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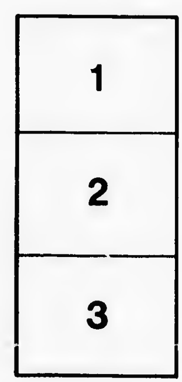
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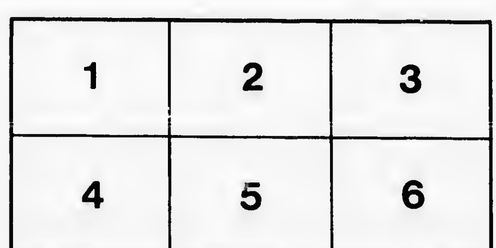
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THE HIGH SCHOOL DRAWING · COURSE

BY

Arthur J. Reading.

LINEAR PERSPECTIVE

Name, ...

Address, ...

Toronto: Grip Printing and Publishing Company.

INTRODUCTORY REMARKS.

All work in Linear Perspective requires to be done mechanically, except in the case of curves which cannot be drawn by means of compasses.

The following are the necessary instruments :—

Pencils—either H or HH, sharpened to a wedge-shaped point, the flat side of which should rest against the ruler in drawing lines. A piece of fine sand paper is about the best thing for keeping the point of the pencil sharp, and saves the blade of the pocket knife.

Ruler—made of hard wood, at least six inches long, with a straight edge, and divided into inches, and halves, quarters, eighths, and sixteenths of an inch.

Compasses—with steel, pencil, and pen points which fit into a socket in one of the legs. The stationary leg should have a needle point if possible, so that its length may be altered to correspond to whichever one of the movable points is in use. The stationary leg should be a trifle longer than the other leg when the pencil or pen point is in use, and exactly the same length when the steel point is in use. The pencil used in the pencil point should be a little softer than that used with the ruler, as F or H, and should be sharpened in the same way. In drawing circles its edge should always be perpendicular to the radius. Properly constructed compasses have a hinge joint in each leg, so that when the pencil or pen point is in use, it can be kept perpendicular to the surface of the paper. If this is not attended to in the case of the pen point, the pen will not work properly. The joint of the compasses can be tightened or loosened by means of a little metal key which accompanies them. The joint should not be so loose that the legs will change their relative position when the compasses are being used, nor should it be so tight as to require any exertion to separate the legs. Practice will teach just how tight it should be. The compasses should be held loosely by the joint only, between the thumb and first finger, with the steel or needle point resting on the paper, without any pressure, and the other leg made to revolve around it. The student should practise until he can draw several concentric circles without puncturing the paper with the steel point. It is absolutely necessary that the steel point should be as sharp as it is possible to make it. India ink only should be used in the pens, as other inks corrode and spoil the points. The two steel points are used together when it is necessary to measure or to set off distances very accurately.

A Drawing Pen for "inking in" straight lines. Its points should be exactly the same length and ground to a sharp rounded edge. In use it should be held nearly vertical, with the handle slightly inclined in the direction of the edge of the ruler, and drawn along the paper at a uniform rate of speed without any stoppages. It should be wiped out with a rag or piece of chamois skin every time it is filled, and before being put away.

Protractor, made of either metal, horn, ivory or wood ; used for measuring angles. It is not absolutely necessary, but most boxes of mathematical instruments contain a protractor. Its form and instructions for constructing one are given in an exercise on problem xiii. in book 2, High School Drawing Course. In using it the centre of the semicircle is placed over the point where the angle is to be constructed, with the diameter coinciding with one line of the angle, and a pencil mark made at the circumference opposite the proper number. A line is then drawn through this point from the centre.

A Set Square, being a triangle of thin wood, will be found useful, though not necessary, for drawing parallel lines and erecting perpendiculars. The ruler is held in position and the set square slid along, with one edge firmly pressed against it. A square about five inches high, having angles of 30°, 60° and 90° will be most convenient.

The importance of being able to change the proportion existing between the object and the drawing of it, will be evident when we consider the limited space our paper offers for a picture of a house, a tree, a street, or even of a room. The method adopted for reducing the size of a drawing is called *working to a scale*, and may be briefly stated as follows: The unit of measurement of the object being taken, it is divided into a convenient number of equal parts, and one of the divisions is used as the unit of measurement in the drawing. If an object is 12 feet long the unit of measurement is one foot, which is divided into any number of parts, say 12. Then one-twelfth of a foot, or one inch becomes the unit of measurement in the drawing, which will be one-twelfth of the natural size of the object, and therefore one foot long. This scale may be expreseed either by the words "scale, 1" to the foot," or by the fraction "$\frac{1}{12}$." In a similar way, if one foot is divided into 16 equal parts and $\frac{1}{16}$ be used as the unit of measurement in the drawing, the scale will be one of "$\frac{3}{4}$" to the foot," or "$\frac{1}{16}$." It must be remembered that if the scale is expressed by a fraction, it indicates the proportion which every portion of the drawing will bear to the corresponding portion of the object drawn.

The sign ′ attached to a figure signifies *foot* or *feet*, and the sign ″, *inch* or *inches :* thus, *1′ 6″* reads 1 foot 6 inches, and *2′ 1″* reads, 2 feet 1 inch, and *Scale 4′ to 1″* reads, Scale 4 feet to 1 inch.

HIGH SCHOOL DRAWING COURSE.

LINEAR PERSPECTIVE.

The term **perspective**, applied to a drawing of an object, indicates that it is a representation of the *apparent* form of that object when viewed from one point.

It has no doubt been noticed, even by the most careless observer, that, except under certain circumstances, objects never appear *as they are*, and that their appearance changes with every change of the spectator's position with regard to them. This difference between appearance and reality is caused partly by the convergence of the rays of light, reflected or transmitted by the objects to the eye,* and partly by the manner in which these rays cut an imaginary transparent plane interposed between the objects and the spectator.

The eye, being opened, admits a flood of light from space, part of which may proceed from objects lying within the range of vision. This light passes through the circular opening in the iris, called the pupil, and the crystalline lens, and excites the optic nerve spread over the inside of the back of the eye, thus producing the sensation which we call vision. The rays composing this volume of light are convergent and meet in the focal point of the crystalline lens, forming a cone, the base of which may be supposed to be at any distance from the eye.†

Only one portion of an object can be seen distinctly at one time. In order to obtain a complete and correct idea of the whole, the gaze is directed at different parts of it until it has all been examined. When the eye is fixed upon one point everything about that point in all directions is seen more and more indistinctly as its distance is increased, so that the angle limiting the field of distinct vision is necessarily comparatively small. In perspective it is fixed, for the sake of convenience, at 60°, and everything lying outside is supposed to be invisible; therefore, in order to make a picture of the whole circle of landscape, the spectator would have to change his position six times, thus dividing his horizon into six different parts, each one of which would be contained by an angle of 60°. * This is called the **visual angle** or **angle of vision.**

The word *perspective* is derived from two Latin words, signifying "to look through," and naturally suggests the thought that there is a "something" through which the spectator is looking. This "something" is the **Picture Plane** (P.P.), or plane of delineation, and is an invisible vertical plane, supposed to be interposed at a given distance between the spectator and the object to be drawn. It is represented by the surface upon which the drawing is made.

A good idea of the picture plane and its use might be obtained by placing upright in front of the eye a pane of glass, and tracing upon it the outline of objects seen through it, taking care that the eye is kept in one position. The drawing thus made would be a true perspective drawing, and could easily be transferred to a sheet of paper.

The position of the eye is called the **Station Point** (S.P.).

The point towards which the eye is directed, being in the centre of the field of vision, is called the **Centre of Vision** (C.V.). When looking straight ahead the eye of the spectator is naturally fixed upon the horizon; the Centre of Vision is therefore in the horizon. If a circle were drawn with a proper radius upon the pane of glass already referred to as representing the picture plane, its circumference would be a picture of the limit of the field of vision, the circle would be the picture plane, its centre would be a picture of the centre of vision, and its horizontal diameter would be a picture of the horizon. This last line representing the horizon is called the **Horizontal Line** (H.L.).

A line drawn from the Station Point to the Centre of Vision represents, not only the distance of the spectator from the picture

* In perspective the spectator is supposed to be looking with only one eye.

† This principle of convergent rays is nicely illustrated by the light issuing from a magic lantern. The diameter of the circle of illumination on the screen (the base of the cone) depends upon the distance of the screen from the lantern. In this case the rays of light diverge, while in the case of the human eye they converge. This shows, too, that the opening through which the light passes, governs the shape of the field of illumination on the screen, in the one case, and of the field of vision in the other.

* This is illustrated in Fig. 5.

plane but also the direction in which he is looking, and is called the **Line of Direction** (L.D.).

Fig. 1 shows the relative position and size of the picture plane with regard to the spectator. It will be seen that the picture plane is the base of the cone of light entering the eye; the apex

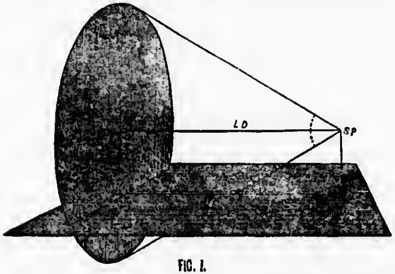


FIG. 1.

of the cone is the station point (S.P.); its axis is the line of direction (L.D.); the centre of the base is the centre of vision (C.V.); and the horizontal diameter of the base is the horizontal line (H.L.). Fig. 1 also shows the fact that the picture plane may be at such a distance from the eye of the spectator as to be wholly visible, but usually it is supposed to be at such a distance

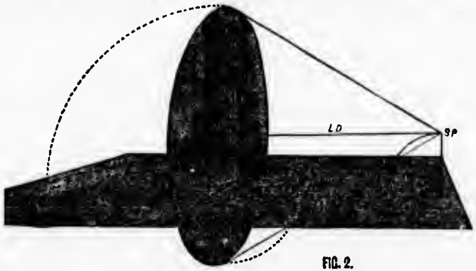


FIG. 2.

that its radius is greater than the height of the eye above the ground and hence a portion of it is hidden by the ground lying between it and the spectator. The line where it cuts the ground is called the **Ground Line** (G.L.). The portion of the picture plane below the ground line may be visible when the spectator is standing on an elevation, or looking down into an excavation. In such a case the ground line is supposed to be in the same place as it would be in if the ground were perfectly level, *i.e.*, it is the line of intersection of the picture plane with a horizontal plane upon which the spectator is standing.

It is supposed that the line of direction is parallel with the ground plane or horizontal plane,* hence the distance from the horizontal line to the ground line is always equal to the height of the eye of the spectator above the ground.

The lines and points thus far explained are in two different planes—the horizontal line, centre of vision and ground line in a

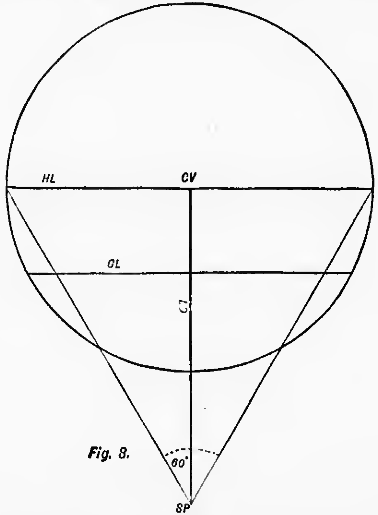


Fig. 8.

vertical plane; and the horizontal line, centre of vision, line of direction and station point, in a horizontal plane—and in order to make use of them in working problems in perspective, they

*This is not actually the case, for as the horizon is the line where earth and sky appear to meet, a line from the eye to the horizon will fall the distance of the eye from the ground in traversing the distance between the eye and the horizon, which, in the case of a person five feet high, is about three miles. Therefore the angle formed by the line of direction produced and the ground plane, is the vertical angle of a triangle having two of its sides three miles long and its base five feet long.

must be supposed to be all brought into one plane without altering the relative positions of the centre of vision, horizontal line and ground line, and of the centre of vision, line of direction and station point. The manner of doing this is shown in fig. 2. The picture plane is supposed to be rotated upon the ground line, and the spectator to be rotated in the same direction upon the point where he stands until the horizontal line, centre of vision, line of direction and station point occupy positions in the horizontal or ground plane. The lines and points mentioned will then appear as shown in fig. 3.

In all probability every one who reads this has, at some time, stood between the rails of a railway and noticed how they appear to approach one another in the distance, and finally meet in the horizon. The same apparent convergence of lines can be seen in any room, or in the street where the sidewalks, the lines of windows and doors, the tops of houses, etc., all appear to approach one another. The different points where these lines would, if produced, ultimately meet are called **Vanishing Points** (V.P.),

Fig. 4.

and experiment will satisfy any one that all parallel retiring lines appear to meet in the same point, and that all parallel horizontal retiring lines appear to meet in the horizon.

