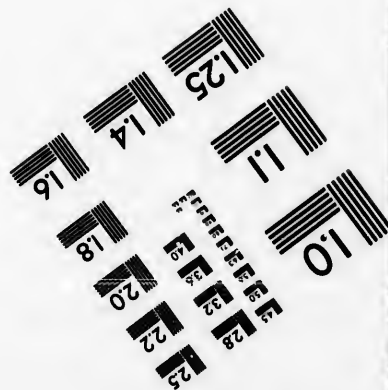
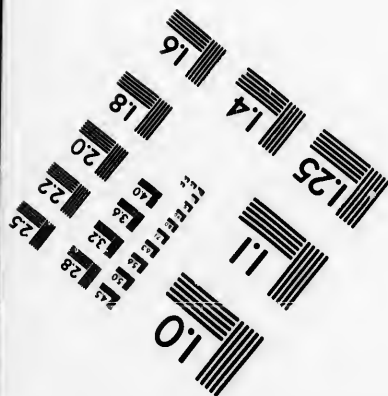
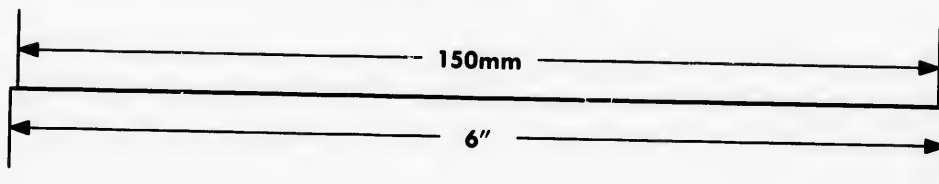
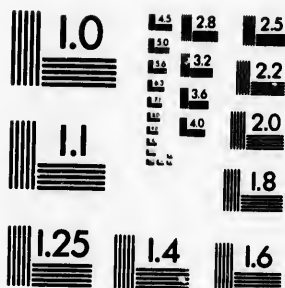
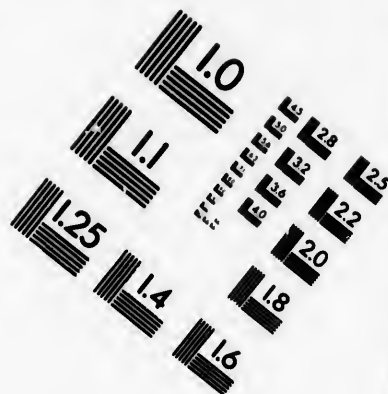
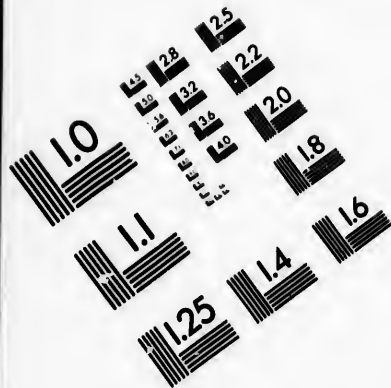


IMAGE EVALUATION TEST TARGET (MT-3)



APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc., All Rights Reserved

**CIHM
Microfiche
Series
(Monographs)**

**ICMH
Collection de
microfiches
(monographies)**



Canadian Institute for Historical Microreproductions / Institut canadien de microreproductions historiques

© 1993

Technical and Bibliographic Notes / Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

- ☒ Coloured covers/
Couverture de couleur
- ☐ Covers damaged/
Couverture endommagée
- ☐ Covers restored and/or laminated/
Couverture restaurée et/ou pelliculée
- ☐ Cover title missing/
Le titre de couverture manque
- ☐ Coloured maps/
Cartes géographiques en couleur
- ☐ Coloured ink (i.e. other than blue or black)/
Encre de couleur (i.e. autre que bleue ou noire)
- ☐ Coloured plates and/or illustrations/
Planches et/ou illustrations en couleur
- ☐ Bound with other material/
Relié avec d'autres documents
- ☐ Tight binding may cause shadows or distortion
along interior margin/
La reliure serrée peut causer de l'ombre ou de la
distorsion le long de la marge intérieure
- ☐ Blank leaves added during restoration may appear
within the text. Whenever possible, these have
been omitted from filming/
Il se peut que certaines pages blanches ajoutées
lors d'une restauration apparaissent dans le texte,
mais, lorsque cela était possible, ces pages n'ont
pas été filmées.

- ☐ Coloured pages/
Pages de couleur
- ☐ Pages damaged/
Pages endommagées
- ☐ Pages restored and/or laminated/
Pages restaurées et/ou pelliculées
- ☒ Pages discoloured, stained or foxed/
Pages décolorées, tachetées ou piquées
- ☐ Pages detached/
Pages détachées
- ☒ Showthrough/
Transparence
- ☐ Quality of print varies/
Qualité inégale de l'impression
- ☐ Continuous pagination/
Pagination continue
- ☐ Includes index(es)/
Comprend un (des) index

Title on header taken from: /
Le titre de l'en-tête provient:

- ☐ Title page of issue/
Page de titre de la livraison
- ☐ Caption of issue/
Titre de départ de la livraison
- ☐ Masthead/
Générique (périodiques) de la livraison

- ☐ Additional comments: /
Commentaires supplémentaires:

This item is filmed at the reduction ratio checked below /
Ce document est filmé au taux de réduction indiqué ci-dessous.

10X	12X	14X	16X	18X	20X	22X	24X	26X	28X	30X	32X
					/						

The copy filmed here has been reproduced thanks to the generosity of:

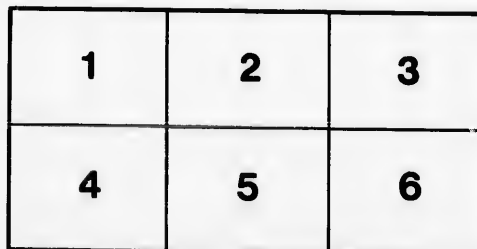
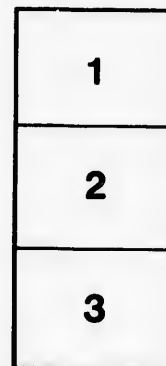
National Library of Canada

The images appearing here are the best quality possible considering the condition and legibility of the original copy and in keeping with the filming contract specifications.

Original copies in printed paper covers are filmed beginning with the front cover and ending on the last page with a printed or illustrated impression, or the back cover when appropriate. All other original copies are filmed beginning on the first page with a printed or illustrated impression, and ending on the last page with a printed or illustrated impression.

The last recorded frame on each microfiche shall contain the symbol → (meaning "CONTINUED"), or the symbol ▼ (meaning "END"), whichever applies.

Maps, plates, charts, etc., may be filmed at different reduction ratios. Those too large to be entirely included in one exposure are filmed beginning in the upper left hand corner, left to right and top to bottom, as many frames as required. The following diagrams illustrate the method:



L'exemplaire filmé fut reproduit grâce à la générosité de:

Bibliothèque nationale du Canada

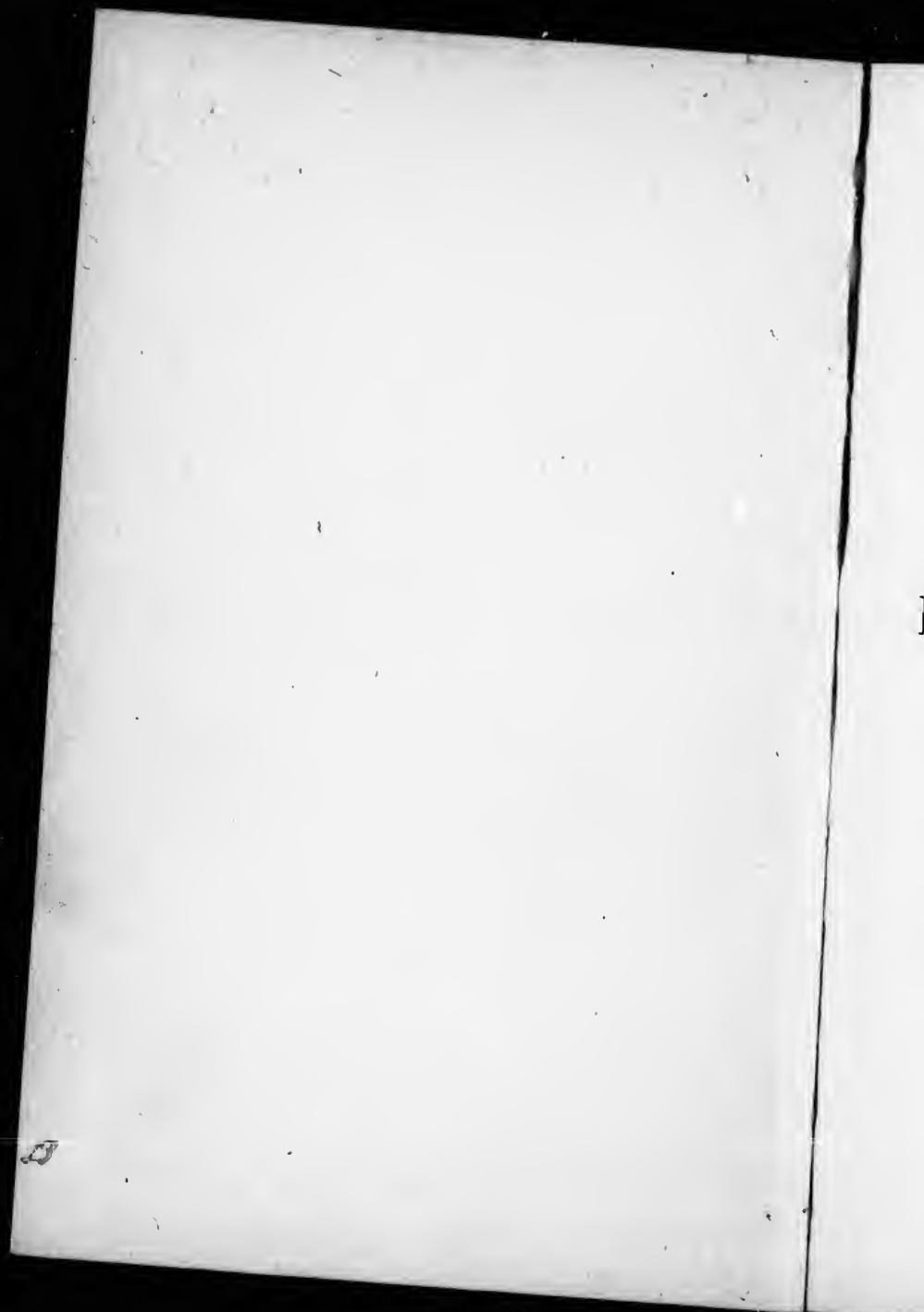
Les images suivantes ont été reproduites avec le plus grand soin, compte tenu de la condition et de la netteté de l'exemplaire filmé, et en conformité avec les conditions du contrat de filmage.

Les exemplaires originaux dont la couverture en papier est imprimée sont filmés en commençant par le premier plat et en terminant soit par la dernière page qui comporte une empreinte d'impression ou d'illustration, soit par le second plat, selon le cas. Tous les autres exemplaires originaux sont filmés en commençant par la première page qui comporte une empreinte d'impression ou d'illustration et en terminant par la dernière page qui comporte une telle empreinte.

Un des symboles suivants apparaîtra sur la dernière image de chaque microfiche, selon le cas: le symbole → signifie "A SUIVRE", le symbole ▼ signifie "FIN".

Les cartes, planches, tableaux, etc., peuvent être filmés à des taux de réduction différents. Lorsque le document est trop grand pour être reproduit en un seul cliché, il est filmé à partir de l'angle supérieur gauche, de gauche à droite, et de haut en bas, en prenant le nombre d'images nécessaire. Les diagrammes suivants illustrent la méthode.





A KEY
TO THE
SOLUTIONS OF PROBLEMS
IN THE
HIGH SCHOOL ARITHMETIC.

TORONTO:
THE G. M. ROSE & SONS COMPANY, LIMITED.
1898.

QA139

K49

Entered according to Act of the Parliament of Canada, in the year one thousand eight hundred and ninety-eight, by THE G. M. ROSE & SONS COMPANY, LIMITED, at the Department of Agriculture.

**THIS KEY to the High School Arithmetic has been
prepared for the use of teachers.**

QA1

K4

-
e
c
-

+

16:

7

the

8

9

= 1

+ 8

10

11

spec

12

m =

SOLUTIONS OF PROBLEMS

IN THE

HIGH SCHOOL ARITHMETIC.

METRIC SYSTEM.

95

1. $1000 \div 100 = \&c.$; $1000 \div 10 = \&c.$; $1000 \div 100 \div 10 = \&c.$
2. $1000 \div 1000$; $1000 \div 10$; 1000×10 .
3. 12×100 ; 15×1000 ; $21 \times 1000 \times 1000$.
4. $123456789 \div 100$; $123456789 \div 1000$; $123456789 \div 1000 \div 1000$.
5. $8.56 \times 1000 \times 100$; 5.632×1000 ; 12468×1000 .

96

6. Expressed in metres these quantities are 14.6, 2.27, 1623, 1634000, then add.
7. Expressed in c.m. these quantities are 567800, 13648.9, then subtract.
8. 12 Km 5 m 8 cm = 12.00508 Km, then multiply by 8×12 .
9. 103 Km = 10300000 cm. $\therefore 1 \text{ yd.} = 64 \text{ miles} \div 64 \div 1760 = 103 \text{ Km} \div 64 \div 1760 = 10300000 \text{ cm} \div 64 \div 1760 = (10300000 \div 8 \div 8 \div 8 \div 2 \div 11) \text{ cm.}$
10. 66000 m in 3600 sec. = $66000 \div 3600 \text{ m per sec.}$
11. In 1 min. the train goes 1 Km = 1000 m. which $\therefore = 20 \text{ spaces.} \therefore 1 \text{ space} = 50 \text{ m.}$
12. The man's height = 5 ft. $10\frac{1}{2} \text{ in.} = 70.5 \text{ in.} = 70.5 \div 39.37 \text{ m} = 70.5 \div 39.37 \times 100 \text{ cm.}$

SOLUTIONS OF PROBLEMS

13. $29.5 \div 39.37 \times 1000$.
14. $10 \text{ m} = 1000 \text{ cm}$. In taking one-half of one part and two-thirds of the other we have taken one-half the whole string and one-sixth of the other part : but one-half the whole string is 500 cm. \therefore one sixth of the other part is 100 cm. \therefore the other part is 600 cm, and the one part is 400 cm.
15. No. minutes = $65 \text{ Km} \div 80 \text{ m} = 65000 \text{ m} \div 80 \text{ m} = 812.5 = \&c$.
16. No. revolutions = distance \div circumference of wheel = $55000 \text{ m} \div (1.4 \times 2 \times \frac{22}{7}) \text{ m} =$
17. $1 \text{ yd.} = \frac{3}{4} \text{ in.} \times 36 \times \frac{5}{2} = 1 \text{ cm} \times 36 \times \frac{5}{2} = 10 \text{ mm} \times 36 \times \frac{5}{2} =$
18. $1 \text{ Km} \div 1 \text{ m } 5 \text{ cm} = 1000000 \text{ mm} \div 1050 \text{ mm}$.
19. No. sq. metres = 6.175×4.12 .
20. Sq. root of $15227.56 = 123.4$.
21. $1200 \div 100$; 1200×100 ; $1200 \div 10$.
22. $12345 \div 1000$; $5678 \div 100 \div 10$; $1 \times 100 \times 100$.
23. These expressed as ares are 12.64, 4.68, 1000 . then add.

97

24. Expressed in dekares those are 10 and .1.
25. $7.5645 \text{ Ha} \times 27 = 204.2415 \text{ Ha}$.
26. The length and breadth in decametres are 40 and 27 ; \therefore area = $40 \times 27 \text{ sq. decametres} = 1080 \text{ ares} = \&c$.
27. $18.49 \text{ Ha} = 1849 \text{ ares} = 1849 \text{ sq. decametres}$. \therefore one side is 43 Dm = 430 m. \therefore perimeter = 1720 m.
28. $100 \text{ dm} = 1 \text{ Dm}$; $\therefore 10000 \text{ sq. dm} = 1 \text{ sq. Dm} = 1 \text{ are} = 1000 \text{ ma}$. $\therefore 1 \text{ sq. dm} = .1 \text{ ma}$.
29. Since the cube is a 3-metre cube the area of each face is 9 sq. m. \therefore whole surface is 54 sq. m = .54 sq. Dm = .54 ares = 54 ca.
30. $1 \text{ a } 5 \text{ da} = 1.5 \text{ a} = 1.5 \text{ sq. Dm} = 1.5 \times 10 \times 10 \text{ sq. m} = 1.5 \times 10 \times 10 \times 100 \times 100 \text{ sq. cm}$.
31. If the dimensions are 2, 3, 4, linear units, the volume is 24 cubic units. $\therefore 8 \text{ cubic units} = 1 \text{ cubic m}$. \therefore linear unit = a half metre. \therefore whole surface = 52 sq. half m. = 13 sq. m = .13 sq. Dm = .13 a = 13 ca.
32. If the block is a metre thick it will cover 4.5 sq. m. \therefore if cut into 100 sections it will cover 450 sq. m = 4.5 Dm = 4.5 a.
33. The surface measures $20 \text{ ha} = 2000 \text{ a} = 2000 \text{ sq. Dm} =$

IN THE HIGH SCHOOL ARITHMETIC.

200,000 sq. m ; and the thickness is .03 m. \therefore vol. = 200,000 \times .03 sq. m.

34. No. of sq. m to be painted = $5 \times 4 + 2(5 + 4) \times 3.5 = 83$;
1 sq. m (= 10 milliares) cost \$7.50. \therefore whole cost = \$622.50.

35. A side of the field measures 10 m. \therefore area of field = 100 sq. m = 1 a.

36. If the length and breadth are 3 and 2 units respectively the area is 6 sq. units. \therefore 1 sq. unit = 4 Ha = 40000 sq. m = $(200)^2$ sq. m. \therefore 1 linear unit = 200 m. \therefore length of field is 600 m, and width 400 m. \therefore diag. is $100\sqrt{5}$ 2, m. = &c.

37. 1 Ha = 100 a = 100^2 sq. m. $100^2 (39.37)^2$ sq. in. = 3937² sq. in. = $3937^2 \div 144 \div 9 \div 4840$ acres = &c.

38. The path is 60 m long and 1.5 m wide, and \therefore has an area of 90 sq. m = 90 ca.

39. Area = $7 \times 7 \times \frac{22}{7}$ sq. m. = 154 ca = 1.54 a ; smaller portion = $\frac{3}{7}$ of this area = .66 a.

40. 10000 sq. m = 1 Ha = $2\frac{1}{2}$ acres = $12100 \times 9 \times 144$ sq. in. \therefore 100 m = $110 \times 3 \times 12$ in. or 1 m = 39.6 in.

41. The vol. = $3 \times 3 \times 3$ cub. m = 27 s.

42. 1 Ks = 1000 cub. m. \therefore edge = 10 m = 1000 cm.

98

43. 1 s = 1 cub. m = $(100)^3$ cub. cm = &c.

44. One million cub. cm = 1 cub. m = 1 s.

45. Vol. = $45 \times 8 \times 1.5$ cub. m = 540 s ; &c.

46. Vol. excavated = $12 \times 10 \times 3.5$ cub. m = 420 s ; &c.

47. 1 ds = .1 s = .1 cub. m = .1 $\times (10)^3$ cub. dm = 100 cub. dm.

48. 1 s = 1 cub. m = $(39.37)^3$ cub. in. = $(39.37)^3 \div 1728$ cub. ft. = &c.

49. 1 s = 35.31 cub. ft. = $35.31 \div 128$ cord = &c.

50. Vol. = $40 \times 30 \times \frac{1}{10}$ cub. m = 24 s.

51. 20 cub. units cost \$54. \therefore 1 cub. unit cost \$2.70 = cost of 27 s. \therefore 1 cub. unit = 27 s = 27 cub. m. \therefore linear unit = 3 m.

52. Vol. of first wall = $15 \times 1 \times 3.4$ cub. m = 51s ; \therefore each stere of wall contains 1000 bricks ; &c.

53. The area of each face of the cube is a centiare \therefore whole surface = 6ca = &c.

54. 1s of earth = $2\frac{1}{2}$ s of water = $2\frac{1}{2}$ cub. m water = $2\frac{1}{2}(100)^3$ cub. cm of water = $2\frac{1}{2}(100)^3$ g = $2\frac{1}{2}(100)^3 \div 1000$ Kg = &c.

55. 1 s ice = $\frac{10}{11}$ s water = $\frac{10}{11}$ of 1000 Kg =.

SOLUTIONS OF PROBLEMS

56. Vol. = $(1.002)^3 s = \&c.$
 57. $1 \text{ cs} = .01 \text{ s} = .01 \text{ cub. m.} \therefore \text{thickness} = .01 \div 80 \text{ m.} = .01 \div 80 \times 1000000 \text{ microns} = \&c.$
 58. A vol. of 1 cub. m has a surface of $\frac{5}{2} \text{ sq. m.} \therefore$ its depth must be $\frac{2}{5} \text{ m} = \&c.$
 59. The vol. when the water is 1 d m deep = $4 \times 2.5 \times 1$ cub m = 1s, wh. requires 1 min.
 60. A section of the tunnel 1 m long contains $7^2 \times \frac{2^2}{7}$ or 154 cub. m = 154s \therefore the number of metres in the length is the quotient of 30800s by 154s.

99

61. A pile 1.5 m wide, 2 m high and 1 m long contains 3s \therefore such a pile 3 m long will last a month, or 36 m long, a year.
 62. 1 cub. m = $(10)^3 \text{ cub. dm.}$ 1s = 1 cub. m = 1000 cub. dm. = 1000 litres.
 63. 1 litre = 1000 cub. cm. wh. weigh 1000g = 1 Kg.
 64. 1 Kl = 1000 l = vol occupied by 1000 cub. dm of water; and $1000 \div 12.5 \div 4 = 20$, and 20 dm = 2m.
 65. 1 cub. cm = .001 cub. dm = .001 l = 1 ml.
 66. 4 cub. dm = $4(.1)^3 \text{ cub. m.} = .004 \text{ cub. m.}$ 4 cub. cm = $4(.01)^3 \text{ cub. m.} = .000004 \text{ cub. m.}$; \therefore the whole quantity = 4.004004 cub. m = &c.
 67. 1000000g = wt. of 1000000 cub. cm = wt. of 1000 cub. dm = wt. of 1000 litres.
 68. The vol. of the solid = vol. of water displaced wh. weighs 1000g and $\therefore = 1000 \text{ cub. cm} = 1 \text{ cub. dm.}$
 69. 1 litre = 1 cub. dm = .001 cub. m = $.001 \times (39.37)^3 \text{ cub. in.} = .001 \times (39.37)^3 \div 1728 \text{ cub. ft.} = \&c.$
 70. 1 gal. = 277.274 cub. in. = $277.274 \div (39.37)^3 \text{ cu. m} = 277.274 \div (39.37)^3 \times 1000 \text{ litres} = \&c.$
 71. From preceding result 1 qt = $\frac{1}{4}$ of 4.5437 litres = 1.1359 l. \therefore dif. = .1359 l = 135.9 ml.
 72. 1s = 1 cu. m = 1000 cu. dm = 1000 l = 1 Kl.
 73. First tap pours in 10 l in 6 sec.; the second 50 l in 20 sec. = 15 l in 6 sec., or 25 l in 6 sec. for both. Vol. to be filled = 9 cu. m = 9000 l. \therefore no. sec. required = $9000 \text{ l} \div 25 \text{ l} \times 6 = \&c.$
 74. If a piece 1 m long be cut off its vol. will be $\frac{2^2}{1} \times (1.4)^3 \text{ cu. m} = 6.16 \text{ cu. m} = 6.16\text{s.}$ \therefore no. of m in the length of the log = $61.6\text{s} \div 6.16\text{s.}$

IN THE HIGH SCHOOL ARITHMETIC.

75. Length of an edge of the box = $1\text{ m} + 2 \times 5\text{ cm} = 1.1\text{ m}$.
 \therefore total vol. = $(1.1)^3\text{ cu. m} = 1.331\text{ cu. m}$.. vol. of box = .331 cu. m.
 76. The cistern = 4 cubes each containing a million litres =
 one million cu. dm = 1000 cu. m \therefore an edge of the cube = 10
 m = depth of cistern.

100

77. 1 cu. ft. = 1728 cu. in. = $1728 \div (39.37)^3\text{ cu. m} = 1728 \div (39.37)^3 \times 1000\text{ cu. dm or l.}$
 78. 1 Kl of water = 1000 l = 1000 cu. dm = 1000000 cu. cm,
 and \therefore weighs 1000000 g = 1000 Kg; \therefore 1 Kl of air weighs $1000 \div 770\text{ Kg} = \&c.$
 79. 1 ca = 1 sq. m. \therefore the box is 1 m long for every cu. m
 in its vol., but the vol. = 1 Kl = 1 cu. m; \therefore length = 1 m.
 80. 1 l = 1 cu. dm = 1000 cu. cm.
 81. 1 Kg = 1000 g = $1000 \times 1000\text{ mg.}$
 82. 1 tonneau = 1000 Kg = $1000 \times 1000\text{ g.}$
 83. 10 cu. m of water = 10000000 cu. cm, and \therefore weighs
 10000000 g = 10000 Kg.
 84. 1 l of water weighs 1 Kg; \therefore 1 l of mercury weighs
 13.5 Kg.
 85. 1 l of water weighs 1000 g; \therefore ratio = $1840 : 1000 = \&c.$
 86. 1 ton = 2000 lbs. = 32000 oz. = $32000 \times 28.35\text{ g} = 32 \times 28.35\text{ Kg.}$
 87. 1 lb. Troy = 5760 grs. = $5760 \div 437\frac{1}{2}\text{ oz. Av.} = 5760 \div 437\frac{1}{2} \times 28.35\text{ g} = \&c.$
 88. Vol. of block = 70 cu. dm = in weight if water, 70 Kg
 = in weight if iron, $70 \times 7\text{ Kg} = \&c.$
 89. Vol. = 4.5 cu. m = 4500 cu. dm, and \therefore weighs 4500 Kg
 = 4.5 T.
 90. Cut off last three digits as decimals; the last six.
 91. 2.679 lb. T. = $2.679 \times 5760\text{ grs} = 2.679 \times 5760 \div 7000\text{ lb. Av.}$
 92. Sides of the triangle, in dm, are 13, 14, 15, \therefore area =
 84 sq. dm = .84 sq. m = .84 ca.
 93. The trapezium = a rectangle of the same width 2 m
 long, \therefore width = .375 m.
 94. The height is 10 cm. \therefore vol. = $(\frac{21}{2})^2 \times \frac{21}{2} \times \frac{10}{3}\text{ cu. cm}$
 = 1155 cu. cm = 1.155 cu. dm = 1.155 l.
 95. The height is 60 cm. \therefore vol. = $(40)^2 \times \frac{60}{3}\text{ cu. cm} =$
 32000 cu. cm, and \therefore its wt. = $32000 \times 11.4\text{ g} = 364.8\text{ Kg.}$

SOLUTIONS OF PROBLEMS

101

96. Radius = 35 cm. \therefore vol. = $\frac{4}{3} \times \frac{22}{7} \times (35)^3$ cu. cm = $\frac{1}{3} \times 539000$ cu. cm, and \therefore its wt. = $\frac{1}{3} \times 539000 \times 7.3\text{g} = 1311.566$ Kg.

97. Area = 1.44 Ha = 144 a = 144 sq. Dm, \therefore side = 12 Dm, \therefore perimeter = 48 Dm = 480 m = .48 Km. \therefore time in hrs. = .48 Km \div 5 Km = .096 = 5.76 min.

98. 5 Km per hour = 500000 cm per 3600 sec. = &c.

99. Vol. of hemisphere = $\frac{2}{3}$ that of its circumscribing cylinder: vol. of cone = $\frac{1}{3}$ &c. \therefore cone must be twice as high as the sphere = .56 \times 2 m = &c.

100. 1 ch. = 66 ft. = 66 \times 12 \div 39.37 m = &c.

101. 250 ac. = 1568160000 sq. in. = 1568160000 \div (39.37)² sq. m. = 156816 \div (39.37)² Ha = &c.

102. 229 miles = 229 \times 5280 \times 12 in. = 229 \times 5280 \times 12 \div 39.37 m = 229 \times 5280 \times 12 \div 39.37 \div 1000 Km.

103. 1 mile = 5280 ft. = 5280 \times 12 in. = 5280 \times 12 \div 39.37 m.

104. 1 tonneau = 1000 Kg. = 1 cu. m. of water = (39.37)³ cu. in. water = (39.37)³ \div 1728 cu. ft. water = (39.37)³ \div 1728 \times 1000 oz. = (39.37)³ \div 1728 \times 1000 \div 32000 tons = &c.

105. 1 Kg. on a sq. cm. = 2.2 lb. on (39.37)² sq. in. = 2.2 \div (39.37)² lb on 1 sq. in.

106. \therefore 8 Km. = 5 miles; but 8 Km. = 8000 m, and 5 miles = 316800 in. \therefore 1 m. = 316800 \div 8000 in.

107. 1 pt. = $\frac{1}{8}$ gal. = $\frac{1}{8} \div .22$ l = $\frac{25}{44}$ l; and \therefore worth $\frac{250}{44}$ francs = $\frac{250}{44} \times \frac{40}{100} = \frac{250}{11} \times \frac{40}{100} \times 240$ pence = etc.

108. 1 Ha = 10000 sq. m = 1000000 sq. dm, and the depth is .1 dm.; \therefore the vol. = 1000000 \times .1 cu. dm = 100000 l.

109. 1 tonneau = 1000 Kg. = the wt. of 1000 l.

110. 1 sq. yd = 1296 sq. in. = 1296 \div (39.37079)² sq. m. or ca. 1 acre = 4840 sq. yd. = 4840 \times .836 ca. = 4840 \times .836 \div 10000 Ha = &c.

111. Area = (4000000)² \div $\frac{22}{7}$ sq. m. = (4000000)² \div $\frac{22}{7}$ \div 10000 Ha. = &c.

112. If a is the length of the edge, and d that of the diag. of a cube, then $3a^2 = d^2 \therefore a = \frac{d}{\sqrt{3}} \therefore$ vol. of the cube = $a^3 = \frac{d^3}{3\sqrt{3}}$; but d = 2m, \therefore vol. = $\frac{8}{3\sqrt{3}}$ cu. m. = $\frac{2}{3}\sqrt{3}$ a. = &c.

SOLUTIONS OF PROBLEMS.

102

1. A convenient arrangement for such divisions is the following:—148.28) 3.003(. 1.4828) .03003 (.02. The advantage of placing the decimal point immediately to the right of the left hand digit in the divisor, is that the place of the first digit in the quotient is at once determined, being either the place of the first significant digit in the dividend or the next lower.

2. $.73256 - .619 = .73256 - .61919 = .11337 = \frac{11337}{100000}$; \therefore exp. becomes $\frac{11337 \times 100000 \times 8827}{8827 \times 100000} \text{ yds.} = \&c.$

3. $1.728 = \frac{1727}{1000}$; $3.146 = \frac{3115}{1000}$; \therefore we have $\frac{1727}{1000} \times \frac{3115}{1000} = \&c.$

4. $\frac{1}{4} - \frac{2}{3}$ of $.16 = \frac{5}{36}$; $\pounds 15, \&c. = \pounds 15 \frac{5}{36} = \$2 \frac{32}{3}$. \therefore we have $\frac{5}{36} \times 16 \frac{1}{11} \times \frac{232}{3} = \&c.$

5. Begin thus:—

523.23857
3 872462

10464 77140
3139 43142

6. $.456) .37848$ $.0456) 3.7848$
 $4.56) 3.7848(.83$ $4.56) 378.48(3$
 $.00456) 3784.8$
 $4.56) 3784800($

7. $\frac{22}{1000} = \frac{1}{32} = \&c.$ In reducing $\frac{1}{17}$ after the first 5 digits have been obtained, the rest may be found by dividing by 3. See Arith. p. 58.

156013789
15601
155998188

9999000
8934
893

8. 21316 31249
 21 312

9. 4141414
 $.0352352$
 6.1011011
 6.5504777

SOLUTIONS OF PROBLEMS

10. $\frac{21772}{999} \div \frac{7891}{999} = \&c.$
 11. $3 + \frac{1}{7\frac{1}{18}} = 3\frac{18}{187} = \frac{565}{187} = 3.141592+.$
 12. $\sqrt{6} = 2.4494897 + ; 1.732 \times 1.4 = 2.4493944 ; \therefore$
difference = .0000953 +.
 13. £34 4s. 6d. = £34 $\frac{9}{20}$; £174 16s. 6d. = £174 $\frac{33}{40}$.
 $\therefore \frac{36}{37} \times \frac{34\frac{9}{20}}{174\frac{33}{40}} = \&c.$
 14. $\frac{1}{188} + \frac{1}{217} = \frac{3}{85}$; $\frac{3}{85} + \frac{2}{285} = \frac{12}{85}$; $\frac{12}{85} + \frac{1}{11} = \&c.$
 15. Second fraction = $\frac{13}{41}$; the third (den. = 1) = $\frac{5 \times 7 \times 19}{4 \times 11 \times 22}$
- 103**
16. Arrange thus:—

325.62534
763472
 17. First num. = $\frac{8}{3} \times \frac{17}{6} = \frac{68}{9}$; First den. = $\frac{56}{9} \times \frac{6}{7} = \frac{16}{3}$;
 \therefore first fract. = $\frac{17}{12}$; second fract. = $\frac{37 \times 111 \times 9}{55 \times 199}$.
 18. (1) The remaining digits may be 0's and \therefore the least value is .8397; (2) the remaining digits may be 9's and thus the greatest value is .8397999... = .83979 = .8398.
 19. Fract. = $\frac{113}{346} = \&c.$
 20. $\frac{23}{99} \div \frac{2875}{9999} = \frac{101}{128} = \frac{808}{1000} = .808.$
 21. $\frac{1}{18} = .076923$; this multiplied by 1.4 gives .1076923; this multiplied by 65 gives 6.9, and this divided by 7 gives .9 = 1.
 22. £120 7s. 10 $\frac{1}{2}$ d. $\div 8\frac{7}{8} =$ £13 14s. 3 $\frac{1}{2}$ d., and this $\times 9\frac{7}{8} =$ £134 2s. 2 $\frac{3}{4}$ d.
 23. 14 wks., &c. = 2519 $\frac{3}{4}$ hrs. = 10079 quarter hrs. 365 $\frac{1}{4}$ days = 3506 $\frac{1}{4}$ quar. hrs.
 24. First denominator = $\frac{4}{9} + \frac{4}{3} - \frac{1}{3} = \frac{10}{9}$, &c.
 25. $1769 \div 5 = 353.8$; this $\div 3 = 117.9333...$; this $\div 7 = 16.8476190...$ = &c.
 26. First two fractions = $\frac{257}{117} - \frac{257}{117} = 0$, &c.
 27. $9\frac{9}{10}$ in. $\div 1$ mile = 9.9 in. $\div 5280 \times 12$ in. = $9.9 \div 10 \div 11 \div 9 \div 8 \div 8 = \&c.$
 28. $\frac{1}{188} + \frac{1}{217} = \frac{2}{81} (\frac{3}{8} + \frac{2}{7}) = \frac{2}{81} \times \frac{31}{56} = \frac{2}{85}$; $\frac{2}{85} + \frac{2}{333} = \frac{2}{87} (\frac{1}{5} + \frac{1}{9}) = \frac{2}{87} (\frac{1}{5} + \frac{1}{9}) = \frac{2}{45}$; $\frac{2}{45} + \frac{2}{857} = \frac{1}{9} (\frac{1}{5} + \frac{1}{85}) = \frac{1}{9} \times \frac{207}{215} = \frac{23}{215}$; $\frac{1}{9} + \frac{23}{215} = \&c.$

IN THE HIGH SCHOOL ARITHMETIC.

29. $73.05d. \div 365.25d. = .2.$

30. Denom. of 2nd. fract. $= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{5}{12}, \&c.$

104

31. $= \frac{1}{2} \div \frac{1}{3} = \&c.$

32. $= \frac{174}{108} \div \frac{60}{108} = \&c.$

33. $= 4 + \frac{1}{2} - \frac{3}{4} + \frac{5}{8} - \frac{7}{16} = \&c.$

34. $17 \div 42 = .40476 + \dots \therefore .4047$ differs from the true quotient by .00006 + , which is less than .0001 ; also .4048 differs from the true quot. by .00003 + , which is less than .0001.

35. $= \frac{\frac{97}{87} + \frac{3}{2}}{\frac{5}{8} + \frac{3}{4}} = \&c.$

36, 38. Reduce to vulgar fractions.

40. $= 7899.114 \times \frac{220.33}{258.33}$ miles = &c.

41. $= 1\frac{1}{8} + 7 - 7\frac{1}{8} = 1.$

42. Reduce to vulgar fractions.

43. The greatest speed will be when the number of fathoms is greatest and the time is least, that is for the greatest value of 65.84.... and the least value of 5.6..., that is for 65.849 fathoms and 5.6 seconds. Similarly the least speed will be 65.84 fathoms in 5.69 seconds.

44. (1) $= \frac{25}{28} \div \frac{25}{28} = 1.$ (2) $= 1\frac{1}{4} + 2\frac{1}{12}.$

45. The given fraction $= \frac{\frac{760}{99} \times 20}{20} = \frac{15\frac{205}{99}}{20}$ which lies between $\frac{15}{20}$ and $\frac{16}{20}$, and being less than $\frac{15\frac{1}{2}}{20}$ is nearer in value to $\frac{15}{20}.$

105

46. These become (5 - 2) of £1 10s. 6d. and (2 + 0) of £1 5s. 6d.

47. See 45. Ans. 38.

48. $\frac{2}{3}$ of £1 + $\frac{2}{7}$ of 1 G. = &c.

50. The sum of these fractions is $2\frac{691}{1170}$, which is less than the next integer, 3 by $\frac{479}{1170}.$

51. Any multiplier which will produce an integer + $\frac{479}{1170}$ will give the desired result. But $\frac{479}{1170}$ when multiplied by 1170 or by any multiple of 1170 will give an integer. Therefore the required multiplier is 1 + 1170 or 1 + any multiple of 1170.

SOLUTIONS OF PROBLEMS

54. $= 5038848 \div 5 \div 7 \div 11 \div 13 \div 17 \div 19 = \&c.$
 55. Express the vulgar fractions as decimals and the answer will be readily obtained.
 56. $= 4\frac{24}{1155} - 1\frac{25}{1155} = 3.$
 57. $= \frac{1}{1} \div 1 = \frac{1}{1}.$

106

58. $.834 \times .623 = .519582.$ This differs from .52 by .000418, and from .519 by .000582 and is \therefore more nearly represented by .52.
 59. First fract. $= \frac{1}{2}$ of $\frac{5}{36} + \frac{17}{18} = 1\frac{19}{108}$; second frac. $= 1\frac{80}{108}$; $\therefore \&c.$
 60. $= \frac{1}{107} \div \frac{107}{107} = \&c.$
 61. Since one factor contains a digit in the tens place, the other factor must be carried to at least the sixth decimal place in order that the product may be correct to the fourth dec. place.
 64. $\sqrt{\frac{1}{2}} = \frac{1}{2}\sqrt{2} = .7071 + ; \&c.$
 66. $\frac{6\frac{1}{2}}{7} = \frac{13}{14} = \frac{1}{\frac{14}{13}} = \frac{17}{17 \times \frac{21}{13}} = \&c.$
 67. $= \frac{\frac{95}{13} - \frac{72}{13}}{\frac{32}{3} - \frac{36}{13}} \times \frac{100}{23} = \&c.$

107

71. In the text .0001 should be .00001. The expressions become respectively .00001369863 and .0000137, and the difference is .000000001369863, which is .0001 of the former.
 73. $1 \div 3.14159 = \&c.$ thus:

$$\begin{array}{r} 3.1415 \overline{)100000} \\ \underline{3813} \\ 5755 \\ \underline{3143} \\ \&c. \&c. \end{array}$$

74. $1 \div 2.302585$ as above $= .4343$ more nearly than. 4342.
 76. L.C.M. of numerators $= 90$, \therefore we have $\frac{15 \times 6}{119 \times 6}$ and $\frac{18 \times 5}{143 \times 5} = \&c.$

IN THE HIGH SCHOOL ARITHMETIC.

$$77. .37892 \div 6146.38 \div .0004675 = .37892 \div 6.14638 \div .4675 = 3.7892 \div 6.14638 \div 4.675 = \&c.$$

$$78. \frac{269}{81} = 4.1230769; \sqrt{17} = 4.123105 \dots$$

$$79. \frac{3}{8} \sqrt{5} = \frac{1}{16} \sqrt{180} = \&c.; \frac{2}{3} \sqrt{2} = \frac{1}{3} \sqrt{8} = \&c.$$

$$80. 1 \text{ m.} = 70 \div 64 \text{ yd.} = 39.375 \text{ in.} \therefore \text{dif.} = \&c.$$

81. The second fract. is formed by dividing the first by 5; the third by dividing the second by 7; the fourth by dividing the third by 9; and so on; hence we proceed thus:

$$\begin{array}{rcl} \frac{1}{3} = .3333333 \dots; & \text{dividing this by } 5 & \\ \text{we have } .0666666 \dots; & \text{"} & 7 \\ \text{"} .00952380 \dots; & \text{"} & 9 \\ \text{"} .00105820 \dots; & \text{"} & 11 \\ \text{"} .00009620 \dots; & \text{"} & 13 \\ \text{"} .0000740 \dots; & \text{"} & 15 \\ \text{"} .00000049 & & \\ \hline & & .410686 \end{array}$$

82. See pp. 69, 70.

108

84. The first three terms are 1, .25, .09375; the fourth is $\frac{5}{12}$ of the third; the fifth is $\frac{7}{12}$ of the fourth; the sixth, $\frac{9}{10}$ of the fifth, &c.

85. These can be more readily calculated as follows:—

	1.25
.09375	.09375
$\frac{1}{2} = .046875$	
$\frac{1}{8} = .007812$	
diff. = .039063	.039063
$\frac{1}{2} = .019531$	
$\frac{1}{8} = .002441$	
.017090	.017090
$\frac{1}{2} = .008545$	
$\frac{1}{10} = .000854$	
.007691	.007691
$\frac{1}{2} = .003845$	
$\frac{1}{12} = .000320$	
.003525	.003525
	<u>1.411119</u>

SOLUTIONS OF PROBLEMS

86. $1 + 1 - 2$

$\frac{1}{1.5} = .5$, the next is $\frac{1}{3}$ of this
 $= .16666 \dots$ the next is $\frac{1}{4}$
of this $= .04166 \dots$

one-fifth $= .00833 \dots$

one-sixth $= .001388 \dots$

one-seventh $= .00019841269$

one-eighth $= \&c.$

87. Kilometer $= 1000 \text{ m.} = 39710 \text{ in.} \therefore \&c.$

88. See No. 61.

89. Numr. $= \frac{3^3 + 1^3}{40^3} = \frac{28}{40^3}$; Denr. $= \frac{3^3 - 3 + 1}{40^3} = \frac{7}{40^3}$; \therefore

fract. $= \frac{28}{40^3} \times \frac{40^3}{7} = \frac{4}{40} = \&c.$

90. The dif. $= \frac{1}{2488}$ of one and $\frac{1}{2488}$ of the other.

91. (1) $\frac{5.12}{33.75} = \frac{512}{3375} = \frac{8^3}{15^3}$; $\therefore \&c.$ (2) $\frac{5.12}{.03375} = \frac{80^3}{15^3}$; \therefore

$\&c.$ (3) $= \frac{8 + .15}{2 - .1} = \&c.$

109

92. Assuming $\frac{5.11}{1000}$ as the value of the fraction its denominator must be $\frac{1000}{5.11}$ of its numerator or $\frac{1000}{5.11}$ of $209 = \&c.$

93. $\text{£}1000 = \frac{1091}{100} \times 4\frac{4}{5} \times 1000 \text{ Canadian dollars} = \&c.$

94. Multiply each term of the fraction by $\sqrt{2} - 1$, and we have $3 - 2\sqrt{2} = \&c.$

95. $\sqrt{5} = 2.236067 + ; \&c.$

96. Metre $= 39.37043 \text{ in.} = 3.28086916 \text{ ft.} = \&c.$

97. Multiply each term by $3\sqrt{5} + \sqrt{3}$.

98. If the full quotient is obtained in each case the first result will be found to differ from the true quotient by .00000111, and the second by .000009; so that the absolute error in the first case is less than in the second. If we take the ratio of remainder to divisor in each case the absolute errors become $\frac{1}{900000}$ and $\frac{1}{111000}$ respectively, giving the same results as before. The relative error in the first case is $.00000111 \div 1.11$, and in the second case $.000009 \div 9.009$, giving in each case .000001.

IN THE HIGH SCHOOL ARITHMETIC.

99. See 83.

100. $\frac{1.04312}{1.0012} \times 25.2282$ cu. in.

101. See Arith. pp. 55-7.

102. Multiplying each term of first fract. by $\sqrt{5} + \sqrt{3}$, and of second fract. by $\sqrt{5} - \sqrt{3}$ we have $\frac{(\sqrt{5} + \sqrt{3})^2 - (\sqrt{5} - \sqrt{3})^2}{2} = 2\sqrt{15} = \sqrt{60} = \&c.$

110

103. From equator to pole = $90 \times 69\frac{1}{4}$ miles = 394099200 in. The metre would = .0000001 of this = 39.40992 in.

104. Arrange the expression thus:

$$\frac{1}{7} \left(1 + \frac{1}{7^2} \left\{ 1 + \frac{1}{7^2} \left(\frac{3}{2} + \frac{1}{7^2} \cdot \frac{5}{2} \right) \right\} \right)$$

and the following solution is suggested: Divide 2.5 by 49, add 1.5, divide by 49, add 1, divide by 49, add 1, divide by 7.

$$\begin{array}{r} 7 \overline{) 2.5} \\ 7 \overline{) 35714285} \\ .05102040 \\ 1.5 \end{array}$$

$$7 \overline{) 1.55102040}$$

and so on, giving true to 8 places .14586487. The square root of $\frac{1}{47} = \frac{1}{47} \sqrt{47} = .14586499$. \therefore difference = .0000001 +.

105. 1 franc = $486\frac{2}{3} \div 24.25$, cents =.

106. A goes 45 miles in $45 \div 3\frac{1}{2}$ hrs., but in this time B goes $45 \div 3\frac{1}{2} \times 2\frac{1}{2}$, miles = $32\frac{1}{2}$ miles, and as he had gone $7\frac{1}{2}$ miles when A started, he still has $45 - 32\frac{1}{2} - 7\frac{1}{2}$ miles to go.

107. $32 \times 25 \times 11\frac{1}{2} \times 1728 \div 100 \times 31 \div 7000$, lbs.

108. If water in freezing expands one-tenth, it follows that equal weights of water and ice are in volume as 10 : 11, therefore equal volumes of water and ice are in weight as 11 : 10; thus a cub. ft. of ice will weigh $\frac{10}{11}$ of 1000 oz.

109. A cubic foot weighs $1728 \div 277.2 \times 10$ lbs. = $997\frac{2}{7}$ oz.; $\therefore \&c.$

110. £12 10s. \div 4s. 2d. = 3000d. \div 50d. = $\&c.$

111. $\frac{3}{4}$ in. = $\frac{1}{16}$ ft. \therefore volume of rain = $4840 \times 9 \times \frac{1}{16}$ cub. ft., and at $6\frac{1}{2}$ gal. to the cub. ft. this gives $17015\frac{5}{8}$ gal.

112. The farm contains 184.48 ac.; the remainder = 36.975 ac., $\&c.$

SOLUTIONS OF PROBLEMS

III

114. If the work which one man can do in an hour be taken as the unit, then the mowing of 15 acres will = 180 units of work; \therefore to mow 11 ac. will require 132 units, and as each man can do but 22 units in the given time, \therefore six men will be needed.

115. A got $\frac{2}{3}$, leaving $\frac{1}{3}$; of this B got $\frac{1}{2}$, leaving $\frac{1}{6}$ of $\frac{1}{3}$; of this C got $\frac{1}{3}$, leaving $\frac{1}{6}$ of $\frac{1}{6}$ of $\frac{1}{3}$ of the farm which, at the price received, must = $\frac{1}{6}$ ac.; \therefore $\frac{1}{6}$ of $\frac{1}{6}$ of $\frac{1}{3}$ of the farm = $\frac{1}{6}$ ac.; \therefore &c.

116. The 17 bush., &c. = 1147 pt., and \therefore contain $1147 \times 90 = 103230$ grs. of barley, and \therefore measure 1147×3000 in., &c.

117. The measure of the wheat with 3 lb. of wheat as the unit is 6860; this measure with 2 lbs. of flour as the unit gives 13720 lbs. flour = 70 bbl.

118. L. C. M. of widths is 15 yd. \therefore hall requires 20 strips each 20 yd. long. = 400 yd. at \$1.10 = &c.

119. $180 \times 96 \div 160 =$ no. ac. = 108. Fencing cost \$6.90 per ac.; ashes and wood yield \$10.90 per ac. \therefore cost per ac. = \$12.50 + \$14.75 + \$6.90 - \$10.90 = \$23.25.

120. The farmer gains \$4.50; the merchant, \$5.

121. When the second train starts the first has gone 80 m., and as the second overtakes the first at the rate of 12 m. an hr. the time required is $6\frac{2}{3}$ hrs.

122. 18s. = $\pounds\frac{18}{20} = \frac{9}{10} \times 1000$ mils = 9fl.; $3\frac{1}{2}$ d. = $\pounds\frac{3\frac{1}{2}}{240} = \frac{13000}{9600}$ mils = 1c. $3\frac{1}{2}$ m.

II2

123. The value of a farthing : value of a mil as 25 : 24, so that 25 m. = 24 fr. = 6d.

124. 4 min. \div 24 hrs. 56 min. = $\frac{4}{3168} =$ &c.

125. $\frac{10}{121}$ ac. sold for \$3000 yields $\$3000 \times \frac{121}{10}$ per ac.

126. 3000 gal. \div $6\frac{1}{2}$ gal. = no. cu. ft. = 480 cu. ft. To have the water 1 ft. deep in the tank requires 70 cu. ft. of water. \therefore total depth = 480 cu. ft. \div 70 cu. ft. = &c.

127. Reaping 1 ac. = 15 units of work; 2880 units are to be done, and as 8 men can do 96 units in a day, 30 days will be required.

128. $1\frac{1}{2} \times 1\frac{1}{2} \times 10 \times 62\frac{1}{2} \times 2.716$, lb.

IN THE HIGH SCHOOL ARITHMETIC.

129. $\frac{1999}{178} \times 19\frac{1}{4}$ oz.
 130. Time = 5 hrs. \times (3 min. + 11 sec.) = $81\frac{9}{11}$ hr.
 131. 6 m. = 9 b. \therefore 6 m. and 10 b. = 19 b. $\therefore \frac{2}{17}$ of 5d. = time required.
 132. See 114.
 133. One cow yields 12 oz. butter per day. \therefore 16 cows in 7 days will yield $12 \times 16 \times 7$, oz = 84 lbs.
 134. $29000 \text{ lb.} \div 14.8 \text{ lb.} = \text{no. required.}$
 135. 3 m. of embankment = 72000 units of work; 120 m. working 1 hr. a day for 24 d. do 2880 units. \therefore to do the work in required time would necessitate working $72000 \div 2880 = 25$, hrs. a day.

113

136. The distance gone in 1 hr. would be $\frac{99}{17}$ of 11220 metres = $\frac{99}{17} \times 11220 \times 39.37 \text{ in.} = \frac{60 \times 11220 \times 39.37}{27 \times 5280 \times 12}$ miles, = &c.
 137. Surface of roller = $28 \times \frac{2}{7} \times 120$ sq. in. \therefore area of field = $\frac{28 \times 22 \times 120 \times 90 \times 70}{7 \times 144 \times 9 \times 4840}$ acres = &c.
 138. $\frac{1}{2}$ of the work = 2100 units : 15 m. working 10 hr. a day do 150 units a day. $\therefore 2100 \div 150 = \&c.$
 139. 10 quar. = bbl. of salt = 280 lb. \therefore 1 quar. = 28 lb, &c.
 140. L.c.m. of 196 and 2240 = 80×196 ; and this $\div 196 = \&c.$
 141. $\frac{\$25}{100} \times \frac{1}{2} \times 20500 = \&c.$

	d	h	'	"	
142.	243	2	11	49	(1)
	157	7	4	7	(2)
	85	19	7	42	(3)
	14	7	11	17	(4)

(1) \div (2) gives remainder (3); twice (3) \div (2) gives rem. (4); (4) divides (3) without rem., and is \therefore g.c.m. of (1) and (2). Again since (1) and (2) are respectively 17 and 11 times (4), \therefore their l.c.m. = $17 \times 11 \times (4) = \&c.$
 143. See 138.
 144. See table of weight, arith. p. 7, 8.
 145. A fall in price of 40c. a gal. on one-sixth of the quantity = a fall of $\frac{1}{6}$ of 40c. a gal. on the whole = $6\frac{2}{3}$ c., reducing the price to $83\frac{1}{3}$ c.

SOLUTIONS OF PROBLEMS

146. It is easily seen that the lot cost A, B, C respectively 160, 180, 153, the unit of price being as yet undetermined; but the cost to C being \$306 fixes the unit at \$2, and \therefore the lot cost A \$320.

147. The value given is that of 780 per. \therefore 1 ac. or 160 per. is worth $\frac{6}{35}$ of this value; but previous value = $\frac{20}{35}$ of present value. \therefore increase in value = $\frac{8}{35}$ of pres. val. = $\frac{8}{35}$ of £1681 17s. 6d. = £45.

114

148. The unit of measurement in selling the coal is $\frac{25}{28}$ of that used in buying it; \therefore the number of short tons is $\frac{25}{28}$ of the number of long tons.

149. A's rate : B's as 20 : 19 = B's : C's; \therefore A's : C's as 400 : 361 = 200 : 180 $\frac{1}{2}$ = a difference of 19 $\frac{1}{2}$ yd. in a 200 yd. race.

150. 1 pound of thread makes 3 $\frac{1}{2}$ sq. yd. \therefore 45 sq. yds. will require 12 pounds.

151. U.S. dol. contains $\frac{9}{10}$ of 412 $\frac{1}{2}$ grs. pure silver; Can. dol. contains $\frac{37}{40}$ of 360 grs. pure silver. \therefore silver in Can. dol. is worth $85 \times \frac{37}{40}$ of $360 \div \frac{9}{10}$ of 412 $\frac{1}{2}$ cts.

152. Area of end of bar = $\frac{1}{40}$ of 1 sq. ft. = 3.6 sq. in. \therefore edge of end of bar = $\sqrt{3.6}$ = 1.8973 in., and if this length is cut from the bar, the cube so cut off will weigh $\frac{1.8973}{480} \times 560$ lbs. = 2.2136 lbs. nearly.

153. 1 gal. = 4.534 litres. 1 pt. = $\frac{1}{8}$ of 4.534 l., 10 fr. = \$1.90 = $1.90 \times \frac{15}{16}$ £ = $\frac{15}{16} \times \frac{15}{16} \times 240$ d. \therefore 1 pt. is worth $\frac{4.534}{16} \times \frac{15}{16} \times 240$ d. = 53.1d.

154. Time = $14\frac{3}{8}$ hr. = $\frac{44}{8}$ hr. Distance = $863 \times 3281 \div 5280$ miles. Speed = $863 \times 3281 \div 5280 \div \frac{44}{8}$ miles per hr.

155. In 5 days 10 min. it loses 10 min., &c.

156. Whole selling price = \$52.10 + \$6.75, and this \div 535 = 11c. = selling price per lb.

157. Enough water must be added to 63 gal. to make 90 gal.

158. Since the time is diminished in the ratio 3 : 2 $\frac{3}{4}$, \therefore the rate of speed must be increased in the ratio 2 $\frac{3}{4}$: 3 = 11 : 12 = an increase of $\frac{1}{11}$.

159. If the unit of val. is one-fourth the val. of the house, then the house will be worth 4 and the lot 3, and the two together, 7 of these units. \therefore the unit is \$300.

115

160. As he received only $\frac{1}{8}$ of the tea bought, he should pay only $\frac{1}{8}$ of \$64.

161. If the quantity of hay eaten by a sheep in a day be the unit, then we have 260 units costing \$95.25, from which to find the cost of 112 units.

162. If $25.8 \times \frac{9}{10}$ grains of pure gold are worth a dollar, then $437\frac{1}{2}$ grs. are worth $437.5 \div 25.8 \div .9$, dollars = &c.

163. In efficiency these taps are as $\frac{1}{2} : \frac{1}{3} : \frac{1}{1\frac{1}{2}} = 21 : 14 : 30$.

\therefore 1st and 2nd together are to the 3rd as 7 : 6, that is, out of every 7 gal. poured in by the first two taps 6 gal. are drawn off by the third, and one gal. remains in the cistern. \therefore when the cistern is finally full, six times as much will have been drawn off, requiring six times as long as to empty it once = $6 \times 1\frac{1}{2}$ hr. = $8\frac{1}{2}$ hr.

164. An increase of $\frac{2}{15}$ in $\frac{3}{4}$ of the price = an increase of $\frac{2}{15}$ of $\frac{3}{4}$ in the whole price = $\frac{1}{10} = 1c$.

165. Divide the whole sum into 91 equal parts, and give them respectively 17, 20, 24 and 30 of these parts.

166. He spends in 20 wks. £176 5s., leaving £188 15s. to be spent in 225 days.

167. $5 \text{ m.} \div 1140 \text{ ft.} = \text{no. of seconds} = \frac{11}{19 \times 90} \text{ hr.} \therefore \text{dist.}$

gone by train = $\frac{11 \times 50}{19 \times 90}$ miles = &c.

168. $31 \div .00122 = 2541$, grs. 100 cu. in. of fresh water will weigh $2541 \div 1.03 = 2467$, grs.

169. He gets for 127 gal. the value of 128 gal. and \therefore gains the value of $\frac{1000}{127}$ gals.

170. For one day's work A should receive $\frac{3}{24\frac{1}{2}}$ and B $\frac{4}{24\frac{1}{2}}$ of the sum ; \therefore working together they should receive $\frac{1}{3\frac{1}{2}}$ for a day's work.

116

171. B's rate is to C's as $420 : 410 = 42 : 41 \therefore$ B will win by $\frac{1}{42}$ of the distance run.

172. Total selling price = \$6133.75; first sale brings \$2027.30 ; \therefore remaining 3135 bushels must sell for \$4106.45.

173. $40 \times 5760 \times \frac{11}{13} \div 1869 = 113$.

SOLUTIONS OF PROBLEMS

174. A and B together earn $9\frac{1}{2}$ s. a day; A and C, 9 s.; B and C, $8\frac{1}{2}$ s. \therefore if each works 2 days their earnings will aggregate 27 s. or $13\frac{1}{2}$ s. in one day; but A and B earn $9\frac{1}{2}$ s. in a day \therefore C earns 4 s.; &c.

175. L.c.m. of $91\frac{1}{2}$ c. and $99\frac{1}{2}$ c. = \$2290.41; &c.

176. $1 \times \frac{1}{10} \times \frac{10}{20} = \frac{1}{200}$; \therefore gain = $\frac{9}{200}$.

177. To build 12 ft. of the sidewalk will require 240 ft. of lumber, or 20 ft. lumber will build a ft. of the walk which \therefore cost 34c. a ft. or $\$.34 \times 1320 = \&c.$

178. Total area to be papered = $1331\frac{1}{2}$ sq. ft.

179. Total length of ditch = $1332\frac{1}{2}$ ft.

180. The difference between the squares of two consecutive numbers is their sum, so that the numbers required are the whole nos. next less and next greater than one half of 691.

181. $7926 \times \frac{2}{3} \div 360$, miles = &c.

117

182. Had his speed been a third greater A would have run 1760 yds. while B ran 1738 yds.; \therefore his real rate : B's rate as $\frac{2}{3}$ of 1760 : 1738 = 60 : 79.

183. 6 yr. = $\frac{1}{3}$ of father's age - $\frac{1}{3}$ of father's age = $\frac{2}{15}$ &c.

184. Theoretically, 49; since no cent will fall off so long as its centre of gravity is supported. As a practical test place 4 cents on an inch cube.

185. $\$.9000 \times \frac{2}{3} \times \frac{5}{4} = \$12937.50.$

186. The min. hand goes 12 times as fast as the hr. hand and \therefore gains 11 min. spaces in 12 min., \therefore it will gain 60 min. spaces in $\frac{11}{12}$ of 60 min. = $65\frac{5}{11}$ min.

187. He is worse off by the loss of his wages, which must \therefore be \$1.25 a day, leaving 50c for his daily expenses.

188. 1 sq. metre = 1550 sq. in. = $1550 \div 144 \div 9 \div 4840$ ac. 1 ac. cost £1200 = 1200×25 francs; \therefore 1 sq. m. cost $1550 \times 1200 \times 25$ francs = 7.4 francs.

189. They will beat 7, 8 and 9 times in one-tenth of a min. or, thus: the time between successive ticks is $\frac{6}{7}$, $\frac{6}{8}$, $\frac{6}{9}$ sec. respectively, and l.c.m. of these is 6 sec.

190. Original weight = $\frac{2}{10}$ of weight of pure gold contained; final weight = $\frac{2}{2}$ of the weight of pure gold; and these are as 11 : 8.

IN THE HIGH SCHOOL ARITHMETIC.

191. $11c. + 2 \text{ loss} = 21c. \therefore \text{loss} = 5c. \therefore \text{cost} = \$1.10 \therefore \text{at } \$1.40 \text{ the gain is } 30c. \text{ and } 30c. \times 800 = \$240.$

192. $(308 \times 204 - 80 \times 60) \times 8 \div 128 = \text{no. cords}.$

193. To excavate 966 yd. requires 1260 units of work (See 114); $\therefore 575 \text{ yd. require } 750 \text{ units; the extra men do } 120 \text{ units, and, as one man does } 90 \text{ units in } 12 \text{ days, the remaining } 630 \text{ units can be done by } 7 \text{ men}.$

118

194. If a child's share is the unit the men will receive 36, the women 72, and the children 72, making 180 units in all; $\therefore \text{each unit is } \$1.60.$

195. Total selling price, \$1089.70; no. gal. to be sold, 306.

196. A man's efficiency is to a woman's as 7 : 6; a woman's to a boy's as 11 : 8; the efficiency of the three = 77 : 66 : 48, or that of 1 m. : 2 w. : 3 b = 77 : 132 : 144, that is, a man does 77 out of every 353 units of work done; $\therefore \text{the time required} = \frac{77}{353} \text{ of one man's time} = \frac{77}{353} \text{ of } 50 \text{ hrs}.$

197. 15 drains 80 rods long at 33c. a rod cost \$396; the increased yield of 2 bush. per ac. on 30 ac. gives \$39.60 a year.

198. The buying prices of A, B, C are as 100 : 110 : 99 or C's is $\frac{99}{100}$ of A's.

199. To make the wages average \$.92 each man would contribute 1c., but each boy must receive 27c.; so that every 2 boys would require the contributions of 3 men; thus for 8 boys 12 men would be required.

200. In 7 hr. B goes 21 m., and A is 7 m. farther on; of this 7 m. B goes 3, making 24 in all.

201. Marked pr. is $\frac{25}{14}$ of cost, $\therefore \text{selling pr. is } \frac{7}{8} \text{ of } \frac{25}{14} \text{ of cost } \therefore \text{gain is } \frac{9}{16} \text{ of cost}.$

202. As the question reads a boy's efficiency is nil and the work could be done by a man in 36 days, &c.

119

203. The clock loses $9\frac{1}{2}$ min. in $106\frac{1}{4}$ hr. and $\therefore \text{loses } 1\frac{1}{2} \text{ min. in } 16\frac{1}{4} \text{ hr}.$

204. $\pounds 405 \text{ } 11s. \text{ } 4d. \div 46 = \pounds 8 \text{ } 16s. \text{ } 4d.; \text{ this } \div 46 = 3s. \text{ } 10d., \text{ and this } \div 46 = 1d. \therefore \text{amt. required} = \pounds 405 \text{ } 11s. \text{ } 4d. + 3(\pounds 8 \text{ } 16s. \text{ } 4d.) + 3(3s. \text{ } 10d.) + 1d. = \pounds c.$

205. Divide each by 1,250,000.

SOLUTIONS OF PROBLEMS

206. 54 men in 13d. dig 1053 ft.; \therefore 46 m. in 11d. dig 759 ft., leaving 77 ft. to be dug by 8 boys in 11d. or 7 ft. in one day.

207. In 35 hr. the first would fill the tank 20 times, the second 14 times and the two together 34 times, or once in $\frac{35}{34}$ hr.

208. $365 \times 400 + 97$, there being 97 leap years.

211. If the cost of the first is the unit, the second cost $1\frac{1}{2}$, the third $2\frac{1}{2}$, and the fourth $3\frac{1}{2}$ = a total cost of 8, making the unit \$3,000; &c.

120

212. In 10 oz. gold, nine-tenths fine, there are 9oz. pure gold; to reduce this pure gold to a fineness of three-fourths will require 3 oz. alloy.

213. A quart = 69.3 cu. in., and \therefore weighs $.693 \times 31$ grs.; a half dime weighs $412.5 \div 20$, grs. \therefore &c.

214. The lots are $\frac{1}{3}\frac{1}{10}$ and $\frac{1}{3}\frac{1}{20}$ of the farm, respectively.

215. $63460.6 \times .82 \div 7000$.

216. It takes 18 min. to row the extra $\frac{3}{4}$ m.; the other 30 min. must be the time of resting, or the steam flows $\frac{3}{4}$ m. in 30 mins.

