# Technical and Bibliographic Notes/Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

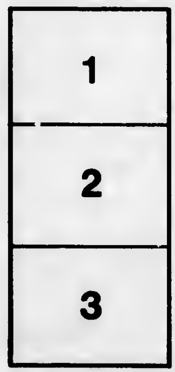
- [x] Coloured covers/ Couverture de couleur
- [ ] Covers damaged/ Couverture endommagée
- [ ] Covers restored and/or laminated/ Couverture restaurée et/ou pelliculée
- [ ] Cover title missing/ Le titre de couverture manque
- [ ] Coloured maps/ Cartes géographiques en couleur
- [ ] Coloured ink (i.e. other than blue or black)/ Encre de couleur (i.e. autre que bleue ou noire)
- [ ] Coloured plates and/or illustrations/ Planches et/ou illustrations en couleur
- [ ] Bound with other material/ Relié avec d'autres documents
- [ ] Tight binding may cause shadows or distortion along interior margin/ La reliure serrée peut causer de l'ombre ou de la distortion le long de la marge intérieure
- [ ] Blank leaves added during restoration may appear within the text. Whenever possible, these have been omitted from filming/ Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.
- [ ] Additional comments:/ Commentaires supplémentaires:

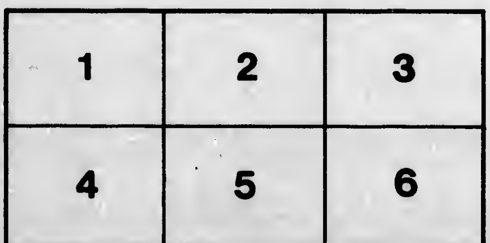
- [ ] Coloured pages/ Pages de couleur
- [x] Pages damaged/ Pages endommagées
- [ ] Pages restored and/or laminated/ Pages restaurées et/ou pelliculées
- [x] Pages discoloured, stained or foxed/ Pages décolorées, tachetées ou piquées
- [ ] Pages detached/ Pages détachées
- [x] Showthrough/ Transparence
- [ ] Quality of print varies/ Qualité inégale de l'impression
- [ ] Includes supplementary material/ Comprend du matériel supplémentaire
- [ ] Only edition available/ Seule édition disponible
- [ ] Pages wholly or partially obscured by errata slips, tissues, etc., have been refilmed to ensure the best possible image/ Les pages totalement ou partiellement obscurcies par un feuillet d'errata, une pelure, etc., ont été filmées à nouveau de façon à obtenir la meilleure image possible.

This item is filmed at the reduction ratio checked below/
Ce document est filmé au taux de réduction indiqué ci-dessous.

| 10X | | 12X | | 14X | | 16X | | 18X | | 20X | | 22X | | 24X | | 26X | | 28X | | 30X | | 32X |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | ✓ | | | | | |


1
2
3


1
2
3


1
2
3
4
5
6

# I.—*A Table of the Cubical Expansions of Solids.*

By Prof. J. G. MACGREGOR, D.Sc.

(Read May 23, 1888.)

It has been found by experiment that in general the volume of any body at temperature $t°$ may be expressed in terms of its volume at any other temperature $\tau°$, and of the difference of temperatures, by means of the expression,

$$V_t = V_\tau\,(1 + A\,(t-\tau) + B\,(t-\tau)^2), \tag{1}$$

where $V_t$ and $V_\tau$ are the volumes at $t°$ and $\tau°$ respectively and $A$ and $B$ are constants for the substance of which the body consists. In some cases an additional term $C\,(t-\tau)^3$ is necessary.. But in general the value of $C$ is so small that it may be neglected.

As the constants $A$, $B$, $C$ have different values for different values of $\tau$, it is convenient to choose some one temperature as a temperature of reference, and for this purpose the temperature $o°C$ is now universally chosen, the above expression becoming therefore,

$$V_t = V_o\,(1 + at + bt^2). \tag{2}$$

The constants $a$ and $b$ (and $c$ also, the coefficient of $t^3$, in cases in which a term in $t^3$ is found necessary) having been determined for any substance, the volume of any body of that substance, whose volume at $o°C$ is known, may be determined at any other temperature within the temperature limits of the experiments by which the values of $a$, $b$ and $c$ were found.

Density may be substituted for volume in the above formula, provided the signs of $a$ and $b$ be changed. For if $\rho_t$ and $\rho_o$ are the densities of a substance at $t°$ and $o°$ respectively, we have. in general,

$$\rho_t / \rho_o = V_o / V_t = 1 / (1 + at + bt^2) = 1 - at - bt^2,$$

since $a$ and $b$ are, in general, small quantities.

In the case of isotropic solids, length may be substituted for volume in the above formula, provided the constants $a$ and $b$ be divided by 3, change of length in any direction in such cases being numerically equal to one third of the corresponding change of volume. The formula in the case of the linear expansion of such solids becomes therefore :—

$$L_t = L_o\,(1 + \frac{a}{3}t + \frac{b}{3}t^2). \tag{3}$$

In the case of æolotropic solids, the linear expansion is different for different directions, and must therefore be specially determined.

To determine the volume at $t'^\circ$ when that at $t^\circ$ is given, we have:—

$$V_t = V_o(1 + at + bt^2),$$

and

$$V_{t'} = V_o(1 + at' + bt'^2).$$

Hence

$$V_{t'} = V_t \cdot \frac{1 + at' + bt'^2}{1 + at + bt^2},$$

$$= V_t(1 + a(t' - t) + b(t'^2 - t^2)), \qquad (4)$$

since $a$ and $b$ are small quantities.

