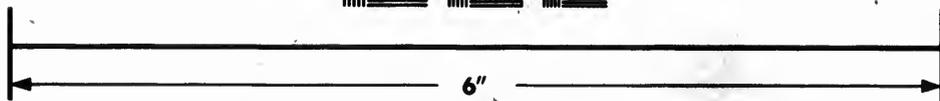
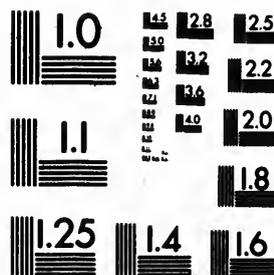


**IMAGE EVALUATION
TEST TARGET (MT-3)**



**Photographic
Sciences
Corporation**

23 WEST MAIN STREET
WEBSTER, N.Y. 14580
(716) 872-4503

15
12.8
13
13.2
16
22
20
11.8

**CIHM
Microfiche
Series
(Monographs)**

**ICMH
Collection de
microfiches
(monographies)**



Canadian Institute for Historical Microreproductions / Institut canadien de microreproductions historiques

11
10
01

© 1991

Technical and Bibliographic Notes / Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

- Coloured covers/
Couverture de couleur
- Covers damaged/
Couverture endommagée
- Covers restored and/or laminated/
Couverture restaurée et/ou pelliculée
- Cover title missing/
Le titre de couverture manque
- Coloured maps/
Cartes géographiques en couleur
- Coloured ink (i.e. other than blue or black)/
Encre de couleur (i.e. autre que bleue ou noire)
- Coloured plates and/or illustrations/
Planches et/ou illustrations en couleur
- Bound with other material/
Relié avec d'autres documents
- Tight binding may cause shadows or distortion along interior margin/
La reliure serrée peut causer de l'ombre ou de la distorsion le long de la marge intérieure
- Blank leaves added during restoration may appear within the text. Whenever possible, these have been omitted from filming/
Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.
- Additional comments: /
Commentaires supplémentaires:

- Coloured pages/
Pages de couleur
 - Pages damaged/
Pages endommagées
 - Pages restored and/or laminated/
Pages restaurées et/ou pelliculées
 - Pages discoloured, stained or foxed/
Pages décolorées, tachetées ou piquées
 - Pages detached/
Pages détachées
 - Showthrough/
Transparence
 - Quality of print varies/
Qualité inégale de l'impression
 - Continuous pagination/
Pagination continue
 - Includes index(es)/
Comprend un (des) index
- Title on header taken from: /
Le titre de l'en-tête provient:
- Title page of issue/
Page de titre de la livraison
 - Caption of issue/
Titre de départ de la livraison
 - Masthead/
Générique (périodiques) de la livraison

This item is filmed at the reduction ratio checked below /
Ce document est filmé au taux de réduction indiqué ci-dessous.

10X	14X	18X	22X	26X	30X
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12X	16X	20X	24X	28X	32X

The copy filmed here has been reproduced thanks to the generosity of:

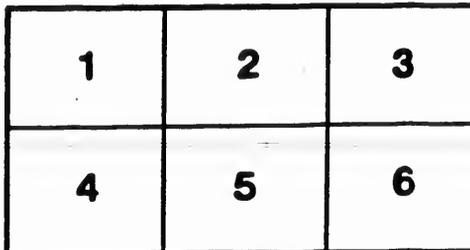
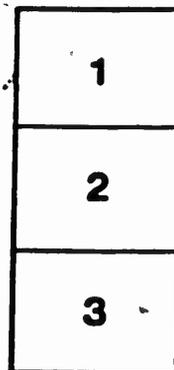
Société du Musée
du Séminaire de Québec

The images appearing here are the best quality possible considering the condition and legibility of the original copy and in keeping with the filming contract specifications.

Original copies in printed paper covers are filmed beginning with the front cover and ending on the last page with a printed or illustrated impression, or the back cover when appropriate. All other original copies are filmed beginning on the first page with a printed or illustrated impression, and ending on the last page with a printed or illustrated impression.

The last recorded frame on each microfiche shall contain the symbol \rightarrow (meaning "CONTINUED"), or the symbol ∇ (meaning "END"), whichever applies.

Maps, plates, charts, etc., may be filmed at different reduction ratios. Those too large to be entirely included in one exposure are filmed beginning in the upper left hand corner, left to right and top to bottom, as many frames as required. The following diagrams illustrate the method:



L'exemplaire filmé fut reproduit grâce à la générosité de:

Société du Musée
du Séminaire de Québec

Les images suivantes ont été reproduites avec le plus grand soin, compte tenu de la condition et de la netteté de l'exemplaire filmé, et en conformité avec les conditions du contrat de filmage.

Les exemplaires originaux dont la couverture en papier est imprimée sont filmés en commençant par le premier plat et en terminant soit par la dernière page qui comporte une empreinte d'impression ou d'illustration, soit par le second plat, selon le cas. Tous les autres exemplaires originaux sont filmés en commençant par la première page qui comporte une empreinte d'impression ou d'illustration et en terminant par la dernière page qui comporte une telle empreinte.

Un des symboles suivants apparaîtra sur la dernière image de chaque microfiche, selon le cas: le symbole \rightarrow signifie "A SUIVRE", le symbole ∇ signifie "FIN".

Les cartes, planches, tableaux, etc., peuvent être filmés à des taux de réduction différents. Lorsque le document est trop grand pour être reproduit en un seul cliché, il est filmé à partir de l'angle supérieur gauche, de gauche à droite, et de haut en bas, en prenant le nombre d'images nécessaire. Les diagrammes suivants illustrent la méthode.

14

e



181



INTRODUCTION
TO THE TREATISE
ON
COMMERCIAL ARITHMETIC

COMBINING THE
ANALYTIC AND SYNTHETIC METHODS;

DESIGNED AS A COMPLETE TEXT-BOOK
ON THIS SCIENCE,

FOR
COMMON SCHOOLS AND ACADEMIES.

BY THE
CHRISTIAN BROTHERS.

Sanctioned by the Council of Public Instruction

SECOND EDITION.



QUEBEC:
G. DARVEAU, PRINTER AND PUBLISHER,
8, MOUNTAIN HILL.

1872.