In the illustration of the railway track (fig. 4) where the spectator is supposed to be standing on one of the rails, the rails appear to meet in one point in the horizon, and this point is the point towards which the gaze is directed (C.V.). The rails in this case are parallel to the line of direction, and their vanishing point is where they disappear on account of the rotundity of the earth. This can be proved by standing upon each of the rails in succession, where there are several parallel to each other.

If the spectator turns either to the right or left until he looks in a direction at an angle of 45° with the tracks, their vanishing

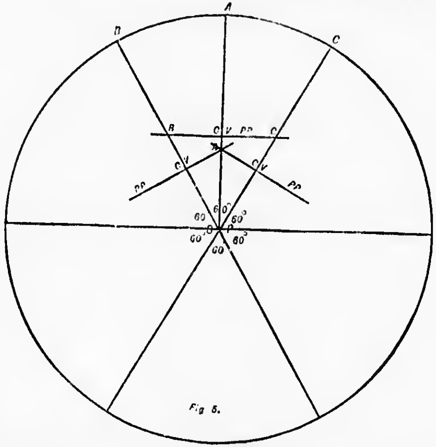


Fig 5.

point will not be changed, but will occupy a new position with regard to the spectator and his line of direction; that is, what was his centre of vision and the vanishing point of the rails, will still be their vanishing point when they form an angle of 45° or any other angle with the line of direction.

From this it is evident that if any horizontal line be followed until it cuts the horizon, it will find there its own vanishing point.

Suppose that the circumference of the circle in fig. 5 represents the complete horizon visible to a spectator stationed at *S P*. When looking towards *A* his centre of vision will be the point *A*, but when looking towards *B* his centre of vision will be the point *B*. The direction of his line of direction, and consequently of his picture plane, which is always perpendicular to the line of direction, changes with every change of his position, and what was his centre of vision when looking towards A becomes, when he looks towards B, the vanishing point for all lines running in the direction $S P A$. In the same way what was his centre of vision when looking towards C, becomes, when he looks towards A, the vanishing point for all lines running in the direction $S P C$. These vanishing points are in the horizon, but when the picture plane is interposed between them and the spectator they are represented by the points A, B and C on the respective picture planes (P. P.).

From what has been said the following rules may be deduced:

I. All retiring lines appear to converge.

II. All parallel retiring lines appear to converge in the same point.

III. All parallel horizontal retiring lines appear to converge in the horizon, represented by $H L$.

IV. All lines perpendicular to the picture plane appear to converge in the centre of vision.

V. The vanishing point of any retiring horizontal line is found by drawing in the proper direction from the station point, a line to cut the horizon, represented by $H L$.

If one edge or face of an object, such as a book or a pencil, be placed against a pane of glass, and its outline traced upon the glass, the drawing of the edge or face will be of the same size and shape as in the object itself; hence it may be inferred that measurements must be taken upon the picture plane (represented by the pane of glass), and that all the points in an object which are in the same vertical plane, will, when occupying positions in the picture plane, be represented by points as far apart as they are in the object. In order to bring any particular point of any object into the picture plane it must be supposed to be moved forward in any direction until it touches the picture plane. The point where it touches the picture plane is called a **Point of Contact** (P.C.). If it be required to find the point of contact of a point situated above the ground plane and away from the picture plane, the point is supposed first to be dropped vertically to the ground plane and then moved towards the picture plane.

For the proper working of a problem in perspective it is necessary that we be able to define with great exactness the size, the shape and the position of the object or objects to be drawn. The position of an object is usually determined by means of some one of its principal points, which is compared, as regards position, with the picture plane, the ground plane, and the line of direction. Thus any point may be required which is 2′ to the right of the line of direction, 3′ back from the picture plane and 10′ above the ground plane; or, a solid object, such as a cube, may be required whose edges are 2′ long, two of whose faces are parallel to the picture plane, and having the near left hand corner of its base touching the picture plane 5′ to the left and 1′ above the ground plane. Broadly stated, the position of objects may be—on, above or below the ground plane; touching, or lying away from the

picture plane, and either directly in front of the spectator, or to the right or left.

Besides knowing the size, shape and position of the objects to be drawn we must also know the height of the eye of the spectator above the ground, his distance from the picture plane, and the *scale* on which the drawing is to be made, or, in other words, the proportion which the drawing will bear to the object.

Referring again to the illustration of the railway track (fig. 4) it will be seen that the ties appear to approach one another, as well as to decrease in size, as their distance from the eye is increased. In the case of a rapidly departing train the decrease in size is plainly seen, and gives to the mind the idea that some mysterious contracting force is acting upon the sides of the rear carriage, causing them to become shorter and closer together until, at a distance of about three or three and a half miles, the whole is reduced to a point on the horizon.

This leads up to the next point which it is necessary to consider, viz.: how vanishing lines can be measured to any required length, or, in other words, how the position of any point lying away from the picture plane can be represented.

The mathematical fact or principle by means of which this is accomplished is, *that a line drawn perpendicular to a line bisecting an angle will intersect both lines of the angle in points equidistant from their point of contact.* In fig. 6 $A B$ is limited, $B C$ is unlimited in length. The line $A E$, perpendicular to the line bisecting the angle $A B C$, makes $B D$ equal to $B A$. In the same way the line $F M$, perpendicular to the bisecting line, $H L$, of the angle $F H K$, makes $H N$ equal to $H F$ and $P T$ makes $H R$ equal to $H P$. Applying this principle to what has been learned concerning the drawing of lines in perspective, let $a b$ in $G L$ (fig. 6) correspond to and represent $A B$. The indefinite line $B C$ is perpendicular to $A B$ and therefore a line to represent it in perspective, drawn from b, must vanish in $C V$ (Rule iv.) These two lines are now represented perspectively, and in order to cut off from $b c$ a part which will represent $B D$ it is necessary to draw from a a line which will be the perspective representation of $A E$. By means of Rule v. the vanishing point for $A E$ is found by drawing from $S P$ a line parallel to it, to cut $H L$ in $R M P$ (**Right Measuring Point**). The line drawn from a to $R M P$ represents $A E$ in perspective, and, cutting $b c$ in d, makes $b d$ the perspective representation of $B D$; that is, $b d$ is the foreshortened or perspective length of $a b$ when perpendicular to the picture plane.

In the case of the line $H K$, which is not perpendicular to the picture plane, its vanishing point and measuring point are found by applying Rule v., that is, by drawing $S P\ V P_1$, parallel to $H K$, and $S P\ M P_1$, parallel to $F M$. After finding these points the method of proceeding is just the same as in the other case. The original retiring line is $h\ V P_1$, and it is measured by means

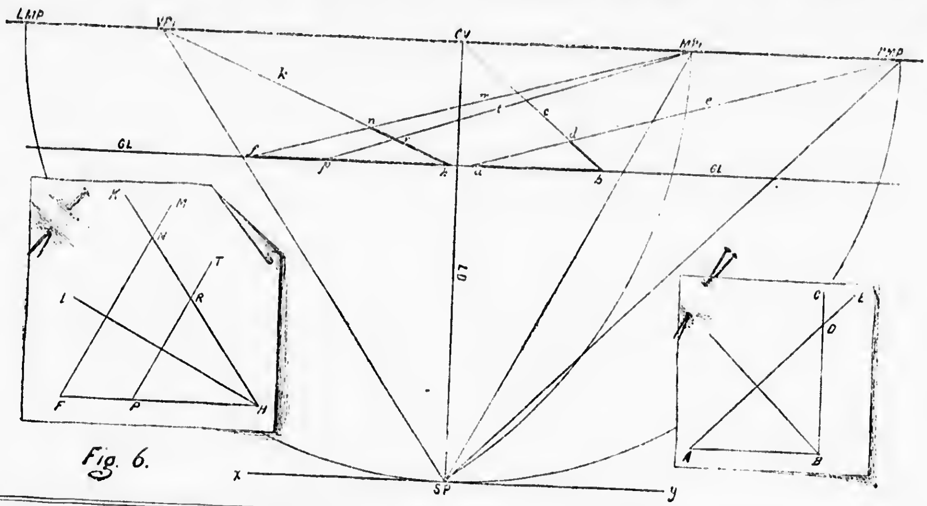


Fig. 6.

of the line $f\,M\,P_1$, making $h\,n$ the perspective length of $h\,f$ which is equal to $H\,F$.

From this illustration it may be seen that every vanishing point has its corresponding measuring point, and that the measuring point for any vanishing point can be found by drawing an arc with the vanishing point as a centre and a radius equal to its distance from the station point. The point where this arc cuts the horizontal line is the measuring point required. If the arc, drawn with $C\,V$ as a centre and $C\,V\,S\,P$ as a radius, be continued to the left, it will find on the horizontal line a point marked $L\,M\,P$ (left measuring point) which can be used as well as $R\,M\,P$* for measuring lines vanishing in $C\,V$. These two points $L\,M\,P$ and $R\,M\,P$ are as far to the right and left of $C\,V$ as the distance of $S\,P$ from the picture plane and are therefore often called

* Every vanishing point has, in reality, two measuring points at an equal distance on each side of it, but, except in the case of the centre of vision, one of them is usually so far to the right or left that it cannot conveniently be used, and the one which is nearer to the centre of vision answers every purpose. It is only on rare occasions that both measuring points will be required.

Distance Points. They are really measuring points, and are so called in this book, to avoid the confusion that might arise from calling the measuring points for $C\,V$, distance points, and calling the measuring points for other vanishing points by their proper name.

There is one point in connection with the measuring of distances on retiring lines, about which the student will need to be very careful. Suppose, instead of being required to measure a certain distance from a point on the picture plane, as h, the point from which the measurement is to be taken, already lies at some distance from the picture plane, as r, and it is required to measure from it, in the direction $h\,k$, a distance equal to $P\,F$. In such a case as this the point of contact of the point r is first found by means of a line from $M\,P_1$, the proper measurement taken on $G\,L$ from p to f and a line drawn from f to $M\,P_1$; then $r\,n$ will be the distance required. The reason for this will be manifest on comparing the lines marked by italic letters with the corresponding lines marked by capital letters.

PROBLEM 1.—Represent properly in perspective the position of a point on the ground plane 2′ to the right of the line of direction and 4′ away from the picture plane. The eye of spectator is 6′ above the ground and his distance from the picture plane is 14′. Scale $\frac{1}{96}$. (Fig. 7.)

The first step is to draw a horizontal line across the paper and mark a point somewhere near its centre to represent the *C V*. From *C V* draw a vertical line equal in length to the distance of the spectator from the *P P*, 14′, and mark it *L D*, and its lower extremity *S P*. The scale in this problem is $\frac{1}{96}$ of the natural size, that is, the unit of measurement in the drawing is to be $\frac{1}{96}$ of 12″ or $\frac{12}{96}$″ or $\frac{1}{8}$″, so that the line of direction will be $\frac{14}{8}$″ or $1\frac{3}{4}$″ long. With *C V* as a centre and *C V S P* as radius draw a semicircle to find the measuring points and letter them *L M P* and *R M P*. Next on *L D* measure from *C V* the height of the eye of the spectator above the ground, 6′, which will be $\frac{3}{4}$″, and through this point draw the *G L* parallel to *H L*.

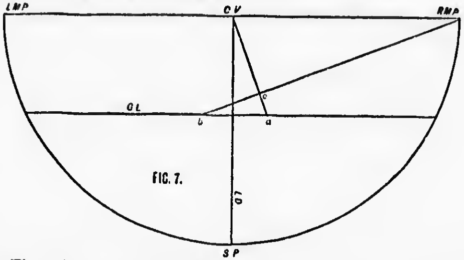


FIG. 7.

The point required in this problem is 2′ to the right. On *G L* measure 2′, or $\frac{1}{4}$″, to the right of *L D* to find a point *a* which will be the *P C* of the point required, when it is moved forward in a direction parallel to *L D* to touch the *P P*. A line from *a* to *C V* will be the representation of a line on the ground plane perpendicular to *P P* and 2′ to the right throughout its entire length, and so we know that the point sought will be in it. From *a* measure the distance of the point from the *P P*, 4′, either to the right or left, as *a b*, and from *b* draw a line to one of the measuring points to cut *a C V* in *c*. Then *c* will be the point required.

PROBLEM 2.—Show the position of a point in the ground plane 2′ to the left and 6′ beyond the picture plane. The eye of the spectator is 7′ from the picture plane and 3′ 6″ above the ground plane. Scale $\frac{1}{48}$. (Fig. 8.)

Measure on *G L*, 2′ to the left of *L D* to find *a* the position which the point required would occupy if brought forward to the *PP*. From *a* draw a line to *C V*. This will represent the track of the point *a* on being moved back along the ground plane to the horizon, in a direction perpendicular to *P P*. From *a* measure 6′ to the right, to *b* and draw *b L M P* to cut *a C V* in *c*. Then *c* will be the point required.

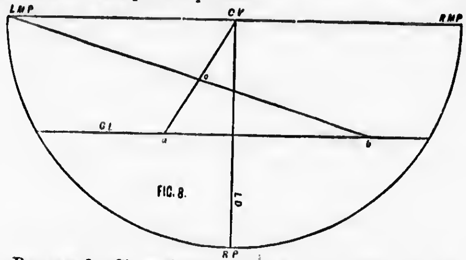


FIG. 8.