218. $64\frac{1}{2} \times 53\frac{1}{2} - 61\frac{5}{8} \times 50\frac{5}{8}$ = &c.; or, $230\frac{3}{4}$ (total length of sidewalk) $\times 1\frac{2}{3}$ = &c.

121

219. If they had gone on as they began they would have shared equally in the sum paid; but the first increases his efficiency $\frac{1}{8}$ for half the time; the second $\frac{1}{4}$; and the third $\frac{1}{6}$; they are \therefore now entitled to share in the ratio $1\frac{1}{8} : 1\frac{1}{4} : 1\frac{1}{6}$ = 765 : 760 : 756. Or, thus: In efficiency the first is to be second as 9 : 8, the second : the third as 10 : 9; \therefore the three rank as 45 : 40 : 36; the times during which they work are as 17 : 19 : 21; \therefore in work done they are as $45 \times 17 : 40 \times 19 : 36 \times 21$ = 765 : 760 : 756.

220. $\$4064.55 \times \frac{26}{7} \times \frac{9}{34} \times \frac{21}{20}$ = \$4073.16.

221. In the first case 704 units of work = $5133\frac{1}{2}$ yd. of road; in the second case there are $19162\frac{1}{2}$ yd. of road (= 2628 units of work) to be done, and since 36 units are done in a day, \therefore 73 days, &c.

222. If the number is a square, each prime factor in the number must occur an even number of times; in 1500 the

IN THE HIGH SCHOOL ARITHMETIC.

factor 2 occurs twice, 3 once, and 5 three times; \therefore another 3 and another 5 must be introduced.

223. In a cube each prime factor must occur 3 or 6 or 9 or, &c., times: in 14175 the factor 5 occurs twice, 7 once and 3 four times; \therefore a 5, two 3's, and two 7's must be introduced, i.e., 14175 must be multiplied by $5 \times 3 \times 3 \times 7 \times 7$.

224. In 1323 are three 3's and two 7's. \therefore a 3 and two 7's must be introduced.

225. 128×1728 cu. in. $\div (2 \times 4 \times 8)$ cu. in. = 3456.

226. The unit is $\frac{1}{27}$ of a cubic ft., or in cu. in. 64.

227. $(3\frac{1}{2} \times 5\frac{1}{2})$ sq. in. $\div (\frac{3}{4} \times \frac{7}{8})$ sq. in. = &c.

229. The trains go respectively $\frac{11935}{342}$ and $\frac{15345}{488}$ miles per hour, which equal $\frac{1705}{448} \times 91$, and $\frac{1705}{448} \times 81$. The rates are \therefore 91 per. hr. and 81 per hr., where $\frac{1705}{448}$ miles is the unit of measurement.

230. See 142.

122

231. A sq. in. represents a block 2 miles sq. = 4 sq. m. = 2560 ac.

232. $\frac{30\frac{1}{2}}{3} \times \frac{19\frac{1}{2}}{3} \div 4840 = \&c.$

233. $5\frac{1}{2} \times 4\frac{7}{2} \times 3\frac{1}{2} \div 27 = \&c.$

234. 5 yd. 2 ft. 6 in. \div 1 mile = $5\frac{5}{8} \div 1760 = \&c.$

235. $7\frac{1}{4}$ s. \div 20s. = $181 \div 480 = \&c.$

236. The quotient in the first case is 19. \therefore the volume of the solid is 19 times the unit volume which must \therefore be 1 cu. yd.

237. The capacity of the second cistern is 305 gals, which = 1220 of the required units. \therefore the unit is a quart.

238. $\frac{1}{104}$ of $(104 \times \frac{1}{2})$ sq. yd. = $\frac{1}{2}$ sq. yd., $\frac{1}{104}$ of $(312 \times 2\frac{1}{2})$ sq. ft. = $6\frac{1}{2}$ sq. ft., $\frac{1}{104}$ of (3744×27) sq. in. = 972 sq. in.

239. $\$10.95 \div \$4.86\frac{2}{3} = \&c.$

240. The measure when the yd. is the unit is $5\frac{1}{2}$ times as great as when the required unit is used. \therefore the unit sought is $5\frac{1}{2}$ times as great as the yd. = 1 rod.

241. First unit = $\frac{66}{100}$ of second = .66 ft. = 1 link.

243. $\$1.085 \times 3\frac{1}{2}$; 7 ft. 4 in. \times 3.1416.

244. L.c.m. of $437\frac{1}{2}$ grs. and 480 grs.

123

245. Length of side of township = $\frac{1}{2}$ in. = 5 miles, &c.

SOLUTIONS OF PROBLEMS

246. L.c.m. of 3 pt., 10 pt., 12 pt., 40 pt.
 247. One sq. in. on the map = 64 sq. miles, $\therefore 1\frac{5}{16} \times 1\frac{1}{8}$ sq. in. = &c.
 248. 4000 ac. = $\frac{4000}{640}$ sq. m. = $\frac{1}{160}$ of $\frac{4000}{640}$ sq. in. on the map = $\frac{1}{16}$ sq. in.
 249. 1 sq. in. on map = 250000 sq. ft. = &c.
 250. 121 sq. units = $\frac{1}{16}$ ac. = 484 sq. yd. \therefore 1 sq. unit = 4 sq. yd. \therefore 1 linear unit = 2 yds.
 251. Sun = 354936E = 10486.9 J, $\therefore J = (354936 \div 10486.9)$ E.
 252. 79.89 M = E = 1.25, $\therefore M = 1.25 \div 79.89 = \&c.$
 253. = $7925.648 \times 111.454 = \&c.$
 254. $2000 \times 2000 \times 50 \div 360 \div 33000 = \&c.$
 255. 320 units of work remain to be done; and 3 w. and 5 b. together do 10 units a day, & \therefore need 32d.

124

256. 6 miles $\div (2$ ft. 4 in. $\times 6\frac{2}{3}) = 6 \times 5280$ ft. $\div \frac{44}{3}$ ft. = 2160.
 257. He receives $62\frac{1}{2}$ c. an hour = $1\frac{1}{2}$ (50c.) = $1\frac{1}{2}$ units.
 258. 100 yd. in 10 sec. = 30 ft. in 1 sec. = 30 units.
 259. The width must be $2\frac{1}{2}$ linear units; but since 104 yd. carpet contain 78 sq. yd., the width must be 27 in. = $2\frac{1}{2}$ ($\frac{27}{108}$ ft.), \therefore the linear unit = $\frac{27}{108}$ ft., and the square unit = ($\frac{27}{108}$)² square ft.
 260. A man in one day will do 10 units, 17 men in 1 day will do 170 units, and in 9 days will do 1530 units.
 261. Not less than 3962.8×59.9643 miles, and not greater than 3962.9×59.9644 miles. See prob. 43, p. 104, arith.
 262. 114 ft. 7 in. = 2750 half inches, and $\frac{1}{80}$ of 2750 half in. = 55 half-in. = 55 units.
 263. If the cub. in. were pure silver it would weigh 10.474, but enough of it has been replaced by gold to increase its weight by 4.249. If the whole had been replaced by gold the weight would be increased by 8.784. $\therefore \frac{4.249}{8.784}$ of the cub. in. is gold. And this fraction of 19.258 is the weight of the gold.
 264. $62.5 \div 1.043$, lbs. = &c.
 265. Platinum is $\frac{21}{11.4}$ times as heavy as lead. \therefore required weight = $\frac{21}{11.4}$ of 56 lb. = &c.

125

266. $\frac{6}{11}$ cu. ft. copper weighs $\frac{6}{11} \times 550$ lbs. = 300 lbs. $\frac{5}{11}$ cu. ft. tin weighs $\frac{5}{11} \times 462$ lbs. = 210 lbs.

IN THE HIGH SCHOOL ARITHMETIC.

267. 180 is divisible by these numbers, and has the prime factors 2, 2, 3, 3, 5; to make this a cube the additional factors 2, 3, 5, 5 must be introduced, making $(2 \times 3 \times 5)^3$.

268. If d is the diameter required, then the volume of the wire = $(\frac{d}{2})^2 \times \frac{\pi}{4} \times 19300 \times 437\frac{1}{2} \times 500 \times 12$ cu. in., which = 1728, giving the value of d .

269. $(.4578)^3 = .0959 +$; $(.4578)^4 = .0439 +$.

270. 42 Scotch ac. = $(24 \times 37.069)^2 \times 10 \times 42 \div 144$, sq. ft.

55 Eng. ac. = $4840 \times 9 \times 55$, sq. ft.
271. Volume of the box = $99 \times 89 \times 51 - 97 \times 87 \times 49 = 35850$ cub. in.; vol. of the cavity = 413511 cub. in., \therefore additional weight = $\frac{413511}{35850}$ of 372 lbs. = &c.

272. The hammered surface is increased to $\frac{11}{10} \times \frac{8}{7}$ of its former area, \therefore the thickness must be reduced to $\frac{7}{8} \times \frac{8}{8}$ of its former measurement, i.e. by $\frac{1}{8}$.

273. In efficiency A : B : C : D as 60 : 40 : 45 : 48. \therefore A is to B, C and D together as 60 : 133, \therefore A can do a piece of work in $\frac{133}{60}$ of the time taken by the other three, or half as much as the others in $\frac{133}{120}$ of their time = $\frac{133}{120}$ of 10 days = &c.

274. In value 1 part alloy = $\frac{1}{235}$ of 1 part gold = $\frac{1}{235}$ of 11 parts gold = $\frac{1}{210}$ of the whole coin; \therefore $\frac{1}{12}$ of 4 oz. troy of alloy = in value $\frac{1}{120}$ of 15 G., &c.

275. A, B, C share the \$3783 in the ratio 441 : 420 : 400. \therefore A receives $\frac{441}{1261}$ of \$3783; &c.

276. That is : What length of rod an inch square will weigh 50 tons? No. feet = $50 \text{ tons} \div 3\frac{1}{2} \text{ lbs.} = \frac{200000}{7} = \&c.$

126

277. After the first drawing $\frac{5}{8}$ of the wine remains; after the second $\frac{7}{8}$ of $\frac{5}{8}$, &c. After the last drawing there will remain $\frac{7}{8}$ of $\frac{5}{8}$ of $\frac{5}{8}$ of $\frac{5}{8}$ of $\frac{5}{8}$ of 120 gal.

278. A hectolitre = $\frac{1}{10}$ cu. metre = $\frac{1}{10}$ (39.37)³ cu. in., and \therefore contains $\frac{1}{10}$ (39.37)³ \times 252.458 grs. of water; this divided by 7000 grs. gives the no. lbs., and this by 10 lbs. gives the no. gal.; and this by 8 gal. the no. of bushels.

279. Since $1397 \div 127 = 11$, the question becomes "What 3 different numbers added together = 11?"

280. Each ton yielded at least $7\frac{1}{2}$ oz. silver = $52\frac{1}{2}$ oz. \therefore 5 tons must have yielded an extra $\frac{1}{2}$ oz. each to make up the 55 oz. \therefore the lead must be $\frac{75}{100}$ of 5 tons + $\frac{75}{100}$ of 2 tons = $\frac{75}{100} \times \frac{5}{4} + \frac{75}{100} \times \frac{2}{4}$ of the whole mass.

SOLUTIONS OF PROBLEMS

281. A convenient method is to work from the boiling point in each case, remembering that $9^{\circ}\text{F} = 5^{\circ}\text{C}$.

282. Divide l.c.m. of 113.002 and 89.604 by 113.002.

283. L.c.m. of 16 ft. and $27\frac{1}{2}$ ft. = 880 ft., 9 miles \div 880 ft. = 54.

284. If the first set do as much in 1330 d. as the second do in 350, then the first set will do as much in 19 d. as the second in 5, or 175×19 m. and 240×19 b. will do as much in 1 d. as 603×5 m. and 1005×5 b. The first has 310 more m. than the second, and the second 465 more boys, $\therefore 310 \text{ m.} = 465 \text{ b.}$ or $2 \text{ m.} = 3 \text{ b.}$

127

285. The loss = $\frac{221^8 - 212^4}{221^8}$ of £1075 = &c.

286. If each coin weighs 120 units, the gold in the first will weigh 110 and the alloy 10; in the second the gold 108, the alloy 12. If the value of a unit of weight is the unit of value, the first coin is worth $110 \times 15 + 10 = 1660$; and the second $108 \times 15 + 12 = 1632$; and these numbers are as 415 : 408.

287. $32 \text{ ft.} \times 813 = \&c.$

288. If the two compositions are added together there will be 18 lb. gold and 18 lb. silver, worth in all £896 8s.; 18 lb. gold are worth £841 1s.; \therefore the silver is worth £55 7s., or 5s. $1\frac{1}{2}$ d. per oz.

289. The work when done by the men costs \$255, and when done by the boys \$319.20.

290. Volume of box = $18 \times 10 \times 6 - 17 \times 9 \times 5$ cu. in. = 315 cu. in. Vol. of sand = 765 cu. in. 1 cu. in. wood weighs $\frac{15}{8}$ lb. $\frac{1}{8}$ lb. 1 cu in. sand weighs $\frac{85}{8}$ lb. = $\frac{1}{8}$ lb. \therefore weights of equal bulks are as $\frac{1}{8} : \frac{1}{8} = 3 : 7$.

291. The average width is $6\frac{1}{2}$ ft.; the depth $3\frac{1}{2}$. \therefore no. cu. yd. = $6\frac{1}{2} \times 3\frac{1}{2} \times 360 \div 27 = 303\frac{1}{3}$.

128

292. The flint occupies the space of $\frac{20000}{2\frac{1}{2}}$ grs. of water = $\frac{20000}{2\frac{1}{2}} \div 8750$ pints; similarly the granite occupies the space of $\frac{10000}{2\frac{1}{2}} \div 8750$ pints; leaving $8 - (\frac{2}{2\frac{1}{2}} + \frac{1}{2\frac{1}{2}}) \frac{10000}{8750}$ pints of water.

293. Fast train goes $\frac{1}{2}$ of the distance while the other should go $\frac{2}{3}$; \therefore ordinary rate of slow train is $\frac{2}{3}$ that of the fast train; \therefore slow train should make the journey in $3\frac{1}{3}$ hrs., but owing

IN THE HIGH SCHOOL ARITHMETIC.

to reduced speed its rate is $\frac{1}{3}$ that of fast train. \therefore it requires 6 hrs.

294. The person walking 4 m. an hr. goes $19\frac{5}{8}$ yd. in 10 sec. \therefore the train in passing him goes 88 yd. + $19\frac{5}{8}$ yd. in 10 sec. \therefore in 9 sec. the train goes 96.8 yd., = 88 yd. + 8.8 yd. \therefore the second person goes 8.8 yd. in 9 sec. = 2 m. an hr.

295. 192 sq. cubits cost 100 nishcas = $100 \times 256 \times 80$ cowrie shells. \therefore $1\frac{1}{2}$ sq. cub. cost $\frac{100 \times 256 \times 80 \times 7}{4 \times 100}$ c. s. = 18666 $\frac{2}{3}$ c. s. = &c.

296. Each of the first benches = $\frac{1}{2} \times \frac{2}{3} \times 14$ cubic cubits. \therefore 30 benches = 140 cu. cub.; in the second case 14 benches = $14 \times \frac{1}{2} \times \frac{1}{2} \times 10$ cu. cub. = 140 c. c. = $\frac{1}{3}$ of the vol. of the first lot, but as they are to be carried 6 times as far the cost will be the same, or 8 dr.

297. The first parcel contains 485640 grs.; the second, 264252 grs.; the third = 32.452 lbs. = 32.452×7000 grs. = 227164 grs.; the g. c. m. of these is 76 grs.

298. The point of the hour hand goes $2 \times \frac{2}{3} \times \frac{2}{3}$ in. in 12 hr. = $\frac{1}{3}$ in. in 1 hr.; the point of the min. hand goes $2 \times \frac{2}{3} \times \frac{2}{3}$ in. in 1 hr.; and that of the sec. hand, $2 \times \frac{2}{3} \times \frac{2}{3}$ in. in 1 min. = $2 \times \frac{2}{3} \times 20$ in. in 1 hr. In whole nos. these become 5 in., 112 in., 2800 in. in $22\frac{3}{11}$ hrs.

129

299. Divide the sum into 60 units; then B and C together receive 36, and A, 24 of these units; but the first sharing gave Band C together 30; \therefore of the balance they received 3 each; \therefore A's share of the balance was 3, and his first share 21.

300. Taking 7, 6, 5 as the length, breadth and height, the area of walls and ceiling = 172; \therefore each sq. unit cost \$2 = cost of 4 sq. yd.; \therefore 1 sq. unit = 4 sq. yd.; but area of floor = 42 sq. units = 168 sq. yd.

301. The first trains pass at the rate of 176 yd. in 6 sec. = 60 m. an hr. \therefore the speed of the first down train is 25 m. an hr. In the second case the trains pass at the rate of 220 yd. in 6 sec. = 75 m. an hr. \therefore speed of second down train is 40 m. an hr. \therefore second down train is overtaking the first at the rate of 15 m. an hr., and is 15 miles 88 yd. behind it; (= sum of the distances gone by the first two trains in 15 min. 6 sec. less the length of the first down train). \therefore the second train will reach the first in 1 hr. 12 sec., that is at 15 min. 18

SOLUTIONS OF PROBLEMS

sec. past 1 o'clock. If these two trains are on parallel tracks they will be abreast in another 12 sec.

302. The first trench cost $\frac{5}{18}$ c. per cu. ft. \therefore the second trench being $\frac{5}{4}$ as deep costs $\frac{5}{4}$ of $\frac{5}{18}$ c. = $\frac{25}{72}$ c. per cu. ft. \therefore total cost = $5 \times 10 \times 360 \times \frac{25}{72}$ c. = \$62.50.

303. In the first case he saves 23 per cent., and in the second, $33\frac{1}{3}$ per cent. of his income. \therefore &c.

130

307. Every even number except 2 has for factors 2 and a number greater than 1, and \therefore cannot be a prime number.

308. The difference between the squares of any two consecutive numbers = twice the less no. + 1 and is \therefore an odd no. Also since $6^2 - 5^2 = 2 \times 5 + 1 = 11$ and $5^2 - 4^2 = 2 \times 4 + 1 = 9$, &c., it is readily seen that the successive pairs of square numbers produce the consecutive odd nos.

309. (1) Since an even no. is a multiple of 2, the sum of any no. of even nos. must be a multiple of 2 and \therefore even.

(2) Any pair of odd nos. makes an even no.; \therefore any no. of pairs of odd nos. will make an even no. These may also be shown thus: The general expression for an even no. is $2n$ where n is any whole no.; and for any odd no. $2n+1$. If \therefore we take the sum of any no. of even nos. as $2m+2n+2p$, we get $2(m+n+p)$ which, being divisible by 2, is an even no.; also, an even no. of odd nos. as $2a+1, 2b+1; 2c+1, 2d+1; 2m+1, 2n+1$; will give $2(a+b)+2; 2(c+d)+2; 2(m+n)+2$; a series of even nos., \therefore the whole sum is even.

(3) The third case is simply the addition of an odd no. to the result in (2) and an odd no. added to an even no. produces an odd no.

310. Since any two consecutive odd nos. differ by 2. \therefore any pair of odd nos. will differ by 2 or by some multiple of 2; i.e., the difference between any two odd nos. is an even no. Or, thus: $2m+1 - (2n+1) = 2m - 2n = 2(m-n)$ = a multiple of 2 = an even no. So that if an odd no. is divided by an odd no. and the quot. is odd the subtrahend must be odd and \therefore the rem. even. Also if the quot. is even the subtrahend is even and the rem. odd.

311. Every even no. is made up of two factors, one-half of itself and 2, and since 2 can contain no part of the odd no,

IN THE HIGH SCHOOL ARITHMETIC.

the one-half must contain it, and \therefore the whole no. must contain it at least twice.

312. (1) An even no. \times an even no. = an even no., for $2m \times 2n = 4mn = 2(2mn)$ = an even no. \therefore if one factor of an even no. is even the other may be even.

(2) An even no. \times an odd no. = even no. for, $2m \times (2n+1) = 2(2mn+m)$ = even no. \therefore if one factor of an even no. is even the other may be odd.

(3) Odd no. \times odd no. = odd no. for, $(2m+1) \times (2n+1) = 4mn+2m+2n+1 = 2(2mn+m+n)+1$ = even no. + 1 = odd no. \therefore when an odd no. is exactly divisible by an odd no. the quot. must be odd. \therefore , also, if divisor and quot. (in exact division) are odd the dividend (their product) must be odd. \therefore when an even no. is exactly divisible by an odd no. the quot. must be even.

From (1) and (2) it appears that the quot. of an even no. by an even, when exact, is either odd or even.

313. The following should now be clear:—

odd) even (even	odd) even (odd
even	odd
even	odd

$$314. 479 \times 125 = 479 \times (1000 \div 8) = 479000 \div 8. \quad 873294 \div 99; \text{ divide by } 3, 3, 11.$$

$$\begin{array}{r} 34687 \\ 320648 \\ \hline 277496 \\ 2219968 \\ 11099840 \\ \hline 11122317176 \end{array}$$

The multiplicand is first multiplied by 8, then this result is multiplied by 80, making 640 times the mult.; then this result is multiplied by 500, making 320000 times the mult. \therefore the mult. has been multiplied by $320000 + 640 + 8 = 320648$.

315. On dividing by 7 we get 548 groups of 7 and 6 units. On dividing 548 by 9 we get 60 groups of nine 7's, or 63 and eight 7's, or 56 left, which, with the 6 units left after the first division, make 62 for the complete remainder.

SOLUTIONS OF PROBLEMS

$$316. 10 \text{ times } .\dot{7} = 7.777\ldots$$

$$\quad \quad \quad .\dot{7} = \underline{.777\ldots}$$

$$9 \text{ times } .\dot{7} = 7$$

$$\text{or } .\dot{7} = \frac{7}{9}.$$

317 & 318. These are particular cases of the Commutative Law that additions and subtractions may be performed in any order, which is generally considered self-evident.

320. It is necessary, for if they are not prime to each other, they must have a common factor, and are \therefore not in their lowest terms; and it is sufficient, for if they are prime to each other they have no common factor and \therefore cannot be reduced to lower terms.

321. If the last two digits be replaced by 0's the no. is a multiple of 100 and is \therefore divisible by 4; \therefore any remainder is due to the last two digits. \therefore if the no. expressed by these is divisible by 4 so also is the whole number.

322. If the primary unit is, say, 1 ft., then $\frac{1}{5}$ ft. is a derived unit, and $\frac{5}{5}$ ft. is a quantity which contains this derived unit 5 times.

131

323. Every number must divide by 3 exactly, or with remainder 1 or with rem. 2, that is, must be of one of the three forms $3n$, $3n+1$, $3n+2$, where n is any whole number. The squares of these nos. are respectively $9n^2$, $9n^2+6n+1$, $9n^2+12n+4$; the first of these leaves no rem. when divided by 3, and each of the others leaves remainder 1.

324. Any no. whose units digit is 0 is a multiple of 10, and \therefore divisible by 2 and 5; \therefore any remainder on dividing a no. by 2 or 5 must be due to the units digit.

325 & 326. If a no. ends in 5 its sq. will end in 5; if a no. ends in 4 or 6. its sq. will end in 6; if in 3 or 7, the sq. must end in 9; if in 2 or 8, the sq. ends in 4; if in 1 or 9, the sq. in 1; if in 0, the sq. in 0; and as nos. can have no endings other than these, squares can have no endings other than 0, 1, 4, 5, 6, 9; \therefore no sq. no. can end in 2, 3, 7 or 8.

329. The third term is $\frac{1}{3}$ of the second, the fourth $\frac{1}{4}$ of the third, etc. See no. 81 arith. p. 107.

330. Three more terms must be taken. It will be well to obtain the last term directly in order to verify the work; this

IN THE HIGH SCHOOL ARITHMETIC.

may be done thus:—

$$\begin{array}{r}
 10) 1 \\
 \hline
 8) .1 \\
 \hline
 8) .0125 \\
 \hline
 9) .0015625 \\
 \hline
 9) .00017361 \\
 \hline
 7) .00001929 \\
 \hline
 .00000275
 \end{array}$$

The divisors used will be found to contain all the factors in the nos. 1 to 9.

331. $2 - (1.4142)^2 = .00003836$, which when divided by 2×1.4142 will give the next 3 digits in the sq. root which to 7 places is 1.4142135.

334. "Multiplications and Divisions may be performed in any order." Commutative Law.

335. These two nos. added together = the sq. of the odd no., and their difference is 1; \therefore their sum multiplied by their dif. = sq. of the odd no. But the third side = sq. rt. of the dif. of the squares of these nos. = sq. rt. of prod. of their sum and dif. = sq. rt. of the sq. of the odd no. = the odd no. \therefore the third side in all such cases is the odd no. itself.

This may be used to form right angled triangles whose sides are required to be whole nos. and will give a triangle for each odd no. thus: 3, 4, 5; 5, 12, 13; 7, 24, 25; 9, 40, 41 : etc.

336. When any no. is multiplied by 11, the last digit in the product must be the same as the last digit in the no.; and since the quot. \times divisor (11) = dividend \therefore dividend and divisor must end in same digit.

132

337. In addition, the units, tens, hundreds, etc., are added separately. In multiplication, the units, tens, etc., in multiplicand and multiplier are practically treated as separate terms, and each term in the former is multiplied by each term in the latter.

338. One disadvantage would be that the different orders must be added separately, and then these sums added to get the final aggregate.

339. This also holds if the same number be subtracted from each.

SOLUTIONS OF PROBLEMS

340. So far as this is true it will be exemplified by a careful inspection of the operation in each case.

341. Any no. of 3 digits is less than 1000, any no. of 4 digits is less than 10000; \therefore their product is less than 10,000,000 and \therefore cannot have more than 7 digits, and so on; or, generally, any no. consisting of m digits is less than 10^m ; any no. of n digits is less than 10^n ; \therefore their product is less than 10^{m+n} and \therefore cannot consist of more than $m+n$ digits since 10^{m+n} is the least no. consisting of $m+n+1$ digits.

342. Consider the dividend as subdivided into units, each equal to the divisor, then the number of these units is the quotient. If this dividend be multiplied by any no., say 3, we may consider the result in either of two ways: (1) as consisting of the same no. of units as before, each unit being 3 times as large as in the former case, when evidently the divisor has been multiplied by 3 and the quotient remains the same; (2) as consisting of 3 times as many units as before, the unit remaining the same. If now we group these units in threes, each group will be equal to the multiplied divisor and the no. of these groups is the same as the original quotient.

If there is still a part of the dividend left after all units equal to the divisor have been taken, then evidently this remainder will be multiplied by whatever no. the dividend has been.

343. This will appear from case (2) above, in which, when the unit remains the same, the quotient becomes 3 times as great.

344. The number added is a larger fraction of the denominator than of the numerator. \therefore the denominator has been increased in a greater ratio than the numerator, \therefore the value of the fraction has been diminished.

345. Multiply any numbers, say 837 by 429, and then divide the product by the multiplicand; and compare the steps in the two operations.

346. The sq. of an even no. is even, the sq. of an odd no. is odd. \therefore the difference between any no. and its sq. is either the difference between two even nos. or the difference between two odd nos., and is \therefore an even no.

347. Every even no. contains 2 as a factor. \therefore its sq. contains 2×2 , or 4 as a factor.

348. Take such a multiplier as 1999, then the last partial product is 1000 times the multiplicand, and the sum of all the other partial prods. is 999 times the multiplicand, \therefore the statement holds in this case, and \therefore *a fortiori* when the left hand digit is greater than 1, or any of the other digits less than 9.

349. See 325.

350. The chance of errors occurring is always diminished when questions are worked by more than one method, but cannot always be guarded against with certainty.

133

351. See Arith. pp. 42, 47.

352. See Arith. p. 66.

353. The side of a 10-acre sq. field is 10 chains in length; it is also 220 yds. in length; \therefore 1000 links = 10 ch. = 220 yds. = 7920 in. \therefore 1 link = 7.92 in.

354. See Arith. pp. 49, 69.

355. See 344.

357. Before two or more quantities can be expressed as a definite aggregate they must have a common unit of measurement. Such a unit must therefore be found as can bear the relation of derived unit to each of the separate units of measurement.

358. See 316.

359. Since the dividend is 9 times the quotient, \therefore the sum of the dividend and quotient is 10 times the quot., \therefore the last digit is 0.

360. See Arith. p. 56.

361. $(99.9899995)^2 = (100 - .01 - .0000005)^2 = 100^2 + .0001 - 2 - .0001 - .00000001 + (.0000005)^2 = 9998 - .00000001 + 25 \text{ to } 12\text{th place.}$

362. 3.14159 divided in this way = .999722 and differs from the true quotient 1 by .000277, \therefore any quotient so obtained will differ from the true quotient by .000277 of the quotient.

363. $a \text{ ft. a min.} = 15a \text{ ft. in } 15 \text{ min.} = 1 \text{ mile. } \therefore a = \frac{1}{15} \text{ mile} = 352 \text{ ft.}$

364. This ultimately reduces itself to the question whether in introducing or in striking out two factors, say 3 and 5,

separately we shall get the same result as we would by at once introducing or striking out the factor 15.
365. See 334.

134

366. See 317.

367. A fraction may be changed in four ways :

1. By the introduction of equal factors in its two terms.
2. " suppression " " " "
3. " introduction of unequal " " " "
4. " suppression " " " "

In (1) it is easily shown that the resulting fraction is equal to the original. (2) does not apply to the present case since the terms are prime to each other. In (3) and (4) the terms of the fraction are increased or diminished in unequal ratios, and \therefore the resulting fraction cannot be equal to the old.

368. If we take any two nos. that have a com. meas., such as 15 and 35, we see that their product is made up of 3, 7 and the sq. of the com. factor 5; their l.c.m. is the product of 3, 7, and 5, and their g.c.m. is the other 5. And the same is true when, instead of single factors, as 3, 5, and 7, there are groups of factors.

369. If $2m+1$, $2n+1$ are any two odd nos. their sqrs. are $4m^2+4m+1$, $4n^2+4n+1$; the sum of their sqrs. = $2(2m^2+2n^2+2m+2n+1) = 2 \times \text{an odd no.}$, and \therefore does not contain the factor 4 which every even sq. must do. See 347.

370. Every sq. no. must end in one of the digits 0, 5, 1, 6, 4, 9; See 325. Nos. ending in 0, 5, are divisible by 5. Nos. ending in 1, 6, are each greater by 1 than a multiple of 5; while those ending in 4, 9, are each less by 1 than a multiple of 5.

371. See arith. p. 55.

372. See 348. The 1 in the thousands place is greater than the sum of 900, 90, and 9, &c.

373. $10 = 9 + 1$; $100 = 11 \times 9 + 1$; $1000 = 111 \times 9 + 1$; and generally any power of 10 is 1 more than some multiple of 9, i.e., will leave 1 for rem. when divided by 9. Again, since $100 = 11 \times 9 + 1$, $\therefore 300 = 33 \times 9 + 3$; similarly $7000 = 777 \times 9 + 7$, i.e., 300 when divided by 9 has 3 for rem., and 7000 has 7 for rem.; and the same is readily seen to hold for any nos. so formed.

But any no., as 7647, may be expressed thus : $7000 + 600 + 40 + 7$; the first of these is 7 more than a multiple of 9, the next 6, the third, 4, so that as a result of the separate divisions there would remain $7 + 6 + 4 + 7$; but these contain 9 twice with rem. 6. \therefore &c.

374. Two quantities are multiplied together when each term in one is multiplied by each term in the other and the partial products collected, and the process here exemplified consists in multiplying 16 by 3 and by $\frac{1}{2}$, and $\frac{5}{8}$ by 3 and by $\frac{1}{4}$ and collecting the products so obtained.

375. The \$18.75 is $\$18.75 \times 1$; the 1.13 is $\$18.75 \times .06$ approximately. $\therefore 19.88 = 18.75 \times 1.06$ and similarly for the other products.

135

376. See 344. Take the fraction $\frac{13}{7}$ and subtract 3 from each term and the fraction becomes $\frac{10}{4} = 2\frac{1}{2}$. Take 1 more from each term and the fract. $= \frac{9}{3} = 3$; take away 1 more and we have $\frac{8}{2} = 4$; take 1 more away and the fraction is increased to 7.

377. It will easily be seen that the remainders are obtained as fast as the products are formed, and the products are \therefore not set down, nothing appearing but the successive dividends.

379. The addition or subtraction of two multiples of any factor or unit cannot cause that factor or unit to disappear. Seven 3's and four 3's are eleven 3's. 7 quarts and 4 quarts are 11 quarts. Seven 3-qts. and four 3-qts. are eleven 3-qts.

381. See arith. p. 60.

382. Whether $6 \times 3 = 18$ is a correct solution or not will depend upon the meaning attached to this expression as well as upon the process of reasoning which led up to the formal solution. If the reasoning is that 3c. is the price of 1 stamp

and \therefore 6 stamps will cost 6 times as much, we shall have $\frac{18c.}{3c.}$ as the solution. If, however, the reasoning were that, at 1c. each, the stamps would cost 6c., \therefore at 3c. each the cost is 3 times as great; we would have $\frac{6c.}{3c.}$ as the solution.

18c.

384. $\frac{1}{11} = \frac{1}{10} + \frac{5}{110}$ where the form of the repeating period is determined by the 7. See arith. p. 59.

385. The civil year is 365d. or 366d.; the solar year is the period of time in which the earth performs a revolution in its orbit round the sun = 365 days, 5 hrs., 48 min., 46 sec., mean solar time. See any good encyclopedia.

386. Silver coinage is legal tender to the amount of \$10; copper to the amount of 25c.

$$387. \frac{5+8}{6+9} = \frac{13}{15} = \frac{5}{3} \times \frac{26}{25} = \frac{8}{5} \times \frac{117}{125}.$$

136

388. See 345, 348, 372.

389. See 347, 369. The sq. of an odd no. assumes the form $(2n+1)^2 = 4n^2 + 4n + 1 = 4(n^2 + n) + 1 =$ a multiple of 4 + 1.

390. Square nos. must contain factors in pairs, if then 3 occurs as a factor it must occur twice, i.e., 9 is a factor.

391. The g.c.m. of two nos. contains all the factors common to these nos.; any other c.m. must be made up of a part only of these same factors. \therefore g.c.m. is a multiple of each c.m. and since the g.c.m. is itself a c.m. it must be the least com. multiple of all the com. measures.

392. 3 and 5 are c.m. of 30 and 45, but neither is a factor of the other.

393. Each com. mult. of two nos. must contain every factor that these nos. contain; thus each com. mult. of 20 and 15 must contain 2, 2, 5, in order to contain 20 and 3, 5, in order to contain 15. \therefore in order to contain them both it must contain 2, 2, 5, 3. The l.c.m. contains these factors only, any other c. m. contains these with additional factors, (either the same or different) and \therefore is a multiple of the l.c.m.

394. 12 and 18 are com. mults. of 2 and 3, but neither is a mult. of the other.

395. By 2 if the last digit is 0, or divisible by 2; by 4 if the last two digits are 0's, or form a no. divisible by 4; by 8, if the last three; by 16, if the last four, &c. By 3 if the sum of the digits is divisible by 3; by 9 if the sum is divisible by 9; by 5 if the last digit is 0 or 5; by 25 if the last two are 0's or form a no. divisible by 25; by 125 if the last three, &c. By 6 if the no. is divisible by 2 and by 3.

396. In the first a quantity and its unit of measurement are given to find the no. of times the quantity contains the unit *i.e.* to find its measure. In the second the quan. and its measure are given to find the unit of measurement.

397. See arith. p. 42.

398. The dividend is 101 times the quot. \therefore if the quot. be subtracted from the dividend the remainder will be 100 times the quot.; *i.e.* will end in two 0's; \therefore the last two digits in dividend and quot. must have been the same. See 336.

399. Every no. must be of one of the forms $6n$, $6n+1$, $6n+2$, $6n+3$, $6n+4$, $6n+5$, *i.e.*, must be divisible by 6 either without rem. or with rem. 1, 2, 3, 4 or 5. But $6n$ is a multiple of 6; $6n+2$ and $6n+4$ are multiples of 2; $6n+3$ of 3, leaving as the only possible primes $6n+1$, $6n+5$.

400. Since the dividend is 9 times the quot., if we can subtract the dividend from 10 times the quot. the rem. will be the quot. If the quot. is multiplied by 10 the last digit in the prod. is either 0, or the first digit to the right of the dec. pt. in the quot. which is always the rem.; thus the full quot. in the example given is 597218.3. \therefore the units digit in 10 times the quot. is 3, *i.e.*, is the remainder after dividing by 9.

401. Since any even power of 10, (100, 10000, &c.) when divided by 11, leaves 1 for rem. \therefore any nos. such as 7400, 380000, &c., when divided by 11 will leave the same rem. as 74, 38, &c. \therefore the rem., after dividing 387426, is the sum of the rems. after dividing 380000, 7400, $26 = \text{sum after dividing } 38, 74, 26 = 5 + 8 + 4 = 17$, which contains 11 once, leaving a final rem. 6. Similarly the rem. left by 5783742 = sum of rems. left by 5, 78, 37, $42 = 5 + 1 + 4 + 9$, giving final rem. 8.

137

402. Apply 401 or the following: if the sum of the 1st, 3d, 5th, &c. digit = sum of 2d, 4th, 6th, &c., there will be no rem.

404. Divide 8 by 2, add 7, divide by 2, add 6, &c., and it will be found that 8 has been divided 9 times by 2; 7, 8 times; 6, 7 times, &c. The last division is by 2^2 which gives .98046875 as the final result.

405. By extracting sq. root, cub. root, &c.

406. See arith. p. 60.

407. See 334.

408. See 364.

SOLUTIONS OF PROBLEMS

409. The dividend is 11 times the quot. \therefore dividend - 10 times quot. = quot. The remainder 8 in the Ex. enables us to find the decimal part of the quot., which is $\frac{8}{11} = .72$. \therefore 10 times the quot. as far as the decimal and units digits = 7.27. Subtracting this from the dividend will give the quot. as far as the dec. and units digits, viz., 7.72. We can now determine 10 times the quot. as far as the tens digit, viz., 77.27; subtracting this new digit gives the tens digit in the quot., &c.

138

410. See 317.

411. If the three nos. are ar, br, cr, then the l. c. m. is a b c r, and the g. c. m. is r; their prod. is ar br cr = a b c r \times r² = &c. But this does not hold in the general case where two of the nos. may have a com. factor not contained in the third. Let A, B, C represent the three nos., then A may have a factor x not contained in either B or C, a factor m contained in B but not in C, or factor n in C, but not in B, and a factor r com. to B and C; thus A = x m n r; similarly B = y m p r and C = z n p r. The l. c. m. of these is x y z m n p r; the g. c. m. is r, and the prod. of A, B and C is x y z m² n² p² r³.

412. Every no. is of one of the forms 9n, 9n+1, &c. to 9n+8. On dividing the cubes of these nos. by 9 the truth of the statement will appear.

413. A no. divisible by 9 and by 11 is divisible by 99.

414. The dividend is 99 times the quot., and if this be subtracted from 100 times the quot., the rem. will be the quot.

415. See 325.

416. Arith. pp. 62, 63, suggest the answer viz., from any no. of p+q digits subtract the no. formed by the first p digits; this gives the numerator of the fraction. The denominator will be q 9's followed by p 0's.

417. 37, 111, 999.

418. It fails when digits have become transposed or when 0 has been set down for 9, or 9 for 0.

419. See 314.

422. These are .576634, .28345, .1654321.

423. See arith. p. 55.

424. \$3.60 is the price of a bushel, \$1.80 being one-half of \$3.60 is the price of half a bushel, or 2 pecks; similarly 45c. is the price of 1 gal., and 22½c. of 2 qts.

PROBLEMS ARISING FROM BUSINESS TRANSACTIONS.

PERCENTAGE.

140

21. 10% of the number = 13. \therefore 100% of the number = 130.

22. Number of boys = 60% of number of girls = $\frac{60}{100}$ of 60 = 36. \therefore number of pupils = 60 + 36 = 96.

23. $112\frac{1}{2}\%$ of av. attendance last term = 225. \therefore 1% of av. attendance last term = 2. \therefore av. attendance last term = 200.

24. 114% of a number = 285. \therefore the number = $285 \times \frac{100}{114} = 250$.

25. $100 - 11\frac{1}{4} = 88\frac{3}{4}$. $88\frac{3}{4}\%$ of a number = 710. \therefore number = $710 \times \frac{1}{88\frac{3}{4}} \times 100 = 800$.

26. A loaf formerly cost 10c. It will now cost 125% of 10c, or $12\frac{1}{2}$ c. \therefore 4 loaves may now be bought for 50c.

27. 90% of remainder after battle = 360 men. \therefore remainder after battle = 400 men. \therefore 80% of original regiment = 400 men. \therefore original regiment = 500 men.

28. 2nd year's earnings = 105% of 1st year's earnings. \therefore 205% of 1st year's earnings = \$6560. \therefore 105% of 1st year's earnings = $\$6560 \times \frac{105}{205} = \3360 .

29. $62\frac{1}{2}$ lbs. = weight of 1 cu. ft. of water. \therefore 2000 lbs. = weight of 32 cu. ft. of water. 32 cu. ft. of water becomes (110% of 32) cu. ft. of ice, or $35\frac{1}{2}$ cu. ft. of ice.

30. A creditor receives 30% of the debt. he loses 70% of the debt. \therefore he receives $\frac{3}{7}$ of amount of loss. i.e., he receives $42\frac{6}{7}\%$ of amount of loss.

141

31. The increased value = 220% of cost = 220% of \$4000 = \$8800.

SOLUTIONS OF PROBLEMS

32. He saves $7\frac{1}{2}\%$ of a year's salary in 1 year. \therefore he saves 100% of a year's salary in $\frac{100}{7\frac{1}{2}}$ years, or $13\frac{1}{3}$ years.
33. 10% of greater number = 20% of smaller number. \therefore $32\frac{1}{2}\%$ of smaller number = 39. \therefore smaller number = $39 \times \frac{100}{32\frac{1}{2}} = 120$. \therefore greater number = 240.
34. First number = 120% of second number. \therefore $12\frac{1}{2}\%$ of first number = 15% of second number. Hence 10% of second number = 10. \therefore second number is 100, and \therefore first number is 120.
35. Each unit of volume becomes $(1.001)^3$ units, or 1.003003001 units. \therefore increase = $.3003001\%$.
36. $33\frac{1}{3}\% = \frac{1}{3}$. A's money = $\frac{1}{3}$ of B's money. \therefore B's money = $\frac{3}{1}$ of A's money, = 75% of A's money.
37. 50% of the silver money = \$1.50. \therefore the silver money was \$3. \therefore the paper money was \$9.
38. A cu. metre = $(39.37)^3$ cu. inches = 61023.377 + cu. in. A cu. yard = 36^3 cu. in. = 46656 cu. in. $61023.377 = 130.7 + \%$ of 46656. \therefore a cu. metre is $30.7 + \%$ more than a cu. yard.
39. The number of boys is $37\frac{1}{2}\%$ of whole number. \therefore the number of girls is $62\frac{1}{2}\%$ of whole number. \therefore 25% of whole number = 50. \therefore whole number = 200. \therefore number of girls = 125.
40. A does 20% of work in two days, and B does $37\frac{1}{2}\%$ of work in three days. Whole amount done is $57\frac{1}{2}\%$ of work. \therefore $42\frac{1}{2}\%$ of work remains to be done.
41. $33\frac{1}{3}\%$ of the man's share was worth \$12,000. \therefore the man's share was worth \$36,000. \therefore 30% of value of mine was \$36,000. \therefore value of mine was \$120,000.
42. 60% of \$4000 = \$2400. 40% of 3 times the value of the house = \$2400. \therefore 3 times the value of the house = \$6000, or value of the house = \$2000.
43. 10% of 1st part = $16\frac{2}{3}\%$ of 3rd part. \therefore 1st part = $166\frac{2}{3}\%$ of 3rd part. $12\frac{1}{2}\%$ of 2nd part = $16\frac{2}{3}\%$ of 3rd part. \therefore 2nd part = $133\frac{1}{3}\%$ of 3rd part. \therefore 1st part + 2nd part + 3rd part = 400% of 3rd part = 1440. \therefore 3rd part = 360, 1st part = 600, 2nd part = 480.
44. $\frac{1}{2}$ is 100% of $\frac{1}{4}$. \therefore 1 is 200% of $\frac{1}{4}$. \therefore $\frac{1}{4}$ is 40% of $\frac{1}{2}$.
45. C receives 4% of selling price. \therefore A and B receive 96% of selling price. \therefore A receives 80% of 96% of selling price, or 76.8% of selling price.

142

46. 126 gal. = 504 qts. 2 gal., $1\frac{3}{4}$ qt. = $9\frac{3}{8}$ qt. $\frac{9\frac{3}{8}}{504} = 1\frac{1}{2}\%$
 47. 100 lbs. of flour is made into 125 lbs. of bread.
 48. 1 lb. Av. = 7000 grs.: 1 lb. Troy = 5760 grs. He sells 5760 grs. for the cost of 7000 grs. \therefore on 5760 grs. he gains the cost of 1240 grs. \therefore on 1 gr. he gains $\frac{1240}{5760}$ of cost of 1 gr. \therefore he gains $21\frac{1}{3}\%$ of cost.
 49. 90% of amount Brown gave = \$15840. \therefore Brown gave \$17600. \therefore 110% of actual value = \$17600. \therefore actual value = 16000.
 50. He drinks 1095 pints of beer in one year. \therefore he drinks 4% of 1095, or 43.8 pints of alcohol in 1 year.
 51. 3% of half of the number is $1\frac{1}{2}\%$ of the number. $\therefore 3\frac{1}{2}\%$ of the number = 21. \therefore the number = 600.
 52. 6% of twice the number = 12% of the number. $\therefore 17\%$ the number = 175. \therefore the number = $1029\frac{1}{7}$.

TRADE DISCOUNT.

1. 10% of \$600 = \$60: \therefore he paid \$540.
 2. 85% of amount of bill = \$850
 \therefore 15% of amount of bill = \$150 = discount.
 3. Original amount of bill was \$200: discount was \$30:
 \therefore rate of discount was 15%.
 4. Marked price was \$5 per ream, \therefore price, after discount, was $87\frac{1}{2}\%$ of \$5 per ream, \therefore cash price was 98% of $87\frac{1}{2}\%$ of \$5, or \$4.28 $\frac{1}{2}$ per ream.
 5. To make a gain of \$3, the article must sell for \$15:
 \therefore 90% of marked price = \$15: \therefore marked price = \$16.66 $\frac{2}{3}$.
 6. $87\frac{1}{2}\%$ of marked price = \$4
 \therefore marked price = \$4.57 $\frac{1}{7}$.

143

7. 80% of marked price = \$3: \therefore marked price = \$3.75.
 8. Reduced price = 90% of marked price. Customer pays 90% of reduced price: \therefore pays 81% of original marked price.
 9. By previous result 10 and 10% off leaves 81% of original price. But 20% off leaves 80% of original price. Hence difference is 1% of original price, or $1\frac{1}{4}$ cents.

SOLUTIONS OF PROBLEMS

10. 95% of marked price = \$7.60
 \therefore marked price = \$8.00
 $133\frac{1}{3}\%$ of cost = \$8.00 : \therefore cost = \$6.00.
11. Marked price = 140% of cost : \therefore selling price = 90% of 140% of cost = 126% of cost. \therefore 26% of cost = \$2.60 : \therefore cost = \$10.
12. Cost of 150 axes at \$2.50 per dozen is \$31.25. \therefore net cost is 90% of \$31.25, or \$28.12 $\frac{1}{2}$.
13. $66\frac{2}{3}\%$ of marked price = 30 cents
 \therefore marked price = 45 cents.
14. 120% of cost of 1st house = \$4,000 : \therefore cost of 1st house = \$3333 $\frac{1}{3}$.
 80% of cost of 2nd house = \$4,000 : \therefore cost of 2nd house = \$5,000.
 \therefore cost of two houses was \$8333 $\frac{1}{3}$: \therefore loss was \$333 $\frac{1}{3}$.
15. Marked price = 140% of cost. Cash price = 70% of 140% of cost = 98% of cost. \therefore loss was 2% of cost.
16. Giving 16 $\frac{1}{2}$ oz. for price of 16 oz. equals a discount of $\frac{1}{2}$ on 16 $\frac{1}{2}$. This is $3\frac{1}{3}\%$.
17. See solution of 16.
18. 95% of usual cash price = \$3.42. \therefore usual cash price = \$3.60. \therefore 90% of marked price = \$3.60. \therefore marked price = \$4.
19. The first reduced price is a certain fraction of the marked price. The cash price is the same fraction of the first reduced price. \therefore the cash price is obtained by multiplying the marked price by the square of the fraction. \therefore the square of the fraction is $\frac{100}{121}$, or the fraction is $\frac{10}{11}$. \therefore rate of discount is $\frac{1}{11}$ or 9 $\frac{1}{11}\%$.
20. Nine gallons of mixture contain 1 gallon of water. \therefore he can throw off $\frac{1}{9}$, or 11 $\frac{1}{9}\%$.

I44

21. The selling price of one article is $\frac{1}{3}$, or $\frac{4}{5}$ of the list price of one article. \therefore discount is $\frac{1}{5}$, or 20%.
22. 90% of marked price = 110% of cost. \therefore marked price = $122\frac{2}{3}\%$ of cost. \therefore 95% of marked price = $116\frac{1}{3}\%$ of cost. \therefore gain is $16\frac{1}{3}\%$.
23. The merchant gives 35.28 in. for 90% of marked price per yard. \therefore he would give 36 in. for $(90 \times \frac{100}{89.28})\%$ of marked price per yard, that is, for 91 $\frac{1}{4}\%$ of marked price. He could give a discount of $8\frac{3}{4}\%$.

24. 80% of $90\% = 72\%$. \therefore 20 and 10% off = 28% discount.
 25. 90% of 1st reduced price = 85% of original price. \therefore 1st reduced price = $94\frac{1}{4}\%$ of original price. \therefore discount = $5\frac{1}{4}\%$.
 26. 90% of reduced price = \$3.60. \therefore reduced price = \$4. \therefore marked price was reduced by \$1, or 20%.
 27. \$21.87 is $\frac{2187}{1000}$, or $\frac{729}{1000}$, of \$30. The first reduced price is a certain fraction of the marked price. The second reduced price is the same fraction of the first reduced price. The third reduced price is the same fraction of the second reduced price. \therefore the selling price is obtained by multiplying the marked price by the cube of the fraction. \therefore the cube of the fraction = $\frac{729}{1000}$. \therefore the fraction = $\frac{9}{10}$. \therefore rate of discount is 10%.
 28. He sold for $88\frac{2}{3}\%$ of 135% of cost; that is, for 120% of cost. \therefore gain = 20%.
 29. Goods sell for 95% of 105% of cost; that is, for $99\frac{1}{4}\%$ of cost. \therefore loss = $\frac{1}{4}\%$.
 30. The loss is equal to the discount on the amount of the marked price above the cost; that is, the loss may be obtained by multiplying the cost by the square of the discount fraction. \therefore the discount fraction = $\frac{1}{8}$, or $12\frac{1}{2}\%$.
 31. The loss is $\frac{1}{8}$ of the cost. \therefore the discount fraction is $\frac{1}{8}$, or 25%. Hence the marked price was \$120.
 32. 50 cents was a certain per cent. of the marked price; 45 cents was the same per cent. of that marked price when diminished by 50 cents. Hence 5 cents was the discount off 50 cents, or rate of discount was 10%. \therefore original price was \$5.00, and selling price \$4.05.
 33. The article must sell for \$1.75. \therefore 75% of marked price = \$1.75. \therefore marked price = \$2.33 $\frac{1}{3}$.

145

34. 5% of selling price = 4% of list price. \therefore selling price = 80% of list price. \therefore 20% of list price = 10 cents. \therefore list price = 50 cents.
 35. 20 and 25% off leaves 80% of 75% of cost, that is, leaves $\frac{6000}{10000}$ of cost. When a rate of discount is taken off, there remains a certain fraction of original price. When the same rate of discount is applied a second time, there remains the result of multiplying the original price by the square of the fraction. Hence the square of the fraction = $\frac{6000}{10000}$. \therefore the fraction = $\frac{77}{100}$. \therefore rate of discount = 22.5 + %.

SOLUTIONS OF PROBLEMS

PROFIT AND LOSS.

6. Gain 1st year = $12\frac{1}{2}\%$ of \$6000 = \$750. 75% of \$750 = \$562 50. \therefore capital 2nd year = \$6562.50. 20% of \$6562.50 = \$1312.50. $\frac{1}{2}$ of \$1312.50 = \$656 25. \therefore capital 3rd year = \$7218.75. \therefore gain in 3rd year = 40% of \$7218.75 = \$2887.50.

7. The sheep cost \$1920. \therefore they are sold for 120% of \$1920, or \$2304. 5% of 240 = 12. \therefore 228 sheep were sold by the drover for \$2304, or each sheep sold for \$10.10 $\frac{1}{2}$.

8. Paper must be sold for $112\frac{1}{2}\%$ of \$1000, or \$1125. 1200 reams = 24000 quires. 24000 quires sell for \$1125, or 1 quire sells for $4\frac{1}{8}$ c.

9. He loses $\frac{1}{8}$ of each gallon bought. \therefore he must sell $\frac{7}{8}$ gal. for 20c, or 1 gal. for 22 $\frac{2}{3}$ c.

10. B paid 115% of amount A paid. \therefore C paid 120% of 115% of amount A paid. \therefore C paid 138% of amount A paid. \therefore 38% of amount A paid = \$190. \therefore A paid \$500.

146

11. A gain of 20% on $\frac{1}{3}$ of the goods is equivalent to a gain of $6\frac{2}{3}\%$ on the whole amount. A loss of 10% on $\frac{2}{3}$ of the goods is equivalent to a loss of $6\frac{2}{3}\%$ on whole amount. \therefore he neither gains nor loses.

12. That which costs 7 cents, is sold for 12 cents. \therefore he gains 5 cents on 7 cents, that is, $\frac{5}{7}$, or $71\frac{3}{7}\%$.

13. $\frac{2}{3}$ of the goods sell for cost of the goods. \therefore the goods sell for $\frac{2}{3}$ of cost of the goods. \therefore the gain is $\frac{1}{3}$ of the cost, or 50% of the cost.

14. He must sell 80% of a gal. for 120% of cost of a gal. \therefore he must sell one gal. for 150% of cost of a gal. \therefore he must increase cost price by 50% .

15. Mr. Smith originally owned the house, worth \$4000. After the two transactions, he still owned the house, but was out \$300. \therefore he lost $7\frac{1}{2}\%$ of value of house.

16. The cost of ice in store houses + the cost of delivery = \$3600. \therefore to make $12\frac{1}{2}\%$, the ice must sell for $112\frac{1}{2}\%$ of \$3600 = \$4050.

17. 90% of cost of flour per hundred = \$2.70. \therefore cost per hundred = \$3.00. Sold at \$2.55, the loss would be 45c. per hundred. 45c. is 15% of \$3.00. \therefore loss would be 15% .

18. 85% of cost = \$4.25. \therefore 115% of cost = \$5.75.

IN THE HIGH SCHOOL ARITHMETIC.

19. 5% of cost = 20c. \therefore cost = \$4.00. \therefore \$4.25 = 106 $\frac{1}{4}$ % of cost, and gain is 6 $\frac{1}{4}$ %.

20. At first sale received 112 $\frac{1}{2}$ % of original sum invested. At second sale received 112 $\frac{1}{2}$ % of 112 $\frac{1}{2}$ % of original sum invested. At third sale received 75% of 112 $\frac{1}{2}$ % of 112 $\frac{1}{2}$ % of original sum invested. And, \therefore lost 5 $\frac{5}{8}$ %.

21. He sells $\frac{9}{10}$ of original number for $\frac{9}{8}$ of total cost. \therefore at same rate he would sell the original number for $\frac{10}{9}$ of $\frac{9}{8}$, or $\frac{5}{4}$ of cost. \therefore he sells each sheep for $\frac{5}{4}$ of \$8, or \$10.

22. $\frac{9}{10}$ of the cargo must be sold for cost of cargo. \therefore at same rate, the cargo would be sold for $\frac{10}{9}$ of cost. \therefore advance is $\frac{1}{9}$, or 11 $\frac{1}{9}$ % of cost.

23. See Solution of No. 14, page 143.

24. The paper cost \$6 per ream. \therefore to gain 20% it must sell for 120% of \$6, or \$7.20 per ream. \therefore 95% of marked price per ream = \$7.20. \therefore marked price per ream = \$7.20 \times $\frac{100}{95}$. \therefore marked price per quire = \$7.20 \times $\frac{100}{95}$ \times $\frac{1}{2}$ = 37 $\frac{1}{5}$ c.

147

25. 125% of cost or 16 qts. = \$1.00. \therefore cost of 16 qts. = 80c. \therefore cost of 15 qts. = 75c. Hence gain on 15 qts. would be 25c., or 33 $\frac{1}{3}$ %.

26. $\frac{2}{3}$ of selling price = $\frac{3}{4}$ of cost price. \therefore selling price = $\frac{9}{8}$ of cost price. \therefore gain = $\frac{1}{8}$ = 12 $\frac{1}{2}$ %.

27. He sells for 80% of 140% of cost; that is, for 112% of cost. \therefore gain is 12%.

28. He sells 11 qts. for 18 times cost of 1 qt. \therefore sells 1 qt. $\frac{18}{11}$ of cost of 1 qt. \therefore he gains $\frac{7}{11}$, or 63 $\frac{7}{11}$ %.

29. 114 $\frac{2}{3}$ % of cost of 1 gal. of mixture = \$1.20. \therefore cost of 1 gal. of mixture = \$1.05. \therefore each gal. of mixture contains $\frac{105}{114}$ gal. of whiskey, and $\frac{20}{114}$ gal. of water. \therefore water is $\frac{20}{105}$, or $\frac{4}{21}$ of amount of whiskey.

30. He sells 15 $\frac{3}{4}$ oz. for the cost of 16 oz. \therefore he sells 1 oz for the cost of $\frac{64}{63}$ oz., or for $\frac{64}{63}$ cost of 1 oz. \therefore he gains $\frac{1}{63}$ or 1 $\frac{57}{63}$ %.

31. 80% of marked price = 120% of cost. \therefore marked price = 150% of cost. \therefore advance is 50%.

32. The selling price is $\frac{7}{8}$ of $\frac{8}{5}$ of cost, or $\frac{43}{25}$ of cost. \therefore loss is $\frac{1}{25}$ or 4%.

33. See Solution of No. 30, page 144.

34. See Solution of No. 30, page 144.

SOLUTIONS OF PROBLEMS

35. For 8 papers he pays 5 cents, and sells the same for 16 cents. He gains $\frac{1}{4}$, or 220%.

36. Man must receive 125% of \$180, or \$225. \therefore horse must be sold for \$225 + \$10, or \$235.

37. She buys 100 apples for 25 cents. Ten of these are lost. She sells the remaining 90 for 54c. \therefore on 25 cents she gains 29 cents, and on 100 cents she gains 116 cents. \therefore she gains 116%.

38. The manufacturer sold for a certain fraction of cost. The wholesaler sold for same fraction of what goods cost him. The retailer sold for same fraction of what goods cost him. \therefore retailer's price is obtained by multiplying manufacturer's cost by the cube of that fraction. \therefore cube of the fraction = $\frac{\$8.64}{\$5} = \frac{1728}{1000}$. \therefore fraction is $\frac{12}{10}$, and rate of profit is 20%.

148

39. A sold to B for $(\frac{21}{20})$ of original cost. B sold to C for $(\frac{21}{20})^2$ of original cost. C sold to D for $(\frac{21}{20})^3$ of original cost.

C's gain was $\left\{ (\frac{21}{20})^3 (\frac{21}{20})^2 \right\}$ or $\frac{441}{8000}$ of original cost. A's gain was $\frac{1}{20}$ of original cost. $\therefore \frac{441}{8000}$ of original cost = \$5.

$\therefore (\frac{21}{20})^3$ of original cost = \$5 $\times \frac{8000}{441} \times \frac{9261}{8000} = \$1,129\frac{1}{4}$.

40. The gain is 20% of cost. \therefore 20% of cost of 1 bush. = 8c. \therefore cost of 1 bush. = 40c. \therefore selling price = 48c.

41. 110% of cost of 1st cow = \$60. \therefore cost of 1st cow = \$54 $\frac{6}{11}$. 125% of cost of 2nd & 3rd cows = \$120. \therefore cost of these two = \$96. 66 $\frac{2}{3}$ % of cost of 4th & 5th cows = \$120. \therefore cost of these two = \$180. \therefore total cost was \$330 $\frac{6}{11}$; the selling price was \$300. \therefore loss was \$30 $\frac{6}{11}$ on \$330 $\frac{6}{11}$, or 9 $\frac{73}{103}$ %.

42. 110% of cost of 1 bush. of mixture = 50c. \therefore cost of 1 bush. of mixture = 45 $\frac{5}{11}$ c. On 1 bush. of oats the loss would be 4 $\frac{5}{11}$ c. \therefore on 100 bush. of oats the loss would be $\frac{5000}{11}$ c. On 1 bush. of corn the gain would be 5 $\frac{5}{11}$ c. \therefore on $(\frac{5000}{11} + 5\frac{5}{11})$ bush. of corn the gain would be $\frac{5000}{11}$ c. \therefore no. bush. of corn required = 83 $\frac{1}{4}$.

43. 3 articles are sold for 4 times cost of 1 article. \therefore 1 article is sold for $\frac{4}{3}$ of cost of 1 article. \therefore gain is $\frac{1}{3}$, or 33 $\frac{1}{3}$ %.

44. A's increased capital = $\frac{1}{5}$ of sum each invested. B's increased capital = sum each invested + \$100. \therefore sum each invested + \$100 = $\frac{1}{5}$ of sum each invested. $\therefore \frac{4}{5}$ sum each invested = \$100. \therefore sum each invested = \$71 $\frac{3}{4}$.

IN THE HIGH SCHOOL ARITHMETIC.

45. The mixture of $(a+c)$ lbs. cost $(a+b+c d)$ cents. \therefore the mixture cost $\frac{a+b+c d}{a+c}$ cents per lb. The gain on each lb. =

$$(e - \frac{a+b+c d}{a+c}) \text{ cents} = \frac{a e + c e - a b - c d}{a+c} \text{ cents. } \therefore \text{rate of}$$

$$\text{gain} = \left(\frac{a e + c e - a b - c d}{a+b+c d} \right) 100\%.$$

46. 115% of cost = \$16.10. \therefore cost = \$14.00. Selling at \$20, the gain would be \$6. \therefore rate of gain would be $\frac{6}{14}$, or $42\frac{2}{7}\%$.

47. The total cost of the molasses was \$88.75. He sells 90% of 315 gals., or $283\frac{1}{2}$ gals. He receives payment for 94% of $283\frac{1}{2}$ gals., or $266\frac{49}{100}$ gals. To make a gain of 40%, he must sell $266\frac{49}{100}$ gals. for 140% of \$88.75, \therefore he must sell 1 gal. for 46.6+c.

48. The article is sold for 105% of actual cost. If the cost had been 95% of actual cost, to gain 10% the selling price would have been 110% of 95% of actual cost, that is, $104\frac{1}{2}\%$ of actual cost. This is \$1 less than former selling price. \therefore $\frac{1}{2}\%$ of actual cost = \$1. \therefore actual cost = \$200.

49. The marked price is 120% of cost. The selling price is 108% of cost. \therefore discount is 12% of cost on 120% of cost. \therefore rate of discount is 10%.

50. Jones' present capital = 120% of \$9600 = \$11520. \therefore Smith's present capital is \$11520.

COMMISSION.

149

1. 2% of \$12000 = \$240.
2. $2\frac{1}{4}\%$ of \$1850 = \$41.62 $\frac{1}{2}$.
3. Potatoes sold for \$6000. He sent to his employer $97\frac{1}{2}\%$ of \$6000, or \$5850.
4. Agent sent employer $98\frac{1}{2}\%$ of selling price. \therefore $98\frac{1}{2}\%$ of selling price of 1 bush. = 65 $\frac{3}{4}$ c. \therefore selling price = 66 $\frac{3}{4}$ c.
5. Cargo sold for \$540. \therefore com. was $1\frac{2}{3}\%$ of \$540, or \$10 $\frac{1}{3}$.
6. Sale amounted to \$1568 + \$32, or \$1600. Com. was \$32. \therefore rate of com. was 2%.
7. See Solution of 6.

SOLUTIONS OF PROBLEMS

8. The com. was \$104. On a sale of \$5200 the com. was \$104. \therefore on a sale of \$100 the com. was \$2. \therefore rate of com. = 2%.

9. For every \$100 received by consignor, agent retained \$4 $\frac{1}{2}$. \therefore on a sale amounting to \$104 $\frac{1}{2}$, the com. is \$4 $\frac{1}{2}$. \therefore on a sale amounting to \$10000, the com. is \$400.

10. 5% of amount returned to consignor was the com. \therefore the com. was $\frac{5}{100}$ of amount of sale. \therefore $\frac{5}{100}$ of amount of sale = \$45. \therefore amount of sale = \$945.

11. The com. for selling 1 acre was $\frac{1}{2}$ % of \$125, or 62 $\frac{1}{2}$ c. \therefore no. of acres sold = $\frac{\$50}{62\frac{1}{2}\text{c.}} = 80$.

12. The com. on \$10000 was \$75. \therefore the com. on \$100 was \$ $\frac{3}{4}$. \therefore rate was $\frac{3}{4}$ %.

150

13. The com. is 2 $\frac{1}{2}$ % of \$650, or \$16.25.

14. The agent must receive the price he pays for the horse, and his commission on this price. He must receive \$100 + \$2, or \$102.

15. The agent receives \$5 com. on \$100 invested. \therefore his com. is $\frac{5}{100}$ of amount received by him. \therefore his com. is $\frac{5}{100}$ of \$525, or \$25.

