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NEW RESEARCH IN ARMS CONTROL VERIFICATION USING GAME THEORY

by

Niall M. Fraser

and

D. Marc Kilgour

Research Report for

Arms Control and Disarmament Division
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Canada

University of Waterloo

Wilfrid Laurier University

New Research in Arms Control Verification Using Game Theory

Final Report

Niall M. Fraser
Department of Management Sciences
University of Waterloo
Waterloo, Ontario N2L 3G1

and

D. Marc Kilgour
Department of Mathematics
Wilfrid Laurier University
Waterloo, Ontario N2L 3C5

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Principal Findings and Conclusions

This document reports on a research project which involved new applications of game theory to arms control and disarmament verification. Many useful results and valuable insights have been developed.

Three specific research directions were explored actively: the development of allocation models in inspection games representing chemical weapons treaty verification problems; the application of agency theory to the bilateral verification problem; and the extension of basic bilateral verification principles to the multilateral case. In the main text of this report, the problems, methods and results for each research direction are described; technical details about the findings and techniques are contained in the appendices.

Following are some of the results, principles and insights arising from this research which will be of value to policy makers:

(1) In negotiating a treaty, it may be necessary to consider trading off measures which increase the detectability of cheating (longer inspections, a larger inspection team, etc.) against measures which increase the penalty for detected cheating (increased negative publicity, the right to control facilities where violations have occurred, the right to destroy stocks and equipment, etc.). The methods developed in this research provide techniques for estimating the net effects of such trade-offs.

(2) It is possible to assess the effects of certain treaty variables such as the total number of inspections allowed and concealment effort on the amount of violation.

(3) Uncertainty over inspections always deters the violator, so that it is better to fix the number of inspections over longer rather than shorter time periods, in order to reduce the total amount of violation. For example, a treaty with k inspections allowed each year for 2 years should be altered to allow 2k inspections over 2 years. This step alone can be expected to reduce substantially the total number of violations over the two years.

(4) Agency theory shows that information asymmetry between the inspector and inspectee is the major cause of compliance disputes. In order to reduce information asymmetry and consequently minimize these disputes, a treaty should be written clearly. Unless ambiguous and uncertain provisions are reduced, the treaty can be even worse than a second-best contract.

(5) Agency theory provides a rigorous explanation why a treaty should contain as many cooperative verification measures as possible. Since it is virtually impossible to attain a "first-best" arms control treaty, it is wiser to concentrate on incorporating cooperative verification measures in the treaty rather than to strive for an ideal treaty with absolute verification.

(6) In the framework of multilateral verification, the ability to detect a violation with certainty may not be enough to ensure that no violations will occur. There are situations where a party can profitably violate an agreement even if violation is sure to be detected. In other words, verification cannot solve all non-compliance problems. In general, however, a party will not violate under perfect detection except in situations involving a high degree of coordination of actions.

(7) Similar to point 6, there are circumstances in which it is in a party's interest to abide by a treaty in which there is no effective verification, despite the immediate benefit gained by violating. Again, coordination among the players is necessary for this phenomenon. In general, a party acting independently will cheat for certain under no detection unless there is nothing to be gained by cheating.

(8) The threshold at which a party is indifferent between violating or not violating a treaty can be calculated as a function of the benefits to be gained by violating, the probability of getting caught violating, the penalty for getting caught, the penalty for being falsely accused of violating, and the probability of a false alarm.

(9) The false alarm frequency and the effect of false alarms on the accused party may have a direct effect on treaty violations. Compliance can be improved by lowering either the false alarm frequency or the penalty (to the accused) for a false alarm.

A brief description of some future research opportunities following up on this project concludes the main body of the report.

Game Theory and Verification

With the negotiation of new arms control treaties covering a broader range of military equipment and activity, new forms of verification of compliance with treaties have been suggested and, in some cases, accepted. Potter's (1985) claim that verification systems are an overriding prerequisite for arms control treaties is just as true now, and will remain so in the future. The development of arms control depends, in large measure, on the development of verification.

The improvement of existing verification systems, and the development of verification systems covering new areas, present problems that are both technological and decision-theoretic. Technological advances in verification increase the amount or reliability of data which can be gathered, enabling the inspecting party to learn more, or in some cases to prove more, about the activity in question.

Decision-theoretic advances in verification have a different function, however. By understanding how decision makers would rationally select actions--such as to cheat, to inspect, or to accuse -- decision theory makes a different, but equally important contribution to the improvement of a verification system. Using decision theory, the effectiveness of proposed provisions of treaties can be evaluated before they are put in place; decisions about when, where, and how to inspect can be optimized so as to maximize the probability of detecting violations which have already occurred, and deterring them in the future; and advances in verification technology can be evaluated.

The branch of decision theory that must be used in the analysis of verification problems is *game theory*, a collection of models and principles dealing with decisions in situations of (at least partial) conflict. Non-cooperative game theory is most appropriate because it emphasizes decision problems in multi-decision maker, multi-objective situations. The central objective of this research project is to contribute to the overall development of game theory models and analysis of the verification problem.

In the last few years, there have been a number of attempts to apply game theory models to verification problems. Among the important early meetings on this subject were the Workshop on the Application of Game Theory to the Arms Control and Disarmament Process, held in Ottawa in April, 1986, and the Workshop on the Application of Game Theory to Arms Control Verification, held at the University of Waterloo in March, 1987. The results of the Waterloo workshop were published as a report (Fraser and Kilgour, 1987).

The recent uses of game theory to model, analyze and understand the verification process are exemplified in these recent works:

(1) Brams and Kilgour (1986, 1987, 1988) indicate that there are minimum quality standards for detection systems, below which verification is incapable of deterring cheating.

(2) Avenhaus (1986) and O'Neill (1988) focus on uncertainty ("ambiguity") as an inevitable feature of verification systems, and on how it affects decision making.

(3) Brams, Davis and Kilgour (1988) develop a simple model of the optimal allocation of cheating and inspection effort under the I.N.F. Treaty.

The research reported here is directed toward increasing both the depth and the breadth of game-theoretic research into verification. Three research directions were taken up:

(a) Optimal allocation of inspection and cheating effort, with particular reference to an inspection problem which might arise under a chemical weapons non-production treaty.

(b) Agency theory applied to characterize an arms control treaty and the verification process in the treaty and to derive strategic implications based on these characterizations.

(c) Multilateral verification, with emphasis on how it differs in principle from bilateral verification.

Allocation of Cheating and Inspection Resources

Under Chemical Weapons Treaties

Allocation is an important problem under arms control agreements because most such agreements rely on the threat of close inspections with little or no warning to deter cheating. Generally, there are many more opportunities to cheat than inspections. Thus, as an inspector, a side faces the problem of how to spread its inspections over its inspection opportunities in such a way as to keep the amount of cheating to a minimum.

It is also important to an inspecting side to know how an opponent who is motivated to cheat would optimally allocate violations over opportunities to violate. One reason this information is useful is that private information about specific situations may become available. How can the inspector tell where the cheater is most vulnerable to inspection without a measure of the "normal" amount of cheating? In other words, an inspector may wish to adjust its inspection pattern to take advantage of violation data when such data is available. To do so, it must have a yardstick with which this other data can be compared.

Knowledge of the optimal allocations of both cheating and inspection effort is therefore essential if a treaty, or a potential treaty, is to be evaluated. In particular, these quantities provide estimates of the amount of violation to be expected when both sides are sophisticated. As well, an appreciation of how the details of a treaty -- inspection frequency and thoroughness, for example -- affect those optimal levels is of great value when treaties are negotiated or renegotiated.

The objective of the allocation direction of this research is to build and analyze an abstract but useful treaty model which allows certain optimal allocation problems, like those indicated above, to be solved formally. Similar work includes Maschler (1966), which is based on quite restrictive assumptions, and Brams, Davis and Kilgour (1988), which is designed to be applicable to the Intermediate-range Nuclear Forces (I.N.F.) Treaty.

The abstract treaty model used here has been chosen in part because it is a good general description of a simple chemical weapons inspection problem. These are discussed in detail in Avenhaus, Fichtner, and Vachon (1987). With some modifications, this model also applies approximately to verification problems under the Stockholm Document.

The specific features of the inspection problem studied here are chosen to represent the problem of inspection of a single chemical plant under a non-production treaty. To take a typical case, suppose that the minimum time for set-up, production, and clean-up of a prohibited chemical is two weeks. Then a treaty might be written so that every two weeks the inspector must be given the opportunity to inspect the facility. However, because inspections can be disruptive and costly in other ways, there are usually severe restrictions on the number of inspections an inspector can make. To return to the typical case, in a 1-year treaty the inspector might be allowed only 5 inspections to cover the 26 2-week time slots. The inspector's strategic problem is then to allocate his few inspections over the many time slots so as to deter cheating as much as possible, and to have the highest possible probability of detecting any cheating that does occur.

The general features of this inspection problem are as follows.

* inspections are limited in number, usually fewer than the number of inspection opportunities (time periods);

* there is a ceiling on the amount of cheating in each time period, but no additional overall restriction;

* if cheating does occur

  - the amount of cheating during a time period is variable;

  - if inspected, the detection probability increases as the amount of cheating increases;

  - the value to the cheater of undetected cheating depends on the amount;

  - the penalty paid by the cheater for detected cheating does not depend significantly on the amount.

* values to the cheater (inspectee) accumulate over time

* values to the inspector are the same as to the inspectee, but opposite in sign.

All but the last of these properties can be understood in the context of a chemical plant, or perhaps a weapons storage facility, which falls under the terms of a hypothetical non-production treaty [see Avenhaus, Fichtner, and Vachon (1987)]. The last property listed above may be problematic because arms control treaties are obviously sometimes in the common interests of the signatories (otherwise they would not be signed). But if, within a treaty, one side for some reason decides to cheat, then the interests of both sides with respect to the amount, detection, and value of cheating are indeed opposite. Thus the last feature listed above can be thought of as characterizing the cheating game within the arms control game.

The additional assumptions which are incorporated into the model to simplify the analysis, but which could be altered if necessary, are as follows:

* false alarms are unlikely, and/or of negligible cost;

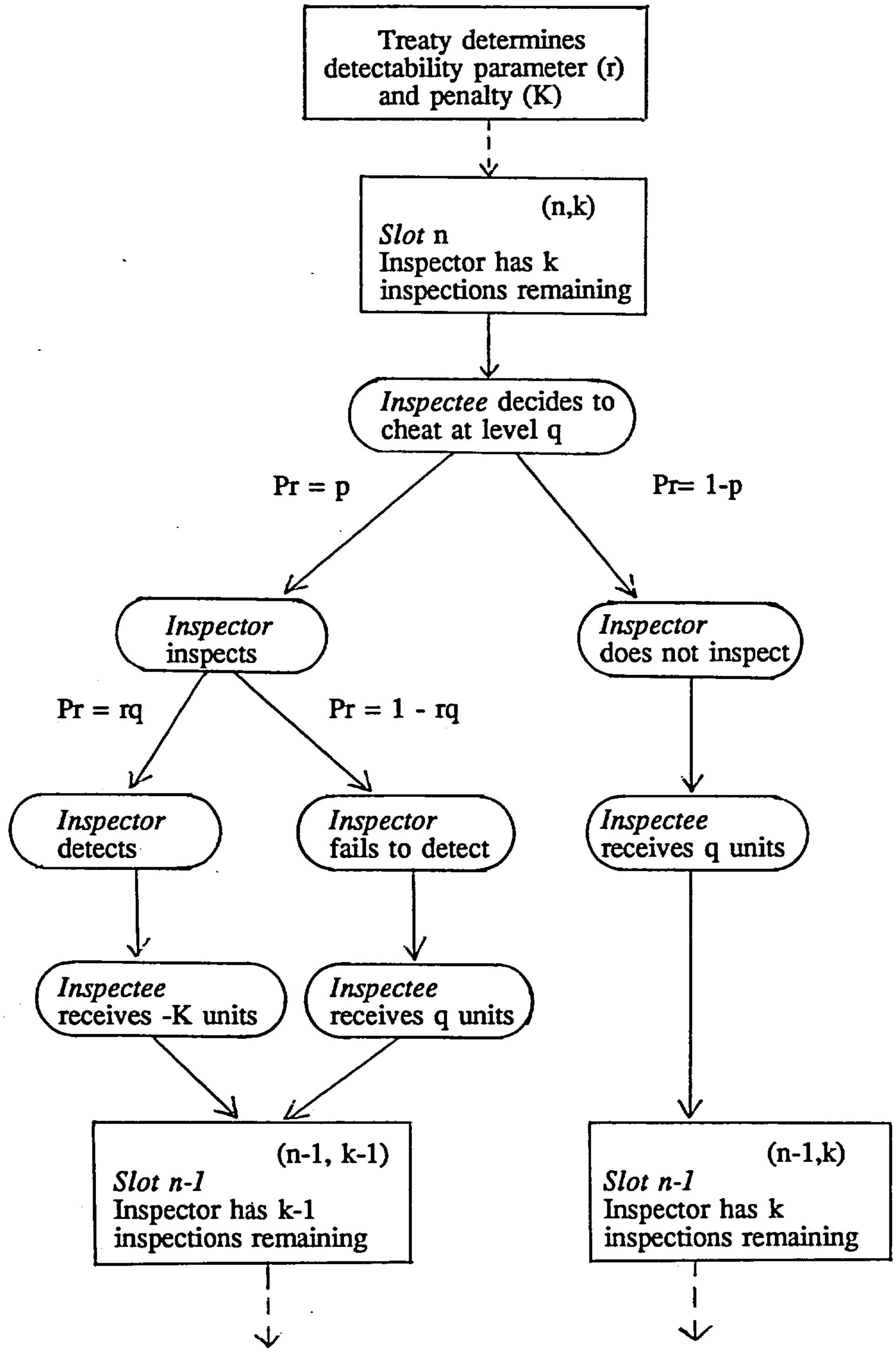
* once the treaty has been agreed to, additional costs per inspection (to either the inspector or the inspectee) are negligible;

* all time periods are identical with respect to the detectability of cheating, the (per unit) value of undetected cheating, and the maximum amount of cheating.

These assumptions and linearity yield the treaty model described schematically in Figure 1.

Figure 1

Sequential Inspection Model from Inspectee's Viewpoint



Here are some comments which should help in the interpretation of Figure 1. The original negotiation of the treaty is thought of as determining the total number of time periods, and the maximum number of inspections to be allocated over them. Other parameters which are determined by the treaty are the detectability of cheating (r) and the penalty for detected cheating (K). Figure 1 shows the possible chains of events at the n^{th} last time slot when the inspector has k inspections remaining. Time slots can be thought of as numbered in descending order. The inspector's strategic variable is p, the probability that this inspection opportunity is selected, and the inspectee's strategic variable is q, the amount of cheating during this time period. When both inspector and inspectee have chosen values for these variables (which of course may depend on the number of time periods, n, and inspections, k, remaining), one of the scenarios shown in Figure 1 plays itself out, and the next time slot [the $(n - 1)^{st}$ from the end] is reached with new accumulated payoffs and, perhaps, one fewer inspection.

Appendix A details the analysis of this model, along with two possible extensions which have been investigated. These extensions concerned the introduction of a more sophisticated detectability function, and the incorporation into the model of concealment (camouflage) effort -- which would reduce detectability but also reduce the value of undetected cheating.

Table 1 is a good introduction to the results of the analysis. It contains the values to the inspectee [Table 1(a)] of all possible inspection problems with 5 or fewer time periods, i.e. $0 \leq k \leq n \leq 5$, when detectability r = 0.5 and the penalty K = 5.0. These are typical parameter values which were used a standard throughout this study. Notice that the value to the inspectee drops rapidly as the number of inspections increases. For example, if there are 5 time periods and no inspections, the inspectee cheats to the maximum at each time period, resulting in a total value of 5.00. If there is 1 inspection, the inspectee's value drops to 2.47, and if there are 2, to 0.90. As the number of inspections approaches 5, the inspectee's value approaches 0.00.

Table 1(b) shows how the inspectee's optimal cheating amounts also decline as the number of inspections increases. When there are 5 time periods, the inspectee's optimal cheating level is 1.00 when there are no inspections, 0.80 when there is 1 inspection, 0.45 when there are 2, and so on down to no cheating at all when there are 5 inspections. But as the number of inspections increases, the optimal inspection probabilities increase, as shown by Table 1(c). When there are fewer inspections than time slots, the inspection probability never exceeds a definite limit, which equals 0.40 for this standard example. The inspector's best strategy is to inspect with greater probability when more inspections are allowed, but the increase in probability is much less than proportionate.

Table 1

Values and Optimal Strategies with r = .5, K = 5, and up to 5 time slots

(a) Values to Inspectee ($V_{n,k}$)

	n = 1	n = 2	n = 3	n = 4	n = 5
k = 5					.0000
k = 4				.0000	.0228
k = 3			.0000	.0572	.2098
k = 2		.0000	.1447	.4503	.8984
k = 1	.0000	.3723	.9554	1.6708	2.4736
k = 0	1.0000	2.0000	3.0000	4.0000	5.0000

(b) Inspectee's Optimal Cheating Amounts ($q_{n,k}$)

	n = 1	n = 2	n = 3	n = 4	n = 5
k = 5					.0000
k = 4				.0000	.0228
k = 3			.0000	.0572	.2098
k = 2		.0000	.1447	.3056	.4481
k = 1	.0000	.3723	.5831	.7155	.8028
k = 0	1.0000	1.0000	1.0000	1.0000	1.0000

(c) Inspector's Optimal Inspection Probabilities ($p_{n,k}$)

	n = 1	n = 2	n = 3	n = 4	n = 5
k = 5					1.0000
k = 4				1.0000	.3964
k = 3			1.0000	.3910	.3770
k = 2		1.0000	.3781	.3564	.3392
k = 1	1.0000	.3482	.3244	.3110	.3028
k = 0	.0000	.0000	.0000	.0000	.0000

Some thresholds relating to the occurrence of cheating can be inferred from Table 1. This model is not an exact representation, and cheating levels of 2% or 3% should probably be interpreted as insignificant. With this interpretation, it takes 4 inspections to effectively deter violations,if there are 5 slots, whereas if there are 10 slots (not shown in Table 1) violation becomes negligible when there are at least 7 inspections.

If the number of time periods (n) is fixed, the factors which affect the inspectee's value and the total amount of violation are the number of inspections (k), the detectability of cheating (r), and the penalty for detected violations (K). A sensitivity analysis was undertaken to assess the relative sensitivity of the inspectee's value (V) to each of these three variables.