SCHOOL-BOOKS

PUBLISHED BY THE

CHRISTIAN BROTHERS

IN THE DOMINION OF CANADA.

~~~~~

**THE FIRST READER**, in two parts, which may be had separately.

This book is carefully illustrated and arranged for the use of schools:

**COMPENDIUM OF SACRED HISTORY**, approved by his Grace the Archbishop of Quebec.

**THE DUTIES OF A CHRISTIAN** towards God, approved by his Lordship the Bishop of Montreal.

**St. PATRICK'S MANUAL**, approved by his Grace the Archbishop of Quebec.

**INTRODUCTION TO THE TREATISE ON COMMERCIAL ARITHMETIC**, 12 mo., of 264 pages. This work was sanctioned by the Council of Public Instruction, Province of Quebec, 1871.

**KEY** to Introduction to the Treatise on Commercial Arithmetic, to which is added a Series of Practical and well graded Questions in Mental Arithmetic, for the use of teachers.

**THE COMMERCIAL ARITHMETIC**, to which are added Treatises on Mensuration and Book-keeping, in 12 mo., of 430 pages.

**KEY** to the Commercial Arithmetic and to the Treatises on Mensuration and Book-keeping, to which is added a Series of Practical and well graded Questions in Mental Arithmetic, for the use of teachers.

**SYLLABAIRE FRANÇAIS**, in 18 mo., of 144 pages.

**EXERCICES ORTHOGRAPHIQUES** (Nouveaux), Cours de 1ère année.

**INTRODUCTION AU TRAITÉ D'ARITHMÉTIQUE COMMERCIALE**, in 12 mo., of 216 pages.

**TRAITÉS D'ARITHMÉTIQUE COMMERCIALE, DE MENSURATION ET DE COMPTABILITÉ**, in 12 mo., of 468 pages. The last two Works were sanctioned by the Council of Public Instruction, Province of Quebec, 1870:

**SOLUTIONS** des Exercices et Problèmes de l'**INTRODUCTION AU TRAITÉ D'ARITHMÉTIQUE COMMERCIALE**, suivies d'un **RECUEIL d'Exercices Pratiques** et bien gradués pour Calcul Mental à l'usage des Maîtres:

**SOLUTIONS** des Exercices, Problèmes, etc., des **TRAITÉS d'Arithmétique Commerciale, de Mensuration et de Comptabilité**, à l'usage des Maîtres seulement.

**COURSE OF COMMERCIAL PENMANSHIP**, in two books, one containing 20, and the other, 32 well graduated models.—Sold separately.

---

Entered, according to Act of the Parliament of Canada, in the year one thousand eight hundred and seventy-one, by EPHREM GAGNON, in the Office of the Minister of Agriculture.

---

KS

DA.

ately.  
hools:  
rase the Arch-  
y his Lordship

op of Quebec.  
RITHMETIC,  
of Public In-

e, to which is  
rithmetic, for

ties on Men-

uration and  
ed Questions

e année.  
RCIALE, in

URATION  
Works were  
1870:

U TRAITÉ  
d'Exercices

ithmétique  
seulement:  
containing

the year  
axon, in

## PREFACE.

---

Although this book is only an Introduction to a higher course of Commercial Arithmetic, we expect that it will be found a sufficiently complete practical treatise for Common Schools and even for Academies; in a word, for the great majority of learners.

Decimals following the same scale as whole numbers, we have chosen to treat of them with the latter; we have then introduced them with Numeration.

We have essentially followed the decimal system, but without neglecting the old method.

Our main object has been to supply our schools with a practical, and, at the same time, a cheap book, within the reach of the laboring classes. We think it contains more examples than other works of the same size. And amongst its particular features, it offers the precious advantage of Mercantile Forms, followed by nu-

PREFACE.

merous examples of application, having for principal object to render the pupil familiar with figures.

Some desire the answers placed immediately after the examples, and others desire them omitted. Both methods have their advantages and their disadvantages. In order, therefore, that pupils may receive the advantages of both methods, the answers to nearly one third of the examples in this book are omitted. They will be found, together with clear solutions of all the examples, in a Key to this work, prepared for the use of teachers and private learners.

L  
S  
R  
A  
N  
R  
R  
D  
A  
  
A  
  
E  
  
T  
F  
  
D  
B  
A  
S  
M  
  
D  
O  
E  
U  
F

r principal  
 res.  
 ately after  
 ed. Both  
 vantages.  
 ne advant-  
 one third  
 They will  
 the exam-  
 ne use of

## CONTENTS.

### SIMPLE NUMBERS AND DECIMALS.

|                                                       | PAGE. |                                                            | PAGE. |
|-------------------------------------------------------|-------|------------------------------------------------------------|-------|
| Definitions.....                                      | 9     | Subtraction.....                                           | 28    |
| Signs.....                                            | 10    | Multiplication.....                                        | 36    |
| Roman Notation.....                                   | 11    | Contractions in Multiplication.....                        | 44    |
| Arabic Notation.....                                  | 13    | Division.....                                              | 50    |
| Numeration Table.....                                 | 14    | Contractions in Division.....                              | 59    |
| Rule for Notation.....                                | 14    | Decimal Currency.....                                      | 63    |
| Rule for Numeration.....                              | 15    | Reduction of Decimal Currency.....                         | 65    |
| Decimals.....                                         | 16    | Practical Problems combining the<br>Fundamental Rules..... | 66    |
| Application of the Principles of Nu-<br>meration..... | 16    | Bills and Accounts.....                                    | 69    |
| Addition.....                                         | 22    | Forms of Bills and Accounts.....                           | 70    |

### PROPERTIES OF NUMBERS.

|                                            |    |                              |    |
|--------------------------------------------|----|------------------------------|----|
| Exact Divisors and Prime Num-<br>bers..... | 78 | Cancellation.....            | 81 |
| Table of Prime Numbers.....                | 79 | Common Divisor.....          | 82 |
| Factoring.....                             | 79 | Greatest Common Divisor..... | 83 |
|                                            |    | Least Common Multiple.....   | 84 |

### FRACTIONS.

|                                  |    |                                         |     |
|----------------------------------|----|-----------------------------------------|-----|
| Definitions, etc.....            | 86 | Division of Fractions.....              | 99  |
| Reduction of Fractions.....      | 88 | Greatest Com. Divisor of Fractions..... | 103 |
| Addition of Fractions.....       | 94 | Least Com. Multiple of Fractions.....   | 104 |
| Subtraction of Fractions.....    | 95 | Practice by Aliquot Parts.....          | 105 |
| Multiplication of Fractions..... | 96 | Miscellaneous Problems.....             | 109 |

### DENOMINATE NUMBERS.

|                          |     |                              |     |
|--------------------------|-----|------------------------------|-----|
| Definitions, etc.....    | 113 | Troy Weight.....             | 115 |
| Old Canadian Money.....  | 114 | Apothecaries' Weight.....    | 116 |
| English Money.....       | 114 | Avoirdupois Weight.....      | 116 |
| United States Money..... | 114 | Linear or Long Measure.....  | 117 |
| French Money.....        | 115 | Surveyors' Long Measure..... | 118 |

CONTENTS.

| PAGE.                                                               | PAGE.                                                               |
|---------------------------------------------------------------------|---------------------------------------------------------------------|
| Square Measure..... 118                                             | Reduction of the Decimal Currency to the Old Canadian Currency. 142 |
| Surveyors' Square Measure..... 120                                  | Addition of Compound Numbers... 142                                 |
| Cubic or Solid Measure..... 120                                     | Subtraction of Compound Numbers. 144                                |
| Liquid Measure..... 122                                             | Multiplication of Comp. Numbers.. 146                               |
| Dry Measure..... 122                                                | Multiplication of Compound Numbers by Aliquot Parts..... 149        |
| Measure of Time..... 123                                            | Division of Compound Numbers... 153                                 |
| Circular Measure..... 124                                           | Longitude and Time..... 155                                         |
| Miscellaneous Tables..... 125                                       | Duodecimals..... 156                                                |
| The Metric System of Weights and Measures..... 126                  | Multiplication of Duodecimals..... 157                              |
| Reduction of Compound Numbers.. 134                                 | Division of Duodecimals..... 158                                    |
| Reduction of the Old Canadian Currency to the Decimal Currency. 141 | Miscellaneous Examples ..... 159                                    |

RATIO, PROPORTION, AND PERCENTAGE.

|                                                  |                                       |
|--------------------------------------------------|---------------------------------------|
| Ratio..... 163                                   | Forms of Notes..... 195               |
| Proportion..... 164                              | Profit and Loss..... 197              |
| Simple Proportion..... 165                       | Commission and Brokerage..... 200     |
| Compound Proportion..... 168                     | Fire and Marine Insurance.. 203       |
| Percentage..... 171                              | Assessment of Taxes..... 205          |
| Miscellaneous Examples in Percentage..... 175    | Custom-House Business..... 207        |
| Simple Interest..... 177                         | Discount and Present Worth..... 209   |
| Partial Payments..... 184                        | Bank Discount..... 212                |
| Problems in Interest ..... 186                   | Promiscuous Examples in Discount. 216 |
| Promiscuous Examples in Simple Interest..... 188 | Stocks..... 218                       |
| Compound Interest..... 191                       | Partnership..... 223                  |
| Promissory Notes..... 193                        | Exchange..... 228                     |
|                                                  | Foreign Exchange..... 230             |
|                                                  | Equation of Payments..... 233         |

MISCELLANEOUS.

|                               |                                   |
|-------------------------------|-----------------------------------|
| Alligation Medial..... 236    | Cube Root..... 245                |
| Alligation Alternate..... 237 | Arithmetical Progression..... 248 |
| Involution..... 241           | Geometrical Progression..... 251  |
| Evolution..... 241            | Measurement of Lumber..... 253    |
| Square Root..... 242          | Miscellaneous Examples..... 254   |

incor  
ber  
5  
or a  
6  
7  
to a  
T  
1st  
eigh  
2nd  
three  
thous  
3rd  
forty  
strac  
8  
some  
yard

1. V  
tude  
are sh

|              |     |
|--------------|-----|
| PAGE.        |     |
| Currency     |     |
| Currency.    | 142 |
| Numbers...   | 142 |
| Numbers.     | 144 |
| Numbers..    | 146 |
| d. Num-      |     |
| .....        | 149 |
| Numbers..... | 153 |
| .....        | 155 |
| .....        | 156 |
| .....        | 157 |
| .....        | 158 |
| .....        | 159 |

|       |     |
|-------|-----|
| ..... | 195 |
| ..... | 197 |
| ..... | 200 |
| ..... | 203 |
| ..... | 205 |
| ..... | 207 |
| ..... | 209 |
| ..... | 212 |
| ..... | 216 |
| ..... | 218 |
| ..... | 223 |
| ..... | 228 |
| ..... | 230 |
| ..... | 233 |

|       |     |
|-------|-----|
| ..... | 245 |
| ..... | 248 |
| ..... | 251 |
| ..... | 253 |
| ..... | 254 |

# INTRODUCTION

TO THE TREATISE

ON

# COMMERCIAL ARITHMETIC.

## DEFINITIONS.

- 1. Arithmetic** is the science of numbers.
- 2. A Number** is a unit, or a collection of units.
- 3. A Unit** is one, or a single thing.
- 4. Magnitude, or Quantity,** is any thing that will admit of increase or decrease.
- 4 bis. An Integral Number, or Integer,** is a whole number; as *three, eight dollars, twenty horses.*
- 5.** Any quantity less than the unit, is a **Fractional Number, or a Fraction**; as  $\frac{1}{2}$ ,  $\frac{3}{4}$  of a foot.
- 6.** Numbers, in general, are either abstract or concrete.
- 7. Abstract Numbers** are numbers used without reference to any particular thing or quantity. Thus, *five, seven, fifteen.*  
They are divided into three classes:
  - 1st. Those which are not accompanied with subdivisions, as *four, eight, &c.*: they are called *abstract integral numbers.*
  - 2nd. Those which are accompanied with decimal subdivisions as *three units fifteen hundredths, — six units two hundred twenty-five thousandths*: they are called *abstract decimal numbers.*
  - 3rd. And lastly, those which contain only decimal subdivisions, as *forty hundredths, — seventy-five thousandths*: they are called *abstract decimal fractions, or simply decimals.*
- 8. Concrete Numbers** are numbers used with reference to some particular thing or quantity. Thus, *seven dollars, nine yards.*

---

1. What is Arithmetic?— 2. What is a number?— 3. Define unit?— 4. Magnitude or quantity?— 5. Fraction?— 6. How are numbers divided?— 7. What are abstract numbers?— 8. What are concrete numbers?

They are also subdivided into three classes :

1st. Those which contain no subdivisions, as *six yards, eleven pounds.*

2nd. Those which are accompanied with decimal subdivisions, as *five dollars twenty-five cents.*

3rd. And lastly, those which contain decimal subdivisions only, as *twenty-five cents (\$0.25).*

**9.** A **Simple-Number** is either an abstract or a concrete number of but one denomination; as, *two, ten dollars, fifteen hats.*

**10.** A **Compound Number** is a collection of concrete units whose subdivisions are not decimals, but represent several denominations, taken collectively; as, *six pounds four shillings nine pence, three feet, five inches, etc.*

**11.** A **Power** is the product arising from multiplying a number or quantity by itself, or repeating it any number of times as a factor.

**12.** A **Root** is a factor repeated to produce a power.

**13.** A **Demonstration** is the process of reasoning by which a truth or principle is established.

**14.** An **Operation** is the process of finding, from given quantities, others that are required.

**15.** A **Problem** is a question requiring an operation.

**16.** A **Rule** is a direction for performing an operation.

**17.** **Analysis**, in Arithmetic, is the process of investigating principles, and solving problems, independently of set rules.

**18.** The **Principal or Fundamental Operations** of Arithmetic are, Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

## S I G N S .

**19.** A **Sign** is a symbol employed to indicate the relations of numbers, or quantities, or operations to be performed upon them.

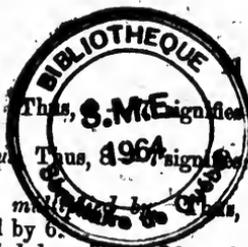
(.) is the *decimal* sign indicating that the number after it is a decimal.

§ means dollar.

---

9. What is a simple number?— 10. What is a compound number?— 11. What is a power?— 12. What is a root?— 13. What is a demonstration?— 14. What is an operation?— 15. What is a problem?— 16. A rule?— 17. What is Analysis?— 18. What are the fundamental operations of Arithmetic?— 19. What is a sign?

NOTATION.



**+**, the sign of *addition*, is read *plus*. Thus,  $8 + 6$  signifies that 7 is to be added to 8.

**-**, the sign of *subtraction*, is read *minus*. Thus,  $8 - 6$  signifies that 7 is to be subtracted from 8.

**×**, the sign of *multiplication*, is read *multiplied by*. Thus,  $9 \times 6$  signifies that 9 is to be multiplied by 6.

**÷**, the sign of *division*, is read *divided by*. Thus,  $32 \div 8$  signifies that 32 is to be divided by 8.

**=**, the sign of *equality*, is read *equal*, or *equal to*. Thus,  $8 + 6 = 14$ , signifies that 8 plus 6 is equal to 14.

**( )**, a parenthesis, the sign of *aggregation*, indicates that all the numbers, or quantities, included within it, are to be considered as a single one. Thus,  $(7 + 4) \times 3$ , indicates that the sum of 7 and 4, or 11, is to be multiplied by 3. A vinculum or bar,  $\overline{\quad}$ , has the same signification. Thus,  $\overline{9 \times 4} + 3 = 12$ .

**[ ]**, brackets or crotchets, are used to indicate that the operations on the quantities contained within the parenthesis have been performed, but before those indicated by the signs outside the brackets. Thus,  $[(8 \times 7) + 14] \div 2$  comes to  $8 \times 7 = 56$ ;  $56 + 14 = 70$ ;  $70 \div 2 = 35$ .

**:** is the sign of *ratio*. Thus,  $5 : 4$  means the ratio of 5 to 4, and is read 5 is to 4.

**::** indicates the sign of *proportion*, or the equality of ratios. Thus,  $6 : 9 :: 8 : 12$ , is read, 6 is to 9 as 8 is to 12.

NOTATION AND NUMERATION.

**20.** Notation is the process of *expressing* numbers by letters or figures; and,

**21.** Numeration is the process of *reading* numbers when expressed by figures.

**22.** Two methods of notation are in common use—the *Roman* and the *Arabic*.

ROMAN NOTATION.

**23.** The *Roman Notation*, so called from its having originated with the Romans, employs *seven* capital letters to express numbers, *vis.* :

|      |       |      |        |          |          |           |
|------|-------|------|--------|----------|----------|-----------|
| I    | V     | X    | L      | C        | D        | M         |
| one, | five, | ten, | fifty, | one      | five     | one       |
|      |       |      |        | hundred, | hundred, | thousand. |

20. What is notation?— 21. What is numeration?— 22. How many methods of notation are now in common use?— 23. Why is the Roman notation so called.

It will be seen from the following Table, that all numbers may be expressed by the use of these letters, either by repetitions or combinations.

1st. Every repetition of a letter repeats its value; thus, II, represents *two*; III, represents *three*; XX, *twenty*, etc.

2nd. When a letter of any value is placed after one of greater value, it *adds* its own value to the greater; but when placed *before*, its value is to be subtracted; thus, VII represents *seven*; XI represents *eleven*; while IX represents *nine*, or one less than ten; XL, *forty*, etc.

3rd. A bar or dash (—) placed over a letter, increases its value a thousand-fold; thus  $\bar{V}$  denotes *five thousand*;  $\bar{IV}$ , *four thousand*;  $\bar{X}$ , *ten thousand*, etc.

TABLE.

|          |                 |                 |                    |
|----------|-----------------|-----------------|--------------------|
| I.....   | is One.         | XXVII           | is Twenty-seven.   |
| II.....  | " Two.          | XXIX.           | " Twenty-nine.     |
| III..... | " Three.        | XXX...          | " Thirty.          |
| IV.....  | " Four.         | XXXVI           | " Thirty-six.      |
| V.....   | " Five.         | XL.....         | " Forty.           |
| VI.....  | " Six.          | XLIX.           | " Forty-nine.      |
| VII..... | " Seven.        | L.....          | " Fifty.           |
| VIII.... | " Eight.        | LX.....         | " Sixty.           |
| IX.....  | " Nine.         | LXX...          | " Seventy.         |
| X.....   | " Ten.          | LXXXI           | " Eighty-one.      |
| XI.....  | " Eleven.       | XC.....         | " Ninety.          |
| XII....  | " Twelve.       | XCIV.           | " Ninety-four.     |
| XIII.... | " Thirteen.     | C.....          | " One hundred.     |
| XIV....  | " Fourteen.     | CCC...          | " Three hundred.   |
| XV.....  | " Fifteen.      | CD....          | " Four hundred.    |
| XVI....  | " Sixteen.      | D.....          | " Five hundred.    |
| XVII..   | " Seventeen.    | DC....          | " Six hundred.     |
| XVIII.   | " Eighteen.     | CM....          | " Nine hundred.    |
| XIX....  | " Nineteen.     | M.....          | " One thousand.    |
| XX.....  | " Twenty.       | MC....          | " Eleven hundred.  |
| XXI....  | " Twenty-one.   | MD....          | " Fifteen hundred. |
| XXII..   | " Twenty-two.   | MM....          | " Two thousand.    |
| XXIII.   | " Twenty-three. | MMM.            | " Three thousand.  |
| XXIV.    | " Twenty-four.  | X.....          | " Ten thousand.    |
| XXV...   | " Twenty-five.  | $\bar{M}$ ..... | " One million.     |

## EXERCISES IN ROMAN NOTATION.

Express the following numbers by letters:

- |                 |                 |                                           |
|-----------------|-----------------|-------------------------------------------|
| 1. Six.         | <i>Ans. VI.</i> | 9. Thirty.                                |
| 2. Eight.       |                 | 10. Forty-six.                            |
| 3. Ten.         |                 | 11. Fifty-four.                           |
| 4. Thirteen.    |                 | 12. Sixty.                                |
| 5. Fifteen.     |                 | 13. Sixty-eight.                          |
| 6. Seventeen.   |                 | 14. Eighty-four.                          |
| 7. Nineteen.    |                 | 15. Ninety-nine.                          |
| 8. Twenty-five. |                 | 16. One hundred and six.                  |
|                 |                 | 17. Four hundred and nineteen.            |
|                 |                 | 18. Eight hundred and seventy-five.       |
|                 |                 | 19. Nine hundred and sixty-five.          |
|                 |                 | 20. Four hundred and forty-one.           |
|                 |                 | 21. Four hundred and eighty-seven.        |
|                 |                 | 22. Six hundred and ninety-five.          |
|                 |                 | 23. One thousand six hundred and fifty.   |
|                 |                 | 24. One thousand eight hundred and forty. |

## ARABIC NOTATION.

24. Arabic Notation employs ten characters, or figures, to express numbers, viz. :

1 2 3 4 5 6 7 8 9 0  
One, two, three, four, five, six, seven, eight, nine, cipher.

25. The first nine characters are called *significant figures*, because each has a value of its own. They are sometimes called *digits*, from the Latin word *digitus*, which signifies *finger*. The *cipher* is called *naught*, or *zero*, because it has no value of its own.

26. In order to reduce the numeral *figures* to a small number, we give each a second value according to the place it occupies. Thus, the first represents the *units*; the second, the *tens*; the third, the *hundreds*; the fourth, *units of thousands*; and so on, each succeeding figure to the left belonging to a distinct order, the unit of which is tenfold the value of a unit of the order to the right.

27. Since the value of a number expressed by any figure depends upon the place the latter occupies, it follows that figures have two **Values**; the one *absolute* or simple, that is, the value expressed by a figure standing alone, or, when in a collection, standing in the right-hand place; the other *relative* or *local*, that

24. How many characters are employed in the Arabic Notation?— 25. What are the first nine characters called?— 26. How can we represent all possible numbers with the ten figures?— 27. How many values have figures?

is, the value expressed by a figure used in combination with other figures and depending upon the place the figure occupies. The cipher becomes significant when connected with other figures only, by filling a place which otherwise, would be vacant (No. 28).

Thus, in 8042, the simple value of the first figure on the left hand is 8, and its local value 8 units of thousands, because it is a figure of the fourth order; the simple value of the third figure is 4, and its local value 4 tens, because it is a figure of the 2nd order; the simple value of the first is 2, and its local value 2 units; the cipher fills the vacant place of the hundreds.

## NUMERATION TABLE.

|                          |                           |                           |                        |                       |                       |                        |                  |
|--------------------------|---------------------------|---------------------------|------------------------|-----------------------|-----------------------|------------------------|------------------|
| Hundreds of Sextillions. | Hundreds of Quintillions. | Hundreds of Quadrillions. | Hundreds of Trillions. | Hundreds of Billions. | Hundreds of Millions. | Hundreds of Thousands. |                  |
| Tens of Sextillions.     | Tens of Quintillions.     | Tens of Quadrillions.     | Tens of Trillions.     | Tens of Billions.     | Tens of Millions.     | Tens of Thousands.     |                  |
| 1 2 7,                   | 8 9 4,                    | 2 3 7,                    | 8 6 7,                 | 5 2 3,                | 6 7 8,                | 4 7 8,                 | 6 3 8.           |
| Period of Sextillions.   | Period of Quintillions.   | Period of Quadrillions.   | Period of Trillions.   | Period of Billions.   | Period of Millions.   | Period of Thousands.   | Period of Units. |

## RULE FOR NOTATION.

**28.** To write in figures any number without difficulty.

*Place as many dots or points as the number contains figures; then begin at the left-hand, and write each figure in the place it must occupy, and if there are any vacant places, supply them with naughts or ciphers.*

*Example.* Given the number *four billions six millions twenty thousand five hundred units.*

Write the 4 in the order of the units of billions, the 6 in the order of the units of millions, the 2 in the order of tens of thousands, the 5 in the order of hundreds of units, and put ciphers in the vacant places. Thus

4 0 0 6 0 2 0 5 0 0

28. What is the rule for notation?

RULE FOR NUMERATION.

29. To read numbers represented by figures.

*Begin at the right hand, and point off the figures into periods of three places each. The first period is called UNITS; the second, THOUSANDS; the third, MILLIONS; the fourth, BILLIONS; the fifth, TRILLIONS, &c. The last may have but one or two figures...*

*Ex. The number 345 678 907 654 326 is read in the following manner: three hundred and forty-five trillions, six hundred and seventy-eight billions, nine hundred and seven millions, six hundred and fifty-four thousands, three hundred and twenty-six units.*

EXERCISES IN NUMERATION OF SIMPLE NUMBERS.

READ AND WRITE THE FOLLOWING NUMBERS.

|    |          |     |           |     |           |
|----|----------|-----|-----------|-----|-----------|
| 1. | 400      | 7.  | 800800003 | 13. | 28754105  |
| 2. | 6004     | 8.  | 87974015  | 14. | 1000500   |
| 3. | 80067    | 9.  | 35000918  | 15. | 3008727   |
| 4. | 670005   | 10. | 30150900  | 16. | 505054045 |
| 5. | 9006014  | 11. | 708000549 | 17. | 78592835  |
| 6. | 92100121 | 12. | 4050300   | 18. | 106405021 |

EXERCISES IN NOTATION AND NUMERATION OF SIMPLE NUMBERS.

EXPRESS BY FIGURES AND READ THE FOLLOWING NUMBERS:

1. Twenty-seven, forty-eight, sixty-five.
2. Seventy-five, ninety-three, eight hundred.
3. One hundred and ten, one hundred and twenty-four.
4. Three hundred and fifty-one, six hundred and two.
5. Four hundred and ninety-one, nine hundred and nine.
6. One thousand and one, three hundred and three.
7. Eight thousand one hundred and twelve, thirty-six thousand.
8. Nine hundred and seventeen thousand five hundred and two.
9. Seven hundred and eighteen thousand three hundred and ten.
10. Two millions six hundred and twenty-five thousand.
11. Seventy-seven millions eight hundred thousand and fifteen.
12. Four hundred millions three thousand four hundred.

EXPRESS THE FOLLOWING ROMAN NUMBERS BY FIGURES.

|    |    |     |      |     |      |     |        |
|----|----|-----|------|-----|------|-----|--------|
| 1. | IV | 7.  | V    | 13. | XXXV | 19. | CD     |
| 2. | X  | 8.  | VII  | 14. | XL   | 20. | CMIV   |
| 3. | L  | 9.  | IX   | 15. | XLIX | 21. | DCCXXX |
| 4. | C  | 10. | XI   | 16. | LXV  | 22. | CMXLIX |
| 5. | D  | 11. | XV   | 17. | XCIX | 23. | XIX    |
| 6. | M  | 12. | XXIV | 18. | CVI  | 24. | MM     |

29. What is the rule for numeration?

with other  
pies. The  
gures only,  
o. 28).

on the left  
ause it is a  
figure is 4,  
order; the  
the cipher

Hundreds.  
Tens.  
Units.  
6 3 8.

Period of  
Units.

figures;  
place it  
mm with

twenty

e order  
the 5  
vacant

## DECIMALS.

**30.** By **Decimals** are meant, parts ten times, a hundred times, a thousand times, etc., smaller than the unit, or which are successively ten times smaller than the other.

**31.** The parts contained ten times in the unit are called *tenths*; the tenths of tenths, *hundredths*, because they are contained a hundred times in the unit; the tenths of hundredths, *thousandths*, because they are contained a thousand times in the unit; the tenths of thousandths, *ten-thousandths*, &c.

**32.** A whole number and decimals, in a single expression, constitute a **Mixed Number**.

Thus, 12.54 is a mixed number, and is read twelve, and decimal fifty-four hundredths; 60.208, read sixty, and decimal two hundred and eight thousandths.

## NUMERATION TABLE

FOR WHOLE NUMBERS AND DECIMALS.

ASCENDING PROGRESSION.

DESCENDING PROGRESSION.

|                        |                       |                       |                        |                  |                        |                       |                       |
|------------------------|-----------------------|-----------------------|------------------------|------------------|------------------------|-----------------------|-----------------------|
| Hundreds of Trillions. | Hundreds of Billions. | Hundreds of Millions. | Hundreds of Thousands. | Hundreds.        | Tenths of Units.       | Ten-Thousandths.      | Ten-Millionths.       |
| Tens of Trillions.     | Tens of Billions.     | Tens of Millions.     | Tens of Thousands.     | Tens.            | Hundredths.            | Hundred-Thousandths.  | Hundred-Millionths.   |
| TRILLIONS.             | BILLIONS.             | MILLIONS.             | THOUSANDS.             | UNITS.           | THOUSANDTHS.           | MILLIONTHS.           | BILLIONTHS.           |
| 1 3 6                  | 3 8 4                 | 9 4 7                 | 8 6 7                  | 2 2 3            | 4 2 0                  | 5 7 8                 | 0 9 3                 |
| Period of Trillions.   | Period of Billions.   | Period of Millions.   | Period of Thousands.   | Period of Units. | Period of Thousandths. | Period of Millionths. | Period of Billionths. |

As is easily seen, decimals, with regard to their order, follow inversely the system of numeration of whole numbers; the *tenth* is ten times smaller than the unit, whereas *ten* is the unit repeated ten times; the *hundredth* expresses the hundredth part of the unit, and a hundred, the unit repeated hundred times, &c.

30. What are decimals?— 31. What names are given to decimal parts?— 32. What is a mixed number?

**33.** The formation of decimal parts is made obvious by the following example :

If an apple be divided into ten equal parts, each piece will represent the tenth part of the unit or the whole apple. Dividing again each tenth into ten equal parts, we obtain hundredths. The result would be the same for a line, a dollar, &c.

**34.** From the foregoing illustrations, we deduce the following rules :

**I.** Write first the whole number, after which place the decimal point ; then from the left to the right, write successively the tenths, the hundredths, the thousandths, &c.

Thus, the number 3 units 25 hundredths is written 3.25.

**II.** If some order of decimals be wanting, fill the vacant places with ciphers.

Thus, the number 12 units 5 hundredths is written 12.05, in placing a cipher to represent the tenths ; and 4 units 3 tenths 8 hundredths and 8 ten-thousandths is represented by 4.3808.

**III.** If there are decimals only, a cipher is put in the place of the units, and the decimals follow in their regular order.

Thus, one tenth is expressed by 0.1 ; 5 tenths 8 hundredths, 0.58 ; 5 thousandths, 0.005.

There is always a figure less in decimals than in a corresponding whole number, because the figure of the unit which is included in the whole number is not included in decimals.

**35.** Annexing ciphers to decimals does not alter their value as long as the decimal point is not displaced ; the parts are made ten, a hundred times more numerous, but they are ten, a hundred times smaller : there is then compensation.

Thus, 0.25 becomes 0.250 by the addition of one cipher, and 0.2500 by the addition of two, but the value of the decimal is always equivalent to 25 hundredths.

### EXERCISES ON DECIMAL NOTATION AND NUMERATION.

WRITE IN FIGURES THE FOLLOWING MIXED NUMBERS.

1. Two hundred and sixteen, and three tenths.
2. Five hundred and seven, and twenty-five hundredths.
3. Twenty-seven, and four hundredths.
4. Three hundred and twenty-one, and nine millionths.
5. Forty-four, and twenty-three hundredths.
6. Three hundred, and forty-two ten-thousandth.
7. Twenty, and forty-eight thousandths.
8. Four hundred and ten, and five hundredths.

**33.** Explain by an example the formation of decimal parts ?— **35.** Is the value of decimals altered by placing ciphers at their right hand side ?

undred times,  
ich are suc-

t are called  
ey are con-  
undredths,  
times in the

expression,

and decimal  
two hundred

PROGRESSION.

ILLIONTHS.  
Ten-Millionths.  
Hundred-Millionths.  
BILLIONTHS.  
3, 0 9 3.

Period of  
Bill-  
ionths.

er, follow  
the tenth  
repeated  
t of the  
o.

arts 1— 32.

9. Two hundred and twenty, and nine hundred-thousandths.
10. One thousand and six, and five ten-thousandths.
11. Four thousand and seven, and three hundred-thousandths.
12. Fifty-nine, and twenty-two millionths.
13. Eighty-two, and thirty-six hundred-millionths.
14. Eight hundred and fifteen, and sixteen thousandths.
15. Twenty-seven, and one hundred and two billionths.
16. Twenty thousand and ten, and thirty millionths.

EXPRESS ORALLY AND WRITE IN WORDS THE FOLLOWING MIXED NUMBERS AND SINGLE DECIMALS.

*Mixed numbers.*

|               |                 |    |             |
|---------------|-----------------|----|-------------|
| 1.   8.90     | 5.   354.0064   | 9  | 41.004064   |
| 2.   9.908    | 6.   352.06046  | 10 | 452.010778  |
| 3.   541.400  | 7.   76.26007   | 11 | 7657.008007 |
| 4.   703.2004 | 8.   375.500506 | 12 | 1898.04     |

*Single decimals.*

|                |                |                   |
|----------------|----------------|-------------------|
| 1.   0.004     | 5.   0.4872    | 9.   0.69804445   |
| 2.   0.000607  | 6.   0.401950  | 10.   0.736050210 |
| 3.   0.005     | 7.   0.9540626 | 11.   0.000500019 |
| 4.   0.0007007 | 8.   0.075003  | 12.   0.00000501  |

APPLICATION OF THE PRINCIPLES OF NUMERATION

AS LAID DOWN IN NOS. 27 & 31.

**36.** According to the principles laid down in Nos. 27 & 31, it follows:

1st. That, to render a *whole* number, ten, a hundred, a thousand times greater, we must write at the right-hand side of the number, one, two, three naughts or ciphers (1).

Thus, the number 26 units, becomes 260 in adding a cipher after the 6, that is, ten times greater than the first, since the units become tens, and the tens, hundreds; or, in other words, the figure of the first order becomes a figure of the second order, and that of the second order, becomes a figure of the third order. If we add another cipher, we obtain 2600, which is a hundred times greater than the first number, since the 260 units have become 26 hundreds.

2nd. That, when the whole number has a decimal annexed, it suffices to remove the decimal point to one, two, three, &c., figures towards the right, to render the number ten, a hundred, a thousand, &c., times greater.

Thus, 26.35 becomes ten times greater if written 263.5, since the tenths become units, the units tens, &c.

36. How can a whole number be rendered ten, a hundred, a thousand times greater?—36. Do. A whole number with a decimal annexed?

(1) This means that the number obtained, equals ten, a hundred times, &c., the first; otherwise this reasoning would lead us to conclude that the unit is ten times greater than the unit, which would be absurd.

3rd. That, when the number of decimals is not sufficient to render the number as required, we must annex to its right-hand side as many ciphers as will answer the purpose.

Thus, to render the number 24.5 a thousand times greater, it would be necessary to remove the point three figures towards the right; but, as there is only one decimal, place two ciphers after the five, and the number becomes 24500, which number is evidently a thousand times greater than the first, since the units of the first order have been changed into units of thousands, or of the second order.

37. From the same principles, it follows also :

1st. That, to render a whole number ten, a hundred, a thousand times, &c., smaller, it suffices to cut off from the right-hand side one, two, three, &c., figures.

Thus, in the number 925; if we cut off two figures by the decimal point we obtain 9.25, which is a hundred times smaller than the first, since the hundreds have become units, the tens, tenths, &c.

2nd. That, if it be a mixed number, the decimal point must be removed one, two, three, &c., figures towards the left.

Thus, to render 26.35 ten times smaller, remove the decimal point one figure towards the left, and it becomes 2.635, that is, ten times smaller than the first, since the tens become units, &c.

3rd. That, if the number, either whole or decimal only, does not contain a sufficient number of figures at the left-hand side of the point, we must write as many ciphers as will answer the purpose of the question, taking care that one remain to take the place of the units.

Thus, to render the numbers 8 and 2.635 a thousand times smaller, place three ciphers on the left-hand side of each of those numbers; the first of these ciphers will hold the place of the units, and the others will reduce the primitive number to the required value; thus, the numbers become 0.008 and 0.002635, which are evidently a thousand times smaller than the first, since the units have been changed into thousandths, &c.

PRACTICAL EXERCISES

ON THE PROPERTIES OF DECIMAL NUMERATION.

I. Render the whole number 38

|                |         |                  |      |          |
|----------------|---------|------------------|------|----------|
| 1 <sup>o</sup> | 10      | } times greater. | Ans. | 380.     |
| 2 <sup>o</sup> | 100     |                  | Ans. |          |
| 3 <sup>o</sup> | 1000    |                  | Ans. | 38000.   |
| 4 <sup>o</sup> | 10000   |                  | Ans. |          |
| 5 <sup>o</sup> | 100000  |                  | Ans. | 3800000. |
| 6 <sup>o</sup> | 1000000 |                  | Ans. |          |

37. How can a whole number be rendered, ten, a hundred, &c., times smaller?  
 37. Do. mixed number?

## THE PROPERTIES OF NUMERATION.

2. Render the mixed number 42.1064231

|    |         |                  |      |            |
|----|---------|------------------|------|------------|
| 1° | 10      | } times greater. | Ans. |            |
| 2° | 100     |                  | Ans. | 4210.64231 |
| 3° | 1000    |                  | Ans. |            |
| 4° | 10000   |                  | Ans. | 421064.231 |
| 5° | 100000  |                  | Ans. |            |
| 6° | 1000000 |                  | Ans. | 42106423.1 |

3. Render the mixed number 4.20

|    |         |                  |      |        |
|----|---------|------------------|------|--------|
| 1° | 10      | } times greater. | Ans. | 42.    |
| 2° | 100     |                  | Ans. |        |
| 3° | 1000    |                  | Ans. |        |
| 4° | 10000   |                  | Ans. | 42000. |
| 5° | 100000  |                  | Ans. |        |
| 6° | 1000000 |                  | Ans. |        |

4. Render the decimal 0.05

|    |         |                  |      |     |
|----|---------|------------------|------|-----|
| 1° | 10      | } times greater. | Ans. | 0.5 |
| 2° | 100     |                  | Ans. |     |
| 3° | 1000    |                  | Ans. | 50. |
| 4° | 10000   |                  | Ans. |     |
| 5° | 100000  |                  | Ans. |     |
| 6° | 1000000 |                  | Ans. |     |

5. Render the whole number 6705415

|    |         |                  |      |              |
|----|---------|------------------|------|--------------|
| 1° | 10      | } times greater. | Ans. |              |
| 2° | 100     |                  | Ans. |              |
| 3° | 1000    |                  | Ans. |              |
| 4° | 10000   |                  | Ans. | 67054150000. |
| 5° | 100000  |                  | Ans. |              |
| 6° | 1000000 |                  | Ans. |              |

6. Render the mixed number 7610438.06

|    |         |                  |      |            |
|----|---------|------------------|------|------------|
| 1° | 10      | } times smaller. | Ans. |            |
| 2° | 100     |                  | Ans. | 76104.3806 |
| 3° | 1000    |                  | Ans. |            |
| 4° | 10000   |                  | Ans. |            |
| 5° | 100000  |                  | Ans. | 76.1043806 |
| 6° | 1000000 |                  | Ans. |            |

7. Render the mixed number 5.45

|    |         |                  |      |            |
|----|---------|------------------|------|------------|
| 1° | 10      | } times smaller. | Ans. |            |
| 2° | 100     |                  | Ans. |            |
| 3° | 1000    |                  | Ans. | 0.00545    |
| 4° | 10000   |                  | Ans. |            |
| 5° | 100000  |                  | Ans. |            |
| 6° | 1000000 |                  | Ans. | 0.00000545 |

THE PROPERTIES OF NUMERATION.

8. Render the decimal 0.05

4210.64231

6423.1

6423.1

|    |         |                  |             |          |
|----|---------|------------------|-------------|----------|
| 1° | 10      | } times smaller. | <i>Ans.</i> | 0.000005 |
| 2° | 100     |                  | <i>Ans.</i> |          |
| 3° | 1000    |                  | <i>Ans.</i> |          |
| 4° | 10000   |                  | <i>Ans.</i> |          |
| 5° | 100000  |                  | <i>Ans.</i> |          |
| 6° | 1000000 |                  | <i>Ans.</i> |          |

42.

9. Render the mixed number 206.007

2000.

|    |         |                  |             |            |
|----|---------|------------------|-------------|------------|
| 1° | 10      | } times smaller. | <i>Ans.</i> | 0.00206007 |
| 2° | 100     |                  | <i>Ans.</i> |            |
| 3° | 1000    |                  | <i>Ans.</i> |            |
| 4° | 10000   |                  | <i>Ans.</i> |            |
| 5° | 100000  |                  | <i>Ans.</i> |            |
| 6° | 1000000 |                  | <i>Ans.</i> |            |

0.5

10. Render the mixed number 1462.309.

50.

|    |         |                  |             |          |
|----|---------|------------------|-------------|----------|
| 1° | 10      | } times smaller. | <i>Ans.</i> | 146.2309 |
| 2° | 100     |                  | <i>Ans.</i> |          |
| 3° | 1000    |                  | <i>Ans.</i> |          |
| 4° | 10000   |                  | <i>Ans.</i> |          |
| 5° | 100000  |                  | <i>Ans.</i> |          |
| 6° | 1000000 |                  | <i>Ans.</i> |          |

11. Render

00.

|     |           |                     |                       |
|-----|-----------|---------------------|-----------------------|
| 12. | 165.      | 10 times greater.   | <i>Ans.</i> 1650.     |
| 13. | 3867.     | 100 " smaller.      | <i>Ans.</i> 38.67     |
| 14. | 2064.15   | 1000 " greater.     | <i>Ans.</i>           |
| 15. | 640.4     | 100 " "             | <i>Ans.</i> 64040.    |
| 16. | 74.       | 1000 " smaller.     | <i>Ans.</i> 0.074     |
| 17. | 746.      | 10000 " greater.    | <i>Ans.</i>           |
| 18. | 9.35      | 100 " smaller.      | <i>Ans.</i>           |
| 19. | 76874.    | 10000000 " greater. | <i>Ans.</i>           |
| 20. | 6.468     | 1000 " "            | <i>Ans.</i>           |
| 21. | 0.45      | 1000 " "            | <i>Ans.</i> 450.      |
| 22. | 9.10      | 1000 " smaller.     | <i>Ans.</i>           |
| 23. | 0.05      | 1000 " "            | <i>Ans.</i> 0.00005   |
| 24. | 9.6786    | 10000 " greater.    | <i>Ans.</i>           |
| 25. | 4.0000007 | 100 " smaller.      | <i>Ans.</i>           |
| 26. | 0.0007    | 100 " greater.      | <i>Ans.</i>           |
| 27. | 14.666    | 10000 " smaller.    | <i>Ans.</i>           |
| 28. | 0.7       | 10 " "              | <i>Ans.</i>           |
| 29. | 674.867   | 10000000 " greater. | <i>Ans.</i>           |
| 30. | 40.6804   | 1000 " "            | <i>Ans.</i>           |
| 31. | 60600867. | 10000 " smaller.    | <i>Ans.</i> 6060.0867 |
| 32. | 9.45      | 10000000 " "        | <i>Ans.</i>           |
| 33. | 46.520    | 1000000 " greater.  | <i>Ans.</i>           |
| 34. | 76840.3   | 100000 " smaller.   | <i>Ans.</i>           |
|     | 0.013     | 10000 " greater.    | <i>Ans.</i>           |

0.00545

0.00000545

04.3806

6.1043806

## ADDITION.

**38.** Addition is the process of uniting together several numbers of the same kind, so as to form a single number called the **Sum** or **Amount**.

**39.** Numbers are of the same kind when they have the same denomination.

For instance, dollars can be added to dollars, pounds to pounds, and yards to yards, &c.; but dollars could not be added to yards, nor shillings to feet.

*Example of an Addition with whole numbers.*

What is the sum of the three following numbers: 428, 635, and 874?

OPERATION.

$$\begin{array}{r} 428 \\ 635 \\ 874 \\ \hline 1937 \end{array}$$

**ANALYSIS.**—Having arranged the numbers, so that all the units of the same order shall stand in the same column, we first add the column of *units*; thus, 8 and five are 13, and 4 are 17 units, = 1 ten and 7 units. We write the 7 units under the column of units, and *carry* or add the 1 ten to the column of tens; thus, 1 added to 2 makes 3, and 3 are 6, and 7 are 13 tens = 1 hundred and 3 tens. We write the 3 tens under the column of tens, and carry the 1 hundred to the column of hundreds; thus, 1 added to 4 makes 5, and 6 are 11, and 8 are 19 hundreds = 1 thousand and 9 hundreds. We write the 9 hundreds under the column of hundreds; and there being no other column to be added, we set down the 1 thousand in the thousands' place, and find the amount of the three numbers to be 1937.

**40.** We begin the addition by the figures of the first column at the right-hand side, so that in whole numbers, we may carry the tens proceeding from the addition of the units to the column of the tens, the hundreds proceeding from the tens to the column of the hundreds, &c.; and also in decimals, carry the tenths proceeding from the hundredths to the column of the tenths, and the units proceeding from the addition of the tenths to the column of the units, and so on.

**41.** From the preceding illustrations we deduce the following:

**RULE.**—I. *Write the numbers to be added so that all the units of the same order shall stand in the same column; that is, units under units, tens under tens, etc.*

II. *Beginning at units, add downward, or upward, each column separately, and write the sum underneath, if it be less than ten.*

III. *If the sum of any column be ten, or more than ten, write the unit figure only, and add the ten or tens to the next column.*

IV. *Write the whole sum of the last column.*

38. What is addition?—39. What is meant by numbers of the same kind?—40. How is addition to be commenced?—41. What is the general rule for addition?

## ADDITION OF DECIMALS.

**42. RULE.**—In adding decimals, we follow the same process as for the addition of whole numbers; but we cut off from the right, by a point, as many decimals as there are in the number which contains the most of all the numbers added.

*Ex.* Given the following numbers to be added: 3579 units 25 hundredths, 4682 units 15 hundredths, 573 units 75 hundredths, and 7856 units 80 hundredths.

OPERATION.

$$\begin{array}{r} 3579.25 \\ 4682.05 \\ 573.75 \\ 7856.80 \\ \hline \end{array}$$

*Ans.* 16691.85  
which is read in the following manner: 16691 units 85 hundredths.

**ANALYSIS.**—We commence by the 1st. column at the right; thus, 5 and 5 are 10, and 5 are 15 hundredths = 1 tenth and 5 hundredths. We write the 5 hundredths under the column of hundredths, and carry the 1 tenth to the column of the tenths; thus, 1 and 2 are 3, and 7 are 10, and 8 are 18 tenths = 1 unit and 8 tenths. We write the 8 under the column of tenths and carry the 1 unit to the column of units; thus, 1 and 9 are 10, &c.

## PROOF.

**43.** The Proof of an Arithmetical operation is another operation to prove the exactness of the first.

**44. RULE.**—Separate the numbers to be added into two parts; add each of these two parts and set down their respective sums. Then add these two sums together; if their amount is equal to the first answer, the work is presumed to be correct.

The numbers could be divided into a greater number of parts than two.

*Example.*

| OPERATION.          | PROOF.     |            | Addition of partial totals. |
|---------------------|------------|------------|-----------------------------|
| 123.24              | 1st. Part. | 2nd. Part. | 1172.91                     |
| 349.00              | 123.24     | 56.25      | 472.24                      |
| 56.25               | 349.00     | 149.34     | 1645.15                     |
| 149.34              | 472.24     | 967.32     |                             |
| 967.32              |            | 1172.91    |                             |
| <i>Ans.</i> 1645.15 |            |            |                             |

which is read 1645 units 15 hundredths.

**USE OF ADDITION.**—Addition serves to find the sum of several numbers: the whole cost when the buying price and other expenses are given. The selling price when the buying price and profit are given, &c.

We know that the resolution or solving of a problem requires an addition, when we must find a number equal to the sum or amount of several others.

43. How do you add decimals?—43. What do you understand by an arithmetical proof?—44. How is the proof of an addition made?

## PRACTICE IN ADDITION.

1.  $600 + 850 + 501 + 49 + 904 + 769 + 215 + 555.$

2.  $604 + 810 + 333 + 1226 + 8004 + 4004 + 5105.$  *Ans. 4433 units.*

3.  $19223 + 125979 + 189023 + 100610 + 3300.$  *Ans. 15086.*

4.  $15879 + 15957 + 100101 + 810799 + 975020 + 100110.$  *Ans. 452135.*

5.  $41 + 64 + 77 + 49 + 64 + 47 + 36 + 1012.$  *Ans. 1390.*

6.  $110200 + 9104 + 4610 + 10110 + 95303 + 8888.$  *Ans. 1390.*

7.  $100989 + 100001454 + 77777707 + 10110000 + 100000090.$

8.  $50319010 + 15015 + 132 + 20000020 + 109909 + 8888888.$  *Ans. 80317134.*

9.  $49 + 97 + 68 + 45 + 54 + 68 + 38 + 97 + 75 + 63 + 49 +$

10.  $49 + 468 + 429 + 47 + 64 + 46 + 36 + 49 + 94 + 39 +$  *Ans. 1238.*

11.  $56 + 48 + 64 + 46 + 57 + 86 + 39 + 47 + 74 + 93 + 57.$

12.  $52 + 34 + 42 + 29 + 423 + 695 + 987 + 429 + 678 + 542.$

13.  $94 + 569 + 439 + 590 + 694 + 678 + 594 + 864 + 684 + 468.$

14.  $249 + 75 + 99 + 88 + 89 + 98 + 36 + 674 + 99 + 89 + 69 +$

15.  $429 + 98 + 103 + 138 + 274 + 391.$

16.  $94 + 95 + 649 + 946 + 495 + 789 + 647 + 963 + 769 + 956 +$

17.  $875 + 708 + 1075 + 3548 + 739.$

14. Express by figures and add up the following numbers: eighteen

units, + ninety-five, + one hundred and one, + one hundred and twenty-

three, + three hundred and ten, + six hundred. *Ans. 1247.*

15. Required the sum of six hundred units, + eight hundred and

fifty, + five hundred and one, + forty-nine, + nine hundred and four,

+ seven hundred and fifty-nine, + two hundred and fifteen, and five

hundred and fifty-five.

16. Express by figures one hundred and ninety-five, + two hundred

and eleven, + one hundred and ten, + one hundred and ninety-nine,

+ eight hundred and one, + seven hundred and seventy-seven, +

nine hundred and one. *Ans. 3194.*

17. Express by figures two thousand one hundred and ninety-

seven, + twenty-three thousand six hundred and fifteen, + twelve

thousand six hundred and ten, + one thousand and fifteen, and make

up the sum.

18. Required the sum of nineteen thousand two hundred and twenty-

three units, + one hundred and twenty-five thousand nine hundred

and seventy-nine, + one hundred and eighty-nine thousand and

twenty-three, + one hundred thousand six hundred and ten, + three

thousand and three hundred.

19. Required the sum of fifteen thousand eight hundred and seventy-

nine units, + fifteen thousand nine hundred and fifty-seven, + one

hundred thousand one hundred and one, + eight hundred and ten

thousand seven hundred and ninety-nine, + nine hundred and seventy-

five thousand, + one hundred thousand and ten? *Ans. 2017746.*

- 555.

s. 4433 units

05.

Ans. 15086

Ans. 438135

+ 100110

Ans. 1390.

10000090.

99 + 8888888

s. 80317134.

+ 63 + 49 +

Ans. 1238.

+ 94 + 39 +

74 + 98 + 57.

95 + 34 + 66

69 + 49 + 95

+ 678 + 542.

+ 89 + 69 +

64 + 68 + 468

769 + 956 +

ers: eighteen

d and twenty-

Ans. 1247.

undred and

red and four,

en, and five

two hundred

nety-nine,

nty-seven, +

Ans. 3194.

and ninety-

undred, + twelve

n, and make

and twenty-

ne hundred

ousand and

en, + three

and seventy-

ven, + one

ten and

and seventy-

2017746.

20. Required the sum of fifty millions three hundred and nineteen thousand and ten units, + fifteen thousand and fifteen, + one hundred and thirty-two, + twenty millions and twenty, + one hundred and nine thousand nine hundred and nine, + eight millions eight hundred and eighty-eight thousand eight hundred and eighty-eight units, + eleven thousand, + eleven hundred and eleven?

21.  $40.05 + 104.8 + 1003.025 + 7.38 + 2.15$ .

Ans. 1157.405 thousandths.

22.  $0.4 + 0.20 + 0.0306 + 0.01 + 0.200 + 0.044 + 0.18$ .

Ans. 1.0646 ten thousandths.

23.  $0.05 + 0.00012 + 0.110 + 0.22 + 0.000015 + 0.014 + 0.0017$ .

24.  $100. + 0.400 + 20.130 + 0.020 + 0.10012 + 0.0001005 + 0.1$ .

25.  $6.96 + 3.99 + 6.78 + 4.39 + 4.79 + 2.98 + 4.67 + 7.69 + 4.42 + 6.81 + 7.59 + 9.76 + 4.36 + 7.95 + 5.35 + 7.77 + 3.79 + 9.99 + 7.889$ .

Ans. 117.929 thousandths.

26.  $4.95 + 9.54 + 8.69 + 4.29 + 24.09 + 4.07 + 7.45 + 3.68 + 9.86 + 7.85 + 7.67 + 3.75 + 47.47 + 9.09 + 4.47 + 6.97$ .

27.  $3.78 + 8.95 + 9.84 + 9.38 + 37.14 + 6.053 + 67 + 4.78 + 4.98 + 5.75 + 7.75 + 5.55 + 47 + 15 + 1.75 + 2.55 + 8.47$ .

28.  $4.24 + 4.70 + 3.65 + 1.95 + 1640 + 49.65 + 3.45 + 2.90 + 9.80 + 1.40 + 3.55 + 7.40 + 4.65 + 9.09 + 7.60 + 55.45 + 2.95$ .

29. Express by figures forty units and five hundredths, + one hundred four units and eight tenths, + one thousand three units and twenty-five thousandths, + seven units and thirty hundredths, + two units and fifteen hundredths, and add them up.

Ans. 1167.405.

30. Required the sum of four tenths, + twenty thousandths, + three hundred ten-thousandths, + one hundredth, + two hundred thousandths, + forty-four thousandths, + eighteen hundredths.

31. Required the sum of four hundredths, + twelve hundred-thousandths, + one hundred ten-thousandths, + eleven hundredths, + fifteen millionths, + fourteen thousandths.

Ans. 0.174135.

32. Required the sum of three hundred-thousandths, + four thousandths, + seven tenths, + three hundred-thousandths, + eight billionths, + nineteen thousandths.

33. Add the following numbers: eight hundred-thousandths, + nine hundred ten-thousandths, + three hundred tenths, + one thousand hundredths, + thirteen ten-thousandths, + twenty millionths, + eight hundredths, + eleven hundred-thousandths, + three thousand and nineteen millionths.

Ans. 40.174529.

34. Required the sum of one thousand tenths, + four hundred thousandths, + two thousand hundredths, + thirteen hundred tenths, + twenty thousand millionths, + ten thousand and twelve hundred-thousandths, + one thousand and five ten-millionths, + one hundred thousand millionths?

35. What is the sum of the following numbers: twenty-five, and seven millionths; one hundred forty-five, six hundred and forty-three thousandths; one hundred and seventy-five, and eighty-nine hundredths; seventeen, and three hundred and forty-eight hundred thousandths?

Ans. 363.536487.

## PRACTICAL PROBLEMS OR QUESTIONS IN ADDITION.

1. I bought a house for \$25840, I paid \$1565 for right of possession, and \$238 for repairs; what did it cost me?

OPERATION.

$$\begin{array}{r} \$ 25840 \\ 1565 \\ 238 \\ \hline \end{array}$$

$$\$ 27643 \text{ Ans.}$$

ANALYSIS.—The whole cost of the house is equal to the amount of the sums expended, that is  $25840 + 1565 + 238 = \$27643$  Ans.

NOTE.—The whole cost is the cost of an object when all expenses are paid.

2. I bought some merchandise for the sum of \$245.65; how much must I sell them to gain \$25.20?

OPERATION.

$$\begin{array}{r} \$ 245.65 \\ 25.20 \\ \hline \end{array}$$

$$\$ 270.85 \text{ Ans.}$$

ANALYSIS.—We must sell the merchandise at a price equal to what it cost, plus the sum we wish to gain; that is,  $245.65 + 25.20 = \$270.85$ , selling price.

3. A family spends on Monday, \$4.75; on Tuesday, \$1.15 more than on Monday, and \$2.05 less than on Sunday: how much was spent during these three days?

ANALYSIS.—First, find the expenditure of Tuesday and Sunday. On Tuesday, they spent  $4.75 + 1.15 = \$5.90$ ; on Sunday,  $5.90 + 2.08 = \$7.98$ . Then,  $4.75 + 5.90 + 7.98 = \$18.63$ , whole expense for the three days.

4. A family owes the baker \$27; the butcher, \$46; the shoe-maker, \$69; the grocer, \$108; and for house-rent, 145; how much does the family owe in all?

Ans. \$395.

5. Louis was born in 1847, in what year will he be 24 years old?

6. The population of Montreal is about 135000 souls, that of Quebec, 64150; Three-Rivers, 8300; St. Hyacinth, 4102; Point-Levis, 5300; Sorel, 5250; Sherbrooke, 4300: what is the whole population of those seven towns?

Ans. 226402.

7. A wholesale merchant sold during the year \$9023 worth of cloth; \$4500 of yellow cotton; \$1592 of Irish linen; \$1790 of calico; \$856 of merino. For how much did he sell during the whole year?

8. A man owed a certain sum of money; he paid the 1st. time \$240.50; the 2nd. \$376.25; the 3rd. \$109.40, after which he yet owes \$92. How much did he owe at first?

Ans. \$818.15.

9. A company of soldiers have fired 29682 cartridges in an engagement and they have still 13403 remaining. How many had they before the engagement?

Ans. 43085.

10. An army consists of three grand divisions; the 1st. contains 8640 men, the 2nd. 7960, and the 3rd. 8490. How many men are there in the army?

Ans. 25090 men.

11. The hind-quarters of an ox weigh 390 pounds each; the fore-quarters 325 pounds each; the skin 97 pounds and the suet 95 pounds. What is the whole weight of the ox?

Ans. 1622 pounds.

12. Andrew bought a horse and carriage for \$310; and in selling both he gained \$176. How much did he sell them for?

Ans. \$486.

## ADDITION.

of possession,

is equal to the  
 $10 + 1665 + 238$

ject when all ex-

65; how much

disise at a price  
 to gain; that is,

, \$1.15 more  
 ow much was

ay. On Tuesday,  
 \$7.98. Then,  
 s.

he shoe-maker,  
 much does the  
*Ans.* \$395.

4 years old?  
 ouls, that of  
 4102; Point-  
 he whole po-  
 s. 226402.

orth of cloth;  
 calico; \$856  
 e year?

the 1st. time  
 hich he yet  
 s. \$818.15.

n an engage-  
 y had they  
*Ans.* 43085.

st. contains  
 y men are  
 5090 men.

h; the fore-  
 95 pounds.  
 2 pounds.

d in selling  
*Ans.* \$486.

13. A man bought three fields for \$7680, he sells them at a profit of \$750. For how much did he sell them? *Ans.* \$8430.

14. John bought a new farm in a township; the 1st. year it yielded 736 bush. of oats; the 2nd. year, 3697 bush.; the 3rd. year, 9982 bush.; the 4th. year, 10065 bush.; the 5th. year, 12760 bush.: how many bushels did it yield in the five years? *Ans.* 37240 bush.

15. How many years elapsed from the taking of Troy, which occurred 1184 years before Christ, till the year 1869 of the Christian era?

16. A person who was born in 1831, died at the age of 37. In what year did she die? *Ans.* 1868.

17. I have four bills to pay; the 1st., of \$1405; the 2nd., \$875.40; the 3rd., \$96.15; and the 4th., \$798. What sum do I require to pay them? *Ans.* \$3174.55.

18. The area of the Dominion of Canada is computed as follows: the Province of Ontario, 180000 square miles; the Province of Quebec, 210000 square miles; the Province of Nova Scotia, 19650 square miles; and the Province of New Brunswick, 27710 square miles. What is the whole area? *Ans.* 437360 square miles.

19. A tanner bought 25 hides for the sum of \$164.80; after having prepared them, he sold them for \$112.60 more than he had paid. How much did he sell them for? *Ans.* \$277.40.

20. A certain sum of money was divided among three persons: the 1st., received \$65; the 2nd., \$26.30 more than the first; the 3rd., \$32.10 more than the second. How much did each receive and what was the sum divided? *Ans.* 1st. \$65; 2nd. \$91.30; 3rd. \$123.40. Whole sum \$279.70.

21. A merchant in selling cloth to the amount of \$6218.50, lost \$143.40 by the bargain; how much did he pay for it?

22. At the census of 1861, the population of Upper Canada was 1409430 inhabitants; that of Lower Canada, 1130800; Nova Scotia, 300000; New Brunswick, 250000. How many inhabitants were there in those four Provinces which compose the present Dominion of Canada? *Ans.* 3090230 inhabitants.

23. The battle of Marathon took place 490 before Christ. How many years since that period to 1868? *Ans.* 2358 years.

24. Eighteen tanned horse-hides weigh 486 pounds; they have lost 324 pounds in being tanned. What was their raw weight?

25. A number is such that if diminished by 6487 there remains but 5976. What is the number? *Ans.* 12463.

26. Raw wool is worth \$0.75 per pound, when prepared it augments \$2.45. What is the price of a pound of prepared wool? *Ans.* \$3.20.

27. The population of Europe consists of 278694707 inhabitants; that of North America, 43879348; that of South America, 22007823; that of Asia, 688700000; that of Africa, 64035000; that of Oceanica, 20600000; that of Australia, 2025000; and that of Polynesia, 419000. What is the whole population of the globe? *Ans.* 1020360878 inhabitants.

## SUBTRACTION.

**45. Subtraction** is the process of finding the difference between two numbers of the same kind.

The larger number, or that which is to be diminished, is called the *Minuend*; and the smaller, or that which is to be subtracted, the *Subtrahend*.

**46.** The result of subtraction is called the **remainder, excess, or difference.**

**CASE I.**—*To subtract when each figure in the subtrahend is less than the figure above it in the minuend.*

*Ex.* From 547 take 324.

| OPERATION. |       |
|------------|-------|
| Minuend    | 5 4 7 |
| Subtrahend | 3 2 4 |
| Remainder  | 2 2 3 |

we write in hundreds' place. Hence we have for the remainder, 2 hundreds, 2 tens, and 3 units, or 223.

**ANALYSIS.**—We write the less number under the greater, so that units of the same order shall stand in the same column; then, we begin at the right and proceed as follows: 4 units from 7 units leave 3 units, which we write in units' place. Two tens from 4 tens leave two tens, which we write in tens' place. Three hundreds from 5 hundreds leave 2 hundreds, which

## EXAMPLES FOR PRACTICE.

|            |            |            |            |            |            |
|------------|------------|------------|------------|------------|------------|
|            | (1.)       | (2.)       | (3.)       | (4.)       | (5.)       |
| Minuend    | 457        | 273        | 936        | 685        | 974        |
| Subtrahend | 325        | 132        | 714        | 423        | 631        |
| Remainder  | <u>132</u> | <u>141</u> | <u>222</u> | <u>262</u> | <u>343</u> |
|            | (6.)       | (7.)       | (8.)       | (9.)       | (10.)      |
| From       | 648        | 376        | 857        | 498        | 645        |
| Take       | <u>234</u> | <u>164</u> | <u>522</u> | <u>175</u> | <u>542</u> |

- |                                |                              |
|--------------------------------|------------------------------|
| 11. 3692 — 1212 = Ans. 2480    | 21. 1243 — 213 = Ans...      |
| 12. 7634 — 3132 = Ans. 4502    | 22. 48673 — 16330 = Ans...   |
| 13. 8742 — 5331 = Ans. 3411    | 23. 34272 — 13051 = Ans...   |
| 14. 41763 — 11522 = Ans. 30241 | 24. 79832 — 57411 = Ans...   |
| 15. 7839 — 5427 = Ans. 2412    | 25. 15475 — 4050 = Ans...    |
| 16. 3724 — 2502 = Ans. 1222    | 26. 15768 — 4327 = Ans...    |
| 17. 2945 — 832 = Ans. 2113     | 27. 982876 — 120341 = Ans... |
| 18. 69524 — 47321 = Ans. 22203 | 28. 217951 — 5430 = Ans...   |
| 19. 56247 — 15123 = Ans. 41124 | 29. 760142 — 570031 = Ans... |
| 20. 72365 — 1243 = Ans. 71122  | 30. 391657 — 141322 = Ans... |

**CASE II.**—*To subtract when any figure in the subtrahend is greater than the figure above it in the minuend.*

45. What is subtraction?—Define minuend,—subtrahend,—46. How is the result called?

M  
S  
R  
  
4  
1  
w.  
so  
to  
6  
n  
h  
le  
or  
  
the  
we  
lea  
or  
unl  
tho  
to  
add  
tho  
for  
henc  
4 h  
befo  
T  
incr  
4  
I  
uni  
47

*Ex.* Find the difference between 853029 and 360475.

## METHOD BY BORROWING.

## OPERATION.

|            | Hund. of Thousands. | Tens of Thousands. | Thousands. | Hundreds. | Tens. | Units. |
|------------|---------------------|--------------------|------------|-----------|-------|--------|
| Minuend    | 8                   | 5                  | 3          | 0         | 2     | 9      |
| Subtrahend | 3                   | 6                  | 0          | 4         | 7     | 5      |
| Remainder  | 4                   | 9                  | 2          | 5         | 5     | 4      |

