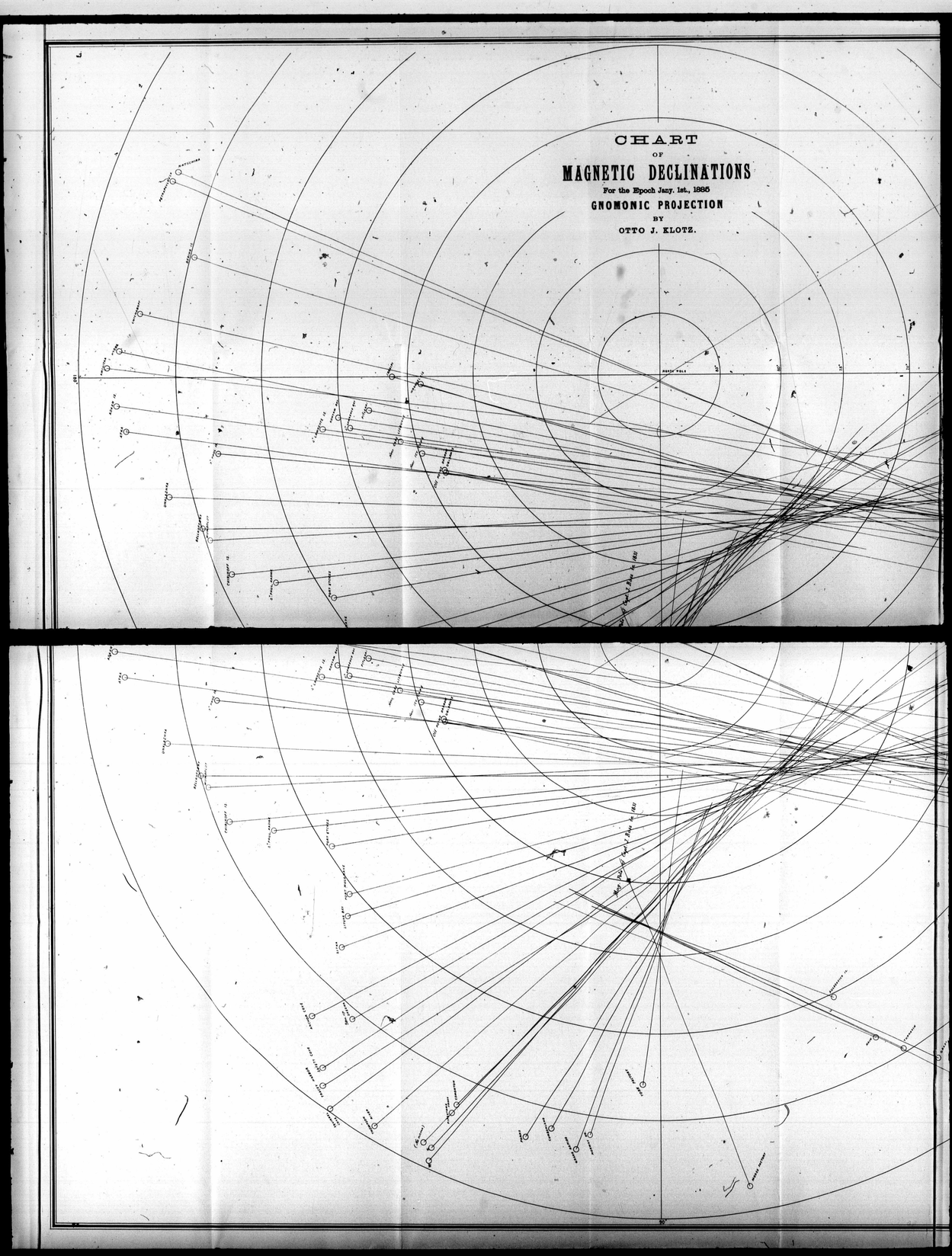
CHART
OF
MAGNETIC DECLINATIONS
For the Epoch Jany. 1st., 1885
GNOMONIC PROJECTION
BY
OTTO J. KLOTZ.



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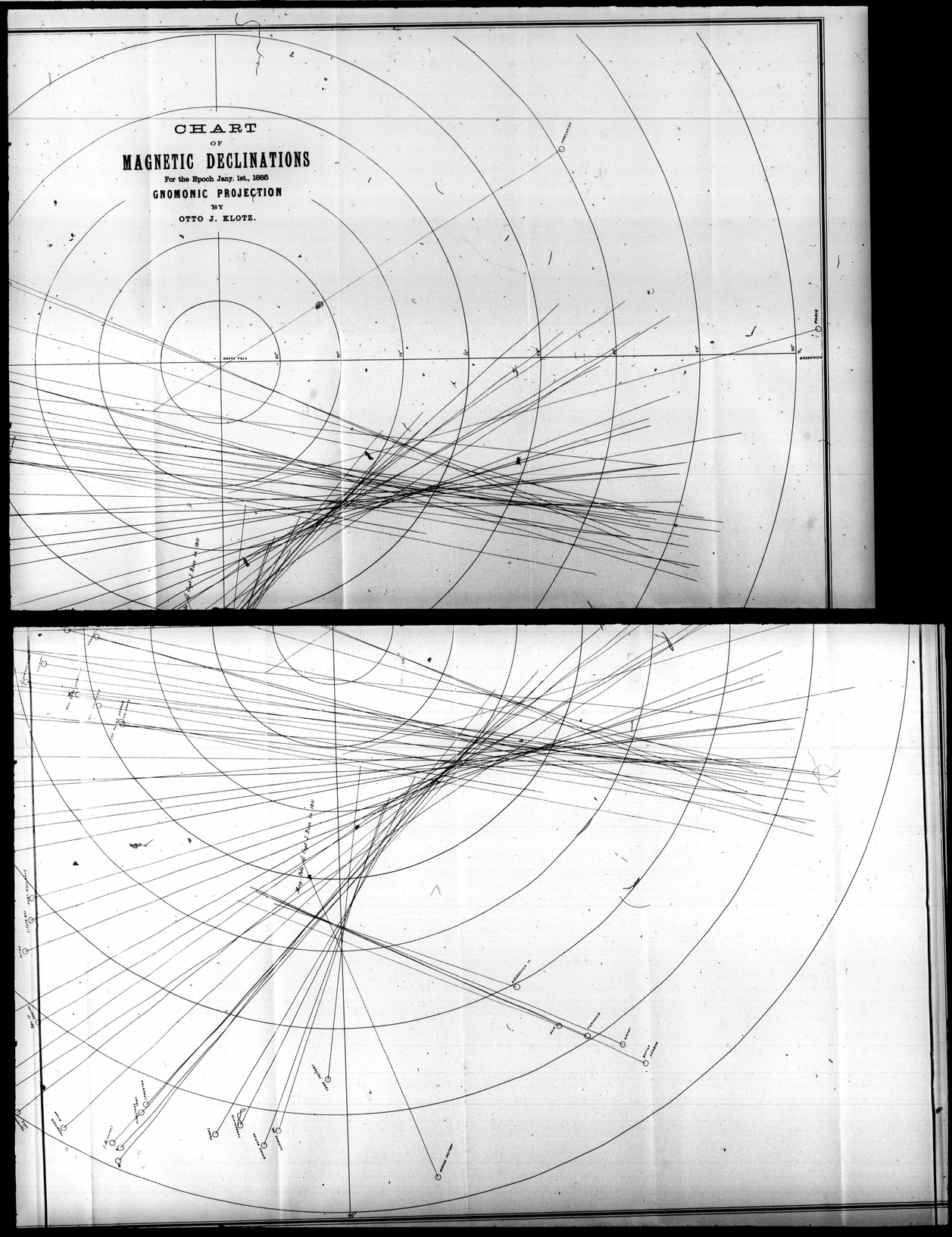

