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Key to Deductions

IN THE

PUBLIC SCHOOL

EUCLID AND ALGEBRA

PROPOSITIONS I.—XXVI.

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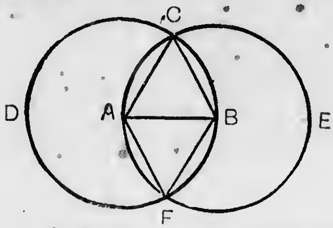
TORONTO

PROPOSITIONS I. TO XXVI.

PROPOSITION I.

1. If the two circumferences intersect also at F, what kind of triangle will be found by joining AF and BF?



AF = AB.	Def. of a circle.
BF = AB.	Def. of a circle.
∴ AF = BF.	Axiom I.

Thus AF, AB, and BF are all equal, and an equilateral triangle ABF has been described on AB.

---

2. What kind of quadrilateral is the figure ACBF?

See figure in No. 1.

Since AC, AF, BC, and BF each equal AB. Def. of a circle.
∴ AC, AF, BC, and BF are all equal. Ax. I.
That is ACBF is equilateral or a rhombus. Def. 25.

PROPOSITION II.

1. Under what circumstances would the point D lie—

(a) Outside the circle CEF?

(b) On the circumference of the circle CEF?

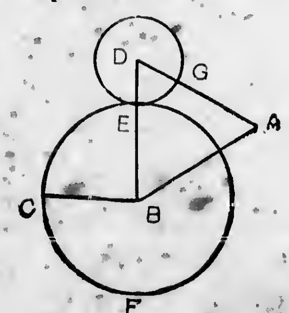
(a) The point D would lie outside the circle CEF if the point A were farther from B than the length of BC. For then AB and BD, which is equal to it, would be longer than BC, the radius of the circle CEF.

(b) The point D would lie on the circumference of the circle CEF if the point A were the same distance from B that the point B is from C.

For then AB and BD, which is equal to it, would be radii of the circle CEF.

---

2. If D were without the circle CEF, would it be necessary to produce DB?



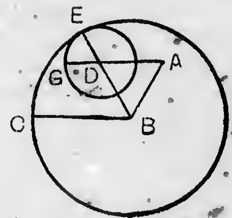
It would not be necessary to produce DB.

BD = AD.	Def. of an equilateral △
DE = DG.	Def. of a circle.
∴ BE = AG.	Ax. 3.
But BE = BC.	Def. of a circle
∴ AG = BC.	Ax. I.

Wherefore from the point A a straight line AG has been drawn equal to BC.

---

3. Could the problem be solved by producing BD instead of DB?



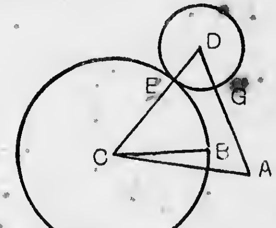
Yes; as in the figure—

BD = AD.	Def. of an equilateral △.
DE = DG.	Def. of a circle.
∴ BE = AG.	Ax. 2.
But BE = BC.	Def. of a circle.
∴ AG = BC.	Ax. I.

Wherefore from the point A a straight line AG has been drawn equal to BC.

---

4. Could the problem be solved by joining AC instead of AB?



Yes; as in the figure—

CD = AD.	Def. of an equilateral △.
DE = DG.	Def. of a circle.
∴ CE = AG.	Ax. 3.
But CE = CB.	Def. of a circle.
∴ AG = CB.	Ax. I.

Wherefore from A a straight line AG has been drawn equal to BC.

---

5. Does the line AG always lie in the same direction, no matter which of the above methods of construction is used?

The line AG does not always lie in the same direction, as may be seen from the diagrams.

A great many excellent exercises may be made from this second proposition, for you may:

(*a*) Draw the △ as in text-book, and then produce DB either down, as in the book, or up, as in question 3.

(*b*) Draw the △ below AB and produce DB each way.

(*c*) Join A to C as in question 4, and make the △ above AC, and produce DB either way.

(*d*) Join AC and make the △ below AC, and produce DB either way.

(*e*) Let these four be drawn, keeping the distance from A to the end of BC, to which it is joined, shorter than the length of BC. Then change all by making this distance greater. This will give, in all, sixteen figures.

NOTE.—You will have no difficulty with any of them if you remember the following:

1. The centre of the first circle is the end of the given line to which the point is joined.

2. The first side produced is that side of the triangle passing through the apex of the triangle and the centre of this circle.

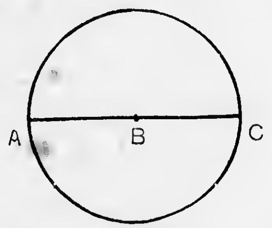
3. The centre of the second circle is always the apex of the triangle.

4. The second side produced is that side which passes through the apex of the triangle and the given point.

These are always true, no matter how the figure may be arranged.

PROPOSITION III.

1. AB is a given straight line. Produce the line, making the whole length double that of AB.



Let AB be the given straight line.

It is required to produce AB, making the whole line thus produced double of AB.

With centre B distance BA describe a circle. Post. 3.

Produce AB to meet this circle in C. Post. 2.

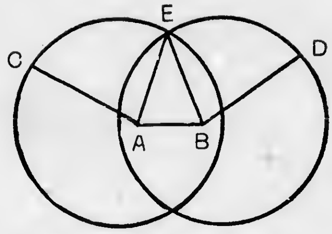
Then AC shall be the line required.

AB = BC. Def. of a circle.

∴ BC = 2 AB

Wherefore the straight line AB has been produced, so that the whole line AC thus produced is double of AB.

2. Describe an isosceles triangle on a given straight line, such that each of the equal sides shall be twice as long as the base.



Let AB be the given straight line.

It is required to describe an isosceles triangle on AB, each of whose sides shall be double of AB.

From A draw AC double of AB. Deduction 1.

From B draw BD double of AB. Deduction 1.

With centre A distance AC describe a circle. Post. 3.

With centre B distance BD describe a circle. Post. 3.

Let the circumferences intersect at E.

Join AE and BE. Post. 1.

Then ABE shall be the triangle required.

AE = AC. Def. of a circle.

AC = 2 AB. Cons.

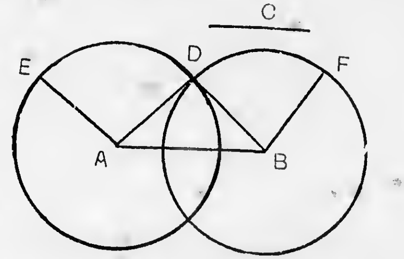
∴ AE = 2 AB. Ax. 1.

In the same manner it may be shown that BE = 2 AB.

∴ AE = BE. Ax. 1.

Wherefore on the base AB an isosceles triangle ABE has been constructed, each of whose sides is double of AB.

3. On a given straight line describe an isosceles triangle having each of the equal sides equal to another given straight line.



Let AB and C be the given straight lines.

It is required to describe on AB an isosceles triangle having each of its sides equal to C.

From A draw AE = C. Prop. 2.

From B draw BF = C. Prop. 2.

With centre A distance AE describe a circle. Post. 3.

With centre B distance BF describe a circle. Post. 3.

Let the circumferences intersect at D.

Join AD and BD. Post. 1.

Then ADB shall be the triangle required.

AD = AE. Def. of a circle.

AE = C. Cons.

∴ AD = C. Ax. 1.

In the same manner it may be shown that BD =C.

∴AD=BD. Ax. 1.

Wherefore on AB an isosceles triangle ADB has been constructed, having each of its sides equal to the given straight line C.

---

4. *Draw a straight line three times as long as a given straight line.*

This may be easily deduced from the first deduction on this proposition.

With C as centre and CB as distance, describe another circle, and produce AC to meet this circle in F. Then AF is the line required.

---

5. *On a given straight line describe an isosceles triangle having each of the equal sides three times as long as the third side.*

This may be easily deduced from the second deduction on this proposition.

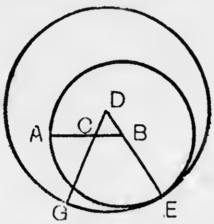
From A draw AC three times as long as AB. Deduction 4.

From B draw BD three times as long as AB. Deduction 4.

Then proceed as in the solution of Deduction 2.

---

6. *From a given point C, in a straight line AB, draw a straight line equal to AB.*



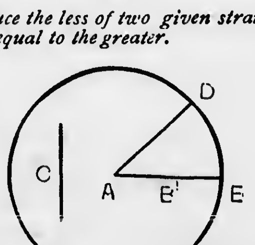
This is a particular case of the second proposition, and if the hints given at the end of the treatment of that proposition be remembered no difficulty will be experienced with this problem.

DG=DE.	Def. of a circle.
DC=DB.	Def. of an equilateral △.
∴CG=BE.	Ax. 3.
But BA=BE.	Def. of a circle.
∴CG=BA.	Ax. 1.

Wherefore from the point C a straight line CG has been drawn equal to the given straight line AB.

---

7. *Produce the less of two given straight lines, making it equal to the greater.*



Let AB and C be the two given straight lines of which AB is the shorter.

It is required to produce AB making it equal to C.

From A draw AD=C. Prop. 2.

With centre A distance AD describe a circle. Post. 3.

Produce AB to meet the circumference in E. Post. 2.

Then AE shall be the line required.

AE=AD.	Def. of a circle.
AD=C.	Cons.
∴AE=C.	Ax. 1.

Wherefore the straight line AB has been produced to E making the whole line thus produced equal to C.

PROPOSITION IV.

1. *The sides of the square ABCD are equal to the sides of the square EFGH.*

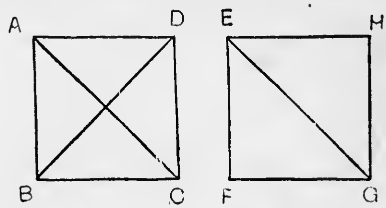
Show that:

(*a*) *The diagonals AC and EG are equal.*

(*b*) *The diagonals AC and BD are equal.*

(*c*) *The diagonal AC bisects, that is, divides into two equal parts, the angle BAD.*

(*d*) *The squares are equal in area.*



(*a*) In the △'s ADC and EHG

AD=EH.	Hyp.
DC=HG.	Hyp.
∠ADC=∠EHG.	Def. of a square.
∴AC=EG.	Prop. 4.

(*b*) In the △'s ADC and DCB

AD=BC.	Def. of a square.
DC=DC.	Common.
∠ADC=∠DCB.	Def. of a square.
∴AC=BD.	Prop. 4.

(*c*) In the △'s ADC and ABC

AD=AB.	Def. of a square.
DC=BC.	Def. of a square.
∠ADC=∠ABC.	Def. of a square.
∴ ∠DAC=∠BAC.	Prop. 4.

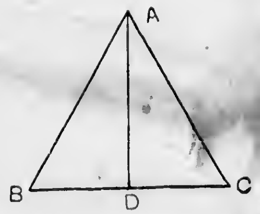
(*d*) In the △'s ADC and EHG

AD=EH.	Hyp.
DC=HG.	Hyp.
∠ADC=∠EHG.	Def. of a square.
∴ △ADC=△EHG.	Prop. 4.

Similarly it may be shown that the △ABC= △EFG.

∴ the square ADCB=the square EHGF. Ax. 2.

2. *A straight line AD bisects the vertical angle BAC of the isosceles triangle ABC, and meets the base at the point D. Show that D is the middle point of the base, and that AD is perpendicular to BC.*



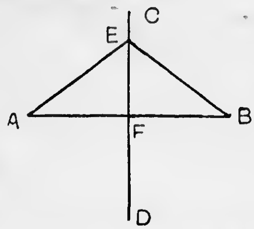
In the △'s ADB and ADC

AB = AC.	Def. of an equilateral △.
AD = AD.	Common.
∠BAD = ∠CAD.	Hyp.
∴ BD = DC.	Prop. 4.

That is, D is the middle point of the base.

Also ∠ADB = ∠ADC	Prop. 4.
∴AD is perpendicular to BC.	Def. 7.

3. If two straight lines bisect each other at right angles, any point in either of them is equidistant from the extremities of the other.



Let AB and CD be the two given straight lines, bisecting each other at right angles in the point F.

In CF take any point E.

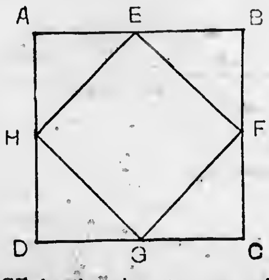
It is required to prove that E is equally distant from A and B.

Join AE and BE. Post. 1.

In the △'s EAF and EBF

AF = BF.	Hyp.
FE = FE.	Common.
∠AFE = ∠BFB.	Ax. 11.
∴AE = BE.	Prop. 4.

4. The middle points of the sides of a square are joined in order. Show that the quadrilateral formed by these joining lines is equilateral.



Let ABCD be the given square, and E, F, G, and H the middle points of the sides, and let EF, FG, GH and HE be joined.