4 from 9 leaves 5, which we write in hundreds' place below. As we have taken 1 thousand from the 3 thousands, 2 thousands remain; naught from 2 leaves 2, which we write under. We cannot take 6 ten-thousands from 5 ten-thousands; so from the 8 hundred-thousands we take 1 hundred-thousand, which equals 10 ten-thousands, and adding them to the 5 ten-thousands, make 15 ten-thousands; 6 ten-thousands from 15 ten-thousands leave 9 ten-thousands, which we write under. Having taken 1 hundred-thousand from the 8 hundred-thousands, 7 hundred-thousands are left; 3 hundred-thousands from 7 hundred-thousands leave 4 hundred-thousands, which we write under; and thus find the difference, or remainder, to be 492554.

## METHOD BY ADDING 10.

## OPERATION.

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 8 | 5 | 3 | 0 | 2 | 9 |
| 3 | 6 | 0 | 4 | 7 | 5 |
| 4 | 9 | 2 | 5 | 5 | 4 |

*ANALYSIS.*—We first take the 5 units from the 9 units, and find the difference to be 4 units, which we write below. As we cannot take 7 tens from 2 tens, we add 10 tens to 2 tens, making 12 tens; 7 tens from 12 tens leave 5 tens. But having added 10 tens, or 1 hundred, to the minuend, we shall have a remainder 1 hundred too large, to compensate, we add 1 hundred to the 4 hundreds of the subtrahend, making 5 hundreds. We cannot take 5 hundreds from 0; so we add 10 hundreds to 0, making 10 hundreds; 5 hundreds from 10 hundreds leave 5 hundreds, which we write below. Now, as we have added 10 hundreds, or 1 thousand, to the minuend, we shall have a remainder 1 thousand too large, unless we add 1 thousand to the 0 of the thousands in the subtrahend, making 1 thousand; 1 thousand from 3 thousands leave 2 thousands. We then proceed to take the 6 ten-thousands from the 5 ten-thousands above it, as we cannot, we add 10 ten-thousands to the 5 ten-thousands, making 15 ten-thousands; 6 ten-thousands from 15 ten-thousands leave 9 ten-thousands. Then, to compensate for the 10 thus added to the 5 in the minuend, we add 1 to the 3 in the subtrahend, making 4 hundred-thousands, and subtract the 4 from the 8, which leave 4 hundred-thousands. Thus, we find the remainder to be 492554, the same as before.

This operation depends on the principle, that, if any two numbers are equally increased, their difference remains the same.

**47.** From the preceding illustrations we derive the following

**RULE.**—I. Write the less number under the greater, so that units of the same order may stand under each other.

**47.** What is the rule for subtraction?

the difference

ished, is called  
be subtracted,

remainder, ex-

trahend is less

under the  
or shall stand in  
the right and  
its leave 3 units,  
tens from 4 tens  
' place. Three  
hundreds, which  
2 hundreds, 2

(5.)

974

631

343

(10.)

645

542

13 = Ans...

30 = Ans...

51 = Ans...

11 = Ans...

50 = Ans...

27 = Ans...

41 = Ans...

30 = Ans...

31 = Ans...

22 = Ans...

trahend is

46. How is the

II. Commencing at the right-hand, take each figure of the subtrahend from the figures above it, and write the result underneath.

III. If any figure in the subtrahend be greater than the corresponding figure above it, add 10 to that upper figure before subtracting, and then add one to the next left-hand figure of the subtrahend.

### PROOF OF SUBTRACTION.

48. We make the **Proof** of Subtraction in adding the remainder to the subtrahend, their sum will be equal to the minuend, if the work is correct.

*Ex.* From 35678 take 27899.

$$\begin{array}{r} 35678 \\ 27899 \\ \hline \text{Rem. } 7779 \\ \text{Proof } 35678 \end{array}$$

**ANALYSIS.**—To prove this operation, we add the remainder 7779 to the subtrahend 27899, and obtain 35678, which sum is equal to the minuend, or greater number. Hence we conclude that the operation is correct.

This method of proof depends on the principle, *that the greater of any two numbers is equal to the less added to the difference.*

**USE OF SUBTRACTION.**—Subtraction serves to find the gain or loss on goods; what we still owe on a sum of money of which we have already paid a part; in general to find the surplus of a number over another; the difference between two numbers, &c.

We know that the solution of a problem requires a subtraction, when we must find the difference between two numbers, or the excess of a number over another; and when it is required to find one of two numbers forming a total, that total or amount, and one of the numbers, being given.

### EXAMPLES FOR PRACTICE.

|            | (1.)  | (2.)        | (3.)        | (4.)         |
|------------|-------|-------------|-------------|--------------|
| Minuend    | 76618 | 57813       | 13042       | 250143       |
| Subtrahend | 49359 | 38675       | 9176        | 176158       |
| Remainder  | 27159 | 19138       | 3866        | 73985        |
| Proof      | 76618 | Proof 57813 | Proof 13042 | Proof 250143 |

48. How do you prove subtraction?

## SUBTRACTION.

31

|     | From | 3900       | take | 357       | Ans. | 3549       |
|-----|------|------------|------|-----------|------|------------|
| 6.  | "    | 49469      | "    | 15574     | "    | 33898      |
| 7.  | "    | 79906      | "    | 16134     | "    | 63772      |
| 8.  | "    | 190540     | "    | 30409     | "    | 160131     |
| 9.  | "    | 478754     | "    | 97125     | "    |            |
| 10. | "    | 426542     | "    | 179127    | "    |            |
| 11. | "    | 740070     | "    | 471097    | "    | 247415     |
| 12. | "    | 577405     | "    | 198576    | "    | 268973     |
| 13. | "    | 405907     | "    | 55595     | "    | 378829     |
| 14. | "    | 897452     | "    | 508578    | "    |            |
| 15. | "    | 8950076    | "    | 4137976   | "    | 388874     |
| 16. | "    | 14003325   | "    | 988827    | "    | 4812100    |
| 17. | "    | 15989700   | "    | 154379    | "    | 13014498   |
| 18. | "    | 21530600   | "    | 737898    | "    | 15835321   |
| 19. | "    | 97660054   | "    | 14550045  | "    | 20792702   |
| 20. | "    | 457652478  | "    | 49876579  | "    | 83110099   |
| 21. | "    | 337008974  | "    | 40073049  | "    |            |
| 22. | "    | 154400000  | "    | 91791994  | "    | 296935925  |
| 23. | "    | 190054009  | "    | 4590489   | "    | 62608006   |
| 24. | "    | 754674895  | "    | 64834795  | "    | 185463520  |
| 25. | "    | 10007549   | "    | 9068073   | "    | 689840100  |
| 26. | "    | 127321155  | "    | 1300475   | "    | 939476     |
| 27. | "    | 418030450  | "    | 27740761  | "    | 126020680  |
| 28. | "    | 945000090  | "    | 1500734   | "    | 390289689  |
| 29. | "    | 809005409  | "    | 3740055   | "    |            |
| 30. | "    | 490009076  | "    | 5475904   | "    | 805265354  |
| 31. | "    | 847653454  | "    | 74375576  | "    | 484533172  |
| 32. | "    | 850070452  | "    | 97050654  | "    | 773277878  |
| 33. | "    | 546807575  | "    | 277451794 | "    | 753019798  |
| 34. | "    | 101010101  | "    | 9737350   | "    | 269355781  |
| 35. | "    | 653405995  | "    | 476294474 | "    | 91272751   |
| 36. | "    | 677454854  | "    | 495647562 | "    |            |
| 37. | "    | 764927074  | "    | 676489572 | "    | 181807292  |
| 38. | "    | 950076074  | "    | 475207454 | "    | 88437502   |
| 39. | "    | 456700750  | "    | 45612495  | "    | 474868620  |
| 40. | "    | 876007054  | "    | 798435495 | "    | 411088255  |
| 41. | "    | 753097507  | "    | 194289778 | "    | 77571559   |
| 42. | "    | 400075546  | "    | 93457897  | "    | 558807729  |
| 43. | "    | 487054554  | "    | 98047775  | "    |            |
| 44. | "    | 432700769  | "    | 71904257  | "    | 389006779  |
| 45. | "    | 954875754  | "    | 577469579 | "    | 360796512  |
| 46. | "    | 453007527  | "    | 276499619 | "    | 377406175  |
| 47. | "    | 400700007  | "    | 203405604 | "    | 178507908  |
| 48. | "    | 974500700  | "    | 93235945  | "    |            |
| 49. | "    | 839457354  | "    | 745689835 | "    | 881264755  |
| 50. | "    | 847654976  | "    | 39787495  | "    | 93767519   |
| 51. | "    | 905207246  | "    | 746855472 | "    | 807867481  |
| 52. | "    | 4184545945 | "    | 178809709 | "    | 158351774  |
| 53. | "    | 9457385700 | "    | 17073959  | "    | 4005736236 |
| 54. | "    | 9707000591 | "    | 19779883  | "    | 9687220708 |

(4.)

250143

176158

73985

roof 250143

## SUBTRACTION OF DECIMALS.

Ex. From 86.7 take 69.354.

OPERATION.

$$\begin{array}{r} 86.700 \\ 69.354 \\ \hline 17.346 \end{array}$$

ANALYSIS.—Having placed the less number under the greater, so that figures of the same decimal place stand in the same column, we write two ciphers at the right of 7, in order that the minuend may have as many decimal figures as the subtrahend; then we subtract as in whole numbers, and finally place the decimal point in the remainder directly under that in the given number.

RULE.—I. Write the less number under the greater, so that the decimal points shall stand directly under each other.

II. Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.

## EXAMPLES FOR PRACTICE.

|         | (1.)         | (2.)   | (3.)         | (4.)   |              |
|---------|--------------|--------|--------------|--------|--------------|
| From    | 12.067       | 8.11   | 36.105       | 1.0062 |              |
| Take    | 9.71         | 6.7519 | 7.11892      | 0.43   |              |
| Ans.    | 2.357        | 1.3581 | 28.98603     | 0.5762 |              |
| 5. From | 90.49        | take   | 39.59        | Ans.   | 50.90        |
| 6. "    | 109.191      | "      | 49.073       | "      | 60.118       |
| 7. "    | 5409.055     | "      | 4045.997     | "      | 1363.058     |
| 8. "    | 764907.05    | "      | 87929.795    | "      | 676977.255   |
| 9. "    | 897450.07    | "      | 98776.095    | "      |              |
| 10. "   | 465742.5     | "      | 76908.075    | "      | 388834.425   |
| 11. "   | 870079.04    | "      | 198789.958   | "      | 671289.082   |
| 12. "   | 400048.2136  | "      | 9372.016     | "      | 390676.1976  |
| 13. "   | 409004.9099  | "      | 100.137      | "      | 408904.7729  |
| 14. "   | 570075.9004  | "      | 4053.509     | "      | 566022.3914  |
| 15. "   | 49.1019      | "      | 35.708       | "      |              |
| 16. "   | 610011.050   | "      | 31971.9999   | "      | 578039.0501  |
| 17. "   | 71079.0013   | "      | 7482.1736    | "      | 63596.8277   |
| 18. "   | 79073.07     | "      | 7398.1204    | "      | 71674.9496   |
| 19. "   | 126001.0001  | "      | 98996.9088   | "      | 27004.0913   |
| 20. "   | 191279.9709  | "      | 50056.0099   | "      | 141223.9610  |
| 21. "   | 401645.1005  | "      | 498.6709     | "      | 401146.4296  |
| 22. "   | 700007.0236  | "      | 79797.0098   | "      |              |
| 23. "   | 411978.10359 | "      | 36730.09671  | "      | 375248.00688 |
| 24. "   | 960945.00005 | "      | 600979.00007 | "      | 359965.99998 |
| 25. "   | 0.0707       | "      | 0.000607     | "      | 0.070093     |
| 26. "   | 0.0006       | "      | 0.0000075    | "      | 0.0005925    |
| 27. "   | 0.90019      | "      | 0.7300007    | "      | 0.1701893    |
| 28. "   | 0.0089       | "      | 0.0070675    | "      | 0.0018325    |
| 29. "   | 0.0904       | "      | 0.00289709   | "      | 0.08750291   |
| 30. "   | 0.7009       | "      | 0.190007     | "      | 0.510893     |
| 31. "   | 0.0991       | "      | 0.004500008  | "      | 0.094599992  |

**SUBTRACTION.**

33

|     |      |             |            |      |            |
|-----|------|-------------|------------|------|------------|
| 32. | From | 0.0779 take | 0.01011001 | Ans. | 0.06778999 |
| 33. | "    | 0.900 "     | 0.0019904  | "    | 0.8980096  |
| 34. | "    | 0.19100 "   | 0.09900035 | "    | 0.09199965 |
| 35. | "    | 0.4500 "    | 0.00550045 | "    | 0.44449955 |
| 36. | "    | 0.09839 "   | 0.09500959 | "    | 0.00338041 |

**PRACTICAL PROBLEMS IN SUBTRACTION.**

1. A field which had cost \$2360 was sold for \$2628. What is the gain?

**OPERATION.**

\$ 2 6 2 8  
 \$ 2 3 6 0  
 \$ 2 6 8

**ANALYSIS.**—The gain is equal to the difference between the cost and selling prices; therefore, in subtracting the cost price \$2360 from the selling price \$2628, we obtain the gain.  
 Ans. \$268 gain.

2. How much does a merchant lose in selling for \$4825.75 goods which cost him \$5174.10?

**OPERATION.**

\$ 5 1 7 4 . 1 0  
 \$ 4 8 2 5 . 7 5  
 \$ 3 4 8 . 3 5

**ANALYSIS.**—The loss is equal to the difference between the buying and selling prices; therefore, in subtracting the selling price \$4825.75 from the buying price \$5174.10, we obtain the loss.  
 Ans. \$348.35 loss.

3. A merchant bought flour for \$5626, and sold the whole of it for \$6853; how much did he gain?

Ans. \$1227.

4. Find the difference between 70401 and 6942?

Ans. 63459.

5. What is the difference between 85450 and 54498?

Ans. 30952.

6. I owed \$1628; I paid \$971; how much do I owe yet?

7. The greater of two numbers is 1302, and their difference 981; what is the smaller?

Ans. 321.

8. A merchant sold in one day \$2517.40 worth of dry goods, and thereby cleared a profit of \$630.95. How much did the goods cost him?

Ans. \$1886.45.

9. To what number must we add 76 to increase it to 740?

10. The city of Quebec was founded by Champlain in 1608; how many years from that period to 1870?

Ans. 262 years.

11. The area of the Province of Quebec is 210000 square miles; that of the Province of Ontario 180000 sq. m.: by how many square miles does the former exceed the latter?

Ans. 30000 sq. m.

12. A father was 28 years old at the birth of his son, what will be the age of the son when the father will be 85 years old?

Ans. 57 yr.

13. What number must be added to 357.75 to have 8000?

14. What will be the age in 1871 of a person who was born in 1792?

Ans. 79 years.

15. What number must be added to 4 units 5 hundredths to have 10 units?

Ans. 5 units 95 hundredths.

16. In 1857, Canada exported to the United States for a value of \$13206436.10, and imported for \$20224650.96. How much did the importations exceed the exportations?

Ans. \$7018244.86.

17. Napoleon I. died in 1821 at the age of 52. In what year was he born?

Ans. 1769.

18. An army consisting of 41500 men, lost during a campaign 14704 men. How many remain? *Ans.* 26796 men.

19. What number must be added to 3 thousandths, to have 12 hundredths? *Ans.* 0.117 thousandths.

20. The population of Paris is 1953262 inhabitants and that of London 2863141; how much does the population of London exceed that of Paris? *Ans.* 909879 inhabitants.

21. Alfred the Great died in 901 at the age of 52, after a reign of 24 years: in what year was he born? *Ans.* 849.

22. Charlemagne was born in 742; he was crowned king of France in 768, emperor of the West in 800, and died in 814. How old was he, 1st. at his coronation as king; 2nd. as emperor; 3rd. at what age did he die; and 4th., how many years elapsed from his death until 1869? *Ans.* 1st. 26 as king, 2nd. 58 as emperor, 3rd. at the age of 72, and 4th. 1055 years.

23. Murillo's picture of the Immaculate Conception, being auctioned, the first bidding was \$30000, but it was finally knocked down at \$117000 and adjudged to the French Government who placed it in the museum of the Louvre. Required the difference between the 1st. and the last bidding? *Ans.* \$87000.

24. The population of Montreal, in 1765, consisted of 7000 inhabitants; in 1851, it was 57715; in 1856, 75000; in 1860, 90000; and in 1868, about 135000. What was the increase of the population from 1851 to 1868? *Ans.* 77285 inhabitants.

25. A farmer reaped 1689 bushels of wheat, and 965 bushels of oats. He sold his neighbor John 890 bushels of wheat and 478 bushels oats, and the remainder to Joseph. How many bushels of each sort did he sell to Joseph? *Ans.* 799 bush. wheat and 487 bush. oats.

26. Two merchants, in commencing business, invested a capital of \$18500; the 1st. invested \$6590.40; how much must he add to his investment to equal that of the second? *Ans.* \$5319.20.

27. Had I \$508.50 more, I could pay a debt of \$1015.80, and would have \$75 left; how much have I? *Ans.* \$582.30.

28. A merchant sold \$11630 worth of cloth, which was \$876 more than cost price; how much did it cost him? *Ans.* \$10754.

29. A house which was sold for \$14360, would have given a profit of \$840 to its owner if he had paid it \$300 less. How much did it cost? *Ans.* \$13820.

30. Gunpowder was invented in the year 1330; how long was this before the invention of printing, which was in 1441? *Ans.* 111 years.

#### PRACTICAL PROBLEMS COMBINING ADDITION AND SUBTRACTION.

1. A retail merchant places \$45.25 in his drawer for change; on Monday he sells for \$75.85; on Tuesday, for \$68.40; on Wednesday, for \$85; on Thursday, for \$128.60; on Friday, for \$54.85; and on Saturday, for \$72.15; after which he pays a Bill of \$95.60, another of \$43.25, and takes \$240.75 for his own expenses, and then there remains to him in cash a sum of \$150. Are his accounts right?

**ANALYSIS.**—First find out what sum he would have had if he had not paid anything, and then what he has paid out.  
 He had,  $45.25 + 75.85 + 68.40 + 85 + 128.60 + 54.85 + 72.15 = \$530.10$ .  
 He has taken out,  $95.60 + 43.25 + 240.75 = \$379.60$ . There should be left  $\$530.10 - \$379.60 = \$150.50$ ; difference  $150.50 - 150 =$  *Ans.*  $\$0.50$  against him.

2. A market woman having 152 eggs, sold to one person 14 of them, to another 27, to another 73, to another 24, and to another 5: how many remain? *Ans.* 9.

3. A gentleman having \$1128, lost \$638, and spent \$172: how much had he remaining? *Ans.* \$318.

4. The waters of the St. Lawrence cover an area of 565000 square miles; two of its tributaries, the Saguenay and St. Maurice, cover, the one an area of 27000 square miles, and the other 21000 square miles. How much does the area of the St. Lawrence exceed those of its two tributaries? *Ans.* 517000 square miles.

5. A man has bought four building lots for the sum of \$16860. For the 1st. he paid \$2070.30; for the 2nd., \$3674.50; for the 3rd., \$4175: how much has he paid for the 4th.? *Ans.* \$6940.20.

6. I deposited in a Savings Bank \$8752.70; the first time I drew from it a sum of \$1286; the second, \$1650.50; the third, \$972.75. How much have I left in the bank? *Ans.* \$1843.45.

7. Moses was born about 1571 years before Christ, he left Egypt with the Hebrews the year 1491 before Christ, and died on Mount Nebo, in the year 1451 before Christ. What age was he, 1st. when he left Egypt; 2nd. at his death; and 3rd. how long from the period of his death to the year 1871 of the Christian era? *Ans.* 1st. 80 years; 2nd. 120 years; 3rd. 3222 years.

8. A speculator gains \$6570, and then loses \$3762.40; at another time he gains \$4545.72, and loses again \$5632.10. Tell how much his gains exceed his losses? *Ans.* \$1721.22.

9. A man deals in grains since 6 years; the 1st. year he lost \$356; the 2nd., he gained \$780.20; the 3rd., he gained \$685.30; the 4th., he lost \$2600; the 5th., he gained \$4320.95; and the 6th., he lost again \$3000. Did he gain or lose, and how much? *Ans.* \$169.55 loss.

10. A owes a sum of \$690, plus \$55.20 for interest. He reimbursed at different times \$87.50, \$210.00, \$318.45; how much does he still owe? *Ans.* \$129.25.

11. A family owing its grocer \$508.75, takes again effects to the amount of \$240.32 and then gives in payment \$704.65; what is yet the balance of its account? *Ans.* \$44.42.

12. My brother owed a certain sum of money; he paid on account \$284, \$570.20, \$210.08 and \$345.30. Finally, in settling, he gave a Bank note of \$1000, on which they returned him \$454 change. What sum did he owe? *Ans.* \$1955.58.

13. Peter has 360 sheep, Maurice 145 more than Peter, and Charles as many as Maurice and Peter together lacking 117. How many sheep has Charles? *Ans.* 748 sheep.

14. A merchant bought a whole cargo of Porto-Rico sugar worth \$12347; he paid \$311.70 for freight expenses, and \$291.30 for commission and storage; after which, he sold his sugar for \$12511.30. Required his gain or loss? *Ans.* \$438.70 loss.

15. If I had sold \$20 more a piece of linen which cost me \$350, I would have gained \$30; how much did I sell it? *Ans.* \$360.
16. A speculator bought 217 cords of wood for \$1085. He gave in payment 1800 pounds of salmon valued at \$144.00; 700 bushels of potatoes worth \$210, and 1200 pounds sugar equal to \$72. How much does he owe yet? *Ans.* \$659.
17. I have three creditors; I owe the 1st. \$2500, the 2nd. \$840, and the 3rd. \$754. On the other hand, I have 2 debtors, the one owes me \$1800, and the other, \$2544. Besides I have \$3768 in cash. Required what sum remains in hand after paying my debts? *Ans.* \$4018.
18. How many pounds of bread will 200 pounds of flour give, knowing that it takes 114 pounds of water to knead them and that 44 pounds evaporate in baking? *Ans.*
19. Three boxes containing 1435 oranges have cost \$17.15, and \$3 each box for drayage; the first contains 240 oranges, the second 80 more; how many does the third contain? *Ans.* 875.
20. In adding \$5.08, the price of an ox hide, to the sum expended by a tanner for 4 calf and 6 horse hides we obtain a sum of \$22.98. Required the price of the 6 horse hides, knowing that the calf hides have cost \$4.40? *Ans.* \$13.50.
21. A cloth merchant bought 60 yards more than he had at first and then sold 140 yards; after which he has left half what he had in his shop before his last purchase. How many yards had he at first?
22. A dyer bought at three different times 109 pounds of dye for the sum of \$3.84. The first time he bought 47 pounds and this quantity exceeded by 15 pounds his third purchase. How many pounds did he buy in his second purchase? *Ans.* 30 pounds.
23. A general starting for an expedition with 18000 men, left 600 of them to garrison a small town; at the same time he received a reinforcement of 800 more, 450 of whom he was obliged to leave in hospitals. Having asked 3500 more, he received only 2730; of these he left 1750 at different posts. Required the number of men he had on reaching his destination? *Ans.* 18730 men.

## MULTIPLICATION.

**49.** Multiplication is the process of taking one number as many times as there are units in another.

**50.** The terms in Multiplication are:

1st. The **Multiplicand**, or number to be taken;

2nd. The **Multiplier**, or number by which we multiply, or which shows how many times the multiplicand is to be taken;

3rd. The **Product**, or the result obtained.

**51.** The multiplicand and multiplier are called **Factors**, because they produce or make the product.

49. What is Multiplication?— Define Multiplicand.— Multiplier.— Product.  
— 51. What are the multiplicand and multiplier called?

## MULTIPLICATION TABLE.

|              |               |               |               |
|--------------|---------------|---------------|---------------|
| 1 × 1 = 1    | 2 × 1 = 2     | 3 × 1 = 3     | 4 × 1 = 4     |
| 1 × 2 = 2    | 2 × 2 = 4     | 3 × 2 = 6     | 4 × 2 = 8     |
| 1 × 3 = 3    | 2 × 3 = 6     | 3 × 3 = 9     | 4 × 3 = 12    |
| 1 × 4 = 4    | 2 × 4 = 8     | 3 × 4 = 12    | 4 × 4 = 16    |
| 1 × 5 = 5    | 2 × 5 = 10    | 3 × 5 = 15    | 4 × 5 = 20    |
| 1 × 6 = 6    | 2 × 6 = 12    | 3 × 6 = 18    | 4 × 6 = 24    |
| 1 × 7 = 7    | 2 × 7 = 14    | 3 × 7 = 21    | 4 × 7 = 28    |
| 1 × 8 = 8    | 2 × 8 = 16    | 3 × 8 = 24    | 4 × 8 = 32    |
| 1 × 9 = 9    | 2 × 9 = 18    | 3 × 9 = 27    | 4 × 9 = 36    |
| 1 × 10 = 10  | 2 × 10 = 20   | 3 × 10 = 30   | 4 × 10 = 40   |
| 1 × 11 = 11  | 2 × 11 = 22   | 3 × 11 = 33   | 4 × 11 = 44   |
| 1 × 12 = 12  | 2 × 12 = 24   | 3 × 12 = 36   | 4 × 12 = 48   |
| 5 × 1 = 5    | 6 × 1 = 6     | 7 × 1 = 7     | 8 × 1 = 8     |
| 5 × 2 = 10   | 6 × 2 = 12    | 7 × 2 = 14    | 8 × 2 = 16    |
| 5 × 3 = 15   | 6 × 3 = 18    | 7 × 3 = 21    | 8 × 3 = 24    |
| 5 × 4 = 20   | 6 × 4 = 24    | 7 × 4 = 28    | 8 × 4 = 32    |
| 5 × 5 = 25   | 6 × 5 = 30    | 7 × 5 = 35    | 8 × 5 = 40    |
| 5 × 6 = 30   | 6 × 6 = 36    | 7 × 6 = 42    | 8 × 6 = 48    |
| 5 × 7 = 35   | 6 × 7 = 42    | 7 × 7 = 49    | 8 × 7 = 56    |
| 5 × 8 = 40   | 6 × 8 = 48    | 7 × 8 = 56    | 8 × 8 = 64    |
| 5 × 9 = 45   | 6 × 9 = 54    | 7 × 9 = 63    | 8 × 9 = 72    |
| 5 × 10 = 50  | 6 × 10 = 60   | 7 × 10 = 70   | 8 × 10 = 80   |
| 5 × 11 = 55  | 6 × 11 = 66   | 7 × 11 = 77   | 8 × 11 = 88   |
| 5 × 12 = 60  | 6 × 12 = 72   | 7 × 12 = 84   | 8 × 12 = 96   |
| 9 × 1 = 9    | 10 × 1 = 10   | 11 × 1 = 11   | 12 × 1 = 12   |
| 9 × 2 = 18   | 10 × 2 = 20   | 11 × 2 = 22   | 12 × 2 = 24   |
| 9 × 3 = 27   | 10 × 3 = 30   | 11 × 3 = 33   | 12 × 3 = 36   |
| 9 × 4 = 36   | 10 × 4 = 40   | 11 × 4 = 44   | 12 × 4 = 48   |
| 9 × 5 = 45   | 10 × 5 = 50   | 11 × 5 = 55   | 12 × 5 = 60   |
| 9 × 6 = 54   | 10 × 6 = 60   | 11 × 6 = 66   | 12 × 6 = 72   |
| 9 × 7 = 63   | 10 × 7 = 70   | 11 × 7 = 77   | 12 × 7 = 84   |
| 9 × 8 = 72   | 10 × 8 = 80   | 11 × 8 = 88   | 12 × 8 = 96   |
| 9 × 9 = 81   | 10 × 9 = 90   | 11 × 9 = 99   | 12 × 9 = 108  |
| 9 × 10 = 90  | 10 × 10 = 100 | 11 × 10 = 110 | 12 × 10 = 120 |
| 9 × 11 = 99  | 10 × 11 = 110 | 11 × 11 = 121 | 12 × 11 = 132 |
| 9 × 12 = 108 | 10 × 12 = 120 | 11 × 12 = 132 | 12 × 12 = 144 |

NOTE.—To repeat the Table by using the second columns as multipliers. Thus, 1 time 2 is 2, 2 times 2 are 4, 3 times 2 are 6, 4 times 2 are 8, etc.

Factors,

Product.

CASE I.—To effect a multiplication when the multiplier does not exceed 12.

*Ex.* Multiply 542 by 7.

OPERATION.

|              |      |  |
|--------------|------|--|
| Multiplicand | 542  |  |
| Multiplier   | 7    |  |
| Product      | 3794 |  |

ANALYSIS.—In this example, it is required to take 542 seven times. If we take the units of each order 7 times, we shall take the entire number 7 times. Therefore, writing the multiplier under the unit figure of the multiplicand, we proceed thus: 7 times 2 units are 14 units = 1 ten and 4 units; we write the 4 units in the units' place, and reserve the 1 ten to add to the next product. Seven times 4 tens are 28 tens, and the 1 ten in reserve, added, are 29 tens = 2 hundreds and 9 tens; we write the 9 tens in the tens' place, and reserve the 2 hundreds to add to the product of hundreds. Seven times 5 hundreds are 35 hundreds, and the 2 hundreds reserved in the last product added, are 37 hundreds, which we write down in full; and the product is 3794.

EXAMPLES FOR PRACTICE.

|              | (1.)  | (2.)  | (3.)  | (4.)  |
|--------------|-------|-------|-------|-------|
| Multiplicand | 3462  | 4276  | 5793  | 8634  |
| Multiplier   | 4     | 5     | 3     | 6     |
| Product      | 13848 | 21380 | 17379 | 51804 |
|              | (5.)  | (6.)  | (7.)  | (8.)  |
| 2893         | 16812 | 48739 | 58607 | 76598 |
| 3            | 5     | 7     | 8     | 9     |

- |                               |         |                                    |  |
|-------------------------------|---------|------------------------------------|--|
| 10. 873 × 3 = <i>Ans.</i>     | 2619    | 21. 76394 × 4 = <i>Ans.</i> ...    |  |
| 11. 946 × 4 = <i>Ans.</i>     | 3784    | 22. 97631 × 5 = <i>Ans.</i> ...    |  |
| 12. 4731 × 4 = <i>Ans.</i>    | 16924   | 23. 266532 × 7 = <i>Ans.</i> ...   |  |
| 13. 5607 × 5 = <i>Ans.</i>    | 28035   | 24. 835456 × 6 = <i>Ans.</i> ...   |  |
| 14. 6924 × 6 = <i>Ans.</i>    | 41544   | 25. 541378 × 8 = <i>Ans.</i> ...   |  |
| 15. 8657 × 8 = <i>Ans.</i>    | 69256   | 26. 367542 × 9 = <i>Ans.</i> ...   |  |
| 16. 27693 × 7 = <i>Ans.</i>   | 193851  | 27. 426985 × 8 = <i>Ans.</i> ...   |  |
| 17. 51786 × 9 = <i>Ans.</i>   | 466074  | 28. 576483 × 11 = <i>Ans.</i> ...  |  |
| 18. 45678 × 11 = <i>Ans.</i>  | 502458  | 29. 6932574 × 9 = <i>Ans.</i> ...  |  |
| 19. 36397 × 9 = <i>Ans.</i>   | 327573  | 30. 397465 × 12 = <i>Ans.</i> ...  |  |
| 20. 634576 × 12 = <i>Ans.</i> | 7614912 | 31. 3745178 × 11 = <i>Ans.</i> ... |  |

CASE II.—To effect a multiplication when the multiplier exceeds 12.

*Ex.* Multiply 478 by 64.

OPERATION.

|                   |       |  |
|-------------------|-------|--|
| Multiplicand      | 478   |  |
| Multiplier        | 64    |  |
| Partial products. | 1912  |  |
| Entire product    | 30592 |  |

ANALYSIS.—We write the multiplicand and multiplier as in Case 1, and proceed thus. Four times 8 units are 32 units = 3 tens and 2 units; we write the 2 units in the place of units, and add the 3 tens to the product of tens. Four times 7 tens are 28 tens, + 3 tens are 31 tens = 3 hundreds and 1 ten; we write the 1 ten in the place of tens, and add the 3 hundreds to the product of hundreds. Four times 4 hundreds are 16 hundreds, + 3 hundreds are

multiplier does

quired to take  
of each order  
number 7 times;  
the unit figure  
times 2 units  
write the 4 units  
to add to the  
reserve, added,  
the tens' place,  
seven times 5  
last product  
product is 3794.

(4.)

8634

6

51804

(9.)

76598

9

= Ans...  
= Ans...

multiplier ex-

and mul-  
Four times  
its; we write  
add the 3 tens  
tens are 28  
is and 4 ten;  
and add the  
Four times  
undreds are

19 hundreds, which we write in its proper place. We then, in like manner, multiply by the 6 tens in the multiplier, taking care to write the first figure obtained by this multiplication, in tens' place directly under the 6 of the multiplier; and, adding the partial products obtained by the two multiplications, we find the whole product of 478 by 64 to be 30592.

**NOTE.**—When there are ciphers between the significant figures of the multiplier, pass over them in the operation, and multiply by the significant figures only, remembering to set the first figure of the product under the figure of the multiplier that produces it.

**52.** From the foregoing illustrations we deduce the following

**RULE.**—I. Write the multiplier under the multiplicand, so that units of the same order shall stand under one another, and draw a line underneath.

II. Multiply each figure of the multiplicand by each figure of the multiplier successively, beginning with the unit figure, and write the first figure of each partial product under the figure of the multiplier used, writing down and carrying as in addition.

III. If there are partial products, add them, and their sum will be the product required.

PROOF OF MULTIPLICATION.

**53.** The Proof of multiplication is generally made by another multiplication (1) in which one of the factors equals the half, the third, or the fourth, etc., of one of the factors of the operation, and the other equals twice, three times, four times, etc., the other factor of the operation. Or,

In multiplying the multiplicand by the multiplier diminished by 1, and to the product adding the multiplicand; if the sum be the same as the product by the whole of the multiplier, the work is correct.

**USE OF MULTIPLICATION.**—Multiplication serves to render any number so many times greater; to take several parts of a number; to find the value of several units or parts of units, when one of them is known; to bring a number expressing units of a certain nature to another number expressing units which are subdivisions of the first, &c.

Generally we know that the solution of a problem requires a multiplication, when the value of the unity is mentioned and that the value of several is required, or that of some parts of the unity.

**52.** What is the general rule for multiplication?—**53.** How is the proof of multiplication made?

(1). In multiplying the multiplier by the multiplicand, the same product must be found.

## MULTIPLICATION.

## EXAMPLES FOR PRACTICE.

|             | (1.)   | (2.)                | (3.)                 |
|-------------|--------|---------------------|----------------------|
| Multiply    | 8621   | 37215               | 167034               |
| By          | 47     | 65                  | 304                  |
|             | 60347  | 186075              | 668136               |
|             | 34484  | 23290               | 5011020              |
| <i>Ans.</i> | 405187 | <i>Ans.</i> 2418975 | <i>Ans.</i> 50778336 |

|     |           |   |        |             |                 |
|-----|-----------|---|--------|-------------|-----------------|
| 4.  | 976       | x | 27     | <i>Ans.</i> | 26352           |
| 5.  | 697       | x | 34     | "           | 23698           |
| 6.  | 749       | x | 46     | "           | 34454           |
| 7.  | 8386      | x | 57     | "           | 478002          |
| 8.  | 753537    | x | 68     | "           | 51240516        |
| 9.  | 134679    | x | 79     | "           | 10639641        |
| 10. | 824956    | x | 387    | "           | 319257972       |
| 11. | 984765    | x | 756    | "           | 744482340       |
| 12. | 6654      | x | 789    | "           | 5250006         |
| 13. | 97248     | x | 865    | "           | 84119520        |
| 14. | 689834    | x | 943    | "           | 650513462       |
| 15. | 867894    | x | 996    | "           | 864422424       |
| 16. | 807497875 | x | 965    | "           | 779235449372    |
| 17. | 84966     | x | 7649   | "           | 649904934       |
| 18. | 543956    | x | 9475   | "           | 5153983100      |
| 19. | 96824     | x | 4696   | "           | 454685504       |
| 20. | 43208     | x | 4962   | "           | 214398096       |
| 21. | 90480     | x | 9007   | "           | 814953360       |
| 22. | 43        | x | 89006  | "           | 3827268         |
| 23. | 76496     | x | 87969  | "           | 6729276624      |
| 24. | 7674      | x | 12478  | "           | 95756172        |
| 25. | 3696      | x | 819162 | "           | 3027622752      |
| 26. | 69421     | x | 21754  | "           | 1510184434      |
| 27. | 4321      | x | 987654 | "           | 4267652934      |
| 28. | 756849    | x | 74323  | "           | 56251288227     |
| 29. | 908708    | x | 70469  | "           | 69109512052     |
| 30. | 4916      | x | 69678  | "           | 342537048       |
| 31. | 7654208   | x | 20963  | "           | 160455162304    |
| 32. | 80097     | x | 74269  | "           | 5948724093      |
| 33. | 900007    | x | 700608 | "           | 630552104256    |
| 34. | 4300407   | x | 700608 | "           | 3012899547456   |
| 35. | 460004    | x | 99804  | "           | 45910239216     |
| 36. | 960076    | x | 90708  | "           | 87086573808     |
| 37. | 690800    | x | 456007 | "           | 315009635600    |
| 38. | 7006924   | x | 540086 | "           | 3784341555464   |
| 39. | 786530746 | x | 357894 | "           | 281494634808924 |
| 40. | 416342505 | x | 987405 | "           | 411098671149525 |
| 41. | 896302456 | x | 943765 | "           | 845888887386840 |
| 42. | 495307429 | x | 936704 | "           | 463956449974016 |
| 43. | 757489007 | x | 900076 | "           | 681797675464532 |
| 44. | 879407854 | x | 698765 | "           | 614499429100310 |

(3.)

167034  
 304  
 668136  
 011020  
 0778336

|     |           |   |         |      |                 |
|-----|-----------|---|---------|------|-----------------|
| 45. | 954907089 | × | 600789  | Ans. | 573697675093221 |
| 46. | 457907842 | × | 796807  | "    | 364864173860494 |
| 47. | 856407809 | × | 305407  | "    | 261552939723263 |
| 48. | 674396856 | × | 285679  | "    | 192661019425224 |
| 49. | 1864321   | × | 609649  | "    | 1136581433329   |
| 50. | 2465783   | × | 3686407 | "    | 9089879711681   |
| 51. | 7240036   | × | 4029008 | "    | 29170162964288  |
| 52. | 908007004 | × | 500123  | "    | 454115186861492 |

26352  
 23698  
 34454  
 478002  
 51240516  
 10639641  
 319257972  
 744482340  
 5250006  
 84119520  
 650513462  
 864422424  
 9235449372  
 649904934  
 5153983100  
 454685504  
 214398096  
 814953360  
 3827258  
 6729276624  
 95756172  
 3027622752  
 1510184434  
 4267652934  
 6251288227  
 9109512052  
 342537048  
 0455162304  
 5948724093  
 0552104256  
 8899547456  
 5910239216  
 7086573808  
 0099635600  
 341555464  
 634808924  
 6711149526  
 887386840  
 449974016  
 675464532  
 429100310

## MULTIPLICATION OF DECIMALS.

Ex. 1. Find the product of 4.35 by 8.26.

OPERATION.

4.35  
 8.26  
 2610  
 876  
 3480

35.9310

Ans. there remains then but to repeat 826 times this hundredth part to obtain the product required. As the number repeated consists of ten-thousandths, the product will be composed of decimals of the same nature; to separate the units it is then necessary to take its ten-thousandth part, that is, out off 4 figures by the insertion of a point at the right side (No. 37). The same reasoning is applicable when there are three, four, &c. decimals in the multiplier.

If the factors are decimals only, we multiply as usual and cut off as many decimals in the product as there are in both factors; but if the product does not contain a sufficient number of figures, we fill up the vacant places by ciphers, placing one also for the units.

Ex. 2. Multiply 0.054 by 0.056.

OPERATION.

54  
 56  
 324  
 270

0.003024

ANSALYSIS.—Multiplying 54 by 56, we obtain 3024; but as there are 6 decimals in the two factors, we place two ciphers at the left side of the product and having put the decimal point, we place another cipher for the units, and thus we find the number 0.003024, which is read 3 thousandths 24 millionths.

54. Hence the following

RULE.—I. Multiply as in whole numbers, and point off as many figures for decimals, in the product, as there are decimals in the multiplicand and multiplier.

II. If there are not as many figures in the product as there are decimal places in the multiplicand and multiplier, supply the deficiency by prefixing ciphers.

NOTE.—To multiply decimals by 10, 100, 1000, etc., (No. 36).

**PROOF.**—The proof is the same as in multiplication of whole numbers.

## EXAMPLES FOR PRACTICE.

|     |         |   |           |             |                |
|-----|---------|---|-----------|-------------|----------------|
| 3.  | 15.27   | × | 9.        | <i>Ans.</i> | 137.43         |
| 4.  | 6.35    | × | 98.       | "           | 622.3          |
| 5.  | 7.41    | × | 675.      | "           | 5001.75        |
| 6.  | 197.19  | × | 56.       | "           | 11042.64       |
| 7.  | 97.85   | × | 975.      | "           | 95403.75       |
| 8.  | 69.78   | × | 596.      | "           | 41588.88       |
| 9.  | 947.    | × | 4.65      | "           | 4403.55        |
| 10. | 869.    | × | 6.96      | "           | 6048.24        |
| 11. | 345.    | × | 3.95      | "           | 1362.75        |
| 12. | 57.     | × | 9.475     | "           | 540.075        |
| 13. | 786.    | × | 7.789     | "           | 6122.154       |
| 14. | 374.    | × | 2.967     | "           | 1109.658       |
| 15. | 9.47    | × | 6.694     | "           | 63.39218       |
| 16. | 39.47   | × | 28.9005   | "           | 1140.702735    |
| 17. | 676.49  | × | 60.705    | "           | 41066.32545    |
| 18. | 401.04  | × | 13001.4   | "           | 521913.456     |
| 19. | 9617.09 | × | 4281.45   | "           | 41175089.9805  |
| 20. | 6789.06 | × | 13808.927 | "           | 93749640.72768 |
| 21. | 3807.45 | × | 5321.807  | "           | 20262510.2547  |
| 22. | 489.04  | × | 37.00845  | "           | 18096.612388   |

## PRACTICAL PROBLEMS IN MULTIPLICATION.

1. If a workman earn \$15 per week: how much will he earn in 9 weeks?

**ANALYSIS.**—In one week he earns \$15; in 9 weeks he will earn nine times more, because he works nine times longer; therefore in multiplying by 9 we obtain the sum required =  $15 \times 9 = 135$ . *Ans.* In 9 weeks he earns \$135.

2. How much will 125 yards of cloth cost at \$3.25 a yard?

**ANALYSIS.**—If one yard cost \$3.25, 125 yards will cost 125 times more; in multiplying \$3.25 by 125, the required sum =  $3.25 \times 125 =$  *Ans.* \$406.25.

3. When a yard of cloth is worth \$2.40, how much will 75 hundredths of a yard cost?

**ANALYSIS.**—The yard being worth \$2.40, the 75 hundredths of a yard will be worth 75 times the hundredth part of \$2.40; therefore, multiplying \$2.40 by 0.75, we find the sum required =  $2.40 \times 0.75 =$  *Ans.* \$1.80.

4. What will 1635 barrels of sugar cost, at \$25 a piece? *A.* \$40875.

5. What will 785 kegs of tobacco cost, at \$36 a keg? *A.* \$28260.

6. What will 5679 bushels of wheat cost, at 85 cents a bushel?

7. How many pounds of flour are there in 387 barrels, there being 198 pounds in each barrel? *Ans.* 76626.

8. How many letters are there in a volume of 719 pages, each page containing 1639 letters? *Ans.* 1106541 letters.

9. A house has 295 windows and each window contains 24 panes of glass, how many panes in the whole edifice? *Ans.* 7080 panes.

n of whole

137.43  
622.3  
001.75  
042.64  
403.75  
588.88  
403.55  
048.24  
362.75  
540.075  
122.154  
109.658  
63.39218  
140.702735  
066.32545  
913.456  
089.9805  
540.72768  
510.2547  
098.612388

ON.

the earn in

nine times  
by 9 we ob-  
tains \$135.

d?

es more; in  
\$406.25.

1 75 hun-

yard will be  
\$2.40 by  
ns. \$1.80.

\$40875.

\$28260.

bushel?

here being

76626.

each page

letters.

24 panes

panes.

10. Required how many trees in a nursery composed of 95 rows, if each row contains 178 trees?  
*Ans.* 16910 trees.
11. The circumference of the earth is divided into 360 degrees and each degree into 69.5 English miles; required how many miles around the earth?  
*Ans.* 25020.
12. Required how many hours in a year of 365 days?  
*Ans.* 8760.
13. How many days in 1000 years?  
*Ans.* 365000.
14. A man deposits \$15 every week in a Savings Bank; how much does he deposit in one year or 52 weeks?  
*Ans.* \$780.
15. A ream of paper contains 20 quires; how many quires are there in 572 reams?  
*Ans.* 11440.
16. If a cask of wine contains 213 quarts; required how many quarts in 136 casks?  
*Ans.* 28968 quarts.
17. How many eggs are there in 37 dozen?  
*Ans.* 444.
18. How many days has a person aged 84 years lived, reckoning 365 days to the year?  
*Ans.* 30660 days.
19. How many pens are there in 200 boxes each containing a gross or 144 pens?  
*Ans.* 28800 pens.
20. How many days elapsed from the birth of J. C. till the 31st. Dec. 1869 inclusively? (Not counting leap years.)  
*Ans.* 682185.
21. Europe produces yearly 3466 pounds of gold; what is the value in dollars knowing that a pound of this precious metal is estimated at \$1718.50?  
*Ans.* \$5956321.
22. A library is composed of 75 shelves and each shelf contains 86 volumes; how many pages are there in all the volumes supposing each volume to contain on an average 420 pages?  
*Ans.* 2709000.
23. A speculator has purchased 268 horses and 274 times as many sheep; how many sheep has he purchased?  
*Ans.* 73432.
24. There are 12 bags of wheat on a truck, each bag containing 3 bushels; how many pounds are there in the whole load, if the bushel weighs 50 pounds?  
*Ans.* 1800 pounds.
25. A workman earns \$8 a week: how much will he earn in 7 years?  
*Ans.* \$2912.
26. How much will 240 pieces of cloth, each containing 44 yds. cost, at \$5.40 per yard?  
*Ans.* \$57024.
27. How many pair of shoes can be made in 265 days, in a factory in which 86 pair can be made in 1 day?
28. If, at one load, a span of horses can draw 2997 pounds; how many pounds can they draw in 327 loads?
29. A field of 7 acres of land yields 45 bushels oats per acre; what is the value of the crops of the 7 acres at \$0.40 a bush.?  
*Ans.* \$126.
30. Supposing a sheep gives 6 pounds of wool a year; how many pounds will 28 sheep give in 3 years and what sum would it bring at 24 cents per pound?  
*Ans.* \$120.96.
31. What is the value of the crop of a field containing 4 acres, if an acre yields 62 bush. oats worth 45 cents per bush.?  
*Ans.* 11160 cts.
32. A laborer thrashes 45 sheaves of wheat per day, giving 15 pecks; how many sheaves could 14 laborers thrash in 9 days, and what would be the quantity of grain obtained?  
*Ans.* 5670 sheaves and 1890 pecks grain.

## CONTRACTIONS IN MULTIPLICATION,

## OR MULTIPLICATION BY FACTORS.

**55.** In many instances, by the exercise of judgment, as it will be seen, the operation may be very much *abridged*.

**56.** Any number that may be produced by multiplying together two or more numbers, is called a **Composite Number**. Thus, 6, 15, 18, are composite numbers; for  $6 = 3 \times 2$ ;  $15 = 5 \times 3$ ;  $18 = 3 \times 3 \times 2$ .

**57.** The **Factors** of a number are the several numbers which, multiplied together, produce the given number. Thus, the factors of 24 are 12 and 2 ( $12 \times 2 = 24$ ); or, 4 and 6 ( $4 \times 6 = 24$ ); or 2 and 3 and 4 ( $2 \times 3 \times 4 = 24$ ).

*NOTE.*—The *factors* must not be confounded with the *parts* of a number. Thus, the *factors* of which 10 is composed, are 5 and 2, ( $5 \times 2 = 10$ ); while the *parts* of which 10 is composed are 6 and 4, ( $6 + 4 = 10$ ). The *factors* are multiplied, while the *parts* are added, to produce the number.

**CASE I.**—To effect multiplication when the multiplier is a composite number.

*Ex. 1.* What will 45 acres of land cost, at \$367 an acre?

OPERATION.

$$\begin{array}{r} 367 \\ 5 \\ \hline 1835 \\ 9 \end{array}$$

\$16515 *Ans.*

*ANALYSIS.*—The factors of 45 are 5 and 9. Now, if we multiply the cost of 1 acre by 5, we obtain the cost of 5 acres; and, by multiplying the cost of 5 acres by the factor 9, we evidently obtain the cost of 9 times 5 acres, or 45 acres, the number bought. Hence the following

**58. RULE.**—I. Separate the multiplier into two or more factors.  
II. Multiply the multiplicand by one of these factors, and that product by another; and so on, till all the factors have been used. The last product will be the one required.

*NOTE.*—The product of any number of factors is the same in whatever order they are multiplied. Thus,  $4 \times 5 = 20$ ; and  $5 \times 4 = 20$ .

## EXAMPLES FOR PRACTICE.

2. Multiply 2745 by  $28 = 4 \times 7$ . *Ans.* 76860.
3. Multiply 65742 by  $35 = 5 \times 7$ . *Ans.* 2291470.
4. Multiply 78036 by  $72 = 3 \times 3 \times 8$ . *Ans.* 5618592.
5. Multiply 36783 by 81. *Ans.* 2979423.
6. What will 56 horses cost at \$178 each? *Ans.* \$9968.
7. What will 435 bushels of potatoes cost, at 32 cents a bushel?
8. What will 64 yards of merino cost, at 75 cents a yard?
9. In 1 mile there are 63360 inches; how many inches, let. in 45 miles?—2nd. in 54 miles? *Ans.* 1st. 2851200;

56. What is a composite number?—57. What are the factors of any number?

10. There are 8766 hours in one year; how many hours, 1st. in 84 years?— 2nd. in 125 years?

Ans. 1st. 736344;

11. A town consumes 12432 pounds of bread in one day; how many pounds will the same consume, 1st. in 72 days?— 2nd. in 96?

Ans. 1st. 897804;

12. An acre of land costs \$475: what will cost, 1st. 15 acres?— 2nd. 70 acres?— 3rd. 144 acres?

Ans. 1st. 7125;

CASE II.—To effect multiplication when the multiplier is 10, 100, 1000, etc. (No. 36, 1st.).

59. RULE.—Annex to the multiplicand as many ciphers as there are in the multiplier.

EXAMPLES FOR PRACTICE (p. 19).

CASE III.—To effect multiplication when there are ciphers at the right-hand of one or both of the factors.

Ex. 1. Multiply 1400 by 80.

OPERATION.

1400  
80  
112000

ANALYSIS.—We resolve the multiplicand into the factors 14 and 100, and the multiplier into the factors 8 and 10. Now, it is evident, (No. 5), that, if these several factors be multiplied together, they will produce the same product as the given numbers, 1400 and 80. Thus,  $14 \times 8 = 112$ , and  $112 \times 100 = 11200$ ; and  $11200 \times 10 = 112000$ , the same result as in the operation.

60. From the preceding illustration we derive the following

RULE.—Write the significant figures of the multiplier under those of the multiplicand, and multiply them together. To their product, annex as many ciphers as there are on the right of both multiplicand and multiplier.

EXAMPLES FOR PRACTICE.

(2.)  
Multiply 3764580  
By 270000  
2635206  
752916  
Ans. 1016436600000

(3.)  
1306950000  
600800  
1045560  
784170  
785215560000000

4. Multiply 610430 by 700500.

Ans. 427606215000.

5. Multiply 3070607 by 7007000.

Ans. 21515743249000.

6. Multiply 2020370 by 40302000.

Ans. 814249517400000.

7. Multiply twenty-eight millions and four thousand, by three hundred and five thousand.

Ans. 8541220000000.

8. Multiply seventy millions seven thousand and six hundred, by eight millions seven hundred and sixty.

Ans. 560114005776000.

60. What is the rule to multiply when there are ciphers at the right-hand of the multiplier or multiplicand, or both?

9. Multiply forty-nine millions and forty-nine, by four hundred and ninety thousand. *Ans.* 24010024010000.

10. Multiply one billion and twenty thousand, by one thousand and one hundred. *Ans.* 1100022000000.

11. Multiply ten billions ninety-six thousand and eight hundred, by thirty thousand and seven hundred. *Ans.* 309971760000.

12. Multiply thirty millions ninety-thousand and eight hundred, by six hundred thousand and eighty. *Ans.* 18056887264000.

**CASE IV.**—*To effect multiplication when a part of the multiplier is a factor of another part.*

*Ex.* 1. Multiply 7439 by 328.

**OPERATION.**

$$\begin{array}{r} 7439 \\ \times 328 \\ \hline 59512 = \text{Prod. by 8 units.} \\ 238048 = \text{Prod. by 32 tens.} \\ \hline 2439992 = \text{Prod. by 328.} \end{array}$$

factor 8 of the other part of the multiplier, we multiply it by 4 tens, obtaining the product of the multiplied by  $8 \times 4$  tens, or 32 tens. These products of the parts, added together, give the true product by 328; and,

**ANALYSIS.**—We consider the multiplier as separated into two parts, 32 tens and 8 units, or  $320 + 8$ ; of which the smaller part is evidently a factor of the larger, since the 32 tens, or 320, is equal to 4 tens  $\times$  8. We next multiply by the 8 units, obtaining the product for that part of the multiplier. Now, as this product is the same as that by the

**61.** From this illustration we derive the following

**RULE.**—*Multiply first by the smaller part of the multiplier; and then that partial product by a factor, or factors, of a larger part; and so on with all the parts. The sum of the several partial products will be the product required.*

**EXAMPLES FOR PRACTICE.**

- |                                 |                          |
|---------------------------------|--------------------------|
| 2. Multiply 6526 by 568.        | <i>Ans.</i> 3706768.     |
| 3. Multiply 3785 by 721.        | <i>Ans.</i> 2728985.     |
| 4. Multiply 85065 by 2432.      |                          |
| 5. Multiply 236428 by 54918.    | <i>Ans.</i> 12984152904. |
| 6. Multiply 397821 by 25125.    |                          |
| 7. Multiply 1146084 by 24816.   | <i>Ans.</i> 28441220544. |
| 8. Multiply 5723605 by 4249784. |                          |

**CASE V.**—*To effect the multiplication of decimals when the multiplier is 10, 100, 1000, etc. (No. 36, 2nd.)*

**62. RULE.**—*Remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers if required.*

61. What is the rule for multiplying when a part of the multiplier is a factor of another part?—62. What is the rule for effecting the multiplication of decimals when the multiplier is 10, 100, 1000, etc.?

EXAMPLES FOR PRACTICE (p. 20 and 21).

**CASE VI.**—To effect the multiplication of decimals when it is not necessary that all the decimal places of the product should be retained.

*Ex.* 1. Multiply 6.5628 by 5.786, retaining only three decimal places in the product.

OPERATION.

$$\begin{array}{r}
 6.5628 \\
 687.5 \\
 \hline
 32814 = 6.562 \times 5 \\
 4594 = 6.56 \times .7 + 2 \\
 525 = 6.5 \times .08 + 5 \\
 39 = 6. \times .006 + 3 \\
 \hline
 37.972 \text{ Product.}
 \end{array}$$

**ANALYSIS.**—We reverse the order of the figures of the multiplier and write them under the multiplicand; and, since thousandths is the lowest decimal figure to be retained in the product, we place the units' figure of the multiplier under the thousandths' figure of the multiplicand. Then, the unit of the product of any figure of the multiplicand by the figure of the multiplier that falls under it will be thousandths. When there are figures in the multiplicand on the right

of that immediately above the figure of the multiplier, their product by the latter figure being expressed in units of lower orders than thousandths, may be neglected, except for the purpose of finding what must be carried to the thousandths' figure from their product.

**63.** From this illustration we deduce the following

**RULE.**—I. Write the multiplier, with the order of its figures reversed, and with the units' place under that figure of the multiplicand which is the lowest decimal to be retained in the product.

II. Find the product of each figure of the multiplier by the figures above and to the left of it in the multiplicand increasing each partial product by as many units as would have been carried from the rejected part of the multiplicand, and one more when the highest figure in the rejected part of any product is 5, or greater than 5; and write these partial products with the lowest figure of each in the same column.

III. Add the partial products, and from the right-hand of the result point off the required number of decimal figures.

**NOTE.**—1. Should the number of decimal places in the multiplicand be less than the number required in the product, supply the deficiency by annexing ciphers.

2. To obtain the number to be carried to each contracted partial product, it is generally necessary to multiply (mentally) only one figure at the right of the figure above the multiplying figure; but when the figures are large, the multiplication should commence at least two places to the right.

3. When the number of units in the highest order of the rejected part of the product is between 5 and 15, we carry 1; if between 15 and 25, we carry 2; if between 25 and 35, we carry 3; and so on.

**63.** What is the rule for effecting the multiplication of decimals, when it is not necessary that all the decimal places of the product should be retained?

## EXAMPLES FOR PRACTICE.

2. Multiply 472.35 by 64.3645; and 3.657389 by 0.0536423, retaining, in the first, 2 decimal places, and, in the second, 5 decimal places.

## OPERATION.

|                            |
|----------------------------|
| 472.350                    |
| 5463.46                    |
| <hr style="width: 100%;"/> |
| 2834100                    |
| 188940                     |
| 14170                      |
| 2834                       |
| 189                        |
| 23                         |
| <hr style="width: 100%;"/> |
| 3040.256                   |

## OPERATION.

|                            |
|----------------------------|
| 3.657389                   |
| 3246350.0                  |
| <hr style="width: 100%;"/> |
| 182869                     |
| 10972                      |
| 2194                       |
| 146                        |
| 7                          |
| 1                          |
| <hr style="width: 100%;"/> |
| 1.96189                    |

3. Multiply 751.2037 by 38.7136, retaining 3 decimal places in the product. *Ans.* 29081.801.

4. Multiply 36.275 by 4.3678, retaining 1 decimal place in the product.

5. Multiply 843.7527 by 8634.175, retaining only the whole numbers in the product. *Ans.* 7285109.

6. Multiply 4256.785 by 0.00564, retaining only 3 decimal places in the product.

7. Multiply 73.27593 by 0.075325, carrying out the product to the seventh decimal place. *Ans.* 5.5195095.

8. Multiply 1.7323152 by 3962.57302, retaining 8 decimal places in the product.

## PRACTICAL PROBLEMS COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION.

1. The hide of an ox costs \$6.15; it requires \$2 worth of bark, 9 quarts of oil at \$0.18 a quart, and \$0.60 for labor to prepare it. Required the gain if it be sold afterwards for \$12.75?

*ANALYSIS.*—The whole cost of the hide = \$6.15 + \$2 + (\$0.18 × 9 = \$1.62) + \$0.60 = \$10.37; \$12.75 - \$10.37 = *Ans.* \$2.38 gain.

2. A muslin manufacturer sold in one year, 540 pieces of it, viz.: 170 pieces to Montreal merchants; 85, to Quebec merchants; 130, to Toronto merchants; and the remainder to Ottawa merchants: what is that remainder? *Ans.* 155 pieces.

3. A man bought 25 barrels of flour at \$5.50 a barrel, and 40 barrels of apples at \$3 a barrel; what was the cost of all? *Ans.* \$257.50.

4. I paid for building my house \$1889, for my farm 3 times as much less \$892, and for my furniture \$140 more than I paid for building my house; how much did I pay for all, and for each? *Ans.* \$4775; \$2029; and \$8693.

5. A young man receives \$1000 salary, and pays \$180 for board,

\$215 for clothing, \$120 for books, and \$165 for other expenses: how much can he save in 4 years?

*Ans.* \$1280.

6. A merchant sold 75 yards of cloth at \$2.47 per yard; he received in payment 132.25 yards of linen at \$0.92, and a bill of \$63.58; how much will the merchant receive?

*Ans.* Nothing.

7. A bookseller made an invoice of books as follows: 125 volumes at \$1.20; 248 at \$0.90; 136 at \$0.67; and 275 at \$0.50; what is the amount of his invoice?

*Ans.* \$601.82.

8. Leo has \$127; Peter, 3 times as much minus \$205; and John has as much as Leo and Peter together: how much have Peter and John respectively, and how much have they all?

*Ans.* Peter, \$176; John, \$303; and all, \$606.

9. A merchant bought 15 pieces of blue cloth, each containing 37 yards, and 12 pieces of black cloth, each containing 34 yards; how many yards of cloth did he buy of the two kinds altogether?

10. If a cow cost \$28, a horse 6 times as much, and a farm 9 times as much as the cow and horse together, minus \$112; how much more will the farm cost than 5 horses and 12 cows, at the same rate?

11. A wholesale grocer bought 95 barrels of salmon at \$10.50 a barrel; he sold 84 barrels of them at \$12 a barrel, and the remainder at \$9 a barrel; how much did he gain or lose? *Ans.* Gained \$109.50.

12. If an acre of land produces yearly 362 pounds of flax and 11 bushels of seed; it is required to know how many pounds of flax and how many bush. of seed will 7 acres produce, and how much will the whole be worth, if the flax be sold at \$0.18 a pound and the seed at \$2.50 per bush.? *Ans.* 2534 pounds flax; 77 bush. seed; \$648.62.

13. In a dairy, there are 27 milch cows which give each, on an average, 108 pounds of butter; what sum will the dairy-man make in selling his butter at \$0.18 a pound?

*Ans.* \$624.88.

14. A farmer desires to manure a field of 12 acres of land with manure worth \$4 the hundred weight, and pays \$1.45 for cartage per hundred weight; how much will it cost him to manure his field, supposing he requires 2 hundred weight per acre? *Ans.* \$130.80.

15. A cabinet-maker earns daily \$1.55; his wife, \$1.20; and his three sons, \$0.65 each; how much can he lay by every week, the daily expenses of the whole family being \$2.68? *Ans.* \$9.44.

16. A lends B \$19560, B lets A have bank stock to the amount of \$3892, a farm 4 times as much as the bank stock — \$1998, and pays the remainder in cash; how much cash did B pay A? *Ans.* \$2098.

17. A jeweller bought a certain quantity of ivory at the rate of \$0.78 per pound; had he bought 6 pounds more, the cost would have been increased one eighth; how much did he pay for his ivory?

18. The repairs and superintendence of a railroad track cost yearly \$993 per mile; the expenses for improvements come to \$4342.60; besides the company pays \$626.40 for administration purposes and other items; required the total yearly expenditure for a track 132 miles long?

19. A plumber furnishes three kinds of zinc pipes: the diameter of the first, is 2 inches at \$0.32 per yard; the second, 5 inches at \$0.64; and the third, 8 inches at \$0.96 per yard. The first kind is

99 yards long which is 20 yards more than the second and 34 more than the third; what sum must be paid to the plumber for his pipes?

*Ans.* \$144.64.

20. A handkerchief manufacturer bought 78 packages of thread, of which 40 are warp, at \$10.90 per package, and 38 weft, at \$10.55. He pays \$0.85 per dozen for weaving and \$26.30 for selling expenses; what will be his gain, knowing that he has made 640 dozen of handkerchiefs, and sold them at the rate of \$2.58 per dozen?

*Ans.* \$244.

### DIVISION.

**64. Division** is the process of finding how many times one number is contained in another; or the process of finding one of the factors, the product and the other factor being known. Thus,

To divide 12 by 3, is to seek a number, which, being multiplied by 3, gives 12 for product; or, to find by what number 3 must be multiplied, to obtain 12 in the product.

The product is called **Dividend**, the known factor, **Divisor**, and the factor sought, **Quotient**.

When the dividend does not contain the divisor an exact number of times, the part of the dividend left is called the **Remainder**, and must be less than the divisor.

**CASE I.**—*To divide when the divisor does not exceed 12.*

**NOTE.**—When the process of dividing is carried on in the mind, and the quotient only is set down, the operation is called *Short Division*.

*Ex.* 1. How many times is 7 contained in 994?

**OPERATION.**

Divisor 7 ) 994 Dividend.

142 Quotient.

**ANALYSIS.**—We write the divisor on the left of the dividend with a line between them and another line beneath the dividend; then, beginning at the left-hand, we say: 7 is contained in 9, 1 time, and 2 hundreds remaining; we write the 1 directly under the 7, its dividend, for the *hundreds'* figure of the quotient. To 9, the next figure of the dividend, which is tens, we unite the 2 hundreds remaining, which equal 29 tens, in which we find the divisor 7 to be contained 4 times, and 1 ten remaining; we write the 4 for the *tens'* figure in the quotient, and the 1 ten remaining, equals 10 units, which, united to 4, the last figure of the dividend, make 14 units; in 14 units, 7 is contained 2 times; writing the 2 for the *units'* figure of the quotient, we have 142 for the entire quotient.

**65. RULE.**—I. *Write the divisor at the left-hand of the dividend, with a line between them, and draw a horizontal line beneath the dividend.*

64. *What is division?—What is the dividend?—The divisor?—The quotient?—The remainder?—65. What is the rule for short division?*

II. *Beginning at the left, divide each term of the dividend by the divisor, and write each quotient figure under its dividend.*

III. *If there be a remainder after dividing any figure, consider it as prefixed to the next term of the dividend, and divide as before.*

IV. *Should any partial dividend be less than the divisor, write a cipher in the quotient, and prefix the number to the figure of the next lower order in the dividend, and divide as before.*

V. *If there be a remainder after dividing the last figure, place it after the quotient, and write the divisor under it.*

NOTE.—Besides the usual sign  $\div$ , division is also indicated by writing the dividend above, and the divisor below a short horizontal line; thus  $\frac{8}{2} = 2$ .

PROOF.—Multiply the divisor and quotient together, and to the product add the remainder, if any; if the result obtained be equal to the dividend, the work is correct.

NOTE.—This method of proof follows from division being the reverse of multiplication. (64).

EXAMPLES FOR PRACTICE.

2. Divide 8154 by 6.

OPERATION.

Divisor 6 ) 8154 Dividend.  
           1359 Quotient.

PROOF.

1359 Quotient.  
    6 Divisor.

8154 Dividend.

(3.)  
 5 ) 714325  
    142865

(4.)  
 3 ) 893763

(5.)  
 7 ) 949112  
    135587

(6.)  
 4 ) 562845

Quotients.

1275.  
 932.  
 12347.  
 19369.  
 136982.  
 4201436.

Rem.

5.  
 2.  
 6.  
 3.  
 4.  
 6.

7. Divide 6375 by 5.
8. Divide 5592 by 6.
9. Divide 98776 by 8.
10. Divide 174321 by 9.
11. Divide 1643784 by 12.
12. Divide 46215796 by 11.
13. Divide 63412632 by 12.

14. Divide 2271582 by 7.
15. Divide 11357912 by 5.
16. Divide 4056360 by 9.
17. Divide 12980400 by 8.
18. Divide 42084795 by 6.
19. Divide 4507060 by 12.
20. Divide 15023520 by 11.

## PRACTICAL PROBLEMS.

1. Nine yards of silk velvet cost \$72; how much did it cost a yard?

**ANALYSIS.**—If the price of a yard were known, in multiplying it by 9, we would obtain \$72; therefore, 72 is a product having for factors 9 and the price of a yard. Then, in dividing the product 72 by the factor 9, we obtain the price of a yard;  $72 \div 9 = \text{Ans. } \$8$ . Or again, as 9 yards cost \$72, 1 yard will cost 9 times less, because there are 9 times less yards; then, in dividing 72 by 9, we obtain the price of a yard.

2. If 5 shillings make a dollar; how many dollars in 8890 shillings?

*Ans.* 1778 dollars.

3. A gentleman divided \$89622 equally among his 9 children; how much did each receive?

*Ans.* \$9958.

4. How many barrels of flour, at \$8 a barrel, can be bought for \$680?

*Ans.* 85 barrels.

5. If 12 inches make one foot; how many feet in 7501464 inches?

6. Eleven horses were sold for \$2531; what was the average sum received for each?

*Ans.* \$231.

7. A boy spent in one month 260 cents for oranges, giving 4 cents for each; how many oranges did he buy?

*Ans.* 65.

8. A carpenter worked 11 months for \$572; how much did he receive a month?

*Ans.* \$52.

9. If maple is worth \$6 a cord; how many cords will be had for \$1152?

*Ans.* 192 cords.

10. A person wishes to distribute 168 apples equally among 4 boys and 3 girls; how many will each of them receive?

*Ans.* 24.

**CASE II.**—To divide when the divisor exceeds 12.

**NOTE.**—When the whole process of division is written, the operation is termed *Long division*.

*Ex.* Divide 4738 by 34.

**OPERATION.**

Divisor. Divd'd. Quotient.

34 ) 4738 ( 139  $\frac{2}{3}$ .

34

2nd. partial dividend  $\overline{133}$

102

3rd. partial dividend  $\overline{318}$

306

$\overline{12}$  Remainder.

which bringing down the next figure of the dividend, we form 133; 34 in 133, 3 times. The 3 we write in the quotient;  $34 \times 3 = 102$ , which we write under the 133;  $133 - 102 = 31$ , to which bringing down the next figure of the dividend, we form 318; 34 in 318, 9 times. The 9 we write in the quotient;  $34 \times 9 = 306$ , which we write under the 318;  $318 - 306 = 12$ , a remainder, or a part of the dividend left undivided, which we write in the quotient with the divisor below it, thus completing the division.