The value to the inspectee, who is assumed to be motivated to cheat, always decreases as k, r, and K increase. But it is the relative rates of change of V with respect to those variables which are of interest. The elasticity of V with respect to r [the ratio of relative (or percentage) rates of change of V and r] was ~1-2. This means, for example, that a 10% increase in r (detectability of cheating) typically results in a 10-20% decrease in V (the value to the inspectee) and in q (the inspectee's optimal cheating level). This elasticity decreases as V decreases (and r increases).

The elasticity of V with respect to K is very similar to, but perhaps slightly less than, the elasticity of V with respect to r. Finally, V appears to be somewhat more sensitive to k than to either r or K, with elasticities in the ~1-3 range. The elasticity of V with respect to k increases as k increases (and V decreases), so that increasing the number of inspections becomes more and more effective at reducing cheating the longer the increase continues. A figure showing typical changes in value is given in Appendix A, along with some details of the calculations.

Certain policy implications can be discerned from this study of elasticities. In negotiating a treaty, it may be necessary to trade off measures which increase the detectability of cheating (longer inspections, a larger inspection team, etc.) against measures which increase the penalty for detected cheating (increased negative publicity, the right to control facilities where violations have occurred, the right to destroy stocks and equipment, etc.). In such a situation, it is important to be able to estimate the net effects of trade-offs of this type. The methods developed here provide estimates of these effects, estimates which can be fine-tuned to some extent.

It is also possible, using these methods, to estimate how other variables, such as the total number of inspections allowed, affect the violation frequency. Yet another variable, concealment effort, is introduced in Appendix A. Concealment refers to activities of the inspectee which camouflage violations; these activities are costly, so that the value of undetected cheating is reduced along with detectability. Appendix A contains some preliminary work involving the incorporation of concealment effort into the model, and the effect of this additional strategic variable controlled by the inspectee.

One additional policy implication is clear from this analysis. Uncertainty over inspections always deters the violator, so that it is better to fix the number of inspections over longer rather than shorter time periods, in order to reduce violation. [For another model, this phenomenon was observed previously by Brams, Davis, and Kilgour (1988).] For example, a treaty with k inspections allowed each year for 2 years might be altered to allow 2k inspections over 2 years. Thinking of the number of time periods (inspection opportunities) as doubling, the appropriate comparison is of the amount of violation with 2n slots and 2k inspections, as compared to twice the amount of violation with n slots and k inspections. For the standard case shown in Table 1, the net amount of violation, as measured by the value to the inspectee under optimal strategies, is as follows:

$n = 2, k = 1, V = .3723$

$n = 4, k = 2, V = .4503$

$n = 8, k = 4, V = .4714$

Notice that the value of V in each line is much less than twice the value in the previous line.

Agency-Theoretic Approach to

Arms Control Verification

Agency theory focuses on the optimal contractual relationship between two individuals whose roles are asymmetric. One, the principal, delegates work or responsibility to the other, the agent. An arms control treaty has important similarities to a contract representing the economic agency relationship. Both are agreements between *remote* parties. The parties are remote in that one party has difficulty observing the other's behaviour related to the agreement. The purpose of this research direction was to apply the agency theory framework to characterize explicitly an arms control treaty and the verification process within the treaty and to derive strategic implications based on these characterizations.

In Appendix B, agency theory is briefly reviewed with particular emphasis on information asymmetry. There are two kinds of information asymmetry: adverse selection and moral hazard. This research focuses on the moral hazard problem, especially on the hidden action agency model. Although there are many other variations of agency models, the *Standard Agency Model*, with rather rigorous assumptions, is presented. The standard agency relationship is modelled mathematically in Appendix B, where the assumptions are carefully interpreted. Any standard agency relationship has one of three broad forms of contracts: first-best, second-best, and second-best with additional information. A first-best contract can be achieved only when the principal has complete knowledge of the agent's performance. A second-best contract results from imperfect information flow. A second-best contract can sometimes be augmented with additional information to improve it for both the principal and agent.

Given specific arms control definitions, it is argued that the Standard Agency Model applies to the arms control situation. Some of the assumptions of the Standard Agency Model are further explained in the arms control context. It is argued that unless there is perfect information flow between the inspector and inspectee, the use of National Technical Means (NTM) guarantees only a second-best contract at most. Here lies the importance of cooperative verification measures which assist NTM (non-intrusive cooperative measures) and independently collect data that NTM cannot (intrusive cooperative measures such as on-site inspections) in order to increase the inspectee's compliance level and consequently to decrease weapon stockpiles. Culminating the presentation in Appendix B is a simple example of a nuclear missile reduction treaty.

First-best arms control treaties are rarely possible except for a few cases such as the 1957 Antarctic Treaty in which an absolute level of verification is feasible. Obtaining additional information through monitoring is regarded as secondary monitoring in arms control, whereas NTM have the role of primary monitoring. The integration of additional information into an arms control treaty is seen as the use of cooperative verification measures. Information obtained through cooperative verification measures is more than what is conveyed by NTM only. Consequently, this information is valuable in the sense that both parties' expected utilities can increase with cooperative measures.

One important practical guideline for writing arms control agreements which is suggested by the application of agency theory is that a treaty should contain as many cooperative verification measures as possible -- this is better for both parties. Since it is difficult to design a first-best arms control treaty in practice, it is wise to concentrate on incorporating cooperative verification measures in a treaty rather than to hold out for an ideal treaty with absolute verification. Agency theory shows that by bringing correlated information into the treaty, a Pareto improvement can be attained. The current trend in arms control negotiations recognizes the importance of cooperative measures. Further, because of the low observability of modern weapons technology, there will increasingly be weapons systems which can no longer be confidently monitored using unaided NTM.

Modelling Multilateral Verification

A key distinction between this project and verification studies conducted previously is a multilateral focus reflecting the verification provisions of many agreements such as the Stockholm Document. Consequently, one research direction concentrated on modelling and analyzing multilateral verification.

There are many possible configurations for a multilateral verification scheme. Four different cases are illustrated in Figure 2. Figure 2a is the situation of bilateral reciprocal inspection, such as seen in existing superpower agreements. Previous game theory analysis has been applied to models based on this structure. Figure 2b represents a multilateral inspection scheme, with an independent inspectorate. An example of this situation is the IAEA. Figure 2c illustrates the structure of multilateral reciprocal inspection. The Stockholm Document specifies this form of multilateral verification. Finally, Figure 2d represents the situation where there are many parties, but each of two (or more) alliances has an inspectorate. A possible example of this form of multilateral inspection may result from the Conventional Arms Reduction Talks.

A comprehensive model of multilateral verification would be very complex. For models including an inspectorate, at least two kinds of players are required. In some cases, each player would have a large number of strategies available. For example, each signatory of the Stockholm Document can inspect each other signatory. Also there may be many types of violation.

Such a complex multilateral analysis is not possible with current tools and methodologies. It is necessary to simplify the problem in an appropriate manner in order to provide insight into the fundamental forces governing the participants' behaviours.

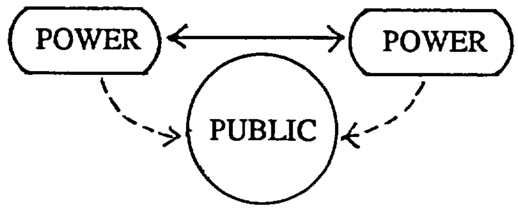
To this end a simple model of multilateral verification was developed. Details about this model are contained in Appendix C. This model has the following characteristics:

* There are n players, each of whom can either violate or not.

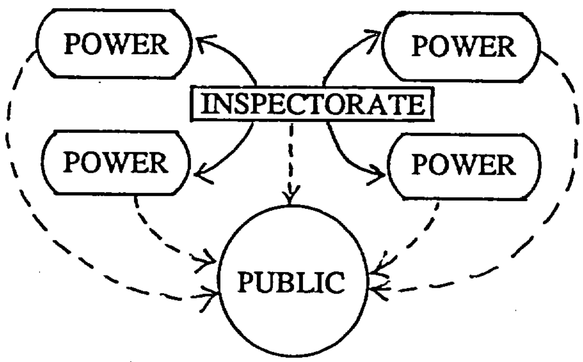
* The inspectorate is not a (strategic) player in the model; rather, it is represented by a probability of detecting violations, which may be different for different violators. This simplification can apply to all of the verification structures shown in Figure 2.

Figure 2 - Bilateral Model and Multilateral Models

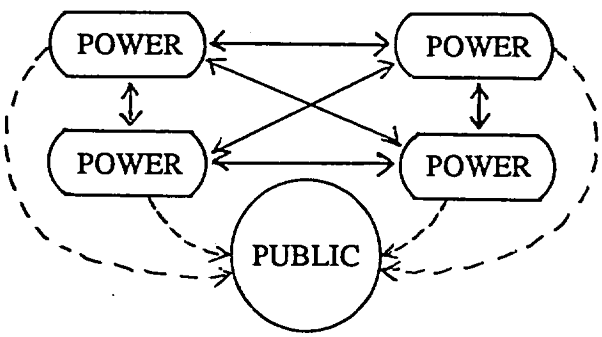
2a: Bilateral Reciprocal Inspection



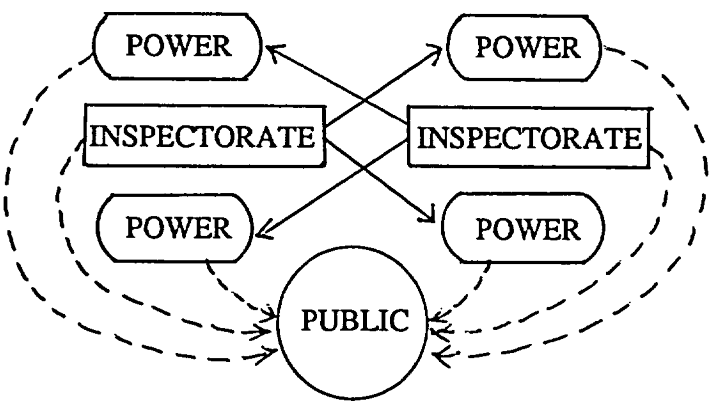
2b: Multilateral with Independent Inspectorate



2c: Multilateral with Reciprocal Inspection



2d: Multilateral with Alliance Inspection



Key: ———— inspection — — — — information

* The value of a particular outcome to a particular player is composed of many factors, including:
  - the benefits to the player or allies of violating without getting caught
  - the benefits when other players violate and get caught
  - the benefits and costs of false alarms
  - the costs of violating and being detected
  - the costs of other players violating without detection.

* There is no benefit or penalty explicitly given for complying with a treaty; the situation where all players comply, and there are no false alarms, has a utility of 0 for all players.

Of course other models of multilateral verification are possible; this field offers considerable scope for future research. However, the chosen model does seem to capture many of the fundamental concerns in multilateral verification.

In addition to the mathematical model, a computer simulation of the model was implemented. The Supercalc spreadsheet program was used (Supercalc is similar to the popular Lotus 123 package). Listings and displays of this program are also available in Appendix C. The spreadsheet program permits the user to specify numerical values for all possible payoffs and probabilities for a multilateral verification system involving four players. The program was useful for examining the behaviour of the model, and can be used to demonstrate all of the analytical results.

The model has a number of interesting consequences which are proven in Appendix C:

1. Even under perfect detection, some players may prefer to violate. Similarly, under no detection, some players may prefer not to violate.

These results are surprising since a basic assumption in the model is that the penalty for a detected violation is never negative, and similarly the benefit of a undetected violation is also never negative. The reason this result is true is that a player's total benefit or penalty (payoff) depends on whether the other players violate or not. If, under perfect detection, the benefit to player i of certain other players violating and getting caught is greater than the penalty to player i for violating and getting caught himself (and this same principle simultaneously applies to all violating players), then the players would jointly prefer to violate than not do so. (Similar logic applies to the case of no detection, where the benefits to player i of the other players not violating must outweigh the costs to player i of not violating). However, note that this requires a certain amount of coordination among the players: in these special circumstances it will never be in player i's *independent* interest to violate (or not violate) -- he requires others to violate (or not violate) simultaneously. (Note that in the case of no detection, such coordination is in some sense equivalent to instituting a detector.) This perspective leads to a second set of results:

2. Under perfect detection, a player acting independently will violate only when there is no penalty for getting caught violating. Similarly, under no detection, a player acting independently will violate whenever there is benefit for violation.

The significance of these results is that they emphasize a property of multilateral verification that is similar to the famous Prisoners' Dilemma of 2-person game theory. In Prisoners' Dilemma there are two generally stable outcomes, one where each player acts in his own immediate self interest, and one, preferred by both players, which requires joint action. There have been many approaches for explaining optimal behaviour in Prisoners' Dilemma; some of these could be transferred to the multilateral case. Unfortunately, coordinating many players simultaneously (since this is a multilateral rather than bilateral problem) has many practical difficulties.

3. The decision to violate or not is never probabilistic -- there is always a single best choice, except in the rare event that two particular combinations of costs and benefits happen to be exactly equal in value. Further, the parameters that determine this choice are the benefits and penalties that accrue to the player as a result of his strategy selection alone.

Each player in the model has two strategies -- violate or not. Result 3 says that no matter what the other players do, one or the other of each of these two strategies is *dominant* -- better under all circumstances. Consequently, the decision to violate is dependent only on the expected benefits of violating without getting caught, the expected penalty for getting caught in a violation, and the expected penalty of being falsely accused of a violation. This result contradicts some results from 2-person models that suggest that there is a probabilistic relationship between detection capability and violation -- that a player should randomly violate at a frequency that depends on the expected detection frequency.

4. The probability of false alarms and the penalty attached are crucial in a player's decision to violate or not. A smaller false alarm penalty results in an increase in the threshold at which a player will choose to violate. Similarly, the smaller the probability of a false alarm, the better.

Intuitively, it seems clear that a player would be likely to violate if the expected benefits from cheating exceed the expected penalties from getting caught. This result points out the direct and significant effect of the false alarm rate.

5. A player acting independently will violate only if the probability of detection is less than or equal to the ratio of the difference between the benefit of violating without getting caught and the expected penalty for false alarms to the difference between the benefit of violating and the penalty after getting caught in a violation.

This textual description is more elegant when written as a mathematical formula, such as, for the case where false alarms are negligible:

$$p \leq \frac{\alpha}{\alpha - \beta}$$

where p is the probability of detection, α is the benefit for violating without getting caught and β is the benefit (negative of penalty) for getting caught violating. This simple formula is a consequence of the dominance result, and provides a method for calculating the circumstances under which violation is optimal, given that some method of measuring or relating the appropriate utilities is employed.

Policy Implications

(1) The model indicates that the decision of a party to violate a treaty or not depends on more than the effectiveness of verification systems. There are natural forces that will lead a party to violate or not that are independent of the verification system. Consequently, rather than placing undue emphasis on verification provisions, a treaty negotiator should concentrate on the basic interests of the parties. The structure of these basic interests will determine whether an agreement is fundamentally viable. Verification should then serve to monitor, rather than control, the ongoing relationship.

(2) Reduction in the frequency of false alarms has a direct effect on violations. Not only should maximum reliability be achieved in technical verification activities, but the intent to minimize false alarms should be clearly communicated to all parties.

(3) Reduction in the penalty for false alarms also has a direct effect on violations. One means for reducing the penalty for false alarms would be to provide a mechanism for an accused party to respond privately to a charge of violation in advance of a public accusation.

(4) The criterion for violating as determined in this model turns out to be a simple test involving five parameters, or only three if the effect of false alarms is ignored. Consequently, is should be possible to use this calculation to determine whether a party will violate. For example, if it is estimated that the penalty for being caught in a violation was 5 times worse than the benefits to be gained, and if false alarms were negligible, then the calculation is:

$$p \leq \frac{\alpha}{\alpha - (-5\alpha)} = \frac{\alpha}{6\alpha} = \frac{1}{6}$$

In other words, the party would likely violate if the probability of getting caught is less than about 17%.

Future Research Opportunities

The research performed in this project has opened up many possibilities for the future. Here are some suggestions for future research projects that could extend these findings and develop further their policy implications.

(1) The effects of dropping the assumptions on which the allocation model is based, particularly the second list, should be assessed. It seems unlikely, but it is possible that some of the conclusions given here do depend crucially on these assumptions.

(2) A useful extension of the allocation model would be the structural incorporation of concealment effort into the model. It is at present difficult to assess the importance of an inspectee's ability to adjust his effort to camouflage violations in determining the level of those violations in the first place.

(3) Another extension of the allocation model which would be extremely valuable in putting the findings into perspective is the development of a non-zerosum (i.e., not strictly competitive) model of an arms control treaty, in which this zerosum (strictly competitive) model would be embedded.

(4) Agency theory is a rich field. The agency theory model defined in this research is only one possible link between agency theory and arms control. Other formulations within the agency theory context can be explored.

(5) Financial auditing as an example of hidden-information agency model can be linked to verification in arms control. Tools and methods used in auditing of financial statements can be applicable to verifying arms control treaties.

(6) There is a great deal of scope for incorporating additional features into the multilateral game model. For example, consideration can be given to

(i) the addition of a parameter providing a penalty or benefit for compliance without false alarm.

(ii) incorporating inspections (of various types) explicitly. In addition to making the decision more complex, this would permit an assessment of the roles of other types of benefits and penalties.

(iii) allowing the benefits and penalties for violation by a particular player to be dependent on whether other players have violated or not. For example, the penalty for violating may be less if another player is also violating.

(iv) allowing a range of possible violations, from minor to major.

Appendix A

Allocation of Cheating and Inspection Resources

Appendix A: Allocation of Cheating and Inspection Resources

This Appendix provides technical details to support the analysis and conclusions in the text concerning the allocation direction of this research project. The text contains an outline of the model, in which the allocation of violations and inspections is treated as a constant-sum game between the players, Inspectee (E) and Inspector (R). E's expected utility (payoff) when there are $n > 0$ time periods (inspection opportunities) and R has k inspections ($0 \leq k \leq n$) is denoted $V_{n,k}$.

Suppose that R has k inspections remaining. R's (current) strategic variable, denoted $p = p_{n,k}$, is the probability that R will inspect during this time period. E's (current) strategic variable, $q = q_{n,k}$, is the level of violation during this time period. Both variables are restricted: $0 \leq p \leq 1$ and $0 \leq q \leq 1$. The parameters are the detectability $r \leq 1$ and the penalty $K > 0$. The conditional probability that a violation at level q will be detected, given that an inspection is carried out, is rq. E's expected payoff is increased by q units if no violation is detected, and decreased by K units if a violation is detected. All of these assumptions are embodied in the iteration equation

$$V_{n,k} = p[rq(-K) + (1-rq)(q) + V_{n-1,k-1}] + (1-p)[q+V_{n-1,k}] \qquad \text{(A1)}$$

which applies for $0 < k < n$. Thus, to find $V_{n,k}$ using (A1), first find $V_{n-1,k-1}$ and $V_{n-1,k}$, then find the optimal choices of p and q , and then apply (A1).