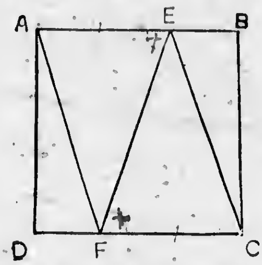
It is required to prove EF = FG = GH = HE.

In the △'s AEH, BFE, CGF and DHG the sides are all equal. Def. of a square, and Ax. 7.

The ∠'s A, B, C and D are all equal. Def. of a square, and Ax. 11.

∴EF, FG, GH and HE are all equal. Prop. 4.

5. ABCD is a square, E is a point in AB, and F is a point in CD, such that AE is equal to CF; EF is joined. Show that the angle AEF is equal to the angle CFE.



Let ABCD be the given square, E and F the given points, and let EF be joined.

It is required to prove that ∠AEF = ∠CFE.

Join AF and EC. Post. 1.

In the △'s ADF and CBE

DF = BE	Ax. 3.
AD = CB	Def. of a square.
∠D = ∠B	Def. of a square, and Ax. 11.
∴ ∠DAF = ∠ECB	Prop. 4.
And AF = EC	Prop. 4.

Now, since ∠DAE = ∠BCF Def. of a square, and Ax. 11.

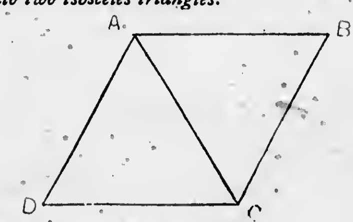
And	∠DAF = ∠ECB	Proved equal.
∴	∠FAE = ∠ECF	Ax. 3.

In the △'s AEF and CEF

AE = CF.	Hyp.
AF = EC.	Proved equal.
∠FAE = ∠ECF.	Proved equal.
∴ ∠AEF = ∠CFE.	Prop. 4.

PROPOSITION V.

1. Prove that the diagonal of a rhombus divides it into two isosceles triangles.



Let ABCD be the given rhombus and AC one of its diagonals.

It is required to prove ABC and ADC are isosceles △'s.

AB = BC.	Def. of a rhombus.
∴ABC is an isosceles △.	
Again, AD = DC.	Def. of a rhombus.
∴ADC is an isosceles △.	

2. Prove that the opposite angles of a rhombus are equal.

From the figure in deduction (1) above we have :

$\angle BAC = \angle BCA.$ Prop. 5.

Also $\angle DAC = \angle DCA.$ Prop. 5.

$\therefore \angle DAB = \angle DCB.$ Ax. 2.

Similarly by joining DB it may be shown that the $\angle ABC = \angle ADC.$

3. Prove that the diagonal of a rhombus bisects each of the angles through which it passes.

From the figure in deduction (1) above we have :
In the Δ's ABC and ADC

AB = AD.	Def. of a rhombus.
CB = CD.	Def. of a rhombus.
$\angle ABC = \angle ADC.$	Proved in deduction 2.

$\therefore \angle BAC = \angle DAC.$ Prop. 4.

And $\angle BCA = \angle DCA.$ Prop. 4.

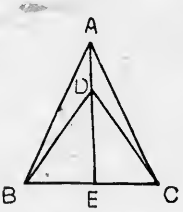
That is, the diagonal AC bisects the angles of the rhombus through which it passes.

4. Two isosceles triangles, ABC and DBC, have the same base BC.

(a) Prove that the angle ABD is equal to the angle ACD.

(b) Prove that the angle BAD is equal to the angle CAD.

(c) Prove that AD, or AD produced, bisects the base BC.



Let ABC and DBC be the two isosceles Δ's on the same base BC, and let AD be joined and produced to meet BC in E.

It is required :

(*a*) To prove $\angle ABD = \angle ACD.$

$\angle ABC = \angle ACB.$ Prop. 5.

$\angle DBC = \angle DCB.$ Prop. 5.

$\therefore \angle ABD = \angle ACD.$ Ax. 3.

(*b*) To prove $\angle BAD = \angle CAD.$
In the $\triangle$'s BAD and CAD

AB = AC.	Def. of isosceles $\triangle$.
DB = DC.	Def. of isosceles $\triangle$.
$\angle ABD = \angle ACD.$	Proved above.

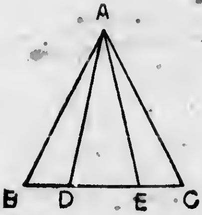
$\therefore \angle BAD = \angle CAD.$ Prop. 4.

(*c*) To prove AD produced bisects BC.
In the $\triangle$'s ABE and ACE.

AB = AC.	Def. of isosceles $\triangle$.
AE = AE.	Common.
$\angle BAE = \angle CAE.$	Proved above.

$\therefore$ base BE = base CE. Prop 4.

5. ABC is an isosceles triangle; and in the base BC two points D and E are taken such that BD = CE; prove that ADE is an isosceles triangle.



Let ABC be the given Δ, let the points D and E be taken such that BD = CE and let AD and AE be joined.

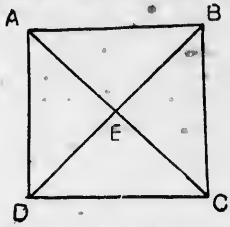
It is required to prove ADE is an isosceles Δ.
In the Δ's ABD and ACE

AB = AC.	Def. of isosceles $\triangle$.
BD = CE.	Hyp.
$\angle ABD = \angle ACE.$	Prop. 5.

$\therefore$ AD = AE. Prop. 4.

That is, ADE is an isosceles $\triangle$.

6. Prove that the diagonals of a square divide the figure into four isosceles triangles.



Let ABCD be the given square, and AC and BD the diagonals intersecting in E.

It is required to prove that AEB, CEB, ADE, and CDE are isosceles $\triangle$'s.
In the $\triangle$'s DAB and ABC

AD = BC.	Def. of a square.
AB = AB.	Common.
$\angle DAB = \angle ABC.$	Ax. 11.

$\therefore$ AC = DB. Prop. 4.

Again, in the $\triangle$'s ABE and CBE

AB = BC.	Def. of a square.
BE = BE.	Common.
$\angle ABE = \angle CBE.$	Proved, deduction 3.

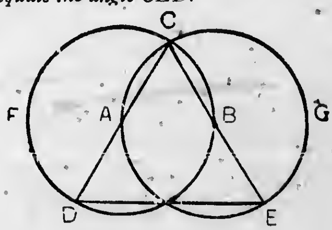
$\therefore$ AE = EC. Prop. 4.

Similarly we may show DE = EB.

$\therefore$ AE = EC = DE = EB. Ax. 7.

That is, AEB, CEB, ADE, and CDE are isosceles Δ's.

7. Two equal circles, whose centres are A and B, intersect at the point C. Join CA and CB, and produce them to meet the circumferences at D and E respectively. Join DE. Prove that the angle CDE equals the angle CED.



Let ACG and BCF be the two equal circles, intersecting at the point C. Let CA and CB be joined and produced to D and E. Let DE be joined.

It is required to prove ∠CDE = ∠CED.

CD is a diameter of the circle BCF.

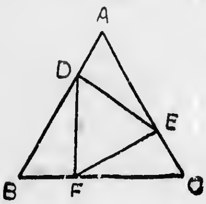
CE is a diameter of the circle ACG.

And since the circles are equal, CD must = CE.

∴ DCE is an isosceles △. Def. of an isosceles △.

And ∠CDE = ∠CED. Prop. 5.

8. ABC is an equilateral triangle; D, E, and F are points in the sides AB, AC, and BC, such that AE = BD = CF. Show that the triangle DEF is equilateral.



Let ABC be the given equilateral △; and let D, E, and F be the given points such that AE = BD = CF. It is required to prove DEF is an equilateral △.

The angles A, B, and C may be shown equal. Prop. 5.

And AD = EC = BF. Ax. 3.

∴ in the △'s ADE and CFE

AD = EC.	Ax. 3.
AE = FC.	Hyp.
∠A = ∠C.	Proved.

∴ DE = FE. Prop. 4.

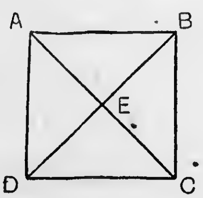
Similarly it may be shown FE = FD.

∴ DF = FE = ED. Ax. 1.

That is, DEF is an equilateral △.

PROPOSITION VI.

1. The diagonals of the square ABCD intersect at E. Use Prop. VI. to prove that the triangle EAB is isosceles.



Let ABCD be the given square, and let diagonals AC and DB intersect at E.

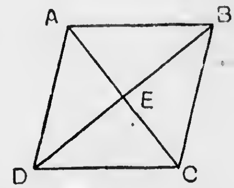
It is required to prove that EAB is an isosceles △.

By deduction 3 in Proposition V., the diagonals AC and DB bisect each of the angles through which they pass.

∴ ∠EAB = half of a rt. ∠.	Proved.
And ∠EBA = half of a rt. ∠.	Proved.
That is, ∠EAB = ∠EBA.	Ax. 1.
∴ AE = BE.	Prop. 6.

Or EAB is an isosceles △.

2. Prove that the diagonals of a rhombus bisect each other at right angles.



Let ABCD be the given rhombus, and let the diagonals AC and DB intersect at E.

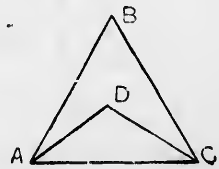
It is required to prove AE = CE, DE = BE, and the ∠'s AEB, AED, CEB, CED all right angles.

In the △'s AEB, AED

AB = AD.	Def. of a rhombus.
AE = AE.	Common.
∠DAE = ∠BAE.	Ded. 3, Prop. 5.
∴ DE = BE.	Prop. 4.
Also ∠AEB = ∠AED.	Prop. 4.
That is, each of them is a rt. ∠.	Def. 7.

Similarly it may be shown by taking the △'s ABE and CBE that AE = CE, and that the ∠'s BEA and BEC are rt. ∠'s.

3. Show that the straight lines which bisect the angles at the base of an isosceles triangle form, with the base, a triangle which is also isosceles.



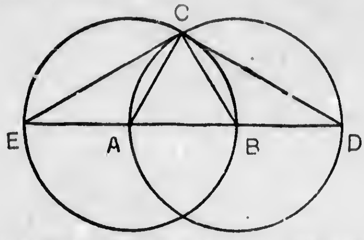
Let ABC be the given isosceles △, and let the lines AD and CD bisect the angles BAC and BCA respectively.

It is required to prove that ADC is an isosceles △.

The ∠BAC = ∠BCA.	Prop. 5.
But ∠DAC = half of BAC.	Hyp.
And ∠DCA = half of BCA.	Hyp.
∴ ∠DAC = ∠DCA.	Ax. 7.
∴ AD = CD.	Prop. 6.

That is, the △ ADC is isosceles.

4. In the figure of Prop. 1, if the straight line AB be produced both ways, to meet the one circumference at D and the other at E, show that the triangle CDE is isosceles.



Let ACD and BCE be the two circles intersecting at C, and let AB produced meet the circumference of the circle ACD in D, and let BA produced meet the circumference of the circle BCE in E, and let CE and CD be joined.

It is required to prove that CE = CD.

BC = AC.	Def. of an equilateral $\triangle$.
$\therefore \angle$ CAB = $\angle$ CBA.	Prop. 5.
Also AE = AB.	Def. of a circle.
And BD = AB.	Def. of a circle,
$\therefore$ AE = BD.	Ax. 1.
And AB = AB.	Common.
$\therefore$ EB = DA.	Ax. 2.

In the $\triangle$'s EBC and DAC

EB = DA.	Proved.
BC = AC.	Proved.
$\angle$ EBC = $\angle$ DAC.	Proved.
$\therefore$ CE = CD.	Prop. 4.

That is, CDE is an isosceles $\triangle$.

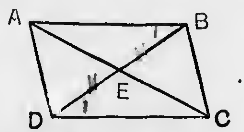
PROPOSITION VIII.

1. The opposite sides of a quadrilateral ABCD are equal. Prove that:

(a) The opposite angles are equal.

(b) The angle ABD is equal to the angle CDB.

(c) The middle point of BD is equidistant from A and C.



Let ABCD be the given quadrilateral, having AB = DC and AD = BC, and let E be the middle point of DB.

It is required to prove:

(*a*) The $\angle$ABC = $\angle$ADC and $\angle$DAB = $\angle$DCB.

(*b*) The $\angle$ABD = $\angle$CDB.

(*c*) That AE = CE.

In the $\triangle$'s ABD and CDB

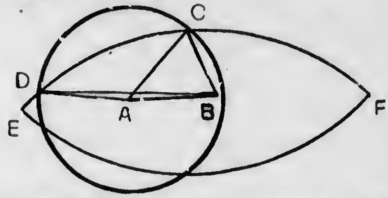
AB = DC.	Hyp.
AD = BC.	Hyp.
DB = DB.	Common.
$\therefore \angle$DAB = $\angle$DCB.	Prop. 8.
And $\angle$ABD = $\angle$CDB. (*b*)	Prop. 8.
And $\angle$CBD = $\angle$ADB.	Prop. 8.
$\therefore \angle$ABC = $\angle$ADC.	Ax. 2.