PROBLEM 3.—Show the position of a point 1′ 3″ to the right, 3′ 6″ distant from the picture plane and 1′ above the ground plane. Height of eye 1′ 3″, distance from the picture plane 3′ 6″, and scale $\frac{1}{24}$. (Fig. 9.)

Find the position which the point would occupy when in the ground plane 1′ 3″ to the right and 3′ 6″ back from *P P*, by measuring 1′ 3″ to the right of *L D* to *a*, and 3′ 6″ from *a* to *b*, and drawing *a C V* and *b R M P* to intersect in *c*. At *a* erect a perpendicular *a d* equal in height to the distance of the point required above the ground plane, 1′, and from *d* draw a line to *C V* to cut a perpendicular from *c* in *e*. Then *e* will be the position of the point required.

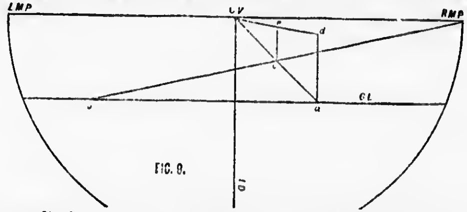


FIG. 9.

It is evident that *c* is the proper distance to the right, and away from the *P P*, and that *d* is the proper distance to the right and above the ground plane, so that if a line be drawn from *d* parallel to *a c* it will pass over *c* at the proper distance. In order to represent it in this direction, it must vanish in *C V*. (Rule iii.)

Exercise 1.—Find the perspective position of a point in the ground plane 6′ to the left of the line of direction and 8′ beyond the picture plane. The eye of the spectator is 6′ above the ground plane, and 15′ from the picture plane; scale $\frac{1}{96}$ or $\frac{1}{8}''$ to 1′.

Exercise 2.—Represent in perspective the position of a point in the ground plane, 2′ to the right of the line of direction, and 3′ away from the picture plane. Height of eye from the ground, 2′; distance from picture plane, 7′; scale $\frac{1}{48}$.

Exercise 3.—Show the perspective position of a point 3′ to the right, 6′ from the picture plane, and 2′ above the ground. Height 3′, distance 5′; scale $\frac{1}{32}$.

Exercise 4.—A ball is suspended in the air, 8′ from the picture plane, 19′ from the spectator, 7′ above the ground and 4′ to the left. The eye of the spectator is 5′ from the ground. Represent the position of the ball by a point; scale $\frac{1}{64}$.

PROBLEM 4.—Represent in perspective the position of a point 3′ to the left, 5′ from the picture plane and 7′ from the ground. Spectator's eye is 4′ from the ground, and 14′ from the picture plane. Scale $\frac{1}{64}$. (Fig. 10.)

It will be noticed that the station point has been used thus far, only to show how the measuring points are obtained. They can be found by measuring on the horizontal line to the right and left of the centre of vision, the distance of the spectator from the picture plane. In the ensuing illustrations the station point will not be shown.

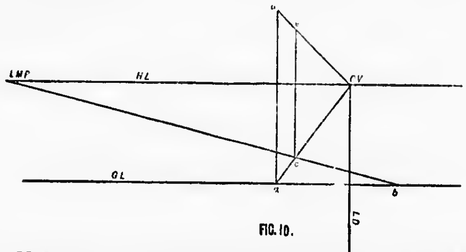


FIG. 10.

Measure 3′ to the left of the line of direction to *a* and draw *a C V.* From *a* measure 5′ to *b* and draw *b L M P.* From *a* draw a vertical line *a d*, 7′ long and draw *d C V.* From *c* draw a vertical line to cut *d C V* in *e* Then *e* will be the position of the point required.

PROBLEM 5.—Show the perspective appearance of a line in the ground plane, parallel to the picture plane. Its left hand end is 1′ to the left and its right hand end is 4′ to the right and 3′ back. Position of spectator's eye 3′ above the ground plane and 7′ 6″ from the picture plane. Scale $\frac{1}{32}$. (Fig. 11.)

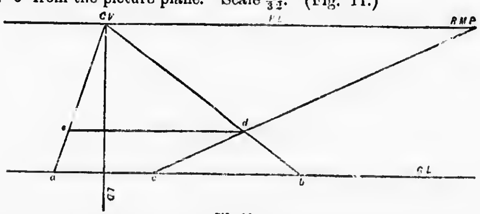


FIG. 11.

Find a point, *a*, on *G L* 1′ to the left of *L D*, and another point, *b*, 4′ to the right of *L D* and draw *a C V* and *b C V*. Then the left hand end of the line required will be in *a C V*, and its right hand end will be in *b C V.* From *b* measure 3′ to the left, to *c* and draw *c R M P* to cut *b C V* in *d.* Lines parallel to the picture plane are represented as they are, without any change of direction, and as the line in this case is in the ground plane, and hence horizontal, therefore if from *d* a horizontal line be drawn to cut *a C V* in *e*, it will be the representation of the line required.

As a straight line is the shortest distance between two points, if the perspective position of the extremities of any line can be found, the line joining them will be the perspective representation of the line required.

PROBLEM 6.—Represent in perspective a line 6′ long, in the ground plane, perpendicular to the picture plane, its nearer end being 4′ to the left and 2′ beyond the picture plane. Height of eye 5′; distance from picture plane 10′; scale $\frac{1}{48}$. (Fig. 12.)

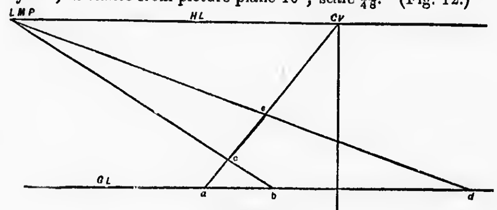


FIG. 12.

On *G L* find *a* 4′ to the left of *L D* and join *a C V.* From *a* measure 2′ to *b*, and from *b* measure 6′ to *d* and join these points with *L M P* by lines cutting *a C V* in *c* and *e.* Then *c e* will be the line required.

PROBLEM 7.—Draw the perspective view of a line in the ground plane having one end 6″ to the left and 2′ from the picture plane, and the other end 4′ to the left and 1′ from the picture plane. Height 2′; distance 5′; scale $\frac{1}{24}$. (Fig. 13.)

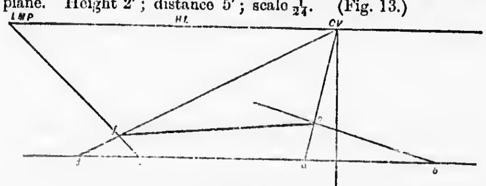


FIG. 13.

Find *a* 6″ to the left and *d* 4′ to the left and draw *a C V* and *d C V.* Measure 2′ from *a* to *b* and 6″ from *d* to *e* and join *b L M P* and *c L M P.* Then *c* and *f* will be the extremities of the line required.

Exercise 5.—Place in perspective a straight line 3′ long in the ground plane, parallel to the picture plane, its right hand extremity being 2′ to the right and 1′ back. Height of eye, 1′ 6″; distance from picture plane, 3′ 9″; scale $\frac{1}{24}$.

Exercise 6.—A person whose eye is 1′ from the ground and 2′ from a pane of plate glass placed in a position perpendicular to his line of direction and the ground plane, is looking at a pole 4′ long lying on the ground, perpendicular to the pane of glass, its near end touching the glass in a point 2′ to the left. Represent the appearance of a tracing of it made upon the glass which is 2″ thick; scale $\frac{1}{12}$.

Exercise 7.—Height of eye 1′; distance 16′; scale $\frac{1}{24}$. Place in perspective two poles 3′ long lying on the ground plane. One is parallel to the picture plane with its right hand extremity 2′ to the right and 4′ back from the picture plane, and the other is perpendicular to the picture plane with its nearer extremity 4′ to the left and 1′ from the picture plane.

PROBLEM 8.—Place in perspective a vertical line 14′ long, when its lower extremity is in the ground plane 12′ to the right and 30′ from the picture plane. Height 8′; distance 36′; scale $\frac{1}{72}$. (Fig. 14.)

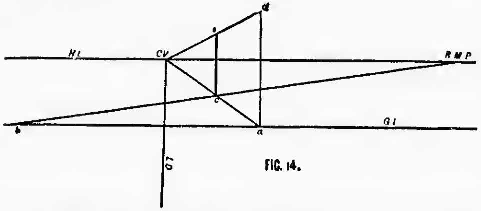


FIG. 14.

By means of the points *a* and *b* find the point *c*, which is the position of the lower extremity of the line required. If it be supposed to be moved forward towards the picture plane, in the direction *C V c*, when in contact with the picture plane it would be represented by the line *a d* 14′ long. From *d* draw a line to *C V* to cut a vertical line from *c* in *e*. Then *c e* will be the line required.

PROBLEM 9.—Place in perspective a square of 3′ side, in the ground plane, having two of its sides parallel to the picture plane, and its nearer left hand corner 2′ to the left and 2′ back. Height 2′; distance 7′; scale $\frac{1}{32}$. (Fig. 15.)

As a line may be said to be generated by a point in motion, so a plane may be said to be generated by a line in motion in a direction other than that of its length. With this fact in view it will be an easy matter to draw a line in a position similar to that in problem 5, and move it back through a distance equal to, and in a direction perpendicular to, its length, and thus obtain a square.

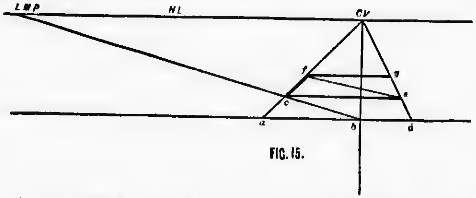


FIG. 15.

In this problem the position of one of the left hand corners of the square is given. In parallel perspective square objects may be said to have near right and left hand corners, and far right and left hand corners. This distinction is sufficient to enable the student to determine accurately which corner is referred to. In the present case if the near left hand corner is 2′ to the left and the side of the square is 3′ long, the near right hand corner will be 1′ to the right of *L D*.

Find *a* 2′ to the left and *d* 1′ to the right, respectively, and from them draw lines to *C V*. From *a* measure 2′ to the right to *b* and draw *b L M P* to cut *a C V* in *c*. Then *c* will be the nearer left hand corner of the square, and a horizontal line drawn from *c* to cut *d C V* in *e* will be its front side.

Referring to what has been said regarding the measurement of retiring lines it will be seen that *all lines vanishing in the measuring points for C V are at an angle of 45° with the P P*, hence the line from *e* to *L M P* will contain the diagonal of the square required, and, cutting *a C V* in *f*, will find *f* the far left hand corner. A horizontal line from *f* to cut *d C V* in *g* will complete the square.

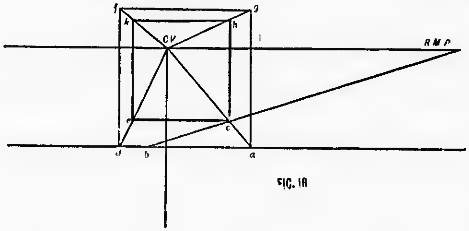


FIG. 16.

PROBLEM 10.—Show the appearance of a square of 4′ side when its plane is parallel to the picture plane, two of its sides being vertical, and its lower right hand corner on the ground plane, $2\frac{1}{2}$′ to the right and 3′ back. Height 3′; distance 9′; scale $\frac{1}{48}$. (Fig. 16.)

Measure to the right of *L D* $2\frac{1}{2}$′ to *a* and draw *a C V*. From *a* measure the distance of the square from the picture plane, 3′, to *b* and draw *b R M P* to cut *a C V* in *c*. From *a* measure 4′ to the left to *d* and draw *d C V*. From *c* draw *c e* parallel to *H L* to cut *d C V* in *e*. This will be the lower edge of the square. Suppose the square to be moved forward until it touches the *P P*. It will be represented there by the square *a d f g* and the track of its upper corners will be in the lines *f C V* and *g C V*. Vertical lines from *c* and *e* to cut these lines in *h* and *k* will give the positions of the upper corners when at the proper distance from the *P P*. Join *h k*.

Exercise 8.—A bar of iron lies on the ground, its left hand extremity being 3′ to the left and 8′ back from the picture plane, and its right hand extremity is 2′ to the right and 1′ back. Show by a line its appearance to a spectator whose eye is stationed 4′ above the ground and 8′ from the picture plane; scale 1″ to 4′.

Exercise 9.—Height 3′; distance 5′; scale $\frac{3}{8}$″ to 1′. Represent in perspective a rectangular vertical plane 3′ wide and 8′ long with one of its long edges in the ground plane perpendicular to the picture plane, its nearer end being 3′ to the left and 1′ back.

Exercise 10.—Place in perspective a square of 2′ side lying on the ground plane touching the picture plane with its front side, its nearer right hand corner being 6″ to the right. Height 2′; distance 5′ 6″; scale 2′ to 1″.

