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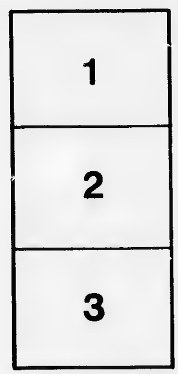

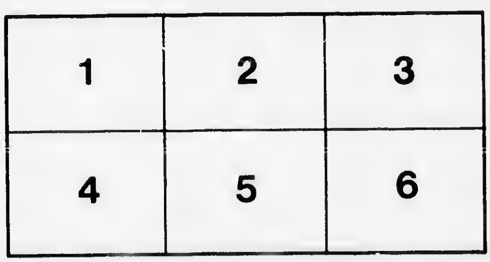

THE ELEMENTS OF GEOMETRICAL OPTICS.

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PREFACE.

In the following pages I have endeavoured to give a concise and at the same time a comprehensive view of the elementary principles of Practical Optics. The method of treating the subject is one of generalization.

I have studied to show the relation between formulæ of reflection and those of refraction by deducing them from one general expression; while at the same time I have employed only positive quantities in the deductions, leaving it to the judgment of the student to determine when any one quantity becomes negative.

I have opened the subject with a short sketch of its history, which, I trust, will be found not uninteresting to the reader; thence follow some general considerations upon the nature and properties of light. The subject of spherical aberration and aplanatic compounds is merely referred to, but otherwise left wholly untouched, it being beyond the scope of the present work. On brightness and illumination, a part of optics to which I conceive too little importance is attached, I have dwelt at some length, and the whole of that chapter I believe to be original. In the last chapter I have given a short sketch of the optical instruments in most common use, limiting myself to the optical principles which they employ.

At the close of each chapter I have collected the most important expressions deduced throughout it, so as to present them conveniently for the purpose of solving the problems which follow.

The work is presented to Canadian Students, and I can only hope that it may be found acceptable, and worthy of their careful perusal. I shall feel amply repaid for the trouble I have experienced in preparing it if it only assists the student in gaining a knowledge of that branch of science so important to the engineer, the surveyor, the astronomer, and in fact to all who employ optical instruments.

In concluding this preface I gladly acknowledge my indebtedness to Professor Williamson, of Queen's College, for many valuable suggestions, as well as for his assistance in correcting the proof.

N. F. D.

Kingston, August, 1868.

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CHAPTER II.—Reflection of Light.

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INTRODUCTION.

Optics is that department of Natural Philosophy which investigates the nature and motions of light.

Optics is divided into two parts: Physical Optics; which deals with the nature and constitution of light, the theoretical laws of its radiation, transmission, absorption and polarization, its relation to heat and electricity, &c. &c.; and Geometrical or Practical Optics; which applies those laws that have been discovered by experiment to the construction and perfection of optical instruments.

There is no doubt that some of the truths of practical Optics were well known and received at a very early date; for mankind could not exist long in a civilized state without discovering that when light falls upon a smooth surface it is reflected or turned back in its course; and the effect of refraction upon a stick plunged obliquely into water would, upon the most superficial observation, become an object of curiosity.

The name of Empedocles, however, is the first of which we have any knowledge, which is associated with a systematic work on Optics.

The famous Euclid composed a treatise on Optics, in which he referred to the well known experiment of placing a coin in the bottom of a cup, in such a manner that it may be just hidden by the edge of the cup, and then rendering it visible by filling the cup with water.

The most noted of the ancient works on this science was written about the middle of the second century by the Astronomer Ptolemy. He seems to have been acquainted not only with the reflection of light, but to a considerable extent with the nature of its refraction; and he succeeded in determining the relation between the angle of refraction and that of incidence in several of the commonest media. He also showed that the refraction occasioned by the atmosphere decreases as we approach the zenith.

Alhazan, an Arabian who wrote about the year 1100, was the next author of any note who turned his attention to this subject. He made many experiments upon the refraction of light from one medium into another, and succeeded in showing that atmospheric refraction increases the altitude of the heavenly bodies; and he advanced the opinion that the stars are sometimes seen above the horizon when they are actually below it—a fact fully established by the subsequent observations of Tycho Brahe, and since his time by all other astronomers.

In 1270 Vitellio published a treatise on Optics, which seems to be, to a great extent, a digest of Alhazan's work.

Contemporary with Vitellio, was Roger Bacon, a man eminent in almost all departments of science, though not advancing in Optics far beyond his predecessors.

In 1575 Maurolycus, a teacher of mathematics at Messina, published his treatise "De Lumine et Umbra," in which he shows that the crystalline lens of the eye collects the rays of light, and thence discovers the cause of short-sightedness and the means of remedying it. Contemporary with him was the famous John Baptista Porta of Naples, who discovered the camera obscura, and was thus led to the true theory of vision.

But there was yet one great desideratum in Practical Optics, a knowledge of the true law of Refraction; and it was not until 1621, when Snell, by laborious and repeated experiments, discovered that the sine of the angle of refraction has a constant ratio to the sine of the angle of incidence, that this branch of Natural Philosophy became truly entitled to be ranked as a separate science.

In more modern times it received the attention of many eminent philosophers, but to none is it more indebted for its present perfection than to Dollond, who by discovering the proper nature of Chromatic Dispersion, was led to the invention of the Achromatic Telescope.

The fundamental principles upon which Practical Optics is based, have been established by a careful observation of the behaviour of light when acted upon by media differing in form or substance, and are entirely independent of any theory in regard to the cause of that behaviour. Thus, without inquiring what light is, or seeking after its origin or mode of propagation, or ascertaining whether it be in motion or at rest, we readily discover, by a very simple experiment, that a sun-beam falling upon a plane mirror is almost wholly reflected, and the direction of the rays after reflection depends altogether upon the relative positions of the sun-beam and the mirror. The same independency of theory characterizes all the other fundamental principles of this science, and it must therefore be distinctly understood that they have not been deduced from any theory in regard to the nature of light, but rather that that theory has been adopted which would give results most in accordance with those furnished by experiment. I make these statements here, in order that the reader may not expect mathematical demonstrations of these first principles, but may be prepared to receive them as given, or to satisfy himself of their correctness by experiment. It may seem to him a very loose way of proceeding, to lay down, in a mathematical work, principles which we are unable to prove by rigorous demonstration; but this inability is not restricted to Optics—it characterizes the whole range of subjects which fall under the general title of "mixed mathematics." We are equally unable to demonstrate the fundamental principles of Mechanics or Electricity, and can hope to arrive at truth in so far only by a series of careful and varied experiments.

1. At the present day two theories are held by which to account for the motions and behaviour of light. The first supposes light to consist in the emission from luminous bodies of exceedingly small particles of matter which possess all the qualities of attraction, repulsion, gravity, etc., which characterize other matter, and which, by falling upon the retina of the eye, produce the sensation of light.

The second makes it to consist in the undulation of an excessively subtle fluid which pervades all space, and is termed ether. The first of these is known by the name of the corpuscular theory of light, and the second by that of the undulatory or wave theory of light.

Either of these theories will suffice to account for many of the practical truths of Optics; but there are certain experimental facts in the polarization and interference of light, which seem to be more in accordance with the results of the wave theory than with those of the other.

2. Experiment teaches us that light proceeds from every visible point of a luminous object in straight lines, and in all directions. That it proceeds from every point, is evident from the fact that we are enabled to see a point of a luminous body only by the rays of light which it emits.

That it moves in straight lines, is shown by the impossibility of seeing through a bent tube, or "around a corner." We suppose, here, that thelight is confined to a homogeneous medium; for where it passes from a medium of one density into a medium of another and different density, it is bent abruptly in its course.

That light proceeds in all directions, follows directly from the fact that we are enabled to see a luminous body equally well in any direction in which we may place it.

3. It was formerly believed that light was propagated instantaneously; but the observations of Roemer upon the eclipses of Jupiter's satellites, of Bradley upon the aberration of the fixed stars, and the more modern ones of M. Fizeau by his ingeniously contrived instruments, prove, beyond a doubt, that light requires time to move from point to point in space, its velocity being about 190,000 miles in one second of time.

4. When light comes in contact with bodies its motion is determined by certain laws; and a careful study of these laws, and their application to the nature of the effect produced, forms the subject of Practical Optics.

These laws are of two kinds:—those which apply to light reflected from a polished surface, and those by which it is governed when it passes through the boundary surface of a medium; hence Practical Optics is sometimes divided into Catoptrics, which considers the motion of light in its reflection; and Dioptrics, which treats of the nature and laws of Refraction.

5. In Optics, bodies are distinguished by their characters of transparency, and non-transparency or opacity.

A transparent body is one which allows the rays of light to pass freely through it; as air, water, clear glass, &c. A non-transparent body opposes an insurmountable barrier to the passage of light.

We know of no body which is perfectly transparent, nor of any which is perfectly non-transparent. The most transparent body with which we are acquainted, atmospheric air, absorbs a considerable amount of light when interposed in extended sheets: a fact which accounts for the dimness of the horizon and of distant objects upon the earth generally. On the other hand, gold, one of the densest substances in nature, admits the passage of a few rays, when used in the form of very thin leaf. A transparent body is termed a medium.

CHAPTER I.

GENERAL PRINCIPLES.

6. Definitions. i. A *ray* is the smallest element of light which we can suppose to proceed continuously from a luminous body in one and the same direction. If the ray comes directly from a luminous point, it is said to be *direct;* if reflected in its course, it is called a *reflected* ray; and if refracted, a *refracted* one.

ii. A *pencil* is a number of rays proceeding from the same luminous point, and practically such that its extreme rays make but a small angle with one another. When the constituent rays proceed parallel to one another, they form a *parallel* pencil; if the rays converge, or tend to converge to a point, the pencil is called *convergent;* and when they proceed or diverge *from* a point, they form the *divergent* pencil. It is obvious that direct rays must, in the strict sense of the word, always be divergent; but when rays come from a great distance, as those of the sun, they may safely be regarded as parallel; hence, a pencil of rays from a very distant point is said to be a parallel pencil. It is also obvious that direct rays can never be convergent; and hence it follows, that a convergent pencil has been acted upon by some Optical Instrument.

iii. Rays, before meeting a given Optical Instrument, are said to be *incident* on that Instrument, and the pencil formed by them is denominated an *incident* pencil. Incident rays must not be confounded with direct rays; the first kind may have previously undergone any amount of change in relative directions, whereas the second can have met with none.

iv. That point at which the rays of a pencil meet, or would meet if produced either way, is called a *focus.* The focus for the incident pencil is termed the *incident* focus, and the focus for the refracted or reflected pencil, the *conjugate* focus.

When these foci are spoken of together, they are termed conjugate foci.

7. Let AB (Fig. 1) be a plane polished surface of a non-transparent substance, and let QP be an incident ray of light meeting the plane at the point P, and after reflection at the surface, moving in a line PQ′. Draw PC perpendicular to the plane at the point P.

Then QP is the incident ray,
PQ′ is the reflected ray,
P is the point of incidence,
QPC is the angle of incidence,
and CPQ′ is the angle of reflection.

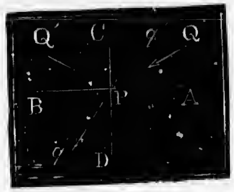

Fig. 1.

Experiment has shown that the angle CPQ′ is in all cases equal to QPC, and that QP, CP and PQ′ are in one and the same plane. Hence we deduce the

Law of Reflection.

When a ray of light is reflected at a plane surface, the angle of reflection is equal to the angle of incidence; and, the incident ray, the reflected ray, and the perpendicular at the point of incidence, are in one and the same plane.

8. Let AB (Fig. 1.) be the surface of a medium, water for example, and let all above AB be vacuum. Let QP be a ray of light falling upon AB at P, and let it, after refraction at the surface of the medium, move along the line Pq. At the point P draw the perpendicular CPD; produce qP backwards to q'.

Then QP is the incident ray,
Pq is the refracted ray,
P is the point of incidence,
QPC is the angle of incidence,
and DPq or CPq' is the angle of refraction.

Numerous experiments have shown that the sine of the angle q'PC has a constant ratio to the sine of the angle QPC; and, that QP, q'P or qP, and CPD are in one plane. Hence we have as a

First Law of Refraction.

When light enters a medium, the sine of the angle of refraction has a constant ratio to the sine of the angle of incidence; and, the incident ray, the refracted ray, and the perpendicular at the point of incidence, are in one and the same plane.

9. Denote the angle of incidence by φ, and that of refraction by φ'; then

$$\frac{\sin\varphi}{\sin\varphi'} = \text{a constant quantity.}$$

This constant quantity is called the *index of refraction*, and is generally represented by the Greek letter μ; hence we may write,

$$\frac{\sin\varphi}{\sin\varphi'} = \mu \dots\dots\dots\dots\dots (1)$$

10. In the case which we have taken for illustration, where the ray passes from a rarer into a denser medium, experiment shows us that, upon entering the surface, the ray is always bent towards the perpendicular, or that φ' is less than φ.

In our supposed case, then, we must have,

$$\frac{\sin\varphi}{\sin\varphi'} > 1, \text{ and } \therefore \ \mu > 1.$$

Experiment proves also that if a ray of light be turned back in its course it will pursue the same path in a reversed direction; i.e. if the ray start from the point q beneath the surface of the water, it will, upon passing that surface, be turned away from the perpendicular, and move along PQ.

In this case qPD becomes the angle of incidence and QPC the angle of refraction, and μ consequently becomes less than unity. Now in order to prevent confusion it has been agreed to adopt that value of μ which is greater than unity, i. e. to consider the ray as moving from rare to dense.

11. Upon the meaning and value of the symbol μ we need to be more explicit.

It is not to be understood that μ represents a fixed invariable quantity, for it has a distinct value for every medium employed.

When light passes from air into water, μ represents a certain quantity, and that quantity is constant whatever be the value of φ; but if instead of water we employ some other medium, as alcohol, glass, diamond, &c., μ will have a different value for each different medium, remaining constant as long as the same medium is used. Hence we see that the value of μ belonging to any given medium is a true index of the *optical* character of that medium; and for this reason, this value is taken as a measure of the density of the medium.

12. If a ray of light passes from vacuum into a medim, μ expresses the *absolute* index of refraction; but if from one medium into another, it expresses the *relative* index.

Since all our Optical Instruments are surrounded by air, and all our experiments carried on within it, it is evident that the value of μ which we adopt is only relative; nevertheless, the refracting power of air is so very small, that these values must approach indefinitely near to absolute ones.

13. It is a fact proved by experiment, that if a ray of light passes successively several media bounded by parallel plane surfaces, the total amount of refraction is the same as would have been produced if the ray had passed directly from the first medium into the last one.

Prob. Given the absolute indices of two media to find their relative index.

In Fig. 2, let AB, GH and KL, be the parallel bounding surfaces, and let the medium GL be denser than AH.

Let QPSTq be the course of a ray of light, and CP, MSN, and TD, perpendiculars at the points P, S and T respectively.

Let μ be the index of the medium AH, and μ' of GL. Now if we suppose all above AB and all below KL to be the same medium, it follows that QP and Tq are parallel, and therefore that the angle QPC is equal to qTD.

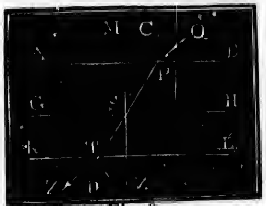

Fig. 2.

Now, $\sin QPC = \mu \sin PSM$,
and $\sin qTD = \mu' \sin TSN$;
but $\sin QPC = \sin qTD$;
$\therefore \mu \sin PSM = \mu' \sin TSN$,

$$\text{Or } \frac{\sin PSM}{\sin TSN} = \frac{\mu'}{\mu}.$$

But it is evident that $\frac{\sin PSM}{\sin TSN}$ expresses the relative index of refraction when the ray passes from AH into GL; therefore, denoting this by μ'', we have,

$$\mu'' = \frac{\mu'}{\mu}. \qquad \text{..............................(2)}$$

14. We may now state the

Additional Laws of Refraction.

i. *When rays of light pass from rarer to denser they are bent towards the perpendicular, but when they pass from denser to rarer they are bent away from the perpendicular.*

ii. *The total amount of refraction caused by any number of media bounded by parallel planes is the same as would take place if the rays were to pass directly from the first medium into the last one.*

iii. *If a ray of light be turned back in its course, it will pursue the same path in a reversed direction.*

Upon the laws of Reflection and Refraction as now laid down depends the whole doctrine of Geometrical Optics.

15. To interpret the expressions $\mu=-1$, and $\mu'=\frac{1}{\mu}$.

I. $\mu=-1$.

$$\text{Since } \mu=-1, \therefore \frac{\sin\varphi}{\sin\varphi'}=-1; \ \therefore \varphi=-\varphi'.$$

Now, what we deduce from this expression is, that the angle of refraction is equal to the angle of incidence, but that it is measured in a negative direction, or upon the opposite side of the perpendicular; that is, the ray moves along PQ′, making the angle Q′PC= QPC. (Fig.1). This result, however, instead of being connected with refraction, is the law of reflection.

Hence we see that *formulæ of refraction are connected with those of reflection by the relation* $\mu=-1$.

Having given, then, a formula of refraction, we transform it to a similar one in reflection, by making μ equal to -1.

II. $\mu'=\frac{1}{\mu}$.

$$\because \mu'=\frac{1}{\mu}, \therefore \mu'=\frac{1}{\frac{\sin\varphi}{\sin\varphi'}}=\frac{\sin\varphi'}{\sin\varphi}.$$

Hence if $\mu'=\frac{1}{\mu}$ *be taken as the index of refraction,* φ' *becomes the angle of incidence and* φ *the angle of refraction;* i. e. the ray is turned back in its course, and proceeds in a direction opposite to that in which it moved when μ was the index of refraction; in other words, *the light moves from a denser medium into a rarer one.*

16. We shall now investigate a general expression for the path of a ray after it has been refracted at one surface of a medium.

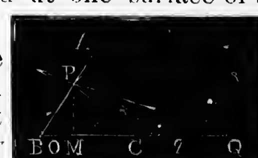

Fig. 3.

In Fig. 3, let AB be a section of the plane surface of a medium situated upon its left; and let BQ be a given straight line meeting the surface in B. Let a ray of light coming from the point Q fall upon the surface at P, and let it, after refraction at this surface, proceed as if coming from q. Draw PC perpendicular to AB, and PM perpendicular to BQ; and with C as a centre and CP as a radius, describe the arc PO cutting QB in O.

What we are in search of is an expression for the relative positions of Q and q. Now it is evident that in order to know the positions of these points we must measure their distances from some given point. We might choose any point upon the line QB, but for our purpose it is the most convenient to take the point O as an origin.

Denote then, QO by D, qO by d,
CO by r, MO by v,
and the angle PCO by θ.

Then from Art. 9, QPC $= \varphi$, and qPC $= \varphi'$.

Since QPC is a plane triangle,

$$PQ : QC :: \sin PCQ : \sin QPC;$$

$$\text{i. e. } \frac{\sin PCQ}{\sin QPC} = \frac{\sin \theta}{\sin \varphi} = \frac{PQ}{QC}.$$

But $PQ^2 = PM^2 + MQ^2$; and $QC = D - r$.
and $MQ^2 = (QO - OM)^2 = (D-v)^2 = D^2 + v^2 - 2Dv$;
also $PM^2 = OM(2{\cdot}OC - OM)$, Euc. VI. 8, $= 2rv - v^2$.

$$\therefore PM^2 + QM^2 = D^2 - 2Dv + 2rv = D^2 - 2v(D-r).$$

$$\therefore \frac{\sin \theta}{\sin \varphi} = \frac{\sqrt{D^2 - 2v(D-r)}}{D-r}.$$

Similarly, $$\frac{\sin \theta}{\sin \varphi'} = \frac{\sqrt{d^2 - 2v(d-r)}}{d-r}.$$

And dividing one of these expressions by the other, and noticing that $\frac{\sin \varphi}{\sin \varphi'} = \mu$, by (1), we obtain by reducing,

$$\frac{D-r}{d-r} = \mu \frac{\sqrt{D^2 - 2v(D-r)}}{\sqrt{d^2 - 2v(d-r)}}. \quad \ldots\ldots\ldots\ldots\ldots (3)$$

17. Equation (3) is rigorously true for every ray refracted at the surface AB, and will become so for those reflected from that surface by writing -1 for μ.

The Equation is perfectly symmetrical and easily remembered, and lies at the foundation of nearly all the particular relations deduced in the course of this work.

CHAPTER II.

ON THE REFLECTION OF LIGHT.

18. When light falls upon the rough surface of a non-transparent body, part of it is absorbed by the body and totally lost, while the other part is reflected in all directions and is said to be scattered. It is by this scattered light that we are enabled to see opaque bodies. If, however, the surface upon which the light falls is polished, the greater portion of it is reflected regularly according to the law of Art. 7, while a small quantity is still absorbed, and a small quantity scattered. By increasing the polish of the surface we may diminish the quantity of scattered light until it is scarcely sufficient to render the surface of the body visible, a fact well exemplified in the difficulty we experience in attempting to examine the surface of a well polished mirror.

19. Bodies with polished surfaces used for the reflection of light, are termed *mirrors* or *specula*, and receive appropriate names depending upon the forms of the surfaces.

Mirrors may be of a great variety of kinds, but only three of these are of any importance in Practical Optics, viz: the plane mirror, the spherical mirror, and the parabolic mirror; these constitute the elementary instruments used in the reflection of light.

The Plane Mirror.

20. The plane mirror has a plane for its reflecting surface, and is the simplest of all Optical instruments. Such is the common looking-glass.

21. Prop. To find the effect of a plane mirror upon parallel rays of light.

Let (Fig. 4,) AB be a section of the reflecting surface; QP, UV incident rays of light, and Pq, Vu their courses after reflection.

Fig. 4.

Draw the perpendiculars PC and VD; and from P and V draw PR and VS perpendiculars upon VU and Pq respectively.

Now, the incident rays being parallel, QPC =UVD; but QPC=CPq and UVD=DVu by the law of reflection, ∴ CPq=DVu; or, *the reflected rays are parallel.*

Again ∵ UVD=QPC=CPq, ∴ qPV=UVP.

But PRV=PSV both being right angles, and PV is common to the triangles PRV and PSV;

∴ PR=VS; or, *the reflected rays are equidistant with the incident rays.*

Lastly, since UV is at the right of the incident rays, but uV at the left of the reflected rays, it follows that *the order of the rays is inverted by reflection.*

Cor. If the rays Pq and Vu were incident upon a second plane mirror it is evident, that, *their* order being inverted, the doubly reflected rays would have the same order as the original ones.

22. Prop. To find the effect of a plane mirror upon divergent rays of light.

In fig. 5, let AB be the section of the reflecting surface; QP, QV rays of light diverging from the point Q, and PU, Vu, their paths after reflection from the surface AB.

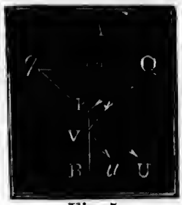

Fig. 5.

Draw QM perpendicular to AB and produce it to q making Mq equal to MQ. Join qP and qV.

Since QMP=PMq, both being right angles; and QM=Mq, and MP is common to the triangles QPM and qPM,

∴ ∠ QPM=qPM. But QPM being the complement of the angle of incidence, is equal to UPB the complement of the angle of reflection; ∴ qPM=UPB, and qPU is a straight line.

In a similar manner it may be shown that qVu is a straight line. Hence *the rays appear after reflection to diverge from the point q, which is as far behind the mirror as Q is in front of it, and which is situated upon the line drawn through Q perpendicular to the mirror.*

Again, since the triangles qMP and PMQ are equal,

∴ qP=PQ; and similarly qV=VQ; but PV is common to the triangles qVP and QVP; ∴ ∠ PqV=PQV; or, *the reflected rays are equally divergent with the incident ones.*

23. If the rays be reversed in their course, or uV and UP become the incident ones, then it may be shown in a manner precisely as in the last article, that the rays after reflection will pass through the point Q.

Hence it appears that *the effects of a plane mirror are to change the order and real directions of the rays without exercising any influence upon their relative directions.*

24. It appears, then, that rays of light diverging from the point Q, proceed after reflection as if coming from the point q; and that if the incident rays converge towards the point q behind the mirror, they will after reflection pass through the point Q. For this reason Q and q are in reality foci of the mirror; but since it is not customary to speak of a plane mirror as having a focus, we designate the points Q and q by the name of *conjugate points.* Thus Q is said to be conjugate to q, and q to Q.

25. Prop. To find the relative positions of the conjugate points to a plane mirror.

From Art. 22, it appears that the points are situated upon the same straight line drawn perpendicularly to the surface of the mirror, and that $QM = qM$.