That is, the opposite $\angle$'s are equal. (*a*).

Again, in $\triangle$'s ABE and CDE

AB = CD.	Hyp.
BE = DE.	Hyp.
$\angle$ABE = $\angle$CDE.	Proved.
$\therefore$ AE = CE. (*c*).	Prop. 4.

2. Show that two circumferences can cut each other in only one point on the same side of the line joining their centres.

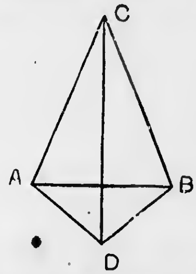


If possible, let the two circumferences CDE and CDF cut each other in two points C and D above the straight line AB which joins their centres.

Join AC, AD and BC, BD.	Post. 1.
Then AC = AD.	Def. of a circle.
And BC = BD.	Def. of a circle.
Which is impossible.	Prop. 7.

Therefore two circumferences cannot cut each other in more points than one above the line joining their centres.

3. Two isosceles triangles are on the same base, and on opposite sides of the base. Prove that the line joining their vertices bisects each of the vertical angles.



Let ACB and ADB be the two isosceles $\triangle$'s on the opposite sides of the base AB, and let CD be joined. It is required to prove that CD bisects the $\angle$ACB and the $\angle$ADB.

In the $\triangle$'s ACD and BCD

AC = BC.	Hyp.
AD = DB.	Hyp.
CD = CD.	Common.
$\therefore \angle$ACD = $\angle$BCD.	Prop. 8.
And $\angle$ADC = $\angle$BDC	Prop. 8.

That is, the $\angle$'s ACB and ADB are bisected.

4. ABC is an isosceles triangle, of which AB and AC are equal sides. Points D and E are taken in AB, and points F and G in AC, such that AD = AF, and AE = AG. CD and BF intersect in H; CE and BG intersect in K.

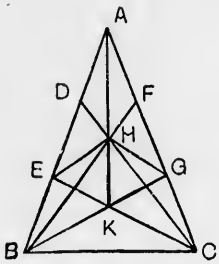
Prove that:

(*a*) *AH bisects $\angle$DAF.*

(*b*) *$\angle$BDH = $\angle$CFH.*

(*c*) *EH = HG.*

(*d*) *A, H and K are in the same straight line.*



Let ABC be the given isosceles △, having AB =AC, and let AD=AF and AE=AG, and let CD and BF be joined, intersecting in H.

(*a*)

It is required to prove that AH bisects the ∠DAF.

In the △'s BAF and CAD

AB=AC.	Hyp.
AF=AD.	Hyp.
∠BAF=∠CAD.	Common.
∴BF=CD.	Prop. 4.
And ∠ABF=∠ACD.	Prop. 4.
But ∠ABC=∠ACB.	Prop. 5.
∴∠HBC=∠HCB.	Ax. 3.
∴ HB= HC.	Prop. 6.
∴ FH= DH.	Ax. 3.

Then in the △'s ADH and AFH

AD=AF.	Hyp.
AH=AH.	Common.
DH=FH.	Proved.
∴∠DAH=∠FAH.	Prop. 8.

(*b*)

It is required to prove ∠BDH=∠CFH.

AB=AC.	Hyp.
AD=AF.	Hyp.
∴DB=FC.	Ax. 3.

Then in the △'s DBH and FCH

DB=FC.	Proved.
DH=FH.	Proved.
HB=HC.	Proved.
∴∠BDH=∠CFH.	Prop. 8.

(*c*)

It is required to prove EH=HG.

Join EH and GH.	Post. 1.
AE=AG.	Hyp.
AD=AF.	Hyp.
∴DE=FG.	Ax. 3.

Then in the △'s DEH and FGH

DE=FG.	Ax. 3.
DH=FH	Proved.
∠EDH=∠GFH.	Proved.
∴EH=HG.	Prop. 4.

(*d*)

It is required to prove A, H and K are in the same straight line.

In the △'s BAK and CAK

AB=AC.	Hyp.
AK=AK.	Common.
∠BAK=∠CAK.	Proved.
∴BK=CK.	Prop. 4.

Therefore in △'s BHK and CHK

BH=CH.	Proved.
HK=HK.	Common.
BK=CK.	Proved.
∴∠BHK=∠CHK.	Prop. 8.

And in △'s BDH and CFH

BD=CF.	Proved.
DH=FH.	Proved.
BH=CH.	Proved.
∴∠DHB=∠FHC.	Prop. 8.

Again, in △'s AHD and AHF

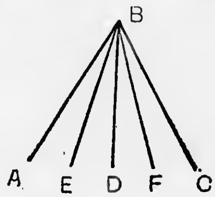
AD=AF.	Hyp.
AH=AH.	Common.
DH=FH.	Proved.
∴∠AHD=∠AHF.	Prop. 8.

∴ ∠'s AHD, DHB and BHK=∠'s AHF, FHC and CHK. Ax. 2.

That is, ∠'s AHD, DHB, and BHK are half of four right angles, or two right angles, and AHK is a straight line.

PROPOSITIONS IX. AND X.

1. To divide a given angle into four equal parts.



Let ABC be the given ∠.

It is required to divide it into four equal parts.

Bisect ∠ABC by BD.	Prop. 9.
" ∠ABD by BE.	Prop. 9.
" ∠DBC by BF.	Prop. 9.

Then ∠'s ABE, EBD, DBF, and FBC are all equal. Ax. 7.

∴the ∠ABC has been divided into four equal parts.

2. To divide a given straight line into eight equal parts.

Let AB be the given straight line.

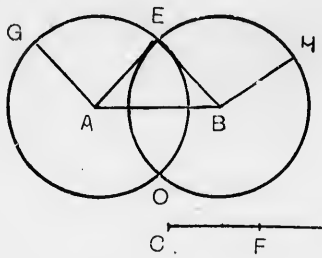
It is required to divide it into eight equal parts.

Bisect AB at C.	Prop. 10.
" AC " D.	Prop. 10.
" CB " E.	Prop. 10.
" AD " F.	Prop. 10.
" DC " G.	Prop. 10.
" CE " H.	Prop. 10.
" EB " I.	Prop. 10.

Then AF, FD, DG, GC, CH, HE, EI and IB are all equal. Ax. 7.

Then the straight line AB has been divided into eight equal parts.

3. On a given base describe an isosceles triangle, such that the sum of its equal sides shall be equal to a given straight line.

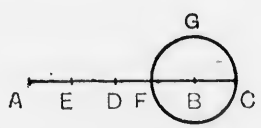


Let AB be the given base, and CD the given straight line.

It is required to describe on AB an isosceles △, the sum of whose sides shall = CD.

Bisect CD at F. — Prop. 10.
Then CF = FD. — Ax. 7.
From A draw AG = CF. — Prop. 2.
" B " BH = FD. — Prop. 2.
With centre A and radius AG describe the circle GEO. — Post 3.
With centre B and radius BH describe the circle HEO. — Post. 3.
Let the circles cut in the point E.
Join EA and EB. — Post. 1.
EAB is the △ required.
GA = CF. — Const.
GA = AE. — Def. of a circle.
∴ AE = CF. — Ax. 1.
BH = FD. — Const.
BH = BE. — Def. of a circle.
∴ BE = FD. — Ax. 1.
But CF and FD = CD.
And CF " FD = GA and BH. — Const.
∴ GA and BH = CD. — Ax. 1.
But GA and BH = EA and EB. — Const.
∴ EA and EB = CD. — Ax. 1.
But CF = FD. — Ax. 7.
∴ GA and BH are equal.
And AE and EB are equal. — Ax. 1.
∴ AEB is isosceles. — Def. of isosceles △.
And AE and BE are equal to CD. — Proved.

4. Produce a given straight line so that the whole line may be five times as long as the part produced.

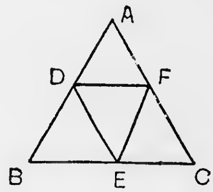


Let AB be the given straight line.

It is required to produce it so that the part produced shall be one-fifth of the whole line thus produced.

Bisect AB at D. — Prop. 10.
Then DB is ½ of AB.
Bisect DB at F. — Prop. 10.
Then FB is ¼ of AB.
Bisect AD at B, then AE is ¼ of AB.
With centre B and distance BF describe a circle FGC. — Post. 3.
Produce AB to meet the circumference in C. — Post. 2.
AC is the line required.
AB is divided into four equal parts: AE, ED, DF, and FB.
But BC = FB.
∴ BC is = to ¼ of AB.
∴ BC is ⅕ of AC.
But BC is the part produced.
∴ AC is the line required.

5. D, E and F are the middle points of the sides of an equilateral triangle; show that the triangle DEF is equilateral.



Let ABC be the equilateral triangle and D,E,F the middle points in the sides AB, BC, CA. And let DE, EF, and FD be joined.

It is required to prove DEF equilateral.

In the △'s DAF and FCE

$$\begin{cases} AF = CF. & \text{Ax. 7.} \\ AD = CE. & \text{Ax. 7.} \\ \angle DAF = \angle ECF. & \angle\text{'s of equil. } \triangle. \end{cases}$$

∴ DF = EF. — I. 4.

In the △'s DBE and CEF

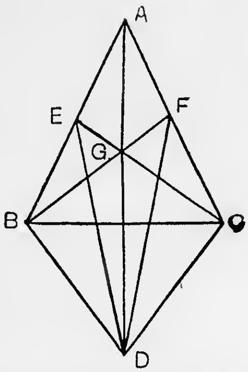
$$\begin{cases} BD = CF. & \text{Ax. 7.} \\ BE = CE. & \text{Ax. 7.} \\ \angle DBE = \angle FGE. & \angle\text{'s of equil. } \triangle \end{cases}$$

∴ DE = EF. — I. 4.
∴ DE, EF and FD are all equal. — Ax. 1.
∴ DEF is an equilateral triangle.

6. Two isosceles triangles ABC and DBC stand on the same base BC but on opposite sides of it. E is the middle point of AB, and F the middle point of AC, and BF and CE intersect in G.

(*a*) Prove that DE = DF.
(*b*) Prove that ∠EDB = ∠FDC.
(*c*) Prove that CE = BF.
(*d*) Prove that ∠DEF = ∠DCE.
(*e*) Prove that △GBC is isosceles.
(*f*) Prove that EG = GF.
(*g*) Prove that ∠EGD = ∠FGD.

(*h*) Prove that A, G and D are in the same straight line.



Let ABC and DBC be two isosceles △'s on the same base BC but on opposite sides of it, having E the middle point of AB and F the middle point of AC. Let BF and CE be joined and intersect at G.

(*a*)

It is required to prove DE = DF.

Join DE and DF.	Post. 1.
∠ABC = ∠ACB.	Prop. 5.
∠DBC = ∠DCB.	Prop. 5.
∴ ∠ABD = ∠ACD.	Ax. 2.
In △'s DBE and DCF	
{ DB = DC.	Def. of an isos. △.
BE = CF.	Ax. 7.
∠EBD = ∠FCD.	Proved.
∴ DE = DF.	Prop. 4.

(*b*)

It is required to prove ∠EDB = ∠FDC.

In △'s DBE and DCF	
{ DB = DC.	Def. of an isos. △.
BE = CF.	Ax. 7.
∠EBD = ∠FCD.	Proved.
∴ ∠EDB = ∠FDC.	Prop. 4.

(*c*)

It is required to prove CE = BF.

In △'s BAF and CAE	
{ BA = CA.	Def. of an isos. △.
AF = AE.	Ax. 7.
∠BAF = ∠CAF.	Common.
∴ CE = BF.	Prop. 4.

(*d*)

It is required to prove ∠DBF = ∠DCE.

In △'s DBF and DCE	
{ DB = DC.	Def. of an isos. △.
BF = CE.	Proved in (*c*).
DF = DE.	Proved in (*a*).
∴ ∠DBF = ∠DCE.	Prop. 8.

(*e*)

It is required to prove that the △ GBC is isosceles.

In △'s BFC and CEB	
{ BF = CE.	Proved in (*c*).
FC = EB.	Ax. 7.
BC = BC.	Common.
∴ ∠FBC = ∠ECB.	Prop. 8.
∴ BG = CG.	Prop. 6.

That is, GBC is an isosceles △.

(*f*)

It is required to prove EG = GF.

BF = CE.	Proved in (*c*).
BG = CG.	Proved in (*e*).
∴ GF = GE.	Ax. 3.

(*g*)

It is required to prove that the ∠EGD = ∠FGD.

Join DG.	Post 1.
In △'s GED and GFD	
{ GE = GF.	Proved in (*f*).
DG = DG.	Common.
ED = FD.	Proved in (*a*).
∴ ∠EGD = ∠FGD.	Prop. 8.

