2 in. and 2¼ in. Make one square equal to the sum of the three. Check by calculation.

3. Draw two squares having sides 1½ in. and 2½ in. Make a third square equal to the difference of the first two. Check by calculation.

4. Draw two squares having sides 9 cm. and 6 cm. Make a third square equal to the difference of the first two. Check your result by calculation.

5. Draw any square and one of its diagonals. Draw the square on the diagonal and show that it is double the first square.

6. Draw a square having one side 4 cm. Draw a second square double the first. Measure a side, and check by calculation.

7. Draw a square having one side 45 mm. Draw a second square three times the first. Measure its side, and check by calculation.

8. Draw three lines in the ratio 1 : 2 : 3. Draw squares on the lines, and divide the two larger so as to show that the squares are in the ratio 1 : 4 : 9.

9. Draw a line-segment $\sqrt{2}$ in. in length.

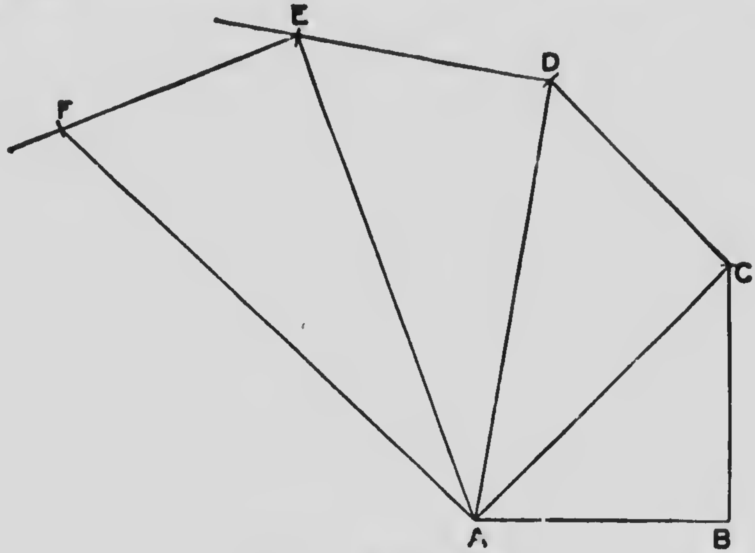
10. Draw a line-segment $\sqrt{3}$ in. in length.

11. Draw a line-segment $\sqrt{5}$ in. in length.

12. Draw any rt.-∠d . Describe equilateral s on the three

sides. Find the areas of the □s and compare that on the hypotenuse with the sum of those on the other two sides.

13. Draw any line-segment AB. Draw line-segments that shall be respectively $\sqrt{2}$ times AB, $\sqrt{3}$ times AB, $\sqrt{5}$ times AB, $\sqrt{6}$ times AB, etc.



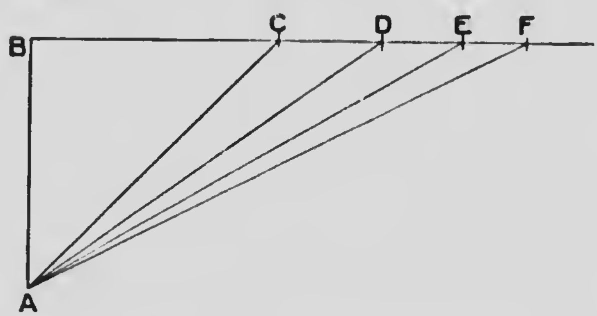
AC = $\sqrt{2}$ times AB

AD = $\sqrt{3}$ times AB

AE = twice AB

AF = $\sqrt{5}$ times AB, etc.

Another method:

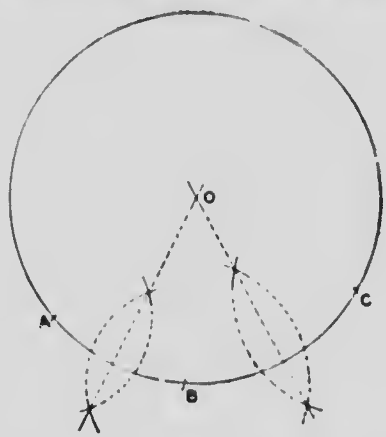


In this diagram BD is cut off equal to AC,
BE " " " AD,
etc.

CHAPTER VIII.

The Circle.

130. Draw a circle, using a coin, or other circular object.



It is required to find the centre, and to test the accuracy of the circle.

Mark three points, A, B and C, on the circumference.

Draw lines that would bisect the chords AB and BC respectively and cut them at right ∠s.

Produce these lines to meet at O.

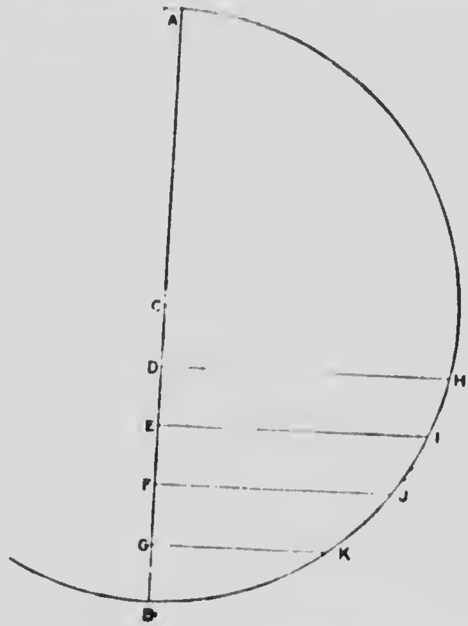
O is the centre of the circle.

Join OA, OB and OC, and prove that these line-segments are equal.

The accuracy of the circle may be tested by describing a circle with centre O and radius OA.

Draw a circular arc, using any round object, and find the centre in the same manner.

131. With centre C and radius 5 cm., draw a circle.



Draw any diameter AB.

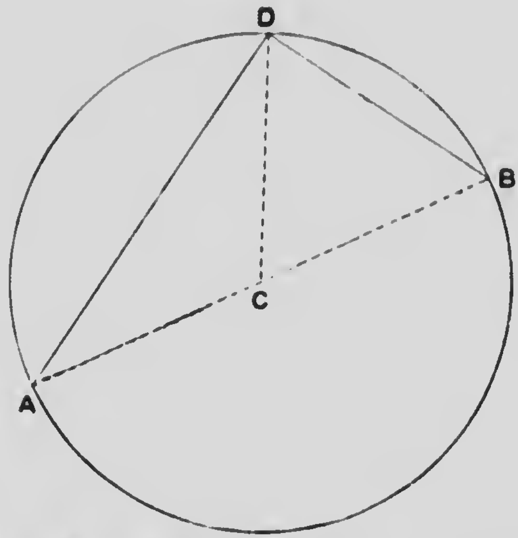
Mark off CD, DE, EF, FG, GB, each one centimetre.

Draw perpendiculars DH, EI, FJ and GK to AB.

Measure these perpendiculars in millimetres, and check your results by calculation. (See § 127.)

132. Draw a circle with radius 6 cm. In the circle place a chord AB 9 cm. in length. Bisect AB at D. From DB measure off DE = 2 cm. From E draw EF perpendicular to AB and meeting the circumference at F and G. Join DF and DG. Measure DF and DG and check your results by calculation.

133. With centre C, draw any circle. Draw any diameter AB. Mark any point D on the circumference. Join DA, DB and DC.

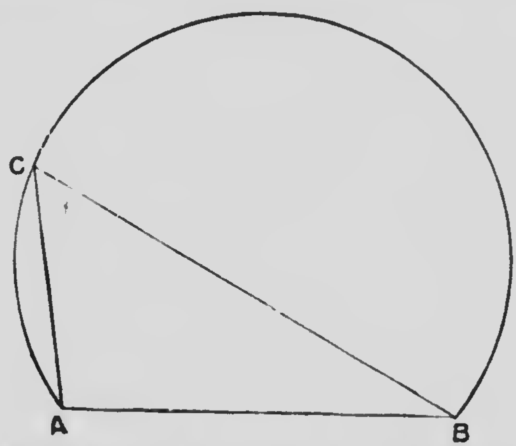


Prove that the ∠ ADB is a rt. ∠.

Thus, **if any point on the circumference of a circle be joined to the two ends of a diameter, the ∠ between the two joining lines is a rt. ∠.**

Test this with the set-square.

134. Any chord of a circle divides the circle into two segments.



Thus, **a segment of a circle** is a figure bounded by a chord and one of the arcs cut off by the chord.

If any point on the arc of a segment be joined to the ends of the chord, the ∠ between the two joining lines is said to be an ∠ **in the segment.**

ACB is a segment bounded by the chord AB and the arc ACB.

Three letters are used in reading a segment, the first and last being those at the ends of the chord.

The ∠ ACB is an ∠ in the segment ACB.