Now taking the point M as an origin, if we consider QM to be positive, we must regard qM, measured in an opposite direction, as negative. Hence denoting QM by D, and qM by d, we have

$$d = -D \ldots\ldots\ldots\ldots\ldots\ldots (4).$$

We shall deduce the same relation from (3), in order to show both the generality of that Equation, and the method of reduction to be employed.

First, since this is a case of reflection we write -1 for μ. Second, since the radius of a plane is infinitely great, we make $r = \infty$. But in fig. 3, when r becomes infinitely great, the point C will be infinitely distant beyond Q, and PC will be parallel to BQ. But, since $v = CO - CM$, it will become zero when C is infinitely distant; for PC being parallel to BC, AB will be perpendicular to BC, and O, B and M will coincide, whatever be the value of r.

Therefore making $v =$ zero, (3) reduces to

$$\frac{D-r}{d-r} = -\frac{D}{d}$$

Now, dividing both numerator and denominator of the left hand member by r, and then making $r = \infty$, we obtain, since $\frac{D}{r}$ and $\frac{d}{r}$ become zero

$$1 = -\frac{D}{d},$$

and therefore $d = -D$; as before.

This gives the relative positions of the points when the reflecting plane is perpendicular to the axis BQ, and passes through the point M. And since one measure is positive and the oth negative, it follows that the conjugate points must be upon opposite sides of the mirror.

26. Since the point Q, in fig. 5, is in front of the mirror, it is possible for all the rays to pass through it; Q is therefore a *real* focus. On the other hand, the point q being behind the mirror, the rays cannot in reality pass through it, but only appear to do so; for this reason q is called a *virtual* focus.

It follows from Equation (4) that, in a plane mirror, one of the foci is necessarily real, and the other virtual.

A good illustration of a virtual focus is had by holding a lighted candle in front of a common looking-glass. The light, after reflection at the surface, comes to our eye as if from a candle behind the glass; and since we judge of the direction of a body by the direction in which the rays from it meet our eye, we apparently see a second candle placed behind the glass in the position of the virtual focus.

We have hitherto supposed that the incident focus is real, but this is not always the case. For, remembering that the incident focus is the focus belonging to the incident rays, and the conjugate focus the one belonging to the reflected rays, if the rays after reflection pass through a real point upon the axis, the conjugate focus must be real, and therefore the incident focus must be virtual.

27. The angle contained between the course of an incident ray and its course after reflection or refraction is termed the *deviation*.

28. Prop. To find the deviation of a ray of light after reflection at one plane surface.

In fig. 6, let AB be the mirror, QP the incident ray, and PQ′ the reflected ray. Produce Q′P backwards to q, and draw PC perpendicular to the mirror. Then the angle contained between QP and PQ′ is the deviation.

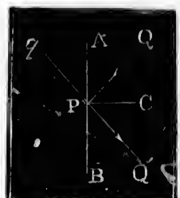

Fig. 6.

Now in order to know whether we are to take the angle QPQ′ or QPq, we have recourse to the following consideration.

By the law of Reflection, $\angle$ QPC = CPQ′; $\therefore$ when QPC becomes a right angle, CPQ′ becomes a right angle. But QP then coincides with AP, and PQ′ with PB; i. e., the deviation then becomes nothing. But the angle QPq then becomes nothing; therefore QPq is the proper expression for the deviation.

$$\text{Now, } \because \angle QPC = CPQ', \therefore \angle QPA = Q'PB = APq;$$

$$\therefore QPq = 2QPA.$$

$$\text{But } QPA = CPA - QPC = \frac{\pi}{2} - \varphi,$$

$$\therefore QPq = 2\left(\frac{\pi}{2} - \varphi\right).$$

Or denoting the deviation by ξ,

$$\xi = \pi - 2\varphi \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

29. Prop. Two points and a plane mirror being given, to find the path of a ray which shall pass from one point to the other, being reflected by the mirror in its course.

In fig. 6, let Q and Q′ be the given points and AB the given mirror.

Find q the conjugate point to Q; draw qQ′ and let it cut AB in P. Join QP. Then QP, PQ′ is the path of the ray.

For, since q is conjugate to Q, the rays which come from Q will after reflection appear to come from q. But the ray PQ′ proceeds as if coming from q; hence PQ′ is the reflected ray required, and therefore QP, PQ′ is the path of the ray.

Two Plane Mirrors.

The principle of Reflection at two plane mirrors enters into the construction of several important Optical instruments, such as the Quadrant, the Sextant, &c. The light is made to undergo two successive reflections at the surfaces of the mirrors, they being either parallel or inclined to one another.

30. Prop. To find the deviation when a ray of light is reflected successively at the surfaces of two plane mirrors inclined at a given angle.

Fig. 7.

Let AB and DB be the mirrors inclined at an angle ABD $= \varepsilon$; and let QP, PP′, P′q be the course of a ray. From the points of incidence P and P′, draw the perpendiculars PV and P′C respectively, and let them meet in C. Then, since the deviations are in the same direction at each of the mirrors, the whole deviation must be equal to the sum of the partial ones. Now, since CPB and CP′B are right angles, ∴ P′CP is the supplement of PBP′, that is of ε.

But VCP′ is the supplement of P′CP,

$$\therefore \text{VCP}' = \varepsilon.$$

Now VCP′ = CP′P + CPP′, (Euc. I. 32.)

$$\therefore \varepsilon = \text{CP}'\text{P} + \text{CPP}';$$

Or, putting CP′P $= \varphi$, and CPP′ $= \varphi_1$,

$$\varepsilon = \varphi + \varphi_1.$$

Again the deviation at P is, (5), $\pi - 2\varphi_1$,

and at P′ it is $\pi - 2\varphi$.

Therefore, denoting the whole deviation by ξ, we have,

$$\xi = \pi - 2\varphi_1 + \pi - 2\varphi = 2\pi - 2(\varphi_1 + \varphi);$$

or putting ε for $\varphi_1 + \varphi$,

$$\xi = 2\pi - 2\varepsilon. \quad \ldots\ldots (6)$$

This measures the angle in that direction in which the deviation takes place at the first mirror; but since 2π is a whole circumference, if we measure the angle in the opposite direction, it is evident that we get for the deviation

$$\xi = 2\varepsilon. \quad \ldots\ldots (7)$$

Hence we see that, disregarding the direction in which the angle is measured, *the deviation is equal to twice the angle contained between the reflecting surfaces of the mirrors.*

If the deviation at the second mirror be different in direction from that at the first; then, since VPB = CP′B, both being right angles, it follows that PCP′ = PBP′ = ε.

Fig. 8.

But PCP′ = UPP′ − PP′C = $\varphi_1 - \varphi$,

$$\therefore \varepsilon = \varphi_1 - \varphi.$$

And since the whole deviation is the difference of the partial ones, we have from (5),

$$\xi = \pi - 2\varphi_1 - (\pi - 2\varphi) = -2(\varphi_1 - \varphi),$$

or,

$$\xi = -2\varepsilon;$$

and, disregarding the negative sign, we have as before.

31. Prop. Given two plane mirrors and a point between them, to determine the positions of the conjugate points.

Let AC and BC be two plane mirrors meeting at C and Q a point between them.

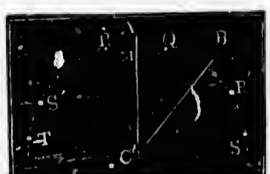

Fig. 9.

From Art. 24, it appears that Q will have a conjugate point at R, found by drawing QMR perpendicular to AC and taking MR = MQ. Again, R, acting as a principal point to BC, will have a conjugate point at S; and in the same manner, in reference to the mirror AC, S will have a conjugate point at T, &c.

But, beginning with the mirror BC, Q will have a conjugate point at R′, R′ at S′, &c.

Now, since QM = MR, and CM is common to the triangles CQM and CRM, and ∠CMQ = CMR, ∴ ∠MCR = MCQ, and CR = CQ.

In the same manner it may be shown that CT, CS, CS′, and CR′, are all equal to CR or CQ.

Therefore, *The conjugate points are situated upon the circumference of a circle passing through the given point, and having its centre upon the line of intersection of the mirrors.*

Again; Denoting the angle ACB by ε, and ACQ by a, we have, $BCR = BCA + ACR = \varepsilon + a$;

But $BCR = BCS$, S being conjugate to R,

$$\therefore BCS = \varepsilon + a;$$

and $ACS = ACB + BCS = \varepsilon + \varepsilon + a = 2\varepsilon + a$.

But T being conjugate to S in respect to the mirror AC, $\angle ACT = ACS$; $\therefore ACT = 2\varepsilon + a$.

In a similar manner by denoting QCB by a' we obtain, $BCR' = a'$ and $\therefore ACR' = \varepsilon + a'$.

But $ACS' = ACR'$, S′ being conjugate to R′;

$\therefore ACS' = \varepsilon + a'$.

But since $a' = \varepsilon - a$, we obtain,

$ACT = 2\varepsilon + a$; $ACS' = \varepsilon + (\varepsilon - a) = 2\varepsilon - a$; $ACR = a$;

$ACQ = a$; $ACR' = \varepsilon + (\varepsilon - a) = 2\varepsilon - a$; $ACS = 2\varepsilon + a$.

But, $TCS' = ACT - ACS' = 2\varepsilon + a - (2\varepsilon - a) = 2a$;

$RCQ = RCA + ACQ = 2a$;

&c., &c.,

Also, $S'CR = ACS' - ACR = 2\varepsilon - a - a = 2(\varepsilon - a)$;

$QCR' = ACR' - ACQ = 2\varepsilon - a - a = 2(\varepsilon - a)$

&c., &c.

Hence, we see that *the conjugate points and the given point form a system of equidistant pairs.*

32. The angular distance of any point from the similarly situated point in the adjacent pair, is 2ε.

For $TCR = ACT - ACR = 2\varepsilon + a - a = 2\varepsilon$.

Hence we infer that *there will be as many pairs in the whole circumference as the number of times 2ε is contained in 360°, or ε in 180°.*

These principles enter into the construction and operation of the Kaleidoscope.

33. Prop. Given two points and two mirrors meeting at an angle, to find the path of a ray which shall pass from one point to the other, being reflected by both mirrors in its course.

Fig. 9*.

Let AC, BC be the mirrors, and Q, q, the given points.

Take Q′, q', conjugate points, to Q and q respectively. Join Q′q' and let the joining line cut AC in P and BC in P′. Join QP, P′q.

Then QP, PP′, P′q is the path of the ray.

For $\angle$ QPA = Q′PA = CPP′;

and $\angle$ qP′B = q'P′B = CP′P;

which satisfies the law of Reflection.

34. When a ray of light falls obliquely upon one of two parallel plane mirrors and is reflected at both mirrors, it suffers no deviation, but is *displaced*.

For, the mirrors being parallel, $\varepsilon = 0$ and $\therefore$ (7) $\xi = 0$.

Prop. To find the displacement.

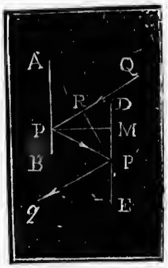

Fig. 10

Let AB and DE be the mirrors, and QPP′q the path of a ray of light.

Draw PM perpendicular to the mirrors, and P′R perpendicular to QP.

Denote PM by D; and P′R, the displacement by d.

Then d = P′R = P′P sin RPP′ = P′P sin 2φ.

But PP′ = PM sec MPP′ = D sec φ;

$\therefore d = D \sec\varphi \sin 2\varphi = 2D \sin\varphi$.......(8)

The Spherical Mirror.

35. A spherical Mirror has a portion of the surface of a sphere for its reflecting surface, and is *concave*, or *convex*, according as the centre of the sphere is in front of or behind the mirror.

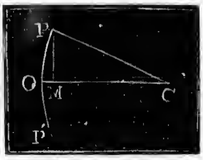

Fig. 11.

In Fig. 11, let PP′ be a section of a spherical mirror having the centre of the sphere at C; and let PO = OP′.

If the reflecting surface be upon the same side as C, the mirror is concave; but if upon the opposite side, it is convex.

Join CO and draw PM perpendicular upon CO.

Then C is the *centre of curvature* of the mirror;

O is the *optical centre*, or centre of the mirror;

CO is the *radius of curvature;*

PM is the *radius of the mirror*,

and $\angle$ PCO is the *semi-aperture* of the mirror.

These terms are in continual use, and should therefore be remembered.

36. The two kinds of spherical mirrors, already described, produce quite different effects on pencils of light incident upon them, and seem to be subject to very different laws; and yet they are so intimately connected, that the peculiar properties of both may be expressed by the same formulae, the difference in effect, being such as to be brought about by a mere change of algebraical sign.

In order that we may not be at a loss to know where and when these changes of algebraic signs take place, we make use of the following system of rules, which apply not only to cases of reflection from mirrors, but also to those of refraction through lenses.

i. Incident rays are supposed to move from right to left across the paper.

ii. All distances are measured from some origin, which will generally be the centre of curvature, the optical centre, or the principal focus.

iii. The radius of curvature is measured *from* the reflecting or refracting surface *to* the centre of curvature.

iv. All lines measured to the right are considered positive, and those measured to the left, negative.

By attention to these rules much trouble in the solution of problems will be avoided.

37. From what has been laid down in the last article, it appears that *the radius of curvature of a concave mirror is a positive quantity, while that of a convex mirror is a negative one.* In order to have positive quantities in our results we take the concave mirror as the typical one, and deduce our general formulae from its properties; these may then be applied to convex mirrors by changing the algebraical sign of the quantity denoting the radius of curvature.

38. The formulae hitherto deduced are rigorously exact, but those which we are now about to investigate are only approximate. Within certain limits, however, which in practice are well known and seldom surpassed, the approximation is remarkably close; and owing to this circumstance, as well as their more simplified forms, these approximate expressions are, except in special investigations, used instead of the exact but more complicated ones.

39. Prop. In a concave spherical mirror, to find the relation between the distances of the foci from the optical centre.

Let O be the optical centre of the mirror, OQ the axis, and C the centre of curvature.

Let a ray of light proceed from Q, a point upon the axis, and after reflection from the mirror at P pass along Pq, cutting the axis again in q. Then, as will shortly be shown, Q and q are the foci of the mirror.

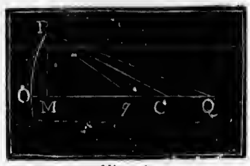

Fig. 12.

As in Art. 16, denote QO by D, qO by d, CO by r, and OM by v.

Then, if PM, the radius of the mirror, be small in comparison with the radius of curvature, it is evident that OM or v becomes a very small quantity, and may be considered as zero without introducing any great error. Therefore, in equation (3) writing -1 for μ, and putting $v = 0$, we reduce it to

$$\frac{D-r}{d-r} = -\frac{D}{d}.$$

multiplying crosswise,

$$Dd - dr = -Dd + Dr,$$

Or $$Dr + dr = 2Dd;$$

and dividing by Ddr, we obtain

$$\frac{1}{d} + \frac{1}{D} = \frac{2}{r}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

the form under which the relation is most generally expressed. Since this equation is independent of v, and consequently of the distance PO, it follows that every ray diverging from Q and meeting the mirror between O and P will, after reflection, pass through q; hence Q and q are foci of the mirror.

40. When the incident rays are parallel, the focus of the reflected rays is called the *principal focus;* and the distance of the principal focus from the mirror is termed the *focal length* of the mirror.

Prop. To find the focal length of a concave spherical mirror.

By moving the point Q, Fig. 12, to an infinite distance, we make the incident rays parallel. Hence making $D = \infty$ in (9), and denoting by f the particular value which d assumes, we have,

$$\frac{1}{f} = \frac{2}{r};$$

and $$\therefore f = \frac{r}{2}. \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Hence, *the focal length of a spherical mirror is one half of the radius of curvature.*

Also, since f depends upon r for its algebraic sign, *the focal length of a concave mirror is a positive quantity, and of a convex one, a negative quantity.*

41. By writing $2f$ for r in (9) and dividing numerator and denominator of the right hand member by 2, we obtain,

$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f} \quad \text{(11)}$$

This equation being perfectly symmetrical is easily remembered, and it presents no difficulties in its application. From it we obtain,

$$d = \frac{Df}{D-f} \quad \text{(12)}$$

42. Prop. To find the relative positions of the conjugate foci when the principal focus is taken as an origin.

Denote by Δ the distance of the incident focus from the principal focus, and by δ the distance of the conjugate focus from the same point. Then, since D and d, in (11), measure the distances from the optical centre, and since the principal focus is at a distance f in front of the optical centre, we must have,

$$D = \Delta + f, \text{ and } d = \delta + f;$$

and writing these values of D and d in (11), we have,

$$\frac{1}{\delta + f} + \frac{1}{\Delta + f} = \frac{1}{f};$$

Reducing,

$$f(\Delta + \delta) + 2f^2 = f(\Delta + \delta) + \Delta\delta + f^2;$$

From which,

$$\Delta\delta = f^2; \quad \text{(13)}$$

which expresses the required relation.

43. Using equation (13) we are enabled to examine very readily the positions of the foci.

i. $\because f^2$ is essentially positive, Δ and δ must have like algebraic signs; and hence, *both foci are upon the same side of the principal focus.*

ii. If $\Delta > f$, $\delta < f$; $\therefore$ *if Q be beyond the centre of curvature, q will be between that centre and the principal focus.*

iii. If $\Delta = f$, $\delta = f$; $\therefore$ *if* Q *be at the centre of curvature, q will be there also, and the foci will coincide.*

iv. If $\Delta < f$, $\delta > f$; $\therefore$ *if* Q *be between the principal focus and the centre of curvature, q will be beyond the centre of curvature.*

v. If $\Delta = 0$, $\delta = \infty$; $\therefore$ *if* Q *be at the principal focus. q is infinitely distant, or the reflected rays proceed parallel.*

vi. If Δ be negative and less than f, δ will be negative and greater than f; $\therefore$ *if* Q *be between the principal focus and the optical centre, q will be behind the mirror, and consequently a virtual focus.*

vii. If $\Delta = -f$, then $\delta = -f$, and *the foci coincide at the optical centre.*

viii. If $\Delta > f$ and negative, $\delta < f$ and negative; $\therefore$ *if* Q *be behind the mirror, and consequently virtual, q will be between the mirror and the principal focus, and consequently real.*

From iii and vii we conclude that the foci coincide at two different points; and from iii and viii, that *a concave spherical mirror may have both foci real, and must have one of them so.*

44. Prop. The effect of a concave spherical mirror upon an incident pencil is to increase the convergency of its rays.

In dealing with this proposition we notice three cases, according as both foci are real, the conjugate focus virtual, or the incident focus virtual.

Case i. *Where both foci are real.*

In this case, since the rays are divergent when they fall upon the mirror, and convergent when they leave it, it is evident that their convergency has been increased.

Case ii. *When the conjugate focus is virtual.*

From 43. vi, it appears that this takes place whenever the incident focus is between the principal focus and the mirror.

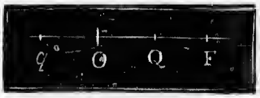

Fig. 13.

In Fig. 13, if O be the mirror, Q the incident, and q the conjugate focus; $D =$ OQ, $d =$ Oq, and $f =$ OF. But from (12),

$$d = D.\frac{f}{D-f} = D.\frac{\text{OF}}{\text{OQ-OF}} = D.\frac{-\text{OF}}{\text{QF}};$$

whence,

$$d > D \text{ or } \text{O}q > \text{OQ};$$

And, since the rays diverge from the point Q, and after reflection, appear to diverge from a point q farther distant from the mirror, it follows that the reflected rays are less divergent than the incident rays, i. e., the incident rays have been rendered more convergent by reflection.

Case iii. *When the incident focus is virtual.*

Reverse the light in its course; then q becomes the incident focus, and Q the conjugate focus.

The incident rays fall upon the mirror converging towards the point q, but after reflection they pass through Q which is nearer to the mirror than q is; hence the convergency of the rays has been increased.

45. Since (Art. 37,) the radius of curvature of a convex mirror is a negative quantity, the principal focus must lie behind the mirror; and hence in order that a focus may be real, it must be at a greater distance from the principal focus than one half the radius of curvature.

But, Δ being this distance, if $\Delta > f$, $\delta < f$;

Hence, *one of the foci must be virtual.*

Moreover, when Δ is negative δ is negative; whence it follows that *both foci may be virtual.*

46. In a similar manner it may be shown that *in a concave mirror, one focus must be real, while both of them may be so.*

47. By reasoning similar to that employed in Art. 44, it may be shown that *the effect of a convex mirror upon a pencil of light, is to decrease the convergency*, or to increase the divergency of its constituent rays.

Reflection from two Spherical Mirrors.

48. We meet with successive reflection from two spherical mirrors in the Cassegrainian and Gregorian telescopes, and in some forms of the reflecting microscope.

The mirrors have a common axis, and may be both concave, both convex, or one concave and the other convex.

The one upon which the rays first fall is termed the *primary*, and the other the *secondary* mirror.

49. Prop. To find the position of the resultant focus when light has been reflected successively at the surfaces of two spherical mirrors.

In Fig. 14, let O O′ be the mirrors, and O Q their common axis.

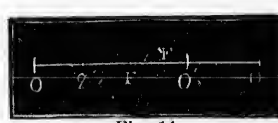

Fig. 14.

In order to have positive quantities only in our result, let both mirrors be concave in respect to the rays incident upon them. Let O be the primary, Q its incident focus, q its conjugate focus, and F its principal focus. Then O′ being the secondary and q its incident focus, let q' be its conjugate focus, and F′ its principal focus.

As before, denote QO by D, qO by d, FO by F; and also, q O′ by D′, q'O′ by d', and $F'O'$ by f.

Now, let OO′ $=l=$ the distance between the mirrors;
and O′q' $=\lambda=$ the distance of the resultant focus from the secondary mirror.

$$\text{Then, } D' = qO' = OO' - Oq = l - d;$$
$$\text{and, } d' = q'O' = \lambda.$$

But from (11),

$$\frac{1}{d'} + \frac{1}{D'} = \frac{1}{f};$$

$$\therefore \frac{1}{\lambda} + \frac{1}{l-d} = \frac{1}{f}; \qquad (14)$$

an equation from which λ may be found in terms of l, d, and f.

This form applies equally well to all cases, by merely changing the algebraic signs of such quantities as require it.

Cor. If the primary rays are parallel, as is the general case with telescopes, $d=F$; and this gives in (14),

$$\frac{1}{\lambda} + \frac{1}{l-F} = \frac{1}{f} \qquad (15)$$

For convenience in reference we here collect the results of this chapter.

Reflection at a plane mirror,

$$d = -D \qquad (4)$$

Deviation at a plane mirror,

$$\xi = \pi - 2\varphi \qquad (5)$$

Deviation at two plane mirrors,

$$\left.\begin{aligned} \xi &= 2\pi - 2\epsilon \\ \text{or } \xi &= -2\epsilon \end{aligned}\right\} \qquad (6)$$

Displacement by two plane mirrors,

$$d = 2D\sin\varphi \qquad (8)$$

Spherical mirror,

$$\frac{1}{d} + \frac{1}{D} = \frac{2}{r} = \frac{1}{f} \qquad (9)\ (11)$$

$$f = \frac{r}{2} \qquad (10)$$

$$d = \frac{Df}{D-f} \text{..................} (12)$$

Two spherical mirrors,

$$\frac{1}{\lambda} + \frac{1}{l-d} = \frac{1}{f'} \text{..............} (14)$$

EXERCISES A.

1. A ray of light is incident upon a plane mirror at an angle of 10°, to find its deviation.

Equation (5) $\xi = \pi - 2\varphi$.

But $\varphi = 10^\circ \therefore \xi = 180^\circ - 20^\circ = 160^\circ$.

2. Two plane mirrors, AC, and BC, meet at C forming an angle ACB = 150°; a ray of light QP meets the mirror AC at P making the angle QPA = 10°; to find the deviation.

It is manifest from the nature of the question, that the deviation takes place in the same direction at both mirrors.

Hence by (6), $\xi = 2\pi - 2\epsilon = 360^\circ - 2 \times 150^\circ = 60^\circ$.

3. Light is incident upon a spherical mirror from a point 10 inches in front of it, and is brought to a focus at a distance of 2 in. in front; determine the focal length of the mirror and, hence its kind.

$\because$ the light is incident from a point in front, $D = +10$; and $\because$ the conjugate focus is in front, $d = +2$;

$$\therefore (11) \quad \frac{1}{2} + \frac{1}{10} = \frac{1}{f}.$$

$$\text{or } f = 1\frac{2}{3},$$

and the mirror is concave.

4. The focal length of a primary mirror is 20 in., and it receives rays from a point 100 in. in front of it: required the mirror which should be placed at a distance of 16 in. in front, so that the resultant focus may be at the primary mirror.

