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Reprinted From The Journal Of
The Royal Astronomical
Society Of Canada,
November-December, 1911

THE ORBIT OF ω URSÆ MAJORIS

BY

T. H. PARKER

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THE star ω Ursæ Majoris, $\alpha = 10^h 48^m$, $\delta = +43° 43'$, phot. mag. 4·8, was announced as a spectroscopic binary by Vogel in 1903.* It was included in a list of 528 stars whose spectra were investigated by Vogel and Wilsing at Potsdam. Vogel states that on one plate he found an indication of the doubling of the K line, and the *Mg.* line λ 4481 doubled on one or two others.

It was first observed here in February 1908, and since then sixty-nine spectrograms have been obtained—fifteen with the old, and the remainder with the new single-prism spectrograph. This star is of the *A* type, according to the Harvard classification, the principal lines being measured being the *Mg.* λ 4481, the hydrogen series and K. Only three of the plates obtained here show definite double lines. This is probably due to the faintness of the secondary component, whose mass, as will be seen later, is only about one sixth that of the primary, as well as to insufficient dispersion in separating the two spectra. The length of exposure required for a star of this magnitude forbade the use of the three-prism instrument. On this account also "Seed 27" plates were used for the majority of the spectrograms. Six were taken however on "Seed 23," and the finer grain gave a much better spectrum. The average length of exposure required for these was 90 minutes. The blending of the lines of the two spectra made the measurement of the plates rather unsatisfactory. In one plate in which the lines were separated those which showed doubling were the *Mg.* line λ 4481 and the two iron lines λ 4325 and λ 4308. In another the lines λ 4308 and λ 4101 ($H\delta$) were found to be doubled, with faint

* *Astronomische Nachrichten*, Vol. CLXIII., p. 145, 1903.

indications also of a secondary spectrum in the iron lines λ 4549, λ 4325 and λ 4260. In the third plate only λ 4308 was measurable. No trace of a doubling of the K line was found on any of our plates.

The lines measured were as follows:

Elements	Wave-Length	No. of Times Measured
Hβ	4861·527	12
Fe	4549·766	46
Mg	4481·400	69
Hγ	4340·634	58
Fe	4325·939	5
Fe	4233·328	7
Si	4128·211	9
Hδ	4101·890	33
Ca (K)	3933·825	39

The hydrogen lines with the exception of *Hγ* are broad and diffuse. The *Mg*. λ 4481 is the best line in the spectrum and was measured on every plate as will be seen in the table above. Metallic lines other than *Mg*. λ 4481, *Fe* λ 4549 and K do not occur frequently. As different lines on the same plate in many cases gave widely differing velocities the determination of the period offered some difficulty. Several such plates were remeasured or checked by other observers, and the resulting means taken. These measures were usually in fair agreement. From the consideration of the velocities of the *Mg*. line alone the period was found to be between fifteen and sixteen days. Several trials using the velocities of whole plates gave 15·84 days as the most satisfactory period.

Following is a table of observations with data of each plate:

