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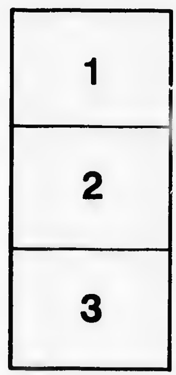

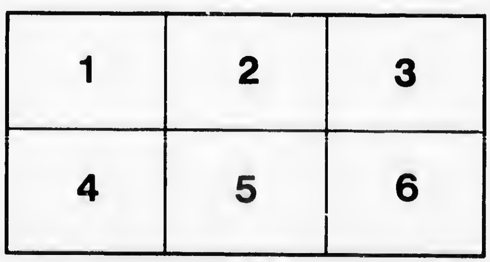

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INCORPORATED 1887.

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DISCUSSION ON TRANSITION CURVES.

By M. W. Hopkins, B.A.Sc., A.M.Can.Soc.C.E.

To be read Friday, 25th November, 1892.

The much vexed question of transition curves is getting pretty nearly settled when such excellent methods as that of Mr. Lordley's are given us and good tables made out which he tells us is being done.

His method is very accurate and not so difficult to use. I have employed a method which is not quite so accurate without corrections, as the one in Mr. Lordley's paper, but it is much more simple to apply, and is, to my mind, sufficiently accurate for 999 cases out of 1000, and with corrections given below can be made as accurate as we please.

It is composed of two separate curves, and is in fact two separate cubic parabolas, one of which is measured along the tangent from the *P. T. C.* to the middle of the offset opposite to the *P. C.* marked *P* in Mr. Lordley's fig. 1, which I will refer to as simply fig. 1 in my following remarks, and the other is measured along the circular arc beginning at the *P. C.*[1] and continuing back to the point *P*, where it meets the first cubic parabola.

The following is the formula employed for the first cubic parabola :—

Let the curvature increase in proportion to x measured along the tangent and not along the curve as in Mr. Lordley's paper.

Then we have

$$\frac{d^2y}{dx^2} = 2\,kx = \frac{1}{r} \qquad (1)$$

$$\frac{dy}{dx} = kx^2 \qquad (2)$$

$$y = \frac{kx^3}{3} \qquad (3)$$

From fig. 1 it will be seen

when $x = FK,\ r = 2\ \text{R}$ and $x = a$

" $x = FK,\ y = \frac{o}{2}$ and $x = a$

where o = the offset

$$\therefore\ 2\,ka = \frac{1}{2\,R} = \frac{D}{2 \times 5730}$$

$$\therefore\ ka = \frac{D}{22920} \qquad (4)$$

$$\text{and } \frac{o}{2} = \frac{ka}{3} = \frac{Da}{3\,(22920)}$$

$$\therefore\ a = 185 \cdot 4 \sqrt{\frac{o}{D}} \qquad (5)$$

Now, this part of the transition is only used from the *P. T. C.* up to the point *P* opposite to the *P. C.* in the circular arc in fig. 1, and the difference of length between '*a*' in this formula and the length of the curve is smaller than is usually measured in the field.

To show how to put in this part of the curve, let us take an example. Suppose we desire to put a transition on a 4° curve and we choose an offset of 2 ft. These numbers are taken as a simple case in order to be better understood. From (5) we see that this gives us "a"=131 ft. Now, the ordinates from the tangent FR (fig. 1) are prop. to x^3, as will be seen from (3), and the ordinate at P will be $\frac{2}{2}$ = 1 ft. The ordinates then will be, beginning at the *P. T. C.*,