The mean coefficient of thermal expansion between two temperatures is by some writers defined as the change of volume per degree and per unit volume at the lower temperature, and by others as the change of volume per degree and per unit volume at the temperature of reference ($o°C$). In terms of the symbols used above, it is in the former case the value of $\frac{V_{t'} - V_t}{V_t(t' - t)}$, and in the latter case the value of $\frac{V_{t'} - V_t}{V_o(t' - t)}$. From (4) it follows at once that:—

$$\frac{V_{t'} - V_t}{V_t(t' - t)} = a + b(t' + t) \qquad (5)$$

provided $a$ and $b$ are so small that their powers and product may be neglected. From (2) and a similar equation with $t'$ substituted for $t$, it follows that:—

$$V_{t'} - V_t = V_o(a(t' - t) + b(t'^2 - t^2))$$

exactly, and therefore that

$$\frac{V_{t'} - V}{V_o(t' - t)} = a + b(t' + t), \qquad (6)$$

whatever the magnitudes of $a$ and $b$ may be. Provided $a$ and $b$ are sufficiently small, therefore, the mean coefficient has the same value between given temperature limits according to both modes of definition; and if $a$ and $b$ are known, this value may be determined for any temperature range.

The "true" coefficient of thermal expansion at any temperature is the rate at which volume varies with temperature at that temperature per unit volume at zero. It is thus $\frac{1}{V_o} \cdot \frac{dV_t}{dt}$, and by differentiation of (2) is seen to have the value $a + 2bt$. The true coefficient at a given temperature is clearly the mean coefficient (per unit volume at $o°C$) between two temperatures indefinitely near one another and including the given temperature; and the above value is also obtained from (6) by noting that ultimately $t' + t = 2t$.

Sometimes, but rarely, the true coefficient at any temperature is defined as the rate at which volume varies with temperature at that temperature per unit volume at that temperature—in symbols, $\frac{1}{V_t} \cdot \frac{dV_t}{dt}$; and by (5), its approximate value, provided $a$ and $b$ be small, is $a + 2bt$, the same as the value of the true coefficient according to the former definition.

The following table contains the values of $a$ and $b$ (and of $c$ also in cases in which $c$ is found to have an appreciable value) in the case of the more important and interesting solids. A table containing an exhaustive list of the determinations of these constants would be so long that in most cases I have thought it well to give only the most recent and most accurate determinations, though in some important cases, as in that of glass, a large number of such determinations have been given. Many observers of thermal expansion have determined, not the values of the constants $a$ and $b$, but the values of the mean coefficient throughout given ranges of temperature. In many cases, especially in cases in which the values of $a$ and $b$ are not known, these results have been given. They are contained in the fourth column, and in the fifth column are stated the temperature ranges to which they apply. In general, also, these temperatures are the ranges throughout which observations were made. A more complete list of the older determinations of mean coefficients of expansion will be found in Part III of the "Constants of Nature," by Prof. F. W. Clarke, published in Vol. XIV of the Smithsonian Miscellaneous Collections, but Prof. Clarke's tables do not give the values of the constants $a$ and $b$.

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Coefficient $\times 10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Aluminium (cast) ....... | 6604 | 3435 | ........ | 7008 | 0—100 | Fizeau |
| Do. (commercial). | ........ | ........ | ........ | 6660 | 0—100 | Calvert, Johnson & Lowe |
| Do. ............. | ........ | ........ | ........ | 7439·4 | 0—100 | L. Pfaff |
| Do. ............. | ........ | ........ | ........ | 7062 | 0—100 | Glatzel |
| Alums (in powder)— Ammonium-Aluminium Alum ...... | ........ | ........ | ........ | 2840 | 0—20 | |
| | ........ | ....... | ........ | 2585 | 0—60 | Spring |
| | ........ | ........ | ........ | 5343 | 0—100 | " |
| Potassium-Aluminium Alum ...... | ........ | ........ | ........ | 2635 | 0—20 | " |
| | ........ | ........ | ........ | 4198 | 0—60 | " |
| | ........ | ........ | ........ | 31118 | 0—90 | " |
| Rubidium-Aluminium Alum ...... | ........ | ........ | ........ | 2070 | 0—20 | " |
| | ........ | ........ | ........ | 2635 | 0—60 | " |
| | ........ | ........ | ........ | 5045 | 0—100 | " |
| Cæsium-Aluminium Alum.... ...... | ........ | ........ | ........ | 2610 | 0—20 | " |
| | ........ | ........ | ........ | 2621 | 0—60 | " |
| | ........ | ........ | ........ | 7699 | 0—100 | " |
| Potassium-Chromium Alum ...... | ........ | ........ | ........ | 2330 | 0—20 | " |
| | ........ | ........ | ........ | 4968 | 0—60 | " |
| | ........ | ........ | ........ | 59503 | 0—80 | " |
| Ammonium Sulphate.. | ........ | ........ | ........ | 8335 | 0—20 | " |
| | ........ | ........ | ........ | 9051 | 0—60 | " |
| | ........ | ........ | ........ | 11191 | 0—100 | " |
| Antimony .............. | 2770 | 3970 | ........ | 3167 | 0—100 | Matthiessen |
| Do. (pure)........ | ........ | ........ | ........ | 2940 | 0—100 | Calvert, Johnson & Lowe |
| Do. (crystal)...... | 3387 | 870 | ........ | 3379 | 0—100 | Fizeau |