Exercise 11.—Show the square of last exercise when in the ground plane with two sides perpendicular to the picture plane, its nearer left hand corner being 1′ to the left and 2′ back. Use the same height, distance and scale as in exercise 10.

PROBLEM 11.—Make a perspective drawing of a square of 6′ side with its plane vertical; one of its sides is in the ground plane, perpendicular to the picture plane; and its near lower corner is 6′ to the left and 3′ from the picture plane. Height 4′; distance 15′; scale $\frac{1}{64}$. (Fig. 17.)

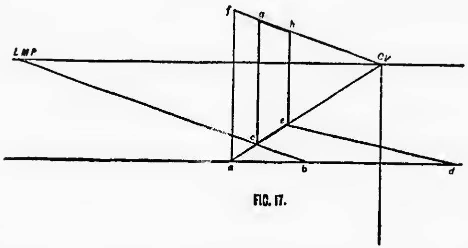


FIG. 17.

Having found the point *c* 6′ to the left and 3′ from *P P*, suppose the square to be moved forward to *P P* in the direction *C V c*. Its front side will then be represented by the line *a f*, and the track of its upper corner will be in the line *f C V*. A vertical line from c to cut *f C V* in *g* will be the near side in its proper position. By means of the measurement *b d* equal to the side of the square, find in *a C V* a point, *e*, 6′ beyond c and from *e* erect a perpendicular to cut *f C V*.

PROBLEM 12.—Place in perspective an equilateral triangle of 5′ side, in the ground plane. Its most distant side is parallel to and 5′ from the picture plane, and the left hand end of this side is 1′ to the right. Height of eye of spectator 4′; distance from picture plane 11′ 9″; scale $\frac{1}{48}$. (Fig. 18.)

The triangle in question when in this position will have two sides at an angle of 60° with the picture plane and so they will not vanish in the centre of vision. But by means of a slight modification of the rules thus far learned the centre of vision and its measuring points may be used for the purpose of obtaining the position of the corners of the triangle. It is necessary therefore to ascertain their position with regard to the picture plane and line of direction, by drawing the triangle and placing it in a position in regard to two perpendicular lines, similar to its position with regard to the picture plane and the line of direction. The ground line, and the line of direction below the ground line may be used for this purpose. Find a point, *a*, 1′ to the right of *L D* and 5′ from *G L*. From *a* draw parallel to *G L*, a line, *a b*, 5′ long which will be the side of the triangle most distant from *P P*. Upon this line construct an equilateral triangle whose vertex, c, will be in the proper position in regard to *P P* and *L D*. If vertical lines be drawn from *a* and *b* to cut *G L* in *d* and *e* these points will indicate the distance to the right of *L D*, of *a* and *b*; and similarly, a vertical line from c will cut *G L* in a point as far to the right of *L D* as c is. If lines be drawn from each of these points in *G L*, to *C V*, the corners of the triangle will be somewhere in them. But *b* is the distance *b e* from *P P*, therefore if an arc be drawn with *e* as a centre and *e b* as radius it will find on *G L* a point *d* that distance to the left of *e*. The line *d R M P* will cut *e C V* in *h* which will be the perspective representation of *b*. A horizontal line from *h* to cut *d C V* in *k* will be the perspective representation of the line *a b* which is the

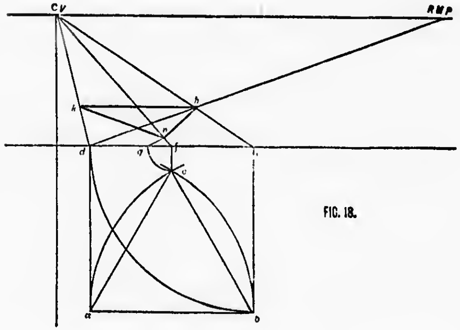
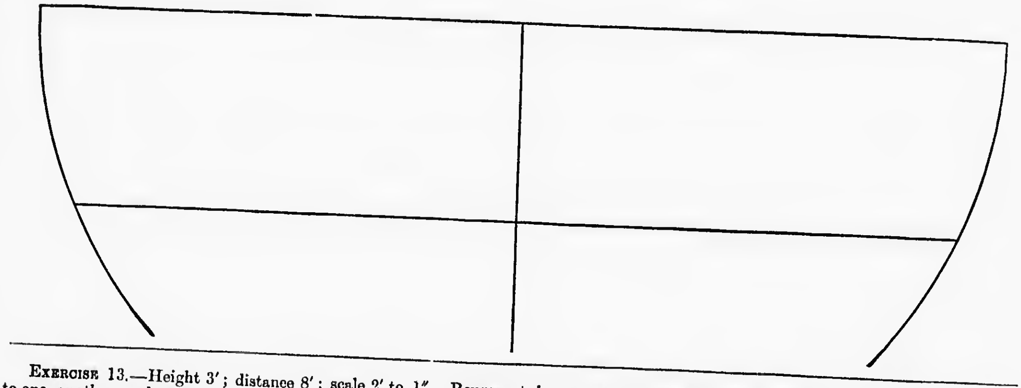


FIG. 18.

most distant side of the triangle. In a similar way the point *n* may be found as the perspective representation of the point *c*. Then the triangle *n k h* will be the perspective representation of the triangle in question.

PROBLEM 13.—Show the triangle of the last problem when its plane is vertical, the edge on which it rests is in the ground plane, perpendicular to the picture plane, and its near end 5′ 9″ to the left, and 2′ 11″ back. Height of eye 5′ 9″; distance from picture plane 13′ 10″; scale $\frac{1}{64}$. (Fig. 19.)

Exercise 12.—From the following diagram drawn on a scale of $\frac{1}{48}$ find the position of spectator's eye with regard to the picture plane and the ground plane, letter the lines and points used, and show the appearance of two squares of 8′ side with their planes vertical and perpendicular to one another. One of them has two sides perpendicular to the picture plane and touches it and the ground plane 5′ to the left. The other square has its four sides parallel to the picture plane and touching the ground plane and the most distant side of the first square.



Exercise 13.—Height 3′; distance 8′; scale 2′ to 1″. Represent in perspective two squares of 4′ side, their planes being parallel to one another, and to the ground plane and two sides of each being parallel to the picture plane. The centre of one square is 1′ from the ground plane, 1′ to the right and 4′ back, and the centre of the other square is vertically above this point at a distance of 3′.

Find a point *a*, 5′ 9″ to the left of *L D*, and draw *a C V*. In it, by means of *L M P* find a point, *c*, 2′ 11″ from *P P*. This will be the near end of the lowest side of the triangle. Anywhere in *G L* select a line *d f*, 5′ long, and on it construct the equilateral tri-

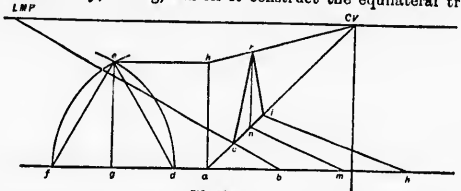


FIG. 19.

angle *d f e* and from its vertex draw a horizontal line to cut a vertical line from *a* in *k*. To the right of *b* measure the distance *d f* to *h* and draw *h L M P* to cut *a C V* in *i* which will be the the far end of the lowest side of the triangle. A vertical line drawn from *e* will bisect the base, *d f*, of the triangle. Bisect

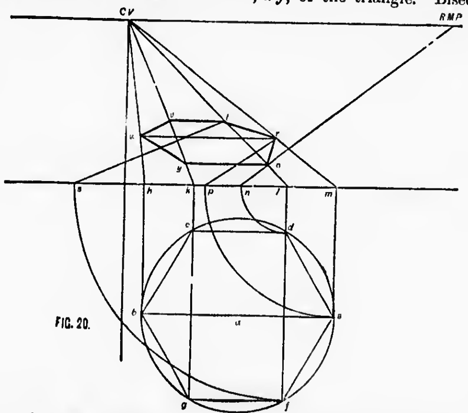


FIG. 20.

b h and draw *m LMP*. Then *n* will be the perspective centre of the line *c i* and the vertex of the triangle will be vertically above it at a distance *a k*. Draw *k CV* to cut a vertical line from *n* in *r* and join *r c*, *r i*.

PROBLEM 14.—Show the appearance of a hexagon of 5′ 9″ side, which is in the ground plane, two of its sides being parallel to the picture plane and its centre being 6′ 10″ to the right and 7′ 9″ from the picture plane. Height 10′; distance 19′ 6″; scale $\frac{1}{96}$. (Fig. 20.)

Find the position of a point, *a*, 7′ 9″ below *G L* and 6′ 10″ to the right of *L D*. With this point as a centre and a radius equal to the side of the hexagon, 5′ 9″, draw a circle, and in the circle construct a hexagon having two sides parallel to *G L*. Draw vertical lines from each of the corners to *G L* and from these points *h k l m* draw lines to *C V*. With *m* as a centre and *m e* as radius draw an arc to cut *G L* in *p*, and from *p* draw a line towards *R M P* to cut *m C V* in *r*. Then *r* will be the perspective position of the point *e*. In the same way by means of arcs with a radius *l d* and *l f* find the position of the point *o* corresponding to *d*, and the point *t* corresponding to *f*. Then as *d* and *c* are the same distance from *P P*, a horizontal line from *o* to cut *k CV*, will give the

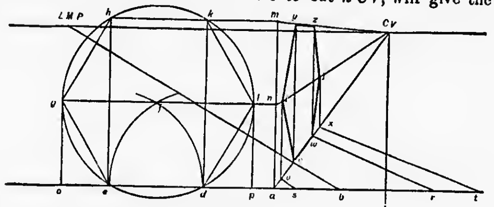


FIG. 21.

perspective position of the point *c*, and similarly, horizontal lines from *r* to cut *h CV*, and from *t* to cut *k CV*, will find the perspective positions of *b* and *g*. The lines joining these six points will represent the sides of the hexagon required.

PROBLEM 15.—Show the appearance of the hexagon of the last problem when its plane is vertical, two of its edges are parallel to the ground plane and perpendicular to the picture plane, and the near end of the side on which it rests is 6′ 10″ to the left and 4′ beyond the picture plane. Height 9′ 9″; distance 19′ 6″; scale $\frac{1}{96}$. (Fig. 21.)

Draw a hexagon of 5′ 9″ side, with one of its sides in the ground line. Draw horizontal lines through *h k* and *g l* to a perpendicular erected at a point on *G L* 6′ 10″ to the left. The perspective position of the near end of the lowest side is found by a measurement 4′ to the right of *a* and a line towards *L M P* to cut *a C V* in the point *c*. The perspective length of the lowest side is next found, and vertical lines drawn from its ends to meet a line from *m* to *CV*. The line marked *y z* will be the top side. It will be seen by the illustration that the distance of *p* to the right of *d*, and the distance of *o* to the left of *e*, is equal to one-half the length of the side of the hexagon, therefore points must be found in *a CV*, 2′ 10½″ nearer than *c*, and 2′ 10½″ beyond *w*. Vertical lines from *v* and *x* to cut *n CV* in *i* and *j* will find the two remaining corners.

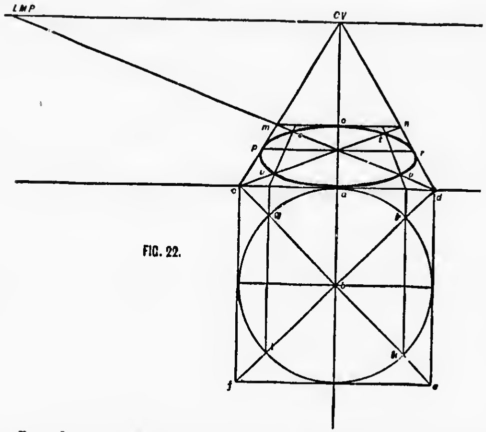
Exercise 14.—Height 5′; distance 8′; scale $\frac{1}{48}$. Represent in perspective an isosceles triangle in the ground plane. Its base, 6′ long, is parallel to the picture plane and 1′ from it. The vertex of the triangle is 4′ to the left, and 12′ beyond the picture plane.

Exercise 15.—Using the same height, distance and scale as in Exercise 14, show the appearance of the triangle mentioned therein when it is in the ground plane, its base is parallel to the picture plane, and its vertex touches the picture plane in a point 1′ to the right.

Exercise 16.—Height 8′; distance 18′; scale $\frac{1}{96}$. Place in perspective an equilateral triangle of 8′ side in the ground plane, its base being perpendicular to the picture plane, touching it in a point 2′ to the left. The vertex of the triangle is to the right of the eye.

Exercise 17.—With the same height, distance and scale as in last problem show the same triangle when its plane is vertical, its base is horizontal and perpendicular to the picture plane, and its vertex is in the ground plane 5′ to the right and 5′ back from the picture plane.

PROBLEM 16.—Represent in perspective a circle 6′ in diameter, lying on the ground plane and touching the picture plane in a point opposite to the eye. Height of the eye 5′; distance from picture plane 10′; scale $\frac{1}{48}$. (Fig. 22.)