(*h*)

Prove that A, G and D are in the same straight line.

Join AG.	Post 1.
In △'s AEG and AFG	
{ AE = AF.	Ax. 7.
EG = FG.	Proved in (*f*).
AG = AG.	Common.
∴ ∠EGA = ∠FGA.	Prop. 8.
∠EGD = ∠FGD.	Proved in (*g*).
∴ ∠'s EGA and EGD = ∠'s FGA and FGD.	Ax. 2.

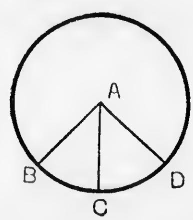
∴ ∠'s EGA and EGD = ½ of the four angles at the point G.

Or ∠'s EGA and EGD = 2 straight angles.

That is, A, G and D are in the same straight line.

PROPOSITION XI.

1. In what line do all points lie which are equally distant from a given point?



Let A be the given point and B, C, and D three points equally distant from A.

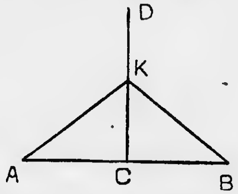
It is required to find the line in which all points would lie which are at the same distance from A as B, C, and D.

AB, AC and AD are all equal. Hyp.

Therefore AB, AC, and AD must be radii of the same circle whose centre is A. Def. of a circle.

Therefore all points which are at the same distance as B, C and D must be on the circumference of the circle BCD.

2. Find a line in which all points lie which are equidistant from two given points.



Let A and B be the two given points.

It is required to find a line, all points in which are equally distant from A and B.

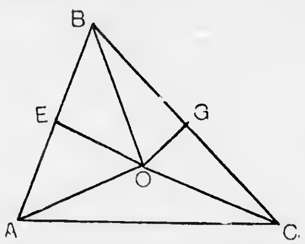
Join AB. — Post. 1.
Bisect AB in C. — Prop. 10.
Draw CD $\perp$ to AB. — Prop. 11.
Then CD is the required line.
In CD take any point K.
Join AK and BK. — Post 1.
Then in the $\triangle$'s ACK and BCK

$$\begin{cases} AC=BC. & \text{Construction.}\\ CK=CK. & \text{Common}\\ \angle ACK=\angle BCK & \text{Ax. 1.} \end{cases}$$

$\therefore AK=BK.$ — Prop. 4.

Similarly it may be shown that any other point in CD is equally distant from A and B.

3. Find, if possible, a point which is equidistant from three given points.



Let A, B and C be the three given points.

It is required to find a point equidistant from A, B and C.

Join AB, BC, AC — Post. 1.
Bisect AB in E. — Prop. 10.
Bisect BC in G. — Prop. 10.
Erect EO $\perp$ to AB. — Prop. 11.
Erect GO $\perp$ to BC. — Prop. 11.
O shall be the required point.
Join AO, BO, CO. — Post. 1.

In the $\triangle$'s AEO and BEO

$$\begin{cases} AE=BE. & \text{Const.}\\ EO=EO. & \text{Common.}\\ \angle OEA=\angle OEB. & \text{Def. }\perp. \end{cases}$$

$\therefore AO=OB.$ — Prop. 4.

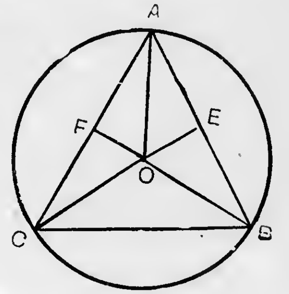
Again, in the $\triangle$'s BOG and COG

$$\begin{cases} CG=BG. & \text{Const.}\\ GO=GO. & \text{Common.}\\ \angle OGC=\angle OGB. & \text{Def. of a }\perp. \end{cases}$$

$\therefore OB=OC$ — Prop. 4.
$\therefore OA=OC.$ — Ax. 1.

That is, O is equidistant for A, B and C.

4. Give a construction for finding the centre of a circle which passes through the three angular points of a triangle.



Let [illegible] and C be the three angular points of $\triangle$ABC.

It is required to give a construction to find the centre of the circle which will pass through A, B and C.

Bisect AB in E and AC in F. — Prop. 1.
Erect EO $\perp$ to AB and FO to AC. — Prop. 11.
O is the centre required.
Join AO, BO and CO. — Post. 1.

In $\triangle$'s AEO and BEO

$$\begin{cases} AE=BE. & \text{Const.}\\ EO=EO. & \text{Common.}\\ \angle AEO=\angle BEO. & \text{Def. of a }\perp. \end{cases}$$

$\therefore AO=BO.$ — Prop. 4.

Again, in the $\triangle$'s AFO and CFO

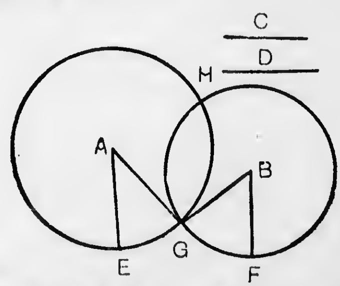
$$\begin{cases} AF=CF. & \text{Const.}\\ FO=FO. & \text{Common.}\\ \angle AFO=\angle CFO. & \text{Def. of a }\perp. \end{cases}$$

$\therefore AO=OC.$ — Prop. 4.
$\therefore BO=OC.$ — Ax. 1.

With centre O and distance OB describe a circle ABC. — Post. 3.

The circumference will pass through A, B, C. — Def. of a circle.

5. Find a point whose distance from a given point A is equal to a given straight line, and whose distance from a given point B is equal to another straight line.



Let C and D be the two given straight lines.

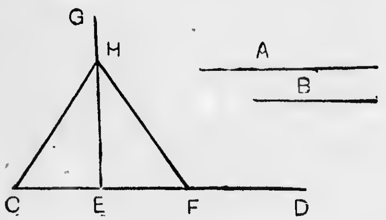
It is required to find a point whose distance from a given point A is equal to a given straight line D, and whose distance from a given point B is equal to another straight line C.

From A draw AE = D. Prop. 2.
From B draw BF = C. Prop. 2.
With centre A and distance AE describe a circle EGH. Post. 3.
With centre B and distance BF describe a circle FGH. Post. 3.
Let the circles intersect at G and H.
Join AG and BG. Post. 1.
Then G shall be the required point.
Because A is centre of circle EGH
AE = AG. Def. of circle.
But AE = D. Const.
∴ AG = D. Ax. 1.
Because B is centre of circle FGH
BF = BG. Def. of circle.
But BF = C. Const.
∴ BG = C. Ax. 1.

This is not possible when the distance between the two points is greater than the sum of the given straight lines.

PROPOSITION XII.

1. Describe an isosceles triangle, having given the base and the length of the perpendicular drawn from the vertex to the base.

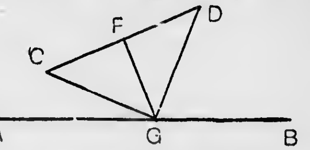


Let A be the given base and B the given perpendicular.
It is required to describe an isosceles triangle having its base equal to A and perpendicular equal to B.
Take any straight line CD limited at C unlimited towards D.
From CD cut off CF = A. Prop. 3.
Bisect CF in E. Prop. 10.
From E erect a perpendicular EG. Prop. 11.
From EG cut off EH = B. Prop. 3.
Join CH and FH. Post. 1.
Then shall HCF be the required triangle.
In the △'s HCE and HFE

CE = FE.	Const.
EH = EH.	Common.
∠ HEC = HEF.	Ax. 11.

∴ CH = FH.
Therefore HCF is an isosceles △ having its base CF = to the given straight line A, and its perpendicular height EH = to the given line B. Def. of isos △.

2. In a given straight line find a point that is equally distant from two given points.

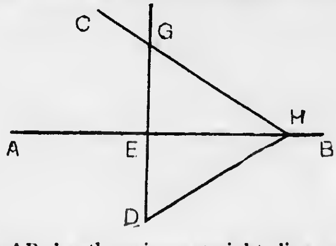


Let AB be the given straight line and C and D the two given points.
It is required to find a point in AB that will be equally distant from C and D.
Join CD. Post 1.
Bisect CD in F. Prop. 10.
From F draw a perpendicular to CD, which meets AB in G. Prop. 11.
Then G shall be the point required.
Join CG and DG. Post. 1.
In △'s CFG and DFG

CF = DF.	Const.
FG = FG.	Common.
∠ CFG = ∠ DFG.	Ax. 11.

∴ CG = DG. Prop. 4.
Therefore a point G has been found in the straight line AB, which is equally distant from C and D.
Note: When the given points are on opposite sides of the given straight line and not equally distant from it, and when the straight line joining them cuts the given straight line at right angles, the proposition is impossible.

3. From two given points on opposite sides of a given straight line, draw straight lines to a point in the given line, making equal angles with it.



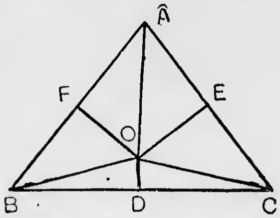
Let AB be the given straight line and C and D the given points on opposite sides of AB.
It is required to draw from C and D straight lines to a point in AB, making equal angles with it.
From the point D draw DE perpendicular to AB. Prop. 12.
Produce DE to F. Post. 2.
From EF cut EG = ED Prop. 3.
Join CG. Post. 1.
Produce CG to meet AB in H. Post. 2.
Then H shall be the required point.
Join DH. Post. 1.
Then in △'s GEH and DEH

EG = ED.	Const.
EH = EH.	Common.
∠ GEH = ∠ DEH.	Ax. 11.

∴ ∠ GHE = ∠ DHE.

Therefore a point H has been found in the straight line AB, such that CH and DH make = ∠'s with AB at that point. Prop. 4

4. ABC is a triangle and D, E and F are the middle points of the sides BC, CA, and AB respectively. From D a straight line is drawn perpendicular to BC, and from E another straight line is drawn perpendicular to CA, meeting the former line in O. Show that OF is perpendicular to AB.



Let ABC be the given triangle, and D, E and F the middle points in BC, CA and AB respectively, and let a line be drawn from D perpendicular to BC, and from E a line perpendicular to CA, and let these lines meet in O.

It is required to prove that OF is perpendicular to AB.

Join AO, BO, CO. Post 1.

Then in △'s ODB, ODC

BD = CD. Hyp.
DO = DO. Common.
∠ODB = ∠ODC. Ax. 11.

∴ BO = CO. Prop. 4.

In △'s OCE and OAE.

AE = CE. Hyp.
OE = OE. Common.
∠AEO = ∠CEO. Ax. 11.

∴ AO = CO Prop. 4.
But since BO = CO. Proved.
And AO = CO Proved.
Then BO = AO. Ax. 1.

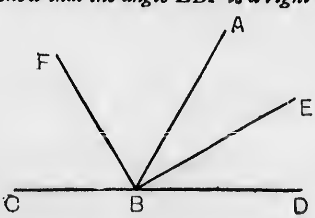
Then in △'s AOF, BOF

AO = BO. Proved.
OF = OF. Common.
AF = BF. Hyp.

∴ ∠OFB = ∠OFA. Prop. 8.
Therefore OF is ⊥ to AB.

PROPOSITION XIII.

1. The angles ABC and ABD, which are made by the straight line AB standing on the straight line CD, are bisected by the straight lines BE and BF. Show that the angle EBF is a right angle.

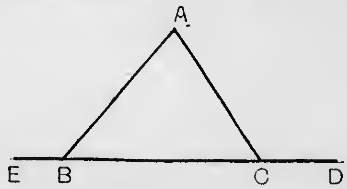


Let ABC and ABD be two angles made by AB standing on straight line CD, and let them be bisected by BF and BE.

It is required to prove ∠FBE is a right angle.

The ∠ABE = ∠EBD. Hyp.
And ∠ABF = ∠FBC. Hyp.
Therefore ∠EBA + ∠ABF = ∠DBE + ∠CBF.
That is, the ∠EBF is half of ∠DBE + ∠EBA + ∠ABF + ∠FBC.
But ∠DBE + ∠EBA + ∠ABF + ∠FBC = 2 rt. ∠'s.
Therefore ∠EBF = ½ of 2 rt. ∠'s.
That is, ∠EBF = one rt. ∠.

2. If two exterior angles of a triangle, made by producing a side both ways, are equal, show that the triangle is isosceles.

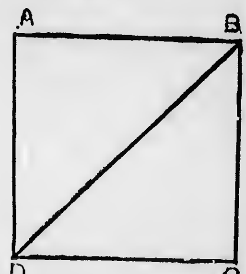


Let ABC be a triangle, having BC produced to D and E, and having the exterior ∠ACD equal the exterior ∠ABE.

It is required to prove that △ABC is isosceles.

The ∠DCA + ∠BCA = 2 rt. ∠'s. Prop. 13.
The ∠CBA + ∠EBA = 2 rt. ∠'s. Prop. 13.
Therefore ∠DCA + ∠BCA = ∠CBA + ∠EBA. Ax. 1.
But ∠ACD = ∠ABE. Hyp.
Therefore ∠ACB = ∠ABC. Ax. 3.
Therefore △ABC is isosceles. I. 6.