In order to use (A1) to find values $V_{n,k}$ recursively, some boundary values must first be determined. First suppose that $n > 1$ and $k = 0$. No inspections are available, so E's value is

$$V_{n,0} = q + V_{n-1,0}$$

Obviously, E maximizes $V_{n,0}$ by choosing $q = 1$. Because $V_{1,0} = 1$, iteration yields

$$V_{n,0} = n$$

Now suppose that $n = k \geq 1$. Then R inspects at every time period, so that

$$V_{n,n} = rq(-K) + (1-rq)q + V_{n-1,n-1} \qquad \text{(A2)}$$

Using calculus it is easy to show that the value of q maximizing $V_{n,n}$ is

$$q^* = \begin{cases} \dfrac{1-rK}{2r} & \text{if } K < 1/r \\ 0 & \text{if } K \geq 1/r \end{cases}$$

Using $V_{0,0} = 0$, $q = q^*$ can be iterated in (A2) to yield

$$V_{n,n} = \begin{cases} n \dfrac{(1-rK)^2}{4r} & \text{if } K < 1/r \\ 0 & \text{if } K \geq 1/r \end{cases}$$

These observations permit the recursive solution of (A1) provided optimal current strategies can be determined. Since $V_{n-1,k-1}$ and $V_{n-1,k}$ must be known to determine $V_{n,k}$, an effective iteration sequence is

(n,k) = (2,1), (3,1), (4,1), ..., (3,2), (4,2), ..., (4,3), (5,3), ..., (5,4), ...

But to determine $V_{n,k}$ from (A1), even when $V_{n-1,k-1}$ and $V_{n-1,k}$ are known, requires that optimal strategies $q = q_{n,k}$ and $p = p_{n,k}$ be determined. Because $V_{n,k}$ can be considered to be the payoff of a zero-sum game on the unit square with continuous kernel, optimal (maximin) strategies and a value must exist (Owen, 1982, pp. 67-72).

To find maximin strategies, $V_{n,k}$ can be rewritten as

$$V = p[a + bq - rq^2] + (1-p)[m+q], \qquad 0 < p, q < 1 \tag{A3}$$

where $a = V_{n-1,k-1}$, $V_{n-1,k} = m$ and $b = 1-rK$. The following theorem applies:

Theorem: For the game V as in (A3), suppose that $a-m>0$ and $r>0$. The equation $(a-m) + (b-1)q - rq^2 = 0$ has exactly one positive root, denoted q_+. Optimal strategies in V are

(i) if $q_+ > 1$, $p^* = 0$, $q^* = 1$

(ii) if $\frac{b}{2r} \leq q_+ < 1$, $p^* = \dfrac{1}{2rq_+ + 1 - b}$, $q^* = q_+$

(iii) if $q_+ < 1$ and $\frac{b}{2r} > q_+$, $p^* = 1$, $q^* = \min \{\frac{b}{2r}, 1\}$

Because the hypotheses of the theorem can be assumed to be satisfied, the determination of all possible values of $V_{n,k}$ is now a matter of iteration as described above, using the Theorem at each step. The results of the theorem were included in a FORTRAN computer program to carry out the recursive solution of (A1). Values such as are shown in Table 1 result from this program. Note that the values of $q_{n,k}$ and $p_{n,k}$ [Table 1(b) and 1(c)] are the optimal strategies in the (n,k) stage. For example, $q_{4,2} = 0.3056$ is E's violation level with 4 time periods and 2 inspections remaining. When only 3 time periods remain, R will have either 1 or 2 inspections left; E will violate accordingly at levels 0.5831 or 0.1447.

Some details illustrating the elasticity calculations reported in the text will be given now. Parameter values $r = 0.5$ and $K = 5.0$ are typical, and the case $n = 10$, $k = 3$ gives a good illustration. Beginning at $V_{10,3} = 2.40$, a 20% decrease in r to $r = 0.4$ (holding K constant) leads to $V_{10,3} = 3.42$, a 36% increase. From the same starting point, a 20% increase in r, to $r = 0.6$, results in $V_{10,3} = 1.87$, a 25% decrease. Thus a change of δ% in r leads to a change of ~-1.5 δ% in V, provided all other parameters are held constant. This case and others justify the statement in the text that the elasticity of V with respect to r is ~1-2.

As a summary of how elasticities are estimated, Figure A1 starts with the standard case r = 0.5, K = 5.0, n = 10, k = 3, and indicates the percentage change in V resulting from (a) a 20% decrease in r, (b) a 20% increase in r, (c) a 20% decrease in K, (d) a 20% increase in K, (e) a 33.33% increase in k, and (f) a 33.33% decrease in k.

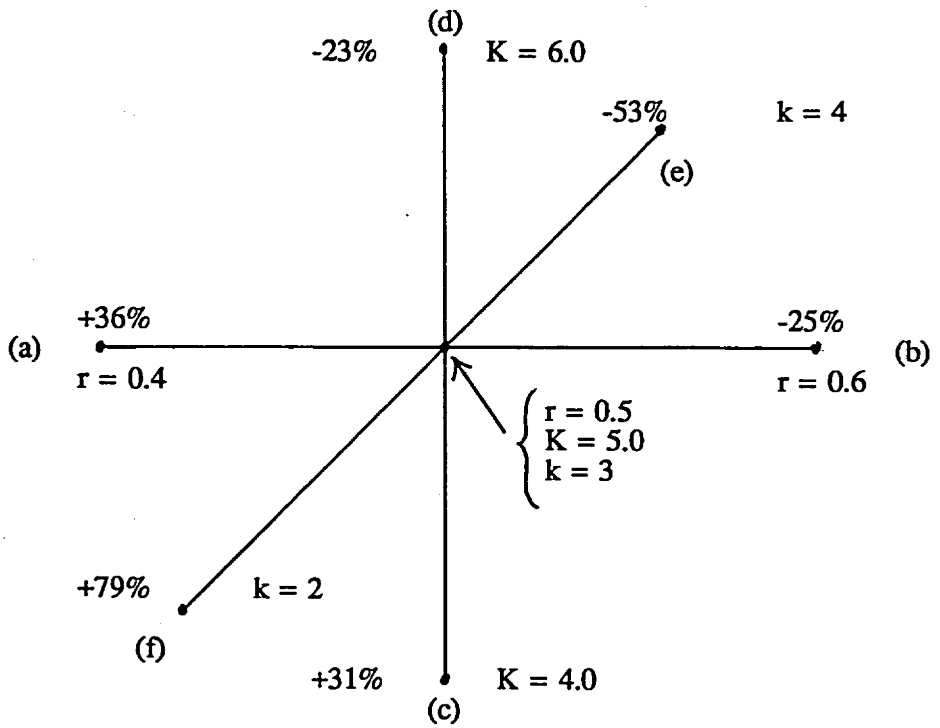


Figure A1: Effects on $V_{10,3}$ of changes in r, K and k, when r = 0.5 and K = 5.0

Based on the investigation of many cases, it was estimated that the elasticity of V with respect to K is ~1-2, and with respect to k ~ 1-3. It should also be noted that the elasticity of V with respect to either r or k tends to increase as r decreases; the elasticity of V with respect to k tends first to decrease, then to increase as k increases.

One potential problem with the model in (A1) is the requirement that the quantity rq be meaningful as a probability. This forces the restriction r < 1, which is equivalent to the assumption that R would never consider any violation level q which, if inspected, would be detected for certain. To explore more general models, consider first the detection probability function $d_r(q)$, defined by

$$d_r(q) = \min\{rq, 1\} \qquad 0 < q < 1$$

This relation is illustrated in Figure A2, for the case where r > 1.

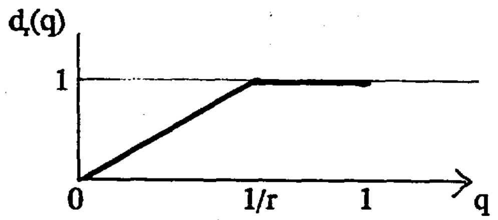


Figure A2: Detection probability function $d_r(q)$, when $r > 1$

If $r \leq 1$ then $d_r(q) = rq$ as before. If $r > 1$, than $d_r(q) = rq$ if $0 < q < 1/r$ and $d_r(q) = 1$ if $1/r < q < 1$. The iteration equation (A1) becomes

$$V_{n,k} = p\ [d_r(q)(-K) + (1-d_r(q))(q) + V_{n-1,k-1}] + (1-p)[q+V_{n-1,k}] \quad \text{(A4)}$$

As in the Theorem, the optimal cheating level $q = q^*$ must make the partial derivative of $V_{n,k}$ with respect to p vanish. But the partial derivative of $V_{n,k}$ with respect to p is

$$V_{n-1,k-1} - V_{n-1,k} - (K+q)d_r(q)$$

It follows that $q^* < 1$ if

$$\frac{V_{n-1,k-1} - V_{n-1,k}}{K + 1/r} < 1,$$

which is certainly true if $V_{n-1,k-1} - V_{n-1,k} < K$. In practice, this sufficient condition was found to be adequate; as long as the value of K was large enough that for all n and k, the expected value difference $V_{n-1,k-1} - V_{n-1,k}$ did not exceed K, then any value of r could be allowed without altering the interpretation of the model, because $q^* < 1/r$. It should be noted that this sufficient condition reflects that intuition that, for E to cheat at a level that makes detection certain (if there is inspection), the gain to E in seeing R use up an inspection must be very large.

As noted in the text, the basic model embodied in (A1) can be modified to include a variable, w, representing concealment effort. Concealment effort refers to activities of the inspectee, E, which reduce the detectability of violations but also reduce the value. Suppose that $w = 1$ is the standard level of concealment effort, and a value $w > 0$ is actually chosen by E, changing detectability from r to $\mathbf{r} = r/w$. Let $\alpha > 0$ be a parameter measuring the ratio of the relative rate of change (with respect to w) of the value q of undetected cheating at level q to the relative rate of change of $\mathbf{r}$ with respect to w. It follows that $\mathbf{q} = q/w^{\alpha}$. Denoting E's expected payoff by $\mathbf{V}$ when concealment effort is included in the model, the recursion equation (A1) must be replaced by

$$\mathbf{V}_{n,k} = p[\mathbf{r}q(-K) + (1 - \mathbf{r}q)\ \mathbf{q} + \mathbf{V}_{n-1,k-1}] + (1-p)[\mathbf{q} + \mathbf{V}_{n-1,k}] \quad \text{(A5)}$$

To see how (A5) can be solved recursively, multiply each term by w^{α} to obtain

$$w^{\alpha}\mathbf{V}_{n,k} = p[rq(-Kw^{\alpha}) + (1-rq)q + w^{\alpha}\mathbf{V}_{n-1,k-1}] + (1-p)[q + w^{\alpha}\mathbf{V}_{n-1,k}] \quad \text{(A6)}$$

Comparison of (A6) and (A1) shows that, if (A1) can be solved replacing r by $r = r/w$ and K by $K = Kw^{\alpha}$, then the solution $V_{n,k}$ can be used to find $V_{n,k}$ by $V_{n,k} = V_{n,k}/w^{\alpha}$.

For example, to find $V_{10,3}$ when $r = 0.5$, $K = 5$, $\alpha = 0.65$ and $w = 2.0$, iterate (A1) using $r = 0.25$ and $K = 7.85$ to obtain $V_{10,3} = 3.73$, so that $V_{10,3} = 2.38$. Notice that optimal strategies p^* and q^* are given by the iteration of (A1); for example, when there are $n = 10$ sites and $k = 3$ inspections remaining, E optimally cheats at level $q^*_{10,3} = .84$, and R inspects with probability $p^*_{10,3} = .42$.

But w is actually a strategic variable controlled by E, the inspectee. E will therefore choose w so as to maximize his expected payoff. E's choice can be identified by finding how V, in this case $V_{10,3}$, depends on w, and maximizing. This process is illustrated in Figure A3, which indicates that, for the case $n = 10$, $k = 3$, $r = .5$ and $K = 5$, E would optimally choose $w = 1.0$.

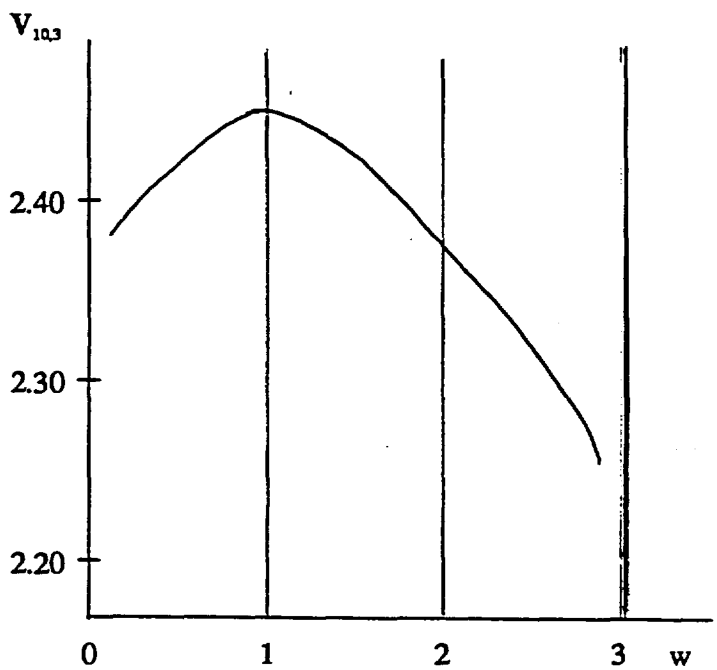


Figure A3: Maximizing E's Payoff with n=10, k = 3, and K = 5

Appendix B

Agency-Theoretic Approach to Arms Control Verification

Appendix B: Agency-Theoretic Approach to Arms Control Verification

(I) INTRODUCTION

In recent years, agency theory in accounting and finance has received much scholarly attention. Agency theory focuses on the optimal contractual relationship between two individuals whose roles are asymmetric. One, the principal, delegates work or responsibility to the other, the agent. An arms control treaty has an important similarity with a contract representing the economic agency relationship. Both are agreements between *remote* parties. The parties are remote because one party has difficulty observing the other's behaviour related to an agreement. The purpose of this research is to apply the agency theory framework to characterize explicitly an arms control treaty and the verification process in the treaty and to derive strategic implications based on these characterizations.

In section II, agency theory is briefly reviewed with particular emphasis on the information asymmetry. Section III models the standard agency relationship with a mathematical formulation. Assumptions used in the mathematical model are also explained in this section. There are, in general, three types of contracts in the standard agency model: first-best, second-best, and second-best with additional information. Each of the contracts are characterized in section IV. Section V applies the agency theory framework to arms control situations. The mathematical formulation of standard agency model is used with different definitions and assumptions reflecting arms control situations. A simple example is also presented in this section. Finally, concluding remarks appear in section VI.

(II) AGENCY THEORY

The agency relationship is a ubiquitous human association in which an individual, designated as the principal, delegates work to another, designated as the agent. As Ross (1973, p. 134) notes:

> The relationship of agency is one of the oldest and commonest codified modes of social interaction... Examples of agency are universal. Essentially all contractual arrangements, as between employer and employee or the state and the governed, for example, contain important elements of agency.

Agency theory, which deals with the economics of asymmetric information, is a field within information economics. Agency theory assumes that individuals are rational, self-interested utility maximizers: the rationality assumption which corresponds to the basic assumption of game theory. Agency theory could also be regarded as part of two person game theory. Although there are many examples of the agency relationship such as society-polluting firm, client-lawyer, and insurer-insuree, a common application of agency theory focuses on the optimal contractual relationship among members of a firm (e.g., employer-employee), where each individual is assumed to be motivated solely by self-interest. One individual acts in his own best interest and, further, expects the other to act solely in his own best interest. Therefore, each chooses his own action based on that expectation. The principal (the employer) chooses, based on his own interest, the payment schedule that best exploits the agent's (the employee's) self-interested behaviour (Baiman, 1982, p. 170). The payment schedule has many equivalent terms: fee schedule, compensation function (schedule), remuneration function (schedule), sharing rule. The agent chooses an optimal level of action contingent on the fee schedule proposed by the principal. Agent's work, effort, decision, performance, responsibility and the amount of attention the agent pays to work are equivalent terms for the agent's action. Also, a work-averse agent can be interpreted as having a tendency to consume excessive perquisites. Baiman (1982, p. 170) argues that "agency theory exploits the power of self-interest as a stabilizing and predictive force". An object of agency theory is to find the contract which specifies how the payoff is shared.

One important issue in agency theory is the asymmetry of information available to the principal and agent. There are two major types of information asymmetry: adverse selection and moral hazard. Further, there are two types of moral hazard problems: hidden-action and hidden-information.

1. Adverse Selection

There is an incentive problem underlying agency models caused by asymmetric information (Hart and Holmström, 1987, p. 76). It is common to distinguish models based on the particular information asymmetry involved. All models in which the agent has pre-contractual information which is unknown to the principal deal with the adverse selection problem. Adverse selection was originally introduced in health insurance. Medical insurance firms which did not examine prospective insurees would have to charge higher premiums. Observing this, only unhealthy people would apply for the insurance and healthy people would drop out of the insurance market. Another typical example is a decentralized socialist economy (Arrow, 1985, p. 39). The individual productive units have more knowledge of productivity and more information about the possibilities of production than available to the central planning unit. The individual units may have incentives to hide their full production potential because it will be easier to work under less taxing requirements. The same problem arises in decentralization within a firm. The adverse selection problem is not considered in this research.

2. Moral Hazard

Models dealing with moral hazard have symmetric information at the time of contracting but asymmetric information concerning the actual productive effort. Moral hazard is further distinguished as two cases (Hart and Holmström, 1987, p. 76):

(1) the Hidden-Action Model where the agent's action cannot be observed and hence contracted on, and

(2) the Hidden-Information Model where the agent's action may be observed but the contingencies under which they were taken are not.

Workers supplying unobservable effort is an example of a hidden action agency model. A bad outcome of a decision in an agency model can result from two cases (Jennergren, 1980, p. 190):

(i) an unfortunate state of the world, and

(ii) an improper choice of action by the agent.

If the agent is effort-averse, he may be tempted to select an improper action (low level of effort). If the principal can observe only the payoff (outcome), the agent may select a very low effort level and claim that the resulting poor payoff was due to the unfortunate random state of nature. The moral hazard (also referred to as adverse incentive by Spence and Zeckhauser, 1971, p. 382) results in the need to monitor the agent's action (effort).

