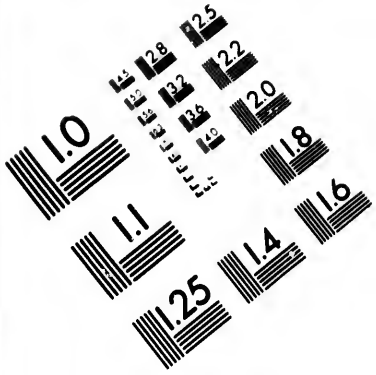

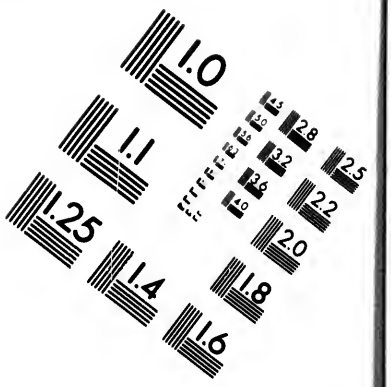

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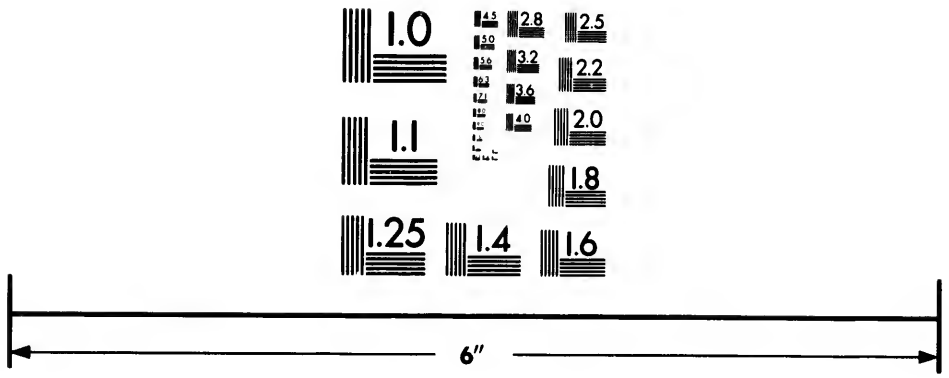

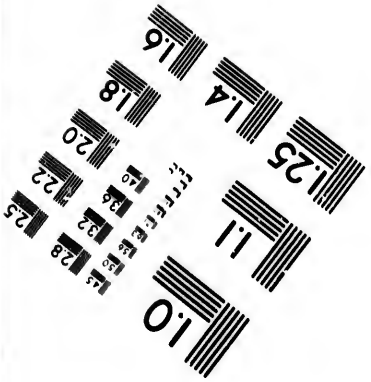

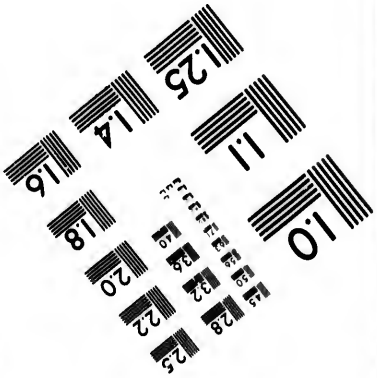

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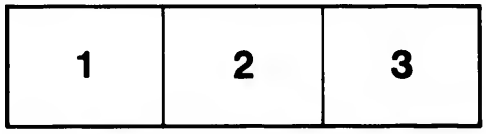

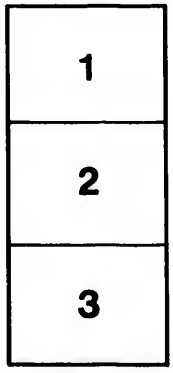

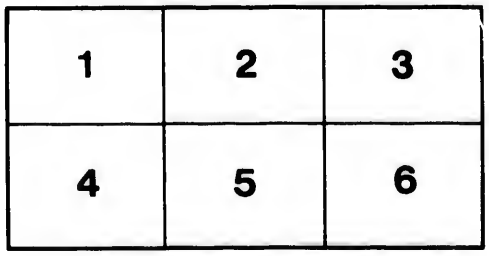

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A CONTRIBUTION TO THE PSYCHOLOGY OF TIME

BY

M. A. SHAW, B.A., AND F. S. WRINCH, M.A.

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I.

HISTORICAL SKETCH OF THE PROBLEM.

It may be said that Kant and Aristotle are culminating points in the history of modern and ancient philosophy respectively. In the comparison of these two, separated by ages, we are reminded of the slow growth of knowledge; for in essentials the philosophy of Aristotle approximates to that of Kant. Both asked the question, "How do we know?" For answer, Kant brought the subjective and objective factors of knowledge face to face, if not together, while in Aristotle's doctrine of the categories and central sentient principle[1] there is a presentation (vague, it is true, and surrounded by crude theories) of the same factors. Prior to each were philosophers who tried to explain the universe from the standpoint of subject or object exclusively; and in succession to them there have been schools based upon the manner of interpreting their master's thought. The history of any particular problem in philosophy follows the same general lines. Arising earlier or later, according as it is included in the first cosmological inquiry or not, it takes its place in successive systems in accordance with the subjective or objective standpoint of the founder.

The time problem was not the subject of particular inquiry up to the period of Aristotle. Plato scarcely touches it, excepting in connection with his doctrine of creation, and then only in a naive manner. Aristotle asks the question, How do we perceive time? and his answer is that it is immediately perceived in connection with any and every sense perception as the number of motion.[2]

[1] Aristotle, *De Sensu et Sensili*, cap. 7, p. 449.
[2] Aristotle, *De Coelo*, c. 9, sec. 8-10.

After Aristotle down to the close of the Middle Ages, the problem was scarcely considered. In modern philosophy, from Descartes to Kant, every philosopher grappled with the question to some extent, but all were governed by their respective presuppositions, and explained time, like all else, as either wholly intellectual or wholly material. Kant's solution of the problem is, in reality, to make time one of the categories (in the Aristotelian sense of the word) or ways in which the mind looks at its content, these categories being contributed by the mind. "With regard to phenomena, we cannot think time away from them," he says, just as we cannot think "quality" and "quantity" away from them. In other words, in that unified or determined assemblage of phenomena which he calls object or fact of experience, there are two factors, the subjective and the objective, each of which is indispensable. In the subjective factor is included the temporal relation of phenomena.

Since Kant, it may be said, speaking generally, there are three schools of interpretation of the time problem; one may be characterized as the idealistic, another the materialistic, and the third is composed of those who make of time a special sensation. The first school adheres to the Kantian presentation, not merely as set forth in his chapter on Time, but as gathered from his whole philosophy. The second school is an off shoot of one which has existed from the beginning of philosophical inquiry. Its adherents would try to show that time has an exclusively sensational basis. The third, as stated above, treats the idea of time as a special sensation. Each of these schools has its representatives in modern psychology. Dr. Mach, of the University of Prague, may be mentioned as an adherent of the last. Among those belonging to the second is Dr. Hugo Münsterberg, while as a representative of the first school the distinguished Professor Wilhelm Wundt, of Leipzig, may be taken. These three psychological theories will now be more fully stated, in the order in which the names of their representatives have been given, and a comparison will be made of their relative merits.

In the presentation of the first theory, we follow the exposition of Dr. Mach in his "Beiträge zur Analyse der Empfindungen." The contention of this school is that "time is a special state of consciousness." Mach says: "The existence of a special specific

time sensation appears to me to be beyond doubt."[1] He draws his conclusion from such facts as these. In the first place, the content of time may be changed without changing the time itself. In contiguous bars of music the rhythm may be exactly the same, but the tone entirely different. The recognition of time intervals as constant, though their content be continually changing, is not a product of the reason, he says, but is rather an immediate sensation. He illustrates this position further from the striking of a clock; the separating of the strokes as first, second, third, fourth, and so on, by which their position as before and after is given, is not, he believes, a thought which accompanies the sensation of hearing, but is rather an immediate "time sensation." To explain the time sensation a little more definitely, Mach notes that it is always present in every conscious state, that it is of necessity organically connected with every conscious state. When the attention is severely strained, time appears to pass slowly; when the exertion is lighter, it passes more quickly. If we are so interested that the exertion of attention is scarcely felt, then the hours pass rapidly away. Where attention is wholly wanting, as in dreamless sleep, time passes quite unconsciously, and the only connection between the two waking periods is an intellectual tie. Besides this influence of attention on the passing of time, a very strict concentration of the attention on some expected event may actually change the time order. The blood may appear to the overstrained attention before the surgeon's lancet has actually pierced the skin. These considerations, Mach thinks, indicate such an organic connection between the time sensation and attention, and such an obedience of the former to the latter, that he concludes that "time is the work of attention." Before examining the validity of the grounds on which the above theory is based, we will state the position of those who maintain that the presentation of time is attached to *certain special* states of consciousness, characterized also by other content. Since both views are affected more or less by the same errors, they may be better discussed together.

In the presentation of the second theory, the exposition of Münsterberg in his "Beiträge zur experimentellen Psychologie" will be followed. He begins with an attack on Wundt's position, that in estimating the relative length of smaller intervals, *e.g.*

[1] Mach, *Beiträge zur Analyse der Empfindungen*, p. 104.

from 0.5 to 3 seconds, the actual time-lengths themselves are compared directly, and not through the medium of their content. Münsterberg holds that these small intervals without content do not normally occur, but that in all time presentations the judgment of the interval depends on muscular tension, that is, on tension of the different organs occasioned by muscular conduction, or by the memory of such. These are the only data at our disposal for the immediate feeling of time. Whence come these feelings of muscular tension? Goldscheider says that they have their seat principally in the joints, in the thin sensitive layer which covers the ball of the bone and the socket in which it works. Münsterberg replies that there are no joints in the eyeball, the tongue, the lips, etc., yet that the feeling of tension is quite as noticeable in these as in the leg or arm, and further, that the feeling may change after the teeth are closed and no further movement takes place in the joint of the maxillary bone. The feeling of tension arises, then, not from the local sign in the joints, but from the muscles; in the muscles lies the whole source of the feelings of tension. In stating the subjective experience on which, according to this theory, an interval is estimated, he says: "If there are given to me impressions of sight; for example, if a point of brightness should appear upon a dark ground irregularly at intervals of from one to three seconds and vanish again, I feel at every appearance of the stimulus the muscles of the eye exert themselves to direct the line of vision to the luminous spot; all the muscles contract to fix the spot firmly, and the accommodation muscle exerts itself to secure a clear impression. In short, the whole organ of vision, as soon as the stimulation begins, comes into a state of exertion, by which the stimulus gains in distinctness, and its perception rises above the other content of presentations."[1] These feelings of tension and relaxation are the whole data for the time idea, and on their nature depends the estimation of intervals. The presentation of time is the synthesis, made up of the perception of the impressions which limit the parts of time together with the sensations of muscular exertion, which increase and decrease in intensity, and which we usually do not refer directly to the muscles. We *believe*, he says, that we refer our special state of consciousness directly to time, we *believe* that we perceive time

[1] Münsterberg, *Beiträge zur experimentellen Psychologie*, Heft 2, p. 22.

sensation directly, just as the man groping in darkness believes that he perceives space directly, whereas he only perceives directly the limitations of space, and infers the rest from these directly perceived data. In the case of time, what is immediately felt is the muscular exertion increasing and diminishing in intensity, and from that time is inferred.