3. Show that the angles of a triangle formed by a diagonal and two sides of a square are together equal to two right angles.



Let AB and AD be two sides of the given square ABCD, and let DB be the diagonal. It is required to show that ∠DAB + ∠ADB + ∠ABD = two rt. ∠'s.

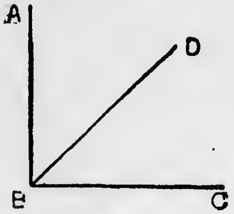
In the △'s DAB and DCB

AB = BC Def. of a square.
AD = DC " " "
∠DAB = ∠DCB Ax. 11.

Therefore ∠ABD = ∠CBD, and ∠ADB = ∠CDB.

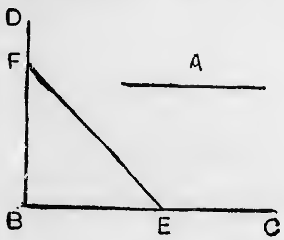
But ∠ ABC is a rt. angle. Def. of a square.
∴ ∠ABD is $\frac{1}{2}$ of a rt. angle.
∠ADC is a rt. angle. Def. of a square.
∴ ∠ADB is $\frac{1}{2}$ of a rt. angle.
∴ ∠ABD + ∠ADB = 1 rt. angle.
And ∠ DAB + ∠'s ABD and ADB = 2 rt. angles.

4. Construct an angle equal to half a rt. angle.



Take a straight line BC.
From B draw a perpendicular BA. Prop. 11.
Bisect ∠ ABC by the line BD. Prop. 9.
∠ABD is the required angle.
∠ABC is a rt. angle. Def. 1.
And ∠ABD = ∠DBC. Const.
Therefore ∠ABD = $\frac{1}{2}$ of a rt. angle.

5. Make an isosceles triangle having each of its base angles equal to half a right angle, and each of the equal sides equal to a given straight line.

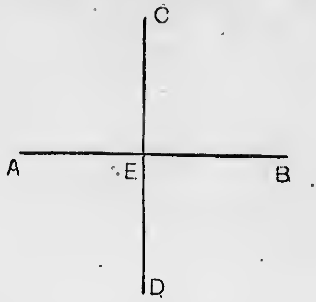


Let A be the given straight line.
It is required to make an isosceles triangle having each of its base angles equal to half a right angle, and each of the equal sides equal to A.
Take any straight line BC terminated at B, but unlimited toward C.
From B draw a ⊥ BD to BC. Prop. 11.
From BC cut off BE = A. Prop. 3.
From BD cut off BF = A. Prop. 3.
Join FE.
Then EBF shall be the triangle required.
In △ EBF

∠FBE = 1 rt. ∠. Def. 10.
∠BFE = $\frac{1}{2}$ rt. ∠. Ded. 3, above.
∠BEF = $\frac{1}{2}$ rt. ∠. Ded. 3, above.
BF = A. Const.
BE = A. Const.

Therefore FBE is an isosceles triangle, having each of its base ∠'s equal to $\frac{1}{2}$ rt. ∠ and each of its equal sides equal to the straight line A.

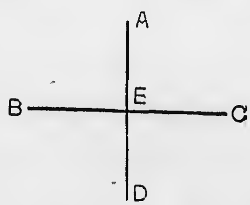
6. If one of the four angles which two intersecting straight lines make with each other be a right angle, all the other angles are right angles.



Let AB and CD be two intersecting straight lines which cut at E, making the ∠ AEC = 1 rt. angle.
It is required to prove ∠ CFB = 1 rt. angle, ∠ BED = 1 rt. angle ∠ DEA = 1 rt. angle.
∠AEC + ∠BEC = 2 rt. angles. Prop. 13.
But ∠AEC = 1 rt. angle. Hyp.
∴ ∠CEB = 1 rt. angle. Ax. 3.
∠CEB + ∠BED = 2 rt. angles. Prop. 13.
But ∠CEB = 1 rt. angle. Proved.
∴ ∠BED = 1 rt. angle. Ax. 3.
∠'s BED + AED = 2 rt. angles. Prop. 13.
But ∠BED = 1 rt. angle. Proved.
∴ ∠AED = 1 rt. angle. Ax. 3.
That is, ∠CEB = 1 rt. angle, BED = 1 rt. angle, ∠AED = 1 right angle.

PROPOSITION XV.

1. If four straight lines meet at a point so that the opposite angles are equal, these straight lines are two and two in the same straight line.



Let AE, BE, CE, DE be four straight lines meeting at E, making ∠AEC = ∠BED, and ∠AEB = ∠CED.
It is required to prove AE and DE, also BE and CE in the same straight line.
∠AEC + ∠CED + ∠DEB + ∠BEA = 4 right angles. Cor. 2, Prop. 13.

But ∠AEC = ∠BED. Hyp.
∠AEB = ∠CED. Hyp.
Therefore ∠AEB + ∠AEC = ∠BED + ∠CED. Ax. 2.

Then ∠AEB + ∠AEC is half of ∠AEC + ∠CED + ∠DEB + ∠BEA

Therefore ∠AEB + ∠AEC = ½ of 4 rt. angles = 2 rt. angles.

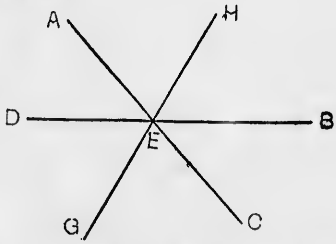
That is, BE and CE are in the same straight line. I. 14.

Similarly it may be shown AE and DE are in the same straight line.

2. In this theorem the angles are given to prove the lines are in the same straight line, while in Prop. 15 the straight lines are given to prove the angles.

Therefore they are converse propositions.

3. Show that the bisectors of either pair of vertically opposite angles in Prop. 15 are in the same straight line.



Let AC and BD be two straight lines intersecting at E, and let EH and EG be the bisectors of ∠AEB and ∠DEC.

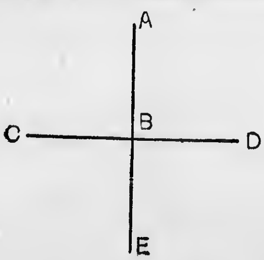
It is required to prove EH and GE are in the same straight line.

The ∠'s AEH, HEB, BEC, CEG, GED, DEA, = 4 rt. angles. Cor. 2, Prop. 14.
But ∠AED = ∠BEC. I. 15.
∠DEG = ∠CEG. Const.
∠AEH = ∠BEH. Const.
Therefore ∠AED + ∠DEG + ∠AEH.
= ∠BEC + ∠CEG + ∠BEH.

Therefore ∠AED + ∠DEG + ∠AEH = ½ of 4 rt. angles = 2 rt. angles.

That is GE and HE are in the same straight line. I. 14.

4. Show that if AB is perpendicular to the straight line CD, which it meets at B, then if AB is produced to E, BE is also perpendicular to CD.

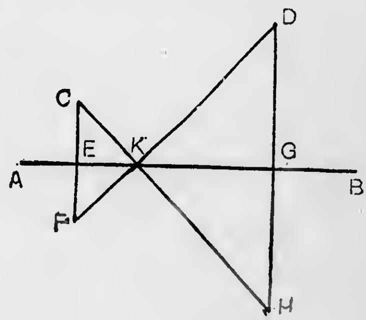


Let AB be a straight line ⊥ to CD and meeting CD in B.

It is required to prove that if AB be produced to E, that EB is ⊥ CD.

∠ABC = ∠ABD. Def. of ⊥.
Then ∠ABC and ∠ABD are each a rt. ∠. Def. of ⊥.
∠ABD = ∠CBE. Prop. 15.
Therefore ∠CBE is a rt. ∠. Ax. 1.
∠ABC = ∠DBE. Prop. 15
Therefore ∠DBE is a rt. ∠. Ax. 1.
Therefore BE is ⊥ to CD. Def. of ⊥.

5. From two given points on the same side of a given straight line, show how to draw two straight lines which meet at a point in the given straight line and make equal angles with it.



Let AB be the given straight line and C and D be two given points on the same side of AB.

It is required to find a point in AB, such that two straight lines drawn from C and D to this point, will make equal angles with AB.

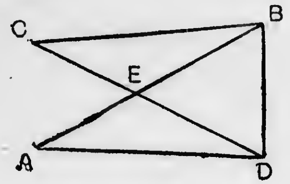
From C draw CE ⊥ to AB. Prop. 12.
Produce CE to F, making EF = CE. Post. 2, and Prop. 3.
From D draw DG ⊥ to AB. Prop. 12.
Produce DG to H, making GH = DG. Post. 2, and Prop. 3.
Join CH. Post. 1.
Let CH cut AB in K.
Then K shall be the point required
Join FK and DK. Post. 1.
Then in the △'s DGK and HGK.

$$\begin{cases} DG = HG. & \text{Cons.} \\ GK = GK. & \text{Common.} \\ \angle DGK = \angle HGK. & \text{Ax. 11.} \end{cases}$$

Therefore ∠DKG = ∠HKG. Prop. 4.
But ∠CKE = ∠HKG. Prop. 15.
Therefore ∠CKE = ∠DKG. Ax. 1.

Wherefore a point K has been found in AB such that straight lines drawn from C and D to this point make equal angles with AB.

6. In the figure of proposition 15, make EB equal to ED, and EC equal to EA, and join AD, DB and BC. Then prove the angle AED equal to the angle CEB, without assuming any proposition after proposition 5.

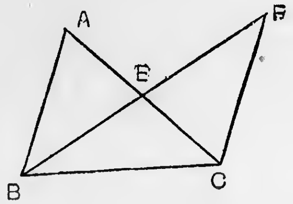


In the figure of proposition 15 let EB=ED and CE=EA, and let CB, BD and AD be joined.

It is required to prove ∠CEB=∠AED.

Because EB=ED,
therefore ∠EBD=∠EDB. Prop. 5.
EB=ED and EA=EC. Hyp.
∴ EB+EA=ED+EC. Ax. 2.
That is, AB=CD. Ax. 2.
In the △'s CBD and ABD.
{AB=CD. Proved.
BD=BD. Common.
∠ABD=∠CDB. Proved.
Therefore CB=AD
and ∠DCB=∠DAB. Prop. 4.
Again in the △'s CEB and AED.
{CE=AE. Hyp.
CB=AD. Proved.
∠ECB=∠EAD. Proved.
Therefore ∠CEB=∠AED. Prop. 4.

7. The side AC of the triangle ABC is bisected at E, and BE is drawn and produced to F, making EF=EB.



Show that:
(*a*) *FC=AB.*
(*b*) *∠FCE=∠BAE.*

Let ABC be a triangle having AC bisected in E and having BE produced to F, making EF=BE.

It is required to prove AB=CF, and ∠BAE=∠ECF.

Join CF. Post. 1.
In the △'s AEB and CEF
{AE=EC. Hyp.
BE=EF. Hyp.
∠AEB=∠CEF. Prop. 15.
Therefore AB=CF
and ∠BAE=∠ECF. Prop. 4.

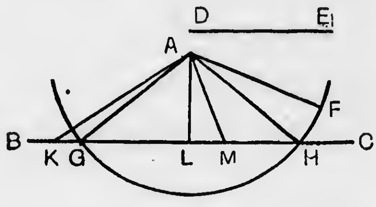
PROPOSITION XVII.

1. A is a given point and BC a given straight line.

(*a*) *Find a point in BC, whose distance from A is equal to the length of another straight line DE.*

(*b*) *Show that two, and not more than two, such straight lines can be drawn.*

(*c*) *Show that only one perpendicular can be drawn from A to BC.*



Let A be the given point, and BC the given straight line.

(*a*) It is required to find a point in BC, whose distance from A is equal to the length of another straight line DE.

(*b*) Show that two, and not more than two, such straight lines can be drawn.

(*c*) Show that only one perpendicular can be drawn from A to BC.

(*a*)

From A draw AF equal to DE. Prop. 2.
And with centre A at the distance AF describe a circle GHF cutting BC in G and H. Post. 3.
Join AG and AH. Post. 1.
Then G or H is the required point.
AF=DE. Const.
AF=AH. Def. of circle.
∴DE=AH. Ax. 1.
Similarly it may be shown DE=AG.

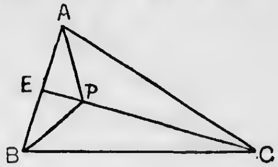
(*b*)

If it be possible let AK be drawn equal to DE.
And since AH equals DE. Proved.
∴AK=AH. Ax. 1.
And ∠AKH=∠AHK. Prop. 5.
But ∠AGH=∠AHG. Prop. 5.
∴∠AKH=∠AGH. Ax. 1.
Which is impossible, since AGH is an exterior angle of the △ AKG. Prop. 16.
∴AK is not equal to AG.
Similarly it may be shown that no lines other than AG and AH can be drawn from A=DE.