The problem in the well-known Prisoners' Dilemma (Rapoport and Chammah, 1965; Snyder and Diesing, 1977) also presents a moral hazard. Both players are better off if neither confesses. But such behaviour is not enforceable because players cannot write an enforceable contract which punishes or rewards each player based on their action choices (Baiman, 1982, p. 163). Each player's action is not observable to the other. However, since there is no exogenous uncertainty involved in game theory each player's action is perfectly inferred after an outcome is realized.

The expert manager making observable investment decisions leads to a typical hidden information agency model (Hart and Holmström, 1987, p. 76). In the auditing of financial statements, the moral hazard of hidden information arises when the agent reports the firm's net income to the principal. The agent's report is the action which is observed by the principal but the contingency of the report (i.e., the true net income) is not. Since both the hidden-action and hidden-information models make complete contracting infeasible (or at least unenforceable) due to the information asymmetries, the resulting inefficiency creates incentives for information gathering (Crawford and Guasch, 1983, p. 373).

(III) MODELLING THE STANDARD AGENCY RELATIONSHIP

1. Defining Notations

a: - agent's action (effort).

θ: - the random state of nature which may be interpreted as the result of any exogenous uncertain event that affects the payoff (e.g., machine breakdown rate, weather, an individual's native ability). The random variable θ captures all relevant uncertainties in the problem.

$P(.,.)$: - production function representing production facilities which are usually assumed to be owned by the principal. Knowing the production function is equivalent to knowing all contingencies of possible (a,θ) pairs.

$x = P(a,\theta)$: - payoff which is a function of agent's effort and random state of nature. This outcome is a surrogate measure of the agent's action and is affected but not completely determined by the action.

$s(x)$: - share of payoff received by the agent.

$x - s(x)$: - the principal's residual share.

$f(x|a)$: - probability density function over payoffs given an action. When the principal and agent makes an agreement, $f(x|a)$ becomes available to both players.

$F(x|a)$: - cumulative distribution function over payoffs given an action.

K : - the agent's reservation utility (or opportunity cost in utilities): the agent's minimum expected utility level guaranteed in the labour market.

$G(x-s(x))$: - principal's utility function.

H($s(x)$,a): - agent's utility function. When the agent has additively separable utility, $H(s(x),a) = U(s(x)) - V(a)$.

2. Mathematical Formulation

The following mathematical formulation and assumptions are based on models presented by Baiman (1982), Baiman and Demski (1980b), Hart and Holmström (1987), Holmström (1979), and Hughes (1982).

$$\max_{s(x),\, a} \int G(x-s(x))f(x|a)dx \qquad (1)$$

$$\text{s.t.} \int U(s(x))f(x|a)dx - V(a) \geq K \qquad (2)$$

$$\int U(s(x))f_a(x|a)dx - V'(a) = 0 \qquad (3), \text{ or}$$

$$\int U(s(x))f(x|a)dx - V(a) \geq \int U(s(x))f(x|a')dx - V(a'), \quad \text{for all } a' \in A \qquad (4)$$

The objective of agency theory is to find the contract which specifies how the payoff is shared between two parties. The objective function (1) indicates that the payment schedule and agent's action are chosen to maximize the principal's expected utility. Since the principal is assumed to know the agent's preferences, he also knows what action the agent will take even though he cannot directly observe it. For each feasible employment contract the principal is seen as deciding on the action he wants the agent to take and picking the least cost remuneration scheme that goes along with that action. The solution to constraint (3) (or (4)) is an argument in the principal's objective function.

Constraint (2) shows that the agent will not join the firm unless his expected utility from doing so is at least as great as his expected utility from selling his services in the labour market. The minimum expected utility level K is determined in the marketplace. Since the principal is assumed to know the agent's beliefs and preferences, he can evaluate each payment schedule $s(.)$ and induced action a combination from the agent's point of view. The principal's search of $s(.)$ can be narrowed to those *self-enforcing combinations* for which the agent would agree to work for the firm. Constraint (2) is called *participation constraint* (Arrow, 1985, p. 44).

Constraint (3) is a first-order condition which maximizes the expected utility of the agent and called the *incentive compatibility requirement* (Baiman and Demski, 1980b, p. 186) or *agent's action self-selection constraint* (Baiman, 1982, p. 173). The function $f_a(x|a)$ is a first-order derivative of the density function $f(x|a)$. The constraint (3) reflects the restriction that the principal can observe x but not a. The sharing rule $s(x)$ must be such that the agent's expected utility maximum occurs at the effort level a that is contracted for. Since the agent is assumed to be a rational decision maker he will want to maximize his expected utility and so will deliver this level of effort. Without this constraint, the agent would be tempted to shirk, because of assumed effort aversion on his part and the lack of effort observability.

Constraint (4) is equivalent to the following expression:

$$a \in \text{argmax } E\{U(s(x)) - V(a') \mid a' \in A\}.$$

There might be multiple optimal actions with this constraint. Constraint (3) is a stronger condition than (4) in that the former is the sufficient condition for optimality for only a subset of situations and the latter is the necessary and sufficient condition for all situations (Baiman, 1982, p. 176). The agency problem is almost always analyzed using a solution approach which requires that the optimal action exist, be unique, and satisfy the incentive compatibility requirement.

3. Assumptions

A1. A two-person single-period principal-agent relationship is considered.

A2. Action $a \in A \subseteq R$ is unobservable to the principal and payoff $x \in X \subseteq R$ is jointly observable.

A3. G, H, U, and V are twice differentiable, real-valued functions.

A4. The agent has additively separable utility in wealth (payoff) and effort, which means that disutility of action is independent of agent's wealth (or the marginal utility of wealth is independent of the action taken).

A5. The agency problem is a game of complete information. Each individual knows the structure of the choice problem and preferences (G(.), U(.)-V(.), Θ, A, and P(.,.)). Only the payoff $x \in X$ is jointly observed and both players share same state belief $\gamma(\theta)$. Typically, the production function P(.,.) is used to define a transformation of variables from states of nature to payoffs parameterized by the agent's effort. In this formulation, the probability measure on X, $f(x|a)$, that is induced by a and $\gamma(\theta)$ is used.

A6. The rationality assumption: the principal and agent are wealth-seeking (meaning that G and U are monotone increasing utility functions) and the agent is work-averse: $dV/da>0$, $dH/da<0$, and $d^2V/da^2>0$ (increased effort-aversion).

A7. The principal is weakly risk-averse ($G'>0$ and $G''\leq 0$) and the agent is risk-averse ($U'>0$ and $U''<0$). When the agent is risk-neutral, the principal gets a constant share C (a rental contract). The agent bears full risk and his payment is $s(x) = x - C$. Such a fee allocates risk in a desirable way and provides the right incentive to the agent (Shavell, 1979, p. 56). Even if the principal has imperfect information y of action in addition to x, nothing is lost if his fee depends on the outcome alone: $s(x,y) = x - C$. Information about efforts has no value (Shavell, 1979, p. 64). In other words, if the agent is risk neutral, any contract which depends on x, a, and θ can be dominated by a contract which depends on x alone (Harris and Raviv, 1979, p. 239). All effects of the agent's performance are internalized, and thus the incentive problem is resolved without using performance-contingent contracts (Harris and Raviv, 1978, p. 24). Observability of a and θ would not lead to a Pareto superior contract.

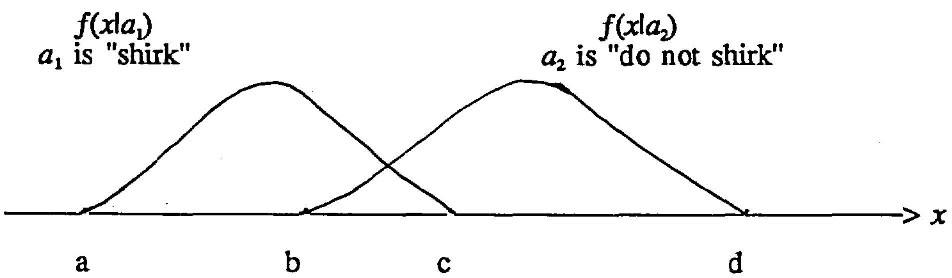
A8. Both parties agree to the probability distribution of θ (i.e., both have the same state belief) and the agent chooses action a before θ is known. The value of θ is unknown to the principal but can be observed by the agent after its realization. The principal observes the outcome but cannot analyze it into its two components, the agent's true effort level and the exogenous uncertainty. When θ is jointly observable *ex post*, regardless of the agent's risk attitude, action a can be inferred *ex post* because x and P(.,.) are also known. Any contract depending on x, a, and θ can be dominated by one which depends on x and θ (Harris and Raviv, 1979, p. 237). Therefore, there are no gains to direct acquisition of information regarding action.

Assumptions (7) and (8) identify two cases where Pareto optimal contracts will not involve the agent's effort: (1) the agent is risk-neutral where the agent bears all the cost of his decision and eliminates the motivational (moral hazard) problem by internalizing it or (2) when θ is jointly observable *ex post* where knowing θ *ex post* and $P(a,\theta)$ allows the principal effectively to infer the agent's effort level. Negation of (1) and (2) provides a necessary condition for the managerial accounts to collect information on the agent's activity such as by installing time clocks, and other supervisory monitoring (Baiman, 1982, p. 185). Therefore, whenever θ is not jointly observable *ex post* and the agent is risk-averse, optimal contracts will depend on the action (if observable) (Harris and Raviv, 1979, p. 239). This result shows that both principal and agent would prefer a situation in which the agent could be perfectly supervised at no cost to one in which information about agent's action is unavailable. This is the case where there are potential gains to *imperfect monitoring* (Harris and Raviv, 1979, p. 247).

A9. $f(x|a)$ has bounded and non-moving support and is twice-differentiable in a, for all x.

Moving support and non-moving support need to be distinguished:

(i) Moving support: If there exist outcomes which can be assured to result only from a certain level of effort, the distribution $f(x|a)$ has a moving support. For example, assume that the outcome set is bounded in [a,d] and two discrete levels of efforts--"shirk" and "do not shirk" exist.



- Figure. B1 -

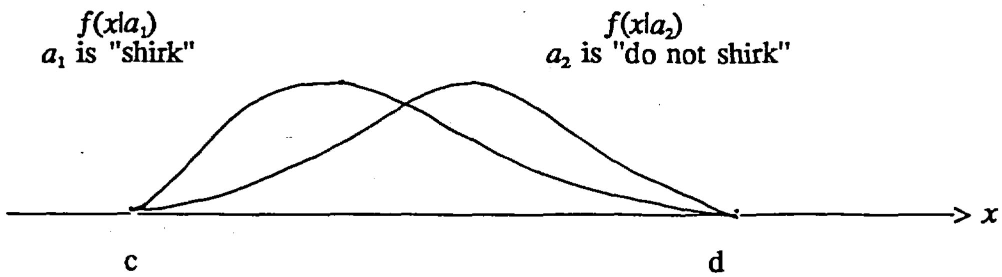
If the principal knows outcomes in [a,b] *ex ante*, he can motivate the agent to exert a_2 by imposing a large penalty to outcomes in [a,b] which can only result from shirking (consequently, x has distribution $f(x|a_2)$). The penalty which is large enough to enforce a first-best contract is assumed to be feasible. A common mathematical assumption is $U'(0)=-\infty$, which means that if all the agent's properties are taken out as a penalty, the agent's marginal utility for that penalty is negative infinity. It does not matter whether x is in [b,c] or [c,d] *ex post* because the principal is sure of $f(x|a_2)$. If the game is one of moving support, the incentive compatibility requirement is not needed. Moving support is simple and has not received much interest. Most agency literature (including Holmström, 1979) considers non-moving support and introduces the incentive compatibility requirement. One simple example of moving support is shown in Figure B2.

		State of Nature	
		good	bad
Agent	do not shirk	10	5
	shirk	5	0

- Figure. B2 -

The agent has two options: "do not shirk" and "shirk". The random state of nature is also assumed to have two possibilities: "good" and "bad" conditions. A "good" condition always means a bigger payoff than in a "bad" condition given an agent's level of effort. As long as there exists some probability of the outcome with payoff 0 and if large penalty is imposed on that outcome, then the agent will never choose shirking.

(ii) Non-moving support: If $f(x|a)$ has a non-moving support, there exists no outcome which can be guaranteed to result only from low level of effort. Assume that there are two discrete levels of effort: "shirk" or "do not shirk". Assume also that the support is bounded, which means that $f(x|a)$ has support that is independent of $a \in A$ (or support of the distribution of x will not change with a).



- Figure. B3 -

As shown in Figure B3, no observed outcome x can assure a particular type of effort. In standard agency relationship, $f(x|a)$ is assumed to have a bounded and non-moving support. That is, $F(c,a)=0$, $F(d,a)=1$, and $F(x,a)>0$ for all $x \in (c,d]$.

A10. $dx/da \geq 0$ which says that greater effort will result in higher (at least not lower) payoffs in every state of nature. This implies that $F_a(x,a) \leq 0$ for all a and strict inequality holds for some value of x. A change in a has a non-trivial effect on the distribution of x. That is, payoff distributions are shifted to the right in the sense of first-order stochastic dominance.

A11. The fee schedule $s(.)$ belongs to the class of differentiable and real-valued functions and $s(x) \in [\underline{x}, \overline{x}]$. This restriction is natural from a pragmatic point of view, since the agent's wealth puts a lower bound, and the principal's wealth including residual payoff a upper bound on $s(x)$.

A12. The first-order constraint (the incentive compatibility requirement) identifies a unique optimal internal solution to the agent's problem of choosing an action for a given sharing rule. The agency problem has a unique solution.

A13. The agency problem is a non-cooperative sequential game and the solution concept used in the agency model is a Perfect Nash equilibrium (Baiman, 1982, p. 170; Antle, 1982, p. 509). The perfect equilibrium in the Standard Agency Model takes an essentially simple form. The agency model is a game of perfect information and each subgame is a single person (the agent) game. The principal moves first subject to the participation constraint (the principal chooses a sharing rule $s(x)$) and the agent observes this move before his move, and the set of moves available to the agent does not depend on the principal's move. The set of perfect equilibria is exactly the set of subgame perfect equilibria (Antle, 1982, p. 509). The agent does not engage in any threat strategy in an attempt to alter the specification of $s(x)$. Among the combinations of $(s(x),a)$, an equilibrium is the one which maximizes the principal's objective function. A contract specified in agency theory is a *self-enforcing contract*. The self-enforcing aspect of the Perfect Nash solution concept comes from the fact that the chosen payment schedule $s(x)$ is such that the agent does what the principal expects him to do (Baiman, 1982, p. 170).

Baiman and Demski (1980b, p. 187) argue that given the sequential play structure of the game, agency equilibrium is also the Stackelberg solution when the principal is the leader and the agent the follower. This is not true because in the agency model, if there are multiple optimal actions given a sharing rule $s(x)$, it is assumed that the agent chooses a^* according to the leader's preferences (Holmström, 1979, note 10; Baiman, 1982, p. 174). On the other hand, in Stackelberg stability, the leader conservatively assumes that the follower, if he has a choice of equally preferred outcomes, will select the one that minimizes the leader's return (Fraser and Kilgour, 1986, p. 108).

(IV) THE STANDARD AGENCY MODEL

1. First-Best Contract

When the payoff and the agent's level of effort are jointly observable, a first-best contract is possible. A first-best contract is the solution of equation (1) the objective function, and equation (2) the participation constraint. Since the agent's action is observable, there is no moral hazard problem in this case. The sharing rule in the first-best contract yields a Pareto optimal sharing of the risk of random state realization between the principal and agent. Effort in the first-best contract is the effort most desired by the principal.

A first-best contract is a forcing contract that penalizes dysfunctional behaviour (Holmström, 1979, p. 74). If the agent is risk-neutral, it is a rental contract. Because managerial effort is observable, the first-best effort level is attainable by having a large penalty if the agent does not choose exactly the contracted-for effort level.

2. Second-Best Contract

When the principal cannot observe the agent's effort, it is necessary to further constrain the agency problem by amending the sharing rule in such a way that the agent's expected utility is maximized at the effort level he contracted for: i.e, by adding the incentive compatibility requirement, equation (3). This constraint has the effect of loading more risk onto the agent than in the first-best solution (Scott, 1984, p. 181). The opportunity to enforce the choice of a as in a forcing contract is no longer available. In addition, there is no direct or indirect supervision of the agent.

The logic behind imposing the incentive compatibility constraint is that the principal is able to get rid of the uncertainty about what action the agent might take, once the sharing rule is set, by agreeing to compensate the agent for no more effort than what the principal anticipates (Hughes, 1982, p. 345). When the incentive compatibility constraint is not binding, a first-best contract is possible. The binding constraint means both a departure from first-best risk sharing in order to motivate the agent by providing appropriate incentive to work (some of the risk sharing benefits are traded off for the incentive provisions), and insufficient incentives to induce the most desired level of effort (first-best effort) (Hughes, 1982, p. 345).

Let δ be the lagrangian multiplier for the participation constraint and μ for the incentive compatibility requirement. An optimal sharing rule is characterized by the following formula (Holmström, 1979, p. 77):

$$\frac{G'(x-s(x))}{U'(s(x))} = \delta + \mu \frac{f_a(x|a)}{f(x|a)} \quad \text{for almost all } x. \qquad (5)$$

If (5) does not have a solution, $s(x)$ will be a upper bound or a lower bound value depending on whether right hand side of (5) is greater or the other way. Furthermore, μ is characterized as following:

$$\mu = \frac{-\int G(x-s(x))f_a(x|a)dx}{\int U(s(x))f_{aa}(x|a)dx - V''(a)} \qquad (6)$$

where the denominator is the second-order condition on the agent's expected utility and so must be ≤ 0 (f_{aa} is a second order derivative of the function f with respect to a). Assume that strict inequality holds.

A first-best solution (perfect risk sharing) is achieved only when $\mu=0$ (Holmström, 1979, p. 78). Based on equation (6) it has been proven that if the effect of the agent's effort corresponds to first-order stochastic dominance ($F_a \leq 0$), then the principal would always desire greater effort of the agent than the agent would be willing to provide under a second-best sharing rule: i.e., $\mu>0$ (Holmström, 1979, p. 78; Hughes, 1982, p. 346). Given that the denominator is negative, $\mu>0$ means that the principal's marginal expected utility under a second-best sharing rule is strictly positive. That is, the expected utility of the principal is increasing with the agent's action. The result implies that the first-best effort -- most desired effort level by the principal -- is greater than the second-best effort.