PROCEEDINGS

—OF THE—

ASSOCIATION OF

Dominion Land Surveyors

—AT ITS—

THIRD ANNUAL MEETING,

—HELD AT—

OTTAWA, FEBRUARY 16 & 17, '86.

OTTAWA:
"EVENING JOURNAL" PRINT, ELGIN STREET.
1886.

ASSOCIATION OF DOMINION LAND SURVEYORS

Organized, April 24th, 1882.

OFFICERS FOR 1886.

PRESIDENT	THOMAS FAWCETT.
VICE-PRESIDENT	G. C. RAINBOTH.
SEC.-TREASURER	A. F. COTTON.
EXECUTIVE COMMITTEE	W. CHIPMAN. W. OGILVIE. A. O. WHEELER.
AUDITORS	J. A. SNOW. J. J. BURROWS.

HONORARY MEMBERS.

The Surveyor General,	Prof. Macoun,	Andrew Russell,
Capt. E. Deville,	Dr. Bell,	E. E. Taché,
W. F. King,	Prof. G. Dawson,	Bolton Magrath.
Prof. Selwyn,	Prof. Harrington,	

CONSTITUTION AND BY-LAWS

—OF THE—

Association of Dominion Land Surveyors.

CONSTITUTION.

ARTICLE I.

NAME OF THE ASSOCIATION.

"The Association of Dominion Land Surveyors."

ARTICLE II.

OBJECTS OF THE ASSOCIATION.

The promotion of the general interests, and elevation of the standard of the Profession.

ARTICLE III.

MEMBERS.

1. The Association shall consist of Active Members and Honorary Members.

2. Active Members must be Dominion Land Surveyors, and only such shall hold office.

3. Honorary Members shall be such persons only who are distinguished for professional attainments. They shall be exempt from dues.

ARTICLE IV.

OFFICERS.

1. The Surveyor-General of Dominion Lands shall be Honorary President of the Association.

2. The Officers of the Association shall consist of an Honorary President, a President, Vice-President, Secretary-Treasurer, and an Executive Committee, all of whom, except the Honorary President, to be elected at the annual general meeting by ballot.

ARTICLE V.

ELECTION OF MEMBERS.

1. Any Dominion Land Surveyor upon being proposed in writing by at least two members shall be eligible for election as a Member of this Association upon payment of the necessary fees.

HONORARY MEMBERS.

2. Honorary Members must be recommended by at least two Members.

VOTING.

3. All voting for the election of Members shall be by ballot and at a general meeting of the Association.

4. The majority of the ballots cast shall decide.

ARTICLE VI.

MEETINGS.

1. The Annual General Meeting shall commence on the third Tuesday in February, in Ottawa; and a General Meeting shall be held on the third Tuesday in April, in Winnipeg.

2. Special Meetings of the Association may be called by the President, or by the President, when requested in writing by three or more Members.

3. Five shall form a quorum at any meeting for the transaction of business.

ARTICLE VII.

AMENDMENTS.

1. This Constitution may be amended at any of the General Meetings by a vote of at least two-thirds of the members present at such meeting, but no amendment shall be considered unless notice in writing thereof shall have been given at the next preceding General Meeting.

2. No By-law or Rule shall be altered, or new one adopted, except at a General Meeting. Notice of such proposed change and of the meeting at which it is to be considered shall be given to the Secretary one month before such meeting, the members to be notified thereof by the Secretary.

ARTICLE VIII.

EXECUTIVE COMMITTEE.

1. The Executive Committee shall consist of the President, Vice-President, Secretary-Treasurer, and three members; and shall have the direction and management of the affairs of the Association. Three members to form a quorum.

2. The Meetings of the Executive Committee to be held at the call of the President or Secretary-Treasurer.

ARTICLE IX.

AUDITORS.

Two Auditors, to be elected by ballot, shall audit the accounts of the Association annually, and present their report of the same at the Annual General Meeting.

ARTICLE X.

SUBSCRIPTIONS.

1. The fee for membership for Active members shall be five dollars, and an annual subscription of two dollars for each subsequent year; both payable in advance.

2. Any member twelve months in arrears shall be struck off the roll, and no member in arrears shall be allowed to vote.

BY-LAWS.

ORDER OF BUSINESS.

1. Reading of Minutes of previous meeting.
2. Reading Correspondence and Accounts.
3. Propositions for Membership.
4. Balloting for Membership.
5. Reports.
6. Unfinished Business.
7. Election of Officers.
8. New Business.
9. Adjournment.

2. All motions must be in writing, and shall contain the names of the mover and seconder, and must be read by the Chair before being discussed.

3. Reports of Committees must be in writing, signed by the Chairman thereof.

4. No member shall speak on any subject more than once, except the introducer of the subject, who shall be entitled to reply; every member, however, shall have the right to explain himself, subject to the discretion of the Chair.

5. When a motion has been finally put to the meeting by the Chairman, all discussion thereon shall be closed.

6. The Chairman shall appoint two Scrutineers when a ballot is taken.

7. Every member while speaking shall address the Chair.

DUTIES OF OFFICERS.

1. The President shall preside at all meetings at which he is present; in his absence the Vice-President; and in the absence of both the meeting shall appoint a Chairman.

2. The presiding officer shall only have the casting vote, but not a deliberate one.

3. The Secretary-Treasurer shall keep an accurate record of all meetings, conduct all correspondence, announce all meetings, receive all fees and subscriptions and other moneys, pay no bills unless sanctioned by the Executive Committee and and signed by their Chairman, make an annual report of all receipts and disbursements, and shall perform such other duties as may from time to time be assigned him by the Executive Committee.