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Coefficient × $10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Aragonite | ........ | ........ | ........ | 6195 | 39·5—40·5 | Fizeau |
| Arsenic (sublimed) | 1158·6 | 6480 | ........ | 1806 | 0—100 | " |
| Augite | 2305 | 1800 | ........ | 2485 | 0—100 | " |
| Beryl | ........ | ........ | ........ | 105 | 0—100 | F. Pfaff |
| Bismuth | 3502 | 4460 | ........ | 3948 | 0—100 | Matthiessen |
| Do. | ........ | ........ | ........ | 3990 | 0—100 | Calvert, Johnson & Lowe |
| Do. (crystal) | 3706 | 4155 | ........ | 4121 | 0—100 | Fizeau |
| Brass (71 % by mass, 66·15 % by volume, of Cu.) | 5161 | 5580 | ........ | 5710 | 0—100 | Matthiessen |
| Do. (71·5 % of Cu, 27·7 of Zn, 0·3 of Sn, 0·5 of Pb) | 5341 | 2940 | ........ | 5637 | 0—100 | Fizeau |
| Bromide of Silver (HgBr) | ........ | ........ | ........ | 10100 | —60— 0 | Rodwell |
| | ........ | ........ | ........ | 10429 | 0—100 | " |
| | ........ | ........ | ........ | 11033 | 0—400 | " |
| Bronze (86·3 % of Cu, 9·7 of Sn, 4 of Zn.) | 5101 | 3060 | ........ | 5406 | 0—100 | Fizeau |
| Do. (25 % of Sn.) | ........ | ........ | ........ | 5532 | 16·6—100 | Daniell |
| Cadmium | 8078 | 14000 | ........ | 9478 | 0—100 | Matthiessen |
| Do. | ........ | ........ | ........ | 9363 | 0—100 | Glatzel |
| Do. (compressed powder) | 8816 | 4890 | ........ | 9306 | 0—100 | Fizeau |
| Calcspar | ........ | ........ | ........ | 1541 | 39·5—40·5 | " |
| Do. | ........ | ........ | ........ | 2010 | 0—100 | F. Pfaff |
| Caoutchouc (commercial grey) | ........ | ........ | ........ | 67000 | 0— 30 | Russnor |
| Do. (vulcanized, unstretched) | ........ | ........ | ........ | 95784 | 14— 37 | Lebedeff |
| Do. do. do. | ........ | ........ | ........ | 67646 | 11—295 | " |
| Do. (do., stretched) | ........ | ........ | ........ | 56158 | 14·5—37·5 | " |
| Do. do. do. | ........ | ........ | ........ | 66906 | 12·5— 30 | " |
| Carbon (graphite) | ........ | ........ | ........ | 877 | 16·6—100 | Daniell |
| Do. do. | ........ | ........ | ........ | 632 | 16·6—350 | " |
| Do. do. | 2237 | 1515 | ........ | 2388 | 0—100 | Fizeau |
| Do. (gas-coke) | 1488 | 1650 | ........ | 1653 | 0—100 | " |
| Do. (diamond) | 181·2 | 2160 | ........ | 396 | 0—100 | " |
| Cassiterite | 925·6 | 1355 | ........ | 1061 | 0—100 | " |

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Coefficient $\times 10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Chloride of Silver (AgCl) | ........ | ........ | ........ | 9245 | —60— 0 | Rodwell |
| | ........ | ........ | ........ | 9492 | 0—100 | " |
| | ........ | ........ | ........ | 9865 | 0—400 | " |
| Cobalt (compressed powder) | 3612 | 1200 | ........ | 3732 | 0—100 | Fizeau |
| Copper | ........ | ........ | ........ | 5150 | 0—100 | Dulong & Petit |
| Do. | ........ | ........ | ........ | 5650 | 0—300 | " |
| Do. | 4443 | 5550 | ........ | 4098 | 0—100 | Matthiessen |
| Do. | 4788 | 3075 | ........ | 5094 | 0—100 | Fizeau |
| Do. (native, Lake Superior) | 4850 | 2740 | ........ | 5124 | 0—100 | " |
| Do. | ........ | ........ | ........ | 5265 | 0—100 | L. Pfaff |
| Do. | ........ | ........ | ........ | 5233 | 0—100 | Rodwell |
| Do. | ........ | ........ | ........ | 5115 | 0—100 | Glatzel |
| Copper-Gold Alloy (66·6 % by mass, 48·06 % by volume, of Gold) | 4015 | 6420 | ........ | 4657 | 0—100 | Matthiessen |
| Copper Oxide (CuO.) | 27 | 3150 | ........ | 342 | 0—100 | Fizeau |
| Corundum | 1443 | 3275 | ........ | 1771 | 0—100 | " |
| Darcet's fusible metal ($Bi_{13} Sn_{10} Pb_8$) | ........ | ........ | ........ | 52143 | 0— 35 | Spring |
| | ........ | ........ | ........ | —36926 | 35— 50 | " |
| | ........ | ........ | ........ | 102534 | 50— 90 | " |
| Diopside | ........ | ........ | ........ | 2330 | 0—100 | F. Pfaff. |
| Ebonite (See Vulcanite) | ........ | ........ | ........ | ........ | ........ | |
| *Emerald | 16 | 1900 | ........ | 206 | 0—100 | Fizeau |
| Epidote | 2627 | 3830 | ........ | 2410 | 0—100 | " |
| Felspar (Adular) | 1399·5 | 1900 | ........ | 1590 | 0—100 | " |
| Fluorspar | ........ | ........ | ........ | 5851 | 0—100 | F. Pfaff |
| Galena | ........ | ........ | ........ | 5578 | 0—100 | " |
| Garnet | ........ | ........ | ........ | 2543 | 0—100 | " |
| German Silver | ........ | ........ | ........ | 5509 | 0—100 | L. Pfaff |
| Glass (crystal of Choisy-le-Roy) | 2231 | 50 | ........ | ........ | 100—280 | Bosscha † |
| Do. do. | 2523 | 70 | ........ | ........ | " | " |
| Do. do. | 2439 | 300 | ........ | ........ | " | " |
| Do. (ordinary) | 2578 | 1330 | 0·37 | ........ | " | " |

* See also Beryl.

† Bosscha's constants were obtained by a recalculation of Regnault's experimental results.

