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THIRD EDITION - REVISED AND ENLARGED

## CANADIAN

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Math. Master, sfafurth H. S.
TORONTO
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1897
** The answers to the Problems in this Book are printed separately in pamphlet form, and supplied in liberal proportion without extra charge, for the use of Teachers in Business Colleges, Schools and Institutions.

Entered according to Act of the Parliament of Canada in the year one thousand eight hundred and ninety-seven, by C. A. Bengouah, in the office of the Minister of Agriculture.

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## PUBLISHER' S PREFACE.

THIS Arithmetic has been so weil recelved by Bushess Collegen, Schools, Aecountants, and the General Public, and fias been found so thorougbly practical and helpful, that a third edttion has been called for. Advantage has been taken of this faet to add severai new features, mong them a fuli and complete Index, for the careful compilation of which we are indebted to Mr. Elven J. Bengouth. By means ef this Index, whieh has heen arranged Topically as well as Aiphabeticaily, the compicte contents of the work can be rapidiy reviewed. A glance over this Index wili ilhstrate, morr foreibly than any words of ours, the comprehensive character of the Canadias Commerclaf, artthmetic.

Tue Metitic System having been legatized in Canada, a chapter has been added dealing with this important subject. This chapter is the most complete and practicad to be found in any work. It has been complled with special reference to commercial usage. and avoids Physles on the one hand and Higber Mathematies on the other. The Ms. was submitted for revision to Sin Henit Joly, Minister of Iniand Reveme for Canada, who is the higriest authority on the subject in the Dominion, having mate a special study of the Metric System, and having gone to the trouble and expense of getting diagrams, etc., from France recentiy. Sir Henrl has exanined our MS. with eare,


Sir Henri Joly Minister of laland Revenue, canada and offered vaiuable suggestions, and he considers that "the chapiter will be very "useful in teaching the Metrieal System, and that the comparison between that "system, which appears so iogleai, with the present systems of measurement, will and intelicetual exercise for students.' of value to all students who uspire to become Charteied Accocntants whli be the Instltute; while the Fxpers of now for the first time inale generaily questions in mercantile arithmetic, scope of the Institute, and will fally pmblie, will give a correet idea as to the This chapter has been revised by the valuable materiat for exeretse by students. Esq., F.C.A., who, with the Preathecretary of the Institute. Harry Vigeon, Treasurer, W. B. Tindalid, Fsq.ent, George: Edwaris, Esq.. F.C.A., and the Canadian Commercial. Arithmetio
valuatic suggestions.
A thoroughy practieal ant selentlie arithmetleat education can be obtained trom this Work, which embraces a treatment of all the suigeets necessary
therefor.

Attention is directed to the following teatures:-

1. To the clearness and conclseness of the definitions, soiutions, and rules. the latter of which are logically deduced from preceding solutions.
2. To the many short methods in addition, multiplieatlon, division, ete.
3. To the numerous solutions, and the large number of exerelses, the practieal character of whlch will doubtless commend them to all teachers. In this important partleular the authors lelleve a long-felt want has been supplied.
4. To the thorough treatment of Percentage, and its applieations in Interest, Discount, Partlal Payments, Equation of Accounts, etc.
5. To the clear statement of the Commercial Law relating to Interest, Discount, etc.

Belleving that this volume in its improved form will find an ever-increasing number of favorites, it is confidently sent forth.

Toronto, November, 1897.

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## INDEX TO ILLUSTRATIONS.



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## CHARACTERS AND ABBREVIATIONS

## USED IN BUSINESS.


a/c
Accoun

- Cents.

Per cent.
Number.
One and one-quarter.
One and one-half.
13 One and three-quarters.
$\sqrt{ }$ Check mark.
$\times$ By, as $14 \times 18$ inches.
\$ Dollars.
$\pm$ Pound sterling.
6/3. English shillings and pence are frequently written in this manner, the shillings on the left of the sloping line, and the pence on the right, the above meaning 6 shillings and 3 pence.
May $18 / 21$. The day of maturity, as expressed in a note, and the last day of grace are indicated by writing the first on the left and the second on the right of the
sloping line.
15 doz. ${ }^{s}$
5 of which ${ }^{5} 5, \frac{5}{515}$. Fifteen doz., @ $\$ 15$, and are $\$ 12$ per doz., 5 doz . @ $\$ 15$, and 5 doz. at $\$ 18$ per doz.
${ }^{1}$ hhd. Sugar.
1100
155945 lbs.
1100 pounds gross weight, 155 lbs. tare, or weight of hhd.
945 lbs., net weight.
The numbers in
4 ps .28 36 138 yds the bracket are the each piece respec. tively.
10 doz. 4 @ 2/-量@3/6. 4 doz. No.
5 @ 2 shillings per doz.: 6 doz.
No. 8 @ 38.6d. per doz.
W. W. and similar charaoters
and letters, are placed on packages to designate a partioular lot or shipment.
(anods are numbered and marked, that they may distinguistied withont minute description.)
$7 \times 9$, or 7 by 9 in. 7 in. wide, 9 in.
long.

|  |
| :---: |
| B.B . . . . . . . . . B |
| Bal .......... . Balanoe |
| Bbl. . . . . . . . . . Barrel. |
| Bdls. . . . . . . . . Bundles. |
| Bgs . . . . . . . . . Bags. |
| Bkts. . . . . . . . . . Baskets. |
| Blk . . . . . . . . . Black. |
| Bls . . . . . . . . . Bales. |
| Bot . . . . . . . . . Bonght. |
| B.L. or B. of L. Bill of Lading. |
| Bills Pay. . . . . . Bills payable. |
| Bnk . . . . . . . . . . . . Bank. |
| Brot . . . . . . . . . Brought. |
| Bque ......... Barque. |
| Br . . . . . . . . . . ${ }^{\text {Brig. }}$ |
| Bus . . . . . . . . . Bushels. |
| Bxs . . . . . . . . . Boxes. |
| C . . . . . . . . . . . Cents. |
| O or centum . . Hundred, |
| C.B.. . . . . . . . Cash-book. |
| Ck. . . . . . . . . . . Check. |
| Cap . . . . . . . . . Capital. |
| Co...... . . . . . . Coinpany. |
| C.O.D . . . . . . . Collent on delivery. |
| Col'd . . . . . . . Colored. |
| Cr. . . . . . . . . . . Creditor. |
| Com. . . . . . . . Commissio |
| Cons't . . . . . . . . Consignment. |
| Cs . . . . . . . . . Cases. |
| Cwt . . . . . . . . . Hinndredweight. |
| C/o ...... . . . Care of. |
| d . . . . . . . . . . . Pence. |
| Dft . . . . . . . . . Draft. |
| Div . . . . . . . . . Dividend. |
| Disct ........ Disoount. |

Adv. ...........Account.
Agt .............. Adventure.
Amt................. Ament.
Ass'd ......... Assorted.
B.B . . . . . . . . . . Bill-book.
Bal
Balanos.

Bbl. .............. . . Balanoe.
Bdls. . . . . . . . . . Bundles.
Bgs . . . . . . . . . . Bags.
Bkts. ........... . . Baskets.
Bls .... . . . . . . . Black.
Bot .............. Bought.
B.L. or B. of L. Bill of Lading.

Bills Rec........ . Bills receivable.
Brot . . . . . . . . . . Bank.
Bque ...........Brought.
Br ............. Barque.
....Brig
Bxs . . . . . . . . . . . Boxes.
C . . . . . . . . . . . . Cents.
C or centum .. Hundred,
..... . . . Cash-book
Cap . . . . . . . . . . Capital.
Co................ Coinpany.
Col'd ....... . . Colleot on delivery.
Cr. . . . . . . . . . . . Creditor.
Com. . . . . . . . . Commission.
. . . . . . . . Consignment.
Cwt . . . . . . . . . . . Handred weight.
d ................ Pence.
Dft . . . . . . . . . . Draft.
Disct . . . . . . . . Disoount.


## ADDITION.

1. Rapidity and accuracy in addition are of the first importance to the commercial student.

These can be acquired only by a thorough familiarity with the simple combinations of numbers, and a proper practice with these combinations.

The following Tables exhibit all the combinations of numbers, and the attention of the student is especially directed to the endings, that is the right-hand figure in such combinations :

Combinations ending with 0.

| 1 | 2 | 3 | 4 | 5 |
| ---: | ---: | ---: | ---: | ---: |
| 9 | 8 | 7 | 6 | 5 |
| 10 | $\frac{5}{10}$ | $\frac{5}{10}$ | $\frac{10}{10}$ | $\frac{1}{10}$ |

Combinations ending with I .

| 1 | 2 | 2 | 4 | 5 |
| ---: | ---: | ---: | ---: | ---: |
| 0 | 9 | 8 | 7 | $\frac{6}{1}$ |
| 1 | $\frac{11}{11}$ | $\frac{11}{11}$ | $\frac{1}{11}$ |  |

Combinations ending with 2.

| 1 | 2 | 3 | 4 | 5 | 6 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $\frac{1}{2}$ | $\frac{0}{2}$ | $\frac{9}{12}$ | $\frac{8}{12}$ | $\frac{7}{12}$ | $\frac{6}{12}$ |

Combinations ending with 3 .

| 2 | 3 | 4 | 5 | 6 |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0 | 9 | 8 | 7 |
| 8 | $\frac{7}{8}$ | $\frac{13}{13}$ | $\frac{13}{13}$ | $\frac{1}{18}$ |

8
ADDITION.
Combinations ending with 4.


Combinations eading with 5 .

$$
\begin{array}{rrrrr}
3 & 4 & 5 & 6 & 7 \\
2 & 1 & 0 & 9 & 8 \\
\hline 5 & \frac{8}{5} & \frac{5}{5} & \frac{15}{15} & \frac{15}{15}
\end{array}
$$

Combinations ending with 6 .


Combinations ending with 7 .

$$
\begin{array}{rrrrr}
4 & 5 & 6 & 7 & 8 \\
3 & 2 & 1 & 0 & 9 \\
\hline 7 & \frac{9}{7} & \frac{7}{7} & \frac{7}{7} & \frac{17}{17}
\end{array}
$$

Combinations ending with 8.

$$
\begin{array}{cccccc}
4 & 5 & 6 & 7 & 8 & 9 \\
4 & 3 & 2 & 1 & 0 & 9 \\
\hline 8 & -8 & \frac{9}{8} & \frac{8}{8} & -8 & \frac{9}{18}
\end{array}
$$

Combinations exding with 9.

$$
\begin{array}{ccccc}
5 & 6 & 7 & 8 & 9 \\
4 & 3 & 2 & 1 & 0 \\
\hline 9 & -9 & -9 & 9 & \frac{9}{9}
\end{array}
$$

After the student becomes familiar with the foregoing cumbinations his attention is directed to the use of the endings. For example:

i.e., the sum of any two numbers, one of which ends with 6 and the other with 7, produces a number ending with 3. A thorough drill of this kind should be given with all the combinations.
2. An effective drill may be given to the student by the use of the following diagram :


The teacher places any number within the circle and requires the pupils to add to it any number or succession of numbers to which he may point.

Rapidity and accuracy in addition can be gained only by adding columns of figures.
3. In adding ledger columns, accountants frequently use the following devices :

## Example 1.-

| $\$ 926.42$ |
| ---: |
| 49.98 |
| 67.84 |
| 876.55 |
| 4867.89 |
| 94674 |
| 6487.45 |
| $\$ 14222.87$ |
| 44543 |

The figure to be carried is placed under the column to which it belonge so that in case of interruption or mistake it may bo used for reference.

## 4. Example 2.-

| 98746 |  |
| :--- | :--- |
| 2385 |  |
| 91642 |  |
| 28735 |  |
| 82614 |  |
| 79186 |  |
| 25738 | 328308 |
| 87264 |  |
| 19285 |  |
| 63127 |  |
| 58432 |  |
| 82691 | 203846 |
| 35417 |  |
| 63529 |  |
| 48763 |  |
| 21734 | 252134 |
|  | 784288 |

The column to be added is divided into several parts. These parts are added and the sum of the results then taken.
5. Addition of two or more columns at the same time.

Example 8-

| 35 |
| ---: |
| 89 |
| 76 |
| 47 |
| 247 |

Method or Addition-
47 \& 6 make $\grave{3} 3, \quad 53 \& 70$ make 123, $123 \& 9$ make 132, 132 \& 80 make 212, 212 \& 5 make 217, 217 \& 30 make 247.
Columns of three or four figures may be added in the same way, or by adding two columns at a time.

The methods 'employed in Examples 2 and 3 are excellent tests of the correctness of addition performed in the ordinary way.
6. To find the sum of any series of numbers which have a common difference.

## role.

Multiply the sum of the first and last terms by the number of terms and divide the result by 2.

Eximpis 1.-Add, 16, 17, 18, 19, 20, 21, 22, 28, 24, 25, 26, 27. Operation. Common difference is 1.

16 first term.
27 last term.
43
12 number of terms.
$2 \longdiv { 5 1 6 }$
Eliample 2.-Add, 48, 56, 64, 72, 80, 88, 96, 104, 112.
48
112 Coinmon difference ip $\mathbf{A}$.
160
9
21440
720

## MULTIPLICATION.

SHORT METHODS IN MULTIPLICATION.
7. To multiply by any of the numbers from II to 19

Multiply 4025 by 14.


The student will observe that we multiply by 4 in the ordinary way, but in addition to the ordinary number to be carried we also carry the figure to the right of the figure multiplied.

## GRCOND IETHOD.

$4625 \times 14$
18500
64750
Multiply by 4 , placing the product one place to the right and add.

Note.-This method may be applied when the multiplier has one or more ciphers between the two figures, by writing the product two or more places to the right, and adding.

## EXERCISE I.

Multiply-

1. 79526 by $11,12,13,14,15,16,17,18,19$.
2. 87295 by $102,104,105,107$.
3. 49278 by $1008,1006,1008,1003$.
4. To multiply by any number of two figures ending with 1 .

Multiply 846 by 41.
FIRET method.

| 846 | $6 \times 1$ | $=6$ |  |
| :---: | :---: | :---: | :---: |
| $\stackrel{4}{34686}$ | $6 \times 4$ |  |  |
|  | $4 \times 4+2$ (carried) | $+4=28$ $+8=26$ | carr |
|  | $8 \times 4+2$ | 34 |  |

The student will observe that we place the units figure of the multiplicand as the units figure of the product. Then multiply by 4 , and in addition to the ordinary number to be carried, we carry the figure to the left of the figure multiplied.
$846 \times 41$
3384
34686
SECOND METHOD.

Multiply by 4, placing the product one place to the left and add.
Note.-This method may be applied when the multiplier has one or more ciphers between the two fignres by writing the product two or more places to the left.

## EXERCISE 2.

Maltiply-

1. 64278 by $21,81,41,51,61,71,81,91$.
2. 87396 by $301,501,601,801$.
3. 93254 by 2001, 3001, 7001, 9001.
4. To multiply two numbers in which the units figures added make 10 , the other figures being the same in each.

Erample 1.-Multiply 74 by 76.

| 74 |
| ---: |
| 76 |
| 5624 |

METHOD.
$4 \times 6=24$ $(7+1) \times 7=56$

Exampler 2.-

| 123 |
| ---: |
| 127 |
| 15621 |

Multiply 123 by 127.
METHOD.
$3 \times 7=21$
$(12+1) \times 12=156$

## 8

## MULTIPLICATION.

## EXERCISE 3.

1. $34 \times 86$.
2. $92 \times 98$,
3. $55 \times 55$. 8. $64 \times 66$.
4. $72 \times 78$.
5. $85 \times 85$
6. $65 \times 65$.
7. $63 \times 67$.
8. $97 \times 93$.
9. $81 \times 39$.
10. $78 \times 72$.
11. $91 \times 99$.
12. $112 \times 118$.
13. $123 \times 127$.
14. $104 \times 106$.
15. $105 \times 105$.
16. $101 \times 109$.
17. $292 \times 298$.
18. $153 \times 157$.
19. $491 \times 499$.
20. $694 \times 696$.
21. $225 \times 225$.
22. $392 \times 398$.
23. $173 \times 177$.
24. To multiply two numbers in which the units figures are the same.
Multiply 46 by 66.

| 46 |
| ---: |
| 66 |
| 3036 |

$$
\begin{aligned}
6 \times 6 & =36 \text { carry } 3 \\
(4+6) \times 6+3(\text { carried }) & =63 \text { carry } 6 \\
4 \times 6+6(") & =30
\end{aligned}
$$

## EXERCISE 4.

1. $21 \times 51$.
2. $53 \times 53$.
3. $64 \times 64$
4. $45 \times 25$.
5. $86 \times 36$
6. $67 \times 57$.
7. $47 \times 87$.
8. $28 \times 38$.
9. $58 \times 48$.
10. $19 \times 29$
11. $27 \times 47$.
12. $105 \times 125$.
13. $81 \times 91$.
14. $36 \times 56$.
15. $113 \times 183$.
16. $92 \times 72$.
17. $42 \times 72$.
18. $84 \times 34$
19. $114 \times 144$.
20. $83 \times 73$.
21. $136 \times 126$.
22. $116 \times 146$.
23. $125 \times 135$.
24. $\quad 117 \times 197$.

## 11. To multiply two numbers in which the units

 figures are unlike, the remaining figures being alike.Example 1.-Multiply 78 by 72.

| 78 |
| ---: |
| 72 |
| 5616 |

$$
\begin{aligned}
\text { Method. } & \\
8 \times 2 & =16 \text { carry } 1 \\
(8+2) \times 7+1 \text { (carried) } & =71 \text { carry } 7 \\
7 \times 7+7 \text { (carried) } & =56
\end{aligned}
$$

Example 2.-Multiply 126 by 122.

$$
\begin{aligned}
& 126 \\
& 122 \\
& \text { Method. } \\
& \begin{aligned}
6 \times 2 & =12 \text { carry } 1 \\
(6+2) \times 12+1 \text { (carried) } & =97 \text { carry } 9 \\
12 \times 12+9 \text { (carried) } & =153
\end{aligned}
\end{aligned}
$$

1. $37 \times$ їб.
2. $54 \times 52$.
3. $69 \times 61$.
4. $74 \times 78$.
5. $75 \times 76$.
6. $85 \times 84$.
7. $48 \times 43$.
8. $83 \times 82$.
9. $91 \times 92$.
10. $27 \times 29$.
11. $74 \times 72$.
12. $26 \times 27$.
13. $67 \times 59$.
14. $116 \times 113$.
15. $74 \times 72$.
16. $38 \times 37$.
17. $124 \times 125$.
18. $46 \times 45$.
19. $61 \times 69$.
20. $147 \times 141$.
21. $78 \times 74$.
22. $157 \times 159$.
23. $323 \times 322$.

## 12. To multiply by means of cross multiplication.

Example 1.-Multiply 56 by 63.
METHOD.
$63 \quad \begin{array}{ll}6 \times 3 \\ 5 & =18, \text { oarry } 1 \\ 5 & 5 \times 1 \text { (carried) }+6 \times 6=52, \quad 4 \quad 5\end{array}$
3528
$5 \times 6+5(\quad$ ( $)$
35
Example 2.-Multiply 346 by 23.
METHOD.


EXERCISE 6.

1. $86 \times 32$.
2. $45 \times 62$.
3. $346 \times 43$.
4. $94 \times 24$.
5. $39 \times 74$.
6. $608 \times 37$.
7. $79 \times 45$.
8. $82 \times 51$.
9. $543 \times 23$.
10. $87 \times 36$.
11. $37 \times 22$.
12. $760 \times 48$.
13. $28 \times 51$.
14. $46 \times 25$.
$15.3268 \times 79$.
15. To multiply by a number ending with 9 . rdee.
Multiply by 1 more than the given multiplier and substract the multipiicand.

Multiply 263 by 69.
operation. 18410 (product by 70)
263 ( " " 1)

18147 (" " 69)
EXERCISE 7.

Multiply-

1. 3764 by 79, 49.
2. 13256 by $119,399,169$.
3. 46251 by 39,59 .
4. 47395 by $290,999,799$.
5. 37284 by 99,69 .
6. 29635 by $89,29$.
7. 27634 by $149,249,189$.
8. 17825 by $9999,499,139$.
9. To multiply by a number which is a little less than 100, 200, 300, 400, etc.
rule.
Multiply the multiplicand by the difference between the multiplier and 100, 200, 300, or etc., and substract the product from the product of the multiplicand by 100, 200, 300. or etc.

Multiply 675 by 97 .

| operation. <br> 67500 (product by 100 ) |
| :---: |
| 2025 ( " ، 3) |
| 65475 ( " " 97) |
| EXERCISE 8. |

Multiply-
5. 36092 by $993,795,990$.

1. 3684 by $98,96,94$.
2. 41523 by $93,95,97$.
3. 27136 by $296,495$.
4. $\quad 3124$ by $794,897$.
5. 14613 by $988,991,791$.
6. 30257 by $989,995,9993$.
7. 17824 by $992,6999,996$.
8. To multiply two numbers, one of which is more and the other less than 100, 1000, etc.

The complement of a number is the difference between that number and the unit of the next higher order.

Multiply the sum $11^{\text {RoLe. }}$ by the unit of comp product of the excess and the complement. Multiply 108 by 94 .

Unit of oomparison is 100.


| Method. |  |
| ---: | :--- |
| $108-6$ | $=102$ |
| or $94+8$ | $=102$ |
| or $108+94-100$ | $=102$ |
| $102 \times 100$ | $=10200$ |
| $8 \times 6$ | $=\frac{48}{10152}$ |

## EXERCISE 9.

1. $107 \times 97$.
2. $105 \times 95$.
3. $112 \times 91$.
4. $113 \times 88$.
5. $115 \times 93$.
6. $103 \times 94$.
7. $108 \times 96$.
8. $114 \times 95$.
9. $104 \times 87$.
10. $1012 \times 994$.
11. $106 \times 92$.
12. $1015 \times 988$.
13. $1032 \times 998$.
14. $1064 \times 993$.
15. $1025 \times 989$.
16. To multiply two numbers of the same number of figures over and near 100, 1000, etc.
role.
From the sum of the numbers substract the unit of comparison, and to the right of the result write the product of the excesses.
notes.
17. When there are fewer figures in the product of the exoesses than ciphers in the unit of oomparison, write ciphers in the result to supply the defloienoy.
18. When there are more figures in the product of the excesses than oiphers in the unit of comparison, add the excess on the left hand to the first part of the resalt.
19. After practice, the writing of the complements or the excesses in examples where they are nsed may be omitted.

Multiply 112 by 106.

| 112 | .. | 12 excess |  |
| :--- | :--- | :--- | :--- |
| 106 | .. | .$^{\prime \prime}$ | 6 |
| 11872 |  |  |  |

METHOD.

| $112+6$ | $=118$ |
| ---: | :--- |
| or $106+12$ | $=118$ |
| or $112+106-100$ | $=118$ |
| $12 \times 6$ | $=72$ |

Unit of comparison is 100 .
$12 \times 6=72$
EXERCISE 10.

1. $112 \times 108$.
2. $114 \times 108$.
3. $1006 \times 1003$.
4. $108 \times 103$.
5. $107 \times 115$.
6. $1017 \times 1008$.
7. $115 \times 105$.
8. $109 \times 107$.
9. $1125 \times 1009$.
10. $113 \times 104$.
11. $116 \times 108$.
12. $1034 \times 1005$.
13. $105 \times 106$.
14. $\quad 112 \times 109$.
15. $1075 \times 1012$.
16. To multiply by means of complements.
rule.
From either number substract the complement of the other, and to the right of the result write the product of the complements.

Note.-The notes of Art 16 apply in these problems if we substitute the word "complements" in place of " exoesses."

Example 1.-Multiply 94 by 98.
$94 . . . .{ }^{6}$ complement.
98
..
98
"
9212

```
и hetrod.
\(94-2=92\)
or \(98-6=92\)
or \(94+98=192\), omit the 1
```

multiplication.


EXERCISE II.
$\begin{array}{lll}\text { Example } & & \\ 685 & \ldots & . \\ 986 & 815 \\ 99 & \ldots & 4 \\ 682260 & & \\ & \text { (see } & \text { Note 2.) }\end{array}$
6. $88 \times 93$.
7. $87 \times 88$.
8. $84 \times 92$.
9. $75 \times 96$.
10. $93 \times 85$.
11. $993 \times 995$.
12. $997 \times 992$.
18. $995 \times 993$.
14. $989 \times 788$.
15. $991 \times 885$.
18. To multiply by means of factors.

The factors of a number are the numbers whose product is equal to that number.
Multiply 865 by 35.


EXERCISE.

1. $626 \times 36$.
2. $296 \times 99$.
3. $1351 \times 42$.
4. $495 \times 48$.
5. $343 \times 72$.
6. $764 \times 56$.
7. $827 \times 45$.
8. To multiply when factor of the other.
role.
Multiply by the part of the multiplier which is a factor of another part, placing the first figure of each partial product under the right hand figure of the multiplier which produced it. Erimple 1-Multiply 467 by 248.

467
248
$\widehat{3736}$ product by 8.
$\frac{11208}{115816} \quad " \quad$ " 24 ( 3 times the produat by 8 ).

Example 2-Multiply 643 by 436.
643
436
$\begin{array}{lc}2572 & \text { product by } 4 . \\ \frac{23148}{280348} & \text { " " } 36 \text { ( } 9 \text { times the product by 4). }\end{array}$
Example 3-Multiply 3247 by 842. 3247 842

- 6494 product by 2.
 2733974


## EXERCISE 12.

1. $364 \times 126$.
2. $3164 \times 427$.
3. $4275 \times 246$.
4. $37281 \times$
5. 
6. $475 \times 279$.
7. $8137 \times 189$.
8. $41325 \times$
9. 
10. $896 \times 142$.
11. $2556 \times 284$.
12. $68587 \times$
13. 
14. $857 \times 857$.
15. $4765 \times 927$.
16. $49126 \times$
17. 
18. $943 \times .426$.
19. $8259 \times 936$.
20. $64273 \times$
21. 
22. $854 \times 369$.
23. $4371 \times 183$.
24. $47821 \times 1682$.
25. $875 \times 632$.
26. $45314 \times 2463$.

## 20. To multiply by a mixed number.

Example 1-Multiply 363 by $6 \underline{3}$.
863
$\begin{array}{lll}\frac{63}{909} & \text { product by } \\ \frac{2178}{\frac{7}{4}} & \text { " } 62 .\end{array}=363 \div 4=90$
Etample 2-Miultiply 3426 by 5 ?
3426
52
$1370 \frac{2}{5}$ product by $\frac{2}{5}=3426 \times 2 \div 6$
17130 " 4.
185003

## EXERCISE 13.

## Multipily


2. 4371 by 157,14 ? , $25 \frac{1}{5}, ~ 33 \frac{3}{4}$.
3. 2137 by $41 \frac{9}{9}, 21 \frac{1}{7}, \quad 36 \frac{1}{10}, 47 \frac{1}{21}$.
4. 4645 by $22 \frac{3}{11}, 355,42 \frac{5}{5}, 52 \frac{9}{8}$.
5. 1316 by $161 \frac{1}{18}, 241 \frac{8}{18}, 110_{1}^{1}, 95 \frac{1}{2}$.
21. In multiplying by a mized number, it is often a shorter method to reduce the mixed number to an improper fraction and to multiply as in fractions. The following exercise contains multipliers of this kind.

Multiply 689 by $88 \frac{3}{3}$.

$$
\begin{aligned}
& 689 \times{ }^{83} \mathrm{y} \\
= & 689 \times \frac{100}{3} \\
= & \frac{68900}{8}=22966 \mathrm{~g}
\end{aligned}
$$

## EXERCISE 14.

Multiply-

2. 1375 by $142, \quad 333, \quad 18 \frac{2}{2}, 222,284$.
3. 4137 by $66 \mathrm{~g}, 429,183 \frac{1}{3}, 57 \%$.
4. 3164 by $444, \quad 86 \frac{4}{11}, \quad 714,55 \frac{1}{6}, \quad 45$ 年

6. 383 by $266 \frac{3}{3}, 1142,88 \mathrm{f}, 72 \mathrm{H}, 128 \%$.


## 22. To multiply by a number which is a convenient

 aliquot part of $10,100,200,300$, etc.Multiply 638 by $2 \frac{1}{2}$.
Since $10 \div 4=2 \frac{1}{2}$, therefore to multiply by $2 \frac{1}{2}$ we multiply by 10 and divide the result by 4, $638 \times 10 \div 4=1595$.

The following list comp: $\quad 3.1$ some of the multipliers that may be used in this way. anditiers used in the preceding exercise are eza

1. $1+=10 \div 8$.
2. $12=10 \div 6$.
3. $2 \frac{1}{5}=10 \div 4$.
4. $5=10 \div 2$.
b. $8 \frac{1}{3}=100 \div 13$.
5. $12 \frac{1}{2}=100 \div 8$.
6. $168=100 \div 6$.
7. $25=100 \div 4$.
8. $37 \frac{1}{2}=300 \div 8$.
9. $75=300 \div 4$.
10. $41 \frac{1}{8}=500 \div 12$.
11. $62 \frac{1}{2}=500 \div 8$.
12. $58 \frac{1}{3}=700 \div 12$.
13. $87 \frac{1}{2}=700 \div 8$.
14. $1162=700 \div 6$.
15. $175=700 \div 4$.
16. $112 \frac{1}{3}=900 \div 8$.
17. $225=900 \div 4$.
18. $83 \frac{1}{s}=1000 \div 12$.
19. $125=1000 \div 8$.
20. $166=1000 \div 6$.
21. $333 \frac{1}{\frac{1}{3}}=1000 \div 3$.

## EXERCISE 15.

## Multiply-

1. 346 by $1 \frac{1}{\text { b }}, 1 \frac{1}{3}, 2 \frac{1}{\text { a }}, ~ 5 \frac{1}{8}$.
2. 258 by $12 \frac{1}{2}, 16 \frac{2}{3}, 25, \quad 37 \frac{1}{2}, 75$.
3. 512 by $41 \frac{1}{3}, 62 \frac{1}{2}, 58 \frac{1}{3}, 57 \frac{1}{2}$.
4. 545 by $16 \frac{2}{3}, 175,112 \frac{1}{2}, 225$.
5. 357 by $83 \frac{1}{3}, 125,166 \frac{2}{8}, 383 \frac{1}{3}$.
6. To multiply by 75 .

Multiply by 100 and subtract one quarter of the product.
Exaypin-Multiply 863427 by 75.
$75=100-25$ (one-fourth of 100)
Oprration-86342700 - product by 100.
21585675 - one-fourth of the produod.
64757025
24. To multiply by 125.
$125=100+25$ (one-fourth of 100 )
Multiply by 100 and add one-fourth of the product.
Examplis-Multiply 1234769 by 125.
Operation-124376900 - product by 100.
31094225 - one-fourth of the product by 100.
$\overline{155471125}$
EXERCISE 16.

1. $367258 \times 66$.
2. $36254 \times 105$.
3. $43729 \times 95$.
4. $27936 \times 133 \mathrm{~s}$.
5. $27364 \times 975$.
6. $478256 \times 150$.
7. $376298 \times 950$.
8. $236471 \times 1025$.

## DIVISION.

## DIVISIbility of NUMBERS.

255. A number is said to be divisible by auother number when the latter will divide the former without a remainder. 26. An even number is a number of which 2 is an exact livisor.
256. An odd number is a number of which 2 is not an sxact divisor.
257. Any number is divisible-
258. By 2 , if it is an even number as $2,4,8,26$.
259. By 3, if the sum of its digits is divisible by 3 , as $744,7+4+4=15,15$ is divisible by 8.
260. By 4, if the two right hand figures are ciphers, or express a number divisible by 4 , as 1500 , 7328.
261. By 5 , if the right hand figure is 0 or 5 , as 60,95 .
262. By 6 , if it is an even number and has the sum of its digits divisible by 3 , as 348 .
263. By 8 , if the three right hand figures are ciphers, or express a number divisible by 8 , as 4000 ,
264. By 9 , if the sum of its digits is divisible by 9 , as 45387.
265. By 11 , if the difference of the sum of the digits in the even places, and the sum of the digits in the odd places is 0 , or is divisible by 11 , as 43263, 459173.
266. By 25 , if the two right hand figares are ciphers or express a number divisible by 25 , as 4700 , 3675.
267. By 75 , the same as for 25 , providing also that the sum of the digits is divisible by 3 , as 8900 , 41475.

## 29. To divide one number by another leaving out the products.

ROLE.
Subtract the several products from the next number greater ending with the corresponding figure in the dividend, and carry each time the left hand figure of the minuend to the next product.

Divide 42343014 by 973.
ordinary method.

978 ) 42343014 ( 43518
3892

3423
2919
5040
4865
1751 973 7784 7784 0000
limaving out the prodiots.

| 42343014 |  |
| :--- | :--- |
| 3423 | $\frac{973}{43518}$ |

METHOD.
The first quotient figure is 4, by whick we multiply. 4 times 3 are 12, whioh, subtracted from 14 (the next number greater ending with 4), leaves 2. Write 2 in the remainder and carry 1. 4 times 7 are 28, and 1 carried makes 29, which, subtracted from 33 (the next number greater
ending with 3), leaves 4. Write 4 in the remainder and carry 3. 4 times 9 are 36 and 3 carried makes 39, which, subtracted from 42 (the next number greater ending with 2), leaves 4. 4 subtracted from 4 leaves 0 Bring down 3 the next figure in the dividend. So proceed until the
division is completed division is completed.

$$
\begin{array}{llll}
\text { 1. } 743297 \div 527 . & \text { EXERCISE } 17 . \\
\text { 2. } 14839 \div 889 . & \text { 4. } & 36287 \div 567 . \\
\text { 3. } & 87(554 \div 743 . & \text { 5. } & 64925 \div 784 . \\
& 7 . & \text { Divide } 3642789 \text { by } 625,436,8173,2106 .
\end{array}
$$

30. To divide by a mixed number.
principle.
Multiplying both divisor and dividend by the same number. does not alter the quotient. livide 736 by $5 \frac{2}{3}$.

$$
\begin{aligned}
& 53) 736 \text { ( } \\
& \frac{3}{17} \frac{3}{2208}\left(129_{18}\right. \\
& 17
\end{aligned}
$$

50
34
$\overline{168}$
153
$\overline{15}$

## Divido

## EXERCISE 18.

1. 475 by $3 \frac{1}{2}, 4 \frac{3}{2}, 7 \frac{3}{3}, \quad 3 \frac{1}{6}, \quad 44,5$

2. 6712 by $7 \frac{1}{6}, 114,2 \frac{1}{13}, 64,43,103$.
3. To divide when all the figures in the divisor except the first on the left hand can be changed to ciphers by using a convenient multiplier.

Example 1-Divide 624395 by 35.

$$
\frac{25}{79} \frac{2}{79} \frac{124395}{124899}
$$

Example 2-Divide 13476 bv 1 i a .

| 16겨) 13476 |  |
| :---: | :---: |
|  |  |
| 190) 80856 |  |
|  |  |
|  | $808 \frac{58}{108}$ |

Notz.-If the true remainder is required it may be obtained by dividing the remainder found by the number by whioh we multiply the divisor.

EXERCISE 19.

|  | $4826 \div 5$ | 7. | 32068 | $\div 12 \frac{1}{2}$. | 13. | 21396 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $3827 \div 25$. | 8. | 68934 | $\div 81$. | 14. | 9201 |  |  |
|  | $9109 \div 75$ | 9. | 32165 | $\div 1 \frac{1}{3}$. | 15. | 7345 |  | 57\%. |
|  | $4863 \div 175$ | 10. | 83271 | $\div 83$. | 16. | 6287 |  |  |
| 5 | $3798 \div 225$ | 11. | 4932 ${ }^{\text {a }}$ | $\div 33 \frac{1}{3}$. | 17. | 31264 |  | 12. |
|  | $8306 \div 45$ | 12. | 9306 | $\div 62 \frac{1}{2}$. | 18. | 31907 |  | $142 \frac{\theta}{7} .$ |

32. To divide by any number that can be changed to a convenient divisor by increasing or diminishing it by an aliquot part of itself.

RULR. •
After dividing by the divisor so increased or diminished, increase or diminish the quotient in the same proportion.

Divide 1920 by 24.
Operation.

| 39 | 1920 |
| ---: | ---: |
| $\lcm{64}$ |  |
| 16 |  |

$\overline{80}$ the quotient.

## Explanation.

$$
\frac{1}{4} \text { of } 24=6 \quad 24+6=30
$$

$1920 \div 30=64$
$\frac{1}{7}$ of $64=16$
80 the quotient.

## EXERCISE 20.

1. $1845+45$.
2. $3640 \div 35$.
3. $15216 \div 48$.
4. $7704 \div 24$.
5. $8343 \div 27$.
6. $41472 \div 81$.
7. $2822 \div 54$.
8. $141120 \div 180$.
9. $24300 \div 184$.
10. $21500 \div 87 \frac{1}{2}$.
11. $337500 \div$ ' 5 .
12. $425100 \div 125$.
13. To divije by means of factors of the divisor.

Eximple 1.-Diviûe 25380 by 108.

$$
\begin{aligned}
& 108=9 \times 4 \times 3 \text { or } 6 \times 6 \times 3 \text { or } 9 \times 6 \times 2 \\
& \text { 108) } 25380(235 \\
& 216
\end{aligned}
$$

| 378 | ${ }^{8} \lcm{25380}$ | 8) 25380 |  |
| :---: | :---: | :---: | :---: |
| 324 | 4) 8460 | 6) 8460 | 8) $8 \longdiv { 2 5 3 8 0 }$ |
|  | $9 \longdiv { 2 1 1 5 }$ | $6 \longdiv { 1 4 1 0 }$ | 2) 21470 |
| 540 540 | 235 | ${ }_{2} 235$ | 2) ${ }^{235}$ |

Erakpliz 2.-Divide 6326 by 75.


Nots 1.-To find the true remainder, take the product of each remainder by all the divisors preceding the one that produced it. The sum of these products with the first remainder will be the true ramainder.
2. Take the product of the last remaindar by the divisor preceding the one that produced it. To this product add the preoeding remainder. Multiply this result by the next divisor and add the next remainder. Continue this process until the first divisor has bean used as a maltiplier.

## EXERCISE 21.

| 1. | $25380+86$. | 6. | $81279 \div 72$. | 9. | $43716 \div 168$. |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 2. | $178584 \div 48$. | 6. | $43827 \div 84$. | 10. | $20378 \div 81$. |
| 3. | $23741 \div 42$. | 7. | $19875 \div 125$. | 11. | $41658 \div 45$. |
| 4. | $43165 \div 64$. | 8. | $41643+185$. | 12. | $28725 \div 96$. |

34. To divide by cancellation.
35. Cancellation is the process of shortening operations by rejecting equal factors from both dividend and divisor. 3B. The sign of cancellation is an oblique mark (/) drawn through the number from which the factor is

## livisor.

## FACTORING.

37. A Factor, a Measure, or an Exact Divisor of a given number is an integral number that will divide the given number without a remainder.
38. A Prime Number is a number that has no factors except itself and 1 , as $3,7,13,19$.
39. A Prime Factor is a prime number used as a factor.
40. A Composite Nuniber is a number that has other factors besides itself and 1 , as $24,32,70$.
41. Factoring is the process of finding the factors of a composite number. 42. To resolve a number into its prime factors. BUTLE.
Diride the number by the least prime number which will divide it exactly. In like manner divide the resulting quotient. Continue this process until a quotient which is a prime number is reached. The several divisors and the last quotient are the prime factors.

Find the prime factors of 420 .

| 2) $\frac{420}{2}$ | $420=2 \times 2 \times 3 \times 5 \times 7$ |
| :--- | :--- |
| 3) $\frac{105}{105}$ |  |
| $5 \frac{35}{7}$ | $2,3,5$ and 7 are the prime faotors. |

## EXERCISE 23.

Find the prime factors of-

1. 1050
2. 2625. 
1. 5985. 
1. 1820. 
1. 4620. 
1. 4802. 
1. 1485. 
1. 5432. 
1. 7000 .
2. 8140. 
1. 8712. 
1. 1320. 
1. 1155. 
1. 1768. 
1. 1848 .

| 16. | 1906. | 21. | 2526. |
| :--- | :--- | :--- | :--- |
| 17. | 1858. | 22. | 2978. |
| 18. | 1478. | 23. | 2992. |
| 19. | 2956. | 24. | 3936. |
| 20. | 2406. | 25. | 3430. |

## HIGHEST COMMON FACTOR.

isor of a livide the
sed as a
as other
ors of a
s. wotient
43. A Common Factor of two or more numbers is a number that will exactly divide each of them ; thus 2, 4, 6, or 12 is a common factor of 24 and 36.
44. The Highest Common Factor, also called the Greatest Common Divisor or Greatest Common Measure, of two or more numbers, is the greatest number that will exactly divide each of them, thus 12 is the H. C. F. of 24 and 36.
45. To find the H. C. F. of two or more numbers : role.
Divide the greater number by the less, and the less number by the remainder, if any, and so continue to divide the last divisor by the last remainder until there is no remainder. The lust divisor will be the H. C. F.
If more than two numbers are given, find the H. C. F. of two of them, then of this factor and the third number and so on.

Find the H. C. F. of 1386 and 2268.

FIRST METHOD.
1386) 2268 ( 1

H. C. F. $\quad \overline{126}) 378$ ( 3

Nots.-Observe that the second method is the same as the first, the work being arranged so as not to necessitate the writing of the divisor more than once.
The column for quotients may be omitted.

| subtract | THIRD METHOD. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1386 | $\begin{gathered} \text { MULTTPLIERS } \\ 2 \end{gathered}$ | $\begin{aligned} & 2268 \\ & 2772 \end{aligned}$ |  |
| downwards. | 1512 | 8 | $\frac{2704}{504}$ | subtract duwnwarida. |
| H. C. F. | 126 | 4 | 504 |  |

In this method we use dact nearest to 2868, that is a multiplier for 1386 as will give a proleaves a remsinder 504. N as will give a pruduct neares 188 a multiplier of 604 such a number 380, that is 3, eto.
FOURTH METHOD.
By means of prime factors.
prime factors found.

| 8 | $\frac{1386}{3}$ |
| :--- | ---: |
| 3 | $\frac{693}{231}$ |
| 7 | $\frac{77}{11}$ |


| 2 | $\frac{2268}{1134}$ |
| :--- | :--- |
| 3 | $\frac{567}{5}$ |
| 3 | 189 |
| 8 | -63 |
| 8 | $-\frac{21}{21}$ |

prime pactors arranogd.
$1386=2 \times 3 \times 3 \times 7 \times 11$ $2268=2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7$

Common prime factors multiplied. $2 \times 3 \times 3 \times 7=126=$ H. C. F.

## RULI.

Resolve the given numbers into their prime factors; the product of all the prime factors common to them is the H.C. $\boldsymbol{F}$. FIFTH METHOD.
By means of common prime factors.

| 2 | 1386 | 2268 |
| :---: | ---: | ---: |
| 8 | 693 | 1134 |
|  | $\frac{231}{}$ | 878 |
| 7 | 77 | 126 |
| 11 | 18 |  |

$$
2 \times 3 \times 3 \times 7=126 \text { H. C.F. }
$$

solm.
Divide the given numbers by the prime factors common to each : the product of these prime factors will be the H.C. $\boldsymbol{F}$. EXERCISE 24.
Find the H. C. F. of

1. 323,425 .
2. $228,399$.
3. 615,735 .
4. 819,945 .
5. 949, 871.
6. 825,960 .
7. $961,1178$.
8. 5355,6545 .
9. 4155, 24720.
10. 7568, 3784.
11. 3876, 1983.
12. $689,1575$.
13. 7956, 7668.
14. $9864,9528$.
15. $45,57,81$.
16. $63,99,90$.
17. 72, 84, 96 .
18. $306,408,510$.
19. $420,462,84$.
20. 546, 462, 882.
21. $900,936,2520$.

## LEAST COMMON MULTIPLE.

give a pro-
2268, which b a number
0.
46. A Multiple of a number is one that is exactly divisible by that number, thus 36 is a multiple of 6 .
47. A Common Multiple of two or more numbers is a number which is exactly divisible by each of them, thus 18 , 36, 72, are common multiples of $2,3,6$ and 9 .
48. The Least Common Multiple of two or more numbers is the least number which is exactly divisible by each of them, thus 18 is the least common multiple of 2,8 , 6 , and 9.

Find the L. C. M. of 18, 28, 42.
TIRST METHOD.
By means of prime factors.

$$
\begin{aligned}
& 18=2 \times 3 \times 3 \\
& 28=2 \times 2 \times 7 \\
& 42=2 \times 3 \times 7
\end{aligned}
$$

$$
28=2 \times 2 \times 7 \quad \text { L. C. M. }=2 \times 2 \times 3 \times 3 \times 7=252
$$

## bule.

Resolve the given numbers into their prime factors; the product of the different prime factors taking each the greatest number of times it appears in any of the numbers will be the L. C. M.

> SECOND METHOD.

$$
\begin{aligned}
& \text { Find the L. C. M. of } 9,15,18,16,12,30,45 \text {; } \\
& \qquad \begin{array}{l}
2 \\
2 \left\lvert\, \frac{9.17,18,16,12,30,45}{9,8,6,1 p, \frac{45}{4}}\right. \\
4,9,45
\end{array} \\
& 2 \times 2 \times 4 \times 45=720 \mathrm{~L} . \mathrm{C} . \mathrm{M} . \\
& \text { or } \\
& 2,2,3 \left\lvert\, \frac{9,15,18,16,12,30,45}{3,} 1\right.,-15 \\
& 2 \times 2 \times 3 \times 4 \times 45=720 \text { L. O. M. }
\end{aligned}
$$

RTLE.
Write the numbers in a horizontal line, cancelling such of the smaller numbers as are factors of the larger, and divide by any prime factor or prime factors that will exuctly divide two or more of the given numbers. Write the quotients and the undivided numbers, if any, in a line beneath.

Continue this process until the results are prime to each other.

The product of all the divisors and the numbers in the last line will be the L. C. M.

## EXERCISE 25.

Find the L. C. M. of

1. $5,6,15$.
2. 7, 14, 21, 28.
3. $4,8,12,16$.
4. $5,7,15,21$.
5. 3, 14, 21, 28.
6. $9,2,6,18,24$.
7. $8,7,12,21,24$.
\& $5,2,15,7,35$.
8. $3,6,9,54$.
9. 7, 9, 12, 14, 36.
10. 27, 24, 15.
11. 63, 27, 84.
12. $12, \quad 51, \quad 68$.
13. $35,63, \quad 72$.
14. $9,12,14,210$.
15. $60,15, \quad 14, \quad 210$.
16. $54, \quad 81, \quad 63, \quad 14$.
17. 19, 27, 36, 63,
18. 22, 27, 54, 108.

## FRACTIONS.

49. A Fraction is one or more of the equal parts of a unit, or $a n y$ thing regarded as a whole; thus, one-half, two-thirds, three-fourths, are fractions.
50. The unit of the fraction is the unit which is divided. One of the equal parts is the fractional unit.
51. Fractions obtained by the division of the unit into tenths, hundredths, thousandths, etc., are called Decimal Fractions. All other fractions are called Common Fractions.
52. A Common Fraction is expressed by two numbers, called the Numerator and the Denominator, the former written over the latter, with a line between them, thus:

| One-third is written | $\frac{1}{8}$ | Five-sixths is written | $\frac{8}{8}$ |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Three-fourths | $"$ | $\frac{5}{4}$ | Seven-thirteenths | " | $\frac{7}{78}$ |
| Three-eighths | $"$ | 8 | Sleven-twentieths | " | $\frac{18}{28}$ |

53. The numerator and the denominator are called the terms of a fraction.
54. The Denominator of a fraction, written below the line, shows the number of equal parts into which the unit is divided and also names the unit: thus in $\frac{7}{8}, 8$ is the denominator and shows that the unit is divided into eight equal parts, named eighths.
55. The Numerator of a fraction, written above the line, shows the number of equal parts taken to form the fraction; thus in $\frac{7}{8}, 7$ is the numerator, and shows that seven of the eight equal parts are taken or expressed by the fraction.
56. Since the denominator of a fraction shows how many fractional units in the numerator are equal to one integral unit, it follows

That a fraction is an expression of unperformed division. The numerator is the dividend, the denominator is the divisor, and the value of the fraction is the quotient.
57. General principles of fractions.
I. Multiplying the numerator or dividing the denominator by any number multiplies the value of the fraction by that number.

If we multiply the numerator of the fraction $\frac{1}{2}$ by 3 , the result is $\frac{3}{2}$, which is three times as great as $\frac{1}{2}$. If we divide the denuminator of $\frac{1}{4}$ by 2 , the reault is $\frac{1}{2}$, which is twice as great as 4 .
II. Dividing the numerator or multiplying the denominator by any number divides the fraction by that number.

If we divide the numerator of the fraction $\frac{7}{7}$ by 2 , the result is $\frac{1}{4}$, which is $\frac{1}{2}$ as great as $\frac{2}{4}$. If the denominator of $\frac{1}{2}$ is multiplied by 2 , the result is $\frac{1}{4}$, which is $\frac{1}{2}$ as great as $\frac{1}{2}$.
III. Multiplying or dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.

If we multiply both the numerator and the denominator of $\frac{1}{2}$ by 2 , the reault is $\frac{2}{4}$, which has the same value as $\frac{1}{2}$. If we divide both numerator and denominator of $q$ by 2 , the result is $\frac{1}{2}$, which has the same value as $\frac{2}{4}$.
58. A Simple Fraction is one whose terms are both integers, as $\frac{7}{9}, \frac{14}{14}$.
59. A Proper Fraction is one whose numerator is less than its denominator ; hence its value is less than 1 , as $\frac{8}{4}$, $\frac{7}{15}$.
60. An Improper Fraction is one whose numerator equals or exceeds its denominator, as $\frac{5}{4}, \frac{10}{7}, \frac{28}{3}$.
61. A Mixed Number is a number composed of an integer and a fraction, as $3 \frac{1}{2}, 5 \frac{3}{7}$.

## EXERCISE 26.

1. Read the following fractions, and tell what each numerator and each denominator shows:

2. Express the following in figures: one third; ten twentieths; thirty one-hundred-and-eighths; three five-thousandths; twelve hundred ninety-thousandths; three sevenths of nineteen forty-fifthe.
3. Write: three and a half; sixty-five and twenty-three forty-eighths : eighteen and eleven eighty-fourths.

## REDUCTION.

62. Reduction of Fractions is the changing of their form without changing their value.
63. To reduce integers or mixed numbers to improper fractions.

Exampur 1.-In 18 units how many fifthe?
Solotion.

> In 1 unit there are 5 fifths
> " 18 units " 18 times 5 ffths or 90 tifths (gog)
> Henoe $18=\frac{g \Omega}{5}$

Exaippre 2.-Rednoe 18g to an improper fraction.
Solution.
$\frac{188}{\frac{88}{5}}$
Explanation.
$18=90$ fifths
$\frac{\frac{g}{8}=3 \text { fifths }}{18 \frac{8}{3}=93 \text { fifths }\left(\frac{g}{8}\right)}$

Multiply the whole number by the denominator of the fraction, to the product add the numerator, and set their sum over the denominator.

## EXERCISE 27.

Reduce to improper fractions-

| I. | 11. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 71. | $2 \frac{1}{6}$. |  | IV. | $\nabla$. |
| 23. | $3{ }^{7}{ }^{\text {r }}$. | 1817. | $27{ }^{7}$ \% | 38. |
| 34. | $8{ }^{\text {P }}$ \% |  | 58. | 5 3 . |
| 43. | 7 ${ }_{\text {g }}$. | ${ }_{33}{ }_{17}{ }^{5}{ }^{5}$. | $95 \frac{1}{3}$. 418. | 1919. |

64. To reduce an improper fraction to an integer or a mixed number.

Example.-Reduoe $\frac{48}{8}$ to a mixed number.

Soldtion.
5) $\frac{48}{98}$

Explanation.
Since $\frac{4}{8}$ expresses an anperformed division (Art. 56), therefore by performing the division we obtain $9 \frac{8}{8}$ for quotient.

## EXERCISE 28.

Reduce to mized numbers-

| 1. | II. | III. |  |
| :---: | :---: | :---: | :---: |
| 38 | 48 | 115. | IV. |
| \% | ${ }^{1284}$ | - ${ }^{38}$ | 219 |
| 4, | 292 | ${ }_{10}^{198}$ | ${ }^{378}$ |
|  | ${ }^{3}{ }_{17} 1^{8}$ | $1{ }_{13}{ }^{37}$ | - |
| ${ }^{4}$ 穝 | $19{ }^{19}$ | ${ }_{4}^{478}$ | \% 4.45 |

## 65. To reduce a fraction to higher terms.

Example.-Reduce a to sixteenths.

Solution
$\frac{3}{4}=\frac{3 \times 4}{4 \times 4}=\frac{3}{4} \times \frac{4}{4}=\frac{12}{16}$

Explanation.
Since it is required to ohange a $^{\text {to }}$ th teenths, (i.e) a fraotion whose denominator is 16 , we must multiply the denominator 4 by 4; then by Art. 57, III., so as not to ohange the value of the fraction, we must multiply the numerator 3 by 4.

RoLe.
To reduce a fraction to higher terms, divide the required denominator by the denominator of the given fraction and multiply both terms by the quotient.

## FRACTIONS.

EXERCISE 29.

## $\nabla$.

89오. 5 s. 1919. 1247.

1teger or
aperformed performing tient.
to six. minator minator $s$ not to we must

1. $\frac{1}{2}, \frac{3}{4}, \frac{1}{6}, \frac{1}{3}$ to twelfths.
2. $\frac{1}{8}, \frac{1}{6}, \frac{1}{2}, \frac{8}{8}$ to eighteenths.
3. $1, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}$ to eighthe.
4. $\frac{2}{3}, \frac{3}{4}, \frac{5}{8}, \frac{7}{8}, 3$
to twenty-fourths.
5. $\frac{3}{3}, \frac{5}{8}, \frac{7}{12}, \frac{7}{7}, \frac{5}{88}, \frac{5}{8}$ to seventy-seconds.
6. $5, \frac{3}{8} \frac{5}{16}, \frac{5}{8}, \frac{9}{2}, 1$ to sixteenths.
7. $\frac{5}{8}$ 香. $\frac{34}{24}, \frac{5}{18}, \frac{1}{2}$ to fifty-fourths,
8. $\frac{2}{3}, \frac{3}{5}, \frac{7}{7}, \frac{8}{15}, \frac{2}{45}$ to forty-fifthe.
9. $\frac{4}{8}, \frac{1}{4}, \frac{9}{18}, \frac{17}{2}, \frac{5}{12}$ to forty-eighths.
10. $\frac{7}{6}, \frac{3}{4}, \frac{7}{12}, \frac{11}{3}, \frac{7}{7}$ to thirty-sixths.
11. To reduce a fraction to its lowest terms.

Solution.
$\frac{12}{16}=\frac{12 \div 4}{16 \div 4}=\frac{3}{4}$
or
$4 \left\lvert\, \frac{12}{16}=\frac{3}{4}\right.$

Explanation.
By Art. 57, III., we may divide both numerator and denominator by 4 without ohanging the value of the fraction.
rude.

Divide both terms of the fraction successively by all the prime factors cominon to the two, or by the continued product of all the prime factors, (i.e) their highest common factor. (H. C. F.)

Notr.-A fraction is in its lowest terms when the numerator and denominator have no common factor.

EXERCISE 30.
Reduce to lowest terms--

| I. | II. | III. | IV. |  |
| :---: | :---: | :---: | :---: | :---: |
| 析 | 88 | H | 17. | $\underset{\sim}{\text { V. }}$ |
| 38 | \% ${ }^{18}$ | \% | 1989 | H\%\% |
| 4 | ${ }^{68}$ | 9 | 488 | +142\% |
| It | $4{ }^{18}$ | 107 |  | H\% |
| \% | 4 | ${ }^{88}$ | \% 818 | 1080 |
| H | 4 | 184 | \% ${ }^{818}$ |  |

67. To reduce two or more fractions which have different denominators to equivalent fractions having a least common denominator.

Notr.-Since the common denominator must be the same for each fraction, and fractions can be ohanged to equivalent fraotions having a
different denominator (Art. 65), therefore the common denominator mast contain eaoh of the denominators of the given fractions exactly. The least number that will contain each of the given denominators is their L. C.M. Therefore the least oommon denominator of the fractions must be the L. C. M. of their denominators.

Eximple.-Change $\frac{1}{2}, z, t$ to equivalent fractions having a least common denominator.

Soldtion.
The least common denominator
$=$ L. C. M. of $2,3,8=24$.

$$
\begin{aligned}
& \frac{1}{2}=\frac{16}{2!} \\
& z=\frac{10}{26} \\
& t=\frac{9}{86}(\text { Art. } 65)
\end{aligned}
$$

Explanation.
We first find the L. C. M. of the given denominstors which is 24 . This nust be the least common denominator to which the given fractions oan be reduced (Note Art. 67.) Redueing each fraction to the denominator 24 (Art. 65), wo obtain $\frac{12}{2}$, $\frac{19}{29}$, 윶, as nesults. mole.
I. Find the L. C. M. of the given denominatore for the least common denominator.

IT. Divide the common demominator by each of the given denominators, and multiply the numerator and denominator of each fraction by the corresponding quotient.

## EXERCISE s.

Reduce to their least common denominator.

1. 3,8 . 9.
2. $t$, in, $\frac{5}{10}$
3. 音, t, t, t, is
4. $\frac{1}{3}$, $\frac{3}{3}, \frac{y_{1}^{2}}{15}$.
5. $\frac{t}{2}$, $\frac{5}{51}, \frac{5}{10}$.
6. 3 , $\frac{1}{8}, \frac{8}{8}$.
7. $\frac{3}{7}$ of $\frac{3}{8}, \frac{14}{81}, 42$.
8. t, $2 \frac{3}{4}, \frac{4}{4}$.





9. s, H2 \&5, th \& 48 .

## ADDITION.

68. Example 1.-Find the sam of $\frac{1}{6}, \frac{7}{8}$, f.

Solution.


## Explanaftos.

In order that fractions may be added they must have like denominators and be parts of like nite.
= 18 twenty-fourths.
$t=21$ twenty-fourths.
$f_{2}^{f}=\frac{10 \text { twenty-fourths. }}{45 \text { twent } y \text {-fourths }}$

FRACTIONS.
Example 2.-Find the sum of 29, 137, 48. Solution.

ROLE.
I. To add Fractions. - When necessary reduce the fractions to their least common denominator: then add the numerators and place the sum over the common denominator.
II. To add Mixed Fractions.-Add the integers and fractions separately, and then add their sums.

Note.-All fractions should be reduced to their lowest terms, and if improper, to whole or mixed numbers.

## EXERCISE 32.

Find the sum of -

1. $\frac{1}{2}, \frac{3}{4}, \frac{1}{3}$.
2. $\frac{1}{8}, \frac{5}{12}, \frac{1}{2}^{2} 5$.
3. $3 \frac{1}{2}, 4 \frac{2}{3}, 2 \frac{2}{2}$.
4. $\frac{2}{8}, \frac{8}{4}, \frac{8}{8}$
5. $\frac{8}{5}, \frac{4}{4}, \frac{4}{23}, \frac{7}{18}$.
6. $\frac{7}{8}, \frac{11}{12}, \frac{17}{8}, \frac{3}{2}, \frac{39}{24}$.
. $1 \frac{1}{2}, 2 \frac{2}{3}, 39,44,5 \frac{4}{8}, 64$.
7. $75,10 \frac{4}{5}, 4 \frac{1}{3}, 7 \frac{7}{3}$.
8. $4 \frac{7}{18}, 8 \frac{5}{21}, 2 \frac{8}{25}$.
9. $\frac{1}{2}, \frac{2}{6}, \mathrm{I}^{3} \mathrm{I}, \frac{7}{18}, \frac{2}{3}$.
10. $\frac{1}{2}, \frac{2}{8}, \frac{5}{6}, \frac{4}{6}, \frac{5}{7}, \frac{7}{8}, \frac{8}{6}, \frac{2}{18}$.
11. $4 \frac{4}{6}, 2 \frac{1}{6}, 1_{\frac{1}{2}}^{\frac{2}{2}}, 2 \frac{8}{2} \frac{8}{4}, 5 \frac{7}{15}$.
12. $24 \frac{9}{3}, 18 \frac{4}{3}, 4 \frac{5}{6}, 70$.
13. $21,15 \frac{2}{3}, 5 \frac{5}{6}, 4 \frac{1}{27}, 6 \frac{5}{6}$.

## SUBTRACTION.

69. Example 1.-Find the difference between $\frac{r^{\frac{7}{8}} \text { and } \frac{3}{8} \text {. }}{\text { B }}$.

Explanation.


In order that fractions may be subtracted, they must have like denominators and be. parts of the same unit.
$\frac{7}{12}=14$ twenty-fourths (Art. 65.)
$\frac{7}{12}-\frac{\frac{3}{8}}{\frac{3}{8}}=9$ twenty-fourths. $\quad 5$ twenty-fourths $=\frac{5}{84}$ Ans.
Example 2.-Find the difference between $83 \frac{5}{12}$ and $45 \frac{1}{3}$. Dolution.



| Solution. |  |
| :---: | :---: |
|  | 18ths |
| 368 | 4 |
| 198 | 15 |
| $16{ }_{1}^{5}$ | $\frac{7}{18}$ |

> Explanation.

You can't take $\frac{15}{18}$ from ${ }^{\text {ts }}$. Borrow unity from 36. Reduce it to eighteenths, and then add result to its which makes if. Hf from 19 leaves $\frac{7}{15}$. 19 from 35 leaves 16. Rosult, 16! ${ }^{\text {r }}$.
role I .
To subtract fractions.-When necessary, reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from the numerator of the minwend, and place the difference over the common denominator. rute in.
To subtract mixed numbers.-Reduce the fractions, if necessary, to a common denominctor, and if the fraction in the subtrahend is smaller than that in the minuend, subtract one jraction from the other, and the smaller whole number from the larger whole number. But if the fraction in the subtraleend is horger than that in the minuend, borrow 1 from the whole oumiver. After changing it to the same denominator as the fraction, add it to the fraction in the minuend. Then subtract as before.

## EXERCISE 33.

Find the difference between-


## MULTIPLICATION.



Solution.
$\frac{3}{4} \times \frac{2}{8}=\frac{3 \times 2}{4 \times 3}=\frac{6}{12}=\frac{1}{2}$

## Explanation.

The numerators are multiplied for a new numerator and the denominators for a new denominator.

Example 2.-Multiply $\frac{1}{2}$ by $\frac{2}{3}$ by ${ }_{5}$ by so

$$
\begin{aligned}
& \text { Solution. } \\
& \frac{1}{2} \times \frac{\%}{\bar{\beta}} \times \frac{6}{8} \times \frac{8}{9}=\frac{2}{9} \quad \\
& \text { Explanation. } \\
& \text { See Art. } 36 \text {. }
\end{aligned}
$$

rule.
Reduce integers and mixed numbers to improper fractions. Multiply the numerators together for a new numerator, and the denominators for a new denominator. Reduce the result to its simplest form.

Nota.-Cancellation often shortens the operation.
EXERCISE 34.
Find the product of-

1. $\frac{1}{2} \times i \times \frac{3}{3}$.
2. $\frac{3}{8} \times \frac{?}{5} \times \frac{2}{5}$.
3. $\frac{2}{3} \times 12 \times \frac{9}{4} \times 16 \times 3$ of 20 .
4. $\frac{5}{8} \times \frac{6}{6} \times \frac{3}{4}$.
5. $\frac{8}{5} \times 15 \times 8 \times 18 \times 4 \times 21$.
6. $\frac{8}{8} \times \frac{3}{7} \times \frac{3}{3}$. $11.21 \times 18 \times 3{ }^{2} \times \frac{3}{8} \times 20 \times \frac{4}{8} \times 27$.

7. $\frac{8}{13} \times \frac{13}{23} \times$ t. $13.8 \frac{1}{4} \times \frac{10}{10} \times 3 \frac{4}{1} \times \frac{18}{18} \times 4 \frac{1}{2} \times 1$.


## DIVISION.

71. To divide 2 fraction by an integer.

Example 1.-Divide a by 3.
Solution.
$\frac{21}{25}+3=\frac{21 \div 3}{25}=\frac{7}{25}$
Example 2.-Divide a by 2.
Solution.
$\frac{3}{4} \div 2=\frac{3}{4 \times 2}=\frac{8}{8}$
Example 3.-Divide 34ă by 11.

Solution.

$$
\begin{aligned}
34+11 & =3, \text { rem. } 1 \\
1 \frac{2}{3} & =\frac{5}{3} \\
\frac{5}{3}+11 & =\frac{5}{33}
\end{aligned}
$$

$$
3 \frac{5}{33} \quad \text { Ans. }
$$

Explanatiou.
Art. 57, 2.
Explanatiom.
Art. 67, 2.

Explanation.
Divide the integer by 11, quotient 3, rem. 1. This rem. prefixed to the fraction makes $1 \frac{1}{3}$, or for yet to be divided. Divide this improper fraction and combine the roseita.

$$
\begin{aligned}
& 34 \frac{2}{3} \div 11 \\
= & \frac{104}{3} \div 11=\frac{104}{3 \times 11} \\
= & \frac{104}{33}=3 \frac{5}{33} \text { Ans. }
\end{aligned}
$$

## EXERCISE 35.

## Divide-

Explanation.
Reduce the mixed number to an improper fraction and proceed as in example 2.

1. $\frac{4}{4}$ by 4.
2. $\frac{13}{23}$ by 11 .
3. $\frac{2}{5}$ by 6 .
4. $\frac{2 t}{2 t}$ by
5. $\mathrm{I}^{8}$ by 8 .
6. $\frac{183}{8}$ by
7. $\frac{5}{6}$ by 7.
8. $16 \frac{1}{4}$ by
9. IT by 3 .
10. $42 \frac{1}{2}$ by 3
11. 
12. 674 by
13. 

-••
7.
3.
12. $19 \frac{1}{8}$ by
8.
14. $11_{I^{1}}^{1}$ by 11 .
15. $24 \frac{5}{6}$ by 6.
72. To divide a fraction by a fraction.

Example.-Divide $\frac{8}{6}$ by $\frac{?}{3}$.

$$
\begin{aligned}
& \text { Explanation. } \\
& \begin{aligned}
\text { Soldtion. } & 3 \text { fifths } \div 2 \text { thirds } \\
\frac{4}{6} \div \frac{2}{3}=\frac{9}{*} \times \frac{3}{2}=\frac{9}{10} \quad & =\frac{9}{10}=\frac{3 \times 3}{5 \times 2}=\frac{3}{5} \times \frac{3}{2}=\frac{9}{10} \text { Ans. }
\end{aligned} \\
& \frac{5}{8} \div \frac{2}{8}= \\
& \begin{aligned}
\text { Soldtion. } & 3 \text { fifths } \div 2 \text { thirds } \\
\frac{4}{6} \div \frac{2}{3}=\frac{9}{*} \times \frac{3}{2}=\frac{9}{10} \quad & =\frac{9}{10}=\frac{3 \times 3}{5 \times 2}=\frac{3}{5} \times \frac{3}{2}=\frac{9}{10} \text { Ans. }
\end{aligned}
\end{aligned}
$$

(i.e) $\frac{3}{8}$ multiplied by $\frac{3}{2}$, (the divisor inverted).

ROLE.
Invert the divisor and proceed as in multiplication of fractions.

Divide-


## EXERCISE 36.

$$
\text { 27. } 1301 \frac{5}{\frac{5}{8}} \text { by } 35 \frac{1}{5} .
$$

## GREATEST COMMON MEASURE.

73. A Measure of a fraction is any nuinber that is contained in the fraction an exact integral number of times; thus $\frac{1}{12}$ is a measure of $\frac{1}{4}$, being contained in it 3 times. Hence,
74. A fraction is a measure of a given fraction when its numerator is a measure of the given numerator, and its denominator is a multiple of the given denominator.
75. A Common Measure of two or more fractions is any number that is contained in each an exact integral number of times; thus, $\frac{1}{24}$ is a common measure of $\frac{1}{3}$ and $\frac{1}{4}$, being contained in $\frac{1}{3} 8$ times, and in $\frac{1}{4} 6$ times.

Hence,
76. A fraction is a common measure of two or more given fractions when its numerator is a common measure of the given numerators, and its denominator is a common multiple of the given denominators.
77. The Greatest Common Measure of two or more given fractions is the greatest number that is contained in each an exact integral number of times; thus, $\frac{1}{12}$ is the greatest common measure of $\frac{1}{3}$ and $\frac{1}{4}$.

Hence,
78. A fraction is the greatest common measure of two or more given fractions when its numerator is the greatest common measure of the given numerators and its denominator is the least common multiple of the civen denominators.

Example.-Find the greatest common measure of $\frac{5}{8}, \frac{5}{18}$, and $\frac{15}{8}$. Soldtion.
The G. C. M. of 5,5 and $15=5$
The L. C. M. of 6,12 and $16=48$
Therefore the G. C. M. of the given fractions is $\frac{3^{5}}{3}$ Ans. Proor.

$$
\begin{aligned}
& \frac{8}{8} \div \frac{8}{88}=8 \\
& \frac{8}{18} \div \frac{8}{18}=4 \\
& 18 \div \frac{8}{88}=9
\end{aligned}
$$

The quotients 8,4 and 9 are prime to eaoh other.

From these principles and illustrations we derive the following rule :
I. Reduce whole rule.
and all fractions to their mixed numbers to improper fractions
II. Find the great lowest terms.
ators for a new numerator, and theasure of the given numerthe given denominators for and the least common multiple of will be the greatest for a new denominator. This fraction

Find the greatest common measure of -

3. $\frac{12}{21}$, $\frac{25}{25}$, 46.
4. $2 \frac{23}{24}, 1 \frac{13}{8}, \frac{28}{185}$

6. $1 \frac{8}{129}, 1 \frac{57}{3}, 4 \frac{2}{2}, 5 \frac{2}{2}, 12 \frac{2}{8}$.


## LEAST COMMON MULTIPLE.

79. A Multiple of a fraction is any number that contains the fraction an exact integral number of times; thus, $\frac{1}{4}$ is a multiple of $\frac{1}{12}$, since $\frac{1}{4}$ contains $\frac{1}{12} 3$ times.

Hence,
80. A fraction is a multiple of a given fraction when its numerator is a multiple of the given numerator, and its denominator a measure of the given denominator.
81. A Common Multiple of two or more given fractions is any number that contains each an exact integral number of times; thus, $\frac{2}{3}$ is a common multiple of $\frac{1}{12}$ and $\frac{1}{6}$, containing $\frac{1}{12} 8$ times, and $\frac{1}{6} 6$ times.

Hence,
82. A fraction is a common multiple of two or more given fractions when its numerator is a common multiple of the given numerators, and its denominator is a common measure of the given denominators.
83. The Least Common Multiple of two or more given fractions is the least number that contains each an exact integral number of times; thus, $\frac{1}{3}$ is the least common multiple of $\frac{1}{12}$ and $\frac{1}{6}$.

Hence,
84. A fraction is the least common multiple of two or more given fractions when its numerator is the least common multiple of the given numerators and its denominator the greatest common measure of the given denominators.

Example.-Find the least common maltiple of $\frac{3}{4}$, $\frac{5}{12}$, and $\frac{18}{18}$. Solution.
L. C. M. of 3,5 and $15=15$
G. C. M. of 4, 12 and $16=4$

Therofore the L.C. M. of the given fractions $=\frac{\mu}{q}$
Proof.

$$
\begin{aligned}
& \frac{18}{4} \div \frac{8}{4}=5 \\
& \frac{18}{4} \div \frac{5}{12}=9 \\
& \frac{18}{4} \div \frac{18}{4}=4
\end{aligned}
$$

The quotients 5,9 and 4 are prime to each other.
From these principles and illustrations we derive the following rule :

## nous.

I. Reduce whole and mixed numbers to improper fractions and all fractions to their lowest terms.
II. Find the least common multiple of the given numerators for a new numerator, and the greatest common measure of the given denominators for a new denominator. This fraction will be the least common multiple sought.

EXERCISE 38.
Find the least common multiple of-

1. \% $\frac{7}{20}$, $\frac{14}{20}$, $\frac{8}{23}$.
2. $\frac{7}{23}, \frac{88}{88}, \frac{49}{8}$, $\frac{29}{298}$.



3. $1 \frac{1}{4}, 2 \frac{1}{3}, 3 \frac{1}{4}, 4 \frac{1}{6}, ~ 5 \frac{1}{6}$.


## DECIMALS.

85. A Decimal Fraction, commonly called Decimal, is one whose denominator is 1 followed by one or more ciphers:

As $\frac{7}{10}, \frac{85}{18}, \frac{8}{80}, 189890$.
86. Since the denominators of decimal fractions increase or decrease by the uniform scale of 10 (the same scale as that used in expressing integers), a system of notation similar to that of integers is employed to express them, thus saving the trouble of writing the denominators.
87. The Decimal Sirn (.) or decimal point determines, by its position, the denominator of the fraction, and, in a number composed of an integer and a decimal, it shows where the decimal part begins.
88. This system of notation will be best explained by the following examples :


The numerator alone is written, and there must be as many figures to the right of the decimal point as there are ciphers in the denominator of the frastion. The vacant orders, if any, must be filled with ciphers.
85. The relation of decimals and integers to each other is clearly shown in the following table :


From this it appears the $t$
$2222.222=2000+200+20+2+\frac{2}{10}+\frac{180}{20}+\frac{1000}{2}$.
90. The method of representing decimal fractions is merely an extension of the method by which integers are represented, since the local value of each digit increases tenfold as we advance from right to left, and also decreases in the same proportion as we advance from left to right.
From the foregoing we derive the following principles :
principles.
91. 1. Decimals are governed by the same law of local value that governs the notation of integers.
2. The different orders of decimal units decrease from left to right and increase from right to left in a tenfold ratio.
3. The value of any decimal figure depends upon the place it occupies at the right of the decimal point.
4. Each removal of a decimal order one place to the left increases its value tenfold.
5. Each removal of a decimal order one place to the right decreases its value tenfold.
6. Prefixing a cipher to a decimal diminishes its value tenfold, since it removes every decimal figure one place to the right.
7. Annexing a cipher to a decimal does not alter its value, since it does not change the place of any figure in the decimal.

## EXERCISE 39.

Express in decimal form and read-

| I. | II. | III. | IV. | V. |
| :---: | :---: | :---: | :---: | :---: |
| \% | 1285000 | $\frac{188080}{180}$ | 285 | ${ }^{2000}$ |
| $\frac{4}{10}$ | $\frac{80}{1000}$ | \% 1000 | ${ }^{\text {in274 }}$ | ${ }^{688880}$ |
| ${ }^{180}$ | $1{ }^{173}$ | ${ }^{\frac{358}{100}}$ | $\frac{4996}{100}$ |  |
| \% ${ }^{\text {\% }}$ | $\frac{50}{100}$ | 788 | ${ }^{3089} 10080$ | ${ }^{81888}$ |
| Express in the form of a fraction and read- |  |  |  |  |
| VI. | VII. | VIII. | IX. | X. |
| . 9 | . 27 | 8.7 | . 0005 | . 0304 |
| . 06 | . 006 | 4.05 | . 81600 | . 00001 |
| . 25 | $.450$ | . 005 | . 0404 | . 15000 |
| . 007 | . 010 | . 136 | . 912 | 35.003 |

Express as decimals-
XI. Five-tenths, eieven ten-thousandthis.
XII. Thirty, and seven-tenths,
XIII. Seventy-four ten-millionths, thirty-six ten-thousandths. hand A Complex. Decimal has a fraction in its right fraction not which is read $12 \frac{1}{3}$ hundredths, the an not being counted as a decimal place.
Express as common fractions in their lowest termsXIV. .75, .72, .625, .024, .0032, .12,


## 83. To reduce a common fraction to a decimal. <br> Reduce f to its equivalent deoimal.

$$
\begin{aligned}
& \text { OPERATION. } \\
& \frac{5.000}{6.25}
\end{aligned} \quad \frac{5}{8}=\frac{5000}{8000}=\frac{\begin{array}{c}
\text { Reason. } \\
\frac{5000}{8} \\
1000
\end{array}=\frac{625}{1000}=.625}{}
$$

94. From this and similar examples we derive the following rule :
rdele.
Annex ciphers to the numerator and divide by the denominator.

Point off as many decimal places in the quotient as there are ciphers annexed.

Note 1.-If the division is not exact, when a sufficient number of decimal places has been obtained, the sign + may be annexed to show that the division is not complete, or it may be expressed as a complex
2. A fraction in its lowest terms aan be reduced to a pure deoimal only when its denominator contains no prime factors bat 2 and 5 . If the denominator contain any other prime factor the division will not end. The decimals thus produced are called Repeating Decimals, and the figures repeated, Repetends.

EXERCISE 40.
Reduce to equivalent decimals-

1. $\frac{3}{4}$
2. 哲
3. $\frac{5}{18}$
4. $\frac{19}{6}$
5. $\frac{8}{8}$
6. 
7. $\frac{7}{8}$
8. $\frac{1}{2}$
$\begin{array}{ll}\text { 3. } & \frac{2}{2} \\ 4 . & \frac{7}{8}\end{array}$
9. $\frac{5}{7}$
10. 8
11. 17
12. 12 畐
.
.
14, $16 \frac{?}{3}$
13. $25 \% \frac{7}{10}$
14. $31 \frac{4}{6}$

## ADDITION.

95. Since integers and decimals increase and decrease uniformly by the scale of 10 , it is evident that decimals may be added, subtracted, multiplied and divided in the same manner as integers.

Add 13.6, 5.034, .3172, 14.52.

## Exphanation.

13.6 5.034 . 3172 14.52 $\overline{33.4712}$

If the decimal points are in the same vertical line it will necessarily bring tenths under tenths, hundredths under hundredths, etc., and the numbers may therefore be added as in integers, RULE.
96. Write the numbers so that their decimal points are in the same vertical line. Add as in integers, and place the decimal point in the result directly under the points in the numbers added.

## EXERCISE 41.

## Add-

1. .3642, 26.035, .0037, 3.4, .017.
2. 41.234, 17.015, $3.3, \quad 400.2$. 0045 .
3. .0126, 40.371, .7251, . $0021,311.5$.
4. $06 \frac{2}{3} . ~ 3.805 \frac{3}{3}, 40.036, \quad 0031 \frac{3}{5}$.
5. . $004 \frac{4}{\frac{4}{8}, ~} 36.02 \frac{3}{8}, \quad 7.34, \quad .37 \frac{1}{8}$.
6. Ninety-seven hundredths, four hundred and three thousandths, thirteen ten-thousandths, sixteen, and fifteen hundredths, forty-seven, three hundred and twelve, and sixty-four thousandthe.

## SUBTRACTION.

97. From 18.65 take 9.3652 .

Note.-The a .ffixing of ciphers to right of the 13.6500
$\frac{9.3652}{4.2848}$ decimal does not alter its valve. In practice we omit the decimals, and merely conceive them to be annexed, subtracting as otherwise.
role.
98. Write the numbers so that the decimal places shall stand directly under each other. Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.

## EXERCISE 42.

Find the difference between-

1. $\mathbf{1 7 . 2 0 5}$ and 13.6
2. 4.037 and .2735.
3. 37.004 and 16.39353
4. 400.7 and . 00362
5. From . 03165 take .0127 .
6. From . 3074 take . 29653 .
7. Subtract 30.365 and 30.3782 .
8. Subtract . 0034715 and .0126 .

## MULTIPLICATION.

99. In multiplication of decimals, the position of the decimal point in the product depends upon the following minciples :
100. The number of ciphers in the denominator of a decimal is equal to the number of decimal places.
101. If two decimals in the fractional form be multiplied together, the denominator of the product must contain as many ciphers as there aro decimal places in both factors. Therefore,
102. The product of two decimals expressed in the decimal form must contain as many decimal places as there are decimals in both factors.

Multiply . 314 by 23 .

100. Multiply as in whole numbers and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.

## CONTRACTIONS IN MULTIPLICATION.

## 0127.

29653. 

and 30.3782 .
and .0126 .
on of the following
tor of a
aultiplied ntain as factors.
decimal here are

1ヵ1. Multiply 62.37416 by 2.34169 so as to retain only 4 places of decimals.

102. It frequently bappens in multiplication that a greater number of decimal figures is obtained in the product than is necessary for practical accuracy. This may be avoided by contracting each partial product to the required number of decimal places.
103. From this principle and illustrations similar to the foregoing example we derive the following:

RULE.
Write the multiplier with the order of the figures reversed, and with the units place under that figure of the multiplicand which is the lowest decimal to be retained in the product.

Find the product of each figure of the multipiier by the figures above and to the left of it in the multiplicand, increasing each partial product by as many units as would have been carried from the rejected part of the multiplicand, and one more when the highest figure in the rejected part of any product is 5 or greater than 5; and write these partial products with the lowest figure of each in the same column.

Add the partial products, and from the right hand point off the required number of decimal figures.
Nots 1.-In obtaining the number to be oarried it is generally necessary to multiply (mentally) only one figure at the right of the figure above the
multiplying figare; but when the figures are large the multiplication should commence at least two places to the right.
2. There is always a liability to an error of one or two anits in the last place.
3. When the number of places in the multiplicand is less than the number to be retained in the produot, supply the deficienoy by annexing oiphers.

| EXERCISE 44. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. | 36.275 | $\times$ | 4.3678 | retaining |  | decimal |  |
| 2. | 41.3075 | $\times$ | 467.32 | ، | 3 |  | places. |
| 3. | 17.0036 | $\times$ | . 08245 | " | 4 | ، | " |
| 4. | . 43261 | $\times$ | . 73158 | " | 3 | " | " |
| 5. | . 003647 | $x$ | . 12739 | " | 4 | " | " |
|  | 700.375 | $\times$ | . 02736 | " | 3 | " | " |
| 7. | . 374825 | $\times$ | . 693847 | " | 5 | " | ${ }_{6}$ |

## DIVISION.

principle.
104. Multiplying both divisor and dividend by the same number aoes not alter thé quotient.
105. Multiplying a decimal expression by 10 , moves the decimal point one place to the right; by 100 , two places to the right; by 1000, three places to the right, etc. Therefore, moving the decimal point in divisor and dividend the same number of places to the right, multiplies each of them by the same number.

Example 1.-Divide 16.578 by 5.4.

$$
5.4) 16.578 \text { ( Explanation. }
$$

Multiply the divisor and dividend
54) 165.78 ( 3.07 $\frac{162}{378}$

378 by 10 and we obtain 54 as divisor

- and 165.78. Now 54 will divide into 165, 3 times, and therefore 3 is the integral part of the quotient.
Example 2.-Divide . 786644 by 234.6 234,6 ) .736644 (

2346) $\begin{gathered}7.36644 \\ 7038\end{gathered}$ (. 00314 Here in dividing we use as tho | 7038 |
| :--- |
| $\mathbf{3 2 8 4}$ |

2346 9384 9384
first partial dividend 7.366 or 7366 thousandths, and hence our first quotient figure 3 thousandths whioh expressed ag e decimal is .003.

RULE.
106. Move the decimal point to the right of the divisor, and the same number of places to the right in the dividend. Divide as in simple division, placing the decimal point in the quotient as soon as the tenths figure is used or brought down.
Nots.-If the dividend doos not contain as many decimal places as the divisor, annex eiphers to the right of the decimal before removing the points.

## EXERCISE 45.

| 1. 48.591 | $\div .96$. | 5. . 0774 | $\div 480$. | 9. 10.66 | . |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2. 2.56 | $\div$. 0032 . | 6. 21.3 | $\div 37.5$. | 10. 15.77 |  |
| 3. 3.1 | $\div$. 025. | 7. 202 | $\div$ - 01. | 11. 134.25 | $\div 7.5$ |
| 4. . 0012 | $\div 1.6$ | 8. 400.8 | $\div \quad .018$. | 12. . 733 | $\div 3$ |
| 13. | Divide 1.21 | 11, 1.1, | .11, .011 | $.0011, .00$ |  |
| 14. | Divide $.03^{n}$ | \% 1610, 180, | 18, . 18 | , .018. |  |

## CONTRACTED DIVISION.

107. Divide 763.14163 by 21.3642 , correct to four places o.' decimals.

| Ordinary Metrod. | Contraoted Meitod. |
| :---: | :---: |
| 218642) 7631416.3 ( 35.7205 | 218642) 76314163 ( $\mathbf{8 5 . 7 2 0 5}$ |
| 640926 | 640926 |
| 1222156 | 122215 |
| 1068210 | 106821 |
| 15394 | 15394 |
| 1495494 | 14955 |
| 439690 | 439 |
| 427 284 | 427 |
| $12 / 40600$ | 12 |
| 1068210 | 11 |
| 172390 | 1 |
|  |  |

108. Compare the highest or left hand figure of the divisor with the units of like order in the dividend, and determine how many figures will be required in the quotient.

For the first contracted divisor take as maty significant figures from the left of the given divisor, as there are pluces
required in the quotient, and, at each subsequent division reiect one place from the righ of the last preceding divisor.

In multiplying by the several quotient figures, carry from the rejected figures of the divisor as in contracted multiplica. tion.

Nore.-Before oommenoing the work, supply oiphers at the right of either divisor or dividend, when necessary.

## EXFRCISE 46.

| $\begin{aligned} & 1 . \\ & 2 . \end{aligned}$ | $27.3788$ | by | 4.3267 | correct |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3. | ${ }^{287.24} 8$ | by | 1.003675 | " | 2 | decim | places. |
| 4. | . 8487564 | by | 75.43 | " | 5 | " |  |
| 5. | 478.825 | by | . 075637 | " | 3 | ، |  |
| 6. | 8972.436 | by 7 | 75¢.3452 | " | 3 | " | " |
| 7. | 1 | by | 1.007633 |  | 4 | - | ، |
| 8. | . 953728 | by | 44.73654 | " | 6 3 | " | " |

## REPEATING, CIRCULATING OR INTERMINATE DECIMALS.

109. In reducing common fractions to equivalent decimals, reference was made in Article 94, Note 1, to the methods of expressing the decimals in cases where the division does not terminate. But if the division were carried far enough (never to number of places in the quotient greater than the number represented by the divisor) a remainder would be obtained which had occurred before, and hence a figure or set of figures in the quotient would be repeated in the same order in a never quotient succession. A decimal of this kind is in a never-ending or circulating decimal, or simply a repetled a repeating 110. When a repetend indicated by a point placed sists of a single figure it is mire than one figure a point is it; when it consists of one over the last figure renented placed over the first and
division reiect isor．
，carry from multiplica the right of places．
＂
＂
＂

## ERMI－

ent deci－ ，to the lere the on were in the by the ccurred uotient ending eating $\theta$ it is sts of $t$ and ating
decimals $.4444+$ and .324324324 ＋are written.$\dot{4}$ and ． $\mathbf{3} 2 \dot{4}$ ．

111．If we take any fractions whose denominator consists of any number of 9 ＇s，as $\frac{4}{8}$ ，$\frac{24}{83}$ ，$\frac{32}{8} \frac{4}{8}$ ，and we reduce each of them to decimals，we obtain

$$
\dot{t}=.444+=\dot{4} ; \dot{74}=.2424+=. \ddot{24} ; 384=.324324+=.324 .
$$

From these and similar examples we infer that all possible repetends can thus be derived from fractions whose numer－ ators are the repeating figures，and whose denominators are． as many 9＇s as there are repeating figures．

Example 1．－Express $\frac{8}{7}$ as a repeating decimal．
7） 5 （．714285714285 $+=.714285^{\circ}$
Example 2．－Express $\frac{1}{2} \frac{9}{8}$ as a repeating decimal．

$$
28) 13(.46428571428571+=46428571 .
$$

112．Decimals in which all the figures do not repeat are called Mixed Circulating Decimals．

$$
\begin{aligned}
& \text { Example 3.-Express. } 2 \boldsymbol{2} \text { as a common fraction. } \\
& . \ddot{2} \overline{5}=3 \frac{3}{8} \\
& \text { Explanation. } \\
& \begin{aligned}
& . \ddot{2} \dot{5}=.252525+ \\
& \because 100 \text { times } \\
& .2 \dot{5}=25.252525+
\end{aligned} \\
& \text { Add } \frac{1 \text { times } . \dot{2 \dot{5}}}{\dot{2}}=.252525+ \\
& \text { Subtracting } \overline{99 \text { times } .2 \overline{5}}=25 \\
& \therefore .2 \dot{5}=25
\end{aligned}
$$

From similar examples to this we derive the following rule ：

## bole．

Omit the points and decimal sign and write the figurcs of the repetend for a numerator and as many 9 ＇s as there are places in the repetend for a denominator．

Example 4．－Express ． $24 \dot{5} \dot{6}$ as a common fraction．
Solution．
2456
$\frac{24}{2432}$ 新考年 Ans

Lxplanation.

$$
\begin{aligned}
& .24 \dot{50} \dot{6}=\quad .2456565656+ \\
& \begin{aligned}
& \because 10000 \text { times } .24 \ddot{5} \ddot{6}=2456.565656+ \\
& \text { And } 100 \text { times } .24 \dot{5} \dot{6}=24.565656+ \\
& \text { Subtracting }+ \\
& 9900 \text { times } .24 \dot{6} \dot{6}=2432=2456-24 \\
& \therefore .24 \ddot{5} \ddot{6}=\frac{2456-24}{9900}
\end{aligned}
\end{aligned}
$$

From examples similar to the preceding, we derive the following rule for reducing mixed circulating decimals (those in which only a portion of the figures in the decimal repeat) to common fractions.

RUle.
Subtract the part of the decimal which does not repeat from the whole decimal as if each were uhole numbers, and place the remainler as a numerator, and for a denominutor as many 9's us there are figures repeating, followed by as many $O$ 's as there are figures in the part which does not repeat.

## EXERCISE 47.

Espress as circulating decimals-

2. $\frac{26}{28}, \frac{3}{13}, \frac{1}{13}, \frac{17}{3}, \frac{1}{2} \frac{5}{2}, \frac{7}{18}, \frac{1}{21}, \frac{13}{3}$. Express as fractions in their lowest terms-
3. $\dot{7}, \quad . \ddot{6} 7, \quad .30 \dot{6}, \quad \ddot{4} 5, \quad .3 \dot{6} 9, \quad . \dot{1} 6 \dot{2}, \quad . \dot{2} 63 \dot{5}$.
4. $2 \dot{7}, \quad .4 \dot{7}, \quad .3 i, \quad .2 \dot{1} \dot{5}, \quad .2 \dot{5}, \quad .3 \dot{7} \overline{34}, ~ .712 \ddot{7} 1$.


