TRANSACTIONS

OF THE

Astronomical and Physical Society of Toronto,

FOR THE YEAR 1894,

INCLUDING FIFTH ANNUAL REPORT.

PRICE ONE DOLLAR.

TORONTO:
ROWSELL & HUTCHISON,
Printers to the Society.

1895.

CHARLES CARPMAEL, M. A. (CANTAB.)

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CORRIGENDA.

Transactions, 1890, page 29, line 8 from bottom, for "Jupiter" read "Neptune."

Transactions, 1891, page 5, line 11 from top, for "B.C. 380" read "B.C. 310."

Transactions, 1893, page 59, line 9 from top, for "7 p.m." read "11.15 p.m."

Transactions, 1893, page 67, line 3 from bottom, value of comp. log *n'* should be "2·2132633."

Transactions, 1894, page 16, line 3 from top, for "1·500th" read "1-500th."

" " " line 5, for "1·250th" read "1-250th."

Transactions, 1894, page 31, line 14 from top, for "two angles" read "three angles."

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Note—Authors are alone responsible for views expressed in papers or abstracts of papers published in this report.

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TRANSACTIONS

OF

The Astronomical and Physical Society

OF TORONTO,

DURING THE YEAR 1894.

FIRST MEETING.

January 23rd. On taking the chair, Dr. Larratt W. Smith, Q. C., thanked the Society for the honour of re-election as Vice-President, and assured the members he would continue to take an active interest in their work. (Applause.)

Mrs. Annie G. Savigny was elected an active member of the Society.

Dr. A. D. Watson reported at some length his observations of Saturn; in his telescope, a 3-inch by Vion Frères, of Paris, he had been able to see Cassini's division very clearly, a test considered severe for the aperture used. He referred also to the approaching conjunction of Venus, and thought it might be of some interest to note how near to the Sun the planet could be seen.

The Chairman announced that short papers on Jupiter had been prepared by several members, and called first upon Mr. W. B. Musson, who read some notes on the "Mythology of Jupiter."

Mr. Musson said that to the astronomer the word "Jupiter" is merely a name, distinguishing the largest planet of our system; it was not so to the ancients; to them the word had a far deeper signification. Jupiter was the chief of all the deities; the lord of the heavens; the god whose favour was to be won by sacrifice and supplication, and whose anger meant destruction. How, then, did the ruler of Olympus, and the beautiful and colossal planet become associated under the same name? Did the observations of the planet, and its supposed influence over human destinies develop in Man the idea of the god, or was man's original conception of the deity personified in the mysterious wanderer of the skies? The study of the heavens stretches so far back into the youth of the human race that its beginnings are shrouded in the gloom of tradition. Astronomy did not originate with any one particular nation, but belongs to all races and all ages. Chinese, Arabians, Egyptians, Chaldeans, Hindoos, Greeks and Romans, among others, were students of the Science, in one or other of its forms, and have bequeathed to us their discoveries, their legends, and their myths. It is true the study only developed into a system when something like civilization had appeared, but the rudest savage must in some degree have observed the appearance and movements of the heavenly bodies. The lonely hunter would, doubtless, note the advancing constellations and superstitiously connect them with the fortunes of the chase. A certain group of stars invariably appeared at the season when the lion, pressed by thirst, most frequented the lakes and pools. Hence, the "Leo" of the Zodiac. When pastoral life began it was still more necessary that Man should have some means of measuring time. He must record the time of breeding in his flocks, the hiring of labour, and the season for sowing seed. What clock, then, more convenient, than the Vault of Heaven? The movements of the "eternal stars" would thus become identified with terrestrial affairs, and might not improbably come to be regarded as their causes. The planets would naturally be included on the heavenly dial, but a very little observation would suffice to mark the fact that their movements were not regular, and they would then become objects of special attention. If the Sun, Moon and stars were found to serve a purpose, why should not the planets exert an influence upon human affairs? Here, then, we arrive at a point where the stars may have become woven into the religious beliefs and confounded with the deities of Man. We know that in the olden time the study of the heavenly bodies certainly formed an important part of religion, and that in the stars men saw their deities visibly enthroned in the mysterious depths of space. So powerful, in fact, was this influence that it is said when Bede and other theologians in the eighth century wished to depose the Olympian gods, they proposed, as the first step, to substitute for the name of the Ram, the first of the Zodiacal groups, that of St. Peter, for the Bull, St. Andrew, and so on. The planet Jupiter, on account of his great lustre, was one of the first single objects to attract attention in this way. This wandering star had various names among different nations; in fact, we know that his Egyptian name corresponds to our word "brilliant"; but he was best known as the Roman Jupiter, or Greek *Ζευς*. Had we time, it would be interesting to recall the various powers and characteristics attributed to this planet. For instance, the Babylonian priests identified Jupiter with the metal tin, Pliny thought him to be the origin of lightning, whilst Albert the Great assigned to him the colour blue, asserted that he was "ruler over honour, riches, wishes and cleanliness," and regarded him as temperate and benign in influence. On the whole, Jupiter appears to have borne a creditable reputation. In the following words, an almanack of 1368 testifies to his innocence:—"Jupiter is hot and moist, does good to all things, and hurts nothing." Surely, this is all that could reasonably be expected of any planet.

Let us now see what manner of god it was that had been evolved by this process of time and of study. Among the Romans, Jupiter was supposed to care for the vintage. The Ides of the month were sacred to him, and he was held to be the "spirit of oaths." No war was undertaken without first seeking his favour, and thanks were always returned to him for victory. His Hindoo name signified "Master of the Faith." The Etruscans supposed him to be the Revealer of Fate. The god Jupiter could claim to be descended from the more primitive Indo-European period, and was thus an exception to the rule that a deity common to Grecians and Romans must necessarily have been derived by the latter from the former people. The influence, however, exerted on Rome by the superior civilization of Greece led the Latin writers to identify Jupiter with Zeus. The Jupiter of worship was a Roman—the Jupiter of literature was half a Greek, and most of the legends attributed to him belong to the Hellenic race. From the Latin this deity emerged as a Roman-Greek, and this character may be regarded as a development of the original Aryan conception of God. The Græco-Italian god was supposed to govern the phenomena and cyclic changes of the heavens, to control the light and darkness, the thunder and the rain, and, it was believed, could be drawn down upon Earth in the form of lightning. These legends and superstitions are now stories of the past; but if we are sometimes inclined to regret that Science, with her "clear, cold light," has dispelled much of the tradition and fable of ancient times, we may console ourselves with the knowledge that she is teaching us, in the continuity and symmetry of law, a truth more beautiful than any myth, and revealing to us in the power which moulds the universe, a deity grander and more sublime, than the giant Thunderer of old. Let us hope that what has been true in the past may also prove true of the future, and that the words of the great poet may be prophetic :—

> "Yet I doubt not thro' the ages one increasing purpose runs,
> And the thoughts of men are widened with the process of the suns."

The following is a synopsis of some remarks by Mr. Thomas Lindsay on Jupiter's density :—

If we assume that the nebular hypothesis does truly outline the history of the solar system, then we may readily calculate the density of this nebula at any given period of its existence. Supposing it to extend to the orbit of Jupiter, we find that its diameter was 1,118 times greater than that of the Sun now ; its volume compared with the volume of the Sun was as the cube of this number to unity ; and as the same mass was in the nebula as is now in the Sun, neglecting the planets since formed, we have the density of the nebula, in round numbers 14,000 million times rarer than the density of the Sun at present. Then a great ring is thrown off from the equatorial regions of the nebula, a great ring which ultimately becomes the planet Jupiter. The ring must have been less dense than the nucleus of the nebula, but having no means of arriving at the difference, we assume the same density throughout. Now, suppose we are standing somewhere in space viewing this phenomenon, we say, "There is a mass destined to become a planet ; there is the parent nebula, contracting, condensing, and destined to throw off more rings. This new planet we see now will be cooled down and condensed into a solid mass long before the great nebula has finished its work." Now, if any one, say on some distant star, looking on the scene reasoned thus, he was altogether wrong ; for notwithstanding the enormous difference in size, the nebula which is now the Sun is of about the same density as Jupiter. We say that the Earth is much younger than Jupiter in years, but older in planet life ; that it is about 300 times less in mass and has cooled down in less time. But the mass of the nebula was 1,000 times greater than that of Jupiter, and both have kept even pace. This seems to require some little explanation, and I offer the following : We are not to suppose that the constitution of a nebula is the same throughout, and when Jupiter was thrown off, the atmosphere of the great parent mass may have had a constitution entirely different from that of the centre. The elements of the atmosphere may have been such that after all these ages of cooling, their mean density is only about one and a half times that of water. I understand that the light from Jupiter is wholly reflected light, and that it is extremely improbable that the planet is glowing like a Sun ; and surely, considering the enormous time that has elapsed since his birth, we would suppose him to be pretty well cooled down, and condensed as much as he ever can be. I conclude, therefore, by venturing the theory that Jupiter is not a hot world, but a very cold one to live in.

In reading some notes upon Jupiter's atmosphere, Mr. R. B. Ellis said it was admitted by good authorities that Jupiter has an atmosphere similar in constitution to that of the Earth, and that spectroscopic examinations showed the presence in it of a large quantity of aqueous vapour. In support of this Mr. Ellis quoted from the chapter on Jupiter in Proctor's Old and New Astronomy, and referred to the astounding results reached on the assumptions that the air of Jupiter is similar to that of the Earth ; that the pressure at the upper part of the Jovian cloud-layers is not less than the pressure above the highest of terrestrial cumulus-clouds, and that the depth of these cloud-layers is one hundred miles. The opinion was expressed that the error lay in the last assumption, and that it was not possible to estimate, with any degree of accuracy, the depth of the clouds at so great a distance.

In the course of some discussion on the atmosphere of the planets generally, Mr. A. F. Miller stated that there was no direct spectroscopic evidence of the existence of oxygen in the atmosphere of Jupiter.

Mr. A. Elvins followed with some diagrams illustrating the distances of the satellites and the phenomena presented by them as they revolved about the planet.

Mr. Arthur Harvey, continuing the subject of Jupiter's satellites, said :—

Kepler declared, long since, that Mars ought to have two satellites, and they were found by Professor Asaph Hall, in August, 1877. Deimos was seen on the 11th ; and on the 17th, by concealing the disc of the planet with a thin metallic plate, Phobus was seen. This set astronomers a-looking for the two missing moons of Jupiter, for Gaussius had formulated a law under which they ought to exist. The following are the real and calculated distances of these bodies :—

	1	2	3	4	5	6
Real	2·50	——	6·05	9·62	15·35	27·00
Calculated	2·19	3·60	5·92	9·72	15·95	26·23

Professor Barnard saw the interior satellite with the Lick telescope on September 15th, 1892. The period being $11^h\ 57^m\ 22{\cdot}56^s$, and the rotation of the planet $9^h\ 55^m\ 37^s$, Laplace's theorem is verified. We may hope that the Yerkes telescope will enable the next yet undiscovered satellite to be seen, when Jupiter comes into a favourable position. The Arago medal was awarded by the Academy of Sciences, Paris, to both the above astronomers.

Dr. Watson described his telescopic work upon Jupiter at the oppositions of 1892 and 1893. He remarked that he could not now see the red-spot upon the disc.

Mr. G. E. Lumsden stated that in his 10-inch reflector, the indentation caused by the red-spot in the belt was plainly visible, and drew on the black-board a sketch of the position of the spot.

Mrs. Savigny read a short paper in a fanciful vein, sketching the conditions under which the inhabitants of Jupiter, if there were any, would exist.

It was announced that at the next meeting Mr. Joseph Allen would, by request, read a paper on "Force and Energy."

SECOND MEETING.

February 6th; Mr. Arthur Harvey presiding. An interesting letter was read from Dr. Sandford Fleming, C.M.G., who had recently returned to Ottawa. Referring to the proposed change in Astronomical Time Reckoning, Dr. Fleming said that, during his sea-travelling, he had conferred with many captains and other sea-officers who, without exception, expressed themselves in favour of the change. He was glad to learn that there had been so many affirmative answers received from the scientific men to whom circulars had been sent, asking their opinions.

Mr. A. Elvins reported some observations of the Sun and Venus. The planet had been well seen, showing a thin crescent gapped along the terminator. It was mentioned by a member that an observer in Barbadoes had recently seen the dark body of the planet, an unusual observation, and corresponding to the familiar "new Moon in the arms of the old."

In a letter read by a member, some doubt had been cast upon the beauty of the well-known constellation of the South, the Southern Cross. Mr. J. Van Sommer wished to vindicate the claims of this conspicuous group, and was surprised that any one should fail to remark its glorious appearance. He said, however, that seeing the constellation low down in the horizon detracts from its beauty.

The paper for the evening was read by Mr. Joseph Allen on

FORCE AND ENERGY.

It was a terse statement of the views of Mr. Grant Allen, his son, as defined in his well-known work, "Force and Energy; a Theory of Dynamics." While Mr. Allen stoutly defended his distinguished son, he did so in a very impartial manner. He mentioned the eminent physicists who had diametrically opposed his theories and done all they could do to crush them out, and stated their reasons for so doing, but added that there was evidence that some of them were now disposed to be less severe in their criticisms. Mr. Allen endeavoured to show that force and energy, always in ceaseless antagonism, divide between them the empire of the universe, and so keep things in wholesome activity; that force is aggregative power, but energy is separative power, thus differentiating force from energy. He sought to show that in no instance was a union between bodies effected under the dominion of force without liberating a vast amount of counteracting energy, and he brought a great array of facts from every department of physics to prove the truth of this theory—from mechanics, from chemistry, from magnetism and electricity. He averred that throughout the whole domain of the sciences there was not one instance to the contrary; that it was only in a state of separateness—as when a body is lifted from the Earth, or oxygen and hydrogen, relieved from their combination in water, when, by being combined, they had lost their specific energies—that bodies manifested their great special powers; that carbon and oxygen, in the act of uniting in our fires, gave up for our benefit the enormous specific energies that belonged to them in their separate state as elements, but, by combining, dropped (as in the case of water before referred to) to the far lower plane of carbonic acid, which could only get back its native energy by being split up by means of Sun energy into its original elements. In dividing all bodies into forces and energies, Mr. Allen contended that the forces were gravitation, cohesion and chemical and electrical affinity; but that heat, light and electricity are energies. He showed, too, that electricity, in a natural state, *i.e.*, in a state of combination of the two electricities, is in a condition of neutrality or dormancy, but that when separated into their elementary state of positive and negative electricities, and then only, they are in a position to yield up their energy of separation to do the work of our horses and steamers.

A cordial vote of thanks, moved by Mr. J. M. Clark, M.A., and seconded by Mr. Elvins, was tendered Mr. Allen at the conclusion of his address.

A short discussion followed, but it being felt that justice to the paper could not be done in the short time remaining, it was decided to set apart the next meeting for the purpose of hearing short papers upon such portions of the subject of Energy generally, and upon Mr. Allen's theory especially, as the members might select.

THIRD MEETING.

February 20th; Mr. John A. Paterson, M.A., in the chair. A letter was read from Mr. G. G. Albery, of Meaford, conveying the information that an Astronomical Society had been organized in that town. Mr. A. Harvey, seconded by Mr. A. Elvins, moved that the Council take into consideration the question of affiliating with this Society any similar associations which might be formed throughout Canada. Carried. Mr. Harvey reported having observed Venus on the evening of February 12th, the planet being then three days from inferior conjunction. Mr. T. Lindsay and Mr. D. G. Ross referred to an observation of the Constantine Cross, which they had made on the evening of February 19th; the Cross was drawn apparently through the Moon, and was fairly well defined. Some drawings of sun-spots made at a small telescope by Mr. G. Wellings were presented. Some discussion arising on the general subject of sun-spots, Mr. C. A. Chant, B.A., stated that the connection between these and magnetic disturbances was still in the realm of doubtful coincidences.

The Chairman announced that short papers on Mr. Grant Allen's theory of Force and Energy would be read. Mr. T. Lindsay read a short review of the subject. He held that Mr. Allen's views of Force and Energy might be of service in giving general instruction, but no advance was made towards reaching the basis of dynamics. Mr. G. F. Hull, B.A., and Mr. C. A. Chant, B.A., followed; both found difficulty in accepting theories which were at variance with those upon which a great science had been built up.

Mr. A. Elvins considered that both Force and Energy were forms of motion, and that it was quite unnecessary to speak of them separately. Mr. D. G. Ross referred in very strong terms to the high standing in the literary and scientific world which was accorded by all to Mr. Grant Allen, and considered that it was no argument against his theories that they were not endorsed by certain great physicists, whose names had been mentioned.

Mr. J. A. Collins read the following remarks on "Force and Energy":—

If it be not considered presumptive on my part to attempt a criticism of the dynamical theories of so distinguished and brilliant an author as Grant Allen, I will endeavour to analyze one or two features of the argument put forward in support of his hypothesis of *dynamical dualism.*

Mr. Allen's position seems to be this: that there are in the material universe but two forces or powers, each the opposite of the other, striving for the mastery; one aggregative, continually drawing together atoms, molecules and masses everywhere throughout the universe—the other seggregative, tearing asunder and dispersing in every direction with equal activity the handiwork of its rival. Thus, the phenomena we see around us every day are but incidents in the great conflict waged by these twin Titans, which will not cease until the weaker has yielded and the stronger asserted universal sway. The combatants in this dual struggle bear the cognomens respectively of "Force" and "Energy,"—Force representing aggregative power and Energy seggregative power, an interpretation differing considerably, it will be seen, from the sense in which those terms are generally understood by physicists of the present day. Mr. Allen cites the directive difference between centrifugal and centripetal force in the rise and fall of a mass of iron (or other body) projected for a short distance from the Earth's surface, and the orbital revolutions of the planets, as evidence of dual powers—that the motion of a body when directed towards the Earth and the motion of the same body when directed away from the Earth are in every case caused by diametrically separate and alien motive powers. If we confine our examinations of the facts in this connection to these two points, I think we shall find that the evidence will not warrant such a conclusion.

Mr. Allen points to the fact that a mass of iron weighing say one ton, when at rest, presses on the surface of the Earth with that weight only; but when let fall a short distance, strikes with a force many times its own weight; and he asks, how does the mass get this extra power unless something has been added, or taken away from it, in its downward flight? His explanation of this sudden acquisition of power seems to be that in this, as in every analogous case, energy has fled and force has triumphed. But if we examine the facts closely, we shall find there is nothing either added or taken away in the shape of power whatever.

In the first place, the weight of the body named is merely the measure of resistance sufficient to keep it at rest while under the full influence of gravity's impulsions, which are continually pulling or exerting power upon it. This power when it reaches the body gives it translatory motion, the direction and intensity of which, in accordance with the well-known laws of motion, are directly as the mass and in a straight line for the centre of attraction, with an acceleration of speed or motion varying inversely as the square of the distance from the attracting centre —consequently, a body falling freely from a position of rest, is, as it were, in a state of suspended animation; meeting with no resistance, the motion continually being put into it by gravity accumulates more and more, until, reaching the surface of the Earth in its onward rush, it delivers at a blow in one unit of time the concentrated energy of hundreds or thousands of units of time.

While the body is at rest on the surface of the Earth, gravity is still continually putting power into it, but this power, as fast as it is being received, is being continually transferred from the body to the Earth's crust and passed along towards its centre in the form of heat, and the weight or energy of the body manifested under these conditions becomes constant, being apparently the measure of the fraction of unit of time elapsing between the receipt and discharge or transference of this power or gravitational energy. But as soon as contact with the Earth ceases and there is no longer a medium capable of resisting and transferring the gravitational impulsions, all escape of power being shut off, the power accumulates in the form of motion in exact proportion to the time units of isolation. Thus, the units of time, space, resistance (or weight), and power, are equal; there is nothing gained and nothing lost but position in the whole transaction.

In the second place, if it can be shown that the mass of iron can be elevated again to its starting place by the force of gravity alone, I think it will be clear that aggregative and seggregative motion are, in this case at least, the result of one and the same power, the difference being merely one of position, when units of time, space, resistance and power are considered. Suppose, for instance, a plank or beam of sufficient strength to be balanced across a fulcrum. On one end of this plank rests the mass of iron that has just fallen. Suppose another equal mass of iron to fall freely, as in the first case, impinging on the free end of the plank; the mass at rest on the other end will be hurled by the intercepted gravitational force high into the air, and, as we all know, if it were not for the loss by friction, it would rise to the height from which the other came. The falling mass would carry the free end of the plank with it in its downward rush, and if the force of impact were not greater than the molecular cohesion of the plank particles, the particles would refuse to part company, conveying the downward motion of one end along and into upward motion of the other, until its motion would be checked. The mass upon it, however, being impelled with the same velocity and having nothing to check it, rushes onward and upward, until the motion put into it is spent in overcoming the gravitational impulsions exerted upon it, in equal units of time to that occupied by the other body in falling, aggregative motion being turned into seggregative or dispersive motion, the difference without friction being one of position only, with but a single motive power in operation.

The Chairman then called upon Mr. Joseph Allen to close the discussion. Mr. Allen defended the theories advanced by his son in spirited language, and offered explanations on many points upon which there was disagreement.

FOURTH MEETING.

March 5th; Mr. John A. Paterson, M.A., Vice-President, in the chair.

Miss Annie Gentle was elected an active member of the Society.

The Society approved of the report of a committee which recommended that the Sir Adam Wilson telescope be set up at No. 23 Walmer Road, the residence of Mr. J. A. Paterson. This site was chosen on the grounds of easy access, excellent horizon, and the desirability of having a good telescope in the western part of the city, the other parts being fairly well supplied with private instruments.

It was announced that this telescope would be available for use on the Tuesday evenings alternating with the regular meetings of the Society, and that a member of the Society would be appointed to assist in using it.

Mr. Harvey read a memorandum, in which he stated that the large sun-spot of the 16-28th of February was visible to the naked eye; that during 1893 there was no month without a spot similarly visible; that during the first half of the year there were 225 sun-spots, and during the second halt 209 spots; that those in the latter half covered a greater area than those of the first half; that he was inclined to the opinion that spots are purple in colour, and may best be photographed by indirect rather than direct methods--for instance, that better results would follow photographing fine sharp images obtained by projection.

Dr. J. C. Donaldson said he, too, had noticed colour in sun-spots, but had attributed it to the use of low powers on his telescope, as the colour seemed to disappear when high powers were substituted. He referred at some length to a series of difficult double-star observations he has been making with a view to preparing, for the use of Canadian amateurs, a list of test-objects for good telescopes of aperture of three and one-half inches and less. He dwelt upon the necessity of buying the very best instrument that one could afford, rather than the largest in size, and said he would prefer a first-class one inch and a half telescope to a three-inch instrument by an inferior maker. Several severe tests he had made successfully had convinced him that the Canadian air is as clear as any and as well adapted for telescopic work.

Several short papers were read by members on

THE AURORA OF FEBRUARY 23RD.

Mr. J. R. Collins, after describing minutely the beautiful appearance of the aurora, said : The phenomenon seemed to be what might be expected either from the passage of a cosmical cloud through the Earth's atmosphere or from discharges of electrically charged clouds of water vapour. The Boreal Crown would indicate the vanishing point of the former, while the rainbow colour-bands, with their close proximity to the Earth's surface, would point to the latter. Perhaps, the most remarkable feature of the observation was that swishing, rustling sounds were heard during the early part of the display (both by myself and my brother, Z. M. Collins), as if sleet were being driven by the wind over crusted snow, as patches of light cloud were whirled apparently near the ground in our immediate vicinity, accompanied by displays of light. As the air was remarkably dry and still at the time, it seemed evident to us that the sounds we heard were produced by an electric discharge of some kind between the light clouds that came near us and the lower stratum of the atmosphere through which they passed.

Mr. S. Hollingworth had not heard the swishing sound during the display, but had been impressed with the apparent proximity of the streamers. He said that as he approached a row of Lombardy poplars, the rays had the appearance of passing through these trees in an upward oblique direction, and to him it appeared as if the streamers were not more than twenty feet away. Particular attention was paid to this, the object being to report the circumstance. It appeared almost like a stratum of fog, only it did not obscure the trees in any way, and the trees appeared to be enveloped in it, with the exception of about twenty feet next the ground.

Mr. John A. Copland reported that on the evenings of Thursday and Friday, February 22nd and 23rd last, he made observations of auroræ. That of Tuesday evening was of the longest duration, but that of Friday night attained the greatest brilliance. At about 8 o'clock on Tuesday evening the entire sky for about 45 degrees above the northern horizon, and well round toward the east, was covered with a dirty-coloured haze, such as is often seen presaging a storm. As the hours passed and the cold intensified, this brownish haze dissipated, and faint streamers began to flicker upward, until about 9.30 the sky was apparently clear. Before

10.10, when the observer next noticed, the aurora had returned with renewed vigour. A peculiar feature which he remarked about 11 o'clock was a pyramidal radiance emanating between due North and about 60 degrees East, which tapered toward the zenith and culminated in a fleecy spiral form high in the starry dome. This panorama gave place to a still more beautiful one of alternating pink and white streamers.

Maximum brilliance may be said to have been reached about 11.40 p.m. Afterward the reddish streamers and markings disappeared, reappearing with less notability between 12.20 and 1.05 a.m. Thenceforward the display became a steady yellowish glow, with sporadic outbreaks of white streamers, until 3 a.m., when it settled into a regular auroral arch, whose apex was about 25 degrees from the horizon and nearly ten degrees East of North. Underneath the arch, which had a thickness of approximately 8 degrees, the sky was perfectly clear and of a deep blue. This arch continued, sometimes considerably agitated by a rolling motion toward the East, until 4.35 a.m., when observation was discontinued.

Friday night's display was somewhat similar to that of the previous evening up till about 10.30 p.m. From that hour until nearly 12 o'clock the display was gorgeously active, more so than on the foregoing evening, a striking phenomenon being the number of red spots and streaks resembling incandescent hydrogen. Whenever a red variation appeared, it became quickly surrounded with white, which eventually took the place of the red. Green and yellow were also noticeable, the former colour always near to the base of the display. As on the former evening, the greatest activity and mass of the aurora was toward the North-east, which the observer states to have noticed is well-nigh invariably the case in important auroral displays, possibly caused by the rotation of the Earth. By about 12.45 a.m. the aurora had faded much, and before 4.30 a.m. it had disappeared, with the exception of a few faint white streaks which intermittently flickered in the North-eastern heavens.

On Saturday and Sunday evenings also there was a dim auroral display from 10 p.m. until 11.30, and on Monday night a green glow suffused the North-eastern heavens from 11 p.m. until 2.30 the following morning. Much light was emitted by the Moon, which interfered with the brilliance of the displays a good deal. A fact mentioned by the observer is that on Friday night whenever one of those sharp interchangings of red and white occurred there appeared to linger in the vicinity a haze somewhat resembling the gaseous material which forms the diaphanous streak in the wake of a swift meteor. If the Earth were being bombarded with a deluge of noxious gases, says he, how comfortable we may feel when we see our atmosphere so completely dissipating them before they can inflict death upon us.

FIFTH MEETING.

March 19th; Mr. John A. Paterson, M.A., Vice-President, in the chair.

A communication was received from the Librarian of the Wisconsin Academy of Sciences, Arts and Letters, who made a formal application for a copy of the recently published "Transactions," and asked the Society to exchange with the Academy, which issues a voluminous and very valuable annual report.

Mr. J. A. Copland presented some notes taken from the report of the Ontario Minister of Education, which were very encouraging and suggestive of the future. The number of books taken from the various Free Libraries and Mechanics' Institutes throughout the Province for the preceding year was given as follows:—In Science and Art, 73,170; Biography, 37,159; History, 58,194; Religion, 31,194; Poetry and the Drama, 15,228. Mr. Copland thought there was undoubted evidence that the study of Science was becoming more and more popular in Canada.

The Chairman read a series of extracts from an article by Sir Robert Ball, recently printed in *The Fortnightly Review*, entitled "Significance of Carbon in the Universe," and discussing the theory of Mr. G. Johnstone Stoney, first enunciated some thirty years ago, and which is being revived as one which had not been sufficiently investigated. Short papers on Magnetism were read by Mr. G. G. Pursey, Mr. J. A. Collins, and Mr. Thomas Lindsay. Some instructive experiments were performed by Mr. Collins, and also by Mr. A. Aronsberg, who brought with him a powerful magneto-machine.

Mr. Collins had constructed one apparatus for the purpose of determining how the strength of a magnet varied at short distances, and stated that he had found the ratio quite the reverse to what is usually accepted. A magnet was brought in contact with a piece of iron attached to the end of a spring balance and was found to bear a pull of a certain weight. When separated from the armature by 1·500th of an inch the power of the magnet was reduced one-half. As the distance was reduced to 1·250th of an inch, the power of the magnet was reduced one-quarter. Mr. Collins held that these experiments proved that the power of a magnet varies directly as the distance for short distances, and also stated that he thought there were some grounds for believing that when a magnet is in apparent contact with an armature it is still really separated from it by an appreciable distance, a point to which Mr. Lindsay had previously drawn his attention.

It was decided to continue short papers and simple experiments in connection with this subject at a future meeting.

SIXTH MEETING.

April 3rd, 1894; held in the Physical Department of University College, Queen's Park, Mr. John A. Paterson, M.A., Vice-President, in the chair. A large number of the members and their friends were present, in response to the invitations sent out by the Secretary.

A letter was read from Mr. E. S. Holden, LL.D., Director of the Lick Observatory, congratulating the Society upon the success it had attained. Mr. Holden had noted with great pleasure the portrait of Mr. A. Elvins, which accompanies as a frontispiece the fourth volume of the "Transactions"; he considered that the compliment to Mr. Elvins as one of the founders of the Society was highly deserved, as was also the recognition of the valuable services of Mr. G. E. Lumsden in materially furthering the Society's interests.

The Chairman then announced that Mr. G. Ferrie Hull, B.A., Fellow in Physics, University of Toronto, would deliver the second of a series of popular lectures, which, through the kindness of the President of the University, would be continued from time to time in the physical lecture room. The first of the series had been given during the preceding session by Mr. C. A. Chant, B.A., on the subject of the "Polarization of Light."

The following is a synopsis of Mr. Hull's lecture, which was profusely illustrated by experiments, most satisfactorily performed with the able assistance of Mr. Plaskett:—

COLOUR.

Very few, indeed, are the persons in this audience who are not appreciative of the beautiful effects which Nature presents in the crimson and gold tints of our sunsets, in the silver and grey of the clouds, or in the shifting panorama of the Aurora Borealis. How often have we stood in silent, reverential admiration, while beholding that most charming of all phenomena, the rainbow in the heavens! How often, also, have our spontaneous exclamations of praise been called forth by the happy and skilful mingling of colours by the hand of man—as seen in the cleverly executed water colour, in the decorations of the hall of a Legislative Assembly, or in the coloured fountains of a World's Fair. True, life would be possible, and we should be able to arrive at a very high state of intellectual and æsthetic culture without this power to perceive colour; but Nature would lose much of her charm and most of her variety.

Shakespeare says of the man who has no music in himself, who is not moved by concord of sweet sounds, that the motions of his spirit are dull as night and his affections dark as Erebus. Might it not also be said of the man who has not the power of perceiving colour, that, being compelled to walk along the shady avenues of life, his nature would adapt itself to its environment, and would in consequence lack that variety and charm that belong to the æsthetically cultured.

But it is not the æsthetic effect of colour that I wish to treat of to-night, but rather of its physical character. And in this connection you will allow me to show some very simple and well-known experiments. The first I wish to show is one which demands our attention, not only on account of its intrinsic worth, but also on account of its historic interest.

In the year 1666 the great Sir Isaac Newton was experimenting with light. Through a small hole in the shutter of the window he allowed a beam of light to pass into the room, giving the round white image of the Sun on the wall. Placing a triangular piece of glass in the path of the beam, he found that the white image of the Sun disappeared, and in a different position there appeared a long band of colours. This experiment, with which every school boy of the present day is acquainted, became the basis of a very large part of optics, and through it of science.

Experiments.—Placing this prism in the path of a narrow beam of light from the electric lamp, we find the spectrum thrown upon the screen; although we have all seen this phenomenon many times, for none of us, I may venture to say, has it lost or will it ever lose its power of fascination. Its beauty is none the less, because of the simple method of obtaining it. The experiment shows that white light is composed of all these colours. That these colours will, when united, produce white light may be shown by placing in the path of the coloured rays this second prism similar, but oppositely situated, to the first. The coloured rays are united into a narrow white band. If we allow the coloured rays to fall upon a concave mirror or to pass through a convex lens and to be brought to a focus, we find them united as before. Causing the rays to fall upon a mirror and from it to the screen, when the mirror is rocked to and fro quickly, we see a long band of white light, with coloured ends.

These experiments show that white light is not a simple quantity but is composed of different parts, which give to the eye the sense of a number of colours, and also prove the reverse of this, that the superposition of these colours produce white light.

There yet remains the question what is colour, and that resolves itself into the question what is light? What is that wonderful agent which, travelling through the universe, gives warmth and life to both the animal and vegetable world? Most of our light comes from the Sun. But how does it travel, and what is it when it arrives? We might at first suppose that light consisted of material particles shot out from the source. This was the theory advanced and ingeniously upheld by Newton himself. He was able to *explain* many of the known properties of light, but in order to explain the law of refraction it was necessary for him to suppose that the velocity of light in a dense medium such as water was greater than its velocity in a rare medium such as air. About 150 years after Newton's time this was shown by direct experiment to be false; and so the emission theory, as it was called, had to be abandoned.

Another theory was probably suggested by analogy. It was known that sound travelled by the particles of air giving up their motion to the particles next them. It was also known that a disturbance on the surface of water, caused for example by the dropping of a stone into it, was communicated to distant parts by the up and down motion of the particles of the water. Hence, it was supposed that light might be due to wave motion or periodic disturbance in some invisible substance.

One experiment was wanting to confirm the hypothesis. If light is due to the vibrations of particles of a substance, then it should be possible to arrange an experiment in which the vibration of a certain particle, due to one source, would be equal and opposite to the vibration of the same particle due to another source; in that case the particle would have no motion, or there would be darkness at that point. This principle of interference is hard to show in the case of light, but it is of common occurrence in sound and in water waves. It is illustrated in the experiment with Crova's disk, which shows the mode of propagation of both transverse and longitudinal waves, and the interference of two sets of waves. (Experiments shown.)

Young and Fresnel both succeeded in showing experimentally these points of rest in the space acted on by two similar sources of light, and were thus able to measure the wave length. Many experiments have since been devised confirming their work, and consequently the wave theory is to-day accepted by nearly all physicists. The wave lengths of the chief colours of the spectrum are approximately as follows:—

Colour.	*No. to the inch.*	*Oscillations per second.*
Red	39,000	477,000,000,000,000
Orange	42,000	506,000,000,000,000
Yellow	44,000	535,000,000,000,000
Green	47,000	577,000,000,000,000
Blue	51,000	622,000,000,000,000
Indigo	54,000	658,000,000,000,000
Violet	57,000	699,000,000,000,000

When the vibrations are fewer than 470 million millions per second, we see nothing, but experience the sensation of heat; when they reach 477 million millions per second, the effect produced on the eye is red light; when 622 million millions is reached, blue is the result, and when more than 470 million millions is reached, no effect is produced on the eye, but the rays then produce chemical effects.

Colour, then, as far as its physical nature is concerned, is a light of definite wave length. And as the wave lengths of the colours in the spectrum vary continuously from the $\frac{1}{38000}$ to $\frac{1}{58000}$ of an inch, we see that the number of colours in white light is not seven, but infinite.

But suppose we are asked to account for the colour of non-luminous bodies. We wish to show that they are due to the absorption of part of the light falling on them. Placing a piece of red glass in the path of the beams producing the spectrum, we find that only red and a small amount of orange are allowed to pass through. With this blue glass we find that blue is the chief colour on the screen, but that red, green and violet also are found. Using these liquids we find similar results; and we find that the colour passing through is the same as the colour of the body. In other words, the colour which a body appears to possess is due to the fact that when white light falls on a body part of it is absorbed and the other part is thrown back to the eye, giving us the impression of a certain colour. We would thus infer that the colours of bodies depend on the incident light. This can be shown by holding these coloured ribbons in the different parts of the spectrum. This fact is recognized in the decorations of our homes, where stained glass windows, coloured curtains, lampshades, etc., etc., are used to produce a pleasing effect.

We might also show that the colours of bodies depend not only on the incident light but also upon the colours of surrounding bodies. But this change of colour due to contact belongs to physiology or psychology, and illustrates the fact that the colour of a body, as usually understood, does not exist apart from ourselves, and is a statement of the eye rather than of the body.

We now wish to show some of the results of the mixture of colours. For this purpose we will use Maxwell's disks, which are circles of coloured cardboard, cut along one of the radii. We are thus able to join two or more disks so as to have any proportion we please of the chosen colours. Placing these red and black disks on the turning table and turning quickly, we find that the colour produced is brown. If we increase the proportion of black to red, the brown is deepened. (A number of mixtures were shown.) If red and bluish-green colours in about equal parts be chosen, the result is seen to be white light. Similarly with orange and light blue, or yellow and deep blue. These pairs of colours producing white light are called complementary colours. Taking Newton's disk—a disk having painted on it the chief colours of the spectrum in their proper proportions—we find when it is rotated rapidly that white light is the result. Placing in front of this disk and on the same axis a brass disk having sectors cut out, and rotating both, a very pretty effect is produced. When the disks rotate with the same velocities, some of the colours are cut out and the remaining ones combine to form a single colour. If the velocities are slightly different, this combination colour is continually changing. Using a brass disk with spiral parts cut out, the effect is still more pleasing.

It should be remarked here that the mixture of coloured pigments is quite different from the mixture of colours. For instance, a mixture of yellow and blue pigments produces a green, while a mixture of the same colours in equal proportions produces white. The reason for the difference is seen immediately, when it is remembered that the colour of a pigment is due to absorption.

It is to be noticed that when yellow and blue-green enter the eye, the result is white light. But the eye is unconscious of there being two colours present; in other words, it is unable to pick out the parts producing any effect. In this it is not as sensitive an instrument as the ear, which, in the case of a trained musician, can pick out the several notes of a chord. Moreover, to the eye, the mixture of red and green colours is the same as the mixture of all the colours of the spectrum. But objectively these mixtures are entirely different. "Each particular kind of yellow may be any one of an infinite combination of homogeneous rays."—(Tait.) From this it would appear that, as far as the detection of colour is concerned, the eye is an imperfect optical instrument.

We have seen that with a few colours, say red, green, and blue-violet, we can produce almost all others. With the same colours in pigments, or with red, yellow and blue, an artist is able to produce almost any desirable tint. Hence, arises the notion that there are only three fundamental or primary colours, and that all others are but mixtures of these. This idea gained the assent and approval of no less an eminent physicist than Sir David Brewster. He maintained that there were three original or fundamental kinds of light—red, yellow and blue—and in fact, thought that he had shown the existence in the spectrum itself of these three sets of rays as well as the absence of all others. In this sense the notion of three primary colours is entirely erroneous. For apart from ourselves there is no such thing as colour, light consisting only of mechanical motions, vibrations of certain wave lengths; and by hundreds of physicists the wave lengths of the different colours in the spectrum have been determined and have been found to vary continuously from $\frac{1}{35000}$ to $\frac{1}{60000}$ of an inch, showing that the number of colours is infinite.

If Brewster's theory be true, the mixture of yellow and blue lights should produce green (as in pigments). We have shown that in equal parts they have produced white. Theoretically the notion is untrue, and experimentally it is untrue.

In another sense, however, the notion of three primary colours may be true. It may be true that on the retina there are three kinds of terminals to the optic nerve, one corresponding to red, one to green, and the other to violet. This is the theory brought forward by the celebrated Thomas Young, and carefully extended and worked out by such giants as Clerk Maxwell and Von Helmholtz. But the three primary colours seem to be arbitrary except under the condition that together they shall produce white light. It should be remarked, however, that physiologists have never discovered the three kinds of terminals necessary for this ingenious theory.

Objectively, then, the notion of three primary colours is entirely wrong, subjectively, it may possibly be right.

Let me turn to another question. What combinations of colours produce a pleasant effect, and why should that effect be produced? Why should some combinations please us and others produce a cold or a jarring effect? I have tried to gain information concerning the harmonizing of colours from a very reliable source, viz., from lady friends, and from the answers received I would infer that the harmonizing of colours has not yet been reduced to a science.

One notion which obtains at present to a large degree, is that there is an analogy between the harmony of colours and the harmony of musical notes. For instance, a note and its *octave*, or its 5th, 4th, or 3rd, will produce harmony. A note and its 2nd or 7th will give discord. Can the same be said of colour? Will the red harmonize with its octave 5th, 4th, or 3rd, and produce an unpleasant effect with its 2nd or 7th. One flaw immediately appears, due to the fact that red has neither *octave* nor seventh.

The intervals in the Diatonic scale are as follows :—

$$C = 1. : D = \tfrac{9}{8}. ; E = \tfrac{5}{4} : F = \tfrac{4}{3} ; G = \tfrac{3}{2} : A = \tfrac{5}{3} : B = \tfrac{15}{8} : C = 2.$$

The ratios of the vibration frequencies of the chief colours are : Red $= 1$: orange $= \frac{17}{16}$: yellow $= \frac{9}{8}$: green $= \frac{29}{24} + (= \frac{5}{4} -)$: blue $= \frac{21}{16}$: indigo $= \frac{11}{8}$: violet $= \frac{3}{2}$. The fifth of red is seen to be violet, which usually does not harmonize with red.

We must first ask the question, what produces discord in sound. The work of Rudolph Koenig leaves no doubt on this point. He shows that discord is due to beats produced not only by the fundamentals but by overtones and combination tones. Now no such thing as an overtone can exist in light, and no combination tone can exist unless its number of vibrations lies between 477 and 499 MM. vibrations per second. Here again the analogy entirely fails.

Moreover, harmony in music is due to combination; in colour it is due to contrast; and it will readily be acknowledged that to produce a pleasant effect not only should there be an agreeable contrast in colour tone, but also in brightness and saturation. These facts tend to show that there is a fundamental difference in the sensations of hearing and vision, the first being connected with time, the second with space.

Our judgment, then, concerning combinations of colours, is probably influenced by many obscure considerations, among which may be mentioned inherited tendencies to like or dislike certain combinations or colours; training; influence of general colour effects by which we are surrounded.

We have shown to-night how colour may be produced by dispersion, by absorption, by the mixture of colours.

We wish now to show another method.

In explaining the colour of bodies it was stated that when white light falls on a body part of it is absorbed. This absorbed part usually causes the body to become heated. In other words, vibrations of between 470 and 700 MM. per second are reduced in number to less than 470 MM. per second. Among the many substances which exist we should find somewhere the reduction is not so great as this. If, for instance, we caused violet light to fall on such a body, and if the rays given off, instead of being heat rays, *i.e.*, rays of 400 or 450 MM. vibrations per second, were rays of 550 or 600 MM. per second, we should receive the sensation of green. Bodies possessing these properties were discovered by Sir George Gabriel Stokes, and this property which bodies have of giving off light differing both from the incident light and from their own light was called by him *fluorescence.* In the general case the emitted light is of a lower order of refrangibility than the incident light.

We wish to call attention to the result when violet light falls on uranium glass, on a solution of sulphate of quinine, on a glass of water into which horsechestnut bark is dropped, on solutions of eosein and fluorescine, and on paper impregnated with any of these solutions (experiment). We may add the interesting fact that one part of fluorescine dissolved in alkali and diluted with 2,000,000 parts of water still shows a fluorescence.

If the effect were not of a temporary nature, but existed even after the incident light was shut off, the phenomenon goes under another name, viz., phosphorescence. This effect is seen when we allow the light from the lamp to fall upon glass tubes containing phosphorescent substances, such as the sulphides of strontium and barium. After the light is shut off they are seen to glow with beautiful tints.

I have tried to explain the physical nature of colour, and to show some of the many modes of producing it. If I have added any interest to the study of colour, or if I have thrown any light upon that study, I shall be pleased.

At the conclusion of the lecture Mr. A. Elvins moved a vote of thanks to Mr. Hull and his able assistant, Mr. Plaskett, who had so admirably succeeded in presenting a most interesting subject, and one which the amateur finds difficulty in understanding until he sees it illustrated as it had been during that evening.

Mr. W. A. Douglas, M.A., in seconding the motion, referred to the comparative ease with which scientific truths can be presented to the mind when given as object lessons, and hoped that the educational authorities would yet see the necessity of adopting this method, even in the teaching of science to the very young.

The Chairman, in putting the motion, which was most heartily received by the meeting, referred to the high standing of the University of Toronto, and thought it a matter of the highest congratulation that those who now give instruction in the various branches of learning were those who had received their education within those walls. This augured well for the future of Canada.

On motion of Mr. G. E. Lumsden, seconded by Mr. J. Van Sommer, a hearty vote of thanks was tendered Professor Loudon, President of the University, for his kindness in granting the use of the lecture room for the occasion. Mr. Hull acknowledged the thanks of the meeting on behalf of Professor Loudon and himself, and tendered his thanks to Mr. Plaskett, whose assistance had enabled him to illustrate the subject chosen

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SEVENTH MEETING.

April 17th; Mr. John A. Paterson, M.A., Vice-President, in the chair.

The Secretary was directed to convey the thanks of the Society to Mr. E. S. Holden, LL.D., of Lick Observatory, for photographs of the Moon received from him; to Mr. John Hollingworth, of Muskoka, and to Mr. E. W. Gardner, both of whom had recently made valuable contributions to the library; and also to Mr. A. Aronsberg, who had kindly mounted and framed the photographs now on the walls of the Society's rooms.

Mr. John A. Copland communicated a minute description of the recent auroral displays, and remarked particularly that the phenomena of April 13th were almost exactly repeated on the evenings of April 14th and 15th.

Mr. A. Elvins, who had brought some electrical apparatus to the rooms, performed some of the experiments designed to illustrate the first principles of electricity and magnetism, for the benefit of the young student. These were quite successful and warmly appreciated. Some special experiments which had been designed were impossible on account of the humidity of the atmosphere in the room. Mr. Elvins was requested to favour the members with these at a future meeting.

Mr. J. A. Collins, who had been requested to continue his experiments in magnetism, presented the following notes on

MAGNETIC STRESS.

