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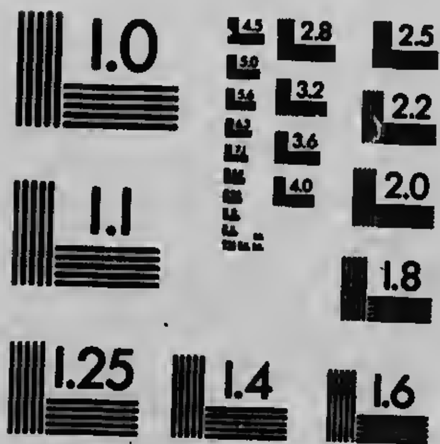

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GRAND TRUNK RAILWAY SYSTEM

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CLASS INSTRUCTION BOOK

IN

MATHEMATICS

OFFICE OF VICE-PRESIDENT OF MOTIVE POWER,
CAR EQUIPMENT AND MACHINERY,
MONTREAL

SEPT., 1918

PREFACE

This textbook has been prepared primarily for the use of apprentices employed on the Grand Trunk Railway System, who, during the term of their apprenticeship, are required, as a part of their education, to so inform themselves in the Subject of Mathematics that at the completion of their term they will be competent tradesmen.

For convenience the word Mathematics is used as a title, but other subjects have been included.

The complete book gives only such matter for study that has been found most suited to the requirements of a machinist along these lines, and is the result of a number of years' observation and experience teaching apprentice classes.

A course in Mechanical Drawing has been prepared which is intended to be studied in conjunction with this book.

The contents of both courses have been so classified as to meet the needs of the apprentice as he progresses through the course of his apprenticeship.

The degree of success attained will be measured by the amount of thought and practice given to the subject. The time is only lost when a student attempts work of this nature without having his mind on the subject in hand.

INDEX

MULTIPLICATION TABLES

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36

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4 × 2 = 8	5 × 2 = 10	6 × 2 = 12
4 × 3 = 12	5 × 3 = 15	6 × 3 = 18
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4 × 11 = 44	5 × 11 = 55	6 × 11 = 66
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7 × 3 = 21	8 × 3 = 24	9 × 3 = 27
7 × 4 = 28	8 × 4 = 32	9 × 4 = 36
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DEFINITIONS AND MATHEMATICAL SIGNS

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1. **Definitions.** Mathematics is the science which treats of quantity, and its fundamental branches are Arithmetic, Algebra, and Geometry.

Quantity is anything which can be increased, diminished, or measured; for example: numbers, lines, space, motion, time, volume, and weight.

A unit is a single thing, or one.

A number is a unit or a collection of units and is either concrete or abstract.

A concrete number is one whose units refer to particular things, as, for example 5 rivets, 7 bolts.

An abstract number does not refer to any particular thing. For example, 5, 23, etc., used without designating any particular objects, are abstract numbers.

2. **Mathematical Signs.** For the sake of brevity, signs are used in mathematics to indicate processes. Those signs most used in Arithmetic are

$$+,\ -,\ \times,\ \div,\ =,\ ()\ \text{and},\ \overline{}.$$

The sign $+$ is read "plus" and is the sign of addition. It shows that the quantities between which it is placed are to be added together. If 2 and 2 are to be added it is expressed thus: $2 + 2$ are 4.

The sign $-$ is read "minus" and is the sign of subtraction. It means that the quantity which follows this sign is to be subtracted or taken away from the quantity which precedes it, thus: 6 — 4 are 2.

The sign × is read "times" and is the sign of multiplication. It means that the quantity which precedes this sign is to be multiplied by the quantity which follows it, thus 2 × 5 are 10.

The sign ÷ is read "divided by" and is the sign of division. It means that the quantity which precedes this sign is to be divided by the quantity which follows it, thus: 4 ÷ 2 are 2.

The sign = is read "equals" or "is equal to" and is the sign of equality. It means that the expressions between which it is placed are identical in value, thus: $4+3=10-3$. This sign is very often misused. Great care should be taken at all times to make sure that the quantities connected by it are equal.

The parenthesis ( ) and vinculum ——— are used to show that two or more quantities are to be treated as one or, in other words, that the operations indicated within the parenthesis or under the vinculum are to be carried out first, thus:

$$(20-5)+3-\overline{2+3}=(15)+3-(5)=13$$

NOTATION

Notation is the art of writing numbers in words, in figures, and in letters.

Arabic Notation. The Arabic notation employs ten characters or figures in expressing numbers. They are:

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	cipher.

The first nine are sometimes called digits; the cipher is also called naught or zero, because it expresses nothing or the absence of a number.

The digits (1, 2, 3, 4, 5, 6, 7, 8, 9) have been termed significant figures, because each has of itself a definite value, always representing so many units or ones as its name indicates. However, the value of the units represented by a figure depends upon the particular position which that figure occupies with regard to other figures. This position is called its place or order.

Example. If three figures are written together to represent a number, as 444, each of these figures, without regard to its place expresses four units, but when considered as part of the number these fours differ in value. The 4 in the first place to the right represents 4 units; the 4 in the second place, represents 4 tens or 4 units each ten times the size or value of a unit of the first place; and the 4 in the third place, represents 4 hundreds, or 4 units each one hundred times the size or value of a unit of the first place. It is readily seen that the value of any figure is increased ten-fold by removing it one place to the left.

4 = 4 Units
40 = 4 Tens
400 = 4 Hundreds
———
444 = Total

The cipher becomes significant when connected with other figures by filling a place that otherwise would be vacant, as in 10 (ten) it gives a ten-fold value to the 1. In 130 (one hundred thirty) it gives a ten-fold value to the 13. A cipher between two or more figures produces the same effect. In 405 the cipher which fills the intervening place between 4 and 5 causes the 4 to represent four hundreds, not four tens.

Hundreds Tens Units

5 = 5 Units
00 = 0 Tens
400 = 4 Hundreds

405 = Total

The following general principles should be firmly fixed in the mind:

All numbers are expressed by the nine digits and zero.

Zero has no value; it is used to fill vacant places only.

A figure has different values according to the place it occupies.

The base of the system of notation is ten; ten units of any order making one unit of the next order.

NUMERATION

Numeration is the art of reading numbers when expressed by letters or figures.

PROBLEMS FOR PRACTICE

Write in words:

1. 18,765,972.
2. 834,769,780.
3. 3,576,879,421.
4. 10,805,056.

Answer: Ten million eight hundred five thousand fifty six.

Write in figures:

5. Seventy-eight million, forty-one thousand seven.
6. One thousand three.
7. Five hundred six thousand.
8. Ninety million, two thousand three hundred twenty-seven.
9. Three hundred five thousand seventy-nine.
10. Eight hundred sixty-four millions, four thousand twenty.

ABBREVIATIONS

Certain words are always recurring in Arithmetic and these are usually abbreviated. The most common of these are:

ins. means inches.
ft. means feet.
yds. means yards.
lbs. means pounds.
gals. means gallons.
dia. means diameter.
sq. ins. means square inches.
sq. ft. means square feet.
sq. yds. means square yards.
cu. ins. means cubic inches.
cu. ft. means cubic feet.
cu. yds. means cubic yards.

There are also many others, the meaning of which will be made plain by studying the question.

TABLES AND DATA

The following tables should be memorized, also the various weights of materials as given.

Linear Measure, or Table of Length

12 inches = 1 foot.
3 feet = 1 yard.
1760 yards = 1 mile.
5280 feet = 1 mile.

Avoirdupois Weight, or Table of Weight

16 ounces (ozs.) = 1 pound (lb.)
100 lbs. = 1 hundredweight (cwt.)
20 cwt. or 2000 lbs. = 1 short ton.
2240 lbs. = 1 long ton.

Note: Unless otherwise stated, short tons are always used.

Square Measure, or Table or Area

144 sq. ins. = 1 sq. ft.
9 sq. feet = 1 sq. yd.
4840 sq. yds. = 1 acre.
640 acres. = 1 sq. mile.

Cubic Measure, or Table of Volume

1728 cu. ins. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.

Board Measure

The unit of board measure is the **Board Foot**, which is one foot square and one inch thick.

The number of board feet contained in any board or number of boards is calculated by multiplying the length in feet by the width in feet by the thickness in inches.

Water Equivalents

One cubic foot of water weighs 62.5 lbs. and contains 7.5 United States gallons and 6.25 Imperial gallons, approximately.

1 U.S. gal. contains 231 cu. ins.
1 Imp. gal. contains 277.274 cu. ins.
1 U.S. gal. weighs 8⅓ lbs.
1 Imp. gal. weighs 10 lbs.
1 cu. in. weighs .03616 lb.

Measure of Angles

60 seconds = 1 minute.
60 minutes = 1 degree.
90 degrees = 1 right angle.
360 degrees = 1 circle.

Weights of Metals

1 cu. in cast-iron weighs .26 lbs.
1 cu. in. wrought-iron weighs .2777 lbs.
1 cu. in. steel weighs .288 lbs.
1 cu. in. brass weighs .3 lbs.
1 cu. in. lead weighs .41 lbs.

ADDITION

Adding is the process of finding a number which is equal to the combined values of two or more given numbers. The result thus obtained is called the sum. Hence, it may be said that the sum of two or more numbers is a number containing as many units as all the numbers taken together. Thus the sum of 5 rivets and 7 rivets is 12 rivets, since 12 contains as many units as 5 and 7 together.

Rules for Addition: Write the numbers under each other, placing them so that units are under units, tens under tens, hundreds under hundreds, and so on.

Add up the column of units; and put the right-hand figure of this sum under the unit column, carrying the remaining figure or figures to the column of tens; add up the tens column, including the carried figures, put down the right-hand figure and carry as before. Continue in this way until the last column is reached, putting down the total of the last column to give the final sum.

Example 1. Find the sum of 567; 141; and 93.

Solution.—Write these numbers under one another so that the units of each shall be in the same vertical column. Then add up as follows: 3 and 1 are 4, and 7 are 11. Place the right-hand figure 1 under the units column and carry 1 ten to the next column. Adding the tens column, 1 (carried) +9 are 10 and 4 are 14 and 6 are 20. Put down the right-hand figure, which is zero, and carry 2 to the next column; then 2 (carried) and 1 are 3 and 5 are 8. Putting down the 8, the total sum is found to be 801.

$$\begin{array}{r} 567 \\ 141 \\ 93 \\ \hline 801 \end{array}$$

Proof. To prove that a sum is correct, begin at the top and add the columns downward in the same manner as they were added upward; if the two sums agree, the work is presumably correct, for adding downward inverts the order of the figures, and therefore any error made in the first addition would probably be detected in the second.

PROBLEMS FOR PRACTICE

Find the sum of:—

1. 56+49+17+36+21.
2. 42+46+43+58+91.
3. 4[illegible]7+536+84+705.
4. 2,008+1,400+706+300+77.
5. 8,950+15,765+7,732.
6. 26,661+8,735+6,877+33,413.
7. 8,792+980+5,607+89.
8. 346+4,682+64+798+21.
9. 26+425,902+3,006+490+36,221.
10. Find the sum of 36,748+6,737,488+891,162 +281,937+743,948+.
11. Find the sum of 348+110,031+7699+67+ 56,300,092.
12. Find the sum of 56839+29938+38348+ 101338+23899.

13. Add

785432	28749	13468
314154	36324	21793
931583	15116	34282
715134	57843	78492

14. Find the sum of seventeen; five hundred and sixty-seven thousand and eight; four thousand and eighteen; nine thousand nine hundred and twelve; four hundred and thirty-six thousand, nine hundred and eighty-nine.

15. Express in figures the following and then find their sum:—nine hundred and six dollars and seventy cents; three hundred and twelve dollars and twenty-five cents; four dollars and seventy-eight cents; and eighty-three cents.

16. A railroad made the following shipments of gravel:—one hundred and twenty-eight tons; seven hundred and three tons; three thousand, nine hundred and fifty tons; and six hundred and seventy-nine tons. What was the total amount of gravel shipped?

17. The following amounts of scrap metal are on hand at a shop:—thirty-nine thousand, four hundred and twelve lbs. of cast-iron; eleven thousand and fifty lbs. of steel turnings; nine hundred and twelve lbs. of boiler plate; and six hundred and seven lbs. of brass. What is the total weight of scrap on hand?

18. A railroad owns 185750 box cars, 75894 coal cars, 93849 flat cars, 12989 dump cars, 8654 passenger cars. Cabooses, boarding cars, baggage cars, etc., total 1489. Find the total number of cars owned by the railroad.

19. A car is loaded with 475831 lbs. of cast-iron, 233950 lbs. of malleable iron, and 8765 lbs. of brass. What is the total weight of the car?

20. Find the number of tons of coal used by a factory if the consumption for the week is as follows:—345 tons, 386 tons, 390 tons, 327 tons, 319 tons, and 401 tons.

21. A car contains 345680 lbs. of cast-iron. What is the total weight contained by the car if 34910 lbs. of malleable iron and 5678 lbs. of brass castings are added to the load ?

22. A shop has 47583 sq. ft. in one department, 67483 in another, 12500 in another, and 34567 sq. ft. in another. If there are 157815 sq. ft. in the balance of the shop, what is the total floor space ?

23. A marine engine during a 3 hours' run makes 9187 revolutions the first hour, 9062 the second, and 9233 the third. How many does it make in the 3 hours ?

24. Coal is fed to a furnace as follows:—Monday, 376 pounds; Tuesday, 307 pounds; Wednesday, 438 pounds; Thursday, 425 pounds; Friday, 399 pounds; Saturday, 301 pounds. Find the total for the week.

25. The items for lumber called for in a contract were: —Frame, 3896 feet; flooring, 6796 feet; finish, 2739 feet. How many feet were used ?

26. A surveying party works six weeks. The first week they survey 151 miles; the second week, 111 miles; the third week, 162 miles; the fourth week, 159 miles; the fifth week, 96 miles; the sixth week, 48 miles. How many miles did they survey ?

27. There are five water wheels installed in a water power plant. The power furnished by the first wheel is 2225 horse-power, and the others furnish, 3150, 4275, 5650 and 8275 horse-power. What is the total capacity of the five wheels ?

28. The weekly capacity of 4 lathes is as follows: 2500 castings, 4175 nuts, 3420 brass boxings, and 2185 finished trimmings. How many pieces do the four lathes turn out per week ?

29. If a 10-inch belt will transmit 17 horse-power at a speed of 1800 feet per minute, and a 16-inch belt will transmit 36 horse-power at the same speed, how much power will be transmitted by the two belts ?

30. Find the total area of a shop's floor space, if the blacksmith shop has 10320 sq. ft., machine shop 6830. sq. ft., carpenter shop, 32415 sq. ft., boiler shop, 65992 sq. ft. and erecting shop 183928 sq. ft.

31. The weekly capacity of four lathes is as follows:—4600 castings, 7461 nuts, 2530 brass boxings, and 2437 turned bolts. How many pieces do the four lathes turn out each week ?

32. A locomotive driving wheel, during a run of three hours, makes 9187 revolutions the first hour, 9062 revolutions the second hour, and 9233 revolutions the third hour. How many revolutions did it make in the three hours ?

33. If a 11" belt will transmit 21 H.P. at a speed of 1700 feet per minute, and a 17" belt will transmit 37 H.P. at the same speed, how much power will be transmitted at the same speed by the two belts ?

34. An engine made the following daily runs during a week:—148 miles, 106 miles, 168 miles, 143 miles, 98 miles, 137 miles and 116 miles. What was its total mileage for the week?

35. The towns along a branch line are the following distances apart:—26, 46, 11, 17, 22, 35, 16, 29, 27, 31, 6, 24, 39, 14, 8 and 19 miles. What is the total length of the line ?

36. The following shipments of iron castings are made by a foundry:—457683 lbs., 340056 lbs., 4958 lbs., 13000 lbs., 3849572 lbs., 55988 lbs., and 17229 lbs. What was the total weight of castings shipped?

SUBTRACTION

Subtraction is the process of finding the difference between two quantities; this difference when added to the smaller will give a result equal to the greater.

For example, the difference between 16 and 7 is 9, since 7 added to 9 makes 16.

It can readily be seen that subtraction is the reverse of addition, and this fact is made use of to prove subtraction as shown in the above question. As in addition write given numbers under each other, placing the smaller number under the larger, taking care to keep the units under units, tens under tens, etc.

Example. Subtract 114 from 237 and prove the result.

Solution. Beginning with the units column, 4 (units) are subtracted from 7 (units) leaving 3 (units), which is set down directly under the column in units place. Proceeding to the next column 1 (ten) is subtracted from 3 (tens) leaving 2 (tens), which is set down in tens place. Proceeding to the next column 1 (hundred) is subtracted from 3 (hundreds) leaving 2 (hundreds), which is set down in hundreds place. The final remainder is found to be 123.

	Proof
237	114
114	123
——	——
23	237

Example 2. From 1000 subtract 621.

Solution. The 0 in the units place of the minuend must first be increased by 10; then 1 subtracted from 10 leaves 9 in the units place of the remainder. In adding 10 to the 0, 1 has been taken from the tens place, but as it was itself 0, it had to borrow from the next place and continue borrowing until a numerical place was reached. Proceeding in this way the total remainder 379 is obtained.

1000	Minuend
621	Subtrahend
379	Remainder.

PROBLEMS FOR PRACTICE

1. From 7282 subtract 4815.
2. From 64037 subtract 5908.
3. From 6231 subtract 3084.
4. From 1740932 subtract 807605.
5. From 71287 subtract 40089.
6. From 1000000 subtract 999999.
7. Subtract:—

7854329	2164325	1689426
2187648	2155537	435607

8. From two hundred and eighty-six thousand, four hundred and twenty-eight, take one hundred and eighty-six thousand, four hundred and twenty-nine.

9. Subtract nine thousand, eight hundred and eighty-nine, from seventy-two thousand, nine hundred and forty-nine.

10. What number must be added to the sum of sixty-four thousand, three hundred and eighty-five; seven million, one thousand and sixty; seventy-seven thousand and seven; and fifty-six thousand, seven hundred and sixty-three to make the result ten millions?

11. Deposit, 850 dollars; checks drawn, 105 dollars, 72 dollars, 183 dollars. Find balance.

12. Deposit, 625 dollars; checks drawn, 130 dollars, 210 dollars, 128 dollars. Find balance.

13. Deposit, 475 dollars; checks drawn, 78 dollars, 162 dollars, 24 dollars. Find balance.

14. From a tank containing 935 gallons of water, 648 gallons were drawn off. Then 247 gallons ran in. How many gallons were then in the tank? (Suggestion: Subtract 648 from 935 and add 247.)

15. A man purchased 8983 bricks, but used only 5363. How many had he left?

16. A coal shed contains 8579 tons. 3243 tons are taken from it. It then receives 4112 tons more. After that 1602 tons are taken out of it. How many tons remain?

17. An electric power plant can generate 2000 horse-power. Of this, 1910 horse-power is used. The manager then agrees to furnish another firm with 784 horse-power. How much more power will be needed? (Suggestion: Add 784 to 1910 and subtract 2000.)

18. An engine develops 147 horse-power. 16 horse-power is used in running the engine itself. How much power is available for running machinery?

19. 1200 gallons are pumped from a tank. Of this, 32 gallons are lost in leakage, etc. How much is discharged by the pump?

20. A 75 horse-power boiler evaporates 2140 pounds of water into steam per hour. One engine uses 1310 pounds, another uses 417 pounds, and a pump requires the remainder. How much steam is used by the pump? (Suggestion: Add 417 to 1310 and subtract from 2140.)

21. How much is the difference between 1628716 and 79019 greater than the sum of 56095, 2800, 10009, 7097, 159, 3000, and 90829?

22. From a coal pile containing 79697 tons of coal the following amounts were removed:—392 tons, 1650 tons, 1175 tons, 485 tons, 1799 tons, and 2366 tons. How much coal remains?

23. In six days a railway's receipts were $5123.25. The expenses were $2495.00 for wages, and $629.89 for repairs. Find the profit.

24. An elevator contains 92428 bushels of wheat. There was sold 7964 bushels and then 72646. How much was left?

25. What is the difference in weight between a class "A" engine which weighs 177772 lbs. and a class "E" engine which weighs 161976 lbs?

26. For five hours work on bolts a boy turns out 30, 32, 38, 36, and 28 bolts respectively. If 85 of these were taken away, how many should he still have on his machine?

27. A pair of scales has on the platform 186986 lbs. of brass and copper, of which 98757 lbs. is copper. What does the brass weigh?

28. What is the difference in the reading of two water meters which read 678480 and 908690 respectively?

29. From a tank containing 1042 gallons of water, 637 gallons were drawn off. Then 138 gallons ran in. How many gallons were left in the tank?

30. A coal shed contains 10321 tons; 3127 tons are taken from it. It then received 3898 tons more, and after that 1327 tons are taken out. How many tons remain?

31. An electric power plant can generate 4500 H.P. Of this 4125 H.P. is used. The manager agrees to furnish another firm with 800 H.P. How much more power will the plant be required to generate?

32. An engine develops 214 H.P. 18 H.P is used in running the engine alone. How much power is available for running machinery?

33. An engine makes 54000 revolutions in a day of 12 hours. A motor makes 72000 revolutions in the same time. By how many revolutions per day does the speed of the motor exceed that of the engine?

34. In a month a coal shed receives shipments of 2112 tons, 2350 tons, 560 tons, and 3486 tons. It delivers 1348 tons, 48 tons, 230 tons, 680 tons, 1148 tons and 1200 tons. How much coal is there left?

35. A casting weighs 6750 lbs. If it contains 340 lbs. of tin, and 1150 lbs. of babbitt, what does the balance of the casting (which is copper) weigh?

36. In five hours running one steam pump delivers 4751 gallons, 3410 gallons, 4864 gallons, 3985 gallons, and 4506 gallons. Another pump delivers 4599 gallons, 4906 gallons, 3966 gallons, 4076 gallons, and 4716 gallons. Find:—

(a) Total number of gallons delivered in the five hours.

(b) Which pump delivered the most water, and how many gallons more?

37. In a shop force of 896 men, 238 are absent, and 165 are hired to replace them. What is the present shop force?

MULTIPLICATION

Multiplication is a short method of adding a quantity to itself a certain number of times;

It is known that 2+2+2+2+2=10; but this same process may be expressed more briefly by the aid of multiplication, thus: 5×2=10. The 5 shows how many twos are used in adding. This last expression is read, "five times two equals ten."

In multiplication three terms are employed—the multiplicand, the multiplier and the product.

The multiplicand is the quantity to be multiplied or taken.

The multiplier denotes the number of times the multiplicand is to be taken.

The product is the result or quantity obtained by the multiplication.

To multiply with accuracy and rapidity, the product of any two quantities, at least from 2 to 12, must be known at sight. The combinations of these should be practiced until they can be given correctly and without hesitation.

Rules for Multiplication. The multiplicand may be either concrete or abstract. The multiplier is always abstract. The product is always like the multiplicand.

When both quantities are abstract, either may be considered as the multiplicand or the multiplier, for the result is the same; thus: 5 times 2 is the same as 2 times 5.

Each figure of the multiplicand is multiplied by each significant figure of the multiplier, and the right-hand figure of each product is placed under the figure of the multiplier used to obtain it. The sum of the several products will be the entire product. When there is a zero in the multiplier, multiply by the significant figures only, taking care to place the right-hand figure of each separate product under the figure used in obtaining it.

Example 1. Find the product 175 and 7.

Solution. Having written the multiplier under the unit of the multiplicand, multiply the 5 units by 7, obtaining 35. Then set down the 5 units directly under the 7 and carry the 3; in other words, reserve the 3 tens for the tens column. Next multiply the seven tens by 7, obtaining 49, and add the 3 which is carried, and obtain 52 tens (which is the same as 5 hundreds and 2 tens). Set down 2 tens and carry the 5 hundreds; and multiply 1 and 7 and add the 5 which was carried, making 12, which can be written down in full. The product then reads, 1225.

```
 175 Multiplicand
   7 Multiplier
 ———
1225 product
```

Example 2. Find the product of 145 and 13.

Solution. Commence with 3, multiply through and write the product 435. Under this write the product 1450 obtained by multiplying by 10. In this latter product the 0 may be discarded but it must be remembered to write the 5 under second place. Adding these two products called partial products, gives the final product 1885.

```
 145
  13
----
 435
145
----
1885
```

Example 3. Multiply 1246 by 235.

Solution. Note that the three partial products are the results of multiplying by 5, 3, and 2 where each successive partial product is set one place further to the left than the preceding one. Note that 0 is under 5, 8 is under 3, and 2 is under 2; in other words, the first figure of each partial product is placed under the digit used to obtain it.

```
  1246
   235
------
  6230
 3738
2492
------
292810
```

Two special cases not covered by the general rules given above should be here considered.

1. When the digits of the multiplier are separated by ciphers:

Example. Multiply 13456 by 2004.

Solution. Although the multiplier contains four figures in the short method only two partial products appear.

The first figures obtained by multiplying by 4 and 2 appear under these respective digits, the zeros simply marking the absence of any other characters in the product.

Regular method.

```
    13456
     2004
    -----
    53824
   00000
  00000
 26912
 --------
 26965824
```

Short method.

```
    13456
     2004
    -----
    53824
 26912
 --------
 26965824
```

2. When ciphers are at the right of multiplier or multiplicand:

Example. Multiply 5760 by 3000.

Solution. In this case it is necessary only to multiply 576 by 3, giving 1728; then annex the total number of ciphers found at the right of both multiplier and multiplicand, in this case, four, giving as the final result 17280000.

```
 5760
     3000
 --------
 17280000
```

PROBLEMS FOR PRACTICE

Multiply:

1. 2928 by 364.
2. 7319 by 394.
3. 5698 by 792.
4. 3186 by 839.
5. 42308 by 692.
6. 876 by itself.
7. 78 by 10.
8. 57 by 1000.
9. 52 by 99.
10. 16 by 25.
11. 92 by 11.
12. 103 by 25.

13. There are 746 watts in a horse-power. How many watts are there in 20 horse-power? (Suggestion: Multiply 746 by 20.)

14. A piston has an area of approximately 113 square inches. If the steam pressure is 47 pounds per square inch, what is the total pressure upon it?

15. If one hundred bolts 1″ in diameter and 16″ long weigh 449 lbs., how much will 5500 bolts of the same size weigh?

16. A wheel 113″ in diameter travels 355 inches every revolution. How far will it travel in 1929 revolutions?

17. How many inches in 76 feet? How many days in 17 common years?

18. A took a railway journey of 93 miles; B travelled 9 times as far; C 12 times as far as A and B together; D 13 times as far as C., less what B travelled. How many miles did all of them travel?

19. The head of a boiler has an area of 11310 square inches. If the pressure per square inch is 40 pounds, what is the total pressure on the head?

20. A concrete mixer delivers 14 cubic yards per hour. A gang of men takes away 12 cubic yards per hour. How many cubic yards will remain unmoved at the end of 4 hours? At the end of 8 hours?

21. If there are 12 threads per inch on a screw, how many threads are there in 4 inches?

22. If a piston moves through 468 feet in one minute, how far does it travel in 45 minutes?

23. If a boiler evaporates 1945 pounds of water in one hour, how many pounds will it evaporate in 9 hours?

24. A certain girder supports 136925 pounds. How much will 65 such girders support?

25. An engine in a certain power plant requires 18 pounds of steam per horse-power per hour. If the engine is developing 640 horse-power, what is the total steam consumption?

26. A gang of men can lay 1 mile of track in 5 days. How long will it require them to lay 35 miles?

27. If a pump discharges 4 gallons of water per stroke and it makes 100 strokes per minute, how many gallons will it discharge in 45 minutes?

28. If one 32 candle-power incandescent lamp costs one cent per hour, what will be the cost of 48 such lamps for 13 hours?

29. A ton of a certain kind of fuel occupies 42 cubic feet. What space will 73 tons occupy?

30. Light travels 185172 miles per second and passes from the sun to the earth in 493 seconds. What is the distance from the sun to the earth?

31. Three towns are in straight line. A is 29 miles east of Toronto, B is three times as far west of Toronto, and C is west of Toronto by 13 miles less than twice the distance from A to B. How far is it from A to C? Make a sketch to help you get the answer.

32. A pulley on the line shaft makes 170 revolutions per minute. How often will it turn in 19 hours?

33. The area of the piston of a 12" brake cylinder is 113 sq. in. What is the total pressure on the piston, if the pressure per sq. in. is 90 lbs.?

34. There are 746 watts in a H.P. How many are there in 375 H.P.?

35. A piston has an approximate area of 244 sq. ins. If the steam pressure upon it is 150 lbs. per sq. in., what is the total pressure upon it?

36. If there are 14 threads per inch on a screw, how many threads would there be in 17" of threading?

37. If a piston moves through 513 ft. of space in one minute, how far does it travel in 1 hr. 35 min.

38. An engine in a power plant requires 19 lbs. of steam per H.P. per hour. If the engine is developing 539 H.P. what is the total steam consumption in four hours?

39. An engine makes 420 revolutions per minute. How many R.P.M. will it make in eighteen minutes?

40. What is the total weight of 79586 castings, each one weighing 458 lbs.?

41. What is the total weight of an army of 19649 men, their average weight being 158 lbs ?

42. If there are twenty trains, each with 34 cars, each car loaded with 715 bushels of grain weighing 45 lbs. to the bushel, what is the total weight of the grain ?

43. Find how many gallons of water three steam pumps would deliver in one day of 16 hours, if the first one delivers 16 gals. per minute, the second one 12 gals. per minute, and the third one 20 gals. per minute.

44. If an engine travels at the average rate of 47 miles per hour, how far will it travel in 67 hours ?

45. Which weighs the most, and by how much, 345 castings weighing 569 lbs. each, or 497 castings weighing 498 lbs. each ?

46. If a piston has an area of 458 sq. ins., and the pressure on it is 116 lbs. per sq. in., what is the total pressure on the piston ?

47. An engine makes 340 revolutions per minute. How many revolutions will it make in 8 hours ? (Suggestion: 8 hours = 480 minutes.)

DIVISION

Division is the process of finding how many times one quantity contains another. In division there are three principal terms, the dividend, the divisor, and the quotient or answer.

The dividend is the quantity to be divided.

The divisor is the quantity which is divided into the dividend.

The quotient is the number of times the divisor is contained in the dividend.

When the dividend does not contain the divisor an exact number of times, the excess is called the remainder. The remainder being a part of the dividend will always be of the same kind as the dividend and must necessarily be less than the divisor.

Division may be indicated in any of the following ways:

$24 \div 2$; $\frac{24}{2}$; 2)24.

Division is the reverse of multiplication, as shown by the following:

$6 \times 7 = 42$	$42 \div 6 = 7$	$42 \div 7 = 6$
$5 \times 8 = 40$	$40 \div 8 = 5$	$40 \div 5 = 8$

There are two distinct methods used, viz., **Long Division** and **Short Division**; in the former all the work is written out but in the latter the process is performed mentally and the result only is written. Short division is generally used when the divisor does not exceed 12.

The following examples illustrate the two processes.

Example 1. Divide 720 by 5.

Solution. In long division it is found that 5 is contained in 7 once. Write 1 as the first figure of the quotient, and subtract, giving a remainder of 2. To the remainder annex the next figure of the dividend, and divide as before, obtaining 4 as the second figure of the quotient. Annex 0 which is the next figure of the dividend, and divide again by 5, obtaining 4 as the last figure of the quotient with no remainder. The division is now complete.

Long division:

```
5)720(144 Quotient
  5
  --
  22
  20
  --
   20
   20
```

Short Division:

```
5)720
  ---
  144 Quotient
```

It often happens, after bringing down a figure from the dividend, that the number is too small to contain the divisor. In this case place a zero in the quotient, and continue bringing down the figures from the dividend until the number thus formed will contain the divisor.

Example 2. Divide 10413 by 13.

Solution.

```
13)10413(801
   104
   ---
     13
     13
```

Rules for Long Division. Write the divisor and dividend in the order named, and draw a curved line between them.

Find how many times the divisor is contained in the left-hand figure or figures of the dividend, and write the number in the quotient over the dividend.

Multiply the divisor by this figure of the quotient, writing the product under that part of the dividend from which it was obtained; subtract, and to the remainder annex the next figure of the dividend.

Find how many times the divisor is contained in the number thus formed, and write the figure denoting it at the right hand of the last figure of the quotient.

Proceed in this manner until all the figures of the dividend are divided. If there is a remainder after dividing all the figures of the dividend, place the remainder over the divisor with a line between them, and annex to the quotient.

Example. Divide 5441 by 26.

Solution. $26)5441(209\frac{7}{26}$

```
     52
     ——
     241
     234
     ———
       7 Remainder.
```

The proper remainder is, in all cases, less than the divisor. If, in the course of the operation, it is found to be larger than the divisor, this indicates that there is an error in the work and that the figure in the quotient should be increased.

In division, the quotient is always abstract, since it shows how many times the divisor is contained in the dividend.

Proof. In order to prove division, multiply the quotient by the divisor, and add the remainder, if there is any. If the quantity thus obtained gives the dividend, the work is correct.

PROBLEMS FOR PRACTICE

Divide 1. 7493, 3246, 2943, 9843, 674; by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

2. 65814 by 6.
3. 3870 by 10.
4. 5321 by 10.
5. 9473 by 100.
6. 13987 by 1000.
7. 563 by 25.
8. (a) 43789 by 387; (b) 764974 by 895.
9. 87643 by 8747.
10. 2037647 by 329.
11. 209376 by 209.
12. 487689 by 3768.
13. (a) 978652 by 652; (b) 392448 by 256; (c) 8917376 by 14914,

14. If coal costs 5 dollars per ton, how many tons may be bought for 275 dollars? (Suggestion: Divide 275 by 5.)

15. A steamer runs 276 miles in 23 hours. What is her average speed per hour? (Suggestion: Divide 276 by 23.)

16. If 9 days' work will pay for 6 tons of coal at 6 dollars per ton, what is the price of a day's labor?

17. A company furnishes equal power to 26 establishments. The total horse-power is 8450. How much does each receive?

18. If a locomotive goes to shop for inspection after every average run of 283 miles, how many times would it be in shop during runs aggregating 10188 miles?

19. A certain boiler supplies steam for heating. If there are 180 square feet of heating surface in the boiler and each radiator requires 12 square feet of heating surface of the boiler, how many radiators can be supplied by the boiler?

20. If 800 cubic feet of air are required for each person, how many people can occupy a room that contains 21600 cubic feet?

21. If 1 foot of 1-inch pipe is allowed for every 90 cubic feet of space in heating a factory, how many feet of the same pipe will be required to heat 225000 cubic feet of space?

22. A man undertakes to excavate a cellar for 29c. per cu. yd. He received $30.24 for the job. How many cu. yds. of earth were removed?

23. If you have 780 inches of wire, how many 12-in. pieces can you cut it into?

24. Glasgow is 3240 miles from New York City. A steamer makes the voyage between these cities in 9 days. Find the steamer's average rate per hour.

25. The seam of a single-rivetted smokestack is 7′–3″ long. If the rivets are 4″ between centres and start 1½″ from the edge of the plate at each end, how many rivets are required?

26. How many 30′ rails are there in a piece of double-track four miles long?

27. If a locomotive goes to the shop for inspection after every run of 210 miles, how many times will it be in the shop during runs aggregating 11760 miles?

28. A boiler supplies steam for heating. If there are 150 sq. ft. of heating surface in the boiler, and each radiator requires 13 sq. ft. of the heating surface of the boiler, how many radiators can be supplied by the boiler?

29. There are 90009 cu. ins. in 39 gals. How many cu. ins. are there in one gallon?

30. In how many hours will a cistern holding 12222 gallons be filled with a pipe that discharges into it at the rate of 194 gallons per hour?

31. If one foot of 1″ pipe is allowed for every 95 cu. ft. of space in a shop, how many feet of the same size pipe will be required to heat 300000 cubic feet?

32. The water main requires to be renewed the entire length of the shop, which is 612 ft. long. If it is made up of 12-ft. lengths of cast-iron pipe which weighs 12-lbs. per ft., how many lengths of pipe will be required, also, what will the total weight of the pipe be?

33. A carload of castings weighs 93688 lbs. If they are all alike and one weighs 98 lbs., find how many there are in the car.

34. 215789 lbs. is to be divided into 705 equal parts. How much will each part weigh?

35. A farm has 256798 sq. ft. of land and is to be divided as nearly as possible into plots containing 28756 sq. ft. Find (a) How many full plots can be had, and (b) How many sq. ft. will be left over.

36. If a locomotive travels 6224 ft. in two minutes, how long will it take to go 297088 feet ?

37. A steam pump delivers 4320 gallons of water in one hour. How long will it take it to deliver 554473280 gallons ?

38. If a train travels at the rate of 52 miles per hour, how long will it take it to go 3952 miles ?

39. A tank has a capacity of 4500 gallons. A feed pipe supplies it at the rate of 11 gals. per minute, and a discharge pipe empties it at the rate of 6 gals. per minute. Starting with the tank empty and all pipes open, how long will it take to fill the tank ? Give answer in hours.

40. A company of 29 men buy 1350 building lots at $417.00 each. These were subsequently sold for $590.00 each. Selling expenses, advertising, etc., amounted to $958.50. If they all held equal shares, how much profit did each man make ?

CANCELLATION

Cancellation. When a series of multiplied factors is to be divided by a second series the operation may be shortened by the process of cancellation as follows:

Example 1.

$$\frac{6\times8\times12\times18\times24}{2\times3\times\ 2\times\ 4\times\ 6}=?$$

$$\frac{\overset{3}{\cancel{6}}\times\overset{4}{\cancel{8}}\times\overset{4}{\cancel{12}}\times\overset{3}{\cancel{18}}\times\overset{6}{\cancel{24}}}{\underset{1}{\cancel{2}}\times\underset{1}{\cancel{3}}\times\underset{1}{\cancel{2}}\times\underset{1}{\cancel{4}}\times\underset{1}{\cancel{6}}}=864 \text{ Answer.}$$

Solution. To cancel, select any two numbers one above and one below the line and find some number that will divide each of them without a remainder. For instance, it is seen that 2 will be contained an exact number of times in both 6 and 2. Then perform the division, crossing out both numbers and placing the results directly over and below the numbers crossed out. Proceed in this manner until there is no longer a number below the line that can be cancelled with one above the line. Then multiply together all the numbers above the line and use this as a dividend; and multiply together those below the line, and use as a divisor.

Example 2.

$$\frac{16\times2\times15\times4}{32\times6\times8\times22} = ?$$

Solution. In an example where the product above the line, after cancelling, is less than the product below the line, the result is allowed to stand as obtained, thus: Since 5 and 22 do not cancel into any of the other numbers the result is the product of the quantities in the numerator divided by the product of the quantities in the denominator.

$$\frac{\overset{1}{\cancel{16}}\times\overset{1}{\cancel{2}}\times\overset{5}{\cancel{15}}\times\overset{1}{\cancel{4}}}{\underset{2}{\cancel{32}}\times\underset{\underset{1}{\cancel{3}}}{\cancel{6}}\times\underset{2}{\cancel{8}}\times22} = \frac{5}{88}$$

Rules. Cancel the common factors from both dividend and divisor.

Next, divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

It is seen, therefore, that cancellation is merely a combination of the processes of factoring and division.

PROBLEMS FOR PRACTICE

Divide:

1. $2 \times 3 \times 8 \times 12 \times 24$ by $6 \times 4 \times 36 \times 4$.
2. $18 \times 24 \times 32 \times 36$ by $9 \times 48 \times 4 \times 18$.
3. $15 \times 20 \times 25 \times 27$ by $15 \times 18 \times 25 \times 10$.
4. $40 \times 48 \times 54 \times 60$ by $30 \times 24 \times 72 \times 3$.
5. $12 \times 60 \times 36 \times 70$ by $28 \times 5 \times 48 \times 6$.
6. $32 \times 36 \times 33 \times 45$ by $24 \times 30 \times 44 \times 9$.
7. 18 piers, each consisting of 5 piles, were set by 10 men in 3 days. What is the cost of driving each pile if each of the men receives 3 dollars a day?
8. A bridge transmits to 3 girders each supported by 5 columns the loads of 2250 people, 45 horses, 69 vehicles. What is the load on each column, the average weights for the people, horses, and vehicles being respectively 150 pounds, 1100 pounds, and 1015 pounds?
9. Each of 40 teamsters hauls 9 yards of sand per day for 4 days at 1 dollar per yard. How many men with wheelbarrows will earn the same amount in two days wheeling 3 yards per day at 1 dollar per yard?

REVIEW QUESTIONS

1. How many times is one hundred and four contained in the sum of:—six thousand, nine hundred and thirty-five; forty-eight; thirteen thousand and eight; seven hundred and sixty-eight; and nine thousand, nine hundred and twenty-one ?

2. Find the sum of 348596, 3300992, 1472, 576829, 4488301, 119, 2216839, 201099 and 4583.

3. From the sum of 39528, 660001, 394857, 49592 and 6601, subtract the sum of 48593, 10009, 34928, 35725 and 625.

4. Divide 6398155575 by 693567.

5. Multiply 34925 by 66259.

6. A firm buys lumber at $39.50 per thousand feet to build into crates, each crate requiring 24 ft. of lumber. If the crates sell for $1.25 each, and labor on each crate amounts to 28 cts., how many must be sold to make a profit of $500.00 ?

7. What is the cost of building four miles of sidewalk, at the rate of 32 cts. per foot ?

8. A tank contains 34 cu. ft. of water. How many U.S. gallons does it contain, if one U.S. gallon contains 231 cu. ins. ?

9. What is the average weight of the following castings: 67 lbs., 341 lbs., 33492 lbs., 675 lbs., 4509 lbs., and 78 lbs. ?