## WEIGHTS AND MEASURES.

## CANADIAN CURRENCY.

113. Money is the measure of value.
114. Currency is the money employed in trade.
115. Coins or Specie are species of metal of known purity and weight, stamped at the Mint, and authorized by the Government to be used as money at fixed value.
116. Bullion is uncoined gold or silver, and includes bars, gold-dust, etc.
117. Paper Money is a substitute for metallic currency. It consists of Dominion Notes issued by the Government and Bank Notes issued by Chartered Banks.
118. Canada money is the legal currency of the Dominion of Canada. It is founded on the Decimal Notation, and its denominations are, Dollars, Cents and Mills.
119. The Silver coins are the fifty-cent piece, the twenty-five-cent piece, the twenty-cent piece, the ten-cent piece and the five-cent piece.
The Copper coin is the cent.
There are no Canadian gold coins; those of England and the United States are a legal tender.


## UNITED STATES MONEY.

121. J. S. Monay is the legal currenoy of the United States, and is often called Federal Money. Its denominations are Eagles, Dollars, Dimes, Cents and Mills.
122. The Gold coine are the double eagle, eagle, half-eagle, quartereagle, three-dollar piece, and dollar.
123. The Silver coins are the dollar, half-dollar, quarter-dollar, and dime.
The Nickel coins are the one-cent and three-cent pieces.
The Bronze coin is the one-cent piece.
124. 10 Mills

TABLE.
$\begin{aligned} 10 \text { Cents } & =1 \text { Cent } \quad . \\ & =1 \text { Dime }-d . \\ 10 \text { Dimes or } 100 \text { Cents } & =1 \text { Dollar }-d . \\ 10 \text { Dollars } . & =1 \text { dol. or } 8\end{aligned}$

## ENGLISH MONEY.

125. English or Sterling money is the currency of Great

## Britain.

126. The unit is the Pound Sterling, which is represented by a gold sovereign, is equal in value to $\$ 4.8665$.

TABLE.
127. 4 Farthings (qr. or far.) $=1$ Penny

12 Pence
$=1$ Shilling -
$=1$ Pound or Sovereign 8.
21 Shillings. $\quad=1$ Pound or Sovereign $\mathcal{E}$
128. The gold coins are the sovereign, and the half-sovereign.
129. The silver coins are the crown $(=58$ ), the half-orown (2s.6d.), the shilling, and the sixpenny piece.
130. The copper coins are the penny, half-penny, and farthing.
131. The standard parity of the gold coins of Great Britain is 22 carats fine; that is $\frac{1}{2}$ pure gold and ${ }_{r^{1}}^{2}$ alloy. That of the silver coins is $\frac{3}{3}$ pure silver and $\frac{3}{80}$ alloy.

TROY WEIGHT.
18\%. Troy Weight is used in weighing gold, silver and jewels; in philosophical experiments.

The measuring unit is the pound.

TABLW.
133. 24 Grains (gr.) $=1$ Pennyweight dvet.

$$
20 \text { Pennyweights }=1 \text { Ounve } . \quad \text { dzo. }
$$

$$
12 \text { Ounces . . }=1 \text { Pouna } . \quad l b \text {. }
$$

134. The value of diamonds and other jewels is estimated by carats.
A carat is the weight of four grains.

## APOTHECARIES WEIGHT.

135. Apothecaries Weight is used by druggists and physicians in compounding medicines, but drugs and medicines are bought and sold by avoirdupois weight.
The measuring unit is the pound.
The pound, ounce, and grain are the same as in troy weight.
136. 

TABLEF.

apothecaries' fluid measure.
137. Apothecaries' Fluid Measure is used in mixing liquid medicines.
thble.
138. 60 Minims, or Drops ( $m$.) $=1$ Fluid Draohm $f 3$


## AVOIRDUPOIS WEIGHT.

139. Avoirdupois Weight is used for all the ordinary purposes of weighing.
The measuring unit is the pound.
ilver and


## LONG TON TABLY.

141. 



SPECIAL AVOIRDUPOIS WEIGHTG.
142.
100 lbs. Nails $\quad=1$ Keg.
100 lbs. Dry Fish $=1$ Quintal.
196 lbs. Flour . $=1$ Rarrel.
200 lbs. Beef or Pork $=1$ Barrel.

COMPARATIVE TABLE OF WEIGHTS.
143.

|  | Troy. |
| ---: | :--- |$\quad$ Avormbupors. $\quad$ Apothecaries.

## GRAIN MEASURE.

TABLE.
144.


## DRY MEASURE.

145. Dry measure is used in measuring substances not hquid, as grain, fruit, salt, roots, etc.

TABLE.
$14 B$.

| 2 Pints (pt.) | $=1$ Quart $\cdot q t$. |
| :--- | :--- | :--- |
| 4 Quarts | $=1$ Gallon $\cdot g a l$. |
| 2 Gallons | $=1$ Peck $-p k$. |
| 4 Pecks | $=1$ sushel $\cdot$ bush. |

147. The Imperial Standard Gallon, for liquids and all dry sub. stanoes, is a measure that will contain 10 pounds avoirdupois of distilled water, weighed in air at $62^{\circ}$ Fahrenheit, the barometer at 30 inohes.
148. The Imperial Gallon contains 277.274 cubic inches.
149. The Imperial Standard Bushel is equal to 8 gallons or 80 oounds of distilled water, weighed in a manner above described.
150. The Standard Bushel contains 2218.192 oubio inches.

## LIQUID MEASURE.

157. Licquid Measure is used in measuring liquids; as liquors, molasses, water, etc.

TABLE.
152.

158. The following denominations are also in use :

42 Gallons . . $=1$ Tierce.
2 Hogsheads, or 126 Gallons $=1$ Pipe, or Butt.
2 Pipes, or 4 Hogsheads $=1$ Tun.
Nore.-The tierce, hogshead, pipe, butt, and tun, are the names of casks, and do not express any fixed definite measures. They are usually gauged, and have their capaoities in gallons marked on them.
154. A Measure is a standard unit established by law or custom, by which extent, dimension, capacity, amount, or value is estimated.

## MEASURES OF EXTENSION.

155. Measures of Extension are those used to ascertain how long a line is, or in calculating the size (sxtent) of a surface or solid.

A iine has only one dimension-length.

## linear or line measure.

In measuring length, linear or line measure is used.

|  | table. |  |
| :---: | :---: | :---: |
| 156. | 12 | Inches (in.) |
|  | 8 | Feet . |
|  | 512 | Yards, or 16 |
|  | 320 | Rods |

equivalents.
1 Mile $=820$ Rods $=1760$ Yards $=5280$ Feet $=63360$ Inches.

## SURVEYORST MEASURE.

157. Gunter's Chain, used by land surveyors, is 4 rods, or 66 feet long, and consists of 100 links, each 7.92 inches long.


## SQUARE MEASURE.

159. Square Mensure is used in measuring surfaces; as of land, boards, painting, plastering, etc.
160. Area or Surface has length and breadth only, and is the space or surface incluted within any given lines.
161. A square inch, square foot or square yard, is a square, each side of which is respectively, 1 inch, 1 foot, or 1 yard in length.

TABLE.
162. 144 Square Inches (sq.in.) $=1$ Square Fuot - aq.ft.

9 Square Feet . $=1$ Square Yard - sq.yd.
301 Square Yards . . = 1 Square Rod . sq. rd.
160 Square Rods . . $=1$ Aore . . . A.
640 Acres - . . $=1$ Square Mile - sq. mi.
Artificers estimate their work as follows:
By the square foot: glazing and ston cuttiag.
By the square yard : painting, plaster nit pavive, ceiling, and paper-hanging.

By the square of 100 square feet : floor:r; partitioning, roofing, slating, and tiling.

Bricklaying is estimated by the thousand bricks, by the square yard, and by the square of 100 square feet.
Notes 1.-In estimating the painting of moldinge, cornices, etc., the measuring-line is carried into all the moldings and cornices.
2. In estimating brick-laying by either the square yard or the square of 100 feet, the work is understood to be 12 inches or $1 \frac{1}{2}$ bricks thick.
3. A thousand shingles are estimated to oover 1 square, being laid inches to the weather.

## SURVEYORS' SQUARE MEASURE.

63. This measure is used by surveyors in computing tharea of land.


## CUBIC MEASURE.

165 Cubic Measure is used in measuring solids or volume
166. solid is that which has length, breadth, and thickness
167. A Cube is a regular solid bounded by six equal squares called faces. Hence length, breadth, and thickness are equal to each other.
table.
168. 1728 Cubic Inches (cu. in.) . $\quad=1$ Cubio Foot - cu.ft.

27 Cubic Feet . . . . $=1$ Cubio Yard - cu.yd.
$\left.\begin{array}{l}40 \text { Cubic Feet of Round Timber, or } \\ 50 \text { Cubio Feet of Hewn " }\end{array}\right)=1$ Ton . . T.
16 Cubic Feet . . . . $=1$ Cord Foot - cd.ft.
8 Cord Feet, or 128 Cubic Feet $=1$ Cord of Wood Cd.
$24 \frac{1}{4}$ Cubic Feet . . . . . $=\underset{\text { Perch of Stone) }}{\text { or Masonry }}$ Pch.
Notes.-1. A oubic yard of earth is called a load.
2. Railroad and transportation companies estimate light freight by the space it ocoupies in cubio feet, and heavy freight by weight.
3. A pile of wood 8 feet long, 4 fcet wide, and 4 feet high, contains 1 cord; and a cord foot is 1 foot in length of suoh a pile.
4. A perch of stone or of masonry is $16 \frac{1}{2}$ feet long, $1_{\frac{1}{2}}$ feet wide, and foot high.
5. Joiners, bricklayers, and masons, make an allowance for window doors, etc., of one half the openinge or vacant spaces. Bricklayers ad masons, in estımating their work by cubic measure, make no allowate for the corners of the walls of houses, cellars, etc., but estimate their .fk by the girt, that is, the entire length of the wall on the outside.
measure of time.
169. Time is the measure of duration. The measting unit is the day.
170. Time is naturally divided into days and years. Thormer are measured by the revolution of the earth on its axis; the latf by its revolution around the sun.
171. 60 Seconds (sec.)

TABLEE.

172. The Civil Year includes both common and leap years, and is divided into 12 Calendar Months, viz. :

173. The numbers of days in each month may be easily remembered from the following lines:

> "Thirty days hath September, April, June and November; Febriary, twenty-eight alone, All the rest have thirty-one, But in leap year, then is the time When February has twenty-nine."

## LEAP YEAR.

174. The period of time required by the sun to pass from one vernal equinx to another, ca!led the vernal or tropical year, is exactly 365 da . 5 hr . $\$ \min .49 .7 \mathrm{sec}$.
175. If 365 days be reckoned as one year, the time lost in the calendar willbe,

$$
\begin{aligned}
& \text { In } 1 \text { Year } \\
& \text { In } 4 \\
& \text { In }
\end{aligned} .
$$

The tim thus lost in 4 years will lack only 44 min. 41.2 sec . of 1 entire day. Hene,
If every furth year be reokoned as leap year, the time gained in the calendar will be,


The time the gained in 100 years will lack only 5 hr .22 min .50 sec . of 1 day. Hence,
If every fourt year be reckoned as leap year, the centennial years excopted, the tim lost in the oalendar will be,

> In 10 Years $\quad . \quad . \quad 5 \mathrm{hr} .22 \mathrm{~min} .50 \mathrm{sec}$.
> In $400 \quad " \quad . \quad$

I'he time thus lostin 400 years laoks only 2 hr .28 min .40 sec . of 1 day. Henoe,

If every fourth year be reckoned as leap year, 3 of every 4 centennial years excepted, the time gained in the oslendar will be,

$$
\begin{aligned}
& \text { In } 400 \text { Years } \quad . \quad . \quad 2 \mathrm{hr} .28 \mathrm{~min} .40 \mathrm{sec} . \\
& \text { In } 4000 \quad \because \quad \text { - } 24 \mathrm{hr} .46 \mathrm{~min} .40 \mathrm{sec} .
\end{aligned}
$$

176. The following rule for leap year will therefore render the calendar correct to within 1 day for a period of 4000 years.

ROLE.
I. Every year that is exactly divisible by 4 is a lsap year, the centerinial years excepted; the other years are common years.
II. Every centennial year that is exactly divisible by 40 , is a leap year; the other centennial years are common year.
177. Circular Measure is used principally in surveying, navigation, astronomy, and geography, for reckoning latitude and longitrde, determining locations of places and of vessels, and in computing diference of time.
178. Every oircle, great or small, is divided into the same nimber of equal parts; as quarters, oalled quadrants; twelfths, oalled signs; three handred and sixtieths, called degrees, eto. Consequently tie parts of different oircles, although having tho eame names, are of (ifferent lengths.

The unit is the degree, whioh is ${ }_{5}$ If $_{\sigma}$ part of the circumfereve of any sirole.
179.

$$
\begin{aligned}
& 60 \text { Seconds (") } \quad=1 \text { Minute } \\
& 60 \text { Minutes } \cdot \\
& 30 \text { Degrees } \quad=1 \text { Degree } \\
& 12 \text { Signs, or } 360^{\circ} \cdot \\
& =1 \text { Cign } \\
& =1 \text { Cicle }
\end{aligned}
$$

## MISCELLANEOUS TABLES,

COUNTING.
180.

12 Things $=1$ Dozen.
12 Dozen $=1$ Gross.
12 Gross $=1$ Great Gross, 20 Things $=1$ Score.

## PAPEf.

181. 

$$
\begin{aligned}
24 \text { Sheets } & =1 \text { Quira } \\
20 \text { Quires } & =1 \text { Ream. } \\
2 \text { Reams } & =1 \text { Bundle. } \\
5 \text { Bundles } & =1 \text { Bale. }
\end{aligned}
$$

B00iks.
182
2 Leaves $=1$ Folio.
4 Leaves $=1$ Quarto, or 4 to.
8 Leaves $=1$ Octavo, or 8vo.
12 Leaves $=1$ Duodecimo, or 12 mo .
The terms folio, quarto, octavo, denotei the number of leaves into which a sheet of paper is folded in making books.

## LONGITUDE AND TIME.

## STANDARD TIME.

18s. During the year 1883 the principal railroads of Canuda and the United States adopted what is known as the "Standard Time System." This system divides Canaria and the United States into four sections or timebelts, each covering 15 degrees of longitude, $7 \pi_{3}^{\circ}$ of which are east and $7 \frac{1}{2}^{\circ}$ are west of the governing or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt.

The governing meridians are the 75th, the 90 th, the 105th, and the 120th, west of the Greenwich Observatory, London, England, and as these meridians are just $15^{\circ}$ apart, there is a difference in time of exactly one hour between any one of them and the one next on the east, or the one next on the west; the standard meridian next on the east being one hour faster, and the one next on the west one hour slower. Hence, the $60^{\circ}$ of longitude is four hours, the $75^{\circ}$ five hours, the $90^{\circ}$ six hours, the $105^{\circ}$ seven hours, and the $120^{\circ}$ eight hours slower than Greenwich time, making five different standards of time between the Atlantic and the Pacific Oceans, viz. : Intercoionial, Eastern, Central, Mountain, and Pacific.
184. Since every circle may be divided into 360 equal parts called degraes, and since the time in which the earth makes one revolution on its axis may be divided into 24
equal parts called hours, it follows that the earth on revolving on its axis passes through $\frac{1}{24}$ of $360^{\circ}$ or $15^{\circ}$ of longitude in one hour ; through $1^{\circ}$ of longitude in $\frac{1}{15}$ of an hour, or 4 minutes, and through $1^{\prime}$ of longitude in $\frac{1}{60}$ of 4 minutes or 4 seconds.
185.
186. To find the difference in time between two places or meridians when the difference of longitude is known.

## Example-

If the difference in longitude of two places be $\mathbf{7}^{\circ} \mathbf{1 8}$, what must be their difference in time?

## Explanation.

Since each minute of distance equals 4

Soldtion.

| $7^{\circ}$ | $18^{\prime}$ |
| :---: | :---: |
|  | 4 |
| 29 min. | 12 sec. | seconds of time, 18 minutes of distance will equal 72 seconds, or 1 minute 12 seconds of time.

Since each degree of distance equal 4 minntes of time, 7 degrees will equal 28 minutes, plus 1 minute, gives 29 minutes.
role.
Multiply the distance between the two places exprissed in degrees and minutes by 4, and the result is the difference in time expressed in minutes and seconds.
Nores.-1. If one place be in east and the other in west longitude, the difference of longitude is found by adding their longitudes, and if the sum be greater than 180 degrees, it must be subtracted from $360^{\circ}$.
2. Since the sun appears to muve from east to west, when it is exactly 12 o'clock at one place, it will be past 12 o'clock at all plaoes east, and before 12 at all places west. Hence, if the difference of time between twe places be subtracted from the time at the easterly place, the result will be the time at the westerly place; and if the difference be added to the time at che westerly place the result w.ll be the time at the easterly place

18\%. To find the difference of longitude between two places or meridians, when the differace of time is known.

## Example-

If the difference of time between two places be 28 minutes, 20 seconds, find the difference in longitude.

Explanation.
Sirce 4 minates oll time equal 1 degree

Soldtion.
$4 \longdiv { 7 ^ { \circ } } \frac { 2 8 \mathrm { min } . 2 0 \mathrm { sec } . } { 5 ^ { \prime } }$ of longitade, 28 minutes of time equal $7^{\circ}$ of longítude.

Since 4 secu da of inne equal 1 minnte of longitude, 20 seconds of time agual $5^{\prime}$ of lougitade.

RULE.
Divide the difference in time between the places expressed in minutes and seconds by 4 and the quotient is the difference in longitude expressed in degrees and minutes.
table of lonaitudes.


## EXERCISE 48

Find the difference in longitude between-

1. Toroato and London (Eng.)
2. Quebec and Calcutta.
3. Ottawa and Viotoria.
4. Hamilton and Berlin.
5. Brantford and Winnipeg.
6. Kingston and Paris.

Find the difference in solar time between-
7. Toronto
8. Kingston
9. Ottawa
10. Montreal and Greenwich. and Winnipeg. and Viotoria.
and Regina.
12. London (Can.) and London (Eng.)
12. Philadelphia and Calcutta.

Find the difference in standard time between-
13. Quebec and Ottawa.
14. Montreal and Victoria.
15. Toronto and Winnipeg.
16. Kingston and Regina.
17. Montreal and Winnipeg.
18. Halifux and Victoria.

Find the difference between the standard time and the solar time in the following cities :

| 19. | Toronto, | Ottawa. |
| :--- | :--- | :--- |
| 20. | Montreal, | Victoria. |
| 21. | Winnipeg, | Halifux. |

22. A navigator finds that when it is noon at his place of observation it is 16 min .34 sec . past $10 \mathrm{p} . \mathrm{m}$. by his chronometer, Greenwich time; what is his longitude?
23. When it is $6: 40$ a.m. at Halifax, what is the time at Victoria?
24. If the difference of solar time between two places is 1 hr .18 min . 4 sec ., what is the difference of longitude?
25. When it is Mondey 10 p m., solar time, at Toronto, what day and time is it in London (Eng.) (Greenwich time) ?

## REDUCTION.

189. Reduction is the process of changing the denomination of a quantity without changing its value. It is of two kinds, Descending and Ascending.
190. Reduction Descending is changing a number of one denomination to another denomination of less unit value:
191. Reduction Ascending is changing a number of one denomination to another denomination of greater unit value.
192. To reduce Higher denominations to Lower.

Examplim.-Reduce 26 bbl. 8 gal. 8qt. to quarts.

Soldtion. 26 bbl. 8 gal. 8 qt.. $\frac{31 \frac{1}{2}}{427}$ $\frac{4}{3311}$ qts. Ans.

Explanation.
Since $31 \frac{1}{2}$ gal. make 1 bbl ., there are, $31 \frac{1}{2}$ times as many gallons as barrels, anā $819+8=827$ gallons. Like. wise, there are 4 times as many quarts as gallons, and $(827 \times 4)+3=3311$ quarts.

## bule.

Multiply the highest denomination by the number requi. ed of the next lower to make a unit of the higher, and to the product add the lower denomination.

Proceed in this manner with the successive denominations, till the one required is reached.

## LONGITUDE AND TIME.

## EXERCISE 49.

1. In 17 dys. 18 hrs. 27 min., how many seconde?
2. Rednce 12 mi .8 rd . $3 y d$. 2 ft , to inches.
3. Reduce 243 lb .3 oz .6 dwt . to grains.
4. In $83 \mathrm{c} . \mathrm{yds}$. how many cubio inches ?
5. £133 68, 8d., how many farthings?
6. How many penoe are there in $£ 16488.0 \frac{1}{2} d . ?$
7. In 481 soveroigns how many penoe?
8. In 4 mi . $120 \mathrm{rd} .2 y d .1 \mathrm{ft}$. 6 int., how many rods? yards? feet? inches?
9. Reduce 16 T. 8 cwt .86 ll . to pounds.
10. Reduce 18 A. $228 q . r d$. $25 \mathrm{sq} . y d$. to square feet.
11. How many grains in 16 lb . Avoirdupois?
12. In 2 mi ., in $3 \frac{1}{2} m i$.. in $\frac{5}{8}$ mi., how many rods? yards? feet? inches?
13. In 47 guineas how many pounds and shillinge?
14. In 12 lb ., Troy, how many drame, Apothecaries?
15. Find the cost of 2 bl . 3 bun .1 rm .4 qr .21 sheets of paper, at $\$ 3.87 \frac{1}{2}$ a ream.

## 193. To reduce Lower denominations to Higher.

Exampris.-Reduce 157540 minutes to weeks.

## Explanation.

Dividing the given number of minutes by 60 , because there are $\frac{{ }^{3}}{3}$ as many hours as minutes, we obtain 2625 hours plus a

Boldtion.
$6 0 \longdiv { 1 5 7 5 4 0 }$ min. $2 4 \longdiv { 2 6 2 5 } h r . + 4 0 \mathrm { min } .$ $7 \longdiv { 1 0 9 } d a . + 9 h r .$
$15 w k .+4 d a$.
15 wk. 4 da. 9 hr .40 min . Ans. emainder of 40 minutes.
We next divide the 2625 hours by 24 , because there are $\frac{1}{2 \pi}$ as many days as hours, and we find that 2625 hours $=$ 109 days plus a remainder of 9 hours. Lastly, we divide the 109 daye by 7, because there are of as many weeks as daye, and we find that $10 y$ days $=15$ weaks plus a remainder of 4 days. The las: quotient and the several remainders arranged in the order of the succeeding denominations form the answer.

## EXERCISE 50.

Reduce-

1. 1913551 ounces
2. 97920 grains
to tons.
3. 43769 inches
4. 27150 pounds
to lbs.
to miles.
5. 3276
pints
6. 184760
seconds
7. 278648
cubic inches
farthings
8. 32459
cubic feet
9. 478960
to long tons.
to gallons.
to days.
to oubic yards
to $£$
to oords.
10. 283546 sheets of prper
11. 2468
pence
12. 23750
grains, Troy,
13. 15630
mills
14. 1800356 links
to reams.
to half.crowns.
to lbs.
to dollarg.
15. 4562
pints
16. 20436 rods
17. $1020300 "$
"
to miles.
to bushels.
to miles.
18. 70
lbs. Avoirdupois to lbs. Troy.
19. 350
oz. Troy
to oz. Avoirdupois.
20. 46030 grains, Apoth. to lbs. Avoirdupois.
21. Find the value of 921640 lbs . of ooal at $\$ 4.75$ per long ton.
22. Find the price of 462 bush. 23 lbs . of wheat at 950 . a bushel.
23. How many bushels are there in 5160 lbs. of timothy seed?
24. What is the freight on 528 bushels of corn at 32 c . a cont.?
25. What is the freight on 16 T .17 cwt .20 lb . of coal at $\$ 1.20$ per ton
of $2240 \mathrm{lbs} . ?$
26. Find the amount of the following bill of grain:


## DENOMINATE NUMBERS.

194. The process of adding, subtracting, multiplying and dividing denominate numbers is based on the same principles that govern similar operations in simple num. bers; the principal difference being that denominate numbers have an irregular scale of increase and decrease, while simple numbers have a uniform decimal scale.

## ADDITION.

Find the sum of $3 \mathrm{lh} .7 \mathrm{oz} .10 \mathrm{dwt} .12 \mathrm{gr} . ; 17 \mathrm{ll} .5 \mathrm{oz}$. 18 dwt. 4 gr. ; and 12 Il .11 oz .9 dwt. 15 gr.

Solution.

| $l b$. | $o z$. | dwt. | $g r$. |
| :---: | :---: | :---: | :---: |
| 3 | 7 | 10 | 12 |
| 17 | 5 | 18 | 4 |
| 12 | 11 | 9 | 15 |
| $34 l b$. | 0 | oz. | 18 dwot. | $\mathbf{7}^{7} \mathrm{gr}$.

## Explasation.

Write the numbers of the same unit value in the same column. Beginning with the lowest denomination, add as in simple numbers, and reduce to higher denominations according to the scale.

Add-
(1)

EXERCISE 51.
(2)

| $\mathscr{L}$ | s. | $d$. |
| :---: | :---: | :---: |
| 145 | 0 | 91 |
| 169 | 17 | 8 |
| 175 | 14 | 74 |
| 166 | 15 | 83 |
| 1199 | 5 | 10 |

(3)

| $h h d$. | $g a l$. | $q t$. | $p t$. |
| ---: | :---: | :---: | :---: |
| 79 | 62 | 3 | 1 |
| 3 | 59 | 2 | 0 |
| 61 | 13 | 2 | 1 |
| 159 | 4 | 1 | 1 |
| 66 | 27 | 0 | 0 |

## Add-

## (4)

| gal. | qt. | pt. | gi. |
| ---: | ---: | ---: | ---: |
| 49 | 2 | 1 | 3 |
| 71 | 3 | 0 | 2 |
| 1 | 1 | 1 | 1 |
| 16 | 3 | 1 | 3 |
| 68 | 3 | 1 | 2 |

(5)

| $m i$. | A. | sq.p. | sq. $y d$ |
| ---: | ---: | ---: | :---: |
| 50 | 75 | 30 | 15 |
| 791 | 11 | 15 | 11 |
| 87 | 345 | 31 | 16 |
| 75 | 473 | 29 | 30 |
| 15 | 29 | 18 | 26 |

(8)

| (6) |  |  |
| ---: | ---: | ---: |
| $T$. | cwt. | lb. |
| 55 | 16 | 17 |
| 14 | 11 | 5 |
| 63 | 19 | 24 |
| 919 | 6 | 20 |
| 89 | 1 | 4 |

7. Add 236 lb .4 oz .15 dvt ., 83 lb .11 oz .21 gr ., $46 \mathrm{lb} .16 \mathrm{dzot} ., 105 \mathrm{lb}$. 9 oz .11 gr .
8. Add 7 T. 14 cwot. 25 lb ., 14 T. 9 cwt. $16 \mathrm{lb} .8 \mathrm{os} ., 36$ cot. 17 lb ., 14 T. 12 cwt ., and 5 cwt . 10 lb .14 oz .
9. Find the sum of 12 wk. $3 \mathrm{da} .5 \mathrm{hr} .20 \mathrm{~min} .42 \mathrm{sec} ., 4 \mathrm{da} .12 \mathrm{hr} .30 \mathrm{~min}$., 8 wh. 1 da .10 hr .40 min ., and 16 hr .36 min .30 sec .
10. Add $6 \mathrm{~cd} .5 \mathrm{~cd} . \mathrm{ft}$., $3 \mathrm{~cd} .6 \mathrm{~cd} f \mathrm{ft} .9 \mathrm{cu} . f \mathrm{ft} ., 4 \mathrm{~cd}$. ft. $14 \mathrm{cu} . f t$, and 5 cd .
$24 \mathrm{cu} . \mathrm{ft}$.
11. Off of one field of wheat were raised 37 bush. 1 pk . $3 q t$.; of a second field, $41 \mathrm{bush} .2 \mathrm{pk} .5 \mathrm{qt}$. ; of a third, $35 \mathrm{bush} .1 \mathrm{pk} .3 \mathrm{qt}$. ; of a fourth, 43 bush. $3 p k$. 1 qt. How mooh was the whole?
12. A grocer received an invoioe of 7 hhd . of sagar; the first weighed 11 cwt . 15 lb. ; the second, 12 cwt . 15 lb .; the third, 9 cwt .16 lb .; the fourth, 12 civt. ; the fifth, 11 cwt. 24 lb .; the sixth, 9 cwt. 24 lb .; the seventh, 13 cwt. How mach did the seven hogsheads contain ?
13. A person has 5 pieoes of ground; the first contains 16 A .8 rd .; the second, $17 \mathrm{~A} .18 \mathrm{sq} . p .45$ sq.ft.; the third, $11 \mathrm{~A} .14 \mathrm{sq} . p .62 \mathrm{sq} . \mathrm{ft}$.; the fourth, 2 A .120 iq. ft.; and the ffith, $41 \mathrm{~A} .7 \mathrm{sq} . \mathrm{p}$. What is the amount of the whole?
14. A person owes several sums of money; to one 178. 6d.; to another, $£ 3 \mathrm{ss} .8 \mathrm{~d}$. ; to another, $£ 25 \mathrm{lls} \mathrm{s}$. $10 \frac{1}{2} \mathrm{~d}$.; to another, $£ 128 \mathrm{~d}$. ; to another, 16 s . 33 $10 \frac{1}{d}$.; to another, $£ 504 s .4 \frac{3}{4} d$. What is the whole amount?
15. A person travelling goes 26 mi . 12 rd ., the first day; 28 mi .5 fur. 9 rd . 9 ft ., the second day ; 31 mi .15 rd .14 ft ., the third day ; 26 mi . $12 f t$., the fourth day; and 33 mi .16 rd .11 ft ., the fifth day. How far does he go during the five days?
16. A jeweller receives on one day 11 lb . 6 oz . of gold; on another day, 10 lb .5 oz .20 gr ; on another, 6 oz .3 dvt . 16 gr .; on another, 6 lb .17 dwt .1 gr ; ; on another, 16 lb .4 oz .15 dwt .15 gr . How much does he reoeive in all?

## SUBTRACTION.

Eruaple.-Subtract $12 \mathrm{lb} .9 \mathrm{oz}, 11 \mathrm{dwt} .15 \mathrm{gr}$. from $27 \mathrm{lb} .50 z$.

## Explanation.

Write the numbers as for simple

| Solution. |  |  |  |
| :---: | :---: | :---: | :---: |
| $l b$. | 08. | divt. | $g r$. |
| 27 | 5 | 16 | 12 |
| 12 | 9 | 11 | 15 |
| $14 l$ | 808. |  | 21 g | subtraction; take each subtrahend term from its corresponding minuend term. In case any subtrahend torm be greater than the minuend term, borrow 1 as in simple subtraction, and reduce it to the denomination required, eto.

## EXERCISE 52.

(1)
(2)

|  | (1) |  | (2) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6. | 2. | d. | lb. | oz. | $d w t$. |  |
| 186 | 4 | 01 | 114 | 3 | 16. | 12 |
| 98 | 11 | 23 | 91 | 4 | 12 | 19 |
|  | (4) |  |  |  | (5) |  |

$\begin{array}{ccc}\text { A. } & 8 q . p . & \text { sq.yd. } \\ 75 & 14 & 11\end{array}$

| 78 | 10 | 16 |
| :--- | :--- | :--- |


| $l b$. | 3 | 3 | $Э$ | $g r$ |
| :--- | :--- | :--- | :--- | :--- |
| 68 | 1 | 7 | 2 | 12 |
| 15 | 0 | 7 | 2 | 15 |


| csot. | $l b$. | os. | $d r$. |
| :---: | :---: | :---: | :---: |
| 58 | 16 | 2 | 5 |
| 27 | 20 | 1 | 6 |

(6)
$r d$. $y d$. ft. in

| 16 | $y d$. | ft. | in |
| :---: | :---: | :---: | :---: |
| 14 | 5 | 1 | 11 |
| 14 | 5 | 2 | 0 |

7. A person owes $£ 78$ 3s. 21 d. ; he pays $£ 17$ 178. 19 ${ }^{\text {d }}$.; how much does he still owe?
8. A. owes B. for 2 invoices of merchandise; one worth $\quad \$ 17 \quad 168$.
9. A farmèr has a farm consisting of 2000 a.ores. He gave his eldest son 109 4. $3 \mathrm{rd} .20 \mathrm{sq} . \mathrm{p}$.; to his second son. 48 A . 1 rd .; the remainder he gave to his third son. What was the remainder?
10. How long is it from June 21st, 1886, to December 14th, 1888 ?
11. The latitade of Hamilton is $43^{\circ} 12^{\prime} 40^{\prime \prime}$, of Quebec, $46^{\circ} 50^{\prime} 10^{\prime \prime}$; how many degrees is Quebeo north of Hamilton?
12. The latitude of Brantford is $42^{\circ} 21^{\prime} 22^{\prime \prime}$; how far is Brantford from
13. A merchant bought 3 pieces of oloth ; the first measured 47 yd. 3 qr ; the second, $43 y d$. ; the third, $41 y d .3 \mathrm{gr}$. ; when he came to Enamine it, he found $13 y d$. worthless; how muoh good oloth

## MULTIPLICATION.

Example,--Each of seven bars of silver weighs 17 lb .5 os .13 dwt .
Find the total weight? 16 gr . Find the total weight?

Solution.

| $l b$. | $o z$. | $d u t$. | $g r$. |
| :---: | :---: | :---: | :---: |
| 17 | 5 | 13 | 16 |
|  |  |  | 7 |
| $122 l 6$. | 3 oz. | $15 d \omega t$. | 16 gr |

$16 \mathrm{gr} . \times 7=112 \mathrm{gr} .=4$ dwt. 16 gr . Pat down 16 under gr. Carry
$13 d w t . \times 7+(4 d w t$. carried $)=95 d w t .=4 \mathrm{oz} .15 d w t . \quad$ Pat down
15 under dwt. Carry 4 to oz.
$5 \mathrm{oz} . \times 7+(4 \mathrm{oz}$. carried $)=39 \mathrm{oz} .=3 \mathrm{lb} .3 \mathrm{oz}$. Put down 3 under oz. Carry 3 to $l b$. $17 \mathrm{lb} . \times 7+(3 \mathrm{lb}$. carried $)=122 \mathrm{lb}$. Put down 122 under $l \mathrm{l}$.

## EXERCISE 53.

1. Multiply 38 lb .6 oz .17 dvt . by 17.
2. Multiply 19 T. 13 cwt . 18 lb . by 19
3. Multiply $3 \mathrm{lb} .4 亏 231$ - 17 gr . by 11.
4. Multiply 15 gd .1 ft .11 il . by 21.
i. Multiply 17 mi .2 rd .16 ft . by 23.
5. Multiply 15 rd .2 yd. 1 ft . by 29.
6. Multiply 144 A. 17 sq.p. 19 sq. yd. by 5.
7. Multiply $17 \mathrm{C} .59 \mathrm{cu} . f t, 718 \mathrm{cu}$. in. by 13.
8. Mulciply 73 hhd .61 gal .3 qt. 1 pt . by 26.

10 If one cold of wood cost $£ 116 \mathrm{y}$. $9 \frac{1}{2} d$., what will 25 cords cost :
11. If you can buy $3 b u, 3 p k$. $3 q t$. for $\$ 1$, how many bushels can be bought for $\$ 79$ ?
12. Bought 17 yards of lace, at $£ 3$ 17s. 1d. per yard; 14 yards of craje, at £2 10 s. per yard. What is the value of both purchases?
13. If you can exohange one acre of wheat for $17 \mathrm{~A} .7 \mathrm{sq} \cdot \mathrm{p}$. of pasture, how many acres of pasture can you get for 41 acres of wheat?
14. Bought 16 pieces of lace, eaoh containing 62 yards, at $£ 1118.2 d$. per yard, and sold 7 pieces for $£ 115 \mathrm{~s}$. per yard, and the rest at $£ 1$ 12r. 10d. per yard; how much was yained?

## DIVISION.

Example.-If 122 lb .3 oz .15 d d t .16 gr . of silver be made into 7 bars of equal weight, what will be the weight of one bar?

Solution.


## Explayation

Write the dividend and divisor as in short division, and divide as in simple numbers, thus : $\frac{1}{7}$ of $122 \mathrm{lb} .=17 \mathrm{ll}$. and an undivided remainder of 3 lb . Reduce this remainder to oz.; add the 3 oz of dividend $=39 \mathrm{oz}$. $\frac{1}{7}$ of $39 \mathrm{oz} .=5 \mathrm{oz}$. and an undivided remainder of 4 oz . Reduce this remainder to dwt.; add the 15 dwt . of dividend $=95 \mathrm{~d} w$. 4 of $95 d w t .=13 d w t$. and an undivided remainder of 4 dwt. Reduce this remainder to gr. ; add the 16 gr . of dividend $=112 \mathrm{gr}$. 4 of $112 \mathrm{gr} .=16 \mathrm{gr}$.

## EXERCISE 54.

1. Divide $£ 91$ 12s. 6 d . by 6.
2. Divide $386 \mathrm{lb} .0 \mathrm{oz} .16 \mathrm{dwt}, 23 \mathrm{gr}$. by 29.
3. Divide 9 T. $16 \mathrm{cwt} .16 \mathrm{lb}, 3 \mathrm{oz}, 4 \mathrm{dr}$. by 17.
4. Divide $61 \mathrm{lb}, 10 \mathcal{Z} 430-16 \mathrm{gr}$. by 36.
5. Divide 78 mi .14 p . by 31 .
6. Divido $4 y d .1$ ft. 11 in . by 15.
7. Divide 19861 sq. mi. 179 A. 20sq.p. 11 sq.yd. by 61.
8. Divids 738 cu yd. $20 \mathrm{cu} . f t .1100 \mathrm{cu}$. in. by 399.
9. Divide 20 hid. 16 gal. 3 qt. 1 gi. by 147.
10. Divide 175 bush. 3 pk. 1qt. 1 pt. by 67.
11. Divide 1 circle by 128.
12. Divide 365 da. 6 hr . by 240.
13. If 16 bushels of oysters cost $£ 7517 \mathrm{~s}$. 4 d , what will one bushel cost?
14. If one yard cost $2 s$. 6d., how many yards oan be bought for $£ 180$ ?
15. If you can buy 15 square rods of land for $£ 1$, for how many pounds can you buy one acre?
16. Divide a square mile into 15 equal parts.
17. A man travelled 1249 mi .36 rd . in 61 days; how far did he travel in a day?
18 A cartman oarricd $117 \mathrm{ca} 110 \mathrm{cu} . f$. in 100 loads; how much did he average a load?

## DENOMINATE FRACTIONS.

195. A Denominate Fraction is a fraction whose integral unit is a denominate number.
Nots.-The prinoiples, analyses, and rules of denominate fractions are essentially the same as those of denominate integers; therefore, no special rules are necessary for their reduction. A sufficient number of examples are given to fully explain the different cases that may arise.
196. To reduce a denominate fraction or decimal to integers of lower denominations.

Example.-Reduce $£_{\text {İ }}^{7}$ (.4370) to integers of lower denominations. Solr rion.

$$
\begin{aligned}
& £_{16}^{7} \times \frac{20}{1} \times 1^{2}=105 d . \\
& 105 d .=8 s .9 d . \\
& \text { or } \\
& \sum_{18}^{7} \times \frac{20}{1}=\frac{38}{2}=8 \frac{1}{2} .
\end{aligned}
$$

$$
\begin{aligned}
£ .4375 \times 20 \times 12 & =105 d . \\
105 d . & =88.9 \mathrm{~d} .
\end{aligned}
$$

or

£. 4375
16) $\frac{\frac{20}{140}}{\frac{128}{12}}(8 s$.
16) $\frac{12}{144}(\Omega d$. 144
197. To change a fraction or decimal of one denomination to a higher or lower denomination.

Example 1. - Reduce $\frac{1}{40}(.025)$ of a yard to the fraction of an inoh.
Operation.
to $y d . \times \frac{4}{2} \times \frac{12}{2}=\frac{2}{2} \mathrm{in}$. Ang. $\quad .025 y d . \times 3 \times 12$ Opration.
Exumpre 2.-Change 9 (.9) of $\times 3 \times 12=.9=\frac{9}{10}$ in. Ans.

Operation.
$\frac{9}{10} \mathrm{in} . \times \frac{1}{12} \times \frac{1}{8}=\frac{1}{40} y d$. Ans.

Operation.
$12) .9 \mathrm{in}$.
3) .075
.025 or $\frac{1}{4} y d$.
198. To change one denominate number to the fraction or to the decimal of another.

Example 1.-Reduce $3 q t$. $\therefore$. pt. to (1) the fraction of a gallon (2), to the decimal of a gallon.

Solution
to a common fraction.
$3 q t .1 p t .=7 p t$.
$1 \mathrm{gal} .=8 \mathrm{pt}$.
$\therefore 7 \mathrm{pt}$. $\quad=\frac{7}{8}$ of a gal.
Esample 2.-Reduce 158. 6d. 3far. (1) to the fraction of a $\mathcal{E}(\mathbf{2})$, to
eimal of a $£$. the deoimal of a $\mathbf{e}$.

Solution.
to a common fraction.

$$
\begin{aligned}
& 15 s .63 d . \\
& \begin{array}{ll}
\text { f1 } & =747 \text { far. } \\
& =960 \text { far. }
\end{array}
\end{aligned}
$$

Solution.

## to 4 decimal.

4) 3 far.

| $1 2 \longdiv { 6 . 7 5 }$ |
| :--- | :--- |
| $2 0 \longdiv { 1 5 . 5 6 2 5 }$ |

.778125 of a $£$. of $£ 1^{\prime} 17 \mathrm{~g} .4 d$.

$$
\begin{aligned}
& \text { Solotion. } \\
£ 1 \quad 3 s .4 d . & =380 d . \\
£ 1 \text { 17s. 4d. } & =448 d . \\
\therefore £ 1 \quad 38.4 d . & =\frac{280}{48} \text { of } £ 1 \quad 17 s .4 d . \\
& =85 \text { of } \\
& =.625 \text { of }
\end{aligned}
$$

EXERCISE 55.

1. Red uoe gity of a mile to the fraction of a yard.
2. What is the value of .8525 of a $£$ ?
3. Reduce fof a pennyweight to the fraction of a pound, Troy.
4. What part of 3 weeks is 4 da .16 hr .30 min ?
5. What part of $1 \pm$ bushels is 45 of a peck?
6. Reduce .425 of a foot to the fraction of a mile.
7. Reduoe 2617 18. 1d. to the decimal of a $£$.
8. What is the value of $\frac{6}{8}$ of $a$ mile ?
9. What part of an inch is fe of a yard?
10. What part of a $l b$. Troy is 75 of a grain?
11. Reduce 3 bush. $1 p k$. $3 q t$. to the decimal of a bushel.
12. Reduoe $2.333 \frac{1}{3}$ years to integers of lower denominations
13. Rednce $£ 14$ 15s. 9d. to the deoimal of $\mathrm{e} \boldsymbol{£}$.
14. Reduoe $\frac{7}{8}$ of a hundredweight to the fraction of an ounce.
15. Reduce $\frac{5}{5}$ of a mile to the fraction of $\frac{3}{1}$ of a rod.
16. Rednce $\mathbf{2} 210 \mathrm{~s}$. Of $d$. to the deoimal of $£ 217 \mathrm{~s} .2 d$.

## ALIQUOT PARTS.

199. An aliquot part of a number or quantity is an exact divisor of that number or quantity. Thus 5 is an aliquot part of $20 ; 33 \frac{1}{3}$ of 100 .

Many business calculations may be shortened by com. bining the values of convenient aliquot parts.

Example 1.-What will 576 yarde of cloth cost at $\$ 1.87 \frac{1}{2}$ a yard ?
Solution.


Example 2.-What will 7 bush. $3 p k .6 q t$. of wheat cost at $\$ 1.60$ a bushel ?

Soletion.
 per $o z$.

Soldtion.


Example 4.-What will 34 bush. $3 p k$. $4 q t$. of clover-seed cost at $\$ 450$ per bushel?

$$
\begin{aligned}
& \text { Solution. }
\end{aligned}
$$

## EXERCISE 56.

What is the cost of

1. 75 lbs . of coffee at $33 \frac{1}{3} \mathrm{c}$. a lb .
2. 120 lbs . of sugar at $12 \frac{1}{2} \mathrm{o}$. a lb. ?
3. 84 yards of carpet at $\$ 1.33 \frac{1}{3}$ a yard?
4. 144 bushels of wheat at $\$ 1.16 \mathrm{3}$ a bushel ?
5. 5386 boxes of oranges at $15 s .9 \frac{1}{2} d$. a box?
6. 886 pieces of silk at $£ 968.7 \frac{1}{4}$ d. a piece?
7. 26 T. 18 cwt .47 lbs . of copper at $\$ 245.20$ a ton $\%$
8. 615 A .152 pr . of land at $\$ 164.80$ an acre?
9. 43 bush. $2 p k .7 q t$. of corn at 58 c . a bushel ?
10. 12 lb .10 oz .14 dvt . of gold at $£ 6312 \mathrm{~s}$, a pound ?
11. $270 y d s$. silk at $£ 15 \mathrm{~s} .6 d$. per $y d$ ?
12. 326 bbls . flour at $\$ 7.87 \frac{1}{2}$ per $b b l$.?
13. 15 A .3 r .20 rd . land at $\$ 60$ per acre?
14. $12 T$ T. 17 cwt . freight at $\$ 4$ per ton?
15. 7 cwot. $3 q r .12 \mathrm{lb}$. at $\$ 61.50$ per long tnn ;
16. $27 y d s$. of cloth at 38.93
17. $84 \mathrm{cu} . \mathrm{yds} .24 \mathrm{cu}$.ft. at $\$ 2.50$ per cu !. d . p
18. 13 gal .1 qt. 1 pt. wine at $\$ 3$ per g.t!?
19. 17 cut. 2 qr. at $\$ 7.50$ per ton?
20. $\frac{3}{f}$ daz. elbows at $\$ 2.75$ per duz. :

## MISCELLANEOUS PROBLEMS.

## EXERCISE 57.

I.

1. Find the total distance around a rectangular field at 1,728 feet long
2. A manufacturer sells $236 \frac{1}{4}$ barrels of flour on Monday, $\mathbf{3 , 1 2 4}$ barrels on Tuesday, 462 barrels on Wodneaday, 3632 barrels on Tharsday, 25.6 barrels on Friday, and ¿3it barrels on Saturday How many barrels did he sell during the week?
3. A certain building oontains 74 windows, each window containing 8 panes of glass. Find the oost of the glass at 14 cents per pane.
4. How many pounds of wire will it require to fence a field 304 feet square, the fenoe being 6 wires high, if 10 feet of the wire weigh one pound?
5. A man deposited in a bank $\$ 8,752$; he drew out at one time $\$ 4,234$, at another. $\$ 1,700$, at another, $\$ 962$, at another, $\$ 49$. How much had he remaining in the bank?

6 A man invests in trade at one time $\$ 680$, at another time, $\$ 820$, at a third time, $\$ 1,580$, and on a fourth occasion, $\$ 420$. How much must he add to the sum of these tbat the amount may be $\$ 5,000$ ?
7. A merohant bought 840 barrels of flour for $\$ 1,920$, and sold it at $\$ 10.50$ a barrel. What did ne gain?
8. A farmer exchanged 754 bushels of wheat, at $\$ 1.25$ a bushel, for 78 barrels of flour, at $\$ 2$ per barrel, and received the balance in money. How much money did he receive?
9. A man bought 45 aores of land at $\$ 38$ an aore, and 76 acres at $\$ 47$ an aore, and sold the whole at $\$ 15$ an acre. Did he gain or lose, and how much ?
10. The cost of the Atlantio Telegraph Cable, as originally made, was 2e follows : 2,500 miles at $\$ 485$ per mile, 10 miles deep sea cable at $\$ 1,450$ per mile, and 25 miles shore ends at $\$ 1,250$ per mile. What was its
botai cost?

## II.

1. How many bags will be required to hold 108 bushels, it 4 bags hold 9 bushels?
2. If 5 barrels of flour cost $\$ 60$, how many cords of wood at $\$ 4$ a cord will pay for 3 barrels of flour?
3. If 12 yards of cloth cost $\$ 60$, for how much a yard must it be sold to gain $\$ 20$ ?
4. A man received $\$ 50$ for 5 barrels of pears, and paid all but $\$ 14$ for 4 ohairs. What did eaob ohair cost?
5. If a man reoeived 16 pounds of sugar in exchange for 20 pounds of cheese at 8 cents a pound. What is the price of the sugar a pound?
6. If a woman pay 60 cents for some lemons at the rate of 10 oents for 6, and sell them at the rate of 9 for 20 cents, how many cents will she gain?
7. What is the smallest sum of money with which I can purchase either sheep at $\$ 3.50$ a head, calves at $\$ 10.50$, cows at $\$ 35$, oxen at $\$ 70$, or horses at $\$ 105$ ?
8. A coal dealer sold 5 tons of coal for $\$ 57.50$, which was $\frac{5}{8}$ as much as he received for all he had left at $\$ 7.66$ per ton. How many tons did
9. How many times is the G. O. M. of 43, 61, 7 $\frac{2}{2}$, and $7 \frac{7}{72}$ contained in the L. C. M. of the same numbers?
10. If $3 \frac{s}{12}$ tons of ooal will last as long as 410 cords of wood, how many tons of coal will last as lonf as $13 \frac{7}{1 r}$ cords of wood?

## III.

1. What will 45 bush. $3 p k$. 1 qt. of wheat cost at $\$ 1.75$ a bushel?
2. Wishing to travel in Great Britain, I exchanged $\$ 1,500$ for English money. How many pounds did I receive?
3. What will 25 T. 6 cwt .94 lbs . of coal cost at $\$ 6.40 \mathrm{a}$ long ton 9
4. From a pile of wood containing $960 \mathrm{cu} . f$ t., was sold at one time $3 \ddagger c d$., at another, $2 \frac{5}{g} c d$. What was the remainder worth at $\$ 43$ a cord?
5. How many aores of land aan be bought for $\$ 25,000$, if a square foot
6. A carriage wheel 12 ft .8 in . in circumference will make how many revolutions in a distance of 65.5 miles ?
7. If $5 \frac{1}{2}$ lbs. of coffee cost $\$ 1 \frac{1}{18}$, what will $27 \frac{1}{d} \mathrm{lbs}$. oost?
8. How many times can a vessel contizining $7^{7}$ of a gallon be filled from I of a barrel conlaining $31 \frac{1}{2}$ gallons?
9. $\frac{8}{11}$ of a certain number exceeds 7 of the same number by 156 . What is the number?
10. A certain number multiplied by 2.5 and divided by 5.2 produces 1 . What is the sumber?

## $1 \nabla$.

1. Divide the sum of .075 and .0075 by the difference of 7.5 and .75 .
2. Find the least common multiple of $\frac{9}{5}, \frac{7}{10}, \frac{14}{\frac{1}{8}}$ and $\frac{8}{25}$.
3. Divide $\$ 2,000$ between two persons so that one should have $\frac{7}{8}$ as much as the other.
4. Bought a cord of wood for $\$ 4.625$, a cheese for $\$ 7.56$, and 14 ig lbs. of butter at 250 . per $l b$. What was the cost of the whole?
5. At $\$ 1 \frac{1}{2}$ a bushel, how many bushels of wheat can be bought for $\$ 37.68 \frac{3}{4}$ ?
6. If a $l l$. of tea be worth $\$ .62 \frac{1}{2}$, what is .8 of a $l b$. worth ?
7. What is the value of 720 pounds of hay at $\$ 12.75$ a ton, and 912 pounds of shorts at $\$ 15 \frac{1}{2}$ a ton?
8. Bought $12 y d 8$. cloth at $\$ .37 \frac{1}{2}$ per $y d$., and agreed to pay $\frac{1}{2}$ the cost in butter, at $\$ .162$ per $l b$.; $\frac{7}{8}$ in money and the remainder in eggs, at $\$ .12 \frac{1}{2}$ a dozen. How many pounds of butter and dozens of eggs were required?
9. What is the value of 1,046 pounds of beef at $\$ 45$ per cwt. $?$
10. How many pairs of pants can be made from 48.6 yds . of cloth. allowing $1.8 y d$. per pair?

## V.

1. Sold 125 equal loads of wood, measuring $115 \mathrm{~cd} .3 \mathrm{~cd} . \mathrm{ft} .7 \mathrm{cu}$. ft., for \$432.50. What is the quantity per loud, and price per cord ?
2. If I buy 120 gallons of rum for $\$ 75$, how much water must be added to it that I may sell it at 60 cents a gallon, and gain $\$ 15$ on the sale of it?
3. What part of a short ton is 5 of a long ton?
4. I have a field 96 rods long and 50 rods wide. How much will it cost to build a fence around it at $\$ .12 \frac{1}{2}$ per foot?
5. A. owns ${ }_{2}^{5}$ of a field, and $B$, the remainder; $\frac{3}{4}$ of the difference between their shares is $5 \mathrm{~A} .3 \mathrm{rds} .16 \frac{1}{2} \mathrm{pr}$. What is B.'s share?
6. What part of a cord of wood is a load $7 \frac{1}{3} f t$. long, $2 \frac{1}{3} f t$. high, $3 \frac{1}{3} f t$. wide?
7. Reduce $\frac{8}{3}$ of a long ton to the decimal of a short ton.
8. A farmer sold 8 loads of potatoes, averaging 27 bush. $3 p k .5$ qt. ewoh, for $\$ .45$ a bushel. How much did he receive?
9. A merchant in selling groceries selle $14 \frac{9}{18} o z$. for a $l b$.; how much does he oheat a costomer who buys of him to the amount of $\$ 38.40$ ?
10. If the longitude of Belleville is $77^{\circ} 26^{\prime} 12^{\prime \prime}$ W., what will be the time ir that place when it is 3 hr .35 min . a.m. in London, Eng.?

## VI.

1. How many bricks, each containing 121d cubic inches, can be puoked in 3 cubic yards?
2. Telegraph posts are placed 66 yards apart; a train passes one every $3^{\prime \prime}$. Find at what rate per hour the irain is travelling?
3. What is the cost per hour of lighting a room with 3 burners, each consuming 5 cubio inches of gas per second, the price of the gas being $\$ 2$ for 1000 cubio feet?
4. A man bought 35 bushels of barley, and sold the whole for $\$ 30$. He made $\$ 5.50$ in the trade. What did he give per buehel?
5. A tailor has 67 g yards of cloth, from which he wishes to out an equal number of coats, pants and vests. What number of each can he cut if they contain respectively $37,23,14$ yards?
6. Bought 12 T. 3czot. 70 lbs . of sugar at $\$ 8.25$ per cwt. What was the cost?
7. How many bales of cotton, of 400 lb . each, at 36 cente per $l b$., are equal in value to 18 hhd . of sugar, of $1,500 \mathrm{lb}$. each, at 8 oents per $l \mathrm{l}$. ?
8. What part of 5 da .23 hr .58 min . is 4 da .6 hr .50 min .?
9. Thirty-two men agree to build 14 mi .234 rd .6 ft . of road. When the work is $\frac{1}{6}$ done, they employ 8 more men. What distance does eaoh man construct?
10. I wish to put 111 bush. $2 p k$. $4 q t$. of grain into bags that should contain 2 bush. $1 p k .4 q t$. each. How many bags will be required?

## VII.

1. If a man travel at the rate of a minute of distance in 10 minutes of time, how long will he be in travelling around the world?
2. St. Thomas is $81^{\circ} 15^{\prime}$, and Halifax $63^{\circ} 36^{\prime}$ Werst Longitude. When it is 12 o'olock noon at St. Thomas, what is the time at Halifax?
3. Theice on a pond, whose area is $\frac{1}{2}$ an acre, is 10 inches thick. How many tons of ice may be taken from the iond, supposing a cubic foot of
ice to weigh 56 poande?
4. If the regular fare on a railway is 3 cents a mile, but $\frac{1}{b}$ is allowed off full fare when return tickets are bonght, find the distance between two places if a return ticket oosts $\$ 1.80$.
5. 450 leaves of a certain kind of paper make an inch of thickness. lind the thickness of a book 6 inches by 4 inches, in which 10 square yards of the paper are used.
6. It costs $\$ 23.10$ to fence a square field at $3 \frac{1}{2}$ cents per yard. How many acres in the ficld?
7. From 10 acres take 8 A .159 pr .30 yld .6 ft .108 in .
8. What is the result, when 500 is divided by .25 , the quotient by .025 , the second quotient by 50 ?
9. Express 3.74976 minutes as the decimal of a week.
10. What is the least number from which 1,224 and 1,656 may each be taken an exact number of tines?

## VIII.

1. If water in freezing expands $\frac{1}{10}$, find the weight of a cubic foot of ice, a cabic foot of water weighing 1,000 ounces.
2. Find the difference between 9 A .159 pr .30 yd .7 . 4 , $3 / \mathrm{il} \mathrm{in}$. and $10 \mathrm{~A} . ?$
3. Divide $\$ 760$ among A. B. and C., so that Il $\mathrm{ir}_{2}$ y have $\$ 160$ more than A., but $\$ 50$ less than $\mathbf{C}$.
4. How far may a person ride in a carriage guived at the rate of 8 miles per hour, so that if he walked back at the rate of 3 miles per hour he may be gone $5 \frac{1}{2}$ hours?
5. What will it cost to dig a ditch on each side of a road 4 miles 80 chains long at 40 cents a rod?
6. Walking 44 miles an hour, I start after a friend whose pace is 3 miles an hour; how long shall I be in overtaking him?
7. How many square rods are there in 100 square chains?
8. A man owns . 1875 of a mine; he sells .17 of his share. What fractional part has he left?
9. Reduce $\frac{2}{3}$ of an hour to the decimal of $\frac{3}{4}$ of 48 minutes.
10. What will it cost to fence a siuare 10 aere feld at 80 oents a rod ?

## IX.

1. At $\$ 2.40$ per rod, what will it cost to fence a piece of land 84.5 rods long by 24.75 rods wide?
2. A ship with its cargo is worth $\$ 340,000$, $\frac{9}{4}$ of the vatue of the dargo is worth ? the value of the ship. Find the value of each?
3. Divide 6 dy .17 hr .11 min . by ${ }_{\mathrm{L}} \mathrm{B}^{5}$.
4. How many reams of paper will bs required to supply 7,500 sub. scribers with a weekly newspaper for a year, allowing a sheet for one copy?
5. Telegraph poles are placed 8 rods apart, and a train passes une every $4 \frac{1}{2}$ seconds. How many miles an hour is the train travelling?
6. A man charged me 15 cents for a scuttle of coal, when coal if selling at $\$ 7$ per ton. How many pounds ought the scuttle to hold?
7. Divide $\$ 82.60$ among 27 men and 37 boys, so that each man may have three times as much as each boy
8. By selling my oloth at $\$ 1.26$ a yard, I gain 11 cents more than I lose by selling it at $\$ 1.05$ a yard. What would I gain by selling 800 yards at $\$ 1.40$ a yard?
9. If $\frac{2}{3}$ of an estate be worth $£ 32$ ), tin 1 the value of $\frac{3}{1}$ of the estate.
10. If a railway train goes 45 miles an hour, how many yards will it go in a second?

## X.

1. How many times will the seconds hund of a watch go around in $12 w \mathrm{k} .2 \mathrm{hr} .15 \mathrm{~min}$. ?
2. Divide $\$ 600$ between two persons, so that one shall have 7 as much as the other.
3. A regiment marching $3 \frac{1}{2}$ miles an hour makes 110 steps in a minnte. What is the length of the step?
4. I bought 20 pounds of opium by Avoirdnpois weight, at 55 cents an ounce, and sold by Troy weight at 60 cents an ounce. Did I gain or lose, and how much ?
5. The G. C. M. of two numbers is 12 ; their L. C. M. is 72 ; one of the numbers is 24 ; find the other?
6. Divide $\$ 345$ among A. B. and C., so that B. will receive $\$ 5$ for A.'s $\$ 4$, while C. receives $\$ 6$ for A.'s $\$ 5$.
7. Which is the greater . 0025 of a mile or 79 of a rod?
8. How long will it take a train 20 rods long, and going at the rate of 15 miles an hour, to cross a bridge 15 rods long ?
9. When an ounce of gold is worth $\$ 19.45$, what is the value of .04 of a pound?
10. A coal dealer bought a quantity of coal at $\$ 6 \mathrm{a}$ tom, and sold it for 48 cents a hundredweight, gaining thercby $\$ 43.20$. How many tons did


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## PE:'CENTAGE.

200. Percentage is the method of calculating by hundredths, or it is the term applied to such computations as involve the number 100 as the basis or unit of measure.
201. Per cent. is an abbreviation of the Latin phrase per centum, and signifies on or by the hundred. Thus 4 per cent. means 4 of every hundred and may signify 4 cents of every 100 cents, $\$ 4$ of every $\$ 100,4$ lbs. of every 100 lbs., etc.
202. The sign \% stands for the phrase per cent. ; thus 8 per cent. is written $8 \%$.
203. To express any per cent. as a decimal or as a common fraction.

Since any per cent. is some number oî hundredths, it is properly expressed by a decimal fraction, or by a common fraction.

Since $6 \%$ means six-hundredths, therefore $6 \%=.06=\frac{g}{180}$.

| table. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| symmots. |  | dectamb. |  | common fractiong. |
| 1\% | $=$ | . 01 | = |  |
| $2 \%$ | = | . 02 | $=$ | $1{ }^{180}=100$ |
| 4\% | = | . $0 \pm$ | $=$ | $\begin{aligned} & 180=\frac{10}{50} \\ & 1 \frac{1}{20}=\frac{1}{285} \end{aligned}$ |
| 5\% | = | . 05 | $=$ | $\frac{5}{180}=2$ |
| 10\% | = | . 10 | $=$ | 100 $=10$ |
| 25\% | = | . 25 | = | ${ }_{25}^{200}=\frac{1}{1}$ |
| 40\% | = | . 40 | = | $\frac{40}{100}=\frac{2}{5}$ |
| 100\% | = | 1.00 | = | ${ }_{180}^{180}$ |
| 125\% | = | 1.25 | = | ${ }_{\frac{1}{1} \frac{1}{5} 5}=\frac{8}{4}$ |
| t\% | $=$ | . $00 \frac{1}{2}=.005$ | = | $\frac{\frac{1}{2}}{100}=\frac{1}{200}$ |
| \% | $=$ | . $00 \frac{1}{2}=.0075$ | = | $\frac{\frac{3}{100}}{100}=\frac{3}{400}$ |
| 122\% | = | . $12 \frac{2}{2}=.125$ | $=$ | $\frac{12 \frac{1}{2}}{100}=\frac{1}{8}$ |
| 33\% \% | = | .337 |  | $\frac{33 \frac{1}{3}}{100}=\frac{1}{8}$ |
| 147\% | $=$ | .14\% |  | $\frac{142}{100}=\frac{1}{7}$ |

The student will observe that any per cent is expressed as a riccimal by removing the decimal point two places to the left in the number expressing the rate per cent., that is, dividin!! the rate ly 100.

## EXERCISE 57.

What decimals and what common fractions are equival.nt to-
 4. $433 \%, 56+\frac{1}{2} \%, 62$ な $\%, 68 \frac{1}{4} \%$. 5. $811 \%, \quad 8 \frac{1}{3} \%, 169 \%, 147 \%$, $222 \%$. 11. $77 \frac{5}{7} \%, 57+\%, 15 \frac{5}{6} \%, 80 \%$. 6. $412 \%, 588 \%, 66 \frac{3}{3} \%, 83 \frac{1}{4} \%$

204. To change a decimal or a common fraction to an equivalent per cent.
205. Since any per cent. is changed to an equivalent decimal or common fraction by expressing it as so many hundredths, that is by dividing it by 100, it follows that any decimal or common fraction can be changed to an equivalent per cent. by multiplying such decimal or fraction by 100 .

Example 1.-What per cent. is equivalent to 06 ?
Soldtion.

$$
.06=(.06 \times 100) \%=6 \%
$$

Exasple 2.-What per cent. is equivalent to the fraction $\frac{3}{4}$ ?
Solution.

$$
\frac{8}{4}=\left(\frac{8}{7} \times 100\right) \%=75 \%
$$

Note.-A decimal is multiplied by 100 by removing the decimal point two places to the left.

EXERCISE 58.
What per cents. are equivalent to the following fractions?

1. $\frac{1}{2}, \frac{3}{3}, \frac{8}{4}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{3}$.
2. $\frac{1}{3}, \frac{7}{8}, \frac{1}{8}, \frac{2}{6}, \frac{8}{8}, \frac{4}{6}, \frac{1}{8}$.
3. $\frac{8}{8}, \frac{7}{4}, \frac{4}{7}, \frac{4}{7}, \frac{5}{7}, \frac{5}{7}$.
4. $\frac{1}{6}, \frac{9}{6}, \frac{4}{6}, \frac{5}{8}, \frac{7}{6}, \frac{8}{8}, i^{2}$.
5. $\frac{3}{10}, \quad \frac{7}{15}, \quad \frac{9}{15}, \quad \frac{8}{15}, \quad \frac{7}{15}, \frac{8}{15}, \quad \frac{0}{15}$,






What per cents. are equivalent to the following decimals?
11. .3, .7, .5, .03, .07, .05, .003, .007, . $005, .75, .055,006$
12. .035, .064, .09, .01, .001, .3875, .0625, .03125, . 0025 .
13. . $03 \frac{1}{3}, .028 \frac{4}{7}, .00 \frac{1}{3}, .00 \frac{1}{2}, .06 \frac{1}{3}, .000 \frac{1}{6}, .33 \frac{1}{8}, .011 \frac{1}{6}$.
206. To find the value of any per cent. of a number or quantity.

Example.-Find 8\% of 625.
Soldition 1. Operation.
$6.25=1 \%$ (rðo) of 625.
$\frac{8}{50.00}=8 \%$ " "
Solotion 2. Operation. Explanation.
$6258 \%$ of $625=.08$ of (or times) $625=50.00$.
$\frac{.08}{50.00}$
Solution 3. Operation.
Explanation.
앙ㅇ $\times 625=50 \quad 8 \%$ of $625=\frac{\text { I }}{80}$ of $625=50$.
The student should use whichever of the preceding methods gives the shortest solution.

Find-

## EXERCISE 59.

1. $20 \%$
of
$5,25,45,75,125,95$.
2. $25 \%$
of
$4,36,76,96,128,240$.
3. $4 \%$ of $25,75,125,250,300,1000$.
4. $12 \frac{1}{2} \%$ of $64,96,160,320,480.500$.
5. $162 \%$ of $6,36,72,84,132,324$.
6. $8 \frac{1}{3} \%$ of $12,72,60,240,252,372$.
7. $37 \frac{1}{2} \%$ of $80,32,48,75,90,724$.
8. $662 \%$ of $9,27,75,335,47,520$.
9. $64 \%$ of $32,64,256,90,750$.
10. $314 \%$ of $48,80,144,75,380$.
11. $87 \frac{1}{2} \%$ of $16,72,108,356,968$.
12. $22_{9}^{2} \%$ of $27,45, \quad 63, \quad 567,656$.
13. $285 \%$ of $21,35,56,987,770$.
14. $7 \frac{\mathrm{~g}}{15} \%$ of $26.39,78,117,273$.
15. $75 \%$
16. $90 \%$
of $24,32,28,264,760$.
of $70,110,40,350,660$
17. $31 \%$ of $86,475,373,254$.
18. $44 \%$ of $374,228,937,8321$.
19. $50 \%$
20. $125 \%$
21. $8 \%$
22. 6\%
of $\frac{1}{2}, \frac{1}{2}, \frac{2}{6}, \frac{8}{12}, \frac{7}{15}$.
of $\$ 7.50, \$ 375,436$ bushels, 328 tons.
of $37 \frac{1}{2}, 62 \frac{1}{2}, 87 \frac{1}{3}, 6 \frac{1}{2} 4 \frac{1}{6}, 33 \frac{1}{3}$.
0 it 350 yds., 450 men, 375 lbs., 580 oz .

## 207. Given the value of any per cent. of a number, to find the number.

Example.-24 is 8\% of what number?

## Solution 1

Operation. .08) 24 (300.

## Explanation.

The question is $08 \times$ what number $=24$. If 24 is the product of two factors, one of which is .08 , the other factor may be found by dividing 24 by .08 .
Solution 2.
Operation.
If $8 \%$ of the number $=\mathbf{2 4}$
$\frac{24}{8} \times 100=300$. then $1 \% " "=\frac{1}{8}$ of $24=3$

$$
" 100 \% \text { " } "=100 \times 3=300
$$

Solution 3.

## Explanation.

The question is $\frac{8}{100}$ of what number
Operation. $\quad=24$. If 24 is composed of two fac$24 \times 180=300$. tors, one of which is $\frac{9}{100}$, the other factor may be found by dividing 24 by 180.

## EXERCISE 60.

Find the numbers of which-

1. 60 is $4 \%, \quad 3 \%, \quad 2 \%, 5 \%, 6 \%$.
2. 96 is $20 \%, \quad 25 \%, \quad 50 \%, \quad 75 \%, \quad 90 \%$.
3. 640 is $125 \%, 150 \%, \quad 225 \%, 160 \%, \quad 80 \%$.
4. 32 is $8 \frac{1}{3} \% \quad 9, \frac{1}{1 H} \%, \quad 79.9 \% \quad 7 \frac{1}{7} \%, \quad 6 \frac{2}{3} \%, \quad 5 \frac{5}{6} \%$.
5. 320 is $16 \frac{2}{3} \%, 35 \%, \quad 444 \%, 14 \frac{2}{7} \%, 12 \frac{1}{3} \%$.
6. 252 is $30 \%, \quad 40 \%, \quad 60 \%, \quad 90 \%, \quad 70 \%$.
7. 105 is $12 \frac{1}{4} \%, \quad 37 \frac{1}{2} \%, 62 \frac{1}{2} \%, \quad 87 \frac{1}{2} \%, \quad 3 \frac{1}{3} \%$.
8. 84 is $41 \frac{3}{3}, \quad 58 \frac{1}{3} \%, \quad 66 \frac{1}{3} \%, \quad 42 \% \%, \quad 15 \frac{5}{6} \%$.
9. 350 is $15 \%, \quad 35 \%, \quad 45 \%, \quad 55 \%, \quad 65 \%$. $85 \%$.
10. 220 is $77 \frac{7}{6} \%, \quad 3 \frac{1}{8} \%, \quad 93 \%, \quad 23 \frac{3}{3} \%, \quad 83 \frac{1}{3} \%$.
11. 48 is $24 \%, 16 \%, 12 \%, 10 \%, 24 \%, 36 \%$.
12. To find what per cent. one number is of another.

Example.-What per cent. of 60 is 15 ?

## Explanation.

Solution 1.
$\frac{15}{8}=\left(\frac{15}{8} \times 100\right) \%=25 \%$.
As 15 is 88 of 60 , and as the frac. tion 88 expressed as \% is ( $18 \frac{8}{8} \times 100$ ) \% $=25 \%$ Art. 205 , it follows that 15 is $25 \%$ of 60 .

Solution 2.
.6) 15 ( 25
$1 \% \times 25=25 \%$.

Solution 3.
60 ) $15(.25$
$.25=25 \%$.

Explanation.
$11 \%$ of $60=.6$, 15 is 25 times .6 and therefore 25 times $1 \%$ of 60 .

## Explanation.

The question is $60 \times$ what $\%$ ? $-\quad=15$. If 15 is the product of two factors, one of which is 60 , the other factor can be fonnd by dividing 15 by 60. $15 \div 60=.25$, and $.25=25 \%$.

## EXERCISE 61.

1. What \% is 30 of 60 ? 12 of 48 ? 15 of $45 ? 7$ of 35 ? 9 of 63 ?
2. What \% of 12 is $2 ? 36$ is 16 ? 35 is 28 ? 49 is $21 ? 75$ is 50 ?
3. What \% of 10 is 1 ? 5 ? 10 ? 20 ? 30 ? 40 ? 50 ? 60 ? $70 ? 80$ ?
4. What $\%$ of 50 is 9 ? 12? 15 ? 18 ? 30 ? 45 ? 5C? 100 ? 125 ? 300 ?
5. What \% of 200 is 25 ? 75 ? 125 ? 260? 12 $\frac{1}{2}$ ? $87 \frac{1}{2}$ ? $16 \frac{\mathrm{~g}}{}$ ? $62 \frac{1}{3}$ ?
6. What $\%$ is $26 \frac{1}{4}, 29$ a $, 33 \frac{1}{4}, 36 \frac{9}{4}, 17 \frac{9}{7}, 29 \frac{7}{8}$ of 175 ?
7. What $\%$ is $49.5,56.25,58.50,63,14.3$ of 225 ?
8. What $\%$ is $.024 \frac{4}{4}, .4 \frac{2}{3}, .06 \frac{8}{8}, .09 \frac{1}{6}, .25$ of $2 \frac{1}{2}$ ?
9. What $\%$ of 1 is $\frac{1}{80}$ ? $\frac{2}{25}$ ? $\frac{1}{10}$ ? 2 ? 条? $\frac{7}{8}$ ? $1 \frac{1}{8}$ ?
10. What \% of $18.79,187 \frac{?}{10}, 281.85,319.43,394.59$ of .1879 ?
11. To find a number, which, if increased or diminished by a certain per cent. of itself, will be equal to a given number.

Exampler 1.-What number increased by $25 \%$ of itself will equal

Solution 1.
$125 \%$ of required number $=300$
Therefore the " " $=\frac{10}{120} \times 300=240$.
See Art. 207.

Solotion 2.
8 of required number $=300$
Therefore the " " $\begin{aligned} & =\frac{4}{6} \times 300=240 .\end{aligned}$

Explanation.
If any number bn $i \cdot$. oreased by $25 \%$ of tself the result will be $(100 \%$ $+25 \%=12 . \%$ of the original number.

## Explanation.

$25 \%=\frac{1}{4}$. A number increased by $\frac{1}{4}$ of itself will be equal to $\frac{5}{4}$ of itself, that is $\left(\frac{4}{4}+\frac{1}{6}\right)$ of itself.

Example 2.-What number deoreased by $20 \%$ of itself will equal 360 ?

Explanation.
If any number be decreased by $20 \%$ of itself
$80 \%$ of required number $=360$ the result will be ( $100 \%$ $-20 \%)=80 \%$ of the original number.

Explanation.
$20 \%=\frac{1}{6} . \quad$ A number decreased by $\frac{1}{8}$ of itself will be $\left(\frac{\pi}{6}-\frac{1}{8}\right)=\frac{4}{6}$ of itself.

## EXERCISE 62.

What number increased by-

1. $10 \%$ of itself equals 110 ?
2. $75 \%$ of itself equals $\$ 420$ ?
3. $62 \frac{1}{2} \%$ of itself equals $\$ 89.37 \frac{1}{2}$ ?
4. $21 \frac{1}{4} \%$ of itself equals $\$ 32.56 \frac{7}{3}$ ?
5. $83 \frac{1}{3} \%$ of itself equals $\$ 87.12$ ?
6. $15 \%$ of itself equuls 345 ?
7. $36 \%$ of itself equals 238 A .?
8. $100 \%$ of itself equals 84.6 cwt ?
9. $6 \%$ of itself equals 1272?
10. $22 \%$ of itself equals $\$ 549$ ?
11. $\quad 2 \%$ of itself equals $\$ 9.06$ ?
12. $\frac{8}{8} \%$ of itself equals $\$ 81.72$ ?

What number diminished by-
13. $65 \%$ of itself equals $\$ 2,590$ ?
14. $50 \%$ of itself equals 28.5 feet?
15. $162 \%$ of itself equals 1,035 miles ?
16. $4 \%$ of itself equals $\$ 465.60$ ?
17. $37 \frac{1}{2} \%$ of itself equals $\$ 203.37 \frac{1}{\mathrm{~g}}$ ?
18. $5 \%$ of itself equals $\$ 6.65$ ?
19. $20 \%$ of itself equals 80 ?
20. $9 \%$ of itself equals $9 \frac{1}{10}$ ?
21. $87 \frac{1}{2} \%$ of itself equals 10 ?
22. $5 \frac{5}{8} \%$ of itself equals $95_{18}^{\frac{5}{8}}$ ?
23. $\quad \$ \%$ of itself equals 67.95 ?
24. $8 \%$ of itself equals 216.38 ?

## PROFIT AND LOSS.

210. Profit and Loss are commercial terms used to express gain or loss in business transactions.
211. Gains and losses are usually estimated at some rate per cent. of the cost of the goods incluling the expenses.
212. To find the Gain, Loss, or Selling Price, the cost and the rate per cent. of gain or loss being given.

Exasple 1.-A merohant sold oloth which cost $\$ 1.75$ per yard, so as to gain $8 \%$ in selling. What was the gain and selling price?
Solution.
Cost $=\$ 1.75$
Gain $=8 \%$ of $\$ 1.75=\frac{.14}{} \quad$ Art. 206.
Selling price . .

Example 2:-Goods whioh cost $\$ 2.40$ are sold at a loss of $5 \%$. Find the loss and the selling price.

Solution.
Cost $\quad=\$ 2.40$
Loss $=5 \%$ of $\$ 2.40=\frac{12}{} \quad$ Art. 206.
Selling price.
213. To find the Cost Price, the Selling Price, and the rate per cent. of gain or loss being given.

Example 1.-By selling goods for \$132, I gain $10 \%$. What is the cost price ?

Solution.

```
\(\begin{aligned} 100 \% \text { Cost price } & =\text { Cost price } \\ 10 \% \text { " } \quad & =\text { Gain } \\ \therefore 110 \% \text { Cost price } & =\text { Gelling price } \\ \therefore 110 \% \text { Cost price } & =\$ 132 \\ \therefore \quad \text { Cost price } & =\text { fig of } \$ 132=\$ 120 . \text { Art. } 207 .\end{aligned}\)
```

Eximple 2.-I find that by selling an artiole for $\$ 1.80$ I lose $10 \%$. What is the cost price?
solution.

| $100 \%$ Cost price | $=$ Cost price |
| ---: | :--- |
| $10 \%$ " " | $=$ Loss |
| $\therefore 90 \%$ Cost price | $=$ Selling price |
| $\therefore \quad 90 \%$ Cost price | $=\$ 1.80$ |
| $\therefore \quad$ Cost price | $=\frac{1 \% 0}{80}$ of $\$ 1.80=\$ 2.00$. Art. 207. |

214. To find the Cost Price, the Gain or Loss and the rate per cent. of gain or loss being given.

Example 1.-By selling a farm at a gain of $20 \%$, I realized a profit of $\$ 850$. Find the cost of farm.

Solution.
$20 \%$ Cost of farm $=\$ 850$
$\therefore$ Cost of farm $=\frac{1800}{30} \times \$ 850=\$ 4,250$.
Examples 2.-A yaohtwas sold for $\$ 1,200$ less than it cost, its owner thereby losing $12 \frac{1}{2} \%$ of the cost. What was the cost?

Solution.
$12 \frac{1}{2} \%$ of the cost $=\$ 1200$
$\therefore \quad$ the cost $=\frac{100}{12 \frac{1}{2}} \times 1200=\$ 9600$. Art 207.
215. To find the rate per cent. of gain or lor $s$, the selling price and the cost price being given.

Example 1.-Goods whioh cost $\$ 5$ are sold for $\$ 7$. What is the sain \%?

Explanation.
Solution.
$\left(\frac{3}{5}\right.$ of 100$) \%=40 \%$. Ans.
$\$ 7-\$ 5=\$ 2$ gain. Since the gain $\%$ is computed on the cost, the question becomes, $\$ 2$ is what $\%$ of $\$ 5$.

By Art. $208 \$ 2$ is ( $\frac{2}{6}$ of 100 ) $\%=40 \%$ of $\$ 5$ (the cost).
Example 2.-Goods whioh oost $\$ 7$ are sold for $\$ 5$. What is the loss \% ?

Explanation.

Soldtion.
$(\times 100) \%=284 \%$. Ans.
$\$ 7-\$ 5=\$ 2$ loss. Since the loss $\%$ is computed on the cost, the question becomes, $\$ 2$ is what $\%$ of $\$ 7$.
By Art. 208 \% 2 is ( $\left(\frac{1}{5}\right.$ of 100) $\%=$ $28 \% \%$ of the oost (\$7).

## 216. To find the Selling Price, the Cost Price and the gain or loss per cent. of the selling price being given.

Example 1.-For what must I sell an article which cost $\$ 2.25$ so as to gain $25 \%$ of the selling price?

> SOLETION.

Selling nrice $=100 \%$ Selling prioe .
Gain $=25$ Cost price
$=75 \%$ Selling price.
$\therefore 75 \%$ Selling price $=\$ 2.25$
$\therefore \quad$ Selling price $=180 \times 2.25=\$ 3.00$. Art. 207.
Example 2.-I sold goods which oost $\$ 2.50$, so that 1 lost $25 \%$ of the selling price. Find the selling prioe.

$$
\begin{aligned}
& \text { Solution. } \\
& \text { Selling price }=100 \% \text { Selling price. } \\
& \frac{\text { Loss }}{\text { Cost }}=\frac{25 \%}{125 \%} \text { Selling price. } \\
& \therefore 125 \% \text { of the selling price }=\$ 2.50 \\
& \text { the selling price }=128 \text { of } 2.50=\$ 2.00 . \text { Art. } 207 .
\end{aligned}
$$

## EXERCISE 68.

Find gain or loss and selling price-

Cost.

1. $\$ 8.00$,
2. $\$ 3.60$
3. $\$ 4.20$,
4. $\$ 5.60$,
5. $\$ 13.20$,
gann \%
$20 \%$.
$10 \%$.
$15 \%$.
$12 \frac{1}{2} \%$.
$40 \%$.
cost.
6. $\$ 15.60$,
7. $\$ 14.75$,
8. $\$ 13.60$,
9. $\$ 10.80$,
10. $\$ 4.50$,
seleino pritar.

Loss \%
37굴 \%.
$4 \%$.
$627 \%$
$16 \frac{3}{3} \%$.
$33 \frac{1}{3} \%$.
Find cost price-
SELLING PRICE.
11. $\$ 7.50$,
12. $\$ 3.90$
13. $\$ 4.59$,
14. $\$ 5.50$,
15. $\$ 4.56$

GAN \%
$50 \%$.
$30 \%$.
$28 \%$.
$22 \frac{3}{8} \%$.
$14 \%$
16. 4.75
17. $\$ 5.64$
18. $\$ 12.60$
19. $\$ 24.30$
20. $\$ 5.61$.

Loss \%.
$5 \%$.
$60 \%$
$429 \%$.
$35 \%$.
$8 \frac{1}{6} \%$

Find gain or loss \%-

| bellino price. | cost. | selling pricre | совт. |
| :---: | :---: | :---: | :---: |
| 21. $\$ 10.00$, | \$8.00. | 26. \$10.40, | \$8.00. |
| 22. \$7.00, | \$5.00. | 27. $\$ 6.50$, | \$7.50. |
| 23. 84.80, | \$4.00. | 28. \$13.50, | \$15.00. |
| 24. \$3.60, | \$4.00. | 29. \$10.60, | \$12.00. |
| 25. \$7.50, | \$5.50. |  |  |

## Find cost-

| OAIN. | gain \%. |  | Loss. | LOSS \%. |
| :---: | :---: | :---: | :---: | :---: |
| \$3.00, | $10 \%$. | 34. | \$2.50, | $30 \%$. |
| 60c., | 121 \% | 3 5. | \$4.80, | $25 \%$. |
| 371 ${ }^{1} 0$. | 16. | 36. | \$1.20, | $81 \%$. |
| \$5.60, | $40 \%$. | 37. | \$3.00, | $64 \%$. |

38. Goods which cost $\$ 2.40$ were sold so as to gain $25 \%$ of the werfing price. Find the selling price.
39. An article which oost $\$ 3.50$ was sold so that $12 \frac{1}{2} \%$ of the proceeds were lost. Find the selling price of the article.
40. What is the selling price of a horse which cost $\$ 125$, ald whioh was sold so as to gain $164 \%$ of the proceeds.

## TRADE DISCOUNT.

217. It is customary for merchants and manufacturers to have fixed price lists of their goods, and when the market varies, instead of changing the fixed price they change the rate of discount.
218. Trade Discount is a percentage deducted from the face of bills, the list prices of goods, or from the amount of a debt without regard to time, and is expressed by the term per cent. off.
219. Thus $20 \%$ off, means a deduction of $20 \%$ from the nominal or asking price. 20 and $5 \%$ off, means a discount of $20 \%$, and then $5 \%$ from the remainder, etc.

The result is not affected by the order in which the discounts are taken.
220. Dealers usually announce their terms upon their bill heads thus, Terms 3 months, or 30 days less $5 \%$, meaning that a credit of 3 months is given, but if the bill be paid within 30 days a discount of $5 \%$ will be allowed.
221. Goods are marked by wholesale dealers or jobbers at a rate \% above, which will allow a certain per cent. of discount from the list or marked price, and still realize a margin of gain.
292. The net price of goods is the list price less the trade discount.

## 223. To find the net price, the list price and discounts

Example.-Goods are invoiced at 20 , with discounts of $\mathbf{2 5}, 10$, and $5 \%$ off. Find cost of goods ?

Solution.

| $\$ 640$ |  |
| ---: | :--- |
| $\frac{160}{\$ 1 s 0}$ | $=25 \%$ of $\$ c: 10$ |
| $\frac{48}{\$ 132}$ | $=10 \%$ of $\$ 4: 30$ |
| $\frac{21.60}{}=5 \%$ of $\$ 432$ |  |
| $\$ 410.40$ | $=$ Net price. |

224. To find the single discount equivalent to two or more discounts.

Example.-Find the direot discount equal to two successive dis. counts of $2 \mathrm{O} \%$ and $10 \%$.

Solution.

| Set list pries | $=\$ 100$ |
| ---: | :--- |
| 1st Discount | $=-\frac{20}{80}=20 \%$ of $\$ 100$ |
| 2nd Discomit | $=-8=10 \%$ of $\$ 80$ |
| Net price | $=\$ 7 \%$. |
| Total discount on $\$ 100$ | $=\$ 100-\$ 72=\$ 28$ |
| $\therefore$ discount | $=28 \%$. |

225. From similar examples we derive the following rule to find a single discount equal to two successive dis. counts.

RULE.
From the sum of the discounts subtract $\frac{1}{10} \sigma$ of their product.
226. Then in the above example the discount $=20+$ $10-\frac{20 \times 10}{100}=28 \%$.

When a third discount is given, combine it with the result obtained from the other two.

Thus, if discounts of $\mathbf{2 0}, 10$ and $5 \%$ off are given.
From the preceding illustration, $20 \%$ and $10 \%$ are equal to a single discount of $28 \%$, combining $28 \%$ and $5 \%$ we get a discount of $28+5-\frac{5 \times 28}{100}=31 \frac{3}{8} \%$, the single discount equal to the discounts of 20,10 and $5 \%$ off.

## 297. To mark goods so that a given per cent. may be deducted and leave a given per cent. profit. .

Example.-At what price must I mark an artiole which cost $\$ 4.00$ so that, after dedncting $20 \%$, I may still have a profit of 25 ?

Solution.
Selling price $=\$ 4.00+25 \%$ of $\$ 4.00=\$ 5.00$,
and $20 \%$ less than the marked price $=$ Selling price $\$ 5.00$ $\therefore 80 \%$ of marked price $=5.00$
$\therefore \quad$ marked price $=1800 \times 5.00=\$ 6.25$.

## EXERCISE 64.

Find cash price of -
list price. trade discount.

1. $\$ 360, \quad 5$ and $20 \%$ off.
2. \$475, 30 and $5 \%$ off.
3. $\$ 800,20$ and $10 \%$ off.
4. $\$ 750,10$ and $8 \%$ off.
5. $\$ 1600,40$ and $20 \%$ off.
6. $\$ 1750,25$ and $10 \%$ off.
7. $\$ 1840,30$ and $\frac{1}{4} \%$ off.
8. $\$ 3200,40$ and $\frac{1}{8} \%$ off.
list phice.
9. $\$ 360.60$,
10. $\$ 2142.45$,
11. $\$ 402.18$,
12. $\$ 675.36$,
13. $\$ 474.25$
14. 8396.60
15. $\$ 4362.50$
16. $\$ 3169.20$,

## TRADE DISCOUNT.

10,5 , and $3 \%$ off. 5, $2 \frac{1}{2}$, and $\frac{1}{8} \%$ off. 20,5 , and $2 \frac{1}{2} \%$ off. $10,8 \frac{1}{3}$, and $\frac{1}{4} \%$ off. 40,10 , and $5 \%$ off. 50,30 , and $1 \%$ off. 20,10 , and $3 \frac{1}{2} \%$ off $33 \frac{1}{3}, 20$, and $10 \%$ off.
What direct discounts are equal to discounts-
17. $5 \%$ and $20 \%$; $30 \%$ and $5 \% ; 20 \%$ and $10 \% ; 10 \%$ and $5 \%$.
18. $40 \%$ and $20 \%$; $25 \%$ and $10 \%$; $30 \%$ and $\downarrow \%$; $40 \%$ and $\frac{1}{8} \%$.
19. $10 \%, 5 \%$ and $3 \%$; $50 \%, 10 \%$ and $5 \%$; $40 \%, 20 \%$ and $10 \%$.
$20.10 \%, 10 \%$ and $10 \%$; $20 \%, 10 \%$ and $5 \%$; $10 \%, 5 \%$ and $5 \%$. 21. $10 \%, 8 \frac{3}{3} \%$ and $\frac{1}{2} \%$; $33 \frac{1}{4} \%, 20 \%$ and $8 \frac{3}{3} \%$; $5 \%, 2 \frac{1}{2} \%$ and $\frac{1}{5} \%$.
22. What is the difference on a bill of $\$ 425$ between a discount of $50 \%$ and a discount of $30 \%$ and $20 \%$ ?
23. A bookseller wishes to mark a book which cost $\$ 2.00$ that he may allow a discount of $25 \%$ and still make a profit $0 \leqslant 20 \%$. What must be the marked price?
24. If the list price of certain goods is $\$ 12$ per gross, what will I gain or lose by buying of Mr. A., whose discounts are $25 \%$ and $10 \%$, instead of from Mr. B., whose discounts are 20,10 and $5 \%$ off?
25. For what must I mark goods which cost $\$ 3.60$, so that I may allow discounts of 20 and $10 \%$ off, and still have a profit of $25 \%$ ?
26. A bookseller wishes to mark up the price of a book which he now sells for $\$ 1.70$, so that he can deduct $15 \%$ and yet receive the present price. What must be the marked price.