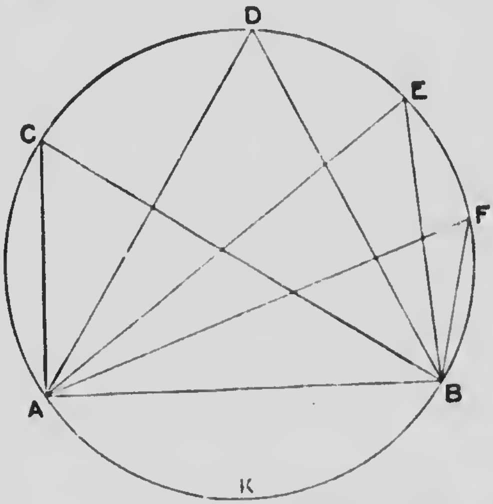
Draw a segment less than a semi-circle.

Draw an ∠ in the segment; read the segment and the ∠.

135. The proposition in § 133 may now be stated as follows:—

The ∠ in a semi-circle is a rt. ∠.

136. Draw any circle, and draw a chord AB cutting off the segment ACB.



Draw any number of ∠s, ACB, ADB, AEB, etc., in the segment ACB.

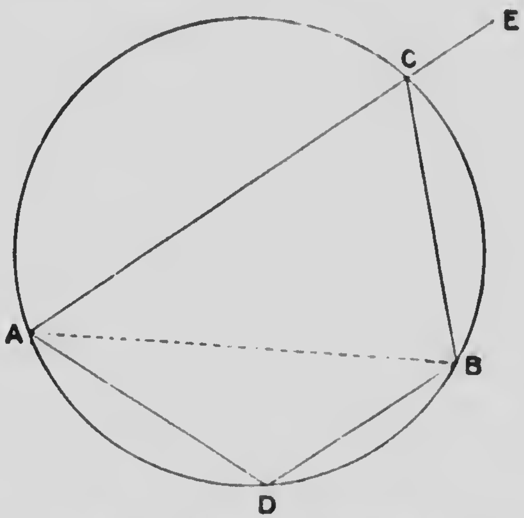
Make a tracing of one of these ∠s, and fit the tracing on each of the others.

If your work is correct the tracing will be found to fit exactly in each case.

Thus: **∠s in the same segment of a circle are equal to each other.**

Draw ∠s in the segment AKB and test their equality in the same way.

137. Draw any circle and divide it into two segments by a chord AB.



Draw an ∠ in each of the segments and produce AC, making the exterior ∠ BCE.

Make a tracing of ∠ ADB and fit the tracing to ∠ BCE. It will be found that ∠ ADB and ∠ ACB together make up the st. ∠ ACE.

Thus: **If a chord divides a circle into two segments, the ∠s in the segments are together equal to two right ∠s.**

Or, another way of stating the same proposition:

If a quadrilateral be inscribed in a circle its opposite ∠s are together equal to two right ∠s.

It follows also, since ∠ ECB is equal to ∠ ADB, that :—

If one side of a quadrilateral, inscribed in a circle, be produced, the exterior ∠ thus formed is equal to the remote interior ∠ of the quadrilateral.

Test these propositions by using the protractor.

138.—Examples

1. Draw a circle, and two chords AB, CD, intersecting at E, within the circle, join AC and BD. Show that the ∠s of △ ACE are respectively equal to the ∠s of △ BDE. Name the segments that contain the equal ∠s.

Join AD and BC, and show that the ∠s of △ AED are respectively equal to the ∠ of △ CEB.

2. From a point P outside a circle draw two st. lines PAB and PCD, cutting the circumference at A, B, C and D. Join AC and BD, and show that the ∠s of △ PAC are respectively equal to the ∠s of △ PBD. Join AD and BC, and show that the ∠s of △ PAD are respectively equal to the ∠s of △ PBC.

3. Draw a circle of any radius. Inscribe an equilateral △ in the circle. Draw an ∠ in each of the three segments outside of the △. Measure the ∠s with the protractor. Show that each ∠ should be exactly 120°.

4. Inscribe any scalene △ in a circle. Draw an ∠ in each of the three segments outside the △. Measure the ∠s with the protractor and add the results. Show that the sum should be exactly 360°.

5. Inscribe any trapezium in a circle. Measure the ∠s in the four segments outside the trapezium and add your results. Show that the sum should be exactly 540°.

6. Draw any circle and divide it into two segments by a chord equal in length to the radius. Measure the ∠ in each segment.

7. Inscribe a regular octagon in a circle. Measure the ∠s in the segments cut off by a side of the octagon.

8. Draw a circle of 5 cm. radius. Place a chord 8 cm. long in the circle. Measure the ∠ in the greater segment.

9. In a circle of 6 cm. radius place a chord 4 cm. long. Measure the ∠s in the two segments.

10. Draw any rt.-∠d △. Circumscribe a circle about the △. Where is the centre of the circle?

11. Make a △ ABC, having A = 65°, B = 90°, and c = 4 cm. Circumscribe a circle about the △. Measure the radius of the circle. What is the measure of the ∠ in the segment ACB?

12. Draw a line-segment, AB, 5 cm. in length. On AB describe a segment containing an ∠ of 35°. Measure the radius of the circle. What is the measure of the ∠ in the smaller segment cut off by AB?

13. On opposite sides of a line-segment, 55 mm. in length, describe segments containing 72° and 108° respectively. Measure the radius.

14. Draw a line-segment AB, 7 cm. in length. On AB describe a segment containing an ∠ of 115°. Measure the radius.

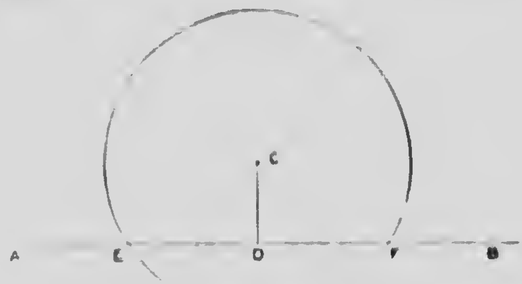
15. Construct an isosceles △, having the base 5 cm., and the ver-
tical ∠ 48°. Measure the radius of the circumscribed circle.

16. Construct a △ ABC, having a = 6 cm., ∠A = [illegible]°, and the median from A to the middle point of BC equal to 5 cm. Measure b and c.

17. Construct a △ ABC, having a = 8 cm., ∠A = [illegible]°, and the perpendicular from A to BC equal to 4 cm. Measure b and c.

Tangents.

139. Draw a st. line AB and mark a point C that is not in the line.



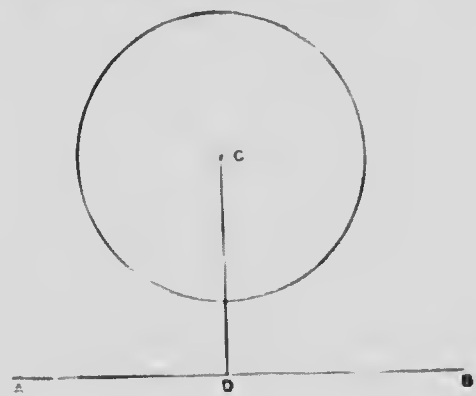
From C draw CD perpendicular to AB.

With centre C and radius greater than AB describe a circle.

The circle cuts AB in two points.

If the distance from the centre of a circle to a st. line is less than the radius of the circle, the circle cuts the st. line in two points.

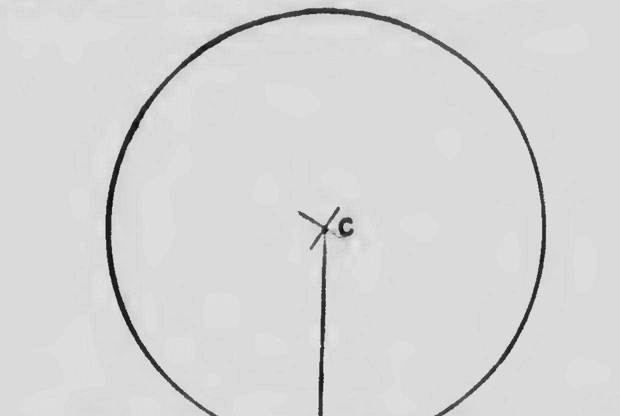
Draw the diagram again, but making the radius of the circle less than the perpendicular.



The circle does not meet the st. line.

If the distance from the centre of a circle to a st. line is greater than the radius of the circle, the circle does not meet the line.

Again draw the diagram and make the radius just equal to the perpendicular.



The circle just meets the line at the point D but does not cut the line.

140. A st. line which meets a circle but does not cut it, is called a **tangent** to the circle; or, is said to touch the circle.