10. If a gas-engine uses 1 cubic foot of gas per horsepower per minute, how many cubic feet of gas are required to run a 32 H.P. engine for one hour ?

11. A tank contains 526 cu. ft. of water. How many cubic inches does it contain?

12. What is the total weight of 560 castings of the same size, if each one weighs 345 lbs.?

13. A car weighing 32500 lbs. is loaded with 5600 lbs. cast-iron, 3769 lbs. wrought-iron, 4586 lbs. of brass, and 9675 lbs. of steel boiler plate. What is the combined weight?

14. At 22 cts. per cubic foot, what is the cost of excavating a cellar if 3498 cu. yds. of earth were removed?

15. A steam plant has two engines of 934 horse-power each. The starboard engine of a steamship develops 3218 horse-power and the port engine 3232 horse-power. How much greater power is developed in the ship than in the steam plant?

16. A pump delivers 33720 cubic feet of water in two hours (60 minutes each). How many cubic feet are delivered per minute?

17. A 9 ft. blower makes 175 revolutions per minute. How many revolutions will it make in 29 minutes?

18. There are 360 rivets in a hundred pounds. How many pounds will be required to forge 64,800 rivets.

19. The heating surface in a locomotive is 88 sq. ft. in the fire box and 792 sq. ft. in the tubes. What is the total heating surface? How many times as much surface is there in the tubes than in the fire box?

20. The weight of a battery of eight marine boilers and their equipments is as follows:

Boilers complete with mountings	295	tons
Water in boilers	73	"
Funnels	50	"
Stoke-hole plates, floors, etc	23	"
Feed pumps	7	"
Fans and fan engines	8	"
Feed regulators	2	"
Tools and fittings	2	"
Spare gear	10	"

What is the weight of two such batteries?

21. At 100 degrees Fahrenheit a cu. ft. of water weighs 62 pounds. At 205 degrees a cu. ft. weighs 60 pounds. What is the difference in weight between 173 cu. ft. of water at 100 degrees and 189 cu. ft. at 205 dgrees?

22. A roof is composed of 11 frames. The weight of one frame in detail is as follows:

2 Rafters each weighing	875	lbs.
5 Rods, each weighing	176	"
16 Bolts, each weighing	5	"
8 Bridle-straps, each weighing	15	"
2 Piers supporting rafters at ridge, average each	11	"
6 Pieces at foot of struts, average each	11	
4 Pieces uniting rafters at junction in strut, with bolts and nuts, each	44	"
2 Rafter shoes, each	144	"
2 Cast-iron struts, each	154	"

What is the total weight of the roof?

23. A double mine ventilating fan runs at the rate of 84 revolutions per minute. It gives 2818 cu. ft. of air

per revolution. How many cu. ft. should it give per minute?

24. If oak-tanned leather belting costs 2 dollars per ft., and four-ply stitched canvas belting costs 1 dollar per ft., what is the difference in the cost of 40 ft.?

25. If there are 9326 heat units in a pound of lignite coal, how many heat units in 287 pounds?

26. If 23 sq. ft. of No. 30 sheet-iron weighs 11 pounds, how much will 23 sq. ft. weigh if three times as thick?

27. The weight of the masonry of a bridge and an engine passing over it is 1698575 pounds. The engine weighs 198560 pounds. What is the weight of the masonry?

28. A pound of Pennsylvania petroleum will theoretically evaporate about 22 pounds of water. How many pounds are necessary to evaporate 4378 pounds of water?

29. A cu. ft. of hemlock weighs 25 pounds. A cu. ft. of iron weighs 450 pounds. Find the difference between the weight of 230 cu. ft. of hemlock and 87 cubic ft. of iron.

30. Three guy ropes are fastened to a stake. The pull on one rope is 560800 pounds; the pull on the second is 118421 pounds; and on the third is 104863 pounds. What is the total pull?

DECIMALS

We have seen how the system of notation as explained on page 1 advanced by tens. This system can be extended so as to include all quantities which are less than a unit.

If we have a quantity of which it requires ten to make a unit each part of this quantity would be called a tenth. Similarly, a quantity, ten of which are required to make a tenth is called a hundredth, and continuing in the same way we obtain a thousandth, a ten thousandth, etc.

For example, to obtain a dollar we require ten mills to make 1 cent, 10 cents to make 1 dime, 10 dimes to make one dollar, which is the unit.

Suppose we had 127 dollars, 64 cents, and 7 mills. It is clear that such a method of writing it would be inconvenient, but if we make a provision whereby it is understood which figures belong in the unit place, the word dollars, etc., could be omitted.

This is done by placing a point (called a decimal point) immediately to the right of the unit's place, and the above number could be written $127.647 and is read "one hundred and twenty-seven decimal, six, four, seven dollars."

ADDITION OF DECIMALS

Rule. Write the numbers down so that the same powers of ten come under each other, or in other words, keep the decimal points under each other and proceed exactly as in simple addition.

Example 1. Add 27.295+.0287+591.68+9.1846

```
 27.295
   .0287
591.68
  9.1846
--------
628.1883  Answer.
```

Place the decimal point in the answer directly beneath those in the question.

QUESTIONS

1. Add 163.6214, 3.851,10.004, and 361.08.
2. Add .05, .0045, .0002, 356.008. and .12.
3. Add 115.3804, .11001, 23.00, 1.50, and 5.246.
4. Add 736.1, 643, 72.141, and 63.001.
5. Add 6.111, 6.11, 611.1 and 6111.
6. Add 6.001, .03, 2.1 and 2.10103.

7. Add seventy-two decimal naught four; six hundred and forty-seven decimal naught eighty-five; decimal six four; decimal four five; eight hundred decimal naught eight four; five hundred and fifty-six.

8. Add decimal naught six four; decimal naught six naught six; eleven decimal four naught four; one thousand three hundred and two decimal six naught naught one three; and sixteen decimal naught nine.

9. Nine castings weigh as follows: 671.128 lbs., 39.0012 lbs., 1428.667 lbs., 267.8948 lbs., 301.9 lbs., 250 lbs., 1777.89 lbs., 449.5226 lbs., and 1000.0012 lbs. What is their total weight?

10. From Montreal to Ottawa is 116.2 miles, from Ottawa to North Bay 244.3 miles, from North Bay to Sudbury 79.2 miles, and from Sudbury to Fort William 556.3 miles. How far is it from Montreal to Fort William?

11. A steamship goes 385.4 miles the first day, 296.57 miles the second day, 347 miles the third day, 389.67 miles the fourth day, and 401.98 miles the fifth day. How far has it gone in the five days?

12. How far does a locomotive go in a week if the daily runs are 145.678 miles, 98.76 miles, 162.9305 miles, 45.6 miles, 78.056 miles, and 187.56 miles?

13. What is the total weight of castings weighing as follows: 5674.6978 lbs., 392.002 lbs., 4500.987 lbs., and 4576.0101 lbs.?

14. How much water does a steam pump deliver in five hours if it delivers 4567.0013 gallons in the first hour, 4003.19 gallons in the second hour, 3490 gallons in the third hour, 4235.625 gallons in the fourth hour, and 3116.10601 gallons during the fifth hour?

15. The following lengths of 1-in. round iron are required for a job: 16.75 ins., 7.1875 ins., 11.125 ins., 16.375 ins., 20.5 ins., 7 ins., 4.75 ins., 5.625 ins., and 19.875 ins. What is the total length of the bar required?

16. A coal shed has received the following shipments of coal: 145.382 tons, 47583.586 tons, 3004.002 tons, 177.801 tons, and 45.667 tons. What is the total amount of coal on hand?

17. A certain machine is composed of parts weighing as follows: 17.025 lbs., 400.5 lbs., 1725.67 lbs., 384.92 lbs., and 65.43 lbs. What is the total weight of the machine?

18. The following lengths of 6-in. cast-iron pipe are used in building a drain: 16.296 ft., 17.383 ft., 6.95 ft., 6.603 ft., 5.275 ft., 13.983 ft., and 8.175 ft. What was the total length of pipe used?

SUBTRACTION OF DECIMALS

Write the smaller number under the larger, keeping its decimal point directly underneath that of the larger number, and proceed as in simple subtraction.

Example 1. Subtract .07295 from 21.651.

The second number being the larger the question is put down thus:

```
21.651
  .07295
--------
21.57805 Answer.
```

Place a decimal point in the answer directly underneath those in the question.

Note. It can be considered that there are naughts above the 9 and the 5.

QUESTIONS

1. Subtract 891.93847 from 972.611.
2. Subtract 29.8 from 33.2675.
3. Subtract 316.23 from 791.647.
4. Subtract seven hundred and eighty-four decimal naught eight, from two thousand, four hundred and twenty-eight decimal eight.
5. Subtract decimal naught naught naught five from decimal naught naught five.
6. Subtract forty-seven decimal four two, from fifty-eight decimal two four six.
7. The shop water tank has a capacity of 100000 gallons. If there are 75626.475 gallons in it at present, how much does it lack of being full?
8. Passenger receipts on a railway for six days amounted to $625.70, $893.10, $700.30, $698.35, $1014.75, and $901.50, respectively. If the expenses amounted to $4690.80, how much profit was made?

9. From the sum of 345.648 2938.087, and 34.005, take the sum of 13.4567, 89.0014, 738.00, and 45.86.

10. A coal pile contains 4400 tons. The following amounts are removed from it: 34.5 tons, 1765.228 tons, 67.9 tons, and 560.125 tons. How much coal remains?

MULTIPLICATION OF DECIMALS

Rule. Proceed as in simple multiplication, but note that the decimal point in the answer has as many figures to the right of it as there are to the right of the decimal point in both numbers which were multiplied together.

Example 1. Multiply 431.552 by 65.49.

```
   431.[illegible]2
     65.49
  ---------
   3884868
  1726608
 2158260
2589912
-----------
28268.88948 Answer.
```

Note. There are three places to the right of the decimal point in the multiplicand (top number) and two places to the right of the decimal point in the multiplier (bottom number) or five in all, therefore there must be five places to the right of the decimal point in the answer.

QUESTIONS

1. Multiply .276×5.641.
2. Multiply .00346×234.698.
3. Multiply 69.007×1246.
4. Multiply 41.372×.195.
5. Multiply 276.428×16.4.
6. Multiply 12.003×248.3.
7. Multiply .01601×.016.
8. Multiply twenty-four decimal naught eight by decimal six one five.

9. Multiply one decimal naught naught naught one by decimal naught naught one.

10. A U.S. gallon of water weighs 8.33 lbs. What is the weight of 17.3 gallons?

11. A cubic foot of water weighs 62.5 lbs. How much will 177.3 cubic feet weigh?

12. A steam pump delivers 26.4 gallons per stroke. A gallon weighs 8.33 lbs. What weight of water will be delivered in 117 strokes?

13. In one pound of phosphor bronze .925 is copper, .07 is tin, and .005 is phosphorus. How much of each is there in 369.5 lbs.?

14. A round bar of rolled iron 2¼ ins. in diameter weighs 11.1 lbs. per foot. What is the weight of a bar of the same dia. and material which is 9.33 ft. long?

15. A train makes an average speed of 1.33 miles per minute. How many miles does it cover in 17.5 minutes?

16. How many feet are there in 34.6758 miles?

17. If the stroke of a 20 in. × 26-in. locomotive is found to be 2.167 feet, how many feet per minute will the piston travel when the engine is making 150 revolutions per minute.

18. What will be the cost of 56.8 tons of coal at $6.85 per ton?

19. If it costs 88.25 cts. per ft. to build a sidewalk, what is the cost of building a sidewalk 5 miles long?

20. Find the value of 4350 board feet of lumber at $45.50 per 1000.

21. The circumference of a driving wheel is 19.75 ft. How many revolutions will the axle have made when the engine has gone 166748.684 miles?

22. What is the weight of 456.785 ft. of round iron at 4.07 lbs. per foot?

23. Find the weight of 104 lots of metal, each lot containing pieces weighing 45.648 lbs., 2340.4 lbs., 34.71 lbs., and 20.019 lbs.

24. If a belt is travelling at the rate of sixty feet per second, how far will it have travelled in one hour, seventeen minutes, and three seconds?

25. A cubic foot of water weighs 62.32 lbs. What is the weight of 347693.23 cu. ft.?

26. In one Imperial gallon there are 277.274 cu. in. How many cu. ins. are there in 3568.013 gallons?

27. The outside diameter of a pipe is 6.125 ins. and the thickness of the metal is .625 ins. What is the inside diameter of the pipe?

DIVISION OF DECIMALS

Rule. Move the decimal point in the divisor as many places to the right as will make it a whole number. Move the decimal point in the dividend an equal number of places to the right, inserting naughts if necessary, and proceed as in simple division, remembering when crossing the decimal point in the dividend in carrying down figures to put the decimal point in the answer.

Example 1. Divide 10.6603 by 7.85.

```
785)1066.03(1.358
     785
    ----
     2810
     2355
     ----
      4553
      3925
      ----
       6280
       6280
       ----
```

Note that 10.6603 and 7.85 become 1066.03 and 785, thus making the divisor a whole number, also when carrying down the naught after dividing by one, the decimal point was placed in the answer before dividing by three.

Example 2. Divide 176.4 by .00012.

Carry the decimal point in the divisor over five places making the whole number 12, and by adding ciphers to 176.4 this becomes 17640000, a whole number. The question now is one in simple division, viz.: Divide 17640000 by 12.

$$
\begin{array}{r}
12)\underline{17640000} \\
1470000
\end{array}
\text{ Answer.}
$$

Example 3. Divide .0000672 by 84.

$$
\begin{array}{l}
84).0000672(.0000008 \\
\qquad\quad\ \underline{672}
\end{array}
$$

This kind of question always seems to present the most difficulty in placing the decimal point correctly. Note that the divisor is already a whole number, and no moving of the decimal point is necessary, also that there is no whole number in the dividend. In dividing it is neccessary to cross the decimal point in the dividend to carry down a figure, so first, place a decimal point in the answer to start with and carry down a naught. 84 into naught cannot go; place a cipher in the answer, take down next figure in the dividend and try that. This again cannot go, so add another cipher to the answer and so on until you get a figure that will divide, which is 8, after having put down six naughts.

QUESTIONS

1. Divide 873.5 by 6520.
2. Divide 673.1489 by .41432.
3. Divide 12.003 by 5690.
4. Divide 451 by .0019.
5. Divide 784.5673 by 9.1.
6. Divide 33000 by .7854.
7. Divide .7854 by 7.854.
8. Divide .01 by 23.65.
9. Divide one hundred and sixty-two decimal one, by forty-eight decimal six naught six.
10. Divide two hundred and ninety-five decimal three by two hundred and twenty-one decimal three naught five.
11. If a freight train runs at the rate of 15.75 miles per hour, how long will it take it to run 189 miles ?
12. The distance between two places in 167.74 miles. A train makes the trip in 5.116 hours. Find its rate in miles per hour.
13. A piece of boiler plate 3 ft. square was found to weigh 183.6 lbs. Find weight per square foot.
14. The weight of 19 steel plates is 1875 lbs. What is the weight of each plate ?
15. If 24 bolts weigh 83.56 lbs., what is the weight of each bolt ?
16. A truck wheel is 7 ft. 2½ ins. in circumference. How often will it turn in going 12.35 miles ?
17. In a U.S. gallon there are 231 cu. in. How many gallons are contained in 38703.309 cu. ins. ?
18. A piston travels 28.65 ins. to a stroke, and it makes two strokes while the engine is going 21.5 ft. How far will the piston have travelled when the engine has gone 234.57 miles ?

19. If 54 bolts weigh 234.89 lbs., what is the weight of each bolt?

20. How often will a truck wheel turn in going 1.33 miles if its circumference is 7 ft. 4 ins.?

21. What is a locomotive's average speed if it covers 3467 miles in a total of 568.97 hours? Answer in miles per hour.

22. Find to two decimal places the average weight of seven boys whose individual weights are 88, 95, 93, 110, 102, 87 and 101 lbs.

23. At a certain mine a ton of iron ore yields .55 tons of pure iron. How much pure iron will there be in 86.4 tons of ore?

24. A man takes 30 inches to a step. How many steps does he take in walking a mile?

25. A locomotive burns 7.25 tons of coal in running 80 miles. How much would it burn in running between Toronto and Montreal, a distance of 334 miles?

26. If the cutting speed of steel be 30 ft. per min., how many R.P.M. must a steel tire 198 ins. in circumference be run to cut at the proper speed?

27. If a truck wheel turns 1765.6 times in travelling 1.5 miles, what is the circumference of the wheel?

REVIEW QUESTIONS

1. A bar of iron 6.976 ft. long weighs 34.107 lbs. What is the weight of a bar 16.073 ins. long of the same material and diameter ?

2. What is the average weight of castings weighing as follows: 16.796 lbs., 105.02 lbs., 3,491 lbs., 190.996 lbs., 34.6701 lbs., 2305.05023 lbs., and 45.9981 lbs. ?

3. Divide 167.001 by .0043.

4. Multiply 996.305 by .00503.

5. Outside diameter of a pipe is 16.093 ins. Thickness of metal is 2.075 ins. What is the inside diameter ?

6. Which weighs the most, and by how much, 167.706 ft. of material at 35.096 lbs. per foot, or 103.45 ft. of material at 51.709 lbs. per ft. ?

7. What is the cost of laying 5.469 miles of sidewalk at an average cost of 76.392 cents per ft. ?

8. How many inches are there in 56.7904 miles ?

9. .26 of a casting's weight has been removed in machining. The finished castings weigh 67.96 lbs. What was the weight of the rough casting ?

10. The high and low diameters of a certain piece of work are 45.6 ft. and 23.056 ins. respectively, What is average diameter, in feet, inches and decimals of an inch ?

11. What is the weight in tons of a steel casting which contains 238375.674 cu. ins. ?

12. The total heating surface of a locomotive is 4500 sq. ft. and .4509 of this is in the firebox. Find the number of square feet of heating surface in the tubes of the boiler.

13. A planer takes 8 cuts per inch. How many cuts must be taken to plane a casting 30.5 ins. wide ?

14. How many revolutions will a boring mill table make in facing off a casting 64.75 ins. wide if the feed is $\frac{1}{4}$in. ?

LEAST COMMON MULTIPLE

To find the L.C.M. or Least Common Multiple of a series of numbers is to find the lowest number into which every one of the numbers in the series will divide evenly.

Example 1. Find the L.C.M. of 32, 16, 64, 8, 4, and 24.

4)	32	16	64	8	4	24
4)	8	4	16	2	1	6
2)	2	1	4	2	1	6
	1	1	2	1	1	3

$4 \times 4 \times 2 \times 2 \times 3 = 192$ Answer.

Explanation. Place all the numbers down in a line as shown and choose a number which will divide into the majority of the numbers in the series without leaving a remainder. In this case 4 was the first chosen as it divided into each of the given numbers. Continue dividing by any number until no more divisions can take place. The second number chosen was another 4, but note that 4 could not divide into 6 evenly, hence 6 was carried down just as it was, and the next number, 2, divided the remaining numbers as far as they could be divided.

The next step is to multiply together all the divisors and the numbers remaining on the bottom row. This gives 192, which is the Least Common Multiple for the series of numbers given.

To prove the correctness of any result each number should divide evenly into the L.C.M. without leaving a remainder, and in this example no other number less than 192 would be correct, and although there could be other numbers greater (as, for instance, 384 or 576) yet, while a number is desired which can be divided evenly by any number in the series, the required number must at the same time be the very lowest, or, as stated, it must be the **least** common multiple.

QUESTIONS

Find the L.C.M. of:

1. 4, 8, 16, 32, 4, 2.
2. 13, 52, 78, 312, 624.
3. 6, 24, 18, 30, 48, 36.
4. 16, 64, 32, 96, 80, 48.
5. 2, 16, 8, 32, 64, 128.
6. 4, 8, 12, 24, 40, 15.
7. 6, 16, 18, 12, 9, 81.
8. 7, 28, 36, 27, 21, 14.
9. 11, 18, 99, 44, 55, 9.
10. A sectionman walking along a track notices a milepost in line with a telegraph post. The telegraph posts are 66 yards apart. How far will he walk before he again finds a milepost and a telegraph post in line with each other?

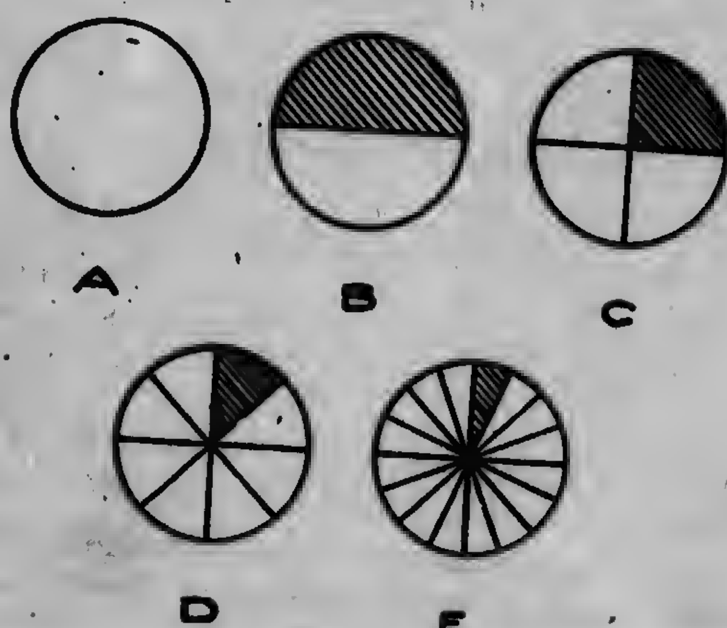
VULGAR FRACTIONS

We have seen how a part of a unit can be expressed by means of decimal fractions by placing a point immediately after the unit place, and while this system of fractions is the most convenient, there are occasions when parts of a unit are not expressed in decimals.

An example of this is in shop work when using a rule to measure a size, say $7\frac{1}{2}$ ins. In this dimension we have 7 one-inch units and half of another one. This method of writing a part of a whole number is known as a vulgar fraction.

Vulgar fractions are expressed thus: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{5}{8}$, $\frac{1}{32}$, etc. The upper number is known as the numerator and the lower number as the denominator.

The sketches show a number of circles all the same size, but each one is divided differently.



It will be seen that the shaded part of B is only one-half the size of the Circle A, yet it is twice the size of the shaded portion of C; this part being only $\frac{1}{4}$ of the circle. Similarly shaded parts D and E are only $\frac{1}{8}$ and $\frac{1}{16}$ the area of the whole circles.

The denominator (bottom number) therefore shows into how many parts the unit (or whole number) has been divided, while the numerator (top number) shows how many of these parts have been taken.

A study of these sketches will also show that one part of B is equal to 2 parts of C, 4 of D, or 8 of E, and these expressed as fractions, would read $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$, showing that fractions of a similar value can be expressed differently.

A rule governing this part is: A numerator and denominator multiplied by or divided by the same number do not change the value of the fraction. Thus:

$$\frac{1\times 4=4}{2\times 4=8} \quad \text{or} \quad \frac{8\div 8=1}{16\div 8=2}$$

Suppose there were three circles cut into shapes as shown in sketch B, that is, each circle is cut into halves, there would be a total of 6 halves. If these were separated into two groups of 5 and 1 these groups would be written thus: $\frac{5}{2}$ and $\frac{1}{2}$, the "2" (or denominator) showing that the unit (circle in this case) had been divided into two parts, and one group had five of them while the other had one.

Fractions are given different names, and any fraction less than a whole number is called a proper fraction and one greater than a whole number is called an improper fraction. In the last example $\frac{1}{2}$, being less than a whole number, is a proper fraction, and $\frac{9}{8}$, which is greater than a whole number, is an improper fraction.

A mixed fraction is a whole number and a proper fraction, thus: $6\frac{5}{8}$ is a mixed fraction.

A mixed fraction can be changed into an improper fraction.

Example. Reduce $6\frac{5}{8}$ to an improper fraction.

Each of the six units contains 8 eighths; therefore the six units contain 48 eighths, to which is added the 5 eighths, giving 53 eighths altogether, or $\frac{53}{8}$.

This can be more simply done by saying mentally 8 times 6 equals 48 and 5 makes 53. Place this down with the denominator 8 and the answer is 53 over 8, or $\frac{53}{8}$.

REDUCTION OF FRACTIONS

Fractions should always be reduced to their lowest terms. For instance, while $\frac{2}{4}$ represents a half, it is not reduced to as low terms as it could be. To do this, choose any number that will divide into both numerator and denominator evenly. In the case of $\frac{2}{4}$ each is divided by 2 and the result is $\frac{1}{2}$. This is simply applying the methods shown in the lesson on cancellation.

Examples. Reduce the following to their lowest terms:

$$\frac{6}{18} \qquad \frac{28}{32} \qquad \frac{55}{88} \qquad \frac{21}{63}$$

$\frac{6}{18}$ can be cancelled (or divided) by 6 and answer is $\frac{1}{3}$
$\frac{28}{32}$ " " " " 4 " " $\frac{7}{8}$
$\frac{55}{88}$ " " " " 11 " " $\frac{5}{8}$
$\frac{21}{63}$ " " " " 21 " " $\frac{1}{3}$

In practice this operation is simplified thus: Reduce $\frac{48}{120}$ to its lowest terms:

$$\frac{48}{120}=\frac{12}{30}=\frac{6}{15}=\frac{2}{5} \text{ Answer.}$$

The first cancellation (or division) was by 4, the second by 2, and the third by 3, all division being completed mentally.

QUESTIONS

1. Reduce one hundred and thirty-two, two hundred and thirty-firsts to its lowest terms.
2. Reduce thirty-six sixty-fourths to its lowest terms.
3. Reduce $\frac{378}{488}$ to its lowest terms.
4. Reduce $\frac{720}{864}$ to its lowest terms.
5. Reduce to its lowest term a fraction, the numerator of which is equal to the sum of 11, 7, 5, and 9, and denominator of which is equal to the sum of 13, 7, 35 and 9.
6. Reduce $\frac{648}{64}$ to a mixed fraction.

ADDITION OF FRACTIONS

If the fractions have the same denominator their sum is obtained by adding the numerators, thus:

$$\frac{1}{7}+\frac{4}{7}+\frac{5}{7}=\frac{1+4+5}{7}=\frac{10}{7} \text{ or } 1\tfrac{3}{7} \text{ Answer.}$$

If the fractions have different denominators each fraction must fiist be expressed as equivalent fractions having the same denominator,

Example 1. Add $\frac{1}{9}+\frac{3}{7}+\frac{5}{21}+\frac{2}{3}$

The L.C.M. of the denominators, 9, 7, 21, and 3, will be the common deuominator. This is 63, and each fraction must be made equivalent in value with 63 as its denominator.

The question is written down thus:

$$\frac{1}{9}+\frac{3}{7}+\frac{5}{21}+\frac{2}{3}=\frac{7+27+15+42}{63}=\frac{91}{63}=1\tfrac{28}{63}=1\tfrac{4}{9} \text{ Answer.}$$

In adding mixed numbers, add the whole numbers and the fractions separately, and their results together.

Example 2. Add $3\frac{1}{8}$, $\frac{7}{24}$, $7\frac{11}{15}$, $4\frac{3}{20}$.

First add the whole numbers $3+7+4=14$.

Then add the fractions:

$$\frac{1}{8}+\frac{7}{24}+\frac{11}{15}+\frac{3}{20}=\frac{15+35+88+18}{120}=\frac{156}{120}=1\tfrac{36}{120}=1\tfrac{3}{10} \quad \text{Ans.}$$

$$14+1\tfrac{3}{10}=15\tfrac{3}{10} \text{ Answer.}$$

QUESTIONS

1. Add $17+13\frac{1}{4}+9\frac{1}{2}+14\frac{5}{8}$.
2. Add $13\frac{9}{16}+17\frac{9}{32}+3\frac{1}{4}+17\frac{5}{8}$.
3. Add $12\frac{3}{4}+14\frac{5}{8}+7\frac{5}{8}+16\frac{7}{64}$.
4. Add $24\frac{1}{2}+16\frac{3}{8}+17\frac{5}{32}+\frac{3}{4}+\frac{7}{8}$.
5. Add $\frac{3}{8}+\frac{5}{16}+1\frac{5}{8}+\frac{1}{8}+\frac{7}{16}$.
6. Add $\frac{7}{32}+\frac{3}{4}+\frac{7}{8}+6\frac{3}{32}$,
7. Add seven-sixteenths, twenty-eight and one-half, thirty-six and one-eighth, and forty-four and three-sixteenths.

8. Add three one hundred and twenty-eighths, seven sixty-fourths, three and three thirty-seconds, and four and seven-sixteenths.

9. Add twenty-four and one-eighth, seventeen and three-eighths, twelve and one-quarter and seven one hundred and twenty-eighths.

10. Add $2\frac{1}{2}$ ins., $5\frac{3}{8}$ins., $3\frac{5}{8}$ ins., 1 ft. $7\frac{1}{4}$ ins., 2ft. $1\frac{7}{8}$ ins.

11. Three castings weigh respectively $225\frac{3}{8}$ lbs., $232\frac{5}{16}$ lbs., and $240\frac{1}{2}$ lbs. What is their total weight ?

12. A room is $32\frac{3}{8}$ ft. long and $29\frac{7}{8}$ ft. wide. What is the distance around the room ?

13. A steel rod is to be cut into 5 pieces. The first is to be $4\frac{5}{8}$ ins. long, the second $3\frac{3}{4}$ ins. long, the third $5\frac{3}{16}$ ins. long, the fourth $4\frac{3}{32}$ ins. long, and the fifth $1\frac{15}{16}$ ins. long. Find the length of rod required.

14. A casting after being machined weighed $18\frac{3}{4}$ lbs., but had $2\frac{1}{2}$ lbs. removed by planer. How much did the rough casting weigh ?

15. The total length of a steel shaft not having been given on a drawing it is required to figure it out by adding together the following dimensions: $4\frac{3}{4}$ ins., $6\frac{3}{8}$ ins., 2 ft. $6\frac{1}{4}$ ins. 2 ft. $2\frac{3}{8}$ ins., and $4\frac{7}{16}$ ins. What is the total length of the shaft?

16. How far will six rails lengthen a siding if they measure respectively 2 at 30 ft. $2\frac{1}{4}$ ins., 2 at 29 ft. $11\frac{3}{8}$ ins., and 2 at 28 ft. $6\frac{3}{4}$ ins.?

17. Six pieces were cut without loss from a bar of iron. The lengths were $16\frac{3}{8}$ ins., $18\frac{1}{4}$ ins., 2 ft. $6\frac{3}{4}$ ins., $9\frac{1}{8}$ ins., 1 ft. $11\frac{3}{4}$ ins., and $6\frac{3}{4}$ ins. What was the length of the bar of iron?

18. The length of a shop is 766 ft. It is required to install hydraulic piping the full length of the shop. Each pipe measures 16 ft. long and it is desired to have a tee every length of pipe to provide machines with branches. Each tee keeps the pipe ends apart $3\frac{1}{2}$ ins. when pipes are tightened into tees. How many tees are required, also how many lengths of pipes? Tees not required at extreme end of pipes. State what clearance space is left between shop wall and end of pipes by using full length pipes only.

SUBTRACTION OF FRACTIONS

In subtraction of fractions the principle is the same as in addition. Reduce the fractions to the least common denominator and subtract the smaller numerator from the larger.

In the case of mixed numbers subtract the whole numbers and the fractions separately.

Example 1.

$$\text{Subtract } \frac{7}{32} \text{ from } \frac{43}{64}$$

The L.C.M. of the denominators is 64, therefore:

$$\frac{43}{64} - \frac{7}{32} = \frac{43-14}{64} = \frac{29}{64} \text{ Answer.}$$

Example 2. Take $4\frac{5}{21}$ from $6\frac{3}{7}$.
First subtract the whole numbers:—$6-4=2$.

Then subtract the fractions: $\frac{3}{7} - \frac{5}{21} = \frac{9-5}{21} = \frac{4}{21}$

Answer is therefore, $2+\frac{4}{21}=2\frac{4}{21}$.

Example 3. From $7\frac{4}{15}$ take $4\frac{11}{25}$.
First subtract the whole numbers, $7-4=3$.
The L.C.M. of the denominators is 75.
$\frac{4}{15}$ is then changed to $\frac{20}{75}$, and $\frac{11}{25}$ becomes $\frac{33}{75}$.

It is apparent that $\frac{33}{75}$ cannot be subtracted from $\frac{20}{75}$, so in order to subtract these fractions one unit must be borrowed from the 3 which is left after subtracting the whole numbers. One unit is equal to $\frac{75}{75}$ths, therefore:

$$\frac{75+20}{75} - \frac{33}{75} = \frac{95-33}{75} = \frac{62}{75}$$

The complete answer is therefore $2\frac{62}{75}$ths.

QUESTIONS

Subtract:

1. $4\frac{7}{16}$ from $9\frac{1}{4}$.
2. $9\frac{11}{32}$ from $17\frac{9}{16}$.
3. $9\frac{5}{8}$ from 1
4. $\frac{7}{16}$ from $1\frac{3}{16}$.
5. [illegible] from 1[illegible].
6. $7\frac{1}{3}$ from $9\frac{3}{8}$.

7. Subtract seven and five-twelfths from nine and three-eighths.

8. Subtract eleven and seven-eighths from sixteen and three-sixteenths.

9. Subtract three-eighths from one hundred and twenty and five-sixteenths.

10. From a shaft 16 ft. $11\frac{3}{4}$ ins. long take 11 ft. $6\frac{1}{2}$ ins.

11. A rod of iron has two pieces cut from it which measure $19\frac{1}{2}$ ins. and $16\frac{3}{4}$ ins. If the rod were twenty feet long how much would be left, supposing no loss took place in cutting pieces off ?

12. In one day there was melted $14509\frac{3}{8}$ lbs. of iron in a furnace and another day there was $16417\frac{3}{4}$ lbs. melted. How much more iron was there melted the second day over what was melted the first day ?

13. There was cast from the total iron melted in Question 12, two cylinder covers each weighing $236\frac{1}{4}$ lbs., two steam pipes together weighing 878.6 lbs., six pistons each weighing $265\frac{5}{8}$ lbs., and 60 brake shoes, each weighing $30\frac{7}{8}$ lbs. How much of the iron would be left for other purposes ?

14. A tank is filled by two pipes which supply 1200 and 2250 gallons per hour respectively, and is being emptied by a pump which delivers 5800 gallons per hour. Starting with 9000 gallons in the tank, how much water will it contain at the end of two hours ?

15. Out of a tank containing $456\frac{11}{45}$ cu. ft. of water there is taken $103\frac{7}{128}$ cu. ft. How much remained ?

16. If four pieces measuring $6\frac{3}{4}$ ins., $4\frac{1}{8}$ ins., $15\frac{5}{8}$ ins., and $7\frac{11}{16}$ ins., were cut from a bar of metal 6 ft. $5\frac{3}{4}$ ins. long, how much would remain if $\frac{1}{16}$ in. was lost on each cut?

17. What would be left of 156 ft. $6\frac{3}{8}$ ins. of iron after the following lengths had been cut from it: $8\frac{1}{8}$ ins., $6\frac{1}{4}$ ins., $11\frac{2}{3}$ ins., 12 ft. 6 ins., $19\frac{7}{8}$ ins., 34 ft. $10\frac{5}{8}$ ins., and 7 ft. $8\frac{3}{4}$ ins.?

18. How much would be left in a tank containing 1200 gallons of water when the following quantities had been taken from it? $7\frac{7}{8}$ gals., $19\frac{3}{8}$ gals., $231\frac{5}{8}$ gals., $45\frac{2}{3}$ gals., and $9\frac{7}{16}$ gals.

19. How much was cut off a bar of iron 5 ft. $5\frac{1}{8}$ ins. long, when 2 ft. $11\frac{2}{3}$ ins. remains?

20. A box-car and contents together weigh $345670\frac{5}{8}$ lbs. If the contents weigh $200310\frac{7}{8}$ lbs., what is the weight of the car?

MULTIPLICATION OF FRACTIONS

To multiply a fraction by a whole number, multiply the numerator by that number, which becomes the new numerator, the denominator remaining the same as the fraction.

Example 1. Multiply $\frac{2}{3}$ by 5.

This really means add $\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$ and the answer is $\frac{10}{3}$ or when reduced to its lowest term $3\frac{1}{3}$.

A simpler way is $\frac{2\times5}{3}=\frac{10}{3}$ or $3\frac{1}{3}$. Answer.

As 5 represents 5 units, or 5 ones, written $\frac{5}{1}$, the above question could be solved thus: $\frac{2}{3}\times\frac{5}{1}=\frac{10}{3}$ or $3\frac{1}{3}$, both numerators being multiplied together and both denominators being multiplied together.

The rule to multiply fractions is thus arrived at and is: Multiply the numerators together, likewise the denominators, and reduce the result to its lowest terms.

Example 2. Multiply $\frac{5}{11}$ by $\frac{13}{20}$.

$$\frac{5}{11}\times\frac{13}{20}=\frac{65}{220}=\frac{13}{44}\text{ Answer.}$$

In multiplication of fractions the work can be considerably shortened by cancelling out common factors in the numerators and denominators. For instance, in Example 2 the numerator 5 could have been divided evenly into the denominator 20 and would then become:

$$\frac{1}{11}\times\frac{13}{4}=\frac{13}{44}\text{ Answer.}$$

Example 3. What is the product of $\frac{3}{4}$ of $\frac{7}{15}$ of $\frac{8}{32}$ of $\frac{1}{2}$?

$$\frac{\overset{1}{\cancel{3}}}{4}\times\frac{7}{\underset{5}{\cancel{15}}}\times\frac{\overset{1}{\cancel{8}}}{\underset{4}{\cancel{32}}}\times\frac{1}{2}=\frac{7}{160}\text{ Answer.}$$

Example 4. Simplify $5\frac{1}{7}\times\frac{11}{27}\times1\frac{11}{24}$.

$$5\tfrac{1}{7}\times\frac{11}{27}\times1\tfrac{11}{24}=\frac{\overset{\cancel{3}}{\cancel{36}}}{\cancel{7}}\times\frac{11}{\underset{9}{\cancel{27}}}\times\frac{\overset{5}{\cancel{35}}}{\underset{2}{\cancel{24}}}=\frac{55}{18}\text{ or }3\tfrac{7}{18}\text{ Answer.}$$

Example 5. What is the product of $7\frac{1}{3}\times9$?

$$7\tfrac{1}{3}\times9=\frac{22}{\underset{1}{\cancel{3}}}\times\frac{\overset{3}{\cancel{9}}}{1}=66.\text{ Answer.}$$

QUESTIONS

1. Multiply $9\frac{3}{4}$ by $5\frac{7}{8}$.
2. What is the product of [illegible] ?
3. What is the product of [illegible] ?
4. Multiply [illegible].
5. Multiply [illegible].
6. Multiply [illegible].
7. Multiply forty-nine and five-twelfths by three and three-quarters by one and eleven-sixteenths by seven thirty-seconds.
8. Multiply one thousand nine hundred and forty-two and three-eighths by three hundred and four and five sixty-fourths.
9. Multiply three and five-sixteenths by four and eleven thirty-seconds by three-eighths by twenty-two and seven-eighths.
10. A bolt was found to weigh 1 lb. 12 oz. How much do 12 of them weigh ?
11. A water tube boiler has a grate surface of $27\frac{9}{16}$ sq. ft. It burns $15\frac{3}{4}$ lbs. of coal per sq. ft. per hour. How much does it burn in $1\frac{1}{2}$ hours ?
12. A point on the rim of an engine fly-wheel travels $16\frac{3}{4}$ feet per revolution. How far does it travel in $29\frac{5}{8}$ revolutions ?
13. What would be the weight of 1998 bolts that are found to weigh 2 lbs. 13 ozs. each ?
14. How far would 4506 rails reach if each is 7 ft. $7\frac{5}{16}$ ins. long ?
15. If the piston stroke is $34\frac{7}{8}$ ins. long, and the driving wheel makes 86 revolutions per minute, how far will the piston have travelled in 19 minutes $12\frac{1}{2}$ seconds ?

16. At $29\frac{1}{4}$ cts. per cu. yd., what is the cost of excavating a foundation containing 3648 cu. ft.?

17. There are $16\frac{1}{2}$ ft. in a rod. How many feet are there in $\frac{7}{8}$ of a rod?

18. A steel rod weighs $63\frac{3}{8}$ lbs. How much would a rod $5\frac{1}{4}$ times as large weigh?

19. A pile of coal weighs $34\frac{5}{8}$ tons. $\frac{1}{5}$ of this pile is removed. How many tons remain?

20. What is the cost of painting a surface containing $360\frac{1}{2}$ sq. ft., at $3\frac{1}{2}$ cts. per sq. ft.?

DIVISION OF FRACTIONS

In division of fractions invert the divisor and change the division sign into a multiplication sign, and proceed as in multiplication of fractions.

Example 1. Divide 30 by $\frac{3}{4}$.

$$\frac{30}{1} \div \frac{3}{4} = \frac{30}{1} \times \frac{4}{3} = \frac{40}{1} \text{ or } 40. \quad \text{Answer.}$$

Example 2. How many times is $3\frac{7}{8}$ contained in $41\frac{11}{32}$. $3\frac{7}{8}$ equals $\frac{31}{8}$, or, inverted, $\frac{8}{31}$. $41\frac{11}{32} = \frac{1323}{32}$.

$$\frac{1323}{32} \times \frac{8}{31} = \frac{1323}{124} \text{ or } 10\frac{83}{124}. \quad \text{Answer.}$$

QUESTIONS

1. (a) Divide 30 by $\frac{3}{4}$. (b) Divide $\frac{13}{16}$ by $\frac{7}{8}$.
2. (a) Divide 24 by $\frac{5}{8}$. (b) Divide $\frac{11}{16}$ by 348.
3. (a) Divide $30\frac{1}{4}$ by $\frac{3}{16}$. (b) Divide $\frac{7}{16}$ by $\frac{4}{16}$.
4. (a) Divide $48\frac{5}{8}$ by $\frac{7}{16}$. (b) Divide $34\frac{3}{4}$ by 11.
5. (a) Divide $3\frac{7}{8}$ by $\frac{15}{16}$. (b) Divide $\frac{3}{8}$ by 20.