## miscellaneous exercise 65.

## I.

1. A man having 1,000 bushels of apples, sold $5 \%$ of them at $\$ 1.25$ per bushel ; $8 \%$ of the remainder at $\$ 1$ per bushel ; $50 \%$ of what was then left at 75 c . per bushel, and the rest at 60c. per bushel, thus receiving $10 \%$ more than he paid; how much did he pay for the whole quantity?
2. Mr. Brooks bought a farm, which was in very poor condition, for $\$ 1,586$ : and, after two years of careful cultivation, which paid for itself with some improvements, he sold it for $65 \%$ more than he paid for it. What did he sell
3. The number of inmates in a workhouse 5 years ago was 110 ; this number has since increased $180 \%$. How many inmates are there now?
4. A merchant bought goods for $\$ 297.70$, and paid an additional sum equal to $7 \%$ of the purchase price for cartage, freight, etc. What must he sell them for to gain $40 \%$ on the whole cost?
5. In a mixture of alcohol and water $85 \%$ is alcohol. How many gills of alcohol in 3 gallons of the mixture, and how many gills of water?
6. 560 bushels of wheat, bought at $\$ 1.10$ per bushel, were sold at a profit of $10 \%$. What did the wheat sell for?
7. Bought a bill of goods amounting to $\$ 875.50$, from which was deducted $5 \%$. What was the percentage allowed, and the amount paid ?
8. Having $\$ 10,720$. I invested $25 \%$ of it in land, and $12 \frac{1}{2} \%$ of the remainder in fencing it. What remained?
9. Two men engaged in trade, each with $\$ 3,540$. One of them gained $33 \frac{1}{3}$ of his capital, and the other gained $60 \%$. How much more did the one gain than the other?
10. A little boy who has 8 apples gives $25 \%$ of thom to his brother, $12 \frac{1}{2} \%$ to his sister, and $50 \%$ to his mother. What per cent. and how many has he left?
11. Charles sold his sled, which had cost him $\$ 1.75$, at $20 \%$ below cost. How much did he get for it ?
12. A lot of damaged calicoes are to be sold at $75 \%$ below the marked price. What prices must be asked for those that are marked $8 \mathrm{c} ., 10 \mathrm{c} ., 12 \frac{1}{2} \mathrm{c} ., 16 \mathrm{c} ., 20 \mathrm{c} ., 30 \mathrm{c}$. ?
13. A grain dealer bought wheat for $\$ 9,38 \pm$, and sold it at a gain of $4 \frac{1}{2} \%$. What did he receive for it ?
14. If a man owes $\$ 2,500$, and agrees to pay it in 4 instalments, the first to be $50 \%$ of the whole. the second $25 \%$, the third $15 \%$, the fourth $10 \%$. What will each instahent be?

## II.

1. A merchant uwes $\$ 6,500$, and his property is worth only $\$ 5,425$. What per cent. of his debt can he pay?
2. A man shipped 3,800 barrels of flour to England, andduring a storm 19 barrels were thrown overboard. What per cent. was lost?
3. If I have $\$ 374.50$ in currency, how much gold can ${ }^{*}$ buy when it sells at a premium of $7 \%$ ?
4. The population of a certain village increased in 5 years from 6,000 to 7,800 . What was the average rate of increase per year?
5. A man bought 350 acres of land, at $\$ 40$ an acre, and sold part of it for $\$ 2,240$ at the same rate. What per cent. of the land did he sell ?
6. An agent received $\$ 67.50$ for collecting $\$ 4,500$. What per cent. was his commissi in?
7. Bought sugar for $\$ 150$ and sold it for $\$ 167.50$. What per cent. was the gain?
8. A merchant owes $\$ 8,250$, his assets are $\$ 3,240$. What per cent. of his debts can he pay?
9. Sold 7 acres of land for what the whole cost. What was the per cent. gain ?
10. What per cent. of 365 days are 30 days?
11. Bought a number of eggs, and sold 11 for the money paid for 18. What per cent. was the gain?
12. A regiment went into battle with 600 men , and came out with 320 . What per cent. were lost?
13. Of 4,000 acres of land, I sell 140 acres. What per cent. do I retain?
14. A grocer sold from a hogshead containing 600 pounds of sugar, $\frac{1}{4}$ of it at one time, and $\frac{1}{8}$ of the remainder at another time. What per cent. of the whole remained ?

## III.

1. A merchant owes $\$ 15,120$, and his assets are $\$ 9,828$. What per cent. of his debts can he pay?
2. If $\$ 52.50$ is paid for the use of $\$ 750$, 1 year, what is the rate per cent. if $\$ 56.70$ is paid for the use of $\$ 1,260$ ?
3. A man shipped 2,600 bushels of grain from Chicago, and 455 bushels were thrown overboard during a gale. What was the rate per cent. of his loss?
4. One number is $6 \%$ of another. What per cent. is the latter number of the former?
5. My furniture is worth $\$ 7,200$, which is $90 \%$ of the value of my lot ; and the value of the lot is $33 \frac{1}{3} \%$ of that of my house. How much are lot, house, and furniture together worth?
6. A gentleman who had a yearly income of $\$ 2,000$ for four years, spent $\$ 1,800$ the first year, $\$ 1,500$ the second, $\$ 1,200$ the third, and $\$ 2,260$ the fourth. What per cent. of his income did he save during the four years?
7. A person expended $16 \%$ of all he was worth in buying $20 \%$ of the stock of a mining company. If the entire stock of the company sold for $\$ 100,000$, how much was the person worth?
8. A merchant, embarking in two speculations, in the first made $£ 379$ s. 3 d., which was $4 \%$ of his investment; in the second he lost $£ 2716 s .8 d$., which was $5 \%$ of his investment. How much had he invested in both enterprises?
9. A.'s yearly income, which is $7 \%$ of $\$ 27,000$, is $150 \%$ of B.'s income. If B. receives an income of $10 \%$ annually from his property, how much is he worth ?
10. A leap year is what per cent. of a common year?
11. C. from an income of $\$ 5,340$, spends $\$ 4,966.20 ; D$. from an income of $\$ 2,790.40$, spends $\$ 2,650.88$; E. on anincome of $\$ 1,559.50$, saves as much per cent. as the rate per cent. that $C$. saves, exceeds the rate per cent. that $D$. saves. How much does E. save?
12. What is the cost of a house which sells at a loss of $7 \frac{1}{2} \%$, the selling price being $\$ 11,500$ ?
13. A merchant owes $\$ 12,575$, and his assets are $\$ 7,500$. What per cent. can he pay?
14. Sold two city lots at $\$ 1,500$ each; on one I made $15 \%$, on the other I lost $15 \%$. What did I gain or lose ?

## IV.

1. Adding to a certain number $11 \%$ of itself, we have 109.335. What would we get, if we subtracted from the same number $11 \%$ of itself?
2. In a certain nursery, $15 \%$ of the trees are pear trees, $1 \%$ cherry trees, $4 \%$ plum trees, and the rest, numbering 480, are apple trees. How many trees in all, and how many pear, cherry, and plum trees does the nursery contain?
3. P. having lost $20 \%$ of his capital, was worth exactiy as much as Q., who had just gained $12 \%$ on his capital. Q.'s capital was originally $\$ 15,000$. How much was P.'s?
4. A railway company sold $12 \%$ of its land, and then mortgaged $5 \%$ of what was left. It then had 250,800 acres unencumbered. How many acres had it originally?
5. What number, increased by $2 \frac{1}{2} \%$ of itself, equals $12 \frac{1}{2}$. diminished by $33 \frac{1}{2} \%$ of itself?
6. What fraction, increased by $21 \%$ of itself, equals $\frac{121}{1} \frac{1}{6}$ ?
7. 240 is $33 \frac{1}{3} \%$ more than what number?
8. A collector who has $8 \%$ commission, pays $\$ 534.75$ for a bill of $\$ 775$. What amount of the bill does he collect?
9. What is $\frac{7}{8} \%$ of $\$ 1,728$ ?
10. What is $94 \%$ of 275 miles?
11. What is the difference between $5 \frac{1}{3} \%$ of $\$ 800$, and $6 \frac{1}{2} \%$ of $\$ 1,050$ ?
12. $25 \%$ of 800 bushels is $2 \frac{1}{2} \%$ of how many bushels?
13. Sold 105 barrels of potatoes, which was $35 \%$ of all I raised. How many did I raise?
14. A farmer sold 7.5 acres of land, which was $15 \%$ of all he owned. How many acres did he own?

## V.

1. What per cent. of a number is $25 \%$ of $\frac{3}{4}$ of it?
2. $\frac{1}{2} \%$ of 1,258 is $\frac{1}{4} \%$ of what number ?
3. What per cent. of a number is $20 \%$ of $\frac{8}{8}$ of it?
4. A man spends $\$ 825.60$, which is $33 \frac{1}{3} \%$ of his salary. How much is his salary?
5. A man drew out $9 \%$ of his bank deposit to pay a debt of $\$ 243.72$. How much had he in bank?
6. If a man, owning $40 \%$ of an iron foundry, sells $25 \%$ of his share for $\$ 1,246.50$, what is the value of the whole foundry?
7. A farmer sold $\mathbf{3 , 1 5 0}$ bushels of grain and had $\mathbf{3 0 \%}$ of his entire crop left. What was his entire crop?
8. If a man owning $45 \%$ of a steamboat sells $16 \frac{2}{3} \%$ of his share for $\$ 5,860$, what is the value of the whole boat?
9. The assets of a business man are $\$ 135,700$, which sum is $43 \%$ of his debts. What is his indebtedness?
10. A fruit dealer sold a lot of oranges for $\$ 337.50$, which allowed him a profit of $12 \frac{1}{2} \%$. What did he pay for them?
11. A city lot was sold for $\$ 25,500$, the gain on the cost being $325 \%$. What was the cost?
12. A grocer sold 300 buithels of potatoes for $\$ 285$, which was $16 \frac{2}{3} \%$ less than he had paid for them. How much did they cost him per bushel ?
13. A. sold goods at a gain of $18 \%$. His profit was \$29.70. How much did he sell them for?
14. By selling a lot of goods for $\$ 380$, I gain 3 times the per cent. that would be gained by selling them for $\$ 340$. What per cent. is gained in the latter case? $\quad(\$ 380-\$ 340=$ 2 times the gain.)
15. In the schools of a village yesterday there were 1,285 pupils present, which was $95 \%$ of the whole number belonging. How many belonged to the schools?

## VI.

1. Sold a horse for $\$ 340$, which was $15 \%$ less than his value. What was his value?
2. A man having increased his bank deposit $40 \%$, it amounted to $\$ 840$. How much had be at first?
3. My income this year is $\$ 2,232$, which is $7 \%$ less than it was last year. How much was it last year?
4. A man sold 160 acres from his farm, which was $12 \frac{1}{2} \%$ less than the number of acres he retained. How many acres in his farm ?
5. The price of a single ticket from Princeton to Woodstock is 30 c., but 20 coupon tickets can be bought for $\$ 5$. What per cent. is saved by buying coupon tickets? What per cent. is lost by buying single tickets?
6. $10 \%$ of a flock of sheep were killed by dogs; $6 \frac{2}{3} \%$ of the rest were lost; $33 \frac{1}{3} \%$ of the remaining number were sold, and 28 then remained. What was the original number?
7. At harvest time a farmer sold 60 bushels of wheat, which was $25 \%$ of the quantity he sent to mill, and what he sent to mill was $40 \%$ of what he kept over till the next spring. How many bushels had he at first?
8. When a merchant sold his goods for $\$ 261$, he gained twice as much as he would have lost had he sold them for \$207. What was his gain per cent.? (How many times the loss is the difference between $\$ 261$ and $\$ 207$ ?)
9. A grocer sold butter at $12 \%$ profit. Had he sold it for 2 c . more per pound, he would bave gained $20 \%$. What did 50 pounds cost him?
10. A boy buys an old pair of skates for 50c. and sells them for 25 c . He then buys a pair for 25 c . which he sells for 50c. What per cent. did he lose on the first pair, what per cent. did he gain on the second?
11. If a dealer buys a hat for $\$ 3$, and sells it for $\$ 4$, what per cent. does he gain? If he buys it for $\$ 4$ and sells it for $\$ 3$, what per cent. does he lose?
12. One hundred pounds of beef were sold for $\$ 6$, having been bought at $4 \mathrm{c} . \mathrm{a} \mathrm{lb}$. What per cent. profit?
13. A retail dealer in boots and shoes sold 50 pairs of boots for $\$ 300$, they cost him $\$ 5$ a pair. What rate per cent. did he gain?
14. A merchant bought goods for $\$ 500$. What per cent. would he gain by selling them for $\$ 530$ ? For $\$ 525$ ? For $\$ 550$ ? For $\$ 540$ ? For $\$ 560$ ? For $\$ 575$ ? For $\$ 600$ ? For $\$ 1,500$ ?

## VII.

1. William buys a penknife for 20c. and sells it to James for 25 c . What per cent. does William gain, and what per cent does James lose?
2. If the $\mathbf{2 5}$ minutes of school time given to recesses are $8 \frac{1}{3} \%$ of the daily session, how many hours in the session?
3. If a book is marked to be sold at $25 \%$ above cost, but it is sold at $20 \%$ below the marked price, what was the gain or loss per cent.?

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4. If 80 pounds of coffee are exchanged for 120 pounds of sugar, what per cent. is the coffee worth per pound more than the sugar?
5. What per cent. do I gain by selling an article for $\$ 3$ for which I paid $\$ 2.25$ ? What per cent. do I lose by buying an article for $\$ 3$ and selling it for $\$ 2.25$ ?
6. A drover sold a horse for $\$ 226$, and thus gained $25 \%$. What did he pay for him?
7. Bought 300 long tons coal at $\$ 3.75$ a ton and sold it at $\$ 460$ a short ton. What is the per cent. profit?
8. Bought a barrel of syrup for $\$ 20$. What must I charge a gallon in order to gain $20 \%$ on the whole?
9. Sold 25 tons of coal at $\$ 5.64$ per ton, and made $\$ 62$. What did the coul cost, and what per cent. was the profit?
10. A quarter section of land was sold for $\$ 4,563$, which was $8 \%$ less than cost. What was the cost per acre?
11. If $15 \%$ of what is received for goods is gain, what is the gain per cent.?
12. Sold goods for $\$ 29,900$ and made $15 \%$ after deducting $5 \%$ for cash. What was the cost and the marked price?
13. A dealer sold 1,600 bbls. beef for $\$ 24,000$, which was a loss of $25 \%$. What did the whole cost, and what did he get a barrel ?
14. A builder sold a house for $\$ 8,250$, which was $12 \%$ more than it cost him. What was the cost?

## VIII.

1. A merchant sold cloth at $\$ 3$ per yard, and thereby gained $20 \%$. What per cent. would he have gained if he had sold the cloth at $\$ 3.75$ ner yard?
2. A person at two auction 'sales bought 1,170 books, buying at the second $30 \%$ of the number purchased at the first. How many did he buy at the second?
3. What number, diminished by $25 \%$ of half of itself, equals 12,600?
4. Mr. A. paid three times as much for his horse as for his gig. If he had paid $15 \%$ more for his gig, and $8 \frac{1}{3} \%$ less for his horse, they would together have cost $\$ 468$. How much did he give for each ?
5. A merchant in 1872 made $3 \%$ on his capital, and in $1878,8 \frac{1}{3} \%$ on his capital thus increased. Capital and profit then equaled $\$ 22,351$. What was his original capital? What was his profit in 1878 ?
6. A. offered B. $\$ 6,045$ for a farm; which B. declined, as it was $2 \frac{1}{2} \%$ less than it cost him. B. afterward sold it for $\$ 6,855$. Did he gain or lose on the farm, and what per cent?
7. A statue was sold for $\$ 753.75$, which was $\frac{1}{2}$ of $1 \%$ more than it cost. Had it been sold for $\$ 700$, what per cent. would have been gained or lost ?
8. Sold goods for $\$ 4,026.75$, at a loss of $3 \frac{1}{4} \%$. What would they have had to sell for to yield a profit of $\mathbf{3} \frac{1}{4} \%$ ?
9. B. bought a horse for $\$ 200$, and sold it at $20 \%$ advance to C., who sold it to D. at a loss of $10 \%$, and D. sold it to E . for $5 \%$ more than it cost him. If E . had paid $\$ 21.60$ less for the horse, would D. have lost or gained, and what per cent.?
10. K. sold X. some goods for $\$ 394$, at a loss of $1 \frac{1}{2} \%$. X. sold them to Y., at a profit of $1 \frac{1}{2} \%$. Did they cost Y. more or less than K., and how much?
11. Sold 40 bush. 1 pk . of apples for $\$ 31.39 \frac{1}{2}$, clearing $4 \%$. What would they have had to sell for per busbel, to yield a profit of $9 \%$ ?
12. A drover laid out equal sums for sheep, cows, and hogs. On the hogs he lost $7 \%$, on the sheep he made $15 \%$, and on the cows he lost $1 \%$. If he received for the whole $\$ 1,535$, and bought 25 hogs, what did each hog cost him? What did all the sheep cost him ?
13. Jones offered his house for $15 \%$ more than it cost him, but afterward sold it for $\$ 15,525$, which was $10 \%$ less than his original offer. How much did his honse cost him?
14. The population of a certain city in 1871 increased $4 \%$ on that of 1870 ; in 1872 it increased $5 \%$ on that of 1871 ; in 1873 it increased $6 \%$ on that of 1872 , and amounted to $1,389,024$. What was its population in 1870 ?
15. If a certain number be increased by $16 \frac{2}{3} \%$ of itself, and the sum is diminished by $50 \%$ of itself, $10 \%$ of the remainder is 14. Required, the number.

## IX.

1. If a merchant who buys goods on 6 months' credit is allowed a deduction of $5 \%$ for paying his bill within 30 days, what can he save on a bill of $\$ 560$ ? How much on $\$ 3,650$ ?
2. If a man fails to pay his water rent until he is charged $12 \%$ for delay, how much will he lose if his water rate is $\$ 18.75$ ?
3. If $1 \%$ per month, counting from the cime of payment, is allowed on all taxes paid before July 1st, and $1 \%$ per month charged on all taxes remaining unpaid thereafter, how much more does A. pay than B., if B. pays his taxes February 1st, and A. pays his taxes November 1st, their tax-bills each being $\$ 180$ ?
4. What is the net amount of a bill of goods, the list price of which is $\$ 435$, sold $5 \%$ off for cash, trade discount $8 \%$ ?
5. Sold borks on 8 mo. amounting to $\$ 854.75$ at a discount of $12 \frac{1}{2} \%$ from retail price, and $10 \%$ off for cash. What is the net value of the bill?
6. The gross amount of a bill is $\$ 236.37$; the rates of discount are $15 \%$ and $8 \%$. What is the net amount?
7. Find a direct discount equal to a discount of $12 \frac{1}{2} \%$ and $8 \%$.
8. What direct discount is equal to a discount of $25 \%$ and $17 \%$ ?
9. On a bill of $\$ 625$, what is the difference between a discount of $30 \%$ and a discount of $25 \%$ and $5 \%$ ?
10. Bought books at a discount of $20 \%$ on the retail price, and sold them at the retail price. What per cent. did I gain?
11. What per cent. would I gain at a discount of $33 \frac{1}{3} \%$ ?
12. With a trade discount of $8 \%$ and $5 \%$ for cash, goods were sold for $\$ 825$ at a profit of $15 \%$. What was the cost ?
13. A bookseller wishes to mark up the price of a book which he now sells for $\$ 2$, so that he can deduct $15 \%$ and yet receive the present price. What must be the marked price?
14. A merchant sells clotbs for $\$ 268$ by which he gains $23 \%$. How must he mark them so that he may deduct 4 . and make the same profit?
15. Bought diamonds at $\$ 920$. How must I mark the price that after abating $5 \%$ the profit may be $25 \%$ ?
16. What : "s s be the price of an article from which you deduct $20 \%$ insa mis 20 sents?

## MARKING GOODS.

228. It is customary in mercantile houses to use a private mark, which is placed on the goods to denote their cost and selling price. A word or phrase containing ten different letters is taken, the letters of which are used to indicate the ten digits. For example, the word "Sutherland" is selected; then the letters represent the figures as follows:

$$
\begin{array}{lllllllllll}
S & u & t & e & r & 1 & \pi & n & d \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0
\end{array}
$$

If it is required to mark $\$ 1.75$, it is done thus, Sle; 47 hl ; 90 nd.
229. The following are among the words and phrases that may be used : Haliburton, Cbelnsford, Cum herland, Blacksmith, Now be smart, Strike hard, Cash profit, Black horse, etc.
230. It sometimes happens that the selling price contains three figures, while the cost price contains but two. To prevent this difference from being noticed, the letter denoting the cipher is prefixed to the cost price. For instance, the cost price was 85 cents, it would be marked dae; and the selling price, sue; thus each price would be indicated by three letters.
231. An extra letter, called a "Repeater," is used to prevent the repetition of a figure. Instead of writing see for 1.55, which would show that the two right hand figures were alike, and thus aid in giving a clue to the key-word, some additional letter is selected for a repeater,-y, for instance-and then the price would be written sey; 387 would be written tyl.

23:2. Arbitrary characters are frequently used instead of letters, thus :
2333. Fractions may be desiguated by additional letters or characters; thus $g$ may represent $\frac{1}{2} ; \mathbf{f}, \frac{1}{3}$, stc.

## EXERCISE 66.

1. What is the profit and what is the selling price of the following:

| Cost $\$ 1.10$, | Freight $10 \%$, | Gain $20 \%$ | Selling price. |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $"$ | 1.50, | $"$ | $8 \%$ | $"$ | $1(1 \%$ |
| $"$ | 4.50, | $"$ | $10 \%$ | $"$ | $25 \%$ |
| $"$ | 1.75, |  |  | 4 | $20 \%$ |
| $"$ | 2.50, | $"$ | $10 \%$ | $"$ | $30 \%$. |

Mark the selling price of the above, using the word "Chelmsford."
2. Knowing a merchant's profit on cloth to be $25 \%$ and his key-word Haliburton, what letters would be used in indicating the cost price per yard, the selling price being hrb?
3. What letters would be used in marking the selling price of single articles which were bought at $\$ 3.50$ per dozen, and sold at a profit of $20 \%$, using the word "Cumberland."
4. A publisher marks each copy of $\varepsilon$ work skd. What mark should he put on each so as to be able to allow the trade $30 \%$ discount? (Key-word "Strike hard.")
5. What would be the selling price of imported articles bought at $\$ 4.60$, on which the charges were $50 \%$ of the purchase price, if they were sold at $40 \%$ profit on total cost? How would the selling price be marked if the phrase "Cash profit" be used, with y as a repeater?
6. A merchant using as his key-word "Chelmsford," indicates the cost per yard of a piece of silk, thus cod. What mark will indicate the selling price so that he may sell it at $10 \%$ less than the marked selling price and still make $20 \%$ profit?
7. A man wishing to sell a web of silk asks $40 \%$ per yard more than it cost him, but he finally sold it at $10 \%$ less than his asking price, and made a profit of 52 cents a yard: Using the phrase "Now ti: smart" indicats the cost price, the asking price and the selling price.

## COUMMISSION AND BROKERAGE.

E.34. Commission is an allowance made to agents or comraission merchants for transacting business. It is usually calculated at so much per cent. on the amount of money received for sales or expended in purchase.
235. A Commission Merchant or Agent is a person engaged in the buying and selling of goods for another, as the purchase or sale of merchandise or real estate, collecting or investing money, etc.
2336. An Agent's Commission for sale is computed on the gross proceeds, and for purchase on the prime cost.
237. A Broker is one who effects purchases or sales in the interest of buyer or seller.

A broker does not generally take possession of the article bought or so'. He usually contracts in the name of the party from whom he receives his compensation.
238. Brokerage is the compensation paid to a Broker.
239. The Principal is the person for whom the business -s transacted.
240. A Consignment is property received to be sold on commis-ion.
241. The Consignor or Shipper is the person who ships the goods to be sold.
242. The Consignee is the person to whom the goods are sent to be sold.
248. A Guarantee is the charge made for assuming the risk of loss from non-payinent by the purchaser.
244. The Gross Proceeds of a sale or collection is the total amount received by the agent before deducting commission or other charges.
245. The Net Proceeds is what remains after all charges have been deducted.
246. An Account Sales is a statement in detail rendered by the Consignee to the Consignor, showing the sales of the consignment, all charges or expenses attending the same, and the net proceeds.
247. An Account Purchase is a detailed statement made by the purchasing agent to his principal, showing the quantity, grade and price of goods bouglit on his account, all expenses incident to the purchase, and the gross amount of the purchase.
248. To find the Commission on a sale of goods, the gross proceeds, and per cent. of commission being given.

Example.-How muoh commission will be due án agent who sold a house and lot for $\$ 6,000$, and charged $3 \%$ for his services?

Solution.
$-\$ 6,000 \times .03=\$ 180$. Ans.
249. To find the Commission on the purchase of goods when the prime cost and the per cent. of commission are given.

Example.-My agent in London bought for me 350 yards of silk, which cost $\$ 2.50$ a yard, at a commission of $20 \%$. Find the amount of his commission.

Soletion.
$\$ 2.50 \times 350=\$ 875.10=$ Cost of silk.

$$
\$ 875.00 \times .02=\$ 17.50 . \text { Ans. }
$$

250. To find the amount of a Sale when the amount of commission and the per cent. of commission are given.

EXAMPLE．－Received $\mathfrak{i} 245$ for selling a shipment of goods at a com． mission of $5 \%$ ．How much did I receive for the goods？

Solution．
$5 \%$ of amount reseived $=\quad \$ 245$

$100 \% \quad " \quad=\frac{\$ 245 \times 100}{j}$
$\therefore$ Amount receivad for goods $=\$ 4,900$ ．Ans．
251．To find the Commission on an investment when the amount sent the agent includes both the amount to be invested and the agent＇s commission．

Eximple 1．－A commission merchant received a check for $\$ 5,150$ ， to be invested in tea after deducting his commission of $3 \%$ ．How much money did he invest，and what was the amount of his commission？

## Solution．

The amount to be invested is $100 \%$ of itself，the commis－ sion is $3 \%$ of amount invested．

| $\therefore 103 \%$ of amount to be invested | $=$ | $\$ 5,150$ |
| ---: | :---: | :---: |
| $1 \%$ | $"$ |  |
|  |  |  |

$100 \% \quad$＂$\quad=\frac{\$ 5,150 \times 100}{103}$
$\therefore$ The amount to be invested $=\$ 5,000$ ．
Commission，$\$ 5,150-\$ 5,000=\$ 150$
Example 2，－Having sold a consignment of cotton on $3 \%$ commis－ sion，I am instructed to invest the proceeds in city property，receiving a commission of $2 \%$ on the price paid for the property．My whole com． mission is $\$ 200$ ．Find the amount for which the cotton sold．

## Solution 1.

Take the amount for which the cotton sold as a unit
then $\frac{{ }^{3}}{}{ }^{3}$ of the amount of sales $=$ first commission．
風等＂＂
1st Com．after deduoting
( n every $\$ 102$ of amount left after deducting 1st Com., the agert receives $\$ 2$ for his second commission.
$\therefore$ The agent's commission $=\gamma^{1} \mathrm{r}$ of the amount to be invested.
Hence tr of $\frac{97}{100}=5 \frac{97}{970}$ of sales $=$ second commission.
$\therefore$ ( $\frac{3}{100}+{ }^{51 \%} \frac{1}{5}$ ) of sales $=$ Agent's total commission. $\frac{5}{102}$ of sales $=8200$.

Sales $=\$ 4,080 . \quad$ Ans.
Solution 2.
$3 \%+2 \%=5 \%$.
If the $5 \%$ commission had been charged on the whole amount of sales, the commission would have been $2 \%$ of $\$ 200=\$ 4$ more, i.c., the entire commission would have been $\$ 200+\$ 4=\$ 204=5 \%$ of sales.

$$
\begin{aligned}
\therefore 5 \% \text { of sales } & =\$ 204 \\
\text { Sales } & =\$ 4,080 . \quad \text { Ans. }
\end{aligned}
$$

Again: If the 5\% commission had been taken on the amount of purchase money, the entire commission would have been $3 \%$ of $\$ 200=\$ 6$ less than it was, i.e., the entire commission would have been $\$ 200-\$ 6=\$ 194$ $=5 \%$ of purchase money.

$$
\begin{aligned}
\therefore 5 \% \text { of purchase money } & =\$ 194 . \\
\text { Purchase money } & =\$ 3,880 .
\end{aligned}
$$

Solution 3.
It will be found that on every $\$ 102$ from sale there is $\$ 5$ entire commission. Suppose we allow for commission for selling, $\$ 2$ of the $\$ 102$, leaving $\$ 100$. For commission for purchasing, $\$ 3$ of the $\$ 100$, leaving $\$ 97$. The entire commission would be $\$ 5$.

In the former case we have charged $2 \%$ of $\$ 3=6$ cents too much. But in the latter case we have charged $3 \%$ of $\$ 2=6$ cents too little, i.e., the excess equale the deficit, and we have still $\$ 5$ entire commission.

$$
\begin{aligned}
\text { Then, } \frac{5}{\delta^{2}} \text { of sales } & =\$ 200 . \\
\text { Sales } & =\$ 4,080
\end{aligned}
$$

> COMMISSIUN .AND BROKERAGE.

$$
\begin{aligned}
& \text { Solorion } 4 . \\
& \text { Let } 100 \%=\text { Sale. } \\
& 3 \% \text { of sale }=\text { First Commission. } \\
& \text { rit of } 97 \%=14 \% \% \text { of sale }=\text { Second } \quad \text {. } \\
& 3 \% \text { of sale }+18 \% \% \text { of sale }=\text { Total } \\
& 4 \frac{185}{8} \% \text { of sale }=\$ 200 . \\
& 100 \% \text { of sale }=\$ 4,080 .
\end{aligned}
$$

From the foregoing solutions we obtain the following
If commission on sale is $4 \%$ and on purchase $3 \%$, the entire commission $=\frac{4+3}{100+3}$, i.e., $\frac{7}{103}$ of sale money, and $\frac{4+3}{100-4}$, i.e., $\frac{7}{96}$ of purchase money.

And generally if we have $m$ per cent. on sales, and $n$ per cent. on purchase, the entire commission $=\frac{m+n}{100+n}$ of sale money, and $\frac{m+n}{100-m}$ of purchase money.

## EXERCISE 67.

Find the commission-

1. On the sale of merchandise for $\$ 8,150$, at $2 \frac{1}{2} \%$.
2. On the sale of a mill for $\$ 8,450$, at $2 \frac{3}{4} \%$.
3. On the sale of 375 bbl . of flour, at $\$ 6.25 \mathrm{a} \mathrm{bbl}$., at $3 \frac{1}{4} \%$.
4. On the purchase of a farm for $\$ 12,370$, at $2 \frac{1}{4} \%$.
5. On the sale of 255 bales of cotton, each weighing 520 lb ., at $14 \frac{3}{4}$ cents a lb., at $1 \frac{1}{2} \%$.
Find the rate of commission-
6. When $\$ 78$ is paid for selling goods for $\$ 5,200$.
7. When $\$ 84$ is paid for collecting a debt of $\$ 4,800$.
8. When $\$ 189$ is paid for selling a farm for $\$ 7,560$.

Find the amount of sales-
9. When a commission of $\$ 360$ is charged, at $2 \frac{1}{2} \%$.
10. When the brokerage charged is $\$ 48$, at $\frac{1}{4} \%$.
11. When the agent charges $\$ 59.60$ commission at $1 \frac{3}{4} \%$.
12. When a commission of $\$ 57.82 \frac{1}{2}$ is charged, at $4 \frac{1}{2} \%$.
13. When the net proceeds are $\$ 38.70$, commission $3 \frac{1}{4} \%$.
14. When the net proceeds are $\$ 2,444.55$, brokerage $\frac{3}{4} \%$.

Find the amount to be invested and commission-
15. If $\$ 4,455$ is remitted, deduciing $1 \frac{1}{4} \%$ commission.
16. If $\$ 9,909.40$ is remitted, deducting $3 \frac{1}{2} \%$ commission.
17. If $\$ 6,500$ is received, and $1 \frac{1}{4} \%$ brokerage deducted.
18. If $\$ 2,846.25$ is remitted, deducting $3 \frac{1}{2} \%$ commission.
19. What weight of wool, at 52 cents a lb ., can be bought for $\$ 1,109.60$, after deducting a commission of $4 \%$.
20. Sent to my agent in Hamilton $\$ 1,508.80$, to invest in flour at $\$ 5.75$ a bbl., after deducting his commission at $2 \frac{1}{2} \%$. How mauy bbls. can he buy?
21. An agent sold a house and lot for $\$ 8,500$, and charged $3 \%$ for his services. How much was his commission?
22. If an agent's charges are $2 \%$, how much commission will he earn by selling property valued at $\$ 10,500$ ?
23. A real estate agent sold a farm of 75 acres at $\$ 85$ an acre, on a commission of $2 \%$; and the stock and implements on the farm for $\$ 3,250$, on a commission of $3 \%$. Find the total amount of his commission.
24. An agent received $\$ 612.50$ for selling grain, on a commission of $1 \frac{1}{4} \%$. What was the amount of his sales?
25. A collector's charges for collecting a note amounted to $\$ 14.10$, at a commission of $5 \%$. What sum was collected ?
26. An agent receives $\$ 12,504.20$ to invest in wheat, on a commission of $3 \%$. Find the amount of money invested in wheat.
27. How many lbs. of wool at 27c. a lb., can be bought ${ }^{\circ}$ for $\$ 8,424$, if the agent is allowed $4 \%$ for purchasing?
28. Paid an agent a commission of $\$ 133.12 \frac{1}{2}$, at $2 \frac{1}{2} \%$, to purchase wheat at \$1.87, a bushel. How many bushels did he buy, and what was the amount of his bill?
29. Paid a broker $\$ 38.10$ for buying 120 shares of railroad stock, at $95 \frac{1}{4} \%$ a share. What was the rate of his brokerage?
30. An agent in Montreal remitted $\$ 8,795.66$ on a sale of 540 barrels of flour, at $\$ 7.25$ a barrel. What was his rate of commission?
81. A real estate broker charges $\$ 182.34$ for investing $\$ 12,156$ in a factory. What was his rate of brokerage?
32. I sell through my broker 7 tons of Brazil nuts at $\$ 7.50$ per cwt. How much do I receive if the broker charges $1 \%$ for selling?
33. Sent $\$ 414$ to an agent in Toronto to be invested in prints, at $12 \frac{1}{2}$ cents a yard, after taking out his commission of $3 \frac{1}{2} \%$. How many yards can he purchase?
34. My attorney collected $80 \%$ of a note for $\$ 1,200$, and charged $5 \frac{1}{2} \%$ commission. What amount should he pay me?
35. An agent sells a consignment of flour for $\$ 7.532 .80$ and then purchases 1,840 bushels of wheat, at $\$ 1.40$ a bushcl, his commission being $2 \frac{1}{4} \%$. What sum must he remit to the consignor?
36. An auctioneer, who charged $2 \%$ for selling, found his commission for the sale of a certain house just sufficient to pay for a Cyclopædia in 16 volumes, worth $\$ 5.50$ a volume. What did the house sell for?
37. A commission merchant received a remittance of $\$ 1,000$ to be invested in sugar, after deducting his commission of $2 \%$. The sugar costing $8 \frac{5}{8} c$. a lb., how many - pounds could he buy?
38. How much does a house bring, for which the owner receives $\$ 24,255,1 \%$ of the purchase money having been first deducted for the agent who sold it?
39. How many barrels of flour, at $\$ 5.60$, can be bought for $\$ 2,545.20$, a commission of $1 \%$ for purchasing having also to be paid out of this sum?
40. A commission merchant sold 500 lbs. of butter at 18c. per lb., and invested the proceeds in oats at 42c. a bushel. He charged $4 \frac{1}{2} \%$ for selling and $1 \frac{1}{2} \%$ for buying. What was his total commission, and how many bushels of oats did he buy?
41. A fruit broker sold $\$ 680$ worth of apples, and after deducting $5 \%$ commission and $20 \%$ for freight and other charges, invested the balance in oranges. How much did he invest in oranges if he charged $2 \%$ for buying?
42. My agent in Brantford sells for me a quantity of dry goods on commission at $6 \%$. How much must be sold that my agent can buy flour with the proceeds to the value of $\$ 5,400$, after retaining his commission, for buying, of $2 \frac{1}{2}$ ?
43. Sold goods at $2 \frac{1}{2} \%$ commission, which I invested in sugars, and sold them at a profit of $15 \%$, realizing a gain of $\$ 240$. How much commission did I receive, and how much did the goods sell for?
44. A merchant purchased an invoice of grain, which, including a commission of $1 \frac{1}{2} \%$, cost $\$ 5,050.65$. The freight charges were $\$ 15.35$. He sold the grain at a profit of $15 \%$ on the entire cost, and invested the proceeds in sugar, which he sold at a profit of $5 \%$. What was the amount paid for commission? What the entire cost of the grain, and how much were his profits?
45. A commission merchant bought goods for which he received $5 \%$ commission for buying and $\$ 63.25$ for charges. The total cost of goods, commission, and charges was $\$ 3,250$. What was paid for the goods?
46. An agent bought coffee at $\frac{5}{8} \%$ brokerage, and received \$350. He afterwards sold the coffee at a profit to his principal of $\$ 5,160$, after deducting $1 \frac{1}{2} \%$ commission on the amount for which it was sold. How much was his commission?
47. I received from Day \& Son, of Chicago, a ship load of corn, which I sold for 60c. per bushel, on a commission of $4 \%$; and, by the shipper's instructions, invested the net proceeds in barley, at 75c. per bushel, charging $5 \%$ for buying; my total commission was $\$ 1,350$. How many bushels of corn did Day \& Son ship, and how many bushels of barley should they receive?
48. A Buffalc brewer remitted $\$ 21,500$ to a Toronto commission merchant, with instructions to invest $40 \%$ of it in barley, and the remainder, less all charges, in hops. The agent paid 60c. per bushel for barley, and 20c. per pound for hops, charging $2 \%$ for buying the barley, $3 \%$ for buying the hops, and $5 \%$ for guaranteeing the quality of each purchase. If his incidental charges were $\$ 187.50$, what quantity of each product did be buy, and what was the amount of his commission ?
49. A Toronto factor received from Cincinatti a consignment of corn, which he sold at 75 c . per bushel, on a commission of $5 \%$; and by instructions of the consignor invested the net proceeds in wool, at 20c. per pound, charging $2 \%$ for buying, and $3 \%$ additional for guaranty of quality. If the total amount of the agent's commission and guaranty was $\$ 1,640$, now many bushels of corn were received?
50. My Memphis agent sends me an account purchase of 350 bales of cotton, averaging 480 lbs . each, bought at 15c. per lb., on a commission of $2 \frac{1}{2} \%$. His charges, other than for commission, were: freisht advanced, $\$ 126.50$,
cartage, $\$ 53.25$, and insurance, $\$ 13.75$. What sum should I remit to pay the account?
51. An agent sells a consignment of goods for $\$ 2,100$. He pays $\$ 33.50$ for freight, and, resorving his commission remits $\$ 2,024.77$. Find the rate of his commission.
52. An agent sells 1,100 barrels of flour, at $\$ 4.50$ a barrel, and charges $2 \frac{1}{2} \%$ commission, He invests the proceeds in stecl, at $1 \frac{1}{2}$ c. a lb ., charging $1 \frac{1}{2} \%$ commission. What is his entire commission, and low many tons of steel ( $2,240 \mathrm{lbs}$. to a ton) does he buy?
53. A commission merchant has consigned to him 5,000 lbs. of cotton, which he sells at 14 c . a lb, and charges $2 \%$ commission. With the net proceeds he buys cotton cloth, at 10 c. a yard, charging $1 \frac{1}{2} \%$ commission for buying. How many yards of cloth does he buy?
54. A commission merchant has consigned to him 5,000 barrels of flour, which he sells at $\$ 5.50$ a barrel, and charges $2 \frac{1}{2} \%$ commission; the expenses for freight, etc., amounted to $\$ 250$. With the net proceeds he buys sugar, at $6 \frac{1}{4} \mathrm{c}$. a lb., charging $2 \frac{1}{2} \%$ commission for buying. How much sugar does he buy, and what is the amount of his commissions?

## CUSTOM HOUSE BUSINESS.

252. Duties or Customs are taxes levied by the Dominion Government on imported goods, for revenue purposes and for the protection of home industry.
253. Duties are of two kinds, ad valorem and specific.
254. An Ad Valorem Duty is a certain per cent. assessed or levied on the actual cost of the goods in the country from which they are imported, as shown by the income.
255. A Specific Duty is a tax assessed at a certain sum per ton, foot, yard, gallon, or other weight or measure, without reference to the value.

Note.-Upon certain goods bnth specific and ad valorem duties are levied.

25\%. A Custom House is an office established by the Dominion Government for the transaction of business relating to duties, and for the entrance and clearance of vessels.
257. Ports of Entry are places at which custom houses are established; and it is lawful to introduce merchandise into a country only at these places.
258. A Clearance is a certificate given by the Collector of a Port after the requirements of law have been complied with, that the vessel has been properly entered.
259. An Invoice or Manifest is a statement made by the seller or shipper, giving a description of the same, showing actual cost, or value of such merehandise; showing also, marks, numbers, quantity, charges, and other details.
230. All invoices are maide out in the weights and measures of the country from which the importation is made.

26i1. All invoices of merchandise subject to an ad valorem duty, are made out in the currency of the country from whieh the importation is made.
2832. When the value of the foreign currency is fixed by law the vulue is to be taken in estimating the duties; when the value is not fixed by law, the invoice must be ncompanied by a consular certificate showing its value.
2833. A Tariff is a schedule of goods, and the rates of import duties imposed by law on the same.
208. The Free List includes classes of goods that are exempt from duty.
985. Tonnage is $a$ tax levied upon $a$ vessel independent of its cargo, for the privilege of coming into a port of entry.

26f5. Allowances are deductions made in estimating Specific Duties, ind are distinguished as Leakage, Breakage, Draft, T'are, etc.
2067. Leakage, determined by gauging, is an allowance for the waste of liquids imported in barrels or casks.
268. Breakage is an allowance made for loss of liquids imported in bottles.
269. Draft is an allowance made for waste or impurities.
270. Tare is an allowance made for the box, bag, crate, or other covering of the goods.
271. Gross weight is the weight before any allowances are made.
289. Net weight is the weight after all allowances are made.
2783. Drawback.-When distilled spirits, fermented liquors, and tolnaces upon which an excise duty has beon paid, and foreign merchandise upon which an import duty has been paid, are exported, the tax or duty upon the same is refunded. Such return of the tax or duty is called a Drawback.
274. An Appraiser is an officer of the customs who examines imported merchandise and determines the dutiable value and the rate of duty of the same.
285. A Bonded Warehouse is a plate for the storage of merchandise on which the duties have not been paid.
Notes 1. -The law requires an entry for goods to be mado within three days after arrival. If no cutry is made the goods may be conveyed to the Quoen's Warehouse, and may be sold after thirty days for duties.
2. In ease goods are warehoused, that is, clained by the importer and transferred by propor entry to some bonded warehouse, they cannot be sold within two yeurs from the date of suoh transfer.
3. When goods arrive at a Port of Entry and are unelaimed, they are taken to the Queen's Warehouse, and are subject to sale by auction within thirty days. The proceeds of the sale, after paying all expenses, are paid over to the Reeeiver Genoral, and may be recovered by proving ownership.
276. A Custom House Broker is a person who makes entries, secures permits, and transacts other business at Custom Houses for merchants. He is familiar with the tariff laws, and the details and regulations of Custom House business. He usually acts under the power of an attorney.

## 27\%. To find Specific Duty.

Example.-What is the specifio duty on 150 easks of alcolol, of ti0 gallons each, at 20c. per gallon; leakage, $5 \%$ ?

Solution.
$60 \mathrm{gal} . \times 150=9000 \mathrm{gal} .=$ Gross quantity.
Less $5 \%$ for leakage $=.450 \mathrm{gal}$.

$$
\begin{array}{r}
8550 \text { gal. }=\text { Net quantity } . \\
\hline 8550=\begin{array}{l}
\$ 1710.00
\end{array}=\text { Specitic duty } .
\end{array}
$$

## 278. To find Ad Valorem Duty.

Example.-What is the ad valorem duty, at $40 \%$, on 120 boxes of brass rivets, at 50 lb . per box, invoiced at 9c. a lb., tare 8 lb . per box? Solution
$50 \mathrm{lb} . \times 120=6000 \mathrm{lb} .=$ Gross weight.
$8 \mathrm{lb} . \times 120=960 \mathrm{lb} .=$ Tare.
$5040 \mathrm{lb} .=$ Net weight.
9 c. $\times 5040=\$ 453.60=$ Net value.
$\$ 453.60 \times .40=\$ 181.44=$ Duty.

## EXERCISE 68 .

## Find the specific duty-

1. On 50 hhd . of sugar, each weighing 480 lb ., at $1 \frac{1}{4} \mathrm{c}$ a lb., tare 78 lb . per hhd.
2. On 360 doz. bottles of porter, duty 50 c . a doz., breakage $10 \%$.
3. On 250 chests of tea, each 75 lb ., invoiced at 15 c . a lh., duty $3 \frac{1}{2} \mathrm{c}$. a lb.
4. On 120 bags of coffee, gross weight 148 lb . each, allowing $3 \%$ tare, at $3 \frac{1}{2} \mathrm{c}$. a lb .
5. On 60 packages of figs, each 16 lb . weight, at $2 \frac{1}{2} \mathrm{c}$. per lb., tare $5 \%$.
6. On $897,120 \mathrm{lb}$. of bituminous coal at 75 c . per ton.
7. On an importation of 200 boxes of plate glass, each box containing 20 plates $24 \times 48 \mathrm{in}$. in size, at 25 c. per sq. ft.
8. On 40 doz. bottles of wine, at $\$ 2$ per doz., on allowance of $10 \%$ for breakage.
9. Oa 1,500 doz. empty bottles, breakage $4 \%$, and rate of duty 10c. per doz.
10. On 6 blocks of marble, each 10 ft . long, 3 ft . wide, 2 ft . high, at 65 c . per cu. ft.

Find the ad valorem duty-
11. On 16 tons of steel, invoiced at 18 c . per lb ., at $25 \%$.
12. On 175 boxes of raisins, 18 lb . per box, at $17 \%$.
13. On 650 doz. kid gloves, invoiced at $\$ 6.50$ a doz., at 52 \%.
14. On 600 gal. sperm oil, of 42 gal. each, invoiced at 45c. a gal., at $20 \%$; $3 \frac{1}{2} \%$ being allowed for leakage.
15. What is the duty at $40 \%$ on an invoice of French jewellery, amounting to 8,560 francs ?
16. What is the duty on an invoice of books from Vienna the value of which was 6,429 florins, at $38 \%$.
17. What is the duty on an invoice of linens amounting to $£ 3,256$ sterling at $27 \%$, allowing $\$ 4.866 \frac{1}{2}$ to a pound?
18. Find the duty on an invoice of woollen cloths from Germany valued at 8,437 Reichmarks, at $45 \%$.
19. What is the duty on 1,000 yd. of brussels carpet, 27 in . wide, invoiced at 6s. 9d. per yd. ; duty 44c. per sq. yd. specific, and $35 \%$ ad valorem ?
20. An invoice of woollen cloth, imported from England, was valued at $£ 9566 \mathrm{~s}$. If its weight was 684 lb ., how much was the duty, at 50 c . per lb. specific, at $85 \%$ ad valorem?
21. I imported from the United States 7,240 bush. of corn and $17 \frac{1}{2}$ tons of hay, invoiced at $\$ 9.50$ per ton. What amount of duties had I to pay, at 15 c . per bush. on the corn and $20 \%$ on the hay?
22. The duty, at $19 \%$ on an importation of satin, is $\$ 309.70$. What is the invoice of the goods?
23. How much duty must be paid on an importation of $27,640 \mathrm{lb}$. of wool, invoiced at $£ 1,49710 \mathrm{~s} .4 \mathrm{~d}$., if the rate of duty is 10 c . per lb . specific, and $11 \%$ ad valorem?
24. What is the duty and total cost of 2,500 pieces bleached calico, 33 yd . each in length, and $1 \frac{1}{2} \mathrm{yd}$. wide; price 6d. per yd., duty 4c. per sq. yd., and expenses at Liverpool $£ 6510 \mathrm{~s}$.? What is the amount of a bill of exchange at $\$ 4.87$ to the $£$ to cover the cost ?
25. Find the duty on 50 cases of tobacco, each weighing 60 lb ., and 50,000 Havanna cigars weighing 55 lb , invoiced at $\$ 75$ per M., the duty being 50c. per lb. specific on the tobacco and $\$ 2.50$ per lb. specific on the cigars, and $25 \%$ ad valorem on both.
26. Paid $\$ 22.40$ duty on 100 bbl . of sugar, each weighing 220 lb ., invoiced at 8c. a lb., tare $4 \%$. What was the rate?
27. Required the duty and total cost of 1 case of French silks, value 3,500 francs, duty $50 \%$ ad valorem; 1 case velvets, value 28,000 francs, duty $50 \%$, expenses, cartage, shipping, etc., 625 francs, and commission $2 \frac{1}{2} \%$.
28. A merchant imported 80 pieces three-ply carpet, 75 sq. yd. in a piece, and paid $\$ 2,591.84$ duty, at 16 c . per sq. yd., and $30 \%$ ad valorem. What was the invoice price per yd., in sterling money?
29. A merchant imported 800 pieces of three-ply carpet, each piece containing 75 sq . yd., invoiced at 3s. 6d. per sq. yd., upon which he paid a duty of 17c. per sq. yd. specific, and $35 \%$ ad valorem. What was the total amount of duty paid?
30. On 40 cases of tobacco, each weighing 65 lb ., and 20,000 Havana cigers, weighing 200 lb ., invoiced at $\$ 45$ per M., the duty on tobacco being $\$ .30$ per lb., and on cigars $\$ 2 \frac{1}{2}$ per lb. specific, and $40 \%$ ad valorem.
31. Find the duty at $33 \%$ ad valorem, on 1 case of shawls valued at £42 5s., 1 case of linens at $£ 8710 \mathrm{~s}$., duty $40 \%$;

1 case prints at $£ 85 \mathrm{~s}$., duty $20 \%$; incidental expenses $£ 15 \mathrm{~s}$., commission $2 \frac{1}{2} \%$; consul's fees 15 s . What is the total cost in Canadian money?
32. W. A. Murray \& Co. imported 10 cases of shawls, averaging 216 lb . a case, invoiced at 24884.10 francs, the duty being $\$ .50 \mathrm{a} \mathrm{lb}$., and $35 \%$ ad valorem. The invoice was paid with a bill of exchange, bought at 5.16 francs to the dollar. What was the duty, and what did the shawls cost, after paying other charges to the amount of $\$ 75.80$ ?

## INSURANCE.

a89. Insurance is a contract by which one party engages for a stipulated consideration to make up a loss which another may sustain. It is distinguisied as Property Insurance, Life Insurance, Accident Insurance, and Health Insurance.
se80. An Insurance Company is a company or corporation which insures against loss or damage.
281. Insurance companies may be classified according to principles of organization as follows:-1. Stock; 2. Mutual; 8. Mixed, or Stock and Mutual.
282. A Stock Insurance Company is one in which the capital stock is owned by the members of the company called stockholders. They alone share the profits and are liable for the losses.

The business of a stock company is managed by directors chosen by the siockholders.
283. A Mutual Insurance Company is one in which the persons insured receive a share or division of the profits.
284. Non-participating policies, the holders of waich do not share in the profits or losses, are issued by certain mutual and mixed companies.

## INSURANCE.

295. A Mixed Insurance Company is one which is conducted upon a combination of the stock and mutual plan.
296. The Insurer or Underwriter is the party who assumes the risk, or agrees to indemnify against loss.
297. The Policy is the name applied to the written agreement of contract between the Insurance Comprny (the Insurer or Underwriter) and the party insured.
298. A Valued or Closed Policy is one in which the amount insured is definitely determined at the time the insurance is effected. Houses, furniture, and goods in a store are insured in policies of this kind.
299. An Open Policy is one upon which additional insurances may be entered at any time from port to port, at rates and under conditions agreed upon.
300. The Premium is the amount paid for the insurance. 291. An Insurance Agent is a person who represents one or more Insurance Companies, and acts for them in soliciting business, collecting premiums, adjusting loses, etc. 292. An Insurance Broker is a person who effects insurance for a compensation called brokerage or commis-

## FIRE INSURANCE.

2933. Fire Insurance refers to insurance against loss or damage by fire. Losses may be total or partial.
2934. Fire Insurance Losses are usually adjusted by the insurance company paying the full amount of the loss, provided that such loss does not exceed the sum insured; if the policy, however, contains the "average clause," the payment made is such proportion of the loss as the amount of insurance bears to the total value of the property.
2935. The Term of Insurance is the period of time for which the risk is taken, or the property insured.
2936. Short Rates are certain rates of premium charged by the companies when the term of insurance is less than a year.
2937. In case a policy is terminated at the request of the insured, he is charged the "short rate" premium; if, however, it be terminated at the option of the company, the lower long rate will be charged, and the company refund the premium for the unexpired time of the policy.
2938. To guard against fraud, property is not usually insured for its full value, and no more can be recovered than the amount of actual loss. The party insured must also have an interest in the property insured.
29)9. Dwelling-houses and permanent property, about the value of which opinions differ, and which deteriorate in time, may generally be insured for from one-half to threefourths their estimated value; goods in store, at their cash value.

Insurance companies usually reserve the privilege of rebuilding, replacing, or repairing damaged property.

## MARINE INSURANCE.

300. Marine insurance refers to insurance of ves, els and their cargoes against the dangers of navigation.
301. Inland or Transit Insurance refers to insurance of merchandise while being transported from place to place either by rail or water routes, or both.
302. Marine Insurance losses are adjusted by the insurance company paying only such a proportion of the loss as the sum insured is to the entire value of the vessel.
303. Policies on Cargoes are issued for a certain voyage, and on vessels, for a voyage, or for a specified time. 3104. Salvage is an allowance made to those rendering voluntary aid in saving vessels or cargoes from marine
304. When the insured ships goods, or receives information of goods shipped to him, he must notify the insurance company as soon as he is in receipt of bill of lading or other advice of shipment, that it may be entered on the open policy.
305. Goods at sea may generally be insured from $5 \%$ to $25 \%$ more than their cost or invoice price, in order to cover the expenses of freight, insurance, and a share of the profits.
306. To find the cost of insurance, the amount insured, and per cent. of premium being given.

Example.-A house and its contents are insured for $\$ 8,500$. What is the cost of insurande for one year at $1 \frac{1}{2} \%$ premium?

Solution.

$$
\$ 8,500 \times .015=\$ 127.50
$$

rule.
Multiply the amount of insurance by the rate per cent. of premium, and the product will be the cost of insurance.
308. To find the amount insured, the premium, and the per cent. of premium being given.

Example,-I paid $\$ 170$ to insure a stock of goods for one year at a premium of $2 \%$. For what amount was the policy insured?

Solution.

| $2 \%$ of amount of policy | $=\$ 170$ |
| ---: | :--- |
| $1 \%$ " " | $=170$ |
| $100 \%$ | $=\frac{170 \times 100}{2}$ |
| $\therefore$ Amount of policy | $=\$ 8,500$. Ans. |
| or $\$ 170 \div .02$ | $=\$ 8,500$. Ans. |
| RULE. |  |

Dicide the premium by the rate per cent. of premium, and the quotient will be the amount insured.
309. To find the rate per cent. of premium, the premium and the amount of insurance being given.

Example.-I paid $\$ 85^{\circ}$ premium on a housc insured for $\$ 6,800$. What was the rate per cent. of insurance?

Solution.
Cost of insuring $\$ 6,800$ is $\$ 85$


Divide the promium by the sum insured, and the quotient will be the rate.
310. To find the sum to be insured that will cover both premium and insurance, in case of loss, the value of the property and the rate being given.

Example.-For what amount must property worth $\$ 7,600$ be insured, at $5 \%$, so that in case of loss, both the premium and the value of the goods may bo recovered?
marine insurance.
Solution.
To realize $\$ 95$ we must insure $\$ 100$ (Since 5 is paid in premium)

- 80

95
$\$ 7,600 \quad$ " $\frac{100 \times 7600}{90}=\$ 8,000$. Ans.
$100 \%-5 \%=95 \%$
$\$ 7,600 \div .9 .5=\$ 8,000$. Ans.
rule.
Divide the value of proprrty by $100 \%$, minus the rate of insurance, and the quotient will te the sum insured.

## 311. To estimate proportionate losses.

Example.-A merchant insured $\$ 2,500$ in the Ontario Mutual, $\$ 1,500$ in the Ploenix, and $\$ 3,500$ in the Western. A loss by tire of $\$ 6,000$ occurred. How much should each company pay?

|  |  | Solution. |
| :---: | :---: | :---: |
| $\begin{array}{r} \$ 2,500 \\ 1,500 \end{array}$ | Ontario Mutual. |  |
| 3,500 | Western. |  |
| \$7,500 | = Sum insured |  |
| \$6,000 | $\div 7,500=.80$ |  |
| 2,500 | $\times .80=\$ 2.000$ | $\begin{aligned} & =\text { Rate of loss on \$1. Ex. } \\ & =\text { Share of Oupt } \end{aligned}$ |
| 1,500 | $\times .80=1,200$ | $=\quad$ = of Ontario Mutual. |
| 3,500 | $\times .80=2,800$ | $\begin{array}{lll} = & " & \text { Phœenix. } \\ = & \text { Western. } \end{array}$ |

Divide the loss by the tutul iusurance, the quotient will be the per cent. which each must pay.

## EXERCISE 69.

1. What will it cost to insure a factory worth $\$ 26,000$ at $\frac{4}{3} \%$, and machinery worth $\$ 16,800$ at $\frac{5}{8} \%$, with $\$ 1.50$ for policy?
2. What premium must be paid for insuring $\$ 6,500$ on a store for 3 years at $2 \frac{1}{2} \%$ ?
3. My house cost me $\$ 8,400$. I insured it for $\frac{3}{4}$ of its value, at $\frac{2}{3} \%$ per year. My books and furniture were insured for $\$ 8,000$ at the same rate. What did I pay annually for insurance on both?
4. If $\$ 125$ are paid annually for insuring $\$ 24,000$, what is the rate per cent?
5. Paid $\$ 350$ on a shipment of goots to insuru $\frac{8}{4}$ the value, at $3 \frac{5}{8} \%$. What was the whole value?
6. A house is insured at $\frac{s}{\delta} \%$, and $d_{n}$ the premium is $\$ 93.60$. For how much is it insured?
7. The cargo of steamer Gallion, 'bound for Liverpool. is insured at $\frac{1}{2} \%$. For what sum is it insured, the premium being \$1,500?
8. A manufacturing company paid $\$ 214.80$ premium for insurance on $\frac{3}{4}$ of the cost of its building and machinery, at 60 c per $\$ 100$. What was their cost?
9. A company had $\$ 125$ preminm for insuring property worth $\$ 18,000$. If similar property worth $\$ 45,000$ were insured at the same rate in another company, what would be the premium?
10. A merchant sent a cargo of goods worth $\$ 25,275$ to Canton. What sum must he get insured at $3 \%$, that he may suffer no loss, if the ship is wrecked ?
11. What sum must be insured, at $3 \%$, on a consignment of tea worth $\$ 4,200$, to sover property and premium?
12. A shipowner insures a ship and cargo for $\$ 89,325$, at $4 \frac{1}{2} \%$, the policy covering both property and premium. What is the value of the property?
13. If a warehouse is worth $\$ 266,250$, what sum must be insured, at $2 \%$, to cover the property and premium?
14. The premiums paid for insuring two stores are $\$ 98.24$ and $\$ 146.50$; the rate is $1 \frac{s}{4} \%$. What sum must be insured to cover the property and preminm?
15. The loss by fire on a store and contents was $\$ 4,525$; the property was insured $\$ 2,500$ in Western, $\$ 4,000$ in British American, $\$ 2,000$ in Provincial, and $\$ 3,000$ ir: Royal Canadian. How much should each pay?
16. The loss by fire on a piece of property was $\$ 8,000$, of which $\$ 2,000$ was insured in the Ottawa Agricultural, $\$ 3,000$ in the London Mutual, and $\$ 3,000$ in the Citizen. How much did each company contribute?
17. A block of stores and contents was insured for $\$ 220,000$, and becane damaged by fire and water to the ameant of $\$ 150,000$. Of the risk, $\$ 40,000$ was taken by the Quebec Co , $\$ 65,000$ by the British American, $\$ 35,000$ by the Western, and the remainder was divided equally between the Royal Canadian and the London Mutuai. What was the net loss of each company, if the premium
paid was $1 \frac{7}{8} \%$ ?
18. A man owing $\frac{2}{5}$ of a ship, insured $\frac{2}{3}$ of his interest at $1 \frac{1}{2} \%$, and paid $\$ 91.50$ for premium, and a policy charge of $\$ 1.50$. If the ship becomes damaged to the extent of $\$ 12,000$, how much can be recovered on the policy?
19. For how much must a house worth $\$ 6,000$, and furniture worth $\$ 2,000$, be insured, at $1 \frac{1}{2} \%$, to cover the cost of the policy, which was $\$ 2$, the amount of premium paid, and $\frac{3}{4}$ of the value of the property?
20. A schooner is valued at $\$ 10,500$, and has a cargo of 3,500 barrels of apples, worth $\$ 2.10$ per barrel. What amount of insurance must be obtained, at $2 \frac{1}{8} \%$, to provide, in case of loss, for the value of the property, the premium, and $\$ 5$ additional which the owner paid for survey and
policy?
21. The furniture in my house is estimated at one-half the value of the house. I get both insured for $\$ 7,687.50$ for 5 years, at $2 \frac{1}{2} \%$, and find that in case of total destruc-
tion the face of the policy will be full indemnity for both the property and premium. Find the value of the house.
22. A factory worth $\$ 45,000$ is insured, with its contents, for $\$ 62,500$; $\$ 30,000$ of the insurance is on the building, $\$ 12,500$ on machinery worth $\$ 20,000$, and $\$ 20,000$ on stock worth $\$ 35,000$. A fire occurs by which the building and the machinery are both damaged, each to the amount of $\$ 15,000$, and the stock is entirely destroyed. How much is the claim against the company, if the risk is covered by an "ordinary" policy? How much if the policy contains the "average clau*e?"
23. A merchant, owning a store worth $\$ 12,000$, and goods to the same amount, insures them both for two-thirds of their value, at the rate of 50 c . on $\$ 100$, through a broker, who allows him a discount of $10 \%$ on the premium and retains $5 \%$ liimself. How much does the insurance cost the merchant, what does the broker get, and what is the net premium received by the company?
24. Three companies insure, at $\frac{3}{4}$ of its value, a building worth $\$ 16,000$. The first company takes $\frac{1}{3}$ the risk, at $\frac{3}{4}$ of $1 \%$; the second, $\frac{2}{6}$ of it, at $\frac{7}{8}$ of $1 \%$; and the third, the remainder, at $\frac{3}{4}$ of $1 \%$. Find the total premium?

## TAXES.

312. A Tax is the sum assessed on the person, property or income of an individual for local improvement, payment of officers, support of schools, and other general purposes.
313. A Poll Tax is a certain sum required of each male citizen liable to taxation, without regard to his property.
314. A Proper Tax is a tax assessed on real or personal estate, and is assessed at a given rate per cent. of the valuation, but usually at so many cents on $\$ 100$, or so many mills on \$1.
315. Property is of two kinds,-Real and Personal.
316. Real Property or Real Estate is immovable property, such as lands and houses.
317. Personal Property is movable property, such as merchandise, furniture, ships, cattle, money, mortgages, etc.
318. An Assessor is an officer appointed to estimate the value of property.
319. An Assessment Roll is a list or schedule containing the names of all the persons liable to taxation in the municipality, and the valuation of each person's taxable property. taxes.
320. To find the tax, the sum assessed and the rate of taxation being given.

Example.-The rate of taxation in a certain city was $11 \underline{\underline{c}}$ mills on the dollar. What tax was paid by a person whose property was assessed for $\$ 12,000$ ?

Soldtion.

$$
\therefore \quad \text { On } \$ 18 \text { the tax is } \$ 12000 \stackrel{.001125 .}{.001125} \times 12000=\$ 135 . \text { Ans. }
$$

Multiply the sum assessed by the rate of taxation, and the product will be the tax.
322. To find the rate of taxation, the sum assessed and the tax being given.

Example 1. -In a oertain village a sohool-house is to be built at a cost of $\$ \overline{5}, 725$, to be paid by a tax upon the assessed property valued at $\$ 229,000$. What rate of taxation will cover the cost ?

Solution.
On $\$ 229,000$ there is a tax of $\$ 5,725$.

$$
\therefore \text { " } \$ 1 \quad \text { " } \quad \frac{5,725}{229,000}=2 \frac{10}{2} 0 . \quad \text { Anm. }
$$

role.
Divide the property tax by the sum assessed, and the $q$ wotient is the rate of taxation.

Example 2.-A tax of $\$ 16,230$ is to be assessed upon the village of Caledonia; the valnation of the taxable property is $\$ 800,000$, and there are 115 polls, to be assessed at $\$ 2$ each. What will be the tax on the dollar, and how mach will be the tax of Mr. Scott, whose property is valued at $\$ 12,500$, and who pays for 2 polls ?

Solution.

| $\$ 2 \times 115$ | $=\$ 230$. | . |
| :--- | :--- | :--- |
| Amount of polit tax. |  |  |
| $\$ 16,230-\$ 230$ | $=\$ 16,000$. | " $\quad$ pruperty tax. |
| $\$ 16,000 \div \$ 800,000$ | $=.02$. | Rate of taxation. |
| $\$ 12,500 \times .02$ | $=\$ 250$. | . |
| $\$ 250+\$ 4(2$ polls $)$ | $=\$ 254$. | Mr. Soott's property tax. |
| $\$ \quad$ " total tex. |  |  |

323. To find the sum assessed, the rate of taxation and the tax being given.

Example.-The tax on a certain property was $\$ 96.10$, and the rate of taxation 7a mills on the dollar. For how much was the property assessed?

Solution.
$\$ 0.00775$ is the tax on $\$ 1$

$$
\begin{array}{llll}
\$ 1 & \text { " } & \text { " } & \frac{1}{.00775} \\
\$ 96.10 & \text { " } & \text { " } & \frac{96.10}{.00775}
\end{array}=\$ 12,400 . \text { Ans. }
$$

Roxs.

## Divide the tax by the rate of taxation, and the quotient will

 be the sum assessed.
## 324. To find what sum must be levied on the assessed

 valuation to raise a given net amount.Exampla. - What sum mast be levied to raise $\$ 38,800$ net, allowing $8 \%$ for collection?
or

$$
\$ 1.00-.03=.97
$$

$$
\$ 38,800 \div .97=\$ 40.000
$$

rule.
Subtract the rate allowed from $\$ 1$, and divide the net amount to be raised by the remainder; the quotient will be the sum to be levied.
325. When the rate of taxation is ascertained, for convenience a Tax Table is usually prepared on that basis. The following is based on the rate of 3 mills on the dollar. By its use much labor and time may be saved.

Tax Tablif at Three Mills per Dollar.


> To raise $\$ 97$
> Solution.
> " $\$ 1$
> - must be levied.
> " $\$ 38,800$ " $\frac{100 \times 38,800}{97}$ " " $=\$ 40,000$. Ans.

Example.-Find, from the tax-table, the amount of taxes Mr. A. has to pay on a property assessed at $\$ 2,475$.

Solution.

| Tax on $\$ 2.000$ | $=\$ 6.00$ |  |
| ---: | :--- | ---: | :--- |
| $"$ | 400 | $=1.20$ |
| $"$ | 70 | $=.21$ |
| $"$ | 5 | $=.01 \frac{1}{2}$ |
| $"$ | $\$ 2,475$ | $=\$ 7.42 \frac{1}{2}$. |

## EXERCISE 70.

1. My property is assessed at $\$ 6,400$. At the rate of $3 \frac{1}{2}$ mills on the dollar, how much tax will I be required to pay :
2. What amount of tax must a man pay who is assessed $\$ 12,000$ for real estate and $\$ 4,500$ for personal property, if he pays a rate of $2 \frac{1}{2} \%$ ?
3. At what rate must property, valued at $\$ 1,250,000$, be assessed to raise a tax of $\$ 15,000$ ?
4. What is the assessed value of a property that pays a tax of $\$ 182$, at the rate of $3 \frac{1}{4}$ mills on the dollar?
5. A tax of $\$ 7,380$ was levied upon the taxable property of a county, valued at $\$ 2,460,000$. What was the rate, and what was the tax on a farm assessed at $\$ 4,000$ ?
6. My entire property is worth $\$ 8,000$, and is assessed at $\frac{3}{4}$ of its value, at the rate of $7 \frac{3}{4}$ mills on the dollar. I pay an additional tax for 2 polls, at $\$ 2$ each. What is my total tax?
7. What sum must be assessed on a school-section to build a school-house, at a cost of $\$ 2,730$, and pay $2 \frac{1}{2} \%$ for collection?
8. What amount of tax nust a man pay who is assessed $\$ 10,500$ for real estate, and $\$ 5,000$ for personal property, if he pays $1 \frac{3}{6} \%$ City tax, $\frac{1}{2} \%$ County tax ?

## taXes.

9. A Town-hall, costing $\$ 12,250$, was built by a tax assessed upon the property of the town. The tax rate was 5 mills on the dollar, and the cost of collection $2 \%$. What was the valuation?
10. If my property is valued at $\$ 2,500$, and the rate of taxation for school purposes is 5 mills on the dollar, what does the tuition of each one of my three children cost me if all of them attend the public schools?

From the table find out bow much-
11. Mr. W. H. Hull pays on $\$ 6,000$
12. Mr. M. Howard " " $\$ 5,583$
13. Mr. H. Brierly "c "\$5,854
14. Mr. E. Munroe " " $\$ 10,000$
15. Mr. W. Galer " "\$ 7,584
16. Mr. D. Turnbril " " $\$ 5,821$
17. Make out a tax table, rate 15 mills on the dollar.
18. Allowing $5 \%$ for taxes uncollectable, and $2 \%$ for collection, what sum must be levied that $\$ 50,000$ may be realized for the building of a school-house ?
19. The Council of the Village of Dunville wish to levy a tax which will net them $\$ 18,979$, after paying the expense of collection, which will be $3 \%$. The assessed value of real and personal property is $\$ 1,260,000$, and there are 323 polls, each taxed $\$ 2$. How much will $\$ 1$ be assessed?
20. For the purpose of building a town-hall, a tax of $\$ 15,961.60$ is to be levied on property valued at $\$ 1,856,000$. What will be the tax on Dr. Burns' property, which in valued at $\$ 8,650$ ?
21. A bridge costing $\$ 18,135$ was built by the proceed of a tax levied upon the property of a town, the rate of taxation being 50 c . on $\$ 100$ ( 5 mills on $\$ 1$ ), the cost of collection being $2 \frac{1}{2} \%$. What was the assessed valuation of the property?
22. If the assessed value of the real and personal property of a city is $\$ 80,000,000$, and a special tax is desired for the construction of sewers, what must be the rate of levy to realize $\$ 188,160$ for the purpose, if $2 \%$ be allowed for collection and $4 \%$ of the levy be uncollectable ?
23. If a tax of $\$ 240$ is assessed upon a cotton mily valued at $\$ 48,000$, what is the valuation of a piece of property that pays a tax of $\$ 35.50$, at the same rate?
24. The assessed valuation of the real estate of $a$ county is $\$ 1,910,887$, of the personal property, $\$ 921,073$. The year's estimated expenditure is:-For schcols, $\$ 8,400$; interesi, $\$ 6,850$; highways, $\$ 7,560$; salaries, $\$ 5,150$; and contingent expenses, $\$ 13,675$. What tax must be levied on a dollar to meet expenses and provide a sinking fund of $\$ 7,000$ ?

## MISCELLANEOUS.

## EXERCISE 7.

## I.

1. A conimission merchant, whose rate both for selling and investing is $5 \%$, receives $24,000 \mathrm{lb}$. of pork, worth 6 c . a lb., and $\$ 3,000$ in cash, with instructions to invest in a shipment of cotton. What will be his entire commission?
2. Having sold a consignment of cotton on $3 \%$ commission I am instructed to invest the proceeds in town lots, after deducting my purchase commission of $2 \%$. My total commisaion is $\$ 265$. How much money did I invest in town lots?
3. I send a quantity of goods into the country to be sold by auction, on a commission of $9 \%$. What amount of goods must be sold, that my agent may buy produce with the avails, to the value of $\$ 3,500$, after retaining his purchase commission of $4 \%$ ?
4. A commission merchant sells a consignment of wheat for $\$ 7,240$. He pays $\$ 40$ for freight and storage, and charges a commission of $2 \frac{1}{4} \%$. What are the net proceeds? 5. A merchant buys, through an agent, 480 yds. of carpet at 80 c. per yd., and pays the agent $\frac{3}{4} \%$ commission. The freight amounted to $\$ 1.92$. At whai price per yard must the carpet be sold to realize a pr.jit of $88 \frac{1}{3} \%$ ?
5. I purchased 6,000 bushels of wheat in Winnipeg at 85 c . a bushel, and shipped the same to my agent at Ottawa, who sold it at $\$ 1.10$ per bushel. How much did I make, after paying $\$ 543$ for expenses and a commission of
6. I remit to my agent at Chicago $\$ 84,650$, to purchase flour. After deducting his commission of $1 \frac{1}{2} \%$ and $\$ 20.25$ for other expenses, how many barrels of flour at $\$ 5$ a barrel will the money purchase?
7. A flour merchant in Montreal remitted to his correspondent in Toronto the proceeds of a consignment amounting to $\$ 2,453.75$ per draft, which he purchased at the expense of the consignor, at $1 \frac{1}{2} \%$ premium. What was the amount of the consignment, his commission being $2 \frac{1}{2} \%$ ?
8. Sold 2,978 bushels of wheat at $\$ 1.05$ a bushel; invested the proceeds in sugar, as per order, reserving my commission of $5 \%$ for selling and $1 \frac{1}{2} \%$ for buying, and the expenses of shipping, $\$ 53.37$. How much did I invest in sugur?
9. Sold goods to a certain amount on a commission of $5 \%$, and having remitted the net proceeds to the owner, received for prompt payment $\frac{1}{3} \%$, which amounted to $\$ 16.15$. What was the amount of commission?

## II.

1. My agent bought tea at $\frac{5}{8} \%$ brokerage, and was paid \$450. He afterwards sold the tea at a profit to me of $\$ 6,150$, deducting $1 \frac{1}{2} \%$ commission on the sale. How much was his commission?
2. 11,500 bushels of wheat were bought through an agent, who charged $\frac{7}{8} \%$ for buying. If the agent paid 85 c . per bushel for the wheat, $\$ 762.50$ freight, and $\$ 12.50$ insurance, what sum should be remitted to him in full settlement?
3. From a consignment of $3,160 \mathrm{lbs}$. of tea, sold by an agent at 30c. per lb., the consignor received as net proceeds $\$ 853.74$. What was the per cent. of commission charged for selling, if the charges for storage and insurance amounted to $\$ 51.60$ ?
4. A man wishes to draw on New York for an amount sufficient to cover expenses of $2 \%$ exchange and $2 \frac{1}{2} \%$ commission, and leave him the sum of $\$ 5,242.50$. For how much must he draw?
5. A farmer received from his city agent $\$ 490$ as the net proceeds of a shipment of butter. If the agent's com. mission is $3 \%$, delivery charges $\$ 6.80$, and $5 \%$ charge is made for guaranty of quality to purchasers, how many pounds, at 27 c . per lb., must have been sold, and how much commission was allowed?
6. Sold by consignee 16,000 bushels of wheat, at 95 c . ; 5,760 bushels of corn, at $86 \frac{1}{2} \mathrm{c}$. ; 9,245 bushels of oats, at 63 c . ; and 1,120 bushels of barley, at $\$ 1.73$. Required, the gross proceeds; also the net proceeds, the charges amounting to $\$ 515.20$, and the commission being $2 \frac{1}{2} \%$ for selling, and $2 \frac{1}{2} \%$ for guaranteeing payment?
7. A broker sold 815 bales of cotton, averaging 395 lbs . to the bale, at $16 \frac{1}{4} \mathrm{c}$. , his commission being $2 \frac{3}{4} \%$, and the charges $\$ 179$. He invested $25 \%$ of the net proceeas in flour for the consiguor, charging a commission of $1 \frac{1}{4} \%$. How much was still due the consignor?
8. An agent bought butter on a commission of $10 \%$, cheese on a commission of $6 \%$, and eggs on a commission of $5 \%$. If his cummission for buying the butter was $\$ 21$, for buying the cheese $\$ 21.60$, and for buying the egge $\$ 22$, and he charges $25 \%$ additional for guaranteeing the freshness of the eggs, what sum should the principal remit to pay for purchases and charges?
9. A merchant sent to his agent in New Orleans a consignment, the gross proceeds of which were $\$ 7,689$, the charges being \$323.5 ), and the commission $3 \frac{1}{2} \%$. He directed the agent to buy sugar with the net proceeds, and pay himself his commission for buying ( $2 \frac{1}{2} \%$ ) out of the same. What was the amount invested, and the agent's commission for both transactions?
10. An agent sold 2,000 bushels Alsike clover seed, at $\$ 7.85$ per bushel, on a commission of $5 \%$; and 1,200 bushels medium red, at $\$ 5.20$ per bushel, on a commission of $2 \frac{1}{2} \%$; taking the purchaser's 3 month's note for the amount of the sales. If the agent charges $4 \%$ for his guaranty of the notes, what amount does he aarn by the transaction?

## III.

1. A consignment of butter was sold for $\$ 1,570$, of which $\$ 1,546.45$ were the net proceeds. What was the rate per cent. of commission?
2. An Australian buyer shipped $33,000 \mathrm{lbs}$. of coarse wool to a London agent to be sold on commission, and gave instructions for the net proceeds to be invested in ieather. If the agent sold the wool at 18c. per liv., on a commission of $2 \%$, and charged $10 \%$ for the purchase and guaranty of grade of the leather, what was the amount of his commissions?
3. What are the net proceeds from the sale of 2,250 bbls. of flour, at $\$ 6.25$ a bbl., if the charges for freight and storage be 50c. a bbl., commission for selling $2 \%$, for guaranteeing paying $1 \frac{1}{2} \%$ ?
4. An agent sold, on commission, $1,750 \mathrm{bbls}$. of messpork, at $\$ 16.50$ per bbl., and 508 bbls. of short-ribs, at $\$ 18$ per bbl., charging $\$ 112.50$ for cartage, and $\$ 5.55$ for adrertising. He then remitted to his principal $\$ 36,000$, the net proceeds. Find the rate of commission.
5. A commission merchant received $\$ 1,640$ with which to buy corn, after deducting a commission of $2 \frac{1}{2} \%$. What is the amount of his commission, and how many bushels of corn, at $62 \frac{1}{2} \mathrm{c}$. a bushel, can he buy?
6. The holder of a douhtful ciaim of $\$ 850$, handed it to an agent for collection, agreeing that, for every dollar sent him by the agent, the agent might keep for himself 20 c. The agent succeeded in collecting but $\varepsilon$ ) \% of the debt. How much did the agent remit, how much commission did he receive, and what was his per cent. of commission?
7. A merchant buys, through an agent, 730 yds . of carpeting, at $\$ 1.25$ a $y d$. , and pays the agent $\frac{3}{4}$ of $1 \%$ commission; the freight amounted to $\$ 7.37$. At what price per yard must the carpeting be sold to realize a profit of $20 \%$ ?
8. I remitted $\$ 10,500$ to a Duluth agent to be invested in wheat, allowing him a commission of $3 \%$ for investing. The agent paid 95 c . per bushel for the wheat, and charged me $1 \frac{1}{2}$ cts. a bushel per month for storage. At the end of 4 months the agent sold the wheat at $\$ 1.10$ per bushel, on a commission of $5 \%$. If I paid $\$ 350$ for the use of the money, did I gain or lose by the operation, and how much?
9. A commission merchant sells a consignment of cotton for $\$ 5,216$. He pays $\$ 51$ for freight and storage, and charges a commission of $2 \frac{1}{4} \%$. What are the net proceeds?
10. The net proceeds of a consignment of wheat was $96 \frac{1}{2} \%$ of the net proceeds of a consignment of oats, and the rate of commission on each was $4 \frac{1}{2} \%$. The sum of the net proceeds on both consignments was $\$ 5,895$, and the sum of the charges, other than commission, was $\$ 330$, of which $\$ 175.00$ was charged to the consignment of wheat. How much was the commission on the consignment of oats?

## IV.

1. Find the duty on 8 dozen clocks, invoiced at $\$ 21.50$ each, and 6 dozen watches, invoiced at $\$ 35$ each, if the ad valorem duty was $35 \%$ on the clocks, and $25 \%$ on the watches.
2. A wine merchant imported 6 casks of wine, and paid $\$ 432$ duty, at $\$ 2$ per gallon, leakage $10 \%$ allowed. How many gallons to each cask, had no leakage been allowed?
3. Paid $\$ 325$ duty on goods which had been damaged; allowance for damage is $24 \%$, and the duty was $24 \%$. What was the invoice price of the goods?
4. An importer paid $\$ 825$ duty on an invoice of silks, the duty being $24 \%$. But damages of $15 \%$ were allowed at the custom-house. What was the entire cost of the goods?
5. A sugar refiner imports 50 hhds . of sugar weighing 480 lbs. each, and 120 hhds. of molasses containing 63 gals. each. What is the amount of the duties, if the sugar pay 3c. a lb. and the molasses 8c. a gal., an allowance being made on the sugar of $10 \%$, and $2 \%$ on the molasses?
6. A liquor dealer receives an invoice of 120 dozen bottles of porter, rated at $\$ 1.25$ per dozen. If $2 \%$ of the bottles are found broken, what will be the duty at $24 \%$ ?
7. A merchant imported 56 casks of wine, each containing 36 gals. net, the duty at $30 \%$ amounting to $\$ 907.20$. At what price per gallon was the wine invoiced?
8. The duty on an invoice of French lace goods at $24 \%$ was $\$ 132$, an allowance of $12 \%$ having been made at the custom-house for damage received since the goods were shipped. What was the cost or invoice of the goods?
9. A quantity of Valencias, invoiced at $\$ 1,654$, cost me $\$ 1,980.50$ in store, after paying the duties and $\$ 12.24$ for freight. What was the rate of duty?
10. A merchant imported 50 casks of port wine, each containing origically 36 gals., invoiced at $\$ 2.50$ per gal. He paid freight at $\$ 1.30$ per cask, and duty at $30 \%, 1 \frac{1}{2} \%$ leakage being allowed at the custom-house, and $\$ 8.50$ for cartage. What did the wine cost him in store?

## V.

1. The duty at $19 \%$ on an importation of Denmark satin was $\$ 619.40$. What was the invoice of the goods?
2. The duty on 600 drums of figs, each containing 14 lbs. , invoiced at $5 \frac{1}{4} \mathrm{c}$. per lb ., was $\$ 35.28$. Required, the rate of duty?
3. The duty on an importation of Bay rum, after allowiug $2 \%$ for breakage, was $\$ 823.20$, and the invoice price of the rum was $\$ .25$ per bottle. How many dozen bottles did the importer receive, duty at $24 \%$ ?
4. A merchant in New York imports from Havana 200 hihds. of W. I. molasses, each containing 63 gals, invoiced at $\$ .30$ per gal.; 150 hhds. of B. coffee sugar, each containing 500 lbs , invoiced at $\$ .05$ per lb .; 80 boxes of lemons, invoiced at $\$ 2.50$ per box; and 75 boxes of sweet oranges, invoiced at $\$ 3.00$ per box. What was the whole amount of duty, estimated at $24 \%$ on molasses and sugar, and at $8 \%$ on lemons and oranges?
5. The duty on an invoice of 300 dozen Buffalo porter, at $30 \%$, was $\$ 190,512$; breakage, $2 \%$. Required, the invoiced price per dozen?
6. Imported 12 casks of wine, each containing 42 gals., invoiced at $\$ 3.25$ per gal.; paid $\$ 96$ for freight, and a duty of $40 \%$. How much shall I gain \% in selling the whole for $\$ 2,747.58$ ?
7. Paid $\$ 63.90$ duties, at the rate of $9 \%$, on 50 casks of raisins, tare, 15 lbs . per cask; allowing the gross weight of each cask to have been 115 lbs ., what was the invoiced value per lb .?

## VI.

1. A man paid $\$ 175$ for insuring his dwelling, at $\frac{7}{8} \%$, and $\$ 100$ for insuring the furniture, at $1 \frac{1}{4} \%$. If both are destroyed by fire, how much is he entitled to receive?
2. A enaal-boat load of 840 bushels of wheat, worth 90c. per bushel, is insured for three-fourths of its value, at $1 \frac{8}{8} \%$ premium. In case of the total destruction of the wheat, how much will the owner lose?
3. A company took a risk at $24 \%$, and re-insured $\frac{8}{8}$ of it in another company at $2 \frac{1}{2} \%$. The premium received exceeded the premium paid by $\$ 72$. What was the amount of the risk ?
4. I insured my grocery store, valued at $\$ 18,500$, and its contents, valued at $\$ 39,000$, and paid $\$ 350$ for premium and policy. If the policy cost $\$ 1.25$, what was the rate per cent. of premium?
5. A merchant shipped a cargo to London, and to cover both the cargo and the premium, he took out a policy of $\$ 100,800$, at $3 \frac{1}{2} \%$. What was the value of the cargo?
6. The steamer Cibola, valued at $\$ 90,000$, is insured for $\$ 75, \subset 00$, at $2 \frac{1}{2} \%$. What will be the actual loss to the insurance company, in case the steamer is damaged to the amount of $\$ 20,000$ ?
7. Insured for their full value 200 barrels of flour, worth $\$ 5.75$ a barrel, and 400 barrels worth $\$ 6.25$, at ${ }^{2}{ }^{2} 0$ $1 \%$. 125 barrels of the first lot and 250 of the second were burned. What was the actual loss to the company?
8. A speculator bought 2,000 barrels of flour, and had it insured for $80 \%$ of its cost, at $3 \frac{1}{2} \%$, paying a premium of \$429. At what price must he sell the flour, to make a net profit of $10 \%$ ?
9. A vessel is so insured that if lost the owner may receive both the value of the vessel and the premium. The value of the vessel is $\$ 96,084$, and the rate of insurance $1 \frac{7}{8} \%$. Find the premium.
10. An underwriter agreed to insure some property for enough more than its value to cover the premium. A policy was issued for $\$ 25,087.81$. The rate being 35 c. on $\$ 100$, what was the property worth?
11. For what sum must a policy be issued, to insure a vessel for $\$ 36,000$ and cover also the premium, the rate being $1 \frac{1}{2} \%$ ?
12. A speculator bought 1,000 bbls. of flour, and had it insured for $80 \%$ of $i$ is cont, at $3 \frac{1}{4} \%$, paying a premium of $\$ 21450$. At wha' price zar? he sell the flour to realize a profit of $20 \%$ ?
13. Four companies iota in insuring a ship and cargo for $\$ 60,000$. One company takes $\frac{1}{3}$, at $\frac{3}{5}$ of $1 \%$; a second takes $\$ 10,000$, at $\frac{8}{4}$ of $1 \%$; a third, $\$ 15,000$, at $\frac{5}{8}$ of $1 \%$; a fourth, the remainder, at $\frac{\pi}{2}$ of $1 \%$. How much is paid for insurance?

## VII.

1 A town containing $\$ 541,250$ taxable real estate and $\$ 15,620$ personal property, levies a tax of $.009 \%$. If $2 \%$ is paid for collecting, what is the net amount realized from the tax?
2. In a school section the valuation of the taxable property is $\$ 752,400$, and it is proposed to repair the school-house and ornament the grounds at an expense of $\$ 5,000$. If old material sells for $\$ 673.70$, what will be the rate per cent. of taxation, and what will be B's tax, whose property was valued at $\$ 9,400$ ?
3. A tax of $\$ 11,466$, besides the cost of collecting at $2 \frac{1}{2} \%$, is to be raised in a certain town. The polls, 560 in number, are taxed $\$ 1$ each. The real estate is assessed at $\$ 1,270,000$, and the personal property at $\$ 130,000$. Determine the rate, make an assessors' table for that rate, and find A's tax for 2 polls, $\$ 2,800$ real estate, and $\$ 1,400$ personal property?
4. The cost of maintaining the public schools of a city during the year 1888, was $\$ 112,000$, and the taxable property of the city was $\$ 44,800,000$. How many mills on a dollar mist be assessed for school purposes? If $10 \%$ of the tax assessed cannot be collected, how many mills on a dollar must then be assessed?
5. The total assessed value of a town, real and personal, is $\$ 630,000$, and the town expenses are $\$ 3,913.95$. How much tax must be collected to provide for town expenses and allow $3 \%$ for collecting? If the same town contains 310 polls, taxed $\$ 1.50$ each, what will be the rate of taxation, and how much will be the tax of a man who pays for two polls and owns property assessed at $\$ 14,500$ ?
6. A tax of $\$ 13,943.20$ is assessed upon a town containing 860 taxable polls; the real estate is valued at $\$ 2,708,000$, and the personal property at $\$ 151,600$. If the polls be taxed $\$ 1.25$ each, what will be the rate of property taxation, and what will be the tax of Peter Parley, who pays for three polls, and has real and personal estate valued at $\$ 28,750$ ?
7. The assessed value of a town is, on real estate, $\$ 1,197,500$, and on personal property, $\$ 432,500$. A poll tax of $\$ .50$ per head is assessed on each of 1,870 persons. The town rotes to raise $\$ 8,000$ for schools, $\$ 1,500$ for highways, $\$ 1,500$ for salaries, $\$ 1,000$ tor support of poor, and $\$ 310$ for contingent expenses. How much tax will a milling company have to pay on a mill valued at $\$ 46,500$, and stock at $\$ 19,750$ ?

## INTEREST.

326. Interest is money paid for the use of money.
327. The Principal is the money for the use of whioh interest is paid.
328. The Amount is the sum of the principal and interest.
329. The Rate is the per cent. of the principal paid for its use for 1 year, or a specified time.

Note.-When the rate is given, it is to be understood in this work to mean rate per annum, unless otherwise specified.
330. Legal Interest is the rate fixed by law for cases in which no rate is specified in the agreement between the parties interested.
In all the Provinces of Canada the legal rate is $6 \%$.
331. Usury is a higher rate than the legal rate.
332. In computing interest, a legal year is 12 months or 365 days.
333. Simple Interest is the interest on the principal only.

## ACCURATE INTEREST.

( 12 monthe or 365 days to a year).

## 334. To find the interest on a sum of money for a given number of years, or fraction of a year, at a given rate.

Example 1.-Find the interest on $\$ 650$ for 2 years at $4 \%$.

Soldtion 1. 650 Principal
.04 $\$ \overline{26.00}$ Int. for 1 yr . $\frac{2}{\$ 52.00}$ " 2 yrs . Solution 2.
$\$ 6.50$ is int. fqr 1 yr. at $1 \%$. $\$ \frac{4}{26.00}$ " " " " $4 \%$. 2
852.00 " 2 yrs. " $4 \%$.

Explanation.
Interest for 1 year is $4 \%$ of the principal $\$ 650=\$ 650 \times .04=$ $\$ 26.00$, and the interest for 2 years is twice the interest for 1 year, or $\$ 26.00 \times 2=\$ 52.00$.

Example 2.-Find the interest on $\$ 960$ for 3 yrs. 4 mos., at $6 \%$.