16. The amount invested is $\frac{100}{102}$ of sum sent to agent. \therefore amount paid by agent for goods = $\frac{100}{102}$ of \$1750 = \$1715 $\frac{35}{51}$.

17. The com. was $\frac{2}{100}$ of \$3570, or \$70.

18. The agent invested \$2400 and retained \$80. \therefore on \$100 invested, his com. was \$3 $\frac{1}{3}$. \therefore rate of com. = 3 $\frac{1}{3}$ %.

19. He paid 95c. for book, and charged 20c. com. At same rate, on 100c. the com. would be 20c. \times $\frac{100}{95}$, or 21 $\frac{1}{19}$. \therefore rate of com. = 21 $\frac{1}{19}$ %.

20. Agent invested $\frac{100}{102}$ of \$1224, or \$1200, in pork. \$4.75 buys 100 lbs. of pork. \therefore \$1 buys $\frac{100}{4.75}$ lbs. of pork. \therefore \$1200

buys $\frac{100}{4.75} \times 1200$, or 25263 $\frac{3}{19}$ lbs. of pork.

21. The price paid for the wheat was \$9600. The agent's com. was 1 $\frac{1}{4}$ % of \$9600, or \$120. \therefore employer must remit \$9720.

22. The agent sold the apples for \$1250. His com. for selling was 2% of \$1250, or \$25. This left for investment

IN THE HIGH SCHOOL ARITHMETIC.

(including com.) \$1225. His com. for buying was $1\frac{1}{2}\%$ of amount paid for sugar. \therefore his com. for buying $\frac{1\frac{1}{2}}{101\frac{1}{2}}$ of amount left for investment. \therefore his second com. was $\frac{1\frac{1}{2}}{101\frac{1}{2}}$ of \$1225, or $\$18\frac{3}{25}$. \therefore his total com. was $\$43\frac{3}{25}$. The sum paid for sugar was $\$1250 - \$43\frac{3}{25}$, or $\$1206\frac{2}{25}$. This bought $30172\frac{12}{25}$ lbs.

23. The coal sold for \$5700. His com. for this was $\frac{1}{2}\%$ of \$5700, or \$28.50. This left \$5671.50 for investment in lumber, and com. Out of \$101, the agent kept \$1 and invested in lumber \$100. \therefore he invested in lumber $\frac{100}{101}$ of \$5671.50. \therefore the number of feet of lumber bought was $\frac{\$5671.50}{\$18} \div \frac{101}{100} \times 1000$, or 311963 + ft.

24. The broker invests $\frac{100}{102}$ of \$12000 in pork. \$1 buys 25 lbs. \therefore he buys $25 \times \frac{100}{102}$ of 12000, or 294117 $\frac{1}{3}$ lbs.

151

25. The flour was sold for \$16000. : the com. for selling was \$240. : the freight charge was \$150. \therefore there remained \$15610 for investment in cotton and for com. The amount paid for cotton was $\frac{100}{100\frac{1}{4}}$ of \$15610. \therefore the no. of lbs. of cotton bought was $\$(\frac{400}{100\frac{1}{4}} \text{ of } 15610) \div \15 , or 103291 $\frac{11}{16}$ lbs.

26. The agent evidently retained \$150 as com. \therefore com. on sale amounting to \$12500 was \$150. \therefore com. on sale amounting to \$100 was \$1 $\frac{1}{4}$. \therefore rate of com. was $1\frac{1}{4}\%$.

27. The agent pays 5c. for 1 lb. of sugar. He pays $\frac{1}{10}$ c. for the freight on the same, and he charges 2% of 5c. or $\frac{1}{50}$ c. for com. \therefore the total cost of 1 lb. of tea is $5\frac{3}{10}$ c. \therefore the number of lbs. bought is $\frac{\$6360}{5\frac{3}{10} \text{ c.}} = 120000$.

28. The hams sell for \$3750. Freight cost \$75. The com. was \$75. \therefore net proceeds from sale of hams was \$3600. The agent pays 45c. for 1 lb. of tea. Charges com. $\frac{2}{10}$ c. for 1 lb. of tea. Pays freight $\frac{1}{2}$ c. for 1 lb. of tea. \therefore the total cost of 1 lb. of tea is $45\frac{7}{10}$ c. \therefore the number of lbs. bought is $\frac{\$3600}{45\frac{7}{10} \text{ c.}} = 7847\frac{1}{3}$ lbs.

SOLUTIONS OF PROBLEMS

29. The agent receives $(2+2\frac{1}{2})\%$ of sales, or \$54.
 30. $4\frac{3}{4}\%$ of amount of sale = \$380. \therefore amount of sale = \$8000.

31. The guaranty com. was $2\frac{1}{4}\%$ of amount of sale. The selling com. was 2% of amount of sale. $\therefore \frac{3}{4}\%$ of amount of sale = \$40. \therefore amount of sale = \$5333 $\frac{1}{3}$.

32. Com. for selling cotton = $\frac{1}{100}$ of selling price of cotton.
 Com. for buying sugar = $\frac{1\frac{1}{2}}{101\frac{1}{2}}$ of $\frac{96}{100}$ of selling price of cotton.

= $\frac{2288}{20300}$ of selling price of cotton. \therefore two coms. amount to $(\frac{1}{100} + \frac{2288}{20300})$, or $\frac{11}{203}$ of selling price of cotton. $\therefore \frac{11}{203}$ of selling price of cotton = \$220. \therefore cotton sold for \$4060.

33. The agent paid \$2.80 for each cwt. of flour. He also paid \$20 freight for each cwt. of flour. He retains also $\$.06\frac{3}{10}$ com. for each cwt. of flour. \therefore it costs $\$.306\frac{3}{10}$ to buy 1 cwt. of flour. But com. on sale of apples is $\frac{3}{100}$ of amount of sale, or $\frac{3}{17}$ of amount left to purchase flour. \therefore com. on sale of apples, sufficient to buy 1 cwt. of flour, is $\frac{3}{17}$ of $\$.306\frac{3}{10}$. \therefore total com. when 1 cwt. of flour is purchased, is $\$.06\frac{3}{10} + (\frac{3}{17} \text{ of } \$.306\frac{3}{10})$, or $1\frac{53}{17}\%$. Hence number of cwt. of flour bought is $\$63 \div 1\frac{53}{17}\%$, or $399\frac{1}{17}$.

152

34. See Solution of 33.

35. The com. for buying tea was $\frac{1}{300}$ of cost of tea. \therefore net proceeds from sale of flour was $\frac{201}{300}$ of cost of tea. The com. for selling flour was $\frac{1}{300}$ of selling price of flour. \therefore the com. for selling flour was $\frac{1}{300}$ of net proceeds from sale of flour, that is, was $\frac{1}{300}$ of $\frac{201}{300}$ of cost of tea. \therefore total com. was $(\frac{1}{300} + \frac{1}{300} \text{ of } \frac{201}{300})$ of cost of tea. $\therefore \frac{5}{807}$ of cost of tea = \$50. \therefore cost of tea = \$5970.

36. Agent's first com. = $\frac{a}{100}$ of selling price. \therefore second com. is $\frac{b}{100+b}$ of $\frac{100-a}{100}$ of selling price. \therefore total com. = $(\frac{a}{100} + \frac{b(100-a)}{100(100+b)})$ of selling price = $\frac{a+b}{100+b}$ of selling price.

37. The first com. is calculated on \$4060. The second com. is calculated on \$3940. \therefore \$120 is the com. on \$4060 + \$3940, or \$8000. \therefore rate is $1\frac{1}{2}\%$.

38. See Solution of 37.

40. The selling com. is calculated on \$4040. \therefore the selling

IN THE HIGH SCHOOL ARITHMETIC.

com. is equivalent to that calculated on $\$4040 \times 1\frac{1}{2}$, or $\$6060$, at the rate of buying com. The buying com. is calculated on $\$2000$. \therefore $\$100$ is com. on $\$6060 + \3940 , or $\$10,000$ at buying rate. \therefore buying rate is 1% , and selling rate $1\frac{1}{2}\%$.

153

41. The selling com. is calculated on $\$1421$. \therefore the selling com. is equivalent to that calculated on $\$1421 \times \frac{4}{3}$, or $\$1894\frac{2}{3}$, at the rate of investing com. The com. for investing is calculated on $\$1421 - \49 , or $\$1372$. \therefore $\$49$ is com. on $\$1894\frac{2}{3} + \1372 , or $\$3266\frac{2}{3}$ at investing rate. \therefore investing rate is $1\frac{1}{2}\%$. \therefore selling rate is 2% .

42. Selling com. is calculated on $\$1734$. Buying com. is calculated on $\$1649$. Selling com. is 1% of $\$1734$, together com. on $\$1734$ calculated at same rate as in buying. 1% of $\$1734 = \17.34 . \therefore $\$85 - \17.34 , or $\$67.66$, is the com. on $\$1734 + \1649 , or $\$3383$, at buying rate. \therefore rate for investment = 2% .

43. See Solution of 42.

44. The com. for buying was calculated on a sum which was $\$100$ less than the sum on which the com. for selling was calculated. \therefore $\$2$ is com. on $\$100$, and rate is 2% .

45. In buying he paid $\$81 + \79 , or $\$160$ less than he received when selling. The com. was $\$2$ less. \therefore $\$2$ is com. on $\$160$. \therefore rate is $1\frac{1}{4}\%$.

46. The flour sold for $\$8500$. The com. was $2\frac{1}{2}\%$ of $\$8500$, or $\$212.50$. The expenses $\$240.80$. \therefore the net proceeds were $\$8046.70$. It costs $\$100\frac{2}{3}$ to send consignor $\$100$. \therefore the consignor receives $\frac{200}{303}$ of $\$8046.70$, or $\$8016.63$.

INSURANCE.

1. The premium = $\frac{3}{4}\%$ of $\$1200 = \9 .

2. The premium for 1 year = $\frac{7}{8}\%$ of $\$4000 = \35 . \therefore premium for 3 years = $\$105$.

3. The premium for 1 year = $\frac{5}{8}\%$ of $\$1800 = \11.25 . \therefore the premium for 5 years = $\$56.25$.

154

4. $\frac{3}{4}$ of value of house = $\frac{3}{4}$ of $\$12000 = \9000 . Premium for 1 year = $\frac{3}{4}\%$ of $\$9000 = \$53\frac{1}{4}$. \therefore premium for 3 years = $\$160$.

SOLUTIONS OF PROBLEMS

5. For \$6000 insurance the premium = \$45. \therefore for \$100 insurance the premium = $\frac{3}{4}$. \therefore rate = $\frac{3}{4}\%$.
6. The premium on \$4000 insurance for 1 year is \$30. \therefore premium on \$100 insurance is $\frac{3}{4}$. \therefore the rate is $\frac{3}{4}\%$.
7. The premium on \$2500 insurance for 1 year is \$16 $\frac{3}{4}$. \therefore the premium on \$100 insurance for 1 year is $\frac{3}{8}$. \therefore rate is $\frac{3}{8}\%$.
8. Two-thirds of value of house is \$4000. The premium on \$4000 insurance for 1 year is \$21. \therefore yearly rate is $\frac{5}{12}\%$.
9. 75% of value of cargo = 75% of \$1250 = \$937 $\frac{1}{2}$. On \$937 $\frac{1}{2}$ the premium is \$12.50. \therefore rate is $1\frac{1}{2}\%$.
10. The yearly premium was \$8. \therefore $\frac{1}{2}\%$ of the amount of insurance = \$8. \therefore amount of insurance = \$1600 \therefore $\frac{3}{4}$ of value of house = \$1600. \therefore value of house = \$2133 $\frac{1}{3}$.
11. \$1 is the premium on \$100 insurance for two years. \therefore \$37.50 is the premium on \$3750 insurance for two years. \$3750 is $\frac{5}{8}$ of \$6000. \therefore $\frac{5}{8}$ of value is insured.
13. The premium on \$10000 insurance at $\frac{3}{4}\%$ is \$75. 40% of 1st company's risk = 40% of \$10000 = \$4000. The premium on \$4000 at $\frac{3}{4}\%$ = \$35. \therefore 1st company received \$75 - \$35, or \$40 more than it paid 2nd company.
15. \$100 insurance cost $\frac{3}{4}$, and therefore covers \$99 $\frac{1}{4}$ value. Goods worth \$99 $\frac{1}{4}$ must be insured for \$100. \therefore goods worth \$7940 must be insured for \$8000.
16. Value to amount of \$99 $\frac{1}{4}$ must be insured for \$100. \therefore value to amount of \$3965 must be insured for \$4000.

155

17. The value of the vessel and \$100 besides amount to \$15900. \$99 $\frac{3}{8}$ is secured when insurance is \$100. \therefore \$15900 is secured when insurance is \$16000.
18. The premium on \$100 insurance is $\frac{3}{4}$. \therefore if amount of policy is \$100, the insurance covers the premium, an additional sum equal to the premium, and \$99 besides. \therefore cargo worth \$99 should be insured for \$100. \therefore cargo worth \$5940 should be insured for \$6000.
19. The company received $1\frac{3}{4}\%$ of \$40000, or \$700 premium. It paid for insurance on \$16000, 2% of \$16000, or \$320 premium, and for insurance on \$10000, 2 $\frac{1}{4}\%$ of \$10000, or \$225 premium. \therefore the company carried a risk of \$14000 for \$155. \therefore rate on amount of risk it retained = $1\frac{3}{8}\%$.

IN THE HIGH SCHOOL ARITHMETIC.

20. The company received $1\frac{1}{2}\%$ of risk as premium. It paid out $1\frac{3}{4}\%$ of 50% of risk, and also $1\frac{1}{4}\%$ of 20% of 50% of risk; that is, it paid 1% of risk for premium on 60% of risk. \therefore it retained $\frac{1}{2}\%$ of risk as premium on 40% of risk. \therefore the rate was $1\frac{1}{4}\%$.

21. Company I. receives 11% of \$25000, or \$312 $\frac{1}{2}$ as premium. Company II. receives 1% of \$40000, or \$400 as premium. Company III. receives $\frac{1}{8}\%$ of \$100000, or \$875 as premium. Divide loss in proportion to amount of risk carried. \therefore divide \$100000 in ratio of 5, 8, 20. Company I. loses $\frac{5}{33}$ of \$100000, or \$1515 $\frac{1}{3}$; etc.

22. Premium on furniture = $\frac{3}{4}\%$ of $\frac{2}{3}$ of value of furniture = $\frac{1}{5}$ of value of furniture. Premium on house = $\frac{3}{4}\%$ of $\frac{3}{4}$ of 5 times value of furniture = $\frac{9}{32}$ of value of furniture. Total premium = $\frac{21}{32}$ of value of furniture = \$12.60. \therefore value of furniture = \$12.60 \times $\frac{32}{21}$. \therefore value of house = \$12.60 \times $\frac{6400}{212} \times 5$ = \$1901 $\frac{4}{7}$.

23. Premium on vessel is $\frac{1}{2}\%$ of $\frac{3}{4}$ of value of vessel, that is, $\frac{3}{800}$ of value of vessel. The premium on the cargo is $\frac{3}{8}\%$ of $\frac{3}{4}$ of 2 times value of vessel, that is, $\frac{9}{1600}$ of value of vessel. The two premiums = $\frac{15}{1600}$ of value of vessel = \$120. \therefore value of vessel = \$12800. \therefore value of cargo = \$25600.

24. Premium on \$20000 insurance at $\frac{3}{4}\%$ = \$150. Premium on \$25000 insurance at $\frac{3}{4}\%$ = \$166 $\frac{2}{3}$. \therefore on \$45000 insurance the premium is \$316 $\frac{2}{3}$. \therefore on \$100 insurance the premium is \$ $\frac{19}{27}$. \therefore rate is $\frac{19}{27}\%$.

25. The problem is ambiguous. If vessel and cargo are insured separately, so as to cover the respective premiums, the solution will be as follows: \$99 $\frac{1}{2}$ value (of vessel) is covered by \$100 insurance. \therefore \$18000 value (of vessel) is covered by \$18136.02. Again \$99 $\frac{1}{2}$ value (of cargo) is covered by \$100 insurance. \therefore \$24000 value (of cargo) is covered by \$24120.60. \therefore the vessel and cargo must be insured for \$42256.62. The premium paid will be \$42256.62 - \$42000, or \$256.62.

26. \$2 $\frac{1}{2}$ is the premium on \$100 insurance. \therefore \$75 is the premium on \$3000 insurance. \therefore $\frac{3}{4}$ of value of flour = \$3000. \therefore value = \$4000. \therefore he must sell the 500 bbls. for \$4000 + \$1000 + \$75. \therefore 500 bbls. sell for \$5075. \therefore 1 bbl. sells for \$10.15.

27. The premium is \$21. \therefore $\frac{3}{4}\%$ of $\frac{3}{4}$ of value = \$21. \therefore value of house = \$4666 $\frac{2}{3}$.

SOLUTIONS OF PROBLEMS

28. The man paid out \$2500 for house, \$9 for insurance, and \$500 for lot: in all, he paid \$3009. He received $\frac{2}{3}$ of \$2500, or \$1500 insurance, and \$750 for lot: in all, he received \$2250. \therefore he lost \$759.

29. See Solution of problem 15, in this section.

30. The premium on \$3000 insurance was \$3000—\$2962.50, or \$37.50. \therefore the rate was $1\frac{1}{4}\%$.

31. $\frac{1}{4}$ of value of vessel = $\frac{1}{4}$ of \$4925 = \$3940. \therefore the premium on \$4000 insurance was \$4000—\$3940, or \$60. \therefore rate was $1\frac{1}{2}\%$.

32. $\frac{2}{3}$ of the value of cargo is \$14750: hence premium on \$15000 insurance is \$150. \therefore rate is 1% .

33. Company I. received as premium $1\frac{1}{4}\%$ of $\frac{2}{3}$ of value, that is, $\frac{1}{10}$ of value. Company II. received as premium $1\frac{1}{2}\%$ of $\frac{1}{4}$ of $\frac{2}{3}$ of value, that is, $\frac{1}{10}$ of value. Company I. lost $\frac{1}{4}$ of $\frac{2}{3}$ of value, or $\frac{1}{6}$ of value. Company II. lost $\frac{1}{4}$ of $\frac{2}{3}$ of value, or $\frac{1}{6}$ of value. \therefore net loss of Company I. is $(\frac{1}{2} + \frac{1}{10} - \frac{1}{10})$, or $\frac{580}{1000}$ of value. And net loss of Company II. is $(\frac{1}{2} - \frac{1}{10})$, or $\frac{480}{1000}$ of value. $\therefore \frac{580}{1000} - \frac{480}{1000} = \frac{100}{1000}$, or $\frac{1}{10}$ of value = \$49000. \therefore value = \$49000 $\times \frac{1000}{100}$. The owners lost $\frac{1}{4}$ of value, and $\frac{1}{10}$ of value (premium). \therefore owners lost \$49000 $\times \frac{1200}{1000} \times (\frac{1}{4} + \frac{1}{10})$, or \$51750.

34. My premium is $\frac{2}{3}\%$ of $\frac{5}{8}$ of value of my house, that is, $\frac{1}{80}$ of value of my house. My brother's premium is $\frac{5}{8}\%$ of $\frac{2}{3}$ of value of his house, or $\frac{5}{8}\%$ of $\frac{2}{3}$ of $\frac{2}{3}$ of value of my house, that is, $\frac{1}{30}$ of value of my house. Difference of premiums is $\frac{1}{80}$ of value of my house: difference is given equal to \$12. \therefore value of my house is \$6283 $\frac{1}{11}$.

35. $\frac{1}{4}$ is premium on \$100 insurance. \therefore \$15 is premium on \$6000 insurance. $\therefore \frac{1}{2}$ of value of flour was \$6000—\$15, or \$5985. \therefore value of flour was \$11970.

36. The cost of the cattle was \$18000. The insurance is to cover \$18084 and the premium. \$98 $\frac{2}{3}$ is covered by \$100 insurance. \therefore \$18084 is covered by \$18312 $\frac{2}{3}$ insurance.

37. 97% of amount of policy = $\frac{2}{3}$ of value of house. \therefore amount of policy = $\frac{200}{97}$ of value of house, that is, $\frac{15}{7}$ of value of house.

TAXES.

157

1. 2% of \$5000 = \$100.
2. On \$1 the tax is $1\frac{1}{2}c.$ \therefore on \$2500 the tax is \$37.50.
3. On \$1 the tax is 2c. \therefore on \$12000 the tax is \$240.
4. On \$1 the tax is 15 mills \therefore on \$2000 the tax is 30000 mills, or \$30.
5. On an assessment of \$1 the tax is \$.012 $\frac{1}{2}$ \therefore on an assessment of \$750000 the tax is \$9375.
6. On \$135,000,000 the tax is \$1,500,000. \therefore on \$1 the tax is \$.011 $\frac{1}{5}$. \therefore rate is 11 $\frac{1}{5}$ mills on the dollar.
7. The tax amounts to \$.018 \times 800000, or \$14400. The cost of collection is 2% of \$14400, or \$288. \therefore net amount received is \$14400 - \$288, or \$14112.
8. Since 2% of tax is paid for collection. \therefore 98% of tax = \$19600. \therefore tax = \$20000. On an assessment of \$1200000 the tax is \$20000. \therefore on an assessment of \$1 the tax is $\frac{2}{100}$, or 16 $\frac{2}{5}$ mills.
9. A's tax is calculated on \$1200 - \$400, or \$800. \therefore A pays 1 $\frac{4}{5}$ % of \$800, or \$14.40.
10. The man's assessment is \$1000. The tax on \$1000 is \$.016 \times 1000, or \$16.
11. I am assessed for \$1400. The tax on \$1400 is \$.015 \times 1400, or \$21. \therefore my net income is \$2000 - \$21, or \$1979.
12. The tax is \$1500 - \$1482.40, or \$17.60. \therefore on \$1100 the tax is \$17.60. \therefore on \$1 the tax is \$.016.
13. 1 $\frac{3}{4}$ % of Mr. Jones' income = \$25. \therefore the income = \$25 \times $\frac{100}{1\frac{3}{4}}$ = $\frac{10000}{7}$. \therefore 25% of the capital = $\frac{10000}{7}$. \therefore capital = \$5714 $\frac{2}{7}$.

158

14. The income tax is \$1800 - \$1779, or \$21. On \$1400 the tax is \$21. \therefore on \$1 the tax is \$.015.
15. The cost of the bridge and the collector's commission, that is, \$10150, was raised by the tax. \therefore 2% of assessment = \$10150. \therefore assessment = \$507500.
16. A man whose gross income is \$1000 pays tax on \$600. \therefore pays \$9. \therefore net income is \$991. The man whose income is \$995 pays no tax, and \therefore is \$4 better off.

SOLUTIONS OF PROBLEMS

17. On \$255000 the tax is \$4250. \therefore on \$1800 the tax is $\$4250 \times \frac{1800}{255000}$, or \$30.

18. On \$930000 the tax is \$15000. Man pays tax on \$900. \therefore on \$900 the tax is $\$14\frac{1}{3}$.

19. \$0.16 is the tax on \$1. \therefore \$24 is the tax on $\frac{1}{.016} \times 24$, or \$1500. \therefore my income was \$1500 + \$400, or \$1900.

20. Smith pays in tax \$66. He pays in insurance \$22.50. He loses interest \$300. \therefore house costs him yearly \$388.50, or monthly \$32.37 $\frac{1}{2}$.

21. 95% of tax = \$5700. \therefore tax = \$6000.

22. See solution of 17 above.

DUTIES AND CUSTOMS.

1. The dealer pays for the book \$2.40, \$.20 and 15% of \$2.40. That is, he pays \$2.96. \therefore he charges me 125% of \$2.96, or \$3.70.

159

2. The cost would be \$2.60. To make same rate of gain he would charge 125% of \$2.60, or \$3.25. The amount of gain in (1) was \$.74. To make the same amount of gain he would charge \$2.60 + \$.74, or \$3.34.

3. The duty is 15% of \$5, that is, \$.75.

4. The specific duty is \$30, the total cost is \$390. \therefore the invoice price and the ad valorem duty = \$360. \therefore 120% of invoice price = \$360. \therefore invoice price = \$300.

5. Invoice price is \$.40 \times 63, or \$25.20. \therefore duty = 15% of \$25.20 = \$3.78.

6. With duty the cost per gallon is 115% of 40c, or 46c. \therefore selling price would be 125% of 46c., or 57 $\frac{1}{2}$ c. Selling price, without duty, would be 125% of 40c., or 50c. \therefore reduction would be 7 $\frac{1}{2}$ c.

7. The area of the section is $(\frac{35}{12} \times \frac{1}{2})^2 \times \frac{22}{7}$ sq. ft. \therefore no. of cords in log = $(\frac{35}{12} \times \frac{1}{2})^2 \times \frac{22}{7} \times 30 \times \frac{1}{128}$. \therefore export duty = \$1.50 \times $(\frac{35}{12} \times \frac{1}{2})^2 \times \frac{22}{7} \times 30 \times \frac{1}{128}$ = \$2.34 +.

8. 1 sq. ft., board measure (i.e., 1 in. in thickness) contains $\frac{1}{12}$ cu. ft., or 1 cu. ft. contains 12 feet, board measure. \therefore the stick contains $(20 \times 3 \times 2 \times 12)$ feet, board measure. \therefore export duty = $\frac{\$2 \times 20 \times 3 \times 2 \times 12}{1000}$ = \$2.88.

9. The specific duty on the tobacco was $30c \times 50$, or \$15.
 \therefore the ad valorem duty was \$2.50. $\therefore 12\frac{1}{2}\%$ of value (invoice)
 $= \$2.50$; \therefore value $= \$20$.

10. 140% of total cost to dealer $= \$304$. \therefore cost to dealer $=$
 $\$217\frac{1}{4}$. The specific duty was \$30. $\therefore 115\%$ of invoice price
 $= \$187\frac{1}{4}$. \therefore invoice price $= \$162.73 +$.

11. If there were no duty, the dealer would sell for 140%
of $\$162.73 +$, or $\$227.82 +$. \therefore price would be reduced by
 $\$304 - \$227.82 +$, or $\$76.17 +$.

12. The whole cost was \$4384, and cartage expenses \$100.
 \therefore invoice price and ad valorem duty $= \$4284$. But duty was
 20% of 95% of invoice price, or 19% of invoice price. $\therefore 119\%$
of invoice price $= \$4284$. \therefore invoice price $= \$3600$. \therefore price
of 3600 bottles was \$3600. \therefore price of 1 bottle was \$1.

13. The duty was $\$1800 - \$1200 - \$75$, or \$525. On \$1200,
invoice, the duty was \$525. \therefore rate $= 43\frac{3}{4}\%$.

160

14. If the duty on cases had been also 20% , the total duty
would have been 20% of \$30, or \$6. But the duty on cases
was 35% . $\therefore 15\%$ of invoice price of case was $\$7.50 - \6 , or
\$1.50. \therefore invoice price of case was \$10. and \therefore invoice price
of instruments was \$20.

15. The number of pounds of raisins was 15000. \therefore the
specific duty was \$150. and \therefore the ad valorem duty was \$120.
The invoice price was \$1200. \therefore rate of ad valorem duty was
 10% .

16. Twice the ad valorem duty, together with $\$24 = \56 .
 \therefore twice the ad valorem duty $= \$32$. \therefore ad valorem duty $=$
 $\$16$, and specific duty $= \$40$. $\$2 =$ specific duty on 1 doz.
 $\therefore \$40$ sp. duty on 20 doz.

17. The invoice price was \$20. \therefore the ad valorem duty was
 $\$3.00$; and hence the specific duty was 75c. Therefore the
bale contained 75 sq. yd.: \therefore the width was $\frac{3}{4}$ yd., or 27 in.

18. The duty on the wine, without water, is \$60 ad valorem,
and \$37 specific: that is, \$97 in all. 30% of the wine is spirits.
 $\therefore 30$ gals. are spirits. $\therefore 26\%$ of the mixture $= 30$ gals. \therefore
mixture $= 115\frac{5}{13}$ gals. The specific duty on the mixture $=$
 $\$.25 \times 115\frac{5}{13} = \$28\frac{11}{13}$. The ad valorem duty $= 30\%$ of $\frac{10}{10}$ of
 $\$200 = \54 . \therefore total duty is $\$82\frac{11}{13}$. \therefore gain in saving of duty
 $= \$97 - \$82\frac{11}{13} = \$14\frac{2}{13}$. Loss in value of wine is \$20. \therefore net
loss $= \$20 - \$14\frac{2}{13} = \$5\frac{11}{13}$.

SOLUTIONS OF PROBLEMS

19. Since each shade is worth twice as much as each roller.
 \therefore 3 times invoice price of 1 roller = \$3. \therefore invoice price of 1 roller is \$1, and invoice price of 1 shade is \$2. The ad valorem duty on 1 roller = 30c., and on 1 shade = 30c. The total duty on 1 shade and 1 roller = $\$9.90 \div 12 = 82\frac{1}{2}$ c. \therefore the specific duty on 1 shade is 22 $\frac{1}{2}$ c. \therefore each shade contains $4\frac{1}{2}$ sq. yds. The width of each shade is $1\frac{1}{2}$ yds. \therefore the length = 3 yds.

20. The invoice price was \$20. \therefore the ad valorem duty was 15% of \$20, or \$3. \therefore the specific duty was \$12.50, or the specific duty on 1 ft. was 12 $\frac{1}{2}$ c. \therefore each foot of hose weighed $2\frac{1}{2}$ lbs.

STOCKS AND INVESTMENTS.

161

4. \$100 stock costs \$90. \therefore \$4500 costs $\$90 \times 45$, or \$4050.

5. \$100 stock costs \$108 $\frac{1}{4}$. \therefore \$2700 stock costs $108\frac{1}{4} \times 27$, or \$2922.75.

6. \$100 stock costs me $\$85\frac{1}{2} + \$\frac{1}{8}$, or $\$85\frac{5}{8}$. \therefore \$5600 stock costs me \$4795.

7. The broker sold 1 share for \$92. \therefore the broker, for each share, gave me $\$92 - \$\frac{1}{8}$, or $\$91\frac{7}{8}$. \therefore for 80 shares he gave me \$7350.

8. \$100 stock cost $\$89\frac{7}{8} + \$\frac{1}{8}$, or \$90. \therefore \$8000 stock cost \$7200.

9. $\$80\frac{5}{8}$ buys \$100 stock. \therefore \$6450 buys \$8000 stock.

10. $\$80\frac{5}{8} + \$\frac{1}{8}$, or $\$80\frac{3}{4}$, pays for \$100 stock. \therefore \$6460 pays for \$8000 stock.

11. \$117 $\frac{3}{4}$ is obtained from sale of \$100 stock. \therefore \$4710 is obtained from sale of \$4000 stock.

12. \$100 stock is sold for \$117 $\frac{3}{4}$, and \therefore owner receives $\$117\frac{3}{4} - \$\frac{1}{8}$, or $\$117\frac{1}{4}$. See solution of 11.

13. Income is 8% of amount of stock. \therefore income is 8% of \$4000, or \$320.

14. \$100 stock pays \$7 dividend. \therefore \$8450 stock pays \$591.50.

15. \$100 stock cost me $\$90\frac{7}{8} + \$\frac{1}{8}$, or \$91, and I sold it for $\$92 - \$\frac{1}{8}$, or $\$91\frac{7}{8}$. \therefore on \$91 of my money I gained $\$7\frac{7}{8}$. \therefore on \$9100 of my money I gained \$87.50.

IN THE HIGH SCHOOL ARITHMETIC.

16. For each share I paid $\$96\frac{1}{2} + \$\frac{1}{8}$, or $\$96\frac{1}{2}$, and sold for $\$95 - \$\frac{1}{8}$, or $\$94\frac{7}{8}$. \therefore on each share I lost $\$96\frac{1}{2} - \$94\frac{7}{8}$, or $\$1\frac{3}{8}$. \therefore on 86 shares I lost $\$118.25$.

17. $\$95$ pays for $\$100$ stock, and this gives $\$5$ annual dividend. \therefore on $\$95$ the gain is $\$5$. \therefore on $\$100$ the gain is $\$5\frac{5}{10}$. \therefore rate is $5\frac{5}{10}\%$.

18. $\$100$ stock costs $\$94\frac{7}{8} + \$\frac{1}{8}$, or $\$95$, and gives $\$5$ annual dividend. See solution of 17.

19. On $\$24000$ stock the dividend is $\$1920$. \therefore on $\$100$ stock the dividend is $\$8$. \therefore rate of dividend is 8% .

20. $\$6$ is annual income from $\$100$ stock. \therefore $\$240$ is annual income from $\$4000$ stock. $\$100$ stock costs $\$75 + \$\frac{1}{4}$. \therefore $\$4000$ stock costs $\$3010$.

162

21. $\$100$ stock gives $\$7$ income. $\therefore 5\%$ of price of $\$100$ stock = $\$7$. \therefore price of $\$100$ stock = $\$140$.

22. $4\frac{1}{2}\%$ of price of $\$100 = \5 . \therefore price paid for $\$100$ stock is $\$111\frac{1}{2}$. \therefore stock is quoted at $\$111\frac{1}{2} - \$\frac{1}{8}$, or $\$110\frac{7}{8}$.

23. The annual income from $\$7200$ stock, paying 5% , is $\$360$. $\$100$ stock yields the owner $\$89\frac{1}{8} - \$\frac{1}{8}$, or $\$89$. \therefore $\$7200$ stock yields the owner $\$6408$. $\$107\frac{7}{8} + \$\frac{1}{8}$, or $\$108$, invested in new stock, yields $\$6$ annually. \therefore $\$6408$ invested in new stock yields $\$356$ annually. \therefore loss in annual income = $\$360 - \$356 = \$4$.

24. See solution of 23.

25. $\$3$ is the income from $\$100$ stock, which sells for $\$67$. \therefore 1st income is $\frac{3}{67}$ of money received for 3% stock. 2nd income is $\frac{1}{100}$ of same sum. $\therefore (\frac{3}{67} - \frac{1}{100})$, or $\frac{32}{6700}$ of this sum = $\$480$. \therefore the sum received for stock sold was $\$100500$. Since 4% stock was at par, $\$100500$ is amount of stock bought.

26. The man receives from the broker $\$67\frac{1}{8} - \$\frac{1}{8}$, or $\$67$, for $\$100$ stock sold, and he pays the broker $\$99\frac{7}{8} + \$\frac{1}{8}$, or $\$100$ for $\$100$ stock bought. $\$100$ of 3% stock returned $\$3$ income. This stock yielded $\$67$. $\$67$ invested in the 4% stock at $\$100$, returned income $\$2.68$. \therefore $\$3 - \2.68 , or $\$.32$ was loss of income on $\$100$ stock sold. \therefore $\$480$ was loss of income on $\$150000$ stock.

27. By selling $\$4800$ stock I gain $\$150$. \therefore by selling $\$100$ stock I gain $\$3\frac{1}{8}$. \therefore I must sell $\$100$ stock for $\$75 + \$3\frac{1}{8}$, or $\$78\frac{1}{8}$.

SOLUTIONS OF PROBLEMS

28. \$120 buys \$100 stock, which yields half-yearly \$4. \therefore stock yields half-yearly $\frac{4}{120}$, or $\frac{1}{30}$, of sum invested in it. The second half-yearly dividend is equal to the first dividend, increased by the gain of first dividend. $\therefore \frac{31}{30}$ of first dividend = \$496. \therefore first dividend was \$480.

29. \$100 consols gives \$3 income. \therefore \$100 consols gives 98% of \$3 net income. $\therefore 3\frac{1}{2}\%$ of price paid for \$100 consols = 98% of \$3. \therefore price paid for \$100 consols = \$84.

30. $\frac{4}{100}$ of sum invested in 4% stock must be equal to $\frac{5}{100}$ of sum invested in 5% stock. \therefore sum invested in 4% stock must be $\frac{5}{4}$ of sum invested in 5% stock. $\therefore \frac{1}{10}$ of sum invested in 5% stock = \$9000 + \$12000 = \$21000. \therefore sum invested in 5% stock = \$11052 $\frac{1}{2}$. \therefore \$12000 - \$11052 $\frac{1}{2}$, or \$947 $\frac{1}{2}$, must be transferred.

31. The amount of the mortgage was \$1,600,000. \therefore the amount of shares (stock) was \$2,400,000. The yearly interest on the mortgage is 6% of \$1,600,000, or \$96000. The yearly dividend to shareholders is 5% of \$2,400,000, or \$120,000. \therefore the net income must be \$96000 + \$120000, or \$216,000. $\therefore 35\%$ of yearly gross receipts = \$216000. \therefore yearly receipts = \$216000 $\times \frac{100}{35}$; \therefore average weekly receipts \$216000 $\times \frac{100}{35} \times \frac{1}{52}$ = \$11868.13.

32. The amount of preference stock is \$500,000, and the dividend on it is 8% of \$500000, or \$40000. The balance \$55000 - \$40000, or \$15000, is dividend on \$500000 ordinary stock; hence on \$100 ordinary stock the dividend is \$3. \therefore rate is 3%.

163

33. In the 6% stock \$1 is the income derived from \$128 $\frac{1}{8}$ \div 6, or \$21 $\frac{1}{4}$ invested. In the 5% stock \$1 is the income derived from \$99 $\frac{1}{4}$ \div 5, or \$19 $\frac{3}{4}$ invested. \therefore the latter stock is the better investment.

34. The net gain is 6% of \$1,000,000, or \$60,000. 2 $\frac{1}{4}\%$ of the whole stock is \$25,000. But preference stock yields 8%. $\therefore 5\frac{1}{2}\%$ of preference stock = \$60000 - \$25,000 = \$35000. \therefore preference stock = \$35000 $\div \frac{8}{100}$ = \$437,500.

35. The dividend was 8% of \$200,000, or \$16,000. $\therefore 5\%$ of increased stock = \$16000. \therefore increased stock = \$320,000. \therefore new stock = \$120,000.

36. \$75 invested in 3% stock yields \$80. \therefore \$6000 invested in 3% stock yields \$6400. \$96 invested in 3 $\frac{1}{2}\%$ stock gives

IN THE HIGH SCHOOL ARITHMETIC.

$\$3\frac{1}{2}$ income. $\therefore \frac{\$6400}{3}$ invested in $3\frac{1}{2}\%$ stock gives $\$77\frac{1}{3}$ income.

$\$100$ invested in 5% stock gives $\$5$ income. $\therefore \frac{\$6400 \times 2}{3}$ invested in 5% stock gives $\$213\frac{1}{3}$ income. \therefore total income = $\$77\frac{1}{3} + \$213\frac{1}{3} = \$291\frac{1}{3}$.

37. On $\$60$ the gain is $\$2\frac{1}{2}$: \therefore on $\$100$ the gain is $\$4\frac{1}{4}$.
rate of gain is $4\frac{1}{4}\%$.

38. The stock sold at $\$90$. $\therefore 112\frac{1}{2}\%$ of buying price = $\$90$.
 \therefore buying price = $\$80$. Stock was bought at a discount of 20% .

39. 4% of price of $\$100$ consols = $\$3$. \therefore the price of $\$100$ consols = $\$75$.

40. $\$80$ pays for $\$100$ stock ($5\frac{1}{2}\%$) and yields $\$5\frac{1}{2}$ income.
 \therefore $\$40000$ pays for $\$50000$ stock ($5\frac{1}{2}\%$) and yields $\$2750$ income. $\$120$ pays for $\$100$ stock ($7\frac{1}{2}\%$) and yields $\$7\frac{1}{2}$ income.
 \therefore $\$60000$ pays for $\$50000$ stock ($7\frac{1}{2}\%$) and yields $\$3750$. \therefore the total income was $\$6500$. The whole amount of stock bought was $\$100,000$. \therefore the brokerage was $\$250$.

41. $\$15000$ stock, paying 5% , yields $\$750$ income. \therefore income from 6% stock was $\$750 + \$60 = \$810$. $\$6$ is income from $\$120$ invested in the 6% stock. \therefore $\$810$ is income from $\$16200$ invested in the 6% stock. \therefore $\$15000$ stock (5%) sold for $\$16200$.
 \therefore $\$100$ stock (5%) sold for $\$108$.

42. $\$1779 - \400 , or $\$1379$. 15 mills on the dollar is $1\frac{1}{2}\%$.
 $\therefore 98\frac{1}{2}\%$ of taxable income = $\$1379$. \therefore taxable income = $\$1400$.
 \therefore whole income = $\$1800$. $\$5$ is income from $\$115$ invested.
 \therefore $\$1800$ is income from $\$41400$.

43. At the end of 5 years, the value of the bond and the yearly payments will be $\$1000 + \$60 + \$60 \times (1.05) + \$60 \times (1.05)^2 + \$60 \times (1.05)^3 + \60×1.05^4 . This amounts to $\$1331.5384$.

5384. The present value is $\frac{\$1331.5384}{(1.05)^5} = \1043.29 .

44. On $\$100$ stock bought at $88\frac{1}{2}$ and sold at $91\frac{1}{2}$, the gain is $\$3\frac{1}{4}$. \therefore $\$100$ is the gain on $\$2666\frac{2}{3}$ stock bought and sold.

45. 8% of price of $\$100$ stock = $\$5$. \therefore price of $\$100$ stock = $\$62\frac{1}{2}$.

46. Income from the 6% stock is $\frac{1}{100}$ of money invested in it. Income from the 5% stock is $\frac{1}{100}$ of money invested in it. \therefore Income from the 5% stock is $\frac{1}{100}$ of $\frac{2}{3}$ of money invested in

SOLUTIONS OF PROBLEMS

6% stock. $\therefore \frac{6}{100} + (\frac{5}{100} \text{ of } \frac{3}{2})$, or $\frac{1}{3}$, of money invested in 6% stock was \$900. \therefore sum invested in 6% stock was \$7200, and sum invested in 5% stock was \$10800.

47. \$5 is income from \$113 invested in 5% stock. \therefore \$1 is income from $\$22\frac{2}{3}$ invested in 5% stock. Similarly \$1 is the income from $\$22\frac{1}{3}$ invested in 6% stock. \therefore 6% stock is better investment.

48. The income from \$1200 stock paying 4% is \$48. The stock sold for \$1080. The income from \$1080 at $4\frac{1}{2}\%$ is \$48.60. \therefore gain in income is 60c.

49. He gains $\pounds\frac{5}{8}$ on £100 consols. \therefore he gains £1 on £160 consols. \therefore he gains £15 15s. on £160 $\times 15\frac{3}{4}$, or £2520 consols. He bought £2520 consols for £2562 10s. \therefore he bought £100 consols for £93 15s.

50. £100 consols costs $\pounds94\frac{5}{8}$. \therefore £1400 consols costs £1324 $\frac{1}{2}$. $\pounds94\frac{1}{2}$ secures £100 Russian stock. \therefore £3150 secures £3333 $\frac{1}{3}$ Russian stock. \therefore total amount of stock = £1400 consols + £3333 $\frac{1}{3}$ loan = £4733 $\frac{1}{3}$. He receives $\pounds95\frac{1}{2} - \pounds\frac{1}{2}$, or £95 for £100 consols. \therefore On £100 consols he gains $\pounds\frac{3}{8}$. \therefore on £1400 consols he gains $\pounds5\frac{1}{2}$. He sells £100 Russian stock for $\pounds96\frac{1}{2} - \pounds\frac{1}{2}$, or £96 $\frac{1}{2}$. \therefore he gains on £100 Russian stock $\pounds96\frac{1}{2} - \pounds94\frac{1}{2}$, or $\pounds1\frac{3}{4}$. \therefore he gains on £3333 $\frac{1}{3}$ Russian stock $\pounds58\frac{1}{3}$. \therefore his total gain is $\pounds5\frac{1}{2} + \pounds58\frac{1}{3}$, or £63 11s 8d.

SIMPLE INTEREST.

165

11. From May 1st to Nov. 14th is 197 days. \therefore the interest is $\$1400 \times \frac{8}{100} \times \frac{197}{365}$, or \$60.449.

12. The time is 1 year and 300 days, or 665 days. \therefore the interest is $\$1650 \times \frac{6}{100} \times \frac{665}{365}$, or \$180.369.

13. The time is 1 year and 195 days, or 560 days. \therefore the interest is $\$1275 \times \frac{5}{100} \times \frac{560}{365}$, or \$156.493. \therefore the amount is \$1275 + \$156.493 or \$1431.493.

14. The interest is $\$5500 \times \frac{4}{100} \times \frac{1}{2}$, or \$110. \therefore the amount of the payment is \$5610.

15. The interest on \$550 for 2 years is \$44. \therefore the interest on \$550 for 1 year is \$22. \therefore the interest on \$100 for 1 year is $\$22 \times \frac{100}{550}$, or \$4.

IN THE HIGH SCHOOL ARITHMETIC.

16. 146 days = $\frac{2}{3}$ of 1 year. The interest on \$840 for $\frac{2}{3}$ year is \$58.80. \therefore the interest on \$650 for 2 years is \$53.80 $\times \frac{5}{7} \times \frac{1}{10} \times 2 \times 650$, or \$65.

17. The interest on \$1440 for $1\frac{2}{3}$ years is \$72. \therefore the interest on \$100 for 1 year is $\$72 \times \frac{3}{8} \times \frac{100}{1440}$, or \$3. \therefore the rate is 3%.

18. The interest on \$400 for 3 years is \$48 - \$400, or \$48. \therefore the interest on \$100 for 1 year is $\$48 \times \frac{1}{4} \times \frac{1}{3}$, or \$4. \therefore the rate is 4%.

19. See Solution of 18.

20. The time is 73 days. The interest on \$1500 for 73 days is \$1515 - \$1500, or \$15. \therefore the interest on \$100 for 365 days is \$5. \therefore the rate is 5%.

21. The interest is \$600 - \$500, or \$100. The interest for 1 year is $\$500 \times .04$, or \$20. \therefore the number of years is $\frac{100}{20}$, or 5.

166.

22. See Solution of 21.

23. The interest is \$100. \therefore the interest on \$100 for 1 year is \$6. \therefore the time is $100 \div 6$, or $16\frac{2}{3}$ years. The amount is double the principal. \therefore \$579.89 amounts to \$1159.78 in the same time.

24. 4% of the principal is the interest for 1 year. \therefore 100% of the principal is the interest for 25 years. \therefore any sum of money will double itself in 25 years.

25. $\frac{3}{100}$ of the principal is the interest for 1 year. \therefore the principal is the interest for $\frac{100}{3}$, or $33\frac{1}{3}$ years. \therefore any sum will double itself in $33\frac{1}{3}$ years.

26. The interest for 1 year is $\frac{4}{100}$ or $\frac{1}{25}$ of the sum lent. \therefore the interest for 3 years is $\frac{3}{25}$ of the sum lent.

27. The interest for 1 year is .05 of the principal. \therefore the interest for 4 years is .2, or $\frac{1}{5}$, of the principal.

28. (a) For 1 year the fraction is $\frac{7\frac{1}{2}}{100}$ or $\frac{15}{200}$. \therefore for 4 years the fraction is $\frac{3}{10}$. (e) For 1 year the fraction is $\frac{8}{100}$, or $\frac{2}{25}$. \therefore for $\frac{1}{2}$ year the fraction is $\frac{1}{25}$. (g) For 1 year the fraction is $\frac{1}{10}$. \therefore for 292 days, or $\frac{4}{5}$ year, the fraction is $\frac{4}{5}$.

29. $\frac{1}{12}$ of the sum lent is the interest for 1 year. \therefore the sum lent is the interest for 12 years.

30. $\frac{5}{100}$ of the sum lent is the interest for 1 year. \therefore the sum lent is the interest for 20 years.

SOLUTIONS OF PROBLEMS

31. $6\frac{1}{4}\%$ of the sum lent is the interest for 1 year. \therefore the sum lent is the interest for $\frac{100}{6\frac{1}{4}}$, or, 16 years. \therefore the sum of money will double itself in 16 years.

32. 4% of the sum lent is the interest for 1 year. \therefore the sum lent is the interest for 25 years. \therefore twice the sum lent is the interest for 50 years. \therefore the sum of money will treble itself in 50 years.

33. The rent for a year amounts to \$240. The rent amounts to $6\frac{1}{4}\%$ of the value and $1\frac{1}{4}\%$ of the value; that is, to $7\frac{1}{2}\%$ of the value. $\therefore 7\frac{1}{2}\%$ of value = \$240. \therefore value = \$3200.

34. The taxes and the interest = 9% of the value = \$720. \therefore rent for 12 months = \$720. \therefore rent for 1 month = \$60.

35. The interest on \$511000 for 5 days = \$280. \therefore the interest on \$511000 for 365 days, or 1 year = $\$280 \times 73$. \therefore the interest on \$100 for 1 year = $\frac{\$280 \times 73}{5110} = \4 . \therefore rate is 4% .

36. The interest on \$500 at double the first rate is equal to the interest on \$1000 at the first rate. \therefore the interest on \$400 + \$1000, or \$1400, for 1 year, at the first rate, is \$28. \therefore the interest on \$100 for 1 year, at first rate, is \$2. \therefore first rate is 2% , and second rate is 4% .

37. The interest on \$300 for 2 years = interest on \$600 for 1 year. The interest on \$600 for 3 years = interest on \$1800 for 1 year. The interest on \$1800 at double first rate = interest on \$3600 at first rate. \therefore interest on \$600 + \$3600, or \$4200, for 1 year, at first rate = \$105. \therefore interest on \$100 for 1 year, at first rate = $2\frac{1}{2}\%$. \therefore first rate = $2\frac{1}{2}\%$.

167

38. The interest on \$250, for 6 months = interest on \$125 for 1 year. The interest on \$450, for $1\frac{1}{2}$ years = interest on \$675 for 1 year. The interest on \$675 for 1 year, at $\frac{3}{4}$ of first rate = interest on \$506 $\frac{1}{4}$ for 1 year at first rate. \therefore interest on \$125 + \$506 $\frac{1}{4}$, or \$631 $\frac{1}{4}$, for 1 year, at first rate, is \$25 $\frac{1}{4}$. \therefore interest on \$100, for 1 year, at first rate = \$4. \therefore first rate = 4% , and second rate = 3% .

39. Interest on \$100 for 2 years at 4% = \$8. \therefore \$100 amounts to \$108 in 2 years.

40. Interest for 2 years at 4% = 8% of principal. \therefore 108% of principal = \$540. \therefore principal = \$500.

IN THE HIGH SCHOOL ARITHMETIC.

41. Interest for 6 months at $5\% = 2\frac{1}{2}\%$ of principal. \therefore $102\frac{1}{2}\%$ of principal = \$820. \therefore principal = \$800.
42. Interest for 9 months at $4\% = 3\%$ of principal. \therefore 103% of principal = \$1339. \therefore principal = \$1300.
43. The amount of taxes $\$.015 \times 50000 = \750 . Interest for 219 days at $6\% = \frac{219}{365}$ of 6% of principal = $3\frac{3}{5}\%$ of principal. \therefore $103\frac{3}{5}\%$ of sum deposited = \$750. \therefore sum deposited = \$723.93+.
44. \$1500 in 9 months at 8% amounts to 106% of \$1500, or \$1590. \therefore \$1650 in 9 months is the better offer.
45. The interest on \$250 in the time = \$25. \therefore the interest on \$250 for half the time = \$12 $\frac{1}{2}$. \therefore \$262 $\frac{1}{2}$ is the amount of \$250 for half the time. \therefore \$275 is the amount of \$250 for half the time.
46. Interest for $3\frac{1}{2}$ years at $8\% = 28\%$ of principal. \therefore 128% of principal = \$320. \therefore principal = \$250.
47. The interest was $(5\% + 6\%)$ of principal. \therefore 111% of sum borrowed = \$166. \therefore sum borrowed = \$149 $\frac{61}{111}$.
48. The interest for 12 months = 6% of principal. \therefore the interest for 2 months = 1% of principal. \therefore the interest for any number of months is equal to 1% of the principal multiplied by one-half of the number of months.
49. A offers \$2180, payable at end of 3 years. B offers \$455 now, \$455 at end of 1 year, \$455 at end of 2 years, and \$455 at end of 3 years. These sums at end of 3 years amount to \$568 $\frac{3}{4}$, \$530 $\frac{5}{8}$, \$492 $\frac{11}{16}$, and \$455: in all \$2047 $\frac{1}{2}$, at end of 3 years. C offers \$1600 cash, this amounts to \$2000 in 3 years. \therefore A's offer is the best.
50. Amount of \$1200 for 1 year at $5\% = \$1260$. Amount of \$1260 for 1 year at $5\% = \$1323$. Amount of \$1323 for 1 year at $5\% = \$1389.15$. \therefore at end of 3 years I must pay \$1389.15.
51. The yearly income from the business was 15% of \$15000, or \$2250. He loaned 90% of \$15000, or \$13500. \therefore yearly interest was 8% of \$13500, or \$1080. \therefore loss in yearly income was \$2250 - \$1080, or \$1170.

52. He sold the flour for $112\frac{1}{2}\%$ of \$2000, or \$2250. The interest on a sum of money for 6 months at 5% is equal to $2\frac{1}{2}\%$ of the sum. \therefore $102\frac{1}{2}\%$ of sum deposited = \$2250. \therefore sum

SOLUTIONS OF PROBLEMS

deposited = \$1951.21 +. \therefore he had left \$2250 - \$1951.21 +, or \$298.78 +.

53. At the end of 6 months he must pay \$2500. In 6 months, at 6%, \$2800 amounts to \$2884. \therefore his gain, at end of 6 months, is \$384.

54. The interest for $7\frac{1}{2}$ years is $\frac{5}{16}$ of the sum lent. \therefore the interest for 1 year is $\frac{5}{16} \div 7\frac{1}{2}$, or $\frac{1}{4}$ of sum lent. \therefore interest for 1 year is $4\frac{1}{4}\%$ of sum lent. \therefore rate is $4\frac{1}{4}\%$.

55. The interest on \$100000 for 1 month = the interest on \$1000 for 100 months = the interest on \$5000 for 20 months. \therefore $\frac{2}{3}$ of the interest on \$100000 for 1 month = the interest on \$5000 for $\frac{2}{3}$ of 20 months = the interest on \$5000 for $1\frac{1}{3}$ years.

56. The interest on \$1.33 $\frac{1}{3}$ for the time is \$33 $\frac{1}{3}$, or \$ $\frac{1}{3}$. The interest on \$1.33 $\frac{1}{3}$ for 1 year at 5% is \$ $\frac{1}{15}$. \$ $\frac{1}{15}$ is interest for 1 year. \therefore \$ $\frac{1}{3}$ is interest for 5 years.

BANK DISCOUNT.

3. Six months after Mar. 1st is Sept. 1st. \therefore day of maturity is Sept. 4th. From June 4th to Sept. 4th is 92 days. \therefore discount is \$1000 $\times \frac{5}{100} \times \frac{92}{365}$, or \$12.60. \therefore proceeds = \$1000 - \$12.60 = \$987.40.

169

4. Sixty days after Feb. 14th, 1890, is April 15th. \therefore day of maturity is April 18th. The note is discounted 63 days before it is due. \therefore discount = \$2356.50 $\times \frac{5}{100} \times \frac{63}{365}$ = \$24.40. \therefore proceeds = \$2332.10.

5. Three months after Jan. 15th is April 15th. \therefore day of maturity is Apr. 18th. From Feb. 1st to Apr. 18th is 76 days. \therefore discount is \$1250 $\times \frac{6\frac{1}{2}}{100} \times \frac{76}{365}$ = \$16.92. \therefore proceeds = \$1233.08.

6. Four months after May 23rd is Sept. 23rd. \therefore day of maturity is Sept. 26th. From July 2nd to Sept. 26th is 86 days. \therefore discount = \$5640.75 $\times \frac{5}{100} \times \frac{86}{365}$ = \$106.32. \therefore proceeds = \$5534.43.

7. Ninety days from Dec. 1st, 1889, is Mar. 1st, 1890. \therefore day of maturity is Mar. 4th, 1890. From Dec. 24th, 1889, to Mar. 4th, 1890, is 70 days. \therefore discount = \$2769 $\times \frac{5}{100} \times \frac{70}{365}$ = \$31.86. \therefore proceeds = \$2737.14.

IN THE HIGH SCHOOL ARITHMETIC.

8. Four months after Ap. 1st is Aug. 1st. \therefore day of maturity is Aug. 4th. From June 4th to Aug. 4th is 61 days. \therefore discount $= \$275 \times \frac{61}{100} \times \frac{5}{100} = \3.68 . \therefore proceeds $= \$271.32$.

9. Since there are only 28 days in February, 1890, this note would be nominally due on Feb. 28th. This is the usual practice at banks. The day of maturity then is Mar. 3rd. From Dec. 1st, 1889, to Mar. 3rd, 1890, is 92 days. \therefore discount $= \$4000 \times \frac{92}{100} \times \frac{5}{100} = \80.66 . \therefore proceeds $= \$3919.34$.

10. Six months after May 5th is Nov. 5th. \therefore day of maturity is Nov. 8th. From June 4th to Nov. 8th is 157 days. \therefore discount $= \$1234.56 \times \frac{157}{100} \times \frac{5}{100} = \31.86 . \therefore proceeds $= \$1202.70$.

170

11. Thirty days after Jan. 29th is Feb. 28th. \therefore date of maturity is Mar. 3rd. From Jan. 29th to Mar. 3rd is 33 days. \therefore discount $= \$400 \times \frac{33}{100} \times \frac{5}{100} = \2.89 . \therefore proceeds $= \$397.11$.

12. Four months after Feb. 3rd is June 3rd. \therefore date of maturity is June 6th. From Mar. 1st to June 6th is 97 days. \therefore discount $= \$576.75 \times \frac{97}{100} \times \frac{5}{100} = \12.26 . \therefore proceeds $= \$564.49$.

13. The date of maturity is May 9th. The interest on \$480 for 92 days at 5% $= \$480 \times \frac{92}{100} \times \frac{5}{100} = \6.05 . \therefore the amount of the note is \$486.05. The time from Feb. 18th to May 9th is 80 days. \therefore discount $= \$486.05 \times \frac{80}{100} \times \frac{5}{100} = \6.39 . \therefore proceeds $= \$486.05 - \$6.39 = \$479.66$.

14. The date of maturity is May 6th. The interest on \$2000 for 63 days at 6% $= \$2000 \times \frac{63}{100} \times \frac{6}{100} = \20.71 . \therefore the amount of the note is \$2020.71. \therefore the discount $= \$2020.71 \times \frac{63}{100} \times \frac{6}{100} = \27.90 . \therefore the proceeds $= \$2020.71 - \$27.90 = \$1992.81$.

15. The date of maturity is Nov. 26th. The interest on \$4200 $= \$4200 \times \frac{7}{100} \times \frac{5}{100} = \74.91 . \therefore the amount of the note is \$4274.91. From Sept. 1st to Nov. 26th is 86 days. \therefore the discount $= \$4274.91 \times \frac{86}{100} \times \frac{5}{100} = \80.58 . \therefore the proceeds $= \$4274.91 - \$80.58 = \$4194.33$.

16. (a) Since there are less than 30 days in February, it is the custom of Canadian banks to treat the note as nominally due on the last day of February, and, therefore, legally due on March 3rd.

17. The interest for 1 year is $\frac{5}{100}$ of the sum lent. \therefore the interest for 73 days is $\frac{73}{365}$ of $\frac{5}{100}$, or $\frac{1}{100}$ of the sum lent.

SOLUTIONS OF PROBLEMS

18. The interest for 1 year is $\frac{6}{100}$ of the amount of the note. \therefore the interest for 95 days is $\frac{95}{365}$ of $\frac{6}{100}$, or $\frac{57}{550}$ of the amount of the note.

171

19. The note is due in 95 days. \therefore the discount is $\frac{95}{365}$ of $\frac{6}{100}$, or $\frac{57}{550}$, of face value of note.

20. The discount is $\frac{73}{365}$ of $\frac{10}{100}$, or $\frac{1}{10}$, of face value of note.

21. The discount is $\frac{90}{365}$ of $\frac{6}{100}$, or $\frac{27}{1825}$, of face value of note. \therefore seller of note receives $\frac{1798}{1825}$, of face value of note.

22. The discount is $\frac{90}{365}$ of $\frac{6}{100}$, or $\frac{27}{1825}$, of face value. $\therefore \frac{1821}{1825}$ of face value = \$360.20. \therefore face value of note = \$365.

23. The note is due Sept. 4th. \therefore it is discounted 95 days before due. The discount is $\frac{95}{365}$ of $\frac{6}{100}$, or $\frac{19}{73}$, of face value. $\therefore \frac{1791}{1825}$ of face value = \$870. \therefore face value = \$888.50.

24. The note is due Sept. 28th. \therefore it is discounted 117 days before it is due. The discount is $\frac{117}{365}$ of $\frac{6}{100}$, or $\frac{351}{1825}$, of the face value. $\therefore \frac{1799}{1825}$ of face value = \$357.98. \therefore face value = \$365.

25. The discount was $\$730 \times \$724.60 = \$5.40$. \therefore the discount for 365 days would be $\$5.40 \times \frac{365}{45}$, or \$43.80. \therefore the discount from \$730 for 1 year is \$43.80. \therefore the discount from \$100 for 1 year is \$6. \therefore rate is 6%.

26. The discount from \$1000 for 73 days is \$10. \therefore the discount from \$1000 for 365 days is \$50. \therefore the discount from \$100 for 1 year is \$5. \therefore rate of discount is 5%.

27. The discount for 58 days was \$20.30. \therefore the discount for 365 days would be \$127.75. \therefore the discount from \$100 for 1 year would be $\$127.75 \times \frac{100}{1825}$, or \$7. \therefore rate of discount is 7%.

28. The discount was $\frac{20}{1480}$, or $\frac{1}{73}$ of the face value of the note. \therefore the discount for 1 year would be $\frac{1}{73} \times \frac{365}{10}$, or $\frac{1}{2}$ of face value. \therefore rate of discount = $12\frac{1}{2}\%$.

29. The rate is due Sept. 2nd. \therefore it is discounted 80 days before it is due. \therefore the discount is $\frac{80}{365}$ of $\frac{6}{100}$, or $\frac{32}{1825}$, of face value. $\therefore \frac{1793}{1825}$ of face value = \$358.60. \therefore face value = \$365.

30. The discount from \$1460 for 1 year would be \$87.60. The discount was \$1460 - \$1448.48, or \$11.52. \$87.60 is discount for 365 days. \therefore \$11.52 is discount for $365 \times \frac{11.52}{87.60}$, or 48 days. \therefore note was due 48 days after May 23rd. \therefore was legally due on July 10th, and nominally due on July 7th.

IN THE HIGH SCHOOL ARITHMETIC.

31. The discount from \$292 for 1 year would be \$23.36. The discount was \$3.88. \therefore the discount was calculated for $365 \times \frac{23.36}{3.88}$, or 45 days. \therefore note was legally due 45 days after Dec. 20th, 1887, that is on Feb. 3rd, 1888. \therefore the note was made 93 days before Feb. 3rd, 1888; \therefore on Nov. 2nd, 1887.

32. The interest on the note = $\$1200 \times \frac{6}{100} \times \frac{63}{365} = \12.43 .
 \therefore amount of note = \$1212.43. \therefore discount = $\$1212.43 \times \frac{6}{100} \times \frac{63}{365} = \12.56 . \therefore proceeds = $\$1212.43 - \$12.56 = \$1199.87$.

PARTIAL PAYMENTS.

172

8. From May 3rd to July 15th is 73 days. The interest is $\$5000 \times \frac{6}{100} \times \frac{73}{365}$, or \$60.

9. The payment made on July 15th was only \$40; this is \$20 less than the interest due.

10. From May 3rd to Sept. 28th is 148 days. \therefore the interest is $\$5000 \times \frac{6}{100} \times \frac{148}{365}$, or \$121.64.

11. The two payments amount to \$1240. This sum exceeds the interest by \$118.36.

12. The balance of the note on Sept. 28th is $\$5000 \times \$121.64 - \$1240$, or \$3881.64. From Sept. 28th to Jan. 1st is 95 days. The interest is $\$3881.64 \times \frac{6}{100} \times \frac{95}{365}$, or \$60.62. \therefore Cole owes Scott on Jan. 1st, $\$3881.64 \times \60.62 , or \$3942.26.

173

13. From Jan. 1st to Mar. 15th is 73 days. The interest is $\$400 \times \frac{6}{100} \times \frac{73}{365}$, or \$4.80. \therefore the note at this date amounts to \$404.80. The payment is \$20. \therefore the balance of the note on Mar. 15th is \$384.80. From Mar. 15th to July 10th is 117 days. The interest on \$384.80 for 117 days is \$7.40, which is more than the payment made. From Mar. 15th to Sept. 20th is 189 days. The interest is $\$384.80 \times \frac{6}{100} \times \frac{189}{365}$, or \$11.96. The amount of the note is $\$384.80 + \11.96 , or \$396.76. The two payments amount to \$156. \therefore the balance of the note on Sept. 20th is \$240.76. From Sept. 20th to Dec. 24th is 95 days. The interest is $\$240.76 \times \frac{6}{100} \times \frac{95}{365}$, or \$3.76. \therefore the balance on Dec. 24th is $\$240.76 + \3.76 , or \$244.52.

SOLUTIONS OF PROBLEMS

14. The interest on \$1000, from Mar. 1st to June 1st, 92 days, is \$20.16. Amount of note on June 1st, 1888, is \$1020.16. The payment is \$300. \therefore the balance on June 1st, 1888, is \$720.16. The interest on \$720.16 from June 1st to Sept. 1st is more than the payment, \$10. The interest on \$720.16 from June 1st to Jan. 1st, 214 days, is \$33.78. \therefore the amount on Jan. 1st, 1889, is \$753.94. The payments amount to \$110. \therefore the balance on Jan. 1st, 1889, is \$643.94. The interest on \$643.94, from Jan. 1st to June 1st, 151 days, is \$21.31. \therefore the amount on June 1st, 1889, is \$665.25. The payment is \$400. \therefore the balance on June 1st, 1889, is \$265.25. The interest on \$265.25 for 1 year is \$21.22. \therefore the balance due on June 1st, 1890, is \$286.47.