On the hypothesis that the estimation of intervals depends on the change in intensity of muscular sensations, Münsterberg accounts for the intelligent division of the continuous flow of time by a physical process which is continually changing, the act of respiration. In this process, the muscles of the organs involved are continuously employed in contracting and expanding, and make a regular alternation of strain and relaxation in muscular tension. Although there are factors which cause irregularities in this periodical division of time, such as the existence of certain central processes, leading us to make an inspiration at the moment we turn to an agreeable impression, and to exhale at the moment when an impression of the opposite nature is experienced, still when the attention is closely occupied inspiration and expiration follow regularly, and thus afford a continuous uniform standard of time. Before explaining the experimental work with which he supports his theory, Münsterberg notes that the discoveries of different seats of temporal sensitivity by other experimenters may not be all wrong; indeed, if the cause of their disagreement were known, it might be shown that they were all right. Their different results may be accounted for by the varying sensibility of the organs involved. In auditory impressions, an interval of 2 σ could be discriminated, but if the two stimuli were separately conducted, one to each ear, the smallest noticeable interval elapsing between the impressions was found to be 64 σ. Similarly, when an auditory and a visual stimulus were given simultaneously, the auditory impression was perceived before the visual, on account of the more rapid transmission of the stimulating process in the labyrinth of the ear. Previous theories, he says, have failed to explain this variation in the point of temporal sensitivity, but he thinks that his own, being based on muscular tension, accounts for it satisfactorily.

In his experiments, Wundt's time-sense apparatus was used, and his calculations were made by the method of average errors. He always used one normal and one comparative time interval,

and in some of his experiments an interval was allowed to elapse between the normal and the comparative intervals. In the first group of his experiments, by touching the lever himself he directly stopped the apparatus, when a time which he judged equal to the normal interval had passed, and thus registered the length of the comparative interval. His trials in this first group were on intervals ranging from six to thirty seconds. First, a series of trials was made on intervals, the length of which was wholly independent of the length of time occupied by any organic function of the body; next, a series of trials was made on intervals that were multiples of his respiration time. The results of his experiments[1] show that in the first series, where the intervals were independent of respiration time, the error was 10.7 per cent., while in the second series, where the intervals occurred just in the same phases of respiration, the error was only 2.9 per cent. In a second group of trials, on intervals ranging from one to sixty seconds, the experiments were divided into two similar series. In the first series, where the phases of respiration were not taken into account, the error was 24 per cent., and in the second series, where the intervals always occurred in the same phases of respiration, the error was only 5.3 per cent. In a third group, the intervals were filled with slow regular muscular exertions and relaxations, independent of respiration. With this content in small intervals, from three to ten seconds, great errors were made, but in the longer intervals the estimation was very accurate.

These experiments prove that there is no special time sense in our consciousness. The changing muscular tension and relaxation, psycho-physically conditioned, form the standard of our time measurement. In the bodily conditioned periodicity of these exertions lies the cause for the phenomena of time. It is erroneous to attribute this periodicity to consciousness itself. "The physically unconditioned transcendental apperception does not function periodically in its apprehension of time, but those physiological excitations, whose centripetal effect is our presentation of time, are subject to periodical change. The apparently most exact proof that our consciousness is not only conscious of the psycho-physically conditioned states, but that it also interferes actively in them and changes its relation to them, has

[1] Münsterberg, *Beiträge zur experimentellen Psychologie*, Heft 2, pp. 66 and 67.

revealed itself as a series of uncritical errors."[1] Not to the *consciousness* must the periodical undulations of the sensation of time be attributed, but to the *content* of consciousness; and no state or content of consciousness changes, unless the bodily state, *i.e.* the tension of the muscles, changes. Hence he concludes that metaphysics must again admit that a property claim of the transcendental consciousness is declared invalid and is transferred to the sensuous reflex apparatus of the body.

In the presentation of his theory, Münsterberg, like Mach, has based his conclusions on what is experimentally insufficient, and hence his theory, like that of Mach, is affected by far-reaching errors. The objection that time intervals without content do not normally occur is urged against Wundt's position that in estimating comparatively small intervals we compare the actual time-lengths themselves. This objection would be valid against Mach's contention that there is a special particular time-sensation, but it scarcely holds against Wundt, who does not maintain that there are time intervals without content, but rather that time, as a property of all states of consciousness, may by abstraction be brought into the gravitation centre of consciousness, and its relation to other properties investigated by experiment. To Münsterberg's conclusion, that the sensations arising from muscular tension and relaxation are the only data at our disposal for the immediate feeling of time, there is the fundamental objection that, since these feelings can only be experienced *in* time, they presuppose time, and must have a position as before and after other sensations. Time is one of the ways through which the mind looks at every fact of consciousness; hence to conclude that certain facts of consciousness are the data of our feeling of time is merely a *petitio principii*. Nichols has shown by careful experiments that sustained attention to a rhythm materially influences the estimation of succeeding intervals.[2] If the estimation depended wholly on muscular sensations, this would not occur. Meumann, also, has demonstrated that the intensity of the limiting sensations influences the estimation of small intervals, and that the content has little influence on the estimation of medium intervals, exercising control only in long intervals. Further, he shows that there is an æsthetic factor

[1] Münsterberg, *Beiträge, etc.*, Heft 2, p. 68.
[2] Nichols, *Psychology of Time*, p. 94.

involved, and that pleasure and pain have an effect upon the estimation of some intervals.[1]

It is far from self-evident that the above theory explains very generally the variations in the position in which different experimenters have placed the point of temporal sensitivity. The fact that an optical impression is slower in coming into consciousness than an auditory is not explained by the muscular tension, but more probably by the different nature of the processes of stimulation in the two sense-organs. The experiments on which Münsterberg bases his theory were not very accurate in method or calculation, but the amount of discrepancy arising from that source would probably not affect the results. If it be granted that his results are correct, still they do not justify the conclusions he draws from them. From the fact that intervals whose limiting sensations fall in similar phases of respiration are much more accurately estimated than others which are independent of any organic function of the body, it can only be inferred that the organic functions of the body provide data which enable the mind more accurately to estimate the intervals in which they occur. It does not justify Münsterberg's inference that the sensations arising from the tension of the muscles are the only data that we have for the feeling of time. Thus, as stated above, the experimental basis of his theory is quite inadequate. His materialistic standpoint involves, further, the fatal objection that the sensorial reflex apparatus of the body is the ultimate basis of the theory. This position is open to all the criticisms that may be urged against the empirical school. Time is nothing without the facts of consciousness, but it is a property of all facts, all processes. The error in both the theories already set forth arises from the fact that they mistake certain factors which influence time presentation for the presentations themselves.

The third position, which is the direct development of the Kantian standpoint, has as its most eminent exponent Professor Wundt of Leipzig. This theory explains time not as a special sense, nor as dependent on certain special states of consciousness, but as a factor in *all* states of consciousness. Wundt criticizes Kant for indicating, through his use of the terms external and internal sense, that there is a double experience;[2] he maintains that there is no such double ex-

[1] *Philosophische Studien*, IX., p. 264.

[2] Wundt, *Outlines of Psychology*, p. 158.

perience. From statements that appear in the *Kritik* there seems to be some ground for the above objection. No doubt Kant's training in the earlier philosophy had biassed his mind strongly in favour of the complete separation of subject and object, so that it was not easy for him to cut himself wholly free from language which might leave the impression that he himself was still enslaved to the conception of a dual experience. But Kant's system hardly justifies that interpretation. Thus, in dealing incidentally with a problem which does not belong to psychology, viz., the explanation of the community of soul and body, he says: "The difficulty which lies in the execution of this task consists, as is well known, in the presupposed heterogeneity of the object of the internal sense (the soul) and the objects of the external senses, inasmuch as the formal condition of the intuition of the one is time, and of that of the other space also. But if we consider that both kinds of objects do not differ internally, but only in so far as one *appears* externally to the other—consequently that what lies at the basis of phenomena, as acting in itself, may not be heterogeneous, this difficulty disappears."[1] This is a direct statement of the *unity of experience* —" both kinds of objects do not differ internally "—but this unity has an internal and external aspect. It is true that there is still left for Kant the " thing in itself," but the charge that he contends for a double experience even in phenomena can hardly be justified, since here, and in other passages, he evidently contends for a single experience with two factors, though it may be questioned whether he saw fully the implications of his own position.

Wundt, then, in the development of his theory, has, as the underlying principle, the " unity of experience," which seems to have been the objective point of Kant's system. Time is nothing apart from the facts of consciousness; it is not even a process, as Nichols says, but is a property of all facts and all processes. But this property can *by abstraction* be brought into the gravitation centre of consciousness, it can be brought before voluntary attention, and then its relation to other properties may be investigated by experiment. This theory is the basis of modern psychology, the science of the immediate experience; and this experience, as stated above, is not the dualism of unrelated subject

[1] Kant, *Critique of Pure Reason*, translated by Meiklejohn, p. 252.

and object, but is composed of processes interconnected by certain laws, each of which processes has two factors, experiencing subject and objective content. The unity of the immediate experience is the self, and each fact of experience is a process connected with others, and hence implying time. Thus we see, once for all, from a psychological standpoint, that time is present in the data from which the science of psychology is constructed. Time is normally present in every fact of experience, time-relation conditions every fact of experience, but time occurs alone only as an abstraction. Time, then, is a constant factor, not only of the self, the unity of the immediate experience, but of every conscious process which goes to make up the self. By the union of sensations and feelings or affective elements, which are obtained by abstraction and analysis of the facts of experience, a psychic compound is formed, whose nature depends not so much on its component elements as upon their union. When these compounds again are united, the result is more than the sum of the compounds.

We find that time-presentations are of the nature of psychic compounds; thus it may be well to look a little more fully into the nature of psychic compounds. In the first place, they embrace all composite components of psychic experience, which have certain peculiar characteristics sufficient to make them relatively independent; next, they are processes, not things—processes which are continually changing, and can therefore only by abstraction be thought of as momentarily constant at all; and further, they are never absolutely independent, but are continuously uniting with other compounds to form a greater interconnection. Compounds are of two classes according as they are formed principally of sensational or of affective elements. Time compounds belong generally speaking to the former class, though not exclusively, for certain affective elements play an important part in their formation. There are two most favourable sources for temporal ideas, the so-called inner-touch sensation, and auditory sensations, though the conditions for their use are present in every sensation. The inner-touch sensations, especially those accompanying the movements of the body which are most involuntary, such as walking, form a basis for the ideas of time. In every step taken there is a series of sensations, repeated for every subsequent step in exactly the same order. The beginning and the end of each step are characterized by outer

tactual sensations, caused by lifting the foot and replacing it again, and between these two limiting points there is a regular series of inner-touch sensations, arising from the movements of the hip joints, and these are stronger at the beginning and the end on account of effort and inhibition. Corresponding to the inner-touch sensations we have a series of affective elements consisting of feelings of strained expectation followed by feelings of fulfilment. Here, then, are two elements of the time idea, sensational and affective, the former predominating. In examining auditory sensations we find that the sensations which go to make up the idea are only at the end of single intervals. But although there is an absence of almost all objective sensation in these intervals, there is a very noticeable affective content, viz., the gradually increasing feeling of strained expectation followed by the feeling of fulfilled expectation. There is also a less noticeable, but still important, sensational element in the content, arising from the tension of the tympanic membrane and other parts of the organ involved; the subjective element is, however, by far the most important in the auditory sphere, and its influence on the time idea has been shown to be quite fundamental. It has been found that an interval of one-fifth of a second is most favourable for the union of successive auditory impressions,[1] a fact which shows that the continuous recurrence of this interval secures the most emphatic alternation of the subjective elements of strained expectation and fulfilment. If the interval be longer, the feeling of strained expectation is so intense as to be unpleasant, and if it be shorter, the rapid alternation of feeling becomes fatiguing. We have in this a proof of the influence of an æsthetic element in the judgment of the time interval. Analysis thus proves that the elements of the time idea are both sensational and affective. They are not mutually exclusive, nor can temporal attributes be ascribed to either of them separately. We therefore conclude that the time idea is a fusion of the two elements with each other and with the objective impression. In conscious life we always find them so fused, so that again after analysis we arrive at the position from which we started, that time is a constant factor of conscious life, and that there is no such thing as isolation in the continuum of consciousness.