**ANALYSIS.**—Taking 47 hundreds for the first partial dividend, we say: 35 is contained in 47, 1 time. The 1 we write in the quotient;  $34 \times 1 = 34$ , which we write under the 47;  $47 - 34 = 13$ , to which bringing down the next figure of the dividend, which is 3, we form 133; 34 in 133, 3 times. The 3 we write in the quotient;  $34 \times 3 = 102$ , which we write under the 133;  $133 - 102 = 31$ , to which bringing down the next figure of the dividend, we form 318; 34 in 318, 9 times. The 9 we write in the quotient;  $34 \times 9 = 306$ , which we write under the 318;  $318 - 306 = 12$ , a remainder, or a part of the dividend left undivided, which we write in the quotient with the divisor below it, thus completing the division.

**66. RULE.**—I. Write the divisor and dividend as in short division, and draw a curved line at the right-hand of the dividend.

66. What is the rule to divide when the divisor exceeds 12?

II. Take for the first partial dividend, the least number of figures on the left that will contain the divisor, and place the quotient on the right.

III. Multiply the divisor by this quotient figure, place the product under the partial dividend, subtract, and to the remainder, annex the next term of the dividend, for the second partial dividend. annex the next term of the dividend, for the second partial dividend.

IV. Divide as before, until all the figures of the dividend have been brought down and divided.

V. If any partial dividend will not contain the divisor, place a cipher in the quotient, and bring down the next figure of the dividend, and divide as before.

VI. If there be a remainder after dividing all the figures of the dividend, it must be written in the quotient, with the divisor underneath.

NOTES.—1. If any remainder be equal to, or greater than the divisor, the corresponding figure in the quotient is too small.

2. If the product of the divisor by the quotient figure be greater than the partial dividend, the quotient figure is too large.

PROOF.—It is the same as in short division.

DIVISION ACCORDING TO THE FRENCH METHOD.

Ex. Divide 11812 by 72.

OPERATION.  
 Dividend 11812 ( 72 Divisor.  

$$\begin{array}{r} 72 \overline{) 11812} \\ \underline{461} \phantom{00} \\ 432 \phantom{00} \\ \underline{292} \phantom{00} \\ 288 \phantom{00} \\ \underline{\phantom{00} 4} \phantom{00} \phantom{00} \\ \phantom{00} 4 \text{ Remainder.} \end{array}$$

OBSERVATION.—We see by the example in the margin, that the divisor is placed on the right of the dividend, and the quotient below it. This mode gives the work a more compact and neat appearance, and possesses the advantage of having the figures of the quotient near the divisor, by which means, the practical difficulty of multiplying the divisor by a figure placed at a distance from it, is removed.

ABBREVIATION OF LONG DIVISION.

67. By the following method, we avoid writing the products in long division, as in the example of Case II, above.

Ex. I. Divide 8764 by 365.

OPERATION.  
 365 ) 876.4 ( 24  
 146 4  
 ... 4 remainder.

ANALYSIS.—In this operation, we say : 3 is contained 2 times in 8; we write 2 at the quotient and multiply the divisor saying: 2 times 5 are 10, which subtracted from 16 (because we increase the 6 by 10), leaves 6 and carry one; 2 times 6 are 12 and 1 is 13, which subtracted from 17 leaves 4 and carry 1; again 2 times 3 are 6 and 1 is

7, which, subtracted from 8, leave 1; we bring down the 4 to form the second partial dividend. Then 3 in 14 is contained 4 times, which we write at the quotient, and multiply 365 by it; we subtract the product from the second partial dividend in a similar manner; there remains 4 which is to be added to the proof. Hence the following

**RULE.—I.** Obtain the first figure of the quotient in the usual manner.

**II.** Multiply each figure of the divisor by this quotient figure, subtract from the first partial dividend, and write underneath the remainder.

**III.** At the right of the first remainder, annex the next figure of the dividend, for a new partial dividend, and proceed as with the former, till the work is finished.

**OBSERVATION 1.**—When, after having employed all the figures of the dividend, there is still a remainder, we may reduce this remainder, firstly in tenths by adding a cipher at the right of it, and continue the division; but then, as we cannot have any more units, we place a point at the quotient. When we continue the division, the second remainder is reduced into hundredths by the addition of another cipher; but place no more points at the quotient, the units being indicated by the order they occupy. (Nos. 27 and 31.)

*Ex.* Divide 679 by 28.

OPERATION.  
 28 ) 679 (24.25  
     119  
       70  
      140  
       ..

ANALYSIS.—After the division, there remains 7; we reduce this remainder to tenths by writing a cipher at the right-hand of it, and we place a point at the quotient, and then proceed as before. But as there remains yet 14 tenths, we reduce this number to hundredths by the addition of another cipher. Multiplying and subtracting as before we find that nothing remains. Hence we conclude that 24.25 is the correct

quotient of 679 by 28, as shown by the proof. Had there been another remainder, we would have added one more cipher. Thus, we can carry the approximation to any order of decimal unity.

**OBSERVATION 2.**—When the dividend is smaller than the divisor, we first place a cipher and a point at the quotient to signify that there are no integers or whole numbers; then we reduce the dividend to tenths, hundredths, &c. (No. 36.), and proceed as before.

*Ex.* Given 6 to be divided by 25; what will be the operation?

OPERATION.  
 25 ) 6.0 ( 0.24  
     1 00  
       0

ANALYSIS.—Having disposed the terms, we say: 25 in 6 is not contained, we write a cipher and a point at the quotient. Then we reduce the 6 units in tenths by placing a cipher at the right-hand of it, and say: 25 in 60 is contained 2 and 10 tenths remain. We reduce them into hundredths by the addition of a cipher, and say: 25 in 100 is contained 4 times, and nothing remain; therefore, 0.24 hundredths is the quotient of 6 divided by 25 units.

2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15

**USE OF DIVISION.**—*Division serves to divide a number into equal parts; to render it a certain number of times smaller; to find how many times a number is contained into another; to find by what number must a given number be multiplied to produce another given number. Division serves also to find the value of the unity when a certain given number of units or parts of units are known, as for instance, the buying, the selling price of a yard, the gain or loss of a yard, &c. &c.; to find the number of units knowing their whole value and that of the unit, as for instance, the number of days that a laborer must work to earn a certain sum; and lastly, to find how many units there are in a number which expresses subdivisions of this unit, such as to find how many hours there are in any given number of minutes, &c.*

*We know generally that the solution of a problem requires a division when the value of several units, or parts of units, are given, to find only one; Or, when the value of the unit, and several units, or parts of units being given, we seek the number of units or parts of units.*

**EXAMPLES FOR PRACTICE.**

1. Find how many times is 72 contained in 23596.

**OPERATION.**

Divid'd.  
 Divisor. 72 ) 23596 ( 327 Quotient.  
           216  
           199  
           144  
           556  
           504  
           52 Remainder.

**PROOF BY MULTIPLICATION.**

327 Quotient.  
 72 Divisor.  
 -----  
   654  
 2289  
 -----  
 23544  
   52 Remainder.  
 -----  
 23596 Dividend.

|     |        |   | Quotients. | Rem. |
|-----|--------|---|------------|------|
| 2.  | 27939  | + | 16         |      |
| 3.  | 38582  | + | 18         | 3    |
| 4.  | 405683 | + | 20         | 8    |
| 5.  | 743241 | + | 25         | 3    |
| 6.  | 954992 | + | 30         | 16   |
| 7.  | 173469 | + | 36         | 2    |
| 8.  | 497699 | + | 40         | 21   |
| 9.  | 218579 | + | 42         | 19   |
| 10. | 611286 | + | 47         | 11   |
| 11. | 41126  | + | 49         | 4    |
| 12. | 432605 | + | 50         | 15   |
| 13. | 845002 | + | 53         | 5    |
| 14. | 867532 | + | 59         | 23   |
| 15. | 876701 | + | 60         | 55   |
|     |        |   |            | 41   |

## DIVISION.

|     |         |   |    | Quotients. | Rem. |
|-----|---------|---|----|------------|------|
| 16. | 4968    | + | 64 |            |      |
| 17. | 940025  | + | 68 | 77         | 40   |
| 18. | 445124  | + | 70 |            | 61   |
| 19. | 4728    | + | 75 | 6358       | 64   |
| 20. | 39006   | + | 79 |            | 3    |
| 21. | 1679407 | + | 80 | 493        | 59   |
| 22. | 4306404 | + | 85 |            | 47   |
| 23. | 167008  | + | 87 | 50663      | 49   |
| 24. | 7456029 | + | 90 |            | 55   |
| 25. | 6717890 | + | 98 | 82844      | 69   |
|     |         |   |    |            | 88   |

To calculate with two decimals in the quotient.

|     |             |   |        | Quotients. | Rem.   |
|-----|-------------|---|--------|------------|--------|
| 26. | 67980       | + | 96     | 708.12     | 48     |
| 27. | 432101      | + | 69     | 6262.33    | 23     |
| 28. | 470896      | + | 72     |            | 16     |
| 29. | 680094      | + | 67     | 10160.65   | 45     |
| 30. | 666648      | + | 441    | 1511.67    | 153    |
| 31. | 767642      | + | 386    |            | 380    |
| 32. | 124674      | + | 126    | 989.47     | 78     |
| 33. | 964321      | + | 216    | 4464.44    | 196    |
| 34. | 7246579     | + | 612    |            | 328    |
| 35. | 7890645     | + | 367    | 21500.39   | 187    |
| 36. | 9120128     | + | 637    |            | 153    |
| 37. | 687621      | + | 4691   | 146.58     | 1422   |
| 38. | 3466604     | + | 1279   |            | 240    |
| 39. | 4268901     | + | 1467   | 2909.95    | 435    |
| 40. | 2486930     | + | 7614   |            | 4532   |
| 41. | 4107129     | + | 7614   | 539.41     | 6126   |
| 42. | 81267904    | + | 6174   |            | 3592   |
| 43. | 69267421    | + | 7186   | 9639.21    | 5794   |
| 44. | 89064010    | + | 7908   |            | 184    |
| 45. | 694735210   | + | 9087   | 76453.74   | 7462   |
| 46. | 468904008   | + | 7064   |            | 6768   |
| 47. | 389006753   | + | 8004   | 48601.54   | 2684   |
| 48. | 86742807    | + | 8906   |            | 5914   |
| 49. | 707070709   | + | 4260   | 165979.03  | 4120   |
| 50. | 654380316   | + | 49060  |            | 37440  |
| 51. | 987654321   | + | 49066  | 20129.09   | 39106  |
| 52. | 8606000041  | + | 60041  |            | 49042  |
| 53. | 61247680241 | + | 74085  | 826721.74  | 13310  |
| 54. | 74238961401 | + | 48647  |            | 11893  |
| 55. | 9649646664  | + | 42867  | 225106.64  | 32712  |
| 56. | 8674289646  | + | 74551  |            | 48424  |
| 57. | 4247698734  | + | 94672  | 44867.52   | 88056  |
| 58. | 6312460086  | + | 59866  |            | 46810  |
| 59. | 45680108007 | + | 300452 | 152037.95  | 185360 |
| 60. | 37894216118 | + | 987684 |            | 88784  |

DIVISION OF DECIMALS.

Rem.  
40  
61  
64  
3  
59  
47  
49  
55  
69  
88

Ex. 1. Divide 3.456 by 2.4.

OPERATION.

$$\begin{array}{r}
 2.4 \overline{) 3.456} \quad \text{Ans.} \\
 \underline{24} \phantom{00} \\
 105 \phantom{0} \\
 \underline{96} \phantom{0} \\
 96 \\
 \underline{96} \\
 0
 \end{array}$$

ANALYSIS.—We divide as in whole numbers; and, since the divisor and quotient are the two factors, which, being multiplied together, produce the dividend, we point off two decimal figures in the quotient, to make the number in the two factors equal to the number in the product or dividend.

Ex. 2. Divide 0.525 by 7.5.

OPERATION.

$$\begin{array}{r}
 7500 \overline{) 525.00} \quad (0.07. \\
 \underline{52500} \\
 0
 \end{array}$$

ANALYSIS.—As the decimal places in the dividend exceed those in the divisor, we make them equal by annexing two ciphers to the divisor; and, having proceeded in the division as in Obs. 2, p. 54, we find the quotient to be 0.07, or 7 hundredths.

Rem.  
48  
23  
16  
45  
153  
380  
78  
196  
328  
187  
153  
1422  
240  
435  
4532  
6126  
3592  
5794  
184  
7462  
6768  
2684  
5914  
4120  
37440  
39106  
49042  
13310  
11893  
32712  
48424  
89056  
46810  
185360  
88784

67. From the preceding illustrations we deduce the following

RULE I.—Divide as in whole numbers, and point off as many decimals in the quotient as the decimals in the dividend exceed those of the divisor; but, if there are not as many, supply the deficiency by prefixing ciphers.

Or,

RULE II.—If the dividend and divisor have not the same number of decimals, annex ciphers at the right-side of the term which has the least, so that it may have as many decimal figures as the other; then divide, without any regard to the point, as in whole numbers.

NOTE 1.—To divide decimals by 10, 100, 1000, etc. (No. 37).

PROOF.—The proof is the same as in division of whole numbers.

EXAMPLES FOR PRACTICE.

|     |         | + |        | Quotients. | Rem. |
|-----|---------|---|--------|------------|------|
| 3.  | 79.1    | + | 2.5    | 31.64      |      |
| 4.  | 67.8632 | + | 16.4   | 4.174      | 96   |
| 5.  | 2.3421  | + | 42.2   | 0.055      | 211  |
| 6.  | 0.338   | + | 0.15   |            | 8    |
| 7.  | 14.     | + | 0.7852 | 17.        | 6516 |
| 8.  | 0.21318 | + | 8.34   |            | 468  |
| 9.  | 10.85   | + | 0.0775 | 140.       |      |
| 10. | 0.1728  | + | 0.012  |            |      |

67. What is the rule for the division of decimals?

To calculate with five decimals in the quotient.

|     |         |   |          | Quotients. | Rem.  |
|-----|---------|---|----------|------------|-------|
| 11. | 16.6    | ÷ | 10.2     | 1.62745    | 34    |
| 12. | 40.72   | ÷ | 16.12    |            | 740   |
| 13. | 46.634  | ÷ | 39.122   | 1.19201    | 18478 |
| 14. | 79.683  | ÷ | 14.244   |            | 6984  |
| 15. | 76.1234 | ÷ | 9.24     | 8.23846    | 296   |
| 16. | 59.2687 | ÷ | 91.42    |            | 1998  |
| 17. | 79.4    | ÷ | 9.04     | 8.78318    | 528   |
| 18. | 70.8    | ÷ | 10.08    |            | 960   |
| 19. | 29.40   | ÷ | 18.126   | 1.62197    | 17178 |
| 20. | 16.74   | ÷ | 17.261   |            |       |
| 21. | 0.7     | ÷ | 3.7      | 0.18918    | 34    |
| 22. | 0.2     | ÷ | 3.2      |            |       |
| 23. | 0.42    | ÷ | 3.07     | 0.13680    | 240   |
| 24. | 0.009   | ÷ | 0.000014 |            | 4     |

### PRACTICAL PROBLEMS.

1. If 45 yards of cloth cost \$123.75; how much will 1 yard cost?

ANALYSIS.—If the price of a yard were known, in multiplying it by 45, we would obtain \$123.75; therefore, \$123.75 is a product having for factors 45 and the price of a yard. Dividing 123.75 by 45, we obtain the price of a yard =  $\$123.75 \div 45 =$

Ans. \$2.75.

2. A laborer earns \$2.65 per day; in how many days will he earn \$47.70?

ANALYSIS.—As many times as \$2.65, the price of a day's labor, are contained in \$47.70, as many days will be required; therefore, in dividing 47.70 by 2.65, we obtain the number of days required =  $47.70 \div 2.65 =$

Ans. 18 days.

3. The product of two numbers is 661045; one of the numbers is 85; what is the other?

Ans. 7777.

4. What is the number that, being multiplied by 72 will give 70344?

5. One of two factors is 4.75 and their product 4222.18. Find the other factor?

Ans. 888.88 hundredths.

6. I paid \$806 for 196 reams of paper; how much is that per ream?

Ans.

7. What number is that which is 25 times smaller than 3575?

8. At \$0.30 per volume, how many volumes can be got for \$69?

9. If a cord of maple wood cost \$4.60; how many cords can be bought for \$989?

Ans. 215.

10. How many sheets of paper in a volume in-8° of 1280 pages? (The sheet in-8° contains 16 pages.)

Ans. 80.

11. How many yards of carpet, at \$4.60 per yard, can be bought for \$676.20?

Ans. 147 yards?

12. If 63 gallons make a hogshead; how many hogsheads will 2016 gallons make?

Ans. 32.

13. In how many days could 35 men accomplish as much work, as one man in 805 days?

Ans. 23.

14. During a cruise of 64 days, a ship sailed 11648 miles; how far did she sail each day?

Ans. 182 miles.

Rem.  
34  
740  
18478  
6984  
296  
1998  
528  
960  
17178

34

240

4

15. Having multiplied 6.55 by a certain number, we obtained 57.3125; what is that number? *Ans.* 8.75.
16. A train on the Grand Trunk Railway runs 62 miles an hour; at the same rate, how long would it take to go round the world, the distance being about 25000 miles? *Ans.* 403 + hours.
17. The large wheels of a coach are 15 feet in circumference, the small ones 6 feet; how many turns will each make in a distance of 140182 feet? *Ans.* Large, 9345 + 7; small, 23363 + 4.
18. Find a number whose product by 0.005 would be 0.00025.
19. I bought a farm containing 175 acres for \$4375; how many dollars did it cost per acre? *Ans.* \$25.
20. A butcher gave \$66 for sheep, at the rate of \$3.30 each; how many sheep did he buy? *Ans.* 20 sheep.
21. How many pair of slippers must be made by a shoemaker, to earn \$1.35 per day, if he be paid \$0.15 for every pair he makes?
22. The annual receipts on a railroad 500 miles long amount to \$360000. Required the average daily receipts, and how much is received per mile annually? *Ans.* Average daily receipts \$9863.01111. Do do per mile \$7200.
23. The air contained in a puncheon weighs 9.75 drachms; the water it would contain would weigh 7507.5 drachms; how many times is the weight of the water greater than that of the air? *A.* 770 times.
24. A charcoal maker places 127 cords of wood in a kiln which cost him \$580; he consumes 13 cords of fuel for the operation, and the value of the charcoal obtained is estimated at \$231/4, at the rate of \$0.28 per bushel. Required how many bushels of coal have been produced by a cord of wood? *Ans.* 6.5 bushels.
25. The population of the globe is about 1300860000 inhabitants; supposed that it is renewed every 33 years (in Canada, the mean length is about 37 years). It is required to know how many persons die yearly, daily, every hour and every minute? *Ans.* 39420000 persons yearly; 10800 daily; 4500 every hour; 75 every minute (or 5 every 4 seconds.)

CONTRACTIONS IN DIVISION,

OR DIVISION BY FACTORS.

CASE I.—To divide when the divisor is a composite number.

Ex. 1. Divide \$1596 equally among 28 persons.

OPERATION.  
4 ) 1596  
7 ) 399

\$7 *Ans.*

ANALYSIS.—The factors of 28 are 4 and 7. We divide \$1596 by 4, and the resulting quotient by 7, and obtain for the final result, \$7, which must be the same as the quotient of 1596 divided by 4 times 7, or 28. We might have obtained the same result by dividing first by 7, and then by 4. Hence the following

68. RULE.—Divide the dividend by one of the factors, and

68. What is the rule for dividing by a composite number?

*the quotient thus obtained, by a second factor, and so on, till every factor of the divisor has been used. The last quotient will be the required quotient.*

**EXAMPLES FOR PRACTICE.**

- |                                              |                    |
|----------------------------------------------|--------------------|
| 2. Divide 4536 by 14 = 2 × 7.                | <i>Ans.</i> 324.   |
| 3. Divide 9774 by 18 = 3 × 6.                | <i>Ans.</i> 543.   |
| 4. Divide 14560 by 35 = 5 × 7.               | <i>Ans.</i> 416.   |
| 5. Divide 126375 by 75 = 3 × 5 × 5.          | <i>Ans.</i> 1685.  |
| 6. Divide 69384 by 42, using its factors.    | <i>Ans.</i> 1652.  |
| 7. Divide 57456 by 72, using its factors.    | <i>Ans.</i> 798.   |
| 8. Divide 246792 by 84, using its factors.   | <i>Ans.</i> 2938.  |
| 9. Divide 2962875 by 125, using its factors. | <i>Ans.</i> 23703. |

*To find the true remainder when there are several in the operation.*

*Ex. 1. Divide 10183 by 105, using the factors 3, 5, and 7, and find the true remainder.*

OPERATION.

|           |                       |
|-----------|-----------------------|
| 3 ) 10183 |                       |
| 5 ) 3394  | ..... 1 rem.          |
| 7 ) 678   | ..... 4 × 3 = 12      |
|           | 96 ... 6 × 5 × 3 = 90 |
|           | 103 true rem.         |

**ANALYSIS.**—Dividing 10183 by 3, we have a quotient of 3394, and a remainder of 1 undivided, which, being a part of the given dividend, must also be a part of the true remainder. The 3394 being a quotient arising from dividing by 3, its units are 3 times as great in value as

the units of the given dividend, 10183. Dividing the 3394 by 5, we have a quotient of 678, and a remainder of 4. As this 4 is a part of the 3394, it must be multiplied by 5 to change it to the same kind of units as the 1. This makes a true remainder of 12 arising from dividing by 5. Dividing the 678 by 7, we have a quotient of 96 and a remainder of 6. This 6 is a part of the 678, the units of which are 5 times as great in value as those of the given dividend, 10183; therefore, to change this last remainder 6, to units of the same value as the dividend, we multiply it by 5 and 3, and obtain a true remainder of 90, arising from dividing by 7. Adding the three partial remainders, we obtain 103 for the true remainder. Hence, the

**69. RULE.**—I. *Multiply each partial remainder by all the divisors preceding the one that produces it.*

II. *Add the several products with the first remainder; the sum will be the true remainder.*

**EXAMPLES FOR PRACTICE.**

- |                                                                                |                 |
|--------------------------------------------------------------------------------|-----------------|
| 2. Divide 3026 by 15, using the factors 3 and 5, and find the true remainder.  | <i>Ans.</i> 11. |
| 3. Divide 34709 by 42, using the factors 6 and 7, and find the true remainder. | <i>Ans.</i> 17. |

**69.** *What is the rule for finding the true remainder ?*

4. Divide 5858 by 84, using the factors 3, 4, and 7, and find the true remainder. Ans. 62.
5. Divide 9078147 by 90, using the factors 3, 5, and 6, and find the true remainder. Ans. 27.
6. Divide 7360481 by 96, using the factors 2, 6, and 8, and find the true remainder. Ans. 65.
7. Divide 10165 by 120, using the factors 2, 3, 4, and 5, and find the true remainder. Ans. 85.
8. Divide 63724 by 135, using the factors 3, 5, and 9, and find the true remainder. Ans. 4.

**CASE II.**—To divide a whole number by 10, 100, 1000, etc. (No. 37, 1st)

**70. RULE.**—From the right-hand of the dividend, cut off as many figures as there are ciphers in the divisor. Under the figures so cut off, place the divisor, and the whole will form the quotient.

EXAMPLES FOR PRACTICE.

1. Divide 87 by 10. Ans. 8 $\frac{7}{10}$ .
2. Divide 5813 by 100. Ans. 58 $\frac{13}{100}$ .
3. Divide 7009 by 1000. Ans. 7 $\frac{9}{1000}$ .
4. Divide 510040 by 10000. Ans. 51 $\frac{40}{10000}$ .
5. Divide 200371 by 100. Ans. 2003 $\frac{71}{100}$ .

**CASE III.**—To divide when there are ciphers on the right-hand of the divisor.

Ex. 1. Divide 85726 by 4500.

OPERATION.

$$\begin{array}{r}
 45 \overline{) 85726} \quad ( 19 \frac{226}{4500} \\
 \underline{45} \phantom{00} \\
 407 \phantom{0} \\
 \underline{405} \phantom{0} \\
 26 \phantom{00} \text{ Remainder.}
 \end{array}$$

ANALYSIS.—The factors of 4500 are 100 and 45. First, dividing by 100, (70), we obtain for a quotient 857, and for a remainder 26. Dividing this quotient by the remaining factor, 45, we obtain for a quotient 19, and for a remainder 2, to which annex 26, the first remainder, and underneath write the divisor, and we have for the entire quotient 19 $\frac{226}{4500}$ .

**71. RULE.**—I. Cut off the ciphers from the divisor, and as many figures from the right of the dividend.

II. Divide the remaining figures of the dividend by the remaining figures of the divisor.

III. Place the entire divisor under the true remainder, and annex it to the integral part of the quotient, for the entire quotient.

70. What is the rule to divide by 10, 100, etc. ?— 71. What is the rule for dividing when there are ciphers on the right-hand of the divisor.

## EXAMPLES FOR PRACTICE.

2. Divide 33100 by 6000.
3. Divide 1047628 by 2400.
4. Divide 72002 by 1200.
5. Divide 96031425 by 92000.
6. Divide 1247701 by 47000.
7. Divide 1703945642 by 4160000.
8. Divide 460352000 by 8100.

Ans. 55180.  
 Ans. 4361188.  
 Ans. . . . 1700.  
 Ans. 2627688.

CASE IV.—To divide a decimal by 10, 100, 1000, etc. (No. 37).

**72. RULE.**—Remove the decimal point as many places to the left as there are ciphers in the divisor, and if there be not figures enough in the number, prefix ciphers.

## EXAMPLES FOR PRACTICE (p. 20 and 21).

CASE V.—To abridge the division of decimals, when the divisor contains a large number of them.

Ex. 1. Divide 675.4563 by 23.54738, extending the quotient to three decimal places.

## CONTRACTED OPERATION.

$$\begin{array}{r}
 23.54738 \overline{) 675.4563} \quad (28.684 \\
 \underline{470 \ 95} = \text{product by } 2, + 1. \\
 204 \ 50 \\
 \underline{188 \ 38} = \text{product by } 8, + 6. \\
 16 \ 12 \\
 \underline{14 \ 13} = \text{product by } 6, + 3. \\
 1 \ 99 \\
 \underline{1 \ 88} = \text{product by } 8, + 4. \\
 11 \\
 \underline{\phantom{1} 9} = \text{product by } 4, + 1. \\
 2
 \end{array}$$

## COMMON OPERATION.

$$\begin{array}{r}
 23.54738 \overline{) 675.4563} \quad (28.684 \\
 \underline{470 \ 94 \ 76} \\
 204 \ 50 \ 870 \\
 \underline{188 \ 37 \ 904} \\
 16 \ 12 \ 9660 \\
 \underline{14 \ 12 \ 8428} \\
 2 \ 00 \ 12320 \\
 \underline{1 \ 88 \ 37904} \\
 11 \ 744160 \\
 \underline{9 \ 418952} \\
 2 \ 325208
 \end{array}$$

**ANALYSIS.**—In the contracted method we first ascertain how many places of figures the above example will have in the quotient. Comparing the entire part of the divisor with the entire part of the dividend, it is evident that the first quotient figure will be of the order of tens, and therefore the quotient will contain two places of whole numbers; and as there are to be three places of decimals, it must contain five figures. Hence, we divide at first by five figures of the given divisor, counting them from the left towards the right, thus, using the 23.547, and rejecting the figures .38, on the right. In multiplying each contracted divisor by its quotient figures, we increase the product by having regard to rejected figures, as in contracted multiplication of decimals (Case VI, p. 57).

By comparing the contracted with the common method, we see the extent of the abbreviation, and the agreement of the corresponding intermediate results. Hence, the

72. What is the rule to divide a decimal by 10, 100, 1000, etc.?

**73. RULE.—I.** Compare the significant figures on the left of the divisor with those on the left of the dividend, and determine how many figures will be required in the quotient.

**II.** For the first contracted divisor, take as many significant figures from the left of the given divisor as there are places required in the quotient; and, at each subsequent division reject one place from the right of the last preceding divisor.

**III.** In multiplying by the several quotient figures, carry from the rejected figures of the divisor as in contracted multiplication.

**NOTE.**—Annex ciphers to either dividend or divisor, when necessary, before commencing the work.

#### EXAMPLES FOR PRACTICE.

2. Divide 487.24 by 1.003675, extending the quotient to 2 decimal places.  
*Ans.* 485.46.
3. Divide 2.3748 by 1.4736, extending the quotient to the third decimal place.  
*Ans.* 1.611.
4. Divide 3.2682 by 2.4736, and carry the quotient to four places of decimals.  
*Ans.* 1.3212.
5. Divide 0.079085 by 0.83497, and carry the quotient to the fifth decimal place.  
*Ans.* 0.09471.
6. Divide 8972.436 by 756.3452, extending the quotient to 4 decimal places.  
*Ans.* 11.8629.
7. Divide 0.4879357 by 0.002963, extending the quotient to the second decimal place.  
*Ans.* 164.69.
8. Divide 12193263.1112635269 by 1234.56789, extending the quotient to as many decimal places, plus one, as there will be whole numbers in it.  
*Ans.* 9876.54321.

#### DECIMAL CURRENCY.

**74. Decimal Currency** is the currency whose denominations increase and decrease in a tenfold ratio.

**75. Currency** is coin, bank bills, treasury notes, etc., in circulation as a medium of trade.

**76. Coin** is money stamped, and has a given value established by law.

**NOTES.**—1. The currency of the present Dominion of Canada is decimal currency; it had been adopted by each of the Provinces before their *Federation*.

2. Decimal currency is also the currency of the United States, and is sometimes called *Federal Money*; it was established by Congress in 1792.

73. What is the rule for abridging the division of decimals?—74. What is decimal currency?—75. Currency?—76. Coin?

**77.** The present **Coins** of the Dominion of Canada are of silver and copper.

The *silver* coins are the fifty-cent piece, the twenty-five-cent piece, the ten-cent piece, and the five-cent piece.

*NOTE.*—The shilling or twenty-cent piece, though still in circulation, is no longer to be coined.

The *copper* coins are the two-cent piece and the cent.

100 cents (*cts.*) make 1 dollar, marked \$1.

**78.** The **Coins** of the United States are of gold, silver, and nickel.

The *gold* coins are the double-eagle, eagle, half-eagle, quarter-eagle, three-dollars, and dollar.

The *silver* coins are the dollar, half-dollar, quarter-dollar, dime, and half-dime.

The *nickel* coins are the 5-cent, 3-cent, 2-cent, and 1-cent pieces.

*NOTE.*—1. The mill is not coined; it is used only in computation.

2. To make the metal of coins more serviceable, *gold* coins contain 9 parts by weight of gold and 1 part of an alloy consisting of silver and copper. *Silver* coins contain 9 parts of silver and 1 part of copper.

#### TABLE OF THE UNITED STATES CURRENCY.

|                        |              |                                  |
|------------------------|--------------|----------------------------------|
| 10 mills ( <i>m.</i> ) | make 1 cent, | marked 1 <i>ct.</i> or <i>c.</i> |
| 10 cents               | " 1 dime,    | " 1 <i>d.</i>                    |
| 10 dimes               | " 1 dollar,  | " \$1.                           |
| 10 dollars             | " 1 eagle,   | " 1 <i>E.</i>                    |

**79.** The **Dollar** is the *unit* of currency in the Dominion of Canada and the United States. Accounts are kept in *dollars*, *cents*, and *mills*.

Dimes, cents, and mills, being fractions of a dollar, are separated from the dollar by the decimal point; thus, four dollars two dimes three cents five mills, or four dollars two hundred thirty-five mills, are written \$4.235.

To express any number of cents less than 10, a cipher must be placed between the figure expressing that number and the decimal point; thus, 8 cents is written .08, or 0.08.

*NOTE.*—1. Business men frequently write cents as common fractions of a dollar; thus, \$3.14 is also written \$3  $\frac{14}{100}$ , read 3 and  $\frac{14}{100}$  dollars.  
2. In business transactions, when the final result of a computation contains 5 mills or more, they are called one cent, and when less than 5, they are rejected.

#### EXAMPLES FOR PRACTICE.

- |                                              |                      |
|----------------------------------------------|----------------------|
| 1. Write fifteen dollars twenty-three cents. | <i>Ans.</i> \$15.23. |
| 2. Write seven dollars six cents.            | <i>Ans.</i> \$7.06.  |
| 3. Write ten dollars nine cents.             |                      |
| 4. Write forty-two cents.                    | <i>Ans.</i> \$0.42.  |

77. What are the coins of the Dominion of Canada?—78. Of the United States?  
—79. What is the unit of currency in the D. C. and U. S?

5. Write five dollars eight mills.
6. Write thirty cents.
7. Write one hundred cents.
8. Write one thousand mills.
9. Write one cent five mills.
10. Write seventeen dollars four mills.
11. Write \$6 and 7 cents.
12. Write 3 eagles 4 dollars 3 dimes 3 mills.

Ans. \$5.008.

## REDUCTION OF DECIMAL CURRENCY.

**80. Reduction** is the process of changing a number of one denomination to another number of a different denomination which shall have the same value.

We see, p. 64, that  $\$1 = 100 \text{ cents} = 1000 \text{ mills}$ ; hence the

**81. RULE.—I.** To change dollars to cents, multiply by 100; that is, annex two ciphers.

**II.** To change dollars to mills, annex three ciphers.

**III.** To change cents to mills, annex one cipher.

**NOTE.**—Dollars, cents, and mills, expressed by a single number, are changed to mills by merely removing the decimal point to the right; and dollars and cents, by annexing one cipher and removing the decimal point to the right.

Conversely,

**RULE.—I.** To change cents to dollars, divide by 100; that is, point off two figures from the right.

**II.** To change mills to dollars, point off three figures.

**III.** To change mills to cents, point off one figure.

## EXAMPLES FOR PRACTICE.

1. In \$7 how many mills?

**ANALYSIS.**—In \$1 there are 1000 mills, and in \$7 there are 7 times 1000 mills = 7000 mills.

2. In 356 cents how many dollars?

**ANALYSIS.**—In \$1 there are 100 cents, therefore,  $\frac{1}{100}$  of the number of cents equals the number of dollars;  $\frac{1}{100}$  of 356 = \$3.56.

3. Change \$464 to cents.
4. Change 612 cents to dollars.
5. Reduce \$3.10 to mills.
6. Reduce 35 cents to mills.
7. Reduce 7045 mills to dollars.
8. Change 10426 cents to dollars.
9. Reduce \$4005 to mills.
10. In 2064 mills how many cents?

Ans. 46400 cts.

Ans. \$6.12.

**NOTE.**—We give no separate rules for Decimal currency, since it may be added, subtracted, multiplied and divided in the same manner as decimals.

80. What is reduction?— What is the rule for changing dollars to cents and mills?— Cents to mills?— 81. What is the rule for changing cents to dollars?— Mills to dollars?— Mills to cents?

## PRACTICAL PROBLEMS COMBINING THE FUNDAMENTAL RULES.

1. A broker bought stocks for \$3729.90, and sold them for \$4168.135; how much did he gain?  
*Ans.* \$438.235.
2. If 1 month's wages amount to \$35.50, what will 12 months' wages amount to?  
*Ans.* \$426.
3. At \$7.40 a barrel, how much flour can be bought for \$111?  
*Ans.* \$0.0625.
4. If 15 quarts of strawberries cost \$0.9375, what will 1/4 quart cost?  
*Ans.* \$5.36.
5. A farmer sold an equal number of chickens, ducks, and geese, for 2130 cents; the chickens at 14, the ducks at 46, and the geese at 82 cents each; how many of each kind did he sell?  
*Ans.* \$1235.
6. Bought 144 acres of land for \$5040, and sold 95 acres of it at \$48 an acre, and the remainder for what it cost; how much did I gain by the bargain?  
*Ans.* \$66.585.
7. A man sold 135 bushels of wheat at \$0.62 a bushel, and received in payment 86 pounds of sugar at \$0.09 a pound, 25 pounds of coffee at \$0.375 a pound, and the remainder in cash; how much cash did he receive?  
*Ans.* \$120.
8. If a gentleman's income be \$3000 a year, and his expenses \$4.20 a day, what will he save at the end of a year, or 365 days?  
*Ans.* \$5.36.
9. A man divided \$360 among three persons; to the first he gave \$130; and to the second, \$20 less than to the first: how much did the third receive?  
*Ans.* \$202.
10. A lumber merchant bought 680 logs for the sum of \$3644.80; what is the price of each log?  
*Ans.* \$255.
11. With a Bank note of \$1000, I paid my tailor's bill of \$348; my shoemaker's of \$75 and my house-rent of \$375; how many dollars have I left?  
*Ans.* \$282.
12. If a hat cost \$4.25, how much will five dozen of similar hats cost?  
*Ans.* \$256.
13. An army composed of 62100 men on the eve of a battle, has 13708 men less after the engagement; how many men are there yet in the army?  
*Ans.* \$7.80.
14. What is the price of a silver cover, if 15 cost \$117? A. \$7.80.  
*Ans.* \$330.
15. How much must I sell goods which cost me \$286, to gain \$76 in giving \$18 commission?  
*Ans.* \$949.
16. Joseph bought 73 casks of syrup at \$39 the cask, and sold them again for \$52; what is his profit?  
*Ans.* \$3830.
17. A Banker is to receive \$13950 in three payments; the first amounting to \$5800, and the second, to \$4320; what will be the amount of the third?  
*Ans.* \$461.60.
18. A silver cover costs \$19.20; how much will 2 dozen of similar ones cost?  
*Ans.* \$250.
19. I bought 150 apples for \$1.05; how many can I buy for \$17.85?  
*Ans.* \$16980.
20. A banker received during the first quarter, \$16980, during the second, \$31940, during the third, \$27674; during the fourth, \$42769. He paid out during the whole year, \$96843; required how much he has left supposing he had \$24375 in his safe at the beginning of the year?  
*Ans.* \$45851.

21. I sold at 65 cents a bushel, 58 bushels of barley for which I had paid 52 cts. per bushel; how much did I gain? *Ans.*
22. Frank was born in 1857, in what year will he be 21 years old? *Ans.* 1878.
23. A father was 34 years old at the birth of his son; what will be the age of the son when the father will be 75 years old? *Ans.* 41.
24. An omnibus able to carry 18 persons makes 12 trips per day; how many travellers will it carry in one year of 365 days supposing that there are 12 persons at each trip? *Ans.* 78840.
25. If we can buy a yard of flannel for \$1.76; how many yards of the same quality can be got for \$626.56? *Ans.* 356.
26. A pedestrian starts out from Quebec to Montreal, the distance being 180 miles; he walks during 5 days at the rate of 27 miles per day; required what distance he has yet to go? *Ans.* 45 miles.
27. Of a certain sum, 82 persons have received each \$24 and there remains yet \$36.40; what is that sum? *Ans.* \$2004.40.
28. I bought 15 yards of linen at \$0.25 a yard, 37 gallons of oil at \$1.30 a gallon, 40 pounds of tea at \$0.80 a pound, and 108 pounds of coffee at \$0.37 a pound; required the amount of my Bill? *A.* \$123.81.
29. A bookbinder has 720 volumes to bind at the rate of \$0.18 a volume; if it takes him 45 days to do the work: how much will he earn per day? *Ans.* \$2.88.
30. Having bought a barrel of oil containing 28 gallons, at \$0.75 a gallon, I lost 7 gallons by leakage and sold the remainder for \$1.20 per gallon; did I lose or gain and how much? *Ans.* Gained \$4.20.
31. Having bought a house for \$3740 and making repairs for the sum of \$1438, I wish to sell it so as to gain \$600: for how much must I sell it? *Ans.* \$5778.
32. What sum of money is required to pay 34 workmen each of whom has worked during 28 days, at \$0.80 per day? *Ans.* \$761.60.
33. I bought 97 barrels of codfish at \$2.10 per barrel, I gave 17 barrels to the poor and sold the remainder at \$2.40 per barrel. Have I lost or gained and how much? *Ans.* Gained \$155.
34. Louis bought 500 acres of land for the sum of \$17876. He afterwards sold it in lots as follows: 127 acres, at \$47; 212 acres, at \$96; and the remainder, at \$37; how much did he gain by his bargain? *Ans.* \$14402.
35. Henry receives 45 cents to buy 6 pounds bread at 3 cents a pound, and 2 loaves at 8 cents a piece; what is his change?
36. The overcoat of Wilfrid costs 3 times as much as the hat of Julius, which is worth \$2.70; what is the cost of Wilfrid's overcoat?
37. A fruit dealer has received 15 dozen oranges in two boxes; one of which contains 50 oranges more than the other; how many oranges are there in each box? *Ans.* 105 and 75.
38. A milliner bought silks at a shop for 36 cents, thread for 20 cents, needles for 9 cents, and cotton for 18 cents; after paying her bill she has 72 cents left; how much money had she? *Ans.* \$1.55.
39. What is the dividend when the divisor is 3061 and the quotient 0.065?
40. A butcher sells a pound of meat for 9 cents which cost him 6 cents; what profit does he make on 175 pounds? *Ans.* \$6.25.

41. A person having an income of \$3285, wishes to lay by \$3 a day. Required how much that person can spend daily, the year being of 365 days. *Ans.* \$6.00.

42. A merchant sold 75 yards of cloth at \$2.70 a yard, and has received in payment 132 yards of linen at 85 cents a yard and a note of \$52.40; how much has he yet to receive? *Ans.* \$37.90.

43. What is that number which, being augmented by 85 and divided by 9, gives 25 for quotient? *Ans.* 140.

44. A millionaire owes a sum of \$6540 which he agrees to liquidate in ten equal payments one every year for ten years. His annual income is \$5925; how much can he spend daily after paying the tenth agreed upon? *Ans.* \$14.44.

45. What number must be divided by 37 so that the quotient may be 13.25 and the remainder 0.35? *Ans.* 490.60.

46. At 39 cents a pound, how much must be paid for 9 bales of wool, each bale containing 317 pounds? *Ans.* \$1112.67.

47. If a pair of boots be sold for \$3.16; how much must be paid for 20 boxes, each containing 60 pairs? *Ans.* \$3792.

48. How much will 3550 laths cost at 22 cents per hundred?

49. How many barrels of apples containing 3 bushels each at 50 cents a bushel can I buy for \$40.50? *Ans.* 27 barrels.

50. A literary work consists of 6 volumes; in each volume there are 560 pages, in each page, 42 lines, and in each line, 40 letters. How many letters are there in the work, if it is divided into 60 chapters, and if 5 blank lines are left between each chapter?

51. How many cords of wood at \$3.25 a cord did I buy for \$136.50? *Ans.* 42.

52. Sold 20 pounds butter for \$3.80, how much will 59 pounds come to at the same price? *Ans.* \$11.21.

53. A cabinet-maker has earned \$45 in a certain number of days by working; had he worked 9 days more, he would have earned \$67.50; how much did he earn per day? *Ans.* \$2.50.

54. The sum of two numbers is 2458, and their difference, 154; what are the two numbers? *Ans.* 1306 and 1152.

55. When a son, who is now 30 years old, was born, his father was 35 and his mother 19; what are the actual ages of the father and mother? *Ans.* 65 and 49.

56. Having some money at my disposition, I bought two farms at the rate of \$1750 each, and 19 shares of Bank Stock at \$103 per share, and I have \$113 left; how much money had I at my command? *Ans.* \$5570.

57. In selling cloth for \$610, a merchant gained as much as the cloth cost him, less \$500; what was the cost? *Ans.* \$555.

58. Although I was robbed of \$25, yet after having paid \$546 which I owed, I have \$17 left; how much money had I?

S  
artic  
who  
No  
ey, g  
2.  
trans  
S  
No  
trans  
action  
2.  
it ma  
S  
the  
S  
ing  
No  
Items  
S  
a pu  
the  
how  
S  
the i  
No  
rent,  
count  
to a p  
occour  
Form  
2.  
a pro  
In  
Dr.  
Cr.  
yd. f  
doz.

82.  
Parcel  
An Ac  
of a B

## BILLS AND ACCOUNTS.

**82. A Bill**, in business transactions, is a written statement of articles bought or sold, together with the prices of each, and the whole cost.

**NOTE.**—1. The party who buys, or who receives money, goods, or services, etc., from another, is a *Debtor*; and the party who sells, or who parts with money, goods, etc., is a *Creditor*.

2. A Bill of goods bought or sold, or of services received or rendered at a single transaction, and containing only one date, is often called a *Bill of Parcels*.

**83. An Account** is a registry of debts and credits.

**NOTE.**—1. An account should always contain the names of both parties in the transaction, the price or value of each item or article, and the date of the transaction.

2. Accounts may have only one side, which may be either debit or credit; or it may have two sides, debit and credit.

**84. The Balance of an Account** is the difference between the amount of the debit and credit sides.

**85. An Account Current** is a full copy of an account, giving each item of both debit and credit sides to date.

**NOTE.**—An account current having only one side is sometimes called a *Bill of Items*.

**86. An Invoice** is a full statement in detail of goods sent to a purchaser or agent at the time the goods are forwarded, giving the marks and contents of each package, the charges paid, and how sent.

**87. The Footing of a Bill** is the total amount or cost of all the items.

**NOTE.**—1. When a creditor receives the amount of a bill or an account current, he acknowledges it to be paid by writing at the bottom of the bill or account "Received Payment," and signing his name. If the payment be made to a person authorized by the creditor to receive it, he should receipt the bill or account by writing the creditor's name first and his own name under it, as in Form I.

2. Bills and accounts are sometimes paid by the debtor giving to the creditor a promissory note for the amount.

In the following bills and accounts the abbreviations are:

*Dr.* for debit or debtor.  
*Cr.* for credit or creditor.  
*yd.* for yard.  
*doz.* for dozen.

*bbl.* for barrel.  
*bush.* for bushel.  
*lb.* for pound.  
*cwt.* for hundred weight.

82. What is a Bill?— What is meant by debtor and creditor?— By a Bill of Parcels?— 83. What is an Account?— 84. The Balance of an Account?— 85. An Account current?— A Bill of Items?— 86. An Invoice?— 87. The Footing of a Bill?

## FORMS OF BILLS AND ACCOUNTS.

(FORM 1.)

MR. G. MURRAY,

KINGSTON, Sept. 8, 1870.

*Bought of E. P. HEALEY & Co.*

|    |       |                         |          |          |          |
|----|-------|-------------------------|----------|----------|----------|
| 32 | yards | Cassimere, . . . . .    | @ \$1.70 | \$ 54 40 |          |
| 15 | "     | Blue Cloth, . . . . .   | @ 3.25   | 48 75    |          |
| 24 | "     | Flannel, . . . . .      | @ .67    | 16 08    |          |
| 15 | "     | Drilling, . . . . .     | @ .12    |          |          |
| 34 | "     | Fine Muslin, . . . . .  | @ .18    |          |          |
| 8  | "     | Gingham, . . . . .      | @ .30    |          |          |
| 4  | doz.  | Shirt Bosoms, . . . . . | @ 5.60   |          |          |
| 2  | "     | Wool Hose, . . . . .    | @ 3.25   |          |          |
|    |       |                         |          |          | \$158 65 |

*Received Payment,*E. P. HEALEY & Co.,  
per N. RYAN.

(FORM 2.)

MR. A. SEYMOUR,

MONTREAL, Sept. 17, 1870.

*Bought of T. MCGREEVY & Co.*

|         |     |         |                             |           |           |  |
|---------|-----|---------|-----------------------------|-----------|-----------|--|
| May 6   | 4   | boxes   | Oranges, . . . . .          | @ \$ 3.55 | \$ 14 20  |  |
| June 10 | 15  | "       | Raisins, . . . . .          | @ 2.90    | 43 50     |  |
| July 21 | 3   | chests  | Black Tea, . . . . .        | @ 25.00   | 75 00     |  |
| " "     | 4   | "       | Green Tea, . . . . .        | @ 28.50   |           |  |
| " 24    | 7   | "       | Imperial Tea, . . . . .     | @ 45.10   |           |  |
| " "     | 15  | bbls.   | Coffee Sugar, . . . . .     | @ 27.20   |           |  |
| Aug. 3  | 10  | sacks   | Coffee, . . . . .           | @ 18.60   |           |  |
| " 12    | 150 | bushels | Corn Meal, . . . . .        | @ .85     |           |  |
| Sept. 2 |     |         | Credited by Cash, . . . . . |           | \$1283 90 |  |
|         |     |         |                             |           | 80 00     |  |
|         |     |         |                             |           | \$1203 90 |  |

*Received Payment,*

T. MCGREEVY &amp; Co.

FORMS OF BILLS AND ACCOUNTS.

71

(FORM 3.)

QUEBEC, June 2, 1870.

MR. D. JOHNSON,

Bought of BYRNE, O'BRIEN & Co.

|     |     |                                  |          |          |
|-----|-----|----------------------------------|----------|----------|
| No. |     |                                  |          |          |
| 2   | 40  | pair Gaiters, . . . . . @ \$2.30 | \$ 92 00 |          |
| 7   | 75  | " Rubbers, . . . . . @ .72       | 54 00    |          |
| 14  | 108 | " Calf Boots, . . . . . @ 3.80   | 410 40   |          |
| 10  | 67  | " Thick " . . . . . @ 2.65       | 177 55   |          |
|     |     | Cooperage and Cartage, . . . . . | 4 37     |          |
|     |     | Insurance, . . . . .             | 1 30     |          |
|     |     |                                  |          | \$739 62 |

By "Canadian Express Line."

(FORM 4.)

TORONTO, Oct. 5, 1870.

L. JACKSON & Co.,

To W. PRIOR & SON. Dr.

|       |    |                                         |           |           |
|-------|----|-----------------------------------------|-----------|-----------|
| 1870. |    |                                         |           |           |
| July  | 3  | To 140 bbls. Flour, . . . . . @ \$ 7.60 | \$1064 00 |           |
| "     | 12 | " 95 " Fish, . . . . . @ 18.50          | 1767 50   |           |
| Aug.  | 9  | " 36 chests Green Tea, . . . @ 31.80    | 1144 80   |           |
|       |    |                                         |           | \$3966 30 |
| 1870. |    |                                         |           |           |
| July  | 20 | By 200 yards Broadcloth, . . . @ \$5.10 | \$1020 00 |           |
| "     | 27 | " 75 " Black Cloth, . . . @ 4.67        | 350 25    |           |
| Aug.  | 4  | " 280 " Red Flannel, . . . @ .72        | 201 60    |           |
| Sept. | 2  | " 24 gross Silk Buttons, . . . @ .43    | 10 32     |           |
|       |    |                                         |           | \$1582 17 |
|       |    | Balance due W. P. & Son . . . .         |           | \$2384 13 |

Received Payment

W. PRIOR & SON.

FORMS OF BILLS AND ACCOUNTS.

(FORM 5.)

S. R. TAYLOR & Co. in % current with C. LYONS & SON.

CR.

DR.

| 1870. |                                        | 1870.     |                                          |
|-------|----------------------------------------|-----------|------------------------------------------|
| Jan.  | 2 To 986 lbs. Cheese, . . . . . \$ .12 | 118 32    | 3 By 85 bush. Potatoes, . . . . . \$ .37 |
| Feb.  | 27 " 780 " Butter, . . . . . .21       | 163 80    | 24 " 71 bbls. Apples, . . . . . 2.40     |
| "     | 28 " 592 " Lard, . . . . . .14         | 82 88     | 2 " 23 " Plums, . . . . . 4.50           |
| May   | 4 " 609 " Tallow, . . . . . .13        | 69 17     | 28 " 70 lbs. Almonds, . . . . . .09      |
| "     | 18 " 17 bbls. Salt, . . . . . 1.70     | 28 90     | 29 " Note at 3 months to Bal. . . . .    |
| "     | 30 " 96 dozen Eggs, . . . . . .16      | 14 40     |                                          |
| July  | 16 " 130 bush. Oats, . . . . . .90     | 117 00    |                                          |
|       |                                        | \$ 594 47 | \$ 594 47                                |

HALIFAX, July 25, 1870.

C. LYONS & SON.

viz  
sug  
L.  
at  
278  
3  
37  
Iris  
at  
4  
187  
Fel  
\$1.  
cre  
\$3.  
acc  
5  
187  
\$1.  
tick  
27 y  
Ren  
187  
sett  
Wh  
6  
tob  
18  
45  
bal

## EXAMPLES TO BE MADE OUT, AS INDICATED.

*On Form 1.*

1. Sold in Montreal, Feb. 2, 1870, by John Hogan, to Mr. A. Larue, viz.: 7 lbs. chocolate, at 25 cts.; 15 lbs. candles, at 22 cts.; 12 lbs. sugar, at 15 cts.; 18 lbs. flour, at 24 cts. Footing of the bill, \$11.17.

*On Form 1.*

2. Edmond O'Shea of Kingston sold to T. Lee, Feb. 10, 1870, and L. Norris, his agent, collected the amount of the bill: 15 lbs. butter, at 17 cts.; 25 lbs. ch ese, at 20 cts.; 750 lbs. maple sugar, at 9 cts.; 278 lbs. coffee, at 36 cts. Footing of the bill, \$175.13.

*On Form 2.*

3. James Owen of Toronto, sold, Jan. 8, 1870, to W. C. Maher, 37 yds. sheeting, at 26 cts.; 43 yds. lace, at 82 cts.; Feb. 3, 75 yds. Irish linen, at 45 cts.; 209 yds. muslin, at 14 cts.; 330 yds. dowlas, at 16 cts. Footing of the bill, \$160.69.

*On Form 4.*

4. Messrs. B. Sharples & Co., Ottawa, sold to D. Hall; Feb. 12, 1870, 110 pair thick boots, at \$3.75; 28 pair buskins, at 86 cts.; Feb. 20, 40 pair slippers, at 85 cts.; March 2, 67 pair gaiters, at \$1.15; 120 pair boys' brogans, at \$1.25. On this are the following credits: Feb. 27, by cash, \$280; March 15, 110 boxes lemons, at \$3.20. What balance was due B. S. & Co., March 23, when the account was settled? *Ans.* \$65.63.

*On Form 5.*

5. L. A. Conroy & Co., Ottawa, sold to G. Morin & Bro., Jan. 2, 1870, 17 yds. broadcloth, at \$5.25; Jan. 15, 29 yds. cassimere, at \$1.62; Feb. 3, 60 yds. bleached shirting at 17 cts.; Feb. 7, 49 yds. ticking, at 27 cts.; Feb. 15, 18 yds. blue cloth at \$3.19; June 17, 27 yds. gray cloth, at \$2.35; Aug. 3, 75 yds. flannel, at 61 cts. Remitted by G. Morin & Co. in part payment, as follows: Jan. 28, 1870, cash, \$83; June 25, 14 bbls. Ontario Flour, at \$7.20; and settled by Note, at 60 days, Aug. 4, the bal. then due L. A. C. & Co. What was the amount of the note? *Ans.* \$153.28.

*On Form 2.*

6. W. Duffy, Montreal, sold to P. Maurice: March 1, 1870, 18 lbs. tobacco, at 32 cts.; 25 lbs. snuff, at 40 cts.; 72 lbs. tobacco leaf, at 18 cts.; 54 lbs. sugar, at 12 cts.; 20 lbs. soap, at 14 cts.; April 2, 45 gallons molasses, at 37 cts. April 6, credited by cash, \$18. What balance was due W. D., April 8? *Ans.* \$36.65.

## On Form 3.

7. Sold, May 2, 1870, by L. T. Nolan, dealer in fruits, to R. S. Lemoine, Toronto: 32 bbls. Montreal apples, marked 4, at \$2.95; 56 bbls. Greenings, marked 5, at \$2.25; 16 bbls. Harveys, marked 6, at \$1.80; 40 bbls. Russets, marked 8, at \$2.75; paid \$7.50 for packing, and \$13.40 for transportation. Find the amount of bill. Forwarded by the "Western Line."

Ans. \$30.10.

## On Form 1.

8. G. A. Parker of Quebec, sold to S. Montigny, May 5, 1870: 20 lbs. Rio coffee, at 24 cts.; 50 lbs. W. I. sugar, at 7 cts.; 75 lbs. Pearl starch, at 13 cts.; 12 gallons syrup, at 65 cts.; 90 lbs. butter crackers, at 9 cts.; 64 lbs. picnic crackers, at 11 cts.—Footing of the bill, \$39.89.

## On Form 6.

9. Philip Doyle, grocer, Toronto, sold to W. Morris & Co.: June 11, 1870, 473 gallons alcohol, at 95 cts.; 308 gal. old rum, at \$1.90; 610 gal. Holland gin, at \$1.05; Aug. 5, 207 gal. rum, at \$1.75; 119 gal. cognac, at \$2.10; Sept. 22, 401 gal. Scotch gin, at \$1.15. On this the following payments were made by W. Morris & Co.: Oct. 4, 30 bbls. salmon, at \$8.75; Nov. 6, cash, \$520; Nov. 22, draft on London, at 30 days, for the balance due P. D. What was the amount of the draft?

Ans. \$1965.85.

## On Form 4.

10. Mr. P. I. Gordon, Kingston, sold to J. Kelly: June 15, 1870, 23 yds. silk, at 95 cts.; 15 yd. ribbon, at 45 cts.; 12 yds. muslin, at 18 cts.; July 10, 4 yds. blue cloth, at \$3.60; 3 yds. broadcloth, at \$4.50; 9 yds. doekin, at \$1.25; 1 cravat, \$1.30; Aug. 15, 5 pair boots, at \$6.50; 3 doz. hose, at \$2.40; 1 doz. sleeve buttons, 50 cts. On this are the following credits: July 20, by 3 bbls. green apples, at \$3.20; 15 bushels potatoes, at 22 cts.; Aug. 20, by cash, \$7.90. What balance was due P. I. G., Aug. 24, when the account was settled?

Ans. \$91.21.

## On Form 2.

11. O. J. Larkin bought of R. Hamilton & Co., Montreal: May 12, 1870, 18 plows, at \$11; 23 handsaws, at \$3.50; 90 spades, at 86 cts.; May 30, 86 shovels, at 50 cts.; 46 iron, at \$12; June 7, 14 hammers, at 62 cts.; 12 mill-saws, at \$12. June 7, credited by cash, \$140; June 15, credited by cash, \$375. What balance was due H. & Co., June 16?

Ans. \$590.02.

## On Form 3.

12. Invoiced by L. Casey & Son, Halifax, to A. C. Samson, May 4, 1870: 12 crates Antwerp ware, marked 3, at \$175; 43 casks House-

illon wine, marked 8, at \$99; 19 bbls. superfine flour, marked 10, at \$7; 23 bbls. peas, marked 3, at \$1.52; 42 chests black tea, marked 5, at \$17.50; 37 chests green tea, marked 1, at \$23.75; coopeage, \$15; cartage, \$6.80; Insurance, \$32.50. Forwarded by the "Maine Express Line." Amount of Invoice, \$8193.01.

## On Form 4.

13. Messrs. Hall & Brothers, St. John, N. B., sold; June 1, 1870, to P. N. Walsh, 15260 lbs. pork, at 5½ cts.; 7265 lbs. cheese, at 8½ cts.; July 3, 11521 bushels corn, at 50 cts.; July 10, 1560 bbls. flour, at \$6.12½. On the above are the following credits: June 25, by 1150 lbs. cotton, at 6½ cts; June 30, by cash, \$750; July 12, 8256 lbs. maple sugar, at 7 cts.; 6450 gallons molaesses, at 37½ cts. What is the amount of cash requisite to balance the account on July 13?  
Ans. \$12953.78.

## On Form 2.

14. B. C. Willis, bought of A. Murphy & Co., publishers, Montreal: Aug. 4, 1870, 75 Juneau's Mental Arithmetic, at 15 cts.; 50 Smith's Practical Arithmetic, at 37 cts.; 2 doz. Miller's Reader, at \$4.50; Aug. 12, 60 Henry's Grammar, at 7 cts.; 36 Kerney's Compendium of History, at 72 cts.; Sept. 1, 30 Walkingame's Primary Algebra, at 18 cts.; Sept. 1, credited by 50 Commercial Arithmetic of the Christian Brothers, at 40 cts. What balance was due A. M. & Co., Sept. 2?  
Ans. \$54.27.

## On Form 5.

15. S. N. Kelly bought of H. Hamel & Co., Quebec, Feb. 3, 1870, 18 yds. cambric, at 14 cts.; 60 yds. calico, at 42 cts.; 39 yds. cassimere, at \$3.75; March 10, 37 yds. cotton, at 35 cts.; 6 yds. velvet, at \$4.70; May 2, 30 yds. linen, at \$2.65; May 4, 24 yds. merino, at 75 cts. S. N. Kelly's credits are: April 1, 50 lbs. coffee, at 25 cts.; April 9, 7 cords of maple, at \$3.50; May 20, draft on Halifax, \$78; June 25, 1 gal. oil, \$1.50. What balance was due Hamel & Co., June 26, 1870?  
Ans. \$196.12.

*Let the pupils make out Bills or Accounts, as the case may be, in proper form, from the following.*

16. Andrew Whelan of Three Rivers, sold to John Gosselin, July 5, 1870, and I. Kane, his clerk, collected the amount of the bill: 36 lbs. maple sugar, at 13 cts.; 16 lbs. coffee, at 15 cts.; 13 lbs. tea, at 98 cts.; 13 lbs. chocolate, at 61 cts.; 7 lbs. ginger, at 17 cts.; 47 lbs. cheese, at 9 cts.; 12 lbs. pepper, at 15 cts.; 20 lbs. butter, at 16 cts.; 2 gal. vinegar, at 68 cts. Footing of the bill, \$40.01.

17. Forwarded per the Eastern Line, June 3, 1870, by B. Ellis & Co., Ottawa, to S. T. Garneau, Quebec: 18 pair worsted stockings, No. 6, at 90 cts.; 15 doz. napkins, No. 10, at 47 cts.; 24 pair men's

kid gloves, No. 7, at 85 cts.; 20 doz. women's kid gloves, No. 2, at 75 cts.; 12 pair silk stockings, No. 16, at \$2.85; 6 pair thread stockings, No. 11, at \$1.12½. Paid for cartage, 75 cts.; charges for packing, \$1.60. Amount \$101.95.

18. Sold by J. M. O'Reilly, Montreal, April 10, 1870, to A. Gauthier: 278 lbs. coffee, at 36 cts.; 1270 lbs. lard, at 13 cts.; 800 lbs. ham, at 11 cts.; 1540 lbs. corned beef, at 8 cts.; 750 lbs. butter, at 17 cts.; 217 lbs. maple sugar, at 7 cts.; 126 doz. eggs, at 12 cts.; 150 bushels oats, at 65 cts. Footing of the bill, \$731.69.

19. Sold in Toronto, April 20, 1870, by Isaac Chambers, to Mrs. Julia Meredith, and the bill paid: 3 doz. silver table forks, at \$43.75 a doz.; 2 doz. silver table spoons, at \$35 a doz.; 2½ doz. silver teaspoons, at \$18.50 a doz.; 1½ doz. ivory handle knives, at \$7.50 a doz.; 1 gold guard chain, at \$136. Footing of the bill, \$394.75.

20. P. Barry & Son, Kingston, sold to H. Miller, March 6, 1870, as follows: 2 loaves white sugar, 52 lbs., at 15 cts.; 4 bbls. extra flour, at \$7.80; 9½ lbs. cheese, at 16 cts.; 15 lbs. raisins, at 15 cts.; 7 lbs. black pepper, at 42 cts.; 20 lbs. butter, at 23 cts.; 3 bushels peas, at 70 cts.; 5 bush. beans, at \$1.10; 14½ lbs. bacon, at 16 cts.; 1 gal. molasses, 60 cts. Footing of the bill, \$60.83.

21. M. Peter Nelson owes D. I. Hogan, Toronto, as follows: June 5, 1870, 3 gross shirt-studs, at 85 cts.; June 17, 15 doz. woolen stockings, at \$3.18½; 3 doz. shirt fronts, at \$5.05; Aug. 2, 12½ yds. ribbon, at 25 cts.; 30 pair silk gloves, at \$1.37½; 4 doz. linen towels, at \$2.85; 22½ yds. ticking, at 45 cts. Footing of the bill, \$131.37½.

22. G. Turner & Son, Quebec, sold to A. I. Green, March 6, 1870, 17 pair boots, at \$3.00; March 18, 19 pair shoes, at \$1.08; April 9, 80 pair hose, at \$1.20; 23 pair gloves, at 75 cts. They received of A. I. Green, the following as credits: April 5, 27 Second Readers, at 20 cts.; 10 Third Readers, at \$3.90; May 11, 7 Brown's Dictionaries, at \$4.75; 19 Golden Manuals, at \$2.93; 20 Christian Duties, at 37 cts. The balance due G. T. & Son, which was paid, May 15, 1870, amounted to \$44.05.

23. Sold by Smith & Watters, Kingston, July 24, 1870, to O. S. Peters: 275 bbls. Patapaco flour, at \$7.16; 150 bbls. Ontario flour, at \$6.25; 170 bbls. Chicago flour, at \$5.87½; 214 bushels corn, at 82 cts.; 326 bush. wheat, at \$1.62½; 300 bush. oats, at 91 cts.; 500 bush. rye, at \$1.06. Footing of the bill, \$5413.48.

24. Joseph R. Simon, bought of C. T. Adams, Montreal, April 20, 1870, as follows: 5 yds. black cloth, at \$3.50; 1 satin waistcoat, \$5.50; Trimmings, \$3.75; 3 yds. yellow linen, at 19 cts.; 10 yds. gray fringe, at 68 cts.; 3 pieces of ribbon, at 31 cts.; 3 yds. black cassimere, at \$2.25; 7½ yds. alpaca, at 55 cts.; 16 yds. cambric, at 10½ cts.; 3 skein silk thread, at 5½ cts.; 4 yds. wadding, at 6 cts.; 9 yds. white flannel, at 90 cts.; 2 cravats, at \$1.12½; 4 yds. green baize, at 58 cts.; 6 cotton shirts, at 65½ cts.; 5 yds. merino, at 80 cts.; 10 yds. muslin, at 14 cts. Footing of the bill, \$70.71.

25. Sold by P. Mayrand & Co., Halifax, to Ed. O'Neil, grocer, as follows: June 8, 1870, 4 pieces muslin, each 37 yds., at \$3.45 a yd.; 8 pieces printed calico, each 47 yds., at 82 cts. a yard; June 27, 5

No. 2, at  
ir thread  
arges for  
01.95.

A. Gau-  
800 lbs.  
utter, at  
12 cts.;  
31.69.

to Mrs.  
at \$43.75  
lver tea-  
\$7.50 a  
94.75.

6, 1870,  
ls. extra  
15 cts.;  
bushels  
16 cts.;  
60.83.

s: June  
woolen  
12½ yds.  
towels,  
1.37½.

6, 1870,  
April 9,  
eived of  
aders, at  
ionaries,  
s, at 37  
5, 1870,

o O. S.  
io flour,  
corn, at  
91 cts.;  
13.48.

April 20,  
aistcoat,  
10 yds.  
s. black  
abric, at  
6 cts.;

s. green  
o, at 80  
70.01.

rocer, as  
5 a yd.,  
e 27, 6

pieces Dutch linen, each 30 yds., at 70 cts. a yard; July 10, 11 pieces serge, each 19 yds., at 56 cts.; Aug. 6, 1750 yds. Lowell cotton, at 20 cts.; 974 yds. Manchester stuffs, at 25 cts. July 30, E. O'Neil, paid in part \$350. What balance was due P. M. & Co., Aug. 2, when the account was made out?

Ans. \$1294.46.

26. Messrs. Fraser, O'Donnell & Co., wholesale dealers, Montreal, sold to Dugal & Lane: Aug. 4, 1870, 18 fine dress coats, No. 52, at \$27.50; 46 cashmere vests, No. 20, at \$4.30 each; Sept. 9, 3 doz. men's black wool hats, No. 22, at \$12.50 per doz.; ¼ doz. men's Pearl hats, No. 54, at \$27 per doz.; 5 umbrellas 28-in., at \$1.75; Oct. 12, 5 doz. men's white cotton hose, No. 7, at \$2.60 per doz.; 3 black leather valises, No. 72, at \$9.50. On this are the following credits: Sept. 10, by cash, \$400; Sept. 30, by cash, \$150; Oct. 7, by 50 bushels corn, at 65 cts. What balance was due F. O'D. & Co., Oct. 18, when the account was settled?

Ans. \$211.55.

27. Bought of L. R. Williams, Quebec, by H. S. Connolly: June 3, 1870, 75 lbs. maple sugar, at 6½ cts.; 9 lbs. green tea, at 65 cts.; 21 gals. maple syrup, at 70 cts.; July 1, 12 lbs. pepper, at 25 cts.; 10 lbs. spice, at 20 cts.; 12 lbs. ginger, at 18 cts.; 15 lbs. coffee, at 12½ cts.; July 12, 20 lbs. dried apples, at 10 cts.; 18 lbs. dried peaches, at 12½ cts.; 2 bushels onions, at 80 cts.; Aug. 1, 13 lbs. mackerel, at 8 cts.; 9 lbs. smoked herrings, at 20 cts.; Aug. 10, 25 lbs. rice, at 5 cts.; 12 lbs. dried beef, at 12½ cts.; Sept. 4, 5 bushels corn meal, at 80 cts.; 5 sacks table salt, at 20 cts.; 17 lbs. soda crackers, at 9 cts. Amount of the bill, \$52.24, which was paid to L. R. Williams, Sept. 7.

28. Sold by L. Trudel, Montreal, to J. B. Poston, as follows: Oct. 20, 1870, 48 pair tongs, at 37½ cts.; 26 doz. pewter-polished bits, at 85 cts. per doz.; 96 doz. hinges, at 18 cts. per doz.; Nov. 3, 32 doz. curry-combs, at 45½ cts. a piece; 20 packets shoemakers' awls, at 58 cts. per packet; Nov. 12, 75 packets 3¼ in. screws, at 95 cts. per packet. L. Trudel received of J. B. Poston on account: Nov. 8, 2 casks Medoc wine, each 45 gal., at 80 cts. per gallon; Dec. 5, cash, \$50. What balance was due L. T., Dec. 6?

Ans. \$38.75.

29. Invoiced by D. Molson, Quebec, to V. R. Lewis, Ottawa, Feb. 1, 1870: 2 cases calf boots, No. 3, each 67 pairs, at \$3.75; 2 cases thick boots, No. 4, each 54 pairs, at \$2.62; 2 cases gaiters, No. 7, each 75 pairs, at \$1.12; 2 cases buskins, No. 10, each 27 pairs, at 86 cts.; 2 cases slippers, No. 14, each 35 pairs, at 70 cts.; 2 cases rubbers, No. 13, each 50 pairs, at \$1.04; charged for packing, cartage, etc., \$3.90.

Footing of the bill, \$1439.76.

30. N. P. Morris & Co., Halifax, sold to U. S. Brown, Sept. 7, 1870, 50 yds. print, at 12½ cts.; 15 yds. cambric, at 9 cts.; 6 yds. cassimere, at \$1.60; Sept. 25, 33 yds. sheeting, at 11 cts.; 3 yards carpet, at \$3.00; 6½ yds. broadcloth, at \$4.37½; Oct. 29, 20 yards french print, at 17 cts.; 15 yds. merino, at 70 cts. On this bill are the following credits: Nov. 1, by 22 lbs. butter at 20 cts.; 6 cords cherry wood, at \$3.00; Dec. 4, by cash, \$16.00; Dec. 10, by 8 days' labor, at \$1.50. What balance was due N. P. M. & Co., Dec. 30, when the account was settled?

Ans. \$21.76½.

PROPERTIES OF NUMBERS.

PROPERTIES OF NUMBERS.

EXACT DIVISORS AND PRIME NUMBERS.

**88.** An **Exact Divisor** of a number is one that divides it without a remainder, or which gives an integer for the quotient.

**89.** All numbers are either *even* or *odd*.

**90.** An **Even Number** is a number of which 2 is an exact divisor; as 2, 6, 8, 24.

**91.** An **Odd Number** is a number of which 2 is not an exact divisor; as 1, 3, 7, 15.

Every number must be either *prime* or *composite*.

**92.** A **Prime Number** is one which can not be resolved or separated into two or more integral factors; as 1, 3, 5, 7.

NOTES.—1. All prime numbers except 2 are odd numbers.

2. Numbers are prime to each other, when they have no common divisor; thus, 7 and 13 are prime to each other, as are also 4, 11, and 15.

**93.** A **Composite Number** is one that has other exact divisors besides 1 and itself; as 6, 9, 14.

**94.** The **Prime Factors** of a number are its exact divisors; thus, 1, 3, and 7, are factors of 21.

**95.** The **Power** of a number is the product obtained by taking the number a certain number of times as a factor; thus, 16 is a power of 4.

NOTE.—When the number is taken once, it is called its first power; when taken twice, as a factor, the product is called its second power; and so on.

**96.** The **Exponent** of a power is a figure written at the right of a number, and a little above it, to show how many times it is taken as a factor; thus, in the expression  $5^2$ , the exponent is 2, and the whole is read 5 second power.

From these principles,

1st. *Any number which will exactly divide one of two numbers will divide their product.*

2nd. *Any number which will exactly divide each of two numbers will divide their sum.*

3rd. *Any number which will exactly divide each of two numbers will divide their difference.*