In this question we have,

$$D = 100, f = 20, l = 16, \lambda = 16;$$

Hence from (14),

$$\frac{1}{16} + \frac{1}{16-d} = \frac{1}{f'}.$$

But from (12),

$$d = \frac{100 \times 20}{100 - 20} = 25;$$

$$\therefore \frac{1}{16} - \frac{1}{9} = \frac{1}{f'} = -\frac{7}{144};$$

$$\therefore f' = -20\frac{4}{7},$$

and the required mirror must be convex.

5. A primary mirror of 24 in. focal length receives parallel rays; where must a secondary of -2 in. focal length be placed so that the resultant focus may be 20 in. in front of the secondary?

In this case, $f = 24, f' = -2$, and $\lambda = 20$, to find l.

From (15) $$\frac{1}{20} + \frac{1}{l-24} = -\frac{1}{2};$$

$$\therefore 24 - l = 1\frac{9}{11},$$

Or $$l = 22\frac{2}{11} \text{ in.}$$

6. If light diverging from a point 20 in. in front of a mirror has its conjugate focus at a distance of 11 in. behind the mirror, determine the mirror employed.

7. The focal length of a mirror is -8 in., and the incident focus is 10 inches in front of it; to find the position of the conjugate focus.

8. The focal length of a mirror is -3, and the reflected rays converge to a point 12 in. in front of it; determine the position of the incident focus.

9. In a combination of two spherical mirrors the secondary has a focal length of 3 in., and is placed 12 in. in front of the primary: if parallel rays upon the primary have their resultant focus half way between the mirrors, determine the primary.

10. Show that if parallel rays falling upon the primary are reflected parallel from the secondary, the distance between the mirrors must be the algebraic sum of their focal lengths.

CHAPTER III.

ON THE REFRACTION OF LIGHT.

50. The general effects produced upon light by the surface of a medium through which it passes, and the laws which govern its motions when so effected, have been laid down in Chapter I; and we now propose to trace these general principles to particular results, confining ourselves to those which are most practical and most simply expressed.

51. The elementary instruments used in the refraction of light are media bounded by one or more geometrical surfaces.

The surfaces most usually employed, and the only ones of which it is necessary to speak in this treatise, are the plane surface and the spherical one.

Refraction at one Plane Surface.

52. Let AB be a plane surface, dividing two media of different densities, of which the more dense is upon the left of the dividing plane.

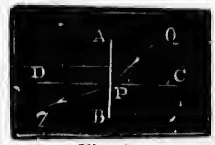

Fig. 15.

Also let CD be a perpendicular or normal to the surface, and QP an incident ray of light.

From the general laws of Refraction (Arts. 8, 14,) we know that the ray QP upon passing the surface AB will be turned towards the perpendicular and pursue a path Pq, making the angle DPq less than CPQ. Moreover, if the ray be reversed in its course, and qP be the first direction, PQ will be the second direction. And lastly, if μ be the relative index of refraction between the two media, sin QPC $= \mu$ sin qPD.

53. Prop. To find the limiting conditions under which a ray of light may pass from any given medium into a denser one.

In fig. 15 it is readily seen that the angle QPC cannot be greater than a right angle. But at this its maximum value the sine is unity. Hence for the angle qPD, under this circumstance, we have

$$\mu \sin qPD = 1,$$

$$\text{Or } \sin qPD = \frac{1}{\mu}.$$

Whence we infer that *any ray of light may pass from the less dense of two media into the other one.*

Also, *the sine of the greatest angle made with the perpendicular by any refracted ray is equal to the reciprocal of the relative index of refraction.*

Hence it follows that when qP has its limiting position, within the angle qPB no ray can be found which has come through the surface AB.

54. Prop. To find the limiting conditions under which a ray of light may pass from a denser medium into a rarer one.

In fig. 15, suppose the ray to be reversed in its course, and to proceed as if coming from the point q, situated within the denser medium, then the equation

$$\sin QPC = \mu \sin qPD,$$

expressing the constant relation between the angles of incidence and refraction, can have neither of its members imaginary. But sin QPC cannot possibly be greater than unity. Hence at a maximum we must have

$$\sin qPD = \frac{1}{\mu} \ldots\ldots\ldots\ldots\ldots (16).$$

What we are to understand by (16) is that when $\sin qPD = \frac{1}{\mu}$ the ray will not pass out into the rarer medium, but will move in a path PQ situated upon the very surface of the medium.

Furthermore, no ray $q'P$ situated between qP and the surface can possibly pass out of the denser medium.

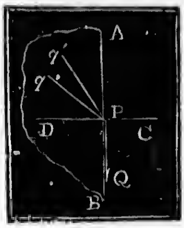

Fig. 16.

For if it pass out we must have

$$\sin Q'PC = \mu \sin q'PD;$$
$$\text{but } \sin QPC = \mu \sin qPD, = 1,$$
$$\text{and } \sin q'PD > \sin qPD;$$
$$\therefore \sin Q'PC > 1, \text{ an impossibility.}$$

It appears, then, that rays which fall between qP and AP cannot come under the law of refraction; and by having recourse to experiment in order to ascertain what becomes of them, we find that they are totally reflected from the surface and thrown back into the denser medium.

This angle qPD when at its maximum is called the *critical angle*, from being the angle of incidence at which refraction ceases; or, *the angle of total reflection*, from being the one at which reflection of this kind commences.

55. This kind of reflection, from the internal surface of a dense medium, comes the nearest to perfection of any known; and when glass is the medium employed, this principle enters into the construction of the Camera Lucida, and one kind of diagonal eyepiece.

This angle is for glass about 42°, and for water about 50°.

56. The principles investigated in Art. 54 give rise to some interesting phenomena, of which one of the most singular is that to a person situated beneath the surface of water the whole horizon and all objects upon it appear to lie in a circle about his zenith having an angular radius of about 50°.

Let OH be the surface of the water, HT a tree in the horizon, and E the position of the eye.

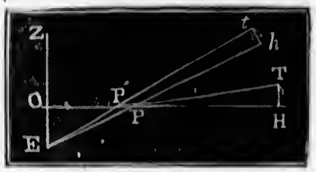

Fig. 17.

Then the rays coming from H are bent at P and enter the eye as if coming from h. Similarly, the rays from T appear to come from t, and upon the whole the tree appears to be suspended in the position ht, such that the angle hEO is about 50°; that is, the critical angle for water.

Another familiar instance is the apparent bending of a straight rod when plunged obliquely into water.

Let AB be the surface of the water, and GH the rod. Then the rays HP, HP′ coming from H, will be bent at P and P′, and move in the paths Pq, $P'q'$, as if coming from a point H′ situated above H; thus the extremity H appears to be raised into the position H′, and with it, the whole immersed part takes the apparent position KH′.

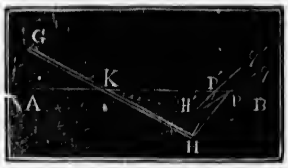

Fig. 18.

57. Prop. If the rays incident upon the plane surface of a medium are parallel, the refracted rays are also parallel.

For let φ be the common angle of incidence, since the rays are parallel, and φ_1, φ_2, the angles of refraction of any two rays.

Then by the laws of Refraction,

$$\text{Sin } \varphi = \mu \sin \varphi_1, \text{ and } \sin \varphi = \mu \sin \varphi_2;$$

$$\therefore \mu \sin \varphi_1 = \mu \sin \varphi_2,$$

Or $\varphi_1 = \varphi_2$, and the refracted rays are parallel.

58. Prop. To find the relative positions of the conjugate points when light is refracted at one plane surface of a medium.

In Fig. 19, let AB be the surface, CD a perpendicular taken as an axis, and QPq a ray of light.

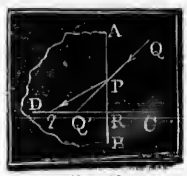

Fig. 19.

Produce QP to meet CD in Q′; then Q′ and q are the conjugate points.

Denote RQ′ by D, and Rq by d; then the angle $PQ'R = \varphi$, and we have,

$$\frac{D}{d} = \frac{RQ'}{Rq} = \frac{RQ'}{RP} \cdot \frac{RP}{Rq} = \tan RPQ' . \tan PqR.$$

But $\tan RPQ' = \cot RQ'P = \cot \varphi$;
and $\tan PqR = \tan \varphi'$;

$$\therefore \frac{D}{d} = \frac{\tan \varphi'}{\tan \varphi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

59. Equation (17) may be transformed as follows:

$$\frac{D}{d} = \frac{\sin \varphi'}{\cos \varphi'} \cdot \frac{\cos \varphi}{\sin \varphi} = \frac{\sin \varphi'}{\sin \varphi} \cdot \frac{\cos \varphi}{\cos \varphi'}.$$

But (1), $\frac{\sin \varphi'}{\sin \varphi} = \frac{1}{\mu}$;

$$\therefore \frac{D}{d} = \frac{1}{\mu} \cdot \frac{\cos \varphi}{\cos \varphi'}.$$

Now when φ and consequently φ' are very small, i.e., when the light meets the surface almost perpendicularly, we have $\cos \varphi = \cos \varphi'$ very nearly; and assuming this relation, we obtain—

$$\frac{D}{d} = \frac{1}{\mu},$$

$$\text{Or} \quad d = \mu D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Cor. If the rays proceed from a point within the denser medium, as is generally the case, we must (Art. 15, II.) write $\frac{1}{\mu}$ for μ in (18).

60. By (18) we are enabled to explain the fact that a river in which the bottom may be seen, is deeper than it really appears when viewed from above the surface.

Since the rays proceed from the bottom of the river they pass from dense to rare, and hence we must write the reciprocal of μ in (18). But for water $\mu = \frac{4}{3}$;

$$\text{hence} \quad d = \frac{3}{4} D,$$

or the river appears three-fourths as deep as it really is.

Refraction at two Plane Surfaces.

61. When the surfaces are parallel, a ray passing them in succession suffers no deviation, but simply a displacement.

Prop. To find the displacement.

Fig. 20.

Let AB, CD, be the parallel surfaces, and $QPSq$ the path of a ray of light.

Produce qS backwards to meet AB in T; draw PR, SN, perpendiculars upon ST and AB respectively.

Denote the displacement PR by D; the distance SN by t; and the angle of incidence TSN by φ.

Then $D = \mathrm{PR} = \mathrm{PS}\sin\mathrm{PSR}$.

But $\mathrm{PSR} = \mathrm{NSR} - \mathrm{NSP} = \varphi - \varphi'$;

$\therefore\ D = \mathrm{PS}\sin(\varphi - \varphi')$.

Again, $\mathrm{PS} = \mathrm{SN}\sec\mathrm{PSN} = t\sec\varphi'$.

$\therefore\ D = t\sec\varphi'\sin(\varphi-\varphi')$.

And this readily reduces to

$$D = t\sin\varphi\left(1 - \frac{\cos\varphi}{\sqrt{\mu^2 - \sin^2\varphi}}\right) \quad \ldots\ldots\ (19)$$

Note.—It is worthy of remark, that this equation is transformed into (8) by writing -1 for μ, and taking the negative root of the surd.

WHEN THE SURFACES ARE INCLINED.

62. A medium bounded by planes whereof two of them meet at an angle, is in Optics termed a *prism*.

The line of intersection of the planes is the *edge*, and the planes which meet to form the edge, are the *faces* of the prism.

The *axis* is any line in the prism parallel to its edge.

Fig. 21.

Let AEB be a section of a prism perpendicular to its axis; then, EA and EB are the faces; E, the edge; and the angle AEB, the refracting angle, or simply the angle of the prism.

Let $QPSq$ be the path of a ray of light; draw the normals CF and DG meeting in F; also, produce QP to meet AE in T, and produce qS backwards to meet QT in R.

QPC is the *angle of incidence*;

and qSD is the *angle of emergence*.

63. Prop. To find the relation between the angles of incidence and emergence, the angle of the prism being given.

Denote AEB by ε, QPC by φ, qSD by ψ;

also, SPF by φ', and PSF by ψ'.

Since EPF and ESF are right angles, SEP is the supplement of SFP; but GFP is also the supplement of SFP, (Euc. I. 13)—

$\therefore$ GFP = AEB = ε.

Again, (Euc. I. 32), GFP = FSP + FPS.

$$\therefore \varepsilon = \varphi' + \psi' \ldots\ldots (a)$$

Again, $\because$ (a), $\psi' = \varepsilon - \varphi'$. $\therefore \sin\psi' = \sin\varepsilon\cos\varphi' - \cos\varepsilon\sin\varphi'$,

and $\mu\sin\psi' = \sin\psi = \mu\sin\varepsilon\cos\varphi' - \mu\cos\varepsilon\sin\varphi'$.

or (1) $\sin\psi = \mu\sin\varepsilon\cos\varphi' - \cos\varepsilon\sin\varphi$;

and writing for $\cos\varphi'$ its value in terms of $\sin\varphi$ and reducing, we obtain—

$$\sin\psi = \sin\varepsilon\left(\sqrt{\mu^2 - \sin^2\varphi} - \sin\varphi\cot\varepsilon\right) \ldots\ldots (20)$$

Cor. If all the angles be very small, we may write the circular measure of the angle for the sine, and reject $\sin^2\varphi$ in comparison with μ^2, and thus reduce (20) to—

$$\psi = \varepsilon\mu - \varphi \ldots\ldots (21)$$

64. To find the deviation.

In fig. 21, TRq is the deviation, which denote by ξ. Then (Euc. I. 32), TRS = RPS + RSP,

= RPF − SPF + RSF − PSF = $\varphi - \varphi' + \psi - \psi'$;

But $\varphi' + \psi' = \varepsilon$, by (a);

$$\therefore \xi = \varphi + \psi - \varepsilon \ldots\ldots (22)$$

Cor. If the angles be small, we have from the corollary to the last Article, $\psi = \mu\varepsilon - \varphi$; and this in (22) gives—

$$\xi = (\mu - 1)\,\varepsilon \ldots\ldots (23)$$

Hence, in a thin prism we may safely assume that *the deviation varies as the angle of the prism.*

65. Equation (23) furnishes a convenient method of ascertaining the index of refraction of a given medium.

Let AEB be a thin prism, and QP a ray of light which would fall upon a screen at q', but by the interposition of the prism is made to fall at q, a point at a small distance from q'.

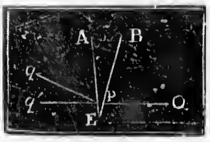

Fig. 22.

Then qPq' is the deviation, and is very nearly equal to qq' divided by $q'P$.

Hence denoting $q'P$ by d, and $q'q$ by δ, we have from (23),

$$\frac{\delta}{d} = (\mu - 1)\,\varepsilon;$$

from which,

$$\mu = \frac{\delta}{d\,\varepsilon} + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

In this manner, by forming a thin prism of the given medium, we are enabled to obtain a very close aproximation to its index of refraction.

66. Prisms are sometimes employed for purposes of Reflection, as stated in Art. 55. When so used it is customary to prevent all refraction by making the angles of incidence and emergence each equal to zero.

In this case the pencil meets the first surface at right angles and consequently suffers no refraction; it then undergoes one or more reflections, entirely within the prism, and finally emerges at right angles to another surface.

The object sought in applying prisms to the reflection of light is to change the direction of the rays with as little loss as possible.


Fig. 23.

Let (fig. 23) ABC be a section of a prism, and QPq the course of a ray of light meeting BC and AB at right angles. Let the angle ABC be denoted by θ.

Since the ray is regularly reflected at P, the angle QPq is double the angle of incidence upon AC. But QPq is the complement of ABC, that is of θ; hence,

$$\theta = \pi - 2\,\varphi.$$

$$\text{But (5)} \quad \xi = \pi - 2\,\varphi,$$

$$\therefore \xi = \theta.$$

From which it appears that the angle of the prism must be equal to the deviation required.

This deviation, and consequently the angle of the prism, is generally a right angle; in which case we find the angle of incidence upon the face AC to be 45°.

But from Art. 55, it appears that the ray cannot possibly pass out of glass when the angle of incidence is greater than about 42°; hence glass answers perfectly in such a prism.

67. In the Camera Lucida the light is twice reflected before leaving the prism, and hence (Art. 21, Cor.) the rays have their original order restored.

Let (fig. 24) ABCD be a section of the prism, and QPP$'q$ the course of a ray of light.

The deviation of the ray being given, to find the angle ADC made by the reflecting faces of the prism.

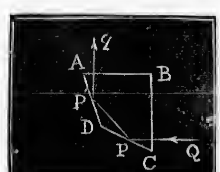

Fig. 24.

Since the deviations at the separate surfaces take place in the same direction, we have for the whole deviation, from (6),

$$\xi = 2\pi - 2\varepsilon;$$

$$\therefore \varepsilon = \text{ADC} = \frac{1}{2}(2\pi - \xi) = \pi - \frac{\xi}{2}.$$

and, ξ being generally a right angle,

$$\text{ADC} = \varepsilon = 135°$$

Refraction at Spherical Surfaces.

68. We investigate here the refraction of light at one spherical surface, merely as a preparatory step towards considering its refraction through lenses.

It must be understood that the system of rules in Art. 36, and the general remarks of Art. 38, are equally applicable here as in the case of spherical mirrors, and that they are therefore applied, without any further explanation, to the investigations which follow.

In fig. 25, let AOB be the surface, OQ its axis, and C its centre of curvature. Let QP be the course of a ray of light before refraction, and qP its direction after it.

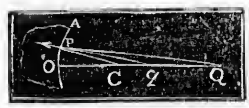

Fig. 25.

Denote QO by D, qO by d, CO by r and the index of the medium by μ. Then D and d are the distances of the conjugate points or foci of the surface from the point O in the surface.

69. To find the relation between D and d.

This relation is given accurately in (3); but assuming that the aperture is small, we have v a very small quantity in (3); and rejecting those terms into which v enters as a factor, as being of no account in comparison with D^2 and d^2, we reduce (3) to—

$$\frac{D-r}{d-r} = \mu \cdot \frac{D}{d}.$$

Hence, $\mu D r - d r = D d (\mu - 1)$,

and dividing by $D d r$,

$$\frac{\mu}{d} - \frac{1}{D} = \frac{\mu - 1}{r} \quad \text{..................} (24)_1$$

Since $(24)_1$ is independent of the aperture of the surface, i. e., of OP, it follows that all rays proceeding from Q, and meeting the surface not far from the axis, proceed after refraction as if coming from q; hence Q and q are the foci.

Of Lenses.

70. A lens is a portion of a medium bounded by two spherical surfaces which have a common axis.

This common axis is termed the axis of the lens. In this definition we consider the plane as the surface of a sphere having its radius infinitely great; hence, a medium bounded by parallel plane surfaces may be considered as a lens, although never used as such in practice.

The different forms which lenses may take are shown in the accompanying diagram, and these are sometimes distinguished by particular names, as follows:—

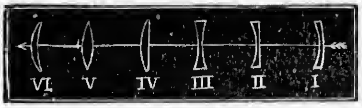

Fig. 26.

I. Convexo-concave,
II. Plano-concave,
III. Double-concave,
IV. Plano-convex,
V. Double-convex,
VI. Concavo-convex, or Meniscus.

It will be readily seen that lenses are naturally divided into two distinct groups; those which are thinnest at the centres and which receive the common name of concave lenses, and those which are thickest at the centres, and which are termed convex lenses.

These names are sufficient to characterize a lens in a general sense; but optically, a lens is known only when the radius of curvature of each surface is given, and its position with respect to the incident light.

In order that we may have only positive quantities in our results, we take as the *standard* lens that one which has the radius of curvature of each surface a positive quantity, and its principal focal length a positive quantity.

71. Prop. To find the relative positions of the foci in a thin lens.

In fig. 27, let PO, P′O′ be sections of the bounding surfaces, O′OQ the common axis, and C, C′ the centres of curvature.

Let a ray of light QP meet the first surface in P, and after refraction proceed to P′ upon the second surface as if coming from Q′; let it be now refracted at the second surface and

Fig. 27.

leave the lens as if coming from q. Then for the first surface, Q is the incident focus and Q′ the conjugate one; and for the second surface, Q′ is the incident focus and q the conjugate one; finally, for the lens Q is the incident focus and q the conjugate one.

Put $QO = D$, $Q'O = d'$, $qO = d$, $CO = r$, $C'O' = r'$.

Since the lens is thin, O′ is very near O, and practically we may consider them as coinciding.

Then for the first surface we have from $(24)_1$,

$$\frac{\mu}{d'} - \frac{1}{D} = \frac{\mu-1}{r} \qquad \text{(a)}$$

Now, in passing the second surface the light moves from a dense medium into a rarer one; hence, by Art. 15, II, we must write $\frac{1}{\mu}$ for the index of refraction.

Therefore, for the second surface, remembering that Q′ is now the incident focus,

$$\frac{\frac{1}{\mu}}{d} - \frac{1}{d'} = \frac{\frac{1}{\mu}-1}{r'},$$

Or
$$\frac{1}{d} - \frac{\mu}{d'} = \frac{1-\mu}{r'} \qquad \text{(b)}$$

Then, adding (a) and (b), $\frac{\mu}{d'}$ disappears, and we have—

$$\frac{1}{d} - \frac{1}{D} = (\mu-1)\left(\frac{1}{r} - \frac{1}{r'}\right), \qquad (25)$$

which is the relation required.

Definition. The *principal* focus of a lens is the focus conjugate for parallel incident rays; and the distance of the principal focus from the lens is called the *focal length* of the lens.

72. Prop. To find the focal length of a lens.

In order that the incident rays may be parallel, the incident focus must be infinitely distant.

Therefore, making $D = \infty$ in (25), and writing f for d, we obtain,

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{r} - \frac{1}{r'}\right) \qquad (26).$$

73. Prop. The focal length of a concave lens is a positive quantity.

A little consideration will suffice to show that a concave lens is characterized by one of the three following conditions, viz:

i. Both radii are positive, and $r < r'$.
ii. Both radii are negative, and $r > r'$.
iii. r is positive, and r' is negative.

Now since μ is always greater than unity, (Art. 10), therefore $\mu-1$ is a positive quantity; hence the algebraical sign of f, or the focal length, depends upon—

$$\frac{1}{r} - \frac{1}{r'}.$$

But this expression is positive under each of the three conditions stated above; hence, f, or the focal length, is positive.

74. By a process of reasoning precisely as in the last article, it may be shown that the focal length of a convex lens is a negative quantity.

75. From Arts. 73 and 74, it appears that in general the two species of lenses are fully distinguished by the algebraic sign of their focal lengths.

Moreover, since the light passes through the lens and is not thrown back as in the case of mirrors, the *real* conjugate focus of a lens is upon that side which is opposite to the incident light. It follows, then, that convex lenses have their principal focus *real*, and correspond to concave mirrors; while on the other hand, concave lenses have their principal focus *virtual*, and consequently correspond to convex mirrors.

76. Prop. To express the relative positions of the foci in terms of the focal length.

Writing, from Equation (26), the value of f in equation (25) we obtain—

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{f} \qquad (27)$$

Cor. From this we readily obtain—

$$d = \frac{Df}{D+f}.$$

77. Prop. To trace the relative positions of the foci.

Taking the concave lens as our standard, since in it only is the focal length positive, we may make the equation $D = nf$ to be generally true, by giving proper values to n.

Writing this value of D in (27) and reducing, we readily obtain—

$$\left.\begin{aligned} D &= nf. \\ d &= f.\frac{n}{n+1}. \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (A)$$

I. Concave Lenses.

i. While n is positive, D is positive, and d is (A) positive and less than f; hence, when the incident focus is real, the conjugate focus is between the lens and the principal focus, and is consequently virtual.

ii. If n is negative and less than unity, D is negative and less than f, while d is negative; hence, when the incident focus is behind the lens at a less distance than the focal length, the conjugate focus is also behind the lens and real.

iii. If n is negative and greater than unity, D is negative and greater than f, and d is positive; hence, when the incident focus is behind the lens at a greater distance than the focal length, the conjugate focus is in front of the lens and virtual.

II. Convex Lenses.

Changing the algebraic sign of f in (A) we obtain—

$$\left.\begin{aligned} D &= -nf. \\ d &= -f.\frac{n}{n+1}. \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots (B)$$

i. When n is positive, D is negative and d is negative: hence, when the incident focus is behind the lens, the conjugate focus is there also and is real.

ii. When n is negative, D is positive, while d is positive for all values less than unity, but negative for all values greater than unity; hence, when the incident focus is in front of the mirror, at a less distance than the focal length, the conjugate focus is also in front and virtual; but if the incident focus be at a greater distance than the focal length, the conjugate focus is behind the lens and real.

In a similar manner many other relative positions might be traced.