My brother, Mr. Z. M. Collins, and myself, undertook some little time ago to determine, if possible, whether *actual contact* takes place between a magnet and its armature when freely acting upon each other. Numerous experiments, conducted by the most direct methods at our command, repeated again and again, seemed to show that a magnet of any size, shape or strength, draws an armature of soft iron to within a distance of about ·001 inch from its surface; at which point the stress or pull is greatest. When the armature was forced nearer than ·001 inch to the magnet's surface it appeared to be repulsed to the neutral zone of maximum magnetic stress, vibrating with a see-saw motion back and forth across this zone until equilibrium was established.

Gold and silver leaf less than ·001 inch in thickness, were interposed between the surfaces without diminishing the force of pull of the magnet on its armature, but when the leaf exceeded ·001 inch in thickness, the pull diminished in equal ratio as the distance was increased; the same results were obtained when one or both poles were brought into action, either with permanent or electro-magnets. The surfaces experimented upon were carefully polished, so as to insure against the possibility of the iron particles' piercing the leaf; other necessary precautions being taken to insure success and freedom from error as far as possible.

At the commencement of these experiments we were handicapped by the failure of the "law of inverse squares" to work out satisfactorily, notwithstanding that our text books and authorities informed us that the law of "inverse squares" as the ratio of magnetic stress, as regards distance, was conclusively established by Coulomb's celebrated torsion balance experiment nearly a century ago; that is, the pull of a magnet upon its armature increases four-fold every time the distance separating them is diminished one-half, under proper conditions. When both poles of two magnets act upon each other the ratio is said to be "the inverse cube."

We had no idea of questioning the universal truth of this until the following results seemed to prove its fallacy for minute distances.

We first began by finding the pull of a magnet on its armature or other magnet at $\frac{1}{4}$ inch to be 1 ounce, while in contact the pull was 14 lbs.; when the distance was worked down in the proportion of inverse squares from surface to surface, we found that if this law were true, in this case a stress of 14 lbs. should be reached when the surfaces were $\frac{1}{60}$ inch apart.

Knowing that such a gap as this could not possibly exist under the conditions, we figured the distance from the centres of mass of armature and poles of magnet without finding agreement with fact, also from the different molecular strata of armature and poles, regarding each molecule as a point in itself—as suggested by Sylvanus Thompson—but found all these methods, when worked out by "inverse squares," to give impossible results.

By the interposition of gold and silver leaf, as described above, we ascertained the actual distance between the two surfaces to be ·001 inch, agreeing with the result calculated by simple proportion, inversely as the distance from surface to surface. When ·001 inch was taken as zero of distance or starting point to measure from, the magnetic stress was found to reduce in simple proportion as the distance was increased. When a film ·002 inch thick was interposed, forcing the armature twice as far away, the pull dropped one half. When two films were interposed, doubling this distance, the pull dropped one-half again, and so on until $\frac{1}{4}$ inch was reached, after passing which point the ratio of stress seemed to gradually merge into 3rds as distance increased, and at about $\frac{3}{4}$ inch the ratio of inverse squares was asserted, and held true as far as could be determined from that point outward. Very thin tissue paper, $\frac{1}{900}$ inch thick, which appeared fuzzy, resembling fungous growth under the microscope, was placed between a magnet pulling 5 lbs. and its armature; the paper was then placed under the microscope, and showed a very slight depression of the tops of the fuzzy surface of the paper where the armature had pressed it; the same paper was then carefully placed on a polished metal block and a similar block of 5 lbs. weight gently placed upon it, and examination with the microscope then showed the fuzzy surface to be flattened out, proving that the surfaces in the latter case were closer together than in the former.

Gold and silver fibres were used for testing the distances inside ·001 inch, and layers of smooth, hard paper, of uniform thickness, preferably of $\frac{1}{500}$ inch, were used for distances outside this point as far as $\frac{1}{32}$ inch, where brass plates of known thickness were substituted. Like results were arrived at with either electro or permanent magnets, large and small, flat, rounded, pointed, thick, broad or thin, with single or double poles, end on or broadside: the shape seeming only to affect the extent of the field of any given ratio of stress.

The torsion balance was also appealed to, and appeared to confirm these results both as regards attraction and repulsion. In this case well tested rubber strand was substituted for wire or fibre, which would not stand the excessive strain of torsion necessary for testing this point, 10,000° of torsion being sometimes used. The needles were suspended as well as being centrally pivoted to keep them in the true axis of rotation; the needles used with this torsion balance were of all shapes and dimensions.

We found that without making allowance for the ·001 inch spoken of, no proper ratio of any kind could be satisfactorily worked out for minute distances. We found it impossible to find the ratio unless ·001 inch was a factor of the thickness of the films used for measuring the distance. This, together with the fact that the ordinary torsion balance will not stand the strain necessary for testing accurately measured minute distances, may be the reason for the failure to secure uniform and satisfactory results on this point since Coulomb is said to have established the universality of the law of "inverse squares."

EIGHTH MEETING.

May 1st; Mr. E. A. Meredith, LL.D., in the chair.

The Council presented the report of their meeting held 24th April, recommending the appointment of an "Assistant-Secretary and Editor of Transactions," to be paid an annual honorarium; and presenting a scheme for the affiliation of other scientific associations with this Society. The report was adopted, the Constitution amended accordingly.

On motion of Mr. Chas. P. Sparling, and Mr. J. Todhunter, Mr. Thos. Lindsay was declared elected to the office of Assistant-Secretary and Editor of Transactions.

Letters were read from Mr. Otto Struve, St. Petersburg, thanking the Society for honorary membership conferred upon him; from T. Cooke & Sons, of York, England, calling attention to the objectives which they had recently made, of triple lenses, and arranged so as to perform equally well for visual and photographic work; from W. H. Wood, of Birmingham, England, objecting to the calculated height of the aurora of the 15th July, 1893, as shown in the Society's last report.

Dr. M. A. Veeder wrote in reference to the auroral research as follows:—The auroral research is progressing. Many excellent reports from such distant localities as Siberia, Finland, Scandinavia, Alaska, etc., are already at hand. The general conclusions which have been brought to your notice from time to time are being confirmed most emphatically. The auroral reports thus far received from the vicinity of the 77th meridian, from South Carolina northward into Canada (March 30th), indicate that at about 8 o'clock p.m. the apex of the luminous mass was at about latitude 40° or 41°, and had an apparent elevation of 25° as seen from points about 600 miles southward on the same meridian. This would give an actual altitude of from 300 to 350 miles above the surface of the Earth. The observations in connection with this display were the best of which I have any knowledge for the determination of altitude.

Continuing the subject, Mr. Arthur Harvey differed from the contentions of Mr. Wood, of Birmingham, and explained the method by which the height of the arch of the 15th of July had been calculated, and referred to the correspondence from Dr. Veeder, just read, to show that it was usefully employed elsewhere. He also alluded, at some length, to the auroral observations of Mr. Cleveland Abbe, of the Weather Bureau, Washington, some of which were of extreme interest, and were described in a report recently received from that officer.

The President, Mr. Charles Carpmael, F.R.A.S., forwarded a most interesting letter, received from Mr. Otto J. Klotz, of the Department of the Interior, Ottawa; Mr. Klotz enclosed two interesting cable records of earth currents obtained at St. Pierre, Miquelon, and referred to very noteworthy disturbances which, on the 28th of February last, damaged one of the recording condensers, and seriously interfered with the working of the cable. He stated that these currents are very little understood, but that a great deal of valuable data could be procured from the cable offices, where the siphon is used, and suggested that the Society might interest itself in collecting earth current records, and in comparing them with other electrical and magnetic phenomena, as well as with solar observations and disturbances, believing the cable companies would cheerfully assist in attempting "to unravel some of that which is unknown." In forwarding the papers, Mr. Carpmael added a memorandum regarding magnetic disturbances affecting Atlantic cables and the magnetic instruments at the Toronto Observatory on certain recent dates, and said he would be glad to see something done in the matter.

The subject of sun-spots, their position—whether elevated above or depressed below the photosphere—their relative brightness, etc., having arisen on the observations reported by Mr. Pursey and others, Mr. A. F. Miller described some of these phenomena, and referred to the contention recently advanced by Mr. Wilson, an English observer, that sun-spots are elevations, and not depressions, a theory not commonly received.

An interesting discussion followed. Mr. Harvey referred to the observations at the Vatican Observatory, which seemed to show conclusively that sun-spots are depressions in the photosphere of the Sun. Mr. J. G. Ridout concurred in this view. Mr. Elvins stated he had, as long ago as 1872, maintained that sun-spots were elevations rather than depressions, and held that the phenomena of spots on the limb corroborated this view, although otherwise interpreted by some observers.

Observations on the aurora of the 27th of April, and the peculiar oval parhelia of the 27th and 28th, were reported by Dr. Donaldson, Mr. J. R. Collins, and others. Mr. Collins also showed four photographs of the Moon taken with a 10-inch reflecting telescope on the 14th of April.

Mr. Pursey reported several additions to the library; besides the current numbers of exchanges, he had received from a member who desired his name withheld, several volumes of scientific works and a geological strata map of the North American Continent. A hearty vote of thanks was passed, and the Librarian directed to convey the same to the donor.

A paper on

FALLACIES IN MATHEMATICS AND ASTRONOMY,

was read by Mr. T. Lindsay.

The errors of the Old World in regard to the form, magnitude and motions of the Earth were discussed at some length, it being pointed out that it is difficult to account for these errors when it is remembered how far advanced the ancients were in pure geometry. The fact that there are still some who cannot grasp the truth in these matters was explained by the supposition that there are minds opaque, as it were, to the fundamental principles of geometry, as some of us are deaf to music and blind to colours. Continuing, Mr. Lindsay said:

We have given but little credit to the old philosophers who did so much thinking and so little observing, but we are not to conclude that we have thoroughly learned how to separate simple truths in pure mathematics from the vague indefinite reasonings of metaphysicians. There is a school to-day—the school of transcendental geometry—which claims, not that our system of geometry is false exactly, but that we cannot prove it to be true. And they have hit upon a great stumbling block in the shape of Euclid's definition of parallel lines. We have a system of geometry of planes; we throw down upon a plane two straight lines; they will either meet if produced or they will not meet; if they do not meet they are parallel. But the difficulty is here conjured up, perhaps if they were produced indefinitely they would always meet. We naturally ask, when, where, why and how, and we find there is at the root of the idea a sort of conception of the universe as finite, pictured mentally by thinking of a vast sphere which, in a certain sense, may be said to have neither beginning nor end, so that what is apparently a straight line upon it may return into itself if produced far enough. There are several very distinguished mathematicians who hold views of that kind; in the opinion of the writer they have simply allowed themselves to wander into metaphysical musings, forgetting that this very course was the cause of the world's losing, at least two thousand years, in learning the most open of Nature's secrets.

They should know, and they do know, that the grand structure of modern exact astronomy is based upon the geometry of planes—that we prove theorems in spherical geometry by tracing back to truths already found in reference to plane figures, and that if a straight line can return into itself, if two parallels may meet, if two angles of a triangle are less than two right angles, then the result of the labours of our mathematical astronomers laid down in our Ephemeris is false and misleading.

So far from there being any difficulty about the theory of parallels, we hold that there is no conception of the human mind which requires less mental effort. The straight line of Newton is the progression of a point in one unchanged direction forever. We claim that there is no mind removed from idiocy which cannot grasp this definition. Lines may return into themselves, but not these lines, for the definition carries within it the condition that they will not. Two lines, in a certain restricted sense parallel, may meet on the surface of a sphere. We have nothing to do with such lines. Our parallels are drawn in a plane; the geometer who says he cannot grasp the idea of a plane is simply deceiving himself. Granted that space is finite, we will not even ask what is on the other side of it, then the straight lines of Newton and the parallels of plane geometry are diameters and chords; transcendental geometry will surely never ask us to imagine a vast sphere all circumference and no diameter. There is positively nothing to be gained by expending one's energies in finding difficulties in the "theory of parallels," or any other of the axioms of plane geometry which are innate concepts—the musings of the mind within itself—truths inseparable from mere consciousness alone.

Of the same order as the fallacy regarding parallels and straight lines is the idea of four-dimensioned space. It has been argued that beings living in one dimension would not be able to comprehend two;

that beings living in two dimensions would not be able to comprehend three; therefore, living in three dimensions as we are, it is no argument, so it is said, against four dimensions, that we cannot readily picture it. It seems to me, however, that the mind which can conceive of beings living in two dimensions and ignorant of a third, can readily conceive of any number of dimensions; one is not more difficult than the other. However ingeniously the argument may be presented, there is still the fundamental concept of space as that in which matter exists, and extended above and below and around him; it is in fact an easy thing to grasp in the mind; it is the unfortunate love of making things purposely difficult that has caused the discussion, aided by an altogether mistaken use of analogy. Another cause is the misapplication of algebraic symbols. We write down a cubic equation and say the root represents the side of a cube; we write down an equation of higher degree and say the root represents the boundary of a higher dimensioned space. But there is no real connection between these at all. Equations of any degree are simply expressions representing the adding together of numbers, nothing more. The root of an equation is so many units, which, repeated so many times, will sum up as many units as there are in the given power. If there were any further connection than this, then as there is no number so great but one may be added to it, we may consider that there is space of an infinite number of dimensions, an idea which any one may hold who pleases.

If investigation upon these lines could in any sense benefit us, it would be well to undertake it; but it can only result in throwing around our pure geometry a metaphysical covering which obscures it. We prefer to hold to our system of exact astronomy, based upon the simplest concepts, and will be content to leave objections with those who desire exercise in the use of words.

NINTH MEETING.

May 15th; Mr. John A. Paterson, M.A., in the chair.

Among communications received were fraternal letters from The Leeds Astronomical Society, and Rev. Walter Sidgreaves, Director of the Stonyhurst Observatory.

Mr. Cleveland Abbe wrote as follows:—

WASHINGTON, D.C., May 12, 1894.

To the President of the Toronto Astronomical and Physical Society:

DEAR SIR,—I have the honour to acknowledge the receipt of the recent volume of your Transactions, containing numerous papers on subjects in which I am much interested. I have particularly to congratulate the Society on the continued activity of the venerable Mr. Elvins, with whom my correspondence began in 1870, and on the activity in auroral studies of Mr. Carpmael, Mr. Harvey, and many others. As the aurora is evidently the optical result of electric discharges, and as its observation is much interfered with by daylight, moonlight, and the various avocations of the voluntary meteorological observers, I take the liberty of urging that the Society initiate a special movement toward the systematic observation of atmospheric electricity, telegraphic earth currents and telephonic phenomena. As Canada has always been conspicuous for its auroral phenomena, and contains within its borders the North Magnetic Pole, and has, moreover, the terminals of both the Atlantic and the Pacific Ocean cables, I think both Europe and America may reasonably combine with the Government of the Dominion in perfecting the great work in terrestrial magnetism that was initiated by the establishment of the Toronto Observatory.

I have elsewhere indicated my conviction (*American Meteorological Journal*, ix., p. 333), that a complete electric observatory should have the means of measuring the differences of potential in its immediate neighbourhood in three directions at right-angles to each other both in the atmosphere and in the Earth; several such observatories, within a hundred miles of each other, would not be too much in order to do the work thoroughly. The general direction of such work should be in the hands of a board whose members represent respectively practical magnetics, electrical engineering, theoretical electricity, and terrestrial physics. It is greatly so be hoped that such work may be organized before death robs us of Lord Kelvin, Mascart, Helmholtz, Wild, Boltzman, Rowland, Preece, Schwendler, Schott, and other workers in this field.

May I suggest that this Society take the initiative by addressing proper memorials to the great national associations that will soon hold their summer meetings in the United States, Great Britain, France and Germany? As I happen to be a member of these I shall be very glad to co-operate with you in this work.

With high regard, I remain, yours truly,

CLEVELAND ABBE.

Dr. E. A. Meredith read some very interesting extracts from a paper in Harper's Magazine for May, and written by Mr. Harrington, Chief of the Weather Bureau at Washington, on the subject (previously discussed by the Society) of the fall of temperature said to be annually experienced in Europe and America from the 12th to the 14th of May. Dr. Meredith explained the theory generally held, that about the date mentioned the Earth is in a part of its orbit crossed by a steam of meteoric matter sufficiently dense to cut off the Sun's heat perceptibly. Mr. R. F. Stupart, Acting Director of the Toronto Observatory, said he had examined the Toronto records for some years, but had failed to find evidence supporting the theory, but promised to consult those of some other stations.

Mr. A. Elvins drew especial attention to a very interesting group of spots then nearly central on the Sun's disc, and remarked that when it came over the Eastern limb the umbræ were seen as soon as the penumbræ, a still further evidence, in his opinion, that spots are elevations rather than depressions in the Sun's visible surface.

Mr. R. F. Stupart then read the text of Mr. Carpmael's communication of April 2nd :—

TORONTO OBSERVATORY, Longitude 5h. 17m. 34·65s. West,

April 2nd, 1894.

Memorandum regarding magnetic disturbances affecting Atlantic cables and the magnetic instruments at the Toronto Observatory on certain recent dates.

On examining our magnetic curves corresponding to dates on cable slips, we find that on the night of the 20th February, at 11.18

p.m. (Toronto mean time), the magnets became considerably disturbed, slow waves of disturbance being registered, the declination magnet tending East, attended by a diminution of the horizontal force. On the 21st there was a moderate disturbance until 7 p.m., when it increased, but after 9 p.m. the magnets became much quieter. At 5.15 p.m. of the 22nd the initiative movements to a large storm appeared, the magnets being suddenly deflected ; but it was not until about 10 p.m. that the full force was felt, which continued throughout the night of the 22nd and early morning hours of the 23rd, becoming one of the most important magnetic storms that we have had for some time. Between 10 p.m. and midnight of the 22nd the storm was at its worst; for about an hour after midnight a lull set in, but after 1 a.m. it burst out afresh, and maintained its activity until the afternoon of the 23rd, appearing again during the evening, and continuing up to 1 a.m. of the 24th as even a more important disturbance than that on the night of the 22nd. On the 25th, after 5 a.m., another large storm set in and continued up to the afternoon of the 25th ; it was particularly active between 5.45 and 8 a.m. After 10 p.m. of the 28th the magnets again became considerably disturbed, and continued so up to midnight.

Some large changes took place during these storms. On the night of the 22nd the declination magnet altered 1° 35′ in about 15 minutes of time ; and the range from 10.33 p.m. of the 22nd to 1.05, a.m. of the 23rd was 1° 55′. The horizontal force changed from 10.37 p.m. to 0.57 a.m. of the 23rd ·0078 C.G.S., and from the afternoon of the 23rd to 9.48 p.m. the force changed ·0089 C.G.S. Both components were above their normal until 5 p.m., when a rapid diminution set in. After 2 a.m. of the 24th the force magnets became very steady. Some extremely rapid oscillations of the V.F. magnet were recorded a little before 10 p.m. of the 23rd. A fall of this component amounting to ·012792 C.G.S. in a short time was well marked. The change of declination on this night was 2° 21′ from 9.35 to 9.50 p.m. On the 25th, at 6 a.m., the declination magnet was vibrating rapidly and considerably West of its mean position, attended by a considerable fall of both forces. A marked Easterly sweep of the declination magnet took place at 6.50 a.m., when, at the same time, both forces commenced to rise. The disturbance on the 28th was particularly active about 10.45 a.m. Both forces during the afternoon were above their normal, and previous to 4 p.m. they commenced to fall.

On examining the curves for the 8th and 9th, we do not notice any important disturbances. Between 7 p.m. of the 8th and 2 a.m. of the 9th, slow waves of disturbance were noticed on the declination curve, and about at 11.15 p.m. of the 8th a steady Westerly increase of declination set in, which was decidedly an abnormal movement. At 2 a.m. of the 9th, or 7.48 a.m. (Paris time) our magnets became exceptionally steady.

In connection with the above disturbances auroral light was observed on the 21st, with faint auroral streamers in North and North-east from 7 to 7.30 p.m.

On the 22nd, a brilliant aurora was observed at 8.15 p.m. At 10.15 o'clock of the same evening a very beautiful display began, showing crimson and green streamers and corona, but no pulsation. This display was preceded by fleecy auroral clouds, and during the time of greatest activity streamers from the corona extended to the celestial equator. The aurora first began in the North-west, with green streamers, and immediately afterwards streamers in the East extended like a scroll, until, meeting the North-west streamers, it became rose-coloured to the Eastward and Westward. On the 23rd there was a brilliant auroral display of first-class. At 4 a.m. patches of yellow aurora were seen in the North and North-east, extending to upwards of 45° in elevation.

On the 23rd, as soon after sunset as possible, auroral light was recognized in the North, but no clouds or haze before sunset could be called auroral in appearance. At 6.15 p.m. there were bright yellow streamers and patches to an elevation of 50°, and extending to 10° South of East. At 6.30 p.m. the auroral light extended and became more active and strongly tinged with red. At 7 p.m. there was bright diffused light in the North-east and East, and at 7.30 p.m. some fine detached streamers in North-east increasing in vigour and extending to zenith. This continued without much change until 9.30, when a sudden burst of activity developed into one of the most magnificent displays seen for some time in Toronto. A marked absence of the mere pencil or streamer was noticed, but patch upon patch, wave upon wave, flashed to the zenith, forming at 9.40 a splendid corona. At this time the auroral display seemed to flow from all parts of the heavens, approaching equally near to the horizon in all directions. In fact we seemed to be in the centre of a vast tent of brilliant colours, yellow, red, green, and bright silver.

At 10 p.m. the display became less active and fainter, and by 10.45 p.m. was much reduced; at 11.30 p.m., however, there was a sudden burst of waves and patches, which rose to the zenith, continuing brilliant and active up to midnight.

After reading the record as above, Mr. Stupart addressed the members on the subject generally, pointing out that investigations upon this line were necessary for the determining of the connection between magnetic disturbances and earth-currents. That a connection exists is well known, but the cause has not yet been explained. Extracts from letters were read outlining the methods by which reliable data might be secured if the co-operation of the cable and telegraph companies were obtained. The Chairman read a recapitulation of a valuable paper on this subject contributed to the Royal Society by Mr. William Ellis, Superintendent of the Magnetic Department of the Observatory at Greenwich; also an extract from the presidential address of Lord Kelvin bearing upon the great advantages which would accrue if the subject were thoroughly studied. A Committee, with Mr. Stupart as chairman, was appointed to enquire into the best means of promoting an investigation of the whole subject.

Mr. G. G. Pursey announced that Mr. R. B. Ellis, who had for some time been an active member of the Society, was about to leave Ontario for British Columbia. Mr. Pursey, seconded by Mr. Musson, moved that the Corresponding Secretary be instructed to convey to Mr. Ellis the best wishes of the Society, and to express the hope that in his new location Mr. Ellis would endeavour to still take as active an interest as possible in the work of the Society. This was carried unanimously.

TENTH MEETING.

May 29th; Mr. E. A. Meredith, LL.D., in the chair.

An invitation from the Canadian Institute to send a deputation to consider the advisability of endeavouring to secure for Toronto the meeting of the British Association for the Advancement of Science for 1897, was accepted, and Mr. John A. Paterson, M.A., and Mr. James Todhunter were appointed to meet the representatives of other bodies.

Mr. Cleveland Abbe, of Washington, D.C., wrote:—

"WEATHER BUREAU, WASHINGTON, May 23, 1894.

"I am sorry to see that Mr. Carpmael is now suffering from ill-health to such an extent as to be obliged to take an extended trip to Europe. But I doubt not that his interviews with foreign scientists and Government officials will result to the advantage of terrestrial magnetism.

"The Astronomical and Physical Society has taken an important step in appointing a Committee, with Mr. Stupart as Chairman, to enquire into the best means of promoting the investigation of terrestrial magnetism and earth currents. This is a matter that has been near to the hearts of many scientists for the past fifty years, and the whole history of the movements in this direction has, I think, but one result, viz., that the governments and large commercial companies that own the telegraph lines and cables must be willing to co-operate with the electrical experts in carrying out some pre-arranged programme."

The Librarian presented a valuable memoir received from the Observatory of the Roman College, and containing the results of exhaustive observations of the prominences on the Sun's limb.

Miss A. A. Gray read the following paper on

THE SOLAR PARALLAX AND SUN'S DISTANCE,

contributed by J. Morrison, M.A., M.B., Ph.D., Washington, D. C., an honourary member.

Of all the numerous astronomical problems which have engaged the attention of astronomers and mathematicians for ages, none is of more importance than that of the solar parallax, or the determination of the Sun's mean distance. It is, in fact, the great fundamental problem of astronomy; it is the "yard-measure," so to speak, or the astronomical unit by which we express the dimensions of the solar system and the sidereal heavens. Thus, when it is stated in our Nautical Almanacs that the distance of a planet from the Sun or from the Earth is, for instance, 1·345, we mean that the distance in question is 1·345 times the Earth's mean distance from the Sun, whatever that may be.

If two observers were to view the Sun at the same instant from any two points—say the northern point of Spitzbergen and the southern point of Tasmania—they would refer the Sun to different positions on the celestial sphere, and the *angular* displacement is the parallax due to these points of observation. The solar parallax is the angle which the

Earth's radius subtends at the Sun's centre, and since this radius is very accurately known, the Sun's distance becomes known when we know the parallax. The measurement of this angle, which is a very small one, is attended with great difficulties; it is not, however, measured directly, but is arrived at by various processes which involve a profound knowledge of mathematics. The object of this paper is not to explain these processes, which are far too abstruse for discussion here, but to trace the history of the problem from the first attempts at its solution down to the present time, and to state the latest results arrived at by the astronomers of our own day.

The first attempt to determine the magnitude and distance of both the Sun and Moon, of which we have any authentic record, was made by Aristarchus of Samos, about 270 years before the Christian Era. It is not known with any degree of certainty how he proceeded to attack the problem, but his method must necessarily have been a very rough one, for he fixed the solar parallax at three minutes of arc, which would place the Sun only about twenty times farther from the Earth than the Moon is. Nothing farther was done in this direction for more than a century, when Hipparchus, by means of the dimensions of the Earth's umbra in lunar eclipses, confirmed the result of Aristarchus. This value was generally accepted among the ancients for about three centuries, or until the year A.D. 140, when Ptolemy, of Alexandria in Egypt, by the same process as that of Hipparchus, reduced the parallax to 2′ 50″, which in turn became the accepted value for more than eight centuries, or during the greater part of the Dark Ages. About the year A.D. 920, an Arabian astronomer, Albategnius, by the same method fixed the parallax at 3′ 7″, and the Hindoo astronomers, as we learn from a work entitled the Surya-Siddhanta, chapter four, determined it to be about 4′. No farther advance was made in this direction for nearly six centuries, when Copernicus, the founder of modern astronomy, fixed the parallax at 3′, and about half a century afterwards, or in A.D. 1602, Tycho-Brahé arrived at the same result. The method employed by the last two astronomers was the same as that of their predecessors, viz., by measuring the diameter of the Earth's shadow, where the Moon crosses it during a lunar eclipse. Thus we see that no advance was made during nearly nineteen centuries; the astronomers during that long period used the same method and reached almost the same results. In A.D. 1618, Kepler, one of the most original men of his time, devised a new method of determining the solar parallax. He was the first to suggest that the planet Mars be observed at the same time from two or more very remote stations, and thence determine both the diurnal parallax of Mars and the parallax of the Sun. In this way, he arrived at a parallax of one minute (of arc), which is greatly in excess of the true value, but a great improvement on the values deduced by his predecessors. The next great advance in this line was made in A.D. 1647, when Wendelin observed that when the Moon is exactly at the "first quarter," or when exactly half her disc, as seen from the Earth, is illumined by the Sun, the lines joining the centres of the Earth, Sun and Moon, form a right-angled triangle, with the right-angle at the Moon, and, therefore, by measuring the angular distance between the Sun and Moon when the latter is exactly half illumined and using the Moon's distance as a base line, the Sun's distance could be easily determined, and thence the solar parallax. This method is quite correct in principle, but not easily put into practice, by reason of the difficulty of determining just when the Moon is actually at the "first quarter." In this way, Wendelin found a parallax of 15″, which was a prodigious advance on all those who preceded him. In A.D. 1672, Flamsteed, the first Astronomer-Royal of England, arrived at the next approximation of the solar parallax. His method was that used by Kepler, viz., the diurnal parallax of Mars. He found a parallax of 10″, which is a closer approximation to the truth, and in the same year Jean Dominic Cassini, of France, by the same method arrived at a still closer value, viz., 9″·5. Subsequently another astronomer, Lahire, of France, found by the same method a parallax of only 6″, the lowest value ever recognized, and quite too small.

Up to the middle of the eighteenth century only three methods were employed, and all of them untrustworthy and incapable of giving accurate results. About the year 1760, it occurred to astronomers that those rare astronomical phenomena known as transits of Venus over the Sun's disc, could be utilized for this purpose with great advantage, and accordingly great efforts were made to observe with the utmost accuracy and in the most favourable positions, the transits of 1761, 1769, and the two subsequent ones of 1874 and 1882.

From the first of these, Pingré, of France, and Short, of England, deduced from different observations of the phenomenon, parallaxes of 10″·6 and 8″·8 respectively. The latter has probably never been surpassed in point of accuracy. It is certainly very near the truth, but unfortunately the results arrived at by other astronomers were so discordant as to cast distrust on all the values thus obtained. From the transit of 1769, Euler obtained a parallax of 8″·8, which is identical with that of Short, but Hornsby, Lalande, Maskelyne (Astronomer-Royal of England), Pingré, Lexell, Wallot and Hell (the last remarkable for his peculiar name and for the notoriety he acquired at the time by reason of his supposed falsification of his observations at Wardhoes, Norway), deduced parallaxes varying from 8″·813 to 8″·53. From the transit of 1761, Encke obtained a parallax of 8″·551, and from that of 1769, one of 8″·5776, and from both transits combined, 8″·571. The second value, viz., 8″·5776, was generally received by astronomers until about thirty years ago, when it was discovered to be quite too small; it makes the Sun's mean distance about ninety-five millions of miles.

The results derived from the transits of Venus have not hitherto proved as satisfactory as was at first anticipated. This is due to several causes. Three methods of observation are employed, viz.: 1st. The transit is observed from points differing widely in latitude so as to shorten or lengthen the duration of the transit as much as possible. 2nd. By observing from points differing very widely in longitude so as to accelerate, or retard, as much as possible the time of internal or external contact. In both of these methods the observations have to be made when the Sun has a tolerably low altitude, but this of itself would not present any great difficulty were it not attended by a far more serious one, viz., the "black drop," the cause of which has yet to be explained. A short time before apparent contact the limbs of Venus and the Sun appear to unite by a ligament or band, which renders it impossible to determine the exact time of apparent geometrical contact. 3rd. There is also the photographic or American method, which is almost free from these disadvantages, but at the same time we are by no means certain that the photographs remain for any length of time unchanged under all atmospheric conditions. For these reasons the results obtained from the transits of Venus are unsatisfactory, and this method will no doubt be abandoned in future.

In the theory of the Moon's motion there occurs a term whose coefficient is the ratio of the solar and lunar parallaxes, and since the latter is known with great exactness, the former can be found when the numerical value of the coefficient becomes known. Meyer was the first to put this method in practice, and obtained a parallax of 8″·6. In

1804, the celebrated Laplace confirmed this value by the same method, and in the same manner. Burg found 8″·62 and Plana 8″·629. These values are all known to be too small, and the method was for a while abandoned, and that by the diurnal parallax of Mars again attempted. In 1833 Henderson, by comparing the observations on Mars made at the Cape of Good Hope with those made in Europe, found a value of 9″·028, and Taylor, by comparing the observations made at Madras with those of Europe, obtained 8″·595. The United States Government sent out an astronomical expedition to Chili, under the command of Lieutenant Gilliss, to observe Mars during the opposition of 1849 and 1850, and from the observations there made and those made at Cambridge, Mass., and Washington, D.C., Lieutenant Gilliss and Dr. Gould obtained a parallax of 8″·495, a value confessedly too small. This was the first attempt to determine this important quantity by an American astronomer. Subsequently Professor Hall of Washington, employing the observations made in Chili, Upsala, Sweden, and Washington, D.C., deduced a parallax of 8″·8415, and Ferguson, another American astronomer, using the observations made in Chili, Albany, N.Y., Upsala, Sweden, and Washington, D.C., obtained a parallax of 8″·778. By comparing the observations of Mars made at the Cape of Good Hope and Williamstown, Australia, with those at Greenwich, Professor Stone, of England, derived a parallax of 8″·943, and Winnecke, using the observations made at the Cape and those at St. Petersburg, Russia, deduced 8″·964, both of which are now regarded as considerably too large, but at that time (1863 and 1865) were very generally accepted, especially as they were in accord with other values deduced from theoretical considerations. About this time, three of the most distinguished mathematicians of modern times, LeVerrier, Hansen, and Sir John Lubbock, made exhaustive researches into the parallactic inequality of the Moon, with the view of deducing the solar parallax. The first found a value of 8″·95, which was for several years adopted in The English Nautical Almanac and The French Connaissance des Temps; the second using two different sets of data obtained two different values, viz., 8″·97 and 8″·9159, while the last found 8″·8103, a value at that time discarded but now considered very accurate. Several other theoretical determinations have been made, thus Powalky, in 1872, by employing the mass of the Earth as indicated by the motion of the node of Venus, compared with the recognized mass of the Sun, obtained a value of 8″·74, and in the same year

LeVerrier, whose analytical conduct knew no bounds, derived three values, one from the motion of the perihelion of Mars, of 8″·866; one of 8″·853 from the motion of the node of Venus, and one of 8″·859 from the secular variations of Venus, resulting from the observations of 106 years. In 1867, three other determinations of the parallax deserve notice, viz., Professor E. J. Stone, of Oxford, England, in revising LeVerrier's value, deduced from the parallactic inequality of the Moon's motion, and correcting an error made by the latter, derived 8″·91, a reduction of 0″·04 from LeVerrier's value. Professor Stone also rediscussed the Transit of Venus of 1769, using more accurate data as regards the Earth, and obtained 8″·91, a value identical with the preceding, and Schultze, employing the observations of Mars made at Santiago, Chili, and Upsala, Sweden, during the favourable opposition of 1862, derived 8″·87.

In 1862, Foucault, of Paris, determined experimentally the velocity of light, and, combining it with the value of the aberration, found a parallax of 8″·86. And in 1874, Cornu, also of Paris, repeated this experiment under more favourable conditions, and, combining his result with the aberration of Struve, obtained 8″·834. Three years afterwards he repeated the experiment under still more favourable circumstances and deduced a parallax of 8″·80; while in the same year (1877) Lindsay and Gill, by the diurnal parallax of the planet Juno, obtained 8″·765; and Airy, from the internal contacts of the transit of Venus of 1874, found 8″·76; and Stone, from the external contacts of the same transit, 8″·88, while the American observations gave 8″·836, all of which results are very discordant.

From the heliometer observations of the transit of 1874, Dr. Auwers, of Germany, obtained 8″·877 ± 0″·043, and from that of 1882, he deduced 8″·879 ± 0″·037, both of which differ very considerably from those derived by Airy, Stone and the American astronomers.

Quite recently, Professor Harkness, of the Naval Observatory at Washington, has made an exhaustive research into the solar parallax and all its related constants, and has deduced a value of 8″·80905, with a probable error of only 0″·00567, from which it follows that the mean distance from the Earth to the Sun is 92,796,900 miles, with a probable error either way of only 59,700 miles. The eccentricity of the Earth's orbit he finds to be 0·0167710, and therefore we are 3,112,600 miles nearer the Sun on or about the 1st of January than we are on the 1st

July. Assuming our theory of aberration to be true (about which there is some doubt), Professor Harkness finds that light requires 498 seconds to travel from the Sun to the Earth, and therefore the velocity of light is 186,337 miles per second.

Professor Harkness's results and methods have not been very generally accepted by astronomers. His method of treating the subject aims at a consistent adjustment of all the so-called "related constants," more or less connected with the solar parallax, rather than at the absolute determination of any one of them or of the parallax itself. Still it must be admitted that his value of the parallax is a close approximation, to say the least.

There remains, now, only one method by which we can ever hope to arrive at an accurate value of the solar parallax, viz., that of the light equation, as it is called, or the method by the velocity of light in connection with the constant of aberration. To show the relation that exists between the velocity of light and the aberration constant, let a denote the Earth's mean distance from Sun, or the semi-major axis of the elliptic orbit; v the true velocity of the Earth in its orbit at any time t; φ the angle which the Earth's radius-vector makes with the tangent or direction of motion; T the periodic time; H the area of the whole ellipse or orbit, and h the area described by the radius vector in the time t, we shall then have

(1) $r = \dfrac{a\,(1 - e^2)}{1 + e \cos \omega}$ the equation of the ellipse,

and by the theory of curves

(2) $\cot \varphi = \dfrac{1}{r} \cdot \dfrac{dr}{d\omega} = \dfrac{e \sin \omega}{1 + e \sin \omega}$ by (1).

(3) We also have $v = r \operatorname{cosec} \varphi \dfrac{d\omega}{dt}$

(4) $h = \dfrac{r^2}{2} \cdot \dfrac{d\omega}{dt}$

(5) and $H = \pi a^2 \sqrt{1 - e^2}$

By the theory of elliptic motion we have also

$$\frac{r^2}{2}\,\frac{d\omega}{dt} = \frac{H}{T} \text{ or } \frac{d\omega}{dt} = \frac{2\,\pi a^2 \sqrt{1 - e^2}}{r^2 T}$$

substituting in (3) we get by the aid of (1)

$$(6)\ v = \frac{2\,a\pi}{\sqrt{1-e^2}} \cdot \frac{(1 + e\cos\omega)}{T}\ \text{cosec}\ \varphi$$

the mean value of which is

$$(7)\ V_m = \frac{a}{\sqrt{1-e^2}} \cdot \frac{2\pi}{T}\ .$$

Now, if c denote the aberration constant $20''{\cdot}2\ldots.$ and V the velocity of light in miles per second, we have

$$(8)\ c \sin 1'' = \frac{a}{V}\ \frac{2\pi}{T\sqrt{1-e^2}}\ ,$$ where T is to be expressed in seconds of time.

When c and V are known, a can be found (8), and thence the solar parallax.

It is now almost universally believed by astronomers that the solar parallax lies between $8''{\cdot}80$ and $8''{\cdot}81$. This is the result of more than twenty-one centuries of observation, of research and of toil.

ELEVENTH MEETING.

June 12th; Mr. John A. Paterson, M.A., Vice-President, in the chair.

A report of the meeting held by representatives of various public bodies to consider the proposal to extend an invitation to the British Association to visit Toronto in 1897, was read and heartily endorsed. It was confidently expected that arrangements would be satisfactorily made, it being upon all sides admitted that such a visit would be of marked advantage to the Dominion.

A report from the Committee on the investigation of Earth Current phenomena was read, and discussed at some length. Mr. R. F. Stupart described how the work might be most advantageously carried on; as the earth current generally or perhaps invariably occurs synchronously with a disturbance of the magnetic needles, it would be quite possible to give from the Magnetic Observatory notification of disturbances to various observers, thus enabling them to take more than ordinary precautions in noting the various changes. Mr. Stupart agreed with the generally received opinion that there is a distinct connection between earth currents, magnetic storms, and the aurora.

The following notes on the subject were received from Mr. John A. Copland, of the staff of the Toronto *Globe* :—

I had a chat recently with Mr. D. Urquhart, the telegrapher in the *Globe* office, some of whose remarks with reference to auroræ and earth currents may be of interest. He says that at times the wires are tampered with, as though some outsider were attempting to transmit a message irrespective of the one coming over the wires and with an extra battery seemingly. This he attributes to the presence of electricity in the atmosphere, snch as at the time of an aurora. He narrated to me several instances of how the wires were interfered with. On one occasion he was taking a long "rush" despatch from Ottawa when the dots and dashes of the ticker became very erratic, often spelling no word at all. There would occur a period when dots and dashes seemed as if they were being hurled at the line promiscuously. Losing his temper, our operator "cut in" and demanded to know what the sender at Ottawa meant. He politely informed the operator down there that nobody except a crazy or a drunken man would send in such a fashion. The Ottawa man wired back that he was sending all right. He could not understand what was the matter. Both men looked for auroras, but none were visible either at Ottawa or Toronto. Shortly afterward, however, the mystery was solved, for there came in despatches from intermediate points saying that a brilliant aurora was in progress. Mr. Urquhart says that often when the wires are thrown down by the wind or otherwise, even though not broken, all the messages run into the ground as it were, and at intervals the instruments will rattle as if a current from a strong battery were turned on. Thunderstorms interfere with the wires greatly. When there is a brilliant and fairly universal aurora in progress, it is often possible to shut off the batteries and send messages across the lines by the aid of the electric current supplied by the aurora. This current, however, is very uncertain and erratic, at times being more than sufficient and again becoming extremely faint, once in a while withdrawing entirely.

Mr. A. F. Miller called particular attention to the importance of observing daylight auroræ, and described a recent observation when light fleecy clouds were seen to form the coronal arch exactly as do the auroral streamers. Mr. Z. M. Collins described a peculiar physical feeling experienced during a recent auroral display.

Part of the evening was taken up by a very interesting discussion on the solar spectrum and the revelations of the spectroscope. A small direct vision instrument, brought by Dr. A. D. Watson, gave excellent definition of the lines of lithium, magnesium and sodium in the flame of a Bunsen burner, readily improvised, thus showing that elaborate apparatus is not always necessary to give a popular illustration of the spectral lines of the various elements.

Mr. C. A. Chant, B.A., and Mr. A. Elvins discussed particularly the cause of the dark lines in the spectrum of the Sun, the latter holding that the usual explanation does not go far enough towards giving a clear idea of what really does take place. The continuous spectrum gives out light of all wave lengths, the vapours of different elements absorb the rays which they themselves emit, and therefore, instead of a bright we have a dark image of the slit imprinted on the spectrum at a particular wave length. Mr. Elvins thought the vibrations under these circumstances might be doubled in frequency, and therefore being too rapid to affect the eye, might be found by the aid of photography in the ultra-violet region of the spectrum.

Mr. Chant dissented from this view as being contrary to the mathematical theory of harmonic wave motion. He reviewed briefly the writings of Tyndall on this subject, and held that the energy apparently lost by the extinction of light was simply transformed into heat.

TWELFTH MEETING.

June 26th; Mr. A. Sinclair, M.A., occupied the chair.

Mr. S. J. Saunders, B.A., of Clinton, N.Y., was elected an associate member, and Mr. P. H. Sims, of Toronto, an active member of the Society.

At a previous meeting Mr. Arthur Harvey had asked whether a complete explanation could be given of the fact that the month of May had been unusually cool and rainy, while the Sun and Earth were in the same positions in relation to each other as in other years at the same time. To this question Mr. A. Elvins replied as follows:—

"To me this is an important question. The Earth is in the same part of its orbit which it occupied at the same date on former years, its axis has the same inclination to its orbit as previously, the Sun seems to radiate the same amount of heat on the whole as it has done in the past, and notwithstanding, there is a great change in a single year.

"I see no explanation except by supposing such weather changes to result from an influx of meteoric matter into our atmosphere from outer space. I think the chemical elements are formed from the ether, and are dissociated and returned to the ether again by solar action. It is said that fogs owe their existence to dust particles, which act as a nucleus around which the moisture of the atmosphere condenses. If so, an influx of fine particles may be a factor in the development of a rain storm. It is also possible that gases in large quantities enter the atmosphere, and I do not think it impossible that oxygen and hydrogen may mingle occasionally in the required quantities to form an explosive mixture, and by their combination form water; the air may be thus saturated from above, as well as by absorption from the water below. It seems a very wild thought, but I regard it as even possible that the aurora may be caused by explosions due to chemical union of H. and O., and even our thunderstorms may be the result of the combination of these two gases on a gigantic scale.

"I may sum up my reply by saying that abnormal weather is the result of the influx into our atmosphere of solid and gaseous matter from outer space, and that the Earth encounters more of such matter at some periods than at others, as when we plunge into the November meteor stream at intervals thirty-three years apart."

Mr. David E. Hadden, of Alta, Iowa, forwarded the following account of an eruptive prominence observed at his station on the evening of June 7th:—

About 5.15 p.m., having finished an observation of the Sun for sun-spots and faculæ, I adjusted the spectroscope to the telescope, and commencing at the North limb, examined the circumference of the Sun around by the West and South towards the East for prominences, with the slit tangential to the limb. On reaching the South-east limb I noticed an intensely brilliant large prominence in the *C* line, and upon opening the slit observed a group of very bright "spikes" and "flames," which seemed to be in violent disturbance, as when I attempted to make a rough pencil sketch of it I found it was changing so rapidly that it was impossible to correctly delineate it. I opened the slit fully $\frac{1}{32}$ of an inch, which at the focus of my 3-inch telescope would represent about 70,000 miles, and I could trace the height of the prominence nearly the entire distance; it was also bright and distinct in D_3 and *F*.

I left the telescope now and returned in about ten minutes, and was astonished at the change which had taken place. The fine spikes had almost disappeared, but a few faint flames remaining and a bright mass of "debris," which rested partly on and within the limb. With a narrow slit the *C* line was noticed to be shifted much towards the blue, indicating a rapid approach towards us in the line of sight. At 6 p.m. but little remained of this great disturbance; but turning my attention farther in on the disc, I found the entire region from the limb where the prominence was, to the Western portion of the group of sun-spots, which was about two days in, to be in a violent state of agitation; a prominent spot of facula was on the South-east limb, and a faculous area surrounded the sun-spots mentioned; at numerous points in this region the *C* line appeared as in the sketches, one or more dark vibrating tongues or blow-pipe looking jets adhering to the line directed towards the red end of the spectrum; these persisted even when the slit was gradually opened, until with a wide slit they appeared as dark points in the brighter opened space. Observations ceased at 6.10 p.m. It would be interesting to know at what time these paroxysms first began and what, if any, effects were noted in the Earth's magnetism.

Mr. Lindsay called the attention of the members to Professor Carnegie's work on Law and Theory in Chemistry, and illustrated the chapter on the Carbon Atom by exhibiting a model to represent the atom, at the centre of a regular tetrahedron, as the new theory supposes it to be; the experiments with polarized light upon different compounds were referred to and the hope expressed that some of these might be repeated at a future meeting.