In order to draw the curve which will represent a circle when viewed obliquely, it is necessary to obtain several points in its circumference, the perspective position of which can be easily ascertained. For this purpose it is enclosed in a square and the diameter and diagonals of the square are drawn, making eight points in the circumference of the circle, viz.: one at each extremity of each diameter, and four others, where the diagonals cut it. The square enclosing the circle must be placed in its proper position below the ground line as in the case of the triangle and hexagon.

In this problem the circle touches the picture plane in a point opposite to the eye. It is evident that excepting when a circle is in a plane, its circumference can touch the plane in only one point, and that a line drawn in the plane of the circle from the point of contact, perpendicular to the line of intersection of the two planes, will pass through the centre of the circle. Applying this to the circle in question, as its point of contact with the picture plane is opposite to the eye, its centre is also opposite to the eye and therefore in the line of direction.

Find on $L D$ a point b distant from a the length of the radius, 3′, of the circle and with b as a centre and b a as radius draw a circle, enclose it in a square and draw the diameters and diagonals of the square. Next place the square, with its diameters and diagonals, in perspective. From the points, g h, k and l, where the diagonals of the square cross the circumference of the circle, draw vertical lines to G L and thence towards C V to cut the diagonals of the perspective square in points s, t, u and v. Through these four points, and the extremities, $a o$ and $p r$ of the diameters, draw an elliptical curve which will be the perspective representation of the circle.

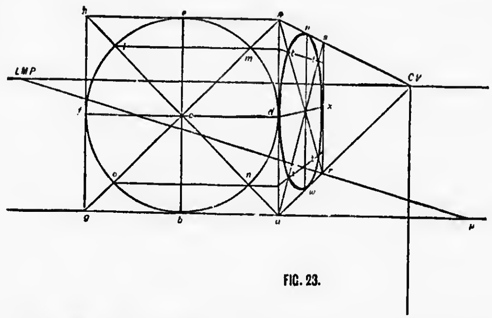


FIG. 23.

PROBLEM 17.—Show the circle of the last problem when its plane is vertical, perpendicular to the picture plane, and its circumference touches the ground plane and the picture plane in points 4′ to the left. Height 4′; distance 11′ 9″; scale $\frac{1}{48}$. (Fig. 23.)

When in this position the centre of the circle will be 3′ from the picture plane, 3′ above the ground plane and 4′ to the left. Find a, 4′ to the left. From a measure 3′ to b, at b erect a perpendicular 3′ long and with c as a centre and c b as radius, draw a circle. Enclose the circle in a square a g h k and draw its diameters and diagonals. Place the square with its diameters and diagonals in perspective. From the points marked l, m, n and o draw horizontal lines to the perpendicular a k and thence towards C V to cut the diagonals of the square in the points t t t t. Through the points d, t, v, t, x, t, w, t draw an ellipse.

PROBLEM 18.—A circle 10′ in diameter stands upright on the ground plane, parallel to the picture plane at a distance of 5′ beyond it. Its centre is 4′ to the left of the eye. Show its appearance. Height 6′; distance 16′; scale $\frac{1}{96}$. (Fig. 24.)

Exercise 18.—Height 8′; distance 20′; scale $\frac{1}{96}$. Represent properly in perspective a hexagon of 6′ side in the ground plane, two of its sides being perpendicular to the picture plane, and its centre being 8′ to the left and 10′ back from the picture plane.

Exercise 19.—Height 5′; distance 11′; scale $\frac{1}{48}$. A hexagon of 4′ side stands on the ground plane with two sides vertical and parallel to the picture plane, and its plane perpendicular to the picture plane. The point on which it rests is 5′ to the right and 4′ back. Show its perspective appearance.

Exercise 20.—Height 4′ 6″; distance 9′; scale $\frac{1}{24}$. Place in perspective a circle 4′ in diameter, lying in the ground plane and touching the picture plane in a point 3′ to the left.

Exercise 21.—Height 4′ 6″; distance 9′; scale $\frac{1}{24}$. Show the appearance of a circle 5′ in diameter, its plane being perpendicular to the picture plane and ground plane, and its circumference touching the ground plane in a point 8′ from the picture plane and 3′ to the right.

Find a point, c, on the ground 4′ to the left and 5′ beyond the picture plane. This will be the point of contact of the circle in question with the ground, and its centre will be directly over this point at a distance of 5′. At a erect a perpendicular 5′ long and draw d $C V$ to cut a vertical line from c in e. Then e c will be the perspective length of one of the radii of the circle. With e as a centre and e c as a radius draw a circle.

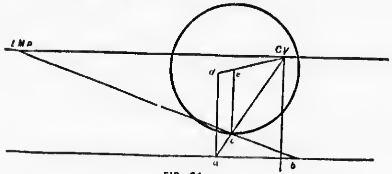


FIG. 24.

PROBLEM 19.—Show the circle of the last problem when it is removed to a distance of 20′ from the picture plane, its centre being 4′ to the left of the eye. Height 6′; distance 20′; scale $\frac{1}{96}$. (Fig. 25.)

If the usual method of measuring vanishing lines were adopted, in order to find a point on a $C V$, 20′ from the picture plane, it would be necessary to measure from a to the right, a distance of $2\frac{1}{2}$″, and the point on the ground line thus found, would be beyond the limit of the paper. The most convenient method of measuring great distances is to use a *Half Measuring Point* found by bisecting the distance between any vanishing point and its measuring point. When the half measuring point is used, one-half of the measurement required is taken on the ground line,

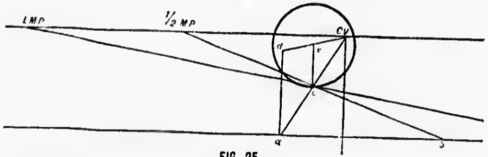


FIG. 25.

and if the measurement is to be taken beyond a point which is itself lying away from the picture plane, a point of contact of this point must be found by a line from the half measuring point, and the half measurement on the ground line must be taken from the point of contact. That is, if the position of the circle in question were given as 15′ beyond the circle of the last problem, instead of 20′ beyond the picture plane, a line would be drawn from $\frac{1}{2}$ $M P$ through c to obtain a point of contact on $G L$, and a measurement of one-half of 15′, or $\frac{15}{16}$″, would be taken to the right of the point of contact, and a line drawn from that point to $\frac{1}{2}$ $M P$ to cut a $C V$. The same result can be obtained by measuring one-half of 20′, or $1\frac{1}{4}$″, to the right of a, to the point marked b, and drawing a line from there to $\frac{1}{2}$ $M P$ to cut a $C V$ in c. Then c will be the lower extremity of the vertical diameter of the circle when at the required distance from the picture plane. Find the perspective position of the centre, e, of the circle as in problem 19, and draw the circle with e as a centre and e c as radius.

PROBLEM 20.—Place in perspective a circle 8′ in diameter, its plane being perpendicular to the picture plane and inclined upwards to the right at an angle of 60° with the ground plane. The diameter parallel to the picture plane touches the ground in a point 3′ to the right and 6′ from the picture plane. Height 6′; distance 9′ 9″; scale $\frac{1}{84}$. (Fig. 26.)

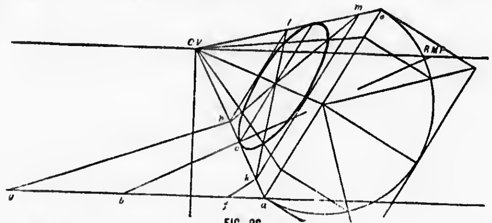


FIG. 26.

Measure 3′ to the right of $L D$, to a and draw a $C V$. Referring to what has been said in connection with problem 16, as to the position of the centre of a circle with regard to the point in which its circumference touches a plane, it will be seen that the line of intersection of the plane of the circle with the ground plane is in a $C V$ and that a line perpendicular to this will contain the centre of the circle. This perpendicular line will be parallel to the picture plane, hence the point of contact of the circle with the ground plane, and the centre of the circle, are the same distance from the picture plane. Measure from a, 6′ to b and draw b $R M P$. Then c will be the point where the circle touches the ground. From a draw a line forming an angle of 60° with $G L$, and on it draw a semicircle with a radius of 4′, making a one extremity of its diameter. Treat this semicircle in the same way as the circle in problem 17. On each side of b measure 4′ to f and g, and from these points draw lines to $R M P$ to obtain h and k. From h and k draw lines parallel to a e to cut e $C V$ in l and m. This will complete the square containing the circle. No difficulty should be experienced in finding the points through which to draw the curve.

Exercise 22.—Height 16′; distance 40′; scale 16′ to 1″. Place in perspective a circle 15′ in diameter, its plane being parallel to the picture plane, and its centre being 60′ beyond the picture plane, 10′ above the ground plane and 5′ to the right.

Exercise 23.—Height 1′ 6″; distance 4′; scale $\frac{1}{24}$. Place in perspective a hexagon of 18″ side. All of its edges are parallel to the picture plane, and two of them are parallel to the ground plane. Its centre is 8′ back from the picture plane, 2′ to the left and 1′ above the ground.

Exercise 24.—Height 5′ 6″; distance 9′; scale $\frac{1}{24}$. Place in perspective a hexagon of 2′ 6″ side, when perpendicular to the ground plane, two of its sides being perpendicular to the picture plane, and its centre is 2′ to the left, 3′ back from the picture plane, and 2′ 6″ above the ground plane.

Exercise 25.—Show the hexagon of exercise 24 when its plane is perpendicular to the picture plane and ground plane, two of its sides are perpendicular to the picture plane, and its centre is 4′ to the right, 3′ from the picture plane and 2′ 6″ above the ground plane. Draw lines joining the corresponding corners of the perspective views of the hexagon in question, and thus obtain the perspective appearance of a hexagonal prism.

Problem 21.—Place in perspective a cube of 4' 10" edge standing on the ground plane with two of its faces parallel to the picture plane and the near left hand corner of its base touching the picture plane 6' 9" to the left. Height of eye of spectator 6'; distance from picture plane 14' 9"; scale $\frac{1}{48}$. (Fig. 27.)

Measure 6' 9" to the left of *L D* to *a*, which will be the position of the near left hand corner of the base of the cube. The near right hand corner will be at *b* 4' 10" to the right of *a*. Draw *a C V*, and *b C V* also *b L M P*, which will contain a diagonal of the base and will therefore cut *a C V* in a point *c* representing the far left hand corner of the base. Draw *c d* parallel to *a b*. On *a b* construct the square *a f g b*, and from *f* and *g* draw lines to *C V* to cut vertical lines from *c* and *d* in *h* and *k*. Join *h k*.

Problem 22.—Place in perspective a block 6' square and 3'

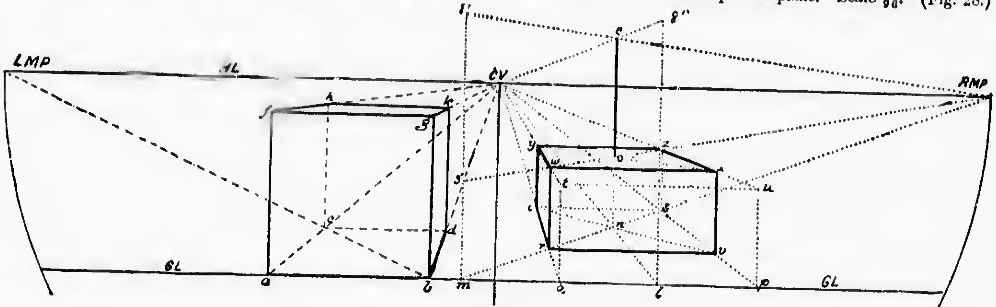


Fig. 27.

thick, resting on the ground plane on one of its large faces, and having two of its small faces parallel to the picture plane. The centre of the base is 5' to the right and 6' away from the picture plane. In the centre of the top face place a vertical pole 5' high. Height 6'; distance 14' 9"; scale $\frac{1}{48}$. (Fig. 27.)

Find the point *n* 5' to the right and 6' beyond *P P*. As this is the centre of the base measure on each side of *l* 3' to *o* and *p*, and draw *o C V* and *p C V* to cut *m R M P* in *r* and *s*. Draw *r v* and *s i* parallel to *G L*. From *o* and *p* draw vertical lines *o t* and *p u* 3' long and join their extremities with *C V*. Draw vertical lines from the corners of the base to cut *t C V* and *u C V* in *w y z* and *x*, which points will be the corners of the top of the block. From *l* draw a vertical line 8' long, and from its upper extremity draw a line to *C V* to cut a vertical line from *n* in *e*. Then *o e* will be the pole required. If a vertical line be drawn at *m* and measurements of 3' and 5' taken on it, lines drawn from these points

to *R M P* will pass through *w* and *z* and *e*. From this it may be inferred that either the *C V* or the measuring points may be used in measuring vertical distances. In this case the measuring point is the better one to use, as by means of it only one vertical line is required for obtaining the height of the block and the height of the pole, while if *C V* be used, at least two vertical lines must be drawn, one from *o* or *p* to find the height of the block, and another from *l* to find the height of the pole.