6. Divide fourteen and eleven thirty-seconds by five and thirty-three sixty-fourths.

7. Divide one thousand and thirty-four and eleven-sixteenths by five thirty-seconds.

8. Divide one hundred and forty-nine and two-thirds by eighty-one and four-sevenths.

9. The speed of a pulley driving a lathe is 165 R.P.M. and it is required to run the machine $\frac{1}{3}$ faster. How many R.P.M. should the pulley run ?

10. A steam pump delivers $2\frac{3}{4}$ gallons per stroke. It delivers 330 gallons in $2\frac{1}{2}$ mins. How many strokes does it make per minute ?

11. A railroad $16\frac{1}{2}$ miles long cost $66937.25. How much did it cost per mile ?

12. Weighing $2\frac{5}{8}$ lbs. each, how many bolts should weigh 998 lbs. ?

13. A bar of machine steel 5 ft. 6 ins. long is to be cut into 13 parts without loss. What will be the length of each part ?

14. How many pieces each 2 ft. $8\frac{1}{2}$ ins. are there in 2 miles, 177 feet ?

15. A machine has turned out 56 articles, which is $\frac{7}{16}$ of its daily production. What is its average daily production ?

16. A man buys 123 bars of metal each weighing $72\frac{15}{16}$ lbs. He melts this down and makes articles out of it each

weighing $7\frac{1}{4}$ lbs. How many articles could he make, and what would be left over ?

17. Of the materials used in constructing a machine, $\frac{3}{5}$ is cast iron, $\frac{1}{15}$ is brass, and the remainder is steel. The steel weighs $849\frac{1}{4}$ lbs. What is the total weight of the machine ?

18. A machine planes off $\frac{3}{35}$ of a casting, and the machined casting weighs 455 lbs. What was the original weight of the casting ?

19. How many castings, each weighing $26\frac{3}{4}$ lbs., could be cast from $468\frac{3}{8}$ lbs. of material, and what would be left over ?

20. It is desired to place electric lights at intervals of 13 ft. 9 ins. along the length of a shop which is 755 ft. long. Lights start 6 ft. 9 ins. from each end of shop. How many lights should be ordered ?

CONVERTING FRACTIONS

It is sometimes necessary to change a decimal fraction to a vulgar fraction, or a vulgar fraction to a decimal fraction.

To change a decimal fraction to a vulgar fraction, draw a line underneath the fraction, put down a 1 under the decimal point, put down a naught for every figure after the decimal point and reduce to its lowest terms.

Example 1. Convert .25 to a vulgar fraction and reduce to its lowest terms.

$$.25 = \frac{25}{100} = \frac{1}{4}. \text{ Answer.}$$

To change a vulgar fraction to a decimal, annex a decimal point and naughts to the numerator and divide by the denominator.

Example 2. Convert $\frac{15}{16}$ to a decimal fraction.

$$
\begin{array}{l}
16)15.0000(.9375 \\
\underline{144} \\
\quad 60 \\
\quad \underline{48} \\
\quad 120 \\
\quad \underline{112} \\
\qquad 80 \\
\qquad 80
\end{array}
$$

QUESTIONS

1. Convert .625 to a vulgar fraction.
2. Convert .565 to a vulgar fraction.
3. Convert $\frac{13}{16}$ to a decimal fraction.
4. Express $4\frac{13}{16}$ in decimals.
5. What is the sum of $33\frac{11}{16}$ and 45.608? Give answer in (a) decimal fractions, and (b) vulgar fractions.
6. From a rod 4 ft. 11 $\frac{3}{4}$ ins. long there is removed 11 pieces each 3.56 ins. long. What remains of the rod ?
7. Divide $3\frac{15}{16}$ by .0425.
8. What is the sum of 34.55 ins., 2 ft. $11\frac{1}{3}$ ins, and 3.45 ft. ?
9. Add $6\frac{7}{8}$ ins., $13\frac{1}{4}$ ins., $3\frac{3}{4}$ ins., $\frac{3}{16}$ ins., $20\frac{5}{8}$ ins and 45.685 ins.

Answer in feet and decimals of a foot.

PERCENTAGE

In the previous lessons fractions have been dealt with, and the student will have a clear idea of using them to express parts of anything. So far only two methods have been used for expressing fractions or parts of a thing. Suppose there are 100 locomotives assigned to a terminal and fifty run west of the terminal and 50 to the east. If you wished to state what fraction or part of the engines were on each division you would readily say one-half, and it could be written in either of the following ways: ½ or .5.

There is a third method of expressing this or any other fractional part known as the "per cent" method, and if we were to apply it to the above case of the engines we would say that 50 per cent (written 50%) of the locomotives ran west and 50% ran east from the terminal. Adding the two per cents gives 100%, which represents the total or whole assignment of engines.

In every case where it is wished to use the per cent method of representing fractions, it is always understood that 100 represents the total or whole amount, and a certain %, say 5%, means 5 out of the hundred, or 5 per hundred. The word "cent," it should be remembered, is derived from a Latin word meaning one hundred, and is related to our common word cent used in money, the cent in this case being the 1/100th part of a dollar. Per cent means per hundred; 6% means 6 per hundred, or 6 out of every hundred. 75% means 75 out of every hundred. 100% of course would represent the entire amount.

When it comes to working out problems in percent it is really only an exercise in fractions, and every problem should be attempted with this in mind. Any percent rate can be changed to a vulgar fraction by writing it over 100 as a denominator. Thus: 5% becomes $\frac{5}{100}$, 25% becomes $\frac{25}{100}$, 89% becomes $\frac{89}{100}$.

The most simple form of question is as follows:

Problem. Find what 3% of 84 pounds will be.

To work out this question change the percent rate to a vulgar fraction; then find that fraction of 84, or in other words multiply that fraction by 84.

Solution.

$$3\% = \frac{3}{100}$$

$$\frac{3}{100} \times 84 = 2.52 \text{ pounds. Answer.}$$

Note. If the above multiplication is not plain to the student, he should review some exercises in multiplication of common fractions, which are based on cancellation. Since the figure 100 will so often occur in questions like the above it should be remembered that to divide a number, such as 84, by 100 it is simply a matter of moving the decimal point toward the left two places, which would give us .84 in the above question: multiplying .84 by 3 gives 2.52, and the amount is named in pounds in accordance with this problem.

PERCENTAGE PROBLEMS OF THE FIRST ORDER

1. 9% of the locomotives assigned to a terminal are held for repairs. How many locomotives will this be if the total assignment is 83 ?

2. A machinist drew a pay check amounting to $60.00 and banked 6% of it. How much money did he bank ?

3. Calculate the following amounts:

   (a) 25% of 300 castings.
   (b) 85% of 649 apprentices.
   (c) 3% of $786.23.
   (d) 11% of 76846 pounds.
   (e) $2\frac{1}{3}\%$ of 4768 tons.
   (f) $31\frac{1}{4}\%$ of 7640 pounds.
   (g) $5\frac{1}{2}\%$ of $87.48.
   (h) $33\frac{1}{3}\%$ of 384 feet.

4. A man is worth $12,000.00 and has 35% of his money invested in property. What is the value of his property ?

5. An apprentice has $87.00 deposited in a bank, and is getting 3% interest each year. What will the interest amount to for a year ?

6. A certain boiler is supposed to generate 110 horse power. Owing to the scale in the boiler it only generates 93% of its power. How many H.P. can it deliver.

7. What percent of the power is lost owing to the scale in the boiler of question No. 6 ?

8. If gun powder consists of 15% charcoal, how much charcoal is required in making 350 lbs. of gunpowder?

9. A machinist earned $73.48 and then was allowed a contract bonus of 3%. What was the amount of the bonus? What were his total earnings?

Since all percent rates can be converted into the form of common fractions, it also follows that they can be written as decimal fractions, for it was shown in a previous lesson that any vulgar fraction can be converted to a decimal fraction or vice versa.

Thus:

$3\% — \frac{3}{100} — .03$
$17\% — \frac{17}{100} — .17$
$25\% — \frac{1}{4} — .25$
$2\frac{1}{2}\% — \frac{1}{40} — .025$
Etc., Etc., Etc.

This offers another method of working out the questions in percent, in that the rate per cent may be converted to a decimal and the problem worked out as a question in multiplication of decimals.

Example. Find 3% of 84 pounds.

Solution. $3\% = .03.$
$.03 \times 84 = 2.52$ pounds. Answer.

For convenience in referring to the different parts of a question in percent they are given names as shown below.

3% of 84 pounds = 2.52 pounds.
(Rate in $\frac{1}{100}$ths) × (Base) = (Percentage).

The quantity of which the percent is taken is called the **base.**

The number of hundredths or % of the base to be taken is called the **rate.**

The result obtained by taking the required percent of the base is called the percentage.

Considering all the questions we have worked so far, it will be seen that the rate multiplied by the base will give the percentage. This rule will help to work problems in some cases, but for the average problem it is well to consider it as a question in simple fractions and follow the outline first explained.

PERCENTAGE PROBLEMS OF THE SECOND ORDER

Problem. A railroad yard contained 480 cars. 120 of these were empty. What percent were empties ?

You will notice that instead of percentage this question asks for percent or the rate. Looking at this problem as a question in fractions ask yourself the question what fraction of the cars are empty. Evidently there would be

$\frac{120}{480}$ or $\frac{1}{4}$ of the cars, or $\frac{1}{4}$ of 100% of the cars.

$\frac{1}{4} \times 100\% = 25\%$ of the cars are empties.

Notice that any fraction is converted to a percent rate by multiplying it by 100%.

Referring to the names of the terms in this problem it is evident that the 480 cars will be the "base," the 120 cars is the "percentage" of empties and the "rate" is the part required.

The rule covering questions of this order is that the percentage divided by the base will give the fraction. Then this multiplied by 100% will give the rate percent.

PRACTICE PROBLEMS

Problems of the Second Order

1. A railroad purchased 152 locomotives of which 19 were standard switchers. What percent of the order were switchers.

2. The total cost of a factory building was $12,646.40, which includes the architect's commission, $296.40. Find the percent of commission.

3. If the base is 25.8 and the percentage is 2.58, what will the percent be ?

4. Write out an explanation of how you worked question No. 3, and give a rule that will work percentage problems of the second order.

5. The premium for insuring an office building worth $12,000 at its full value, was $80. What was the percent ?

6. Plaster is made from a mixture of 5 bushels of lime and 7 bushels of screen sand. What percent of the whole mixture is sand ? Also state what percent will be lime.

7. An engine was bought for $8000, and later sold for $6000, what was the loss in percent ?

8. The total weight of a freight car when loaded is 148600 lbs., and the weight of the empty car is 41700 lbs.; what percent of the entire weight is the weight of the empty car ? What percent of the entire weight is the freight carried ?

9. A drawing called for a special bushing to be bored out to 4.25 ins. The work was turned out bored to a dimension 4.18 ins. What was the percent of error ?

PERCENTAGE PROBLEMS OF THE THIRD ORDER

Problem. If 5% of the weight of a casting is 65 lbs., what is the total weight ?

Solution. 5% of weight is 65 lbs.
1% " " " $\frac{65}{5}$ "
100% " " " $\frac{65}{5} \times 100 = 1300$ lbs. Ans.

After you understand the above it can be written out in a shorter manner as follows:

5% of weight is 65 lbs.
100% " " " $\frac{65}{5} \times 100 = 1300$ lbs. Ans.

Note. The last line reads as follows: 100% of weight is 65 lbs., divided by 5, then multiplied by 100, equals 1300 lbs.

Whenever the percent (rate) and the percentage are known, the base, which is 100%, can easily be found. The rule is to find the base, divide the percentage by the rate and multiply by 100.

PRACTICE PROBLEMS

1. A barrel of valve oil was damaged and there leaked out 5.04 gals., which is 8% of what it orginally contained. What was the original No. of gals. in the barrel ?

2. Find the base in each of the following cases:

(a) 3%......... 47 tons.
(b) 12½%................ 260 feet.
(c) 57%................. 496 lbs.
(d) 66⅔................... 573 cars.

3. The cargo of a steamer was insured for $\frac{1}{8}$% and the premium paid amounted to $1500. What was the value of the cargo?

4. Changes in operating a stationary boiler resulted in a saving of 0.48 ton of coal per day, which was a saving of 4%. What was the previous consumption of coal? What will the present consumption be? Note. The saving referred to was figured on the previous consumption of coal.

5. A certain kind of coal contains 11% ash. How many lbs. of ash does this represent after 5 short tons of coal have been burned?

GENERAL REVIEW OF PERCENTAGE

If, out of 100 pieces made, 12 do not pass inspection, it is said that 12% (12 out of every hundred) are rejected. If a quantity of steel is bought for $100 and sold for $140, the profit is 40%. The percent of gain or loss is found by dividing the amount of the gain or loss by the "original" number of which the percentage is wanted, and multiplying the quotient by 100.

Example 1. Out of a total output of 280 castings a day, 30 castings are rejected. What is the percent of bad castings?

Solution. $\frac{30}{280} \times 100 = 10.7\%$. Answer.

2. If by a new process 100 pieces can be made in the same time as 60 could formerly be made, what is the gain in output of the new process over the old, expressed in percent?

Original number, 60; gain is $100-60=40$ pieces.

Hence $\frac{40}{60}\times100=66.7\%$. Answer.

Care should be taken always to use the "original" number, or the number of which the percent is wanted, as the divisor in these percentage calculations. In the example just given it is the percent of gain over the old output 60 that is wanted, and not the percent of the new output. Mistakes are often made by overlooking this important point.

Example 3. What number, increased by 87% of itself, is equal to 1122?

Solution. Represent the number as 100%.
87%—the increase.
187%—the number after it is increased.
187%—1122.
$100\%—\frac{1122\times100}{187}=600$. Answer.

Example 4. What number, decreased by 35% of itself, is equal to 2600?

Solution. Represent the number as 100%.
35%—the decrease.
$100\%-35\%=65\%=$ the number after decrease.
$65\%=2600$

$$100\%=\frac{2600}{65}\times100=4000.$$ Answer.

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)

1.0 1.1 1.25 1.4 1.6 1.8 2.0 2.2 2.5 2.8 3.2 3.6 4.0 4.5 5.0 5.6 6.3 7.1

Example 5. Suppose you are burning coal that contains 11% ash and you wanted to buy enough so that you would have just a ton of real heat producer (long ton 2240 lbs.). How many pounds of coal would you buy?

At first you are apt to take 11% as the additional amount that is needed but a little thought will show that to be the wrong method.

The whole amount or 100% is made up of ash and combustible together, and if the ash is 11%, then the combustible part of the coal is 89%.

The required amount of coal would then be found as follows:

$$\frac{2240}{89} \times 100 = 2517 \text{ pounds. Answer.}$$

GENERAL REVIEW PROBLEMS IN PERCENTAGE

1. Write 50%, 25½%, 1¼%, and 550% as (a) decimal fractions, and (b) vulgar fractions reduced to their lowest terms.

2. What per cent of 100 is .5? What per cent of 200 is .75?

3. If the heating surface of a locomotive is 3600 sq. ft. and 160 sq. ft. of this is in the firebox, what per cent of the total surface is in the tubes?

4. A hydraulic crane can lift 10,000 lbs. when in good order. Through want of care in repairing leaks, etc., 25% of its efficiency is lost. How much can it now lift?

5. Through want of work a shop is ordered to reduce its staff 5%. 75 men are laid off. How many were originally employed?

6. An apprentice is paid 65% of the contract price paid a machinist. His total contract for two weeks amounted to $16.90. How much would the machinist have been paid for this work ?

7. During an eight-hour test 3110 lbs. of coal are used, the ashes removed at the end of this time weighing 65½ lbs. What is the percent of ash?

8. The average number of revolutions of an engine is 150 per minute. The maximum speed is 16% more revolutions. What is the maximum number of revolutions per minute ?

9. A furnace consumes 130 bushels of coal per day. If it is improved so that 30% of the coal is saved, how many bushels are saved ?

10. A man engaged on an old type planing machine turns out eight truck boxes per day, and the machine is repaired and brought up to date so that the man can turn out 11 boxes in the same time. What is the per cent of increase ?

11. A locomotive boiler carries a pressure of 220 lbs. per sq. in. It is submitted to a hydrostatic test, which calls for a 25% increase in pressure above normal conditions. What pressure will the steam guage show ?

12. A firm purchasing machinery gets 6½% off an order amounting to $46,575.25 for prompt cash settlement. What saving is thereby effected ?

13. During a month a machinist works 240 hours. His rate was 28 cts. per hour. He got $16.20 extra as contract balance. What per cent. did he make ?

14. An engine that when in good working order delivered 148 H.P. is found to be only 67% efficient. How many H.P. is it now delivering ?

15. A man's total wages amounted to $73.71, including contract. He made 35% contract. How many hours did he work if his rate was 26 cts. per hour?

16. A locomotive whose regular run is 117 miles has gone 89 miles. What percent of the day's run has it completed?

17. In a train composed of fifty cars (both box-cars and flat-cars) 64% are box cars. How many flat-cars are there?

18. What is the weight of two castings, if the first one weighs 67 lbs. and the second one is 115% heavier?

19. In four days one man does the same amount of work that another man would do in six days. What per cent more work would the first man do than the second in the same amount of time?

20. Of a machine 456 lbs. are steel and 990 lbs. are iron. What per cent of the total weight does the steel comprise?

21. A crane that is only 65% efficient is able to lift 16000 lbs. How much would it lift if 90% efficient?

INVOLUTION AND EVOLUTION

Involution and evolution treat upon the powers of numbers.

A number raised to the second power is simply a number multiplied by itself: thus, 9 is the second power of 3, as $3 \times 3 = 9$.

Similarly $4 \times 4 \times 4 = 64$, showing that four raised to the third power equals 64, or 64 is the third power of four.

To indicate what power a number has been raised to, a small figure (called the exponent) is placed immediately above the number, to the right, thus 6^2 means that six is multiplied by itself, and the exponent shows that 6 is to be raised to the second power. This is often called "squaring" a number.

The raising of a number to a certain power is called involution. The opposite of involution is evolution, and involves extracting a root. Thus, 36 is the second power of 6, and 6 is the square root of 36.

Raising a number to a certain power (involution) presents no difficulties, multiplication being all that is necessary, but extracting a root (evolution) is more complicated.

The Radical sign $\sqrt{}$ placed before a number indicates that some root of that number is to be extracted, and when the sign alone is used it is understood that the square root is to be extracted. All other roots have an index placed above the sign, thus:

$\sqrt{16}$ means to extract the square root of 16, and

$\sqrt[3]{27}$ means that the cube root of 27 is to be found.

The index number shows which root is required, but any root can be found by a combination of square and cube root. Thus, extracting the square root of a square root will give an answer to the fourth root, and so on.

Example 1. A square shop has a floor area of 10588516 sq. ins. What is the length of one side ?

```
                  —  —  —  —
                  10 58 85 16 (3254 inches.  Answer.
  3               9
  3             ____
  —
                  1 58
 62               1 24
  2              _____
 ---
                   34 85
645                32 25
  5               ______
 ____
                    2 60 16
6504                2 60 16
```

3254 inches, squared (or raised to the second power) would give 10588516, which shows how to prove that a question has been correctly solved.

The operations used in extracting the above root are as follows:

1st. Point off the number into groups of two figures each, commencing from the unit's place.

2nd. Look at the left period and say, "What number multiplied by itself will be ten or less ?" 3 meets this requirement, therefore 3 is put down in the answer, and also used as a divisor (note working of example) in the left-hand column. Place 9 (the result of 3 times 3) under the first period, 10, and subtract.

3rd. Bring down the next period, 58, beside the remainder 1, and add another figure of similar value to the unit figure which was used in the divisor, which in this case was 3, therefore 3 added to 3 equals 6.

4th. Say "What figure added to the 6 and used as a divisor would be equal to or less than 158 ?" 2 does this, so

2 is the second figure in the answer, and is placed beside the six in the left-hand column and becomes 62. $2 \times 62 = 124$; therefore, 124 is placed beneath the 158 and subtracted from it.

5th. Bring down the next period of two figures (85) and always double the last figure of the divisor to obtain part of the next divisor, and so on to conclusion of question.

QUESTIONS

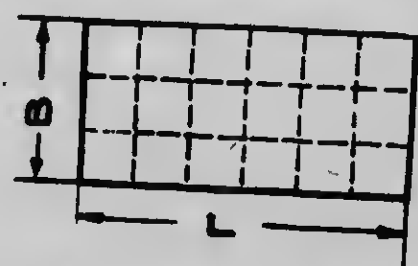
1. Raise 3467 to the second power.
2. Raise 374 to the fourth power.
3. Extract the square root of 104976.
4. What is the area in square feet of a field 280 ft. long on one side, the field being square ?
5. Extract the square root of 2815684.
6. Find the fifth power of 9.
7. Extract the fourth root of 20736.
8. A square field contains 20816 sq. ft. What is the length of one side ?
9. Extract the square root of 41616.
10. Simplify $\sqrt{16^3}$.

MENSURATION

The area of any plane figure is contained within its boundaries and is estimated by the number of square inches, square feet, square yards, or square miles it contains. A square is a four-sided figure which has four equal sides and four equal angles, and for comparison of area every figure, no matter of what shape, must be expressed in squares of the same unit.

Unless this were done it would be impossible to tell accurately whether a circle 6 ft. in diameter or a square having 5 ft. sides were the larger.

The rectangle is the most common figure, and sketch shows one which measures 3 units by 6 units. To illustrate the number of square units contained within this figure,



lines have been drawn dividing the figure into 6 parts on its length and 3 parts on its breadth, and by actual count there are 18 square units as shown.

A simple way to obtain this result is to multiply the length by the breadth, or length × breadth = area.

Thus: $6 \times 3 = 18$ square units.

The unit size depends on the unit of length given in the question. Had the rectangle been 6 in. long by 3 ins. wide, the answer would have been 18 square inches.

Note: Length and breadth must be expressed in the same units.

In mensuration (this word means measurement of surfaces and volumes), in place of writing a long rule in words it is usual to abbreviate by means of letters and signs.

Thus the rule stated to find the area of a rectangle or square would be $L \times B = A$. This is simply using the first letter of each word and is read "Length multiplied by breadth equals area."

Since it has been proved by the diagram that the above rule is correct, it follows that if given the area, the length could be found by dividing the area by the breadth, and to find the breadth divide the area by the length, therefore these three rules pertaining to a rectangle or square could be expressed in a short manner, thus:

$$L \times B = A. \qquad A \div L = B. \qquad A \div B = L.$$

Such a method of expressing a rule is known as a formula. These, by reason of their simplicity, are used throughout all textbooks to express the various rules for solving different kinds of questions. The letters used are the initial letters of the words under consideration and the signs are the usual arithmetical signs and symbols.

Example 1. Find area of a rectangle measuring 42 ft. by 16 ft.

$$A = L \times B.$$

Therefore $A = 42 \times 16 = 672$ sq. ft. Answer.

Example 2. Find the length of a rectangular shop which is 36 ft. wide and contains 3240 sq. ft.

$$L = A \div B.$$

Therefore $L = 3240 \div 36 = 90$ ft. Answer.

Example 3. What is the length of the side of a square which contains 68644 sq. ft. ?

Here there is only one size given, viz., the area, but as the question states that the figure is a square the solution is found by simply extracting the square root of the number.

$$\sqrt{68644} = 262 \text{ ft.} \quad \text{Answer.}$$

QUESTIONS

1. What is the area of a piece of plate 14 ins. × 56 ins. ?

2. What is the area of a field 456 ft. long and 98 ft. wide ?

3. How many square feet of roofing are required for a roof, each side of which measures 20 ft. × 48 ft. ?

4. What is the area in square feet of a shop 280 ft. long on one side and 125 ft. long on the adjacent side ?

5. How many square feet would 124 pieces of plate cover, dimensions of the plate being 2 ft. 6½ ins. × 1¾ ins. ?

6. Area of rectangular shop is 6975 sq. ft. It is 48 ft. wide. How long is it ?

7. Find the area in square feet of the walls of a building 80 ft. × 120 ft., the height of the walls being 16 ft.

8. What size would a square piece of plate require to be in order to have an area 45834 sq. ins. ?

9. A square cast-iron plate has an area, on one side, of 64284 sq. ins. What is the length of one of its sides ?

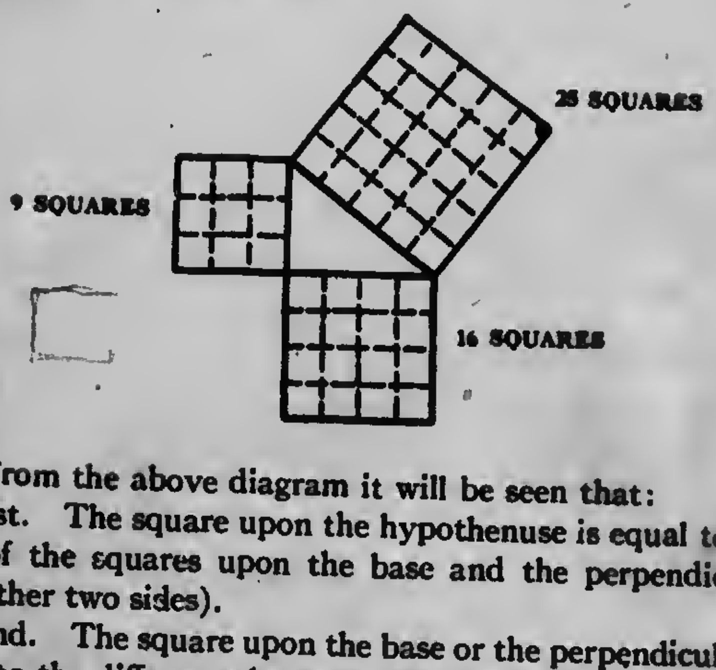
10. The floor of a building is to be renewed. It measures 72 ft. 6 ins. × 18 ft. What will be the cost of renewing it at 12½cts. per sq. ft. ?

11. It is desired to replace 3 scrap platforms, one of which is 15 ft. square and the remaining two 24 ft. square, with a single large one having the same area as the other three combined. What will be the dimensions of the large platform if it is built square ?

TRIANGLES

Triangles are three-sided figures of various shapes, and are named sometimes from their sides and sometimes from their angles.

In every triangle the greater side is opposite the greater angle, and in the case of a right-angled triangle the greater side is called the "hypothenuse." The other two sides are called the "Base" and the "Perpendicular."



From the above diagram it will be seen that:

1st. The square upon the hypothenuse is equal to sum of the squares upon the base and the perpendic (the other two sides).

2nd. The square upon the base or the perpendicular is equal to the difference between the other two squares.

From the foregoing facts it will be seen that the length of any side of a right-angled triangle can be found when the other two sides are given.

Example 1. The sides of a right-angled triangle are 45 ins. and 60 ins. respectively. Find length of hypothenuse.

The square upon the short side $= 45^2 = 2025$ sq. ins.
" " " longer $= 60^2 = 3600$ " "
Therefore, the square on the hypothenuse $= 5625$ " "

From the previous lessons upon squares it was shown that if given the area of a square to find the length of its side, it was necessary to extract the square root. Therefore $\sqrt{5625} = 75$ ins., which is the length of the hypothenuse.

Example 2. The length of the hypothenuse of a right-angled triangle is 65 ins. and one of its sides is 39 ins. What is the length of the other side ?

The square upon the hypothenuse $= 65^2 = 4225$ sq. ins.

The square upon the given side $= 39^2 = 1521$ sq. ins.

Therefore, the square on the other side $= 2704$ sq. ins.

Length of other side $= \sqrt{2704} = 52$ ins. Answer.

These rules expressed as formulas would be:

$H = \sqrt{B^2 + P^2}$ $P = \sqrt{H^2 - B^2}$ $B = \sqrt{H^2 - P^2}$.

QUESTIONS

1. Find the hypothenuse of a right-angled triangle, the sides being respectively:

1. 24 and 32.
2. 45 and 60.
3. 57 and 76.
4. 399 and 532.
5. 271 and 314.
6. 316 and 365.
7. 365 and 402.
8. 147 and 206.
9. 306 and 390.
10. 603 and 670.

Find one side of a right-angled triangle, the hypothenuse and other side being respectively:

1.	45 and 27.	2.	55 and 44.
3.	75 and 60.	4.	440 and 274.
5.	365 and 313.	6.	563 and 603.
7.	635 and 603.	8.	536 and 47.
9.	653 and 236.	10.	356 and 114.

In a few of the preceding questions it will have been noted that in some cases the third side comes out an exact number. When such is the case the sides are either, 3, 4, and 5 (see diagram) or some multiple of those numbers, so that whenever the sides of a triangle are in that proportion the angle opposite the hypothenuse is a right angle. This is worthy of note as frequently a right angle can be struck in this manner.

QUESTIONS

1. The hypothenuse of a right-angled triangle is 234 ft. and the base is 98 ft. long. What is the perpendicular height?

2. How long is the diagonal of a rectangular shop 153 ft. × 105 ft.?

3. The side of a square is 15 ft. Find to three places of decimals the length of its diagonal.

4. A square piece of plate contains 2564 sq. ins. What is the length of the diagonal?

5. The hypothenuse of a right-angled triangle is 348 ft. and the perpendiculat height is 296 ft. What is the length of the base?

6. The foot of a ladder 50 ft. long is 14 ft. from the wall of a house, and its other end just reaches top of window. The foot of ladder is moved 30 ft. more, the end now touching bottom of window. What is the distance from top to bottom of window?

7. A jib-crane requires a tie-rod. The height of the mast is 6 ft. 3 ins., and the length of the trolley runway is 18 ft. What length does the tie-rod require to be?

8. An engine-room measures 69 ft. across corners and is 58 ft. long. How wide is it?

9. A roof of 22 ft. span measures 14 ft. down each of its slopes, from the ridge to the eaves. How high is the ridge of the roof above the level of the eaves?

AREA OF TRIANGLES

In the following sketches note that the shaded portion representing different shaped triangles is exactly half the size of the rectangular figure enclosed by the dotted lines, therefore in any triangle the area is equal to half the area of a rectangle which has the same base and perpendicular height, or, if expressed as a formula where

B = Base

P = Perpendicular height.

A = Area

$$A = \frac{B \times P}{2}$$

Example 1. Find the area of a triangle having a base 27 inches long and a perpendicular height of 34 inches.

$$A = \frac{B \times P}{2} = \frac{27 \times 34}{2} = 459 \text{ sq. ins. Answer.}$$

To find the base (or the perpendicular height) of any triangle when the perpendicular height (or the base) is given, double the area and divide by the given dimension, expressed as a formula, thus:

$$B = \frac{A \times 2}{P} \qquad P = \frac{A \times 2}{B}$$

Example 2. A steel plate shaped like a right-angled triangle has an area of 90534 sq. ins., and the perpendicular height is 764 ins. What is the length of the base ?

$$B = \frac{A \times 2}{P} = \frac{90534 \times 2}{764} = 237 \text{ inches. Answer.}$$

QUESTIONS

1. What is the area of a triangle 66 ft. wide on the base and being 108 ft. in perpendicular height ?
2. What is the perpendicular height of a triangle containing 3048 sq, ft., its base being 56 ft. wide ?
3. What is the base of a triangle when the area is 2680 sq. ft. and the perpendicular height is 298 ft. ?
4. What is the area of a rectangular steel plate which is 66 ins. wide and the diagonal of which is 256 ins. long ?

5. A ladder is leaning against a wall. Top end is 24 ft. from the ground and bottom end is 12 ft. from the wall. How long is the ladder ?

6. What is the area of an equilateral triangle (one having three equal sides and three equal angles), the length of one side being 16 ins. ?

7. What is the area of a right-angled triangle, the hypothenuse being 48 ins. long and the base 16 ins. long ?

8. A right-angled triangle contains 2648 sq. ins. and the perpendiculat height is 116 ins. What is the length of the hypothenuse ?

CIRCLES

From the lessons in Drawing we learn what a circle is, also that the boundary of a circle is called the circumference and a line measuring through the centre from the circumference to the circumference is the diameter.

In all circles the circumference has a fixed ratio to the diameter, and this is 3.1416 to 1.

This can be proved by measurement and hence to find the circumference of any circle multiply the diameter by 3.1416, or to find the diameter divide the circumference by 3.1416.

Expressing these rules as formulas:—

Where D = Diameter

and C = Circumference

$$C = D \times 3.1416$$

$$D = C \div 3.1416$$

Example 1. A circle is $7\frac{1}{8}$ ins. in diameter. Find its circumference.

$$C = D \times 3.1416$$

Therefore $C = 7.125 \times 3.1416 = 22.3839$ ins. Ans.

Example 2. Find the diameter of a circle when the circumference measures 31.0233 ins.

$$D = C \div 3.1416$$

Therefore $D = 31.0233 \div 3.1416 = 9.875 = 9\frac{7}{8}$ ins. Ans.

Note. The fraction equivalent for 3.1416 is $3\frac{1}{7}$, which is not so correct as 3.1416 but is often used where an approximate answer is considered close enough. 3.1416 should be used where greater accuracy is required.

AREA OF CIRCLES

If a circle be drawn inside a square so that the circumference touches the sides of the square, it will be seen that the area of the circle is less than that of the square by reason of losing the amount cut off by the four corners.

The area of these corners is equal to the area of the square times .2146, and subtracting this from one we obtain .7854, which is the constant used in finding the area of a circle.

The area of a square is found by squaring its side. In a circle, therefore, the diameter is squared and the result miltiplied by .7854.

Expressed as a formula this is: $D \times D \times .7854$.

Example 1. Find the area of a circle $6\frac{1}{2}$ ins. in diameter.

$$A = D \times D \times .7854$$

Therefore $A = 6.5 \times 6.5 \times .7854 = 18315$ sq. ins.

Answer.

From the foregoing formula and example it will be seen that if it was required to find the diameter when the area was given, the formula would be as follows:

$$D = \sqrt{A \div .7854}$$

That is, to find the diameter divide the area by .7854 and extract the square root of the result.

A study of the following formulas will show the relation one part bears to another, and how various problems relating to the circle could be solved when certain parts are given.

D = Diameter	C = Circumference
A = Area	R = Radius

$C = D \times 3.1416$	$D = C \div 3.1416$
$A = D \times D \times .7854$	$D = \sqrt{A \div .7854}$
$C = \sqrt{A \div .7854} \times 3.1416$	$A = R \times 2 \times R \times 2 \times .7854$
$R = (C \div 3.1416) \div 2$	$R = \sqrt{A \div .7854} \div 2$

QUESTIONS

1. How far does a driving wheel 60 ins. in diameter advance in making one revolution ?

2. What is the area of a piston 22 ins. in diameter ?

3. The circumference of a pulley is 79.65 ins. Find the diameter, to two decimal places.

4. The cylinder in an air hoist is 14 ins. in diameter. If the air pressure is 90 lbs. per square in., what weight is the hoist capable of lifting ?

5. A driving wheel advances 19 ft. 11½ ins. in making one revolution. What is its diameter ?

6. With a pressure of 90 lbs. per square inch, what diameter cylinder is required in order that an air hoist lifts 13840 lbs. ?

7. What is the total pressure on a locomotive piston 28 ins. in diameter, the mean effective pressure being 160 lbs. per sq. inch ?

8. What is the circumference of a circle which has an area of 4576 sq. ins. ?

9. What is the circumference of circles $4\frac{1}{2}$ ins., $5\frac{3}{4}$ ins., and $9\frac{5}{8}$ ins. in diameter ?

10. Find area of circles 6 ins., $3\frac{7}{8}$ ins., $9\frac{5}{8}$ ins. and 11 ins. in diameter.

11. What is the diameter of a circle which is 278 ins. in circumference ?

12. Find diameter of circles having an area of 34 sq. ft. and 6 sq. ft. 8 sq. ins.

13. What is the area of a circle having a 4.563 ins. radius ?

14. What is the circumference of a circle that contains 3472 sq. ins.

15. The cylinder in a hydraulic press is 16 ins. in diameter and the press exerts a total pressure of 100,500 lbs. It is desired to reduce this to 70000 lbs. What is the inside and outside diameter of the bush that must be put in the cylinder ?

16. What pressure will be on the cylinder head of a locomotive, diameter of cylinder at counterbore being 25 ins. and the mean effective pressure being 100 lbs. per sq. in. ?

17. What diameter safety valve is required in order that it acts when a pressure of 2[illegible]bs. per sq. in. is reached, the total pressure then upon the valve being 1790 lbs. ?

18. What length of a band will be required to go round a cylinder 18 ins. diameter and overlap 2 ins. ?

19. How much can an air hoist lift if its cylinder is 12 ins. dia. and pressure exerted is 80 lbs. per sq. in. ?

20. What is the area of a section through the centre of a cast-iron ball that is 14 ins. in diameter ?

AREA OF RINGS

The sketch shown herewith illustrates the ring, which is frequently met in practice in the shape of the end of a pipe, bush, etc., and to find the area it is plain that if the area of the inner or smaller circle be subtracted from the area of the outer or larger circle, the result will be the area of the ring.

Expressing this as a formula where D = outside diameter and d = inside diameter:

$A = \overline{(D \times D) - (d \times d)} \times .7854$

While this is the formula generally used, it frequently happens that the following method is easier. Note that in sketch there is a dotted line midway between inside and outside diameter. This is called the "mean" or average diameter, and if MD = Mean Diameter and T = Thickness of Metal, as shown in sketch at T, then A = MD×3.1416×T.

Example 1. The end section of a pipe measures 6 ins. outside diameter and 4 ins. inside diameter. Find area of pipe by both methods.

First Method: $A = \overline{(D \times D) - (d \times d)} \times .7854$

$(6 \times 6) - (4 \times 4) \times .7854$

$(36 - 16) \times .7854$

$20 \times .7854 = 15.708$ sq. ins. Answer.

Second Method: $A = MD \times 3.1416 \times T$

$5 \times 3.1416 \times 1 = 15.708$ sq. ins. Answer.

QUESTIONS

1. What is the area of one side of a hub liner that is 28¼ ins. outside diameter and 14 ins. inside diameter ?

2. What is the area of the end of a bush, outside diameter being 24 ins. and inside diameter 18 ins. ?

3. What is the area of a circular racetrack 378 yds. inside diameter and 16 ft. wide ?

4. What is the total surface, inside, outside, and both ends, of a pipe 6 ft. long, 12 ins. inside diameter, and 18 ins. outside diameter ?

5. What is the area of the end of a cast-iron pipe that is 12 ins. outside diameter and 1 in. thick ?

6. What is the area of the end of a pipe 8.9208 ins. outside diameter and 6.07 ins. inside diameter ?

7. The area of the end of a pipe is 30.6306 sq. ins. The outside diameter is 8 ins. What is the inside diameter ?

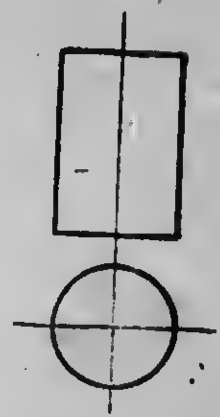
8. What is the area of the end of a bush that is 4¼ ins. outside diameter, and has a 1¼ in. hole running through the centre of it ?

9. What pressure acts upon the back cylinder head of a locomotive, diameter at counterbore being 28 ins., piston rod being 4 ins. in diameter, and the mean effective pressure in the cylinder being 140 lbs. per sq. in. ?

10. The total pressure acting on a locomotive piston in the back end of a cylinder is 19327.48 lbs. If the cylinder is 28 ins. in diameter and the piston rod is 4 ins. diameter, what is the mean effective pressure per sq. in. ?

CYLINDERS

Sketch shows plan and elevation of a cylinder.



Suppose there was paper placed around a cylinder which was 4 ins. in diameter and 6 ins. long. There would be required two circles, each 4 ins. in diameter, to cover the ends, while the body would require a rectangular shaped piece 6 ins. wide and having a length equal in distance to the circumference of a 4 ins. circle, hence the rule for finding the area of the surface of a cylinder is: Add the area of the two ends to the length times the circumference, the formula being as follows:

$$A = (D \times D \times .7854 \times 2) + (L \times D \times 3.1416)$$

Example 1. Find the surface of a cylinder 18 ins. diameter by 30 ins. long.

$$A = (18 \times 18 \times .7854 \times 2) + (30 \times 18 \times 3.1416) = 2205.4 \text{ sq. in}$$

Answer.

QUESTIONS

1. What is the area of the surface of a cylinder 28 ins. high and 9½ ins. in diameter?
2. What is the area of the surface of a cylinder 2 ft. 3½ ins. diameter and 6 ft. 1 in. long?
3. What is the total area of the surface of a pipe 6 ins. inside diameter and 8 ins. outside diameter, by 16 ins. long?
4. What is the area of the surface of a cylinder 17½ ins. long and 9¼ ins. in diameter?

5. The total surface of a cylinder is 348 sq. ins. The cylinder is 8 ins. in diameter. How long is it ?

6. At 21 cts per sq. yd., what is the cost of painting a circular tank 16 ft. in diameter and 18 ft. high ?

7. What is the area of the surface of a cylinder $1\frac{7}{8}$ ins. diameter and $2\frac{1}{4}$ ins. long ?

8. What is the area of the surface of a cylinder $16\frac{1}{2}$ ins. in diameter and 22 ins. long ?

9. What is the area of the surface of a cylinder 6 ft. 5 ins. in diameter and 11 ft. 9 ins. long ? Give answer in sq. ft. and sq. ins.