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Coefficient $\times 10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Glass (crystal of Choisy-le-Roy) (ordinary)..... | 2507 | 1920 | 0·51 | ........ | 100—280 | Bosscha * |
| Do. do. .... | 2281 | 1540 | 0·46 | ........ | " | " |
| Do. (green)...... | 2143 | 890 | 0·37 | ........ | " | " |
| Do. (Swedish) ... | 2218 | 3570 | 1·15 | ........ | " | " |
| Do. (crystal, Choisy-le-Roy) | ........ | ........ | ........ | 2270 | 0— 10 | Regnault |
| | ........ | ........ | ........ | 2270 | 0— 50 | " |
| | ........ | ........ | ........ | 2280 | 0—100 | " |
| | ........ | ........ | ........ | 2300 | 0—150 | " |
| | ........ | ........ | ........ | 2310 | 0—200 | " |
| | ........ | ........ | ........ | 2320 | 0—250 | " |
| | ........ | ........ | ........ | 2330 | 0—300 | " |
| | ........ | ........ | ........ | 2340 | 0—350 | " |
| Do. (ordinary). | ........ | ........ | ........ | 2628 | 0— 10 | " |
| | ........ | ........ | ........ | 2687 | 0— 50 | " |
| | ........ | ........ | ........ | 2761 | 0—100 | " |
| | ........ | ........ | ........ | 2835 | 0—150 | " |
| | ........ | ........ | ........ | 2908 | 0—200 | " |
| | ........ | ........ | ........ | 2982 | 0—250 | " |
| | ........ | ........ | ........ | 3056 | 0—300 | " |
| | ........ | ........ | ........ | 3131 | 0—350 | " |
| Do. (rod) .......... | 2410 | 1700 | ........ | 2580 | 0—100 | Dulong & Petit |
| | | | | 2750 | 0—200 | " |
| | ....... | ....... | ........ | 3040 | 0—300 | " |
| Do. (tube) ............ | ........ | ........ | ........ | 2435 | 0—100 | Lavoisier & LaPlace |
| Do. (plate)............ | ........ | ........ | ........ | 2073 | 0—100 | " |
| Do. (crown plate)...... | ........ | ........ | ........ | 2627 | 0—100 | " |
| Do. do. ...... | ........ | ........ | ........ | 2603 | 0—100 | " |
| Do. do. ...... | ........ | ........ | ........ | 2753 | 0—100 | " |
| Do. (thin tube)........ | 588 | 31500 | ........ | ........ | 0— 30 | Hällström |
| Do. (white, French). .. | ........ | ........ | ........ | 2553 | 0—100 | Kopp |
| Do. (white, tube)...... | ........ | ........ | ........ | 2648 | 0—100 | Regnault |
| Do. ( do. globule).... | ........ | ........ | ........ | 2592 | 0—100 | " |
| Do. ( do. do. ).... | ........ | ........ | ........ | 2514 | 0—100 | " |

* Bosscha's constants were obtained by a recalculation of Regnault's experimental results.

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Co-efficient × $10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Glass (green, tube)....... | ........ | ........ | ........ | 2209 | 0—100 | Regnault. |
| Do. ( do. globule).... | ........ | ........ | ........ | 2132 | 0—100 | " |
| Do. (Swedish, tube).... | ........ | ........ | ........ | 2363 | 0—100 | " |
| Do. ( do. globule)... | ........ | ........ | ........ | 2441 | 0—100 | " |
| Do. (hard, French, tube) | ........ | ........ | ........ | 2142 | 0—100 | " |
| Do. ( do. globule) | ........ | ........ | ........ | 2242 | 0—100 | " |
| Do. (ordinary), from ... | ........ | ........ | ........ | 2431 | 0—100 | " |
| to... | ........ | ........ | ........ | 2758 | 0—100 | " |
| Do. (ord. crystal, tube). | ........ | ........ | ........ | 2101 | 0—100 | " |
| Do. ( do. globule) | ........ | ........ | ........ | 2330 | 0—100 | " |
| Do. (Choisy-le-Roy crystal) from........ | ........ | ........ | ........ | 2144 | 0—100 | " |
| to.. | ........ | ........ | ........ | 2442 | 0—100 | " |
| Do. (St. Gobain)....... | 2141 | 2370 | ........ | 2378·4 | 0—100 | Fizeau |
| Do. (3 pts. of sand, 2 of Pb, and 1 of alkali). . | 2187 | ........ | ........ | ........ | 0—100 | Matthiessen |
| Do. (Thuringian soft, tube) ...... ........ | ........ | ........ | ........ | 3585 | 0—100 | Weinhold |
| Do. ( do. vessel)..... | ........ | ........ | ........ | 3590 | 0—100 | " |
| Do. (before heating)... | ........ | ........ | ........ | 2784 | 0—100 | Crafts |
| Do. (after having been heated for 100 hours in boiling sulphur).. | ........ | ........ | ........ | 2741 | 0—100 | " |
| Gold.................... | 4075 | 3361 | ........ | 4411 | 0—100 | Matthiessen |
| Do. (cast)............. | 4229 | 1245 | ........ | 4353 | 0—100 | Fizeau |
| Do. (pure)............ | ........ | ........ | ........ | 4140 | 0—100 | Calvert, Johnson & Lowe |
| Do. .................. | ........ | ........ | ........ | 4401 | 0—100 | L. Pfaff |
| Gutta-percha (purified).. | 49606 | 496000 | ........ | 69500 | 0— 40 | Russner |
| Gypsum ............... | 6700 | 6940 | ........ | 7394 | 0—100 | Fizeau |
| Do. ............... | ........ | ........ | ........ | 7500 | 0—100 | F. Pfaff |
| Hornblende............. | 2355 | 3025 | ........ | 2658 | 0—100 | Fizeau |
| Ice..... .............. | ........ | ........ | ........ | 11250 | −20— 0 | Brunner |
| Do ................... | ........ | ........ | ........ | 15850 | −20— 0 | Plücker & Geissler |
| Do ................... | ........ | ........ | ........ | 3500 | −12— 0 | Marchand |
| Indium (cast)........... | 7424 | 63570 | ........ | 13782 | 0—100 | Fizeau |
| Iodine.................. | ........ | ........ | ........ | { 23500 (?) 31231 | 40·3—107 | Billet |

