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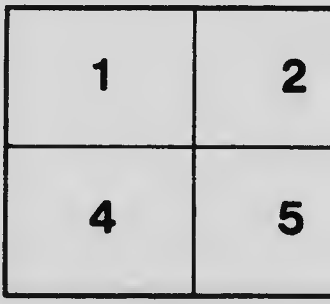
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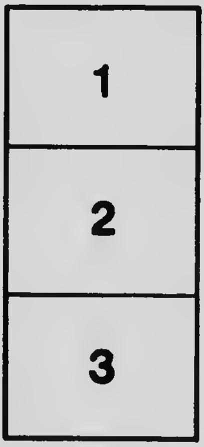

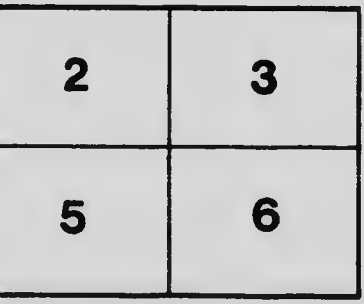

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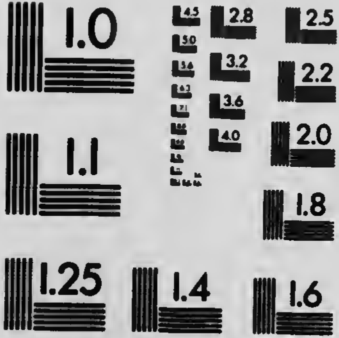

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PAPERS FROM THE PHYSICAL LABORATORIES

No. 42: ON THE RELATION BETWEEN THE ADIABATIC AND ISOTHERMAL YOUNG'S MODULI OF METALS, BY E. F. BURTON.

(Reprinted from Transactions of the Royal Society of Canada, 3rd Series, Vol. v.)

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On the Relation Between the Adiabatic and Isothermal Young's Moduli of Metals.

E. F. BURTON, B.A. (CANTAB.), PH. D. (TOR.)

Presented by Professor J. C. McLennan.

(Read May 17, 1911.)

In the ordinary equation for the velocity of sound in solids, viz., $V = \sqrt{\frac{q'}{d}}$, where q' is Young's modulus for the substance and d is the density, the Young's modulus considered is that which comes into play when the change in strain takes place so rapidly that the heat produced or absorbed during the strain has not time to escape. Lord Kelvin[1] has shown that this Young's modulus should be connected with the Young's modulus (q) found by statical methods such as stretching wires or bending rods, by the following equation:—

$$\frac{1}{q'} = \frac{1}{q} - \frac{w^2 T}{J K d}$$

where w is the coefficient of linear expansion of the metal, T, the absolute temperature, J, the mechanical equivalent of heat, K, the specific heat of the substance, and d its density.

The above equation gives at once the ratio of q' to q. In Table I the values of q for several metals experimented on by Wertheim [2] are given in column I, and the corresponding values of q'/q deduced from the above equation in column 2.

TABLE I.

Substance.	q in dynes per sq. cm. × 10^{-11}	q'/q deduced from the above equation.
Zinc	8·56	1·008
Tin	4·09	1·00362
Silver	7·22	1·00315
Copper	12·20	1·00325
Lead	1·74	1·0031
Glass	6·02	1·0006
Iron	18·24	1·00259
Platinum	16·7	1·00129

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[1] Article on Elasticity, Encyclopaedia Britannica.

[2] Wertheim, Annales de chim. et de phys., 1844. Pogg. Ann. 77, 427, 1849.

Each of the above values for the q's, except that for tin, was determined by Wertheim by a statical method; in the case of tin the value of q was determined from transverse vibrations.