---

88. What is an exact divisor?—89. What are all numbers?—90. What is an even number?—91. An odd number?—92. A prime number?—When are numbers prime to each other?—93. What is a composite number?—94. What are prime factors?—95. What is the power of a number?—96. What is an exponent?

We derive the following properties :

- I. *Two* is an exact divisor of all even numbers.
- II. *Three* is an exact divisor of every number the sum of whose digits it will exactly divide.
- III. *Four* is an exact divisor when it will exactly divide the tens and units of a number.
- IV. *Five* is an exact divisor of every number whose unit figure is 0 or 5.
- V. *Six* is an exact divisor of every even number, the sum of whose digits it will exactly divide, or that 3 will exactly divide.
- VI. *Eight* is an exact divisor when it will exactly divide the hundreds, tens, and units of a number.
- VII. *Nine* is an exact divisor when it will exactly divide the sum of the digits of a number.
- VIII. *Ten* is an exact divisor when 0 occupies the units' place.
- IX. *Eleven* is an exact divisor of every number whose sum of the digits, standing in the *even* places is equal to the sum of the digits standing in the *odd* places.

TABLE OF PRIME NUMBERS FROM 1 TO 1109.

|    |     |     |     |     |     |     |     |     |      |      |
|----|-----|-----|-----|-----|-----|-----|-----|-----|------|------|
| 1  | 59  | 139 | 233 | 337 | 439 | 557 | 653 | 769 | 893  | 1013 |
| 2  | 61  | 149 | 239 | 347 | 443 | 563 | 659 | 773 | 887  | 1019 |
| 3  | 67  | 151 | 241 | 349 | 449 | 569 | 661 | 787 | 907  | 1021 |
| 5  | 71  | 157 | 251 | 353 | 457 | 571 | 673 | 797 | 911  | 1031 |
| 7  | 73  | 163 | 257 | 359 | 461 | 577 | 677 | 809 | 919  | 1033 |
| 11 | 79  | 167 | 263 | 367 | 463 | 587 | 683 | 811 | 929  | 1039 |
| 13 | 83  | 173 | 269 | 373 | 467 | 593 | 691 | 821 | 937  | 1049 |
| 17 | 89  | 179 | 271 | 379 | 479 | 599 | 701 | 823 | 941  | 1051 |
| 19 | 97  | 181 | 277 | 383 | 487 | 601 | 709 | 827 | 947  | 1061 |
| 23 | 101 | 191 | 281 | 389 | 491 | 607 | 719 | 833 | 953  | 1063 |
| 29 | 103 | 193 | 283 | 397 | 499 | 613 | 727 | 839 | 967  | 1069 |
| 31 | 107 | 197 | 293 | 401 | 503 | 617 | 733 | 851 | 971  | 1087 |
| 37 | 109 | 199 | 307 | 409 | 509 | 619 | 739 | 857 | 977  | 1091 |
| 41 | 113 | 211 | 311 | 419 | 521 | 631 | 743 | 859 | 983  | 1093 |
| 43 | 127 | 223 | 313 | 421 | 523 | 641 | 751 | 863 | 991  | 1097 |
| 47 | 131 | 227 | 317 | 431 | 541 | 643 | 757 | 877 | 997  | 1103 |
| 53 | 137 | 229 | 331 | 433 | 547 | 647 | 761 | 881 | 1009 | 1109 |

FACTORING.

**97. CASE I.**—To resolve a number into its prime factors.

**NOTE.**—The process of factoring numbers depends upon the following principles:

When is 2 an exact divisor?—3?—4?—5?—6?—8?—9?—10?—11?

- I. Every prime factor of a number is an exact divisor of that number.  
 II. The only exact divisors of a number are its prime factors, or some combinations of its prime factors.

*Ex.* What are the prime factors of 1596 ?

OPERATION.

$$\begin{array}{r} 2 \overline{) 1596} \\ \underline{2} \phantom{96} \\ 2 \phantom{96} \\ \underline{2} \phantom{96} \\ 2 \phantom{96} \\ \underline{2} \phantom{96} \\ 0 \phantom{96} \\ 19 \phantom{6} \\ \underline{19} \\ 0 \end{array}$$

ANALYSIS.—We divide by 2, the least prime factor, and the result by 2; this gives an odd number, 399, for a quotient. We then divide by the prime numbers 3, 7, and 19, successively, and the last quotient is 1. The divisors, 2, 2, 3, 7, and 19, are the prime factors required. Hence, the

**98. RULE.**—Divide the given number by the smallest prime factor; divide the quotient in the same manner, and so continue the division until the quotient is a prime number. The several divisors and the last quotient will be the prime factors required.

**PROOF.** The product of all the prime factors will be the given number.

#### EXAMPLES FOR PRACTICE.

Required the prime factors of

|         |                      |           |             |            |             |
|---------|----------------------|-----------|-------------|------------|-------------|
| 1. 28.  | <i>Ans.</i> 2, 2, 7. | 6. 1140.  | <i>Ans.</i> | 11. 12673. | <i>Ans.</i> |
| 2. 36.  | <i>Ans.</i>          | 7. 3420.  | <i>Ans.</i> | 12. 12496. | <i>Ans.</i> |
| 3. 86.  | <i>Ans.</i>          | 8. 2445.  | <i>Ans.</i> | 13. 21504. | <i>Ans.</i> |
| 4. 144. | <i>Ans.</i>          | 9. 2431.  | <i>Ans.</i> | 14. 13981. | <i>Ans.</i> |
| 5. 360. | <i>Ans.</i>          | 10. 2205. | <i>Ans.</i> | 15. 17199. | <i>Ans.</i> |

**99. CASE II.**—To find the prime factors common to two or more numbers.

*Ex.* What are the prime factors common to 84, 126, and 210 ?

OPERATION.

$$\begin{array}{r} 2 \overline{) 84, \quad 126, \quad 210.} \\ \underline{2} \phantom{,} \phantom{,} \phantom{,} \\ 3 \overline{) 42, \quad 63, \quad 105.} \\ \underline{3} \phantom{,} \phantom{,} \phantom{,} \\ 7 \overline{) 14, \quad 21, \quad 35.} \\ \underline{7} \phantom{,} \phantom{,} \phantom{,} \\ 2, \quad 3, \quad 5. \end{array}$$

ANALYSIS.—We find 2 to be an exact divisor of all the numbers, and is, therefore, a common factor; 3 is an exact divisor of the first set of quotients, and 7 of the second set of quotients, therefore, 3 and 7 are also common factors of the numbers. There is no exact divisor of the third set of quotients. Hence, 2, 3, and 7 are the only prime factors common to 84, 126, and 210.

**100. RULE.**—I. Divide each of the numbers by the smallest prime number which is an exact divisor of each.

II. Divide each set of quotients in the same manner, until they become prime to each other. The divisors will be the common prime factors.

98. What is the rule to resolve a number into its prime factors?—100. What is the rule to find the prime factors common to two or more numbers?

EXAMPLES FOR PRACTICE.

Required the prime factors common to

1. 12, and 24. Ans. 2, 2, and 3.
2. 48, 96, and 120.
3. 42, 63, and 105. Ans. 3 and 7.
4. 225, 435, and 540.
5. 48, 72, and 96.
6. 140, 210, and 280. Ans. 2, 5, and 7.
7. 252, 336, and 420.
8. 960, 1568, and 5824.
9. 330, 495, and 165.
10. 2340, 11934, 12987, and 14859.

CANCELLATION.

**101. Cancellation** is the process of rejecting equal factors from numbers sustaining to each other the relation of dividend and divisor.

*Ex. 1.* Divide 112 by 56.

OPERATION.

$$\frac{112}{56} = \frac{\cancel{7} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2}{\cancel{7} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}} = \frac{2}{1} = 2.$$

**ANALYSIS.**—The factors of the dividend are 7, 2, 2, 2, and 2. The factors of the divisor are 7, 2, 2, and 2. Rejecting the common factors 7, 2, 2, and 2, we obtain 2 for the quotient.

**NOTES.**—1. When a dividend contains a divisor an exact number of times, there is a factor in the dividend equal to the divisor.

2. When a factor is cancelled, 1 is supposed to take its place.

*Ex. 2.* Divide the product of 7 × 10 × 12 × 5 by the product of 14 × 18 × 6.

OPERATION.

$$\frac{\text{Dividend, } \overset{5}{\cancel{7}} \times \overset{10}{\cancel{14}} \times \overset{12}{\cancel{18}} \times \overset{5}{\cancel{6}}}{\text{Divisor, } \underset{2}{\cancel{14}} \times \underset{9}{\cancel{18}} \times \underset{6}{\cancel{6}}} = \frac{25}{9} = 2\frac{7}{9}.$$

**Ans.**—We have performed this division without factoring the dividend and divisor, by rejecting the factors that are common to both dividend and divisor, and writing the remaining factors in their proper places.

**102. RULE.**—I. Write the dividend above and the divisor below a horizontal line.

II. Cancel all the factors common to both dividend and divisor.

III. Divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.

101. What is cancellation?—102. What is the rule for cancellation?

## EXAMPLES FOR PRACTICE.

3.  $16 \times 24 \times 48 \div 32 \times 36 \times 38 =$  *Ans. 3.*  
 4.  $12 \times 7 \times 5 \div 2 \times 4 \times 3.$  *Ans. 17 1/2.*  
 5.  $16 \times 5 \times 10 \times 18 \div 8 \times 6 \times 2 \times 12.$  *Ans. 124.*  
 6.  $84 \times 12 \times 18 \div 21 \times 24 \times 9.$  *Ans. 4.*  
 7.  $72 \times 18 \times 16 \div 24 \times 16 \times 9.$  *Ans. 6.*  
 8.  $22 \times 9 \times 12 \times 5 \div 3 \times 11 \times 6 \times 4.$  *Ans. 15.*  
 9.  $76 \times 34 \times 96 \div 17 \times 51 \times 32.$   
 10.  $25 \times 7 \times 14 \times 36 \div 4 \times 10 \times 21 \times 54.$   
 11.  $184 \times 145 \times 80 \div 23 \times 29 \times 60.$  *60*  
 12.  $28 \times 27 \times 21 \times 15 \times 18 \div 7 \times 54 \times 7 \times 3 \times 9.$  *5-5 1/2*  
 13.  $12 \times 5 \times 183 \times 18 \times 70 \div 3 \times 14 \times 9 \times 5 \times 20 \times 6.$  *8 2/3*  
 14.  $213 \times 84 \times 190 \times 264 \div 30 \times 56 \times 36.$  *12839*

## DIVISORS OF NUMBERS.

**103.** A **Common Divisor** or **Measure** of two or more numbers is any number that will exactly divide each of them.

**104.** The **Greatest Common Divisor** of two or more numbers is the greatest exact divisor of each of them.

**105.** GENERAL PRINCIPLES.—I. *One is a divisor of all integers.*

II. *Every number is a divisor of itself.*

III. *Every prime factor of a number is a divisor of that number.*

IV. *Every product of any two or more prime factors of a number is a divisor of that number.*

V. *Every number equals the product of its prime factors.*

VI. *A number has no divisors except its prime factors and any product of every two or more of them. Hence, the product of all the prime factors common to two or more numbers is their greatest common divisor.*

## COMMON DIVISOR.

**106.** *To find a common divisor of two or more numbers.*

*Ex.* Required a common divisor of 9, 15, and 21.

OPERATION.

$$9 = 3 \times 3$$

$$15 = 3 \times 5$$

$$21 = 3 \times 7$$

ANALYSIS.—We resolve each of the given numbers into two factors, one of which is common to all of them. In the operation 3 is the common factor, and is therefore a common divisor of the numbers.

**107.** RULE.—*Resolve the given numbers into their prime factors, then if any factor be common to all, it will be a common divisor.*

103. What is a common divisor?—104. What is the greatest common divisor?

EXAMPLES FOR PRACTICE.

Find the common divisors of the following numbers:

- |                        |         |                         |         |
|------------------------|---------|-------------------------|---------|
| 1. 10, 15, and 25.     | Ans. 5. | 5. 28, 14, 42, and 35.  | Ans. 7. |
| 2. 15, 18, 24, and 36. | Ans. 3. | 6. 10, 35, 50, and 75.  | Ans. 5. |
| 3. 3, 9, 18, and 24.   |         | 7. 4, 12, 16, and 28.   |         |
| 4. 21, 77, 35, and 42. |         | 8. 2, 118, 48, and 146. |         |

GREATEST COMMON DIVISOR.

**108.** To find the greatest common divisor of two or more numbers.

*Ex.* What is the greatest common divisor of 168, 210, and 252?

FIRST METHOD.

OPERATION.

|   |     |     |     |
|---|-----|-----|-----|
| 2 | 168 | 210 | 252 |
| 3 | 84  | 105 | 126 |
| 7 | 28  | 35  | 42  |
|   | 4   | 5   | 6   |

**ANALYSIS.**—First find the prime factors common to the numbers; (99), which are 2, 3, and 7. Therefore the greatest common divisor is  $2 \times 3 \times 7 = 42$ . (105, VI.)

**109. RULE.**—Find the prime factors common to all the numbers (99), and their product will be the greatest common divisor.

SECOND METHOD.

OPERATION.

The prime factors of  $\left\{ \begin{array}{l} 168 = 2 \times 2 \times 2 \times 3 \times 7 \\ 210 = 2 \times 3 \times 5 \times 7 \\ 252 = 2 \times 2 \times 3 \times 3 \times 7 \end{array} \right.$

**ANALYSIS.**—The prime factors common to the three numbers are 2, 3, and 7. Therefore the greatest common divisor is  $2 \times 3 \times 7 = 42$ . (105, VI.)

**110. RULE.**—Resolve the numbers into their prime factors, and find the product of the common prime factors.

THIRD METHOD.

**111. PRINCIPLES.**—I. If the less of two numbers is a divisor of the greater, it is their greatest common divisor.

II. A divisor of a number is a divisor of any number of times that number.

III. A common divisor of two numbers is a divisor of their sum, and also of their difference.

IV. The greatest common divisor of the difference of two numbers and one of them, is the greatest common divisor of the two numbers.

109. What is the rule to find the greatest common divisor, first method?—Second method?—Third method?

*Ex.* Required the greatest common divisor of 117 and 1365.

$$\begin{array}{r}
 \text{OPERATION.} \\
 117) 1365 \quad (11 \\
 \underline{117} \\
 195 \\
 \underline{117} \\
 78) 117 \quad (1 \\
 \underline{78} \\
 39) 78 \quad (2 \\
 \underline{78} \\
 0
 \end{array}$$

**ANALYSIS.**—Since 117 is the greatest divisor of 117, if it be a divisor of 1365, it will be their greatest common divisor. By trial, 117 is found not to be a divisor of 1365, since there is a remainder, 78.

If 78, the greatest divisor of itself, is a divisor of 117, it is the greatest common divisor of 78 and 117, and also, of 117 and 1365. (111, IV.) By trial, 78 is found not to be a divisor of 117, since there is a remainder, 39.

If 39, the greatest divisor of itself, is a divisor of 78, it is the greatest common divisor of 39 and 78, also of 78 and 117, and of 117 and 1365. By trial, 39 is found to be a divisor of 78, and is, therefore, the greatest

common divisor of 117 and 1365.

**Obs.**—A knowledge of the Principles (111), will render the above analysis plain, since 39 is an exact divisor of 78, it is a divisor of  $117 = 39 + 78$ , and  $1365 = 11 \times 117 + 78$ .

**112. RULE.**—*Divide the greater number by the less, and the divisor by the remainder, and so on, till there is no remainder. The last divisor will be the greatest common divisor sought.*

**NOTE.**—The greatest common divisor of three or more numbers can be found by finding the greatest common divisor of two of the numbers, then the greatest common divisor of this greatest common divisor and a third number, and so on. The last common divisor will be the greatest common divisor of all the numbers.

#### EXAMPLES FOR PRACTICE.

Find the greatest common divisors of the following numbers :

- |                        |                 |                               |                |
|------------------------|-----------------|-------------------------------|----------------|
| 1. 72 and 168.         | <i>Ans.</i> 24. | 10. 16, 20, and 24.           | <i>Ans.</i> 4. |
| 2. 175 and 455.        | <i>Ans.</i> 35. | 11. 78, 234, and 468.         |                |
| 3. 169 and 866.        | <i>Ans.</i> 1.  | 12. 2041 and 8476.            |                |
| 4. 84, 126, and 210.   | <i>Ans.</i> 42. | 13. 286, 429, and 715.        |                |
| 5. 12, 18, 24, and 30. | <i>Ans.</i> 6.  | 14. 1649 and 5423.            |                |
| 6. 385, 462, and 154.  |                 | 15. 92, 116, and 124.         |                |
| 7. 12, 15, and 18.     |                 | 16. 252, 630, 1134, and 1386. |                |
| 8. 210, 350, and 770.  |                 | 17. 49373 and 147731.         |                |
| 9. 70, 105, and 245.   |                 | 18. 3013, 2231, and 2047.     |                |

#### LEAST COMMON MULTIPLE.

**113. A Multiple** is a number exactly divisible by a given number, thus, 15 is a multiple of 3.

**114. A Common Multiple** is a number exactly divisible by two or more given numbers; thus, 24 is a common multiple of 2, 3, 4, 6, 8, and 12.

113. What is a multiple?— 114. A common multiple?

**NOTE.**—It is evident that the product of two or more numbers, or any number of times their product, must be a common multiple of the numbers. Hence, *A common multiple of two or more numbers may be found by multiplying the given numbers together.*

**115.** The **Least Common Multiple** is the least number exactly divisible by two or more given numbers; thus, 30 is the least common multiple of 10 and 15.

**NOTE.**—The least common multiple of two or more numbers contains all the prime factors of each of those numbers, and no other factors.

**116.** *To find the least common multiple.*

FIRST METHOD.

**Ex.** What is the least common multiple of 9, 12, 16, and 20?

OPERATION.

$$\begin{aligned} 9 &= 3 \times 3 \\ 12 &= 2 \times 2 \times 3 \\ 16 &= 2 \times 2 \times 2 \times 2 \\ 20 &= 2 \times 2 \times 5 \\ 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 &= 720 \text{ Ans.} \end{aligned}$$

**ANALYSIS.**—Resolving the numbers into their prime factors, we find these to be 2, 3, and 5. The greatest number of times the 2 occurs as a factor in any of the given numbers is 4 times; the greatest number of times 3 occurs in any of the given numbers is twice; and the greatest number

of times the 5 occurs in any of the given numbers is once. Hence, 2, 2, 2, 2, 3, 3, and 5, must be all the prime factors necessary to produce in composing 9, 12, 16, and 20. Therefore 720, the product of these factors, is the least common multiple required.

**117. RULE.—I.** *Resolve the given numbers into their prime factors.*

**II.** *Take all the prime factors of the largest number, and such prime factors of the other numbers as are not found in the largest number, and their product will be the least common multiple.*

SECOND METHOD.

**Ex.** What is the least common multiple of 10, 16, 24, and 32?

OPERATION.

|        |     |     |     |
|--------|-----|-----|-----|
| 2) 10, | 16, | 24, | 32. |
| 2) 5,  | 8,  | 12, | 16. |
| 2) 5,  | 4,  | 6,  | 8.  |
| 2) 5,  | 2,  | 3,  | 4.  |
|        | 5,  | 1,  | 3,  |
|        |     |     | 2.  |

**ANALYSIS.**—We first write the given numbers on a horizontal line, then we divide by 2, a prime number that will divide all of them without a remainder, and write the quotients in a line underneath. Now, since some of the numbers in the second line contain the factor 2, we again divide by 2, and write the quotients, and the undivided number,

5, in a line underneath as before. We continue to divide by a prime number, till the divisor and remainders are all prime to each other; the product of both divisors and remainders gives 540 as the least common multiple.

**115.** *What is the least common multiple?—117. What is the rule for finding the least common multiple first method?*

**118. RULE.—I.** Divide by the smallest prime number that is an exact divisor of two or more of the numbers, and write the quotients and the undivided numbers underneath.

**II.** Proceed with the resulting numbers in like manner, until there is no exact divisor of any two of them.

**III.** The product of the divisors and the resulting numbers will be the least common multiple sought.

**NOTES.—1.** When numbers are prime to each other, their product is their last common multiple.

**2.** When any of the given numbers is a factor of any of the others it may be canceled.

#### EXAMPLES FOR PRACTICE.

Required the least common multiples of the following numbers :

- |                        |           |                             |            |
|------------------------|-----------|-----------------------------|------------|
| 1. 24, 36, and 20.     | Ans. 360. | 9. 10, 45, 75, and 90.      | Ans. 450.  |
| 2. 7, 14, 21, and 15.  | Ans. 210. | 10. 12, 15, 18, and 35.     | Ans. 1260. |
| 3. 14, 19, 38, and 57. | Ans. 798. | 11. 25, 60, 100, and 125.   |            |
| 4. 8, 12, 16, and 20.  |           | 12. 22, 12, 44, and 11.     |            |
| 5. 32, 34, and 36.     |           | 13. 18, 27, 36, and 40.     |            |
| 6. 20, 36, 48, and 60. |           | 14. 270, 189, 297, and 243. |            |
| 7. 9, 18, 27, and 54.  |           | 15. 64, 84, 96, and 216.    |            |
| 8. 12, 15, 42, and 60. |           | 16. 84, 100, 224, and 300.  |            |

#### FRACTIONS.

**119. A Fraction** is one or more of the equal parts of a unit.

**120.** Two integers are required to write a fraction, one to express the number of parts into which the whole number is divided, and the other to express the number of these parts taken. Thus,

If an apple be divided into 2 equal parts, one of the parts is called *one half*; if divided into 3 equal parts, one of the parts is called *one third*, two of the parts *two thirds*; if divided into 4 equal parts, one of the parts is called *one fourth*, etc.; if divided into 5 equal parts, one of the parts is called *one fifth*, etc.

The parts are expressed by figures; thus,

|                     |               |                          |               |
|---------------------|---------------|--------------------------|---------------|
| One half is written | $\frac{1}{2}$ | Three fourths is written | $\frac{3}{4}$ |
| One third    "      | $\frac{1}{3}$ | One fifth       "        | $\frac{1}{5}$ |
| Two thirds   "      | $\frac{2}{3}$ | Four fifths     "        | $\frac{4}{5}$ |
| One fourth   "      | $\frac{1}{4}$ | Five sevenths  "         | $\frac{5}{7}$ |

118. What is the rule for finding the least common multiple second method?  
 —119. What is a fraction?—120. How many numbers are required to write a fraction?

**121.** The two integers of a fraction are its **Terms**; the one below the line, the *Denominator*; and the one above, the *Numerator*.

**122.** The **Denominator** names the parts, and shows how many of them are equal to a unit.

**123.** The **Numerator** numbers the parts, and shows how many of them are taken or expressed by the fraction.

**124.** From the foregoing definitions, it follows,

I. That the value of a fraction in units, is the quotient of the numerator divided by the denominator.

II. That fractions indicate division, the numerator being a dividend and the denominator a divisor.

**125.** To *Analyze* a fraction is to name the unit or quantity divided, the value of one of its equal parts and the number of parts expressed.

*Ex.* Analyze  $\frac{5}{6}$  of a yard.

In  $\frac{5}{6}$  of a yard, the unit of the fraction is 1 yard; the part or fractional unit,  $\frac{1}{6}$  of a yard; and the number of fractional units expressed or numbered is 5. Six is the denominator, and shows that the yard is considered as 6 equal parts. Five is the numerator, and shows that 5 of these equal parts are enumerated. 5 is the dividend, and 6, the divisor. Hence,  $\frac{5}{6}$  of a yard expresses 5 equal parts of such value that 6 of them equal 1 yard, the unit of the fraction.

**126.** Fractions are classified as *Simple*, *Compound*, and *Complex*.

**127.** The *Simple fraction* is distinguished as *Proper* and *Improper*.

**128.** A **Simple Fraction** is one whose terms are integral; as  $\frac{3}{4}$ ,  $\frac{5}{7}$ .

**129.** A **Proper Fraction** is one whose numerator is less than its denominator; as  $\frac{1}{2}$ ,  $\frac{3}{8}$ ,  $\frac{13}{15}$ .

**130.** An **Improper Fraction** is one whose numerator equals or exceeds its denominator; as  $\frac{3}{2}$ ,  $\frac{4}{3}$ .

**131.** A **Compound Fraction** is a fraction of a fraction; as  $\frac{2}{3}$  of  $\frac{4}{5}$ ,  $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7}$ .

**132.** A **Complex Fraction** is one having a fraction or a mixed number in either or both of its terms; as,  $\frac{2}{3}$ ,  $\frac{6}{9}$ ,  $\frac{9\frac{1}{2}}{13}$ ,  $\frac{5\frac{1}{8}}{9\frac{3}{4}}$ .

**133.** A **Mixed Number** is an integer and a fraction united in the same expression; as  $5\frac{1}{2}$ .

121. What are the terms of a fraction?—122. Define the denominator?—123. The numerator?—126. How are fractions classified?—128. What is a simple fraction?—129. What is a proper fraction?—130. An improper fraction?—131. A compound fraction?—132. A complex fraction?—133. A mixed number?

**134.** Since fractions are expressions indicating the division of one number by another, it follows,

1st. *That, if the numerator be multiplied, or the denominator be divided, by any number, the fraction is multiplied by the same number.*

2nd. *That, if the numerator be divided, or the denominator multiplied, by any number, the fraction is divided by the same number.*

3rd. *That, if the numerator and denominator be both multiplied, or both divided, by the same number, the fraction will not be changed in value.*

### REDUCTION OF FRACTIONS.

**135.** The **Reduction** of a fraction is the process of changing its terms, or its form, without altering its value.

**136.** CASE I.—*To reduce a whole or mixed number to an equivalent improper fraction.*

*Ex. 1.* Reduce 12 yards to fifths.

OPERATION.  
 $5 \times 12 = 60$ , Ans.

ANALYSIS.—In 1 yard there are 5 fifths, and in 12 yards there are 12 times 5 fifths = 60.

**137.** RULE.—*Multiply the whole number by the given denominator; take the product for a numerator, under which write the given denominator.*

*Ex. 2.* To reduce  $15\frac{1}{2}$  to fourths.

OPERATION.  

$$\begin{array}{r} 15\frac{1}{2} \\ 4 \\ \hline 62 \end{array}$$
, Ans.

ANALYSIS.—In 1 there are 4 fourths; therefore, 4 times the number of whole ones equals the number of fourths; therefore,  $15 = 60$ , to which add  $\frac{1}{2}$  and we have  $15\frac{1}{2} = 62$ .

**138.** RULE.—*Multiply the whole number by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.*

### EXAMPLES FOR PRACTICE.

- |                                      |                                         |
|--------------------------------------|-----------------------------------------|
| 1. Reduce 9 to thirds. Ans. $27$ .   | 6. Reduce 16 to ninths. Ans. $144$ .    |
| 2. Reduce 12 to eighths. Ans. $96$ . | 7. Reduce 70 to tenths.                 |
| 3. Reduce 25 to fourths.             | 8. Reduce 52 to fifteenths.             |
| 4. Reduce 36 to fifths.              | 9. Reduce 35 to sevenths.               |
| 5. Reduce 40 to thirteenths.         | 10. Reduce $8\frac{1}{2}$ to elevenths. |

135. What is reduction of a fraction?— 137. What is the rule for reducing a whole number to an equivalent improper fraction?— 138. For reducing a mixed number to an equivalent improper fraction?

Reduce the following mixed numbers to improper fractions.

|                        |                         |                          |                            |
|------------------------|-------------------------|--------------------------|----------------------------|
| 21. $37\frac{3}{4}$ .  | Ans. $148\frac{3}{4}$ . | 19. $125\frac{9}{14}$ .  | Ans. $1734\frac{9}{14}$ .  |
| 22. $45\frac{7}{8}$ .  | Ans. $367\frac{7}{8}$ . | 20. $172\frac{26}{30}$ . | Ans. $2142\frac{13}{15}$ . |
| 23. $92\frac{9}{10}$ . |                         | 21. $260\frac{2}{5}$ .   |                            |
| 24. $23\frac{7}{8}$ .  |                         | 22. $171\frac{1}{2}$ .   | Ans. $1129$ .              |
| 25. $132\frac{2}{3}$ . |                         | 23. $167\frac{10}{10}$ . |                            |
| 26. $134\frac{1}{2}$ . | Ans. $1214$ .           | 24. $209\frac{1}{2}$ .   |                            |
| 27. $96\frac{7}{10}$ . |                         | 25. $331\frac{1}{5}$ .   |                            |
| 28. $44\frac{1}{10}$ . |                         | 26. $116\frac{3}{8}$ .   |                            |

**139. CASE II.**—To reduce an improper fraction to an equivalent whole or mixed number.

Ex. In  $\frac{37}{8}$  of a yard, how many yards?

OPERATION.

$\frac{37}{8} = 37 \div 8 = 4\frac{5}{8}$ , Ans.

ANALYSIS.—Since 8 eighths make 1 yard, there will be as many yards in 37 eighths of a yard as 37 contains times 8, or  $4\frac{5}{8}$  yards.

**140. RULE.**—Divide the numerator by the denominator, and the quotient will be the number required.

EXAMPLES FOR PRACTICE.

Reduce the following improper fractions to whole or mixed numbers:

|                        |                        |                          |                          |
|------------------------|------------------------|--------------------------|--------------------------|
| 1. $\frac{10}{3}$ .    | Ans. 3.                | 10. $\frac{1321}{108}$ . | Ans. $14\frac{7}{108}$ . |
| 2. $\frac{47}{12}$ .   | Ans. $6\frac{1}{12}$ . | 11. $\frac{277}{54}$ .   | Ans. $5\frac{17}{54}$ .  |
| 3. $\frac{124}{5}$ .   | Ans. $24\frac{4}{5}$ . | 12. $\frac{109}{18}$ .   | Ans. $6\frac{1}{18}$ .   |
| 4. $\frac{202}{15}$ .  |                        | 13. $\frac{2079}{140}$ . |                          |
| 5. $\frac{348}{11}$ .  |                        | 14. $\frac{327}{14}$ .   |                          |
| 6. $\frac{100}{9}$ .   |                        | 15. $\frac{607}{10}$ .   |                          |
| 7. $\frac{1009}{11}$ . |                        | 16. $\frac{2802}{11}$ .  |                          |
| 8. $\frac{1712}{11}$ . |                        | 17. $\frac{4442}{108}$ . |                          |
| 9. $\frac{2111}{10}$ . |                        | 18. $\frac{2121}{108}$ . |                          |

**141. CASE III.**—To reduce fractions to their lowest terms.

NOTE.—A fraction is in its lowest terms, when its numerator and denominator are prime to each other.

Ex. Reduce  $\frac{144}{33}$  to its lowest terms.

OPERATION.

2)  $\frac{144}{33} = \frac{72}{16\frac{1}{2}}$   
 2)  $\frac{72}{16\frac{1}{2}} = \frac{36}{8\frac{1}{4}}$   
 3)  $\frac{36}{8\frac{1}{4}} = \frac{12}{2\frac{1}{3}}$  Ans.

Or,

12)  $\frac{144}{33} = \frac{12}{2\frac{1}{3}}$

ANALYSIS.—Dividing both terms of the fraction by the same number does not alter the value of the fraction (134, 3rd.); hence, we divide both terms of  $\frac{144}{33}$  by 2, both terms of the result,  $\frac{72}{16\frac{1}{2}}$ , by 2, both terms of this result by 3, and obtain  $\frac{12}{2\frac{1}{3}}$  for the final result. As 3 and 7 are prime to each other, the lowest terms of  $\frac{144}{33}$  are  $\frac{12}{2\frac{1}{3}}$ .

Instead of dividing by the factors 2, 2, and

140. What is the rule for reducing an improper fraction to a whole or mixed number?

## REDUCTION, OF FRACTIONS.

3, successively, we may divide by the greatest common divisor of the given terms, and reduce the fraction to its lowest terms in a single operation. Hence, the

**142. RULE.**—Cancel or reject all factors common to both numerator and denominator. Or,  
Divide both terms by their greatest common divisor.

## EXAMPLES FOR PRACTICE.

Reduce the following fractions to their lowest terms :

|                       |                      |                        |                    |                        |                      |
|-----------------------|----------------------|------------------------|--------------------|------------------------|----------------------|
| 1. $\frac{5}{10}$     | Ans. $\frac{1}{2}$   | 9. $\frac{10}{87}$     | Ans. $\frac{1}{9}$ | 17. $\frac{81}{87}$    | Ans. $\frac{3}{29}$  |
| 2. $\frac{17}{46}$    | Ans. $\frac{17}{46}$ | 10. $\frac{168}{108}$  | Ans. $\frac{4}{9}$ | 18. $\frac{124}{288}$  | Ans. $\frac{31}{72}$ |
| 3. $\frac{34}{74}$    | Ans. $\frac{17}{37}$ | 11. $\frac{128}{128}$  | Ans. $\frac{1}{1}$ | 19. $\frac{288}{360}$  | Ans. $\frac{4}{5}$   |
| 4. $\frac{72}{120}$   | Ans. $\frac{3}{5}$   | 12. $\frac{14}{104}$   | Ans. $\frac{1}{8}$ | 20. $\frac{91}{110}$   |                      |
| 5. $\frac{90}{144}$   | Ans. $\frac{5}{8}$   | 13. $\frac{208}{104}$  |                    | 21. $\frac{128}{144}$  |                      |
| 6. $\frac{144}{432}$  |                      | 14. $\frac{312}{312}$  |                    | 22. $\frac{112}{128}$  |                      |
| 7. $\frac{288}{864}$  |                      | 15. $\frac{616}{916}$  |                    | 23. $\frac{288}{360}$  |                      |
| 8. $\frac{900}{1530}$ |                      | 16. $\frac{403}{1087}$ |                    | 24. $\frac{881}{8116}$ |                      |

**143. CASE IV.**—To reduce a fraction to a decimal.

*Ex.* Reduce  $\frac{7}{8}$  to its equivalent decimal.

FIRST OPERATION.

$$\frac{7}{8} = \frac{7000}{8000} = \frac{875}{1000} = 0.875, \text{ Ans.}$$

SECOND OPERATION.

$$\begin{array}{r} 8 \overline{) 7.000} \\ \underline{0.875} \end{array}$$

we have the equivalent decimal 0.875.

In the second operation, we omit the intermediate steps, and obtain the result, practically, by annexing the three ciphers to the numerator, 7, and dividing the result by the denominator, 8.

**144. RULE.**—I. Annex ciphers to the numerator, and divide by the denominator.

II. Point off as many decimal places in the result as there are ciphers annexed.

NOTE.—If the division is not exact when a sufficient number of decimal figures have been obtained, the sign, +, may be annexed to the decimal to indicate that there is still a remainder.

## EXAMPLES FOR PRACTICE.

Reduce the following fractions to equivalent decimals.

|                  |              |                    |           |                     |              |
|------------------|--------------|--------------------|-----------|---------------------|--------------|
| 1. $\frac{1}{2}$ | Ans. 0.5     | 6. $\frac{1}{25}$  | Ans. 0.04 | 11. $\frac{1}{3}$   | Ans. 0.333 + |
| 2. $\frac{3}{4}$ | Ans. 0.75    | 7. $\frac{1}{8}$   |           | 12. $\frac{1}{8}$   |              |
| 3. $\frac{3}{4}$ | Ans. 0.8     | 8. $\frac{1}{8}$   |           | 13. $\frac{1}{8}$   |              |
| 4. $\frac{1}{2}$ |              | 9. $\frac{17}{20}$ | Ans. 0.85 | 14. $\frac{17}{20}$ |              |
| 5. $\frac{7}{8}$ | Ans. 0.714 + | 10. $\frac{1}{80}$ |           | 15. $\frac{19}{32}$ |              |

142. What is the rule for reducing fractions to their lowest terms?—144. What is the rule for reducing a fraction to a decimal?

**145. CASE V.**—To reduce a decimal to a fraction.

*Ex. 1.* Reduce 0.875 to an equivalent fraction.

OPERATION. ANALYSIS.—Writing the decimal figures, .875, over the common denominator, 1000, we have  $\frac{875}{1000} = \frac{7}{8}$ . Hence, the

**146. RULE.**—Omit the decimal point, and supply the proper denominator.

OPERATION.

*Ex. 2.* Reduce  $0.5\frac{1}{2}$  to a fraction.  $5\frac{1}{2} = \frac{5\frac{1}{2}}{10} = \frac{16}{3} = \frac{16}{30} = \frac{8}{15}$ .

**147. RULE.**—Omitting the decimal point, write the denominator under the decimal, and reduce the fraction to its lowest terms (142).

EXAMPLES FOR PRACTICE.

Reduce the following decimals to equivalent fractions:

- |            |                          |                           |                          |
|------------|--------------------------|---------------------------|--------------------------|
| 1. 0.06.   | Ans. $\frac{3}{50}$ .    | 9. 0.000125.              |                          |
| 2. 0.75.   | Ans. $\frac{3}{4}$ .     | 10. 0.3 $\frac{1}{2}$ .   | Ans. $\frac{3}{2}$ .     |
| 3. 0.12.   |                          | 11. 4.00075.              | Ans. $4\frac{3}{4000}$ . |
| 4. 0.125.  |                          | 12. 0.66 $\frac{2}{3}$ .  | Ans. $\frac{2}{3}$ .     |
| 5. 0.024.  |                          | 13. 0.57 $\frac{1}{2}$ .  | Ans. $\frac{11}{20}$ .   |
| 6. 0.655.  | Ans. $\frac{131}{200}$ . | 14. 0.16 $\frac{1}{2}$ .  | Ans. $\frac{1}{6}$ .     |
| 7. 0.0008. | Ans. $\frac{1}{12500}$ . | 15. 5.625 $\frac{1}{2}$ . |                          |
| 8. 0.68.   |                          |                           |                          |

**148. CASE VI.**—To reduce a compound fraction to a simple one.

*Ex. 1.* Reduce  $\frac{2}{3}$  of  $\frac{3}{4}$  to a simple fraction.

OPERATION. ANALYSIS.—By multiplying the denominator of  $\frac{3}{4}$  by 3, the denominator of  $\frac{2}{3}$ , it is evident we we obtain  $\frac{1}{4} = \frac{1}{4}$ , since the parts into which the number is divided are 3 times as many, and consequently only  $\frac{1}{3}$  as large as before; and, since  $\frac{1}{4}$  of  $\frac{3}{4} = \frac{3}{16}$ ,  $\frac{2}{3}$  of  $\frac{3}{4}$  will be twice  $\frac{3}{16} = \frac{3}{8}$ .

*Ex. 2.* Reduce  $\frac{2}{3}$  of  $\frac{4}{5}$  of  $\frac{3}{4}$  of  $\frac{2}{3}$  of  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $3\frac{1}{2}$  to a simple fraction.

OPERATION.

$$\frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} \times \frac{3}{4} \times \frac{11}{2} = \frac{11}{2}, \text{ Ans.}$$

146, 147.. What is the rule for reducing a decimal to a fraction?

**149. RULE.—I.** Cancel the factors common to the numerators and denominators, if any.

**II.** Multiply the remaining numerators together for a new numerator, and the remaining denominators for a new denominator.

**NOTE.**—All whole and mixed numbers that occur in compound fractions must be reduced to improper fractions, before the required reduction is performed.

## EXAMPLES FOR PRACTICE.

1. What is  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$ ?
2. What is  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{3}{4}$ ?
3. What is  $\frac{1}{3}$  of  $\frac{2}{11}$  of  $\frac{3}{7}$  of  $\frac{4}{5}$ ?
4. Required the value of  $\frac{2}{3}$  of  $\frac{1}{2}$  of  $\frac{3}{11}$  of 21.
5. Reduce  $\frac{2}{3}$  of  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$  of  $\frac{7}{8}$  to a simple fraction.
6. What is the value of  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$  of  $\frac{7}{8}$  of  $\frac{9}{10}$  of  $\frac{11}{12}$ ?
7. Reduce  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$  to a simple fraction.
8. Reduce  $\frac{2}{3}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$  of  $9\frac{3}{4}$  to a whole number.
9. What is the value of  $\frac{1}{2}$  of  $2\frac{1}{2}$  of  $1\frac{1}{11}$ ?
10. What is the value of  $\frac{1}{3}$  of  $\frac{2}{3}$  of  $\frac{3}{11}$  of  $\frac{4}{5}$  of  $8\frac{3}{4}$ ?
11. Reduce  $\frac{1}{2}$  of  $\frac{3}{4}$  of  $\frac{5}{6}$  of  $3\frac{3}{4}$  to a simple fraction.
12. Required the value of  $\frac{1}{2}$  of  $7\frac{1}{2}$  of  $1\frac{1}{5}$  of  $\frac{3}{11}$  of  $3\frac{1}{2}$ .

Ans.  $\frac{35}{132}$   
 Ans.  $\frac{1}{4}$   
 Ans.  $\frac{275}{176}$   
 Ans.  $2\frac{1}{11}$

Ans.  $\frac{3}{4}$

Ans.  $\frac{7}{12}$

**150. CASE VII.**—To reduce fractions to a common denominator.

**151. A Common Denominator** is a denominator common to two or more fractions.

*Ex.* Reduce  $\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{3}{4}$  to other fractions of equal value, having a common denominator.

## FIRST OPERATION.

$$\begin{array}{r} 2 \times 4 \times 5 = 40 \\ \frac{2}{3} \times 4 \times 5 = \frac{40}{3} \\ 3 \times 3 \times 5 = 45 \\ \frac{2}{4} \times 3 \times 5 = \frac{60}{4} \\ \frac{4}{5} \times 3 \times 4 = \frac{48}{5} \\ 5 \times 3 \times 4 = 60 \end{array}$$

## SECOND OPERATION.

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4} = \frac{20}{60}, \frac{40}{60}, \frac{45}{60}$$

**153. RULE.**—Multiply the terms of each fraction by all the denominators but its own (for new numerators and a common denominator).

**NOTE.**—Mixed numbers must first be reduced to improper fractions, and compound fractions, to simple ones:

149. What is the rule for reducing a compound fraction to a simple one.—  
 153. What is the rule for finding a common denominator?

R  
 1.  
 2.  
 3.  
 7  
 8  
 9  
 10  
 11  
 12  
 13  
 14  
 15  
 I  
 deno  
 I  
 fract  
 duce  
 inat  
 E

15  
 denom  
 II.  
 nomin  
 quotie

Red  
 1.  
 2.

EXAMPLES FOR PRACTICE.

Reduce the following fractions to their common denominator:—

- |                                                                       |                                     |                                      |                                                                 |
|-----------------------------------------------------------------------|-------------------------------------|--------------------------------------|-----------------------------------------------------------------|
| 1. $\frac{2}{3}$ and $\frac{3}{4}$ .                                  | Ans. $\frac{8}{12}, \frac{9}{12}$ . | 4. $\frac{1}{2}$ and $\frac{3}{8}$ . | Ans. $\frac{4}{8}, \frac{3}{8}$ .                               |
| 2. $\frac{1}{2}$ and $\frac{3}{4}$ .                                  | Ans. $\frac{2}{4}, \frac{3}{4}$ .   | 5. $\frac{2}{3}$ and $\frac{1}{4}$ . | Ans. $\frac{8}{12}, \frac{3}{12}$ .                             |
| 3. $\frac{1}{2}$ and $\frac{3}{4}$ .                                  | Ans. $\frac{2}{4}, \frac{3}{4}$ .   | 6. $\frac{1}{3}$ and $\frac{1}{4}$ . | Ans. $\frac{4}{12}, \frac{3}{12}$ .                             |
| 7. $\frac{1}{2}, \frac{3}{4},$ and $\frac{1}{3}$ .                    |                                     |                                      | Ans. $\frac{6}{12}, \frac{9}{12}, \frac{4}{12}$ .               |
| 8. $\frac{1}{2}, \frac{3}{4},$ and $\frac{1}{6}$ .                    |                                     |                                      |                                                                 |
| 9. $\frac{1}{2}, \frac{3}{4},$ and $\frac{1}{12}$ .                   |                                     |                                      | Ans. $\frac{6}{12}, \frac{9}{12}, \frac{1}{12}$ .               |
| 10. $\frac{1}{2}, \frac{3}{4}, \frac{1}{3},$ and $\frac{1}{6}$ .      |                                     |                                      |                                                                 |
| 11. $\frac{1}{2}, \frac{3}{4}, \frac{1}{3},$ and $\frac{1}{6}$ .      |                                     |                                      | Ans. $\frac{6}{12}, \frac{9}{12}, \frac{4}{12}, \frac{1}{12}$ . |
| 12. $1\frac{1}{2}, 5\frac{1}{4},$ and $\frac{1}{3}$ .                 |                                     |                                      |                                                                 |
| 13. $2\frac{1}{2}, \frac{3}{4}, \frac{1}{3},$ and $8$ .               |                                     |                                      |                                                                 |
| 14. $\frac{1}{2}, \frac{1}{3},$ and $\frac{2}{3}$ of $7\frac{1}{2}$ . |                                     |                                      |                                                                 |
| 15. $\frac{1}{2}, \frac{1}{3}$ of $6,$ and $2\frac{1}{4}$ .           |                                     |                                      | Ans. $\frac{3}{12}, \frac{1}{12}, \frac{6}{12}$ .               |

154. CASE VIII.—To reduce fractions to their least common denominator.

155. The Least Common Denominator of two or more fractions is the least denominator to which they can all be reduced, and it must be the least common multiple of their denominators.

Ex. Reduce  $\frac{2}{3}, \frac{3}{4},$  and  $\frac{1}{12}$  to their least common denominator.

|                    |   |      |
|--------------------|---|------|
| OPERATION.         |   |      |
| 5                  | 3 | 7    |
| 2) 6               | 8 | 12   |
| 2) 3               | 4 | 6    |
| 3) 3               | 2 | 3    |
| 1                  | 2 | 1    |
| 2 × 2 × 3 × 2 = 24 |   |      |
|                    |   | = 24 |
|                    |   | = 12 |
|                    |   | = 12 |
|                    |   | = 12 |

Ans.

ANALYSIS.—We find the least common denominator, by (117), to be 24. We then take such a part of it as is expressed by each of the fractions separately for their respective new numerators. Thus, to get a new numerator for  $\frac{2}{3}$ , we take  $\frac{2}{3}$  of 24, the least common denominator, by dividing it by 3, and multiplying the quotient by 2. We proceed in like manner with each of the fractions, and write the numerators thus obtained over the least common denominator. Hence, the

156. RULE.—I. Find the least common multiple of the given denominators, for the least common denominator.

II. Divide this common denominator by each of the given denominators, and multiply each numerator by the corresponding quotient. The products will be the new numerators.

EXAMPLES FOR PRACTICE.

Reduce the following fractions to their least common denominator.

- |                                                                  |                                                                    |
|------------------------------------------------------------------|--------------------------------------------------------------------|
| 1. $\frac{2}{3}, \frac{5}{8}, \frac{7}{9},$ and $\frac{1}{12}$ . | Ans. $\frac{14}{72}, \frac{49}{72}, \frac{56}{72}, \frac{6}{72}$ . |
| 2. $\frac{3}{4}, \frac{1}{12}, 6,$ and $\frac{1}{2}$ .           | Ans. $\frac{9}{12}, \frac{1}{12}, \frac{72}{12}, \frac{6}{12}$ .   |

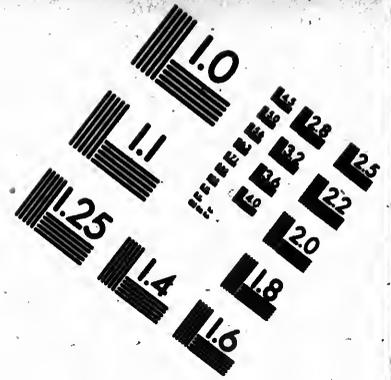
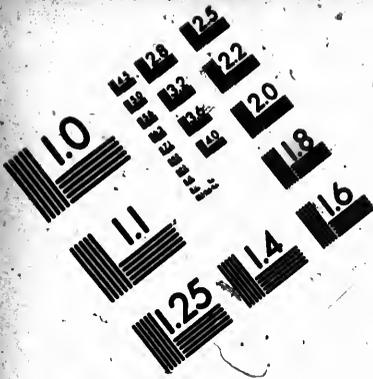
156. What is the rule for reducing fractions to their least common denominator?



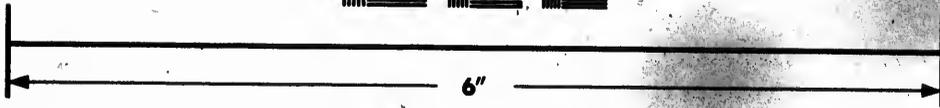
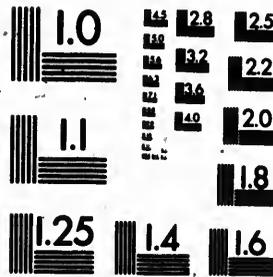








**IMAGE EVALUATION  
TEST TARGET (MT-3)**



**Photographic  
Sciences  
Corporation**

23 WEST MAIN STREET  
WEBSTER, N.Y. 14580  
(716) 872-4303

18  
20  
22  
25

10

3.  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and  $\frac{4}{5}$ .  
 4.  $\frac{1}{3}$ ,  $\frac{2}{4}$ ,  $\frac{3}{5}$ , and  $\frac{4}{6}$ .  
 5.  $\frac{1}{15}$ ,  $\frac{2}{24}$ ,  $\frac{3}{35}$ , and  $\frac{4}{48}$ .  
 6.  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and  $\frac{4}{5}$ .  
 7.  $\frac{1}{3}$ ,  $\frac{2}{4}$ ,  $\frac{3}{5}$ , and  $\frac{4}{6}$ .  
 8.  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and  $5\frac{1}{2}$ .  
 9.  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and  $7\frac{1}{2}$ .  
 10.  $6\frac{1}{2}$ ,  $7$ ,  $7\frac{1}{2}$ , and  $8$ .  
 11.  $\frac{1}{10}$ ,  $\frac{2}{3}$ ,  $3\frac{1}{2}$ , and  $1\frac{1}{2}$ .  
 12.  $\frac{1}{2}$ ,  $9$ ,  $7$ ,  $5$ , and  $4$ .  
 13.  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ , and  $\frac{7}{10}$ .  
 14.  $\frac{1}{15}$ ,  $\frac{2}{24}$ ,  $\frac{3}{35}$ , and  $\frac{4}{48}$ .  
 15.  $\frac{1}{15}$ ,  $\frac{2}{11}$ ,  $\frac{3}{14}$ , and  $\frac{4}{10}$ .

$$\text{Ans. } \frac{1488}{1980}, \frac{1920}{1980}, \frac{1440}{1980}, \frac{1920}{1980}.$$

$$\text{Ans. } \frac{55}{120}, \frac{35}{120}, \frac{74}{120}, \frac{112}{120}.$$

$$\text{Ans. } \frac{115}{168}, \frac{104}{168}, \frac{155}{168}, \frac{161}{168}.$$

$$\text{Ans. } \frac{34}{48}, \frac{48}{48}, \frac{48}{48}, \frac{485}{48}.$$

$$\text{Ans. } \frac{15}{36}, \frac{18}{36}, \frac{25}{36}, \frac{36}{36}.$$

$$\text{Ans. } \frac{118}{144}, \frac{140}{144}, \frac{145}{144}, \frac{144}{144}.$$

## ADDITION OF FRACTIONS.

NOTES.—1. Fractions, to be added or subtracted, must be abstract or of like denomination, and must have a common denominator.

2. Only units of the same kind, whether fractional or integral can be added together.

Ex. 1. What is the sum of  $\frac{1}{2}$ ,  $\frac{2}{3}$ , and  $\frac{7}{12}$ ?

OPERATION.

$$\frac{1}{2} + \frac{2}{3} + \frac{7}{12} = \frac{6 + 8 + 7}{24} = \frac{21}{24} = \frac{7}{8} = 1\frac{1}{8}, \text{ Ans.}$$

ANALYSIS.—We first reduce the given fractions to a common denominator. And as the resulting fractions,  $\frac{6}{24}$ ,  $\frac{8}{24}$ , and  $\frac{7}{24}$  have the same fractional unit, we add them by uniting their numerators into one sum, making  $\frac{21}{24} = 1\frac{1}{8}$ , the answer.

Ex. 2. Add  $7\frac{1}{2}$ ,  $8\frac{1}{12}$ , and  $1\frac{1}{3}$ .

OPERATION.

$$\begin{aligned} 7 + 8 + 1 &= 16 \\ \frac{1}{2} + \frac{1}{12} + \frac{1}{3} &= 1\frac{1}{4} \\ \hline 17\frac{1}{4}, \text{ Ans.} \end{aligned}$$

ANALYSIS.—The sum of the integers, 7, 8, and 1, is 16; the sum of the fractions,  $\frac{1}{2}$ ,  $\frac{1}{12}$ , and  $\frac{1}{3}$ , is  $1\frac{1}{4}$ . Hence, the sum of both fractions and integers is  $16 + 1\frac{1}{4} = 17\frac{1}{4}$ . Hence the

157. RULE. I. To add fractions.—When necessary, reduce the fractions to their least common denominator; then add the numerators and place the sum over the common denominator.

II. To add mixed numbers.—Add the integers and fractions separately, and then add their sums.

NOTE.—All fractional results should be reduced to their lowest terms, and if improper fractions, to whole or mixed numbers.

157. What is the general rule for adding fractions?

## EXAMPLES FOR PRACTICE.

1. What is the sum of  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{4}$ ? Ans.  $2\frac{1}{12}$ .
2. What is the sum of  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{6}$ ? Ans.  $2\frac{1}{12}$ .
3. What is the sum of  $\frac{1}{12}$ ,  $\frac{1}{12}$ ,  $\frac{1}{12}$ , and  $\frac{1}{12}$ ? Ans.  $2\frac{1}{3}$ .
4. Add  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{6}$ . Ans.  $2\frac{1}{12}$ .
5. Add  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{6}$ .
6. Add  $\frac{1}{4}$ ,  $\frac{1}{6}$ , and  $\frac{1}{12}$ .
7. Find the sum of  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{6}$ . Ans.  $3\frac{7}{12}$ .
8. Find the sum of  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{6}$ .
9. Add  $14\frac{1}{2}$ ,  $3\frac{1}{3}$ ,  $1\frac{1}{6}$ , and  $1\frac{1}{2}$ . Ans.  $21\frac{11}{6}$ .
10. Add  $4\frac{1}{2}$ ,  $8\frac{1}{3}$ , and  $2\frac{1}{6}$ .
11. Add  $4\frac{1}{2}$ ,  $1\frac{1}{3}$ , and  $4\frac{1}{6}$ . Ans.  $16\frac{2}{3}$ .
12. What is the sum of  $7$ ,  $1\frac{1}{2}$ ,  $5$ , and  $10\frac{1}{2}$ . Ans.  $14\frac{1}{2}$ .
13. Add  $\frac{1}{2}$  of  $\frac{1}{3}$ ,  $\frac{1}{3}$  of  $\frac{1}{4}$ , and  $\frac{1}{4}$ .
14. Add  $\frac{1}{2}$  of  $\frac{1}{3}$ ,  $\frac{1}{3}$  of  $\frac{1}{4}$  of  $\frac{1}{5}$ , and  $\frac{1}{5}$ .
15. Add  $41\frac{1}{2}$ ,  $105\frac{1}{3}$ ,  $300\frac{1}{4}$ ,  $241\frac{1}{5}$ , and  $472\frac{1}{6}$ . Ans.  $1161\frac{11}{60}$ .
16. Add  $125\frac{1}{2}$ ,  $327\frac{1}{3}$ , and  $25\frac{1}{6}$ .
17. Add  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{6}$ , and  $\frac{1}{12}$ . Ans.  $1\frac{1}{12}$ .
18. Find the sum of  $17\frac{1}{2}$ ,  $18\frac{1}{3}$ , and  $11\frac{1}{6}$ . Ans.  $12\frac{11}{12}$ .
19. Add  $\frac{1}{2}$  of  $18\frac{1}{3}$ , and  $\frac{1}{3}$  of  $\frac{1}{4}$  of  $6\frac{1}{2}$ .
20. Add  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $1\frac{1}{2}$  to  $\frac{1}{4}$  of  $\frac{1}{5}$ .
21. Add  $16\frac{1}{2}$ ,  $8\frac{1}{3}$ ,  $9\frac{1}{4}$ ,  $3\frac{1}{6}$ , and  $1\frac{1}{12}$ . Ans.  $40\frac{1}{12}$ .
22. Add  $3\frac{1}{2}$  of  $5\frac{1}{3}$ ,  $\frac{1}{3}$  of  $7\frac{1}{4}$ , and  $\frac{1}{4}$  of  $\frac{1}{5}$ .

## SUBTRACTION OF FRACTIONS.

Ex. 1. From  $\frac{3}{4}$  take  $\frac{1}{2}$ .

OPERATION.

$$\frac{3}{4} - \frac{1}{2} = \frac{9}{12} - \frac{6}{12} = \frac{3}{12} = \frac{1}{4}$$

ANALYSIS.—We reduce the given fractions to a common denominator, and have  $\frac{9}{12}$  and  $\frac{6}{12}$  which express fractional units of the same value. Then 9 twelfths less 6 twelfths equal 3 twelfths =  $\frac{1}{4}$ , the answer.

Ex. 2. From  $24\frac{1}{2}$  take  $16\frac{1}{3}$ .

FIRST OPERATION.

$$\begin{array}{r} 24\frac{1}{2} = 24\frac{3}{6} \\ 16\frac{1}{3} = 16\frac{2}{6} \\ \hline 7\frac{1}{6} \text{ Ans.} \end{array}$$

ANALYSIS.—We first reduce the fractional parts  $\frac{1}{2}$  and  $\frac{1}{3}$  to a common denominator, 6. Since we cannot take  $\frac{2}{6}$  from  $\frac{3}{6}$ , we add 1 =  $\frac{6}{6}$  to  $\frac{3}{6}$ , making  $\frac{9}{6}$ , and  $\frac{2}{6}$  from  $\frac{3}{6}$  leaves  $\frac{1}{6}$ . We next add 1 to the 6 in the subtrahend, and subtracting, we have  $7\frac{1}{6}$  for the entire remainder.

SECOND OPERATION.

$$\begin{array}{r} 24\frac{1}{2} = 17\frac{1}{2} = \frac{34}{2} \\ 16\frac{1}{3} = 9\frac{1}{3} = \frac{19}{3} \\ \hline 7\frac{1}{6} = 7\frac{1}{6} \end{array}$$

ANALYSIS.—In this operation, we reduce the mixed numbers to improper fractions, and these fractions to a common denominator. We then subtract the less fraction from the greater, and, reducing the remainder to a mixed number, obtain  $7\frac{1}{6}$ , as before.

**158. RULE I.** To subtract fractions.—When necessary, reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference of the new numerators over the common denominator.

II. To subtract mixed numbers.—Reduce the fractional parts to a common denominator, and then subtract the fractional and integral parts separately. Or,—Reduce the mixed numbers to improper fractions, then to a common denominator, and subtract the less fraction from the greater.

## EXAMPLES FOR PRACTICE.

- |                                                                                                                                  |                        |                                         |                           |
|----------------------------------------------------------------------------------------------------------------------------------|------------------------|-----------------------------------------|---------------------------|
| 1. $\frac{3}{4} - \frac{1}{2} =$                                                                                                 | Ans. $\frac{1}{4}$ .   | 17. $71\frac{1}{15} - 13\frac{2}{15} =$ | Ans. $57\frac{11}{15}$ .  |
| 2. $\frac{1}{3} - \frac{1}{12} =$                                                                                                |                        | 18. $75 - 7\frac{1}{2} =$               |                           |
| 3. $\frac{2}{3} - \frac{1}{6} =$                                                                                                 | Ans. $\frac{1}{2}$ .   | 19. $18\frac{1}{2} - 5\frac{1}{2} =$    | Ans. $12\frac{1}{2}$ .    |
| 4. $\frac{1}{10} - \frac{1}{20} =$                                                                                               |                        | 20. $\frac{1}{2} - \frac{1}{10} =$      |                           |
| 5. $\frac{1}{8} - \frac{1}{16} =$                                                                                                | Ans. $\frac{1}{16}$ .  | 21. $28\frac{1}{4} - 3\frac{1}{2} =$    | Ans. $24\frac{1}{4}$ .    |
| 6. $\frac{1}{11} - \frac{1}{22} =$                                                                                               |                        | 22. $9\frac{1}{2} - 2\frac{1}{2} =$     |                           |
| 7. $\frac{1}{11} - \frac{1}{10} =$                                                                                               | Ans. $\frac{1}{110}$ . | 23. $165\frac{1}{2} - 77\frac{1}{2} =$  | Ans. $87\frac{1}{2}$ .    |
| 8. $\frac{1}{12} - \frac{1}{18} =$                                                                                               |                        | 24. $14\frac{1}{2} - 13\frac{1}{2} =$   |                           |
| 9. $\frac{1}{10} - \frac{1}{15} =$                                                                                               | Ans. $\frac{1}{30}$ .  | 25. $3\frac{1}{2} - \frac{1}{10} =$     | Ans. $3\frac{4}{10}$ .    |
| 10. $9\frac{1}{2} - 1\frac{1}{4} =$                                                                                              |                        | 26. $17\frac{1}{2} - 8\frac{1}{2} =$    |                           |
| 11. $10\frac{1}{2} - 4\frac{1}{2} =$                                                                                             |                        | 27. $7\frac{1}{2} - 2\frac{1}{2} =$     | Ans. $4\frac{1}{2}$ .     |
| 12. $14\frac{1}{2} - 3\frac{1}{2} =$                                                                                             | Ans. $10\frac{1}{2}$ . | 28. $47\frac{1}{2} - 1\frac{1}{2} =$    |                           |
| 13. $4\frac{1}{2} - \frac{1}{2} =$                                                                                               |                        | 29. $9\frac{1}{2} - \frac{1}{2} =$      | Ans. $9\frac{1}{2}$ .     |
| 14. $9\frac{1}{2} - 3\frac{1}{2} =$                                                                                              |                        | 30. $101\frac{1}{2} - 93\frac{1}{2} =$  |                           |
| 15. $8\frac{1}{2} - 6\frac{1}{2} =$                                                                                              |                        | 31. $624 - 342 =$                       | Ans. $281\frac{1}{2}$ .   |
| 16. $19\frac{1}{2} - 3\frac{1}{2} =$                                                                                             | Ans. $16\frac{1}{2}$ . | 32. $256 - 128 =$                       |                           |
| 33. From $\frac{3}{4}$ of $\frac{1}{2}$ take $\frac{1}{4}$ of $\frac{1}{2}$ .                                                    |                        |                                         | Ans. $\frac{1}{4}$ .      |
| 34. From $\frac{1}{2}$ of $\frac{1}{3}$ take $\frac{1}{6}$ of $\frac{1}{3}$ .                                                    |                        |                                         | Ans. $\frac{1}{6}$ .      |
| 35. From $\frac{1}{3}$ of $\frac{1}{4}$ take $\frac{1}{12}$ of $\frac{1}{4}$ .                                                   |                        |                                         | Ans. $\frac{1}{12}$ .     |
| 36. From $\frac{1}{2}$ of $\frac{1}{3}$ take $\frac{1}{6}$ of $\frac{1}{3}$ .                                                    |                        |                                         |                           |
| 37. From $\frac{1}{2}$ of $\frac{1}{3}$ of $3\frac{1}{2}$ take $\frac{1}{6}$ of $1\frac{1}{2}$ .                                 |                        |                                         |                           |
| 38. What is the value of $\frac{1}{2}$ of $3 - \frac{1}{2}$ of 2.                                                                |                        |                                         | Ans. $\frac{1}{2}$ .      |
| 39. From 72 lbs. there were taken at one time $17\frac{1}{2}$ lbs., and at another, $28\frac{1}{2}$ lbs.; what quantity remains? |                        |                                         | Ans. $25\frac{1}{2}$ lbs. |
| 40. From \$15, \$3, were given to A, \$4 to B, \$2 to C, and the remainder to D; what did D receive?                             |                        |                                         |                           |

## MULTIPLICATION OF FRACTIONS.

**159. CASE I.**—To multiply a fraction by an integer.

Ex. Multiply  $\frac{1}{3}$  by 3.

FIRST OPERATION.

$$\frac{1}{3} \times 3 = \frac{1 \times 3}{3} = 1$$

ANALYSIS.—In the first operation, we multiply the numerator of the fraction by the integer, 3, and obtain 3 for the answer. It is evident that

158. What is the rule for subtracting fractions?

SECOND OPERATION.

$$\frac{7}{3} \times 3 = \frac{7}{1} = 7$$

THIRD OPERATION.

$$\frac{7}{3} \times \frac{3}{1} = \frac{7}{1} = 7$$

by dividing its denominator by 3, since the parts into which the fraction is divided are only  $\frac{1}{3}$  as many, and consequently 3 times as large, as before, while the parts taken remain the same. Hence,

*Multiplying the numerator or dividing the denominator of a fraction by any integer multiplies the fraction by that integer.*

NOTE.—In the third operation, we express the multiplier in the form of a fraction, indicate the multiplication, and obtain the result by cancellation.

**160. CASE II.**—To multiply an integer by a fraction, or to find a fractional part of an integer.

*Ex.* Multiply 24 by  $\frac{5}{4}$ .

FIRST OPERATION.

$$24 \times \frac{5}{4} = 12 \times 5 = 60$$

SECOND OPERATION.

$$24 \times \frac{5}{4} = 4 \times 5 = 20$$

THIRD OPERATION.

$$\frac{24}{1} \times \frac{5}{4} = 20$$

the fraction  $\frac{5}{4}$  is multiplied by multiplying its numerator by 3, since the parts taken, 21, are 3 times as many as before, while the parts into which the unit of the fraction is divided remain the same.

In the second operation, we divide the denominator of the fraction by the integer, 3, and obtain 21 for the answer, as before. It is evident, also, that the fraction  $\frac{5}{4}$  is multiplied by 3

since the parts into which the unit of the fraction is divided are only  $\frac{1}{3}$  as many, and consequently 3 times as large, as before, while the parts taken remain the same. Hence,

ANALYSIS.—In the first operation, we first multiply the integer, 24, by the numerator of the fraction, then divide the product by the denominator, and obtain 20 for the answer.

In the second operation, we divide the integer, 24, by the denominator of the fraction, and obtain  $\frac{1}{4}$  of 24 = 4, which multiplied by 5, the numerator of the fraction, gives  $\frac{5}{4}$  of 24 = 20. Hence,

*Multiplying by a fraction is taking the part of the multiplier denoted by the multiplier.*

NOTE.—In the third operation, we express the integer, 24, in the form of a fraction, indicate the multiplication, and obtain the result by cancellation.

**161. CASE III.**—To multiply a fraction by a fraction.

NOTE.—To multiply a fraction by a fraction is to find a fractional part of a fraction.

*Ex.* Multiply  $\frac{5}{12}$  by  $\frac{4}{3}$ .

FIRST OPERATION.

$$\frac{5}{12} \times \frac{4}{3} = \frac{20}{36} = \frac{5}{9}$$

SECOND OPERATION.

$$\frac{5}{12} \times \frac{4}{3} = \frac{5}{9}$$

ANALYSIS.—To multiply  $\frac{5}{12}$  by  $\frac{4}{3}$  is to take  $\frac{4}{3}$  of the multiplicand,  $\frac{5}{12}$ . Now, to obtain  $\frac{4}{3}$  of  $\frac{5}{12}$ , we simply multiply the numerators together for a new numerator, and the denominators together for a new denominator (150). Therefore,

necessary, re-  
Subtract  
of the min-  
ver the com-

ional parts  
tional and  
numbers to  
and subtract

Ans.  $57\frac{2}{3}$ .

Ans.  $12\frac{1}{3}$ .

Ans.  $24\frac{1}{3}$ .

Ans.  $87\frac{2}{3}$ .

Ans.  $3\frac{4}{5}$ .

Ans.  $4\frac{1}{2}$ .

Ans.  $5\frac{3}{4}$ .

Ans.  $291\frac{1}{2}$ .

Ans.  $\frac{11}{5}$ .

Ans.  $\frac{1}{3}$ .

Ans.  $\frac{1}{3}$ .

Ans.  $\frac{1}{3}$ .

s., and at

$15\frac{1}{2}$  lbs.

, and the

ver.

we multiply

integer, 3,

evident that

*Multiplying one fraction by another is the same as reducing compound fractions to simple ones.*

From the foregoing we deduce the following general

**162. RULE.—I.** Reduce all integers and mixed numbers to improper fractions.

**II.** Multiply together the numerators for a new numerator, and the denominators for a new denominator.

NOTES.—1. Cancel all factors common to numerators and denominators:

2. The word *of* between fractions is equivalent to the sign of multiplication.

#### EXAMPLES FOR PRACTICE.

- |                                          |                       |                                                                               |                         |
|------------------------------------------|-----------------------|-------------------------------------------------------------------------------|-------------------------|
| 1. $\frac{3}{4} \times 7 =$              | Ans. $5\frac{1}{4}$ . | 16. $3\frac{3}{4} \times \frac{2}{3} =$                                       | Ans. $2\frac{1}{2}$ .   |
| 2. $\frac{5}{8} \times 4 =$              |                       | 17. $\frac{2}{3} \times 15 =$                                                 |                         |
| 3. $\frac{3}{4} \times 8 =$              | Ans. $5\frac{3}{4}$ . | 18. $1\frac{1}{2} \times 1\frac{1}{2} =$                                      | Ans. $2\frac{1}{4}$ .   |
| 4. $\frac{7}{10} \times 5 =$             |                       | 19. $9 \times 8\frac{1}{2} =$                                                 |                         |
| 5. $\frac{1}{2} \times 6 =$              | Ans. $1\frac{1}{2}$ . | 20. $7\frac{1}{2} \times 8\frac{1}{2} =$                                      | Ans. $60\frac{3}{8}$ .  |
| 6. $12 \times \frac{3}{4} =$             |                       | 21. $\frac{1}{2} \times \frac{7}{8} =$                                        |                         |
| 7. $13 \times \frac{5}{8} =$             | Ans. $7\frac{3}{8}$ . | 22. $\frac{1}{2} \times 7\frac{1}{2} =$                                       | Ans. $63\frac{1}{4}$ .  |
| 8. $16 \times \frac{1}{2} =$             |                       | 23. $4\frac{1}{2} \times 9\frac{1}{2} =$                                      |                         |
| 9. $19 \times \frac{3}{4} =$             | Ans. $5\frac{3}{4}$ . | 24. $12\frac{1}{2} \times 11\frac{1}{2} =$                                    | Ans. $147\frac{1}{4}$ . |
| 10. $21 \times \frac{1}{3} =$            |                       | 25. $4\frac{1}{2} \times \frac{2}{3} =$                                       |                         |
| 11. $\frac{1}{2} \times \frac{1}{4} =$   | Ans. $\frac{1}{8}$ .  | 26. $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4} =$                     | Ans. $\frac{1}{8}$ .    |
| 12. $\frac{1}{2} \times \frac{1}{3} =$   |                       | 27. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} =$  |                         |
| 13. $\frac{1}{2} \times \frac{1}{3} =$   | Ans. $\frac{1}{6}$ .  | 28. $\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} =$ | Ans. $\frac{3}{8}$ .    |
| 14. $\frac{1}{2} \times \frac{1}{3} =$   |                       | 29. $1\frac{1}{2} \times \frac{1}{2} \times 2 \times 5\frac{1}{2} =$          |                         |
| 15. $2\frac{1}{2} \times 2\frac{1}{2} =$ | Ans. $5\frac{1}{4}$ . | 30. $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} =$  | Ans. $\frac{1}{120}$ .  |
31. Find the value of  $2\frac{1}{2}$  times  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{1}{4}$ .
32. Find the value of  $\frac{2}{3}$  of  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $\frac{1}{4}$ .
33. What is the product of  $\frac{1}{2}$  of  $\frac{1}{3}$  of  $\frac{1}{4}$  by 11?
34. What is the product of  $12\frac{1}{2}$  by  $5\frac{1}{2}$  times  $6\frac{1}{2}$ ? Ans.  $3\frac{3}{4}$ .

#### PRACTICAL PROBLEMS.

NOTE.—In business transactions it is customary to add 1 cent when the fraction is equal to or greater than a half of a cent, and to omit it when it is less than the half of a cent. The fraction is retained in the following answers.

Required the cost of

- |                                                                      |                            |
|----------------------------------------------------------------------|----------------------------|
| 1. $6\frac{1}{2}$ lbs. of ham, at $12\frac{1}{2}$ cts. per lb.       | Ans. $\$0.81\frac{1}{4}$ . |
| 2. $7\frac{1}{2}$ yds. of tape, at $5\frac{1}{2}$ cts. per yard.     |                            |
| 3. $9\frac{1}{2}$ quarts of plums, at $7\frac{1}{2}$ cts. per qt.    | Ans. $\$0.75\frac{1}{4}$ . |
| 4. $56$ lbs. of chalk, at $\frac{1}{2}$ of a cent per lb.            |                            |
| 5. $7\frac{1}{2}$ yards of muslin, at $9\frac{1}{2}$ cts. per yard.  | Ans. $\$0.74\frac{1}{2}$ . |