Mr. Arthur Harvey, who had been requested to prepare a resumè of the recent work of Professor Dewar in connection with the

LIQUEFACTION OF GASES,

read the following notes :—

The method adopted is to lower boiling points by exhaustion. You know the principle. It comes to our notice practically in mining at or above the summer snow line in the mountains. There are several camps in America so high that boiling water will not cook potatoes or other vegetables so as to make them palatable. Carbonic acid, which boils under ordinary atmospheric pressure at —112 degrees, will, in a vacuum such as the air-pump can be made to give, boil at —166 degrees. At this temperature nitrous-oxide liquefies, and, itself boiled in vacuo, lowers the temperature and liquefies ethyline, which in turn runs down the thermometer to —229 degrees. At this point pressure is resorted to, and the pressure of 1,500 lbs. to the inch (100 atmospheres), forces oxygen into a liquid state. The evaporation of liquid oxygen, also in vacuo, liquefies, under pressure, air and nitrogen, while these again, worked upon in double receivers by powerful air-pumps, will produce solid nitrogen. This was first shown in January of this year 1894. Liquid oxygen is 900 times less in volume than the gas at ordinary temperatures—blue in colour, because it stops many red, yellow and orange rays. That is apparently why the sky is blue. Like the gas it is magnetic, springs from a cup of rock-salt to the poles of an electro-magnet when the circuit is turned on, and stays there pending its rapid evaporation. Nitrogen seems to be an inert body, with no striking qualities, good to be a diluter or absorbent of the more energetic oxygen. Hydrogen remains now the only body unsubdued by cold and pressure, so a hydrogen thermometer is used to indicate these extremely low temperatures. If hydrogen be, as Faraday thought, a metal, water is a metallic oxide, and it is remarkable how easily this oxide liquefies, while oxygen only becomes fluid under the severest compulsion, and hydrogen resists it with success.

Gases contract $\frac{1}{460}$ for each degree of temperature. What is to happen when a temperature of -460 degrees is reached? At present it seems below the limit of possibility. All gases will liquefy and solidify before this is obtained, so the method of successive reductions above described must fail to achieve such a minimum. But if this absolute zero is reached, will matter vanish through the total deprivation of heat? Heat is the life of matter; the more heat the more energetic the molecules. Metals become stiffer and tougher under cold—remarkably so at Professor Dewar's low temperatures—become better conductors of electrical currents; but chemical affinity is diminished so that alloys do not behave in the same way as pure metals, while carbon and some other substances act quite differently. We know from the everyday experience of the incandescent electric light that heat increases the conductivity of carbon, while it reduces that of metals—a corollary of which property of the latter it seems to be that iron at 1,400 degrees is not magnetic at all; nickel at 340 degrees is also inert to the strongest magnets. If the Sun is a magnetic centre at all, it is not because of its iron or other metals, and this consideration leads me to doubt if the aurora has any connection with the spots on the Sun, either as they pass the centre or appear on his Eastern limb, or with their maximum or minimum frequency.

What is the cold of space? We approximate to it in these experiments. Is it permissible to think that this cold—even without pressure—would liquefy and solidify gases and so facilitate the condensation of dispersed matter into suns and planets, and *forbid the existence of a gas in space which would retard the motions of these orbs?* Will cold, rather than gravity, thus fix a limit to the atmospheres, permitting no gas to exist outside the calorific influence of the bodies which are still hot from condensation? *Has the air there was upon the Moon settled down to be a transparent sheet of ice over her surface, fixing her features in an almost eternal setting as hard as adamant?* One more singular point. Molecular convection of heat ceases as the molecules die of cold, but energy still passes through the frozen mass. A burning-glass which concentrates heat and light can be made with a spherical vessel full of liquid oxygen. Radiant or ethereal heat and light encounter no resistance on account of extreme cold, when molecular heat can scarcely creep from particle to particle.

Cold affects colours. Sulphur (at —314 degrees) turns white, vermilion fades to orange, iodine in alcohol loses its violet, my authority states, but as alcohol freezes at —202 degrees, the phenomenon must be seen in the solid.

Is the Earth homogeneous? When it was intensely hot, too hot to hold any but elementary forms of matter, a time came when it was cooled as to its gaseous envelope, and oxygen, if not hydrogen, combined with its materials to a certain depth. The outer shell thus is alone composed of oxides or rusts, for such we may call all the rocks and other substances that contain oxygen. A time may come when the aqueous vapour and carbonic acid of the air will come down as snow, just as oxygen and hydrogen at a given stage form water, just as carbonic acid and calcium have formed the limestones—and, after that, the interstellar cold will be free to act, and the residual oxygen and nitrogen will form an ice case of eleven or twelve yards in thickness. When, in due course, something like this happens even to the Sun, and absolute zero is reached, will matter be loosened from its affinities and disperse? If so, there must be fewer dark stars than Sir Robert Ball thinks possible.

THIRTEENTH MEETING.

July 10th; John A. Paterson, M.A., Vice-President, in the chair.

The Librarian presented a copy of the report of the Sun Section of The British Astronomical Association, and drew the attention of the members to the minute record of the appearance of the solar disc which the volume contained. It had been noted with special interest that drawings of some sun-spot groups had been made by Mr. Elvins on days when English observers had been similarly engaged; laid side by side the drawings appeared almost exactly alike. Mr. A. F. Miller, reporting some observations, stated that he had been able to see the two distant companions to Alpha Herculis in his four-inch telescope, and wished other observers to note whether they could be seen with the same aperture, as they are given in star lists usually as objects within the power of not less than a six-inch objective.

Mr. G. G. Pursey presented drawings of sun-spot groups made during the previous month, and having noted that he had never seen faculæ unconnected with sun-spots, desired observers of the Sun to make special note as to whether the former are ever seen alone.

A monograph by Rev. T. E. Espin, M.A., in the series of publications of the Wolsingham Observatory, was read. The paper dealt with the distribution of stars in space, according to their types of spectra.

A part of the evening was spent in replying to some queries which had been made by some of the younger members of the Society. Mr. Arthur Harvey took up the subject of optics in so far as it related to the construction of the telescope and microscope. Mr. A. F. Miller reviewed the method of determining the wave-lengths of rays of light, and the rapidity of vibration. Mr. Thomas Lindsay discussed the problem of determining the masses of binary systems, known to be such by the evidence of the spectroscope only.

FOURTEENTH MEETING.

July 24th; Rev. C. H. Shortt, M.A., in the chair.

Rev. A. U. DePencier and Mr. W. A. Campbell, both of Toronto, were elected active members. On the subject of Earth Current observations, a letter was read from Mr. Otto J. Klotz, who wrote from Fort Wrangell, Alaska. Mr. Arthur Cottam, F.R.A.S., London, forwarded some sheets of star maps, showing the general style of his admirable "Star Atlas." These were much admired, especially for the large amount of detail shown.

Rev. D. J. Caswell, Ph.B., President of The Meaford Astronomical Society, wrote an interesting account of the formation of that Association, which, it was gratifying to learn, held regular meetings for the purpose of hearing lectures, etc., on scientific subjects.

Mr. Caswell reported also an interesting observation as follows:—

In conversation with two young ladies who have attended our meetings as visitors, and who knew I would be interested, I learned that they had observed in daylight a large meteor. As I had read in the Toronto *Mail* that a meteor had been observed at the Island and at Kew Beach, the day and hour being given, I asked the young ladies to try to recall the circumstances of what they had observed. I learned that it was seen by them on Tuesday, 3rd July, about seven o'clock in the evening, and was due south. They remembered the circumstances, as they were going to play tennis at that hour, and they joked about it, saying that it "must be one of the rockets coming down." A display of fireworks had been given the previous evening in celebration of Dominion Day. I have no doubt their description was perfectly correct and it corresponds with that reported in the *Mail* of July 5th and 6th.

Mr. R. F. Stupart, acting-director of the Magnetic Observatory, Toronto, then read the following paper on

THE DEVELOPMENT AND PROGRESS OF AREAS OF DEPRESSION.

The phenomena of weather changes and of storms have in all ages and in all civilized countries attracted the attention of intelligent and thinking men. The importance of a knowledge of the laws which govern and regulate, not only those more destructive motions of the atmosphere which so often interfere with those engaged in maritime pursuits, but also of the more ordinary conditions which produce rain and changes of temperature, increases in proportion to the increase of agriculture, commerce and navigation. If we glance at the long list of disasters which year after year have occurred on our own great lakes alone, we shall see that a knowledge of the nature of and the laws which govern the phenomena is of more than scientific interest, for it enables us in the majority of instances to warn the mariner of coming bad weather, while for the benefit of agriculture, the farmer may be advised to house his fresh cut crops in order to escape damage from an approaching rain storm. A knowledge of the approach of great changes in temperature during winter may be of great value to those engaged in trade, as the safe shipment of perishable fruits, etc., may often be expedited and secured by such information as can even now be given by the various meteorological services of the world.

In this paper I shall discuss more especially the development and progress Eastward of areas of depression in the middle latitudes, and the relationship that the mean track of areas of depression bears to variations in the character of seasons.

The general and special causes which lead to the formation of atmospheric disturbance may be found in the ever-changing position of the Sun relatively to our globe, and from the inequalities in the surface of the latter which produce differences of temperature and moisture between places not far distant from each other.

In the opinion of most modern meteorologists the position and movements of all other celestial bodies may be ignored and expunged in the study of meteorology, as their united influence is probably but as a drop in the ocean compared with that exerted by the Sun.

That there is a periodicity in the recurrence of great storms and of similar types of seasons is a thing that has certainly never yet been clearly proved, yet the subject is one well worthy of study, and may one day lead to great discoveries.

For my own part, and I think I speak in unison with other professional meteorologists, both scientific and practical, I must confess my inability to forecast the weather for extended periods, and only for short terms with the assistance of charts of synchronous observations of the state of the weather over a large area of country.

Most of you are, doubtless, aware that the intense heating up of land and sea near the equator causes an updraught of air, which, ascending into the upper regions of the atmosphere, then flows away towards higher latitudes. Just North of the tropics we find a belt of high pressure outward, from which on the Southern side the trade winds blow towards the equator, while on the Northern side there is a general drift of the atmosphere from the W.S.W. over the middle latitudes. The position of this belt of high pressure alters considerably as the Sun changes in declination, and hence the mean limits of the drift must also vary with the season, besides, also, according to whether it is over land or ocean. It is obvious that, having this general flow of air from the S.W. over the middle latitudes, there must be return currents, and we find a large proportion of these in the anti-cyclones, which over America generally move South-eastward. The formation of these anti-cyclones in winter occurs principally over the more Northerly portions of the Continent, where the air, being intensely cooled by radiation, becomes dense and heavy and contracts, and in consequence there is a continual tendency of the air in the upper regions of the atmosphere over the comparatively warm oceans to flow towards the Continents; it is probably not wrong to surmise that the almost permanent high pressure over Asia during the winter months draws its supply from the Atlantic, and that the Pacific supplies America.

The mean limits of the South-west drift at the surface of the Earth may practically be identified with the mean track of a vast and persistent system of areas of low pressure or cyclones, which can be traced pretty well round the globe, from the coast of China to the North of Siberia, and which varies more or less in position from season to season, and in different years. In these cyclones the warm air of the South-west drift ascends to the upper regions, and either returns to the South-west as an upper counter current, or goes to supply anti-cyclones over the Northern Continents; as the storms move on in the general flow, ordinarily a cold Northerly current, moving South in the form of anti-cyclones, closes in behind them and finally becomes merged and blended with the subtropical belt of high pressure.

Now, as regards these cyclones, what are they?

From the study of weather charts we can form some ideas as to the conditions most favourable to their development. It is very easy to see how a slight area of depression may be formed during the day over an island or over a portion of the country more susceptible to receiving solar heat than that surrounding it, but why in some cases a small depression, having been formed, should continue to develop until the barometer falls an inch or more below the normal, is a more complex study. It is a matter of fact that, during the first stages of the formation of a storm centre, abnormal pressure and temperature will be found to prevail at varying distances from the district where the depression first appears; there is generally an area of high pressure, accompanied by abnormally low temperature to the West and North of it, while East and South of it is another high area attended by much higher temperature. Such a distribution of pressure naturally causes an inflowing of the air on all sides towards the region of lesser pressure; these currents as they move are deflected to the right, owing to the rotation of the Earth, and thus approach the centre spirally; they will also have a tendency to rise, owing to becoming lighter through expansion; this rising up causes still further expansion and dynamic cooling, which in turn causes condensation of a portion of their moisture, forming clouds and subsequently rain, the heat thereby liberated tending to retard the cooling, which goes on at a slower rate than were no moisture condensed.

It is obvious that the upward movement of the air produced by this sequence of events will take place to the greatest extent in the direction from which come the warmest and moistest winds, while on the opposite side the colder and drier winds will tend to cause a recovery of pressure.

Excessive rainfall generally accompanies the rapid development of a storm centre. In America the rain area frequently extends from 500 to 600 miles in advance of the actual centre, being heaviest, however, in its immediate neighbourhood.

Temperature is evidently also an important factor in the development, as we find in practice that depressions do not usually develop unless there is a good deal of difference between the temperature in advance and in the rear of them.

Now, the development of a depression clearly shows that the upward movement near the centre is in excess of the horizontal inflow towards it, for notwithstanding the enormous quantity of air brought towards the centre by the inblowing winds, the barometer continues falling. The updraught must, on reaching the upper limits of the storm-whirl, spread out laterally, and probably tends to reinforce the high pressure areas on all sides, but during continued development (at all events in the middle latitudes), it seems to help that in the rear to the greatest extent, as this more especially frequently follows the storm for some days without showing signs of dispersion, notwithstanding outblowing West and North gales at the surface of the Earth. We see then, that, a depression having been formed, the tendency is for the pressure to give way, most readily in the direction of most fully saturated air, and we find in practice that it is in such a direction, viz., East, that storms generally move over Canada and the United States.

The majority of American storms have their birth on the East side of the Rocky Mountains; some, however, come from the Pacific, and a fair percentage first become visible over the Gulf of Mexico. Leaving the storms bred within the tropics out of the question, we find that nearly all move in an Easterly direction; those which form over the North-western States and Territories generally pass over the Lake Region and thence to the Gulf of St. Lawrence and Newfoundland; those which form over the South-west States move to the Lake Region and then spread out over Northern Canada; those which first appear over the Gulf of Mexico usually move to the middle Atlantic States, and thence either over, or to the South of, the Maritime Provinces.

The direction and rate of travel of these storms has for many years been carefully registered, and it has been found that the average rate of movement over this Continent is about twenty-nine miles an hour—in winter it is thirty-five, and in summer twenty-three miles. The average rate of travel of depressions across the Atlantic Ocean is less than over this Continent, and the rate over Europe is less than over the sea. The late Professor Loomis gave 28·2 as the average yearly rate over America, 18·0 over the Atlantic, and 16·7 over Europe.

It might appear at first sight, after studying the movement of areas of depression across America, that moisture is the chief thing that governs their course, but I think, on considering the fact that their average course is still Easterly over the Atlantic and Europe, we may conclude that moisture in itself cannot be the prime cause, although it probably materially assists to modify or intensify some other influence or influences.

About the oldest theory on the subject is that storms follow the general drift of the atmosphere, much as eddies move with the current in a river. Within the tropics they move in a direction a little North of West, which direction does not, however, altogether tally with the general flow of the atmosphere, but the agreement is so close that we may well consider whether the direction of movement is not primarily caused by the general flow, and modified by other influences.

We may then regard the regular system of atmospheric circulation as the primary cause of the Eastward progress of areas of depression in the middle latitudes. I have already shown that moisture must influence the direction of movement of areas of depression, and I think the figures shewing rate of travel over America, the sea and Europe, warrant the conclusion that it assists or retards the rate, according to position. Over the Continent the greatest moisture is generally to the Eastward of depressions, hence the progress of development works with the general drift, thereby causing a faster movement than would be produced by the drift alone. Over the Atlantic, where the air is on all sides comparatively moist, the influence of moisture as a factor in producing an Easterly movement will be materially lessened, and a slower average rate of motion is the result. Then again as depressions approach and move over Europe, the region of greatest moisture relatively to their centres is shifted from the Eastward to the South and West, and instead of being a factor assisting the Easterly progress, it becomes rather than otherwise a retarding influence.

Having now considered briefly the general wind circulation of this Hemisphere, and the formation of cyclones like eddies in a flowing stream, I will now endeavour to explain how the weather of a district is affected very differently, according to whether a storm centre passes to the South or North of it. Take the example of a storm centred over Lake Michigan: if it moves North-east and passes far North of Northern Ontario, in Toronto we at first get Easterly winds and a moderate rain of short duration, followed by South-west and West winds, and warm, clearing weather; but let this same centre move South-eastward and pass to the South of the lakes, we in Toronto experience Easterly backing to North-east and North winds, and a cold protracted rain storm. Consider the weather of our Canadian North-West. Alberta has a very changeable climate, in winter intense cold giving place to balmy breezes in a short interval. What is the relationship between these changes and the storm tracks?

During the winter months, as I have stated, the normal atmospheric pressure conditions are anti-cyclonic over the interior of the Continent, while over the North Atlantic and North Pacific there is tolerably persistent low pressure. This distribution of pressure gives prevailing

North-westerly and Westerly winds over Canada from Manitoba to the Gulf of St. Lawrence; along the Eastern slope of the Rocky Mountains they are generally Northerly; but as we cross to the Pacific Coast the barometric gradient alters its direction in a way to produce a prevalence of Easterly to Southerly winds near the British Columbia coast, and as we go still further West, we find in crossing the Pacific, South of latitude 50 degrees, a prevalence of South-westerly winds blowing from a sub-tropical belt of high pressure towards the low to the Northward.

Now, the normal low pressure area over the North Pacific is a region frequently traversed by barometric depression or storm centres, many of which move to the coast of California or British Columbia and thence force their way into the continent, temporarily displacing the high pressure there prevalent. It is when one of these areas of depression is forcing its way inland over Northern British Columbia that the winter chinook is felt in Alberta. Then is Alberta within the South-west barometric gradient in connection with the storm centre, and Southerly to Westerly winds are blowing over British Columbia. These winds blowing off the sea are moist; they ascend the mountain slopes and are gradually deprived of their moisture, rain or snow falling on the mountain sides; this condensation of moisture sets free latent heat, which retards the dynamic cooling, due to diminished pressure; and afterwards, as the dry air descends the Eastern slopes, it dynamically regains heat, and the result is a mild, dry wind over the Alberta prairies.

It will easily be understood from this that the character of the seasons in the far West must depend greatly on the track that storm centres take in passing into the Continent from the ocean; in some seasons the majority of them pass over Northern British Columbia, and then is there a marked persistence of the mild South-westerly winds in Alberta; in other years the majority pass into the Continent further South, perhaps only a few moving North of the latitude of Calgary, then is Alberta favoured with but a small quantity of balmy breezes.

When a storm centre passes over Washington or Oregon States and thence perhaps East or South-east, Alberta and the Territories, in lieu of the mild South-wester, experience bitter North and North-east winds, and perhaps a snowfall, and at times even Vancouver Island itself is subjected to a Northerly gale with snow.

Yet another example. The average track of depressions lies to the North of the British Isles; the result is a preponderance of South-westerly winds; but in some exceptional seasons—1881 is an example—the majority of storms pass into the Bay of Biscay; the result is North-easterly winds in England, and unusually bad weather in Western Europe.

I am convinced that on the position of the mean tracks of barometric depressions, and on their persistency and rate of movement, depends very greatly the character of seasons; again, I think it not improbable that the position of the mean track of depressions, and their rate of movement, may depend very largely on the energy of the South-westerly drift over the middle latitudes. A question, then, to be considered is, what causes most probably lead to a difference in the character of this drift in corresponding seasons of different years.

It seems not improbable that a changing amount of heat received from the Sun, owing to solar disturbance, may be the secret agency we are in search of, as an overplus or deficiency of heat must have an effect on the atmospheric currents, especially as the Equator may well be regarded as the initial point from which the general circulation begins.

Scientists have clearly demonstrated a connection between the magnetic declination and sun-spot periods, and several well-known men, notably Mr. Elvins, Professor Smyth and Mr. Meldrum, have done some splendid work, and shown almost conclusively a connection between the sun-spots and rain-fall. Certainly, if the mean track of storms is in any degree effected by the spots, then so must be the rain-fall; and it is more than possible that the failure to study the rain-fall in connection with storm tracks may have caused the agreement between maximum rain-fall and maximum and minimum sun-spot periods to appear less evident.

There is another aspect of the question of the connection between sun-spot periods and storm tracks that may prove interesting and worthy of study. We know that the mean track of storms varies in the corresponding season of different years, and have endeavoured to show that on its position must greatly depend the character of seasons. Is there nothing in the fact that the spots or solar storms at maximum periods are almost invariably found between the parallels of 8 and 15 solar latitude, and that just before its minimum they begin to appear in higher latitudes, and that after the minimum has passed there is often an outbreak of spots far from the equator?

FIFTEENTH MEETING.

August 7th; Mr. E. A. Meredith, LL.D., in the chair.

It was moved by Mr. Arthur Harvey, seconded by Mr. A. Elvins, That the Corresponding Secretary be requested to communicate with the Secretary of the Royal Society of Canada with a view to the affiliation of this Society with that body. Carried.

A letter from the Governor-General's Secretary was read, and also copies of letters from the Secretary of State for the Colonies, and from the Secretary of the Admiralty, to the effect that the discussion of the subject of change in astronomical time reckoning would shortly be taken up, at a meeting of the Admiralty.

The Librarian reported the receipt of a collection of publications of the Royal Society of Belgium, from Dr. François Terby, to whom the thanks of the Society were due.

Mr. Harvey and Mr. Elvins presented drawings of Mars taken at the telescope. These agreed very closely. Mr. A. F. Miller reported having observed a disturbance on the Sun in the neighbourhood of a spot on August 5th. An uprush of hydrogen was indicated; this particular phenomenon in such a region of the Sun's surface Mr. Miller had not observed on any former occasion. Discussing the spectrum of Mars, Mr. Miller stated that it is wanting in the violet end; he suggested very careful spectroscopic observations of different regions of the surface of Mars as an aid to our learning more of the atmosphere of the planet.

Mr. Arthur Harvey then read a paper on

THE SOUTH POLAR REGIONS OF THE EARTH AND MARS,

of which the following is a synopsis:—

The view we now have of the Southern polar regions of Mars leads us to enquire if we cannot find some light reflected from his Antarctic regions upon our own. Singular as it may seem, we really know little more of the one than of the other. No one has yet wintered within the Antarctic circle of the Earth—nothing is known of its winter climate, the distribution of land and water, the laws of storms, the formation of ice and its movement, or the magnetic phenomena of high Southern latitudes. It is, however, of the utmost importance that enquiries should be made into these subjects, to enable us to arrive at true conclusions as to the figure of the Earth, the causes of atmospheric and oceanic circulation, and the correct theory of the Earth's magnetism.

Looking at Mars, we now see a great white patch around his Southern pole, and a similar appearance would present itself to an astronomer who from Mars would turn a telescope upon the Earth when our South pole is turned to him. It would even be larger in proportion, and would certainly not be reduced to the small size to which, in October, the Martian ice-cap will, after a full twelve-months' summer, be brought down.

Our Antarctic regions were first described for us by Captain Cook, the great navigator, who was acquainted with Simcoe, the first Governor of Upper Canada, and whose name survives in Cook's Bay, of Lake Simcoe, fifty miles from the good city of Toronto. He twice crossed the Antarctic circle, and once reached latitude 71° 15′, where he found ice barring his way, and said he believed no higher Southern latitude would ever be attained.

In 1823, however, Captain James Weddell sailed to 74° 15′, in latitude 34° 17′ West, where he found a sea clear of field ice, with only three icebergs in view, plenty of whales, petrels and other living things—and this after going through 120 miles of water with (at least on one occasion) "not a particle of ice of any description visible."

Most of our knowledge is, however, derived from the voyage of Sir James Clark Ross, with the "Erebus" and "Terror," he having spent the years between 1839 and 1842 in the survey of the regions around the South magnetic pole.

The "Challenger," in 1874, ran down to just within the circle—the only steam vessel which has ever been there.

In 1892-3 Norway and Scotland sent out a whaling fleet, with a small scientific staff, who have added something to the common stock of information.

The great chain of the Andes, broken across by the ocean at Cape Horn, rises again above the sea in the South Shetland Islands. Then, also in the range, we get Joinville Island, Palmer Land and Trinity Land, and next—still just without the Antarctic Circle—Graham's Land, which may be the Northern apex of a Southern continent, only 10° of latitude from South America. Here our Southward journey stops. Turning Westward we skirt a formidable ice barrier, from 62° West longitude to 170° East longitude, where in latitude 78° Ross found

Mounts Erebus and Terror, and a shore which he called Victoria Land. Whether the intermediate space, apparently covered with ice, is land or water, no one knows. Dr. John Murray (of the "Challenger") supposes it to be the coast line of a great Southern Continent, and a mountainous region analogous to that on the West of South America. He thinks the other side of it is much lower, with a gradual slope towards broad plains. The coast line he thinks continuous, under the names of Adélie Land, Clarie Land, North Land, Sabrina Land, etc. It runs almost along the Antarctic Circle, facing the Southern coasts of Australia, as far as 110° East longitude, when it bows with a great curve towards the South, and has been seen as Kemp's Land in 68° West. Enderby's Land, again, in 50° West, is on the Circle. Then, in another great curve, within which Weddell made his Southern dash, this imagined Antarctica is supposed to continue until it reaches Graham's Land—our original point of departure.

If this view be correct, the Antarctic Continent covers four million square miles, and is larger than all Australia, while the ice covered region may be much larger in winter, during which no man has ventured to behold it.

The shores of the mountainous side of this land are very bold, "rising sharply from the ocean in a stupendous mountain range, peak above peak, enveloped in perpetual snow." The surgeon of the "Erebus," Robert McCormack, who gives this account, says :—"One very remarkable peak, in shape like a huge crystal of quartz, rose to the height of 7,867 feet, another to 9,096, and a third to 8,444 feet above the level of the sea. * * In latitude 77° 31′ and longitude 167° 1′ the burning volcano (Mount Erebus) was discovered, covered with ice and snow from its base to its summit, from which a dense column of smoke towered high above the numerous other lofty cones. * * Its height is 12,367 feet; and Mount Terror, an extinct crater adjoining it, * * attains an altitude little inferior, being 10,884 feet in height and ending in a cape from which a vast barrier of ice extended in an Easterly direction, checking all progress South. This continuous perpendicular wall of ice, varying in height from 200 to 300 feet, its summit presenting an almost unvarying level outline, we traced for about 300 miles."

Sir James Ross himself says: "In a few places the rocks broke through their icy covering, by which alone we could be assured that land formed the nucleus of this, to all appearance, enormous iceberg."

On February 2nd (answering to our August), he speaks of the ice barrier "about 160 feet high, extending as far to the East and West as the eye could discern, continuing in one unbroken line from Cape Crozier, a distance of 250 miles."

In approaching the barrier and the land, Captain Ross had driven his ships through 200 miles of pack ice, yet there was often a broad space—100 or 200 miles—of comparatively open water between this outer ice-stream, or pack ice, and the ice barrier proper. The winds, perhaps, account for this, for in January there were 75 units of wind-force with Southing and only 14 with Northing; in February, 42 of Southerly and 38 of Northerly winds. The January mean temperature of the sea at the surface was 29° 18′, and of the air in the shade 29° 02′. In February the sea was the same, the air only 24° 28′. The maximum of heat of the air in the shade was on January 8th, 41° 5′, and the minimum in January was 19° 5′, and in February 13°. Small wonder, then, that the land is covered with snow!

Dr. C. A. Donald, of the Dundee whaler "Active," speaks of "a dark expanse of open water between the bergs and the floe. To the North was the loose, scattered ice, small bergs and dark water-channels, through which we had just steamed." What I infer is that while the edge of the true barrier is fairly permanent, there are, in summer, broad channels and patches of open water to the immediate Northward of it quite large enough to be seen with a telescope from Mars. Now, on Mars we see an unmistakable edge to the Polar Continent, especially at the North Pole. It is an irregular ring, in latitude about 70°. Next to it appears a belt of 5, 8 or 10 degrees in width, called Schröter Sea; then an irregular belt called LaPlace Land; then another belt of fluid called Bradley Sea, Deslambre Sea, etc., after which we get the several continents. Around the South polar cap we have in the same way the Phillips Sea and some encircling belts of land. (All this is nearer to the poles than Schiaparelli's channels, of which much has of late been written.) If the drift or pack ice, which (speaking now of the Antarctic Pole) would be driven by Southerly winds a long distance from the ice barrier, were caught on Mars, as seems to be the case on Earth, by a prevailing Westerly current in a lower latitude, we have a possible explanation for the long lanes, resembling gulfs, or straits, or channels, called after Phillips and others. There would be a high barometer at the South Pole of Mars, and outward flowing winds, just as there is upon

Earth. Around the South Pole of the Earth, and from 5° to 8° from it, Dr. Murray draws an isobar for February of 29·50 inches. Ten degrees from the Pole he marks 29·40; fifteen degrees, 29·30; twenty degrees, 29·10; and around the Antarctic Circle, 29. In all this region we should then have Southerly winds. Mr. Bruce and Dr. Donald's experience in the Dundee whalers confirms this view. But, a few degrees further North, we get into the prevailing low pressure, and as far as 40° latitude we have strong westerly gales all around the globe. This little ray of light on the possible cause of the narrow belts observed around the Martian Poles, is the only one our very imperfect knowledge of our own Antarctica helps to reflect on the other planet, except that the glittering brightness and crystalline appearance of snow-covered mountains which Ross alludes to may explain the strange brightness of some of the spots —bright white upon white—which have been observed on Mars. We may, perhaps, get more reflected information in return.

The cooling process is supposed to have gone farther in Mars than in the Earth, but the phenomena of the melting of the ice around his poles—the fact that the greater part of it is regularly thawed away each season, shows that his ice, whether it be ice formed from water or from some other liquid, cannot have any enormous thickness—not by any chance a depth of miles. And we begin to question from this analogy whether there was ever a depth of ice upon the Earth which it took miles to measure. Certainly it does not seem to be the case now, notwithstanding the evident fact that Antarctica is going through a glacial period. There, indeed, we may learn what an ice-cap really is—such as they say existed hereabouts. In Alaska and in Greenland we have plenty of mountain glaciers, but no true ice-cap. Our North Atlantic icebergs announce their origin by their appearance; they are pinnacled like the Alps, most irregular in outline. They come from glaciers which discharge from valleys into the sea direct. The Southern bergs are, however, quite different.

Ross says (January 21st, 1841), "The land ice, although not more than five or six feet above the surface, and therefore probably not more than forty feet in thickness, blends so imperceptibly with the snow which descends from the mountains at this part and extends far into the sea, that it was almost impossible to form any idea of the exact position of the coast line." Again, near Mount Erebus (January 28th):—"As we approached the land, we perceived a low white line extending * * as far as the eye could discern to the Eastward. It presented an extraordinary appearance, gradually increasing in height as we got nearer to it, and proving at length to be a perpendicular cliff of ice, between 150 and 200 feet above the level of the sea, perfectly flat and level at the top, and without any fissures or promontories on its even seaward face. * * We might with equal chance of success try to sail through the cliffs of Dover as penetrate such a mass. * * It is impossible to conceive a more solid looking mass of ice; not the smallest appearance of any rent or fissure could we discover throughout its whole extent, and the intensely bright sky beyond it but too plainly indicated the great distance to which it reached to the Southward. * * Having sailed along this curious wall of ice in perfectly clear water, a distance of upwards of 100 miles, by noon, we found it still stretching to an indefinite extent in an ESE direction. * * We sounded in 410 fathoms. The temperature of 300 fathoms was 34·2°, and at 150 fathoms 33°, that of the surface being 31°, and the air 28°. Current S. by E., 12 miles per diem. So great a depth of water seemed to remove the supposition that had been suggested of this great mass of ice being formed upon a ledge of rock, and to show that its outer edge at any rate could not be resting upon the ground." In the Arctic regions, he adds, "we have witnessed the almost magical power of the sea in breaking up land ice or extensive floes of from twenty to thirty feet thick. * * Here the extraordinary barrier of ice, of probably more than a thousand feet in thickness, crushes the undulations of the waves and disregards their violence: it is a mighty and wonderful object, far beyond anything we could have thought of or conceived." A few days later (January 31st) Ross saw several icebergs; they were chiefly of the tabular form, perfectly flat on the top, precipitous in every part, and from 150 to 200 feet high. They had evidently been detached from the barrier, and had grounded on a bank, sixty miles from the edge of the barrier. Soundings on this bank gave 250 fathoms, or say 1,500 feet.

Dr. Murray, of the "Challenger," says:—"The ice and snow which form on the slopes of the mountain ranges forming the interior of Victoria Land, descend to the lower reaches of the Continent, where they accumulate in vast undulating fields and plains, hundreds of feet in thickness, and ultimately this great glacier or ice-cap is pushed out over all the low lands into the ocean, forming there the true ice-barrier, a solid, perpendicular wall of ice, probably from 1,200 to 1,500 feet in thickness. * * When the forefronts of this great creeping glacier are pushed into depths of about 200 or 400 fathoms, large stretches are broken off and float away, as the oft-described perpendicularly-faced, horizontally-stratified, table-topped icebergs of the Antarctic and Southern Oceans, which may be miles in length, and usually float from 150 to 200 feet in height, above the sea surface."

Dr. Donald, of the "Active," says:—"These Southern glaciers differ in many important aspects from those of the North, and many problems in regard to them still require solution. I merely wish to state, however, that some of those we saw around here produced a berg more than sixty or seventy feet in height."

The flat-topped figure of the Southern icebergs, which results from the level surface of the ice-cap, is not the only point in which Antarctic glaciers differ from Arctic ones. They also differ in stratification. This is invariably horizontal; for the ice is not thrust from between narrow valleys into the ocean, nor has it rapidly travelled down steep slopes, its centre outstripping the sides in its daily progress of several feet, its whole mass suffering distortion as it splits up into blocks when passing over a ledge, and breaking into separate sections as frequent changes in temperature make it contract—to expand the next day or night with irresistible force. Whether it moves at all with a general movement seawards is quite a question, and it may be that Dr. Murray is unduly impressed by the theories of glacialists here—now beginning to be questioned as having been carried to absurd extremes. There can have been no movement of ice for hundreds of miles over level country. It rather seems that *annual snowfall adds layer after layer to the thickness of the Southern ice-cap where it lies*, that it began to form where it now is, accumulating up to the possible limit of thickness, while some laws, not yet understood, remove either as water or through ice-streams within its substance, whatever excess may be due to hundreds of years of such increment. It seems, at any rate, that no such thickness as spoken of by the late Dr. Croll existed, nay, that no ice ten or twenty miles in thickness can be formed, for if it could for a time be supposed, the base would melt, from the necessarily increasing temperature of the crust below it. If the snow-cap rests on rock of a temperature half a degree below the freezing point, the greatest thickness of ice formed would not be likely to exceed 1,600 or 1,800 feet. Such a layer of ice, however, such as that which we know to exist in the South, presents for examination most interesting considerations, and one may feel sure that when it has been studied, in all its phenomena of formation, regelation and movement, much light will be thrown on the theories of our Northern glacialists, and I believe that much will have to be recanted. It has always seemed to me incredible that ice, miles thick, should have moved in a sort of semi-fluid state across enormous areas of land. If physical conditions are found to limit the thickness of an ice-cap to 2,000 feet (only about one-third of a mile), it is still more inconceivable that any such ice-caps as are said to have covered this part of North America and the North of Europe should have moved, almost independently of gravity and across a generally level country. The true explanation of ice-cap formation, motion and erosion, may be found quite different from the methods which pronounced glacialists with assumed certainty tell us of.

Ross' map differs much from Murray's. He gives facts only, and his chart shows but a few lines of coast. Murray connects these by dotted lines, and colours his Antarctica in the approved hydrographic way. But what if instead of a great Continent, with the broad features he imagines, across which no ice could be pushed, there is an irregular complex, with several mountain ranges or spurs of the chief range, with fairly steep slopes in most cases? We can then imagine without violence to common sense a flow of ice down their valleys which will give some thrust to the main field. A uniformly high barometer at the Pole means very little snowfall, and I do not suppose this thrust is ever very great; still the ice must move outwards more or less, until it reaches the point at which temperature abruptly limits its extent. In reaching that point it must move upon the sea floor for some distance. Cliffs must have disappeared, and even the sea-bottom must be smoothed in some places and striated in others. When the ice-fields reach a latitude where their accretions from above are not enough to sink them to the bottom, and where the warmer bottom layer of water can penetrate, portions are detached and ice islands sail away. Dr. Murray thinks the horizontally stratified appearance of these Southern ice-fields and ice-bergs is due to "the constant melting and regelation which go on throughout the ice-cap; in the deeper parts of the bergs these layers are not thicker than wafers." Surely Dr. Murray would re-write this after a visit to even our land in winter. We see similar strata formed by successive snowfalls, ice-crusts, occasional warm days, and not at all by constant melting and regelation. Stratification occurs, I suggest, for the most part, after the ice-cap has been launched into the sea, and while the lower ice becomes abraded by its slow movement over the ocean floor, the upper layers are added to each season by frequent snowfalls. Such was, I imagine, the ice-cap in Northern regions, too. It was not formed throughout, nor did it perform its work on land while the land was above water but below it. Behind us there were the Laurentian hills; perhaps the Alleghanies were above the sea. The rest of this part of the Continent was below sea level. The ice extended over the region that now shows the signs of ice action in a sheet, not of miles but of 500 or 1,000 feet in thickness. In valleys alone could it be more. It would stretch into the sea like the Antarctic sheet, and rivers would flow from under it in summer time. Its movement would be very slight, and always along the valleys, never up them or across them. Under its calm protection, clays would be deposited, with here and there a boulder, which had been slowly carried within or upon it. How otherwise can we account for the beautifully level stratification of our extensive tracts of boulder clay?

In estimating the dangers of an Antarctic expedition, to elucidate such problems as here alluded to, it must be considered that the temperature is probably much more uniform than at the North. Mr. Bruce, of the "Balœna," inferred this from the singular evenness of the temperatures in December, January and February, which gave means of 31·14°, 31·10°, 29·65°, and asked, "might not this indicate similar uniformity throughout the year?" No doubt it is a daring thing to face the unknown, and it may be found very difficult even to effect a landing in a desirable locality for observation. But steam has minimized the dangers of the ice-pack, and though self-sacrifices would have to be made, yet, if England or Australia (which is especially called on for the task) were to provide the vessels, there would be no lack of volunteers to man them, and Canada would esteem it a privilege to provide her quota.

SIXTEENTH MEETING.

August 21st; Mr. E. A. Meredith, LL.D., in the chair.

Mr. J. Maughan, of Toronto, was elected an active member of the Society.

The Librarian reported the receipt of the Proceedings of the Institute of Nova Scotia, and was instructed to place that Society upon the list of exchanges. Mr. J. R. Collins presented some copies of a photograph of the full Moon, taken on August 16th at the Wilson telescope. The prints had been made by Mr. W. B. Musson. During a general discussion on the subject of electric phenomena, Rev. C. H. Shortt described a case of lightning stroke in which it seems the lightning had ascended. Some of the members had met with similar cases, but rarely.

Mr. Z. M. Collins described some experiments he had recently made with magnetized steel discs, and presented photographs of the lines of force. His method of preparing these was simply to place sensitized paper over the disc, sprinkle steel filings upon it, and then expose to direct sun-light, the action of which upon the paper was marked where the filings were absent. As a result of noting the lines of force, Mr. Collins thought it was an error to speak of the pole of a magnetized sphere as being localized. Each hemisphere is a pole, and the magnetism equally distributed around it. What is commonly called the pole is merely the centre of the field.

The following paper by Mr. W. F. Denning, F.R.A.S., of Bristol, England, a corresponding member of the Society, on

COMETS OF SHORT PERIOD,

was read by Rev. C. H. Shortt, M.A.

The remarkable increase effected by new discoveries, during the last ten years, in the Jovian family of Comets must have been noticed by everyone interested in this branch of Astronomy. Between 1884 and 1894 our knowledge of these bodies has been enriched to the extent of ten new members as follows:—

Year and Number.	*Name.*	*Perihelion Passage.*	*Period in Years.*	*Next Return.*
1884 II.	Barnard	August 16	5·398	1895, June.
1884 III.	Wolf	September 27	6·821	1898, June.
1886 IV.	Brooks	June 6	5·595	1897, August.
1886 VII.	Finlay	November 22	6·627	1900, February.
1889 V.	Brooks	September 30	7·073	1896, October.
1889 VI.	Swift	November 29	8·534	1898, June.
1890 VII.	Spitaler	October 26	6·378	1897, March.
1892 III.	Holmes	June 13	6·909	1899, May.
1892 V.	Barnard	December 11	6·309	1899, April.
1894 I.	Denning	February 9	7·698	1901, October.

This average addition of one Comet annually gives us the clearest possible intimation that this family is an exceedingly numerous one, and probably comprises some hundreds of members, for it is evident we are still only acquainted with a small proportion of the aggregate number which exist. It might be supposed that Comets of this class returning so frequently, would soon all be discovered, but this is far from being the case as they are, in general, small and often pass perihelion under circumstances which prevent their being detected. Thus Comet I., 1894, though it returns every seven years and two hundred and fifty-five days, can only be seen at one return out of three (*i.e.*, once in twenty-three years) as the position of the Earth relatively to the Comet is too distant for the latter to be seen except on occasions when it reaches perihelion (longitude 130°) in January, February, or March.

The majority of recent discoveries show that the average period is about six and three-fourth years, and that the perihelion distance lies outside the Earth's orbit while the aphelion is just outside that of Jupiter. The perihelion of Finlay's Comet lies, however, slightly inside the Earth's orbit, but that of Brooks' (1889 V.), Spitaler's (1890 VII.), and Holmes' (1892 III.), is situated far outside the orbit of Mars, and at about twice the Earth's mean distance from the Sun.

Two of the Comets enumerated in the table (*viz.*, Wolf's and Finlay's) have been seen at a second return, but it is not likely that all the others will be fully verified in future years, though there can be little doubt as to the character of their orbits at the particular apparition when their discovery was effected. The individual members of this Comet family are doubtless constantly undergoing disturbances by the attraction of Jupiter, and fresh members appear to be added from time to time by the capture of long-period Comets which happen to pass near that planet.

The fact that the perihelia in nearly all cases lie outside the Earth enables these Comets to be often well observed in a quarter of the sky opposite to the Sun's place. Whenever the perihelion of one of these bodies occurs at a time when the Earth is on the same side of the Sun it will be comparably near the Earth and projected on a region of dark sky so that it is presented under the best conditions. This is really a fortunate circumstance, for some of the short-period Comets are really faint and difficult objects, and can only be picked up when the conditions are highly favourable.

From the numerous and interesting results accumulated during the last decade, we may expect that the immediate future will yield a plentiful harvest of new discoveries of Comets belonging to Jupiter's family.

SEVENTEENTH MEETING.

September 11th; Mr. E. A. Meredith, LL.D., in the chair.

A communication was received from the Honourary Secretary of the Royal Society of Canada, conveying the information that upon the list of the associates of that body, the Astronomical and Physical Society of Toronto had been placed, and would be called upon to take part in all meetings of the Royal Society.

Dr. François Terby, of Louvain, Belgium, was elected a corresponding member of the Society.

Moved by Mr. J. Todhunter and seconded by Mr. A. Harvey, and *Resolved*, that the next meeting of this Society take into consideration the advisability and practicability of erecting an astronomical observatory in the city of Toronto and maintaining the same by public or private enterprise or both. Carried.

A member read a letter from Mr. J. Connon of Elora, giving some valuable hints to the amateur engaged in lunar photography. Referring to the difficulty of getting a sharp image, Mr. Connon said:— "The better way is not to use an eyepiece at all, but when you enlarge for photographing use what is called by photographers a 'half-size portrait lens.' This is a style of lens not much used now-a-days in ordinary work, and can be found in any gallery. With this you can get a perfectly flat image, and if it is a good lens, and clear, the focus will be very fine in quality."

Mr. Thomas Lindsay reported having made a series of observations with a telescope by Bardou, of Paris, recently imported by Mr. James Foster. Of 3-inch aperture and bearing easily a power of 150, the instrument had performed extremely well. The definition was particularly sharp, many objects being well seen that are considered severe tests for the aperture. Mr. Foster had kindly offered to place the telescope at the service of the members on any special occasions when interesting phenomena might be observed. The mounting was admirably adapted for field observations.

Mr Elvins reported having observed quite successfully the spectrum of the lightning flash on the night of September 2nd. The literature on this subject is not extensive, but it seems that other observers have noted bright lines in the spectrum. Mr. Elvins succeeded in seeing the flash in the spectroscope fully forty times, with the constant result of a continuous spectrum, no bright lines being seen ; the colours were clear and distinct from the red to the violet. As the spectrum of an incandescent solid, or of a gas at very high pressure, is continuous, the question is an open one as to why it was continuous in this case. Without pressing any explanation, Mr. Elvins thought that the particles of the dust and smoke-laden atmosphere might be rendered incandescent by the lightning, and so produce a spectrum without bright lines.

The following notes on

THE FORM OF THE AURORA

were received from Mr. J. Van Sommer, writing from Southampton, on the Lake Huron shore, where he had made many observations :—

Allowing for the streamers, an aurora is always seen in the form of a crescent or semi-circle, the highest point due North from the spectator. I am not aware that it is ever seen edgeways, as if it were facing East or West. That being so, we arrive at the conclusion that the crown of the aurora follows a circle round the North Pole. All sides may not be equally illumined at the same time, but the shape would be that of a "band." The centre of the auroral band of light under the North Star would then be nearest to us, and the sides East and West further away. The greatest height due North would be due to the nearest distance, and the lower and less distinct appearance of the sides would be due to the greater distance. So the first point in regard to the shape we must decide is that the crescent or semi-circular form, as we see it, is due to how it appears to us and not to an actual shape the light possesses; that is, the aurora is not a semi-circle set upon a base, but a portion of a complete circle.

Next, in regard to the position of the light and streamers as compared with the Earth's surface. I do not think the light radiates from the Earth's surface towards its zenith. If it did, the light and streamers would incline away from a spectator. Now, if the base of an aurora were from twenty to sixty miles nearer to a spectator than the crown, the base should appear the brightest. Any light would do so. The streamers shooting up and away from the spectator, according to the certain laws of perspective, would look both smaller and dimmer at their points than at their base. But as a matter of fact, *the reverse is the case*, the crown of the aurora and head of the streamers are the brightest and largest, leading us to the conclusion that their movement is towards or approaching us. I think, therefore, that the auroral light does not radiate from the Earth's surface, but from a point above the Earth, and the streamers shoot out parallel with the Earth's surface, the light being spread out in the same way; in fact, that the form and position of an aurora is that of an open umbrella, extending over the Northern and Southern regions of the Earth. This is the only shape I can think of that is capable of giving the form and appearance of the aurora to two or more spectators wherever they may be. Any other form would give different appearances to spectators at different points at the same time. Of course, the light may be brighter in one direction or another. If this idea be correct, it would follow that we should look for the cause of the aurora outside of the Earth, whatever that cause may be, especially if, as has been suggested, they appear and reappear at definite intervals.