[1] Wundt, *Outlines of Psychology*, p. 149.

In the time idea the elements bear a fixed unchangeable relation to one another; none can be changed without changing the whole. But in addition to this the elements bear a relation to the ideating subject, so that when any element is changed in relation to another it also changes its relation to the ideating subject. These two relations are always found connected in actual experience, but when we isolate them for the purpose of investigation, we find that from the relation of the elements to one another we get the so-called modes of time, viz., brief, short, long, etc., and from their relation to the ideating subject we get the temporal stages, past, present and future. If a series *a*, *b*, *c*, *d*, etc., represent points in space, they may all be perceived at once; the fixation point may rest on any one of them, and the rest be grouped from that centre. If it be a time series, they may also be perceived at once, if they be points in the past, but the fixation point of time is always the present moment, and from this point the other points in time are always arranged. Since the present moment is always judged in relation to the totality of the past, then it follows that no two moments are identical, for the totality of the past with which one moment is related is increased by that moment and thereby altered before the next moment is related to it. Thus there is a characteristic belonging to every point in time, which distinguishes it as past in relation to the present moment, and makes this present the fixation point from which all the rest are arranged. The fact that the affective element in time is conditioned by the totality of previous experience, and so made different, produces an ever-changing impression of the present moment. This impression of the present moment is called the inner fixation point, and the never-ceasing change in the inner fixation point is called the continuous flow of time. This continuous change means that no moment of time is like any other moment, and that no moment can ever return. There is no such thing as time in which no change takes place, and since the fixation point is continually moving forward it follows that time is of a one-dimensional character.

The above is in substance Wundt's theory of time; it is the theory of modern experimental psychology. And let it be observed that although by abstraction and analysis the time idea has been reduced to its elements for the purpose of investigating its nature, still it is only by artificial analysis that it can be so

separated into its elements. In nature the time idea does not appear so separated, but only as a property of all facts of consciousness. This theory avoids the errors which Mach falls into in making the time idea a special particular sensation, and also the errors of which Münsterberg is guilty in making the time idea the property of a special sense only, and in working out his theory ultimately from a materialistic standpoint. Wundt's theory, starting as it does from what we hold to be the only consistent standpoint, viz., immediate experience, will, we believe, account for all the phenomena of time.

II.

THEORY OF TIME ESTIMATION.

In all states, the percipient subject is conscious of a temporal relation; no experience is without duration. Both sensations and the intervals between them occupy time; each has a content, but the content of a sensation is more evident than that of an interval. The duration of sensations has not been experimentally investigated in regard to its temporal value for consciousness. Experiments on the estimation of time in intervals, by Külpe, Meumann, and others, show that below an interval of a certain length the limiting stimuli exercise an important influence. Above this interval, where longer periods are involved, the content of the interval, *e.g.* respiration, heart-beat, or some other organic function, enters as a factor influencing the judgment.

The interval that lies between the short and long groups, which has been found to be the most accurately estimated interval, we believe to be the "unit of time," and the basis of all time estimation. It differs in length more or less in different individuals, but in all cases the constant error and mean variation are less as we approximate to it. Here, neither the limiting impression, which influences the judgment of shorter intervals, nor the organically functioned content, which influences the judgment of longer intervals, modifies the estimation to any marked degree. This remarkable fact in connection with the estimation of intervals, that a more or less definite interval is estimated most accurately, and the further fact that as we depart

from it in either direction errors arise in judgment, lead us to believe it to be the basis of the estimation of intervals in all cases—a temporal unit. This theory of a unitary basis of estimation, moreover, is in accordance with that demand for single principles, which is fundamental in the human mind. With dual principles the mind is never satisfied, and it was for this reason, in part, that the theory of a three-fold basis of estimation advanced by Külpe was rejected. The theory of a temporal unit is therefore in harmony with the scientific demand for explanation by as few principles as are consistent with facts. That this theory has an experimental basis, we shall now proceed to show in the three cases of the unitary interval, of intervals below, and of intervals above the temporal unit.

1. In the estimation of the unitary interval, the interval itself is the basis of estimation, as Külpe, Meumann, and others have said. Here, also, its more or less definite content (obtained by analysis) of feelings of expectation and relief, and of muscular sensation, is scarcely noticed. Some investigators would make this content the basis of time estimation. Among these, Schumann, whose theory[2] will be examined later, may be mentioned. The estimation of this interval, too, as would be naturally expected, is more accurate than that of any other.

2. In the estimation of intervals below the temporal unit, the experiments of Külpe and Meumann revealed a tendency to reproduce these intervals longer. Such was also the case in our own experiments.[3] This error in estimation seems to be due to the tendency of the temporal unit to complete itself, it being the individual basis of estimation. It will then be made up of both the empty interval and the limiting stimuli; and the judgment will be affected more or less by the nature of these, but the overestimation becomes less and less as the interval approaches the length of the unit of time. This tendency to complete itself, on the part of the unit, is confirmed by results of other investigations. In experiments on the time relations of poetical metre, a tendency to complete the foot, where the metre was irregular, was noticed.[4] In this connection, we may note that Leuba[5]

[1] Külpe, *Outlines of Psychology*, sec. 65, 3.

[2] *Zeitschrift für Psychologie*, Bd. xviii., p. 1.

[3] Cf. Tables III., IV. and V., *infra*.

[4] See "*Experiments on the Time Relations of Poetical Metres*," by Messrs. Hurst and McKay, in this volume.

[5] *Psychological Review*, 1898, p. 483.

suggests, as an explanation of the quantitative change in memory, that there is a typical representative for each class of experiences (conscious processes), which is a residuum left over from all the experiences we have had. Towards this middle type or unit all our memory images are drawn.

As was said above, the nature of the limiting stimuli is a factor which enters into the estimation of intervals smaller than the unit, and has a bearing on the accuracy of their reproduction. It is found, for example, that intervals which are bounded by a fairly loud, sharp sensation are judged to be shorter than intervals bounded by a weak sensation,[1] while on the other hand they are judged to be shorter likewise than intervals whose end-signals are still stronger.[2] This difference is accounted for by the subjective strain of attention or expectation in the weak and also in the very strong sensations, which tends to lengthen the intervals. From these facts, Külpe[3] concludes that the limiting stimuli are the basis for the estimation of short intervals, whereas they are but a disturbing factor in the estimation. Schumann, in the same way, makes the feelings of strained expectation and surprise negotiate the estimation of the interval. Intervals shorter than the unit of time, however, cannot be estimated and reproduced individually, but only by being grouped into a series. This is confirmed by the fact that they are thrown into a subjective rhythm, which divides itself according to the temporal unit or multiples of it. Rhythmic reproduction of short intervals is established by the results of Meumann, Külpe, Müller and others. We do not believe that such rhythmic grouping is confined to intervals limited by auditory impressions alone, as Külpe says.[4] Experiments were made by students under the direction of Professor Kirschmann, in the laboratory of the University of Toronto during the session 1894-95, with the following recorded results.[5] Mr. Warren, under the heading *Pendulum Experiments*, writes: "In the first experiment, a striking rhythmic interval occurs, showing itself from the first and continuing with the same regularity during the reproduced time, the periodic return of under-estimation and over-estimation increasing during the

[1] Cf. Tables IV., V. and VI., *infra*.

[2] Wundt, *Outlines of Psychology*, p. 151.

[3] Külpe, *Outlines of Psychology*, sec. 65, 3 (i).

[4] Külpe, *Outlines of Psychology*, sec. 65, 5, 2a.

[5] These results are unpublished, but they are preserved in the Archives of the Psychological Department.

progress of the experiments." Mr. Crawford's experiments show a noticeable, although not a constant, periodicity in visual impressions. Under the heading *Points to be Noticed*, he says: "In experiments with visual impressions (pendulum vibration), a periodicity is noticeable in some series of experiments, but not in all." Mr. F. W. Varley, in a very carefully prepared paper, summing up results of experiments with both visual and auditory stimuli, says: "On viewing the graphic representations of these experiments, it may be seen that they have been characterized by waves of attention. Periodically there has been a comparatively great aberration, indicating a relaxation." In our own experiments with visual impressions, the intervals ranged from half a second to nine seconds, and the tendency to rhythmic accentuation was not noticeable. In nine series of experiments with auditory (metronome) impressions, there was a marked periodicity in some, while in others it was not apparent.

From the above statements we do not think that the exclusion from rhythmic accentuation of all intervals except those bounded by auditory impressions is warranted. On the contrary, it seems to be a constant characteristic in the reproduction of all short intervals, and is due, in part at least, to the fact that such short intervals cannot be estimated individually, but are thrown into a subjective grouping. Thus the experimental facts in connection with the estimation of the class of short intervals supply a basis for the theory of a temporal unit.

3. In the estimation of intervals above the unit of time, observers find great, almost invincible difficulty in excluding the influence of the content. This content is generally some regularly recurring organic function, and upon it depends largely the accuracy of the judgment. The observer K. attempted to exclude the influence of content as far as possible. The results show that this attempt vitiated his judgment to a very marked extent. The explanation is not, however, that the content is the basis of judgment, but that the mind estimates intervals longer than the unit in terms of the unit itself, and that the unit or the multiple of it is here represented by some regularly recurring organic function. As in short intervals reproduced in groups the quality of the limiting sensation is a factor entering into the estimation of the interval, so in long intervals the character of the content plays a part in the estimation. An interval which has a full and varied content is judged to be comparatively

longer than one which is not so crowded, because of the strain required to keep the attention on the interval, when there are other things to distract it. If, however, the content becomes so interesting as to absorb the attention entirely, the time passes more quickly and is judged to be shorter. The part thus played by attention does not make it a basis of estimation, as Mach thinks, but only shows its influence as a single factor.

We may now sum up our conclusions as follows:

(1) A certain interval, which varies considerably in different individuals and which may vary in the same individual at different times, is the unit of time and the psychological basis of the estimation of intervals in all three classes.

(2) In the first and third classes of intervals factors enter which influence the judgment, but they are only modifying conditions, not the bases of our estimation. There are, in our opinion, not three different kinds of temporal judgment, but three applications of the one form of temporal judgment.

(3) Although time must be considered as an elementary property of consciousness, incapable of further analysis, the unit of time, nevertheless, which seems to be the basis of the estimation of intervals, is of the nature of a psychic compound. It contains, as may be seen by analysis, both subjective and objective factors. Of these two factors, the objective seems to have, at least genetically, the predominating influence. It may be some organic function, or multiple of such, which has accustomed us to the duration which is our unit more than to any other, or the duration of some objective impression or interval to which we have been always accustomed may have played the fundamental part in its formation.

Before proceeding to give any account of our experimental work in the laboratory of the University of Toronto, it may be well to consider briefly two recent articles on this subject of time judgment by F. Schumann.[1] The views there presented might be classed with the second of the time theories given above (p. 7) and passed without further comment, for they partake of the same general nature, being one-sided and materialistic in tendency. The views are, however, the most recent exposition of the school, and for that reason we consider them a little more fully.

[1] *Zeitschrift für Psychologie*, Bd. xvii., p. 106, and Bd. xviii, p. 1.