78. Prop. To find the deviation caused by a lens.

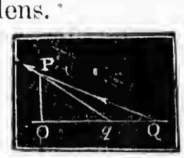

Fig. 28.

Let OP be a semilens, QP an incident ray of light, and qP the course of the refracted ray.

Then $\angle$ QPq = the deviation.

But $\angle$ QPq = PqO − PQO;

and $\angle \text{P}q\text{O} = \frac{\text{PO}}{\text{O}q} = \frac{\text{PO}}{d}$ nearly;

and $\angle \text{PQO} = \frac{\text{PO}}{\text{OQ}} = \frac{\text{PO}}{D}$ nearly.

Hence, denoting PO by a, and the deviation by ξ, we have,

$$\xi = \frac{a}{d} - \frac{a}{D} \text{ nearly.}$$

But (27), $\frac{1}{f} = \frac{1}{d} - \frac{1}{D}$;

$$\therefore \ \xi = \frac{a}{f} \qquad \text{(28)}$$

79. There is a point within or near every lens, such that if the direction of a ray, while within the lens, passes through that point, the ray suffers no deviation. That point is called the *optical centre* of the lens.

Prop. To find the Optical centre of a lens.

Let AB be a double convex lens having the axis COC′; and let C, C′, be the centres of curvature of the first and second surfaces respectively. Draw the radii CP and C′P′ parallel to one another. Join PP′. The point O, where PP′ cuts the axis CC′, is the centre.

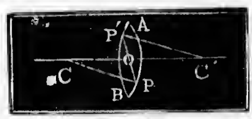

Fig. 29.

Since CP is perpendicular to the tangent at P, and C′P′ to the tangent at P′, and since CP is parallel to C′P′; therefore, the tangent at P is parallel to that at P′: hence, a ray entering the lens at P and passing through O will leave the lens at P′, and being refracted through the lens by parallel surfaces (Art. 61) it suffers no deviation.

80. From the preceding article it follows that every pencil of light which passes through a lens may have one of its rays so situated as not to suffer deviation, and this ray, whether situated centrically or not, is the axis of the pencil.

81. Prop. If the conjugate foci of a convex lens be upon opposite sides of the lens; then, the least distance between such foci is four times the focal length of the lens.

Since the foci are upon opposite sides, we have by putting v to represent their distance apart—

$$v = D + d.$$

Also, by remembering that d is a negative quantity from being measured behind the lens, and f is negative in convex lenses, we obtain from (27)—

$$d = \frac{Df}{D-f}.$$

Hence, $v = D + \frac{Df}{D-f}$;

or, $$D^2 - Dv = -vf;$$

and $$D = \frac{v}{2} \pm \frac{v}{2}\sqrt{1 - \frac{4f}{v}} \quad \ldots\ldots\ldots\ldots (29)$$

Now, in the part affected by the radical sign, we cannot have $4f$ greater than v without leading to an impossibility; hence the least value of v is $4f$.

This relation is very conveniently employed in the process of finding the focal length of a convex lens by experiment.

Of Refraction at two Lenses.

The doctrine of refraction at two lenses successively is an important one inasmuch as its principles enter into the working of all refracting telescopes, and the better class of microscopes.

In dealing with this subject there are, as a matter of course, many details into which we cannot enter in an elementary work, but we shall seek to develope those relations which are most important, and which form the groundwork of the whole subject.

82. Prop. Light being refracted successively at two lenses having a common axis; to find the position of the resultant focus.

Here, as elsewhere, it is necessary to adapt our formulae to positive quantities; hence, we must consider both lenses as concave, since (Art. 73) that species has a positive focal length.

Let then PO, P′O′ be sections of two concave lenses having O′OQ as their common axis; and let a ray of light, QP, fall upon the first or *primary* lens, and by refraction be turned into the direction PP′ as if coming from Q′; let it now meet the *secondary* lens at P′, and proceed, after leaving that lens, as if coming from q. Then Q is the incident focus; Q′ is the focus conjugate to Q but incident to the lens P′O′; and finally, q is the second conjugate or *resultant* focus.

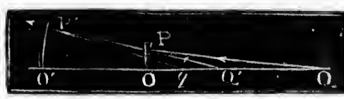

Fig. 30.

Denote QO by D, Q′O by d, Q′O′ by D', qO′ by λ, and OO′ by l.

Then, $D' = \text{Q}'\text{O}' = \text{Q}'\text{O} + \text{OO}' = d + l.$

Now putting F and f for the focal lengths of the first and second lenses respectively, we obtain from (27)—

$$\frac{1}{q\text{O}'} - \frac{1}{\text{Q}'\text{O}'} = \frac{1}{f},$$

$$\text{Or} \quad \frac{1}{\lambda} - \frac{1}{d+l} = \frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(30)}$$

and d is known from (27)—

$$\frac{1}{d} - \frac{1}{D} = \frac{1}{F}; \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(C)}$$

hence, λ is fully known.

In (30), l is necessarily a positive quantity, but λ, d and f may be either positive or negative.

83. Equations (30) (C) may undergo different transformations so as to be adapted to particular cases.

i. If the incident rays be parallel, we have from (C), $d = F$; and this relation in (30) gives—

$$\frac{1}{\lambda} - \frac{1}{F+l} = \frac{1}{f}; \quad \ldots\ldots\ldots\ldots\ldots \text{(31)}$$

a formula particularly adapted to the telescope.

ii. If the incident rays be not parallel, but if the lenses be in contact, we have $l =$ zero in (30), which gives—

$$\frac{1}{\lambda} - \frac{1}{d} = \frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots \text{(32)}$$

iii. If the incident rays be parallel, and if the lenses be in contact, we have, by writing F for d in (32)—

$$\frac{1}{\lambda} - \frac{1}{F} = \frac{1}{f},$$

$$\text{Or} \quad \frac{1}{\lambda} = \frac{1}{F} + \frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots \text{(33)}$$

But in this case λ is the focal length of the combination, since it determines the focus for parallel incident rays.

Therefore, if we call the reciprocal of the focal length of a lens its *power*, we have as a general rule;

The power of two lenses in contact is the sum of their individual powers.

And the same may be shown for any number of lenses.

84. It should here be distinctly understood that although Equation (30) gives the position of the resultant focus, it furnishes no knowledge as to the *power* of the combination.

This power depends upon the size of the image which would be formed at that focus, and will consequently be treated of in Chapter IV.

For convenience in reference, we here collect the principal formulae of this Chapter.

Critical angle; $\sin\varphi = \dfrac{1}{\mu}$ ....(16)

One plane surface;

φ large, $\dfrac{D}{d} = \dfrac{\tan\varphi'}{\tan\varphi}$ ....(17)

φ small, $\dfrac{D}{d} = \dfrac{1}{\mu}$ ....(18)

Displacement; $D = t\sin\varphi\left(1 - \dfrac{\cos\varphi}{\sqrt{\mu^2 - \sin^2\varphi}}\right)$ ..(19)

Prism; in general—

$\sin\psi = \sin\varepsilon\,(\sqrt{\mu^2 - \sin^2\varphi} - \sin\varphi\cot\varepsilon)$ ....(20)

$\xi = \varphi + \psi - \varepsilon$ ....(22)

angles small,

$\xi = (\mu - 1)\,\varepsilon$ ....(23)

Lens; $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{r} - \dfrac{1}{r'}\right)$ ....(26)

$\dfrac{1}{d} - \dfrac{1}{D} = \dfrac{1}{f}$ ....(27)

$\xi = \dfrac{a}{f}$ ....(28)

Two lenses; $\dfrac{1}{\lambda} - \dfrac{1}{d + l} = \dfrac{1}{f}$ ....(30)

Examples. B.

1. Required the angle of total reflection for a medium in which $\mu = 1{\cdot}020$.

From (16) $\sin\varphi = \dfrac{1}{\mu} = \dfrac{1}{1{\cdot}020}$;

$\therefore\ \operatorname{cosec}\varphi = 1{\cdot}020$,

and $\varphi = 78^\circ\ 38'$.

2. A plate of glass bounded by parallel surfaces is 3 inches thick, and its index of refraction is $\mu = 1{\cdot}5$; find the displacement of a ray which meets it at an angle of 60°.

Taking Equation (19) we have, $\mu = 1{\cdot}5$, $t = 3$, $\varphi = 60^\circ$;

$$\therefore D = 3 \times \frac{\sqrt{3}}{2} \left\{ 1 - \frac{\frac{1}{2}}{\sqrt{\left(\frac{3}{2}\right)^2 - \frac{3}{4}}} \right\},$$

$$= \frac{3\sqrt{3}}{2} \left(1 - \frac{1}{\sqrt{6}}\right),$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{2}}{4}, = 1{\cdot}537 \text{ inches.}$$

3. A prism in which the refracting angle is 45° is formed of a medium in which $\mu = 1{\cdot}2$, and a ray of light is incident upon one face at an angle of 30°; find the deviation.

First, from equation (20), $\epsilon = 45°$, $\mu = 1{\cdot}2$, $\varphi = 30°$;

$$\therefore \sin \psi = \frac{1}{\sqrt{2}} \left(\sqrt{1{\cdot}44 - \left[\frac{1}{2}\right]^2} - \frac{1}{2}\right),$$

$$= \frac{1}{\sqrt{2}} \left(\sqrt{1{\cdot}19} - \frac{1}{2}\right),$$

$$= {\cdot}4178;$$

hence, $\psi = 24° \, 42'$.

Second, from equation (22)—

$$\xi = 30° + 24° \, 42' - 45°,$$

$$= 9° \, 42'.$$

Note.—If in this example we use the approximative form given in (23) we obtain—

$$\xi = {\cdot}2 \times 45° = 9°;$$

a result which differs from the true one by 42′, or only about one thirteenth of the whole, although the angle of the prism is by no means a small one.

4. To find the focal length of a lens in which $r = -3$, $r' = 12$, and $\mu = 1{\cdot}5$.

From equation (26)—

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right) \left(\frac{1}{-3} - \frac{1}{12}\right),$$

$$= -\frac{5}{24},$$

$$\therefore f = -\frac{24}{5}, = -4{\cdot}8,$$

and the lens is convex.

5. A watch glass of five inches curvature is filled with spirits of turpentine, and a pencil of rays coming from a point 20 inches above it, is brought to a focus at a distance of 28 in. below it; determine the index of refraction of the fluid.

Taking equation (25), $d = -28$, $D = 20$, $r = \infty$ since the first surface is a plane, and $r' = 5$.

$$\therefore \frac{1}{-28} - \frac{1}{20} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{5}\right);$$

$$\therefore -\frac{3}{35} = (\mu - 1) \times -\frac{1}{5};$$

$$\therefore \mu - 1 = \frac{3}{7}, = 0{\cdot}428;$$

and $\mu = 1{\cdot}428.$

6. A screen is placed at a distance of 12 feet from a luminous point; where must a convex lens of 2 feet focal length be placed so that the conjugate focus may be at the screen?

Taking equation (29) we have, $v = 12$, $f = 2$, to find D.

$$\therefore D = 6\left(1 \pm \sqrt{1 - \frac{8}{12}}\right)$$

$$= 6 \pm 3.464,$$

$= 9.464$ ft. or 2.536 ft. from the luminous point.

7. The index of refraction for glass being 1·5, find φ' when $\varphi = 60°$.

8. When light falls upon a certain medium, $\varphi = 50°$, and $\varphi' = 35°$; find its refractive index.

9. If a vessel 12 inches deep be filled with alcohol how deep will it appear to be?

10. What must be the refracting angle of a prism of crown glass, that the deviation may be 5°?

11. Find the curvature of a plano-convex lens of water when the focal length is 4 inches.

12. One surface of a crown glass lens has a radius of curvature of 3 inches; find the radius of curvature of the second surface, when rays coming from a point 10 feet in front of the lens are brought to a focus 2 feet behind it.

13. In a combination of two lenses, the focal length of the primary is — 3 feet, and of the secondary 8 inches, and they are 2 feet apart; if the incident focus be 6 feet in front of the primary, what is the position of the resultant focus.

14. Find the position of the resultant focus in 13, when the lenses are in contact.

CHAPTER IV.

ON THE FORMATION AND SIZE OF IMAGES.

85. When a body is visible to us we see it by means of the rays of light which every point of it transmits to our eye, and we naturally refer the body to that position from which the rays appear to come, although it may at the same time be placed in a position quite different from the apparent one.

This is well exemplified in the case of the common looking-glass, where an object which is really and necessarily in front of the glass appears to be situated behind it.

Every point in front of the glass has its conjugate point behind the glass, and the rays really come to the eye as if from that conjugate point, and as if there was a second object placed in that position. Hence, if by any means we cause the rays which emanate from any point of a body to pass through a given point before reaching the eye, we naturally refer the real point to that through which the rays pass before entering the organ of vision.

This is the principle of the formation of an image, and may be illustrated as follows:—

Fig. 31.

Let AB be a convex lens, and MN an object in front of it. The rays proceeding from M have M as their incident focus, and, by principles established in Chapter III, have a conjugate focus at some point *m* where the rays cross before passing on to E; in like manner N has its conjugate focus at *n*, and every point between M and N has a conjugate point between *m* and *n*; moreover, these rays enter the eye at E precisely as if they came from a body placed at *mn*; for this reason we call *mn* the *image* of MN.

Again, since no ray of light coming from MN by way of the lens can reach *m* except those which come from M, it follows that the point *m* must possess all the peculiarities of brilliancy, colour, &c., which characterize the material point M. And the same may be shown for every pair of corresponding points in the object and its image.

If a screen be placed at $m\,n$, and the light from the object be of sufficient intensity, the concentration of rays at $m\,n$ will produce a picture of the object upon the screen, and this picture becomes visible by having its light scattered in all directions.

86. Prop. To establish an approximate relation between the linear dimensions of an object and the linear dimensions of its image.

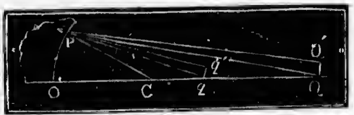
Fig. 32.

Let P (fig. 32) be a portion of surface bounding a dense medium upon the left, OQ a straight line passing not far from P, PC a perpendicular to the surface at P, and PO an arc of a circle described from C as a centre.

Then, since QO passes not far from P, PCO is a small angle.

Let QP be a ray of light meeting the surface of the medium at P, and after refraction let it proceed as if coming from q.

From what we saw in the last Chapter, Q and q are conjugate points.

Now, if QP be turned through a small angle QPQ′, qP will be turned through a small angle qPq'.

Put, then, $QQ' = O$, $qq' = I$, $\angle\, QPQ' = \delta$, $\angle\, qPq' = \delta'$.

And by previous notation, $QO = D$, and $qO = d$; also,

$$\angle\, QPC = \varphi \text{ and } qPC = \varphi'.$$

$$\text{Then} \quad Q'PC = Q'PQ + QPC = \delta + \varphi;$$
$$\text{and} \quad q'PC = q'Pq + qPC = \delta' + \varphi'.$$

But by the law of refraction, Art. 8,—

$$\text{Sin } Q'PC = \mu \sin q'PC.$$

$$\therefore \sin(\delta + \varphi) = \mu \sin(\delta' + \varphi');$$

Or $\sin\delta\cos\varphi + \cos\delta\sin\varphi = \mu\sin\delta'\cos\varphi' + \mu\sin\varphi'\cos\delta'$.

But since δ and δ' are very small angles we may write the circular measures for the sines, and put the cosines equal to unity, without material error;

$$\text{Hence, } \delta\cos\varphi + \sin\varphi = \mu\,\delta'\cos\varphi' + \mu\sin\varphi'.$$

But (1) $\sin\varphi = \mu\sin\varphi'$;

$$\therefore\ \delta\cos\varphi = \mu\,\delta'\cos\varphi'.$$

But since φ and φ' are both small, we may safely put $\cos\varphi$ and $\cos\varphi'$ each equal to unity, and we obtain—

$$\delta = \mu\,\delta' \ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

Finally, since $\delta = \frac{QQ'}{QP} = \frac{QQ'}{QO} = \frac{O}{D}$ nearly;

and $\delta' = \frac{qq'}{qP} = \frac{qq'}{qO} = \frac{I}{d}$ nearly;

we obtain from (D), $\frac{O}{D} = \mu . \frac{I}{d}$;

$$\text{Or} \qquad \frac{O}{I} = \mu . \frac{D}{d} \dots\dots\dots\dots\dots\dots (34)$$

This is the fundamental relation from which, in the course of this treatise, all the particular ones are drawn.

87. As will appear hereafter, I is sometimes affected with a negative sign; the meaning of this we shall now explain.

In our investigation we have taken qq' in the same direction as QQ′, both being upon the same side of the axis OQ. If by any means, when QP takes the position QP′, qP moves in the opposite direction so as to bring qq' below the axis, then, as we have considered qq' positive in its present position, it would be necessary to distinguish it as negative in the other position.

In order to distinguish these cases and prevent confusion, we adopt the following system of rules and definitions.

i. The Object is taken to represent by its position the positive quantity.

ii. When I is positive, the image lies in the same position as the object, and is said to be *erect*.

iii. When I is negative, the image is reversed in respect to the object, and is said to be *inverted*.

iv. Images formed at real foci are termed *real* images, while those formed at virtual foci are denominated *virtual* ones.

Images Formed by Reflection.

88. In Equation (34) making μ equal to -1 we adapt it to reflection, and obtain—

$$\frac{O}{I} = -\frac{D}{d} \dots\dots\dots\dots\dots\dots (35)$$

(a) PLANE MIRROR.

89. In plane mirrors, we have from (4),

$$D = -d;$$

and this relation in (35) gives—

$$\frac{O}{I} = 1 .$$

$\therefore\ O = I$; i.e., the image is equal to the object, and erect.

(b) Spherical Mirror.

90. Since the object must necessarily be before the mirror, we have D a positive quantity.

But from (12), $d = \frac{Df}{D-f}$.

And writing this for d in (35) gives—

$$\frac{O}{I} = \frac{f-D}{f}; \quad \text{.............} (36)$$

an equation which is true for concave mirrors, the focal length in that species being positive.

91. If we would adapt (36) especially to convex mirrors, we must change the algebraic sign of f, which gives—

$$\frac{O}{I} = \frac{f+D}{f} \quad \text{..................} (37)$$

It may here be remarked that although (37) is convenient in some investigations which follow, the student should not accustom himself to employ such in the solution of problems; but that he should rather have recourse to those containing positive quantities only, that being the sole means by which he can hope to arrive at accuracy and certainty in his results.

92. Prop. To trace the ratio between the linear dimensions of the Object and Image in a concave mirror.

Taking equation (36) we readily obtain—

$$\frac{O}{I} = 1 - \frac{D}{f}.$$

i. If $D < f$, $\frac{D}{f}$ is a proper fraction;

$\therefore\ \frac{O}{I}$ is a positive proper fraction;

$\therefore\ I > O$ and erect.

ii. If $D = f$, $\frac{D}{f} = 1$, and $1 - \frac{D}{f} = 0$;

$\therefore\ I$ is infinitely great.

iii. If $D > f$, then $\frac{D}{f}$ is an improper fraction, and

$1 - \frac{D}{f}$ is a negative quantity.

Hence, I is negative, or the image is inverted.

iv. If $D = 2f$, $\frac{D}{f} = 2$, and $1 - \frac{D}{f} = -1$.

$\therefore I = -O$, or the image is equal to the object, and inverted.

v. If $D > 2f$, $\frac{D}{f} > 2$, and $1 - \frac{D}{f}$ is negative and greater than unity; hence the image is less than the object, and inverted.

Hence, it appears that while the object remains at a less distance than the focal length, the image is erect; but when the object is at a greater distance from the mirror, the image is inverted.

93. Prop. To trace the ratio between the linear dimensions of the object and image in a convex mirror.

Taking equation (37), we readily perceive that $f + D$ is greater than f, and consequently that the object is greater than the image, both being positive; hence, in a convex mirror the image is less than the object and erect.

Images formed by Refraction.

(a) One Plane Surface.

94. Equation (34) applies directly to images formed by refraction at one surface of a medium. If, however, the surface be a plane, we must suppose that the incident rays meet it nearly at right angles.

Upon this supposition we obtain from (18)—

$$d = \mu D;$$

and writing this value of d in (34) we readily get—

$$O = I.$$

Hence, the image is equal to the object, and erect.

(b) Single Lens.

95. Let I' denote the linear dimensions of the image formed by the action of the first surface of the lens upon the incident light; I, of the image formed by the mutual action of both surfaces, or by the lens; and O, of the object.

Now it is evident that the image formed by the first surface becomes an object to the second surface, and that in passing the second surface the light passes from a dense medium into a rarer one.

Hence, from (34)—

$$\frac{O}{I'} = \frac{\mu D}{D'};\ldots\ldots\ldots\ldots\ldots\ldots\ldots(E)$$

D' being the distance of the first image I' from the origin.

Also,
$$\frac{I'}{I} = \frac{\frac{1}{\mu}\cdot D'}{d}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(F)$$

and multiplying together (E) and (F)—

$$\frac{O}{I} = \frac{D}{d}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(38)$$

Now taking the concave lens as our standard, since it has a positive focal length, we obtain from (27)—

$$\frac{D}{d} - 1 = \frac{D}{f};$$

hence,
$$\frac{O}{I} = \frac{f + D}{f}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(39)$$

96. Prop. To determine the nature of the image formed by a concave lens.

Taking equation (39) we readily perceive that f and D being essentially positive quantities, $f + D$ is greater than f; hence, O is greater than I; or, *the image is less than the object, and erect.*

Moreover, the image, being formed at (Art. 77. I. i.) a virtual focus, is *virtual.*

97. To trace the variations in the image formed by a convex lens.

Since the focal length of this species of lens is negative (Art. 74), we must change the algebraic sign of f in (39), which gives—

$$\frac{O}{I} = \frac{f - D}{f}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(40)$$

From equation (40) it appears:—

i. I is positive only when D is less than f; hence, *the image is erect whenever the object is at a less distance from the lens than its focal length.*

ii. I is negative when D is greater than f, since O is always positive; hence, *the image is inverted whenever the object is beyond the principal focus.*

iii. The conditions which render I positive (Art. 77. II. ii.), make it also virtual, while those which render it negative denote it to be real.

98. We have now considered the ratio of the linear dimensions of the image to those of the object, whether that image be formed by means of a mirror or a lens.

If, however, D becomes very great, or the object be inaccessible, or its distance or linear dimensions be unknown, the forementioned formulae are inapplicable.

In this case we may resort to *angular* measurements, and determine the ratio between the angles subtended, at the origin, by the object and by its image.

By rejecting the negative sign in (35) it becomes identical with (38); but, as has been stated (Art. 87), the negative sign is useful only in giving the position of the image; hence, rejecting it, we have for objects and their images the proportion—

$$\frac{O}{I} = \frac{D}{d};$$

$$\therefore \frac{O}{D} = \frac{I}{d}.$$

But $\frac{O}{D}$ measures, very nearly, the angle subtended, at the centre of the lens or mirror, by the object, and $\frac{I}{d}$ measures the corresponding angle of the image; hence, *at the centre of the lens or mirror employed the object and its image subtend equal angles.*

This relation is an important one, and is true for all practical values of D and d.

99. Prop. To find the linear dimensions of the image when the angular dimensions of the object are known.

Denote the angle subtended by the object by ω.

Then, angle subtended by the image equals ω by Art. 98—

$$\text{Or,} \qquad \frac{I}{d} = \omega;$$

$$\therefore I = d\,\omega \qquad (40)$$

Cor. If the object be very distant, as the sun or moon, we have $d = f$, and hence—

$$I = f\,\omega \qquad (41)$$

On Magnification.

100. We have seen (Art. 98) that when the lens is taken as an origin, the object and its image subtend equal angles; this is not so, however, when any other point upon the axis is taken as an origin. Thus, to an eye placed in some given position upon the axis, the object and image generally subtend very different angles, and consequently appear of very different sizes.

The effects thus produced are known by the general name of *magnification;* and when the image subtends an angle greater than that subtended by the object, the object is said to be *magnified.*

101. Magnification is the ratio of the angle subtended by the image to that subtended by the object, when a given point upon the axis is taken as an origin.

Let e be the distance of the object from the given origin, and e' the distance of the image.

Then, O and I denoting as before,

$$\frac{O}{e} = \text{the angle subtended by the object,}$$

$$\text{and } \frac{I}{e'} = \text{the angle subtended by the image.}$$

$\therefore$ Denoting the magnification by M—

$$M = \frac{I}{e'} \div \frac{O}{e};$$

$$\therefore M = \frac{I}{O} \cdot \frac{e}{e'} \qquad \text{(42)}$$

102. Prop. To obtain an expression for the magnification when the image is formed by reflection.

Taking all our quantities as positive, let O be a mirror, OS its axis, and S the given origin or position of the eye.