Problem 23.—Show the perspective appearance of a block of stone 8' x 8' x 16' standing on the ground plane with its axis vertical and two of its large faces parallel to the picture plane. The near right hand corner of the base is 12' to the right and 4' beyond the picture plane. The eye of spectator is 5' 6" from the ground, and 30' from the picture plane. Scale $\frac{1}{96}$. (Fig. 28.)

First find the point *b*, 12' to the right of *L D*, from *b* measure 4' to the left, to *c*, and draw *c R M P*. Then *d* will be the near right hand corner of the base of the block. The near left hand corner of the base will be 8' to the left of *d*. Find *a*, 8' to the left of *b*, and draw *a C V*. From *d* draw a horizontal line to cut *a C V* in *e*. Then *d e* will be the front side of the base of the block. From *e* draw a line to *R M P* to cut *b C V* in *f*, and draw *f g* parallel to *d e*. Then *e g f d* will be the base of the block. If the block be supposed to be moved forward until its front face touches the picture plane, its vertical edges will be represented by vertical lines from *a* and *b*. From either of these points, as *b*, draw a vertical line equal in length to the height of the block, and from its upper extremity draw a line to *C V* to cut perpendiculars from *d* and *f* in *k* and *l*. From *k* draw a horizontal line to cut a perpendicular from *e* in *m*, and join *m* with *C V* by a line cutting a perpendicular from *g* in *n*. Join *n l*.

Exercise 26.—In the illustration below, $ABCDEF$ is a hexagon drawn on a scale of $\frac{1}{48}$. Give in your own words its position and size, and the position of the spectator. Place the hexagon in perspective with its plane in the ground plane, and make it the base of a pyramid whose altitude will be 7′.

Exercise 27.—Place in perspective a circle, 8′ in diameter when its plane is inclined upwards to the left at an angle of [illegible] with the ground plane, and is perpendicular to the picture plane. One of its diameters touches the ground plane in a point 3′ to the left and 5′ back from the picture plane.

Exercise 28.—Height 4′; distance 8′; scale $\frac{1}{24}$. Represent properly in perspective a cube of 3′ edge resting on the ground plane with four of its edges parallel to both picture plane and ground plane, and its near right corners 1′ to the left and 2′ back from the picture plane. Represent also a triangular prism 4′ long, resting on the ground plane upon one of its oblong faces, the long edges of which are parallel to the picture plane with their left hand ends 3′ to the right of the right hand face of the cube. The edges of the ends of this prism are all 2′ 6″ long.

Problem 24.—Show the block mentioned in the last problem, when it is lying on the ground upon one of its oblong faces, its two ends being parallel to the picture plane, and its near right hand corners being 5′ to the left and 4′ beyond the picture plane. Height 5′ 6″; distance 30′; scale $\frac{1}{50}$. (Fig. 28.)

Find a point *r*, 5′ to the left and 4′ beyond the picture plane. Measure 8′ from *o* to *s*, draw *s CV* and a horizontal line from *r* to cut it. At *s* erect a perpendicular, *s x*, 8′ long and draw *x CV*. Draw a vertical line from the left hand end of the horizontal line from *r*, to cut *x CV* in *y*. Complete the near end of the block by a horizontal line from *y* and a vertical line from *r*, to intersect in *z*. Find the lower right hand corner of the far end of the block by a measurement of 16′ to the right of *p* to *t*, and a line *t LMP* cutting *o CV* in *v*. Draw a line *v w* parallel to *GL*. This will be the lower edge of the far end of the block. On *v w* construct a square, and join its upper corners with the upper corners of the square representing the near end of the block.

Problem 25.—Represent in perspective a block of stone 3′ x 3′ x 1′ lying on the ground plane upon one of its square faces, two edges of which are parallel to the picture plane, its far left hand corner being 2′ to the right and 4′ from the picture plane.

Centrally upon this place a cube of 2′ edge whose sides are parallel to the corresponding sides of the block on which it rests.

Make the top face of the cube the base of a pyramid 2′ high. Height of the eye 3′; distance from the picture plane 7′ 6″; scale $\frac{1}{24}$ (Fig. 29.)

Find *a*, 2′ to the right of *LD*, to the right of *a* measure 4′ to *b*, and draw *a CV* and *b LMP* intersecting in *c* which will be the far left hand corner of the base of the block. Measure 3′ from *a* to *d* and draw *d CV*. The line through *c* vanishing in *LMP* is

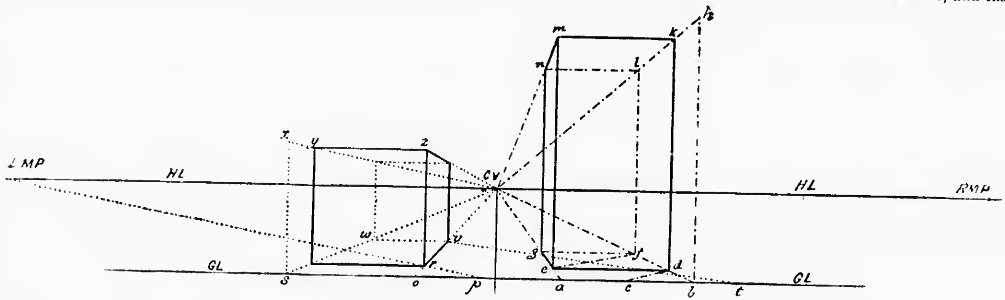


Fig. 28.

at an angle of 45° with *a c* and therefore contains one diagonal of the base, and will cut *d CV* in the near right hand corner, *e*, of the base. Complete the base by drawing horizontal lines from *c* and *e* cut to *a CV* and *d CV*.

Before proceeding to measure the height or thickness of the block, it will be well to consider that the centres of the three objects under consideration are in the same vertical line, and that their sides are parallel. Therefore a diagonal of the base of the block will pass vertically beneath two corners of the base and two corners of the top of the cube, also two corners of the base and the vertex of the pyramid. From this it is manifest that if lines be drawn parallel to this diagonal and at proper distances vertically above it, one will pass through two corners of the top of the block and two corners of the base of the cube, another will pass through two corners of the top of the cube which are also corners of the base of the pyramid, and another will pass through the vertex of the pyramid.

One of the diagonals, *c e*, of the base of the block is already produced to cut the ground line in *b*. At *b* erect a perpendicular, on it measure 1′ to *g* and draw *g LMP*. Vertical lines from *c* and *e* to cut this will find the near right hand and the far left hand corners of the top of the block. A vertical line from the near left hand corner of the base will cut a horizontal line from *h* in the near left hand corner of the top, and a line from it to *CV* to cut a vertical line from *c* will be the left hand edge of the top. Having obtained these points and lines, the block can easily be completed. As the edges of the cube are 1′ shorter than the edges of the top and bottom of the block, its right and left hand faces will be 6″ to the left and right of the corresponding faces of the block, therefore measure 6″ to the right of *a*, to *k*, and 6″ to the left of *d*, to *l*, and draw *k CV* and *l CV* cutting the diagonal

Exercise 29.—Height 6′; distance 16′; scale $\frac{1}{48}$. Place in perspective a cube of 4′ edge when its two horizontal faces and two of its vertical faces are perpendicular to the picture plane and the near right hand corner of its base is 2′ to the left and 3′ back. In the centre of the top face place an upright pole 4′ long.

Exercise 30.—Height 6′; distance 16′; scale $\frac{1}{48}$. Show the perspective appearance of a block 5′ x 5′ x 10′ standing on end with its axis vertical, and two of its oblong faces parallel to the picture plane. The centre of the base is 6′ to the right and 5′ back.

Exercise 31.—Height 8′; distance 15′; scale $\frac{1}{48}$. Represent the block mentioned in exercise 30 when resting with one of its long edges in the ground plane parallel to the picture plane and one of the diagonals of each end vertical. The centre of the edge on which it rests is 2′ to the left and 4′ back. Show the appearance of a vertical pole 10′ long resting against the centre of the far horizontal edge of the block.

e c m p and n. These points will be the far left hand and the near right hand corners of a square representing the base of the cube when resting on the ground. Find the other two corners of this square, and from the points m, p, o and n, draw vertical lines to cut the diagonals of the top of the block in r, v, t and s. Join these points and thus obtain the base of the cube. The top of the cube when it is in the position mentioned will be on a level with the eye, and therefore its top face will be represented by a straight line in H L. From the points r, v, t and s, draw vertical lines to cut H L. These lines will complete the cube. Next, from w, measure on the perpendicular erected at b, 2' to x, and draw x

First find a point, D, on the ground plane 1' to the right and 3' from the picture plane. Measure 5' to the left of A to B, and draw B CV. From D draw a horizontal line to cut B CV in E. Then D E will be the top edge of the near wall of the excavation.

From what has been said in connection with the explanation of the picture plane and its use, and from the statement of the fact that the portion of the picture plane below the ground line can be rendered visible, it may be inferred that measurements on the picture plane can be taken below the ground line as well as on it or above it, and consequently, that if vertical lines $1\frac{1}{2}'$ or $\frac{3}{4}''$ long be drawn from A and B, and their lower extremities F and G

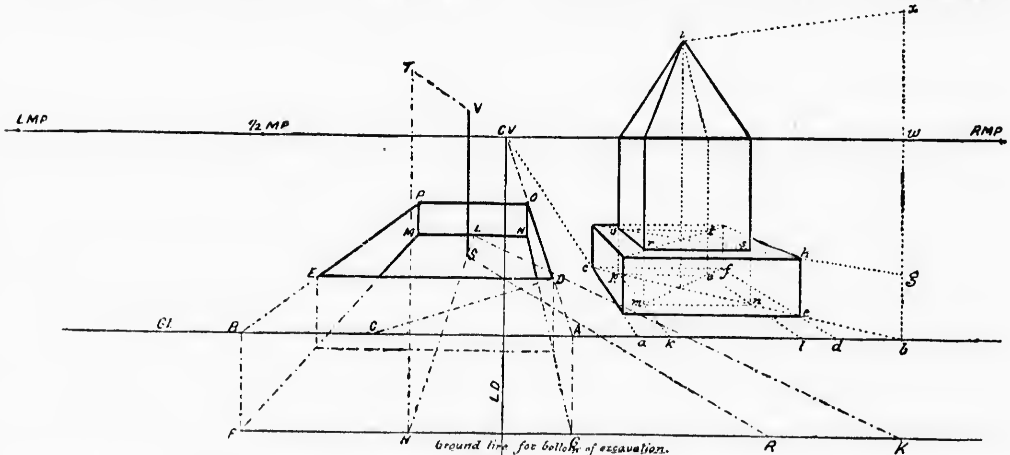


Fig. 29.

L M P to cut a vertical line from the centre of the base of the block, in z. This will be the vertex of the pyramid. Join it with the points representing the corners of the top of the cube.

PROBLEM 26.—The spectator is looking into an excavation 5' wide, 12' long, and 1' 6" deep. Its long sides are perpendicular to the picture plane, the near top corner of the right hand face being 3' from the picture plane and 1' to the right. (Fig. 29.)

Show the appearance of the excavation, and represent by a line the position and size of a man 5' 6" high, standing in the excavation midway between the side walls and 4' from the far end. Height 3'; distance 7' 6"; scale $\frac{1}{24}$. (Fig. 29.)

be joined, the oblong B F G A will represent the appearance of the near wall of the excavation if it were moved forward to touch the picture plane. Therefore lines from F and G to CV will find the lower edges of the left and right hand walls. In order to measure the distance of the far wall from the picture plane, 15', it will be most convenient to use a half measuring point, found as explained in problem 19. Produce F G indefinitely to the right, and on it measure from H one-half of 15', or $3\frac{3}{4}''$, and from K draw a line to ½ M P to cut H CV in L. The line H CV is used instead of F CV or G CV, because by means of it the position of the man in the problem can be ascertained, as well as

EXERCISE 32.—In the illustration below $A\ B\ C\ D$ is the perspective view of a square of 7′ side. By means of it find the horizontal line, centre of vision, and measuring points, and the height, distance and scale. Make it the base of a block 3′ thick. Centrally on this block place a cube of 5′ edge whose edges will be parallel to the corresponding edges of the block.