15. The interest on \$1500 from April 1st, 1889, to Sept. 1st, 1889, 153 days, is \$37.73. \therefore principal and interest amount to \$1537.73. The payment is \$500. \therefore the balance on Sept. 1st, 1889, is \$1037.73. The interest on \$1037.73 from Sept. 1st, 1889, to Jan. 1st, 1890, 122 days, is \$20.81. \therefore principal and interest amount to \$1058.54. The payment is \$600. \therefore the balance on Jan. 1st, 1890, is \$458.54. The interest on \$458.54 from Jan. 1st, 1890, to June 1st, 1890, 151 days, is \$11.38. \therefore the balance due on June 1st, 1890, is \$469.92.

16. The interest on \$950, from Jan. 25th to March 2nd, 1888, 37 days, is \$6.74. Principal and interest amount to \$956.74. The payment is \$225. \therefore the balance is \$731.74. The interest on \$731.74, from Mar. 2nd to May 5th, 64 days, is \$8.98. \therefore the amount on May 5th is \$740.72. The payment is \$174.19. \therefore the balance on May 5th is \$566.53. The interest on \$566.53, from May 5th to June 29th, 55 days, is \$5.98. \therefore the amount on June 29th is \$572.51. The payment is \$187.50. \therefore the balance on June 29th is \$385.01. The interest on \$385.01, from June 29th, 1888, to Jan. 1st, 1889, 186 days, is \$13.73. \therefore the amount due on Jan. 1st, 1889, is \$398.74.

17. The interest on \$3400, from Sept. 13th, 1886, to Ap. 20th, 1887, 219 days, is \$102. \therefore the amount on Ap. 20th, 1887, is \$3502. The payment is \$800. \therefore the balance is \$2702. The interest on \$2702, from Ap. 20th to July 2nd, 73 days, is \$27.02. \therefore the amount is \$2729.02. The payment is \$600. \therefore the balance is \$2129.02. The interest on \$2129.02,

IN THE HIGH SCHOOL ARITHMETIC.

from July 2nd, 1887, to July 2nd, 1888, is \$106.45. \therefore the amount is 2235.47. The payment is \$1000. \therefore the balance is \$1235.47. The interest on \$1235.47, from July 2nd, 1888, to Jan. 2nd, 1889, 184 days, is \$31.14. \therefore the balance due on Jan. 2nd, 1889, is \$1266.61.

18. The balance on Jan. 7th, 1889, after paying \$1200, was \$800. The interest on \$800, from Jan. 7th to Ap. 7th, 3 months, is \$16. The balance is \$516. The interest on \$516 from Ap. 7th to June 7th, 2 months, is \$6.88. The balance is \$322.88. The interest on \$322.88, from June 7th to Dec. 7th, 6 months, is \$12.92. \therefore the amount due on Dec. 7th was \$335.80.

174

19. The interest on \$600, from June 30th, 1888, to Sept. 11th, 1888, 73 days, is \$9. The balance after payment is deducted is \$409. The interest on \$409, from Sept. 11th, 1888, to June 30th, 1889, 292 days, is \$24.54. The balance after payment is deducted is \$283.54. The interest on \$283.54, from June 30th, 1889, to Jan. 31st, 1891, 1 yr. 215 days, is \$33.79. \therefore the balance due on Jan. 31st, 1891, is \$317.33.

EQUATION OF PAYMENTS.

7. The interest on \$62.50 for 12 days = the interest on \$750 for 1 day = the interest on \$50 for 15 days.

8. The interest on \$200 for 4 months = the interest on \$800 for 1 month = the interest on \$160 for 5 months.

9. The interest on \$600 for 5 months = the interest on \$3000 for 1 month = the interest on \$1000 for 3 months.

10. The interest on \$300 for 4 months = the interest on \$1200 for 1 month. The interest on \$500 for 3 months = the interest on \$1500 for 1 month. The interest on \$450 for 2 months = the interest on \$900 for 1 month. \therefore Smith gave me the interest on \$3600 for 1 month. \therefore I must let Smith have \$3600 for 1 month.

11. The interest on \$400 for 6 months = the interest on \$2400 for 1 month. The interest on \$100 for 11 months = the interest on \$1100 for 1 month. \therefore I am entitled to the interest on \$3500 for 1 month. \therefore I must keep the \$500 for 7 months.

SOLUTIONS OF PROBLEMS

12. The interest on \$500 for 6 months = the interest on \$3000 for 1 month. The interest on \$800 for $1\frac{1}{2}$ months = the interest on \$1200 for 1 month. \therefore I owe the interest on \$4200 for 1 month, or the interest on \$1300 for $3\frac{2}{3}$ months.

175

13. The interest on \$1500 for 20 days = the interest on \$30000 for 1 day. The interest on \$1700 for 40 days = the interest on \$68000 for 1 day. \therefore Morton & Co. must allow the purchaser the interest on \$98000 for 1 day, that is, the interest on (or use of) \$4900 for $\frac{98000}{4900}$, or 20, days.

14. The interest on \$100 for 30 days = the interest on \$3000 for 1 day. The interest on \$800 for 40 days = the interest on \$32000 for 1 day. The interest on \$600 for 60 days = the interest on \$36000 for 1 day. Total interest = interest on \$71000 for 1 day = interest on \$1500 for $47\frac{1}{3}$ days. \therefore equated time is 48 days.

15. The interest on \$1200 for 10 days = the interest on \$12000 for 1 day. The interest on \$800 for 30 days = the interest on \$24000 for 1 day. Eaton & Co. must allow the interest on \$36000 for 1 day, which is equal to the interest on \$2400 for 15 days.

16. See solution of 14.

17. The interest on \$800 for 3 months = the interest on \$2400 for 1 month. The interest on \$600 for 5 months = the interest on \$3000 for 1 month. The man ought to have the interest on \$2400 for 6 months, which equals the interest on \$14400 for 1 month. \therefore He must keep the balance \$1000 until the interest on it equals the interest on \$14400—\$5400, or \$9000, for 1 month. \therefore the remainder, \$1000, becomes due in 9 months.

18. See solution of 15.

19. The interest on \$2400 for 30 days = interest on \$72000 for 1 day. The interest on \$800 for 60 days = interest on \$48000 for 1 day. \therefore White & Co. must allow me the interest on \$120,000 for 1 day. \therefore the note, for \$1000, is given for 120 days.

20. The interest on \$2000 for 15 days = interest on \$30000 for 1 day. The interest on \$1500 for 12 days = interest on \$18000 for 1 day. The balance, \$1500, must be kept after debt was due until the interest on it equals the interest on

\$48000 for 1 day. \therefore the balance should be paid 32 days after debt was due.

21. The interest on \$400 for 30 days = the interest on \$12000 for 1 day. The interest on \$600 for 40 days = the interest on \$24000 for 1 day. The interest on \$500 for 60 days = the interest on \$30000 for 1 day. The interest on \$66000 for 1 day = the interest on \$1500 for 44 days. \therefore average term of credit is 44 days, and equated time of payment is July 15th.

22. From March 4th to June 15th is 103 days. \therefore counting from March 4th, the \$1200 is due in 138 days. The interest on \$800 for 30 days = the interest on \$24000 for 1 day. The interest on \$1200 for 138 days = the interest on \$165600 for 1 day. The interest on \$189600 for 1 day = the interest on \$2000 for $94\frac{1}{2}$ days. \therefore equated time is 95 days from March 4th, that is, June 7th.

23. The time is counted from June 1st. The interest on \$400 for 30 days = interest on \$12000 for 1 day. The interest on \$850 for 49 days = interest on \$41650 for 1 day. The interest on \$1200 for 62 days = interest on \$74400 for 1 day. The interest on \$128050 for 1 day = interest on \$2450 for 52 days. \therefore equated time is 52 days after June 1st, that is, July 23rd.

24. I owe the friend the interest on \$16000 for 1 day. He must allow me the interest on \$30000 for 1 day. \therefore I ought to have the use of \$14000 for 1 day. The whole debt is \$2000. \therefore the equated time is 7 days.

176

25. The interest on $\frac{1}{2}$ of the debt for 10 days = interest on 2 times debt for 1 day. The interest on $\frac{3}{10}$ of the debt for 20 days = interest on 6 times debt for 1 day. The debt must be paid when the interest on it is equal to the interest on 4 times the debt for 1 day. \therefore the debt should be paid in 4 days.

26. Count time from June 20th. \$500 is due in 30 days; \$600 in 29 days; \$450 in 102 days; \$800 in 141 days. The equated time is 82 days; that is, Sept. 10.

27. The equated time of the Debit side is $63\frac{2}{3}$ days, counting from May 1st. The equated time of the Credit side is $27\frac{2}{3}$ days, counting from May 1st. Jones should pay \$2400 in $63\frac{2}{3}$ days: the payments made are equivalent to

SOLUTIONS OF PROBLEMS

\$1500 paid in $27\frac{1}{2}$. The interest on \$2400 for $63\frac{1}{2}$ days = interest on \$153000 for 1 day. The interest on \$1500 for $27\frac{1}{2}$ days = interest on \$41500 for 1 day. Jones should have the interest on \$111500 for 1 day, or the interest on the balance, \$900, for 124 days. \therefore balance should be paid 124 days after May 1st; that is, Sept 2nd.

28. The amount of \$900 at 8% per annum, from Sept. 2nd to Jan. 1st, is \$923.87.

29. Counting from Jan. 5th, Smith is entitled to: (1) The interest on \$840 for 30 days, or interest on \$25200 for 1 day; (2) The interest on \$900 for 45 days, or interest on \$40500 for 1 day; (3) The interest on \$750 for 57 days, or interest on \$42750 for 1 day; (4) The interest on \$800 for 71 days, or interest on \$56800 for 1 day; in all, the interest on \$165250 for 1 day. He has had (1) The interest on \$1500 for 27 days, or interest on \$40500 for 1 day; (2) The interest on \$500 for 46 days, or interest on \$23000 for 1 day; in all, the interest on \$63500 for 1 day. \therefore Smith is still entitled to the interest on \$165259 for 1 day. \therefore he is entitled to the use of the balance, \$1290, for 79 days. \therefore balance is due on March 25th.

30. The amount of \$1290, at 6% per annum, from March 25th to June 1st, is \$1304.42.

COMPOUND INTEREST.

177

13. The interest for 1 year is .05 of the principal. \therefore the amount for 1 year is (1.05) of the principal. \therefore the amount for 2 years is $(1.05)^2$ of the principal. \therefore the amount for 3 years is $(1.05)^3$, or 1.157625, of the principal.

14. The interest for 1 year is .04 of \$525.35. \therefore the amount for 1 year is (1.04) of \$525.35. \therefore the amount for 2 years is $(1.04)^2$ of \$525.35. \therefore the amount for 3 years is $(1.04)^3$ of \$525.35. \therefore the amount for 4 years is $(1.04)^4$ of \$525.35. The required fraction is $(1.04)^4$, or, 1.16985856.

15. The amount of \$100, at 4% , for 6 years, is \$126.532. The amount of \$100, at 4% , for 5 years, is \$121.665. The amount of \$100, at 4% , for 4 years, is \$116.986. The amount

IN THE HIGH SCHOOL ARITHMETIC.

of \$100, at 4%, for 3 years, is \$112.486. The amount of \$100, at 4%, for 2 years, is \$108.160. The amount of \$100, at 4%, for 1 year, is \$104.000. \therefore the total amount is \$689.83.

16. In 3 years at 4% per annum the amount is $(1.04)^3$ of the principal; that is, 1.124864 of the principal. \therefore the interest is .124864 of the principal, and this is \$150. \therefore the principal = $\$ \frac{150}{.124864} = \1201.31 .

178

17. The amount of \$1 is $(1.04)^4 + (1.04)^3 + (1.04)^2 + (1.04)$ dollars, that is, \$4.4163. \therefore the amount of \$150 is $\$4.4163 \times 150$, or \$662.45.

18. The amount of simple interest is 1.15 of the principal. The amount at compound interest is 1.157625 of the principal. \therefore the difference is .007625 of the principal. \therefore difference on \$1275 is $\$1275 \times .007625$, or \$9.72.

19. The amount at simple interest is 1.24 of the principal. The amount at compound interest is 1.26247696 of the principal. \therefore the difference of interest is .02247696 of the principal. This difference is \$100. \therefore the principal = $\$100 \div .02247696 = \4449.00 .

20. In 4 years, at 4% per annum, the amount is $(1.04)^4$ of the principal. \therefore the principal = $\$1200 \div (1.04)^4 = \1025.765 .

21. The interest each half-year is .03 of the principal for that half-year. \therefore in four half-years the amount is $(1.03)^4$ of the principal. \therefore the amount of \$1200 = $\$1200 \times (1.03)^4 = \1350.61 .

22. 1 year and 6 months = 3 half-years. The interest each half year is .025 of the principal for that half-year. \therefore the amount = $\$1450 \times (1.025)^3 = \1561.49 . \therefore the interest = $\$1561.49 - \$1450 = \$111.49$.

23. The amount at end of 2 years is $\$1460 \times (1.06)^2$. The amount for a half-year is 1.03 of the principal for that half-year. \therefore the amount at end of 2 years and 6 months is $\$1460 \times (1.06)^2 \times (1.03)$, or \$1689.67.

24. The interest for 73 days, or $\frac{1}{4}$ of a year, is .01 of the principal. \therefore the amount of \$1 in 2 years and 73 days is $\$1 \times (1.05)^2 \times (1.01)$, or \$1.113525. \therefore the interest is 11.3525 cents.

25. In 2 years and 3 months, at 4% per annum, the amount is $(1.04)^2 \times (1.01)$ of the principal; that is, 1.092416 of the

SOLUTIONS OF PROBLEMS

principal. \therefore the interest is .092416 of the principal. \therefore the principal = $\$400 \div .092416 = \4328.25 .

26. The population at the end of 4 years is $(1.1)^4$ of that at the beginning; that is, 1.4641 of that at the beginning. The increase is .4641 of original population. \therefore the original population was $13923 \div .4641$, or 30000. \therefore present population is 43923.

179

37. The amount in $2\frac{1}{2}$ years at 4%, is $(1.04)^2 \times (1.02)$, or 1.103232 of the principal. \therefore the principal = $\$16989.7728 \div 1.103232 = \15400 .

38. The amount in 4 half-years, at 2% each half-year, is $(1.02)^4$, or 1.08243216, of the principal. \therefore the principal = $\$10824.3216 \div 1.08243216 = \10000 .

39. The amount is $(1.05)^2 \times (1.01)$, or 1.113525, of the principal. \therefore the interest is .113525 of the principal. \therefore the principal = $\$82.82 \div .113525 = \729.53 .

40. In 7 years the amount is $(1.1)^7$, or 1.9487+, of the principal. In 8 years the amount is $(1.1)^8$, or 2.1435+, of the principal. \therefore the sum of money will double itself in a little more than 7 years.

41. The difference between the interest for the first year and that for the second year is the interest on the first year's interest. The difference between the interest for the second year and that for the third year is the interest on the second year's interest; that is, is the interest on the first year's interest and the interest on \$1. \therefore \$.05 is the interest on \$1 for 1 year. \therefore the rate is 5%.

42. At 3% per half-year, the amount in 1 year is $(1.03)^2$, or 1.0609 of the principal. \therefore 6.09% compounded yearly equals 3% per half-year.

180

43. The amount at the end of a half-year is obtained by multiplying the principal by a certain fraction. \therefore the square of this fraction = 1.06. \therefore the fraction = $\sqrt{1.06} = 1.029+$. \therefore rate per half-year is 2.9+ %.

44. \$129600 amounts to \$178506.25 in two years. \therefore the amount in 2 years is obtained by multiplying the principal by the fraction $\frac{178506.25}{129600}$. \therefore the amount for 1 year is obtained by multiplying the principal by $\sqrt{\frac{178506.25}{129600}}$, or $\frac{4225}{3600}$; that is,

IN THE HIGH SCHOOL ARITHMETIC.

by $\frac{169}{144}$. \therefore the interest fraction is $\frac{25}{144}$, and the rate is $17\frac{1}{3}\%$ per annum. Again, the original sum = $\$129600 \div \frac{17850625}{12980000} = \9409.28 .

45. The difference, \$.525, is evidently the interest on \$10.50 for 1 year. Hence the rate is $\frac{.525}{10.50} \times 100$, or 5%. \therefore 5% of amount at end of first year = \$10.50. \therefore amount at end of first year = \$210. \therefore principal = $\$210 \div 1.05 = \200 .

46. $\$2.1632 - \2.08 , or \$.0832, is the interest on \$2.08 for 1 year. \therefore the rate is 4%. Since \$50 is the interest for one year, the original sum is \$1250.

47. The difference between the interest of the first year and that of the third year is the interest for one year on the sum of the first year interest and the second year interest. The difference between the interest of the second year and that of the third year is the interest for one year on the sum of the second year interest and the third year interest. Hence the difference between these two differences is interest for one year on the difference of interest in the first and third years. \therefore $\$.509184 - \4.896 , or \$.19584 is the interest for one year on \$4.896. \therefore the rate is 4%. $\therefore \{ (1.04)^4 - 1 \}$ of principal = $\$254.78784$. \therefore principal = \$1500.

48. The interest is $\frac{34481}{100000}$ of the principal. \therefore the amount is $\frac{194481}{100000}$ of the principal. The time is 4 years. \therefore the amount for 1 year is $\sqrt[4]{\frac{194481}{100000}}$, or $\frac{21}{20}$, of the principal. \therefore the rate is $\frac{1}{20}$, or 5%.

49. The amount in 2 years, at 10% yearly, is $(1.1)^2$, or 1.21, of the principal. The amount in 2 years, at 5% half yearly, is $(1.05)^4$, or 1.21550625. \therefore the difference, or .00550625 of principal = \$55.0625. \therefore the principal = \$10000.

50. $(1.07)^{10} = 1.967151 + \therefore$ amount of \$1000 is \$1967.15+.

PRESENT WORTH AND TRUE DISCOUNT.

181

7. The amount in 8 months at 6% per annum is 1.04 of the principal. \therefore 1.04 of the present worth = \$800. \therefore present worth = $\$800 \div 1.04 = \769.23 .

SOLUTIONS OF PROBLEMS

8. The amount in 6 months at 8% per annum is 1.04 of the principal. \therefore 1.04 of the present value = \$1. \therefore present value = 96.15 + cents.

9. The amount in 1 year and 8 months, at 6% per annum, is $(1.06) \times (1.04)$ of the principal. $\therefore (1.06) \times (1.04)$ of the present value = \$1. \therefore present value = $\$1 \div \{(1.06) \times (1.04)\} = 90.71c$.

10. The amount in 2 years at 4 per cent per annum is $(1.04)^2$ of the principal. $\therefore (1.04)^2$ of the present worth = \$8000. \therefore the present worth = $\$8000 \div (1.04)^2 = \7396.45 .

11. The amount of \$100 in 6 months at 4% per annum is \$102. \therefore from \$102 the discount is \$2. \therefore from \$1200 the discount is \$23.53.

12. The amount of \$100 in 9 months at 5% per annum is \$103.75. \therefore the discount from \$103.75 due in 9 months is \$3.75. \therefore the discount from \$1350 due in 9 months is \$48.79 +.

13. (a) \$100 amounts to \$103 in the time. \therefore the discount is $\frac{3}{103}$ of the amount, or $\frac{3}{103}$ of \$485.50, or \$14.14. (b) \$100 amounts to \$108 $\frac{1}{2}$ in the time. \therefore the discount from \$108 $\frac{1}{2}$ is

\$8 $\frac{1}{2}$. \therefore the discount from \$1250.60 is $\$1250.60 \times \frac{8\frac{1}{2}}{108\frac{1}{2}}$, or

\$97.97. (c) \$100 amounts to \$108.16 in the time. \therefore the discount from \$108.16 is \$8.16. \therefore the discount from \$1234.56 is \$93.14. (d) \$1 amounts to $(\$1.05)^3 \times (1.03)$ in the time. \therefore the present value of $(\$1.05)^3 \times (1.03)$ is \$1. \therefore the present value of \$17684.95 is $\$17684.95 \div (1.05)^3 (1.03)$, or \$14831.97. \therefore the discount is \$2852.98. (e) In 5 years at 5% per annum, the amount is $(1.05)^5$ of the principal. \therefore present worth of \$1, due in 5 years, = $\$1 \div (1.05)^5 = \$.7835 +$. \therefore discount from \$1 = 21.65c.

14. The present value of \$400 due in 1 year is $\frac{\$400}{1.05}$, or \$380.952. The present value of \$400 due in 2 years is $\frac{\$400}{(1.05)^2}$, or \$362.811. The present value of \$400 due in 3 years is $\frac{\$400}{(1.05)^3}$, or \$345.535. \therefore the present value of three years' rent is \$1089.298, or \$1089.30.

15. \$400 paid at end of 1 year = $\$400 \times (1.04)$, or \$416 at end of 2 years. \$400 paid at end of 2 years = \$400 at end of 2 years. \$400 paid at end of 3 years = $\$400 \div (1.04)$, or \$384.62 at end of 2 years. \therefore the required sum = $\$416 + \$400 + \$384.62 = \1200.62 .

16. The P. W. of \$100, due in 2 years, = $\$100 \div (1.05)^2 = \90.702 . The P. W. of \$150, due in 3 years, = $\$150 \div (1.05)^3 = \129.575 . The P. W. of \$200, due in 4 years, = $\$200 \div (1.05)^4 = \164.540 . \therefore the debt would be paid now by \$384.82.

17. We first find the value of rental at the end of 1 year. The first quarter's rent bears interest for 9 months, the second for 6 months, the third for three months. \therefore the amount of rental at end of one year is \$100, together with the simple interest on \$25 for 18 months: that is, in all, \$101.50. \therefore the yearly rental paid in advance is P. W. of \$101.50, due in 1 year, which is $\$101.50 \div 1.04$, or \$97.60 nearly.

182

18. The share of the second heir is the amount of the share of the first heir in 2 years at 5% per annum. \therefore share of second heir = $(1.05)^2$ of share of first heir. Also share of third heir = $(1.05)^4$ of share of first heir. \therefore the three shares = 3.31800625 of share of first heir = \$10000. \therefore share of first heir = $\$ \frac{10000}{3.31800625} = \3013.86 . Hence, the others may be found.

19. The present value of \$a, due in 3 years, is \$b. \therefore the present worth for 3 years is $\frac{b}{a}$ of the amount. \therefore the present worth for 1 year is $(\frac{b}{a})^{\frac{1}{3}}$ of the amount. \therefore the present worth for 2 years is $(\frac{b}{a})^{\frac{2}{3}}$ of the amount. \therefore the present worth of \$c, due in 2 years, is $\$ (\frac{b}{a})^{\frac{2}{3}} c$.

20. The discount is \$b. \therefore the P. W. is $\$(a-b)$. \therefore the P. W. of \$a, due in 2 years, is $\$(\frac{a-b}{a})^{\frac{2}{3}} a$. \therefore the discount is $\$ \{ a - (\frac{a-b}{a})^{\frac{2}{3}} a \}$, or $\$ \{ 1 - (\frac{a-b}{a})^{\frac{2}{3}} \} a$.

SOLUTIONS OF PROBLEMS

23. The interest for 3 years at 5% is $\frac{15}{100}$ of the principal. \therefore the discount is $\frac{15}{115}$, or $\frac{3}{23}$, of the debt.

24. The amount is $(1.02)^4$, or $\frac{108243216}{100000000}$, of the principal. The interest is $\frac{8243216}{100000000}$, or $\frac{515201}{6250000}$, of the principal. \therefore the discount is $\frac{515201}{6765201}$ of the amount.

25. The amount = the present worth and the discount.

26. \therefore the interest on the amount = the interest on the present worth, and the interest on the discount.

27. By 26, \$8.20 is the interest on the discount for 2 years at 5% per annum, compounded yearly. For 2 years at 5%, the amount is $(1.05)^2$, or 1.1025, of the principal. \therefore the interest is .1025 of the principal. \therefore .1025 of the discount = \$8.20. \therefore the discount = \$80. \therefore the interest = \$88.20. \therefore .1025 of the sum of money = \$88.20. \therefore the sum of money = \$860.48.

28. The amount is $(1.02)^5$ of the principal. \therefore the interest is $\{(1.02)^5 - 1\}$ of the principal. Since discount is interest on P. W. \therefore P. W. of required sum = $\left\{ \frac{\$360}{(1.02)^5 - 1} \right\}$ \therefore the sum = $\$360 + \left\{ \frac{\$360}{(1.02)^5 - 1} \right\} = \3818.85 .

183

29. \$250 - \$240, or \$10, is the interest on \$240, for same time and rate. \therefore \$250 is the interest on \$6000, for same time and rate. \therefore \$6000 is the sum.

30. The discount is $\frac{400}{441}$ of the interest. \therefore the interest is $\frac{441}{400}$ of the discount. But the interest is the amount of the discount, for 2 years, at the same rate. \therefore the amount for 1 year is $\sqrt{\frac{441}{400}}$, or $\frac{21}{20}$, of the principal. \therefore the rate is 5%.

31. The interest for 3 years = $\frac{1331}{1000}$ of the discount for 3 years. \therefore the interest for 1 year = $\sqrt[3]{\frac{1331}{1000}}$, or $1\frac{1}{10}$, of the discount for 1 year. \therefore the rate is $\frac{1}{10}$, or 10%.

32. \$30 is the interest on \$240 for 6 months. \therefore $\$30 \times \frac{270}{240}$, or \$33.75, is the interest on 270 for 6 months. \therefore \$240 amounts to \$303.75 in 1 year. \therefore \$303.75 has \$63.75 discount for 1 year. \therefore \$270 has \$56.66 $\frac{2}{3}$ discount for 1 year.

IN THE HIGH SCHOOL ARITHMETIC.

33. P. W. of \$243, due in 1 year, is \$192. \therefore amount in 1 year is $\frac{243}{1.12}$, or $\frac{81}{4}$, of the principal. \therefore amount in 6 months is $\sqrt{\frac{81}{4}}$, or $\frac{9}{2}$, of the principal. \therefore P. W. is $\frac{9}{2}$ of amount due in 6 months. \therefore discount is $\frac{1}{2}$ of amount due in 6 months, or \$27.

34. The difference, \$1.9968, is the interest for 1 year on the second year's interest, that is, on \$49.92. \therefore the rate is 4%. \therefore the amount at end of first year is \$49.92+.04, or \$1248. \therefore 1.04 of the original sum = \$1248. \therefore the original sum = \$1200.

35. The first difference, \$2.40, is the interest for 1 year on the first year's interest. The second difference, \$2.496, is the interest for 1 year on the second year's interest. \therefore \$.096 is the interest for 1 year, of the difference of the first and second year's interest, that is, is the interest on \$2.40 for 1 year. \therefore the rate is 4%. The first year's interest is .04 of the original sum. The second year's interest is .04 of 1.04, or .0416 of the original sum. \therefore .0016 of original sum = \$2.40. \therefore original sum = \$1500.

36. The amount at the end of the second year is $(1.05)^2$ of the original principal. The difference between the interest for the third year and that for the sixth, is the same as the difference between the interest for the first and fourth years, on $(1.05)^2$ of the original principal. \therefore the difference is \$12.61 \times $(1.05)^2$, or \$13.902+.

37. The discount at 10%, for 1 year, is $\frac{1}{11}$ of the sum. The discount at 8%, for 1 year, on one-half of the sum, is $\frac{2}{27}$ of $\frac{1}{2}$ of the sum, or $\frac{1}{27}$ of the sum. The discount at 12% for 1 year, on one-half of the sum, is $\frac{3}{25}$ of $\frac{1}{2}$ of the sum, or $\frac{3}{50}$ of the sum. \therefore $(\frac{1}{11} - \frac{1}{27} - \frac{3}{50})$ of the sum = \$3 $\frac{58}{187}$. \therefore the sum = \$11000.

38. I pay now \$5000, and \$5300 at the end of a year. The P.W. of \$5300, due in 1 year, is $5300 \div 1.05$, or \$5047.62. \therefore the present cost of the farm is \$10047.62. I receive at the end of 3 months \$12120. The present worth of \$12120 due in 3 months, is $12120 \div 1.0125$, or \$11970.37. \therefore my present gain is \$1922.75.

39. The amount = P.W. + discount. \therefore The interest on the amount = the interest on P.W. + interest on discount. But interest on P.W. = the discount. \therefore the interest on the

SOLUTIONS OF PROBLEMS

amount = the discount + the interest on the discount. \therefore the discount is the P.W. of the interest.

40. The present value of the instalment due in 1 year is $(\frac{100}{108})$ of the instalment. The present value of the instalment due in 2 years is $(\frac{100}{108})^2$ of the instalment. The present value of the instalment due in 3 years is $(\frac{100}{108})^3$ of the instalment. The present value of the instalment due in 4 years is $(\frac{100}{108})^4$ of the instalment. $\therefore \left\{ (\frac{100}{108}) + (\frac{100}{108})^2 + (\frac{100}{108})^3 + (\frac{100}{108})^4 \right\}$ of the instalment = \$12000. \therefore the instalment = \$3463.09.

41. \$1389.15 is the amount of \$1200 in 3 years. \therefore the amount is $\frac{138915}{100000}$, or $\frac{9261}{6000}$, of the principal, in 3 years. \therefore the amount in 1 year is $\frac{21}{20}$ of the principal. \therefore the rate is $\frac{1}{20}$, or 5%. \$1200 in 1 year, at 5%, amounts to \$1260. \therefore \$1260 is the sum.

42. The P.W. of a given sum, due in 3 years, is equal to the P.W. of a sum due in 1 year, which latter sum is the P.W. of the given sum, due in 2 years. Similarly, the P.W. of the given sum, due in 5 years, is equal to the P.W. of a sum due in 1 year, which latter sum is the P.W. of the given sum due in 4 years. \therefore taking the differences, the P.W. of \$5.10 due in 1 year = \$5.00. Hence the rate is 2% per annum.

 ANNUITIES.

184

7. \$250 is the interest on \$6250 for 1 year at 4%. \therefore \$6250 is the value of the perpetuity, when the first payment is to be made at the end of 1 year. The present worth of \$6250, payable in 5 years, is $\$6250 \div (1.04)^5$, or \$5137.04.

8. \$200 is the interest on \$5000 for 1 year at 4%. \therefore the value of a perpetual annuity of \$200, beginning now, is \$5000. \therefore The value, when the annuity begins after 4 years, is $\$5000 \div (1.04)^4$, or \$4274.02.

9. "Deferred 7 years" means that the first payment is made at the end of 8 years. \$100 is the interest for 1 year at 5% on \$2000. \therefore the value of the annuity is $\$2000 \div (1.05)^7$, or \$1421.36.

10. The value is $\$2000 \div (1.05)^3$, or \$1727.68.

11. N.B.—It is important to notice that the value of a limited annuity may be expressed as the difference of values of perpetuities.

12. The value of a perpetuity beginning now is 20 times the annuity. If the perpetuity is deferred 1 year, its value is $\frac{20}{1.05}$ times the annuity. If the perpetuity is deferred 5 years, its value is $\frac{20}{(1.05)^5}$ times the annuity. \therefore the value of the

limited annuity is $\left\{ \frac{20}{1.05} - \frac{20}{(1.05)^5} \right\}$ times the annuity, or 3.377 times the annuity.

13. The present value of the first payment is $\frac{\$80}{1.05}$, or \$78.

431. The present value of the second payment is $\frac{\$80}{(1.02)^3}$, or

\$75.385. The present value of the third payment is $\frac{\$80}{(1.02)^5}$, or \$72.458. The present value of the fourth payment is $\frac{\$80}{(1.02)^7}$, or \$69.644. \therefore the present value of the payments is \$295.92.

14. The perpetual annuity will cost \$4000, at the end of 4 years. \therefore the sum invested now must be $\$4000 \div (1.05)^4$, or \$3290.81.

15. The present value of a perpetuity of \$800 is $\$800 \div .05$, or \$16000. The present value of the perpetuity, if deferred 10 years, is $\$16000 \div (1.05)^{10}$, or \$9822.61. \therefore the annual payments are worth \$6177.39.

16. The value of a perpetuity, beginning now, is 25 times the annuity. The value of a perpetuity, beginning in 6 years, is $\frac{25}{(1.04)^6}$ times the annuity, or 19.757863 times the annuity.

The difference, 5.242137 times the annuity = \$12000. \therefore the annuity = \$2289.15.

17. The value of a perpetuity of \$40, beginning immediately, is \$800. The value of a perpetuity, if it begins in 10 years, is $\$800 \div (1.05)^{10}$. The present value of the ten payments is $\left\{ 1 - \frac{1}{(1.05)^{10}} \right\} \times 20$ times the payment. Hence

SOLUTIONS OF PROBLEMS

the annual payment will be $\frac{\$800}{(1.05)^{10}} \div \left[\left\{ 1 - \frac{1}{(1.05)^{10}} \right\} \times 20 \right]$,
or $\left\{ \frac{\$800}{20 \{ (1.05)^{10} - 1 \}} \right\}$, which is, \$63.60.

18. The perpetual annuity, beginning in 7 years, is worth now $\{ \$50 \div .06 \} \div (1.06)^7$, or \$554.214. The present value of the payments is $\left\{ 1 - \frac{1}{(1.06)^5} \right\} \times 16\frac{2}{3}$ times the payment. \therefore

the payment will be $\$554.214 \div \left[\left\{ 1 - \frac{1}{(1.06)^5} \right\} \times 16\frac{2}{3} \right]$, or \$131.57.

19. The present value of the six deposits will be $25 \times \left\{ 1 - \frac{1}{(1.04)^6} \right\}$ times the amount deposited each year. The

present value of the partial annuity will be $\$12500 \left\{ \frac{1}{(1.04)^8} - \frac{1}{(1.04)^{13}} \right\}$. \therefore the annual deposit = $\frac{\$12500}{25} \times \frac{\frac{1}{(1.04)^8} - \frac{1}{(1.04)^{13}}}{1 - \frac{1}{(1.04)^6}}$
= $\$500 \times \frac{(1.04)^5 - 1}{(1.04)^{13} - (1.04)^7} = \310.26 .

186

22. The value is the difference of value of a perpetuity of \$1600, beginning now, and of the same perpetuity when deferred 5 years. \therefore the value = $\frac{\$1600}{.04} \times \left\{ 1 - \frac{1}{(1.04)^5} \right\} = \7122.92 .

23. A perpetuity of \$200, the first payment to be made at the end of 1 year, is worth $\frac{\$200}{.03}$. If deferred 1 year, its present value is $\frac{\$200}{.03} \times \frac{1}{1.03}$. If deferred 13 years, its present value is $\frac{\$200}{.03} \times \frac{1}{(1.03)^{13}}$. \therefore present value of the partial annuity is $\frac{\$200}{.03} \left\{ \frac{1}{1.03} - \frac{1}{(1.03)^{13}} \right\}$ or \$1932.82.

24. "An annuity of \$400, payable half-yearly," means a payment of \$200 each half-year. If the first payment were made in 6 months, the present value would be $\$200 \div .02$, or

\$10000. But since it is deferred six months, the present value is $\$10000 \div (1.03)$, or $\$9803.92$.

25. 5% per annum, payable half-yearly, equals 5.0625% per annum, payable yearly. The present value of \$5000 due in 10 years is $\$5000 \div (1.050625)^{10}$, or $\$3051.35$. The present value of the ten payments of interest is $\$.300 \left\{ 1 - \frac{1}{(1.050625)^{10}} \right\}$, or $\$2309.50$. \therefore the total present value is $\$5360.85$.

26. Consider first the value 3 months ago. The value of \$4000 due in 5 years and 6 months would be $\$4000 \div (1.025)^{11}$, or $\$3048.578$. Each payment of interest is $\$110$. The value of the 11 payments would be $\$.110 \left\{ 1 - \frac{1}{(1.025)^{11}} \right\}$, or $\$1046.562$. \therefore the total value, 3 months ago, was $\$4095.14$. Hence the value now is $\$4857.285 \times 1.0125$, or $\$4146.33$.

27. A perpetuity, equal to the yearly instalment, beginning now, would have present value 20 times the instalment. Such a perpetuity, deferred 20 years, would be worth $\frac{20}{(1.05)^{20}}$ times the instalment. \therefore the difference $20 \left\{ 1 - \frac{1}{(1.05)^{20}} \right\}$ times the instalment is equal to $\$12000$. \therefore the instalment

$$= \$600 \div \left\{ 1 - \frac{1}{(1.05)^{20}} \right\} = \$962.91.$$

28. The value is $\frac{\$1000}{.04} \left\{ 1 - \frac{1}{(1.04)^{15}} \right\}$, or $\$11118.39$.

29. The value is $\frac{\$600}{.05} \left\{ \frac{1}{(1.05)^4} - \frac{1}{(1.05)^{23}} \right\}$, or $\$5965.57$.

30. The value of a perpetuity of \$450, first payment at end of 1 year, is $\frac{\$450}{.045}$, or $\$10000$. The value of the perpetuity, first payment at end of 16 years, is $\frac{\$10000}{(1.045)^{16}}$, or $\$5167.20$.

187

31. The present value of \$2500, due in 4 years, is $\$2500 \div (1.03)^4$, or $\$1973.52$. The interest is \$100 for 6 months. The present value of the payments of interest is $\frac{\$100}{.03} \left\{ 1 - \frac{1}{(1.03)^8} \right\}$ or $\$701.97$. \therefore the present value of mortgage is $\$2675.49$.



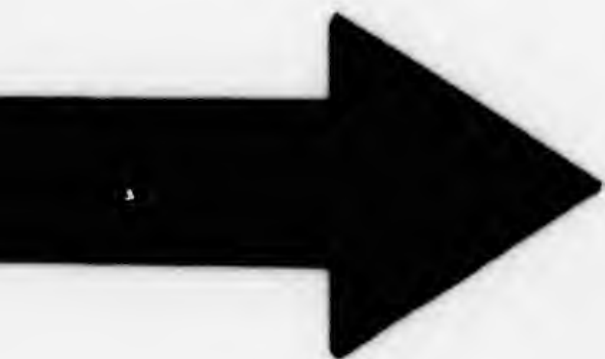
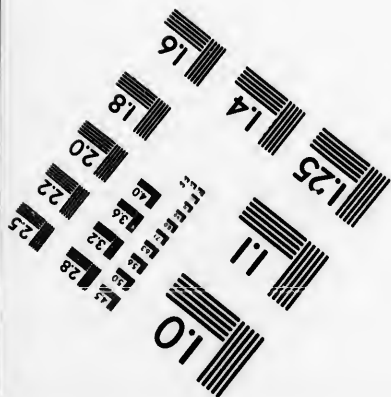
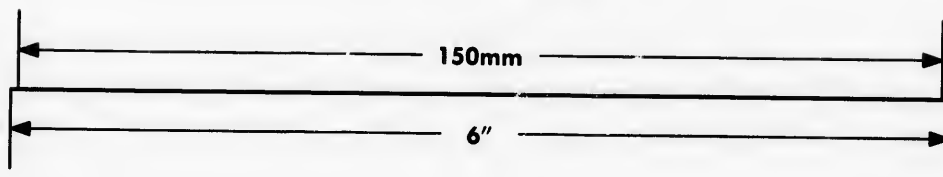
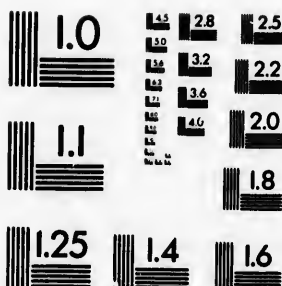
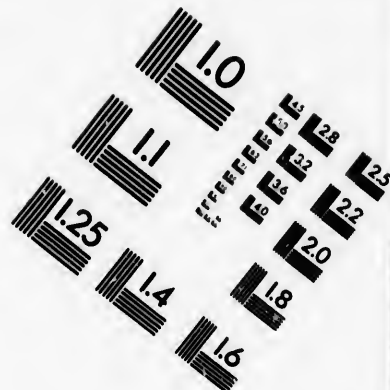
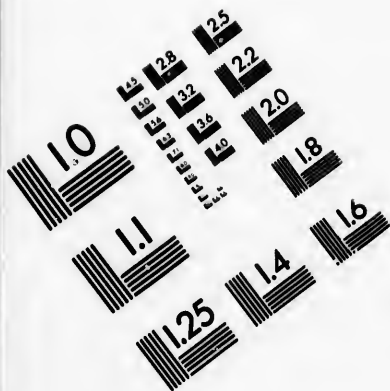
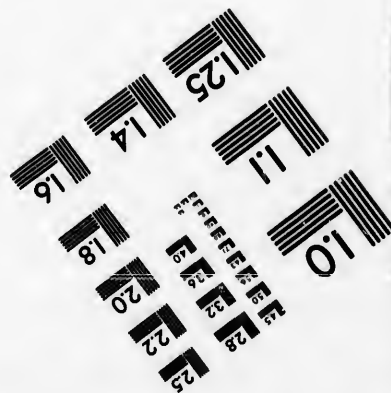


IMAGE EVALUATION TEST TARGET (MT-3)



APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc., All Rights Reserved



0
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
7.1
8.0
9.0
10.0
11.2
12.5
14.0
16.0
18.0
20.0
22.4
25.0
28.0
31.5
36.0
40.0
45.0
50.0
56.0
63.0
71.0
80.0
90.0
100.0

10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100

SOLUTIONS OF PROBLEMS

32. The amount evidently will be \$200 $\{ (1.02)^{10} + (1.02)^{20} + (1.02)^{30} + \dots + (1.02)^{100} \}$. Working to five decimal places, the result will be \$4178.91.

33. The interest is \$120 each half-year. If the rate were reduced to 5%, the interest would be \$75 each half-year. \therefore the sum paid must be equivalent to ten half-yearly payments of \$45 each. The present value of these payments is $\$4\frac{5}{6}$

$$\left\{ 1 - \frac{1}{(1.02)^{10}} \right\} \text{ or } \$404.21 +.$$

34. Each son receives the income every fourth year; that is, a perpetuity beginning now, in 1 year, in 2 years, in 3 years respectively. The present values of such perpetuities are proportional to $1, \frac{1}{1.04}, \frac{1}{(1.04)^2}, \frac{1}{(1.04)^3}$, or to $(1.04)^3, (1.04)^2, (1.04), 1$.

35. The value, one year before the payments begin, is $\$1\frac{50}{100} \left\{ 1 - \frac{1}{(1.06)^{15}} \right\}$ or \$1456.837. \therefore the value at the end of 15 years (16 years after the time above), is \$1456.837 $\times (1.06)^{15}$, or \$3700.88.

36. The value of the 15 payments, one year before the first payment is made, is $1 - \frac{1}{(1.05)^{15}}$ times the annual payment. \therefore the amount of the payments at the end of the time is $\left\{ 1 - \frac{1}{(1.05)^{15}} \right\} \times (1.05)^{16}$ times the annual payment; that is, $\frac{(1.05)^{16} - (1.05)}{.05}$ times the payment, or 22.65749 times the payment. This amount is \$5000. \therefore the annual payment is $\$5000 \div 22.65749$, or \$220.67.

37. We shall first find the annual payment, which will amount in 15 years to \$4000. From the previous solution it will be seen that the amount of the payments will be $(1.06)^{16} - (1.06)$ times the annual payment, or $\frac{(1.06)^{16} - (1.06)}{.06}$ times the annual payment. \therefore the annual payment = $\$4000 \div 24.67252 = \162.12 . \therefore he is paying each year \$240 - \$162.12, or \$77.88 more than enough to amount to \$4000.

38. First find the value six months ago. The present value of the mortgage and interest would have been $\frac{\$5000}{(1.05)^6} +$

\$300 $\left\{ \frac{1}{(1.05)^6} + \frac{1}{(1.05)^5} + \frac{1}{(1.05)^4} + \frac{1}{(1.05)^3} + \frac{1}{(1.05)^2} + \frac{1}{1.05} \right\}$
 or \$5253.784. This is the value six months ago. \therefore the
 value now is $\$5253.784 \times (1.025)$, or \$5385.12.

39. The present value of a \$100 Bond and its interest is
 $\frac{\$100}{(1.04)^6} + \$5 \left\{ \frac{1}{(1.04)^6} + \frac{1}{(1.04)^5} + \frac{1}{(1.04)^4} + \frac{1}{(1.04)^3} + \frac{1}{(1.04)^2} + \frac{1}{1.04} \right\}$, or \$79.03 + \$26.21, or \$105.24.

40. He makes the following payments at the end of the
 respective quarters: (1) \$25, (2) \$25, (3) \$25, (4) \$29.13,
 (5) \$28.75, (6) \$28.38, (7) \$28., (8) \$27.63, (9) \$27.25,
 (10) \$26.88, (11) \$26.50, (12) \$26.13, (13) \$25.75, (14)
 \$25.38. At the end of the first quarter, the value of the 1st,
 5th, 9th, and 13th payments is $\$25 + \frac{\$28.75}{1.05} + \frac{\$27.25}{(1.05)^2} + \frac{\$25.75}{(1.05)^3}$. This amounts to \$99.34. The P. W. of this is
 $\$99.34 \div (1.0125)$, or \$98.11. The value of the 2nd, 6th,
 10th and 14th payments, at end of second quarter, is $\$25 + \frac{\$28.38}{1.05} + \frac{\$26.88}{(1.05)^2} + \frac{\$25.38}{(1.05)^3}$. This is \$98.33. Its P. W.
 is $\$98.33 \div (1.025)$, or \$95.93. The value of the 3rd, 7th,
 and 11th payments, at end of third quarter, is $\$25 + \frac{\$28.}{1.05} + \frac{\$26.50}{(1.05)^2}$, or \$75.70. The P. W. of this is $\$75.70 \div (1.0375)$
 or \$72.96. The value of the 4th, 8th, and 12th payments, at
 end of fourth quarter, is $\$29.13 + \frac{\$27.63}{1.05} + \frac{\$26.13}{(1.05)^2}$, or
 \$79.14. The P. W. of this is $\$79.14 \div (1.05)$, or \$75.37. \therefore
 the total P. W. of the payments is \$342.37.

41. \$1 paid each year will at the end of ten years amount
 to $\$ \left\{ (1.06)^{10} + (1.06)^9 + \dots + 1.06 \right\}$, or \$13.9716.
 \therefore the yearly payment, which will in ten years amount to

SOLUTIONS OF PROBLEMS

\$1000, is $\$1000 \div 13.9716$, or $\$71.57$. \therefore he paid $\$105 - \71.57 , or $\$33.43$ more than enough to amount to $\$1000$.

42. The heirs receive $\$1000$ insurance, and $\$82.36 \times \{(1.04)^5 + (1.04)^4 + (1.04)^3 + (1.04)^2 + 1.04\}$, or $\$1463.93$ in all. Had he taken the endowment policy, the heirs would have received only $\$1000$. \therefore the heirs gain $\$463.93$.

43. If the man live the ten years his bank account will amount to $\$82.36 \times \{(1.04)^{10} + (1.04)^9 + \dots + 1.04\}$ or $\$1028.38$. \therefore he is $\$28.38$ better off than if he had taken the endowment policy.

44. The amount of $\$1$, deposited each year, at the end of 15 years, is $\$ \{(1.025)^{30} + (1.025)^{28} + \dots + (1.025)^2\}$ or $\$22.77791$. \therefore the semi-annual payment is $\$1000 \div 22.77791$, or $\$43.90$.

45. By calculating the interest, compounded each half year, and subtracting $\$7500$ from the amount at the end of *each year*, it will be found that the sinking fund will pay the debt, and leave a balance at the end of ten years.

46. The interest on $\$200000$ for 1 year at 5%, is $\$10000$. Hence the fund pays only the interest.

47. The P. W. of 20 instalments of equal amount, paid at the end of each year, is $25 \times \left\{ 1 - \frac{1}{(1.04)^{20}} \right\}$ times the instalment. Hence the instalment is equal to $\$250000 \div \left[25 \times \left\{ 1 - \frac{1}{(1.04)^{20}} \right\} \right]$ that is, equal to $\$18395.44$.

PARTNERSHIP.

2. On $\$10000$ capital the gain was $\$2800$. \therefore on $\$4000$ capital the gain was $\$1120$, and on $\$6000$ capital the gain was $\$1680$.

189

3. The whole gain was $\$1200$. $\$1200$ was the gain on $\$12000$ capital. \therefore $\$400$ was the gain on $\$4000$ capital.

4. After paying Sykes $\$1200$ for managing the business, there remained $\$2000$ gain on the whole capital. Of this remainder Smith received $\$1500$ and Sykes $\$500$. $\$2000$ was

IN THE HIGH SCHOOL ARITHMETIC.

the gain on \$8000 capital. \therefore \$500 was the gain on \$2000 capital. \therefore Sykes invested \$2000.

5. B invested \$4000 more than A, and his gain was \$1000 more than A's. \therefore \$1000 was the gain on \$4000 capital. \therefore \$5000 was the gain on \$20000 capital. \therefore D invested \$20000.

6. A gave to the business the use of \$4500 for two months, or the use of \$9000 for 1 month. B gave the use of \$4000 for 3 months, or the use of \$12000 for 1 month. For the use of \$21000 for 1 month the gain was \$2800. \therefore for the use of \$9000 for 1 month A should receive \$1200.

7. The use of \$1600 for 3 months = the use of \$4800 for 1 month. The use of \$1100 for 2 months = the use of \$2200 for 1 month. The use of \$3000 for $1\frac{1}{2}$ months = the use of \$4500 for 1 month. The gain will be divided in the proportion 48 : 22 : 45. \therefore A's share = $\frac{48}{115}$ of \$2400 = \$1001.74. B's share = $\frac{22}{115}$ of \$2400 = \$459.13. C's share = $\frac{45}{115}$ of \$2400 = \$939.13.

8. The manager received \$100 a month, and the book-keeper \$87.50. \therefore C should receive \$600 as manager and \$700 as book-keeper. The net gain, after paying for manager and book-keeper, was \$8640 - \$1200 - \$1050, or \$6390. This sum must be divided in proportion to use of capital. A gave the use of \$2400 for 6 months, or the use of \$14400 for 1 month. B gave the use of \$3000 for 4 months, or the use of \$12000 for 1 month. C gave the use of \$4000 for 12 months, or the use of \$48000 for 1 month. The use of \$74400 for 1 month gave gain \$6390. \therefore the use of \$48000 for 1 month gave gain \$4122 $\frac{1}{2}$. \therefore C should receive altogether \$600 + \$700 + \$4122 $\frac{1}{2}$, or \$5422 $\frac{1}{2}$.

9. A gave the use of \$4000 for 4 months, and the use of \$2000 for 8 months, \therefore in all the use of \$32000 for 1 month. B gave the use of \$4000 for 6 months, and the use of \$1000 for 6 months, \therefore in all the use of \$30000 for 1 month. C gave the use of \$4000 for 12 months, or the use of \$48000 for 1 month. The use of \$110000 for 1 month gave gain \$6000. \therefore the use of \$48000 for 1 month gave gain \$2618.18 +

10. On \$3500 capital the gain was \$1120. \therefore on \$2200 capital, for the same time in business, the gain would be \$1120 $\times \frac{2}{5}$, or \$604. But B's capital gained \$880. \therefore On \$2200 for 2 months the gain was \$176. \$176 was the gain on B's capital for 2 mos. \therefore \$880 was the gain on B's capital for 10

SOLUTIONS OF PROBLEMS

mos. \therefore B's capital was in trade for 10 months, and \therefore A's capital for 8 months. Since the gain on \$2200 for 2 months was \$176. \therefore the gain on \$100 for 1 month was \$4. \therefore the gain on \$2500 for 1 month was \$100. \therefore the gain on \$2500 for 12 months was \$1200. \therefore C's capital was in the business 12 months.

11. A gained \$600 in 2 months, \therefore \$300 in 1 month. B gained \$500 in $2\frac{1}{2}$ months, \therefore \$200 in 1 month. C gained \$800 in 4 months, \therefore \$200 in 1 month. \$300 was the monthly gain from \$3000 invested. \therefore \$200 was the monthly gain from \$2000 invested. \therefore B and C invested \$2000 each.

190

12. The capital for the first four months was \$27000. The capital for the fifth and sixth months was \$24000. The capital for the last six months was \$20000. Hardy received, as manager, \$800 for the first four months, $200 \times \frac{2}{3} \times 2$, or \$355 $\frac{5}{6}$, for the fifth and sixth months, and $200 \times \frac{2}{3} \times 6$, or \$888 $\frac{2}{3}$ for the last six months. In all, he received \$2044 $\frac{4}{9}$ for managing the business. There was left \$3955 $\frac{5}{6}$ net gain, to be divided in proportion to use of capital. Hardy gave the use of \$12000 for 4 months and \$9000 for 8 months, which equals the use of \$120000 for 1 month. Jones gave the use of \$15000 for 6 months and \$11000 for 6 months, which equals the use of \$156000 for 1 month. The use of \$276000 for 1 month yields \$3955 $\frac{5}{6}$. \therefore the use of \$120000 for 1 month yields \$1719 $\frac{167}{207}$. \therefore Hardy receives $2044\frac{4}{9} + 1719\frac{167}{207}$, or \$3764.25.

13. Lock gave the use of \$2500 for 10 months, or the use of \$25000 for 1 month. Smith gave the use of \$2300 for 11 months, or the use of \$25300 for 1 month. Knight gave that which was equal to the use of \$2000 for 12 months, or the use of \$24000 for 1 month. The use of \$74300 for 1 month yielded \$2972. \therefore the use of \$25000 for 1 month yielded \$1000, the use of \$25300 for 1 month yielded \$1012, and the use of \$24000 for 1 month yielded \$960.

14. B's workmen did 4000 days' work. C's workmen did 3600 days' work. \therefore they should receive shares in the ratio of 10 to 9. \therefore B's share was $\frac{10}{19}$ of \$12000, or \$6315.79, and C's share was $\frac{9}{19}$ of \$12000, or \$5684.21.

15. A owned $\frac{1}{4}$ of the value of the vessel. He lost $\frac{1}{4}$ of his share, or $\frac{1}{16}$ of the value of the vessel. This amounted to

IN THE HIGH SCHOOL ARITHMETIC.

\$1000. \therefore the vessel was worth \$16000. B lost $\frac{1}{4}$ of $\frac{1}{4}$ of the value of the vessel, or \$1333.33 $\frac{1}{3}$. C lost $\frac{1}{4}$ of $\frac{1}{1\frac{1}{2}}$ of the value of the vessel, or \$1666.66 $\frac{2}{3}$.

16. Resources.	Liabilities
Goods on hand, \$40000	Credits.....\$47000
Cash on hand.. 22000	
Debits..... 25000	

Total Resources, \$87000 \therefore Net Capital.....\$40000

The original capital was \$20000. There has been a gain equal to capital. \therefore the gains are: Smith \$8000, Jones \$9000, Cook \$3000.

17. The capital for the first year is \$45000; for the second year \$41700; for the third year \$38400; for the fourth year \$35100; for the fifth year \$31800. The gain for the period has been \$11100. A's capital was equivalent to \$34500 for 1 year. B's capital was equivalent to \$70200 for 1 year. C's capital was equivalent to \$105500 for 1 year. \therefore B's share of the gain was $\frac{702}{2102}$ of \$11100, or \$3707.04. B's capital at end of fifth year was \$10200. \therefore B's share was \$13907.04.

18. C's capital, \$1200, gained \$288. \therefore A's capital, \$1500, gained \$360 in the same time. \therefore \$1500 gained \$600 - \$360, or \$240, in 4 months. \therefore B's capital, \$1000 gained \$160 in 4 months. \therefore B's capital gained \$320 in 8 months.

191

19. The rent is \$40 a month. Terry paid the first three months' rent, \$120. Terry and Tucker paid the next four months' rent, that is, \$80 each. Terry, Tucker and Taylor paid the next four months' rent, that is, \$53 $\frac{1}{3}$ each. Terry and Taylor paid the last month's rent, that is, \$20 each. \therefore Terry paid \$120 + \$80 + \$53 $\frac{1}{3}$ + \$20, or \$273 $\frac{1}{3}$. Tucker paid \$80 + \$63 $\frac{1}{3}$, or \$133 $\frac{1}{3}$. Taylor paid \$53 $\frac{1}{3}$ + \$20, or \$73 $\frac{1}{3}$.

20. A had \$4000 in business for 183 days, which is equivalent to \$732000 for 1 day. B had \$3000 in the business for 105 days, which is equivalent to \$315000 for 1 day. The use of \$1047000 for 1 day yields \$2400. \therefore the use of \$315000 for 1 day yields \$722.06.

SOLUTIONS OF PROBLEMS

EXCHANGE.

1. He pays \$4000 and $\frac{1}{4}\%$ of \$4000, that is, \$4000 + \$10, or \$4010.
2. The draft cost \$2500 + $\frac{3}{8}\%$ of \$2500, that is, \$2500 + \$9.37 $\frac{1}{2}$, or \$2509.37 $\frac{1}{2}$.
3. The draft costs \$800 less $\frac{1}{2}\%$ of \$800, that is, \$800 less \$4, or \$796.
4. The draft costs \$12000 and $\frac{1}{4}\%$ of \$12000, or \$12090.
5. A bill of £1 costs \$4.80 $\frac{3}{4}$. \therefore a bill of £1200 costs \$4.80 $\frac{3}{4}$ \times 1200, or \$5764.50.
6. \$100 $\frac{1}{4}$ buys a draft of \$100. \therefore \$7500 buys a draft of \$7481.29.
7. \$4.80 buys a bill of £1. \therefore \$1350 buys a bill of £ $\frac{1350 \times 1}{4.80}$, or £281 5s.
8. A bill of 5.16 francs costs \$1. \therefore a bill of 1500 francs costs \$290.69 +.
9. \$4.86 $\frac{2}{3}$ - \$4.44 $\frac{4}{9}$ = \$.42 $\frac{2}{3}$. On \$4.44 $\frac{4}{9}$ the increase is \$.42 $\frac{2}{3}$. On \$100 the increase is 9 $\frac{1}{2}\%$. \therefore the increase is 9 $\frac{1}{2}\%$.
10. The cost of £1, in exchange, is 108 $\frac{1}{4}\%$ of \$4.44 $\frac{4}{9}$, or \$4.80 $\frac{3}{4}$. \therefore the cost of £3000, in exchange, is \$14433 $\frac{1}{4}$.

192

11. The cost of a bill of £1 is 109 $\frac{1}{2}\%$ of \$ $\frac{40}{9}$. \therefore the cost of a bill of £1500 is \$(1500 \times $\frac{873}{800}$ \times $\frac{40}{9}$), or \$7275.
12. $\frac{108}{100}$ of \$ $\frac{40}{9}$, or \$4.80, buys a bill of £1. \therefore \$2400 buys a bill of £500.
13. £1500 costs \$7300. \therefore £1 costs \$ $\frac{73}{15}$. \$ $\frac{40}{9}$ is 100% of old par of exchange. \therefore \$ $\frac{73}{15}$ is ($100 \times \frac{9}{40} \times \frac{73}{15}$)% of old par of exchange, that is, 109 $\frac{1}{2}\%$ of old par of exchange. \therefore exchange is quoted at 9 $\frac{1}{2}$.
14. The direct exchange will cost 100 $\frac{1}{4}\%$ of \$4000, or \$4010. In Chicago a bill of \$4000 on New York will cost 100 $\frac{3}{4}\%$ of \$4000, or \$4030. In Winnipeg a bill of \$4030 on Chicago will cost 99 $\frac{1}{2}\%$ of \$4030, or \$4009.85.
15. A bill for \$2700 cost \$2673: \therefore a bill for \$100 cost \$99. \therefore exchange is at 1% discount.
16. The amount added because exchange is at a premium of $\frac{1}{4}\%$, is $\frac{1}{4}\%$ of \$2750, or \$6.875. The discount, which is usually calculated on the face of the bill, is $\frac{73}{800}$ of $\frac{8}{100}$ of \$2750, or \$44. \therefore I received \$2750 + \$6.875 - \$44, or \$2712.875.

IN THE HIGH SCHOOL ARITHMETIC.

17. When exchange is at $9\frac{3}{4}$, £1 yields, in Canada, $109\frac{3}{8}\%$ of $\$4\frac{9}{16}$, or $\$1\frac{17}{32}$. \therefore 25.30 francs yields, in Canada, $\$1\frac{17}{32}$. \therefore 294000 francs yields, in Canada, $\$ \left\{ \frac{175}{36} \times \frac{100}{2530} \times 294000 \right\}$

On $\$ \left\{ \frac{175}{36} \times \frac{100}{2530} \times 294000 \right\}$ the interest is \$2920. \therefore on

\$100 the interest is $\$ \left\{ 2920 \times \frac{36}{175} \times \frac{2530}{294000} \right\}$ or \$5.16+.

\therefore rate is 5.16+ %.

18. £3 17s. $10\frac{1}{2}$ d. = $934\frac{1}{2}$ d. Gold valued at $934\frac{1}{2}$ d. weighs 1 oz. \therefore gold valued at (1869×240) d. weighs 1869×240

$\frac{934\frac{1}{2}}{1869 \times 240}$ oz., or 480 oz., or 40 lbs. Troy.

19. The eagle (\$10) contains $\frac{255}{480} \times \frac{9}{10}$ oz. pure gold. "22 carats fine" means $\frac{22}{24}$, or $\frac{11}{12}$, pure gold. But $\frac{11}{12}$ oz. pure gold is valued at $\frac{1869}{2}$ d. $\therefore \frac{255}{480} \times \frac{9}{10}$ oz. pure gold is valued at $(\frac{1869}{2} \times \frac{255}{480} \times \frac{9}{10} \times \frac{11}{12})$ d. $\therefore (\frac{1869}{2} \times \frac{255}{480} \times \frac{9}{10} \times \frac{11}{12})$ d = \$10. \therefore 240d, or £1, = \$4.866+. But \$4.866+ is very nearly $109\frac{1}{4}\%$ of \$4.444. \therefore "By the new par of exchange sterling money is worth $9\frac{1}{2}\%$ more than by the old par."

193

20. The agent's commission is 7% of \$7800, or \$546. The duty is $\$(\frac{1}{100}$ of 616×4.86), or \$359.25. \therefore the agent remits to the merchant \$6894.75, or £1418.673. \therefore the merchant's gain is £802.673.

21. By the circuitous exchange, $\$10000 = 10000 \times 5.40$ francs = $10000 \times 5.40 \times \frac{100}{185}$ marcs = $10000 \times 5.40 \times \frac{100}{185} \times \frac{17\frac{1}{2}}{17\frac{1}{2}}$ stivers = $10000 \times 5.40 \times \frac{100}{185} \times \frac{35}{2} \times \frac{1}{240}$ pounds = £2321 17s 4+d. By direct exchange £1 costs $\$(\frac{40}{100} \times \frac{110}{100})$, or $\$4\frac{4}{5}$. \therefore \$4.8 produces £1, \therefore \$10000 produces £2045 9s 1d. \therefore gain is £276 8s 3d.

22. The agent's commission was $2\frac{1}{2}\%$ of \$12500, or \$312.50. The net amount is \$12187.50. \$991 pays for a draft for \$100, \therefore \$12187.50 pays for a draft for \$12279.60.

23. 11520 marcs banco = $11520 \times 2.12\frac{1}{2}$ francs = $11520 \times 2.12\frac{1}{2} \times \frac{1}{5.18}$ dollars = \$4744.186.

24. 1800 francs = $\frac{1800}{5.20}$ pounds = $\frac{1800}{5.20} \times \frac{108}{100} \times \frac{40}{9} = \342.857 .

25. 1 rouble = 38.177 pence = $38.177 \times \frac{25.2215}{240}$ francs = 4.012 francs.

26. \$1 = $\frac{1}{4.866666}$ pounds = $\frac{12.1071}{4.866666}$ florins = 2.48 + florins.

MENSURATION.

195

1. The ladder, in its new position, forms with the wall and ground a right-angled \triangle . Ladder 30 ft., wall to the top of ladder 24 ft. \therefore dist. of the bottom of ladder from the foot of the wall = $\sqrt{30^2 - 24^2}$ ft. = 18 ft.

2. See 1.

3. Perp. bisects chord. The \perp , $\frac{1}{2}$ chord and radius form a right-angled \triangle . $\therefore \frac{1}{2}$ chord = $\sqrt{26^2 - 10^2}$ in = 24 in. \therefore chord = 48 in.

4. Diameter of \odot = diagonal of square. The dia. of sq. = $\sqrt{8^2 + 8^2}$ ft. = $8\sqrt{2}$ ft. \therefore area of \odot = $\frac{\pi}{4} (4\sqrt{2})^2$ sq. ft. = $100\frac{4}{7}$ sq. ft.

5. The line joining pt. to centre, the tangent and the radius to the point of contact of tangent and circle, form a right-angled \triangle . \therefore tangent = $\sqrt{7^2 - 4^2}$ ft. = 5.744 ft.

6. Apply Euc. 1. 47.

7. Let A = jet., AB \perp to ground, CD = height man. BD = ground and DE = shadow, then ECA is a st. line. By similar \triangle 's $\frac{BE}{AB} = \frac{DE}{CD}$ $\therefore BE = \frac{11.7}{5\frac{2}{3}}$ ft. = $13\frac{1}{3}$ ft. $\therefore DE = (13\frac{1}{3} - 7)$ ft. = 6.2 ft.

8. The diagonal of the end of the stick equals the diameter of the end of the tree. Diameter = $(12 \div \frac{22}{7})$ ft. = $\frac{42}{11}$ ft. Now side of stick : the diagonal : : 1 : $\sqrt{2}$ \therefore side = $(\frac{42}{11} \div \sqrt{2})$ ft. = 2.699 ft.