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Coefficient $\times 10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Iodides— | | | | | | |
| AgI (crystal)......... | −206·9 | −754·5 | ........ | −282 | 0—100 | Fizeau |
| Do. (compressed precipitate)......... | −218 | −2400 | ........ | −458 * | 0—100 | " |
| Do. (cast)........... | −249 | −2100 | ........ | −249 | 0—100 | " |
| Do............... | ........ | ........ | ........ | −1749 | 70—142 | Rodwell |
| | ........ | ........ | ........ | −450000 | 142—145·5 | " |
| | ........ | ........ | ........ | 2844 | 145·5—300 | " |
| Do............... | ........ | ........ | ........ | −8610 | −60—142 | " |
| | ........ | ........ | ........ | 2843 | 142—400 | " |
| $PbI_2$ (cast)........... | 9378·6 | 8760 | ........ | 10255 | 0—100 | Fizeau |
| Do................. | ........ | ........ | ........ | 7614 | 0—205 | Rodwell |
| Do............... | ........ | ........ | ........ | 8317 | 0—205 | " |
| | ........ | ........ | ........ | 63780 | 205—253 | " |
| | ........ | ........ | ........ | 18000 | 253—M.P. | " |
| $HgI_2$ (cast)........... | ........ | ........ | ........ | 9523 | 0—200 | Rodwell & Elder |
| Iridium (cast)........... | 2005 | 1185 | ........ | 2124 | 0—100 | Fizeau |
| Iron (forged)......... | ........ | ........ | ........ | 3661 | 0—100 | Lavoisier & LaPlace |
| Do. (wire drawn)...... | ........ | ........ | ........ | 3705 | 0—100 | " |
| Do. (soft)............ | 3408 | 2775 | ........ | 3684 | 0—100 | Fizeau |
| Do. (cast, grey)....... | 3019 | 2055 | ........ | 3235 | 0—100 | " |
| Do. (cast)............ | ........ | ........ | ........ | 3360 | 0—100 | Calvert, Johnson & Lowe |
| Do. (wrought).......... | ........ | ........ | ........ | 3570 | 0—100 | " |
| Do. ................. | ........ | ........ | ........ | 3742 | 0—100 | L. Pfaff |
| Do. ................. | ........ | ........ | ........ | 3307 | 0—100 | Rodwell |
| Do. ................. | ...... | ........ | ........ | 4161 | 0—100 | Glatzel |
| Iron-glance (oxide, rhombohedric)............ | 2241 | 3215 | ........ | 2565 | 0—100 | Fizeau |
| Iron-stone (magnetite)... | ........ | ........ | ........ | 2862 | 0—100 | F. Pfaff |
| Iron Pyrites............ | ........ | ........ | ........ | 3025 | 0—100 | " |
| Do. ............ | ........ | ........ | ........ | 2721 | 39·5—40·5 | Fizeau |
| Lipowitz's Fusible Metal ($Bi_{11}Pb_6Cd_4Sn_5$) ... | ........ | ........ | ........ | 15400 | 0— 25 | Spring |
| | ........ | ........ | ........ | −39514 | 25— 40 | " |
| | ........ | ........ | ........ | 38381 | 40— 60 | " |
| Lead .................. | 8177 | 2220 | ........ | 8399 | 0—100 | Matthiessen |

* The negative coefficient shewn qualitatively to extend to—10°*C.*

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Co-efficient × $10^9$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Lead (pure)........... | ........ | ........ | ........ | 9030 | 0—100 | Calvert, Johnson & Lowe |
| Do. (cast)............. | 8485 | 3585 | ........ | 8844 | 0—100 | Fizeau |
| Do. ................. | ........ | ........ | ........ | 8725 | 0—100 | L. Pfaff |
| Do. ................. | ........ | ........ | ........ | 9063 | 0—100 | Rodwell |
| Do. ................. | ........ | ........ | ........ | 8808 | 0—100 | Glatzel |
| Lead - Antimony alloy ($Sn_2Pb$, 22·28 % by vol. of Pb).............. | 6200 | 9880 | ........ | 7188 | 0—100 | Matthiessen |
| Do. ($Sn\,Pb_6$, 82·09 % by vol. of Pb)........... | 8087 | 3320 | ........ | 8419 | 0—100 | " |
| Magnesium (cast)....... | 7261 | 10260 | ........ | 8286 | 0—100 | Fizeau |
| Do. ............. | ........ | ........ | ........ | 7042 | 0—100 | L. Pfaff |
| Nickel (comp. powder)... | 3752 | 1005 | ........ | 3858 | 0—100 | Fizeau |
| Osmium (semi-fused).... | 1709 | 3270 | ........ | 2037 | 0—100 | " |
| Palladium.............. | 3032 | 2800 | ........ | 3312 | 0—100 | Matthiessen |
| Do. (forged, annealed) | 3370 | 1980 | ........ | 3567 | 0—100 | Fizeau |
| Paraffin.............. | 58400 | 99200 | ........ | ........ | 0— 33 | Russner |
| | ........ | ........ | ........ | 260000 | 33·5—37·7 | " |
| | ........ | ........ | ........ | 666000 | 37·7— 41 | " |
| | 115000 * | 690000 * | ........ | ........ | 41— 52 | " |
| Do. (high boiling point)..... | ........ | ........ | ........ | 31985 | 0—15·55 | Rodwell |
| | ........ | ........ | ........ | 39090 | 15·55—37·6 | " |
| | ........ | ........ | ........ | 143118 | 37·6—48·85 | " |
| | ........ | ........ | ........ | 244358 | 48·85—61·11 | " |
| Do. (Rangoon, melting at 56°C) | 71651 | 148890 | ........ | 70989 | 0— 56 | Fizeau |
| Pewter.................. | ........ | ........ | ........ | 6099 | 16·6—100 | Daniell |
| Do. ................. | ........ | ........ | ........ | 5982 | 16·6—206 | " |
| Phosphorus............. | 37988 | ........ | ........ | ........ | 0—37·5 | Erman |
| Do. ............ | 38300 | ........ | ........ | ........ | 0— 44 | Kopp |
| Do. ............ | 20000 | 11500 | ........ | 20506 | 0— 44 | Pisati & de Franchis |
| Platinum ............... | ........ | ........ | ........ | 2644 | 16·6—100 | Daniell |
| Do. .............. | ........ | ........ | ........ | 2650 | 0—100 | Dulong and Petit |
| Do. .............. | ........ | ........ | ........ | 2750 | 0—300 | " |
| Do. .............. | 2554 | 1040 | ........ | 2658 | 0—100 | Matthiessen |
| Do. (cast).......... | 2603 | 1170 | ........ | 2721 | 0—100 | Fizeau |

* These constants are for the formula :— $V_t = V_0 (1 + at + bt^2)$.