| 6. $7\frac{1}{2}$ lbs. of beef, at 5 cts. per lb.                    |                            |
| 7. $6\frac{1}{2}$ bush. of apples, at $74\frac{1}{2}$ cts. per bush. | Ans. $\$4.84\frac{1}{2}$ . |
| 8. $12\frac{1}{2}$ bush. of oats, at $62\frac{1}{2}$ cts. per bush.  |                            |

162. What is the rule for the multiplication of fractions?

Div  
fracio

is reducing

numbers to

operator, and

ators:  
Multiplication.

Ans.  $2\frac{1}{2}$ .

Ans.  $2\frac{1}{2}$ .

Ans.  $60\frac{3}{8}$ .

Ans.  $63\frac{1}{15}$ .

Ans.  $147\frac{1}{4}$ .

Ans.  $\frac{10}{10}$ .

= Ans.  $\frac{3}{4}$ .

Ans. 2.

Ans.  $3\frac{1}{4}$ .

9. 79 bush. of salt, at  $\frac{7}{8}$  of a dollar per bush. Ans. \$69 $\frac{1}{4}$ .
10.  $5\frac{1}{2}$  quarts of nuts, at  $9\frac{1}{2}$  cts. per quart. Ans. \$2.27 $\frac{1}{2}$ .
11.  $2\frac{1}{2}$  yards of cloth, at  $\frac{7}{8}$  of a dollar per yd.
12. 9 barrels of vinegar, at  $6\frac{1}{2}$  per bbl. Ans. \$1.42 $\frac{1}{2}$ .
13. 15 lbs. of almonds, at  $9\frac{1}{4}$  cts. per lb.
14.  $8\frac{1}{2}$  yds. of cloth, at \$5 per yard. Ans. \$3.99.
15. 15 yds. of ribbon, at  $26\frac{1}{2}$  cts. per yd.
16.  $7\frac{1}{2}$  lbs. of coffee, at  $\frac{3}{4}$  of a dollar per lb. Ans. \$22 $\frac{1}{10}$ .
17.  $8\frac{1}{2}$  cords of wood, at  $2\frac{1}{2}$  per cord.
18. 12 cords of wood, at  $6.37\frac{1}{2}$  per cord. Ans. \$26.58.
19. 42 bush. of apples, at  $63\frac{1}{2}$  cts. per bush.
20. 11 cwt. of sugar, at  $39\frac{1}{2}$  per cwt. Ans. \$0.96 $\frac{1}{4}$ .
21.  $7\frac{1}{2}$  doz. of eggs, at  $12\frac{1}{2}$  cts. per doz.
22.  $11\frac{1}{2}$  bbbls. of salmon, at  $33\frac{1}{2}$  per bbl. Ans.
23.  $12\frac{1}{2}$  bush. of potatoes, at  $37\frac{1}{4}$  cts. per bush.
24.  $22\frac{1}{2}$  yds. of selicis, at  $87\frac{1}{2}$  cts. per yard. Ans. \$41 $\frac{1}{4}$ .
25.  $7\frac{1}{2}$  cords of maple, at  $35\frac{1}{2}$  per cord.
26.  $4\frac{1}{2}$  bush. of rye, at \$1.75 per bush. Ans. 1.74 $\frac{3}{10}$ .
27.  $10\frac{1}{2}$  yds. of calico, at  $15\frac{1}{2}$  cts. per yd.
28.  $35\frac{1}{2}$  lbs. of raisins, at  $13\frac{1}{2}$  cts. per lb. Ans. \$26 $\frac{1}{4}$ .
29.  $7\frac{1}{2}$  yds. of cloth, at  $33\frac{1}{2}$  per yd.
30.  $75\frac{1}{2}$  bush. of wheat, at  $1\frac{1}{2}$  per bush. Ans. \$95 $\frac{1}{2}$ .
31. 9 doz. of adzes, at  $10\frac{1}{2}$  per doz.
32.  $6\frac{1}{2}$  bush. of turnips, at  $37\frac{1}{4}$  cts. per bush.
33.  $23\frac{1}{2}$  cords of wood, at  $33\frac{1}{2}$  per cord. Ans. \$5.85 $\frac{1}{4}$ .
34.  $75\frac{1}{2}$  lbs. of sugar, at  $7\frac{1}{2}$  cts. per lb.
35.  $212\frac{1}{2}$  lbs. of beef, at  $7\frac{1}{4}$  cts. per lb.
36.  $3\frac{1}{2}$  tons of hay, at  $12\frac{1}{2}$  per ton.
37.  $14\frac{1}{2}$  bbbls of vinegar, at  $10\frac{1}{2}$  per bbl. Ans. \$1.52 $\frac{1}{4}$ .
38.  $6\frac{1}{2}$  gal. of molasses, at  $23\frac{1}{2}$  cts. per gal.
39. 18 handkerchiefs, at  $\frac{1}{2}$  of a dollar each.
40.  $13\frac{1}{2}$  lbs. of fish, at  $9\frac{1}{2}$  cts. per lb.

DIVISION OF FRACTIONS.

163. CASE I.—To divide a fraction by an integer.

Ex. Divide  $\frac{11}{12}$  by 6.

FIRST OPERATION.

$$\frac{11}{12} \div 6 = \frac{11}{72}$$

SECOND OPERATION.

$$\frac{11}{72} \div 6 = \frac{11}{432} = \frac{11}{72}$$

ANALYSIS.—In the first operation, we divide the numerator of the fraction by 6, and write the quotient, 2, over the denominator.

In the second operation, we multiply the denominator of the fraction by the divisor, 6, and write the product under the numerator, 12. Hence,

*Dividing the numerator or multiplying the denominator of a fraction by any number divides the fraction by that number (134).*

**164. CASE II.**—*To divide an integer by a fraction.**Ex.* How many times will 24 contain  $\frac{2}{3}$ ?

FIRST OPERATION.

$$24 \div \frac{2}{3} = 168 \div 6 = 28.$$

SECOND OPERATION.

$$24 \div \frac{2}{3} = 4 \times 7 = 28.$$

ANALYSIS.—The integer 24 will contain  $\frac{2}{3}$  as many times as there are sevenths in 24, equal 168 sevenths. Now, if 24 contains 1 seventh 168 times, it will contain  $\frac{2}{3}$  as many times as 168 will contain 6, or 28.

In the second operation, we divide the integer by the numerator of the fraction, and multiply the quotient by the denominator, which produces the same result as in the first operation. Hence,

*Dividing by a fraction consists in multiplying by the denominator, and dividing by the numerator of the divisor.*

**165. CASE III.**—*To divide a fraction by a fraction.**Ex.* Divide  $\frac{7}{8}$  by  $\frac{2}{3}$ .

OPERATION.

$$\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16}.$$

ANALYSIS.—We invert the terms of the divisor, and then proceed as in multiplication of fractions (162). The reason of this process will be seen, if we consider that the divisor,

$\frac{2}{3}$ , is an expression denoting that 2 is to be divided by 3. Now, regarding 2 as an integer, we divide the fraction  $\frac{7}{8}$  by it, by multiplying the denominator; thus,  $\frac{7}{8} \times 2 = \frac{7}{4}$ . But the divisor, 2, is 3 times as large as it ought to be, since it was to be divided by 3, as seen in the original fraction; therefore the quotient,  $\frac{7}{4}$ , is  $\frac{1}{3}$  as large as it should be, and must be multiplied by 3; thus,  $\frac{7}{4} \times 3 = \frac{21}{4}$ , the answer. By this operation we have multiplied the denominator of the dividend by the numerator of the divisor, and the numerator of the dividend by the denominator of the divisor.

From the foregoing we derive the following general

**166. RULE.—I.** *Reduce integers and mixed numbers to improper fractions.*

**II.** *Invert the terms of the divisor, and proceed as in multiplication of fractions (162).*

NOTES.—1. The dividend and divisor may be reduced to a common denominator, and the numerator of the dividend be divided by the numerator of the divisor; this will give the same result as the rule.

2. Apply cancellation where practicable.

## EXAMPLES FOR PRACTICE.

1.  $\frac{2}{3} \div 3 =$

Ans.  $\frac{2}{9}$ . | 4.  $28 \div \frac{1}{2} =$

2.  $\frac{3}{4} \div 6 =$

6.  $\frac{2}{3} \div \frac{1}{4} =$

Ans.  $3\frac{1}{2}$ .

3.  $4 \div \frac{2}{3} =$

Ans. 6. | 6.  $\frac{1}{2} \div \frac{1}{3} =$

166. What is the general rule for dividing fractions?

7.  
8.  
9.  
10.  
11.  
12.  
13.  
14.  
15.  
16.  
17.  
18.  
31.

The  
by a  
32.  
33.  
34.  
35.  
36.  
37.  
38.  
39.  
40.  
41.

$$6\frac{3}{4} = \frac{27}{4}$$

fraction,  
divide a

42. V  
43. V  
44. V

- |                                        |                              |                                         |                               |
|----------------------------------------|------------------------------|-----------------------------------------|-------------------------------|
| 7. $17\frac{1}{2} \div 7 =$            | <i>Ans.</i> $2\frac{1}{2}$ . | 19. $\frac{3}{4} \div \frac{2}{10} =$   | <i>Ans.</i> $2\frac{3}{4}$ .  |
| 8. $\frac{2}{3} \div \frac{4}{5} =$    | <i>Ans.</i> $3$ .            | 20. $11\frac{1}{2} \div 5\frac{1}{2} =$ | <i>Ans.</i> $1\frac{1}{2}$ .  |
| 9. $2\frac{1}{2} \div \frac{1}{3} =$   | <i>Ans.</i> $5\frac{1}{2}$ . | 21. $\frac{1}{2} \div \frac{1}{12} =$   | <i>Ans.</i> $3\frac{1}{2}$ .  |
| 10. $3\frac{1}{2} \div 3\frac{1}{2} =$ | <i>Ans.</i> $2\frac{1}{2}$ . | 22. $6\frac{1}{2} \div 6\frac{1}{2} =$  | <i>Ans.</i> $31\frac{1}{2}$ . |
| 11. $75 \div 13\frac{1}{2} =$          | <i>Ans.</i> $117$ .          | 23. $\frac{1}{2} \div \frac{1}{101} =$  | <i>Ans.</i> $31\frac{1}{2}$ . |
| 12. $\frac{1}{2} \div \frac{1}{3} =$   | <i>Ans.</i> $1\frac{1}{2}$ . | 24. $15 \div \frac{1}{12} =$            | <i>Ans.</i> $31\frac{1}{2}$ . |
| 13. $7\frac{1}{2} \div 3\frac{1}{2} =$ | <i>Ans.</i> $117$ .          | 25. $19 \div \frac{1}{12} =$            | <i>Ans.</i> $31\frac{1}{2}$ . |
| 14. $\frac{1}{2} \div 167 =$           | <i>Ans.</i> $117$ .          | 26. $\frac{1}{2} \div 19 =$             | <i>Ans.</i> $\frac{1}{2}$ .   |
| 15. $63 \div \frac{1}{2} =$            | <i>Ans.</i> $117$ .          | 27. $9\frac{1}{2} \div 47 =$            | <i>Ans.</i> $\frac{1}{2}$ .   |
| 16. $3\frac{1}{2} \div 7\frac{1}{2} =$ | <i>Ans.</i> $117$ .          | 28. $4\frac{1}{2} \div 1\frac{1}{2} =$  | <i>Ans.</i> $8\frac{1}{2}$ .  |
| 17. $\frac{1}{2} \div 28 =$            | <i>Ans.</i> $117$ .          | 29. $81\frac{1}{2} \div 9\frac{1}{2} =$ | <i>Ans.</i> $8\frac{1}{2}$ .  |
| 18. $\frac{1}{2} \div 49 =$            | <i>Ans.</i> $117$ .          | 30. $\frac{1}{2} \div 7\frac{1}{2} =$   |                               |

31. Divide  $\frac{2}{3}$  of  $\frac{1}{7}$  by  $\frac{1}{3}$  of  $\frac{2}{3}$ .

$$\frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$$

$$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

$$\frac{2}{21} \times \frac{9}{2} = \frac{3}{7} = 18\frac{1}{2}, \text{ Ans.}$$

Or,

$$\frac{2}{3} \times \frac{1}{7} \times \frac{3}{2} \times \frac{3}{2} = 18\frac{1}{2}.$$

**ANALYSIS.**—The dividend, reduced to a simple fraction, is  $\frac{2}{21}$ ; the divisor, reduced in like manner, is  $\frac{2}{9}$ ; and  $\frac{2}{21}$  divided by  $\frac{2}{9}$  is  $18\frac{1}{2}$ , the quotient required. Or, we may apply the general rule directly by inverting both factors of the divisor.

The second method of solution has the twofold advantages of giving the answer by a single operation, and of affording greater facility for cancellation.

- |                                                                                                                   |                               |
|-------------------------------------------------------------------------------------------------------------------|-------------------------------|
| 32. Divide $\frac{1}{12}$ of $\frac{1}{2}$ by $\frac{1}{3}$ of $\frac{1}{12}$ .                                   | <i>Ans.</i> $1\frac{1}{2}$ .  |
| 33. Divide $\frac{1}{2}$ of $\frac{1}{12}$ by $\frac{1}{12}$ of $\frac{1}{12}$ .                                  |                               |
| 34. Divide $\frac{1}{2}$ of $\frac{1}{12}$ by $\frac{1}{3}$ of $\frac{1}{2}$ .                                    |                               |
| 35. Divide $\frac{1}{2}$ of $7\frac{1}{2}$ by $\frac{1}{12}$ of $17\frac{1}{2}$ .                                 | <i>Ans.</i> $3\frac{1}{2}$ .  |
| 36. Divide $\frac{1}{2}$ of $4$ by $\frac{1}{3}$ of $3\frac{1}{2}$ .                                              |                               |
| 37. Divide $\frac{1}{2}$ of $1\frac{1}{2}$ of $\frac{1}{2}$ by $\frac{1}{3}$ of $1\frac{1}{2}$ of $\frac{1}{2}$ . | <i>Ans.</i> $\frac{1}{2}$ .   |
| 38. Divide $\frac{1}{2}$ of $5\frac{1}{2}$ of $7$ by $\frac{1}{3}$ of $3\frac{1}{2}$ .                            | <i>Ans.</i> $11\frac{1}{2}$ . |
| 39. Divide $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ by $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{12}$ .  |                               |
| 40. Divide $\frac{1}{2}$ of $\frac{1}{2}$ of $36$ by $1\frac{1}{2}$ of $\frac{1}{2}$ .                            |                               |
41. What is the value of  $\frac{6\frac{1}{2}}{8\frac{1}{2}}$ ?

OPERATION.

$$\frac{6\frac{1}{2}}{8\frac{1}{2}} = \frac{5\frac{1}{2}}{3\frac{1}{2}} = \frac{56}{9} + \frac{26}{3} = \frac{56}{9} \times \frac{3}{13} = \frac{28}{39}, \text{ Ans.}$$

**Ans.**—This example is only another form for expressing division of fractions; it is called a *complex fraction*. We simply reduce the upper number or dividend to an improper fraction, and then

fraction, and the lower number, or divisor, to an improper fraction, and then divide as before.

- |                                                                |                              |
|----------------------------------------------------------------|------------------------------|
| 42. What is the value of $\frac{4\frac{1}{2}}{3}$ ?            | <i>Ans.</i> $6\frac{1}{2}$ . |
| 43. What is the value of $\frac{1\frac{1}{2}}{4\frac{1}{2}}$ ? | <i>Ans.</i> $2\frac{1}{2}$ . |
| 44. What is the value of $\frac{7\frac{1}{2}}{8}$ ?            |                              |

24 will con-  
ere are sec-  
nths. Now,  
68 times, it  
es as 168 will  
e, we divide  
ient by the  
on. Hence,  
e denomi-  
n.

terms of the  
ultiplication  
7 this process  
the divisor,  
arding 2 as  
enominator;  
ought to be,  
herefore the  
y 3; thus,  
the denomi-  
rator of the

ers to im-  
in multi-  
non denomi-  
ator of the

*Ans.*  $3\frac{1}{2}$ .

45. What is the value of  $\frac{3}{4}$  of  $\frac{1}{2}$ ? Ans. 1.
46. What is the value of  $\frac{4}{5}$  of  $3\frac{1}{2}$ ?
47. What is the value of  $\frac{8}{9}$  of  $\frac{1}{3}$ ?
48. What is the value of  $\frac{2}{3} \times \frac{1}{4}$ ?
49. Reduce  $\frac{6}{7}$  of  $\frac{3}{4}$  to its simplest form. Ans.  $\frac{11}{14}$ .
50. Reduce  $\frac{3}{7} \times 5\frac{1}{2} \times 3\frac{1}{2}$  to its simplest form.

## PRACTICAL PROBLEMS.

- If  $\frac{1}{4}$  of an acre of land sell for \$63, what will an acre sell for at the same rate? Ans. \$147.
- At \$ $\frac{3}{4}$  per bushel, how many bushels of onions can be bought for \$12? Ans. 16.
- How many times will  $16\frac{1}{2}$  gallons of vinegar fill a vessel that holds 3 gallons? Ans.  $5\frac{1}{3}$ .
- At  $\frac{1}{2}$  of a cent each, how many apples can be bought for  $8\frac{1}{2}$  cents? Ans. 11.
- If 15 pounds of raisins can be obtained for \$3 $\frac{1}{2}$ , what will 1 pound cost? Ans. \$0.21 $\frac{1}{3}$ .
- A butcher expended \$56 $\frac{1}{2}$  for sheep, giving \$1 $\frac{1}{2}$  per head; how many sheep did he buy? Ans. 47.
- At \$5 per yard of broadcloth, what part of a yard can be bought for  $\frac{1}{2}$  of a dollar? Ans.  $\frac{1}{10}$ .
- If I pay 6 $\frac{1}{2}$  cents for riding 1 mile, how many miles can I ride for 113 $\frac{1}{2}$  cents? Ans. 20.
- How many pounds of tea, at \$1 $\frac{1}{2}$  per pound, can be obtained for \$13 $\frac{1}{2}$ ? Ans. 12.
- If 9 men consume  $\frac{1}{2}$  of 9 $\frac{1}{2}$  pounds of meat in a day, how much does each man consume? Ans.  $\frac{1}{2}$  of a lb.
- A man bought 37 $\frac{1}{2}$  yards of calico for \$5.61, how much did it cost per yard? Ans. \$0.15.
- How many tons of coal, at \$5 $\frac{1}{2}$  per ton, can be bought for \$57?
- A horse eats  $\frac{1}{4}$  of a bushel of oats in a day, in how many days will he eat 16 $\frac{1}{2}$  bushels? Ans. 42.
- A merchant bought 97 sheep for \$100 $\frac{1}{2}$ , how much did he give per head? Ans. \$1.04.
- If a boy earn  $\frac{1}{4}$  of a dollar a day, how many days will it take him to earn \$9 $\frac{1}{2}$ ? Ans. 26.
- Peter paid \$543 $\frac{1}{2}$  for a farm, giving \$21 $\frac{1}{2}$  per acre; of how many acres did the farm consist? Ans. 25.
- If \$2 $\frac{1}{2}$  is paid for 5 $\frac{1}{2}$  pounds of grapes, how much is that per pound? Ans. \$0.50.

11  
per  
19  
20  
for \$  
21  
get f  
22  
9 ya  
23  
24  
woul  
25  
\$31  
26  
cover  
27  
each  
28  
chase  
29  
4 $\frac{1}{2}$  ya  
30  
each,  
did h  
GI  
10  
tions  
them  
10  
fracti  
Ex  
Great  
Least  
ANAR  
least co  
tors 11  
fish, t  
fish;  
\$1 on t  
107.

18. How many tons of hay can be purchased for \$119 $\frac{1}{10}$ , at \$9 $\frac{1}{2}$  per ton? *Ans.* 12 $\frac{7}{10}$ .
19. At \$1 $\frac{1}{2}$  per day, how many days must a man work for \$37 $\frac{7}{10}$ ? *Ans.* 2 $\frac{3}{10}$  gal.
20. At  $\frac{1}{2}$  of  $\frac{1}{4}$  of a dollar per gallon, how much beer can be bought for \$ $\frac{1}{10}$ ? *Ans.* 2 $\frac{3}{10}$  gal.
21. If 2 $\frac{1}{2}$  apples are worth 3 $\frac{1}{2}$  cents, what part of an apple can you get for 1 cent? *Ans.*  $\frac{1}{5}$ .
22. If 2 yards of merino cost \$3 $\frac{1}{2}$ , how much less than \$17 will 9 yards cost? *Ans.* \$2 $\frac{1}{2}$ .
23. If 3 turkeys cost \$4 $\frac{1}{2}$ , how many can be bought for \$38 $\frac{1}{2}$ ? *Ans.* 7.
24. If 3 horses eat 3 $\frac{1}{2}$  bushels of oats in a day, how many horses would 8 $\frac{1}{2}$  bushels supply for the same time? *Ans.* 2 reams.
25. A young man, having \$10, gave  $\frac{2}{3}$  of his money for paper at \$3 $\frac{1}{2}$  per ream; how much did he buy? *Ans.* 2 reams.
26. How many feet of carpet 2 $\frac{1}{2}$  feet in width will be required to cover a floor 1 $\frac{1}{2}$  feet in length and 10 $\frac{1}{2}$  feet in width? *Ans.* 8 $\frac{1}{2}$  gal. of wine, if each bottle will hold  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$  of 1 gal.?
27. How many bottles will be required to hold 8 $\frac{1}{2}$  gal. of wine, if each bottle will hold  $\frac{1}{2}$  of  $\frac{1}{2}$  of  $\frac{1}{2}$  of 1 gal.?
28. If 5 barrels of flour cost \$48 $\frac{1}{2}$ , how many barrels can be purchased for \$263 $\frac{1}{2}$ ? *Ans.* 31.
29. How much more than 8 $\frac{1}{2}$  yards of tape, at 4 cts. a yard, will 4 $\frac{1}{2}$  yards of calico cost at 11 cts. a yard? *Ans.* 27.
30. A farmer gave \$46 for some calves, 5 of which he sold for \$5 $\frac{1}{2}$  each, and traded the rest for 5 $\frac{1}{2}$  bbls. of flour, at \$4 a bbl.; how much did he gain? *Ans.* \$4.

## GREATEST COMMON DIVISOR OF FRACTIONS.

**167.** The Greatest Common Divisor of two or more fractions is the greatest number which will exactly divide each of them, giving a whole number for a quotient.

**168.** To find the greatest common divisor of two or more fractions.

*Ex.* What is the greatest common divisor of  $3\frac{1}{2}$ ,  $1\frac{1}{4}$ , and  $\frac{1}{2}$ ?

## OPERATION.

$$3\frac{1}{2}, 1\frac{1}{4}, \frac{1}{2} = \frac{7}{2}, \frac{5}{4}, \frac{1}{2} = \frac{14}{4}, \frac{5}{4}, \frac{2}{4}$$

Greatest common divisor of the numerators = 4 } Greatest common  
Least common denominator of the fractions = 35 } divisor required.

**ANALYSIS.**—Having reduced the fractions to equivalent fractions having the least common denominator, we find the greatest common divisor of the numerators 14, 5, and 2 to be 1. Now, since the 14, 5, and 2 represent thirty-fifths, their greatest common divisor is not 1, a whole number, but 4 thirty-fifths; therefore we write the 4 over the least common denominator, 35, and have  $\frac{4}{35}$  as the answer.

**167.** What is the greatest common divisor of fractions?

**169. RULE.**—Reduce the fractions, if necessary, to their least common denominator. The greatest common divisor of the numerators, written over the least common denominator, will give the greatest common divisor required.

EXAMPLES FOR PRACTICE.

Required the greatest common divisor of

- |                                                                        |                       |                                                             |                       |
|------------------------------------------------------------------------|-----------------------|-------------------------------------------------------------|-----------------------|
| 1. $\frac{2}{3}$ , $\frac{1}{2}$ , and $\frac{1}{4}$ .                 | Ans. $\frac{2}{12}$ . | 5. $3\frac{1}{2}$ , $5\frac{1}{10}$ , and $2\frac{1}{15}$ . | Ans. $\frac{1}{30}$ . |
| 2. $\frac{3}{4}$ , $\frac{1}{2}$ , and $1\frac{1}{4}$ .                |                       | 6. $2\frac{1}{2}$ , $4\frac{1}{8}$ , and $5\frac{1}{4}$ .   |                       |
| 3. $\frac{1}{2}$ , $\frac{1}{3}$ , and $\frac{2}{5}$ .                 | Ans. $\frac{1}{30}$ . | 7. $8\frac{1}{2}$ , $12\frac{1}{4}$ , and $9\frac{1}{2}$ .  | Ans. $\frac{1}{4}$ .  |
| 4. $\frac{1}{2}$ , $\frac{1}{3}$ , $\frac{1}{4}$ , and $\frac{1}{5}$ . |                       | 8. $2\frac{1}{2}$ , $4\frac{1}{10}$ , and $2\frac{1}{5}$ .  |                       |

LEAST COMMON MULTIPLE OF FRACTIONS.

**170.** The Least Common Multiple of two or more fractions is the least number which can be exactly divided by each of them, giving a whole number for a quotient.

**171.** To find the least common multiple of two or more fractions.

Ex. What is the least common multiple of  $7\frac{1}{2}$ ,  $5\frac{1}{4}$ , and  $3\frac{1}{8}$ ?

OPERATION.

$$7\frac{1}{2}, 5\frac{1}{4}, 3\frac{1}{8} = 9\frac{3}{8}, 5\frac{2}{4}, 3\frac{1}{8}.$$

Least common mult. of the numer. = 63  
 Greatest com. div. of the denom. =  $\frac{1}{4}$  } Least common multip. required.

**ANALYSIS.**—Having reduced the fractions to their simplest form, we find the least common multiple of the numerators, 63, 21, and 63, to be 63. Now, since the 63, 21, and 63 are, from the nature of a fraction, dividends, of which their respective denominators, 8, 4, and 16, are the divisors (118), the least common multiple of the fractions is not 63, a whole number, but so many fractional parts of the greatest common divisor of the denominators. This common divisor we find to be 4, which, written as the denominator of the 63, gives  $9\frac{3}{8} = 15\frac{3}{8}$  as the least number that can be exactly divided by the given fractions.

**172. RULE.**—Reduce the fractions, if necessary, to their lowest terms. Then find the least common multiple of the numerators, which, written over the greatest common divisor of the denominators, will give the least common multiple required. Or,

Reduce the fractions, if necessary, to their least common denominator. Then find the least common multiple of the numerators, and write it over the least common denominator.

169. What is the rule for finding the greatest common divisor of fractions?—170. What is the least common multiple of fractions?—172. What is the rule for finding the least common multiple of fractions?

## EXAMPLES FOR PRACTICE.

Required the least common multiple of

- |                                                                           |                        |                                                                                  |                         |
|---------------------------------------------------------------------------|------------------------|----------------------------------------------------------------------------------|-------------------------|
| 1. $\frac{2}{3}$ , $\frac{3}{4}$ , and $2\frac{1}{2}$ .                   | Ans. 8 $\frac{1}{2}$ . | 5. $5\frac{1}{2}$ , $\frac{2}{3}$ , and $7\frac{1}{2}$ .                         | Ans. 10 $\frac{1}{2}$ . |
| 2. $\frac{1}{2}$ , $\frac{3}{8}$ , and $\frac{1}{12}$ .                   |                        | 6. $1\frac{1}{2}$ , $\frac{3}{4}$ , and $2\frac{1}{2}$ .                         |                         |
| 3. $\frac{1}{3}$ , $\frac{2}{5}$ , and $\frac{1}{2}$ .                    | Ans. 24.               | 7. $\frac{1}{3}$ , $\frac{2}{3}$ , and $\frac{1}{4}$ .                           | Ans. 3 $\frac{1}{2}$ .  |
| 4. $\frac{1}{10}$ , $\frac{1}{15}$ , $\frac{1}{20}$ , and $\frac{1}{3}$ . |                        | 8. $\frac{1}{10}$ , $5$ , $6\frac{1}{2}$ , $\frac{1}{15}$ , and $2\frac{1}{2}$ . |                         |

## PRACTICE, OR ANALYSIS BY ALIQUOT PARTS.

**173.** An **Aliquot Part** of any number or quantity is such a part as will exactly divide that number or quantity; thus, 2, 3, 4, and 6 are aliquot parts of 12.

*NOTE.*—An aliquot part may be a whole or a mixed number, while a factor must be a whole number.

## ALIQUOT PARTS OF ONE DOLLAR.

|                                                     |                                                     |
|-----------------------------------------------------|-----------------------------------------------------|
| 50 cents = $\frac{1}{2}$ of 1 dollar.               | 12 $\frac{1}{2}$ cents = $\frac{1}{8}$ of 1 dollar. |
| 33 $\frac{1}{3}$ cents = $\frac{1}{3}$ of 1 dollar. | 10 cents = $\frac{1}{10}$ of 1 dollar.              |
| 25 cents = $\frac{1}{4}$ of 1 dollar.               | 8 $\frac{1}{2}$ cents = $\frac{1}{12}$ of 1 dollar. |
| 20 cents = $\frac{1}{5}$ of 1 dollar.               | 6 $\frac{1}{2}$ cents = $\frac{1}{15}$ of 1 dollar. |
| 16 $\frac{2}{3}$ cents = $\frac{1}{6}$ of 1 dollar. | 5 cents = $\frac{1}{20}$ of 1 dollar.               |

**174.** To find the cost of any number or quantity, when the price of a unit is an aliquot part of one dollar.

*Ex.* At 12 $\frac{1}{2}$  cents a yard, what will 416 yards of muslin cost?

OPERATION.

$$\begin{array}{r} 8 \overline{) 416} \\ \underline{64} \phantom{0} \\ 800 \end{array}$$

Ans. \$52

*ANALYSIS.*—If the price were \$1 a yard, the cost would be as many dollars as there are yards. But since the price is  $\frac{1}{8}$  of a dollar a yard, the whole cost will be  $\frac{1}{8}$  as many dollars as there are yards; or,  $\frac{1}{8}$  of 416 = 416  $\div$  8 = \$52. Hence, the

**175. RULE.**—Take such a fractional part of the given number as the price is part of one dollar.

## EXAMPLES FOR PRACTICE.

1. What will be the cost of 724 pounds of coffee at 33 $\frac{1}{3}$  cts. a pound?  
Ans. \$241.33 $\frac{1}{3}$ .
2. What cost 376 yards of calico, at 25 cts. per yd.?
3. At 6 $\frac{1}{2}$  cts. a pound, what will 1056 lbs. of nails cost? A. \$66.
4. At 8 $\frac{1}{2}$  cts. a dozen, what cost 387 doz. of eggs?
5. What cost 384 yards of cloth, at \$4.33 $\frac{1}{3}$  per yd.? Ans. \$1664.
6. At \$3.16 $\frac{2}{3}$  each, what will 93 hats cost?

**173.** What is an aliquot part of a number?—**175.** What is the rule for finding the cost of any number or quantity, when the price of a unit is an aliquot part of a dollar?

## QUESTIONS

INVOLVING THE RELATION OF PRICE, COST, AND QUANTITY.

**176. CASE I.**—The price and the quantity being given, to find the cost.

ANALYSIS.—The cost of 5 units must be 5 times the price of 1 unit; of 6 units, 6 times the price of 1 unit; of  $\frac{1}{2}$  of a unit,  $\frac{1}{2}$  times the price of 1 unit, etc. Hence, the

**177. RULE.**—*Multiply the price of ONE by the quantity.*

**178. CASE II.**—The cost and the quantity being given, to find the price.

ANALYSIS.—By Case I, the cost is the product of the price multiplied by the quantity. Now, having the cost, which is a product, and the quantity, which is one of two factors, we have the product and one of two factors given, to find the other factor. Hence, the

**179. RULE.**—*Divide the cost by the quantity.*

**180. CASE III.**—The price and the cost being given, to find the quantity.

ANALYSIS.—Reasoning as in Case II, we find that the cost is the product of two factors, and the price is one of the factors. Hence, the

**181. RULE.**—*Divide the cost by the price.*

**182. CASE IV.**—The quantity, and the price of 100 or 1000, being given, to find the cost.

ANALYSIS.—If the price of 100 units be multiplied by the number of units in a given quantity, the product will be 100 times the required result, because the multiplier used is 100 times the true multiplier. For a similar reason, it will be the same if the given price be 1000 units. The true value will be obtained either by dividing the product by 100 or 1000, as the case may be, or, by reducing the given quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand. Hence, the

**183. RULE.—I.** *Reduce the given quantity to hundreds and decimals of a hundred, or to thousands and decimals of a thousand.*

**II.** *Multiply the price by the quantity, and point off in the result as in multiplication of decimals.*

**184. CASE V.**—To find the cost of articles sold by the ton of 2000 pounds.

ANALYSIS.—If the price of 1 ton or 2000 pounds be divided by 2, the quotient will be the price of  $\frac{1}{2}$  ton or 1000 pounds. We then have the quantity and the price of 1000 to find the cost. Hence, the

**177.** What is the rule for finding the cost of articles, the price and the quantity being given?—**179.** For finding the price of articles, the cost and the quantity being given?—**181.** For finding the quantity, the price and the cost being given?—**183.** For finding the cost of articles, the quantity, or the price of 100 or 1000, being given?

**185. RULE.**—*Divide the price of 1 ton by 2, and multiply the quotient by the number of pounds expressed as thousandths.*

**EXAMPLES FOR PRACTICE IN THE PRECEDING CASES.**

1. At \$7.50 per barrel, how many barrels of flour can be obtained for \$217.50 ? *Ans.* 29 barrels.
2. If 1 yard of calico cost 23 cents, what will  $31\frac{1}{2}$  yards cost ?
3. What cost 15 tubs of butter, each containing  $70\frac{1}{2}$  lbs., at  $\$1\frac{1}{2}$  a pound ?
4. What is the freight on 1244 $\frac{1}{2}$  pounds from Montreal to Quebec, at \$0.85 per 100 lbs. ? *Ans.* \$10.578 +.
5. If board for a family be \$342.18 $\frac{1}{2}$  for 1 year, how much is it per day ? *Ans.* \$0.93 $\frac{1}{2}$ .
6. How many dozen of eggs can be bought for \$9.24, at 10 $\frac{1}{2}$  cts. a dozen ? *Ans.* 88.
7. What will 3921 feet of pine boards cost, at \$17.25 per 1000 ?
8. What is the value of 210 kegs of nails, each weighing 162 $\frac{1}{2}$  lbs., at \$17 $\frac{1}{2}$  a ton ?
9. At \$1 $\frac{1}{2}$  a bushel, how many bushels of oats can be bought for \$113.06 $\frac{1}{2}$  ? *Ans.* 75 $\frac{1}{2}$  bushels.
10. At 5 cents a pound, how many barrels of codfish, each containing 90 lbs., can be purchased for \$94.50 ? *Ans.* 21 $\frac{1}{2}$  bbls.
11. What will be the cost of 1620 apple trees at \$16 $\frac{1}{2}$  per hundred ?
12. At \$7 $\frac{1}{2}$  cts. a bushel, what will  $\frac{1}{2}$  of 456 bushels of potatoes cost ?
13. How much must be paid for 486 feet of boards, at \$20.25 per 1000; 787 $\frac{1}{2}$  feet of scantling, at \$2.87 $\frac{1}{2}$  per 100; and 4378 feet of lath, at \$7.50 per 1000 ? *Ans.* \$65.317 $\frac{1}{2}$ .
14. What will be the cost of 2244 pounds of Paris plaster, at \$3.87 $\frac{1}{2}$  per ton ? *Ans.* \$8.416 $\frac{1}{2}$ .
15. If 32 $\frac{1}{2}$  barrels of Montreal apples cost \$97.50, what is the price per barrel ? *Ans.* \$3.
16. How many acres of land can be bought for \$2117.18 $\frac{1}{2}$ , at \$5 $\frac{1}{2}$  an acre ? *Ans.* 376 $\frac{1}{2}$ .
17. At 57 $\frac{1}{2}$  cts. per bushel, how many barrels of potatoes, each containing 24 bushels, can be purchased for \$60.5217 ? *Ans.* 54.
18. If 32 $\frac{1}{2}$  of a barrel of oil be worth \$6.42, what is a barrel worth ?
19. What must be paid for 323 lbs. of meat, at \$4 $\frac{1}{2}$  per hundred pounds ? *Ans.* \$24.18 $\frac{1}{2}$ .
20. What cost 1000 lbs. of hay, at \$12.75 a ton, and 1368 lbs. of mill meal at \$15.25 a ton ? *Ans.* \$17.487.
21. What will be the cost of 554 feet of boards, at \$15 $\frac{1}{2}$  per 1000; 1344 feet of siding, at \$1.82 $\frac{1}{2}$  per 100; and 2216 bricks, at \$4 $\frac{1}{2}$  per 1000 ? *Ans.* \$41.39 $\frac{1}{2}$ .
22. A grocer bought 100 gallons of oil for \$145.50, and lost 12 gal. of it by leakage. He sold the remainder at \$1.70 per gallon; how much did he gain ?

186. What is the rule for finding the cost of articles by the ton of 2000 lbs.?

23. A lumber dealer bought 1000 feet of lumber at \$14.375 per 1000, and retailed it out at \$1.75 per 100; how much was his whole gain?  
*Ans.* \$332.03+

24. A load of plaster weighing 3360 pounds cost \$5.71 $\frac{1}{2}$ , how much will a ton cost?

25. If \$6.97 $\frac{1}{2}$  be paid for 0.93 of a hundred pounds of beef, how much will one hundred pounds cost?

26. A farmer exchanged 42 $\frac{1}{2}$  bushels of barley worth 37 $\frac{1}{2}$  cts. per bushel, and 679 $\frac{1}{2}$  lbs. of hay worth 75 cts. per hundred, for 18760 lbs. of plaster; how much was the plaster worth per ton?

27. If 42 yards of cassimere cost \$147, what will be the cost of 34 $\frac{1}{2}$  yards?  
*Ans.* \$121.80.

28. What is the value of 12 pieces of black cloth, each piece containing 27 $\frac{1}{2}$  yards, worth \$2 $\frac{1}{2}$  a yard?  
*Ans.* \$954.50.

29. At \$ $\frac{1}{2}$  per bushel, how many bushels of wheat may be bought for \$18.90?  
*Ans.* 21 $\frac{1}{2}$ .

30. A farmer sold to a merchant three loads of hay weighing respectively 2739, 2217, and 2881 $\frac{1}{2}$  lbs., at \$8.80 per ton, and 421 $\frac{1}{2}$  lbs. of pork, at \$5.25 per hundred. He received in exchange 46 $\frac{1}{2}$  yards of muslin at \$0.09, 9 $\frac{1}{2}$  yards of carpet at \$4.50, and the balance in money; how much money did he receive?

*Let the pupils make out, in proper form, as the case may be, the following:*

1. Sold by R. S. Graham, Montreal, to E. Dudley, as follows: 1870, Jan. 3, 109 $\frac{1}{2}$  yds. calico, at 18 $\frac{1}{2}$  cts.; Feb. 11, 430 yds. muslin, at 15 $\frac{1}{2}$  cts.; March 2, 37 $\frac{1}{2}$  yds. sheeting, at 23 $\frac{1}{2}$  cts.; May 16, 75 $\frac{1}{2}$  yds. Irish linen, at 4 $\frac{1}{2}$  cts.; 49 $\frac{1}{2}$  yds. lace, at 78 $\frac{1}{2}$  cts.

Footing of the bill, \$161.507 $\frac{1}{2}$ .

2. T. H. Clark bought of F. Larocé & Co., Quebec, 1870, June 10, 73 $\frac{1}{2}$  gal. Irish whisky, at 86 cts.; 108 $\frac{1}{2}$  gal. fine old rum, at \$2.12 $\frac{1}{2}$ ; 67 $\frac{1}{2}$  gal. Holland gin, at \$1.45; Aug. 14, 69 $\frac{1}{2}$  gal. old cognac, at \$2.67 $\frac{1}{2}$ ; 107 gal. brandy, at \$1.37 $\frac{1}{2}$ ; Sept. 7, 201 $\frac{1}{2}$  gal. Champagne, at \$1.30. T. H. Clark gave in part payment, Sept. 11, 4 chests green tea, each 67 $\frac{1}{2}$  lbs., at 56 cts. per lb. What balance was due to F. Larocé & Co., Sept. 12, when the bill was made out?

3. J. N. Webster, butcher, Kingston, sold to A. O'Rourke, Montreal, 1870: A fillet of veal, weight 158 lbs., at 16 $\frac{1}{2}$  cts.; a loin of same weight 7 $\frac{1}{2}$  lbs., at 17 $\frac{1}{2}$  cts.; a leg of mutton, weight 138 lbs., at 21 $\frac{1}{2}$  cts.; a leg of pork, weight 161 lbs., at 9 $\frac{1}{2}$  cts.; a pig, weight 75 lbs., at 12 $\frac{1}{2}$  cts.; a buttock of beef, weight 87 $\frac{1}{2}$  lbs., at 14 $\frac{1}{2}$  cts.

Footing of the bill, \$344.75.

4. H. Lemay & Co. bought of Messrs. J. O'Rourke & Co., Montreal, Sept. 3, 1870: 123 $\frac{1}{2}$  lbs. raisins, at \$1.15; 24 $\frac{1}{2}$  lbs. currants, at \$14.10; 107 $\frac{1}{2}$  lbs. rhubarb, at \$2.50; 180 $\frac{1}{2}$  lbs. muslin, at 18 $\frac{1}{2}$  cts.; 25 $\frac{1}{2}$  lbs. madder, at 97 $\frac{1}{2}$  cts.

5. Sold by S. H. Porter, Ottawa, to Miss E. Walker, Aug. 26, 1870: 27 $\frac{1}{2}$  yds. Decaden lace, at \$5.00; 19 $\frac{1}{2}$  yds. French lace, at \$1.93 $\frac{1}{2}$ ; 83 $\frac{1}{2}$  yds. gauze, at 45 $\frac{1}{2}$  cts.; 56 $\frac{1}{2}$  yds. muslin, at 18 $\frac{1}{2}$  cts.; 50 pair kid gloves, at 89 cts.; 25 $\frac{1}{2}$  dozen stockings, at \$5.12 $\frac{1}{2}$ .

Footing of the bill, \$355.36.

6. Invoiced, per Canadian Express, by S. Blanchard & Co., Quebec, to J. Butler, Kingston, July 6, 1870: 25 sacks tares, No. 3, each 2½ bush., at 54 cts. per bush.; 32 sacks pease, No. 4, each 3 bush., at 87½ cts. per bush.; 20 sacks oats, No. 6, each 3½ bush., at 56½ cts. per bush.; 8 sacks malt, No. 5, each 2½ bush., at \$1.37½ per bush.; 16 sacks beans, No. 7, each 2½ bush., at 86 cts. per bush. Insurance and cartage, \$3.40. Amount of Invoice, \$221.56.

7. T. McCullen & Co., wholesale merchants, Halifax, sold to Lenoir & O'Neil, Montreal, as follows: May 19, 1870, 85 pieces Norwich crapes, at \$8.32; 102 pieces Liverpool cottons, at \$7.63; June 5, 175½ yds. Antwerp sheeting, at 24½ cts.; 698½ yds. Amiens velvet, at \$1.80; Aug. 8, 375½ yds. Yorkshire drab, at 65 cts.; 872½ yds. Abbeville merino, at \$1.12½. On this are the following credits: July 10, by 18 bbls. Canadian flour, at \$7.50; Aug. 12, by draft, at 3 day's sight, for \$500. What balance was due T. McC. & Co., Sept. 3, when the account was settled? Ans. \$3377.01.

8. C. N. Stonehouse of Montreal, sold to Mrs. F. Stephens, April 6, 1870, and Ed. Noonan, his clerk, collected the amount of the bill: 39½ yds. cambric, at 81½ cts.; 47½ yds. shalloon, at 32 cts.; 27½ yds. druggat, at 45½ cts.; 19½ yds. calico, at 1½ cts.; 4½ yds. ohints, at 90½ cts.; 34½ yds. callimanco, at 37½ cts. Amt. of the bill, \$93.02½.

9. L. Rogers & Son, Quebec, sold to Messrs. O. Cooper & Co., Sorci, as follows: 1870, April 5, 12½ doz. palm sack, at \$9.42; May 12, Port wine, red, 66½ gal., at \$1.68; 42½ gal. Claret, at \$2.17½; June 10, Lisbon wine, white, 31½ gal., at 45 cts.; 32½ gal. Rhenish wine, at 56½ cts.; July 8, 26½ gal. Sherry wine, at \$1.32. Received in part payment, July 9, 160 bush. oats, at 57½ cts., and \$50 in cash. What was the balance due to L. R. & Son, July 10? Ans. \$240.63.

10. T. J. Binnet, bought of Tessier & Gray, Montreal, as follows: 1870, June 18, 43 pieces muslin, each 37½ yds., at \$2.15; 7½ pieces chints, each 4½ yds., at 32½ cts.; July 12, 44 pieces Holland linen, each 2½ yds., at 57½ cts.; 10½ pieces serge, each 19½ yds., at 48 cts.; Aug. 2, 176½ yds. Rosal cottons, at 17 cts.; 987½ yds. Lowell cotton, at 18½ cts. What was the amount due, Aug. 3, to T. & G? Ans. \$1335.47.

## MISCELLANEOUS PROBLEMS.

1. What will be the cost of 104 pounds of honey, at 16½ cts. per pound? Ans. \$17.06.

2. At \$44 per yard, how many yards may be bought for \$114? Ans. 2.59.

3. Reduce 1/2 to a mixed number. Ans. 1 1/2.

4. Reduce 1/2, 1/3, and 1/4 to equivalent fractions having a common denominator. Ans. 1/2 = 2/4, 1/3 = 1.33, 1/4 = 0.25.

5. The sum of two numbers is 2378, and their difference 411; what is the greater number? Ans. 2424.5.

6. At 70 cts. per bushel, how many bushels of barley may be bought for 167 cts. 7/10? Ans. 2.39 bush.

7. John has 25 cents, James has 25 cents, and Sam has 25 cents; how much more has John than James? Ans. 0.

8. How much more has John than James? Ans. 0.

8. What will  $15\frac{1}{2}$  cords of wood cost at  $\frac{1}{2}$  of \$9 $\frac{1}{2}$  per cord?
9. How many pounds in 4 bags, the first containing 360 $\frac{1}{2}$ , the second 580 $\frac{1}{2}$ , the third 296 $\frac{1}{2}$ , and the fourth 375 $\frac{1}{2}$ ? *Ans.* 1614 $\frac{1}{2}$ .
10. Andrew spent  $\frac{2}{3}$ ,  $\frac{1}{4}$ , and  $\frac{1}{5}$  of his money, and had \$54.50 left; how much had he at first? *Ans.* \$384.70 $\frac{1}{2}$ .
11. A servant had  $\frac{1}{3}$  of his savings in one bank,  $\frac{1}{4}$  in another, and the remainder, which was \$77, in a third bank; how much money had he? *Ans.* \$140.
12. Leo had  $\frac{2}{3}$  of  $\frac{2}{3}$  of 7 $\frac{1}{2}$  times \$7862, and paid  $\frac{1}{4}$  of  $\frac{1}{4}$  of it for a farm; how much had he remaining? *Ans.* \$35379.
13. In 6 hogheads of sugar containing, respectively, 945 $\frac{1}{2}$  lbs., 1054 $\frac{1}{2}$  lbs., 963 $\frac{1}{2}$  lbs., 901 $\frac{1}{2}$  lbs., and 899 $\frac{1}{2}$  lbs., how many pounds?
14. Henry bought a bale of cloth for \$36.37 $\frac{1}{2}$ ; he disposes of it for  $\frac{2}{3}$  of the cost, and by so doing, loses \$2 on a yard; required the number of yards in the bale. *Ans.* 18 $\frac{1}{2}$ .
15. What is the value of 3761 $\frac{1}{2}$  acres of land, at \$7 $\frac{1}{2}$  per acre?
16. If the transportation of 18 $\frac{1}{2}$  tons of iron costs \$48.15 $\frac{1}{2}$ , what is it per ton? *Ans.* \$2.62 $\frac{1}{2}$ .
17. A man purchased  $\frac{1}{4}$  of a yard of velvet at the rate of \$3.62 $\frac{1}{2}$  per yard; what did it cost him? *Ans.* \$3.17 $\frac{1}{2}$ .
18. Charles has 634 sheep, which is 94 more than  $\frac{2}{3}$  of 3 $\frac{1}{2}$  times David's number; how many has David? *Ans.* 243.
19. A man travels 4 miles in  $\frac{2}{3}$  of an hour, how far will he travel in  $1\frac{1}{2}$  hours at the same rate? *Ans.* 10 miles.
20. A merchant owned  $\frac{2}{3}$  of a ship, and sold  $\frac{1}{4}$  of  $\frac{1}{4}$  of his share for \$2400. At that rate, what was the whole worth? *Ans.* \$19200.
21. What will  $\frac{1}{4}$  of 10 $\frac{1}{2}$  tons of coal cost, at  $\frac{1}{4}$  of \$42 per ton?
22. If  $\frac{1}{4}$  of  $\frac{2}{3}$  of 3 $\frac{1}{2}$  be multiplied by  $\frac{1}{4}$  of itself, and the product divided by  $\frac{1}{4}$ , what will be the result? *Ans.* 144.
23. B and C own 3144 sheep; how many has each, if B has 1 $\frac{1}{2}$  times as many as C? *Ans.* B 1694, C 1450.
24. Edward has  $\frac{2}{3}$  of a dollar; he gives Louis  $\frac{1}{4}$  of this amount, and then divides the remainder equally among three boys; what part does each of the 3 boys receive? *Ans.*  $\frac{1}{12}$ .
25. James obtains from two fields 344 bushels of oats; if the first yielded  $\frac{2}{3}$  as much as the second, required the yield of each field?
26. How long will it take a man to travel 553 miles, provided he travels 3 $\frac{1}{2}$  miles per hour, and 9 $\frac{1}{2}$  hours per day? *Ans.* 14 days.
27. I bought 15 loads of wood, each containing 112 feet, cord measure, and divided it equally among 2 persons; what did each receive?
28. A tree, whose length was 136 feet, was broken into two pieces by falling;  $\frac{1}{4}$  of the length of the longer piece equalled  $\frac{1}{4}$  of the length of the shorter; What was the length of each piece? *Ans.* 73 and 63.
29. How many bushels of wheat worth 50 cts. a bushel, will pay for  $\frac{1}{4}$  of a barrel of flour at \$7 $\frac{1}{2}$  a barrel? *Ans.* 7 $\frac{1}{2}$  bush.
30. Bought  $\frac{2}{3}$  of  $\frac{1}{4}$  of 5 $\frac{1}{2}$  yards of blue cloth at the rate of \$2 $\frac{1}{2}$  per yard; what is the cost? *Ans.* \$5.62 $\frac{1}{2}$ .
31. If  $\frac{1}{4}$  of a barrel of oats costs \$5, how much will 3 tubs of the same containing  $\frac{2}{3}$  of a barrel, and the other  $\frac{1}{4}$  of a barrel?
32. If  $\frac{1}{4}$  of a gal. of porter is worth  $\frac{1}{4}$  of a gal. of ale, and ale is worth \$3 per gal., how many gal. of porter will \$20 buy? *Ans.* 24.

33. A certain quantity of apples is to be divided among 5 boys; William is to have  $\frac{1}{4}$ , John  $\frac{1}{5}$ , Peter  $\frac{1}{10}$ , Thomas  $\frac{1}{20}$ , and Paul the remainder, which is 24; what is the whole quantity to be divided?

34. What will be the cost of 7 $\frac{1}{2}$  yds. of calico, at 12 $\frac{1}{2}$  cts. per yd., and 12 $\frac{1}{2}$  yds. of muslin, at 18 $\frac{1}{2}$  cts. per yard? *Ans.* \$3.28 $\frac{1}{2}$ .

35. Philip owns  $\frac{1}{4}$  of a ship's cargo, valued at \$493000; Daniel owns  $\frac{1}{5}$  of the remainder; Joseph owns  $\frac{1}{10}$  as much as Philip and Daniel; and Henry owns the remainder. How much does each own? *Ans.* P, owns \$87000; D, \$210000; J, \$89100; and H, \$9000.

36. I own  $\frac{1}{3}$  of a steamboat, and sell  $\frac{1}{4}$  of my share to Owen for \$45000. What part of the steamboat have I left, and what is it worth at that rate? *Ans.*  $\frac{1}{4}$  left, worth \$15000.

37. If 4 $\frac{1}{2}$  pounds of maple sugar cost 34 $\frac{1}{2}$  cts., how much must be paid for 80 $\frac{1}{2}$  pounds?

38. A grocer bought 9 $\frac{1}{2}$  tons of coal at \$5 $\frac{1}{2}$  per ton, and paid for it in coffee at  $\frac{1}{4}$  of a dollar a pound; how many pounds were required to pay for the coal? *Ans.* 133 lbs.

39. I have \$800 and wish to lay out \$346 $\frac{1}{2}$  of it in sugar at 8 $\frac{1}{2}$  cts. a pound, and the remainder in tea at 52 $\frac{1}{2}$  cts. a pound; how many pounds of tea do I buy? *Ans.* 859 $\frac{1}{4}$  lbs.

40. A merchant expended \$840 for dry goods, and then had remaining only  $\frac{1}{15}$  as much money as he had at first; how much money had he at first?

41. A farmer has three fields; the first contains 73 $\frac{1}{4}$  acres, the second 88 $\frac{1}{4}$  acres, the third 139 $\frac{1}{4}$  acres. What is the largest-sized house-lots of the same extent into which the three fields can be divided, and also the number of lots? *Ans.* Size of each lot, 7 $\frac{1}{4}$  a.; 41 lots.

42. A man owning 135 $\frac{1}{2}$  acres of land, sold  $\frac{1}{4}$  of it, and gave  $\frac{1}{5}$  of it to his son; what was the value of the remainder, at \$57.80 per acre? *Ans.* \$2288.51 $\frac{1}{2}$ .

43. A merchant owns  $\frac{1}{3}$  of a factory worth \$48000. He sells  $\frac{1}{4}$  of his share to A, and  $\frac{1}{4}$  the remainder to B. How much does he receive from A and B respectively, and what part has he remaining? *Ans.* From A, \$25200; From B, \$8400; has left,  $\frac{1}{6}$ .

44. A drover bought 267 sheep, at \$2.25 per head; he afterward bought 348 at \$1.87 $\frac{1}{2}$  per head; then sold  $\frac{1}{4}$  of the whole number at \$1.75 per head, and the remainder at \$2.12 $\frac{1}{2}$ ; did he gain or lose, and how much? *Ans.* Lost \$35.87 $\frac{1}{2}$ .

45. A mother divided a basket of oranges among her three daughters; to the first she gave 12 oranges, to the second  $\frac{1}{3}$  of the whole, and to the third as much as to the other two; how many oranges did the third have? *Ans.* 48 oranges.

46. What is the smallest sum of money with which a farmer could purchase a number of sheep at \$2 $\frac{1}{2}$  each, a number of calves at \$4 $\frac{1}{2}$  each, and a number of yearlings at \$9 $\frac{1}{2}$  each? and how many of each could he buy with this money? *Ans.* \$112.50, 50 sheep, 25 calves, 12 yearlings.

47. In selling 46 $\frac{1}{2}$  yards of merino for \$50 $\frac{1}{2}$  I lost  $\frac{1}{4}$  of the buying price. What was the cost of one yard? *Ans.* \$1.318 $\frac{1}{2}$ .

48. Bought  $\frac{1}{4}$  of a yard of cotton for  $\frac{1}{3}$  of 20 cents, and gave in payment  $\frac{1}{5}$  of a yard of cloth worth \$3 a yd. Did I gain or lose by the bargain? *Ans.* \$1.318 $\frac{1}{2}$ .

49. The  $\frac{5}{11}$  of a farm are sown with corn; the  $\frac{2}{11}$  with barley; and the remainder, containing  $10\frac{1}{2}$  acres, planted with potatoes; how many acres does the farm contain? *Ans.*  $30\frac{27}{11}$  acres.

50. How many bushels of oats at  $62\frac{1}{2}$  cents per bushel are required to pay for 31 yards of cotton at  $8\frac{1}{2}$  cents a yd., and  $7\frac{1}{2}$  yards cloth, at \$2.75 per yard? *Ans.*  $37\frac{27}{11}$  bush.

51. If it required  $3\frac{1}{2}$  days for a mason and his son to make  $2\frac{1}{2}$  cubic yds. of masonry, how long will it take them to make a cubic yard?

52. If the  $\frac{3}{4}$  of a hundred bottles of Rhenish wine cost \$9.36; how much will 3482 bottles come to? *Ans.* \$543.192.

53. What will be the price of  $97\frac{1}{2}$  bushels of rye, if  $17\frac{1}{2}$  bushels of the same quality cost \$5 $\frac{1}{2}$ ? *Ans.* \$30.66 +.

54. A piece of silk velvet would bring \$210 were it  $\frac{1}{2}$  longer; knowing the price of a yard to be \$7.50, required the length of the whole piece? *Ans.* 24 yds.

55. A market woman sold the  $\frac{3}{4}$  of a basket of eggs, in adding 28 eggs to the remainder, the number she had at first would be augmented  $\frac{1}{2}$ : how many had she? *Ans.* 35 eggs.

56. A man has an income such, that if it were augmented by the price he paid for a mahogany writing desk, that is \$54, he could spend \$2.02 $\frac{1}{2}$  per day. What is his income? *Ans.* \$686.12 $\frac{1}{2}$ .

57. A weaver can weave a yard of linen in  $1\frac{1}{2}$  hours; how long will it take him to weave: 1st. 15 yds.; 2nd.  $2\frac{1}{2}$  yds.; 3rd.  $4\frac{1}{2}$  yds.; 4th.  $\frac{1}{2}$  of a yd.; 5th.  $\frac{1}{3}$  of a yd.? *Ans.* 1 $^{\circ}$  28 $\frac{1}{2}$  h.; 2 $^{\circ}$  6 $\frac{1}{2}$  h., etc.

58. What is the price of a lb. of sponge, if the difference between the  $\frac{1}{2}$  and the  $\frac{3}{4}$  of the sum paid for  $9\frac{1}{2}$  lbs. be 60 cts.? *Ans.* \$2.25.

59. In mixing 10 lbs. of bismuth with 6 lbs. of pewter and 4 lbs. of lead, we obtain an alloy which melts at the temperature of boiling water; required 1st. what quantity of each metal enters into the mixture of 2 lbs.; 2nd.  $1\frac{1}{2}$  lbs.; 3rd.  $3\frac{1}{2}$  lbs.; 4th.  $1\frac{1}{2}$  lbs.; 5th.  $27\frac{1}{2}$  lbs.; 6th. 1 lb.; 7th.  $1\frac{1}{2}$  lbs.; 8th.  $43\frac{1}{2}$  lbs.; 9th.  $144\frac{1}{2}$  lbs.; 10th.  $97\frac{1}{2}$  lbs.? *Ans.* 1 $^{\circ}$  1 lb. of bismuth,  $\frac{3}{4}$  lb. of pewter, and  $\frac{1}{2}$  lb. of lead; 2 $^{\circ}$   $\frac{3}{4}$  lb. of bismuth,  $\frac{1}{2}$  lb. of pewter, and  $\frac{1}{4}$  lb. of lead, etc.

60. A weaving machine makes  $13\frac{1}{2}$  yards of cloth per day; how many yards will it make, 1st. in 3 days; 2nd. in  $\frac{1}{2}$  of a day; 3rd. in  $4\frac{1}{2}$  days; 4th. in  $1\frac{1}{2}$  days; 5th. in  $32\frac{1}{2}$  days; 6th. in  $47\frac{1}{2}$  days; and 7th. in  $274\frac{1}{2}$  days? *Ans.* 1 $^{\circ}$   $41\frac{1}{2}$  yds.; 2 $^{\circ}$   $6\frac{1}{2}$  yds., etc.

61. It would require 1800 yards of cloth  $\frac{1}{2}$  yds. wide to make clothes for a regiment; but, on delivery, the cloth is found to be too narrow and the purveyor is obliged to buy 2000 yards: what is the width of the cloth? *Ans.*  $1\frac{1}{2}$  yds.

62. Paid \$2235.45 for 8 pieces of broadcloth of equal length and a remnant of  $15\frac{1}{2}$  yards: required the length of a piece knowing that one yard costs \$10.50? *Ans.* 24.7 yds.

63. The breadth of a painting is but the  $\frac{1}{7}$  of its height. If the breadth equal the  $\frac{1}{4}$  of  $2\frac{1}{2}$  yards, what is the height? *Ans.*  $2\frac{1}{2}$  yds.

64. A teacher of a select school has 60 pupils; 24 of them pay \$1.25 a month each, the  $\frac{1}{3}$  of the remainder, \$1.75, and the rest \$2.50. How much does he receive from his pupils in 8 months? *Ans.* \$840.

65. The difference of time between two watches is  $\frac{1}{2}$  of an hour; one of them gains  $4\frac{1}{2}$  minutes per day, while the other loses  $5\frac{1}{2}$  in the same time: in how many days will they again mark the same time?

66  
sold  
the v  
of 80  
67  
sells  
main  
that  
68  
he p  
betw  
per t  
69  
rate  
of  
\$19.2  
70  
chasi  
itself

18  
nate  
orang  
18  
of dif  
6 our

Nor  
increa  
decrea

18  
expre

18  
integr  
Thus  
being  
fracti

18  
and 18

186.  
nomina  
numbe

66. How many herrings were there in a barrel of which 243 were sold at one time, then the  $\frac{1}{2}$ , and if there still remain  $\frac{1}{3}$ . Required also the value of the whole barrel if the herrings were sold on an average of 80 cents per hundred? *Ans.* 1080 herrings; \$8.64.

67. A dealer in porcelain bought a certain quantity of plates; he sells  $\frac{1}{2}$  of them at 36 cents a doz.,  $\frac{1}{3}$  at 38 cents a doz., and the remainder at 41 cents. How many dozen of plates did he buy, knowing that he paid 31 cents per doz n and gained \$1.05 by his bargain?

68. A man having bought 84 bushels potatoes, forgets how much he paid per bushel; but remembers that there was a difference of \$4 between the  $\frac{1}{2}$  and the  $\frac{1}{3}$  of the sum laid out. How much did he pay per bushel? *Ans.* \$0.37 $\frac{1}{2}$ .

69. A dealer in furs sold a certain number of astrakhan skins at the rate of \$1.70 a piece. Now, in adding to the proceeds of his sales the  $\frac{1}{4}$  of the same proceeds less \$9.60, he could buy 25 fox skins at \$19.20. How many astrakhan skins did he sell?

70. A farmer sold 4 sheep and expended the  $\frac{2}{3}$  of the sum in purchasing 5 lambs; the remainder of his money is equal to  $\frac{1}{4}$  of the sum itself less \$2.00. Required the price of a sheep and of a lamb?

*Ans.* \$9, the price of a sheep; \$4, the price of a lamb.

### DENOMINATE NUMBERS.

**186. A Simple Number** is either an abstract or a denominate number of but one denomination; as 18, \$12, 40 rods, 15 oranges (8).

**187. A Compound Number** is a collection of concrete units of different denominations (10); as, 3 feet 4 inches, 5 pounds 6 ounces, 2 days 8 hours 24 minutes.

*NOTE.*—In simple numbers and decimals the scale is uniform, and the law of increase and decrease is by 10. In compound numbers, the scale of increase and decrease is varying.

**188. A Denominate Number** is any concrete number which expresses some particular kind or quantity; as 3 yards, 7 dollars.

**189. A Denominate Fraction** is a concrete fraction whose integral unit is one of a denomination of some compound number. Thus,  $\frac{2}{3}$  of a bushel is a denominate fraction, the integral unit being one bushel; so are  $\frac{1}{2}$  of a day,  $\frac{1}{4}$  of a yard, etc., denominate fractions.

**190. Denominate Numbers express Currencies, Weights, and Measures.**

---

186. What is a simple number?—187. A compound number?—188. A denominate number?—189. A denominate fraction?—190. What do denominate numbers express?

## CURRENCIES.

## I. DOMINION OF CANADA MONEY (77).

## II. OLD CANADIAN MONEY, OR HALIFAX CURRENCY.

TABLE.

|             |      |             |            |
|-------------|------|-------------|------------|
| 4 farthings | make | 1 penny,    | <i>d.</i>  |
| 12 pence    | "    | 1 shilling, | <i>s.</i>  |
| 5 shillings | "    | 1 dollar,   | <i>\$.</i> |
| 4 dollars   | "    | 1 pound,    | <i>£.</i>  |

|            |           |                 |
|------------|-----------|-----------------|
|            | <i>d.</i> | <i>qr.</i>      |
|            | <i>s.</i> | 1 = 4.          |
| <i>\$.</i> | 1 =       | 12 = 48.        |
| <i>£</i>   | 1 = 5 =   | 60 = 240.       |
|            | 1 = 4 =   | 20 = 240 = 960. |

NOTE:—Every 3*d.* of the old coinage is equal to 5 cents of the new:

## III. ENGLISH MONEY.

TABLE.

|                                           |      |                      |                         |
|-------------------------------------------|------|----------------------|-------------------------|
| 4 farthings ( <i>far.</i> or <i>qr.</i> ) | make | 1 penny              | <i>d.</i>               |
| 12 pence                                  | "    | 1 shilling           | <i>s.</i>               |
| 20 shillings                              | "    | 1 pound or sovereign | <i>£</i> or <i>sov.</i> |

|          |           |             |
|----------|-----------|-------------|
|          | <i>d.</i> | <i>far.</i> |
|          | <i>s.</i> | 1 = 4.      |
| <i>£</i> | 1 =       | 12 = 48.    |
|          | 1 = 20 =  | 240 = 960.  |

NOTE.—1: Farthings are generally expressed as fractions of a penny; thus, 1 *far.*, sometimes called one quarter, (*qr.*) =  $\frac{1}{4}$  *d.*; 3 *far.* =  $\frac{3}{4}$  *d.*

2. The old *s.*, the original abbreviation for shillings, was formerly written between shillings and pence, and *d.*, the abbreviation for pence, was omitted. Thus 3*s.* 6*d.* was written 3*s.* 6*d.* A straight line is now used in place of the *s.*, and shillings are written on the left of it, and pence on the right. Thus, 3*s.* 6*d.*, etc.

3. The present value of the sterling pound in the Dominion of Canada is \$4.8666, and hence the value of an English shilling is 24 cents.

4. The coins of England in general circulation are: the sovereign (= £1), and the half-sovereign (= 10*s.*), made of gold; the crown (= 5*s.*), the half-crown (= 2*s.* 6*d.*), the florin (= 2*s.*), the shilling, the six-pence, the four-pence, and the three-pence, made of silver; the penny, the half-penny, and the farthing, made of copper.

5. The standard gold coin of England is 11 parts pure gold and 1 part alloy. The standard silver coin is 37 parts  $\frac{11}{16}$  = .825 pure silver and 5 parts  $\frac{1}{16}$  = .075 copper. 24 pence, in copper coin, weigh a pound avoirdupois.

## IV. UNITED STATES MONEY (78).

## V. FRENCH MONEY.

**191. French Currency** is decimal. The *Franc* is the unit of the currency, and is equal in value to \$0.186 Dominion of Canada money.

## TABLE.

10 millimes make 1 centime.  
 10 centimes " 1 decime.  
 10 decimes " 1 franc.

COINS.— { Gold pieces of 100, 50, 20, 10, and 5 francs.  
 { Silver pieces of 5, 2, and 1 francs; 50 and 20 centimes.  
 { Copper or bronze pieces of 10, 5, 2, and 1 centimes.

DOMINION OF CANADA, ENGLISH, AND FRENCH MONEYS  
COMPARED.

| ENGLISH. | D. C.       | FRENCH.     | D. C.       |
|----------|-------------|-------------|-------------|
| 1d. =    | \$0.020275. | 1 millime = | \$0.000186. |
| 1s. =    | \$0.2433.   | 1 centime = | \$0.00186.  |
| £1 =     | \$4.866.    | 1 franc =   | \$0.186.    |

## WEIGHTS.

**192. Weight** is the measure of the quantity of matter a body contains, determined according to some fixed standard. Three scales of weight are used in the Dominion of Canada, Great Britain, and the United States, viz.: Troy, Apothecaries', and Avoirdupois.

## I. TROY WEIGHT.

**193. Troy Weight** is used in weighing gold, silver, and jewels; in philosophical experiments, &c.

## TABLE.

24 grains (*gr.*) make 1 pennyweight, *pwt.* or *dwt.*  
 20 pennyweights " 1 ounce, *oz.*  
 12 ounces " 1 pound, *lb.*

*pwt.* *gr.*  
*oz.* 1 = 24.  
*lb.* 1 = 20 = 480.  
 1 = 12 = 240 = 5760.

- NOTES.—1. Diamonds, etc., are weighed by carats, and fractions of a carat. A carat weighs 4 grains Troy weight.  
 2. In speaking of the purity of gold, a carat means  $\frac{1}{24}$  part; as, 18 carats fine, meaning  $\frac{18}{24}$  pure gold and  $\frac{6}{24}$  alloy.  
 3. A Troy pound is equal to 372.965 French grammes.

## II. APOTHECARIES' WEIGHT.

194. Apothecaries' Weight is used by apothecaries and physicians in mixing medicines; but medicines, in the quantity, are bought and sold by Avoirdupois weight.

TABLE.

|                          |      |            |          |            |       |            |
|--------------------------|------|------------|----------|------------|-------|------------|
| 20 grains ( <i>gr.</i> ) | make | 1          | scruple, | <i>sc.</i> | or    | 3.         |
| 3 scruples               | "    | 1          | dram,    | <i>dr.</i> | or    | 3.         |
| 8 drams                  | "    | 1          | ounce,   | <i>oz.</i> | or    | 3.         |
| 12 ounces                | "    | 1          | pound,   | <i>lb.</i> | or    | lb.        |
|                          |      |            |          | <i>sc.</i> |       | <i>gr.</i> |
|                          |      | <i>dr.</i> | 1        | =          | 3     | = 20       |
|                          |      | <i>oz.</i> | 1        | =          | 8     | = 60       |
| <i>lb.</i>               |      | 1          | =        | 12         | = 24  | = 480      |
| 1                        | =    | 12         | =        | 96         | = 288 | = 5760     |

## III. AVOIRDUPOIS WEIGHT.

195. Avoirdupois Weight is used for all the ordinary purposes of weighing.

TABLE.

|                         |      |                                  |                 |                         |                  |
|-------------------------|------|----------------------------------|-----------------|-------------------------|------------------|
| 16 drams ( <i>dr.</i> ) | make | 1                                | ounce,          | <i>oz.</i>              |                  |
| 16 ounces               | "    | 1                                | pound,          | <i>lb.</i>              |                  |
| 25 pounds               | "    | 1                                | quarter,        | <i>qr.</i>              |                  |
| 4 quarters              | "    | 1                                | hundred weight, | <i>cwt.</i>             |                  |
| 20 cwt., or 2000 lbs.,  | "    | 1                                | ton,            | <i>T.</i>               |                  |
|                         |      |                                  |                 | <i>oz.</i>              |                  |
|                         |      | <i>lb.</i>                       | 1               | =                       | 16.              |
|                         |      | <i>qr.</i>                       | 1               | =                       | 16 = 256.        |
|                         |      | <i>cwt.</i>                      | 1               | =                       | 25 = 400 = 6400. |
| <i>T.</i>               |      | 1                                | =               | 4 = 100 = 1600 = 25600. |                  |
| 1                       | =    | 20 = 80 = 2000 = 32000 = 512000. |                 |                         |                  |

NOTE.—The long or gross ton, hundred weight, and quarter, were formerly in common use; but they have now fallen into disuse among merchants in Canada. The Custom-Houses continue to use it. Farmers and others weigh still some few articles by the long ton.

## LONG TON TABLE.

|                     |      |   |                 |        |             |
|---------------------|------|---|-----------------|--------|-------------|
| 28 lbs.             | make | 1 | quarter,        | marked | <i>qr.</i>  |
| 4 qr. = 112 lbs.    | "    | 1 | hundred weight, | "      | <i>cwt.</i> |
| 20 cwt. = 2240 lbs. | "    | 1 | ton,            | "      | <i>T.</i>   |

## COMPARATIVE TABLE OF WEIGHTS.

|         |                |                |                |
|---------|----------------|----------------|----------------|
|         | Troy.          | Apothecaries.  | Avoirdupois.   |
| 1 pound | = 5760 grains, | = 5760 grains, | = 7000 grains. |
| 1 ounce | = 480 "        | = 480 "        | = 437.5 "      |
|         | 175 pounds,    | = 175 pounds,  | = 144 pounds.  |

**Notes.**—1. 7560 grains Troy make one pound maro weight, or old French weight. This pound contains 16 ounces; the ounce, 8 drams; and the dram, 72 grains of the maro weight. It is also divided into two maros of 8 ounces each. 100 lbs. maro weight make 108 lbs. Avoirdupois weight, or 131½ lbs. Troy; and, 16 lbs. maro weight make 21 lbs. Troy. It is proper to remark that the old French weights and measures are yet in general use and legally recognised in the Province of Quebec.

2. To reduce or change the English pound into French, multiply by 100 and divide by 108, and vice versa.

3. To change a quantity from one weight to its equivalent in another weight, reduce the given quantity to Troy grains, and then find their value in denominations of the weight required.

MEASURES.

**196. Measure** is that by which extent, dimension, capacity or amount is ascertained, determined according to some fixed standard. It may be properly divided into two classes.—Measures of Extension, and Measures of Capacity.

MEASURES OF EXTENSION.

**197. Extension** has three dimensions—length, breadth and thickness.

A **Line** has only one dimension—length.

A **Surface** or **Area** has two dimensions—length and breadth.

A **Solid** or **Body** has three dimensions—length, breadth, and thickness.

I. LINEAR OR LONG MEASURE.

**198. Linear or Long Measure**, is used in measuring lines or distances.

TABLE.

|                         |                                |            |
|-------------------------|--------------------------------|------------|
| 1 inch (in.)=           | 0.3363 French inch.            |            |
| 12 inches               | make 1 foot,                   |            |
| 3 feet                  | " 1 yard,                      | ft.        |
| 5½ yd., or 16½ ft.      | " 1 rod,                       | yd.        |
| 40 rods                 | " 1 furlong,                   | rd.        |
| 8 furlongs, or 320 rods | " 1 mile,                      | fur.       |
| 3 miles                 | " 1 league,                    | mi.        |
| 69½ miles (nearly)      | " 1 degree on the equator,     | lea.       |
| 360 degrees             | " 1 great circle of the earth. | deg. or °. |

|     |      |       |        |        |
|-----|------|-------|--------|--------|
|     |      | yd.   | ft.    | in.    |
|     |      | 1 =   | 3 =    | 12     |
|     | fur. | 1 =   | 5½ =   | 16½ =  |
| mi. | 1 =  | 40 =  | 220 =  | 660 =  |
| 1 = | 8 =  | 320 = | 1760 = | 5280 = |
|     |      |       |        | 63360  |

NOTES.—1. For the purpose of measuring cloth and other goods sold by the yard, the yard is divided into halves, fourths, eighths, and sixteenths. The old table of cloth measure is practically obsolete.

2. In Mariners' Measure, 12 lines make 1 inch; 4 inches, 1 hand; 4 feet, 1 fathom; 120 fathoms, 1 cable-length; 7½ cable-lengths, 1 mile;  $\frac{1}{360}$  of a degree of the circumference of the earth, 1 knot, or geographical mile, equal to 1½ statute miles.

3. The length of a degree of latitude varies, being 69.72 miles at the equator, 68.9 to 69.05 miles in middle latitudes, and 66.30 to 69.24 miles in the polar regions. The mean or average length is as stated in the table. A degree of longitude is greatest at the equator, where it is 69.16 miles, and it gradually decreases toward the poles, where it is 0.

### TABLE OF THE OLD FRENCH LINEAR MEASURES.

|                  |      |                   |
|------------------|------|-------------------|
| 1 line           | =    | 0.089 Engl. inch. |
| 12 lines (l.)    | make | 1 inch, in.       |
| 12 inches        | "    | 1 foot, ft.       |
| 6 feet           | "    | 1 toise, to.      |
| 3 toises         | "    | 1 perch, per.     |
| 10 perches       | "    | 1 arpent, arp.    |
| 84 arpents       | "    | 1 league, lea.    |
| 1000 French feet | "    | 1068 Engl. feet.  |

NOTES.—1. The French linear measures are in frequent use in the Province of Quebec.

2. The Engl. league = 15840 Engl. feet, and the French league of Canada = 15120 French ft., or 16148.16 Engl. ft.; the difference between the two = 308.16 Engl. ft., or 288½ French ft.

### SURVEYORS' LINEAR OR LONG MEASURE.

199. A Gunter's Chain, used by land surveyors, is 4 rods or 66 feet long, and consists of 100 links.

#### TABLE.

|             |      |            |      |
|-------------|------|------------|------|
| 7.92 inches | make | 1 link,    | l.   |
| 25 links    | "    | 1 rod,     | rd.  |
| 4 rods      | "    | 1 chain,   | ch.  |
| 10 chains   | "    | 1 furlong, | fur. |
| 8 furlongs  | "    | 1 mile,    | mi.  |

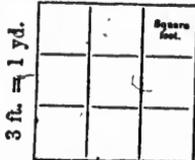
|     |      |     |    |     |      |   |      |   |       |   |        |
|-----|------|-----|----|-----|------|---|------|---|-------|---|--------|
|     |      |     | l. | =   | in.  |   |      |   |       |   |        |
|     |      | rd. | 1  | =   | 7.92 |   |      |   |       |   |        |
|     |      |     | 1  | =   | 198. |   |      |   |       |   |        |
|     |      | ch. | 1  | =   | 792. |   |      |   |       |   |        |
|     | fur. | 1   | =  | 100 | =    |   |      |   |       |   |        |
| mi. | 1    | =   | 10 | =   | 40   | = | 1000 | = | 7920. |   |        |
|     | 1    | =   | 8  | =   | 80   | = | 320  | = | 8000  | = | 63360. |

### II. SQUARE MEASURE.

200. A Square is a figure bounded by four equal lines, perpendicular to each other. It is the *Unit of Measure* for com-

puting areas or surfaces; as of land, boards, painting, plastering, paving, etc.