1 ft. from the *P. T. C.*			$\left(\frac{1}{131}\right)^3 \times 1 =$	00
2 "	"	"	$\left(\frac{2}{131}\right)^3 \times 1 =$	00
10 "	"	"	$\left(\frac{10}{131}\right)^3 \times 1 =$	00
20 "	"	"	$\left(\frac{20}{131}\right)^3 \times 1 =$	00
30 "	"	"	$\left(\frac{30}{131}\right)^3 \times 1 =$	·01
35 "	"	"	$\left(\frac{35}{131}\right)^3 \times 1 =$	·02
40 "	"	"	$\left(\frac{40}{131}\right)^3 \times 1 =$	·03
45 "	"	"	$\left(\frac{45}{131}\right)^3 \times 1 =$	·04
50 "	"	"	$\left(\frac{50}{131}\right)^3 \times 1 =$	·06, etc.
125 "	"	"	$\left(\frac{125}{131}\right)^3 \times 1 =$	·87
130 "	"	"	$\left(\frac{130}{131}\right)^3 \times 1 =$	·97

This is a very convenient length, 131 ft., of transition, which being doubled gives us 262 ft. of transition curve. If we always make the offset half the degree of curvature, that is $\frac{o}{D}=\frac{1}{2}$, it will be seen from (5) that we will always get this length, and indeed it can be nearly always used to good advantage. But as we will sometimes require other lengths or other offsets, we can prepare, say, half a dozen such little tables as the above, and put them in the back of the field-book.

We can have, perhaps,

$$\frac{o}{D} = 1 \quad \therefore \quad a = 186 \text{ ft.}$$

$$\frac{o}{D} = \frac{1}{2} \quad \therefore \quad a = 131 \text{ ft.}$$

$$\frac{o}{D} = \frac{1}{4} \quad \therefore \quad a = 93 \text{ ft.}$$

$$\frac{o}{D} = 2 \quad \therefore \quad a = 262 \text{ ft.}$$

$$\frac{o}{D} = 4 \quad \therefore \quad a = 372 \text{ ft.},$$

and tabulate the ordinates as above for the transition curve for every 5 ft., say, from the *P. T. C.* to the *P. C.* One table does for each length of 'a.' Make it out for an offset of 2 ft., and then, if your offset is any other multiple of 2, multiply the ordinates so tabulated by this multiple mentally when you are putting in the transition curve to get the proper ordinates.

So far we have only given the curve for the first half of the total, that is from the *P. T. C.* to the *P. C.*

I intend to show that an exactly similar formula can be used for the second part of the transition curve, only it will be measured from the *P. C.*[1] backwards from the point B in fig. 1 to the *P. C.* at the point P. But the curvature will be supposed to vary as the distance from B measured *along the circular arc towards A* and up to the *P. C.*

Now, since the curvature of the straight line FK is constant (and always zero), its curvature can properly be represented by a horizontal straight line as a base of no curvature. Let $A\ B$ (fig. 4) represent this.

FIG. 4.

Again, since the curvature of the transition varies as the distance from the $P.\ T.\ C.$, its curvature can be properly represented by a straight line beginning at A and making an angle with $A\ B$ depending on the constant $2\ K$ in (1). Let such a line be $A\ C$ and let $A\ B = a$ in fig. 1. Then the ordinate from $A\ B$ to $A\ C$ at any point x from A will represent the curvature at that point, and of course will represent the difference of curvature between the tangent and the transition curve at that point.

Now, a similar diagram will apply to the second part of the transition curve. For since the curvature of the circular arc is constant, its curvature can properly be represented by a horizontal straight line at a distance of $\frac{1}{R}$ above the base line $A\ B$ of no curvature, since the curvature of the circle is $\frac{1}{R}$. Let this be represented by the straight line $D\ E$. Now make $D\ E$ equal to the length of the circular arc between the $P.\ C.$ and the $P.\ C.^1$ and also equal to the length of tangent from the $P.\ T.\ C.$ to the $P.\ C.$, and also, of course, equal to $A\ B$ in fig. 4.

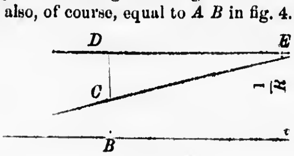

FIG. 5.

