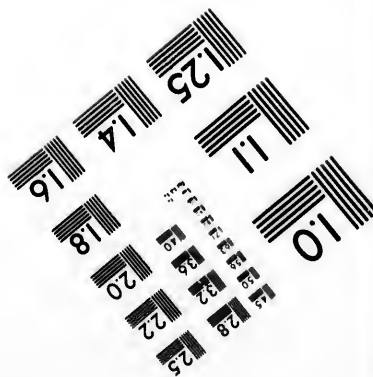
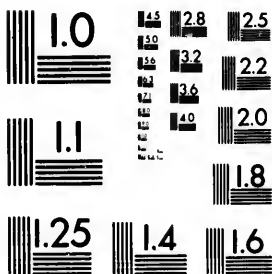


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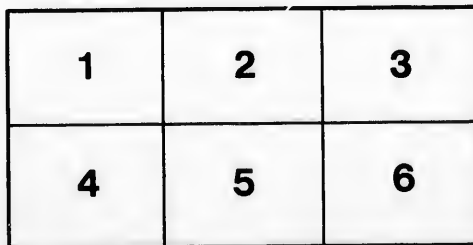
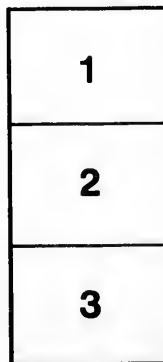
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EXTRAORDINARY
GEOMETRICAL DISCOVERY
TRISECTION
OF ANY RECTILINEAL ANGLE

— BY —

ELEMENTARY GEOMETRY
AND SOLUTIONS OF OTHER PROBLEMS

Considered impossible except by aid of the higher Geometry.

— BY —

ANDREW DOYLE.



OTTAWA:
A. BUREAU, PRINTER, SPARKS STREET.

1880

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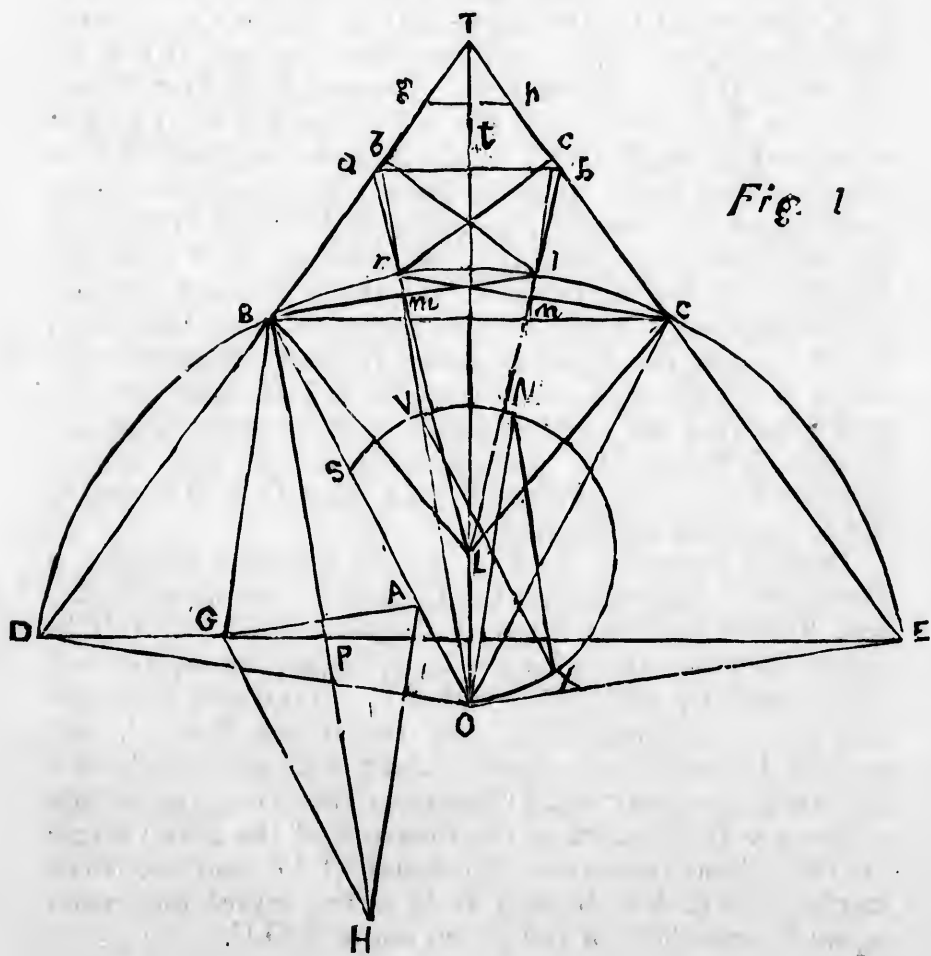
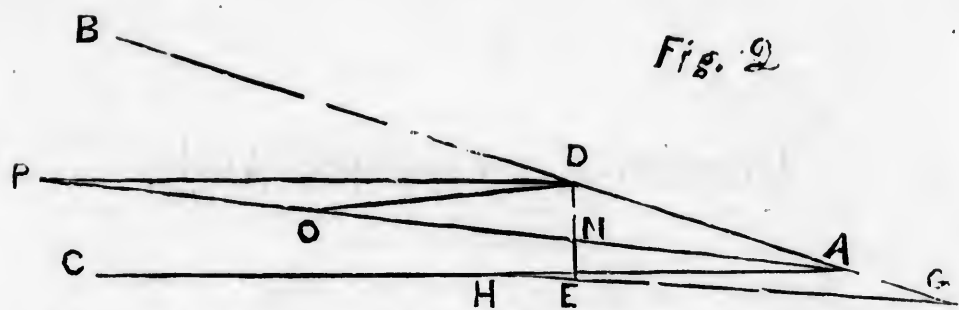
Entered according to Act of Parliament of Canada, in the year
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Trisection of a Rectilineal Angle.