(*c*)

From A draw AL perpendicular to BC. Prop. 12.
And if it be possible draw another line AM perpendicular to BC.
Then ∠ALM+∠AML will together equal two right angles.
Which is impossible. Prop. 17.
Therefore from A only one perpendicular can be drawn to the straight line BC.

3. P is any point within the triangle ABC, and PA and PB are joined. Show that the angle APB is greater than the angle ACB.



Let ABC be the given triangle and P the given point within it, and let PA and PB be joined.

It is required to prove the angle APB greater than the angle ACB.

Join CP. Post. 1.

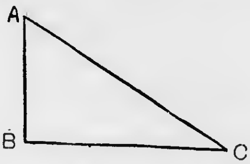
Produce CP meeting AB in E. Post. 2.

Then the angle APE is greater than the angle ACP. Prop. 16.

And the angle BPE is greater than the angle BCP. Prop. 16.

∴The whole angle APB is greater thnn the whole angle ACB.

4. Show that two angles of every triangle must be acute angles.



Let ABC be the given triangle.

It is required to prove any two of its angles are acute.

The angle BAC+ ∠ABC are together less than 2 right ∠'s. Prop. 17.

If the ∠ABC be an obtuse angle or a right angle, then the angle BAC must be acute.

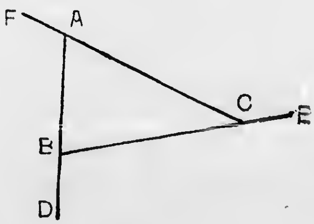
The angles ABC+ACB is together less than 2 right ∠'s. Prop. 17.

If the angle ABC be an obtuse angle or a right angle, then the angle ACB must be acute.

Wherefore two angles of the △ are acute.

5. Show that two exterior angles of every triangle must be obtuse angles.

Of what triangle will the three exterior angles be obtuse?



Let ABC be the given △ having AB, BC, and CA produced to D, E, and F.

It is required to prove that two exterior angles thus formed are obtuse.

By deduction 4, two ∠'s of the △ must be acute.

Let ∠'s ABC and ACB be each acute.

∠ABC + ∠DBC = two right angles. Prop. 13.

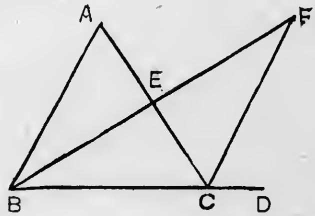
∠ACE + ∠ACB = two right angles. Prop. 13.

But, since the ∠'s ABC and ACB are each acute, the ∠DBC and ∠ACE must each be obtuse.

(*b*)

Each of the exterior angles of an acute-angled triangle would necessarily be obtuse.

6. In the figure of Prop. 16, show that the area of the triangle ABC is equal to the area of the triangle FBC.



Let ABC and BFC be the two triangles.

It is required to prove that the area of triangle ABC is equal to the area of triangle BFC.

In the △'s ABE and FEC

$$\begin{cases} BE=EF. & \text{Cons.} \\ AE=EC. & \text{Cons.} \\ \angle AEB=\angle FEC. & \text{I. 15.} \end{cases}$$

∴ △ABE = △FEC. I. 4.

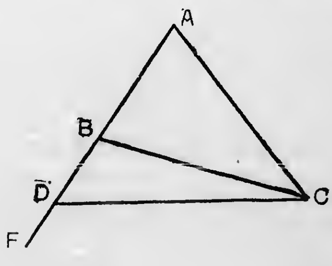
To these equals add the △BEC.

Then △ABE + △BEC = △FEC + △BEC. Ax. 2.

That is, △ABC = △BFC.

PROPOSITION XVIII.

1. Prove Prop. 18 by producing the shorter side.



Let ABC be the triangle, and let the side AC be greater than the side AB.

It is required to prove $\angle$ABC greater than $\angle$ACB.

Produce AB to F. Post. 2.

Cut off AD = AC. Prop. 3

The $\angle$ABC is greater than $\angle$ADC. Prop. 16.

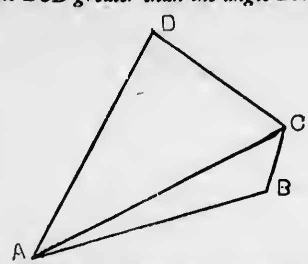
$\angle$ADC = $\angle$ACD. Prop. 5.

Therefore $\angle$ABC is greater than $\angle$ACD.

But $\angle$ACD is greater than $\angle$ACB. Ax. 8

Therefore $\angle$ABC is much greater than $\angle$ACB.

2. ABCD is a quadrilateral of which AD is the longest side, and BC the shortest; show that the angle ABC is greater than the angle ADC, and the angle BCD greater than the angle BAD.



Let ABCD be the quadrilateral, having AD the longest side and BC the shortest.

It is required to prove that $\angle$ABC > $\angle$ADC.

And $\angle$BCD > $\angle$BAD.

Join AC. Post. 1.

In $\triangle$ABC, $\angle$BCA is greater than $\angle$BAC. Prop. 18.

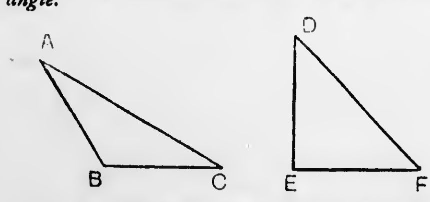
In $\triangle$DAC, $\angle$DCA is greater than $\angle$DAC. Prop. 18.

Therefore $\angle$BCD is greater than $\angle$DAB. Ax. 4.

Similarly, by joining BD it may be shown that $\angle$ABC is greater than $\angle$ADC.

PROPOSITION XIX.

1. In an obtuse-angled triangle the greatest side is opposite the obtuse angle; and in a right-angled triangle the greatest side is opposite the right angle.



(*a*) Let ABC be the obtuse-angled triangle, and let $\angle$ABC be the obtuse angle.

It is required to prove that AC is greater than either AB or BC.

In $\triangle$ABC the $\angle$'s A + B are less than 2 rt. angles. Prop. 17.

But $\angle$B is an obtuse angle. Hyp.

$\therefore$ $\angle$A must be less than a rt. angle.

Similarly we may show the $\angle$C less than a rt. angle.

That is, $\angle$B is greater than either $\angle$A or $\angle$C.

$\therefore$ AC is greater than AB or BC. Prop. 19.

(*b*) Let DEF be the right-angled triangle having angle DEF a rt. angle.

It is required to prove that DF is greater than DE or EF.

In $\triangle$DEF the $\angle$'s D + E are less than two rt. angles. Prop. 17.

But $\angle$E is a rt. angle. Hyp.

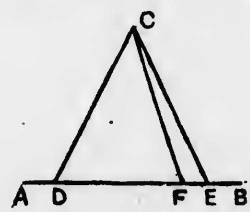
$\therefore$ $\angle$D must be less than a rt. angle.

Similarly we may show that $\angle$F is less than a rt. angle.

That is, $\angle$E is greater than $\angle$D or $\angle$F.

$\therefore$ DF is greater than DE or EF. Prop. 19.

2. Show that three equal straight lines cannot be drawn from a given point to a given straight line.



Let AB be the given straight line and C the given point, and let CD and CE be two equal straight lines drawn from C to AB, and if it be possible let CF be drawn from C to AB equal to CD or CE.

It is required to prove CF is not equal to CD or CE.

In the $\triangle$DCE because CD = CE the $\angle$CDE = $\angle$CED. Prop. 5.

In the $\triangle$CDF because CD = CF the $\angle$CDF = $\angle$CFD. Prop. 5.

Therefore the $\angle$CFD = $\angle$CED. Ax. 1.

But $\angle$CFD is greater than $\angle$CED. Prop. 16.

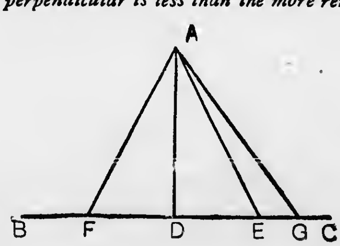
That is, the $\angle$CFD is both equal to and greater than $\angle$CED.

Which is impossible.

Therefore CF is not equal to CD or CE.

Similarly it may be shown that no other straight line can be drawn from C to AB equal to CD or CE.

3. The perpendicular is the shortest straight line that can be drawn from a given point to a given straight line; and of others, that which is nearer to the perpendicular is less than the more remote.



From the given point A let there be drawn to the given straight line BC (1) the perpendicular AD, (2) AE and AF equally distant from the perpendicular, that is, so that DE = DF, (3) AG more remote than AE or AF.

It is required to prove AD the least of these straight lines, and AG greater than AE or AF.

In the triangles ADE and ADF

$$\begin{cases} AD = AD. & \text{Common.} \\ DE = DF. & \text{Hyp.} \\ \angle ADE = \angle ADF. & \text{Ax. 1[illegible]} \end{cases}$$

∴ AE = AF. Prop. 4.

Because ∠ADE is right ∴ AED is acute. Prop. 17.

∴ AE is greater than AD. Prop. 19.

Hence also AF is greater than AD.

Because ∠AEG is greater than ∠ADE. Prop. 16.

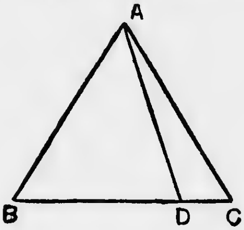
∴ AEG is obtuse.

∴ AGE is acute. Prop. 17.

∴ AG is greater than AE. Prop. 19.

Hence also AG is greater than AF, and than AD.

4. Any straight line drawn from the vertex of an isosceles triangle to a point in the base is less than either of the equal sides.



Let ABC be an isosceles triangle, and let AD be the straight line drawn from the vertex to the base.

It is required to prove AD is less than AB or AC.

The ∠ADC is greater than the ∠ABD. Prop. 16.

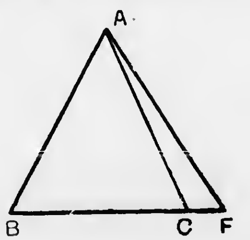
But ∠ABD = ∠ACD. Prop. 5.

Therefore ∠ADC is greater than ∠ACD.

That is, AC is greater than AD. Prop. 19.

Similarly it may be shown that AB is greater than AD.

5. Enunciate and prove a theorem similar to Ex. 4 when the point is taken in the base produced.



Enunciation.—Any straight line drawn from the vertex of an isosceles triangle to a point in the base produced is greater than either of the equal sides.

Let ABC be the given isosceles triangle, having BC produced to F, and let AF be joined.

It is required to prove that AF is greater than AB or AC.

In △ABC the ∠ABC = ∠ACB. Prop. 5.

The ∠ACF is greater than ∠ABC. Prop. 16.

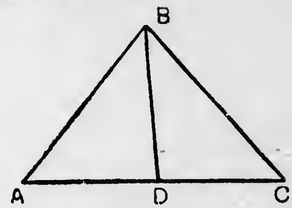
Therefore ∠ACF is greater than ∠ACB.

But ∠ACB is greater than ∠AFC. Prop. 16.

Therefore ∠ACF is much greater than ∠AFC.

Then AF is greater than AC or AB. Prop. 19.

6. The vertical angle ABC of the triangle ABC is bisected by the straight line BD, which meets the base in D. Show that AB is greater than AD, and CB is greater than CD.



Let ABC be the given triangle, having ∠ABC bisected by BD, which meets AC in D. It is required to prove that AB is greater than AD, and that CB is greater than CD.

The ∠ABD = ∠CBD. Const.

And ∠BDA is greater than ∠CBD. Prop. 16.

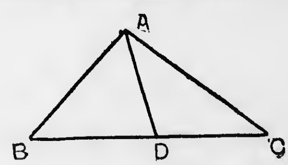
Then ∠BDA is greater than ∠ABD.

Therefore AB is greater than AD. Prop. 19.

Similarly it may be shown that CB is greater than CD.

PROPOSITION XX.

7. Prove Prop. 20 by bisecting the vertical angle by a straight line which meets the base.



Let ABC be the triangle and let ∠BAC be bisected by the straight line AD, which meets BC at D.

It is required to prove that BA + AC is greater than BC.

The ∠ADB is greater than ∠CAD. Prop. 16.

But ∠CAD = ∠BAD. Hyp.

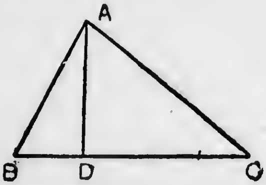
∴ ∠ADB is greater than ∠BAD.

∴ AB is greater than BD. Prop. 19.

∠ADC is greater than ∠DAB: Prop. 16.

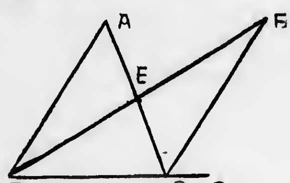
But $\angle DAB = \angle DAC$. Hyp.
$\therefore \angle ADC$ is greater than $\angle DAC$.
$\therefore$ Therefore AC is greater than CD. Prop. 19.
$\therefore AB+AC$ is greater than $BD+DC=BC$.
Ax. 4.