Mr. J. Phillips then read the following paper on

THE NEBULAR HYPOTHESIS OF LA PLACE.

After the Lunar Theory was completed and gravity established as a law of the universe, La Place began to collect and arrange for publication all the advancements mathematical science and physical astronomy had made since the time of Newton. In 1796 appeared his *Système du Monde*, the sixth and last volume of his great work *La Mécanique*

Céleste. At the end of that volume he propounds what is commonly known as his Nebular Hypothesis, but more properly his Atmospheric Hypothesis, though he, himself, gives it no name. In that hypothesis La Place seeks to account, *not* for the origin of the whole universe, nor for the Sun even, the centre of our own system, but merely for the planets, satellites and comets accompanying him. And first of all, he postulates the Sun as already formed, somehow or other—nearly spherical in shape, rotating on an axis, and surrounded, *not* by a nebulous mass—but by an *atmosphere*, extending farther than the present bounds of the solar system. This atmosphere he conceives to be rotating along with that central mass, the Sun, but more slowly, and to derive its rotatory motion from him—to be so rare that he is solid in comparison, and in mass to be but a very small fraction of his mass.

La Place's idea of the probably prior condition of the Sun he gives in three sentences. What he means by an atmosphere he thus defines: "A thin, *transparent*, compressible and elastic fluid, surrounding a planet while being supported on it, is what we call an atmosphere." His hypothesis is founded on certain propositions demonstrated in his *Mécanique Céleste*, especially in the second volume. He therein shows that such an atmosphere surrounding the Sun as supposed, could not extend indefinitely, but must have had certain limits. And that its form must have been an ellipsoid of revolution, with its shortest axis coinciding with the axis of rotation, and which ellipsoid became more and more oblate as more and more rotatory motion was given it by the central mass. He then demonstrates by a set of the most elegant equations, that when the equatorial and polar diameters became in proportion to each other as 3 to 2, the particles along the equator lost all weight, and the great spheroid had reached its centrifugal limit. Soon after this the contiguous inner parts, still being attracted Sunward, would break away, leaving the outer belt as a great ring revolving around the Sun and his remaining atmosphere. Call this ring the embryotic mass of the outermost primary planet, say that of Neptune, and *his* initial career as a separate identity is begun.

On the shrinking away of the inner atmosphere the ellipsoid is more spherical in shape than before, its equatorial diameter having just been shortened. But, as it keeps on condensing and gaining in rotatory motion, it grows more and more oblate as before, until the 3 to 2 centrifugal limit is again reached. Then another belt or ring is in like manner left behind—call this the making of the future Uranian system.

In this manner proceeds La Place, from differentiation to integration or *vice versa*, and from one equation to another, until the whole mechanism of the heavens is revealed in one great mathematical poem, matchless for its elegance, beauty of style, and power of convincing all who stop to read. On such basis is the La Placean hypothesis founded.

The Kantian or real nebular hypothesis was thought out and published by Immanuel Kant when La Place was but six years old and Herschell (afterwards Sir William) only seventeen. Kant, in his *Natural History of the Heavens*, seeks to account for the origin of the whole universe, and supposes the *substance* of the Sun and planets, for instance, to have been at first so diffused as to fill a space greater than that now occupied by the solar system. La Place begins with the Sun already formed and rotating, and surrounded as at present by a transparent atmosphere, but *extending* farther. The Kantian and La Placean hypotheses bear much the same relation to each other that the Cartesian and Newtonian philosophies do—the former emanating from the inner consciousness of Des Cartes, the latter founded on experiment and geometry. Yet, strange to say, through some misconception, the Kantian hypothesis is generally attributed to La Place by all English-speaking people who know anything about it, but not by the French or Germans.

In setting out La Place seeks to account for these five phenomena which the solar system presents, to which there was only one known exception in his time, viz., the Uranian moon system.

Here are the five phenomena:—(1) The motions of the planets in the same direction and nearly in the same plane. (2) The motions of the satellites in the same direction as the planets. (3) The rotations of all and of the Sun in the same direction as their projectile movements, and in planes that differ but little from each other. (4) The nearly circular orbits of all the planets and of their satellites. (5) And finally, the great eccentricity of the orbits of the comets, although their inclinations and directions seem as if left to chance.

In laying down his own hypothesis, La Place first refutes that of Buffon, which was older than Kant's, and which La Place says was the only attempt at a cosmogony he knew of being made since the establishment of the Copernican system. This statement plainly shows that he never heard even of Kant's speculation, though thought by many to be its author. But in proceeding with his own hypothesis La Place unconsciously refuted Kant's as well as Buffon's, and proved *it* to be untenable also. He sought to account for the above five phenomena only—*nothing else.* And, considering the state of chemical science in his day, he succeeded well. Though he knew all about the retrograde motions and the orbital positions of the Uranian moons and treated of them elsewhere, yet he did not try to explain or account for either phenomenon, or allude to them at all in his hypothesis. And though he lived *thirty-one years*, which was more than a third of his life time, *after* the publication of his *Systême du Monde*—yet never once did he attempt to reconcile the peculiarities of the Uranian system with his hypothesis. Herein he showed his wisdom.

There are other phenomena also, unknown in his time, which his hypothesis fails to explain. There is the retrograde motion of Neptune's moon in an orbital plane not much inclined to the ecliptic. If Neptune be found hereafter to have a retrograde rotation it will account for this ; but the other horn of the dilemma is just as bad—for, how came he to have a retrograde rotation ? La Place's hypothesis does not pretend to account for any such rotation. Neptune's moon does not comply with (2), and all the Uranian moons run counter to both (2) and (3) of the aforesaid phenomena which La Place deals with. But the short period of the inner Martian moon, only a third of its primary's rotation time, is probably the most fatal blow of all given this hypothesis. Had La Place known of it, his hypothesis would likely never have seen the light. The late Professor Mitchell, of Cincinnati, in defence of this hypothesis, once said:—"If the nearest moon of Jupiter or Saturn were found revolving in less time than its primary rotates, that would falsify the theory." No one disputed Mitchell's statement then, and never would if the Martian or no other moon had been found fulfilling the conditions. No sooner, however, did little Phrobos manifest himself sweeping round Mars every 7½ hours, than up sprang analysts declaring this did not affect the nebular hypothesis at all ! That Mars rotated in 30 hours when his first lunar ring was thrown off, in 7½ when his second broke loose, and then his young hopeful Phobos slowed down his rotation period for him to 24 hours 37 minutes ! Verily this looks like analysis gone mad. Only think of a wilful pebble attracting a mountain, whirling the mountain off its base, and twirling the mountain around him !

No theory yet announced accounts for all the phenomena the solar system is now known to present, unless "the volcanic theory of planetary projection" does. Let us briefly apply this theory to Phobos and see how it fits in there. That little satellite revolves round Mars, 4,000 miles from his surface and 6,100 from his centre, in 7h. 39 min. Conceive Mars' atmosphere extending 4,000 miles in height around him, resting upon him and rotating along with him, just as La Place supposes. Then, instead of that tiny planet being detached in a gaseous ring, let us suppose it to be projected from Mars himself in a molten mass by volcanic eruption, and projected with such force as to drive it through the atmosphere and out into the non-resisting medium beyond. What would be the future career of that projectile?

Generally speaking, it would of course fall down again upon Mars after its projectile force was spent. Or, if its initial velocity were great enough to carry it off beyond the influence of the Martian system, it would evidently go on and never return. Ultimately it would likely fall into the Sun, find itself an orbit around him, or wander off into stellar space. But let us bear in mind the projectile, after clearing the Martian atmosphere, must not *necessarily* either quit the Martian system or return to Mars. It might and likely would take a medium course. If its velocity or direction, then, were not such as to enable it to find a permanent orbit, it would sweep round for a while, but its recapture by Mars would only be a question of time. If, on the other hand, by design or chance, its velocity then were about the square root of half the square of the speed that would drive it off for ever—and if the direction of its course, or, which is the same thing, if the tangent to its trajectory there and then, made a right angle or nearly that with its radius vector—its trajectory just then would merge into a circular path, with Mars in its centre, or into a slightly eccentric curve, with Mars in one of the foci. And the said tangent to the trajectory would be tangent also to the circular or elliptic path at that point, the common point of contact where both curves would unite. The projectile would therefore leave, as it were, the trajectory leading from Mars all the way up through his atmosphere, would enter the circular or elliptic path just found, would take possession of the same as its orbit, and begin as a new-born satellite, its course of perpetual revolutions therein, round and round its primary and parent planet—Mars! Its period of revolution would depend entirely on its distance from its primary, and not on the time of Mars' rotation on his axis. It might be longer, shorter, or precisely equal to his rotation time. If shot out from Mars' equatorial regions it would revolve around those regions; if from a high latitude, over that latitude crossways it would revolve. If projected from either pole or very near there, it would likely depart for ever or return soon. Its orbital motion, whether direct or retrograde, would depend on which latitude projected from, North or South, and on the hour of the Martian day when projected.

Let any one interested in this matter apply this theory to every satellite in the system, leaving Neptune's for the last, before falling *in* with or falling OUT with the theory. No one need attempt this unless he can "pocket" his prejudices and preconceived notions for a while—a mental feat which but few are able to perform, though most people think they can. Sir John Hershell truly says: "On beginning a new enquiry we have not only much to learn, but much to *unlearn*, the latter often the more difficult."

EIGHTEENTH MEETING.

September 18th; Rev. C. H. Shortt, M.A., in the chair.

After some discussion, it was moved by Mr. Arthur Harvey, seconded by Mr. G. E. Lumsden, That the proposal to establish a popular observatory be referred to the Council, with a recommendation to take immediate action. Carried.

The Librarian reported having received from Mr. E. Stanton, of Toronto, two prints of photographs of the lightning flash. It was interesting to note that in one of the prints a distinct loop could be seen in the streak.

Miss Katharine E. Vale, an associate member, who had recently removed to Davenport, Iowa, had been requested to favour the Society with any items of interest pertaining to the advancement of science in her locality. Miss Vale had forwarded the following account of a visit to

THE CHAMBERLIN OBSERVATORY,

which was received and read:—

The Chamberlin Observatory has just been erected, by the munificence of one of the citizens of Denver, the Hon. H. B. Chamberlin, and has cost, altogether, nearly $60,000. It is a beautiful building, replete with all the wonderful and delicate mechanism that modern science requires, or can suggest, situated in the University Park, on a piece of ground fourteen acres in extent, on which no trees are to be planted that might obstruct the view. Four blocks away is the University College Campus, and Iliff School of Theology. Although Denver is built in the midst of a plain, formerly a vast prairie, seventeen miles from the foot-hills of the Rocky Mountains, still it is 5,196 feet above sea level (the altitude of many of the Alps in Switzerland); consequently the air is exceedingly pure and clear, and observation remarkably good. University Park is on a higher level than Denver; the ground rises gently into a rounded hill, upon the highest point of which the Observatory is built; it is more than four miles from town, and a longer distance from the large smelting works, so that the smoke of the city interferes very little with observation. The main building is sixty-five feet long, fifty feet deep, lighted throughout by electricity and heated with steam, built of red sandstone from the Archalow quarries, and very ornate in appearance; it is crowned by an iron dome, the apex of which is more than fifty feet from the ground. The principal room is, of course, the spacious Dome room; the wings contain the Transit room, Library, Computing room, Director's office, Clock room, etc.; in addition are the Janitor's quarters, Photographic room, Store room, etc. Besides the large Observatory, close by is a smaller building, called the "Students' Observatory," in which is a very good six-inch equatorial refractor, made in Dublin, and a two-inch transit instrument; these are wholly for the use of the University students in mathematical astronomy, who will be allowed to use the large telescope when they have fully learnt the use of the smaller one. The *outside* appearance of the great telescope, with all its array of shining wheels and circles, is chaste and beautiful; the large steel tube, over twenty-five feet long, is painted or enameled pure white, while the fittings shine like burnished gold; the twenty-inch object glass, finished by Alvan Clark, is a very fine lens, and most accurate in its work. We looked at object after object, each one more beautiful than the last,—ε Lyræ was perfectly separated, and the Ring Nebula beautifully defined. The great cluster in Hercules was so distinct, it seemed as if I could count the stars, and so plainly defined, shewing beautifully the star-fish arrangement of the principal stars. The object glass is arranged for celestial photography by reversing the outside lens. Mr. Saegmuller, of Washington, D.C., is the maker of the mounting, which is of the highest order of mechanical excellence. It is astonishing how easily the great bulky thing is moved about,—rapidly, noiselessly,—and objects so quickly and easily found. All this heavy machinery is poised most accurately and solidly upon a 320-ton pedestal of the toughest Colorado sandstone, and anchored to it by steel bolts nine feet long, three inches in diameter, while the pedestal itself rests upon the bed rock.

I cannot speak too highly of the kindness and courtesy of the Director, Dr. Howe, and his assistants, in showing me everything that laid within their power the evening I went there. It is by far the finest telescope in the West, excepting, of course, the great Lick telescope, and I shall always look back with delight at the exquisite pleasure I felt on observing the beauty of the Heavens in one of the most beautiful of modern telescopes.

The Observatory is easily reached by electric car from Denver, and Dr. Howe kindly gives up two evenings a week to visitors, for he desires to make the telescope a factor in the educational life of the people of Denver, not only by original research done with it, but also by the direct instruction, and astronomical enlightenment, which will come to those who choose to avail themselves of the opportunity of using the instrument. Dr. Howe is most accurate in his methods, and patiently describes everything to the smallest detail, so that "he who runs may read," and the most ignorant cannot fail to learn much of that most beautiful and noble of sciences, Modern Astronomy.

Mr. G. E. Lumsden read the following paper on

SPECTROSCOPIC BINARY STARS,

contributed by Mr. J. E. Gore, F.R.A.S., M.R.I.A., etc., of Ballysodare, Ireland, a corresponding member of the Society.

The term "Spectroscopic Binary" has been given to those remarkable and interesting stars which spectroscopic observation have shown to consist of two components so close together that the highest powers of our largest telescopes fail to show them as anything but single stars. The doubling of the spectral lines in some of these objects indicates that the components are both bright bodies, but in the case of Algol as the lines are merely shifted from their normal position, not doubled, it seems that one of the components is a dark body. In either case, the motion in the line of sight can be measured by the spectroscope, and hence we can calculate the actual dimensions of the system in miles, and its mass in terms of the Sun's mass, although the star's distance from the Earth is unknown.

Judging, however, from the light of the star and the character of its spectrum, we can make an estimate of its probable distance from the Earth.

Let us first take the case of Algol. The spectrum of this famous variable star is, according to the Draper Catalogue, of the first or Sirian type (A). It may, therefore, be comparable with Sirius in density and intrinsic brightness of surface. Assuming the mass of Sirius at 2·2 times the mass of the Sun, as found by Dr. Auwers, and that of the bright component of Algol at $\frac{4}{9}$ of the Sun's mass, as given by Dr. Vogel, I find that, for the *same distance*, Sirius would be about 2·8 times brighter than Algol. But the Oxford photometric measures show that Sirius is about 22 times brighter than Algol, from which it follows—since light varies inversely as the square of the distance—that Algol is 2·77 times further from us than Sirius. Assuming the parallax of Sirius at $0''\cdot39$, this would give for the parallax of Algol $0''\cdot14$, or a light journey of about 23 years. From the dimensions of the system, as given by Vogel, this parallax would give an apparent distance between the components of less than $\frac{1}{200}$th of a second of arc, a quantity much too small to be visible in the largest telescopes, or probably in any telescope which men can ever construct. From a consideration of irregularities in the proper motion of Algol and in the period of its light changes, Dr. Chandler infers the existence of a third dark body, and a parallax of $0''\cdot07$. As this is exactly one-half the parallax found above, it implies a distance just double of what I have estimated, and would, of course, indicate that Algol is intrinsically four times brighter than Sirius. This greater brilliancy would suggest greater heat, and would agree with its small density, which—from its diameter in miles, as given by Vogel—I find to be only one-third of that of water.

Let us now consider the case of β Aurigæ, which the spectroscopic observations show to be a close binary with a period of about four days, and a distance between the components of about eight millions of miles. This period and distance imply that the mass of the system is about five times the mass of the Sun. As in this case the spectral lines are doubled and not merely shifted, as in the case of Algol, we may conclude that both the components are bright bodies, and we may not be far wrong in supposing that they are of equal mass, each having two and one-half times the mass of the Sun. As the spectrum of β Aurigæ is of the same type as Sirius, we may compare it with that star as we did in the case of Algol. Assuming the same density and intrinsic brightness of surface for both stars, it follows that the light of β Aurigæ should be for *equal distances*, $2 \times \left(\frac{2 \cdot 5}{2 \cdot 2}\right)^{\frac{2}{3}}$, or 2·178 times brighter than Sirius. Now, according to the Oxford photometric measures, Sirius is 2·89 magnitudes, or 14·32 times brighter than β Aurigæ. Hence, it follows that the distance of β Aurigæ should be $\sqrt{14 \cdot 32 \times 2 \cdot 178}$, or 5·584 times greater than the distance of Sirius. Hence, assuming the parallax of Sirius at 0″·39, we have for the parallax of β Aurigæ $\frac{0'' \cdot 39}{5 \cdot 584} = 0'' \cdot 061$. From actual measures of the parallax of β Aurigæ, made at Oxford by the late Professor Pritchard, he found from one comparison star a parallax of 0″·065 and from a second 0″·059, results in remarkably close agreement with that computed above from a consideration of the stars' mass and light compared with that of Sirius. As the actual distance between the components of β Aurigæ is equal to the Sun's distance from the Earth divided by 11·625, we have the maximum angular separation between the components equal to $\frac{0 \cdot 061}{11 \cdot 625}$ or only 0″·005, or nearly the same as in the case of Algol.

Another object of this class is the bright star Spica (α Virginis) which has also a spectrum of the Sirian type, and for which Vogel finds a period of four days, and a mass of 2·6 times the mass of the Sun, with a distance between the components of 6,260,000 miles, on the assumption that the components are of equal mass and moving in a circular orbit. This would give each of the components 1·3 times the mass of the Sun, and it follows that the light of Spica should—for the *same distance*—exceed that of Sirius by $2 \left(\frac{1 \cdot 3}{2 \cdot 2}\right)^{\frac{2}{3}}$, or 1·4 times. Now the photometric measures at Oxford show that Sirius is 1·91 magnitudes, or 5·8 times brighter than Spica. Hence, it follows that the distance of Spica should be $\sqrt{5 \cdot 8 \times 1 \cdot 4}$, or 2·85 times the distance of Sirius. This would make the parallax of Spica 0″·137. So far as I know, a measurable parallax has not yet been found for this star. Brioschi, observing with a vertical circle of four inches aperture in 1819-20, found a negative parallax, which would imply that its parallax is too small to be measurable. Still the above results would seem to indicate that its parallax might be measurable by the photographic method.

We now come to ζ Ursæ Majoris (Mizar), which has also a spectrum of the Sirian type, and for which the spectroscopic measures indicate a period of about 104 days, and a combined mass equal to about 40 times the mass of the Sun. Proceeding as before, we find that the light of Mizar would be, for the same distance $2(\frac{20}{22})^{\frac{2}{3}}$, or 8·712 times that of Sirius. But the Oxford photometric measures show that Sirius is 3·04 magnitudes, or 16·45 times brighter than Mizar. Hence, the distance of Mizar should be $\sqrt{16{\cdot}45 \times 8{\cdot}712}$, or nearly 12 times the distance of Sirius. This gives for the parallax of Mizar $\frac{0''{\cdot}39}{12} = 0''{\cdot}032$. Klinkerfues found a parallax of 0·0429″ to 0·0477″, which does not differ very widely from the above result. As the velocity of the orbital motion shown by the spectroscope indicates a distance between the components of about 143,000,000 of miles, or about one and one-half times the Sun's distance from the Earth, the maximum distance between the components would be $0''{\cdot}032 \times 1\frac{1}{2}$, or about 0″·048, a quantity beyond the reach of our present telescopes.

It should be mentioned that in the case of β Aurigæ, Spica, and ζ Ursæ Majoris, as there is no variation of light, as in Algol, the plane of the orbit is probably inclined to the line of sight. This would have the effect of increasing the computed mass of the system and thus diminishing the calculated parallax. As the above calculations are made on the assumption that the orbital plane passes through the Earth, it follows that the computed parallaxes are a maximum, and that these remarkable objects may be really farther from the Earth than even the minute parallaxes found above would indicate. As the parallaxes of the nearest stars, such as α Centauri, 61 Cygni, and some others, are considerably greater than those found above, it would seem that our solar system is not situated in a region of close binaries, and that these wonderful objects lie beyond our immediate neighbourhood. It is also remarkable that they have all spectra of the first, or Sirian type, including the Algol variables whose spectra have been examined.

By the aid of the parallaxes found above we can compare the relative brightness of the Sun compared with that of the spectroscopic binaries. Assuming that the Sun is $26\frac{1}{2}$ magnitudes brighter than an average star of the first magnitude, and taking the parallax of Algol at 0″·07, I find that the Sun placed at the distance indicated by this parallax would be reduced to a star of 6·85 magnitude, that is 4·45 (6·85—2·40) magnitudes fainter than Algol, which implies that Algol is sixty times brighter than our Sun. Placed at the distance indicated by the parallax of 0″·061 found for β Aurigæ, the Sun would be reduced to a star of 7·15 magnitude, or 5·21 magnitudes (7·15 — 1·94) fainter than β Aurigæ, which implies that β Aurigæ is 121 times brighter than the Sun. In the case of Spica we have the Sun reduced to a star of 5·4 magnitude, or 5·44 magnitudes fainter than Spica, indicating that Spica is 150 times brighter than the Sun, although the mass of Spica is only 2·6 times the Sun's mass. Finally, in the case of ζ Ursæ Majoris, we have the Sun reduced to a star of 8·54 or 6·45 magnitudes (8·54 — 2·09) fainter than Mizar, which indicates that Mizar is no less than 380 times brighter than the Sun! These results show the great relative brilliancy of stars with a spectrum of the first or Sirian type when compared with that of the Sun, a conclusion which has already been arrived at from other considerations.

NINETEENTH MEETING.

October 2nd; Mr. John A. Paterson, M.A., Vice-President, in the chair.

The Librarian reported the receipt of several volumes donated by Lady Wilson, and which were a valuable addition to the Library. The Secretary was directed to convey to Lady Wilson the thanks of the Society. The third volume of the Lick Observatory reports was received and was particularly admired. Mr. A. Elvins presented some drawings of the planet Mars which he had made at the telescope of the Toronto Observatory. He had not been able to see the rectilinear markings, but his drawings showed the varying colours of the different parts of the surface very clearly.

Mr. Thomas Lindsay then read a paper on

THE TRANSIT OF MERCURY OF NOVEMBER 10, 1894.

After briefly sketching the history of the transits as observed since Kepler's time, and illustrating the conditions under which the phenomenon is possible, the calculations for Toronto were given as follows:—

To determine the moment of contact we must apply to the true positions of the Sun and planet the corrections for parallax, and as these depend upon the point from which the phenomenon is viewed, it follows that the contact will not be observed simultaneously at two different stations. The correction, however, is very small, and this necessitates very careful calculation if we wish to make a prediction very near to the truth. For this purpose we refer to the Greenwich ephemeris for the elements of the Sun and Mercury at the transit, and from the data there given may determine approximately the times of contact by the graphic method explained in a former paper on the subject. We shall find that the first contact occurs about 3h. before and the last contact about 2h. 20m. after geocentric conjunction.

With these times as assumed, we proceed to investigate the problem rigorously, by the same method as employed in the calculation of eclipses of the Sun.

The observer is supposed to be stationed at the centre of the Sun, and a straight line from that somewhat inaccessible position is drawn to Toronto Observatory in latitude North 43° 39′ 35″, longitude West 5h. 17m. 34s.

He sees a circle in space of the radius of the Sun, and when Mercury comes into the field, at the moment when the distance from Toronto to its centre is equal to the sum of the radii of Sun and planet, first external contact occurs. The Earth is seen as a disc, and the position of Toronto will be determined as in eclipse or occultation problems. The relative radii of Earth and Mercury will be :—

For the Earth : Parallax Mercury — parallax Sun = 13″ — 8″·9 = 4″·1.

For Mercury : Its semi-diameter = 4″·9.

A 3 hours before conjunction we have

	H.	M.	SEC.
Greenwich mean time, Nov. 10	3	54	47·6
Toronto difference	5	17	34·6
Toronto mean time, Nov. 9, astronomical reckoning	22	37	13·0
R. A. Meridian	13	56	23·5
Sun's R. A.	15	3	14·4
Hour Angle —	1	6	50·9
In arc h =	16°	42′	44″·5

Adopting the usual notation we have: For Toronto,

$$\varphi' = 43^\circ\ 28'\ 6'',\ \rho = 9{\cdot}9993104,\ \rho \cos \varphi' \sin h = -{\cdot}20838$$

of the radius of disc, and in seconds of arc

$$({\cdot}20838)\ (4''{\cdot}1) = 0''{\cdot}854 = -\ \xi.$$

Variation of ξ in one minute =

$$(7{\cdot}63992)\ \rho \cos \varphi' \cos h = {\cdot}00303$$
$$({\cdot}00303)\ (4''{\cdot}1) = 0''{\cdot}0124 = +\ \xi'$$

$$\rho \cos \varphi' \sin \delta \cos h = {\cdot}20617$$
$$\rho \sin \varphi' \cos \delta = {\cdot}65686$$

$${\cdot}86303 = 3''{\cdot}538 = +\ \eta.$$

Variation of η in one minute =

$$(7{\cdot}63992)\ \xi \sin \delta = {\cdot}00027 = 0''{\cdot}0011 = +\ \eta'.$$

Mercury's Decl............	17°	19′	21″·9	
Sun's Decl................	17°	16′	53″·9	
Co-ordinate of planet South..		2′	28″	= 148″ = − y

	H.	M.	SEC.
Mercury's R. A.	15	4	22·19
Sun's R. A.	15	3	14·47
		1	7·72

reduced to arc of great circle $969''{\cdot}72 = +\ \chi$ = co-ordinate of Mercury East.

From the elements we have

$$\text{Variation of } y \text{ in one minute} = 2''{\cdot}45 = +\ y',$$
$$\text{Variation of } \chi \text{ in one minute} = 5''{\cdot}64 = -\ \chi'.$$

Then the position of the planet is determined by

$$\chi - \xi = 970''{\cdot}574 = m \sin M$$
$$y - \eta = 151''{\cdot}538 = m \cos M$$

and in one minute of time

$$\chi' - \xi' = -\ 5''{\cdot}6524 = n \sin N$$
$$y' - \eta' = +\ 2''{\cdot}4489 = n \cos N$$

from which we deduce

$$(M - N) = 14^\circ\ 33'\ 19''\ ;\ \log m\ 12{\cdot}992266\ ;\ \log n\ 10{\cdot}789564\ .$$

Putting L for sum of semi-diameters $976''{\cdot}8$, we have

$$\sin \psi = \frac{m \sin (M - N)}{L},\ \psi = 14^\circ\ 38'\ 23''.$$

Then the moment of contact, n being one minute motion, is given by the correction

$$\frac{m \cos (M - N)}{n} - \frac{L \cos \psi}{n} = 55{\cdot}8\text{sec.}$$

added to assumed time.

Therefore Greenwich mean time of first contact at Toronto is 3h. 55m. 43·4sec. Eastern standard time is 10h. 55m. 43·4sec.

For first internal contact we have L' = difference of semi-diameters = 967″, $\psi = 14° 47' 42''$, and the correction to assumed time is 2m. 34·2 sec. Therefore Eastern standard time first internal contact is 10h. 57m. 21·8sec. a.m.

For last contact we assume 2h. 20m. after conjunction, at which time we have

$$\begin{array}{ll} \xi = 2''{\cdot}647 & \xi' = 0''{\cdot}005 \\ \eta = 3''{\cdot}086 & \eta' = 0''{\cdot}003 \\ y = 634''{\cdot}7 & y' = 2''{\cdot}45 \\ \chi = 754''{\cdot}23 & \chi' = -\ 5''{\cdot}64 \\ \psi = 16° 33' 19'', & \end{array}$$

and the correction becomes 1m. 18sec. to be subtracted

(2h. 20m. — 1m. 18sec.) + 6h. 54m. 47sec. = 9h. 13m. 29sec.

Greenwich mean time; or Eastern standard time of last contact, 4h. 13m. 29sec. p.m.

Last internal contact occurs 4h. 11h. 50sec. p.m.

Mr. Lindsay then reviewed the chapter in Newcomb & Holden's "Astronomy" on the solar parallax as determined from observations of the transit of an inferior planet, and sketched the method by which the final equations are reached, involving the errors of the ephemeris and the Sun's parallax.

TWENTIETH MEETING.

October 16th; Mr. E. A. Meredith, LL.D., in the chair.

Miss May Bambridge was elected an active member of the Society. On motion of Mr. G. E. Lumsden and Mr. Thomas Lindsay, The Meaford Astronomical Society was formally enrolled as a body in affiliation with the Astronomical and Physical Society of Toronto under the terms and conditions proposed by the Council and accepted by the Meaford Society.

Mr. J. Todhunter addressed the meeting, referring to the death of Mr. Stephen Huebner, an active member of the Society since the year of incorporation, and one who had been at all times most assiduous in furthering its interests. Mr. Huebner had been for some time before his decease residing in Montreal, and some of the members who had but lately become associated with the Society had not met him, but those who remembered Mr. Huebner in Toronto knew of the zeal he had always displayed in the cause of science and education generally. Several members present recalled the pleasant associations enjoyed with the deceased. It was resolved to communicate with the relations of the deceased in Europe and acquaint them of the Society's regret at his loss.

Mr. Arthur Harvey, in the course of a discussion on the subject of sun-spots, referred to the supposed rotation of these objects, and asked whether any member had ever observed this. Mr. A. Elvins stated that he had on one occasion particularly observed the turning round of a sun-spot, and had communicated the observation to the *Astronomical Register*. It was seen also by one other observer in England, and has been referred to by the editor of "Webb's Celestial Objects." Mr. G. G. Pursey described a very simple method which the amateur might adopt to determine the dimensions of sun-spots. The image of the Sun is projected on a screen, so that it covers a disc of a certain diameter. The part covered by a group of spots may then be measured, and will very approximately bear the same proportion to the disc as the linear breadth of the group to the Sun's diameter, which latter is known.

Mr. Z. M. Collins presented two silver-on-glass specula of 6-in. and 4-in. diameter respectively, which he had constructed. Mr. Collins, who was warmly congratulated on the success of his work, announced his intention of continuing the work, with a view to the construction of still larger mirrors, and thought that reflecting telescopes might be very successfully introduced among Canadian observers. He had followed essentially the method described by the late Dr. Henry Draper, and was indebted also for some valuable hints on silvering to Dr. J. A. Brashear, of Allegheny.

Mr. Thomas Lindsay then read the following paper

ON A LENS FOR ADAPTING A VISUALLY CORRECTED REFRACTING TELESCOPE TO PHOTOGRAPHIC OBSERVATIONS WITH THE SPECTROSCOPE,

by Professor James E. Keeler, D.Sc., of Allegheny, Pa., Director of the Allegheny Observatory, and a corresponding member of the Society.

Everyone who has any familiarity whatever with the telescope knows that the so-called achromatic objective is by no means exactly what its name implies. The differently coloured rays from a star are not accurately united in a single point, as they would be by a parabolic reflector, but are converged to different foci, distributed along the axis over a line of considerable length. The lenses of a visual telescope are so figured that the brightest rays are brought as nearly as possible to the same focus, while in the photographic telescope the rays united are those which most actively affect the ordinary dry-plate. Every visual objective made of two lenses has a minimum focus for the brightest rays of the spectrum in the yellow or greenish-yellow (the position of the minimum varies somewhat for different instruments), the focus of all other rays being outside this point. The most widely aberrant rays are the violet, which do not greatly affect the eye.

Thus the image of a star formed by the brighter rays is surrounded by a violet halo of outstanding colour, well known to all observers with the telescope. It is not specially conspicuous with small instruments, but becomes positively offensive in one of the great refractors of modern times, at least to perceptions which have not become blunted by custom. The observer who habitually uses one of these great telescopes learns to disregard the secondary spectrum, and after a while is hardly aware of its existence, but the colours around a bright object are invariably noticed at once by the novice, who expresses his admiration for what the astronomer regards as a defect. A well-known observer once remarked to me, while we were testing a remarkably fine reflector by Brashear, that if he should accustom himself to the use of such an instrument he would never be able to tolerate the images seen in a refractor.

To diminish the effect of chromatic aberration, the astronomers of the seventeenth century gave their single objectives a very great focal length. In our modern great telescopes we notice a tendency of the opticians to revert to this practice, as the focal length of such a telescope is generally something like nineteen or twenty times the aperture instead of fifteen times, which is the usual ratio for small instruments

The distribution of foci in the axis of a telescope is best illustrated by the "colour-curve," which is a curve representing the focal length of the objective as a function of the wave-length of light. It can be computed if the constants of the objective are known with sufficient exactness (which is seldom the case), or it may be determined by observation with a spectroscope. In No. 9 of the *Publications of the Astronomical Society of the Pacific*, I gave a popular account of the latter method, and its application to the case of the Lick telescope. Figure 1 is the colour-curve platted from the data there published. The objective is supposed to be about 57 feet distant, directly below the curve, so that the lowest point of the curve represents the minimum focus. It will be noticed that the minimum focus falls at about λ 5650, that the B and F lines of the spectrum are united, and that the focus for the extreme violet rays is some eight centimetres or more than three inches beyond that of the D line.

The colour-curve of any other modern telescope would be quite similar to that shown in the figure; the difference would be mainly in the scale, or in the lengths of the ordinates, which would be pretty nearly proportional to the length of the telescope. The observer, having continually in mind the form of the colour-curve, can readily understand all phenomena depending upon the chromatic aberration of his objective, and in the case of a large telescope they are sometimes curious and interesting.

Large telescopes are sometimes provided with a third lens, called a photographic corrector, which is placed over the visual objective to adapt it to photographic purposes. The colour-curve of the combination has a minimum near the hydrogen line $H\gamma$, where the photographic action is greatest. At the same time the focal length of the telescope is considerably shortened.

However annoying the chromatic aberration of a great telescope may be in visual observation, it is found to be still more objectionable when the attempt is made to photograph the upper part of a star spectrum with a spectroscope adjusted in the focal "plane" of the telescope. The difficulty then encountered will be readily understood from a consideration of the colour-curve. To take the case of the Lick telescope, suppose that the slit of the spectroscope is in the focus for the $H\gamma$ rays. It is then 36·8 millimetres beyond the focus for $H\beta$ and 33·3 millimetres inside the focus for $H\delta$. The cone of rays corresponding to the former line has a diameter of 1·9mm. where it is intersected by the slit-plate, and that of the $H\delta$ rays has a diameter of 1·7mm. As the slit is only about one-fiftieth of a millimetre wide, very few of these rays can enter. The spectrum, instead of being linear and of nearly uniform intensity between $H\beta$ and $H\delta$, is very narrow at $H\gamma$, widens rapidly on both sides of that line, and falls off very rapidly in intensity. Hence, only an extremely short range of the spectrum can receive the proper exposure. Not only is the extent of the photograph thus greatly lessened, but reliable estimates of the relative intensities of lines are made almost impossible. These difficulties, which are so readily understood when pointed out, are perhaps fully realized only by one who has actually attempted to make the photographs.

It is, therefore, highly desirable to find some way of changing the chromatic aberration of a visual telescope, so as to unite the photographically active rays. We have seen that this end can be attained by the use of a specially corrected third lens, but the photographic corrector, besides being expensive, shortens the focus so greatly that the spectroscope cannot be placed in the proper position. In the case of the Lick telescope the focus is inside the tube, about nine feet above the eye end, where it is inaccessible for spectroscopic purposes.

What is required, therefore, is a small lens, which, placed in the cone of rays from the large objective, will effect the desired change in the colour-curve. Such a lens, it may be observed, would be useless for ordinary celestial photography, as the distortion at a short distance from the axis would be very great. For stellar spectroscopy, however, a field two or three hundredths of an inch in diameter is quite sufficient, and the question of distortion need not be considered.

In photographing star spectra with the Allegheny refractor, the difficulties already mentioned naturally forced themselves on my notice, and led me to consider what form should be given to a correcting lens that would facilitate the observations. Some of the results which I have reached are given in the rest of the present article. Although the considerations are simple, I have not been able to find anything in print on the subject, probably because it is only quite recently that the necessity for a device of the kind referred to has arisen. I believe, however, that correctors of some kind are actually in use, but do not know whether they are satisfactory. I have been told that one of them consists of a thick spherical shell of glass with short radii, the convex surface being turned toward the objective. A little consideration will show that the effect of such a shell is in the proper direction, and it might answer for a small telescope, but the thickness of glass required in the case of a large telescope would be too great. I have seen the suggestion somewhere that a plate of glass of sufficient thickness would effect the desired change, but this is evidently a mistake, for the action of such a plate is just the reverse of what is necessary.

Although in what follows I have employed the approximate formulæ for lenses, in which the thickness of the glass and the spherical aberration are neglected, the results are quite sufficiently accurate for the purpose, which is to determine the best general form of correcting lens. It would hardly be worth while to make a more accurate computation, since this would in any case have to be repeated for the special kinds of glass used, and for the particular telescope under consideration. I have chosen for illustration the case of the Lick telescope, because its colour-curve is known. The other data required are the refractive indices of crown and flint-glass, and these I have taken (to four decimal places) from the tables of Hopkinson. The "dense flint" of the tables is nearly the same as the glass of a prism in my possession, which is only slightly coloured. The following table contains all the necessary data. In the column under "colour-curve" are given the distances in metres of the different foci of the great telescope from a point on the axis one metre above the focus for the *F* line :—

LINE OF SPECTRUM.	REFRACTIVE INDICES.		COLOUR-CURVE.
	Hard Crown.	Dense Flint.	Distance of Focus from Lens.
			m.
B	1·5136	1·6157	1·0000
C	1·5146	1·6175	0·9939
D_2	1·5171	1·6224	0·9886
E	1·5203	1·6289	0·9905
b_4	1·5210	1·6302	0·9914
F	1·5231	1·6347	1·0000
$H\gamma$	1·5280	1·6453	1·0368
h	1·5309	1·6518	1·0701

The lens may be placed in the cone of rays at any convenient point. We shall first suppose that it is placed one metre above the focus for the *F* line, in which case its aperture must be (at least) 53mm.

In order to make the spectrum of a star linear at $H\gamma$, which is the line most used for photographic determinations of motions in the line of sight, it will probably be sufficient to unite the rays F ($H\beta$) and h ($H\delta$); at least we are not likely to find a more promising condition in the beginning. The problem is, therefore, to find a lens which, placed one metre above the focus for the F line, will unite the rays F and h at some point on the axis.

Let us first take the case of a single lens. Since the h rays must be deviated more than the F rays in order that both may meet at the same point, the lens must be convex; further, since it is desirable to produce this dispersive effect with the least change of refraction, the lens should preferably be of flint-glass. The formulæ, which are only first approximations, tell us nothing about the *shape* of the lens, or relation between the radii of its surfaces. It is, however, easy to see, on general principles, that the best form is very nearly a plano-convex lens, with the curved surface turned toward the objective.

Let u be the distance from the lens at which the two rays are united, v and v' the distances from the lens of the foci for F and h respectively, as given in the table. If the point whose distance is u is regarded as a source of light, the F and h rays, after refraction by the lens, would appear to diverge from points whose distances are respectively v and v'. Hence, u and v are conjugate foci for the F rays, and u and v' are conjugate foci for the h rays.

The general equation connecting the conjugate foci of a thin lens, the refractive index, and the radii of the surfaces, is

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$$

in which all lines measured from the lens toward the source of light (at u, in this case toward the focus of the objective) are positive.*

If the lens is plano-convex, $r = \infty$, and the formula becomes

$$\frac{1}{v} - \frac{1}{u} = -\frac{\mu - 1}{s}$$

If the symbols in this formula are regarded as referring to the F line, we may write for the h line,

$$\frac{1}{v'} - \frac{1}{u} = -\frac{\mu' - 1}{s}$$

* According to this convention, the focal length of a convex lens is negative.

and since u is the same in both cases,

$$\frac{1}{v} + \frac{\mu - 1}{s} = \frac{1}{v'} + \frac{\mu' - 1}{s}, \text{ or } \frac{1}{v} - \frac{1}{v'} = \frac{\mu' - \mu}{s}.$$

From the table of data we have

$$\mu = 1{\cdot}6347, \mu' = 1{\cdot}6518, v = 1{\cdot}0000, v' = 1{\cdot}0701,$$

and substituting these values we find for the radius of the lens, $s = + 0{\cdot}2611$ metres.

For the focal length of the lens (for the F line)

$$f = - \frac{s}{\mu - 1}$$

we find $f = - 0{\cdot}4113$ metre. The focal length for the h line is $- 0{\cdot}4005$ metre. The distance u of the point at which the rays are united is given by

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

and is found to be 0·2915 metre.

We now wish to obtain the form of the colour-curve, as modified by the correcting lens. To do this we substitute successively for v the values in the last column of the table of data, and compute the corresponding values of u, using for each ray its appropriate index. The values of u so obtained are the distances of points on the modified colour-curve. The results are given in the following table:—

LINE.	μ	v	u
		m.	m.
B	1·6157	1·0000	0·2978
C	1·6175	0·9939	0·2966
D_2	1·6224	0·9886	0·2945
E	1·6289	0·9905	0·2925
b_4	1·6302	0·9914	0·2922
F	1·6347	1·0000	0·2915
Hγ	1·6453	1·0368	0·2910
h	1·6518	1·0701	0·2915

The resulting colour-curve is shown in Figure 2, which is drawn on the same scale as Figure 1. It shows an enormous advantage over the latter, not only for the part of the spectrum considered in its determination, but for the lower spectrum as well. Nevertheless, the arrangement is impracticable, for the following reasons:—In the first place, the focus of the telescope is shortened about 0·7 metre, which would make the focal plane inaccessible, with the ordinary construction of the eye end of a telescope tube. In any case, the change of focus would be extremely inconvenient. In the second place, the convergence of the cone of rays would be increased from 1 : 19 to 1 : 5·53, which would necessarily also be the ratio of the aperture of the collimator to its focal length. With a given effective aperture the collimator would, therefore, have to be very short, a construction which is very disadvantageous in every respect; it would, moreover, be difficult or impossible to make a sufficiently good collimator lens with such a very large angular aperture.

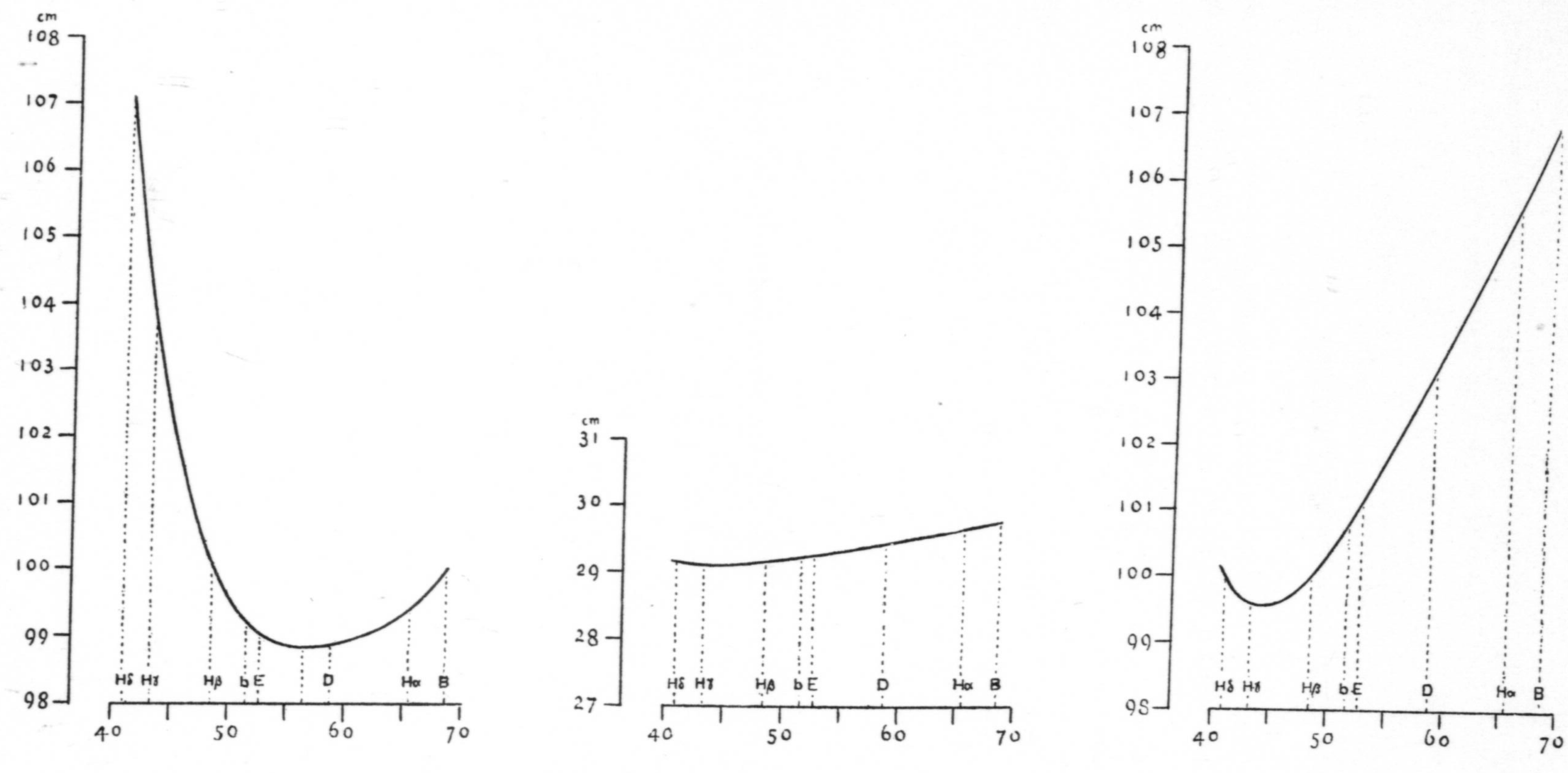
A correcting lens placed two metres above the focal plane of the great objective would have an aperture of 105·3mm.; its focal length would be 1·591m., and it would shorten the focal length of the telescope by 1·114 metres. The ratio of aperture to length of the transmitted cone of rays would be 1 : 8·42. A lens of the same kind placed close to the telescope objective becomes the ordinary photographic corrector. The greatest shortening of the focus is then produced; at the same time the angle of the convergent cone of rays is the least possible.