Solution 1.
$\$ 960$
.06
$\$ 57.60$ Int. for 1 yr .
$\overbrace{12}^{31}$ $3^{\frac{1}{3}}$
$\$ 192.00$ " " 31 yrs.(3 yrs. 4 mos.) $\$ 1 \overline{92.00}$
Note 1.-1 \% of a number is found by removing the decimal point in the number, 2 places to the left.
2. The result will be the same in Lix. 1, whether we multiply by 4 and then by 2, as in Solution 2, or by $8(4 \times 2)$, as in Solation 8 .

## EXERCISE 72.

Find the interest for one year of -

1. $\$ 450$ at $4 \frac{1}{2} \%$.
2. $\$ 680$ at $3 \frac{1}{2} \%$.
3. $\$ 960$ at $7 \frac{1}{2} \%$.
4. $\$ 840$ at $5 \frac{1}{2} \%$.
5. 31,720 at $6 \frac{1}{2} \%$.
6. $\$ 2,630$ at $4 \frac{1}{2} \%$.
7. $\$ 4,920$ at $5 \%$.
8. $\$ 5,000$ at $33 \%$.
9. $\$ 3,720$ at $3 \frac{1}{2} \%$.
10. $\$ 4,680$ at $4 \frac{1}{2} \%$.
11. 87,428 at $5 \frac{1}{2} \%$.
12. $\$ 9,654$ at $6 \%$.
13. 89,613 at $6 \frac{1}{2} \%$.
14. $\$ 9,643$ at $7 \%$.
15. $\$ 5,430$ at $5 \%$.

Solution 3.
$\$ 6.50$
$\$ \frac{8}{\$ 52.00}=4 \times 2$

Find the interest and amount-

|  | cipal | katm. | time. |
| :---: | :---: | :---: | :---: |
| 16. | \$600.00, | 5\%, | 2 yrg . |
| 17. | \$700.00, | 6\%, | 2 yrs. 6 mos. |
| 18. | \$500.00, | 7\%, | 5yrs. |
| 19. | \$950.00, | 8 ', | $3 \mathrm{yrs}$.3 mos , |
| 20. | \$800.00, | 9\%, | 6 yrs. |
| 21. | \$740.00, | 812\%, | 7 yrs . |
| 22. | \$1,320.00, | 10\%, | 2 yrs. 10 mos. |
| 23. | \$960.50, | $12 \%$, | ¢y yrs. 9 mos. |
| 24. | \$475.80, | $64 \%$ | 4 yrs. 6 mos. |
| 25. | \$363.20, | 21\% | 3 yrs .8 mos . |
| 26. | \$1,020.00, | 39\%, | 1 yr .7 mos . |
| 27. | \$4,075.00, | $6 \%$, | 2 yrs .4 mos. |
| 28. | \$4,028.75, | $4 \%$, | 5 yrs. |
| 29. | \$4,026.00, | $8 \%$, | 3 yrs. 2 mos. |
| 30. | \$270.36, | $3 \frac{1}{2} \%$, | 1 yr .11 mos . |
| 31. | \$840.00, | $9 \%$, | 1 yr .9 mos . |
| 32. | \$100.00, | $6 \%$, | 2 yrs 7 mos . |
| 83. | \$900.00, | 5\%, | 3 yrs .6 mos . |
| 84. | \$360.00, | $7 \%$, | 5 yrs. 4 mos. |
| 35. | \$750.80, | $4 \%$, | 2 yrs .7 mos . |
| 36. | \$475.30, | $3 \%$, | 6 yrs . 3 mos . |
| 87. | \$328.00, | $64 \%$, | 2 yrs .5 mos. |
| 88. | \$474.90, | 812\%, | $4 \mathrm{yrs}$.6 mos . |
| 89. | \$640.80, | 51 \%, | 1 yr .3 mog . |
| 40. | \$143.33, | 51 \% , | 6 yrs . |
| 41. | \$360.96, | $12 \%$, | 2 yrs . |
| 42: | \$796.00, | $11 \%$, | 37 yrs . |
| 43. | \$1,800.00, | $13 \%$, | 47 yrs . |
| 44. | \$1,080.00, | 10\%, | $2 \frac{1}{6} \mathrm{yrs}$. |
| 45. | \$894.00, | 41 \% , | 312 yrs. |

335. To find the interest on a sum of monev. for a given number of days, at a given rate.

Example 1.-Find the interest on $\$ 850$ for 62 days at $5 \%$.
Solution 1.
$\$ 8.50$
$\$ 42.50$
$365) \frac{62}{2635.00}$

Solution 2.
canoellation method.

or
$\$ 772$.

## Explayation.

Sixty-two days is ${ }_{86}{ }^{2} 5$ of 1 year. The interest for 62 days is therefore $\frac{62}{885}$ of the interest for 1 year, and this may be found by multiplying the interest for 1 year ( $\$ 42.50$ ) by 62 and dividing the result by 365 , as in Solution 1, or by cancellation, as in Solution 2.

Example: 2.-Find the interest on $\$ 3,250$ from April 16th. 1889, to June 18th, 1891, at $6 \%$ per annum.
(From April 16th, '89, to June 18th, '91, is 2 years and 63 days.)
Solution 1.
$\$ 32.50$
$\frac{6}{\$ 195.00}$
$\frac{289}{888}$ \$423.66.

$$
\begin{aligned}
& \text { Solution 2. } \\
& 32.50 \times 6 \times \frac{83}{886}=\begin{array}{l}
33.66 \text { Int. for } 63 \mathrm{ds} \\
32.50 \times 6 \times 2= \\
\frac{390.00}{\$ 423.66} \quad \text { ". } 2 \text { yrs. }
\end{array}
\end{aligned}
$$

336. It is the custom with banks when the time is given in months, to consider them calendar months in reference to the maturity of the paper, but even then they compute the discount by days.
Time table, showing the number of days:

|  | To the Correrponding day oi |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jan. | ${ }_{\text {Feb }}{ }^{2}$ | Mar. | Apr. | $\begin{gathered} 5 \\ \text { May } \end{gathered}$ |  |  |  | pt. |  |  |  |  |
|  |  |  |  |  |  |  |  |  | $\begin{aligned} & \text { Ben } \\ & \hline \end{aligned}$ |  |  |  |  |

1. How many days from May 13th to August 23rd?

## Explanation.

Find "May in the column of months at the left; and on tho same line nnder "August" find 92, which is the number of days from any day in May to the same day in August. But August 23 is 10 days more than August 13, and $92+10=102$ days. Ans.

Note 1.-If the required date be earlier in the month than the date from which time is counted, subtract the difference from the tabular number.
2. If in Leap Year, and the month of Febraary be included in the time reckoned, add 1 day to the number of days found by the table.

EXERCISE 73.

## Find interest on-

| principal. | time. | Rate. | principal. | time. | RATE. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. \$3,600, | $65 \mathrm{da} .$, | 5\%. | 7. $\$ 340.80$, | 130 da., |  |
| 2. $\$ 4,500$, | 80 da., | $7 \%$. | 8. $\$ 424.40$, | 67 da., | $6 \%$. |
| 3. $\$ 800$, | 90 ds., | 8\%. | 9. \$625.30, | 48 da., | 3\% $\%$. |
| 4. \$750, | 45 da., | $4 \frac{1}{2} \%$. | 10. \$426.50, | 292 da., | 32 $4 \%$. |
| 5. $\$ 9,360$, | 135 da., | 6\%. | 11. $\$ 370.75$, | 73 da., | $7 \%$. |
| 6. $\$ 4,350$, | 219 da., | $3 \frac{1}{2} \%$ 。 | 12. $\$ 420.80$. | 60 da., | 8\%. |

Find the amount-
principal. bate.
13. $\$ 542.00, \quad 7 \%, \quad$ From 1888, Oct. 27, to 1890, May 12.
14. $\$ 684.00,8 \%$, " 1887, Sept. 19, to 1889, June 1.
15. $\$ 96000, \quad 9 \%$, 1882, Deo. 31, to 1892, Oot 1 .
16. $\$ 1,100.00$, $10 \%$, " 1889, Jan. 1, to 1892, Deo. 20.
17. $\$ 1,186.20$, $11 \%$, " 1885, April 1, to 1886, July 28.
18. $\$ 1,260.48, \quad 12 \%$ " 1888 , Aug. 31 , to 1893,' Nov. 1 .
19. $\$ 1,040.25, \quad 8 \%, \quad$ " 1890 , Feb. 20, to 1891, May 10.
20. $\$ 1,097.76, \quad 6 \%, \quad$ " 1885. Mar. 15, to 1885, Jan. 15.
21. $\$ 976.80, \quad 7 \%$ - $\quad$ 1R85 June 19, to 1889, April 7.
22. $\$ 896.84, \quad 9 \%$ " 1887 , Nov. 24 , to 1887, Nov. 30
23. $\$ 1,272.24, \quad 10 \%$ " 1891, Sept. 27, to 1892, Dec. 9 .
24. $\$ 1,284.96, \quad 12 \%$ " 1890 , Dec. 8 , to 1891, May 1.
25. $\$ 1,200.00$, $11 \%$ " 1888 , Deo. 25 , to 1890, May 28 .
26. $\$ 989.00, \quad 12 \%$ " 1889, Mar. 21, to 1890, June 30.
27. A note for $\$ 560.60$, dated May 5 th, 1881, was paid Dec. 31st, 1882, with interest at $7 \%$. What was the amount?
28. If I have the use of $\$ 275$ for 4 years 10 months from Jan. 12th, 1883, what amount must I return to the owner, allowing $6 \%$ interest, and what will be the date of maturity?
29. Required the amount of $\$ 408.60$ from Aug. 20th to Dec. 18th, 1886, at $10 \%$ ?
30. What is the interest on a note for $\$ 515.62$, disted March 1st, 1883, and payable July 16th, 1885, st $7 \%$ ?
31. What is the value of a note of $\$ 65.75$, due with interest for 1 year 2 months, at $6 \frac{1}{2} \%$ ?
32. If a person borrow $\$ 875$ at $5 \%$, what will be dus the lender at the end of 2 years 6 months?
33. A man sold his house and lot for $\$ 12,500$; the terms were, $\$ 4,000$ in cash on retivery, $\$ 3,500$ in 9 months, $\$ 2,600$ in 1 year 6 montha, and the balance in 2 years 4 months, with $6 \%$ interest. What was the whole amount paid?

## SIX PER CENT. METHOD.

3337. The Six Per Cent. Method is formed on a basis of 360 days to the year and 30 days to the month.
3:3s. At $6 \%$ per annum the interest of $\$ \mathrm{I}$.
For 1 yr. 12 mo., or 360 da ., is $6 \mathrm{c} .=.06$ of the principal.
For $\frac{1}{6}$ yr. 2 mo ., or 60 da., is $1 \mathrm{c} .=.01$ of the principal.
For $\frac{1}{1} \frac{1}{2} \mathrm{yr} .1 \mathrm{moo}$., or 30 da ., is $5 \mathrm{~m} .=.005$ of the principal.
For $\quad \frac{1}{5} \mathrm{mo}$. , or 6 du. , is $1 \mathrm{~m} .=.001$ of the principal.
For $\frac{1}{3}$ mo., or 1 da. , is $\frac{1}{\frac{1}{2} \mathrm{~m}} .=.000 \frac{1}{\frac{1}{d}}$ of the principal.
Hence the following-
privcipies.
3338. 3339. The interest of $\$ 1$ at $6 \%$ is half as many cents as there are months in the given time.
1. The interest of $\$ 1$ at $6 \%$ is one-sixth as many mills as there are days in the given time.
2. The interest for 60 days at $6 \%$ is found by removing the decimal point two places to the left in the principal.
3. The interest for 30 days at $6 \%$ is found by removing the decimal point two places to the left in the principal and dividing the result by 2.
4. The intersest for 6 days at $6 \%$ is found by removing the decimal point 3 places to left in the principal.
5. The interest for 1 day at $6 \%$ is found by removing the decimal point 3 places to right in the principal and dividing the result by 6.
6. To find the interest for any number of years, months and days at $6 \%$

Example 1.-What is the interest on $\$ 450.75$ for 1 yr .3 mos. 21 da . at $6 \%$ ?

Soltition 1
Int. on $\$ 1$ for 15 mos. $=\$ .075$. (Principle 1) " ${ }^{\prime}$ " $\$ 1$ " $21 \quad "=0.0035 . \quad$ (Princıple 2)
Int. on $\$ 1$ for 1 yr .3 mos. $21 \mathrm{da} .=\$ .0785$.
$\therefore$ Int. on $\$ 450.75$ for 1 yr. 3 mos. 21 da. $=\$ 450.75 \times .0785=\$ 35.383875$

## Solution 2.

| $1 \mathrm{yr} .3 \mathrm{mos} .21 \mathrm{da} .=471 \mathrm{da}$. |  |  |  |  | Shorter Phocesb.$\$ 4.508$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| \$4.5075 | $=$ Int. f | 60 d |  | (Principlo 3) |  |
| \$31.5525 | = " | 420 | " | $(60 \times 7)$ | \$31.556 |
| 2.25375 | " | 30 | " | (60 $\div 2$ ) | 2.254 |
| 1.126875 | $="$ | 15 | " | (30 $\div 2$ ) | 1.127 |
| . 45075 | $={ }^{\prime}$ | 6 | " | (Prinoiple 5) | . 451 |
| \$35.383875 | $=$ Int. | 71 |  |  | \$35.388 |

Notr 1.-For business purposes it is sufficiently exact to carry the work to mills, as in the shorter process.
2. In this process when the decimal in the fourth places is less than 5 it is rejected; when 5 or greater than 5 , the figure in the third deoimal place is increased by one, and the deoimals to the right of the third decimal place are rejected.
341. To find the interest at any other rate than $6 \%$ by this method, first. find the interest at $6 \%$, and then increase or diminish the result by as many sixths as the given rate is units greater or liess than $6 \%$. Thus, for $7 \%$ add $\frac{1}{8}$, for $8 \%$ add $\frac{2}{6}$ or $\frac{1}{3}$, for $4 \%$ subtract $\frac{2}{6}$ or $\frac{1}{3}$, etc.

## EXERCISE 74.

Find the interest at $6 \%$ of-

1. $\$ 267.27$ for 6 mo .24 da .
2. $\$ 146.18$ for 1 yr .21 da .
3. $\$ 256.84$ for 2 yr .4 mo .12 da .
4. $\$ 597.25$ for $7 \mathrm{mo}, 18 \mathrm{da}$.
5. $\$ 418.75$ for $1 \mathrm{mo} .2 \overline{\mathrm{da}}$.
6. $\$ 309,18$ for 2 yr .24 da.
7. $\$ 38.90$ for 1 yr .1 mo .6 da .
8. $\$ 146.48$ for 9 mo .10 da .
9. $\$ 275.50$ for 11 mo .13 da .
10. $\$ 1,298$ for 3 yr .1 mo .27 da .
11. $\$ 2,000$ for 2 yr. 7 mo. 24 da.
12. $\$ 4,010$ for 1 yr .1 mo .13 da
13. $\$ 680$ for 2 yr .6 mo .10 da .
14. $\$ 1,805$ for 1 yr 7 mo .7 da .
15. $\$ 468$ for 5 yr .5 mo .1 du .
16. $\$ 1,000$ for 11 yr .1 mo .20 da .
17. $\$ 645$ for 4 yr .4 mo .5 du .
18. $\$ 500$ for 3 yr .1 mo. 27 da .
19. $\$ 895$ for 5 yr .11 mo .11 da .
20. $\$ 1650$ for 1 yr. 10 mo .23 da .
21. $\$ 1,463$ for 9 yr .1 mo .9 da .
22. $\$ 365$ for 4 yr. 1 mo. 25 da.

Find the interest and amount-
PRINOIPAL. RATE. TIME. PRINCIPAL RATE. TIME.
23. $\$ 1,080.50,7 \%, 1$ yr. 9 mo. 35. $\$ 1,248.00,9 \%, 9$ mo. 25 da.
24. $\$ 420.25,8 \%, 2 \mathrm{yr} .9 \mathrm{mo}$.
25. $\$ 960.00,9 \%, 3 \mathrm{yr} .4 \mathrm{mo}$,
26. $\$ 576.48,10 \%, 3$ yr. 6 mo.
36. $\$ 740.00,6 \%, 1$ yr. 9 mo .15 da .
37. $\$ 960.00,7 \%, 1$ yr. 9 mo. 24 da .
27. $\$ 645.00,12 \%, 5 \mathrm{yr} 10 \mathrm{mo}$. $81,29.00,8 \%, 2 \mathrm{yr} .3$ mo. 9 da.
28. $\$ 1,200.00,5 \%, 6 \mathrm{yr} .3 \mathrm{mo}$.
29. $\$ 1,200.00,10 \%, 12$ yr. 6 mo.
30. $\$ 828.00,6 \%, 8 \mathrm{mo} .16$ da.
31. $\quad \$ 972.36,8 \%, 17 \mathrm{mo} .18 \mathrm{da}$.
32. $\$ 600.60,10 \%, 23 \mathrm{mo} .14 \mathrm{da}$.
33. $\$ 1,165.17,12 \%, 40$ mo. 6 da.
34. $\$ 894.00,7 \%, 14 \mathrm{mo} .17 \mathrm{da}$.
$9 \%, 2$ yr. 9 mo .21 da.
40. $\$ 1,800.00,10 \%, 3$ yr. 6 rno. 15 da.
41. $\$ 600.00,11 \%, 4$ yr. 7 mo .18 da .
42. $\$ 796.00,12 \%, 5 \mathrm{yr} .10 \mathrm{mo} .6$ da.
43. $\$ 976.28,7 \%, 7 \mathrm{yr} .9 \mathrm{mo} .27 \mathrm{da}$.
44. $\$ 869.44,9 \%, 8 \mathrm{yr} .4 \mathrm{mo} .17 \mathrm{da}$.
45. $\$ 1,126.56,11 \%, 10 \mathrm{y}=5 \mathrm{mo} .1$ da.
46. $\$ 1,295.28,8 \%, 13 \mathrm{yr} .4 \mathrm{mo} .29 \mathrm{da}$.
342. To find the interest for any number of days at $6 \%$

Example 1.-Find the interest on $\$ 672$ for 216 days at $6 \%$.
Solution 1.

| 86.72 |  | Int. | r 60 |  |  | Principle 3) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \$20.16 | = | " | 180 | ' |  | $(60 \times 3)$ |
| 3.36 | = | " | 30 |  |  | (60 $\div 2$ ) |
| . 672 | $=$ | " | 6 | " |  | Principle 5) |

Solution 2.
$\$ 672$
$\frac{.036}{4032}$
2016
\$24.192.

## Explanation.

By Principle 2, the interest on $\$ 1$ for 216 days $=36$ mills $=\$ .036 . \quad \therefore$ Interest on $\$ 672$ for 216 days $=. \$ 672 \times .036=3$ \$24.192.

Solution 3.
$8.672 \div 6=\$ .112=$ Int. for 1 da. (Principle 6)
$\therefore \$ .112 \times 216=\$ 24.192=\quad$ " 216 da.
Example 2.-Find the interest on $\$ 760.48$ for 174 days at $6 \%$. Solution.


## EXERCIGE 75．

Find the interest on－

1．$\$ 1,750.00$ ，for 15 days，at $6 \%$ ．
2．$\$ 1,125.00$ ，for 24 days，at $7 \%$ ．
3．$\$ 742.50$ ，for 30 days，at $6 \%$ ．
4．$\$ 900.00$ ，for 95 days，at $7 \frac{1}{2} \%$ ．
6．$\$ 660.00$ ，for 63 days，at $8 \%$ ．
6．$\$ 136.42$ ，for 33 days，at $9 \%$ ．
7．$\$ 1,000.00$ ，for 21 llyys，at $10 \%$ ．
8．$\$ 2,000.00$ ，for 12 days，at $5 \%$ ．
9．$\$ 351.00$ ，for 40 d ys ，at $4 \frac{1}{2} \%$ ．
10．$\$ 1,368.00$ ，for 50 days，at $3 \%$ ．
11．$\$ 93.00$ ，for 130 days，at $6 \%$ ．
12．$\$ 550.00$ ，for 75 dzys ，at $7 \%$ ．
13．$\$ 842.50$ ，for 45 days，at $6 \%$ ．
14．$\$ 800.00$ ，for 27 days，at $5 \%$ ．
15．$\$ 1,725.00$ ，for 57 days，at $9 \%$ ．
16．$\$ 125.00$ ，for 55 days，at $6 \%$ ．
17．$\$ 3,741.85$ ，for 6 days，at $7 \%$ ．

18．$\$ 5,178.00$ ，for 9 days，at $9 \%$ ．
19．$\$ 732.00$ ，for 11 days，at $6 \%$ ．

21．$\$ 340.00$ ，for 70 days，at $10 \%$ ．
22．$\$ 1,478.00$ ，for 80 days，at $6 \%$ ．
23．$\$ 2,150.00$ ，for 96 days，at $4 \frac{1}{2} \%$ ．
24．$\$ 1,200.00$ ，for 53 days，at $6 \%$ ．
25．$\$ 1,500.00$ ，for 87 days，at $7 \%$ ．
26．$\$ 420.00$ ，for 41 days，at $5 \%$ ．
27．$\$ 360.00$ ，for 81 days，at $6 \%$ ．
28．$\$ 2,347.50$ ，for 18 days，at $7 \%$ ．
29．$\$ 1,112.4^{n}$ ，for 25 days，at $8 \%$ ．
30．$\$ 1,300.00$ ，for 13 days，at $6 \%$ ．
31．$\$ 17,000.00$ ，for 3 days，at $5 \frac{1}{2} \%$ ．
32．\＄195．50，for 33 तays，at $10 \%$ ．
33．$\$ 1,050.00$ ，for 43 days，at $7 \%$ ．
34．$\$ 1,560.00$ ，for 44 days，at $7 \frac{1}{2} \%$

Find the interest in－

|  | PRINCIPAL | FROM |  |
| ---: | ---: | :--- | ---: |
| 35． | $\$ 35.61$, | Nov． | 11,1891, |
| 36． | $\$ 50.00$, | Sept． | 4,1890, |
| 37． | $\$ 97.86$, | May | 17,1886, |
| 38． | $\$ 325.28$, | June | 20,1882, |
| 39． | $\$ 154.75$, | April | $10,1888$. |
| 40． | $\$ 861.50$, | June | 3,1889, |


| TO |  | RATE． |
| :--- | ---: | ---: |
| Deo． | 15,1893, | $6 \%$ |
| Jan． | 1,1892, | $3 \frac{1}{2} \%$. |
| Dec． | 20,1893, | $7 \%$. |
| Sept． | 4,1884, | $8 \%$ |
| Nov．24，1888， | $6 \%$. |  |
| March．25，1890， | $5 \%$. |  |

Find the amount of－

41．\＄450．80，March 6，1893，
42． $81,500.00$ ，liay 5,1894 ，
43．\＄127．36，Deo．12，1889，

Dow．20，1893，$\quad \mathbf{~ \%}$ ．
Јลน．20，18\＆5， $4 \%$ ．
Julv 3，1891，4 \％

## ACCURATE INTEREST.

$$
\text { ( } 12 \text { months or } 365 \text { days to a year.) }
$$

343. Since interest in Canada is reckoned upon a basis of 365 days to a year, the interest found by the "Six Per Cent. Method," which is based upon the supposition that 360 days make a year and 30 days a month, is not strictly accurate.
344. Since the year contains 365 days, the interest, found by the Six Per Cent. Method for 360 days to the year, is $\frac{5}{385}$ or $\frac{1}{7}$ part of itself too large.
345. In many States of the American Union interest is reckoned on the basis of $\mathbf{3 6 0}$ days to the year, and many people in Canada still reckon the interest on small amounts on this basis.
346. On account of the shortness of the Six Per Cent. Method, many accountants prefer to reckon interest by this method, and to then make the necessary deduction of $\frac{1}{\frac{1}{8}}$ of elf.

Example.-Find the accurate interest on $\$ 750$ for 96 days at $8 \%$. Solution.


EXERCISE 76.
Find the interest at $6 \%$ on-

1. $\$ 2,500$ for 75 days.
2. $\$ 8,360$ for 78 days.
3. $\$ 750$ for 48 days.
4. $\$ 4,780$ for 51 days.

5. $\$ 3,654$ for 43 days.
6. $\$ 4,525$ for 47 days.
7. $\$ 9,875$ for 158 days.

Find the interest and amount of-
9. $\$ 850.00$ for 63 days at $6 \%$.
10. $\$ 945.50$ for 33 days at $6 \%$.
11. $\$ 378 . f 8$ for 75 days at $6 \%$.
12. $\$ 354.75$ for 130 days at $6 \%$.
13. $\$ 510.00$ for 63 days at $7 \%$.
14. $\$ 615.00$ for 93 days at $6 \%$.
15. $\$ 450.00$ for 78 days at $5 \%$.
16. $\$ 120.00$ for 96 days at $7 \frac{1}{2} \%$.
17. $\$ 353.00$ for 80 days at $10 \%$.
18. $\$ 670.00$ for 78 days at $5 \%$.
19. $\quad \$ 785.00$ for 45 days at $7 \%$.
20. $\$ 1,200.00$ for 68 days at $5 \%$.
21. $\$ 2,500.00$ for 93 days $n$ t $8 \%$.
22. $\$ 1,935.50$ for 75 days it $5 \%$.
23. $\$ 2,136.88$ for 70 days at $4 \%$.
24. $\$ 1,000.00$ for 73 days at $6 \%$.
25. $\$ 2,000.00$ for 146 days at $9 \%$. 26. $\$ 1,500.00$ for 219 days at $4 \frac{1}{2} \%$.

Find the interest of-

PRINCIPAL.
27. $\$ 450$,
28. $\$ 720$,
29. 8960 ,
30. $\$ 540$,
31. $\$ 100$,
32. 8900 ,
33. \$240,
34. \$333,
35. \$672,
36. \$60,
37. $\$ 600$,
38. $\$ 630$,
39. $\$ 480$,
40. \$270,
41. \$386,

TIME.
From Aug. 10 to Nov. 8, 1885,
" Jan. 25 to April 7, 1885,
" Feb. 3 to Mar. 19, 1884,
" April 8 to May 18, 1890,
" Jan. 30 to Mar. 6, 1892,
" Feb. 12 to Mar. 4, 1893,
" May 31 to Nov. 27, 1895,
" Aug. 1 to Nov. 29, 1886,
" Feb. 28 to Oct. 25, 1880,
" June 19 to Nov. 10, 1881,
" July 4 to Oct. 20, 1889,
" Feb. 1 to Aug. 20, 1889,
" Jan. 21 to Dec. 2, 1891,
" May 10 to July 29, 1894,
" Oot. 13 to Dec. 12, 1895,
rati.
$6 \%$.
$7 \%$.
$8 \%$.
$9 \%$.
$4 \%$.
$7 \frac{1}{2} \%$.
$10 \%$.
$5 \%$.
$42 \%$.
$12 \%$.
$3 \%$.
$54 \%$.
$5 \%$.
$6 \%$.
$9 \%$.
42. A person borrows $\$ 3,754.45$, being the property of a minor who is 15 years 3 months old. He retains it until the owner is 21 years old. How much money will then be due at $6 \%$ ?
43. A note for $\$ 710.50$, with interest after 3 months at $7 \%$ was given Jan. 1st, 1884, and paid Aug. 12th, 1886. What was the amount due?
44. A speculator borrowed $\$ 9,675$, at $6 \%$, April 15th, 1884, with which he purchased flour at $\$ 6.25$ a barrel. May 10th, 1885, he sold the flour at $\$ 7 \frac{3}{8}$ a barrel, cash. What did he gain by the transaction?
45. A man, engaged in business with a capital of $\$ 21,840$, is making $12 \frac{1}{2} \%$ per annum on his capital ; but on account of ill health he quits his business, and loans his money at $\frac{3}{4} \%$ a month. How much does he lose in 2 years 5 months by the change?
46. Bought 4,500 bushels of wheat at $\$ 1.12 \frac{1}{3}$ a bushel, payable in 6 months; I immediately realized for it $\$ 1.06$ a bushel, cash, and put the money at interest at $10 \%$. At the end of the 6 months I paid for the wheat. Did I gain or lose by the transaction, and how much?

## 347. To find the principal, the rate, time, and interest being given.

 4 mos. at $4 \%$ ?Solution 1. $\$ 1.00$

$$
\begin{array}{ll}
\frac{.04}{.04} & \left..09 \frac{2}{3}\right) 44.80 \\
.04 \\
\frac{2 \frac{1}{3}}{.09 \frac{1}{3}} & \left.\frac{3}{28}\right) \frac{3}{13440}(\$ 480 .
\end{array}
$$

Solution 2.
$4 \% \times 2 \underset{y}{2}=93 \%$
$97 \%$ of the principal $=\$ 44.80$
$\begin{aligned} \therefore \quad \text { the principal } & =4480 \times \frac{100}{9 \frac{1}{8}} \\ = & \$ 480 .\end{aligned}$

Explanation.
The interest on $\$ 1$ for 2 yrs . 4 mos. at $4 \%$ is $\$ .09 \frac{1}{3}$, therefore $\$ 44.80$ must be the interest on as many dollars at $\$ .09 \frac{1}{3}$ is con. tained in $\$ 44.80$ or $\$ 480$. Ans.

## Explanation.

The interest each year $=4 \%$ of the principal, and for $2 \frac{1}{3}$ years $=4 \% \times 2 \frac{2}{3}=9 \frac{1}{3} \%$ of the principal, and therefore 9 \% of the principal $=\$ 44.80$.

Example 2.- On what sum of money is $\$ 45.60$ the interest for 76 days at $5 \%$.

Solution.
$5 \% \times{ }_{36}^{765}=17 \mathrm{~s} \%$ $183 \%$ of the principal $=\$ 4.5 .60$
$\begin{aligned} \therefore \quad \text { the principal } & =\$ 45.60 \times \frac{100}{179} \\ & =\$ 4,380 .\end{aligned}$

Explanation.
Interest for each year $=5 \%$ of the principal, and for 76 days $=5 \% \times{ }_{365}^{765}=17_{73}$ of the principal and therefore $1 \frac{3}{7} \%$ of the principal $=\$ 45.60$.
nole.
Divide the given interest by the interest on $\$ 1$ for the givern time and rate.

## EXERCISE 77.

## Find the principal-

RATE. TIME. TNTEREGT.

| , | , | , | mite. | TIMM | nfteres |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1. $3 \frac{1}{2} \%$, | 1 yr., | \$451. | 7. $5 \%$, | 7 yrs. | 829.75. |
| 2. $5 \frac{1}{2} \%$, | 1 " | \$41. | 8. $31 \%$, | 412 ${ }^{\text {" }}$ | \$94.50. |
| 3. $4 \frac{1}{2} \%$, | $\frac{1}{2}{ }^{\prime \prime}$ | \$25ㄱ․ | 9. $4 \%$, | $1{ }^{3}$ | \$68.25. |
| 4. $3 \frac{3}{4} \%$, | 交" | \$39. | 10. $4 \frac{1}{2} \%$, | 174 | \$47.25. |
| 5. $8 \%$, | 显" | \$18. | 11. $6 \%$, | $5{ }^{2}$ " | \$170.00. |
| 6. $2 \frac{1}{2} \%$, | 6 " | \$521. | 12. $31 \%$, | $44^{\circ}$ | \$136.00. |

Find the principal-

|  | Interest. | Rate. |
| :--- | ---: | ---: |
| 13. | $\$ 42.70$, | $7 \%$, |
| 14. | $\$ 197.80$, | $8 \%$, |
| 15. | $\$ 26.08$, | $6 \%$, |
| 16. | $\$ 60.75$, | $5 \%$, |
| 17. | $\$ 987.75$, | $9 \%$, |
| 18. | $\$ 366.32$, | $10 \%$, |
| 19. | $\$ 90.06+$ | $11 \%$, |
| 20. | $\$ 561.56$, | $12 \%$, |
| 21. | $\$ 445.19$, | $7 \%$, |
| 22. | $\$ 277.76$, | $8 \%$, |
| 23. | $\$ 315.64+$ | $5 \%$, |
| 24. | $\$ 95.97$, | $6 \%$, |
| 25. | $\$ 700.70$, | $9 \%$, |
| 26. $\$ 1,150.86$, | $12 \%$, |  |

time.
From Jan. 1, 1886, to Sept. 1, 1887.
" Jan. 1, 1887, to July 12, 1889.

* Jan. 1, 1888, to Sept. 9, 1890.
" Jan. 1, 1890, to Oct. 10, 1891.
" Jan. 1, 1890, to July 1, 1891.
"JJan. 1, 1888, to Oct. 18, 1890.
" Jan. 1, 1892, to July 1, 1894.
" Jan. 1, 1889, to Oct. 1, 1893.
" Jan. 1, 1888, to July 24, 1893.
" Jan. 1, 1892, to Nov. 15, 1895.
* Jan. 1, 1887, to Aug. 6, 1892.
"Jan. 1, 1891, to Nov. 1, 1893.
" Jan. 1, 1890, to Oot. 10, 1899.
" Jan. 1, 1880, to July 20, 1887.


## 34s. To find the principal, the amount, time and rate being given.

Etimple 1.-What prinoipal will amount to $\$ 760.20$ in 2 yrs .7 mos . at $8 \%$ ?

Soldtion 1.

$$
\begin{aligned}
& \$ 1.00 \text { Explanation. } \\
& \text { The amount of } \$ 1 \text { for } 2 \mathrm{yrs} \text {. } \\
& 7 \mathrm{mos} \text {. at } 8 \% \text { is } \$ 1.20 \text {, therefore } \\
& \text { the prinoipal will be as many } \\
& \text { dollars as } \$ 1.20 \frac{2}{3} \text { is contained } \\
& \text { times in } \$ 760.20 \text { or } \$ 630 \text {. Ans. }
\end{aligned}
$$

aCCURATE INTEREST.

Solution 2.
$100 \%+8 \% \times 2 \frac{7}{12}=120 \%$ $1202 \%$ of the principal $=\$ 760.20$
$\begin{aligned} \therefore \quad \text { the principal } & =\$ 760.20 \times \frac{100}{120.2} \\ & =\$ 630 . \text { Ans. }\end{aligned}$
$\square$
 prinoipal, hence the amount $=$ $100 \%$ of the principal $+202 \%$ of the principal $=1202 \%$ of the principal, therefore $1202 \%$ of the principal $=\$ 760.20$.

Example 2.-What principal will amount to $\$ 2,235.60$ in 152 days at $5 \%$ ?

Solution.
$100 \%+5 \times \frac{15}{36} 5=10283 \%$ $102{ }_{7}^{\mathrm{e}} \mathrm{e} \%$ of the principal $=\$ 2,235.60$
$\therefore$ the principal $=\$ 2,235.60 \times$

$$
\frac{100}{102_{\sigma_{s}}}=\$ 2,190 . \quad \text { Ans. }
$$

## Explanation.

Interest for each year $=5 \%$ of the principal, and for 152 days $=$ $5 \% \times \frac{152}{365}=2{ }^{6} 3 \%$ of the principal, ana therefore $102{ }_{7}{ }^{3} 3 \%$ of the principal $=\$ 2,235.60$, the amount. role.
Divide the given amount by the amount on $\$ 1$ for the given time and rate.

## EXERCISE 78.

What sum must be put out at interest for-

1. 2 years at $4 \%$ to amount to $\$ 540$.
2. 4 " $6 \%$ " $\$ 2,480.00$.
3. $6 \quad 2 \frac{1}{2} \% \quad$ " $\$ 2,760.00$.
4. 3 " $3 \%$ " $\$ 87.20$.
5. 10 " $7 \%$ " $\$ 342.00$.
6. 8 " $5 \%$ " $\$ 616.00$.
7. $2 \frac{1}{2}$ " $2 \%$ " $\$ 53.00$.
8. 3 子 " $6 \%$ " $\$ 120.00$.
9. $7 \frac{1}{2}{ }^{\circ} \quad 8 \% \quad$ " $\$ 960.00$.
10. $4 \frac{1}{4} \quad 3 \% \quad$ " $\$ 1,353.00$.
11. 98 " $1 \%$ " $\$ 175.60$.
12. 6\% " $6 \%$ " $\$ 360.00$.
13. 3 yr .1 mo , at $4 \%$ " $\$ 1,011.00$.
14. 2 yr .5 mo " $6 \%$ " $\$ 11450$.
15. 3 yr .7 mo " $8 \%$ " $\$ 386.00$.
16. 1 ₹゙ 8 mo. " $3 \%$ " $\$ 945.00$.
17. 2 yr .2 mo. " 6 \% " $\quad \$ 1,03000$.
18. 1 yr .6 mo . at $3 \frac{1}{3} \%$ to amount to $\$ 840.00$.
19. 2 yr .8 mo . $8 \%$ " $\$ 1,092.00$.
20. 1 yr .9 mo " $10 \%$ " $\$ 940.00$.
21. 5 yr. 2 mo. " $12 \%$ " $\$ 972.00$.
22. 3 yr. 1 mo. " $6 \%$ " $\$ 1,185.00$.
23. 45 da. " $6 \%$ " $\$ 1,470.80$.
24. 16 da. " $7 \frac{7}{2} \%$ " $\$ 1,098.60$.
25. 12 da. " $3 \frac{1}{4} \%$ " $\$ 2,923.36$.
26. 87 da " $4 \%$ " $\$ 1,842.40$.
27. 102 da. " $5 \%$ " $\$ 4,441.20$.
28. 318 da. " $3 \%$ " $\$ 2,246.70$.
29. 75 da " $6 \%$ " $\$ 2,586.50$.
30. 150 da " $2 \frac{1}{2} \%$ " $\$ 3,318.75$.
31. 200 da. " $64 \%$ " $\$ 755.00$.
32. 85 da. " $5 \%$ " $\$ 516.95$.
33. 174 da
" $3 \frac{1}{3} \%$ " $\$ 593.28$.
34. 312 da. " $62 \%$ " $\$ 462.96$.

## 349. Find the time, the principal, interest and rate being given.

 $8 \%$ ?Example 1.-In what time will $\$ 607.50$ produce $\$ 125.55$ interest at

## Solution.

6.075
$\frac{8}{48.60}=$ Int. for 1 yr . at $8 \%$.
48.60 ) $125.55\left(2 \frac{7}{12}\right.$
$1 \mathrm{yr} . \times 2 \frac{7}{12}=2 \mathrm{yrs} .7$ mos. Ans.
Example 2. -In what time will $\$ 584 \mathrm{p}$ oduce $\$ 6.72$ interest at $4 \%$ ?

Exilanation
The interest for 1 year at 8\% is $\$ 48.60$, but the interest is $2{ }^{3} 2$ tines $\$ 48.60 . \quad \therefore$ the time $=$ $2 \frac{7}{12}$ times 1 year $=2$ yrs. 7 mos.

$$
\begin{aligned}
& \text { Solvtion. } \\
& 5.84 \\
& \frac{4}{23.36}=\text { Int. for } 1 \text { yr. at } 4 \% \text {. } \\
& \frac{6.72}{23.36} \times 365=105 \text { days. Ans. } \\
& \text { RULE. }
\end{aligned}
$$

## Explanation.

The interest for 1 year at $4 \%$ is \$23.36, but the interest is only ${ }_{28}^{652}$ of this a and $\therefore$ the time is $\frac{{ }_{2} 872}{2336}$ o yoar $=$ $\frac{872}{2388}$ of 365 days $=105$ days.

Divide the given intercst by the interest of the principal for 1 year at the given rate.

Notes 1.-If the quotient consists of a fraction, or of a whole number and a fraction, reduce the fractioual parts to days by multiplying the fraction by 36 :
2. If the amount is given instead of the interest, find the part omitted and proceed as above.
5. At $190 \%$, any sum of money will double itself in 1 year; therefore, any per cent. will require as many years to double the prinoipal as the given per cent. is contained times in $100 \%$.

EXERCISE 79.

## Find the time-


27. B. loaned $\$ 1,600$ at $6 \%$ until it amounted to $\$ 2,000$. What was the time?
28. Mr. Roper paid $\$ 48$ interest. For what period did he pay it, the principal being $\$ 640$, and the rate $5 \%$ ?
29. Borrowed Jan. 1st, 1889, 860 at $6 \%$, to be paid as soon as the interest amounted to one-half the principal. When is it due?
30. May 18th a speculator bought 1,606 bushels of wheat at $\$ 1.00$ a bushel. He afterwards sold the whole for $\$ 1,658.80$ cash, his profit being equivalent to $6 \%$ per annum on the amount invested. What was the date of the sale?
31. In what time will any sum of money double itself at $4 \%, 5 \%, 6 \%, 8 \%$ and $10 \%$ per annum ?
350. To find rate, when principal, interest, and time are given.

Example,-At what rate will $\$ 1,248$ in 2 years 5 months produce $\$ 135.72$ interest?

## Solution.

$$
\begin{gathered}
1 \geq .48=\text { Int. for } 1 \mathrm{yr} \text {. at } 1 \% \\
\frac{2 \frac{5}{12}}{\$ 30.16}=\text { Int. for } 2 \frac{5}{12} \text { yrs. at } 1 \% \text {. } \\
\$ 30.16){ }^{\$ 135.72\left(4 \frac{1}{2}\right.} \\
1 \% \times 4 \frac{1}{2}=4 \frac{1}{2} \% \text { Ans. }
\end{gathered}
$$

## Explanation.

The interest on \$1,248 for 2 yrs. 5 mos. at $1 \%=\$ 30.16$, bnt the interest is $4 \frac{1}{2}$ times as great as $\$ 30.16$. $\therefore$ the rate per cent. is $4 \frac{1}{2}$ times $1 \%=4 \frac{1}{2} \%$.

Explanation.
$\frac{\$ 135.72}{\$ 1,248.00}$ expresses what frac-
tion the interest is of the prinoition the interest is of the prinoipal for $2 \frac{5}{12}$ years; this fraction divided by $2 \mathrm{r}^{\frac{5}{2}}$ expresses what fraction of the principal the interest is for 1 year; this latter fraction is expressed as per cent. by multiplying by 100 .

Example 2.-At what rate will $\$ 4,380$ in 76 days, produce $\$ 45.60$ interest?

Solution 1.


## Explanation.

Interest on $\$ 4,380$ for 76 days at $1 \%=\$ 9.12$, but the interest is 5 times as great as 9.12 .. the rate is $\bar{o}$ times $1 \%=5 \%$.

$$
\begin{aligned}
& \text { Explanation. } \\
& \frac{\$ 45.60}{\$ 4,380.00} \text { expresses what frac. } \\
& \text { tion of the principal the interest is } \\
& \text { Ans. }{ }^{7885} \text { year; this fraction divided } \\
& \text { by }{ }^{78} 88 \text { expresses what fraction of } \\
& \text { the principal is for } 1 \text { year; this } \\
& \text { latter fraction is expressed as per } \\
& \text { cent. by multiplying by } 100 \text {. }
\end{aligned}
$$

Divide the given interest by the interest of the principal at $1 \%$ for the given time.
Notr.-If the amount be given instead of the interest, find the part omitted and proceed as above.

## EXERCISE 80.

Find the rate-


## COMPOUND INTEREST.

:Ens. Compound Interest is the interest of the principai and of the unpaid interest after it becomes due.

Notes 1.-The simple interest may be added to principal annuelly semi annually, or quarterly, as the parties may agree.
2. Compound interest can not be collected by law, except as per written agreement, bat a creditor may receive it without incurring the penalty of usury.
3. In the Post Office Savings Banks, interest is calculated to the thirtieth day of June in every year, and is then added to and becomes part of the principal money, unless withdrawn.
4. Some Savings and Loan Companies compound interest semiannually.

## 352. To compute compound interest when the principal, rate and time are given.

 at $5 \%$Example 1.-Find the compound interest on $\$ 2,000$ for 3 years
Solution 1.

| Principal . . . . . .. .. .. .. .. .. .. \%2,000.00 |  |
| :---: | :---: |
| Int. for 1st yr. (\$2.000 $\times$.05) .. .. .. .. .. | $2,000.00$ 100.00 |
| Amt. for 1 yr., or 2 nd Principal | $\frac{100.00}{\$ 2,100.00}$ |
| Int. for 2 nd yr . $(2,100 \times, 05)$. Amt. for 2 yrs , | \$2,10.00 10500 |
| Amt. for 2 yrs ., or 3rd Principal Int. for 3rd. | \$2,205.00 |
| Amt. for 3rd yr. .. . | 110.25 |
| Original Principal to be subtr | \$2,315.25 |
| Compound Interest for 3 yrs. .. | $\frac{2,000.00}{8,315.25}$ |

Solution 2.

| $\$ 2,000$ <br> $\frac{1.05}{\$ 2.100}$ | Amt. of $\$ 1$ for $1 \mathbf{~ y r}$. |
| :--- | :--- |
| $\frac{1.05}{\$ 2,205}$ | Amt. of $\$ 2,000$ for $1 \mathbf{y r}$. |
| $\frac{1.05}{\$ 2,315.25}$ | Amt of $\$ 2,100$ for 1 yr. |
| $\frac{2,000}{\$ 315.25}$ | Principal. |
|  | Compound Interest. |

Solution 3.
1.05
1.05
1.1025
1.157625
$\$ 2,315.25$
$\frac{2,000}{4315.25}$

Example 2.-Find the compound interest on $\$ 1,000$ for 2 years 3 months at $8 \%$.

Solution 1.

| $\$ 1,000$ | Principal. |
| :--- | :--- |
| 80 | Int. 1st yr. |
| $\$ 1,080$ | Amt. Ist yr. |
| 86.40 | Int. 2nd yr. |
| $\$ 1,166.40$ | Amt. 2nd yr. |
| $\frac{23.328}{}$ | Int. for 3 mos. |
| $\$ 1,189.728$ | Int. for 3 yrs. 3 mos. |
| $\frac{\$ 1,000}{\$ 189.728}$ | Principal. |
| Compound Interest. |  |

Solution 2. $\begin{aligned} & \$ 1,000 \\ & \frac{1.08}{\$ 1,080}\end{aligned} \quad$ Amt. of $\$ 1$ for 1 yr.
$\frac{1.08}{\$ 1,166.40}$
$\frac{1.02}{\$ 1,189.728}$
$\frac{1,000}{\$ 189.728}$

Solution 3.
1.08
1.08
$1 . \overline{1664}$
1.02
1.189728

1000
$\$ \overline{1189.728}$
$\$ 1000$
$\$ 180.728$
353. The use of the following table will greatly shorten calculations in compound interest.

## Tabli.

Showing the amount of $\$ 1$ or $\mathbf{2 1}$, at different rates for any number of years from 1 to 40.

| Yrs. | 1 per et. | $1 \frac{1}{2}$ per ct. | 2 per ct. | $2 \frac{1}{2}$ per ct. | 3 per ct. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.0100000 | 1.0150000 | 1.0200000 | 1.0250000 | 1.0300000 |
| 2 | 1.0201000 | 1.0302250 | 1.0404000 | 1.0506250 | 1.0609000 |
| 3 | 1.0303010 | 1.0456784 | 1.0612080 | 1.0768906 | 1.0927270 |
| 4 | 1.0406040 | 1.0613636 | 1.0824321 | 1.1038128 | 1.1255088 |
| 5 | 1.0510101 | 1.07728 .10 | 1.1040808 | 1.1314082 | 1.1592788 1.1592 |
| 6 | 1.0615202 | 1.0934433 | 1.1261624 | 1.1596834 | 1.1910523 |
| 7 | 1.0721354 | 1.1098450 | 1.1486856 | 1.1886857 | 1.2298738 |
| 8 | 1.0828567 | 1.1264926 | 1.1716593 | 1.2184029 | 1.2667700 |
| $\stackrel{9}{9}$ | 1.0936853 | 1.1433900 | 1.1950925 | 1.2438629 | 1.3047731 |
| 10 | 1.1046221 | 1.1605408 | 1.2189944 | 1.2800845 | 1.3439163 |
| 11 | 1.1156683 | 1.1779489 | 1.2433743 | 1.3120866 | 1.3842338 |
| 12 | 1.1268250 | 1.1956182 | 1.2682417 | 1.3448888 | 1.4257608 |
| 13 | 1.1380933 | 1.2135524 | 1.2936066 | 1.3785110 | 1.4685337 |
| 14 | 1.1494742 | 1.2317557 | 1.3194787 | 1.4129738 | 1.5125897 |
| 15 | 1.1609690 | 1.2502321 | 1.8458683 | 1.4482981 | 1.5579674 |
| 16 | 1.1725756 | 1.2689855 | 1.3727857 | 1.4845056 | 1.6047064 |
| 17 | 1.1843044 | 1.2850203 | 1.4002414 | 1.5216182 | 1.6528476 |
| 18 | 1.1961475 | 1.3073406 | 1.4282462 | 1.5596587 | 1.7024330 |
| 19 | 1.20814100 | 1.3269507 | 1.4568111 | 1.5986501 | 1.7535060 |
| 20 | 1.2201900 | 1.3468550 | 1.4859474 | 1.6386164 | 1.8061 112 |
| 21 | 1.2323919 | 1.3670578 | 1.5156663 | 1.6795818 | 1.8602945 |
| 22 | 1.2447159 | 1.3875637 | 1.5459796 | 1.7215714 | 1.9161084 |
| 23 | 1.2571630 | 1.4083772 | 1.5768992 | 1.7646106 | 1.9735865 |
| 24 | 1.2697346 | 1.4295028 | 1.6084372 | 1.8087259 | 2.0327941 |
| 25 | 1.2824320 | 1.4509454 | 1.6406059 | 18539441 | 2.0937779 |
| 2 | 1.2925563 | 1.4727095 | 1.6734181 | 1.9002927 | 2.1563912 |
| 27 | 1.3082089 | 1.4948002 | 1.7068864 | 1.9478000 | 2.2212890 |
| 28 | 1.3212910 | 1.5172222 | 1.7410242 | 1.9964950 | 2.2879276 |
| 29 | 1.3345039 | 1.5399805 | 1.7758446 | 2.0464073 | 2.3565655 |
| 36 | 1.3478490 | 1.5630802 | 1.8113615 | 2.0975675 | 2.4272624 |
| 31 | 1.3613274 | 1.5865264 | 1.8475888 | 2.1500067 | 2.5000803 |
|  | 1.3749407 | 1.6103243 | 1.8845405 | 2.2037569 | 2.5750827 |
|  | 1.3886901 | 1.6344792 | 1.9222314 | 2.2588508 | $2.6523 \quad 352$ |
| 4 | 1.4025770 | 1.6589964 | 1.9606760 | 2.3153221 | 2.7319053 |
| 5 | 1.4166028 | 1.6838813 | 1.9998895 | 2.3732051 | 2.8138624 |
| 8 | 1.4307688 | 1.7091395 | 2.0398873 | 2.4325353 | 2.8982783 |
| 8 | 1.4450765 | 1.7347766 | 2.0806850 | 2.4933187 | 2.9852266 |
| 8 | 1.4595272 | 1.7607983 | 2.1222987 | 2.5556824 | 3.0747884 |
| 9 | 1.4741225 | 1.7872103 | 2.1647447 | 2.6195744 | 8.1670269 |
| 16 | 1.4888637 | 1.8140184 | 2.2080396 | 2.6850638 | 3.2620377 |

Table.


Table.


Notrs 1.-It each of the numbers in the table be diminished by 1 , the remainder will denote the compound interest of $\$ 1$, instead of its 2. If interest is compounded semi-annually, take $\frac{7}{7}$ the given rate and twice the number of years; if componnded quarterly, take 1 the given rate for 4 times the number of years, eto.
3. The amount for any number of years not .
computed by finding the products of the amou in the table may be of years whose sum equals the given time.
4. To find the amount of any given pris
multiply the principal by the amount of $\$ 1$ foral at compound interest,
5. If the time contains parts of a for the time and rate.
amount due for the full periods, and to this add its inters, find the months or days.

## EXERCISE 81.

Find the amount and the compound interest of-

1. $\$ 312$ for 3 years at $6 \%$; $\$ 800$ for 4 years at $4 \%$.
2. $\$ 640$ for 4 years at $5 \%$; $\$ 376$ for 3 years 8 months and 15 days at $6 \%$.
3. $\$ 1,200$ for two years 4 months at $4 \frac{1}{2} \%$; for 3 years 8 months at $7 \%$.
4. $\$ 400$ for 1 year 6 months at $7 \%$, payable semiannually.
5. $\$ 2,000$ for 1 year at $8 \%$, payable quarterly.
6. $\$ 1,000$ for 23 years at $7 \%$.
7. $\$ 750$ for 12 years at $3 \%$.
8. $\$ 920$ for 8 years at $5 \%$.
9. $\$ 2,600$ at $6 \%$, from Jan. 1st, 1870, to Jan. 1st, 1894.
10. $\$ 1,410$ at $8 \%$, from March 30th, 1889, to August 15th, 1894.
11. What is the amount of $\$ 3,500$ for 5 years at $5 \%$
mpound interest? 12. What is the amount of $\$ 1,350$ for 12 years at $7 \%$ ?

IMAGE EVALUATION TEST TARGET (MT-3)




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13. What is the compound interest of $\$ 1,469$ for 15 years at at $\mathbf{3 \%}$.
14. What is the compound interest of $\$ 2,500$ for 24 years at $6 \%$.
15. What is the compound interest of $\$ 1,650$ for 30 years at $3 \frac{1}{2} \%$.
16. What is the amount of $\$ 1,800$ for 3 years at $6 \%$ compound interest, payable semi-annually?
17. What is the amount of 1,500 for 2 years, at $12 \%$ compound interest, payable quarterly?
18. What is the compound interest of $\$ 5,000$ for 2 years, at $6 \%$, if the interest is due annually? If the interest is payable half-yearly? If the interest is payable quarterly?
19. By how much does interest compounded semi-annually exceed simple interest, on $\$ 400$, for 2 years 6 months at $7 \%$ ?
20. What is the amount of $\$ 2,400$ from May $1 \mathrm{st}, 1887$, to Jan. 14th, 1890, interest compounded half-yearly, at $5 \%$ ? What is the amount, if the interest is compounded yearly? What is the amount, at simple interest?
21. What is the compound interest on $\$ 7,325$ for 2 years 2 months at $7 \%$ ?
22. Find the compound interest on $\$ 3,333$ at 3$\} \%$ semiannually for 1 year 7 months.
23. What amount was due March 25th, 1886, on $\$ 1,512$ hocrowed Jun. 25th, 1885, with compound interest at $1 \frac{1}{2} \%$ quarterly?
24. What is the amount of $\$ 4,615$ at compound interest for 2 years 5 months at $8 \%$ ?
25. Find the amount of $\$ 3,500$ at compound interest from Oct. 29 th, 1888 , to Nov. 15 th, 1889 , at $2 \%$ quarterly.
26. How much greater, at compound than at simple interest, would be the amount of $\$ 1,568$ in 3 years 8 months at $6 \%$ ?
27. Find the amount due Sept. 18th, 1889, on $\$ 450$, loaned Sept. 18th, 1886. Interest compounded annually at $4 \frac{1}{2} \%$.
28. What is the interest, compounded every six months, of $\$ 600$ from July 1st, 1890 , to July 1st, 1894, at $8 \%$ ?
29. What will $\$ 16,000$ invested Jan. 14th, 1888, amount to Sept. 16th, 1893, at $10 \%$, interest payable half-yearly?
30. How much must a lady invest when her son is 12 years old, that, on arriving at 21 , he may have $\$ 25,000$, the rate being $6 \%$ and the interest compounded semiannually?

To find the principal or present worth of an amount at compound interest, divide the given amount by the amount of 81 for the given time and rate at compound interest, (i. e., the saine method for finding the present worth as in simple interest.)
81. What is the present worth of $\$ 6,086.25$ due in 8 years, at $6 \%$ compound interest?
32. What principal at compound interest will amount to $\$ 2,375.92$ at $5 \%$ in 14 years ?
83. What is the present worth of $\$ 2,521.81$ due in 14 years, at $\dot{6} \%$ compound interest?
34. What principal at $10 \%$ will amount to $\$ 265.83$ in 10 years, interest payable semi-annually?
85. What sum at compound interest at $4 \%$, interest due annually, will amount to $\$ 1,000$ in three jears?
36. What sum would have to be put out at $6 \%$, interest payable every six months, to produce $\$ 548.8456$ compound interest in 5 years?
37. At what rate would $\$ 500$ have to be loaned, to amount to $\$ 1079.46$ in 10 years, the interest being compounded annually?

## DISCOUNT.

354. Discount is an abatement or allowance made from the amount of a debt, a note or other obligation.
355. The term discount is often used without reference to time to imply an abatement at a certain rate per cent. on a price asked.
356. When Time enters in as an element, two kinds of Discount are distinguished, viz.: True Discount and Bank Discount.

## TRUE DISCOUNT.

357. The Present Worth of a debt, note or other obligation, payable at a future time without interest, is such a sum as, being placed at interest at a legal rate, will amount to the given sum when it becomes due.
358. True Discount is the difference between any sum of money payable at a future time and its present worth, and is equal to the interest on the present worth.
Infustration.-Suppose A. owes B. $\$ 106$ payable a year hence without interest. The ourrent rate of interest being $6 \%$, the present worth of the debt is $\$ 100$, because that sam would amount to $\$ 106$ in 1 year at $6 \%$.

The true discount is $\$ 106-\$ 100$ or $\$ 6$, which is evidently the interest on the present worth $\$ 100$, for 1 year at $6 \%$.
359. To find the present worth and true discount, the face of the debt, rate per cent. per anaum, and time being given.

Erample.-Find the present worth and true disoount of a debt of $\$ 1,860$ for 6 years at $6 \%$.
solution.
Interest on $\$ 1.00$ for 6 years at $6 \%=\$ .06$
$\therefore \quad \$ 1.36$ has for its present worth $\$ 1.00$


1. Divide the face of the delt by the amount of $\$ 1$ for the given time, and the quotient will be the present worth.
2. Subtract the present worth from the face of the debt, and the remainder will be the true discount.
3. To find the true discount of a debt. interest at $5 \%$.

Solution.
Interest on $\$ 1.00$ for 4 yars at $5 \%=\$ .20$.
$\therefore \quad \$ 1.20$ has for its discount $\$ .20$

$$
\begin{array}{llll}
\$ 1 & \text { " } & & \text { " } \\
\$ 1,781.40 & \text { " } & \text { " } & \frac{.20}{1.20} \\
& & & \frac{1,781.49 \times 20}{1.20} \\
& & \$ 296.90, \text { discount. }
\end{array}
$$

Divide the interest of the debt for the given time and riate by the amount of $\$ 1$, and the quotient will be the discount.

## EXERCISE 82.

Find the present worth and true discount -

1. Of $\$ 475.50$ at $7 \%$ due in 2 yr .9 mo .
2. Of $\$ 661.50$ at $7 \%$ due in 3 yr .9 mo .
3. Of $\$ 500$ at $5 \%$ due in 11 mo.
4. Of $\$ 4,261.33$ at $4 \frac{1}{2} \%$ due in 1 yr .6 mo .
5. Of 31,575 at $7 \%$ due in 1 yr .3 mo. 15 da.
6. Of $\$ 860$ at $6 \frac{1}{2} \%$ due in 90 da .
7. Of $\$ 67840$ at $4 \frac{1}{2} \%$ due in 16 mo .
8. Of $\$ 715.20$ at $3 \frac{1}{2} \%$ due in 1 yr .4 mo .
9. Of $\$ 990.75$ at $10 \%$ due in 73 da.
10. Of $\$ 1,215.45$ at $8 \%$ due in 219 da.

Find the true discount on-
11. $\$ 1,500$ due in 3 yr .6 mo . at $6 \%$.
12. $\$ 3,550$ due in 90 da. at $7 \%$.
18. $\$ 4,960.75$ due in 18 mo . at $6 \frac{1}{2} \%$.
14. $\$ 960.40$ due in 73 da. at $10 \%$. 15. \$625.13 due in 8 mo. at $7 \frac{3}{10} \%$.
16. Which is the better, to buy flour at $\$ 8$ a barrel, on 6 months' credit, or $\$ 7.50$ cash, money being worth $8 \%$ ?
17. What is the difference between the interest and true discount of $\$ 1,650$, at $6 \%$, due in 8 months?
18. Which is worth the most, $\$ 640$ in 12 months, $\$ 620$ in 6 months, or $\$ 600$ cash, money being worth $8 \%$ ?
19. Bonght a farm for $\$ 2,964.12$ ready money, and sold it again for $\$ 3,665.20$, payable in 1 year, 6 months. How much would be gained in ready money, reckoning true discount at $8 \%$ ?
20. Having bought a house for $\$ 5,048$ cash, $I$ at once sold it for $\$ 7,000$, to be paid in 18 months without interest. If money is worth $8 \%$ per annum, did I gain or lose, and how much?
21. A man bought a flouring mill for $\$ 10,000$ cash, or for $\$ 12,000$ payable in 6 months, or $\$ 15,000$ payable in 1 year 3 months. He accepted the latter offer; did he gain or lose, and how much, money being worth to him $10 \%$.
22. Goods to the amount of $\$ 510$ were sold on 6 months' credit. If the selling price was $\$ 30$ less than the goods cost, and money is worth $6 \%$ per annum, how much was the loss and the per cent. of loss?
23. A speculator bought 120 bales of cotton, each bale containing 488 pounds, at 9 cents a pound, on a credit of 9 months for the amount. He immediately sold the cotton for $\$ 6,441.60$ cash, and paid the debt at $8 \%$ discount ; how much did he gain?
24. How much must be discounted for the present payment of a debt of $\$ 8,741.50, \$ 2,000$ of which is on credit for 5 months; $\$ 3,000$ for 8 months, and the remainder for 15 months, money being worth $10 \%$ per annum?
25. A merchant bought a bill of goods for $\$ 2,150$, on 6 months' credit, and the seller offered to discount the bill at $5 \%$, for cash. If money is worth $7 \frac{1}{2} \%$ per. annum, how much would the merchant gain by accepting the seller's offer.
26. A merchant bought a bill of goods on 6 months' credit amounting to $\$ 1,450$. What will he gain by present payment of the bill, if allowed $5 \%$ off, money being worth $\frac{3}{4} \%$ a month ?
27. A dealer bought grain to the amount of $\$ 2,700$, on 4 months' credit, and immediate. $y$ sold it at an advance of $10 \%$. If from the proceeds of the sale he paid the present worth of his debt at a rate of discount of $8 \%$ per anuum, how much did he gain?
28. After carrying a stock of sill for 4 months, I soli it at an advance of $30 \%$ on first cost, extending to the purchaser a credit of one year without interest. If money is worth $5 \%$ per annum, what was my per cent. of profit or loss ?
29. Bought a house for $\$ 3,786$ ready money, and sold it for $\$ 5,250$, payable in 1 year 6 months. How much would be gained in ready money, discounting at the rate of $8 \%$ ? How much, discounting at the rate of $6 \%$ ?
30. The asking price of a hardware stock is $\$ 5,460$, on which a trade discount of $25 \%, 15 \%$, and $10 \%$ is offered, and a credit of 90 days on the selling price. If money is worth $5 \frac{1}{2} \%$, what sum should be discounted for the payment of the bill ten days after its purchase?

## BANIK DISCOUNT.

361. Bank Discount is a deduction usually made by banks for paying a note before it is due. This deduction is the interest on the face of the note for the time it has to run, including three additional days, called Days of Grace.
362. Days of Grace are three days usually allowed for the payment of a note, after the expiration of the time specified in the note.
363. The Proceeds of a note is the amount received by the holder from the bank when the note is discounted. It is the amount of the note at maturity less the interest on that amount for the term of discount.

Illostration.-A person holds a note.for $\$ 1,000$ payable in 73 days, including the days of grace. Wishing to use the money immediately he indorses the note and offers it to his bank for discount. If both maker and indorser are considered responsible, the bank retains the note, and if the legal rate is $6 \%$, deducting $\$ 12$ (the interest of $\$ 1,000$ for 73 days) pays over the balance $\$ 988$ to the holder. The note is thus discounted; the bank discount is $\$ 12$; the proceeds are $\$ 988$.
364. Negotiable paper commonly includes all orders and promises for the payment of money, the property interest in which may be negotiated or transferred by indorsement.
365. A Promissory Note is a written, or partly written and partly printed, agreement to pay a certain sum of money, either on demand or at a specified time.
366. The Face of a Note is the sum for which it is given.
367. The Maturity of a note is the expiration of the time including days of grace.
368. The Time in bank discount is always the number of days from the date of discounting to the date of maturity. 369. The Term of Discount is the time the note has to run after being discounted.
370. Value of a note at its maturity is its face, if it does not bear interest; if the note is given with interest, its value at maturity is its face plus the interest for the time, including days of grace.

Notes 1. In Ontario and Manitoba promissory notes and drafts are governed by commercial huw as to days of grace, protest and notice. In these provinces notes do not bear interest, unless expressed on the face, until after maturity, from which period however, the legal rate-six per cent.-con be recovered. If a rate of interest is named on face of note, and in addition to the words, " with interest at the rate of -," the words "until paid" are added, the named rate can be oollected till date of payment of note, otherwise, without these words (or others of similar import), the expressed rate only during period of currency, and legal rate from maturity till payment can be collected.
2. The person who promises to pay is oalled the maker: the person in whose favour the promise is made is called the payee, and the person who writes his name on the back of the note is called an endurser and is responsible for the payment of the note.
3. A note is non-negotiable when it is made payable only to the person whose name is mentioned in it. Such a note cannot be transferred; it must be held by the payee until it falls due.
4. A note is transferable only when it is made payabls to the order of the payee or to bearer.
5. The maker of a note is the one primarily liable, but each endorser is liable to succeeding holders in default of payment by the maker. A subeequent endorser is not liable to a prior endorser.
6. When the payee writes only his name it is termed a blank endorsement, and it has the effect of rendering the note payable afterwards to any bona fide holder. The payee thus becomes the endorser and as such is responsible for payment.
7. If the payee writes above his signature, "Pay to the order of A. B." it is called a full endorsement. In this case A. B. will have to endorse it before he can negotiate it.
8. If the payee writes above his signature, "Pay to A. B. only," it is termed a restrictine endorsement.
9. If the endorser does not wish to render himseit liable for payment he should write, " Without recourse to me," above his name. This is called a qualifed endorsement.
10. When a note is made payable to bearer it is negotiabie without endorsement, delivery being all that is necessary.
11. In calculating the date of matarity of a note, the three days grace must be allowed after the time expressed, that is, it falis due on the third day after its term has expired. The day on whioh the note is dated is not counted in computing the date of maturity.
12. When a note becomes due whioh happens on the third day after the time expressed, it must be presented for payment during business huurs at the place mentioned in it. If no plase is stated it should be presented at the maker's place of business or at his residence.
13. Should the maker refuse to pay it, the proper demand being made, it is the duty of the hoider to give due notice to all the parties to it. He may have it protested if he chooses.
371. A Protest is a declaration in writing by a Notary Public, giving legal notice to the maker and endorsers of a note of its non-payment. In Ontario a note must be protested on the day of its maturity, otherwise the endorsers are released from all obligation to pay the note.

Notes 1. When a note becomes due on Sunday or a legal holiday, it must be paid on the day following.
2. The person paying a note has a right to a receipt, which is usually written on the back of the note.
3. The person who pays a note has a right to it as his voncher, if it is negotiable, but not otherwise.
4. When a note is made payable with interest it bearsinterest from the date of it, and not merely from ite maturity. In suoh a oase the interest is part of the debt.
5. When a note bears interest, the discount is computed on the face of the now with the interest added.
6. When the term of a note is given in months, calendar months are meant and no allowance is made for a defioiency in the number of days in any month. This being the case the student will see that four notes drawn at 2 months and bearing dates, Dec. 28, Dec. 29, Deo. 30, Dec. 31, respectively, will become due on the same day, viz. : Mareh 3rd, of next year.
7. When the time is expressed in days, the day of maturity is fonnd by counting forward from the date of the note the number of days named in the note, and the three days of grace. When the time is in months, the day of maturity is found by connting the number of calendar months, and the three days of grace.
372. Banks in discounting notes always reckon discount for an exact number of days from the time of discounting to date of maturity. Thus on a note maturing July 5th, and discounted May 25th, the term of discount would be reckoned as follows: 6 days in May, +30 days in June +5 days in July $=41$ days.

## 373. To find the bank discount and proceeds of a

Example 1.-Find the bank discount and proceeds of a note for $\$ 684$, due 90 days hence, at $7 \%$.

Solution.
The term of discount is 93 days. Interest of $\$ 684$ for 93 days at $7 \%=\$ 12.20=$ Bank discount.

$$
\$ 68 t-\$ 1220=\$ 671.80=\text { Procoeds } .
$$

Example 2.-A note of $\$ 375$ dated October 23rd, payable in 30 daya, with interest at $7 \%$, is disconnted at a bank November 12th at $8 \%$. Find the proceeds.

Soldtion.
The date of raturity is November 26th. The note bears interest for 34 days.
$\$ 2.479$ ( 360 da. int.) less $\frac{1}{78}$ of $\$ 2.479=\$ 2.45$ (actual int.)
The amount of note at matarity is $\$ 375+\$ 2.45=\$ 377.45$. or 14 day.

## BANK DISCOUNT.

$\$ 1.174$ ( 360 da . int.) less $\frac{1}{3}$ of $\$ 1.174=\$ 1.15$ (actu.ll int.)
$\$ 377.45=$ Amt. of note at maturity.
$1.15=$ Disot. for time held by bank.
$\overline{\$ 376.30}=$ Proceeds.
Example 3.-A note of $\$ 750$ dated August 4th, 1888, payable in $\epsilon$ Find the proceeds.

Solution.
The date of maturity is February 7th, 1889.
The note bears interest from August 4th, 1888, to February 7th. 1889, of 187 days.

| $\$ 7.50$ | $=$ | Int. for | $\frac{60}{}$ da. at $6 \%$ |
| ---: | :--- | ---: | :--- |
| 22.50 | $=$ | $"$ | 180 |
| .75 | $=$ | $"$ | 6 |
| $\frac{.125}{}$ | $=$ | $"$ |  |
| $\$ 23.375$ | $=$ | $"$ | $\frac{1}{187}$ |

$\$ 23.375$ ( 360 da . int.) less ts of $\$ 23.375=\$ 23.06$ (actual int.)
The amount due at maturity is $\$ 750+\$ 23.06=\$ 773.06$.
The note is held by the bank from October 20th, '88, to February 7th, '89, or 110 days.

| $\$ 7.7306$ | $=$ | Int. for | 60 da. at $6 \%$ |
| ---: | :--- | ---: | :--- |
| 3.8653 | $=$ | $"$ | 30 |$\quad$ "

$\$ 16.53$ ( 360 da. int.) less $\frac{1}{7 s}$ of $\$ 16.53=\$ 16.31$ (atotual int.) $\$ 773.06=$ Amt. of note at maturity. $16.31=$ Disct. for time held by bank. $\overline{\$ 756.75}=$ Proceeds.

Example 4.-Find day of maturity, the time to run, the discount, and proceeds of the following note: $\$ 1,800$.

Otrawa, February 3rd, 1889. mise to pay John Craig, or ordel; interest at $6 \%$ Thousand Eight Hundred Dollars, value reoeived, with

Discounted May 22nd, 1889, at $7 \%$.
Thomas Cowan.

# BdNK DISCOUNT. 

Solution.
Date of muturity will be 5 months and 3 days from February 3rd, 1880,or July 6th, 1889.

The time to run will be the interval between the date of disoount, May 22 nd , and July 6 th, -or 45 days.

As the note bears interest, the disoonnt must be oomputed on the amount of $\$ 1,800$, from February 3 rd to Jnly 6 th, or 153 days.
Interest on $\$ 1,800$ for 153 days at $6 \%=\$ 45.27+$
The amount of note at maturity $=\$ 1,800+\$ 45.27=\$ 1,845.27$.
The note is held by the bank from May 22nd, to July 6th, or 45 days.
Interest on $\$ 1,845.27$ for 45 duys at $7 \%=\$ 15.92=$ discount.
Proceeds $=\$ 1,845.27-\$ 15.92=\$ 1,829.35$.
BOLE.

1. For the bank discount, find the interest on the face of the note (or, if the note bears interest, on the amount due at maturity), at the given rate, from the date of discount to the date of maturity.
2. For the proceeds, subtract the bank discount from the face of the note (or, if the note bears interest, from its amount).

## EXERCISE 83.

Find the bank discount and proceeds of a note for-

1. $\$ 440.00$, payable in 90 da., discounted at $6 \%$ on the day drawn.
$\begin{array}{lrrrr}\text { 2. } & 8500.00 \\ \text { 3. } 81,000.00 & \text {, } & \text { " } & 60 & 45\end{array}$
2. $\$ 140.25$,
$5 \%$
64
47\% "
Find the date of maturity and proceeds of the following notes:

|  | Date of Note. | Time. | Face. | DATE 0 if Discountr. | Rate or Discount. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 5. | January 20.. | $90 \mathrm{da} .$. | \$2,500 . |  |  |
| 7. | $\begin{array}{lll}\text { May } \\ \\ \text { June } & . . & . . \\ \end{array}$ | 60 "... | \$2,500.. | January 20.. | 6\%. |
| 8. | July $27 . .$. | 60 das. . . |  | July 18... ... | 8\%. |
| 9. 10. | November May 27 | 90 ".. | \$8,200.. $\$ 6.000 .$. | September2. | 6\%. |
| 10. | May $27 .$. | $6 \mathrm{mos} ..$. | \$ $\%$ \%, 8800 | November 28 August $15 .$. | $8 \%$ $5 \%$ |

Find the proceeds and date of maturity of the following notes discounted through a broker, his commission being $\frac{1}{4} \%$ of the face of the notes:

|  | Date of Note. | Time. | Face. | Date of Discount. | Rate or <br> Discount. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 11. | February 18. | $4 \mathrm{mos}$. . | \$2,000 | February |  |
| 13. |  | 90da... | \$6,000.. | February 18. June $12 .$. | 5\%. |
| 14. | March $3 .$. | 120 " 6 mos. | 85,500.. | January 10.. | 7\%. |
| 15. | May $18 .$. |  |  | April 30.. .. | $8 \%$. |
| 16. | Jinuary 3 .. | 60 da . | \$9,280 .. | May $18 .$. | $\begin{aligned} & 4 \% . \\ & 6 \% . \end{aligned}$ |

17. Find the proceeds of a note of $\$ 850$, due in 3 months, at $6 \%$ ?
18. Find the proceeds of a draft of $\$ 885$, on 60 days, at. $6 \%$ ?
19. Find the maturity, the term of discount and the proceeds of a note of $\$ 5,250$, on 60 days, dated July 1st, 1889, and discounted August 21st, 1889, at $5 \%$.
20. Find the difference between the true and bank discount on $\$ 6,000$ for 1 year, allowing each 3 days grace, at $7 \%$ ?
21. A merchant bought $\$ 6,800$ worth of goods for cash, sold them on 4 montiss, at $15 \%$ advance, and got the note discounted at $6 \%$ to pay the bill. How much did he make?
22. \$652.45.

Five monthe after Ottawa, Jan. 25th, 1889. Charles Barm after date I promise to pay to the order of value rearrett siz hundred and fifty-two and $\frac{45}{100}$ dollars. Diser with interest at six per cent.
Discounted at $4 \frac{1}{2} \%$ Mar. 15 . William Kimball.
23. $\$ 215$.

Peterborough, Jan. 28th, 1889.
Thirty days after date, I promise to pay to the order of James Fogg two hundred and fifteen dollars, value received. Discounted at $6 \%$, Feb. Brd.

John Rogers.
24. \$2,017.85.

Three months after date I Gait, Jan. 14th, 1889. John Brown two thousand and mise to pay to the order of value received.
Discounted at $10 \%$, Mar. 1st.
Timothy Bruce.
25. $\$ 4,760$

Ninety days after date I Guelph, Jan. 1st, 1889. James Pike four thousand seromise to pay to the order of value received.

Discounted at $7 \frac{1}{2} \%$ Feb. 15 th.
Wifi I Clement.
26. $\$ 5,000$.

Six months after date I promise to pay to John Adams or order five thousand dollars, value received, with interest at seven per cent.

Discounted at $8 \%$, Dec. 81st.
Whliam Dunno.
27. \$9,040.

Sixty days from date I London, Jan. 19th, 1889. Charles Carroll nine tho mise to pay to the order of received.

Discounted $5 \frac{1}{2} \%$, Feb. 16th.
James Monroe.
28. \$650.

Six months from date we join Berlin, Nov. 3rd, 1888. pay to the order of Charles Fall and severally promise to lars, value received, with interest hundred and fifty dolDiscounted at $7 \%$, Jan. Ard, 1889.

## john Henderson. James Hendricks.

29. A note for $\$ 3,600$ with interest, dated Jan. 15th, 1889, and payable 3 months after date, was discounted at a bank Feb. 15th, the legal rate being $7 \%$; with the procoeds was paid on account $40 \%$ of at bill due that day. How much remained due on the bill?
30. A merchant sold some goods that cost him $\$ 840$, at a profit of 12 , and took in payment a four-month note dated May 15th, which after 52 days he got discounted at a bank for $7 \%$. How much did he receive from the bank;
31. A merchant, having sold 200 barreis of flour at $\$ 6.80$ a barrel, and having taken in pay sent a 30 -day note, found, on getting the note discounted at a bank the day of its date for $7 \%$, that he had realized on the transaction a cash profit equal to $300 \%$ on the bank discount. What had the flour cost him per barrel?
32. A person owing for 117 A .5 sq . rd, of land, which he had bought for $\$ 32$ an acre, paid on account the proceeds of a sixty-day note for $\$ 2,000$, which he got discounted at a bank, for $7 \%$, on the day it was drawn. How much remained due?
33. I paid in cash $\$ 950$ for an engine, and sold it the same day for $\$ 975$, taking a 60 -day note, which I discounted at a bank at $8 \%$. What was my gain or loss?
34. Perkins, Ince \& Co's bank account is overdrawn $\$ 11,546.19$; they now discourt, at $6 \%$, a 90 -day note for $\$ 3,975.21$; a 60 -day note for $\$ 5,514.25$; a 30 -day note for $\$ 1,546.19$; a 20 -day note for $\$ 2,546.85$; proceeds of all to their credit at the bank. What is the condition of their bank account after they receive credit as above?
35. W. Darling \& Co.'s benk account is overdrawn $\$ 12,915.47$; they now discount, at $6 \%$, a 90 -day note for $\$ 2,428.40$; a 60 -day note for $\$ 6,311.25$; a 30 -day note for $\$ 1,120.50$; a 20 -day note for $\$ 4,500$; à 10 -day note for $\$ 1,550.50$; Proceeds of all to their credit at the bank. What is the condition of their bank account after they receive the above credits ?
n \$840, at lonth note sounted at the bank:
flour at day note, the day masaction What
which be proceeds inted at w much it the I disss ? drawn te for ote for $f$ all to their
drawn note note te for bank. they
bANK DISCOUNT.
36. To find the face value of duce a given sum whe value of a note that shall pro-Example-For discounted at bank.
days, that, when discount muoh must a note be drawn, paynble in 70 Solotion.
Bank discount of $\$ 1$ for 73 days at $8 \%=\$ .016$.

$$
\begin{aligned}
& \$ 1-\$ .016=\$ .984 \quad \text { proceeds of } \$ 1 . \\
& \$ .984=\text { proceeds of } \$ 1 \\
& \$ 1=\quad " \quad \frac{1}{.984} \\
& \$ 1,968=\quad \text { " } \quad \frac{1,968}{.984}=\$ 2,000 . \quad \text { Ans. } \\
& \text { rule. }
\end{aligned}
$$

Divide the given sum by the proceeds of $\$ 1$ for the :iren rate and time, and the quotient will be the fuce value of the note.

## EXERCISE 84.

Find the face of note or draft-

13. What sum, due 73 days hence, at $7 \%$, should be discounted, so that the present payment may be $\$ 900$ ?
14. What is the face of a note at 60 days, the proceeds of which, when discounted at bank at $6 \%$, ar6 $\$ 1,275$ ?
15. If a merchant wishes to draw $\$ 5,000$ at bank, for what sum must he give his note at 90 days, discounting at $6 \%$ ?
16. The avails of a note having 3 months to run, discounted at a bank at $7 \%$, were $\$ 276.84$. What was the face of the note?
17. For what sum must a note be drawn at 80 days, to net $\$ 1,200$ when discounted at $5 \%$ ?
18. Find the face of a 6 months' note, the proceeds of which, discounted at $2 \%$ a month, are $\$ 496$.
19. Owing a man $\$ 575$, I give him a 60 day note. What should be the face of the note, to pay him the exact debt, if discounted at $1 \frac{1}{2} \%$ a month ?
20. James T. Fisher buys a bill of merchandise in Montreal at cash price, to the amount of $\$ 1,486.90$, and gives in payment his note at 4 months at $7 \frac{1}{2} \%$. What must be the face of the note?

## 375. Given, the rate of interest to find the corresponding rate of bank discount.

Example.-A broker buys a 70 day note at such a discount thet his money earns him $10 \%$. What is his rate per cent. of discount?

> Solution.

70 day note $=73$ days' time.
Interest on $\$ 100$ for 73 days at $10 \%=\$ 2$.
$\therefore$ Amount of $\$ 100=\$ 102$.
$\$ 102$ in 73 days gives $\$ 2$ interest.
$\therefore 100$ " 365 " $\$ 9$ 告 "
$\therefore$ Rate of disoount $=987 \%$. Ans.

## 376. Given, the rate of bank discount, to find the corresponding rate of interest.

Example.-What rate of interest is paid, when note payable in 70 days is discounted at $10 \%$ ?
t bank, for ounting at run, dis$t$ was the

0 days, to
oceeds of
e. What act debt,
ndise in .90, and What
corresthat his

## BANK DISCOUNT.

Solution.
70 day note $=73$ days' time.
Interest on $\$ 100$ for 73 days at $\mathbf{1 0 \%}=\mathbf{\$ 2}$,
$\therefore$ Proceeds of $\$ 100=\$ 98$.
$\$ 98$ in 73 days gives $\$ 2$ interest.
$\therefore 100$ " 365 " $\$ 1010$ "
$\therefore$ Rate of interest $=1019 \%$. Ans.

## EXERCISE 85.

1. What rate of interest is paid, when a note payable in 30 days is discounted at $6 \%$ ?
2. A speculator discounted a note due in 90 days, at $12 \%$ per annum, what was the actual rate of interest received on the sum invested?
3. If a note payable in 3 months without grace be discounted at $10 \%$ per annum, what will be the rate of interest?
4. If a note for $\$ 500$, maturing in 96 days, without grace, can be purchased for $\$ 12$ less than its face, what is the rate of interest.
5. A broker discounted a note rlue in 4 months, without grace, at the rate of $6 \%$ per annum, what was the actual rate of interest realized on the sum advanced?
6. At what rate should a 3 month's note be discounted to produce $8 \%$ interest?
7. What rates of bank discount on 30 day notes correspond to $5,6,7$, and 10 per cent. interest.

## 3'7'\%. The bank discount exceeds the true discount

 by the simple interest on the true discount.Bank discount $=$ Interest on prinoipal.
True discount = Interest on present worth of principal.
" " = Interest on (principal-true discount).
" $\quad$ " $\quad=$ (Bank discount)-(interest on prestest on true disoount).
$=$ (Bank discount)-(interest on true discount).

Let $P=$ Principal, or,
$\therefore \quad \mathbf{P t r}=$ Interest, or bank discount.

$$
\frac{P t r}{1+t r}=\text { True discount. }
$$

$\mathbf{P t r}_{\mathrm{t}}-\frac{\mathbf{P}_{\mathrm{t}} \boldsymbol{r}}{1+\boldsymbol{t r}}=$ Difference B. D. and T. D.
$=\left(P-\frac{P}{1+t r}\right) t r$.
$=\left(\frac{P t r}{1+t r}\right) t r$.
= Simple interest on the true disoount.
B. D. on $\$ 100$ for 1 yr . at $6 \%=\$ 6$
T.D. $\quad$. $\quad \| \quad=\frac{6}{1.06}$

Difference $=\$ 6-\frac{6}{1.06}=\$ \frac{36}{106}$
But $\$ \frac{86}{106}$ is the simple interest on $\$ \frac{6}{1.06}$ for 1 year at $6 \%$.
$=$ Simple interest on the true discount.
378. If the bank discount or simple interest on a sum of money for a given time and rate is $\frac{a}{b}$ of that sum, then the true discount will be $\frac{a}{a+b}$ of the sum.
$\dot{\text { If interest }}=\frac{\mathrm{a}}{\mathrm{b}}$ of prinoipal, then sa is intereat on $\$ \mathrm{~b}$.
$\therefore \$ \mathrm{~b}$ (i.e. principal) $+\$ a$ (i.e. interost) $=\$(\mathrm{a}+\mathrm{b})=$ Amount.
$\therefore$ b is present worth of $\$(a+b)$, and $\$ a$ is the true disoonnt of $8(a+b)$.
$\therefore$ True discount is $\frac{a}{a+b}$ of principal.
Thus:
Simple interest on $\$ 100$ for 1 yr . at $6 \%=\$ 6$.
i.e., the in 3 rest is $\frac{8}{80}$ of prinoipal.

Then $\$ 6$ is interest on $\$ 100$.
$\therefore \$ 100$ of principal $+\$ 6$ of interest $=\$ 106$. Amt.
$\therefore \$ 100$ is present worth of $\$ 108$,
and $\$ 6$ is true discour't of $\$ 106$.
$\therefore$ True discount $=\frac{6}{106}$ of prinoipal, i.e., $\frac{6}{100+6}$ of principal.

## EXERCISE 86.

1. The interest is $\frac{2}{5}$ of the principal, and the difference between the interest and discount is $\$ 8$. Find the principal.
2. The interest is $\$ 5$, the discount for the same time and rate is $\$ 4$. Find the principal.
3. The interest is $\$ 2$, and the difference between the interest and discount is $16 \frac{3}{3}$ cents. Find the principal.
4. If the interest is $\mathbf{1}_{4}^{8}$ of the principal, what fraction of the principal is the true discount?
5. The interest of a certain sum is $\&$ of the principal. The amount is $\$ 640$. Find the principal.
6. The difference between the interest and the discount on a sum of money for $1 \frac{1}{2}$ years at $8 \%$ is $\$ 18$. Find the sum of money.
7. Reckoning bank discount at $5 \%$, a person would receive $\$ 21$ less than the nominal value of a note which has a year to run. What would he receive for the note if true discount were deducted.
8. I have two notes (1 year to run) amounting to $\$ 38$; both are discounted at $20 \%$ one at bank discount, the other at true discount, the entire discount being $\$ 7$. Find the face of the note on which bank discount was allowed.
9. The interest on a certain sum for 6 years is $\$ 261$, and the discount for the same time is $\$ 180$. Find the sum and rate per cent.
10. If $\$ 4$ is allowed as 12 months' discount off a bill for$\$ 76$, and at the same rate $\$ 7$ be allowed off a bill for $\$ 91$, for how long was the latter sum discounted?
11. The interest on a sum of money for 5 years is $\$ 140$, and the discount for the same time and rate $\$ 100$. Find the sum and rate per cent.
12. The interest on a certain sum of money is $\$ 180$, and the discount on the same sum for the same time and same rate is $\$ 150$. Find the sum.

## PARTIAL PAYMENTS.

379. Partial Payments are part payments made at different times of notes, acceptances, bonds, mortgages or other written and interest-bearing instruments which the debtor is obliged to pay.
380. Indorsements are the acknowledgments of the payments written on the back of the note, acceptance, etc., stating the amount and date of the payment.
Special receipts are sometimes given for partial payments mrde, instead of writing the acknowledgment on the back of the obligation.
381. The method of computing interest when partial payments have been made is based on the following principles:
382. Payments must be applied first to discharge accrued interest, and then the remainder, if any, towart; the rlischarge of the principal.
383. Only unpaid principal can draw interest.

Exasple 1.-A note the face of which was $\$ 3,600$, bearing interest at $6 \%$, was given October 17th, 1884, and settled February 14th. 1889. Find the balanoe due, the following indorsements having been made: March 3rd, 1885, $\$ 500$; October 25th, 1836, $\$ 1,000$; Deoember 6th, 18.34, $\$ 2,400$.

## Solution.

| Solution. |  |
| :---: | :---: |
| Face of note |  |
|  | 81.07 |
| First payment (Mar. 3rd, 1885) .. .. .. .. .. .. | \$3,681.07 |
| Remainder after dedneting first payment ... .. ... .. ${ }^{\text {a }}$.. | $\stackrel{600.00}{83,081.07}$ |
| Interest to date of second payment ( 1 yr .236 da.$)$.. .. .. | $\begin{array}{r}304.39 \\ \hline\end{array}$ |
| Amount due at time of second payment Seoond payment (Oct. 25th, 1886) | \$3,385.46 |
| Remainder after deducting second paymer ${ }^{\text {t }}$ | \$ $\mathbf{\$ , 0 0 0} \mathbf{2 , 3 8 5}$ |
| Interest to date of third payment (2 yr. 4. (ᄂ.) .. .. ... | $302.72$ |
| Amount due at time of third payment Third payment (Dec. 6th, 1888) | \$2,688.18 |
| Remainder after deducting third payment | 2,400.00 |
| Interest to time of eettlement ( 70 da.) .. |  |
| Balance due at time of settlement (Feb, 11th, 18s, i, |  |

Example 2.-
nade at ages or ich the of the ce, etc., ts made, atim.
partial llowing scharge
$\$ 1,000$.
Two years after date for Tonto, May 15th, 1881. interest "t $7 \%$.

On this note were indorsed the following payments :
September 20th, 1882 .. .. .. .. $\$ 150.60$
October 25th, 1884 .. .. .. ..
200.90
July 1!th, 1886 .. .. .. .. .. ${ }^{200.90} \begin{array}{r}75.20\end{array}$
September 20th, 1887 .. .. .. 112.10
December 5th, 1888 .. .. .. .. 105.00
What remained due May 20 th, $\ddot{1889}$ ?
Solution.
Face of note

|  | $\$ 1,000.00$ 94.55 |
| :---: | :---: |
| First payment (Sept. 20th, 1882) .. ${ }^{\text {a }}$ (ime of first payment .. | \$1,094.55 |
| Remainder after deducting first payment | 150.60 |
|  | \$943.95 |
| Amount due at time of second payment .. .. .. .. .. | 138.49 |
| Second payment (Oct. 25th, 1884) .. .. .. ${ }^{\text {Remainder }}$.. ... .. | $\$ 1,082.44$ 200.90 |
| Remainder after deducting second payment ...".. | 200.90 $\$ 881.54$ |
| Amount due at time of fifth peym. 5th, 1888 ( 4 yrs. 41 da.) | 253.76 |
| Third payment, less than interest due .. .. .. .. .. | \$1,135.30 |

Fourth "
Sum of third and fourth payments, less than interest $\quad . \quad . \quad . \quad . \quad .2 .10$
due .. .. .
Fifth payment .. .. .. .. .. ." .. .. .. .. $\$ 187.30$
Sum of third, fourth, and fifth payments ${ }^{\circ}$... .. 105.00
Remainder after deducting third, fourth, and fifth payments. . $\quad \$ 292.30$
Interest from fifth payment to May 20th, 1889 (166 da.) .. $\$ 843.00$
Balance due at time of settlement (May 20th, 1889) ... .. $\frac{2669.84}{\$ 84}$
382. 1. Compute the interest on the given principal from the date of the note to the time of the first payment. If this payment equals or exceeds the interest due, subtract the paymont from the amount, and treat the remainder as a new minsipal.
2. If any payment is less than the accrued interest, compute the interest on the same principal, to a date when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from the amount, and regard the remainder as a new principal.
3. Proceed in the same manner with the remaining payments, until the date of settlement.