**201. An Area or Surface** is that which has length and breadth, without thickness.



The square in the margin is called *three feet square*, as it is three feet on each side. Each of the small squares, within the large square, represents 1 square foot, or 1 foot square. Since there are 3 square feet in each row, and 3 rows in the square, there are 3 lines 3 square feet, equal to 9 square feet in 3 feet square. Hence,

3 ft. = 1 yd.

*The area of a square or rectangle is found by multiplying its length by its width.*

NOTE.—From the above it will be observed that the difference between 3 feet square and 3 square feet is 6 square feet.

TABLE.

|                                  |                                    |
|----------------------------------|------------------------------------|
| 1 square inch ( <i>sq. in.</i> ) | = 0.8767 French inch.              |
| 144 square inches                | make 1 square foot, <i>sq. ft.</i> |
| 9 square feet                    | " 1 square yard, <i>sq. yd.</i>    |
| 30¼ square yards                 | " 1 square rod, <i>sq. rd.</i>     |
| 40 square rods                   | " 1 rood, <i>R.</i>                |
| 4 roods                          | " 1 acre, <i>A.</i>                |
| 640 acres                        | " 1 square mile, <i>sq. mi.</i>    |

|                |           |                |                |                |                |
|----------------|-----------|----------------|----------------|----------------|----------------|
|                |           | <i>sq. rd.</i> | <i>sq. yd.</i> | <i>sq. ft.</i> | <i>sq. in.</i> |
|                |           | 1 =            | 30¼ =          | 9 =            | 144 =          |
|                | <i>R.</i> | 1 =            | 1210 =         | 272¼ =         | 39204 =        |
|                | <i>A.</i> | 1 =            | 4840 =         | 10890 =        | 1568160 =      |
| <i>sq. mi.</i> | 1 =       | 4 =            | 160 =          | 43560 =        | 6272640 =      |
|                | 1 =       | 640 =          | 2560 =         | 102400 =       | 3097600 =      |
|                |           |                |                |                | 27878400 =     |
|                |           |                |                |                | 4014489600 =   |

TABLE OF THE OLD FRENCH SQUARE MEASURES.

|                                  |                                    |
|----------------------------------|------------------------------------|
| 1 square inch ( <i>sq. in.</i> ) | = 0.007921 Engl. foot.             |
| 144 square inches                | make 1 square foot, <i>sq. ft.</i> |
| 36 feet                          | " 1 square toise, <i>sq. to.</i>   |
| 9 toises                         | " 1 square perch, <i>sq. per.</i>  |
| 100 perches                      | " 1 square arpent, <i>sq. arp.</i> |
| 7056 arpents                     | " 1 square league, <i>sq. L.</i>   |

NOTE.—1. Artificers estimate their work as follows, viz.: glazing and stone-cutting, by the square yard; painting, plastering, paving, ceiling, and paper-hanging, by the square yards; flooring, partitioning, roofing, slating, filling, by the square of 100 square feet; brick-laying is estimated by the thousand bricks, by the square yard, and by the square of 100 square feet.

2. In estimating the painting of mouldings, cornices, etc., the measuring-line is carried into all the mouldings and cornices.
3. In estimating brick-laying by either the square yard or the square of 100 feet, the work is understood to be 12 inches or  $1\frac{1}{2}$  brick thick.
4. A thousand shingles are estimated to cover 1 square, being laid 5 inches to the weather.

## SURVEYORS' SQUARE MEASURE.

**202.** This measure is used by surveyors in computing the area or contents of land.

## TABLE.

|                                    |                   |                |
|------------------------------------|-------------------|----------------|
| 625 square links ( <i>sq. l.</i> ) | make 1 pole,      | <i>P.</i>      |
| 16 poles                           | " 1 square chain, | <i>sq. ch.</i> |
| 10 square chains                   | " 1 acre,         | <i>A.</i>      |
| 640 acres                          | " 1 square mile,  | <i>sq. mi.</i> |
| 36 square miles (6 miles square)   | " 1 township,     | <i>Tp.</i>     |

NOTES.—1. Canal and railroad engineers commonly use an engineer's chain, which consists of 100 links, each 1 foot long.

2. The contents of land are commonly estimated in square miles, acres, and hundredths; the denomination, *rood*, is rapidly going into disuse. A square mile of land is also called a *section*.

## III. CUBIC OR SOLID MEASURE.

**203.** A **Cube** is a solid, or body, bounded by six equal square sides or faces. The sides of the squares are called its *edges*.

**204.** **Cubic Measure** is used in estimating the contents of solids, or bodies; as timber, wood, stone, etc.

**205.** The **Contents**, or **Solidity**, of a volume, is the number of times it contains a given unit of measure.

The measurements for computing solidity are always taken in the denominations of linear measure.

If each of the sides of a cube is 1 foot, it is called a *cubic foot*. If each of the sides of a cube is 3 feet = 1 yard, it is called a *cubic yard*.



The annexed cube represents a cubic yard. Since each of the edges of a cubic yard is 3 feet, each of its faces will contain 3 times 3 equal to 9 square feet. If, from one face of this cube, we cut off a piece 1 foot in thickness, we evidently have 9 *solid feet*; and as the whole block is 3 feet thick, it must contain 3 times 9 = 27 solid feet. Hence,

To find the solid contents of a cube, multiply its length, breadth, and thickness together.

1728  
27  
40  
50  
16  
8  
128  
24

1728  
216  
1000  
1000

NOTES

the sp  
2. A  
and a

3. A

4. Jo  
etc., of  
mating  
walls o  
entire l

5. Ex  
dimensi  
comput  
yards,  
crown

6. In  
contents  
that wil  
by meas  
sawed t  
timber,

7. Sa  
solid by

8. A  
sea, and  
Avoirdun

20  
ity and  
nifies e  
20  
two cla

TABLE.

|                                   |                     |                             |
|-----------------------------------|---------------------|-----------------------------|
| 1728 cubic inches (cu. in.)       | make 1 cubic foot,  | cu. ft.                     |
| 27 cubic feet                     | " 1 cubic yard,     | cu. yd.                     |
| 40 cubic feet of round timber, or | } " 1 ton or load,  | T.                          |
| 50 " " " hewn "                   |                     | " 1 cord foot,              |
| 16 cubic feet                     | } " 1 cord of wood, | cd.                         |
| 8 cord feet, or                   |                     | " 1 { perch of stone } Pch. |
| 128 cubic feet                    | " 1 { or masonry. } |                             |
| 24½ cubic feet                    |                     |                             |

TABLE OF FRENCH MEASURES.

|                        |                                 |         |
|------------------------|---------------------------------|---------|
| 1728 cubic inches      | make 1 cubic foot,              | cu. ft. |
| 216 cubic feet         | " 1 " toise,                    | cu. to. |
| 1000 French cubic feet | " 1218. 186432 Engl. cub. feet. |         |
| 1000 cubic toises      | " 9745. 491456 cub. yd.         |         |

**NOTES.**—1. Railroad and transportation companies estimate light freight by the space it occupies in cubic feet; and heavy freight, by weight.

2. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains one cord; and a cord foot is one foot in length of such a pile.

3. A perch of stone or of masonry is 16½ feet long, 1½ feet wide, and 1 foot high.

4. Joiners, bricklayers, and masons, make an allowance for windows, doors, etc., of one half the openings or vacant spaces. Bricklayers and masons, in estimating their work by cubic measure, make no allowance for the corners of the walls of houses, cellars, etc., but estimate their work by the *girt*, that is, the entire length of the wall on the *outside*.

5. Engineers, in making estimates for excavations and embankments, take the dimensions with a line or measure divided into feet and decimals of a foot. The computations are made in feet and decimals, and the results are reduced to cubic yards. In civil engineering, the cubic yard is the unit to which estimates for excavations and embankments are finally reduced.

6. In scaling or measuring timber for shipping or freighting,  $\frac{1}{8}$  of the solid contents of round timber is deducted for waste in hewing or sawing. Thus, a log that will make 36 feet of hewn or sawed timber, actually contains 45 cubic feet by measurement; but its market value is only equal to 36 cubic feet of hewn or sawed timber. Hence, the cubic contents of 36 feet of round and 45 feet of hewn timber, as estimated for market, are identical.

7. Sawed timber, joists, planks, and scantlings are now generally bought and sold by what is called *board measure*.

8. A cubic foot of distilled water at the maximum density, at the level of the sea, and the barometer at 30 inches, is equal in weight to 62½ lbs. or 1000 os. Avoirdupois.

## MEASURES OF CAPACITY.

**206.** Measures of Capacity are all cubic measures, solidity and capacity being referred to different units. *Capacity* signifies extent of space.

**207.** Measures of capacity may be properly subdivided into two classes, *Measures of Liquids* and *Measures of Dry Substances*.

## I. LIQUID MEASURE.

**208. Liquid Measure**, also called **Wine Measure**, is now used for measuring all kinds of liquids.

TABLE.

|                         |      |             |             |
|-------------------------|------|-------------|-------------|
| 4 gills (gi.)           | make | 1 pint,     | <i>pt.</i>  |
| 2 pints                 | "    | 1 quart,    | <i>qt.</i>  |
| 4 quarts                | "    | 1 gallon,   | <i>gal.</i> |
| 31½ gallons             | "    | 1 barrel,   | <i>bb.</i>  |
| 2 barrels               | "    | 1 hogshead, | <i>hhd.</i> |
| 2 hogsheads             | "    | 1 pipe,     | <i>pi.</i>  |
| 2 pipes, or 4 hogsheads | "    | 1 tun,      | <i>tun.</i> |

|             |             |             |            |            |            |
|-------------|-------------|-------------|------------|------------|------------|
|             |             | <i>gal.</i> | <i>qt.</i> | <i>pt.</i> | <i>gi.</i> |
|             |             | 1 =         | 4 =        | 1 =        | 4 =        |
|             | <i>bb.</i>  | 1 =         | 4 =        | 8 =        | 32 =       |
|             | <i>hhd.</i> | 1 = 31½ =   | 126 =      | 252 =      | 1008 =     |
|             | <i>pi.</i>  | 1 = 2 =     | 63 =       | 252 =      | 504 =      |
| <i>tun.</i> | 1 = 2 =     | 4 = 126 =   | 504 =      | 1008 =     | 4032 =     |
|             | 1 = 2 =     | 4 = 8 =     | 252 =      | 1008 =     | 8064 =     |

Notes.—1. The English Imperial gallon contains 277.274 cubic inches or 10 lbs, Avoirdupois of pure distilled water, weighed at a temperature of 62° Fahrenheit, and under a barometer pressure of 30 inches.

2. In the United States the wine gallon contains 231 cubic inches, and the beer gallon 282 cubic inches. The gallon of England is therefore about equal to 1.2 gallons United States Wine Measure.

3. By an Act of the Imperial Parliament, 1826, the Imperial gallon of 277.274 cubic inches, was adopted as the only gallon, and is therefore the standard for both liquid and dry measures.

4. Beer is usually sold by the gallon; sometimes, however, in casks of 5, 10, 20 gals. etc. The beer barrel contains 36 gallons, and the hogshead, 54 gallons.

## II. DRY MEASURE.

**209. Dry Measure** is used in measuring articles not liquid, as grain, salt, fruit, roots, &c.

TABLE.

|                        |      |             |              |
|------------------------|------|-------------|--------------|
| 2 pints ( <i>pt.</i> ) | make | 1 quart,    | <i>qt.</i>   |
| 4 quarts               | "    | 1 gallon,   | <i>gal.</i>  |
| 2 gallons              | "    | 1 peck,     | <i>pk.</i>   |
| 4 pecks                | "    | 1 bushel,   | <i>bush.</i> |
| 36 bushels             | "    | 1 chaldron, | <i>ch.</i>   |

|            |              |             |            |            |
|------------|--------------|-------------|------------|------------|
|            |              | <i>gal.</i> | <i>qt.</i> | <i>pt.</i> |
|            |              | 1 =         | 4 =        | 16 =       |
|            | <i>pk.</i>   | 1 =         | 8 =        | 32 =       |
|            | <i>bush.</i> | 1 =         | 2 =        | 8 =        |
| <i>ch.</i> | 1 =          | 4 =         | 8 =        | 32 =       |
|            | 1 =          | 36 =        | 144 =      | 288 =      |
|            |              |             |            | 1152 =     |
|            |              |             |            | 2304 =     |

Notes.—  
sun's leavi  
365 d. 5h.

Notes.—1. The English or Winchester bushel is an upright cylinder whose internal diameter is 18½ inches, and depth 8 inches. It contains 2150.4 cubic inches, or 77.627 lbs. Avoirdupois of pure distilled water, at 62° Fahr. and 30 in. barometer. The bushel of Canada is 18½ inches in diameter, and 8.701 inches deep, and must contain 2338,917 Engl. cubic inches, or 1920 French cubic inches. The standard unit of Dry Measure in the United States is the Winchester bushel: The standard unit of Dry Measure in Great Britain is the Imperial bushel, which is 18.739 inches in diameter, and 8 inches deep. It contains 2218.192 cubic inches, or 80 lbs. Avoirdupois of pure distilled water at 62° Fahr. and 30 in. barometer.

2. Grain is frequently bought and sold by weight. The standard per bushel is, viz.: of wheat, 60 lbs.; of rye, 56 lbs.; of Indian corn, 56 lbs.; of barley, 48 lbs.; of oats, 34 lbs.; of peas, 60 lbs.; of beans, 50 lbs.; of buckwheat, 40 lbs.; of flax-seed, 56 lbs.; of Timothy-seed or red clover-seed, 60 lbs.

3. The old French Weights and Measures are legal in the Province of Quebec:

MEASURE OF TIME.

210. Time is the measure of duration. The unit is the day, and the table is made up of its divisors and multiples.

TABLE.

|                    |      |                |      |
|--------------------|------|----------------|------|
| 60 seconds (sec.)  | make | 1 minute,      | min. |
| 60 minutes         | "    | 1 hour,        | h.   |
| 24 hours           | "    | 1 day,         | da.  |
| 7 days             | "    | 1 week,        | wk.  |
| 4 weeks            | "    | 1 lunar month, | mo.  |
| 365 days           | "    | 1 common year, | yr.  |
| 366 days           | "    | 1 leap year,   | yr.  |
| 12 calendar months | "    | 1 year,        | yr.  |
| 100 years          | "    | 1 century,     | ∨ C. |

The calendar year is divided as follows:—

| No. of months. | Seasons. | Names of months. | Abbreviations. | No. of days. |
|----------------|----------|------------------|----------------|--------------|
| 1              | Winter,  | January,         | Jan.           | 31.          |
| 2              |          | February,        | Feb.           | 28 or 29.    |
| 3              |          | March,           | Mar.           | 31.          |
| 4              | Spring,  | April,           | Apr.           | 30.          |
| 5              |          | May,             | May.           | 31.          |
| 6              |          | June,            | Jun.           | 30.          |
| 7              | Summer,  | July,            | July.          | 31.          |
| 8              |          | August,          | Aug.           | 31.          |
| 9              |          | September,       | Sept.          | 30.          |
| 10             | Autumn,  | October,         | Oct.           | 31.          |
| 11             |          | November,        | Nov.           | 30.          |
| 12             | Winter,  | December,        | Dec.           | 31.          |

Notes.—1. The true Solar or Tropical Year is the time measured from the sun's leaving either equinox or solstice to its return to the same again, and is 365 d. 5h. 48 min 49 1/10 sec.

asure, is now

pt.  
qt.  
gal.  
bbl.  
hhd.  
pi.  
tun.

gi.  
= 4.  
= 8.  
= 32.  
= 1008.  
= 2016.  
= 4032.  
= 8064.

io inches or 10 lbs,  
of 62° Fahrenheit,

s inches, and the  
ore about equal to

l gallon of 277.274  
the standard for

in casks of 5, 10,  
shead, 64 gallons.

icles not liquid,

qt.  
gal.  
pk.  
bush.  
ch.

pt.  
= 2.  
= 8.  
= 16.  
= 64.  
= 2604.

2. The *Julian Year*, so called from the calendar instituted by Julius Cæsar, contains 365½ days, as a medium; three years in succession containing 365 days, and the fourth year 366 days; which, as compared with the true solar year, produces a yearly error of 11m. 10<sup>s</sup>., or of 1 whole day in about 120 years.

3. The *Gregorian Year*, or that instituted by Pope Gregory XIII, in the year 1582, and which is now the *Civil* or *Legal Year* in use among the different nations of the earth, contains 365 days for three years in succession, and 366 days for the fourth, *excepting centennial years* whose number cannot be exactly divided by 400. The *Gregorian year* gives an error of only 1 day in 3866 days.

4. The *civil day* begins and ends at 12 o'clock, midnight. The *astronomical day*, used by astronomers in dating events, begins and ends at 12 o'clock, noon.

6. In most business transactions 30 days are called 1 month.

TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH IN THE SAME YEAR.

| FROM ANY DAY OF | TO THE SAME DAY OF |      |      |      |     |      |      |      |       |      |      |      |
|-----------------|--------------------|------|------|------|-----|------|------|------|-------|------|------|------|
|                 | Jan.               | Feb. | Mar. | Apr. | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
| January         | 365                | 31   | 60   | 90   | 120 | 151  | 181  | 212  | 243   | 273  | 304  | 334  |
| February        | 334                | 365  | 29   | 59   | 89  | 120  | 150  | 181  | 212   | 242  | 273  | 303  |
| March           | 306                | 337  | 365  | 31   | 61  | 92   | 122  | 153  | 184   | 214  | 245  | 275  |
| April           | 275                | 306  | 334  | 365  | 30  | 61   | 91   | 122  | 153   | 183  | 214  | 244  |
| May             | 245                | 276  | 304  | 335  | 365 | 31   | 61   | 92   | 123   | 153  | 184  | 214  |
| June            | 214                | 245  | 273  | 304  | 334 | 365  | 80   | 61   | 92    | 122  | 153  | 183  |
| July            | 184                | 215  | 243  | 274  | 304 | 335  | 365  | 31   | 62    | 92   | 123  | 153  |
| August          | 153                | 184  | 212  | 243  | 273 | 304  | 334  | 365  | 31    | 61   | 92   | 122  |
| September       | 122                | 153  | 181  | 212  | 242 | 273  | 303  | 334  | 365   | 30   | 61   | 91   |
| October         | 92                 | 123  | 151  | 182  | 212 | 243  | 273  | 304  | 335   | 365  | 31   | 61   |
| November        | 61                 | 92   | 120  | 151  | 181 | 212  | 242  | 273  | 304   | 334  | 365  | 30   |
| December        | 31                 | 62   | 90   | 121  | 151 | 182  | 212  | 243  | 274   | 304  | 335  | 365  |

For example, to find the number of days from April 4th to November 4th, we look for April in the left vertical column, and November at the top, and, where the lines intersect, is 214, the number sought. Again, to find the number of days from June 10th to September 16th, we find the difference between June 10th and September 10th to be 92 days, and add 6 days for the excess of the 16th over the 10th of September, so we have 98 days as the exact difference.

If the end of February be included between the points of a time, a day must be added in leap year.

When the time exceeds one year, there must be added 365 days for each year.

## CIRCULAR MEASURE.

**211. Circular Measure**, called also *Angular Measure*, is used principally in surveying, navigation, astronomy, and geography; for reckoning latitude and longitude, determining locations of places and vessels, and computing difference of time.

A

A

An  
The  
center  
is one  
which

o.  
1  
NOTE.  
of 90°;

12 units  
12 dozen

24 sheets  
20 quires

A  
2 leave  
4 "  
8 "  
12 "

by Julius Cæsar, containing 365 days, the solar year, protracted 120 years.

XIII, in the year of the different nation, and 366 days not to be exactly divided in 3866 days.

The astronomical 12 o'clock, noon.

MONTH TO THE YEAR.

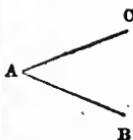
| t. | Oct. | Nov. | Dec. |
|----|------|------|------|
| 8  | 273  | 304  | 334  |
| 2  | 242  | 273  | 303  |
| 4  | 214  | 245  | 275  |
| 3  | 183  | 214  | 244  |
| 3  | 153  | 184  | 214  |
| 2  | 122  | 153  | 183  |
| 2  | 92   | 123  | 153  |
| 1  | 61   | 92   | 122  |
| 5  | 30   | 61   | 91   |
| 5  | 365  | 31   | 61   |
| 4  | 334  | 365  | 30   |
| 4  | 304  | 335  | 365  |

November 4th, where the top, and, where the numbers of days between June 10th and the 16th over the

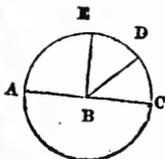
a time, a day must

days for each year.

Measure, is any, and geographical locations time.



**212. An Angle** is the difference of direction of two lines which meet at a point; thus, A, B, C, is an angle. The lines are called the sides of the angle, and the point where they meet is called the vertex.



**213. A Circle** is a plane figure bounded by a curved line, all the parts of which are equally distant from a point within called the center.

A circumference is the curve line which bounds a circle, and always contains 360 degrees.

An arc is any part of the circumference, as C D, D E.

The arc within the sides of an angle whose vertex is on the center of a circle is the measure of the angle; thus, the arc C E is one fourth of the circumference, and measures the angle E B C, which contains 90 degrees.

TABLE.

|                   |                                 |          |
|-------------------|---------------------------------|----------|
| 60 seconds (")    | make 1 minute,                  | '        |
| 60 minutes        | " 1 degree,                     | °        |
| 30 degrees        | " 1 sign,                       | S.       |
| 12 signs, or 360° | " 1 circle,                     | C.       |
|                   | "                               | "        |
|                   | °                               | 1. = 60. |
|                   | 1 = 30 = 1800 = 108000.         | 3600.    |
|                   | 1 = 12 = 360 = 21600 = 1296000. |          |

NOTE.—A quadrant, or right angle, is one-fourth of a circumference, or an arc of 90°; as A B. 60° is called a sextant, or  $\frac{1}{6}$  of a circle.

MISCELLANEOUS TABLES.

COUNTING.

|          |               |          |                     |
|----------|---------------|----------|---------------------|
| 12 units | make 1 dozen. | 12 gross | make 1 great gross. |
| 12 dozen | " 1 gross.    | 20 units | " 1 score.          |

PAPER.

|           |               |           |                |
|-----------|---------------|-----------|----------------|
| 24 sheets | make 1 quire. | 2 reams   | make 1 bundle. |
| 20 quires | " 1 ream.     | 5 bundles | " 1 bale.      |

BOOKS.

|                   |                             |                             |
|-------------------|-----------------------------|-----------------------------|
| A sheet folded in | 2 leaves is called a folio. | 16 leaves is called a 16mo. |
| 4 "               | " a quarto, or 4to.         | 18 " " an 18mo.             |
| 8 "               | " an octavo, or 8vo.        | 24 " " a 24mo.              |
| 12 "              | " a 12mo,                   | 32 " " a 32mo.              |

## THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

The metric system of weights and measures—so called, because the metre is the unit from which the other units of the system, whether of length, area, solidity, capacity, or weight, are derived—originated in France in 1790. It was determined and established as follows: a very accurate survey of that portion of the terrestrial meridian, or north and south circle, between Dunkirk in the north of France, and Barcelona in Spain, was made under the direction of Government, and from this measurement the exact length of a quadrant of the entire meridian, or the distance from the equator to the north pole, was computed. The ten millionth part of this arc was denominated a metre, and from this all the standard units of measure and weight are derived and determined.

The metric system was finally made the only legal system throughout the whole of France in 1841. Since that time, it has been adopted by Spain, Belgium, and Portugal, to the exclusion of other weights and measures. In Holland, other weights are used only in compounding medicines. In 1864, the system was legalized in Great Britain; and its use, either as a whole or in some of its parts, has been authorized in Greece, Italy, Norway, Sweden, Mexico, Guatemala, Venezuela, Ecuador, United States of Columbia, Brazil, Chili, San Salvador, and Argentine Republic. In 1866, the use of the metric system of weights and measures, was authorized by Congress for the whole of the United States.

## TABLES AUTHORIZED BY CONGRESS OF THE UNITED STATES.

## MEASURES OF LENGTHS.

| Metric Denominations and Values. |                                 | Equivalents in Denominations in use.    |
|----------------------------------|---------------------------------|-----------------------------------------|
| Myriametre,...                   | 10,000 metres,.....             | 6.2137 miles.                           |
| Kilometre,.....                  | 1,000 metres,.....              | 0.62137 miles, or 3280 feet, 10 inches. |
| Hectometre,...                   | 100 metres,.....                | 328 feet and 1 inch.                    |
| Decametre,....                   | 10 metres,.....                 | 393.7 inches.                           |
| Metre,.....                      | 1 metre,.....                   | 39.37 inches.                           |
| Decimetre,....                   | $\frac{1}{10}$ of a metre,...   | 3.937 inches.                           |
| Centimetre,....                  | $\frac{1}{100}$ of a metre,...  | 0.3937 inch.                            |
| Millimetre,....                  | $\frac{1}{1000}$ of a metre,... | 0.0394 inch.                            |

THE METRIC SYSTEM.

MEASURES OF SURFACES.

| Metric Denominations and Values. |                       | Equivalents in Denominations in use. |
|----------------------------------|-----------------------|--------------------------------------|
| Hectare, .....                   | 10,000 square metres, | 2.471 acres.                         |
| Are, .....                       | 100 square metres,    | 119.6 square yards.                  |
| Centiare, .....                  | 1 square metre,       | 1550 square inches.                  |

MEASURES OF SOLIDS.

| Metric Denominations and Values: |                        | Equivalents in Denominations in use. |
|----------------------------------|------------------------|--------------------------------------|
| Decastere, .....                 | 10 cubic metres, ..... | 13.079 cubic yards.                  |
| Stere, .....                     | 1 cubic metre, .....   | 0.2759 of a cord of wood.            |
| Decistere, .....                 | 100 cubic decimetres,  | 3.53144 cubic feet.                  |

MEASURES OF CAPACITY.

| Metric Denominations and Values. |                  |                                         | Equivalents in Denominations in use. |                         |
|----------------------------------|------------------|-----------------------------------------|--------------------------------------|-------------------------|
| Names.                           | No. of litres    | Cubic Measure.                          | Dry Measure.                         | Liquid or wine measure. |
| Kilolitre, or stere,             | 1000             | 1 cubic metre, .....                    | 1.308 cubic yd.                      | 264.17 gallons.         |
| Hectolitre, .....                | 100              | $\frac{1}{10}$ of a cubic metre, ....   | 2 bu. 3.35 pk..                      | 26.417 gallons.         |
| Decalitre, .....                 | 10               | 10 cubic decimetres, ....               | 9.08 quarts, ....                    | 2.6417 gallons.         |
| Litre, .....                     | 1                | 1 cubic decimetre, .....                | 0.908 quart, .....                   | 1.0567 quarts.          |
| Decilitre, .....                 | $\frac{1}{10}$   | $\frac{1}{10}$ of a cubic decimetre, .. | 6.1022 cubic in.                     | 0.845 gill.....         |
| Centilitre, .....                | $\frac{1}{100}$  | 10 cubic centimetres, ....              | 0.6102 cubic in.                     | 0.338 fluid oz...       |
| Millilitre, .....                | $\frac{1}{1000}$ | 1 cubic centimetre, .....               | 0.061 cubic in..                     | 0.27 fluid dr...        |

WEIGHTS.

| Metric Denominations and Values. |                    |                                                     | Equivalents in Denominations in use. |
|----------------------------------|--------------------|-----------------------------------------------------|--------------------------------------|
| Names.                           | Number of grammes. | Weight of what quantity of water at maximum density | Avoirdupois weight.                  |
| Millier, or tonneau,             | 1,000,000          | 1 cubic metre, .....                                | 2204.6 pounds.                       |
| Quintal, .....                   | 100,000            | 1 hectolitre, .....                                 | 220.46 pounds.                       |
| Myriagramme, .....               | 10,000             | 10 litres, .....                                    | 22.046 pounds.                       |
| Kilogramme, or kilo,             | 1,000              | 1 litre, .....                                      | 2.2046 pounds.                       |
| Hectogramme, .....               | 100                | 1 decilitre, .....                                  | 3.5274 ounces.                       |
| Decagramme, .....                | 10                 | 10 cubic centimetres, .....                         | 6.3527 ounces.                       |
| GRAMME, .....                    | 1                  | 1 cubic centimetre, .....                           | 15.432 gr. Tr. W.                    |
| Decigramme, .....                | $\frac{1}{10}$     | $\frac{1}{10}$ of a cubic centimetre, ...           | 1.5432 grains.                       |
| Centigramme, .....               | $\frac{1}{100}$    | 10 cubic millimetres, .....                         | 0.1543 of a grain.                   |
| Milligramme, .....               | $\frac{1}{1000}$   | 1 cubic millimetre, .....                           | 0.0154 of a grain.                   |

MEASURES.

alled, because  
f the system,  
t, are deriyed  
and established  
the terrestrial  
ankirk in the  
le under the  
ont the exact  
distance from  
ten millionth  
n this all the  
nd determined.  
legal system  
at time, it has  
the exclusion  
r weights are  
e system was  
a whole or in  
taly, Norway,  
United States  
ntine Republic.  
and measures,  
nited States.

THE UNITED

inuations in use.

0 feet, 10 inches.

## MEASURES OF ANGLES.

| Metric Denominations and Values. |                                 | Equivalents in Denominations in use. |
|----------------------------------|---------------------------------|--------------------------------------|
| Circle,.....                     | 400 grades, .....               | 1 circle or 360°.                    |
| Quadrant, ..                     | 100 grades,.....                | 1 quadrant or 90°.                   |
| Grade, .....                     | 1 grade,.....                   | 64 minutes.                          |
| Minute,.....                     | $\frac{1}{100}$ of a g.,.....   | 32.4 seconds.                        |
| Second,.....                     | $\frac{1}{10000}$ of a g.,..... | 0.324 of a second.                   |

## NOMENCLATURE AND TABLES.

There are eight kinds of quantities for which tables are usually constructed; viz., Lengths, Surfaces, Volumes or Solids, Capacities, Weights, Values, Times, and Angles or Aros. The table for Times is the same in the metric as in the ordinary system. The table for Angles is constructed upon a centesimal scale. The tables for the other six kinds of quantities are constructed upon a decimal scale. In each of the tables for Lengths, Surfaces, Volumes, Capacities, and Weights, there are eight denominations of units,—one principal and seven derivative. The principal units are the *metre*, which is the base of the system, and those derived directly from it. The two following tabular views present the facts regarding the principal and derivative units, which should be fixed in the memory.

## PRINCIPAL UNITS.

- |                                                                                 |   |                                                                                                            |   |                                |
|---------------------------------------------------------------------------------|---|------------------------------------------------------------------------------------------------------------|---|--------------------------------|
| I. METRE, . . . . .                                                             | } | 1. Principal unit of lengths.                                                                              |   |                                |
|                                                                                 |   | 2. The base of the metric system, and nearly one ten-millionth part of a quadrant of the earth's meridian. |   |                                |
|                                                                                 |   | 3. Equivalent, 39.3708 inches.                                                                             |   |                                |
|                                                                                 |   | II. ARE, . . . . .                                                                                         | } | 1. Principal unit of surfaces. |
|                                                                                 |   | 2. A square whose side is ten metres.                                                                      |   |                                |
| 3. Equivalent, 119.6 square yards.                                              |   |                                                                                                            |   |                                |
| III. STERE, . . . . .                                                           | } | 1. Principal unit of volumes or solids.                                                                    |   |                                |
| 2. A cube whose edge is one metre.                                              |   |                                                                                                            |   |                                |
| 3. Equivalent, 1.308 cubic yards.                                               |   |                                                                                                            |   |                                |
| IV. LITRE, . . . . .                                                            | } | 1. Principal unit of capacities.                                                                           |   |                                |
| 2. A vessel whose volume is equal to a cube whose edge is one-tenth of a metre. |   |                                                                                                            |   |                                |
| 3. Equivalent, .908 quart dry measure, or 1.0567 quarts wine measure.           |   |                                                                                                            |   |                                |
| V. GRAMME, . . . . .                                                            | } | 1. Principal unit of weights.                                                                              |   |                                |
| 2. The weight of a cube of pure water whose edge is .01 of a metre.             |   |                                                                                                            |   |                                |
| 3. The water must be weighed in a vacuum 4° C., or 39.2° F.                     |   |                                                                                                            |   |                                |
| 4. Equivalent, 15.432 grains.                                                   |   |                                                                                                            |   |                                |

## DERIVATIVE UNITS.

## I. HOW DERIVED.

1. Three orders of small units, or submultiples of each kind, are formed by dividing each of the principal units into tenths, hundredths, and thousandths.
2. Four orders of larger units, or multiples of each kind, are formed by considering as a unit ten times, one hundred times, one thousand times, and ten thousand times, each of the principal units.

## II. NAMES HOW FORMED.

1. General Principle. { The names of derivative units are formed by attaching a prefix to the name of the principal unit from which they are derived, which indicates their relation to the principal unit.
2. For Submultiples, Latin Ordinals are used as Prefixes.
  1. Millesimus, one thousandth, contracted Milli. *Example*, Millilitre =  $\frac{1}{1000}$  of a litre; 8 millilitres =  $\frac{8}{1000}$  of a litre.
  2. Centesimus, one hundredth, contracted centi. *Ex.*, Centiare =  $\frac{1}{100}$  of an are; 4 centiares =  $\frac{4}{100}$  of an are.
  3. Decimus, tenth, contracted deci. *Ex.*, Decimetre =  $\frac{1}{10}$  metre; 3 decimetres =  $\frac{3}{10}$  metre.
3. For multiples, Greek Cardinals are used as Prefixes.
  1. Deca, ten. *Example*, Decametre, = 10 metres; 5 decametres = 50 metres.
  2. Hecaton, one hundred, contracted hecto. *Ex.*, Hectolitre = 100 litres; 7 hectolitres = 700 litres.
  3. Kilioi, one thousand, contracted kilo. *Ex.* Kilogramme = 1000 grammes.
  4. Myria, ten thousand. *Ex.*, Myriastere = 10,000 steres; 3 myriasteres = 30,000 steres.
  5. The  $\alpha$  in deca and myria, and the  $o$  in hecto and kilo, are dropped when prefixed to are.

## III. ORDER OF PROGRESSION IN TABLES.

- { The tables being constructed upon a decimal scale, ten units of a lower order make one of the next higher, thus: 10 millimetres = 1 centimetre; 10 centimetres = 1 decimetre; 10 decimetres = 1 metre; 10 metres = 1 decametre, &c.

The facts in the preceding views being mastered, the tables can be constructed by the pupil at sight. For example: The names of the derivative units are formed by attaching the seven prefixes,

in their order, to the principal units of the tables. The order of progression being ten, the table of capacities will be written thus:—

|                                |                               |
|--------------------------------|-------------------------------|
| 10 Millilitres = 1 Centilitre. | 10 Litres = 1 Decalitre.      |
| 10 Centilitres = 1 Decilitre.  | 10 Decalitres = 1 Hectolitre. |
| 10 Decilitres = 1 Litre.       | 10 Hectolitres = 1 Kilolitre. |
| 10 Kilolitres = 1 Myrialitre.  |                               |

All the tables peculiar to the Metric System are presented together in a convenient form in the two following tables:—

TABLE OF SUBMULTIPLES AND PRINCIPAL UNITS.

| NAMES OF UNITS.                        |        | PRONUNCIATION.         | SYMBOLS.       |
|----------------------------------------|--------|------------------------|----------------|
| PREFIX.                                | BASE.  |                        |                |
| 10 Milli-<br>Equal<br>1 Centi-         | Metre  | Mill' - e - mee' - ter | <sub>3</sub> M |
|                                        | Are    | Mill' - e - âre        | <sub>3</sub> A |
|                                        | Stere  | Mill' - e - stêr       | <sub>3</sub> S |
|                                        | Litre  | Mill' - e - li' - ter  | <sub>3</sub> L |
|                                        | Gramme | Mill' - e - gram       | <sub>3</sub> G |
| 10 Centi-<br>Equal<br>1 Deci-          | Metre  | Sent' - e - mee' - ter | <sub>2</sub> M |
|                                        | Are    | Sent' - e - âre        | <sub>2</sub> A |
|                                        | Stere  | Sent' - e - stêr       | <sub>2</sub> S |
|                                        | Litre  | Sent' - e - li' - ter  | <sub>2</sub> L |
|                                        | Gramme | Sent' - e - gram       | <sub>2</sub> G |
| 10 Deci-<br>Equal<br>1 Principal Unit. | Metre  | Des' - e - mee' - ter  | <sub>1</sub> M |
|                                        | Are    | Des' - e - âre         | <sub>1</sub> A |
|                                        | Stere  | Des' - e - stêr        | <sub>1</sub> S |
|                                        | Litre  | Des' - e - li' - ter   | <sub>1</sub> L |
|                                        | Gramme | Des' - e - gram        | <sub>1</sub> G |
| 10 Principal Units<br>Equal<br>1 Deca- | Metre  | Mee' - ter             | M              |
|                                        | Are    | Are                    | A              |
|                                        | Stere  | Stêr                   | S              |
|                                        | Litre  | Li' - ter              | L              |
|                                        | Gramme | Gram                   | G              |

To  
adopt  
sary t  
pronou  
differ  
they s  
The  
univer

TABLE OF MULTIPLES.

The order of  
then thus:—

- 1 Decalitre.
- 1 Hectolitre.
- 1 Kilolitre.

represented to  
be:—

UNITS.

| SYMBOLS. |   |
|----------|---|
| 3        | M |
| 2        | A |
| 1        | S |
| 1        | L |
| 2        | G |
| 2        | M |
| 2        | A |
| 2        | S |
| 2        | L |
| 2        | G |
| 1        | M |
| 1        | A |
| 1        | S |
| 1        | L |
| 1        | G |
| 1        | M |
| 1        | A |
| 1        | S |
| 1        | L |
| 1        | G |

| NAMES OF UNITS.               |        | PRONUNCIATION.   |                |
|-------------------------------|--------|------------------|----------------|
| PREFIX.                       | BASE.  |                  |                |
| 10 Deca-<br>Equal<br>1 Hecto- | Metre  | Dek'-a-mee-ter   | <sup>1</sup> M |
|                               | Are    | Dek'-âre         | <sup>1</sup> A |
|                               | Stere  | Dek'-a-stêr      | <sup>1</sup> S |
|                               | Litre  | Dek'-a-li'-ter   | <sup>1</sup> L |
|                               | Gramme | Dek'-a-gram      | <sup>1</sup> G |
| 10 Hecto-<br>Equal<br>1 Kilo- | Metre  | Heo'-to-mee-ter  | <sup>2</sup> M |
|                               | Are    | Heo'-târe        | <sup>2</sup> A |
|                               | Stere  | Heo'-to-stêr     | <sup>2</sup> S |
|                               | Litre  | Heo'-to-li'-ter  | <sup>2</sup> L |
|                               | Gramme | Heo'-to-gram     | <sup>2</sup> G |
| 10 Kilo-<br>Equal<br>1 Myria- | Metre  | Kill'-o-mee-ter  | <sup>3</sup> M |
|                               | Are    | Kill'-âre        | <sup>3</sup> A |
|                               | Stere  | Kill'-o-stêr     | <sup>3</sup> S |
|                               | Litre  | Kill'-o-li'-ter  | <sup>3</sup> L |
|                               | Gramme | Kill'-o-gram     | <sup>3</sup> G |
| Myria-                        | Metre  | Mir'-e-a-mee-ter | <sup>4</sup> M |
|                               | Are    | Mir'-e-âre       | <sup>4</sup> A |
|                               | Stere  | Mir'-e-a-stêr    | <sup>4</sup> S |
|                               | Litre  | Mir'-e-a-li'-ter | <sup>4</sup> L |
|                               | Gramme | Mir'-e-a-gram    | <sup>4</sup> G |

ABBREVIATED NOMENCLATURE.

To secure the fullest advantage to business men by the universal adoption of the new system of weights and measures, it is necessary that the names used should be short and easy to write and pronounce, that they should express clearly the relation of the different denominations of the same table to each other, and that they should be identical in all languages.

The last two of these requirements would be secured by the universal use of the nomenclature adopted by the French. It is

cosmopolitan in its character: it belongs to their language no more than to any other. The former, however, is not secured. It is evident to all, that, for business purposes, the long names of the metric system are inconvenient, and that to shorten them would prove a great advantage. Efforts have been made to introduce short names; but these efforts have invariably sacrificed their universal and expressive character, which is of more importance to the business world than their shortness.

The only true course which seems to be open, is to abbreviate the names already introduced, in such a way as to retain their peculiar characteristics.

To secure this, the following plan of abbreviation is suggested:—

*First.* Let the prefixes be abbreviated thus: Myr, kil, hect, dec, des, cent, mil.

*Second.* Let the initial letter of the names of the five principal units be used, instead of the names themselves, thus: For metre, use a capital M; for are, use a capital A; for stère, a capital S; for litre, a capital L; and, for gramme, a capital G.

*Third.* For the names of multiples and sub-multiples, attach to these initial capital letters the abbreviated prefixes, thus: Kil M, pronounced kill-em'; Kil S, pronounced kill-ess', &c.

By this method of abbreviation, the elements of the original terms are retained in such a form that each part is clearly indicated. The capital letter used after the prefix will always point to the base-word of which it is the initial, although the pronunciation is changed.

## TABLES WITH ABBREVIATED NOMENCLATURE.

### MEASURES OF LENGTHS.

| Written.   | Pronounced. |      |           |
|------------|-------------|------|-----------|
| 10 Mil M,  | Mill-em'    | make | 1 Cent M. |
| 10 Cent M, | Cent-em'    | "    | 1 Des M.  |
| 10 Des M,  | Des-em'     | "    | 1 M.      |
| 10 M,      | Em,         | "    | 1 Dec M.  |
| 10 Dec M,  | Dek-em'     | "    | 1 Hect M. |
| 10 Hect M, | Hect-em'    | "    | 1 Kil M.  |
| 10 Kil M,  | Kill-em'    | "    | 1 Myr M.  |
| Myr M,     | Mir-em'     |      |           |

MEASURES OF SURFACES.

| Written.   | Pronounced. |      |           |
|------------|-------------|------|-----------|
| 10 Mil A,  | Mill-a',    | make | 1 Cent A. |
| 10 Cent A, | Cent-a',    | "    | 1 Des A.  |
| 10 Des A,  | Des-a',     | "    | 1 A.      |
| 10 A,      | A,          | "    | 1 Dec A.  |
| 10 Dec A,  | Dek-a',     | "    | 1 Hect A. |
| 10 Hect A, | Hect-a',    | "    | 1 Kil A.  |
| 10 Kil A,  | Kill-a',    | "    | 1 Myr A:  |
| Myr A,     | Mir-a'.     |      |           |

MEASURES OF VOLUMES, OR SOLIDS.

| Written.   | Pronounced. |      |           |
|------------|-------------|------|-----------|
| 10 Mil S,  | Mill-ess',  | make | 1 Cent S. |
| 10 Cent S, | Cent-ess',  | "    | 1 Des S.  |
| 10 Des S,  | Des-ess',   | "    | 1 S.      |
| 10 S,      | Ess,        | "    | 1 Dec S.  |
| 10 Dec S,  | Dek-ess',   | "    | 1 Hect S. |
| 10 Hect S, | Hect-ess',  | "    | 1 Kil S.  |
| 10 Kil S,  | Kill-ess',  | "    | 1 Myr S.  |
| Myr S,     | Mir-ess'.   |      |           |

MEASURES OF CAPACITY.

| Written.   | Pronounced. |      |           |
|------------|-------------|------|-----------|
| 10 Mil L,  | Mill-ell',  | make | 1 Cent L. |
| 10 Cent L, | Cent-ell',  | "    | 1 Des L.  |
| 10 Des L,  | Des-ell',   | "    | 1 L.      |
| 10 L,      | Ell,        | "    | 1 Dec L.  |
| 10 Dec L,  | Dek-ell',   | "    | 1 Hect L. |
| 10 Hect L, | Hect-ell',  | "    | 1 Kil L.  |
| 10 Kil L,  | Kill-ell',  | "    | 1 Myr L.  |
| Myr L,     | Mir-ell'.   |      |           |

MEASURES OF WEIGHTS.

| Written.   | Pronounced. |      |           |
|------------|-------------|------|-----------|
| 10 Mill G, | Mill-gee,   | make | 1 Cent G. |
| 10 Cent G, | Cent-gee',  | "    | 1 Des G.  |
| 10 Des G,  | Des-gee',   | "    | 1 G.      |
| 10 G,      | Gee,        | "    | 1 Dec G.  |
| 10 Dec G,  | Dek-gee',   | "    | 1 Hect G. |
| 10 Hect G, | Hect-gee',  | "    | 1 Kil G.  |
| 10 Kil G,  | Kill-gee',  | "    | 1 Myr G.  |
| Myr G,     | Mir-gee'.   |      |           |

REDUCTION OF COMPOUND DENOMINATE  
NUMBERS.

**214.** Reduction is the process of changing numbers from one denomination to another, without altering their value.

Reduction is of two kinds, *Descending* and *Ascending*.

**215. Reduction Descending** is changing numbers to lower denominations without altering their value; as pounds to shillings, yards to feet, etc. It is performed by *Multiplication*.

**216. Reduction Ascending** is changing numbers to higher denominations without altering their value; as farthings to pence, inches to feet, etc. It is performed by *Division*.

## REDUCTION DESCENDING.

**217. CASE I.**—To reduce a compound number to lower denominations.

*Ex.* Reduce £45 7s. 8d. to pence.

OPERATION.

£45 7s. 8d.

20

907s.

12

10892d.

ANALYSIS.—There are 20s. in £1; therefore, 20 times the number of £ = the number of shillings. 20 times 45 = 900s., to which we add 7s., and obtain 907s. There are 12d. in 1s.; therefore, 12 times the number of shillings equal the number of pence. 12 times 907 = 10884d., to which we add 8d., and obtain 10892d. Hence the following

**218. RULE.**—I. Multiply the highest denomination of the given number by that number of the scale which will reduce it to the next lower denomination, and add to the product the given number, if any, of that lower denomination.

II. Proceed in like manner with the results obtained in each lower denomination, until the reduction is brought to the denomination required.

## EXAMPLES FOR PRACTICE.

- In £35 6s. 8d., how many pence? *Ans.* 8480.
- In £28 12s. 8½d., how many farthings?
- In 14lb. 10oz. 18pwt. 22gr., how many grains? *Ans.* 85894.
- In 165T. 13cwt. 3qr. 19lb. 14oz., how many ounces?
- In 23h 9s 0s 2s 13 gr., how many grains?
- In 12rd. 8yd. 2ft., how many feet? *Ans.* 224.
- How many inches in 2mi. 4fur. 32rd. 1yd.?
- In 60arp. 1per. 1to. 5ft., how many feet?

214. What is reduction?—How many kinds of reduction?—215. What is reduction descending?—216. Reduction ascending?—218. What is the rule for reduction ascending?

9. How many links in 7mi. 5fur. 6ch. 30l. ? *Ans.* 61630.
10. In 4mi. 49ch. 72l., how many links?
11. Reduce 12A. 3R. 24sq. rd. 144sq. ft. 72sq. in., to square inches? *Ans.* 80937864 square inches.
12. In 10A. 1R. 25sq. rd. 16 sq. yd. 4sq. ft. 136sq. in., how many square inches? *Ans.* 65296108 square inches.
13. How many square links in 75A. 4sq. ch.-18P. 118sq. l. ?
14. How many poles in 3 townships of land?
15. In 7sq. arp. 30sq. per. 4sq. to. 9sq. ft. 40sq. in., how many square inches? *Ans.* 34080952.
16. How many cubic feet in 67 cords and 74 cubic feet of wood?
17. In 30 cords of wood, how many cubic inches?
18. In 4½gal. 4.25qt. 4.75gi., how many gills? *Ans.* 190½.
19. In 57tuns 3hd. 50gal. 3qt., how many pints?
20. How many pints in 10bu. 3.5pk. 7½qt. 1pt. ?
21. How many quarts in 676 chaldrons, of 36 bushels each?
22. In 4da. 4h. 45mi., how many seconds? *Ans.* 362700.
23. In 3wk. 2da. 1h. 1min., how many minutes?
24. How many days from March 17th., 1870, to May 16th. 1871?
25. In 44S. 18° 57' 23", how many seconds? *Ans.* 4820243".
26. How many minutes in 1¼C. 1S. 1° 1'?
27. Reduce 38lb, 6s 3s lb, to grains.
28. How many days from August 30th 1771, to June 1st. 1872?
29. Louis has a lump of pure silver weighing 13lb. 9oz. What is its value at \$1.385½ per ounce? *Ans.* \$228.640½.
30. Change 13lb. 6oz. Avoirdupois weight to Troy weight.
31. Purchased 3A. 1R. 30rd. of land, at \$1.25 per square foot; what did I pay for the land? *Ans.* \$187171.87½.
32. Bought 2 hogsheads of sirup at 40 cts. per gal., and sold it at 12 cts. per quart, what did I gain by the bargain? *Ans.* \$10.08.

**219. CASE II.**—To reduce a denominate fraction to one of a lower denomination.

*Ex.* Reduce  $\frac{1}{4}$  of a gallon to the fraction of a gill.

**OPERATION.**

$$\frac{1}{4} \text{ gal.} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64} \text{ gi.}$$

**ANALYSIS.**—To reduce gallons to gills, we multiply successively by 4, 2, and 4, the numbers in the scale. And, since the given number is a fraction, we indicate the process, as in multiplication of fractions; and, after cancelling, obtain  $\frac{1}{64}$  the answer. Hence, the

**220. RULE.**—Multiply the fraction of the higher denomination by the numbers in the descending scale successively, between the given and the required denominations.

**EXAMPLES FOR PRACTICE.**

1. What part of a farthing is  $\frac{1}{1000}$  of a £? *Ans.*  $\frac{1}{40000}$  far.
2. Reduce  $\frac{1}{1000}$  of a week to the fraction of a minute.

ATE

bers from  
lue.

g.  
ers to lower  
ds to shil-  
ation.

ers to higher  
ys to pence,

lower de-

l; therefore,  
e number of  
which we add  
12d. in 1s.;  
billings equal  
7 = 10884d.,  
892d. Hence

ion of the  
reduce it to  
the given

d in each  
the denom-

ns. 8480.

ns. 85894.

Ans. 224.

What is the  
rule for

3. What part of a square foot is  $\frac{1}{14400}$  of an acre? *Ans.*  $\frac{1}{36}$  sq. ft.
4. Reduce  $\frac{1}{1000}$  of a lb. Troy to the fraction of a grain.
5. Reduce  $\frac{1}{10}$  of a £ to a fraction of a penny. *Ans.*  $\frac{1}{d}$ .
6. Reduce  $\frac{1}{1000}$  of a cwt. to the fraction of an ounce.
7. What part of a pound is  $\frac{1}{16000}$  of a ton?
8. What part of a link is  $\frac{1}{7}$  of a rod? *Ans.*  $\frac{1}{l}$ .
9. Reduce  $\frac{1}{1000}$  of a furlong to a fraction of a foot.
10. What part of a pint is  $\frac{1}{160}$  of a bushel? *Ans.*  $\frac{1}{16}$  pt.
11. Reduce  $\frac{1}{2}$  of  $\frac{1}{2}$  of 2lb. to the fraction of an ounce Troy.
12. What part of a square rod is  $\frac{1}{1600}$  of  $4\frac{1}{2}$  times  $\frac{1}{2}$  of an acre?
13. What fraction of a yard is  $\frac{1}{2}$  of  $\frac{1}{2}$  of a rod?
14. What part of a dram is  $\frac{1}{10000}$  of a hundred weight? *A.*  $\frac{1}{1000}$  dr.
15. Reduce 0.03125 of a mile to feet.

**221. CASE III.**—*To reduce a denominate fraction to integers of lower denominations.*

*Ex.* What is the value of  $\frac{1}{4}$  of a £?

| OPERATION.                         | ANALYSIS.—                                          |
|------------------------------------|-----------------------------------------------------|
| £ s. d. far.                       | $\frac{1}{4}$ of £1 is the same as $\frac{1}{4}$ of |
| 7) 3 0 0 0                         | £3 = 8s. 6d. $3\frac{1}{2}$ far. Hence, the         |
| 0 8 6 $3\frac{1}{2}$ , <i>Ans.</i> |                                                     |

**222. RULE.**—*Consider the numerator of the fraction as so many units of the given denomination, and divide them by the denominator.*

**EXAMPLES FOR PRACTICE.**

What is the value of

1.  $\frac{1}{4}$  of a £? *Ans.* 5s. 5d.  $1\frac{1}{2}$  far.
2.  $\frac{1}{4}$  of a bushel? *Ans.* 1pk. 4qt.  $1\frac{1}{2}$  pt.
3.  $\frac{1}{4}$  of a shilling?
4.  $\frac{1}{4}$  of a cwt. *Ans.* 3qr. 2lb. 12oz.  $7\frac{1}{2}$  dr.
5.  $\frac{1}{4}$  of a yard?
6.  $\frac{1}{4}$  of a lb. Avoirdupois? *Ans.* 7oz.  $1\frac{1}{2}$  dr.
7.  $\frac{1}{4}$  of a day?
8.  $\frac{1}{4}$  of 15 cwt. *Ans.* 12cwt. 85lb. 11oz.  $6\frac{1}{2}$  dr.
9.  $\frac{1}{4}$  of 2½ pounds Apothecaries' weight?
10.  $\frac{1}{4}$  of an acre? *Ans.* 2R. 6rd. 4sq. yd. 5sq. ft.  $127\frac{1}{2}$  sq. in.
11.  $\frac{1}{4}$  of 5½ tons?
12.  $\frac{1}{4}$  of a hhd. of wine? *Ans.* 6gal. 2qt. 1pt.  $0\frac{1}{2}$  gi.
13.  $\frac{1}{4}$  of 3½ cords of wood?
14.  $\frac{1}{4}$  of a sign? *Ans.* 12° 51' 25"  $\frac{1}{4}$ .
15. From a piece of velvet containing 8yd. 3qr. I cut 2yd. 2qr. What part of the whole piece did I take?

**223. CASE IV.**—*To reduce a denominate decimal to integers of lower denominations.*

*Ex.* Reduce 0.628125 of a £ to shillings and pence.

OPERATION.  
 £0.628125  
 20  
 12.562500s.  
 12  
 6.750000d.  
 4  
 3.000000far.

£0 12s. 6½d. *Ans.*

ANALYSIS.—We first multiply the given decimal, 0.628125 of a £, by 20 to reduce it to shillings, and the result is 12s. and the decimal .5625 of a shilling. We then multiply this decimal by 12 to reduce it to pence, and obtain 6d. and .75 of a d. This last decimal we multiply by 4, to reduce it to far. or gr., and the result is 3 far. or ¾ of a d. Hence, the answer is £0 12s. 6½d.

**224. RULE.—I.** Multiply the given decimal by that number in the scale which will reduce it to the next lower denomination, and point off as in multiplication of decimals.

**II.** Proceed with the decimal part of the product in the same manner until reduced to the required denominations. The integers at the left will be the answer required.

EXAMPLES FOR PRACTICE.

What is the value of

- |                            |                                        |
|----------------------------|----------------------------------------|
| 1. 0.46¾ of a £?           | <i>Ans.</i> 9s. 1d. 2½far.             |
| 2. 0.748 of a bushel?      | <i>Ans.</i> 2pt. 7qt. 1pt. 3.4888gi.   |
| 3. 0.765 of a pound Troy?  |                                        |
| 4. 0.7525 of a mile?       | <i>Ans.</i> 6fur. 0rd. 4yd. 1ft. 2½in. |
| 5. 0.659 of a week?        |                                        |
| 6. 0.217°?                 | <i>Ans.</i> 13' 1.2".                  |
| 7. 0.875 of a hhd.?        |                                        |
| 8. 0.865 of an acre?       | <i>Ans.</i> 3R. 18¾sq. rd.             |
| 9. 7.88125 acres?          |                                        |
| 10. 0.625 of a fathom?     | <i>Ans.</i> 3½ft.                      |
| 11. 0.78875 of a long ton? | <i>Ans.</i> 15cut. 3qr. 2lb. 12.8oz.   |
| 12. 0.8469 of a degree?    |                                        |

REDUCTION ASCENDING.

**225. CASE I.—**To reduce a denominate number to a compound number of higher denominations.

*Ex.* In 78692gr., how many pounds Troy weight?

OPERATION.  
 24 ) 78692gr.  
 20 ) 3275pwt. 20gr.  
 12 ) 163oz. 18pwt.  
 13lb. 7oz.  
 13lb. 7oz. 18pwt. 20gr., *Ans.*

ANALYSIS.—24gr. = 1pwt.; therefore, ¼ of the number of grains = the number of pennyweights. ¼ of 78692 = 3275pwt., and 20gr. remaining. 20pwt. = 1oz.; therefore, ⅓ of the number of pennyweights = the number of ounces. ⅓ of 3275 = 163oz., and 18pwt. remaining. 12oz. = 1lb.; therefore, ⅓ of the number

of ounces = the number of pounds.  $\frac{1}{4}$  of 163 = 13lb., and 7os. remaining; therefore, 78692gr. = 13lb. 7os. 18pwt. 20gr. Hence, the

**226. RULE.—I.** Divide the given number by that number of the ascending scale which will reduce it to the next higher denomination.

**II.** Divide in like manner the quotient thus obtained, and so proceed until it is brought to the denomination required. The last quotient, with the several remainders annexed in a reversed order, will be the answer.

## EXAMPLES FOR PRACTICE.

1. In 16452*far.*, how many £? *Ans.* £17 2s. 9d.
2. In 90720 pence, how many £?
3. How many pounds in 4253? *Ans.* 4lb 5s 1s.
4. In 78692*gr.*, how many pounds Troy weight?
5. A physician who averages daily 5 prescriptions of 20 grains each, how many pounds of medicine will he use in one year, or 365 days? *Ans.* 6lb 4s 1s.
6. How many pounds of standard silver can be purchased for \$1099.88, at the rate of \$0.062 per *pwt.*?
7. In 87320*lb.*, how many tons? *Ans.* 43T. 13*cwt.* 20*lb.*
8. How much will 230*lb.* of hay cost, at \$10 per ton?
9. In 1265 pints, how many bushels? *Ans.* 19bu. 3pk. 1pt.
10. At 6 cts. a *pt.*, how much sirup can be bought for \$3.84?
11. How many francs in \$176.70? *Ans.* 950.
12. In 2468 pence, how many half crowns?
13. In 90060 seconds, how many days? *Ans.* 1d. 1h. 1mi.
14. What would be the cost of plastering a room 18*ft.* long, 16½*ft.* wide, and 9*ft.* high, at 22 cts. a *sq. yd.*? *Ans.* \$22.44.
15. In a pond measuring 28*ft.* 6in., how many fathoms deep is there?
16. How many bushels of oats in 27072*qt.*? *Ans.* 846bush.
17. How many days in 98960 seconds?
18. The extent of a certain farm is found, by survey, to be 1377*sq. ch.* How many acres does it contain? *Ans.* 137A. 2R. 32*per.*
19. A load of wood is 12 feet long and 3 feet wide, how high must it be to make a cord? *Ans.* 3½*ft.* high.
20. How many tons of round timber in 622080 *cu. in.*?
21. A cellar wall, 32*ft.* by 24*ft.* is 6*ft.* high and 1½*ft.* thick. How much did it cost at \$1.25 a perch? *Ans.* \$50.909 + .
22. Reduce 16936 links to miles. *Ans.* 2mi. 9ch. 36l.
23. In 161384 inches, how many miles?
24. How many beer gallons is there in 1*bb.* 1*gal.* 2*qt.* wine measure? *Ans.* 27½.
25. In 5832000 square inches, how many roods?
26. Reduce 20937 minutes to signs. *Ans.* 11S. 18° 57'.
27. Change 16*lb.* 3oz. 1*pwt.* 1*gr.* Troy weight, to Avoirdupois weight. *Ans.* 13lb. 6oz. 1*pwt.* 1*gr.*

remaining;

number of  
er denom-

l, and so  
The last  
red order,

28. A ship, during 3 days' storm at sea, changed her latitude 412 geographical miles; how many degrees and minutes did she change?

29. How many acres of land can be purchased in the city of Montreal for \$147500, at 65 cts. a square foot? *Ans.* 5A. 33per. 15sq. yd. 3sq. ft. 119.44sq. in.

30. In 13360128 drams, how many tons?

**227. CASE II.**—To reduce a denominate fraction from a lower to a higher denomination.

*Ex.* Reduce  $\frac{3}{4}$  of a farthing to the fraction of a £.

OPERATION.

$$\begin{array}{r} \text{far.} \\ \frac{3}{9} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{2160} \text{ Ans.} \end{array}$$

ANALYSIS.—There are 4far. in 1d., therefore  $\frac{1}{4}$  of the number of farthings equals the number of pence. There are 12d. in 1s., therefore  $\frac{1}{12}$  of the number of pence equals the number of shillings.

There are 20s. in £1, therefore  $\frac{1}{20}$  of the number of shillings equals the number of £. Hence  $\frac{3}{4}$ far. =  $\frac{3}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{2160}$  of a £.

**228. RULE.**—Divide the fraction by the numbers in the scale, successively, between the given and the required denomination.

EXAMPLES FOR PRACTICE.

What part of

- |                                                                                               |                                   |
|-----------------------------------------------------------------------------------------------|-----------------------------------|
| 1. a pound Troy is $\frac{1}{3}$ of a grain?                                                  | <i>Ans.</i> $\frac{1}{2700}$ lb.  |
| 2. a pound is $\frac{1}{2}$ of a scruple?                                                     |                                   |
| 3. a rod is $\frac{1}{4}$ of a foot?                                                          | <i>Ans.</i> $\frac{1}{12}$ rd.    |
| 4. a mile is $\frac{1}{4}$ of a rod?                                                          |                                   |
| 5. a hundred-weight is $\frac{1}{2}$ of an ounce?                                             | <i>Ans.</i> $\frac{1}{3200}$ cwt. |
| 6. an hour is $\frac{1}{2}$ of 20 seconds?                                                    |                                   |
| 7. an acre is $\frac{1}{4}$ of a square foot?                                                 | <i>Ans.</i> $\frac{1}{1152}$ A.   |
| 8. 3 hhd. is $\frac{1}{4}$ of a quart?                                                        |                                   |
| 9. 4 days is $\frac{1}{2}$ of a minute?                                                       | <i>Ans.</i> $\frac{1}{720}$ .     |
| 10. a cord of wood is a pile $7\frac{1}{2}$ ft. long, 2ft. high, and $3\frac{1}{2}$ ft. wide? |                                   |
| 11. a rod is $2\frac{1}{2}$ of $\frac{1}{2}$ of an inch?                                      | <i>Ans.</i> $\frac{1}{16}$ .      |
| 12. an acre is $\frac{1}{4}$ of $\frac{1}{4}$ of 9 $\frac{1}{2}$ square rods?                 |                                   |
| 13. Reduce 9.312far. to the decimal of a £.                                                   | <i>Ans.</i> £0.0097.              |
| 14. Reduce 517.44ft. to the decimal of a mile.                                                |                                   |

**229. CASE III.**—To reduce a compound number to a fraction of a higher denomination.

*Ex.* Reduce 8s. 6d. 2far. to the fraction of a £.

OPERATION.

$$\begin{array}{r} 8s. 6d. 2far. = 410far. = 41 \\ 1£ = 960far. = 96 \text{ £.} \end{array}$$

ANALYSIS.—By reduction of denominate numbers (217), we find 8s. 6d. 2far. = 410far., and that £1 = 960far. One farthing is  $\frac{1}{4}$  of a £, and 410far. = 410 times  $\frac{1}{4}$  =  $\frac{410}{4}$  =  $\frac{1025}{11}$  of a £!

2s. 9d.

lb 5s 15.

20 grains  
ear, or 365  
lb 4s 1b.  
chased for

wt. 20lb.

3pk. 1pt.  
3.847  
Ans. 950.

1h. 1mi.  
ong, 16 $\frac{1}{2}$ ft.  
\$22.44.  
ep is there?  
846bush.

be 1377sq.  
2. 32per.  
high must  
ft. high.

nick! How  
0.909 +.  
9ch. 36l.

wine meas-  
ns. 27 $\frac{1}{2}$ .

18° 57'.  
Avoirdupois  
wt. 1gr.

**230. RULE.**—Reduce the given number to its lowest denomination for the numerator, and a unit of the required denomination to the same denomination for the denominator of the required fraction.

## EXAMPLES FOR PRACTICE.

What part of

- |                                         |                           |
|-----------------------------------------|---------------------------|
| 1. a £ is 10s. 10d. ?                   | Ans. $\frac{11}{200}$ .   |
| 2. a ton is 4cwt. 3qr. 12lb. ?          | Ans. $\frac{187}{2000}$ . |