THIRD ANNUAL MEETING

—OF THE—

ASSOCIATION OF DOMINION LAND SURVEYORS

—HELD AT—

Ottawa, February 16 and 17, 1886.

THE ASSOCIATION assembled Tuesday, February 16, at 3 P.M., at St. Andrew's Hall, in Ottawa.

OTTO J. KLOTZ, President, in the Chair.

PROGRAMME:

Tuesday, February 16—3 *p.m.*

Routine business.

President's Address.

Election of Officers.

7.30 *p.m.*—A Paper on the method of "Least Squares," by W. F. King, Chief Inspector of Surveys.

A paper on the subject of "Government Map-Making," by R. Bell, LL.D., M.D. & C.E.

Wednesday, 17th—10 *a.m.*

New and General business.

On motion the minutes of the preceeding meeting were confirmed.

The following Dominion Land Surveyors were then duly elected members of the Association:—

J. S. Dennis,

J. J. McArthur.

The President then delivered his Annual Address:—

GENTLEMEN,—Reviewing the events and incidents of the busy world during the past year since we last met, the one touching our profession most was the Rebellion in the North-West, but which was happily soon quelled. Among those who sacrificed their blood for their country was Alexander W. Kippen, D.L.S., from Perth, a promising young man who, scarce a year held his

commission, when in the very bud of development he was cut down at Batoche. Beside the one death, several of our members were wounded while gallantly defending their country and the homes of fellow-countrymen, but I am glad to believe that none of the wounds were of a serious nature.

Looking at the Rebellion in another respect, we must admit that it had anything but an encouraging effect upon the prosecution of surveys in the North-West our *sine qua non*. At the same time, we must remember that the conditions of 1885 were vastly different from those ten years previous. In 1875 we had a tract of country north of the 49th parallel nearly one thousand miles long and three hundred miles wide, scarcely known and but lightly trodden by the foot of the surveyor. In that year there were sub-divided 30 townships and 1,020 miles of block work done. For any of our older provinces thirty townships would be an exceedingly large survey for one year, but in our large domain of the North-West it forms but a small corner. But the great impetus to surveys was given when, in 1880, the Canadian Pacific Railroad Company assumed the construction of the transcontinental railroad and pushed it with such vigor and success that the rapidity of construction stands unrivalled in the history of railroading.

This great national undertaking became an accomplished fact on the 7th November last, when the last rail was laid in the Gold Range in British Columbia, establishing continuous rail communication from Halifax on the Atlantic to Port Moody on the Pacific, and exclusively over Canadian territory.

While speaking of transcontinental railways, I will say, and without fear of contradiction, having travelled over the various lines northward from Mexico, that along no line west of the Missouri, say longitude 96°, is there as much, no, not nearly as much, available agricultural land as along the C. P. R. As yet, Canadians are not in that measure cognizant of the undeveloped wealth of the North-West and British Columbia to appreciate their future greatness, which is sure to follow since life has begun to flow through the iron artery. But I must not dilate on this subject here, although of vital interest to us as Canadians.

In 1880 the surveys had risen to the sub-division of 194 townships and a total mileage of 14,638. But it remained for the year 1883 to give us the grand record of 1,221 townships sub-divided, 1,360 outlined, and a total mileage of lines surveyed of 81,300 miles, a length sufficient to encircle our globe more than three times, and covering an area in which we could drown many of the kingdoms of Europe. During this year there were 119 parties in the field, making a small army of 1,200 men and nearly as many horses. Regarding the work done, I will quote the words of the Inspector of Surveys, Mr. W. F. King, "that the surveys of the North-West as regards accuracy, economy, and the rapidity of their execution, can compare favourably with those of any other country in the world." Never before in the history of any government have such extensive land surveys been prosecuted, nor will ever hereafter such be made; the best reason therefor

being that there does not exist on this earth such vast unsurveyed lands fit for cultivation. But we saw in that year too, only too well, the handwriting on the wall.

We have surveyed our heads off. Block and outline surveyors vied with each other in doing a big season's work, without any special financial benefit accruing therefrom, and the sub-divider who worked on contract naturally completed as much as possible, all tending to hasten the approach of the inevitable, of having devoured so to speak the whole country with the instrument and chain. There will still be for some years a little to do, but the days of outfitting in Winnipeg and camping at Moosejaw are things of the past, never to return. The functions of the Dominion Land Surveyor are about over and he must turn his face to pastures new and in another capacity. In this respect doctors and lawyers have the advantage over us; operating on a patient or client once does not necessarily kill him, in fact it is bad policy to kill him, but the public land admits of sub-division but once and then we are done. For Provincial Land Surveyors the circumstances are very different and more favorable, inasmuch as their practice is confined almost exclusively to private work; and nearly every P.L.S. now-a-days combines engineering in a greater or less degree with his surveying practice. This opens a pretty wide field of action, although the practice is mostly of a local character, that is confined within certain geographical or, more frequently, municipal limits. On the other hand the surveys of Dominion Land Surveyors outside of government work are very limited. The Dominion Land Surveyor precedes civilization, the Provincial Land Surveyor succeeds it.

There are several resolutions of last year to which I shall allude:

Firstly. The one regarding the survey of Dominion Lands in British Columbia.—The Government held an examination in Victoria last May for the first time, and under very lenient terms, to afford surveyors in British Columbia an opportunity of acquiring commissions. I regret to state that the surveyors in general there did not receive it in that spirit in which it was intended, and did not look upon the examination favorably, but considered it wholly unnecessary, in fact preposterous; apparently ignorant of the fact that all Dominion Land Surveyors from the other provinces were, prior to their receiving commissions, subjected (and more stringently) to the same regulations as laid down by statute. The action of the Government in giving surveys of Dominion lands only to such as hold commissions as D.L.S. must meet the approval of every fair-minded person. It is not right to make fish of one and flesh of the other.

Respecting the filling of the vacancy of Chief Inspector of Surveys, I am happy to state that Mr. W. F. King has received the appointment.

Regarding incorporation, as I anticipated, nothing effectual was done, for the Government did not look favorably upon the scheme, especially that part relating to the power of examining and granting commissions. In this respect we are placed in a far different position from Provincial Land Sur-

veyors seeking incorporation ; to the latter, undoubtedly, such rights belong and should be given.

Our association, if it continue to exist, which I earnestly hope it will, must from force of circumstances become more and more of a scientific body, treating of such subjects as come within range of the Dominion Land Surveyor while acting in that capacity. Such subjects would include say :— Instruments, field work, stellar, magnetic and meteorological observations, mathematical solutions, geology and natural history. Up to the present there has certainly not been a dearth of material for our members to write upon, if they would only avail themselves thereof.