Plate	Julian Date	Phase from Final T	Velocity	No. of Lines	Weight	O − C
1340	2417993·801	2·701	− 12·1	6	5	+ 5·1
1386	8010·713	3·772	− 24·1	4	6	+ 0·9
1489	047·695	9·074	− 43·5	4	4	− 12·1
1499	049·605	10·979	− 6·0	1	2	+ 18·3
1537	080·661	10·360	− 19·2	1	2	+ 8·0
1579	098·656	12·515	− 27·5	6	3	− 13·2
1637	119·621	1·799	− 22·3	2	2	− 14·3
2021	285·941	9·720	− 38·3	2	2	− 8·8
2037	292·938	0·877	− 2·7	4	4	− 4·2
2063	297·934	5·873	− 29·3	6	6	+ 3·4
2099	313·897	5·996	− 26·9	5	5	+ 5·9
2232	341·807	2·229	− 12·7	6	7	0·0
2259	346·795	7·214	− 22·9	2	2	+ 10·9
2299	360·757	5·136	− 26·0	6	4	+ 5·6
2321	369·750	14·329	+ 0·7	5	4	− 0·4
2340	374·741	3·479	− 31·7		2	− 8·3
2354	378·681	7·420	− 34·5	4	4	− 0·8
2369	379·694	8·433	30·4	4	6	+ 2·4
2411	388·699	1·598	13·8	2	4	− 8·2
2431	389·719	2·618	− 6·4	3	3	+ 9·6
2447	397·671	10·570	− 17·5	2	5	+ 8·7
2466	398·786	11·685	− 21·5	2	4	− 1·0
2480	405·625	2·679	− 20·7	5	6	− 3·6
2494	413·633	10·692	− 32·4	2	2	− 6·8
2500	416·510	13·569	+ 2·3	2	4	+ 7·8
2508	420·443	1·662	− 6·4	3	3	− 0·6
2520	423·685	4·901	− 34·1	4	6	− 3·6
2525	425·621	6·840	− 48·5	2	2	− 14·3
2535	430·577	11·796	− 6·9	2	3	+ 12·8
2549	451·640	1·179	+ 0·2	3	6	+ 1·0
2551	453·688	3·227	− 5·1	2	2	+ 16·0
2552	418458·692	8·231	− 37·9	3	1	− 5·8
2557	460·598	10·137	− 39·3	3	2	− 11·3
2571	473·667	7·366	− 41·7	2	1	− 8·0
2583	482·037	0·496	0·0	2	5	− 4·9
2878	588·906	11·725	− 20·0	3	4	+ 0·1
2959	626·899	2·198	− 12·5	6	6	+ 0·3
3112	686·822	14·601	0·0	3	6	− 3·3
3144	697·833	9·772	− 28·2	6	7	+ 1·2
3161	703·715	15·654	+ 13·0	4	6	+ 5·5
3198	721·687	1·946	− 18·2	2	3	− 8·6
3205	724·702	4·961	− 34·2	4	6	− 3·8
3212	726·723	6·982	− 42·0	2	2	− 8·3
3248	731·687	11·946	− 26·3	4	4	− 8·0
3256	733·688	13·947	− 9·4		2	− 7·2
3282	734·807	15·066	− 4·5	5	3	− 10·6
3321	742·636	7·055	− 25·7	5	4	+ 8·0
3340	749.566	13·985	− 5·3	4	5	− 3·3
3353	747·697	12·161	− 12·5	4	5	+ 4·7
3357	754·674	3·253	− 25·9	3	3	− 4·2
3364	759·680	8·259	− 27·9	3	5	+ 4·9
3375	768·639	1·379	+ 8·4	5	4	+ 11·2
3377	770·666	3·409	− 20·0	5	6	+ 2·9
3388	774·817	7·556	− 33·1	5	5	+ 0·2
3391	775·611	8·350	− 30·8	3	4	7·0

Plate	Julian Date	Phase from Final T	Velocity	No. of Lines	Weigh	O - C
3395	2418776·646	9·385	– 29·6	6	5	– 0·9
3397	782·666	15·405	+ 9·9	6	6	+ 2·5
3406	787·625	4·524	– 29·3	5	6	– 0·2
3407	789·627	6·526	– 31·5	6	4	+ 2·0
3416	790·594	7·493	– 40·8	5	6	– 7·1
3422	797·549	14·448	+ 2·8 } * – 145·1 }	5	6	+ 0·7
3441	803·639	4·698	– 32·7	3	5	– 2·9
3454	811·653	12·712	– 13·0	5	4	0·0
3866	9018·965	14·104	+ 5·2	6	2	+ 6·0
3893	027·880	7·179	+ 114·6 } * – 34·0 }	3	4	– 0·2
4094	106·826	6.925	+ 83·7 } * – 36·8 }	4	3	– 3·1
4182	137·786	6·205	– 31·1	5	5	+ 2·0
4231	148·700	1·279	– 8·1	6	4	– 6·1
4267	153·771	6·350	– 32·3	5	3	+ 0·9

* Double spectrum.

The phases are computed from the final value of *T*, and the residuals are scaled from the corrected curve. The plates were grouped into seventeen normal places, according to phase, and each weighted as in table below.

NORMAL PLACES 1ST SOLUTION

No.	Julian Date	Phase	Velocity	Weight	Residuals O - C
1	2418393·130	1·210	+ 1·50	1·0	+ 4·53
2	742·924	1·733	– 1·75	2·0	– 4·49
3	743·191	2·793	+ 11·44	1·0	+ 1·68
4	419·903	4·224	– 2·87	2·0	– 3·96
5	448·513	5·567	– 13·22	2·5	+ 1·86
6	754·423	6·105	– 21·02	1·0	– 1·74
7	020·966	6·155	– 19·24	1·5	+ 1·30
8	770·127	7·831	– 32·05	2·0	– 2·19
9	349·629	8·623	– 29·49	2·0	+ 2·68
10	771·435	10·293	– 34·45	2·0	– 0·45
11	391·880	10·847	– 33·95	1·5	– 0·13
12	746·138	12·140	– 30·70	2·0	+ 1·52
13	058·684	12·600	– 35·40	·5	– 4·17
14	391·700	13·459	– 27·95	1·0	+ 0·68
15	079.036	15·040	– 18·90	·5	+ 2·82
16	476·591	14·834	– 16·97	1·0	+ 4·75
17	762·449	15·349	– 16·90	1·5	+ 2·23

A velocity curve was drawn through the normal places by the graphical method of Dr. King, giving the following preliminary elements :

P = 15·84 days,
e = ·30,
ω = 10°,
K = 22 km.,
γ = − 18·50 km.,
T = 2417991·168 J. D.