In the paper referred to Wertheim gives results for Young's modulus for the various metals by methods involving (1) direct elongation, (2) transverse vibrations, and (3) longitudinal vibrations. The agreement between the values obtained for the same metal by direct elongation and by vibration differ in some cases by as much as 20%. Regarding these results Lord Kelvin wrote:—"It will be seen that Young's moduli obtained by Wertheim by vibrations, longitudinal or transverse, are generally in excess of those which he found by static extension; but the differences are enormously greater than those due to heating and cooling effects of elongation and contraction and are certainly to be reckoned as errors of observation. It is probable that his modulus determinations by static elongation are minutely accurate; the discrepancies of those found by vibrations are probably due to imperfection of the arrangements for carrying out the vibrational method."[3]

A glance at the published tables of the elastic constants of various substances would suffice to show the utter uselessness of trying to test the formula showing the relation between q and q′ for any substance by any method except finding directly q and q′ for the same specimen of a given material. The purpose of the experiment described below was to find these two Young's moduli for a given specimen by direct methods in order to get the value of the ratio q : q′. The adiabatic Young's modulus was found by determining the velocity of sound in a brass rod by means of Kundt's well known dust-tube method and applying the formula:—$V = \sqrt{\frac{q'}{d}}$. The static method used was that based on the observation of the bending produced in the rod when it was supported on two knife-edges and a weight was applied to the middle of the bar.

1. *Determination of the Adiabatic Young's Modulus.*

The Kundt method is so well known as to need no description here. The air and powder in the closed tube were carefully dried by blowing a slow current of air through a tube containing phosphorus pentoxide and afterwards through the dust tube. The distances between the 1st and 7th, 2nd and 8th, etc., dust heaps were measured by means of a microscope which was placed so as to view also a standard yard placed just below the tube. The microscope was provided with a scale and

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[3] *Loc. cit.*

vernier so that the distance between the edge of any dust heap and the nearest division of the standard yard could be accurately determined. The mean of 24 observations on the value of $\frac{\lambda}{2}$ in air gave 7·1757 cms., the greatest divergence from this number being the values 7·1788 and 7·1723 cms.

The mean of three readings of the length of the rod gave 75·0484 cms. The specific gravity of the rod was determined by finding its volume and weight, corrections being carefully made for the very small holes made in the rod in attaching the disc on the end inserted in the dust-tube. The average of 15 readings on the diameter of the rod gave 1.2675 cms. The corrected volume of the rod was 94.6949 ccs.; the weight of the solid rod was 805·19 grams, which brings the density equal to 8·503.

If V_1 denotes the velocity of sound in brass, V_2, that in air, λ_1, the wave-length in brass, λ_2, the wave-length in air,

$$V_1 = V_2 \cdot \frac{\lambda_1}{\lambda_2} \cdot$$

This gives from the above results $V_1 = V_2 \times \frac{75\cdot0484}{7\cdot1757}.$

The results given for the value of V_2 for air in tubes are as follows:—

Kayser	(1877)	332·5	metres	per	sec.	at 0°C.
Wüllner	(1878)	331·9	"	"	"	"
Müller	(1902)	331·9	"	"	"	"

One is doubtless justified in taking as the most probable value of V_2, 331·9 metres per second. We have still to correct for the temperature as the above result refers to air at 0° C. and the equation connecting the velocity at 0° C. and t° C. is, for small values of t, $V_t = V_0 (1 + \frac{1}{2} at)$, where "a" is the coefficient of expansion of air. Therefore the value of V_2, the velocity of sound in air at 15° C., the temperature at which the above experiments were carried out, is

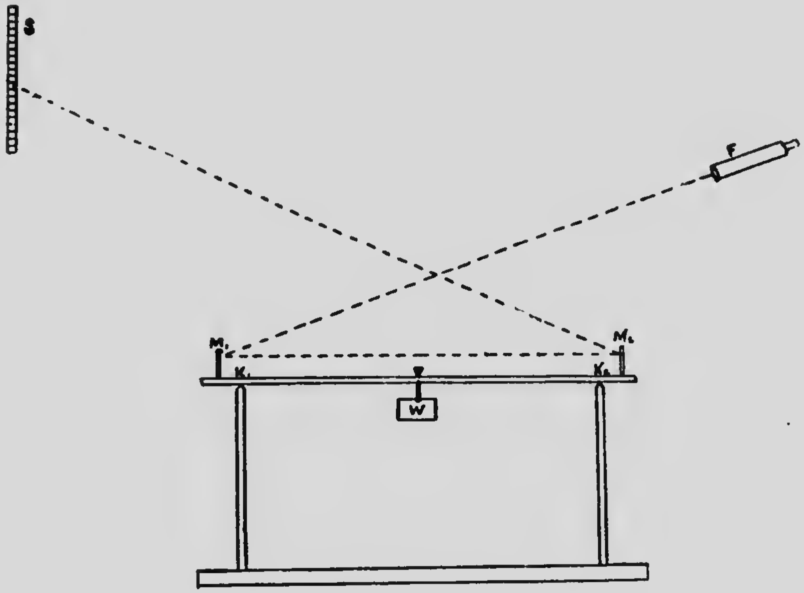
34105 cms. per second.