EXERCISE 33.—The line $E\ F$ is the lowest edge of the left hand square face of a block 2′ thick. Ascertain and state in your own words its size and position and show the appearance of a hole 6′ square passing horizontally through it from face to face. The hole passes through the centre of the square faces, its top and bottom edges being horizontal

EXERCISE 34.—Spectator stands with his eye 5′ above the ground and 15′ from the centre of an opening 6′ x 10′ in a floor 1′ thick. The centre of the opening is 1′ to the right of the eye and 6′ from the picture plane, and its long edges are perpendicular to the picture plane. Hinged upon the left hand edge of the opening is a door 6′ x 10′ x 3″ inclined upwards to the left at an angle of 45° with the floor, supported by a stick placed on the floor perpendicular to the plane of the door, and resting against the near end of its upper edge. Show the perspective appearance of the opening and the door. Scale $\frac{1}{24}$.

the position of the far wall of the excavation, and thus a line is saved. Through *L* draw a horizontal line to cut *F CV* in *M* and *G CV* in *N*. This will be the far end of the bottom of the excavation. From *M* and *N* draw the vertical lines *MP* and *NO* and join *OP*. These, with vertical lines from *E* and *D* to cut *F CV* and *G CV*, will complete the drawing of the excavation. From *K*, measure *KR*, the distance of the man from the far end of the excavation, 4′, and draw *R* ½ *MP*. Then *S* will be the point where he is standing. At *H* erect a perpendicular *HT*, 5′ 6″ long, and draw *T CV* to cut a vertical line from *S* in *V*. Then *SV* will be the representation of the man.

PROBLEM 27.—Place in perspective a flight of five steps, each one of which is 5′ 6″ long, 11″ high and 22″ wide. The front face of

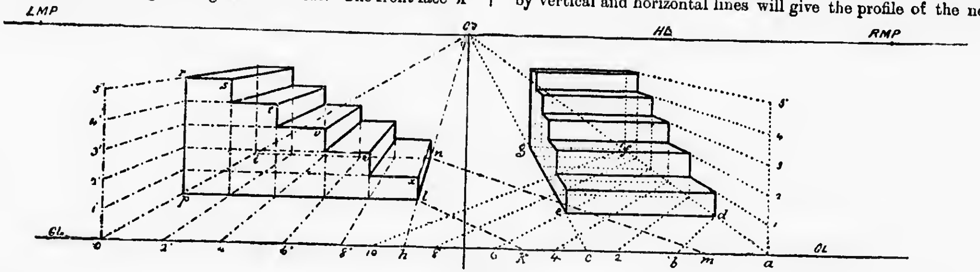


Fig. 30.

each step is parallel to and facing the picture plane, and the near right hand corners of the lowest step are 9′ 3″ to the right, and 2′ 9″ beyond the picture plane. Height of eye 6′ 6″; distance from picture plane 13′; scale ¼″ to the foot. (Fig. 30.)

Find *d* the near right hand corner of the bottom of the front step. On *G L*, from *b* measure five distances of 22″ each, and draw lines from the points 2, 4, 6, 8 and 10 towards *R M P* to cut *a C V*. In this way the perspective width of each step is obtained on *a C V*. At *a* erect a perpendicular 55″ long, divide it into five equal parts, and from these points of division draw lines towards *C V* to cut the vertical lines shown, drawn from *a C V*. These lines will give the perspective appearance of the right hand end of the flight of steps. From *a* measure 5′ 6″ to the left to *c*, draw *c C V*, and from *d* and *f* and the intervening points of division in *d f*, draw horizontal lines to obtain corresponding points in *c C V*. The remainder of the work is clearly shown. The long edges of each step are horizontal, the ends of the top face of each step vanish in *C V*, and the ends of the front face of each step are vertical.

PROBLEM 28.—Show the steps mentioned in the last problem, when the ends are parallel to the picture plane. The steps ascend towards the left from a line 1′ 10″ to the left, the near end of which is 3′ 8″ from the picture plane. Height 6′ 6″; distance 13′; scale $\frac{1}{48}$. (Fig. 30.)

First, obtain the position of the corners of the oblong space 5′ 6″ x 9′ 2″ covered by the steps. Divide the space *o h* into five equal parts, and draw lines from 2′, 4′, 6′ and 8′ towards *C V* as far as *n i*. At *o* erect a perpendicular 4′ 7″ high, divide it into five equal parts, and from the points of division draw lines towards *C V* as far as *p r*. From the points of division in *p r* draw horizontal lines to intersect vertical lines from the points of division in *p l*. The points of intersection, *s*, *t*, *v*, *w* and *x*, of these lines connected by vertical and horizontal lines will give the profile of the near end of the steps. The manner of obtaining the corresponding lines of the far ends of the steps is evident.

PROBLEM 29.—A cross stands upright on the ground plane, the axis of its shaft being 5′ 6″ to the right and 3′ 9″ from the picture plane. Its shaft is 2′ 10″ square and 9′ 3″ high. Its two arms are each 2′ 10″ square, projecting 2′ 3″ on each side of the shaft, and the top of the arms is 1′ 10″ from the top of the shaft. The face of the cross is parallel to the picture plane. Height 5′ 6″; distance 13′ 9″; scale $\frac{1}{48}$. Fig. (30.)

Find the perspective position, *c*, of the axis of the shaft of the cross. On each side of *a* measure 1′ 5″ to *d* and *e*, and from these points draw lines to *C V*, to cut *b R M P* in *g* and *h* two corners of the base of the shaft. The extremity of each arm of the cross is 2′ 3″ beyond the shaft, therefore measure that distance to the left of *d* and to the right of *e*, and draw lines from these points towards *C V*, to cut a horizontal line through *g* and *f* in *x* and *y*, and a horizontal line through *k* and *h* in *v* and *w*. At *e* erect a perpendicular 9′ 3″ long. On it measure 1′ 10″ from *l* to *m*, and 2′ 10″ from *m* to *n*. Lines from *l*, *m* and *n* towards *C V* will transfer these measurements to *f o*. At *g* erect a perpendicular,

Exercise 35.—Height 5′; distance 18′; scale $\frac{1}{4}''$ to the foot. Show the perspective appearance of two flights of stairs, the first one containing 4 steps, the long edges of which are perpendicular to the picture plane, and the second containing 8 steps, the long edges of which are parallel to the picture plane. The first flight ascends towards the left to a landing 6′ square, and the second flight ascends from the far side of the landing. The near corners of the front face of the lowest step are 3′ from the picture plane and 4′ to the right. The steps are 18″ wide, 9″ high, and 6′ long.

Exercise 36.—Height 10′; distance 36′; scale $\frac{1}{96}$. A block of stone 4′ x 10′ x 15′, stands on the ground plane upon one of its 4′ x 15′ faces, the near left hand corner of which is 8′ to the right and 4′ back. This block supports another one 6′ x 10′ x 20′, having one of its 10′ edges in the ground plane perpendicular to the picture plane 8′ to the left. The centres of both blocks are the same distance from the picture plane. Show them in perspective.

g s, equal to *f o* and join *s o*. Through *p* and *t*, and *r* and *u* draw horizontal lines to be cut by vertical lines from *x* and *y*. These lines will complete the front face of the cross. The back face can be obtained by lines towards *C V* from each of the angles of the front of the cross, cut by vertical lines from *v*, *k*, *h* and *w*.

Problem 30.—Show the cross referred to in the last problem, when the axis of the shaft is vertical, the ends of the arms are parallel to the picture plane, and the near right hand corner of the base of the shaft is 3′ 6″ from the picture plane and 4′ 6″ to the left. Height 5′ 6″; distance 13′ 9″; scale $\frac{1}{15}$. (Fig. 31.)

Find the perspective shape and position of the base of the shaft *F E G C*. Find on *A C V* the point *K*, 2′ 3″ nearer than *C*, and on *D C V* the point *H*, 2′ 3″ beyond *E*. Draw *H I* and

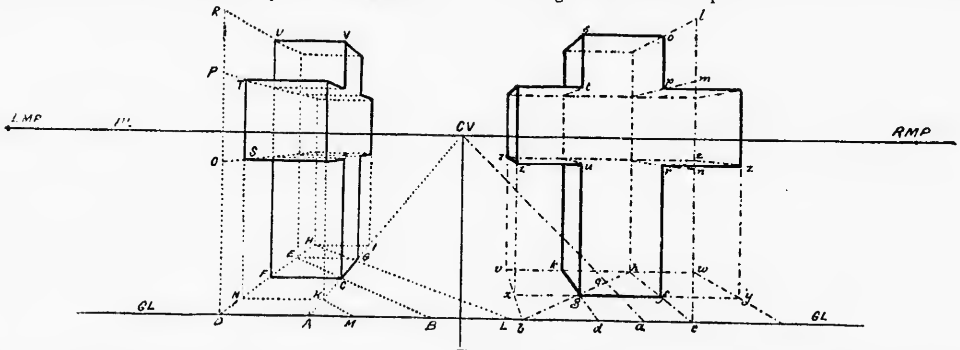


Fig. 31

K N. At *D* erect a perpendicular 9′ 3″ long, on it take the measurements required, and from the points *R*, *P* and *O*, draw lines towards *C V* to cut vertical lines from *N*, *F*, *E* and *H*. Then *U F* will be the height of the shaft at its near left hand edge, and *T S* will be the length of the edges of the near end of the near arm. From *C* draw a vertical line *C V* equal and parallel to *F U*. Horizontal lines from the corners of the left hand face of the cross, to cut vertical lines from *K*, *C*, *G* and *I* will complete the figure.

Problem 31.—Represent in perspective a cylinder 9″ high, 1′ 9″ in diameter, lying on the ground on one of its circular faces, the centre of which is 1′ from the picture plane, and 1′ to the left. Make its top face the base of a cone 2′ high. Height 1′ 6″; distance 3′ 6″; scale $\frac{1}{12}$. (Fig. 32.)

Find the position of the centre, *A*, of the base of the cylinder 1′ from *G L* and 1′ to the left of *L D*, and with *A* as a centre, and a radius of $10\frac{1}{2}$″ draw a semicircle and find in its circumference the points *F* and *G*. Measure to the left of *B*, by means of an arc, the distance of the centre of the semicircle from the picture plane, and draw *H R M P*. After placing the square in perspective find in it the points *h*, *f*, *l*, *g*, *k*, etc., as explained before, and draw the ellipse.

Measure on a perpendicular from *H*, 9′ to *K*, and draw *K R M P*. This will find two corners of the square which will contain the upper face of the cylinder. Draw its diameters and diagonals, and from the points *h*, *f*, *l*, *g*, *k*, etc., draw vertical lines to find in the upper square the points *h′*, *f″*, *l′*, *g′*, *k′*, etc. Through these draw an ellipse.

From *K* measure the height of the cone, 2′, to *L*, and draw *L R M P* to cut the vertical line from *a′* or *a* in *m*. Join the right and left hand extremities of the ellipses representing the top and bottom faces of the cylinder, and the extremities of the ellipse representing the top of the cylinder with the vertex, *m*, of the cone.

Problem 32.—Show the appearance of a hemisphere 2′ in diameter resting upon its flat face on the ground plane. Its centre is 1′ 6″ from the picture plane, and 1′ 3″ to the right. Height 1′ 6″; distance 3′ 6″; scale $\frac{1}{12}$. (Fig. 33.)

Find the point *O* 1′ 6″ back and 1′ 3″ to the right of *L D*, and draw the semicircle with a radius of 1′. By means of this draw the perspective form of the circular side of the hemisphere.

It is clear to all that a sphere will be represented by a circle, but the position of the centre of this circle will not represent the

EXERCISE 37.—Place in perspective a cross formed by a cube of 12″ edge upon each face of which is attached a similar cube, the corresponding edges of all the cubes being parallel to one another, and four edges of each cube parallel to both picture plane and ground plane. The centre of the lowest face of the cross is 6″ above the ground plane, 2′ from the picture plane, and 1′ to the right. Height 1′ 6″; distance 4′; scale $\frac{1}{12}$.

EXERCISE 38.—Place in perspective a square pyramid standing with its vertex in the ground plane and its axis vertical, the edges of the base are 18″ long and its altitude is 2′ 6″ long. The centre of the base is 15″ to the left and 4′ back, two edges of the base are parallel to the picture plane.

EXERCISE 39.—Using the same height and distance as in Exercise 37, and a scale of $\frac{1}{16}$, show the cross of Exercise 37 when resting with two of its edges in the ground plane perpendicular to the picture plane, the right hand edge in the ground plane being opposite to the eye. One face of the cross is in the picture plane.

EXERCISE 40.—Height 1′ 6″; distance 4′; scale $\frac{1}{16}$. A cross lies on the ground with the axis of its shaft horizontal and perpendicular to the picture plane. Its shaft is 1′ 4″ x 1′ 4″ x 4′. The arms are cubes of 1′ 4″ side attached to the shaft with their top faces 1′ 4″ from the top of the shaft. The centre of the top of the shaft is 3′ 6″ to the right and 5′ from the picture plane.

centre of the sphere. This is illustrated in the small drawing to the right, showing the relative position of the sphere and the spectator, drawn to a scale of $\frac{1}{24}$. The line $y z$ shows the diameter of the circle which will represent the sphere, and the point x', its centre, is the distance $x' x$ nearer than the centre of the sphere. Measure twice $x' x$ from O on $O P$, and represent the point S in perspective at s. With s as a centre, and a radius equal to its distance from either of the extremities of the ellipse representing the flat face of the hemisphere, draw a semicircle. The perspective centre of the hemisphere is shown at o.