Let $B O C$ be the angle to be trisected.

At the point O , make (23.1) the angles $B O D$ and $C O E$, each equal to the angle $B O C$. Take O as centre with any radius $O B$, and describe the arc $D B C E$; join $D B$, $B C$, $C E$, and $D E$. Produce $D B$, and $E C$ to meet in T ; join $O T$. By 4.1., the angle $O D B$ is equal to the angle $O E C$, and (5.1) the angle $O D E$ is equal to the angle $O E D$; take away the two latter angles and the remaining angles $E D T$ and $D E T$ are equal: therefore (6.1) $D T$ is equal to $E T$. In the triangles $O D T$ and $O E T$, the three sides of the one are respectively equal to the three sides of the other, by (8.1) they are equal in every respect; therefore the angle $D T O$ is equal to the angle $E T O$, then the line $O T$ bisecting the vertical angle of an isosceles triangle, bisects $D E$ and $B C$ perpendicularly. It also bisects the arc $B C$; it is evident by 4.1 that $D B$, $B C$, and $C E$ are equal to one another.

Draw $a d$ parallel to $B C$, so that $a d$ shall be equal $a B$ and $d C$. This is done by bisecting the angles $T B C$ and $T C B$ by lines meeting the sides $B T$ and $C T$ in a and d , when the triangle is isosceles, or equilateral and proved by (29.1, 6.1, and 4.1). Find the diameter of a circle circumscribing the trapezium $B a d C$, and make $t L$ equal to it; join $a L$ and $d L$, cutting the arc $B C$ in the points r and l : these are the trisecting points of the arc $B C$ which is the measure of the given angle $B O C$. Draw the lines $O r b$ and $O l c$, and the three angles $b O c$, $b O B$, and $C O c$ are equal and each equal to one third of the given angle $B O C$.



FIRST DEMONSTRATION.

Through the point B , draw BG parallel to Ol or Oc , and meeting DE in G ; draw also BP parallel to Or or Ob ; make (3.1) BA equal to BG ; through G draw GH parallel to BO , and through the point A draw AH parallel to BG ; join BH , and BH passes through the point P . We have now, the straight line BP and the line BH , having the two points B and P in common; therefore (Prop. 2, Legendre,) they coincide throughout the whole, or they are in one and the same straight line.

AB is parallel to GH , and AH is parallel to BG ; therefore $ABGH$ is a parallelogram, having its opposite sides equal and parallel, and AB is equal to BG by construction; therefore $ABGH$ is a rhombus and the diagonal BH bisects its opposite angles; therefore the angle ABP is equal to the angle GBP .

BG is parallel to Ol and BO a line meeting them, by (29.1), the angle ABG is equal to the angle BOl ; for the same reason, the angle ABP is equal to the angle $Bo r$; therefore the remaining angle GBP is equal to the angle $ro l$ but GBP and ABP are equal; $Bo r$ and $ro e$ similarly it can be proved that the angle lOC is equal to the angle $ro l$; but things which are equal to the same thing, are equal to one another; therefore the angles $Bo r$, $ro l$, and lOC are equal, and the angle BOC is trisected.