10. What is the area of the surface of a cylinder 14 ins. in diameter and 20 ins. long ?

MENSURATION OF SOLIDS
CUBES

Previous questions have taken into consideration the mensuration of surfaces only, in which two dimensions were usually given, namely, length and breadth.

In mensuration of solids, thickness (or depth) is considered in conjunction with the above, and of all solids the cube (or the rectangular prism) is the most simple. In the case of the cube all three dimensions are equal and each of its six faces are equal squares.

The unit of measurement of any solid is the cubic inch, foot, or yard, and to find the volume of a cube multiply the length by the breadth by the depth, or, expressed as a formula:

$$V = L \times B \times D$$

Note. Sizes of Length, Breadth, and Depth must be expressed in the same units.

Example 1. Find the number of cu. ft. contained in a cube having 16-in. sides.

$$V = L \times B \times D$$

$$V = \frac{16 \times 16 \times 16}{1728} = 2 - 10/27 \text{ cu. ft.} \quad \textbf{Answer.}$$

Example 2. What is the volume of a rectangular tank, inside dimensions being 8 ft. × 8 ft. × 16 ft. ?

$$V = L \times B \times D$$

$$V = 8 \times 8 \times 16 = 1024 \text{ cu. ft.} \quad \textbf{Answer.}$$

QUESTIONS

1. What is the volume of a cube 11 ins. long on one side ?
2. What is the surface of a cube 14 ins. square ?
3. What is the volume of an 18-in. cube ?
4. How many U.S. gallons will a tank contain whose inside dimensions are 6 ft × 3 ft. × 3 ft. ?
5. A rectangular tank contains 908 U.S. gallons. If the end of it measures 14 ins. × 16 ins., how long is the tank ?
6. What is the weight of a steel plate $20\frac{3}{4}$ ins. wide, 6 ft, $5\frac{1}{2}$ ins. long, and $\frac{3}{8}$ in. thick ?
7. What is the weight of a bar of .784 ins. square wrought-iron, its length being 12.25 ins. ?
8. A rectangular tank is 6 ft. long, contains 180 U.S. gallons, and is square on the ends. How deep is it ?
9. What is the weight of the water enclosed in a rectangular tank measuring 5 ft. × 3 ft. × $2\frac{1}{2}$ ft. ?
10. If a tank contains 345839 lbs. of water when two-thirds full and is 14 ft. long and 4 ft. deep, how wide is it ?
11. How many cu. ft. are contained in a tank $11\frac{1}{2}$ ins. × $22\frac{3}{4}$ ins. × 6 ins. ?

12. How many cu. ft. are contained in a tank 9 ft. 4 ins. long by 5 ft. 0½ ins. wide by 5 ft. 11 ins. deep, deducting 8% for angles, stays, rivet-heads, etc ?

13. How many U.S. gallons does a rectangular tank contain that measures 6 ft.×4 ft.×3 ft., when it is 67% f. 1 ?

14. At 25cts. per cu. yd., what is the cost of excavating a foundation measuring 68 ft.×96 ft.×16 ft. ?

CYLINDERS

The volume of a cylinder is found by multiplying the area of the base by the height, or, expressed as a formula:

$$V = D \times D \times .7854 \times L$$

Example 1. Find the volume, in cu. ft., of a cylinder which is 18 ins. diameter by 42 ins. long,

$$V = D \times D \times .7854 \times L$$

$$V = 1.5 \times 1.5 \times .7854 \times 3.5 = 6.185 \text{ cu. ft. Answer,}$$

Example 2. A cylindrical vessel of the same dimensions given in Example 1 is filled with water. How many U.S. gallons will it contain ?

$$V = D \times D \times .7854 \times L$$

1 cu. ft. = 7.5 U.S. gallons

No. of U.S. gals. $= 1.5 \times 1.5 \times .7854 \times 3.5 + 7.5 = 46.3875$.

Answer.

QUESTIONS

1. How many cu. ft. are contained in a cylindrical tank 16 ins. in diameter and 3 ft. 9 ins. long ?

2. How many U.S. gallons are contained in a cylindrical tank measuring 7 ft. in diameter and 12 ft. high ?

3. What is the weight of a steel cylinder 6 ins. outside diameter, 10 ins. long, with a hole 1¼ in. diameter drilled through the centre ?

4. What is the weight of a wrought-iron bar 1¾ in. in diameter and 16¾ ins. long ?

5. What is the weight of a cast-iron pipe, 24 ins. outside diameter, 16 ins. inside diameter, and 48 ins. long ?

6. A cylindrical bar of steel weighs 67 lbs. It is 3 ins. in diameter. How long is it ?

7. What is the weight of a brass siderod bush, 6 ins. outside diameter, 4½ ins. inside diameter, and 4 ins. wide ?

8. How many U.S. gallons are contained in a cylindrical tank measuring 18 ft. 6 ins. diameter by 16 ft. 0 in. deep, if a post 18 ins. in diameter is running through the centre of the tank ?

9. How many cu. ins. are contained in a cylindrical vessel 6.25 ins. in diameter and 8.374 ins. deep ?

RATIO

Ratio is the comparison of two numbers or quantities of the same kind, and indicates how much greater or less one is when compared with the other.

The sign for ratio is a single colon (:) but more often it is expressed as a fraction, or by a division sign.

If the ratio of 12 to 6 were required it could be written 12:6, 12÷6, or $\frac{12}{6}$, the answer being 2 to 1.

This is known as a direct ratio and is considered as two to one, but if the ratio of 6 to 12 were required, then the result would be an inverse ratio and would be 6:12, 6÷12, or $\frac{6}{12}$, and is equal to one to two. Unless otherwise stated, all ratios are understood to be direct.

The best comparison of ratio is with the unit as shown above, but this is only possible when the two numbers compared divide evenly.

When they will not divide evenly, the best way is to reduce them to their lowest terms by cancellation. Thus what is the ratio of 77 to 49 ? This, when cancelled by 7 equals 11 to 7, which is easier to understand than if 77 had been divided by 49, the answer to which would have been 1.551 or, when compared with the unit, as 1.551 to 1.

Example 1. If a boiler having a grate 4 ft. wide by 7 ft. long has a heating surface of 896 sq. ft., what is the ratio of grate area to heating surface ?

Grate Area $= 7 \times 4 = 28$ sq. ft.

Ratio $= 28:896 =$ 1 to 32. Answer.

Example 2. The ratio of the length to the width of a field is as 5 to 2. The width is 144 ft. What is the length ?

Length $= \frac{5}{2}$ of $\frac{144}{1} = 360$ ft. Answer.

QUESTIONS

1. The heating surface of a locomotive is 3600 sq. ft. and the grate measures 6 ft. by 8 ft. What is the ratio of grate area to heating surface ?

2. What is the ratio of the number of teeth in the driver to the number of teeth in the driven, in the following gears, used in a lathe ?—

	Driver	Driven
A	20	40
B	80	50
C	20	30
D	70	25

3. The diameter of a high-pressure cylinder is 20 inches and the diameter of the low-pressure cylinder is 30 inches. What is the ratio of their areas?

4. An indicator diagram shows the mean effective pressure per sq. in. in an engine's cylinders to be 95 lbs., and the steam gauge shows boiler pressure to be 220 lbs. per sq. in. What is the ratio of the mean effective pressure to the boiler pressure?

5. A boiler has a grate area of 4½ ft. × 5 ft., and has 745 sq. ft. of heating surface. What is the ratio of grate area to heating surface?

6. The ratio of a chimney's height to its width is as 11 to 2. The height is 38 ft. 6 ins. What is the width?

7. In a mixture of copper and tin the ratio is 3 of copper to 1 of tin. How much of each is contained in a casting weighing 176 lbs?

8. If a casting contains 384 lbs. of copper and 128 lbs. of tin, what is the ratio of the metals?

9. A planer has a cutting speed of 32 feet per minute and a return speed of 150 ft. per minute. What is the ratio of the cutting speed to the return speed?

10. The ratio of the height of a certain door to its width is 3: 2. It is 4½ ft. wide. How high is it?

11. Two air hoists have cylinders measuring 6 ins. and 8 ins. respectively. If the pressure in each case is 90 lbs. per sq in., what is the ratio of the weight that can be lifted by the first hoist as compared with the second?

12. The ratio of the height to the width of a certain tank is as 7 to 3. Diameter of the tank is 12 ft. How many U.S. gallons will the tank hold?

13. The ratio of two gears connected together is as 5 to 3. The first makes 105 revolutions per minute. How many does the second gear make?

PROPORTION

Proportion is the comparison of two ratios, and by its use numerous problems can be solved.

There are always three terms given and the solution calls for finding the fourth.

The signs of proportion are the single and double colons placed thus:— : :: : . This leaves four spaces in which the terms are placed, these being known by their number from left to right, as under:

1st term : 2nd term :: 3rd term : 4th term.

The fourth term is always required and has an × placed in it which means that this term is the unknown quantity and is to be found.

The third term is always of the same denomination as the answer will be, and is the only one of its kind mentioned in the given question.

The first and second terms are always given and are of the same denomination, but which one is to be taken as the second term depends upon whether the answer will be more or less than the given third term. If a larger answer is required, then the larger quantity is put down as the second term, but if the answer will be less than the third term, then take the lesser quantity of the two remaining terms as the second term. The remaining one is used as the first term.

The outside terms (1st and 4th) are the extremes and the inside terms (2nd and 3rd) are the means, and in any proportion the product of the means equals the product of the extremes.

Rule. Divide the product of the second and third terms by the first term, and the result will give the fourth term, which will be the answer required.

Example 1. If two men can machine a certain number of castings in 24 days, how long will it take 12 men to do the same amount of work ?

It is evident that the answer will be in days, therefore the third term must be 24 days, since it is the only term given in the question in days, and as 12 men will do the work faster than 2 men, the answer will be less than 24 days. The lesser number of men (2) will, therefore, be the second term, and the greater number of men (12) will be the first.

Solution:—

$$12 : 2 :: 24 : \times$$

$$\frac{2 \times 24}{12} = 4 \text{ days. Answer.}$$

QUESTIONS

1. If 15 tons of coal cost $60.00, what will 18 tons cost?

2. If 12 men can do a job in 4 days, how long will it take 8 men to do it ?

3. A column of water 18 ft. high exerts a pressure of 8.8 lbs. per sq. in. What pressure will a column 72 ft. high exert ?

4. A pump discharges 20 gal. per min., and fills a tank in 24 hrs. How long would it take to fill the tank with a pump discharging 42 gals. per min. ?

5. Find the cost of 28125 ft. of lumber at $13.50 per 1000 ft.

6. If the freight charges on a shipment be $40.50 for 216 miles, what should it be for 300 miles ?

7. 40 men accomplished a certain work in 35 days. How many men could have done the same work in 28 days?

8. If 416 bolts weigh 978 lbs., what will be the weight of 560 bolts?

9. A does a piece of work in 60 hours. B can do it in 40 hours. In what time will B do a piece of work which takes A 215 hours?

10. A machinist gets $3.00 per day and a helper $1.50 per day. How much would the helper receive when the machinist gets $75.00, providing both work the same number of days?

11. If a yardstick held upright casts a shadow 3 ft. 9 in. long, how long a shadow would be cast at the same time by a chimney 66 ft. 8 in. high?

12. A labourer can build a wall in 16 days by working 10 hours per day. How many days would it take him to do it by working 8 hours a day?

EQUATIONS AND FORMULAS

Throughout the previous lessons all rules have been stated and then expressed shortly by using the initial letter of the words in conjunction with certain signs, as, for instance, $L \times B = A$ meant length multiplied by breadth equals area.

As previously stated, this short method of expressing a rule is known as a formula, and it will have been noted that this was a very convenient method of remembering a rule when the quantities considered have only been three in number.

When, however, a fourth or more terms are added, and two or more are equal to a similar or greater number, the expression is called an equation and some care is necessary in evolving a formula from it to suit the problem in hand.

To illustrate this the next subject taken up is on transmission of motion and the equation for this is $D \times ND = F \times NF$. (See next lesson for meaning of letters.)

The first two quantities form a couplet and their product must equal the product of the second couplet.

It is unnecessary to remember more than this equation for transmission of motion questions, as all formulas can be built up from it and the method of doing so can be applied to all equations.

Rule.—When one term of a couplet is required, divide the product of the other couplet by the remaining term of the first couplet, thus:

$$D = \frac{F \times NF}{ND} \quad ND = \frac{F \times NF}{D} \quad F = \frac{D \times ND}{NF} \quad NF = \frac{D \times ND}{F}$$

A little study of the solutions of the questions on the next subject will make this clearer, and no trouble should be experienced if the correct formula is chosen to suit the requirements of the particular question under consideration.

TRANSMISSION OF MOTION
PULLEYS

It is not always possible to have an individual electric motor or a separate engine for each machine in a shop, and to overcome this, an arrangement of pulleys and belts, is often used to transmit the power from one point to another.

Generally where this is done a main shaft is run along the length of the shop supported upon hangers and which in turn retain bearings in which the shaft rotates.

The motor, or engine, as the case may be, has a pulley keyed (fastened by means of a key) to its shaft, and upon this main line of shop shafting there is another pulley keyed to it. These pulleys are connected by means of a belt and thereby the main shaft is revolved. From this shaft power is transmitted to all machines by a similar arrangement of pulleys and belts.

The ratio of movement of each shaft is inversely proportional to the diameters of the different pulleys.

The name given the pulley which gives motion is the "driver," and the name given to the one receiving motion is the "follower," sometimes spoken of as the "driven."

When D = Diameter of the driver

ND = No. of R.P.M. of the driver

F = Diameter of the follower

NF = No. of R.P.M. of the follower

Then $D \times ND = F \times NF$

Upon examination of the sketch shown below where both pulleys are the same diameter, it will be seen that if the circumference of the driver moves one foot the belt will likewise move the same distance, and will turn the circumference of the follower an equal space. One revolution of the driver must therefore transmit through the

movement of the belt one revolution to the follower. Hence, the number of revolutions of the driver multiplied by its circumference is equal to the number of revolutions of the follower multiplied by its circumference.

The circumference of all circles are in ratio to their diameter, and as diameters are always easier to measure in place of the circumference, in both cases call them diameters.

Expressing this rule as an equation:

$$D \times ND = F \times NF$$

Formulas can be made from this equation to solve any question.

Example 1. The shaft of a motor revolves at 840 R.P.M., and to it is keyed a pulley 24 ins. in diameter. Find how many R.P.M., a shaft upon which there is fastened a 36 in. pulley will revolve, if connected by a belt to the first pulley.

Solution:

$$D \times ND = F \times NF$$

$$24 \times 840 = 36 \times NF$$

$$NF = \frac{24 \times 840}{36} = 560 \text{ R.P.M.} \quad \text{Answer.}$$

QUESTIONS

1. A pulley 24 ins. in diameter is belted to another pulley 30 ins. in diameter which runs at the rate of 180 R.P.M. What R.P.M. does the first pulley make ?
2. The diameters of two pulleys are 18 ins. and 30 ins. The sum of their revolutions per minute is 250. Find R.P.M. of each.

3. A pulley 42 ins. in diameter drives another 18 ins. in diameter at a speed of 250 R.P.M. Find belt velocity in feet per second and R.P.M. of first pulley.

4. R.P.M. of the driver 180, of the follower, 100, Circumference of follower is 5½ ft. Find diameter of driver.

5. The difference between the number of revolutions of two pulleys in a given time is 60, and the ratio of their radii is as 2 to 3, the sum of the diameters being 30 ins. Find diameters and number of revolutions of each pulley in the given time.

6. Diameter of driver is 12 ins., and diameter of follower is 9 ins. What is the ratio of comparison between number of revolutions of each pulley ?

7. A pulley, 28 ins. diameter, is keyed to a main shaft making 200 R.P.M. This pulley is connected to another on a countershaft which makes 120 R.P.M. If the driving pulley on a lathe driven from the countershaft is 36 ins. in diameter, what R.P.M. does the spindle of the lathe make ?

8. The sum of the diameters of two pulleys is 42 ins. One makes 80 R.P.M. and the other 140. What are the diameters ?

9. The diameter of the driver pulley is 12 ins., and its speed is 400 R.P.M. What is the speed of the driven pulley, whose diameter is 3 ins. ?

10. A pulley 36 ins. in diameter drives another pulley 14 ins. in diameter. The belt velocity is 22 ft. per second. What are the R.P.M. of the pulleys ?

11. A pulley on a countershaft making 110 R.P.M. drives an 18-in. pulley on a drilling machine at the rate of 265 R.P.M. What is the diameter of the pulley on the countershaft ?

12. The diameter of the driving pulley is 9 ins. and its speed is 1000 R.P.M. At what speed will the follower run if its diameter is 4 ins. ?

13. The surface speed of an emery wheel is 3000 ft. per minute and its R.P.M. is 600. What is its diameter ?

14. The fly wheel of an engine is 12 ft. 6 in. in diameter and revolves at 96 R.P.M. It is belted to a 48 in. pulley on the main line shaft. Find the speed of the shaft.

15. A 36-in. pulley making 143 R.P.M. is belted to another making 396 R.P.M. Find the diameter of the latter.

16. A grindstone has a surface or rim speed of 800 ft. per minute. At how many R.P.M. would it run if its diameter is 4 ft. 8 ins. ?

17. The surface or rim speed of a grindstone is 500 ft. per minute and its diameter is 30 in. If the line shaft to which it is belted makes 300 R.P.M. and the driving pulley on this shaft is 3 ins. in diameter, what must be the diameter of the driven pulley on the grindstone shaft ?

18. The diameter of a piece that is being turned is two inches. It the piece makes 2000 R.P.M., what is its cutting speed ?

GEARS.

If shaft D in figure 1 has fastened to it a disc with a smooth circumference, the disc being in contact at the point P with another disc on shaft F, the rotation of one shaft will cause the other to rotate, provided there is sufficient friction between the two surfaces to prevent slipping.

Such friction discs will not transmit much power without slipping, and they cannot be depended upon to drive positively, as the least wear or loss of adjustment is liable to make them slip. Hence teeth are provided on each disc which, being made to mesh together, will transmit motion from disc to disc, in a positive manner, and without excessive friction in the bearings of shafts D and F.

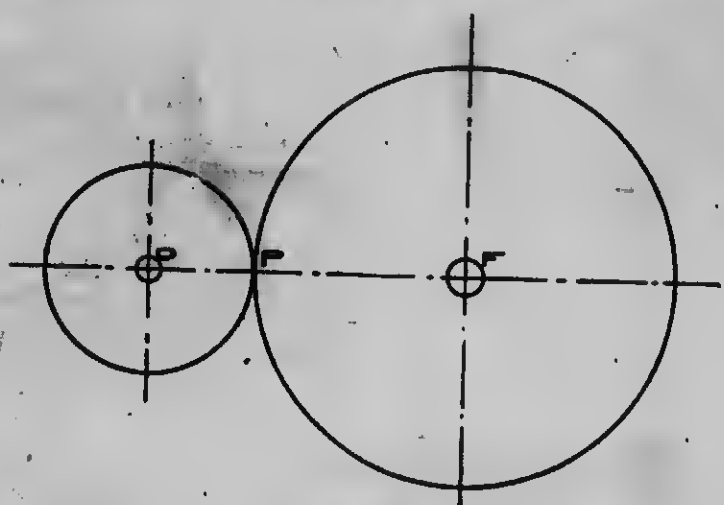


FIG. 1.

These discs having been given teeth (see Fig. 2, page 116) are called gears. The teeth may have any one of several forms and the gears may connect shafts which are parallel or intersecting at some angle. This part of the subject is studied in detail in the Drawing Course, and will not be considered here. These pages are devoted to presenting the various calculations relative to Gearing.

In the previous paragraphs on Pulleys, it was shown that the ratio of movement of each shaft is inversely proportional to the diameters of the different pulleys connected. The same law applies to gears in mesh, so that we are again able to make use of the equation,

$$D \times ND = F \times NF$$

in which D = the pitch diameter of the driver and F = the pitch diameter of the follower.

The term "pitch diameter" is derived from the fact that the two circles shown in Fig. 1 are used as the foundation of a pair of gears and the diameters of the circles become known as the "pitch diameters" of the gears. It will be noticed that these circles must always be tangent, and the point where they touch is known as the pitch point.

When two gears are running in mesh, the smaller is often referred to as the pinion.

The following examples and questions will illustrate some of the features pertaining to the movement, ratio, etc., of gears.

Example 1. A gear wheel 40 ins. pitch diameter meshes into another 25 ins. pitch diameter. The latter makes 56 R.P.M., how many will the first gear make?

Solution:

$$ND = \frac{F \times NF}{D} = \frac{25 \times 56}{40} = 35 \text{ R.P.M. (ans.)}$$

QUESTIONS

1. The pitch diameter of a gear is 24 ins. and it makes 80 R.P.M. What must be the pitch diameter of another gear connected with it in order that it may make 96 R.P.M.?

2. A gear 80 ins. pitch diameter makes 165 R.P.M. and is connected to another gear which makes 45 R.P.M. What is the pitch diameter of the second gear?

3. The pitch diameter of driving gear is 12 ins. and the pitch diameter of the pinion is 8 ins. What is the comparison between the number of revolutions of each gear?

4. A gear 74 ins. pitch diameter makes 160 R.P.M. and is connected to another gear which makes 40 R.P.M. What is the pitch diameter of the second gear?

5. The pitch diameter of the driving gear is 9 ins. and its speed is 270 R.P.M. At what speed will the follower gear run if its diameter is 4 ins.?

6. The sum of the pitch diameters of two gears is 42 ins. One makes 80 R.P.M. and the other 120. What are the diameters?

7. The difference between the number of revolutions of two gears in a given time is 40, the ratio of their pitch diameters is as 2 to 3, and the sum of the diameters is 30 inches. Find the pitch diameters of the gears and the number of revolutions of each gear in a given time.

Discussion of Terms. In Fig. 2, the two circles drawn around the centers D and F and in contact at P correspond to the two circles in Fig. 1, and the gears may be said to be derived from these circles, which are called the "pitch circles." The diameters of these circles are called the "pitch diameters" of the gears. The point P, where the two pitch circles touch each other, is called the "pitch point."

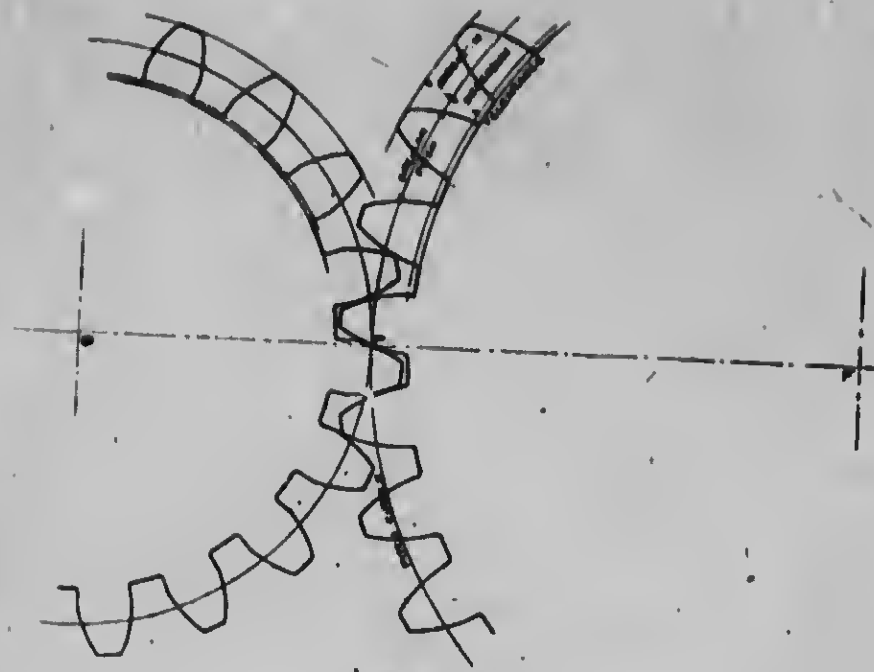


FIG. 2.

The circle drawn through the outer ends of the teeth of a gear is called the "addendum circle," and the circle drawn at the bottom of the teeth is called the "root circle". The distance, measured radially, from the pitch circle to the beginning of the fillet at the bottom of the tooth, is called the dedendum; and the circle drawn through this point is called the "dedendum circle". The distance between the dedundum circle and the root circle is known as the "clearance". These circles are indicated on the figure.

That part of the tooth outlined between the pitch circle and the addendum circle is called the face of the tooth, and that part between the pitch circle and the root circle is called the flank.

The distance from the center of one tooth to the center of the next, measured on the pitch circle, is called the "circular pitch," and is equal to the circumference of the pitch circle divided by the number of teeth. This expressed as a formula would be.

$$\text{Circular Pitch} = \frac{\text{Diameter} \times 3{,}1416}{\text{No. of teeth.}}$$

In order to run together, two gears must have the same circular pitch. The number of teeth on two gears of the same pitch is proportional to the circumferences, and consequently, to the diameters of their pitch circles. From this it will be seen that since the speeds of two shafts are inversely proportional to the pitch diameters of the gears connecting them, then their speeds will also be inversely proportional to the number of teeth in each gear. Thus the relative number of revolutions made by two gears in mesh can be calculated by knowing the number of teeth in each gear and the number of R.P.M. made by one gear.

Example. Two gears in mesh have 20 and 30 teeth respectively. If the first gear makes 400 R.P.M. what will the second make ?

Solution. D × DN = F × FN (Note, instead of using diameters in ins., use the No. of teeth).

$$20 \times 400 = 80 \times FN$$

$$FN = \frac{8000}{80} = 100 \text{ R.P.M. Answer.}$$

On rough cast gears, the width of the tooth is made a little less than the width of the tooth space, to allow for irregularities in construction. This difference in the width of the two is called "back-lash."

Circular and Diametral Pitches. Although the circular pitch is frequently used in connection with gear calculations, there is another form of pitch that is more often used owing to the fact that the calculations are more simple. This is "diametral pitch." Diametral pitch is not a distance like circular pitch, but is the number of teeth for each in. of the pitch diameter of the gear. For example, if the diameter of the pitch circle of a gear of 60 teeth were 20 ins., the number of teeth per in. of diameter would be $\frac{60}{20} = 3$, and the gear would be described as a "60-tooth, 3 diametral pitch gear."

Diametral pitch is very convenient to use, as the calculation is simpler than with circular pitch, and the pitch diameters of gears come in even figures, or even fractions of the pitch. For machine cut gears it is universal practice to use diametral pitch in the specifications. For cast gears, where the teeth are fashioned by the pattern-maker, it is common to use circular pitch.

The product of the circular pitch and the diametral pitch is always equal to the constant, 3.1416; that is, if we have one kind of pitch, and wish to change to the other, we divide 3.1416 by the given pitch.

Example. 4 diametral pitch is equal to $\frac{3.1416}{4} = .7854$ circular pitch.

Again, 2 in. circular pitch is equal to $\frac{3.1416}{2} = 1.57$ diametral pitch.

Note carefully that diametral pitch is not a Dimension, but a Ratio which gives the number of teeth for each inch of Pitch Diameter.

Formulas for Gear Calculatious. The following group of formulas cover the calculations relative to the majority of gear problems.

In the formulas:

P = diametral pitch.
P_1 = circular pitch.
D = pitch diameter.
D_1 = outside diameter.
N = number of teeth in one gear.
n = number of teeth in mating gear.
A = addendum.
T = thickness of tooth at pitch line.
E = full depth of tooth.
C = distance between centres.
F = clearance.

$$P = \frac{3.1416}{P_1} \qquad T = \frac{P_1}{2} \qquad P = \frac{N + 2}{D_1}$$

$$P_1 = \frac{3.1416}{P} \qquad T = \frac{1.5708}{P} \qquad D = D_1 - \frac{2}{P}$$

$$D = \frac{N}{P} \qquad E = \frac{2.157}{P} \qquad D = \frac{D_1 \times N}{N + 2}$$

$$A = \frac{1}{P} \qquad E = 0.6866P_1 \qquad N = P \times D$$

$$D_1 = \frac{N + 2}{P} \qquad C = \frac{N + n}{2P} \qquad N = (D_1 \times P) - 2$$

$$F = \frac{0.157}{P} \qquad P = \frac{N}{D} \qquad D_1 = \frac{D + 2}{P}$$

To find the diameters of two gears in mesh with each other, when the velocity ratio, of the two gears and the distance between centers (c) is given.

Rule. Divide the distance between the centers by the sum of the terms of the ratio; the pitch diameters will be twice the quotient multiplied by each term of the ratio.

Example. Find the diameters of two gears 16 ins. between centers and which have a velocity ratio of 3 to 4.

$3 + 4 = 7.$

$\frac{16}{1} \times \frac{1}{7} \times \frac{3}{1} \times \frac{2}{1} = 13.71$ ins.—Pitch diameter of pinion.

$\frac{16}{1} \times \frac{1}{7} \times \frac{4}{1} \times \frac{2}{1} = 18.29$ ins.—Pitch diameter of other gear.

PROBLEMS

Note. In the following problems frictions of moving parts will not be considered.

1. What is the circular pitch of an 8-pitch gear of 40 teeth ?

2. What is the circular pitch of a 6-pitch gear of 108 teeth ?

3. What is the pitch of a 75-tooth gear when the circular pitch is .7854 ?

4. A gear of 45 teeth has a circular pitch of .3142 in. Find the diametral pitch of gear.

5. What is the diametral of a gear blank that has 6 pitch teeth and 18 in. diameter ?

6. A gear blank is 3¾ ins. diameter and is to have 13 teeth. What is the pitch ?

7. What is the number of teeth in a 6-pitch gear 18 in. diameter ?

8. What is the number of teeth in a 4-pitch gear 19¼ in. blank diameter ?

9. What is the thickness of tooth for a 75-tooth gear, .7854 in. circular pitch.

10. What is the thickness of tooth for a 4-pitch gear of 40 teeth ?

11. Find diameter of blank for a 16-pitch gear with 32 teeth.

12. What is the diameter of blank of a 10-pitch gear with 28 teeth ?

13. What is the pitch of a 75-tooth gear 19¼ in. outside diameter ?

14. Find the number of teeth in a 16-pitch gear 8 in. diameter.

15. Find the number of teeth in a 10-pitch gear 64 in. diameter.

16. Find the number of teeth in a 20-pitch gear 10.1 in. outside diameter.

17. Find the sizes of two gears when the velocity ratio is 1 to 2 and the distance between centres is 12 ins.

18. Find the diameters of two gears 12 in. between centres, with velocity ratio 2 to 4.

19. Find the diameters of two gears 28 in. between centres, when the velocity ratio is 3 to 5.

20. When the distance between centres of two gears is 27 in. and the velocity ratio is 4 to 5, find the diameters of the gears.

21. When velocity ratio of two gears is 4 to 5, and the distance between the centres is 36 ins., find the diameters of the gears.

22. The velocity ratio of two gears is 2 to 3 and distance between the centres 30 ins. Find the diameters of gears.

23. What is the clearance of a 4-pitch gear of 75 teeth ?

24. Find the clearance of teeth for a 2-in. diameter gear with 32 teeth.

25. Find the diameter of a 6-pitch gear with 108 teeth.

26. What is the circumference of a gear blank with 32 teeth 16 pitch ?

27. Two gears in mesh have 30 and 24 teeth respectively, and are 6 pitch. What is the distance between centres ?

28. Two gears running together have 120 and 80 teeth respectively and are 10 pitch. Find the distance between the centres.

GEARING FOR LATHES

Note. This lesson on Lathe Gearing is Supplementary and need not be worked in Class, except by advanced students.

To cut threads in a lathe, changes of gear are made according to how many threads per inch are required.

Simple and sometimes Compound Gears are necessary.

These are placed on the Lead Screw and Stud respectively, and are connected by means of intermediate gears.

Note. If in doubt as to how many threads per in. on Lead Screw, select any two equal gears and use as driver and driven to cut a thread, which thread will be equal to the thread on the Lead Screw (sometimes called the Driving Screw).

If we desire the cutting tool to travel one (1) inch while the stud has revolved a certain number of times, there must be a certain relation between the gears L and S on the lead screw and stud respectively. The revolutions of the stud would determine the number of threads cut by the cutting tool while the carriage travelled along one inch.

Suppose we wished to cut a thread having 12 threads per in., then the stud and its gear must turn 12 revolutions. The lead screw having 4 threads per in., it follows that it must turn 4 times to move the carriage along one (1) inch. The ratio therefore is 12 to 4, or 3 to 1, and any gears capable of being applied which have that ratio would cut 12 threads. From this statement a simple formula can be built which would solve all cases of simple gears.

Let T = Threads per inch. L.S. = Lead Screw.
C.N. = Increase in number of teeth of gears (called common number).

$$\frac{LS \times CN}{T \times CN} = \frac{\text{Gear on Stud.}}{\text{Gear on Lead Screw.}}$$

Applying this to the above case, there is obtained:

$$\frac{4 \times 5}{12 \times 5} = \frac{20}{60} \quad \frac{\text{Gear on Stud.}}{\text{Gear on Lead Screw.}}$$

A gear having the number of teeth found in the numerator is the driver and would be placed on the stud. One having an equal number to that of the denominator is the driven and would be attached to the Lead Screw.

Note. If a right hand thread is required, use one intermediate gear, and if a left hand thread, use two intermediate gears. Intermediate gears can be of any convenient size.

Simple gears will cut many kinds of threads, but sometimes a thread is wanted which calls for a gear on the stud which would be very small, while that on the Lead Screw would be inconveniently large.

It would even be difficult to find such a pair of gears in some shops.

In all such cases it is more convenient to use four gears called "Compound Gears."

First. Find two gears as in previous method.

Second. Find any two numbers which when multiplied together equal the numerator, and any other two which when multiplied together would equal the denominator. Multiply each of these new numbers by the common number and the result will give the necessary gears.

Example. Find gears necessary to cut 20 threads per inch.

$$\frac{LS \times CN}{T \times CN} = \frac{4 \times 6}{20 \times 6} = \frac{24}{120} = \frac{\text{Gear on Stud.}}{\text{Gear on Lead Screw.}}$$

6 and 4 multiplied = 24.
12 and 10 multiplied = 120.

These factors when multiplied by the common number which in this case we will assume to be 6 gives 36 and 24 for the Stud Gears and 72 and 60 for the Lead Screw Gears.

QUESTIONS

1. What are the number of teeth required on the Lead Screw and Stud to cut 2 threads per in. with a 4 thread lead screw ?

2. What are the number of teeth required on the Lead Screw and Stud to cut 5½ threads per in. with a 4 thread lead screw ?

3. What change gears can be used to cut an 11½-pitch thread, when lead screw is 5 pitch ?

4. What change gears can be used to cut a 12-pitch thread, when lead screw is 5-pitch and stud gear has 25 teeth?

5. Find the stud gear to use to cut 16 threads per in. when lead screw is 5-pitch and screw gear has 80 teeth.

6. Find the screw gear to cut 18-pitch screw thread, when lead screw is 6-pitch and stud gear has 24 teeth.

7. When the velocity ratio of driving spindle and stud is 2 to 1, and lead screw is 8-pitch, what is the number of teeth in screw gear to cut 11 threads per in. when stud gear has 96 teeth ?

Note. The gears on inside end of spindle are arranged on a pivoted lever so that a train of three gears shall be in mesh, or by throwing over lever a train of four gears can be put into mesh, thus giving a feed motion of carriage in either direction according as lever is thrown up or down; this mechanism is called the reversing feed gears.

8. When the reversing feed gears are in the ratio of 2 to 1 with driving spindle, what stud gear will be required to cut 16 threads per in., if lead screw is 8-pitch and screw gear has 48 teeth ?

9. The gears furnished with a 14-in. Reed lathe have 24, 32, 40, 44, 48, 52, 56, 60, 64, 72, and 110 teeth, and the lead screw is 6 pitch. Find the compound gears required for cutting a $3\frac{1}{2}$ pitch thread.

10. With the same set of gears and same pitch lead screw as given in problem 1, find compound gearing required for cutting a 25 pitch screw. A 28-pitch screw. A $7\frac{1}{2}$-pitch screw. A screw of $\frac{3}{5}$ in. lead.

11. What stud gear will be used to cut a 9-pitch thread, when lead screw is 6-pi. and the screw gear has 36 teeth ?

12. What screw gear will be used to cut a 16-pitch thread, when lead screw is 6-pitch and stud gear has 24 teeth ?

13. The change gears furnished with a 13-in. Ree lathe have 25, 30, 35, 40, 45, 50, 55, 60, 65, 69, 70, 80, 90, 100, 110, and 120 teeth, and lead screw has 5 threads to the inch. Find the compound gearing required for cutting a $11\frac{1}{2}$-pitch pipe thread.

14. What change gears can be used to cut a 5-pitch thread when lead screw is 6-pitch ?

15. With the same lathe and same set of gears, find the compound gearing required for cutting a 40-pitch screw. A 23-pitch screw. A 27-pitch screw. A screw of $\frac{2}{11}$ in. lead. A screw of $\frac{3}{16}$ in. lead.

TIME AND DISTANCE, OR UNIFORM MOTION

The speed or velocity at which an object is moving is measured by the distance through which it moves in a given time.

When a person at a stated time is said to be walking at the rate of four miles per hour, it is meant that if he continues at the same pace he will have covered four miles in an hour.

If he continued walking at the same velocity he will in two hours have covered a distance or space of twice four miles, or eight miles. Hence Space = Time × Velocity, expressed $S = T \times V$. This formula can be changed around to find any one term when the other two terms are given, the quantities bearing the same relation to each other as those used in finding area of a rectangle, as is shown below, using the area to represent space, the length to represent velocity, and the breadth to represent time.

These quantities are applied in a similar manner as the length, breadth, and area of a rectangle, and are

Space = Time × Velocity	expressed	$S = T \times V$
Time = Space ÷ Velocity	expressed	$T = S \div V$
Velocity = Space ÷ Time	expressed	$V = S \div T$

In many questions it is necessary to convert "miles per hour" into "feet per second," and for this purpose it is convenient to remember the following method, based upon the feet per second of a speed which is equal to 60 miles per hour.

Applying the formula V = S + T to a speed of 60 miles per hour, the equivalent rate of speed per second is 88ft. per second, as per following solution:

$$V = S + T$$
$$S \text{ (in feet)} = 60 \times 5280$$
$$T \text{ (in seconds)} = 60 \times 60$$
$$\text{Therefore } V = \frac{60 \times 5280}{60 \times 60} = 88 \text{ ft. per second.}$$

Remembering that 60 miles per hour is equal to 88 ft. per second, other rates can easily be found by taking so many 60th parts of 88 ft.

Example 1. Find speed in feet per second of a train running at 20 miles per hour.

Solution:—

$$\frac{20}{60} \times \frac{88}{1} = \frac{88}{3} = 29\tfrac{1}{3} \text{ ft. per second.}$$

Had the answer been required in yards per second:—

$$\frac{20}{60} \times \frac{88}{3} = 9\tfrac{7}{9} \text{ yards would be the answer.}$$

Example 2. The piston speed of a steam engine is 10 ft. per second. How many miles will it travel in one hour ?

$$S = T \times V$$
$$V = 10$$
$$T = 60 \times 60$$

Therefore $S = (10 \times 60 \times 60)$ ft., but the question asks how many miles will the piston travel in one hour. This is obtained by dividing the result by 5280, the number of feet in one mile, giving:—

$$\frac{10 \times 60 \times 60}{5280} = 6\text{-}9/11 \text{ miles. Answer.}$$

In the case of two objects moving towards each other their distance apart is decreased by the sum of their rates of speed, while in the case of one overtaking the other the distance they are apart is diminished by the difference in their speeds.

Example 3. The distance between Montreal and Toronto is 334 miles. At 9 a.m. a train leaves Montreal at a speed which averages 46 miles per hour. A train leaves Toronto at 8 a.m. for Montreal and runs at 34 miles per hour. At what time and at what distance from Montreal will they meet?

Solution.—At 9 a.m. the Toronto train has travelled 34 miles, so that trains are then $334 - 34 = 300$ miles apart. They approach each other at $46 + 34 = 80$ miles per hour. Therefore, the time they will meet will be $300 \div 80 = 3.75$ hours after 9 a.m., which was the time the train left Montreal. In 3.75 hours this train will have travelled $3.75 \times 46 = 172.5$ miles, and will, therefore, meet the other train 172.5 miles from Montreal at 45 minutes past noon.

It will be noticed that the foregoing example simply required careful reasoning and that a formula was not used. Certain questions do not permit of these being directly applied, the terms of the question requiring solving first.

In the case of a train of certain length passing a certain point, or of two trains passing each other, the distances travelled have to be considered.

In the first case the train travels a distance equal to its own length, so that knowing the length of the train and its velocity the time taken to pass a given point can easily be found. In the second instance, the train must gain a distance equal to the sum of the lengths of the trains.

Example 4. Two trains (A) 77 yds. and (B) 88 yds. long are moving on parallel tracks. A runs at the rate of 45 miles and B at 30 miles per hour. Find, 1st, how long they will take to pass each other when going in opposite directions, and, 2nd, when moving in the same direction.

First Solution. When moving in opposite direction their relative speed is $45+30=75$ miles per hour, which is equal to $\frac{75}{60}\times\frac{88}{3}$ yards per second. The combined length of trains is $77+88=165$ yards. Hence they will pass each other in:—

$$\frac{165\times 60\times 3}{75\times 88}=4\tfrac{1}{2}\text{ secs. Answer.}$$

Second Solution. When moving in the same direction their relative speed is $45-30=15$ miles per hour, or 22/3 yards per second. Therefore they pass each other in $165\div 22/3$ or:—

$$\frac{165\times 3}{22}=22\tfrac{1}{2}\text{ seconds. Answer.}$$

In problems concerning the speed of a boat with or against the stream the principle is the same.