It appears, therefore, that a single correcting lens is not a practical arrangement; a little consideration will show, however, that the difficulties which have been pointed out can be obviated, to some extent at least, by the use of a double lens. If the curvature of the convex flint lens is increased, and the excess of dispersive effect neutralized by a concave lens of crown glass, the superior refractive power of the latter will give the combination a greater focal length than that of the equally dispersive single lens, and it will, therefore, not converge the rays so strongly. Whether this principle of construction can be carried so far as to leave the focal length of the telescope unchanged is a question we may now seek to determine. As before, we shall regard the compound lens as placed one metre above the focal plane for the *F* line, and use formulæ which hold only to the first approximation.

For thin lenses in contact, the relation between the conjugate foci and the focal lengths of the lenses is expressed by the equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

In this and in the following formulæ, quantities relating to the crown lens will be indicated by the subscript 1, those relating to the flint lens by the subscript 2.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$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{r_1} - \frac{1}{s_1}\right),$$

if μ_1 changes to $\mu_1 + \Delta\mu_1$, the corresponding change produced in $\frac{1}{f_1}$ is

$$\Delta\left(\frac{1}{f_1}\right) = \Delta\mu_1\left(\frac{1}{r_1} - \frac{1}{s_1}\right) = \frac{\Delta\mu_1}{\mu_1 - 1}\cdot\frac{1}{f_1}$$

and in the same way the alteration of $\frac{1}{f_2}$ due to an alteration $\Delta\mu_2$ of μ_2 is

$$\Delta\left(\frac{1}{f_2}\right) = \frac{\Delta\mu_2}{\mu_2 - 1}\cdot\frac{1}{f_2}$$

u is the distance of the point at which the F and h rays are united by the lens, and hence is the same for both. The total effect of the change in the refractive indices is, therefore,

$$\Delta\left(\frac{1}{v}\right) = \frac{\Delta\mu_1}{\mu_1 - 1}\cdot\frac{1}{f_1} + \frac{\Delta\mu_2}{\mu_2 - 1}\cdot\frac{1}{f_2}\,.$$

If the symbols in the formula relate to the F line, the change of v, to satisfy the conditions for the proposed lens, must be equal to the difference between the ordinates at h and F in the colour-curve of the objective.

From the table of data we take

$\mu_1 = 1{\cdot}5231,\ \Delta\mu_1 = {\cdot}0078,\ v = 1{\cdot}0000$ for the F line.
$\mu_2 = 1{\cdot}6347,\ \Delta\mu_2 = {\cdot}0171,\ v = 1{\cdot}0701$ for the h line.

Also, $\Delta\frac{1}{v} = -\,0{\cdot}06551$, and substituting these values the condition becomes

$$\frac{{\cdot}014911}{f_1} + \frac{{\cdot}026942}{f_2} = -\,{\cdot}06551.$$

If we wish to determine the lens so that the position of the focus for the F line shall remain unchanged, then for these rays the lens must act like a thin glass plate, and we have the further condition $f_1 = -f_2$. Combining this with the preceding one,

$$\frac{{\cdot}014911}{f_1} - \frac{{\cdot}026942}{f_1} = -\,{\cdot}06551$$

from which

$f_1 = +\,0{\cdot}18365$ metres
$f_2 = -\,0{\cdot}18365$ "

If we make the flint lens double convex with equal radii, and the inner surface of the crown lens to fit the flint, the radius of each of these surfaces will be 0·233 metres, and the radius of the back concave surface of the crown lens 0·163 metres. Such a lens would be perfectly easy to make. The surfaces in contact could be cemented to diminish loss of light by reflection, and a lighter flint could be used than that chosen for illustration. With the flint lens turned toward the objective, the lens would, moreover, have very little spherical aberration, although determined by these approximate formulæ.

In order to find the form of the colour-curve when this lens is placed in position, we first require the focal lengths of each lens of the combination for different rays of the spectrum,—or rather the reciprocals of the focal lengths, which are most conveniently obtained by means of the relation

$$\frac{1}{f} = \frac{1}{f^1} \cdot \frac{\mu - 1}{\mu^1 - 1}$$

where u^1, f^1, are respectively the refractive index and focal length for the F line, and u, f, the corresponding values for any other line. For any one line of the spectrum we then substitute the values so found, together with the value of v for the same line, as given in the table of data, in the equation

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f_1} - \frac{1}{f_2}$$

and find u, which is the distance at which the same rays are united after passing through the lens. The following table contains the results :—

LINE.	v	$\frac{1}{v}$	$\frac{1}{f_1}$	$\frac{1}{f_2}$	$\frac{1}{u}$	u
B	1·0000	1·0000	5·3464	—5·2821	0·9357	1·0687
C	0·9939	1·0061	5·3567	—5·2976	0·9470	1·0560
D_2	0·9886	1·0115	5·3827	—5·3396	0·9684	1·0326
E	0·9905	1·0096	5·4160	—5·3954	0·9890	1·0111
b_4	0·9914	1·0087	5·4234	—5·4065	0·9918	1·0083
F	1·0000	1·0000	5·4452	—5·4452	1·0000	1·0000
Hγ	1·0368	0·9645	5·4962	—5·5361	1·0044	0·9956
h	1·0701	0·9345	5·5264	—5·5919	1·0000	1·0000

From the last column of this table the colour-curve shown in Figure 3 has been platted. It is obviously much better adapted to photographic work than the original curve. With the spectroscope slit in the focal plane of the $H\gamma$ rays, the F and h rays would be only 4·6mm. out of focus. The spectrum would be practically linear for a considerable distance on each side of $H\gamma$.

In practice, the lens could be mounted in a cell provided with suitable adjusting screws, on a swinging arm within the telescope tube, so that it could be pushed into place by a rod projecting through the tube, and held centrally by adjustable stops. When the telescope was required for ordinary visual observation, the lens could be withdrawn from the cone of rays. I believe that in the future a lens of this kind will be regarded as a necessary adjunct to every large refractor.

Mr. A. Elvins read in outline a paper on

METEOROLOGY.

For certain phenomena a cosmic origin was sought, and records of observations made at various places were read, tending to prove that the magnetic needle is disturbed by the coming into the atmosphere of meteoric matter. This being established, and also the connection between auroral and magnetic disturbances, Mr. Elvins proceeded to give his reasons for believing that sun-spots are caused by the inrush of meteoric matter upon the Sun. To uphold this theory, it was necessary not only to account for the periodicity of the sun-spots, but also for the irregularities of this periodicity. The former was held to be due to the gravitating influence of the planet Jupiter in drawing meteoric matter into the solar system from outer space when in a particular part of its orbit. The irregularities were thought to be due to the fact that Jupiter would at times be aided by the other major planets, and at other times would exert its attractive force alone.

Mr. Elvins held that the motion of the solar system in space is an important factor in this investigation, and that more meteoric matter is encountered from the direction in which the Sun moves than from any other. Each of the planets must cross this direction once in a revolution, so Saturn's period would be about thirty years, Jupiter's about twelve, the Earth's one year, that of the Moon twenty-seven and one-third days, and meteorological changes might be due to these varying configurations. As much research and comparison of data would be necessary to fully establish the cosmic origin sought, Mr. Elvins announced that he would at a future meeting present his views in more extended form.

TWENTY-FIRST MEETING.

October 30th; Mr. J. A. Paterson, M.A., Vice-President, in the chair.

Mr. Paterson referred feelingly to the loss sustained by the Society in the death of Mr. Charles Carpmael, M.A., F.R.A.S., etc., who had been President since the date of incorporation. The high attainments of the deceased in the scientific world and the active interest he had taken in this Society's work so long as his health permitted, were well-known by those who had had the pleasure of a more or less intimate acquaintance with him, and his genial personality would be long remembered. He was always at the service of the amateur who wished to consult him upon some difficult subject, and was known to be a most conscientious critic, quick to point out the errors into which the beginner in scientific work is so liable to fall, but always ready to impart information gathered from his own great store of knowledge in physical science. Mr. A. Elvins, who had been one of the committee which had waited upon Mr. Carpmael in 1890 to ask him to take the leadership of the then small society about to be incorporated, paid a high tribute to the late President's zeal and kindly desire to further the work in every possible manner. In his death the Society lost an earnest friend and an accomplished astronomer and physicist. Other members present recalled instances illustrating Mr. Carpmael's unvarying kindness to the young enquirer. The Vice-President appointed a committee, consisting of the gentlemen who had formed the nucleus of the Society in 1890, to prepare a suitable memorial to the late President.

Mr. M. Turnbull presented a most carefully executed diagram illustrating the approaching transit of Mercury. Being drawn to scale, it showed the path of the planet across the Sun and its position at stated intervals. A short address on the subject of the transit was given, after which Mr. Turnbull was asked to favour the Society by leaving the drawing in its possession, and, having consented to do so, was warmly thanked by the meeting.

The Chairman then announced that Mr. D. J. Howell would exhibit on the screen the lantern slides in possession of the Society. Many of these were from the original negatives donated by the Lick and Paris Observatories, and prepared for the lantern by Mr. Howell. They included views of the Moon in different phases, views of sun-spots and faculæ, and of many of the nebulæ and star clusters. A most interesting feature was the exhibition side by side of a photograph of the Sun's corona as seen at total eclipse, and a drawing of the same. A photograph of the solar prominences, made at the Kenwood Observatory by Professor Hale, was much admired; this was taken without an eclipse, and was considered a very great triumph for the distinguished observer. Drawings and photographs of the Saturnian system were also highly appreciated. A vote of thanks was tendered Mr. Howell, who had always been most painstaking in collecting and reproducing whatever could be made an interesting subject for exhibition.

TWENTY-SECOND MEETING.

November 13th; Mr. John A. Paterson, M.A., Vice-President, in the chair.

Mr. John Bertram, of Toronto, was elected an active member of the Society.

The Committee appointed to prepare a memorial to the late President reported as follows:—

To the Vice-President and Members of The Astronomical and Physical Society of Toronto.

Your Special Committee appointed to consider the action proper to be taken with respect to the death of the President of your Society, begs leave to report as follows:

Your Committee would recommend that the following resolution be inscribed on the minutes of your Society:

Resolved, that The Astronomical and Physical Society of Toronto has learned with deep sorrow of the untimely death of its President, Mr. Charles Carpmael, M.A., F.R.A.S., Director of the Toronto Observatory and Superintendent of the Meteorological Service of Canada, and, in common with other scientific bodies, deplores the loss thus sustained; that this Society desires to pay a tribute to the memory of one who, apart from the obligations he conferred upon it as a whole, endeared himself to many of its members by the willingness he always displayed in assisting to remove the difficulties which the Society had to encounter in its Astronomical and Physical work; that this Society gratefully acknowledges the readiness with which Mr. Carpmael, at an important stage of its existence, acceded to its request to become its presiding officer and to lend the benefit of his name and influence in opening to it avenues by which it could come into official relationship with similar bodies in the Mother Land as well as in foreign countries; that it was a subject of equal regret to this Society as it was to its late President that the state of his health precluded his regular attendance at its meetings and prevented him from giving to its interests that personal supervision which it would otherwise have enjoyed; that, whether publicly or privately, the loss to the general community, of a citizen possessing such natural gifts and acquired qualifications as Mr. Carpmael possessed is greatly to be lamented, and that in recording its sense of this loss this Society begs leave to tender its condolence to the bereaved family.

All of which is respectfully submitted.

(Signed) A. ELVINS, *Chairman.*

On motion of Mr. G. G. Pursey, seconded by Mr. Thomas Lindsay, the report was adopted and a copy ordered to be engrossed and forwarded to the relatives of the deceased.

Rev. C. H. Shortt, M.A., addressed the meeting on the subject of appointing a successor to the Presidential chair, and nominated Larratt W. Smith, Q.C., D.C.L. Mr. Arthur Harvey seconded the nomination, which was made unanimous. Dr. Larratt W. Smith was, therefore, declared elected President of the Society. (Applause.)

A committee was appointed to assist Mr. G. G. Pursey in the work of compiling a complete catalogue of the works in the Society's library, which had assumed very large proportions. An interesting discussion followed regarding particular branches of work which the members of the Society might engage in. The field of sidereal astronomy was thought to be the most inviting of all, and arrangements were made towards the preparation of a series of papers in which all might take part. Mr. Harvey wished the members to study the nebulæ and the problems connected with their dimensions, distances, etc. He thought it was an open question whether the nebulæ or stars were nearer to our system.

Reports from various observers were then received on

THE TRANSIT OF MERCURY, NOVEMBER 10TH.

Arrangements had been made to send telescopes to some of the public schools of the city, that the pupils might have an opportunity to observe the phenomenon. Mr. G. H. Meldrum and Mr. A. Elvins had placed 3-inch refractors at Wellesley School, and under the management of Miss A. A. Gray, very satisfactory observations had been made of the planet on the disc. Messrs. Michael Bros. had placed one of their 3-inch telescopes by Vion Frères, of Paris, at the York Street School; two others of these refractors were successfully used, one by Dr. A. D. Watson, and one by Mr. C. T. Gilbert, who had taken charge of the arrangements at the Jesse Ketchum School. Messrs. Michael Bros. had announced their willingness to place their telescopes at the service of the School Board on any special occasions. A Gregorian reflector had been sent to the Ryerson School by Dr. Watson, and also a small refractor.

Mr. G. E. Lumsden had observed in his 10-inch Newtonian reflector, and several members availed themselves of the Wilson telescope at the residence of Mr. J. A. Paterson. The sky at first contact was so clouded as to render accurate observations impossible, a circumstance much regretted; and at the last contact the Sun was buried in dense clouds. During the day, however, many observations of the planet had been made. Mr. Lumsden reported having distinctly noted a bright halo around the planet. Messrs. J. and T. Clougher had observed in a 3-inch refractor, and made several sketches of the planet and sun-spots at different hours.

At the Toronto Observatory, Mr. R. F. Stupart had made efforts to note the contacts, but failed, as did all others who reported from Toronto and vicinity. In anticipation of possible cloudy weather at the transit, Mr. F. L. Blake, chief observer at the Observatory, had made meridian observations of Mercury at the Eastern elongation, and was successful in noting the meridian passage of the planet when on the Sun's disc. He reported the following

MERIDIAN TRANSITS OF MERCURY.

October 18th.—Owing to the low altitude of Mercury great difficulty was experienced in obtaining transits across the meridian near the elongation; only one observation was taken, the planet being very faint and unsteady. The transit of the centre was taken by estimation as far preferable to that of the limb on account of its tremulous motion. The following is the result for October 18th, 1894, giving clock time of transit of centre of planet, the instrumental errors in azimuth, collimation and level being allowed for.

H.	M.	SEC.	
15	21	55·80	
	16	6·80	clock fast.
15	5	49·00	R. A. of Mercury by observation.
15	5	49·33	R. A. of Mercury by Almanac.
		0·33	difference.

November 10th, 1894.—Transit of Mercury across the meridian while on the face of the Sun.

1st Limb.	*2nd Limb.*
H. M. SEC.	H. M. SEC.
15 20 34·91.	15 20 35·75.

Mean of the two gives—

H.	M.	SEC.	
15	20	35·33	centre of Mercury.
		—0·77	instrumental corrections for azimuth, collimation and level.
15	20	34·56	
	–16	26·43	clock error.
15	4	8·13	R. A. of Mercury by observation.
15	4	7·956	R. A. of Mercury by Almanac.
		0·174	difference.

The above transit was the mean of 9 wires on each limb. The clock error for observations may be taken as correct within the limits of ± 0·025 sec.

Reports of observations from other stations were encouraging. Mr. David E. Hadden, of Alta, Iowa, reported as follows :—

Mercury's transit across the disc of the Sun was fairly well observed in Alta. The morning of the 10th opened with the sky entirely overcast, with light snow falling and a very strong cold wind blowing from the North-west; about 9 a.m. the sky partly cleared, so that the Sun could be seen at intervals, but the air was filled with flying frost particles, which entirely prevented accurate observations of the I. and II. contacts. Owing to the atmospheric conditions the Sun's limb was in a "boiling" condition, and when the planet was just within the disc it appeared of all shapes but a round disc—being lengthened out two or three times its diameter. However, I noted the times at which I could detect a ring of light between the limb and the planet, but this is fully two minutes late, which will serve to indicate the unsteady air I had to contend with.

Towards noon the conditions were much better, and definition quite good; the image of Mercury was well defined and sharp, and appeared of a uniformly jet black colour, without any lighter markings or haziness. III. contact was quite carefully observed; the error in time will not exceed one or two seconds, I think. At IV. contact there was a slight unsteadiness of the Sun's limb, and the time recorded for this phase is a little late.

I used a 3-inch Jena glass telescope, diagonal eye-piece, magnifying power 85. Time used is Central Standard, which was compared on 9th and 10th with noon telegraphic signals.

Following are times of contacts:—

	Central Standard Time.			*Remarks.*
	H.	M.	SEC.	
I. contact..	..	..	..	Could not observe owing to unsteady air.
II. contact..	9	59	45	Fully 2m. late; unsteady atmosphere.
III. contact..	3	11	30	Quite accurate.
IV. contact..	3	13	03	A little late.

Dr. J. C. Donaldson, of Fergus, Ont., reported as follows:—

I managed to get some views of the transit of Mercury on the 10th instant, although the sky was very cloudy. I was unable from clouds intervening to see the first external contact of the planet with the Sun's disc, but with a low power (50) noted the time of the first internal contact, which by my watch was about 10.58 E. S. T. I was not able to get a very prolonged view owing to clouds intervening again, until the planet had got some little distance on the Sun's disc, about 11.05 E. S. T., when I was for a very short time able to use a higher power (130), which showed the planet quite round and distinct to my sight. Another person who was present stated that there seemed to be a grayish spot about the centre of Mercury; this I did not notice, and it might arise from the fact that our eyesight is different, my eyes being near-sighted whereas the other person to whom I refer has eyesight requiring convex lenses to correct ocular defects. At various times during the day Mercury was plainly visible in my small finder telescope of $1\frac{1}{8}$-inch aperture, power 7, as a very small black spot on the Sun's disc. I tried to get a view of the last contacts of the planet with the Sun's disc, but was unable to catch either the last internal or external contact in consequence of the Sun passing along behind the edge of a bank of cloud at 4.13 p.m. E. S. T. I got a glimpse of the Sun when a very small fraction of the disc of Mercury could be seen, which had wholly disappeared when I got the next glimpse of the Sun at 4h. 13m. 30sec. E. S. T. by my watch.

Mr. J. Connon, of Elora, had been quite successful in photographing Mercury when on the disc of the Sun, and forwarded several prints, which were highly appreciated.

TWENTY-THIRD MEETING.

Nov. 27th; Larratt W. Smith, Q.C., D.C.L., President, occupied the chair. On taking the chair, the President returned thanks for the honour of election to the office rendered vacant by the death of Professor Carpmael, and paid a high tribute to the memory of the deceased, whose place in the scientific field in Canada it would be difficult, indeed, to fill. Referring to his recent extended absence from Toronto, Dr. Smith read some interesting notes of observations made in various places. He had been particularly impressed with the beauty of the Zodiacal light as seen by him for the first time under the clear skies of Florida; and also with the brilliancy of the auroral displays as observed on the North Atlantic seaboard. While in England rather unfavourable weather for observation had been experienced; it was a matter of congratulation for Canadian observers that they have many more nights in a given year when telescopic work can be carried on than can possibly be hoped for in the British Islands. The President's assurance that he would continue to take an active interest in the Society's welfare was warmly received.

For the office of Vice-President, rendered vacant by the election of Dr. Smith to the Presidential chair, Mr. E. A. Meredith, LL.D., was nominated and unanimously elected.

Mr. A. Elvins reported recent observations of Mars. He had been able to note the formation of the snow-cap about the North Pole of the planet. Mr. M. Winger, of Cleveland, Ohio, forwarded some interesting notes on observations of the unilluminated part of the disc of Venus, early in the year.

Mr. Lindsay, by request, then read the following paper, contributed by Mr. S. E. Peal, F.R.G.S., of Sibsagar, Asam, India, a corresponding member of the Society,

ON PROF. WEINCK'S ENLARGEMENTS OF THE LICK PHOTOGRAPHS OF LUNAR CRATERS.

Soon after the appearance of Professor Weinck's enlargements of the "Lick Photographs" of Lunar Craters, speculation was rife as to the nature of the fine linear markings seen on many of them. For a short time the view was hazarded by some, that they might possibly be due to imperfections in the photographic film. Others, again, suggested that they might be the relics or markings of former river beds.

At the present time neither of these views appears to have found favour with astronomers; their nature appears so far to be unknown, though it is generally conceded that they are real markings in, or on, the Lunar surface.

On examining the photographs carefully with the aid of a large hand lens, we find that there are several singular features held in common, and very noteworthy in regard to these fine linear markings—features, too, which can be grouped under quite distinct headings.

Firstly, there seems to be fairly good proof that most of them are fissures, cracks or crevasses, in the outer crust, whatever its nature may be, and this is demonstrated by the fact that the light and shade (revealing them) fall so consistently in the same general direction, as that of the light falling on the main "crater" or "walled plain," near which, or on which they lie. If in a percentage of them the direction of incident light appeared to be reversed, this test, of course, would not hold good; but, in the great majority of cases, the light falling on these finer lines so closely coincides with that falling on the main crater, that it becomes obvious to us that they are real fissures or crevasses in the surface, and hence may be valuable aids in elucidating the nature of that surface.

Secondly, a very general feature in regard to these fine linear markings or fissures, is the fact that they pass over the outer detail quite regardless, or generally regardless, of the surface modelling, that, like the larger and well-known fissures called "clefts," they impartially cross hill and dale, and at all angles, thus disposing conclusively of all attempts to attribute their origin to river action, new or old. In other words, they are not structurally related to the local peculiarities of the present surfacing, but are obviously a *later* feature.

Thirdly, their modes of branching, crossing, or junction, are very similar to those of the larger so-called "clefts," being independent of elevation. The junction is as often on a ridge or hill top as in a valley bottom. In some cases there is a very marked resemblance to a river with tributaries falling in, but instead of these flowing downwards, as with us, they look as if the river and its tributaries were flowing up-hill.

This is precisely the appearance which would be due to surface fissures due to elevation of some central area, and this view it is necessary to bear in mind.

In crossing each other, we also notice that in some instances a parallel pair of fissures will quite cross another one, or even another pair, a feature which effectually disproves their fluvial origin.

Last, but not least, there is a distinctly marked tendency to a general parallelism, to run in, or mainly in, one particular direction, *i.e.*, to S.W. or N.E., cutting the ramparts at all angles. The general direction of these minute fissures in or near Petavius, Vendelinus, and Langrenus are practically identical, with, of course, occasional exceptions.

This characteristic, which is of the utmost importance, is obviously not entirely, or even largely, due to the direction of the light, it being favourable to their visibility, as might be inferred from a hasty inspection. There actually is a very distinct parallelism, and over an enormous area, so large, indeed, that the cause must be either deeply seated or very widespread. But in many instances, however, there is such a marked contiguity and parallelism that it has caused, or at least intensified, the very singular vermiform character of the surfacing seen in many places, as to the East of Vendelinus and Langrenus.

In some cases this repeated parallelism of minute fissures, at a short distance apart, has obviously given rise to well defined parallel ridges, like those, for instance, crossing the oblong elevation on the Western side of the floor of Vendelinus. This particular instance is noteworthy, indeed, for another reason; *i.e.*, in it we see what looks like a sequence in *time*—the fissure bordering the pair of ridges to the S.E. is quite clear and distinct, the other two fissures presumably having become closed. As the closing of the larger fissures called "clefts" has occasionally been suspected, this would be another feature tending to show that these fine markings and "clefts" were of the same character, *i.e.*, crevasses.

Now, bearing in mind the several peculiarities above noted, as we look over these beautiful photo enlargements, what do they suggest to us; what legitimate conclusions may we draw from them in regard to the nature of and cause of these markings and the character of the lunar surface generally?

In the first place, they appear to be "gapes," or linear fissures, in the outer crust, minute "clefts" in fact, and in that case—like the well-known group of "clefts" on the incline near Plinius—must be due to *lateral movement* of the *outer* layers of the crust. If they are fissures, and demonstrably not river beds, there is little or no option left to us, except to view them as gapes or crevasses, due to lateral movement. Whether this latter is due to contraction, caused by dessication, or to a slow movement, as in our crevassed glaciers, it matters not. The thing to realize is, that they are due to lateral *movement* of the surface layers; that over a vast area of the Lunar surface, at least from S. latitude 5° to 30°, on the meridian of 60° W., this lateral movement of the outer crust is in one general direction: either to the S.W. or N.E., unless, indeed, this region is a slightly raised one, with inclines running down East and West.

On our Earth, geologists are now quite familiar with vast slow lateral movements of the crust, due to cooling and contraction of the nucleus, which crumples the outer strata and forms all our mountain systems. It is so well known that it is needless to detail the proofs here.

This system of mountain formation by lateral pressure seems to have ended on our satellite ages ago, and to have been succeeded by another of a totally different character, resulting in small and large piled rings. From "craters" twenty miles in diameter to the great "walled plains," 100 and even 150 miles across, or even to the circular "seas," 300 to 700 miles in diameter, around which we now find, as a (structural) border, the greatest ranges.

But there are not wanting proofs that the era of mountain-ring formation on our Moon is also over, and that another era, one of levelling, has supervened and is in full swing.

As on our Earth so on the Moon, the ocean floors appear to be the sites of slow, permanent subsidence. On or around the lunar "seas," especially marked in the cases of Serenitatis and Humorum, we see the proofs of this subsidence in the vast tangential crevasses called "clefts."

On the great incline S.W. border of Serenitatis we see the proofs of the *movement of the surface layers* down the incline in the great clefts near Plinius and Dawes. They are huge gapes or crevasses, and the direction of movement is obvious—it is necessarily at right angles to the line of fracture.

But what does all this teach us, or suggest, in relation to the extensive series of minute fissures over the region from Petavius down to Langrenus? Simply this, that over a vast area, and quite regardless of the outer features, there is a slow drift or movement of the surface layers *in one general direction*, at right angles to the fissures, and secondly, that this movement is not due to lateral compression, but to its opposite, *i.e.*, extension, and hence is a result of gravitation, as in the "crevasses" of terrestrial glaciers.

TWENTY-FOURTH MEETING.

December 11th; the President, Dr. Larratt W. Smith, Q.C., in the chair.

Mr. R. Tyson, of Toronto, was elected an active member of the Society.

Circular No. 41, from the Wolsingham Observatory, was read, announcing the discovery of a very red 8th mag. IV. Type star, not in D. M. Position, R. A. 17h. 54·3m.; Decl. + 58° 14′ (1900).

The subject of the possibility of there being forms of life on other planets having occasioned considerable discussion in the periodicals of the day, the chairman was requested to read the exhaustive article by Sir Robert Ball, published in the *Fortnightly Review*, and dealing with this question. The paper was discussed by several members present. Mr. John A. Paterson agreeing with the writer on many points, outlined the reasons, apart from the purely physical, for believing that the universe really teems with life. Mr. Elvins and Mr. Lindsay took exception to the view that upon large planets pigmy forms must exist and on small globes gigantic forms. They held that there was but one argument required to show that this was erroneous: upon this globe we have all forms of life from the monad to the huge pachyderm, and they all seem to enjoy life to the utmost. It is true that the energy which each form possesses depends upon the proportion it bears to the Earth itself; but it is quite impossible to point to any one form and say it is better adapted for its work than may be another for what it may be destined to accomplish.

TWENTY-FIFTH MEETING.

December 27th; Mr. Arthur Harvey in the chair.

Mr. J. F. Lash, of Toronto, was elected an active member of the Society.

A communication from Messrs. Poole Bros. was received, announcing the publication of their map of the Moon. Some notes were received from Dr. J. C. Donaldson, of Fergus, on

STAR OBSERVATIONS.

Last evening (the 17th instant) was very fine here from about 9 until 11.30 o'clock, and as it was the only fine night we have had here for a very long time I took out my glass to try some difficult stars with rather close small companions, some of them supposed to be variables. The first one I tried was k Geminorum, a very difficult star for a small glass. I have never before been able to see the *comes* to k steadily, but last night I succeeded in seeing it very distinctly and steadily with the full aperture of my glass, $3\frac{1}{2}$ inches, using the 75 power. I then tried the group σ Orionis, and saw seven stars in the group, the 10·3 (or 11 as it is sometimes called) magnitude star being quite bright and distinct. The same was the case with Σ 701, Σ 750 (Σ 754 near ι Orionis), ρ^1 Orionis and δ Geminorum, all the companions to these stars appearing quite sharp and distinct; also 30 Tauri, another difficult star, which I had only seen once before. I then tried some of these stars with aperture contracted to $2\frac{1}{2}$ inches, but could not catch any of the companions except those of δ Geminorum and Σ 701. I then tried an aperture of 2·7 inches, and with this aperture could see seven stars in group σ Orionis and the companions to 30 Tauri, ρ^1 Orionis, Σ 754, Σ 750, δ Geminorum, and Σ 701, but could not be certain of the *comes* to *k* Geminorum, although I glimpsed it once or twice, but did not see it sharply and steadily as I did with the full aperture of 3½ inches. From my observations of these stars I do not think there is any reasonable ground to suspect that any of the companions are variable. With small glasses they may be seen if the air is very good, but if the air or the glass is inferior, then the companion fails to show up. Ex. Gr. I have noticed that when I could see the companion to Σ 754 nicely, I had not much difficulty with the 10·3 magnitude *comes* to σ Orionis; but if the air was not good enough to show the companion to Σ 754, then I might look in vain for the 10·3 magnitude *comes* of σ Orion.

Mr. G. E. Lumsden read an extract from a letter received from Professor James E. Keeler, to whom a question regarding the Orion nebula had been referred:—"My spectroscopic observations (and those of Mr. Campbell, which are substantially the same) merely show that there is no spectroscopic evidence that the trapezium stars are *in* the Orion nebula. They may be anywhere in the line of sight, either on this or on the other side of the nebula. According to the earlier observations (notably those of Huggins), the nebular lines were bright in the trapezium stars, and the star lines blended with the nebular lines, which would indicate that the stars were actually within the nebula. It seems to me probable that such *is* actually their position; there is simply no *spectroscopic* evidence in favour of this view. Taking the evidence of every kind into account, the distances of the stars and of the nebulæ are probably of the same order."

Mr. A. Elvins by request gave a short account of the discovery of the 5th Satellite of Jupiter, and read extracts from correspondence with Mr. O. E. Cartwright, of Detroit, who had announced an observation of the shadow of a 5th satellite before the discovery of the satellite itself by Professor Barnard at Lick Observatory.

The following paper, contributed by J. Morrison, M.A., M.B., Ph.D., of Washington, D.C., was received:—

SPECIAL PERTURBATIONS.

The orbit described by a body moving round a centre of force which varies inversely as the square of the distance, is a conic section, that is, a circle, an ellipse, a parabola or a hyperbola, but in the case of a single planet, it is an ellipse with the Sun in one of the foci—and the radius vector describes equal areas in equal times. The true place of such a body at any time is easily found by applying the elliptic inequality or "equation of the centre" to its mean place, or that which it would have if it moved uniformly with its mean motion in a circle whose radius is equal to the semi major axis of its orbit. When another body is added to the system, the equable description of areas no longer exists; each body disturbs the motion of the other, and the computation of the true place of either becomes a problem of prodigious difficulty—it is, in fact, "the famous problem of three bodies" which has severely taxed the analytical skill of mathematicians since the discovery of the law of universal gravitation. The computation of perturbations, even in its most simple form, finds no place in our ordinary treatises on general or practical astronomy; in fact, we must refer to the writings of the most distinguished votaries of astronomical science for a full development of the subject—to the works of La Place, La Grange, Euler, Le Verrier, Hansen, Pontecoulant, and other eminent continental mathematicians—works which are rarely accessible to the ordinary astronomer. In this short, but very imperfect article, we purpose to develop some of the more important formulæ for the computation of special perturbations and to make an application of them in the case of one of the secondary planets, and in order, too, that the article may be as complete in itself as practicable, we shall first deduce from elementary principles, the general formulæ of both disturbed and undisturbed motion.

Take any fixed plane passing through the centre of gravity of the Sun, as the plane of reference, the centre of gravity of the Sun as the origin, x, y, z, the co-ordinates of a planet or comet, r its radius vector, k^2 the mass of the Sun, and mk^2 the mass of the planet—m will then be the ratio of the planet's mass to that of the Sun, and let x', y', z', r', and $m'k^2$ denote similar quantities for a second planet; x'', y'', z'', r'', and $m''k^2$ those of a third, and so on. Again, let ξ, η and ζ be the co-ordinates of the centre of gravity of the Sun referred to any fixed point in space, the co-ordinate planes of the latter system being parallel to those of x, y and z, respectively. The attraction of m on the Sun will be $\frac{mk^2}{r^2}$, and, if we resolve this force along the axes, we shall have the three components,

$$mk^2\,\frac{x}{r^3},\quad mk^2\,\frac{y}{r^3}\quad\text{and}\quad mk^2\,\frac{z}{r^3}$$

since $\frac{x}{r}$, $\frac{y}{r}$ and $\frac{z}{r}$ are the direction cosines. The action of m' on the Sun will be

$$m'k^2\,\frac{x'}{r'^3},\; m'k^2\,\frac{y^1}{r'^3} \text{ and } m'k^2\,\frac{z'}{r'^3}$$

and similarly for the action of m'' and all the other bodies, whatever the number may be. Therefore, the combined action of all the bodies of the system on the Sun, resolved in directions parallel to the co-ordinate axes, will be

$$mk^2\,\frac{x}{r^3} + m'k^2\,\frac{x'}{r'^3} + m''k^2\,\frac{x''}{r''^3} + \ldots\ldots = k^2\Sigma\frac{mx}{r^3},$$

$$mk^2\,\frac{y}{r^3} + m'k^2\,\frac{y'}{r'^3} + m''k^2\,\frac{y''}{r''^3} + \ldots\ldots = k^2\Sigma\frac{my}{r^3}$$

$$mk^2\,\frac{z}{r^3} + m'k^2\,\frac{z'}{r'^3} + m''k^2\,\frac{z''}{r''^3} + \ldots\ldots = k^2\Sigma\frac{mz}{r^3}$$

Now, in order that the entire system may be at rest, we must have

$$\frac{d^2\xi}{dt^2} - k^2\Sigma\frac{mx}{r^3} = 0\,;\; \frac{d^2\eta}{dt^2} - k^2\Sigma\frac{my}{r^3} = 0\,;\; \frac{d^2\zeta}{dt^2} - k^2\Sigma\frac{mz}{r^3} = 0, \qquad (1)$$

Let Δ denote the distance of m from m'; Δ' its distance from m'' and so on, then will the combined attraction of the bodies m', m'', etc., on m be

$$\frac{k^2m'}{\Delta^2} + \frac{k^2m''}{\Delta'^2} + \ldots\ldots = k^2\Sigma\,\frac{m'}{\Delta^2},$$

and this resolved parallel to the co-ordinate axes gives

$$k^2\Sigma m'\,\frac{x'-x}{\Delta^3},\; k^2\Sigma m'\,\frac{y'-y}{\Delta^3},\; k^2\Sigma m'\,\frac{z'-z}{\Delta^3}$$

because $\frac{x'-x}{\Delta}$, $\frac{y'-y}{\Delta}$ and $\frac{z'-z}{\Delta}$ are the direction cosines.

The attraction of the Sun on m, resolved in the same directions, is

$$-k^2\frac{x}{r^3},\; -k^2\frac{y}{r^3} \text{ and } -k^2\frac{z}{r^3},$$

because the Sun's force tends to diminish the co-ordinates x, y and z, therefore the combined action of all the other bodies of the system on m, is

$$-\frac{k^2x}{r^3} + k^2\Sigma m'\,\frac{(x'-x)}{\Delta^3},$$

$$-\frac{k^2y}{r^3} + k^2\Sigma m'\,\frac{(y'-y)}{\Delta^3}$$

$$\text{and } -\frac{k^2z}{r^3} + k^2\Sigma m\,\frac{(z'-z)}{\Delta^3},$$

but the co-ordinates of m referred to the *fixed* origin are

$$\xi + x,\ \eta + y \text{ and } \zeta + z$$

therefore we must have

$$\begin{aligned}
\frac{d^2\xi}{dt^2} + \frac{d^2x}{dt^2} &= -\frac{k^2x}{r^3} + k^2\Sigma m'\frac{(x'-x)}{\Delta^3},\\
\frac{d^2\eta}{dt^2} + \frac{d^2y}{dt^2} &= -\frac{k^2y}{r^3} + k^2\Sigma m'\frac{(y'-y)}{\Delta^3},\\
\frac{d^2\zeta}{dt^2} + \frac{d^2z}{dt^2} &= -\frac{k^2z}{r^3} + k^2\Sigma m'\frac{(z'-z)}{\Delta^3}.
\end{aligned} \tag{2}$$

Eliminating $\frac{d^2\xi}{dt^2}$, $\frac{d^2\eta}{dt^2}$ and $\frac{d^2\zeta}{dt^2}$ by (1) we get

$$\begin{aligned}
\frac{d^2x}{dt^2} + k^2(1+m)\frac{x}{r^3} &= k^2\Sigma m'\left(\frac{x'-x}{\Delta^3} - \frac{x'}{r'^3}\right)\\
\frac{d^2y}{dt^2} + k^2(1+m)\frac{y}{r^3} &= k^2\Sigma m'\left(\frac{y'-y}{\Delta^3} - \frac{y'}{r'^3}\right)\\
\frac{d^2z}{dt^2} + k^2(1+m)\frac{z}{r^3} &= k^2\Sigma m'\left(\frac{z'-z}{\Delta^3} - \frac{z'}{r'^3}\right)
\end{aligned} \tag{3}$$

which are the equations of disturbed motion in which the symbol of summation Σ applies to the masses and co-ordinates of the bodies which act on m, excluding the Sun.

By a simple transformation the second members of (3) may be put in a more convenient form ; (4) thus let

$$\theta = \frac{m'}{1+m}\left(\frac{1}{\Delta} - \frac{xx'+y'y+zz'}{r'^3}\right) + \frac{m''}{1+m}\left(\frac{1}{\Delta'} - \frac{xx''+yy''+zz''}{r''^3}\right) + \text{etc.},$$

and differentiate with respect to x, and we have for the partial differential co-efficient

$$\left(\frac{d\theta}{dx}\right) = \frac{m'}{1+m}\left(-\frac{1}{\Delta^2}\frac{d\Delta}{dx} - \frac{x'}{r'^3}\right) + \frac{m''}{1+m}\left(-\frac{1}{\Delta'^2}\frac{d\Delta'}{dx} - \frac{x''}{r''^3}\right) + \ldots.$$

$$\text{but } \Delta^2 = (x'-x)^2 + (y'-y)^2 + (z'-z)^2$$

$$\text{and } \Delta'^2 = (x''-x)^2 + (y''-y)^2 + (z''-z)^2$$

$$\text{and } \frac{d\Delta}{dx} = -\frac{x'-x}{\Delta}, \qquad \frac{d\Delta'}{dx} = -\frac{x''-x}{\Delta'}$$

therefore we easily find

$$\left(\frac{d\theta}{dx}\right) = \frac{m'}{1+m}\left(\frac{x'-x}{\Delta^3} - \frac{x'}{r'^3}\right) + \frac{m''}{1+m}\left(\frac{x''-x}{\Delta'^3} - \frac{x''}{r''^3}\right) + \ldots.$$

$$\text{or } (1+m)\left(\frac{d\theta}{dx}\right) = \Sigma m'\left(\frac{x'-x}{\Delta^3} - \frac{x'}{r'^3}\right)$$

and similarly for the partial differential co-efficients with respect to y and z. Therefore, equation (3) may be written

$$\begin{aligned}
\frac{d^2x}{dt^2} + k^2 (1 + m) \frac{x}{r^3} &= k^2 (1 + m) \left(\frac{d\theta}{dx}\right) \\
\frac{d^2y}{dt^2} + k^2 (1 + m) \frac{y}{r^3} &= k^2 (1 + m) \left(\frac{d\theta}{dy}\right) \qquad (5) \\
\frac{d^2z}{dt^2} + k^2 (1 + m) \frac{z}{r^3} &= k^2 (1 + m) \left(\frac{d\theta}{dz}\right)
\end{aligned}$$

If the second members of (5) be neglected, we shall have the equations of undisturbed motion; hence, it follows, that the second members express the difference between the action of the Sun on m and of the other bodies m' and m'', etc., on the same, resolved, of course, in directions parallel to the co-ordinate axes.

The integration of the equations of undisturbed motion, viz., eq. (5) when the second members become zero, introduces six arbitrary constants, which are the elements which determine the motion of m around the Sun; but when the disturbing forces are taken into account, these constants of integration or elements of the orbit are no longer constant, but become variable, and their variations, which are generally very small, are called *perturbations*. These constants or elements are M, the mean anomaly at any given epoch; π, the longitude of the perihelion, or instead, ω, the distance from the node of the orbit to the perihelion; Ω, the longitude of the ascending node; i, the inclination; e, the eccentricity, and a, the semi axis major of the orbit.

The quantity θ is called the *Perturbative* Function of Planetary Motion. This function is usually developed into a converging series proceeding in ascending powers of the eccentricities, inclinations, and some function of the mean anomalies, so as to include an indefinite number of revolutions, and its final integration will then give what are called absolute or general perturbations. If, however, the eccentricities and inclinations are very great, as is sometimes the case with comets and some of the asteroids, and if also the ratio of the mean distances from the Sun of the disturbed and disturbing bodies exceeds ·75, this development is no longer practicable, and we must then have recourse to methods of approximation known by the name of *mechanical quadrature*, by which we determine the variations of the elements from one epoch to another. This latter method is applicable in every case, and may be used with advantage even where the determination of the absolute perturbation is possible; it is known as the method of special perturbations, and is the one which we propose to partially develop in this paper.

In order to facilitate the development of the perturbative function, it is put in the following form :—Considering only two planets, the disturbed and disturbing—and we can only consider two at any one operation—we have from (4)

$$\theta = \frac{m'}{1+m}\left(\frac{1}{\Delta} - \frac{xx' + yy' + zz'}{r'^3}\right)$$

Let the axis of X pass through the common node of the orbits of m and m', i the angle between the orbits, v and v' the distances of m and m' from the common node, and V the angular distance between them, then in polar co-ordinates we shall have

$$\begin{aligned} x &= r\cos v & x' &= r'\cos v' \\ y &= r\sin v\cos i & y' &= r'\sin v' \\ z &= r\sin v\sin i & z' &= 0 \end{aligned}$$

therefore

$$\begin{aligned} \theta &= \frac{m'}{1+m}\left(\frac{1}{\Delta} - \frac{r(\cos v\cos v' + \sin v\sin v'\cos i)}{r'^2}\right) \\ &= \frac{m'}{1+m}\left(\frac{1}{\Delta} - \frac{r\cos V}{r'^2}\right) \\ &= \frac{m'}{1+m}\left\{(r^2 - 2rr'\cos V + r'^2)^{-\frac{1}{2}} - \frac{r\cos V}{r'^2}\right\} \end{aligned} \qquad (6)$$

By putting $1 - 2\sin^2\frac{1}{2}i$ in the place of $\cos i$ and

$$\cos(v'+v) - \cos(v'-v) = \gamma \text{ and } \sin^2\tfrac{1}{2}i = \eta$$

the above becomes

$$\begin{aligned} \theta &= \frac{m'}{1+m}\Big\{(r^2 - 2rr'\cos(v'-v) + r'^2 - 2rr'\gamma\eta)^{-\frac{1}{2}} - \\ &\qquad \frac{r}{r'^2}(\cos(v'-v) + \gamma\eta)\Big\} \\ &= \frac{m'}{1+m}\left\{(\Delta_1 - 2rr'\gamma\eta)^{-\frac{1}{2}} - \frac{r}{r'^2}(\cos(v'-v) + \gamma\eta)\right\} \\ &= \frac{m'}{1+m}\Big\{\Delta_1^{-\frac{1}{2}} - \frac{r}{r'^2}\cos(v'-v) \\ &\qquad + rr'\gamma\eta\left(\Delta_1^{-\frac{3}{2}} - \frac{1}{r'^3}\right) \\ &\qquad + \frac{3}{2}r^2r'^2\gamma^2\eta^2\Delta_1^{-\frac{5}{2}} \\ &\qquad + \frac{5}{2}r^3r'^3\gamma^3\eta^3\Delta_1^{-\frac{7}{2}} \\ &\qquad + \ldots\ldots\ldots\ldots\ldots \Big\} \end{aligned} \qquad (7)$$

Where we put $\Delta_1 = r^2 - 2rr'\cos(v'-v) + r'^2$.

The development of the several terms of (7) will form the subject of a subsequent paper, and we will now return to the consideration of special perturbations.