Then, since at the $P.\ C.^1$ the curvature of the transition curve is equal to that of the circular arc, its curvature at this point can properly be represented by the ordinate from E to the base line of no curvature, the same as that of the circular arc. And since the decrease in the curvature of the transition curve is proportionate to the distance from the $P.\ C.^1$ measured backwards towards the $P.\ C.$, this decrease of curvature can properly be represented by a straight line drawn from the point E in (fig. 5) below $D\ E$, and making an angle with it depending on the constant K in (1), and this angle will, of course, be equal to the angle $B\ A\ C$ in (fig. 4) and the curvature of the transition at any point x from the $P.\ C.^1$ will be represented by the ordinate between this inclined line and the base line of no curvature. Then let the curvature of the transition curve be represented by the strait line $C\ E$. Then the difference of curvature between the circle and transition at any point x from E will be represented by the ordinate from $D\ E$ to $C\ E$ in fig. 5.

Now, on examining figs. 4 and 5, it will at once be seen that they nicely fit into one another, and that one is not complete without the other. Of course $B\ C$ in fig. 4 is equal to $B\ C$ in fig. 5; that is, the curvature of the two parts of the total transition is the same at the $P.\ C.$ or at point P in fig. 1. And as the second part of the transition curve recedes from the circular arc at any point with the same angle as the first part of the transition recedes from the tangent at the corresponding point, both depending equally on the constant k in (2), and as the length of the circular arc is equal to the length of the tangent from the $P.\ T.\ C.$ to the $P.\ C.$, it follows that the inclination of the first part at F to the tangent $F\ K$ is equal to the inclination of the second part at P to the tangent of the circular arc at $P.\ C.$, and consequently the first and second parts of the transition have a common tangent at the point P, and from (5) it is evident that the point P must bisect the offset at the $P.\ C.$

It will be seen from the explanation above that the same equations will apply to the second part of the transition curve as for the first part, with this distinction: the formulæ for the second part represent not the total curvature, or tangent, or ordinate, as in the formulæ for the first part, but represent the *difference* between these functions of the circular arc and the transition curve. Then the above equations are identical for the two parts.

Now, since the length of the tangent from the *P. T. C.* is so very nearly equal to that of the first part of the transition, and the length of the circular arc is so very nearly the same length as the second part of the transition curve, I am pretty safe in asserting that a passenger riding in the train passing over this part of the track would not be able to say whether the variation in curvature was calculated according to the length of the transition curve or of the combined lengths of the tangent and the circular arc.

The second part of the transition is put in exactly as the first part excepting that you commence at the *P. C.*[1] and measure *backwards* towards the *P. C. along the circular arc*, and measure the ordinates outward from the circular arc in fig. 1. But in practice the circular arc will be the distance of the offset "*o*" outside of the circular arc (which we will call the inner circular arc), and will be a continuation of the tangent *F K* instead of the line *D A*, and will have its *P. C.* at the point *K* instead of at *A*. Consequently it will be necessary to measure the ordinates for the transition curve *inwards the distance of the offset, minus the tabulated ordinates*, or, if "*y*" is the tabulated ordinate from the inner circular arc to the transition curve, it will be necessary to measure the distance (*o—y*) *inwards* instead of the distance "*y*" *outwards*. I always run in the regular circular curve in location just as if there were to be no transition, and then, just before construction, go along the line and, with an assistant, pull up the stakes in the circular curve and move them in the distance (*o—y*) for the second part of the transition. But for the first part of the transition the stakes are moved in only the distance "*y*."

Two men can do this as fast as they can walk along, and one pull up the stakes and move them in the proper distance, while the other gives that distance from the little tables. If the stakes are set 50 ft. apart in a transition of 262 ft., only five stakes will have to be moved on each end of the circular curve, which will give an idea of the rapidity with which this can be done. A number of miles can be done in a few hours even where there is considerable curvature, as in a hilly country.

It must be remembered that this transition is composed of two distinct parts which happen to fit in together nicely. But one is measured *along a circular arc backwards*, while the first is measured *along the tangent forwards*.