Schumann maintains that in the estimation of time intervals the judgment is mediate.[1] Whether it is mediate or immediate must be answered, he says, by experimental psychology, and not from theoretical principles, and he concludes from his experiments that the feelings of expectation and surprise mediate or negotiate the estimation of the interval. Without following the steps by which he reaches this conclusion, to be noted later, we shall first examine the basis from which he starts and his theory of judgment. The starting point for psychology, according to Schumann, must be "inner perception."[2] The latter term is used for the perception which arises on the occasion of a sense stimulation. In making this the basis of certain knowledge Schumann is unduly emphasizing the subjective, almost to the exclusion of the objective element of the immediately given fact of experience. The immediately given is the idea-object, as Wundt has shown, and not the idea, which must be the element of inner perception. The idea and the object are secondary products, the result of the analysis of the immediately given fact. Hence Schumann's "inner perception," which he assumes to be the immediately given, and therefore the basis of certainty, is not the immediately given, but an abstraction. However valuable the accuracy of his experimental investigation may be, the value of the conclusions he arrives at will be, if not vitiated, at least much impaired, on account of this erroneous basis of certain knowledge from which he starts. Schumann's state of confusion as to the nature of the immediately given is again apparent in his theory of the judgment of time intervals. In his conclusion[3] that expectation and surprise mediate or negotiate the judgment of time intervals, he is using two complex temporal processes as the basis of his judgment. Hence, on his own showing, time estimation is not a judgment mediated by untemporal processes, differing entirely in nature from time itself, but it is rather of the nature of an immediate comparative judgment. For those processes, which he says mediate the judgment, in so far as they play a part in that judgment, do it through their own time element, which in the act of judgment is compared with the respective time intervals under discussion.

Schumann seems to leave out of consideration, in his theory of

[1] *Zeitschrift für Psychologie*, Bd. xviii.
[2] *Zeitschrift für Psychologie*, Bd. xvii.
[3] *Zeitschrift für Psychologie*, Bd. xviii.

judgment, the relativity of the whole content of consciousness. He maintains that there is no comparative activity in judgment, on such grounds as that the intensity of an impression advances a number of units before the judgment of increased intensity arises. But, if there be no comparison, to say that the intensity is increased means nothing. It must be increased in relation to some other intensity. Further, if there be no comparison in judgment, then Schumann's theory that expectation and surprise mediate the judgment cannot be sustained, for in this process of mediation there is comparison involved. Either Schumann must give up his theory of judgment being entirely passive, or else he cannot maintain his theory of time estimation. A clear comprehension of experimental psychology, the standard to which Schumann appeals, should show that the most fundamental and primary judgment involves activity from one point of view, and passivity from another. For the more or less relatively isolated immediate experience may be viewed from one point as the active knowing, and from another as the passive known. These are not two different experiences, but two ways of looking at the one unitary experience. The most fundamental judgment, then, is both active and passive, and every other judgment must partake more or less of the same nature.

Having shown what we believe to be the errors of Schumann's theory of judgment, and that too by an appeal to the same standard which he himself applies, viz., the decision by experimental research, we shall now consider his theory of the estimation of small intervals. Schumann says: [1] "Ich habe nun die Ansicht ausgesprochen und zu beweisen gesucht, dass diese Nebeneindrücke der Erwartungsspannung und der Ueberraschung die Schätzung der Intervalle vermitteln und zwar in der Weise dass ein Intervall, vor dessen Endsignal eine lebhaftere Erwartungsspannung auftritt, länger erscheint als ein Intervall, bei welchem sich nur eine schwächere Erwartungsspannung geltend macht, und dass jedes durch Erwartungsspannung ausgefüllte Intervall für länger gehalten wird als ein Intervall dessen Endsignal unerwartet kommt." We have stated in a previous part of this paper what place these feelings of strained expectation and surprise occupy in the estimation of intervals. That they are not the basis of our estimation seems evident from the following considerations:

[1] *Zeitschrift für Psychologie.* Bd. xviii., p. 2.

1. There is one fact in connection with the estimation of intervals which has been experimentally established almost beyond a doubt, viz., that there is a certain more or less definite interval which is most accurately estimated. It is admitted by investigators that in the estimation of this interval these accompanying impressions of expectation and surprise are less marked than at any other. Why is it then, if they negotiate the estimation of the interval, that where the sensible discrimination is most accurate we are least conscious of these feelings? If they negotiated the estimation of intervals they should be most marked where that estimation is most accurate. That this is not the case has been experimentally established. Not only are they not marked where the estimation is most accurate, but according to Schumann[1] himself, when the strain of attention is strongest, and surprise greatest, there the greatest errors in judgment are made. This error, he goes on to say, becomes less as the feelings of expectation and surprise decrease. Hence we must look deeper than these for the basis of the estimation.

2. It is as clearly established as the fact of a most accurately estimated interval, that above and below this interval there is a gradually increasing error in judgment. This fact of the over-estimation of intervals shorter than the one we have called the unit of time, and the under-estimation of intervals longer than the unit, is not explained by Schumann's theory. If these negotiated the estimation of intervals, it would follow that wherever they are as little noticed as at the unit of time the estimation should be just as accurate. But that this accuracy does not occur has been established by experiment; for, both above and below the unit of time, after the strain consequent upon the change of normal time has passed away, when the attention has adjusted itself and the end-signal enters just when it is expected, there is ever found an error in judgment which increases as we depart in either direction from the most accurately estimated interval. Hence we conclude that the feelings of expectation and surprise do not mediate the estimation of the intervals, either when these feelings are most marked, or when they are at the mean of alternation. We do not deny their presence, but they are only factors in the estimation, not its basis.

3. In addition to the above reasons for rejecting the theory that makes time estimation depend on feelings of strained expec-

[1] *Zeitschrift für Psychologie.* Bd. xviii., p. 2.

tation and surprise, there is another, and not one of the least important. It has been mentioned above in connection with the criticism of Schumann's theory of judgment. Wherever a feeling of strained expectation is experienced a complicated process takes place, which has temporal relations and can itself be estimated. This feeling of strained expectation is one of the affective accompaniments of any content of consciousness which is being actively apperceived. It is exceedingly complicated and may be accompanied with excitement, pleasure, pain and the like, and its temporal relations may be of longer or shorter duration. Yet we are asked to take as the negotiator of temporal estimation a complicated affective process which is itself as temporal as that which we are estimating!

III.

EXPERIMENTS ON THE EFFECT OF A LAPSE OF TIME BETWEEN THE NORMAL AND COMPARATIVE INTERVALS.

The purpose of the experiments now to be described was to investigate the effect of a lapse of time between the normal and the recording of the comparative interval upon the accuracy of the reproduction. The work on these experiments was done at regular times of the day and week, so as to secure as far as possible uniform conditions in the subject, and the experiments were continued over a period of four months. Everything was removed that might create any counter-interest, or in any way disturb the subject whilst making the judgments, so that, as far as objective conditions were concerned, there was nothing to hinder the subject from giving his undivided attention to the work.

The apparatus used was a Charles Verdin kymograph (Paris, 1892), with a number of fixtures designed by Professor Kirschmann, for the purpose of securing visual and auditory stimulation, and to insure accurate recording of the judgments of the subject. Visual stimulation was used exclusively in these experiments, and the method by which the light-flash was communicated to the observer may be seen by referring to the half-tone cut of the kymograph (Fig. 1.) The flame of an incandescent lamp was reflected from a small mirror (C in Fig. 2) at the end of the drum

of the kymograph, through holes cut at equal distances in a band-shaped ring which was fitted on to the end of the drum. As the drum revolved the reflected light was seen through a telescope by the observer, who sat about fifteen feet away. The light appeared in successive flashes as the holes passed the mirror. The length of the interval was varied by changing the number of open holes. The field of vision of the observer for the flash of light was entirely obstructed except for what appeared through a graduated aperture in the screen A (see Fig. 1.), in front of the revolving band. The lamp B was enclosed, so as to throw not much light out into the room, which was darkened during the experiments; the light flashes would therefore be sharp. The comparative intervals were registered on the carboned paper which covered the drum of the kymograph, by means of a rigid steel wire, one end of which the observer held between his fingers, and the other end, passing round a pulley, was connected with the pointer C in contact with the paper. At the pull of the observer the pointer slid along steel rods, thus making a curve in the otherwise regular line, and by this means the estimation of the subject was recorded. A weight (D, Fig. 1) connected with the pointer brought it back to its place as soon as the strain on the wire was released, after each reaction.

The use of simple mechanical transmission might at first glance seem primitive, but, in accord with the request of the director of the laboratories, we wished to avoid all complications arising from the use of electricity. The current and silently accepted view that electricity, since it travels so swiftly, requires also but an exceedingly short time to accomplish work, *i.e.*, to overcome resistance, is utterly fallacious. Wherever electricity has to accomplish a mechanical labour, be it in the form of chemical or electro-magnetical action, or simply the discharge of a spark, it requires time; and this time is not independent of the duration of previous conditions.[1] In the case where longer periods have to be recorded, and where efficient controlling apparatus can be applied, as in Wundt's chronograph, the errors mentioned do not amount to much; but wherever short and varying intervals have to be recorded, the errors produced by electric marking will materially impair if not wholly invalidate the results.

[1] Külpe and Kirschmann, *Ein neuer Apparat zur Controlle zeitmessender Instrumente.* (*Phil. Stud.*, Bd. viii., p. 145.)

In order to control the speed of the revolution of the drum, and to serve as a basis of measurement of the normal and the recorded intervals, the vibrations of a König tuning-fork were excited by an electric current from a storage battery, and transferred to the carboned paper on the drum by a Deprez signal (D, Fig. 2). The steel point of the Deprez signal, which was used to record the vibrations of the tuning-fork, was mounted on the same rods, but just behind the pointer which the observer used to record his judgments. The lines made by the two points thus follow each other side by side round the drum, except where the straight track of the pointer is interrupted by the curve marking the reaction. The Deprez signal and the pointer passed gradually from one end of the revolving drum to the other, by means of a screw connection (E, Fig. 1) worked by the assistant.

The experiments of this investigation, the results of which are given in the tables below, were made by S. on intervals of half a second, three-quarters of a second, and a second and a half. In order to avoid the errors of reaction time involved in experiments where a single normal and comparative interval are used, a successive series of intervals was used in both cases. The observer was allowed before each series of judgments to see the objective stimulation for half a minute, and in order to assist in getting the rhythm of the successive stimuli, where necessary, some movement of the hand or foot was used. When the half minute had elapsed the objective stimulus was turned out, and the observer began immediately to record the series of comparative intervals, continuing until stopped by the assistant. The same series was exactly repeated twice again for each interval tested, with only this change, that half a minute and one minute respectively were allowed to elapse between the cessation of the objective stimuli and the beginning of the recording of the compared intervals. During this lapse of time the physical movement referred to above was discontinued. With the intervals of half a second and one and a half seconds three groups of the series were made, but only two with the three quarters of a second interval. In the first table below is given a summary of the experiments. Only the first twelve trials in each case are counted, making in all thirty-six trials at the "immediate," half minute, and one minute pauses with the half-second and second and a half intervals respectively, and twenty-four trials at the same three pauses with the three-quarters of a second interval. The

second table is a summary of all the trials made. The normal, average, constant error, and mean variation are given in terms of one-hundredth of a second. The first table is given with an equal number of trials in each group of the series, lest the experiment should be vitiated by having an unequal number of trials at the various lengths of pause.

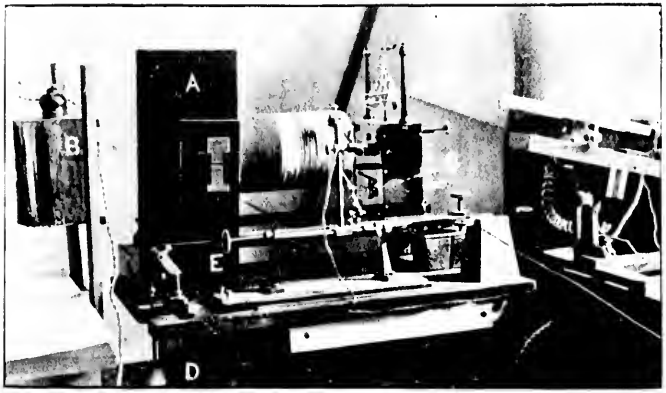

FIG. 1.