PROBLEM 33.—Place in perspective a square of 5′ side in the ground plane. Its sides are at an angle of 45° with the picture plane, and its nearest corner touches the picture plane 3′ to the right. Height 5′; distance 14′; scale $\frac{1}{12}$. (Fig. 33.)

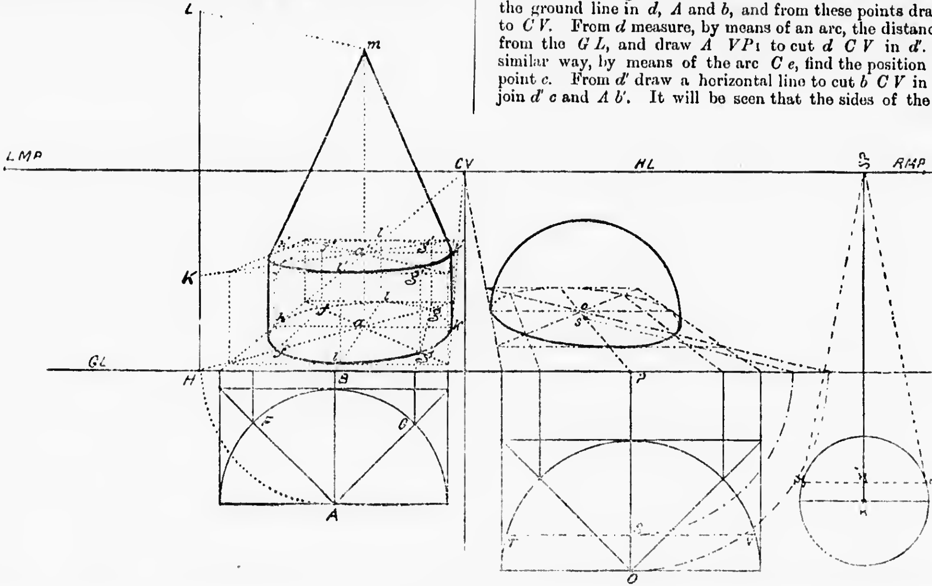


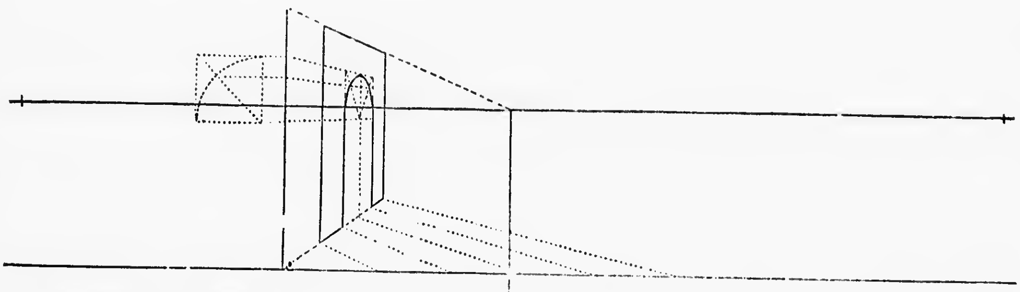
Fig. 32.

It will be noticed that this square is in a different position with regard to the picture plane than the objects treated of by previous problems; that its sides are neither parallel nor perpendicular to the picture plane. However, it can be treated in the same way as the triangle or hexagon, although in this case this is not the most convenient method.

First draw the square $A B C D$ with one corner touching $G L$ at A, 3′ to the right. From the corners draw vertical lines to cut the ground line in d, A and b, and from these points draw lines to $C V$. From d measure, by means of an arc, the distance of D from the $G L$, and draw A VP_1 to cut d $C V$ in d'. In a similar way, by means of the arc $C e$, find the position of the point c. From d' draw a horizontal line to cut b $C V$ in b', and join $d' c$ and $A b'$. It will be seen that the sides of the square vanish in VP_1 and VP_2, which points are also the measuring points for $C V$. But from fig. 6, and the remarks made thereon we have learned that every vanishing point has its corresponding measuring point, and we have also learned how the position of any measuring point can be found. With VP_1 as a centre and its distance from $S P$ as a radius drawn an arc to cut $H L$ in the

Exercise 41.—Below is given the perspective view of the left hand face of a wall in which is a semicircular arch drawn on a scale of $\frac{1}{48}$. Determine and state the height and distance and the measurements of the wall and arch, and show the appearance of the right hand face of the arch, if the wall were 3′ thick.

Exercise 42.—Using the same height, distance and scale as in Exercise 41, represent in perspective a cylinder 8′ long, 6′ in diameter, resting on the ground plane with its axis horizontal and parallel to the picture plane. The centre of the left hand end is 3′ to the right and 4′ back.



Exercise 43.—Height 3′; distance 7′; scale $\frac{1}{24}$. Show in perspective a hemisphere 4′ in diameter resting with its rounded surface touching the ground plane in a point 3′ to the left and 3′ back. Its flat surface is horizontal.

Exercise 44.—Height 3′; distance 7′; scale $\frac{1}{24}$. Place in perspective a rectangular block 2′ x 4′ x 6′ resting on one of its largest faces, the long edges of which retire towards the left at an angle of 45° with the picture plane. The nearest corner is 2′ 6″ to the left and 1′ back.

point marked MP_1. In a similar way find the position of MP_2. Having obtained these two measuring points it is reasonable to suppose that they will give the same result as has been obtained by means of the square $ABCD$ and CV as a vanishing point, and which result we know to be correct. To the right and left of A measure the length of the side of the square, 5′, to f and g. From f draw a line to MP_1 to cut $A\ VP_1$ in d', and from g draw a line to MP_2 to cut $A\ VP_2$ in b'. But these are the points already obtained as the left and right hand corners of the square, and we can therefore assume that the method of measuring lines vanishing in VP_1 by means of MP_1, and those vanishing in VP_2 by means of MP_2 is correct. From d' and b' draw lines towards VP_2 and VP_1 to intersect in c. It will be seen that this point is in $A\ CV$ perpendicular to the picture plane, which is the position of one of the diagonals, AC, of the square.

PROBLEM 34.—Show the appearance of the same square when it is in the ground plane, its sides form angles of 45° with the picture plane, and its nearest corner is 4′ to the left and 2′ back from the picture plane. Height 5′; distance 14′; scale $\frac{1}{48}$. (Fig. 33.)

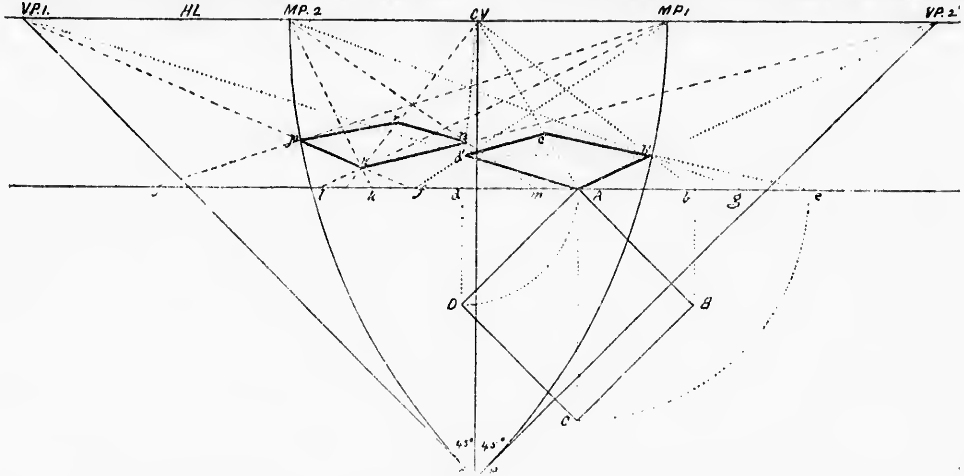


Fig. 33.

Find the position h of the nearest corner of the square, and from h draw lines to VP_1 and VP_2. Through h draw lines from MP_1 and MP_2, to cut the picture plane in k and l. From the point of contact k measure 5′ to the right to m, and draw $m\ MP_2$ to cut $h\ VP_2$ in n. From the point of contact l measure to the left 5′ to o and draw $o\ MP_1$ to cut $h\ VP_1$ in p. Then p and n will be the left and right hand corners of the square. From these points draw lines towards VP_2 and VP_1 to intersect in r.

PROBLEM 35.—Place in perspective a cube of 9″ edge resting on the ground plane upon one of its faces, all the edges of which are at an angle of 45° with the picture plane, and the nearest corner of which is 8″ to the right and 6″ beyond the picture plane.

Make the top of this cube the base of a pyramid 8″ high. Height 1′ 3″; distance 2′ 3″; scale $\frac{1}{8}$. (Fig. 34.)

First find the vanishing points and measuring points required. Find the position of the point a, and from a draw lines towards VP_1 and VP_2. Draw lines from MP_1 and MP_2 through a to obtain points of contact e and b. From e measure 9″ to f, and draw $f\ MP_1$ cutting $a\ VP_1$ in g. From b measure 9″ to c and draw $o\,MP_2$ cutting $a\,VP_2$ in d. Draw $g\ VP_2$ and $d\,VP_1$ to intersect in h.

Next notice that the vertex of the pyramid will be vertically above the centre of the base of the cube, and that a diagonal of the base of the cube will pass through the centre as well as two corners. At k erect a perpendicular, and on it measure 9″ to l and 8″ from l to s. Draw $l\ CV$ to cut vertical lines from a and h in m and p. Draw $m\ VP_1$ and $m\ VP_2$ to cut vertical lines from g and d in o and n. Join op and np. Find the centre r of the base of the cube by means of the diagonals, from r draw a vertical line to intersect $s\ CV$ in t. Join to, tm, tn and tp.

PROBLEM 36.—A model of an obelisk 8″ square at the base, 6″ square at the top, stands on the ground plane with its axis vertical and the nearest corner of the base 12″ to the right and 4″ from the picture plane. The edges of the base are at an angle of 45° with the picture plane. The top of the shaft of the obelisk is finished with a pyramid 6″ square and 6″ high. The total height is 2′. Show its perspective appearance. Height 1′ 3″; distance 2′ 3″; scale $\frac{1}{8}$. (Fig. 34.)

Find the perspective position of the corners of the base, as in problems 33 and 34, and draw its diagonals. Centrally between D and B, and C and E, take the measurement of the edges of the top of the shaft and transfer these measurements to the front edges of the base of the shaft, in L, M, N and O. From these points draw lines towards VP_1 and VP_2 to cut the diagonals of the large square, and thus obtain a smaller square representing the top of the shaft when in the ground plane. At P erect a perpendicular, and on it measure 2′ to W, and from W measure 6″ to R. Draw $W\ CV$ and $R\ CV$. Draw vertical lines from the corners of the smaller square, two of them to cut $R\ CV$ and the other two to cut $S\ VP_1$ and $S\ VP_2$. In this way the appearance of the top of the shaft is obtained. Join its corners with A, F, H and G, and also with the point X where a vertical line from K cuts $W\ CV$.

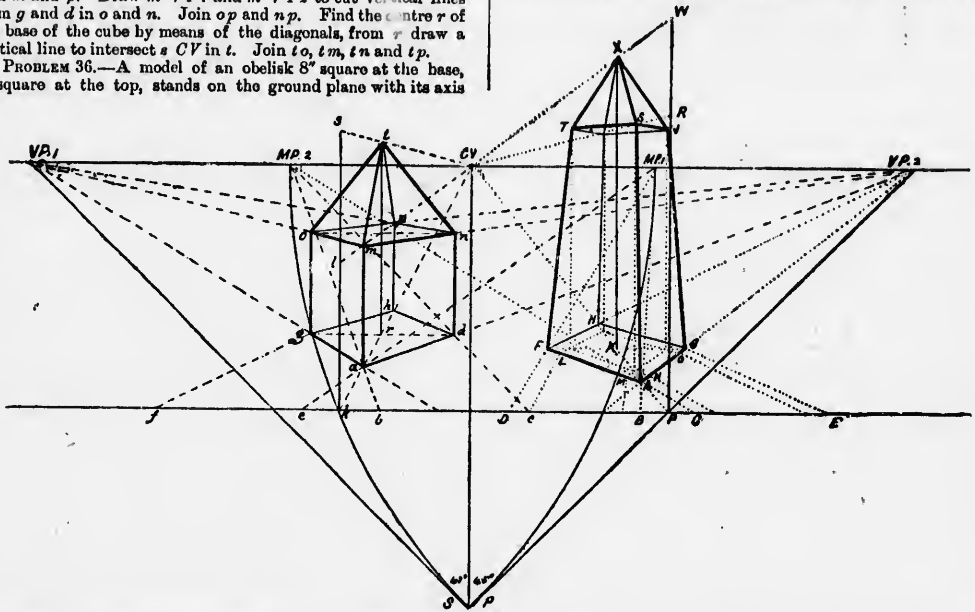


Fig. 34.