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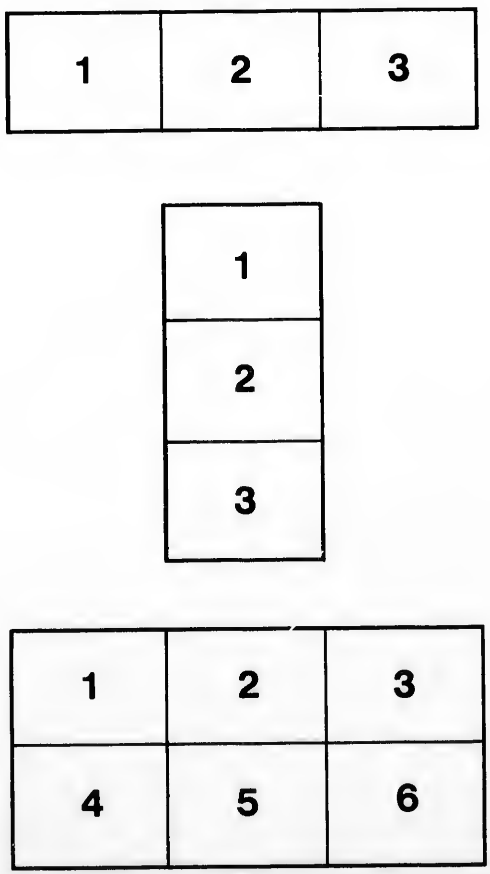
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EXTRAORDINARY

GEOMETRICAL DISCOVERY

TRISECTION

OF ANY RECTILINEAL ANGLE

— BY —

ELEMENTARY GEOMETRY

AND SOLUTIONS OF OTHER PROBLEMS

Considered impossible except by aid of the higher Geometry.

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Trisection of a Rectilineal Angle.

Let B O C be the angle to be trisected.

At the point O, make (23.1) the angles B O D and C O E, each equal to the angle B O C. Take O as centre with any radius O B, and describe the arc D B C E; join D B, B C, C E, and D E. Produce D B, and E C to meet in T; join O T. By 4.1., the angle O D B is equal to the angle O E C, and (5.1) the angle O D E is equal to the angle O E D; take away the two latter angles and the remaining angles E D T and D E T are equal: therefore (6.1) D T is equal to E T. In the triangles O D T and O E T, the three sides of the one are respectively equal to the three sides of the other, by (8.1) they are equal in every respect; therefore the angle D T O is equal to the angle E T O, then the line O T bisecting the vertical angle of an isosceles triangle, bisects D E and B C perpendicularly. It also bisects the arc B C; it is evident by 4.1 that D B, B C, and C E are equal to one another.

Draw *a d* parallel to B C, so that *a d* shall be equal *a* B and *d* C. This is done by bisecting the angles T B C and T C B by lines meeting the sides B T and C T in *a* and *d*, when the triangle is isosceles, or equilateral and proved by (29.1, 6.1, and 4.1). Find the diamater of a circle circumscribing the trapizium B *a d* C, and make *t* L equal to it; join *a* L and *d* L, cutting the arc B C in the points *r* and *l*: these are the trisecting points of the arc B C which is t measure of the given angle B O C. Draw the lines O *r b* ard O *l c*, and the three angles *b* O *c*, *b* O B, and C O *c* are equal and each equal to one third of the given angle B O C.