For any epoch t_0, let M_0, π_0, Ω_0, i_0, a_0, and e_0, represent the elements of the orbit of the planet whose perturbations are required. After an infinitesmal interval dt, the planet will begin to describe a new orbit whose elements will differ slightly from the former which are called the osculating elements, because they represent an orbit which *touches* the disturbed orbit at the assumed epoch. The osculating elements are *constant,* while those of the disturbed orbit vary continually. Let the osculating orbit be taken as the fundamental plane, and let θ denote the angle which the projection of the disturbed radius vector on the plane of xy, makes with the axis of x and λ the latitude of the disturbed planet with respect to this same plane; then x, y, z denoting the co-ordinates of the body at any interval $t_0 + dt$ we shall have

$$x = r \cos\lambda \cos\theta$$
$$y = r \cos\lambda \sin\theta \qquad (8)$$

and

$$z = r \sin\lambda.$$

Let the forces represented by the second members of (5) be denoted by X, Y and Z respectively, then (5) may be written thus

$$\frac{d^2x}{dt^2} + k^2(1+m)\frac{x}{r^3} = X$$
$$\frac{d^2y}{dt^2} + k^2(1+m)\frac{y}{r^3} = Y \qquad (9)$$
$$\frac{d^2z}{dt^2} + k^2(1+m)\frac{z}{r^3} = Z$$

From the first and second of these we find

$$x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2} = Yx - Xy, \text{ and integrating}$$

$$x\frac{dy}{dt} - y\frac{dx}{dt} = \int (Yx - Xy)\,dt + C. \qquad (10)$$

Differentiating the first and second of (8) we have after substituting in (9) and combining with (8)

$$r^2\cos^2\lambda\frac{d\theta}{dt} = \int (Yx - Xy)\,dt + C.$$

Now let P represent the component of the disturbing force along the disturbed radius vector and Q the component perpendicular to it and parallel with the plane of xy, then we have

$X = -Q \sin\theta$ and $Y = +Q \cos\theta$ and $Yx - Xy = Qr \cos\lambda$

therefore

$$r^2 \cos^2\lambda \frac{d\theta}{dt} = \int Qr \cos\lambda dt + C.$$

When the disturbing force becomes 0, both Q and λ become zero and we have

$$r^2 \frac{d\theta'}{dt} = C = \text{twice the area described in a unit of time}$$
$$= kp_0^{\frac{1}{2}} (1 + m)^{\frac{1}{2}}$$

where p_0 is the semi-parameter of the undisturbed orbit. Therefore we have finally

$$r^2 \cos^2\lambda \frac{d\theta}{dt} = \int Qr \cos\lambda dt + kp_0^{\frac{1}{2}} (1 + m)^{\frac{1}{2}} \qquad (11)$$

which is the differential equation of the longitude.

From (9) we also find

$$\frac{1}{r} \cdot \frac{xd^2x + yd^2y + zd^2z}{dt^2} + \frac{k^2 (1 + m)}{r^2} = X\frac{x}{r} + Y\frac{y}{r} + Z\frac{z}{r}$$
$$= P,$$

the component of the disturbing force in the direction of the disturbed radius vector.

But

$$xd^2x + yd^2y + zd^2z = d(xdx + ydy + zdz) - (dx^2 + dy^2 + dz^2)$$

Differentiating (8), substituting and reducing we get

$$\frac{d^2r}{dt^2} - r\cos^2\lambda \frac{d\theta^2}{dt^2} - r\frac{d\lambda^2}{dt^2} + \frac{k^2 (1 + m)}{r^2} = P \qquad (12)$$

which is the differential equation of the radius vector.

From the last of (9) we have

$$\frac{d^2z}{dt^2} = Z - \frac{k^2 (1 + m)}{r^3} z \qquad (13)$$

which is the differential equation of the heliocentric latitude.

We also have $\sin\lambda = \frac{z}{r}$.

We must now determine the values of P, Q and Z in equations (11), (12) and (13), but before this can be done the position of the orbit of the *disturbing* planet must be referred to the fundamental plane, or that of the disturbed planet at the epoch t_0. In the spherical triangle formed by the intersection of the plane of the ecliptic, the plane of the orbit of the disturbing body and the fundamental plane, we have i' and i_0, the inclination of the latter two planes to the ecliptic respectively, and Ω' and Ω_0 the longitude of their nodes with reference to the ecliptic, or two angles and the included side to find the remaining parts by Napier's Analagies, or better by Delambre's equations, that is, the distances n' and n from the nodes of the disturbing and disturbed orbits on the ecliptic respectively to the common node of the orbits of the two planets, and I their inclination. Then if λ' be the heliocentric latitude of the disturbing planet referred to the fundamental plane, θ' its longitude in this plane from the axis of x and u'' its argument of latitude with reference to same plane. We shall have

$$\begin{aligned} \tan(\theta' - n) &= \tan u'' \cos I \\ \tan \lambda' &= \tan I \sin(\theta' - n) \end{aligned} \qquad (14)$$

and if u' denote the argument of latitude of the disturbing planet referred to the ecliptic, then

$$u'' = u' - n'$$

These formulæ determine u'' and then θ' and λ' from (14). Now, let x', y', z' denote the co-ordinates of the *disturbing* planet referred to the plane of the *disturbed* at the epoch t_0, then we have

$$\begin{aligned} x' &= r' \cos \lambda' \cos \theta' \\ y' &= r' \cos \lambda' \sin \theta' \\ z' &= r' \sin \lambda' \end{aligned} \qquad (15)$$

Also
$$P = X\frac{x}{r} + Y\frac{y}{r} + Z\frac{z}{r}$$

but
$$\begin{aligned} X &= k^2(1+m)\left(\frac{d\theta}{dx}\right) = k^2 m'\left(\frac{x'-x}{\Delta^3} - \frac{x'}{r'^3}\right) \\ Y &= k^2(1+m)\left(\frac{d\theta}{dy}\right) = k^2 m'\left(\frac{y'-y}{\Delta^3} - \frac{y'}{r'^3}\right) \\ Z &= k^2(1+m)\left(\frac{d\theta}{dz}\right) = k^2 m'\left(\frac{z'-z}{\Delta^3} - \frac{z'}{r'^3}\right) \end{aligned}$$

Substituting the values of x, y, z given by (8) and the values of x', y', z' given by (15) in the above, we find

$$\begin{aligned} X &= k^2 m'\left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right) r' \cos\lambda' \cos\theta' - k^2 m' \frac{r \cos\lambda \cos\theta}{\Delta^3} \\ Y &= k^2 m'\left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right) r' \cos\lambda' \sin\theta' - k^2 m' \frac{r \cos\lambda \sin\theta}{\Delta^3} \\ Z &= k^2 m'\left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right) r' \sin\lambda' - k^2 m' \frac{r \sin\lambda}{\Delta^3} \end{aligned} \qquad (16)$$

Multiplying these respectively by the values of $\frac{x}{r}$, $\frac{y}{r}$ and $\frac{z}{r}$

deduced from (8) and adding we find after some easy reductions are made

$$P = k^2 m' r' \left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right)(\cos\lambda' \cos\lambda \cos(\theta' - \theta) + \sin\lambda' \sin\lambda) - k^2 m' \frac{r}{\Delta^3} \quad (17)$$

Resolving the components X and Y perpendicular to the disturbed radius vector and parallel to the fundamental plane, we have

$$Q = Y\frac{x}{r} - X\frac{y}{r},$$

therefore, multiplying the first and second of (16) by $\frac{y}{r}$ and $\frac{x}{r}$ respectively, we have, using the values of $\frac{y}{r}$ and $\frac{x}{r}$ as given by (8)

$$Q = k^2 m' \left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right) r' \cos\lambda' \cos\lambda \sin(\theta' - \theta) \quad (18)$$

and the third of (16) gives the value of Z. When we substitute these values of P, Q and Z in equations (12), (11) and (13) respectively, we shall have the differential equations of the longitude, radius vector and latitude of the disturbed planet rigorously determined. If, however, we neglect the squares and higher powers of the disturbing force, that is, small quantities of the second, third, etc., orders, and retain only the first power of the disturbing force, the expressions for P, Q and Z assume a similar form. Thus the latitude λ is always extremely small, and therefore we may without any appreciable error put $\cos\lambda = 1$ and $\sin\lambda = 0$; also for r and Δ we may write r_0 and Δ_0, the values that belong to the osculating orbit or that which the disturbed planet was describing at the epoch t_0.

Therefore, we shall have, with the above restrictions,

$$\begin{aligned} P &= k^2 m' r' \left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right) \cos\lambda' \cos(\theta' - \theta) - k^2 m' \frac{r_0}{\Delta_0^3} \\ Q &= k^2 m' r' \left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right) \cos\lambda' \sin(\theta' - \theta) \qquad (19) \\ Z &= k^2 m' r' \left(\frac{1}{\Delta^3} - \frac{1}{r'^3}\right) \sin\lambda' \end{aligned}$$

The distance Δ is found from

$$\Delta^2 = (x' - x)^2 + (y' - y)^2 + (z' - z)^2$$

and introducing the values of x, x', etc., from (8) and (15), we have

$$\Delta^2 = r'^2 + r^2 - 2r'r \cos\lambda' \cos\lambda \cos(\theta' - \theta) - 2r'r \sin\lambda' \sin\lambda$$

or, neglecting terms of the second order,

$$\varDelta_0^2 = r'^2 + r_0^2 - 2r_0r' \cos\lambda' \cos(\theta' - \theta) \qquad \text{(19 bis)}$$

and hence $\dfrac{1}{\varDelta_0^3} = (r'^2 + r_0^2 - 2r_0r' \cos\lambda' \cos(\theta - \theta))^{-\frac{3}{2}}$

The second member of the last equation may be developed into an infinite series by the ordinary processes of plane trigonometry, but in such cases we first put $r' = a'$, $r_0 = a_0$, $\theta' = l'$ and $\theta = l$; l and l' denoting the *mean* longitudes $nt + \varepsilon$ and $n't + \varepsilon'$ of the two planets, and after the development pass to the case of elliptic orbits by attributing to the quantities a', a_0, l and l', the increments $a_0\mu$, $a'\mu'$, ν and ν' respectively, μ, μ', ν and ν' being very small quantities, depending on the eccentricities of m and m'. Writing a_0 and a for r_0 and r', l' and l for θ' and θ, and neglecting for the present $\cos\lambda'$, which is always very nearly equal to unity, we have in the case under consideration

$$\begin{aligned}
\varDelta^{-3} &= (a_0^2 - a^2 - 2a_0a\cos(l' - l))^{-\frac{3}{2}} \\
&= a_0^{-3}\Big\{1 + \frac{3^2}{2^2}\left(\frac{a}{a_0}\right)^2 + \frac{3^2\cdot5^2}{2^8}\left(\frac{a}{a_0}\right)^4 + \frac{3^2\cdot5^2\cdot7^2}{2^8\cdot3^2}\left(\frac{a}{a_0}\right)^6 + \ldots\ldots \\
&\quad + \left(3\left(\frac{a}{a_0}\right) + \frac{3^2\cdot5}{2^3}\left(\frac{a}{a_0}\right)^3 + \frac{3^2\cdot5^2\cdot7}{2^6\cdot3}\left(\frac{a}{a_0}\right)^5 + \ldots\ldots\right)\cos(l' - l) \\
&\quad + \left(\frac{3\cdot5}{2^4}\left(\frac{a}{a_0}\right)^2 + \frac{3\cdot5\cdot7}{2^4}\left(\frac{a}{a_0}\right)^4 + \ldots\ldots\ldots\ldots\ldots\right)\cos 2(l' - l) \\
&\quad + \left(\frac{5\cdot7}{2^3}\left(\frac{a}{a_0}\right)^3 + \frac{3_2\cdot5\cdot7}{2^7\cdot3}\left(\frac{a}{a_0}\right)^5 + \ldots\ldots\ldots\ldots\ldots\right)\cos 3(l' - l) \\
&\quad + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\Big\}
\end{aligned}$$

This formula is generalized thus

$$\varDelta^{-\mu} = \tfrac{1}{2}\Sigma A^{(i)} \cos i\,(l' - l)$$

where Σ is extended to all integral values of i, positive or negative including 0, and $A^{(i)}$ denotes the co-efficients such that $A^{(-i)} = A^{(i)}$.

The development, however, is practically useless unless $\dfrac{a}{a_0}$ is a small proper fraction, in which case the series which are the co-efficients of $\cos(l' - l)$, etc., converge rapidly and only a few terms may be taken for an approximate value of the expression. But if $\dfrac{a}{a_0}$ exceeds $\frac{1}{2}$, the series converge very slowly and a great number of terms is required.

It is for this reason that the method of general perturbations in which this general development is employed, frequently fails to give satisfactory results.

The differential equations (11), (12) and (13) represent the motion of a planet or comet about the Sun when acted upon by disturbing forces, and when integrated give the values θ, r and z, or the latitude, at any time t, but they cannot be integrated directly, and therefore we must proceed to deduce formulæ which when integrated, will give the values of the perturbations $\delta\theta$ and δr.

In the case of the undisturbed or osculating orbit, equations (11) and (12) become

$$r_0^2 \frac{d\theta_0}{dt} = k\sqrt{p_0(1+m)}$$

and

$$\frac{d^2 r_0}{dt^2} - r_0 \frac{d\theta_0^2}{dt^2} + \frac{k(1+m)}{r_0^2} = 0 \qquad (20)$$

because P, Q and λ are then each zero.

Let $\theta = \theta_0 + \delta\theta$ and $r = r^0 + \delta r$ where θ and r denote the longitude and radius vector in the *disturbed* orbit, θ_0 and r_0 the same quantities in the undisturbed orbit and $\delta\theta$ and δr the perturbations arising from the action of the disturbing force. Then

$$\frac{d\theta}{dt} = \frac{d\theta_0}{dt} + \frac{d\delta\theta}{dt}$$

$$= \frac{k\sqrt{p_0(1+m)}}{r_0^2} + \frac{d\delta\theta}{dt}$$

and substituting in (11) we have

$$\frac{r^2\cos^2\lambda k\sqrt{p_0(1+m)}}{r_0^2} + r^2\cos^2\lambda\frac{d\delta\theta}{dt} = \int Qr\cos\lambda dt + k\sqrt{p_0(1+m)}$$

whence we have

$$\frac{d\delta\theta}{dt} = \frac{1}{r^2\cos^2\lambda}\int Qr\cos\lambda dt - \left(1 - \frac{r_0^2}{r^2\cos^2\lambda}\right)\frac{k\sqrt{p_0(1+m)}}{r_0^2} \qquad (21)$$

which will give $\delta\theta$ when integrated.

Also,

$$dr = dr_0 + d\delta r \text{ and } \frac{d^2r}{dt^2} = \frac{d^2r_0}{dt^2} + \frac{d^2\delta r}{dt^2}$$

which substitute in (12) and we find with the aid of the second of (20)

$$\frac{d^2\delta r}{dt^2} = P + r\cos^2\lambda\frac{d\theta^2}{dt^2} + r\frac{d\lambda^2}{dt^2} + k^2(1+m)\left(\frac{1}{r_0^2} - \frac{1}{r^2}\right) - r_0\left(\frac{d\theta_0}{dt}\right)^2 \qquad (22)$$

Put $k\sqrt{p_0(1+m)} = r_0 2h$ and since $r = r_0 + \delta r$ we have

$$\frac{1}{r_0^2} - \frac{1}{r^2} = \frac{r^2 - r_0^2}{r^2 r_0^2} = \frac{2\delta r}{r^2 r_0} + \frac{\delta r^2}{r^2 r_0^2} = \frac{2\delta r}{r^2 r_0},$$

because we can neglect $\frac{\delta r^2}{r^2 r_0^2}$ being an infinitesimal of a very high order and therefore from (21) we have

$$\left(h + \frac{d\delta\theta}{dt}\right) r^2 \cos^2 \lambda = \int Qr \cos \lambda dt + r_0^2 h$$

$$= r^2 \cos^2 \lambda \frac{d\theta}{dt} \text{ by (11)}$$

and hence

$$r \cos^2 \lambda \frac{d\theta^2}{dt^2} = r\,(1 - \sin^2 \lambda) \left(h + \frac{d\delta\theta}{dt}\right)^2$$

$$= r \left(h + \frac{d\delta\theta}{dt}\right)^2 - r \sin^2 \lambda \left(h + \frac{d\delta\theta}{dt}\right)^2$$

Substituting in (22) and reducing we have

$$\frac{d^2\delta r}{dt^2} = P + 2rh \frac{d\delta\theta}{dt} + r \left(\frac{d\delta\theta}{dt}\right)^2 + r \frac{d\lambda^2}{dt^2} - r \sin^2 \lambda \left(h + \frac{d\delta\theta}{dt}\right)^2 + h^2\delta r + \frac{2k^2\,(1 + m)\,\delta r}{r^2 r_0} \qquad (23)$$

Equations (21) and (23) are perfectly rigorous; but when we neglect the squares and products of the disturbing forces and retain only terms of the first order of small quantities—which we generally do, especially in the first approximation or when extreme accuracy is not required—these equations become considerably simplified. The latitude λ is always small, and therefore we may put $\cos \lambda = 1$ and $\sin \lambda = 0$; again,

$$r_0 = r - \delta r, \text{ and hence } \frac{r_0^2}{r^2} = 1 - \frac{2\delta r}{r} + \frac{\delta r^2}{r^2}$$

$$\text{and } 1 - \frac{r_0^2}{r^2} = \frac{2\delta r}{r} = \frac{2\delta r}{r_0}$$

approximately, since $r_0 = r$ very nearly; therefore (21) and (23) become

$$\frac{d\delta\theta}{dt} = \frac{1}{r_0} \int Qr_0 dt - \frac{2h\delta r}{r_0}$$

and
$$\frac{d^2\delta r}{dt^2} = P + \frac{2h}{r_0} \int Qr_0 dt + \left(\frac{2k^2\,(1 + m)}{r_0^3} - 3h^2\right) \delta r \qquad (24)$$

In the practical application of these formulæ, dt is not considered an infinitesimal but a definite portion of time, such as a fraction of a day or even several days. Let τ denote this interval; then it is evident that in determining $\delta\theta$ from the first of the above equations we must multiply *each* term by τ, the representative of dt, but as the first term of the second member contains the integral $\int Qr_0 dt$, it will be necessary to form the quantity $\tau^2 Qr_0$, which when differentiated and then put under the sign of integration will give $\tau \int Qr_0 dt$, the form required.

In solving the second equation for δr, we must multiply each term by τ^2, but as Qr_0 in the second term of the second member is supposed to be already multiplied by τ^2, it will only be necessary to multiply the integral

$$\frac{2h}{r_0}\int Qr_0 dt \text{ by } \tau.$$

The integration of (24) is performed by the aid of the ordinary formulæ for the summation of finite differences, but the values of $\delta\theta$ and δr can be obtained with equal accuracy without them. Equations (24) can be solved only by an indirect process, since the unknown quantity δr occurs in both members of the second equation. In the first approximation to the value of δr, the last term must be omitted, and with this approximate value the last term can be computed and the solution repeated, when a very accurate value of δr will be obtained. After a few consecutive values of δr have been thus found, we can then anticipate the next succeeding value of δr with sufficient accuracy to compute the second member of the second equation, and then the value of $\delta\theta$ from the first equation.

If very great accuracy is demanded, we may now employ (21) and (22) or (23), in which we put $r = r_0 + \delta r$ and perform the integration over again. With this value of r we also determine z by the indirect process from

$$\frac{d^2z}{dt^2} = Z - \frac{k^2(1+m)}{r^3}z,$$

and thence

$$\sin\lambda = \frac{z}{r} = \frac{z}{r_0 + \delta r}. \tag{25}$$

When the perturbations $\delta\theta$, δr and δz are known we may then find the magnitude of the disturbed orbit. If v denote the true anomaly in the disturbed orbit we shall evidently have from the infinitesimal right angled triangle formed by the projection of $d\theta$ on the fundamental plane, $d\lambda$ and dv.

$$\frac{dv^2}{dt^2} = \frac{k^2p(1+m)}{r^4} = \cos^2\lambda\frac{d\theta^2}{dt^2} + \frac{d\lambda^2}{dt^2}$$

and in the undisturbed orbit we also have

$$\frac{dv_0^2}{dt^2} = \frac{k^2p_0(1+m)}{r_0^4} = h^2.$$

From these last two equations we find

$$p = \frac{r^4}{r_0^{\,4}} \cdot \frac{p_0}{h^2} \cdot \frac{dv^2}{dt^2} = \frac{r^4}{r_0^{\,4}} \cdot \frac{p_0}{h^2} \left(\cos^2\lambda \, \frac{d\theta^2}{dt^2} + \frac{d\lambda^2}{dt^2} \right)$$

$$= \frac{r^4}{r_0^{\,4}} \cdot \frac{p_0}{h^2} \left(\cos^2\lambda \left(h + \frac{d\delta\theta}{dt} \right)^2 + \frac{d\lambda^2}{dt^2} \right) \quad (26)$$

because

$$\frac{d\theta}{dt} = \frac{d\theta_0}{dt} + \frac{d\delta\theta}{dt}$$

and by (20) we have

$$\frac{d\theta_0}{dt} = \frac{k}{r_0^{\,2}} \sqrt{p_0 \, (1 + m)} = h \, .$$

From (26) p the semi-parameter of the disturbed orbit may be found. We also have

$$\frac{dv}{dt} = \frac{k \sqrt{p \, (1 + m)}}{r^2}$$

and from the equation of the ellipse

$$\frac{dr}{dv} = \frac{r^2 e \sin v}{p} \text{ where } p = a \, (1 - e^2)$$

whence

$$\frac{dr}{dt} = \frac{k \sqrt{1 + m}}{\sqrt{p}} e \sin v = \frac{k \sqrt{1 + m}}{\sqrt{p_0}} e_0 \sin v_0 + \frac{d\delta r}{dt} \quad (27)$$

and also $e \cos v = \frac{p}{r} - 1$ from which e and v may be computed.

We then have

$$a = \frac{p}{1 - e^2} \, ; \; \mu = \frac{k \sqrt{1 + m}}{a^{\frac{3}{2}}} \, ; \; \tan \frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}} \tan \frac{v}{2} \quad (28)$$

and $M = E - e \sin E$ where μ is the mean motion, E the eccentric and M the mean anomaly.

There are still three elements of the disturbed orbit to be determined, viz., the longitude of the node Ω, the inclination i, referred to the Ecliptic and ω the distance from the node to the perihelion, or if π denote the longitude of the perihelion then $\pi - \Omega = \omega$. Let Ω_0 and i_0 denote the longitude of the node of the osculating orbit on the Ecliptic and its inclination to that plane and let ψ denote the longitude of the ascending node of the disturbed orbit on the plane of the osculating orbit measured from the same origin as θ and β its inclination to the same plane, then from the right-angled spherical triangle whose base is $\theta - \psi$, its perpendicular λ and the opposite angle β, and hypothenuse u' where u' is the argument of latitude of the disturbed planet measured on the osculating orbit—we have, by the ordinary formulæ of Spherical Trigonometry,

$$\tan u' = \tan(\theta - \psi)\sec\beta. \qquad (29)$$

and
$$\tan\beta\sin(\theta - \psi) = \tan\lambda \qquad (30)$$

Differentiating the last we have

$$\tan\beta\cos(\theta - \psi)\frac{d\theta}{dt} = \sec^2\lambda\frac{d\lambda}{dt}$$

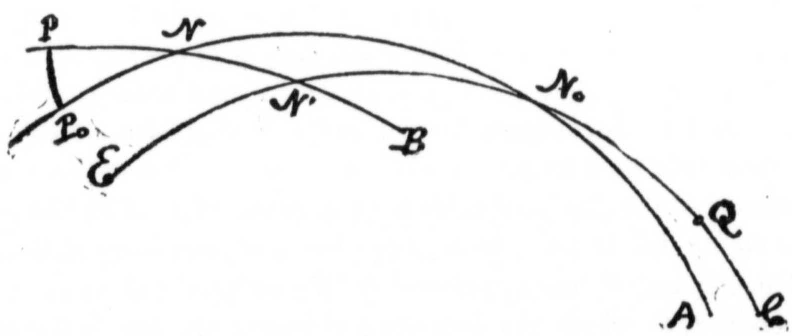
$$\text{whence } \tan(\theta - \psi) = \tfrac{1}{2}\sin 2\lambda.\frac{\frac{d\theta}{dt}}{\frac{d\lambda}{dt}}$$

$$= \tfrac{1}{2}\frac{\sin 2\lambda}{\frac{d\lambda}{dt}}\left(\frac{d\theta_0}{dt} + \frac{d\delta\theta}{dt}\right)$$

$$= \tfrac{1}{2}\frac{\sin 2\lambda}{\frac{d\lambda}{dt}}\left(\frac{k\sqrt{p_0(1+m)}}{r_0^2} + \frac{d\delta\theta}{dt}\right)$$

$$= \tfrac{1}{2}\frac{\sin 2\lambda}{\frac{d\lambda}{dt}}\left(h + \frac{d\delta\theta}{dt}\right) \qquad (31)$$

from which ψ may be found and then u' from (29). It must be observed here, however, that $\sin(\theta - \psi)$ and $\tan\lambda$ must have the same sign which will determine the magnitude of ψ, and also u' and $\theta - \psi$ must be, of course, in the same quadrant.



In the diagram let EC, ANP_0, and BP represent portions of the ecliptic, the osculating orbit and the disturbed orbit as projected on the celestial sphere and let NN_0N' be the spherical triangle formed by their intersection; also let P be the position of the planet in its disturbed or instantaneous orbit PB at any time t and PP_0 perpendicular to P_0NA the osculating orbit, and C any fixed point on the ecliptic from which the longitude θ is reckoned. Then we shall have,

$QN_0 = \Omega_0$, the longitude of the ascending node of the osculating orbit

$QN' = \Omega$, the longitude of the ascending node of the disturbed orbit

$QN_0 + NN_0 = \psi$, the longitude of the ascending node of the disturbed orbit on plane of the osculating orbit

$PN = u'$, the argument of latitude of the disturbed orbit in reference to the plane of the osculating orbit

$PN' = u$, the argument of latitude in reference to the ecliptic.

$PP_0 = \lambda$, the latitude

$PNP_0 = \beta$, the inclination of the disturbed orbit to the osculating orbit

$PN'E = i$, its inclination to the ecliptic

and $NN_0N' = i_0$, the inclination of the osculating orbit to the ecliptic, then we shall have

$$P_0N = \theta - \psi \qquad NN' = u - u'$$
$$N'N_0 = \Omega - \Omega_0 \qquad NN_0 = \psi - \Omega_0$$

that is in the triangle $NN'N_0$ we have given the two angles N_0 and N and the included side NN_0, to find the remaining parts, hence by Delambre's Equations we have

$$\left.\begin{aligned}
\cos \tfrac{1}{2}i \sin \tfrac{1}{2}\left((u - u') + (\Omega - \Omega_0)\right) &= \cos \tfrac{1}{2}(i_0 - \beta) \sin \tfrac{1}{2}(\psi - \Omega_0) \\
\cos \tfrac{1}{2}i \cos \tfrac{1}{2}\left((u - u') + (\Omega - \Omega_0)\right) &= \cos \tfrac{1}{2}(i_0 + \beta) \cos \tfrac{1}{2}(\psi - \Omega_0) \\
\sin \tfrac{1}{2}i \sin \tfrac{1}{2}\left((u - u') - (\Omega - \Omega_0)\right) &= \sin \tfrac{1}{2}(i_0 - \beta) \sin \tfrac{1}{2}(\psi - \Omega_0) \\
\sin \tfrac{1}{2}i \cos \tfrac{1}{2}\left((u - u') - (\Omega - \Omega_0)\right) &= \sin \tfrac{1}{2}(i_0 + \beta) \cos \tfrac{1}{2}(\psi - \Omega_0)
\end{aligned}\right\} \quad (32)$$

from which we can find i, $u - u'$ and $\Omega - \Omega_0$ and since u' and Ω_0 are known, u and Ω also become known and v having been already determined from (27) we have $\omega = u - v$ and $\pi = \omega - \Omega$ and, therefore, all the elements of the disturbed orbit for any epoch t for which the perturbations $\delta\theta$, δr and δz have been computed, are completely determined. The elements are, of course, referred to the ecliptic and mean equinox of the epoch t_0 to which the osculating elements are also referred. A check on the accuracy of the computation may be had by computing the heliocentric co-ordinates of the planet for the epoch to which the elements belong, and also computing the same co-ordinates from the fundamental osculating elements; the difference of the two results should agree exactly with the computed perturbations.

We will now deduce the formulae for the variation of the elements in terms of the quantities already computed, and for this purpose it will be necessary to refer the motion of the disturbed planet to the plane of its disturbed or instantaneous orbit. We shall then have $\lambda = o$ and equations (11) and (12) become

$$r^2 \frac{d\theta}{dt} = \int Qrdt + k\sqrt{p_0(1+m)}$$

and
$$\frac{d^2r}{dt^2} - r\frac{d\theta^2}{dt^2} + \frac{k^2(1+m)}{r^2} = P \qquad (33)$$

in which Q now denotes the component of the disturbing force acting in the plane of the disturbed orbit and at right angles to the radius vector. The effect of P and Q on the orbit is to vary its form and the motion of the perihelion, while that of Z which acts perpendicular to the plane of the disturbed orbit, will be to change the elements which fix the position of the orbit in space. Let L denote the *orbit* longitude of the planet reckoned from a point whose angular distance φ from the ascending node on the Ecliptic is equal to the longitude of the node, then so long as we consider only the effects of the components P and Q, the position in space of the orbit will remain unchanged and we shall have

$$\varphi = \Omega$$

and
$$L - \varphi = v + \omega = u$$

If, however, the position of the node is changed by the quantity $d\Omega$ in consequence of the action of Z, we shall evidently have

$$d\varphi = \cos i \, d\Omega$$

and
$$d\omega = dL - d\varphi \qquad (34)$$

and also
$$u = v + \omega.$$

Now, if V be the linear velocity of the planet we have

$$V^2 = k^2(1+m)\left(\frac{2}{r} - \frac{1}{a}\right)$$

and if we regard a and V as the only variables we have

$$2V\frac{dV}{dt} = \frac{k^2(1+m)}{a^2} \cdot \frac{da}{dt}$$

whence

$$\frac{da}{dt} = \frac{2a^2 V}{k^2(1+m)} \frac{dV}{dt}$$

in which $\frac{dV}{dt}$ is the increment of the accelerating force due to the action of P and Q in the direction of the tangent of the orbit, and if we designate by a the angle which the tangent makes with the radius vector, we shall have by resolving the components P and Q along the tangent

$$\frac{dV}{dt} = P \cos a + Q \sin a$$

therefore

$$\frac{da}{dt} = \frac{2a^2 V}{k^2 (1 + m)} (P \cos a + Q \sin a)$$

If we now resolve the velocity V along the radius vector and perpendicular to it we have

$$V \cos a = \frac{dr}{dt},$$

along the radius vector, and

$$V \sin a = \frac{rdv}{dt}.$$

perpendicular to radius vector, but

$$\frac{dr}{dt} = \frac{k\sqrt{1+m}}{\sqrt{p}} e \sin v \text{ by (27)}$$

and

$$r \frac{dv}{dt} = \frac{k\sqrt{p(1+m)}}{r}$$

Making these substitutions we find

$$\frac{da}{dt} = \frac{2a^2}{k\sqrt{p(1+m)}} \left(Pe \sin v + \frac{p}{r} Q\right)$$

$$= \frac{2a^2}{k\sqrt{p(1+m)}} (Pe \sin v + Qe \cos v + Q) \qquad (35)$$

Differentiating the first of (33) we get

$$\frac{d}{dt}\left(r^2 \frac{dv}{dt}\right) = Qr, \text{ because } d\theta = dv$$

or

$$\frac{d}{dt} (k\sqrt{p(1+m)}) = Qr \text{ or } \frac{k\sqrt{1+m}}{2\sqrt{p}} \cdot \frac{dp}{dt} = Qr$$

whence

$$\frac{dp}{dt} = \frac{2\sqrt{p}}{k\sqrt{1+m}} Qr \qquad (36)$$

But

$$p = a(1 - e^2) \text{ and } \frac{dp}{dt} = \frac{da}{dt} - e^2 \frac{da}{dt} - 2ae\frac{de}{dt}$$

$$\text{or } \frac{dp}{dt} = \frac{p}{a} \cdot \frac{da}{dt} - 2ae \frac{de}{dt}$$

therefore by (35) and (36) we have, after some reductions,

$$\frac{de}{dt} = \frac{p}{k\sqrt{p(1+m)}} (P \sin v + Q(\cos v + \cos E)) \qquad (37)$$

where E is the eccentric anomaly and may be found from

$$\cos E = \frac{e + \cos v}{1 + e \cos v} = \frac{r}{p} (e + \cos v) \qquad (38)$$

The orbit longitude is $L = v + \varphi + \omega$; and the equation of the ellipse is

$$\frac{p}{r} = 1 + e\cos(L - \varphi - \omega)$$
$$= 1 + e\cos(L - \pi)$$

where π is the longitude of the perihelion.

Differentiating this last equation with reference to p, e and π we have

$$\frac{1}{r}\frac{dp}{dt} = \cos(L - \pi)\frac{de}{dt} + e\sin(L - \pi)\frac{d\pi}{dt}$$

or
$$\frac{2\sqrt{p}}{k\sqrt{1+m}}Qr = r\cos v\frac{de}{dt} + er\sin v\frac{d\pi}{dt}\cdot \quad \text{by (36)}$$

whence

$$\frac{d\pi}{dt} = \frac{2\sqrt{p}}{k\sqrt{1+m}}\cdot\frac{Q}{e\sin v} - \frac{\cos v}{e\sin v}\cdot\frac{de}{dt}$$
$$= \frac{2\sqrt{p}}{k\sqrt{1+m}}\cdot\frac{Q}{e\sin v} - \frac{\cos v}{e\sin v}\left\{\frac{p}{k\sqrt{p(1+m)}}\left(P\sin v + Q(\cos v + \cos E)\right)\right\}$$
$$= \frac{1}{k\sqrt{p(1+m)}}\cdot\frac{1}{e}\left\{-Pp\cos v + \frac{pQ}{\sin v}(2 - \cos^2 v - \cos v\cos E)\right\}$$

But by (38) $p\cos E = r(e + \cos v)$

and $\quad p\cos v\cos E = er\cos v + r\cos^2 v$
$$= er\cos v + r - r\sin^2 v$$
$$= p - r\sin^2 v$$

or $p(1 - \cos v\cos E) = r\sin^2 v$ or $1 - \cos v\cos E = \frac{p}{r}\sin^2 v$

substituting in the above we have

$$\frac{d\pi}{dt} = \frac{1}{k\sqrt{p(1+m)}}\cdot\frac{1}{e}(-Pp\cos v + Q(p + r)\sin v) \qquad (39)$$

$$= \frac{d\omega}{dt},$$ when the effect of Z is not considered, or is zero.

When it is necessary to take the component Z into account, we have

$$\omega = \pi - \varphi \text{ and } \frac{d\omega}{dt} = \frac{d\pi}{dt} - \frac{d\varphi}{dt}$$
$$= \frac{d\pi}{dt} - \cos i\frac{d\Omega}{dt} \qquad (40)$$

In these formulæ p, e and v are the semi-parameter, eccentricity and true anomaly respectively in the disturbed or instantaneous orbit.

The computation of (40) will require the variation of the node which has not yet been considered, but it will be better to determine Ω and i as well as π and ω by the aid of (32) and the two following equations.

PERTURBATIONS OF HYPERION BY TITAN.

We shall now proceed to give an application of the preceding formulæ in the case of the action of Titan on Hyperion.

The former is a body of considerable magnitude, having a diameter of about 3,500 miles, and, therefore, capable of exercising a great influence on its nearest neighbour, Hyperion, whose motion is unique in the solar system.

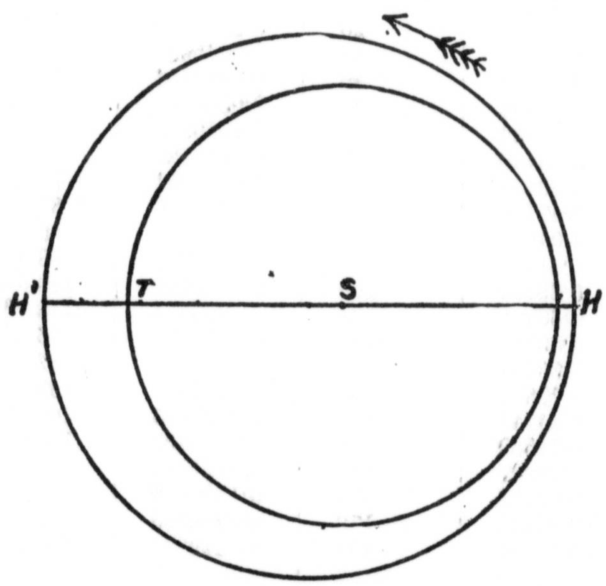
Hyperion, the latest addition to Saturn's family, was discovered in September, 1848, by Bond, of Cambridge, Mass., and independently by Lassell, of Liverpool, England, two days afterwards. It is one of the smallest of Saturn's satellites, its mass being estimated at about $\frac{1}{400}$ of that of Titan. By reason of the close approach of the two satellites at conjunction the orbit of Hyperion is made very eccentric, and the line of apsides has a retrograde motion—a necessary result of the disturbance of Titan.

Observation has established the fact that conjunction always takes place when Hyperion is in apo-saturnium and opposition when in peri-saturnium. The planes of their orbits very nearly coincide, and in the following computations the orbits are regarded as being in the same plane.

The mean daily motion of Hyperion, $n = 16^\circ{\cdot}919883$; mean daily motion of Titan, $n' = 22^\circ{\cdot}576974$; $a = 213''{\cdot}98$; $a' = 176''{\cdot}552$; regarding the latter as unity for convenience of computation we shall have $a = 1{\cdot}212$ and $a' = 1$; we also have $e = {\cdot}1$, and although Titan's orbit has an eccentricity of ·028 we shall regard it as circular; the *assumed* mass of Titan, $m' = \frac{1}{4850}$, that of Saturn being unity, and $m = 0$, Hyperion's mass being so small that it may be neglected.

The synodic period is, therefore, 63·6369492 days, during which Hyperion describes 1076°·72974767 and Titan 1436·72974767, and accordingly when they next arrive at conjunction each will lack only

$3°·27025233$ of having performed an entire number of revolutions, or in other words the conjunction point or the line of apsides will have retrograded $3°·27025233$ in a synodic period, and it also appears that four revolutions of Titan are very nearly equal to three of Hyperion. The motion of the line of apsides in half a synodic period is, therefore, $-1°·635$ or $-5886''$. The Gaussian constant for the Saturnian system is $\frac{n'}{\sqrt{1+m'}} = k$, hence, we find $\log k = 9·5954983$ and $\log m'k^2 = \bar{5}·5052549$ where n' is of course expressed in *circular* measure. Let the line of apsides at any given epoch t_0 be taken as fixed or as the initial line from which longitude is reckoned and let the satellites start from *opposition*, that is, from the points H and T in the annexed diagram; Hyperion



will then be in *peri-saturnium*, and at the end of half a synodic period they will be in *conjunction* H', T, and Hyperion will be in *apo-saturnium*, having made about a revolution and a half, and Titan two revolutions. The circumstances of the motion during the latter half of the synodic periods will be precisely similar to that in the former but in the reverse order.

We must now compute the distance between the satellites at equal intervals during half a synodic period or $31·81847$ days, and in the present instance, it will be found convenient to take for this interval

·47128 day $= \tau$ during which Hyperion's mean anomaly increases by 8°, and Titan's mean motion by 10°38′·4.

We then form the following table of the values of r_0 and v_0, employing for this purpose the ordinary formulæ which may be found in most works on Astronomy, or in the writer's article in the Monthly Not. Royal Astronomical Society, vol. xliii., page 345.

HYPERION.

DAYS.	M.	$\log r_0$	$v_0 = \theta_0$
	°		° ′ ″
− ·23564	− 4	0·0379288	355 5 15·3
+ ·23564	+ 4	0·0379288	4 54 44·7
·70692	12	0·0390800	14 42 40·2
1·17820	20	0·0413339	24 25 59·4
1·64948	28	0·0445991	34 1 55·3
2·12076	36	0·0487484	43 28 4·9
2·59204	44	0·0536305	52 42 33·1
3·06332	52	0·0590810	61 43 57·3
3·53460	60	0·0649325	70 31 25·2
4·00588	68	0·0710231	79 4 33·0
4·47716	76	0·0772028	87 23 21·7
4·94844	84	0·0833362	95 28 11·9
5·41972	92	0·0893054	103 19 41·6
5·89100	100	0·0950096	110 58 40·2
6·36228	108	0·1003841	118 26 5·4
6·83356	116	0·1052998	125 43 0·9
7·30484	124	0·1097602	132 50 35·0
7·77612	132	0·1137001	139 49 57·2
8·24741	140	0·1170845	146 42 18·2
8·71869	148	0·1198859	153 28 48·9
9·18997	156	0·1221133	160 10 50·3
9·66125	164	0·1236641	166 49 2·7
10·13253	172	0·1246160	173 25 6·1
10·60381	180	0·1249337	180 0 0

It is not necessary to produce this table any farther, as the quantities r_0 and v_0 or θ_0 recur in the reverse order in the remaining half of the orbit. We can now for every date form the angle $\theta' - \theta_0$ required in computing Δ.

We now compute Δ from (19 bis), in which $\cos \lambda = 1$ and $r' = a' = 1$, and P and Q from (19); Z vanishes since $\sin \lambda = 0$. We thus form the following table of values of Δ, P, Q, the function $\tau^2 Q r_0$ and the integral $\tau \int Q r_0 dt$, the last two being expressed in units of the *seventh decimal* place.

DAYS AFTER ☊	Δ	log *P*.	log *Q*.	$\tau^2 Q r_0$.	$\tau \int Q r_0 dt$.
− 0·23564	2·091256	$\bar{5}$·392488	$\bar{7}$·320778*n*	− 0·5073	+ 0·1266
+ 0·23564	2·091256	5·392488	7·320778	+ 0·5073	0·1266
0·70692	2·094021	5·392708	7·806972	1·5582	1·1533
1·17820	2·099453	5·393100	6·046110	2·7165	3·2788
1·64948	2·107280	5·393565	6·216276	4·0499	6·6447
2·12076	2·117140	5·393957	6·354401	5·6199	11·4575
2·59204	2·128571	5·394083	6·473680	7·4799	17·9813
3·06332	2·141034	5·393718	6·579869	9·6727	26·5284
3·53460	2·153929	5·392562	6·675838	12·2287	37·4479
4·00588	2·166620	5·390340	6·763183	15·1645	51·1126
4·47716	2·178956	5·386710	6·842904	18·4817	67·9043
4·94844	2·188791	5·381318	6·915711	22·1664	88·1988
5·41972	2·197000	5·373784	4·982137	26·1882	112·3497
5·89100	2·202497	5·360875	5·042649	30·5025	140·6727
6·36228	2·204807	5·350607	5·097630	35·0516	173·4333
6·83356	2·203290	5·334005	5·147411	39·7575	210·8279
7·30484	2·197719	5·313324	5·192294	44·5430	252·9752
7·77612	2·187720	5·287859	5·232538	49·3151	299·9094
8·24741	2·173114	5·256748	5·268373	53·9782	351·5694
8·71869	2·153583	5·218806	5·299984	58·4320	407·7958
9·18997	2·128851	5·172544	5·327539	62·5828	468·3402
9·66125	2·100046	5·115505	5·351211	66·3283	532·8256
10·13253	2·066129	5·044280	5·371079	69·5894	600·8283
10·60381	2·027681	6·952783	5·387245	72·2856	671·8160
11·07509	1·984988	6·829711	5·399774	74·3516	745·1894
11·54637	1·938406	6·647349	5·408705	75·7352	820·2912
12·01765	1·888410	6·311711	5·414061	76·4071	896·4237
12·48893	1·835365	7·594407*n*	5·415787	76·3245	972·8518
12·96021	1·779960	6·459022*n*	5·413868	75·5053	1048·8279
13·43149	1·722759	6·729266*n*	5·408205	73·9548	1123·6180
13·90277	1·664112	6·894451*n*	5·398669	71·7014	1196·5031
14·37405	1·605611	5·012550*n*	5·385089	68·7901	1266·8011
14·84533	1·547094	5·103529*n*	5·367253	65·2852	1333·8858
15·31661	1·489445	5·176638*n*	5·344850	61·2462	1397·1921
15·78789	1·433497	5·236765*n*	5·317604	56·7766	1456·2355
16·25917	1·379888	5·287163*n*	5·285143	51·9732	1510·6337
16·73045	1·329235	5·329557*n*	5·247084	46·9488	1560·1081
17·20173	1·282100	5·365354*n*	5·203060	41·8269	1604·4988
17·67301	1·238937	5·395527*n*	5·152756	36·7359	1643·7722
18·14429	1·200086	5·420840*n*	5·095960	31·8030	1678·0234
18·61557	1·165725	5·441945*n*	5·032621	27·1452	1707·4701
19·08685	1·135874	5·459450*n*	6·962873	22·8596	1732·4382
19·55813	1·110316	5·473964*n*	6·886980	19·0117	1753·3357
20·02941	1·088852	5·486120*n*	6·805109	15·6267	1770·6165
20·50069	1·070810	5·496575*n*	6·716836	12·6856	1784·7375
20·97197	1·055584	5·505987*n*	6·620211	10·1272	1796·1161
21·44325	1·042350	5·515039*n*	6·509228	7·8426	1805·0835
21·91453	1·030247	5·524361*n*	6·369146	5·6946	1811·8468
22·38581	1·018412	5·534501*n*	6·159092	3·5286	1816·4673
22·85709	1·005940	5·546010*n*	7·673259	+ 1·1613	1818·8367
23·32837	0·991994	5·559204*n*	7·808714*n*	− 1·6014	1818·6591
23·79966	0·975741	5·578793*n*	6·296995*n*	− 4·9843	1815·4249
24·27094	0·956732	5·592246*n*	6·556275*n*	− 9·1681	1808·4271

DAYS AFTER ☍	Δ	log P.	log Q.	$\tau^2 Q r_0$.	$\tau \int Q r_0 dt$.
24·74222	0·934217	$\bar{5}$·612586n	$\bar{6}$·747978n	− 14·4471	+ 1796·7215
25·21350	0·907860	5·635800n	6·906151n	− 21·0864	1780·0808
25·68478	0·877387	5·662188n	5·044538n	− 29·4126	1753·9869
26·15606	0·842669	5·692161n	5·169893n	− 39·8102	1719·5661
26·62734	0·803727	5·726303n	5·286116n	− 52·7430	1673·5217
27·09862	0·760740	5·765448n	5·395676n	− 68·7723	1613·0555
27·56990	0·714083	5·810746n	5·500703n	− 88·6758	1534·6717
28·04118	0·664186	5·863726n	5·600638n	− 112·8884	1434·3000
28·51246	0·611917	5·926221n	5·697132n	− 142·4294	1307·1008
28·98374	0·558287	4·000315n	5·788617n	− 177·4275	1147·6137
29·45502	0·504739	4·087078n	5·871755n	− 216·5400	950·6939
29·92630	0·453251	4·186465n	5·938792n	− 254·3134	714·8696
30·39758	0·406656	4·293565n	5·972697n	− 276·3625	447·2353
30·86886	0·368222	4·396902n	5·937956n	− 256·0105	+ 175·9823
31·34014	0·342532	4·475193n	5·739423n	− 162·4156	− 39·8371
31·81142	0·333318	4·505290n	7·673071n	− 1·3955	− 125·0492
32·28270	0·342217	4·476171n	5·733780	+ 160·2582	− 42·4004
32·75398	0·367644	4·398517n	5·936674	+ 255·0635	+ 171·8637
33·22526	0·405877	4·295502n	5·973065	+ 276·2852	+ 442·6320
	0·452334	4·188316n	5·940003	+ 254·9500	+ 712·60

The characteristics of log P and log Q are negative.