The point where the tangent touches the circle is called the **point of contact.**

141. **The radius drawn to the point of contact of a tangent is perpendicular to the tangent.**

142. With centre C draw any circle. Mark any point A on the circumference.

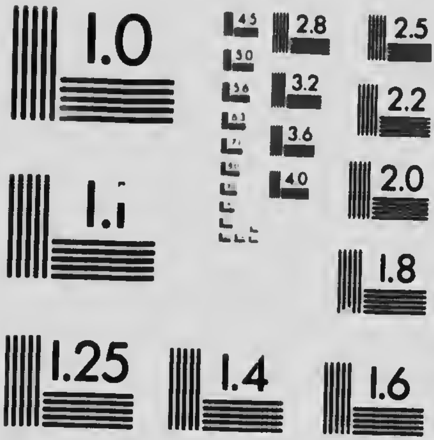
Join AC.

Through A draw a st. line BAD perpendicular to AC.

Then BD is a tangent to the circle, drawn from a point on the circumference.

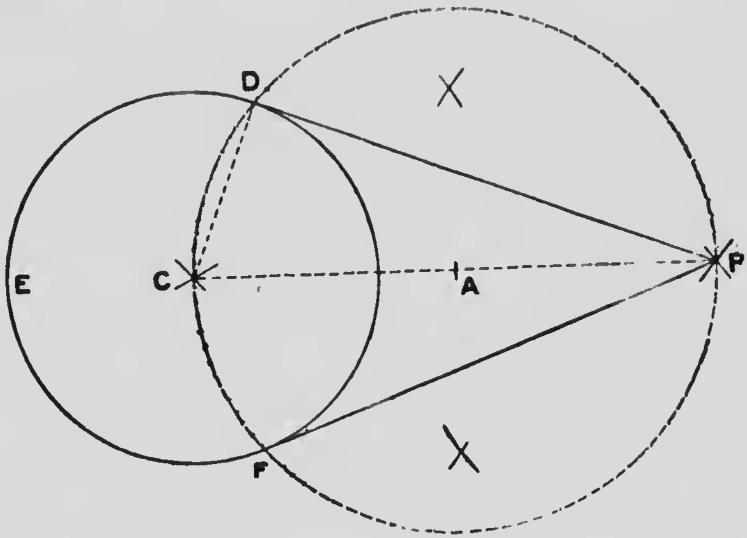
MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



143. PROBLEM: To draw a tangent to a circle from an external point.

With centre C draw any circle. Mark a point P outside the circle.



It is required to draw a tangent to the circle from the point P.

Join CP.

Bisect CP at A.

With centre A and radius AP, describe a circle, cutting the first circle at D and F.

Join PD and PF.

Then PD and PF are both tangents drawn from the point P to the circle.

Join CD.

The ∠ CDP is a rt. ∠ (§ 135), and therefore PD is a tangent to the circle DEF (§ 141).

144. Examples.

1. Draw a circle of radius 4 cm., and in the circle place a chord 6 cm. in length. Draw a perpendicular from the centre to the chord, and measure the length of the perpendicular. Check your result by calculation.

2. In a circle of radius 5 cm., place a chord 4 cm. long, and measure its distance from the centre. Check your result.

3. Draw a circle of radius 45 mm. Draw a chord at a distance of 3 cm. from the centre. Measure the length of the chord and check your result.

4. In a circle of radius 55 mm. place a chord at a distance of 35 mm. from the centre. Measure the chord and check your result.

5. Draw a circle of radius 4 cm. Take a point 9 cm. from the centre of the circle. From this point draw two tangents to the circle. Measure the length of each tangent and check your result by calculation.

6. Draw a circle of radius 5 cm. Mark a point 7 cm. from the centre. From this point draw two tangents to the circle and measure the ∠ between the tangents.

7. Draw a circle with a radius of 3 cm. Mark any point A on the circumference, and from this point draw a tangent AB 4 cm. long. Measure the distance of B from the centre and check your result.

8. Draw a circle with 43 mm. radius. Draw any st. line through the centre, and find a point, in this line, from which the tangent to the circle will be 5 cm. in length. Measure the distance of the point from the centre and check your result.

9. Mark two points A and B 7 cm. apart. Draw two st. lines from A such that the length of the perpendicular from B to either of them is 4 cm.

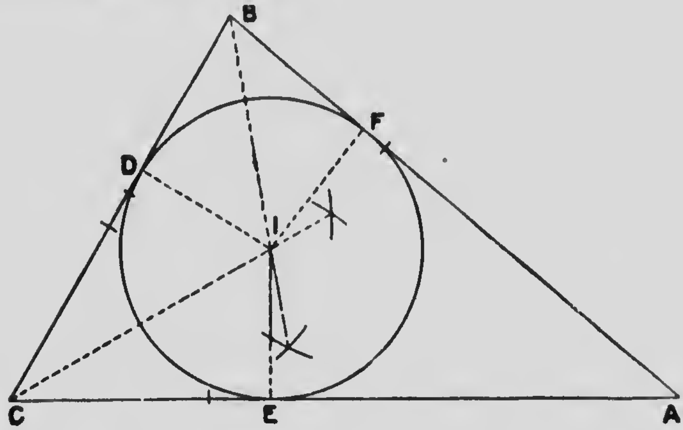
10. Draw a circle of radius 6 cm. Mark a point P 4 cm. from the centre. Draw a chord through P such that the perpendicular from the centre to the chord is 3 cm. in length. Measure the length of the chord and check your result by calculation.

11. Draw a circle of radius 36 mm. Mark any point P without the circle. Draw a st. line from P such that the chord cut off on it by the circle is 4 cm. in length.

12. Draw a circle of radius 47 mm. Mark a point P 4 cm. from the centre. Draw two chords through P, each of which is 65 mm. in length.

INSCRIBED AND ESCRIBED CIRCLES.

145. Draw any △ ABC.



Bisect the ∠s B and C, and produce the bisectors to meet at I. From I draw perpendiculars ID, IE, IF to BC, CA and AB respectively.

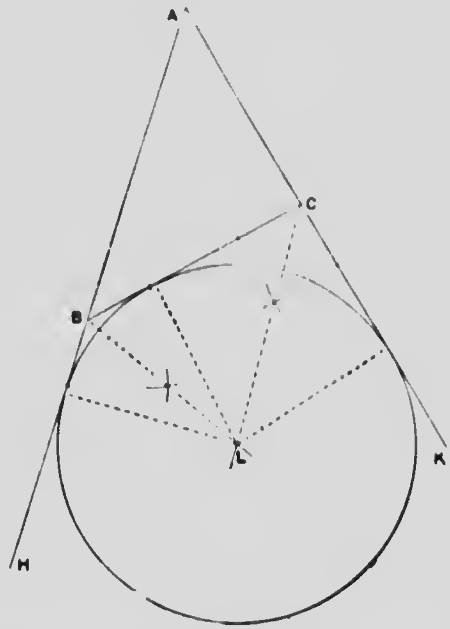
Show that these three perpendiculars are equal to each other. (§ 116.)

Draw a circle with centre I to pass through the three ·nts D, E, F.

Then, because the ∠s at D, E and F are right ∠s, the sides of the △ ABC are tangents to this circle. (§ 141.)

146. When a circle is drawn within a △, such that the three sides of the △ are tangents to the circle, it is said to be **inscribed in the △**, and is called the **inscribed circle of the △**.

147. Draw any △ ABC.



Produce AB and AC to H and K. Bisect the exterior ∠s HBC and KCB and produce the bisectors to meet at L.

Draw perpendiculars from L to the three sides of the △, and show that the three perpendiculars are equal to each other.

With centre L draw a circle that touches the three sides of the △.

148. When a circle lies without a △, and touches one side and the other two sides produced, it is called an **escribed circle of the △**

Show that a △ has three escribed circles.

149.—Examples.

1. Draw a △ with sides 52, 56 and 60 mm. Inscribe a circle in the △, and measure the radius.

Draw the three escribed circles, and measure the radii.

2. Draw a △ with sides 3, 4 and 5 inches. Draw the four circles that touch the three sides of the △, and measure their radii.

3. Draw a △ with sides 40, 40 and 48 mm. Draw the inscribed and three escribed circles, and measure the radii.

4. Make an ∠ of 43°. Describe a circle of 3 cm. radius that has the two arms of the ∠ for tangents. Measure the distance from the vertex of the ∠ to the centre of the circle.

5. Make an ∠ of 57°. Describe a circle of 4 cm. radius to touch both arms of the ∠. Measure the distance from the vertex of the ∠ to the centre of the circle.

6. Make an ∠ BAC of 45°. Find a pt. P such that its distance from AB is 3 cm., and its distance from AC is 4 cm. Measure PA.

7. Make an ∠ BAC of 60°. Find a point P such that its distance from AB is 4 cm. and its distance from AC is 5 cm. Measure AP.

8. Make a rectangle 48 mm. by 64 mm. Circumscribe a circle about the rectangle, and measure the radius.

9. Draw a circle of radius 5 cm. In the circle place a chord 35 mm. in length. Find two points in the circumference equidistant from the ends of the chord. Measure the distances of the points from the ends of the chord, and check your results by calculation.