## EXERCISE 87.

1. A note of $\$ 4,560$, dated Jan. 22nd, 1887, and drawing interest at $7 \%$, had payments endorsed upon it as follows : Jan. 10th, 1888, \$2,000; Aug. 31st, 1888, \$500; Jan. 15th, 1889, $\$ 1,200$; Mar. 4th, 1889, $\$ 860$. Find the balance due June 15th, 1889.
2. On a claim for $\$ 8,000$, dated Aug. 12th, 1885, and bearing interest at $7 \%$, payments were made as follows: Dec. 15th, 1885, $\$ 30$; April 1st, 1887, $\$ 550$; Jan. 20th, 1888, $\$ 85$; June 12th, 1888, \$1,651.50. How much was due May 30th, 1889 ?
3. I held a bond against Ira Fox, dated May 1st, 1885, for $\$ 4,000$, on interest at $6 \%$. The following payments were endorsed on this bond: May 21st, 1886, $\$ 800$; June 10th, 1887, \$1,200; Aug. 10th, 1888, \$1,500. What was due May 1st, 1889 ?
4. On a mortgage for $\$ 5,500$, dated Aug. 18th, 1882, and bearing $6 \%$ interest, the following payments were made: Jan. 1st, 1883, \$100; Mar. 2nd, 1883, \$25; Aug. 13th, 1885, $\$ 2,500$; Dec. 19th, 1887, $\$ 2,500$; Mar. 1st, 1889, $\$ 500$. How much was required for full settlement, Mar. for $\$ 4,119.82$, at $6 \%$, dated June 25th, 1888, on which a payment of $\$ 450.25$ was made Aug. 1st, 1888, and a payment of $\$ 21.19$ on the 15 th of each subsequent month.
5. On a loan of $\$ 2,000$, made Mar. 19 th, 1885 , and bearing $6 \%$ interest, payments were made as follows : Nov. 1st, 1886, \$500; May 3rd, 1888, \$700; Feb. 1st, 1889, \$1,000. How much will be required for settlement in full, Mar. 2nd,
6. I gave a mortgage for $\$ 10,000$, May 9 th, 1882 , bearing $6 \%$ interest, and made thereon the following payments : Sept. 19th, 1882, \$500; Jan. 1st, 1883, \$500; April 25th, $1884, \$ 4,000$; Oct. 15 th, 1884, $\$ 4,000$; May 1st, 1889 , $\$ 3,525$. How much was due at final settlement, June 2nd,
1889 ?
7. A bond was given Mar. 3rd, 1883, for $\$ 8,650$, with interest at $8 \%$. The following payments were made on account : April 17th, 1884, $\$ 1,000$; May 10th, 1885, $\$ 550$; June 23rd, 1885, \$540; Dec. 22nd, 1886, \$803; Feb. 15th, 1887, \$25; Mar. 18th, 1887, \$25; April 19th, 1887, \$115; April 25th, 1888, \$146. How much remained due, May 7th,
1889 ?
8. A note of $\$ 1,520$, dated May 20th, 1888, and drawing interest at $6 \%$, had payments endorsed upon it as follows: Oct. 2nd, 1888, $\$ 300$; Feb. 26th, 1889, $\$ 25$; April 2nd, 1889, $\$ 570$; Aug. 8th, 1889, $\$ 600$. Find the amount due Dec. 6th, 1889.
9. A note of $\$ 2,000$, dated Jan. 22nd, 1889, and drawing interest at $6 \%$, had payments endorsed upon it as follows: May 20th, 1889, $\$ 100$; July 20th, 1889, $\$ 325$; Nov. 2nd, 1889, $\$ 20$; Dec. 23rd, 1889, \$125. Find the balance due Mar. 1st, 1890.
10. A note of $\$ 1,662.50$, dated Jan. 15th, 1888 , and drawing interest at $61 \%$, had payments endorsed 7pon it as follows : April 30th, 1888, \$25 ; June 24th, 1888, \$25; Sept. 2nd, 1888, \$625; Jan. 31st, 1889, \$700. Find the balance due May 12th, 1889.
11. Oct. 1st, 1885, a note for $\$ 1,000$ was given, payable in 4 years, with $6 \%$ interest. A payment of $\$ 50$ was made 1 year from date; a payment of $\$ 250$ was made 1 year 6 months from date; a payment of $\$ 224$ was made 2 years from date; a payment of $\$ 20$ was made 2 years 8 months from date ; a payment of $\$ 110$ was made 2 years 10 months from date. How much remained due at the maturity of the note?
12. A mortgage for $\$ 5,400$ was dated Strathroy, Jan. 1st. 1886 ind endorsed as follows: May 22nd, 1887, $\$ 1,200$; Feb. 9th, 1888, \$150; Oct. 28th, 1888, \$1,500. What was due Mar. 1st, 1889, interest $5 \%$ ?
13. A note of $\$ 302.25$, dated Aug. 4th, 1887, and drawing interest at $6 \frac{1}{2} \%$, had payments endorsed upon it as follows: Oct 14th, 1887, $\$ 100$; July 21st, 1888, $\$ 100$; Oct. 11th, 1888, \$50; Jan. 18th, 1889, \$50. Find the amount due July $22 \mathrm{nd}, 1889$.
14. On the following note, payments were endorsed as follows; Nov. 3rd, 1887, \$50; Mar. 16th, 1888, \$50; Oct. 1st, 1888 , $\$ 50$; Dec. 30th, 1888, $\$ 1,000$; April 1st, 1889, $\$ 625$. How much was due, if paid in full, May 8th, 1889, money being worth $6 \%$ ? $\$ 1,600.00$.
Three years after date, I Brantrord, April 1st, 1887. Silas Hopkins, one thou promise to pay to the order of received.

Jas, Murray. follows: Aug. 1st, 1833 , $\$ 35$ nems were made as follows: Aug. 1st, 1833, $\$ 350$; Wr. 3rd, 188 $\$ 1,000$;

1888, and I 7pon it 388, \$25 ; Find the
partial payments.
Mar. 20th, 1885, $\$ 600$; Mar. 31st, 1885, $\$ 2,500$; Dec. 11 th, $1888, \$ 2,000$. What was the balance due Jan. 30th, 1889 ?

$$
\$ 6,500.00 .
$$

Brockville, Mar. 19th, 1882. six thousand dive hume to pay to the order of T. Gilmour, six thousand live hundred dollars, with interest at $6 \%$.
W. Hindson.
17. The following note was settled Oct. 13th, 1888 ; a payment of $\$ 25$ having been made Jan. 15th, 1887 ; one of $\$ 300$, July 12th, 1887 ; and one of $\$ 200$, April 1st, 1888. If money be worth $8 \%$, how much was due at final settlement?
$\$ 585.50$.
Six months after date, I promis, Aug. 1st. 1886. Buchanan, or order, five hundomise to pay to Alex. dollars, value received. F. MoHIARDY.
18. $\$ 500$.

For value received, I promi. Thomas, Feb. 1, 1888. order, five hundred, promise to pay D. E. Broderick, or interest at $7 \%$. Endorsed as follows, James Monioe.
Endorsed as follows, May 1, 1887, $\$ 40$.
"، Nov. 14, 1888, \$8.
". April 1, 1889, \$12.
" May 1, 1889, \$30.
How much was due Sent. 16. 1889 ?
19. $\$ 5,000$.

Six months after date I promise to may 1st, 1887. order, five thousand dollars, with to pay G. T. Smith, or value received. Endorsed, Oct. 1st, John Adams. c $1887, \$ 700$.
c Feb. 7th, 1888, $\$ 45$.
What was the balance due. 13th, $1888, \$ 480$.

## PARTIAL PAYMINTS.

20. $\$ 2,460$.

Trenton, April 10th, 1887.
Four months after date I promise to pay W. H. Austin, or order, two thousand four hundred sixty dollars, with interest at 6 per cent., value received.

George G. Williams.
Endorsed, Aug. 20th, 1888, $\$ 840$.
" Dec. 26th, 1888, $\$ 400$.
" May 2d, 1889, \$1,000.
How much was due Aug. 20th, 1889 ?
21. $\$ 650$.

For value received
Guelph, Jan. 1st, 1887. Kenzie, or order, six promise to pay Alexander Mcinterest at 6 per cent.

Endorsed, Aug. 18th, George Law.
" Ap. 18th, 1C87, \$100.
What was due on the April 13th, 1888, $\$ 120$.

## EQUATION OF ACCOUNTS.

383. Equation of Accounts, also called Equation of Payments, and Averaging Accounts, is the process of finding the time at which several debts due at different times may be paid in one sum without loss of interest to either party. It is al 30 the process of finding the time when th: 'alance of an account having both debits and credits, may be paid without loss of interest to either party.
384. The Equated Time is the date at which the several debts due at different times may be equitably paid in one sum.
385. The Term of Credit is the time between the contraction of a debt and its maturity.
386. The Average Term of Credit is the time to elapse before several debts due at different times may all be paid at once without loss to debtor or creditor.
387. The Focal Date is any assumed date of settlement, with which the dates $c$ the several accounts are compared for the purpose of finding the equated time. Nores 1.-Any oonceivable date may be taken as the foosl date most common dates used being, the earliest due as the fooal date; the the first day of the month of the earliest duede ate, the latest due date, month preceding the month of the earliest due date, and the last day of the
388. In Equation Triar date.
389. Interest may be calculate 1 st, or Jan. 1st, is taken for all oxamples. day basis, or a 365 day basis, withouny rate per cent., and either on a 360 that a uniformity in rate and manner varying the result, providing only throughout.
390. The student is recommended to ohoose one method of equating accounts, that method being uniform regarding ohoice of fooal date, rate, and form of solution.
391. Equation of accounts depends upon the following principles:
392. The rate and time remaining the same. Double the principal produces turice the interest. Half the principal produces half the interest, etc.
393. The rate and principal remaining the same. Double the time produces twice the interest. Half the time produces half the interest, etc.
394. Hence, the interest on any given principal for 1 year, 1 month, or 1 day, is the same as the interest of $\$ 1$ for as many years, months, or days, as there are dollars in the given principal.
395. Hence, the interest on any given principal for any number of years, months, or days, is the same as the interest for 1 year, 1 month, or 1 duy, on as many dollars as is expressed by the product of the given principal multiplied by the given number of years, months, or days.
396. The several rules in equation of accounts are based upon the principle of bank discount, for they imply that the discount of a sum paid before it is due equals the interest of the same amount paid after it is due.
397. To find the average time when the items are all debits or all credits, having the same date and different terms of credit.

Example.-A. bought a farm June 24th and was to pay $\$ 500$ down, $\$ 300$ in 2 months, $\$ 400$ in 6 months, and $\$ 600$ in 8 months. Find the average term of oredit and the equated time.

Solution 1.
By the interest method.
Interest on $\$ 500$ for 0 mo, at $6 \%=\$ 0.00$.

| $"$ | $\$ 300$ for $2 "$ | $"$ | $=3.00$. |
| ---: | :--- | ---: | :--- |
| $"$ | $\$ 400$ for $6 "$ | $"$ | $=12.00$ |
| $"$ | $\$ 600$ for 8 | $"$ | $=24.00$. |

Amount of payments $=\$ 1,800 \quad$ Interest $=39.00$.
Interest on $\$ 1,800$ for 1 month at $6 \%=\$ 9 . \quad \$ 39 \div \$ 9=4 \frac{1}{3}$.
$1 \mathrm{mo} . \times 4 \frac{1}{8}=4 \frac{1}{8} \mathrm{mo}$. the average term of credit. June 24th $+4 \frac{1}{\mathbf{i}}$ mo. $=$ Nov. 3rd, the equated time.

Double the e principal

2e. Double ne produces for 1 year, f $\$ 1$ for as in the given al for any the interest llars as is ultiplied by
counts are they imply equals the

## items are date and

ay $\$ 500$ down, hs. Find the

## Explanation.

If we take June $24 t \mathrm{li}$ as the time for payment of all the items, A. would lose the interest of $\$ 300$ for 2 months, $\$ 400$ ior 6 months, and $\$ 600$ for 8 months, in all $\$ 39$ interest. He is therefore entitled to the use of $\$ 1,800$, the amount of the debt, for such a time as the interest on it would be equal to $\$ 39$, and which is shown above to be $4 \frac{1 子}{\frac{1}{3}}$ months, and $4 \frac{1}{2}$ months, from June 24th, gives the equated time Nov. 3rd. A. could therefore pay the amount of the debt, $\$ 1,800$, on Nov. 3rd, without loss of interest either to himself or his creditor.

Rele for Interest Method.
Find the interest on each item for its term of credit, and diride the sum of these interests by the interest of the sum of thi' items for 1 day, 1 month or 1 year as the case may be.

I'he quotient will be the number of months or days from the, focal date to the equated time of payment.

Add this number to the focal date and the result will be the equitable date of pmyment.
Notes 1.-In compntir? by the interest method the rate forms no element of the calculation, hence any rate may be used. The most convenient rates are $6 \%$ and $12 \%$.
2. The result will be the same whether we reokon 365 days to the
year or 360 days to the year.

Solution 2.

## By the product method.

$$
\begin{aligned}
& 7,800 \div 1,800=\begin{array}{l}
7,800 \mathrm{~m} \\
4 \mathrm{~g} \mathrm{mo} \\
\mathrm{~m}
\end{array}
\end{aligned}
$$

Explanation.
This method is the same in prinoiple as the interest method. The interest on $\$ 300$ for 2 months is the same as the interest on \$1 for 600 months ; the interest on $\$ 400$ for 6 months equals the interest on $\$ 1$ for 2,400 months ; and the inter. sst on $\$ 600$ for 8 months equals the interest on $\$ 1$ for 4,800 months. A. would therefore lose the interest on $\$ 1$ for 7,800 months. He would therefore be entitled to the use of $\$ 1,800$ for suoh a time as the interest on it would equal the interest on $\$ 1$ for 7,800 months, or $4 \frac{1}{\mathrm{~g}}$ months.

Rule for Prodect Method.
Multiply each item by its term of credit, and divide the sum of the products by the sum of the items; the quotient uill be the avcrage term of credit.

## EXERCISE 88.

1. On a certain day A. bought a horse for $\$ 175$ on 30 days, 3 cows for $\$ 120$ on 45 days, 80 sheep for $\$ 250$ on 60 days, and 5 tons of bay for $\$ 130$ on 90 days. What is the average term of credit?
2. Bought a ship for $\$ 30,000$; the payments were $\$ 5,000$ cash, $\$ 8,000$ in 4 months, $\$ 7,500$ in 6 months, $\$ 4,500$ in 8 months, and the balance in a year. What is the average term of credit?
3. Sept. 1st, 1891, I bought goods, as follows: $\$ 200$ on 2 months' time, $\$ 400$ on 3 months, and $\$ 450$ on 4 months. What was the average term' of credit, and the average date of maturity?
4. On the first day of December, 1890, a man gave 3 notes, the first for $\$ 500$, payable in 3 months; the second for $\$ 750$, payable in 6 months ; and the third for $\$ 1,200$, payable in 9 months. What was the average term of credit, and the equated time of payment?
5. Bought merchandise Jan. 1st, 1898, as follows: \$350 on 2 months, $\$ 500$ on 3 months, $\$ 700$ on 6 months. What is the equated time of payment?
6. Jan. 15 th, I bought a bill of goods amounting to $\$ 900$, $\$ 275$ of which was on 30 days' credit, $\$ 300$ on 60 days, and $\$ 325$ on 90 days. What was the equated time of payment?
7. James Hudson, Jane 12th, owes $\$ 317.75$ due in 4 months, $\$ 216.38$ due in 5 months, and $\$ 170$ due in 6 months. Find the average time of payment and date of maturity.
8. Dec. 1st. 1894, bought goods to the amount of $\$ 1,200$, on terms as follows : $25 \%$ in cash, $30 \%$ in 3 months, $20 \%$ in 4 months, and the balance in 6 months. Find the equated time of payment.
9. May 1st, 1890 , I purchased property for $\$ 8,500$, paid

3 on 30 days, on 60 days, What is the were $\$ 5,000$ is, $\$ 4,500$ in 3 the average
xs : \$200 on n 4 months. average date gave 3 notes, s second for $\$ 1,200$, paym of credit,
llows: \$350 aths. What
ting to $\$ 900$, on 60 days, time of pay-
.75 due in 170 due in and date of it of $\$ 1,200$, onths, $20 \%$ Find the cash $\$ 1,500$, and gave notes, one for $\$ 3,000$, payable in 2 years, and another for $\$ 4,000$, pavable in 4 years. Find the average term of credit on the notes.
10. Bought a bill of goods April 20th amounting to $\$ 6,000$, on the following terms: $\frac{1}{8}$ cash, $\frac{1}{4}$ in 4 months, and the balance in 6 months. At what date may the whole be justly paid?
11. A stock of groceries was purchased Jan. 1st, 1889, the purchase price payable as follows: $\frac{1}{4}$ in 1 month, $\frac{1}{4}$ in 3 months, $\frac{1}{6}$ in 4 months, $\frac{1}{3}$ in 5 months. When may the whole amount be equitably paid in one sum?
12. William Owens bought a farm of 320 acres at $\$ 68$ per acre, $\frac{1}{4}$ payable in cash, $\frac{1}{4}$ in 1 year, $\frac{1}{3}$ in 3 years, and the remaindor in 5 years. What was the average term of credit?
391. To find the average time when the items have different dates and different terms of credit, all the items being on the same side of the account.

Example.-L. C. Hill bought goods of Wm. Grant as follows: June 1st, 1890, amonnting to $\$ 350$, on 2 months' oredit; July 15th, 1890, $\$ 400$ on 3 months; Aug. 10th, 1890, $\$ 450$, on 4 months ; Sept. 12th, 1890 , 3600 on 6 months. What is the equated time?

Solution 1.
Interest method.


Interest on 1,800 for 1 day at $6 \%=\$ .30$.

$$
37.12 \frac{1}{2} \div 30=123 \frac{3}{4} \text { days. }
$$

Aug. $1+124$ days $=$ Deo. 3. Explanation.
If we take Aag. 1 st as the time for payme
would lose the interest on $\$ 400$ eor payment of all the items, $2 . \boldsymbol{O}$. Hill 8600 for 223 days, in all $\$ 37.12 \frac{1}{2}$. In justice he $\$ 450$ for 131 days, and on
of $\$ 1,800$ for such time as the interest will amount to $\$ 37.12 \frac{1}{2}$, or as shown above for 124 days.

Henoe the equated time is 124 days, after Aug. 1st or Deo. 3rd.
Rule for Intenest Metiod.
Take as the focal date the eurliest due date. Find the interest on each item from the strndard date to the date of its maturity and diride the sum of the interests by the interest of the sum of the items for 1 day.

The quotient will be the number of days from the standard date to the average date of payment. Add this number to the standard date and the result will be the equated time of payment.

Notrs 1.-If the earliest or latest due date is the focal date, its item has no interest, bat such item must be included in the sum of the debts.
2. If the fraction in the quutient is $\frac{1}{2}$ day or more, 1 day is added; if less than $\frac{1}{2}$ day it is rejected.
3. Any date may be assumed as the focal date, the most prefera.ble being the earliest or latest due date.
4. In business practice, odd cents and even odd dollars are rejected from the items in the interest calculations.
5. In the solution given above the gain of interest to the ! ayee on the first two bills, which are to be paid after they are due, equale the loss of interest on the last two which are to be paid before they are due.
6. In regard to the foregoing problem, it may be urged that a debt osn not be paid before it is contracted, but, it must be remembered, that the object of the solution is really to find at what date a note, given in settlement of the account, should be dated, in order that neither party would lose interest.
7. When terms of credit are given in months, calendar montlis are meant.

## Solution 2.

By the product method.

## Assu:ne August 1st as the focal date.

| DUs. | items. |  | tame. | pronucts. |
| :---: | :---: | :---: | :---: | :---: |
| Aug. 1, | \$350 | $x$ | 0 da . | Provocts. |
| Oct. 15, | 400 | $x$ | 75 " | 300.00. |
| Dec. 10, | 450 | $x$ | 131 | $=589.50$ |
| Mar. 12, | 600 | $\times$ | 223 " | $=1,338.00$. |
| 1800)222750 ( 1239. \$2,227.90. |  |  |  |  |
|  |  |  |  |  |
|  | 1 |  | ys $=$ | Dec. 3 |

37.12 $\frac{1}{2}$, or as

3rd.
Find the late of its e interest stumdard umber to $d$ time of
te, its item $f$ the debts. added; if prefera he re rejected yee on the the loss of ue. a debt can i, that the , given in her party

EQUATION OF ACCOUNI'S.
Explanation.
This method of solution may be explained in a manner similur to that given to Solution 2, Art. 390.

Rule for Phodjot Metrod.

1. Find the date at which each item matures, and find the number of days letween the focal dute and the date of maturity of each item.
2. Multiply each item ly its number of days, and divide the sum of the products by the sum of the items. The quotient will be the average term of credit.
3. Add this quotient to the focal date, and the result witl be the equitable date of par, ment.

## Solution 3.

## Interest method.

date.
Assume the latest date, March 12th, 1889, as the focai
onths are

## Solution 4. <br> By product method.

Assume March 12th as the fooal date.
DUE. ITEME. DAYS. PRODUOT
Aug. 1. $\$ 350 \times 223=\$ 78,050$.
Oot. 15. $400 \times 148=59,200$.
Deo. 10. $450 \times 92=41,400$.
Mar.12. $600 \times 0=00$.
Amount $\$ \overline{1,800} \$ 1,800) \$ \overline{178,650}$.
Mar. 12, 1891 - 99 991 dayn.
Explanation.
The number of days is found as in Solution 3.
If the debt is settled on Mar. 12th, 1891, William Grant will lose the interest on $\$ 350$ for 223 days, or the interest on $\$ 78,050$ for 1 day; on $\$ 400$ for 148 days, or the interest on $\$ 59,200$ for 1 day; and on $\$ 450$ for 92 days, or the interest on $\$ 41,400$ for 1 day. The total loss of interest is therefore the interest on $\$ 178,650$ for 1 day. We bave then to determine for how many days the interest on $\$ 1,800$ will equal the interest on $\$ 178,650$ for 1 day, whioh is found to be 991 days. Therefore the debt is due 99 daya before Mar. 12th, 1891, or Deo. 3rd, 1890.

## EXERCISE 29.

1. A merchant bought goods as follows:

Sept. 5, 1890, a bill of $\$ 2.0$ on a credit of 6 mos.


What is the average date for the payment of the whole?
2. John E. Lewis purchased goods of Isaac S. Smyth \& Co. to the amount of $\$ 5,000, \$ 1,250$ to be paid June 2nd, 1889, $\$ 1,000$ to be paid July 5th, $\$ 2,000$ to be paid Aug. 15th; the balance, $\$ 750$, will become due Aug. 30th. At what date must a single note for the whole amount be drawn, payable in 3 months, that it may become due at the average date?
3. Bought goods as follows:


What is the average date of payment?
4. When shall a note to settle the following account be made payable?
Henry Field.

| 1892. |  |  |
| :---: | :---: | :---: |
| $\mathrm{Mar.}^{3}$ | To Mdse. @ 3 mos., as per bill rendered. <br>  |  |
| Apr. ${ }_{\text {a }}{ }^{4}$ |  |  |
| May 1 |  | 10000 |
|  |  | 30000 |
|  |  |  |  |
|  |  |  | $\$ 10700$ |

5. Average the following statem.3nt of account :

Mar. 6, To Mdse. @ 30 days $\$ 315.00$


6. The following items 400.00 each. What is the average whole amount?

> Apr. 1, 20 bbls. ex. fam. flour "" 11, 500 bush. Manitoba wheat
> @ $\$ 8.50$
> " 21,30 bbls. Ontario flour " 1.25
> " 26, 100 bush. oats
> 6.75
> 7. Find the average of the following: " . 45
3. May 5, Mdse. @ 60 days $\$ 600.00$ " 16, " " 30 " june 10, Cash July 7, Mdse. (net) Aug. 14, " @ 60 days
396.40
250.00
420.00
538.28
$\$ 2,204.68$
9. A young man, having money advanced to help him pay his way through college, received :

Sept. 1, 1888, \$75. Feb. 15, 1890, \$86.
Feb. 15, 1889, $\$ 80$.
Aug. 31, 1889, $\$ 95$.

$$
\text { Sept. } 20,1890, \$ 128
$$

$$
\text { Aug. } 30,1891, \$ 175 .
$$

What was the equated time at which he should date a single interest bearing note for the whole sum?
10. Five years from the date of the first loan, the above mentioned note was paid, with interest at $4 \%$. What was the amount?
11. What is the average time at which the following bills become due? Feb. 10th, 1892, $\$ 400$ on 2 months' credit; May 10th, 1892, $\$ 300$ on 4 months' credit; June 16th, 1892, $\$ 350$; Aug. 6th, 1892, $\$ 150$.
12. Find the equitable date for a single note given on the following bills for merchandise: June 1st, 1895, \$20, July 1st, $\$ 30$, Aug. 1st, $\$ 30$, Sept. 1st, $\$ 20$, each on 2 months' credit.
18. Bought goods of Messrs Holt \& Co., as follows: Mar. 11th, $\$ 35$, on 30 days' credit ; July 20th, $\$ 95$, on 2 months' credit; Sept. 8th, $\$ 215$, on 3 months' credit. What was the average term of credit?
392. To find the extension of credit to which the balance of a debt is entitled when partial payments have been made before they are due. amounting to $\$ 1,740$; July 10th, B. paid him $\$ 500$; Aug. 6th B. paid $\$ 700$. To what additional credit is B. entitled on the balance?

## Solution 1.

## Interest method.

Bill is due Mar. $12+6$ months $=$ Sept. 12.
netrazet at $6 \%$ on
frost
July 10,
Aug. 6,

To

$$
\begin{array}{ll}
\text { Sept. 12, } & 64=\$ 5.331 . \\
\text { Sept. } 12, & 37=4.31 \mathrm{~g} .
\end{array}
$$

Balance $=\$ 1,740-\$ 1,200=\$ 540 \quad \$ 9.55$ interest. Interest on $\$ 540$ for 1 day, at $6 \%$ $\$ 9.65 \div .09=107_{2}^{2}$ days.
Supt. $12+107$ daye $=$ Dec. 28 , the equitable time of payment.
isxplanation.
If a partial payment is made before a debt is due, equity requires that the debtor should have an extension of credit on the balance, equivalent to the interest of the pre-payment.
B., by paying a portion of his debt before it is due, loses the interest on $\$ 500$ for 64 days, and the interest on $\$ 700$ for 37 days, in all $\$ 9.65$ interest. A. should therefore allow B. the ase of the balance, $\$ 540$, until the interest on it amounts to $\$ 9.65$, and which is slown above to

Nort.-Equity requires an extension of oredit, but the creditor is not always willing to allow this and is not required to do so by law.

## Solution 2.

By the product method.
ITEMS.

$\$ 500 \times 64=$| Days. |
| :---: |
| $\$ 700$ |
| $\$ 37$ |$=\frac{25900}{57900}$

$\$ 1,740-\$ 1,200=\$ 540$
540 ) 57900 ( $107 \frac{2}{3}$ days.
Sept. $12+107$ days $=$ Dec. $^{28}$.
Explanation.
A similar explanation to that, given in Solution 4, Art. 391, wry bo given

## EXERCISE 90.

1. P. owed me $\$ 1,300$ due in 1 year. At the end of 4 months he paid me $\$ 500$, and at the end of 7 months $\$ 300$, on condition that I would let the balance stand an equit. able time in consideration of these pre-payments. What was the balance, and when should it be paid?
2. A man bought a bill of goods on 90 days, amounting to $\$ 2,340.75$; if he pays $\$ 1,000$ down, what extension ought he to have on the balance?
3. A man owes $\$ 1,569.75$, payable in 90 days; 60 days before it is due he pays $\$ 350.86$, and 30 days later $\$ 211.89$ more; what extension ought he to have on the balance?
4. A person owes a debt of $\$ 1,680$ due in 8 months, of which he pays $\frac{1}{3}$ in 3 months, $\frac{1}{4}$ in 5 months, $\frac{1}{6} 6$ months, and $\frac{1}{8}$ in 7 months. When is the remainder due?
5. Bought a bill of goods, amounting to $\$ 1,500$ on 6 months' credit. At the end of 2 months, I paid $\$ 300$ on account, and 2 months afterward, paid $\$ 400$ on account, giving my note for the balance. For what time was the note drawn?
6. The following sums are due from E. to F.:-\$500, at the present time; $\$ 600$, in 30 days; $\$ 400$, in 40 days; and $\$ 900$, in 60 days. If E. pays F. $\$ 500$ to-day, and $\$ 1,000$ in 10 days, how long from the present time should the rest stand, to balance the pre-payments?
7. A debt of $\$ 2,000$ is due in 1 year from Jan. 1st, 1890. In consideration of the payment of $\$ 400$ March 2 nd, and $\$ 800$ April 1st, till what date should the balance be allowed to stand?
8. $\frac{1}{3}$ of a certain debt is paid 78 days before it is due; $\frac{1}{4}, 60$ days $; \frac{1}{5}, 27$ days. What extension should the debtor be allowed for the payment of the balance?

## EQUATION OF ACCOUNTS.

9. A. sold B. a bill of goods March 12th, on 6 months, amounting to $\$ 1,740$; July 10th, B. paid him $\$ 500$; Aug. 6th he paid $\$ 700$ more; to what additional credit is B. entitled on the balance?
10. On a debt of $\$ 2,500$ due in 8 months from Feb. 1st, the following payments were made : May 1st, $\$ 250$; July $1 \mathrm{st}, \$ 300$; and Sept. 1st, $\$ 500$. When is the balance due?
11. Find the average term of credit, and the equated time of pryment from Dec. 15th, of $\$ 225$ due in 35 days, $\$ 350$ due in 60 days, and $\$ 750$ due in 90 days.

## 392. To find the equated time for the payment of

 the balance of an account having both debit and credit items.Example.-What is the equated time and date of paying the following account : Dr.

| $\begin{aligned} & 1890 . \\ & \text { May } 21 \end{aligned}$ | To Mdse. 3 mos. |  | 1890 |  | Cr |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| June 9 | " 3 mos. | 8250 | June 8 | By Cash | \$300 |
|  |  | \$160 | July 21 | .، Sush ${ }^{\text {a }}$ - | \$400 |

Solution 1.
Interest method.

| DUE. | Items. | time. | chest. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aug. 21 | \$500 | 112 da . | \$9.337 | Muy 24 | TTEMS. $\$ 300$ | time. | Interest. |
| July 9 | \$160 | 119 da . | \$4.955 | Aug. 7 | 840 | 23 98 | \$1.15 |
|  |  |  | \$1.84 | July 21 | \$100 | 81 | ¢ $6.5 .53{ }^{\text {d }}$ |
|  | $\$ 10$ |  | \$16.131 |  |  |  | \$1.35 |
|  | \$800 |  | $9.03 \frac{1}{3}$ |  | \$800 |  | \$9.031 |
|  | \$110 |  | \$7.098 |  |  |  |  |

Int. on $\$ 110$ for 1 day at $6 \%=\$ .018 \frac{1}{8}$

$$
\begin{aligned}
& \$ 7.09 \frac{5}{8} \div \$ .018 \frac{1}{\mathrm{~d}}=3871^{2} \mathrm{r} \text { days. } \\
& \text { May 1. } 1890
\end{aligned}
$$

$$
\text { May 1, } 1890+387 \text { days = May 23, 1891. }
$$

Notes 1.-May 1 st is chosen as the focal date. Any date niay be
2. In this example the balance of interest on May 1st is in favor of H. Brierley, hence he is entitled to the interest on the balance of the account for 387 days after May 1st.

Had the balance of interest been on the credit side of the account, we should then have subtracted the equated time from the focal date.

> Role for Intenest Method.

1. Find the interesl on each item for the time from the focal date to the maturity of the respective items, and divide the balance of the interests by the interest of the balance of the itcms for 1 day or 1 month; the quotient will be the number of days or months, as the case may be, between the standard date and the time of settlement.
2. When the balance of an account and the balance of interest are both on the same side, add the quotient to the focal date; if on opposite sides, subtract it; the result will be the date of settlement.

Notes.-1. In finding the maturity of notes and drafts 3 days of graoe should be added to the specificd tine of payment.
2. When no time of oredit is mentioned the transaotion is understood to be for oash, and the payment due at once.

Solution 2.
Dr.
By the product method.


1. Finu the number of days from the focal date to the maturity of each item.
in favor of lance of the
account, we date.
from the ıd divide alance of $l l$ be the veen the
lance of the focal $l$ be the
of grace
derstood
$C r$.

EqUATION OF ACCOUNTS.
2. Multiply each item by its number of days, and divide the difference between the sums of products by the difference between the sums of items; the quotient will be the equated time.
3. If the greater sum of items and the greater sum of pro. ducts are both on the same side of the account, add the equated time to the focal date; if on opposite sides subtract it; the result will be the date when the balance of the account will be equitably due.

## EXERCISE 91.

1. When did a note given in settlement of the following account begin to bear interest?
$D r$.
L. R. Clem.

2. When did interest begin on the following account, and what was due on settlement, Jan 1st, 1892, interest $5 \%$ ? Dr.
 equation? Dr.

3. What is the balance of the following account, and when due by equation?
Dr.
Benj. F. Hawkins.

|  | To mdse. |  | 1 l |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Feb. ${ }^{28} 8$ | "، | $\$ 600$ $\$ 300$ | Jan. 20 Feb. 10 | By cash, | \$1,000 |
| " 15 | " | 8 |  |  | \$700 |

5. Balance the following account by two methods : Dr. J. H. Strong \& Co. in acct. with Smith \& Crane. Cr.

| 1893. Mar. 25 | To mdse., 60 ds. |  | 1893. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Apr. 7 |  | $\$ 560$ | Apr. ' 30 |  |  |
| May 2 |  | $\$ 830$ $\$ 730$ | July 13 | ": cash |  |
|  |  | \$730 | Oct. 31 | " draft, 30 da. | $\begin{array}{r} \$ 500 \\ \$ 260 \end{array}$ |

Nots.-In this example the balance of items and excess of products being on opposite sides, the average time is subtracted from the standard
date.
6. What is the balance of the following account and when due?

Dr.
H. Morgan in acet. with Lockwood \& Co.

| 1890. <br> July 20 | To sundries, " |  | 1890. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Aug. 10 |  | \$760 | Aug. 26 |  |  |
| Sept. 15 |  | \$8540 | Sept. 12 | "، stocks, 30 da. |  |
| Sep. |  | \$850 | Oot. 1 | $\begin{aligned} & " \begin{array}{l} \text { stocks, } 30 \text { da. } \\ \text { cash, } \end{array} \\ & \hline \end{aligned}$ | \$300 |

7. Find the average time of paying the following account:

Dr.
Georae Jenking.


## EQUATION OF ACCOUNTS.

8. Find the equated time for the payment of the balance due on the following account : Dr.
W. T. Dawes.
 Dr.

| 1892 | James Green \& Co. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Jan. 10 | To mdse., 3 mos. $\begin{array}{ll}\text { " } & 30 \mathrm{da} . \\ & 3 \text { mos. }\end{array}$ |  | 1892. |  |  |
| " 25 <br> 10 |  | \$450 | June 1 | By bal. of acct |  |
| Apr. 20 |  | \$865 | Feb. 10 | " note, 3 mos. | \$485 |
|  |  |  | Mar. 1 | " draft, 30 da. | \$2,500 |

10. Balance the following account: Dr.

11. Find the balance of the following account and when due: Dr.

12. Find the balance of the following account and when due:
A. B. in acet. with C. D.

Dr.

18. When is the balance of the following account due by equation?

| Dr. | Samuel Peok \& Son. |  |  |  | $C r$. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1889. Mar. 3 |  |  |  |  |  |
| Apr. 24 |  | \$60 $\$ 100$ | ${ }^{\text {Appr. }}$ June 1 | By cash, | \$150 |
| ${ }_{\text {May }} 1$ | " | \$150 |  | " | \$150 |
| Aug. 17 | " | \% $\begin{array}{r}\$ 90 \\ \$ 200\end{array}$ | Oct. 1 | " | \$150 |

14. Find 1st, the balance of the following account, 2nd, when due by equation :
Dr。

15. Find when the following account is due by equation: ${ }^{\prime} I_{r}$.

Jofn Montgomery \& Co.
$C r$.


## AVERAGING ACCOUNT SALES.

 commission agent, of goods sold on account of a consignor, and contains a statement of the sales, attendant charges, and the net procseds due the owner.Notes.-1. The charges inclade freight, oartage, storage, advertising, insurance, commission, guaranty, etc.
2. The sales form the credit side of the account and the charges and advances the debit side.
394. Guaranty is a charge made in addition to the commission, for insuring the owner against the risk of nonpayment in case of goods sold on credit.
395. The charges for transportation, cartage, advertising, storage and insurance are considered due at the time of payment of the same.
398. The commission, guaranty, and other after charges of the commission merchant are considered due by some at the average date of sales; by others at the average due date of sales; while some merchants enter the commission at the date the account sales is rendered.
Notrs.-1. When the oommission is small oompared with the gross sales, either of these methods produoe a resnit, which is progs sufficiently acourate.
2. In this work they will be considered due at the average due date of the sales.
3. Of course the due date of the commission must be a matter of agreement between the parties concerned.
397. The method of averaging an account sales is the same as that for averaging an account having both debits and credits, except in the matter of adjusting the date for the commission and other charges.
398. To average an account sales, and find when the net proceeds are due.

Example.-Average the following, and find the due date of the ret proceeds:

Received on consignment 1,000 barrels of flour from Scott. Bros. Caledonia.


Solution.

1. Find average date of sales-Fooal date, July 1st.

2. Find average date of charges, focal date July 1st.


$\begin{array}{lrrr}\text { July 4.: } & 150,00 & 0 . & .00 .\end{array}$ Sept. 7. 148.00. $2 . \quad .05$. 148.00. 68.
\$779.00.
Int. on $\$ 779$ for 1 day at $6 \%=\$ .12 \frac{8}{8}$.
Uharges due July $1+\frac{8.12 \text { gig }}{}=13$ days.

## and find when

e due date of the
rom Scott. Bros.

| 1,100.00 | 35,920,00 |
| :---: | :---: |
| ,170.00 |  |
| ,500.00 |  |
| ,150.00 |  |
| 45025 |  |
| 30.75 |  |
| 15000 |  |
| 148.00 | \$779.00 |
|  | 5,141.00 |

: 6\%
y 1st.

## AVERAGING ACCOUNT SALES.

3. Averaging sales and expenses, they now stand as follows : Focal date July 1st.
due. items. days. produot.


bule.
4. Find the amount and the average date of sales. The date of the sales will be the date of the commission and guaranty.
5. Find the amount and the average date of the charges.
6. Make the charges the debits and the sales the credits, and find the average date for paying the balarice.

## EXERCISE 92.

1. Put the following items into the form of an account sales, find the net proceeds and date of payment :
A. B. Harrison, of Montreal, sold a consignment of goods from Chase \& Co., Toronto, as follows: Nov. 15th, 1889, 135 chests tea at $\$ 45$, on 30 days; Nov. 20th, 75 sacks coffee at $\$ 28$, on 2 months: Dec. 1st, 256 kegs lard at $\$ 4.50,30$ days; same date 285 tubs butter at $\$ 18.37$ on 2 months. Paid freight Dec. 1st, $\$ 23.75$; at $\$ 18.37$ on storage, Dec. $10 \mathrm{th}, \$ 7.80$; commi $\$ 23.75$; cartage, $\$ 5.40$; 2 S $2 \frac{1}{2} \%$.
2. Same parties sold Sept. 1st, on 3 months, $3,520 \mathrm{lb}$. $\$ .98$, on 2 m.12 ; Sept. 15th, 25 chests tea, each 85 lbs ., at 42 lbs. each, at $\$ 1.05$ ctober 2nd, 28 half-chests Oolong tea, paid October 15th, freigh 2 months. The charges were and guaranty $5 \%$.
3. Average the four following account salea :

4. Average the following account of sales :

Ac sount sales of 500 barrels of porl received from Conover \&
Drowne, of Cincinnati, to be sold on their account and risk.

July 3, Freight on 500 bbls. at 750
" 3, Cartage 500 bbls., at 75 c ., ..
" 28 , Storage .. ... .. .. .. ..
28, Storage and insurance, "... .. .. .. .. .. 43.50
, Commission on $\$ \longrightarrow$, at $2 \frac{1}{2} \ddot{\%}, \quad$. ... ... .. .. 12.50
Total charges
…
Net proceeds, due as per average,

## ACCOUNTS CURRENT.

399. An Account Current is an itemized record of the mercantile transactions between two parties, showing the cash balance due at a certain date.
Nores.-1. An account current is a transcript of the ledger account, entry, and is arran of certain details taken from the books of original
400. Whether in a differen's form.
ment between the parties.
401. It is customary for merchants, bankers, and brokers to render their accounts at stated times, as monthly, quarterly, semi-annually, or
402. Among retail dolare
bear irterest ; herice, in settling such ars, farmers, eto., the items seldom the merchandise balance. 5. In the illastrative
basis, the necessary ohange to 365 interest is calculated on the 360 days' 6. In Ontario and Manitoba days' basis being afterwards made. accounts from and after demand interest may be recovered on open will be charged. .
403. The Commercial or Merchandise Balance is the difference between the debit and credit items.
404. The Cash Balance is the sum required to settle an account at a given dara
405. To find the cash balance of an account at a given date.

Erample.-Fiad the oash balance of the follewing account, due on July 15th, 1890, at $\delta \%$ intersst :
Dr.


## ACCOUNTS CURRENT.

ord of the wing the

3r aocount, of original

1 or agree.
nder their lually, or

8 seldom find only

60 days uade.
on open interest
is the

He an
riven
ae ou

Cr.

300

Roldtion.

| DOE. | DAPB. | ITEM8 | interest. | UE. | $\begin{gathered} \text { DAYs. } \\ 86 \\ -30 \\ 44 \end{gathered}$ | iteme. | inteses |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Apr. } 9 \\ & \text { July } \mathbf{1} \end{aligned}$ | 197 | \$650 | \$10.51 | Apr. 20Aug. 14June 1 |  |  |  |
|  | -13 | ${ }_{1260} 100$ | ${ }^{17.50}$ |  |  | ${ }_{9}^{\$ 500} 9$ | \$7.17 |
|  |  | \$2910 | $4.70+$ |  |  | 1000 | 7.33 |
|  |  | 2440 | \$32.71 |  |  | \$2440 | 2.73 * |
| Bal. | items | \$470 | 17.23 |  |  |  | \$17.23 |

$\longdiv { \$ 1 5 . 4 8 }$ Interest. 360 days to year.
$\$ 15.48$ - $7_{3}^{\prime}$ of $\$ 10.48=\$ 15.27$. Actual interest. $\$ 470+\$ 15.27=\$ 485.27$. Cash balance.

## Explanation.

The third item on ths Dr. side is not due until 13 days (indioated by

- 13) after the date of settlement. and therefore J. M. Doyle is entitled from the ir to $\$ 1,260$ for 13 days. This amount may either be deduoted side as in the problem Jr. side or added to the interest on the Cr. the Cr. side.

Notrs.--1. The reason: for placing the interest of an item on its own side, when it becomes due before the time of settlement, is because it is entitled to interest for the intervening time.
2. In like mainner, if a credit extends beyond the settlement, equity for that that interest should be allowed on that item. Hence, its interest to the opposite. The be subtracted from its own side or be added adopted.
3. Interest tables are much used in making out accounts carrent.
4. If the aocount has been averaged, the amount due at a given date may be found by calculuting the interest on the balance of the account from the time it is due to the date of settlement. If the date of settlement is earlier than the average date, subtract the interest from the balance of the account; if later than the average date, add the interest. because in gives method of finding a Casn Balance is recommended stood, it is mora satisfactory to theount on eaoh item, it is readily underthan the product method, and when inter whom accounts current are sent than any other method,

Rule for Interest Method.

1. Find the due date of each item of the account. Then find the interest on each item from the date it becomes due to the day of settlement. The difference between the sums of the debit and the credit interests will be the balance of interest.
2. To find the cash balance due, when the balance of interest and the balance of items are on the same side, take their sum; when on opposite sides, take their difference.

## EXERCISE 93.

1. Find the cash balance of the following account, Aug. 5th, 1892, at $6 \%$ :
Dr.

| $\begin{array}{cc}1892 . \\ \text { June } & 10 \\ \text { " } & 30\end{array}$ | To mdse ${ }^{\text {a }}$ | \$200 | June 18 | By cash, | \$100 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| July 11 | , | 300 120 |  |  | \$150 |
| "24 | " | 250 | July $\begin{array}{rr}6 \\ & 30\end{array}$ | " | 200 300 |

2. Find the cash balance of the following account, Oct. 80th, 1892, at $6 \%$ :
Dr. J. S. Carson in acct. with James Ferguson. Cr.

| 1892. 5 | To mdse., 60da. |  | 1892. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Feb. 12 |  | \$182 | Feb. 1 | By bal. of acct, | \$300 |
| Mar. 7 | soda. | 430 | Mar. 30 | "" oash, | 250 |
| Apr. 15 | 60 da. | 640 | Apr. 20 |  | 200 |
| May 9 | " | 530 | Aug. 1 | " " cash, case | 300 |

3. Find the cash balance of the same account at $8 \%$.
4. Find the balance due Aug. 1st, 1892, at $8 \%$.
5. Find the balance of the same account due Jan. 1st, 1893, at $6 \%$.
6. Find 1st, the balance of the following account; 2nd, when due by equation ; 3rd, cash balance due Jan. 1st, 1888, if money be worth $6 \%$ per annum. Prove the result.

| Dr. | John McMillan \& Co. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{\text {May }} 18$ |  |  |  |  |  |
| June 3 |  | $\$ 300$ 200 | May 31 | By 2 mo. note (no |  |
| July | 2 mos . | 400 | July 15 | interest), 30 da. note a | \$ 240 |
|  |  |  | 1888 | interest), (no | 50 |
|  |  |  |  | " oash. |  |

inter their

| 1892. |  |
| :---: | :---: |
| $\begin{array}{r} \text { May } \\ \text { "4 } \end{array}$ | To mdse., 3 mos |

10. Find 1st, the balance of the following account ; 2 n 1 , when due by equation ; 8 rd, the cash balance due March 1st, 1889, if money be worth 5\% per annum. Prove the result. Dr.
S. S. Coor.

| 1888. | 13 y mdse., 1 mo . | \$150 | $\left\lvert\, \begin{aligned} & 1888 . \\ & \text { Oct. } 2 \mid \end{aligned}\right.$ | By 30 da. note (no interest), |  | \$100 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{\text {Aug. }} \mathbf{3 1}$ |  |  |  |  |  |  |
| Oct. 31 | 4 mo . | 600 |  |  |  |  |
| Dec. 19 | $1{ }^{1} 30 \mathrm{da}$. | 150 |  |  | cash, | 200 |
| Jan. 1880. | * 1 mo . | 100 |  |  | 60 da. note (no interest), | 300 |
|  |  |  |  |  | 1 mo atept. (no interest). | 500 |

11. Find cash balancé due Jan. 1st, 1898, interest $6 \%$ :

> J. Bradfield \& Co.
$C r$.

| 1892. |  |  | 1892. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | To mdse., 00 da. | $\begin{array}{r}\$ 150 \\ 350 \\ \hline\end{array}$ | Aug. 25 Scpt. 20 | By mdse. ${ }_{40}^{30}$ da. |  |
|  | " draft, 30 da . | 250 |  |  | 350 |

12. Find the cash balance of the following account, due Nov. 3rd, 1893, interest $8 \%$ :
A. B. bought of C. D., July 16th, 1993, merchandise $\$ 350$; Aug. 11th, $\$ 465$; Sept. 9th, $\$ 570$; Sept. 14th, $\$ 850$; Oct. 18th, $\$ 780$. The former paid August 1st, $\$ 360$; Sept. 30th, in grain $\$ 340$; Oct. 5 th, cash $\$ 500$; Oct. 21 st, $\$ 625$.
13. Reduce the following memoranda to the form of an account, and find the cash balance due Jan. 1st, 1889 :

Aug. 1st, 1888, A. bought goods of B. amounting to $\$ 560$; Aug. 26th, $\$ 840$; Sept. 21st, $\$ 1,000$; Oct. 12th, $\$ 1,370$; and Nov. 1st, $\$ 600$. A. sold B. Sept. 11th, 1888, wheat amounting to $\$ 350$; Oct. 1st, wool amounting to $\$ 760$; Oct. 31st, $\$ 400$ worth of butter ; Nov. 16th, paid him $\$ 1,000$ cash.
14. What is the cash balance of the following account, Dec. 81 st, 1889 , at $7 \%$ ?

Dr. S. Morgan in acct with J. D. Bigsonnette. Cr.

| $\begin{aligned} & 1889 \\ & \text { Sept.10 } \end{aligned}$ | To mdse., 30 da. | \$1,250,15 S89. |  |  |
| :---: | :---: | :---: | :---: | :---: |
| $\text { Oct. } 1$ | To mase., 30 da . | \$1,250.15 Sept. 25 | By mdse., 60 da. | \$1,560 50 |
| $\begin{array}{r}4 \\ \text { Nov } \\ \hline\end{array}$ | " 460 da . | $1,051.60$ Oct. 10 <br> $1,500.85$ -. 30 | " 4090 da . | $1,048.30$ 1,480 |
| Nov. 15 | 60 da . | $1,7 \pm 3.44$ Dec. 15 <br> 15  | " 4040 da . | 1,430 65 |

15. What is the cash balance on the following account, Jan. 10th, 1892 ?

Dr. W. R. Telford in acct with A. T. Stewart. Cr.

| 1891. | To sundries, ${ }_{\text {a }} \mathbf{3} \mathrm{mos}$. | \$1,400 | ${ }_{\text {July }}^{1891 .} 5$ | By mdse., 3 mos. | \$685 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{16} 420$ |  |  |  |  |  |
| Sept. 10 | - | 1,000 780 | " 18 | " ${ }^{\text {" }}$ | 840 |
| " 24 | " " | 1,310 | Aug. 11 | " ${ }^{\text {draft, } 30}$ | 960 |

16. Reduce the following transactions to the form of an account bearing interest at $6 \%$, and find the cash balance :

Feb. 11th, 1890, C. bought goods of D. amounting to $\$ 1,250$; March 14th, a bill of $\$ 2,160$; Apr. 10 th, a bill of $\$ 1,700$; Apr. 30th, a bill of $\$ 1,070$; May 6 th, a bill of $\$ 2,000$. March 1st, 1890 , C. sold a bill to D. of $\$ 1,640$, March 20th, a bill of $\$ 1,160$; Apr. 15th, a bill of $\$ 1,600$; May 1st, a bill of $\$ 1,340$; May 21st, a bill of $\$ 1,000$. What was the cash balance June 10th, 1890 ?
17. What was the cash balance due July 20th, 1889, on the following account, at $7 \%$ interest ?

Dr. C. W. Harrison in acet with L. Congdon. Cr.

| 1889. Mar. 1 | For midse., $3 \mathrm{mos}$. | $\begin{array}{r} \$ 500 \\ 750 \\ 410 \\ 600 \end{array}$ | $\|$1889.  <br> Apr. 5 <br> " 20 <br> May 1 <br> $"$ 22 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| " 20 |  |  |  | By mdse., 3 mes. | \$350 |
| Apr. 10 | 5 mos. |  |  | " ${ }^{\text {c }} 2 \mathrm{mos}$. | 900 |
| May 21 | c. 1 mo . |  |  | " cash, 4 mos. | 620 |

## STORAGE.

403. Storage is a provision made for keeping goods in a warehouse for a time agreed upon, or for an indefinite time, subject to accepted conditions.
The term storage is used also to designate the oharges for keeping the goods stored.

Rates of storage may be fixed by agreement of the parties to the contract, but are often regulated by Boards of Trade, Chambers of Commerce, or Warehouse Companies, and are estimated at a certain price per barrel, bale, bag, bushel, etc., per storage term.
404. A storage term is the number of days for which the storage is charged. The storage term is usually one week, 10 days, 20 days or 30 days. The rates of storage often vary for grains, or goods of different grades or values, and also on account of different modes of shipment.
405. Cash storage is a term applied to cases in which the payment of charges is made on each withdrawal or shipment, at the time of such withdrawal or shipment, notwithstanding the fact that the owner may still have goods of the same kind in store at the warehouse.
406. Credit storage is a term applied to cases in which sundry deposits or consignments are received, from which sundry withdrawals or shipments are made; and all charges adjusted at the time of final withdrawal.
407. A grain elevator is a building erected for the convenience of storing and shipping grain.
408. Storage receipts, especially of grains, are frequently bought and sold under the name of "warehouse receipts " or "elevator receipts," as representing so much value by current market reports.
Note. - When deposits or consignments, and withdrawals or shipments, are made at different times, debit is to be given for the amount of each,
deposit or consigument, from dnte to its final withdrawal or shipment, and credit given to the owner or consignor for each withdrawal or shipment, from date up to the time of settlement.

## 409. To find the average storage when goods have

 been receiven at different dates, but none delivered.Example.-There was received at a storage warehouse: Oct. 15th, 500 bbls. flour ; Oct. 24th, 120 bbls. apples ; Nov. 5 thl, 1.25 bbls. potatoes; Nov. 20th, 200 bbls. quinces ; Nov. 24th, 340 bbls. apples. The merchandise was all deiivered Dec. 12th. If the storage oharge was 40 . per bbl. for a period of 30 days average storage, what was the storage bill? The storage of parties Trade, nd are jushel, which ly one torage alues,
which val or ment, have

A farmer received for pasture: April 30th, 12 head of cattle; May 15th, 14 head of cattle; May $23 \mathrm{rd}, 27$ head of cattle; June 9th, 5 head of cattle ; June 30th, 8 head of cattle; July 16th, 40 head of catile. All were delivered

July 25th, and the charges wer. 75c. per head for each week of 7 days' average pasturiage. How much was his
bill?
3. The following produce was received at a warehouse: Oct. 19th, 250 bbls. flour; Oct. 27th, 160 bbls. potatoes; Nov. 2nd, 240 bbls. apples; Nov. 24th, 60 bbls. onions; Dec. 6th, 180 bbls. flour. The merchandise was all delivered Dec. 8th. What was the eorage bill, the charge being $2 \frac{1}{2} \mathrm{c}$. per bbl. per term of 30 days?
410. To find the average storage when goods have been received and delivered at different times.

Eximple.-A warehouseman received and delivered the following:

## Received.

Jan. 19, 300 bbls. Feb. 24, 200 " Mar. 8, 150 " Apr. 21, 400 "

What was paid for storage at 2c. a bbl., for a period of 30 "
delivered.
Feb. 9, 150 bbls.
Mar. 18, 200 "
Api. 4, 150 " storage, a settlement having been mede May 7th ? First Method.

## Solution.

From Jan. 19 to Feb. $9=21$ da.; 300 bbl . stored for $21 \mathrm{da} .=6,300$ for 1 da .
Feb. 9 .. .. .. .. 150 bbl . delivered.
From Feb. 9 to Fob. $24=15$ da.; $\overline{150} \mathrm{bbl}$. rem'g fcr $15 \mathrm{da} .=2,250$
Feb. 24. . . . . .. 200 received.
From Feb. 24 to Mar. $8=12 \mathrm{da} . ; \overline{350} \mathrm{bbl}$. stored for $12 \mathrm{da} .=4,200$
Mar. 8.. .. .. .. 150 bbl received.
From Mar. 8 to Mar. $18=10 \mathrm{da} . ; 500 \mathrm{bbl}$. stored for $10 \mathrm{da},=5,000$
Mar. 18 .. .. .. 200 bbl . delivered.
From Mar. 18 to Apr. $4=17 \mathrm{da} ; 300 \mathrm{bbl}$. stored for $17 \mathrm{da}=5,100$
Apr. 4
. .. .
. 150 bbl . delivered.
From Apr. $\leq$ to Apr. $21=17 \mathrm{da}$; $\overline{150} \mathrm{bbl}$. rom'g for $17 \mathrm{da}=2,550$
Apr. 21 .. .. . 400 bbl recejved.
From Apr. 21 to May $7=16 \mathrm{da} ; \overline{550} \mathrm{bbl}$. stored for $16 \mathrm{da}=8,800$
May 7 .. .. .. 550 bbl delivered.

$1,140 \mathrm{bbl}$. @ 2c. a bbl. $=80 \mathrm{~m}=1,140 \mathrm{bbl}$. for 80 da .
$1,140 \mathrm{bbl}$ @ 2c. a bbl. $=\$ 22.80$. Cost of storage.

REGETVED.
Jan. 19, 300 bbl. $\times 108=32,400$
Feb. 24, $200 \mathrm{bbl} . \times 72=14,400$ Mar. 8, $150 \mathrm{bbl} . \times 60=9,000$ Apl. 21, 400 bbl. $\times 16=6,400$

RULE.

1. Multiply the number of barrels, bales, etc., by the number of days between the date of their receipt and the date of the next receipt or delivery; add the number of articles of such next reccipt, or subtract the number of such delivery, as the case may be, and so proceed to the time of the final delivery.
2. Divide the sum of the products thus found by the number of days in the sirrage term, and the quotient will be the Average Storage for that term.

## Second method.

Solution.

$$
\begin{aligned}
& -62,200 \\
& \begin{array}{r}
28,000 \\
34,200
\end{array} \\
& \begin{array}{r}
28,000 \\
34,200
\end{array} \\
& 34,200 \div 30=1,140 . \\
& 1,140 \mathrm{bbl} \text {. @ 20. por bhl. }=\$ 22.80 \text {. Cost of storage. }
\end{aligned}
$$

## EXERCISE 95.

1. What will be the storage charge, at 4c. per bbl., for a term of thirty days average, on the following transaction? 1889.-June 12, 200 bbls., potatoes 1889 DELIVERED.
" " 20,15 's " apples. 1889.-June 17,75 bbls. potatoes.
" July 18, 60 " apples.
$\begin{array}{lllll}\text { " } & \text { " } & 25,125 & \text { " } & \text { " } \\ \text { " } & \text { " } & 30,90 & \text { " } & \text { apples. }\end{array}$
(1) July 5,60
" " 4 Aug. 9,20 " 40 " $\quad$ "
" " 15,90 " onions.
2. What will be the storage charge, at $4 \frac{15}{25}$ e. per bbl., for a term of thirty days average, in the following transaction?

Received.
1889.-Feb. 8, 180 bbls. flour.
" " 27, 100 " apples. 1889.-Mar. 1, 100 obls. apples.
"Mar. 8, 60 " potatoes " " 28, 190 " flour.

" " " 60 " flour.
" " 29,230 " "
3. What is the storage on the following account to Dec. 81st, 1889, at $2 \frac{1}{2} \mathrm{c}$. per bbl., for 30 days ?

RECEIVED.
1889.-Aug. 17, 250 bbls. mdse.
" " 25, 90 " "
6. Sept.19, 200 • 4
"Oct. 12, 800 " "
" Nov. 18, 200 " "
" Dec. 17, 400 " "

DELIVERED.
1889.-Aug. 23, 200 bble. mdse.
" Sept. 25, 240 " "
"Oot. 13, 300 "
"Nov. 20, 150 " *

* Dec. 25, 550 * *

411. To find the Cash Storage on goods received and delivered at different dates, when charges vary. Example.-At a warehouse there was received and delivered
merohandise as follows :

RECEIVED.
Jan. 3, 150 bbl .
Jan. 20, 200 bbl.
Feb. 1, 300 bbl .

DELITERED.
Jan. 23, 250 bbl .
Mar. 1, 400 bbl.

How much must be paid for storage on the above, at the rate of 5c. per bbl. for the first 10 days, or part thereof, and 3c. per bbl. for each subsequent 10 days, or part thereof?

Soletion.
Date. Receipta and Deliveries.
Jan 3, received 150 bbl .
" 20, " 200 "
350 bbl . in store.

3 da. or 1 term, 5c. $=5.00$
Feb. 1, received 300 bbl .
400 bbl in store.
Mar. 1, delivered $400 \mathrm{bbl} .\left\{\begin{array}{l}100 \mathrm{bbl} \text { stored } 40 \mathrm{da} \text {. or } 4 \text { terms, } 14 \mathrm{c} .=\$ 14.00 \\ 300\end{array}\right.$
Total cost of storage, $\begin{array}{ccc}28 & 3 & \\ \$ 64.00\end{array}$

## EXERCISE 96.

1. How much must be paid for storage on the following account at the rate of 5 cents per bbl. for the first 10 days, or part thereof, and 3 cents per bbl. for each subsequent 10 days, or part thereof?

RECEIVED

$$
\begin{aligned}
& \text { 1889.-May 7, } 350 \text { bbl. flour. } \\
& \text { " " 26, } 150 \text { " " } \\
& \text { June 15, } 200 \text { " " }
\end{aligned}
$$

delivered.
1889.-May 26. 250 bbl . flour.

2. The receipts and deliveries at a certain warehouse on the following account were as follows :

REUEIVED.

deifivered.
1889.-July 10, 90 bbl. pork.
" Aug. 15, 100 " "
" " 25, 250
"
" " 20, 300 " "
What was the total storage paid, the rate being 5 cents per bbl. for the first 10 days, and 3 cents for eash subsequent 10 days, or part thereof?
3. Find the cash storage on the following storage account :
recarved.

| 1889.-Sept. | 2, | 100 | bbl. |  |
| :---: | :---: | :---: | :---: | :---: |
| " | $" 4$ | 25, | 200 | $"$ |
| $"$ | Oct. | 19, | 350 | $"$ |
| $"$ | $"$ | 31, | 150 | $"$ |
| $"$ | Nov. | 7, | 200 | $"$ | present that required the payment of 6c. per bbl. for the bbl . for f of 80 days or fraction thereof, and 8c. per thereof.

delivered.
1889.-Sept. 20, 100 bbl.
" " 30, 100 "
Oct. 10, 100 "
" 20, 100 "
"30, 100 "
Nov. 20, the remainder.

## Miscellaneous.

## EXERCISE 97.

1. The interest on $\$ 1,805$, loaned on May 13th, at $5 \frac{4}{4} \%$ per annum is $\$ 37.905$; on what day was the money returned?
2. A sum of money at simple interest has in four and one-half years amounted to $\$ 735$, the rate of interest heing 5 per cent. per annum; what was the sum at first, and in how many years more will it amount to $\$ 1,140$ ?
3. I am offered a house that rents for $\$ 27$ per month, at such a price that, after paying $\$ 67.20$ taxes, and other yearly expenses amounting to $\$ 24.85$, my net income will be $8 \frac{1}{2} \%$ on my investment. What is the price asked for
the house?
4. In order to engage in business, I borrowed $\$ 3,750$ at $6 \%$, and kept it until it amounted to $\$ 4,571.25$. How long did I keep the money?
5. October 12th, 1889, I purchased 2,700 bushels of wheat, at $\$ 1.05$ per bushel, and afterwards sold it at a profit of $6 \%$. On what date was the wheat sold, if my gain was equivalent to $10 \%$ interest on my investment?
6. December 11th, 1888, a lumber dealer borrowed money and bought shingles at $\$ 4.50$ per M.; September 17th, 188.9, he sold the shingles and paid his debt, and $8 \%$ interest, amounting to $\$ 3,462.60$. How many thousand shingles did he buy?
7. I loaned a bridge builder $\$ 17,500$ for seven years, at $10 \%$ per annum, compound interest payable quarterly, and took a bond and mortgage to secure the debt and its interest. Nothing having been paid until the end of the seven years, how much was required in full settlement?
8. Harry is ten, and Fred seven years old. If $7 \%$ compound interest investments can be secured by their father, for what amounts must such investments be made in order that at the age of twenty-one the boys may each have $\$ 12,500$ ?
9. The day Charles was six years old, his father deposited for him in a savings bank such a sum of money that, at $4 \%$ interest, compounded quarterly, there will be $\$ 7,500$ to his credit on the day he attains his majority. What sum was deposited?
10. Having purchased July 15 th 1,150 barrels of pork, at $\$ 16$ per barrel, on four months' credit, the dealer, thirty days later, sold it at $\$ 17.50$ per barrel, receiving therefor 9 six months' note without interest. When the purchase money became due, he discounted the note on a basis of $7 \%$, and paid his debt. How much was gained?
11. I loaned a friend a sum of money for nine months, at $6 \%$ per annum, and when the loan was due he paid $\$ 851.50$ in cash, which was $75 \%$ of the amount due me; the remainder was paid six months, fifteen days later, with interest at the rate of $10 \%$. Find the amount paid at final settlement.
12. Having bought a mill for $\$ 12,000$, I paid cash $\$ 4,000$ on delivery, and gave a bond and mortgage for eight years without interest to secure the balance; to secure the interest, which was to be paid semi-annually, at the rate of $7 \%$ per annum, I gave sixteen non-interest bearing notes, without grace, for $\$ 280$ each, one maturing at the end of each six months for the eight years. If the four of the notes first maturing were paid when due, and no other payment was made until the mortgage became due, how much was required for fúll settlement?
13. The discount on $\$ 566.50$ for nine months is $\$ 16.50$ : find the rate of interest.
14. Bought 5,000 bushels of wheat at $\$ 1.25$ a bushel, payable in six months; I immediately realized for it at $\$ 1.20$ cash, and put the money out at interest at $10 \%$. At the appointed time I paid for the wheat; did I gain or lose by the transaction, and how much?
15. Jones loaned $\$ 2,400$ at $6 \%$ simple interest, until it amounted to $\$ 3,000$. For what time was the loan made ?
16. A man invested $\$ 16,000$ in business, and at the end of three years, three months, withdrew $\$ 22,880$, which sum included investment and gains. What yearly per cent. of interest did his investment pay?
17. Sold an invoice of crockery on two months' credit; the bill was paid three months, eighteen days, after the date of purchase, with interest at $8 \%$, by a chetk for $\$ 1,963.45$. How much was the interest?
18. A bond, bearing interest at $8 \%$, and dated May 1st, 1881, was settled in full November 16th, 1889, by the payment of $\$ 17,685$. For what face amount was the bond
given?
19. What sum will be due January 18th, 1892, on a debt of $\$ 5,100$, dated March 17th, 1885 , bearing interest at $7 \%$ per annum, payable semi-annually, if the first five paywents were made when due and no subsequent payments were made?
20. A merchant sold a stock of glassware on one month's credit; the bill was not paid until three months, twentyone days after it became due, at which time the seller received a draft for $\$ 4,716.21$ for the bill, and interest thereon at the rate of $5 \%$. Find the selling price of the goods.
21. A tradesman who is ready to allow $5 \%$ per annum, compound interest, for ready money, is asked to give credit for two years. If he charged $\$ 110.25$ in his bill, what ought the ready money price to have been?
22. A speculator borrowed $\$ 6,250$, at $7 \frac{1}{2} \%$ interest, and with the money bought a note, the face of which was $\$ 7,500$, maturing in nine wonths without interest, bui which was not paid until two years from the date of its purchase. If the note drew $6 \%$ interest after maturity, did its parchaser gain or lose, and how much?
23. A jobber bought 6,000 yards of Axminster carpet, at $\$ 2.80$ per yard, payable in six months, and immediately sold it at $\$ 3.15$ per yard, giving a credit of two months; at the expiration of the two nonths he anticipated the payment of his own paper, getting a discount off of $10 \%$ per annum. How much did he gain by the transaction?
24. Un the 20th of March, 1889, I borrowed $\$ 13,500$, at $5 \%$ interest; on April 5th, I loaned $\$ 5,000$ of the money until December 20th, 1889, at $8 \%$; April 15th, I purchased with the remainder a claim for $\$ 10,000$, due August 1st, but which. not being paid at maturity, was extended until the $\$ 5,000$ became due, at the rate of $6 \%$. How much did I gain, both claims having been paid on the day the loan of $\$ 5,000$ became due?
25. Find the present worth of $\$ 842.70$ for two years at $6 \%$, compound interest.
26. If $\$ 20$ be allowed off a bill of $\$ 420$ due in six months, how much should be allowed off the same bill due in twelve months, reckoning true discount?
27. If $\$ 15$ be the interest on $\$ 115$ for a given time, what should be the true discount off $\$ 115$ for the same time?
28. If $\$ 10$ be allowed off a bill of $\$ 110$ due eight months hence, what should be the bill from which the same sum is allowed as four months' discount?
29. How much may be gained by hiring money at $5 \%$ to pay a debt of $\$ 6,400$, due in eight months, allowing the present worth of this debt to be reckoned by deducting $5 \%$ per annum discount?
30. The discount on a certain sum due nine months hence is $\$ 20$, and the interest on the same sum for the same time is $\$ 20.75$. Find the sum and the rate of interest.
31. Having bought goods to the amount of $\$ 2,481.80$ cash, I gave my 60 day note in settlement. If discount be at $7 \%$ what should have been the face of the note?
32. A note dated September 1st, 1889, payable in 90 days, with interest at $7 \frac{1}{2} \%$, was discounted twenty-one days after date, at $10 \%$. If the proceeds were $\$ 690.52$, what must have been the face?
33. If, on a note made for $\$ 700$, bearing interest at $6 \%$, and dated January 1st, $1889, \$ 50$ is paid on the first of every month, commencing February 1st, following the date, what is due January 1st, 1890 ?
34. F. J. Ramsay \& Co. bought goods of John Hope \& Co. as follows: July 1st, $\$ 150$, at three months; July 20th, $\$ 200$, at four months ; August 16th, $\$ 300$, at two months; and October 4th, $\$ 250$ at four months. Find the equated time of payment, and what would be due on the account March 15th following, at $6 \%$ interest.
35. I owe $\$ 480$ payable in ninety days, and $\$ 320$ payable in sixty days. My creditor consents to an extension of time to one year, and offers to take my note for the
whole amount on interest at $6 \%$ from the equated time, or a note for the true present worth of both debts, on interest from date. How much will I gain if I choose the latter condition?
36. I sell goods to A. at different times, and for different terms of credit, as follows:

Sept. 12, 1859, a bill on thirty days' ,redit, fur \$180

| Oct. 7, " | " | thirty | $"$ | 300 |
| :--- | :--- | :--- | :--- | :--- |
| Nov. 16, " | " | sixty | $"$ | 150 |
| Dec. 20, " | " | ninety | $"$ | 350 |
| Jan. 25, 1860, | " | thirty | " | 130 |
| Feb. 24, " | " | thirty | $"$ | 140 |

If I take his note in settlement; at what time should interest commence?
37. A person owes $\$ 350$, due in three months, and $\$ 750$, due in six months; but at the end of two months he pays $\$ 200$, and three months afterwards, $\$ 500$. When is the remainder due?
38. A note for $\$ 1,000$, dated April 1st, 1889, payable on demand, with interest at $7 \%$, bears the following endorsements: May 6th, $\$ 200$; July 5th, $\$ 225.37$; October 18th, $\$ 322$. What is due January 1st, 1889 ?
39. Bought goods to the amount of $\$ 10,000$, of which $\$ 2,000$ was to be paid in one month; $\$ 2,000$ in two months; $\$ 4,000$ in three months, and the balance in six months. If a note is given for the whole amount, how long should it run?
40. Four notes, made by J. Simpson, and payable as follows: $\$ 560$, due September 10th, 1883; \$800, due October 15th, 1888; \$1,100, due December 1st, 1888 ; \$900, due February 1st, 1889, were exchanged for a single note. When will it fall due?
41. Asa May has given three notes; one for $\$ 300$, due May 1st ; one for $\$ 350$, due June 15th ; and one for $\$ 550$, due August 1st. Desiring to exchange them for two notes of $\$ 600$ eacb; he makes one payable June 15th; when should the other fall due?
42. Bonght a bill of goods amounting to $\$ 1,200$, on six months' credit. Paid cash on account $\$ 100$; at the end of three months paid $\$ 300$ more; and two months afterwards paid $\$ 400$, giving a note for the balance. For what time was the note drawn?
43. A note for $\$ 835.25$, dated July 1st, 1838 , payable on demand, with interest at $6 \frac{1}{2} \%$, bears the following endorsements: August 20th, $\$ 157.50$; September 21st, $\$ 180.25$; October 5th, $\$ 200$; December 1st, $\$ 80$. What is due January 1st, 1889 ?
44. On a bill of goods bought Mareh 1st, amounting to $\$ 1,500$, on eight months' credit, the following payments were made: May 1st, $\$ 350$; August 1st, $\$ 500$; September 1 st, $\$ 150$. What is the equated time for the payment of the balance?
45. A note for \$618.75, dated April 17th, 1889, payable on demand, bears the following endorsements: June 5th, $\$ 126.50$; August 20th, $\$ 127 .{ }^{\circ} 5$; November 17th, $\$ 210$. What is due 'anuary list, 1889, reckoning interest at 6 ,'is?
46. Bought of A. T. Stewart \& Co., the following bills of goods on five montlis' cre lit: Fehruary 10th, 1888, $\$ 900$; March 15th, 1888, \$2,000; May 10th, 1888, \$750; June 12th, 1888, $\$ 3,000$. Find the present worth of a note drawn July 1st, in payment of the while, discounted at $6 \%$.
17. Bought goods at different dates, as follows :

Aug. 15, amounting to $\$ 475$, on 6 months' credit.
$\begin{array}{lll}\text { Sept. 10, "6 } & \text { "، } & \text { " } 5 \\ \text { Oct. 5, " }\end{array}$
Oct. 5, " 750, " 4 " Nov. 1, " 450, " 3 "
What sum will equitably discharge the whole debt November 10th, allowing true discount at $7 \%$ ?
48. Purchased merchandise of W. Duncan \& Co., as follows :

Jan. 1, a bill amounting to $\$ 375.50$, on 4 months' credit.

| Jan. 20, | $"$ | 168.75, | 5 | $"$ |
| :--- | :--- | :--- | :--- | :--- |
| Feb. 4, | $"$ | 386.25, | 4 | $"$ |
| Mar. 11, | $"$ | 144.60, | 5 | $"$ |
| Apr. 7. | $"$ | 386.90, | 3 | $"$ |

What is the present worth of a note made May 1st, in payment of the whole, discounted at $6 \%$ ?

## PERCENTAGE.

## STOCKS.

412. Stocks represent the capital or property of incor. porated companies.
413. An Incorporated Company is an association authorized by law to transact business, and having the same rights and obligations as a single individual.
414. A Share is one of the equal parts into which the capital stock of a corporation is divided.
Notr.-The par value of a share varies in different companies. It is usually $\$ 100$, and will be so regarded in this work unless otherwise stated. Shares of $\$ 50$ and $\$ 25$ are called half-stock and quarter-stock respectively.
415. A Certificate of Stock is a paper issued by a corporation specifying the number of shares to which the holder is entitled, and the par value of each share.
416. The par value of a stock is the sum named in the certificate.
417. The Market Value of stock is the sum for which it can be sold.
Nots.-When shares sell tor ther nominal value, they are at par; when they sell for more, they are above par, at a preminm, or at an advance; when they sell for less, they are below par, or at a discount.
When stocks se! at par they are quoted at 100 ; when at $5 \%$ above par they are quoted at 105 ; when at $10 \%$ discount they are quoted at 90 .
418. A Dividend is a sum divided among the stockholders from the net profits of the company, and is a certain percentage computed on the par value of the stock.
Nots.-Companies sometimes deolare a Scrip Dividend, entitling the holder to the sum named payable in stock at par value.
419. A Preferred Stock is one which is entitled annually to a stated per cent. dividend out of the net profits before the common stock dividond is declared.
420. A Stock Broker is one who buys and sells stocks for others, on a commission called brokerage which is always a certain percentage computed on the par value of the stock purchased or sord.
421. A Stock Jobber is one who buys and sells stocks on his own account.
422. An Instalment is a payment of part of the capital.
423. An Assessment is a sum required of stockhoiders to meet the losses or the business expenses of the company.
424. The Gross Earnings of a company are its entire receipts from its ordinary business.
425. The Net Earnings is the remainder after all expenses are deducted.
426. A Bond or Debenture is a written agreement to pay a sum of money, with a fixed rate of interest, at or before a specified time. The term is applied to the Dominion, Provincial, County, Township, City, Town, Village, Railroad Bonds, etc.

Note.-Bonds or Debentures are named from the oorporations who issue them, the rate of interest they bear, the date at which they are payable or from a oombination of any of these.

Bonds are also known, First Mortgage, Second Mortgage, eto., Income Bonds, Consols, Sinking Fund, ete.
427. Coupon Bonds are those having small certificatee attached representing the different instalments of interest payable at the times specified, and which are to be cut off when paid, as a receipt.
Note.-1. Bonds are also issued without ooupons, in what is known as the registered form. In $t$.. s case the bond is only payable to the rogistered owner, or his assignee, and the interest is paid by oheque or in casb to the owner or to his atto:ney.
2. Bonds are sometimes issued with coupons attaohed payable to bearer, but the principal of which may or may not be registered at the choioe of the owner.

4ss. The principai United States government bonds are the $4 \frac{1}{2}$ 's of 91 , redeemable at the option oi the govern. ment after Sept. 1st, 1891 ; 4's of 1907, redeemable at the option of the Government after July 1st, 1907 ; Refunding Certificates of the denomination of $\$ 10$, bearing interest at $4 \%$, and convertible at any time with accrued interest, into $4 \%$ bonds; Currency 6 's, issued to aid in the construction of Pacific railroads, payable in thir'y years after date, and maturing at different dates from 1895 to 1899.

Consols are the leading funded arcurities of the Inglish Government, bearing $3 \%$ interest, payable half-yearly, and redeemable only at the pleasure of the Government.

The funded debt of France bears the title of Rentes, bearing usually, interest at the rate of $5 \%$

The German Empire has a funder debt bearing $4 \%$ interest, known as $4 \%$, Imperial bonds.

The funded debt of Austria is known as the Austrian Consols, the largest part of which bears $5 \%$ interest.

Russia has a debt which bears a nominal interest of $5 \%$, or $5 \frac{1}{2} \%$. The bonds are known as Oriental Loans, and are helow par.

The homas in Italy are cailed Rentes, and bear interest of $3 \%$, or $5 \%$.

## STOCK EXCHANGE.

429. Stock Exchanges are associations organized for thaying and selling stocks, wonds, and other similar securities.
430. Quotations are usually made at so much per cent. on the basis of a par value of $\$ 100$ per share.
431. Stocks are usually bought or sold either "cash," "regular way," "seller three," "buyer three."

Note.-1. A stock sold "cash" is deliverable the day sold, a stock sold "regular way" is deliverable next day, or if bought "recular way" is to be paid for the next day. "Seller three" means deliverable on either of three days at the option of the seller. "Buyer three" means the buyer can demand delivery within three days, but must take and pay for it the third day.
2. Quotations are termed "flat" when the accrued interest is included in the price named.
3. Transactions on any of the above terms carry no interest.
4. If the option is over three days, interest on the selling value of the stock is paid by the buyer to the seller.
5. One day's notice is required of intention to terminate an option of s longer period than three days.
6. Should the stock pay a dividend during the pendency of a contract, the dividend belongs to the purchaser of the stook, unless otherwise previously agreed.
432. Margin is cash or other security deposited with a broker on account of either the purchase or sale of securities, and to protect the oroker against loss, in case the market price of the securities, brught or sold, varies so as to be against the interosts of the castomur. It is nyually $10 \%$ of the par value of the stock.

Note.-1. Hrokers oharge intorest on the amount faraished by them for " carrying the stook."
2. The margin depouited with the broker is simply to protect the broker against losing any money should the stock inove in the wrong direotion. In case of the stook so doing, the margin must be mude good by the deposit of an additional amount, otherwise the broker will sell the stook to protect himself from losing any of the money he has alvanced It is usnally $10 \%$ of the par value of the stock.
433. 1. A Bear is an operator who is "short" of stock. He wishes to buy at a lower rate, and tries to depress the price of the stock of which he is "short."
2. A Bull is an operator who in hoiding stock for an advance. He is said to be "long" stool. Bulls try to advance the price of the stock of which bhey aro "long."
3. Collaterals. Stocks, bonds, notes, or other value given in pledge as secmarity, when money is borrowed.
4. Hyphothecating Stocks and bonds, is depositing them as collaterals.
5. B. C. "between calls." The sale not taking place on the call of the stock but after the first call and befors the second call.
6. Short. When one has sold stock which he does not own hoping to realize a profit by buying it at lower prices, he is said to be "short."
7. A "Put" is a contract which secures to the holder the privilege of delivering to the person named therein a number of shares of stock at a specified price per share, within a limited time (usually thirty days), without the obligation to deliver it. The holder of a "put" is not required to pay interest.
8. A "Call" is a contract which secures to the holder the privilege of buying a number of shares of stock at a specified price, within a limited time without the obligation to purchase it. The holder of the "call" must way interest on the purchase price of the stocks to day of delivery.
9. $A$ "Spread" is a contract which secures to the holder the prinilege of either buying or selling within a limited time, a number of shares of stock, at a specified price. without the obligations of taking or delivering it.
10. A"Straddle" is a contract which secures to the holder the privilege of either buying or selling, within a limited time a number of shares of stock, not only at the price mentioned in the contract, but, also at the market price of the stocks at the date the privilege was purchased.
11. Puts, Calls, Spreads and Straddles, are privileges not recognized by the Stock Exchange.
12. Cover, to "cover one's shorts." Where stock has been sold short and the seller buys it in to realize his profit or to protect himself from loss, or to make his delivery, he is said to be " covering short sales."
13. Ex.-Div. or Ex--Dividend. When the price of stock does not include, and the stock does not carry to the buyer a recently declared dividend.
14. Difference. When the price at which a stock is bargained and the price of the stock on the day of delivery are not the same, the broker against whom the variation exists, frequently pays the " difference" in money, instead of furnishing or receiving the stock.
15. Watering Stock is increasing the number of shares of an incorporated company without a corresponding increase of their value. This is usually done in the re-organization of a railroad or in the consolidation of two or more railroads.
16. A "Corner" is produced when one or more operators owning or controlling all the stock of a company are able to purchase still more for either immediate or future delivery, from one who is " short." When they demand the stock, the sellers are unable to find it in the market.
17. Brokerage. 'The usual brokerage for baying and selling stucks is $\frac{1}{8} \%$, and is calculated on the par value of the stock.

4is4. Given number of shares, the par value of a share. To find the stock, or vice versa.

Example 1. - What amount of stook is represented by 40 shares of Bank of Montreal stook, par value $\$ 200$ per share? Solution.
40 shares at $\$ 200$ each $=\$ 200 \times 40=\$ 8,000$ stock.
Example 2.-How many shares, par value $\$ 200$ each, are represented by $\$ \diamond, 000$ Bank of Montreal stock ?

Solotion.
$\$ 200=$ value of 1 share.

$$
\therefore 8,000=" \quad \text { " }
$$

Example 3. - What is the par value of a share, when 40 shares of Bank of Montreal stock represent $\$ 8,000$ stock?

Solution,
40 shares represent $\$ 8,000$ stock
$\therefore 1$ share represents 8 gefe $=\$ 200$ stook.

## EXERCISE 98.

What amount of stock is represeated by-

1. 120 shares Western Assurance, par value $\$ 40$ per sharez
2. 60 " Bank of Montreal, " $\$ 200$ "
$\begin{array}{llll}\text { 3. } 200 & \text { " } \\ \text { 4. } 150 & \text { " } & \text { " } \\ \text { " }\end{array}$
3. 175 " " Commerce, " $\$ 50$ "
4. 240 " " Hamilton, " $\$ 100$ "

| 7. | 98 | " | Dominion Bank, | " |
| ---: | ---: | ---: | ---: | ---: |
| . 75 | " | Stank | " |  |

Find the par value of a share when -
9. 40 shares Imperial Bank represent $\$ 4,000$ stock 9
10. 75 " Merchants' Bank " $\$ 7,500$ "
11. 90 " Ontario Bank " $\$ 9,000$.
12. 120 " Standard Bank " $\$ 6,000$
13. 300 " Western Assurance Co. $\$ 12,000$
14. 70 " Imp. S. \& Invest. " $\$ 12,000$
15. 80 " B. \& L. Association " $\$ 7,000$ *
15. 110 " Dominion Telegraph " $\begin{array}{llll} & \$ 2,000 & \text { " } \\ \$ 5,500 & \text { c }\end{array}$
ying and value of
de of a shares of

How many shares are represented by-.
17. $\$ 8,500$ stook Merchants' Bank, par value $\$ 100$.
18. $\$ 9,600$ " Bank of Montreal, " $\$ 200$
19. $\$ 7,525$ " Lon. \& Can. L. \& A., "
20. \$2,640 " Western Assuranco Co., " $\$ 40$.
21. $\$ 3,150$ " Bank of Toronto, " $\$ 200$.
22. $\$ 3,175$ " B. \& L Association, " $\$ 25$.
$\begin{aligned} & \text { 23. } \\ & \text { 24. } \$ 6,400 \text { " } \text { North-West Land Co., " } \\ & \text { £5. }\end{aligned}$
24. $\$ 6,400$ " Imperial Bank. " $\$ 100$.
435. To find the cost price or selling price of any number of shares, the market value of the shares being given, and vice versa.

Example 1.-What is the oost of 60 shares of Bank oi Coumerce Stock at $121 \%$, brokerage $4 \%$ ?

$$
\begin{aligned}
& \text { Solution. } \\
& \text { Cost of } 1 \text { share }=\$ 121+\$ 4=\$ 1214 \\
& \text { " } 60 \text { shares }=\$ 121 \frac{1}{4} \times 60=\$ 7,275 .
\end{aligned}
$$

Example 2.-What will be received as proceeds of a sane of 60 shares of Bank of Commerce Stock at $121 \%$, brokerage $\frac{1}{4} \%$ ?

$$
\begin{aligned}
& \text { Solution. } \\
& \text { Selling price } 1 \text { share }=\$ 121-\$ 1=\$ 1209 \\
& \text { " }=\$ \text { shares }
\end{aligned}=\$ 1203 \times 60=\$ 7,245 .
$$

Example 3.-If 60 shares of Bank of Commerce Stock cor 5 57,275, find the market value, brokerage $\frac{1}{4} \%$.

Solution.
60 shares oost $\$ 7,275$
$\therefore 1-$ share costs $\frac{7,275}{60}=\$ 121$ $\$ 121$ - $\$ 1$ brokerage $=\$ 121=$ markat valne.
Example 4.-If 60 shares Bauk of Commeroe Stock sold for $\$ 7,245$, find the market vaine of the stook, brokerage $\frac{1}{4} \%$.

Solution.
60 shares sold for $\$ 7,245$
$\therefore 1$ share sold for $\frac{\$ 7,245}{60}=\$ 120 \frac{8}{4}$
$\$ 1203+\$ 1$ brokerage $=\$ 121=$ market valne.
Example 5.-How many shares Jiarli of Oommerce Stock at 121 oan be bought for $\$ 7,275$, brokerage $\frac{1}{4} \%$

Solutio:.
Cost of 1 share $=\$ 121+\$ 1=\$ 121 \$$
$\$ 7,274 \div \$ 1214=60$ shares. Ans.

Example 6.-How many shares Bank of Cummorus Stuck at $\mathbf{1 2 1}$ must I sell to realize \$7,245, brokerage $\frac{1}{1} \%$ ?

Solution.
Receipts from sale 1 share $=\$ 121-\$ 1=\$ 120$ ?
$\$ 7,245 \div 1203=60$ shares. Ans.

## EXERCISE 99.

Find the amount of cash required to purchase-


|  | simareg. | mar. val. | brok. |
| :---: | :---: | :---: | :---: |
| 9. | 135 | 11: $\frac{1}{4}$ | 1\%. |
| 10. | 46 | 874 | \% |
| 11. | 130 | 938 | + |
| 12. | 200 | 751 | $\frac{1}{6} \%$. |
| 14. | 75 170 | 869 | $\frac{1}{8} \%$. |
| 15. | 800 | 1229 | 4\%. |
|  | 360 | 2643 873 | $\frac{3}{7} \%$. |

Find the cash received from the sale of-
shares. hab. vaf. broí

| 17. | 160 | $96$ | brok. |  | shares. | mar. val. | Broz. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18. | 70 | 47 | $\frac{1}{8}$ | 25. | 200 | 110 $\frac{1}{2}$ | \% |
| 19. | 200 | 135 | IR\%. | 26. | 48 | 223 | \% |
| 20. | 96 | 120 | 4\%. | 27. | 120 | 2608 | \% |
| 21. | 148 | 110 | 4\%. | 28. | 36 | 1501 | $\frac{1}{2} \%$. |
| 22. | 250 | 80 | \% | 29. | 45 | 751 | 者\% |
| 23. | 36 | 84 | \% $\%$ | 30. | 160 | 378 | \% \% |
| 24. | 87 | 120 | \% $\%$ \% |  | 240 | 1451 ${ }^{\frac{1}{4}}$ | 8\% |
|  |  |  | 8 \%. | 32 | 60 | 754 | , \% |

Find the market value of the siock when-

| shares. |  |  |  | broz. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | shares. | Brox |
| 84. | 40 | " |  |  |  | + \% | 41. | 70 sold for \$5,600 |  |  |
| 35. | 50 | ${ }^{\prime}$ | 2,405 | \% | 42. |  |  |  |  |
| 36. | 60 | " | 3,795 | $\frac{1}{2} \%$. | 43. | 100 | " | 6,720 | ${ }^{1} \%$ |
| 37. | 80 | ${ }^{\prime}$ | 7,215 | $4 \%$. | 44. | 60 | 36 | ,025 | $\frac{1}{4} \%$ |
| 38. | $120{ }^{\circ}$ | " | -14,510 | $\frac{1}{8} \%$ | 45. | 48 | ، | 4,890 | $\frac{1}{2} \%$. |
| 39. | 360 | " |  | . 8 \%. | 46. | 56 | " | 3,858 | $\frac{1}{8}$ \% |
| 40. | 90 | " | 25,245 | \% | 47. | 75 | ، | 3,962 | $\frac{1}{4} \%$ |
|  |  |  | 6,750 | \% \% | 48. | 80 | " | 4,500 | $\frac{1}{8} \%$. |

## STOCK EXChange.

How many shares may be bought for-

| 49. | $\$ 18,155$ | Mar. VAL. 225 | вrox. |  | cost. | uar. VAL. | \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50. | \$9,760 | 1214 | 5\%. | 53. | \$1,923 | 80 | \%. |
| 51. | \$5,610 | 140 | 1\%. | 64. | \$3,850 | 96 | + \% |
| 52. | \$13,620 | 140 | \% | 65. | \$12,025 | 240 | \%. |
|  |  |  | $\frac{1}{8} \%$. | 56. | \$4,134 | 86 | $\frac{1}{8} \%$. |

How many shares must be sold to realize-

## впох.

$4 \%$

| 57. | s. P. <br> 88,505 | mar. Vals. | bro | 8. P . | mar. $\mathrm{var}_{\text {\% }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\square$ | \$10,245 | 1219 854 | 4\%. | 61. $\$ 19,755$ | 220 | \%. |
| 59. | \$4,314 | ${ }_{90}$ | \#1\%. | 62. 82,400 | 96 | 1\%. |
| 60. | \$4,350 | 87\% |  | 63. $\$ 8,336$ | 1301 | i\% |
|  |  |  | \% \%. | 64. \$10,548 | 110 | \% \% |

436. Given the number of shares or amount of stock held and tate per cent of dividend, to find income, of vice versa.

Example 1.- wh at income will be derived from 60 shares G. T. R Stook paying $6 \%$ dividends ?