I am fully aware of the difficulties which now lie at the door of our future existence as an association. In former years when nearly all the members had Government surverys, the coming to Ottawa to attend the meetings was always combined with business, but since that has almost stopped, there is little inducement for members to incur annually an expense of nearly $50, as it costs many of us to come to Ottawa to attend a meeting at which the discussions, although they may be interesting, treat of subjects in which but a few are practically interested.

It is probable that during the coming season some surveys of Dominion lands in British Columbia will be made. Let me assure you that, though the system of survey will be the same or similar to that in the North-West, the execution thereof will be very much different, that is, attended with more physical labor, and in many cases in no small measure. Surveying at best, even on the plains, has its attendant hardships, but in the mountains this is increased many fold. Near the sea-coast the extremely heavy timber, where trees obtain a thickness of many feet, is a drawback to rapid work, to say nothing of cliffs and gulches where chaining is impossible, triangulation impracticable, and micrometic measurements have to be resorted to. The use of carts is out of the question. In the interior, open, or park country, pack horses are most advantageous, and in the woods the most reliable animal of all—man, with a pack strap is necessary. The surveyors' coat-of-arms should undoubtedly bear the inscription "*Per aspera ad astra*"; but while we have all of the *per aspera*, little of the *ad astra* is granted us.

During the last week in January last I attended the annual meeting of the Michigan Engineering Society held at Ann Arbor. I was very kindly received, and the courtesies of the floor were extended to me. Their scope of topics is, of course, very much wider than ours could be. The society experimented this year by dividing into three seperate sections, surveying, civil engineering and mechanical engineering. The idea is a good one, provided a sufficient number can be obtained who take a special interest in each one particular section, but as it is, most of the members are no "specialists," but prosecute a general engineering and surveying business, and therefore a separation into sections, especially of civil engineering from surveying of doubtful advantage, which was so expressed by

some of the members. One interesting feature of their discussions is the one pertaining to the Act—how to establish lost corners and boundaries. This would be also an interesting topic for a Provincial Association in Ontario.

A scheme is now developing among the various engineering societies of the United States for laying before Congress, whereby a "Civil Bureau of Public Works" is to be established, and which is to have "an absolutely non-political service." At Cleveland, Ohio, the preliminary convention was held last December, and from the committee's report I make the following extract: "The distinguishing feature of these organizations is the employment of the best available talent, of men especially fitted by education, training and experience to design and execute such works. Under our present methods, the Government can not possibly secure such trained skill. The reason is that no emoluments or reputation, commensurate with the service rendered, or equal to what can be obtained on other civil works, are attained on the public works of our Government. These facts deter men of experience from entering the service of the Government, and also those who have gained experience therein, from remaining in its employ." It is pleasant to know that in the realization of this scheme our great neighbor to the south of us can advantageously take a leaf out of our book. Let us hope that their efforts will be crowned with success, for its ultimate result will be not only an advantage to the profession, but in a greater measure to the country's welfare and the economical expenditure of its finances, saving hundreds of thousands, nay, millions, to a now over-indulgent people.

Reverting now to some of my remarks to you in my last year's address on terrestrial magnetism, I lay before you a magnetic declination chart which I prepared shortly after the meeting.

I was prompted to prepare the same from a desire to know within what area the magnetic meridians of northern stations intersected each other; whether even approximately thereby a magnetic pole became apparent, and if so, where its approximate location is. As I stated last year, the whole question of terrrestrial magnetism is an intricate problem. It is impossible to separate local magnetism (not of a visible isolated body) from terrestrial magnetism.

It was my intention to embrace the whole of the northern hemisphere north of the 49th parallel on my chart, showing the magnetic declinations throughout this vast segment, but I failed to obtain the expected data throughout the great Russian domain, although I received from H. Wild, Director of the Central-Observatorium, St. Petersburg, much information, but not what I especially wanted.

In preparing such a chart, it is necessary to reduce for comparison all declinations to a common epoch. For this epoch, I have taken Jan. 1, 1885, and for projection the gnomonic in which all great circles are projected as straight lines, hence both the projected astronomic and magnetic meridians are straight lines, and the parallels of latitude are projected as circles.

Assuming L for the latitude, d for the declination, and O for the projected angle, we have tan. O=sin. L tan. d.

I believe that from the data and the graphic results obtained therefrom I am safe in concluding that there is no physical magnetic pole.

That the magnetic pole of Capt. J. Ross, in 1831, was not the resultant of purely terrestrial magnetism, but attributable more to local or circumscribed causes. Look on the chart at the many intersections and you will find scarcely a single one that is as far west (long. 96° 46′) as Ross' pole was in 1831; and since then the magnetic declination has been moving westward, how much farther would this supposed pole be away now. If there be a magnetic pole we certainly would expect to find it within the medley of intersections of magnetic meridians and not far beyond this area.

Assuming a uniform change in declination between Moose Factory and York Factory, which are the two nearest stations at which the declination is of opposite signs, we find the agonic line to be situate about longitude 90°, similarly another agonic line west of Petropavlovsk approximately in longitude 203°, and a third agonic line east of St. Petersburg, about longitude 32° E. This gives three distinct agonic lines in the northern hemisphere, and which are very nearly symmetrically situate, that is 120° apart. One point more and I shall have done with this subject at present, as I expect to enter into it more fully when I have the data to cover the northern part of the eastern hemisphere, and that is, that there appears to be a magnetic area as a resultant from the declination of places west of the Cordilleras different from that of the remainder of the northern part of this continent.

To the United States coast and geodetic survey I am indebted for the declinations in the Territory of Alaska; and at the same time acknowledge the universal courtesies that have been extended to me by the various departments at Washington. This brings vividly before me the superior official reports issued at Washington, especially as regards their make-up. In this respect there is considerable room for improvement in our official reports.

This closes my address, gentlemen, and I now resign the presidency which through your kindness and forbearance I have occupied during the past four years, since the first organization of the Association.

The following is a list of stations employed in preparing the accompanying chart:—

Stations used for the accompanying Magnetic Declination Chart (Gnomonic Projection).