9. Surface = $\left\{ \left(\frac{18+25}{2} \div 12 \right) \times 16 \right\}$ sq. ft. = 28.6 sq. ft.

10. Cu. ft. = 1728 cu. in. 3 ac. = $(3 \times 4840 \times 9 \times 144)$ sq. in. \therefore thickness = $\left\{ 1728 \div (3 \times 4840 \times 9 \times 144) \right\}$ in. = .0000918 in.

196

11. Length of string = $\sqrt{24^2 + 18^2 + 7^2}$ ft. = 30.805 ft.

IN THE HIGH SCHOOL ARITHMETIC.

12. Dimensions inside the road are 77 yds. and 48 yds. respectively. \therefore area road = $(85 \times 56 - 77 \times 48)$ sq. yds. = 1064 sq. yds. \therefore cost = $\$(1064 \times .25) = \266 .

13. Perimeter of semi-circle = semi-circumference + diameter, circumference = $2.2\frac{1}{2} \times 2$ ft. \therefore semi-circumference = $6\frac{1}{2}$ ft. \therefore &c.

14. $(2^2r + 2r)$ in. = 80 in. \therefore &c.

15. $(\text{Side})^2 = (2\frac{1}{2} \times 4840 \times 9)$ sq. ft. \therefore &c.

16. Altitude bisects base. The alt., $\frac{1}{2}$ base and side form a right-angled Δ . \therefore &c.

17. See 15.

18. Apply Euc. 1. 47.

19. See 8.

20. $\frac{1}{3}$ ac. = $\frac{4840}{3}$ sq. yds. Length of ground = 60 yds. \therefore

width = $(\frac{4840}{3} \div 60)$ yds. See 12.

21. $(25000 \div 360)$ miles = $69\frac{1}{4}$ miles.

22. (15×9) sq. ft. $\div (27 \times 18)$ sq. in. = &c.

23. The diagonal divides the quadrilateral into 2 triangles, whose sides are 20, 30, 40 and 25, 32, 40 chains, respectively, &c.

24. See 1.

197

25. When the complete figure is drawn, there will be two equal right-angled triangles. \therefore the width of the street is $(24 + 18)$ ft.; and the ladder = $\sqrt{24^2 + 18^2}$ ft. = 30 ft.

26. Apply formula.

27. No. of revolutions = 1 mile $\div (2\frac{2}{3} \times 30)$ in. = &c.

28. Inner radius = $(420 \div \frac{4}{7})$ ft. and the outer radius = $(560 \div \frac{4}{7})$ ft. \therefore breadth of road = $(560 - 420) \div \frac{4}{7}$ ft. = 22.27 ft.

29. $(\frac{4}{7}r - 2r)$ ft. = 12 ft. where r = radius, &c.

30. Apply formula.

31. See 9.

32. Radius of circular base = $4.7 \div \frac{4}{7}$ miles. \therefore length of slant side = $\sqrt{(\frac{329}{440})^2 + (1\frac{1}{3})^2}$ miles. \therefore area = $\frac{1}{2} \sqrt{(\frac{329}{440})^2 + (1\frac{1}{3})^2} \times 4.7 \times 640$ acres = 2005.3 ac.

33. Length of side = $\sqrt{150}$ yds. \therefore dimensions of new

SOLUTIONS OF PROBLEMS

space are ($\sqrt{150}$ yds. + 4 ft. 3 in.) and ($\sqrt{150}$ yds. - 3 ft. 4 in.) &c.

34. 1000 times the circumference of the wheel equals one mile. \therefore the circumference = 5.28 ft. \therefore dia. = $(5.28 \div \frac{22}{7})$ ft. = 1.68 ft.

35. See 22.

36. The diagonals of a rhombus bisect each other at right angles, &c.

37. Radius of pond = $(220 + \frac{44}{7})$ yds = 35 yds. Area of pond = $(\frac{22}{7} \times 35^2)$ sq. yds. = 3850 sq. yds. \therefore area of outer circle, including both path and pond is $(3850 + 120)$ sq. yds. = 3970 sq. yds. \therefore radius of this circle = $(\sqrt{3970 \div \frac{22}{7}})$ yds = 35.541 yds. \therefore width of road = $(35.541 - 35)$ yds. = .541 yds.

38. Length of pole : 29 ft. 8 in. = 5 ft. 10 in : 7 ft. 5 in., etc.

39. Apply the following rule :

If h be the height of the frustum of a cone, R the radius of the bottom and r the radius of the top, the volume is $\frac{1}{3} \pi h (R^2 + Rr + r^2)$; or if A and a be the areas of the bottom and top respectively the volume is $\frac{1}{3} h (A + \sqrt{Aa} + a)$. Proof: Complete the cone. Let h' denote the height of the part required to complete it. Then $h' + h$ = the height of the completed cone. By similar triangles $\frac{h + h'}{h'} = \frac{R}{r}$

$\therefore h' = \frac{h r}{R - r}$ and $h + h' = \frac{h R}{R - r}$. Now the volume of the frustum = the volume of the completed cone - the volume of the cone required to complete it. \therefore the vol. of the frustum = $\frac{1}{3} (h + h') \pi R^2 - \frac{1}{3} h' \pi r^2 = \frac{1}{3} \pi h \left(\frac{R^3 - r^3}{R - r} \right) = \frac{1}{3} \pi h (R^2 + Rr + r^2) = \frac{1}{3} h (A + \sqrt{Aa} + a)$.

198

40. The length contains 3 ft. as many times as the width contains 2 ft. Area of a rectangle 3 ft. by 2 ft. = 6 sq. ft. \therefore 240 sq. ft. contains $(240 \div 6)$ of these rectangles = 40. \therefore the length = $(\sqrt{40} \times 3)$ ft. = 18.973 ft.

41. In 88 days it goes $37000000 \times \frac{44}{7}$ miles, &c.

42. See 1.

43. See 7.

44. See 9.

45. Circumference of circular field = $(\frac{1}{2} \times 15)$ rods = $47\frac{1}{2}$ rods, and perimeter of square field = (4×14) rods = 56 rods. \therefore square field by $8\frac{1}{2}$ rods.

46. Radius of base of circular cistern = $(20 \div \frac{4}{7})$ ft. = $35\frac{1}{2}$ ft.

Volume of water = $\left\{ 7 \times 2\frac{1}{2} \times (\frac{35}{11})^2 \right\}$ cu. ft. = $24\frac{5}{11}$ cu. ft.

The side of the square base = $(20 \div 4)$ ft. \therefore the area of the base = 25 sq. ft. \therefore the depth of water = $24\frac{5}{11}$ cu. ft. \div 25 sq. ft. = $8\frac{1}{11}$ ft.

47. See 3. Let x = perp. on chord 12 units in length, then $14 - x$ = length of other perp. $\therefore x^2 + 6^2 = \text{rad.}^2 = (14 - x)^2 + 8^2$. $\therefore x = 8$. \therefore rad. = $\sqrt{8^2 + 6^2} = 10$.

48. Field is to contain $(\frac{3}{4} \times 10)$ sq. chains. \therefore length of the field = $\left\{ (\frac{3}{4} \times 10) \div 2\frac{1}{2} \right\}$ chains = 3 chains.

49. Area of quad. = $\left\{ \frac{1}{2} \cdot 40.12\frac{1}{2} + \frac{1}{2} \cdot 40.9 \frac{3}{8} \right\}$ sq. ft. = etc.

50. Side of square = $\sqrt{14}$ in. \therefore dia. = $\sqrt{14}$. $\sqrt{2}$ in. = $\sqrt{28}$ in. = 5.291 in.

199.

51. Side of sq. = $\sqrt{80}$ in. \therefore length = $(\sqrt{80} \div 8)$ in. = 1.118 in.

52. Apply Euc. I. 47.

53. Area field = $(40 \times 5\frac{1}{2} \times 3 \times 30 \times 3)$ sq. ft. \therefore side of sq. = $\sqrt{40 \times 5\frac{1}{2} \times 3 \times 30 \times 3}$ ft. = 243.721 ft.

54. See 11.

55. See 39.

56. Ground passed over = $(3 \times 3\frac{5}{8} \times 5\frac{3}{8})$ sq. ft. = $64\frac{1}{2}$ sq. ft.

57. 3 (side of cube) $^2 = 1$ sq. in. \therefore side = $\sqrt{\frac{1}{3}}$ in. = $.577$ in.

58. Cost = $\left\{ (21\frac{1}{2} \times 13\frac{1}{2}) \times 6 \right\}$ d. = etc.

59. See 15, 40.

60. Rad. outer circle = $\frac{7}{4} \times 110$ yds. = $17\frac{1}{2}$ yds., and rad. inner circle = $\frac{7}{4} \times 88$ yds. = 14 yds. \therefore area outer circle = $\frac{7}{4} \times (17\frac{1}{2})^2$ sq. yds., and area of inner circle = $\frac{7}{4} \times 14^2$ sq. yds. \therefore area path = $\frac{7}{4} \left\{ (17\frac{1}{2})^2 - (14)^2 \right\}$ sq. yds. = $(\frac{7}{4} \times 31\frac{1}{2} \times 3\frac{1}{2})$ sq. yds. = 346.5 sq. yds.

61. 1 gal. = 10 lbs., and $62\frac{1}{2}$ lbs. = 1 cu. ft. \therefore no. gals. = $(8 \times 10 \times 9 \times 62\frac{1}{2} \div 10) = 4500$.

SOLUTIONS OF PROBLEMS

62. Rad. of circ. field = $\sqrt{\frac{7}{22} \times 2 \times 4840 \times 9}$ ft. \therefore circumference = $\frac{44}{7} (\sqrt{\frac{7}{22} \times 2 \times 4840 \times 9})$ ft. = 1046.529 ft. Side of sq. field = $\sqrt{2 \times 4840 \times 9}$ ft. = 295.1609 ft. \therefore peri. of field = 4×295.1609 ft. = 1180.643 ft. \therefore diff. = 134.114 ft.

63. See 22.

64. Cistern contains $(\frac{22}{7} \times 4^2 \times 4)$ cubic feet of water. See 61.

200

65. Number of cu. ft. emptied per hour = $(\frac{1}{4} \times 2000 \div 62\frac{1}{2})$.
Area of base = $\left\{ \frac{22}{7} \times (\frac{7}{2})^2 \right\}$ sq. ft. \therefore water must rise per hour $\left[(\frac{1}{4} \times 2000 \div 62\frac{1}{2}) \div \left\{ \frac{22}{7} \times (\frac{7}{2})^2 \right\} \right]$ ft. = $\frac{16}{77}$ ft.

66. Surface of earth = $4\pi \cdot 4000^2$ sq. miles, and surface of globe = $4\pi \cdot 6^2$ sq. inches. \therefore 1 sq. inch on globe represents $4000^2 \div 6^2$ sq. miles = 444444.4 sq. miles.

67. Rad. of end = $(\frac{7}{4} \times 22)$ ft. = $3\frac{1}{2}$ ft. \therefore area of end = $\left\{ \frac{22}{7} \times (3\frac{1}{2})^2 \right\}$ sq. ft. \therefore no. of cu. ft. in stick = $\left\{ \frac{22}{7} \times (3\frac{1}{2})^2 \times 40 \right\}$ cu. ft. \therefore no. cords = $\frac{22}{7} \times (3\frac{1}{2})^2 \times 40 \div 128 = 12\frac{1}{8}$.

68. Apply formula.

69. Let r = rad. of pond. $\therefore \frac{22}{7} r^2 = 2\frac{1}{2} \times 4840$ sq. yds. $\therefore r = 5\sqrt{154}$ yds. Area of walk = $\frac{22}{7} \left\{ (r \times 2)^2 - r^2 \right\}$ sq. yds. = $\frac{22}{7} (4r + 4)$ sq. yds. = $\frac{22}{7} (5\sqrt{154} + 1)$ sq. yds. = 792.608 sq. yds.

70. Area of rim = $\frac{22}{7} (3^2 - 2^2)$ sq. in. = $\frac{22}{7} \cdot 5$ sq. in. Area of outer surface = $2 \cdot \frac{22}{7} \cdot 3^2$ sq. in. Area of inner surface = $2 \cdot \frac{22}{7} \cdot 2^2$ sq. in. = $\frac{22}{7} \cdot 8$ sq. in. \therefore whole surface = $\frac{22}{7} (5 + 18 + 8)$ sq. in. = $97\frac{3}{7}$ sq. in.

71. Theory.

72. Length of tree = $\left\{ 12 + \sqrt{12^2 + 34^2} \right\}$ ft. = 48.055 ft.

73. Distance around the pond = $(\frac{22}{7} \times 2)$ miles. \therefore time required to drive = $(\frac{44}{7} \div 10)$ hrs., and time required to row = $(4 \div 6)$ hrs. &c.

SOLUTIONS OF PROBLEMS

74. Let x = width and y the length of field in rods. $\therefore x + 4$ and $y + 4$ rods equal respectively the width and length to the outside of the road. $\therefore xy = 480$ sq. rods and $(x + 4)(y + 4) - xy = 516$ sq. rods. Solve for x and y .

75. Area field = 36^2 sq. rods. The area of large field = $(36^2 \times 3)$ sq. rods. \therefore the side = $\sqrt{36^2 \times 3}$ rods = $36\sqrt{3}$ rods, 62.353 rods.

76. Wt. of sphere of water = $(\frac{4}{3} \cdot \frac{22}{7} \cdot 3^3 \div 1728 \times 62\frac{1}{2})$ lbs. \therefore wt. of iron = $8 \times (\frac{4}{3} \cdot \frac{22}{7} \cdot 3^3 \div 1728 \times 62\frac{1}{2})$ lbs. = $32\frac{31}{4}$ lbs.

201

77. Vol. of cube = (5×2^3) cu. in. \therefore side = $\sqrt[3]{5 \times 2^3}$ in. = $2\sqrt[3]{5}$ in.

78. If side of cube be 1 in., when diminished it is $\frac{7}{8}$ in. \therefore new vol. = $(\frac{7}{8})^3$ cu. in. = $\frac{343}{512}$ cu. in. \therefore it is diminished by $\frac{169}{512}$ cu. in., &c.

79. If radii be 2 and 3 in. respectively, their vols. are $\frac{4}{3}\pi 2^3$ and $\frac{4}{3}\pi 3^3$ cu. inches respectively, or as $2^3 : 3^3$.

80. See 47.

81. If 60 ft. be represented by 5 in. \therefore 25 ft. will be represented by $2\frac{1}{2}$ in.

82. 1 sq. yd. = (9×144) sq. in. \therefore length = $(9 \times 144 \div 8)$ in. = 13 ft., 6 in.

83. Area = $2 \{ (9 \times 10) + (10 \times 7\frac{1}{2}) + (9 \times 7\frac{1}{2}) \}$ sq. ft. = &c.

84. $\frac{1}{8}$ alt. = $(4840 \div 90\frac{1}{8})$ yds. = &c.

85. See 12.

86. Draw the diagram.

87. Side of sq. = 10 in., and sides of rectangle 5 and 15 inches respectively, &c.

88. Outer dimensions 48", 60", 72", and inner dimensions 46", 58", 71".

Vol. of whole box = $(48 \times 60 \times 72)$ cu. in. = 207360 cu. in.

Vol. of interior of box = $(46 \times 58 \times 71)$ cu. in. = 189428 cu. in.

\therefore Vol. of iron = $(207360 - 189428)$ cu. in. = 17932 cu. in.

Wt. of water = $(189428 \div 1728 \times 62\frac{1}{2})$ lbs., and wt. of iron = $(17932 \div 1728 \times 7 \cdot 7 \times 62\frac{1}{2})$ lbs. \therefore whole wt. = $(189428 + 17932 \times 7 \cdot 7) \div 1728 \times 62\frac{1}{2}$ lbs. = 11845.5 lbs.

89. Area of field = $(132 \cdot 24 \div 12)$ ac. = 11.02 ac. \therefore side of

field = $\sqrt{11.02 \times 4840}$ yds. \therefore cost = $\$(4 \times \sqrt{11.02 \times 4840} \times .35) = \323.326 .

90. Page 81.

91. Let r be the rad. of inner boundary. $\therefore \frac{22}{7} (14)^2 - \frac{22}{7} r^2 = 462$ sq. in. $\therefore 14^2 - r^2 = 147$ sq. in. $\therefore r^2 = 49$ sq. in., &c.

202

92. See 12.

93. Dia. stick = $\frac{7}{8} \times 120$ in. = $38\frac{1}{4}$ in. If we deduct 10 in. for slabs and $\frac{1}{4}$ in. for cut, there will be left as many planks as cuts. \therefore every plank will require $2\frac{1}{4}$ in. \therefore no. planks equals the greatest whole no. in $(38\frac{1}{4} - 10\frac{1}{4}) \div 2\frac{1}{4} = 12$.

94. See 72.

95. Slant height = $\sqrt{4^2 + 9^2}$ ft. = $\sqrt{97}$ ft. and circumference of base = $\frac{4}{7} \times 4$ ft. = $1\frac{2}{7}$ ft. \therefore no. of sq. yds. = $(\frac{1}{2} \sqrt{97} \times 1\frac{2}{7} \div 9) = 13.75$.

96. See 39, 61.

97. No. of sq. ft. of wall to be papered = $2(18 + 12)11 = 660$ sq. ft. \therefore no. yds. paper = $(660 \div \frac{22}{7}) \div 3 = 97.7$.

98. See 87.

99. See 89, 87.

100. 12 ac. = 120 sq. ch. \therefore width = $(120 \div 24)$ ch. = 5 ch.

101. Area of circle = $\frac{22}{7} \times 8^2$ sq. ft., area of small circle = $\frac{22}{7} \times \frac{8}{5}^2$ sq. ft. \therefore rad. = $\sqrt{\frac{64}{5}}$ ft. = $\frac{8}{\sqrt{5}}$ ft. = $\frac{8}{5} \sqrt{5}$ ft. \therefore circumference

= $\frac{44}{5} \times \frac{8}{5} \sqrt{5}$ ft. = $\frac{352}{5} \sqrt{5}$ ft. = 22.488 ft.

102. $\frac{1}{2}$ base = $(3\frac{1}{2} \div 2\frac{1}{4})$ metres, etc.

103. Apply formula.

203

104. Let x = length of corner cut off. $\therefore x\sqrt{2}$ = side octagon

$\therefore 2x + x\sqrt{2} = 40$ yds. $\therefore x = \frac{40}{2 + \sqrt{2}}$ yds. = $20(2 - \sqrt{2})$ yds.

and $2x^2$ = area of corners cut off = $800(6 - 4\sqrt{2})$ sq. yds. = 1600 $(3 - 2\sqrt{2})$ sq. yds., but area of sq. = 1600 sq. yds. \therefore area of octagon = diff., etc.

105. 3 (side)² = 16 sq. ft. \therefore side = $\frac{4}{\sqrt{3}}$ ft. = $\frac{4}{3}\sqrt{3}$ ft. \therefore vol. = $(\frac{4}{3}\sqrt{3})^3$ cu. ft. = $\frac{64}{9}\sqrt{3}$ cu. ft.

106. Inner rad. = $\frac{7}{4}$. 1050 yds. \therefore outer rad. = $(\frac{7}{4} \cdot 1050 +$

SOLUTIONS OF PROBLEMS

$\frac{35}{8}$ yds. \therefore outer circumference = $\frac{1}{2} (\frac{7}{11} \cdot 1050 + \frac{35}{8})$ yds. = $(1050 + 73\frac{1}{2})$ yds. = $1123\frac{1}{2}$ yds.

107. No. of patterns in one strip = $(12 \div 1\frac{1}{2}) = 9\frac{3}{4}$, or, as paper must be matched, 10 patterns to a strip will be required, or $12\frac{1}{2}$ ft. of paper. No. of strips required = $2(20 + 12) \div \frac{3}{2} = 24$. \therefore No. yds. paper = $12\frac{1}{2} \times 24 \div 3 = 100$.

108. Side of field = $\sqrt{10 \times 4840}$ yds. = 220 yds. Length of wire = $(5 \times 4 \times 220)$ yds. = 4400 yds. Cost of wire = $\$(4400 \times .03) = \132 . No. of posts to a side = 84. \therefore no. of posts required = $4 \times 84 - 4 = 332$. Cost of posts = $\$(332 \times .08) = \26.56 . \therefore total cost = $\$(132 + 26.56) = \158.56

109. Circumference = $\frac{4}{7} \times 12$ in. \therefore length of arc = $\frac{7.5}{360} \times \frac{4}{7} \times 12$ in. = $15\frac{5}{7}$ in.

110. No. of cu. ft. of ice = $4 \times 4840 \times 9 \times \frac{1}{2}$. \therefore no. of cu. ft. of water in it = $4 \times 4840 \times 9 \times \frac{1}{4} \times \frac{10}{11}$. \therefore wt. = $(4 \times 4840 \times 9 \times \frac{1}{2} \times \frac{10}{11} \times 62\frac{1}{2} \div 2000)$ tons = 2475 tons.

111. Vol. of earth = $(40 \times 32 \times 8)$ cu. ft. \therefore wt. = $\{ (40 \times 32 \times 8) \times 2 \times 62\frac{1}{2} \div 2000 \}$ tons = 640 tons.

112. See 12.

113. Vol. = $(12 \times 1760 \times 7 \times \frac{5}{8})$ cu. yds. \therefore time = $\{ 12 \times 1760 \times 7 \times \frac{5}{8} \div (20 \times 400) \}$ days = $30\frac{1}{2}$ days.

114. No. of sq. ft. = $(90 \times 16 \times 8 \times 12 + 60 \times 12 \times 7 \times 14) \div 144 = 1450$, etc.

115. Rad. = $\frac{7}{4} \times 55$ in. = $\frac{385}{4}$ in. \therefore area of circle = $\frac{\pi}{4} (\frac{385}{4})^2$ sq. in. \therefore side of sq. = $\sqrt{\frac{22}{7} \cdot (\frac{385}{4})^2}$ in. = $\frac{385}{4} \sqrt{\frac{22}{7}}$ in. = $\frac{385}{4} \sqrt{3.142857}$ in. = 15.512 in.

204

116. Side of sq. = $\sqrt{1296}$ yds. = 36 yds. Sides of rectangle are as 1 to 2. \therefore Sides are 24 and 48 yds. respectively. \therefore area = (24×48) sq. yds. = 1152 sq. yds.

117. Area plate = $(\frac{22}{7} \times 12^2)$ sq. in. \therefore whole pressure = $(\frac{22}{7} \times 144 \times 15)$ lbs. = $6788\frac{4}{7}$ lbs.

118. Area of outer boundary = $\frac{\pi}{4} (56 + \frac{22}{7})^2$ sq. ft. and of the inner boundary = $\frac{\pi}{4} (56)^2$ sq. ft. \therefore no. of sq. ft. in base = $\frac{\pi}{4} (56 + \frac{22}{7})^2 - \frac{\pi}{4} (56)^2 = \frac{\pi}{4} \cdot \frac{22}{7} \cdot (112 + \frac{22}{7}) = \&c.$

119. Area of sq. = 1600 sq. in. \therefore rad. = $\sqrt{\frac{7}{22} \times 1600}$ in. = 22.563 in.

120. See 49.

121. Apply formula.

IN THE HIGH SCHOOL ARITHMETIC.

122. Let $r = \text{rad.}$ $4\pi r^2 = 616 \text{ sq. in.} \therefore r^2 = \frac{7}{8} \times 616 \text{ sq. in.}$
 $= 49 \text{ sq. in.} \therefore r = 7 \text{ in.}$ Vol. of sphere $= \frac{4}{3} \cdot \frac{22}{7} \cdot 7^3 \text{ cu. in.}$
 $= 1437\frac{1}{3} \text{ cu. in.}$

123. See 64.

124. See 116.

125. Area = area of sq. - area of 4 quadrants of circles =
 $(16 - \frac{22}{7} \cdot 4) \text{ sq. ft.} = 3\frac{3}{7} \text{ sq. ft.}$

126, 127, 128. See 148, 149. Draw figure.

205

129. Space = equilateral \triangle (side 6 ft.) - 3 equal sectors
 (angle 60°) $= (9\sqrt{3} - \frac{1}{2} \cdot \pi \cdot 9) \text{ sq. ft.}$

130. Hyp. = 13 in. Space $= (13^2 + 12^2 + 5^2 + \frac{1}{2} \cdot 5 \cdot 12) \text{ sq. in.}$
 Peri $= 3(13 + 12 + 5) \text{ in.}$

131. Circumference of \odot whose rad. is 40 ft.

132. Aisle + row of desks equal at least 44 in. Take one
 of the outside aisles off the width of the room. \therefore no. of rows
 $= (22 \times 12 \div 44) = 6.$

133. Room requires 9 strips each 18 ft. 8 in. long. Price
 $= 9 \times \frac{18\frac{2}{3}}{3} \times \$1.75 = \$98.$

134. Sector $= \frac{50}{360}$ of the area of \odot . $\therefore \pi r^2 = \frac{50}{360} \times 230 \text{ sq. ft.}$
 $\therefore r = 22.9545 \text{ ft. and peri.} = 2r + \frac{5}{36} \cdot 2\pi \cdot r.$

135. Slant side of cone $= \sqrt{1 + (\frac{3}{2})^2} \text{ ft.} = \frac{1}{2} \sqrt{13} \text{ ft.}$ Circumference of base $= \frac{22}{7} \cdot 2 \text{ ft.}$ Slant surface $= \frac{1}{2} \cdot \frac{22}{7} \cdot 2 \times \frac{1}{2} \sqrt{13} \text{ sq. ft.}$
 Base $= \frac{22}{7} \times 1 \text{ sq. ft.} \therefore$ whole surface = sum.

136. Whole area = sq. + 4 equal semicircles $= (36 + 4 \cdot \frac{1}{2} \cdot \frac{22}{7} \times 9) \text{ sq. ft.}$ Peri. $= 4$ semi-circumferences $= 4 \cdot \frac{1}{2} \cdot \frac{22}{7} \cdot 6.$

137. See 130.

138. Whole area = original \triangle + 3 equilateral \triangle 's.

139. Area of zone = areas of 2 sectors (angle 120°) and of 2 equilateral \triangle 's (side 8 ft.).

206

140. $r = \text{rad. inner } \odot, R = \text{rad. outer } \odot. 2\pi r = 360 \text{ yds.}$
 $\therefore r = \frac{630}{11} \text{ yds. similarly } R = \frac{735}{11} \text{ yds. Area outer } \odot = \frac{22}{7} \times (\frac{735}{11})^2 \text{ sq. yds., and area inner } \odot = \frac{22}{7} \times (\frac{630}{11})^2 \text{ sq. yds.} \therefore$ area of road $= \frac{22}{7} \{ (\frac{735}{11})^2 - (\frac{630}{11})^2 \} \text{ sq. yds.} = 123.00 \text{ sq. yds.}$

141. Similar vols. are as the cubes of their like dimensions. \therefore dia. of Saturn is $\sqrt[3]{1000}$ times dia. of earth.

IN THE HIGH SCHOOL ARITHMETIC.

142. Dia. of sphere = edge of cube, etc.

143. Solid contents = $\left\{ \frac{4}{3} \cdot \frac{22}{7} \cdot 2^3 - \frac{4}{3} \cdot \frac{22}{7} \cdot 1^3 \right\}$ cu. in. = $29\frac{1}{3}$ cu. in.

144. Vol. by first pipe : vol. by second pipe :: $3^2 : (4\frac{1}{2})^2$ \therefore
time = $2 \times \frac{3^2}{(4\frac{1}{2})^2}$ hrs. = $\frac{8}{9}$ hrs.

145. Side of court = $\sqrt{196}$ yds. = 14 yds. = 42 ft. Side of walk = $(42 + 4\frac{1}{2})$ ft. \therefore area of walk = $\left\{ (46\frac{1}{2})^2 - (42)^2 \right\} = (4\frac{1}{2} \times 88\frac{1}{2})$ ft. \therefore cost = $\$ \left\{ 4\frac{1}{2} \times 88\frac{1}{2} \times 20 \right\} = \79.65 .

146. See 139.

147. Area = 10000 plan \therefore linear measurements = $\sqrt{10000}$ of plan. = 100. \therefore 24 yds = $\frac{24}{100}$ yds. in plan = 8.64 in.

148. Space = sq. + $\frac{1}{4}$ circle (rad. 3 in.) + $\frac{1}{4}$ circle (rad. 6 in.) + $\frac{1}{4}$ circle (rad. 9 in.) + $\frac{1}{4}$ circle (rad. 12 in.) = etc. Peri. = $\frac{1}{4}$ circumference (rad. 3 in.) + etc.

149. Space = sq. + $\frac{1}{4}$ circle (rad. $1\frac{1}{2}$ in.) + $\frac{1}{4}$ circle (rad. $4\frac{1}{2}$ in.) + etc.

150. Sector = $\frac{115}{275}$ circle, and arc of sector = $\frac{115}{275}$ circumference \odot , but $\pi r^2 = 275$ sq. in. $\therefore r = 5\sqrt{\frac{7}{2}}$ in. \therefore sector = $2 \cdot \frac{22}{7} \cdot 5 \sqrt{\frac{7}{2}} \times \frac{115}{275}$ in. = 24.589 in.

151. 1 gal. = 10 lbs. \therefore 6 gals. = 60 lbs., and $62\frac{1}{2}$ lbs = 1 cu. ft. = 1728 cu. in. \therefore 6 gals. = $\frac{60}{62\frac{1}{2}} \times 1728$ cu. in. Area base = $\frac{22}{7} \cdot 6^2$ sq. in. \therefore depth = $\left\{ \left(\frac{60}{62\frac{1}{2}} \times 1728 \right) \div \left(\frac{22}{7} \cdot 6^2 \right) \right\}$ in. = 14.661 in.

152. Side $\Delta = 12$ in. \therefore area = $6 \times 6 \sqrt{3}$ sq. in. In \odot $2\pi r = 36$ in. $\therefore r = \frac{1}{2} \cdot \frac{7}{22} \cdot 36$ in. = $\frac{63}{11}$ in. \therefore area = $\frac{22}{7} \cdot \left(\frac{63}{11} \right)^2$ sq. in. Diff. in area = $\left\{ \frac{22}{7} \cdot \left(\frac{63}{11} \right)^2 - 36 \sqrt{3} \right\}$ sq. in. = 40.737 sq. in.

207

153. Inner rad. = r , outer rad. = R . Now $2\pi r = 796$ yds. $\therefore r = \frac{1}{2} \cdot \frac{7}{22} \cdot 796$ yds. = $\frac{1393}{11}$ yds. $\therefore R = \left(\frac{1393}{11} + 13 \right)$ yds. = $\frac{15336}{11}$ yds. \therefore area moat = $\frac{22}{7} \left\{ \left(\frac{15336}{11} \right)^2 - \left(\frac{1393}{11} \right)^2 \right\}$ sq. yds. = 108794 sq. in.

SOLUTIONS OF PROBLEMS

154. Degree = $(\frac{2\pi}{7} \times 7913 \div 360)$ miles = 69.08 miles.

155. The length of the degree depends on the length of the parallel of latitude at 60° north latitude. The dia. of this small circle is $\frac{7913}{2}$ miles. \therefore length of degree of longitude = $(\frac{2\pi}{7} \times \frac{7913}{2} \div 360)$ miles = 34.54 miles.

156. See 155.

157. See 151.

158. External dimensions are 36 in., 24 in., 18 in. Internal dimensions are 34 in., 22 in., 17 in. No. of cu. in. of material = $\{36 \times 24 \times 18 - 34 \times 22 \times 17\}$ cu. in. = 2836 cu. in.

159. Area of end = $\left\{ \frac{15+25}{2} \times 20 \right\}$ sq. ft. = $\frac{400}{9}$ sq. yds.
Contents wall = $(\frac{400}{9} \times 1500 \times 1760)$ cu. yds. = etc.

160. No. cu. ft. = $\left\{ \frac{2\pi}{7} \cdot (\frac{3}{2})^2 \times 30 \right\} = 212\frac{1}{2}$.

161. Space = area $\triangle + \frac{1}{3}$ circle (rad. 5 in.), etc. See 148.

162. See 149, 161.

163. Sector = $\frac{1}{2}$ arc \times 15 sq. ft. = 90 sq. ft. \therefore arc = 12 ft.

164. Whole area = area of 2 ends + area of 3 sides.

165. r ft. = rad. front wheel $\therefore (r + \frac{1}{2})$ ft. = rad. hind wheel.
Circumferences are $2 \cdot \frac{2\pi}{7} r$ ft. and $2 \cdot \frac{2\pi}{7} (r + \frac{1}{2})$ ft. respectively.
No. of revolutions made by wheels in going a mile are

$$\frac{5280}{2 \cdot \frac{2\pi}{7} r} \text{ and } \frac{5280}{2 \cdot \frac{2\pi}{7} (r + \frac{1}{2})} \text{ respectively. } \therefore \frac{5280}{2 \cdot \frac{2\pi}{7} r} - \frac{5280}{2 \cdot \frac{2\pi}{7} (r + \frac{1}{2})} = 50 \text{ etc.}$$

208

166. Let h = height of part cut off. r = rad. of its base, and R = rad. of base of whole cone. $\therefore \frac{h}{14} = \frac{r}{R}$ (sim. \triangle 's); and

$$\frac{1}{3} \pi r^3 \cdot h = \frac{1}{3} \cdot \pi R^2 \cdot 14 \therefore \frac{r^2}{R^2} = \frac{7}{h} = \frac{h^2}{14^2} \therefore h = 7 \sqrt[3]{4}.$$

167. h ft. = height cylinder, \therefore vol. = $\frac{2\pi}{7} (\frac{h}{2})^2 \cdot h$ cu. ft. See 151.

168. Area of semi-circular section = $(\frac{1}{2} \cdot \frac{2\pi}{7} \cdot 10^2)$ sq. ft.
No. of cu. yds. = $(\frac{1}{2} \cdot \frac{2\pi}{7} \cdot 10^2 \div 9 \times 80) = 1396.825$.

169. Vol. of cube = 3^3 cu. in. = 27 cu. in., and vol. of one piece of money = $\left\{ \frac{2\pi}{7} \cdot (\frac{3}{8})^2 \cdot \frac{1}{8} \right\}$ cu. in. \therefore no. of coins = $27 \div \left\{ \frac{2\pi}{7} (\frac{3}{8})^2 \cdot \frac{1}{8} \right\} = 488\frac{8}{11}$. Ans. 488.

IN THE HIGH SCHOOL ARITHMETIC.

170. Inner rad. = $\frac{3}{4}$ in., and outer rad. = $1\frac{1}{4}$ in. Area of end
 = $\frac{\pi}{7} \left\{ (1\frac{1}{4})^2 - (\frac{3}{4})^2 \right\}$ sq. in. = $\frac{\pi}{7}$ sq. in. No. cu. ft. = $(\frac{\pi}{7} \div$
 $144 \times 20)$; \therefore weight = $(\frac{2^2}{7} \times \frac{20}{144} \times \frac{11500}{18})$ lbs. \therefore cost = $\$ \left\{ \frac{2^2}{7} \times \right.$
 $\left. \frac{20}{144} \times \frac{11500}{18} \times .08 \right\}$ = \$25.099.

171. Side of \triangle : altitude = 2: $\sqrt{3}$ \therefore side = $\frac{28}{\sqrt{3}}$ ft. \therefore area
 = $\frac{1}{2} \times \frac{28}{\sqrt{3}} \times 14$ sq. ft. = etc.

172. See 25.

173. Dist. trav. in 1 rev. = $2 \cdot \frac{\pi}{7} \cdot \frac{1}{2}$ ft. = 22 ft., and in $1\frac{1}{2}$ rev.
 = 33 ft. Train goes in 1 sec. 33 ft. \therefore in 1 hr. it goes $(\frac{33}{3600}$
 $\times 60 \times 60)$ miles = $22\frac{1}{2}$ miles.

174. Dia. of \odot = 4 in. \therefore no. in. of wire = $(2 \cdot \frac{2^2}{7} \cdot 2 + 4 \cdot 4)$
 = 28.

175. In 1 hr. it trav. $2 \cdot \frac{2^2}{7} \cdot 11$ ft. \therefore in 1 day it trav. $(2 \cdot$
 $\frac{2^2}{7} \cdot 11 \div 3 \times 24)$ yds. = $553\frac{1}{3}$ yds.

176. See 151.

177. Vol. sov. = $\left\{ \frac{2^2}{7} \cdot (\frac{7}{18})^3 \cdot \frac{1}{18} \right\}$ cu. in. \therefore vol. 80000
 sovs. = $80000 \times \frac{2^2}{7} \cdot (\frac{7}{18})^3 \cdot \frac{1}{18}$ cu. in. \therefore edge cube =
 $\sqrt[3]{80000 \times \frac{2^2}{7} \times (\frac{7}{18})^3 \cdot \frac{1}{18}}$ in. = etc.

209

178. Height cone = $\sqrt{12^2 - 8^2}$ ft. = $\sqrt{80}$ ft. Vol. = $\frac{1}{3} \cdot \frac{2^2}{7} \cdot 8^2$
 $\sqrt{80}$ cu. ft. = &c.

179. Apply formula.

180. Length of cylinder = $(42 - 2 \times 2\frac{1}{2})$ in. = 37 in. Vol. of
 cylinder = $\frac{2^2}{7} (\frac{5}{2})^2 \cdot 37$ cu. in. Vol. of ends = $\frac{4}{3} \cdot \frac{2^2}{7} (\frac{5}{2})^3$ cu. in.
 \therefore &c.

181. Vol. of sphere = $\frac{4}{3} \cdot \frac{2^2}{7} (\frac{9}{2})^3$ cu. in. \therefore wt. of gunpowder
 = $\left\{ \frac{4}{3} \cdot \frac{2^2}{7} (\frac{9}{2})^3 \div 30 \right\}$ lbs. = 12.728 lbs.

182. Vol. of water replaced by stone = $\frac{2^2}{7} \cdot 14^2 \cdot 4$ cu. in. \therefore
 wt. of stone = $(8 \cdot \frac{2^2}{7} \cdot 14^2 \cdot 4 \div 1728 \times 1000)$ oz. = &c.

183. Circumference of wheel = $\frac{2^2}{7} \times 14$ ft. = 44 ft. Distance
 traversed in 1 hr. = $(44 \times 50 \times 60 \div 5280)$ miles = 25 miles.

184. Distance travelled in 1 second = $(35 \times 5280 \div 3600)$ ft.

SOLUTIONS OF PROBLEMS

\therefore diameter of wheel = $\{ (35 \times 5280 \div 3600) \div 4 \times \frac{7}{22} \}$ ft. = 49 in.

185. See. 39.

186. Wt. of water = $(480 - 31)$ lbs. = 449 lbs. \therefore no. gals. = 44.9.

187. Alt. of one of faces = $\sqrt{(34\frac{2}{3})^2 + (17\frac{1}{2})^2}$ ft. = $23\frac{3}{8}$ ft.
Whole area = $(2.35.23\frac{3}{8} + 35^2)$ sq. ft. = $3943\frac{1}{2}$ sq. ft.

188. Let. r = rad. of sphere. $\therefore 4.2\frac{2}{7}.r^2$ = surface of cylinder = $\{ \frac{4}{7} \cdot 4 \cdot 12 + 2 \cdot 2\frac{2}{7} \cdot 4^2 \}$ sq. in. = $\frac{4}{7} \cdot 64$ sq. in. $\therefore r = 4\sqrt{2}$ in. \therefore vol. sphere = $\frac{4}{3} \cdot 2\frac{2}{7} (4\sqrt{2})^3$ cu. in. = 758.556 cu. in.

210.

189. See 188.

190. Draw the figure. The area = 2 sectors ($\angle 150^\circ$ and rad. 100 ft.) + 2 sectors ($\angle 30$ and rad. 60 ft.) + 2 sectors ($\angle 90^\circ$ and rad. 20 ft.) + equilateral triangle (side 40 ft.) = $\{ \frac{4}{7}(100^2 \cdot \frac{5}{12} + 60^2 \cdot \frac{1}{12} + 20^2 \cdot \frac{1}{4}) + 400\sqrt{3} \}$ sq. ft. = 29397.58 sq. ft.

191. Vol. of sun = $\frac{4}{3} \cdot 2\frac{2}{7} \cdot 441500^3$ cu. miles; and vol. of earth = $\frac{4}{3} \cdot 2\frac{2}{7} \cdot 3956^3$ cu. miles. \therefore vol. of sun : vol. of earth :: $441500^3 : 3956^3$, etc.

192. See 186.

193. See 191.

194. No. of cu. in. of lead = $(208 \times 160 \times \frac{1}{16}) = 2080$. \therefore cost = $\$(2080 \times 6.5 \div 16 \times .07) = \59.15 .

195. External vol. = $(40 \times 30 \times 20)$ cu. in., and the internal vol. = $(37 \times 27 \times 17)$ cu. in. \therefore plank contains $\{ (40 \times 30 \times 20) - (37 \times 27 \times 17) \}$ cu. in. = 7017 cu. in. But a sq. ft. of plank contains $(12 \times 12 \times 1\frac{1}{2})$ cu. in. = 216 cu. in. \therefore no. sq. ft. = $(7017 \div 216) = 32\frac{3}{4}$.

196. Apply formula.

197. See 72.

198. See 194.

211

199. Vol. of water in 5 min. = $(2\frac{1}{2} \times 5280 \times \frac{1}{2} \times 20\frac{1}{2} \times 12\frac{1}{2})$ cu. ft. \therefore no. of gals. = etc. Ans. = $1738229\frac{1}{8}$ gals.

IN THE HIGH SCHOOL ARITHMETIC.

200. Area of face of stone which can be used = $\frac{2^2}{7}$
 $\left\{ \left(\frac{6.5}{2} \right)^2 - \left(\frac{5}{2} \right)^2 \right\}$ sq. in. = $\frac{2^2}{7} \cdot 35 \cdot 30$ sq. ft. Each man uses
 $\frac{1}{2} \cdot \frac{2^2}{7} \cdot 35 \cdot 30$ sq. ft. = $\frac{2^2}{7} \cdot 175$ sq. ft. Let r = rad. of stone
 when first man has used his share. \therefore his share = $\frac{2^2}{7}$
 $\left\{ \left(\frac{6.5}{2} \right)^2 - r^2 \right\} = \frac{2^2}{7} \cdot 175$ sq. ft. $\therefore r = \sqrt{81.25}$ ft. = etc.

201. Let 25 ft. pole be AB, 31 ft. pole CD, and the middle
 pole NM. BMD on the ground. NC = 20 ft. NA = 26 ft.,
 and MD = 12 ft. Through N draw ENF horizontal, and
 cutting AB at E and CD at F. NFC is a right-angled tri-
 angle, and NF = MD. \therefore CF = 16 ft. \therefore DF = 15 ft. = MN =
 BE. \therefore AE = 10 ft. AEN is a right-angled triangle. \therefore
 EN = 24 ft. = BM. \therefore BD = (24 + 12) ft. = 36 ft. Through
 A draw AG horizontal, and cutting CD at G. AG = BD =
 36 ft. CG = CD - AB = (31 - 25) ft. = 6 ft. \therefore AC = $\sqrt{36^2 + 6^2}$
 ft. = 36.496 ft.

202. $6(\text{edge})^2 = 2$ sq. ft. \therefore edge = $\sqrt{48}$ in. = etc.

203. See 186.

204. 1 cu. ft. of iron weighs $(62\frac{1}{2} \times 7.7)$ lbs. \therefore 64 lb.
 shot is equal to $\left\{ 64 \div (62\frac{1}{2} \times 7.7) \right\}$ cu. ft. Let r = the rad.
 of ball. $\therefore \frac{4}{3} \cdot \frac{2^2}{7} \cdot r^3 = \left\{ 64 \div (62\frac{1}{2} \times 7.7) \right\}$ cu. ft. $\therefore r = \&c.$

205. Page 81.

206. Vol. of pulp = $(4 \times 3 \times \frac{3}{4})$ cu. ft. = 9 cu. ft.; vol. of
 paper = $(2700 \times 3 \times 2\frac{1}{2} \times .004 \div 12)$ cu. ft. = $6\frac{3}{4}$ cu. ft. \therefore
 $2\frac{1}{4}$ cu. ft. lost in drying, &c.

GENERAL PROBLEMS.

212

1. 9 of A's days = 10 of B's. \therefore 10 of A's days = $\frac{100}{9}$ of B's.
 \therefore the money must be divided in the ratio of $\frac{100}{9} : 11$ or as 100 : 99. \therefore A receives $\frac{100}{199}$ of \$59.70 or \$30.
2. Amount of policy = $\frac{1}{5}$ of \$4000 = \$2400. Amount of premium = $\frac{2}{100}$ of \$2400 = \$48. \therefore owners lose (\$4000 - \$2400) + \$48 or \$1648.
3. Page 167 (40).
4. Page 188 (2).
5. 3 lbs. of tea at 40 cts. per lb. = \$1.20
 5 " " 48 " " = \$2.40 } = \$3.60.
- \therefore 8 lbs. of tea cost \$3.60. \therefore 8 lbs. of tea will sell for $\frac{100}{80}$ of \$3.60. \therefore 1 lb. of tea will sell for $\frac{1}{8}$ of $\frac{100}{80}$ of \$3.60 or 48 $\frac{3}{4}$ c.
6. \$31 is the income from \$84 invested. \therefore \$7 is the income from $\frac{\$4 \times 7}{3}$ invested or \$196.
7. Page 149 (3).
8. 4000 yds. of carpet at 4 $\frac{1}{2}$ s. per yd. = $\pounds \frac{4000 \times 4\frac{1}{2}}{20} =$
 $\pounds \left(\frac{4000 \times 4\frac{1}{2}}{20} \right) \times 4.87 = \$4383.$
9. Page 199 (58).
10. If \$12 be deducted from \$201, the balance may be divided between A and B in the ratio of 1:2. \therefore A gets $\frac{1}{3}$ of \$(201 - 12) + \$12 = \$75.
11. The first man goes 5 times around, while the second goes 4 $\frac{1}{2}$ times. \therefore the first gains $\frac{3}{4}$ of a round in 5. \therefore the first gains 1 round in $\frac{4}{3}$ of 5 or 6 $\frac{2}{3}$. As he must gain one round before they are together, they will be first together $\frac{2}{3}$ of the way around; or, as the track is circular, $\frac{1}{3}$ of the way, measuring the distance in the opposite direction.
12. 12 $\frac{1}{2}\%$ = $\frac{1}{8}$. \therefore $\frac{9}{8}$ of cost = 78 cts. \therefore cost = $\frac{8}{9}$ of 78 cts. = 69 $\frac{1}{3}$ cts.
13. The dial is divided into 60 spaces. The hour hand goes 1 space in 12 min. The min. hand goes 12 spaces in 12 min. Place an extra hand on the clock pointing to figure 4. If this hand travels at one half the rate of the hour hand it will al-

SOLUTIONS OF PROBLEMS

ways bisect the space between the figure 4 and the hour hand. The rate of the extra hand is $\frac{1}{2}$ space in 12 min. The minute hand must gain 20 spaces before it coincides with the extra hand, but it gains $11\frac{1}{2}$ spaces in 12 min. \therefore it will gain 20 spaces in $\frac{12}{11\frac{1}{2}}$ of 20 min. or $20\frac{2}{3}$ min. When these two hands coincide the minute is doing what is required of it in the example.

14. The work requires 36 days' labor. One man works $\frac{36-5}{2}$ days or $15\frac{1}{2}$ days. Now 36 days' labor cost \$126. \therefore $15\frac{1}{2}$ days' labor cost $\$126 \times 15\frac{1}{2} \div 36$ or \$54.25.

213

15. In selling 60 cent tea for $57\frac{2}{5}$ cents I lose $2\frac{3}{5}$ cents on each lb. \therefore on 34 lbs. I lose $(2\frac{3}{5} \times 34)$ cents. In selling 54 cent tea for $57\frac{2}{5}$ cents I gain $3\frac{2}{5}$ cents on each lb. \therefore to make up the whole loss on the 60 cent tea I must sell $\left\{ (2\frac{3}{5} \times 34) \div 3\frac{2}{5} \right\}$ lbs. of 54c. tea or 26 lbs.

16. Page 188 (2).

17. \$85 invested gives \$4 $\frac{1}{2}$ income. \therefore \$5100 invested gives $\frac{4\frac{1}{2} \times 5100}{85}$ income or \$270.

18. Page 181 (7).

19. " 147 (35).

20. " 212 (13).

21. " 199 (58).

22. " 212 (10).

23. $\frac{3}{4} \div \frac{1}{2}$ of $\frac{5}{8} = \frac{3}{4} \times \frac{2}{1} \times \frac{5}{8} = \frac{9}{8}$, and $\frac{3}{4} \div \frac{1}{2} \times \frac{5}{8} = \frac{3}{4} \times \frac{2}{1} \times \frac{5}{8} = \frac{9}{8}$.
Diff. $= (\frac{9}{8} - \frac{5}{8}) = \frac{1}{2}$.

24. A & B can do $\frac{1}{8}$ of the work in 1 day. B & C can do $\frac{1}{10}$ of the work in 1 day. \therefore 2 (A & B & C) can do $(\frac{1}{8} + \frac{1}{10} + \frac{1}{10})$ of the work in one day. \therefore A & B & C can do $\frac{3}{10}$ of the work in one day. \therefore A & B & C can do the whole work in $\frac{3}{10} \div \frac{3}{10} = 1$ day or $6\frac{1}{3}$ days.

25. Page 146 (12).

26. Page 165 (14).

$$27. .714285 = \frac{714285}{999999} = \frac{142857 \times 5}{142857 \times 7} = \frac{5}{7}$$

28. For every dollar A has, B has \$2 and C \$3. \therefore A & B together get \$2700. \therefore A gets $\frac{1}{4}$ of \$2700 or \$900.

29. Out of every \$100 worth of wheat sold, the agent keeps \$4 and remits \$96. \therefore when \$96 is remitted the agent gets \$4 com. \therefore when \$1872 is remitted the agent gets $\$(\frac{4}{96} \times 1872)$ or \$78.

30. Page 161 (17).

214.

1. Page 212 (5).

2. With carpet $\frac{3}{4}$ yd. wide and a room 11 ft. wide, it will require $(\frac{11}{\frac{3}{4}} \div \frac{3}{4})$ strips or $4\frac{8}{9}$, i.e., 5 strips. With a pattern every 8 ft. in a 20 ft. room it will require 24 ft. of carpet in a strip. \therefore no. of sq. yds. carpet required = $\frac{24}{3} \times \frac{3}{4} \times 5 = 30$; and no. sq. yds. in room = $(20 \times 11 \div 9) = 24\frac{4}{9}$. \therefore no. sq. yds. wasted = $(30 - 24\frac{4}{9}) = 5\frac{5}{9}$.

3. Page 212 (14).

4. See page 212 (13). Rate of extra hand is 6 spaces in 12 min. Place this extra hand midway between the min. hand and the fig. 3. When the hour hand and this extra hand coincide or lie in the same straight line the hour hand will bisect the space between the min. hand and the fig. 3. The extra hand is $7\frac{1}{2}$ spaces behind the hour hand and gains 5 spaces on it in 12 min. It gains 5 spaces in 12 min. \therefore it will gain $7\frac{1}{2}$ spaces in $\frac{12 \times 7\frac{1}{2}}{5}$ min. or 18 min.

5. Total expenses = $\$(164 + 206) = \370 . A's expenses = $\frac{2}{3}$ of \$370 = \$148. But he pays \$164. \therefore B must pay A $\$(164 - 148) = \16 .

6. Let \$100 = cost, $10\% = \frac{1}{10}$. \therefore $\frac{3}{4}$ selling price = $\frac{9}{10}$ of \$100 = \$90. \therefore selling price = \$135. \therefore gain = \$35 or 35% of cost.

7. Page 165 (21).

8. Page 161 (7).

9. Page 182 (8).

10. Page 181 (12).

11. Page 154 (15).

12. Suppose A has \$12. \therefore B will have $\frac{4}{3}$ of \$12 or \$16, and C will have $\frac{5}{4}$ of \$12 + \$16 or \$35. All will have $\$(12 + 16 + 35)$ or \$63. \therefore A should get $\frac{1}{3}$ of \$31.50 or \$6.

13. Page 203 (108).

14. Page 157 (11).

215

15. The 3rd brother gets $(1 - \frac{2}{5} - \frac{1}{3})$ of the farm = $\frac{4}{15}$ of the farm. Now $\frac{4}{15}$ of the farm costs \$1884. $\therefore \frac{2}{5}$ will cost $\$(1884 \div \frac{4}{15} \times \frac{2}{5})$ or \$2826.

16. $\frac{304}{1000} \times \frac{2}{1000} \times \frac{18}{10} \times \frac{10000}{1000} \times \frac{1000}{38} = 32$.

17. A can do $\frac{1}{6}$ of the work in 1 day. B can do $\frac{1}{8}$ of the work in 1 day. C can do $\frac{1}{7}$ of the work in 1 day. \therefore the money should be divided in the proportion of $\frac{1}{6}, \frac{1}{8}, \frac{1}{7}$, or 42, 35, 30. \therefore A gets $\frac{42}{107}$ of \$21.40 or \$8.40

18. Page 167 (40).

19. Page 161 (6).

20. Minute hand gains 11 spaces in 12 min. \therefore min. hand will gain 25 spaces in $(12 \times 25 \div 11)$ min. = $27\frac{3}{11}$ min.

21. If house be worth \$500, it is insured for $\frac{3}{5}$ of \$500 = \$300. Premium = $\frac{3}{100}$ of \$300 = \$9. Loss = $\$(500 - 300) + \$9 = \$209$. The loss is \$209 on a house worth \$500. \therefore the loss is \$522.50 on a house worth $\$ \frac{500 \times 522.50}{209}$ or \$1250.

22. Cost of keep = $\$1.75 \times 11 \times 17 = \327.25 . \therefore total cost = $\$327.25 + \$253 = \$580.25$. Selling price = $\$48 \times 11 = \528 . \therefore loss = $\$(580.25 - 528) = \52.25 .

23. Page 199 (58).

24. Page 212 (5).

25. Page 116 (178).

26. $\frac{12}{100}$ of 1st sum = $\frac{15}{100}$ of 2nd sum = $\frac{16}{100}$ of 3rd sum. If 3rd sum = \$3000. \therefore 1st sum = $\$(\frac{16}{100} \text{ of } 3000 \div \frac{12}{100}) = \4000 ; and 2nd sum = \$3200. \therefore total sum = $\$(4000 + 3200 + 3000) = \10200 . But the total sum is \$612. \therefore 1st sum = $\frac{4000}{10200}$ of \$612 = \$240.

27. Page 188 (2).

28. Loss on one bushel = $(87\frac{1}{2} - 77)$ cents = $10\frac{1}{2}$ cents. \therefore on $87\frac{1}{2}$ cents he loses $10\frac{1}{2}$ cents. \therefore on \$1.00 he loses $\frac{10\frac{1}{2}}{87\frac{1}{2}} \times 100$ cents = 12 cents.

216

1. A and B can do $\frac{1}{4}$ of work in 1 day, A can do $\frac{1}{8}$ of work in 1 day. \therefore B can do $(\frac{1}{8} - \frac{1}{8})$ or $\frac{3}{40}$ of work in 1 day. A and B do $\frac{3}{40}$ of work in 3 days. B can do $\frac{2}{5}$ of work in $\frac{2}{3} \times \frac{40}{3}$ days or $5\frac{1}{3}$ days.

2. Page 142 (1).

3. Page 146 (17).

IN THE HIGH SCHOOL ARITHMETIC.

4. Page 189 (6).

5. If \$72 is invested, income is \$3. If \$3370 is invested, income is $\frac{3}{72}$ of \$3370 = \$140.41 $\frac{2}{3}$.

6. B's cost = 106% of \$2500 = \$2650. C's cost = 95% of B's cost = 95% of \$2650 = \$2517 $\frac{1}{2}$.

7. $(\$100 \times \frac{4}{100} \times 3) + \100 or \$112 is amount of \$100 in 3 yrs. at 4%. \therefore \$336 is amount of \$300 in 3 yrs. at 4%. Int. on \$300 for 4 yrs. at 4% = \$48. \therefore amount = \$348.

8. Page 196 (12).

9. 1 day A's work = $\frac{2}{3}$ of day B's work } \therefore 6 of A's = 4 of B's.
1 day C's work = $\frac{5}{4}$ of day B's work } \therefore 8 of C's = 10 of B's.
7 of B's = 7 of B's.

Divide \$42 in the proportion of 4, 7 and 10. A gets $\frac{4}{21}$ of \$42 or \$8. B gets $\frac{7}{21}$ of \$42 or \$14. C gets \$20.

10. A's share = \$4.20 + 14% of B's share. A's share + B's share = \$369. \therefore 114% of B's share + \$4.20 = \$369. B's share = $\frac{100}{114}$ of \$364.80 = \$320. \therefore A's share = \$49.

11. S.P. = 92% of cost. 92% of cost + \$1.05 = 107% of cost. \therefore 15% of cost = \$1.05. \therefore 92% of cost or S.P. = $\frac{92}{105}$ of \$1.05 = \$6.44.

12. A's share = 110 acres. B's share = $\frac{2}{3}$ of farm. C's share = $1\frac{2}{3}$ (110 acres + $\frac{2}{3}$ of farm). 110 ac. + $\frac{2}{3}$ of farm + $1\frac{2}{3}$ (110 ac. + $\frac{2}{3}$ of farm) = farm. $293\frac{1}{3}$ ac. + $\frac{1}{3}$ of farm = farm. \therefore $\frac{1}{3}$ of farm = $293\frac{1}{3}$ ac. \therefore farm = 720 acres.

217

13. 10 of A's steps = 11 of B's. \therefore 8 of A's steps = $8\frac{4}{5}$ of B's. A takes 8 steps while B takes 9. \therefore B goes 9 yds. while A goes $8\frac{4}{5}$ yds. B goes 100 yds. while A goes $97\frac{1}{5}$ yds. B wins by $2\frac{4}{5}$ yds.

14. \$100 in $4\frac{1}{2}$ yrs. at 6% amounts to \$127. \therefore true disc. off \$127 is \$27. \therefore true disc. off \$508 is $\frac{27}{127}$ of \$508 or \$108. Bank disc. off \$508 due in $4\frac{1}{2}$ yrs. at 6% = \$508 $\times \frac{6}{100} \times \frac{9}{2} =$ \$137.16. Difference = \$137.16 - \$108 = \$29.16

15. Page 203 (107).

16. Gain = 14% of \$600. \therefore cost = 86% of \$600 = \$516. S.P. = 114% of \$516 = \$588.24.

17. Page 179 (38).

18. \$100 is present value of \$103 $\frac{3}{4}$ due in 9 mos. at 5% \therefore present value of \$7470 due in 9 mos. = $\frac{100}{103\frac{3}{4}}$ of \$7470 = \$7200. Amount of stock sold at 72 to produce \$7200 is \$10000.

SOLUTIONS OF PROBLEMS

19. If he had sold 135 bbls. at \$4 per bbl., he would have realized \$540. He makes \$1 more on each bbl. he sells for \$5. \therefore he makes \$(615 - 540) more on each 75 bbls. he sells for \$5.

20. Sells $35\frac{3}{4}$ in. as 36 in., or $\$35\frac{3}{4}$ worth as \$36. Sells $\$35\frac{3}{4}$ worth for 120% of \$36 or \$43.20. \therefore gain = \$(43.20 - 35.75) = \$7.45. \therefore \$7.45 is gain on \$35.75 cost \therefore \$134.10 is gain on \$643.50 cost.

21. Page 144 (29).

22. On an investment of \$400 he realizes \$32. \therefore he realizes 8%.

23. If he had 3 \$10 bills; 4 \$5 bills; 6 \$4 he would have \$74. But he has \$518. $518 \div 74 = 7$. \therefore he has 21 tens, 28 fives and 42 four dollar bills.

24. $\frac{5}{4}$ of distance B travels in 1 hr. = 7 mls. \therefore distance B travels in 1 hr. = $\frac{4}{5}$ of 7 mls. = $5\frac{3}{5}$ mls.

25. $\frac{105}{100}$ of amount of stock = \$4200. \therefore amount of stock = $\frac{100}{105}$ of \$4200 or \$4000. He had 40 shares.

218

1. Page 142 (48).

2. Page 205 (133).

3. Page 198 (40).

4. Page 212 (1).

5. Page 212 (1).

6. Let \$1 = cost per lb. Sold 5 lbs. for 112% of \$5 or \$5.60. Afterward, sold $4\frac{1}{2}$ lbs. for \$5.60, or sold 1 lb. for \$1.24 $\frac{2}{3}$. \therefore gain 24 $\frac{2}{3}$ %.

7. Selling price = 130% of cost of $\frac{3}{4}$ of goods. \therefore selling price = ($\frac{3}{4}$ of 130)% or 97 $\frac{1}{2}$ % of cost of goods. Loss of 2 $\frac{1}{2}$ %.

8. Page 215 (16).

9. Amt. paid for material = 3 times amt. paid for labor. 110% of 3 times amount paid for labor + 94% of amt. paid for labor = \$3637.92. \therefore 424% of amt. paid for labor = \$3637.92. \therefore 400% of amt. paid for labor = \$34.32.

10. Page 196 (15).

11. \$87 invested in the 4 per cents gives an income of \$4. \$87 invested in the 5 per cents gives an income of $\frac{87}{100}$ of \$5, or \$4 $\frac{9}{10}$. $\$4\frac{9}{10}$ is difference when \$87 is invested. \$27 is difference when $\frac{27 \times 34}{100}$ of \$87 or \$8874 is invested.

12. Hour hand goes 1 space in 12', (1). Minute hand goes 12 spaces in 12', (2). Second hand goes 720 spaces in 12', (3).

IN THE HIGH SCHOOL ARITHMETIC.

Place at 7 o'clock an extra hand bisecting the space between hr. and sec. hands and start it with the rate which is average of (1) and (2) or, $(\frac{720+1}{2})$ spaces or $360\frac{1}{2}$ spaces in 12'. It will always bisect the space between those hands. \therefore when minute hand is exactly opposite it the min. hand will also bisect the space. Extra hand is $17\frac{1}{2}$ spaces ahead of min. hand and when it gains $(30 - 17\frac{1}{2})$ spaces or $12\frac{1}{2}$ spaces it will be opposite min. hand. Extra hand gains on min. hand $(360\frac{1}{2} - 12)$ spaces in 12'. \therefore extra hand gains on min. hand $12\frac{1}{2}$ spaces in $\frac{360}{397}$. $\frac{360}{397}$ past seven.

219

13. Page 189 (6).

14. Policy = $\frac{3}{5}$ of value + 2% of policy. \therefore 98% of policy = $\frac{3}{5}$ of \$4900 = \$2940. \therefore policy = $\frac{100}{98}$ of \$2940 = \$3000.

15. Page 171 (19).

16. Area = 40 ac. or 6400 sq. rods. \therefore length of side = $\sqrt{6400}$ rods = 80 rods. Length of diagonal = $80\sqrt{2}$ rods.

17. Page 161 (17).

18. Cost = 125% of invoice price. Selling price = 90% of cost = \$2925. \therefore $\frac{90}{100}$ of 125% of invoice price = \$2925. \therefore Invoice price = \$2600.

19. Let \$400 be cost of goods bought. He wants to gain 6% on whole. \therefore S.P. of \$400 worth = $\frac{106}{100}$ of \$400 or \$424. But he sells \$100 worth for $\frac{94}{100}$ of \$100 or \$94. \therefore he sells \$300 worth for \$424 - \$94 or \$330. \therefore he sells \$100 worth for \$110. What formerly sold for \$94 now sells for \$110. What formerly sold for \$100 now sells for $\frac{110}{94}$ of \$100 or \$117.17. \therefore increase is $17\frac{1}{47}\%$.

20. Assets = $\frac{1}{2}$ of liabilities. $\frac{1}{3}$ of assets are found to be worth 50 cts. on dollar. \therefore he realizes on $\frac{5}{6}$ of assets. \therefore $\frac{5}{6}$ of assets = $\frac{5}{6}$ of $\frac{1}{2}$ of liabilities or $\frac{5}{12}$ of liabilities. \therefore he can only pay $\frac{1}{2}$ of liabilities or $41\frac{2}{3}$ cts. on dollar.

21. 8 gals. of wine @ \$1.12 $\frac{1}{2}$ = \$9.00.

12 " " " 1.25 = 15.00.

14 " " " 1.50 = 21.00.

16 " " water

50 gals. of mixture cost \$45.00.

50 " " " sold for 50.00.

\therefore Gain

= \$5.

SOLUTIONS OF PROBLEMS

On \$45 gain is \$5. On \$100 gain is $\frac{5}{100}$ of \$100 = \$1. \therefore gain = $11\frac{1}{10}\%$.

22. For 12 mos. he receives \$40 + suit. But for 9 mos. he receives \$25 + suit. \therefore 3 mos.' work is worth \$15. \therefore 1 mo. work is worth \$5. \therefore 9 mos.' work is worth \$45 = \$25 + suit. \therefore suit is worth \$20.

23. 98% of income taxed is \$(865.50 - 400) or \$465.50. \therefore 100% of income taxed is $\frac{100}{98}$ of \$465.50 = \$475. \therefore total income is \$475 + \$400 or \$875.

24. Page 215 (20).

220

1. He deducts \$25 and invests \$4300 in stock. For investing \$4300 broker receives \$25. For investing \$86 broker receives $\frac{86}{4300}$ of \$25 = $\frac{1}{2}$. \therefore broker charges $\frac{1}{2}\%$.

2. Page 196 (15).

3. S.P. of 27¢ yds = 115% of cost of 276 yds.

(1) \therefore S.P. of 276 yds. = $276 \times 115\%$ or 31740% of cost of 1 yard. S.P. of 398 yds. = 107% of cost of 398 yds.