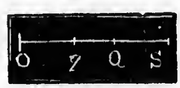

Fig. 33.

Denote SO by E; QO and qO as before.

Then $e = \text{SQ} = \text{SO} - \text{OQ} = E - D,$

and $e' = \text{S}q = \text{SO} - \text{O}q = E - d.$

Hence, from (42) we obtain—

$$M = \frac{I}{O} \cdot \frac{E-D}{E-d}.$$

But from (35) $\frac{I}{O} = -\frac{d}{D};$

$$\therefore M = -\frac{d}{D} \cdot \frac{E-D}{E-d} \qquad \text{(43)}$$

In applying (43) D is known, and d may thence be found from (12).

Cor. If the object be very distant, as the heavenly bodies, we obtain, by dividing by D, and then making D equal to infinity—

$$M = \frac{f}{E - f} \quad \ldots\ldots\ldots\ldots\ldots\ldots (44)$$

103. Prop. To find an expression for the magnification when the image is formed by refraction.

In the case of refraction the light is not thrown back but passes through the surface of the medium; hence the eye and the object must be upon opposite sides of that surface.

Let, then, O be a positive or concave surface or lens, SQ its axis, and S the position of the eye, or the origin.

Fig. 34.

Denote, as in the last article, SO by E.

Then, $e = \text{SQ} = \text{SO} + \text{OQ} = E + D$;

and, $e' = \text{S}q = \text{SO} + \text{O}q = E + d$.

And these values in (42) give—

$$M = \frac{I}{O} \cdot \frac{E + D}{E + d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (45)$$

In the case of lenses we obtain from (38)—

$$\frac{I}{O} = \frac{d}{D};$$

$$\therefore\ M = \frac{d}{D} \cdot \frac{E + D}{E + d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

104. If the rays come from below the surface of water, which is a very common case, we have from Art. 59. Cor.,

$$d = \frac{1}{\mu} \cdot D,$$

and from Art. 94, $O = I$; these relations in (45) give—

$$M = \frac{E + D}{E + \frac{D}{\mu}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

Since μ is greater than unity, $E + \dfrac{D}{\mu}$ is less than $E + D$; hence M is greater than unity.

From this it appears that bodies at the bottom of a vessel of water are magnified; and, upon the same principle, if we lay a thick plate of glass upon the page of a book the letters appear larger than before.

105. If in (46) D becomes indefinitely great, we obtain by a reduction similar to that employed in Art. 102, Cor.,

$$M = \frac{f}{E+f}; \qquad (48)$$

which is an expression adapted to concave or positive lenses.

Of the Image formed by a combination of Mirrors or Lenses.

106. In Articles 49 and 83 we determined the position of the resultant focus when light is successively reflected from two mirrors, or refracted through two lenses. We now propose to find an expression for the size of the resultant image, or the image formed at the resultant focus.

In addition to the notation of Arts. 48 and 82, we will, for convenience, term the combination a *compound* mirror or lens, the image formed by the first mirror or lens the *primary* image, and that formed by the combination the *resultant* image.

107. Prop. To determine the linear dimensions of the resultant image formed by a compound mirror.

Let O be the linear dimension of the object; I', of the primary image; and I, of the resultant image.

Also, let D and d be the distances of the object and primary image respectively from the primary mirror; and D' the distance of the primary image from the secondary mirror.

Then, remembering that the primary image becomes an object to the second mirror, we obtain from (36)—

$$\frac{O}{I'} = \frac{F-D}{F}, \text{ and } \frac{I'}{I} = \frac{f-D'}{f}.$$

And by multiplication—

$$\frac{O}{I} = \frac{O}{I'} \cdot \frac{I'}{I} = \frac{F-D}{F} \cdot \frac{f-D'}{f}.$$

But, as in Art. 49, we have, $D' = l - d$,

$$\therefore \frac{O}{I} = \frac{F-D}{F} \cdot \frac{f-l+d}{f} \qquad (G)$$

And writing for d the value given in equation (12), we finally reduce (G) to—

$$\frac{O}{I} = 1 - \frac{l}{f} - \frac{D}{F} - \frac{D}{f} + \frac{Dl}{Ff} \qquad (49)$$

108. When the incident rays are parallel, the single mirror, which will form an image in every respect equal to the resultant one, is termed the *equivalent* mirror.

109. Prop. To find the focal length of the equivalent.

Divide both members of (49) by D, and we have—

$$\frac{O}{D} \cdot \frac{1}{I} = \frac{1}{D}\left(1 - \frac{l}{f}\right) - \frac{1}{F} - \frac{1}{f} + \frac{l}{Ff}.$$

But $\frac{O}{D}$ = the angular size of the object $= \omega$; and making $D = \infty$, since the incident rays are parallel—

$$\frac{\omega}{I} = \frac{l}{Ff} - \frac{1}{F} - \frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

Again, if f' be the focal length of the equivalent, we must have (41), $I = f'\omega$;

and writing this in (II) we have—

$$\frac{1}{f'} = \frac{l}{Ff} - \frac{1}{F} - \frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

110. Prop. To determine the linear dimension of the resultant image formed by a compound lens.

Taking positive or concave lenses, and using the notation of Art. 107, we have from (39)—

$$\frac{O}{I'} = \frac{F + D}{F}, \text{ and } \frac{I'}{I} = \frac{f + D'}{f}.$$

$$\text{Hence, } \quad \frac{O}{I} = \frac{O}{I'} \cdot \frac{I'}{I} = \frac{F + D}{F} \cdot \frac{f + D'}{f}.$$

But from Art. 82, $\quad D' = d + l$;

and from (27), $\quad d = \frac{DF}{D + F}$.

and these relations finally reduce our equation to—

$$\frac{O}{I} = 1 + \frac{l}{f} + \frac{D}{F} + \frac{D}{f} + \frac{Dl}{Ff} \quad \ldots\ldots (51)$$

111. Prop. The elements of a compound lens being given to find the focal length of the equivalent lens.

Dividing both members of (51) by D, noticing that $\frac{O}{D} = \omega$; and making $D = \infty$, we obtain—

$$\frac{\omega}{I} = \frac{1}{F} + \frac{1}{f} + \frac{l}{Ff}.$$

And denoting the focal length of the equivalent by f', we have, since $I = f'\omega$,

$$\frac{1}{f'} = \frac{1}{F} + \frac{1}{f} + \frac{l}{Ff} \text{..............} (52)$$

It must be remembered that this is an expression for concave lenses, and that in applying it, the focal lengths must be taken with their proper signs.

Equations (50) and (52) find an important application in treating of the telescope.

For convenience in reference, we here collect the principal relations of this Chapter.

Spherical mirror; $\frac{O}{I} = \frac{f-D}{f}$ .... (36)

Lens; $\frac{O}{I} = \frac{f+D}{f}$ .... (39)

$I = f\omega$ .... (41)

Magnification;

Reflection, $M = -\frac{d}{D} \cdot \frac{E-D}{E-d}$ .... (43)

$D = \infty, \quad M = \frac{f}{E-f}$ .... (44)

Refraction, $M = \frac{d}{D} \cdot \frac{E+D}{E+d}$ .... (46)

$D = \infty, \quad M = \frac{f}{E+f}$ ...... (48)

Compounds;

Mirrors, $\frac{O}{I} = 1 - \frac{l}{f} - \frac{D}{F} - \frac{D}{f} + \frac{Dl}{Ff}$ ... (49)

$\frac{1}{f'} = \frac{l}{Ff} - \frac{1}{F} - \frac{1}{f}$ .... (50)

Lenses, $\frac{O}{I} = 1 + \frac{l}{f} + \frac{D}{F} + \frac{D}{f} + \frac{Dl}{Ff}$ .... (51)

$\frac{1}{f'} = \frac{l}{Ff} + \frac{1}{F} + \frac{1}{f}$ .... (52)

EXAMPLES C.

1. The focal length of a mirror is 6 inches, and an object 3 inches in length is placed at a distance of 6·25 inches from the mirror; determine the nature and size of the image.

Taking equation (36), we have $f = 6$, $D = 6{\cdot}25$, $O = 3$.

Hence, $$\frac{3}{I} = \frac{6 - 6{\cdot}25}{6} = -\frac{1}{24}$$

$$\therefore I = -72 \text{ inches.}$$

∴ The image is 72 inches in length, and inverted.

2. A mirror placed in front of an object 1 inch in diameter forms an image having its diameter one-fifth of an inch; the distance of the object from the mirror is 6 inches; determine the mirror employed.

Taking equation (36), $O = 1$, $I = \frac{1}{5}$, $D = 6$.

Hence, $$1 \div \frac{1}{5} = \frac{f - 6}{f}$$

$$\therefore 5f = f - 6$$

and $$f = -\frac{3}{2}.$$

∴ The mirror is convex, and its focal length is 1.5 inches.

3. If in the preceding example the diameter of the image were $I = -\frac{1}{5}$, what would be the nature of the mirror?

In this case we have,

$$1 \div -\frac{1}{5} = \frac{f-6}{f};$$

$$\therefore -6f = -6,$$

$$\text{or } f = 1.$$

And the mirror would be concave.

4. While looking at the page of a book from a distance of 12 inches, I place upon it a plate of glass 3 inches thick; determine the magnification which takes place, the value of μ being 1.5.

In equation (47) we have, $E = 9$, $D = 3$ and $\mu = 1.5$.

$$\therefore M = \frac{9 + 3}{9 + \frac{3}{1{\cdot}5}} = \frac{12}{11} = 1\frac{1}{11}.$$

Hence the letters appear one-eleventh larger than before.

5. The focal length of a convex lens is 3 inches, and an object one inch in length is placed at a distance of 2 inches

from the lens: determine the length of the image, and whether it is real or virtual.

In equation (39) we have, $O=1, f=-3$ since the lens is convex, and $D=2$.

$$\therefore \frac{1}{I}=\frac{-3+2}{-3}=+\frac{1}{3};$$

and $I=3$ inches.

And from Art 97, iii, it appears that the image is virtual.

6. When I place a convex lens of 6 feet focal length between my eye and the moon, at a distance of $7\frac{1}{2}$ feet from the eye; determine the magnification produced.

In Equation (48) we have, $f=-6$, $E=7\frac{1}{2}$.

$$\therefore M=\frac{-6}{7\frac{1}{2}-6}=-\frac{6}{1\frac{1}{2}}=-4;$$

Or the image appears four times as large as the moon.

7. An eye placed 6 feet above the surface of a pond sees a fish at the bottom; how much larger does it appear than it would if no water were interposed?

8. While looking at an object 18 inches distant, I bring a convex lens, having a focal length of 8 inches, between the eye and the object so as to be equally distant from each; do I gain or lose by so doing, and to what extent?

9. The object glass of a microscope consists of two convex lenses separated by an interval of 2 inches; the focal length of the primary is 3 inches, and of the secondary 8 inches. An object .2 inches in length is placed at a distance of 4 inches in front of the primary. Determine the position and size of the image.

Use Equations (30) and (51).

10. A person in reading a book at a distance of 12 inches, uses a convex eyeglass, the focal length being 20 inches; he holds this glass 2 inches from the eye. Required the magnification.

11. A certain lens held at a distance of 3 feet from the eye, causes the moon to appear double its former size. Determine the lens.

CHAPTER V.

ON ABERRATION.

112. It was discovered by Sir Isaac Newton, that when a small pencil of white light, such as may be received from the sun, passes obliquely through the surface of a medium it is not only bent out of its former course, but it is also decomposed or separated into rays possessing different degrees of refrangibility and being highly coloured.

The diagram will illustrate this. Let EF be a white screen, and ST a small pencil of light coming from the sun and gaining admission by a hole in the shutter of a darkened room. The rays will produce a small bright spot at T where they meet the screen. If now the prism ABC be interposed in a position as shown in the diagram, the pencil is bent out of its former course; but instead of making a single bright spot as might be expected, its rays are scattered or dispersed in a direction at right angles to the axis of the prism, and form upon the screen an oblong spot which exhibits a regular gradation of colours from red, which is the lowest or least refracted, to violet, which is the most refracted.

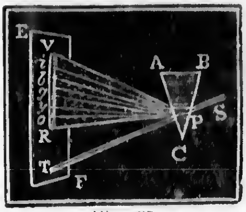

Fig. 35.

This coloured oblong spot is called the *solar spectrum*, and the phenomenon of its formation, is called the *chromatic aberration* or *dispersion* of light.

113. The solar spectrum contains seven principal colours disposed in the following order—commencing with the least refrangible—red, orange, yellow, green, blue, indigo, violet. These colours, however, are so nicely blended at their adjacent borders, and the gradation is so exceedingly regular that it is difficult to say where one colour ends and another begins.

Besides the solar spectrum, every luminous or incandescent body gives off rays of light which form its own peculiar spectrum.

By careful observations upon different spectra it has been discovered that they are crossed by many fine dark and bright lines, and that the positions and characters of these lines fully determine the nature of the light experimented upon.

This discovery gave rise to the science of *spectrum analysis*—a science which, though in its infancy, has done wonders in solving many physical problems.

114. From what has been said, it follows that in the same medium the differently coloured rays cannot have the same index of refraction; and, although the difference is in all cases small, the index will be greatest for the violet rays, and least for the red. The mean index given in the table is an arithmetical mean between these two.

Def. The angle (VPR, fig. 35) contained between the direction of the red ray and that of the violet one is called the *dispersion.*

115. Prop. To find the dispersion when a ray of white light passes through a thin prism.

Put u = the dispersion;
μ' = the index for red rays;
and μ'' = the index for violet rays.

Then, we have from (23),

$(\mu' - 1)\,\varepsilon$ = the deviation of the red rays,
and $(\mu'' - 1)\,\varepsilon$ = the deviation of the violet rays.

But the dispersion is the difference of the deviations;

hence, $u = (\mu'' - 1)\,\varepsilon - (\mu' - 1)\,\varepsilon$;

Or $$u = (\mu'' - \mu')\,\varepsilon \ldots\ldots\ldots\ldots\ldots (53)$$

116. Prop. In one and the same medium the dispersion varies as the mean deviation.

From (53) $u = (\mu'' - \mu')\,\varepsilon$;
and from (23) $\xi = (\mu - 1)\,\varepsilon$;
and eliminating ε, we obtain,

$$u = (\mu'' - \mu') \cdot \frac{\xi}{\mu - 1}.$$

But in the same medium, μ', μ'', and μ are constants; hence u varies as ξ.

117. From Art. 116, it appears that for any one medium,

$$\frac{u}{\xi} = \text{a constant quantity},$$

$$= \frac{\mu'' - \mu'}{\mu - 1}$$

This constant is called the *Dispersive Power* of the medium; and it is, like the index of refraction, different in different media, but constant for the same medium.

A table of Dispersive Powers is appended to this treatise.

118. Representing the dispersive power by U, we have from the last Article—
$$U = \frac{u}{\xi};$$
$$\therefore u = U\,\xi \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

i. e. *The dispersion is equal to the product of the dispersive power and the deviation.*

119. Prop. To find the dispersion produced by a thin prism when its refracting angle and dispersive power are known.

From (23) we obtain for the deviation—
$$\xi = (\mu - 1)\,\varepsilon.$$

And writing this in (54),
$$u = U\,\varepsilon\,(\mu - 1) \ldots\ldots\ldots\ldots\ldots\ldots (55)$$

120 Prop. To find the dispersion when a ray of white light passes through a thin lens.

If the ray passes centrally there is no deviation, and consequently no dispersion; but if it does not pass centrally, put a for the distance from the axis at which it meets the lens.

Then we have from (28), $\xi = \frac{a}{f}$.

And this in (54) gives
$$u = \frac{U\,a}{f} \ldots\ldots\ldots\ldots\ldots\ldots (56)$$

121. The general effect of chromatic dispersion upon the nature of the image may be shown as follows:—

Let OP be a convex lens, OO′ its axis, and MN an object in front of it. Let NOR be the ray which passes centrally, NP a ray meeting the lens at P, and after refraction, let PR be the course of the red ray, and PV that of the violet ray.

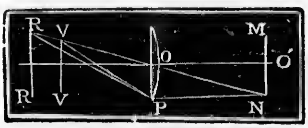

Fig. 36.

Since PV is more refracted than PR, it meets the ray NR at a point V nearer to the lens than that in which the red ray meets it. And the same holds good for the red and violet constituents of every ray which goes to form the image. The violet rays form their image at VV, and the red form theirs at RR; and since all the other colours are intermediate between these two, they form their several images between VV and RR.

These images increase in size from VV to RR, so that when viewed from the side opposite the lens the red image appears to overlap the orange one, the orange the yellow, &c.; the general effect being to render the image somewhat confused, and to fringe its borders with the prismatic colours.

On the means of Correcting Chromatic Aberration.

122. The ill effects of Chromatic Aberration in the working of lenses being far greater in magnitude than those arising from all other sources, it was found highly important in the construction of the better class of instruments to devise, if possible, some means of correcting it. To effect this, Newton laboured long and unsuccessfully, and at length gave it up in despair, turning his attention to the use of mirrors in the construction of his telescopes.

Newton arrived at the conclusion, which has been found to be erroneous, that the dispersion was in all cases proportional to the deviation; and hence, that the construction of a lens which should correct the dispersion was impossible.

Many years later Mr. Dollond, an optician of London, by repeating the experiments of Newton, but with different media, found that the dispersive power has no connection with the index of refraction: but that two media having the same index of refraction may have quite different dispersive powers, and *vice versa*.

This discovery led to the construction of a compound lens consisting of two, or sometimes three, simple ones composed of different media, and so related that while their joint effect produced a certain amount of deviation, it destroyed very nearly the whole of the dispersion.

Such a compound lens is said to be *achromatic*, and it consists generally of a convex lens of crown glass united with a convex one of flint glass; or, in the case of a compound consisting of three simple lenses, of two convex lenses of crown glass having a concave one of flint glass between them.

123. Prop. To find the ratio between the refracting angles of two prisms formed of different media, so that a ray of white light passing through both in close succession may not suffer dispersion.

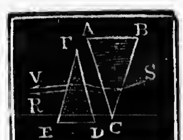

Fig. 37.

Let ABC, DEF, be two prisms placed with the two faces AC and FD very close to one another, and having their refracting angles turned in opposite directions.

A ray of white light meeting the first prism at P undergoes a certain amount of dispersion;

but on passing the second prism the dispersion takes place in the opposite direction, and if the prisms be properly proportioned, the whole dispersion may be reduced to zero.

Denote by ε, μ, U the refracting angle, index of refraction, and dispersive power of the first prism;

and by ε', μ', U' the like quantities in the second prism. Then, by (55),

dispersion at first prism $= U \varepsilon (\mu - 1)$;

dispersion at second prism $= U' \varepsilon' (\mu' - 1)$.

But these dispersions being in opposite directions, when they are equal the whole amount must be zero.

Hence, $U \varepsilon (\mu - 1) = U' \varepsilon' (\mu' - 1)$;

Or,

$$\frac{\varepsilon}{\varepsilon'} = \frac{U' (\mu' - 1)}{U (\mu - 1)} \ldots\ldots\ldots\ldots\ldots (57)$$

124. The total deviation produced by the combination of the last Article is the difference of the partial deviations.

Hence, from (23),

total deviation $= (\mu - 1) \varepsilon - (\mu' - 1) \varepsilon'$.

And eliminating $(\mu' - 1) \varepsilon'$ between this and (57), we obtain—

$$\xi = \varepsilon (\mu - 1) \left(1 - \frac{U}{U'}\right) \ldots\ldots\ldots\ldots\ldots (58)$$

From which it appears that if U and U' have different values, i. e., if the prisms have different dispersive powers, there will be *some* deviation when the dispersion becomes zero.

125. Prop. To find a relation between the focal lengths of the components of an achromatic lens.

From equation (56) we learn that the algebraical sign of the dispersion is the same as that of the focal length, and hence, that it is positive in concave and negative in convex lenses. Therefore, in order that the total deviation may be zero, we must combine the two species of lenses.

Moreover, since the combination must be upon the whole convex, we must form the convex constituent of that medium which has the lowest dispersive power.

Let, then, U, F be the dispersive power and focal length of the convex lens; and U', f the same quantities in the concave one.

From (56) we obtain,

dispersion at convex lens, $= \dfrac{U a}{F}$;

dispersion at concave lens, $= \dfrac{U' a'}{f}$.

But since the lenses are in contact, the ray meets each at very nearly the same distance from the axis;

$$\therefore a = a';$$

and since the dispersions are in opposite directions, we must have—

$$\frac{Ua}{F} = \frac{U'a}{f},$$

in order that the whole dispersion may be zero.

$$\therefore \frac{f}{F} = \frac{U'}{U} \dots\dots\dots\dots\dots\dots (59)$$

∴ *The focal lengths must be proportional to the dispersive powers of the media employed.*

126. Although the achromatic lens is an immense improvement upon the simple lens of equal dimensions, yet in its general form, as being composed of lenses of crown and flint glass, it is not perfect. The cause of this imperfection may be explained as follows:—

If two spectra of equal dimensions, one formed by a prism of crown glass and the other by one of flint glass, be placed side by side, it will be seen, upon careful observation, that the positions occupied by the various colours do not exactly correspond; but that while the extreme rays hold the same relative positions in each spectrum, the intermediate and central ones are a little higher or lower in one spectrum than in the other.

On account of this, that proportion between the focal lengths of the constituent lenses which totally corrects the extreme rays can not fully do so in the case of the central ones. Hence, in the best lenses consisting of two simple ones of crown and flint glass, the images of bright objects are always slightly tinged about the borders with green or blue, they being the central colours of the spectrum.

To correct this slight imperfection, recourse is had to a compound composed of a concave lens of flint glass placed between two convex ones of crown glass, they being made to differ slightly in their effects by varying the constituents of the glass. Such a lens is termed a *triplet;* it is not however generally used, since the gain in the perfection of the image does not compensate for the extra loss of light arising from an addition in the number of surfaces, and the increased errors of workmanship.

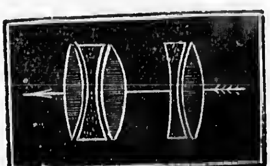

Fig. 38.

The two kinds of achromatic lenses are shown in the margin.

127. An achromatic combination may be formed by two convex lenses of the same material separated by a given interval. Such combinations are used as eye-pieces, as objectives for common microscopes, &c.

Let AB, CD represent sections of semi-lenses having the common axis QR.

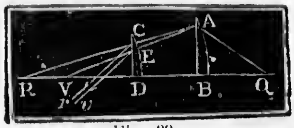

Fig. 39.

Let a ray of light coming from Q meet the first lens at A, and there suffering dispersion, let AR and AV be the directions of the red and violet constituents respectively. Then the red constituent, meeting the lens CD at a point more remote from the axis than that at which the violet portion meets it, undergoes a greater deviation; and if the lenses be separated by a proper interval, the red and violet portions after passing CD become parallel, and are thus deprived of any effective colour.

Prop. To find the interval DB.

Put QB = D, BD = l, AB = a; also, denote the focal length of AB for violet rays by F, and for red rays by $n\,F$; then, since the lenses are of the same material, if f denote the focal length of CD for violet rays, nf will be its focal length for red ones.

The dispersion being in all cases a small quantity, n is very nearly unity, and F and f are very nearly the mean focal length of the lenses.

Since the red and violet constituents are parallel before meeting AB, and also parallel after leaving CD, they must have undergone the same amount of deviation.

By (28) we have,

$$\text{deviation of red at AB} = \frac{a}{n\,F};$$

$$\text{deviation of red at CD} = \frac{\text{C D}}{n\,f};$$

$\therefore$ for the total deviation of the red ray we have—

$$\frac{a}{n\,F} + \frac{\text{C D}}{n\,f}.$$

Similarly, for the total deviation of the violet we obtain—

$$\frac{a}{F} + \frac{\text{E D}}{f}.$$

And since these are equal, we have—

$$\frac{a}{nF} \therefore \frac{CD}{nf} = \frac{a}{F} + \frac{ED}{f},$$

which reduces to

$$f(1-n) = F\left(n \cdot \frac{ED}{a} - \frac{CD}{a}\right) \ldots\ldots (K)$$

Again, from (27), $\frac{1}{BR} = \frac{1}{nF} - \frac{1}{D}$;

and from similar triangles

$$\frac{CD}{a} = \frac{CD}{AB} = \frac{DR}{BR} = \frac{BR - BD}{BR} = 1 - \frac{l}{BR}.$$

Hence,

$$\frac{CD}{a} = 1 - \frac{l}{nF} + \frac{l}{D}.$$

Similarly,
$$\frac{ED}{a} = 1 - \frac{l}{F} + \frac{l}{D}.$$

Writing these values in (K), and reducing, we obtain—

$$l = \frac{F + f}{\frac{n+1}{n} - \frac{F}{D}}.$$

But n being very nearly unity, and F and f very nearly the focal lengths of the lenses for the central rays, we may introduce those approximate values without material error, and hence obtain—

$$l = \frac{F + f}{2 - \frac{F}{D}} \ldots\ldots\ldots\ldots\ldots\ldots (60)$$

Cor. If D is great in comparison with F, we have very nearly,

$$l = \frac{1}{2}(F + f) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (61)$$

$\therefore$ *The interval between the lenses must be one half the sum of their focal lengths.*

Spherical Aberration.