QUESTIONS

1. A train travels 54 miles in an hour. Find the velocity in feet per second.

2. How long will it take a train to travel $3\frac{1}{4}$ miles at a velocity of 88 ft. per second?

3. How far will a train go in 15 minutes, travelling at the rate of 47 miles per hour?

4. A man walks $5\frac{3}{4}$ miles in 67 minutes. How many miles per hour is he walking?

5. A pulley 28 ins. in diameter makes 280 revolutions per minute. What is the velocity at rim of pulley?

6. Two trains start at the same time, one from Toronto and one from Stratford (a distance of 88.3 miles). The train leaving Toronto for Stratford travels at the rate of 55 miles per hour and the one leaving Stratford for Toronto goes at the rate of 48 miles per hour. How soon will they meet, and at what distance from Toronto?

7 A boat whose rate of speed is 16 miles per hour makes a trip 67 miles down a river and back. What time will it take to make the trip, the current of the river amounting to 5 miles per hour?

8. Two trains are travelling in the same direction on a double track. One is 230 yards long and makes 42 miles per hour, and the other is 415 yards long and goes at rate of 22 miles per hour. How long will it take the first train to pass the second?

9. A train leaves Montreal at 9 a.m. for Toronto (a distance of 334 miles) and travels at the rate of 48 miles per hour. Another train leaves Montreal at 10 a.m. for Toronto and makes 36 miles per hour. How far are the trains apart when the first one arrives in Toronto?

10. A flywheel 8 ft. in diameter has a rim velocity of 8 ft. per second. How many revolutions per minute is it making?

11. The piston speed of an engine going 50 miles per hour is 12 feet per second. How far will the piston have travelled when the engine has gone 32 miles?

12. How far will a train go in 2 hours and 45 minutes when travelling at the rate of 56 ft. per second?

WORK

The term WORK as used in mechanics is the amount of resistance overcome through space by force.

The unit of work is the **Foot-Pound,** which means that one pound has been raised one foot vertically.

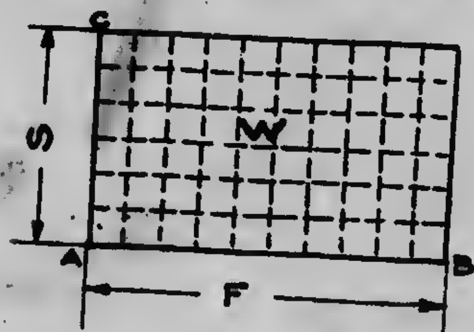
Foot-Pounds, written ft.-lbs., is the result of multiplying feet by pounds.

There are three terms in Work problems, namely Work, Space and Force, and if ten pounds be raised vertically 6 ft. the work performed is $10 \times 6 = 60$ ft.-lbs.

Expressing this as a formula $W = F \times S$. Therefore, $F = W \div S$, and $S = W \div F$.

A very clear way of obtaining the understanding of work problems is to represent such by diagrams, known as graphical representations.

Sketch shows a rectangular diagram which graphically



represents uniform resistance, illustrating the 10-lb. weight raised 6 ft. vertically. The line AB represents the weight, in this case represented by ten divisions, and the vertical line AC represents the space through which the weight was lifted, each division on AC representing one foot. The product of F+S=W.

Example 1. A crane lifts 120 lbs. to a height of 16 ft. Find amount of work done.

$$W = F \times S$$
$$F = 120 \text{ lbs.}$$
$$S = 16 \text{ ft.}$$

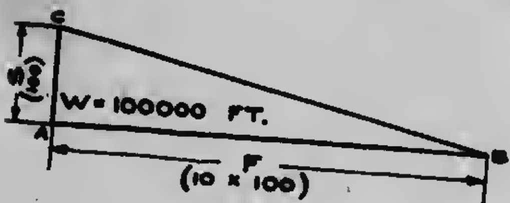
Therefore $W = 120 \times 16 = 1920$ ft.-lbs. Answer.

Example 2. A hole is punched through a ½ in. plate, the pressure being 36 tons. Find amount of work performed

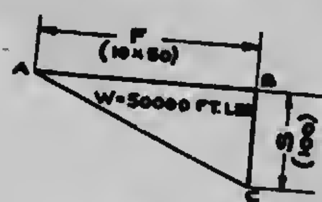
$$W = F \times S$$
$$F = 36 \times 2000$$
$$S = \tfrac{1}{2} \text{ of } \tfrac{1}{12} = \tfrac{1}{24} \text{ ft.}$$

$$\text{Therefore } W = \frac{36 \times 2000}{24} = 3000 \text{ ft.-lbs. Answer.}$$

These examples can be proved by means of the rectangular diagrams by substituting the figures stated in the question for the letters shown on diagram. The area of the rectangle equals work performed. In all cases the total amount of work done is not affected by the length of time doing it, whether it be accomplished in one hour or one second.



A



B

Other diagrams in the shape of a right-angled triangle standing upon its base and another inverted illustrate graphically how work questions can be represented when the resistance increases or decreases uniformly, the triangle's area being equal to the work done.

Example 3. A chain 100 ft. long hangs from a drum to which one end of the chain is fastened. Weight of chain is 10 lbs. per foot. Find (1st) how much work is done coiling it up, and (2nd) in raising the lower end to meet the upper.

Solution. The area of the left-hand diagram A solves the first part and the area of the right-hand diagram B solves the second part, the figures within the brackets being substituted for the letters. When solving these mathematically it is necessary to first find the average weight lifted, by considering the weight of the chain at the beginning and at the end of the work performed.

QUESTIONS

1. An elevator weighing 20 tons is lifted 16 ft. How many foot-pounds of work are used ?

2. A horse performs 60000 ft. lbs. of work in 5 minutes and pulls continually with a force of 100 lbs. Find the distance it moves in miles per hour.

3. The plunger of a force pump is $8\frac{3}{4}$ ins. in diameter, the length of the stroke is 30 ins., and the pressure of the water on the plunger is 50 lbs. per square inch. Find work performed per stroke.

4. A mine is 238 ft. deep, and a certain quantity of water is pumped from it, work performed in doing so being equal to 5764000 ft. lbs. Find weight of water, also number of cu. ft. of water raised.,

5. A belt has a velocity of 800 ft. per minute and the effective pull or tension upon it is 330 lbs. Find work transmitted per minute.

6. In a machine a force of 20 lbs. is applied to a handle 16 ins. long, and the handle (or crank) makes 30 revs. per minute. Find the work performed per minute.

7. How much work is performed in one stroke by a steam engine, diameter of cylinder being 24 ins., length of stroke 28 ins., and average steam pressure in cylinder being 165 lbs. per sq. in.?

8. The ram of a piledriver weighing 13 cwts., and the drop being 17 ft., how much work is done in raising the ram?

9. A steam pump is 9 ins. in diameter, length of stroke 12 ins., and mean effective pressure is 160 lbs. per sq. in. How much work is performed per stroke?

10. How much work is performed in raising a weight of 2200 lbs. 7 ft. in 3 minutes?

11. A belt has a velocity of 750 ft. per minute, and the effective pull or tension upon it is 450 lbs. Find work performed per minute.

12. How much work does a man perform when he lifts a weight of 40 lbs. to a height of 3 ft.?

13. The work performed in lifting a certain weight 21 ft. is 474600 ft. lbs. What is the weight?

14. An elevator exerts 161245 ft. lbs. or work in lifting a certain weight to a height of 35 ft. What is the weight?

15. A mine is 198 ft. deep and a certain quantity of water is hauled up from it, work performed in doing so being equal to 4586000 ft. lbs. Find the weight of the water and number of cu. ft. of water raised.

16. How much work is done by a steam derrick when it lifts 22000 lbs. 4 ft. high?

HORSEPOWER

Power is the rate at which work is done. When it became necessary to fix a unit of work done in one minute, the strength of the horse was chosen.

From a number of experiments James Watt estimated this as being equal to raising 33000 lbs. vertically one foot in one minute. Whether this power is greater or less than that of a horse is immaterial while it is a power so well defined, and has been accepted as such since his time.

Any machine, therefore, which is equal to overcoming a resistance of 33000 foot-pounds in one minute is considered as being equal to one horsepower, written H.P.

Unlike questions upon work, where time is not considered, a horsepower is always understood to be the equivalent of exerting 33000 ft.-lbs. in one minute, but in questions relating to steam engines, etc., time is seldom mentioned, one minute always being understood to be the time taken.

From these remarks and remembering the formula for work questions an equation can be made from which formulas can be built up to solve all horsepower questions (from Work) and this is:

Where H.P. = Horsepower
T = Time
F = Force (as in Work)
S = Space (as in Work)

$$H.P. \times T \times 33000 = F \times S$$

Example 1. How many horsepower will be exerted in raising an iron ball used for breaking cast-iron, if the ball weighs 11000 lbs. and is raised 30 ft. in one minute?

From lessons given in previous formulas it is apparent that where horsepower is required, as is the case in this question, a formula can be compiled from the equation given which will solve this and similar problems and would be:—

$$H.P = \frac{F \times S}{T \times 33000}$$

Substituting the figures given in the example this would be:—

$$H.P. = \frac{11000 \times 30}{1 \times 33000} = 10 \text{ H.P. Answer.}$$

Example 2. Find the horsepower necessary to draw a train of 120 tons on the level at the rate of 40 miles per hour against a resistance of 18 lbs. per ton.

$$H.P. = \frac{F \times S}{T \times 33000}$$

$$F = 120 \times 18 = 2160 \text{ lbs.}$$

$$= 40 \times 5280$$

$$H.P. = \frac{2160 \times 40 \times 5280}{60 \times 33000} = 230.4 \text{ H.P. Answer.}$$

From the equation upon Horsepower-from-Work problems the following formulas can also be made to solve Force, Space, or Time, and applied as in Examples 4, 5, and 6:—

$$F = \frac{H.P. \times 33000 \times T}{S}$$

$$S = \frac{H.P. \times 33000 \times T}{F}$$

$$T = \frac{F \times S}{H.P. \times 33000}$$

Example 4. An engine exerting 5 H.P. lifts a weight 30 ft. in 2 mins. What is the weight ?

$$F = \frac{H.P. \times 33000 \times T}{S}$$

$$F = \frac{5 \times 33000 \times 2}{30} = 11000 \text{ lbs.} \quad \text{Answer.}$$

Example 5. How high will a 16 H.P. steam derrick lift 45000 lbs. in 3 minutes ?

$$S = \frac{H.P. \times 33000 \times T}{F}$$

$$S = \frac{16 \times 33000 \times 3}{45000} = 9.6 \text{ ft.} \quad \text{Answer.}$$

Example 6. How long will it take a 36 H.P. engine to lift 52000 lbs. 30 ft. ?

$$T = \frac{F \times S}{H.P. \times 33000}$$

$$T = \frac{52000 \times 30}{36 \times 33000} = 1.313 \text{ minutes.} \quad \text{Answer.}$$

QUESTIONS

1. The velocity of a belt is 600 ft. per minute and the effective tension on it is 385 lbs. Find the work transmitted by it in one minute, also the H.P. that is being exerted.

2. Find the H.P. of an engine that is required to raise 2 tons to a height of 85 ft. in a quarter of a minute.

3. How high could a 12 H.P. steam engine raise 3450 lbs. in 3 minutes ?

4. How much weight could a 72 H.P. engine lift in two minutes to a height of six feet ?

5. What is the H.P. required to lift a weight of 72000 lbs. 18 ft. in three minutes ?

6. What would be the resistance overcome when the H.P. available was 7, and the space passed over was one-third of a mile in an hour ?

7. A punching machine is so arranged that 7 holes can be punched in three minutes through a plate ½ in. thick. The pressure required is 20 tons, which may be assumed to be uniform. Find how much work is performed by the machine in one minute, and the H.P. required.

8. An endless cord, stretched and running over grooved pulleys with a linear velocity of 3000 ft. per minute, transmits 5 H.P. Find the tension on the cord in pounds.

9. A locomotive of 120 H.P. has to pull with a force of 800 lbs. in order to draw its load. Find its speed in miles per hour.

10. An engine performs 138,000 ft.-lbs. of work in 4 minutes. What is the H.P. of the engine ?

11. What H.P. is required to lift a weight of 72000 lbs. 4 ft. in one minute ?

12. How many cu. ft. of water can be raised 100 ft. in 20 minutes with a 24 H.P. engine ?

13. The crank of an engine is 2 ft. long, pressure on crank is 3000 lbs., and the crank makes 90 revolutions per minute. What is the horsepower of the engine ?

14. What H.P. is required to raise 3450 lbs. to a height of 22 ft. in one minute.

15. A 36 H.P. engine is required to raise 18000 lbs. In what time does it lift same to a height of 18 ft. ?

HORSEPOWER OF A STEAM ENGINE

To find horsepower of a steam engine the equation is expressed differently, yet it is the same in principle as that given for work, a subdivision of the terms being the only change.

In Steam Engine questions, Force becomes Pressure and Area, and Space becomes length of stroke and number of strokes.

The relation of these will be seen at a glance, as Pressure ×Area equals the total force pressing against the piston, and length × number of strokes is equivalent to the total space through which the piston moves.

Expressing these new terms as an equation, where

P = Pressure in lbs. per sq. in.
L = Length of stroke in feet.
A = Area of piston in sq. ins.
N = Number of strokes per minute.

$$H.P. \times 33000 = P \times L \times A \times N$$

It is important to remember that these terms must be expressed as stated, as it is common to find mistakes made in this way.

L is always in feet, and N means number of strokes per minute, not revolutions. These two are the cause of more mistakes than are the others. If question states length of stroke is 26 ins., simply put it down as 26 over 12, thus $\frac{26}{12}$.

If number of revolutions is given, always multiply by two, as there are two strokes to every revolution. By watching these points no difficulty should be experienced in obtaining correct solutions.

It is evident that frequently occasions will arise when any of the terms besides H.P. may be required. By remembering the equation $H.P.\times 33000 = P\times L\times A\times N$ formulas can at any time be built from it, thus:

$$P = \frac{H.P.\times 33000}{L\times A\times N} \qquad L = \frac{H.P.\times 33000}{P\times A\times N} \qquad A = \frac{H.P.\times 33000}{P\times L\times N}$$

$$N = \frac{H.P.\times 33000}{P\times L\times A} \qquad H.P. = \frac{P\times L\times A\times N}{33000}$$

Example 1. Find the horsepower of an engine when the piston has an area of 3 sq. ft., steam pressure is 125 lbs. per sq. in., length of stroke 33 ins., and the engine makes 115 revolutions per minute.

$$H.P. = \frac{P\times L\times A\times N}{33000}$$

$P = 125$
$L = 33 \div 12$
$A = 3\times 144$
$N = 115\times 2$

Therefore,

$$H.P. = \frac{125\times 33\times 3\times 144\times 115\times 2}{12\times 33000} = 1035\ H.P.$$

Answer.

Example 2. The piston of a 280 H.P. steam engine is 20 ins. in diameter. It makes 100 R.P.M., and the length of its crank is 18 ins. Find the average steam pressure throughout the stroke.

$$P = \frac{H.P. \times 33000}{L \times A \times N}$$

$L = 36 \div 12 = 3$ ft.
$A = 20 \times 20 \times .7854$
$N = 100 \times 2$
$H.P. = 280$

Therefore,

$$P = \frac{280 \times 33000}{20 \times 20 \times .7854 \times 3 \times 100 \times 2} = 49.019 \text{ lbs.}$$

Answer.

Example 3. Find the number of revolutions per minute made by a steam engine of 100 H.P. when the piston is 30 ins. diameter and average pressure is 60 lbs. per sq. in., length of stroke being 48 ins.

$$N = \frac{H.P. \times 33000}{P \times L \times A \times 2}$$

Note.—2 is used as a divisor because question asks for number of revolutions, and one rev. equals two strokes.

$H.P. = 100$
$L = 4$ ft.
$P = 60$ lbs.
$A = 30 \times 30 \times .7854$

Therefore,

$$N = \frac{100 \times 33000}{60 \times 4 \times 30 \times 30 \times .7854 \times 2} = 9.0726 \text{ R.P.M.}$$

Ans.

Example 4. What is the diameter of the cylinder of a 480 H.P. engine when steam pressure is 100 lbs. per sq. in., it makes 110 R.P.M., and length of stroke is 2 ft. 6 in.

Note.—The area of the piston must be found first.

$$A = \frac{H.P. \times 33000}{P \times L \times N}$$

$P = 100$ lbs.

$L = 2.5$ ft.

$N = 110 \times 2$

Therefore $A = \frac{480 \times 33000}{100 \times 2.5 \times 110 \times 2} = 288$ sq. ins.

and Diameter $= \sqrt{288 \div .7854} = 19\frac{1}{8}$ ins. Answer.

Example 5. An engine transmits 28 H.P. Its average pressure is 30 lbs. per sq. in., dia. of cylinder is 12 ins., and it makes 40 R.P.M. Find length of Stroke.

$$L = \frac{H.P. \times 33000}{P \times A \times N}$$

$P = 30$

$A = 12 \times 12 \times .7854$

$N = 40 \times 2$

Therefore,

$$L = \frac{28 \times 33000}{33 \times 12 \times 12 \times .7854 \times 40 \times 2} = 3.4 \text{ feet. Ans.}$$

QUESTIONS

1. The area of the piston of an engine is 1282 sq. in., the mean effective pressure is 40 lbs. per sq. in., length of stroke 72 ins., number of strokes per minute, 42. Find the H.P. of the engine.

2. H.P. transmitted 70, average steam pressure 35 lbs. per sq. in., length of stroke 36 ins., number of strokes per minute 160. Find the diameter of the piston.

3. Steam pressure per sq. in. in a 28 H.P. engine is 30 lbs., diameter of piston is 12 ins., and it makes 40 R.P.M. Find the length of the stroke.

4. The diameter of the piston of an engine is 12.355 ins., the mean effective pressure is 55 lbs. per sq. in., length of stroke is 16 ins., and it makes 180 R.P.M. Find the horsepower of the engine.

5. What is the diameter of the cylinder of a 120 H.P. engine, making 50 R.P.M., steam pressure being 170 lbs. per sq. in., and length of stroke 30 ins. ?

6. What R.P.M, should a 160 H.P. engine make if diameter of cylinder is 28 ins., length of stroke 34 ins., and steam pressure 120 lbs. per sq. in. ?

7. How many footpounds of work are performed per stroke by an engine having a 14 in. piston, 18 in. stroke, and 80 lbs. pressure per sq. in. ?

8. What H.P. is an engine whose piston is 20 in. dia., length of stroke 22 ins., pressure 70 lbs. per sq. in., and it makes 60 strokes per minute ?

9. A 6 H.P. engine has a piston 16 ins. diameter, 18 in. stroke, and makes 100 strokes per minute. What pressure is required ?

10. What is the horsepower of an engine whose cylinder is 48 ins. in diameter, has a stroke 56 ins. long, makes 48 revolutions per minute, and the mean effective pressure in the cylinder is 180 lbs. per sq. inch ?

11. What is the size of the cylinder of a 48 H.P. engine, stroke being 24 ins., engine making 100 revolutions per minute, and pressure equalling 90 lbs. per square inch ?

12. What length of a stroke does a 24 H.P. engine make, cylinder being 8 ins. diameter, pressure 100 lbs. per sq. in., and it makes 110 R.P.M. ?

13. A locomotive of 156 H.P. has to pull with a force of 800 lbs. in order to draw its load. Find its speed in miles per hour.

14. What H.P. is required to lift a weight of 72000 lbs. 18 ft. high in 3 minutes ?

15. What is the H.P. of an engine whose cylinder measures 28 ins. in diameter, has a 26 in. stroke, makes 90 R.P.M., and pressure in cylinder is 120 lbs. per sq. in. ?

16. What is the pressure per sq. in. in the cylinder of an engine 14 ins. diameter, 16 in. stroke, 180 R.P.M., and 24 H.P. ?

17. What H.P. is an engine with a 28 in. cylinder, 30 in. stroke, 90 R.P.M., and steam pressure of 160 lbs. per sq. in. ?

18. The crank of an engine is 18 ins. long, pressure on crank is 3000 lbs., and crank makes 90 R.P.M. What is the H.P. of the engine ?

19. How many R.P.M. does an engine make that has a 28 in. cylinder, 30 in. stroke, and is rated at 240 H.P., pressure per sq. in. being 110 lbs.

20. What is the diameter of the cylinder of an engine that makes 110 strokes per minute, has a 20 in. stroke, steam pressure is 124 lbs. per square in., and engine is rated at 60 H.P.?

21. What H.P. is an engine with a 9 in. cylinder, a 15 in. stroke, when it makes 120 R.P.M., and has a steam pressure of 140 lbs. per sq. in.?

22. What is the H.P. of an engine that has a 4 in. diameter piston, makes 120 R.P.M., has a 6 in. stroke, and steam pressure is 24 lbs. per sq. in.?

23. How long will it take a 28 H.P. engine to lift a weight of 24000 lbs. to a height of 16 ft.?

24. What H.P. is exerted in lifting a weight of 5400 lbs. to height of 27 ft. in 3 minutes?

FRICTION

Work and horsepower problems solve the resistance which is overcome.

The principal resistance is due to friction, as, for instance, if it were desired to move a casting from one part of the shop to another, there would be a great difference between dragging it along the floor and pulling it on a shop truck that had its axle journals well oiled. A locomotive pulling a train along the track has to overcome the friction of the wheels against the rails, the resistance through the air, etc., and if the least upgrade is met there is the force of gravity to be overcome, which is equivalent to raising the dead weight of the train the difference in height between the high and the low level.

A train illustrates rolling friction, while a sleigh being pulled over bare ground or ice illustrates sliding friction.

The resistance due to friction is always proportional to the weight, and is always a fraction of the weight. If the force (P) necessary to move a certain weight is divided by that weight (W), there is obtained what is known as the coefficient of friction, which is always expressed as a decimal fraction.

Where $C = $ Coefficient of friction.

$P = $ Power of force required to overcome friction.

$W = $ Weight moved.

Then $C = P \div W$

$P = C \times W$

$W = P \div C$

Example 1. A stone weighing 600 lbs. is moved along a platform. What is the force required, when the coefficient of friction is .166 ?

$$P = C \times W$$

Therefore $P = .166 \times 660 = 99.6$ lbs. Answer.

A study of this question will show that the weight of 600 lbs. requires a force of 99.6 lbs. to move it, and that 99.6 lbs. (the Force) if divided by 600 lbs. (the Weight) gives .166, which is the coefficient of friction.

Had the question stated that the stone was moved through a space of five feet, the answer would then have been in ft.-lbs., and the work done would have been $99.6 \times 5 = 498$ ft.-lbs.

Example 2. A traction engine rated at 32 H.P. pulls a load behind it weighing 38 tons, the resistance being 40 lbs. per ton. Find speed in miles per hour, the weight of the engine being 12 tons.

$$\text{Force} \times \text{Space} = \text{H.P.} \times 33000 \times \text{T (in min.)}$$

$$\text{Therefore S} = \frac{\text{H.P.} \times 33000 \times \text{T}}{\text{F}}$$

$$\text{F} = (38+12) \times 40 = 50 \times 40$$
$$\text{H.P.} = 32$$

$$\text{Therefore S (in miles)} = \frac{32 \times 33000 \times 60}{5280 \times 50 \times 40} = 6 \text{ miles per hr.}$$

QUESTIONS

1. Weight of a locomotive and train is 4850 tons. Coefficient of friction is .16. What force is necessary to move train ?

2. A man pulling a truck exerts 480 ft.-lbs. in moving a truck 10 ft. Weight of truck is 600 lbs. What is the coefficient of friction ?

3. What is the coefficient of friction when it requires 20 lb. energy to pull a sleigh weighing 320 lbs. ?

4. What is the coefficient of friction when it requires 140 lbs. of energy to move a 5400 lbs. load ?

5. Weight of a locomotive and train is 1350 tons. Resistance is 16 lbs. per ton. What force is necessary to move train, and what H.P. is exerted when train is running at the rate of 45 miles per hour ?

6. A loaded box-car weighs 34560 lbs. and the co
efficient of friction is .12. What H.P. will be required t
haul it at a speed of 35 miles per hour ?

7. What H.P. is a locomotive exerting in hauling
train at the rate of 36 miles per hour, weight of train bein
2200 tons, and resistance being 17 lbs. per ton ?

8. The weight of an engine and train is 100 tons, H.P
available is 80, and the speed in miles per hour is 32. Find
the resistance per ton of load.

MECHANICAL POWERS

A machine, no matter of what nature or how complicated it may be, is an instrument by which force applied at one point is transferred to another point by one or another of the six elements of machines, known as the mechanical powers.

Of these, six are usually reckoned upon, namely, the lever, wheel and axle, pulley, inclined plane, wedge, and screw.

The force necessary to overcome the resistance in any one of these is expressed by the letter P, meaning "power," and the resistance by the letter W, meaning "weight."

LEVERS

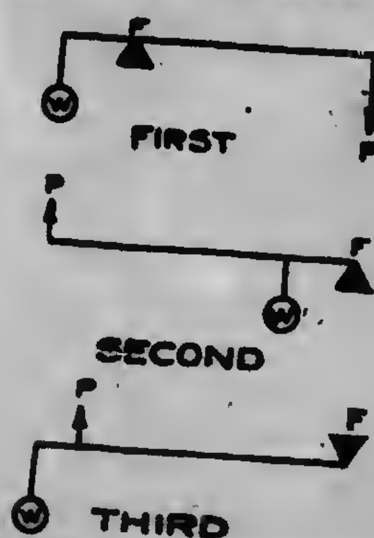
A lever is a rigid rod free to turn about a fixed point called the fulcrum.

Levers are divided into three kinds according to the relative positions of the Power, Fulcrum, and Weight. The following diagrams illustrate the three kinds:

P = Power.

W = Weight.

F = Fulcrum.



Each lever has two arms, the power arm (that on wh the power acts), and the weight arm (that on which t weight acts). Each arm moves around the fulcrum as centre, and the condition necessary to balance a lever that the movement of the power around the fulcrum equal to and opposite that of the weight, or, in other word the amount of power multiplied by the length of its ar must be equal to the amount of weight multiplied by th length of its arm.

Power and weight must be expressed in the same unit of force. The length of both arms must also be expresse in the same units of length.

In the diagram that part of the lever shown as PF is th power arm and the part WF is the weight arm. It has also been stated that P = Power and W = Weight, therefore

$$P \times PF = W \times WF$$

From this equation, which is the only one necessary to remember, there are built up four formulas in the same manner as already shown in previous subjects, thus:

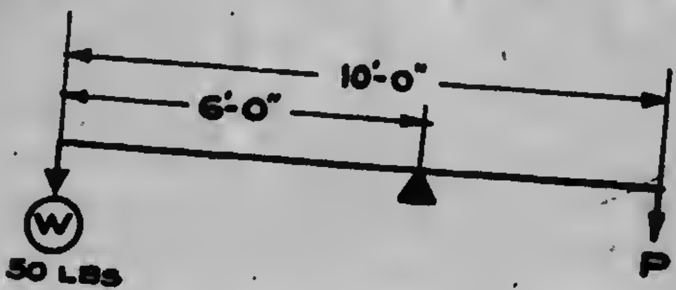
$$P = \frac{W \times WF}{PF} \qquad PF = \frac{W \times WF}{P}$$

$$W = \frac{P \times PF}{WF} \qquad WF = \frac{P \times PF}{W}$$

It will be noticed that no matter which term is required, the divisor is always that term which is coupled with the required one in the general equation.

Example 1. A lever 10 ft. long has a weight of 50 lbs. on one end. Find how much power should be exerted on the other end so as to balance the lever when fulcrum is 6 ft. from the weight end.

Solution. In every case draw the diagram first and dimension it to suit the question. Arrowheads should be used pointing in the direction that the power and the weight act.

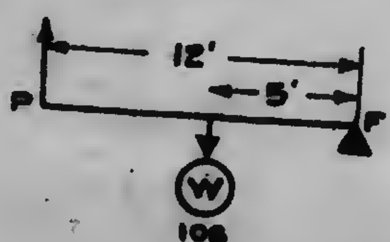


$$P = \frac{W \times WF}{PF}$$

$$P = \frac{50 \times 6}{4} = 75 \text{ lbs.} \quad \text{Answer.}$$

Note. This is a lever of the first kind, and is in frequent use throughout the shops in the shape of a pinch bar or in stores as a balance scale.

Example 2. A lever 12 ft. long with a fulcrum at one end has a weight of 108 lbs. hung on lever 5 ft. from fulcrum. What force must be applied at the free end so as to maintain the lever in a horizontal position?

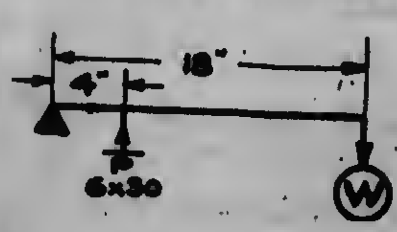


$$P = \frac{W \times WF}{PF}$$

$$P = \frac{108 \times 5}{12} = 45 \text{ lbs.} \quad \text{Answer.}$$

This is a lever of the second kind. A wheel barrow is practical illustration of this kind of lever, the centre of th wheel being the fulcrum, W where the load acts, and P th handles where the power, or force, lifts upwards.

Example 3. A safety valve has a surface of 6 squar ins. on the steam side. The lever is 18 ins. long and th distance from the fulcrum (which is at one end of the lever to the point on the lever where the safety valve acts is 4 ins What weight should be hung at the end of lever in order that the steam may blow off at 30 lbs. pressure per square in. ?



$$W = \frac{P \times PF}{WF}$$

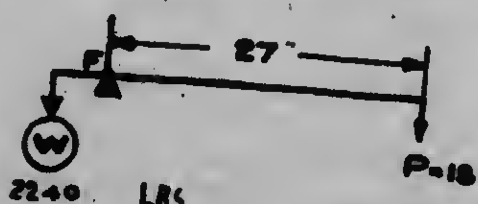
$$W = \frac{6 \times 30 \times 4}{18} = 40 \text{ lbs.} \quad \text{Answer.}$$

This is a lever of the third kind and as stated is in practical use on safety valves of the lever kind.

In all three kinds of levers there is pressure on the fulcrum and is in the first kind the sum of the forces or P+W. In the second kind of lever the pressure is equal to the excess of weight over power, or W−P, and in the third kind the pressure is equal to the excess of power over weight, or P−W.

Example 4. In a lever of the first kind a power of 18 lbs. is used to balance a weight of 2240 lbs. Power arm is 27 ins. long. What is the length of the lever, also what is the pressure upon the fulcrum ?

Note. To find the length of the lever, it is only necessary to find the length of the weight arm and add it to the length of the power arm, which is 27 ins.



$$WF = \frac{P \times PF}{W}$$

Therefore,

$$\text{Total length} = \frac{18 \times 27}{2240} + 27 = 27\ .217 \text{ ins. Answer.}$$

The strain on the fulcrum is P+W or 18+2240=2258 lbs.

Example 5. A lever has a weight of 32 lbs. on or end; the fulcrum is 7½ ins. from the weight. The pow applied is 6.234 lbs. What is the length of the power arm

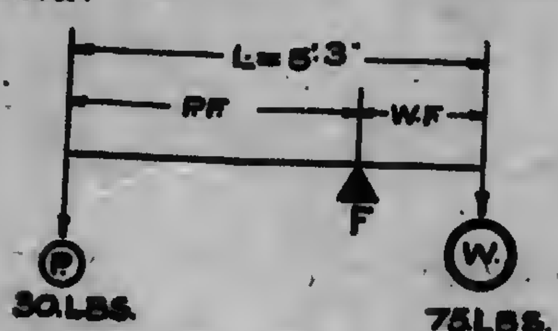
$$PF = \frac{W \times WF}{P}$$

$$\text{Therefore } PF = \frac{32 \times 7.5}{6.234} = 38\tfrac{1}{2} \text{ ins. nearly. Ans.}$$

TO FIND POSITION OF THE FULCRUM

Questions often arise making it necessary to find the position of the fulcrum, and to do so draw the diagram and letter it as is done in lever questions.

The following formula will solve the length of the weight arm from the fulcrum, "L" representing total length of lever:—



$$WF = \frac{P \times L}{P + W}$$

Example 6. A lever is 5 ft. 3 in. long; 30 lbs. act on one end and 75 lbs. on the other. Find the position of the fulcrum in order that the lever may be balanced.

$$WF = \frac{P \times L}{P + W}$$

Therefore,

$$WF = \frac{30 \times 63}{30 + 75} = 18 \text{ ins. from the weight end. Ans.}$$

In all the foregoing examples the weight of the lever has not been considered.

While its weight has some effect on lever questions, the difference in results is so slight that, unless in very important calculations, it is never considered.

In the case of a beam or bridge being supported at two ends half the weight comes on each end support, the weight of the beam being usually uniform throughout.

Questions therefore where the beam is supported at two ends and having the weight of the beam considered, would have the weight part solved first, and then the weight of that part of the beam resting on each support added afterwards.

To find the pressure on the supports of a bridge with a locomotive or other weight upon it use this equation from which to build up necessary formulas, and always draw a diagram:

$P \times L = W \times WF$

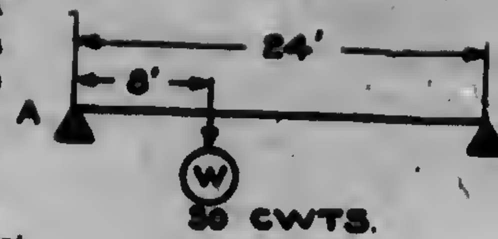
P = Pressure on support B.

L = Length of beam.

W = Weight upon beam.

WF = Distance of weight from support A.

Example 7. A bridge 24 ft. wide has a load weighing 30 cwts. placed upon it at a distance of 8 ft. from one end. Find pressure on each support.



Solution:—$P \times L = W \times WF$.

Therefore $P = \dfrac{30 \times 8}{24} = 10$ cwts.

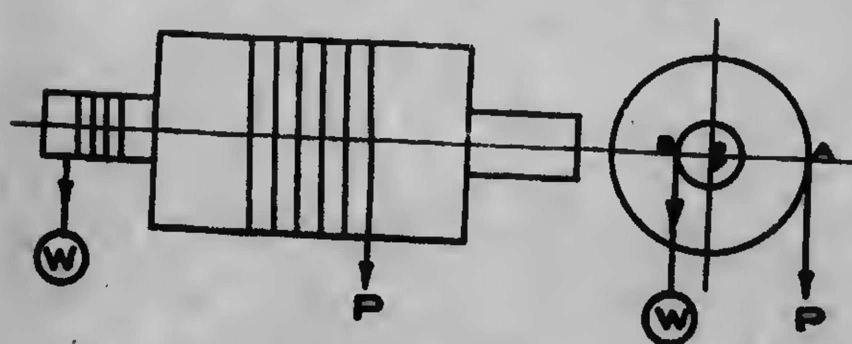
10 cwts. Pressure on support B

3−10=20 cwts. Pressure on support A. Ans.

WHEEL AND AXLE

The wheel and axle is merely a modification of the lever. Practically, it consists of two cylinders turning about a common centre, the large cylinder being called the wheel, and the smaller one the axle.

A pulley keyed to a shaft illustrates this clearly. In practice the large cylinder is often merely a crank, such as the handle of a windlass or hand-crane. Ropes are coiled round both wheel and axle but in opposite directions, so that when one winds up the other is unwinding.



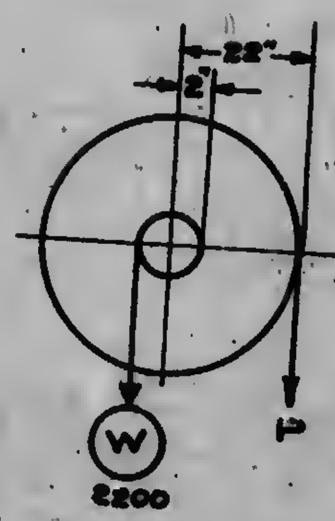
Looking at this sketch this will be apparent and from the end view a lever of the first kind will be observed, AC being the power arm and CB the weight arm. F is the fulcrum, which is the common centre of both wheel and axle.

The power and weight act upon the outside of the wheel and axle respectively, the greater leverage being obtained by power by reason of it always being applied at the rim of the wheel.

As before stated, the wheel is not always a cylinder or pulley, as power is sometimes applied by means of a handle, toothed gearing, rack pinion, etc., but the principle is the same, and the solution is found by using the equation given for levers, i.e., P×PF=W×WF.

Example 1. If the radius of a wheel be 22 ins., and the diameter of the axle be 4 ins., what force would be required to lift 2200 lbs.?

Note.—Remember that force is always applied at rim (radius) of wheel, and that weight is always lifted at the outside of the shaft (its radius).



$$P = \frac{W \times WF}{PF}$$

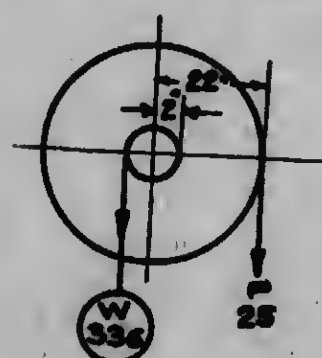
Therefore $P = \frac{2200 \times 2}{22} = 200$ lbs. Answer.

Example 2. What is the diameter of a shaft when a weight of 336 lbs. on the axle can be raised by a force of 25 lbs. applied on a wheel which is 56 ins. in diameter?

$$WF = \frac{P \times PF}{W}$$

$$\text{Therefore } WF = \frac{25 \times 28}{336} = 2.083 \text{ ins. radius}$$

or $2.083 \times 2 = 4.166$ ins. diameter. Answer.



QUESTIONS

1. What power will be required to balance a weight of 48 lbs. at one end of a lever 8 ft. long, fulcrum being located 15 ins. from the weight?

2. With a lever 14 ins. long having a fulcrum at one end and a power of 36 lbs. on the other, what weight can be lifted if acting on lever 2 ins. from the fulcrum?

3. Length of a lever is 108 ins.; fulcrum is located 24 ins. from a weight of 348 lbs. at one end. What power is required to balance lever?

4. What must be the length of a lever of the 2nd order to lift 9500 lbs. at 40 ins. from the fulcrum with a force of 1195 lbs.?

5. Give the dimensions of a lever to lift 1640 lbs. wi a force of 240 lbs., the fulcrum to be between weight ar power, total length of lever is 7 ft. 10½ ins.

6. At what point will two weights balance if they a 12 ft. apart, and weigh 400 and 150 lbs. ?

7. A lever is 10 ft. long. If the fulcrum is at one en and the power is applied at the other end, how much forc will it take to lift a weight of 12540 lbs., hung 29 ins. fror the fulcrum ?

8. What push will an 8 in. dia. brake cylinder exer acting through a lever 42 ins. long if the pressure in it i 90 lbs. per sq. in. ? Fulcrum is 22 ins. from cylinder pusl rod.

9. A lever is 65 ins. long with the fulcrum at one enc and 35 lbs. weight at the other end. Find what power at 7 ins. from the fulcrum will balance it.

10. A wheel is 98 ins. in diameter and the axle is 4 ins. in diameter. What power is required to raise 16000 lbs. ?

11. On a 4 in. diameter axle it requires a power of 48 lbs. to raise a weight of 756 lbs. What is the radius of the wheel ?

12. How much can be lifted by a power of 574 lbs., radius of wheel being 56 ins., and the axle being 16 ins. radius ?

13. A crank shaft is 2 ins. in diameter and the crank itself is 18 ins. long. How much power is required on end of crank to lift 360 lbs. ?

14. A lever for a safety valve is 18 ins. long; safety valve is 2½ ins. diameter and acts 3 ins. from the fulcrum, which is at one end. What weight is required at the other end of lever in order that the safety valve may blow off at 60 lbs. pressure per sq. in. ?

THE PULLEY

The pulley is a wheel whose circumference is grooved to prevent the rope (called the tackle) which passes around it from slipping off.

A combination of pulleys supported by framework (called the block) is known as a "block and tackle," and is in frequent use around the shops in various shapes, from the hand tackle fitted with a rope to those in use on cranes.

4 CORD PULLEY

Sketch illustrates a block and tackle fitted with a rope. The one end of the rope is attached to the bottom framework, and the other, after being wound through the pulleys (called "reeving a tackle") in a certain way finally passes around a top pulley so as to be free for pulling upon.

Pulleys shown on sketch are drawn on different centres so as to clearly show how the rope is reeved, but in practice the top pulleys are all on the same centre and the bottom ones also. The pulleys are also usually the same diameter in practice. Had the first end of the rope been fastened to the bottom block an extra pulley would have been required on the top and the number of ropes would have been six, in place of five.

To find the weight a certain power will lift, count the number of ropes and multiply this number by the power exerted, and if N = number of ropes, P = power, and W = weight, then:

$$W = P \times N \qquad P = W \div N \qquad N = W \div P$$

The rope upon which the power acts is not counted as it does not give any mechanical advantage, but simply changes direction of motion. Sketch of fixed pulley having an equal weight on each end of rope will illustrate this. The one weight being equal to the other the rope is stationary, but add extra weight (sometimes weight is substituted by power) then the lesser weight will rise.

Example 1. Given a power of 90 lbs. what weight can be supported by a 4-cord block-and-tackle? Also, how many pulleys would tackle have at top and bottom, and to which block would the rope be fastened?

$$W = P \times N$$

Therefore $W = 90 \times 4 = 360$ lbs. Answer.

There would be two pulleys each at top and bottom.

The rope would be fastened to the top block.

Example 2. A pulley has 7 cords and it is desired to raise a weight of 3 tons.