(2) \therefore S.P. of 398 yds. = $398 \times 107\%$ or 42586% of cost of 1 yard. \therefore S.P. of 674 yds. = 74326% of cost of 1 yard, and S.P. of 674 yds. = 111% of cost of 674 yds. \therefore S.P. of 674 yds. = $674 \times 111\%$ or 74814% of cost of 1 yd. Difference 488% of cost of 1 yard = \$4.88. \therefore cost of 674 yds. = \$674.

4. Selling price = 120% of cost. 2nd cost = 1st cost + \$40. \therefore selling price = 95% of 2nd cost = 95% (1st cost + \$40). \therefore 120% of cost = 95% of cost + 95% of \$40. \therefore 25% of cost = $\frac{95}{100}$ of \$40. \therefore cost = $\frac{100}{25}$ of $\frac{95}{100}$ of \$40 = \$152.

5. If discount is 40%, reduced price = 60% of M.P.; 1st discount = 20% of M.P., 1st reduced price = 80% of M.P.; 2nd discount = 10% of 1st reduced price, 2nd reduced price = 72% of M.P.; 3rd discount = 10% of 2nd reduced price, 3rd reduced price = 64.8% of M.P. Difference is $64\frac{4}{5}\% - 60\% = 4\frac{4}{5}\%$.

6. From \$75 invested income is \$3. From \$7950 invested income is $\frac{100}{75}$ of \$3 = \$318. From \$106 invested income is \$4. From \$7950 invested income is $\frac{100}{106}$ of \$4 = \$300. Amount saved = \$318 - \$300 = \$18.

7. 14 lbs. sugar @ $6\frac{1}{2}$ cts. = \$.91
 7 lbs. tea " 35 " = 2.45
 3 brooms " 25 " = .75

IN THE HIGH SCHOOL ARITHMETIC.

20 yds. cotton @	8cts. =	1.60
4 pkgs. pins "	5 " =	.20
3 spools "	3 " =	.09

Amount of bill.....\$6.00

There is a discount of 5% for cash. $\therefore \frac{1}{20}$ of amount paid = \$1.90. \therefore amount paid = $\frac{19}{20}$ of \$1.90 = \$2.00

Cash = \$2.00

5 doz. eggs @ 12cts. = .60

2 lbs. butter @ 20cts. = .40

Amt. paid in all..... = \$3.00

Balance booked..... = 3.00

8. Discount off \$130 is \$10. \therefore interest on \$120 for a certain time is \$10. Interest on \$1 for twice time is $(\frac{1}{120} \times 2) = \frac{1}{60}$. \therefore off \$1 $\frac{1}{60}$ true discount for twice time is $\frac{1}{60}$. \therefore off \$130 true discount for twice time is $(\frac{1}{60} \times \frac{6}{7} \times 130) = \$18.57\frac{1}{2}$.

9. Page 143 (15).

10. A runs 12 yds. when B runs 10 yds. A runs 150 yds. when B runs $\frac{12}{10}$ of 150 yds. or 125 yds. A has gone half distance; B has 175 yds. further to go. B runs 10 yds. when A runs 8 yds. B runs 175 yds. when A runs $\frac{8}{10}$ of 175 yds. or 140 yds. A has gone 290 yds., while B has gone 300 yds. B wins by 10 yds.

11. Simple interest on \$400 for 3 yrs. at 7% per annum = $\$400 \times 3 \times \frac{7}{100} = \84 . Compound interest on \$400 for 3 yrs. at 7% per annum = $\$400 (1.07)^3 - \$400 = \$90.01\frac{1}{2}$. Difference is $\$6.01\frac{1}{2}$.

12. Page 217 (14).

13. Page 158 (19).

221

14. On \$76 $\frac{1}{2}$ sent to invest, commission is $\frac{1}{8}$. On \$5176.50 sent to invest, commission is $\frac{1}{8}$ of $\frac{1}{76\frac{1}{2}}$ of \$5176.50, or \$8.50.

15. Page 219 (21).

16. In 4 days $\left\{ \begin{array}{l} \text{A works } 4 \times 6 \text{ hrs., or } 24 \text{ hrs.} \\ \text{B works } 4 \times 7 \text{ hrs., or } 28 \text{ hrs.} \\ \text{C works } 4 \times 8 \text{ hrs., or } 32 \text{ hrs.} \end{array} \right.$

In 2 days B and C each work 20 hrs. \therefore A works 24 hrs. B works 48 hrs. C works 52 hrs. Divide \$62 in proportions of

SOLUTIONS OF PROBLEMS

24, 48, 52. A gets $\frac{24}{124}$ of \$62, or \$12. B gets $\frac{48}{124}$ of \$62, or \$24. C gets $\frac{52}{124}$ of \$62, or \$26.

17. Page 146 (13).

18. Page 155 (21).

19. Gain = 30% of selling price. \therefore cost = 70% of selling price. $\therefore \frac{7}{10}$ of selling price = 100% of cost. $\therefore \frac{10}{7}$ of selling price = $\frac{10}{7}$ of cost or 142 $\frac{2}{7}$ % of cost. \therefore gain = 42 $\frac{2}{7}$ % of cost.

20. Customer receives 35 in., but pays for 36 in. \therefore On every \$36 worth the customer buys he loses \$1. \therefore on every \$120 worth the customer buys he loses $\frac{120}{36}$ of \$1 or \$3.33 $\frac{1}{3}$.

21. Page 196 (12).

22. Page 144 (29).

23. A can do work in 7 days, B in 10 days, and C in 14 days. \therefore A can do $\frac{1}{7}$ of work in 1 day, B can do $\frac{1}{10}$ of work in 1 day, and C can do $\frac{1}{14}$ of work in 1 day. In two days A and B do $(\frac{2}{7} + \frac{2}{10})$ of work or $\frac{17}{35}$ of work. \therefore C has $\frac{18}{35}$ of work to do. C does $\frac{1}{14}$ of work in 1 day. \therefore C does $\frac{18}{35}$ of work in $\frac{14 \times 18}{35}$ days or $7\frac{1}{5}$ days.

24. Page 148 (42).

25. Page 220 (11).

26. Page 144 (24).

222

1. 1 lb. Av. = 7000 grs. 1 lb. Troy = 5760 grs. Marked price = 150% of cost. Sells 7000 grs. for M.P. of 5760 grs. \therefore sells 7000 grs. for 150% of cost of 5760 grs. \therefore sells 5760 grs. for $\frac{5760}{7000}$ of 150% of cost of 5760 grs., or sells 5760 grs. for 123 $\frac{3}{7}$ % of cost of 5760 grs. \therefore gain is 23 $\frac{3}{7}$ %.

2. Selling price = 130% of \$40. A discount of 25% has been allowed. $\therefore \frac{3}{4}$ of marked price = selling price. $\therefore \frac{4}{3}$ of marked price = 130% of \$40. \therefore marked price = $\frac{4}{3}$ of $\frac{130}{100}$ of \$40, or \$69.33 $\frac{1}{3}$.

3. Page 199 (59).

4. Page 149 (11).

5. Page 161 (7).

6. Policy = value of property + 2% of policy (premium). \therefore 98% of policy = \$7140. \therefore 2% of policy = $\frac{2}{98}$ of \$7140 or \$145.71 $\frac{1}{7}$. Premium paid is \$145.71 $\frac{1}{7}$.

7. Page 188 (2).

8. Page 146 (24).

9. Page 212 (13).

10. $\frac{2}{3}$ of gain expected = \$80. $\therefore \frac{2}{3}$ of gain expected = $\frac{2}{3}$ of \$80 or \$120. Selling price = \$5300. \therefore value of farm \$5300 - \$120 or \$5180.

11. Page 179 (31).

12. Page 198 (40).

13. Page 161 (4).

14. 140% of price of one lot = \$1400. \therefore 100% of price of one lot = $\frac{100}{140}$ of \$1400 or \$1000. \therefore the price of the other lot = \$1400 - \$1000 or \$400. Sold cheaper at a gain of 50%. \therefore S. P. of cheaper = 150% of cost = 150% of \$400 = \$600. Sold dearer at a loss of 30%. \therefore S. P. of dearer = 70% of cost = 70% of \$1000 = \$700. \therefore Total S. P. = \$1300. Total cost = \$1400. \therefore loss = \$100.

223

15. 80% of total profits is divided between A and B in proportion of 27 to 37. \therefore A's share = $\frac{27}{64}$ of 80% of total profits. $\therefore \frac{27}{64}$ of 80% of profits = \$675. \therefore profits = $\frac{64}{27}$ of $\frac{100}{80}$ of \$675 = \$2000. On \$6400 capital profits are \$2000. \therefore \$100 capital profits are $\frac{1}{64}$ of \$2000 or \$31 $\frac{1}{4}$. \therefore profits = 31 $\frac{1}{4}\%$.

16. Page 220 (4).

17. Page 148 (50).

18. Average price paid was 92 cts. a day. Average price paid 8 boys = 92 cts. \times 8 or \$7.36. But 8 boys received 50 cts. \times 8 or \$4.00. Difference = \$3.36. \therefore \$3.36 is amount paid to men over the average price paid. Diff. between price paid and average price is \$(1.40 - .92) or 48 cts. for each man. 48 cts. is diff. for 1 man. \$3.36 is diff. for 7 men.

19. Page 141 (43).

20. 1 share cost \$79 $\frac{1}{4}$. 150 shares cost 150 of \$79 $\frac{1}{4}$ = \$11887.50.

21. Page 219 (19).

22. Let \$1 per lb. be the cost and 4 lbs. the quantity. He intended to gain $\frac{3}{10}$ of \$4 or \$1.20, but gained only $\frac{2}{5}$ of \$1.20 or \$1.06 $\frac{2}{3}$. He gained $\frac{3}{10}$ of \$3 on the 3 lbs. or 90c. \therefore he gained 16 $\frac{2}{3}$ c. on the last lb., or threw off 13 $\frac{1}{3}$ c. \therefore he lowers the price 13 $\frac{1}{3}$ c. when tea is \$1 lb. \therefore he lowers the price 10c. when tea is 75c. a lb.

23. Page 182 (18).

24. When he has \$100 invested at 4%, income = \$4. When he has \$300 invested at 6%, income = \$18. Total income =

SOLUTIONS OF PROBLEMS

\$22. \$22 is income when he has \$400 invested. \therefore \$76.42 $\frac{4}{5}$
is income when he has $\frac{76.42\frac{4}{5}}{22}$ of \$400 invested, or \$1389.60.

224

1. Gain = \$715. A's share = \$275 + $\frac{1}{2}$ (\$715 - \$275), or \$495. \therefore B's share = \$715 - \$495, or \$220. \therefore A's contract used $\frac{495}{1300}$ of stock = $\frac{495}{1300}$ of \$1300 = \$900. \therefore B used \$1300 - \$900 or \$400.

2. Page 198 (40).

3. Page 143 (15).

4. Page 221 (14).

5. Page 217 (19).

6. 252 lbs. @ 6 $\frac{3}{4}$ cts. a lb. cost \$16.06 $\frac{1}{2}$. But he paid 17 $\frac{1}{2}$ cts. more than this, or \$16:24. If he had bought 252 lbs. @ 5 $\frac{1}{2}$ cts., it would have cost \$13.86. Difference = \$16.24 - \$13.86, or \$2.38. But diff. is (7 $\frac{1}{4}$ - 5 $\frac{1}{2}$) cts., or 1 $\frac{3}{4}$ cts., when there is 1 lb. at 7 $\frac{1}{4}$ cts. \therefore diff. is \$2.38 when there are 2.38
 $\frac{.01\frac{3}{4}}{1}$ lbs. at 7 $\frac{1}{4}$ cts., or 136 lbs. in hind quarter.

7. 97% of debt = \$1008.80. \therefore 96% of debt = $\frac{96}{97}$ of \$1008.80 or \$998.40. \therefore after a reduction of 4% debt becomes \$998.40.

8. Page 215 (26).

9. Invest \$187 $\frac{1}{2}$ in each. From \$134 invested in 6% income is \$6. \therefore from \$187 $\frac{1}{2}$ invested in 6% income is $\frac{187\frac{1}{2}}{134}$ of \$6

or \$8 $\frac{53}{134}$. From \$187 $\frac{1}{2}$ invested in 8 $\frac{1}{2}$ % income is \$8 $\frac{1}{2}$. Difference in income = \$8 $\frac{1}{2}$ - \$8 $\frac{53}{134}$ or \$ $\frac{14}{134}$. \$ $\frac{14}{134}$ is diff. when amt. invested is \$187 $\frac{1}{2}$. \therefore \$10.60 is diff. when amt. invested is \$10.60 \times $\frac{134}{14}$ \times 187 $\frac{1}{2}$ or \$19023.21 $\frac{3}{7}$.

10. Let \$2 = price paid per bush. Sold 435 bush. for 113% of cost or 113% of \$870 or \$983.10. Sold 325 bush. for 111% of cost or 111% of \$650 or \$721.50. Total S. P. is \$1704.60. If sold 760 bush. for 112% of cost, S. P. = \$1702.40. Diff. is \$1704.60 - \$1702.40 = \$2.20. \$2.20 is diff. when \$2 is cost per bush \therefore \$1.10 is diff. when \$1 is cost per bush.

225

11. Page 222 (14).

12. Marked price = 75% of cost. Selling price = 125% of marked price = $\frac{125}{100}$ of 75% of cost = 93 $\frac{3}{4}$ % of cost. \therefore loss is 6 $\frac{1}{4}$ %.

13. Page 225 (8).

14. When he gave 5 cts. to each he had 14 cts. left. When he gave 8 cts. to each he had 22 cts. too little. \therefore when he gave 3 cts. more to each the difference in the amount required was 36 cts. \therefore There were $36 \div 3$ or 12 beggars.

15. Amount in 9 mos. = \$334.40. Amount in 16 months = \$345.60. \therefore interest on sum for 7 mos. = \$11.20. \therefore interest on sum for 9 mos. = $\frac{2}{7}$ of \$11.20 or \$14.40. \therefore sum = \$334.40 - \$14.40 = \$320. interest on \$320 for 9 mos. is \$14.40. \therefore interest on \$100 for 12 mos. is $\frac{12}{9}$ of $\frac{1}{2}$ of \$14.40 or \$6. \therefore rate is 6%.

16. Page 198 (40).

17. Page 148 (46).

18. Rates are 5, 7 and 9 mls. per hour. 2nd man gains 2 miles over 1st while 1st goes 5 miles. 2nd man gains 80 rods over 1st while 1st goes $\frac{5}{4}$ mls. or 200 rods. \therefore 1st and 2nd will first be together when 1st has gone 200 rods. 3rd man gains 80 rods over 1st in going $\frac{5}{3}$ mile or 100 rods. \therefore 1st and 3rd will first be together when 1st has gone 100 rods. L.C.M. of 100 and 200 is 200. \therefore the three men will first meet when 1st man has gone 200 rods or $2\frac{1}{2}$ rounds. \therefore they meet $\frac{1}{2}$ way round.

19. Page 218 (9).

20. Wholesale price = 115% of cost. Retail price = 110% of wholesale price. \therefore retail price = $\frac{110}{115}$ of 115% of cost = $126\frac{1}{2}\%$ of cost. \therefore gain on cost is $26\frac{1}{2}\%$.

21. Page 217 (19).

226.

1. Page 161 (9).

2. A invests \$2 for B's \$3; B invests \$3 for C's $\$1\frac{1}{4}$; C invests $\$1\frac{1}{4}$ for D's $\$2\frac{3}{4}$. \therefore the money should be divided in the proportion of 2, 3, $1\frac{1}{4}$ and $2\frac{3}{4}$ or 16, 24, 30, 35. \therefore A gets $\frac{16}{105}$ of \$3150 or \$480.

3. Page 215 (26).

4. Page 121 (219).

5. Page 146 (17).

6. Page 162 (23).

7. A difference of 20c. a lb. in the selling price makes a difference of \$10 on the total. \therefore the number of lbs. is $\$10 \div 20c.$ or 50. On 50 lbs. the gain is \$7.00. \therefore on 1 lb. the gain is 14c. \therefore cost = $(70 - 14)c.$ = 56c.

SOLUTIONS OF PROBLEMS

8. $45\% - 39\% = 6\%$. 6% of A's money + 2 times. A's money = \$164.80 or $2\frac{2}{5}$ of A's money = \$164.80. \therefore A's money = $\frac{600}{170}$ of \$164.80 = \$80.

9. $\frac{4}{5}$ of last army = 6000 men. \therefore last army = 7500 men.
 \therefore number before reinforcement = (7500 - 2500) = 5000.
 $\therefore \frac{3}{4}$ of 1st army - 4000 men = 5000 men. \therefore 1st army = 12000 men.

10. 25 acres @ \$120 an ac. = \$3000. \therefore had he sold the whole he would have gained \$(3000 + 200) or \$3200. On 1 ac. he gains \$80. \therefore no. acres = 40.

11. Page 178 (21).

227

12. Page 193 (23).

13. Page 171 (24).

14. Page 159 (12).

15. 95% of total taxes = \$9690. \therefore total taxes = \$10200.
 \$17 are taxes on \$1000 assessment. \therefore \$10200 are taxes on \$600000 assessment.

16. $\frac{3}{100}$ of $\frac{3}{4}$ value flour = \$36. \therefore value of flour = \$1800.
 \therefore value of 1 barrel = \$6. \therefore selling price per bbl. = $1\frac{1}{10}$ of \$6 + $\frac{1}{10}$ of \$36 = \$(6.90 + .12) = \$7.02.

17. Page 126 (277).

(18) Salary left after paying board = $\frac{7}{10}$ of \$1200 = \$840.
 Salary left after paying rent = $\frac{4}{5}$ of \$840 = \$672. Salary left after paying clothes = $\frac{6}{100}$ of \$672 = \$571.20. Salary left after paying books = (\$571.20 - 71.20) = \$500. Salary left after paying loan = $\frac{6}{10}$ of \$500 = \$300. \therefore per cent left = $\frac{3}{12} = 25\%$.

19. The number of bush. squared = 1849. \therefore the number of bush. = $\sqrt{1849} = 43$.

20. Time past noon + time till midnight = 12 hours, and $\frac{2}{3}$ of time past noon = $\frac{2}{3}$ of time till midnight. \therefore time till midnight = $\frac{6}{3}$ of time past noon. $\therefore \frac{6}{3}$ of time past noon = 12 hrs. \therefore time past noon = $4\frac{1}{2}$ hrs., or 4.30 p.m.

21. $\frac{1}{10}$ of cost = \$11700. \therefore cost = \$9000. $\therefore \frac{3}{8}$ of $\frac{4}{5}$ of a vessel cost \$9000. \therefore vessel cost \$30000.

22. Page 184 (41).

228

1. Page 146 (13).

2. Page 162 (25).

3. B has to pay at end of 9 mos. \$5000 + int. on \$5000 for

IN THE HIGH SCHOOL ARITHMETIC.

9 mos. at 7% or \$5262.50. Cash B pays now = $\frac{1}{10}$ of \$5000 = \$4750. B receives at end of 9 mos. \$250 + int. on \$250 for 9 mos. at 10% or \$268.75. Balance B has to pay at end of 9 mos. = \$5262.50 - \$268.75 = \$4993.75. By first offer B would have had to pay at end of 9 mos. \$5,000. B's gain at end of 9 mos. = \$6.25.

4. S. P. of remainder ($\frac{9}{100} \times \frac{5}{4}$) cost of goods = $\frac{43}{100}$ cost of goods. \therefore gain = $\frac{3}{100}$ cost of goods = \$360. Cost = \$48.00.

5. Page 215 (20).

6. Int. on 1st part for 4 yrs. at 6% = $\frac{6}{100}$ of 1st part. Int. on 2nd part for 6 yrs. at 5% = $\frac{3}{100}$ of 2nd part. $\therefore \frac{6}{100}$ of 1st part = $\frac{3}{100}$ of 2nd part + \$12. \therefore 1st part = $\frac{5}{4}$ of 2nd part + \$50. $\therefore \frac{5}{4}$ of 2nd part + 2nd part = \$450. 2nd part = \$200. 1st part = \$300.

7. A works 30 hrs., B 63 hrs., and C 110 hrs. 1 hr. B's work = $\frac{7}{8}$ hrs. C's. 63 hrs. B's work = $\frac{147}{8}$ hrs. C's. 1 hr. A's work = ($\frac{4}{3} \times \frac{7}{8}$) hrs. C's. 30 hrs. A's work = $\frac{140}{8}$ hrs. C's. \therefore required proportion is $\frac{140}{8} : \frac{147}{8} : 110 = 280 : 441 : 660$. \therefore A's wages = \$28. B's = \$44.10. C's = 66.

8. Proposed gain = $\frac{8}{100}$ of \$600 = \$48. Actual gain = $\frac{1}{12}$ of \$600 = \$50. Latter gain greater by \$2.

9. $\frac{4}{100}$ total rate on investment = $5\frac{1}{4}\%$. \therefore total rate = $7\frac{5}{8}\%$. $\$7\frac{5}{8}$ is income on \$100 money, \$3 is income on \$56 money. \therefore market price = $56 - \frac{1}{2} = 55\frac{1}{2}$.

10. Page 216 (11).

229

11. Page 182 (18).

12. Total money owned by A and B = \$78. At end of game 4 times B's money = \$78 \therefore B's money = \$19 $\frac{1}{2}$, A's = \$58 $\frac{1}{2}$. \therefore A has won \$2.50.

13. Page 215 (17).

14. Page 216 (11).

15. Page 199 (60).

16. Page 161 (15).

17. Page 226 (10).

18. Dishonest gain = $\frac{2}{354}$ of marked price = $\frac{1}{47}$ of marked price. Actual S. P. = ($\frac{5}{4} \times \frac{4}{7}$) cost = $\frac{5}{7}$ cost. \therefore entire gain = $\frac{13}{47}$ cost = \$124.80. Cost = \$451.20. \therefore dishonest gain = $\frac{1}{47}$ of \$451.20 $\times \frac{5}{4}$ = \$12.

SOLUTIONS OF PROBLEMS

19. $\frac{3}{2}$ distance B travels in 1 hr. = 5 miles, \therefore distance B travels in 1 hr. = $3\frac{1}{2}$ miles.

20. 3 hound leaps = 6 hare leaps. Hound gains distance of 1 hare leap when he takes 3 leaps. Hound gains distance of 75 hare leaps when he takes 225 leaps.

21. Page 218 (12).

230

1. Suppose he sold 1st lot for \$200 and 2nd lot for \$300, cost of 1st lot = $\frac{100}{92}$ of \$200 or $\frac{\$20000}{92}$ and cost of 2nd lot = $\frac{100}{108}$ of \$300 or $\frac{\$30000}{108}$. \therefore gain = $\$(500 - \frac{\$20000}{92} - \frac{\$30000}{108}) = \frac{\$1000}{207}$. Gain is $\frac{\$1000}{207}$ when proceeds are \$500. \therefore gain is \$16 when proceeds are \$1656.

2. Average gain on whole = $5\% = \frac{1}{20}$. $\therefore \frac{1}{20}$ cost = \$88.50. \therefore cost = \$1270.

3. Page 217 (25).

4. Page 144 (29).

5. Page 178 (22) (23).

6. Page 199 (60).

7. Page 215 (26).

8. Page 215 (26).

9. Let \$100 = the cost. \therefore \$12 = gain and \$112 = selling price. $\frac{12}{100}$ of \$112 = \$13.44. Per cent. = $\frac{13.44}{112} \times 100 = 12$.

10. Apply formula.

11. Page 161 (17).

12. Page 220 (5).

13. Commerce dividend = $\$8 \times 250 = \2000 . When money is worth 7%, 8% stock is worth $\frac{\$2000}{7} = \$114\frac{2}{7}$. S.P. of 250 shares at $\$(114\frac{2}{7} - \frac{1}{2}) = \$(\frac{1593}{14} \times 250)$. No. shares Toronto stock bought = $\$(\frac{1593}{14} \times 250) \div \$205\frac{1}{2}$. \therefore Toronto dividend = $\$(\frac{1593}{14} \times 250 \times \frac{2}{11} \times 12) = \1661.105 . Difference in incomes = \$338.895.

231

14. B's s.p. of 1st lot = 120% B's cost. B's s.p. of 2nd lot = 75% B's cost. \therefore 45% B's cost = \$153. \therefore B's cost = \$340. $\frac{17}{20}$ of A's cost = \$340. \therefore A's cost = \$400.

15. S.P. of remainder = $\frac{2}{3}$ of $\frac{1}{2}$ cost = $\frac{2}{3}$ cost. \therefore loss 4%.

16. At end of 1 hr. 3rd is midway between 4 and 6 miles. \therefore he goes 5 miles per hour.

17. Page 226 (10).

18. Taking bankrupt stock worth \$1 at wholesale price.

IN THE HIGH SCHOOL ARITHMETIC.

Jobber's cost = 75c., and s.p. = \$1.10. Jobber's net s.p. = $\frac{94}{100}$ of \$1.10 = \$1.034. \therefore gain = \$.284. \therefore gain % = $\frac{28.4}{75} \times 100 = 37\frac{2}{3}$.

19. $\frac{1}{2}$ total votes polled = 240. \therefore total votes polled = 1440.

No. who did not vote = 1800 - 1440 = 360.

20. Fast train gains 10 miles in 1 hr. Fast train gains 187 yds. in $38\frac{1}{4}$ sec.

21. Consider a sale of goods that cost \$1 and sold for \$1.40. No. lbs. butter received = $\frac{1.40}{\frac{1}{2}} = 5\frac{2}{5}$. Actual value of butter = $(\frac{2}{3} \times \frac{9}{10} \times 15)c. = 75\frac{3}{5}c.$ Loss = $24\frac{2}{5}\%$.

22. Let \$a = A's stock. \therefore \$(150 + a) = B's stock, and \$(a - 306) = A's gain. Gain on \$a for 3 mo. = gain on \$3a for 1 mo. Gain on \$(150 + a) for 4 mo. = gain on \$(600 + 4a) for 1 mo. $\therefore \frac{3a}{600 + 7a} = \frac{a - 306}{400}$. $\therefore a = 450$.

23. Page 214 (12).

232

1. Cost of 2 apples, one of each kind = $(\frac{1}{2} + \frac{1}{3})c. = \frac{5}{6}c.$ S. P. of 2 apples = $\frac{1}{2}c.$ Loss = $(\frac{5}{6} - \frac{1}{2})c. = \frac{1}{3}c.$ To lose 1c. he must have bought 30 apples at each price.

2. Page 146 (13).

3. Page 165 (11).

4. Page 162 (21).

5. Page 229 (20).

6. Incomes below £100 + incomes above £100 = £500,000. $\therefore \frac{1}{20}$ incomes below £100 + $\frac{1}{20}$ incomes above £100 = £25,000. But $\frac{7}{40}$ incomes below £100 + $\frac{1}{20}$ incomes above £100 = £18,750. $\therefore \frac{5}{40}$ incomes below £100 = £6,250, and $\frac{7}{40}$ incomes below £100 = $\frac{7}{5}$ of £6,250 = £8,750.

7. Page 218 (12).

8. Page 144 (24).

9. P. W. of \$1654 in 9 mos. at $4\frac{1}{2}\% = \$1654 \times \frac{829}{827} = \1600 . Amount of stock sold = $\$1600 \times \frac{106}{96} = \$1666\frac{2}{3}$.

10. Page 231 (20).

11. P.W. of \$200 in 8 mos. at $8\% = \$200 \times \frac{75}{79} = \189.87341

" " 12 " " = $\$200 \times \frac{25}{27} = \185.18518

Total P.W. = \$375.05859. P.W. of \$400 in 10 mos. at $8\% = \$400 \times \frac{15}{16} = \375 . Loss = 5.86 cents.

12. Page 189 (7).

233

13. Page 156 (33).

14. 6% cost = \$200. \therefore cost = $\$3333\frac{1}{3}$.

SOLUTIONS OF PROBLEMS

15. Page 217 (19).
16. Page 215 (26).
17. B's contribution = \$30 more than A's. C's contribution = \$70 more than A's. \therefore A's, B's and C's contribution = \$100 more than 3 times A's. \$100 more than 3 times A's = \$3100. \therefore A's contribution = \$70, B's = \$100, C's = \$140.
18. Page 217 (19).
19. Gain is \$6 on \$75 = 8%.
20. Length of side = $\frac{3}{4}$ yds. = $\frac{3}{2}$ yds. Area = $(\frac{3}{2} \times \frac{3}{2})$ sq. yds. = $240\frac{1}{4}$ sq. yds.
21. Page 221 (19).
22. Page 215 (20).
23. Page 204 (24).
24. Original cost of material = 2 times cost of labor. Total first estimate = 3 times cost of labor. Actual cost of 1st half of material = cost of labor. Actual cost of 1st third of labor = $\frac{1}{3}$ cost of labor. Actual cost of 2nd half of material = $\frac{2}{10}$ cost of labor. Actual cost of 2 thirds of labor = $\frac{4}{5}$ cost of labor. Total actual cost = $\frac{8}{10}$ cost of labor. \therefore amt. saved = $\frac{1}{10}$ cost of labor = \$10. \therefore cost of labor = \$3000. Total first estimate = \$9000.

234

- (1) \$4850 st. at $87\frac{1}{2}$ yields $\frac{7}{8}$ of \$4850 cash = \$4243 $\frac{3}{4}$. Amount of money invested in 2nd stock = \$(4243 $\frac{3}{4}$ - 46) = \$4197 $\frac{3}{4}$. Amount of 2nd stock = \$4197 $\frac{3}{4}$ $\times \frac{100}{96\frac{1}{2}}$ = \$4350. Total amount of st. handled by broker = \$(4850 + 4350) = \$9200 or 92 shares. For handling 92 shares the broker receives \$46. \therefore for handling 1 share the broker receives $\frac{1}{2}$.
- (2) Take a risk of \$400. Premium = $\frac{4}{100}$ of \$400 = \$16. Amount reinsured = $\frac{3}{4}$ of \$400 = \$300. Premium = $\frac{3}{100}$ of \$300 = \$9. Difference in premium = \$(16 - 9) = \$7. \$7 is the difference on a \$400 risk. \therefore \$27 is the difference on a $\frac{27}{7}$ of \$400 risk = \$1542.85 $\frac{5}{7}$ risk.
- (3) Page 189 (7).
- (4) " 153 (42).
- (5) " 175 (22).
- (6) " 217 (19).
- (7) 40 days' work at 40c. a day = \$16.00. But the workman received only \$7.60. \therefore through being idle he loses \$(16.00 - 7.60) = \$8.40. For every idle day he loses (40 + 16)

IN THE HIGH SCHOOL ARITHMETIC.

cents. = 56 cents. \therefore no. of days idle = $\frac{840}{56} = 15$. \therefore no. of days he worked = 25.

(8) Page 178 (22).

(9) $\frac{5}{4}$ cost of 1st horse = \$200. \therefore cost = \$160. \therefore gain = \$40. \therefore cost of second horse = $\$(200 + 40) = \240 . \$240 On he lost \$40. \therefore on \$100 he lost $\$16\frac{2}{3}$.

(10) Page 161 (8).

235

11. Page 229 (21).

12. $\frac{28}{100}$ of \$2700 = \$2646 = amount spent in paying liabilities. \therefore $\frac{86.15}{100}$ of the liabilities = \$2646. \therefore liabilities = $\frac{100}{86.15}$ of \$2646 = \$4000.

13. Suppose 100 lbs. consumed, \therefore duty = \$6. After reduction in duty consumption = 150 lbs., and duty = $\frac{2}{3}$ of \$6 = \$4. On 150 lbs. the duty is \$4. \therefore on 1 lb. the duty is $\frac{4}{150}$ cents = $2\frac{2}{3}$ cents.

14. Price of st. = $\$(72 \times 46\frac{5}{8}) = \3357 . Cash val. of draft = $\frac{99\frac{1}{2}}{100}$ of \$2500 = \$2478.12 $\frac{1}{2}$. Cash balance = $\$(3357 - 2478.12\frac{1}{2}) = \$878.87\frac{1}{2}$.

15. Page 176 (27).

16. $\frac{7}{100}$ of \$4200 = \$294.

17. Page 157 (13).

18. The int. on the sum for the given time at $(8\frac{1}{2} - 6)\%$ = $\$(616.35 - 558.60) = \57.75 . The int. at $2\frac{1}{2}\%$ = \$57.75. \therefore the int. at $6\% = \frac{6}{2\frac{1}{2}}$ of \$57.75 = \$138.60. \therefore sum = $\$(558.60 - 138.60) = \420 . Int. on \$420 for 1 yr. at $6\% = \$25.20$. \therefore time = $\frac{138.60}{25.20}$ yrs. = $5\frac{1}{2}$ yrs.

19. The 2 extra pay $\$(4 \times .75) = \3.00 . \therefore one pays \$1.50. \therefore total rent = $\$1.50 \times 6 = \9.00 .

20. Let \$100 = cost. \therefore \$70 = new cost. Let r = rate of gain. \therefore selling price = $100(1 + r) = 70(1 + 2\frac{1}{2}r)$. \therefore $100 + 100r = 70 + 175r$. \therefore $75r = 30$. \therefore $r = \frac{2}{5} = 40\%$.

21. 3 lengths require 2 cuts and 4 lengths 3 cuts. For 2 cuts the cost per cord is \$2. \therefore for 3 cuts the cost per cord is \$3.

22. Rate of man + rate of stream is 3 miles in 30 min. or 6 miles per hr. Rate of man - rate of stream is 3 miles in 45 min. or 4 miles per hr. Twice rate of man is $(6 + 4)$ miles per

SOLUTIONS OF PROBLEMS

hr. Rate of man is 5 miles per hr. \therefore rate of stream is (6 - 5) miles = 1 mile per hr.

23. 4 times the number = 36. \therefore the number = 9.

236

1. Page 217 (19).

2. Page 178 (22).

3. Page 164 (47).

4. Page 221 (19).

5. Page 229 (21).

6. A gains 2 yds. on B in 1 sec. \therefore A gains 10 yds. on B in 5 sec. B gains 3 yds. on C in 1 sec. \therefore B gains 15 yds. on C in 5 sec. \therefore all will be together in 5 sec.

7. Page 234 (7).

8. The profit on \$3200 for 6 mos. = the profit on \$19200 for 1 mo. The profit on \$4000 for 5 mos. = the profit on \$20000 for 1 mo. The profit on \$2500 for 7 mos. = the profit on \$17500 for 1 mo. \therefore the total profits = the profit on \$56700 for 1 mo. \therefore C's share of profits on account of the money invested = $\frac{17.5}{56.7}$ of net total profits = $\frac{17.5}{56.7}$ of $\frac{9}{10}$ total profits = $\frac{31.5}{113.4}$ of total profits. \therefore C's whole profit = $(\frac{1}{10} + \frac{31.5}{113.4})$ total profits = \$428.40. \therefore total profits = \$1134. A receives $\frac{18.2}{56.7}$ of $\frac{9}{10}$ of \$1134 = \$345.60.

9. Let the amount of st. in each be \$(96 \times 101)\$. Income from 4% st. = $\frac{4}{100}$ of \$(96 \times 101) = \\$387.84\$. Income from 3% st. = $\frac{3}{100}$ of \$(96 \times 101) = \\$290.88\$. \therefore total income = \$(387.84 + 290.88) = \\$678.72\$. \$(96 \times 101)\$ st. transferred from 3%'s at 96 to 4%'s at 101 = \$(96 \times 101) \times \frac{96}{101} = \\$9216\$ st., and a similar transfer from the 4%'s to the 3%'s = \$10201 st. Income from 4% st. = $\frac{4}{100}$ of \$9216 = \\$368.64\$. Income from 3% st. = $\frac{3}{100}$ of \$10201 = \$306.03. Total income = \$(368.64 + 306.03) = \$674.67. Diff. in incomes = \$(678.72 - 674.67) = \$4.05. \therefore \$4.05 is the diff. in incomes when \$(96 \times 101)\$ st. is held in each. \therefore \$12.15 is the diff. in incomes when \$(3 \times 96 \times 101)\$ st. is held in each. \$100 st. costs \$96. \therefore \$(3 \times 96 \times 101)\$ st. costs \$(\frac{96}{100} \times 3 \times 96 \times 101) = \\$27924.48\$.

10. (\$600 - \$500) is the int. for 3 yrs. on \$500. \therefore \$600 is the int. for 3 yrs. on \$3000. Again, the int. on \$500 for 3 yrs. is \$100. \therefore the int. on \$100 for 1 yr. is \$6 $\frac{2}{3}$.

237

11. Page 217 (19).

12. Page 216 (11).

IN THE HIGH SCHOOL ARITHMETIC.

13. 2000 lbs. at 6 cents a lb. = \$120 = cost. Quantity sold = $\frac{3}{4}$ of 2000 lbs. = 1500 lbs. 15 oz. sell for 6 cents. \therefore 1500 lbs. sell for $\frac{6}{100} \times 15 \times 1500$ cents or \$122.88. \therefore gain = \$(122.88 - 120) = \$2.88.

14. Page 201 (86).

15. Amount of debt at end 1st year = $\frac{10}{100}$ of \$15000 - \$2500 = \$13250. Amount of debt at end of 2nd year = $\frac{10}{100}$ of \$13250 - \$2500 = \$11412.50. Amount of debt at end of 3rd year = $\frac{10}{100}$ of \$11412.50 - \$2500 = \$9483.12 $\frac{1}{2}$.

16. Price paid by consumer = \$9875 \times $\frac{11}{100} \times \frac{9}{10} \times \frac{5}{4}$ = \$16293.75.

17. Let value of goods consigned = \$1020. Selling commission = $\frac{2}{100}$ of \$1020 = \$20.40. Net proceeds = $\frac{98}{100}$ of \$1020. Buying commission = $\frac{2}{100}$ of $\frac{98}{100}$ of \$1020 = \$19.60. Diff. in commissions = (\$20.40 - \$19.60) = \$.80. \$.80 is the diff. when \$1020 goods are consigned. \therefore \$12 is the diff. when \$($\frac{12}{.80} \times 1020$) goods are consigned = \$15300.

18. 2 men earn \$2 \times 1.25 = \$2.50. 1 woman earns = \$.75. \therefore all earn \$3.25. \therefore the no. of men = $\frac{45.50}{3.25} \times 2 = 28$.

19. He rides 1 mile in $\frac{1}{8}$ hr., and walks 1 mile in $\frac{1}{4}$ hr. \therefore he rides 1 mile and returns walking in ($\frac{1}{8} + \frac{1}{4}$) hrs. = $\frac{3}{8}$ hrs., or he rides 24 miles and returns walking in 11 hrs.

20. Page 196 (12).

21. Income from \$4470 st. in $4\frac{1}{2}\%$'s = $\frac{-2}{100}$ of \$4470 = \$201.15.

\therefore Income from 3% 's = \$(201.15 - 16.87 $\frac{1}{2}$) = \$184.27 $\frac{1}{2}$. The money received from sale of $4\frac{1}{2}\%$ st. = \$(4470 \times $\frac{102\frac{3}{8}}{100}$) = \$(4470 \times $\frac{819}{800}$). \$184.27 $\frac{1}{2}$ is the income from \$(4470 \times $\frac{819}{800}$) invested. \therefore \$3 is the income from \$74 $\frac{1}{2}$ invested. \therefore market price = \$(74 $\frac{1}{2}$ - $\frac{1}{8}$) = \$74 $\frac{3}{8}$.

238

1. If A had received \$7 from B they would have had equal sums. \therefore the difference in the sums is \$14, &c.

2. 50 acres would occupy 1 man for 144 days and 1 boy for 168 days. \therefore 7 $\frac{1}{4}$ acres would occupy 1 man for 20.88 days and 1 boy for 24.36 days. But 7 $\frac{1}{4}$ acres would occupy 1 man for 24 days and 1 boy for 15 days. \therefore 3.12 days' work for a man = 9.36 days' work for a boy. \therefore work of 1 man = work of 3 boys, &c.

SOLUTIONS OF PROBLEMS

3. He sells $388\frac{1}{2}$ yds. for the cost of $\frac{177}{2} \times \frac{36}{5} \times \frac{5}{4}$ yds. \therefore he must sell the rem. for the cost of $\{777 \times \frac{5}{4} - (\frac{177}{2} \times \frac{36}{5} \times \frac{5}{4})\}$ yds. But he sells the remainder as $\frac{177}{2} \times \frac{36}{5}$ yds. \therefore each yard of the rem. is sold for the cost of $1\frac{25}{64}$ yds. \therefore he must mark it at $1\frac{25}{64}$ of 100% above cost.

4. 105% of the premium = \$63. \therefore \$60 is prem. on \$9000.

5. $\frac{2}{5}$ of the profits = gain on \$36000 invested for 1 month.

$\therefore \frac{1}{10}$ of the profits = gain on \$9000 invested for 1 month. \therefore

B's investment = \$2250 for 4 mos.

6 The value of the 70 lbs. of the inferior kind would be \$24 50. \therefore the inferior is worth 35 cts. per lb.

7. 22% on 80% of the goods = 17.6% on all. \therefore 17.6% of the invoice = \$633.60.

8. Page 217 (19).

9. " 150 (16).

239

10. The interest on \$120 is \$5, and \therefore \$10 for twice the time. \therefore the discount off \$130 is \$10, and \therefore off \$125 is \$9 $\frac{5}{8}$.

11. Page 212 (1).

12. " 215 (26).

13. " 146 (14).

14. 14 lbs. of each cost \$10.15. \therefore 14 lbs. of green cost \$4.90, &c.

15. Page 204 (124); 196 (12).

16. Find the cost and selling price of an equal number of oranges.

17. He sells the barrel as $(\frac{8}{15} + \frac{8}{17})$ or $\frac{256}{55}$ of a barrel, and \therefore gains $\frac{1}{255}$ or $\frac{20}{551}\%$.

18. The premium is \$33.75 on \$2700.

19. Page 189 (9).

20. " 219 (21).

240

1. Page 163 (40); 157 (5).

2. Since the duty was 5 cts. per bottle, there were 1464 bottles unbroken.

3. 1 cu. ft. of water weighs $1000 \times \frac{4371.1}{480}$ ozs. troy. 1 cu. metre = $(\frac{5280}{1770})^3$ cu. ft. \therefore 1000 kilogramme = $1000 \times \frac{4371.1}{480} \times (\frac{5280}{1770})^3$ ozs. troy.

4. Page 217 (9).

5. Page 213 (29).
6. " 147 (38).
7. Had there been 3 women less the earnings would be \$24.75. \$2.75 is the wage of 2 women and 1 man. \therefore \$24.75 is the wage of 18 women and 9 men.
8. Page 148 (42).
9. They approach each other at the rate of $2\frac{3}{4}$ mls. per hour.
10. Page 196 (12) ; 198 (40).

241

11. The value of $\frac{227}{800}$ of $\frac{54}{8}$ of the patent is \$756, &c.
12. The net earnings are \$290054.57, which is $4\frac{1}{3}\%$ of the capital.
13. The cost per lb. was $27\frac{7}{8}$ cts. \therefore total cost = $\$77\frac{7}{8}$. Total s.p. = $\$85\frac{5}{8}$. \therefore $\$61\frac{5}{8}$ = s.p. of 200 lbs.
14. When he sells \$63 worth of goods he charges \$64. \therefore gains $\frac{1}{63}$ of \$64.
15. Former rate is 21% . \therefore he must pay 3% on \$2500.
16. B's equivalent capital for 1 month is \$78000. \therefore A's must be \$58000. \therefore A's withdrawal for the 7 mos. must be equivalent to \$14000 for 1 month.
17. Each gal. contains $\frac{5}{8}$ of a gal. of wine, and \therefore $\frac{1}{8}$ of a gal. of water.
18. Cost of the unbroken = \$189. \therefore duty is 24% of this.
19. If both rates had been the same as first the interest would have been \$8.24 less. \therefore interest on \$787 at the rate of the first = \$39.35.
20. Page 234 (7).

242

1. Page 182 (27).
2. His assets are 65% of \$3000 + \$50.
3. Page 185 (3).
4. " 204 (122).
5. The net taxes on \$1000 are \$18.62.
6. Page 191 (2).
7. " 178 (18).
8. If they can pay 12% on 10% of their capital, they can pay only $\frac{10}{14}$ of 12% on 14% of their capital.
9. He invests $\$34920 \times \frac{800}{825}$ at \$2.95 per yd. The total outlay = $\$34920 + \$1997.13 + 2\frac{1}{2}\%$ of $\$34920 \times \frac{800}{825}$, etc.
10. If he buys \$65 worth of goods he pays only \$64. He

SOLUTIONS OF PROBLEMS

sells \$63 worth of goods for \$64. He sells \$65 worth of goods for \$66 $\frac{2}{3}$. \therefore he gains \$2 $\frac{2}{3}$ when he pays \$64. He gains \$15 when he pays \$372.50.

11. \$7.50 is the premium on \$1000. \therefore \$25.50 is the premium on \$3400.

243

12. Page 236 (8).

13. " 192 (13).

14. 68 days' wages = sum and int. for 4 days. 72 days' wages = sum and int. for 6 days. \therefore int. for 2 days = 4 days' wages. \therefore int. for 4 days = 8 days' wages, or the wages of 2 men for 4 days.

15. Suppose 12 lbs. cost \$1.00. \therefore s.p. = \$1.12. At the increased rate he sells 12 lbs. for \$1.49 $\frac{1}{3}$, or 9 lbs. for \$1.12.

16. Page 175 (20).

17. \$30 will buy 60 bush. of oats or $60 \times \frac{5}{14}$ bush. of rye, or $60 \times \frac{5}{14} \times \frac{7}{10}$ bush. wheat.

18. Page 158 (19).

19. The 6 boys earn \$.63 \times 6, or \$3.78 below the average \therefore the men must earn \$3.78 above the average. But each man earns 12 cts. above. \therefore there are 31 $\frac{1}{2}$ men, or 31 men full time, and 1 man half time.

20. Page 232 (1).

21. He held $\frac{100}{108}$ of \$1317.58 stock before the dividend. \therefore cost is $\frac{100}{108}$ of \$1317.58 \times $\frac{100}{108}$ = \$1267.86.

244

1. Page 185 (20).

2. On \$100 worth of wheat sold he got \$5 $\frac{1}{4}$ com. \therefore the net proceeds are \$(100 - 5 $\frac{1}{4}$) or \$94 $\frac{3}{4}$. When the net proceeds are \$94 $\frac{3}{4}$, the com. is \$5 $\frac{1}{4}$. \therefore when the net proceeds are \$9790, the com. is \$210. 40 ft. frontage is bot. for \$210. \therefore 1 ft. frontage is bot. for \$5.25.

3. Selling price with a just measure = $\frac{5}{4}$ of \$1500 = \$1875. Real selling price = \$(1500 + 750) or \$2250. By dishonesty he disposes of \$1875 worth of goods for \$2250. \therefore by dishonesty he disposes of 1875 gals. (true) as 2250 gals. (false). \therefore by dishonesty he disposes of $\frac{1875}{2250}$ gals. (true) as 1 gal. (false), or $\frac{5}{6}$ gal. as 1 gal.

4. Suppose \$400 is the value of the house. 1st premium = $\frac{5}{100}$ of $\frac{3}{4}$ of \$400 = \$4.50. 2nd premium = $\frac{9}{100}$ of $\frac{1}{4}$ of \$400 =

\$1.12 $\frac{1}{2}$. Total premium = \$5.62 $\frac{1}{2}$ or \$5 $\frac{5}{8}$. On \$400 the premium is \$5 $\frac{5}{8}$. \therefore on \$100 the premium is \$1 $\frac{3}{8}$.

5. Page 215 (26).

6. " 193 (21).

7. " 145 (35).

8. " 243 (14).

9. (40 - 24) cents on the dollar are paid by \$500. \therefore 100 cents on the dollar will be paid by \$ $\frac{100}{100} \times 500 = \312

10. The capital is invested in the proportion 6, 10, 14. \therefore 3 men's capital = $\frac{14}{30}$ of \$13200 or \$6160 = total gain. A's share = $\frac{6}{30}$ of \$6160 or \$1232.

245

11. Page 182 (27).

12. $\frac{3}{100}$ of taxes = \$172.80. \therefore taxes = \$5760. On \$320000 the taxes levied are \$5760. \therefore on \$1 the taxes levied are 18 mills.

13. Alloy required contains 13 oz. nickel, 12 oz. lead, 14 oz. tin. Alloy obtainable contains 14 oz. nickel, 9 oz. lead, 12 oz. tin, or alloy required contains 182 oz. nickel, 168 oz. lead, 196 oz. tin, and alloy obtainable contains 182 oz. nickel, 117 oz. lead, 156 oz. tin. \therefore we must add to the alloy obtainable (168 - 117) oz. lead or 51 oz. lead, and (196 - 156) oz. tin or 40 oz. tin. The alloy obtainable contains (182 + 117 + 156) oz. or 455 oz. 455 oz. alloy + 51 oz. lead + 40 oz. tin = 546 oz. alloy required. \therefore $\frac{455}{546}$ of 1092 oz. = 910 oz. alloy, &c.

14. Page 175 (23).

15. " 195 (8).

16. Preference stock receives (8% - 6 $\frac{1}{10}$ %) of \$150000 = \$2850 more than the average. \therefore the ordinary stock receives \$2850 less than the average. \$(6 $\frac{1}{10}$ - 5%) is the amt. less than average for \$100 ordinary st. \therefore \$2850 is the amt. less than average for \$350000 ordinary st. \therefore total st. = \$(150000 + \$350000) = \$500000.

17. Let $a\%$ = rate. Selling com. = $\frac{a}{100}$ or \$5330 = \$ $\frac{5330a}{100}$. Total com. = $\frac{2a}{100 + a}$ of \$5330 = \$ $\frac{5330 \times 2a}{100 + a}$. See page 152 (36).

Diff. in coms. = 2 selling com. - total com. \therefore \$ $\frac{2 \times 5330a}{100}$ -

\$ $\frac{5330 \times 2a}{100 + a}$ = \$6 $\frac{1}{2}$, or $\frac{a}{100} - \frac{a}{100 + a} = \frac{1}{1640}$. $\therefore a = 2\frac{1}{2}$.

18. Page 244 (3).
 19. " 155 (21).
 20. " 217 (19).

246

1. Page 192 (13).
2. " 220 (4).
3. $\frac{7}{100}$ of price paid for stock = $\frac{7}{100}$ of \$5. \therefore price paid for stock = \$70. \therefore market price = $\$(70 - \frac{1}{8}) = \$69\frac{7}{8}$.
4. Page 152 (37).
5. He sells $15\frac{3}{4}$ oz. as 16 oz. \therefore he sells \$15 $\frac{3}{4}$ worth for \$16. \therefore he sells \$1050 worth for $\$16 \div 15\frac{3}{4} \times 1050 = \$\frac{84}{3} \times 1050$. Selling price of $\frac{84}{3} \times 1050 = \frac{112}{3}$ of $\frac{84}{3} \times 1050 = \$1226\frac{2}{3}$. \therefore gain = $\$(1226\frac{2}{3} - 1050) = \$176\frac{2}{3}$.
6. Let amount of insurance = \$1500. $\therefore \frac{1}{2}$ of \$1500 = \$300, $\frac{1}{3}$ of \$1500 = \$500, and balance = \$700. 2% of \$300 = \$6. 2 $\frac{1}{2}$ % of \$500 = \$12.50. 2 $\frac{1}{4}$ % of \$700 = \$15.75. \therefore total premium = \$34.25. \$34.25 is the premium when the amount of insurance is \$1500. \therefore \$342.50 is the premium when the amount of insurance is \$15000.
7. Page 221 (16).
8. 20 weeks' work is worth \$20 and a coat, and 12 weeks' work is worth \$9 and a coat. \therefore 8 weeks' work is worth \$11. \therefore 12 weeks' work is worth $\frac{12}{8}$ of \$11 or \$16.50. \therefore coat is worth $\$(16.50 - 9)$ or \$7.50.
9. If receipts = \$100. Outlay for material = \$60, expenses = \$20. \therefore cost = \$80. Gain = \$20, or 25% on cost. 2nd outlay = $\frac{7}{10}$ of \$60 or \$42, expenses = $\frac{11}{10}$ of \$20 or \$22, cost = $\$(42 + 22)$ or \$64. Selling price = $\frac{5}{4}$ of \$64 or \$80. \therefore 1st selling price: 2nd selling price as 100 : 80, or as 5 : 4. \therefore loaf will sell for $\frac{4}{5}$ of 7 $\frac{1}{2}$ c. or 6c.
10. Example incorrect. Change \$82.50 to \$45. Interest for 3 years is \$45. \therefore int. for 1 year is \$15. Interest for 5 $\frac{1}{2}$ years is \$110. \therefore int. for 1 year is \$20. \therefore 2% of the sum = $\$(20 - 15)$ or \$5. \therefore sum = \$250. \$15 is the int. on \$250 for 1 year. \therefore rate = $\frac{15}{250}$ or 6%.
11. Page 175 (16).

247

12. Assignee's charges = $\frac{3}{100}$ of \$3347.50 = \$97.50. \therefore amount distributed = $\$(3347.50 - 97.50)$ or \$3250. On \$5000 he pays \$3250 to his creditors. \therefore on \$1 he pays 65 cents.

IN THE HIGH SCHOOL ARITHMETIC.

13. Total amount raised = $\frac{3}{10000}$ of \$450000 or \$1350. Collector's charges = \$(1350 - 1296) or \$54. On \$1350 he gets \$54 for collection. \therefore on \$100 he gets \$4 for collection.

14. Cost = (2000 of $4\frac{1}{2}\text{c.}$) = \$90. Ad. val. duty = $\frac{1}{4}$ of \$90 or \$18. Sp. duty = 20 of 14c. or \$2.80. \therefore total duty = \$20.80.

15. Page 245 (3).

16. " 195 (1).

17. " 237 (21).

18. Selling price of wheat = \$(2000 \times .85) or \$1700. Draft = \$(1700 \times $\frac{97}{100}$ \times $\frac{99\frac{1}{2}}{100}$ - 130 $\frac{1}{4}$) \times $\frac{100}{101\frac{1}{4}}$ = \$1491.856.

19. Page 242 (10).

20. If value of property = \$500. \therefore value of policy = $\frac{3}{4}$ of \$500 or \$300. Premium = $\frac{2}{100}$ of \$300 or \$6. Expenses = $\frac{5}{100}$ of \$6 or \$.30. Total cost of insuring is \$6.30 when \$500 is val. of property. \therefore total cost of insuring is \$75.60 when \$6000 is val. of property.

248

1. Gain on \$2500 for 12 mos. = gain on \$30000 for 1 mo. Gain on \$3000 for 7 mos. = gain on \$21000 for 1 mo. B gains \$1333 $\frac{1}{3}$ on \$(30000 + 21000) capital for 1 mo. \therefore A gains \$1066 $\frac{2}{3}$ on \$40800 capital for 1 mo. Gain on \$5000 for 4 mos. = gain on \$20000 for 1 mo. \therefore 8 mos' gain on the capital A left in = the gain for 1 mo. on \$(40800 - 20000). \therefore capital A left in = \$2600, or he withdrew \$2400.

2. 16 bush. corn and 20 bush. rye sell for \$30, or 8 bush. corn and 10 bush. rye sell for \$15. Similarly 8 bush. corn and 3 $\frac{1}{2}$ bush. rye sell for \$9. \therefore (10 - 3 $\frac{1}{2}$) bush. rye sell for \$(15 - 9), or 1 bush. rye sells for 90c.

3. There are 28 days' provisions when 300 men are killed. \therefore $\frac{700}{100}$ of 28 days = 49 days.

4. Page 246 (10).

5. " 247 (12).

6. " 175 (6).

7. 97 $\frac{3}{4}\%$ of taxes = \$9775. \therefore taxes = $\frac{100}{97\frac{3}{4}}$ of \$9775. Rate = $\frac{100}{97\frac{3}{4}}$ of \$9775 \div \$5000 or 2.

8. 1000 barrels at \$8 per barrel = \$8000. 4000 bushels wheat at \$.90 per bushel = \$3600. Total sales = \$(8000 + 3600) or \$11600. Storage of flour = \$(1000 \times $\frac{1}{8}$) or \$125. Storage of

SOLUTIONS OF PROBLEMS

wheat = $\$(4000 \times .03)$ or \$120. Freight and duty = \$67.48.
 Commission for selling flour = $\frac{1\frac{1}{2}}{100}$ of \$8000 = \$120. Commis-
 sion for selling wheat = $\$(4000 \times .02)$ = \$80. Total expenses =
 \$512.48. \therefore total amount remitted = $\$(11600 - 512.48)$ or
 \$11087.52.

9. Draft on New York costs $\frac{100}{100}^{1\frac{1}{2}}$ of \$4500 or \$4522.50.
 Draft on Chicago costs $\frac{100}{100}^{1\frac{1}{2}}$ of \$4500 or \$4522.61. 1st
 method by 11c

249

10. Page 237 (21).

11. \$750 will pay $(\frac{27}{40} - \frac{3}{8})$ of his liabilities. \therefore liabilities =
 \$10000.

12. Page 238 (3).

13. " 156 (34).

14. " 188 (2).

15. B gains a round on A every 6 days. C and A meet
 every 4 days. \therefore in 12 days they are all together again.

16. Page 188 (2).

17. Interest on \$100 for $2\frac{1}{4}$ years at 4% = \$9. \therefore \$62.10 is
 the int. on \$690. \therefore 1st sum equals \$230. Int. on \$230 for
 1 yr. at 6% = \$13.80. \therefore time = $\frac{62.10 - 13.80}{13.80}$ yrs. or $4\frac{1}{2}$ yrs.

18. Page 248 (9).

19. Total taxes = $\frac{100}{100}$ of \$893 or \$900. \therefore assessed value =
 $\frac{\$900 \times 1000}{2\frac{1}{4}}$ or \$400000.

250

1. Page 175 (4).

2. Different solutions may be obtained for this problem.
 The sheep at \$5 and \$6 will balance each other. A sheep at
 \$4 gains \$1 $\frac{1}{2}$, and a sheep at \$8 loses \$2 $\frac{1}{2}$. \therefore 5 sheep at \$4
 will balance 3 sheep at \$8.

3. Insurance = $\frac{100}{3\frac{1}{2}}$ of \$58.80 = \$1680. Total cost = $\frac{100}{17}$ of
 \$1680 = \$2400. Cost per barrel = $\$2400 \div 400$ = \$6. Selling
 price = $\frac{2}{7}$ of \$6 = \$7.20. This gain is computed on first cost,
 and leaves out the cost of insuring.

4. Page 185 (11).

5. Area sq. field = $(10)^2$ sq. chs. = 10 ac. Perimeter = 40
 chs. Rad. circle = $\frac{70}{11}$ chs. \therefore area = $\frac{22}{7} \cdot (\frac{70}{11})^2$ sq. chs. = $12\frac{8}{11}$
 ac. Diff. = $2\frac{8}{11}$ ac.

IN THE HIGH SCHOOL ARITHMETIC.

6. Page 237 (21).

7. With guarantee merchant receives $\frac{94}{100}$ of \$2300 = \$2162. Without guarantee merchant receives ($\frac{97\frac{1}{2}}{100}$ of \$2300) - \$90 = \$2152.50. Diff. = \$9.50.

8. In 10 min. A runs 10×450 yds. - $(2 + 4 + 6 + 8 + 10 + 14 + 16 + 18)$ yds. = 4410 yds. In 10 min. B runs 10 times rate + $(2 + 4 + 6 + 8 + 10 + 14 + 16 + 18)$ yds. = 4410 yds., or 10 rate = 4320 yds. Rate = 432 yds.

9. Page 189 (4).

10. " 145 (7).

251

11. Page 212 (1).

12. " 192 (15).

13. $\frac{15}{100}$ of cost = selling price. $\frac{6}{100}$ of $\frac{15}{100}$ of cost = amount received = $\frac{9}{100}$ of cost. \therefore loss = $\frac{6}{100}$ of cost or 10%.

14. Page 166 (24).

15. Duty on 1 bag = $\frac{15}{100}$ of $\frac{95}{100}$ of 180 of $12\frac{1}{2}$ c. = $\frac{\$51.3}{100}$. No. bags = $\$961\frac{7}{8} \div \frac{\$51.3}{100} = 300$.

16. Page 242 (10).

17. Page 156 (27).

18. $\frac{5}{7}$ of work is done in 20 days. $\therefore \frac{25}{8}$ of the work could be done by the men in 25 days. $\therefore \frac{5}{8}$ of work is done in 5 days by 3 men. $\therefore \frac{5}{7}$ of work is done in 20 days by 5 men.

19. Area = $\frac{1}{2} \cdot \frac{27}{2} \cdot 40^2 + \frac{1}{2} \cdot \frac{27}{2} \cdot 10^2$ } sq. ft. = 2592 $\frac{9}{2}$ sq. ft.

20. $\frac{5}{8}$ cost of a pint = 90c. \therefore cost = 75c. \therefore wine in 1 gal. = $\frac{75}{100}$ gal. = $\frac{3}{4}$ gal. \therefore water in gal. = $\frac{1}{4}$ gal.

252

1. Page 235 (22).

2. $\frac{6}{100}$ of $\frac{1}{2}$ liabilities + $\frac{1}{2}$ of $\frac{2}{3}$ liabilities = \$4800. \therefore liabilities = \$9000.

3. No. of yds. = $20000 \div 3$. No. of metres = $\frac{20000}{3} \times \frac{36}{100} = \frac{128000}{3}$. Expense of carriage = 20000d = £83 $\frac{1}{3}$. Duty = $\frac{128000}{21}$ of .42 francs = 2560 francs. He sells the cloth at the average rate of 7 francs per metre. \therefore selling price = $\frac{128000}{21} \times 7$ francs = $\frac{128000}{3}$ francs. Gain = $\frac{128000}{3}$ francs - £83 $\frac{1}{3}$ - £1000 } = \$2348.04 $\frac{4}{5}$.

4. Every time a share of 4% st. is sold it increases the original capital by \$3 $\frac{1}{3}$. \therefore if we invest \$3 $\frac{1}{3}$ in 3 $\frac{1}{4}$ % st., and subtract the income derived from it from \$4, we may alter the example so as to have \$15510 invested in each kind of stock,

SOLUTIONS OF PROBLEMS

and the difference in income $\$16.40\frac{5}{8}$. The income from $\$3\frac{1}{2}$ invested in $3\frac{1}{4}\%$ st. at 77 = $\$3\frac{1}{2} \times 3\frac{1}{4} = \$3\frac{325}{164}$. For every share of 4% st. held $\$3\frac{325}{164}$ is added to the $3\frac{1}{4}\%$ st. income. \therefore if the 4% st. yielded only $\$(4 - \frac{325}{164})$, or $\$3\frac{139}{164}$, the $\$3\frac{325}{164}$ would not be required to increase the $3\frac{1}{4}\%$ income, and the difference would still be $\$16.40\frac{5}{8}$ or $\$5\frac{75}{32}$. The income from $\$15510$ invested in the $3\frac{1}{4}\%$ s at 77 = $\frac{\$15510 \times 1\frac{1}{4}}{77} = \frac{\$18330}{77}$. \therefore the income from the stock paying $3\frac{2139}{404}\%$ is $\$(\frac{\$18330}{77} - \frac{525}{32}) = \frac{\$142965}{224}$. \therefore no. of shares = $\frac{\$142965}{\frac{\$18330}{77} \div \frac{\$32139}{164}} = 165$. \therefore price per share = $\$15510 \div 165 = \94 . (Another solution.) Let x = no. sh. 4% st. \therefore inc. = $4x$. Stock sold for $\$(15510 + 3\frac{1}{2}x)$. because each share brings $\$3\frac{1}{2}$ more than was paid for it. No. of sh. of $3\frac{1}{4}\%$ st. = $\frac{15510 + 3\frac{1}{2}x}{77}$. Inc. = $\frac{15510 + 3\frac{1}{2}x}{77} \times 1\frac{1}{4}$. Diff. = $\frac{15510 + 3\frac{1}{2}x}{77} \times 1\frac{1}{4} - 4x = \$16.40\frac{5}{8}$. $\therefore x = 165$. \therefore price =

$\$15510 \div 165 = \94 .

5. Net sales, when no guarantee = $\frac{96}{100}$ of $\$3400 = \3298 . \therefore net sale with guarantee = $\$(3298 - 96 + 19.50) = \3221.50 . \therefore com. with guarantee = $\$(3400 - 3221.50) = \178.50 . \therefore rate = $\frac{178.50}{3400} \times 100 = 5\frac{1}{4}$.

6. Page 191 (5).

7. A's gain = $\$(4600 - 1000) = \3600 . $\$3600$ is the gain in 12 mos. on $\$12000$ capital. \therefore $\$1000$ is the gain in 8 mos. on $\$5000$ capital. \therefore value per acre = $\$5000 \div 37\frac{1}{2} = \$133\frac{1}{3}$.

8. Whole gain % = $(12\frac{1}{2} + 7) = 19\frac{1}{2}$. $\frac{19\frac{1}{2}}{100}$ of $\frac{9\frac{5}{6}}{100}$ of sales = $\frac{119.5}{100}$ of cost. \therefore sales = 1.2956 of the cost. \therefore advance = 29.56%.

253

9. Page 246 (8).

10. " 216 (1).

11. Dist. by coach = $\frac{1}{3}$ of $\frac{3}{20}$ dist. by sea = $\frac{1}{20}$ dist. by sea. Dist. by rail = $\frac{3}{20}$ dist. by sea. \therefore dist. by coach, rail and sea are as 1, 3, 20. \therefore dist. by coach = $\frac{1}{24}$ of 480 miles = 20 miles. Dist. by rail = 60 miles, and dist. by sea = 400 miles. Fare by rail = $\frac{3}{2}$ of 8c. = 12c., and fare by sea = $\frac{1}{2}$ of 12c. = 40c. Total cost = $(20 \times 8 + 60 \times 12 + 400 \times 40)$ cents = $\$168.80$.

12. 5 hrs. 12 min. diff. in the times. For 1 hr. diff. the places are distant 15° from each other. \therefore for 5 hrs. 12 min. diff. the places are distant 78° from each other, and as the time at Callao is behind Greenwich, Callao is 78° west.