128. Chromatic Aberration arises from the nature of light and media, and is present only when refraction takes place; but there is another kind known as *spherical* aberration, which depends upon the form of mirrors or lenses, and which is common to both reflection and refraction.

For any given position of the incident focus there is some certain form which, if it were possible to apply it, would totally overcome this species of aberration; but owing to the mechanical difficulties which present themselves, attempts at the formation of such surfaces have not succeeded well; and the form generally used for the surfaces of mirrors and lenses is the spherical, as being the one which admits of the greatest accuracy in its formation. Hence the name of spherical aberration.

129. The general effects of spherical aberration may be shown as follows:—

Let OP be a convex lens, COc its axis, and MN an object in front of it. Let $m\,n$ be the image of MN formed by those rays which pass centrally.

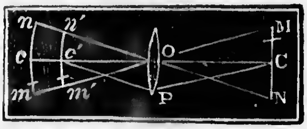

Fig. 40.

The points m, c, and n, will be nearly equi-distant from the point O, and hence the image will be a curve, approaching to an arc of a circle.

Again, the ray CP which passes through the extreme portions of the lens has its conjugate focus at c', a point nearer to the lens than c: hence, at $m'\,n'$ there will be an image formed by the extreme rays, within the one formed by the central rays; and the lens being continuous, there will be a continuity of images filling up the space between $m\,n$ and $m'\,n'$; and these images increasing in size in proportion to their distances from the lens, the outer ones overlap the inner ones.

Hence, the effect of spherical aberration is to make the image curved, and to produce an indistinctness or confusion of outline.

This species of aberration is many times less in amount than chromatic aberration, and in lenses of small size and great focal length it becomes almost imperceptible. But when the diameter of a lens or mirror is large in comparison with its focal length, the ill effects of this aberration are so decided as to render these instruments entirely useless in the formation of images; and even with small lenses it becomes necessary, in fine instruments, to correct the aberration as far as possible by using combinations.

A combination which corrects the effects of spherical aberration is said to be *aplanatic*.

The condition of perfect aplanatism is never fulfilled in practice, for the same compound must at the same time be achromatic, and it usually happens that both aberrations are only partially, although nearly, corrected.

130. For a full investigation into the nature and amount of spherical aberration the student must consult some of the larger works upon this subject, since all our limits will allow is to give a mere abstract of some of the most important results.

Mirrors.

i. The aberration of concave mirrors is opposite to that of convex ones; so that whenever these two kinds occur in combination the aberration of one tends to correct that of the other.

ii. The aberration decreases in amount as the incident focus approaches the centre of curvature, and at that point it becomes zero.

iii. When the incident rays are parallel, the aberration is nearly equal to the square of the radius of the mirror divided by four times the radius of curvature.

Lenses.

iv. The aberration of concave lenses is opposed to that of convex ones, so that a combination of these tends to correct the resultant aberration.

v. The aberration in lenses varies with the focal length and also with the radii of curvature; hence, lenses of the same focal length may give rise to very different amounts of aberration.

For convenience in reference, we here collect the principal forms of this chapter.

Dispersion; $u = (\mu'' - \mu')\varepsilon$ .... (53)

$u = U\xi$ .... (54)

$u = U\varepsilon(\mu - 1)$ .... (55)

$u = U \cdot \frac{a}{f}$ .... (56)

Achromatic lens; $\frac{f}{F} = \frac{U'}{U}$ .... (59)

$l = \frac{1}{2}(F + f)$ .... (61)

Examples D.

1. The dispersive power of crown glass being about one-twenty-eighth, determine the deviation of a ray when its dispersion is 10′.

Taking (54) we have, $u = 10'$, $U =$ one-twenty-eighth.

$$\therefore\ 10' = \xi \times \frac{1}{28};$$

$$\therefore\ \xi = 280', = 4^\circ\ 40'.$$

2. A prism of crown glass produces a dispersion of 15′, what is its refracting angle, μ being 1·5?

In (55) we have, $u = 15'$, $U = \dfrac{1}{28}$, $\mu = \dfrac{3}{2}$

$$\therefore\ 15' = \frac{1}{28}\left(\frac{3}{2} - 1\right)\varepsilon = \varepsilon \times \frac{1}{56}.$$

$$\therefore\ \varepsilon = 840', = 14^\circ.$$

3. It is required to make an achromatic lens of crown and flint glass which shall have a focal length of 10 feet; determine the focal lengths of the constituents.

This is solved by combining (33) with (59).

The whole focal length is $\lambda = 10$ feet; and for the dispersive powers we have from the table $U' = \cdot 048$, $U = \cdot 036$.

$$\therefore\ (33) \qquad \frac{1}{10} = \frac{1}{F} + \frac{1}{f};$$

$$\text{and (59)} \quad \frac{f}{F} = \frac{U'}{U} = \frac{\cdot 048}{\cdot 036} = \frac{4}{3}:$$

$$\therefore f = \frac{4}{3}\ F;$$

$$\text{and} \qquad \frac{1}{10} = \frac{1}{F} + \frac{3}{4\,F};$$

whence, $F = 17\frac{1}{2}$ feet, and $f = 23\frac{1}{3}$ feet.

4. Find the dispersion produced by a lens of rock salt $1\frac{1}{2}$ inches aperture and 10 inches focal length.

5. Determine the lens of crown glass which will achromatize the lens of problem 4.

6. Determine the angles of the constituents in a compound prism composed of flint glass and diamond when, being achromatic, it causes a deviation of 5°.

CHAPTER VI.

ON ILLUMINATION AND BRIGHTNESS.

131. Bodies are sometimes divided into luminous, which shine by their own inherent light, and non-luminous, which become visible by reflecting or scattering the light received from a luminous body.

This division, though convenient at times, is not adapted to our purpose, for it is immaterial, as far as the light itself is concerned, whether that light be direct or reflected. Thus the moon, although in reality an opaque body, sends forth her rays in the same manner as if she were luminous. On this account we shall, in what follows, consider and speak of all bodies which radiate light as being luminous.

132. Every luminous body is considered to be made up of luminous particles, each of which radiates light equally in all directions. Moreover, these particles are considered to form a luminous surface placed at right angles to the direction of the central ray of the pencil with which we have to deal. Finally, since the general effects of a luminous surface do not depend upon its form, we shall, for the sake of simplicity, consider every luminous surface as having a circular outline.

133. When rays of light fall upon any surface, that surface is said to be *illuminated;* and the illumination is measured by the number of rays which impinge upon each unit of that surface.

Hence, if n rays fall upon s units of surface, and if l denote the illumination, we have,

$$l = \frac{n}{s} \qquad (62)$$

134. Light emanates from a luminous particle in conical pencils, each of which has its vertex at the particle. Let us take for a unit that pencil whose semi-vertical angle is α, it being in all cases a very small angle, and let such a pencil be composed of n rays.

Let ASB (fig. 41) be such a conical pencil falling upon the screen MN placed at right angles to the axis SC. The pencil will illuminate a circular spot at AB.

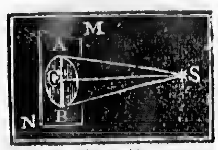

Fig. 41.

Denote SC by d; then also, ASC $= \alpha$; hence, AC $= d\,\alpha$ very nearly, α being very small; the area of the circle AB is $\text{AC}^2\,\pi$, or $(d\,\alpha)^2\,\pi$, and it receives n rays; hence, denoting the illumination by l, we get from (62),

$$l = \frac{n}{a^2 \pi} \cdot \frac{1}{d^2}; \quad \ldots\ldots\ldots\ldots (63)$$

and π being constant, and n and a constant for the same units, we have, as a result, the following:—

Other things being the same, the illumination varies inversely as the square of the distance.

And this being true for each individual particle, is true for any luminous surface formed of such particles.

135. From the foregoing article it appears that any two luminous bodies may be made to give the same amount of illumination by properly choosing the distances. Hence, in order to compare the values of luminous bodies as sources of illumination, we have recourse to a constant element, in the body itself, denominated *the illuminating power.*

This element is the illumination given at the unit of distance; hence, making d equal to unity, and denoting the illuminating power by L, we obtain from (63),

$$L = \frac{n}{a^2 \pi}; \quad \ldots\ldots\ldots\ldots (64)$$

and this relation in (63) gives,

$$l = \frac{L}{d^2} \quad \ldots\ldots\ldots\ldots (65)$$

From (65) we infer that *the illumination is directly proportional to the illuminating power;* and also, that *the illuminating power is measured by the product of the illumination into the square of the distance.*

136. Article 135 furnishes the means of comparing two lights as to their value for illuminating purposes.

The lights being so placed as to produce the same amount of illumination, let L and d be the illuminating power and distance of the first, and L', d', of the second.

Then (65) illumination given $= \frac{L}{d^2}$;

and it also equals $\frac{L'}{d'^2}$;

$$\therefore \frac{L}{d^2} = \frac{L'}{d'^2};$$

Or $$L : L' :: d^2 : d'^2 \quad \ldots\ldots\ldots\ldots (66)$$

That is, *the illuminating powers are to one another directly as the squares of the distances from the illuminated screen.*

137. Prop. To find a point of equal illumination between two luminous bodies.

Let L, L' be their illuminating powers, and D their distance apart. Denote by x the distance of the point from the first body; then $D - x$ is its distance from the second.

From (66) we obtain,

$$L : L' :: x^2 : (D - x)^2,$$

$$\therefore \sqrt{L} : \sqrt{L'} :: x : D - x;$$

From which $$x = \frac{D\sqrt{L}}{\sqrt{L} + \sqrt{L'}} \quad \text{..............} (67)$$

Cor. Since the surds may take either plus or minus, we may consider $\sqrt{L'}$ as positive or negative. But if $\sqrt{L'}$ is negative, and is less than $\sqrt{L}$, it is evident that x must be greater than D. That is, there will be a second point situated beyond the weaker light which satisfies the condition of being equally illuminated, but not of being *between* the luminous bodies.

138. Prop. To find how the illumination varies according to angle of incidence.

In fig. 42, let S -- AB be a pencil of rays meeting the plane AC at right angles, and let the illumination produced be denoted by l.

Fig. 42.

Now let the plane be turned through an angle φ so as to take the position AB. Then, since the same number of rays is distributed over a greater space, the illumination of AB is less than that of AC. Denote this second illumination by l'. Then, if n be the number of rays in the pencil,

(62), $$l' : l :: \frac{n}{AB} : \frac{n}{AC} :: AC : AB;$$

But, $$AC = AB \cos\varphi;$$

$$\therefore l' : l :: \cos\varphi : 1,$$

and $$l' = l \cos\varphi:$$

and, writing for l its value from (65), we get,

$$l' = \frac{L\cos\varphi}{d^2} \quad \text{..................} (68)$$

Hence, it follows that, other things being the same, *the illumination varies directly as the cosine of the angle of incidence.*

Co-efficients of Transmission and Reflection.

139. Rays of light incident upon a medium are partly transmitted, partly reflected, and partly absorbed; in like manner, those which fall upon the surface of a reflector are partly absorbed and partly reflected. Hence, neither lenses nor mirrors render available the whole quantity of light received.

The ratio of the number of rays rendered available to the number received, is called, in the case of a lens, the *co-efficient of transmission*, and in that of mirrors, the *co-efficient of reflection.*

Denote this co-efficient by c. Then it follows that c is always less than unity. In the best specula it is scarcely ever greater than ·6 while in a well polished lens formed of a highly transparent and homogeneous medium it may be as high as ·9, although it varies to a considerable extent with the nature of the light.

It appears, then, that in the construction of Optical Instruments it is always desirable to employ as few lenses or specula as possible consistently with the object to be attained; and that in the choice of these, lenses have a preference over specula from rendering available a greater proportion of the incident rays.

Illumination by a Lens.

140. Let AB (fig. 42*), be a luminous body, PQ a lens enclosed in an aperture in a non-transparent wall, and MN a screen placed in the conjugate focus.

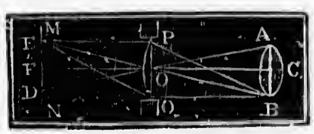

Fig. 42*.

The image of AB falls upon the screen at DE and illuminates the portion upon which it is depicted.

Denote the area of AB by A, of the lens PQ by a, and of the image DE by A'; also, denote CO by d, OF by δ, and the illumination of the screen by λ.

Then, the object and its image being similar, we have,

$$A' : A :: ED^2 : AB^2 :: FO^2 : OC^2 :: \delta^2 : d^2;$$

$$\therefore A' = A \cdot \frac{\delta^2}{d^2}.$$

Denoting by N the whole number of rays which fall upon the lens, we obtain,

$$N = l\,a,$$

l being the illumination of the lens, i. e., the number of rays which fall upon each unit of its surface.

And writing for l its value from (65),

$$N = \frac{L}{d^2} \cdot a \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (69)$$

Now, the number of rays which leave the lens and fall upon the screen at DE is,

$$c\,N = \frac{L}{d^2} \cdot ca$$

Therefore, according to (62), dividing this quantity by that expressing the area of the image, we obtain,

$$\lambda = \frac{c\,a}{\delta^2} \cdot \frac{L}{A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

If the focal length of the lens be small compared with the distance of the luminous object, δ is very nearly equal to the focal length and sensibly constant; also, c and a are constant for the same lens, and L and A for the same luminous body.

Hence, *The illumination of a screen placed at the conjugate focus of a given lens is sensibly constant, provided the focal length of the lens is small compared with the distance of the luminous object: and with different lenses it is proportional to their areas.*

141. Lenses are frequently used after the manner of Art. 140, for the purpose of intensely illuminating a small object which is to be viewed by the microscope, or for other purposes.

When thus employed, we infer from (70) that with the same light the illumination increases as the area of the lens, i. e., as the square of its diameter, and also inversely as the square of its focal length. Hence, the advantage, in such cases, of employing lenses of large diameters and short focal lengths.

Brightness.

142. The brightness of a body is a term generally employed to denote the effect of its light upon the eye.

The eye consists essentially of a lens, by which the rays are transmitted and an image formed, and a screen, termed the *retina*, upon which the image is received.

Brightness, then, is but another expression for the illumination of the retina, and as such is measured by equation (70).

The focal length of the lens of the eye is always many times less than the distance at which objects can be viewed, and hence δ is so nearly equal to that focal length, that we may write f for δ without introducing any sensible error.

Denoting then the brightness by B, we obtain from (70),

$$B = \frac{c\,a}{f^2} \cdot \frac{L}{A} \ldots\ldots\ldots\ldots\ldots\ldots (71)$$

Now, in this expression for the brightness, we notice that it consists of two factors, of which one depends for its variations entirely upon the eye itself, while the other is as entirely dependent upon the luminous object. But with the same eye and the same object, the only element which we can suppose to undergo any change is the size of the pupil. Hence, denoting the *radius* of the pupil by p, we must write $p^2\pi$ for a, and we thus obtain,

$$B = \frac{c\,\pi}{f^2} \cdot p^2 \cdot \frac{L}{A}.$$

Now, since our units are perfectly arbitrary, we simplify this expression by making the first factor our unit of measure; and we thus obtain,

$$B = \frac{L}{A} \cdot p^2 \ldots\ldots\ldots\ldots\ldots\ldots (72)$$

Hence, we infer that *the brightness of a given object is independent of its distance, and varies as the square of the radius of the pupil of the eye.*

143. When we view an object with the naked eye, the pupil takes in a pencil of rays which completely fills it; and although by the interposition of various instruments we may reduce the size of the pencil which enters the eye, yet we cannot increase it beyond the size of the pupil. Thus, by viewing an object through a small hole pierced in a plate of metal, we reduce the pencil which enters the eye to the size of the aperture, and the effect upon the brightness is the same as if the pupil of the eye itself suffered a like reduction.

Denoting by b this brightness, and by t the radius of the aperture, we obtain from (72),

$$b = \frac{L}{A} \cdot t^2 = \frac{L}{A}\, p^2 \cdot \frac{t^2}{p^2}$$

Or

$$b = B \cdot \frac{t^2}{p^2} \ldots\ldots\ldots\ldots\ldots\ldots (73)$$

Now, the effective value of t can never be greater than that of p, although it may be indefinitely less; for if t becomes greater than p, the extra light admitted is totally lost, since the largest pencil which can possibly enter the eye has a radius equal to p.

When t is very small in comparison with p, b is very small in comparison with B. This explains the cause of our being able to gaze at the noon-day sun through a small hole in a card.

144. Prop. To determine the size of the pencil transmitted from any point of an image.

Let AB (fig. 43) be a lens forming an image at P of a given object in front, and let ED be the position of the pupil of the eye.

Fig. 43.

Denote CA, the radius of the lens, by R; PF, the distance of the eye from the image, by e; FD, the radius of the pencil proceeding from a point in the image, by t, and PC by d.

Then from similar triangles,

$$\text{DF} : \text{FP} :: \text{AC} : \text{CP}$$

Or

$$t : e :: R : d$$

$$\therefore t = R \cdot \frac{e}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (74)$$

145. Prop. To determine the brightness of an image.

In fig. 42, the lens PQ is illuminated by the object AB, and the measure of that illumination is from (65) and (72),

$$l = \frac{L}{d^2} = \frac{A \cdot B}{p^2\, d^2}\,.$$

Similarly, if we suppose the rays to be reversed in their course, the lens will then be illuminated by the image; and denoting this illumination by l', the brightness of the image by b, and the radius of the pencil which it transmits to the eye by t, we have,

$$l' = \frac{A'\, b}{t^2\, \delta^2}\,.$$

But the number of rays which are transmitted through any portion of the lens is equal to the number incident upon that portion multiplied into the co-efficient of transmission; hence,

$$l' = c\, l.$$

Or

$$\frac{A'\, b}{t^2\, \delta^2} = c \cdot \frac{A \cdot B}{p\, d^2}\,.$$

Writing for A' its value as found in Art. 140, we reduce this expression to,

$$d = c\,B\,.\,\frac{t^2}{p^2} \qquad (75)$$

In equation (75), when t becomes equal to p, which is its greatest effective value, we have $b = c\,B$; but c being always less than unity, we infer that *there is always a loss of brightness in the formation of an image.*

146. We have seen (Art. 143) that as far as brightness is concerned, it is useless to have t greater than p; and that the brightness of an image continually increases as t^2 increases, until t becomes equal to p, when the brightness becomes constant, and cannot be increased by any further augmentation of t.

Therefore, writing p for t in (74), and denoting the particular value which R will then take by k, we obtain,

$$k = \frac{dp}{e} \qquad (76)$$

The particular value of R which we have here denoted by k is termed the *limit of efficiency;* and it expresses the greatest radius which it is profitable to give to any lens, in as far as that radius affects the brightness of images formed by that lens. If the radius be less than this, the pencil from any point of the image is not sufficiently large to fill the eye; whereas, if it be greater, the whole of the pencil cannot be admitted, and the extra light is therefore lost to the eye and rendered useless.

147. Let AB (Fig. 44) be a lens, and PQ the position of an image formed by it. Let EG be the pupil of the eye, placed centrically upon the axis FC; and ED the position of a pencil coming from the point P.

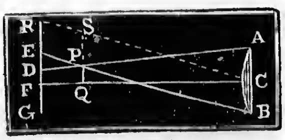

Fig. 44.

Now, it is evident that the pencil EPD enters the eye, as also pencils coming from every point between P and Q. But from any point beyond P, part of the pencil, at least, must fall without the pupil and be lost, thus producing a corresponding diminution in the brightness of that part of the image from which it proceeds. All points of the image, then, between P and the axis will appear equally bright, and that part of the image will thus present a uniform brightness; but beyond P the brightness of the image grows gradually less until it finally disappears.

Prop. To find the radius of the circle of uniform brightness.

Through C draw CR parallel to BE, and produce GE to meet it in R, and QP to meet it in S.

In addition to the former notation, put PQ, the radius of the circle of uniform brightness, $= V$.

Then, RFC and SQC being similar,

RF : FC :: SQ : QC;

i. e., $R + p : d + e :: R + V : d$;

from which we obtain,

$$V = \frac{dp - Re}{d + e} \quad \text{.................(77)}$$

Writing the value of dp as obtained from (76), we reduce (77) to the more convenient form,

$$V = \frac{e}{d + e} \cdot (k - R) \quad \text{..........(78)}$$

In instruments, where a second lens takes the place of the eye in fig. 44, it is usual to mark out this circle by an annular stop termed the *diaphragm*, thus cutting off the edges or *ragged* part of the image.

For convenience in reference we here collect the principal formulae of this Chapter.

Illumination; $l = \frac{L}{d^2}$; ....(65)

$$l' = \frac{L \cos \varphi}{d^2} \quad \text{..................(68)}$$

Illuminating power; $\frac{L}{L'} = \frac{d^2}{d'^2}$ ....(66)

Point of equal illumination;

$$x = \frac{D\sqrt{L}}{\sqrt{L} + \sqrt{L'}} \quad \text{............(67)}$$

Brightness; $B = \frac{L}{A} \cdot p^2$; ....(72)

$$b = B \cdot \frac{t^2}{p^2} \quad \text{.................(73)}$$

Brightness of images;

$$b = cB \cdot \frac{t^2}{p^2} \dots\dots\dots\dots\dots\dots (75)$$

Radius of transmitted pencil;

$$t = R \cdot \frac{e}{d} \dots\dots\dots\dots\dots\dots (74)$$

Limit of efficiency; $k = \frac{dp}{e} \dots\dots\dots\dots\dots\dots (76)$

EXAMPLES E.

1. Two lights, whose illuminating powers are as 3 and 4, are placed upon a table at distances of 5 and 6 feet respectively from a wall, and upon it they cast two separate shadows of an opaque object; compare the illumination of the wall with that of the portion covered by each shadow.

Let $L = 3$, $L' = 4$, $d = 5$ and $d' = 6$.

Then for the illumination given by the first light we have from (65),

$$l = \frac{3}{25} = 0{\cdot}12;$$

and for that of the second,

$$l' = \frac{4}{36} = 0{\cdot}111 \cdots.$$

∴ Illumination of the wall is $0{\cdot}12 + 0{\cdot}111 = 0{\cdot}231$; and the illumination of the shadow cast from the first light is evidently that of the second light, and *vice versa*.

Hence, the shadow from the first light, that from the second, and the wall, have their illuminations in the proportion of,

$$0{\cdot}111 : 0{\cdot}12 : 0{\cdot}231.$$

2. If the lights of the last problem be 12 feet apart, find two points of equal illumination.

Here we have, $L = 3$, $L' = 4$, and $D = 12$.

∴ From (67), $x = \frac{12\sqrt{3}}{\sqrt{3} \pm \sqrt{4}} = 5{\cdot}568$, or $-77{\cdot}568$

Hence, it is between the lights at a distance of 5·568 feet from the first one, or 77·568 feet in front of the first one.

3. A lens having a radius of 1 inch, forms an image at the distance of 6 feet. An eye is placed at the distance of 12"

inches from the image, and upon the axis of the lens. Determine the limit of efficiency, the circle of uniform brightness, and the brightness of that circle, when the radius of the pupil is .2 inches, and the co-efficient of transmission is .85.

Here, $R = 1$, $d = 72$, $e = 12$, $p = \cdot 2$, and $c = \cdot 85$.

First, taking (76), $k = \dfrac{72 \times \frac{1}{5}}{12} = 1\frac{1}{5}$ inches.

Second, taking (78), $V = \dfrac{12}{84}\left(1\frac{1}{5} - 1\right) = \dfrac{1}{35}$ inches.

Lastly, from (74) we obtain,

$$t = 1 \times \frac{12}{72} = \frac{1}{6};$$

and then from (75),

$$b = \cdot 85 \times B \times \frac{\left(\frac{1}{6}\right)^2}{\left(\frac{1}{5}\right)^2} = B \times \cdot 5902.$$

4. At a place on the equator on March 21st, compare the illumination at 9 A. M. with that at noon.

5. Venus being 68,000,000 miles from the sun, and the earth 95,000,000, compare their respective illuminations.

6. Assuming that the diameter of the pupil is one-fifth of an inch, what would be the brightness of the sun when viewed through an aperture having a diameter of ·0125 inch?