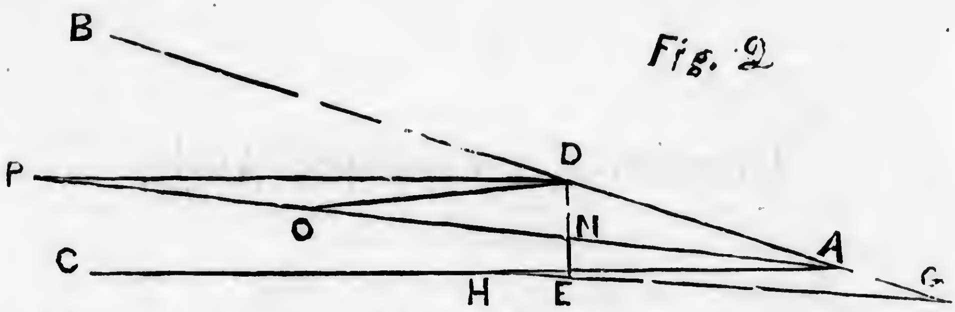

Fig. 2

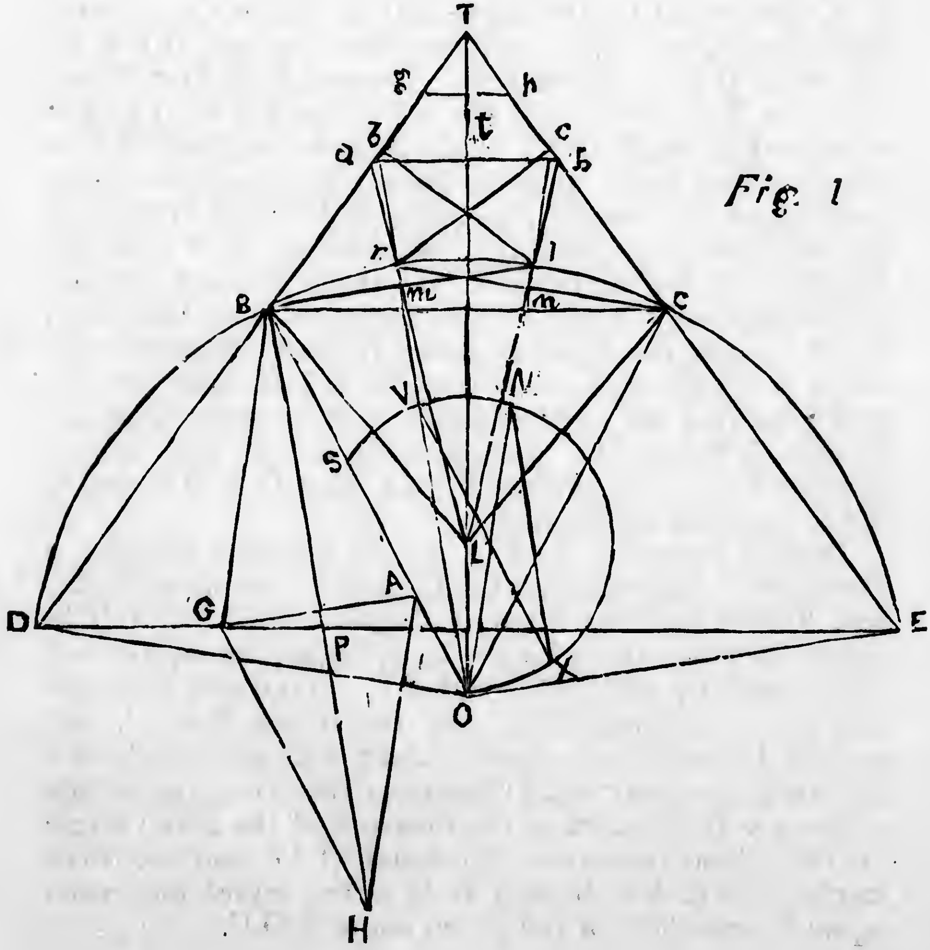

Fig. 1

FIRST DEMONSTRATION.

Through the point B, draw B G paralled to O *l* or O *c*, and meeting D E in G; draw also B P parallel to O *r* or O *b*; make (3.1) B A equal to B G; through G draw G H parallel to B O, and through the point A draw A H parallel to B G; join B H, and B H passes through the point P. We have now, the straight line B P and the line B H, having the two points B and P in common; therefore (Prop. 2, Legendre,) they coincide throughout the whole, or they are in one and the same straight line.

A B is parallel to G H, and A H is parallel to B G; therefore A B G H is a parallelogram, having its opposite sides equal and parallel, and A B is equal to B G by construction; therefore A B G H is a rhombus and the diagonal B H bisects its opposite angles; therefore the angle A B P is equal to the angle G B P.