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Co-efficient × $10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Platinum............... | ........ | ........ | ........ | 2583 | 0—100 | L. Pfaff |
| Do. (commercial)... | ........ | ........ | ........ | 2040 | 0—100 | Calvert, Johnson & Lowe |
| Platinum-iridium alloy (10% of Ir.) (cast) ..... | 2561 | 1140 | ........ | 2675 | 0—100 | Fizeau |
| Porcelain (Bayeux)..... | ........ | ........ | ........ | 4950 | 1000—1400 | Deville & Troost |
| Do. do. ...... | ........ | ........ | ........ | 6000 | about and above 1500 | Weinhold |
| Do. .............. | ........ | ........ | ........ | 806 | 0— 99 | " |
| Potassium ............ { | 24070 | 17980 | ........ | } 24990 | 0— 50 | Hagen |
| | 23800 | 23870 | ........ | | | |
| Potassium Chromate... { | ........ | ........ | ........ | 10570 | 0— 20 | Spring |
| | ........ | ........ | ........ | 10731 | 0— 60 | " |
| | ........ | ........ | ........ | 11344 | 0—100 | " |
| Potassium Sulphate.... { | ........ | ........ | ........ | 9520 | 0— 20 | " |
| | ........ | ........ | ........ | 10037 | 0— 60 | " |
| | ........ | ........ | ........ | 12645 | 0—100 | " |
| Quartz ....... ......... | 3358 | 3265 | ........ | 3685 | 0—100 | Fizeau |
| Do. .............. | | | ........ | 3840 | 0—100 | F. Pfaff |
| Rhodium (semi-fused).... | 2453 | 1215 | ........ | 2574 | 0—100 | Fizeau |
| Rock Salt.............. | 11578 | 6735 | ........ | 12252 | 0—100 | " |
| Rose's Fusible Metal ‡ .. | 6784·7 | —181580 | 553·07† | ........ | 0— 90 | Kopp |
| Do. ($Bi_7\ Sn_6\ Pb_4$). { | ........ | ........ | ........ | 33575 | 0— 40 | Spring |
| | ........ | ........ | ........ | —129922 | 40— 55 | " |
| | ........ | ........ | ........ | 54545 | 55— 90 | " |
| Rubidium Sulphate.... { | ........ | ........ | ........ | 9760 | 0— 20 | " |
| | ........ | ........ | ........ | 10020 | 0— 60 | " |
| | ........ | ........ | ........ | 11148 | 0—100 | " |
| Ruthenium (semi-fused). | 2552 | 4215 | ........ | 2973 | 0—100 | Fizeau |
| Rutile ................. | 2196 | 2225 | ........ | 2419 | 0—100 | " |
| Selenium (cast). ........ | 9702 | 16725 | ........ | 11376 | 0—100 | " |
| Do. (crystallized) { | ........ | ........ | ........ | 14780 | 0— 20 | Spring |
| | ........ | ........ | ........ | 17430 | 0— 60 | " |
| | ........ | ........ | ........ | 19810 | 0—100 | " |

† Kopp uses the formula:—$V_t = V_o (1 + at + bt^2 + ct^3 + dt^4)$; $a$, $b$, and $c$ are given in the table; $d = 10^{-13} \times 5256$.

‡ Erman gives the formula:—$V_t = V_o(1 + 10^{-8}.\,21864t - 10^{-8}.\,93485 \sqrt{(t - 34°.9)(78°.5 - t)}$, the temperatures being expressed in Reaumur degrees and the imaginary values of the irrational factor being put equal to 0.

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Co-efficient × $10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Selenium (com. powder) | ........ | ........ | ........ | 13070 | 0— 20 | Spring |
| | ........ | ........ | ........ | 16440 | 0— 60 | " |
| | ........ | ........ | ........ | 17510 | 0—100 | " |
| Silicon (cast) ........... | 2086 | 2535 | ........ | 2340 | 0—100 | Fizeau |
| Silver ................. | 5426 | 4050 | ........ | 5831 | 0—100 | Matthiessen |
| Do. (cast) ........... | 5587 | 2205 | ........ | 5808 | 0—100 | Fizeau |
| Do. ................. | ........ | ........ | ........ | 5516 | 5— 26 | Plantamour & Hirsch |
| Do. ................. | ........ | ........ | ........ | 5573 | 0—100 | L. Pfaff |
| Do. ................. | ........ | ........ | ........ | 5792 | 0—100 | Rodwell |
| Silver-platinum alloy (33·4 % by mass, and 19·65 % by vol., of Pt.) ....... | 4246 | 3220 | ........ | 4568 | 0—100 | Matthiessen |
| Sodium ................ | 14178 | 52000 | 26 | 16674 | 0— 40 | de Lucchi. |
| Do. ................. | ........ | ........ | ........ | 23670 | 0— 90 | " |
| Do. ................. | 20510 | 22730 | ........ | 21630 | 0— 50 | Hagen |
| | 20280 | 25730 | ........ | | | |
| Solder (2 parts Pb, 1 Sn.) | ........ | ........ | ........ | 7524 | 0—100 | Smeaton |
| Speculum Metal ......... | ........ | ........ | ........ | 5799 | 0—100 | " |
| Steel (not tempered) ..... | ........ | ........ | ........ | 3238 | 0—100 | Lavoisier & LaPlace |
| Do. (tempered yellow) . | ........ | ........ | ........ | 4158 | 0—100 | " |
| Do. (French cast, tempered) ........... | ........ | ........ | ........ | 4086 | 0—100 | Fizeau |
| Do. (French cast, annealed) .......... | ........ | ........ | ........ | 3339 | 0—100 | " |
| Do. (English cast, annealed) .......... | 3103 | 2280 | ........ | 3331 | 0—100 | " |
| Do. (wire drawn) ...... | ........ | ........ | ........ | 3768 | 0—100 | Glatzel |
| Do. (soft) ............ | ........ | ........ | ........ | 3090 | 0—100 | Calvert, Johnson & Lowe |
| Do. (bar at max. softness) ........... | ........ | ........ | ........ | 3270 | 0—100 | " |
| Do. (same bar at max. hardness) .......... | ........ | ........ | ........ | 4260 | 0—100 | " |
| Sulphur ............... | 10458 | 265880 | —146·73 | ........ | 0— 78 | Kopp |
| Do. ................. | —83804 | 7878840 § | ........ | ........ | 78—115 | " |
| Do. (Sicilian crystal) . | 15222 | 50440 | ........ | 20244 | 0—100 | Fizeau |
| Do. (crystallized from solution in Carbon Bi-sulphide) .... | ........ | ........ | ........ | 21220 | 0— 20 | Spring |
| | ........ | ........ | ........ | 24380 | 0— 60 | " |
| | ........ | ........ | ........ | 35410 | 0—100 | " |