13. Suppose the apples are a penny each, we get 240 apples for a sov., and at the reduced price $\frac{5}{4}$ of 240, or 300 apples for a sov. \therefore 60 apples more are obtained for a sov. when apples are 1d. each. \therefore 120 apples more are obtained for a sov. when apples are $\frac{1}{2}$ d. each.

14. Page 192 (15).

15. " 179 (38).

16. " 188 (2).

17. Crew pumps out $6 \times 2\frac{3}{4}$ tons in 1 hr. = $16\frac{1}{2}$ tons. \therefore the ship retains $(20 - 16\frac{1}{2})$ tons per hr. = $3\frac{1}{2}$ tons. $3\frac{1}{2}$ tons remain after 1 hr. \therefore 70 tons remain after 20 hrs. \therefore distance from shore (12×20) miles or 240 miles.

18. Page 242 (10).

19. " 159 (5).

254

1. To make 4c. there are $\frac{4}{13}$ of 16 apples sold, or $\frac{64}{13}$ apples. \therefore $(90 - \frac{64}{13})$ apples when sold bring equal prices. At the 1st price $\frac{9}{13}$ apples sell for 1c. At the 2nd price $\frac{1}{13}$ apples sell for 1c. \therefore the apples left must be sold in the proportion of $\frac{9}{13}$ and $\frac{1}{13}$, or 78 and 80, or at the 1st price $\frac{78}{13}$ of $(90 - \frac{64}{13})$ apples sold = 42 apples.

2. Page 199 (61).

3. 1st, the number must be a multiple of 2, 3, 5, 6, 8 and 12, or 120. 2nd, it must be a perfect square, i.e., every prime in 120 must be squared. The prime factors of 120 are $2^3, 3, 5$. \therefore the least number is $2^4, 3^2, 5^2$ or 3600.

4. The 3rd hand's rate is $6\frac{1}{2}$ sp. in 12 min. At 4 o'clock it must bisect the space if it points to fig. 2. \therefore it must travel 5 spaces. It goes $6\frac{1}{2}$ sp. in 12 min. \therefore it goes 5 sp. in $9\frac{3}{4}$ min.

5. Vols. = $2\frac{2}{3} \times (\frac{9}{2})^2 + 6^2 \times 1$ cu in. $\therefore 2\frac{2}{3}(\text{rad.})^2 \times \frac{9}{2} + 6^2 \times 1$ cu. in. $\therefore (\text{rad.})^2 = \frac{225 \times 2}{4}$ $\therefore \text{rad.} = \frac{15}{2} \sqrt{2}$ in. $\therefore \text{dia.} = 15\sqrt{2}$ in.

6. L.C.M. = $11\frac{54}{13}$ or 858.

7. Sale = $\$(\frac{103.50}{8} \times 75) = \2587.50 . Stock purchased = $\frac{2587.50}{115}$ shares = $22\frac{1}{2}$ shares. Income from latter stock =

$130\frac{10}{13}\%$ of $\$103.50 = \135 . Rate = $\frac{135}{22\frac{1}{2}}$ or 6%.

8. By accepting the latter offer he should gain $3\frac{1}{2}\%$ of $\$2700$ or $\$94.50$. But he loses $\$21.50$. \therefore the worthless sales are $\$(94.50 + 21.50)$ or $\$116$.

SOLUTIONS OF PROBLEMS

9. Page 251 (13).

10. Page 215 (17).

255

11. Page 244 (3).

12. Expenses = $$(1100 + 400 - 1187.50)$ or $$312.50$. 2% of $$1000 = 20 , and $\frac{1}{1000}$ of $$1100 = 1.10 . $\therefore \frac{1}{1000}$ of $\frac{2}{3}$ of val. farm + $\frac{1}{1000}$ of $\frac{2}{3}$ of val. of farm = $$(312.50 - 36.50)$. $\therefore \frac{1}{1000}$ of val. of farm = $$276$. \therefore val. of farm = $$6000$.

13. Page 208 (170).

14. The true discount is the interest on the present worth, and bank discount is calculated similar to the interest on a sum of money equal to the amount.

15. Page 253 (12).

16. 2nd number = $\frac{224 \times 12}{84} = 132$.

17. Page 191 (5).

18. " 213 (29).

19. Cash cost = $\frac{100}{104\frac{1}{2}}$ of 20 of 25 of $$4.37\frac{1}{2} = 2093.30 .

Cash sale = $\frac{100}{102}$ of 20 of 25 of $$4.62\frac{1}{2} = 2267.15 . \therefore cash gain = $$173.85$.

20. Int. on $$100$ is $\frac{1}{100}$ of $\frac{2}{3}$ of $$100 = 17.10 . $\therefore $38.90\frac{1}{4}$ is int. on $\frac{38.90\frac{1}{4}}{17.10}$ of $$100 = 227.50 .

21. As assignee, A receives $3\frac{1}{2}\%$ of $$7290$ or $$255.15$, and as creditor, $\frac{4}{5}$ of $$(7290 - 255.15)$ or $$3126.60$.

256

1. $1\frac{1}{2}$ lb. troy gold and $\frac{1}{2}$ lb. troy alloy are worth 45 guineas, but $\frac{1}{2}$ lb. alloy is worth $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ lb. gold. $\therefore (1\frac{1}{2} + \frac{1}{4})$ of $\frac{1}{2}$ lbs. troy gold are worth 45 guineas, or $\frac{2}{3} \times \frac{2}{3}$ lbs. troy gold are worth 45 guineas, or $\frac{2}{3} \times \frac{2}{3}$ lbs. troy alloy are worth $\frac{1}{2}$ of 45 guineas, or 1 lb. troy alloy is worth $2\frac{1}{2}$ guineas. \therefore 1 lb. avoirdupois alloy is worth $\frac{7}{8} \times \frac{9}{8}$ of $2\frac{1}{2}$ guineas or $2\frac{7}{8}$ guineas.

2. The sum of their rates is 39 miles in 4 hrs., or $9\frac{3}{4}$ miles per hour. The diff. of their rates is $5\frac{1}{4}$ miles in 7 hrs., or $\frac{3}{4}$ miles per hour. Twice rate of faster man = $10\frac{1}{2}$ miles per hr., or rate of faster man = $5\frac{1}{4}$ miles per hr.

3. Let x = perp. \therefore hypo. = $50 - x$. $\therefore (50 - x)^2 = x^2 + 32^2$, $\therefore x = 14.76$ in. \therefore area = $\frac{1}{2} \times 14.76 \times 32$ sq. in. = 236.16 sq. in.

4. At the end of the 1st year he is worth $\frac{7}{8}$ of original capital - $$1000$. At the end of the 2nd year he is worth $(\frac{7}{8})^2$ of

IN THE HIGH SCHOOL ARITHMETIC.

original capital - $\frac{7}{8}$ of \$1000 - \$1000. At the end of the 3rd year he is worth $(\frac{7}{8})^3$ of original capital - $(\frac{7}{8})^2$ of \$1000 - $\frac{7}{8}$ of \$1000 - \$1000. At the end of 4th year he is worth $(\frac{7}{8})^4$ of original capital - $\{ (\frac{7}{8})^3 + (\frac{7}{8})^2 + \frac{7}{8} + 1 \}$ of \$1000. At the end of the 5th year he is worth $(\frac{7}{8})^5$ of his original capital - $\{ (\frac{7}{8})^4 + (\frac{7}{8})^3 + (\frac{7}{8})^2 + (\frac{7}{8}) + 1 \}$ of \$1000 = 5 times original capital. $\therefore \{ (\frac{7}{8})^5 - 5 \}$ original capital = $(\frac{7}{8})^5 - 1$ of \$1000. \therefore original capital = \$28938.24+.

5. Page 112 (130).

6. One cent is the unit. Int. on \$325 for $6\frac{1}{2}$ years at 6% per annum = \$126.75. \therefore with one cent as unit the int. is represented by 12675.

7. Income from 1st stock = $\frac{3\frac{1}{2}}{82}$ of \$10947 = \$467.25. In-

come from 2nd inv. in 3% st. = $\frac{3}{75}$ of $\frac{2}{3}$ of \$10947 = \$291.92. Income from 2nd inv. in 4% st. = $\frac{4}{80}$ of $\frac{1}{2}$ of \$10947 = \$164.00.

\therefore alteration in income = \$(467.25 - 291.92 - 164) = \$11.33.

8. Net receipts = $\frac{90}{100}$ of $\frac{75}{100}$ of \$2500 + $\frac{90}{100}$ of $\frac{40}{100}$ of $\frac{25}{100}$ of \$2500 = \$2025. \therefore percentage obtained = $\frac{2025}{2500} = 81\%$.

9. Take 30 gals. in mixture, or 25 gals. of wine and 5 gals. water, and 30 gals. in 2nd mixture, or 18 gals. of wine and 12 gals. water. \therefore 7 gals. of wine must be drawn off, and with it $\frac{1}{5}$ of 7 gals. of water, or $8\frac{2}{5}$ gals. of mixture, or $\frac{82}{30}$ of the mixture or $\frac{7}{35}$.

257

10. Page 236 (18).

11. Asking price = $\frac{5}{4}$ of cost, and selling price = $\frac{80}{100}$ of asking price = $\frac{80}{100}$ of $\frac{5}{4}$ of cost = $\frac{43}{100}$ of cost. $\therefore \frac{3}{10}$ of cost = \$170. $\therefore \frac{43}{100}$ of cost = $\frac{43}{100}$ of \$170 = \$2436.66 $\frac{2}{3}$.

12. Rent = \$360. Int. = \$180. Insurance = \$37.50. Taxes = \$57. Water rates = \$15. Loss on sale = \$60. Diff. = \$10.50.

13. Page 193 (21).

14. A mows $\frac{6}{11}$ ac. in 1 day, B $\frac{9}{11}$ ac. in 1 day, C $1\frac{2}{3}$ ac. in 1 day. \therefore jointly they mow $(\frac{6}{11} + \frac{9}{11} + 1\frac{2}{3})$ ac., or $\frac{2547}{935}$ ac. in 1 day. \therefore they can mow the field in $(\frac{1547}{100} \div \frac{2547}{935})$ days = 5.679 + days.

15. Page 253 (12).

16. " 183 (39).

SOLUTIONS OF PROBLEMS

17. Cost = $\frac{98\frac{1}{2}}{100}$ of 500 of \$3.77 = \$1856.72 $\frac{1}{2}$.

18. Page 236 (10).

258

1. Page 234 (7).

2. " 217 (19).

3. " 157 (7).

4. " 215 (26).

5. $\frac{5}{9}$ remainder - \$1200 = \$1800. \therefore remainder = \$5400, and $\frac{3}{5}$ money - \$600 = \$5400. \therefore money = \$10000.

6. Time required to plough 1 strip of 40 rods = $\frac{60 \times 40}{9 \times 60}$ min. = 2 $\frac{1}{2}$ min. \therefore time required to plough 1 strip and to turn =

4 $\frac{1}{2}$ min. \therefore in 10 hrs. he ploughs in length $\frac{40 \times 600}{4\frac{1}{2}}$ rods = $\frac{16000}{3}$ rods. \therefore no. of acres ploughed = $\frac{16000}{3} \times \frac{8}{12 \times 3 \times 5\frac{1}{2}}$

$\times \frac{1}{160}$ = ans.

7. By selling with true weight he should get 123 $\frac{1}{2}$ $\frac{7}{11}$ % of 15 $\frac{1}{16}$ of 40c. By selling with false weight he gets 123 $\frac{1}{2}$ $\frac{7}{11}$ % of

40c. Gain by dishonesty = 123 $\frac{1}{2}$ $\frac{7}{11}$ % of $\frac{3}{8}$ of 40c. = $\frac{6}{5}$ c. $\frac{6}{5}$ c. is the gain when he buys 15 $\frac{1}{16}$ oz. \therefore 3250c. is the gain when

he buys $\left(\frac{15\frac{1}{16}}{16} \times \frac{3250}{1} \div \frac{65}{28}\right)$ lbs. = 1334 $\frac{2}{3}$ lbs.

8. A gains 1 mile in 34 miles. \therefore A gains in 10 miles $\frac{10}{34}$ mile = $\frac{5}{17}$ mile.

9. Page 253 (12).

10. Selling price of wheat = \$(5000 \times .70) or \$3500. Selling commission = $\frac{3}{100}$ of \$3500 or \$105. Net proceeds = \$(3500 -

105 - 380) or \$3015. Val. of draft = $\frac{100}{100\frac{1}{2}}$ of \$3015 or \$3000.

259

11. The two pipes fill $\left(\frac{1}{30} + \frac{1}{35}\right)$ of the cistern in one min. \therefore the two pipes fill $\frac{2}{35}$ of the cistern in 12 min. \therefore there is $\frac{9}{35}$ of the cistern yet to fill. The first pipe fills the cistern in 30 min. \therefore the first pipe fills $\frac{9}{35}$ of the cistern in $\frac{9}{5}$ of 30 min. = 7 $\frac{5}{7}$ min. \therefore the time required = $(12 + 7\frac{5}{7})$ min. = 19 $\frac{5}{7}$ min.

12. Take 205 shares of stock. \therefore dividend = \$205 \times 3. \therefore

no. of shares purchased by income = 6. \therefore total number now held = 211 shares. \therefore next dividend = \$633, but the dividend was \$316.50. \therefore the shares held at first were $\frac{633}{316\frac{1}{2}}$ of 205 = $102\frac{1}{2}$. \therefore money invested was $\$(102\frac{1}{2} \times 103) = \10557.50 .

13. No. of times equals the integral part of $23.846 \text{ quarts} \div .04679 \text{ gals., \&c.}$

14. Or 27 horses, 42 cows, 120 sheep for 1 week. 8 horses eat as much as 12 cows, or as 21 sheep. \therefore 27 horses eat $\frac{27 \times 21}{8}$ times as much as 1 sheep, and 42 cows eat $\frac{42 \times 21}{12}$ times as much as 1 sheep, and 120 sheep eat 120 times as much as 1 sheep. \therefore money divided in the proportion of $\frac{27 \times 21}{8}$, $\frac{42 \times 21}{12}$, 120, or as 567, 588 and 960. \therefore A pays $\frac{567}{2117}$ of $\$132.18\frac{3}{4} = \$35.13\frac{1}{4}$.

15. Page 146 (23).

16. " 154 (15).

17. The sum of their rates in yds. per sec. = $\frac{99 + 132}{6\frac{3}{4}}$. The diff.

of their rates in yds. per sec. = $\frac{99 + 132}{47\frac{1}{4}}$. \therefore twice the rate of the faster train = $\frac{352}{9}$ yds. per sec. = $\frac{352}{9} \times \frac{1}{1760} \times 3600$ miles per hr. = 80 miles. \therefore the rate = 40 miles.

18. The men and the boys do $\frac{1}{3}$ of work in 1 day, and $\frac{2}{3}$ of men and the boys do $\frac{1}{10}$ of work in 1 day. \therefore $\frac{1}{3}$ of men do $(\frac{1}{3} - \frac{1}{10})$, or $\frac{1}{40}$ of work in 1 day. \therefore the men do $\frac{24}{40}$ of the work in 8 days. \therefore the boys do $\frac{16}{40}$ of the work in 8 days. \therefore the works are as 3 : 2.

19. Page 175 (17).

260

1. Page 253 (12).

2. A dividend of \$100 produces in Detroit by direct exchange $\$(100 - 1\frac{1}{4}) = \98.75 . It produces in New York $\$(100 - 1 - \frac{1}{2}) = \$98\frac{1}{2}$. Again, \$100 in New York yields $\$(101\frac{1}{2} - \frac{1}{2}) = \101 in Detroit. \therefore the dividend of \$100 produces via New York in Detroit $\$(\frac{101}{100} \times 98\frac{1}{2}) = \$99.48\frac{1}{2}$.

3. Duty on 1 gal. = $\frac{1}{8}$ of $\frac{5}{8}$ of \$1.75 = $\$.31\frac{1}{8}$. \therefore no. gals = $\$268.80 \div \$.31\frac{1}{8} = 864$. \therefore no. gals in 1 cask = $864 \div 24 = 36$.

4. Page 166 (31).

5. " 227 (20).

6. " 217 (22).

SOLUTIONS OF PROBLEMS

7. Page 155 (19).
8. $\frac{2}{5}$ of $\frac{5}{7}$ of the cost = \$714. \therefore the cost = \$734.40.
9. With all the taps running ($\frac{1}{24} + \frac{1}{18} - \frac{1}{20} - \frac{1}{30}$) of the cistern is emptied in 1 min. \therefore the whole cistern is emptied in 72 min.
10. $\frac{37}{100}$ of an ounce of silver is worth $61\frac{1}{2}$ d. $\therefore \frac{9}{10}$ of $\frac{41}{10}$ oz. of silver is worth 2s. 11.5 + d.
11. If \$4000 gains \$960 for a certain time. \therefore \$6500 gains \$1560 for a certain time. \therefore gain on \$6500 is \$(2340 - 1560) for 3 mos. \therefore gain on \$6500 is \$2340 for 9 mos., &c.

261

12. Page 253 (12).
13. Suppose the cask contains 4 gals. wine and 3 gals. water. After the operation the cask will contain 3 gals. wine and 4 gals. water. \therefore 1 gal. of wine must be drawn off, and with it $\frac{3}{4}$ gal. water, or $1\frac{3}{4}$ gals. altogether. \therefore the part drawn off equals $1\frac{3}{4} \div 7$ or $\frac{1}{4}$.
14. Income at first = $\frac{9}{100}$ of \$11771.25 = \$706.27 $\frac{1}{2}$. \therefore income after transfer = \$706.27 $\frac{1}{2}$ + \$6.57 = \$712.84 $\frac{1}{2}$. \therefore money invested in latter stock = $\frac{107\frac{1}{2}}{7}$ of \$712.84 $\frac{1}{2}$. \therefore price of 1st stock = $\frac{107\frac{1}{2}}{7}$ of \$712.84 $\frac{1}{2} \div \$117.7125 = 93$. After the rise amount of sale = $\frac{\$11771.25}{100} \times 95$. Income from 2nd stock = $\frac{7}{109\frac{1}{2}}$ of $\frac{\$11771.25}{100} \times 95 = \$714.87\frac{1}{2}$. \therefore the diff. in income = $\$(714.87\frac{1}{2} - 706.27\frac{1}{2}) = \8.60 .
15. Draft cost $101\frac{1}{4}\%$ of \$8000 = \$8100. $\frac{96}{100}$ of the sale - \$300 = \$8100. \therefore sale = \$8750. \therefore cost per barrel = \$8750 \div 2500 = \$3.50.
16. He travels a mile at first rate in $\frac{2}{3}$ hr., and he travels a mile at 2nd rate in $\frac{4}{17}$ hr. \therefore in going a mile at 2nd rate he gains ($\frac{4}{17} - \frac{2}{3}$) hrs. \therefore he gains 24 min. in going $7\frac{1}{4}$ miles.
17. Page 188 (2).
18. " 179 (38).
19. Let \$100 be the face of the note. Bk. disc. = $\frac{3}{100} \times \frac{1}{5}$ of \$100 (as 73 days = $\frac{1}{5}$ year) = \$1 $\frac{3}{5}$. \therefore P. W. = \$98 $\frac{2}{5}$. \therefore int. on \$98 $\frac{2}{5}$ for $\frac{1}{5}$ year is \$1 $\frac{3}{5}$. \therefore int. on \$100 for 1 year is \$8 $\frac{16}{125}$. \therefore per cent. = $8\frac{16}{125}$.
20. Wt. of silver : wt. of gold :: 1 : 4. 1 oz. silver is worth

IN THE HIGH SCHOOL ARITHMETIC.

\$1.10. 4 oz. gold is worth \$68. \therefore 5 oz. coin is worth \$69.10.
 \therefore \$10 coin weighs $\frac{50}{89.10}$ oz. = $\frac{500}{891}$ oz.

21. Theory.

262

1. After 1st drawing $\frac{2}{3}$ cask is wine. After 2nd drawing $\frac{2}{3}$ of $\frac{2}{3}$ cask is wine, &c. After 5th drawing $(\frac{2}{3})^5$ cask is wine = $\frac{32}{243}$.

2. Page 253 (12).

3. $(1\frac{1}{2} + 1\frac{1}{4})$ days = $2\frac{3}{4}$ days. In 15 days A does $\frac{1}{4}$ of the work. \therefore in $2\frac{3}{4}$ days A does $\frac{11}{10}$ of the work. Altogether A does $(\frac{3}{4} + \frac{11}{10})$ of the work = $\frac{71}{20}$ of the work. \therefore B does $\frac{9}{20}$ of the work in $1\frac{1}{2}$ days. \therefore B can do the whole work in $13\frac{1}{3}$ days.

4. A draft for \$100 cost $\$(100 + \frac{1}{2} + \frac{1}{4})$, or $\$100\frac{3}{4}$. \therefore a draft for \$4000 cost \$4030. \therefore 98% of total proceeds = \$4030. \therefore proceeds = \$4112.24.

5. Selling price = $\frac{11}{10}$ of $\frac{4}{3}$ of cost = $\frac{44}{30}$ of cost. \therefore gain per cent. = $\frac{14}{30} = 46\frac{2}{3}\%$.

6. For 75 days' work of 1st, and 40 days' work of 2nd, they receive \$305, and for 24 days' work of 1st, and 40 days' work of 2nd, they receive \$152. \therefore for 51 days' work the 1st receives \$153. \therefore for 1 day's work the 1st receives \$3.

7. If 1st had the same salary as 2nd he would be $\frac{97\frac{1}{2}}{100}$ of \$20 worse off than he is. \therefore (25 - 6) mill rate on 2nd salary = $\frac{97\frac{1}{2}}{100}$ of \$20 + \$1.40. \therefore 2nd salary = \$1100.

8. Cash price = $(\frac{100}{103}$ of \$510) $\times \frac{4}{5} \times \frac{9}{10} = \360 .

9. Assignee's charges = $\frac{5}{100}$ of \$8850 = \$265.50. \therefore amount to be divided = $\$(8850 - 265.50 - 59) = \8825.50 . A gets $\frac{125}{177}$ of \$8825.50 = \$3612.50.

263

10. Numerator = $\frac{2}{5}$ denominator. \therefore $\frac{11}{12}$ denominator = 352. \therefore denominator = 160. \therefore numerator = 192. \therefore fraction = $\frac{192}{160}$.

11. 1 lb. troy sells for $\frac{2}{5}$ of \$2.40 = \$2.88. \therefore 5760 grains sell for \$2.88. \therefore 437 $\frac{1}{2}$ grains sell for $21\frac{7}{8}$ ¢.

12. Page 253 (12).

13. " 259 (17).

14. Sale of 1st stock = $\frac{77\frac{1}{2}}{100}$ of \$9144. Inc. from 1st stock = $\frac{8}{100}$ of \$9144 = \$365.76. \therefore inc. from 2nd stock = \$365.76 +

SOLUTIONS OF PROBLEMS

\$6.24 = \$372. \therefore no. shares of 2nd stock = $\frac{372}{74\frac{2}{5}}$ or $74\frac{2}{5}$. \therefore price of 1st stock = $\frac{77\frac{1}{2}}{100}$ of $\$9144 \div 74\frac{2}{5} = \$95\frac{1}{4}$. Under the new conditions the sale of 1st stock = $\frac{75\frac{1}{2}}{100}$ of $\$9144$, and income from 2nd stock = $\frac{5}{8}$ of $\frac{75\frac{1}{2}}{100}$ of $\$9144 = \$359.56\frac{1}{8}$. Diff. in income = $\$(365.76 - 359.56\frac{1}{8}) = \$6.19\frac{1}{8}$.

15. Selling price of apples = $\$7.25 \times 3200 = \23200 . Cost of draft = $99\frac{1}{2}\%$ of $\$22500 = \22387.50 . Selling price of apples less com. = $\$22387.50 + \$232.50 = \$22620$. \therefore com. = $\$(23200 - 22620) = \580 . \therefore Rate = $\frac{\$580}{\$22620} = 2\frac{1}{2}\%$.

16. The men do $(7 \times 20 + 6 \times 16)$ days' work = 236 days' work. In 20 days' work for 1 man, $32\frac{1}{2}$ yds. are dug. \therefore in 236 days' work for 1 man, $377\frac{3}{8}$ are dug. \therefore the boys dig $(452 - 377\frac{3}{8})$ yds = $74\frac{5}{8}$ yds. In 90 days' work for 1 boy, $74\frac{2}{5}$ yds. are dug. In 20 days' work for 1 boy, $16\frac{4}{5}$ yds. are dug. \therefore work of a man : work of a boy :: $32 : 16\frac{4}{5}$, or as 60 : 31.

17. Page 260 (11).

18. $\frac{7}{8}$ of cost of 2nd house = $\$3640$. \therefore 2nd house cost $\$4160$. \therefore $\frac{1}{10}$ of cost of 1st house = $\$4160$. \therefore 1st house cost $\$3200$.

19. The interest in each case is the same. \therefore the diff. in the amounts equals the principal. \therefore the principal = $\$(741 - 416) = \325 .

264

1. $\$480$ will buy 1500 yards. \therefore selling price of 220 yds. = gain on 1500 yds. But selling price of 220 yds. = $\$(220 \times .32)$ + gain on 220 yds. \therefore gain on 1280 yds = $\$(220 \times .32)$. \therefore gain on 1 yd. = $5\frac{1}{2}\text{c}$. \therefore gain per cent. = $\frac{5\frac{1}{2}}{32}$ or $17\frac{3}{8}\%$

2. Duty = $(\frac{1}{2}\text{c.} \times 7200 + \frac{1}{4} \text{ of } 6\text{c.} \times 7200) = \$36 + \$108 = \144 .

3. Page 253 (12).

4. Bk. disc. = $\frac{1\frac{1}{2}}{100}$ of bill True disc. = $\frac{1\frac{1}{2}}{101\frac{1}{2}}$ of bill. \therefore $(\frac{1\frac{1}{2}}{100} - \frac{1\frac{1}{2}}{101\frac{1}{2}})$ of bill = 50c. \therefore bill = $\$2255\frac{5}{8}$.

5. No. barrels = $\frac{100}{102\frac{1}{2}}$ of $\$6150 \div \$6 = 1000$. Total cost = $\$6150 + \$250 = \$6400$. Selling price = $\frac{2}{3}$ of $\$6400 = \7360 . Price per barrel = $\$7.36$.

IN THE HIGH SCHOOL ARITHMETIC.

6. Amount received = $\frac{100}{100\frac{1}{4}}$ of $\frac{98\frac{3}{4}}{100}$ of \$8000 = \$7880.29.
7. Whole capital = \$13000. C gets $\frac{1}{3}$ of \$13000 = \$4333 $\frac{1}{3}$.
 \therefore he pays A \$8000 - 4333 $\frac{1}{3}$ = \$3666 $\frac{2}{3}$.
8. 10 of A's steps = $10 \times \frac{10}{9}$ of B's = $11\frac{1}{9}$ of B's steps. \therefore A wins the race. In running $11\frac{1}{9}$ yards A gains $\frac{1}{9}$ yd. \therefore in running 100 yards A gains 1 yd.
9. Page 165 (7).
10. Cost of farm at end of the year = $\frac{100}{100}$ of \$4000 = \$4240. Cost of taxes at the end of year = $\$ \left(\frac{104\frac{1}{2}}{100} \times \frac{100}{1000} \times \frac{3}{8} \times 4000 \right)$ = \$45.14 $\frac{1}{2}$. Cost of repairs at the end of the year = $\frac{100}{100}$ of \$500 = \$515.00. Total cost = \$4800.14 $\frac{1}{2}$. Gain = \$(5500 - 4800.14 $\frac{1}{2}$) or \$699.85 $\frac{1}{2}$.
11. Premium received = $\frac{7}{100}$ of risk. Premium paid = $\frac{2}{100}$ of $\frac{3}{8}$ of risk = $\frac{3}{400}$ of risk. \therefore net premium = $\frac{1}{100}$ of risk = \$4.30. \therefore risk = \$1720.

265

12. Duty = $2\frac{1}{2}\% \times \frac{88}{100} \times 3825$ = \$84.15. Total cost = 4c. \times 3825 + \$36.25 + \$84.15 = \$273.40.
13. Page 260 (9).
14. " 260 (10).
15. No. who receive honors = $8\frac{1}{3}\%$ of 120 = 10. No. who pass = 80% of 110 = 88. \therefore No. who fail = 22. \therefore percentage = $\frac{22}{110}$ or $18\frac{1}{5}\%$.
16. Page 253 (12).
17. P. W. of 1st tender = \$11000 \div 1.02 5 . P. W. of 2nd tender = $\$ \frac{10000 \left\{ \frac{1.02^5 - 1}{.02} \right\}}{1.02^5}$. Diff. = \$340 nearly.
18. Page 261 (14).
19. $99\frac{1}{2}\%$ of $97\frac{3}{4}\%$ of total sales = \$2736.25 + \$179.10. \therefore total sales = \$3000. \therefore no. barrels sold = 4000.
20. Page 224 (6).

266

1. A at the rate of 6 miles an hour goes 12 miles in 2 hrs., the distance B travelled before he met A. \therefore the time it took B to walk 12 miles = $(12 \div 4\frac{1}{2})$ hrs. or $2\frac{2}{3}$ hrs. A walks $(5 \times 2\frac{2}{3})$ miles before he meets B = $13\frac{1}{3}$ miles. B takes $(13\frac{1}{3} \div 4)$ hrs. to go this distance or $3\frac{1}{3}$ hrs. A's time = $(2\frac{2}{3} + 2)$ hrs. = $4\frac{2}{3}$

SOLUTIONS OF PROBLEMS

hrs. B's time = $(2\frac{2}{3} + 3\frac{1}{3})$ hrs. = 6 hrs. Distance = $(12 + 13\frac{1}{2})$ miles = $25\frac{1}{2}$ miles.

2. Page 148 (47).

3. " 236 (10).

4. $\frac{9}{100}$ of cost = $\$(210 - 199.50)$. \therefore cost = $\$175$.

5. Page 253 (12).

6. The square of the greater number = 35643×3 . \therefore the greater number = 327.

7. 6 mos. credit price = $\frac{7}{8}$ of cost. 12 mos. credit price = $\frac{100}{92\frac{1}{2}}$ of $\frac{7}{8}$ of cost = $\frac{56}{87}$ of cost. Cash price = $\frac{7}{8}$ of $\frac{56}{87}$ of cost = $\frac{49}{87}$ of cost. \therefore the cost, cash and 12 mo. credit prices are in the proportion of 37, 49, 56. Cost of goods = $\frac{37}{49}$ of $\$6.66$. 12 mo. credit price = $\frac{56}{49}$ of $\$6.66$. Gain = $\frac{19}{100}$ of $\$6.66$ = $\$2.58 +$.

8. No. lbs. sugar = $\frac{100}{102}$ of $\frac{99\frac{3}{4}}{100}$ of $\frac{99\frac{1}{2}}{100}$ of $\$4000 \div 6c.$ = 64870 lbs., 2 oz.

9. There are (48×30) days' work = 1440 days' work. 48 men in 3 days do 144 days' work. \therefore at the end of 3 days there are $(1440 - 144)$ days' work to do = 1296. \therefore at the end of 6 days there are $(1296 - 138)$ days' work to do, &c. At the end of 39 days there are 36 days' work to do and 22 men to do it. \therefore it takes them $1\frac{7}{11}$ days. \therefore total time = $40\frac{7}{11}$ days.

267

10. Page 216 (11).

11. Int. on $\$1$ at 1% for 25 yrs. = $\$1$. \therefore int. on $\$1$ at 4% for 25 yrs. = $\$1$.

12. Premium on $\$4000$ at 2% = $\$80$. Premium on $\$6000$ at $2\frac{1}{4}\%$ = $\$135$. Premium paid = $\$215$. Premium received = $\$215 + \$26.50 = \$241.50$. $\frac{7}{100}$ of risk = $\$241.50$. \therefore risk = $\$13800$.

13. P.W. = $\frac{\$700}{.05} (1 - \frac{1}{1.05^4}) = \$2482.16 +$.

14. C's stock = $\$(1534 - 456 - 546) = \532 . $\$532$ in 8 mo. gains $\$108.80$. \therefore $\$456$ in 7 mo. gains $\$81.60$.

15. True disc. off $\$1$ for 4 yrs. at $3\frac{1}{2}\%$ = $\$1\frac{1}{4}$. True disc. off $\$1$ for 6 yrs. at $2\frac{1}{2}\%$ = $\$1\frac{1}{2}$. \therefore If the true discounts are to be equal, the 1st sum must be to the 2nd sum as $\frac{1}{1\frac{1}{4}} : \frac{1}{1\frac{1}{2}}$ or as 171 : 161. \therefore 1st sum = $\frac{171}{322}$ of $\$1660$ or $\$855$.

16. Let a = one side, and $a + 8$ the other. Then $a^2 + (a + 8)^2 = 1600$; $a^2 + 8a = 768$; but $a(a + 8)$ = twice area. \therefore area = 384 sq. in.

17. P. W. of cost = $\frac{100}{104}$ of \$520 = \$500. Cost of goods at end of 3 mo. = $\frac{100}{108}$ of \$500 = \$510. $\frac{10}{11}$ of \$575.96 = \$523.60 = cost at the end of the term of credit. \therefore \$510 at 8% amounts to \$523.60. \therefore time of credit = 4 months.

18. $\frac{97}{100}$ of total taxes = \$33950. \therefore total taxes = \$35000. Rate = $\$35000 \div \$2800000 = 1\frac{1}{4}\%$. 12 $\frac{1}{2}$ mills on the dollar.

19. Time at Picton is same as that of a place 75° west. Diff. in longitude 77°; diff. in time $\frac{77}{15}$ hrs. = 5 hrs. 8 min. 3.45 a.m. = 15.45 p.m. of day before. Time at Picton = (15.45 - 5.08) p.m. = 10.37 p.m.

20. Take 4 lbs. of better quality of tea at \$1 a lb. Cost of mixture = \$4 + \$4 = \$8. \therefore selling price = $\frac{11}{10}$ of \$8 = \$8.80. \therefore gain = \$0.80. \therefore gain per cent. = $\frac{80}{800} = 10\%$.

268

1. Quantity of coal oil = $\frac{3}{4}$ of 210 gals. = 157 $\frac{1}{2}$ gals. \therefore $\frac{7}{8}$ of remaining liquid = 157 $\frac{1}{2}$ gals. \therefore remaining liquid = $\frac{8}{7}$ of 157 $\frac{1}{2}$ gals. = 180 gals. \therefore 30 gals. leaked away.

2. Page 236 (9).

3. " 262 (7).

4. $\frac{2}{3}$ of $\frac{1}{4}$ of invoice price + $\frac{7}{100}$ of $\frac{1}{4}$ of invoice price = \$810. \therefore $\frac{9}{20}$ of invoice price = \$810. \therefore invoice price = \$2250.

5. If he sold both at 2% com. he would realize \$16, but as he got \$26, he made the \$10 by selling one lot at 4% com. Every \$100 that he handles at 4% he increases the \$16 com. by \$2. \therefore he increases it by \$10 when he handles \$500. Lots \$300, \$500.

6. Cost of 1 lb. of the 1st kind = 30c. Cost of 1 lb. of the 2nd kind = 34c. Cost of 2 lbs. = 64c. Selling price of 2 lbs. = 66 $\frac{1}{2}$ c. Gain per cent. = $\frac{2\frac{1}{2}}{64} = 3\frac{3}{8}\%$.

7. Suppose they travel m hrs. before meeting. Then A travels in 16 hrs. the distance that B travelled in m hrs., and B travels in 25 hrs. the distance A travelled in m hrs. \therefore As rate : B's rate = $m : 16 = 25 : m$. $\therefore m = 20$. \therefore A's time = (16 + 20) hrs. or 36 hrs.

8. Page 143 (16).

9. " 267 (19).

SOLUTIONS OF PROBLEMS

269

10. Cost = \$3. Cash selling price = \$4. Credit selling price = $\frac{9}{8}$ of \$4 or \$4 $\frac{1}{2}$. Gain = \$1 $\frac{1}{2}$. \therefore gain % = $\frac{1\frac{1}{2}}{3}$ or 50%.

11. $\frac{99}{100}$ of selling price of house = $\frac{100\frac{3}{4}}{100}$ of \$6000. \therefore selling price of house = \$6168.36 $\frac{3}{4}$.

12. Page 148 (48).

13. " 262 (3).

14. " 267 (11).

15. 4 $\frac{3}{8}$ % of policy = \$87.50. \therefore policy = \$2000. \therefore val. house = \$2000 - \$87.50 - \$125 = \$1787.50.

16. $\frac{3}{8}$ of $\frac{97}{100}$ of assets + $\frac{3}{100}$ of assets = \$1575. \therefore assets = \$4000.

17 Rate of train for 8 sec. - rate of man for 8 sec. = 88 yds. \therefore rate of train in miles per hr. = 26 $\frac{1}{2}$.

18. Page 267 (20).

19. Val. of tea = \$/1956 - 129 $\frac{1}{4}$ = \$1826 $\frac{3}{4}$. \therefore buying com. = $\frac{1}{20}$ of \$1826 $\frac{3}{4}$ = \$91 $\frac{13}{16}$. \therefore com. for selling butter = \$(129 $\frac{1}{4}$ - 91 $\frac{13}{16}$) = \$37 $\frac{1}{4}$. \therefore val. of butter sold = $\frac{100}{9}$ of \$37 $\frac{1}{4}$ = \$756.

270

1. Area of square = 9 sq. in. \therefore rad. of circle = $\sqrt{\frac{7}{22} \times 9}$ in. \therefore circ. of circle = $\frac{44}{7} \sqrt{\frac{7}{22} \times 9}$ in. = 10.63 in.

2. A's rate = 5.2 $\frac{3}{4}$ \times 2.4 \div 3.7 miles per hr. = ($\frac{471}{90} \times \frac{24}{37}$) miles per hr. Time A takes to go 10 miles = $10 \times \frac{90}{471} \times \frac{37}{24}$ hrs. Time B takes to go 10 miles = $10 \times \frac{90}{471}$ hrs. \therefore no. of sec. start = $10 \times \frac{90}{471} \times \frac{1}{2} \times 3600$ or 3752.

3. Page 236 (9).

4. Cost = \$1.15 \times 3000 = \$3450. Sale = $\frac{11}{10}$ of \$1.04 \times 3000 = \$3432. \therefore loss = \$18.

5. A's gain at the end of the year = $\frac{7}{27}$ of $\frac{51}{4}$ of \$4050 = \$1443.75.

6. $(\$8 \times \frac{101\frac{1}{2}}{100})$ = am't realized from 1 share. \therefore \$406 = am't realized from 50 shares.

7. The Bk. disc. off \$100 for 63 days at 8% = $\frac{\$594}{388}$. \therefore P. W. = \$(100 - $\frac{594}{388}$) = $\frac{\$388596}{388}$. \therefore $\frac{\$388596}{388}$ has a face value of \$100. \therefore \$719.92 has a face value of \$730.

8. Page 179 (37).

9. 40 lbs. at $37\frac{1}{2}$ c. a lb. = \$15. 64 lbs. at 45 c. a lb. = \$28.80.
 \therefore total cost = \$43.80. \therefore total selling price = $\frac{5}{4}$ of \$43.80
 = \$54.75. 24 lbs. at 50 c. a lb. = \$12.00. \therefore 80 lbs. must sell
 for \$42.75, or 1 lb. for 53.4375 c.

10. When the customer pays for 16 oz. he is cheated $\frac{1}{4}$ oz.
 \therefore when he pays out \$16 he is cheated \$1. \therefore when he pays
 out \$50 he is cheated $78\frac{1}{2}$ c.

11. Cost of hats = 325 of \$2.75 = \$893.75. Selling price =
 $\frac{4}{3}$ of $\frac{3}{2}$ of 325 of \$2.75 + $\frac{2}{3}$ of 325 of \$2 = \$903.50. \therefore gain =
 \$(903.50 - 893.75) = \$9.75.

271

12. Net proceeds = $96\frac{1}{2}\%$ of \$9800 - 1400 of 25 c. = \$9107.

13. 1000000 units = $\frac{1}{360} \times \frac{355}{113} \times 7913 \times 63360$ in. \therefore 1 unit
 = 4.375 + in.

14. Page 264 (11).

15. Income after transfer = \$($\frac{5}{8}$ of $\frac{80}{100}$ of 4850) = \$200.
 \therefore income from 1st stock = \$194. \therefore rate = $\frac{194}{4850} = 4\%$.

16. Invoice price = \$100. \therefore cash price = \$90, and selling
 price = $\frac{16}{15\frac{1}{2}}$ of $\frac{121\frac{1}{2}}{100}$ of \$100 = \$125 $\frac{2}{3}$. \therefore gain = \$35 $\frac{2}{3}$, and
 gain per cent = $\frac{35\frac{2}{3}}{90}$ or 39 $\frac{1}{3}\%$.

17. Page 188 (2).

18. Loss = $\frac{1}{20}$ of book debts + $\frac{1}{3}$ of \$24000 = $\frac{7}{20}$ of book debts.
 \therefore book debts = \$26666 $\frac{2}{3}$.

19. \$93 invested at 6% yields \$5.58 inc. Again \$9 is the
 income from \$100 invested. \therefore \$5.58 is the income from \$62
 invested. \therefore stock sells at a discount of \$(100 - 62) = \$38.

20. Rate of man + rate of stream = twice the distance in 1 hr.
 Rate of man - rate of stream = $\frac{3}{2}$ the distance in 1 hr. \therefore rate
 of stream = $\frac{1}{4}$ the distance in 1 hr. = $\frac{1}{2}$ mile per hr. \therefore distance
 = 2 miles.

21. Page 223 (18).

272

1. $\frac{1}{8}$ val. house = $\frac{1}{2}$ val. lot. \therefore val. house = $\frac{5}{2}$ val. lot. \therefore
 $\frac{7}{8}$ val. lot = \$1400. \therefore val. lot = \$400.

2. $\frac{82}{100}$ of cost = \$24.60. \therefore cost = \$30. \therefore gain per cent.
 = $\frac{6}{30}$ or 20%.

SOLUTIONS OF PROBLEMS

3. 5 cows cost \$150, and 1 horse costs \$120. \therefore 5 cows and 1 horse cost \$270. \therefore no. of such groups = $\$10800 \div \$270 = 40$.
 \therefore cows = 200, horses = 40.

4. The extra man works 4 days, and does 8 days' work. The whole work takes 320 days for 1 man. \therefore the fraction of the work performed in 4 days by extra man = $\frac{1}{80}$, or, in 1 day $\frac{1}{80}$ of the work.

5. Page 212 (10).

6. The int. on \$100 for 16 years at 5% = \$80. The int. is \$20 less when sum is \$100. \therefore the int. is \$90 less when sum is \$450.

7. Dividend = $\frac{11}{96}$ of \$2304 or \$36. Money received from sale = $\frac{94\frac{3}{8}}{96}$ of \$2304 or \$2265. Loss = \$(2304 - 2265 - 36) or \$3.

8. Rad. of base = $\frac{7}{4}$ of 9 ft. = $\frac{63}{4}$ ft. \therefore vol. cone = $\frac{1}{3} \cdot 4 \cdot \frac{27}{4}$.
 $(\frac{63}{4})^2$ cu. ft. = $81\frac{3}{4}$ cu. ft.

9. Cost = \$100, gain = \$20, selling price = \$120. \therefore gain % = $\frac{20}{100}$ or 16 $\frac{2}{3}$ %.

10. No. gals. coal oil in 15 gals. mixture = $\frac{4}{5}$ of 15 gals. or 12 gals. \therefore he cheats the customer 3 gals., or 45c.

11. Page 225 (18).

12. " 218 (12).

13. Net assets = $\frac{97}{100}$ of \$540 = \$523.80. \therefore no. cents on the dollar = $\frac{523.80}{30} = 17.46$.

273

14. Suppose each contributed 66 $\frac{2}{3}$ c., the amount would be \$16. \therefore the \$8 must be raised by some of the people contributing \$1.20 each. When 1 person contributes \$1.20 instead of 66 $\frac{2}{3}$ c., 53 $\frac{1}{3}$ c. of the \$8 is raised. \therefore no. of people required to contribute \$1.20 each = $\frac{800}{53\frac{1}{3}}$ or 15.

15. They approach each other at the rate of $(4 + 2\frac{1}{2})$ miles per hr. or 6 $\frac{1}{2}$ miles. \therefore time = $\frac{39}{6\frac{1}{2}}$ hrs. or 6 hrs. The traveller from Toronto will have gone $(2\frac{1}{2} \times 6)$ miles = 15 miles.

16. Page 272 (7).

17. Diff. = \$46 $\left\{ \begin{matrix} 12\frac{1}{2} & 6\frac{1}{4} \\ (1.045) & - (1.09) \end{matrix} \right\} = 92c.$

IN THE HIGH SCHOOL ARITHMETIC.

18. $\frac{5}{8}$ of $\frac{2}{3}$ of the number = 165. \therefore the number = 110.
19. Page 223 (22).
20. " 218 (12).
21. " 225 (18).
22. On \$80 invested he gets \$8 a year. \therefore on \$100 invested he gets \$10 a year. 10%.
23. Page 271 (20).
24. " 231 (19).

1. Page 116 (171).
2. \$400 in 6 mo. gains \$240. \therefore \$400 in 7 mo. gains \$280. If A had kept his money in 7 mo. his share would have been \$680. \therefore A's : B's :: \$680 : \$510 or as 4 : 3. \therefore B's money = $\frac{3}{4}$ of \$400 or \$300.
3. $\frac{9888}{10000}$ of assessable income = \$345.80. \therefore assessable income = \$350. \therefore total income = \$400 + \$350 = \$750.
4. Page 164 (48).
5. Rate down stream = $(12 + 3\frac{1}{2})$ miles per hr. Rate up stream = $(12 - 3\frac{1}{2})$ miles per hr. He can go 1 mile up stream in $\frac{2}{17}$ hr. \therefore he goes up and down 1 mile in $(\frac{2}{17} + \frac{2}{17})$ hrs. = $\frac{4}{17}$ hrs. \therefore in 7 hrs. 24 min. he can travel up and down $40\frac{22}{48}$ miles.
6. The value of one 50c., two 25c., three 10c., four 5c., = \$1.50. \therefore no. of such groups = $\$10.50 \div \$1.50 = 7$.
7. He gives 480 grains for $437\frac{1}{2}$ grains. \therefore loss per cent. = $\frac{42\frac{1}{2}}{480} = 8\frac{1}{8}\%$.
8. Page 212 (1).
9. " 146 (13).
10. 96% of assets = $\frac{96}{100}$ of \$4000. \therefore assets = \$875.
11. Page 218 (9).
12. Net income from 1 share = $\frac{96}{100}$ of \$5 = \$4.90. \$6 is int. on \$100. \therefore \$4.90 is int. on $\$81\frac{2}{3}$. Price of stock = $\$81\frac{2}{3}$.

275

13. Page 274 (5).
14. Loss = $\frac{3}{8}$ of \$3200 - $\frac{1\frac{1}{2}}{100}$ of $\frac{3}{8}$ of \$3200 = \$1179.
15. Page 274 (2).
16. " 239 (14).

SOLUTIONS OF PROBLEMS

17. 40 days' work are worth \$120. \therefore 1 days' work is worth \$3. \therefore 7 days' work are worth \$21. $\therefore \frac{1}{4}$ of $\$(120 - 21) = \$49.50 =$ one man's wages. $\therefore \$49.50 + \$21 = \$70.50 =$ the other man's wages.

18. Page 217 (16).

19. Cost of \$400 st. = $\$(4 \times 104\frac{7}{8}) = \419.50 , and $\frac{1}{2}$ year's income = \$8. \therefore amount received by sale of stock = $\$419.50 - \$8 + \$105 = \516.50 . \therefore amt. recd. for 1 sh. = $\$516.50 \div 4 = \$129\frac{1}{8}$. \therefore market value = $\$(129\frac{1}{8} + \frac{1}{8})$ or $\$129\frac{1}{4}$.

20. $\$250 (1+r)^2 = \300 . $\therefore 1+r = \sqrt{1.2} = 1.095+$. \therefore rate = $9.5+$.

21. When one cistern is empty the other is $\frac{3}{4}$ full. When the first cistern contains the required part, the second contains $\frac{4}{7}$ of this part + $\frac{3}{4}$ of the whole cistern. \therefore 4 times this part = $\frac{4}{7}$ of this part + $\frac{3}{4}$ of the whole cistern. \therefore this part = $\frac{1}{8}$ of the whole cistern. \therefore the part of the first cistern which has run out = $\frac{7}{8}$ of the whole cistern. \therefore time required = $(\frac{7}{8} \times 4)$ hrs. = $3\frac{1}{2}$ hrs.

22. $\frac{4}{5}$ of selling price = $\frac{3}{4}$ of cost. \therefore selling price = $\frac{15}{8}$ of cost. \therefore loss = $\frac{1}{8}$ of cost, or $6\frac{1}{4}\%$.

276

1. The person walks $1\frac{2}{3}$ miles in 20 min. Coach travels $1\frac{2}{3}$ miles in 10 min., or 10 miles an hr. Coach might have travelled 5 miles while it is resting. Suppose they are together first at C, and afterwards at D. $BD = 11\frac{1}{3}$ miles, and as the coach travels twice as fast as the man, $CD = (22\frac{2}{3} + 5)$ miles. $\therefore AB = (1\frac{2}{3} + 22\frac{2}{3} + 5 + 11\frac{1}{3})$ miles = 40 miles.

2. Page 274 (5), 237 (19).

3. " 214 (12).

4. Sale of stock = $\$500 \times 75 \times \frac{64}{100}$. No. of shares bought = $\$500 \times 75 \times \frac{64}{100} \div \$105 = 228\frac{4}{7}$.

5. Page 178 (22).

6. " 251 (19).

7. " 235 (20).

8. Let b = breadth. Area of 4 walls = $(2b + 50) 12\frac{1}{2}$ sq. ft. Area floor = 25b. $\therefore 25b = \frac{4}{5} (2b + 50) \frac{25}{2}$. $\therefore b = 20$ ft. No. of sq. yds. of floor = $\frac{500}{2}$, but 1 yd. carpet contains $\frac{10}{9}$ sq. yds. \therefore no. yds. of carpet = 50. 50 yds. at 80c. a yd. = \$40.

9. Page 268 (7).

10. Assignee's charges = $\frac{1}{20}$ of \$4500 or \$225. \therefore amount of assets for ordinary creditors = \$(4500 - 2000 - 225) = \$2275. On \$4000 he pays \$2275. \therefore on \$1 he pays 56.875c.
 11. \$112 invested yields \$8. \therefore \$100 invested yields $\$11\frac{2}{3}$ = \$7 $\frac{1}{3}$ or 7 $\frac{1}{3}\%$.

277

12. Page 275 (20).
 13. " 219 (19).
 14. $\frac{1}{2}\frac{2}{3}$ (\$150 - $\frac{7}{8}$ B's money) = $\frac{1}{2}$ B's money. \therefore B's money = \$25.
 15. Page 235 (20).
 16. The fast train goes 20 miles farther than the other, when both together go (30 + 50) miles. \therefore the fast train goes 135 miles farther when both together go 540 miles.
 17. Page 216 (1). 188 (2).
 18. 1st Int. - 1st Disc. = \$11 $\frac{2}{3}$. 2nd Int. - 2nd Disc. = \$40. But 2nd Int. = 2 (1st Int.) \therefore 2 (1st Disc.) - 2nd Disc. = \$16 $\frac{2}{3}$. Again Int. on 1st Disc. : Int. on 2nd Disc. = 11 $\frac{2}{3}$: 40, or Int. on 1st Disc. for 8 yrs. : Int. on 2nd Disc. = 23 $\frac{1}{3}$: 40. \therefore 1st Disc. : 2nd Disc. = 7 : 12. \therefore 2 (1st Disc.) - $\frac{1}{2}$ (2nd Disc.) = \$16 $\frac{2}{3}$. 1st Disc. = \$350. \therefore int. on \$350 for 4 yrs. is \$11 $\frac{2}{3}$. \therefore int. on 100 for 1 yr. is \$5. \therefore rate = 5%. Int. for 4 yrs. at 5% is \$350 + \$11 $\frac{2}{3}$ = \$361 $\frac{2}{3}$. The int. on \$100 is \$20. \therefore amount = $\frac{7}{10}$ of \$100 = \$350.
 19. One share costs \$92, and 8% of \$92 = \$7.36. \therefore one share of stock has a dividend of \$7.36.
 20. Compound int. = \$32 { (1.02)¹⁵ - 1 } = \$11.06 +.
 21. $\frac{1}{4}$ of 1st cistern is emptied in 1 hr. $\frac{1}{4}$ of 2nd cistern is emptied in 1 hr. \therefore $\frac{1}{4} - \frac{1}{4}$, or $\frac{1}{20}$ of volume of 1st cistern remains in 1st more than in 2nd at the end of 1 hour. But the difference at the end of the required time is equal to the amount remaining in 2nd cistern. \therefore the no. of hours \times ($\frac{1}{4} + \frac{1}{20}$) = 1. \therefore no. of hours = 3 $\frac{1}{3}$. See page 275 (21).
 22. Draw figure, drop \perp from top of small wheel on the vertical dia. of large wheel. Diameter of large wheel = ($19\frac{5}{8}$ + $10\frac{1}{2}$) in = 63 in. Circumference of large wheel = ($2\pi \times 63$) in. = 198 in. No of revolutions of large wheel in a mile = $\frac{5280}{198}$ = 320. No of revolutions of small wheel in a mile = $\frac{43360}{63360}$ = 1920. Difference = 1600 revolutions.
 $\frac{2\pi}{7} \times \frac{21}{2}$

1. The problem has 3 distinct stages. 1st. C's age = $5\frac{2}{3}$ B's age. 2nd. C's age now = C's age at 1st + A's age, and B's age now = B's age at 1st + A's age. \therefore C's age at 1st = B's age at 1st + 2 A's age. \therefore A's age = $4\frac{2}{3}$ B's age at 1st. 3rd. A's age + 1 year = $\frac{6}{7}$ B's age at 2nd = $\frac{6}{7}$ (B's age at 1st + A's age). \therefore $\frac{1}{7}$ A's age + 1 yr. = $\frac{6}{7}$ B's age at 1st. \therefore $\frac{2\frac{2}{3}}{7}$ B's age at 1st + 1 yr. = $\frac{6}{7}$ B's age at 1st. \therefore B's age at 1st = 5 yrs., &c.

2. Page 235 (20).

3. Front wheel gains $3\frac{1}{2}$ ft. while hind wheel goes 11 ft. Front wheel gains $7\frac{1}{2}$ ft., or 1 revolution while hind wheel goes $1\frac{2}{3}$ ft. Front wheel gains 250 revolutions while hind wheel goes $(250 \times 1\frac{2}{3})$ ft. = 589 $\frac{2}{3}$ ft.

4. Page 164 (7).

5. " 277 (20).

6. " 275 (21).

7. Intended to sell 1 lb. for $\frac{6}{7}$ of 10c. or 12c. Really sold 1 lb. for $\frac{16}{16\frac{1}{2}}$ of $\frac{6}{7}$ of 10c. or $11\frac{7}{11}$ c. Gained $\frac{4}{11}$ c. less on 1 lb. \therefore gained \$6 less on 1650 lbs.

8. Firstly $\frac{1}{6}$ of cost = selling price. Secondly $\frac{1}{10}$ of cost - $\frac{1}{10}$ of 10c. = selling price + 10c. \therefore $\frac{1}{10}$ of cost - 16c. = $\frac{1}{6}$ of cost + 10c. \therefore $\frac{1}{30}$ of cost = 26c. \therefore cost = 60c.

9. A walks 25 miles in $(25 \div 5\frac{1}{4})$ hrs. or $4\frac{6}{11}$ hrs. \therefore B walks 25 miles in $4\frac{6}{11}$ hrs. + $\frac{5}{6}$ hr. or $5\frac{2}{3}$ hrs. \therefore B's rate = $(25 \div 5\frac{2}{3})$ miles per hr. = $\frac{23}{7}$ miles per hr. = $4\frac{4}{7}$ miles per hr. A walks 30 miles in $(30 \div 5\frac{1}{4})$ hrs. or $5\frac{5}{11}$ hrs. B is walking $(5\frac{5}{11} + \frac{5}{6})$ hrs. or $\frac{41}{6}$ hrs. \therefore dist. B walks = $\frac{41}{6} \times \frac{23}{7}$ miles = $29\frac{1}{7}$ miles. \therefore B is beaten by $\frac{5}{7}$ of a mile.

10. Page 112 (130).

11. " 161 (7).

12. " 275 (20).

13. $\frac{3}{100}$ of his liabilities = \$480. \therefore liabilities = \$16000. \therefore assets = $\frac{27}{100}$ of \$16000 or \$4320.

14. Page 261 (19).

15. In clay A digs 3 rods and B 2 rods a day, and in sand A digs 4 rods and B 5 rods a day. If A worked in clay all the time and B in sand they would dig $(3+5) \times 17$ rods = 136 rods. But as they dig only 120 rods, B must have worked in

IN THE HIGH SCHOOL ARITHMETIC.

clay part of the time. Every day he works in clay he does 3 rods less. \therefore He will do $(136 - 120)$ rods less in $(16 \div 3)$ days or $5\frac{1}{3}$ days. \therefore B worked in sand $(17 - 5\frac{1}{3})$ days or $11\frac{2}{3}$ days. \therefore the length of the ditch which was sand $= 11\frac{2}{3} \times 5$ rods or $58\frac{1}{3}$ rods.

16. Int. on \$20 for certain a time is \$5. Int. on \$20 for $\frac{1}{2}$ of the time $= \$1\frac{2}{3}$. \therefore disc. off $\$21\frac{2}{3}$ is $\$1\frac{2}{3}$. \therefore disc. off \$25 is $\$11\frac{2}{3}$.

17. Page 217 (19).

18. The A train takes 8 hrs. and the B train 6 hrs. to go the whole distance. \therefore the rate of A train : rate of B train as 3 : 4. At 11 a.m. the A train has gone $\frac{5}{8}$ of the distance. \therefore $\frac{3}{8}$ of the distance left after B train starts. B train covers $\frac{4}{3}$ of this distance before they meet. \therefore B train covers $\frac{4}{3}$ of $\frac{3}{8}$ of whole dist. or $\frac{1}{2}$ of the dist. B train goes $\frac{3}{4}$ distance in 1 hr. $17\frac{1}{2}$ min. Time $= 17\frac{1}{2}$ min past 12.

19. Liabilities $= \$1200$, and assets $= \$400$. But \$100 of assets realize only \$40. \therefore on \$1200 of a liability he pays only \$340, on \$1 of a liability he pays only $28\frac{1}{3}\text{c}$.

20. Page 158 (19).

21. $\frac{1}{100}$ of greatest sum $= \frac{1}{100}$ of smallest sum. \therefore greatest sum $= \frac{1}{3}$ of smallest sum. \therefore $\frac{1}{3}$ of smallest sum $= \$47.50$. \therefore smallest sum $= \$142.50$. \therefore greatest sum $= \$ (142.50 + 47.50) = \190 , and $\frac{1}{100}$ of other sum $= \frac{1}{100}$ of \$190 or other sum $= \frac{4}{5}$ of \$190 $= \$152$.

22. Page 218 (19).

23. If both A and B worked as long as C $(\frac{3}{10} + \frac{2}{12})$ more of the work would have been done, i.e., if all worked C's time $(1 + \frac{3}{10} + \frac{2}{12})$ of the work would have been done or $\frac{11}{6}$ of the work. But A, B and C do $(\frac{1}{10} + \frac{1}{12} + \frac{1}{15})$ of the work in 1 day. \therefore A, B and C do $\frac{11}{6}$ of the work in $5\frac{1}{2}$ days. C worked $5\frac{1}{2}$ days, or the work occupied $5\frac{1}{2}$ days.

280

1. $\frac{1}{10}$ of cost of 1st horse $= \frac{9}{10}$ of cost of 2nd horse. And $\frac{1}{10}$ of cost of 2nd horse $= \frac{1}{10}$ cost of 1st horse $= \$4$. \therefore cost of 1st horse $= \$180$.

2. Page 280 (1).

3. " 273 (14).

4. " 260 (9).

SOLUTIONS OF PROBLEMS

5. Duty = £72. \therefore total cost = £792. Net amount of sales = $\frac{9}{100}$ of \$4200 or \$3990. \therefore gain = \$3990 - £792 = \$3990 - \$792 \times 4.86 $\frac{2}{3}$ or \$135.60.

6. Net income from 1 share = $\frac{9}{100}$ of \$4 or \$3.92. \therefore no. shares = $\frac{3.92}{.08}$. Amount realized from sale = $\frac{9}{100} \times 98 =$ \$15000. Net income from new stock = $\frac{9}{100}$ of $\frac{5}{112}$ of \$15000 = \$656.25. \therefore diff. = \$56.25.

7. Page 261 (14).

8. $\frac{1}{200}$ of $\frac{9}{100}$ of sales = \$6.41 $\frac{1}{4}$. \therefore sales = \$1350. Agent received $\frac{5}{100}$ of \$1350 + \$6.41 $\frac{1}{4}$ = \$73.91 $\frac{1}{4}$.

9. $\frac{2}{3}$ of 9 gal. 1 qt. 1 pt. is alcohol or 72 pts. 72 pints is 84% of $\frac{100}{84}$ of 72 pts. = 85 $\frac{5}{7}$ pts. = 10 gal. 2 qt. 1 $\frac{5}{7}$ pts. \therefore water added = 1 gal. 1 qt. $\frac{5}{7}$ pt.

10. Selling price of oats = $\frac{9}{100}$ of \$1500 = \$1800. \therefore selling price of wheat and barley = \$7596. He sells \$100 worth of wheat for \$94. He sells \$300 worth of barley for \$327. \therefore he receives \$421 for wheat and barley when wheat cost \$100. He receives \$7596 for wheat and barley when wheat cost \$1804.27.

281

11. Page 155 (21).

12. " 217 (4).

13. A and C do $\frac{1}{4}$ of the work in 3 days. \therefore there is $\frac{3}{4}$ yet to do. A, B and C do $\frac{1}{2}$ ($\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$) in 1 day. \therefore A, B and C do $\frac{3}{4}$ in 4 $\frac{3}{4}$ days. \therefore whole time = 7 $\frac{3}{4}$ days.

14. Page 112 (130).

15. P. W. of \$5 = $\frac{100}{103\frac{1}{2}}$ of \$5 = \$4.83. \therefore credit is the better way.

16. Page 212 (10).

17. $(\frac{9}{100})^4$ of the original value = \$4197.61 $\frac{1}{2}$. \therefore original value = \$5859.37 $\frac{1}{2}$.

18. $\frac{9}{100}$ of $\frac{4}{5}$ of selling price per lb. = $\frac{1}{100}$ of 5c. \therefore selling price per lb. = 8 $\frac{1}{2}$ c.

19. Amount distributed = \$74537.50 + \$94567.50 - \$107-963.00 - \$7397.00 = \$53745.00. Rate = \$53745 \div \$895750 or 6%.

20. Page 218 (9).

282

1. Income from latter stock = \$(4 \times $\frac{9}{100}$ \times 156) = \$480. \therefore

IN THE HIGH SCHOOL ARITHMETIC.

income from former stock = $$(480 - 12) = \468 . \therefore rate = $\frac{\$468}{\$15000}$ or 3%. See page 261 (14).

2. Cash value paid for goods = $\frac{100}{101\frac{1}{2}}$ of \$304.50. Selling price = $\frac{125}{100}$ of $\frac{104}{100}$ of $\frac{100}{101\frac{1}{2}}$ of \$304.50 = \$390.

3. Av. of first two = $76\frac{1}{2}$ c. Av. of last two = $96\frac{1}{2}$ c. By selling a bush. of 1st mixture for 80c., there is a gain of $3\frac{1}{2}$ c., and a bush. of the 2nd mixture a loss of $16\frac{1}{2}$ c. \therefore $16\frac{1}{2}$ bush. of 1st mixture will balance $3\frac{1}{2}$ bush. of 2nd mixture, or 33 bush. to 7 bush., or 33 bush. of each of 1st two kinds, and 7 bush. of each of remaining kinds.

4. Cash price = $\frac{98}{100}$ of $\frac{11}{10}$ of cost = $\frac{1078}{1000}$ of cost. \therefore gain = $\frac{78}{1000}$ of cost, or $7\frac{8}{10}\%$.

5. Page 214 (5).

6. P.W. of \$618 for 4 mos. at 9% = $\frac{109}{100}$ of \$618 or \$600. Cash price = $\frac{96}{100}$ of \$618 or \$593.28. \therefore diff. = \$6.72.

7. Page 116 (171).

8. " 158 (13).

9. $\frac{7}{100}$ of \$150 = \$6.09 = $\frac{9}{100}$ of B's money. \therefore B's money = \$49.

10. A receives $\frac{35}{100}$ of the divided profits = \$2100. \therefore the divided profits = \$6000. \therefore total profits = $$(6000 + 800)$ or \$6800. Per cent. = $\$6800 \div \10000 or 68%.

283

11. Let \$100 = invoice price. Purchase val. (cash) = $\frac{100}{104\frac{1}{2}}$ of \$100 = $\frac{20000}{209}$. Gain = $$(100 - \frac{20000}{209}) = \frac{900}{209}$. Gain % = $\frac{\$900}{\$2000} \div \frac{\$2000}{\$209}$ or $4\frac{1}{2}\%$.

12. Page 275 (22).

13. Suppose he borrows \$120. Int. for a year = \$7.20. Gain on stock = $$(7 + 5)$ or \$12. \therefore total gain = $$(12 - 7.20) = \4.80 . \therefore net gain = $\frac{98}{100}$ of \$4.80. \therefore $\frac{98}{100}$ of \$4.80 is the net gain from \$120 borrowed. \therefore \$51.74 $\frac{1}{2}$ is the net gain from \$1320 borrowed.