(a) What force is required?

(b) How many pulleys are there in top and bottom sheaves?

(c) Where would the rope be fastened?

(a) $P = W \div N$

$$P = \frac{3 \times 2000}{7} = 875\tfrac{1}{7} \text{ lbs.}$$

(b) There would be four pulleys on top and three on bottom.

(c) Rope would be fastened to the lower block.

Example 3. The force available is 76 lbs., weight to be lifted is 456 lbs. How many pulleys are required on top and bottom of pulley ? Also, where will rope be fastened ?

$$N = W \div P$$

Therefore $N = 456 \div 76 = 6$ ropes.

There being six ropes, there would be three pulleys on top and three pulleys on the bottom, and the fixed end would be fastened to the top block.

In every case where the number of pulleys in the upper and lower blocks are the same, the rope is secured to the upper block, and the number of ropes will be even number, equal to the total number of pulleys.

When the number of pulleys is odd, the extra pulley is placed in the upper block, the rope is fastened to the lower block and the number of ropes is an odd number, equal to the total number of pulleys.

QUESTIONS

1. Given a power of 560 lbs., what weight can be supported by a six-cord pulley ?

2. A pulley has 4 cords, and it is desired to raise a weight of 57486 lbs. What force is required ?

3. With a power of 465 lbs. and a weight of 1860 lbs. (a) how many ropes will there be, (b) how many pulleys in both top and bottom sheaves, and (c) where will end of rope be secured, on the top or bottom sheave ?

4. What weight can be supported by a five-cord pulley if the force exerted be 180 lbs. ?

5. Given a weight of 57684.687 lbs., what force is necessary to balance it with a five-cord pulley ?

6. A force of 220 lbs. is exerted on the free end of the rope of a five-cord pulley. What weight will this lift ?

7. With a force of 24 lbs. and a weight of 96 lbs., how many ropes are required? Also, how many pulleys, top and bottom?

8. What force is required to lift a weight of 576849.0576 lbs. with a seven-cord tackle?

9. What force is required to lift a weight of 3400 lbs. with a four-cord tackle, if the tackle is only 75% efficient?

10. If a force of 136 lbs. will lift a weight of 612 lbs. with a tackle that is only 75% efficient, how many ropes are in the tackle? Draw a sketch of same.

11. A weight of 36700 lbs. is to be dragged along a skidway by means of a tackle. Coefficient of friction on the skidway is .18. The tackle is only 75% efficient, and force necessary to apply to tackle is 2202 lbs. How many cords has the tackle?

12. If a weight has been lifted 6 ft. by means of a four-cord pulley, how far has the power end of the rope been moved?

13. How many foot-pounds of work is done by dragging a weight of 3650 lbs. along the ground with a four-cord tackle, coefficient of friction being .20, and distance weight was dragged being 24 ft.?

14. With a six-cord tackle, how much can a force of 340 lbs. raise?

15. What force is necessary to lift a weight of 5600 lbs. with a seven-cord tackle?

16. With a force of 45 lbs., how much can a four-cord tackle lift?

17. Using a four-cord rope tackle that is only 84% efficient, what weight can be lifted with a force of 186 lbs.?

18. Using a seven-cord tackle that is only 67% efficient, what force is required to lift 2000 lbs.?

19. What size tackle is required in order that a force of 210 lbs. may lift a weight of 1050 lbs. ?

20. What force is required with a four-cord tackle to lift 1456.38492 lbs. ?

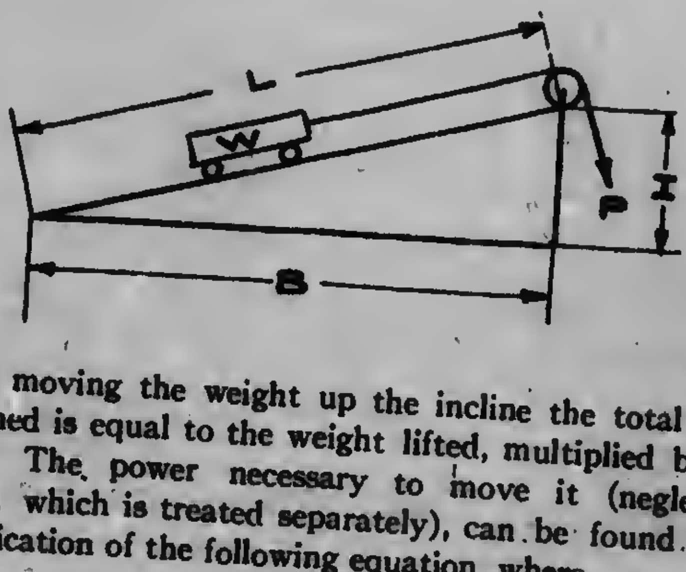
21. What weight will a force of 48 lbs. lift with a five-cord tackle that is only 83% efficient ?

22. A team of horses apply the power to a four-cord tackle by means of a fixed pulley on the ground. How many foot-pounds of work do the horses exert in lifting a weight of 3680 lbs. to a height of 20 ft. ?

THE INCLINED PLANE

The inclined plane is a plane inclined to the horizontal, and by it objects are raised to a higher level by means of a smaller force acting through a longer distance than would be the case in a perpendicular lift.

An example of this is a train ascending a grade, and in so doing uses less power at any given time than would be the case if the train were lifted perpendicularly to the same height.



In moving the weight up the incline the total work performed is equal to the weight lifted, multiplied by the height. The power necessary to move it (neglecting friction, which is treated separately), can be found from an application of the following equation, where

P = Power
W = Weight
L = Length of grade
H = Height of grade
(L and H must be in the same units).

$$P \times L = W \times H$$

Example 1. A train weighs 1160 tons and ascends a grade of 3 in 100; that is, for every 100 ft. of base the train is raised vertically 3 ft. What power (neglecting friction) would be necessary to move the train?

Note.—For a correct answer to this the length of the hypothenuse formed by the base and height should be taken, but as the difference between it and the base in these questions is so small the length of the base is considered close enough for practical purposes.

$$P = \frac{W \times H}{L}$$

$W = 1160 \times 2000$ lbs. $H = 3$ ft. $L = 100$ ft.

$$\text{Therefore } P = \frac{1160 \times 2000 \times 3}{100} = 69600 \text{ lbs. Ans.}$$

In the case of a locomotive this force would be reckoned as drawbar pull, and while this would be just sufficient to counteract the weight and maintain the load in equilibrium, in practice more force would be required to move the load and overcome friction. The latter details, however, would be taken care of in separate questions.

THE SCREW

The screw, upon close examination, will be seen to be the same in principle as the inclined plane (see sketch).

A thread is cut around a circular piece of metal, which for convenience we will call a cylinder, and the distance between centres of threads is known as the pitch. Cut out in paper an inclined plane having as its base a length equal to the circumference of the cylinder around which it is proposed to cut a thread and having a height equal to the pitch of the threads. Place this around the cylinder and the screw thread will correspond to the side L on the inclined plane.

Various forms of threads are used, which, however, do not affect the question. One thread upon the cylinder in sketch illustrates the principle, but in practice the thread is continuous for a certain length according to the particular requirements of the work, but must at least run out at one end to allow a nut or fixed collar being placed upon it. This nut is made to fit the screw, having a hole in it the same size as the cylinder (measured at root of thread) and internal grooves cut around the hole to allow the thread of the screw fitting into it.

Generally this nut or collar is fastened, and by revolving the screw by means of a lever fastened to the head of the screw for every complete revolution made by the screw it will advance or recede a distance equal to the pitch of the thread. The mechanical advantage of the screw is increased by increasing the length of the lever or decreasing the pitch of the thread on the screw.

The work performed is equal to the amount of power multiplied by the circumference of the circle which has as its radius the distance from the centre of the screw to the place on the lever where the power is applied. At the same time the work done by the screw in moving through the distance of the pitch of the thread (T) equals the weight being lifted multiplied by the pitch of the thread, or W×T.

Expressing this as an equation:

$$P \times C = W \times T$$

Where P = Power applied on lever.

C = Circumference, radius of which measures from centre of screw to point on lever where the power is applied (express in inches).

W = Weight being lifted, or resistance being overcome.

T = Pitch of thread.

Example 1. It is desired to raise a weight by means of a screw-jack having 5 threads per inch. A power of 40

lbs. is applied at a distance of 14 ins. from centre of screw. How much weight can be iifted ?

$$W=\frac{P\times C}{T}$$

P = 40 lbs.
C = 14×2×3.1416
T = 1/5 or .2

$$\text{Therefore } W=\frac{40\times 14\times 2\times 3.1416}{.2}=17592.96\text{lbs.}$$

Answer

Example 2. What power is required to be applied 16 ins. from the axis of a screw-jack having 6 threads per inch, in order to raise a weight of 24000 lbs. ?

$$P=\frac{W\times T}{C}$$

W = 24000 lbs.
T = 1/6 in. or .1666 in.
C = 16×2×3.1416

$$P=\frac{2400\times .1666}{16\times 2\times 3.1416}=39.77\text{ lbs.}$$ Answer.

QUESTIONS

1. It is desired to raise a weight with a screw-jack having 4 threads per inch. How great a weight can be lifted if a power of 100 lbs. is applied 12 ins. from centre of screw ?

2. A power of 140 lbs. is used to lift a weight of 36400 lbs. How many threads per inch are on the screw, if power is applied 28 ins. from centre of screw ?

3. What power is required to raise a weight of 34500 lbs. with a six-thread screw-jack, power being applied 26 ins. from centre of screw ?

4. A power of 45 lbs. applied 15 ins. from centre of screw lifts a weight of 25500 lbs. How many threads per inch are there on the screw ?

5. A power of 36 lbs. lifts a weight of 60000 lbs. with a screw-jack having 7 threads per inch. At what distance from centre of screw was the power applied ?

6. A machinist in tightening up a nut exerts a force of 40 lbs. at end of spanner which is 10 ins. from centre of bolt. If the bolt has 9 threads per in., what pressure is the nut sustaining ?

7. A press, manipulated by a screw with a handwheel on the end of it, exerts a pressure of 7000 lbs. If the screw has 8 threads per inch, and the hand wheel is 20 ins. in diameter, what power is required to operate press ?

8. A power of 48 lbs. is applied 15 ins. from the centre of a screw-jack. What weight will it lift if the screw has 6 threads per inch ?

9. A weight of 28 tons is to be moved along the ground 3 ft. by means of a screw-jack supported against a wall. The coefficient of friction is .70. The screw has 6 threads per in., and its lever has a 30 in. radius. Find (a) power required, and (b) work performed.

10. A screw-press is used for pressing in cylinder bushes. There are four threads per inch on the screw. It is operated by a wrench, having a power of 300 lbs. applied 44 ins. from centre of screw. What is the total pressure pressing in the bush ?

HYDRAULICS

Hydraulics treat of the power derived from water in motion. The principles involved could be applied to other liquids, but in these lessons water only will be dealt with.

The weight of water per cubic foot is taken as being 62.5 lbs., and while this is not strictly correct, it is close enough for all practical purposes.

The weight of one cubic inch of water is .03616 lbs., and a column of water 12 inches high and having a cross-sectional area of one square inch would weigh 12 × .03616 = .43392 lbs. This number is simplified to 434 lbs., and as such is generally used.

The pressure upon any point under water which has its surface free to the atmosphere is the same in all directions, and varies according to the depth it is below the surface of the water. This depth of water is called the "head," and at a depth of one foot each square inch of surface would have a pressure of .434 lbs. upon it. Similarly, at a depth of two feet the pressure would be double and so on, and if

H = Head, expressed in feet
P = Pressure per square inch
.434 = Weight of a column of water 12 ins. high having a sectional area of 1 sq. in.,

Then $P = H \times .434$

$\therefore H = P \div .434$

It is sometimes necessary to find the total pressure upon a surface greater than one square inch but less than one square foot.

The solution of such a problem includes the consideration of so many more square inches, or the foregoing formula multiplied by the area expressed in square inches to find total pressure, or divided by the area in square inches to find the head. This is so apparent that a formula need not be made to possibly confuse the simplicity of the previous one.

When, however, the area of the surface in the problem is greater than one square foot, the solution is simplified by keeping the question in feet and taking the weight of one cubic foot of water as shown in the following formula:—

When $T.P. = $ Total pressure on surface
$H = $ Head, expressed in feet
$A = $ Area of surface, expressed in sq. ft.
$62.5 = $ Weight of one cu. ft. of water.

Then $T.P. = H \times A \times 62.5$
$H = T.P. \div (A \times 62.5)$
$A = T.P. \div (H \times 62.5)$

Example 1. A tank is filled with water to a depth of 3 ft., and has a base measuring 2 ft. by 4 ft. Find the pressure per sq. in. on its base.

Solution:—$P = H \times .434$
$H = 3$ ft.
Therefore $P = 3 \times .434 = 1.302$ lbs. pressure per sq. in on base of tank. Answer.

Example 2. Find the total pressure on the base of the tank in Example 1.

Solution:—$T.P. = A \times H \times 62.5$
$A = 2 \times 4 = 8$ sq. ft.
$H = 3$ ft:
Therefore $T.P. = 8 \times 3 \times 62.5 = 1500$ lbs., total pressure on base of tank.
Answer.

Example 3. A G.T.R. water tank used for watering engines measures 30 ft. diameter by 20 ft. high. Find the pressure per sq. in. on base of tank.

Solution:—$P = H \times .434$
$H = 20$ ft.
Therefore $P = 20 \times .434 = 8.68$ lbs. pressure per sq. in. on base of tank. Answer.

Example 4. Find the total pressure upon the base of the tank in Example 3.

Solution $T.P. = A \times H \times 62.5$
$A = 30 \times 30 \times .7854$ sq. ft.
$H = 20$ ft.
Therefore $T.P. = 30 \times 30 \times .7854 \times 20 \times 62.5 =$ 873575 lbs. Answer.

In all questions similar to these examples it will have been noted that the pressure is equal to the weight of a vertical column of water which is supported by the area of the given base.

PRESSURE UPON VERTICAL SURFACES

The pressure upon the sides of a retaining wall or tank is at right angles to their surfaces and becomes greater as the depth increases. At the surface the pressure is zero, while the maximum pressure is at the lowest depth.

From these two dimensions an average is struck and is substituted for the centre of gravity, which is the correct value, but the difference is so little (the centre of gravity is always slightly lower) and the average depth is so much easier obtained that it is used in all but very close calculations.

The letters CG, meaning "centre of gravity," will be retained in formulas pertaining to the solution of questions involving pressure upon vertical sides, and when

T.P. = Total pressure on surface
A = Area of surface, in square feet
C.G. = Average depth of water
62.5 = Weight of one cubic foot of water
Then T.P. = A × C.G. × 62.5

Example 5. The lock gates of a canal are 36 ft. wide and 28 ft. deep. The water on one side is 8 ft. deep while on the other it is 24 ft. deep. What is the total pressure upon the gates ?

It is evident that the depth of the gates does not affect the question, and that the problem is a double one, inasmuch as the pressure on the low water side must be found and subtracted from the pressure on the high water side.

Solution.—(for low water side):
T.P. = A × CG × 62.5
A = 8 × 36
C.G. = 8 ÷ 2 = 4
Therefore T.P. = 8 × 36 × 4 × 62.5 = 72000 lbs.

Solution. (for high water side):
T.P. = A × CG × 62.5
A = 24 × 36
C.G. = 24 ÷ 2 = 12
Therefore T.P. = 24 × 36 × 12 × 62.5 = 648000 lbs.
648000 lbs.—72000 lbs. = 576000 lbs., excess pressure on one side. Answer.

HORSEPOWER REQUIRED TO RAISE WATER

To find the horsepower necessary to raise water to a given height is simply a question in work, and the same equation, HP×33000=W×H, will, when reconstructed into formulas suitable to the different questions, solve all problems. W represents weight in lbs. raised per minute, and H is height water is to be raised in feet.

Example 1. Find the horsepower necessary to raise 1200 gallons (U.S.) of water per minute to a height of 22 feet.

Solution:—$HP = \dfrac{W \times H}{33000}$

$W = 1200 \times 8\tfrac{1}{3}$

$H = 22$ ft.

Therefore $HF = \dfrac{1200 \times 25 \times 22}{3 \times 33000} = 6\tfrac{2}{3}$ H.P. Ans.

Example 2. How many U.S. gallons of water can be delivered per hour by a pump which has a water cylinder 4 ins. in diameter by 9 in. stroke, when the number of strokes per minute is 33. The pump is 75% efficient.

Note. In all pump questions, the number of strokes is taken as meaning the number of discharges per cylinder per minute.

Solution:—Volume of cylinder $= 4 \times 4 \times .7854 \times 9$ cu. ins.

Therefore $\dfrac{4 \times 4 \times .7854 \times 9 \times 33}{231} =$ No. of U.S. gals. delivered in one minute.

The question asks how much per hour, when the efficiency is 75%,

$$\text{Therefore } \frac{4\times4\times.7854\times6\times33\times60\times75}{231\times100} = 727.05564 \text{ gals. per hr. Answer.}$$

Note. It is not necessary to detail questions out as fully as the above has been shown, which was done simply to make the explanation clear.

Example 3. Find the horsepower of a pump required to deliver 9600 Imp. gallons of water per hour into a storage tank, if the delivery pipe be 33 ft. above the pump and the pump's efficiency be 80%. **Note.** One Imperial gallon weighs 10 lbs.

Solution. $$H.P. = \frac{W\times H}{33000}$$

$$W = \frac{9600\times10}{60}$$

$$H = 33 \text{ ft.}$$

$$\text{Efficiency} = 100/80$$

$$\text{Therefore } \frac{9600\times10\times33\times100}{33000\times60\times80} = 2 \text{ horsepower.}$$

Answer.

QUESTIONS

1. A tank is 13 ft. sq. and 4 ft. 6 ins. deep. What is the total pressure upon the base when tank is filled with water?

2. In Question 1, what would be the pressure upon, 1st, the end, and, 2nd, on one side ?

3. The water level in a city reservoir is 156 ft. above the hydrants on the street. What is the pressure per square inch at the hydrant ?

4. A cylindrical tank is 6 ft. deep and 5 ft. diameter. What will be the pressure on the base when the tank is filled to within 6 ins. from the top of tank ?

5. A valve 12 ins. in diameter has its face free to a vertical pressure of water, the water level being 16 ft. above the valve's centre. Find the total pressure upon the valve face.

6. A canal lock gate is 18 ft. wide by 12 ft. deep. The water level is 9 ft. on one side and 18 ins. on the other. Find the pressure on the lock gate.

7. How many U.S. gallons of water will a single-action pump deliver in 40 minutes, when the water piston is 8 ins. dia. by 12 ins. stroke, and the number of strokes per minute is 45. 20% is lost through slippage.

8. How many Imperial gallons of water can be raised in one hour from a mine 180 ft. deep by a single acting pump of 40 H.P.

9. From what depth can a 7 H.P. pump raise 20 long tons of water in one hour ?

10. A rectangular tank which is 16 ft. deep by 26×18 ft. base has a hole at end to permit fastening an 8 in. dia. valve by means of bolts. The centre of the valve is 18 ins. up from bottom of tank. Find total pressure upon valve face when water is within six in. from top of tank.

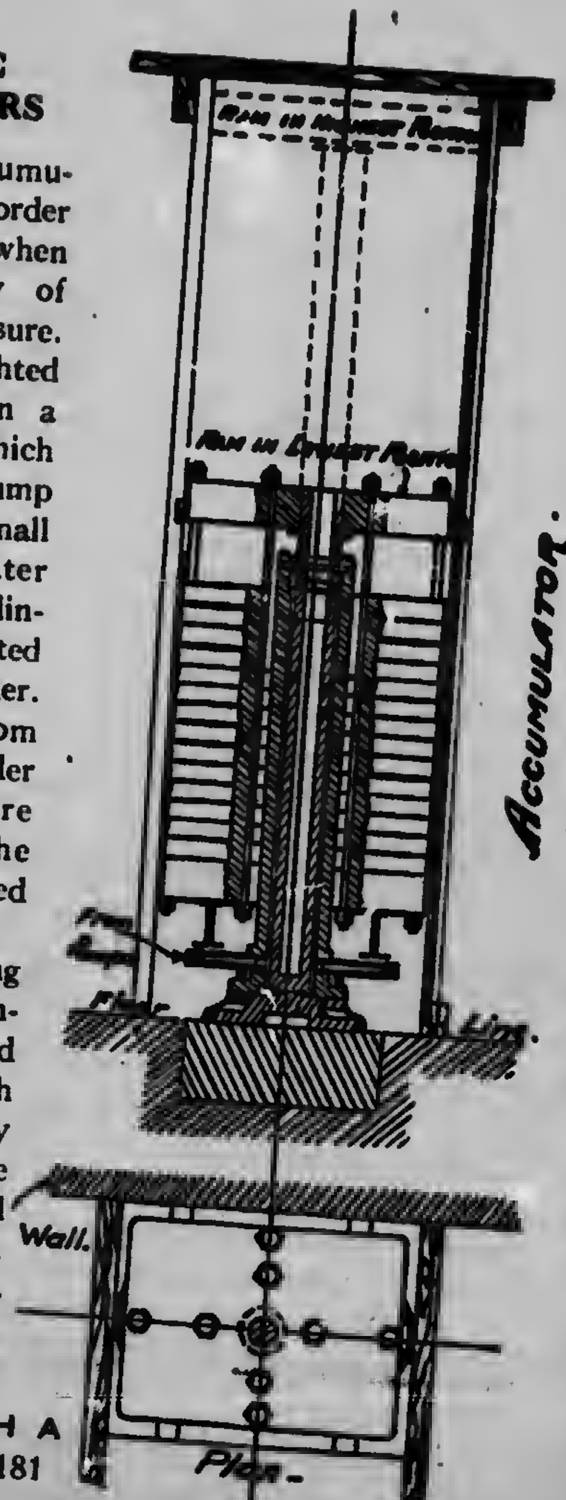
11. A water gauge shows a pressure of 18 lbs. per sq. in. It is attached by means of piping to a tank of water. Find the height of water above the gauge.

HYDRAULIC ACCUMULATORS

Hydraulic accumulators are used in order to have on hand when required a supply of water at high pressure. It consists of a weighted piston acting within a cylinder into which water is forced by pump pressure through a small opening. This water gradually fills the cylinder, lifting the weighted piston in the cylinder. The pipe leading from it is thereafter under a head of pressure depending upon the amount of load carried by the piston.

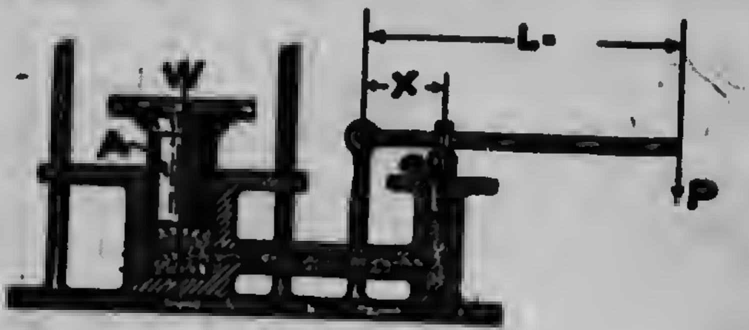
The accompanying sketch A shows the general arrangement, and the convenience of such will be seen when by simply adding more weight the pressure will correspondingly increase.

Sketch B illustrates very clearly the working principles of a hand



SKETCH A

pressure press, which are exactly the same as those in use in an accumulator.



SKETCH B

Example 1. The diameter of the large ram of an hydraulic press is 8 ins. and the small ram 2 ins. The lever arm is 30 ins. long. From fulcrum to small ram is 6 ins. If 20 lbs. be applied at end of lever, find weight that can be raised by large ram.

A = Area of large ram = 50.26 sq. ins.
a = Area of small ram = 3.14 sq. ins.

$$\frac{L \times P}{A} = \text{power at small ram} = \frac{30 \times 20}{6} = 100 \text{ lbs.}$$

$$\frac{\text{Power at small ram} \times A}{a} = \frac{100 \times 50.26}{3.14} = 1600 \text{ lbs.}$$

-Answer.

QUESTIONS

1. The diameter of the small ram of a hydraulic press is 3 ins. and the large ram 7½ ins. The lever arm is 26 ins. long and distance from the fulcrum to the small ram is 4¾ ins. What power is necessary to apply at end of lever to raise 1675 lbs. by the large ram?

2. The load in an hydraulic accumulator is 65 tons. The diameter of the ram is 9½ ins. What pressure will this give per sq. in.?

FORMULAS

HOW TO USE THEM

A formula may be defined as a mathematical rule expressed by signs and symbols instead of in actual words. In formulas, single letters (symbols) are used to represent numbers or quantities, the term "quantity" being used to designate any number involved in a mathematical process. The use of symbols in formulas, in place of the actual numbers, simplifies the solution of problems, and makes it possible to condense into small space the information that otherwise would be imparted by long and cumbersome rules. The symbols used in formulas are mainly the letters in the alphabet, and the signs are the ordinary signs used in arithmetic. Letters from the Greek alphabet are frequently used to designate angles, and the Greek letter π (pi) is always used to indicate the ratio between the circumference and the diameter of a circle; π, therefore, is always, in mathematical expressions, equal to 3.1416.

Knowledge of algebra is not necessary in order to make successful use of formulas of the general type, such as are found in engineering handbooks; it is only necessary to thoroughly understand the use of letters or symbols in place of numbers, and to be well versed in the methods, rules, and processes of ordinary arithmetic.

Use of Formulas. As mentioned, the symbols or letters used in formulas designate or "stand for" actual figures or numerical values. The figures or values for a given problem are inserted in the formula according to the requirements in each specific case. When the values are thus inserted, in place of the letter, the result or answer is obtained by ordinary arithmetical methods. There are two reasons why a formula is preferable to a rule expressed in words;

1. The formula is more concise, it occupies less space, and it is possible for the eye to catch, at a glance, the whole meaning of the rule laid down.

2. It is easier to remember a brief formula than a long rule, and it is, therefore, of greater value and convenience. It is not always possible to carry a handbook or reference book about, but the memory must be relied upon to store up a number of the most frequently occurring mathematical and mechanical rules. The use of formulas can be explained most readily by actual examples. In the following, therefore, a number of simple formulas will be given, and the values will be inserted so as to show in detail the principles involved.

Example 1. When the diameter of a circle is known, the circumference may be found by multiplying the diameter by 3.1416. This rule, expressed as a formula, is:

$$C = D \times 3.1416,$$

in which,

C = circumference of circle;
D = diameter of circle.

This formula shows at a glance that the circumference of a circle is always equal to the diameter times 3.1416. The diameter may be any number of inches, feet, or miles, the relation stated by the formula always exists.

Let it be required to find the circumference of a circle 22 ins. in diameter. Insert 22 in the formula in place of D, then:

$$C = 22 \times 3.1416 = 69.1152 \text{ ins.}$$

By simple multiplication, the formula thus gives C, the circumference, equal to 69.1152 ins.

The diameter of a circle is 3.72 ins. Find the circumference.

Insert the value 3.72 in place of D in the formula. Then:

$$C = 3.72 \times 3.1416 = 11.6867 \text{ ins.}$$

Example 2. In spur gears, the outside diameter of the gear can be found by adding 2 to the number of teeth, and dividing the sum obtained by the diametral pitch of the gear. This rule can be expressed very simply by a formula. Assume that D is the outside diameter of the gear; N the number of teeth; and P, the diametral pitch. Then the formula would be:

$$D = \frac{N+2}{P}$$

This formula reads exactly as the rule given above; that is, the outside diameter (D) of the gear equals 2 added to the number of teeth (N), and this sum is divided by the pitch (P).

If the number of teeth in a gear is 16 and the diametral pitch 6, simply insert these figures in the places of N and P in the formula, and find the outside diameter as in ordinary arithmetic.

$$D = \frac{16+2}{6} = \frac{18}{6} = 3 \text{ ins.}$$

In another gear, the number of teeth is 96; the diametral pitch is 7. Find the outside diameter.

$$D = \frac{96+2}{7} = \frac{98}{7} = 14 \text{ ins.}$$

From the examples given the following general rule may be expressed: In formulas each letter stands for a certain dimension or quantity; when using a formula for solving a problem, replace the letters in the formula by the figures given for a certain problem, and find the required answer as in ordinary arithmetic.

Example 3. The formula for the horsepower of a steam engine is as follows:

$$H = \frac{P \times L \times A \times N}{33000}$$

in which,

H = indicated horsepower of engine;
P = mean effective pressure on piston in pounds per sq. in.;
L = length of piston stroke, in ft.;
A = area of piston, in sq. ins.;
N = number of strokes of piston per minute.

Assume that P = 90; L = 2; A = 320; and N = 110; what would be the horsepower?

Inserting the given values in the formula:

$$H = \frac{90 \times 2 \times 320 \times 110}{33000} = 192$$

Omitting Multiplication Signs in Formulas. In formulas, the sign for multiplication (×) is often left out between letters the values of which are to be multiplied. Thus AB means A×B, and the formula:

$$\frac{P \times L \times A \times N}{33000} \text{ can also be written } \frac{PLAN}{33000}$$

Thus, if A = 3, and B = 5, then:

$$AB = A \times B = 3 \times 5 = 15.$$

If A = 12; B = 2; and C = 3, then:

$$ABC = A \times B \times C = 12 \times 2 \times 3 = 72.$$

It is only the multiplication sign (×) that can be thus left out between the symbols or letters in a formula. All other signs must be indicated the same as in arithmetic. The multiplication sign can never be left out between two figures: 35 always means thirty-five, and "three times five" must be written 3×5; but "three times A" may be written 3A. As a general rule, the figure in an expression such as "3A" is written first, and is known as the coefficient of A. If the letter is written first, the multiplication sign is not left out, but the expression is written "A×3."

Use of Parenthesis. Parenthesis ( ) or brackets [ ] in a formula, or in an algebraic expression in general, indicate that the expression inside the parenthesis or brackets should be considered as one single symbol, or, in other words, that the calculation inside the parenthesis or brackets should be carried out by itself, before other calculations are carried out.

Example 1. $6 \times (8+3) = 6 \times 11 = 66.$

Example 2. $5 \times (16-14) + 3(2.25-1.75) = 5 \times 2 + 3 \times 0.5 = 10 + 1.5 = 11.5.$

In the last example it will be seen that 5 is multiplied by 2 and 3 by 0.5, and then the products of these two multiplications are added. From the order of the numbers 5×2+3×0.5, one might have assumed that the calculation should have been carried out as follows: 5 times $2=10$, plus $3=13$, times $0.5=6.5$. This latter procedure, however, is not correct, as explained in the following paragraph.

Order of Operations. When several numbers or expressions are connected by the signs $+$, $-$, $\times$, and $\div$, the operations are carried out in the order written, except that all multiplications should be carried out before the other operations. The reason for this is that numbers connected by a multiplication sign are only factors of the product thus indicated, which product should be considered by itself as one number. Divisions should be carried out before additions and subtractions, if the division is indicated in the same line with these other processes and if there are no parentheses indicating that other operations must first be worked out.

Examples.

$$5\times6+4-6\times4=30+4-24=34-24=10.$$
$$5+3\times2=5+6=11.$$
$$100\div2\times5=100\div10=10$$
$$3.5+16.5\div3-1.75=3.5+5.5-1.75=7.25.$$

But

$$5\times(6+4)-6\times4=5\times10-24=50-24=26.$$
$$(5+3)\times2=8\times2=16.$$
$$(100\div2)\times5=50\times5=250.$$
$$(3.5+16.5)\div(3-1.75)=20\div1.25=16.$$

Exponents. The square of a number is the product of that number multiplied by itself. The square of 2 is $2\times2=4$, and the square of 10 is $10\times10=100$; similarly, the square of 177 is $177\times177=31329$. Instead of writing

4×4 for the square of 4, it is often written 4^2 which is read four square, and means that 4 is multiplied by 4. In the same way 128^2 means 128×128. The small figure (^{2}) in these expressions is called the exponent.

The cube of a number is the product obtained if the number itself is repeated as a factor three times. The cube of 2 is $2 \times 2 \times 2 = 8$, and the cube of 12 is $12 \times 12 \times 12 = 1728$. Instead of writing $2 \times 2 \times 2$ for the cube of 2, it is often written 2^3, which is read two cube. In the same way 128^3 means $128 \times 128 \times 128$. The small figure (^{3}) in these expressions is called the exponent, the same as in the case of the figure (^{2}) indicating the square of a number. An expression of the form 18^3 may also be read the "third power of 18."

In the same way as $2 \times 2 = 2^2$, and $2 \times 2 \times 2 = 2^3$, the fourth power may be written $2^4 = 2 \times 2 \times 2 \times 2$. Similarly, the expression 2^5 means that 2 is repeated as a factor five times, or: $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.

The expression 2^4 is read "the fourth power of 2," and the expression 6^5, "the fifth power of 6." Thus, the expression "seventh power" would mean that the exponent equals 7.

From these examples, it is evident that the object of the exponent is to indicate how many times the number to which the exponent is affixed is to be taken as a factor. Exponents may be affixed to letters or symbols as well as to figures. For example, a^5 indicates that a is to be taken as a factor five times, or:

$$a = aaaaa = a \times a \times a \times a \times a.$$

In an expression of the form $7a^3$, the exponent applies only to the symbol a to which it is affixed, and not to the coefficient "7." If $a=3$, then:

$$7a^3=7\times3^3=7\times3\times3\times3\times=189.$$

The exponent can, however, be made to apply to the coefficient also, by the use of parentheses enclosing the expression to which the exponent applies:

$$(7a)^3=(7\times3)^3=(21)^3=21\times21\times21=9261.$$

If the exponent applies only to the coefficient, the expression would be written "7^3a." The value, if $a=3$, would be:

$$7^3a=7\times7\times7\times3=1029.$$

The meaning of exponents will be made still clearer by the following examples:

$$a^2b^3=a\times a\times b\times b\times b.$$
$$(ab)^2=a^2b^2=a\times a\times b\times b.$$
$$3\,(ab)^3=3\,a^3b^3=3\times a\times a\times a\times b\times b\times b.$$

Roots.—The square root of a number is that number which, when multiplied by itself, will give a product equal to the given number: Thus, the square root of 4 is 2, because 2 multiplied by itself gives 4. The square root of 25 is 5; of 36, 6, etc. The square root is the reverse of the square, so that if the square of 24 is 576, then the square root of 576 is 24. The mathematical sign for the square root is $\sqrt[2]{}$, but the index figure (²) is generally left out, making the square root sign simply $\sqrt{}$, thus:

$\sqrt{4}=2$ (the square root of four equals two).

$\sqrt{100}=10$ (the square root of one hundred equals ten).

The operation of finding the square root of a given number is called extracting the square root. Squares and square roots as well as cubes and cube roots of all numbers up to 1000 are generally given in all standard handbooks.

In the same way as square root means the reverse of square, so cube root means the reverse of cube; that is, the cube root of a given number is the number which, if repeated as a factor three times, would equal the number given; thus the cube root of 27 is 3, because $3\times3\times3=27$. If the cube of 15 is 3375, then the cube root of 3375 is 15. The mathematical sign for the cube root is $\sqrt[3]{\ }$ thus:

$\sqrt[3]{64}=4$ (the cube root of sixty-four equals four).

$\sqrt[3]{4096}=16$ (the cube root of four thousand ninety-six equals sixteen).

In the same way as the square root means the reverse of square, and the cube root the reverse of cube, so also the fourth root is the reverse of the fourth power; that is, if a number is required which, when repeated as a factor four times, will give as a product a given number, the fourth root, or $\sqrt[4]{\ }$, must be obtained. Thus:

$\sqrt[4]{81}=3$, because $3\times3\times3\times3\times=81$.

The fifth root is written $\sqrt[5]{\ }$; and, as an example:

$\sqrt[5]{32}=2$, because $2\times2\times2\times2\times2=32$.

Symbols may be used as well as figures. For example:

$\sqrt[3]{ab}$; $\sqrt[5]{n}$; $\sqrt{a+b}$; $\sqrt[n]{a}$; $\sqrt[n]{17}$.

In the same way as $3^2=3\times3$, so $a^2=a\times a$; and as $\sqrt{9}=3$, so also $\sqrt{a^2}=a$, because $a\times a=a^2$.

The principles of roots applied to algebraic expressions are shown by the examples below:

$\sqrt[3]{a^3}=a$; $\sqrt[3]{a^3}=a$; $\sqrt{a^2b^2}=ab$. $\sqrt{4a^2}=2a$;

$\sqrt[3]{27a^3}=3a$; $\sqrt{16\,a^2b^2}=4ab$.

Expressions, such as $\sqrt{a}$, cannot be further simplified, because the root cannot be extracted from a quantity consisting of a letter without an exponent, except by introducing fractional exponents, which belongs to a more advanced stage of algebraic study.

The square and cube roots of numbers may be extracted by methods known from arithmetic. In practice, however, the tables of squares and cubes, and square roots and cube roots, given in standard handbooks, are used to avoid the time consumed and cumbersome methods otherwise necessary to employ.

Examples Involving Roots and Exponents.—A number of examples, containing exponential expressions and roots, will make the preceding explanations clearer.

Example 1. Find the value of A in the formula:

$$A = \frac{\sqrt{B} \times C}{D}$$

Assume $B = 36$; $C = 3.5$; $D = 10.5$. By inserting these values in the formula:

$$A = \frac{\sqrt{36} \times 3.5}{10.5} = \frac{6 \times 3.5}{10.5} = \frac{21}{10.5} = 2 \text{ Ans.}$$

Example 2. Find the value of A in the formula:

$$A = \frac{B^2 + C^2}{D^2}$$

If $B = 10$; $C = 14$; and $D = 4$.

$$A = \frac{10^2 + 14^2}{4^2} = \frac{10 \times 10 + 14 \times 14}{4 \times 4} = \frac{100 + 196}{16} = 18.5$$

Example 3:—Find the value of A in the formula:

$$A=\sqrt{B^2+C^2}$$

if B = 8 and C = 6.

Inserting the given values in the formula:

$$A=\sqrt{8^2+6^2}=\sqrt{8\times8+6\times6}=\sqrt{64+36}=\sqrt{100}=10.$$

The examples given indicate the principles involved in the use of formulas, and show also how easily formulas may be employed by anyone who has a general understanding of arithmetic.

Transposition of Formulas. An important method for facilitating the use of formulas is known as transposition; by this is meant the method for moving or transposing one or more terms or symbols from one side of the equals sign to the other. As an example, the formula for the horsepower transmitted by belting may be written:

$$H.P.=\frac{SVW}{33000}$$

in which,

H.P. = horsepower transmitted.
S = working stress of belt per in. of width, in lbs.
V = velocity of belt, in feet per minute.
W = width of belt, in ins.

If the working stress S, the velocity V, and the width W are known, the horsepower can be found directly from this formula by inserting the given values. Assume S = 33; V = 600; and W = 5. Then:

$$H.P.=\frac{33\times600\times5}{33000}=3$$

Assume, however, that the horsepower, the stress S, and the velocity V are known, and that the width of belt, W, is to be found. The formula must then be transposed so that the symbol W will be on one side of the equals sign and all the known quantities on the other. The transposed formula is as follows:

$$\frac{H.P. \times 33000}{SV} = W$$

The quantities (S and V) that were in the numerator on the right side of the equals sign are transposed to the denominator on the left side, and "33000" which was in the denominator on the right side of the equals sign is transposed to the numerator on the other side. This is in conformity with the general rule for transposition. Symbols which are not part of a fraction, like "H.P." in the formula first given, are to be considered as being numerators (having the denominator 1).

The general rules for transposition are:

1. An independent term preceded by a + sign may be transposed to the other side of the equals sign if the + sign is changed to a − sign.

2. An independent term preceded by a − sign may be transposed to the other side of the equals sign if the − sign is changed to a + sign.

3. A term which multiplies all the other terms on one side of the equals sign may be transposed to the other side if it is made to divide all the terms on that side.

4. A term which divides all the other terms on one side of the equals sign may be transposed to the other side if it is made to multiply all the terms on that side.

Example of Rules 1 and 2:

$$W = N - T; \qquad T = N - W.$$

Example of Rule 3:

$$W = NvR \qquad \frac{W}{Nv} = R.$$

Example of Rule 4:

$$W = \frac{NR}{S} \qquad SW = NR, \text{ and, by Rule 3; } S = \frac{NR}{W}.$$

According to the rules given, any formula of the form $A = \frac{B}{C}$ can be transposed as below:

$$A \times C = B, \text{ and } C = \frac{B}{A}$$

Suppose a formula to be of the form:

$$A = \frac{B \times C}{D}$$

Then: $D = \frac{B \times C}{A}$; $\frac{A \times D}{C} = B$; $\frac{A \times D}{B} = C.$

The rules given for the transposition of formulas also apply to the transposition of the terms of equations.

Mensuration—Summary of Formulas.

Diameter of circle × 3.1416 = Circumference.
Square of the radius of a circle × 3.1416 = Area.
Square of diameter of circle × .7854 = Area.
Half the circumference of a circle × half the diameter = Area.

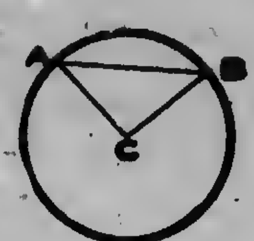
Diameter of circle × 0.7071 = Side of an inscribed square.

Diameter of circle × 0.282 = Side of an equal square.

To find the area of any triangle. Multiply base by half the altitude; or, from the half sum of the three sides subtract each sum separately, multiply the half-sum and the three remainders continuously together and extract the square root of the product.

Circumference of a sphere × the diameter = Surface.
Square of diameter of sphere × 3.1416 = Surface.
Cube of diameter of sphere × 0.5236 = Volume.
Area of base × $\frac{1}{3}$ of its altitude = Volume of cone or pyramid.