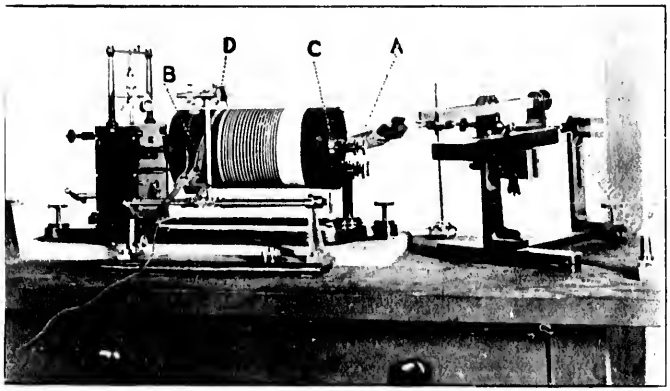

FIG. 2.

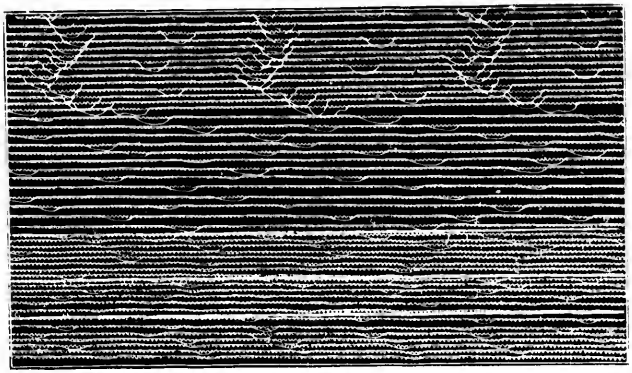

FIG. 3.

TABLE I.

TABLE I.

Normal.	No. of Trials.	Length of Pause.	Average.	Const. Error.	Mean Variation.
50	36	Immediate	47.27	— 3.73	1.90
50	36	½ min.	45.98	— 5.02	2.57
50	36	1 min.	47.69	— 3.31	1.90
75.5	24	Immediate	66.91	— 8.09	2.60
75.5	24	½ min.	65.12	— 9.88	2.91
75.5	24	1 min.	66.91	— 8.09	3.52
153.66	36	Immediate	132.60	—21.06	4.02
153.66	36	½ min.	136.26	—17.40	6.12
153.66	36	1 min.	132.60	—21.06	10.25

TABLE II.

Normal.	No. of Trials.	Length of Pause.	Average.	Const. Error.	Mean Variation.
50	64	Immediate	45.96	— 5.04	1.96
50	87	½ min.	45.31	— 5.69	2.40
50	87	1 min.	47.42	— 3.58	2.25
75.5	54	Immediate	67.33	—10.17	3.05
75.5	60	½ min.	65.7	—11.81	2.40
75.5	41	1 min.	96.59	—10.91	3.49
153.66	38	Immediate	131.96	—21.60	4.04
153.66	47	½ min.	134.79	—18.87	6.38
153.66	54	1 min.	129.47	—24.19	9.65

Results. (Taken from Table I.)

(1) The interval of half a second approximates to what has been called above, the unit of time. The constant error and mean variation are less at this interval than at any of the others. Here also the effect of pause is less than with intervals above it. The mean variation is the same for "immediate" and "1 min.", while the constant error is nearly the same for both.

(2) With the intervals above the unit of time the mean variation increases with the length of pause. Sensibility decreases with intervals above the unit.

(3) The constant error is practically the same throughout for "1 min." and "immediate."

(4) After the lapse of half a minute the results are not regular; with the longest normal interval the constant error is less than for "immediate," but with the other normal intervals it is greater. The mean variation is always greater for "half min." than for "immediate."

The results from Table II. do not vary much from the above.

IV.

WEBER'S LAW AND TIME ESTIMATION.

The limit of the sphere in which Weber's law holds accurately is still an open question. Sanford[1] says that it holds "tolerably" for medium stimuli of several kinds, with very large and small stimuli only imperfectly, and with some it cannot be demonstrated. Külpe[2] says that with pressure, auditory, and strain sensation, also with light intensities, a constancy of sensible discrimination is experienced. The scanty investigation of the senses of temperature, taste, and smell has not yet reached a stage at which definite conclusions can be drawn. Wundt[3] says that the law has been shown to hold for the intensity of sensations, and within certain limits for the comparison of extensive compounds, especially temporal ideas; also to some extent for spacial ideas of sight and for motor ideas. On the other hand it

[1] Sanford, *Experimental Psychology*, Ch. viii., p. 334.
[2] Külpe, *Outlines of Psychology*, p. 126.
[3] Wundt, *Outlines of Psychology*, p. 255.

does not hold for spacial ideas of external touch, obviously on account of the complexity of the local signs, and it cannot be verified for sensational qualities.

The above authorities do not differ much among themselves, but there is a great disparity in the direct testimony of those who have investigated the validity of the law in reference to time estimation; and on this account our own series of experiments was undertaken. The great difference in the testimony of earlier investigators has probably arisen from their having failed to take into account correctly the many disturbing factors which may vitiate the results of experiments.

One such factor is the general condition of the operator; fatigue or nervousness may be the source of much divergence in the testimony of subjects at different times. The state of attention also is a source of considerable variation; a concentrated, well-governed attention will give more accurate judgment than a musing, listless or uncontrolled attention. This factor has been treated in detail above. The method of registering experiments has led to erroneous results, as noted in connection with the work of Mach and Münsterberg. A lack of appreciation of the true function of rhythm in time estimation has led many experimenters to regard it as a factor which detracts from the accuracy of the estimation of intervals, instead of as a condition which makes the estimation of certain very short intervals possible. The effect of aesthetic factors too has been often misinterpreted, as has also the part played by the content in the estimation of larger intervals. These factors, which are all involved in the estimation of time intervals, are mentioned to show the many sources from which confusion may arise in the treatment of the question. But if these, and perhaps others not noted, could be carefully eliminated from the experiments, there would probable be no such disparity in the testimony of investigators as to the validity of Weber's law in the estimation of time intervals.

Ernst Mach, in a series of experiments which are neither fully nor definitely enough stated to sanction his conclusion, finds that Weber's law is not valid. His experiments were made from 1860 to 1865, extending over a period of five years, during which the conditions were not very uniform, his rather primitive methods of stimulation and recording being changed from time to time. He professes to have tested intervals from .016 sec. upward, but he only gives figures for a few of the intervals. Taking such a

small interval as the lower limit, and testing the validity of the law from the whole range of intervals, from one so very short to a long one, he could scarcely have come to any other conclusion, since such small intervals, even if they could be individually estimated, could not be recorded with any degree of accuracy. Hence little importance can be attached to the results at which he arrived.[1]

Karl Vierordt maintains that the law does not hold. He thinks that the fact of contrast between long and short intervals, making the former longer and the latter shorter, is sufficient to cause a discrepancy, and that this, together with some other factors, renders uniformity in time estimation impossible. Though Vierordt is right in maintaining that after a succession of short intervals a long one seems unduly long, still that fact is not sufficient to destroy the correctness and uniformity of time estimation. His experiments, moreover, were too limited in number to be very conclusive.[2] Volkmar Estel concludes with Mach and Vierordt that the law does not hold. He experimented on intervals ranging from 1.5 to 8 seconds, and his recording was done with the earlier form of Wundt's time-sense apparatus. His experiments were too scanty to be of real value; some of the intervals received only three trials by each subject, and few were treated more thoroughly. In addition to this, his work is coloured by such a strong controversial spirit, arising from his dispute with Vierordt as to the effect of a pause between the normal and comparative intervals, that his results cannot be greatly relied on.[3] Fechner subjects Estel's work to a severe criticism, and exposes its weakness. He himself thinks that nothing is to be discovered which is irreconcilable with Weber's law.[4] Mehner comes to a conclusion midway between Estel and Fechner; he finds that the law holds approximately above the interval of 7.1 seconds, where the sensibility is constant, but that below 7.1 seconds, where the sensibility is rhythmic, it does not hold. He maintains that in the results of Estel and others the disparity of results arises from lack of practice, strain of atten-

[1] *Untersuchen über den Zeitsinn des Ohres.* (Moleschott's *Untersuchungen*, 1866, p. 181.)

[2] *Der Zeitsinn.* (Tübingen, 1868).

[3] *Neue Versuche über den Zeitsinn.* (*Phil. Studien*, Vol. ii., p. 37.)

[4] *Über die Frage des Weber'schen Gesetzes und Periodicitätsgesetzes im Gebiet des Zeitsinns.* (*Abhandl. d. mathemat.-phys. Classe d. kgl. Sächs. Gesell. d. Wiss.*, Vol. viii. p. 3.)

tion, etc.[1] Richard Glass found that there was a tendency to follow the law,[2] and the results of Ejner also show that the increase of the average error with the length of the interval approximately follows Weber's law. A weakness in his experiments is that they were practically all conducted on one subject, but otherwise they were sufficiently comprehensive.[3] Thorkelson maintains that Weber's law is entirely valid throughout in the judgment of time intervals.

We proceed now to our own experiments. We first conducted a series, extending over four months, for the purpose of investigating the influence exerted on the reproduction of an interval by a lapse of time between the normal time magnitude and the recording of the comparative interval, the results of which are stated above.[4] The work on that problem practised us and gave us a thorough acquaintance with the apparatus used for recording the estimated length of intervals, and with the method of conducting experiments similar to those required for the testing of Weber's law. On this account a smaller number of trials was made in the latter investigation than would have been necessary to draw any valid conclusions, had the ground been entirely new. Our first aim in the experiments was to remove as far as possible all factors which might have a disturbing influence on the accuracy of the judgment of the subject; since these seem to have been responsible in many cases, probably to some extent in all the work of previous experimenters, for the disparity of their testimony in regard to the validity of Weber's law. To this end, in order to secure the same physical state, and avoid the influence of fatigue with its concomitants of musing, listless or laboured attention, the experiments in 1897-98 were made regularly between the hours of eleven and one. In the following academic year 1898-99 the experiments were made regularly between the hours of two and four in the afternoon. All disturbing factors which might interrupt the subject during the course of his work were removed, and there were, as far as could be ascertained, no special or unusual aesthetic influences present to bias his judgment.

[1] *Zur Lehre vom Zeitsinn.* (*Phil. Studien*, Vol. ii., p. 546.)

[2] *Kritisches und Experimentelles über den Zeitsinn.* (*Phil. Studien*, Vol. v., p. 423.)

[3] *Experimentelle Untersuchungen über den Zeitsinn.* (Inaugural Dissertation No. 137, Dorpat, 1889).

[4] *Supra*, p. 30.

For the last division of the three following tables, the method of visual stimulation and the apparatus for recording the estimation described above were used. But in the other divisions of the tables, where the intervals were too short to admit of the same system, an auditory stimulation was arranged, and also a modified method of recording. The stimulation was made by means of little pegs fitted into sockets corresponding to the holes in the band on the end of the drum through which the light flashes were reflected. As the drum revolved, these pegs, in passing, struck a small aluminum plate (A, Fig. 2), and so produced a series of regular clicks. The length of the interval between the clicks was varied by changing the number of pegs. The method of registering these intervals may be seen by referring to Fig. 2. Instead of the pointer described above for writing the records, another was substituted, having the same connection, and the same relation to the Deprez signal, but extending below the rods on which it travelled. This pointer (B, Fig. 2) moved on a pivot through its centre, so that the observer, by striking the lower end with his finger, made a slight movement of the other end of the pointer, which was in contact with the carboned paper, thus causing a slight divergence in the line and so recording his judgment. The pointer was immediately drawn back to its place by a spring.