Introducing this value in the equation for V_1, we get as the value for the velocity in the brass rod at 15° C.—358,080 cms.per second.

From the equation $q' = V^2_1.d$, we obtain $q' = 10\cdot902 \times 10^{11}$ dynes per square centimetre.

2. *Determination of the Isothermal Young's Modulus.*

The statical method used depends on observing the depression produced at the centre of a bar, supported on two knife-edges, when a known weight is added at the centre of the bar. The depression was found by a method, due to König, illustrated in the figure below.



The rod rests on two knife-edges, K_1 and K_2, and mirrors, which are at right angles to the rod, are rigidly attached to it. The vertical scale S is reflected first from the mirror M, then from M_1 and read through the telescope F. The weight is applied at a knife-edge which is just midway between the knife-edges K_1 and K_2. On looking through the telescope we see one division on the scale coinciding with the cross hair of the telescope; on loading the beam another division of the scale will come on the cross hair, and by measuring the distance between these divisions we can determine the angle φ through which each free extremity of the bar has been bent. If d_1 is the distance between the mirrors, D the distance of the scale S from the mirror M_2, and v the total alteration in the scale reading,

$$\varphi = \frac{v}{2d_1 + 4D}$$

But the angle φ is also given by

$$\varphi = \frac{W}{2q.Ak^2} . \frac{l^2}{8}$$

where W is the weight added at the centre of the bar, l the distance between the knife-edges, q Young's modulus, and Ak^2 the moment of inertia of the cross section about a diameter.

This gives finally $$q = \frac{(d_1 + 2D).W.l^2}{8\ Ak^2 \cdot v}.$$

In the experiment performed the bar was supported on knife-edges which were placed on two stone piers in the basement of the Physical Laboratory. The values of the quantities involved in the above equation were as follows:—

$d_1 = 71{\cdot}9941$ cms.
$D = 45{\cdot}300$ cms.
$l = 69{\cdot}9941$ cms.
$W = 1{,}000$ grams.
$v = 7{\cdot}211$ cms. (mean of 45 readings).

Ak^2 ($= \frac{\pi}{4} a^4$) may be determined from the diameter recorded above, 1·2675 cms.

Supplying these numbers in the above formula we obtain

$$q = 10{\cdot}667 \times 10^{11} \text{ dynes per sq. cm.}$$

3. *Ratio of the Adiabatic and Isothermal Moduli.*

The recorded experimental values of the ratio $q' : q$ is

$$q'/q = 1{\cdot}022.$$

The value of $1/q - 1/q' = {\cdot}0200 \times 10^{-12}$.

The value for this difference deduced thermodynamically by Lord Kelvin is (see above)

$$w^2 .T /J.K.d.$$

For this experiment we may put

$w = \cdot 0000188$
$T = 288.$
$J = 4 \cdot 19 \times 10^7$
$K = \cdot 0917$
$d = 8 \cdot 503$

The substitution gives

$$1/q - 1/q' = \cdot 0031 \times 10^{-12}.$$

The agreement between theory and experiment leaves no such large gap to be filled as did the results of Wertheim; that the approach is so good suggests that it might be profitable to extend such experiments over a much larger range of substances.

I have to thank Messrs. J. D. Buchanan and C. W. Robb of the class of 1912, for making many of the observations recorded above.

Department of Physics,
University of Toronto,
May 15th, 1911.