I always keep the same hubs as were used for the ordinary circular curve, and when the transition is put in no other hubs are required, but the old ones must be preserved, and if it is necessary to run in the line again the circular arc is run in first as if no transition were going to be put in, and at any time convenient the stakes moved as directed above. In the transit book it is only necessary to note down the offset chosen or "*a*," as the degree of curvature will have been already noted for the ordinary circular curve. Then the transition curve can be put in any time afterwards.

And then we have to consider that by this method we can lay down the transition curve from lines already established, and without any calculation more than can easily be done mentally by any one who can do ordinary multiplication.

As said above, this method, without using the corrections, is only an approximation, but a very close approximation, so close, in fact, that as far as making the trains ride easily over it, it is as good as if the variations of curvature were calculated to vary with the distance from the *P. C.* along the transition curve itself. But as there may be times when it will be necessary to know the exact difference between the length of the total transition and

"2 a", and as this is a very easy matter to find, and the resulting formula is very simple, the solution is given in what follows.

For the first part of the transition let

s = distance of any point from the $P. T. C.$ measured along the transition curve, and let

x = the corresponding distance measured along the tangent.

Then we have

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + k^2 x^4}\, dx \text{ from (2)}$$

$$= \left(1 + \frac{k^2 x^4}{2} - \frac{k^4 x^8}{8} + \frac{k^6 x^{12}}{16} - \&c.\right) dx$$

$$\therefore s = \left(x + \frac{k^2 x^5}{10} - \frac{k^4 x^9}{72} + \frac{k^6 x^{13}}{208} - \&c.\right)$$

When $x = a$, $s = s_1''$ = length of first half of the transition curve

$$\therefore s_1'' = \left(a + \frac{k^2 a^5}{10} - \frac{k^4 a^9}{72} + \frac{k^6 a^{13}}{208} - \&c.\right)$$

$$s_1'' = a\left(1 + \frac{D^2 a^4}{10\,(22920)^2} - \frac{D^4 a^8}{72\,(22920)^4} + \frac{D^6 a^{12}}{208\,(22920)^6} - \&c.\right)$$

$$= a\left\{1 + \frac{1}{10}\left(\frac{D\,a}{22920}\right)^2 - \frac{1}{72}\left(\frac{D\,a}{22920}\right)^4 + \frac{1}{208}\left(\frac{D\,a}{22920}\right)^6 - \&c.\right\}$$

Writing b for $\left(\frac{D\,a}{22920}\right)$ we have

$$s_1'' = a\left\{1 + \frac{b^2}{10} - \frac{b^4}{72} + \frac{b^6}{208} - \&c.\right\}$$

$$\therefore \frac{s_1''}{a} = 1 + \frac{b^2}{10} - \frac{b^4}{72} + \frac{b^6}{208} - \&c.$$

$$\text{and } s_1'' - a = a\left\{\frac{b^2}{10} - \frac{b^4}{72} + \&c.\right\} \quad \ldots\ldots\ (6)$$

Thus we see that the ratio $\frac{s_1''}{a}$ depends only on the product $D\,a$.

If we make this product 1310,

then $b = \cdot057155$

$$\therefore \frac{s_1''}{a} = 1 + \cdot000326669 - \cdot0000001482 + \&c.$$

$$= 1\cdot00032652 \text{ ft.}$$

Now if $a = 131$ ft. and $D = 10°$

$$s_1'' - a = .043 \text{ ft}$$

= half an inch.

It will be seen that it is a very simple matter to find the value of the ratio $\frac{s_1''}{a}$ from the above formula, as with a table of logarithms one logarithm will do for all the terms, and when this logarithm is multiplied by the respective exponents of "b" all the values of b^2, b^4, &c., can be found at once. But it is evident that b^2 and b^4 are all the terms that need to be considered to secure the greatest accuracy ever required.