In experiments which have been made with single, normal and comparative intervals, the reaction time of the subject materially reduces the accuracy of the judgment registered. This source of error was avoided in our experiments by making both the normal and the comparative consist of a series of immediately succeeding intervals. Four persons were tested on intervals ranging from 0.1 to 9 seconds. A large number of trials were made on each of the intervals, by each subject; in all six thousand nine hundred and thirty-five judgments were recorded. The exact distribution of the judgments and intervals may be seen from the tables below, in which the experiments are summarized. Throughout the experiments, on both short and long intervals, in order to get the interval definitely into consciousness, before recording the judgments, twenty practice reactions were made immediately before each series, followed by from twenty to forty reactions recorded on the drum of the kymograph. This plan was followed throughout the experiments, and it acted as a safeguard against the errors which so frequently arise from

diffused attention, and from the nervous confusion frequently involved in the first few reactions. The accompanying cut (Fig. 3) represents parts of three sample sheets taken from the drum of the kymograph, containing the tracings of a tuning-fork of one hundred vibrations to the second, by which the length of the interval was registered, and the tracings of the pointer which marked the limits of the estimated interval.

We give next a sample sheet containing the readings in detail of the length of each estimation of the interval, together with their average and mean variation, the latter representing the average error. In the tables that follow appear only the summed results of the different series of estimations made by each subject in the experiments.

Reading of Sheet 17, Observer K.

Visual stimulation. Two holes open in the drum of the kymograph, through which the light stimulation flashed to the observer at each revolution of the drum.

Three series of trials, a short stimulation each time, then twenty practice reactions with objective stimulation, followed by twenty reactions without objective stimulus, recorded on the carboned paper on the drum.

Normal interval 0.75 seconds.

Series I.			*Series II.*			*Series III.*		
68	−	$8\frac{2}{20}$	73	−	$2\frac{1}{18}$	83	+	$8\frac{1}{5}$
87	+	$10\frac{13}{18}$	73	−	2	75	+	
77	+		81	+	$5\frac{1}{18}$	68	−	$6\frac{1}{2}$
75	−	1	78	+	2	71	−	3
74	−	2	76	+		77	+	2
72	−	4	73	−	2	76	+	1
82	+	5	77	+	1	72	−	2
70	−	6	70	−	5	78	+	3
76	−		73	−	2	70	−	4
76	−		74	−	1	72	−	2
72	−	4	75	−		83	+	8
82	+	5	74	−	1	69	−	5
75	−	1	73	−	2	73	−	1
82	+	5	76	+		79	+	4
80	+	3	81	+	5	75	+	
73	−	3	75	−		75	+	
79	+	2	73	−	2	76	+	1
73	−	3	76	+		68	−	6
74	−	2				76	+	1
80	+	3	18)1351		18)$34\frac{2}{5}$	80	+	5
20)1527		20)$76\frac{7}{5}$	Average = $75\frac{1}{18}$		M.V. = $2\frac{29}{182}$	20)1496		20)$70\frac{4}{5}$
Average = $76\frac{7}{5}$		M.V. = $3\frac{33}{100}$				Average = $74\frac{4}{5}$		M.V. = $3\frac{54}{100}$

Table III contains the records of the estimations of observer K. In the first column the stimulus intensity is stated; in the second, the number of separate judgments that the subject made on the intervals; the third contains the length of the normal interval in terms of a thousandth of a second, the fourth, the length of the average estimation of that normal, also in terms of a thousandth of a second. The fifth column contains the constant errors for each series, the sixth, the mean variations, and the seventh, the relation of the mean variation to the normal in percentages. In order to show exactly the average deviation from a perfect equality in percentage of the mean variations, and hence the deviation from the absolute validity of the law, the mean variation of those percentages is worked out and placed in the last column in each of the divisions of the tables. The summary of results contained in these tables represents practically the whole of the estimations made, and not a few of the best, or those which best suited the theory of the experimenter.

Table III is divided into three divisions. The first division represents results of experiments made during the academic year of 1897-98. These experiments were made for the double purpose of testing Weber's law, and also to examine the influence of a change of intensity in the limiting impressions which marked the beginning and close of the intervals. In them an auditory stimulus was used. The second division records experiments similar to the first set, except that no change in the intensity of the limiting impressions was made, and these experiments were made in the following academic year, 1898-99. The third division contains the estimations of intervals of much greater length, and in these the limiting impressions were made by visual stimulation, instead of auditory as in the former series. The records in the table are arranged with the interval increasing constantly from the top to the bottom of the column, so that in reading down the column of the constant error the point of maximal accuracy may be easily noted where the constant error is smallest and just where the plus error of the short interval changes into the minus error of the long interval. That point seems to be what is called above the unit of time, and by this arrangement its relative length for the different subjects may be easily seen.

Table IV is a similar record of the estimations made by W. They were made in the same years and under circumstances similar to those which obtained in the case of K.

Table v differs from the above two only in that the first division was more comprehensive in the case of S., so that the second division which contained a number of series of experiments on approximately the same intervals as division one, in the above tables, was omitted in his case.

Table VI is a set of trials made by B. on the short intervals only, viz., from 99σ to 254σ.

The gentlemen who acted as subjects in these experiments, represented respectively as K., S., W., and B., are Professor Kirschmann, Mr. M. A. Shaw, Mr. F. S. Wrinch, and Mr. S. Blumberger. Here we wish to acknowledge with gratitude our indebtedness to Professor Kirschmann, under whose direction our work was done, for the generous and learned assistance tendered us, both in our study of the literature of the subject and in the conduct of the experiments.

TABLE III.—Observer K.

Division 1.

Stimulus Intensity.	Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the °/₀'s of the M. V's.
Low	120	93	113	+20	4.5	4.8	
High.....	120	93	114	+21	5.1	5.3	
Low	60	101	117	+16	3.6	4.8	
High.....	60	101	114	+13	4.8	4.5	
Low	60	125	128	+ 3	4.5	3.6	0.82%
High.....	60	125	135	+10	4.5	3.6	
Low	60	150	152	+ 2	5.1	3.4	
High.....	60	150	150	.....	7.9	5.2	
Low	60	187	189	+ 2	5.2	2.7	
High.....	60	187	190	+ 3	5.8	3.1	

Division 2.

Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the °/₀'s of the M. V's.
90	97	122	+25	5.10	5.28	
120	111	126	+15	5.60	5.05	
90	132	139	+ 6	4.96	3.75	0.81%
90	156	158	+ 2	6.49	4.19	
90	189	185	+ 4.52	6.48	3.41	
90	225	257	+ 5.05	5.58	2.48	

Division 3.

The following trials were made during the academic year 1896–97, on intervals of increasing length :

Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the %'s of the M. V's.
60	503	491	-12	15	3.11	0.99%
60	750	752	+ 2	31	4.15	
40	1500	1331	- 1.69	48	3.46	
60	2950	2802	- 1.48	183	6.21	6.67%
40	4600	5363	+ 7.63	803	17.46	
20	9050	10785	+17.55	1905	21.016	

NOTE.—In the estimations of the two longest intervals, viz., 4600σ and 9050σ, records of which appear in the last two lines of the table above, the mean variations are abnormally large, compared with estimations of the same length of interval made by the other two subjects at the same time; and also compared with estimations made by the same subject at a later date. These later estimations were made on long intervals divided up into unequal rhythmical divisions, and hence they are not stated here.* We give in the last column of this division the mean variation of the percentages of the mean variations of the first four intervals, which we regard as normal, in addition to the mean variation of the whole six intervals, which we regard as abnormal.

TABLE IV.—Observer W.

Division 1.

Stimulus Intensity.	Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the %'s of the M. V's.
Low	60	101	125	+24	7.8	7.2	0.92%
High.....	60	101	121	+20	6.	5.5	
Low	40	125	129	+ 4	6.1	4.8	
High.....	40	125	127	+ 2	6.5	5.2	
Low	40	150	167	+17	5.4	3.6	0.72%
High.....	40	150	159	+ 9	5.9	3.9	
Low	60	187	180	- 7	7.5	4.	
High.....	60	187	180	- 7	10.9	5.8	

* See below, p. 47 *et seq.*

Division 2.

As in the previous table the above intervals were also investigated again in the next year, with results as follows:

Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the %'s of the M. V's.
150	98	131	+33	8.38	8.59	
90	108	136	+28	8.67	8.03	
90	128	149	+21	7.93	6.19	1.51%
90	157	149	− 8	5.79	3.69	0.67%*
180	197	181	−16	9.71	4.93	
90	250	248	− 2	12.75	5.1	

* In Division 2 the first two intervals, i.e., 98σ and 108σ, are regarded as abnormal (see below). On this account we have given the mean variation of the percentages of the last four intervals, in which the recording was satisfactory, as well as that of the whole six.

Division 3.

The following trials were made during the academic year of 1896-97 on longer intervals:

Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the %'s of the M. V's.
60	500	512	+ 12	34	4.48	
40	750	730	− 20	49	3.89	
40	1490	1414	− 76	47	2.98	0.727%
40	3030	2895	−135	142	4.47	
40	4500	4454	− 46	102	2.28	
20	9150	9825	+675	376	4.02	

TABLE V.—Observer S.

Division 1.

Stimulus Intensity.	Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the %'s of the M. V's.
Low	180	107	141	+34	5.2	4.8	
High.....	180	107	140	+33	4.3	4.	
Low	180	125	152	+27	4.2	3.4	
High.....	180	125	150	+25	5.1	4.1	
Low	80	150	167	+17	6.8	4.5	0.62%
High.....	80	150	176	+26	6.	4.	
Low	120	187	188	+1	7.7	4.1	
High.....	120	187	186	−1	9.6	5.1	
Low	120	214	213	−1	6.3	2.5	
High.....	120	214	212	−2	6.7	2.8	

Division 2.

Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the %'s of the M. V's.
60	500	497	− 3	[illegible]	6.4	1.5%
60	745	728	− 17	[illegible]	5.2	
40	1505	1251	−254	74.4	4.9	
40	2920	3271	+351	238.8	8.1	
20	4500	4444	− 56	351.4	7.7	
20	8870	7884	−986	633.4	7.5	

TABLE VI.—Observer B.

Number of trials.	Normal.	Average estimation.	C. E.	M. V.	% of M. V.	M. V. of the %'s of the M. V's.
90	99	151	+52	8.41	8.45	1.73%
90	109	158	+49	7.25	6.63	
90	128	169	+41	8.07	6.29	
90	153	164	+11	6.65	4.34	1.34%
90	192	195	+ 3	7.3	3.71	
90	254	249	− 5	7.42	2.95	

Subject B. also had difficulty in registering the shortest intervals as fast as he heard them. On this account in the last column of his records the mean variation of the percentages of the last five intervals is given as well as that for the whole six.

We proceed now to the results indicated by the above tables. The effect produced by changing the intensity of the limiting impression has already been noted. In the trials made by K. the law of Weber holds very closely throughout all the intervals investigated except the last two (Table III., Div. 3). In these two intervals, 4600σ and 9050σ, the percentage of the mean variation is about four times larger than the average percentage for the other intervals, and the constant error is proportionately large, and is plus, where, according to the tendency shown in his other judgments and also in the judgments of other subjects on this same interval, it should be minus. This anomaly in the case of the two longest intervals very probably arose from the fact that the subject, K., in his anxiety to exclude any external factor which

might influence his estimation of the interval, interrupted the regularity of the organic functions which naturally make up, at least in part, the content of longer intervals. Hence in attempting to exclude this content, which normally has its part in the estimation of longer intervals, his judgments became abnormal, and in addition to this source of error, which in a measure accounts for the irregularity, the subject was somewhat distracted by the pressure of other duties[1] at the time when these experiments were made. Leaving out those two intervals then, we find that the mean variation of the whole of the percentages of the mean variations for the various intervals in all three divisions of the trials made by K. amounts to less than one, the mean variation for the first division being 0.82, for the second 0.81, and for the third, apart from the two last intervals, only 0.99. The very close uniformity in the results of the different divisions of K.'s estimations has greater significance from the fact that the experiments were made in successive years—division three in 1896-97, division one in 1897-98, and division two in 1898-99. This circumstance, together with the fact that the experiments of division three were made with a visual stimulation and those of divisions one and two with auditory stimuli, and also the large number of judgments, speak very conclusively for the validity of Weber's law.