SECOND DEMONSTRATION.

Suppose LC and LB to be joined, it is easily proved that LC is equal to LB , then by 5.1, the angle LCB is equal to the angle LCB , and the angle $BC T$ is equal to the angle $CB T$; therefore the whole angle

$\angle C d$ is equal to the whole angle $\angle B a$. In the triangles $d C L$ and $a B L$, $a B$ and $B L$ are equal to $d C$ and $C L$, and their contained angles equal, by (4.1), $a L$ is equal to $d L$, and the angle $B a L$ equal to the angle $C d L$, by (13.1), the angle $r a b$ is equal to the angle $l d c$. In the triangle $r L O$ and $l L O$, the three sides of the one are respectively equal to the three sides of the other, by (8.1), $\angle r L O$ is equal to the angle $\angle O l L$, and the angle $r O L$ equal to the angle $l O L$; by (15.1) the angle $a r b$ and $d l c$ are equal but it has been proved that the angle $b a r$ is equal to $c d l$; therefore (32.1), the two triangles are equiangular, and $a r$ equal to $d l$; therefore, by (26.1), $a b$ is equal to $c d$, but $a B$ is equal $d C$; therefore $b B$ is equal to $c C$. Then, in the triangles $b B O$ and $c C O$, we have $b B$ and $B O$ equal to $c C$ and $C O$ and their contained angles equal, by (4.1), the angle $B O b$ is equal to the angle $C O c$. Then, as in the first demonstration it may be proved that each of these angles, is equal to the angle $b O c$; therefore the three angles $B O r$, $r O l$, and $l O C$, are equal, and the angle $B O C$ is trisected.

THIRD DEMONSTRATION.

Join $B l$ and $C r$, cutting $O l$ and $O r$ in n and m . If on $O B$ we describe a semicircle, it passes through the point m ; therefore (31.3), the angles at m are right angles, and (3.3) $B l$ is bisected perpendicularly by $O m$; hence $B m$ and $m O$ are equal to $O m$ and $m l$ and the angles at m equal, by (4.1), the angle $B O m$ is equal to the angle $l O m$. This may also be proved by letting fall a perpendicular from O , on $B l$ and this perpendicular always falls on the point m ; then we have two straight lines which coincide in part they must coincide throughout the whole (Legendre, Prop. 2). It

may be similarly proved that $C r$ is bisected perpendicularly by $O c$ in n , and that the angle $C O n$ is equal to the angle $r O n$; therefore the three angles, $B O r$, $r O l$, and $l O C$, are equal; therefore the angle $B O C$ is trisected.

FOURTH DEMONSTRATION.

In $O T$, take any point, as L , and with $L O$ as radius describe the circle $O U V S$. Through V draw $V X$ parallel to $S O$, meeting the circle in X ; and through u draw $u X$ parallel to $V O$.

Because $S O$ is parallel to $V X$ and $V O$ meeting them, (29.1) the angle $S O V$ is equal to the angle $O V X$; for the same reason, the angle $O V X$ is equal to the angle $V X U$; but $V X U$ is equal to $V O U$; therefore the angle $S O V$ is equal to the angle $V O U$. Similarly the angle $C O U$ may be proved equal to $V O U$; therefore $S O V = V O U = U O C$, and the angle $B O C$ is trisected. The lines $V X$ and $U X$, in the trisection of every angle, always meet the circumference of the circle in the same point. This evidently, can never happen but when the three arcs $B r$, $v l$, and $l C$ are equal.

ANDREW DOYLE.

The following problems also have been considered by all mathematicians to be beyond the range of elementary geometry.

1. Let BAC (Fig. 2) be any angle; take any point D in AB and let fall the perpendicular DE ; through D , draw DP parallel to AC ; it is required to draw the line $ANOP$, so that NP may be equal to twice AD .

Trisect the angle BAC by AP and NP is the line required. Bisect NP in O , and join OD .

2. Let CAG be any angle, (Fig. 2) it is required to draw a line GH cutting the legs of the angle, so that the angle AGH may be equal to twice the angle AHG .

Trisect BAC by AP ; take any point G and draw GH parallel to AP , meeting AC in H .

ANDREW DOYLE.

In the next Edition will appear the method of finding two geometrical means between two straight lines, and also the duplication of the cube.

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