Solution.
Income rom 1 share is $\$ 6$
60 res is $\$ 6 \times 60=\$ 360$.
Example 2.-What would a stockholder, who owns $\$ 4,000$ Bank of Commerce Stock, receive from a $5 \%$ dividend?

> Solution.
> $\$ 4,000$ stock $=40$ shares 40 shares at $\$ 5$ income per share $=\$ 200$.

Example 3.-What number of shares does a person hold who receives $\$ 300$ income, from a $6 \%$ dividend?

Solution.
$\$ 6$ income is derived from 1 share
$\therefore \$ 300$

$$
300 \div 6=50 \text { shares. }
$$

Example 4.-What amount of stock must be held to obtain $\$ 200$ income from a $4 \%$ dividend

Soldtion.
$\$ 4$ inoome is derived from 1 share
$\therefore$ 终 20
50 shares $=50 \times 100=\$ 200 \div 4=50$ shares

Exampin 5.-What is the rate per cent. dividend when 40 shares yield an mome of \$240?

Sozution.
40 shares yield an income of $\$ 240$
$\therefore 1$ share yields an inoome of $\$ 6$
$\therefore$ rate per cent, dividend is $6 \%$.
EXAMPLE 6.- $\$ 300$ income is derived from $\$ 3,750$ stoak ; find the rate per cent. of dividend.

Solution.
$\$ 3,750$ stock $=87 \frac{1}{2}$ sharea $37 \frac{1}{2}$ shares yield an inoome of $\$ 300$
$\therefore 1$ share yields an income of $\frac{300}{37 \frac{1}{2}}=98$
$\therefore$ rate per cent. dividend $=8 \%$.

## EXERCISE 100.

What income will be derived from-

| SHARES. <br> 1. 70 | DIV. |  | shares. | DIV. |  | mar | rv. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2. 120 | 6\%. | 5. | 120 | $3 \%$ | 9. | 130 | $52 \%$ |
| 3. 150 | 6i\% | 6. | 110 | 812\% | 10. | 145 | 6\%. |
| $\begin{array}{rrr}\text { 5. } \\ \text { 4. } & 65\end{array}$ | $4 \%$ $8 \%$ | 7. | 75 | $9 \%$. | 11. | 64 | 7\%. |
| 4. 65 | 8\%. | 8. | 126 | 81\% | 12. | 87 | $84 \%$. |

What income will be derived from-
ETOCK. DIV.


What number of shares and what stock must be held to obtain-


## STOCK EXCHANGE.

What is the rate per cent. of dividend when-

| shareg. | income. |  | shares. |  | incoure |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 37. 50 | yield | \$275. |  |  |  |  |
| 38. 60 | " | \$300. | 42. | 36 | yield | \$198. |
| 89. 90 | " | \$390. | 43. | 42 | " | \$189. |
| 40. 75 | 4 | \$450. | 44. | 80 | " | \$500. |
| 41. 34 | ${ }^{\prime}$ | \$170. | 45. | 54 | " | \$351. |
|  |  |  | 46. | 120 | " | \$900. |

What is the rate per cent. of dividend when-

| 47.48. | stocz. | INCOME.yields $\$ 245$. |  | 62. | stock. | income. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | \$3,500 |  |  |  |  |  |  |
| 48. | \$3,640 | " | \$182. | 02. | \$4,500 | yields | \$135. |
| เ9. | \$2,250 | * | \$225. | 58. | \$7,550 | " | \$45: |
| 50. | \$4,000 | - | \$225. | 54. | \$8,600 | * | \$301 |
| 51. | \$2,300 | d | \$380. | 55. | \$3,275 | " | $\$ 131$ |
|  |  |  |  | 56. | \$4,125 | " | \$330 |

487. Given cash invested, market value of stock and rate per cent. dividend to find income, or vice versa.

Example 1. What inoome will be derived from investing $\$ 6,315$ in the 6 per cents at 105, brokerage $\frac{1}{2} \%$ ?

$$
\begin{aligned}
& \frac{6815}{105!}=\text { Nolution. } \\
& \frac{6815}{1051} \times 6=\$ 360 . \text { Inoome. Art. } 436 .
\end{aligned}
$$

Example 2.-What sum must be invested to secare an income of $\$ 860$ from the 6 per cents at 105 , brokerage $\frac{1}{4} \%$ ?

$$
\begin{aligned}
& \frac{860}{6}=60, \text { Solumber of shares held. } \\
& 1057 \times 60=8,815, \text { Art. } 436 . \\
&
\end{aligned}
$$

## EXERCISE 101.

1. What income is derived from investing-

| 1. $\$ 4,210$ | $\begin{gathered} \text { RATR. } \\ 5 \% \end{gathered}$ | $\begin{gathered} \text { MAR. } \\ \hline 105 \end{gathered}$ | Bror. |  | OABH. | M:IR. | KA 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2. $\$ 5,715$ | 4t\% | 95 | \$\% | 9. | \$8,510 | $4 \%$ | 1064 |  |
| 8. $\$ 1,683$ | 3\% | 70 | \% ${ }^{1} \%$ | 10. | \$5,811 | 51\% | $110 \frac{1}{2}$ | \%. |
| 4. $\$ 3,524$ | 6\% | 110 | \% \% |  | \$23,070 | 67\% | 96 | \% \% |
| 5. $\$ 15,025$ | $7 \%$ | 150 | $\stackrel{1}{8} \%$ |  | \$27,820 | 31\% | 854 | 8\% |
| 6. \$7,988 | 8\% | 220 | \% \% | 13. | \$5,049 | 8\% | 140 | + |
| 7. $\$ 24,050$ | 9\% | 240 | 年\% |  | \$13,025 | 7\% | 130 | \% |
| R. $\$ 10.189$ | 71 \% | 140 | \% \% |  | \$15,785 | 10\% | 225 | \% \% |
|  |  |  |  |  | \$6,090 | 4.1 \% | 76 | \% \% |

2. What amount of cash must be invested in order to derive an-
ingome. rate. mar. val. brok.

| 1. | \$200 | 5 \% | 105 |  |  | INOOM | Rate. | Ar. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2. | \$270 | 41 \% | 95 | 4\%. | 10 | \$320 | 4\% | 1064 | $18 \%$ |
| 3. | \$72 | 31 $\frac{1}{2}$ \% | 70 | \% | 11. | \$264 | 57 \% | 110. | $\frac{1}{8} \%$. |
| 4. | \$192 | 6\% | 110 | 8\%. |  | \$1,500 | $64 \%$ | 96 | \% $\frac{1}{8} \%$. |
| 5. | \$700 | $7 \%$ | 150 | \% | 12. | \$112 | 31 \% | 851 | \% \% |
| 6. | \$288 | $8 \%$ | 220 | , | 13. | \$288 | 8\% | 140 | 4\% |
| 7. | $\$ 900$ | 9\% | 240 |  | 14. | \$700 | $7 \%$ | 130 | 4\%. |
| 8. | \$540 | 712\% | 140 | $\frac{1}{8} \%$. | 16. |  | 10\% | 225 | $\frac{1}{2} \%$. |
| 8 ${ }^{2}$ \% 16. \$360 $4 \frac{1}{2} \% \quad 76 \quad \frac{1}{8} \%$. |  |  |  |  |  |  |  |  |  |

438. To find the per cent. of income from a given investment without regard to its maturity.

Example,-What per cent. of my investment shall I secure by prechasing Ontario Benk stock at 105 , paying $7 \%$ dividends ?

On $\$ 105$ investiont Solution.
" $\$ 1$ " $\$ 7$ income is derived.
" $\$ 100$ " $100 \quad$ "
$\therefore$ rate per cent $=62 \%$.
439. To find how stock must be bought, which pays a given per cent. dividend, to realize a specified per cent. : in the investment.

Example.-At what price must I buy stook which pays $6 \%$ dividend to ruslize $8 \%$ on my investment?

Solution.
Since the income derived from 1 share is $\$ 6, \$ 6$ must therefore be $8 \%$ of my investment for 1 share

$$
\begin{aligned}
8 \% \text { of parchase price of } 1 \text { share } & =\$ 6 \\
0 \% \quad " \quad " \quad & =102 \times 6=\$ 75 .
\end{aligned}
$$

EXERCISE 102.
What per cent. of my investment will be derived from investing in the -

| 1. 4 per cents at 120. |  |  | 5. 8 per cents at 125. |  |  | 9. $3 \frac{1}{2}$ per cents at |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2. 5 | " | 80. |  |  |  | 70. |
| 3. 6 | " | 110. | 7. 10 | " | 175. |  |  | 10. $1 \frac{1}{2}$ | " | 76. |
| 4. 3 d | " | 90. | 8. 19 | * | 225. | 11. 53 | " | 110. |
|  |  |  |  | \% | 240. | 12. 6 | 4 | 90. |

At what price must I buy stock which pays-
13. $6 \%$ dividends to realize $9 \%$ on my investment?
14. $4 \%$ " "
$5 \%$ "
15. $5 \%$ " " $6 \%$ " "
16. $8 \%$ " " $4 \frac{1}{2} \%$ " "
17. $3 \frac{1}{2} \%$ " " $5 \%$ " "
18. $4 \frac{1}{2} \%$ " " $3 \frac{1}{2} \%$ " "
$\begin{array}{lllcl}\text { 19. } 7 \% & \text { " } & \text { " } & 4 \% & \text { " } \\ \text { 20. } & \text { " } & \text { " } & \text { " } & 4 \%\end{array}$
20. $9 \%$ " " $10 \%$ ،
440. To find the per cent. income derived from investing in bonds or debentures payable in a given time.

Example.-What per cent. income will be received if I buy Dominion 6's at 120, payable at par in 16 years?

Solution 1.
Cost price of $\$ 100$ of bonds $=\$ 120$
Selling " " $\quad$ " $=\$ 100=$ par value.
1 year $=\$ 14$
Income each year from $\$ 100$ of bonds $=\$ 6$
$\therefore$ Gain eanh year on $\$ 100$ of bonds $=\$ 6-\$ 1 \frac{1}{4}=\$ 49$
On $\$ 120$ invested, the income cleare $=\$ 4$.
$\therefore$ On $\$ 100 \quad$ " $\quad=\frac{49}{120} \times 100=\$ 329$
$\therefore 3_{2} 3^{2}$ is dorived from the investment.
Solotion 2.
Receipts of $\$ 100$ of bonds $=\$ 100$ par value at end of 16 years

| Income | $"$ | $"$ | $=96, \$ 6$ per year for of 16 year |
| ---: | :--- | :--- | :--- |
| Total years |  |  |  |
| roceipts | $"$ | $"$ | $=\$ 196$ at end of 16 years |

$\therefore$ Gain on $\$ 120$ investment $=\$ 76$ for 16 years
$\because 100 \quad=\$ 32_{2}$ for 1 year
$\therefore 323 \%$ of intereat is derived from the investinent.
441. To find how bonds must be bought, which have several years to run, and which pay a given per cent. dividend, to realize a specified per cent. on the investment.

Exampla.-At what price must $6 \%$ bonds, payable in 10 years, be bought so as to realize $5 \%$ on the investment?

Solution 1.

## By simple interest.

Amount of $\$ 100$ of bonds in 10 yrs at $6 \%=\$ 160$.
In order to realize $5 \%$ on the investment we oan afford to pay the present worth of $\$ 160$ due in 10 years, reckoning interest at $5 \%$.
Present worth of $\$ 160$ for 10 yrs . at $5 \%=188 \times 160=\$ 106 \mathrm{~g}$.
We can therefore afford to pe.y $\$ 1062$ for $\$ 100$ of bonds.
Soldtion 2.

## By compound interest.

If $\$ 6$ income be invested at compound interest as soon as received each year at $5 \%$, the income at the end of 10 years will amount to $\$ 75.467$ (see Table of Annnities).
$\therefore$ Amonat of $\$ 100$ of bonds at end of 10 years $=8175.467$, and the present worth of this amount for 10 years at $5 \%$, compound interest $=$ $\$ 1 \% 5.467 \div \$ 1.6289+=\$ 107.72+$ Ans.

## EXERCISE 103.

1. What per cent. of the investment is received as income by purchasing C. P. R. 5's at 105, payable at par in twenty years?
2. What per cent. income will be received if I buy Dominion 4's at 112, payable at par in sixteen years?
3. Bought Intercolonial Railway bonds at 90, bearing $4 \%$ interest, having twenty-five years to run. What per cent. will be realized if they are paid at par at maturity?
4. What per cent. income will be gained from $8 \%$ bonds, bought at 80, and payable at par in twenty years?
5. In 1882, Intercolonial 6's, due at par in 1930, were bought for 108. What interest will this pay?
6. If I pay 108 for Dominion 4's, having fifteen years to run, what per cent. will I receive if I keep them till they mature, and they are paid at par?
7. At what price must $6 \%$ debentures, payable at par in tight years, be brought to realize $4 \%$ on the investment?
8. Bought railroad bonds payable in five years, and expect to realize $7 \%$ on the investment. What did I pay?
9. What must I pay for $5 \%$ debentures, which mature in fifteen years, that my investment may yield $4 \%$ ? (Both simple and compound interest).
10. What shall I pay for a bond of $\$ 500$ having twelve years to run, with interest at $6 \%$, in order to make it an $8 \%$ investment? (Both methods).
11. What must be paid for a $\$ 600$ debenture, due in five years, with interest annually at $4 \%$, so as to realize $5 \%$ on the investment?

## EXERCISE 104.

1. What income will $\$ 19,650$ invested in Dominion $9 \frac{1}{2}$ 's at $97 \frac{3}{4}$ yield, brokerage $\frac{1}{2} \%$ ?
2. If $\$ 48,000$ is invested, $\frac{1}{2}$ in $5 \%$ stock, at $95 \frac{1}{2}$, and $\frac{1}{2}$ in $6 \%$ stock at 112, brokerage $\frac{1}{2} \%$ in each case, what annual income is secured?
3. A farm which rents for $\$ 411.45$ per annum, is sold for $\$ 8,229$, and the proceeds invested in $5 \%$ bonds at 105 , brokerage $\frac{1}{2} \%$. Is the yearly income increased or diminished, and how much ?
4. How much must a gentleman invest for his daughter in $7 \%$ bonds, selling at 95 , to secure to her a semi-annual income of \$315 ?
5. Bought 300 shares of Michigan Central at 101; held them twenty days, paying interest at $7 \%$ on the purchasemoney, and sold them at 1027. Deducting interest, and brokerage $\frac{1}{8} \%$, for purchase and sale, what was the net profit?
6. A man bought 100 shares Canadian Pacific at $79 \frac{1}{4}$, and sold the same at $82 \frac{3}{8}$. What was the gain, less $\frac{1}{8} \%$ brokerage?
7. Governments yielding $\$ 240$ income a year at $4 \%$ interest, were sold at 108, and the proceeds invested in land at $\$ 75$ an acre. How many acres were bought?
8. Which is the better investment, R. R. stock at $25 \%$ discount, and paying a semi-annual dividend of $4 \%$, or money loaned at $10 \%$, interest payable annually? What per cent. better?
9. What per cent. of his money will a man obtain by investing in $6 \%$ stock at 108 , at a discount of $16 \%$ ?
10. If stock paying $10 \%$ dividends is at a premiun: of $12 \frac{1}{2} \%$, what per cent. of income will be realized on an investment in it?
11. Which will yield the better income, $8 \%$ bonds at 110 , or 5 's at 75 ; 5's at 70 , or 6 's at 80 ?
12. Which is the more profitable, and bow much, to buy B. \& L. H. 7's at 105 , or $6 \%$ bonds, at 84 ?
13. If a man buys stock at $17 \%$ above par, what per cent does he receive on his investment, if the stock pays a dividend of $8 \frac{1}{2} \%$ on its par value ( $\$ 100$ )?
14. A man bought 8 shares of stock at 1083, and after keeping it eleven months received a dividend of $\$ 7$ a share, and sold the stock then at $109 \frac{1}{8}$. What per cent. did he receive on his investment?

1 ; held rrchaseest, and the net at $79 \frac{1}{4}$, ess $\frac{1}{8} \%$ in land $84 \frac{1}{8}$, can be bought for $\$ 12,000$, brokerage $\frac{1}{8} \%$ ?
16. Bought Oct. 12th, 400 G. W. R. at $42 \frac{1}{4}$, and 200 Michigan Central at $92 \frac{1}{2}$; Nov. 10th, sold the former at $42 \frac{7}{8}$, and the latter at $98 \frac{3}{4}$. What was my gain, money being worth $5 \%$ ?
17. Which would be the better investment, $\$ 12,120$ in Michigan Central at 84, paying $3 \%$ annual dividends, or the same invested in Canada Bank stock at 2,020, paying $15 \%$ every two months?
18. On 84 shares of stock two semi-annual dividends were diclared, one at $5 \%$, the other at $4 \%$, the investment paid $10 \%$. What did the stock cost?
19. A man's income from $\$ 2,000$ worth of stock is $\$ 75$ semi-annually. What is the per cent. per annum?
20. At what per cent. discount must $6 \%$ stock be bought, that the investment may pay $9 \%$ ?
21. If a stock yields $15 \%$ per annum, what is its value when money is worth $8 \%$ ?
22. Which is the more profitable investment, a stock at $6 \%$ annually? .
23. At what price must $6 \%$ bonds, payable in eight years, be bought to realize $4 \%$ on the investment.
24. How many shares of a half stoch, standing at $5 \%$ above par, should be given in exchange for 700 shares of the stock of an express company, at $25 \%$ below par?
25. A man subscribed for 300 shares of stock in a manufacturing company, the par value of which was placed at \$50 per share; but, after paying three instalments, amounting to $75 \%$ of the par value, a dividend of $3 \%$ was declared. How much will he receive, and tht what rate per:
26. The gross earnings of a stock company with capital of $\$ 3,500,000$ are $\$ 420,000$; their expenses are $60 \%$ of their gross earnings. What per cent. dividend can they declare, after putting aside $\$ 28,000$ as a surplus?
27. The receipts of a mining company in one year are $\$ 170,000$, clear of all expenses. The company has a capital of $\$ 500,000$, divided into shares of $\$ 10$ each, reserving $\$ 50,000$ as a contingent fund. What rate of dividend can it declare for the jear? what per month? and how much can be paid on each share of stock ?
28. March 4th, deposited with my broker $\$ 500$ margin, for purchasing 50 shares Canada Pacific R. R. stock at 924. The stock was sold March 28th at 963 . Allowing $6 \%$ interest on the deposit, and eharging $6 \%$ interest on the purchase, and $\frac{1}{8} \%$ brokerage, what was the net profit on the transaction?
29. Sold "short" through my broker 200 shares Michigan Central at 90, and "covered" my "short" at $86 \frac{5}{8}$. Allowing $\frac{1}{8} \%$ commission for buying and selling, what was my net profit?
30. May 6th, I bought through my broker 300 shares of a certain stock at $93 \frac{4}{4}$, depositing with him $\$ 3,000$ as " margin," for his security against loss by a fall of price. On the first of the following month, he sold them for my account at 95. How much does he owe me besides the $\$ 3,000$, if he charges $\frac{1}{8} \%$ brokerage for each transaction, interest, at $6 \%$ (for the exact number of days) on the money used in excess of my deposit?
31. Three companies, A, B, and C , are to be consolidated on the basis of the relative market values of their stock.
Thus, A's capital $\$ 1,000,000$, Market value $100 \%$;

| ${ }^{\text {B }}$ |  | \$1,500,000, | " |
| :---: | :---: | :---: | :---: |
|  | " | \$625,000, | " |

The capital of the consolidated company is to be $\$ 2,000,000$, in 20,000 shares of $\$ 100$ each. What proportion and what amount oi the capital should be allotted to cach of the old companies; and how much stock in the new company should the holder of 1 share of the stock of each of the old companies be entitled to ?
32. A customer deposited $\$ 500$ margin with a broker November 23 rd , who purchased for him 50 shares Michigan Central at 80. He sold the same stock November 30th, at 98. What was the gain, brokerage $\frac{1}{8} \%$ ?
38. Aug. 30th, a broker purchased for the account of a customer 300 shares of Railroad Stock at 78. He deposited as a margin $\$ 3,000$. On Sept. 22nd , the stock was sold at 743. What was the loss? Interest $6 \%$, and commission $\frac{1}{8} \%$.
34. May 10th, a speculator deposited with his broker $\$ 5,000$ as a margin, and directed him to purchase for his account 500 shares l'ominion Saving \& Loan, preferred at 908 . May 20th, the stock was sold at $94 \frac{1}{8}$. What was the gain? Interest $6 \%$, brokerage $\frac{1}{8} \%$.
35. Sept. 10th, I deposited with my broker $\$ 5,000$ as margin, and he purchased for me 200 shares, C. P. R. at $90 \frac{1}{2}, 200$ shares, Lon. \& Can. L. \& A. (half stock) at 1224, and 200 shares Intercolonial Railway Stock at $49 \frac{3}{4}$. The stocks on Sept. 30th were quoted as follows: C. P. R. $80 \frac{3}{4}$, Lon. \& Can. L. \& A., 1201 ${ }_{8}$, Intercolouial Railway 415 $\frac{5}{8}$. How much should I have deposited with my broker to make my margin of $10 \%$ good, and to cover commission of $\frac{1}{8} \%$ for buying and selling, and interest at $6 \%$ ? If I had been unable to have made an additional deposit, and the broker had "sold me out," what would have been my loss?

## EXCHANGE.

442. Exchange is the svstem by which merchants in distant places discharge their debts to each other without the transmission of money.
Suppose for example that A. of Toronto owes B. of Halifax $\$ 2,000$ for grain, and C. of Halifax owes D. of Toronto $\$ 2,000$ for dry goods. The two debts may be discharged by means of one draft or bill of exchange without the transmission of money. Thus B. of Halifax draws on A. of Toronto for $\$ 2,000$ and sells the draft to C. of Halifax, who remits it to D. of Toronto, D. of Toronto presents the draft to A. of Toronto for acceptance or payment, and thus both debts are cancelled. There is in effect a setting off or exchange of one debt for the other.
443. A Bill of Exchange is a written order, drawn by one party on another, to pay a specified sum of money to a party named therein, or to his order, or to bearer.
444. Bills of Exchange are of two kinds, viz. : Inland or Domestic, and Foreign.
445. An Inland Bill of Exchange is one which is drawn and made payable in the same country.
446. A Foreign Bill of Exchange is one which is drawn in one country and made payable in another country.

47\%. Inland Bills of Exchange are usually called Drafts, and are distinguished as Time Drafts and Sight Drafts.
448. A Sight Draft is one which is made payable upon presentation or on demand.

44!. A Time Draft is one which is made payable at a certain specified time after date or after time of presentation for acceptance.
450. A Bill of Exchange is negotiable when it may be transferred from one person to another by endorsement or assignment.
ints in rithout
451. The Rate of Exchange is the rate per cent. which is computed on the Bill of Exchange.
452. The Course of Exchange is the current price paid in one place for bills of exchange on another place. This price varies, according to the relative conditions of trade and commercial credit at the two places, between which exchange is made.

The course of exchange between two oountries, depends on their relative amount of indebtedness to each other; and these, in turn, are largely dependent on "the balance of trade," or comparative amonnt of exports and imports. Thus, if the United States owes Great Britain more than Great Britain owes the United States, which is likely to be the case if it has imported from Great Britain more than it has exported thither, exchange on that country will be in demand, and will consequently command a premium. If, on the other hand, the balance of trade is in favor of the United States-that is, if the exports exceed the imports, Great Britain will be indebted to the United States, the supply of bills on Great Britain will more than meet the demand, and exchange will fall below par.

The premium for exchange on any country oan not long exceed the cost of shipping specie thither; for merchants will transmit coin to pay their indebtedness abroad, if it is oheaper so to do than to bny exchange.
453. The Par of Exchange is the estimated value of the coins of one country as compared with those of another, and is either intrinsic or commercial.
454. The Intrinsic Par of Exchange is the comparative value of the coins of different countries, as dete:mined by their weight and purity.

Thus, according to the mint regulations of Great Britain and France, \&1 sterling is equal to 25 fr .20 cent., which is said to be the par between Iondon and Paris. Exohange between the two countries is said to be at nar whea bills are negotiatoã at this rate; that is, when a bill for $£ 100$ drawn in London is worth 2,520 franos in Paris, and conversely. When

## EXCHANGE.

$\mathcal{L 1}$ in London bays a bill on Paris for more than 25 fr .20 cent., the exchange is said to be in favor of London and against Paris; when $£ 1$ in London will not bay a bill on Paria for 25 fr. 20 cent., exchange is against London and in favor of Paris.

Exchange is made to diverge from par by any discrepancy between the actual weight or fineness of the coins and the mint standard, and by the variations in the demand and supply of bills of exchange.
455. The Commercial Par of Exchange is the comparative value of the colns of different countries, as determined by their nominal or market price.
Note.-The intrinsio par is always the same while the coins remain unchanged; bat the commeroial par, being determined by commercial

45̄6. When exchange sells for more than the face of the draft, it is above par, or at a premium, and below par, or at a discount, when sold for less than if ifsee.

0 cent., the when $£ 1$ in is againgt y between rd, and by
he comis deter-

## INLAND OR DOMESTIC EXCHANGE.

## 457. To find the cost of a draft at sight.

Example 1.-How mush mast be paid for a sight draft of $\$ 1,000$, on the Bank of Montreal, at a premiam of $1 \frac{1}{2} \%$ ?

Solution.
$\$ 1+\$ .015=\$ 1.015$, course of exohange
$\therefore \$ 1$ costs $\$ 1.015$
$\therefore \$ 1,000$ cost $\$ 1.015 \times 1,000=\$ 1,015$. Ang.
Example 2.-How much must be paid for a sight draft of $\$ 600$, on the Bank of Ottawa, at a discount of $1 \%$ ?

> Soldtion.
\$1 $-\$ .01=\$ .99$, course of exchange
$\therefore \$ 1$ costs $\$ .99$
$\therefore \$ 600$ cost $\$ .99 \times 600=\$ 594$. Ans.

## 45\%. To find the cost of a time draft.

Example 1.-What will be the cost of the following draft, exohange on Hamilton being in Toronto at $24 \%$ preminm?
$\$ 600$.
Toronto, July 18th, 1889.
Seventy days after sight, pay to J. S. Carson, or order, six hundred doliars, value received, and charge the same to my account.

Janks Feraugon.
To Bank of Montreal, Hamilton.

## Soldtion.

$\$ 1+\$ .0225=\$ 1.0225$, course of exohange
.012, bank discount of $\$ 1$ for 73 da. at $6 \%$ (legal rate) \$1.0105, cost of exohange of \$1
$\$ 1$ cost $\$ 1.0105$
$\$ 600$ " $\$ 1.0105 \times 600=\$ 606.30$.


## IMAGE EVALUATION

 TEST TARGET (MT-3)




Photographic Sciences


Example 2.-Find the cost of a 60 days' draft on the Bank of Quebec, Toronto. for $\$ 900$, at a discount of $2 \frac{1}{2} \%$.

Solution.
$\$ 1-\$ .025=\$ .975$, course of exchange
$.0104+$, bank disoount of 81 ( 63 da.), at $6 \%$ (legal rate) $\$ .9646$, cost of exohange of $\$ 1$
$\$ 1$ cost $\$ .9646$
$\therefore \$ 900$ " $\$ .9646 \times 900=\$ 868.14$.

## EXERCISE 105.

1. Find the cost of a draft on Montreal for $\$ 1,100$, at $\frac{1}{4}$ of $1 \%$ premium.
2. Find the cost of a draft on Winnipeg for $\$ 1,350$, at $\frac{1}{4}$ of $1 \%$ discount.
3. What is the cost of a draft on Chatham for $\$ 1,800$, at $1 \frac{3}{4} \%$ premium?
4. Exchanged $\$ 600$ in bank notes for gold at $5 \%$ premium. How much did I receive?
5. Sold $\$ 375$ uncurrent money at $2 \frac{1}{4} \%$ discount. How much did I receive? How much did I lose?
6. What was the cost of a bill for $\$ 240$ on Belleville, puischased at $14 \%$ premium?
7. Required the amount to pay for a draft to be remitted to Hart \& Denton, Kingston, for $\$ 1,250$, exchange at $\frac{3}{4} \%$ discount.
8. Shipped goods to Winnipeg, and received a draft for $\$ 2,500$, which gave me a profit of $20 \%$; sold the draft at $4 \frac{1}{2} \%$ premium. How much did I gain by both transactions?
9. Bought goods for $\$ 1,250$, and sold them at a profit of $25 \%$; purchased a draft on Fredricton with the proceeds, at a diecount of $\frac{3}{4} \%$. What was the amount of the तraft?
10. A commission merchant sold goods, the net proceeds of which were $\$ 2,750$. How large a draft can he buy tc remit to his consignor, if he pays $2 \frac{1}{2} \%$ premium for the draft ? How large a draft if he purchases at $2 \frac{1}{2} \%$ discount \&
11. Find the cost of a draft for $\$ 1,600$, payable 30 days after sight, when exchange is 4 of $1 \%$ premium, and interest $6 \%$.
12. Find the cost of a draft for $\$ 950$, payable in 30 days, when exchange is at par and interest $4 \frac{1}{2} \%$.
13. Find the cost of a draft for $\$ 500$, payable 60 daye after sight, when exchange is $\frac{1}{2}$ of $1 \%$ discount, and interest $7 \%$.
14. Find the cost of a draft for $\$ 1,200$, payable in 90 days after sight, when exchange is $\frac{1}{2}$ of $1 \%$ premium, and interest $7 \%$.
15. Find the cost of a draft for $\$ 810$, payable in 90 days, when exchange is at $\frac{1}{4}$ of $1 \%$ premium, and interest $5 \frac{1}{2} \%$.
16. Find the cost of a draft for $\$ 725$, payable in 60 days, when exchange is at $\frac{1}{4}$ of $1 \%$ discount, and interest $5 \%$.
17. What must be paid in Toronto for a draft on Victoria at 90 days, for $\$ 4,800$, the course of exchange being 101 옵 \% ?
18. A firm in Toronto bought a 60 days' draft on Montreal for $\$ 2,500$, at $\frac{8}{8} \%$ premium, $6 \%$ interest. What did the draft cost?
19. A broker in Montreal bought a 90 days' draft on Halifax for $\$ 1,299$ at $\frac{1}{2} \%$ discount. He paid $\frac{1}{8} \%$ additional for brokerage. How much did he pay for the draft?
20. A commission merchant in Winnipeg sold for a firm in Hamilton a consignment of cotton. The sales amounted to $\$ 12.240$, and his commission was $5 \%$ on the sales. He bought and remitted a 30 days' draft at E\% discount for the proceeds due the firm. How much did the draft cost?

## 45\%. To find the face of a draft at sight.

Example 1.-I paid $\$ 652.86$ for a sight draft on the Bank of Commerce, Winnipeg, at a premium of $\frac{3}{4} \%$. What was the amount of its face?

Solution.
$\$ 1+\$ .0075=\$ 1.0075$, course of exchange
$\$ 1.0075$ is paid for $\$ 1$ faoe

$$
\begin{aligned}
& \$ 1 \text { ". " } \$ \frac{1}{1.0075} \text { " } \\
& \$ 652.86 \text { " " } \$ \frac{652.86}{10.075} \text { " } \\
& \therefore \text { Face of draft }=\$ 648 \text {. }
\end{aligned}
$$

Example 2.-A commission merohant in Belleville wishes to remit to his employer at Halifax a sight draft purchased with $\$ 7,202.70$. What is the faoe of the draft, exchange at $\%$ discount ?

Solution.
$\$ 1-\$ .00625=\$ .99375$, course of exchange
$\$ .99375$ is paid for $\$ 1$ face
$\$ 1 \quad$ " $\$ \frac{1}{.99375}$ "
$\$ 7,202.70$ " $\quad \$ \frac{7,202.70}{.99375}$ "
$\therefore$ Face of draft $=\$ 7.248$.

## 460. To find the face of a time draft.

Examply 1.-The cost in London of a 70 days' draft on Ottawa, exchange $\frac{7}{8} \%$ premium, was $\$ 797.40$. What was the face of the draft?

Boldtion.
$\$ 1+\$ .00875=\$ 1.00875$, course of exohange
.012, bank disoount of $\$ 1$ for 73 da. at $6 \%$
$\$ .99675=$ cost of 31
$\$ .99675$ is paid for $\$ 1$ faoc
$\$ 1 \quad$ " 1 $\$ \frac{1}{.99675}$ "
3799.40 " $\quad \mathbf{7 9 7 : 4 0} .99675$
$\therefore$ Fruce of araft $=\$ 800$.

Example 2.-A commission merchant in Stratford wishes to remit to his employer in Montreai $\$ 987.10$ by a draft at 30 days. What is the face of the draft which he oan purchase with this sum, exchange being et a discount of $\frac{8}{4} \%$ ?

Soldtion.

$$
\begin{aligned}
\$ 1-\$ .0075= & \$ .9925, \text { conrse of exohange } \\
& : 0054+, \text { bank discount for } 33 \text { da. at } 6 \%
\end{aligned}
$$

$$
\$ .9871=\text { cost of } \$ 1
$$

$\$ .9871$ is paid for $\$ 1$ faoe
$\$ 1$ " " $\frac{1}{.9871}$ "
$\$ 937.10$ " $\quad \$ \frac{987.10}{.9871}$ "
$\therefore$ Face of draft $=\$ 1,000$.

## 461. To ind the rate of exchange on a sight draft

Example 1.-The cost of a sight draft on Winnipeg for $\$ 1,200$ was \$1,213.50. Find the rate of exchange.

> Solution.

$$
\begin{array}{ll}
\text { Cost } & =\$ 1,213.50 \\
\text { Face } & =\$ 1,200.00 \\
\text { Premium } & =\frac{\$ 13.50}{}
\end{array}
$$

$\$ 1,200$ was purchased at a premium of $\$ 13.50$

| $\$ 1$ | " | " | " | $\$ \frac{13.50}{1,200}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\$ 100$ | " | " | " | $\$ \frac{13.50 \times 100}{1,200}=\$ 1$ 3 |

Example 2.-The cost of a sight draft on Victoria for $\$ 600$ was $\$ 594.75$. What was the rate of exchange?

Solution.

$$
\begin{aligned}
& \text { Face }=\$ 600.00 \\
& \text { Cost }=\$ 594.75 \\
& \text { Discount }=\$ 5.25
\end{aligned}
$$

$\$ 600$ was purohased at a disoount of $\$ 5.25$
$\$ 1$
$\$ 100$
4
4
" $\$ \frac{5.25}{600}$
" $\$ \frac{5.25 \times 100}{600}=1 \%$
$\therefore$ Rate of exchange $=$ \% discount.

## 462. To find the rate of exchange on a time draft.

Exampla 1. -The cost in Oollingwood of a 70 days' draft for $\$ 1,000$ is $\$ 1,020$. Interest being $6 \%$, what was the rate of exohange ?

$\$ 1,000$ was purchased at a premium of $\$ 32$
$\$ 1$
"
" $\frac{32}{1,000}$
$\$ 100$
"
c $\quad \$ \frac{32 \times 100}{1,000}=\$ 84$
$\therefore$ Rate of exchange $=37 \%$ preminm.
Example 2.-The cost in Quebec of a 70 days' draft for $\$ 6,000$ is $\$ 5,910$. Interest being $6 \%$, what is the rate of exohange?
$\$ 6,000$ was purchased at a discount of
$\therefore$ Rate of exchange $=\frac{3}{10} \%$ discount.

## EXERCISE 106.

1. A sight draft was purchased for $\$ 550.62$, exchange being at a premium of $8 \frac{1}{2} \%$; what was the face?
2. What is the face of a sight draft bought for $\$ 7,500$ at a premium of $\$ 2.50$ ? ( $\$ 2.50$ on $\$ 1,000=4 \%$.)
3. Find the largest draft payable 30 days after date that can be bought for $\$ 4,985.00$, exchange being at a premium of $\frac{1}{4} \%$.
4. What per cent. of its face is the cost of a 90 days' draft, if exchange is $1 \%$ premium, and interest is allowed at $4 \%$ ?
5. Find the face of a 60 days' draft, bought for $\$ 620.75$, if exchange is $\$ 2.50$ discount, and interest $6 \%$.,
6. Find the face of a draft, payable 60 days after date, that can be bought for $\$ 1,125$, when exchange is at $\frac{1}{4}$ of $1 \%$ discount, and interest $5 \frac{1}{2} \%$.
7. Find the face of a draft, payable 30 days after date, that can be bought for $\$ 520$, when exchange is at $\frac{1}{2}$ of $1 \%$ premium, and interest $4 \%$.
8. Find the face of a draft, payable 60 days after sight, that can be bought for $\$ 1,250$, when exchange is at $\frac{1}{4}$ of $1 \%$ premium, and interest $7 \%$.
9. Find the face of a draft, payable 80 days after sight, that can be bought for $\$ 274$, when exchange is at par, and interest $6 \%$.
10. Find the faice of a draft, payable 90 days after date, that can be bought for $\$ 10,000$, when exchange is at par, and interest $4 \frac{1}{2} \%$.
11. A commission merchant in Detroit wishes to remit to his employer in St. Louis, $\$ 512.36$ by draft at 60 days; what is the face of the draft which he can purclase with this sum, exchange being at $2 \frac{1}{2} \%$ discount?
12. An agent in Halifax having $\$ 1,324.74$ due his employer, is instructed to purchase with the same a draft drawn at 30 days; what will be the face of the draft, exchange being at $1 \frac{3}{4} \%$ premium ?
13. My agent in Winnipeg sells a house and lot for $\$ 7,500$, on commission of $1 \frac{1}{2} \%$, and remits to me the proceeds in a draft purchased at $\frac{1}{2} \%$ premium; what sum do I receive from the sale of my property?
14. The Merchants' Bank of New York having declared a divid, ud of $6 \frac{1}{4} \%$, a stockholder in Toronto drew on the bank for the sum due him, and sold the draft at a premilum of $1 \frac{3}{4} \%$, thus realizing $\$ 508.75$ from his dividend; how many shares did he own?
15. A man in Owen Sound has $\$ 4.800$ due him in Quebec; how much more will he realize by making a draft for this sum on Quebec aud selling it at $\frac{1}{2} \%$ discount, than by having a draft on Owen Sound remitted to nim, purchased in Quebec for this sum at $\frac{3}{4} \%$ preminm?
16. A man in Brantford purchased a draft on Montreal for $\$ 5,320$, drawn at 60 days, paying $\$ 5,141.78$; what was the course of exchange?
17. An agent, owing his principal $\$ 5,059.20$, was directed so buy a draft with this amount, and remit it. The prinsipal received $\$ 4,960$; what was the rate of exchange?
18. Sight exchange on Toronto for $\$ 5,000$ cost $\$ 5,075$ : what was the course of exchange?

## FOREIGN EXCHANGE.

463. Foreign Exchange is the exchange-which is carried on between different countries, and is distinguished as direct and circuitous.
Exchange with Europe is effected mainly through the great financial centres, Lundon, Paris, Antwerp, Berlin, Hamburg, Frankfort, and Amsterdam.
464. Direct Exchange is confined to the two places between which the money is to be remitted.
465. There are always two methods of transmitting money between two places. Thus, if A. is to receive money from B.,

1et. A. may draw on B. and sell the draft;
2nd. B. may remit a draft made in favor of A.
Note.-One person is said to draw on another person when he is the maker of a draft addressed to that person.
466. A set of exchange is a bill usually drawn in triplicate, each bearing the same date, payable to the same party, and so expressed that when one of the bills is paid the others become void.

The object of drawing Bills of Exchange in sets of three is to provide against loss in transmitting from one country to another. The bills are sometimes sent by different routes or by the same route at different dates. Some merchants send only the first and second and preserve the third.

## SET OF EXCHANGE.

$$
£ 1,000 \text {. }
$$

(1.)

Sixty days after sight of thi Toronto, July 23, 1889. and Third of the same order of $H$. $E$. value f. E. Clarke, One Thousand Pounds Sterling, value received, and charge the same to account of To Brown, Shipley \& Co.,

John MoDonald \& Co. London, England.
No. 179.
(2.)
£1,000.
Toronto, July 23, 1889.
Sixty daye after sight of this Second of Exchange (First and Third of the same tenor and date unpaid), pay to the order of H. E. Clarke, One Thousand Pounds Sterling, value received, and charge the same to account of To Brown, Shipley \& Co., John MoDonald \& Co. No. 179. London, England.

> (8.)
£1,000.
Sixty days after sight Toronto, July 23, 1889. and Second of the same of this Third of Exchange (First order of H. E. Clarke, Onor and date unpaid), pay to the value received, and charge the to account of To Brown, Shipley \& Co., John MoDonald \& Co. No. 179.
467. Foreign Bills of Exchange are usually drawn in the currency of the country in which they are payable. Thus drafts on England are usually drawn in pounds, shillings, and pence; on France, Belgium, and Switzerland, in francs ; on Germany in marks, etc.
468. Foreign Bills of Exchange are usually drawn at sight ( 3 days); or at sixty ( 63 days) days' sight.
469. Quotations for 3 days refer to sight exchange, on the theory that 3 days' grace are allowed on sight drafts, though custom varies in this respect.
470. Sight drafts are frequently called "short" exchange, and 60 day drafts, " long " exchange.
471. "Long" exchange is sold at a rate below that for "short" exchange, sufficient to equalize the difference in interest between the dates of maturity of the two classes of bills, the banker having the use of the money from the time the draft is drawn till it is paid.
472. A Letter of Credit is a draft made by a banker in one country, addressud to foreign bankers, by which the holder may draw funds at different places to any amount not exceeding the limits of the letter of credit.
473. Exchange on England (sterling exchange) is quoted by giving the value of $£ 1$ in dollars and cents.

Thus, when exchange is 484 , a draft of $£ 1$ will cost $\$ 4.84$; of $\$ 100, \$ 484$.
474. By Act of Parliament the value of the pound sterling was fixed at $\$ 4 \frac{4}{0}(9 £=\$ 40)$. This is much below its intrinsic value, which is now fixed at $\$ 4.86 \frac{2}{3}$. The rates of exchange usually quoted in commercial papers are
calculated at a certain per cent. on the old par of exchange.

Exchange is at par between Great Britain and Canada when the old par of exchange is at a premium of $9 \frac{1}{2}$ per cent., for $\$ 4 \frac{4}{0}$ increased by $9 \frac{1}{2}$ per cent., equals $\$ 4.86 \frac{4}{3}$.
475. Sterling quatations usually range between 4.80 and 4.91 ( $i$ e. $\$ 4.80$ to $\$ 4.91$ to the $£$ sterling). Two quotations are menti ned for each kind of exchange, and indicate the highest and lowest price paid on the same day. Thus 60 days' sterling 4.86 @ 4.87 , means that the lowest quotation to the $£$ was $\$ 4.86$, and the highest \$4.87.

Quotations are frequently given with reference to the old par of exchange. Thus 60 days' sterling $9 \frac{1}{4}$ to $9 \frac{1}{2}$ means that the old par of exchange ( $£ 1=\$ 4 \frac{4}{9}$ ) ranges from $9 \frac{1}{4} \%$ to $9 \frac{1}{2} \%$ premium, $i$. e. the lowest course of exchange is $\$ 4 \frac{4}{8} \times 1.09 \frac{1}{4}$; the highest, $\$ 4 \frac{4}{8} \times 1.09 \frac{1}{2}$.
476. Exchange on France, Belgium, and Switzerland, is quoted by giving the value of $\$ 1$ in francs and centimes. Thus, when exchange is $5.27 \frac{1}{2}, \$ 1$ will buy 5 francs and $27 \frac{1}{2}$ centimes.
477. Exchange on Amsterdam, (Netherlands), is quoted by giving the value of one guilder or florin in Canadian currency.

The intrinsic par value of one guilder is $40_{\frac{1}{1 \sigma}}^{2}$ cents.
478. Exchange on Germany is quoted by giving the value of 4 marks (reichsmarks) in cents.

The iutrinsic par value of 1 mark is $23 \frac{8}{10}$ cents.

## 480. To find the cost of a foreign bill of exchange.

Example 1.-How much must be paid in Toronto for a bill of exchange on Liverpool for $£ 1,200$, exchange being quoted at $\$ 4.86 \frac{3}{2}$ to the £ sterling ?

Solution.


Example 2.-How much must be paid in Hamilton for a draft on Paris for 2,072 franos, exchange being quoted at 5.18 ?

Solution.
5.18 francs $=\$ 1$
$1 \quad$ franc $=\$ \frac{1}{5.18}$
$2,07 \%$ francs $=\$ \frac{2,072}{5.18}=\$ 400$. Ans.
Example 3.-What will be the cost in Montreal of the following bill of exchange on Liverpool, at $9 \frac{1}{2} \%$ premium? £432.

At sight of this first of exchange (second 22nd, 1889. same tenor and date unpaid), pay to the order of W. R. Telford, Montreal, four hundred and thirty-two pounds, value received, and charge the same to the account of,

To Alex. Grant \& Son., Liverpool, England.

$$
\begin{aligned}
& \text { Solution. } \\
& \mathbf{£ 9}=\$ 40 \times 1.095 \\
& \mathbf{£ 1}=\$ \frac{40 \times 1.095}{9} \\
& £ 432=\$ \frac{40 \times 1.095 \times 432}{9}=\$ 2,102.40 \text { Ans. }
\end{aligned}
$$

J. P. Hume \& Co.

EXERCISE 107.

1. Sold to a broker 480 English sovereigns at 4.86. I was paid in currency when gold was quoted at 1.054. How much did I receive?
2. An importer purchased a lill of exchange on London, at 3 days' sight, for $£ 48816 \mathrm{~s}$. 6 d ., at $4.85 \frac{1}{2}$. What was the cost?
3. Find the cost of a bill of exchange on Manchester, for $£ 485$ 12s. 6 d . at the par value.
4. An exporter sold a draft for $£ 540$ 3s. on Liverpool, proceeds?
5. What is the cost in $\mathrm{K}^{\prime}$ ston of a bill on London, Eng.; for $£ 425$ 6s. 8d., at $9 \frac{3}{4} \%$ premium ?
6. How much will a draft on Berlin for 2,400 marks cost, exchange being quoted at $94 \frac{1}{2}$ ?
7. Bought a bill of exchange on Paris for $3,760.20$ francs, when exchange was $5.22 \frac{1}{4}$. What did the bill cost?
8. What is the cost in Toronto of a bill of exchange on St. Petersburg for 3000 roubles at $1 \frac{1}{4} \%$ premium, the par of exchange being $\$ .754$ for 1 rouble ?
9. What is the cost of a bill of exchange on New York for $\$ 7,200$, at $8 \%$ premium ?
10. Bought at par, 260 rupees of India, 560 Austrian florins, and 480 crowns of Denmark. How much did I pay for all?
11. Sold a bill of exchange on Amsterdam for 1,440 guilders. Exchange 393. What was the sum obtained ?
12. Sold exchange on Geneva, through a broker, for 8,000 francs at 60 days' sight. What were the proceeds of the draft, exchange being $5.20 \frac{5}{8}$, brokerage $\frac{1}{8} \%$ ?
13. What will it cost to remit 8,750 francs to Antwerp at par value?
14. What were the proceeds of a draft, sold through a broker, for 8,748 marks (Reichsmarks), at $94 \frac{3}{8}$, brokerage 8\% ?
15. What are the proceeds of a draft on Paris for 12,420 frencs, at $5.19 \frac{3}{4}$, brokerage on exchange $\frac{1}{8} \%$ ?

## 481. To find the course of exchange.

Example 1.-The cost of a bill of exchange on Liverpool for $£ 500$, including a brokerage of $\frac{1}{8} \%$, was $\$ 2,443.05$. What was the quotation?

$$
\begin{aligned}
& \quad \text { Solotion. } \\
& \quad 100 \%+\frac{1}{8} \%=100 \frac{1}{8} \% \\
& 100 \frac{1}{8} \% \text { of cost of bill }=\$ 2,443.05 \\
& \therefore \text { Cost of bill } \quad=\frac{2,443.05 \times 100}{100 \frac{1}{2}}=\$ 2,440 . \\
& \therefore \text { £500 are worth } \\
& £ 1 \text { is worth } \\
& \quad \frac{\$ 2,440}{500}=\$ 4.88, \text { course of exahange. }
\end{aligned}
$$

Example 2. -The cost of a bill of exohange on Hamburg for 4,400 marks, including brokerage of $\frac{4}{8} \%$, was $\$ 1,057.32$. What was the course of exchange on Hamburg?

Solution.

$$
\begin{aligned}
& 100 \%+\frac{1}{8} \%=100 \frac{1}{8} \% \\
& 100 \frac{2}{8} \% \text { of cost of bill }=\$ 1,057.32 \\
& \therefore \text { Cost of bill }=\frac{\$ 1,057.32 \times 100}{100}=\$ 1,056 . \\
& \therefore 4,400 \text { marks are worth } \$ 1,056
\end{aligned}
$$

1 mark is worth 240.
24c. $\times 4=96 \mathrm{c}$. $=$ course of exchange. Art. 478.

## EXERCISE 108.

Find the course of exchange of a bill.

11. A draft. on Dublin for $£ 360$ cost $\$ 1,786$. What was the course of exchange?
12. The cost in currency, when gold was at $104 \frac{1}{4}$, for a bill of exchange for 12,800 guilders on Amsterdam was $\$ 6,245.80$, including $\frac{1}{8} \%$ brokerage. What was the course of exchange?
13. I paid $\$ 5,817$ for a bill of exchange for $£ 1,200$ on Liverpool. What was the course of exchange, exclusive of brokerage?
14. The cost, including $\frac{1}{8} \%$ brokerage, for a draft on Antwerp for 833 francs was $\$ 161$. What was the course of exchange?
15. A merchant paid $\$ 755$ for a bill of exchange for 3,200 marks on Frankfort. What was the course of exchange, no charges for brokerage being made?

## 482. To find the Face of a Foreign Bill of Exchange.

Example 1.-A bill of exchange on Manchester, England, cost $\$ 1194.94$ when exchange was 4.88 . What was the face of the bill?

$$
\begin{aligned}
& \text { Soldtion. } \\
& \$ 4.88=\text { cost of } £ 1 \\
& \$ 1 \quad=\quad \text { " } £ \frac{1}{4.88} \\
& \$ 1194.94=\quad \text { = } \quad \frac{1194.99}{4.88}=£ 244.875 . \\
& =£ 244 \text { 17s. 6d. Face of bill. }
\end{aligned}
$$

Example 2.-The cost of a bill of exchange on Bremen was 8570, when exchange was 95. What was the face of the bill?

Solution.

Hxample 8.-The cost of a bill of exohange on Paris was \$51)0, when exchange was at 5.18. What was the face of the bill? Solution.

$$
\begin{aligned}
& \$ 1=\text { cost of } 5.18 \quad \text { franos. } \\
& \text { 6.18×500 " } \\
& =2,590 \text { francs, Fape of bill. }
\end{aligned}
$$

## EXERCISE 109.

1. A bill of exchange on Montreal, cost $£ 125$ in Birmingham, England, exchange being at $8 \%$ premium for sterling; required the face of the bill?
2. Bought a bill of exchange on London, when exchange was 4.90 and gold $102 \frac{1}{2}$. I paid $\$ 37,668.75$ in currency. What was the face of the bill?
3. An agent remitted to his principal a draft on Toronto from Amsterdam at $\frac{1}{8} \%$ brokerage, exchange being at 40. The cost of the draft in Amsterdam, including brokerage, was 960 guilders. What was the face of the draft?
4. A broker invested $\$ 1,158$ in Paris francs at par. How many francs did he purchase?
5. What will be the face of a bill on Hamburg, exchange being quoted at $94 \frac{1}{2}$ and the cost of the draft $\$ 756$ ?
6. An agent in Boston, having $\$ 7,536.30$ due his employer in England, is directed to remit by a bill on Liverpool. What is the face of the bill which he can purchase for this money, exchange being at $11 \%$ premium ?
7. A merchant in Chatham has 9.087 guilders, 10 stivers, due him in Amsterdam, and requests the remittance by draft. What sum will he receive, exchange on Canada being in Amsterdam at $2 \frac{1}{2}$ guilders for $\$ 1$ ? ( 1 guilder $=20$ stivers.)
8. What is the face of a 8 days' draft on Bremen, that was purchased in Hamilton for $\$ 3,261.60$, exchange $94 \frac{3}{8}$ ? 9. A trader in London, Eng., wishes to invest £2,500 in merchandise in Lisbon. If he remits to his correspondent at Lisbon a bill purchased for this sum at the rate of $64 \frac{1}{2}$ d. sterling, per milree. What sum in the currency of Portugal will the agent receive?
9. G. B. Smith \& Co., Toronto, instructed their agent at Berlin to draw on them for $a$ bill of goods of 4,500 marks, exchange at $97 \frac{1}{8}$, brokerage $\frac{1}{4} \%$. What did they pay in Canadian money for the goods?

## FOREIGN CIRCUITOUS EXCHANGE.

183. Arbitration of Exchange is the process of finding the cost of exchange between two places, if remittance be made through one or more intermediate places.
Note.-1. When there is only one intermediate exehange, the process is called Simple Arbitration; when there are two or more intermediate exchanges, the process is called Compound Arbitration.
184. The objeot of arbitration is to ascertain the most adventageous ronte for making drafts or remittances.
185. There are always three methods of receiving money from a place, or of transmitting money to a place, by means of indirect exchange through one intervening place. Thus,

If $A$. is to receive money from C. through B., 1st. A. may draw on B., and B. draw on C. ; 2nd. A. may draw on B., and C. remit to B. ; 8rd. B. may draw on C., and remit to A .

If $A$. is to transmit to $C$. through $B ., 1$ st. A. may remit to B., and B. remit to C.; 2nd. A. may remit to B., and C. draw on B.; 3rd. B. may draw on A., and remit to C.

Example 1.-A man in Toronto paid a demand bill in Paris of 5,400 francs, by remitting to Amsterdam at the rate of 21 cents for 10 stivers, and thence to Paris at the rate of 28 stivers for 3 francs. How
muoh Canadian muah Canadian money was required?

$$
\begin{aligned}
& \text { SoLu I ion. } \\
& 28 \text { stivers }=3 \text { francs } \quad \therefore \frac{28}{8} \text { stivers }=1 \text { frano. } \\
& 21 \text { cents }=10 \text { stivers } \quad \therefore \frac{14}{4} \text { cents }=1 \text { stiver. } \\
& 5,400 \text { francs } \quad=\frac{5,400 \times 28}{8} \text { stivers } \\
& \begin{aligned}
\frac{5,400 \times 28}{3} \text { stivers } \quad & =\frac{5,400 \times 28 \times 21}{810} \text { oent } \\
& =81,058.40 . \text { Ang. }
\end{aligned}
\end{aligned}
$$

## Explanation.

 as many stivers as there are franos.

To reduce stivers to cents, multiply by $\frac{21}{2}$, because there are $\frac{21}{2}$ times as many cents as there are stivers.

Example 2.-A Montreal merchant remits $5 \overline{5}, 880$ floring to Amsterdam by way of London and Paris, at a time when the exchanse of Moutreal on London is $\$ 4.885$ for $£ 1$, of London on Paris is 25.4 francs for £1, and of Paris on Amsterdam is 212 francs for 100 florins; $t$ per cent. brokerage being paid in London and in Paris. How many dollars will purchase the bill of exchange?

## Solution.

100 florins $=212$ francs
25.4 francs $=£ 1$
$\frac{85,880 \text { florins } \times}{\frac{55,880 \times 212 \times 801}{100 \times 800} \text { francs }}$

$$
\frac{55,\{9.9 \times 212 \times 801 \times 801}{100 \times 800 \times 25.4 \times 800}
$$

$$
\begin{aligned}
& \therefore \quad\left(\frac{212}{100} \times \frac{1001}{100}\right) \text { francs }=1 \text { florin. } \\
& \therefore f\left(\frac{1}{25.4} \times \frac{100}{100}\right)=1 \text { franc. } \\
& \$ 4.885=£ 1 . \\
& =\frac{55,880 \times 212 \times 801}{100 \times 800} \text { francs. } \\
& \quad=£ \frac{55,880 \times 212 \times 801 \times 801}{100 \times 800 \times 25.4 \times 800} \\
& \quad=\$ \frac{55,880 \times 212 \times 801 \times 801 \times 4885}{100 \times 800 \times 25.4 \times 800} \\
& \quad=\$ 22,840.634+\text { Ans. }
\end{aligned}
$$

Explanation.
To reduce florins to franos, multiply by $\frac{212}{100} \times \frac{100 \frac{1}{8}}{100}$, because there are $\left(\frac{212}{100} \times \frac{100 \frac{1}{8}}{100}\right)$ times as many irsncs as there are florins.

To reduce franos to $£$, multiply by $\left(\frac{1}{25.4} \times \frac{100 \frac{1}{8}}{100}\right)$, because there are $\left(\frac{1}{254} \times \frac{100 \frac{1}{4}}{100}\right)$ times as many $£$ as there are francos.

To reduce $£$ to \$, multiply by 4.885 , because there are 4.885 timen as many as there are $\boldsymbol{f}$.

Example 3.-A banker in New York remits $\$ 3,000$ to Liverpool, by arbitration, as follows: First to Paris at 5 francs 40 ceitimes per $\$ 1$; thence to Hamburg at 185 francs per 100 marcs; thence to Amsterdam at 85 ativers per 2 maros; thence to liverpool at 220 stivers per 21 sterling. How mnch sterling money will he have in liank at Liverpool, and what will be his gain over direot exchange at $10 \%$ preminm?
re are $\frac{28}{3}$ times
e arts $\frac{2}{2} \frac{1}{6}$ times

80 florins to the exchange Paris is 25.4
r 100 florins; How many
florin.
3.
os.
$\begin{array}{r}\times 801 \times 801 \\ \hline 25.4 \times 800\end{array}$
$\frac{801 \times 4885}{\times 800}$
se there are
ause there

5 timen as
rerpool, by es per \$1; msterdam n per 21 Liverpool, n 1 ?

$$
\begin{aligned}
& \text { Soldtion. } \\
& 220 \text { stivers } \quad=£ 1 \quad \therefore £_{\frac{1}{2} \sigma} \quad=1 \text { stiver. } \\
& 185 \text { franos } \quad=35 \text { stivers } \quad \therefore \quad \frac{3}{2} \text { stivers }=1 \text { maro. } \\
& 6 \text { francs } 40 \text { cont }=\$ 10 \text { maros } \quad \therefore \quad \begin{array}{l}
109 \text { marcs }=1 \text { franc. }
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{3,000 \times 540}{100} \text { francs }=\cdots \quad \frac{3,000 \times 540 \times 100}{100 \times 185} \text { marcs. } \\
& \frac{3,000 \times 540 \times 100}{100 \times 185} \text { marcs. } \quad=\frac{8,000 \times 540 \times 100 \times 35}{100 \times 185 \times 2} \text { etivers. } \\
& \frac{3,000 \times 540 \times 100 \times 35}{100 \times 185 \times 2} \quad=£ \frac{3,000 \times 540 \times 100 \times 35}{100 \times 185 \times 2 \times 220} \\
& =\mathbf{x} 696 \text { 11s. 2d. Circuitous exohange. } \\
& \text { § }\left(\frac{40}{8} \times \frac{110}{180}\right)=21
\end{aligned}
$$

$$
\begin{aligned}
& \text { Gain. Ans. }
\end{aligned}
$$

EXERCISE 110.

1. When exchange at New York on Paris is 5 francs 16 centimes per $\$ 1$, and at Paris on Hamburg $2 \frac{1}{8}$ frances per marc banco, what will be the arbitrated price in New York of 7,680 marc banicos of Hamburg?
2. The exchange at Paris upon London is at the rate of 25 francs 70 centimes for $£ 1$ sterling, and the exchange at Vienna upon Paris is at the rate of $40 \frac{1}{2}$ Austrian florins for 20 francs: find how many Austrian florins should be paid at Vienna for a $\mathbf{5 5 0}$ note.
3. An agent in Boston, having $\$ 7,536.80$ due his employer in England, is directed to remit by a bill on Liverpool. What is the face of the bill, which ho can purchase for this money, exchange being at $11 \%$ premium?
4. Bills on Amsterdam, bought in London at 12 florins 15 cents per $£ 1$ sterling, are sold in Paris at $57 \frac{1}{2}$ florins for 120 francs. What is the course of exchange between London and Paris?
5. If at Philadelphia, exchange on Liverpool is at $98 \%$ premium, and at Liverpool on Paris 26 francs 86 centimes per $£ 1$; what is the arbitrated course of exchange between Philadelphia and Paris, through Liverpool?
6. A resident at Naples having a bequest of $\$ 8,720$ made him in Boston, orders the remittance to be made to his agent in London, who remits the proceeds to Naples, reserving his commission of $\frac{1}{2} \%$ on the draft sent. If exchange on London is $9 \%$ in Boston, and the rate between London and Naples is $£ 1$ for 5 scudi, how much does the man realize from his bequest?
7. A merchant of Toronto wishes to transmit 2,400 mares banco to Hamburg. He finds exchange between Toronto and Hamburg to be $\mathbf{3 5}$ cents for 1 marc. The exchange between Toronto and London is $\$ 4.83$ for $£ 1$; that between London and Paris is 26 francs for $£ 1$; and that of Paris on Hamburg is 47 francs for 25 marcs. By what way should the Toronto merchant remit?
8. A person in London owes another in St. Petersburg 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 25.15 francs for $£ 1$, and between Paris and St. Petersburg 1.2 francs for 1 rouble. The remittance is delayed until the rates are 25.35 francs for $£ 1$ and 1.15 francs for 1 rouble. What does the broker gain or lose by the delay?
9. A merchant in New York wishes to pay $£ 3,000$ in London. Exchange on London is at par; on Paris, 5

12 florins Gorins for a London

8 at $9 \frac{8}{8} \%$ centimes between
\$8,720 made to Naples, ent. If between loes the

2,400 etween The or $£ 1$; ; and 3. By
rsburg $\mathrm{H}_{\mathrm{e}}$ n the or $\mathrm{x}^{2} 1$, for 1 25.35 sthe

## foreign exchange.

francs 25 centimes per $\$ 1$; and on Amsterdam, 40 cents to a guilder. The exchange between France and England at the same time 25 francs to $£ 1$, that of Amsterdam on England is $12 \frac{1}{8}$ guilders to $£ 1$. Which is the most through Amsterdam?
10. When the course of exchange between London and Paris is $9 \frac{1}{2} d$. per franc, and 3.63 francs are equivalent to 1 Prussian thaler, and 24.5 thalers to 34 Austrian florins, and 25 Austrian florins to 12.6 Venetian ducats,-if a London merchant owe to one in Venice 1,000 ducats, will it be more advantageous to remit by way of Paris, Berlin, and Vienna, or direct to Vevice, supposing a ducat to be equivalent to 4 s . 2 d . ?

## RATIO.

485. Ratio is the relation between two members of the same denomination, expressed by the quotient of the first divided by the second.
Thus the ratio of 9 to 6 is $(9 \div 6)$; the ratio of 6 to 9 is $(6 \div 9)$.

4丹6. The Sign of ratio is the colon (:).
The ratio of 9 to 6 is expressed $9: 6$, or $9+6$, or as a fraction $\frac{7}{8}$.
487. The Terms of a ratio are the numbers compared.
488. The Antecedent is the first term, or the dividiend, or, if expressed as a fraction, the numerator.
489. The Consequent is the second term, or the divisor, or, if expressed as a fraction, the denominator.
490. The two terms together form a Couplet.
491. A Direct Ratio is the quotient of the antecedent ${ }^{\prime}$ divided by the consequens.
492. An Inverse Ratio or Reciprocal Ratio is the quotient of the consequent divided by the antecedent.
493. Ratios are compared by comparing the fractions by which they are represented.
494. Ratios are compounded by multiplying together the fractions by which they are represented, and expressing the resulting fraction as a ratio.
Thus the ratio compounded of $3: 5$ and $7: 9$, is $\frac{8}{8} \times \frac{7}{8}=$ $\frac{27}{8}=21: 45$.

## PROPORTION.

495. Proportion consists in the equality of two ratios.

For example, the ratio of 27 yds . to 9 yds . is $\mathrm{yz}=3$; the ratio of $37 \frac{1}{2}$ cts. to $12 \frac{1}{2}$ ots. is $\frac{87 \frac{1}{2}}{12 \frac{1}{2}}=3$ and, therefore, the ratio of 27 yds . to 9 yds . is equal to the ratio of $37 \frac{\mathrm{z}}{\mathrm{f}}$ ots. to $12 \frac{\mathrm{~d}}{\mathrm{~d}}$ ots., since each ratio is equal
to 8 .
This is expressed thus : -27 yds. : 9 yds. $=37 \frac{1}{2}$ ots. : $12 \frac{12}{}$ cts., or $2^{2 \prime} \mathrm{yds}$ : 9 yds. :: $87 \frac{1}{2}$ cts. : $12 \frac{1}{4}$ ots., the double colon (::) being used instead of the sign of equality ( $=$ ), or it may be expressed $\frac{27 \mathrm{yds} \text {. }}{9 \mathrm{yds} .}=\frac{37 \frac{1}{\mathrm{~h}} \mathrm{cts} .}{12 \mathrm{c} \text { cts. }}$
496. The arithmetic test of proportion is, therefore, that the two fractions representing the ratios must be equal.

497. The two terms 6 and 8 are called the extremes. The two terms 12 and 4 are called the means.
6 is called the first proportional. 12 is called the second proportional, 4 is called the third proportional. and 8 is oalled the fourth proportional.
498. Where the two means are the same number, that number is said to be a mean proportional between the two extremes.

Thus, in the proportion $4: 6:$ : $6: 9,6$ is the mean proportional between 4 and 9.
499. When two quantities are connected in such a way, that, when the first is increased any number of times, the second is increased the same number of times, they are said to be in direct proportion.

## PROPORTION.

For example, if 1 lb . of sugar oost $\$ \mathrm{cts}$.
2 lbs. will cost 2 times 8 ots.

eto., eto.
That in, if we increase the weight any number of times we increase the oost the same number of times, i.e., the cost of the sugar is directly proportional to its weight and vice versa.
Hence, 1 lb . : 7 lbs. :: 8 ots. : 7 times 8 ots.
500. When two quantities are connected in such a way, that, when the first is increased any number of times, the second is decreased the same number of times, they are said to be in inverse proportion.

For example, if one man can do a pieoe of work in 12 days,
2 men will do the work in 12 days $\div 2$
$\begin{array}{llll}3 & " & " & 12 \text { days } \div 3\end{array}$
ato., eto.
That is, if we increase the number of men any number of times, we decrease the time the same number of times, i.e., the number of men requirel to do the work is inversely proportional to the number of mays,
and vice verea.
Hence, 1 man : 4 man :: $\frac{12}{4}$ days $: 12$ days.
501. The student will obtain from the foregoing illustrations the following principles.

1. T'he product of extremes is equal to the product of the means.
2. Hence, the product of the extremes, divided by eitwer mean, will give the other mean.
3. The product of the means, divided by either extreme will give the other extreme.

## SIMPLE PROPORTION.

## 502. A Simple Proportion is an expression of equality between two simple ratios.

Example 1.-Find the term omitted in the following proportion $3: 16::$ no. required : 48.

$$
8 \times 48+16=9, \text { no. required. Principle } 2 .
$$

Example 2.-If 5 lbs. of sugar cost 60 ots, find the cost of 11 lbs solution.
Here more requires more, (i.e., more weight requiree more cost) hence the oost is directly proportional to the weight.
$\therefore 5 \mathrm{lbs} .: 11 \mathrm{lbs} .:: 60 \mathrm{cts} .:$ required cost. $\therefore$ required cost $=\frac{11 \times 60}{5}=\$ 1.32$ Ans. Principle 3.
Example 3.-If 3 men can do a piece of work in 25 days, how long will it take 5 men to do the same work?
golution.
Here more requires lase (i.e., more men require less time to do the same work) hence, the time is inversely proportional to the number of men.
$\therefore 3$ men : 5 men :: time required for 5 men : 25 daye (time required for 3 men).
$\therefore$ time required for 5 men $=\frac{3 \times 25}{\dot{j}}=15$ days Ans. Principle 0.
5 men : 8 or, required,
$\therefore$ no. days required $=\frac{3 \times 25}{5}=15$ days. Principle 3.
Example 4.-If 6 men oan do a piece of work in 12 days, in what time will 4 men do the same work?
solution.
Here less requires more (i.e., less men require more time to do the same quantity of work), hence the time is inversely proportional to the number of men.
$\therefore 6$ men : 4 men :: time required for 4 men: 12 ( $t$ l) required for $C$
$\therefore$ time required for $4 \mathrm{men}=\frac{6 \times 12}{4}=18$ days, Ans. Principle .8.

Notes 1.-If the terms of any conplet are of different denominations, they must be reduced to the same denomination.
2. If the odd term is a compound number reduce it to its lowest unit.
3. If the divisor and dividend contain factors common to both, cancel them.

## EXERCISE III.

Find the term omitted, and represented by $x$, in each of the following proportions :

1. $8: 52=20: x$.
2. $12: x=1: 144$.
3. $x: 20:: 120: 50$.

4. $\$ 80: \$ 4=x: 8$.
5. $4 \frac{1}{2}$ yd. $: x$ yd. $:: \$ 93: \$ 274$.
6. $2.5: 62.5:: 5: x_{0}$
7. $\frac{x}{9.01}=\frac{16.05}{5.35}$.
8. s yd. : $x$ yd. :: \$音: $\$ 5$.
9. If 12 gallons of wine cost $\$ 30$, what will 63 gallons cost?
10. If 9 bush. of wheat make 2 bbl . of flour, how many barrels of flour will 100 bush. make?
11. If $6 \frac{1}{2}$ bush. of oats cost $\$ 3$, what will $9 \frac{1}{4}$ bush. cost ?
12. What will 87.5 yd . of cloth cost, if $1 \frac{3}{4} \mathrm{yd}$. cost $\$ .42$ ?
13. If by selling $\$ 1,500$ worth of dry goods I gain $\$ 275.40$, what amount must I sell to gain $\$ 1,000$ ?
14. What will $11 \frac{\mathrm{l}}{\mathrm{l}} \mathrm{lb}$. of tea cost, if 8 lb .12 oz . cost $\$ 8.50$ ?
15. If a speculator in grain gain $\$ 26.32$ by investing \$325, how much would he gain by investing \$2,275 ?
16. In canning 5 lb . of raspberries 3 lb . sugar are needed, how many pounds sugar for 38 lb . of berries?
17. If with the money I have, I can buy 84 lb . of coffee at 25 c a lb., how many pounds can I buy for the same money at 80 c a lb?
18. If wall paper be 20 inches wide, I shall need 7 rolls to paper a room. How many rolls will suffice if the paper be 24 inches wide? If 30 inches wide?
19. If $\$ 750$ will yield $\$ 120$ interest in a certain time, what interest will $\$ 600$ yield in the same time?
20. A man, whose step measures $\frac{5}{8}$ yard, counts 1,200 steps fro'n his house to his office. How many steps will his son have to take, whose step measures $\frac{1}{2}$ yd ?
21. If each man on board ship consumes daily $1 \frac{1}{4} \mathrm{lb}$. bread, their bread will last $5 \frac{1}{8}$ months. How much will each man get per day if it is to last $6 \frac{1}{2}$ months ?
22. The rate of two perlestrians is as $5: 4$. How many miles will the first travel in the same time in which the second travels $84 \frac{1}{2}$ miles?
23. At the rate of $\$ 180$ for $\frac{3}{10}$ acre, what will 5 acres cost ?
24. The heat produced by a cubic yard of beech-wood is to that produce ! by a cu. yd. of pine as $9: 7$. How many cu. yd. of oeech-wood are needed to produce the heat of $50 \mathrm{cu} . \mathrm{yd}$. of pine?
25. If $1 \frac{8}{4}$ yards of velvet cost $\$ 5 \frac{1}{2}$, what will 9 yd . cost?
26. A farmer sowed 8 bush. of buckwheat on $2 \frac{2}{6}$ acres. How much would he need for a field containing $4 \frac{1}{2}$ acres ?
27. ? of a sum of money is $\$ 800$, how much is $\frac{5}{8}$ of it ?

## COMPOUND PROPORTION.

503. A Compound Proportion is an expression of equality between two ratios, one or both of which are compound.

Thus $3: 4\}:: 14: 28$ is a proportion composed of a $00 m$ a simple ratio, and may be expressed, $3 \times 6.4 \times 9$ to a simple proportion, $18: 36:: 14: 28$.
504. The terms of a proportion have not only the relations of magnitude, but also the relations of cause and effect.
503. Causes, in proportion, are considered as things that produce a certain result: as, men at work, money lent, horses, time, etc.
506. Effects are the result of causes : as, work done, interest drawn, cost, distance travelled.
507. Every problem in proportion may be considered as a comparison of two causes and two effects; these causes and effects being themselves either simple or compound.
Thus if 4 tons ot hay as a cause, will bring, when sold, $\$ 24$ as an effect, 12 tons, when sold, as a cause, will bring $\$ 72$ as an effect. Or, if 6 horses as a cause, draw 10 tons as an effect, 9 horses as a cause, will draw 15 tons as an effect.
508. Since like causes produce like effects, the ratio of two like causes must equal the ratio of two like effects produced by these causes.

Hence every question in proportion must give one of the following statements :
xpression of ich are commpound and a : 28, equivalent
ot only the f cause and
d as things ork, money

1st cause : 2 nd cause : : 1 st effect $: 2 n d$ effect.
or 1st effect : 2nd effect :: 1st cause : 2 nd cause.
Example 1.-If 4 horses consume 24 bushels of oats in 12 days, how many bushels will 20 horses eat in 16 days?

Soldtion.
1st cause : 2nd canse :: 1st effect : 2nd effeot. $\left.\begin{array}{c}42 \text { herses } \\ 12 \text { days }: 16 \text { days }\end{array}\right\}::\{24$ bush. : No. bush. required. $\therefore$ No. bush. required $=\frac{20 \times 16 \times 24}{4 \times 12}=160$ bush. Ans. Prin. 3.

Exampla 2.-If 2 workmen dig a ditoh 24 yards long and 3 feet wide, and 2 feet deep, in 5 days, how long will it take 3 workmen to dig a diloh 30 yards long, 4 feet wide, and 3 feet deep?

## Soletion.

| 1st cause | 2nd cause | $\because$ 1st elfect | 2nd effec |
| :---: | :---: | :---: | :---: |
| 2 workmen : 5 days | 3 workmen <br> No days required | $\}::\left\{\begin{array}{c} 24 \text { yards } \\ 3 \text { feet } \\ 2 \text { feet } \end{array}\right.$ | $\begin{array}{cc} : 30 \text { yards. } \\ : & 4 \text { feet. } \\ : 3 \text { feet. } \end{array}$ |

Here one part of the means is missing, and it may be found by dividing the product of the extremes by the product of the given parts of the means.

Hence, required time $=\frac{2 \times 5 \times 30 \times 4 \times 3}{3 \times 24 \times 3 \times 2}=8 \frac{1}{3}$ days. Ans. Prin. 2.

## EXERCISE 112.

Find the term omitted and represented by $x$ in the following proportions.

1. $\left.\begin{array}{c}8: 9 \\ 3: 4\end{array}\right\}=40: x$.
2. $\left.\begin{array}{r}6 \\ 14: 12\end{array}\right\}=\left\{\begin{array}{l}38: 48, \\ 56: 54 .\end{array}\right.$
3. $\left.\begin{array}{c}480: x \\ 30: 15\end{array}\right\}:$ : $84: 21$.

4. Five clerks uge 25 quires of paper in 8 days. At the same rate, how much paper will 6 clerks use in 10 days?
5. Six lamps consume 2 gallons of petroleum in 8 days. How many lamps will consume 3 gallons in 12 days?
6. Two workmen dig a ditch of 24 yds. in length and 8 ft . in width in 5 days. How long will it take 3 workmen to dig a ditch 30 yds. long and 4 ft . wide ?
7. Eight persons spend $\$ 296$ on a journey of 7 days. How long will $\$ 300$ last 7 persons at that rate?
8. If a block of marble 5 ft . long, 3 ft . wide, 2 ft . thick, weighs $4,850 \mathrm{ll}$., what will a block weigh measuring 7 ft . in length, 4 ft . in width, and 8 ft . in thickness?
9. Ten ewt. of merchandise cost $\$ 2 \frac{1}{2}$ freight for 245 miles. What will 5 cwt . cost for 210 miles?
10. If $\$ 700$ at interest amounts to $\$ 770$ in 15 months, what sum must be put at the same rate to amount to $\$ 845$ in the same time?
11. From the milk of 20 cows, each giving 18 qts. daily, $16 \frac{1}{2}$ cheeses of 50 lb . each are made in 42 days. How many cows, giving but 16 qts. daily, will be needed to make 33 cheeses of 60 lb . each in 28 days?
12. Being asked to find the number of bricks in a wall 10 ft . high, 922 ft . long, and 16 in . thick, I found that a part of the wall, 4 ft . high, 4 ft . long, and 16 in . thick, contained 448 bricks. How many in the whole wall?
13. If $\$ 750$ gain $\$ 202.50$ in 4 years 6 months, what sum will gain $\$ 155.52$ in 1 year 6 months?
14. If it require $1,200 \mathrm{yds}$. of cloth $\frac{5}{4}$ wide to clothe 500 men, how many yards which is $\frac{7}{8}$ wide will clothe 960 men?
15. If by travelling 6 hours a day at the rate of $4 \frac{1}{2}$ miles an hour, a man perform a journey of 540 miles in 20 days, in how many days, travelling 9 hours a day at the rate of $4 \frac{2}{3}$ miles an hour, will he travel 600 miles?
16. What sum of money will produce $\$ 300$ in 8 months, if $\$ 800$ produce $\$ 70$ in 15 months?
17. How many days will 21 men require to dig a ditch 80 ft . long, 3 ft . wide, and 8 ft . deep, if 7 men can dig a ditch 60 ft . long, 8 ft . wide, and 6 ft . deep, in 12 days?
18. How many men will be required to dig a cellar 45 ft . long, 34.6 ft . wide, and 12.3 ft . deep, in 12 days of 8.2 hours each, if 6 men can dig a similar one 22.5 ff . long, 17.3 wide, and 10.25 ft . deep, in 3 days of 10.25 hours each?
19. If a bin 8 ft . long, $4 \frac{1}{8} \mathrm{ft}$. wide, and $2 \frac{1}{2} \mathrm{ft}$. deep hold $67 \frac{1}{2}$ bush., how deep must another bin be made, that is 18 ft . long and $3 \frac{5}{6} \mathrm{ft}$. wide, to hold 450 bush. ?
20. How long should $A$. lend $B$. $\$ 1,175$, to balance loans from B. to A. of $\$ 100$ for 3 months, $\$ 400$ for 2 months, and $\$ 600$ for 6 months? How much should A. lend B. for 10 months, to balance these loans?
qts. daily, ays. How ed to make $s$ in a wall ound that in. thick, wall ?
what sum
clothe 500 960 men? f $4 \frac{1}{2}$ miles 120 days, he rate of

## DISTRIBUTIVE PROPORTION.

509. Distributive or Partitive Proportion is the method of dividing a number, or quantity, into parts which are proportional to given numbers.
510. The principle of this rale can be applied to the solution of numerous questions of a practical nature, such as determining the profits and losses of partuers in trade, apportioning shares of participators of prize money, finding the relative proportion of ingredients requisite to form a given quantity of a compound, apportioning taxes, school rates, averaging, etc.

Example 1.-Divide $\$ 600$ ameng A. B. C. and D., so that their shares may be in the proportion of $3,4,5$ and 6 .

Solution 1.
$3+4+5+6=18$
$18: 3:: \$ 600:$ A.'s share $\quad \therefore$ A.'s share $=\$ 100$
$18: 4: \$ 600:$ B.'s share $\quad \therefore$ B.'s share $=\$ 133_{\mathrm{B}}$
$18: 5:: \$ 600:$ C's share $\quad \therefore$ C.'s share $=\$ 1668$
$18: 6:: \$ 600:$ D.'s share $\quad \therefore$ D.'s share $=\$ 200$.

Explanation.
Altogether there are 18 shares, of which A. gets 3, B. 4, C. 5, D. 6, and the problem then becomes: If 18 shares represent $\$ 600$, what is represented by 3 shares? by 4 shares? by 5 shares $?$ by 6 shares? These give rise to the preoeding proportions.

## Solution 2.

A. 3 shares
B. 4 "
C. 5 "
D. 6 "

Total 18 shares.

18 shares $=\$ 600$
$\therefore 1$ share $=\frac{800}{18}$
A. gets 3 shares $=\frac{802}{1 / 2} \times 8=\$ 100$
B. gets 4 shares $=\frac{899}{18} \times 4=\$ 133 \mathrm{~h}$, etc
on is the arts which
ied to the ture, such s in trade, эy, fiuding to form a es, school

0 that their

5, D. 6, and epresented give rise to

Solution 3.
A. 3 shares
B. $4 \quad 1$
C. 5 "
D. 6 "
A. gets ${ }^{3} 8$ of the whole and $\therefore$ s ${ }^{8} 8$ of $\$ 600=\$ 100$
B. geta
eto.

Total 18 shares.
The stadent is recommended to use either the second or third method
Eximple 2.-Divide $\$ 2,000$ among A., B., C., so that B. may have - 500 more than A., and $0 . \$ 200$ more than B.

## Solution.

$$
\begin{aligned}
& \text { A.'s share }=\text { A.''s share } \\
& \text { B.'s share }=\text { A.'s share }+\$ 300 \\
& \text { C.'s share }=\text { A.'s share }+\$ 300+\$ 200 \\
& \hline \text { Total }=3 \text { times A.'s share }+\$ 800 \\
& \therefore 8 \text { times A.'s share }+\$ 800=\$ 2,000 \\
& \therefore 8 \text { " }=\$ 1,200 \\
& \therefore \text { A.'s share }=\$ 400 \\
& \text { B.'s share }=\$ 400+\$ 300=\$ 700 \\
& \text { C.'s share }=\$ 700+\$ 00=\$ 900 .
\end{aligned}
$$

## EXERCISE 113.

1. Divide $\$ 60$ into two parts proportional to 11 and 9 .
2. Divide $\$ 2,500$ into parts proportional to $2,3,7,8$.
3. Divide $\$ 8,470$ into parts proportional to $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ and $\frac{1}{4}$.
4. Gunpowder is made of saltpetre, sulpiur and charcoal in parts proportional to 75,10 and 15 ; how many pounds of each are contained in 12 cwt. of gunpowder ?
5. The sides of a triangle are as $3,4,5$, and the sum of the lengths of the sides is 480 yards : find the sides.
6. Divide $\$ 640$ among A., B. and C., so that A. may have three times as much as B., and C. as much as A. and B. together.
7. Divide the number 582 into 4 such parts that the second may be twice the first, the third 21 more than the second, and the fourth 54 more than the first.
8. If C. has twice as much money as B., and if $\$ 12$ be taken from A.'s money, it will be equal to $\frac{1}{3}$ of B.'s; how much has each, the sum of their moneys being $\$ 645$ ?
9. A man left his property to be divided among his 3 sons in proportion to their ages, which are 21, 18, and 12 years. The share of the youngest is $\$ 1,440$. What was the value of the property?
10. A., B., C., and D. commenced business with a capital of $\$ 18,500$; A. invested $\$ 800$ less than B., and C. invested $\$ 1,000$ more than A., and D. $\$ 900$ less than C. ; how mueh did each invest?
11. Divide 560 into parts, so that the second may bf 4 times the first.
12. A force of police 1,921 strong is to be distribute among 4 towns in proportion to the number of inhabitants in each; the population being $4,150,12,450,24,900$, and 29,050 respectively. Determine the number of men sent to each.
13. Divide 450 shares of stock among 3 persons, in proportion to the number of shares owned by each ; $A$. holds 400, B. 200 , and C. 300 ; how many shares will each receive?
14. A piece of land of 200 acres is to be divided among 4 persons, in proportion to their rentals from surrounding property. Supposing these rents to be $£ 500, £ 350, £ 800$, and £ 90 , how many acres must be allotted to each?
15. If $\frac{2}{3}$ of A.'s money, and $\frac{3}{4}$ of B.'s equal $\$ 900$, and $\frac{3}{4}$ of B.'s is twice $\frac{2}{3}$ of A.'s, what sum has each ?
16. A father divided $\$ 18,500$ among 3 children, so that the portion of the second was greater by one-half than that of the first, and $\frac{1}{2}$ the first was equal to $\frac{1}{6}$ of the third; what was the share of each?

## PARTNERSHIP.

a capital . invested ow mueh
nay be 4
stributeu 1abitants $y 00$, and nen sent s , in proA. holds vill each

0 , and $\frac{3}{4}$ , so that ban that e third;
511. A Partnership is an association of two or more persons, who combine their capital, skill or labor, or all of them, for the purpose of carrying on some lawful business, ard for participating in the profits or losses arising there. from, according to the terms of their agreement.
512. The busin $3 s$ association is called a Firm, House, or Company: and each individual of the association. is called a Partner.
513. Partners may be classified as-

1. Active partners.
2. Silent or dormant partners.
3. Nominal partners.
4. Special partners.
5. An Active Partner is one who has an interest in the business, and is known to the public as a partner.
6. A Silent or Dormant Partner is one who has an interest in the business, but is unknown to the public as a partner.
7. A Nominal Partner is one who allows his name to be used for the benefit of the firm, without having any pecuniary interest in its business.
8. A Special Partner is one who is held liable for only a specified amount.
9. In an ordinary partnership, each member is tiable to the full extent of his means for the liabilities of the firm ; but in a joint stock company, each shareholder is liable only for the amount of his unpaid capital. This explains the meaning of the term "Limited," which is added to the names of companies, as for example, "The Canada Publishing Co." (Limited).
10. Capital is the money or property invested in the business.
11. The Resources or Assets of a firm consist of the property it owns and the debts due the firm.
12. The Liabilities of a firm embrace all the debts or obligations due by the firm to its creditors.
13. The Investment is the aggregate of the cuoney or property jointly contributed by the partners.
14. The Net Capital is the excess of the Assets or Resources over all Liabilities.
15. The Net Insolvency is the amount which the liabilities exceed the resources.
16. The Net Investment of a firm is the difference between the total sum invested and the total withdrawals.
17. The Net Gain is the excess of the gains over the losses, during a certain time.
18. The Net Loss is the excess of the losses over the gains, during a certain time.
19. A Partnership Settlement is an adjustment of the partners' accounts setting forth the net investment, liabilities assumed, withdrawals, gains, losses, and showing his net capital or net insolvency at closing or settling the partnership's interests.

## PARTNERSHIP.

529. To divide the Gain or Loss, when each partner's capital has been employed for the same period of

Example.-A. and B. formed a partnership; A. furnished $\$ 3,000$, B. $\$ 5,000$; they gained $\$ 2,000$, and agreed to share the profit or loss in proportion to the capital of each; what was each partner's share?

Solutio:
A.'s oapital $=3,000$
B.'s " $=5,000$

Total " $=\$ 8,000$
$\therefore$ A. furnishes $\frac{3809}{8} 90$ or ${ }^{8}$ of capital.
B. " $\frac{58}{8} 8 \frac{80}{8}$ or 8
$\therefore$ A.'s share of gain $=\frac{3}{3}$ of $\$ 2,000=\$ 750$

$$
\begin{aligned}
& =\frac{6}{8} \text { of } \$ 2,000=\$ 1,250 . \\
& \text { or, }
\end{aligned}
$$

Total gain
$\therefore$ A, 000$)=\frac{3988}{8} 888$ or 1 of capital $=.25$ of capital.
$\begin{aligned} \text { B.'s ure of gain } & =\$ 3,000 \times .25=\$ 750 . \\ & =\$ 5,000 \times .25=\$ 1,250 .\end{aligned}$

## EXERCISE 114.

1. A. and B. buy a store which rents for $\$ 950$ a year ; A. advanced $\$ 3,500$, B. $\$ 4,800$; how much rent should each receive?
2. A. and B. form a partnership, A. furnishing $\$ 2,200$ and B. $\$ 2,500$; they lose $\$ 800$; what is each one's share of the loss?
3. A. put $\$ 7,500$, and B. $\$ 6,000$ into a land speculation; and A.'s share of the loss was $\$ 225$; what was B.'s share?
4. Two men formed a partnership, the former furnishing 3 times as much capital as the latter; they gained $\$ 12,500$; what was each one's share of the gain?
5. The net gains of A., B., and C. for a year are $\$ 12,800$; A. furnishes $\$ 25,000$, B. $\$ 18,000$, and C. $\$ 15,000$; how should the profit be divided?
6. X., Y. and Z. bought a ship on speculation ; X. put in $\$ 30,000$, Y., $\$ 20,000$, and Z., $\$ 15,000$; they sold it at a loss of $\$ 7,500$; what was each man's share of the loss?
7. A., B., C. and D. form a partnership with a capital of $\$ 57,000$; A. furnishing $\$ 10,000$, B. $12,000, \mathrm{C} . \$ 5,000$, and D. the remainder; they gain $15 \%$ of the joint stock; what is each partner's share of the profit?
8. A., B. and C. entered into partnership ; A. furnishing $\frac{1}{2}$, B. $\frac{1}{3}$, and C. the rest of the capital. On winding up the business, U.'s share of the profit was $\$ 4,518$; what were the respective dividends of A. and B.?
9. A. invested $\$ 12,000$ and B. $\$ 8,000$ in a business. A.'s share of the gain or loss is to be $\frac{2}{3}$ and B.'s $\frac{1}{3}$. At the close of the year their resources are $\$ 25,000$ in goods and cash, and liabilities $\$ 15,000$; what is the net capital, and what each partner's share of the gain or loss?
10. Four persons engage in the lumber trade, and invest jointly $\$ 22,500$; at the expiration of a certain time, A.'s share of the gain is $\$ 2,000$, B.'s $\$ 2,800.75$, C.'s $\$ 1,685.25$, and D.'s $\$ 1,014$; how much capital did each put in ?
11. Three persons enter into partnership for the manu, facture of coal oil, with a joint capital of $\$ 18,840$. A. puts in $\$ 3$ as often as B. puts in $\$ 5$, and as often as C. puts in \$7. Their annual gain is equal to C.'s stock; how much is each partner's gain?
12. A., B. and C. are employed to do a piece of work for $\$ 26.45$. A. and B. together are supposed to do $\frac{3}{4}$ of the work, A. and C. $\frac{8}{10}$, and B. and C. $\frac{18}{28}$, and are paid proportionally ; how much must each receive?
13. Three men trade in company. A. furnishes $\$ 8,000$, and B. $\$ 12,000$. Their gain is $\$ 1,680$, of which C.'s share is $\$ 800$; required, C.'s stock. and A.'s and B.'s gain.
14. Six persons are to share among them $\$ 6,300$; A. is to have $\&$ of it, B. $\frac{1}{6}$, C. $\frac{3}{6}$, D. is to have as much as A. and C. together, and the remainder is to be divided between E . and $F$. in the ratio of 3 to 5 . How much does each receive?
15. A., B. and C. form a oompany for the manufacture of woollen cloths. A. puts in $\$ 10,000$, B. $\$ 12,800$, and C. $\$ 3,200$. C. is allowed $\$ 1,500$ a year for personal attention to the business; their expenses for labor, clerk hire, and other incidentals for 1 year are $\$ 3,400$, and thoir receipts during the same time are $\$ 9,400$. What is A.'s, B.'s and C.'s income respectively from the business?