In the shorter intervals K. uniformly reproduces the comparative interval longer than the normal. Up to 225σ there are none but plus signs in the constant errors. Above that, the intervals, with the exception mentioned above, are almost uniformly judged shorter. At the position where the constant error changes from plus to minus, the percentage of the mean variation is smallest, though in K.'s results there is no great variation in the percentages of the mean variation. K.'s unit of time lies probably not much above 225σ, since at 503σ the constant error has changed very considerably from plus to minus.

The testimony of W. does not differ much from that of K. In the second division of Table IV., in the first two intervals, as well as in the first interval of division one of the same Table, W. was conscious of being unable to react fast enough to record the intervals as he heard them. This accounts for the marked and abnormal increase in the percentage of the mean variation for

[1] Annual Examinations.

these intervals. Apart from these shortest intervals, the mean variation of the percentages of the mean variations differs very slightly in the three groups, being for the first division 0.72, for the second 0.67, and for the third 0.72. These are even more uniform than the results of the trials of K., and the amount of the variation in the percentages is smaller. The circumstances under which the experiments were made were similar in the case of both subjects, the different divisions being made in different years, and one with visual stimulation, the other with auditory. Hence in the results of W.'s estimations the validity of Weber's law is more accurately established than in those of K., although the absolute difference in the two results is very small, so that their testimony is practically at one. The sign of the constant error changes from plus to minus at about the same position in both the groups of experiments, of which one was made in 1897-98, the other in 1898-99. The comparative interval is reproduced longer almost without exception up to a normal of 150σ, above that with a couple of exceptions it is reproduced shorter. The mean variation also reaches its minimum at the same point, which being the point of greatest accuracy in judgment indicates the length of W.'s unit of time. This unit for W. is much shorter than for K., and much shorter than the usual point of maximal accuracy, which lies between 200σ and 300σ. But the results of our experiments do not justify the theory that the point of maximal accuracy in the estimation of intervals is at all uniform; on the contrary it appeared to differ considerably in different individuals.

In Table v., which records the judgments made by S., exceptions similar to those noted above, in the longest intervals of K.'s work and in the shortest of W.'s, do not appear. The mean variation of the percentages of the mean variations of the several intervals tested in division one is only 0.62, an amount smaller than in the case of either K. or W., and the work in that division was very comprehensive, comprising over 1350 separate estimations. In the second division the variation is a little larger though still comparatively small, being only 1.5. Hence S.'s estimations confirm the testimony of the two previous subjects to the validity of the psychophysical law. S.'s time unit is quite definitely marked. It lies between the units of the previous two subjects, a little longer than that of W. and much

shorter than K.'s, at about 187σ. There are almost no exceptions to the longer reproductions of the comparative in the intervals shorter than 187σ, and at only one point in the longer intervals is the average reproduction in the comparative not shorter than the normal.

B. did not complete the whole series of intervals which were investigated by the other three subjects. But in the trials made on shorter intervals, recorded in Table VI., his results do not vary much from the testimony of the others. He also, as in th case of W., found difficulty in recording the shortest interval viz., 99σ, as fast as he heard the limiting impressions, and hence we omit the record of that interval in calculating the mean variation of the percentages of the mean variations. The variation in the percentages of B.'s trials is a little larger than for the other subjects, but it is still comparatively small and points in the same direction, towards the validity of Weber's law. So far as relates to the intervals with which B. worked, his time unit is quite definitely marked. It lies at about 200σ, longer than that of W. and S., but still considerably shorter than K.'s.

To sum up the results of our experiments in brief—the testimony of the whole four subjects inclines clearly towards a confirmation of the validity of the law of Weber, at least within the limits covered by the intervals investigated, viz., from 93σ up to 9150σ. The mean variation of the percentages of the mean variations for all the intervals in the 1600 judgments made by K. is only 0.87, in the 1330 judgments made by W. it is only 0.70, in the 1600 judgments made by S. it is only 1.06, and in the 540 judgments made by B. it is 1.34. As to the comprehensiveness of the ground covered by the intervals tested, our apparatus, as noted above, is arranged so that with the least possible exertion the subject records his judgments; and since with this delicately adjusted apparatus the subjects found it difficult or impossible to record the smallest intervals as quickly as they heard the limiting impressions, we may justly assume that the series starts with the shortest intervals for which it is possible to record judgments. Also, in the longest intervals of the series a point is reached where the judgment depends very largely on the content of the interval; hence if longer intervals were tested, the same principle would be involved in their estimation, and the same results would thus naturally ensue. Thus we think that the results of

the investigation of the above series of intervals practically cover the whole scope of time estimation in as far as it is accessible to a direct test of Weber's law. With each of the subjects the point of maximal accuracy of judgment, or the time unit, is quite definitely marked, and does not vary materially in the experiments made by the same subject in different years. But this point differs in its position for each of the subjects; for K. it was an interval of about 300σ, for B. about 250σ, for S. 200σ, and for W. about 175σ. Thus the time unit differs in length for different individuals, but it is a definitely marked interval in each case.

V.

REPRODUCTION OF COMPLEX RHYTHMICALLY ARRANGED GROUPS OF INTERVALS.

A further series of experiments was undertaken to investigate the degree of accuracy that obtains in the reproduction of complex rhythmically arranged groups of intervals of unequal length. This series was also undertaken in part for the purpose of testing further Weber's law, by applying it to somewhat varied conditions of reproduction. In these experiments, instead of using as stimulus a uniform series of intervals, limited by visual or auditory impressions, as was done in the previous experiments, an unequal series was used. In the first two groups for each of the three subjects the stimulus was given by clicks made as before, but in this case one of the intervals between the clicks was twice as long as each of the others. In the remaining groups of intervals, which were of increasing complexity, no mechanical stimulation was given. Each subject chose some grouping which was familiar to him, and reproduced it a certain number of times, making the records on the kymograph as described above for intervals limited by auditory impressions. The groups of intervals were chosen quite arbitrarily; whatever was familiar to the subject and sufficiently complex to satisfy the requirements of the experiment was allowed, *e.g.*, the rhythm of the beats of a drum, a couple of bars of some popular air, or a group made up impromptu and easily retained in consciousness.

Before inserting the tables which contain the summed results of the experiments, one sheet of the readings in full of such a series is given. A cut of the records on one of the sheets made in this series, taken from the drum of the kymograph, is also included in Fig. 3 above.

Table VII. contains the records of judgments made by K. It is almost self-explanatory. There are eight groups of intervals, the simplest containing only three intervals, the most complex nineteen. The first line in each group shows the number of intervals and the number of judgments made on each. The second gives the average length, in order, of each of the respective intervals of the group. The third line gives the amount of the mean variation of each of the intervals, and the fourth the relation in percentages which the mean variations bear to the length of the intervals. This last is included for convenience in comparing the relative accuracy of the judgments on the different lengths of intervals. In the last line is given the average of the total duration of all the intervals of the group summed, the total mean variation, and the percentage of the mean variation. Tables VIII. and IX. are similarly arranged records of the judgments made by the other two subjects, W. and B. The length of all the intervals in these, as in the other tables, is stated in terms of one thousandth of a second.

(One sheet of the readings in full.)

Observer K.

Stimulation, twenty intervals from the "Boccaccio March."

435	340	115	250	255	455	440	315	135	775
445	310	125	230	240	435	435	320	145	760
435	310	120	240	240	435	450	325	140	785
435	320	110	235	245	465	420	300	130	830
430	330	120	235	220	435	435	300	120	775
445	320	110	245	240	435	435	310	140	785
455	340	95	255	240	435	440	325	125	785
440	325	115	255	230	440	440	320	145	840
445	325	120	250	235	450	435	350	125	780
465	330	125	265	210	435	420	330	130	810
10)4430	3250	1155	2460	2355	4430	4355	3195	1335	7925
443.	325.	115.5	246.	235.5	443.	435.5	319.5	133.5	792.5
M. V. = 8.	8.	6.5	9.	9.4	10.2	5.6	5.6	7.5	20.5
% 1.8	2.5	5.6	3.6	3.9	2.2	1.2	1.7	5.6	3.5

Total duration of the 10 intervals = 3489σ.

Total mean variation = 90.3 = 2.5%.

Mean variation of the percentages of the M. V.'s = 1.25.

TABLE VII.—Observer K.

Group 1 :—No. of intervals=3. No. of judgments on each=13.

Average length.....	379	192	182
Mean variation.....	11.98	8.28	4.08
% of M. V	3.1	4.3	2.2

Total duration of group 752σ. Total M. V. 24.26=3.2%.

Group 2 :—No. of intervals=3. No. of judgments on each=14.

Average length.....	402	189	189
Mean variation.....	16.53	5.71	7.55
% of M. V	4.2	3	4

Total duration of group 772σ. Total M. V. 29.79=3.8%

Group 3 :—No. of intervals=5. No. of judgments on each=17.

Average length.....	284	288	130	137	352
Mean variation.....	13.04	12.56	10.59	14.58	18.37
% of M. V.....	4.6	4.3	8.1	10.6	5.2

Total duration of group 1192σ. Total M.V. 69.15=5.8%.

Group 4 :—No. of intervals=5. No. of judgments on each=22.

Average length.....	548	546	522	267	264
Mean variation.....	24.21	20.45	16.05	9.77	10.2
% of M. V	4.4	3.7	3.1	3.6	3.8

Total duration of group 2103σ. Total M. V. 80.6=3.8%.

Group 5 :—No. of intervals=7. No. of judgments on each=15.

Average length.....	374	128	287	258	263	263	544
Mean variation.....	12.8	15.24	16.44	10.8	10.93	8.91	30.13
% of M. V	3.1	4.3	2.2	4.2	4.1	3.4	5.5

Total duration of group 2117σ. Total M. V. 105.27=4.9%.

Group 6 :—No of intervals=8. No. of judgments on each=12.

Average length.....	212	560	214	617	203	235	218	623
Mean variation.....	15	38.33	12.43	28.89	15.27	12.15	11.25	31.38
% of M. V	7.1	6.8	5.9	4.7	7.4	5.2	5.1	5.1

Total duration of group 2886σ. Total M. V. 164.72=5.7%.

Group 7 : No. of intervals=10. No. of judgments on each=10.

Average length..	443	325	115	246	235	443	435	319	133	792
Mean variation..	8	8	6.5	9	9.4	10.2	5.6	5.6	7.5	20.5
% of M. V	1.8	2.5	5.6	3.6	3.9	2.2	1.2	1.7	5.6	3.5

Total duration of group 3489σ. Total M. V. 90.3=2.5%.

Mean variation of the percentages of the M. V.'s=1.25.

Group 8 :—No. of intervals = 19. No. of judgments on each = 6.

Average length.....	287	311	131	130	328	291	286	121	128	
Mean variation.....	10.55	22.77	17.77	11.11	25	11.11	10.55	5	11.5	
% of M. V.....	3.7	7.3	13.6	3.8	7.5	3.8	3.7	4.1	10	
Average length..	339	288	297	284	294	294	286	127	130	337
Mean variation..	12.22	5.55	7.5	9.66	6.5	5.55	13.88	4.15	5	10
% of M. V.....	3.6	1.9	2.5	3.4	2.5	1.9	4.8	3.3	3.8	3

Total duration of group 4697σ. Total M. V. 207.42 = 4.4%.

Table VIII.—Observer W.

Group 1 :—No. of intervals = 3. No. of judgments on each = 16.

Average length.....	181	198	414
Mean variation.....	3.48	11.72	30.03
% of M. V.....	1.9	5.9	7.3

Total duration of group 789σ. Total M. V. 45.23 = 5.7%.

Group 2 :—No. of intervals = 3. No. of judgments on each = 18.