No.	Station.	Latitude.	Longitude.	Declination.
1	Battle Harbor, Labrador	52° 16′ .3	55° 34′ .5	+36° .81
2	Grady, "	53° 48′ .2	56° 25′ .3	+38° .65
3	Turnavik, "	55° 14′ .9	59° 19′ .0	+39° .97
4	Nain, "	56° 32′ .7	61° 40′ .7	+44° .43
5	Aulezavik Island	59° 47′ .5	64° 13′ .2	+48° .45
6	Moose Factory	51° 15′	80° 40′	+16° .21
7	York Factory	57° 00′	92° 26′	− 6° .59
8	Norway House	53° 59′ .6	98° 03′	−14° .97
9	Grand Rapids	53° 08′	99° 27′	−15° .62
10	Cumberland House	53° 56′ .7	102° 19′ .2	−20° .14
11	Forks Saskatchewan	53° 13′	104° 51′ .6	−21° .82
12	Edmonton	53° 32′	113° 30′ .4	−26° .36
13	A, Pipestone Creek	53° 04′	113° 35′	−25° .20
14	W, N. W. Territory	50° 22′	113° 49′	−22° .02
15	X, "	51° 02′	114° 00′	−24° .19
16	"	51° 05′	115° 00′	−23° .85
17	Thompson River	50° 41′	120° 12′	−23° .96
18	Squirrel Cove	50° 08′	124° 57′	−23° .91
19	Tracy Harbor	50° 51′	126° 53′	−26° .64
20	Safety Cove	51° 32′	127° 57′	−23° .58
21	50 miles up the Skeena	54° 30′	128° 35′	−26° .44
22	Anchor Cove	53° 12′	132° 14′	−24° .88
23	Sitka	57° 03′	135° 20′	−28° .84
24	Lituya Bay	58° 37′	137° 40′	−29° .25
25	Port Mulgrave	59° 34′	139° 46′	−29° .06
26	Port Etches	60° 21′	146° 38′	−27° .21
27	St. Paul, Kadiak	57° 48′	152° 21′	−24° .42
28	Chirikof Is.	55° 48′	155° 43′	−22° .17
29	Humboldt Harbor	55° 19′	160° 31′	−19° .96
30	Bellkoffski	55° 05′	162° 00′	−21° .16
31	Unalashka	53° 53′	166° 32′	−18° .80
32	St. Paul Is.	57° 07′	170° 19′	−17° .40
33	Atka Island	52° 11′	174° 15′	−16° .57
34	Adakh Is.	51° 49′	176° 52′	−13° .47
35	Amchitka	51° 24′	180° 48′	− 6° .85
36	Kyska	51° 59′	182° 30′	−10° .67
37	Chicogoff Harbor	52° 56′	186° 48′	− 7° .28
38	Bering Is.	55° 14′	194° 08′	− 3° .63
39	Petropavlovsk, Siberia	53° 01′	201° 19′	− 0° .83
40	Natchika	53° 07′	202° 35′	− 0° .74
41	Off Point Barrow	71° 20′	156° 15′	−36° .69
42	Uglaamie	71° 18′	156° 40′	−35° .16
43	Near Icy Cape	70° 13′	162° 15′	−29° .32
44	Near Cape Lisburne	68° 53′	166° 06′	−24° .93
45	St. Lawrence Bay	65° 35′	170° 44′	−19° .38
46	St. Lawrence Is.	63° 43′	171° 23′	−18° .08
47	Konyam Bay	64° 50′	172° 57′	−16° .87
48	Pitlekai	67° 05′	173° 30′	−18° .62
49	Wrangell Is., south coast	70° 57′	178° 10′	−19° .38
50	Irkaipi	68° 50′	180° 00′	−16° .80
51	Paris, France	48° 50′ 13″	− 2° 20′ .2	+15° .8
52	Pawlowsk, near St. Petersburg.	59° 41′ 13″	−30° 29′	+ 0° .6

The officers for the ensuing year were then elected:—

President	THOMAS FAWCETT
Vice-President	G. C. RAINBOTH.
Sec.-Treasurer	A. F. COTTON.
Executive Committee	W. OGILVIE. W. CHIPMAN. A. O. WHEELER.
Auditors	J. A. SNOW. J. J. BURROWS.

It was moved by T. Drummond and seconded by C. E. Wolff, and resolved, that a vote of be tendered to Otto J. Klotz, the retiring President, for the efficient manner and zeal in which he has filled the chair since the organization of the Association.

The meeting then adjourned, to meet again at 7:30 p.m.

Tuesday, Feb. 16th, 7:30 p.m.

THOMAS FAWCETT, President, in the Chair.

W. F. King, Chief Inspector of Surveys, read the following paper on the method of "Least Squares."

SOME REMARKS UPON THE METHOD OF LEAST SQUARES.

"Precision of measurement," said Herschel, "is the soul of science."

We may know that the earth and the planets move round the sun, that heat expands metals, that an electric current deflects a magnetic needle, we may be in possession of the knowledge of a multitude of such facts. Such general information is, however, of little use. When we come to consider the hidden causes of these phenomena or to make applications of them, it is necessary to measure accurately the effects produced, and the progress of science ever demands more and more accurate measurements. This is especially the case in astronomy and geodesy.

All measurements are subject to many errors, some of which may be foreseen and either avoided or eliminated by the process of observation, or corrected for afterwards in the final result.

As examples of the avoidance of error may be cited—the shading of thermometers from radiant heat from surrounding objects, when the temperature of the air is desired—the choice of a level spot for measuring a base line for a trigonometrical survey—the selection of a star near the prime vertical in observation of time by altitudes, for the purpose of avoiding as much as possible the effect of instrumental error—and the use of the most perfect instruments available in any kind of work.

As examples of elimination of error by the method of observation we may give:—The elimination of the effect of refraction in zenith telescope observations; the elimination of collimation error by observations in reversed positions of the transit instrument; in longtitude determinations by electric telegraph the elimination of the time of passage of the electric fluid by sending signals both ways, and of the personal equation of the observers by exchange of stations; and in the observation of magnetic inclination the obviating of errors due to inequality of the needle pivots, eccentricity of the graduated circle, and want of balance of the needle, by successive reversals of the position polarity of the needle.

Some examples of computed corrections applied to the result after the observation are:—The usual method of applying the correction for refraction

to an observer altitude; of calculating the level correction in the transit instrument; the determination of the personal equation in longitude determinations by comparison of the observers' results at the same place; the correction of a chain measurement for expansion of the chain by heat.

Astronomy stands at the head of the sciences for the facility and perfection with which such constant errors can be got rid of.

But the most perfect observations, by the most careful observers, and with best instruments, exhibit discrepancies which can only be accounted for as "accidental" errors. Thus, if several observations for latitude be taken with an instrument which reads to less than one second of arc, the results may be found to differ from another over a range of three or four seconds. Such differences may be accounted for as due to imperfect manipulation of the instrument, tremors caused by wind, irregularity of refraction, and so on. This last mentioned cause, irregularity of refraction, is particularly noticeable about sunset, and in changeable weather. But even in the finest weather, and in apparently the most favorable circumstances, these irregular errors appear.

For obtaining as nearly as possible the truth from these divergent results, recourse is had to the theory of probability, which, working from the assumption that there is no reason that errors in excess or defect should predominate in number or magnitude, leads us to the conclusion that the most probable result is that which makes the sum of the squares of the residual errors of the several observations the least possible. Hence this mode of working is called the "Method of Least Squares."