§ These constants are for the formula:— $V_t = V_{78}\left(1 + a(t-78) + b(t-78)^2\right)$

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Co-efficient $\times 10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Sulphur (Sicilian) ..... | ........ | ........ | ........ | 24300 | 0— 20 | Spring |
| Do. .............. | ........ | ........ | ........ | 25000 | 0— 60 | " |
| Do. .............. | ........ | ........ | ........ | 26000 | 0—100 | " |
| Do. (crystallized from solution in Carbon Bi-sulphide) | 12800 | 186000 | 153 | 29468 | 0— 60 | Russner |
| Do. (crystal) ......... | ........ | ........ | ........ | 17596 | 20·5—21·5 | Schrauf |
| Do. (natural crystal) | ........ | ........ | ........ | 19991 | 0— 60 | Scichilone |
| | ........ | ........ | ........ | —4410 | 62— 70 | " |
| | ........ | ........ | ........ | 28483 | 70—110 | " |
| Do. (after heating to 140°C) ..... | ........ | ........ | ........ | 20958 | 0— 82 | " |
| | ........ | ........ | ........ | —13731 | 82— 85 | " |
| | ........ | ........ | ........ | 201700 | 85—100 | " |
| Do. (after heating to 240°C) ...... | ........ | ........ | ........ | 18246 | 0— 90 | " |
| | ........ | ........ | ........ | —22[illegible]85 | 90— 92 | " |
| | ........ | ........ | ........ | 195180 | 92—100 | " |
| Tellurium (crystallized) | ........ | ........ | ........ | 10320 | 0— 20 | Spring |
| | ........ | ........ | ........ | 11210 | 0— 60 | " |
| | ........ | ........ | ........ | 11060 | 0—100 | " |
| Do. (comp. powder) | ........ | ........ | ........ | 10410 | 0— 20 | " |
| | ........ | ........ | ........ | 10110 | 0— 60 | " |
| | ........ | ........ | ........ | 10630 | 0—100 | " |
| Do. (cast) ........... | 4335 | 8625 | ........ | 5196 | 0—100 | Fizeau |
| Thallium (cast) ......... | 7694 | 17115 | ........ | 9405 | 0—100 | " |
| Tin .................... | 6100 | 7890 | ........ | 6889 | 0—100 | Matthiessen |
| Do. (Malacca, compres'd powder) ........... | 6281 | 5265 | ........ | 6807 | 0—100 | Fizeau |
| Do. (pure) ............ | ........ | ........ | ........ | 6567 | 0—100 | L. Pfaff |
| Topaz .................. | ........ | ........ | ........ | 2137 | 0—100 | F. Pfaff |
| Tourmaline ............. | ........ | ........ | ........ | 2181 | 0—100 | " |
| Type Metal ............. | ........ | ........ | ........ | 6099 | 16·6—100 | Daniell |
| Do. ............. | ........ | ........ | ........ | 5856 | 16·6—264 | " |
| Vulcanite ............. | 18300 | 114000 | ........ | 24239 | 16·7—35·4 | Kohlrausch |
| Do. ............... | ........ | ........ | ........ | 24600 | 18— 19 | Fuess |
| Do. (22·2 per cent. of Sulphur) ..... | ........ | ........ | ........ | 40000 | 20— 60 | Russner |

| SUBSTANCE. | $a \times 10^8$ | $b \times 10^{11}$ | $c \times 10^{10}$ | Mean Coefficient × $10^8$ | Temperature Range. (°C.) | OBSERVER. |
|---|---|---|---|---|---|---|
| Vulcanite (27 per cent. of Sulphur) ..... | ........ | ........ | ........ | 35000 | 20— 60 | Russner |
| Do. (30·7 per cent. of Sulphur) ..... | ........ | ........ | ........ | 23000 | 20— 60 | " |
| Do. (31 per cent. of Sulphur) ..... | ........ | ........ | ........ | 41000 | 20— 60 | " |
| Do. (36·4 per cent. of Sulphur) ..... | ........ | ........ | ........ | 38000 | 20— 60 | " |
| Wood's Fusible Metal ($Bi_4$ Pb $Cd_2$ $Sn_2$)... | ........ | ........ | ........ | —16600 | 0— 25 | Spring |
| | ........ | ........ | ........ | 27827 | 25— 70 | " |
| Zinc .................. | ........ | ........ | ........ | 8919 | 16·6—100 | Daniell |
| Do. .................. | 8222 | 7000 | ........ | 8928 | 0—100 | Matthiessen |
| Do. (pure, rolled)...... | ........ | ........ | ........ | 6600 | 0—100 | Calvert and Lowe |
| Do. (compres'd powder) | 8906 | 1905 | ........ | 8715 | 0—100 | Fizeau |
| Do. .................. | ........ | ........ | ........ | 8663 | 0—100 | Rodwell |

## LIST OF PAPERS BY OBSERVERS CITED IN ABOVE TABLE.

BILLET.—L'Institut, XXIII. (1855), p. 292 (Jahrsber. d. Chem. 1855, p. 46).