10. Draw a square with 6 cm. side. Inscribe a circle in the square. Measure the radius of the circle.

11. Draw a circle of radius 4 cm. Draw two tangents, to the circle, to contain an ∠ of 60°. Measure the distance from the centre to the point of intersection of the tangents.

12. Draw a circle of 45 mm. radius. Draw two tangents to contain an ∠ of 45°. Measure the st. line-segment joining the points of contact.

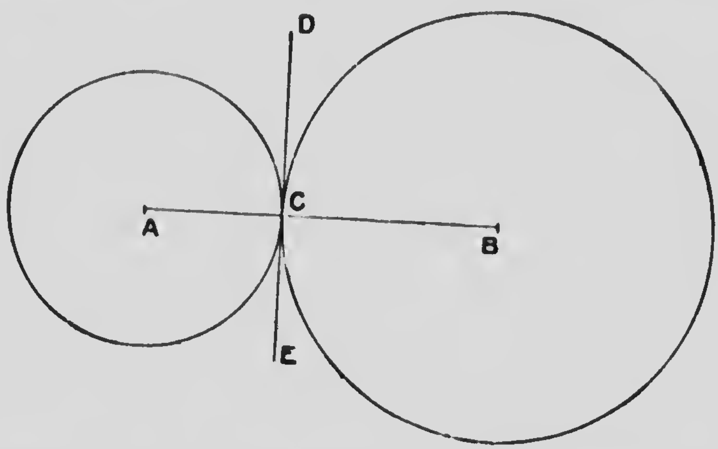
The st. line joining the points of contact of two tangents to a circle is called the **chord of contact**.

13. Draw a line-segment. AB 7 cm. in length. With centre A and radius 6 cm. describe a circle. With centre B and radius 4 cm. describe a circle. The two circles intersect at C and D. Draw the common chord CD, and measure its length.

14. Draw a line-segment AB 5 cm. in length. With centre A and radius 8 cm. describe a circle. With centre B and radius 4 cm. describe a circle. Draw the common chord of the circles and let it cut AB produced at C. Measure BC.

Contact of Circles.

150. Draw any line-segment AB. Mark any point C in AB.



With centre A and radius AC describe a circle, and with centre B and radius BC describe a circle.

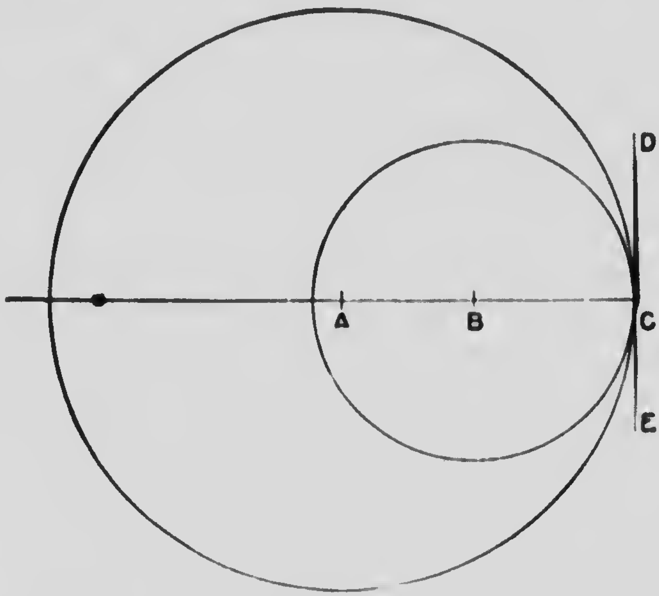
Through C draw DE perpendicular to AB.

DE is a common tangent to the two circles. One circle is on one side of DE and the other on the other side, except at the point C where they touch DE and also touch each other.

The two circles are said to touch **externally** at the point C.

If two circles touch externally, the distance between their centres is equal to the sum of their radii.

151. Draw any line-segment AB. Mark a point C in AB produced.



With centres A and B, and radii AC and BC respectively, describe circles.

Through C draw DE perpendicular to AC.

The two circles touch each other **internally,** and each touches the common tangent, at the point C.

If two circles touch internally, the distance between their centres is equal to the difference of their radii.

152. Examples.

1. Draw a line-segment AB 6 cm. in length. Draw two circles, with centres at A and B, and radii in the ratio 4:5, that touch externally.

2. Draw a line-segment AB 2 cm. in length. With centres at A and B, and radii in the ratio 4:5, draw two circles that touch internally.

3. Draw three circles with radii 23, 32 and 43 mm. each of which touches the other two externally.

4. Draw a circle of radius 9 cm., and within it draw two circles of radii 3 cm. and 4 cm., to touch each other externally, and each of which touches the first circle internally.

5. Draw a circle of radius 85 mm., and within it draw two circles of radii 25 mm. and 35 mm., to touch each other external., and each of which touches the first circle internally.

6. Draw a △ ABC with sides 5, 12 and 13 cm. Draw three circles with centres A, B and C respectively, each of which touches the other two externally. Measure the radii.

7. Construct the △ ABC having a = 5 cm., b = 4 cm., and c = 3 cm. Draw three circles with centres A, B and C respectively, such that the circles with centres B and C touch externally, and each touches the circle with centre A internally. Measure the radii.

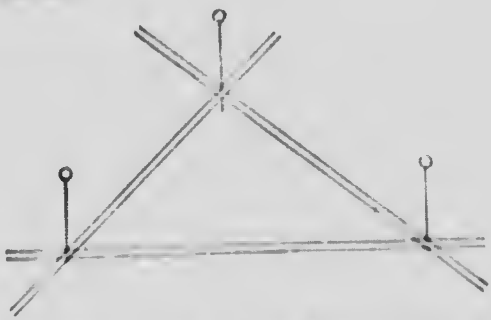
8. Mark two points A and B 10 cm. apart. With centres A and B, and radii 4 cm. and 3 cm., describe two circles. Draw a circle of radius 5 cm. which touches each of the first two circles externally. Measure the distance of the centre from AB.

CHAPTER IX.

SIMILAR TRIANGLES.

153. Cut a number of strips of wood, or of stiff paste board, of different lengths.

Fasten three of them together to form a △, with one pin at each corner.



The figure is rigid, that is, its shape cannot be changed without bending or twisting the strips.

The size of the ∠s cannot be changed without changing the lengths of the sides of the △.

Thus: **When the sides of a △ are of fixed length, the ∠s are of fixed size.**

154. In the same manner, fasten four of the strips together to form a quadrilateral.

The quadrilateral is not rigid. Its shape may be changed by pressing two of the opposite corners towards each other, or by drawing them apart, and thus the size of each ∠ may be changed, although the sum of the four ∠s is always equal to four rt. ∠s.

If, for a quadrilateral, only the lengths of the four sides are given, the quadrilateral may be made of different shapes, but when, in addition to the lengths of the sides, one diagonal, or one ∠, is given, the shape of the figure is fixed.

If a figure has more than four straight sides of given lengths, is its shape fixed?

155. Examples.

1. Draw the △ ABC, a = 10, b = 8, c = 6. Measure the ∠s.

2. Draw the △ ABC, a = 9, b = 7, c = 5. Measure the ∠s.

3. Draw a quadrilateral ABCD, having AB = 4 cm., BC = 5 cm., CD = 6 cm., DA = 8 cm., and ∠ ABC = 120°. Measure the other ∠s.

4. Draw a quadrilateral ABCD, with the same sides as in Ex. 3, but ∠ ABC = 60°. Measure the other ∠s.

5. Draw a pentagon ABCDE, having sides 4, 5, 6, 7, 8 cm. and any ∠s you choose at A and B. Measure all the ∠s and check your results by using the fact that the sum of the ∠s of a pentagon must be equal to six rt. ∠s.

6. If the lengths of the sides of a quadrilateral are given, and if either a diagonal or an ∠ is also given, show that the quadrilateral consists of two fixed △s.

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156. Draw any △ ABC and another DEF having DE twice AB, EF twice BC and FD twice CA.

Thus $\frac{DE}{AB} = \frac{EF}{BC} =$ [illegible], each fraction being equal to 2.

Compare the ∠s, using [illegible] paper or the protractor.

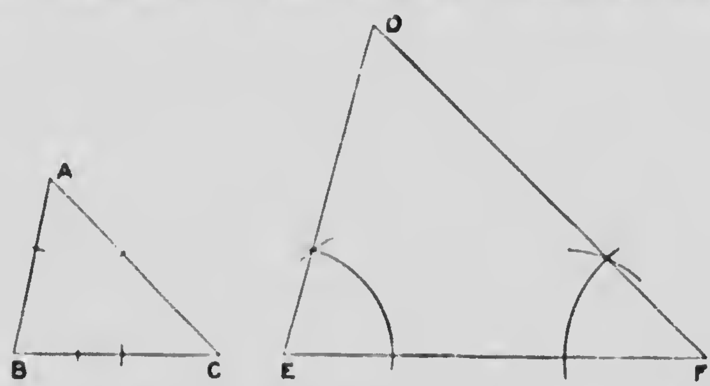
It will be found that ∠ D = ∠ A, ∠ E = ∠ B,
∠ F = ∠ C.