The ordinary method of means, *i.e.*, the taking of the arithmetical mean or average of a number of direct measurements of the same quantity, is a particular case of the method of least squares, the arithmetical mean being, in this case, the most probable result. In other cases, however, where two or more unknown quantities are involved together in the measurements, the general formulæ must be used, the arithmetical mean here not being applicable.

This method has been objected to in that it depends upon the theory of probabilities or chances. How can we reason about results subject to errors of pure chance? and there being a doubt of each result, in fact a certainty that no result is true, how from these divergent observations can we expect to obtain a result more probable than any of them?

Chance or probability, howevever, does not exist in nature, everything is subject to fixed laws, and if we knew those laws we could explain every effect of them. The doubt only exists in the mind, and the theory of probabilities simply teaches us how to sift our knowledge of a particular matter, and to regulate our belief in such a way as will in the long run lead to the least disappointment.

If we have a large series of direct observations, measurements of an angle, for instance, equally well determined, the mean of all will be the most probable result. The difference between each of the observations and this mean is called the residual. We may represent these residuals graphically by means of a curve, the abscissal representing the magnitudes of the residuals and the ordinates their relative frequency. The mathematical theory again shews that the frequency of errors of various amounts may be represented by a continuous curve. Now the curve given by any actual series of good observations will very nearly coincide with this theoretical curve, thus supporting in the most satisfactory manner the assumption upon which the mathematical processes rest, and confirming the theory of probabilities as applied to these matters.

An example of this may be given—470 observations made by Bradley and reduced by Bessel exhibit the following numbers of errors of various magnitudes, as in the following table:

Magnitudes of the Errors in parts of a Second.				Number of Errors of each Magnitude.	
				Observation.	Theory.
Between	0.0″	and	0.1″	94	95
do	0.1″	do	0.2″	88	89
do	0.2″	do	0.3″	78	78
do	0.3″	do	0.4″	58	64
do	0.4″	do	0.5″	51	50
do	0.5″	do	0.6″	36	36
do	0.6″	do	0.7″	26	24
do	0.7″	do	0.8″	14	15
do	0.8″	do	0 9″	10	9
do	0.9″	do	1.0″	7	5
Above,	1.0″	..	....	8	5

The correspondence betweeu theory and observation is seen to be remarkably close.

Formulæ and tables for the calculation of the number of theoretical errors as above are given in works upon the method of Least Squares. (See for instance, Chauvenet's Astronomy, vol. 2.)

By summation of the squares of the residuals, dividing the sum by the number of the observations diminished by unity, taking the square root and multiplying it by the constant 0.6745, we get what is called the *probable error* of an observation.

The meaning of this term is that error which theoretically stands midway in the series of residuals, so that there are as many residuals arithmetically greater than it as there are less—or, as it is sometimes expressed, it is an even chance that the error of an observation is less than the probable error.

The probable error of an observation, multiplied by the square root of the number of observations, gives the *probable error of the mean*. This is usually employed as the test of the precision of the observations; but in using it as such, it must be remembered that it is a measure of the precision of those observations only which are employed in its computation, and that these observations may be all subject to some constant source of error, which will, of course, equally affect their mean, so that the true result may not in fact tie within the limits of the probable error.

Thus, suppose a number of observations to give, as their mean, latitude 52° 40′ 30″.73, with a probable error of 0″.25. It is, then, an even chance, so far as these observations are concerned, that the latitude of the place is between 50° 40′ 30″.98 and 50° 40′ 30″.48. But if the observations have been, for example, made on all of the stars south of the zenith, a constant refraction or instrumental error may enter into all of them, and we may find by other observations made in such a way as to cut out these errors, a result very different from the above. Still we may assert that if a similar series of other observations be made in like manner as the first, that it is an even chance that the result of them will lie between 52° 40′ 30″.98 and 52° 40′ 30″.48.

The probable error is thus a very useful test of precision, but it must be supposed that a small probable error relieves the observer from the task of looking into the possibility of his observations being vitiated by a constant error, which he may have overlooked, and of renewing his observations under as many different conditions as possible.

In a good series of observations there should be as many errors less than the probable error of an observation as there are greater than it. Also, the greatest residual should not exceed about five times this probable error. If the actual series shews results differing much from these rules, it may be assumed in general that there is some particular cause of error drawing the observations apart or together as the case may be, and they should be investigated. For instance, sometimes it happens that there is a want of small residuals, and that the results are grouped into two series, about equally distant from the mean. This would indicate a cause of error working in one way during part of the observations and in the opposite way during the rest. These observations are then in the position of observations with the transit face right and face left, which differ by the amount of the collimation error, and the formula of the probable error is not applicable until the error has been corrected.

An example of the inapplicability of the probable error from a similar cause is to be found in the mean annual temperature of a place. Suppose an observation of the temperature taken every hour during a whole year and the mean of all taken. This would be the mean annual temperature very nearly, but it would be absurd to take the differences of the several observations from the mean in the usual way to determine the probable error of an observation or of the mean for the heat of summer and the cold of winter act as regular errors pulling the observations apart.

In cases also, where the results are not independent, the probable error cannot be applied. For instance, in target shooting, the marksman is governed in his aim by the results of his previous attempts, and besides the moral effect of success or failure, may show itself in his subsequent trials. In this case the successive events are really not independent, as is supposed in the least square theory. But if the aim were made independently each time, the marksman not knowing the result of his shot, which would be subject only to the purely accidental errors of aim and wind, etc., we should no doubt find on examining the target that the shots were distributed according to the law of error.

The same remarks may be made of any observations in which the observer allows himself to have a bias, or tries to make his results accordant. In such a case he may succeed in making the results agree, but the probable error found from them will not represent the real precision of his observations, and the results very likely will be quite far from the truth. In order to keep the mind free from this bias, it is better not to allow oneself to speculate on the results already obtained or to think of the effect of one's measurements until all have been completed.

Closely allied with this subject is that of mean values of functions. Thus the mean of all the values of the common logarithm of x for all possible values of x from 1 to 10 is to seven places of decimals 0.6768166. This result is found by the integral culculus, but anyone can easily satisfy himself, by taking the logarithms of 2, 3, 4, etc. up to 10, that their mean is nearly the number stated, and by taking more logarithms of intermediate numbers equally distant, such as $2\frac{1}{2}$, $2\frac{1}{4}$, $2\frac{3}{4}$, etc., the mean will always come nearer and nearer to the theoretical value above.