2. Prove Prop. 20 by drawing a perpendicular from the vertex to the base.



Let ABC be the triangle, having AD the perpendicular drawn from A to the base BC.
It is required to prove that $BA+AC$ is greater than BC.
$\angle ADB$ is a right angle. Hyp.
$\therefore \angle DAB$ is an acute angle. Prop. 17.
$\therefore \angle ADB$ is greater than the $\angle DAB$.
$\therefore$ BA is greater than BD. Prop. 19.
$\angle ADC$ is a right angle. Hyp.
$\therefore \angle DAC$ is an acute angle. Prop. 17.
$\therefore \angle ADC$ is greater than the $\angle DAC$.
$\therefore$ AC is greater than DC. Prop. 19.
$\therefore AB+AC$ is greater than $BD+CD=BC$.
Ax. 4.

3. (*a*) *In the figure of Prop. 16 prove that CF is equal to AB.*



Let $EB=EF$ and $AE=EC$.
It is required to prove that $CF=AB$.
In the Δ's ABE and FCE

$$\begin{cases} AE=EC. & \text{Hyp.} \\ BE=EF. & \text{Hyp.} \\ \angle AEB=\angle FEC. & \text{Prop. 15.} \end{cases}$$

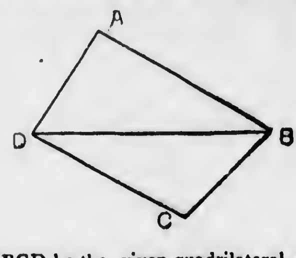
$\therefore CF=AB$. Prop. 4.

3. (*b*) *Hence prove that the sum of any two sides of a triangle is greater than twice the straight line drawn from the middle point of the third side to the opposite vertex.*

Use the same figure as 3 (*a*).

Let $CF=AB$. Proved in (*a*).
It is required to prove $AB+BC$ is greater than twice BE.
$BC+CF$ is greater than BF. Prop. 20.
But $BC+CF=BC+AB$, since $CF=AB$.
And $BF=$ twice BE, since $BE=EF$.
That is, $AB+BC$ is greater than twice BE.

4. (*a*) *Prove that the sum of the sides of any quadrilateral is greater than twice either diagonal.*



Let ABCD be the given quadrilateral and DB and AC its diagonals.
It is required to prove that $AB+BC+CD+DA$ is greater than twice AC or twice DB.
In the ΔADB the sides $AD+AB$ are greater than DB. Prop. 20.
In the ΔCDB the sides $CB+CD$ are greater than DB. Prop. 20.
That is, $AB+AD+BC+DC$ is greater than twice DB.
Similarly it may be shown that $AB+BC+CD+DA$ is greater than twice AC.

4. (*b*) *Hence prove that the sum of the sides is greater than the sum of diagonals.*

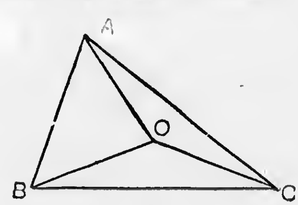
Use same figure as 4 (*a*).

The sides $AB+BC+CD+DA$ are greater than twice DB. Proved.
The sides $AB+BC+CD+DA$ are greater than twice AC. Proved.
That is, twice $(AB+BC+CD+DA)$ is greater than twice $(DB+AC)$.
Therefore $AB+BC+CD+DA$ is greater than $DB+AB$.
That is, the sum of the sides of the quadrilateral is greater than the sum of the diagonals.

5. Take any point O, and join to the angular points of the triangle ABC.

(*a*) *Prove that the sum of OA and OB is greater than AB.*

(*b*) *Prove that twice the sum of OA, OB and OC is greater than the sum of the sides.*



(*a*)

Let O be the given point and ABC the given triangle of which the angular points A, B and C are joined to O.

It is required to prove that the sum of OA and OB is greater than AB.

AO and OB are any two sides of the triangle AOB.

Therefore AO+OB is greater than AB. Prop 20.

(b)

Use same figure as in (*a*).

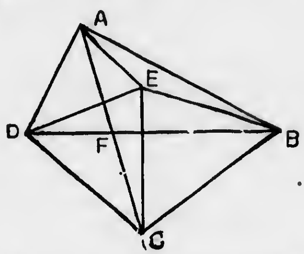
Let O be the given point and ABC the given triangle, of which the angular points A, B and C are joined to O.

It is required to prove that twice the sum of OA, OB and OC is greater than the sum of AB, AC, BC.

AO+OB is greater than AB. Prop. 20.
OB+OC is greater than BC. Prop. 20.
OA+OC is greater than AC. Prop. 20.

That is, twice OA+OB+OC is greater than once AB+BC+CA.

6. *If a point be taken within a quadrilateral and joined to each of the angular points, show that the sum of these joining lines is the least possible, when the point taken is the point of intersection of the diagonals.*



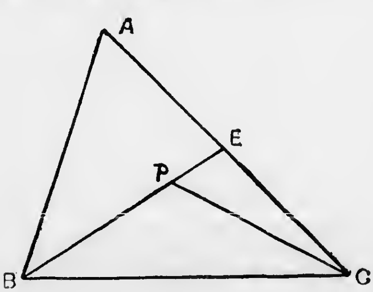
Let ABCD be the quadrilateral, and E any point in it, and F the point of intersection of the diagonals.

It is required to prove that AE, BE, CE and DE are greater than DB and CA.

DE+EB is greater than DB. Prop. 20.
AE+EC is greater than AC. Prop. 20.

Therefore AE+BE+CE+DE is greater than AC and DB.

7. *A point P is taken within the triangle ABC. Show that the sum of the sides AB and AC is greater than the sum of PB and PC.*



Let ABC be the given triangle and P the given point within it, and let BP and CP be joined.

It is required to prove BA+AC is greater than BP+PC.

Produce BP to meet AC in E. Post 2.

Then BA+AE is greater than BE. Prop. 20.

Add EC to each.

Then BA+AC is greater than BE+EC. Ax. 4.

Again, PE+EC is greater than PC. Prop. 20.

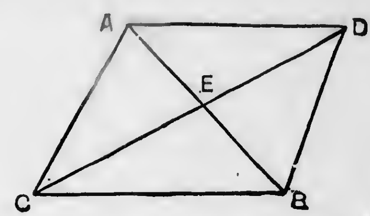
Add PB to each.

Then BE+EC is greater than BP+PC. Ax. 4.

But BA+AC is greater than BE+EC. Proved.

Much more then is BA+AC greater than BP+CP.

8. *Four points lie in a plane, no one of them being within the triangle formed by joining the other three. Find the point the sum of whose distances from these four points is the least possible.*



Since no one of the points is within the triangle, the four points when joined form a quadrilateral.

Let ADBC be the quadrilateral.

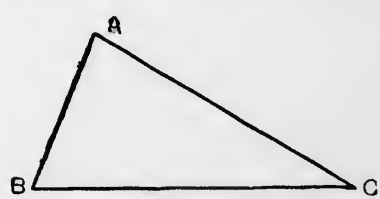
It is required to find a point the sum of whose distances from A, B, C, D is the least possible.

Join AB and CD. Post. 1.

Let E be the point where they intersect.

Then E is the point required. Deduction 6.

9. *In any triangle, the difference between any two sides is less than the third side.*



Let ABC be the given triangle.

It is required to prove BA greater than the difference between BC and AC.

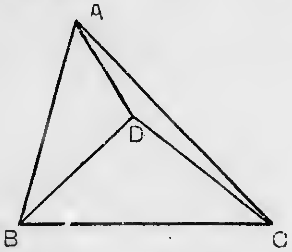
BA+AC is greater than BC. Prop. 20.

Take AC from each side.

Then BA is greater than BC − AC. Ax. 5.

PROPOSITION XXI.

1. In the figure of Prop. 21, join DA, and show that the sum of DA, BD and DC is less than the sum of the sides of the triangle ABC, but greater than half the sum.

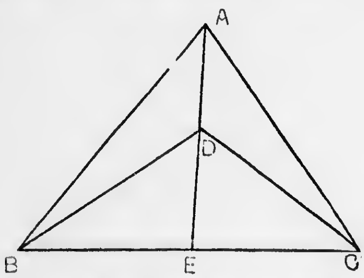


Let ABC be the triangle, and from B and C let BD and CD be drawn to any point D within the triangle, and let AD be joined.

It is required to prove (1) DA+BD+DC less than BA+AC+CB. (2) DA+DB+DC greater than $\frac{BA+AC+CB.}{2}$

(1) AD+DB is less than AC+BC. Prop. 21.
BD+DC is less than BA+AC. Prop. 21.
AD+DC is less than BA+BC. Prop. 21.
That is, 2 (AD+BD+DC) is less than 2 (BA+AC+BC), or AD+BD+DC is less than BA+AC+BC.
(2) AD+DB is greater than AB. Prop. 20.
BD+DC is greater than BC. Prop. 20.
DC+AD is greater than AC. Prop. 20.
That is, 2 (AD+DC+DB) is greater than AB+BC+AC, or AD+DC+BC is greater than $\frac{AB+BC+AC.}{2}$

2. In the figure of Proposition 21, show that the angle BDC is greater than the angle BAC, by joining AD and producing it towards the base.



Let ABC be a triangle, and from B and C let BD and CD be drawn to any point D within the triangle and let AD be joined and produced to meet the base BC in E.

It is required to prove that the angle BDC is greater than the angle BAC.

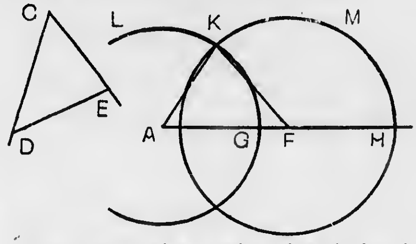
The ∠BDE is greater than the ∠BAD. Prop. 16.

The ∠CDE is greater than the ∠CAD. Prop. 16.

Therefore the whole ∠BDC is greater than the whole ∠BAC. Ax. 4.

PROPOSITION XXIII.

1. Prove proposition 23, giving all the construction instead of assuming proposition 22.



Let AB be the given straight line, A the given point in it, and ∠DCE the given angle.

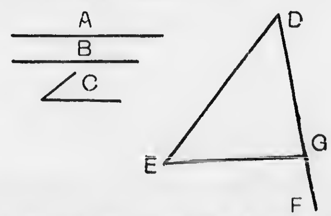
It is required to make at A an angle equal to ∠DCE.

In CD, CE take any points D, E.
Join DE. Post 1.
From AB cut off AF=CD, AG=CE, and from FB cut off FH=DE. Prop. 3.
With centre A and distance AG, describe circle GKL. Post. 3.
With centre F and distance FH, describe circle HKM. Post 3.
Let the circles cut each other at K.
Join KF, AK. Post. 1.
Then AFK is the required triangle.
AF=CD. Const.
Because AG=AK. Def. 1.
And AG=CE. Const.
∴AK=CE. Ax. 1.
Because FH=FK. Def. 1
And FH=DE. Const.
∴FK=DE. Ax. 1.
∴ΔAFK has its sides respectively equal to CD, CE and DE.

In the Δ's CDE, AFK $\begin{cases} CD=AF & \text{Const.} \\ CE=AK & \text{Proved.} \\ DE=FK & \text{Proved.} \end{cases}$

∴∠FAK=∠DCE. Prop. 8.

2. Construct a triangle, having given the two sides and the angle between them.



Let A and B be the two given sides and C the given angle.

It is required to construct a triangle having its two sides equal to A and B, and the contained angle equal to C.

Take any straight line DE=A.

At the point D make the angle EDF= ∠C. I. 23.

From DF cut off DG=B. I. 3.

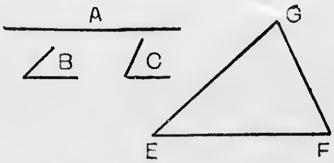
Join EG. Post. 1.

Then EDG is the required triangle.

In the triangle EDG $\begin{cases} DE=A & \text{Const.} \\ DG=B & \text{Const.} \\ \angle EDG=\angle C & \text{Const.} \end{cases}$

Therefore the triangle EDG has its two sides respectively equal to the two given sides and the contained angle equal to the given angle.

3. Construct a triangle, having given the base, and having the angles adjacent to the base equal to two given angles. Is this always possible?



Let A be the given base and B one angle at the base and C the other angle at the base.

It is required to construct a triangle, having the base equal to A and the angles at the base equal to B and C.

Take any straight line EF=A.

At E make an angle equal to B. I. 23.

At F make an angle equal to angle C. I. 23.

Produce EG and FG, meet at G.

Then EFG shall be the triangle required.

The angle at E=angle B. Const.

The angle at F=angle C. Const.

And these are the angles adjacent to the base.

And the base EF was made equal to the given line A.