Make any other △, the sides of which are respecti
in the same ratio to the sides of △ ABC.

For example, if GHK be a △ having $\frac{GH}{AB}$ =
= $\frac{GK}{AC}$ = $\sqrt{3}$, it will be found that ∠ H = ∠ B, ∠
= ∠ A and ∠ K = ∠ C.

Thus: **If △s have their sides in the same p portion, the ∠s of one △ are respectively equal the ∠s of the other, the equal ∠s being oppos to the corresponding sides.**

157. Draw any △ ABC, and any line-segment EF.



At E and F make ∠s DEF and DFE respectively eq to ∠s B and C. Then ∠ D = ∠ A.

Measure the sides of the two △s and compare the lengths of those which are opposite the equal ∠s.

It will be found that $\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$.

Thus: **If the ∠s of one △ are respectively equal to the ∠s of another, the sides are in the same proportion, corresponding sides being those opposite equal ∠s.**

158. Two △s, which have the ∠s of one respectively equal to the ∠s of the other, are called **similar △s.**

159. Draw a square, and another rhombus having its sides equal to twice the sides of the square but its ∠s not rt. ∠s.

We see that two quadrilaterals may have the sides of one respectively proportional to the sides of the other, but the ∠s of one not equal to the corresponding ∠s of the other.

Draw two quadrilaterals which have the ∠s of one equal to the ∠s of the other, but their sides not in the same proportion.

160. △s are the only rectilineal figures in which, if the sides of one are in the same proportion as the sides of the other, the ∠s of one must be respectively equal to the ∠s of the other; or, if the ∠s of one are respectively equal to the ∠s of the other, the sides must be in the same proportion.

161. Two polygons that have the ∠s of one respectively equal to the ∠s of the other, and also have their corresponding sides in the same proportion, are called **similar polygons.**

162.—Examples.

1. Draw a △ ABC. Bisect AB at D. Through D draw DE ∥ BC meeting AC at E. Show that △ ADE is similar to △ ABC. Measure DE and check your result by comparing it with the length of BC. Measure AE and check your result.

2. Draw any △ ABC. In AB take any point D. Draw DE ∥ BC meeting AC at E. Measure the sides of the △s ADE and ABC and check your results.

3. Draw any △ ABC. Produce AB to any point D. Through D draw DE ∥ BC meeting AC produced at E. Measure the sides of △s ADE, ABC and check your results.

Through D draw DF ∥ CA meeting CB produced at F. Measure the sides of △ BDF and check your results.

4. Draw any circle and mark a point E within the circle. Through E draw any two chords AEB, CED. Join AC and BD. Show that △ AEC is similar to △ BED. Measure AE, EB, CE, ED and check your results.

Construct a rectangle having its adjacent sides equal to AE and EB; and another having its adjacent sides equal to CE and ED. Show that the two rectangles are equal in area. Give a general statement of which this last result is an illustration.

5. Repeat the last example throughout, taking the point E outside of the circle.

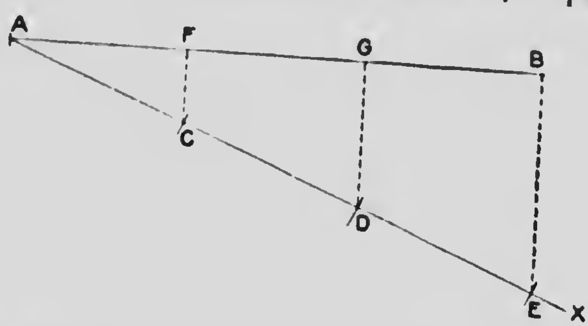
6. Draw a △ ABC having B ∠ = 57°, ∠ C = 85° and a = 5 cm. Make a similar △ having the shortest side 6 cm.; and check your result by measuring and comparing the remaining sides of the △s.

7. Draw a △ ABC, having ∠ B = 30°, ∠ C = 45° and a = 3 inches. Make a similar △ having the shortest side one inch; and check your result.

Division of a Line-Segment into Equal Parts.

163. Draw any line-segment AB.

It is required to divide AB into three equal parts.



From A draw any st. line AX, and with any unit mark off three equal parts AC, CD and DE.

Join EB.

From C and D draw CF and DG each ∥ BE, cutting AB in F and G.

Show that AF equals one-third of AB; and that AG equals two-thirds of AB.

164.—Examples.

1. Draw a line-segment 9 cm. in length, and divide it into three equal parts by the method of § 163. Check your result by measuring the parts.

2. Draw any line-segment and divide it into five equal parts. Test the result with the dividers.

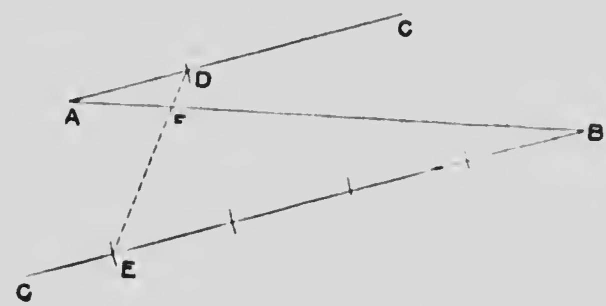
3. Draw a line-segment 10 cm. in length, and cut off three-sevenths of it.

4. Make a line-segment $\frac{\sqrt{2}}{3}$ inches in length.

5. Make a line-segment $\frac{\sqrt{3}}{5}$ inches in length.

6. Make a line-segment $\frac{3\sqrt{5}}{7}$ inches in length.

7. Draw any line-segment AB.



Through A draw any st. line AC; and through B draw BG ∥ A
From AC cut off any convenient unit AD, and from BG cut
BE equal to four times AD. Join DE cutting AB at F. Sh
that AF equals one-fifth of AB.

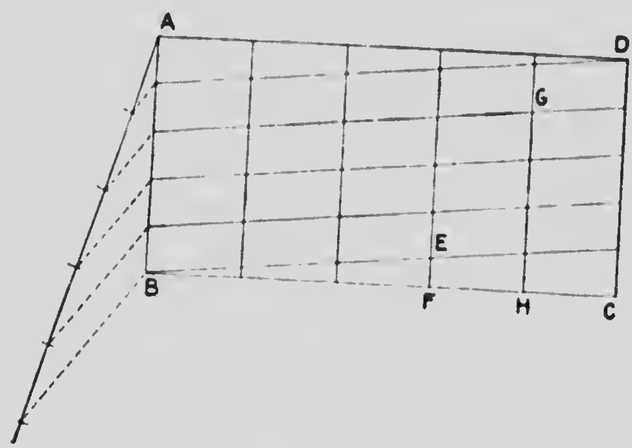
8. Draw a line-segment 72 mm. in length; and cut off one-six
of it, using the method of Example 7.

9. Draw a line-segment 65 mm. in length; and divide it in t
ratio 2 : 3, by the method of Example 7.

10. Draw any line-segment; and divide it in the ratio 1 : √
Check your result by constructing a line-segment $\sqrt{2}$ times t
smaller part, and comparing it with the larger part.

11. Draw any three line-segments AB, C and D. Divide
into two parts AE and EB such that $\frac{AE}{EB} = \frac{C}{D}$. Measure t
line-segments and check your work by calculation.

165. Draw a rectangle ABCD having the width AB one inch.



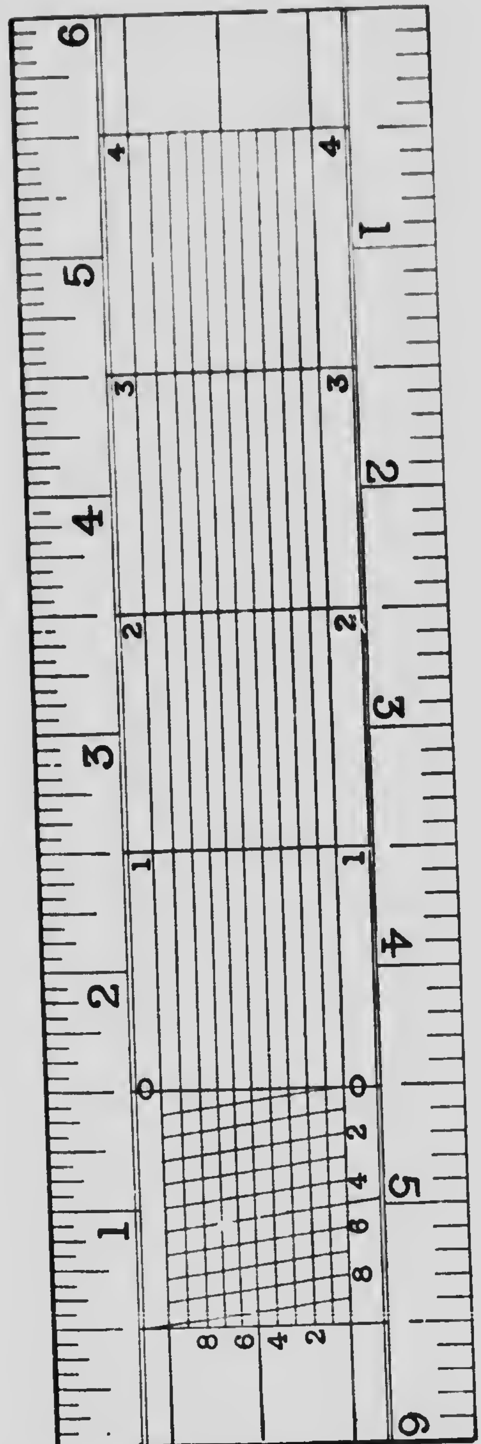
Divide each side into five equal parts; join the points of division of AD and BC directly, and those of AB and CD diagonally, as in the diagram.