B G is parallel to O *l* and B O a line meeting them, by (29.1), the angle A B G is equal to the angle B O *l*; for the same reason, the angle A B P is equal to the angle B O *r*; therefore the remaining angle G B P is equal to the angle *r* O *l* but G B P and A B P are equal; B *o r* and *r o e* similarly it can be proved that the angle *l* O C is equal to the angle *r* O *l*; but things which are equal to the same thing, are equal to one another; therefore the angles B O *r*, *r* O *l*, and *l* O C are equal, and the angle B O C is trisected.

SECOND DEMONSTRATION.

Suppose L C and L B to be joined, it is easily proved that L C is equal to L B, then by 5.1, the angle L B C is equal to the angle L C B, and the angle B C T is equal to the angle C B T; therefore the whole angle

L C *d* is equal to the whole angle L B *a*. In the triangles *d* C L and *a* B L, *a* B and B L are equal to *d* C and C L, and their contained angles equal, by (4.1), *a* L is equal to *d* L, and the angle B *a* L equal to the angle C *d* L, by (13.1), the angle *r a b* is equal to the angle *l d c*. In the triangle *r* L O and *l* L O, the three sides of the one are respectively equal to the three sides of the other, by (8.1), O *r* L is equal to the angle O *l* L, and the angle *r* O L equal to the angle *l* O L; by (15.1) the angle *a r b* and *d l c* are equa but it has been proved that the angle *b a r* is equal to *c d l*; therefore (32.1), the two triangles are equiangular, and *a r* equal to *d l*; therefore, by (26.1), *a b* is equal to *c d*, but *a* B is equal *d* C; therefore *b* B is equal to *c* C. Then, in the triangles *b* B O and *c* C O, we have *b* B and B O equal to *c* C and C O and their contained angles equal, by (4.1), the angle B O *b* is equal to the angle C O *c*. Then, as in the first demonstration it may be proved that each of these angles, is equal to the angle *b* O *c*; therefore the three angles B O *r*, *r* O *l*, and *l* O C, are equal, and the angle B O C is trisected.

THIRD DEMONSTRATION.

Join B *l* and C *r*, cutting O *l* and O *r* in *n* and *m*. If on O B we describe a semicircle, it passes through the point *m*; therefore (31.3), the angles at *m* are right angles, and (3.3) B *l* is bisected perpendicularly by O *m*; hence B *m* and *m* O are equal to O *m* and *m l* and the angles at *m* equal, by (4.1), the angle B O *m* is equal to the angle *l* O *m*. This may also be proved by letting fall a perpendicular from O, on B *l* and this perpendicular always falls on the point *m*; then we have two straight lines which coinside in part they must coinside throughout the whole (Legendre, Prop. 2). It may be similarly proved that C r is bisected perpendicularly by O c in n, and that the angle C O n is equal to the angle r O n; therefore the three angles, B O r, r O l, and l O C, are equal; therefore the angle B O C is trisected.

FOURTH DEMONSTRATION.

In O T, take any point, as L, and with L O as radius describe the circle O U V S. Through V draw V X parallel to S O, meeting the circle in X; and through u draw u X parallel to V O.

Because S O is parallel to V X and V O meeting them, (29.1) the angle S O V is equal to the angle O V X; for the same reason, the angle O V X is equal to the angle V X U; but V X U is equal to V O U; therefore the angle S O V is equal to the angle V O U. Similarly the angle C O U may be proved equal to V O U; therefore S O V=V O U=U O C, and the angle B O C is trisected. The lines V X and U X, in the trisection of every angle, always meet the circumference of the circle in the same point. This evidently, can never happen but when the three arcs B r, v l, and l C are equal.

ANDREW DOYLE.

The following problems also have been considered by all mathematicians to be beyond the range of elementary geometry.

1. Let B A C (Fig. 2) be any angle; take any point D in A B and let fall the perpendicular D E; through D, draw D P parallel to A C; it is required to draw the line A N O P, so that N P may be equal to twice A D.

Trisect the angle B A C by A P and N P is the line required. Bisect N P in O, and join O D.

2. Let C A G be any angle, (Fig. 2) it is required to draw a line G H cutting the legs of the angle, so that the angle A G H may be equal to twice the angle A H G.

Trisect B A C by A P; take any point G and draw G H parallel to A P, meeting A C in H.

ANDREW DOYLE.

In the next Edition will appear the method of finding two geometrican means between two straight lines, and also the duplication of the cube.

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