## 533. To divide the gain or loss according to the

 amount of capital invested and time it is employed.Example.-A., B. and C. are partners in a business; A. invested $\$ 3,000$ for four years, B. invested $\$ 5,000$ for three years, and $C$. invested $\$ 4,500$ for two years. How should a gain of $\$ 18,000$ be divided ?

Solution.
A.'s investment of $\$ 3,000$ for 4 yrs. $=$ an investment of $\$ 12,000$ for 1 yr .
B.' B

A. furnishes is of investment $\therefore$ his gain $=\frac{4}{1^{4}}$ of $\$ 18,000=\$ 6,000$.

or,
Total gain $(\$ 18,000)=\frac{1}{3} 88 \% 8$ or $\frac{1}{2}$ of investment $=.5$ of investment.
$\therefore$ A.'s share of gain $=\$ 12,000 \times .5=\$ 6,000$.
B.' $\quad$. $\quad \$ \$, 000 \times .5=\$ 7,500$.
C. " $\quad=\$ 9,000 \times .5=\$ 4,500$.
bule.
Multiply each partner's capital by the time it is employed, consider these products as their respective capitals and proceed as in Art. 529.

## EXERCISE 115.

1. A., B. and C. form a partnership; A. furnishing $\$ 3,000$ for 9 months, B $\$ 3,400$ for 10 months, and $C$. $\$ 2,800$ for 15 months; they lose $\$ 3,200$; what is each man's share of the loss?
2. January 1st, 1889, A., B. and C. form a partnership; A. puts in $\$ 8,000$, but after six months withdrew $\$ 2,000$; B. puts in $\$ 6,000$, and adds $\$ 500$ after 4 months; C. puts in $\$ 4,000$ for the year; they gain $\$ 3,600$; what is the share of each ?
3. Three men hire a pasture for $\$ 175$. A. put in 20 cows for 7 months, B. 120 sheep for 5 months, and C. 24 horses for 8 months; 5 sheep being considered equal to 1 cow, and 4 horses equal to 5 cows; how much should each pay?
4. A. and B. are partners, A. putting in $\$ 4,500$ and B. $\$ 2,500$; after 6 months they take in C., who furnished $\$ 10,000$; their gain for the year was $\$ 5,000$; what was the share of each?
5. X., Y. and Z. formed a partnership; X. putting in. $\$ 3,000$ for 1 year, Y. $\$ 4,500$ for 8 months, and Z. $\$ 5,000$ for 6 months; they lost $\$ 4,000$; what was each man's share of the loss?
6. A. and B. formed a partnership and divided the gain or loss in proportion to their average investments. A. put in $\$ 6,000$ for 12 months, and afterwards $\$ 4,000$ for 6 months. He withdrew $\$ 3,000$ for 4 months, then $\$ 6,000$ for 2 months, before the close of the partnership. B. put in $\$ 7,000$ for 12 months, then 6,000 for 8 months. He withdrew $\$ 4,000$ for 5 months, then $\$ 8,000$ for 2 months. They gained $\$ 4,560$; what was each partuer's share?
7. A., B. and C. began business Jan. 1st, when A. put in $\$ 7,500$, and July 1st he put in $\$ 2,500$ more ; B. put in

Jan. 1st $\$ 12.000$, and May 1st withdrew $\$ 4,000 ;$ C. put in Jan. 1st $\$ 10,000$, Aug. 1st he added $\$ 3,000$, and Oct. 1st he withdrew $\$ 7,000$. At the close of the year the profit was $\$ 8,500$; how much ought each to have, the gains being divided according to their average investment?
8. Howard \& Salter commenced business with a capital of which Hioward furnished $\$ 2$ to Salter's $\$ 1$. At the end of 3 months, Howard withdrew half of his capital, and Salter increased his $25 \%$. At the end of 9 months, they had $\$ 3,150$ to divide. What was the share of each?
9. Mills, Ross and McAdams, having been in partnership for one year, under an agreement to divide the profit proportionally to their respective shares of capital, have made $\$ 2,408$. On the first day of the year, each put in $\$ 10,000$; but Ross in 4 months withdrew $20 \%$ of his share, and McAdams at the end of six months put in $\$ 2,000$ more. Find each partner's share of the profit.
10. R. E. Walker and John Lawson engaged in a lumber business on January 1st, 1889. Mr. Walker invested $\$ 6,000$, and Mr. Lawson invested $\$ 6,000$. On March 1st, Mr . Lawson made an additional investment of $\$ 3,000$, and Mr. Walker withdrew $\$ 1,500$. July 1st, Mr. Walker invested $\$ 2,900$, and Mr. Lawson withdrew $\$ 3,000$. The profits for the year were $\$ 4,620$. What was each partner's average investment and share of the profits, if the profits were divided in proportion to the capital invested and the time it was employed ?
11. S. Morgan, J. R. Street and R. C. Cheswright formed a co-partnership, and invested respectively, $\$ 9,600, \$ 8,400$ and $\$ 7,200$. At the end of four months, Mr. Morgan invested $\$ 2,000, \mathrm{Mr}$. Street $\$ 1,400$, and Mr. Cheswright $\$ 800$. The net profits for the year were $\$ 12,800$. What was each partner's share, the gains and losses being divided in proportion to their average investments?
12. Three men take an interest in a coal mine. B. invests his capital for 4 months, and claims $\frac{1}{10}$ of the profits; C.'s capital is in 8 months; and D. invests $\$ 6,000$ for 6 months, and claims $\frac{2}{5}$ of the profits; how much dia B. and C. put in?
13. A. and B. are partners. A.'s capital is to B.'s as 5 to 8 ; at the end of 4 months A. withdraws $\frac{1}{2}$ of his capital, and B. $\frac{2}{3}$ of his; at the end of the year their whole gain is $\$ 4,000$; how much belongs to each ?
14. Three men engage in trade. A.'s money was in 10 months, for which he received $\$ 456$ of the profits ; B.'s was in 8 months, for which he received $\$ 342.20$ of the profits; and C.'s was in 12 months, for which he received $\$ 750$ of the profits. Their whole capital invested was $\$ 14,345$; how much was the capital of each ?
15. A., B. and C. engage in manufacturing shoes. A. puts in $\$ 1,920$ for six months ; B. a sum not specified for 12 months ; and C. $\$ 1,280$ for a time not specified. A. received $\$ 2,400$ for his stock and profits, B. $\$ 4,800$ for his, and C. $\$ 2,080$ for his. Required, E.'s stock and C.'s time?
16. B. commenced business with a capital of $\$ 15,000$. Three months afterward C. entered into partnership with him, and put in 125 acres of land. At the close of the year their profits were $\$ 4,500$, of which C . was entitled to $\$ 1,800$; what was the valu? of the land per acre?
17. B., C. and D. form a manufacturing company, with capitals of $\$ 15,800, \$ 25,000$, and $\$ 30,000$ respectively. After 4 months, B. draws out $\$ 1,200$, and in two months more he draws out $\$ 1,500$ more, and four months afterwards puts in $\$ 1,000$. C. draws out $\$ 2,000$ at the end of 6 months, and $\$ 1,500$ more 4 months afterwards, and a month later puts in $\$ 800$. D. puts in $\$ 1,800$ ar the end of

7 months, and 8 months after draws out $\$ 5,000$. If their gain at the end of 18 months be $\$ 15,000$, how much should each receive?
18. July 1st, 1886, A. and B. commencod business with a capital of $\$ 7,500$, for which $A$. furnished $\frac{2}{6}$ and B. the remainder; May 1st, 1887, B. invested $\$ 1,500$, and A. with. drew $\$ 600$; Oct. 1st, 1887, they admitted C. as a partner, with an investment of $\$ 4,500$; Jan. 1st, 1888, each partner invested $\$ 1,000$, and on Jan. 1st, 1889, each partner withdrew \$500. On closing business, Oct. 1st, 1889, it is found that a net loss of $\$ 3,000$ has been sustained. Find each partner's proportion of the loss.
19. Gibson and Montague dissolved a three-years partnership Aug. 1st, 1888, having resources of $\$ 16,500$, and liabilities of $\$ 2,150$. At first Gibson invested $\$ 2,750$, and Montague $\$ 2,500$; at the end of the first year Gibson drew out $\$ 1,500$, and Montague invested $\$ 3,000$; six months later each invested $\$ 1,200$. No interest account being kept, what has been the gain or loss, and the share of each partner, if apportioned according to average investments? 20. Day, Scott and Oarruthers, each invested $\$ 15,500$ in a business that gave the firm a profit of $\$ 21,000$ in one year. Nine months before dissolution, Day increased his investment $\$ 3,000$, and Scott and Carruthers each withdrew $\$ 3,000$; six months before dissolution, Scott invested $\$ 2,000$, and Day and Carruthers each drew out $\$ 2,000$; three months before dissolution, Carruthers invested $\$ 1,000$, and Day and Scott each drew out $\$ 1,000$. If no interest account was kept, and the gain be divided according to average investment, what is each partner's share?
21. A. and B. formed a co-partnership for 3 years, A. investing $\$ 7,200$, and $B$. investing $\$ 5,400$. At the end of 6 months $A$. increased his investment by $\$ 1,500$,
and B. withdrew $\$ 900$; one year before the expiration of the partnership, each withdrew $\$ 1.000$, and six months later each invested $\$ 500$. The net loss was $\$ 2,400$. How much should be sustained by each, if the partners receive credit for interest at the rate of $6 \%$ on all investments, and are charged interest on all sums drawn out, and the loss be sustained in proportion to average investment?
22. April 1st, 1884, Craig and Cowan commenced business as partners, Craig investing $\$ 8.000$, and Cowan $\$ 6,000$; six months later each increased his investment $\$ 1,500$; and on Jan. 1st, 1885, Allan was admitted as a partner with an investment of $\$ 2,400$. On Oct. 1st, 1885 , each partner drew out $\$ 1,500$; on Apr. 1st, 1886, Craig and Cowan each drew out $\$ 1,000$, and Allan invested $\$ 6,000$. On Jar. 1st, 1889, it was found that a net gain of $\$ 37,500$ had been realized. What was the share of each, if by agreement Craig, at final settlement, was to be allowed $\$ 1,200$ per year for keeping the books of the concern?

## 531. To find the net gain or loss, the net resources or the liabilities of a partnership.

Example 1.-A. and B. commenced business with a capital of $\$ 10,000$ cash ; merchandise as per inventory, $\$ 5,000$; bills payable; $\$ 1,500$. At the end of the ycar they had cash $\$ 6,500$; merchandise as per inventcry, $\$ 5,400$; bills receivable, $\$ 3,200$; debts owed by firm, $\$ 650$. What was the net gain or loss of the firm?

Solution.


Examplas 2.-A. and B. are partuers, A. sharing $\frac{2}{3}$ of the gain or loes and B. 1. A. invests $\$ 5,000$, and B. $\$ 2,350$. At the end of the year thoir resouross and liabilities are as follows : merchandise on hand, as per inventory, $\$ 2,000$; real estste, $\$ 7,000$; oash on hand and in bank, $\$ 1,532$; due on personal accounts, $\$ 1,640.25$; bills receivable, $\$ 1,000$. bills payable, $\$ 800$; owing by the firm to sundry persons, $\$ 4,471.69$. What is the amount of net resources belonging to each partner ?

Solution.
arsouroes.

| M'dse. on hand . . | . | .. | .. | $\$ 2,000.00$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Real estate .. | . | .. | .. | .. | $7,000.00$ |
| Cash on hand and in bank | .. | $1,532.00$ |  |  |  |
| Personal account | .. | .. | .. | $1,640.25$ |  |
| Bills receivable .. | .. | .. | .. | $1,000.00$ |  |
|  | $\$ 13,174.25$ |  |  |  |  |


$\$$ of $\$ 550.56=\$ 367.04$, A.'s share of gain.
t of $\$ 550.56=\$ 183.52$, B.'s "。
A.'s investment $=\$ 5,000.00$
A.'s gain .. .. $=367.04$
A.'s present worth .. .. .. $\$ 5,867.04$
B.'s investment $=\$ 2,350.00$

B,'s gain .. .. $=183.52$
B.'s present worth .. .. .. \$2,583.52

Present worth as before .. .. $\$ 7,900.58$
532. To find each partner's interest, when each partner is allowed to withdraw a certain sum, and when no interest account is kept.

Example.- A. and B. are partners, each invested $\$ 6,000$, and sgreed to share the gains and losses equally. A. drew out $\$ 1,200$ and B. 11,000. Reqnired their gains at the end of the year, their booke showing the following result:


| A. invested.. .. .. .. $\$ 6,000$ | B. invested |
| :---: | :---: |
| withdrew .. .. .. .. 1,200 | B. withdrew .. .. .. 1.00 |
| \$4,800 | \$5.000 |
| A.'s $\frac{1}{2}$ net grin .. .. .. 3,600 | B.'s $\frac{1}{2}$ net gain .. .. .. 3,6ico |
| A.'s net capital at olosing \$8,400 | B.'s net capital at closing $\$ 88,600$ |
| \$8,400 $+\$ 8,600$ |  |

533. To find each partner's interest, when one or more partners are allowed a fixed salary and no interest arcount is kept.

Example.-A., B. and O. entered into partnership January 1st, 1899. A. invested $\$ 14,000$, B. $\$ 14,000$, and C. $\$ 28,000$. A. to share $\frac{1}{4}$ of the gains and losses; B. $\frac{1}{2}$, and C. $\frac{1}{2}$. A. was to receive a salary of $\$ 1,000$ per year, B. $\$ 1,200$, anc C. $\$ 600$ for services. A. drew out $\$ 1.300$, B. $\$ 900$, and C. \$1,800. What was each partner's interest in the firm Jannary 1st, 1890, when their resouroes were $\$ 108,000$, and their liabilities की:27.000?

Sowerms.

A.'s investment .. .. .. $\$ 14,000$
A. B salary.. .. .. .. .. $\frac{1,000}{15000}$

Lsss amoant withdrawn .. $\begin{array}{r}\$ 15,000 \\ 1,300 \\ \hline\end{array}$
A.'s oredit balance.. .. .. $\$ 13,700$
B.'s investment .. .. .. $\$ 14,000^{\circ}$
B.'s salary .. .. .. .. $\frac{1,200}{\$ 15,200}$

Less amount withdrawn .. $\quad 900$
B.'s oredit balance.. .. .. $\$ 14,300$
C.'s investment
$\$ 28,000$
C.'s salary.
$\frac{600}{\$ 28,600}$
Less amount withdrawn . 1,800
$\$ 26,800$
Firm's net gain
854,800
$\frac{1,000}{85.000}$
g $\quad \frac{3,610}{\$ 8,600}$
one or interest
A. s credit bal. $\$ 13,700$
A.'s $\frac{1}{}$ gain
A.'s net oapital \$20,250
B.'s oredit bal. $\$ 14,300$
B.' $\frac{1}{2}$ gain .. $\frac{6,550}{\$ 20,850}$
B.'s net capital \$20,850
proor.
A.'s net capital $\$ 20,250$
B.'s ". 20,850
O.'s " 39,900

Firm's net capital $\$ \overline{\$ 81,000}$
534. To find each partner's interest at the end of the year or close of partnership when amounts withdrawn are averaged, and interest is charged and allowed.

Example.-A. and B. entered into partnership Jannary 1st, 1889, and agreed to share the gains or losses equally. A. invested $\$ 6,000$, and B. $\$ 7,250$; each partner was allowed $6 \%$ on his investment and was cherged $6 \%$ for the sums withdrawn. A. drew as follows: March 1st, $\$ 300$; July 9th, $\$ 250$; September 10th, $\$ 200$; December 18th, $\$ 150$. R. drew, April 17th, $\$ 100$; August 4th, $\$ 400$; November 23 rd, $\$ 250$. What was each pertner's intereat in the business Jonuary ist, 1890, their remerces and liabilities boing, as follows:

| resodrces. |  | linbilitieg. |  |
| :---: | :---: | :---: | :---: |
| Cash .. .. .. | .. \$1,800 | Personal debts firm owe. | \$5,750 |
| Personal debts due firm | .. 8,000 | Bills payable .. .. .. | $\$ 5,750$ 250 |
| Bills receivable .. .. | .. 700 | Total liabilities .. .. .. | \$6,000 |
| M'dee. as per inventory | .. 18,000 | Total Habinties .. | \$6,000 |
| O. P. R. Railway Stock | . 3,000 | Firm's net capital .. .. | \$20,500 |
| Total resources .. | . $\$ 26,500$ |  | \$26,500 |

A.'s amount withdrawn $\$ 900$; average date July 7th. From July 7th to January 1st $=178$ days.
B.'s amount withdrawn \$750; average date August 27th. From August 27th to January 1st $=127$ days.

| A.'s investment .. .. .. <br> Less withdrawn .. .. .. .. | $\begin{array}{r} \$ 6,000.00 \\ 900.00 \end{array}$ | \$5,100.00 |
| :---: | :---: | :---: |
| Int. on investment for 1 year | \$360.00 |  |
| Less int. on $\$ 900$ for 178 da. at $6 \%$ | 26.33 | \$5,433.67 |
| A.'s credit balance |  |  |
| B.'s investment | 7,250.00 |  |
| Less withdrawn | 750.00 | \$6,500.00 |
| Int. on investment for 1 year | \$435.00 |  |
| Less int. on \$750 for 127 da. at 6\% | 15.66 | 419.34 |
| B.'s oredit balance. . |  | $\$ 6,919.34$$\$ 20,500.00$ |
| Firm's net capital .. |  |  |
| A.'s credit balance. . | \$5,433.67 |  |
| B.'s | 6,919 $3 \pm$ | 12.353.01 |
| Firm's net gain .. .. |  | \$8,146.99 |

A.'s credit balance .. $\$ 5,433.67 \mid$ B.'s credit balance. . .. $\$ 6,919.34$
A.'s $\frac{1}{2}$ gain.. .. .. $\frac{4,073.49 \frac{1}{2}}{\text { B.'s } \frac{1}{2} \text { gain .. .. .. 4,073.49 } \frac{1}{2}}$
A.'s net capital .. .. \$9,507.16 $\frac{1}{2}$
B.'s net capital .. $\$ 10,992.83 \frac{1}{2}$

Firm's net capital $\$ 20,500$.

## EXERCISE 116.

1. At the expiration of a year from the commencement of their business, Baker, Morgan \&. Co., after taking an account of stock, find the amount of merchandise, as per inventory, to be $\$ 17,450$; cash on hand, $\$ 10,250$; debts due the firm, $\$ 11,300$; amount of firm's indebtedness,
$\$ 15,500$. Make out a statement, showing the resources and liabilities of the firm, with net capital and gain; and find each partner's share of the latter, the respective shares of capital being as follows: J. Baker, $\$ 8,000$; S. Morgan, $\$ 5,000$; and J. Murray, $\$ 3,000$.
2. A. put $\$ 10,000$ into a partnership and B. $\$ 5,000$. They agreed to divide the gain or loss in proportion to their original investments, and to keep no interest account. During the year A. withdrew $\$ 800$ and B. $\$ 500$; what was the net capital of each at the close of the year, their resources being $\$ 25,800$ and their liabilities $\$ 18,500$ ? What per cent of their investment was the gain or loss?
3. Duff, Fry \& Rowat became partners, each investing $\$ 15,000$, and each to have one-third of the gains or sustain one-third of the losses. Duff withdrew $\$ 2,100$ during the time of the partnership, Fry $\$ 1,800$, and Rowat $\$ 2,000$. At close of business their resources were : cash, $\$ 3,540$; mdse., $\$ 14,785$; notes, acceptances, and accounts receivable, $\$ 16,250$; real estate, $\$ 28,500$. They owed on their outstanding notes $\$ 8,125$, and on sundry personal accounts, $\$ 1,950$. Find the present worth of each partner at closing.
4. A., B., and C. formed a partnership ; A. put in $\$ 5.000$, B. $\$ 4,000$, and C. $\$ 2,500$. A. withdrew $\$ 1,000$, B. $\$ 800$, and C. $\$ 500$. They agreed to share the gain or loss in proportion to their original investments, no interest account being kept. At the close, what was each partner's share of gain or loss, and the net capital of each, as shown by the following statement :

Resourcers.

5. At the time of closing business, the resources of a firm were: cash, $\$ 981.50$, mdse., per inventory, $\$ 18,196.25$; notes and accounts due it, $\$ 8,154$; interest on same, $\$ 211.50$; real estate, $\$ 11,150$. The firm owed, on its nutes, acceptances and bills outstanding, $\$ 7,142$, and interest on the same, $\$ 348.50$; and there was an unpaid mortgage on the real esta'e of $\$ 2,500$, with interest accrued thereon of $\$ 88.50$. If the invested capital was $\$ 22,500$, what was the net solvency or net insolvency of the firm at closing, and how much bas been the net gain or net loss?
6. The firm of A. \& B. formed 乞 partnership Jan. 1st for 1 year, investing $\$ 8,000$ each. They were to have $6 \%$ interest on their capital and be charged $6 \%$ on sums withdrawn. The gains or losses were to be shared equally. April 4th, A. Arew out $\$ 500$, July 10th, $\$ 400$, and Sept. 5th, $\$ 200$. B. drew out May 6th, $\$ 700$, Aug. 12 th, $\$ 300$, and Oct. 4th, $\$ 400$. What was each partner's net capital on closing, the net gains being $\$ 3,850$ ?
7. Johnston and Atkinson became partners April 1st, 1888, under an agreement that each should be allowed $6 \%$ simpla interest on all investments, and that, on final settlement, Johuston should be allowed $10 \%$ of the net gains, before other division, for superintending the business, but that otherwise the gains and losses be divided in proportion to average investment. April 1st, 1888, Atkinson invested $\$ 18,000$, and Johnston, $\$ 4,000$; Jan. 1st, 1889, Atkinson withdrew $\$ 5,000$, and Johnston invested $\$ 8,000$; Aug. ist, 1889, Atkinson withdrew $\$ 1,500$; Dec. 1st, 1889 , the partners agreed upon a dissolution of the partnership, having resources and liabilities as follows:


If, of the accounts receivable, only $80 \%$ prove to be good, what has been the net gain or loss? What has been the gain or loss of each partner? What is the firm's net insolvency at dissolution? What is the net insolvency of each ?
8. A., B., and C., formed a co-partnership for 2 years, investing equal sums, with the agreemeat that each shall receive interest at the rate of $6 \%$ on all sums invested, be charged interest at the same rate on all sums withdrawn, and the gains or losses shown on final settlement be apportiowed according to average net investment. Three months after the formation of the partnership A. drew out $\$ 1,200$, and cix months later B. and C. each drew out $\$ 1,000$, and A. invested $\$ 6,000$; at the end of the first year each drew out $\$ 500$. On closing the affiurs of the firm, the following stafement was made : net gain, $\$ 15,000$; present worth, $\$ 75,000$. What was the original investment of each? What was the present worth of each at the time of the dissolution? What was each partner's share of the gain?
9. A. and B. became partners for one year ; A. investing $\frac{8}{8}$ of the capital, and B. $\frac{9}{8}$; the agreement being that the gains or losses shall be apportioned according to average net investment, and that each partner be allowed $6 \%$ interest per annum on all investments, and be charged interest at that rate on all sums withdrawn. At the end of the year the firm had as resources: mdse., per inventory, $\$ 21,460$; real estate, $\$ 15,000$; cash, $\$ 1,950$; bills receivable, $\$ 18,146.50$; interest accrued on the same, $\$ 519.25$; accounts due it, $\$ 11,218.50$; store furniture, $\$ 1,3 \Delta 0$; delivery wagons and horses, $\$ 2,100$. The liabilities were : mortgage on real estate, $\$ 7,000$; interest on same accrued, $\$ 210$; notes outstanding, $\$ 26,950$; interest accrued on same, $\$ 811.75$. The firm owes H. W. Darling $\&$ Co., Toronto, $\$ 33,560$. It is found that $33 \frac{1}{3} \%$ of the
accounts due the firm are uncollectable. If the firm's losses during the year have been $\$ 12,000$, how much was invested by each partner? What is the present worth or net insolvency of the firm, and of each partner, at closing?
10. Sills and Jones became partners July 1st, 1886, under a 3 -year's contract, which provided that Sills should have $\$ 1,500$ each year for superintending sales, and that Jones should have $\$ 1,000$ each year for keeping the books of the concern, and that these salaries should be adjusted at the end of tach year, and before other apportionment of gains or losses was made. July 1 st, 1886 , each invested $\$ 12,500$. Six months later, each increased his investment $\$ 5,000$. July 1st, 1887, Sills drew out $\$ 3,600$ and Jones drew out $\$ 3,000$. Oct. 1st, 1857, Sills withdrew $\$ 1,000$ and Jones invested $\$ 2,000$. July 1 st, 1888 , each drew out $\$ 1,500$. At the expiration of the time of the contract, the resources exceeded ali liabilitics by $\$ 17,280$. What was the gain of each, and the present worth of each?
11. A. and B. commenced business as partuers. A. invested $\$ 20,000$, and $B . \$ 10,000$, A. sharing $\frac{2}{3}$ and $\mathrm{B} . \frac{1}{3}$ of the gains and losses. No interest account was kept. A. drew out $\$ 1,700$, and B . $\$ 2,150$. Their assets at the close of the year consisted of-cash, $\$ 4,200$; bills receivable, $\$ 8,800$; mdse., $\$ 26,000$; and personal debts, $\$ 16,000$. $10 \%$ of the personal debts are considered bad. Their liabilities are-bills payable, $\$ 3,250$; personal accounts, $\$ 11,250$. If B . should retire from the firm, how much ought he to recive?
12. On January 1st, 1889, A. E. Brock, W. McMa.ter and H. Crawford entered into a co-partnership. Brock was to i. $\mathrm{ve}^{2}$ 年 of the capital and share $\frac{5}{3}$ of the gains. McMaster was to invest $\frac{3}{8}$ of the capital and share $\frac{5}{8}$ of the gains, and Crawford was se invest $\frac{2}{y}$ of the capital and share $\frac{2}{9}$ of

## partnenship.

the gains. Interest at the rate of $10 \%$ per annum was to be allowed to each partner should he invest more than his proportion; and interest, at the same rate, was to be charged each partner if he failed to invest his proportion. A settlemient was made at the end of the year, and the net gain was $\$ 3,600$. Find Brock's and McMaster's net interest, and Crawford's insolvency Jan. 1st, 1890, the following being a statement of each partner's account. Dr.

> A. E. Brock.

| 1889.-April 23, Drew out |  | \$3,000 | 1889.-Jan. 1, Invested |  |  | Or. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jane 16, " | 1,600 |  |  |  | \$32,000 |
|  | Aag. 17, | 1,800 | ${ }^{\prime \prime}$ | Oct. 20, | " | 4,800 |
|  | Total withdrawn | 86,400 | ${ }^{\prime \prime}$ | Total in | vestme | 6,000 |

Dr.
W. MoMaster.

A.
B. $\frac{1}{3}$ of pt. A. e close sivable, 16,000. Their counts, much ier and was to Master gains, :a $\frac{2}{\square}$ of

## BANKRUPTCY.

535. Bankruptcy is the formal acknowledgement in accordance with the law, by a person or firm, of inability to pay indebtedness.
536. A Bankrupt is a person who is insolvont, or unable to pay his debts.
537. After the assets of a bankrupt have been applied to meet his liabilities, he still remains liable for them unless discharged, or unless a compromise has been effected with his creditors.
538. The Assets of a bankrupt are his entire property.
539. The Liabilities of a bankrupt are the debts and oblin utions due by him to his creditors.
540. The Net Proceeds are the assets less the expense of settlement. They are divided among the creditors according to their claims.

The claims of a certain class of creditors, as employees and others, are paid in full np to a certain amount. These are called "Preferred Creditors."
541. An Assignee is a person appointed in accordance with the law, to take charge of the bankrupt's property, to make collections of debts due the estate, and after deducting the expenses of the assignment, to pay such proportion of the debts due the creditors as the available assets will allow.
542. To find each creditor's dividend, the liabilities and net proceeds being given.

Example.-A merchant failing in business gave the following statement of his assets and liabilities: Assets, oash, $\$ 5,474$; real estate, $\$ 3,000$; merohandise, $\$ 4,000$; personal accounts, $\$ 1,900$. Liabilities, bills payable, $\$ 2,400$; due R. E. Walker \& Co., $\$ 5,000$; due A. Boyle \& Co, $\$ 17,500$. The expenses of assignment were $\$ 430$. How muoh did eaoh oreditor receive?

Solution.
 1 unless ed with operty. bts and expense réditors
thers, are ed Credi-

## EXERCISE 117.

1. A bankrupt owes A. $\$ 6,500$, B. $\$ 4,600$, and D. $\$ 3,800$; his assets are $\$ 5,9,50$, and the expenses of settling $\$ 1,700$; what per cent. and how much will each creditor receive?
2. J. Gould \& Co. failed with liabilities amounting to $\$ 300,000$. The assets of the firm were $\$ 186,294$. How much should each creditor receive on the dollar, and what sum was allowed J. P. Hume \& Co., whose claim was $\$ 17,314$, the expenses of settling being $\$ 6,294$ ?
3. J. Wild \& Co., went into bankruptey, owing $\$ 48,500$, and having $\$ 13,300$ assets; the expense of settling was $5 \%$ of the amount distributed to creditors. What per cent. and how much did a creditor receive on $\$ 8,350$ ?
4. A grain firm failed with liabilities amounting to $\$ 24,500$. The assets were : cash, $\$ 1,080$; real estate, $\$ 8,250$; notes on hand, $\$ 1,170$. The expenses of settling were $2 \%$ of the assets. How much should W. H. Hull \& Co. receive, whose claim against the firm was $\$ 6,308.50$ ?
5. A manufacturer failed, owing A. $\$ 12,260$, B. $\$ 18,850$, and C. $\$ 14,560$; his assets were $\$ 28,350$, and the expenses of settling were $\$ 1,250$. He owed $\$ 850$ to employees who were to be paid in full; what per cent. and how much did the other creditors receive?
6. The real estate of a hankrupt firm was sold by an assignee for $\$ 24,000$, goods in store for $\$ 12,244$. There were collected on notes due the firm $\$ 4,214$, and on personal accounts $\$ 5,346$. The total liabilities of the firm were $\$ 54,067.50$, and the expenses of settling $\$ 1,350$. How much on the dollar can be paid, and what should Howard Bros. receire, whose claim is $\$ 12,430$ ?
7. A. Reid's claim against a bankrupt firm was $\$ 7,200$, and J. Taylor's $70 \%$ of that of A. Reid's. After the expenses of the assignment were deducted from the assets, there remained $\$ 18,260$. The total liabilities were $\$ 24,480$. How much did A. Reid and J. Taylor respectively receive?
8. A firm failed with liabilities amounting to $\$ 26,125$. The assets of the firm exclusive of real estate were $\$ 1,52125$. The assignee obtained for a warehouse and three building lots the sum of $\$ 15,675$. The expenses for settling the bankruptcy was $\$ 287.50$. W. Alexander's claim against the firm was $\$ 8,642$; J. Moblo's, $\$ 3,191$; R. A. Harrison's, $\$ 2,897$; D. McGregor's, $\$ 2,388.50$; W. Ayer's, $\$ 1,982$. How much did each of these creditors receive?

## ANNUITIES.

d by an There personal m were ). How Howard
$\$ 7,200$, xpenses s , there 24,480 . eceive?

26,125. 52125. uilding ng the against risou's, $\$ 1,982$.
543. An Annuity is a specified sum of money paid annually, or at equal periods as, half-yearly, quarterly, etc., to continue a given number of years, for life, or for ever.
544. A Certain Annuity is one which begins and ends at a Arad time.
continues A Perpetual Annuity or Perfetuity is one which
546. A Contingent Annuity is one whose time of commencement or ending, or both, is uncertain, and which depends upon some unforeseen event, as the death of an individual, or his arrival at a certain age. Life insurance, pensions, dowers, leases, etc., belong to this class of incomes.

## 547. An Annuity in Possession or an Immediate Annuity is one that begins immediately.

548. A Deferred Annuity or an Annuity in Reversion is one that begins at some future time, it may be at some specified time, or at the occurrence of some event.
549. An Annuity in Arrears or Forborne is one on which the payments were not made when due.
550. The Amount or Final Value of an annuity is the sum to which all its payments with interest on each, will amount at its termination.
551. The Present value of an Annuity is the sum which at th" given rate of interest, will amount to its final value.

Notr 1.-The present value of a deferred annuity is that principal whioh will amount, at the time the reversion expires, to what will then be the present value of the annuity.
2. The present value of a perpetual annuity is the sum whose interest equals the annuity.
3. Annuities and their values are oomputed by simple interast or by oompound interest.
552. To find the amount of an annuity at simple interest when the time and rate are given.

Example-What is the amount of $\$ 500$ annuity for 5 years at $6 \%$ simple interest?

Solution.

| annoitr. int. | amt. |
| ---: | ---: |
| $\$ 500+120=$ | $\$ 620$ |
| $500+90=$ | 590 |
| $500+60=$ | 560 |
| $600+30=$ | 530 |
| $500+0=$ | 500 |
| Amount $\$ 2,800$ |  |

## Explanation.

The interest on $\$ 500$ for 1 year at $6 \%$ $=\$ 30$. The first annuity is not due until the end of the first year, and hence draws interest for only 4 years $=$ \$120. The seoond is not due until the end of the second year, and hence draws interest for only 3 years, eto.
553. To find the present worth of an annuity at simple interest.

Examplis.-What is the present value of an aunuity of 8500 for 5 years, when money is worth $6 \%$ simple interest?

## Solution.

By the preceding example the final value of the annuity is $\$ 2,800$. The rresent worth of $\$ 2,800$ due in 5 years at $6 \%=138$ of $\$ 2,800=\$ 2153.846$.

## EXERCISE 118.

1. What is the amount of an annuity of $\$ 150$ for 8 years, when money is worth $6 \%$ simple interest?
2. A man works for 1 year and 6 montins at $\$ 20$ per month, payable monthly; and these wages remain unpaid until the expiration of the whols time of service. How much is due to the workman, allowing simple interest at the rate of $6 \%$ per annum?
3. A father deposits $\$ 50$ a year for his son, commencing on the son's 10 th birthday. What amount will the son have on his 21st birthday, if the payments draw simple interest at the rate of $8 \%$ per annum?
4. A lady has $\$ 300$ a year left to her for 10 years. What is its present cash value, at $7 \%$ simple interest?
5. What is the present worth of an annuity of $\$ 600$ for 4 years, money being worth $6 \%$ simple interest?
6. How much will an annuity of $\$ 100$ amount to in 8 years at $8 \%$ simpie interest?
7. An annuity of $\$ 200$ for 12 years is in reversion for 6 years. What is its present worth, simple interest at $6 \%$ ?

## ANNUITIES AT COMPOUND INTEREST.

554. The labor of computing the values of annuities at compound interest is greatly dimmished by the use of the following tables. The tables are always used in practice.

Table 1.
Amount of $\$ 1$ annuity at compound interest, from 1 year to 40 , inclusive.


## REST.

mnuities at use of the practice.
rom 1 year

| 7\% | Yrs. |
| :---: | :---: |
| 1.000000 | 1 |
| 2.070 000 | 2 |
| 4.439 933 | - |
| 7.155 201 |  |
| ${ }_{803}$ | 8 |
| -977 989898 | 10 |
| \%88599 | 11 |
| .1480813 | ${ }_{13}^{12}$ |
| . 1250 | 15 |
| 888054 |  |
| 840 847 | 18 |
| 378 9.5 | 19 |
|  |  |
| 005 73814 | ${ }_{23}^{22}$ |
| 671 | ${ }_{24}^{23}$ |
|  |  |
| 46770 | 26 |
| ${ }_{6}^{691}$ | ${ }^{28}$ |
|  | 29 |
| 7304 |  |
| 18154 | 32 |
|  | ${ }^{34}$ |
| 38878 | 35 |
| 3460 <br> 100 | 36 |
| 1030 | 38 |
| 5112 | $\begin{array}{r}39 \\ \hline\end{array}$ |

Table 8.
5.55. Present worth of $\$ 1$ annuity at compound interest, from 1 year to 40 , inclusive.

556. To find the final value of an annuity by compound interest.

Example 1.-What is the final value of an annuity of $\$ 500$ for 6 years at $5 \%$.

## Solution.

By Table 1 the final value of an annuity of
$\$ 1$, at $5 \%$ for 6 years $=\$ 6.801913$.
$\therefore$ final value of an annuity of $\$ 500=6.801913 \times \dot{0} 00=\$ 3400.9565$.
Note.-When payments are made half-yearly, take from the table double the time, and $\frac{1}{2}$ the rate.
557. To find the present value of an Annuity.

Example.-What is the present worth of an annuity of $\$ 500$ for 5 years at $6 \%$.

Solution.
By Table 2 the present worth of an annuity is $\$ 1$ for 5 years at $6 \%$ is \$4.212364.
$\therefore$ The p. w. of an annuity of $\$ 500=\$ 4.212364 \times 500=\$ 2106182$.
55\%. To find the present worth of an annuity in reversion.

Example.-What is the present worth of an annuity in reversion of $\$ 500$ at $6 \%$, whioh begins in 4 years, and then terminates after 6 ycars. Solotion.
The p. w. of an annuity of

$$
\$ 1, \text { at } 6 \% \text { for } 10 \mathrm{yrs} .=\$ 7.360087 .
$$

$\therefore$ The present worth of an annuity of $\$ 1$ to

$$
\text { " } 4 \mathrm{yrs}=\$ 3.465106 \text {. }
$$ begin in 4 years, and then to continue 6 yerrs $=\$ 3.891931$, the difference

$\therefore$ p.w. of $\$ 500=3.894991 \times 500=\$ 1947.4905$.
559. To find the present worth of a perpetual annuity.

Example 1.-A perpetual scholarship oi $\$ 150$ per year is established at Queen's University. What sum must be invested at $5 \%$ to yield this inoome.

Solotion.
$5 \%$ of the investment $=\$ 150$.
$\therefore$ the investment $=\frac{150}{8} \times 100=\$ 3,000$ Ans.
Example 2.-What is the present worth of a perpetual annaity of $\$ 300$ in arrears for 20 years, allowing $5 \%$ compound interest.

## Solotion.

There is now due the amount of $\$ 300$ for 20 years at $5 \%$ compound interest, together with the present worth of the perpetual annuity of $\$ 300$.

The p. w. of the perpetual annnity of
$\$ 300$, by Example $1=280 \times 100=\$ 6,000$.
Amount of annuity of

$$
\$ 1 \text { for } 20 \text { years at } 5 \%=\$ 33.065954 \text {. (Table 1). }
$$

$$
\begin{aligned}
\$ 300 & =\$ 33.065954 \times 300=\$ 9919.7862 .
\end{aligned}
$$

$\therefore$ total present worth $=\$ 9919.7862+\$ 6,000=\$ 15919.7862$. Ans.
EXERCISE 119.

1. Money being wo ${ }^{-{ }^{+1} \cdot} 6 \%$, how much must be presented to a college, to insure $\$ 50$ a year forever, for an annual prize?
2. A person left $\$ 5,000$ for the poor of his native town. How great was the perpetuity realized from it, at $6 \%$ ?
3. What is the final value of an annual pension of $\$ 150$ for 15 years at $4 \%$ compound interest?
4. A widow is entitled to $\$ 140$ year for 18 years, at $10 \%$ semi-annual compound interest: what is its final value?
5. An annuity of $\$ 350$ was left to A., and one of $\$ 550$ to B., by the same person; both were to run 12 years. Allowing compound interest, at $6 \%$, by how much would the amount of A.'s exceed that of B.'s in the given time?
6. How much will an annuity of $\$ 100$ amount to, in 8 years, at $8 \%$ simple interest? How much at $6 \%$ com. pound interest?
7. A soldier 57 years old, having a pension of $\$ 80$ a year, agreed to sell it for cash at $10 \%$ less than its present value, compound interest being allowed at $7 \%$. How much should he receive, his expectation of life being 74 years of age?
8. A lawyer collected for a client an annuity of $\$ 700$, in arrears for 4 years, the legal rate of interest being $6 \%$. He charged $15 \%$ on the amount collected. At this rate, how much greater would have been his fee had he been able to colleot compound interest?
9. A clerk saves from his salary $\$ 50$ every year, and puts it in a savings bank which allows interest compounded annually at $6 \%$. If he draws no checks on the bank, how much will he have there at the end of 10 years?
10. A person aged 54 has a life annuity of $\$ 400$. What is its present value, allowing compound interest at $4 \%$, his expectation of life being 19 years?
11. At the age of 20 , and every year after, a young man places $\$ 200$ at compound interest at $5 \%$. How much will he have at the age of $30 ?$ At the age of 40 ?
12. How much a year must be invested for a boy 11 years old, that the sums thus invested, with compound in. terest at $5 \%$, may make a total of $\$ 10,000$ by the time he becomes of age?
13. What is the present worth of an annuity of $\$ 500$ for 8 years, at $4 \%$ compound interest?
14. What is the present worth of an annuity of $\$ 3,00 \mathrm{C}$ for 20 years at $3 \%$ compound interest?
15. What is the present worth of an annuity in reversion of $\$ 1, \mathrm{c} 00$, at $5 \%$ compound interest, which begins in 3 yeare and then terminates in 8 years?
16. The reversion of a lease of $\$ 450$ per year, at $5 \%$, begins in 3 years and continues 16 years. What is its present worth?
17. A father bequeathed to his son, 11 years of age, a $5 \%$ annuity of $\$ 1,000$, to begin in 3 years and continne 10 years. What would be the amount when the son was 21 years old? What is its present worth?

- 18. What is the present worth of a perpetual annuity of $\$ 250$, in arrears for 10 years, allowing $3 \%$, compound interest?

19. What is the present worth of a perpetuity of $\$ 500$ in arrears for 30 years, allowing compound interest at $5 \%$ ?
ar, and puts compounded o bank, how
20. What at $4 \%$, his young man much will
a boy 11 pound in. te time he f $\$ 500$ for of $\$ 3,00 \mathrm{C}$ reversion in 3 yeare r, at $5 \%$, at is its
of age, a tinue 10 n was 21
nuity of ound in-

## SINKING FUNDS.

560. Sinking Funds are sums of money set apart at segular intervals for the payment of indebtedness.
561. Sinking Fund Bonds are securities issued by sorporations, based on the pledge of a special income, which is funded for their redemption.
562. To find what sum wust be set apart annua!ly, as a sinking fund, to pay a debt in a given time.

Example,-The Town of Woodstook borrowed $\$ 20,000$ to build a High Sohool, and agreed to pay $5 \%$ oompound interest. What sum must be set apart annusilly, as a sinking fund, to pay the debt in 12 years?

Amount of Solution.
$\$ 1$ at $5 \%$ compound interest for 12 years $=\$ 1.795856$. $\therefore \$ 20,000=\$ 1.795856 \times 20,000=\$ 35,917.12$.
Amount of annual payment of $\$ 1$ for 12 years at $5 \%=\$ 15.917127$. Art. 554 . $\therefore$ amount necessary to pay a debt of
$\$ 35,917.12$ will require $\$ 35,917.12 \div 15.917127=\$ 2,256.68$. Ans. RULE.
Diride the amount of the debt at its maturity at compound interest, by the amount of an annuity of $\$ 1$ for the given time and rate, and the quotient will be the sinking fund required.
563. To find the number of years required to pay a given debt, by a given annual sinking fund.

Example.-The Town of Port Hope builf a Court Eense at a cost Ji $\$ 15,000$, and raised $\$ 1,300$ a year to pay for it. Allowing $6 \%$ compnund anterest, how many years will it require to cancel the debt?

## Soldtion.

A sinking fund of $\$ 1,300$ has a present worth of $\$ 15,000$ for a certain tit at 6\%.
$\therefore$ A sinking fund of $\$ 1$ has a present worth of $188000=\$ 11.538461$, for the required time at $6 \%$

Looking in Table 2, Art. 555, in the column B\% we find tho nearest number less than 11.538461 , to be 11.469421 , the present worth of $\$ 1$ annuity for 20 years.

20 years is therefore the number of whole years required. Again:
The amount of the debt $\$ 10,090$ at $\mathbf{6} \%$ compound interest
for 20 years
The amount of a ginking fund 81,300 a $\%$.. intercst

$$
\begin{aligned}
& . \\
& \text { Balance due at ond of } 20 \text { years }=\frac{17,821.27}{\$ 286.1 \mathrm{~s}}
\end{aligned}
$$

rule.
Divide the debt by the given sinking fund, and the quotient will be the present worth of $\$ 1$ annuity for the given time.

Look for this number in Table 2, Art. 555, in the column denoting the given rate, and opposite in the column of time will be found the number of whole years.
Notes 1.-II the exact number is not found in the column, take the years atanding opposite the next smaller number.
2. To ascertain the balance due at the end of the number of whole years, find the difference between the amount of the debt, at the given rate, for the time taken, and the amount of the sinking fund for the same time and rate.

## EXERCISE 120.

1. If a railroad company sets apart an annual sinking fund of $\$ 20,000$, and loans it at $5 \%$ compound interest. What will be its amount in 12 years?
2. What will be the amount in 15 years of a sinking fund of $\$ 12,000$, yielding $4 \%$ compound interest ?
3. What sum must be set apart annually to rebuild a bridge costing $\$ 30,000$, estimated to last 17 years, allowing $5 \%$ compound interent?
4. A railroad company bought $\$ 102,000$ worth of rolling stock, payable in 5 years with $6 \%$ compound interest; what sum must be set apart annually as a sinking fund to discharge the debt?
5. A man buys a farm for $\$ 5,000$, and agrees to pay for it in six equal annual instalments. What is the amount of each payment, moircy berars worth $5 \%$ compound interest?
o. A railroad ec minny ise sed sinking fund bouds at $6 \%$ for $\$ 200.000$, payatin in 10 vars. If at compound interest, what sum must be sex spait annually to meet interest and principul when due?
6. What would be the amount in 10 years, at $6 \%$ simple interest?
7. If the funded securities were drawing an annual income of $4 \%$ compound interest, by how much would the amount necessary to meet principal and interest at $6 \%$ be reduced?
8. With the above reduction, what sum would be needed anuually as a sinking fund to pay the amount when due at $4 \%$.
9. A man buys a farm for $\$ 6,000$, and agrees to pay $\$ 700$ each year until paid, allowing $6 \%$ compound interest, both on the debt and on the payments. How many number of whole years will he be in paying for the farm? What is the balance then due?
10. A village built a school-house costing $\$ 12,000$, and raised $\$ 1,700$ a year to pay for it ; allowing $6 \%$ compound interest. How many whole years will it require to cancel the debt? What will be the balence then due?

## GROUND RENTS.

564. Ground Rents is a term applied to leases of building lots, the rent of which is considered equal to the interest on the valuation of the land. The payment is generally secured by a claim on the building erected on the land occupied.
565. When the party who rents the ground has the privilege of purchasing it, the Ground Rent is said to be redeemable; otherwise, it is irredeemable. The rentor of the land usually erects buildings thereon in his own right and pays a specified sum quarterly, semi-annually, or yearly, for the use of the ground. In some cities the issue of irredeemable ground rents is prohibited.
566. Building lots are sometimes sold at so much per foot frontage ground rent. Thus, a lot valued at $\$ 4,000$, with a frontage of 20 feet, drawing interest at $8 \%$, is said to be worth $\$ 16$ per foot. The interest on $\$ 4,000$ for 1 year at $8 \%$ is $\$ 320$, which, veing divided by 20 , the number of feet on the front, gives $\$ 16$ as the price.

Finen a $6 \%$ ground rent yields the owner $\$ 180$ per year, the value of the ground is estinated at $\$ 3,000$, since $\$ 180$ is the interest on $\$ 8,000$ for 1 year at $6 \%$.

## EXERCISE 121.

1. What is the capitalized value of ground, which at $5 \%$ ground rent, yields the owner $\$ 600$ per year 8

## GROUND RENTS.

2. What will be received as ground rent for a lot valued at $\$ 5,000$, leased at a ground rent of $8 \%$ ?
3. What is the ground rent price per foot frontage of a lot 30 feet front, valued at $\$ 12,000$ and paying a ground rent of $7 \%$.
4. If $\$ 192$ be received yearly from a ground rent bought for $\$ 3,840$, what is the rate per cent. ground rent?
5. I bought three lots, each 25 feet front-áad 140 feet in depth, at $\$ 50$ per foot frontage, and leased them at $4 . \frac{1}{2} \%$ ground rent. What income do I receive from my investment?
6. A real estate owner sold a ground rent of $\$ 75$ at $6 \%$. What did he receive for it?
7. The annual income received on a $6 \%$ ground rent was $\$ 540$. If the ground rent be sold at its value and the proceeds applied to the purchase of a mining stock at $\$ 50$ per share, how many shares can be bought?
8. Find the prosent worth of a ground rent of $8 \%$ on a lot valued at $\$ 4,500$, to commence in 3 years and to then continue 15 years; if money be worth $5 \%$ compound interest.

## LIFE INSURANCE.

567. Life Insurance is a contract by which a company (the insurer), in consideration of certain payments, agrees to pay to the heirs of a person, when he dies, or to himself, if living at a specified age, a certain sum of money.
568. The principal kinds of policies issued by Life Insurance Companies are the following : Ordinary Life, Limited Payment Life, Endowment, and Annuity.
569. An Ordinary Life Policy is one on which a certain premium is to be paid every year until the death of the insured, when the policy becomes payable to the persons named in the policy as the beneficiaries.
570. A Limited Payment Life Policy is one on which the premium is paid annually for a certain number of years, fixed upon at the time of insuring, or until the death of the insured, should that occur prior to the end of the selected period. The policy is payable on the death of the insured.
571. An Endowment Policy is one which is payable to the person insured, if he survives a certain number of years, or to bis heirs, if he should die before the expiration of such period, in consideration of certain regular payments from the person insured.
572. An Annuity Policy is one which secures to the holder the payment of a certain sum of money every year during his life-time. It is secured by a single payment.
$57 \%$ A Non-Forfeiting Policy is one which does not become void on account of non-payment of premium.
573. The Surrender Value of a policy is the amount of cash which the Company will pay the holder on the surrender of the policy. It is the legal reserve less a certain per cent. for expenses.
574. The Reserve of Life Insurance Policies is the present value of the amount to be paid at death less the present value of all the net premiums to be paid in the future.
comnents, or to im of Life, ich a death 0 the vhich er of 1 the ad of th of vable or of tion ients the year t.
not
junt the ss a
575. The Reserve Fund of a Life Insurance Company is that sum on hand which invested at a given rate of interest together with future premiums on existing policies, should be sufficient to meet all obligations as they become due. It is the sum of the separate reserves of the several policies outstanding.

57\%. The Premium is the sum paid for the insurance of a person's life. It is paid annually, semi-annually, or quarterly.
578. The Premium consists of three elements: 1st. The Reserve, or that portion of each premium which must be kept and improved by interest, to pay the policy at its certain maturity.

2nd. An estimated amount for each man's share of the annual losses of the company.

3rd. Loading, or a certain per cent. to be added to the net premium to cover the general expenses of the business, and to provide against unusaal contingencies.
579. The Sum Insured is the sum which is payable by the company upon the conditions mentioned in the contract.
580. Tables of Mortality are tables showing the average rate cf deaths in every ten thousand persons.
581. Expectation is the average number of years which a person of a certain age is expected to live, based on a Table of Mortality.
582. The Rates of premium for Life Insurance, as fixed by different companies, are based on the probabilities of life, determined by a table of mortality, and the probable rates of interest which money will bear, and a loading or margin for expenses.

## 583.

Expectation of Life.
The following table shows the number living, the number dying, anl the expectation or duration of life of each individual, calculated from the Combined Experience Mortality Table:

| Age. | Living. | Dying. | Expectation. | Age. | Living. | Dying. | Expectation. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 100000 | 676 | 48.36 | 55 | 63469 |  |  |
| 11 | 99324 | 674 | 47.68 | 56 | 62094 | 1436 | 16.86 16.22 |
| 12 | 98650 | 672 | 47.01 | 57 | 60658 | 1497 | 16.22 15.59 |
| 13 | 97978 97307 | 671 671 | 46.33 | 58 | 59161 | 1561 | 14.97 |
| 15 | 96636 | 671 | 45.64 44.96 | 59 | 57600 | 1627 | 14.37 |
| 16 | 95965 | 672 | 44.27 | 60 | 55973 54975 | $\xrightarrow{1698}$ | 13.77 |
| 17 | 95293 | 673 | 43.58 | 62 | ${ }_{5256}^{54 .}$ |  | 13.18 |
| 18 | 94920 | 675 | 42.88 | 63 | ${ }_{5}^{525661}$ | 1 | 12.61 12.05 |
| 19 | 93945 | 677 | $4: 19$ | 64 | 48744 | 1990 | 12.05 |
| 210 | 93268 | 680 | 41.49 | 65 | 46754 | 2061 | 11.51 |
| 22 | 92558 | 683 | 40.7) | 66 | 44693 | 2128 | 10.46 |
| 28 | 91905 | 686 | 40.09 | 67 | 42565 | 2191 | 9.96 |
| 24 | 90529 | 694 | 39.39 38.68 | 68 | $4037 \pm$ | 2246 | 9.47 |
| 25 | 89835 | 698 | 37.98 | 69 | 38128 | 2291 | 9.00 |
| 26 | 89137 | 703 | 37.27 | 71 | 35837 33510 | 2327 | 8.54 |
| 27 | 88434 | 708 | 36.56 | 78 | 31159 | 2362 | 8.10 7.67 |
| 28 | 87726 | 714 | 35.86 | 78 | 28797 | 2358 | 7.26 |
| 29 | 87012 | 720 | 35.15 | 74 | 26439 | 2339 | 6.86 |
| 30 | 86292 | 727 | 3443 | 7.5 | 2410 | 2303 | 6.48 |
| 31 | 85565 | 734 | 33.72 | 76 | $217{ }^{\circ}$ | 2249 | 6.48 |
| 32 | 84831 | 742 | 3301 | $7 \%$ | 19548 | 2179 | 5.76 |
| 33 | 84089 | 750 | 32.30 | 78 | 17369 | 2092 | 5.42 |
| 35 | 82581 | 758 767 | 31.58 | 79 | 15277 | 1987 | 5.09 |
| 36 | 81814 | 776 | 30.87 30.15 | 80 | 13290 | 1866 | 4.78 |
| 37 | 81038 | 785 | 29.44 | 88 | 11424 | 1730 | 4.48 |
| 38 | 80253 | 795 | 28.72 | 88 | 9694 8112 | 1582 1427 | 4.18 |
| 39 | 79458 | 805 | 28.00 | 84 | 8685 | 1427 | 3.90 3.63 |
| 10 | 78653 | 815 | 27.28 | 85 | 5417 | 1111 | 3.63 3.36 |
| 11 | 77838 | 826 | 26.56 | 86 | 4306 | ${ }_{958}$ | 3.36 8.10 |
| 2 | 77012 | 839 | 25.84 | 87 | 3348 | 811 | 3.10 2.84 |
| 8 | 76173 | 857 | 25.12 | 88 | 2537 | 673 | 2.84 2.59 |
| 4 | 75316 | 881 | 24.40 | 89 | 1864 | 545 | 2.55 |
| 5 | 74435 | 909 | 23.69 | 90 | 1319 | 427 | 2.11 |
| 7 | 73526 | 944 | 22.97 | 91 | 892 | 322 | 1.89 |
| 8 | 72582 71601 | ${ }^{981}$ | 22.27 | 92 | 570 | 231 | 1.67 |
| 9 | 70580 | 1053 | 21.56 | 93 | 839 | 155 | 1.47 |
| 0 | 69517 | 1108 | 20.87 | 94 | 184 | 95 | 1.28 |
| 1 | 68109 | 1156 | 19.50 | 85 | 89 | 52 | 1.12 |
| 2 | 672:3 | 1207 | 18.80 | 86 | 37 | 24 | 0.97 |
| 3 | 66046 | 1261 | 1816 | 98 | 13 | 9 | 0.89 |
| 1 | 6478 ¢̃ 1 | 1316 | 17.50 | 8 | 4 | 3 | 0.75 |

the num. fe of each ixperience

### 6.86

6.48
6.11
5.76
5.42
5.09
4.78
4.48
4.18 3.90

### 3.63

3.10
2.84
2.59
2.35
2.11
1.89
1.47
1.28
1.12
0.97
0.89
0.75
0.75
0.50
584.

Table of Rates.
RATES FOR WHOLE LIFE INSURANCE.
Premiums to Insure $\$ 1,000$ payable at Death, with Plinfits.

| Age. | Annual Premiums | $\begin{gathered} \text { Single } \\ \text { Pramiums. } \end{gathered}$ | $\left\lvert\, \begin{gathered} \text { Annual } \\ \text { Promiums } \\ \text { for } \\ 5 \text { Yoers. } \end{gathered}\right.$ | Annual Promiums for <br> 10 Years | Annual Promiums for <br> 15 Years. | Annual Premium 20 Yor | Aga. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 17.80 | 265.17 | 60.22 |  |  |  |  |
| 21 | 18.20 | 270.07 | 61.34 | 35.03 35.69 | 26.95 27.46 | 23.10 23.59 | 20 |
| 22 | 18.62 | 275.11 | 62.50 | 36.38 | -8.00 | 24.59 | 21 |
| 24 | 19.51 | 280.38 | 63.71 | 37.09 | 28.55 | 24.54 | 23 |
| 2.5 | 19.99 | 291.39 | 64.95 66.24 | 37.82 | 29.13 | 25.04 | 24 |
| 26 | 20.49 | 297.17 | 67.24 67.57 | 38.58 39.37 | 29.72 | 25.55 | 25 |
| 27 | 21.01 | 303.15 | 69.94 | 39.37 40.18 | 30.34 30.97 | 26.09 | 28 |
| 28 | 21.56 | 309.32 | 70.36 | 40.18 41.02 | 30.97 $\mathbf{8 1 . 6 4}$ | 26.65 | 27 |
| 29 | 22.13 | 315.70 | 71.83 | 41.92 | 31.64 32.32 | 27.23 | 28 |
| 30 | 22.78 | 322.28 | 73.35 | 41.90 42.80 | 32.32 33.03 | 27.83 | 29 |
| 31 | 23.36 | 329.08 | 74.92 | 43.83 43.73 | 33.03 33.76 | 28.45 | 30 |
| 32 | 24.02 | 336.10 | 76.55 | 44.70 | 33.76 $\mathbf{8 4 . 5 2}$ | 29.10 39.78 | 31 |
| 34 | 24.71 | 343.33 | 78.22 | 45.70 | 35.31 | 30.48 | 32 |
| 35 | 26.21 | 350.81 358.53 | 79.95 | 46.73 | 36.13 | 31.21 | 34 |
| 36 | 27.01 | 366.50 | 883.59 | 47.80 | 36.98 | 31.97 | :35 |
| 37 | 27.86 | 374.73 | 88.50 | 48.90 | 37.87 | 32.77 | 36 |
| 38 | 28.76 | 383.23 | 87.48 | 50.05 | 38.79 | 33.60 | 37 |
| 39 | 29.71 | 392.02 | 88.53 | 51.24 | 39.75 | 34.47 | 38 |
| 10 | 30.71 | 401.10 | 91.67 | 63.48 | 40.76 | 3539 | 39 |
| 11 | 31.78 | 410.49 | 93.84 | 63.77 | 41.81 | 36.35 | 40 |
| 2 | 32.91 | 420.19 | 96.13 | 55.12 | 42.92 | 37.37 | 41 |
| 8 | 34.11 | 430.22 | 98.50 | 68.01 | 44.08 | 38.45 | 42 |
| 4 | 35.39 | 440.54 | 100.96 | 59.55 | 45.30 | 39.58 | 48 |
| 15 | 36.74 | 451.13 | 103.51 | 61.15 | 46.59 47.93 | 40.78 | 44 |
| 6 | 38.17 | 461.96 | 106.13 | 62.82 | 47.93 49.33 | 42.01 | 45 |
| 7 | 39.67 | 472.99 | 108.81 | 64.53 | 49.33 50.79 | 43.37 | 46 |
| 8 | 41.26 | 484.23 | 111.57 | 66.31 | 50.79 62.32 | 44.76 46.29 | 47 |
| 19 | 42.93 | 495.68 | 114.89 | 68.04 | 62.32 53.90 | 46.22 47.75 | 48 |
| 1 | 44.70 | 507.27 | 117.28 | 70.05 | 53.90 55.56 | 47.75 49.37 | 49 |
| 2 | 46.56 | 519.06 | 120.24 | 72.01 | 55.56 57.30 | 49.37 51.07 | 50 |
| 3 | 48.53 | 531.01 | 123.28 | 74.05 | 59.11 | 52.86 | 51 |
| 4 | 50.61 | 543.10 <br> 55583 | 126.38 | 76.16 | 61.00 | 54.75 | 53 |
| 5 | 55.14 | 567.70 | 129.55 | 78.33 | 63.00 | 56.75 | 54 |
| 6 | 57.61 | 580.17 | 136.11 | 80.61 | 65.09 | 58.86 | 55 |
|  | 60.22 | i92.74 | 139.51 | 82.97 | 67.29 | 61.11 | 56 |
|  | 63.00 | 60.41 | 143.00 | 85.43 | 69.61 | 63.49 | 57 |
|  | 65.94 | 618.17 | 146.58 | 88.00 | 72.07 | 66.03 | 58 |
| (8) | 69.07 | 830.98 | 150.26 | ${ }_{93.51}^{90.09}$ | 74.68 | 68.74 | ${ }^{8}$ |

585. RATES ROR ENDOWMENT INSURANOE.

Annoal Premioms to Insure $\$ 1,000$, Payablas af Death de gay Expilation of the following Trrms, whet Profits.


## EXERCISE 122.

1. Find 'he amount of premium for an ordinary life policy of $\$ \pm, 000$, 1ssued to a person 40 years of age. (Art. 583.)
2. Find the annual premium for a 10 -payment life policy of $\$ 5,000$, issued to a person 35 years of age. (Art. 585.)
3. When 40 years of age, a person took out a 20 -year endowment policy of $\$ 10,000$. He survived the endowment period. How much less did he receive than he paid as premiums, not reckoning interest?
4. The annual premium, without profits, on a life policy of $\$ 5,000$ at the nge of 35 is $\$ 111$. How much would be necessary to invest at $6 \%$ interest to secure the payment of the annual premium?

5 Mr . A., age 30 , insures his life for $\$ 10,000$, ordinary life plan, with profits. How much must he place in trust so that the interest at $5 \%$ will be sufficient to pay the premiums on the policy?
6. A single premiuin for an insurance of $\$ 1,000$, without profits, for q person 32 years of age, is $\$ 300$. What would be the excess of the insurance over the amount produced by placing the money at compound interest at $4 \%$, supposing the insured to live 20 years?
7. Mr. A., aged 36 , insured his life for $\$ 5,000$, and paid an annual premium of $\$ 185$; supposing he died at the age of 68, how much did the premiums he paid exceed the face of his policy, money being worth $6 \%$ compound interest?
8. Mr. A., at the age of $\mathbf{3 5}$, takes out a 20 -year endowment policy for $\$ 3,000$ and pays an annual premium of \$141. By what amount will the premiums exceed the face of the policy at the end of the endowment period, money being worth $5 \%$ compound interest ?

## MISCELLANEOUS.

## EXERCISE 122.

## I.

1. Which is the better investment, a $\$ 3,0007 \%$ bond, or a bouse which rents for $\$ 240$ a year, taxes being $\$ 30.50$, and annual repairs $\$ 40$ ?
2. A person exchanges 250 shares of $6 \%$ stock, at 70 , for stock bearing $8 \%$, at 120 ; what is the difference in his income?
3. A gentleman has been receiving $12 \%$ on his capital in Canada. He goes to England to resile, and invests it in the 3 per cents. at $94 \frac{3}{8}$, and his income in England is $£ 2,400$. What was his income in Canada, the $£$ being equal to $\$ 4.86 \frac{3}{8}$ ?
4. Find the alteration in income occasioned by shifting $£ 3,200$ atoci from the 3 per cents. at $86 \frac{3}{8}$, to 4 per cent. stock at 1147 : the brokerage being $\frac{1}{8} \%$.
5. Suppose a railroad stock, actually worth $\$ 100$ a sha: 3 , to be " watered" by the issue of a stock dividend of $20 \%$ to the stockbolders, what would the watered stock be worth?
6. A person bought stock at $95 \frac{1}{4}$, and after receiving the half yearly dividend at the rate of $7 \%$ per annum, sold out at 923 and made a profit of $\$ 37.50$. How much stock did he buy?
7. At what price must U. S. $4 \frac{1}{2}$ 's be bought, to yield the interest on the investment that $5 \%$ bonds will at 110 ?

What amount of the latter bonds (par value) must be sold at 109, leaving brokerage out of account, that with the proceeds a sufficient amount of $4 \frac{1}{2}$ 's miy be bought, at par, to yield a semi-anntal income of $\$ 364.50$ ?
8. A person invests the proceeds of a note for $\$ 9,607.50$, due 18 months hence, discounted (true discount) at $4 \frac{1}{8} \%$, in $6 \%$ stock at 91 , brokerage $\frac{1}{4} \%$. Find his net annual income from this investment after deducting an income tax of $2 \frac{1}{2} \%$.

习. The present income of a railway company would justify a dividend of $3 \frac{3}{4} \%$, if there were no preference shares; but as $\$ 1,200,000$ of the stock consists of such shares, which are guaranteed $5 \%$ per annum, the ordinary shareholders receive only $3 \%$. What is the whole amount of stock ?
10. A gentleman has $\$ 25,000$ of Bank of Commerce stock which pays a dividend of $8 \%$. When money is worth $7 \%$ he sells out, and invests in Bank of Toronto stock at 205, which pays a dividend of $12 \%$. What difference in his income after allowing his agent $\frac{1}{2} \%$ commission for each transaction?
11. A man invests $\$ 19,450$ in Bank of Montreal stock at 194, and $\$ 19,850$ in Bank of Toronto stock at 198, paying his broker in each case $\frac{1}{2} \%$ on the amount of stock purchased. If the former pays a half-yearly dividend of $6 \frac{1}{2} \%$, and the latter a half-yearly dividend of $6 \frac{1}{4} \%$, find his total income for the half-year.
12. A man invested a certain sum in Bank of Commerce stock, which is at 120 , and pays $4 \frac{3}{8} \%$ half-yearly dividends; and $62 \frac{1}{2}$ per cent. more than that sum in Dominion Bank stock, which is at 180, and prys $4 \frac{1}{2} \%$ half-yearly dividends; his income from both isvestments is $\$ 222.50$. Find the amount of money invested in each kind of stock.

## II.

1. Jan. Ist, 1889, three persons began business. A. put in $\$ 1,200$, B. put in $\$ 500$, and May 1st $\$ 800$ more, C. put in $\$ 700$, and July 1st $\$ 400$ more. At the end of the year the profits were $\$ 875$. How shall it be divided ?
2. A. B. and C. commence business; A. puts in 250 firkins of butter, B. puts in $\$ 2,500$, and C. $\$ 4,100$. Their profits amounted to $\$ 2,210$, of which A. took $\$ 560$. How much was his butter a pound, and to ? much were B. and C. entitied?
3. A building worth $\$ 28,500$ is insured in the Etna for $\$ 3,200$, in the Western for $\$ 4,200$, and in the Mutual for $\$ 6,500$. It having been parially destroyed, the damage is set at $\$ 10,500$. What should each company pay?
4. A. had $\$ 3,800$ at interest for 60 days; B. had $\$ 4,100$ at interest for 45 days; and 0 . had $\$ 4,950$ at interest for 70 days. They received $\$ 162$ interest money. What did each get, and what was the rate per cent?
5. A. and B. formed a partnership Jan. 1st, 1889. A. put in $\$ 6,000$, and at the end of 3 months $\$ 900$ more, and at the end of 10 months drew out $\$ 300$; B. put in $\$ 9,000$, and 8 months after $\$ 1,500$ more, and drew out $\$ 500$ Dec. 1st. At the end of the year the net profits were $\$ 8,900$. Find the share of each.
6. Two persons commence trade with the same amount of money. The first man spends $43 \%$ of his money yearly, and the second spends a sum equal to $25 \%$ of what both had at first. At the end of the year they both together had $\$ 3,468$. How much had each at the end of the year?
7. A. commenced business with a capiial of $\$ 10,000$, on the 1st of January, 1889 ; on the 1 st of May, B. entered into partnership with him, and, in 1,500 barrels of flour. On the first of January, 1890 eir ofits were $\$ 5,100$, of which B. was entitled to $\$ 2,100$. What was the value of the flour per barrel?
8. Three persons formed a partnership, with a eapital of $\$ 4,600$. The first man's stock was in trade 8 months and
gained $\$ 752$; the second man's stock was in trade 12 months and gained $\$ 600$; and the third man had his stock in 16 months and gained $\$ 640$. What was each man's stock?
9. Three men engaged in the manufacture of pails; $A$. put in $\$ 2,550$ for 8 months; B., a sum not specified for 12 munths ; C., $\$ 1,080$ for a time not specified. A. received for his stock and profit $\$ 3,400$; B., $\$ 4,200$ for his; C., $\$ 1,485$ for his. Required, B.'s stock and C.'s time.
10. On the 1 st of January, 1889, James Wilson opened a hardware store with a stock of $\$ 17,200$; on the 1 st of April, Joseph Brooks entered into partnership with him, and advanced $\$ 12,000$; on the 1st of July, Abraham Miller put in goods to the amount of $\$ 16,000$; on the 1 st of January, 1890, when the balance sheet was exhibited, there appeared a net profit of $\$ 8,060$. To how much was each partner entitled ?

11 A., B. and C. engaged in business. A. puts in $\$ 400$ at $f_{1}$, and $\$ 400$ more at the end of 6 months ; B. puts in $\$ 900$ at first, and withdraws one-third of his capital at the end of 6 months; C. puts in $\$ 200$ at the end of every 6 months. At the end of two years they have gained $\$ 6,700$. What share of the profits should C. receive in addition to $25 \%$ of the total profit for managing the business?
12. A., B. and C. formed a partnership for 2 years; A. put in $\$ 10,000$, B. $\$ 5,000$, and C. $\$ 2,500$; it was agreed that C. should receive $\$ 1,500$ a year for superintending the business. A. drew out $\$ 1,000$ at the end of each quarter for one year, and at the ond of 18 months put in $\$ 15,000$ more ; B. withdrew $\$ 600$ at the end of each quarter. At the time of settiement tine net gain was $\$ 22,500$. Required each one's share.

## III.

1. A draft on Winnipeg bnught at $\frac{3}{4} \%$ premium for $\$ 12,000$, was sent to an agent to pay for cotton purchased at $2 \frac{1}{2} \%$ commission; what was the value of the cutton?
2. A commission merchant in Peterborough wishes to remit to his employer in Belleville $\$ 512.36$ by draft at 60 days; what is the face of the draft that he can purchase with this sum, exchange being at $2 \frac{1}{2} \%$ discount, interest $7 \%$ ?
3. Shipped to Liverpool, 2,000 barrels of flour, which cost in Montreal $\$ 4.50$ per barrel ; it was sold at $£ 118 \mathrm{~s} .6 \mathrm{~d}$. per barrel, when the premium was $8 \frac{1}{2} \%$; how much was the gain?
4. A grain dealer bought 10,000 bushels of corn, at $38 \frac{8}{8}$ cts. a bushel. He sent it to London, where it brought 28s. 9d. a quarter, when the premium was $9 \frac{1}{2} \%$; the cost of transportation was $12 \frac{1}{2}$ cts. per bushel ; how much was gained?
5. A person in Barrie received $£ 1,000$ sterling, from England, when the premium was $9 \%$. He put it out at interest for 9 months, 18 days at $6 \%$ per annum; to how much did it amount?
6. A merchant sent his agent in London 425 bales of cotton weighing 356 lbs . apiece, which cost him $9 \frac{1}{2}$ cents a lb .; the agent paid $\frac{3}{8} d$. a lb. for freight, $£ 43$ for cartage, sold it at 8 d . a lb ., and charged $2 \frac{1}{2} \%$ commission. If the merchant sells a bill of exchange for the amount, at $10 \frac{1}{2} \%$, will he make or lose by the operation. How mrch ?
7. Received from my correspondent in New York $\$ 6,150$ U. S. currency, with instructions to deduct my commission at $2 \frac{1}{2} \%$, and invest the remainder in Canadian Tweeds worth $\$ 1.08 \frac{1}{2}$ per yard. How many yards should I send bim, gold being quoted at 115 ?
8. An importer bought 1,565 yards of silk, at 5 s .6 d . per yard; paid $£ 7$ 12s. for freight, $25 \%$ duties, and remitted a bill on London at $94 \%$ premium; how must he sell it per yard on 6 months, in order to make $12 \frac{1}{2} \%$, allowing. $7 \%$ interest?
9. Exchange between Paris and Amsterdam being at the rate of 2 francs 20 centimes to the guilder, that between London and Paris at the rate of 25 francs 80 centimes to the $£$, and that from New York on London at $9 \frac{1}{2} \%$ premium, what will be the cost of a remittance for 1,000 guilders from New York to Amsterdam by bills of exchange through London and Paris?
10. A merchant in Toronto wishes to pay $£ 8,000$ in London. Exchange on London is $9 \frac{1}{2} \%$ premium ; on Paris, 5 francs 25 centimes per $\$ 1$; and on Amsterdam, 40 cents to a guilder. The exchange between France and England at the same time is 25 francs to £1, and that of Amsterdam on England 12t guilders to $£ 1$. Which is the most advantageous, the direct exchange, or through Paris, or through Amsterdam ?
11. A Hamilton mezchant, owing 2,400 florins in Amsterdam, can buy exciange on that city for $41 \frac{1}{4}$. Is it better for him to do so, or to remit to London, and thence to Amsterdam,-exchange on London being 4.87 in Hamilton, exchange on Amsterdam being 12 florins to the pound sterling in London, and brokerage for purchasing. the exchange in London being $\frac{1}{8}$ of $1 \%$ ?
12. A banker in Toronto remits $\$ 1 \hat{u}, 000$ to Liverpool as follows : First to Paris, at 5 francs 40 centimes per $\$ 1$; thence to Hamburg, at 185 franos per 100 mares; thence to Amsterdam, at $17 \frac{1}{2}$ stivers per mare ; thence to Liverpool, at 220 stivers per $£$ sterling; how much sterling money will he have in bank at Liverpool, and what will be his gain over direct exchange at $10 \%$ premium?

## IV.

1. Allowing $6 \%$ compound interest on an annuity of $\$ 200$ which is in arrears 20 years, what is its present amount?
2. What is the present worth of an annuity of $\$ 500$ for 7 years, at $6 \%$ compound interest?
3. Find the annuity whose amount for 25 years is $\$ 16,459.35$, allowing componnd interest at $6 \%$.
4. The present worth of an annuity to be continued 10 years at $6 \%$, compound interest, compounded annually, is $\$ 7,360.08$. What is the annuity?
5. A. man bought a farm for $\$ 4,500$, and agreed to pay prinoipal and interest in 4 equal annual instalments ; how much was the annual payment, interest being $6 \%$ ?
6. A mar bought a piece of property for $\$ 10,000$, and agreed to pay principal and interest in 3 equal annual instalments. How much was the annual payment, interest being 7. \% ?
7. H father !bequeathed /his won, 11 years af age, $2: 6 \%$ annuity of $\$ 2 ; 500$, to begin in 3 years sand ceontinue 10 years; what mould be the amount when cthe anon was 21. years old?
8. A man took out a life policy for $\$ 3,000$, at the rate of $\$ 21.50$ per $\$ 1,000$. What sum must he deposit in a saviugs bank, the compound interest of which, at $5 \%$, payable semi-annually, shall discharge his annual premium?
9. A man died leaving $\$ 5,000$ to be divided between his three sons, aged 13, 15, and 16 years respectively, in such a proportion that the share of each being put at simple interest at $6 \%$, should amount to the same sum when they should arrive at the age of 21 . How much was each one's share?
10. A man paid annually $\$ 10$ for tobacco from the age of 14 until he was 50 , when he died, he left $\$ 1,000$ for his heirs. What sum might he have left them had he dispensed with tohacco, and loaned the money thus saved at the end of each year at $6 \%$ compound interest?
11. A mortgage of $\$ 1,000$, repayable in 5 years at $\$ 200$ a year with interest at $6 \%$ on the unpaid principal, is sold; what is its value allowing the purchaser $8 \%$ for his money?
12. A mortgage on a farm is payable in four equal annual instalments of $\$ 1,000$ each. When the first instalment falls due the mortgagor offers in part payment $\$ 2,000$ in $6 \%$ municipal debentures upon which interest is due, and which mature in one year. What balance in cash should the mortgagor demand in exchange for the mortgage, money boing worth $10 \%$ ?

## POWERS AND ROOTS.

586. A Power of a number is the number itself, or the product of equal factors, each of which is that number. Thus, 8 is a power of 2 , since $8=2 \times 2 \times 2$.
5*7. The First Power is the number itself.
58\%. The Second Power is the product of a number taken twice as a factor, and is called a Square.

Thus, 16 is the square of 4 , since $16=4 \times 4$.
589. The Third Power is the product of a number taken three times as a factor, and is called a Cube.

Thus, 125 is the cube of 5 , since $125=5 \times 5 \times 5$.
590. A Root is one of the equal factors of a number. Note-Roots are named from the number of equal faotors they contain.
591. The Square Root is one of the two equal factors of a number.

Thus, 7 is the square root of 49 , since $49=7 \times 7$.
592. The Cube Root is one of the three equal fuctors of a number.
Thus, 7 is the cube root of 348 , since $343=7 \times 7 \times 7$.
593. The Radical Sign is the character $V$, which, placed before a number, indicates that its root is to be found.
594. The Index of the root is the figure placed above the radical sign to denote what root is to be taken. When no index is written, the index 2 is always understood.
Nors.-The names of the roots are derived fro ocorresponding pawers, and are denoted by the indices of the radical agn.
Thus $\sqrt{9}$ denotes the square root of 9 , the $\sqrt[3]{9}$ denotea the onbe root of 9 , eto.
595. A Perfect Square is one whose exact square root can be found ; as $9,16,96$, etc.
596. A Perfect Cube is one whose exact cube root can be found; as 27, 64, 216, etc.

## SQUARE ROOT.

597. Extracting the Square Root of a number is the process of finding one of the two equal factors of a number. Nore.-The etudent should memorize the equares of the first nine digits.
The squares of $1,2,3,4,5,6,7,8,9$, are respectively $1,4,9,16,25,36$, 49, 64, 81.

## 5!)8. To extract the square root of a number.

Example 1.-Extract the square root of 5,625 .

> Process.
> $56 \mid 25(75$
> $\frac{49}{725}$
> $\underline{725}$

145

Explanation of the Metriod.
Separate the given number into periods of two figures each, beginning at the units' figure.

Find the greatest square in the first period (56), whioh is 49 , and place it under 56, also write the root of 49, which is 7, as the first figure in the required root.

Subtract 49 from 56, and to the remainder (7) affix the next period (25), giving 725 for a dividend. At the left of the dividend (726), write twice the root alreidy found (7), which gives 14.

Divide 72 by 14, which gives a quotient (5).
Affix 5 to 14 , giving 145 , also place 5 , as the seoond figure of the root.
Multiply 145 by 5 , giving 725, which subtracted from the dividend (725), leaves no remainder.
75 is the required root.
Example 2.-Extraot the square root of $6,838,225$.
Process.

| 46 | $\left.\right\|_{4} ^{6}\|83\| 82 \mid 25(2615$ |
| :---: | :---: |
|  | $2 \overline{83}$ |
|  | 276 |
| 521 | 782 |
|  | 521 |
| 5225 | $2 \hat{2125}$ |
|  | 26125 |

## Explanation of the Method.

Separate the given number into periods of two figures each, commencing at the units' figure.

Find the greatest square in the first period (6), which is 4, and place it under 6, also write the root of 4 , which is 2 , as the first tigare of the required root.

Subtract 4 from 6, and to the remainder (2) affix the next period (83), giving 283 as the dividend.
At the left of the dividend (283), write twice the root already found (2), which gives 4.
Divide 28 by 4 , which gives 7 as a quotient.
Affix 7 to 4 , giving 47, also place 7 as the second figure of the root, and multiply 47 by 7 , which gives 329 , a number greater then the dividend (283), showing that 7 is too large a number.

We next try 6 as the second figure of the root.
Affix 6 to 4 , giving 46 ; and place 6 as the second figure of the root.
Multiply 46 by 6 , giving 276 , which subtracted from the dividend 283 , leaves a remainder 7, to which affix the next period (82), giving as the next dividend 782.

Multiply the part of the root already found (26) by 2 , obtaining 52 , which place to the left of the dividend 782.

Divide 78 by 52 , which gives a quotient of 1 .
Affix 1 to 52, giving 521, also place 1 as the third figare of the root. Multiply 521 by i and subtract from the dividend 782, after which pro. ceed as bsfore.

No es 1.-If there is a remainder after the root of the last period is found, annex periods of ciphers, and proceed as before. The figures of the root thus obtained will be decimals.
2. If the trial divisor is not contained in the dividend, annex a cipher both to the root and to the divisor, and bring down the next period.
3. It sometimes happens that the remainder is larger than the divisor; but it does not necessarily follow that the figure in the root is too small.
599. To extract the square root of a decimal.

ROIE.
Begin at the units' place, and proceed towards the left and right, to separate into perionls of two figures each, then extract the root as in whole numbers.

Notes 1-The left hand period in whole numbers may lave but one figure; but in decimals, each period must have tivo figures. Hence, if the number of decimals is odd, a cipher must be annexed to complete the
2. It must be kept in mind that no period should contain an integer and decimal, and that, if there is an odd number of decimal places in the given number, the last period must be completed by annexing a cipher.

## 600. To extract the square root of a fraction.

## RUle.

Reduce the fraction to its simplest form and find the square root of each term separutely.

Notes 1.-If the denominator of the given fration, when reduced, is an imperfect square, reduce the fraction to a decimal, and proceed as above.
2. Mixed numbers should be reduced to improper fractions, or the fractional part to a decimal.

## EXERCISE 123.

Find the square root of--

1. 36864. 
1. 244036. 
1. 81225. 
1. 258064
2. 579121. 
1. 168921 .
2. 396900. 
1. 734449 .
2. 212521. 
1. 499849. 
1. 820836 .
2. 966289
.
3. 850625. 
1. 1081600. 
1. 1177225. 

Find one of the two equal factors of-
17. 6838225.
18. 9048064.
19. 6885376.
20. 296356225
21. 3196944.
22. 19228225.
23. 44502241.
24. 616855. .
25. $17958 \cup 801$.

## Extract the square root of -

| 26. | .0961. | 30. | 28867. | 34. | 3819.24 | 38. | 5416.96. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 27. | 15.21. | 31. | 33489. | 35. | 1.338649. | 39. | 50.1264. |
| 28. | 22.09. | 32. | 4.2849. | 36. | 226.8036. | 40. | .00720801. |
| 29. | .0004. | 33. | 17.3056. | 37. | .00001024. | 41. | 200.225296. |

Extract the square root of -

| 42. 5. | 46. 2. | 60. 204. | 54. 3 . |
| :---: | :---: | :---: | :---: |
| 43. .5. | 47. . 06. | 51. 1537. | 55. 35‥ |
| 44. . 05. | 18. \%. | 52. $1{ }_{168}^{56}$ | 56. 278. |
| 45. .00\%. | 49. . 02. | 53. 23.1. | 57. 364. |

Find the square root of-

| 58. $\frac{7}{8}$. | 61. 8738. | 64. ${ }^{2} 25$. | 67. |
| :---: | :---: | :---: | :---: |
| 69. $\frac{1}{3}$. |  | 65. 1718 d | 68. 3 \% |
| 60.8 | 63. H48988. | 66. 119\% | 69.3815 |

## CUBE ROOT.

601. Extracting the Cube Root of a number is the process of finding one of the three equal factors of the number.

Nore.-The student should memorize the cubes of the first nine digits. The oubes of $1,2,3,4,5,6,7,8,9$ are respectively $1,8,27,64,125,216$

## 602. To find the cube root of a number.

Example.-Find the cube root of 32768 .

> Process.
I. II. III.

Explanation of tae Method.
First separate the given number into periods of three figures each, beginning at the units' figure.
Then take the nearest perfect cube not greater than 32 , which is 27 , and set down its cube root, which is 3 in columı II., in line with 32768 .
Then subtract 27 from 32, and to the remainder (5) annex the next period (768), giving 5768.
Next place 3 times the first figure (3) of the root, already found, whioh
is $(3 \times 3) 9$ in oolumn I, and 3 times the square of the root (3) already found which gives $(3 \times 3 \times 3) 27$, with two ciphers annexed to it, in colunn III., each opposite 5768 .

Divide 5768 by 2700 , whioh gives a quotient of 2.
Pl ce 2 in column II., opposite 9.
Read 9-2 as one number 92, multi
184 under 2700 , add and multiply tiply this by 2 , and place the produot product 5768 under 5768, and subtreir sum, 2884 by 2, and place their is a perfect cube.

The figures in column II, taken in order give the cube root 32.

Example 2.-Extract the cobe root of 122615327232.
Prockes.
is the rs of the aine digits $4,125,216$


## Explanation of the Metiod.

Separate the given number into periods of three figures, each beginning at the units' figure.

Then as in Example 1, take the nearest perfect cube not greater than 122, which is 64, and set down its cube root whioh is 4 in column II., in line with the given number.

Subtract 64 from 122, and to the remainder (58) annex the next period (615), giying 58615.

Next place 3 times 4 (the first figure of the root), that is 12 in column I; and 3 times $4 \times 4$ (the square of 4 ), which equals 48 in column III; each in line with 58615, and annex two ciphers to 48 giving 4800.

Divide 58615 by 4800 , and a quotient 12 is obtained.
Now 9 is the largest namber we can have as a figure of the root, and we therefore use 9 , plaoing it in column II. opposite 12.

Read 12..9 as one number 129. Multiply 129 by 9 , and place the product 1161 under 4800, to which it is then added, giving as a result 5961.
Multiply 5061 by 9 , and place the product 53649 under 58615 and subtract, and to the remainder 4966 annex the next period 327.
Next place the equare of 9 , whioh is 81 , under 5961, add the three numbers connected by the bracket, and to their sum 7203 annex two ciphers.

Then place 3 times 49 (the part of the root already found), wh oh is 147, in column I., in the position indicuted in the solution.

Divide 4966327 by 720300 , and a quotient 6 is obtained. Place 6 in column II. opposite 147.

Read 147--6 as one number 1476. Multiply 1476 by 6 , and add the product 8856 to 720300. Multiply their sum 729156 by 6 , and place their product 4374936 in the position given in the solution, ete.

The attention of the student is directed first to the method of obtain. ing the numbers in column 1 . from those in column II; $12=4 \times 3$; $147=49 \times 3 ; 1488=496 \times 3$; etc.
Second, to the formation of the following numbers :
$1161=129 \times 9 ; 8856=1476 \times 6 ; 119104=14888 \times 8$.
Third, to the formation of trial divisors marked T.D.
Fourth, to the formation of complete divisors marked O.D.
Notes 1.-If there is a remuinder after the root of the last period is found, annex periods of ciphers, and proceed as before. The root figures thus obtained will be decimals.
2. If a trial divisor is not contained in the dividend, put a cipher in the root, tuo ciphers on the right of the divisor, and bring down the next period.
8. If the product of the divisor completed into the figure last placed in the root exceeds the dividend, the root figure is too large. Sometimes the remainder is larger than the divisor completed; but it does not necessarily follow that the root figure is too small.
603. To extract the cube root of a decimal.

RULE.
Begin at the units' place, and proceed both toward the left and right to separate into periods of three figures each, then extract the root us in whole numbers.
Note.-The left hand period in whole numbers may have but one or two fignres, but in decimals each period must have three figures. Hence, ciphers mast be annexed to the right of the decimal to complete the periods, when necessary.

## 604. To extract the cube root of a fraction.

role.
Reduce the fraction to its lowest terms, then extract the root of its numerator and denominator.
Notes 1. When the denominator is not a perfect cube, the fraction should be reduced to a decimal, and the root of the dscimal found as above.
2. A mixed numeter should bo reảuced to an improper fraction.

## CUBE RUOT

add the proI place their d of obtain. $1=4 \times 3$;
8.
st period is root figures
pher in the on the next
st placed in aetimes the necessarily
: the left ach, then nplete the
the root
fraction found as

Find the cube root of -

1. 6859. 
1. 12167. 
1. 27000. 
1. 2406104. 
1. 3869898. 
1. 5545233 .
2. 49027896. 
1. 66430125. 
1. 929714176. 

Extract the cube root of -
10. 1412467848.
11. 1865409391.
12. 3341362375.
14. 3616805375.
18. 2857243059.

Find the cube root of-
16. 830.584 .
18. 1.092727.
17. . 970299.
19. .002197.
20. . 000175616.
21. . 007645373.

Find the cube root of the following numbers carrying incomplete roots to three or five decimal places, as may be required :
22. 1
24. . 01 .
$\begin{array}{ll}26 . & .001 . \\ 27 & .002 .\end{array}$
28. 2.
25.
393.
31. $\ddagger$.

## PRACTICAL MENSURATION.

605. Mensuration treats of the measurement of lines, surfiaces and solids.
606. Lines are measured by expressing their length in inches, feet, yards, etc. (Linear Measure), or in links, chains, etc. (Surveyors' Measure.)
607. A Surface is that which has length and breadth only.
608. Surfaces ace measured by expressing the number of times they wothin the units of surface measure, i.e., the sq. inch, sq. yard, etc. (Square Measure), or the sq. link, Fy. chain (Surveyors' Square Measure).
609. If a straight edge laid anywhere upon a suriace touches at every point, the surface is a plane surface.
610. A Polygon is a plane surface bouniled by straight lines.
611. The Area of a plane surface is the spacs enclosed by the lines which bomnd it
612. A polygon takes its name from the number of sides which bound it, thus :


Tringle. quadrilateraí. Pentagon.