Show that EF = $\frac{3}{25}$ of an inch.

What fraction of an inch is GH?

Show that the diagram gives each twenty-fifth of an inch from one twenty-fifth up to twenty-five twenty-fifths.

By drawing a rectangle one inch in width, and dividing each side into ten equal parts, in a similar manner to the above, measurements may be made to the hundreth part of an inch.



Diagonal Scale.

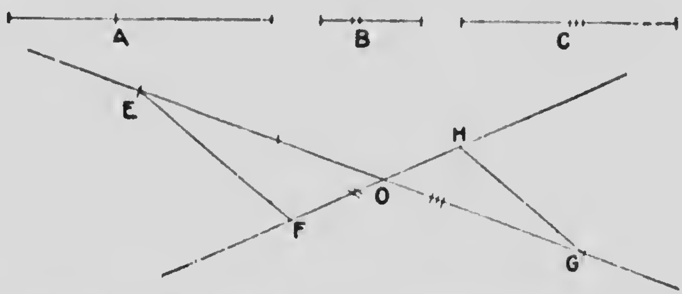
166.—Examples.

1. Draw a square each side of which is two inches. Measure the diagonal of the square to the hundredth part of an inch by using the diagonal scale. Check your result by calculation.

2. Draw a line-segment 8 cm. in length, measure its length in inches, using the diagonal scale, and thence calculate the number of inches in a metre.

Applications of Similar Triangles.

167. Draw three line-segments A, B and C, of different lengths.



It is required to find a fourth line-segment D, such that $\frac{A}{B} = \frac{C}{D}$.

Mark a point O. Draw OE = A; and, with any ∠ at O, draw OF = B.

Produce EO to G, making OG = C.

Through G draw GH ∥ FE, meeting FO produced at H.

Show that $\frac{EO}{OF} = \frac{OG}{OH}$.

Thus OH is a line-segment such that

$$\frac{A}{B} = \frac{C}{OH}.$$

Measure A, B, C and OH; and check your work by calculation.

The line-segment OH is the fourth proportional to the line-segments A, B and C.

168. Draw any rt.-∟d △ ABC, having C the rt. ∠.

From C draw CD perpendicular to AB.

Show:

1. That △ BCD is similar to △ ABC.

2. That △ ACD is similar to △ ABC.

3. That △ BCD is similar to △ ACD.

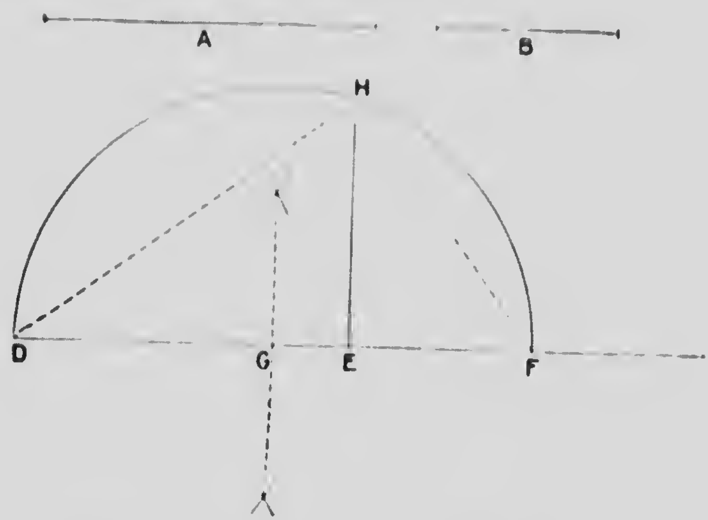
4. That $\frac{BC}{CA} = \frac{BD}{DC} = \frac{CD}{DA}$.

5. That $\frac{BA}{AC} = \frac{CA}{AD} = \frac{BC}{CD}$.

Thus, from (1), (2), and (3): **If a perpendicular be drawn to the hypotenuse of a rt-∟d △ from the vertex of the rt. ∠, the perpendicular divides the rt.-∟d △ into two △s that are similar to the original △ and to each other.**

Give general statements corresponding to (4) and (5).

169. Draw any two line-segments A and B.



It is required to find a third line-segment C such that $\frac{A}{C} = \frac{C}{B}$.

Draw a st. line and cut off DE = A, EF = B.

Bisect DF at G.

With centre G and radius GD describe a semi-circle.

From E draw EF perpendicular to DF to meet the circumference at H.

EH is the required line.

Join DH and HF.

Show that $\frac{DE}{EH} = \frac{EH}{EF}$.

Thus EH is a line-segment such that $\frac{A}{EH} = \frac{EH}{B}$.

The line-segment EH is the mean proportional between the line-segments A and B.

170.—Examples.

1. Draw a rectangle ABCD, 8 cm. by 5 cm. Draw a line-segment EF, 10 cm. in length. Draw, by the method of § 167, a line-segment GH such that $\frac{GH}{AB} = \frac{BC}{EF}$. Make a rectangle having its length equal to EF and its breadth equal to GH. How does the area of this rectangle compare with that of ABCD?

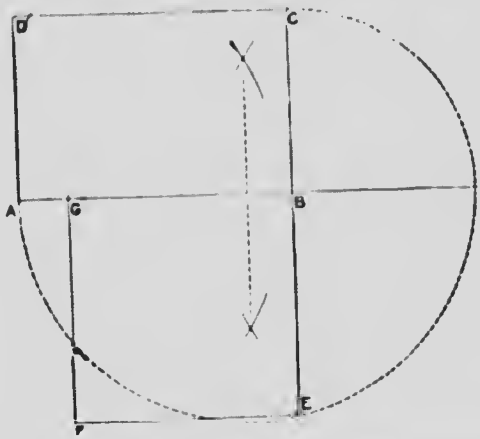
If a, b, c, d represent four numbers such that $\frac{a}{b} = \frac{c}{d}$, show that ad = bc. State this algebraic theorem in words.

2. Draw a rectangle 7 cm. by 5 cm. Make another rectangle of the same area having its length 8 cm. Measure its breadth and check your result by calculation.

3. Draw a square ABCD having its side 6 cm. Make a rectangle equal in area to the square and having its length equal to the diagonal AC. Measure its breadth.

4. Draw AB = 7 cm. and BC = 6 cm. in the same st. line. Find, by the method of § 169, BD such that $\frac{AB}{BD} = \frac{BD}{BC}$. Measure BD and check your result by calculation.

5. Make a rectangle ABCD, 9 cm. by 6 cm. Make a line-segment



BE such that $\frac{AB}{BE} = \frac{BE}{BC}$. Describe a square BEFG on BE; and show that the area of the square is equal to that of the rectangle.

6. Make a ||gm. ABCD, having AB = 5 cm., BC = 8 cm., and ∠ B = 75°. Construct a square equal in area to the ||gm; and measure the side of the square.

7. Make an equilateral △ on a line-segment 7 cm. in length. Construct a square equal in area to the △; and measure a side of the square.

8. Make a △ ABC, having a = 11 cm., b = 6 cm., and ∠ C = 30°. Construct a square equal in area to the △; and measure a side of the square.

9. An upright rod 5½ ft. high stands 4 ft. from a lamp-post, and the shadow of the pole cast by the lamp is 3½ ft. long. Draw a diagram to scale and find the height of the lamp. Check by calculation.

10. Make a rectangle having a diagonal 87 mm., and one side 45 mm. Construct a square equal in area to the rectangle, and measure a side of the square.

Miscellaneous Examples.

NOTE.—The first twenty-eight of these examples are from papers in geometrical drawing set at the Entrance Examinations of the Royal Military College, Kingston.

1. Draw an ∠ of 60° and bisect it.

2. Draw a line 5·5 inches long and divide it into nine equal parts.

3. Divide a line 5 inches long into three parts in the proportion of 7 : 5 : 3.

4. Draw three lines 2·5, 2·75, 3·5 inches long respectively, and find their fourth proportional.

5. Find a mean proportional between two lines of length 2·5 and 4·25 inches respectively.

6. Taking 2·5 inches as unit, determine the st. lines respectively equal to $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$.