The mean value of the sum of the latitudes and departures of a traverse calculated in the usual way from bearing and distance for all possible bearings 0° to 90° is $\frac{\pi}{4}$ × distance or 1.273 × distance, π representing the ratio of the circumference of a circle to its diameter.

Hence if we had a traverse of an infinite number of courses, the bearings being of all possible values, we should find that

sum of latitudes + sum of departures =
1.273 × sum of distances,

and in any actual traverse of a finite number of courses we shall find this law

to hold more or less exactly according as the bearings are more or less equally distributed through the quadrant. This rule forms a curious, though of course rather rough check upon the calculation of a long traverse. The sums must be the arithmetical sums without regard to sign.

In concluding, in order to show the inherent impossibility of perfect measurement, I may refer to the fact that in theory it is infinitely more probable that two given quantities are incommensurable than commensurable. Thus in comparing a rod with a standard measure, it is infinitely unlikely that the rod and the standard will have a common measure. Hence the rod cannot be exactly expressed in terms of the standard. If they appear to be commensurable it is because of the imperfection of our senses in not perceiving the small differences.

If we improve our means of measurement as by increasing the power of our microscopes, then small quantities may become perceptible. So that increased power of observation by no means does away with the theoretical inaccuracy of measurement.

ROBERT BELL, B.A.Sc., M.D., LL.D., Assistant Director of the Geological Survey, then read a paper, of which the following is a report:

GOVERNMENT MAP-MAKING.

In addressing the meeting on the subject of the production of maps by the various Departments of the Government at Ottawa, Dr. Bell said it was not his intention to express any views of his own, but to endeavor to elicit the opinions of the Association in regard to this important subject. He would mention a few points which had pressed themselves on his attention, in the hope that they would be discussed by the members present. One of these was the question of uniformity of scales. Was it desirable that, instead of the great diversity of scales now in use, a certain fixed and uniform system or grade in scales should be established? Different departments selected different systems of scales, and even in the same department there might be no uniformity of practice. The scales adopted seemed to be capriciously chosen, and they were changed from year to year. No regular set of numbers of miles or chains to the inch appeared to be in use, and even fractions were introduced. In fact, the scale of the map was often made to fit the size of the particular stone on which it was about to be engraved. It was hardly to be expected that we would at an early day construct our maps on natural scales, as was being done in European countries, but, in the meantime, could not some improvement be made in our method—or, rather, want of method? On the Geological Survey, Sir William Logan had adopted a scale of four miles to the inch, and multiples and sub-multiples of this; and nearly all our maps were on 4, 8, 2, 1 and ½ miles to the inch. This gave sufficient scope for delineating every variety of work, and made it easy in compiling to reduce or enlarge from one scale to another, to say nothing of the convenience in comparing areas, measuring distances, etc. For general small-scale maps, Sir William had used 25 and 125 miles to the inch. Could not some such plan be introduced with advantage into all the Departments?

A second point which had occurred to him was the question whether the same maps, printed in black, might not be used for a variety of objects by afterwards printing on them in colors—or even in black—the particular things to be represented, instead of, in every case, having a fresh map engraved, often very much like one already executed, as far as the scale and topography were concerned. In this way much expense and delay might be avoided. For example, the same foundation-map might thus be utilized by the Departments of Railways and Canals, the Post Office, Telegraphic Service, Marine, Agriculture, Dominion Lands, Geological Survey, and Indian Affairs, and also for a variety of highly useful purposes for which such maps are now used

in the United States, such as the representation of statistical, religious and political information, the distribution of the economic minerals, the different kinds of timber, etc. If the money spent on half-a-dozen or more of hurriedly compiled and poorly executed lithographs were concentrated in the production of a carefully drawn and skilfully engraved copper-plate sheet, we might perhaps see some maps of which we should have more reason to feel proud. Another advantage of this method would be that we should secure uniformity in the spelling of geographical names. The various ways of spelling names, due to the caprice or carelessness of draughtsmen, which one may now see on the maps of the various Departments, all issued on the authority of the one Government must be rather perplexing to strangers, if not to some of our own people. We have, as yet, no recognized authority on this subject, and it would be very desirable to have some standard to guide us, especially in these days, when such extensive new districts are being opened up and large numbers of names are being constantly added to the geography of the Dominion. Some names are seldom spelled alike in any two maps or descriptions. For instance, the original, correct and simple spelling of the word Nipigon—which had been established for more than a hundred years—is now fancifully varied to Nepigon, Nipegon, Neepigon, Neepegon, Nipeegon, etc., etc.

Let us suppose that we admit the desirability of some uniformity as to scales and nomenclature, the question arises, would there be any advantage in making a general rule that all our maps should be published in sheets of a fixed size and shape, like the Ordnance maps in Great Britain? And here Dr. Bell believed there was room for a difference of opinion, and thought that the majority would lean to the view that there would be no advantage in such a plan, as the very nature of the country made it undesirable to attempt any such regulation. It might be applicable to certain limited districts, where the population was dense and the land valuable, such as the inter-lake peninsula of Ontario. We should not be carried away with the notion that everything which was suitable for England must be suitable for us. People who knew little or nothing about the subject were apt to imagine that they could take any general map of half a continent and by ruling it with straight lines into rectangles like the bricks in the side of a house, they would have a true index map, and that the actual areas on the ground would all be found to fit themselves into their arbitrary "sheets," forgetting altogether that the surface of the earth is not a plane. These persons do not seem to imagine that there would be any trouble about correctly projecting on a single map such an extensive part of the surface of the planet. No matter how the arbitrary boundaries of a set of such divisions may be selected, it will be found that whole sheets have to be printed in order to show certain obstinate points or corners that refuse to come into any system of sheets. Would it not be better to make the size of our sheet according to the ground which it was actually required to represent, always adhering to one or another of the scales which had been resolved upon? As a matter of fact, we all knew that there were vast areas of the Dominion which will never be worth accurately surveying or mapping at all, as far as we can reasonably judge at present. On the other hand, there are certain natural districts, more or less isolated, either by water or great wastes of worthless ground, which might require to be well surveyed, and which could be conveniently shown within the borders of a map of the proper dimensions to cover the whole of such a districts and no more. Such a map could not be made to form one in any system of an immense number of sheets arbitrarily laid down, both as regards position, and size and shape. In illustration of the folly of cast-iron rules about the size of sheets, Dr. Bell related an an anecdote about a geologist whose cabinet was divided into pigeon-holes of a uniform size, and who was in the habit of smashing valuable fossils and minerals, because nature had not made them according to his

ideas of size, or, as he said, "Because they would not go into the box." In the United States, the Navy Department makes its charts of all sizes, according to the purpose for which they are intended to be used, and not to fit into anybody's "box," and probably we cannot do better than follow their example in regard to our maps.