BOSSCHA.—Archiv. Neerland. IV. (1869), p. 197 (Pogg. Ann. Ergänz. Bd. V. (1871), p. 469).

BRUNNER.—Pogg. Ann., LXIV. (1845), p. 113.

CALVERT, JOHNSON & LOWE.—Chem. News, III. (1861), p. 315.

CRAFTS.—Comptes Rendus, XCI., p. 413.

DANIELL.—Phil. Trans., 1830, p. 257 and 1831 p. 443.

DE LUCCHI.—Atti d. R. Ist. Veneto, (5), V. (1880).

DEVILLE & TROOST.—Jahresber. d. Chem. (1864), p. 70.

DULONG & PETIT.—Ann. de Chim. et de Phys. 2 ser., T. II. (1816), p. 254 (Gilbert's Ann. LVIII. (1818), p. 270); Ann. de Chim. et de Phys., 2 ser., T. VII. (1817), p. 113.

ERMAN.—Pogg. Ann., IX. (1827), p. 557.

FIZEAU.—Comptes Rendus, T. LVIII. (1864), p. 923 (Ann. de Chim. et de Phys., 4 ser., T. II. (1864), p. 143; Pogg. Ann., CXXIII. (1864), p. 515); C. R., T. LX. (1865), p. 1161 (Pogg. Ann., CXXVI. (1865), p. 611); C. R. LXII. (1866), p. 1101 (Pogg. Ann., CXXVIII. (1866), p. 564; Ann. de Chim. et de Phys., 4 ser., T. VIII. (1866), p. 335); C. R., LXIV. (1867), pp. 314 and 771 (Pogg. Ann., CXXXII. (1867), p. 292); C. R., LXVI. (1868), pp. 1005 and 1072 (Pogg. Ann., CXXXV (1868), pp. 372 and 381); C. R., LXVIII. (1869), p. 1125 (Pogg. Ann., CXXXVIII. (1869), p. 26).

FUESS.—Zeitchr. f. Instrumentenkunde, I. (1881), p. 390.

GLATZEL.—Pogg. Ann., CLX. (1877), p. 497.

HAGEN.—Wied. Ann., XIX. (1883), p. 436.

HÄLLSTRÖM.—Pogg. Ann., I. (1824), p. 149.

KOHLRAUSCH.—Pogg. Ann., CXLIX. (1873), p. 577 (Carl's Repertorium, IX., p. 408; Phil. Mag., 4 ser., XLVII., p. 159).

KOPP.—Ann. Chem. u. Pharm. LXXXI. (1852), p. 1. (Pogg. Ann. LXXXVI. (1852), p. 156; Phil. Mag. 4 ser., III. (1852), p. 268); Ann. Chem. u. Pharm. XCIII. (1855), p. 129.

LAVOISIER & LAPLACE.—Œuvres de Lavoisier, T. II., p. 739 (Biot, Traité de Physique (1816) T. I., p. 151).

LEBEDEFF.—Journ. d. Russ. phys. Ges., XIII. (1881), p. 246.

MARCHAND.—Journ. Prakt. Chem., XXXV. (1845), p. 254.

MATTHIESSEN.—Phil. Trans. CLVI. (1866), pp. 231 and 861 (Phil. Mag. XXXI. (1866), p. 149; Pogg. Ann. CXXVIII. (1866), p. 513 and CXXX. (1867), p. 50).
PFAFF, F.—Pogg. Ann., CIV. (1858), p. 175 and CVII. (1859), p. 150.
PFAFF, L.—Erlangen. Besold. 1872 (Fortschritte d. Physik, XXIX. (1873), p. 579).
PISATI & DE FRANCHIS.—Ber. d. deutsch. chem. Ges., VIII. (1875), p. 70.
PLANTAMOUR & HIRSCH.—Arch. Sc. phys. (2), XXXVIII. p. 37.
PLÜCKER & GEISSLER.—Pogg. Ann., LXXXVI. (1852), p. 265.
REGNAULT.—Ann. de Chim. et de Phys., 3 ser., T. IV. (1842), p. 64 (Mem. d. l'Acad., T. XXI. (1847), p. 191; Pogg. Ann. LV. (1842), p. 585.
RODWELL.—Proc. Roy. Soc. Lond., XXIII., p. 108; XXV., p. 272; XXXI. (1881), p. 291; XXXII. (1881), pp. 23 and 540; and XXXIII. (1882), p. 143.
RODWELL & ELDER.—Chem. News, XXXIX. (1879), p. 56; Proc. Roy. Soc. Lond., XXVIII., p. 284.
RUSSNER.—Carl's Repertorium, XVIII. (1882), p. 152.
SCHRAUF.—Wied. Ann., XXVII (1886), p. 315.
SCICHILONE.—Beiblätter Wied. Ann., V. (1881), p. 496.
SMEATON.—Phil. Trans. XLVIII. (1754), p. 598.
SPRING.—Ann. de Chim. et de Phys., 5 ser., VII. (1876), p. 178; Bull. de l'Ac. Roy. des Sc. de Belg., 3 ser., II. (1881), p. 88; Bull. de l'Ac. Roy. des Sc. de Belg., 3 ser., III. (1882), p. 331; Chem. Ber. XV. (1882), pp. 1254 and 1940
WEINHOLD.—Pogg. Ann., CXLIX. (1873), p. 186.