7. Draw any △ (not equilateral) and inscribe a circle in it.

8. Draw a regular pentagon on a side AB equal to 1·5 inches.

9. Inscribe a regular pentagon in a circle of 1·5 inches radius.

10. Draw a st. line to touch a circle of radius 1 inch:

(*a*) From any point on the circumference.

(*b*) From any point outside the circumference.

11. Draw an ∠ of 69°, and bisect it by construction.

12. By construction divide a line 6·9 inches long into 11 equal parts.

13. Construct a rhombus on a side of 3 inches, each of one pair of opposite ∠s being 60°.

14. Construct a rt.-∠d △, hypotenuse 4·3 inches, and one of the ∠s 48°.

15. Inscribe a regular pentagon in a circle of 1·75 inches radius.

16. Construct a regular pentagon on a side of 3 inches.

17. Through any point on the arc of a circle draw a tangent without using the centre.

18. From a circle of radius 1·5 inches cut off a segment to contain ∠s of 30°.

19. The chord AC of an arc of a circle is 3 inches long, and the perpendicular distance BD from the middle point B of this chord to a point on the arc is 0·9 inches long, find O, the centre of the circle.

Also find a point E in AD, or AD produced, which shall be equidistant from B and C.

20. Three lines AB, BC and AC are 3·25 inches, 3·75 inches, 4·1 inches long respectively. Construct a △ having sides equal to these lines; and on the other side of BC construct a rt.-∠d △ BCD (having rt. ∠ at D); the point D being a perpendicular distance of 1·7 inches from BC.

21. On a line AB, 2·4 inches long, construct an isosceles △, having the vertical ∠ equal to 45°.

22. On a st. line AB, 11·2 feet long, construct a regular heptagon. Scale: 7 feet equals 1 inch.

23. Through a point A, near [illegible] site the end of, and not less than 2 inches from a st. line BC, draw a line ∥ BC. Construct an

equilateral [illegible], having an altitude AD, the perpendicular from A to the line BC.

24. Draw a st. line 2·5 inches long; from [illegible] extremity raise by construction a perpendicular AC 1·3 inches long. From the extremity B drop a perpendicular BD, join CD, cutting AB in E. With centre E and radius EC describe a circle. Measure and write down the length of its diameter.

25. Draw a st. line 5·25 inche[illegible] divide [illegible] into four equal parts. On the first describe an [illegible] on the fourth an isosceles △ with altitude 2 in[illegible]

26. Describe a [illegible], with a pe[illegible] 18 inches [illegible] sides in the proportion 3 : 4 : 5.

27. The line joining the angu[illegible] pentagon to the centre of the opposite side is [illegible] inches [illegible] construct the pentagon.

28. Draw a square with sides 2·5 inche[illegible] regular octagon by cutting off its ∠s.

29. A man is 5 ft. 11 in. in height. [illegible] from your ruler, the number of millimetres in 2 in[illegible] calculate the man's height in centimetres.

30. There are three towns [illegible] B and C [illegible] distance from B to C is 10 miles, from C to A [illegible] 7 miles, [illegible] A [illegible] B is 6 miles. Draw a plan showing their positions. [illegible] miles to an inch.]

Find the point P that is equally d[illegible] A, B and C; and give the distance PA in miles.

31. The minute-hand of a clock is 4·6 [illegible] long and the hour hand 3·8 inches. Find the distance apart of their extremities at seven o'clock.

32. Make a △, with sides 5·4, 9·5 and 11·5 cm. [illegible] P, Q and R, the middle points of the sides, and draw the three medians. Measure the longest median.

Join QR, RP, PQ and measure the sides of the △ PQR. What relationship do you observe between the lengths of the sides of the △ PQR and of △ ABC?

33. Make a △ having its base 50 mm. and base ∠s 60° and 45°. Measure the shortest side.

34. Make a rhombus with each side 82 mm. and one ∠ 105°. Measure the longer diagonal.

35. Make an isosceles △ having its vertical ∠ 45° and the length of the perpendicular from the vertex to the base 6 cm. Measure one of the equal sides.

36. Construct △ ABC having a = 7·9 cm., B = 64°, C = 50°. On a base 5·2 cm. long make a ||gm equal in area to the △ and having one ∠ 40°. Measure the side of the ||gm.

37. A man travelling north along a st. road observes a windmill at an ∠ of 30° west of north. When he has gone five miles the windmill is exactly south-west. What is the distance of the windmill from the road?

38. The height of a house, observed from a window on the opposite side of the street, subtends an ∠ of 75°, the top being 30° above the horizontal st. line. Find the height of the house, the breadth of the street being 66 ft. [Scale: 25 ft. to the inch.]

39. A fortress was observed from a ship at sea to bear east-north-east (east-north-east bisects the ∠ between east and north-east), and after sailing four miles to the east it was observed to bear north-north-east. Find the distance of the ship from the fortress at each observation. [Scale: 2 miles to an inch.]

40. A ship sailing towards the north observes two lighthouses in a line due west; and after an hour's sailing the bearings of the lighthouses are observed to be south-west and south-south-west. The distance between the lighthouses being 8 miles, find the rate at which the ship is sailing. [Scale: 4 miles to an inch.]

LIST OF DEFINITIONS.

Acute Angle: An ∠ which is less than a rt. ∠. § 51.

Acute-Angled Triangle: A △ which has all three of its ∠s acute. § 55.

Alternate Angles: Two interior ∠s which are on opposite sides of a transversal. § 93.

Angle: The figure which is formed by two st. lines drawn from the same point. § 23. The inclination of two st. lines.

Angle in a Segment: The ∠ which is contained by two st. lines joining a point on an arc of a segment to the ends of the chord. § 134.

Arc: An arc of a circle is a part of the circumference. § 16.

Arms of an Angle: The two st. lines which meet and form the ∠. § 23.

Axis of Symmetry: When a figure is such that if it be folded along a straight line, the part on one side exactly fits the part on the other side, this line is an axis of symmetry of the figure. § 18.

Centre of a Circle: The point within the figure from which all st. lines drawn to the circumference are equal. §§ 12 and 13.

Centroid: The point where the medians of a △ intersect each other. § 64, Ex. 9.

Chord of a Circle: A st. line which is drawn from one point on the circumference to another. § 14.

Chord of Contact: The st. line which joins the points of contact of two tangents to a circle. § 149, Ex. 12.

Circle: A figure enclosed by one curved line, the circumference, and such that all st. lines drawn from a point within to the circumference are equal to each other. §§ 10 to 13.

Circumference: The curved line which bounds a circle. § 11.

Circumscribed Circle: A circle which passes through the vertices of a , quadrilateral or polygon. § 79.

Complement of an Angle: When the sum of two ∠s is a right ∠, each ∠ is the complement of the other. § 70.

Congruent: Figures that are equal in all respects, so that one may be made to fit the other. § 38.

Decagon: A polygon which has ten sides. § 80.

Diagonal: A st. line which joins opposite vertices of a quadrilateral. § 34. A st. line which joins any two non-adjacent vertices of a polygon.

Diameter of a Circle: A chord which passes through the centre and is terminated both ways by the circumference. § 15.

Dodecagon: A polygon which has twelve sides. § 80.

Equilateral Triangle: A , which has its sides all equal to each other. § 31.

Escribed Circle: A circle which touches one side of a , and the other two sides produced. § 148.

Exterior Angle of a Triangle: The ∠ which is contained by one side of a , and the produced part of another side. § 87.

Heptagon: A polygon which has seven sides. § 80.

Hexagon: A polygon which has six sides. § 80.

Hypotenuse: The side which is opposite the rt. ∠ in a rt.-∠d △. § 55.

Inscribed Polygon: A polygon which has its vertices on the circumference of a circle. § 79.

Isosceles Triangle: A triangle which has two sides equal to each other. § 31.

Line: That which has length but neither breadth nor thickness. § 2.

Line-Segment: A part of a st. line terminated by two fixed points. § 8.

Median: A line which joins a vertex of a △ to the middle point of the opposite side. § 64, Ex. 8.

Nonagon: A polygon which has nine sides. § 80

Obtuse Angle: An ∠ which is greater than a rt. ∠. § 51.

Obtuse-Angled Triangle: A △ which contains an obtuse ∠. § 55.

Octagon: A polygon which has eight sides. § 80.

Orthocentre: The point where the perpendiculars from the vertices of a △ to the opposite sides intersect each other. § 68, Ex. 5.

Parallel Straight Lines: St. lines which are in the same plane and which do not meet when produced to any finite distance in either direction. § 92.

Parallelogram: A quadrilateral which has both pairs of opposite sides ∥. § 96.

Pentagon: A polygon which has five sides. § 80.