14. Page 282 (1).

15. 2 lbs. at 30c. and 1 lb. at 60c. make a mixture worth 40c. 4 lbs. at 45c. and 1 lb. at 60c. also make a mixture worth 40c. \therefore 2 lbs at 30c., 4 lbs. at 45c. and 2 lbs. at 60c., or any mixture in the proportion of 1, 2 and 1 make a mix-

SOLUTIONS OF PROBLEMS

ture worth 40c. a lb. \therefore for 144 lbs. of mixture $\frac{1}{4}$ of 144 lbs. or 36 lbs at 30c. &c. (Different sets of answers may be obtained for this problem.)

16. $98\frac{3}{4}\%$ of the policy = the value of the house. $\frac{4}{5}$ of the policy = value of the house - \$750. $\therefore \frac{39}{100}$ of the policy = $\frac{4}{5}$ of the policy + \$750. \therefore policy = \$4000. \therefore premium = \$50 &c.

17. Page 212 (1).

18. A goes (2×9) miles = 18 miles. \therefore B goes 18 miles in $1\frac{1}{2}$ hrs. \therefore rate = 12 miles per hour.

19. Selling price = $\$(617\frac{1}{2} \times 1.87\frac{1}{2}) + \$260.62\frac{1}{2} + \$711.93\frac{3}{4}$
= \$2130.37 $\frac{1}{2}$. \therefore selling price per yd. = \$3.45.

284

1. Page 116 (171).

2. $\frac{98\frac{5}{6}}{100}$ of taxable income = \$470.36. \therefore taxable income = \$477 $\frac{103}{197}$. \therefore total income = \$877 $\frac{103}{197}$. $\therefore \frac{6}{100}$ of investment = \$877 $\frac{103}{197}$. \therefore the investment = $\frac{100}{6}$ of \$877 $\frac{103}{197}$ = £ $\frac{1}{4.86\frac{3}{4}}$

of $\frac{100}{6}$ of 877 $\frac{103}{197}$ = £3005.4s.3.6 + d.

3. Page 168 (3).

4. " 212 (10).

5. " 270 (7).

6. " 154 (15).

7. " 238 (3).

8. " 214 (5).

9. " 282 (3).

10. " 163 (36).

285

11. Page 155 (17).

12. " 212 (1).

13. " 155 (9).

14. For 2 lbs. coffee he receives 48c. \therefore this sum must be balanced by the higher price of tea. He takes in 8c. more for tea, when he sells 1 lb. of each. \therefore he takes in 48c. more for tea when he sells 6 lbs. of each. \therefore 6 lbs. tea. 8 lbs. coffee.

15. Page 216 (11).

16. " 116 (171).

17. " 282 (2).

18. " 282 (9).

19. $\frac{1}{5}$ of cost of one lot - $\frac{1}{5}$ of cost of the other = \$3. \therefore the difference in the cost of the two lots = \$10. $\frac{6}{5}$ of the cost

IN THE HIGH SCHOOL ARITHMETIC.

of the first lot + $\frac{1}{4}$ of the cost of the second lot = \$208, or $\frac{5}{8}$ of the cost of the first lot + $\frac{1}{4}$ of (the cost of the first lot—\$40) = \$208. \therefore the cost of the first lot = \$120.

20. Page 118 (196).

21. " 150 (22).

286

1. Page 158 (16).

2. " 145 (10).

3. " 282 (3).

4. " 282 (2).

5. " 163 (36).

6. " 171 (19).

7. Cash value = $\frac{100}{103}$ of \$2100 + $\frac{100}{104\frac{1}{2}}$ of \$2100 = \$4048.40.

8. A's wages for $14\frac{1}{2}$ days + B's wages for $14\frac{1}{2}$ days = A's wages for 25 days. \therefore A's wages for $10\frac{1}{2}$ days = B's wages for $14\frac{1}{2}$ days. \therefore A's wages for 25 days = B's wages for $34\frac{1}{2}$ days.

9. Page 281 (18).

10. Av. cost per bush. = $\frac{5}{8}$ of $\frac{100}{102}$ of $76\frac{1}{2}$ c. or $62\frac{1}{2}$ c. See also page 282 (3).

287

11. Page 212 (10).

12. Selling price = $\frac{108}{100}$ of $\frac{93}{100}$ of cost. \therefore gain = $\frac{44}{10000}$ of cost = \$22. \therefore cost = \$5000.

13. Page 146 (12).

14. " 216 (11).

15. 8 times A's capital : 6 times B's : 5 times C's = \$72 : \$90 : \$112.50. A's capital = $\frac{2}{3}$ C's capital and B's capital = $\frac{3}{4}$ C's capital. \therefore whole capital = $(\frac{2}{3} + \frac{3}{4} + 1)$ C's capital. \therefore $\frac{17}{12}$ C's capital = \$1550. C's capital = \$750.

16. $\frac{982}{1000}$ of taxable salary = \$491. \therefore taxable salary = \$500. \therefore whole salary = \$900.

17. \$31 is the income from \$100 invested. \therefore \$83.12 is the income from \$2557.69 $\frac{2}{3}$ invested. The stock and the income tax do not affect the solution.

18. Page 189 (9).

19. " 151 (32).

20. " 216 (9).

288

1. $\frac{5}{4}$ of $\frac{5}{9}$ of cost + $\frac{85}{100}$ of $\frac{4}{9}$ of cost = $\frac{193}{180}$ of cost, and $\frac{115}{100}$ of

SOLUTIONS OF PROBLEMS

$\frac{5}{8}$ of cost + $\frac{3}{4}$ of $\frac{4}{5}$ of cost = $\frac{85}{88}$ of cost. Now $\frac{43}{100}$ of cost = \$260.
 $\therefore \frac{8}{88}$ of cost = \$100 loss.

2. Page 112 (130).

3. A receives $\frac{15}{100}$ of original flock + 20 sheep. \therefore there are $\frac{85}{100}$ of original flock - 20 sheep left. B receives $\frac{13}{100}$ ($\frac{85}{100}$ of original flock - 20 sheep) + 70 sheep. \therefore there are $\frac{87}{100}$ ($\frac{85}{100}$ of original flock - 20 sheep) - 70 sheep left. C receives $\frac{10}{100}$ ($\frac{87}{100}$ of original flock - 20 sheep) - 70 sheep } + 72 sheep. \therefore there are $\frac{84}{100}$ ($\frac{87}{100}$ of original flock - 20 sheep) - 70 sheep } - 72 sheep left, which equals $\frac{1}{2}$ of original flock. \therefore original flock = 1200 sheep.

4. Cash selling price = $\frac{100}{102}$ of 68c. or 66 $\frac{2}{3}$ c. \therefore gain = 11 $\frac{2}{3}$ c.

5. Page 248 (3).

6. " 212 (10).

7. Draw figure. 10 meas. of length + 20 meas. of width + 20 \times 10 = 1700, and meas. length = 3 meas. width. \therefore 50 meas. width = 1500. \therefore meas. width = 30, and meas. length = 90. \therefore area of field = 2700 sq. yds.

8. Total receipts = $\frac{10}{100}$ of \$20000000 + \$3500 + \$1760 = \$15260. Teachers' salaries = \$15260 - \$1150 - \$1000 = \$13110. Taxes collected from salaries = $\frac{2}{100}$ of $\frac{3}{4}$ of \$13110 = \$157.32. \therefore net amount expended by city for teachers' salaries and running expenses = \$(13110 - 157.32 - 3500 - 1760 + 1150) = \$8842.68.

9. Bot. 50 oz. for 49c. Sold 50 oz. for $\frac{50}{49}$ of 50c. \therefore on 49c. he gains $\frac{2500}{49}$ c. - 49c. = $\frac{9}{49}$ c. \therefore gain % = $\frac{9}{49}$ c. \div 49c. = $\frac{2296}{4901}\%$.

10. Average to each = \$3. On one man he loses \$2. On 1 woman he gains \$2. On 1 boy he gains \$2 $\frac{1}{2}$. \therefore the loss on 1 man is bal. by the gain on 1 woman, and the loss on 5 men is bal. by the gain on 4 boys, i.e. 11 laborers in all. \therefore 5 such sets will make 55 laborers at \$3 each, or 30 men, 5 women, and 20 boys.

289

11. No. of shares of 3 $\frac{1}{4}\%$ st. = \$4690 \div \$87 $\frac{1}{2}$. \therefore selling price = \$($\frac{4690}{87\frac{1}{2}} \times 90$) = \$4824. Income from 3 $\frac{1}{4}\%$ st. = \$($\frac{4690}{87\frac{1}{2}} \times 3\frac{1}{4}$) = \$174.20. \therefore income from 3% st. = \$(174.20

1. Finally his fortune = $\frac{4}{5}$ of $\frac{4}{5}$ of $\frac{180}{100}$ of original fortune = $\frac{144}{125}$ of original fortune. \therefore rate of gain = $\frac{19}{125}$ or $15\frac{1}{5}\%$.

SOLUTIONS OF PROBLEMS

2. Income from $\frac{1}{4}$ of \$3750 st. at 5% = \$140.62 $\frac{1}{2}$. Income from 3% st. = $\frac{3}{4}$ of $\frac{1}{4}$ of \$3750 $\times \frac{117}{100}$ = \$121.62 $\frac{1}{2}$. \therefore alteration in income = \$9.

3. Net earnings = $\frac{14}{100}$ of $\frac{9}{10}$ of sale - $\frac{1}{10}$ of sale = $\frac{26}{1000}$ of \$(450 \times 125) = \$1462.50.

4. 1 ox sold for \$28, 2 cows sold for \$34, 6 sheep sold for \$45. \therefore 1 lot sold for \$107. \therefore 7 lots sold for \$949. \therefore the drove = 7 oxen, 14 cows, 42 sheep.

5. Total assets = \$(1 $\frac{1}{2}$ \times 365) + \$100 = \$647.50. Expenses = \$60 + \$25 + \$33 + \$11 \times 12 + \$17.50 + \$2 \times 12 = \$291.50 \therefore net assets = \$356.

6. Wt. of water = ($\frac{19}{11}$ \times 30 \times 5 \times $\frac{9}{12}$ \times 1000) oz. = 113636 $\frac{4}{11}$ oz.

7. Page 289 (14).

8. " 145 (10).

9. " 145 (35).

10. Cost of sugar = 6c. \times 150 + 25c. = \$9.25. Selling price = ($\frac{93}{100}$ \times 150) of 8c. = \$11.16. Rate of gain = $\frac{\$11.16 - 9.25}{\$9.25}$ = 20 $\frac{3}{4}$ %.

291

11. 9 marksmen score 12 counts. \therefore the remaining 11 must score 28 counts. If 11 centres are scored the count would be 33. \therefore there were 5 outers and 6 centres.

12. Page 214 (12).

13. 2 men and 5 boys do $\frac{1}{20}$ work in 1 day. 2 men and 16 boys do $\frac{2}{15}$ work in 1 day. \therefore 11 boys do $\frac{11}{180}$ work in 1 day. \therefore 1 boy does the whole work in 180 days. Again 5 boys do $\frac{1}{36}$ work in 1 day. \therefore 2 men can do $\frac{4}{180}$ work in 1 day. \therefore 1 man can do the whole work in 90 days.

14. Page 214 (12).

15. Clear gain = 82 $\frac{1}{2}$ % of $\frac{2}{100}$ Oz \$1700 = \$27.50.

16. From March 10th to Oct. 25th = 229 days. Cost = ($\frac{24}{1000}$ of \$12 + $\frac{3}{8}$ of 40c) \times 229 = \$67.32 $\frac{3}{4}$.

17. Prime cost = \$(25 \times .75) = \$18.75. Freight = \$(25 \times 1.75) = \$43.75. Specific duty = \$($\frac{1}{4}$ \times 25 \times .60) = \$3.75. Ad val. duty = $\frac{1}{4}$ of \$18.75 = \$3.75. \therefore total cost = \$70. Selling price = \$(25 \times 3.50) = \$87.50. \therefore gain = \$17.50.

18. 14 bush. wheat at \$1.50 = \$21. 19 bush. barley at \$4.48 = \$9.12. \therefore 33 bush. mixture cost \$30.12, and by selling 33 bush. at 65 $\frac{1}{2}$ c he gets \$21.61 $\frac{1}{2}$, \therefore he loses \$8.50 $\frac{1}{2}$. By selling

IN THE HIGH SCHOOL ARITHMETIC.

1 bush. oats at $65\frac{1}{2}c.$ he gains $31\frac{1}{2}c.$ \therefore no. of bush. oats = $\$8.50\frac{1}{2} \div 31\frac{1}{2}c. = 27.$
19. Page 248 (9).

292

1. $\$1200$ st. at $87 = \$1044.$ \therefore he can hold as much stock in the 4% 's as the $4\frac{1}{2}\%$, and still have $\$1044$ money. It takes $\$12$ less to buy 1 share of 4% st. than to buy 1 share of $4\frac{1}{2}\%$ st. \therefore it takes $\$1044$ less to buy 87 shares of 4% st. than to buy 87 shares of $4\frac{1}{2}\%$ st. \therefore he can buy 87 shares of $4\frac{1}{2}\%$ st. or $(87 + 12)$ shares of 4% st. with his money. Income from 4% st. = $\$(99 \times 4) = \396 , and income from $4\frac{1}{2}\%$ st. = $\$(87 \times 4\frac{1}{2}) = \391.50 , \therefore difference = $\$4.50$.

2. Page 281 (13).

3. " 145 (35).

4. " 290 (6).

5. 3% of cost = $\$10$, \therefore cost = $\$333.33\frac{1}{3}$.

6. Page 282 (9).

7. If the successful candidate had been promised as many votes as his opponent, and had received 75% of the extra number, he would have added to his majority 75% of 200, or 150 votes. \therefore 5% of the votes promised the defeated candidate = 300 votes. \therefore the total number promised the defeated candidate = 6000 votes. \therefore total number of votes promised = 11800. \therefore $73\frac{1}{2}\%$ of total number of voters = 11800 \therefore total number of voters = 16000.

8. Page 212 (1).

9. It is assumed in the problem that the society is allowed to reduce the amount of its obligation by $\$11200$, the price obtained for the 4 houses. The taxes will be collected on $\$20000$, the cost of the remaining 8 houses. Int. due at the end of the year = $\frac{5}{100}$ of $\$18800 = \940 ; taxes due at the end of the year = $\frac{6}{100}$ of $\$20000 = \300 . \therefore total outlay in expenses = $\$1240$, and rent = $\$(12 \times 6 \times 25) = \1800 . \therefore gain = $\$560$.

10. Page 163 (42).

293

11. $\frac{26}{100}$ of $\frac{86}{100}$ of the invoice price = $\$111.80$. \therefore the invoice price = $\$500$.

SOLUTIONS OF PROBLEMS

12. Duty = $\$(1452.38 - 14.63 - 1278) = \159.75 . \therefore rate of duty = $\frac{\$159.75}{\$1278}$ or $12\frac{1}{2}\%$.
13. Page 289 (20).
14. 88% of proceeds = $\$550$. \therefore the proceeds = $\$625$.
15. Page 217 (19).
16. " 275 (20).
17. " 146 (13).
18. 88% of $\$700$ = the cost, $\therefore 112\%$ of the cost = $\frac{112}{100}$ of $\frac{88}{100}$ of $\$700$ or $\$689.92$.
19. Page 177 (15).
20. No. of square yds. of broad cloth after sponging = $(\frac{95}{100}$ of $50 \times \frac{95}{100}$ of $1\frac{1}{2}$), and one yd. of lining after sponging is $(1\frac{1}{2}$ yds. - $2\frac{3}{4}$ in.) or $1\frac{1}{8}$ yds. wide and contains $\frac{1}{8} \times 1\frac{1}{8}$ sq. yds. \therefore the no. of yds. of lining = $(\frac{95}{100}$ of $50 \times \frac{95}{100}$ of $1\frac{1}{2}$) sq. yds. \div $(\frac{1}{8} \times 1\frac{1}{8})$ sq. yds. = $42\frac{90}{101}$.
21. Page 143 (9).
22. " 287 (15).

294

1. Page 156 (33).
2. " 290 (1).
3. " 217 (13).
4. Suppose he invests $\$(78 \times 112\frac{1}{2})$. The number of shares of $3\frac{1}{2}\%$ st. = $112\frac{1}{2}$, and the number of shares of 5% st. = 78. \therefore inc. from $3\frac{1}{2}\%$ st. = $\$(112\frac{1}{2} \times 3\frac{1}{2}) = \393.75 , and inc. from 5% stock = $\$(78 \times 5) = \390 . \therefore diff. in inc. = $\$3.75$, but the real diff. in inc. = $\$7.50$. \therefore the sum he had to invest = $\$(78 \times 112\frac{1}{2}) \times 2 = \17550 .
5. Page 214 (5).
6. $\frac{5}{4}$ of the cost = $\$3.15$. \therefore the cost = $\$2.52$. \therefore the gain at the latter price = $\$.81$. \therefore the part gained = $\frac{\$.81}{\frac{5}{4}} = \frac{9}{25}$.
7. Page 260 (9).
8. " 275 (20).
9. " 146 (12).
10. " 218 (12); 212 (13).

295

11. Page 143 (11).
12. " 177 (15).
13. " 179 (38).

IN THE HIGH SCHOOL ARITHMETIC.

14. Amount due the teacher at the end of the year = $(\frac{100}{100} + \frac{104}{100} + \frac{106}{100} + 1)$ of \$125 = $\frac{410}{100}$ of \$125 = \$515. \therefore total amount collected = \$(515 + 124.81) = \$639.81 \therefore the value of assessable property = $\frac{1000}{4\frac{1}{2}}$ of \$639.81 = \$142180.

15. The av. price of the first two kinds = 42c. He loses in selling 10 lbs. of 60c. tea at $49\frac{1}{2}$ c. a lb. \$1.05. He gains $7\frac{1}{2}$ c. by selling 1 lb. of first mixture at $49\frac{1}{2}$ c. \therefore no. of lbs. = $\frac{105}{7\frac{1}{2}}$ = 14, or 7 lbs. of each.

16. He gains \$(93\frac{1}{8} - 86\frac{1}{4}) + \\$4\$ or \$10\frac{7}{8}\$ on 1 share of stock which cost \$86\frac{1}{4}\$. \therefore he gains \$108.75 on stock which cost \$862.50.

17. Page 257 (12).

18. Wine cost \$3720. By adding water he obtains (1240 + 248) gals. = 1488 gals. No. of bottles required = 1488 gals. $\div 1\frac{1}{2}$ pts. = 7936. Cost of bottles = \$396.80. \therefore total outlay = \$(3720 + 396.80) = \$4116.80. \therefore selling price per bottle = $\frac{4}{5}$ of \$4116.80 $\div 7936 = 64\frac{2}{3}$ c.

19. Page 239 (14).

296

1. Page 274 (3).

2. " 178 (22).

3. Let cost be \$1 a yard. 480 yds. sell for $\frac{100}{100}$ of \$480 = \$518.40; 240 yds. sell for $\frac{110}{100}$ of \$240 = \$268.80; 180 yds. sell for $\frac{110}{100}$ of \$180 = \$205.20; 540 yds. sell for $\frac{90}{100}$ of \$540 = \$491.40. \therefore selling price = \$1483.80, and selling price at 5% av. gain = $\frac{105}{100}$ of \$1440 = \$1512. \therefore gain at \$1 a yd. = \$28.20. \therefore selling price per yd. = $\frac{28.20}{20} = 83\frac{1}{2}$ c.

4. Page 296 (3).

5. P. W. annuity. = $\$ \frac{25}{(1.005)^{30}} \left\{ \frac{(1.005)^{30} - 1}{.005} \right\} = \694.72 .

6. The 5 men have a certain work to do in 9 days. 15 men can do it in 3 days. \therefore the 5 men may stay away 6 days.

7. Income from rent = \$300. Expenses are: Interest \$60, insurance \$25, taxes on house \$47.50, or total \$132.50. \therefore income from house = \$167.50. \therefore $\frac{90}{100}$ of taxable salary = \$(1156.10 - 400 - 167.50). \therefore taxable salary = \$600 on total salary = \$1000.

8. Page 189 (8, 9).

SOLUTIONS OF PROBLEMS

9. " 283 (13).

297

10. 16 gal. at \$1.80 a gal. = \$28.80. 46 gals. at \$.90 a gal. = \$41.40. \therefore 62 gals. cost \$70.20, and 62 gals. at \$1.14 $\frac{1}{2}$ a gal. = \$70.73 $\frac{1}{2}$. \therefore by selling these kinds he gains 53 $\frac{1}{2}$ c. He loses $\frac{1}{2}$ c. by selling 1 gal. of \$1.15 wine. \therefore he loses 53 $\frac{1}{2}$ c. by selling 68 gals of \$1.15 wine.

11. Page 175 (22).

12. A lb. of mixture consisting of equal parts is worth 40c. The first mixture is worth (12×45) c. = \$5.40, and 3 lbs. of each kind (mixed) are worth \$2.40. \therefore 6 lbs. of one kind are worth \$3.00. \therefore 1 lb. is worth 50c.

13. Page 237 (19).

14. " 178 (22).

15. " 288 (9).

16. 88% of the crockery sells for 125% of the cost. \therefore rate = $\frac{37}{88}$ or $42\frac{1}{2}\%$.

17. Page 296 (5).

18. " 216 (11).

19. " 212 (13).

20. " 212 (1).

298

1. Income from 1st chance = \$2000 + $\frac{6}{1000}$ of \$10000 = \$2600. Duty = $\frac{24}{100}$ of \$8800 = \$2112. Sales = $\frac{100}{1000}$ of \$8800 + $\frac{20}{100}$ of \$1200 = \$14760. Net receipts = $\frac{90}{100}$ of \$14760 = \$2112 = \$11769.78. \therefore gain = \$1769.78. \therefore loss by not accepting first offer = \$(2600 - 1769.78) = \$830.22.

2. Page 290 (1).

3. The money must be divided in the ratio of 4 to 9.

4. No. shares of consols = \$720 \div \$3 = 240. He sells 80 shares at 84, which bring \$6720. \$6720 invested in st. at 120 will buy 56 shares. The income from this st. = \$(240 + 40) = \$280. \therefore if 56 shares give \$280 income, 1 share will give \$5 in come.

5. $\frac{1}{10}$ of army - $\frac{6}{1000}$ of $\frac{9}{10}$ of army = 504 men. \therefore the army = 18000 men.

6. In A, $\frac{3}{5}$ of the mixture is wine and $\frac{2}{5}$ water. In B, $\frac{3}{4}$ of the mixture is wine and $\frac{1}{4}$ water. \therefore $\frac{2}{5}$ of the quantity from A + $\frac{3}{4}$ of the quantity from B = the wine drawn off = 1 gal., and $\frac{3}{5}$ of the quantity from A + $\frac{1}{4}$ of the quantity from B = the water drawn off = 1 gal. \therefore the quantity from A = $1\frac{3}{7}$ gals.

IN THE HIGH SCHOOL ARITHMETIC.

7. 4 cu. ft, 113 cu. in. = 7025 cu. in. Area of base = $(7025 \div 14)$ sq. in. = 501.78 sq. in. \therefore side of base = $\sqrt{501.78}$ in. = 22.4 in.

8. P.W. of annuity = $\frac{\$8000}{12 \times .05} \left(1 - \frac{1}{1.05^{12}}\right) = \5908.84 \therefore the annuity is better for the purchaser.

9. Page 178 (22).

299

10. 9842 roubles = 3.39×9842 francs = $\pounds 3.39 \times 9842 \div 25.35 = \pounds 1316$ 2s 11.7d. Afterwards 9842 roubles = 3.37×9842 francs = $\pounds 3.37 \times 9842 \div 25.625 = \pounds 1294$ 6s 10.3d, \therefore gain by delay = $\pounds 1316$ 2s 11.7d - $\pounds 1294$ 6s 10.3d = $\pounds 21$ 16s 1d nearly.

11. One gal. of the mixture contains $\frac{5}{8}$ gal. of coal oil, and is sold for 16c. $\frac{5}{8}$ gal. is worth $\frac{5}{8}$ of 18c., or 15c. \therefore when the customer buys one gal. he is cheated 1 cent, \therefore when he buys 40 gals. he is cheated 40c.

12. Page 281 (19).

13. " 223 (22).

14. Rate of dividend is $\frac{10\%}{100}$ of 6, or $6\frac{3}{10}$, \$85 invested yields \$6.30, \therefore \$100 invested will yield $\frac{100}{85}$ of \$6.30 or \$7.17.

15. Com. = $5\frac{1}{2}\%$ of sales. Net com. = $(5\frac{1}{2} - 1\frac{1}{4})\%$ of sales = $4\frac{1}{4}\%$ of sales. Amount remitted = $94\frac{1}{2}\%$ of sales. But $4\frac{1}{4}\%$ of sales = \$51, \therefore $94\frac{1}{2}\%$ of sales = \$1134.

16. Fixed rent of 450 ac. at \$1.50 an ac. = \$675.00. 100 bush. wheat at 75c. a bush. = \$75.00. 40 bush. barley at 48c. a bush. = \$19.20. 75 bush. oats at 35c. a bush. = \$26.25. \therefore total rent = $\$(675.00 + 75.00 + 19.20 + 26.25) = \795.45 .

17. Money invested in bank stock = $\frac{9\%}{100}$ of \$1680 or \$230.40. \$60 buys 1 railway share, \therefore \$230.40 will buy 384 railway shares.

300

1. One inch = $\frac{1}{12}$ of a foot = .083 ft.

2. Page 235 (13).

3. " 178 (22).

4. " 193 (21).

5. " 184 (6).

6. " 189 (4).

7. " 289 (14).

SOLUTIONS OF PROBLEMS

8. One doz. @ \$18 = \$18. 4 doz. @ \$10 = \$40. \therefore 5 doz. cost \$58. \therefore 1 doz. costs \$11.60. \therefore gain = \$(14.50 - 11.60) or \$2.90. \therefore gain per cent. = $\frac{2.90}{11.60}$ or 25%.
9. B receives $\frac{4}{5}$ of \$2931.25 or \$2696.75. Assignee's charges = \$(6000 - 2931.25 - 2696.75 - 132) or \$240. Rate = $\frac{240}{6000}$ or 4%.

301

10. Page 300 (8).
11. The broker makes \$ $\frac{1}{4}$ on every share of st. handled, \therefore he makes \$56 on \$22400 st. handled.
12. Page 218 (9).
13. A quart contains 69.3185 cu. in. \therefore a quart of water weighs $\frac{69.3185}{1728}$ of 1000 oz. or 41.114 oz.
14. Page 216 (12).
15. There are 27 qts. drawn from the first cask and 60 qts. from the second. \therefore there are 33 qts. more left in the first than in the second. \therefore there are 33 qts. left in the second. \therefore there were (33 + 60) qts. or 93 qts. originally in each.
16. Total area = $\{ 4(30 + 21) \times 8\frac{1}{2} + 2(30 \times 21) \}$ sq. ft. or 2994 sq. ft. Cost = $\frac{2994}{9}$ of 12c. = \$39.92.
17. Page 275 (20).
18. " 193 (21).
19. " 185 (11).

302

1. Page 81.
2. Total cost = \$(120 + 625 + 12.50) or \$757.50. Gain = $\frac{1}{5}$ of \$625 or \$125. \therefore selling price = \$882.50. \therefore selling price per head = $\frac{1}{7}$ of \$882.50 or \$11.76 $\frac{2}{3}$. Cost per head = $\frac{1}{7}$ of \$625 or \$8.33 $\frac{1}{3}$. \therefore gain per head = \$(11.76 $\frac{2}{3}$ - 8.33 $\frac{1}{3}$) or \$3.43 $\frac{1}{3}$.
3. 12% of the total cost = \$600. \therefore the cost = \$5000. 125% of the invoice price = \$5000. \therefore the invoice price = $\frac{100}{125}$ of \$5000, or \$4000.
4. Page 164 (49).
5. " 230 (9).
6. Cost of the land at the end of the year = \$($\frac{194}{100} \times 368 \times 57.50$) or \$22006.40. Selling price = \$($\frac{3}{4} \times 368 \times 63 + \frac{1}{4} \times 368 \times 50$) or \$21988. \therefore loss = \$(22006.40 - 21988) or \$18.40.
7. Page 299 (10).
8. Suppose the cost price = \$1.00 a lb. \therefore cost of 10 lbs. = \$10.00. Selling price = $\frac{115}{100}$ of \$10.00, or \$11.50. Afterwards

IN THE HIGH SCHOOL ARITHMETIC.

8 lbs. are sold for \$11.50. \therefore afterwards 1 lb. is sold for \$1.43 $\frac{1}{2}$. \therefore gain % = 43 $\frac{1}{2}$.
9. Page 217 (25).

303

10. Suppose each child receives \$200, and each brother \$100 after the duty has been paid. \therefore each child is left $\frac{100}{100}$ of \$200, and each brother $\frac{100}{100}$ of \$100. \therefore in this case the estate was worth $5 \times \frac{100}{100}$ of \$200 + $3 \times \frac{100}{100}$ of \$100 = \$($\frac{100000}{100} + \frac{30000}{100}$) = \$12670. \therefore out of the estate worth \$12670 each child receives \$($12670 \div \frac{1267000}{100} \times 200$) or \$1920.60.

11. Total cost = \$160 \times 2 + \$13 $\frac{1}{2}$ = \$333 $\frac{1}{2}$. Let r = rate, \therefore total cost = $\frac{160}{1+r} + \frac{160}{1-r} = \$333\frac{1}{2}$. $\therefore r = \frac{1}{5}$ or 20%.

12. 20 English miles = $20 \times .2136$ German miles, or 4.272 German miles. The train goes 4.272 German miles in 1 hr. \therefore it goes 3 $\frac{1}{2}$ German miles in 45', 38 $\frac{8}{9}$ ".

13. Page 275 (20).

14. " 193 (21).

15. A's profits = \$(2400 - 1920), or \$480. C's profits = \$(2080 - 1280), or \$800. \$480 are the profits on \$1920 for 6 mos. \therefore \$800 are the profits on \$1280 for 15 mos. Again \$1920 gain in 6 mos. \$480. \therefore \$1920 will gain in 12 mos. \$960. \therefore \$(1920 + 960) or \$2880 are the stock and profits for 12 mos. from \$1920 stock. \therefore \$4800 are the stock and profits for 12 mos. from \$3200 stock.

16. Stock cost $\frac{2}{3}$ of \$3500, or \$4200. \$4000 draft is worth \$4200. \therefore \$100 draft is worth \$105. \therefore rate of premium = 5%.

17. Page 242 (5).

18. " 145 (10).

19. " 246 (9).

\therefore 5 doz.
- 11.60)

Assignee's
Rate =

ndled, \therefore

t of water

nd 60 qts.
n the first
he second.
each.

sq. ft. or

Gain = $\frac{1}{5}$ of
selling price
head = $\frac{1}{5}$ of
- 8.33 $\frac{1}{3}$) or

000. 125%
ice = $\frac{100}{125}$ of

$\frac{104}{100} \times 368 \times$
 $63 + \frac{1}{2} \times 368$
or \$18.40.

of 10 lbs. =
Afterwards

SOME PROPERTIES OF NUMBERS.

304

1. Any no. ending in 0 is a multiple of 10 and is \therefore divisible by 2 and by 5; \therefore when any no. is divided by 2 or 5 the remainder is determined by the digit in the units place, \therefore if the units digit is 2, 4, 6 or 8 the no. is divisible by 2, and if the units digit is 5 the no. is divisible by 5.

2. Since $10 \div 3 = 3 + \frac{1}{3}$, $\therefore 7 \times 10 \div 3 = 7 \times 3 + 7 \times \frac{1}{3}$, $\therefore 70 \div 3 = 21 + \frac{7}{3}$, i.e., the rem. in dividing 70 by 3 is the same as that obtained in dividing 7 by 3; and similarly for any other multiple of 10. Again, $100 \div 3 = 33 + \frac{1}{3}$, $\therefore 700 \div 3 = 7 \times 33 + \frac{7}{3}$; \therefore the same holds true for 100, and similarly for 1000, and so on for any power of 10. \therefore if any no. as 5724 is expressed thus: $5000 + 700 + 20 + 4$ it will readily be seen that whatever rem. may be left after dividing 5724 by 3 will result from dividing 5 and 7 and 2 and 4 by 3, but $\frac{5}{3} + \frac{7}{3} + \frac{2}{3} + \frac{4}{3} = \frac{5+7+2+4}{3}$ and \therefore the rem. is that obtained by dividing the sum of the digits 5, 7, 2, 4 by 3.

3. When the last two digits of a no. are 0's the no. is a multiple of 100 and is \therefore divisible by 4 and by 25; \therefore when any no. is divided by 4 or by 25 the rem. is determined by the digits in the tens and units places. \therefore if these digits form a no. divisible by 4, the whole no. is divisible by 4, and if these digits are 25, 50 or 75 the whole no. is divisible by 25.

4. See 1.

5. Any no. is divisible by 8 which is divisible by both 2 and 3. See 1 and 2.

6. Powers of 10 when divided by 7 give remainders as follows:— 10^0 gives 1; 10^1 , 3; 10^2 , 2; 10^3 , 6; 10^4 , 4; 10^5 , 5; 10^6 , 1; &c., the rems. 1, 3, 2, 6, 4, 5 being repeated in the same order continuously. Now take any one of these, as 10^5 , and we have $100000 = 14285 + \frac{5}{7}$. $\therefore 8 \times 100000 = 8 \times 14285 + \frac{8 \times 5}{7}$, that is, the remainder obtained by dividing 8 times 100000 by 7 is the rem. got by dividing 8 times 5 by 7; and similarly for any other multiple of 100000. In the same way

it may be shown that the rem. in dividing 8 times 10^4 (10000) by 7 is that got by dividing 8 times 4 by 7; and similarly for any other multiple of 10000. Also the rem. after dividing any multiple of 10^3 by 7 is that got by dividing that mult. of 6 by 7; and similarly for the other powers of 10. Now take any no., say 435826: since this = $400000 + 30000 + 5000 + 800 + 20 + 6$, the rems. will be $\frac{4 \times 4}{7}, \frac{4 \times 3}{7}, \frac{6 \times 5}{7}, \frac{2 \times 8}{7}, \frac{3 \times 2}{7}, \frac{1 \times 6}{7} = (5 \times 4 + 4 \times 3 + 6 \times 5 + 2 \times 8 + 3 \times 2 + 1 \times 6) \div 7 = 90 \div 7$, and \therefore the final rem. is 6. \therefore 435826 when divided by 7 will give rem. 6. \therefore in any no. when the sum of once the units digit, 3 times the tens, twice the hundreds, &c., is divided by 7 the rem. so obtained will be the rem. after dividing that no. by 7.

7. When the last 3 digits of a no. are 0's, the no. is a mult. of 1000 and is \therefore divisible by 8 and by 125. \therefore when any no. is divided by 8 or by 125 the rem. is determined by the digits in the last 3 places. \therefore if these digits form a no. divisible by 8 the whole no. is div. by 8, and if by 125, so is the whole no.

8. See 2. The reasoning in no. 2 will, with the necessary changes, apply here.

9. If a whole no. contains no units it must consist of tens or multiples of 10; \therefore &c.

10. Every odd power of 10 when divided by 11 gives 10 as rem.; and every even power gives a rem. 1. \therefore any no. divided by 11 gives the same rem. as once the units digits + 10 times the tens + once the hundreds + 10 times the thousands + &c. = sum of the 1st, 3rd, 5th, &c. digits + 10 times the sum of the 2nd, 4th, 6th, &c. digits = $a + 10b$ suppose = $a + 11b - b = 11b + a - b$, which, since $11b$ is a multiple of 11, gives the same rem. as $a - b$ = sum of digits in the odd places - sum of those in the even places.

11. Any no. is divisible by 12 which is divisible by both 3 and 4. See 2 and 3.

12. Since $10^0 \div 13$ gives rem. 1; and 10^1 gives 10; and $10^2, 9 \therefore$ the rem. on dividing any no. of 3 digits by 13 is that obtained by dividing the units dig. + $10 \times$ tens dig. + $9 \times$ hundreds dig. by 13; but (if u, t, h are the units, tens, &c. digits, respectively) $u + 10t + 9h = u + (13 - 3)t + (13 - 4)h = 13(t + h) - (4h + 3t - u)$; $13(t + h)$ being a mult. of 13 leaves no rem. \therefore if $4h + 3t - u$ is divisible by 13 so also is the no. Although this is a test of the divisibility of a no. of 3 digits by 13 it does not give the rem. when the no. is not divisible. Thus the no.

SOLUTIONS OF PROBLEMS

846 when so treated gives 12, but as this is really - 12 the rem. is $13 - 12 = 1$.

13. Let $a, b, c, d, \&c.$, represent the 1st, 2nd, 3rd, 4th, &c. periods (i.e., if the no. is 736428579361 then $a=361, b=579, c=428, d=736$); then since $10^3, 10^6, 10^9, \&c.$, divided by 13 each leaves 1 for rem.; and $10^3, 10^6, \&c.$ each leaves 12 for rem. \therefore rem. on dividing any no. by 13 is that obtained by dividing $a + 12b + c + 12d + e + 12f + \&c. = a + c + e + \&c. + 12(b + d + f + \&c.) = 13(b + d + f + \&c.) + a + c + e + \&c. - (b + d + f + \&c.) = a$ mult. of 13 + sum of the odd periods - sum of the even periods, \therefore if this diff. is divisible by 13 so also is the no.

If the sum of the odd periods exceeds the sum of the even, their diff. $\div 13$ leaves the same rem. as the no. $\div 13$. If not the rem. so obtained when subtracted from 13 will give the same rem. as the no. $\div 13$. See 12.

305

14. Since $10^0, 10^6, \&c.$ divided by 7 leave 1 each for rem.; and $10^3, 10^9, \&c.$ leave 6 for rem. \therefore we shall get by similar reasoning the same result as in 13. In the case of 11 the rems. are 1 and 10 alternately with the same result.

15. If the test of ex. 6 is satisfied the no. is div. by 7; and since the no. is even it is div. by 2, \therefore it is div. by 14.

16. See 12. In this case the 3 rems. are 1, 10, 2. \therefore the rem. is obtained by dividing 14 into $u + 10t + 2h$, which $= u + (14 - 4)t + 2h = 14t + 2h + u - 4t$. \therefore since $14t$ is a mult. of 14 there will be no rem. if $2h + u - 4t$ is div. by 14. In practice $2h + 10t + u$ gives the readier test for since $10t + u$ is simply the no. expressed by the last 2 digits we have merely to add to this twice the digit in the hundreds place; thus in 749 the test is given by $49 + 14$: also in 1749 by $49 + 34$.

17. These are the tests for 5 and 3, and any mult. of 5 and 3 is a mult. of 15.

18. See 12 and 16. The rem. is obtained by dividing 16 into $8th + 4h + 10t + u$ which $= (16 - 8)th + 4h + (16 - 6)t + u = 16(th + t) - (8th - 4h + 6t - u)$; &c.

19. The rem. is obtained by dividing 17 into $14th + 15h + 10t + u = 17(th + h + t) - (3th + 2h + 7t - u)$; &c.

20. An even no. is div. by 2 and if the test for 9 is satisfied it is a mult. of both 2 and 9 and \therefore of 18.

21. The rem. is obtained by div. 19 into $12\text{ th} + 5\text{ h} + 10\text{ t} + u = 19(\text{th} + \text{t}) - (7\text{ th} - 5\text{ h} + 9\text{ t} - u)$; &c.
 22. See 3.
 23. See 7.
 24. Any such no. is a mult. of 11, 1001, 100001, 10000001, or &c. and \therefore of 11.
 25. Any such no. is a mult. of 1001, and \therefore of 7 and 13.
 26. Any such no. is a mult. of 10001, and \therefore of its two factors 73 and 137.
 27. Since the two nos. consist of the same digits they must leave the same rem. when divided by 9; let the first no. contain 9 a times with rem. r, and the second contain 9 b times with rem. r; then one no. $= 9a + r$, the other $= 9b + r$, and their dif. $= 9(a - b) = a$ mult. of 9.

306

28. A no. and the sum of its digits when divided by 9 leave the same rem. \therefore &c. as in 27.
 29. The rem. is obtained by dividing 8 into $4\text{ h} + 2\text{ t} + u$. See 7 and 12.
 30. The rem. is obtained by div. 19 into $5\text{ h} + 10\text{ t} + u$, but $10\text{ t} + u$ is the no. formed by the last 2 dig.; \therefore &c.
 31. The rem. is obtained by div. 9 into $\text{h} + \text{t} + u$ which $= 9(\text{u} - \text{h}) + 10\text{ h} + \text{t} - 8\text{ u} = a$ mult. of $9 + 10\text{ h} + \text{t} - 8\text{ u}$; \therefore if $10\text{ h} + \text{t} - 8\text{ u} = 0$, there will be no rem.
 32. The rem. is obt. by div. 13 into $9\text{ h} + 10\text{ t} + u$ and if this is a mult. of 13 so also is $9(9\text{ h} + 10\text{ t} + u) = 91(\text{h} + \text{t}) - (10\text{ h} + \text{t} - 9\text{ u}) = \&c.$, as in 31.
 33. Any such no. is a mult. of 21 and \therefore a mult. of 3 and 7.
 34. Any such no. is a mult. of 91 and \therefore a mult. of 7 and 13.
 35. Any such no. is divisible by 14 (see 16), and \therefore by 7.
 36. If the whole no. of digits is odd the odd digits remain the odd digits, and the even the even, and the case is precisely that of no. 10; but if the no. of digits is even the digits in the odd places become those in the even places and *vice versa*, but their respective sums are the same, and \therefore the same result holds.
 37. That is, the first $n - 1$ digits followed by half the middle digit represent a number divisible by 11; we have \therefore to prove that the remaining half of the middle dig. followed by the

SOLUTIONS OF PROBLEMS

$n - 1$ dig. to the right represent a no. div. by 11; but this second no. is simply the first with its digits in reverse order, and is \therefore div. by 11. See 36. Thus suppose 3874783 to be the no., on dividing 3874 by 11 the rem. is 2, \therefore 3872 is div. by 11; \therefore (by 36) 2783 is div. by 11; but $3874783 = 3872000 + 2783$; and since ea. of these is div. by 11, the whole no. must be.

38. If t be the tens, and u the units digit, the two nos. will be $10t + u$ and $10u + t$, and the dif. between the squares of these is $99(t^2 - u^2)$ or $99(u^2 - t^2)$ and \therefore divisible by 99.

39. Affixing two 0's multiplies any no. by 100, \therefore we have the no. $\times \frac{100}{4} = \text{no.} \times 25$.

40. $= \text{no.} \times \frac{1000}{8} = \text{no.} \times 125$.

41. $= \text{no.} \times (100 + \frac{1}{4} \text{ of } 100) = \text{no.} \times 125$.

42. The first partial product is 8 times the multiplicand, and 40 times this is $40 \times 8 (= 320)$ times the mult'd., and \therefore the sum of these is $320 + 8 (= 328)$ times the mult'd.

307

43. First multiply by 12, placing the prod. 7 places to the left ($= 120000000$ times the mult'd.); next mult. this prod., omitting the 0's, by 12, placing units under units, &c. ($= 144$ times the mult'd.); lastly, mult. this second prod. by 12, placing the result 3 places to the left ($= 1728000$ times the mult'd.); and the sum of these will be the complete product.

44. If n is any whole no. then $n + 1$, $n + 2$ are the two nos. next greater, and it will be found that $n(n + 1)(n + 2) + n + 1 = (n + 1)^3$. Or, if n is the middle no. the other two are $n - 1$ and $n + 1$, and $(n - 1)n(n + 1) + n = n^3$.

45. This will appear from a consideration of two such fractions, say, $\frac{5}{7}$, $\frac{7}{5}$. Here the $\frac{5}{7}$, the smaller or proper fraction, is less than 1 by $\frac{2}{7}$, i.e., by a fraction whose num. is the dif. of the two terms, 5 and 7, and whose den. is the greater of these terms, whereas $\frac{7}{5}$, the greater or improper fraction, is greater than 1 by $\frac{2}{5}$, i.e., by a fraction whose num. is also the dif. of the terms, but whose den. is the smaller of these terms; \therefore the sum of the fractions exceeds $1 + 1$ by the excess of $\frac{2}{5}$ over $\frac{2}{7}$, i.e., by a fraction whose num. is twice the excess of the greater term over the less, and whose den. is the prod. of the terms.

46. If n and $n + 1$ are the two nos., the dif. of their sqrs. is $2n + 1 = n + (n + 1) = \text{sum of the nos.}$

but this
rse order,
783 to be
72 is div.
= 3872000
whole no.

o nos. will
squares of
y 99.
. we have

ltiplicand,
'd., and ∴
d.

ces to the
this prod.,
c. (= 144
y 12, plac-
e mult'd.);

ne two nos.
+ 2) + n +
o are n - 1

such frac-
r fraction,
is the dif.
greater of
fraction, is
is also the
ese terms;
excess of $\frac{2}{3}$
cess of the
rod. of the

their sqrs. is

47. Any no. ending in 5 is of the form $10m + 5$, and its sq. takes the form $100m(m+1) + 25 = m(m+1)$ with 25 placed to the right.

Or, consider a case, 75, the sq. is made up of the four partial prods., 5×5 , 5×70 , 70×5 , 70×70 ; the second and third of these make 10 times 70, and this with the fourth makes 80 times 70, and this with 5×5 makes 8×7 with 25 placed to the right.

48. The sq. of $m + \frac{1}{2} = m(m+1) + \frac{1}{4}$; &c. Also consider the partial products in squaring any such no., say $7\frac{1}{2}$.

49 to 53. See Arith. p. 54 et seq.

54. $\frac{5}{271} = \frac{5 \times 366}{99998}$; &c.

55. $\frac{5}{271} = \frac{5 \times 2151}{99998}$; &c.

56. $\frac{1010101}{1010101} = \frac{1010101 \times 99}{99999999}$; &c.

57. Multiply both terms by 2997; &c.

58. Multiply both terms by 37683; &c.

59. Let $\frac{m}{n}$ be the fraction; then (Arith. p. 56) the no. of digits in the repetend cannot exceed $n - 1$; $\therefore n$ divides $m10^{n-1}$ with rem. m ; and \therefore divides 10^{n-1} with rem. 1; and \therefore divides the no. consisting of $n - 1$ 9's without rem.; and \therefore (n being prime and not a mult. of 9) divides the no. consisting of $n - 1$ 1's. Now suppose n to be a factor of r 1's where r is less than $n - 1$ and prime to it; then n must be a factor of p r 1's and of $q(n - 1)$ 1's (where p, q , are any whole nos.); but p, q can be taken such that the dif. between p r and $q(n - 1)$ is 1; $\therefore n$ must be a factor of 1 or of some power of 10 which is impossible.

If r and $n - 1$ are not prime to each other but have a g.c.m. a , then the same reasoning will show that n must be a factor of the no. consisting of a 1's, in which case the no. of digits in the repetend is a factor of $n - 1$.

308

60. If the no. of digits is even, the no. can be arranged in periods thus 99, 99, &c., and each period is divisible by 11. If the no. of digits is odd then 11 will divide without rem. all to the left of the units digit; the 9 in the units place is \therefore the rem. i.e., if 2 were added to the no. it would be a mult. of 11.

61. $\frac{1}{10^n + 1} = \frac{1}{10^{2n}} \cdot \frac{1}{10^n + 1} = \frac{10^{2n} - 1}{10^{2n}} \cdot \frac{1}{10^n + 1} = \frac{1}{10^{2n} + 1} \cdot \frac{1}{10^n + 1} \therefore$
the repetend consists of n 0's followed by n 9's.

SOLUTIONS OF PROBLEMS

62. See Arith. p. 56.

63. $\frac{1}{7} = .142857 = \frac{142857}{1000000}$, $\therefore 999999 \div 7 = 142857$.
 $\frac{1}{7} = \frac{142857}{1000000} = \frac{15873}{111111}$, $\therefore 111111$ is divisible by 7.

64. Multiplying 111111 by 4 introduces no factor common to 7 \therefore there must be six 4's.

65. Every six 4's will divide by 7 \therefore the no. of 4's must be 6, 12, 18, &c.

66. We know that $\frac{1}{17}$ gives a quotient of 16 digits before the rem. 1 occurs. \therefore the least no. composed of 9's which will contain 17 is $10^{16} - 1$.

67. In $\frac{1}{27}$ the rem. 1 occurs after 28 digits in the quot. have been obtained; i.e., $10^{28} - 1$ (= 28 9's) is the least no. of 9's that will contain 29: \therefore since 9 contains no factor com. to 29, a no. consisting of 28 1's will be a mult. of 29.

68. 37 is a factor of 111, \therefore of 999, \therefore of $10^3 - 1$, \therefore in reducing $\frac{1}{37}$ the rem. 1 occurs after 3 digits have been found in the quot.

69. Since $16 = 17 - 1$, this is a particular case of 76.

71. See 6 and 12. The successive rems. are 1, 10, 26, 1, 10, 26, &c. \therefore the successive multipliers of the digits are 1, 10, -11, 1, 10, -11, &c. These give 172, and -209 and $172 - 209 = -37$, a mult. of 37, \therefore the gn. no. is a mult. of 37.

72. See Arith. p. 60. Dividing by 17 we get 29 in the quot. \therefore the repeating period is .2941176470588235.

73. By 59 the no. of digits in the repetend must be 2, 4, 67 or 268, we find by trial that it must be more than 4, \therefore it cannot be less than 67.

74. The no. must be either 2, 23, or 46; \therefore &c., as in 73.

75. Suppose the sq. root produced to be a terminating or repeating decimal; $\therefore = 9 +$ a proper fraction = an improper fraction (reduced to its lowest terms); \therefore the sq. of this root would be an improper fraction in its lowest terms, and \therefore could not produce a whole no. That the root cannot terminate will be evident from the fact that finally each minuend ends in two 0's while each subtrahend ends in one of the digits which can end a sq. no. (omitting 0).

309

77. See Arith. p. 53. Obtain the first digit, 6, in the quot. and divide by 4.

78. Obtain the first digit 7 in the quot. and divide by 12.

IN THE HIGH SCHOOL ARITHMETIC.

79. Obtain 94 in the quot. and divide by 8.

80. Obtain the first digit and divide by 7.

81. See 78.

82. Every even no. contains 2 as a factor. \therefore any power of an even no. contains that power of 2 as a factor, and is \therefore an even no. If 2 occurs as a factor of a complete power it must have been a factor of the root, since no new (simple) factor can be introduced by raising a no. to any power, \therefore if the power is even the root must be.

83. The dif. must be a mult. of 9 (see 27), \therefore the other digit must be 4, making the dif. 54 or 45, 93 and 39 will give one; 94 and 49 the other.

310

84. This may be inferred from 82. If 2 does not occur as a factor in the root, it cannot be introduced by multiplying the root by itself any no. of times, &c.

85. The diagonal form results from the omission of the 0's which properly belong to the right of these partial products, but are omitted because the value of the digits is indicated by reference to the first partial product which gives the position of the units digit.

86. The final product is the sum of the partial products and must be the same in whatever order these products are arranged.

87.

46987

4967

187948000

42288300

2819220

328909

233384429

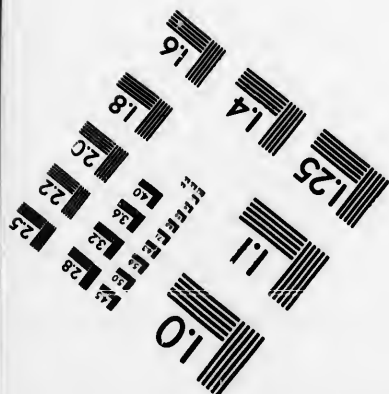
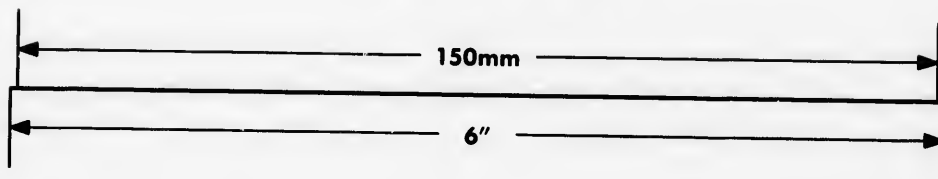
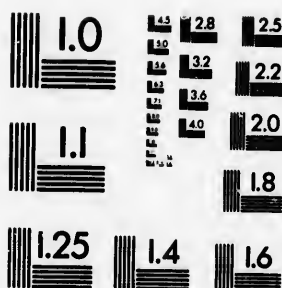
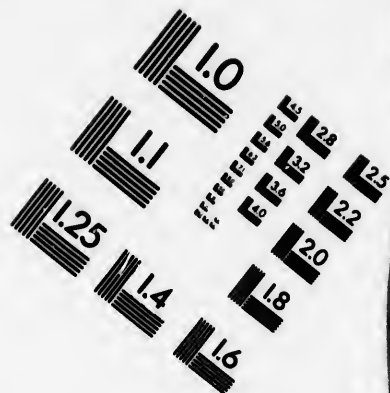
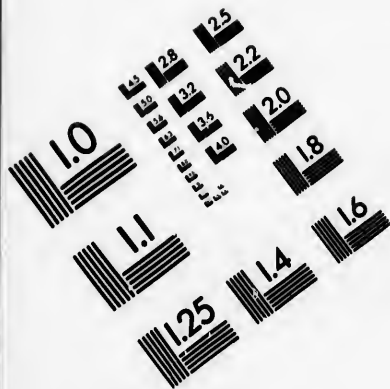
Remember that the 4 in the multiplier is 4000; the 9, 900; &c.

89. The greatest no. containing 3 digits is 999 whose sq. 998001 is less than 1000000 (the least no. containing 7 digits) by 1999. Or thus: The greatest no. containing 3 dig. is $10^3 - 1$, whose sq. $10^6 - 2 \cdot 10^3 + 1$ is less than 10^6 by $2 \cdot 10^3 - 1$. The second part follows necessarily from the above.

90. The total no. subtracted from the original no. is the sq. of the no. represented by the digits thus far obtained in the

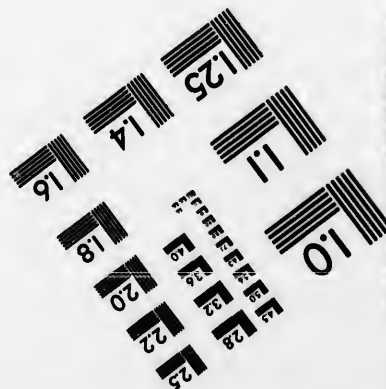


IMAGE EVALUATION TEST TARGET (MT-3)



APPLIED IMAGE, Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
Fax: 716/288-5989

© 1993, Applied Image, Inc., All Rights Reserved



0
F F F F F F
E E E E
1.8 2.0 2.2 2.5

ii
oi
E
3

SOLUTIONS OF PROBLEMS

sq. root, (see Arith. p. 73.) But the subtrahend immediately producing a remainder $= ab + b^2$, where b is the no. represented by the digit last obtained (thus far) in the root and a , those previously obtained.

91. Between the sq. of the part of the root already found and the no. whose sq. root is to be obtained.

92. Between the cube of the part of the root already found and the no. whose cube root is to be obtained. See Arith., p. 77.

93. Disregarding the dec. pt., the first 6 digits in the root are 331662, i.e. the part of the root already found is 33166200000 ($= a$ suppose); and if we denote the no. whose rt. is required (11 and 20 0's) by N , the next complete rem. is $N - a^2 (= 317756$ and 10 0's), and the next trial divisor is $2a (= 66332400000)$. Now if x denote the rest of the root ($= 47903$), $\therefore N = (a + x)^2$, $\therefore N - a^2 = 2ax + x^2$, and we are required to show that the rest of the root (x) may be obtained by dividing $2ax + x^2$ by $2a$ instead of continuing the ordinary

process. The quot. so obtained is $x + \frac{x^2}{2a}$ which gives the remaining part of the root x provided $\frac{x^2}{2a}$ is a proper fraction.

Now since x contains 5 digits and a , 11, $\therefore x^2$ must always be less than a , $\therefore \frac{x^2}{2a}$ is less than $\frac{1}{2}$. In this division by $2a$ the contracted method may of course be used and the whole operation, retaining remainders only, is as follows:—(The 30974 under the divisor is the quotient in reversed order. See Arith., p. 70.)

	11	(3.3166247903
63	200	
661	1100	
6626	43900	
66326	414400	
663322	1644400	
663324	3177560	
30974	524264	
	59940	
	243	
	45	

94. A no., as 6, may divide the product of two nos. 4 and 9 without dividing either of them, for the reason that 6 is made up of two factors one of which is contained in 4, and the other in 9; but a prime no. cannot be broken up into factors and \therefore must be wholly contained in one or other of any two factors whose product is a multiple of it.

311

95. If a no. has been resolved into prime factors, any other set of factors can be formed only by breaking up or combining the factors already obtained, but prime factors cannot be broken up, nor can any combination of factors produce a prime factor.

96. See Arith., p. 135, no. 379, 380.

97. See Arith. p. 67, 68.

98. See Arith. p. 69, 70.

99. $\frac{1}{9} = .1111\&c.$ On dividing this by 9 it is easily seen that the successive rems. are 1, 2, 3, 4, 5, 6, 7, 8. \therefore the successive quots. are 0, 1, 2, &c., until the dividend 81 is reached when we have a quot. 9 instead of quot. 8 and rem. 9; &c.

100. Since 2 indicates 8 times as many units as it would indicate in the units place \therefore it must indicate 16 units wh. added to the 3 in the units place give 19.

101. 3 indicates 3 units; 2 indicates 4×2 units; and 1, 4×1 units; = 27 units.

312

102. $1 + 5 \times 5 + 5 \times 5 \times 5 \times 5 = 651.$

103. $5 + 4 \times 6 + 3 \times 6^2 + 2 \times 6^3 + 6^4 = 1865.$

104. $\frac{125}{4} = 31 + \frac{1}{4}$; $\therefore \frac{125}{4^2} = 7 + \frac{3}{4} + \frac{1}{4^2}$; $\therefore \frac{125}{4^3} = 1 + \frac{3}{4} +$

$\frac{3}{4^2} + \frac{1}{4^3}$; $\therefore 125 = 1 \times 4^3 + 3 \times 4^2 + 3 \times 4 + 1.$

105. From 104 this is 1331.

106. Dividing by 5 the successive rems. are 0, 4, 3, 3, 4, 3. \therefore the no. is 343340.

107. The successive rems. are 5, 7, 3, 6, 2.

108. 4321 in scale of $7 = 4(7^3) + 3(7^2) + 2(7) + 1 = 1534.$ Or, divide by 10 successively and we get the rems. 4351. In dividing 10 into 43 bear in mind that this 43 is $4 \times 7 + 3 = 31$, and that the next rem. 1 taken with the 2 $= 1 \times 7 + 2 = 9$, and

SOLUTIONS OF PROBLEMS

as 10 will not divide into 9 the next dividend is $9 \times 7 + 1 = 64$, giving the first rem. 4; &c.

109. The sum is 27376 in the scale of 9.

110. The dif. is 2767 in the scale of 9.

111. The prod. is 11578813 in the scale of 9.

112.
$$\begin{array}{r} 41625 \\ 254 \\ \hline \end{array}$$

$$\begin{array}{r} 230436 \\ 302364 \\ \hline 113553 \\ \hline 14643006 \end{array}$$

113. The partial prods. are 3421, 12540, 255200.

114. The first rem. is 288.

115. 110)1009002(3030

$$\begin{array}{r} 330 \\ \hline 1000 \\ 330 \\ \hline 102 \end{array}$$

116.	1	2	3	4	5	6
	2	4	6	11	13	15
	3	6	12	15	21	24
	4	11	15	22	26	33
	5	13	21	26	34	42
	6	15	24	33	42	51

117. $8 = 2^3 = 1000$; $10 = 2^3 + 2^1 = 1010$; &c.

118. This is a particular case of the general theorem established in the algebras that the sum of the digits of any whole no. (radix r) divided by $r - 1$ will have the same rem. as the whole no. divided by $r - 1$. Or, the reasoning in no. 2 may with the necessary changes be applied.

The same will be true of 2 and 3 since they are factors of 6.

119. Place the 1-lb. wt. in the scale pan with the sugar and the $\frac{1}{2}$ -lb. and the 4-lb. wt. in the other scale pan.

121. Place with the quantity to be weighed the wts. 3^3 , 3^4 , 3^5 , and in the other pan 1, 3, 3^3 , 3^6 .

313

122. Since 6×6 ends with 6 and the successive powers of any no. ending in 6 necessarily involve the multiplication of 6 by 6. \therefore every power of a no. ending with 6 will end with 6.

IN THE HIGH SCHOOL ARITHMETIC.

$$9 \times 7 + 1 =$$

123. Let n and $n+2$ be the two nos. ; their prod. $n+2n$ is less by 1 than n^2+2n+1 , which is the sq. of $n+1$.

124. See Arith. p. 131, no. 323.

$$125. 1\frac{1}{2} - \sqrt{3} = \frac{7-4\sqrt{3}}{4} = \frac{1}{4(7+4\sqrt{3})} \text{ which is less than}$$

$\frac{1}{8}$ since $\sqrt{3}$ is greater than $1\frac{1}{2}$ (since the sq. of $1\frac{1}{2} = 2\frac{1}{4}$ only).

126. The sq. of 12345 exceeds that of 12344 by $2 \times (12344) + 1$; subtracting this leaves 152374336.

127. If the even no. is in the right hand the mult. gives even by even wh. is even, and odd by odd which is odd ; and the sum of odd and even is odd. If the odd no. is in the right hand the mult. gives even by odd which is even, and odd by even which is even, and the sum of even and even is even.

128. The first of the two nos. must be of one of the forms $3m, 3m+1, 3m+2$; and the second, of one of the forms $3n, 3n+1, 3n+2$, now one of the first can be taken with one of the second in 9 different ways, viz.: (denoting the nos. respectively by a, b, c, x, y, z) $ax, ay, az, bx, by, bz, cx, cy, cz$. In 5 of these cases ax, ay, az, bx, cx one of the nos. is divisible by 3 ; in 2 of the cases bz, cy , the sum, and in the remaining two, by, cz , the difference, is divisible by 3.

129. See Arith. p. 131, no. 335. Let $2n+1$ represent any odd no. ; its sq. is $4n^2+4n+1$; the two nos. nearest the half of this sq. are $2n^2+2n, 2n^2+2n+1$; and these are the two sides and the hyp. of a rt. angled tri. The ratio of the greater of these sides to the less is $\frac{2n^2+2n}{2n+1} = n + \frac{n}{2n+1}$, which by giving n the successive values 1, 2, 3, &c., produces the series $1\frac{1}{3}, 2\frac{2}{5}, \&c.$

$$130. \text{ This is equivalent to multiplying the no. by } \frac{1}{1+\frac{1}{20}} (1 + \frac{1}{11} + \frac{1}{11 \cdot 20} + \frac{1}{11 \cdot 20 \cdot 11} + \dots) = \frac{1}{1+\frac{1}{20}} (1 + \frac{1}{11}) (1 + \frac{1}{220} + \frac{1}{220^2} + \dots) = \frac{1}{1+\frac{1}{20}} \times \frac{1}{11} \times \frac{1}{1-\frac{1}{220}} = \frac{1}{314}.$$

314

131. This is equivalent to multiplying the no. by $(1 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300}) \times \frac{1}{100000} = \frac{411}{3000000}$; but $\frac{1}{78000} = \frac{411}{30000000} \times \frac{1}{1+\frac{1}{10000}} = \frac{411}{30000000} \times (1 - \frac{1}{10000} + \frac{1}{10000^2} - \&c.) \therefore \text{ the re-}$

SOLUTIONS OF PROBLEMS

sult obtained by the method given exceeds the true result by less than $\frac{1}{10000}$ of either. If the result obtained be diminished by $\frac{1}{10000}$ of itself a closer approximation will be obtained.

$$132. \sqrt[3]{137} = \frac{137}{10000} = \frac{137}{10000} \times \frac{10000}{10001} = \frac{137}{10000} \times \frac{1}{1 + \frac{1}{10000}} = \frac{137}{10000} (1 - \frac{1}{10000}) \text{ very nearly; hence the rule.}$$

$$133. \frac{1}{137} = \frac{1}{10000} = \&c. \text{ as above.}$$

134. (1) $\frac{2N+a^3}{N+2a^3}$ a is greater than a, if $2N+a^3$ is gr. than $N+2a^3$, i.e. if N is gr. than a^3 . (2) Let $\sqrt[3]{N} = a+x$; then $3\sqrt[3]{N}$ is greater than $\frac{2N+a^3}{N+2a^3}$ if $\frac{a+x}{a}$ is gr. than $\frac{2(a+x)^3+a^3}{(a+x)^3+2a^3}$ i.e. if $\frac{x}{a}$ gr. than $\frac{3a^2x+3ax^2+x^3}{(a+x)^3+2a^3}$, if $(a+x)^3+2a^3$ is gr. than $3a^3+3a^2x+x^3$, if $2a+x$ is greater than 0.

135. If we take the whole no. in the rt. as the first value of a we get $\frac{10+1}{5+2} = \frac{11}{7}$, \therefore the next approximation is

$$\frac{10+(\frac{11}{7})^3}{5+2(\frac{11}{7})^3} \times \frac{11}{7} = \frac{52371}{30335} = 1.709 +$$

136. In the algebras it is shown that the sum of the natural nos. from 1 to n is $\frac{n}{2}(n+1)$; and that the sum of their cubes is $\frac{n^2}{4}(n+1)^2$; \therefore &c.

result by
diminish-
obtained.

$$\frac{1}{10000} =$$

gr. than

x; then
 $(x)^3 + a^3$
 $(x)^3 + 2a^3$

gr. than

rst value

nation is

e natural
eir cubes