In the second-best contract, each party attains a lower level of expected utility than in the first-best solution. Thus the second-best solution is strictly Pareto inferior to the first-best solution (Holmström, 1979, p. 78). There are positive gains to observing the agent's action, since in that case a first-best solution can be attained by using a forcing contract. This provides the basis for discussing ways to realize part of these gains by using *imperfect monitoring* (Holmström, 1979, p. 79). This implies that the agent's effort level with monitoring is greater than that of second-best contract, and with perfect monitoring, first-best effort is achieved. In this respect, monitoring is interpreted as a way to move the agent's second-best effort level to the first-best effort and achieve Pareto superior expected utilities.

Let y be a signal, which in addition to x, is observed to both parties and hence can be used in constructing the sharing rule. Let $h(x,y|a)$ be the joint density function of x and y given a. The corresponding mathematical formulation is:

$$\max_{s(x,y),\, a} \int\int G(x-s(x,y))h(x,y|a)dxdy \qquad (7)$$

$$\text{s.t.} \int\int U(s(x,y))h(x,y|a)dxdy - V(a) \geq K \qquad (8)$$

$$\int\int U(s(x,y))h_a(x,y|a)dxdy - V'(a) = 0 \qquad (9)$$

An optimal sharing rule is characterized by the following formula (Holmström, 1979, p. 77):

$$\frac{G'(x-s(x,y))}{U'(s(x,y))} = \delta + \mu \frac{h_a(x,y|a)}{h(x,y|a)} \text{ for almost all } x. \qquad (10)$$

An example of getting additional information y is that in many cases of torts, a damager is not liable if due care has been exercised. Therefore, the plaintiff is required to show negligence on the part of the defendant, so that additional knowledge beyond the outcome is available (Arrow, 1985, p. 45). Similarly, the custom of paying lawyers on the basis of time spent as well as by a contingent fee is an example of monitoring.

3. Value of Information

A signal y is said to be *valuable* if both the principal and agent can be made strictly better off with a contract of the form $s(x,y)$ than $s(x)$ (Holmström, 1979, p. 83). From equation (10), y will be valuable if and only if it is false that

$$\frac{h_a(x,y|a)}{h(x,y|a)} = \frac{f_a(x|a)}{f(x|a)} \quad \text{for all } a \text{ and almost all } (x,y). \quad (11)$$

The reason is that when (11) holds, a contract $s(x)$ will satisfy (10) and when (11) is false (i.e., y is valuable), a Pareto optimal contract must necessarily take the form of $s(x,y)$. Equation (11) is equivalent to

$$h(x,y|a) = \frac{h_a(x,y|a)}{f_a(x|a)} f(x|a). \qquad (12)$$

When solved as a differential equation in a, it follows that

$$h(x,y|a) = g(x,y)f(x|a), \text{ for almost every } (x,y). \qquad (13)$$

Equation (13) is equivalent to (11). When (13) holds, x is a *sufficient statistic* for the pair (x,y) with respect to a, which means that x carries all the relevant information about a and y adds nothing; when (13) is false, y contains some information about a beyond that conveyed by x (Holmström, 1979, p. 84). When equation (13) holds, a signal y is called *informative* and it has been shown that there exists a sharing rule $s(x,y)$ which is strictly Pareto superior to $s(x)$ if and only if the signal y is informative (Holmström, 1979, p. 84).

Thus a signal y is valuable if and only if it is informative (the cost of information is not considered). Only when the imperfect monitor of action conveys information which is not conveyed by the payoff x, can a strict Pareto improvement be achieved by gathering additional information y, regardless of the beliefs and preferences of the principal and agent. The value of obtaining y derives solely from its motivational influences on the agent (Baiman, 1982, p. 188).

While the imperfect monitoring of action may reinforce the agent's work incentive, additional uncertainty is introduced by imperfect monitoring -- the inaccurate perception of the agent's true effort (Baiman, 1982, p. 187). The introduction of this new risk is undesirable to the risk-averse agent and if the principal is risk-averse, it is undesirable to him, too (Shavell, 1979, p. 56). However, Holmström (1979, p. 87) argues that:

> The more important part of the proposition [3] is the result that any informative signal, regardless of how noisy it is, will have positive value (if costlessly obtained and administered into the contract).

If the principal has information which reflects something of the truth about the agent's action, then no matter how imprecise the information, it has value and ought to be incorporated into the terms of a contract. Benefits from new information y of the agent's action override the new risk of imperfectness and noisiness given by the information.

(V) APPLICATION OF STANDARD AGENCY MODEL TO ARMS CONTROL

1. Agency-Theoretic Modelling of Arms Control Treaties

The following model is an attempt to transfer arms control situations into the agency theory framework. It is recognized that there might be other variations depending on different interpretations of analogies between the two areas. The principal in the Standard Agency Model is equivalent to *the inspector* in arms control and the agent *the inspectee*. For a more understandable interpretation, it is convenient to look at particular treaties such as limitation of nuclear missiles or non-production of prohibited chemical (or biological) material.

(1) Defining Notations

a: - while a represents the agent's production effort (action) in economics, in an arms control treaty, a corresponds to inspectee's effort of complying with the treaty. If the treaty says to eliminate a certain number of missiles or amount of chemical material, a will represent the amount of compliance effort required or resources which must be spent in order to achieve that goal.

θ: - the random state of nature which is unknown at least to the principal after its realization: for example, exogenous factors such as weather or failure rate of inspector's monitoring facilities.

$P(.,.)$: - the function P in arms control is the verification technology function representing monitoring facilities owned by the inspector. Knowing the verification technology function P is equivalent to knowing all contingencies of possible (a,θ) pairs.

$x = P(a,\theta)$: - the outcome of an arms control agreement, measured in terms of number of missiles reduced or amount of chemical material reduced. x is a function of the inspectee's compliance effort and the random state of nature. Thus x corresponds to the estimated outcome of a, showing the inspectee's compliance effort, assessed by the inspector's verification technology and jointly observed. Due to the random θ, there are risks of showing low x given that the true effort level is high (risk of *false alarms*) and showing high x given that the true effort level is low. Nevertheless, both parties are bound by the observed x.

$s(x)$: - In economics, $s(x)$ is the agent's share based on the outcome x. It is interpreted in a slightly different way in arms control. The function $s(x)$ is assumed to be the sanction imposed to the inspectee based on the joint observation of x. Note that the units of $s(x)$ must be the same as x in order to be consistent with the economic agency theory models. Finding a proper $s(x)$ through mathematical formulation is a difficult exercise. As a simple example, it can be assumed that $s(x)$ is linear to x. Since very few restrictions can be placed on the shape of the sharing rule $s(x)$ in Standard Agency Models (Hart and Holmström, 1987, pp. 80-2), the linearity assumption is relevant. That is, if x shows that the inspectee is properly complying with the treaty, $s(x)$ will have the value of zero. Otherwise, $s(x)$ will indicate further destruction of inspectee's missiles or chemical material. $s(x)$ is assumed to be non-positive.

$x - s(x)$: - the principal's residual payoff in Standard Agency Models. In arms control, it is interpreted as an estimated total number of missiles destroyed or amount of chemical material destroyed after a sanction is imposed.

$f(x|a)$: - probability density function over payoffs given an effort level a.

$F(x|a)$: - cumulative distribution function over payoffs given an effort level a.

K: - the inspectee's reservation utility (or opportunity cost in utilities): utility of not reaching (or abandoning) an arms control treaty.

$G(x-s(x))$: - inspector's utility function.

$H(s(x),a)$: - inspectee's utility function. $H(s(x),a) = U(s(x)) - V(a)$. A more general interpretation of the arguments of the functions G(.) and U(.) is national security, which is a function of x and $s(x)$. It can be assumed that a country's national security function (n_1 for the inspector and n_2 for the inspectee) is increasing with respect to the values of x-$s(x)$ for the inspector and $s(x)$ for the inspectee. Thus $G(x-s(x))$ becomes $G\{n_1(x-s(x))\}$ and $U(s(x))$ matches $U\{n_2(s(x))\}$. With this interpretation, the results of this research would still apply. We write utility as a function of x only for simplicity.

(2) Mathematical Formulation

The mathematical formulation (consisting of equations (1), (2), and (3)) used in the Standard Agency Model holds in an arms control application, given the definitions of the previous section. The objective function (1) shows that the sanction $s(x)$ and the inspectee's compliance effort are chosen so as to maximize the inspector's expected utility. Even if the problem is converted to the maximization problem of the inspectee's expected utility, the results do not change. The solution to constraint (3) is an argument in the inspector's objective function.

The participation constraint (2) shows that the inspectee will not join any arms control treaty unless his expected utility from doing so is at least as great as his expected utility from not being a member of the treaty. The incentive compatibility requirement (3) reflects the restriction that the inspector can observe x but not a. The sanctioning scheme $s(x)$ must be such that the inspectee's expected utility maximum occurs at the level a that is contracted for. Thus the sanction must be reasonable enough to be accepted by the inspectee. Without this constraint, the inspectee would be tempted to cheat, because of assumed compliance effort aversion on his part and the lack of effort observability.

(3) Assumptions

The basic assumptions used in Standard Agency Models hold in the arms control formulation. Some of assumptions in section 3.3. need further explanation in the arms control context:

A1. Only two countries are considered. Neither the inspector nor the inspectee can be multiple independent players. Also, no long-term relationship is considered. Since the agency relationship as modeled here is for a single-period, it is reasonable to assume that the inspector and inspectee are short-sighted players. Both consider only benefits and losses directly resulting from the treaty. No higher level of consideration such as political stability is made.

A5. The verification technology function P(.,.) is jointly known. This means that even if P is not directly known to the inspectee, the inspectee is presented with convincing evidence of the observed value x. The dilemma of intelligence compromise can occur, where the inspector cannot convince the inspectee of the value x because he is not willing to reveal his secret intelligence sources. This is not modeled in this research. If an independent inspectorate is introduced, the inspectee may be assumed to be subject to what the independent inspectorate observes.

A6. The inspectee is compliance effort-averse: $dV/da>0$, $dH/da<0$, and $d^2V/da^2>0$ (increased compliance effort-aversion).

A7. $dG/dx>0$; reduction of the inspectee's missiles or chemical material results in increased benefits to the inspector. $dU/ds(x)>0$; a larger value of $s(x)$ (which corresponds to less reduction in missiles or chemical material by the inspectee because $s(x)$ is assumed to be non-positive, see A11 following) -- i.e., lower absolute value -- leads to increased benefits to the inspectee. It should be noted that functions G and H are proxies for the utility functions of countries and therefore are group utility functions. We do not consider here the complications that result from this. As in most game theory models, a country is treated as an individual player. Although there might be debates within the bureaucracy, the policies of a state are assumed to include all opinions raised from many government branches.

A8. Both parties agree on the probability distribution of θ and the inspectee chooses action a before θ is known.

A9. $f(x|a)$ is assumed to have bounded and non-moving support. That is, $F(c,a)=0$, $F(d,a)=1$, and $F(x,a)>0$ for all $x \in (c,d]$. Since the number of missiles or amount of chemical material the inspectee has must be bounded, so must the number of missiles reduced or amount of chemical material reduced.

A10. $dx/da \geq 0$: the marginal increase of reduced number of missiles or amount of chemical material with respect to compliance effort is non-negative. The greater compliance effort, the more compliance result.

A11. $s(x)$ is bounded as [-m,0] where m is a total number of missiles or amount of chemical material the inspectee has.

(4) Similarities and Differences between Economic Contracts and Arms Control Treaties

Economic contracts and arms control treaties have both similarities and differences. Some organizational similarities are mentioned here.

A. Similarities

An economic contract and an arms control treaty are both agreements among *remote* parties. Parties are remote because activities of each party are rarely transparent and consequently, it is not easy for each party to perceive the true action of the others -- effort is unobservable. The problem of information asymmetry naturally occurs. This is the most important structural similarity. In addition to this, other analogies are identified and properly reflected in the mathematical formulation and assumptions of the previous sections.

B. Differences

In an economic contract, the two parties have asymmetric roles; one as principal delegates work to the other, the agent. However, in arms control treaties, each country has a dual role both as principal and agent. The country is concerned with the other's compliance activities as an inspector and also needs to exert compliance effort as an inspectee. In economic contracts, for example, the agent exerts effort in exchange for a share of the payoff when the outcome is a profit or, as in the polluting firm-society relationship, in order to avoid a fine imposed by the principal when the outcome is loss to society -- pollution. But in arms control agreements, the inspectee exerts compliance effort in exchange for the reciprocal compliance effort of the inspector. This feature of the relationship is not captured in the compensation function $s(x)$. Instead, this research models that the inspectee complies with the treaty so as to avoid sanctions.

The second difference is that only one contract between the principal and agent is considered in agency theory. In arms control, on the other hand, two parties may have many different treaties covering different areas, as shown by the numerous treaties made between the superpowers. So there might be interdependent effects among different treaties.

The agreed-on measure for the evaluation of outcomes is the third major difference. Here, we use x, the number of missiles reduced. In economics, monetary units are used. As

Arrow (1985, p. 50) notes, it is a major limitation of current principal-agent models that the reward or penalty system is virtually always stated in terms of monetary payments because there is a wide variety of rewards and penalties that take social rather than monetary forms. In arms control, however, there is nothing like money as a measure. For instance, security, tension, or peace are all not easily quantifiable. In spite of this difficulty, as noted in section 5.1.1., national security which is a function of reduction of a weapon stockpile may replace money as a unit.

In arms control, an agreement on sanctions or rewards is not always realistic, especially in a treaty made among opposing parties. A central authority which imposes sanctions or rewards, or agreement among the parties to do so, rarely exists in an anarchic international society. However, there is at least one exception. There is an obvious sanction against violation in the Non-Proliferation Treaty (NPT). In this case, the sanctioning provision appears in each country's (especially the nuclear suppliers') domestic laws. Smith (1987, pp. 259-60) notes:

> ... according to the International Financial Institution Act of 1977, the United States [Nuclear Weapon State] must suspend any export-import bank loans to any country that terminates safeguards on U.S.-exported material or nuclear equipment; this law also prohibits the extension of any new credits to a NNWS [Non-Nuclear Weapon State] that subsequently detonates a nuclear device.

2. An Arms Control Treaty Between First and Second-Best Contracts

In real life full information (costless and perfect) rarely is freely available to all parties. The nature of effort is so complicated that full observation of action is either impossible or prohibitively costly. The first-best solution could be achieved only in the unrealistic world of costless information flow. (There are a few exceptional cases. For example, consider a hockey player's contract. His action is perfectly and costlessly observable and his remuneration may be proportional to the number of goals he scores.) In a second-best situation, the problem becomes how to structure an agreement that will induce agents to serve the principal's interest even when their actions and information are not observed by the principal (Pratt and Zeckhauser, 1985, p. 2).

(1) First and Second-Best Contracts

As just mentioned, the challenge in the agency relationship arises whenever the principal cannot perfectly and costlessly monitor the agent's action and information; the problem of inducement and enforcement comes to the fore (Pratt and Zeckhauser, 1985, p. 3). Given information asymmetries, any agency relationship cannot be expected to function as well as it would if all information were costlessly shared or if the preferences of the principal and agent could be costlessly aligned. This shortfall -- the difference between realized utilities in the first and second-best contracts -- is called the *agency loss* or *agency cost* (Pratt and Zeckhauser, 1985, p. 3). The challenge in structuring an agency relationship is to minimize it.

It may be argued that any *agreed* contract between the principal and agent under imperfect information flow is an example of a second-best contract. Salespeople contracted solely on a commission system are under a second-best contract. If some *additional information* about the agent's action such as the number of miles he drove or the number of people he met are incorporated in the fee schedule, it may be a better contract than just a pure commission system. Consequently, additional information holds out the possibility of reducing the imperfections and shortfalls of the second-best contract. In both first and second-best contracts, note that there are no ambiguities about what action the agent is going to take since the agent's action is fully observable in the former and in the latter, the principal accurately motivates and anticipates the agent's effort level from equation (3): the incentive compatibility requirement. However, if additional information correlated with effort can be obtained, this can be used to improve the contract.

In the arms control application of agency theory, the production function P is assumed to represent the inspector's verification technology. In real arms control situations, the monitoring function P is assessed as the capability of inspector's national technical means (NTM). The U.S. State Department (1983, p. 65) has provided a very comprehensive definition of NTM:

> Assets under national control for monitoring compliance with the provisions of an agreement. NTM include photographic reconnaissance satellites, aircraft-based systems (such as radars and optical systems), as well as sea- and ground-based systems such as radars and antennas for collecting telemetry.

NTM provide the foundation for judgments of monitoring confidence. Walter Slocombe (1983, pp. 85-6) remarks that:

> It is important to realize that agreed or "cooperative" verification procedures can never be more than a backup to national technical means... For in a critical sense, we always rely ultimately on NTM, i.e., our own intelligence, and increased assurance of access by NTM may make risks acceptable by reducing uncertainties.

While many technical systems come within the scope of NTM, there seems to be general agreement that satellite systems form the primary NTM system. There are two types of limitations on NTM. First is the imperfectness of NTM themselves. NTM are not panaceas for resolving the difficulties inherent in the verification of compliance; it is circumscribed by very significant limitations. Such limitations include those relating to:

(i) resolution capabilities

(ii) satellite launch and orbital constraints

(iii) launch vehicle payload considerations

(iv) range and coverage

(v) processing restrictions on massive amounts of data

(vi) real-time transmission of data constraints, and

(vii) cost and availability of technological expertise (Canadian Government, 1986, p. 10).

Second, there are random factors which disturb proper function of NTM. Two important factors are:

(i) environmental factors such as cloud coverage, amount of available light (*external randomness*), and

(ii) *internal randomness* of NTM technology such as NTM breakdown rate.

The first limitation, NTM imperfection, determines the maximum number of missiles or amount of chemical material which is observed to have been eliminated. It is assumed that the more accurate the NTM are, the more missiles will be observed to have been eliminated. It is also assumed that random factors are incorporated into the Standard Agency Model through the random variable θ.

As in economic contracts, the first-best solution is rarely possible in arms control treaties. Only if the inspectee's action is fully observable, the first-best arms control treaty can be achieved. One real example can be argued to be the 1957 Antarctic Treaty. The treaty provides for a theoretically absolute level of verification (Canadian Government, 1986, annex 6). The Antarctic Treaty guarantees each observer to have complete freedom of access at any time to any or all areas of Antarctica. The first-best level of compliance effort can be assured in this case.

Arms control treaties that solely depend on the inspector's NTM are second-best contracts. Compliance questions are not raised in the second-best contract since the inspector is sure of the contracted level of compliance effort to be exerted by the inspectee. The treaty guarantees a certain level of security, which cannot be achieved without the agreement. A treaty based on NTM for verification leads only to the inspectee's second-best compliance effort and consequently, assures the second-best security for the inspector. As an example of the importance of NTM in treaty negotiations, concerns over verification influenced Carter Administration officials to emphasize tailoring substantive provisions of the SALT II Treaty to the capability to verify using NTM (Rowell, 1986, p. 95).