It is impossible to construct the triangle if the given angles are right angles or greater than right angles. (I. 17.)

PROPOSITION XXV.

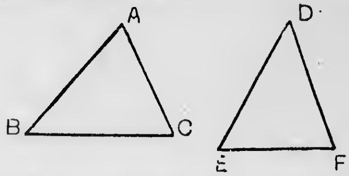
1. Show that Propositions 24 and 25 are converse propositions.

Converse propositions are those in which the conclusions of each becomes the hypothesis of the other.

In the 24th proposition we are given the angles contained by the equal sides of two triangles, unequal; and are required to prove the base of the triangle having the greater contained angle greater than the base of the other triangle.

In the 25th proposition we are given the bases of two triangles, unequal; and are required to prove that the angle contained by the equal sides of the triangle that has the greater base is greater than the angle contained by the equal sides of the other triangle.

2. Assuming the truth of proposition 25; deduce the truth of proposition 24.



Let ABC and DEF be two triangles, having AB=DE; AC=DF, but ∠A greater than ∠D.

It is required to prove BC greater than EF.

If BC is not greater than EF, it must be either equal to, or less than, EF.

But BC is not=EF.

For then ∠A would equal ∠D. Prop. 8.

Which it does not. Hyp.

Also BC is not less than EF.

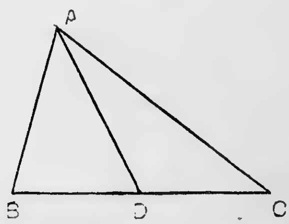
For then ∠A would be less than ∠D. Prop. 25.

Which it is not. Hyp.

Therefore BC is greater than EF.

3. D is the middle point of the side BC of a triangle ABC. Prove that the angle ADB is greater or less than the angle ADC according as AB is greater or less than AC.

CASE I.



Let ABC be the given triangle, having D the middle point of BC, and let AD be joined, and let AB be less than AC.

It is required to prove ∠ADB is less than ∠ADC.

In the triangles ADB and ADC,

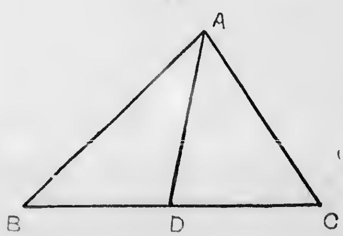
BD=CD. Hyp.

AD=AD. Common.

But AB is less than AC. Hyp.

Then ∠ADB is less than ∠ADC. Prop. 25.

CASE II.



Let ABC be the given triangle, having D the middle point of BC, and let AD be joined, and let AB be greater than AC.

It is required to prove ∠ADB greater than ∠ADC.

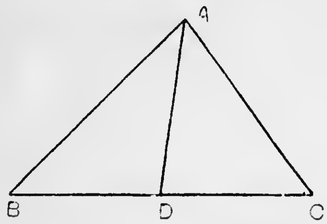
In the triangles ADB and ADC,

BD = CD.	Hyp.
AD = AD.	Common.
But AB is greater than AC.	Hyp.
Then ∠ADB is greater than ∠ADC.	Prop. 25.

∴ ∠ADB is greater or less than ∠ADC according as AB is greater or less than AC.

4. D is the middle point of side BC of a triangle ABC. Prove that, if ∠ADB is greater than ∠ADC, then AB is greater than AC, and, if ∠ADC is greater than ∠ADB, then AC is greater than AB.

CASE I.



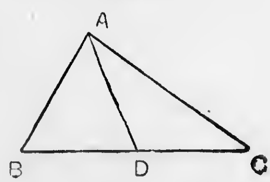
Let ABC be the triangle, having D the middle point of the side BC, and let AD be joined, and let ∠ADB be greater than ∠ADC.

It is required to prove that AB is greater than AC.

In △'s ADB and ADC,

AD = AD.	Common.
BD = CD.	Hyp.
But ∠ADB is greater than ∠ADC.	Hyp.
Then AB is greater than AC.	Prop. 24.

CASE II.



Let ABC be the triangle, having D the middle point of the side BC, and let AD be joined, and let ∠ADC be greater than ∠ADB.

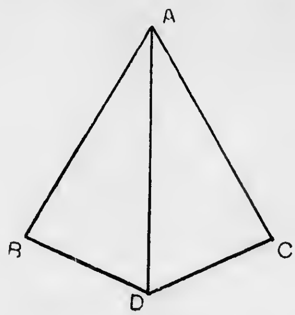
It is required to prove that AC is greater than AB.

In △'s ADB and ADC,

AD = AD.	Common.
BD = CD.	Hyp.
But ∠ADC is greater than ∠ADB.	Hyp.
Then AC is greater than AB.	Prop. 24.

PROPOSITION XXVI.

1. The angle BAC is bisected by the straight line AD. From D the lines BD and DC are drawn making the angles ADB and ADC equal. Prove that DB is equal to DC.



Let BAC be the angle and let it be bisected by AD. From D let DB and DC be drawn, making ∠ADB = ∠ADC.

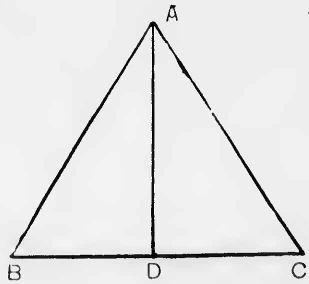
It is required to prove DB = DC.

In △'s ADB and ADC {

AD = AD.	Common.
∠DAB = ∠DAC.	Hyp.
∠ADB = ∠ADC.	Hyp.

∴ DB = DC. Prop. 26.

2. The bisector AD of the angle BAC of the triangle ABC meets BC in D. Prove that, if the angles ADB, ADC are equal, the triangle is isosceles.



Let ABC be the given triangle, having its angle BAC bisected by AD, which meets BC at D, and let ∠ADB = ∠ADC.

It is required to prove that AB = AC.

In the △'s ABD, ACD {

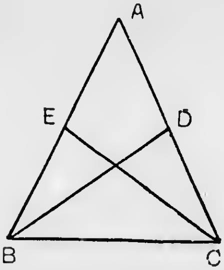
∠BAD = ∠CAD.	Hyp.
∠ADB = ∠ADC.	Hyp.
AD = AD.	Common.

∴ AB = AC. Prop. 26.

That is, △ABC is isosceles.

Def. of an isosceles triangle.

3. The equal angles of an isosceles triangle ABC are bisected by BD and CE, which meet the opposite sides in D and E. Prove that BD is equal to CE.



Let ABC be the given isosceles triangle, and let the angles ABC and ACB be bisected by BD and CE, which meets the opposite sides in D and E.

It is required to prove BD = CE.

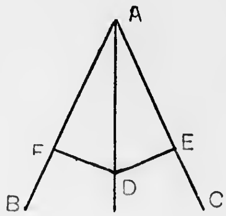
The $\angle ABC = \angle ACB$. Prop. 5.

Therefore $\angle ABD = ACE$. Ax. 7.

In the triangles ACE and ABD $\begin{cases} \angle ABD = \angle ACE. & \text{Proved.} \\ \angle BAD = \angle CAE. & \text{Common.} \\ AB = AC. & \text{Hyp.} \end{cases}$

$\therefore$ BD = CE. Prop. 26.

4. Any point in the bisector of an angle is equidistant from the arms of the angle.



Let BAC be the angle and AD the bisector and D any point in it.

It is required to prove that D is equi-distant from AB and AC.

From D drop a perpendicular DF on AB meeting AB in F.

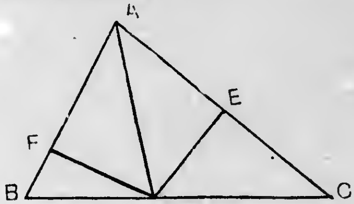
From D drop a perpendicular DE on AC meeting AC in E.

In $\triangle$'s AFD and AED $\begin{cases} \angle FAD = \angle EAD. & \text{Hyp.} \\ \angle AFD = \angle AED. & \text{Ax. 11.} \\ AD = AD. & \text{Common.} \end{cases}$

Therefore DF = DE. Prop. 26.

That is, the point D is equi-distant from AB and AC.

5. Find the point in the base of a triangle which is equidistant from the sides.



Let ABC be the given triangle. It is required to find a point in BC equidistant from the sides AB and AC.

Bisect $\angle$ BAC by the straight line AD which meets BC at D. Prop. 9.

Then D is the required point.

From D drop the perpendiculars DF and DE which meet the sides AB and AC in F and E. Prop. 11.

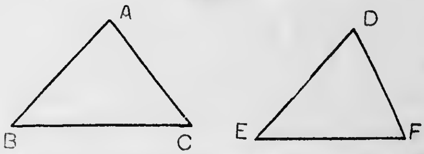
Then in the $\triangle$'s AFD and AED

$\begin{cases} AD = AD & \text{Common.} \\ \angle DAF = \angle DAE & \text{Const.} \\ \angle AFD = \angle AED & \text{Ax. 11.} \end{cases}$

Therefore FD = ED Prop. 26.

6. Prove 26th proposition by superposition.

CASE I.



In triangles ABC and DEF let the angle ABC equal the angle DEF, angle ACB equal angle DFE, and BC equal EF.

It is required to prove AB equal DE, AC equal DF, and angle BAC equal angle EDF, and the triangle ABC equal triangle DEF.

Apply the triangle ABC to the triangle DEF, so that B will fall on E and BC on EF.

Then C will fall on F because BC = EF. Hyp.

BA will fall on ED because $\angle B = \angle E$. Hyp.

CA will fall on FD because $\angle C = \angle F$. Hyp.

Then A will fall on D because D is the only common point in ED and FD.

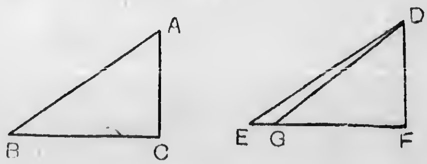
Then $\angle A$ will coincide with $\angle D$ and be equal to it. Ax. 8.

And AB will coincide with and equal DE. Ax. 8.

And AC will coincide with and equal DF. Ax. 8.

The triangle ABC will coincide and equal triangle DEF.

CASE II.



In triangle ABC and DEF let angle ABC equal angle DEF, angle ACB equal angle DFE and AC equal DF. It is required to prove AB equal DE, BC equal EF, angle A equal angle D and triangle ABC equal triangle DEF.

Apply the triangle ABC to the triangle DEF so that A will fall on D, and AC on DF.

Then C will fall on F because AC = DF. Hyp.

Then BC will fall on EF because $\angle C = \angle F$. Hyp.

Then B will fall on E. If it does not let it fall otherwise as on G.

Join DG. Post. 1.

Then $\angle DGF = \angle ABC$. Ax. 8.

But $\angle DEF = \angle ABC$. Hyp.

Therefore $\angle DGF = DEF$. Ax. 1.

Which is impossible since $\angle DGF$ is greater than $\angle DEF$. Prop. 16.

Therefore B will not fall off the point E.

That is, it will fall on E.

Then because the point A falls on D and B on E the straight line BA will fall on the straight line ED. Ax. 10.

Then BC coincides with and is equal to EF. Ax. 8.

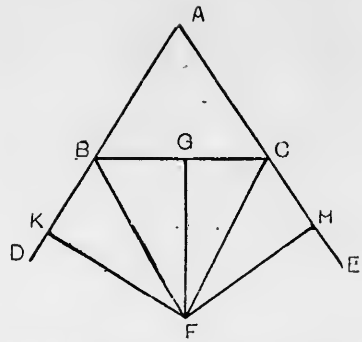
AB coincides with and is equal to ED. Ax. 8.

$\angle A$ coincides with and is equal to $\angle D$. Ax. 8.

$\triangle ABC$ coincides with and is equal to $\triangle DEF$. Ax. 8.

7. *If two sides of a triangle be produced, prove that the two bisectors of the angles so formed meet at a point equidistant from the three sides of the triangle.*

Let ABC be the given triangle having AB and AC produced to D and E, making the angles DBC and ECB. And let BF and CF, the bisectors of



the angles DBC and ECB, meet at F. It is required to prove F equidistant from AB, AC, and BC.

From F drop FK perpendicular to AD, FG perpendicular to BC, FH perpendicular to AE. Prop. 12.

Then in $\triangle$'s FKB and FGB

$$\left\{\begin{array}{ll} \angle FKB = \angle FGB & \text{Ax. 11.} \\ \angle FBK = \angle FBG & \text{Hyp.} \\ FB = FB & \text{Common.} \end{array}\right.$$

Therefore FK = FG. Prop. 26.

Again, in the $\triangle$'s FGC and FHC

$$\left\{\begin{array}{ll} \angle FGC = \angle FHC & \text{Ax. 11.} \\ \angle FCG = \angle FCH & \text{Hyp.} \\ FC = FC & \text{Common.} \end{array}\right.$$

Therefore FG = FH. Prop. 26.

But FK = FG. Proved.

Therefore FK = FH Ax. 1.

This is, F is equidistant from AD, AE, BC.

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