It is also easy to find the difference in length between the second part of the transition curve and that of the circular arc, from which it is measured off, in the following manner:

Let s be the length of any point from the $P. C.^1$ measured along the transition curve, and x the length of the corresponding point measured along the inner circular arc. As before, let y = ordinate measured along the inner circular arc, and let R_1 = radius of curvature of the inner circular arc, and D_1 its degree of curvature, and a_1 its length measured along the inner circular arc. Then,

$$y = \frac{k\,x^3}{3}$$

$$\therefore \frac{ds}{dx} = \frac{R_1 + y}{R_1} = \frac{R_1 + \frac{k_1 x^3}{3}}{R_1} = 1 + \frac{k\,x^3}{3\,R_1}$$

$$\therefore s = x + \frac{k\,x^4}{12\,R_1}$$

when $x = a$, $s = s_2''$

$$\therefore s_2'' = a_1\left(1 + \frac{k\,a_1^3}{12\,R_1}\right) = a_1\left(1 + \frac{D_1^2\,a_1^4}{3\,(22920)^2}\right)$$

$$\therefore \frac{s_2''}{a_1} = 1 + \frac{D_1^2\,a_1^2}{3\,(22920)^2} \quad \therefore s_2'' - a_1 = a\,\frac{b^2}{3} \ldots (6\tfrac{1}{2})$$

But as in practice the last half length of the transition curve is measured along the outer circular arc, which lies just the distance of the offset "o" radially outward from the inner circular arc, what we really want is not the ratio $\frac{s_2''}{a_1}$ but the ratio $\frac{s_2''}{a}$. If we let D, a and R stand for similar functions of this outer circular arc, as follows, we can easily transform the above equation to give us this requirement, for $D_1\, a_1 = D\, a$ since the factors in each vary inversely as one another,

$$\text{and } a_1 = \frac{R-o}{R}\, a = \frac{5730 - o\,D}{5730}\, a$$

$$\therefore \frac{s_2''}{a} = \frac{5730 - o\,D}{5730}\left(1 + \frac{D^2\, a^2}{3\,(22920)^2}\right)$$

or using "b" to stand for $\frac{D\,a}{22920}$ we have

$$\frac{s_2''}{a} = \frac{5730 - o\,D}{5730}\left(1 + \frac{b^2}{3}\right)$$

$$\therefore a - s_2'' = a\left\{\frac{o\,D}{5730}\left(1 + \frac{b^2}{3}\right) - \frac{b^2}{3}\right\} \quad \ldots\ldots (7)$$

It will be noticed that, although in our formula for $\frac{s_1''}{a}$ for the first part of the transition curve, we have taken D to be the degree of curvature of the inner circular arc instead of that of the outer circular arc, it makes so very little difference in the result that we need not consider it when the offset is small, but when the offset is large it becomes considerable. For in that formula

$\frac{s_1''}{a}$ depends only on $D\,a$ and constants, and since

$$a = 185{\cdot}4\sqrt{\frac{o}{D}} \qquad \text{from (5)}$$

$$D\,a = 185{\cdot}4\sqrt{o\,D}$$

Hence the correction for "b" in that formula would take the form

$$b^1 = \sqrt{\frac{5730}{5730 - o\,D}}\; b \quad \ldots\ldots (8)$$

This is only a correction of a correction, it will be noticed, and a small one at that if '$o\,D$' is kept small. Of course this correction would seldom need to be applied to more than the second term to give the greatest accuracy required. For instance, if we take the example given of the application of that formula where $D = 10$ and $a = 131$, and consequently 'o' $= 5$, the correction is only

$$\cdot 0000287\, a$$
$$= \cdot 0037 \text{ ft.}$$

when applied to the second term, and the correction from the succeeding terms is practically infinitely small. But for larger values of '$o\,D$,' it will be seen from equation (8), the correction increases very rapidly.

Hence the total difference between '$2\,a$' and the total length of the two parts of the transition curve can now be put in one single formula, which will be very simple.