A least squares solution with these elements gave the following corrections:

δP = + 00008 days,
$\delta \gamma$ = + 0·17 km.,
δK = − 2·03 km.,
δe = − ·060,
$\delta \omega$ = + 4° 13,
δT = + 0·018 days.

The value of Σpvv was reduced from 193 to 137. On substitution in the observation equations it was found that the computed and ephemeris residuals did not agree closely. A second solution was accordingly made. The velocities of six additional plates were included which had been obtained after the first solution was made. The number of normal places was reduced to ten and the period taken as fixed at 15·8401 days. The normal places for the second solution follow. In the last column will be found the residuals from the final curve.

	Julian Date	Phase	Velocity	Weight	Residual
1	2418682·660	1·541	− 0·44	3	− 1·0
2	743·191	2·791	+ 11·45	1	+ 3·9
3	568·528	4·280	− 2·76	3	− 1·8
4	379·090	5·464	− 14·59	3	− 0·8
5	450·809	6·616	− 24·09	2	− 0·2
6	537·746	8·269	− 30·62	4	+ 0·4
7	740·258	10·118	− 34·32	4·5	− 0·5
8	574·889	12·012	− 32·45	3	− 0·6
9	343·848	13·460	− 26·91	1	+ 0·5
10	536·191	15·092	− 17·25	3	+ 1·1

The solution of these gave as further corrections :

$$\delta\gamma = + \cdot 51 \text{ km.},$$
$$\delta K = + \cdot 39 \text{ km.},$$
$$\delta e = + \cdot 024,$$
$$\delta\omega = - 2^\circ \cdot 177,$$
$$\delta T = - \cdot 085 \text{ days}.$$

The definitive elements of the orbit now were :

$$P = 15 \cdot 8401 \text{ days},$$
$$e = \cdot 264,$$
$$\omega = 11^\circ \cdot 95,$$
$$K = 20 \cdot 64 \text{ km.},$$
$$\gamma = - 18 \cdot 45,$$
$$T = 2417991 \cdot 101 \text{ J. D.}.$$

The value of Σpvv was reduced from 43 to 33, and the agreement between the computed and ephemeris residuals was now satisfactory, the greatest difference being ·08 km. The table below gives a summary of the values of the elements after each solution.

Element	Preliminary Values	First Corrected Values	Final Values
P	15·84 days	15·8401	15·8401 days
e	·30	·24	·264 ± ·024
ω	10°	14°·13	11°·95 ± 5°·57
K	22 km.	20·25	20·64 ± 0·40
γ	18·50 km.	− 18·96	− 18·45 ± 0·32
T	2417991·168 J. D.	. . . . 991·186 J. D.	. . . . 991·101 J. D. ± ·208
$a \sin i$			4,336,000 km.

In the column of final values is also given the probable error for each element. The probable error of a normal place of unit weight was ± 1·7 km., and that of a plate of average weight was computed from the residuals scaled from the final curve and found to be ± 4·1 km.

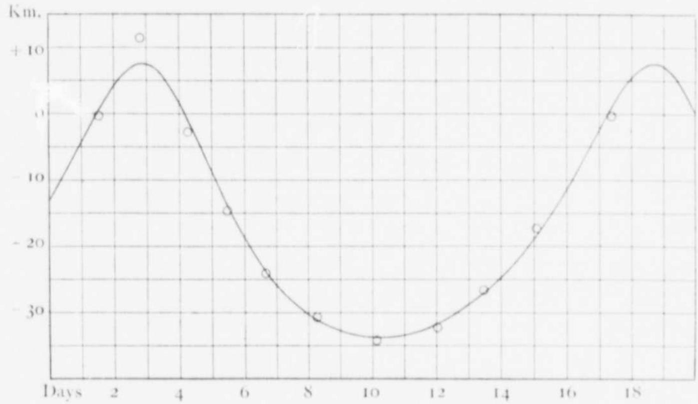
Although there are only three measures of the secondary component an approximation to the value of K was arrived at by substitution in the equation :

$$\frac{dz}{dt} = \gamma + K \left\{ \cos u + e \cos \omega \right\}$$

giving the velocity at any point in the orbit. The values of e, ω and γ being known, that of u was determined in the usual way from the mean anomalies at the observed velocities. Successive trials of the value of K in the above equation gave 120 km. as the most satisfactory. Hence a comparison of the masses of the system may be had from the relation :

$$M_1 : M_2 = K_2 : K_1 = 120 : 20{\cdot}6 = 5{\cdot}8 : 1.$$

It is interesting to note that if further measures of the secondary substantiate this value of K this proportion of the



VELOCITY CURVE OF ω URSÆ MAJORIS

masses is one of the highest yet obtained. It is probably due to the resulting faintness of the companion that more plates showing the double spectrum were not obtained.

In conclusion I wish to acknowledge with thanks the kindly interest shown by the Director throughout this work.

DOMINION OBSERVATORY,
OTTAWA, CANADA.
December, 1911