Area of Sector of a Circle. This equals area of circle divided by 360 and multiplied by the number of degrees contained within the sector. To find the area of the segment of a circle, first find area of sector and subtract area of triangle shown at ABC.



Formula for Tractive Power or Drawbar Pull of a Locomotive.

$$\frac{C^2 \times S \times P}{D} = \text{tractive power in lbs.}$$

Where

C = diameter of cylinder in ins.
S = stroke of piston in ins.
P = mean effective or average pressure during stroke in pounds taken at 85% of boiler pressure.
D = diameter of driving-wheels in ins.

Example. Find the tractive power of a locomotive having cylinders 18 in. diameter, 24 in. stroke, 160 lbs. boiler pressure, and 5 ft. 8 in. driving-wheels.

$$\frac{18^2 \times 24 \times (85\% \text{ of } 160)}{68} = 15552 \text{ lbs. Answer.}$$

QUESTIONS

1. What is the tractive power of a locomotive with cylinders 22 ins. diameter × 28 in. stroke, driving-wheels 6 ft. 1 in. diameter, and boiler pressure 200 lbs. per square inch?

2. Find the tractive power of a locomotive when the cylinders are $18\frac{1}{2}$ in. diameter and 26 in. stroke, boiler pressure 165 lbs., and driving-wheels 5 ft. 3 in. diameter.

HAULING CAPACITY AND ADHESIVE WEIGHT

Formula for Haulage Capacity of Locomotives.

Multiply the number of 1000 lbs. weight on driving-wheels × 32 − the weight of engine and tender in tons.

The following figures are based on straight track, slow speed, adhesion taken at $\frac{9}{40}$ and fractional resistance at 7 lbs. per ton.

Example. What is the hauling capacity of a locomotive having a weight of 180000 lbs. on driving-wheels, the total weight of engine and tender loaded being 352000 lbs.?

352000 lbs. = 176 tons.

180 × 32 − 176 = 5584 tons H.C. Answer.

QUESTIONS

1. Find the haulage capacity of a locomotive with 133500 lbs. weight on driving-wheels, and the weight of engine and tender loaded being 318000 lbs.

2. Find the haulage capacity of a locomotive having a total weight of engine and tender of 294000 lbs., the weight on drivers being 140000 lbs.

Adhesive Weight of a Locomotive.

Note. The adhesive weight of a locomotive is generally taken at $\frac{9}{40}$ of the weight on the drivers.

Example. The weight on the driving-wheels of a locomotive is 140744 lbs. Find the adhesive weight.

$$\frac{9}{40} \times \frac{140744}{1} = 31667\tfrac{2}{5}.$$ Answer.

QUESTIONS

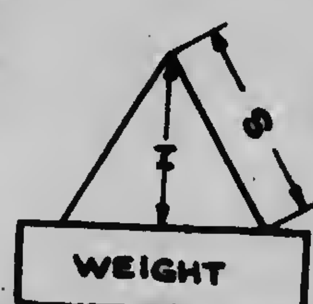
What is the adhesive weight of a locomotive whose weight on the driving-wheels is 167376 lbs.?

CHAINS

The size of a chain is known by the size of iron from which it is made.

Size of chain inches	Safe working load in lbs. B. B. B.	Breaking strain in lbs. B. B. B.	Weight of common coil chain per 100 ft.
3/16	625	2500	50
1/4	1175	4700	75
5/16	1650	6600	110
3/8	2600	10400	155
7/16	3270	13080	200
1/2	4275	17100	265
9/16	5400	21600	325
5/8	7400	29600	420
11/16	8500	34000	500
3/4	11000	44000	590
13/16	12000	48000	700
7/8	13550	54200	800
15/16	14600	58400	900
1	17050	68200	1000
1 1/16	19050	76200	1100
1 1/8	22065	88260	1250
1 3/16	23565	94260	1400
1 1/4	26080	104320	1600
1 5/16	28100	112400	1750
1 3/8	31100	124400	1900
1 7/16	33500	134000	2100
1 1/2	37060	148240	2100

In chains used as slings with a double leg, as per sketch, note that the greater the distance between the legs the greater is the strain on each chain.



The formula $\frac{W}{2} \times \frac{S}{H}$ will determine the strain on one leg in any problem.

ROPES

The breaking strain for hemp rope is 6000 lbs. per sq. in. of cross section area; and for manila rope the breaking strain is 3000 lbs. per sq. in. of cross section area.

To find the breaking strength of any rope apply the following formula:

Area × 6000 = breaking strength for hemp rope.
Area × 3000 = breaking strength for manila rope.

A factor of safety of not less than 3 to 1 should be allowed for safe handling.

Weight of Rope. A 1 in. diameter rope weighs .3 lbs. per foot of length. Weight of other sizes can be found by the following formula:

$$W = D^2 \times .3.$$

KEYS

Pulleys, etc., are secured to shafting to prevent them slipping by means of short lengths of steel, usually rectangular in section, called keys.

These are inset into the shaft to half their depth, and into the pulley the other half. In fitting them $\frac{1}{8}$ in. taper per ft. is sufficient.

Generally the width of the key for shafting up to 3 in. in diameter is taken as $\frac{1}{4}$ of the diameter. For large shafting this is excessive, and $\frac{1}{6}$ of the diameter is better practice, sizes being considered to the nearest sixteenths of an inch.

The thickness should be $\frac{2}{3}$ of the width, plus $\frac{1}{16}$ in.

ELECTRIC UNITS

The "volt" is the unit of electric pressure.

The "ampere" is the unit of current strength, or rate of flow.

The "ohm" is the unit of resistance.

The "watt" is the unit of power.

1000 watts = 1 kilowatt.

746 watts = 1 electric horsepower.

The following equation is applicable to a direct current generator:—

$$\text{H.P.} \times 746 = \text{volts} \times \text{amperes current.}$$

GASOLINE ENGINE

The horsepower of a gasoline engine is not computed from the regular formula given for engines, as considerable difficulty is met with in determining the mean effective pressure.

The following formula has been adopted:

$$H.P. = \frac{D^2 \times N}{2.5}$$

Where D = Diameter of cylinder.
N = Number of cylinders.

Standard Dimensions of Wrought Iron Pipe.

Note. Size of iron pipe is measured on the inside diameter.

Size of Pipe Inches	Actual Outside Diameter Inches	Actual Inside Diameter Inches	Number of Threads per Inch of Screw
1/8	0.405	0.270	27
1/4	0.54	0.364	18
3/8	0.675	0.494	18
1/2	0.84	0.623	14
3/4	1.05	0.824	14
1	1.315	1.048	11½
1¼	1.66	1.380	11½
1½	1.9	1.611	11½
2	2.375	2.067	11½
2½	2.875	2.468	8
3	3.5	3.067	8
3½	4	3.548	8
4	4.5	4.026	8
4½	5	4.508	8
5	5.563	5.045	8
6	6.625	6.065	8
7	7.625	7.023	8
8	8.625	7.982	8

Decimal Equivalents of Fractional Parts of One Inch.

To change any fraction to a decimal of equivalent value, divide the numerator by the denominator.

Fractions	Decimals	Fractions	Decimals
1/64	0.015625	[illegible]	0.515625
[illegible]	0.03125	[illegible]	0.53125
[illegible]	0.046875	[illegible]	0.546875
[illegible]	0.0625	[illegible]	0.5625
[illegible]	0.078125	[illegible]	0.578125
[illegible]	0.09375	[illegible]	0.59375
[illegible]	0.109375	[illegible]	0.609375
[illegible]	0.125	[illegible]	0.625
[illegible]	0.140625	[illegible]	0.640625
[illegible]	0.15625	[illegible]	0.65625
[illegible]	0.171875	[illegible]	0.671875
[illegible]	0.1875	[illegible]	0.6875
[illegible]	0.203125	[illegible]	0.703125
[illegible]	0.21875	[illegible]	0.71875
[illegible]	0.234375	[illegible]	0.734375
[illegible]	0.25	[illegible]	0.75
[illegible]	0.265625	[illegible]	0.765625
[illegible]	0.28125	[illegible]	0.78125
[illegible]	0.296785	[illegible]	0.796875
[illegible]	0.3125	[illegible]	0.8125
[illegible]	0.328125	[illegible]	0.828125
[illegible]	0.34375	[illegible]	0.84375
[illegible]	0.359375	[illegible]	0.859375
[illegible]	0.375	[illegible]	0.875
[illegible]	0.390625	[illegible]	0.890625
[illegible]	0.40625	[illegible]	0.90626
[illegible]	0.421875	[illegible]	0.921875
[illegible]	0.4375	[illegible]	0.9375
[illegible]	0.453125	[illegible]	0.953125
[illegible]	0.46875	[illegible]	0.96875
[illegible]	0.484375	[illegible]	0.984375
[illegible]	0.5	1	1.0

Decimal Equivalents of Parts of One Foot.

Example. What decimal equivalent of a foot is $\frac{1}{4}$ of an inch?

$$\frac{1}{4} \times \frac{1}{12} = \frac{1}{48} = .02083$$

In.	0	1/16	1/8	3/16	In.
0	0.0000	0.0052	0.0104	0.0156	0
1	0.0833	0.0885	0.0937	0.0990	1
2	0.1667	0.1719	0.1771	0.1823	2
3	0.2500	0.2552	0.2604	0.2656	3
4	0.3333	0.3385	0.3437	0.3490	4
5	0.4167	0.4219	0.4271	0.4323	5
6	0.5000	0.5052	0.5104	0.5156	6
7	0.5833	0.5885	0.5937	0.5990	7
8	0.6667	0.6719	0.6771	0.6823	8
9	0.7500	0.7552	0.7604	0.7656	9
10	0.8333	0.8385	0.8437	0.8490	10
11	0.9167	0.9219	0.9271	0.9323	11

In.	1/4	5/16	3/8	7/16	In.
0	0.0208	0.0260	0.0313	0.0365	0
1	0.1042	0.1094	0.1146	0.1198	1
2	0.1875	0.1927	0.1979	0.2031	2
3	0.2708	0.2760	0.2813	0.2865	3
4	0.3542	0.3594	0.3646	0.3698	4
5	0.4375	0.4427	0.4479	0.4531	5
6	0.5208	0.52 0	0.5313	0.5365	6
7	0.6042	0.6094	0.6146	0.6198	7
8	0.6875	0.6927	0.6979	0.7031	8
9	0.7708	0.7760	0.7813	0.7865	9
10	0.8542	0.8594	0.8646	0.8698	10
11	0.9375	0.9427	0.9479	0.9531	11

In.	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{11}{16}$	In.
0	0.0417	0.0469	0.0521	0.0573	0
1	0.1250	0.1302	0.1354	0.1406	1
2	0.2083	0.2135	0.2188	0.2240	2
3	0.2917	0.2969	0.3021	0.3073	3
4	0.3750	0.3802	0.3854	0.3906	4
5	0.4583	0.4635	0.4688	0.4740	5
6	0.5417	0.5469	0.5521	0.5573	6
7	0.6250	0.6302	0.6354	0.6406	7
8	0.7083	0.7135	0.7187	0.7240	8
9	0.7917	0.7969	0.8021	0.8073	9
10	0.8750	0.8802	0.8854	0.8906	10
11	0.9583	0.9635	0.9688	0.9740	11

In.	$\frac{3}{4}$	$\frac{13}{16}$	$\frac{7}{8}$	$\frac{15}{16}$	In.
0	0.0625	0.0677	0.0729	0.0781	0
1	0.1458	0.1510	0.1563	0.1615	1
2	0.2292	0.2344	0.2396	0.2448	2
3	0.3125	0.3177	0.3229	0.3281	3
4	0.3958	0.4010	0.4063	0.4115	4
5	0.4792	0.4844	0.4896	0.4948	5
6	0.5625	0.5677	0.5729	0.5781	6
7	0.6458	0.6510	0.6563	0.6615	7
8	0.7292	0.7344	0.7396	0.7448	8
9	0.8125	0.8177	0.8229	0.8281	9
10	0.8958	0.9010	0.9063	0.9115	10
11	0.9792	0.9844	0.9896	0.9948	11

Areas and Circumferences of Circles.

To find the area of a circle, square the diameter and multiply by .7854.
To find the circumference of a circle, multiply the diameter by 3.1416.

Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.
1/4	.785398	.04990						
5/16	.981748	.07670						
3/8	1.17810	.11045						
7/16	1.37445	.15033						
1/2	1.57080	.19635	3	9.42478	7.0686	5½	17.2788	23.758
9/16	1.76715	.24850	1/16	9.62113	7.3663	9/16	17.4751	24.301
5/8	1.96350	.30680	1/8	9.81748	7.6699	5/8	17.€715	24.850
11/16	2.15984	.37122	3/16	10↑0138	7.9798	11/16	17.8678	25.406
3/4	2.35619	.44179	1/4	10.2102	8.2958	3/4	18.0642	25.967
13/16	2.55254	.51849	5/16	10.4065	8.6179	13/16	18.2605	26.535
7/8	2.74889	.60132	3/8	10.6029	8.9462	7/8	18.4569	27.109
15/16	2.94524	.69029	7/16	10.7992	9.2806	15/16	18.6532	27.688
1	3.14159	.78540	3½	10.9956	9.6211	6	18.8496	28.274
1/16	3.33794	.88664	9/16	11.1919	9.9678	1/8	19.2423	29.465
1/8	3.53429	.99402	5/8	11.3883	10.321	1/4	19.6350	30.680
3/16	3.73064	1.1075	11/16	11.5846	10.680	3/8	20.0277	31.919
1/4	3.92699	1.2272	3/4	11.7810	11.045	1/2	20.4204	33.183
5/16	4.12334	1.3530	13/16	11.9773	11.416	5/8	20.8131	34.472
3/8	4.31969	1.4849	7/8	12.1737	11:793	3/4	21.2058	35.785
7/16	4.51604	1.6230	15/16	12.3700	12.177	7/8	21.5984	37.122
1½	4.71239	1.7671	4	12.5664	12.566	7	21.9911	38.485
9/16	4.90874	1.9175	1/16	12.7627	12.962	1/8	22.3838	39.871
5/8	5.10509	2.0739	1/8	12.9591	13.364	1/4	22.7765	41.282
11/16	5.30144	2.2365	3/16	13.1554	13.772	3/8	23.1692	42.718
3/4	5.49779	2.4053	1/4	13.3518	14.186	1/2	23.5619	44.179
13/16	5.69414	2.5802	5/16	13.5481	14.607	5/8	23.9546	25.664
7/8	5.89049	2.7612	3/8	13.7445	15.033	3/4	24.3473	47.173
15/16	6.08684	2.9483	7/16	13.9408	15.466	7/8	24.7400	48.707
2	6.28319	3.1416	4½	14.1372	15.904	8	25.1327	50.265
1/16	6.47953	3.3410	9/16	14.3335	16.349	1/8	25.5224	51.849
1/8	6.67588	3.5466	5/8	14.5299	16.800	1/4	25.9181	53.456
3/16	6.87223	3.7583	11/16	14.7262	17.257	3/8	26.3108	55.088
1/4	7.06858	3.9761	3/4	14.9226	17.712	1/2	26.7035	56.745
5/16	7.26493	4.2000	13/16	15.1189	18.190	5/8	27.0962	58.426
3/8	7.46128	4.4301	7/8	15.3153	18.665	3/4	27.4889	60.132
7/16	7.65763	4.6664	15/16	15.5116	19.147	7/8	27.8816	61.862
2½	7.85398	4.9087	5	15.7080	19.635	9	28.2743	63.617
9/16	8.05033	5.1572	1/16	15.9043	10.129	1/8	28.6670	65.397
5/8	8.24668	5.4119	1/8	16.1007	20.629	1/4	29.0597	67.201
11/16	8.44303	5.6727	3/16	16.2970	21.135	3/8	29.4524	69.029
3/4	8.63938	5.9396	1/4	16.4934	21.648	1/2	29.8451	70.882
13/16	8.83573	6.2126	5/16	16.6897	22.166	5/8	30.2378	72.760
7/8	9.03208	6.4918	3/8	16.8861	22.691	3/4	30.6305	74.662
15/16	9.22843	6.7771	7/16	17.0824	23.221	7/8	31.0232	76.589

Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.
10	31.4159	78.540	15	47.1239	176.71	20	62.8319	314.16
1/8	31.8086	80.516	1/8	47.5166	179.67	1/8	63.2246	318.10
1/4	32.2013	82.516	1/4	47.9093	182.65	1/4	63.6173	322.06
3/8	32.5940	84.541	3/8	48.3020	185.66	3/8	64.0100	326.05
1/2	32.9867	86.590	1/2	48.6947	188.69	1/2	64.4026	330.06
5/8	33.3794	88.664	5/8	49.0874	191.75	5/8	64.7953	334.10
3/4	33.7721	90.763	3/4	49.4801	194.83	3/4	66.1880	338.16
7/8	34.1648	92.886	7/8	49.8728	197.93	7/8	65.5807	342.25
11	34.5575	95.033	16	50.2655	201.06	21	65.9734	346.36
1/8	34.9502	97.205	1/8	50.6582	204.22	1/8	66.3661	350.50
1/4	35.3429	99.402	1/4	51.0509	207.39	1/4	66.7588	354.66
3/8	35.7356	101.62	3/8	51.4436	210.60	3/8	67.1515	358.84
1/2	36.1283	103.87	1/2	51.8363	213.82	1/2	67.5442	363.05
5/8	36.5210	106.14	5/8	52.2290	217.08	5/8	67.9369	367.28
3/4	36.9137	108.43	3/4	52.6217	220.35	3/4	68.3296	371.54
7/8	37.3064	110.75	7/8	53.0144	223.65	7/8	68.7223	375.73
12	37.6991	113.10	17	53.4071	226.98	22	69.1150	380.13
1/8	38.0918	115.47	1/8	53.7998	230.33	1/8	69.5077	384.46
1/4	38.4845	117.86	1/4	54.1925	233.71	1/4	69.9004	388.82
3/8	38.8772	120.28	3/8	54.5852	237.10	3/8	70.2931	393.20
1/2	39.2699	122.72	1/2	54.9779	240.53	1/2	70.6858	397.61
5/8	39.6626	125.19	5/8	55.3706	243.98	5/8	71.0785	402.04
3/4	40.0553	127.68	3/4	55.7633	247.45	3/4	71.4712	406.49
7/8	40.4480	130.19	7/8	56.1560	250.95	7/8	71.8639	410.97
13	40.8407	132.73	18	56.5487	254.47	23	72.2566	415.48
1/8	41.2334	135.30	1/8	56.9414	258.02	1/8	72.6493	420.00
1/4	41.6261	137.89	1/4	57.3341	261.59	1/4	73.0420	424.56
3/8	42.0188	140.50	3/8	57.7268	265.18	3/8	73.4347	429.13
1/2	42.4115	143.14	1/2	58.1195	268.80	1/2	73.8274	433.74
5/8	42.8042	145.80	5/8	58.5122	272.45	5/8	74.2201	438.36
3/4	43.1969	148.49	3/4	58.9049	276.12	3/4	74.6128	443.01
7/8	43.5896	151.20	7/8	59.2976	279.81	7/8	75.0055	447.69
14	43.9823	153.94	19	59.6903	283.53	24	75.3982	452.39
1/8	44.3750	156.70	1/8	60.0830	287.27	1/8	75.7909	457.11
1/4	44.7677	159.48	1/4	60.4757	291.04	1/4	76.1836	461.86
3/8	45.1603	162.30	3/8	60.8684	294.83	3/8	76.5763	466.64
1/2	45.5531	165.13	1/2	61.2611	298.65	1/2	76.9690	471.44
5/8	45.9458	167.99	5/8	61.6538	302.49	5/8	77.3617	476.26
3/4	46.3385	170.87	3/4	62.0465	306.35	3/4	77.7544	481.11
7/8	46.7312	173.78	7/8	62.4392	310.24	7/8	78.1471	485.98

Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.
25	78.5398	490.87	30	94.2478	706.86	35	109.956	962.11
1/8	78.9325	495.79	1/8	94.6405	712.76	1/8	110.348	969.00
1/4	79.3252	500.74	1/4	95.0332	718.68	1/4	110.741	975.91
3/8	79.7179	505.71	3/8	95.4259	724.64	3/8	111.134	982.84
1/2	80.1106	510.71	1/2	95.8186	730.62	1/2	111.527	989.08
5/8	80.5033	515.72	5/8	96.2113	736.62	5/8	111.919	996.78
3/4	80.8960	520.77	3/4	96.6040	742.64	3/4	112.312	1003.8
7/8	81.2887	525.84	7/8	96.9967	748.69	7/8	112.705	1010.8
26	81.6814	530.93	31	97.3894	754.77	36	113.097	1017.9
1/8	82.0741	536.05	1/8	97.7821	760.87	1/8	113.490	1025.0
1/4	82.4668	541.19	1/4	98.1748	766.99	1/4	113.883	1032.1
3/8	82.8595	546.35	3/8	98.5675	773.14	3/8	114.275	1039.2
1/2	83.2522	551.55	1/2	98.9602	779.31	1/2	114.668	1046.3
5/8	83.6449	556.76	5/8	99.3529	785.51	5/8	115.061	1053.5
3/4	84.0376	562.00	3/4	99.7456	791.73	3/4	115.454	1060.7
7/8	84.4303	567.27	7/8	100.138	797.98	7/8	115.846	1068.0
27	84.8230	572.56	32	100.531	804.25	37	116.239	1075.2
1/8	85.2157	577.87	1/8	100.924	810.54	1/8	116.632	1082.5
1/4	84.6084	583.21	1/4	101.316	816.86	1/4	117.024	1089.8
3/8	86.0011	588.57	3/8	101.709	823.21	3/8	117.417	1097.1
1/2	86.3938	593.96	1/2	102.102	829.58	1/2	117.810	1104.5
5/8	86.7865	599.35	5/8	102.494	835.97	5/8	118.202	1111.8
3/4	87.1892	604.71	3/4	102.887	842.39	3/4	118.596	1119.2
7/8	87.5719	610.27	7/8	103.280	848.83	7/8	118.988	1126.7
28	87.9646	615.75	33	103.673	855.30	38	119.381	1134.1
1/8	88.3573	621.26	1/8	104.065	861.79	1/8	119.773	1141.6
1/4	88.7500	626.80	1/4	104.458	868.31	1/4	120.166	1149.1
3/8	89.1427	632.36	3/8	104.851	874.85	3/8	120.559	1156.6
1/2	89.5354	637.94	1/2	105.243	881.41	1/2	120.951	1164.2
5/8	89.9281	643.55	5/8	105.636	888.00	5/8	121.344	1171.7
3/4	90.3208	649.18	3/4	106.029	894.62	3/4	121.737	1179.3
7/8	90.7135	653.84	7/8	106.421	901.26	7/8	122.129	1186.9
29	91.1062	660.52	34	106.814	907.92	39	122.522	1194.6
1/8	91.4989	666.23	1/8	107.207	914.61	1/8	122.915	1202.3
1/4	91.8916	671.96	1/4	107.600	921.32	1/4	123.308	1210.0
3/8	92.2843	677.71	3/8	107.992	928.06	3/8	123.700	1217.7
1/2	92.6770	683.49	1/2	108.385	934.82	1/2	124.093	1225.4
5/8	93.0697	689.30	5/8	108.788	941.61	5/8	124.486	1233.2
3/4	93.4624	695.13	3/4	109.170	948.42	3/4	124.878	1241.0
7/8	93.8551	700.98	7/8	109.563	955.25	7/8	125.271	1248.8

Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.
40	125.664	1256.6	45	141.372	1590.4	50	157.080	1963.5
	126.056	1264.5		141.764	1599.3		157.472	1973.3
	126.449	1272.4		142.157	1608.2		157.865	1983.2
	126.842	1280.3		142.550	1617.0		158.258	1993.1
	127.235	1288.2		142.942	1626.0		158.650	2003.0
	127.627	1296.2		143.335	1684.9		159.043	2012.9
	128.020	1304.2		143.728	1643.9		159.436	2022.8
	128.413	1312.2		144.121	1652.9		159.829	2032.8
41	128.805	1320.3	46	144.513	1661.9	51	160.221	2024.8
	129.198	1328.3		144.906	1670.9		160.614	2052.8
	129.591	1336.4		145.299	1680.0		161.007	2062.9
	129.993	1344.5		145.691	1689.1		161.399	2073.0
	130.376	1352.7		146.084	1698.2		161.792	2083.1
	130.769	1360.8		146.477	1707.4		162.185	2093.2
	131.161	1369.0		146.869	1716.5		162.577	2103.3
	131.554	1377.2		147.262	1725.7		162.970	2113.5
42	131.947	1385.4	47	147.655	1734.9	52	163.363	2123.7
	132.340	1393.7		148.048	1744.2		163.756	2133.9
	132.732	1402.0		148.440	1753.5		164.148	2144.2
	133.125	1410.3		148.833	1762.7		164.541	2154.5
	133.518	1418.6		149.226	1772.1		164.934	2164.8
	133.910	1427.0		149.618	1781.4		165.326	2175.1
	134.303	1435.4		150.011	1790.8		165.719	2185.4
	134.696	1443.8		150.404	1800.1		166.112	2195.8
43	135.088	1452.2	48	150.796	1809.6	53	166.504	2206.2
	135.481	1460.7		151.189	1819.0		166.897	2216.6
	135.874	1469.1		151.582	1828.5		167.290	2227.0
	136.267	1477.6		151.975	1837.9		167.683	2237.5
	136.659	1486.2		152.367	1847.5		168.075	2248.0
	137.052	1494.7		152.760	1857.0		168.468	2258.5
	137.445	1503.3		153.153	1866.5		168.861	2269.1
	137.837	1511.9		153.545	1876.1		169.253	2279.6
44	138.230	1520.5	49	153.938	1885.7	54	169.646	2290.2
	138.623	1529.2		154.331	1895.4		170.039	2300.8
	139.015	1537.9		154.723	1905.0		170.431	2311.5
	139.408	1546.6		155.116	1914.7		170.824	2322.1
	139.801	1555.3		155.509	1924.4		171.217	2332.8
	140.194	1564.0		155.902	1934.2		171.609	2343.5
	140.586	1572.8		156.924	1943.9		172.002	2354.3
	140.979	1581.6		156.687	1953.7		172.395	2365.0

Areas and Circumferences of Circles.—Con.

Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.
55	172.788	2375.8	60	188.496	2827.4	65	204.204	3318.3
	173.180	2386.6		188.888	2839.2		204.596	3331.1
	173.573	2397.5		189.281	2851.0		204.989	3343.9
	173.966	2408.3		189.674	2862.9		205.382	3356.7
	174.358	2419.2		190.066	2874.8		205.774	3369.6
	174.751	2430.1		190.459	2886.6		206.167	3382.4
	175.144	2441.1		190.852	2898.6		206.560	3395.3
	175.536	2452.0		191.244	2910.5		206.952	3408.2
56	175.929	2463.0	61	191.637	2922.5	66	207.345	3421.2
	176.322	2474.0		192.030	2934.5		207.738	3434.3
	176.715	2485.0		192.423	2946.5		208.131	3447.2
	177.107	2496.1		192.815	2958.5		208.523	3460.2
	177.500	2507.2		193.208	2970.6		208.916	3473.2
	177.893	2518.3		193.601	2982.7		209.309	3486.3
	178.285	2529.4		193.993	2994.8		209.701	3499.4
	178.678	2540.6		194.386	3006.9		210.094	3512.5
57	179.071	2551.8	62	194.779	3019.1	67	210.487	3525.7
	179.463	2563.0		195.171	3031.3		210.879	3538.8
	179.856	2574.2		195.564	3043.5		211.272	3552.0
	180.249	2585.4		195.957	3055.7		211.665	3565.2
	180.642	2596.7		196.350	3068.0		212.058	3578.5
	181.034	2608.0		196.742	3080.3		212.450	3591.7
	181.427	2619.4		197.135	3092.6		212.843	3605.0
	181.820	2630.7		197.528	3104.9		213.236	3618.3
58	182.212	2642.1	63	197.920	3117.2	68	213.628	3631.7
	182.605	2653.5		198.313	3129.6		214.021	3645.0
	182.998	2664.9		198.706	3142.0		214.414	3658.4
	183.390	2676.4		199.098	3154.5		214.806	3671.8
	183.783	2687.8		199.491	3166.9		215.199	3685.3
	184.176	2699.3		199.884	3179.4		215.592	3698.7
	184.569	2710.9		200.277	3191.9		215.984	3712.2
	184.961	2722.4		200.669	3204.4		216.377	3725.7
59	185.354	2734.0	64	201.062	3217.0	69	216.770	3739.3
	185.747	2745.6		201.455	3229.6		217.163	3752.8
	186.139	2757.2		201.847	3242.2		217.555	3766.4
	186.532	2768.8		202.240	3254.8		217.948	3780.0
	186.925	2780.5		202.633	3267.5		218.341	3793.7
	187.317	2792.2		203.025	3280.1		218.733	3807.3
	187.710	2803.9		203.418	3292.8		219.126	3821.0
	188.103	2815.7		203.811	3305.6		219.519	3834.7

Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.
70	219.911	3848.5	75	235.619	4417.9	80	251.327	5026.5
1/8	220.304	3862.2	1/8	236.012	4432.6	1/8	251.720	5042.3
1/4	220.697	3876.0	1/4	236.405	4447.4	1/4	252.113	5058.0
3/8	221.090	3889.8	3/8	236.798	4462.2	3/8	252.506	5073.8
1/2	221.482	3903.6	1/2	237.190	4477.0	1/2	252.898	5089.6
5/8	221.875	3917.5	5/8	237.583	4491.8	5/8	253.291	5105.4
3/4	222.268	3931.4	3/4	237.976	4506.7	3/4	253.684	5121.2
7/8	222.660	3945.3	7/8	238.368	4521.5	7/8	254.076	5137.1
71	223.053	3959.2	76	238.761	4536.5	81	254.469	5153.0
1/8	223.446	3973.1	1/8	239.154	4551.4	1/8	254.862	5168.9
1/4	223.838	3987.1	1/4	239.546	4566.4	1/4	255.254	5184.9
3/8	224.231	4001.1	3/8	239.939	4581.3	3/8	255.647	5200.8
1/2	224.624	4015.2	1/2	240.332	4596.3	1/2	256.040	5126.8
5/8	225.107	4029.2	5/8	240.725	4611.4	5/8	256.433	5232.8
3/4	225.409	4043.3	3/4	241.117	4626.4	3/4	256.825	5248.9
7/8	225.802	4057.4	7/8	241.510	4641.5	7/8	257.218	5264.9
72	226.195	4071.5	77	241.903	4656.6	82	257.611	5281.0
1/8	226.587	4085.7	1/8	242.295	4671.8	1/8	258.003	5297.1
1/4	226.980	4099.8	1/4	242.688	4686.9	1/4	258.396	5313.3
3/8	227.373	4114.0	3/8	243.081	4702.1	3/8	258.789	5329.4
1/2	227.765	4128.2	1/2	243.473	4717.3	1/2	259.181	5345.6
5/8	228.158	4142.5	5/8	243.866	4732.5	5/8	259.574	5361.8
3/4	228.551	4156.8	3/4	244.259	4747.8	3/4	259.967	5378.1
7/8	228.944	4171.1	7/8	244.652	4763.1	7/8	260.359	5394.3
73	229.336	4185.4	78	245.044	4778.4	83	260.752	5410.6
1/8	229.729	4199.7	1/8	245.347	4793.7	1/8	261.145	5426.9
1/4	230.122	4214.1	1/4	245.830	4809.0	1/4	261.538	5443.3
3/8	230.514	4228.5	3/8	246.222	4824.4	3/8	261.930	5459.6
1/2	230.907	4242.9	1/2	246.615	4839.8	1/2	262.323	5476.0
5/8	231.300	4257.4	5/8	247.008	4855.2	5/8	262.716	5492.4
3/4	231.692	4271.8	3/4	247.400	4870.7	3/4	263.108	5508.8
7/8	232.085	4286.3	7/8	247.793	4886.2	7/8	263.501	5525.3
74	232.478	4300.8	79	248.186	4901.7	84	263.894	5541.8
1/8	232.871	4315.4	1/8	248.579	4917.2	1/8	264.286	5558.3
1/4	233.263	4329.9	1/4	248.971	4932.7	1/4	264.679	5574.8
3/8	233.656	4344.5	3/8	249.364	4948.3	3/8	265.072	5591.4
1/2	234.049	4359.2	1/2	249.757	4963.9	1/2	265.465	5607.9
5/8	234.441	4373.8	5/8	250.149	4979.5	5/8	265.857	5624.5
3/4	234.834	4388.5	3/4	250.542	4995.2	3/4	266.250	5641.2
7/8	235.227	4403.1	7/8	250.935	5010.9	7/8	266.643	5657.8

Areas and Circumferences of Circles.—Con.

Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.	Diam. Ins.	Circum. Ins.	Area Sq. Ins.
85	267.035	5674.5	90	282.743	6361.7	95	298.451	7088.2
1/8	267.428	5691.2	1/8	283.136	6379.4	1/8	298.844	7106.9
1/4	267.821	5707.9	1/4	283.529	6397.1	1/4	299.237	7125.6
3/8	268.213	5724.7	3/8	283.921	6414.9	3/8	299.629	7144.3
1/2	268.606	5741.5	1/2	284.314	6432.6	1/2	300.022	7163.0
5/8	268.999	5758.3	5/8	284.709	6450.4	5/8	300.415	7181.8
3/4	269.392	5775.1	3/4	285.100	6468.2	3/4	300.807	7200.6
7/8	269.784	5701.9	7/8	285.492	6486.0	7/8	301.200	7219.4
86	270.177	5808.8	91	285.885	6503.9	96	301.593	7238.2
1/8	270.570	5825.7	1/8	286.278	6521.8	1/8	301.986	7257.1
1/4	270.962	5842.6	1/4	286.670	6539.7	1/4	302.378	7276.0
3/8	271.355	5859.6	3/8	287.063	6557.6	3/8	302.771	7294.9
1/2	271.748	5876.5	1/2	287.456	6575.5	1/2	303.164	7313.8
5/8	272.140	5893.5	5/8	287.848	6593.5	5/8	303.556	7332.8
3/4	272.533	5910.6	3/4	288.241	6611.5	3/4	303.949	7351.8
7/8	272.926	5927.6	7/8	288.634	6629.6	7/8	304.342	7370.8
87	273.319	5944.7	92	289.027	6647.6	97	304.734	7389.8
1/8	273.711	5961.8	1/8	289.419	6665.7	1/8	305.127	7408.9
1/4	274.104	5978.9	1/4	289.812	6683.8	1/4	305.520	7428.0
3/8	274.497	5996.0	3/8	290.205	6701.9	3/8	305.913	7447.1
1/2	274.889	6013.2	1/2	290.597	6720.1	1/2	306.305	7466.2
5/8	275.282	6030.4	5/8	290.990	6738.2	5/8	306.698	7485.3
3/4	275.675	6047.6	3/4	291.383	6756.4	3/4	307.091	7504.5
7/8	276.067	6064.9	7/8	291.775	6774.7	7/8	307.483	7523.7
88	276.460	6082.1	93	292.168	6792.9	98	307.876	7543.0
1/8	276.853	6099.4	1/8	292.561	6811.2	1/8	308.269	7562.2
1/4	277.246	6116.7	1/4	292.954	6829.5	1/4	308.661	7581.5
3/8	277.638	6134.1	3/8	293.346	6847.8	3/8	309.054	7600.3
1/2	278.031	6151.4	1/2	293.739	6866.1	1/2	309.447	7620.1
5/8	278.424	6168.8	5/8	294.132	6884.5	5/8	309.840	7639.5
3/4	278.816	6186.2	3/4	294.524	6902.9	3/4	310.232	7658.9
7/8	279.209	6203.7	7/8	294.917	6921.3	7/8	310.625	7678.3
89	279.602	6221.1	94	295.310	6939.8	99	311.018	7697.7
1/8	279.994	6238.6	1/8	295.702	6958.2	1/8	311.410	7717.1
1/4	280.387	6256.1	1/4	296.095	6976.7	1/4	311.803	7736.6
3/8	280.780	6273.7	3/8	295.488	6995.3	3/8	312.196	7756.1
1/2	281.173	6291.2	1/2	296.881	7013.8	1/2	312.588	7775.6
5/8	281.565	6308.8	5/8	297.273	7032.4	5/8	312.981	7795.2
3/4	281.958	6326.4	3/4	297.666	7051.0	3/4	313.374	7814.8
7/8	282.351	6344.1	7/8	298.059	7069.6	7/8	313.767	7834.4
						100	314.159	7854.0

PHYSICAL SCIENCE

A study of the following lessons which contain in condensed form a "General Discussion of Physical Science" will more than repay the time spent in doing so, touching as they do on many points well worth knowing to the average railway employee. These are not intended to be taken up as classwork, but as home study.

Physical Science is divided into two departments known as Physics and Chemistry, and by means of a few simple illustrations we will see how to discern between actions that are of physical nature and those which come under the head of chemical changes.

In Science everything that has weight is known as matter. This includes everything that we would ordinarily call "substances." Thus iron, coal, air, brass, wood, water, and oils are known as different forms of matter. Matter can neither be created nor destroyed, but is capable of undergoing many changes, some of which are physical and others chemical. The following will afford several examples of physical changes in the state of matter. Take a piece of ice in a dish, heat it, and it will slowly melt, changing from the solid to the liquid state; continue to heat it and the liquid will commence to vaporize or change to steam; the change this time being from a liquid to a gas. If this steam is passed through a cooling tube it will condense or change back to a liquid form, and if the temperature is lowered below the freezing point it will solidify. It would be possible to take a certain amount of water and pass it through these states as many times as desired, and we would in the end still have the same water and, moreover, exactly the same amount of water.

Similar changes of state could be produced in nearly all substances by altering the temperature or amount of heat present; but it must be noticed that the "changing points" are different for every substance. Thus, within the range of our ordinary natural temperatures we find different substances existing naturally in different states. For instance: iron occurs naturally as a solid; air occurs as a gas; water as a liquid; coal as a solid; oil as a liquid; and mercury as a liquid. All matter, regardless of its state, is made up of very minute particles called molecules; and any substance could be halved in quantity and subdivided again and again until the smallest existible quantity was reached, and this would be defined as one molecule of the substance. "A molecule would then be the smallest portion of any kind of matter that could be obtained by mechanical means." But the stopping point is not here, for it can be reduced still further by chemical means. Taking water again as an example: if a current of electricity is passed through a certain quantity of water, it gradually disappears as a liquid, changing into two gases known as oxygen and hydrogen. The molecules of water are said to have broken up into atoms, one molecule of water actually containing two atoms of hydrogen and one of oxygen. Actions of this kind come under the head of chemistry. It should be noted that the hydrogen and oxygen will now remain as such, and will not change back by mechanical mixing or cooling, as in the case of steam.

As a further example let us consider iron. Iron can be heated and melted, and when cooled will solidify, and still be iron; thus passing through physical changes. But leave a piece of iron exposed to air and moisture for a considerable time, and a heavy coating of rust will form. No amount of coaxing will get this rust to form into iron again, and if the rust is removed the iron will be found to have lost in weight. A chemical action has taken place, and some of the iron has united with the oxygen of the atmosphere, forming an entirely new substance called iron oxide or "rust." The change which iron undergoes when it is melted is of a different order entirely to the change which it undergoes when exposed to moist air. In the former case the change is temporary, and the iron is still iron. On the other hand, when exposed to the moist air it undergoes a permanent change, being converted into a new substance essentially different from iron. Actions in which the composition of substances is changed are called chemical actions, while changes that do not affect the composition of the substance are called Physical changes. Another example of a chemical change is afforded by the burning of a match, or the combustion of coal in a firebox. As it burns it evidently undergoes a permanent change, some of the solid constituents are changed into a gas, and the whole mass unites with the oxygen of the air, or as we would commonly say, it burns. This type of chemical action is known as combustion.

SOME PROPERTIES OF MATTER (Solids)

All solids are endowed with different and distinct properties which render them suited for some certain purpose. In like manner, different metals have properties peculiar to themselves that make them better adapted to perform certain duties. Thus, brass would be a very poor metal from which to manufacture rails; and cast iron would never do for locomotive side rods. In this chapter the general properties will be discussed; that is, we will consider properties that belong to solids in general.

The following is a list of the more important properties belonging to solids: Cohesion, Tenacity, Hardness, Elasticity, Malleability, Ductility, Brittleness, Lustre, Conductivity, Fusibility, and Specific Gravity.

Cohesion. Cohesion is the name given to the bond which holds the molecules of a body together. This property belongs to all matter, but is greatest and most apparent in the case of solids. The cohesion of solids varies, and the varying strength gives us the different degrees of rigidity, tenacity, and hardness in bodies. When we break a substance we conceive that the molecules become so far separated that their cohesion is overcome. Once separated, cohesion can only be restored by bringing the molecules very close together again by some agent such as heat. This is done when two pieces of wrought iron are welded together; this welding is a process in which the molecules of two different masses are brought into such intimate contact that the bond of cohesion is again restored. Some substances can be made to weld together much more easily than others. Clean surfaces of metallic lead when pressed together cohere so that it requires considerable force to pull them apart; and powdered graphite (the substances used in lead pencils), when submitted to a very great pressure becomes once more a solid mass. We will again define cohesion as the natural attraction of the molecules of a body for one another. The strength of materials depends upon their cohesion. This brings us to the term **Tenacity**, which is the strength or resistance offered by a body when some force is tending to pull it apart. More often we use the term Tensile Strength, and it is measured in pounds or tons per square inch. That is, if the ultimate tensile strength of a bar of iron is 40,000 lbs. per sq. in., that means that a load of 40,000 lbs. suspended at the end of a bar one inch square in cross section would just suffice to tear the bar asunder. The tensile strengths of all metals and alloys differ, and they even vary in one substance according to the temperature, and the presence in minute quantities of any other substance. Steel and iron possess the highest tensile strength, while zinc, tin, and lead possess the least.