7. Determine the relative positions of two luminous bodies when with illuminating powers represented by 8 and 10 they give equal amounts of illumination.

CHAPTER VII.

ON OPTICAL INSTRUMENTS.

In this Chapter we propose to deal with the essentials of Optical Instruments, rather than with the peculiarities or varieties in their construction.

148. The simple Optical Instruments are three in number, viz: the Mirror, the Prism, and the Lens.

Every compound optical instrument is composed of two or more of these simple ones, combined in such a manner as to produce the desired effect. Besides these, many instruments are furnished with diaphragms, screens, &c.; but these are mechanical rather than optical arrangements.

The Camera Obscura.

149. The Camera Obscura consists essentially of a dark box or chamber, in one wall of which is placed a convex lens.

This lens forms images of exterior objects, and these images are received upon a screen placed in the proper position within the chamber. If the distances of the objects be great compared with the focal length of the lens employed, their images will be very nearly at equal distances from the lens, and may be received upon the same screen. The picture thus formed, although inverted, is wonderfully life-like, partaking of all the motions and changes which characterize a landscape; and in fineness it far surpasses the most accurately executed painting.

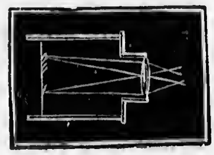

Fig. 45.

The instrument is variously modified in practice, the object to be attained being to bring the picture into a convenient position for copying or contemplating.

When finely constructed and supplied with an achromatic lens, this instrument constitutes the camera of the Photographer; in it the place of the screen is occupied by a prepared plate, upon which the rays of light, by their chemical action, produce a copy of the picture.

The brightness of the picture depends upon the radiating power of the screen and its illumination.

For the first reason, a white screen is preferable to one of any other colour; and for the second, a large lens is superior to a small one, since the illumination is as the square of its radius, or as its area (Art. 140).

The Eye.

150. Leaving it to Physiologists to describe the various parts of the eye and their exact uses, we shall confine ourselves to a consideration of its optical principles.

The eye is a natural camera obscura, having in front the *crystalline lens* by which an image of any exterior object is formed, and at the back the *retina* or screen upon which that image is received.

The space between the lens and the retina is filled up with transparent layers called *humours*, by which the eye is rendered achromatic, and at the same time freed from any spherical confusion or aberration.

The eye is in fact a perfect camera, and the only one known.

In front, the lens of the eye is supplied with an annular stop called the *iris*, which cuts off the outer rays, and which is capable of being dilated or contracted within certain limits. The circular opening so formed is termed the *pupil*.

The motions of the iris, and consequently the size of the pupil, are governed by a tendency to equalize the brightness of objects; thus, it contracts in the presence of an intensely luminous body, and dilates when exposed only to the rays proceeding from a body of feeble brightness. This explains the cause why the moon, which appears as a fleecy cloud when surrounded by the bright atmosphere of day, is conspicuous by her brightness as contrasted with the evening sky.

151. The lens of the eye being convex, the image of an object at a distance from the lens greater than its focal length is (Art. 77, II. ii) behind the lens, or within the eye; and for its distance from the pupil we get from equation (27),

$$d = \frac{Df}{D - f}.$$

From this it appears that d increases as D decreases, and *vice versa*; i. e., the retina must be farther from the lens when viewing near objects than when viewing distant ones. The eye in its normal condition has the power of making this adjustment for any value of D, from about 10 inches at its minimum to infinity at its maximum. In some eyes, however, the limits are quite different from the normal ones, and give rise to what is termed

Defective Vision.

152. The first kind of defective vision that we shall notice is *short-sightedness* or *myopia*. In this the minimum limit is too low, being often not more than 4 or 5 inches; and the eye being capable of adjusting itself for such near objects, has not sufficient range of adaptation to enable it to do so for distant ones.

This defect may be remedied by using such a lens as will increase to a proper extent the minimum limit.

Let s be the natural minimum limit of the eye, and S the normal one required. Then, when any object is placed at a distance of S from the eye, its image as formed by the lens must be brought to a distance from the eye equal to its minimum limit of vision, that is s. And since both distances are in front of the eye, they are positive; and we accordingly get from (27),

$$\frac{1}{s} - \frac{1}{S} = \frac{1}{f};$$

$$\therefore f = \frac{S\,s}{S - s}.$$

And since in this case s is less than S, f is positive, or the lens must be concave.

153. The second species of defective vision is termed *long-sightedness*, or *presbyopia*, from the fact that it almost universally attends old age.

In this defect the minimum limit is too great, so that the eye is incapable of adjusting itself to view near objects. To remedy this defect we must employ a lens which reduces the minimum limit.

Let, as before, s be the natural limit of the eye, and S the normal limit required. Then, we obtain as in the last article,

$$f = \frac{S\,s}{S - s}.$$

But s being now greater than S, f becomes negative, and the lens must be convex.

The Camera Lucida.

154. This instrument, invented by Wollaston, is used as an assistant in drawing and copying.

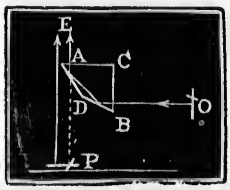

Fig. 46.

It consists essentially of a prism, ACBD, as described in Art. 67, through which the light in its passage is twice reflected but not refracted. Rays of light coming from an object at O (fig. 46) are reflected at the face BD, and again at DA, and enter the eye at E as if coming from P. The eye seeing at the same time past the edge of the prism views the paper at P directly. By using lenses the image of O may be made to coincide with the surface of the paper, and by seeing at the same time both the paper and the image superimposed upon it, we are enabled to trace its outlines with a great degree of accuracy.

The advantage of the double reflection is fully explained in Art. 67. Were it not for this arrangement the instrument would be nearly useless, as every picture would be reversed.

The Goniometer.

155. Goniometers are instruments employed for the purpose of measuring the dihedral angles of a crystal, and are of different forms. The only one that employs optical principles, and consequently requires notice here, is that known as *Wollaston's* or the *reflecting* Goniometer.

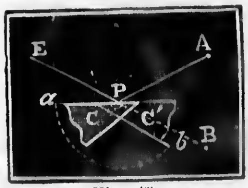

Fig. 47.

The principle of its operation is as follows: The crystal (C) is so arranged upon the instrument as to allow of its being revolved upon an axis parallel to the line of intersection of the faces forming the angle to be measured.

Two given signals, A and B, are taken such that the plane passing through them may be at right angles to the axis of motion, and the crystal is so adjusted that the image of A, as seen reflected from P, appears superimposed upon B. In this position it is manifest that the face of the crystal bisects the angle APB. The instrument, and with it the crystal, is now turned about its axis until the adjoining face is brought to bisect the same angle, which is known by the image of A being again superimposed upon B as seen from E. Then, it is evident that since the two faces have been made to occupy similar positions, the crystal has been turned through the angle $a\ b$, which is the supplement of the angle made by the faces. The instrument is provided with a graduated circle by which the angle through which it has been turned is measured, and denoting it by β we have for the dihedral angle $180° - \beta$.

The Kaleidoscope.

156. The Kaleidoscope is an Optical toy, by means of which an endless variety of pleasing Geometrical figures can be formed.

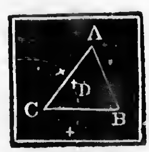

Fig. 48.

It consists essentially of two plane mirrors, AC and BC, nicely joined at the angle C, which is generally made to be some integral part of the whole circle.

Between the mirrors, as at D, any object being placed, its successive images (Art. 31) arrange themselves in pairs in the circumference of a circle having its centre at C; thus constituting a figure of geometrical regularity.

The instrument is generally composed of three strips of glass of equal width nicely joined at their edges, thus forming a tube the section of which is an equiateral triangle. The objects are bits of coloured glass enclosed between two glass plates at the end of the tube.

When turning the tube around its axis while looking through it, the objects by falling from side to side give rise to some of the most elegant patterns which it is possible to imagine.

In its common or triangular form, it follows from Art. 32 that the figures will be hexagonal.

The Optical Square.

157. The optical square is an instrument sometimes employed in surveying, for the purpose of setting off right angles. It consists of a plane horizontal table upon which two mirrors, A and B (fig. 49), are fixed with their planes at right angles to that of the table.

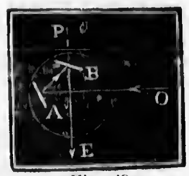

Fig. 49.

These mirrors are inclined to one another at an angle of 45° and admit of accurate adjustment in case of accidental displacement. The light coming from an object in the direction of O, after being twice reflected, will enter the eye at E, as if coming from an object placed in the direction of P.

The angle between the lines EB and OA is by Art. 30 equal to twice the angle at which the mirrors are inclined, i. e., a right angle or 90°.

The Sextant, Quadrant, and Reflecting Circle.

158. These three instruments employ the same Optical principles, and differ only in the details of their mechanical parts, and therefore an explanation of the principles which enter into one will suffice for all. We shall choose the sextant, as being the one most frequently employed.

H and I (fig. 50) are two plane mirrors having their reflecting surfaces accurately perpendicular to the plane of the instrument. I, which is denominated the *index glass*, is fully silvered, and, by the motion of the arm IL to which it is fixed, may be turned about an axis at right angles to the instrument's plane. The amount of its motion is read off from the

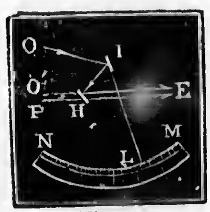

Fig. 50.

IMAGE EVALUATION
TEST TARGET (MT-3)

6″

graduated arc MN. The mirror H, termed the *horizon glass*, is silvered only upon its lower half, and is fixed immovably, except for minute adjustments, to the body of the instrument.

Rays of light coming from any object at O are successively reflected from the two mirrors and enter the eye as if coming from O′. But through the unsilvered portion of the horizon glass the eye sees at the same time any natural object as at P. The angular distance of any two objects, which are thus apparently brought together, is evidently equal to the deviation of the reflected ray; but that is, from Art.30, equal to twice the inclination of the mirrors, and this inclination being measured upon the arc MN, the deviation is known.

Usually the arc MN is so numbered that every degree of angle at its own centre is counted as two degrees upon the divided limb, thus giving at once the whole angle of deviation, or the angular distance of the objects brought into apparent coincidence.

To improve the action of the instrument and render it more delicate, it is often furnished with a small telescope directed towards the horizon glass, by means of which the eye sees two images superimposed in the field of the instrument, and can thus bring them into coincidence with a considerable degree of accuracy.

The Lens.

159. The different forms of lenses, their division into classes, and their general properties have been given in Chapter III: we here propose to deal with their properties as constituting the whole or a part of an Optical Instrument. In the construction of instruments, concave lenses are seldom employed, and we shall consequently confine ourselves mostly to a consideration of convex ones.

160. Prop. To find the focal length of a convex lens by experiment.

I. Expose the lens to the direct rays of the sun and receive his image upon a smooth screen held behind the lens. Then, the distance from the lens to the screen is the focal length.

This is evident from equation (27), where D, denoting the distance of the sun, may be taken as infinitely great without any perceptible error.

II. Place a taper, the lens and a screen in a straight line, and at any distance so as to have an image of the taper's flame formed upon the screen. Call the distance from the taper to the lens D, and that from the lens to the screen, d; then we have from (27), since d and f are both negative,

$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f}$$

and $$\therefore f = \frac{D\,d}{D + d}.$$

161. Prop. To find the focal length of a concave lens by experiment.

Choose a convex lens whose focal length is known to be less than that of the concave one, and place the lenses in contact so as to form a compound. This compound will be convex. Find its focal length by Art. 160, and denote it by f'. Denote the focal length of the convex lens employed by F, and of the concave one by f; then we obtain from equation (33), by noticing that F and f' are negative,

$$f = \frac{f'\,F}{f' - F}$$

162. In order to see objects distinctly it is necessary that the constituent rays of every pencil entering the eye shall converge to a focus exactly upon the retina; but this condition can be fulfilled, in normally constituted eyes, only when the rays incident upon the eye are parallel or nearly so. As before stated, objects *may* be seen distinctly when placed at a distance of 10 inches, but in the use of instruments such as Telescopes and Microscopes, the fatigue is much less and the view much more satisfactory when the point of divergence is much further off, or when the incident rays become sensibly parallel.

It appears from Art. 75 and Art. 87 iv, that convex lenses may form either a real or virtual image. Now, in using a convex lens for assisting our vision, we may view either of these images; but since the real image is at a real focus, the rays which pass from it, and by which it must be seen, diverge as if from an object placed in its position; hence, in order to view it directly, it would be necessary for the eye to treat it as an object, and hold it at a distance of at least ten inches.

Now this is not the case with a virtual image, for when an object is placed very near the focus of a convex lens, the rays of every pencil which leaves the lens are very nearly parallel, and therefore well fitted to produce distinct vision. But in using a lens in this manner, we view not the object, but its virtual image removed to a great distance and proportionally enlarged.

It is in this way that single lenses assist the vision, by allowing us to bring objects within the minimum limit, while at the same time they remove the virtual image far beyond this limit.

For reasons now stated the most important class of Optical Instruments are so constructed that the eye may examine directly the *virtual image*.

163. Optical Instruments used to assist the vision are divided into *simple*, which act without the formation of a *real* image; and *compound*, which employ this image as a second object.

The compound ones are further divided into *Refractors*, which form the real image by means of lenses; and *Reflectors*, which form it by specula.

These instruments are moreover divided into *microscopes*, which are employed to examine small but accessible objects; and *telescopes*, by which we view distant or inaccessible objects.

This last is a division in name and intent rather than in reality, for both act upon the same principles and consist essentially of the same parts. These divisions, with the instruments included under them in this work, are given in the annexed scheme:—

		MICROSCOPES.	TELESCOPES.
SIMPLE.......		Single Lens Wollaston's Doublet	Galileo's
COMP.	REFR....	Common	Common
	REFL.	Brewster's Smith's	Gregory's Cassegrain's Newton's Herschel's

Another instrument, known as the Solar microscope, will be treated of in connection with the Magic Lantern (Art. 193).

SIMPLE INSTRUMENTS.

164. Single Lens. To find the magnifying power.

The Magnifying Power is the ratio of the angle under which the image is seen to that under which the object may be seen by the naked eye, both angles being small.

Let f be the focal length of the lens, D the distance of the object in front, d the distance of the virtual image in front, e

the distance of the eye behind the lens, and P the magnifying power.

The distance of the eye from the image is $e + d$, and the angle under which the image is seen is, very nearly,

$$\frac{I}{e+d}.$$

Also, the angle under which the object may be seen is at a maximum, nearly $\frac{O}{s}$.

Hence, $$P = \frac{I}{e+d} \div \frac{O}{s} = \frac{I}{O} \cdot \frac{s}{e+d}.$$

But the lens being convex we have from (40),

$$\frac{I}{O} = \frac{f}{f-D};$$

and from (27), $$d = \frac{Df}{f-D};$$

and these relations reduce our equation to,

$$P = \frac{fs}{(f-D)e + fD} \dots\dots\dots\dots (79)$$

But since for distinct vision (Art. 162) D must be equal to f or very nearly so, we may without sensible error put $f = D$, which gives,

$$P = \frac{s}{f} \dots\dots\dots\dots\dots\dots (80)$$

From (80) it appears that under the most favourable condition, that is when the object is placed in the focus, the *magnifying power is independent of the distance of the eye from the lens, and is equal to the minimum limit of distinct vision divided by the focal length of the lens.*

From this it appears that the magnifying power is less for a short-sighted person than for one who is not so.

165. Wollaston's Doublet. This is the combination of lenses generally employed as a simple microscope, and consists of two plano-convex lenses having their plane surfaces turned towards the object. The lens nearest the eye has a focal length three times as great as the other, and they are separated by an interval equal to about one half the sum of their focal lengths, (Art. 127).

Prop. To find the magnifying power.

Since the pencil which passes into the eye must consist of rays which are parallel or nearly so, (Art. 162) we may replace the compound by its equivalent (Art. 111), and obtain for its magnifying power, equation (80),

$$P = \frac{s}{f'}.$$

And writing for f' its value from (52), and noticing that the lenses are all convex, we get,

$$P = s\left(\frac{1}{F} + \frac{1}{f} - \frac{l}{Ff}\right); \ldots\ldots\ldots\ldots(81)$$

where F and f are the focal lengths of the lenses, and l their distance apart.

Now, in Wollaston's doublet, putting f to denote the focal length of the lens next the object, we have, $F = 3f$, and $l = \frac{1}{2}(F+f) = 2f$; hence by substitution,

$$P = \frac{2}{3} \cdot \frac{s}{f} \ldots\ldots\ldots\ldots\ldots\ldots(82)$$

Therefore, the magnifying power is two-thirds of that belonging to the lens of shorter focal length, or twice that of the other.

166. Galilean Telescope. This instrument, invented by Galileo, is interesting from being the form in which the telescope first existed; now, however, it is seldom employed except as an opera glass, and is always constructed of small dimensions.

It may be considered as a compound lens in which the primary is convex and the secondary concave, and from which parallel incident rays emerge parallel.

Making $\lambda = \infty$ in (31), and changing the sign of F, since the primary is convex, we obtain by reducing,

$$l = F - f.$$

That is, *the distance between the lenses must be equal to the difference of their focal lengths.*

This being the case, no real image can be formed, for the rays coming from the primary are intercepted before reaching the focus, and are transmitted to the eye in a fit state to produce distinct vision.

Prop. To find the magnifying power.

Let ω be the angle under which the object is seen, (Art. 99); then, since the object is very distant, we have, from (41),

$$\omega = \frac{I}{F}.$$

But the angle under which the image is seen is $\frac{I}{f}$, since the image would be formed in the focus of the secondary.

Therefore, $$P = \frac{I}{f} \div \omega = \frac{I}{f} \cdot \frac{F}{I} = \frac{F}{f}$$

Or, *the magnifying power is equal to the focal length of the primary divided by the focal length of the secondary.*

This telescope has a very small field of view, and cannot take a very high power conveniently, but it has the advantage for terrestrial objects of showing them erect.

Compound Instruments.

167. In these instruments a lens or a speculum, either simple or compound, is used to form a real image, and this image is then viewed by means of a simple microscope termed the *ocular* or *eyepiece.*

The instrument by which the image is formed is called the *objective*, or *object glass*, and in reflectors sometimes *the speculum* or *metal.* The character of this part gives the name to the microscope or telescope, and divides them into Refracting and Reflecting.

168. The Ocular or Eyepiece. However the Objective may vary, the Ocular consists of a lens or lenses in all instruments.

Formerly this part consisted of a single convex lens of short focal length, but modern improvements have furnished us with the *Huyghenian* or *negative*, and *Ramsden's* or the *positive* eyepiece.

169. Huyghenian Eyepiece. This is the one most commonly used, because it corrects the aberrations more nearly than any other.

It consists of two plano-convex lenses, F and E, fig. 51, having their convex sides turned towards the objective, and separated by a distance equal to one half the sum of their focal lengths.

Fig. 51.

The lens F is termed the *field* lens or *amplifying* lens, because it enlarges the field of view; and the other is the *eye* lens.

If f be the focal length of the eye lens, that of the field lens is $3f$, and their distance apart is consequently $2f$.

Since the image must be formed at the focus of the eye lens, (Art. 162), it is half way between the two lenses; hence the field lens, although generally considered a part of the *Ocular*, is in reality a part of the *Objective*, for it assists in forming the image rather than in viewing it.

170. Ramsden's Eyepiece. When the image is to be received upon a system of *cross wires*, or to be measured by a micrometer, as is the case in many Astronomical Instruments, the Huyghenian eyepiece becomes inapplicable, since it contains the image between its lenses; in this case we have recourse to Ramsden's or the positive eyepiece, with which the image is left entirely without the Ocular.

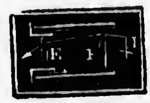
Fig. 52.

It consists, like the last, of two plano-convex lenses, but they are of equal focal length, they have their convex sides turned towards each other, and they are separated by two-thirds the focal length of either.

Putting f for the focal length of one of the lenses, we have for the focal length of the equivalent, from (52),

$$f' = \frac{3}{4} f$$

and for the position of the image we get, from (30) and (C), by making $\lambda = \infty$,

$$D = \frac{1}{4} f.$$

171. Both of the eyepieces now mentioned sometimes undergo a modification, by which the light, while passing from one lens to the other, is brought into contact with a plane mirror inclined at an angle of 45° to the axis of the tube, or transmitted through a prism, as in Art. 66, by which it is turned through an angle of 90° and thus made to emerge from the side of the telescope or microscope instead of along the axis. Such an eyepiece is named a *diagonal* one, and is used merely as a matter of convenience.

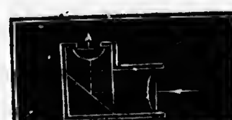
Fig. 53.

172. Sometimes a *compound* microscope of low power is employed to examine the image, and is known as the *terrestrial* eyepiece, because it inverts the image, which has already been placed in an inverted position by the Objective, and causes Objects to appear *erect*.

This eye-piece is generally applied to telescopes which are to be used in viewing terrestrial objects.

173. Compound Refracting Microscope. The objective in this instrument generally consists of more than one lens, but varies in its construction with the quality of the instrument and the work it is intended to do. For low powers the objective generally consists of a Wollaston's doublet, but for high ones, it is formed of achromatic lenses (Art. 122) so united as to correct as far as possible both species of aberration.

Owing to the number of lenses generally employed in the Objective, it is not convenient to find, very accurately, the *magnifying power* of the compound microscope by calculation; but we may approximate very closely to it, as follows :—

Let F be the focal length of the equivalent for the Objective, and f of that for the eye-piece; also, let l be their distance apart. Then we may consider the whole microscope as a *compound lens* from which the rays emerge parallel, and we thus get for its magnifying power, as in equation (81),

$$P = s\left(\frac{1}{F} + \frac{1}{f} - \frac{l}{Ff}\right) \text{........(83)}$$

A final negative sign in this result is of no account, and may be rejected, since magnifying power cannot be considered as a negative quantity.

174. Prop. To find the magnifying power by experiment.

Direct the eye-end of the Instrument towards a strong light, and receive the circular image, formed near the other extremity, upon a white paper screen. When this image is sharply defined, measure its diameter with a delicate scale, and let v be the number of divisions over which it extends. With the same scale measure the diaphragm, and let its diameter be V.

Then we have, $$\frac{I}{O} = \frac{V}{v}.$$

Now let f be the focal length of the *eye-glass* in the case of a Huyghenian Ocular, or of the whole eye-piece in the case of a Ramsden's; then,

$$P = \frac{I}{f} \div \frac{O}{s} = \frac{I}{O} \cdot \frac{s}{f};$$

Or, $$P = \frac{V}{v} \cdot \frac{s}{f} \text{....................(84)}$$

175. Let R be the radius of the Objective; l its distance from the image, which will be nearly the microscope's length; and f the focal length of the Ocular.

Then, for the radius of the pencil transmitted to the eye we readily obtain,

$$t = \frac{f R}{l}.$$

But f and R being both small quantities in comparison with l, t is also a small quantity, and the brightness of the field is from (75), taking B as a unit,

$$b = c \cdot \frac{f^2 R^2}{l^2 p^2}, \qquad \text{(85)}$$

which is a small quantity. Hence the necessity, in using the microscope, of illuminating the object strongly.

If, for example, we have $f = 1$, $R = \cdot 5$, $l = 6$, $F = \cdot 5$; we obtain for the magnifying power (Art. 173),

$$P = 10 \left(\frac{1}{\cdot 5} + \frac{1}{1} - \frac{6}{\cdot 5}\right) = 90;$$

and for the brightness of the field,

$$b = c \cdot \frac{1^2 \times \cdot 5^2}{6^2 \times p^2} = \frac{c}{p^2} \cdot \frac{1}{144}.$$

And taking p as one-tenth of an inch, which is about its mean value, and c as $\cdot 8$, we get, nearly,

$$b = \frac{5}{9}.$$

176. Compound Reflecting Microscope. Of this class of microscopes a great many forms have been invented from time to time. Many of these are but modifications of one another, and differ only in the method of illuminating the object or some such mechanical details. In all, however, the image is formed by means of a speculum instead of a lens, and this forms the distinctive characteristic of the group.

This class of microscopes is not at present in common use, having been almost entirely superseded by the common or Refracting Microscope, which is neater in appearance, more compact, and more easily managed.

Among the instruments belonging to this class, we may notice *Brewster's* and *Smith's*.

Fig. 54.

177. Brewster's Microscope. In fig. 54, AB is the tube of the microscope. At one extremity a small tube screws in, which contains a concave speculum S, and a small plane speculum *s*, supported upon a thin arm. The large speculum is pierced with a hole at its centre, and the rays, coming from an object at O, after reflection at the plane mirror and again at the concave one, form an image at I, where it is viewed by the Ocular.