Hexagon.


Heptagon.


## QUADRILATERALS.

613. A Right Angle is an angle furwed by two lines perpendicular to each other.
614. Parallel Lines are lines in the same plane, which being produced both ways never meet, and which are therefore the same distance apart throughout their entire length.
615. Quadrilaterals are of three kinds, as follows:



Trapezoid.


Trapezium.
616. A Parallelogram has its opposite sides parallel; a Trapezoid has only two sides parallel; a Trapezium has has no two sides parallel.
617. Parallelograins are of four kinds, as follows:


8quare.


Rectangle.


Rhomboid.


Rhombus.

61\%. A Square has all its sides equal and all its angles right angles; a Rectangle has its opposite sides equal, and all its angles right angles; a Rhomboid has its opposite sides equal, and none of its angles right angles; a Rhombus has all its sides equal and none of its angles right angles.


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619. The Altitude of a parallelogram or trapezoid is the perpendicular distance between the parallel sides.
620. The Diagonal of a quadrilateral is a straight line joining two opposite corners.

## 621. To find the area of a rectangle or square.

Example 1.-Find the area of the rectangle whose sides are 3 inches and 5 inches in length.

Soldtion.
5 sq . in. $\times 3=15 \mathrm{sq}$ in. Ans.


## Explanation.

In the figure $A B C D$, let $A B$ be 5 inches, and A D be 3 inches. Let A B be divided into 5 equal parts, each 1 inch in length, and let A.D be divided into 3 equal divisions each, 1 inch in length. Draw through these divisions the lines represented in the figure. The whole figure will then be divided into squares, each of whose sides is 1 inch in length, and hence each square is a square inch. In each horizontal row there are 5 square inches, and in the three horizontal rows there will be 3 times 5 square inches, or 15 square inches, and hence the solution, 5 sq . in. $\times 3=15 \mathrm{sq}$. in.

Example 2.-Find the area of a square whose side is 8 inches.

Solution.
8 sq. in $\times 8=64 \mathrm{sq}$. in. Ans.

## Explanation.

Same as Eifample 1.
rolm.
Multiply the length by the breadth and the result will be the area.

Notes 1.-The student.will observe that the rule is only a shortened form of expressing the longer rule. Multiply the measure of the length expressed in units of square messure by the measure of the breadth.
2. All the following rules will be exprasped in a shortened form.

The convarie of the preceding rule must he, true:
If the area, of a reotangle be divided by aside, the quotient will be the other side, ar if the square soot of the area of a square be extracted, the result will be the length af $a_{i}$ side.
622. To find the area of a rhomboid or rhombus, the length of a pair of opposite sides and the perpendicular distance between them being given.

Example.-Find the area of a rhomboid, one peir of whose opposite sides are 10 feet in length, and the distance between them 6 feet. Solution.

Explanation
A. Ans. It is proved in Euclid, Book I, pro. lelogram is equal to the area of a position 35 , that the area of a paral. same altitude, and hence the solution given.

## bule.

Multiply the length of one of the parallel sides by the per. pendicular distance between them.
623. To find the area of a trapezoid, the lengths of the parallel sides and the perpendicular distance between them being given.

Example.-Find the area of a trapezoid, the lengthe of the parallel sides being 6 feet and 10 feet, and the perpendicular distance between Solution. is 8 inches.
$\triangle$ tion.
kample 1.
lt will be the
ly a shortened : of the length to preadth. form.
rue:
the quotient $e$ area of a $f a, s i d e$.

## TRIANGLES.

624. A Triangle is the space enclosed by three straight lines.
625. Triangles are named according to their sides, and also according to their angles, as follows:

626. An Equilateral Triangle has its three sides equal.
627. An Isosceles Triangle has only two sides equal.
628. A Scalene Triangle has all of its sides unequal.
629. A Right Angled Triangle has une of its angles a right angle.
630. The Base of a triangle is any side or a triangle upon which a perpendicular is let fall from the opposite angle.
631. The Altitude of a triangle is the length of the perpendicular let fall from an angle on the opposite side or the
 opposite side produced.


Nots.-Dotted lines represent the altitude.
632. To find the area of a triangle.

Examply 1.-Find the are ria triangle whose base is 16 feet, and whose altitude is 9 feet.

## Solution.

( $18 \mathrm{sq} . \mathrm{ft}+2$ ) $\times 9=72 \mathrm{sq} . \mathrm{ft}$.
it is proved in Euclid, Book I, proposition 41, that the area of a at frengle is half the area of a parallelogram on the same base and of the same altitude, hence the solution givan.
ane-half the base by the altitude.
The fnllowing rule is also necessary when three sides are given.
rule.
From half the sum of the sides subtract each side separatoly; then multiply half the sum and the three remainders together, and extract the square root of the product.

Example 2.-What is the area of a triangle whose sides are $1:$ feet, 16 feet, and 18 feet?

Solution.

$$
\begin{aligned}
& (12+16+18) \div 2=23 \\
& 23-18=5 \\
& 23-16=7 \\
& 23-12=11
\end{aligned} \quad 23 \times 5 \times 7 \times 11=8,855
$$

4333. It is proved in Eaclid, Book I, proposition 47, that in any right angled triangle the area of the square described on the side opposite the right angle, is equal to the sum of the reas of the squares described on the sides containing the , ight angle.

In the accompanying figure, if A B C be a triangle having a right wngle at $C$, the area of the square described on $A B$ is equal to the sum of the areas of the squares described on A C and BC.


A B, the side opposite the right angle, is called the hypothenuse ; B C the bass; and A C the perpendicular. Hence, the square on the hypothenuse $=$ square on the hase + the square on the perpendicular.

Example 1.-If the base of a right angled triangle be $\varepsilon$ s $38 t$, and the perpendioular be 6 feet, what is the length of the hypothenmen $\cdot$

Solution.
In the preceding figure,

$$
\begin{aligned}
\text { sq. on A B } & =8 \times 8+6 \times 6 \\
& =100 \mathrm{sq} . \mathrm{ft} . \\
\therefore \text { A B } & =\sqrt{100}=10 \mathrm{ft} .
\end{aligned}
$$

Example 2.-The hypothenuse of a right angled triangle is 35 faet and the perpendicular is 28 feet, find the base.

Solution.
$35 \times 35=$ sq. on the base $+28 \times 28$
$\therefore$ sq. on the base $=35 \times 35-28 \times 28=441$
$\therefore$ the base $\quad=\sqrt{441}=21 \mathrm{ft}$. Ans.

## 634. To find the area of a trapezium.

A trapezium may be divided into two triangles dy joining two opposite corners, and hence it is only necessary to find the areas of the two triangles and to take their sum.

Example.-Find the area of a trapezinm whose sides are 10 feet, 11 feet, 12 feet, and 15 feet, the length of the line joining opposivo ourners being 18 feet.

Soldtion.


# pOLYGONS. 

## POLYGONS.

## To find the area of a regular polygon containing

Role.
Multiply the perimeter (sum of all the sides) of the base by one-half the peryendicular distance from the centre to one of the sides.

Example.-What is the area of a hexagon, side 8 feet, the perpen. dioular distance from the centre to one of the sides being $6.928+$ feet.

Solution.
Perimeter $=8 \mathrm{ft} . \times 6=48 \mathrm{ft}$
Area $=48 \mathrm{sqft} \times \frac{6.928}{2}=166.272+$ sq. ft.
The area of an equilateral triangle equals the square of a side multiplied by 438 , and the area of a heragon, which is made up of 6 equilateral triangles, is therefore $6 \times .483$ umes the square of a side.

## THE CIRCI.E.

636. A Circle is a plane figure bounded by a curve line called the circumference, every point of which is equally distant from a point called the centre.
637. The Diameter of a circle is a line drawn through the centre, and terminated at both ends by the circumference.
638. A Radius is a straight line drawn from the centre to the circumference and is equal to half the diameter.

Note.-From the definition of a circle, it follows that all the radii are equal ; also, that all diameters are equal.
639. Prindiples. 1. The circumference $=$ the : iameter 3.1416 nearly.
2. Therefore the diameter $=$ the encunterence +31416 nearly.
3. The area of a circle $=$ the square of the radius $\times 3.1416$ nearly.
4. The area of $a$ circle $=$ the circumference $\times$ half the radius.
5. Therefore the radius of a circle $=8 q$. root of (the area $\div 3.1416$ ) nearly.

Note.-The fraotion 37 is commonly used in place of the decimal 3.1416, and is near enough for common practical operations, and will be used in this work.

Example 1. What is the circumference of a circle whose radius is Scletion.
$7 \mathrm{ft} . \times 2=14 \mathrm{ft}$. diameter, $14 \mathrm{ft} . \times 3 \frac{1}{7}=44 \mathrm{ft}$. Ans. (Prin. 1.)

Example 2. The circumference of a circle is 176 feet. What is the diamotar ?

Solution.

$$
176 \div 3 \frac{1}{7}=56 \mathrm{ft} . \quad \text { Ans. (Prin. 2.) }
$$

Example 3. What is the area of a circle whose diameter is 14 fect $\%$
Solution 1.
$14 \mathrm{ft} . \div 2=7 \mathrm{ft}$. the rading, $7 \times 7 \times 3\}=154$ sq. ft. Ans. (Prin. 3.)

Solution 2.
$14 \mathrm{ft} . \times 34=44 \mathrm{ft}$. the circumference. (Prin. 1.) $14 \mathrm{ft} . \div 2=7 \mathrm{ft}$. the radius.

$$
44 \times \frac{7}{2}=154 \text { sq. ft. Ans. (Prin. 4.) }
$$

Example 4. The area of a circle is 616 square feet. Find the rud'ts, fiameter, and circumference.

Solution.
Radins $=\sqrt{616 \div 3 \frac{1}{4}}=14 \mathrm{ft} . \quad$ (Prin. $\dot{\text { e.; }}$ $14 \mathrm{ft} . \times 2=28 \mathrm{ft}$. the diameter.


## MISCELLANEOUS.

## EXERCISE 125.

1. How may acres in a piece of woodland 220 yards in length and 40 rods in width?
2. How many square miles in a township 5 miles and 40 chains square?
3. How many square feet in a floor 20 feet long and 5 yards wide?
4. Find the surface of a pane of glass measuring $37 \frac{1}{2}$ inches long and 23 inches wide.
5. How many square yards in the four walls of a room 15 ft .6 in . high and 80 feet in compass?
6. A rectangular pavement, 50 ft .9 in . long and 12 ft. 6 in . wide, was laid with a central line of stone 5 feet wiừ at $\$ 1.75$ a running foot; the sides were flanked with brick at 80 cents per square yard. What did the paving cost?
7. How many square feet in a surface 24 feet long 20 feet wide? How many in another surface of half these dimensions?
8. Two fields contain 10 acres each ; one is in the form of a square, the other is 4 times as long as it is wide. What would be the difference in expense of fencing them at $\$ 2.25$ per rod ?
9. If the fence were built $4 \frac{1}{2}$ feet high, of boards 8 inches wide, the lower one raised 2 inches above the ground, and a space of 8 inches between the boards, how many square feet of boards would be required for both fields?
10. How many more for one than for the other?

11 A piece of land containing 2 acres is 5 times as long as it is broad. What is its length and breadth?
12. How many bricks 8 inches long and 4 inches wide will pave a yard that is 100 feet by 50 ?
18. What will it cost to pave a roadway 80 feet long and 15 feet wide, at $\$ 1.50$ per square yard ?
14. I have a box without a lid; it is 5 feet long, 4 feet wide, and 3 feet deep, interior dimensions. How many square feet of zine will it take to line the bottom and sides of the box ?
15. Find the area of a rhomboid whose length is 1 yd . 3 ft .6 in ., and whose width is 2 ft .3 in .
16. The base of a rhombus is 10 ft .6 in., and its altitude 8 feet. What is its area?
17. How many acres in a piece of land in :he form of a rhomboid, the base being 8.75 ch . and altitude 6 ch ?
18. A man bought a farm 108 , rods long and 150 rods wide, and agreed to give $\$ 32$ an acre. What did the farm cost?
19. A certain rectangular piece of land measures 1,000 links by 100. How many acres does it contain?
20. How many square feet in a board 16 feet long 18 inches wide at one end and 25 inches wide at the other end ?
21. Required the area of a trapezoid whose parallel sides are 178 and 146 feet, and the altitude 69 feet.
22. One side of $n$ quadrilateral field measures 38 rods; the side opposite and parallel to it measures 26 rods, and the distance between the two sides is 10 rods. Find the area.
23. The parallel sides of a trapezoid measure respectively $8 \frac{1}{3}$ feet and 6 inches; the perpendicular distance between them is 2 feet. Wbat is the area?
24. Find the area of a trapezium whose diagonal is 168 , and one perpendicular 42, the other $E 6$.
25. Find the area of a trapezium whose diagonal is 35 ft .6 in ., and the perpendiculars to this diagonal 9 feet and $12 \frac{1}{2}$ feet.
26. How many acres in a quadrilateral field whose diagonal is 30 rods, and the perpendiculars to this diagonal 20.453 and 50.832 rods. ?
27. What is the base of a triangle whose area is 156 square feet, and its altitude 12 feet?
28. What is the base of a triangle whose area is 144 acres and its altitude 60 rods?
29. Find the base of a triangle whose area is 5,280 square yards, and altitude 240 yards.
30. What is the area of a triangle whose three sides are 18,14 , and 15 feet?
81. What is the area in acres of a triangular field whose three sides measure 1 uspectively 47,58 , and 69 rods?
32. What is the area of a triangle whose base is 24 feet and altitude 16 feet?
33. The base of a triangle is 28 inches and the altitude 16 inches; what is the area?
34. A board 16 feet long is 22 inches wide at one end, and tapers to a point; what is the vilue at $4 \frac{1}{2}$ cents a square foot?
35. Find the area of a triangle whose base is 12 ft .6 in . and altitude 6 ft .9 in .
36. Whose base is $\Sigma 5.01$ chains and altitude 18.14 chains.
37. What is the cost of a trinngular piece of land whose base is 15.48 ch . and altitude 9.67 ch . at $\$ 60$ an acre?
38. At $\$ .40$ a square yard, find the cost of paving a triangular court, its base being 105 feet, and its altitude 21 yards?
39. Find the area of a circular pond, its circumference being 200 chains.
40. The distance around a circular park is $1 \frac{1}{2}$ miles. How many acres does it contain?
41. How much land in a circular garden that requires 84 rods of fencing to inclose it:
42. Find the difference in cost at $87 \frac{1}{2}$ cts. per rod between fencing a square field of 10 acres and a rectangular field 32 rods wide of the same area.
43. Draw a square containing 81 square inches; inscribe a circle in this square. What is the superficies of this circle in square inches?
44. A cow is tethered to a post driven in the centre of a lot 100 feet square ; the tether is just long enough for her to reach the fence. How much of the surface of the field is she unable to crop?
45. If the diameter of an iron column is 3 ft .5 in ., what is the circumference? If the girth of a tree is 5 ft .9 in ., what must be its diameter?
46. If the equatorial diameter of the earth is 7,925 miles, how long in miles and rods is the equator?
47. The distance from the centre of the hub of a wheel to the outer edge of the felly is 15 inches. How long must the tire be?
48. Ir the length of an oar from the thole-pin to the end of the blade is 5 feet, how many feet would the end of the blade travel in the water during 6,000 strokes, each describing an arc of $60^{\circ} ?\left(60^{\circ}=\frac{f}{f}\right.$ of the circumference.)
49. If the circumference of a circular pond is 628.318 rods, what part of a mile must I row to pass from shore to shore across the centre of the pond?
50. If a horse is tethered to the middle post of a fence, from which he can graze out into the field in a curved line $78.5393 i 4$ feet long, how long is the tether?
51. What will be the circumference of the largest nircle that can be drawn on a sheet of paper 12 inches wide and 18 inches long?
b of a wheel w long must $n$ to the end e end of the rokes, each sumference.)
d is 628.818 om shore to
; of a fence, curved line
argest sircle es wide and

## SOLIDS.

640. A Solid is that which has length, breadth, and thickness.
641. A Prism is a solid whose bases are similar, equal, and parallel polygons, and whose side~ qre parallelograms.
642. Prisms take their names from the forms of their bases, as triangular, rectangular, pentagonal, hexagonal, etc.
643. A Cube is a rectangular prism whose faces are all equal squares.
644. A Cylinder is a circular body of uniform diameter whose ends are equal and parallel circles.
645. The Altitude of a prism or cylinder is the perpendicular distance between its bases.


Triangular prlsm.
Rectangular prism.


Pentagonal prism. Hexagonal prism.

646. To find the convex surface of a prism or cylinder.

Suppose a block of the shape of one of the preceding prisms to have been fitied with a piece of paper so as to exactly cover its convex surface. Now if the paper be unrolled it will be found to be the shape of a rect. angle, one side being equal to the height, and the other side equal to the perimeter of the base. Hence, the following rule.

ROLE.

1. Multiply the perimeter (sum of all the sides) of th, base by the altitude.
2. To find the entire surface, ada the areas of the base: to the convex surface.

Example 1. Find the convex surfaoe and also the entire surface of a reotangular prism whose ends are .5 inches by 7 inches, and whose altitude is 12 inches.

## Solution

Perimeter of the base $=(5+7+5+7) \mathrm{in} .=24 \mathrm{in}$.

$$
\text { Altitude }=12 \mathrm{in} .
$$

$\therefore$ Convex surface $=24 \mathrm{sq}$. in. $\times 12=288 \mathrm{sq} . \mathrm{in}$. Again, area of base $=7 \mathrm{sq}$. in. $\times 5=35 \mathrm{sq}$. in.
$\therefore$ Entire surface $=35 \mathrm{sq} . \mathrm{in} .+35 \mathrm{sq} . \mathrm{in} .+238 \mathrm{sq} . \mathrm{in} .=358 \mathrm{sq} . \mathrm{in}$
Example 2. Find entire surface of a cylinder the diameter of whose base is 14 inches, and whose altitude is 20 inches.

Solution.
Perimeter of base $=14 \times 34=44 \mathrm{in}$.
$\therefore$ Convex surface $=44 \mathrm{sq}$. in $\times 20=880 \mathrm{sq}$. in.
Again, area of base $=7 \times 7 \times 34=154$ sq. in.
$\therefore$ Entire surface $=(154+154+880)$ sq. in. $=1183$ sq. in.

## 647. To find the volume of a prism or cylinder.

ROLE.
Multiply the area of the base by the altitude.
Example. 1. Find the volume of a reotangular prism whose base is 4 inohes by 6 inches, and altitude 10 inches.

## Soldtion.

Area of base $=6 \mathrm{sq}$. in. $\times 4=24 \mathrm{sq}$. in. Volume $=24 \mathrm{oub}$. in. $\times 10=240 \mathrm{cub}$ in.

Explanation.
The base can be divided into 24 squares each side of which is 1 innh . If a piece of the prism 1 inch in thickness be cut off by a plane paraliel to the base it can be divided in 24 small blocks, corresponding to :the 24 squares into which the base can be divided, each of these small blocks will therefore be 1 inch long, 1 inch wide, and 1 inch in thicl ness. Hence the part cut off will contain 24 cubio inohes, 10 such pieces oan be cut off the whole block, and the whole block therefore contains

$$
24 \mathrm{cub} \text { in. } \times 10=240 \mathrm{cub} . \text { in }
$$

Example. 2. What is the volume of a triangular prism rizose base is an equilateral triangle each side 8 inchos, and whose altitule is 12 inches?

## Solution.

$$
\begin{aligned}
& \text { Area of base }=\sqrt{12 \times 4 \times 4 \times 4}=27.712+\mathrm{sq} . \mathrm{in} . \\
& \text { Volume } \\
& =27.712 \text { cub. in. } \times 12=332544 \text { cub. } \mathrm{in} .
\end{aligned}
$$

Example. 3. Find the volume of a cylinder, the diameter of whose base is 14 inches and altitude 20 inches.

Solution.

$$
\begin{array}{ll}
\text { Area of base } & =7 \times 7 \times 37=154 \text { sq. in. } \\
\text { Volume } & =154 \text { cub. in. } \times 20=3080 \text { cub. in. }
\end{array}
$$

648. A Pyramid is a solid whose base is a polygon and whose sides terminate in a point called the vertes.
649. A Cone is a solid which has a circle for its base, and terminates in a point called the vertex.
650. The Altitude of a pyramid or cone is the perpendicular distance from the base to the vertex.
651. The Slant Height of a pyramid is the distance from the vertex to the middle point of any side of the base.
652. A Frustrum of a pyramid or cone is the part which is left after the top is cut off by a plane parallel to the base.

6i33. The Altitude of a frustrum is the perpendicular distance between its ends.
654. The Slant Height of a frustrum of a pyramid is the distance between the middle points of two parallel sides of one of its faces.


Pyramid.


Cone.
 Frustrum of a pyramid. Frustrum of a cone.
655. To find the convex surface of a pyramid or cone.
ROLE.

1. Multiply the perimeter by one-half the slant height.
2. To find the entire surface add the area of the base to the area of the convex surface.

Example 1.-Find the entire surface of a pyramid whose base is a square side 6 inches, and whose slant height is 10 inches.

Soletion.
Perimeter of base $=16 \mathrm{in}$. Convex surface $=16 \mathrm{sq}$. in. $\times \frac{10}{2}=80 \mathrm{sq}$. in. Area of base $\quad=4 \mathrm{sq} . \mathrm{in} . \times 4=16 \mathrm{sq} . \mathrm{in}$.
$\therefore$ Entire surface $=(80+16)$ sq. in. $=96 \mathrm{sq} . \mathrm{in}$.
Example 2.-Find entire surface of a cone, the diameter of the base being 14 inches, and slant height 30 inches.

Soldtion.
Perimeter of base $=14 \mathrm{in} . \times 3 \frac{7}{7}=44 \mathrm{in}$. Convex surface $=44 \mathrm{sq}$. in $\times \frac{\mathrm{s}_{2}^{2}}{2}=660 \mathrm{sq}$. in. Area of base $=7 \times 7 \times 3 \frac{1}{7}=154 \mathrm{sq}$. in. Entire surface $=(660+154)$ sq. in. $=814$ sq. in.
656. To find convex surface of a frustrum of a cone or pyramid.

## ROLE.

1. Multiply one-half the sum of the perimeters of the ends by the slant height.
2. To find the entire surface, add the areas of the endo to the area of the convex surface.

SOLIDS.

Example.-Find entire surface of the frustrum of a cone, the diameters of whose ends are 7 inches and 14 inches, and whose slant height is 20 inches.

## Solution.

Perimeter of ends $=7 \mathrm{in} . \times 37=22 \mathrm{in}$., and $14 \mathrm{in} . \times 37=44 \mathrm{in}$.
Convex surface $=\left(\frac{44+22}{2}\right)$ sq. in. $\times 20=660$ sq. in.
Area of smallerend $=\frac{7}{7} \times \frac{7}{3} \times 3 \frac{1}{7}=38 \frac{1}{2} \mathrm{sq}$, in.
Area of larger end $=7 \times 7 \times 31=154 \mathrm{sq}$. in.
Entire surface $=\left(660+38 \frac{1}{2}+154\right)$ sq. in. $=852 \frac{1}{2}$ sq. in.
657. To find the volume of a cone or pyramid.

RULE.
Multiply area of the base by one-third the altitude.
Example.-Find volume of a cone, whose base is 14 inches in diameter, and whose altitude is 24 inches.

Solution.
$7 \times 7 \times 3 \frac{1}{4}$ (area of base) $\times \frac{24}{3}=1232 \mathrm{cub}$. in
658. To find the volume of the frustrum of a cone or nyramid.
$(A+a+\sqrt{A \times a}) \times h \times \frac{1}{3}$, where ' $A$ ' stands for the area of the larger end, ' $a$ ' for the area of the smalle, end, and ' $h$ ' for the perpendicular height.

Example.-Find the volume of the frustrum of a cone, whose end diameters are 7 feet and 14 feet, and whose altitude is 12 feet.

## Solution.

659. A Sphere or Globe is a solid terminated by a curve surface, every part of which is equally distant from a point within, called the centre.
660. The Diameter of a sphere is a straight line drawn through its centre and terminated at both ends by the sur-
 face
661. A Hemisphere is one-half a sphere.
662. The Radius of a sphere is a straight line drawn from its centre to any point in its surface.
663. To find the surface of a sphere.

ROLE.
Multiply the square of the diameter by $3 \frac{1}{7}$.
Example.-What is the surface of a sphere whose diamates is 7 feet?

Solution.
$7 \times 7 \times 34=154$ sq. ft. Ans.
664. To find the volume of a sphere.

HULE.
Multiply the cube of the diameter by $3 \frac{1}{7}$, and divide the result ly 6.
Exaynre.-What is the volume of a sphere whose diameter 18 Solution.
$7 \times 7 \times 7$ (oube of the diameter) $\times 27 \times 1=179$ cub ft.

## CISTERNS AND BINS.

665. To find the number of gallons in a cistern.

Find the volume in cubic inches and divide the result by 231. Norn.-There are 231 cabio inohes in one gallon.

Erumpus.-Find the number of gallons in a reotangular oistern, ofeot by 6 feet, and 3 feet deep.

Solution.
Volume $=(8 \times 6 \times 3)$ oub. ft. $=(8 \times 6 \times 8) \times 1728$ cub. in.
$\therefore$ No. gallons $=8 \times 6 \times 3 \times 1,728 \div 231=1,07714 \mathrm{gal}$.
666. To find the number of bushels of wheat in a bin or pile.

## nole.

Find the volume in cubic inches and divide the result by 8150.48.

Nots,-There are 2160.42 oubio inches in one bushel.
Exumpin,-How many bushels of grain in a bin 4 feet by 6 feet, Soldition.
Volume $=4 \times 6 \times 8 \times 1,728 \mathrm{oub}$ in.
$\therefore$ No. bushels $=4 \times 6 \times 4 \times 1,728 \div 2150.42=58$ bush. nearly.

## GAUGING OF CASKS.

667. Gaugnis is the process of finding the capacity or volume of casks and other vessels.

Nore.-A cask is equivalent to a oylinder, having the same length ind a diameter equal to the mean diameter of the cusk.

## 668. To find the mean diameter of a cask (nearly).

 RULE.Add to the head diameter $\frac{2}{3}$, or, if the staves are but little curced, $\frac{8}{b}$ of the difference between the head and bung diameters.
669. To find the volume of the cask in galions.

RULE.
Multiply the square of the mean diameter by the length (bnth in inches), and this product by .0034 .

Example.-How many gallons in a cask whose head diameter is 34 inches, bung diameter 30 inches, and length 34 inches?

Solution.
Mean diameter $=\{24+(30-24) \times 3\}=28 \mathrm{in}$. Capacity $\quad=28 \times 24 \times 34 \times .0034=90.63$ gal.

## EXERCISE 126.

1. What is the solidity of a triangular prism whose length is 12 feet, and one of the equal sides of one of its equilateral ends is $\mathbf{3}$ feet?
2. How many gallons of water would a cylindrical boiles sontain if 25 inches high and 12 inches in diameter?
3. Find the cubic inches in the largest cone that can be cut from a cylinder 2 ft .6 in . high and 14 inches in diameter.
4. A sphere 8 inches in diameter is placed in a cubical box whose interior dimensions are 8 inches. How much vacant space is left?
5. I have a cylindrical tank which contains 160 gallons; it is 6 ft .5 in . in diameter. How deep is it?
6. How many square feet of canvas will be required to cover a cylinder $16 \frac{1}{2}$ feet in circumference and 25 feet long?
7. How many square inches of surface in a stove pipe 22 inches in circumference and 12 feet long?
8. What is the convex surface of a $\log 25$ feet in circum. ference and 18 feet long?
9. What is the convex surface of a cylinder 3 feet long and $1 \frac{1}{2}$ feet in diameter? What is its entire surface?
10. What are the contents of a $\log 15$ feet long and 2 feet in diameter?
11. The standard liquid gallon is 281 cubic inches; how many gallons in a can 22 inches in diameter and 3 feet high ?
12. How many cubic feet in a triangular prism, the area of whose base is 920 square feet and height 20 feet?
13. What are the contents of a quadrangular prism whose length is 25 centimeters, and the base a rectangle 3 by 5 centimeters?
14. What is the lateral surface of a regular pyramid whose slant height is 15 feet, and whose base is 30 feet square?
15. What is the surface of a pyramid whose base is an equilateral triangle measuring 4 feet on each side, and slant height 16 feet?
16. What is the convex surface of a cone, the diameter of whose base is 7 feet and its slunt height 12 feet ?
17. What is the entire surface of a triangular pyramid whose slant height is 25 feet, and each side of the base 10 feet?
18. What is the entire surface of a right cone, the diameter of the base and the slant height being each 40 feet ?
19. Find the cubic feet in a $\log 30$ feet long and 2 feet in diameter at the larger and 1 ft .10 in . at the smaller end.
20. Find the cubic contents of a pyramid, base 300 feet square, and altitude 80 feet.
21. How many cubic feet in a circular mound 48 feet high, and having a diameter of 86 feet at the top, and a circumference of 471.24 feet at the bottom?
22. How many cubic miles in the earth, supposing it to be a perfect sphere 8,000 miles in diameter?
23. How many barrels of oil in a tank 60 feet in diameter if the oil is 5 feet deep. $?$ ( 40 gal . to the barrel.)
24. A monument in the form of a square pyramid, is 2 ft. 10 in. square at base, and 11 feet high; at 175 pounds to a cubic foot what is its weight?
25. What are the contents of a round log whose length is 20 feet, diameter of larger end 12 inches, and smaller end 6 inches?
26. The altitude of a frustrum of a pyramid is 27 feet, the ends are 4 feet and 3 feet square; what is its solidity?
27. What are the contents of a pyramid whose base is 144 square feet, and its altitude 33 feet?
28. Find the solidity of a sphere whose diameter is 12 inches.
29. What are the contents of a cone the area of whose base is $1,865 \mathrm{sq}$. feet, and its altitude 36 feet ?
30. Find the convex surface of a frustrum of a cone whose slant height is 15 feet, the circumference of the lower base 30 feet, and of the upper base 16 feet.
31. What will it cost to gild a ball 12 inches in diameter, at 10 cents a square inch ?
32. The standard bushel of the United States is $18 \frac{1}{2}$ inches in diameter and 8 inches deep; how many cubic inches does it contain?
33. How many square yards in the convex surface of a frustrum of a pyramid, whose bases are heptagons, each side of the lower base being 8 feet, and of the upper base 4 feet, and the slant height 55 feet?
34. Find the contents in gallons of a cask whose length is 54 inches, its bung diameter 42, and head diameter 36 inches.
35. Required the contents in gallons of a rectangular cistern $4 \frac{1}{3}$ feet long, $3 \frac{1}{4}$ feet wide, and 6 feet deep.
36. What are the contents in gallons of a cask 36 inches long, its bead diameter 26 inches, and bung diameter 32 inches?
37. How many gallons in a cask whose head diameter is 24 inches, bung diameter 30 inches, and its length 34 inches?
38. What is the volume of a cask whose length is 40 inches the diameters 21 and 30 in . respectively?
39. How many gallons in a cask of slight curvature, 8 ft. 6 in. long, the head diameter being 26 inches, the bung diameter 81 inches?

## MEASUREMENT OF CARPETING.

6ifo. Carpet is sold by the linear yard, and is of various widths. The more common widths are 27 inches and 36 inches.
671. In determining the number of yards of carpet that will be required to cover a room, it is first necessary to decide whether the strips of carpeting shall run lengthwise of the room or crosswise. Economy in matching usually decides this.
672. In determining the length of each strip of carpet, allowance must be made for waste in matching.

## 673. To find the number of yards of carpeting required for a room of given dimensions.

Examplar 1.- How many yards of carpet 27 inches wide will be required for a reotangular room 21 feet long and 18 feet wide, if the strips run lengthwise and no waste in matching?

Solotion.
$18 \mathrm{ft} .=216 \mathrm{in}$.
$216 \div 27=8$, No. strips of carpet. 1 strip is 21 ft . or 7 yds . long. 8 strips are, 7 yds $\times 8=56$ yds. Ans.

Example 2.-How many yards of carpet 36 inches wide will be required for a rectangular room 20 feet 6 inches long, and 16 feet 9 inches wide, if the strips run crosswise, and 4 inches per strip be allowed for matching?

## Solution.

$16 \mathrm{ft} .9 \mathrm{iz}, \cdots$ ? 9 in.
$201 \mathrm{in} . \div 5 \mathrm{i}$, $\quad$, imes and 21 in remaining.
$\therefore$ It will take ástast ne agyot.
Length of earbl at: $=20 \mathrm{ft} .6 \mathrm{in} .: 4 \mathrm{in} .=0 \mathrm{ft} 10 \mathrm{in}$.
1 strip is 20 it. 10 is lung.
$\therefore 6$ strips are, $20 \mathrm{ft} .10 \mathrm{in} . \times 6=125 \mathrm{ft}$. or 412 yds . Ans.

## EXERCISE 127.

NG.
s of various nches and
carpet that ecessary to lengthwise ng usually
of carpet, carpeting
wide will be $e$, if the strips
11. Find the cost of carpeting a room 28 ft .10 in . long, by 17 ft .8 in . wide, with carpet $\frac{3}{4}$ of a yard wide, at $\$ 1.80$ per yard, if the strips run lengthwise of the room, and 9 inches per strip be wasted in matching.
12. Find the cost of the carpet for a stair of $\mathbf{1 7 - 1 2}$ inch steps, each rising 8 inches, at 90 cents a yard.
13. Find the cost of the stair carpet at $\$ 1.20$ a yard, for a flight of stairs of 22 steps, 11 inches wide, with 7 in 'les rise, allowing 1 yard extra at the top.
14. Find the cost of covering the floor of a hall 24 feet long by 8 feet wide, with oil-cloth 4 feet wide, no waste in matching. le, at \$1.80 room, and

## MEASUREMENT OF WALL PAPER.

674. Wall paper is sold by the roll, any part of a roll being countsd as a whole roll.
675. Canadian and American wall papers are 18 inches wide, and have 8 yards in a roll. For convenience wall paper is done up in double rolls of 16 yards.
676. In estimating the number of rolls necessary for a certain room, paper-hangers ascertain the height of the room and its perimeter, making an allowance in the perimeter of 3 feet for each door or window.

67\%. The exact cost of papering a room can be ascertained only by taking account of the number of rolls of paper actually used in doing the work.
678. To find ate number of rolls of paper required for a room.

Example 1:-How many rolls of wall paper will be required for the walls of a rectangular room 20 feet by 16 feet, with a 12 foot ceiling, there being one door 3 feet 8 inches vide, and 2 windows each 4 feet 2 inches wide?

Solution.
Perimeter of room is $\quad(20 \mathrm{ft} .+16 \mathrm{ft}) \times 2=72 \mathrm{ft}$. Width of door, $\quad 3 \mathrm{ft} .8 \mathrm{in}$.
Width of 2 windows ( 4 ft .2 in .) $\times 2=8 \mathrm{ft} .4 \mathrm{in} . \quad 12 \mathrm{ft}$.
Perimeter after dednoting width of door and windows $=60 \mathrm{ft}$.
$60 \mathrm{ft} .=720$ inches.
$790 \mathrm{in} . \div 18 \mathrm{in}$. (width of paper) $=40$, number of strips.
1 strip is 12 ft . long.
$\therefore \quad 40$ strips are 480 ft . or 160 yds . long.
160 yards $\div 8 \mathrm{yds}$. (No. yds. in a roll) $=20$, No. of rolls. Ans.

Example 2.-Find the cost of the wall paper at 80 cents a roll and bordering at 7 cents a yard for a room 18 feet 9 inches long by 16 feet 5 inches wide, with the ceiling 10 feet 9 inches above the base boards, allowing for 2 doors each 3 feet 8 inches wide, and 3 windows each 3 feet 6 inches wide, also an allowance of 9 inches on each strip for matching. (In reckoning the cost of the bordering no allowance is made for the doors and windows.)

## Solution.

Perimeter of room is ( $18 \mathrm{ft} .9 \mathrm{in} .+16 \mathrm{ft} 5 \mathrm{in}.) \times 2=70 \mathrm{ft} .4 \mathrm{in}$.
Width of doors $\quad(3 \mathrm{ft} .8 \mathrm{in}$.) $\times 2=7 \mathrm{ft} .4 \mathrm{in}$.
Width of windows ( $3 \mathrm{ft}$.6 in .) $\times 3=10 \mathrm{ft} 6 in .17 ft .10 in.$.
Perimeter of room after deducting width of doors and windows $=$ 52 ft. 6 in.
$52 \mathrm{ft} .6 \mathrm{in} .=630 \mathrm{in}$.
$630 \mathrm{in} . \div 18 \mathrm{in} .=35$, No. of strips.
To allow for matching, the paper will cut into strips of $(10 \mathrm{ft} .9 \mathrm{in} .+9 \mathrm{in})=.11 \mathrm{ft} .6 \mathrm{in} . \mathrm{in}$ length.

One roll will practically cut into 2 strips.
$\therefore$ No. of rolls $=35 \div 2=17 \frac{1}{2}$
$\therefore$ It will take 18 rolls
1 roll is worth 80 cents
$\therefore 18$ rolls are worth 80 cents $\times 18=\$ 14.40$, Cost of wall paper. $70 \mathrm{ft} .4 \mathrm{in} .=24$ yds. nearly
1 yard is worth 7 cents
$\therefore 24$ yds. are worth 7 cents $\times 24=\$ 1.68$, Wost of border. \$16.08. Total cost.

## EXERCISE 128.

1. How many strips of paper will go around a room 18 feet by 24 feet?
2. How many strips of paper are required for a room 30 feet by 24 , if there are 4 windows and 2 doors? (Art. 676.)
3. How many rolls will paper a ceiling 24 feet by 18 feet?
4. How many double rolls are required for a hall 21 feet long and 13 feet high, with a cornice 1 foot deep?
5. Find the cost of the paper for a room 36 feet by 24 feet and 11 feet high, with a cornice 1 foot deep, and a wainscoting 2 feet deep, at 50 cents per double roll.

80 oents a roll nches long by 16 above the base 2, and 3 windows on each strip for lowance is made
$0 \mathrm{ft} . \quad 4 \mathrm{in}$.
17 ft .10 in. and windows $=$ trips.
of wall paper.
of border. al cost.
d a room 18
a room 30 (Art. 676.) $t$ by 18 feet? hall 21 feet

36 feet by deep, and a roll.
6. How many double rolls of wall paper will be requirerl for a room 18 ft .6 in . by 15 ft .4 in ., the ceiling 8 feet above the base-boards, allowance being made for 1 door 3 ft .8 in . wide and 2 windows each 4 feet wide?
7. If a roll of paper cuts into two strips, and 10 strips be allowed for doors and wimlows, find the cost of papering a room 24 ft .8 in . long by 16 feet wide with paper at 45 cents a roll and bordering at 7 cents a yard, the haiging of the paper costing 15 cents a roll.
8. Find the cost of paper for a hall 72 feet by 44 feet, 14 feet high, below the cornice, allowing for 8 windows each 4 ft . 2 in . wide and 2 doors each 3 ft .8 in . wide, the paper costing 45 cents per double roll.
9. With paper at $12 \frac{1}{2}$ cents per roll, and border at 8 cents a yard, what is the cost of paper and border for a room 24 feet by 20 feet and $12 \frac{1}{2}$ feet high, with cornice 6 inches deep, there being 5 openings of an average width of 3 feet?
10. If the paper-hanger charges $\$ 3$, and the paper costs 80 cents a double roll and the border 4 cents a yard, find the cost of papering a room 18 ft .9 in . long, 16 ft .8 in . wide, with a ceailing 18 ft .6 in . high, allowing for two doors, each 3 ft .9 in . wide, and 3 windows, each 4 ft .2 in . wide ; also for a base-board 18 inches deep.

## MEASUREMENT OF SAW-LOGS.

## 679. TABLE OF LUMBER AND LOG MEASUREMENT.

Showing net proceeds (fraotions of feet omitted) of logs in 1 inch boards, deducting saw kerf and slabs. The length will be found in the left hand column, and the diameter in inches on the heal of the other columns.

|  |  |
| :---: | :---: |
|  | ${ }_{10}^{\text {Ditm. }}$ |
|  | $\underset{11}{\text { Diam. }}$ |
|  | Diam. 12 |
|  | Diam. $13$ |
|  | ${ }_{14}{ }_{14}$ |
|  | Diam. 15 |
|  | $\underset{16}{\text { Diam. }}$ |
|  <br>  | ${ }_{17}^{\text {Diam }}$ |
|  <br>  | $\begin{array}{\|c} \text { Diam. } \\ 18 \\ \hline \end{array}$ |
|  <br>  | $\underset{19}{\text { Diam. }^{2} .}$ |
|  | $\begin{gathered} \text { Diam. } \\ 20 \end{gathered}$ |

s．

LOGS．

ASUREMENT．
of logs in 1 inch ll be found in the head of the other

| っ尸ち $\vdots \vdots$ $\vdots \vdots$ $\vdots$ |  |
| :---: | :---: |
| N0 | Diam． 10 |
| 島这 | $\underset{11}{\text { Diam. }}$ |
| 出念出 | Diam． |
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| ¢9909 | Diam． |
| ¢¢0 | Diam． 15 |
| ＇\％ | Diam． <br> 16 |
|  | Diam． 17 |
| 込盗式 | Diam． 18 |
|  | ${ }_{19}^{\text {Diam. }^{2}}$ |
| 或式家 | $\begin{gathered} \text { Diam. } \\ 20 \end{gathered}$ |

MEASUREMENT OF SAW－LOGS．

6\$0. in some parts of Canada saw-logs are bought and sold by the Standard, in other parts with reference to the number of feet of inch lumber which they will produce.
681. A Standard Log is 12 feet long and 21 inches in diameter, and will produce 1,085 feet of inch lumber.
682. The measurement of a $\log$ is always taken at the small end and betweer the bark.
683. To find the number of standards in a given number of saw-logs.

Exampie 1.-How many standards are there in 4 saw-logs, each 12 feet long, the diameters of which are 16 inches, 20 inohcs, 22 inches and 25 inches respeotively?

Solótion.

$$
\begin{aligned}
16^{2} & =256 \\
21,{ }^{2} & =400 \\
22^{2} & =484 \\
25^{2} & =625 \\
\text { Sum } & =1,765
\end{aligned}
$$

$$
1,765 \div 212=1,765 \div 441=4 . \text { No. standard. Ans. }
$$

Exam"le 2.-How many standards are there in 5 loge, each 16 feet long, the dianeters of which are $18,20,21,24$, and 30 inches respec. tively ?

Soldtion.

| $18^{2}$ | $=324$ |
| ---: | :--- |
| $20^{2}$ | $=400$ |
| $21^{3}$ | $=441$ |
| $24^{2}$ | $=576$ |
| $30^{2}$ | $=900$ |
| Sum | $=9,641$ |

$2,641 \div 441=6$ nearly. No. of standards 12 feet long.
$16=1 \frac{1}{8}$ times 12
$\therefore$ No. of standards $=6 \times 1 \frac{1}{3}=8$. Ans.

## EXERCISE 129.

1. How many standards are there in 6 saw-logs, each 12 feet long, the diametors of which are 12, 16, 20, 25, 26 and 28 inches respectively?
2. How many standards are there in 5 logs, each 18 feet long, the diameters of which are 14, 20, 22, 24 and 30 inches respectively?
3. What is the side of the largest square piece of timber which can be sawn from a log, the diameter of which is 28 inches?
4. From the Table, Art. 679, find out the quantity of inch lumber that can be sawn from the following :

| 3 | logs | 10 | feet long, diameters | 15,20 and 32 | inches respectively. |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2 | $" 1$ | 14 | $"$ | $"$ | 18 and 24 | " | " |
| 4 | $"$ | 16 | $"$ | $"$ | $16,20,22$ and 30 | $"$ | $"$ |
| 2 | $"$ | 18 | $"$ | $"$ | 20 and 26 | $"$ | $"$ |

5. A man wishes a piece of timber 18 inches square, what is the diameter of the smallest log from which it may be rawny

## MEASUREMENT OF LUMBEK.

684. Lumber, as the term is used here, includes alt kinds of sawed boards, plank, scantling, joists, etc.
685. A foot of lumber, or a board foot, is the unit of measurement. It is 1 foot long, 1 foot wide, and 1 inch thick.
686. The term scantling is given to lumber 3 or 4 inches wide, and from 2 to 4 inches thick.

Joist is usually from 2 to 4 inches thick, and from 6 to 16 inches wide.

Lumber heavier than joist or scantling is called timher. A broad piece of lumber thicker than a board,-usually from $1 \frac{1}{2}$ to 4 inches thick, is called a plank.
687. All lumber less than one inch in thickness is considered inch lumber in measuring.
688. In measuring the width of a board a fraction greater than a half inch is called a half, and if less than a half it is rejected. Thus a board $5 \frac{7}{2}$ inches wide would be considered 6 inches wide. a board 93 inches wide would be considered 9 inches was.
689. The price of lumber is usually quoted at a certain rate per thousand feet, board measure.
690. To find the number of board feet or feet of lumber in a board, plank, joist, etc.

Example 1.-Find the number of feet of lumber in a board 14 feet long, 12 inches wide, and 1 inch thick.

Solution.
$(14 \times 12 \times 1) \div 12=14$ feet. Ans.

Example 2.-Find the number of feet of lumber in a plaus 16 feet long, 14 inches wide, and 3 inohes thick.

> Soldtion.
> $(16 \times 14 \times 3) \div 12=56$ feet. Ans.
> nule.

Multiply the length in feet by the width and thickness in inches, and divide the product by 12, and the result will be the number of board feet of lumber.

## EXERCISE 130.

1. Find the number of feet of lumber in 24 boards 14 feet long and 10 inches wide.
2. Find the cost of fifty 2 -inch plank 16 feet long and 10 inches wide at $\$ 18$ per thousand.
3. How many square feet are there in the surface of a board 16 feet by 9 inches?
4. How many feet of lumber are there in a board 12 feet long, 6 inches wide and 1 inch thick ?
5. How many feet of lumber are there in the following bill ?- 24 joists 16 feet by 10 inches, 2 inches thick; 210 pieces of siding, 12 feet long, 4 inches wide, $\frac{3}{4}$ inch thick; 14 beams 20 feet long, and 9 inches square; 16 scantling, 2 inches by 4 inches, 16 feet long.
6. How many feet of lumber in a 140 pieces of siding, each 12 feet long, 6 inches wide, and $\frac{7}{2}$ inch thick?
7. How much lumber is there in eighty $2 \times 4$ scantling 14 feet long?
8. Find the cost of 2,250 feet of lumber at $\$ 20$ per
9. Find the cost of $1 \frac{1}{4}$ inch flooring required to lay a floor 42 feet by 24 feet at $\$ 24$ per thousand.
10. Find the cost of flooring a bridge 320 yards long by 20 feet wide with 3 inch oak planks, at $\$ 22$ per thousand.
11. If $2 \times 4$ studs are used, and they are placed 16 inches apart, from centre to centre, how many feet of lumber are there in the studding of a wall 20 feet long and 12 feet high?
12. How many 12 foot boards 6 inches wide are required to put a wainscoting 3 feet high around a kitchen 12 feet by 16 feet, allowing for 2 doors, each $3 \frac{1}{2}$ feet wide?
13. Find the cost of the lumber for two floors of a house 24 feet long and 18 feet wide, if the lower floor is $1 \frac{1}{2}$ inches thick, and the upper floor 1 inch, at $\$ 20$ a thousand.
14. A bain is 64 feet long and 40 feet wide, and 20 feet high to the eaves; the gables are 8 feet high, and the rafters 22 feet, 6 inches long. Find the number of feet of inch boards necessary to inclose the two sides, allowing for two doors 12 feet by 16 feet.
15. In No. 5 , find the number of feet of lumber in the ends and gables.
16. In No. 5, find the number of feet of lumber required to sheet the roof.
17. In No. 5, find the cost of the lumber for the doors at $\$ 20$ a thousand.
18. In No. 5, find the cost of the 2 inch plank needed for the floor at $\$ 24$ a thousand.
19. If $4 \times 5$ rafters are used, and they are placed 30 inches apart, from centre to centre, how many feet of lumbar are there in the 20 foot rafters of a doubie roof 40 feet long?
20. Find the price of the following bill of lumber at $\$ 24$ per thousand:-

1202 -inch plank 10 inches wide, 14 feet long.
125 boards 10 inches wide, 16 feet long.
$802 \times 4$-inch scantling, 14 feet long.
$503 \times 4$-inch " 12 "
$1203 \times 10$-inch joist, 16 feet long.
21. How many feet of lumber are there in the $2 \times 4$-inch studs of a partition wall 32 feet long and 14 feet high? Note.-The studs of partition walls are usually placed 16 inohes apart
22. How many 12 -foot strips $2 \frac{1}{2}$ inches wide will lay a walk 4 feet wide and 80 yards long, allowing half an inch between the strips?
23. If lumber 10 inches wide is used in sheeting the roof in No. 19, and the boards are placed two inches apart, allowing for a projection of one foot at each end, how many feet of lumber will be required?
24. How many feet of lumber are there in the 12 -inch base board of a square 10 acre field?
25. Find the cost of the lumber for the dressed door facings of 18 doors, each 7 feet high and 2 feet 8 inches wide, the facings being 6 inches wide, at $\$ 30$ per thousand

## MEASUREMENT OF SHINGLING.

691. Shingles are sold by the bunch, each bunch contains a quarter thousand. A bunch of shingles is 20 inches wide, and has 25 courses on each side. Dealers will not sell a part of a bunch.
692. Ordinary shingles have an average width of 4 inches, and are generally laid 4 inches to the weather.
693. Allowing for waste, 1000 shingles will cover a surface of 100 square feet (a square of shingling), 4 inches to the weather ; laid $4 \frac{1}{2}$ inches to the weather, 900 shingles are required.

## EXERCISE 131.

1. How many shingles are there in 24 bunches?
2. How many bunches are there in $15 \frac{1}{2}$ thousand ?
3. How many thousand are there in 48 bunches?
4. Laid 4 inches to the weather, how many square inches are covered by the exposed part of one shingle?
5. How many shingles are required for a roof having a surface of 2,400 square feet?
6. How many bunches of shingles will shingle a roof 32 feet by 24 feet ?
7. How many shingles are required for a double roof 36 feet long, with 20 -foot rafters?
8. Find the cost of laying a double roof 48 feet long, rafters 24 feet long, with shingles 4 inches to the weathry at $\$ 3.20$ per thousand.
9. Find the cost of shingles for a double roof 36 feet long, rafters 21 feet long, at 60 cents a bunch, if the shingles are laid $4 \frac{1}{2}$ inches to the weather.
10. At $\$ 3.60$ per thousand, find the cost of the shingles for a roof of a building 60 feet long, 40 feet wide, having a gable 12 feet high, and the rafters having an 18 -inch heel.

## G.

unch con. 20 inches $s$ will not idth of 4 ther.

## FENCING.

## EXERCISE 132.

1. How many fence posts are required for a fence 80 rods long, if the posts are placed 8 feet apart?
2. How many posts are required for a fence around a field 40 rods square, if they are placed 8 feet apart?
3. How many posts are required for a square 10 -acre $f$ ald, if they are placed 8 feet apart?
4. Find the cost of the posts for a ience around a garaen plot 250 yards by 220 yards, if the posts are placed 6 feet apart and cost 10 cents each.
5. In No. 4, how many $2 \times 4$ scantling, 12 feet long will be required for the 2 stringers of the fence?
6. In No. 8, find the cost of $2 \times 4$ scantling, 16 feet long, that will be required for the 2 stringers of the fence, if the lumber is worth $\$ 18$ par thousand.
7. How many feet on sumver are required for a 10 -inch base board around the field in No. 2?
8. How many 2 -inch pickets are required for a fence 40 rods long, if the pickets are placed 2 inches apart?
9. How many $2 \frac{1}{2}$-inch pickets, nlaced 2 inches apart, are required for a fence around a garden 200 yards by 150 yards?
10. How much lumber is there in a common board fence 40 rods long, consisting of 5 rounds of 6 -inch boards?
11. What will it cost to fence 5 miles of railway, both sides, with 6 rounds of 6 -inch boards, at $\$ 12$ per thousand feet ?
12. What will it cost at $\$ 10$ per thousand to fence a field 40 rods by 60 rods with 1 round of 12 -inch boards, and 5 of 6 -inch boards?
13. What will be the cost per mile to fence a railway with 5 strands of barbed wire, which weighs 1 lb . per rod, at 8 cents a pound ?
14. Find the cost of a quarter mile of fence with the posts 8 feet apart, a 12 -inch base, a $2 \times 4$ rail at top, and 4 strands of barbed wire ; the posts cost 10 cents each, the lumber $\$ 12$ per thousand, and the wire at 7 cents a pound. (A pound stretches $16 \frac{1}{2}$ feet.)

## MEASUREMENT OF PAINTING, KALSOMINING AND PAVING.

694. The unit of measurement of painting, kalsomining, and paving is the square yard.

## EXERCISE 133.

1. How many square yards of painting are there in a floor 30 feet by 28 feet?
2. Find the cost of kalsomining the ceiling of a hall 64 feet long and 36 feet wide, at 20 cents a square yard.
3. What will it cost to paint a close board fence 6 feet high around a lot 36 yards long by 24 yards wide?
4. What will it cost to paint a house 36 feet by 30 feet, which bas an average height of 18 feet, at 18 cents a square yard?
5. What will it cost to kalsomine a room 20 feet by 18 feet and 10 feet high, at 7 cents a square yard?
6. Find the cost of painting a double roof 44 feet long by 24 feet, at 12 cents a square yard.
7. What will it cost to tuckpoint the front of a brick house 36 feet long and 22 feet high, allowing for half the openings which form one quarter of the surface, $\$ 1.25$ per square yard?
8. Find the cost of paving a street half a mile long and 60 feet wide, at 30 cents a square yard.
9. Find the cost of paving a street one-eighth of a mile long and $1 \frac{1}{2}$ chains wide, at 25 cents per square yard.
10. A circular plot of ground, 4 chains in diameter, has a walk 8 feet wide, formed around the outer edge. Find the cost of gravelling the walk, at 15 cents a square yard.

## MEASUREMENT OF LATHING AND PLASTERING.

695. Laths are sold by the bunch. There are 50 laths in a bunch, each lath being 4 feet long and $1 \frac{1}{2}$ inches wide. They are usually laid about three-eights of an inch apart.
696. Allowing for waste, contractors reckon that a bunch of laths will cover 3 square yards of surface.
697. Lathing and plastering are estimated by the square yard. Only one-half the surface of openings is allowed.

## f988. To find the cost of lathing and plastering a room of given dimensions.

Example.-A rectangular room 24 feet by 18 ft .9 in ., and 10 ft . 10 in . high. The base board is 10 inches high; there are two doors 8 feet by 4 ft .3 in . eaoh, and three windows 6 ft .4 in . by 4 feet each. Find the cost of lathing and plastering the walls and oeiling at 30 cents a square yard.

## Solution.

Perimeter of room $\quad=(34 \mathrm{ft} .+18 \mathrm{ft} .9 \mathrm{in}) \times 2=.85 \mathrm{ft} .6 \mathrm{in}$. Height of walls above base board $-10 \mathrm{ft} .10 \mathrm{in} .-10 \mathrm{in} .=10 \mathrm{ft}$.
Area of walls $=85 \mathrm{ft} .6 \mathrm{in} . \times 10 \mathrm{ft} .=\ldots \quad . . .855 \mathrm{sq} . \mathrm{ft}$.
Area of ceiling $=24 \mathrm{ft} . \times 18 \mathrm{ft} .9 \mathrm{in} .=. . \quad . . \quad \underline{450 \mathrm{sq} . \mathrm{ft}}$. Total gross area $=\ldots \quad . . \quad 1,305$ sq. ft .
Area of 2 doors $=(8 \mathrm{ft} . \times 4 \mathrm{ft} .3 \mathrm{in}) \times 2=.68 \mathrm{sq}$. ft .
Area of 3 windows $=(6 \mathrm{ft} .4 \mathrm{in} . \times 4 \mathrm{ft}) \times 3=.76 \mathrm{sq} . \mathrm{ft}$.
Total area of ${ }^{3}$ rors and windows $=144 \mathrm{sq} . \mathrm{ft}$.
Half of 144 sq . ft . is allowed
= .. .. .. .. 72 sq. ft.
Net area to be lathed and plastered $=\ldots . . . . \quad 1,233 \mathrm{sq} . \mathrm{ft}$.

$$
\begin{aligned}
& 1,233 \text { sq. } \mathrm{ft} .=137 \mathrm{sq} . \mathrm{yds.} \\
& \text { 1 sq. yd. is worth } 30 \text { cents. } \\
& 137 \mathrm{sq} . \text { yds. are worth } 30 \text { cents } \times 137=\$ 41.10 \text {. Ans. }
\end{aligned}
$$

## EXERCISE 134.

 ch apart.1. Including one of the spaces between the laths, how many square inches does one lath cover?
2. How many square feet will a bunch of laths cover?
3. How many bunches of laths will be required for a wall 36 feet long and 12 feet high ?
4. How many bunches of laths will be required for the ceiling of a room 32 feet by 28 feet?
5. How many bunches of laths are required for the walls and ceiling of a room 15 feet by 18 feet, and 9 feet high?
6. How many bunches of laths are required for a ball 84 feet long, 52 feet wide, and 24 feet high, allowing for 4 doors and 10 windows, each having an average surface of 32 square feet. Art. 696.
7. At 80 cents a bunch, find the cost of the laths for a roum 20 feet by 24 feet and 15 feet high, there being 8 windows and 2 doors, each 8 feet by 4 feet.
8. At 25 cents a bunch, find the cost of the laths for a room 24 feet by 16 feet and 10 feet high, allowing for a door 8 feet by 8 ft .6 in ., and a window 7 feet by 4 feet.
9. How many square yards of plastering are there in the ceiling of a room 60 teet bv 82 feet?
10. How many square yards of plastering are there in the walls and ceiling of a room $\mathbf{3 6}$ feet by 24 feet and 12 feet high ?
11. Allowing for an 18 -inch base-board, find the number of yards of plastering in a room 36 feet by 30 feet and 14 feet high.
12. Find the cost of plastering the ceiling of a room 36 feet by 32 feet, at 9 cents per square yard.
13. Find the cost of plastering the walls and ceiling of a room 18 feet by 24 feet, 12 feet high, at $12 \frac{1}{2}$ cents a square yard.
14. At 15 cents a square yard, ñod the cost of plastering the walls and ceiling of a room 21 feet long, 14 feet wide, and 12 feet high, with 4 openings, each 8 feet by 4 feet.
15. At $12 \frac{1}{2}$ cents a square yard, find the cost of plastering a room 20 feet by 16 feet and 12 feet high, with an 18 inch wase, and having 4 openings, averaging 32 square feet each.
16. Find the cost of lathing and plastering a room 16 feet by 18 feet and 12 feet high, with laths at 30 cents a bunch, and plastering at 15 cents a square yard.
17. Find the cost of cementing a circular cistern 8 feet in diameter and 9 feet high, at 8 cents per square foot.

## MEASUREMENT OF STONE-WORK.

699. A cord of stone is of the same size as a cord of wood. In estimating stone-work no smaller part than quarter-cords is allowed.
700. A cord of stone will make about 100 cubic feet of wall.
701. In estimating the cost of mason-work, it is customary to take the outside measurement of the wall, and make no allowance for openings, except they are large.
702. It takes about three bushels of lime and a cubic yard of sand to lay a cord of stone.
703. Stone-work is usually estimated by the perch.
704. A perch of stone-work is 1 rod long, $1 \frac{1}{2}$ feet thick, and 1 foot high. It contains $24 \frac{3}{4}$ cubic feet.

## EXERCISE 135.

1. How many cubic feet of stone are there in a pile 38 feet long, 6 feet wide, and 4 feet high ?
2. How many cubic feet of stone are there in wagon-box 9 feet long, $8 \frac{1}{2}$ feet wide, and $1 \frac{1}{2}$ feet high? What part of a cord does it contain?
3. How many cords of stone are there in a pile 20 feet long, 8 feet wide, and 3 feet high ?
4. In No. 3, how many cubic feet of wall will the stone build?
5. How many cords of stone will build a wall 200 feet long, 6 feat high, and 8 feet thick?
6. How many cords of stone will build a wall 60 yards long, 6 feet high, and 18 inches thick? How many perch of stone-work in the wall?
7. Find the cost of the stone in a wall 42 feet long, 8 feet high, 18 inches thick, at $\$ 6$ per cord.
8. How many cords of stone are required for a cellar 36 feet long, 30 feet wide, if the wall be built 8 feet high, and two feet thick? Find the cost of the mason work at 50 cents a perch.
9. How many cords of stone are required for the foundation of a bank barn 60 feet long, by 36 feet wide, if the foundation wall be 7 feet high and 3 feet thick? Find the cost of building the foundation at 60 cents a perch.
10. At 60 cents per perch, what is the cost of the stonework for the basement of a house which has an outside perimeter of 160 feet, the wall being 8 feet high and 20 inches thick?
11. How much lime and sand will be required for the mortar of an 18 -inch wall 8 feet high, under a house 40 feet by 30 feet?
i2. In No. 9, find the cost of the material at $\$ 6$ per cord for the stone, 30 cents a bushel for the lime, and $\$ 1.20$ per cubic yard for the sand.
12. A stone house is 36 feet by 24 feet; the cellar walls are 9 feet high and 3 feet thick; the walls of the ground floor are 12 feet high and 2 feet thick; the walls of the second floor are 8 feet high and 18 inches thick; the gable walls are 7 feet high and 12 inches thick; find-
1 sl . Number of perches of mason work in the building, and cost of labour at $\$ 1.10$ a perch.
2nd. Cost of the stone at $\$ 5$ a cord.
3 rd. Cost of the lim? at 35 cents a bushel.
4th. Cost of the sand at $\$ 1.10$ per cubic yard.

## MEASUREMENT OF BRICK-WORK.

705. Bricks vary so much in size and style, that to give the exact dimensions of the different styles is impracticable. Ordinary bricks are 8 inches long, 4 inches wide, and $2 \frac{1}{2}$ inches thick.

7(16. It is sufficiently accurate, in making an estimate of the number of brick needed for a certain work, to reckon 20 bricks to the cubic foot laid dry.
707. In half-brick walls, such as in veneering wooden houses, each brick, with the mortar required to lay it, has an external surface of $8 \frac{1}{2} \times 3$, or for about every 25 square inches of surface.
708. In single-brick walls, each brick, with the mortar required to lay it, has an external surface of $4 \frac{1}{2} \times 3$, or one brick is required for about every 18 square inches of surface.
ll at $\$ 6$ per te lime, and
cellar walls the ground walls of the k ; the gable
the building,
709. In a brick-and-a-half wall, a brick is required for about every $8 \frac{2}{3}$ square inches.
710. In double-brick walls, a brick is required for about every $6 \frac{1}{2}$ square inches of surface.
711. In estimating material, corners are measured once, and allowance is made for doors and windows.

In estimating labor, the corners are measured twice, that is, the outside measurement is taken, and allowance is usually made for one-half the openings.

## EXERCISE 136.

1. A pile of ordinary bricks is 8 feet 6 inches high, 14 feet long, and 15 feet wide. What is the pile worth at $\$ 8$ per thousand ?
2. How many bricks are there in a wall 36 feet long, 12 feet high, and half a brick thick?
3. How many bricks are required to veneer the front of a house 18 feet wide and 25 feet high?
4. How many bricks are requirel for a single brick partition between two houses, 40 feet deep and 24 feet high ?
5. How many bricks are required to build a house 30 feet by 24 feet, and 18 feet high, with single brick walls?
6. How many bricks are required for a double brick wall of a basement, 48 feet by 32 feet, and 10 feet high ?
7. What will it cost to lay the brick of a house 40 feet by 32 feet, and 21 feet high, with a flat roof and double walls, at \$2.75 per thousand?
8. Find the cost of the brick in the wall around a garden, 400 feet by 200 feet, 6 feet high, and a brick and a hicil thick at $\$ 7$ per thousand.
9. At $\$ 8$ per thousand, find the cost of the brick in the front walls of a terrace block, 120 feet long and 22 feet ligh. There are 6 doors, each 8 feet by $3 \frac{1}{2}$, and 20 windows, each 8 feet by 4 feet, the wall being a brick and a half thick.
10. How many bricks will be required for a house 40 feet by 30 feet; the basement walls are 8 feet high and 2 brick thick, one door 4 feet by 6 feet; the ground floor is 11 feet between the floors, and the walls a brick and a half thick, 2 doors and 4 windows, each 8 feet by $8 \frac{1}{2}$ feet; the second floor is 10 feet high between the floors, and the the walls one brick thick, 6 windows, each 8 feet by $3 \frac{1}{2}$ feet; the gables are 10 fect high and half a brick thick.

## the metric system of measurement.

 round floor brick and a by $3 \frac{1}{2}$ feet ; ors, and the feet by $3 \frac{1}{2}$ thick."The real beginning of exact knowledge, or sctence, lies in measuring, and the falthful observer of nature is always occupled in measuring."-Earl.


This engraving is a Decimetre (exact size), or the tenth part of a METRE The large white-and-black squares at top show its division into 10 Centimetres, each of which is sub-divided into 10 Millimetres, as shown at bottom by the smal! white-and-black strips. A complete Metre can be easily constructed from this
712. The Metric System (pronounced Met'-ric) is a system of weights and measures expressed in the decimal scale. It is now legal in nearly all civilized countries. It was legalized in Canada by Act of Dominion Parliament in 1886 (chap. 104, sec. 21), and all contracts based upon it are now enforcible at law. It was legalized in the United States in 1866, and copies of the standard metre furnished to all the States. This system of measurement is used in all countries for scientific purposes on account of its exactness, and in many countries it is used for ordinary purposes. Since 1840 the metric measures have been the only ones in common use in France.
713. The Standard Metre, which is the basis of the Metric System of Measurement, is a bar of platinum 39.37 inches long. This length was chosen becanse it was supposed to be one ten-millionth (i0006000 or .0000001 ) of a quarter of the earth's circumference measured by a line passing through Paris, France, from the equator to the pole. The
original bar, or metre, was made by Borua in 1795 at Paris, where it is carefully preserved, accurate copies being furnished to the governments of all civilized nations. Its length being nearly $3 \mathrm{ft} .3 \frac{3}{8} \mathrm{in}$., the metre may be remembered as the pule of the three threes.
714. The Standards used in a general scheme of measurement are called Units. Thus, the Mctre in France forms the foundation and starting-point of every measure in existence.
715. All the Units of measures are derived in a simple manner from the Metre. Thus:

The Metre is the unit of Length. It is a bar 39.37 inches long.

The Ar (or Are) is the unit of Land Measure. It is a square whose side is 10 metres. $1 \mathbf{A r}=119.6$ sq. yds.

The Litre (Lecter) is the unit of Capacity. It is a cubic decimeter; that is, a cube whose edge is a decimetre long. A Litre $=1.76$ pint.

The Gram is the unit of Weight. It is the weight of a cubic centimetre of water.

As the terms used in the Metric System are derived from the Greek, Latin and French languages, we have thought it best to give them English spellings, dropping the final "me" in "gramme," etc.
716. The Metre is sub-divided always into tenths, hundredths, thousandths, \&c., or decimal parts, thus :

Decimetre (dm) Lacin decem, ten=1/ or .1 metre (m).
Centimetre ( cm )
"illimetre $(\mathrm{mm})$
centum, hundred $=\frac{10}{10}$ or .01 metre.
mille, thousand $=\frac{1}{100}$ or .001
The nomes of these lower denominations are formed by prefixing Latin numerals (deet, centi, milli,) and writing the abbreviations (dm, cm, mm,) without Capital letters. All the compound names are accented on the first syllable thus, mil'limetre.

Therefore : 1 metre $=10$ decimetres $=100$ centimetres $=1000 \mathrm{~mm}$.
1 decimetre $=10$ centimetres $=100 \mathrm{~mm}$.
1 centimetre $=10 \mathrm{~mm}$.

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eight of a Greek, LatIn lsh spellings, to tenths, Is:
re (m).
.01 metre. 001 "
effxing Latin mm, ) without syllable thus,
$=1000 \mathrm{~mm}$.
$;=100 \mathrm{~mm}$.
$=10 \mathrm{~mm}$.
717. Multiples of the Metre are as follows: Decametre ( Dm ) Greek Deka, ten $=10$ metres. Hectometre (Hm) " Hekaton, hundred=100 metres. Kilometre (Km) "Kilioi, thousand $=1000$ metres. Myriametre (Mm) " Myria, ten thousand $=10,000$ metres. Megametre (Mgm) " Mega. million $=1,000,000$ metres.
The names of these higher denominations are formed by pretixing Greek numerals (deka, hekto, kilo, myria, mega) and writing the sbbreviations ( $\mathrm{Dm}_{r}$ $\mathrm{Hm}, \mathrm{Km}, \mathrm{Mm}, \mathrm{Mgm}$ ) with Capital letters.
718. A person who wished to buy 125 metres of cloth would not ask for " 1 hectometre, 2 decametres, 5 metres," any more than a Boston merchant would tell a person who owes him $\$ 25.96$ that his bill is 2 eagles, 5 dollars, 9 dimes, 6 cents.
719. Comparative Lengths are as follows:

|  |  | Inches. | Feet. | Yards. |
| :--- | :--- | ---: | :---: | :---: |
| 1 Matre $=$ | 39.37079 | 3.2808992 | 1.09336331 |  |
| 1 Decimetre $=$ | 3.93708 | .3280899 | .1093633 |  |
| 1 Centimetre $=$ | .39371 | .032809 | .0109363 |  |
| 1 Millimetre $=$ | .03937 | .0032809 | .0010936 |  |

720. The Metre, like the yard-stick, is used in measuring cloth and short distances; the Kilumetre is used in measuring long distances.
721. Since, in the Metric System, $10,100,1000$, etc., units of a lower denomination make a unit of a higher denomination, it follows that any one of the metric measures may be expressed in terms of another measure by simply moving the decimal point to the right or left.
722. A number is reduced to a LOWER denomination by removing the decimal points as many places to the RIGHT as there are ciphers in the multiplier.
723. A number is reduced to a HIGHER denomination by removing the decimal point as many places to the LEFT as there are ciphers in the divisor.

Thus, $12,465,687^{m m}$ may be written as Kilo-metres by observing that Milli-metres are changed to metres by mor-
ing the point three places to the left, and metres are changed to Kilo-metres by carrying the point three places further, making in all six places.

Therefore $12,465,687^{\mathrm{mm}}=12.465687^{\mathrm{km}}$
RULE.-First count the number of places needed to convert the given measures into terms of the principal unit; then the number needed to convert the principal into the required units.

Before adding or subtracting, the quantities must be written in the same unit of measure.

## 722.

## MEASURES OF LENGTH.

| millimetres, | rked | mm. | are | re, |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 centimetres, | , | cm . | " | 1 decimutre, |  |  |
| 10 decimetres, | " | dm. | " | 1 metre, |  | m. |
| 10 metres, | " | m. | " | 1 dekametre, | , | Dm. |
| 10 dekametres, | / | Dm. | " | 1 hektometre, |  | Hm. |
| 10 hektometres, | ' | Hm. | " | 1 Kilometre, | ' | Km. |
| 0 Kilometres, |  | Km . |  | 1 Myriametre, |  | M |

## 723. To Reduce 3.825 m . to cm .

Solution.-To reduce metres to centimetres, multiply by 100 . Write 3825 , and place the decimal point between 2 and 5 , two orders farther to the right than it is in 3.825. Ans. 382.5 cm .

## 724. To Reduce 1025.5 m . to Km .

Solution.-To reduce metres to kilometres, divide by 1000. Write 10255, and place the decimal point between 1 and 0 , three orders farther to the left than it is in 1025.5. Ans. 1.0255 Km .

## 725. To Reduce 2.15 Dm. to centimetres.

Soldtion.-To reduce dekametres to centimetres, multiply $10 \times 100=1000$. Write 215 and annex a cipher. Ans. 2150 cm .

## LAND OR SQUARE MEASURE.

726. The Are is the unit of Land measure (or Area). It is legal at 119.6 sq. yd.s. The Are is the principal unit of
surface of small plots of land. The area of a farm is expressed in Hektars ; of a country in square Kilometres.

Table.
100 centiares, marked ca., are 1 Are, marked a. 100 ares " a., " 1 hektar " Ha.
727. An Are is 100 square metres, marked $\mathrm{m}^{2}$. The Hektar is nearly $2 \frac{1}{2}$ acres (2.47).
728. For measuring other surfaces, squares of the metre and its sub-divisions are used.

1. Reduce 397.8 a. to hektars.
2. $\quad$ A. $\quad 3.8$ a. to squard metres. $\quad$ A.978 Ha.

## MEASURES OF CAPACITY.

729. The Litre is the unit of capacity. It is legal at 1.0567 quarts, Liquid me, ure.

Table.
10 centilitres, marked cl., are 1 decilitre, marked dl.
10 decilitres, " dl., " 1 litre, " 1.

| 10 litres, | "., " | 1 dekalitre | " | Dl. |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 10 dekalitres, | ". | Dl., " | 1 hektolitre | " | H1. |

730. The measures commonly used are the litre and the hektolitre. The litre is very nearly a quart; it is used in measuring milk, wine, etc., in moderate quantities. The hektolitre is about $2 \mathrm{bu} .3 \frac{1}{3} \mathrm{pk}$.; it is used in measuring grain, fruit, roots, etc. in large quantities.
731. For measuring wood the Stere is used; it is a cubic metre ( $=35.316$ cub. ft.)

## MEASURES OF WEIGRT.

7 732. The Gram is the unit of Weight ; it is legal at 15.132 grains Troy.

## 733.

Table.
10 milligrams, marked mg., are 1 centigram, marked cg.


10 quintals or 1000 kilograms are 1 Metric ton, marked M.T.


1 Kilogram $=10$ (n) grama, (exnet size), commonly called the "Kilo."
734. The weights commonly used are the Gram, Kilogram, and Metricton. The Gram is used in mixing medicines, in weighing the precious metals, and in all cases where great exactness is required. The Kilogram, (communly called the "Kilo"), is the usual weight for Groceries and coarse articles generally ; it is very nearly $2_{5}^{1}$ ibs. Avoir. The metric ton is used for weighing hay and other heavy articles; it is about 204 tbs, more than our ton.
735. Legal and Approximate Values are as follows:
Denomination. Legal Value. Approximate Value.
$\qquad$
Centimetre
39.37 inches $3 \mathrm{ft} .8{ }_{8}^{3}$ inches.
............................... inch.
鬲 miló. Square Metre .............. 1.196 sq. yards...... .......... $10 \frac{3}{4}$ sq. feet.

Legal and Approximate Values (contInued).

Dg.
Hg.
Kg. Mg.
Q. ked M.T. hts comare the Im, and eGram $g$ mediing the and in reat exd. The mmunly '), is the Grocerarticles is very oir. The used for Id other is about ian our
3. What is the value of 20 Km .?
4. How many hektars in 160 acres?

Approximate Value.
119.6 sq. yards................. 4 sq. rods.
2.471 acres . . . . . . . . . . . . . . . . . . . . $2 \frac{1}{2}$ acres.
.1.308 cub. yds................ $85 \frac{1}{\frac{1}{8}}$ cub. feet.
.2759 cord
4 cord.

| Denomination. | Legal Value. | Approximate Value. |
| :---: | :---: | :---: |
| Are........... | .119.6 sq. yards | Approximate Value. |
| Hektar..... | 2.471 acres | . . ........... $2 \frac{1}{2}$ acres. |
| Cubic Metre | .1.308 cub. yds. |  |
| Stere. | 2759 cord. |  |

Litre....................... 1.0567 quarts.......... $\left\{1{ }_{2}{ }^{2} d\right.$ liquid quart.
Hektolitre . .............. 2.8375 bushels . ${ }_{10}^{9} d$ dry quart.
Gram ................... 15.432 pr. Tro ........... 2 bush. $3 \frac{1}{8}$ pk.
Kilogram ................ 2.2046 tb . Avoir. . . . . . . . . . . . . $15_{2}^{\frac{1}{2}}$ grains
Metric Ton (or tonneau). . 2204.6 tb . "................. 1 T. 204 tbs.
Stere...................... 0.27590 cord....... .................. $\frac{1}{4}$ cord.
736. The legal value is used in solving the following examples.
737. MISCELLANEOUS EXAMPLES.

1. How many yards, feet, etc., in 4 M. ?

Solution. - In one metre there are 39.37 in . ; in 4 metres there are 4 times 89.37 in ., which are 157.48 in . ; 157.48 in . reduced to integers of higher denominations are 4 yds. 1 ft .1 .48 in.
2. What is the value of 36 fbs . in kilograms?

Solution.-In one kilogram there are 2.2046 tbs ; in 36 tbs. there are as many kilograms as 2. 2046 are contained times in 36, which are $16.329+$.

| $2.2046) 36.0000(16.3$ |
| :---: |
| $\frac{22046}{139540}$ |
| $\frac{132276}{72640}$ |
| $\frac{66138}{65020}$ |
| $\underline{44092}$ |
| 209280 |
| 198414 |

12.4274 miles.
$64.75+\mathrm{Ha}$.
5. What is the value of 49 m ? ? $\quad 9 \mathrm{rd} .4 \mathrm{yd} .3 .13 \mathrm{in}$.
6. How many hektolitres in 42 bu. ?
$14.8+\mathrm{Hl}$.
7. How many square yards in a roll of paper 9 m . long and 5 m . wide?
$5.382 \mathrm{sq} . \mathrm{yd}$.
8. The five-cent piece weighs 5 grams; how much will 100 such pieces weigh? .5 Kg .
9. Ten litres of a certain liquid weigh 92 Kg . ; what is the weight of a decilitre?
.92 Kg .
10. One hektogram of goods costs 85.35 ; what costs one kilogram?
$\$ 53.50$
11. A piece of money weighs 10 g. ; how many such pieces in a bag weighing 1 Kg .?

100
12. A hektolitre of wheat costs $\$ 6.25$; what is the price of a dekalitre?
$\$ .625$
13. A hektolitre of wine costs $\$ 25.10$; what is the price of a litre?
14. A kilogram of wool costs $\$ 1.875$; what is the cost of 100 kilograms?
$\$ 187.50$
15. A litre of wine weighs 880 g. ; what is the weight of a hektolitre?

88 Kg .
16. Add 45 kilograms, 4 hektograms, 5 dekagrams; 35 kilograms, 8 dekagrams, 7 grams; and 45 hektograms, 4 grams. $\quad 85.041 \mathrm{Kg}$.
17. A wine merchant sold 1270 litres, 487 litres, 1563 litres, 1000 litres, and 2345 litres; how many hektolitres did he sell?
66.65 Hl .
18. A vase, weighing 24.67 hektolitres, contains 18.79 hektolitres of liquid; what is the weight of the empty vase?
5.88 Hl .
19. From a barrel containing 147 litres of wine, 42.75 litres leaked out; how much remained?
104.25 .
3.13 in.
$.8+\mathrm{Hl}$.
n. long sq. yd. ch will . 5 Kg . at is the .92 Kg . osts one $\$ 53.50$ h pieces 100 price of $\$ .625$ e price $\$ .251$ cost of $\$ 187.50$ eight of 88 Kg . ms; 35 rams, 4 041 Kg . s, 1563 tres did 6.65 Hl .
20. How much will 135.60 m . of cloth cost at $\$ 1.16$ a metre?
$\$ 157.296$
21. A grocer bought 3845 Kg . of sugar at 19 cents a kilogram ; how much did it cost?
$\$ 730.55$
22. Bought 25 hogsheads of wine, of 225 litres each, at the rate of $\$ .156$ a litre ; how much did it coast? $\$ 877.50$
23. What is the cost of 21 pieces of cloth of 42 m . each, at $\$ 5.69$ a metre?
$\$ 5018.58$
24. I have an article that sells for 26 cents a pound ; how much i.: vorth a kilogram?
$\$ .573+$
25. . . san bought 25 lbs . of tea at $\$ 1.8 \mathrm{~J}$ a pound; he exchanged it for five times its weight in coffee, which he sold at $\$ .86$ a kilogram ; did he gain or lose by the bargain, and how much ?
$\$ 3.76+$
26. How many metres of carpeting, .75 m . wide, will cover a floor 8 m . long and 5 m . wide ? $53.33+\mathrm{m}$.
27. I paid $\$ 13$ for a barrel of vinegar containing 1401 .; I lost 22 l. by leakage, and sold the remainder at 20 cents a litre ; how much did I gain?

## institute of chartered accountanis.



## ORGANIZATION.

738. This Institute, which received its charter from the Ontario Legislature in 1883, comprises in its membership the leading Accountants of Canada. The chief aim of the Institute is to raise the standard of accountancy; and in order to increase the knowledge, skill and proficiency of its members, it is empowered to establish classes, lectures and examinations; to prescribe tests of competency; to grant diplomas entitling members to use the distinguishing letters F.C.A. (Fellow of the Chartered Accountants); and to affliate with any other similar bodies for mutual benefit.

## AFFILIATION.

739. Business Colleges and other Educational institutions having a department devoted to the study of Accounts may become affiliated with the Institute, and may conduct the Intermediate Examinations in connection therewith, on terms fixed from time to time by the Council.
740. Students-at-Accounts, of the age of 16 years or over, are admitted to registration under two classes: (1) Primary Students and (2) Intermediate Students or Book* keepers. Such Students are entitled to attend the meetings of the Institute and take part in discussion of papers. Students may form an Association for the better advancement of their studies and professional knowledge, and for making recommendations to the Council affecting their joint interests.
741. The Primary Examination required of students on entrance comprises Business Composition and Correspondence, Spelling and Punctuation, Arithmetic, Penmanship, Elementary Book-keeping, Common Latin Terms and Roots, British and Canadian History, Geography, Stenography (the last optional). This examination may be conducted in any affiliated institution, or the Council may waive this examination on students showing that they have passed one equivalent, or have had practical experience at accounts which may be deemed equivalent. The object of the Primary Examination is to reasonably ensure that future candidates for membership shall be men of good general education, the Council holding the view that the comparatively slow progress made hitherto, towards obtaining recognition from the public of the claims of accountancy to be considered as a profession, has been due in no small measure to the superficial character of the education deemed to be liccessary to fit a man for intelligently undertaking the duties of an accountant, or even of a book-keeper (understood in the sense of one versed in one branch only of accountancy). While it may be true that every accountant will find his own level, on the ground of natural ability alone, it is equally certain that the accountant who has had the initial advantage of a good general education, supplemented by a judicious course of special training for his calling or profession, will out-distance the accountant who has not had these advantages, everything else being equal.
742. The Intermediate Examination is open to any one who has registered as a Student-at-Accounts, 19 years of age or over, after one year from passing the Primary or equivalent Examination. The Intermediate Examination comprises Mercantile Arithmetic, Negotiable Instruments, Book-keeping, Auditing, Shareholders' and Partners' Accounts, Insolvency. This examination may be held in affiliated institutions. Evary person passing the Intermediate Examination is entitled to a Certificate to that effect, p:-d setting forth in suitable terms his attainments as a book-keeper. The Intermediate Examinations are intended to afford to students who desire to take up accountancy as a profession, an opportunity to test their general progress in professional knowledge, to enable the Council to form an estimate of their capabilities, and to advise upon and direct, so far as may be, their course of preparation for the Final Examination, which qualifies for admission to membership as an Associate. There is the further intention to provide recognition of the attainments of those candidates who do not purpose attempting the Final Examinations, but desire to have the Certificate of the Institute of competency to undertake the duties of a bookkeeper. The scope of the Intermediate Examinations, therefore, will, generally speaking, be limited to a thorough comprehension of the duties of one required to undertake the duties of chief book-keeper in a first-class business.
743. Final Examinations. Any person who has passed the intermediate may apply for membership in the Institute, and if of the age of 21 or over, the Council will set a Final Examination comprising Book-keeping, Auditing, Insolvency, Joint Stock Companies, Mercantile Law, Partnerships and Executorships. This Final Examination shall be held in Toronto, and any who pass, upon being admitted to the Institute by ballot, shall receive a Certificate of membership, and right to use the appellation "Chartered Accountant," and to be styled "Associate." aay be, , which There attainting the e of the a booknations, horough take the s passed nstitute, a Final olvency, ips and held in to the bership, nt," and

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" \text { F. C. A." }
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744. A Chartered Accountant who has been in continuous practice as snch for three years after admission as a member may be admitted a "Fellow of the Chartered Accountants" upon passing the tests, viz. : (1) Known standing and reputation as a Public Accountant, and (2) a thesis upon some subject to be approved by the Conncil. Upon passing these tests a "Diploma of Hellowship" is issued to the candidate, giving him the right to use the letters "F.C.A."
745. Every Commercial Student should aim to secure membership in thls Institute of Chartered Accountants, and to pass through the various grades above outlined till the goal is reached-the high honors and privileges of a "Fellow of the Chartered Accountants," upon whom the stamp of this honorable I.stitute is placed in the letters "F.C.A." In order to help our readers to reach this end, the above information is given and the following Examination Papers are quoted.

## MERCANTILE ARITHMETIC.

## Problems set for Candidates in Intermediate Examination, Institute of Chartered Accountants, Maj, 1897.

1. A nail manufacturer has 3 grades of nails which he wants to net him per keg $\$ 2.75, \$ 2.80, \$ 2.85$. He desires to make a list of prices to sell at $50 \%, 10 \%, 5 \%$ discount to net the above prices. Give the list prices and show how it is worked out.
2. A Trustee invests $\$ 4,000$ in Ontario Bank stock at 80 , paying $6 \%$, and $\$ 1,000$ in Dominion Bank stock at 200 paying 10\%. After two years he sells the former at 86 and the latter at 180. What rate of interest has he received during the period of investment and how has the value of the capital changed?
3. Convert $£ 8555 \mathrm{~s}$. 10d. into currency, exchange being 9.78.
4. Convert $\$ 750$ into Francs, Sterling exchange being at 94, $25 \frac{1}{4}$ Francs representing \&1 Sterling.
5. Find the equated time of paying the balance of the following account on basis of 860 days to the year. Interest, $6 \%$.

| 1896 |  | 1896 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Jan. 3 | Goods 4/m, | \$175 | Feb. 9 | By Cash, | \$100 |
| Jan. 20 | " $2 / \mathrm{m}$, | 75 | Mar. 2 | By Cash, | 50 |
| Mar. 1 | 1/m, | 125 |  |  |  |
| Mar. 14 | Net, | 50 | Apr. 3 | By Cash, | 60 |
| Apr. | 3/m, | 200 | May 7 | By Cash, | 200 |
| May | 2/m, | 100 |  | Balance, | 315 |
|  |  | \$725 |  |  | \$725 |
| May 7th, | Balance, | 8815 |  |  |  |

Adjust the interest and state what amount is due in Cash May 7th.
6. A merchant has a line of tweeds which he is selling in 50 yd . ends, for $\$ 75$ per end, a profit of $25 \%$ on cost. His clerk, in order to make quick cash, sales, sells for $15 \%$ cash discount. What advance over cost did he net?
7. A note of $\$ 500$, dated April 1st, 1895, payable July 1st (without grace) with interest at $6 \%$ was discounted May 1st at $8 \%$. Find the proceeds. Iaterest on basis of 360 days to the year.
8. A Board of School Trustees desire to issue Debentures to the amount of 82,500 . Interest $5 \%$ payable annually 1st January each year, the whole amount with interest to be paid in five equal annual payments. Divide the amount into five debentures, one to mature each year.

Find the face amount of each debenture numbering them 1, 2, $3,4,5$, and the amount of coupons due each year.
9. A merchant has 6 chests (of 30 lbs . each) of Tea at the following prices:-

| 1 | at | 80 c. | per |
| :--- | :--- | :--- | :--- |
| 1 | lb | 75 c. | $"$ |
| 1 | $"$ | 50 c. | $"$ |
| 1 | $"$ | 60 c. | $"$ |
| 1 | $"$ | 25 c. | $"$ |
| 1 | $"$ | 20 c. | $"$ |

He desires to make 1 chest of a blend containing all these grades to sell at $\$ 1.00$ per lb. which will give him an advance over costs of $100 \%$. Find how many pounds of each he must use.
10. If the profits are divided in proportion to the Capital invested and the time it was employed, at the end of a year what would be each partner's average investment and share of the profits from the following accounts. Net profit $\$ 500$.

Problems set for Candidates in Intermediate Examination, Institute of Chartered Accountants, Nov. 1895.

## MERCANTILE ARITHMETIC.

1. A merchant buys a sort of wine at $\$ 2$ per gallon, and another at $\$ 1.50$ per gallon. At what price must he sell a blend of 7 parts of the former and 3 parts of the latter to realize 20 per cent. profit?
2. You manage an estate, and receive as your remuneration 5 per cent. of the net amount paid to the beneficiaries. Taxes, repairs and sundry expenses in a given year are 8540. Your commissions amount to $\$ 850$. Find the gross revenue of the estate.
3. Find the present value of $\$ 3,250$ due 3 years and 6 months hence at 4 per cent. per annum. Show working.
4. Average the following account:

5. A certain stock pays a semi-annual dividend of $3 \frac{1}{2}$ per cent What is it worth to an investor who wants a return of $4 \frac{1}{2}$ per cent. per annum upon his investment?
6. Convert $\$ 1,000$ into sterling at ten and one-half per cent.
7. Find the cost of papering a room $30 \times 22$ feet, and 12 feet high, with pape: 18 inches broad, costing eighty cents per roll of 12 yards, deducting 20 yards of paper for window and door spaces.
8. A merchant imports as follows:

850 yards sheeting at 5 cents; 1,400 yards flannel at 13 cents.
The duty on sheeting is 20 per cent. ad val., and 5 cents per lb . ( 9 yards to 2 lbs .) ; the duty on flawnel is 30 per cent. ( 4 yards to the lb.) Packages are charged at \$4. Freight \$6.50. Cartage \$1. Find the cost per yard of each laid down in his warehouse.
9. An insolvent estate realized, after payment of expenses, 81,840.72. The claims to rank are as follows: $\mathbf{A}, 83,400.60 ; B$, $\$ 1,847.85 ;$ C, $\$ 890.96 ;$ D, $\$ 870.42 ;$ E, $\$ 391.80 ;$ F, $\$ 102$; G, $\$ 84.58$. Prepare a dividend sheet showing the rate per cent. and the amount coming to each.
10. You are being charged interest monthly at 7 per cent. per annum on an overdraft at your bankers. They offer to discount your bills at three months at $6 \frac{1}{2}$ per cent. per annum. Which is the more profitable transaction, and by how much ?

6 months rehouse.
expenses, , 400.60 ; B, G, \$84.58.
$t$. and the
r cent. per o discount Which is