The fact that there is no ambiguity concerning the agent's action in economic contracts is not convincing in arms control treaties. The treaty itself can be written ambiguously. It may be difficult to assure the inspectee's action because of a treaty's inherent characteristics (e.g., due to the low observability of modern weapon technologies such as cruise missiles, mobile ICBMs, it will be difficult to see exactly how many missiles the inspectee possesses). Agreements are also sometimes purposely vague because treaty partners may not wish to lose possible military options or because they cannot reach a mutually agreeable solution (Krepon, 1985, p. 145). These agreements are worse than second-best when they depend only on NTM. Ambiguity inherent in the treaty, which gives the inspectee room for violating the treaty at the margin, is not considered in agency theory.

(2) Unconditional Imperfect Monitoring and Cooperative Verification Measures

In Standard Agency Models, the term *monitoring* refers to gathering additional information y. Since the jointly known x is assumed to be obtained by the inspector's NTM in arms control, obtaining additional y may be thought as a secondary monitoring whereas NTM have the role of primary monitoring. It should be noted that the production function P and the random state of nature θ are not functionally related to the additional information y in Standard Agency Models. The new information y is correlated just with the inspectee's action. In arms control treaties, it is argued that collecting y and incorporating the value of y into the sharing rule is equivalent to obtaining additional information by providing cooperative measures of verification in the treaty provisions.

Cooperative verification measures -- intrusive and non-intrusive -- are voluntary or negotiated measures designed to enhance the verifiability of arms control agreements (Rowell, 1986, p. 55). Non-intrusive (or passive) cooperative measures are designed to assist (or facilitate) the task of verification by NTM. Rowell (1986, p. 56) notes that specific non-intrusive cooperative measures can be as simple as agreeing not to deliberately interfere with the other side's NTM (e.g, SALT I, II Treaties), or as complex as agreeing to design final-assembly and basing facilities for mobile missile systems in such a way as to assist monitoring by NTM. Schear (1984) classifies four different types of cooperative measures:

(i) Designation Measures: Designation measures involve each side's designating the location and function of certain types of military facilities or basing areas.

(ii) Transparency Measures: Whereas designation measures primarily help focus NTM, transparency measures are intended to increase the visibility of the systems and activities that NTM monitors. For instance, to facilitate counting ALCM (Air Launched Cruise Missile)-equipped heavy bombers, the SALT II Treaty required such bombers to be equipped with Functionally Related Observable Differences (FRODs) (Rowell, 1986, p. 57). Designation and transparency measures ease the task of NTM to assess permitted systems and activities, thus raising the risk of cheating.

(iii) Collateral Measures: Collateral measures are designed to cut off the most likely routes of evasion and to prevent permitted activity from threatening the intent of the treaty. Collateral measures can help distinguish permitted and non-permitted activities, and reduce the likelihood of ambiguous activities. For instance, the ABM Treaty requires that all future radars for early warning of strategic ballistic missile attack be deployed along the periphery of a nation's territory and oriented outward. This provision was designed to prevent either side from deploying an ABM battle management radar under the guise of an early warning radar, which is permitted (Rowell, 1986, p. 57).

(iv) Compliance Measures: Compliance measures are designed to improve the effectiveness of compliance-dispute resolution by (1) establishing mutual recognition on what sorts of ambiguities would give rise to suspicions and (2) agreeing on what force levels constitute a formal baseline for determinations of compliance.

The intrusive cooperative measures (on-site inspections, or OSI) independently generate data unobtainable or very difficult to obtain through NTM. There are two types of OSI -- continuous/periodic and demand/challenge (Rowell, 1985, pp. 60-1):

(i) Continuous/Periodic OSI: Continuous/periodic OSI involve an in-country monitoring device such as a seismic station which monitors seismic events, a passive electronic sensing device monitoring activity at a closed missile assembly plant, or human inspectors permanently stationed at the portal of a land-based mobile missile final assembly plant.

(ii) Demand/Challenge OSI: Demand/challenge inspection is initiated by the inspecting party and involves agreeing on such major issues as: (1) notification, justification, and approval procedures, (2) permitted frequency of inspections, (3) facilities or classes of facilities that may be inspected.

Gathering additional information at the second-best contract and reflecting the new information into the sharing rule in economic contracts (i.e., adopting $s(x,y)$ instead of $s(x)$) is equivalent to using cooperative verification measures to sign a better treaty than just the second-best in arms control. In arms control treaties, the outcome x which is already monitored by the inspector's NTM is publicly known. Compliance effort level at the second-best contract must be less than that of first-best contract and of the contract with additional information (see, section 4.2). Additional information y through cooperative measures must be *informative* (correlated with a) because it is not conveyed by the result obtained through NTM only. Informative signal y must increase the inspectee's compliance effort level. Here lies the importance of the cooperative verification measures. Informative signal y is valuable and a treaty with y is Pareto superior to the second-best agreement. Thus if the inspector has information which reflects something of the truth about the inspectee's compliance activity, then no matter how imprecise the information, it has value. Benefits from new information y of the inspectee's action override the new risk of imperfectness and noisiness given by the information.

However, one thing we should note is that in this value calculation, the cost of information is not considered. It was assumed that additional information is obtained and administrated at no cost. In reality, trade off between value and cost of the information need to be considered. Here, the cost is interpreted more broadly by considering not only monetary cost but also technology, time, and so on. As long as cost permits, additional information needs to be reflected in the treaty. This additional verification information may require more intrusive monitoring techniques, and sometimes the inspectee's agreement to inspection as shown in on-site inspections (OSI). The process of gathering additional information y (procedures exercising cooperative measures) must also be understood by the inspectee in order for the new information to improve both players' payoffs. Additional information could also be obtained conditional on x. Conditional imperfect monitoring (see, Baiman and Demski, 1980a; 1980b) seems similar to the inspector's decision whether to exercise further intrusive verification methods after initially observing x.

Unless a party is willing to disclose all details related to the treaty, he should agree with various cooperative measures as much as he can. This is a rationale of parties who emphasize verification. On the other hand, the fact that additional monitoring makes both parties better off does not consider other aspects of two countries' relations. The inspectee may be very risk-averse to disclose his country's military secrets and may give up benefits of valuable information resulting from further monitoring especially when the benefits of not revealing his military secrets are greater than those resulting from additional monitoring information.

Even if agency theory suggests that a violation does not occur in optimal economic contracts, violations can occur in arms control treaties. An economic contract, as described in agency theory, only considers benefits or losses caused by complying with the contract. In economic contracts, it is assumed that the sanctions are so large that the agent will always choose to comply. The benefits from violating the contract are not significant in economic situations because any agent who violates a contract may not be accepted in the labour market after such a violation. However, in arms control, a treaty is considered in a more comprehensive perspective. For instance, benefits and/or losses of national security, foreign relations, and political stability by complying with the treaty, which are not represented in the incentive compatibility requirement of agency theory as well as the treaty per se, are considered. A party is tempted to violate especially when an agreement is made among opposing parties and the stakes are important.

A violation may occur when benefits from complying with the treaty are overridden by violating it. Cheating is not supposed to happen but if occurs successfully, it may cause serious damage to non-cheaters. On the other hand, if any cheating is detected, it may have contagious effect and jeopardize not only the treaty itself but also the overall political climate. Thus, it may be argued that the compliance problem is a most important aspect in arms control treaties.

> The long history of debate over compliance and related verification issues, in both bilateral and multilateral contexts, demonstrates not only their intractability but amounts to an explicit acknowledgment of the centrality of verification to the successful negotiation and effective implementation of arms control and disarmament agreements (Canadian Government, 1986, p. 14).

3. A Simple Example

The purpose of this simple illustration is to show that it is possible to find out two contracts: second-best and second-best with additional information which becomes a Pareto improved contract. All variables are arbitrarily chosen. Suppose that John and Peter have five intermediate-range nuclear missiles, each in inventory. For convenience, John is the inspectee and Peter the inspector in this example. It should be noted that the reciprocal inspection relationship (one player both as an inspector and an inspectee) is not modeled in agency theory. An extension of this example can be regarded as the I.N.F. Treaty made between two superpowers. Both have NTM as their major monitoring facilities. Since NTM are not perfect, reduction of only 0, 1, or 2 of five missiles can be observed using NTM; consequently, the probability of observing 3 or more missiles reduced is zero.

If only NTM are permitted as means of verifying compliance with the treaty, an optimal treaty is second-best at most. The second-best treaty specifies compliance activities which lead to a probability distribution of x given second-best effort level a_2: $f(x|a_2)$. If cooperative verification measures are incorporated in the treaty, then an optimal treaty with additional information y through cooperative measures is better than the second-best contract. The treaty with additional y causes the probability distribution $h(x,y|a_y)$ to shift to the right of $f(x|a_2)$ in the sense of first-order stochastic dominance. With additional information y, reduction of 0, 1, 2, or 3 of five missiles, for example, can be observed. Thus a maximum number of missiles which can be observed as eliminated is determined by Peter's (the inspector's) monitoring capacity. If John's (the inspectee's) compliance behaviour is fully observable either through complete information flow or through perfect additional information, the first-best treaty with high penalty for observed violations is feasible.

x : observed number of missiles reduced, $x \in X$ where $X = [0,1,2,3,4,5]$.

$s(x)$: sanction imposed to John (the inspectee), based on x. There might be various forms of $s(x)$. Here assume that $s(x)$ = - (the largest observable value of reduced missiles - x).

$x-s(x)$: estimated total missiles reduced after sanction.

$H(s(x),a) = U(s(x)) - V(a) = 3(10+s(x))^{1/2} - a^2$, where the number 10 is used to make the argument of the square root positive. The square root is taken to model risk averse behaviour. 10 can be interpreted as military strength or total number of nuclear missiles John possesses. From the assumption A6, $V(a)$ must be a convex function. For example, let $V(a)$ be a^2.

$G(x-s(x)) = (x-s(x))^{1/2}$

(1) Second-Best Contract:

A treaty specifies various compliance efforts which lead to the distribution $f(x|a_2)$. Assume that the inspectee's compliance effort can be quantified as $a_2 = .1$ and $V(a_2) = (a_2)^2 = (.1)^2$. Assume that $f(x|a_2) = .2$ for $x = 0, 1$ and $.6$ for $x = 2$.

x	$f(x\|a_2)$	$s(x)$	x-$s(x)$	$3(10+s(x))^{1/2}$ - .01	$(x-s(x))^{1/2}$
0	.2	-2	2	$3(8)^{1/2}$ - .01	$(2)^{1/2}$
1	.2	-1	2	$3(9)^{1/2}$ - .01	$(2)^{1/2}$
2	.6	0	2	$3(10)^{1/2}$ - .01	$(2)^{1/2}$
3	0	0	3	$3(10)^{1/2}$ - .01	$(3)^{1/2}$
4	0	0	4	$3(10)^{1/2}$ - .01	$(4)^{1/2}$
5	0	0	5	$3(10)^{1/2}$ - .01	$(5)^{1/2}$

- Table. B1 -

The expected utility for John E(H) = 9.179 and Peter's expected utility E(G) = 1.414.

(2) Contract with Additional Information y:

The treaty with additional information can specify more detail and comprehensive compliance activities and consequently leads to the distribution $h(x,y|a_y)$ which (first-order) stochastically dominates $f(x|a_2)$. Additional information encourages John to exert more compliance effort and consequently leads to the Pareto superior contract which eliminates more missiles than at the second-best treaty. Assume that a_y is .2, greater than a_2 (see, section 4.2) and $V(a_y) = (.2)^2$. Assume also that the sanctioning scheme $s(x,y) = s(x)+\alpha$. The additional information y results in increased compliance effort by John. Consequently, for any observed x, the corresponding sanction $s(x,y)$ will be no more severe than the sanction without y, $s(x)$. In this example, α is chosen as $-s(x)/2$. Assume that $h(x,y|a_y) = .1$ for $x = 0$, 1 and .4 for $x = 2, 3$. Note that $h(x,y|a_y)$ shifts to the right of $f(x|a_2)$ in the sense of first-order stochastic dominance.

x	$h(x,y\|a_y)$	$s(x,y)$	$x\text{-}s(x,y)$	$3(10+s(x,y))^{1/2} - .04$	$(x-s(x,y))^{1/2}$
0	.1	-3+1.5=-1.5	1.5	$3(8.5)^{1/2} - .04$	$(1.5)^{1/2}$
1	.1	-2+1.0=-1.0	2.0	$3(9.0)^{1/2} - .04$	$(2.0)^{1/2}$
2	.4	-1+0.5=-0.5	2.5	$3(9.5)^{1/2} - .04$	$(2.5)^{1/2}$
3	.4	0	3	$3(10)^{1/2} - .04$	$(3)^{1/2}$
4	0	0	4	$3(10)^{1/2} - .04$	$(4)^{1/2}$
5	0	0	5	$3(10)^{1/2} - .04$	$(5)^{1/2}$

- Table. B2 -

The expected utility for John E(H) = 9.228 and the expected utility for Peter E(G) = 1.589. Thus both players' expected utilities with additional information are strictly greater than those at the second-best treaty.

(VI) CONCLUDING REMARKS

Agency theory in information economics has recently offered promise for explaining the characteristics of an optimal contractual relationship among individuals associated in some forms of agency relationship (e.g., manager-shareholder, lawyer-client, insuree-insurer). An economic contract between two or more rational wealth-seeking individuals has some analogies to an arms control treaty between nations seeking national security. This research is an attempt to transfer the existing theoretical body of agency theory to arms control situations and to use the insights of agency theory to explain and characterize complex international arms control and disarmament processes.

The first half of the appendix reviews agency theory. Although there are many other variations of agency models, in this research, the *Standard Agency Model* with rather rigorous assumptions is presented. Section II emphasizes the importance of information asymmetry in agency relationship. There are two kinds of information asymmetry: adverse selection and moral hazard. This research focuses on the moral hazard problem and especially on the hidden-action agency model. The standard agency relationship is mathematically modeled in section III. Assumptions are carefully interpreted. Any standard agency relationship has one of three broad forms of contracts: first-best, second-best with additional information, and second best (section IV). The inspectee's compliance effort decreases with each respective form of contract.

The application of agency theory to arms control and verification is made in the second half of the appendix (section V). Given specific arms control definitions, it is argued that the mathematical formulation of the Standard Agency Model holds for the arms control application. Some of the assumptions of Standard Agency Models are also further explained in the arms control context. It is argued that unless there is a perfect information flow between the inspector and inspectee, using NTM only can guarantee a second-best contract. Here lies the importance of the cooperative verification measures which assist NTM (non-intrusive cooperative measures) and independently collect data that NTM cannot (on-site inspections) in order to increase the inspectee's compliance level and consequently to decrease the weapon stockpiles. Finally, a simple example of nuclear missile reduction treaty is presented.

This research views an arms control treaty depending solely on the inspector's NTM as a second-best contract. First-best arms control treaties are rarely possible except for a few cases such as the 1957 Antarctic Treaty where the absolute level of verification is feasible. Obtaining additional information through monitoring in agency theory is regarded as a secondary monitoring in arms control whereas NTM have the role of primary monitoring. In this research, the importance of cooperative measures for verification is emphasized. The integration of additional information y into arms control verification procedures is seen as the use of cooperative measures. Information obtained through cooperative verification measures is more than that conveyed by NTM only. Consequently, the information is valuable in the sense that both parties' expected utilities can increase with cooperative measures. One important practical guideline for writing arms control agreements suggested by the application of agency theory is that a treaty should contain as many cooperative verification measures as possible. Since it is difficult to sign a first-best arms control treaty with perfect verification in practice, it would be wise to concentrate on incorporating cooperative verification measures in the treaty rather than to argue for an ideal treaty with absolute verification. Agency theory shows that by bringing y into the treaty, a Pareto improvement would be attained. This points out how important it is to agree on additional cooperative measures for verification. The current trend in arms control negotiations recognizes the importance of cooperative measures. Further, due to the low observability of modern weapons technology, there will increasingly be weapon systems which can no longer be confidently monitored using unaided NTM.

Appendix C

Modelling Multilateral Verification

Appendix C: Modelling Multilateral Verification

Definitions:

α_{ij} = the benefit (penalty) to player i of an undetected violation by player j.

β_{ij} = the benefit (penalty) to player i of the detection of a true violation by player j.

Γ_{ij} = the benefit (penalty) to player i of the apparent detection of a violation by player j due to a false alarm (i.e. no actual violation).

p_i = the probability of detection of a violation by player i when player i is actually violating.

q_i = the probability of an apparent detection of a violation by player i when player i is not violating. This is the probability of a false alarm against player i.

v_i = the strategy of violating for player i.

$\bar{v}_i$ = the strategy of not violating for player i.

s = a particular outcome in the game, representing a set of strategies, one of v_i or $\bar{v}_i$ for each player i.

s_o = the outcome when all players select the $\bar{v}_i$ strategy.

s_i = the outcome formed from outcome s when player i takes the alternative strategy to that taken by i in s.

V(s) = the set of players who have selected the strategy v_i instead of $\bar{v}_i$ at the outcome s.

$V_j(s)$ = the set of players, other than player j, who have selected the strategy v_i instead of $\bar{v}_i$ at the outcome s.

$u_i(s)$ = payoff (utility) for player i for outcome s.

Basic Game Model

A **multilateral verification game** is a game in which there are n players, each of whom has two strategies, v_i and $\bar{v}_i$. An outcome s in the game has payoff for player i of $u_i(s)$, composed of the benefits of violation of player i if he is violating and not getting caught, the penalties or benefits to player i of other players violating and not getting caught, the penalties to player i of violating and getting caught, and the benefits or penalties of other players violating and getting caught. An additional factor is the possibility of a player not violating but apparently getting caught due to a false alarm.

Thus:

$$u_i(s) = \sum_{j \in V(s)} [\alpha_{ij}(1 - p_j) + \beta_{ij}(p_j)]$$

$$+ \sum_{j \notin V(s)} [\Gamma_{ij}(q_j)]$$

Assumptions

1. $\alpha_{ii} \geq 0$ A player benefits by violating without getting caught.

2. $\beta_{ii} \leq 0$ A player is penalized for being detected in a true violation.

3. $\Gamma_{ii} \leq 0$ Player i suffers when unjustifiably accused of a violation.

4. $p_i > q_i$ The chance of detection when violating is greater than the chance of apparent detection when not violating.