The rays evidently meet the Objective as if coming from an object at *r*, situated as far behind the plane mirror as the object is in front of it.

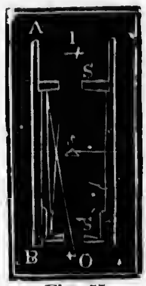

Fig. 55.

178. Smith's Microscope. In fig. 55, AB is the tube of the microscope, S is a concave mirror pierced with a hole, and S′ is a convex mirror also pierced at its centre. Rays of light coming from an object at O, near the centre of curvature of S, would, after reflection from the surface of S, be brought to a focus at some point near O; but, meeting the mirror S′, they are reflected back and caused to converge more slowly so as to form their image at I, in the focus of the Ocular. A stop *s*, placed in the body of the tube, prevents the direct rays from passing from the opening in S′ to the eye-piece.

Although these instruments have the advantage of forming a *perfectly* achromatic image, yet they labour under the insuperable disadvantage of wasting a great portion of the light which enters them, thus effectually prohibiting the use of very high powers with any degree of satisfaction.

179. Compound Refracting Telescope. The Objective in this instrument consists either of a single lens, or of an achromatic compound, as described in Art. 122.

Being intended for distant objects, the object lens is always of large size and long focal distance as compared with the lenses of the Ocular; and in its general application the image is formed at the focus of the object lens.

The Ocular employed is one of those already described; the Huyghenian for distinct view, the Ramsden's for micrometrical measurements, the terrestrial for common *land* or *sea* telescopes, and the diagonal form of any of these when convenience requires it.

180. Prop. To find the magnifying power.

However compound the parts may be in the telescope, since the rays enter it parallel and emerge parallel, we may replace both the Objective and the Ocular by their equivalent lenses. Let, then, F be the focal length of the Objective, and f of the Ocular.

The angle under which the object is seen is $\omega = \frac{1}{F}$, (41).

The angle under which the image is seen by the naked eye is $\frac{I}{s}$, and the ocular magnifies $\frac{s}{f}$ times, (80); therefore, the angle under which the image is seen by means of the Ocular is

$$\frac{I}{s} \cdot \frac{s}{f} = \frac{I}{f}.$$

Hence, for the magnifying power we have,

$$P = \frac{I}{f} \div \omega = \frac{I}{f} \cdot \frac{F}{I} = \frac{F}{f} \quad \ldots\ldots\ldots (86)$$

Or, *the magnifying power is the quotient arising from dividing the focal length of the Objective by the focal length of the Ocular.*

181. Prop. To find the magnifying power by experiment.

Direct the object end of the instrument towards a bright sky and receive upon a paper screen the small ring of light formed near the other extremity. When sharply defined, measure its diameter with a finely divided scale.

Let r be its radius thus found, and R the radius of the clear opening at the object end of the instrument.

Then, $$P = \frac{R}{r}.$$

For, the small circle is but the image of the objective aperture, formed by means of the Ocular. Hence, we have from equation (27), by noticing that the Ocular is convex.

$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f};$$

and $$\therefore \frac{D}{d} = \frac{D}{f} - 1.$$

But from (38), $$\frac{D}{d} = \frac{O}{I} = \frac{R}{r};$$

$$\therefore \frac{R}{r} = \frac{D}{f} - 1.$$

Now, D being the distance of the Objective from the Ocular, we have evidently,

$$D = F + f;$$

$$\therefore\ \frac{R}{r} = \frac{F + f}{f} - 1 = \frac{F}{f};$$

which is the magnifying power, from Art. 180.

182. Prop. To find the angular field of view.

The *angular field of view* is the angle subtended by two extreme points which may be seen in the telescope at one view of a distant object.

In fig. 56, let O be the equivalent for the objective of the telescope, and E, for the eye-piece; and let PQ be the position of the diaphragm, P being in its circumference. Denote PQ by V, and the angle POQ by θ. Then $2\,\theta$ is the angular field of view.

Fig. 56.

But,

$$\tan\theta = \frac{PQ}{OQ} = \frac{V}{F};$$

and the angle being in all cases small, we may write θ for $\tan\theta$; hence,

$$2\,\theta = \frac{2\,V}{F} \text{ ....} (87)$$

That is, *the diameter of the angular field of view is equal to the diameter of the diaphragm divided by the focal length of the equivalent for the objective.*

183. With a Ramsden's eye-piece the objective and its equivalent are identical; but in the Huyghenian eye-piece (Art. 169) the focal length of the objective is influenced by that of the field lens, and hence the equivalent is not of the same focal length as the object glass itself.

Prop. To find the equivalent lens.

Denote the focal length of the equivalent by F', of the eye-lens by f, and of the field lens by $3f$ (Art. 169). Then, since the image is to be formed half way between the lenses of the eyepiece, it must be at a distance equal to f from the field lens.

Therefore, writing $-f$ for λ, $-F$ for F, and $-3f$ for f in equation (31), and reducing, we get,

$$l = F - \frac{3}{2}f;$$

which expresses the distance between the object and field lenses when adjusted for distinct vision.

Then, writing this value of l in (52), and also putting $-F$ for F, $-3f$ for f, and $-F'$ for f' we obtain,

$$F' = \frac{2}{3} F.$$

Or, *the focal length of the equivalent is two-thirds that of the object lens.*

Hence, for the angular field of view in a telescope furnished with a Huyghenian eye-piece, we have,

$$\theta = V \div \frac{2}{3} F,$$

or

$$\theta = \frac{3}{2} \cdot \frac{V}{F} \quad \text{..................(88)}$$

Hence it appears that with the same diaphragm and object lens the Huyghenian eyepiece takes in a greater field than Ramsden's.

184. Prop. To find the brightness of the field of view.

Since every pencil, after leaving the eye-glass, consists of parallel rays, it is evident that the same number of rays, belonging to any one pencil, will enter the pupil when placed behind the eye-glass, as if placed in its position.

But if the pupil were placed in the position of the eye-glass, its distance from the image would be f.

Hence, writing f for e in (74), and noticing that R will be the radius of the object lens, and d the focal length of its equivalent, we obtain,

$$t = R \cdot \frac{f}{F} = \frac{R}{P}, \text{ equation (86).}$$

And from (75), by calling B the unit of brightness, and c the coefficient of transmission for all the lenses of the telescope taken together, we have,

$$b = c \cdot \frac{R^2}{P^2 p^2} \quad \text{..................(89)}$$

Equation (89) may be transformed as follows:—

From (76) we have for the limit of efficiency,

$$k = \frac{Fp}{f} = Pp \quad \text{..................(90)}$$

and this in (89) gives

$$b = c \cdot \frac{R^2}{k^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (91)$$

From (89) we infer that *the brightness of the field increases directly as the square of the radius of the object lens, and inversely as the square of the magnifying power;* and from (91), since it is useless to have R greater than k (Art. 146), *the brightness of the field can never become equal to unity.*

185. In the Huyghenian eye-piece, the Ocular, in the strict sense of the word, consists of a single lens, viz. the eye-glass, and it is the focal length of this lens that is represented by f in equation (86), provided we take F to denote the focal length of the equivalent to the objective.

But we obtain an identical result if we take F to denote the focal length of the object glass, and f of the equivalent to the eye-piece considered as a compound lens.

For, writing $-3f$ for F, $-f$ for f, and $2f$ for l in (52), and reducing, we obtain,

$$-f' = \frac{3}{2} f.$$

That is, the eye-piece is equivalent to a convex lens whose focal length is one-and-a-half times as great as that of the eye-glass.

But we have for the magnifying power,

$$\frac{F}{f'} = \frac{F}{\frac{3}{2}f} = \frac{\frac{2}{3}F}{f} = \frac{F'}{f}.$$

Or, the two forms give the same result. Hence, when the focal length of the eye-piece is known, we may most conveniently make use of the focal length of the object lens.

To illustrate, let us take the following example.

In a compound Refracting Telescope the object lens is 6 inches in diameter, and has a focal length of 6 feet. It is furnished with a Huyghenian eye-piece in which the focal length of the eye lens is ·3 inch, and the diameter of the diaphragm is ·2 inch. Assume $p =$ ·1 inch.

Equivalent to the objective, (Art. 183), $F = 48$ inches.

Magnifying power, (86), $P = \frac{48}{\cdot 3} = 160.$

Field of view, (87), $2\,\theta = \frac{\cdot 2}{48} = 14'$ nearly.

Limit of efficiency, (90), $k = 160 \times \cdot 1 = 16$ inches.

Brightness, (91), $b = c \,.\, \frac{3^2}{16^2} = \frac{c}{28}$ nearly;

and if c be $\cdot 8$, which we may suppose,

$$b = \cdot 028 \text{ or about } \frac{1}{35}\,.$$

This great reduction in brightness is upon the supposition that the pupil remains continually of the same size; but this is not true, for the eye itself endeavours, by dilating and contracting the pupil, to equalize as far as possible the brightness of bodies. Thus, if the object upon which the telescope was employed was very bright, the pupil would probably have twice the diameter, when viewing it through the instrument, that it has when looking at it without, which would increase the brightness to about one-eighth. But if the object be faint, the pupil undergoes a very slight change. Hence we see the reason why in viewing such objects as nebula or comets, we are under the necessity of employing low powers and large apertures. With the fixed stars, on the other hand, we have an exception to our formula for brightness. For, since with the highest powers available they undergo no perceptible magnification, their brightness, or, as it is better termed, *intensity*, is independent of the magnifying power, while it increases as the square of the radius of the object glass until the limit of efficiency is reached. Hence, in viewing these objects the highest powers are employed with advantage; for, simultaneously, the brightness of the ground-work of the sky is diminished, and the intensity of the light of the star is increased. Hence also, high powers reveal stars perfectly invisible to the naked eye, and even bring to view moderately bright ones at noon-day.

186. Compound Reflecting Telescope. In instruments of this class the Objective consists of a single or compound mirror (Art. 106), and the image being thus formed without any refraction taking place, is perfectly achromatic (Art. 128).

Hence, these telescopes, when well constructed, have a decided advantage over Refractors in the sharpness of the image produced and the magnifying power which they will consequently bear; but on the other hand, they yield to Refractors in the proportion of light rendered useful, in stability and convenience in working, and in their liability to become deranged. They are now seldom constructed except upon a gigantic scale, but even then they are not so generally useful as good Refractors.

187. Gregory's Telescope. In this telescope the Objective consists of a compound mirror, composed of two concave ones.

In fig. 57, A is a large concave speculum pierced at its centre. Rays of light, coming from a distant object in the direction of O, after reflection at the surface of the mirror, converge and form an inverted image at I; the rays after leaving this image meet the second concave speculum B, and are again converged to a focus and caused to form a second image at I′, in the focus of the eye-piece.

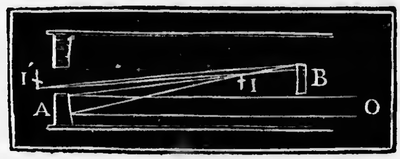

Fig. 57.

Let F be the focal length of the large mirror, F' of the small one, and f of the eye-piece.

For the distance from the small mirror B, at which the image I′ is formed, we may without material error put $\lambda = F + F'$; and this relation in (15) gives for the distance between the mirrors,

$$l = F + F' + F' \cdot \frac{F'}{F} \ldots\ldots\ldots\ldots (92)$$

and writing this value of l in (50), and reducing, we obtain for the focal length of the equivalent to the objective,

$$f' = \frac{F^2}{F'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (93)$$

But since the magnifying power is $\frac{f'}{f}$, we have for it in the Gregorian telescope,

$$P = \frac{F^2}{F'f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (94)$$

The field of view, limit of efficiency, brightness &c., will be the same as that given for the refracting telescope, by using the equivalent of (93) for the Objective.

188. Cassegrain's Telescope. The Objective in this instrument is compound, and consists of a large concave mirror and a small convex one.

In fig. 58, A is a large concave speculum pierced at the centre. Rays of light coming from a distant object in the direction of O would, after reflection at the surface of the mirror, converge and form an image at the focus *i*. But, before reaching this focus, they are intercepted by the convex mirror B, by which their convergence is decreased, and they are thus made to form their image at I, which is then viewed by the eye-piece.

Fig. 58.

Using the same notation as before, and noticing that the focal length of the small mirror is negative (Art. 40), we obtain by a method as in the last article,

$$l = F - F' + F' \cdot \frac{F'}{F}; \qquad (95)$$

$$f' = \frac{F^2}{F'}; \text{ (rejecting the negative sign)}$$

and

$$P = \frac{F^2}{F'f} \qquad (96)$$

189. It will be noticed that these two telescopes come under the same general formulae, and are therefore but modifications of one another. The Gregorian has the advantage for terrestrial objects of presenting them erect; for the first image being inverted, the second one is re-inverted or made erect. On the other hand, Cassegrain's has the advantages of being shorter for the same power; also, the mirrors being of opposite kinds, the aberration is partly corrected (Art. 130, i); and there being but one image formed, that image is much less confused than the second image of Gregory's telescope.

If we let λ denote the distance from the secondary mirror to the image, formed without the intervention of an eye-glass, we get the accurate formulae,

$$l = F + F' \cdot \frac{\lambda}{\lambda - F'}; \qquad (97)$$

$$f' = \frac{F}{F'} \cdot (\lambda - F'); \qquad (98)$$

and $$P = \frac{F}{F'f} \cdot \left(\lambda - F'\right) \ldots\ldots\ldots (99)$$

These are true for both instruments by taking F' with the proper sign.

190. Newton's Telescope. The Objective in this telescope consists of a single concave speculum.

A (fig. 59) is a concave mirror. Rays of light coming from a distant object in the direction of O would, after reflection at the mirror, form an image at i in the focus; but before reaching this point they meet the small plane mirror B, inclined at an angle of 45° to the axis of the instrument, by which they are turned out of their course and caused to form the image at I, without the tube of the instrument, and in the focus of the eye-piece.

Fig. 59.

The mirror B has no influence over the convergency of the rays (Art. 23), and hence, B i is equal to BI, or the plane mirror must be placed at a distance in front of the focus equal to the radius of the tube.

Frequently a prism, as described in Art. 66, is employed instead of the mirror B to change the direction of the pencil, and it has the advantages of wasting less light and of retaining its polish much better.

191. Herschel's Telescope. This telescope is a modification of Newton's, in which the small speculum is dispensed with, and the image formed at i (fig. 59) is viewed by the eye-piece. This saves the loss of light at the second mirror, but is applicable only to instruments of such dimensions that the diameter of the speculum is large in comparison with that of the human head. To bring the image into a convenient position the speculum is slightly inclined to the axis of the tube, thus throwing the focus close to one edge of the opening.

This is the form of telescope given by Sir William Herschel and Lord Rosse to their mammoth instruments.

In regard to magnifying power, field of view, &c., it is manifest that Newton's and Herschel's telescopes are measured precisely as in the Refractor (Art. 179), making the speculum to represent the Object lens.

The Magic Lantern.

192. This is an instrument by which a small object or painting may have its magnified image thrown upon an illuminated screen so as to be exhibited to an audience. It consists of a light, L (fig. 60), a large lens, C, of short focal length called the *condenser*, and a small lens, M, termed the *magnifier*. The object is placed at O, a little without the focus of M, and its magnified image is received upon a screen S S′. The condenser, by collecting a large pencil of rays from the lamp, L, transmits, through the instrument, a strong light which serves to illuminate the screen, thus causing the image to appear as a darkish or coloured painting upon a light ground.

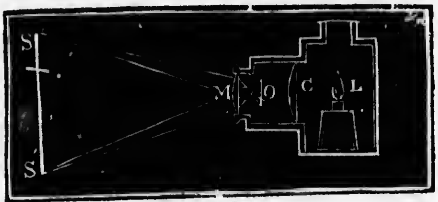

Fig. 60.

The objects are generally pictures painted upon glass slides in transparent colours, or those formed by means of the Photographer's Camera.

Since the image is inverted in regard to the object, it is necessary to place the slides in the instrument in an inverted position, in order to have an erect picture upon the screen.

The lenses and the lamp are enclosed in a box from which no light can escape, except by way of the magnifier, and the exhibition is conducted in a darkened room.

193. Solar Microscope. This is but a Magic Lantern in which the magnifier is accurately constructed and compounded so as to be achromatic and aplanatic, or nearly so. The resultant focal length is short, and in order to get sufficient light, the rays of the sun, rendered horizontal by a plane mirror, are condensed by a lens and transmitted through the instrument.

The same instrument when illuminated by the oxy-hydrogen light is known as the oxy-hydrogen microscope.

In these microscopes it is not easy to arrive at any definite conclusion in regard to the magnifying power, for the image will not generally admit of a very close inspection. We can, however, obtain an approximate value as follows. Denote the distance from the magnifier to the screen by d, the focal length of the magnifier by f, and the minimum distance at which the picture upon the screen will bear satisfactory inspection, by E.

Then from (27), since d and f are negative, we obtain,

$$\frac{d}{D} = \frac{d-f}{f}.$$

Hence, from (38), $\dfrac{I}{O} = \dfrac{d-f}{f}.$

But the angle under which the image is seen is $\dfrac{I}{E}$; and that under which the object may be seen is $\dfrac{O}{S}$.

Therefore, $$P = \frac{I}{E} \div \frac{O}{S} = \frac{I}{O} \cdot \frac{S}{E}.$$

Or, $$P = \frac{S}{E} \cdot \frac{d-f}{f} \ldots\ldots\ldots\ldots (100)$$

Photometry.

Photometry is the application of instruments, called photometers, to measuring or rather comparing the illuminating powers of two or more luminous bodies. The methods of photometry most commonly employed are known as Ritchie's, Rumford's, and "the extinction of shadows."

194. *Ritchie's method.* This method consists in causing two luminous bodies to give equal amounts of illumination.

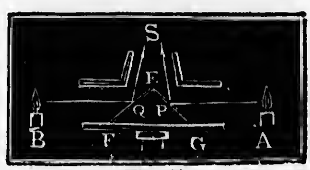

Fig. 61.

The triangular prism EFG (fig. 61) is nearly covered with white paper, and so placed that its faces, EG and EF, have the same inclination to the horizon. The two bodies, whose illuminating powers are to be compared, are placed in a horizontal line with the centre of the prism, and in such positions, A, B, that they may illuminate equally (as detected by the eye looking down from S) the respective faces EG and EF.

Denote the illuminating powers of the bodies by L and L', and their respective distances from the instrument by d and d'. Then, the angle of inclination being the same in each, we reject it, and obtain from (66),

$$L : L' :: d : d'^2 \ldots\ldots\ldots\ldots (101)$$

195. *Rumford's method.* This method differs from Ritchie's in details and in simplicity of apparatus, although it employs the same principle.

The two luminous bodies, which we may denote by A and B, are made to cast shadows of the same non-transparent body upon a white screen, and the lights are so placed that the shadows may be side by side, and appear of the same depth of colour. Now it is evident that the illumination of the shadow cast by A is due to B, and of that cast by B to A, while the illumination of the screen is due to both. Hence, in this method as in the other, the luminous bodies are caused to give the same amount of illumination, and we consequently compare them as before.

196. *Method of extinction of shadows.* This depends upon the principle that the eye is not able to detect the effects of a light when brought into the presence of one about sixty-four times as powerful.

One of the luminous bodies, A, is made to cast a shadow upon a white screen, and the body, B, is then made to approach the screen, until the shadow can no longer be distinguished. In this state we infer that the illumination given by B is about 64 times as great as that given by A.

Hence, denoting the illuminating powers of A and B by L and L' respectively, and their distances by D and d, we have from (65),

illumination given by $$A = \frac{L}{D^2},$$

and illumination by $$B = \frac{L'}{d^2};$$

$$\therefore \frac{L'}{d^2} = 64 \,.\, \frac{L}{D^2}.$$

Or, $$L' : L :: (8\,d)^2 : D^2 \dots\dots\dots (102)$$

The numerical factor of $8\,d$ is for normal eyes, but as its value may vary slightly in different persons, it must be found experimentally by some of the other methods.

If, however, we wish to compare a third body, C, with B, we eliminate this factor entirely,

For, $$L'' : L :: (8\,d')^2 : D^2,$$

and, $$\therefore L' : L'' :: d^2 : d'^2,$$

in which the factor, being constant for the same eye, has disappeared.

A TABLE OF THE VALUES OF μ AND U FOR THE MOST COMMON SUBSTANCES.

	μ	U		μ	U
Alcohol	1·370	0·029	Iceland Spar	1·655	0·040
Alum	1·457	0·036	Nitric Acid	1·406	0·045
Beryl	1·598	0·037	Oil of Turpentine	1·470	0·042
Canada Balsam	1·545	0·045	Plate Glass	1·510	0 032
Crown Glass	1·530	0·036	Rock Crystal	1 560	0.026
Diamond	2·440	0·038	Rock Salt	1·557	0·053
Ether	1·366	0·037	Sapphire	1·780	0·026
Feldspar	1·536	0·042	Sulphide of Carbon	1·768	0·115
Flint Glass	1·580	0·048	Sulphuric Acid	1·435	0·031
Fluor Spar	1·435	0·022	Water	1.336	0·035

MISCELLANEOUS PROBLEMS.

1. What is the relative index when light passes from water into flint glass?

2. Find the deviation when the angle of incidence upon a plane mirror is $22° \; 30'$.

3. At what angle must two plane mirrors be inclined so that a ray incident parallel to one of them may, after reflection at both, be parallel to the other?

4. Rays falling upon a mirror from a distance of 10 feet are brought to a focus at a distance of 6 feet in front; determine the mirror.

5. Parallel rays fall upon a concave mirror having a focal length of 3 feet, and thence upon a convex one of 3 inches focal length. If the mirrors be 38 inches apart, find the position of the resultant focus.

6. In Problem 5, find the distance between the mirrors when the resultant focus is at the primary.

7. What is the critical angle for diamond?

8. An equilateral triangular prism is to be employed for the purpose of total reflection without producing refraction; determine the lowest index necessary for the substance forming the prism.

9. A stone at the bottom of a pond is seen obliquely at an angle of 40°, and appears to be 3 feet below the surface; determine the depth of the pond.

10. In a double-convex lens of crown glass the radii of curvature are 3 and 4 inches respectively; find its focal length when used under water.

11. The primary lens of a compound has a focal length of 20 inches, and receives rays from a point 12 feet distant; determine the secondary which, being placed at a distance of two feet from the primary, may have the resultant focus two inches behind itself.

12. A convex lens is placed at a distance of 2 feet from an object 1 inch long, and the image is found to be 2·25 inches in length; determine the focal length of the lens.

13. An object 5 inches in diameter is placed 18 inches in front of a convex lens of 7 inches focal length; find the position and size of the image.

14. Compare the size of the image with that of the object in Problem 11.

15. Find the dispersion in a lens of crown glass 3 feet in focal length, and 4 inches in diameter.

16. Determine the distance between the focus for red rays and that for violet ones in the lens of Problem 15.

17. Determine the constituents of an achromatic prism of water and sulphide of carbon, when causing a deviation of 5°.

18. The first face of a flint glass lens has $r = 12$ inches; what must be the radius of curvature of the second face to achromatize a convex crown glass lens of 3 feet focal length?

19. A simple microscope consists of two lenses, the first being 1 inch focal length, and the second 2 inches. What must be their distance apart in order to be achromatic when viewing an object 6 inches from the first lens?

20. A lamp is placed 6 inches from a plane wall. At a point on the wall 12 inches from the lamp compare the illumination with the greatest received by any point on the wall.

21. The least distance at which a person can see distinctly is 45 inches; determine the lens he should use.

22. In viewing a small object with a convex lens of 1 inch focus, the lens is one-half inch from the eye, and seven-eighths of an inch from the object. Determine the magnifying power under these conditions.

23. Determine the magnifying power of a Ramsden's eye-piece.

24. In a given compound microscope we have, $F = \cdot 8$, $f = 1 \cdot 5$, $R = \cdot 1$ and $l = 10$; find the magnifying power and the brightness of the field.

25. In a given telescope the radius of the object glass is 1·5 inches, and its focal length 2 feet. Required the highest power which can be used without diminishing the brightness of the field.

26. If in the instrument of Problem 25 a power of 120 be used, and the diaphragm have a radius of ·1 inch, what will be the field of view and its brightness?

27. How much shorter would Cassegrain's telescope be than Gregory's, if in each the focal length of the large speculum were 4 feet, of the eye-piece 2 inches, and if the magnifying power were 100?

28. A luminous body, A, extinguishes the shadow cast by another, B; compare their illuminating powers, the distances from the screen being, for A 8 inches, and for B 10 feet.

GLOSSARY OF TERMS AND PHRASES.

The number refers to the article in which the term is found explained.