For the first part,

$$s_1'' - a = a\left(\frac{b^2}{10} - \frac{b^4}{72}\right)$$

For the second part,

$$a - s_2'' = a\left\{\frac{o\,D}{5730}\left(1 + \frac{b^2}{3}\right) - \frac{b^2}{3}\right\}$$

$$\therefore \text{ total difference} = d_2 - d_1$$

$$= d = 2\,a - s_1'' - s_2'' = a\left\{\frac{o\,D}{5730}\left(1 + \frac{b^2}{3}\right) - \frac{b^2}{3}\right\} - a\left(\frac{b^2}{10} - \frac{b^4}{72}\right)$$

As the term $\frac{b^4}{72}$ will only have to be used when 'b' is exceptionally large indeed, it can be neglected for railroad work, and still the equation will give the correct result to a small fraction of an inch when small offset is used.

$$\therefore d = a\left\{\frac{o\,D}{5730}\left(1 + \frac{b^2}{3}\right) - \frac{b^2}{3}\right\} - a\,\frac{b^2}{10}$$

$$= a\left\{\frac{o\,D}{5730}\left(1 + \frac{b^2}{3}\right) - \frac{b^2}{3} - \frac{b^2}{10}\right\}$$

If the maximum degree of curvature used be 10° then the last 'd' will be correct to less than half an inch, even when 'a' is as great as 262 ft.

It will be evident from the above that it would be waste of time to make out a set of tables for these corrections, as they will not have to be applied in more than one case in a thousand, and for that isolated case it would be better to calculate the correction than to carry a needless load around so long.

As for a set of tables for the ordinates along 'a' for the transition curve, a dozen such as the one given above would be enough and to spare. In every case where it is not desirable to choose a particular offset so as to fit the ground, it is likely that the one where 'a' = 131 and $\frac{o}{D} = \frac{1}{2}$ will almost always be employed. This method has this in its favor, among other things, that it is so simple and easy to put in on the ground that, no matter how indolent or ill-informed an engineer might be, he would in all probability put it in. And then, after it is in, it is as good as the best. I can't help but think that it is the one that will always be used when all engineers come to understand the importance of transition curves, and to see how easy it is to put this one in on the ground. It is simplicity itself, is as good as if it were better, and can be accurately calculated as to length, &c., from the formulæ given above, when such calculation is necessary, which will not be once in a thousand times. Enough is as good as a feast and much more wholesome. And last but not least, it can be put in at any time after location by simply moving the stakes as directed above.

If, as might happen sometimes, it is necessary to have a result infinitely correct, the combination of (6), (7) and (8) will give it, and then we have

$$d = a\left\{\frac{o\,D}{5730}\left(1+\frac{b^2}{3}\right)-\frac{b^2}{3}\right\} - a\left(\frac{b'^2}{10}-\frac{b'^4}{72}+\frac{b'^6}{208}-\&c.\right)$$

$$= a\left[\frac{o\,D}{5730}\left(1+\frac{b^2}{3}\right)-\frac{b^2}{3}-\left\{\frac{\left(\sqrt{\frac{5730}{5730-o\,D}}.\,b\right)^2}{10} - \frac{\left(\sqrt{\frac{5730}{5730-o\,D}}.\,b\right)^4}{72} + \&c.\right\}\right] \quad \ldots\ldots (9)$$

With this correction the method is practically correct, and when it is required to find the difference in length between the total length of the transition curve and '$2\,a$' this last formula (9) should always be used, unless the offset 'o' is small. For accuracy it should be run out the same number of terms as is done above.

If it should be desired to run in new tangents from an already established circular curve, this circular curve would then become the inner circular arc, and the second part of the transition would be measured along it. If it should be required to know the difference in length between the whole transition and '$2\,a$,' this can be found by combining (6) and (6½), and we get

$$d = a\left\{\frac{b^2}{10}-\frac{b^4}{72}+\&c.\right\}+a\frac{b^2}{3}$$

$$= a\left\{\frac{13\,b^2}{30}-\frac{b^4}{72}+\&c.\right\}$$

$$= a\,\frac{13\,b^2}{30} \text{ very approximately.}$$

The transition should be put in as in the other case, of course, except the ordinate 'y' for the transition curve is measured *outward from the circular arc*, and (o—y) *outward from the old* tangent or 'y' *inward from the new* tangent in all cases in same direction as the other case.