Perpendicular: Either arm of a rt. ∠ is perpendicular to the other. § 50. A st. line which stands on another st. line making the adjacent ∠s equal to each other.

Point: That which has position but no size. § 1.

Point of Contact: The point where the tangent touches the circle. § 140.

Polygon: A closed figure bounded by any number of st. lines more than four. § 80.

Quadrilateral: A figure which is formed by four st. lines. § 34.

Quindecagon: A polygon which has fifteen sides. § 80.

Radius: A st. line which is drawn from the centre of a circle to the circumference. § 13.

Rectangle: A ∥gm that has its ∠s rt. ∠s. § 104.

Regular Polygon: A polygon which has all its sides equal and all its ∠s equal. § 81.

Rhombus: A quadrilateral which has its four sides equal to each other. § 35.

Right Angle: An ∠ which is half a st. ∠. § 50. When one st. line stands on another st. line making the adjacent ∠s equal to each other, each of these ∠s is rt. ∠.

Right-Angled Triangle: A △ which contains a rt. ∠. § 55.

Scalene Triangle: A △ which has no two of its sides equal to each other. § 31.

Segment of a Circle: A figure which is bounded by a chord and one of the arcs cut off by the chord. § 134.

Side of a Triangle: A line-segment joining two vertices of a △. § 28.

Similar Polygons: Polygons which have the ∠s of one respectively equal to the ∠s of the other, and also have their corresponding sides in the same proportion. § 161.

Similar Triangles: △s which have the ∠s of one respectively equal to the ∠s of the other. § 158.

Solid: That which has length, breadth and thickness. § 4.

Square: A rectangle that has all its sides equal. § 107.

Straight Angle: An ∠ which has its arms in the same st. line and extending in opposite directions from the vertex. §§ 47, 48.

Surface: That which has length and breadth but no thickness. § 3.

Symmetrical Figure: A figure that can be folded along a st. line so that the part on one side exactly fits the part on the other side. § 18. This is axial symmetry. Central symmetry is indicated in § 20 and in § 115.

Tangent to a Circle: A st. line which meets a circle but does not cut it. § 140.

Transversal: A st. line which cuts two other st. lines. § 93.

Triangle: The figure which is formed by three st. lines which intersect each other. § 28.

Vertex: The point from which the arms of an ∠ are drawn is the vertex of the ∠. § 23. The points of intersection of the sides of a △, quadrilateral or polygon.

ANSWERS.

Page 14.—8—Each line 99 mm. nearly.

Page 15.—3—69 mm. nearly.
4—52·9 mm. nearly.
5—62 mm. nearly.
6—55 mm. nearly.
7—66 mm. nearly.

Page 19.—3—BD 23, DC 27, AD 30 mm. nearly in each case.
4—27 and 36 mm. exactly.

Page 21.—1—6 cm.
2—58 mm. nearly.
3—$b' = 11·25$ cm., $c' = 13·5$ cm.
4—104 mm. nearly.

Page 25.—7—8 cm.
8—47·2 ft. nearly.
9—38·7 mm. nearly.
10—12 miles.
11—13·9 miles nearly.
12—45 ft.
13—7 miles nearly.

Page 32.—7—$48\frac{3}{4}$ mm.
8—42 mm. nearly.
13—34 mm.

Page 35.—2—36 mm.
3—43 mm. nearly.
4—AX = 31 mm. nearly.
AD = 43 mm. nearly.
AH = 32 mm. nearly.

Page 36.— 6—BX = 33 mm., XC = 42 mm.
CY = 45 mm., YA = 25 mm.
AZ = 27 mm. nearly.
ZB = 38 mm. nearly.
AO = 31 mm. nearly.
BO = 41 mm. nearly.
CO = 49 mm. nearly.

7—64 mm., 94 mm., 96 mm. nearly.

8—108 ft. nearly.
10—4·2 miles nearly.
11—About 950 yds.

Page 39.— 1—70°.
2—59°.
3—30°.
8—89·6 mm. nearly.
9—b = 29·5 mm., c = 80·6 mm. nearly.
10—b = 86 mm., c = 70 mm. nearly.

Page 40.— 2—$\angle$ ADF = 120°, $\angle$ BDE = 150°,
$\angle$ CDE = 75°, $\angle$ CDF = 60°,
$\angle$ ADB = 15°.

Page 41.— 3—108°.
4—120°.
5—135°.
6—10 rt. $\angle$s.

Page 42.— 7—86·6 mm. nearly, 120°.
10—10·4 cm. nearly, 12 cm.
12—$53\frac{1}{2}$ mm. nearly.
13—65 mm. nearly.

Pr[illegible].— 3—25 mm.
4—A = [illegible]$\frac{1}{2}$°, B = $32\frac{1}{2}$° nearly.
5—B = $71\frac{3}{4}$°, C = $48\frac{1}{4}$° nearly.

Page 44.— 6—c = 111 mm., A = 41°, B = 19° nearly.

7—90°.

8—59° nearly.

9—104½° nearly.

10—132½° nearly.

11—25 mm., 114 mm. nearly.

12—1·7 or 3·7 nearly.

13—32 mm.

14—42 ft. nearly.

15—134 ft. nearly.

16—237 ft. nearly.

17—46 ft., 87 ft. nearly.

Page 45.—18—137 ft. nearly.

19—87 yds. nearly.

20—9·8 miles, 48½° nearly.

Page 52.— 3—86 mm., 106 mm., 124 mm.

4—46 mm., 111 mm. nearly.

5—97·3 mm. nearly.

6—52½° nearly.

Page 53.— 8—33 mm. and 56 mm. nearly.

9—83° and 97° nearly.

Page 55.— Art. 106—13 cm. each.

Page 56.— 1—76·4 mm. nearly.

2—3·54 in. nearly.

3—63·6 mm. nearly.

4—1·41 in. nearly.

5—70·7 mm. nearly.

6—53 mm. nearly.

Page 60.— 2—24 mm. nearly.

3—12·6 mm., 54·5 mm. nearly

5—27° nearly, 73 yds. nearly.

Page 60. 7 -81 mm. nearly.

8 75 mm. nearly.

Page 61.—11 24 mm., 63 mm., 72 mm., 84 mm.

Page 63.— 1 21 sq. cm.

2 23 sq. cm. nearly.

3—13·7 sq. cm. nearly.

4—28·8 sq. cm.

5 23·4 sq. cm. nearly.

6—15·8 sq. cm. nearly.

Art. 122—9 sq. yds. 56·25 sq. m.

Art. 123—80 chains. 10 sq. chains.

1—36 acres.

2—73 acres nearly.

Page 64.— 4—20·16 sq cm.

5—57·7 mm. nearly.

6—38° nearly.

7—64 mm. nearly.

8—64 mm. nearly.

9—69 mm. nearly.

Page 67.— 7—78 mm. nearly.

Page 75.— 6—30°, 120°.

7—22½°, 157½°.

8—53° nearly.

9—19½°, 160½° nearly.

11 47 mm. nearly, 25°.

12—43·6 mm., 145°.

13—29 mm. nearly.

14—38·6 mm. nearly.

15—33·6 mm. nearly.

16—33 mm, 75·6 mm. nearly.

17—84 mm., 40 mm. nearly.

Page 79.—6—91° nearly.

Page 82.—1—16, 42, 48, 56 mm.
2—1, 2, 3, 6 in.
3—12, 32, 32, 48 mm.
4—82 mm. nearly.
5—84 mm. nearly.
6—91·6 mm. nearly.
7—9 cm. nearly.
8—4 cm.
10—3 cm.
8 cm.
12—83 mm. nearly.

Page 83.—13—68 mm. nearly.
14—23 mm.

Page 85.—6—2 cm., 3 cm., 10 cm.
7—6 cm., 3 cm., 2 cm.
8—68·4 mm. nearly.

Page 87.—1—A = 90°, B = 53° nearly, C = 37° nearly.
2—A = $95\frac{3}{4}$°, B = $50\frac{3}{4}$°, C = $33\frac{1}{2}$° nearly.
3—∠ BCD = 96°, ∠ CDA = 66°, ∠ DAB = 78° nearly.
4—∠ BCD = $146\frac{1}{2}$°, ∠ CDA = $34\frac{1}{2}$°, ∠ DAB = 119° nearly.

Page 98.—3—42·4 mm.

Page 99.—6—62 mm. nearly.
7—46 mm. nearly.
8—40·6 mm. nearly.
10—58 mm. nearly.

Page 101.—29—180·3 cm. nearly.
30—5·08 miles nearly.
31—8·12 inches nearly.

Page 101. 32 102 mm. nearly.

33—36·6 mm. nearly.

Page 102. 34 13 cm. nearly.

35—65 mm. nearly.

36 73 mm. nearly.

37 1·83 miles nearly.

38 104 ft. nearly.

39 5·2 miles, 2·2 miles nearly.

40 13·66 miles per hr. nearly.

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