When there are exactly two players, or when there are more than two players who are all strictly competitive, then the following assumptions also hold, where $i \neq j$:

5. $\alpha_{ij} \leq 0$ Player i suffers a penalty when player j violates without getting caught.

6. $\beta_{ij} \geq 0$ Player i benefits when player j violates and is detected.

7. $\Gamma_{ij} \geq 0$ Player i benefits if player j is accused of a violation unjustifiably.

Result 1: If no player violates, payoffs are based only on false alarms.

Proof: follows from the basic model definition.

Result 2: Under perfect detection, it may be preferable to violate.

Proof:

Under perfect detection, $p_j = 1$, $q_j = 0$, for all j. Therefore:

$$u_i(s) = \sum_{j \in V(s)} [\beta_{ij}]$$

It is possible to choose values for β_{ij} such that for a particular outcome s*, where $v_i \in s^*$,

$$|\beta_{ii}| \leq \sum_{j \in V_i(s^*)} [\beta_{ij}] \text{ for all i, with strict inequality for at least one.}$$

Since

$$u_i(s^*) = \beta_{ii} + \sum_{j \in V_i(s)} [\beta_{ij}]$$

then

$u_i(s^*) \geq 0$ for all i, with strict inequality for at least one.

But since $q_j = 0$, for all j, from Result 1:

$u_i(s_o) = 0$, for all i.

Consequently, s_o is **Pareto inferior** to s* when these assumptions hold. (All players at outcome s_o are as badly off as they are at outcome s* and at least one player is worse off.)

Result 3: Under no detection, it may be preferable to not violate.

Proof: Similar to the proof of Result 2. Outcome s_o can be **Pareto superior** to s* for particular values of the α_{ij} parameters. (All players at outcome s_o are as well off as they are at outcome s* and at least one player is better off.)

<u>Result 4:</u> Every player has a dominant strategy.

Proof:

From the basic game model

$$u_i(s) = \sum_{j \in V(s)} [\alpha_{ij}(1 - p_j) + \beta_{ij}(p_j)] + \sum_{j \notin V(s)} [\Gamma_{ij}(q_j)]$$

Let $$\delta = \sum_{j \in V_i(s)} [\alpha_{ij}(1 - p_j) + \beta_{ij}(p_j)] + \sum_{\substack{j \notin V_i(s) \\ j \neq i}} [\Gamma_{ij}(q_j)]$$

Assume, without loss of generality, that $v_i \in s$

$$u_i(s) = \alpha_{ii}(1-p_i) + \beta_{ii}(p_i) + \delta$$

and

$$u_i(s_i) = \Gamma_{ii}(q_i) + \delta$$

(if $\bar{v}_i \in s$ then outcomes s and s_i would be interchanged)

The there are two cases:

<u>Case 1</u>: $[\alpha_{ii}(1-p_i) + \beta_{ii}(p_i)] > \Gamma_{ii}(q_i)$

Because δ is constant for a given s and s_i, in this case player i will always prefer to violate (v_i) because this strategy always gives him a higher payoff, for every s and s_i.

Case 2: $[\alpha_{ii}(1-p_i) + \beta_{ii}(p_i)] < \Gamma_{ii}(q_i)$

Because δ is constant for a given s and $s_{\bar{i}}$, in this case player i will always prefer to not violate ($\bar{v}_i$) because this strategy always gives him a higher payoff, for every s and $s_{\bar{i}}$.

In the special situation where $[\alpha_{ii}(1-p_i) + \beta_{ii}(p_i)] = \Gamma_{ii}(q_i)$, both strategies weakly dominate the other.

Consequently, player i acting independently will choose to violate iff $[\alpha_{ii}(1-p_i) + \beta_{ii}(p_i)] \geq \Gamma_{ii}(q_i)$, otherwise he will choose not to violate.

Note two further consequences of this result:

1) Γ_{ii} and q_i are critical in the violation decision. For given α_{ii} and β_{ii}, Γ_{ii} and q_i control the threshold value for p_i at which the player will prefer to violate.

2) If the expected penalty from false alarms ($\Gamma_{ii}(q_i)$) exceeds the expected penalty from violating and getting caught ($\beta_{ii}(p_i)$), a player will always prefer to violate.

Result 5: A player acting independently will only violate if the probability of detection is less than or equal to the ratio between the benefits of violating without getting caught minus the expected penalty for false alarms and the difference between the benefits of violating without getting caught and the penalty of getting caught in a violation.

From Result 4, a player will violate when

$$\alpha_{ii}(1-p_i) + \beta_{ii}(p_i) \geq \Gamma_{ii}(q_i)$$

$$\alpha_{ii} - p_i\alpha_{ii} + p_i\beta_{ii} \geq \Gamma_{ii}(q_i)$$

$$p_i(\beta_{ii} - \alpha_{ii}) \geq \Gamma_{ii}(q_i) - \alpha_{ii}$$

$$p_i \leq \frac{\alpha_{ii} - \Gamma_{ii}(q_i)}{\alpha_{ii} - \beta_{ii}}$$

(the inequality changes since $\beta_{ii} - \alpha_{ii} \leq 0$ from assumptions 1 and 2)

If there are no false alarms ($q_i = 0$), or no penalty for false alarms ($\Gamma_{ii} = 0$), then a player will violate when

$$p_i \leq \frac{\alpha_{ii}}{\alpha_{ii} - \beta_{ii}}$$

<u>Result 6:</u> Under perfect detection, a player acting independently will violate only when there is no penalty for getting caught violating.

Proof:

From Result 4, Player i will violate only when

$[\alpha_{ii}(1-p_j) + \beta_{ii}(p_j)] \geq \Gamma_{ii}(q_j)$

Under perfect detection, $p_j = 1$, $q_j = 0$, for all j. Therefore Player i will violate only when

$\beta_{ii} \geq 0$

but from assumption 2,

$\beta_{ii} \leq 0$

Thus a player, acting independently, will violate only when

$\beta_{ii} = 0$.

Note that this result does not contradict Result 2. Result 6 is based on the concept of dominance, which is related to independent action by the player. Result 2 is based on Pareto superiority, which is related to cooperative or joint actions by more than one player.

<u>Result 7:</u> Under no detection, a player acting independently will violate whenever there is benefit to violating.

Proof:

Similar to the proof for Result 6, player i will not violate only when

$\alpha_{ii} = 0$

Computer simulation:

Program Display:

```
Multilateral verification simulation

Alpha (undetected violation)                    Violation (1=yes, 0=no)  Payoff                      Gamma
                 violator                        1   2   3   4             1   2   3   4              1   2   3   4
                 1   2   3   4                   0   0   0   0            .2  .2  .2  .2             .2  .2  .2  .2
             1  .5  -1  -1  -1                   1   0   0   0            .2 -.1 -.1 -.1             .3  .1  .1  .1
Benefiter    2  -1  .5  -1  -1                   0   1   0   0           -.1  .2 -.1 -.1             .1  .3  .1  .1
             3  -1  -1  .5  -1                   1   1   0   0           -.1 -.1 -.4 -.4             .2  .2   0   0
             4  -1  -1  -1  .5                   0   0   1   0           -.1 -.1  .2 -.1             .1  .1  .3  .1
                                                 1   0   1   0           -.1 -.4 -.1 -.4             .2   0  .2   0
Beta (detected violation)                        0   1   1   0           -.4 -.1 -.1 -.4              0  .2  .2   0
                 violator                        1   1   1   0           -.4 -.4 -.4 -.7             .1  .1  .1 -.1
                 1   2   3   4                   0   0   0   1           -.1 -.1 -.1  .2             .1  .1  .1  .3
             1  -1   1   1   1                   1   0   0   1           -.1 -.4 -.4 -.1             .2   0   0  .2
Benefiter    2   1  -1   1   1                   0   1   0   1           -.4 -.1 -.4 -.1              0  .2   0  .2
             3   1   1  -1   1                   1   1   0   1           -.4 -.4 -.7 -.4             .1  .1 -.1  .1
             4   1   1   1  -1                   0   0   1   1           -.4 -.4 -.1 -.1              0   0  .2  .2
                                                 1   0   1   1           -.4 -.7 -.4 -.4             .1 -.1  .1  .1
Gamma (false alarm)                              0   1   1   1           -.7 -.4 -.4 -.4            -.1  .1  .1  .1
                 violator                        1   1   1   1           -.7 -.7 -.7 -.7              0   0   0   0
                 1   2   3   4
             1  -1   1   1   1  dominant:
Benefiter    2   1  -1   1   1                   0   0   0   0
             3   1   1  -1   1
             4   1   1   1  -1


Detection probabilities

                      player detected
                       1   2   3   4
  p (detection)       .4  .4  .4  .4
  q (false alarm)     .1  .1  .1  .1
```