It may also be remarked that the distance Δ, between the satellites actually increases from opposition until 6·36 days thereafter, contrary to what would be expected. This is owing to the form of Hyperion's orbit and causes a slight apparent irregularity in the value of δr and $\delta\theta$.

We must now compute the value of the function $\tau^2 \dfrac{d^2\delta r}{dt^2}$ for each of the preceding dates, by the second of (24), omitting for the first approximation to δr, the last term in the second member. With the approximate value of δr thus obtained from two integrations of $\tau^2 \dfrac{d^2\delta r}{dt^2}$ we recompute the value of $\tau^2 \dfrac{d^2\delta r}{dt^2}$ by the second of (24), retaining the last term of the second member and then after two integrations we obtain a very accurate value of δr with which we can now compute $\dfrac{d\delta\theta}{dt}$ from the first of (24) and thence $\delta\theta$ by a single integration. But if we want to retain the terms involving the squares and products of the disturbing forces, that is, all the terms which have any appreciable influence, we must employ equations (21) and (23), which in the present case become

$$\frac{d\delta\theta}{dt} = \frac{1}{r^2}\int Qr dt - \left(1 - \frac{r_0^2}{r^2}\right)h$$

and

$$\frac{d^2\delta r}{dt^2} = P + 2rh\frac{d\delta\theta}{dt} + r\left(\frac{d\delta\theta}{dt}\right)^2 + h^2\delta r + \frac{2k^2(1 + m)\,\delta r}{r^2 r_0} \quad (41)$$

in which $r = r_0 + \delta r$, and $m = 0$.

From the last equation we now re-compute for each of the preceding dates the value of the function $\tau^2 \frac{d^2\delta r}{dt^2}$, using for this purpose the second value of δr, and thus form the following table. The integration is performed between the limits $-\frac{1}{2}$ and n_0, where n_0 is the number of any term; and of course the values of δr and $\delta\theta$ are 0 for the date zero, or at the moment of opposition.

DAYS AFTER 8	$\tau^2 \frac{d^2\delta r}{dt^2}$.	$\tau \frac{d\delta r}{dt}$.	δr.
− 0·23564	+ 54·513	− 27·312	− 6·831
+ 0·23564	54·513	+ 27·312	+ 6·831
0·70692	53·159	81·260	61·231
1·17820	50·502	133·200	168·681
1·64948	46·640	181·843	326·531
2·12076	42·266	226·300	530·978
2·59204	38·291	266·552	777·810
3·06332	34·451	302·902	1062·776
3·53460	31·064	335·598	1382·314
4·00588	28·648	365·380	1735·000
4·47716	27·031	393·154	2112·399
4·94844	26·158	419·679	2518·890
5·41972	26·179	445·810	2951·618
5·89100	26·226	471·960	3410·519
6·36228	27·481	498·745	3895·757
6·83356	28·924	526·958	4408·486
7·30484	30·217	556·508	4950·127
7·77612	32·128	587·682	5522·034
8·24741	33·381	620·447	6126·019
8·71869	34·827	654·563	6763·398
9·18997	35·974	689·984	7435·580
9·66125	36·921	726·474	8143·720
10·13253	37·088	763·300	8888·716
10·60381	37·042	800·635	9670·780
11·07509	35·989	837·217	10489·804
11·54637	34·317	872·441	11344·762
12·01765	31·513	905·457	12233·950
12·48893	27·506	935·056	13154·547
12·96021	22·011	960·130	14102·548
13·43149	14·493	978·718	15072·661
13·90277	+ 4·932	988·586	16057·090
14·37405	− 7·038	987·766	17046·270
14·84533	− 21·900	973·547	18028·166
15·31661	− 39·832	+ 942·952	+ 18987·910

DAYS AFTER ☊	$\tau^2 \frac{d^2 \delta r}{dt^2}$.	$\tau \frac{d\delta r}{dt}$.	δr.
15·78789	− 61·119	+ 892·758	+ 19907·535
16·25917	− 85·681	819·608	20765·769
16·73045	− 112·886	720·527	21538·101
17·20173	− 142·162	593·130	22197·375
17·67301	− 172·107	435·960	22714·421
18·14429	− 200·306	249·500	23059·507
18·61557	− 224·150	+ 35·820	23204·651
19·08685	− 241·219	− 196·653	23126·202
19·55813	− 247·321	− 441·903	22807·456
20·20941	− 240 218	− 686·851	22242·494
20·50069	− 218·542	− 917·461	21438·533
20·97197	− 182·425	− 1119·099	20417·242
21·44325	− 133·725	− 1278·087	19214·582
21·91453	− 76·336	− 1383·670	17878·923
22·38581	− 14·390	− 1429·197	16467·310
22·85709	+ 46·891	− 1412·703	15041·252
23·32837	103·200	− 1337·030	13661·667
23·79966	150·367	− 1209·508	12384·495
24·27097	189·049	− 1038·945	11257·107
24·74222	215·810	− 835·550	10317·558
25·21350	231·984	− 610·783	9593·036
25·68478	237·871	− 375·050	9099·637
26·15606	+ 234·806	− 138·113	8843·365
26·62734	223·730	+ 91·912	8821·234
27·09862	204·827	+ 306·817	9022·180
27·56990	178·589	+ 499·145	9427·346
28·04118	144·326	+ 661·338	10010·439
28·51246	100·016	+ 784·452	10737·023
28·98374	+ 42·733	+ 857·063	11562·549
29·45502	− 31·767	+ 864·228	12429·378
29·92630	− 129·087	+ 785·926	13262·525
30·39758	− 251·732	+ 597·542	13964·423
30·86886	− 389·696	+ 276·845	14413·190
31·34014	− 501·982	− 174·014	14474·272
31·81142	− 519·809	− 695·208	14041·373
32·28270	− 403·954	− 1167·582	13100·115
32·75398	− 189·572	− 1469·840	+ 12263·2
33·22526	+ 55·476	− 1526·5	

The quantities in the last three columns are expressed in units of the seventh decimal place; thus the last value of δr in the table is ·0012263. It will be noticed that the perturbations of the radius vector are all positive, the disturbed orbit, therefore, lies outside of the osculating orbit. We now compute the values of the function $\tau \frac{d\delta\theta}{dt}$ by the first of (41), and thence $\delta\theta$ by a single integration.

DAYS AFTER ☊	$\tau \frac{d\delta\theta}{dt}$.	$\delta\theta$	$\delta\theta$ SECONDS OF ARC.	DIFF.
			″	
— 0·23564	— 2·627	— 0·674	— 0 01	
+ 0·23564	— 2·627	— 0·674	— 0·01	
0·70692	— 17·964	— 9·725	— 0·20	— 0·19
1·17820	— 48·122	— 41·522	— 0·86	— 0·66
1·64948	— 92·152	— 110·649	— 2·28	— 1·42
2·12076	— 145·796	— 229·064	— 4·72	— 2·44
2·59204	— 203·973	— 403·594	— 8·32	— 3·60
3·06332	— 266·646	— 638·624	— 13·17	— 4·85
3·53460	— 331·435	— 937·662	— 19·34	— 6·17
4·00588	— 394·624	— 1300·822	— 26·83	— 7·49
4·47716	— 456·322	— 1726·428	— 35·61	— 8·78
4·94844	— 516·157	— 2212·861	— 45·64	— 10·03
5·41972	— 573·406	— 2757·840	— 56·88	— 11·24
5·89100	— 628·756	— 3359·043	— 69·28	— 12·40
6·36228	— 683·052	— 4015·025	— 82·81	— 13·53
6·83356	— 736·641	— 4724·840	— 97·45	— 14·64
7·30484	— 791·358	— 5488·768	— 113·21	— 15·76
7·77612	— 845·802	— 6307·729	— 130·11	— 16·90
8·24741	— 904·228	— 7182·987	— 148·16	— 18·05
8·71869	— 965·832	— 8117·625	— 167·44	— 19·28
9·18997	—1032·932	— 9116·546	— 188·04	— 20·60
9·66125	—1105·669	— 10185·224	— 210·08	— 22·04
10·13253	—1187·654	— 11331·091	— 233·72	— 23·64
10·60381	—1278·595	— 12563·317	— 259·14	— 25·42
11·07509	—1382·054	— 13892·666	— 286·56	— 27·42
11·54637	—1497·412	— 15331·358	— 316 23	— 29·67
12·01765	—1626·062	— 16891·804	— 348·42	— 32·19
12·48893	—1772·312	— 18589·505	— 383·44	— 35·02
12·96021	—1936·585	— 20442·286	— 421·65	— 38·21
13·43149	—2122·719	— 22470·145	— 463·48	— 41·83
13·90277	—2329·845	— 24694·625	— 509·36	— 45·88
14·37405	—2559·438	— 27137·338	— 559·75	— 50·39
14·84533	—2812·617	— 29821·414	— 615·11	— 55·36
15·31661	—3088·483	— 32793·977	— 676·42	— 61·31
15·78789	—3385·074	— 36005·326	— 742·66	— 66·24
16·25917	—3697·623	— 39545·661	— 815·69	— 73·03
16·73045	—4017·742	— 43403·050	— 895·25	— 79·56
17·20173	—4336·287	— 47580·701	— 981·42	— 86·17
17·67301	—4639·214	— 52070·522	—1074·03	— 93·61
18·14429	—4906·989	— 56847·402	—1172·56	— 98·53
18·61557	—5119·486	— 61865·950	—1276·08	—103·52
19·08685	—5258·800	— 67062·297	—1383·26	—107·18
19·55813	—5299·206	— 72350·245	—1492·33	—109·07
20·02941	—5227·093	— 77623·167	—1601·09	—108·76
20·50069	—5035·889	— 82764·700	—1707·14	—106·05
20·97197	—4726·607	— 87655·358	—1808·02	—100·88
21·44325	—4313·930	— 92183·319	—1901·42	— 93·40
21·91453	—3822·427	— 96256·978	—1985·44	— 84·02
22·38581	—3280·018	— 99811·037	—2058·75	— 73·31
22·85709	—2721·034	—102811·586	—2120·64	— 61·89
23·32837	—2176·435	—105257·949	—2171·10	— 50·46
23·79966	—1671·913	—107177·948	—2210·70	— 39·60
24·27097	—1225·012	—108620·986	—2240·47	— 29·77

DAYS AFTER ☍	$\tau \frac{d\delta\theta}{dt}$.	$\delta\theta$	$\delta\theta$ SECONDS OF ARC.	DIFF.
			″	
24·74222	— 847·783	—109651·439	—2261·72	— 21·25
25·21350	— 541·853	—110340·125	—2275·93	— 14·21
25·68478	— 310·079	—110760·159	—2284·59	— 8·66
26·15606	— 144·702	—110982·321	—2289·17	— 4·58
26·62734	— 38·631	—111069·302	—2290·97	— 1·80
27·09862	+ 14·707	—111077·241	—2291·13	— 0·16
27·56990	+ 24·635	—111054·342	—2290·66	+ 0·47
28·04118	— 0·086	—111039·440	—2290·35	+ 0·31
28·51246	— 53·175	—111064·052	—2290·86	— 0·51
28·98374	— 125·536	—111152·205	—2292·68	— 1·82
29·45502	— 206·743	—111318·064	—2296·10	— 3·42
29·92630	— 283·594	—111564·350	—2301·39	— 5·29
30·39758	— 333·533	—111876·450	—2307·62	— 6·23
30·86886	— 319·045	—112210·307	—2314·50	— 6·88
31·34014	— 182·286	—112474·085	—2319·94	— 5·44
31·81142	+ 141·915	—112511·892	—2320·72	— 0·78
32·28270	+ 684·245	—112116·415	—2312·57	+ 8·15
32·75398	+1415·134	—111080·528	—2291·20	21·37
33·22526	+2279·105	—109239·500	—2253·23	+ 37·97

From the preceding table it is seen that all the values of $\delta\theta$ are negative, which is in accord with the values of δr previously found. We can now interpolate the values of δr and $\delta\theta$, for any integral number of days or for any convenient fraction of a day such as $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of a day. It is also seen that the values of the functions

$$\tau^2 \frac{d^2\delta r}{dt^2} \text{ and } \tau \frac{d\delta\theta}{dt}$$

vary rapidly as the satellites approach conjunction. More accurate values of the perturbations δr and $\delta\theta$ near this point would be obtained by stopping the integration at, say the twenty-fourth day after opposition, and computing the value of these functions for half the period, and then integrating again. This has been done, but the difference is so small that it may be neglected without appreciable error.

We will now proceed to find the elements of the disturbed orbit for any of the preceding dates, and for this purpose we first compute p from (26) which in the present case becomes

$$p = \frac{r^4}{r_0^4} \cdot \frac{p_0}{h^2}\left(h + \frac{d\delta\theta}{dt}\right)^2 \text{ or } \sqrt{p} = \frac{r^2}{r_0^2} \cdot \frac{\sqrt{p_0}}{h}\left(h + \frac{d\delta\theta}{dt}\right)$$

$$= \frac{r^2}{k}\left(h + \frac{d\delta\theta}{dt}\right)$$

in which $r = r_0 + \delta r, \; p_0 = a_0(1 - e_0^2) = 1{\cdot}212 \times {\cdot}99$

and $h = \frac{k\sqrt{p_0(1+m)}}{r_0^2} = \frac{k\sqrt{p_0}}{r_0^2}$

We also compute $e \cos v$ and $e \sin v$ from (27) whence we obtain $\sqrt{p}$, e and v as in the following Table :

DAYS AFTER δ	log $\sqrt{p}$	log e sin v	log e cos v	v ° ′ ″	e
— 0·23564	0·0395881	7·9334569n	8·9983962	— 4 55 18·0	·10000
+ 0 23564	5881	7·9334567	8·9983962	+ 4 55 18·0	·10000
0·70692	5884	8·4055614	8·9855013	14 44 18·5	·10001
1·17820	5890	8·6174387	8·9591936	24 28 37·8	·10002
1·64948	5896	8·7487544	8·9182799	34 5 27·6	·10004
2·12076	5905	8·8384010	8·8605554	43 32 21·2	·10006
2·59204	5920	8·9015402	8·7819828	52 47 19·5	·10009
3·06332	59 9	8·9457368	8·6747610	61 48 59·1	·10013
3·53460	5961	8·9753290	8·5218726	70 36 28·4	·10016
4·00588	5991	8·9930212	8·2753655	79 9 17·4	·10020
4·47716	6027	8·0005708	7·6477946	87 27 31·2	·10023
4·94844	6070	8·9991154	7·9846501n	95 31 28·9	·10026
5·41972	6121	8·9893383	8·3651134n	103 21 18·7	·10029
5·89100	6182	8·9715394	8·5554790n	110 59 22·2	·10032
6·36228	6212	8·9456540	8·6793516n	118 26 27·7	·10035
6·83356	6329	8·9112129	8·7672301n	125 40 17·9	·10033
7·30484	6420	8·8672279	8·8333193n	132 45 55·8	·10033
7·77612	6518	8·8119653	8·8838777n	139 43 19·6	·10032
8·24741	6632	8·7424933	8·9227135n	146 33 39·9	·10030
8·71869	6755	8·6536546	8·9522062n	153 18 14·2	·10027
9·18997	6885	8·5355544	8·9741690n	159 59 11·2	·10023
9·66125	7026	8·3662397	8·9887457n	166 35 7·4	·10018
10·13253	7172	8·0761248	8·9974156n	173 9 52·5	·10012
10·60381	7322	6·6743892	9·0002397n	179 43 46·0	·10006
11·07509	7479	8·0402862n	8·9973327n	186 17 59·0	·09999
11·54637	7637	8·3488014n	8·9885822n	192 53 42·0	·09992
12·01765	7804	8·5235668n	8·9739126n	199 31 15·6	·09987
12·48893	7968	8·6446372n	8·9518786n	206 14 19·2	·09982
12·96021	8138	8.7352530n	8·9223102n	213 1 32·3	·09975
13·43149	8294	8·8059118n	8·8834400n	219 54 46·3	·09969
13·90277	8454	8·8620228n	8.8328739n	226 55 16·8	·09964
14·37405	8609	8·9066518n	8·7668152n	234 4 7·8	·09961
14·84533	8755	8.9416078n	8·6790725n	241 21 1·3	·09960
15·31661	8896	8·9679226n	8·5554838n	248 51 0·8	·09959
15·78789	9028	8·9860996n	8·3658538n	256 31 4 6	·09959
16·25917	0·0399154	8·9962303n	7·9883966n	264 23 27·0	·09961
16·73045	9270	8·9980359n	7·6347291	272 28 43·9	·09964
17·20173	9378	8.9908570n	8·2712087	280 47 48·1	·09968
17·67301	9478	8·9735832n	8·5189546	289 20 37·4	·09973
18·14429	9569	8·9444934n	8·6723870	298 7 17·5	·09978
18·61557	9660	8·9009412n	8·7800185	307 7 28·4	·09984
19·08685	9741	8·8386743n	8·8589406	316 20 10·9	·09990
19·55813	9823	8·7503337n	8·9170403	325 44 10·2	·09996
20·02941	9908	8·6212499n	8·95[illegible]3886	335 17 32·6	·10002
20·50069	9992	8·4143123n	9·9852053	344 57 54·7	·10008
20·97197	0·0400081	7.9653151n	8·9987117	354 42 34.5	·10013
21·44325	0179	7 8930412	8·9994533	4 28 31·2	·10018
21 91453	0279	8 3909926	8·9874517	14 12 40·0	·10022
22·38581	0396	8·6081219	8·9622779	23 51 57·6	·10025
22·85709	0522	8·7418692	8·9228188	33 23 41·6	·10027

DAYS AFTER ☊	$\log \sqrt{p}$	$\log e \sin v$	$\log e \cos v$	v	e
				° ′ ″	
23·32837	0·0400664	8·8330266	8·8670496	42 45 28·7	·10028
23·79966	0809	8·8972589	8·7912217	51 55 34·2	·10027
24·27097	0977	8·9423277	8·6883165	60 52 28·9	·10024
24·74222	1156	8·9726610	8·5432547	69 35 33·4	·10019
25·21350	1347	8·9910077	8·3157086	78 4 27·1	·10012
25·68478	1551	8·9991530	7·8701357	85 45 2.8	·10008
26·15606	1766	8·9982508	7·8790729*n*	94 20 46·4	·09988
26·62734	1990	8·9889948	8.3222941*n*	102 9 26·6	·09974
27·09862	2225	8·9716931	8·5273807*n*	109 46 22·9	·09956
27·56990	2458	8·9462759	8·6578629*n*	117 14 11·9	·09938
28·04118	2701	8·9122723	8·7494085*n*	124 30 0·4	·09915
28·51246	2934	8·8686713	8·8178742*n*	131 39 24·6	·09892
28·98374	3157	8·8136942	8·8700576*n*	138 42 27·3	·09868
29·45502	3367	8 7442955	8·9100553*n*	145 40 40·8	·09843
29·92630	3564	8·6550821	8·9403640*n*	152 35 41·7	·09824
30·39758	3782	8·5355532	8·9628022*n*	159 29 57·9	·09800
30·86886	4088	8·3619593	8·9773958*n*	166 22 24·2	·09768
31·34014	4623	8·0562226	8·9853797*n*	173 17 10·2	·09736
31·81142	5522	6·6138894*n*	8·9864562*n*	180 14 34·7	·09693
32·28270	7012	8·0857137*n*	8·9804683*n*	187 15 41·9	·09638
32·75398	8898	8·3755475*n*	8·9672699*n*	194 21 37·9	·09573
33·22526	0·0411122	8.5341797*n*	8·9467469*n*	201 32 47·8	·09511

By means of (28) we can now find a, μ, E and M for any date required, and from the data furnished by the preceding table we can compute the function $\tau \frac{dw}{dt}$ from (39) and thence ω by a single integration, as in the following table:

DAYS AFTER ☊	$\tau \frac{d\omega}{dt}$.	ω.	ω in arc.	DIFF.
			″	″
— 0·23564	— 3218·400			
+ 0·23564	— 3218·400	— 1614·608	— 33·30	
0·70692	— 3088·933	— 4779·016	— 98·57	— 65·27
1·17820	— 2832·000	— 7749·956	— 159·85	— 61·28
1·64948	— 2452·200	— 10402·039	— 214·56	— 54·71
2·12076	— 1957·236	— 12615·922	— 260·22	— 45·66
2·59204	— 1358·805	— 14281·957	— 294·59	— 34·37
3·06332	— 673·212	— 15304·432	— 315·68	— 21·09
3·53460	+ 78·606	— 15606·237	— 321·90	— 6·22
4·00588	+ 870·804	— 15133·723	— 312·16	+ 9·74
4·47716	+ 1674·007	— 13860·919	— 285·90	+ 26·26
4·94844	+ 2455·927	— 11792·700	— 243·24	+ 42·66
5·41972	+ 3183·013	— 8966·950	— 184·96	+ 58·28
5·89100	+ 3815·167	— 5459·500	— 112·61	+ 72·25
6·36228	+ 4343·460	— 1369·280	— 28·24	+ 84·37
6·83356	+ 4715·530	+ 3173·800	+ 65·46	+ 93·70
7·30484	+ 4919·644	+ 8006·118	+ 165·14	+ 99·68
7·77612	+ 4938·312	+ 12951·000	+ 267·13	+ 101·99

DAYS AFTER 8	$\tau\frac{d\omega}{dt}$.	ω.	ω in arc.	DIFF.
			″	″
8·24741	+ 4762·643	+ 17817·838	+ 367·52	+100·39
8·71869	+ 4391·097	+ 22410·925	+ 462·26	+ 94·74
9·18997	+ 3829·430	+ 26536·302	+ 547·35	+ 85·09
9·66125	+ 3096·820	+ 30013·080	+ 619·06	+ 71·71
10·13253	+ 2211·078	+ 32678·044	+ 674·03	+ 54·97
10·60381	+ 1206·879	+ 34391·800	+ 709·38	+ 35·35
11·07509	+ 118·220	+ 35063·956	+ 723·25	+ 13·87
11·54637	— 1013·737	+ 34617·738	+ 714·04	— 9·21
12·01765	— 2139·738	+ 33038·950	+ 681·48	— 32·56
12·48893	— 3220·702	+ 30352·455	+ 626·06	— 55·42
12·96021	— 4197·622	+ 26632·495	+ 549·33	— 76·73
13·43149	— 5039·734	+ 22001·169	+ 453·81	— 95·52
13·90277	— 5693·829	+ 16617·243	+ 342·75	—111·06
14·37405	— 6129·061	+ 10686·227	+ 220·42	—122·33
14·84533	— 6317·232	+ 4441·904	+ 91·62	—128·80
15·31661	— 6247·910	— 1862·637	— 38·42	—130·04
15·78789	— 5913·120	— 7964·981	— 164·29	—125·87
16·25917	— 5326·162	— 13604·760	— 280·62	—116·33
16·73045	— 4513·225	— 18541·829	— 382·45	—101·83
17·20173	— 3515·775	— 22569·127	— 465·52	— 83·07
17·67301	— 2395·633	— 25533·530	— 526·67	— 61·15
18·14429	— 1194·007	— 27331·560	— 563·75	— 37·08
18·61557	— 0·548	— 27925·420	— 576·00	— 12·25
19·08685	+ 1126·033	— 27354·850	— 564·23	+ 12·37
19·55813	+ 2134·027	— 25712·832	— 530·36	+ 33·87
20·02941	+ 2977·773	— 23141·950	— 477·34	+ 53·02
20·50069	+ 3632·334	— 19820·445	— 408·83	+ 68·51
20·97197	+ 4085·555	— 15944·543	— 328·88	+ 79·95
21·44325	+ 4336·000	— 11716·965	— 241·68	+ 87·20
21·91453	+ 4387·164	— 7338·990	— 151·38	+ 90·30
22·38581	+ 4242·713	— 3007·888	— 62·04	+ 89·34
22·85709	+ 3904·874	+ 1082·360	+ 22·32	+ 84·36
23·32837	+ 3370·236	+ 4735·050	+ 97·67	+ 75·35
23·79966	+ 2662·448	+ 7769·380	+ 160·25	+ 62·58
24·27097	+ 1687·042	+ 9963·860	+ 205·52	+ 45·37
24·74222	+ 520·503	+ 11085·100	+ 228·65	+ 23·13
25·21350	— 874·364	+ 10924·014	+ 223·26	— 5·39
25·68478	— 2441·611	+ 9289·270	+ 191·61	— 31·65
26·15606	— 4378·625	+ 5902·750	+ 121·75	— 69·86
26·62734	— 6499·500	+ 481·210	+ 9·93	—111·82
27·09862	— 8874·900	— 7183·800	— 148·18	—158·11
27·56990	—11522·716	— 17360·290	— 358·08	—209·90
28·04118	—14435·206	— 30315·370	— 625·30	—267·22
28·51246	—17659·638	— 46335·710	— 955·74	—330·44
28·98374	—21222·634	— 65747·480	—1356·14	—400·40
29·45502	—25144·814	— 88901·410	—1833·72	—477·58
29·92630	—29404·182	—116148·660	—2385·74	—552·02
30·39758	—33921·600	—147798·280	—3048·56	—662·82
30·86886	—38353·342	—183975·580	—3794·77	—746·21
31·34014	—41824·971	—224186·720	—4624·18	—829·41
31·81142	—43366·410	—266976·340	—5506·78	—882·60
32·28270	—42440·637	—310078·580	—6395·83	—889·05
32·75398	—39479·109	—351172·500	—7243·46	—847·63
33·22526	—35401·227	—388672·000	—8016·93	—773·47

From the values of ω it is seen that the line of apsides regresses and progresses alternately; thus for the first three and a-half days after opposition, it regresses, then advances until about the 11th day, then again regresses to 18·6 days, again advances to the 24·7 day, and finally regresses rapidly until after conjunction. The amount of regression, however, exceeds that of progression to such an extent that the line of apsides makes an entire revolution in about 20·45 years. The line of apsides of our own Moon behaves in a similar manner, but its advance exceeds its regression to such an extent that it completes a revolution in about 8·855 years.

If we interpolate the value of ω for 31·81847 days or half a synodic period we have $\omega = -5520''$, but according to observation it should be $-5886''$, and since the amount of disturbance is proportional to the mass we shall have for the corrected mass of Titan $\frac{1}{4548}$.

We can now deduce the elements at any date after opposition; thus at conjunction we have $e = \cdot 09694$, $p = 1\cdot 2053$, $a = 1\cdot 2167$ and $\mu = 16^\circ\cdot 8217$.

It will not be difficult now to form Tables of the motion of Hyperion which will give its position in its orbit for any date after opposition or before conjunction. For this purpose the following approximate elements are here given

Epoch	1888 January 1·0 G. M. T.	
M	$268^\circ 34'\cdot 9$	
N	$120^\circ 42'\cdot 3$	Mean equator and equinox Jan. 1·0
I	$6^\circ 8'\cdot 6$	
ω	$166^\circ 18'\cdot 2$	
a	$213''\cdot 98$	
e	0·1	
μ	$16^\circ\cdot 9199$	

FIFTH ANNUAL MEETING.

January 8th, 1895; the Vice-President, Mr. John A. Paterson, M.A., in the chair. A note was received from Dr. Larratt W. Smith to the effect that he was unable to attend.

Mr. Archibald Blue, of Toronto, was elected an active member of the Society.

The following officers of the Society for 1895 were declared duly elected by acclamation:—Honourary President, the Honourable George W. Ross, LL.D., Minister of Education; President, Larratt W. Smith, D.C.L., Q.C.; Vice-Presidents, E. A. Meredith, LL.D., and John A. Paterson, M.A.; Treasurer, James Todhunter; Corresponding Secretary, George E. Lumsden; Recording Secretary, Charles P. Sparling; Assistant Secretary and Editor, Thomas Lindsay; Librarian, G. G. Pursey; Assistants to the Librarian, Miss M. Bambridge and Miss Jeane Pursey.

The following gentlemen were elected members of the Council, with the officers, for 1895:—Messrs. Arthur Harvey, C. A. Chant, B.A., D. J. Howell, R. F. Stupart, Rev. C. H. Shortt, M.A.

The Treasurer's report and certified statement of accounts was received and adopted. The balance on hand was sufficient to meet all existing liabilities.

A cordial vote of thanks was directed to be tendered the Editors of the *Empire*, *Globe*, *Mail* and *World* for their unvarying courtesy to the Society, in allowing the use of their columns for the Society's announcements, reports of meetings, etc.

Mr. R. F. Stupart, Chairman of the Committee on the investigation of Earth Current phenomena, communicated a brief report of the work which had so far been accomplished. In August, 1894, an outline of the plans of the Committee had been laid before the directorate of the Commercial Cable Company of New York, and Mr. G. G. Ward, Vice-President and General Manager, courteously consented to render such assistance as might be possible. Mr. Stupart was accordingly placed in communication with Mr. S. S. Dickenson, Superintendent at the company's station at Canso, Nova Scotia, and to whom the thanks of the Society were due for the active interest he had taken in the work.

Following is a tabulated statement of the reports received from Canso, from the date when arrangements were finally made for the regular transmission of the records to the Toronto Observatory:—

19

EARTH CURRENT READINGS.

The Commercial Cable Company's Station,
Canso, Nova Scotia.

Date, 1894.	Time.	*No. Three.** E.C. in volts.	*Southern.†* E.C. in volts.	*Coney Is'd.‡* E.C. in volts.	Weather, etc.
Oct. 14	5 a.m.	− 1·69005	− 1·01550	+ 0·57795	Overcast, South-east.
15	"	− 7·80345	− 7·96282	0·57492§	Fine, West, fresh.
16	"	+ 1·82357	+ 0·65011	− 0·52618	Fine, West wind.
17	"	+ 0·37317	+ 0·10685	− 0·12167	Overcast, S.-W.
18	"	+ 1·61028	+ 0·81616	− 0·15208	Fine, West.
19	"	− 0·02850	+ 0·72150	− 1·22800	Fine, North-west.
20	"	− 0·05200	− 0·86200	− 0·06000	Dull, North-west.
21	"	− 0·39400	− 0·41000	− 0·71400	Dull, North-east.
22	"	+ 0·41200	− 0·37000	− 0·19800	Fine, North-west.
23	"	+ 0·73100	+ 0·53100	+ 0·61400	Fine, North-west.
24	"	− 0·30200	− 0·73700	+ 2·51000	Fine, North.
25	"	+ 0·76200	+ 0·18800	+ 0·47300	Fine, North.
26	"	+ 4·57000	+ 0·18700	+ 0·59600	Fine, North-east.
27	"	+ 0·37000	+ 0·09400	+ 0·12400	Fine, South-east.
28	"	− 0·98740	− 0·14170	+ 0·07340	Overcast, North-east.
29	"	+ 0·19810	+ 0·03400	+ 0·01120	Fine, North.
30	"	+ 0·31810	− 0·15390	− 0·53700	Fine, North.
31	"	− 1·98850	− 4·08700	+ 0·45670	Fine, North.
Nov. 1	"	− 1·09800	− 0·72040	− 0·24540	Fine, North.
2	"	− 1·96800	− 0·32280	− 0·01336	Overcast, N.-W.
3	"	+ 1·72740	+ 0·14820	− 1·47150	Overcast, N.-W.
4	"	− 3·27700	− 0·84000	− 1·17300	Fine, North-west.
5	"	+ 1·74000	+ 0·38700	− 0·31400	Fine, North-west.
6	"	....	....	....	Fine, North-west.
7	"	− 1·24100	− 0·3900	+ 0·41400	East'rly gale and rain.
8	"	+ 1·74000	+ 0·58800	....	Fine, North.
9	"	+ 0·97500	+ 0·06000	+ 0·03000	Fine, North.
10	"	+ 1·90400	+ 0·72500	+ 0·68800	Snowy, East.
11	"	....	....	....	Wet, South.
12	"	+ 1·43700	+ 0·31400	− 0·10400	Fine, West.
13	"	− 3·27700	− 0·84000	− 1·17300	Overcast, West.
14	"	−38·97000	−20·75500	+ 0·93900‖	Rain, S.-S.-E.
15	"	− 2·02990	− 1·18500	− 1·26700	Overcast, West.
16	"	−10·76400	− 1·18600	+ 1·50100	Overcast, S.-W.
17	"	− 0·33200	− 1·16300	− 1·83100	Fine, West.
18	"	....	....	....	
19	"	+ 2·44900	+ 1·63300	+ 2·65800	Overcast, South.
20	"	+ 0·82900	+ 0·57900	+ 0·68800	Fine, North-west.
21	"	+ 1·09800	+ 1·44900	− 0·34400	Fine, North-west.
22	"	+ 1·25400	+ 1·63000	− 0·29500	Fog, South-west.
23	"	+ 0·85390	+ 0·81400	− 0·51600	North-east, Fine.
24	"	− 6·95400	− 5·87000	+ 2·06500	North-east, Overcast.
25	"	....	....	....	
26	"	+ 1·93500	+ 1·51300	− 1·26400	North, Fine.
27	"	+ 1·63000	+ 0·44400	− 0·56400	West, Light snow.
28	"	+ 2·74500	+ 1·71000	− 2·01000	West, Rain.
29	"	+ 4·27000	+ 2·72500	− 1·83900	North-west, Fine.
30	"	− 0·80300	− 0·30200	− 0·68900	N.-W., Very fine.

Date, 1894.	Time.	*No. Three.** E.C. in volts.	*Southern.*† E.C. in volts.	*Coney Is'd.*‡ E.C. in volts.	Weather, etc.
Dec. 1	5 a.m.	+ 0·48700	– 0·42300	+ 0·45900¶	West, Fair.
2	"	....	....	....	
3	"	+ 0·93500	+ 1·51300	– 1·26400	North-east, Snow.
4	"	– 0·48500	– 0·54700	+ 0·33300	North-west, Fair.
5	"	+ 1·10400	+ 0·89300	– 0·27400	Nort-west, Fine.
6	"	+ 5·61200	– 0·60500	– 0·22900	North, Overcast.
7	"	....	....	....	
8	"	+ 0·73200	+ 0·54400	– 0·20500	North-west, Cloudy.
9	"	+ 1·21900	+ 0·48300	– 1·14900	North, Fine.
10	"	....	....	....	
11	"	+ 0·42400	+ 0·20200	+ 0·34300	North, Fresh, cloudy.
12	"	– 1·00500	– 1·08900	– 0·22900	North, Fine.
13	"	+ 0·88400	+ 0·36100	– 1·00200	North, Overcast.
14	"	+ 0·77100	+ 0·43300	– 0·54400	South-west, Strong.
15	"	+ 2·58400	+ 4·72100	– 5·17600	West, Overcast.
16	"	– 3·46500	– 0·40000	– 1·72500	North, Fair.
17	"	....	....	....	South, Fog.
18	"	+ 0·43500	+ 0·73200	+ 0·91700	North-west, Fog.
19	"	+ 1·23100	+ 0·80200	+ 0·87000	West, Cloudy.
20	"	– 0·54900	– 0·27300	– 0·30900	North, Fair.
21	"	+ 0·62200	+ 0·45800	+ 1·08800	North, Fine.
22	"	+ 0·96300	+ 0·63000	+ 0·65200	South, Cloudy.
23	"	....	....	....	
24	"	+ 3·46500	+ 0·40000	+ 1·72500	North, Snow.
25	"	....	....	....	
26	"	+ 1·37100	+ 0·93000	– 0·31800	North, Fresh, clear.
27	"	– 0·57400	+ 0·17200	– 0·18100	South-east, Overcast.
28	"	+ 0·28400	+ 0·04200	+ 0·33300	Fog.
29	"	+ 0·20200	+ 0·24100	+ 0·57400	South, Rain.
30	"	+ 0·40600	+ 0·54300	– 0·80400	North-west, Fine.
31	"	....	....	....	
1895.					
Jan. 1	"	....	....	....	
2	"	+ 0·68900	+ 0·78700	– 0·77200	East, Rain.
3	"	– 0·07500	– 0·60400	+ 0·22800	North-west, Cloudy.
4	"	+ 1·21900	+ 0·24100	– 0·22800	West, Cloudy.
5	"	+ 0·68800	+ 0·31600	– 0·36300	North-west, Fine.
6	"	....	....	....	
7	"	+ 0·87100	+ 0·39800	– 0·11400	South-west, Fog.
8	"	+ 0·57400	+ 0·81300	– 0·12100	West, Fog.
9	"	– 0·71200	– 0·04200	– 0·97600	North-west, Cloudy.
10	"	+ 1·21000	+ 2·43000	+ 1·60500	North, Fine.
11	"	– 2·75700	+ 5·00100	+ 1·33200	North-east, Sleet.
12	"	+ 0·91500	– 0·69600	– 2·01400	South-west, Fog.
13	"	....	....	....	
14	"	....	....	....	
15	"	....	....	....	

* *No. 3* runs eastward from Cape Canso.
† "*Southern*" first runs south-east and then east.
‡ *Coney Island* runs south 180 miles and then west to New York.

§ Reversing current.
‖ Aurora very bright.
¶ Reversing current.

S. S. DICKENSON, *Superintendent.*

On presenting the record, Mr. Stupart stated that the Earth Current values had been plotted on millimetre paper, together with the values of the magnetic elements, in order to determine whether the curves of the former resembled in any degree the magnetic curves. At a later date the result would be placed before the Society.

Mr. John A. Paterson, M.A., Vice-President, then read the following address:—

THE PROGRESS OF ASTRONOMY IN 1894.

The days of 1894 have closed their record,—the youngest born of all eternity, heiress of all the past ages and mother of all the future has given place to another year, and now in these earliest days of 1895 we turn back and ask the hand-maid Science what she has thought, what she has uttered, and what she has wrought for the betterment of the age we live in; and not only that, but let us also ask her what foothold she has cut for the upward climber, with what promise she has inspired the toiling student, and what guerdon she holds out for those who "scorn delights and live laborious days."

All discovery is not truth, and all knowledge is not wisdom. Scientific truth comes to us from the years gone by, sifted from the alloy of error as gold dust sparkling and precious is washed down from the rocks and filtered through the sands mingled with the debris of centuries. A great philosopher has left us this to ponder—"the inquiry of truth, which is the love-making or wooing of it,—the knowledge of truth, which is the presence of it, and the belief of truth, which is the enjoying of it, is the sovereign good of human nature." That was Bacon looking for wisdom, and not Bacon looking for the seals. To rest is to rust; to rust is to rot; when truth rots it becomes a lie, and the corruption of a false philosophy, be it moral, mental or scientific, is baneful, it is pestiferous, liberty hides herself and civilization retrogrades.

When we ask Truth to let us read that page of her record of the past year marked "Science," we find that she has not rested, and that, therefore, she has not perished; that she has been wooed by many a lover, and her presence has been the enjoyment of many a student, and thus the sovereign good of human nature has commanded a widespread loyalty. We deal with that department of Truth called scientific, and with that class of scientific truth called astronomical. What new principles, then, has 1894 demonstrated? What new evolution of former principles has she recorded? What new triumphs of astronomical research has she emblazoned on her banner?

Great, brilliant leaps of astronomical progress were made when the world welcomed Copernicus, Kepler, Newton, LaPlace, Adams, Fraunhoffer, the Herschells, Struve, Proctor, and others. The invention of the telescope, the spectroscope and the photographic plate mark great epochs in astronomical work. The history of astronomy has been more like a variable star, now flashing, then waning, but yet not like a variable star, but constantly and steadily increasing. This "variable" did not flash more brightly in 1894 than in some former years; it shone with a steady lustre and increasing strength, but with no startling radiance. The coral architects build deeply and strongly far beneath the sunlight—their foundations are laid to bear a crown of verdure and fruitful beauty in the far off future; we hail this crest of fertility, but we think little of the laboured structure down in the dark depths on which it stands. Students and observers of the stars may toil for years, nay, even for centuries, to reach some general law from the vastness and variety of calculation and observation, building and rebuilding, testing and retesting, receiving and rejecting; by laborious induction extracting some great conclusions from a bewildering multiplicity of details; and the rest of the world hails the crowning result, and thinks little of the structure of laborious lives, successes and defeats, cherished theories uprooted and truer propositions upbuilt in the buried depths of past years. The year 1894 has been a year of building, broadly and deeply building; not far hence may we see the pyramid of her labour and those of her sister years crowned with some glorious consummation. We must not expect to be garnering in splendid harvests of astronomical achievement yearly, else the cohorts of scientists, like so many philosophic Alexanders, would have no more problems to conquer, and lest, indeed, their observatories be deserted, their spectroscopes and sensitized plates rest and rust, and their telescopes show more cobwebs than the threads their artificers have placed therein.

LORD SALISBURY'S ADDRESS.

One of the scientific events of the present year is the address of the President of the British Association for the Advancement of Science, the Right Hon. the Marquis of Salisbury, Chancellor of the University of Oxford, on the "Unsolved Problems of Science." It bears largely on that as yet undiscovered country to whose borders many a scientist, from that ancient astronomer of the land of Uz down to the last of them has gone with girt-up loins and bouyant hope, and after gazing wistfully through the dim has returned baffled and weary of conjecture, but yet to go again and yet again, and reach out still farther and farther. He speaks much of our own special problems ; the triumphs of the spectrum analysis, but also its failure to discern the nature of the atom ; the nebular theory, but yet the want of nitrogen and oxygen in the Sun ; the unsuccessful quest after the mystery of ether, that imponderable vacuous entity. He might also have challenged a solution of the problem as to the nature of gravity. He tells us of the vast distance between the jelly fish, lying upon the primeval shore, and man as we know him now, and the feeble answer that "Evolution" makes when overborne by Kelvin's arithmetic, dealing with the age of organic life here from the necessary gradual refrigeration of the Earth. The address was a remarkable one, coming as it did from a lay scientist, a noble son of a hundred earls and a distinguished statesman, "conveying most unworthily," as he said, with a modesty that honoured the coronet he wore, "the voice of English science." Equally striking were the criticisms which it evoked. Lord Kelvin said, "throughout there was the spirit of the student, the spirit of the man of science." Professor Karl Pearson, on the other hand, addresses heart-cutting words to the nobleman and reviles his address at this "the great annual palaver of the scientific tribe," as he calls it, and speaks of it as conveying a message of despair and ignorance, which finds not the least justification in the facts. He accords Lord Salisbury the credit, as he thinks, of discarding the "old bigotry," by which he means the inspired character of the Old Testament, and then indicts him for advancing a new and a more dangerous bigotry. Moreover, he finds a deep political significance buried in its sentences—a cunningly devised conspiracy against the liberties of England. This is surely a new engine of politics. We have not yet reached that advancement in Canada when politicians use the pure atmosphere of scientific study to hatch out microbes wherewith to corrupt the liberties of the people. We would readily run that risk here, and are prepared for the experiment. Scientific discussion would surely illuminate with a heavenly light the dark meanderings of the professional politician. Imagine if we can a Government being forced out by an ante-election speech on the mystery of the atom, or the ether, or an inquiry how the nebular theory is consistent with the absence of nitrogen and oxygen in the Sun! A careful and unprejudiced reading of Salisbury's address brings us to far different conclusions; its undertone is no Ichabod lamentation, neither does it swell with any foolish enthusiasm. It is true he says "*Ignoramus*," but he does not mean "*Ignorabimus*" and so end it, but rather "*laborandum est*," and, it may be added, at the risk of offending Karl Pearson and being called "bigot," "*orandum est*," and so Science is called on to sweep on to higher reaches, to sound deeper depths and achieve higher triumphs. Napoleon said "There shall be no Alps," and his eagles soon appeared on the sunny plains of Italy. Salisbury is not so drastic; he says: "Here are the Alps of scientific difficulty,—let us fairly realize our position,—let us not imagine that we are upon the sunny plains of Italy until these ice cliffs are behind us, and then let us reach out our hands for the fruits of conquest." He evinces the true scientific spirit in that he stands by "the everlasting law of honour to face fearlessly every problem which can fairly be presented to Science." As to matters of belief that appertain not to sensation, the "*Ignoro*" of the true scientist must be followed by "*sed credendum est.*" Karl Pearson to the contrary, notwithstanding, we venture to affirm that.

A disciple of the "old bigotry" may yet be an evolutionist. The Creator with His omnipotent fiat may in an instant create a line of battle ship, with all its equipment, armament and panoply, and we would say, here is wondrous power. But if the Creator, not only with omnipotence, but with omniscience, creates a cockle-shell boat and devises and gifts it with some wondrous internal mechanism whereby the shell can in ages advance through all intermediate gradations and climb the scale of naval architecture until grown a monster and equipped with armour plates, and batteries, and decks, with its mighty engines in its heart, it rides the sea an "evolution"; then would we say, here is not only wondrous power, but here is wondrous wisdom—not only is Jupiter here with right hand clutching the emblems of power, but here is Minerva with her far-reaching, deep-sounding intellect. If "evolution" can be placed upon a solid, scientific basis (which, however, is much doubted), then no Christian man need fear for the foundations of his Christianity. Scientific truth is mighty and loveable, because, and only when God is behind her,—there is no valid antithesis between what man discovers and what God reveals. Says Dr. Josiah Strong, "Science discovers natural laws and processes, and if God is really the ruler of the Universe, the laws and processes of nature are only the Divine purposes and methods. Science is therefore as truly a revelation from God as are the Scriptures, as really a revelation of His will, as was the Decalogue and one which is to have as real a part in the coming of His Kingdom among men as the New Testament—God's will expressed in what we call natural law is as benevolent and as sacred as His will expressed in what we call moral law."

1. THE SUN.

"See the Sun !
God's crest upon His azure shield the Heavens."