Hardness. By the hardness of a body, we mean the difficulty of penetrating between its particles. Talc (soapstone) possesses very little hardness, as it can be dug away with the finger nail. Lead can be cut with a piece of glass; a piece of steel will cut glass; carborundum or emery will grind steel; while diamonds stand alone, being able to resist all other substances. Some metals have considerable range of hardness. Steel, for instance, as we find it in mild bars or as used in a locomotive main rod, can be easily dinted with a hammer; but yet it is used to manufacture ball bearings of extreme hardness.

Crystalline Form. Many solids exhibit a peculiar appearance when broken, being made up of regularly shaped masses or crystals. It is not known why substances take this crystallized form. Sugar, salts, ice, rocks, quartz, etc.—these always take a crystallized form. But steel will sometimes change from the fibrous to the crystallized form when subject to constant vibration and repeated stresses.

Ductility. Ductility is that property of metals that makes it possible to pull them out into wires. Steel and copper are very ductile. Ductile metals become hardened and crystallized by wire-pulling and it is then necessary to anneal them. This is done by heating the metal and allowing it to cool very slowly, the effect of the heat being to produce a natural rearrangement of the molecules, and the softening of the metal. Copper does not follow this law, however, for to anneal copper it is necessary to cool it suddenly in a bath after being heated. The reason for this has never been ascertained.

Malleability. Malleability is not identical with ductility, though it is very much akin to it. It is the property that enables the shape of a metal to be changed by hammering or rolling. Copper is the most malleable of the commercial metals. Gold stands at the top of the list. Sheet iron and steel can be bent and rolled, but cannot be raised under the hammer or in dies to anything like the same extent as copper. This malleability forms the basis of the sheet-metal operations. There is quite a difference, though, in the operations of the boilermaker and sheet-metal worker, due to the difference in the thickness of the sheets. The first-named artisan requires the aid of heat; the second works in the cold metal. A thick plate cannot be bent to a quick curvature unless it is heated. The effect of the bending is to extend the outer layers and compress the inner layers. The layers in the centre of the plate are neither extended nor compressed, and this neutral plane of bending is called the neutral axis. The effect of the heating is to allow the molecules to move over one another and to adjust themselves to the new shape without remaining in a state of stress, such as would occur if the bending was done cold. This heating has the effect of making metals malleable that ordinarily are not so when cold.

Elasticity. When a body has its form altered by a pull or by pressure, the result is called a "strain," and the pressure applied is called a stress. If when the stress is removed the strain also disappears, the body is said to be elastic. Such is the case with rubber and spring steel. If the strain always disappears when the stress is removed, no matter how great, then the body is said to be "perfectly elastic." Fluids such as water, oil, air, and steam are the only bodies that fulfill this condition. No solids are perfectly elastic. The degree of their elasticity is specified by a figure found by experiment, and called the coefficient of elasticity.

Brittleness. When a substance can be easily broken by a blow it is said to be brittle, like glass, sugar crystals, etc. This property is one that is much avoided in materials intended for use in manufacturing machinery.

Conductivity of heat is a property which renders the metals so valuable for heating purposes. The conducting power of different substances varies. In general, the more compact the structure the greater the conductivity. Hence, the metals are the best conductors; wood and stone are poor conductors, also cloth, fur, etc. The best conductor is silver; the worst one is air. The following table shows the relative conductivity of the metals in regards to silver:

Silver	1.000	Iron	.119
Copper	.736	Steel	.116
Gold	.532	Lead	.085
Brass	.231	Platinum	.084
Zinc	.190	Bismuth	.018

Liquids are very poor conductors. A test tube full of water can easily be made to boil at the top, and be quite cool below.

Conductivity of Electricity. Different substances have different powers of conducting electricity. Some substances will not conduct it at all, and are known as insulators. The different metals all conduct electricity, also carbon and dilute acids. Among the metals, copper is the best conductor from a commercial standpoint, although gold and silver stand at the top of the list.

Adhesion. Adhesion is the general name given to the bond existing between unlike molecules; that is, between different substances. Place a pane of glass on the surface of a tub of water and it will require quite an effort to lift it clear of the water, holding it parallel to the surface. This is caused by the force of adhesion. Mucilage and glue adhere strongly to substances, due to their adhesive force. Thus, we have adhesive tape, shellac, paint, etc., as examples of the force of adhesion.

Viscosity. When fluids are poured there is a certain amount of internal friction, so that some fluids pour more readily than others. Light liquids are easily and quickly formed into drops, while heavy oils like valve oil form drops slowly. The application of heat to oils makes them less viscous, or thinner. If we continue to apply heat they will reach a temperature at which they will vaporize, or change into a gas. The following table shows the vaporizing points for some of the ordinary liquids, at atmospheric pressure:

Alcohol	173 deg. F.	Petroleum	316 deg. F.
Water	212 "	Paraffin	536 "
Salt Water	226 "	Mercury	648 "
Turpentine	315 "	Linseed Oil	600 "

These boiling points vary according to the pressure exerted on the liquid. Although the boiling point of water is 212° in an open vessel, in a locomotive boiler under steam pressure of 200 lbs. per sq. in., it rises to 387°. The presence of salts, such as boiler compounds, will raise this boiling point to a still higher figure. Accordingly it takes more heat to boil a cubic foot of water in a locomotive boiler than it does in a kettle open to the atmosphere. When iron is heated it turns red and increases to a dazzling white at the point of fusion. The following table shows the color indicative of the temperatures:

Dull Red	977 deg. F.	Bright White	2552 deg. F.
Cherry Red	1652 "	Dazzling White	2732 " to 2912 "
Orange	2192 "		
White Heat	2372 "		

Specific Gravity. The specific gravity of a substance is its relation in weight to a similar bulk of pure water at a temperature of 62° F. When we say that the specific gravity of zinc is 7, we mean that any quantity of zinc is seven times heavier than the same volume of water.

Cork	.24	
Wood	.434 to 1.33	depending on kind of wood.
Ice	.917	
Wax	.96	
Alcohol	.8062	
Olive Oil	.91	
Water	1.000	
Sea Water	1.026	
Glass	2.52	
Diamond	3.5	
Iron	7.8	
Brass	8.5	
Copper	8.9	
Gold	19.5	
Lead	11.4	
Platinum	21.5	
Silver	10.5	
Tin	7.3	
Zinc	7.0	
Aluminum	2.56	

SOME PROPERTIES OF MATTER (FLUIDS)

A Fluid is a body which offers no permanent resistance to forces tending to change its shape. Water and alcohol are examples of fluids, and so is air. But fluids can be divided into two main classes, having almost distinct properties. Some fluids, such as water, oils, etc., can only be compressed a very slight amount in volume, even though subjected to enormous pressures; such fluids are known as liquids. Other fluids, such as air, hydrogen, etc., are easily compressed; these are called gases. For all ordinary purposes liquids may be considered as incompressible. A gas is considered as a fluid because it has no rigidity, or permanent shape; and it differs from a liquid because it readily expands and contracts under changes of pressure. A liquid, if placed in a dish, will accommodate itself to the shape of the dish and then assume a level surface. This surface is the dividing line between the liquid and the fluid air above. A gas, on the other hand, is always tending to expand in all directions and has no surface. The difference between a liquid body, a rigid body, and a gas can be expressed as follows:

A rigid body has a definite size and shape.

A liquid has a definite size, but no definite shape.

A gas has no definite size or shape.

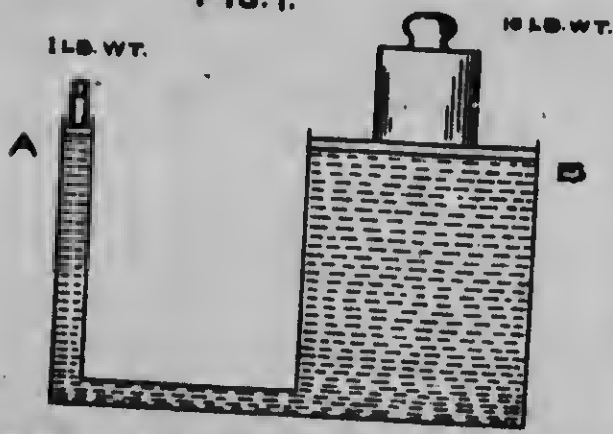
There is a law known as Pascal's Principle which is the fundamental law governing actions of liquids; and the practical applications of this law are used every day. Whenever a liquid is enclosed in a space and a pressure applied at any point, the same pressure will be transmitted in all directions throughout the liquid. This principle, which is true for gases as well as liquids, may be stated concisely as follows:

> "Pressure exerted anywhere on the mass of a liquid is transmitted undiminished in all directions and acts with the same force on all equal surfaces, in a direction at right angles to them."

SOME PRACTICAL APPLICATIONS OF THIS PRINCIPLE OF PASCAL'S

By the application of this principle a force may be multiplied for performing some practical duty much the same as a force can be multiplied by the use of a lever.

FIG. 1.



Referring to Fig. 1 it will be seen that a 1-lb. pressure on the small piston is counterbalancing the 10 lb. weight on the large piston. This is owing to the pistons A and B being in the same ratio as the weights. Piston A has a ratio to piston B of 1 sq. in. to 10 sq. ins., hence the 1-lb. pressure transmitted to the liquid by the small piston will cause a similar force of 1 lb. to be exerted at right angles to every sq. in. of surface, enclosing the liquid. The large piston B comprises 10 of these sq. inches, so there will be an upward pressure on this piston of 10 lbs., which will just support the 10-lb. weight above. In this calculation we have assumed that both pistons have the same weight, and that there is no escape of liquid anywhere from the vessel.

Hydraulic jacks, presses, and elevators are operated on this principle. Piston A is replaced by a small pump having a small plunger which forces more liquid into the vessel. The large piston B doing the heavy work is forced up to make way for the additional liquid. It must not be imagined that we can get more work out of this machine than we put in, for that is not so of any machine. The pressures exerted by the pistons are directly in proportion to their areas, but the distance through which they act must also be considered, and the small piston moves through a far greater distance than the large one. If we multiply the area of the small piston by the distance through which it acts, we find that it is equivalent to the area of the large piston multiplied by the distance through which it moves. This gives us the equilibrium of forces pointed out in the explanation of Pascal's Principle.

Gauge Testers using standard weights are constructed to act on the above principle. The action of compressed air is more or less based on this principle also, but will come under the head of gases. In the practical application of this principle to gases the feature of compression comes into the calculation, but this is neglected in liquids, which are practically incompressible. We have all seen the disastrous effect of this law on a cylinder head when the engine is moved quickly with the cylinder cocks closed and the cylinder full of water, especially in a piston valve engine.

PRESSURE DUE TO WEIGHT

Our common experiences teach us that even when a liquid is at rest there are forces existing within it. For example, pierce a hole in a tin can below the water level and the water will squirt out in a small solid stream. If a hole is put below this, and another below it again, it will be seen that the further down in the liquid the greater must be the internal force, for the lowest hole projects the stream of water a much greater distance than the one above it, and so on. Careful experiments have shown that this pressure increases from the surface downward and in direct proportion to the depth. Another important point has also been found out—that the pressure is the same at any point in a horizontal plane in a liquid and that this pressure is equal in all directions; that is, upwards, sideways, or at any angie from the point. Only one thing determines this pressure, and that is the height of the liquid above. It makes no difference what the shape of the containing vessel may be, for it is not the total volume of liquid that makes the pressure; it is only the weight of the liquid perpendicularly above a given aiea that determines the pressure at a given depth. Then remember this point, as stated above—that the pressure is the same at all points horizontally around a given point. This must be thoroughly understood to see through the following experiment, which has been performed and proved countless times.

In the figures shown here we have four vessels having the same sized bases, and the water stands at the same height in all.

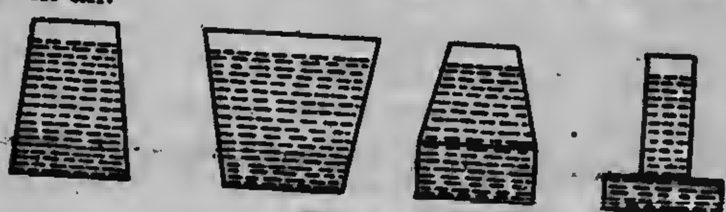


FIG. 2.

The volume of water in each of the different vessels is manifestly not the same, but the pressure exerted by the liquid on the base is the same in each case. This may not be apparent at the first glance, but it will be understood if we apply the laws just given for the pressure due to weight, viz., that the pressure at a given point in a liquid is due to the head or height of water, and independent of other considerations, such as the shape of the vessel containing the liquid; and that the pressure is the same at all points of the same depth, or in other words, in the same horizontal plane.

With a 1-ft. head of water the pressure will be 0.43302 lbs. per sq. in., or to produce a pressure of 1 lb. per sq. in. will require a head of 2.30947 ft. Our ordinary air pressure is 14.7 lbs. per sq. in., and will therefore balance or support a column of water 33.94 ft. high.

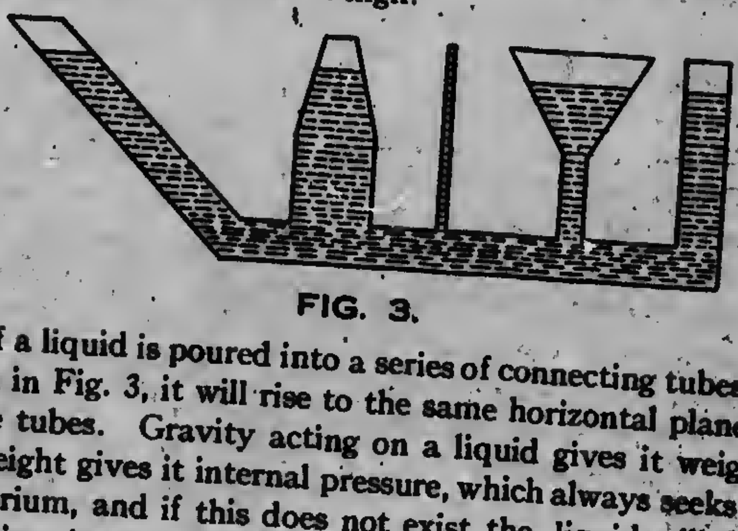


FIG. 3.

If a liquid is poured into a series of connecting tubes, as shown in Fig. 3, it will rise to the same horizontal plane in all the tubes. Gravity acting on a liquid gives it weight; this weight gives it internal pressure, which always seeks an equilibrium, and if this does not exist the liquid will flow and adjust itself until the pressure is the same at all points in the same horizontal plane. This law is sometimes expressed rather crudely by the expression "water always seeks its own level," but like lots of other shop expressions it is somewhat vague.

Before passing, let us notice the behaviour of the liquid in the very small tube (Fig. 3). If the tube is very small, a new force of Nature enters into action with an effect of its own. It is called Capillarity, and is closely related to Adhesion. It will cause a liquid in the small tube to rise above the normal level in the larger tubes. This is providing the liquid is one that wets the tube. The smaller the bore of the tube the greater will be the rise in level. It is in accordance with this law that oil will rise in a lamp wick, or travel along a "trimmer" or packing to lubricate a bearing. This is commonly called siphoning by some mechanics, but that is the wrong term to use, for a siphon is an entirely different instrument of Physics. It will be explained in a later paragraph.

BUOYANCY

When a body is immersed in a liquid, every point of its surface is subjected to an upward pressure which varies as the depth of the point below the surface of the liquid. Imagine a 1-in. cube of wood (half the weight of a cubic inch of water) immersed until it is covered by 1 in. of water (see Fig. 4). We can disregard the forces acting on the vertical faces, for they will be equal and opposite and will have no effect on the cube. But there will be a downward pressure acting on the top face equal to the weight of the cubic inch of water above it; and there will be an upward pressure acting on the lower face twice as great, for it is

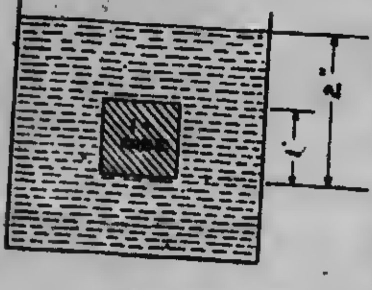


FIG. 4.

two inches below the surface. There will also be the downward pressure due to the weight of the cube, but since this is only half as much as a cubic inch of water there will be a greater upward pressure than the total downward pressure, and the cube will rise until the two forces are equalized. This will occur when the cube is half immersed in the water, and it will then be said to float.

The principle governing Flotation is as follows:

A floating body sinks until it displaces its own weight of water; in other words, the volume of water displaced has a weight equal to the floating body. Accordingly, a battleship sinks until it displaces a volume of water equal in weight to the battleship complete.

Even though a body is too heavy to float, the upward pressure exerted on it will equal the weight of water displaced, and the apparent weight of the body will be less by that amount. This is a matter of common experience, for nearly everyone has noted the difference in weight of a stone immersed in water and when lifted clear of the water. The loss in apparent weight is the same for all substances having the same volumes and at all ordinary depths.

These actions of a liquid are often referred to as the Law of Archimedes, and are summed up in these few words:

> "When any object is wholly or partially immersed in a liquid it is buoyed up by a force equal to the weight of the displaced liquid."

SPECIFIC GRAVITY OF LIQUIDS

The relative density of a substance as compared with some standard substance is known as its Specific Gravity. Solids and liquids are usually compared with water as a standard, while gases are often referred to air or hydrogen. The specific gravity of a solid or liquid referred to water (the common standard) is found by dividing the weight of the given substance by the weight of an equal volume of water (pure water at 4 degrees Centigrade).

The specific gravity of a substance is a "ratio," and is therefore the same whatever system of weights or kinds of units are used in finding it. An instrument called a "hydrometer" allowed to float in a liquid will sink until a scale on the instrument registers the specific gravity of the liquid. This is the common method used in finding the specific gravity of liquids.

Tables showing the specific gravity of all ordinary substances have been prepared, and these should be consulted when information of this kind is needed.

SOME PROPERTIES OF GASES

Every mechanic should understand the general physical laws of gases, for now-a-days they are made use of in countless ways for operating devices in mechanical work. Compressed air is used in all railroad shops and on all trains, and is quite as indispensable as steam. While some of the laws of gases are not used directly in our work, yet it is well to be acquainted with them. In speaking of gases we particularly refer to the common gases used in the shop under pressure as air, oxygen, acetylene, etc.

Air has Weight. If we were asked the question, "How much does air weigh?" we should probably answer, "Air has no weight." A very simple experiment, however, will show us that air has weight, and that it would take three or four strong men to lift a weight equal to that of the air in a large room. The experiment is as follows: A glass flask fitted with a stopper and tap is attached to an air pump and the air exhausted as much as possible. While its interior is in this state of partial vacuum, place it on a delicate pair of scales and exactly counterbalance it. After this, open the tap and allow the outside air to flow in and restore natural pressure in the flask, and it will then be noticed that the scales register a greater weight on the side holding the flask.

At ordinary atmospheric pressure and temperature, one cu. ft. of air weighs approximately 1¼ oz.

Atmospheric Pressure. We have found that air has weight; it is also a fact that we all live at the bottom of an ocean of air (the atmosphere), which is some miles deep. It is then easy to understand that this mass of air on top of us exerts a pressure on everything at the earth's surface. The following are two ways of illustrating this fact: Fasten a thin sheet of rubber over the end of a small cylinder and exhaust some of the air from the cylinder, and the rubber will be pushed in by the outside air pressure as shown in sketch.

FIG. 5.

Take two hemispheres fitted together with a flat ground seat and equipped with a tap. Ordinarily the two hemispheres will not cling together, but if the air is exhausted from them and the tap closed, it will take a great force to pull them apart. When they are filled with air, the pressure on the outside and the inside is equal; when the pressure within is lessened by exhausting some of the air, then the greater pressure on the outside presses them tightly together.

Another experiment to prove this same law: Get an air-tight can and put some water in it. Boil the water with the plug out. Replace the plug tightly and let cool. As it cools the can will collapse inwardly, being crushed in by the great outside pressure.

Another experiment is to completely fill a tumbler with water, then lay a piece of paper over the top. Invert the tumbler and the water will remain in it, due to the atmospheric pressure being greater than the downward weight of the water.

Note: These last two experiments should be performed by the reader.

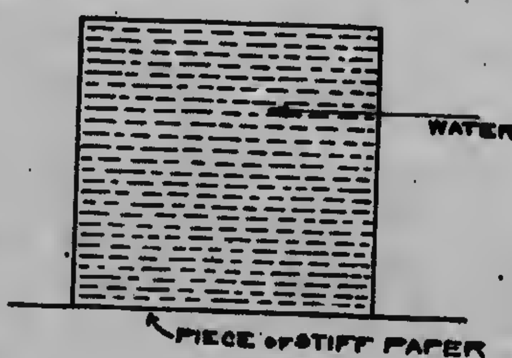


FIG. 6.

Torricelli. An Italian named Torricelli in 1640 was the first to prove that the atmosphere exerts pressure, and to measure this pressure. It was noticed when trying to operate a deep well, that no matter how good a pump was used, the water could not be lifted over 33 ft. Torricelli concluded that it was the atmospheric pressure that forced the water up from the well, and that when a height of 33 ft. was reached, the column of water just counterbalanced the pressure of the air. Then knowing the weight of the water, it gave the measure of the air pressure. He also reasoned that, if this be true, a column of any liquid heavier than water would rise to a height less than 33 ft. Experimenting with mercury, which is 13.6 times heavier than water, he found it to rise to 1/13.6 of 33 ft. (Appr.); or nearly 30 inches. If this experiment is performed at the base and at the top of a high mountain, the height of the mercury column will vary three or four inches, being less at the top of the mountain.

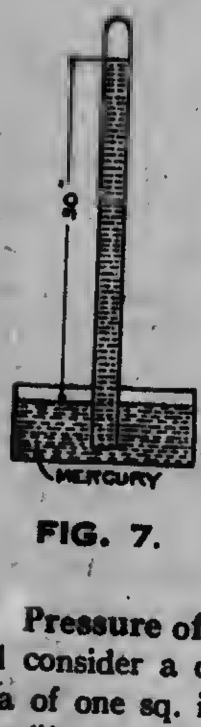


FIG. 7.

Here we have the principle of the ordinary mercury barometer used to register the air pressure and foretell the weather. (See Sketch.)

If we repeat Torricelli's experiment using a number of tubes of different sizes and shapes, standing in vessels of different sizes and shapes, we find in each case that the height of the mercury is the same, showing that the height of the column is independent of the size or shape of the tubes or vessels, providing that the tube is over 30 ins. long and the vessel open to the atmosphere.

Pressure of Atmosphere to the Square Inch. We will consider a case where the tube has a cross sectional area of one sq. in., then it is evident that the atmosphere is holding up a column of mercury 30 × 1 × 1, or 30 cu. inches. One cu. in. of mercury weighs .49 lbs. Therefore the atmosphere is sustaining a weight of .49 × 30, or

14.7 lbs. for each sq. in. of area exposed. We now see that everything on the surface of the earth is under a continual pressure of nearly 15 pounds.

We will now proceed to notice how some of the laws of liquids are applied to gases. We know now that air has weight; that, like liquids, it exerts pressure on everything immersed in it, and that this pressure increases with the depth of the air. In liquids the pressure is directly proportional to the depth of liquid above. This is not true, however, in the case of gases, because while liquids are nearly incompressible, all gases as we shall learn later are very compressible, and therefore the gas near the bottom of a column is more dense than gas near the top. For example, a cu. ft. of air at the base of a mountain has a greater weight than a cu. ft. near the top. Gases then have weight and exert pressure on objects immersed in them. The other laws of liquids also apply to gases, namely, Pascal's Principle and the Law of Archimedes.

Pascal's Law applied to gases. Pressure on a confined gas is exerted equally and undiminished in all directions. This is shown by the fact that two gauges set at different points in a boiler (but at same level) will both register the same pressure per sq. in.

The Law of Archimedes applied to gases. All bodies immersed in a gas are buoyed up by a force equal to the weight of the gas displaced. The buoyant force on a balloon is an illustration of this law. For example, if a balloon displaces 12,800 cu. ft. of air, the total buoyant force on it is equal to the weight of the 12,800 cu. ft. of air. This is 12,800 × 1¼ oz. or 16,000 ozs. or 1,000 lbs. The balloon could then lift 1,000 lbs. (Note, the weight of a cu. ft. of air is 1¼ oz.)

We will now consider two new laws that apply to gases, namely, Boyle's Law and Henry's Law.

Boyle's Law. The volume of a gas varies inversely as the pressure on it. This is an important law that applies to all gases. It is named after an Englishman who discovered it in 1666. This law means that if a gas is confined under a certain pressure and we double this pressure, then the gas is compressed to one half of its first volume. Or, if the volume is reduced to one half its former volume, the pressure of the confined gas will increase to double the former pressure. Notice also that if the volume is doubled then the pressure will decrease to one half its former amount. This law will be gone into fully in the paper on Compressed Air. It is sufficient now to understand this law in a general way.

Henry's Law. If a gas stands in contact with a liquid, part of the gas dissolves in the liquid. An American Scientist, Henry, in 1803, investigated and announced this law, namely, that if a gas does not combine chemically with a liquid, the amount of gas dissolved is directly proportional to the pressure. In other words, doubling the pressure will double the amount of gas dissolved.

Siphon. If a bent tube is filled with a liquid and one end is introduced into a vessel of the liquid while the other end is open and held at a lower level than the surface, the liquid will flow from the vessel through the tube. Such an arrangement is known as a siphon and as an easy experiment is one that should be tried by everyone. See Fig. 8.

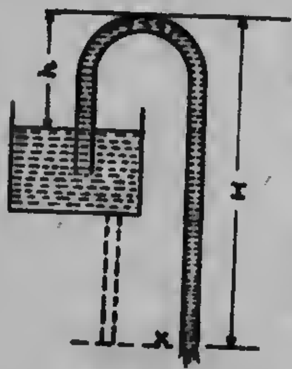


FIG. 8.

The upper surface of the liquid in the vessel, and also the open end of the siphon are subject to the atmospheric pressure, but this is partly balanced on the short side by the column of liquid of height "h", while on the other it is opposed by the longer column "H". The pressure which is effective in causing the flow is therefore that of a column of liquid of height "H" minus "h". From this you will see that the velocity of escape of liquid through a siphon is the same as from an opening directly into the bottom of the vessel and piped as indicated by dotted lines. Practically though you would find it to be slightly less owing to the friction of the extra piping in the siphon.

The height of the pipe "h" is limited to 34 ft. if water is being siphoned, and if heavier liquids are used, this height decreases in proportion to their specific gravity which is their relative weight to water. The reason for this is that the atmosphere must sustain the weight of liquid in the short tube "h". It is also clear that the surface of liquid in vessel must be above point "x" or there will be no flow. This will clear up a common misconception among some mechanics who often state that water can be siphoned to a higher level. Such is impossible as a little study will soon show, or we would then have perpetual motion in defiance of the laws of gravitation and resistance.

Many shop men use the term siphon when speaking of oil trimmers conveying oil to the bearings. This is wrong, and when you see a trimmer conveying oil to a bearing above it, keep in mind that this is an example of the law of capillarity, of which the ordinary lamp wick is a good example.

COMMON LIFT OR SUCTION PUMP

Fig. 9 is a diagram of an ordinary lift pump. In this pump there are two valves opening upward, one in the piston and one at the bottom of the cylinder. As the piston raises, its valve closes, and a partial vacuum is formed in the cylinder; then atmospheric pressure acting on the surface of the water in the cistern forces water from the cistern to rise up the pipe and enter the cylinder past the lower valve, filling up the cylinder after the piston. As the piston descends, the lower valve closes, preventing return to the cistern, and the valve on the piston opens, allowing the water to pass through. The next upward stroke will do as noted above, and in addition will lift what water is above the piston up and out of the spout. Since atmospheric pressure must sustain the weight of water in the feed pipe, it follows that a suction pump cannot raise water from a level more than about 34 ft. below the pump piston. As in the case of the siphon, this height varies with weight of the liquid that is being pumped. As no pump is absolutely perfect and there is always some leakage and resistance, in practice very few pumps will draw up water more than 20 or 22 ft. In deep wells the plunger is placed in a cylinder, and the cylinder is placed on the end

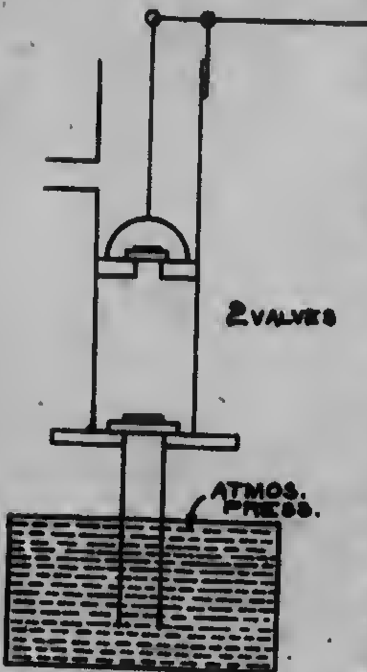


FIG. 9.

of a pipe long enough to bring it within 20 ft. of the water. The water is then brought to the surface by being actually lifted by the piston, which carries the weight of the entire column of water on its upward stroke, but is relieved by a check valve on the downward stroke.

The Force Pump. Fig. 10 shows a diagram of a force pump. When we desire to force water above the level of the pump spout we generally use this kind of a pump which is the same as the lift pump previously described, except that the top of the barrel is closed. The pump rod passes through a stuffing box which is water-tight. The air bell or chamber on the discharge pipe serves to prevent strains on the pump, and also to keep up a steady stream in the pipe. While the plunger is moving up, part of the water is forced into the discharge pipe, and part into the air chamber, compressing the air in the chamber and increasing its pressure. While the plunger is moving down, the air in the chamber expands forcing what water there is in the chamber out into the discharge pipe. Thus a steadier stream is maintained. (Note extra valve in force pump.)

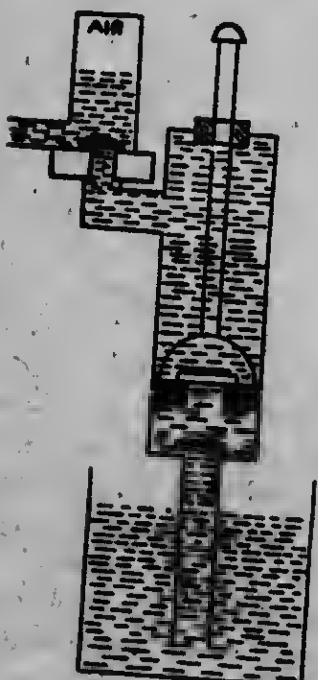


FIG. 10.

Double-acting Force Pump. Fig. 11 shows the arrangement of valves in a double-acting force pump. We will now see how the above principles are applied on a larger scale, and how they are really the fundamental principles upon which much of our present day pumping machinery is based. Study the two sketches shown here and you will see what is meant by the term "double-acting." In all pumps of this kind the same operations are performed on each side of the piston, only at different times. The movement of the piston is more fully utilized to perform work. When pumping is to be done on a large scale, the pumps are always double acting and power driven. Either steam, compressed air or electricity may be used as the motive power. The water pumps of a boiler plant are usually steam driven, so also is the air pump on a locomotive. A steam driven pump generally has the pump and steam pistons both on the same shaft or rod. A similar arrangement is found on the hydrostatic pumps used for testing boilers. Here we have a water pump driven by a compressed air engine, the piston of both being on the same shaft.

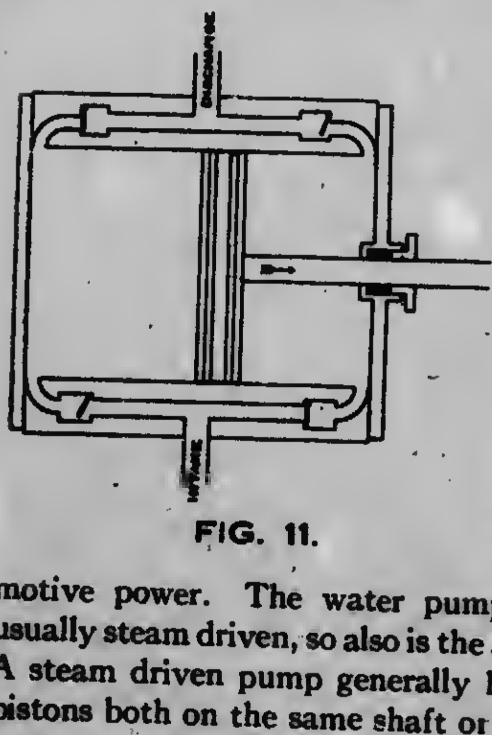


FIG. 11.

All the pumps so far mentioned are known as single stage or single pumps. In the case of air pumps or compressors it is possible to have them double stage or compound. That is, the discharge of the first pump goes to the suction of a second pump which in turn discharges at the final pressure. This is the principle of the Westinghouse 8½″ Compound Pump, used on some of the locomotives. In regard to this compressor it might be well to notice that the steam portion works on the compound expansion principle, but this is independent and has nothing to do with the air end compressing on the compound principle.

Some Uses of Compressed Air. Now-a-days there are countless uses of compressed air. We will mention a few of them in a general way. There are all the uses on a modern train such as the all-important air brake, the air train signal, water service in coaches, air sanders on locomotives, also air bell ringers, air fire-doors, air reverse gears, etc. Another useful application is the pneumatic drill used for boring holes in rock; in it a steel drill is attached to a piston which is made to move back and forth in a cylinder by allowing compressed air to act alternately on its two faces. The Pneumatic hammer, which is similar in principle, is used for rivetting and in general foundry and boilermaker's work. Steam could be used, but the pipes conveying it would be hot and water would be formed from it. By means of a blast of sand, projected by a jet of air, castings and paint surfaces are cleaned. Boiler tubes are blown clean with compressed air, and small motors are operated in the same way.

COMPRESSED AIR

The following chapter deals with some of the practical calculations that are necessary in working out problems in compressed air. We have already stated one of the most important laws governing this in a previous paper on Gases. It is known as Boyle's Law, and is as follows:—

> "The temperature remaining the same, the volume of a given mass of gas varies inversely as the pressure acting upon it."

This means that if a cylinder is full of air at atmospheric pressure and the piston moved to mid stroke, the pressure (presuming no leakage) will rise to two atmospheres. Moving the piston two-thirds of the stroke will compress the air into one-third of its original space and raise its pressure to three (3) atmospheres, and so on.

Now we will start at the beginning and go into the subject in a more complete manner. Remember this point from the start: that **compression produces heat**, and expansion (the opposite of compression) lowers the temperature of the gas expanded, and if these operations take place too quickly for the heat to be radiated or absorbed it will have an effect on the pressures. If it were possible to compress air without generating heat, then a great loss of power could be avoided, and the calculations would exactly follow Boyle's Law. Engineers have a name for this ideal condition of compression and expansion; it is called **Isothermal** Compression. As a matter of fact there is always heat formed, and when it is considered, the calculations follow the laws of **Adiabatic Compression.** An effort is always made to conduct away some of the heat so that actual results secured in the best compressors is somewhere between the Isothermal and the Adiabatic Compression. To simplify the calculations as much as possible we will presume all out pressures to follow the laws of Isothermal compression and expansion, and the results will be quite accurate enough from a practical standpoint.

Let us first consider the characteristics of free air:—

Atmospheric Pressure. By atmospheric pressure is meant the pressure exerted by the atmosphere on all things, due of course to its weight. While one cu. ft. of air only weighs appr. 1¼ ozs., yet the pressure exerted at the earth's surface by the ocean of air above amounts to 14.7 lbs. per sq. in., measured at the sea level. In all ordinary calculations the pressure of the atmosphere is assumed to be 15 lbs. per sq. in., so as to simplify the calculations. The higher above the sea level we go, the less is the atmospheric pressure as will be seen from the following figures:—

Sea Level	14.7	lbs. per sq. in.
5,000 ft. above S.L.	12.1	"
10,000 "	10.0	"
15,000 "	8.3	"
6 miles "	4.4	"

Note: The atmosphere has been calculated to extend upward for 40 miles, but it becomes rarer the higher it goes.

Measuring Air Pressures. The pressure of the atmosphere is measured by a barometer or column of mercury. It has been found that the atmosphere will sustain a column of mercury 30 ins. high, having an area of one sq. in. and weighing 14.7 lbs. The pressure of the atmosphere cannot be measured by a gauge. This can be proved by opening the drain cock in the main reservoir and letting the pressure drain down to atmospheric pressure. The gauge will then show no lbs. per sq. in. It is right here that you can see the difference between **gauge pressure** and **absolute pressure,** two entirely different scales of pressure.

Close the drain cock in the main press reservoir, and pump up the pressure until the gauge shows 20 lbs. The **absolute pressure** in the reservoir will then be atmospheric pressure (15 lbs.) plus the 20 lbs. as shown on the gauge, which equals 35 lbs. absolute pressure in the main reservoir. No matter what pressure the gauge shows there will always be 15 lbs. more than shown, and if we add this 15 lbs. to the gauge pressure it gives us the **absolute pressure.** Now this is important, for in working out problems in air pressure, we will always have to work with **absolute pressures,** and not gauge pressures. This is no hardship, for, as seen above, it is simply a matter of adding 15 lbs. to the gauge pressure to give you the absolute pressure, and substracting from absolute to give you gauge pressure.

Air pressures are sometimes measured by **atmospheres** instead of pounds per sq. in. One **atmosphere** represents an absolute pressure of 15 lbs. and a gauge pressure of no pounds per sq. in. Consult the following table:

No. of Atmospheres	Absolute Press.	Gauge Press.
0 (perfect vacuum)	0.	0.
1	14.7	0.
2	29.4	14.7
3	44.1	29.4
4	58.8	44.1
5	73.5	58.8
6	88.2	73.5
7	102.9	88.2
8	117.6	102.9
9	132.3	117.6
10	147.0	132.3
etc.		etc.

Action of Air under Compression and Expansion.

Through experiments in 1662, the relationship between the volume and the pressure of a gas was found to be as follows:

The volume of a gas varies inversely with the absolute pressure, providing the temperature is kept the same.

This means that if the volume of a certain mass of air is decreased a certain amount, the absolute pressure will be increased the same amount. For example, if the volume be reduced one-half, the absolute pressure will be doubled; if compressed to one-third of its original volume, the pressure will be trebled; if expanded to twice the volume, the absolute pressure will be one-half the original amount, and so on. This relationship of volume and pressure may then be stated as follows: if the **absolute pressure** and the **volume before** compression or expansion takes place be multiplied together, the product will always equal the product of the absolute pressure and volume **after** the change.

For example, suppose that 4 cu. ft. at 30 lbs. (this is 45 lbs. absolute) is expanded into 8 cu. ft.

Since this is twice the former volume, we know that the absolute pressure will be one-half the original absolute pressure, or 22½ lbs.

Notice the equation:

(original absolute pressure) × (original volume) equals (final absolute pressure) × (final volume)

45 × 4 equals 22½ × 8 or 180 equals 180.

Now we come to the rules for finding the pressure or volume after compression of expansion has taken place.

Rule 1. After a volume of air has either expanded or compressed, the **resulting absolute pressure** will equal the product of the absolute pressure and volume before the change, divided by the final volume.

The reader should check over the following examples, keeping in mind that you must use absolute pressures in working out the problems, and not gauge pressures.

Example No. 1. If 4 cu. ft. of air at 50 lbs. pressure is expanded to 10 cu. ft., what will be the resulting pressure?

Solution: Absolute Pressure = 50 + 15 = 65 lbs.

From Rule 1, the absolute pressure $= \frac{4 \times 65}{10} = 26$ lbs.

Changing this to gauge pressure gives 26 — 15 = 11 lbs.

Example No. 2. If. 10 cu. ft. of air at 11 lbs. pressure is compressed into 4 cu. ft., what will be the resulting pressure?

Solution: Absolute pressure will be 11 + 15 = 26 lbs.

By Rule 1, the final absolute press. $= \frac{26 \times 10}{4} = 65$ lbs.

Changing this to gauge pressure gives 65 — 15 = 50 lbs.

Rule 2. After a volume of air has been either expanded or compressed, the **Final Volume** is equal to the product of the original absolute pressure, and the original volume divided by the final absolute pressure.

Example 3. If a volume of 4 cu. ft. of air at 50 lbs. pressure is expanded until its pressure is 11 lbs., what will be the volume ?

Solution. Absolute pressure before equals 65 lbs., after expansion 26 lbs. From Rule 2, the final volume equals $\frac{4 \times 65}{26} = 10$ cu. ft.

Example 4. If 10 cu. ft. at 11 lbs. is compressed until it exerts a pressure of 50 lbs., what will be its volume ?

Solution. Original absolute pressure is $11 \times 15 = 26$ lbs. Final absolute pressure is $50 + 15 = 65$ lbs.

By Rule 2, the final volume equals

$$\frac{10 \times 26}{65} = 4 \text{ cu. ft.}$$

Calculation of Piston Effort. When it is desired to figure out the effort exerted by a piston, say in a 14″ brake-cylinder, it is necessary to know the area of the piston in sq. ins. and the air pressure in lbs. per in. gauge pressure. These two amounts are multiplied together to give the piston effort in pounds.

Example 5. 60 lbs. air pressure acts in a 14″ brake-cylinder. What is the piston force ?

Solution. Area of piston is $14 \times 14 \times .7854 = 154$ sq. ins. $154 \times 60 = 9240$ lbs. $= 4.62$ tons.

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