In the preparation of the general topographical maps already suggested, would it be desirable to have a central map bureau, so that the labors of the numerous skillful draughtsmen now in the service of the Government might be more effectively applied? If this plan were adopted, the collections of maps now scattered among the various departments might be brought together and classified, and so made available for the perfecting of any compilation which might be required. As it is now, maps may be produced in one department which might have been made much more complete and accurate had the draughtsmen been aware of the existence of the necessary data in other departments.

The subject of a general mapping bureau naturally led to that of a Government engraving and printing establishment, such as those which exist in some other countries, and have been found to work satisfactorily. This matter should not be looked upon altogether as a question of greater or less cost, but regard should also be had to the quality of the work which might be done.

Another important point in regard to Government map-making to which Dr. Bell referred was the matter of projections. At present there is no uniformity in this matter. Different draughtsmen are using different systems, the conical, polyconical and cylindrical being the commonest. But on most of these it was impossible to find the true distance between widely separated points. Captain Deville, our accomplished Surveyor-General, had informed Dr. Bell that at the next meeting of the Royal Society he intended to propose a new system which he had devised, as suitable for the Dominion, and which might be called an oblique-cylindrical projection. It would be desirable hereafter to adhere to some one system to be adopted after mature consideration.

Lastly, we might now very properly begin to consider the advisability of commencing a proper trigonometrical survey of the whole Dominion, but which should only be made in detail where the population was most dense, or where the requirements of the country otherwise warranted it. Until such a survey was made it would be impossible to correctly map even such a well-known region as the country between Lakes Huron, Erie and Ontario. This fact was forced upon Dr. Bell's attention, as he was just now engaged in preparing a geological map of that region. The plan of each of the large number of townships comprised in this tract appears to have been plotted as if the area which it covered were a plane, and when all the separate plans are placed together they cannot be crowded into the space between the boundaries of the lakes. No amount of skill on the draughtsman's part can make the matter right. Adjust the townships as you will, discrepancies are found; the railway lines will not cut the lines of the township surveys where they should cut them, and the compiled townships will not fit to the shores of the lakes as laid down on the charts, according to any system of projection. The expense of a general trigonometrical survey, such as now proposed would, no doubt, be ultimately very considerable, and although the land would not, after it were done, produce any more wheat to the acre nor sell for a dollar more than without such a survey, still there are other ways, as you all know, in which it would, in the end, be worth much more than the cost. This has been abundantly proved in the United States in regard to the admiral Coast Survey. The outlay would be spread over many years, and would never be felt. It was clearly the duty of an enlightened government to begin this work.

Dr. Bell's address was followed by an interesting discussion, in which several of the members of the Association took part. At its close, Mr. W. F.

King, D.L.S., and Dr. Bell were appointed a Committee to enquire into the whole subject of Government map-making, and to report at the next meeting, in order that the Association may be in a better position to advise the Government in this important matter.

The meeting then adjourned, to meet again on Wednesday the 17th, at 3 p.m., in the office of Messrs. Wolff & Cotton.

Wednesday, Feb. 17th—3 p.m.

THOMAS FAWCETT, President, in the Chair.

It was moved by J. S. Dennis and seconded by E. Bray, and

Resolved, that the officers and Executive Committee of the Association be instructed to take any steps necessary to find out whether it is the practice for Dominion Land Surveyors to sign plans of surveys made by persons who are not Dominion Land Surveyors, and in the event of their being able to prove such action on the part of any surveyor, that they report the same to the Board of Examiners for Dominion Land Surveyors and request that action may be taken by the Board to cancel the commission of such Surveyor.

Moved by J. S. Dennis and seconded by E. Bray, and

Resolved, that a committee be appointed to draw up a memorial to be presented to the Department, setting forth the injustice done to Dominion Land Surveyors by allowing civil engineers to sign plans of right of way surveys for railways in the North-West Territories, and asking that the Department refuse to accept or fyle any plans signed in this way.

Moved by J. S. Dennis and seconded by C. E. Wolff, and

Resolved, that E. Bray, Wm. Ogilvie and the president be a committee to draft a memorial to the Department of the Interior, as provided in the motion regarding right of way surveys in the North-West Territories.

Moved by T. Drummond aud seconded by T. S. Gore, and

Resolved that the mover and seconder be added to the Committee.

Moved by T. Drummond, seconded by E. Bray and

Resolved, that the President, Otto J. Klotz, Wm. Ogilvie and J. S. Dennis be a committee to draft a memorial in extenso on the advisability and public benefit to be derived from a trigonometrical survey, to extend over the older Provinces of the Dominion, and that such memorial be laid before the Minister of the Interior.

Moved by W. F. King, seconded by J. S. Dennis, and

Resolved, that Messrs. Drummond, Bray, Garden and Cotton be added to the Committee appointed to draft a memorial with respect to the advisability of a trigonometrical survey ot the older Provinces of the Dominion.

Moved by Otto J. Klotz, seconded by Wm. Ogilvie, and

Resolved, that Dr. Bell and Mr. King be a Committee to enquire into the best means for having a uniform system of map-making in the various

Departments of the Government, and that the conclusion at which they may arrive be communicated to the various Departments.

Mr. Ogilvie then gave some notes of his experience in the use of the micrometer.

Moved by J. Dennis, seconded by T. Drummond,

That if it is found that the paper promised by Mr. Ogilvie on "Micrometric Measurements" is not ready in time to be issued with the Annual Report, that the Secretary be authorized to get the paper printed in sheet form and issue the same to the members.

The Treasurer then presented his accounts.

Moved by G. C. Rainboth, seconded by E. Bray, and

Resolved, that the accounts of the Secretary-Treasurer, as audited, be accepted.

Moved by E. Bray, seconded by J. F. Garden, and

Resolved that the sum of twenty dollars ($20.00) be paid Mr. Cotton from the funds of this Association as a slight token of our appreciation of his services as Secretary and Treasurer.

NOTICES OF MOTION.

J. S. Dennis gave notice that at the next annual meeting of the Association he will move that the Constitution be amended so as to provide that the election of officers shall be the last business of the meeting, before adjournment.

Mr. Ogilvie gave notice that at the next annual meeting of the Association he will move that an article be inserted in the Constitution of this Association, providing that no member shall be allowed to fill the office of President of it for more than two consecutive years.

Mr. Ogilvie gave notice that at the next annual meeting of the Association he will move that the article in the Constitution, which requires one year's notice of any change in the Constitution, be struck out and the following inserted :—

Any member of this Association, who may desire any change in the Constitution of the Association, shall give notice of such contemplated change to the Secretary of this Association at least two months before the next annual meeting, and the Secretary shall in his notice of such meeting to the members, notify them of the name of the party proposing such change and the nature thereof.

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