From the dawn of history man has uplifted his wondering gaze to the Sun and sought to wrest from him his secret and write his story; slowly but surely are astronomers with busy brain and skilful hand plucking out the heart of his mystery. During 1894 sun-spot activity was especially active, in April, July and August. In contrast with 1893 auroræ have been numerous and brilliant, in January, February and March especially so. Dr. Veeder's work in North Greenland goes bravely on. Solar spectroscopy has made an emphatic advance. The well known researches of Langley on the heating effects of the spectrum, and particularly the infra red region, have been hitherto carried on by laborious micrometrical measurements of the deflections of the bolometer, but they are now being conducted automatically by a very ingenious instrument recently constructed by the Smithsonian Astro-Physical Observatory, and thus our knowledge of the invisible rays beyond the red, and the important part played by the Earth's atmosphere in absorbing them, will certainly be largely extended by its means, seeing that in a single day this apparatus will accurately record more details in the spectrum than can be obtained with a year's assiduous labour with the old method. This most wonderful mechanism may be thus briefly described :—The spectrum is made to slowly revolve by clock-work past the bolometer thread, which consists of a narrow strip of metal put in a position parallel to the lines on the spectrum, and which form part of an electric circuit. As the dark lines of the spectrum pass over it very small variations of temperature occur, and thus the electric resistance is altered, producing corresponding variations in the current passing through it, which in turn affect the needle of an extremely sensitive galvanometer; to this needle a minute mirror is attached, which reflects a ray of light upon a sensitized sheet of paper, and thus every movement of the needle is permanently recorded. Thus the Sun, that great mystery, is dominated by the intellect of man and compelled to write its own story, and, like a sentient being, discourses of its own nature. The solar prominences, those fascinating flames which previously could only be drawn in haste during the four or five fleeting minutes which the totality of an eclipse lasts, are now mapped out at will, and have been photographed day by day since 1891; especially in Italy has this work been carried to great perfection, the shell of the Sun has practically been taken to pieces and each layer separately photographed. Deslandres of Paris and Hale of Chicago seek the further triumph of photographing day by day, as well, the faint invisible *corona*, that mysterious lustrous agglomeration of matter which surrounds our bright star and extends two or three millions of miles into space—and most probably they will succeed. Deslandres follows the method of Wm. Huggins, the founder of the spectral analysis of the stars, which consists in absorbing all the luminous rays emanating from the Sun and its corona with the exception of the ultra-violet rays, and photographing the violet image thus obtained. On one of his photographs the Sun really appears surrounded by a "glory" sharply separated from the diffuse light of the sky, and most probably representing the *corona;* this, however, is difficult to test, as it is really the photograph of an invisible object. Professor Hale has made a further trial of his spectroscopic method of photographing the corona in full sunshine from the top of Pike's Peak and also from Mount Ætna. Last year there were two solar eclipses, one an annular on the 5th April, with a duration of from 30″ on the Eastern coast of India to less than a single second in North-western China; the other, on the 28th September, was total, with a maximum duration of not quite 2′, but its path lay across the Indian and Antarctic Oceans, and offered no good stations for observation.

The theory of sun-spots—the period of their appearances, their connection with the form of the corona and the magnetic disturbances observed on the Earth have compelled much observation during the past year.

2. JUPITER.

"Hail, mighty chief!"

Next to the Sun, the Jovian system commands our interest. The discussion—we were almost going to write "quarrel"—over the form of the satellites still rages. Professor Pickering from his Andean Observatory at Arequipa, surrounded by the purest atmospheric conditions, is certain he is right, that in fact the first satellite is a prolate-spheroid or lemon-shaped, and revolves end on end reversely to its orbital motion. The second is an ellipsoid of three unequal axes, like a squeezed lemon or fish-shaped, and revolves round her intermediate axis. The third, the larger, is orange-shaped, keeps her face always to the planet; the fourth is the brunette of the family (the rest are blondes), and orange-shaped like her sister next her, but revolves so that her edge is kept towards the planet, her progenitor. And thus they seem to be clouds of gaseous matter, and not solid globes, and writhe as they circle round their gigantic centre. E. E. Barnard, from Mount Hamilton, fails to confirm these observations, but admits that the satellites when crossing the disk appear oval at some part of their transit, which is occasioned by their poles being lost in the light of the planet, and so the middle part of the satellite, being all that is visible, appears egg-shaped and lying on its side.

Professors Holden and Schaeberle, and Burnham also, differ from the Peru observers, and present their views, as they say, "courteously, but very plainly." If astronomical questions are to be settled by the principle of majorities, then the much discussed satellites are surely circular. But Galileo was at one time in the minority very largely. Mayhap the observations at Jupiter's opposition on 22nd December may advance the truth, at least so Professor Pickering promised last June. Professor Holden says "everything is not yet settled with respect to Jupiter's satellite system."

3. A NEW STAR.

"A single star sparkles new-set in heaven."

The discovery of Nova Normæ affords another of the many examples of the remarkable value of photography as an aid to astronomical investigation. A spectrum plate taken at Arequipa was months after being examined by Mrs. Fleming at Harvard, when it was found to contain the impression of a new star of about the 7th magnitude. It afterwards, in May, fell to the 14th magnitude, and in June it had risen to the 10th magnitude. The spectrum at its first discovery showed the bright and dark lines, but later it dwindled to a nebula. The Nova Aurigæ of 1893, and the Nova Cygni of 1876, ran through the same evolution. Only five new stars have been discovered since the application of the spectroscope to astronomical investigation, and that three of those should have similar spectroscopic histories is a remarkable fact and pregnant with possibilities. A Mr. Bicherton, of New Zealand University, has a theory of new and variable stars, which may thus be concisely stated :—Let space be occupied by a vast number of dark bodies, almost all in motion ; some may be as large as the whole solar system ; two of them may come into collision, the impact being probably oblique. In this event, a portion common to the two will be broken off and remain. The two "wounded" stars may go off into space, or, controlled by their attractions for each other, or for the third star, may form a binary or multiple system. The middle part will be a temporary star, or may survive as a nebula or system. The "wounded" stars revolve on their axes, and, the side of collision being hotter than the rest, will present the phenomenon of variable stars. It would, therefore, be expected that variable stars should frequently occur in pairs, and that variable stars should remain where temporary stars have appeared. For this reason it is evident that variable stars might gradually lose their variability and become ordinary stars. But it is better to have many observations and few theories than to have few observations and many theories.

4. COMETS.

"And from his horrid hair shakes pestilence and war."

Kepler said "there are as many comets in the heavens as fish in the sea." The observers' net in 1894 has not been cast in vain. Two have been discovered, and two periodic comets have been re-found. Mr. W. F. Denning, of Bristol, England, has again demonstrated his skill and industry, and to the roll call of the planets has been added *a*, 1894, first seen on March 26th, a month after its perihelion passage. It is of short period, about eight years, and is inconspicuous in the telescope, the most noticeable of its features being the very eccentric position of the nucleus, and an apparently very short, broad, fan-shaped tail. Dr. Hind says this comet will approach the orbit of Jupiter; it may there share the fate of others of its family, either be broken up or be wheeled off in some mighty ellipse or hyperbola. The second, *b*, 1894, is claimed by Mr. Gale, of Sydney, N.S.W., first seen on 1st April, and in Europe visible to the naked eye in May. Professor Barnard, of the Lick Observatory, succeeded in developing beautiful photographs of this comet, and demonstrated the advantage of this way of observing the structure of the tail over ordinary observation through the eye-piece. On May 10th, Mr. Finlay, of the Cape of Good Hope Observatory, succeeded in finding Temple's comet, known as 1873 II. Encke's comet has also been detected both visually and photographically. In addition to this catalogue, it may be observed that Dr. Schaeberle recognized a comet near the Sun on the photographic plate of the eclipse of 16th April, 1893. This was for some time debated, until the record of the African photographic plate verified that of the South American, and so the "*coronal comet*" became a reality. For his discovery the Astronomical Society of the Pacific, over the names of Holden and Schaeberle, awarded the Comet Bronze Medal to Mr. Denning. But the Englishman, thinking, perhaps, that his fame as a comet finder was more enduring than American metal, declined the honour of the gift, which he rather contemptuously calls "a bit of bronze." We have not yet heard of any consequent international difficulties. As an example of dogged perseverance, it may be recorded that Denning's work of five new comets covered nearly 600 hours clear sky sweeping, which is equivalent to two months' work for two hours a night of unclouded weather for each comet. Yet another comet was discovered on 20th November by Edward Swift, by his 16-inch telescope, on the top of Echo Mountain, near Los Angeles, Cal. It was a very faint comet, and seemed to be moving in a path very like that of De Vico, known as 1844 I. If this turns out to be the truth, we have the phenomena of a bright comet lost for fifty years (whose proper period is about five and a-half years), appearing now shorn of much of its brilliance.

5. ASTEROIDS.

> "And teach me how
> To name the bigger lights, and how the *less*
> That burn by day and night."

The number of the asteroids that are dodging about between Mars and Jupiter is becoming embarrassing. Before the application of photography to star hunting the asteroid population could be kept within reasonable limits ; but now we know not if a thousand or more of these fifteen or twenty mile worlds may not yet be accurately catalogued. The old order of things was tedious chart-making, and star-by-star comparisons of chart and sky, and so strangers were discovered in the upper galleries. But now a photographic plate is strapped to a telescope covering a large field of view. The plate is exposed for some hours, and by clock-work kept in motion at the same rate as the diurnal rotation of the heavens. Then scrutinize the plate. Each one of the many thousand stars will appear as a round defined white dot. But look again : a white line marks the plate, and lo, an asteroid ! Its own inherent motion produces a white line ; but, let us pause ; he may be an old friend, and so let his place be compared with former catalogues ; and he may now be a little stranger ; if so, he is recorded by his proud discoverer and duly named by his glad god-father. And thus it is that these wandering children of the Sun are by the chemical effect of sunlight on the photographic film picked out from the bewildering maze of star, and nebula, and comet, entered on the roll of the solar system ; and so another strip of the unknown desert of the sky is fertilized by the work of the astronomer, and Science has planted her foot one step farther on.

Charlois Wolf and Courty are the principal workers at this research. At least twenty have been discovered in 1894.

6. THE R. A. S. MEDAL.

"Palmam qui meruit ferat."

The gold medal of the Royal Astronomical Society was awarded to Mr. S. W. Burnham for his discovery and measurement of double stars. This gentleman has had a very distinguished career as an astronomer ; beginning life as a stenographer, he became director of the Chicago Observatory, and, on the establishment of the Lick Observatory, was appointed to a post therein. He now holds the position of Professor of Practical Astronomy at Chicago University. Of double-star observers the world over he is *facile princeps.* The Royal Astronomical Society in awarding to him the highest distinction it could confer, acknowledged through Captain Abney, its President, the honour which it reflected on the Society.

7. TRANSIT OF MERCURY.

"First in his silver circle wheels
Fair Maia's swiftly flying son."

True to its hour, on 10th November, 1894, as predicted by Ferguson, the Scottish astronomer, more than 100 years ago, Mercury appeared on the disc of the Sun, and, true to its law, made its transit. The phenomenon is not one of great importance, but its rarity gives it consequence, and it is useful to verify and correct astronomical tables and to elucidate some problems relative to the planet's atmosphere. There is not, as a rule, very much interest taken in the younger brother, but when Venus, the elder sister, makes her periodic sweep across her parent's face, then truly the astronomical world palpitates with excitement to watch what particular path she will take, so that Terra, the next older sister, may know how far she has been placed from her fiery sire.

Mr. Henry C. Mayne, a writer on sun-spots, from Rochester, observed a brownish, transparent, oval shadow just in advance of the planet. Tests were made with the lenses to secure perfect optical alignment, and so make sure there was no optical illusion. The possibility presents itself of a large gaseous satellite of Mercury picked up by him in its rapid revolution round the Sun, not dense enough to cut off the sunlight completely. It is not, however, recorded how far this satellite moved during the six hours of the transit.

8. MARS.

"The Warrior of the skies."

The opposition of Mars in the latter part of October brought round in their orbit, as regular as that of Mars, the swarm of newspaper meteorites, wherein the editorial "we" and the unknown quantity "they" did duty as most veracious chroniclers of the most wondrous phenomena. Our forefathers ascribed a peculiar influence of the full Moon upon not overly strong-minded people, but in these later days it would seem that such an adverse influence should be transferred to Mars, and our asylums should be called "Martian" and not the other kind. This was especially so in the last opposition in 1892, when the press became so gorged with nursery tales about Mars, the fiery-eyed, that the Lick Observatory people were compelled to publish a statement that no notice was to be taken of any reported observation from Mount Hamilton unless it was authenticated by the signature of one of the astronomers. This year the "silly season" recurred in full force. We have heard of a class of citizen in Toronto called "Aqueducter." It is a small and privileged class; but in Mars every one must be an "Aqueducter," and they can dig canals as large as the Red Sea or the Gulf of California; and not only that, but they have the insolence to flash bright signals to their less favoured brethren on this Earth, and so glorify their work and jeer at the small success attending that of terrestrial toilers. Brightly flashing points have doubtless been seen beyond the general line of the terminator, but they are doubtless occasioned by the Sun's setting or dawning rays gilding a vast extent of cloud floating at a great height in the Martian atmosphere, if such there is, or irradiating a mountain peak rising with steep slope from the surrounding plains. The question of the existence of a Martian atmosphere drew forth much discussion in the 1894 journals. The spectroscope has been consulted, and its answer is that it cannot detect any appreciable atmosphere. The problem is soluble by a comparison of the Martian and Lunar spectrum taken when these bodies are at the same altitude, when the telluric lines would have the same influence in each case. Mr. Campbell, of the Lick Observatory, finds they are precisely similar. We know the Moon has no appreciable atmosphere, and so therefore neither has Mars. But what then are we to do with the phenomena of the snow-cap, with its diameter of 47 degrees last June and its gradual disappearance, bordered by a dark streak, at least 100 miles in width, perhaps a polar sea formed by the melting Sun? With Mars the critical speed is about three miles a second, and with that he can, like the Earth, hold on to his oxygen and water vapour. The summation of the evidence would justify a conclusion that the atmosphere at the surface of the planet is about half as thin as ours is on the summit of the Himalayas, but more heavily charged with watery vapour, and such that no terrestrial lung could successfully breathe as a steady occupation. Before we leave Mars we would call attention to Flammarion's ingenious suggestion that at the proper relative position of the three bodies the Sun's disc should be mirrored in the lakes of Mars and be seen by an observer on the Earth.

9. THE MOON.

"Swear not by the Moon, the inconstant Moon,
That monthly changes in her circled orb."

In September, 1894, for the first time in nearly nineteen years, the Harvest Moon appeared in her full splendour. The Moon's ascending node was very near the vernal equinox on the 14th September, the date of the full Moon, and consequently the inclination of her orbit to the horizon was very nearly the same as if the node were exactly at the equinox, and therefore the inclination was a minimum. Hence, the intervals between the times of the Moon's rising near the September full Moon were a minimum. The average interval between the Moon's rising on successive nights is about 51 mins., but for a week, between 13th September and 20th September, it was only about 25 mins.

10. UNIFICATION OF TIME.

In the onward march of astronomical events and discussion, the part that our own Society has taken must not be overlooked. To ourselves she is "a little, but a dear one." We are not rich in professional scientists or in astronomical renown, but when we recollect what successful work patient amateurs have done in the past and what great discoveries small telescopes have made, let us not despair. Dr. Sandford Fleming suggested a unification of the Civil and Astronomical Day, and our Society, aided by the Canadian Institute, has taken a leading part in the movement for effecting that change. It must be remembered that the civil day begins at midnight and ends at midnight following; the astronomical day begins at noon of the civil day and continues until the following noon; the nautical day commences at the preceding noon and ends at noon of the civil day; and thus any given date extends over or into three different days. In May, 1893, a circular was sent out to astronomers of all nations, inviting replies to the following questions:—"Is it desirable, all interests considered, that on or after 1st day of January, 1901, the astronomical day should everywhere begin at mean midnight?" The last of the replies came in December, 1894, from Signor Denza, Chief Astronomer of the Vatican Observatory, signifying his assent to the change. This made 171 replies in all, of which 108 were in the affirmative and 63 in the negative. By countries they voted as follows:—For the change—Austria, Australia, Belgium, Canada, Columbia, England, France, Greece, Italy, Ireland, Jamaica, Madagascar, Mexico, Roumania, Russia, Scotland, Spain, and United States of America—18 in all. Against the change—Germany, Holland, Norway, and Portugal—4 in all. The eighteen countries in favour of the change represent 85 per cent. of the tonnage of the world's marine. His Excellency the Governor-General of Canada, was asked to bring the whole matter to the attention of the Imperial Government, so that some common international understanding might be reached. The British authorities desire to know whether the observatories who publish nautical almanacs desire the change, and accordingly the British Ambassadors of foreign Courts have been requested to bring the matter to the notice of the several Governments to which they are accredited, and report. The coming change in 1901 is on the highway of progress. The Davidic sling once more moves the old Goliath of scientific conservatism. Modest Toronto points the way of unity and simplicity to the kingdoms, empires and thrones of the Earth. The housewife in the kitchen, the sailor in his cabin on the Atlantic, and the astronomer in his observatory, sweeping the greater Atlantic of space, can be, and should be, regulated by one time system. Conservation of areas is scientific, so, too, is conservation of energy, but verily conservation of time is not.

Before closing this reference to our own Society's work, we would further record that the torch of astronomic study has, during the past year, been lighted at the town of Meaford, where a flourishing Society has been formed, and bears affiliation to our own.

11. ARE OTHER WORLDS INHABITED?

> "Who can believe that the Great Architect
> With all these fires the heavenly arches decked
> Only for show?"

But to turn for a moment or two from the rigorously scientific to what borders upon the romantic, but is also scientific. The imagination of the astronomer still finds much to feed upon in the intensely interesting question of the inhabitability of other worlds. Sir Robert Ball finds time to discuss it most scientifically in a recent number of the *Fortnightly Review*, and we have Camille Flammarion writing a clever conceit under the heading "Can organic life exist in the solar system anywhere but in the Planet Mars?" being a letter from a citizen of Mars, found in a meteorite, wherein it is conclusively proved that only in Mars can there be life; that the most elementary common sense teaches that the other planets are either too near or too far from the Sun, and that "our own is alone at the golden mean * * * What would they make of our seventeen senses? The sense of magnetic direction will certainly be wanting, and their dull intelligence will surely be incapable of communicating by psychic force alone. Unquestionably they can enjoy no more than five or six senses."

The Creator made man for wise purposes a restless enquirer. For ages scientist has reasoned and poet has dreamed as to this problem, for man is a gregarious animal, and his social desires are not bounded by this globe. A voice comes from Westminster Abbey, the ever eloquent Laureate, of whose words death cannot rob humanity—

"Venus near her! Smiling downward at this earthlier earth of ours,
Closer on the Sun, perhaps a world of never fading flowers.
Hesper, whom the poet call'd the Bringer home of all good things;
All good things may move in Hesper; perfect peoples, perfect kings.
Hesper—Venus—were we native to that splendour, or in Mars,
We should see the globe we groan in fairest of their evening stars.
Could we dream of wars and carnage, craft and madness, lust and spite
Roaring London, raving Paris in that point of peaceful light?
Might we not in glancing heavenward on a star so silver fair,
Yearn, and clasp the hands, and murmur "Would to God that we were there!"

12. THE CYPRESS BLOOMS ABOVE THE LAUREL.

"Twilight and evening star
And after that the dark."

The Father gives to every man to read in the Star book of Nature that which He has written, and to reveal what is still unread in these "Manuscripts of God"; and when the man has read these marvellous tales and interpreted them to his less gifted fellow man, then the Father calls him home to read other volumes with brighter vision and clearer mind. Among those called upward last year we have to record A. Cowper Ranyard, the editor of *Knowledge*, who succeeded Richard Proctor, and caught his inspiration. He finished for him his great work, "Old and New Astronomy." Peters, the astronomer, also ended his earthly labours. Both were painstaking men and eminent for their research. The great Italian astronomer, Signor Denza, of the Vatican

Observatory, in December last, and not long after he had written his assent to the plan for unifying terrestrial time, as above noted, passed away to where time and eternity are united.

We bear in our saddened memories the fact of our own President and friend, Charles Carpmael, having passed away. A full record of this will appear elsewhere in our Transactions for the past year.

Science lost one of her most gifted sons on the 9th September last, when Herman Ludwig Ferdinand Von Helmholtz died in the 74th year of his age. He was not an astronomer in the narrow sense of the term, but in the department of Physics he shone with great lustre. On his 70th birthday the crowned heads of Europe and the scientific institutions of the world vied in conferring honours upon him, at a great international demonstration. "Science," says *Nature*, "has had few investigators who have furthered her interests more than Helmholtz. He was constantly exploring new fields of research or bringing his keen intellect to bear upon old ones. With his contributions he helped to raise science to a higher level."

THE FUTURE.

The year 1895 is now with us, and astronomers are still building upwards to the sunlight of perfect knowledge, and slowly conquering inch by inch the unknown, which is not unknowable. The myriad songs which Nature sings to all her children are being year by year more clearly interpreted, and the jarring discord of error is being separated from the Creation's diapason of praise. What not long ago was insoluble is now taught to our children in the schools as a matter of common knowledge. The records of the sky are year by year more truly read, and the mountain tops commence to glow with the dawn of coming triumphs.

> "Build to-day, then, strong and sure,
> With a firm and ample base,
> And ascending and secure
> Shall to-morrow find its place.
> Thus alone can we attain
> To those turrets where the eye
> Sees the world as one vast plain
> And one boundless reach of sky."

OBITUARY.

Charles Carpmael.

CHARLES CARPMAEL, M.A., F.R.A.S., F.R.S.C., was born September 19th, 1846, at Streatham Hill, Surrey, England, and was educated at Clapham Grammar School. He gave his attention particularly to the study of mathematics and natural science, and in 1865 obtained a scholarship at St. John's College, Cambridge, going into residence in that institution in the same year.

In 1868, he obtained a foundation scholarship, and in January, 1869, entered for the mathematical tripos, and was classed sixth in the list of wranglers. In 1870, Mr. Carpmael was elected Fellow of St. John's College, and in the same year accompanied the British Eclipse Expedition to Spain. He first visited the United States and Canada in 1871, and remained until 1872. This visit led him to ultimately settle in Canada.

On the superannuation of Prof. Kingston, in 1876, Mr. Carpmael was appointed Director of the Magnetic Observatory, at Toronto, and Superintendent of the Dominion Meteorological Service, and to him was very largely due the great efficiency of the Weather Bureau.

In 1876 he was married to Julia, youngest daughter of the late Walter McKenzie, of Castle Frank, Toronto. Mr. Carpmael was a member of the first Council of the Royal Society of Canada, appointed by the Marquis of Lorne, and in 1886, became President of Section 3 of that Society. In 1888 he was elected President of the Canadian Institute, and in 1890, at the earnest request of the gentlemen who were then seeking incorporation for The Astronomical and Physical Society of Toronto, he accepted the Presidency of that Society, which office he held until his decease.

For some time before Mr. Carpmael's death his health had become impaired and he was advised to spend some time in the South of England, in the hope of ultimately recovering.

The journey was made but did not result in improvement, and on October 21st, 1894, news of his death, which had taken place at Hastings, Eng., was cabled to Toronto. Mr. Carpmael's loss was most severely felt, and particularly by those, young in scientific study, who had enjoyed the pleasure of his kindly aid and genial personality.

Almost his last act in connection with The Astronomical and Physical Society was the arranging for systematic observations of magnetic disturbances, more especially in connection with Earth Current phenomena. A memorandum on the subject appears in the report for 1894. A likeness of Mr. Carpmael faces the title page of this volume.

UNIFICATION

OF

THE ASTRONOMICAL, CIVIL, AND NAUTICAL DAYS.

Report of the Joint-Committee of The Canadian Institute and The Astronomical and Physical Society of Toronto.

COMMITTEE:

SANDFORD FLEMING, C.M.G., LL.D., C.E.—*Chairman.*

ARTHUR HARVEY, ESQ.
GEORGE KENNEDY, M.A., LL.D.
ALAN MACDOUGALL, M. INST., C.E.
CHARLES CARPMAEL, M.A., F.R.A.S.
JOHN A. PATERSON, M.A.
G. E. LUMSDEN, ESQ.

The Joint-Committee, appointed by The Canadian Institute and The Astronomical and Physical Society of Toronto, have the honour to report on that branch of the subject of Time-reckoning specially referred to them.

The unification of the reckoning of the day has long been under consideration. Sir John Herschell, in his "Outlines of Astronomy," alluded to the advantages which would result from bringing into agreement the Civil, the Astronomical, and the Nautical Days. He pointed out that the adoption of the Civil Day for Astronomical purposes would but slightly inconvenience Astronomers, and that in a question which concerns all other classes of men, Astronomers should resolve to act on general principles and cheerfully submit to a small inconvenience in view of the far wider interests which would be benefited. "Uniformity," he said, "in nomenclature and mode of reckoning in all matters relating to time, space, weight, measures, etc., is of such vast and paramount importance in every relation of life as to outweigh every consideration of technical convenience or custom."

The Civil Day begins at midnight and ends at the midnight following. The Astronomical Day begins at noon of the Civil Day and continues until the following noon. The Nautical Day concludes at noon of the Civil Day, having commenced at the preceding noon.

It is obvious that any given date extends over, or into, three different Days. Take for example, Wednesday, June 13th. By Astronomical and Nautical reckonings, only half of this date in each case is on Wednesday; the first half of June 13, according to Nautical reckoning, is on Tuesday, June 12, while the second half of the same date, (June 13th), according to Astronomical reckoning, is on Thursday, June 14th, Civil Time.

In this we have the elements of confusion, and it is not surprising that The Washington International Conference of 1884 recommended that the Civil Day should take the place of the Astronomical and Nautical Days for all purposes. The recommendations of the Washington Conference must be held to carry weight, as this assembly comprised representatives of science from twenty-five nations specially called together to consider questions of Time-reckoning. Among them were Astronomers of world-wide fame, as well as men who held high rank as navigators. They were unanimous in the opinion that as soon as practicable the Astronomical and Nautical Days should be arranged everywhere to coincide with the Civil Day.

The Civil Day is the reckoning used by the generality of mankind. It is the exact mean between the Astronomical and Nautical Days, and differs precisely twelve hours from both. To effect a complete coincidence, it is only necessary to shift Astronomical and Nautical Days each twelve hours, and this shifting will bring both to the Civil Day. Many ships have already abandoned Nautical Time and date their logs according to Civil reckoning; all ships would use the one reckoning only, if the Nautical Almanac and Ephemerides generally were arranged for Civil Time. There can be no doubt whatever that the marine of all nations would benefit by the change.

If we consider the subject simply in its relation to the Nautical Almanac and Navigation, the unification of Time-reckoning would simplify the calculations of mariners and reduce the chances of error. One correspondent (Dr. Johnston of McGill University) points out very truly "that the omission of even a single step in an oft-repeated process of calculation has an obvious advantage; when the simplification removes at the same time that most dangerous source of error, an ambiguous expression, it becomes a great gain." He says that the subject resolves itself unto a question of practical utility, viz.,what is the greatest good of the greatest number? The Nautical Almanac, as its name implies s for the use primarily of navigators, who are very numerous and yearly, increasing. Compared with the men who guide the floating tonnage of the world, astronomers are extremely few in number, and Astronomers as a class are skilled calculators; moreover, Astronomers can make their calculations under the most favourable circumstances, consequently with the least liability to error, as they are removed from the disturbing influences to which seamen are frequently exposed.

The Joint-Committee considered it important to ascertain how far Astronomers generally would support the proposal which would practically abolish the Astronomical Day. On April 21st, 1893, a Circular was issued to Astronomers of all nations, inviting replies to the following question, viz.: "Is it desirable, all interests considered, that on and after the first day of January, 1901, the Astronomical Day, should everywhere begin at Mean Midnight?" The Circular was sent to every Astronomer whose name appears in the general list of Observatories and Astronomers prepared by Mr. Lancaster, of the Royal Observatory of Brussels, with the following result. 171 replies in all have been received, a complete list of which is appended; of these 108 are in favour and 63 are not in favour of the proposed change. Many of the former are strongly and earnestly in favour of the adoption of the Civil Day for Astronomical purposes, while the writers of some of the latter seem to have been under a misapprehension. They object to the adoption of the Civil Day on the ground that its division into two series of 12 hours, designated A. M. and P. M., would be inconvenient for Astronomers. It is obvious that this objection has no weight, as the 24 Hour-notation would remain associated with Astronomical reckonings as at present; moreover, indications are not wanting that the Astronomical practice of counting the hours in a single series from 1 to 24, will gradually win its way into general favour in civil life. The 24 Hour-notation has already been introduced into use over wide districts in Canada, in the whole of Italy, and throughout the Indian Empire, and there is a movement in Europe, in Australia, as well as in the United States of America, especially among Railway men, to bring this mode of reckoning the hours into general use.

In classifying the replies from Astronomers according to the countries from which they have been received, the votes for or against the change, stand as follows:—

IN FAVOUR OF THE CHANGE.

AUSTRIA.	AUSTRALIA.	BELGIUM.
CANADA.	COLOMBIA.	ENGLAND.
FRANCE.	GREECE.	ITALY.
IRELAND.	JAMAICA.	MADAGASCAR.
MEXICO.	ROUMANIA.	RUSSIA.
SCOTLAND.	SPAIN.	UNITED STATES.

UNFAVOURABLE TO THE CHANGE.

GERMANY.	HOLLAND.
NORWAY.	PORTUGAL.

According to this classification of the Astronomers heard from, those of eighteen countries are in favour, and those of four are against, the adoption of the recommendations of The Washington International Conference of 1884 with respect to the Astronomical and Nautical Days. If we compare the shipping of the countries thus classified, (and the shipping has an important relation to the Nautical Almanac), we find that the first list, that is to say, the countries in favour of adopting the Civil Day for Astronomical purposes, represents $\frac{17}{20}$, or 85 per cent., of the tonnage of the world's marine.

Thus it appears that there is a preponderating weight of opinion among Astronomers themselves, that a change should be made in the Astronomical Day. The Joint-Committee, therefore, feel warranted in recommending that the Home Authorities be informed of the facts and that a respectful appeal be made to have the Nautical Almanac adapted to the change, proposed to take effect at the beginning of the coming Century. The Joint-Committee are of opinion that the proper course is to lay before His Excellency the Governor General a respectful Memorial asking His Excellency to bring the whole matter to the attention of the Imperial Government in order that some common international understanding may be reached, by which all nations shall assent to the change; and in order that the Nautical Almanac which has to be prepared four or five years in advance may be made conformable to the change.

All which is respectfully submitted.

SANDFORD FLEMING.

Chairman,

Joint-Committee of The Canadian Institute and The Astronomical and Physical Society of Toronto.

Toronto, 10th May, 1894.

Replies received to the following Question sent April 21st, 1893, to the Astronomers of all Nations:—

"Is it desirable, all interests considered, that on and after the first day of January, 1901, the Astronomical Day should everywhere begin at Mean Midnight?"

Name.	Observatory, Etc.	Place.	Country.	Ans.
Abbe, Cleveland....	United States Weather Bureau	Washington	United States ...	Yes.
Anguiano, Angel...	National Astronomical Observatory	Tacubayo.......	Mexico........	Yes.
Anton, Dr. Ferdinand	Marine Observatory........	Trieste	Austria.........	Yes.
Arcimis, A. F.....	Meteorological Institute	Madrid..........	Spain	Yes.
Ashley, Miss Mary..	Private Observatory.......	Bath	England	No.
Auwers, Dr. A.....	Academy of Science........	Berlin..........	Germany	No.
Backhouse, F. W..	Private Observatory	Sunderland	England........	Yes.
Bacon, Chas. A....	Smith Observatory.........	Beloit, Wis.	United States...	Yes.
Bardwell, Elizabeth.	Mount Holyoke College Observatory......	S. Hadley, Mass.	United States...	Yes.
Barnes, Willis S....	Private Observatory........	Charlestown, Ind.	United States...	Yes.
Bauschinger, Dr. J..	Royal Bogenhausen........	Munich, Bavaria.	Germany	No.
Becker, Prof. Dr. E.	University Observatory.....	Strasburg.......	Germany	No.
Böe, A. de........	Private Observatory........	Antwerp	Belgium	Yes.
Börgen, Prof. Dr. C.	Marine Observatory..... ..	Wilhelmshaven .	Germany	Yes.
Braun, Dr. Chas. ..	Kalocsa Observatory.......	Kalocsa, Hungary	Austria.........	Yes.
Brown, M. V	McKim Observatory..	Greencastle, Ind.	United States...	No.
Bruns, Dr. H..... .	University Observatory.....	Leipzig	Germany	No.
Burckhalter, Chas.	Chabot Observatory..... ..	Oakland, Cal. ..	United States...	Yes.
Carpmael, Chas....	The Observatory	Toronto	Canada.........	Yes.
Chambers, G. F....	Northfield Grange Observatory....................	Eastbourne	England	Yes.
Chree, Chas.	Kew Observatory..........	Richmond	England	Yes.
Christie, W. H. M.	Royal Observatory.........	Greenwich	England	Yes.
Cobb, John N......	....	Philadelphia	United States...	Yes.
Colton, A. L.......	Lick Observatory..........	Mount Hamilton, Cal...........	United States...	No.
Combe, F. P.......	Royal Observatory.........	Tananarivo	Madagascar	Yes.
Comstock, Geo. C..	Washburn Observatory.....	Madison, Wis. ..	United States...	No.
Contarino, Francesco	Capo di Monte............	Naples..........	Italy	Yes.
Deichmuller, Prof. Dr. F..........	University Observatory.....	Bonn	Germany	No.
Deville, E.........	Surveyor General..........	Ottawa	Canada.........	Yes.
Egnitis, D.........	Royal Observatory	Athens	Greece	Yes.
d'Engelhardt, Dr. Baron	d'Engelhardt Oservatory....	Dresden	Germany	No.
Epstein, Dr. Th....	Private Observatory........	Frankfort, A. M.	Germany	Yes.
Esmond, Darwin W.	Geraldine Observatory......	Newburgh, N.Y.	United States...	Yes.
Ewell, Marshall D..	Private Observatory........	S. Evanston, Ill..	United States...	Yes.
Fenyi, J...........	Haynald Observatory	Kalocsa, Hungary	Austria.........	Yes.
Fergoler, Em......	Capo di Monte Observatory.	Naples	Italy..........	No.
Flint, A. L........	Washburn Observatory	Madison, Wis. ..	United States...	No.
Folie, F...........	Royal Observatory	Uccle	Belgium	Yes.
Fulton, Robt. B...	University Observatory.....	University, Miss.	United States...	Yes.
Fuss, V.	School for Pilots..........	Kronstadt	Russia	Yes.

Name.	Observatory, Etc.	Place.	Country.	Ans.
Galle, Dr. Andreas	Royal Institute of Geology	Potsdam	Germany	No.
Galle, Dr. J. G.	University Observatory	Breslau	Germany	No.
Gaudibert, C. M.	Private Observatory	Vaison	Paris	Yes.
Gautier, R.	Geneva Observatory	Geneva	Switzerland	No.
Gedeonow, D.	Astro-Physical Observatory	Tashkend	Russia	Yes.
Geelmuyden, Dr. H.	University Observatory	Christiania	Norway	No.
Giacomelli, Dr. Fr.	Capitol Observatory	Rome	Italy	Yes.
Giovannozzi, Dr. G.	Ximenian Observatory	Florence	Italy	Yes.
Glauser, J.	Railway Engineer	Zurich	Switzerland	Yes.
Gogow, Prof. Cons.	University Observatory	Bucharest	Roumania	Yes.
Gonzales, José M.	Flammarion Observatory	Bogota	Colombia	Yes.
Gore, J. Ellard	Private Observatory	Ballysodare	Ireland	Yes.
Gruss, Prof. Dr. G.	Imperial Observatory	Prague	Bohemia	No.
Hadden, David E.	Private Observatory	Alta, Iowa	United States	Yes.
Hall, Maxwell	Government Meteorologist	Montego Bay	Jamaica	Yes.
Hanig, Dr. C.	Hamburg Observatory	Hamburg	Germany	No.
Hartwig, Dr. Ernest	C. Rameis Observatory	Bamberg, Bavaria	Germany	No.
Harzer, Prof. Dr. Paul	Ducal Observatory	Gotha	Germany	No.
Hastings, Chas.	Yale University Observatory	New Haven, Conn	United States	Yes.
Haywood, John	Otterbein " Observatory	Westerville, Ohio	United States	Yes.
Hess, F.	Private Observatory	Fort Dodge, Iowa	United States	Yes.
Holden, Dr. E. S.	Lick Observatory	Mount Hamilton, Cal	United States	No.
Hopkins, B. J.	Private Observatory	London	England	Yes.
Horr, Dr. Asa	Private Observatory	Dubuque, Iowa	United States	Yes.
Hoxie, Capt. R. L.	Field Observatory	Willets Pt., N.Y.	United States	Yes.
Jacoby, Harold	Columbia College Observatory	New York	United States	No.
Johnson, Rev. S. J.	Private Observatory	Bridport	England	Yes.
Johnston, Alex.	McGill University	Montreal	Canada	Yes.
Kammerman, A.	Geneva Observatory	Geneva	Switzerland	No.
Kirk, Ed. Bruce	Private Observatory	Barrhead	Scotland	Yes.
Knobel, Ed. B.	Late President Royal Astronomical Society	London	England	Yes.
Knopf, Dr. Otto	Grand Ducal Observatory	Jena, Saxe Weimar	Germany	Yes.
Kobold, Dr. H.	University Observatory	Strasbourg	Germany	No.
Kortazzi, J.	Naval Observatory	Nicolaïeff	Russia	Yes.
Kreutz, Prof. Dr. H.	Royal Observatory	Kiel	Germany	No.
Krone, Herman	Royal Technical School	Dresden	Germany	Yes.
Krueger, Prof. Dr. A.	Royal Observatory	Kiel	Germany	No.
Küstner, Dr. F.	" "	Bonn	Germany	No.
Laughton, J. K.	Royal Naval College Observatory	Greenwich	England	Yes.
Ledger, Rev. E.	Gresham College Observatory	London	England	Yes.
Legge, Dr. Alf. di	Capitol Observatory	Rome	Italy	Yes.
Lehmann, P.	Royal Observatory	Berlin	Germany	No.
Leite, Duarte	Polytechnical Academy	Porto	Portugal	No.
Lenahan, Henry A.	Sydney Observatory	Sydney	Australia	Yes.
Lewis, Thomasi	Royal Observatory	Greenwich	England	Yes.
Lindelöf, Dr. L.	Counsellor of State	Helsingfors, Finland	Russia	Yes.
Lohse, Dr. O.	Astro-Physical Observatory	Potsdam	Germany	Yes.
Lorentzen, Dr. G.	Rameis Observatory	Bamberg	Germany	No.
Mayer, Lt. Chas.	Private Observatory	Parenzo	Austria	No.
Mazelle, Edouard	Marine Observatory	Trieste	Austria	Yes.

NAME.	OBSERVATORY, ETC.	PLACE.	COUNTRY.	ANS.
Micknik, H	University Observatory.....	Breslau.........	Germany	No.
Monnichmeyer, Dr. C	University Observatory.....	Bonn	Germany	No.
Naccari, Prof. Dr. Joseph	Naval Observatory.........	Venice	Italy	Yes.
Niesl, Prof. G. von.	Technical University Observatory..................	Brünn, Moravia .	Austria	No.
Niesten, L.........	Royal Observatory..........	Uccle	Belgium	Yes.
Nobile, A.	Capo di Monte Observatory.	Naples	Italy	No.
Noble, Capt. Wm..	Private Observatory.........	Mansfield, Uckfield	England.........	Yes.
Numsen, W. H....	Denmore Observatory	Baltimore, Ind...	United States ...	Yes.
Nyrien, M..........	Imperial Observatory	Pulkova........	Russia	No.
Oppenheim, Prof. Dr. H	Private Observatory........	Berlin..........	Germany	Yes.
Oudemans, Prof. J. A. C...........	University Observatory.....	Utrecht	Holland........	No.
Parkhurst, Henry M	Private Observatory........	Brooklyn, N.Y..	United States ...	Yes.
Pasquier, Prof. Dr. E. L. J.	Royal Observatory...... ..	Louvain........	Belgium	Yes.
Pavey, Henry A ...	Private Observatory........	Hillsboro', Ohio.	United States...	Yes.
Penrose, F. C......	Coleby Field Observatory...	Wimbledon......	England.........	Yes.
Peter, Dr. B.......	University Observatory.....	Leipzig..........	Germany	No.
Peters, Prof. C. F. W.	" "	Kœnigsberg.....	"	No.
Pettit, H..........	Private Observatory...	Belmont, Ont ...	Canada..... ...	Yes.
Pittei, Dr. Constantine	Royal del Museo..	Florence	Italy...........	Yes.
Plassman, J........	Professor of Astronomy	Warendorf, Westphalia.........	Germany	No.
Pluvinel, Ay de la Baume..........	Meudon Observatory.......	Paris...	France	Yes.
Pond, Lt. Chas. F	Navy Yard	Mare Island, California	United States ...	Yes.
Porro, F.....	University..................	Turin	Italy...........	Yes.
Pritchett, H. S......	Washington University Observatory..................	St. Louis, Mo ..	United States....	No.
Quimby, Alden W..	Private Observatory........	Philadelphia	United States ...	Yes.
Quintana, —	National Astronomical Observatory............	Tacubayo..	Mexico	Yes.
Rambaut, Prof. A. A.	Dunsink Observatory..... .	Dublin..........	Ireland	Yes.
Rechenberg, G.....	University Observatory	Breslau	Germany	No.
Renz, F...........	Imperial Observatory	Pulkova........	Russia	No.
Rey, F. R	National Astronomical Observatory..................	Tacubayo.......	Mexico.........	Yes.
Riggenbach, Prof. Dr. A...........	Bernoulliam Observatory..	Basle..........	Switzerland	Yes.
Riggs, Joseph......	Creighton Observatory......	Omaha, Neb....	United States ...	Yes.
Rivero, F. D	National Astronomical Observatory............	Tacubayo.......	Mexico.........	Yes.
Rizzo, Dr. J. B .	University Observatory.....	Turin	Italy............	Yes.
Roberts, Isaac......	Private Observatory.........	Crowborough....	England	Yes.
Rockwell, Chas....	Private Observatory.........	Tarrytown, N.Y.	United States...	No.
Romberg, Hermann.	Imperial Observatory.......	Pulkova........	Russia,.........	No.
Safarik, Dr. A....	Bohemian University Observatory..................	Prague.........	Austria.........	No.
Scherbner, Prof. Dr. W.............	Professor of Mathematics...	Leipzig.	Germany	No.

NAME.	OBSERVATORY, ETC.	PLACE.	COUNTRY.	ANS.
Schiaparelli, J. V.	Royal de Brera	Milan	Italy	No.
Schorr, Dr. Richard	Hamburg Observatory	Hamburg	Germany	No.
Schur, Prof. Dr. W.	Royal Observatory	Göttingen	"	No.
Searle, G. M	Catholic University Observatory	Washington	United States	Yes.
Serviss, Garrett P.	Private Observatory	Brooklyn	United States	Yes.
Seyboth, J.	Imperial Observatory	Pulkova	Russia	No.
Sidgreaves, Walter	Jesuit College Observatory	Stonyhurst	England	No.
Smith, H. L.	Hobart College Observatory	Geneva, N. Y.	United States	Yes.
Solar Physics Committee, *per* Capt. Abney	South Kensington Department of Science	London	England	Yes.
Stechert, Dr. C.	Hamburg Observatory	Hamburg	Germany	No.
Sternock, Lt.-Col. R. von	Military Institute	Vienna	Austria	Yes.
Stockwell, John	Private Observatory	Cleveland, Ohio	United States	No.
Stone, E. J	Radcliff Observatory	Oxford	England	No.
Stoney, G. Johnstone	Formerly Assistant to Earl of Rosse	Dublin	Ireland	Yes.
Stroobant, Dr. P.	Royal Observatory	Uccle	Belgium	Yes.
Struvé, Otto	'Ancien Directeur' Pulkova Observatory	St. Petersburg	Russia	Yes.
Swift, Lewis	Warner Observatory	Rochester, N. Y.	United States	Yes.
Tatlock, John		New York	United States	Yes.
Tennant, Lt. Gen. J. F.	Private Observatory	London	England	No.
Thirion, J.	Jesuit Observatory	Louvain	Belgium	Yes.
Tillo, Gen. Alexis de	Corresponding Member Academy of Science, Paris	St. Petersburg	Russia	Yes.
Trouvelot, E. L.	Astro-Physical Observatory	Meudon	France	Yes.
Turner, H. H	Royal Observatory	Greenwich	England	Yes.
Valle, F.	National Astronomical Observatory	Tacubayo	Mexico	Yes.
Veeder, Dr. A. M.	Private Observatory	Lyons, N. Y.	United States	Yes.
Very, Frank W.	Alleghany Observatory	Alleghany	United States	Yes.
Vinot, J.	Astro-Physical Observatory	Paris	France	Yes.
Vogel, Prof. Dr. H. C.	Astro-Physical Observatory	Potsdam	Germany	No.
Wanach, Dr. B.	University Observatory	Strasburg	Germany	No.
Weinek, Dr. Ladislaus	Imperial & Royal Observatory	Prague, Bohemia.	Austria	Yes.
Weyer, Dr. G. D. E.	University Observatory	Kiel	Germany	No.
White, E. J.	Melbourne Observatory	Melbourne	Australia	Yes.
Williamson, Prof. J.	Kingston Observatory	Kingston	Canada	Yes.
Wilson, Wm. E.	Private Observatory	Rathowen	Ireland	Yes.
Wittram, Prof. Dr. Th.	Imperial Observatory	Pulkova	Russia	No.
Wittstein, Dr. A.	Private Observatory	Leipzig	Germany	No.
Wolf, Dr. Max	University Observatory	Heidelberg	Germany	Yes.
Yendell, P. S.	Private Observatory	Dorchester, Mass.	United States	No.
Zenger, Chas. Venc.	Polytechnic School Observatory	Prague	Austria	Yes.
Zelbr, Dr. Karl	Private Observatory	Brünn	Austria	No.

REPLIES.

Classified according to the Countries from which they have been received.

	Totals.	Yeas.	Nays.	Majorities.	
Austria	12	7	5	In favour	
Australia	2	2	0	In favour	
Belgium	6	6	0	In favour	
Canada	5	5	0	In favour	
Colombia	1	1	0	In favour	
England	20	16	4	In favour	
France	4	4	0	In favour	
Germany	38	7	31		Against.
Greece	1	1	0	In favour	
Holland	1	0	1		Against.
Italy	11	8	3	In favour	
Ireland	4	4	0	In favour	
Jamaica	1	1	0	In favour	
Madagascar	1	1	0	In favour	
Mexico	5	5	0	In favour	
Norway	1	0	1		Against.
Portugal	1	0	1		Against.
Roumania	1	1	0	In favour	
Russia	11	6	5	In favour	
Scotland	1	1	0	In favour	
Spain	2	2	0	In favour	
Switzerland	4	2	2		Equal.
United States	38	28	10	In favour	
Totals	171	108	63	18	4