Program Contents Listing:

```
SuperCalc ver.  1.00
Multilateral verification simulation
A1        = "Multilateral verification simulation
A3        = "Alpha (undetected violation)
H3        = "Violation (1=yes, 0=no)
M3        = "Payoff
R3        = "Gamma
D4        = "violator
H4        = 1
I4        = 2
J4        = 3
K4        = 4
M4        = 1
N4        = 2
O4        = 3
P4        = 4
R4        = 1
S4        = 2
T4        = 3
U4        = 4
C5        = 1
D5        = 2
E5        = 3
F5        = 4
H5        = 0
I5        = 0
J5        = 0
K5        = 0
M                                                                            5                    =
H5*(C6*(1-C32)+(C14*C32))+I5*(D6*(1-D32)+(D14*D32))+J5*(E6*(1-E32)+(E14*E32))+K5*(F6*(1-F32)+(F14*F32))+R5
N                                                                            5                    =
H5*(C7*(1-C32)+(C15*C32))+I5*(D7*(1-D32)+(D15*D32))+J5*(E7*(1-E32)+(E15*E32))+K5*(F7*(1-F32)+(F15*F32))+S5
O                                                                            5                    =
H5*(C8*(1-C32)+(C16*C32))+I5*(D8*(1-D32)+(D16*D32))+J5*(E8*(1-E32)+(E16*E32))+K5*(F8*(1-F32)+(F16*F32))+T5
P                                                                            5                    =
H5*(C9*(1-C32)+(C17*C32))+I5*(D9*(1-D32)+(D17*D32))+J5*(E9*(1-E32)+(E17*E32))+K5*(F9*(1-F32)+(F17*F32))+U5
R5        = (1-H5)*C22*C33+(1-I5)*D22*D33+(1-J5)*E22*E33+(1-K5)*F22*F33
S5        = (1-H5)*C23*C33+(1-I5)*D23*D33+(1-J5)*E23*E33+(1-K5)*F23*F33
T5        = (1-H5)*C24*C33+(1-I5)*D24*D33+(1-J5)*E24*E33+(1-K5)*F24*F33
U5        = (1-H5)*C25*C33+(1-I5)*D25*D33+(1-J5)*E25*E33+(1-K5)*F25*F33
B6        = 1
C6        = .5
D6        = -1
E6        = -1
F6        = -1
H6        = 1
I6        = 0
J6        = 0
K6        = 0
M                                                                            6                    =
H6*(C6*(1-C32)+(C14*C32))+I6*(D6*(1-D32)+(D14*D32))+J6*(E6*(1-E32)+(E14*E32))+K6*(F6*(1-F32)+(F14*F32))+R6
```

```
N
                                                                                  6                    -
H6*(C7*(1-C32)+(C15*C32))+I6*(D7*(1-D32)+(D15*D32))+J6*(E7*(1-E32)+(E15*E32))+K6*(F7*(1-F32)+(F15*F32))+S6
O
                                                                                  6                    -
H6*(C8*(1-C32)+(C16*C32))+I6*(D8*(1-D32)+(D16*D32))+J6*(E8*(1-E32)+(E16*E32))+K6*(F8*(1-F32)+(F16*F32))+T6
P
                                                                                  6                    -
H6*(C9*(1-C32)+(C17*C32))+I6*(D9*(1-D32)+(D17*D32))+J6*(E9*(1-E32)+(E17*E32))+K6*(F9*(1-F32)+(F17*F32))+U6
R6        = (1-H6)*C22*C33+(1-I6)*D22*D33+(1-J6)*E22*E33+(1-K6)*F22*F33
S6        = (1-H6)*C23*C33+(1-I6)*D23*D33+(1-J6)*E23*E33+(1-K6)*F23*F33
T6        = (1-H6)*C24*C33+(1-I6)*D24*D33+(1-J6)*E24*E33+(1-K6)*F24*F33
U6        = (1-H6)*C25*C33+(1-I6)*D25*D33+(1-J6)*E25*E33+(1-K6)*F25*F33
A7        = "Benefiter
B7        = 2
C7        = -1
D7        = .5
E7        = -1
F7        = -1
H7        = 0
I7        = 1
J7        = 0
K7        = 0
M
                                                                                  7                    -
H7*(C6*(1-C32)+(C14*C32))+I7*(D6*(1-D32)+(D14*D32))+J7*(E6*(1-E32)+(E14*E32))+K7*(F6*(1-F32)+(F14*F32))+R7
N
                                                                                  7                    -
H7*(C7*(1-C32)+(C15*C32))+I7*(D7*(1-D32)+(D15*D32))+J7*(E7*(1-E32)+(E15*E32))+K7*(F7*(1-F32)+(F15*F32))+S7
O
                                                                                  7                    -
H7*(C8*(1-C32)+(C16*C32))+I7*(D8*(1-D32)+(D16*D32))+J7*(E8*(1-E32)+(E16*E32))+K7*(F8*(1-F32)+(F16*F32))+T7
P
                                                                                  7                    -
H7*(C9*(1-C32)+(C17*C32))+I7*(D9*(1-D32)+(D17*D32))+J7*(E9*(1-E32)+(E17*E32))+K7*(F9*(1-F32)+(F17*F32))+U7
R7        = (1-H7)*C22*C33+(1-I7)*D22*D33+(1-J7)*E22*E33+(1-K7)*F22*F33
S7        = (1-H7)*C23*C33+(1-I7)*D23*D33+(1-J7)*E23*E33+(1-K7)*F23*F33
T7        = (1-H7)*C24*C33+(1-I7)*D24*D33+(1-J7)*E24*E33+(1-K7)*F24*F33
U7        = (1-H7)*C25*C33+(1-I7)*D25*D33+(1-J7)*E25*E33+(1-K7)*F25*F33
B8        = 3
C8        = -1
D8        = -1
E8        = .5
F8        = -1
H8        = 1
I8        = 1
J8        = 0
K8        = 0
M
                                                                                  8                    -
H8*(C6*(1-C32)+(C14*C32))+I8*(D6*(1-D32)+(D14*D32))+J8*(E6*(1-E32)+(E14*E32))+K8*(F6*(1-F32)+(F14*F32))+R8
N
                                                                                  8                    -
H8*(C7*(1-C32)+(C15*C32))+I8*(D7*(1-D32)+(D15*D32))+J8*(E7*(1-E32)+(E15*E32))+K8*(F7*(1-F32)+(F15*F32))+S8
O
                                                                                  8                    -
H8*(C8*(1-C32)+(C16*C32))+I8*(D8*(1-D32)+(D16*D32))+J8*(E8*(1-E32)+(E16*E32))+K8*(F8*(1-F32)+(F16*F32))+T8
P
                                                                                  8                    -
H8*(C9*(1-C32)+(C17*C32))+I8*(D9*(1-D32)+(D17*D32))+J8*(E9*(1-E32)+(E17*E32))+K8*(F9*(1-F32)+(F17*F32))+U8
R8        = (1-H8)*C22*C33+(1-I8)*D22*D33+(1-J8)*E22*E33+(1-K8)*F22*F33
S8        = (1-H8)*C23*C33+(1-I8)*D23*D33+(1-J8)*E23*E33+(1-K8)*F23*F33
T8        = (1-H8)*C24*C33+(1-I8)*D24*D33+(1-J8)*E24*E33+(1-K8)*F24*F33
U8        = (1-H8)*C25*C33+(1-I8)*D25*D33+(1-J8)*E25*E33+(1-K8)*F25*F33
B9        = 4
C9        = -1
D9        = -1
E9        = -1
F9        = .5
H9        = 0
I9        = 0
J9        = 1
K9        = 0
M
                                                                                  9                    -
H9*(C6*(1-C32)+(C14*C32))+I9*(D6*(1-D32)+(D14*D32))+J9*(E6*(1-E32)+(E14*E32))+K9*(F6*(1-F32)+(F14*F32))+R9
N
                                                                                  9                    -
H9*(C7*(1-C32)+(C15*C32))+I9*(D7*(1-D32)+(D15*D32))+J9*(E7*(1-E32)+(E15*E32))+K9*(F7*(1-F32)+(F15*F32))+S9
O
                                                                                  9                    -
H9*(C8*(1-C32)+(C16*C32))+I9*(D8*(1-D32)+(D16*D32))+J9*(E8*(1-E32)+(E16*E32))+K9*(F8*(1-F32)+(F16*F32))+T9
P
                                                                                  9                    -
H9*(C9*(1-C32)+(C17*C32))+I9*(D9*(1-D32)+(D17*D32))+J9*(E9*(1-E32)+(E17*E32))+K9*(F9*(1-F32)+(F17*F32))+U9
R9        = (1-H9)*C22*C33+(1-I9)*D22*D33+(1-J9)*E22*E33+(1-K9)*F22*F33
S9        = (1-H9)*C23*C33+(1-I9)*D23*D33+(1-J9)*E23*E33+(1-K9)*F23*F33
T9        = (1-H9)*C24*C33+(1-I9)*D24*D33+(1-J9)*E24*E33+(1-K9)*F24*F33
U9        = (1-H9)*C25*C33+(1-I9)*D25*D33+(1-J9)*E25*E33+(1-K9)*F25*F33
H10       = 1
I10       = 0
J10       = 1
K10       = 0
M
                                            1                                     0                      -
H10*(C6*(1-C32)+(C14*C32))+I10*(D6*(1-D32)+(D14*D32))+J10*(E6*(1-E32)+(E14*E32))+K10*(F6*(1-F32)+(F14*F32))+R10
N
                                            1                                     0                      -
H10*(C7*(1-C32)+(C15*C32))+I10*(D7*(1-D32)+(D15*D32))+J10*(E7*(1-E32)+(E15*E32))+K10*(F7*(1-F32)+(F15*F32))+S10
O
                                            1                                     0                      -
H10*(C8*(1-C32)+(C16*C32))+I10*(D8*(1-D32)+(D16*D32))+J10*(E8*(1-E32)+(E16*E32))+K10*(F8*(1-F32)+(F16*F32))+T10
P
                                            1                                     0                      -
H10*(C9*(1-C32)+(C17*C32))+I10*(D9*(1-D32)+(D17*D32))+J10*(E9*(1-E32)+(E17*E32))+K10*(F9*(1-F32)+(F17*F32))+U10
R10       = (1-H10)*C22*C33+(1-I10)*D22*D33+(1-J10)*E22*E33+(1-K10)*F22*F33
S10       = (1-H10)*C23*C33+(1-I10)*D23*D33+(1-J10)*E23*E33+(1-K10)*F23*F33
T10       = (1-H10)*C24*C33+(1-I10)*D24*D33+(1-J10)*E24*E33+(1-K10)*F24*F33
U10       = (1-H10)*C25*C33+(1-I10)*D25*D33+(1-J10)*E25*E33+(1-K10)*F25*F33
A11       = "Beta (detected violation)
H11       = 0
I11       = 1
J11       = 1
K11       = 0
M
                                            1                                     1                      -
H11*(C6*(1-C32)+(C14*C32))+I11*(D6*(1-D32)+(D14*D32))+J11*(E6*(1-E32)+(E14*E32))+K11*(F6*(1-F32)+(F14*F32))+R11
```

```
N                                                  1                                                1           =
H11*(C7*(1-C32)+(C15*C32))+I11*(D7*(1-D32)+(D15*D32))+J11*(E7*(1-E32)+(E15*E32))+K11*(F7*(1-F32)+(F15*F32))+S11
O                                                  1                                                1           =
H11*(C8*(1-C32)+(C16*C32))+I11*(D8*(1-D32)+(D16*D32))+J11*(E8*(1-E32)+(E16*E32))+K11*(F8*(1-F32)+(F16*F32))+T11
P                                                  1                                                1           =
H11*(C9*(1-C32)+(C17*C32))+I11*(D9*(1-D32)+(D17*D32))+J11*(E9*(1-E32)+(E17*E32))+K11*(F9*(1-F32)+(F17*F32))+U11
R11        = (1-H11)*C22*C33+(1-I11)*D22*D33+(1-J11)*E22*E33+(1-K11)*F22*F33
S11        = (1-H11)*C23*C33+(1-I11)*D23*D33+(1-J11)*E23*E33+(1-K11)*F23*F33
T11        = (1-H11)*C24*C33+(1-I11)*D24*D33+(1-J11)*E24*E33+(1-K11)*F24*F33
U11        = (1-H11)*C25*C33+(1-I11)*D25*D33+(1-J11)*E25*E33+(1-K11)*F25*F33
D12        = "violator
H12        = 1
I12        = 1
J12        = 1
K12        = 0
M                                                  1                                                2           =
H12*(C6*(1-C32)+(C14*C32))+I12*(D6*(1-D32)+(D14*D32))+J12*(E6*(1-E32)+(E14*E32))+K12*(F6*(1-F32)+(F14*F32))+R12
N                                                  1                                                2           =
H12*(C7*(1-C32)+(C15*C32))+I12*(D7*(1-D32)+(D15*D32))+J12*(E7*(1-E32)+(E15*E32))+K12*(F7*(1-F32)+(F15*F32))+S12
O                                                  1                                                2           =
H12*(C8*(1-C32)+(C16*C32))+I12*(D8*(1-D32)+(D16*D32))+J12*(E8*(1-E32)+(E16*E32))+K12*(F8*(1-F32)+(F16*F32))+T12
P                                                  1                                                2           =
H12*(C9*(1-C32)+(C17*C32))+I12*(D9*(1-D32)+(D17*D32))+J12*(E9*(1-E32)+(E17*E32))+K12*(F9*(1-F32)+(F17*F32))+U12
R12        = (1-H12)*C22*C33+(1-I12)*D22*D33+(1-J12)*E22*E33+(1-K12)*F22*F33
S12        = (1-H12)*C23*C33+(1-I12)*D23*D33+(1-J12)*E23*E33+(1-K12)*F23*F33
T12        = (1-H12)*C24*C33+(1-I12)*D24*D33+(1-J12)*E24*E33+(1-K12)*F24*F33
U12        = (1-H12)*C25*C33+(1-I12)*D25*D33+(1-J12)*E25*E33+(1-K12)*F25*F33
C13        = 1
D13        = 2
E13        = 3
F13        = 4
H13        = 0
I13        = 0
J13        = 0
K13        = 1
M                                                  1                                                3           =
H13*(C6*(1-C32)+(C14*C32))+I13*(D6*(1-D32)+(D14*D32))+J13*(E6*(1-E32)+(E14*E32))+K13*(F6*(1-F32)+(F14*F32))+R13
N                                                  1                                                3           =
H13*(C7*(1-C32)+(C15*C32))+I13*(D7*(1-D32)+(D15*D32))+J13*(E7*(1-E32)+(E15*E32))+K13*(F7*(1-F32)+(F15*F32))+S13
O                                                  1                                                3           =
H13*(C8*(1-C32)+(C16*C32))+I13*(D8*(1-D32)+(D16*D32))+J13*(E8*(1-E32)+(E16*E32))+K13*(F8*(1-F32)+(F16*F32))+T13
P                                                  1                                                3           =
H13*(C9*(1-C32)+(C17*C32))+I13*(D9*(1-D32)+(D17*D32))+J13*(E9*(1-E32)+(E17*E32))+K13*(F9*(1-F32)+(F17*F32))+U13
R13        = (1-H13)*C22*C33+(1-I13)*D22*D33+(1-J13)*E22*E33+(1-K13)*F22*F33
S13        = (1-H13)*C23*C33+(1-I13)*D23*D33+(1-J13)*E23*E33+(1-K13)*F23*F33
T13        = (1-H13)*C24*C33+(1-I13)*D24*D33+(1-J13)*E24*E33+(1-K13)*F24*F33
U13        = (1-H13)*C25*C33+(1-I13)*D25*D33+(1-J13)*E25*E33+(1-K13)*F25*F33
B14        = 1
C14        = -1
D14        = 1
E14        = 1
F14        = 1
H14        = 1
I14        = 0
J14        = 0
K14        = 1
M                                                  1                                                4           =
H14*(C6*(1-C32)+(C14*C32))+I14*(D6*(1-D32)+(D14*D32))+J14*(E6*(1-E32)+(E14*E32))+K14*(F6*(1-F32)+(F14*F32))+R14
N                                                  1                                                4           =
H14*(C7*(1-C32)+(C15*C32))+I14*(D7*(1-D32)+(D15*D32))+J14*(E7*(1-E32)+(E15*E32))+K14*(F7*(1-F32)+(F15*F32))+S14
O                                                  1                                                4           =
H14*(C8*(1-C32)+(C16*C32))+I14*(D8*(1-D32)+(D16*D32))+J14*(E8*(1-E32)+(E16*E32))+K14*(F8*(1-F32)+(F16*F32))+T14
P                                                  1                                                4           =
H14*(C9*(1-C32)+(C17*C32))+I14*(D9*(1-D32)+(D17*D32))+J14*(E9*(1-E32)+(E17*E32))+K14*(F9*(1-F32)+(F17*F32))+U14
R14        = (1-H14)*C22*C33+(1-I14)*D22*D33+(1-J14)*E22*E33+(1-K14)*F22*F33
S14        = (1-H14)*C23*C33+(1-I14)*D23*D33+(1-J14)*E23*E33+(1-K14)*F23*F33
T14        = (1-H14)*C24*C33+(1-I14)*D24*D33+(1-J14)*E24*E33+(1-K14)*F24*F33
U14        = (1-H14)*C25*C33+(1-I14)*D25*D33+(1-J14)*E25*E33+(1-K14)*F25*F33
A15        = "Benefiter
B15        = 2
C15        = 1
D15        = -1
E15        = 1
F15        = 1
H15        = 0
I15        = 1
J15        = 0
K15        = 1
M                                                  1                                                5           =
H15*(C6*(1-C32)+(C14*C32))+I15*(D6*(1-D32)+(D14*D32))+J15*(E6*(1-E32)+(E14*E32))+K15*(F6*(1-F32)+(F14*F32))+R15
N                                                  1                                                5           =
H15*(C7*(1-C32)+(C15*C32))+I15*(D7*(1-D32)+(D15*D32))+J15*(E7*(1-E32)+(E15*E32))+K15*(F7*(1-F32)+(F15*F32))+S15
O                                                  1                                                5           =
H15*(C8*(1-C32)+(C16*C32))+I15*(D8*(1-D32)+(D16*D32))+J15*(E8*(1-E32)+(E16*E32))+K15*(F8*(1-F32)+(F16*F32))+T15
P                                                  1                                                5           =
H15*(C9*(1-C32)+(C17*C32))+I15*(D9*(1-D32)+(D17*D32))+J15*(E9*(1-E32)+(E17*E32))+K15*(F9*(1-F32)+(F17*F32))+U15
R15        = (1-H15)*C22*C33+(1-I15)*D22*D33+(1-J15)*E22*E33+(1-K15)*F22*F33
S15        = (1-H15)*C23*C33+(1-I15)*D23*D33+(1-J15)*E23*E33+(1-K15)*F23*F33
T15        = (1-H15)*C24*C33+(1-I15)*D24*D33+(1-J15)*E24*E33+(1-K15)*F24*F33
U15        = (1-H15)*C25*C33+(1-I15)*D25*D33+(1-J15)*E25*E33+(1-K15)*F25*F33
B16        = 3
C16        = 1
D16        = 1
E16        = -1
F16        = 1
H16        = 1
I16        = 1
J16        = 0
```

```
K16       = 1
M                                              1                                              6          =
H16*(C6*(1-C32)+(C14*C32))+I16*(D6*(1-D32)+(D14*D32))+J16*(E6*(1-E32)+(E14*E32))+K16*(F6*(1-F32)+(F14*F32))+R16
N                                              1                                              6          =
H16*(C7*(1-C32)+(C15*C32))+I16*(D7*(1-D32)+(D15*D32))+J16*(E7*(1-E32)+(E15*E32))+K16*(F7*(1-F32)+(F15*F32))+S16
O                                              1                                              6          =
H16*(C8*(1-C32)+(C16*C32))+I16*(D8*(1-D32)+(D16*D32))+J16*(E8*(1-E32)+(E16*E32))+K16*(F8*(1-F32)+(F16*F32))+T16
P                                              1                                              6          =
H16*(C9*(1-C32)+(C17*C32))+I16*(D9*(1-D32)+(D17*D32))+J16*(E9*(1-E32)+(E17*E32))+K16*(F9*(1-F32)+(F17*F32))+U16
R16       = (1-H16)*C22*C33+(1-I16)*D22*D33+(1-J16)*E22*E33+(1-K16)*F22*F33
S16       = (1-H16)*C23*C33+(1-I16)*D23*D33+(1-J16)*E23*E33+(1-K16)*F23*F33
T16       = (1-H16)*C24*C33+(1-I16)*D24*D33+(1-J16)*E24*E33+(1-K16)*F24*F33
U16       = (1-H16)*C25*C33+(1-I16)*D25*D33+(1-J16)*E25*E33+(1-K16)*F25*F33
B17       = 4
C17       = 1
D17       = 1
E17       = 1
F17       = -1
H17       = 0
I17       = 0
J17       = 1
K17       = 1
M                                              1                                              7          =
H17*(C6*(1-C32)+(C14*C32))+I17*(D6*(1-D32)+(D14*D32))+J17*(E6*(1-E32)+(E14*E32))+K17*(F6*(1-F32)+(F14*F32))+R17
N                                              1                                              7          =
H17*(C7*(1-C32)+(C15*C32))+I17*(D7*(1-D32)+(D15*D32))+J17*(E7*(1-E32)+(E15*E32))+K17*(F7*(1-F32)+(F15*F32))+S17
O                                              1                                              7          =
H17*(C8*(1-C32)+(C16*C32))+I17*(D8*(1-D32)+(D16*D32))+J17*(E8*(1-E32)+(E16*E32))+K17*(F8*(1-F32)+(F16*F32))+T17
P                                              1                                              7          =
H17*(C9*(1-C32)+(C17*C32))+I17*(D9*(1-D32)+(D17*D32))+J17*(E9*(1-E32)+(E17*E32))+K17*(F9*(1-F32)+(F17*F32))+U17
R17       = (1-H17)*C22*C33+(1-I17)*D22*D33+(1-J17)*E22*E33+(1-K17)*F22*F33
S17       = (1-H17)*C23*C33+(1-I17)*D23*D33+(1-J17)*E23*E33+(1-K17)*F23*F33
T17       = (1-H17)*C24*C33+(1-I17)*D24*D33+(1-J17)*E24*E33+(1-K17)*F24*F33
U17       = (1-H17)*C25*C33+(1-I17)*D25*D33+(1-J17)*E25*E33+(1-K17)*F25*F33
H18       = 1
I18       = 0
J18       = 1
K18       = 1
M                                              1                                              8          =
H18*(C6*(1-C32)+(C14*C32))+I18*(D6*(1-D32)+(D14*D32))+J18*(E6*(1-E32)+(E14*E32))+K18*(F6*(1-F32)+(F14*F32))+R18
N                                              1                                              8          =
H18*(C7*(1-C32)+(C15*C32))+I18*(D7*(1-D32)+(D15*D32))+J18*(E7*(1-E32)+(E15*E32))+K18*(F7*(1-F32)+(F15*F32))+S18
O                                              1                                              8          =
H18*(C8*(1-C32)+(C16*C32))+I18*(D8*(1-D32)+(D16*D32))+J18*(E8*(1-E32)+(E16*E32))+K18*(F8*(1-F32)+(F16*F32))+T18
P                                              1                                              8          =
H18*(C9*(1-C32)+(C17*C32))+I18*(D9*(1-D32)+(D17*D32))+J18*(E9*(1-E32)+(E17*E32))+K18*(F9*(1-F32)+(F17*F32))+U18
R18       = (1-H18)*C22*C33+(1-I18)*D22*D33+(1-J18)*E22*E33+(1-K18)*F22*F33
S18       = (1-H18)*C23*C33+(1-I18)*D23*D33+(1-J18)*E23*E33+(1-K18)*F23*F33
T18       = (1-H18)*C24*C33+(1-I18)*D24*D33+(1-J18)*E24*E33+(1-K18)*F24*F33
U18       = (1-H18)*C25*C33+(1-I18)*D25*D33+(1-J18)*E25*E33+(1-K18)*F25*F33
A19       = "Gamma (false alarm)
H19       = 0
I19       = 1
J19       = 1
K19       = 1
M                                              1                                              9          =
H19*(C6*(1-C32)+(C14*C32))+I19*(D6*(1-D32)+(D14*D32))+J19*(E6*(1-E32)+(E14*E32))+K19*(F6*(1-F32)+(F14*F32))+R19
N                                              1                                              9          =
H19*(C7*(1-C32)+(C15*C32))+I19*(D7*(1-D32)+(D15*D32))+J19*(E7*(1-E32)+(E15*E32))+K19*(F7*(1-F32)+(F15*F32))+S19
O                                              1                                              9          =
H19*(C8*(1-C32)+(C16*C32))+I19*(D8*(1-D32)+(D16*D32))+J19*(E8*(1-E32)+(E16*E32))+K19*(F8*(1-F32)+(F16*F32))+T19
P                                              1                                              9          =
H19*(C9*(1-C32)+(C17*C32))+I19*(D9*(1-D32)+(D17*D32))+J19*(E9*(1-E32)+(E17*E32))+K19*(F9*(1-F32)+(F17*F32))+U19
R19       = (1-H19)*C22*C33+(1-I19)*D22*D33+(1-J19)*E22*E33+(1-K19)*F22*F33
S19       = (1-H19)*C23*C33+(1-I19)*D23*D33+(1-J19)*E23*E33+(1-K19)*F23*F33
T19       = (1-H19)*C24*C33+(1-I19)*D24*D33+(1-J19)*E24*E33+(1-K19)*F24*F33
U19       = (1-H19)*C25*C33+(1-I19)*D25*D33+(1-J19)*E25*E33+(1-K19)*F25*F33
D20       = "violator
H20       = 1
I20       = 1
J20       = 1
K20       = 1
M                                              2                                              0          =
H20*(C6*(1-C32)+(C14*C32))+I20*(D6*(1-D32)+(D14*D32))+J20*(E6*(1-E32)+(E14*E32))+K20*(F6*(1-F32)+(F14*F32))+R20
N                                              2                                              0          =
H20*(C7*(1-C32)+(C15*C32))+I20*(D7*(1-D32)+(D15*D32))+J20*(E7*(1-E32)+(E15*E32))+K20*(F7*(1-F32)+(F15*F32))+S20
O                                              2                                              0          =
H20*(C8*(1-C32)+(C16*C32))+I20*(D8*(1-D32)+(D16*D32))+J20*(E8*(1-E32)+(E16*E32))+K20*(F8*(1-F32)+(F16*F32))+T20
P                                              2                                              0          =
H20*(C9*(1-C32)+(C17*C32))+I20*(D9*(1-D32)+(D17*D32))+J20*(E9*(1-E32)+(E17*E32))+K20*(F9*(1-F32)+(F17*F32))+U20
R20       = (1-H20)*C22*C33+(1-I20)*D22*D33+(1-J20)*E22*E33+(1-K20)*F22*F33
S20       = (1-H20)*C23*C33+(1-I20)*D23*D33+(1-J20)*E23*E33+(1-K20)*F23*F33
T20       = (1-H20)*C24*C33+(1-I20)*D24*D33+(1-J20)*E24*E33+(1-K20)*F24*F33
U20       = (1-H20)*C25*C33+(1-I20)*D25*D33+(1-J20)*E25*E33+(1-K20)*F25*F33
C21       = 1
D21       = 2
E21       = 3
F21       = 4
B22       = 1
C22       = -1
D22       = 1
E22       = 1
F22       = 1
G22       = "  dominant:
A23       = "Benefiter
B23       = 2
C23       = 1
D23       = -1
```

```
E23     = 1
F23     = 1
H23     = IF(C6*(1-C32)+C14*C32>C22*C33,1,0)
I23     = IF(D7*(1-D32)+D15*D32>D23*D33,1,0)
J23     = IF(E8*(1-E32)+E16*E32>E24*E33,1,0)
K23     = IF(F9*(1-F32)+F17*F32>F25*F33,1,0)
B24     = 3
C24     = 1
D24     = 1
E24     = -1
F24     = 1
B25     = 4
C25     = 1
D25     = 1
E25     = 1
F25     = -1
A28     = "Detection probabilities
D30     = "player detected
C31     = 1
D31     = 2
E31     = 3
F31     = 4
A32     = "  p (detection)
C32     = .4
D32     = .4
E32     = .4
F32     = .4
A33     = "  q (false alarm)
C33     = .1
D33     = .1
E33     = .1
F33     = .1
```

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