Average length.....	191	197	359
Mean variation.....	9.16	7.22	27.46
% of M. V.....	4.8	3.6	7.6

Total duration of group 746σ. Total M. V. 43.85 = 5.8%.

Group 3 :—Number of intervals = 5. No. of judgments on each = 15.

Average length.....	557	601	564	285	308
Mean variation.....	18.75	22.80	27.15	13.02	8.44
% of M. V.....	3.4	3.7	4.8	4.5	2.7

Total duration of group 2311σ. Total M. V. 90.16 = 3.9%.

Group 4 :—No. of intervals 7. No. of judgments on each = 13.

Average length.....	645	199	165	356	335	358	673
Mean variation.....	21.14	10.94	7.39	17.57	8.16	12.25	28.58
% of M. V.....	3.3	5.5	4.4	4.9	2.4	3.4	4.2

Total duration of group 2733σ. Total M. V. 106.05 = 3.8%.

Group 5 :—No. of intervals = 8. No. of judgments on each = 12.

Average length.....	388	199	180	393	387	378	405	696
Mean variation.....	22.08	14.02	7.22	12.22	22.5	16.11	9.65	22.36
% of M. V.....	5.7	7.1	4	3.1	5.8	4.2	2.4	3.2

Total duration of group 3029σ. Total M. V. 126.17 = 4.1%.

Group 6 :—No. of intervals = 8. No. of judgments on each = 9.

Average length.....	499	268	935	198	201	277	654	722
Mean variation.....	24.56	16.42	51.11	4.07	9.01	14.56	31.23	23.70
% of M. V.....	4.9	6.1	5.4	2.1	4.4	5.2	4.7	3.2

Total duration of group 3752σ. Total M. V. 174.68 = 4.6%.

Group 7 :—No. of intervals = 10. No. of judgments on each = 7.

Average length..	667	473	223	597	427	264	688	175	214	932
Mean variation..	36.53	32.24	16.94	23.47	20.2	15.1	17.09	2.85	12.04	74.89
% of M. V.....	5.5	6.8	7.3	3.9	4.7	5.7	2.5	1.6	5.6	8

Total duration of group 4660σ. Total M. V. 256.37 = 5.4%.

Group 8 :—No. of intervals = 15. No. of judgments on each = 8.

Average length.....	517	340	165	785	528	331	175	642	195
Mean variation	21.87	22.5	8.75	46.87	10	16.15	4.37	69.37	12.03
% of M. V.....	4.2	6.6	5.3	5.9	1.9	4.9	2.5	10.8	6.2
Average length.....	360	180	507	514	520	919			
Mean variation.....	6.25	6.25	13.12	14.53	17.5	27.5			
% of M. V.....	3.1	6.2	2.5	2.8	3.3	5.5			

Total duration of group 6683σ. Total M. V. 332.31 = 4.9%.

TABLE IX.—Observer B.

Group 1 :—No. of intervals = 3. No. of judgments on each = 21.

Average length.....	373	198	178
Mean variation.....	17.19	7.17	8.02
% of M. V.....	4.6	3.6	4.4

Total duration of group 751σ. Total M. V. 32.39 = 4.3%.

Group 2 :—No. of intervals = 3. No. of judgments on each = 22.

Average length.....	330	187	170
Mean variation.....	20.08	7.72	9.54
% of M. V.....	6.1	4.1	5.6

Total duration of group 693σ. Total M. V. 37.35 = 5.4%.

Group 3 :—No. of intervals = 5. No. of judgments on each = 16.

Average length.....	527	471	252	254	500
Mean variation.....	20.51	18.12	9.61	8.98	32.18
% of M. V.....	3.8	3.8	3.8	3.5	6.7

Total duration of group 2005σ. Total M. V. 89.41 = 4.5%.

Group 4 :—No. of intervals = 7. No. of judgments on each = 12.

Average length.....	393	214	203	433	408	369	453
Mean variation.....	25.41	26.18	31.04	29.16	29.58	22.91	33.12
% of M. V.....	6.4	12.2	15.2	6.7	7.2	6.1	7.3

Total duration of group 2476σ. Total M. V. 197.42 = 7.9%.

Group 5 :—No. of intervals = 8. No. of judgments on each = 15.

Average length.....	400	290	530	224	198	228	557	588
Mean variation.....	22.76	19.33	43.33	9.86	12	11.82	39.77	32.13
% of M. V.....	5.6	6.6	8.1	4.5	6	5.2	7.1	5.4

Total duration of group 2616σ. Total M. V. 192.52 = 7.3%.

Group 6 :—No. of intervals = 15. No. of judgments on each = 7.

Average length.....	403	312	178	706	391	313	177	554	188
Mean variation.....	16.53	14.69	9.18	47.96	18.98	20.61	13.06	26.32	13.87
% of M. V.....	4.1	4.7	5.1	6.7	4.8	6.3	7.3	4.7	7.3
Average length....	317	175	403	414	424	661			
Mean variation....	18.16	8.57	14.49	23.47	24.39	47.22			
% M. V.....	5.7	4.9	3.6	5.6	5.8	7.1			

Total duration of group 5625σ. Total M. V. 318.02 = 5.6%.

To turn now to the results of the experiments. We have seen that in reproducing one uniform interval in a successive series, the average mean variation in the whole of K.'s judgments was about 4.1 per cent., and that there was not much deviation from this mean in intervals of different length (cf. Table III.). In the present experiments, where instead of a series of reproductions of one interval of uniform length a series of groups of intervals of unequal length rhythmically arranged is reproduced, the average mean variation for all the groups and intervals is 4.2 per cent., which is only one-tenth per cent. greater than for the simple interval. Although there is some deviation from this average in individual intervals, still in the averages for the whole groups the mean variation is very uniform. This is another illustration of a phenomenon noted by Pringle in experimental investigation of poetical metre,[1] where there was a distinct tendency to complete a defective line, so as to make a time unit for the respective lines in a stanza. The phenomenon in these experiments shows itself in the fact that although there is a deviation within a narrow limit among the individual intervals of the more complicated groups, there is very little deviation in the average mean variation of each complete group.

The relation of the different lengths of intervals to one another throughout the judgments of the different groups is very constant, there being no distinct or regular falling off in the accuracy from the first reproduction of the group to the last. This is well illustrated in the detailed readings of the sheet on page 46, which is a fair average representative of the other sheets. There is if anything rather an increase in accuracy than a falling off; in the above sheet the last two reproductions of the group are a little more constant, and a little nearer the average, than the first two. We do not wish to infer from this that there is

[1] The records of his work are preserved in the archives of the Psychological Laboratory, University of Toronto.

a constant increase in accuracy in the reproduction of the groups, but that within certain limits they are comparatively uniform.

Referring again to the above sheet we find in the ten intervals four distinct types. The first is about 440σ in length, the second about 320σ, the third about 120σ, and the fourth about 240σ. The last interval in the group, averaging 792σ, is probably in part composed of the pause at the end of the line, and thus it does not represent a definite length of note used in the composition. In the above four typical lengths of intervals, there is not the definite relation of one, a half, a fourth and an eight, as in written musical compositions. The shortest interval, 120σ, is about half of the next shortest, but it is not a third of the next, nor a fourth of the longest. The subject who made the judgments in the above sheet is neither a true musical artist, nor a mechanical musician who would be dominated by the measure written on the sheet of music before him, but one who thoroughly appreciates melody and rhythm. Our experiments do not justify any definite conclusion as to whether the relative lengths of intervals in musical rhythm, as represented in the interpretations of the true artist or of the appreciative reproducer, are the same as are represented on the written sheet. Such a conclusion would need the support of a special set of investigations. But the indications from our own experiments are that the relative length of musical notes in the production of a true musician is not in the exact ratio of one to a half, a quarter, an eighth, and so on. A further point illustrated by the above sheet is the indifference of the length of the interval to the pitch of the tones composing the melody. The intervals recorded on this sheet are intended to represent twenty intervals from the "Boccaccio March;" the rhythm in the first ten is similar to that of the second ten, but the pitch of the tones is quite different. In the records the second group of ten is placed directly under the first, and in that order through all the judgments made; there is no noticeable regular difference between the lengths of the intervals in the respective groups, notwithstanding the great difference of pitch.

In the first two groups of the table, containing three intervals each, a mechanical auditory stimulation was used to give the subject the rhythm, but this stimulation ceased before he began to record. In the physical stimulation one interval was equal in

length to the sum of the other two, and it was repeated in a regular succession so that either short or long interval might come first. The judgments of K. in the first group come definitely in the rhythm of the dactyl, *i.e.*, the long interval comes first, retaining its relation as almost exactly equal to the sum of the other two, the second interval is lengthened at the expense of the third, being 10σ longer, and the third is the shortest. This is the normal relation of the intervals of the dactyl, the second interval being longer than the third, though the relative lengths be not quite those given above.[1] The second group does not fall quite so definitely into dactylic rhythm, since on the average the two shorter intervals are exactly equal, but in the individual judgments seven out of fourteen are in the dactylic rhythm and two have the two shorter intervals equal. So that with K. there is a marked tendency to throw groups of three intervals, one long and two short, into the rhythm of the dactylic poetic foot.

The other Tables, VIII. and IX., very generally confirm the tendencies noted in connection with Table VII., with the exception of the last point, in reference to the dactylic form of the grouping of the three intervals. In regard to the comparative accuracy of the successive reproduction of the single uniform interval and of the groups of unequal rhythmically arranged intervals, with K. there was an average difference of one-tenth per cent. in favour of the accuracy of the judgment of the single interval; the average percentage of the mean variation for the latter was 4.1 per cent., and for the groups 4.2 per cent. For W. the average difference is three-tenths per cent., being in the one case 4.4 per cent., and in the other 4.7 per cent. The difference for observer B. is greater; for the single interval his average was 4.8, and for the groups 5.8. But the work of B. was not so comprehensive as that of the other two subjects, either in the number of judgments made or in the scope of the intervals tested, so that his testimony is not so reliable as theirs.

In the groups of three intervals, where one was equal in length to the sum of the other two, W.'s results differ from those of K. Instead of the dactylic rhythm, W. definitely relates them in the rhythm of the anapæstic foot. In the first group his

[1] Cf. Hurst and McKay, *Experiments on the Time Relations of Poetical Metres*, *infra*.

judgments are 181σ, 198σ and 414σ, and in the second 191σ, 197σ and 359σ. In both cases the longer of the two shorter intervals comes before the longest instead of after. B. again in these groups reverts to the same order that was noted in K.'s judgments, but they are more pronounced in their dactylic rhythm than were his, since in both of B.'s groups the longer of the two shorter intervals comes after the longest, and is about 20σ longer than the shortest. What is involved for the question of poetic metre in this fact, that one subject hears a certain series of sounds in the order of the anapæst, and that another hears the same series in the order of a dactyl, we leave for a more extended investigation of poetic rhythm.

The results in the reproduction of these grouped intervals do not vary much in their testimony to the validity of Weber's law from the results of the previous series of experiments. With the interval represented by the sum of each of the groups, they are not quite so uniform in K.'s judgments as in the previous experiments. But, in spite of little irregularities, for the whole of the percentages of the mean variations of the eight groups of K.'s judgments, the mean variation is only 1.14. For the eight groups of W.'s judgments the figures are just as uniform as in his earlier experiments, the mean variation of the percentages of the eight groups being only 0.67. For B. it was a little larger, 1.12. In the individual intervals of the several groups of each subject there were some cases of irregularity which would seem to vary the testimony, but these are so few as to make it very probable that they have their source in some irregularity in recording the judgments, or in some slight confusion on the part of the subject. Generally the percentages of the mean variation for the different lengths of intervals in the groups vary little, the average length of deviation in many of them being less than one per cent. Thus in this series of experiments, as well as in the previous one, the results point quite definitely towards the validity in time estimation of the relation stated in Weber's law.