| 3. an acre is 2R. 20per. ?              |                           |
| 4. a mile is 1fur. 12rd. 4yd. 2ft. ?    | Ans. $\frac{108}{1000}$ . |
| 5. a hogshead of wine is 18gal. 2qt. ?  |                           |
| 6. a square rod is 144ft. 19in. ?       | Ans. $\frac{9}{17}$ .     |
| 7. 2cwt. 3qr. is 1cwt. 2qr. 20lb. ?     |                           |
| 8. 30 days is 8da. 17h. 20min. ?        | Ans. $\frac{157}{240}$ .  |
| 9. a bushel is $1\frac{1}{2}$ pecks ?   |                           |
| 10. a pound Troy is 10oz. 13pwt. 8gr. ? |                           |

**231. CASE IV.**—To reduce a compound number to a decimal of a higher denomination.

*Ex.* Reduce 12s. 9d. 3far. to the decimal of a pound.

|            |                                   |
|------------|-----------------------------------|
| OPERATION. |                                   |
| 4          | $\frac{3.00\text{far.}}{\quad}$   |
| 12         | $\frac{9.7500\text{d.}}{\quad}$   |
| 20         | $\frac{12.81250\text{s.}}{\quad}$ |
|            | $0.640625\text{£. Ans.}$          |

Or, 12s. 9d. 3far. = 645far.  
 £1 = 960far.  
 $\frac{645}{960} = \text{£}0.640625, \text{Ans.}$

ANALYSIS.—Since there are 4 farthings in 1d.,  $\frac{1}{4}$  of the number of farthings equals the number of pence.  $\frac{1}{4}$  of 3 = 0.75d. which added to 9d. = 9.75d. There are 12d. in 1s., therefore,  $\frac{1}{12}$  of the number of pence equals the number of shillings.  $\frac{1}{12}$  of 9.75d. = 0.8125s. which added to 12s. = 12.8125s. There are 20s. in £1, therefore,  $\frac{1}{20}$  of the number of shillings equals the number of pounds,  $\frac{1}{20}$  of 12.8125s. = £0.640625. Hence, the

**232. RULE.**—Divide the lowest denomination given by that number in the scale which will reduce it to the next higher denomination, and annex the quotient as a decimal to that higher. Proceed in the same manner until the whole is reduced to the denomination required. Or,

Reduce the given number to a fraction of the required denomination, and reduce this fraction to a decimal.

## EXAMPLES FOR PRACTICE.

What decimal part of

- |                                           |                       |
|-------------------------------------------|-----------------------|
| 1. a gallon is 3qt. 1pt. 2gt. ?           | Ans. 0.9375gal.       |
| 2. a week is 5da. 9h. 46min. 48sec. ?     |                       |
| 3. a mile is 5fur. 36rd. 2yd. 2ft. 9in. ? | Ans. 0.13603219 + mi. |
| 4. a bushel is 3pk. 6qt. 1pt. ?           |                       |

5. a pound Troy is 10oz. 12*wt.* 18*gr.*? *Ans.* 0.886458*lb.*  
 6. a fathom is 3*ft.*?  
 7. a ton is 16*cwt.* 3*qr.* 16.45*lb.*? *Ans.* 0.8857257.  
 8. 1*½* bushels is 0.45 of a peck?  
 9. Reduce 12*T.* 3*cwt.* 2*qr.* 20*lb.* to hundred-weights and the decimal of a hundred-weight. *Ans.* 243.7.  
 10. Reduce to the decimal of a pound, 19*s.* 11*d.*, 16*s.* 9*d.*, and 17*s.* 5*d.*, and find their sum. *Ans.* £2.710416 +.

REDUCTION OF THE OLD CANADIAN CURRENCY TO THE NEW OR DECIMAL CURRENCY.

*Ex.* Reduce £72 13 9*½* to cents.

OPERATION.

$$\begin{aligned} £72 \times 400 &= 28800 \text{ cents.} \\ 13s. \times 20 &= 260 \text{ " } \\ 9\frac{1}{2} = 39\text{far.} \times 5 \div 12 &= 16\frac{1}{4} \text{ " } \\ £72 \ 13 \ 9\frac{1}{2} &= \underline{29076\frac{1}{4}} \text{ " } \\ \text{or } \$290.76\frac{1}{4}, \text{ Ans.} \end{aligned}$$

ANALYSIS.—We multiply £72 by 400, because each pound is equal to 4 dollars or 400 cents; next we multiply 13, the number of shillings, by 20, because each shilling is equal to 20 cents; lastly, we multiply the number of farthings in the pence

and farthings by 5, and divide the remainder by 12, because each farthing is equal to  $\frac{5}{12}$  of a cent:

That each farthing is equal to  $\frac{5}{12}$  of a cent, is evident from the fact that 48 farthings (or one shilling) are equal to 20 cents; or 12 farthings equal 5 cents, and one farthing equals  $\frac{5}{12}$  of a cent. Hence, the following

**233. RULE.**—I. Multiply the pounds by 400, the shillings by 20, and take five-twelfths of the number expressing how many farthings there are in the given pence and farthings.

II. Add the three results together, and their sum will be the number of cents required.

III. Consider the last two figures as cents, and the result will be dollars and cents. \*

EXAMPLES FOR PRACTICE.

How many dollars and cents in

- |                                                        |                                                        |
|--------------------------------------------------------|--------------------------------------------------------|
| 1. £ 4 3 1 <i>½</i> ? <i>Ans.</i> \$16.52 <i>½</i> .   | 10. £16 6 2? <i>Ans.</i> \$65.23 <i>½</i> .            |
| 2. 27 16 3 <i>¼</i> ? <i>Ans.</i> \$111.38 <i>¾</i> .  | 11. 97 3 11 <i>½</i> ? <i>Ans.</i> \$187.52 <i>½</i> . |
| 3. 27 16 11 <i>½</i> ? <i>Ans.</i> \$111.38 <i>¾</i> . | 12. 46 17 7 <i>½</i> ? <i>Ans.</i> \$187.52 <i>½</i> . |
| 4. 69 15 6? <i>Ans.</i> \$2.94.                        | 13. 121 0 7? <i>Ans.</i> \$49.98 <i>½</i> .            |
| 5. 0 14 8 <i>¼</i> ? <i>Ans.</i> \$2.94.               | 14. 12 9 11? <i>Ans.</i> \$49.98 <i>½</i> .            |
| 6. 77 19 4 <i>¼</i> ? <i>Ans.</i> \$71.29 <i>½</i> .   | 15. 1 12 9 <i>½</i> ? <i>Ans.</i> \$694.66 <i>¾</i> .  |
| 7. 17 16 5 <i>½</i> ? <i>Ans.</i> \$71.29 <i>½</i> .   | 16. 173 13 4? <i>Ans.</i> \$694.66 <i>¾</i> .          |
| 8. 18 18 10 <i>½</i> ? <i>Ans.</i> \$36.69 <i>½</i> .  | 17. 91 8 8? <i>Ans.</i> \$78.27 <i>½</i> .             |
| 9. 9 3 5 <i>¼</i> ? <i>Ans.</i> \$36.69 <i>½</i> .     | 18. 19 11 4 <i>¼</i> ? <i>Ans.</i> \$78.27 <i>½</i> .  |

## REDUCTION OF THE DECIMAL CURRENCY TO THE OLD CANADIAN CURRENCY.

*Ex.* Reduce \$246.88 to the old Canadian currency.

OPERATION.

$$\begin{array}{r}
 4) \ 246.88 \\
 \underline{£61.72} \\
 \quad 20 \\
 \quad \underline{14.40s.} \\
 \quad \quad 12 \\
 \quad \quad \underline{4.80d.} \\
 \quad \quad \quad 4 \\
 \quad \quad \quad \underline{3.20far.}
 \end{array}$$

*Ans.* £61 14 4 $\frac{1}{2}$  +  $\frac{2}{10}$  =  $\frac{1}{2}$  far.

*ANALYSIS.*—We divide 246.88 by 4, the number of dollars in a pound, and the result is £61 and 72 hundredths of a pound. We multiply 72 by 20 (224), the number of shillings in a pound, and the result is 14s. and 40 hundredths of a shilling. Again, we multiply 40 by 12, the number of pence in a shilling, and the result is 4d. and 80 hundredths of a penny. Lastly, we multiply 80 by 4, the number of farthings in a penny, and the result is 3far. and 20 hundredths or  $\frac{1}{2}$  of a farthing. Hence, the

**224. RULE.**—*Divide the given number by 4, and the quotient will be pounds and decimals of a pound. Then proceed as in No. 224.*

### EXAMPLES FOR PRACTICE.

Reduce to the old Canadian currency :—

- |                                                             |                                                                 |
|-------------------------------------------------------------|-----------------------------------------------------------------|
| 1. \$162.30 = <i>Ans.</i> £40 11 6                          | 10. \$319.13 $\frac{1}{2}$ = <i>Ans.</i> £79 15 8 $\frac{1}{2}$ |
| 2. 716.12                                                   | 11. 933.04 $\frac{1}{2}$                                        |
| 3. 391.37 = <i>Ans.</i> 97 16 10 $\frac{1}{2}$              | 12. 601.53 = <i>Ans.</i> 150 7 7 $\frac{1}{2}$                  |
| 4. 537.37 $\frac{1}{2}$                                     | 13. 293.17                                                      |
| 5. 82.19 = <i>Ans.</i> 20 10 11 $\frac{1}{2}$               | 14. 39.06 $\frac{1}{2}$ = <i>Ans.</i> 9 15 3 $\frac{1}{2}$      |
| 6. 207.16                                                   | 15. 436.99                                                      |
| 7. 569.09 $\frac{1}{2}$ = <i>Ans.</i> 142 5 5 $\frac{1}{2}$ | 16. 152.18 $\frac{1}{2}$ = <i>Ans.</i> 38 0 11 $\frac{1}{2}$    |
| 8. 17.35 $\frac{1}{2}$                                      | 17. 846.07 $\frac{1}{2}$                                        |
| 9. 924.08 = <i>Ans.</i> 231 0 4 $\frac{1}{2}$               | 18. 719.11 = <i>Ans.</i> 179 15 6 $\frac{1}{2}$                 |

## ADDITION OF COMPOUND NUMBERS.

**225.** Addition, Subtraction, Multiplication, and Division of Denominate Numbers are performed by the same general methods, as are employed for like operations in Abstract Numbers. The only difference arises from *varying*, instead of *uniform scales*.

*Ex.* 1. What is the sum of £5 10s. 4d., £6 16s. 10d., £8 15s. 6d., and £4 13s. 9d. ?

OPERATION.

$$\begin{array}{r}
 £ \quad s. \quad d. \\
 5 \quad 10 \quad 4 \\
 6 \quad 16 \quad 10 \\
 8 \quad 15 \quad 6 \\
 4 \quad 13 \quad 9 \\
 \hline
 25 \quad 16 \quad 8
 \end{array}$$

*ANALYSIS.*—Having written units of the same denomination in the same column, we find the sum of pence in the right-hand column to be 29 pence = 2s. 5d. We write the 5d. under the column of pence, and carry the 2s. to the column of shillings; the sum of which is 5s. = £2 10s. Having written the 16s. under the column of shillings, we carry the £2 to the column of pounds, and find the entire amount sought to be £25 16s. 8d.

Ex. 2. Add  $\frac{1}{15}$  of a £ to  $\frac{1}{4}$  of a shilling.

OPERATION.

$$\begin{aligned} \frac{1}{15} \text{ of a } \pounds &= 9\text{s. } 4\text{d.} \\ \frac{1}{4} \text{ of a s.} &= 0\text{s. } 8\text{d. } 2\frac{1}{2}\text{ far.} \\ \text{Ans. } &10\text{s. } 0\text{d. } 2\frac{1}{2}\text{ far.} \end{aligned}$$

Or,

$$\begin{aligned} \frac{1}{15} \times \frac{1}{4} &= \frac{1}{60} \pounds. \\ \frac{1}{15} + \frac{1}{4} \pounds &= \frac{41}{60} \pounds; \\ \frac{41}{60} \pounds &= 10\text{s. } 0\text{d. } 2\frac{1}{2}\text{ far.} \end{aligned}$$

ANALYSIS.—We first find the value of each fraction in integers of less denominations (221), and then add the resulting or equivalent compound numbers.

Or, we may reduce the given fractions to fractions of the same denomination (219), then add them, and find the value of their sum in lower denominations. Hence, the following

**236. RULE.—I.** *If any of the numbers are denominats fractions, or if any of the denominations are mixed numbers, reduce the fractions to integers of lower denominations.*

**II.** *Write the numbers so that units of the same denominations will stand in the same column.*

**III.** *Beginning with the lowest denomination, add as in simple numbers, carrying to each succeeding denomination one for as many units as it takes of the denomination added, to make one of the next higher denomination.*

EXAMPLES FOR PRACTICE.

(1.)

| T.  | cwt. | qr. | lb. | oz. | dr. |
|-----|------|-----|-----|-----|-----|
| 71  | 19   | 3   | 27  | 14  | 13  |
| 14  | 13   | 2   | 15  | 15  | 15  |
| 14  | 13   | 1   | 11  | 13  | 12  |
| 11  | 17   | 3   | 16  | 15  | 11  |
| 13  | 18   | 2   | 13  | 11  | 13  |
| 127 | 3    | 2   | 11  | 8   | 0   |

(2.)

| yr. | da. | h. | min. | sec. |
|-----|-----|----|------|------|
| 12  | 10  | 13 | 42   | 27   |
| 16  | 102 | 18 | 24   | 36   |
| 19  | 8   | 21 | 54   | 57   |
| 23  | 13  | 19 | 49   | 48   |
| 29  | 18  | 23 | 58   | 56   |

(3.)

| deg. | mi.           | fur. | rd. | ft.           | in. |
|------|---------------|------|-----|---------------|-----|
| 18   | 19            | 7    | 15  | 11            | 1   |
| 61   | 47            | 6    | 89  | 10            | 11  |
| 78   | 32            | 6    | 14  | 9             | 9   |
| 17   | 159           | 7    | 36  | 16            | 10  |
| 28   | 56            | 1    | 30  | 16            | 1   |
| 205  | 8             | 5    | 17  | 14            | 8   |
|      | $\frac{1}{2}$ | 4    |     | $\frac{1}{2}$ | 6   |
| 205  | 9             | 1    | 17  | 15            | 2   |

(4.)

| A.  | R. | per. | sq. yd. | sq. ft. |
|-----|----|------|---------|---------|
| 140 | 3  | 17   | 27      | 6       |
| 320 | 1  | 30   | 14      | 2       |
| 111 |    | 7    | 3       |         |
| 214 | 2  | 15   | 22      | 7       |
| 100 | 3  |      | 6       | 1       |
| 25  | 1  | 36   |         | 8       |
| 104 | 2  | 9    | 1       | 4       |

5. What is the sum of 20lb. 9oz. 19pwt. 23gr., 10lb. 7oz. 15pwt. 13gr., 11oz. 8gr., and 1lb. 8oz. 17pwt. 21gr. ? Ans. 34lb. 1oz. 13pwt.

6. Find the sum of 81lb 11s 6s 1s 19gr., 75lb 10s 7s 2s 13gr., 14lb 9s 7s 1s 12gr., 37lb 8s 1s 1s 11gr., 61lb 11s 3s 2s 3gr.  
*Ans.* 272lb 4s 3s 18gr.
7. Add 197sq. yd. 4sq. ft. 104½sq. in., 122sq. yd. 2sq. ft. 27½sq. in., 5sq. yd. 8sq. ft. 2½sq. in., and 237sq. yd. 7sq. ft. 128½sq. in.  
*Ans.* 663sq. yd. 4sq. ft. 118.825sq. in.
8. What is the sum of 17mi. 5fur. 8ch. 3rd. 24l., 16mi. 3fur. 7ch. 1rd. 21l., 47mi. 7fur. 9ch. 3rd. 19l., 19mi. 6fur. 6ch. 1rd. 16l., 31mi. 7fur. 1ch. 20l.?  
*Ans.* 133mi. 7fur. 4ch.
9. Add 3S. 22° 50', 24° 36' 25.7", 17° 18.2", 1S. 3° 12' 15.5", 12° 36' 17.8", and 57.3".  
*Ans.* 6S. 3° 33' 14.5".
10. Find the sum of  $\frac{1}{4}$  of a mile,  $\frac{1}{2}$  of a mile,  $\frac{1}{4}$  of a furlong, and  $\frac{1}{4}$  of a yard.  
*Ans.* 6fur. 29rd. 3yd. 1ft. 0¼in.
11. Add  $\frac{1}{4}$  of a ton to  $\frac{1}{2}$  of a cwt.
12. Add  $\frac{1}{8}$  of a week to  $\frac{1}{2}$  of a day. *Ans.* 2da. 9h. 18min.
13. What is the sum of  $\frac{1}{2}$  of an acre and  $\frac{1}{2}$  of a rood?  
*Ans.* 3R. 10sq. rd. 8sq. yd. 5sq. ft. 113½sq. in.
14. Find the sum of  $\frac{1}{2}$  of a cwt., 8½lb., and 3½oz. by long ton table.
15. A farmer received 60cts. a bushel for 4 loads of corn; the first contained 42.4bu.; the second, 2866lb.; the third, 36½bu.; and the fourth, 39bu. 29lb. How much did he receive for the whole? *Ans.* \$100.83 +.
16. Add  $\frac{1}{2}$  of a yard,  $\frac{1}{3}$  of a yard, and  $\frac{1}{4}$  of a quarter.

## SUBTRACTION OF COMPOUND NUMBERS.

*Ex.* 1. From £35 6s. 10d. 1far. take £14 15s. 8d. 3far.

## OPERATION.

|      | £  | s. | d. | far. |
|------|----|----|----|------|
| From | 35 | 6  | 10 | 1    |
| Take | 14 | 15 | 8  | 3    |
| Rem. | 20 | 11 | 1  | 2    |

**ANALYSIS.**—Writing the subtrahend under the minuend, placing units of the same denomination under each other, we begin at the right-hand; since we cannot take 3far. from 1far., we add 1d. or 4far. to 1far., making 5far.; and taking 3far. from 1far., we write the remainder, 2far., underneath the column of farthings. Having

added 1d. or 4far. to the minuend, we now add 1d. to the 8 in the subtrahend, making 9d.; and 9d. from 10d. leaves 1d., which we write in the remainder. Next, as we cannot take 15s. from 6s., we add £1 or 20s. to 6s., making 26s., and taking 15s. from 26s., we write the remainder, 11s., under the denomination of shillings. Adding £1 to £14, we subtract £15 from £35, as in simple numbers, and write the remainder, £20, under the column of £.

*Ex.* 2. From  $\frac{1}{2}$  of a mile subtract  $\frac{1}{4}$  of a furlong.

## OPERATION.

|                    |   |       |       |      |      |       |
|--------------------|---|-------|-------|------|------|-------|
| $\frac{1}{2}$ mi.  | = | 4fur. | 17rd. | 4yd. | 0ft. | 10in. |
| $\frac{1}{4}$ fur. | = | 22    | 4     | 2    | 1    | 4     |
| <i>Ans.</i>        |   | 3     | 34    | 4½   | 1    | 8½    |

**ANALYSIS.**—We perform the same reduction as in addition of denominate fractions, (234), and then subtract the less value from the greater.

$$\text{Or, } \frac{1}{2} \text{mi.} \times 8 = 4 \text{fur.}$$

$$40 - 4 = 36 =$$

$$/3 \text{fur. } 34 \text{rd. } 4 \frac{1}{2} \text{yd. } 1 \text{ft. } 8 \frac{1}{2} \text{in.}$$

**237. RULE.—I.** Write the subtrahend under the minuend, so that units of the same denomination shall stand under each other.

**II.** Beginning at the right-hand, subtract each denomination separately, as in simple numbers.

**III.** If any term of the minuend is less than the corresponding term of the subtrahend, add to that term as many units as are required of that denomination to make one of the next higher, and from the sum take the term of the subtrahend, and add 1 to the next term of the subtrahend before subtracting.

**IV.** Proceed in like manner with each denomination.

EXAMPLES FOR PRACTICE.

(1.)

|       |      |     |     |     |     |
|-------|------|-----|-----|-----|-----|
| T.    | cwt. | qr. | lb. | oz. | dr. |
| 71    | 18   | 1   | 13  | 1   | 13  |
| 19    | 19   | 2   | 16  | 8   | 5   |
| <hr/> |      |     |     |     |     |
| 51    | 18   | 2   | 21  | 9   | 8   |

(2.)

|       |   |   |   |     |
|-------|---|---|---|-----|
| H     | h | s | d | gr. |
| 15    | 7 | 3 | 1 | 14  |
| 11    | 9 | 7 | 2 | 19  |
| <hr/> |   |   |   |     |
| 3     | 9 | 3 | 1 | 15  |

(3.)

|       |     |               |     |     |     |               |
|-------|-----|---------------|-----|-----|-----|---------------|
| deg.  | mi. | fur.          | rd. | yd. | ft. | in.           |
| 95    | 3   | 7             | 31  | 1   | 1   | 3             |
| 18    | 17  | 1             | 39  | 1   | 2   | 7             |
| <hr/> |     |               |     |     |     |               |
| 76    | 55  | $\frac{1}{2}$ | 5   | 31  | 4   | $\frac{1}{2}$ |
|       |     | $\frac{1}{2}$ | =1  | 13  | 1   | 2             |
| <hr/> |     |               |     |     |     |               |
| 76    | 55  | 7             | 5   | 1   | 1   | 2             |

(4.)

|       |    |    |     |                   |
|-------|----|----|-----|-------------------|
| A.    | R. | p. | ft. | in.               |
| 96    | 1  | 13 | 100 | 113               |
| 89    | 3  | 17 | 200 | 117               |
| <hr/> |    |    |     |                   |
| 6     | 1  | 35 | 171 | $\frac{1}{2}$     |
|       |    |    |     | $\frac{1}{2}$ =36 |
| <hr/> |    |    |     |                   |
| 6     | 1  | 35 | 172 | 32                |

5. From £23 18s. 3 $\frac{1}{2}$ d. take £13 14s. 10 $\frac{1}{2}$ d. Ans. £10 3s. 5 $\frac{1}{2}$ d.
6. From 71lb. 3oz. 12pwt. 15gr. take 16lb. 10oz. 17pwt. 20gr.
7. Subtract 3h 8s 2s 2s 18gr. from 10h 7s 4s 1s 15gr.
8. From 171T, 3hhd. 8gal. 1qt. 1pt. 1gi. take 99T. 1hhd. 19gal. 3qt. 1pt. 3gt. Ans. 72T. 1hhd. 51gal. 1qt. 1pt. 2gi.
9. From 56A. 1R. 19p. 119ft. 110in. take 17A. 3R. 13p. 127ft. 113in. Ans. 38A. 2R. 5p. 264ft. 33in.
10. From 16mi. 7fur. 18rd. 3ft. lin. take 9mi. 7fur. 19rd. 16ft. 8in. Ans. 6mi. 7fur. 38rd. 2ft. 11in.
11. From  $\frac{1}{2}$  of a bushel take  $\frac{1}{8}$  of a peck. Ans. 1pt. 4qt. 1pt.
12. From  $\frac{1}{4}$  of a week take  $\frac{1}{7}$  of a day. Ans. 4da. 3h.
13. Subtract  $\frac{1}{4}$  of 9cwt. from  $\frac{1}{2}$  of 5 tons.
14. From 51bbl. take  $\frac{1}{4}$  of a hogshead. Ans. 4bbl. 11gal. 1qt.
15. Subtract 0.659 week from 2 weeks 3 $\frac{1}{2}$  days.
16. From a hogshead of sirup containing 100 gallons,  $\frac{1}{4}$  of it leaked out, and  $\frac{1}{2}$  of the remainder was sold; what quantity still remained for sale? Ans. 24gal. 0qt. 1 $\frac{1}{2}$ pt.

## PRACTICAL PROBLEMS IN COMPOUND ADDITION AND SUBTRACTION.

1. I had 10A. 3R. 10per. of land; and I have sold two house-lots, one containing 1A. 1R. 13per., the other 2A. 2R. 5per.; how much have I remaining?
2. An excavation 58ft. long 37ft. wide, and 6ft. deep is to be made for a cellar; after 471cu. yd. 16cu. ft. 972cu. in. of earth have been removed, how much still remains to be taken out?
3. Bought a hogshead of sugar weighing 9cwt. 3qr. 21lb.; sold John 1cwt. 2qr. 15lb.; to Bernard 2cwt. 3qr. 24lb.; and to Thomas 3cwt. 1qr. 15lb.; how much remains unsold? *Ans.* 1cwt. 3qr. 17lb.
4. Joseph and Henry start from two places 120 miles apart, and travel toward each other; after Joseph travels  $\frac{2}{3}$ , and Henry  $\frac{1}{3}$ , of the distance, how far are they apart? *Ans.* 41mi. 1fur. 9rd. 8ft. 7 $\frac{1}{2}$ in.
5. A man agrees to build 136 rd. and 15ft. of stone fence; at one time, he builds 36rd. 2ft.; at another time, 56rd. 3ft.; and at another time, 10rd. 1ft. How much still remains to be built?
6. A merchant sold goods to the amount of £397 18s. 6 $\frac{1}{2}$ d.; and received in payment £199 19s. 10 $\frac{1}{2}$ d.; how much remains due?
7. A hogshead of wine, lost by leakage, on an average, for 5 years, including two leap years, one gill of wine a day; how much remained? *Ans.* 5gal. 3qt. 1pt. 1gi.
8. Suppose a person was born February 29, 1792; how many anniversaries of his birthday will he have had on Feb. 29, 1844?
9. How long has a note to run, dated April 23, 1870, and made payable Dec. 9, 1874? *Ans.* 4yr. 7mo. 16da.
10. From a mass of silver weighing 106lb., a goldsmith made 36 spoons, weighing 5lb. 11oz. 12pwt. 15gr.; a tankard, 3lb. 0oz. 13pwt. 14gr.; a vase, 7lb. 11oz. 14pwt. 23gr.; how much unwrought silver remains? *Ans.* 88lb. 11oz. 18pwt. 20gr.
11. From a pile of wood containing 423 cords, I sold at one time, 56C. 112cu. ft.; at another time, 97C. 113cu. ft.; at another time, 126C. 97cu. ft. How many cords remain unsold?
12. Suppose a note given Sept. 10, 1856, to be paid March 5, 1868. How long was the note on interest, if we count 30 days to the month? How long, if the time is computed by days? *Ans.* 1st. 11yr. 5mo. 25da.; 2nd. 4135 days.

## MULTIPLICATION OF COMPOUND NUMBERS.

*Ex.* 1. Multiply £8 9s. 5d. by 6.

| OPERATION. |      |     |
|------------|------|-----|
| £          | s.   | d.  |
| 8          | 9    | 5   |
|            |      |     |
| 6          |      |     |
| £50        | 16s. | 6d. |

*ANALYSIS.*—6 times 5d. are 30d. = 2s. 6d. We write the 6d. under the pence, and add the 2s. with the product of shillings. 6 times 9s. are 54s. and 2s., are 56s. = £2 16s. We write the 16s. under the shillings and add the £2 with the product of pounds. 6 times £8 are £48, and £2 = £50 which we write under pounds. Therefore 6 times £8 9s. 5d. = £50 16s. 6d.

23

ination

II

of com

NOTE

visible

2. W

solved 1

Ex.

10

£ 97

23

multip

Ea

1 bb

10 bb

100 bb

ANALY

factors;

arately?

and obtai

the valu

last prod

10 barre

barrel by

answer.

240

ber, res

multipl

obtaine

**238. RULE.—I.** Write the multiplier under the lowest denomination of the multiplicand.

**II.** Multiply as in simple numbers, and carry as in addition of compound numbers.

NOTE.—1. When the multiplier is large, and is a composite number, it is advisable to multiply by the component factors.

2. When the multiplier is large, and is not a composite number, it may be resolved into any convenient parts, and multiplication made by these several parts.

*Ex.* 2. What will 45 yards of cloth cost, at £2 3s. 6d. per yard ?

| OPERATION. |      |                        |
|------------|------|------------------------|
| £          | s.   | d.                     |
| 2          | 3    | 6                      |
|            |      | 6 = price of 1 yard.   |
|            | 1    | 5                      |
| 10         | 17   | 6                      |
|            |      | 6 = price of 5 yards.  |
|            |      | 9                      |
| £ 97       | 17s. | 6d. = price of 45 yds. |

**ANALYSIS.**—We find the number 45 equal to the product of 5 and 9; we therefore multiply the price of 1 yard by 5, and then that product by 9; and the last product is the answer. Hence the

**239. RULE.**—When the multiplier is a composite number, multiply by its factors in succession.

*Ex.* 3. What cost 643 barrels of flour, at £2 5s. 7d. per bbl. ?

| OPERATION. |     |    |    |       |      |    |                       |
|------------|-----|----|----|-------|------|----|-----------------------|
|            | £   | s. | d. |       | £    | s. | d.                    |
| 1 bbl. =   | 2   | 5  | 7  | × 3 = | 6    | 16 | 9                     |
|            |     |    | 10 |       |      |    | 9 = value of 3 bbl.   |
| 10 bbl. =  | 22  | 15 | 10 | × 4 = | 91   | 3  | 4                     |
|            |     |    | 10 |       |      |    | 4 = value of 40 bbl.  |
| 100 bbl. = | 227 | 18 | 4  | × 6 = | 1367 | 10 | 0                     |
|            |     |    |    |       | 1465 | 10 | 1                     |
|            |     |    |    |       |      |    | 1 = value of 600 bbl. |
|            |     |    |    |       |      |    | 1 = value of 643 bbl. |

**ANALYSIS.**—Since 643 is not a composite number, we cannot resolve it into factors; but we may separate it into parts, and find the value of each part separately: thus, 643 = 600 + 40 + 3. In the operation, we first multiply by 10, and obtain the value of 10 barrels, and this product we multiply by 10, and obtain the value of 100 barrels. Then, to find the value of 600 barrels, we multiply the last product by 6; and to find the value of 40 barrels, we multiply the value of 10 barrels by 4; and to find the value of 3 barrels, we multiply the value of 1 barrel by 3. Adding the several products, we obtain £1465 10s. 1d. for the answer. Hence the

**240. RULE.**—When the multiplier is not a composite number, resolve it into any convenient parts, as of units, tens, etc., multiply by these several parts, and add together the products thus obtained for the required result.

## EXAMPLES FOR PRACTICE.

|             |             |            |            |            |            |             |                |                |             |            |             |            |
|-------------|-------------|------------|------------|------------|------------|-------------|----------------|----------------|-------------|------------|-------------|------------|
| (1.)        |             |            |            | (2.)       |            |             |                | (3.)           |             |            |             |            |
| <i>cut.</i> | <i>qr.</i>  | <i>lb.</i> | <i>oz.</i> | <i>lb.</i> | <i>oz.</i> | <i>pwt.</i> | <i>gr.</i>     | <i>lb.</i>     | <i>s.</i>   | <i>5.</i>  | <i>9.</i>   | <i>gr.</i> |
| 18          | 3           | 17         | 10         | 82         | 8          | 17          | 12             | 33             | 10          | 5          | 2           | 14         |
|             |             |            | 6          |            |            |             | 8              |                |             |            |             | 11         |
| <hr/>       |             |            |            | <hr/>      |            |             |                | <hr/>          |             |            |             |            |
| 113         | 2           | 5          | 12         | 261        | 11         | 13          | 3              | 427            | 10          | 0          | 2           | 14         |
| (4.)        |             |            |            | (5.)       |            |             |                | (6.)           |             |            |             |            |
| <i>mi.</i>  | <i>fur.</i> | <i>rd.</i> | <i>ft.</i> | <i>A.</i>  | <i>R.</i>  | <i>p.</i>   | <i>sq. yd.</i> | <i>sq. ft.</i> | <i>deg.</i> | <i>mi.</i> | <i>fur.</i> | <i>rd.</i> |
| 14          | 6           | 36         | 14         | 7          | 1          | 33          | 21             | 7              | 18          | 12         | 6           | 18         |
|             |             |            | 9          |            |            |             |                | 6              |             |            |             | 8          |
| <hr/>       |             |            |            | <hr/>      |            |             |                | <hr/>          |             |            |             |            |

7. How much cloth will it take for 8 suits of clothes, if each suit require 8yd. 1qr. 3na. ? *Ans.* 67yd. 2qr.

8. A man gives each of his 9 sons 23A. 3R. 19½p., what do they all receive? *Ans.* 214A. 3R. 12p.

9. How long will it take a man to saw eleven cords of wood, if it take him 8h. 45min. 50sec., to saw 1 cord?

10. If 1 share in a certain stock be valued at £13 8s. 9½d., what is the value of 96 shares? *Ans.* £1290 4s. 0d.

11. If a family consume 12gal. 3qt. 1pt. of molasses in one week, what quantity will they consume in 1 year?

12. If a man be 2da. 5h. 17min. 19sec. in walking 1 degree, how long would it take him to walk round the earth, allowing 365½ days to a year? *Ans.* 2y. 68da. 19h. 54min.

13. What will be the value of 1 dozen gold cups, each cup weighing 9oz. 13pwt. 8gr., at \$212.33 a pound?

14. If a ship sails 3° 24' 10" per day, how far will she sail in 60 days? *Ans.* 204° 10'.

15. One ton of copper ore will buy 17T. 14cut. 3qr. 18lb. 14oz. of iron ore; how much will 451 tons buy?

16. If \$30 will buy 4A. 3R. 26sq. yd. 3sq. ft. of land, how much will \$4800 buy? *Ans.* 275A. 10sq. yd.

17. If 1 cask of oil contains 86gal. 2qt. 1pt., how much will 100 casks of the same size contain?

18. What is the cost of a board 18ft. 9in. long, and 2ft. 3½in. wide, at \$0.053 per foot? *Ans.* \$2.277½.

19. Bought 17 bags of hops, each weighing 4cut. 3qr. 7lb., at \$5.87½ per cwt.; what was the cost?

20. What cost 27T. 15cut. 1qr. 3½lb. of hemp, at \$183.62 per ton? *Ans.* \$5098.07 +.

21. At \$125.75 per acre, what cost 37A. 3R. 35rd. ?

22. What cost the construction of 17mi. 6fur. 36rd. of railroad, at \$3765.60 per mile? *Ans.* \$67263.03 +.

23. Bought a farm containing 144A. 3R. 30per., at \$97.62½ per acre; what was the cost of the farm? *Ans.* \$14149.52 +.

24. At \$9.25 per cwt., what cost 19cut. 3qr. 14lb. of iron?

Par

10s.

6s.

5s.

4s.

3s.

2s.

1s.

1s.

1s.

1s.

Par

6 d

4 d

3 d

2 d

1½ d

1 d

(1)  
the ali

24

pence,

or shi

farth

Ex

½d. =

¼d. =

MULTIPLICATION OF COMPOUND NUMBERS

SOLVED BY ALIQUOT PARTS.

TABLE OF ALIQUOT PARTS (173).

| Parts of £1.             | Parts of a cwt. (1) of 112lb. | Parts of 1lb. Avoirdupois.  | Parts of 1oz. Troy.        | Parts of a year.          |
|--------------------------|-------------------------------|-----------------------------|----------------------------|---------------------------|
| 10s. = $\frac{1}{2}$     | 56 lb. = $\frac{1}{2}$        | 8oz. = $\frac{1}{2}$        | 5pwt. 0gr. = $\frac{1}{4}$ | 6 months = $\frac{1}{2}$  |
| 6s. 8d. = $\frac{1}{3}$  | 28 lb. = $\frac{1}{3}$        | 4oz. = $\frac{1}{3}$        | 4 " 0 " = $\frac{1}{3}$    | 4 " " = $\frac{1}{3}$     |
| 5s. = $\frac{1}{4}$      | 16 lb. = $\frac{1}{4}$        | 2oz. = $\frac{1}{4}$        | 3 " 8 " = $\frac{1}{4}$    | 3 " " = $\frac{1}{4}$     |
| 4s. = $\frac{1}{5}$      | 14 lb. = $\frac{1}{5}$        | 1oz. = $\frac{1}{5}$        | 2 " 12 " = $\frac{1}{5}$   | 2 " " = $\frac{1}{5}$     |
| 3s. 4d. = $\frac{1}{6}$  | 8 lb. = $\frac{1}{6}$         |                             | 2 " 0 " = $\frac{1}{6}$    | 1 1/2 " " = $\frac{1}{6}$ |
| 2s. 6d. = $\frac{1}{8}$  | 7 lb. = $\frac{1}{8}$         |                             | 1 " 16 " = $\frac{1}{8}$   | 1 " " = $\frac{1}{8}$     |
| 2s. = $\frac{1}{10}$     | 4 lb. = $\frac{1}{8}$         | Parts of 1lb. Troy.         |                            |                           |
| 1s. 8d. = $\frac{1}{12}$ | 3 1/2 lb. = $\frac{1}{8}$     |                             | Parts of 1 acre.           | Parts of a month.         |
| 1s. 4d. = $\frac{1}{15}$ | 2 lb. = $\frac{1}{8}$         | 6oz. = $\frac{1}{2}$        | 2R. = $\frac{1}{2}$        | 15 days = $\frac{1}{2}$   |
| 1s. 3d. = $\frac{1}{16}$ |                               | 4oz. = $\frac{1}{3}$        | 1R. = $\frac{1}{3}$        | 10 " " = $\frac{1}{3}$    |
| 1s. = $\frac{1}{20}$     |                               | 3oz. = $\frac{1}{4}$        | 20per. = $\frac{1}{3}$     | 7 1/2 " " = $\frac{1}{3}$ |
|                          | Parts of a quarter of 28lb.   | 2oz. = $\frac{1}{5}$        | 16per. = $\frac{1}{4}$     | 6 " " = $\frac{1}{4}$     |
|                          |                               | 1oz. 10pwt. = $\frac{1}{5}$ |                            | 5 " " = $\frac{1}{5}$     |
|                          |                               | 1oz. = $\frac{1}{8}$        | Parts of 1 rood.           | 3 " " = $\frac{1}{5}$     |
|                          |                               |                             |                            | 2 " " = $\frac{1}{6}$     |
|                          |                               | Parts of 1oz. Troy.         |                            | 1 " " = $\frac{1}{6}$     |
|                          |                               |                             |                            |                           |
| 6 d. = $\frac{1}{4}$     | 14 lb. = $\frac{1}{4}$        | 10pwt. 0gr. = $\frac{1}{2}$ | 10per. = $\frac{1}{2}$     |                           |
| 4 d. = $\frac{1}{3}$     | 7 lb. = $\frac{1}{3}$         | 6 " 16 " = $\frac{1}{2}$    | 8per. = $\frac{1}{3}$      |                           |
| 3 d. = $\frac{1}{4}$     | 4 lb. = $\frac{1}{4}$         |                             |                            |                           |
| 2 d. = $\frac{1}{6}$     | 3 1/2 lb. = $\frac{1}{4}$     |                             |                            |                           |
| 1 1/2 d. = $\frac{1}{8}$ | 1 1/2 lb. = $\frac{1}{8}$     |                             |                            |                           |
| 1 d. = $\frac{1}{12}$    |                               |                             |                            |                           |

(1) The aliquot parts of the short ton or new cwt. of 100lb. are the same as the aliquot parts of £1 (p. 105).

**241. CASE I.**—When the given price is: 1<sup>o</sup> farthings; 2<sup>o</sup> pence, or pence and farthings; 3<sup>o</sup> shillings, shillings and pence, or shillings, pence and farthings; 4<sup>o</sup> pounds, shillings, pence and farthings.

*Ex.* Find the price of 944 pens, at 1/4 d. per pen.

OPERATION.

944 pens at 1d. = 944d. = £3 18 8

1/4 d. = 1/4 of 1d.; 1/4 of £3 18 8 = £1 19 4 = price of 944 pens at 1/4 d.

1/4 d. = 1/4 of 1/4 d.; 1/4 of £1 19 4 = " " " " " 1/4 d.

Ans. £2 19 0 = " " " " " 1/4 d.

ANALYSIS.—In this example, the price being *farthings*, we multiply the given number by a penny; but, as  $\frac{3}{4}d.$  is not an even part of a penny, we decompose it into  $\frac{1}{4}d.$  and  $\frac{1}{2}d.$ ;  $\frac{1}{4}d.$  is the half of a penny, and  $\frac{1}{2}d.$ , the fourth of a penny, or the half of  $\frac{1}{4}d.$  We then take the  $\frac{1}{4}$  of £3 18 8 for  $\frac{1}{4}d.$ , giving for result £1 19 4; then  $\frac{1}{2}d.$ , or  $\frac{1}{2}$  of  $\frac{1}{4}d.$ , that is, one half of £1 19 4 = 19s. 8d., which we add to £1 19 4; the sum then gives £2 19 0, for the answer.

Ex. 2. What cost 1638lb. of sugar, at  $8\frac{1}{4}d.$  per lb.?

OPERATION.

1638lb. at 1s. = 1638s. = £81 18 0  
 $6d. = \frac{1}{2}$  of 1s.;  $\frac{1}{2}$  of £81 18 0 = £40 19 0 = price of 1638lb. at 6 d.  
 $2d. = \frac{1}{2}$  of 6d.;  $\frac{1}{2}$  of £40 19 0 = £13 13 0 = " " " " 2 d.  
 $0\frac{1}{4}d. = \frac{1}{4}$  of 2d.;  $\frac{1}{4}$  of £13 13 0 = £ 3 8 3 = " " " "  $\frac{1}{4}d.$   
 Ans. £58 0 3 = " " " "  $8\frac{1}{4}d.$

ANALYSIS.—The price being *pence* and *farthings*, we multiply the given number by a shilling. Now, as  $8\frac{1}{4}d.$  is not an aliquot part of a shilling, we decompose it into 6d., 2d., and  $\frac{1}{4}d.$ , and then proceed as in the foregoing example.

Ex. 3. Find the price of 252 yards of merino, at 3s.  $9\frac{1}{4}d.$  per yd.

OPERATION.

252 yards at £1 = £252  
 $3s. 4d. = \frac{1}{4}$  of £1 £42 0 0 = price at 3s. 4 d. per yd.  
 $0s. 5d. = \frac{1}{4}$  of 3s. 4d. 5 5 0 = " " 0s. 5 d. " "  
 $0s. 0\frac{1}{4}d. = \frac{1}{16}$  of 5d. 0 10 6 = " " 0s. 0 $\frac{1}{4}d.$  " "  
 Ans. £47 15 6 = " " 3s.  $9\frac{1}{4}d.$  " "

ANALYSIS.—Here, the price being *shillings*, etc., we multiply the given number by a pound; then, we decompose 3s.  $9\frac{1}{4}d.$  into 3s. 4d., 5d., and  $0\frac{1}{4}d.$ , and proceed as in the preceding examples:

Ex. 4. What cost 694 cwt. of butter, at £5 11  $6\frac{1}{4}$  per cwt.?

OPERATION.

£694.....=price of 694cwt. at £1 a cwt.  
 $694cwt. \times £5 = £3470$  0 0 = " " 694 " " £5 " "  
 $10s. 0d. = \frac{1}{2}$  of £1 347 0 0 = " " " " 0 10 0 " "  
 $1s. 3d. = \frac{1}{4}$  of 10s. 43 7 6 = " " " " 0 13 " "  
 $0s. 3d. = \frac{1}{4}$  of 1s. 3d. 8 13 6 = " " " " 0 03 " "  
 $0s. 0\frac{1}{4}d. = \frac{1}{8}$  of 0s. 3d. 1 8 11 = " " " " 0 0 0 $\frac{1}{4}$  " "  
 Ans. £3870 9 11 = " " " " £5 11  $6\frac{1}{4}$  " "

EXAMPLES FOR PRACTICE.

|                                   | Answers. |    |         |                                   | Answers. |    |                |
|-----------------------------------|----------|----|---------|-----------------------------------|----------|----|----------------|
|                                   | s.       | d. | £ s. d. |                                   | s.       | d. | £ s. d.        |
| 1. $664 \times 0 0\frac{1}{4} =$  | 0        | 13 | 10      | 5. $1078 \times 0 0\frac{1}{4} =$ | 1        | 2  | $6\frac{1}{4}$ |
| 2. $1732 \times 0 0\frac{1}{4} =$ | 3        | 12 | 2       | 6. $1683 \times 0 2\frac{1}{4} =$ |          |    |                |
| 3. $1984 \times 0 0\frac{1}{4} =$ |          |    |         | 7. $2142 \times 0 5\frac{1}{4} =$ | 51       | 6  | $4\frac{1}{4}$ |
| 4. $1896 \times 0 0\frac{1}{4} =$ | 2        | 12 | 8       | 8. $1053 \times 0 5\frac{1}{4} =$ | 23       | 0  | $8\frac{1}{4}$ |

9.  
10.  
11.  
12.  
13.  
14.  
15.  
16.  
17.  
18.  
19.  
20.  
21.  
22.  
23.  
24.  
25.  
26.  
27.  
28.  
29.  
30.  
31.  
32.

24  
tity.

Ex.

2s.  
5d.  
 $\frac{1}{4}$   
 $\frac{1}{8}$

ANALYSIS  
parts of  
 $\frac{1}{4}$  yard,  
the sum

2s. 6d.  
0s. 5d.

ply the given  
we decompose  
of a penny, or  
result £1 19 4;  
which we add to

8lb. at 6 d.  
" " 2 d.  
" " 1/2 d.  
" " 8 1/2 d.

e given num-  
e, we decom-  
example.

d. per yd.

per yd.  
" "  
" "  
" "

given number  
d., and pro-

cwt. ?

a cwt.  
" "  
0 " "  
13 " "  
3 " "  
0 0 1/2 " "  
6 1/2 " "

|     |                   | Answers. |      |              |     |                      | Answers. |      |             |
|-----|-------------------|----------|------|--------------|-----|----------------------|----------|------|-------------|
|     |                   | £ s. d.  |      |              |     |                      | £ s. d.  |      |             |
| 9.  | 5728 × 0 7 1/2 =  | 173      | 0    | 8            | 33. | 1893 × 0 4 10 1/2 =  | 248      | 10   | 5           |
| 10. | 5430 × 0 3 =      | 67       | 17   | 6            | 34. | 604 × 0 8 2 1/2 =    | 868      | 14   | 6           |
| 11. | 2436 × 0 6 1/2 =  | 31       | 6    | 2 1/2        | 35. | 2916 × 0 5 11 1/2 =  | 1960     | 15   | 0           |
| 12. | 2147 × 0 3 1/2 =  | 241      | 11   | 9            | 36. | 5348 × 0 7 8 1/2 =   | 1094     | 0    | 6           |
| 15. | 7028 × 0 8 1/2 =  | 171      | 11   | 10 1/2       | 37. | 3720 × 0 10 6 1/2 =  | 3113     | 19   | 10 1/2      |
| 14. | 2708 × 0 6 1/2 =  | 115      | 11   | 10           | 38. | 1509 × 0 14 6 =      | 61       | 8    | 6           |
| 15. | 5491 × 0 7 1/2 =  | 129      | 15   | 2 1/2        | 39. | 878 × 0 11 4 1/2 =   | 71       | 6    | 7           |
| 16. | 4936 × 0 8 1/2 =  | 75       | 19   | 4 1/2        | 40. | 4571 × 0 13 7 1/2 =  | 88       | 2    | 6           |
| 17. | 4967 × 0 10 1/2 = | 105      | 3    | 9            | 41. | 54 × 1 2 9 =         | 3785     | 9    | 4           |
| 18. | 2522 × 0 11 =     | 51       | 0    | 0            | 42. | 62 × 1 7 4 1/2 =     | 14179    | 18   | 8 1/2       |
| 19. | 2897 × 0 10 1/2 = | 84       | 5    | 7 1/2        | 43. | 17 × 4 3 11 =        | 48       | 972  | 3 15 10     |
| 20. | 7509 × 0 11 1/2 = | 892      | 1    | 2 1/2        | 44. | 42 × 5 10 3 1/2 =    | 8198     | 13   | 3           |
| 21. | 1870 × 0 9 1/2 =  | 382      | 4    | 4            | 45. | 472 × 5 10 3 1/2 =   | 10023    | 18   | 7 1/2       |
| 22. | 2244 × 0 11 1/2 = | 32       | 4    | 4            | 46. | 1958 × 1 18 8 =      | 51       | 1415 | 4 11 10 1/2 |
| 23. | 392 × 1 8 =       | 892      | 1    | 2 1/2        | 47. | 2471 × 5 14 9 1/2 =  | 21033    | 8    | 9           |
| 24. | 576 × 1 9 1/2 =   | 892      | 1    | 2 1/2        | 48. | 1077 × 7 12 3 =      | 53       | 2150 | 9 16 1 1/2  |
| 25. | 465 × 3 7 1/2 =   | 1740     | 6    | 8            | 49. | 1077 × 7 12 3 =      | 12818    | 18   | 1 1/2       |
| 26. | 425 × 4 11 1/2 =  | 55       | 6494 | 6 19 5 1/2 = | 50. | 3714 × 2 13 11 1/2 = | 54       | 7251 | 8 7 7 1/2   |
| 27. | 1349 × 5 8 =      | 863      | 13   | 9 1/2        | 51. | 1415 × 4 11 10 1/2 = | 55       | 6494 | 6 19 5 1/2  |
| 28. | 7045 × 0 7 =      | 56       | 7122 | 9 13 4 1/2 = | 52. | 2150 × 9 16 1 1/2 =  | 68860    | 16   | 9           |
| 29. | 2426 × 7 4 1/2 =  |          |      |              | 53. | 2175 × 5 17 10 1/2 = |          |      |             |
| 30. | 1454 × 6 5 1/2 =  |          |      |              | 54. | 2175 × 5 17 10 1/2 = |          |      |             |
| 31. | 3632 × 9 7 =      |          |      |              | 55. | 7251 × 8 7 7 1/2 =   |          |      |             |
| 32. | 6741 × 2 6 1/2 =  |          |      |              | 56. | 6494 × 6 19 5 1/2 =  |          |      |             |

**242. CASE II.**—When there is a fraction in the given quantity.

*Ex.* Required the price of 158 1/2 yards of cloth, at £1 2 11 per yd.

OPERATION.

158 1/2 yards, at £1 2 11

|                          |                                       |
|--------------------------|---------------------------------------|
| 2s. 6d. = £ 1/2          | 19 15 0 = price of 158 yd. at 2s. 6d. |
| 5d. = 1/4 of 2s. 6d.     | 3 5 10 = " " " " " 0s. 5d.            |
| 1/2 of £1 2 11           | 0 11 5 1/2 = " " " " " 1/2            |
| 1/4 of 11s. 5 1/2 d.     | 0 5 8 1/2 = " " " " " 1/4             |
| Ans. . . . £181 18 0 1/2 |                                       |

**ANALYSIS.**—In this process, the price of 158 yards is first found (or rather the parts composing it are found) according to the method of Case I.; and then, for 1/2 yard, the half of £1 2 11 is taken, and for 1/4 yard, the half of that is found: the sum of all which parts, is £181 18 0 1/2, the result required.

ANOTHER METHOD.

158 1/2 yards, at £1 2 11.

|                                            |                                  |
|--------------------------------------------|----------------------------------|
| £158 15 0 = price at £1. . . . . per yard. |                                  |
| 2s. 6d. = £ 1/2                            | 19 16 10 1/2 = " " 0 2s. 6d. " " |
| 0s. 5d. = 1/4 of 2s. 6d.                   | 3 6 1 1/2 = " " 0 0s. 5d. " "    |
| Ans. . . . £181 18 0 1/2 = " " £1 2 11 " " |                                  |

MULTIPLICATION BY ALIQUOT PARTS.

ANALYSIS.—In this method, we first find the price of 158½ yards, at £1 per yard. This is £158 15 0; for the price of 158 yards is £158, and, the price of a quarter of a yard being evidently 5s. 0d., that of ½ of a yard is 15s. Then, the price at £1 per yard being £158 15, the price at 2s. 6d. will be one eighth of this, or £19 16 10½; and the price at 5d., one sixth of the price at 2s. 6d., or £3 6 1½. The sum of these is £181 18 0½, the whole price, as before.

EXAMPLES FOR PRACTICE.

|          | £ | s. | d. |     | £    | s.   | d. |     |
|----------|---|----|----|-----|------|------|----|-----|
| 1. 187½  | × | 1  | 17 | 8   | Ans. | 353  | 2  | 6   |
| 2. 328   | × | 0  | 6  | 6   | Ans. | 106  | 16 | 10½ |
| 3. 208   | × | 0  | 13 | 10  |      |      |    |     |
| 4. 971   | × | 3  | 15 | 2   | Ans. | 3650 | 5  | 7½  |
| 5. 675   | × | 1  | 7  | 2   | Ans. | 917  | 11 | 1   |
| 6. 371   | × | 3  | 14 | 7½  |      |      |    |     |
| 7. 538   | × | 0  | 4  | 8   | Ans. | 125  | 14 | 2   |
| 8. 495   | × | 3  | 5  | 9½  | Ans. | 1630 | 13 | 9   |
| 9. 917   | × | 4  | 18 | 10½ |      |      |    |     |
| 10. 515  | × | 2  | 9  | 4   | Ans. | 1272 | 7  | 9½  |
| 11. 63   | × | 3  | 18 | 9   | Ans. | 249  | 12 | 9   |
| 12. 85   | × | 2  | 7  | 6   |      |      |    |     |
| 13. 172½ | × | 3  | 15 | 10  | Ans. | 654  | 16 | 5   |
| 14. 176½ | × | 0  | 11 | 8   | Ans. | 103  | 2  | 1   |
| 15. 785  | × | 7  | 6  | 3½  |      |      |    |     |
| 16. 239½ | × | 1  | 10 | 10  | Ans. | 369  | 12 | 3½  |
| 17. 375½ | × | 4  | 19 | 11½ | Ans. | 1877 | 7  | 8½  |
| 18. 759½ | × | 2  | 15 | 9½  |      |      |    |     |
| 19. 774½ | × | 9  | 11 | 6   | Ans. | 7416 | 15 | 10½ |
| 20. 749½ | × | 8  | 19 | 10½ | Ans. | 6736 | 5  | 9½  |

243. CASE III.—When the given quantity and price are both of several denominations.

Ex. What is the cost of 94cwt. 2qr. 15lb. of tobacco, at £5 12 6 per cwt.?

OPERATION.

|                     |      |    |    |                         |          |
|---------------------|------|----|----|-------------------------|----------|
| 94cwt. × £5 =       | £470 | 0  | 0  | = cost of 94cwt. at £5  | per cwt. |
| 10s. 0d. = ¼ of £1. | 47   | 0  | 0  | = " " " " 10s.          | " "      |
| 2s. 6d. = ½ of 10s. | 11   | 15 | 0  | = " " " " 2s. 6d.       | " "      |
| 2qr. = ¼ of 1cwt.   | 2    | 16 | 3  | = " " 2qr. at £5 12 6   | " "      |
| 10lb. = ¼ of 2qr.   | 0    | 11 | 3  | = " " 10lb. " " " " " " | " "      |
| 5lb. = ½ of 10lb.   | 0    | 5  | 7½ | = " " 5lb. " " " " " "  | " "      |
| Ans. . . .          | £532 | 8  | 1½ | = cost required         |          |

ANOTHER METHOD.

|                   |      |    |    |                                      |
|-------------------|------|----|----|--------------------------------------|
| £5 12 6 × 94 =    | £528 | 15 | 6  | = cost of 94cwt. at £5 12 6 per cwt. |
| 2qr. = ¼ cwt.     | 2    | 16 | 3  | = " " 2qr. " " " " " "               |
| 10lb. = ¼ of 2qr. | 0    | 11 | 3  | = " " 10lb. " " " " " "              |
| 5lb. = ½ of 10lb. | 0    | 5  | 7½ | = " " 5lb. " " " " " "               |
| Ans. . . .        | £262 | 8  | 1½ | = " " 94 2 15 at £5 12 6 per cwt.    |

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.  
10.  
11.  
12.  
13.  
14.  
15.  
  
Ed  
1 bar  
  
24  
numb  
if the  
II.  
reduc  
ber of  
III.  
several  
NOTE  
shorten  
2. Wh  
reduced  
ple num  
  
Ex.  
is the p  
  
6) 57  
4) 9  
2



**245. RULE.**—Divide by the factors of the composite number in succession.

*Ex. 3.* Divide £360 8 4 by 173.

|            |           |             |
|------------|-----------|-------------|
| OPERATION. |           |             |
| 173)       | £ s. d.   | 360 8 4 (£2 |
|            | 14        |             |
|            | 20        |             |
| 173)       | 288 (1s.  |             |
|            | 173       |             |
|            | 115       |             |
|            | 12        |             |
| 173)       | 1384 (8d. |             |
|            | 1384      |             |

**ANALYSIS.**—We divide the pounds by 173, and obtain £2 for the quotient, and £14 remaining, which we reduce to shillings, and add the 8s.; and again, divide by 173, and obtain 1s. for the quotient. The remainder, 115s., we reduce to pence, and add the 4d., and again divide by 173, and obtain 8d. for the quotient. Thus, the method is the same as by general rule (244). By uniting the several quotients, we obtain £2 1 8, for the answer.

*Ex. 4.* Divide £24 3 8 by £3 0 5½.

|                   |             |      |
|-------------------|-------------|------|
| OPERATION.        |             |      |
| £24 3s. 8d. 0far. | 23216 far.  |      |
| £ 3 6s. 5d. 2far. | = 2902 far. | = 8. |

**ANALYSIS.**—Reducing both dividend and divisor to the lowest denomination mentioned in either, and then dividing as in simple numbers, we have 8 for the quotient.

#### EXAMPLES FOR PRACTICE.

|             |             |                   |
|-------------|-------------|-------------------|
| (1.)        | (2.)        | (3.)              |
| T. cwt. lb. | lb. oz. dr. | hhd. gal. qt. pt. |
| 7) 45 15 26 | 9) 143 5 5  | 12) 9 28 2        |
| 6 10 75     | 15 14 13    | 49 2 1            |

4. A man in 1 month travels 746mi. 5fur., how far does he go in 1 day?  
*Ans. 24mi. 7fur. 4rd.*
5. If 21 yards of cloth cost £10 8 3, what is the price of 1 yard?
6. If 35 loads of coal weigh 72T. 14cwt. 2qr. 10lb., what will 1 load weigh?  
*Ans. 2T. 1cwt. 2qr. 6lb.*
7. Divide 28° 51' 27.756" by 2.754.  
*Ans. 10° 28' 42½"*
8. Divide 1275A. 2R. 16per. 22yd. 8ft. 32in. equally among 32 persons.  
*Ans. 39A. 3R. 17per. 30yd. 8ft. 100in.*
9. When 96 shares of a certain stock are valued at £1290 4s. 6d., what would be the cost of 1 share?
10. If a town 4 miles square be divided equally into 124 farms, how much will each farm contain?  
*Ans. 82A. 2R.. 12½per.*
11. Divide 57T. 19cwt. 42lb. 14oz. by 123.
12. If a man walk round the earth in 2yr. 68da. 19h. 54min., how long would it take him to walk 1 degree, allowing 365½ days to a year?  
*Ans. 2da. 5h. 17min. 19sec.*
13. Divide 916mi. 3fur. 30rd. 10ft. 6in. by 47.
14. How many times are £5 10 10 contained in £537 10 10?

15. Divide 336bu. 3pk. 4qt. by 4bu. 3pk. 2qt. Ans. 70.  
 16. Divide 121lb 3s 2s 1s 4gr. by 13½.  
 17. A merchant sold to each of a certain number of farmers 6bu. 1pk. 7qt. of grass seed, and to them all he sold 71bu. 5qt. How many farmers were there? Ans. 11.  
 18. Divide 3794cu. yd. 20cu. ft. 709½cu. in. by 33½.

LONGITUDE AND TIME.

**246. Meridians of Longitude** are direct lines on the globe, from the north pole to the south pole, crossing the equator at right angles.

**247. Longitude** is distance on the globe, east or west of a determined meridian. In the British Isles and on this continent, also generally on the ocean, the meridian of Greenwich Observatory, England is the determined meridian. All parts of the earth on this line are considered to have no longitude.

The highest longitude any place on the earth can have is 180° east, or 180° west from the determined meridian.

**248. The Equator and parallels of latitude** being circles, are divided into 360°, called degrees of *Longitude*.

NOTES.—1. The earth revolves on its axis from west to east once in 24 hours, which constitute a solar day. The middle of this day is 12 noon. When the sun is directly over the meridian of a place, it is noon at that place, and at places west of this meridian the time is before noon; at those east, the time is after noon.

2. The whole circle of the earth = 360° which pass under the sun in 24 hours, and in one hour passes  $\frac{1}{24}$  of 360° = 15°. One hour = 60 minutes; hence, in 1 minute passes  $\frac{1}{60}$  of 15° =  $\frac{15}{60}$  = 15' = 15'. One minute = 60 seconds; hence, in 1 second passes  $\frac{1}{60}$  of 15' =  $\frac{15}{60}$  = 15" = 15". Hence, the following

COMPARISON OF LONGITUDE AND TIME.

|                  |   |                   |
|------------------|---|-------------------|
| 15° of longitude | = | 1 hour of time.   |
| 15' of longitude | = | 1 minute of time. |
| 15" of longitude | = | 1 second of time. |

**249. RULE.—I.** The difference of longitude between two places, expressed in degrees, minutes, and seconds, divided by 15 will give their difference in time expressed in hours, minutes, and seconds.

**II.** The difference of time in two places, expressed in hours, minutes, and seconds, multiplied by 15 will give their difference in longitude expressed in degrees, minutes, and seconds.

NOTES.—1. If one place be in east, and the other in west longitude, the difference of longitude is found by adding them; and, if the sum be greater than 180°, it must be subtracted from 360°.

2. Since the sun appears to move from east to west, when it is exactly 12 o'clock at one place, it will be *past* 12 o'clock at all places east, and *before* 12 at all places west. Hence, if the difference of time between two places, be *subtracted* from the time at the *easterly* place, the result will be the time at the *westerly* place; and, if the difference be *added* to the time at the *westerly* place, the result will be the time at the *easterly* place.

## EXAMPLES FOR PRACTICE.

1. Quebec is in longitude  $71^{\circ} 16'$  west, and Toronto,  $79^{\circ} 21'$  west. When it is 12 o'clock at Toronto, what is the time at Quebec?

|            |                   |
|------------|-------------------|
| OPERATION. |                   |
|            | $79^{\circ} 21'$  |
|            | $71^{\circ} 16'$  |
| 15 )       | $8^{\circ} 5'$    |
|            | 0h. 32mi. 20sec.  |
|            | 12                |
|            | 12h. 32mi. 20sec. |

ANALYSIS.—The difference of longitude is  $8^{\circ} 5'$ . Dividing by 15 and changing to time gives 32mi. 20sec. for the difference of time between the two places; and, as Quebec is east of Toronto, the time is later, and we add the difference of time, which gives 12h. 32mi. 20sec. the time at Quebec.

2. The longitude of Halifax is  $63^{\circ} 35' 30''$  west, and that of Ottawa is  $75^{\circ} 41'$  west; when it is 10 o'clock 12min. A. M. in Halifax, what time is it at Ottawa?

3. The longitude of Valparaiso is  $71^{\circ} 37'$  west, and the longitude of Rome is  $20^{\circ} 30'$  east; when it is 11 o'clock 15min. A. M. at Valparaiso, what is the time at Rome? *Ans.* 23min. 28sec. past 5 P. M.

4. The longitude of New Orleans is  $90^{\circ} 7'$  west, of Philadelphia,  $75^{\circ} 10'$  west; what is the time at N. O. when it is 8 o'clock 20min. 40sec. at Philadelphia? *Ans.* 7h. 20min. 52sec.

5. When it is noon at St. Paul's, Minnesota, longitude  $93^{\circ} 5'$  west, it is at Bangor 1h. 37min. 12sec. P. M.; what is the longitude of Bangor, Maine? *Ans.*  $68^{\circ} 47'$  west.

6. The longitude of Jerusalem is  $35^{\circ} 32'$  east, and the longitude of Montreal  $73^{\circ} 25'$  west; when it is 10 o'clock A. M. at Jerusalem, what time is it at Montreal? *Ans.* 2h. 44min. 12sec. A. M.

7. The longitude of Boston is  $71^{\circ} 4' 9''$  west, and when it is 10 o'clock A. M. in Boston, it is 8 o'clock 53min. 57sec. in Chicago; what is the longitude of Chicago? *Ans.*  $87^{\circ} 34' 45''$ .

8. The longitude of Constantinople is  $28^{\circ} 48'$  east, and of Kingston, Canada,  $75^{\circ} 41'$  west; when it is 3 o'clock P. M. at the latter place, what time is it at the former? *Ans.* 9h. 57min. 56sec. P. M.

9. A captain at sea finds by his chronometer that it is 3h. 40min. 30sec. P. M., at Greenwich, when it is 1h. 10min. 45sec. by solar time on board his vessel; in what longitude is the vessel? *Ans.*  $37^{\circ} 26' 15''$  west.

## DUODECIMALS.

**250.** Duodecimals are denominate numbers, the denominations of which increase according to the *scale* of 12; or denom-

ina  
pra  
the

2  
ma

E

74/.

2  
der t  
II  
mult  
the n  
first  
the p  
rying  
II  
be the

1. 1  
4ft. 3"  
2.  
and w  
3. 1  
1ft. 8"

inate fractions, whose denominators are 1, 12, 144, 1728, etc. In practice, duodecimals are applied to the measurement of extension, the foot being taken as the unit.

TABLE.

|                            |                     |             |
|----------------------------|---------------------|-------------|
| 12 fourths, marked ("'''), | make 1 third,       | marked 1''' |
| 12 thirds                  | " 1 second,         | " 1''       |
| 12 seconds                 | " 1 prime, or inch, | " 1'        |
| 12 primes, or inches,      | " 1 foot,           | " ft.       |

The marks ', ', ', ', ', are called *indices*.

**251.** Duodecimals are added and subtracted in the same manner as compound numbers.

MULTIPLICATION OF DUODECIMALS.

*Ex.* How many square feet in a floor 9ft. 7' long and 7ft. 9' wide ?

OPERATION.

$$\begin{array}{r}
 9ft. 7' \\
 7ft. 9' \\
 \hline
 7ft. 2' 3'' \\
 67ft. 1' \\
 \hline
 74ft. 3' 3''
 \end{array}$$

ANALYSIS.—Beginning at the right,  $7' \times 9' = 63'' = 5' 3''$ ; writing the 3'' one place to the right, we reserve the 5' to be added to the next product. Then,  $9ft. \times 9' + 5' = 86' = 7ft. 2'$ , which we write in the places of feet and primes: Next, multiplying by 7ft., we have  $7' \times 7ft. = 49' = 4ft. 1'$ ; writing the 1' in the place of primes, we reserve the 4ft. to be added to the next product. Then,  $9ft. \times 7ft. + 4ft. = 67ft.$ , which we write in the place of feet. Adding the partial products, we have

74ft. 3' 3'' for the product required. Hence, the

**252. RULE.—I.** Write the several terms of the multiplier under the corresponding terms of the multiplicand.

II. Multiply each term of the multiplicand by each term of the multiplier separately, beginning with the lowest denomination in the multiplicand, and the highest in the multiplier, and write the first figure of each partial product one place to the right of that of the preceding product, under its corresponding denomination, carrying 1 for every 12.

III. Finally, add the several partial products; their sum will be the required answer.

EXAMPLES FOR PRACTICE.

- How many square feet in a piece of marble 12ft. 7' long, and 4ft. 3' wide ?  
*Ans.* 53ft. 6' 9''
- What is the area of a floor, the length of which is 9ft. 8' 11'', and width 3ft. 7' ?  
*Ans.* 34ft. 10' 11'' 5'''
- How many square feet in 10 boards, each 12ft. 10' long and 1ft. 8' wide ?  
*Ans.* 313ft. 10' 8''

4. If a block of granite be 7ft. 6' long, 3ft. 3' wide, and 1ft. 10' thick, what are the solid contents? *Ans.* 44ft. 8' 3".
5. How many square feet of boards will it take to inclose a piece of land 80ft. 10in. long, and 60ft. 8in. wide, with a close fence 7ft. 6in. high? *Ans.* 2122ft. 6'.
6. What will the plastering of a room cost, at 18 cents a square yard, the length of which is 30ft. 10in., width 24ft. 6in., and height of ceiling 8ft. 4'? *Ans.* \$33.55.

## DIVISION OF DUODECIMALS.

*Ex.* There are 8ft. 5' 3" in the surface of a marble slab, the length of which is 3ft. 9'; what is its width?

OPERATION.

$$\begin{array}{r} 3ft. 9' \ ) \ 8ft. 5' 3'' \quad (2ft. 3', \text{ Ans.} \\ \underline{7ft. 6''} \\ 11' 3'' \\ \underline{11' 3''} \\ 0 \end{array}$$

ANALYSIS.—3ft. is contained in 8ft., 2 times. Multiplying the whole divisor by 3ft. gives 7ft. 6' for the product, which we subtract from the corresponding denominations of the dividend, and obtain 11' for a remainder, to which we annex the next denomination of the dividend, and have 11' 3". 3ft. is contained in 11', 3 times. The divisor being multiplied

by this 3' gives 11' 3", which being subtracted from the last remainder, leaves nothing. Therefore, the marble slab was 3ft. 3' in width.

**253. RULE.**—I. Divide the highest term of the dividend, by the highest term of the divisor; multiply the divisor by this term of the quotient, and subtract the product from the dividend.

II. To the result bring down the next term of the dividend, and divide as before.

## EXAMPLES FOR PRACTICE.

1. Divide 184ft. 3' by 40ft. 11' 4". *Ans.* 4ft. 6".
2. Divide 41ft. 8' 7" 6" by 7ft. 4". *Ans.* 5ft. 7' 6".
3. A table whose length is 6ft. 9' 7", has an area of 28sq. ft. 3' 11' 2"; what is its width? *Ans.* 4ft. 2".
4. What is the length of an alley whose area is 792ft. 6' 9" 2", and width, 12ft. 7' 8"? *Ans.* 62ft. 8' 6".
5. A block of marble contains 64ft. 2' 5"; its width is 2ft. 6', and its thickness 3ft. 7"; what is its length? *Ans.* 7ft. 2".
6. What is the width of a rectangular pond, whose length is 43ft. 9' 6", and area, 1075sq. ft. 3' 6" 6"? *Ans.* 24ft. 6' 7".
7. A stick of timber is 3ft. 2' wide, 2ft. 11' thick, and contains 135cu. ft. 10' 2" 1". What is its length?
8. It required 334sq. ft. 3" of board to cover the side of a certain building. The height was 17ft. 9in.; what was the length of the side? *Ans.* 47 feet.

1. bought  
2.  
3.  
of each  
4.  
5.  
15rd.  
6.  
7. C  
cubit h  
8. F  
9. A  
of it, a  
barrel  
10.  
and so  
at 15 c  
11.  
3in. w  
12. V  
at \$64  
13.  
14. T  
pleting  
man co  
15. I  
how ma  
96 feet  
16. A  
was sen  
How mu  
17. P  
per barr  
How mu  
18. A  
of it, an  
76lb. an  
19. A  
remaind  
maind f  
20. Fi  
weight.  
21. H  
12gal. 3  
22. T  
and 1 1/2

## MISCELLANEOUS EXAMPLES.

1. At 1s. 6d. sterling per yard, how many yards of linen may be bought for £5 6 6? *Ans.* 71yd.
2. Reduce 456575 grains to pounds, Apothecaries' weight.
3. If 22½ gallons of wine be bought for £30 2 10½, what is the cost of each gallon? *Ans.* £1 6 6.
4. What is the value of 15cwt. 3qr. 14lb. of tea, at \$950 per cwt.?
5. What would be the expense of making a turnpike 87mi. 3fur. 15rd., at \$578.75 per mile? *Ans.* \$50695.41½
6. What cost 7hd. 47gal. of gin, at \$87.25 per hoghead?
7. Goliath was 6½ cubits high; what was his height in feet, the cubit being 1ft. 7.168in. ? *Ans.* 10ft. 4.592in.
8. Reduce 3cwt. 1qr. 7lb. of hay to the decimal of a long ton.
9. A farmer having 17cwt. 2qr. 19lb. of pork, sold 4cwt. 3qr. 21lb. of it, and the remainder he put into 6 barrels; how much did each barrel contain? *Ans.* 2cwt. 12½lb.
10. Bought by Avoirdupois weight, 15lb. opium, at 4cts. a dram, and sold the same by Apothecaries' weight, in doses of 10gr. each, at 25 cts. per dose; how much did I gain? *Ans.* \$2471.40.
11. How many solid feet in a stick of timber 34ft. 9in. long, 1ft. 3in. wide, and 1ft. 6in. deep? *Ans.* 65.15625ft.
12. What is the value of a field 15ca. 75l. long, and 12ch. 50l. wide, at \$64 per acre? *Ans.* \$1260.
13. What part of 4gal. 3qt. is 2qt. 1pt. 2gi. ? *Ans.* ½.
14. Thirty-two men construct 28mi. 4fur. 32rd. of road; after completing ¼ of it, ¼ of the number of men left. What distance did each man construct before and after ¼ of the men left? *Ans.* 3fur. 23rd. before, and 4fur. 30rd. after.
15. If it require 3h. 20min. for a man to cut 1 cord of wood, how many days of 8 hours each will be required to cut 746 cords, 96 feet? *Ans.* 311da. 1h. 10min.
16. A housebreaker, having stolen property to the value of £9 12 9½, was sentenced to pay at the rate of £9 12 9½ for every pound stolen. How much was the fine? *Ans.* £92 18 9½ +.
17. Bought 4 barrels of cranberries, each containing 2½bu. at \$8 per barrel, and retailed the same at 12½ cts. per quart, wine measure. How much was my profit? *Ans.* \$14.54½.
18. Andrew received ¼ of a certain quantity of flour, Edward ⅓ of it, and Louis the remainder. Now it is found that Andrew has 76lb. and 8oz. more than Edward. How much did each receive? *Ans.* A received 210½lb., E 133½lb., and L 1128½lb.
19. A man having a hogshead of sirup, sold ¼ of it to F, ¼ of the remainder to G, and ¼ of the residue to J. How many gallons remained? *Ans.* 19gal. 1qt. 1pt. 4gi.
20. Find the value in Troy weight of 13lb. 8oz. 11.4dr. Avoirdupois weight. *Ans.* 16lb. 5oz. 10pwt. 11.7 + gr.
21. How much butter, at 18½ cents a pound, must be given for 12gal. 3qt. of molasses, at 37½ cents a gallon?
22. The wall of a cellar is 20 feet square on the inside, 8 feet high, and 1½ feet in thickness; how many perches of masonry are there?

23. The total yield of nine copper mines in 1868, was 3942 *T.* 12 *cwt.* 1 *qr.* 1 *lb.*; in 1869, the same mines yielded 4101 *T.* 8 *cwt.* 3 *qr.* 3 *lb.*; if copper was worth 20 cts. per lb., of how much greater value was the amount produced in 1869, than 1868? *Ans.* \$63530.40.
24. Sold 15 *cwt.* 22 *lb.* of rice at \$3.75 a *cwt.*, and 7 *cwt.* 36 *lb.* of pearl barley, at \$4.25 a *cwt.* How much would be gained by selling the whole at 4½ cts. a pound? *Ans.* \$13.25½.
25. Bought a lot 25 rods long and 20 rods wide for \$10000, and sold the same at 25 cts. per square foot. How much was my gain?
26. Sold 72 yds. carpeting at \$1.37½ a yd., and gained \$18. How much did it cost me per yard? *Ans.* \$1.12½.
27. How many square yards in the walls of a room 40 feet long, 31½ feet wide and 12 feet high?
28. How many tons of hay, at \$0.75 per *cwt.*, must be given for 35 cords of wood, at \$0.60 per cord foot? *Ans.* 11½ tons.
29. Purchased a farm, containing 176A. 3R. 25rd., at \$75.37½ per acre; what did it cost? *Ans.* \$13334.308 +.
30. What will be the expence of plastering a room 40ft. long, 36½ft. wide, and 22½ft. high, at 18 cents a sq. yd., allowing 1375sq. ft. for doors, windows, and base board? *Ans.* \$69.78½.
31. When it is 11 A. M. at a place, 30° east of Greenwich, it is 3h. 44min. 20sec. A. M. at Buffalo, United States; what is the longitude of Buffalo? *Ans.* 78° 55' west.
32. Nineteen lots of equal size contain 159A. 2R. 17sq. rd. 25sq. yd. 8sq. ft. 130sq. in. What is the value of one lot, the land being worth 50 cts. per square foot? *Ans.* \$182965.32.
33. Sold 4 building lots of ground; the first contained ¼ of ⅓ of an acre; the second, 40½ rods; the third, ⅓ of an acre; and the fourth, ⅓ of ⅓ of an acre. How much land in the four lots? *Ans.* 3R. 7¼per.
34. How much beef, at 7d. per pound, ought I to receive for 27lb. 12oz. of butter at 1s. 9d. per lb.? *Ans.* 83½lb.
35. The difference in longitude between London and St. Louis, Mo., is 90° 20'; at a certain time each day it is as much past noon in London as it lacks of noon in St. Louis. What is the time in St. Louis? *Ans.* 8h. 59min. 20sec. A. M.
36. Express in acres and the decimal of an acre the area of 49 square lots, each measuring 5rd. 8ft. 3in. on a side.
37. On an acre of ground there were erected 21 buildings occupying on an average 3sq. rd. 112sq. ft. 8sq. in. How much of the acre remained unoccupied? *Ans.* 88per. 97sq. ft. 12sq. in.
38. Reduce ⅔ of ⅓ of 45½lb. to the decimal of a short ton.
39. A person lived in Montreal until he was 18yr. 8mo. 24da. old: in Toronto, ¼ as long; in Kingston, ⅓ as long as in Toronto, and ¼ as long in Quebec as in Kingston. What was his age? A. 31yr. 2mo. 20da.
40. A farmer owning 195A. 3R. 38sq. rd. of land, divided ⅓ of it equally among his four sons. How much did each son receive, and how much had the father remaining? *Ans.* 36A. 2R. 36sq. rd. each, and 48A. 3R. 39sq. rd. remaining.
41. A steamer, going from Halifax to Liverpool, traversed 10½ degrees of longitude daily. What length of time was it from one noon to the next? *Ans.* 23h. 18min.

42. What cost 0.01975 of a ton of steel at 20 cents per pound?
43. A man having a field 30 rods square, sold 25 square rods to one of his neighbors, and 20 rods square to another. What is the value of the remainder at \$175 per acre? *Ans.* \$519.53 $\frac{1}{2}$ .
44. A man paid \$46.50 for a certain pile of wood. Measuring it he found that it contained 5cd. 6cd. ft. 12cu. ft. What did the wood cost him per cord? *Ans.* \$7.957 +.
45. A grocer lost from  $\frac{7}{8}$  of a hogshead of molasses,  $\frac{1}{4}$  of a gallon and  $\frac{1}{2}$  of a quart. How much of the hogshead, expressed decimally, leaked out, and how much remained? *Ans.* 0.008 $\frac{1}{2}$  gal. leaked out, and 54.11 $\frac{1}{2}$  gal. remained.
46. A merchant spent  $\frac{1}{2}$  of his money and gave away £5; he then spent  $\frac{1}{3}$  the remainder and gave away £2, 10 less than  $\frac{1}{4}$  of what then remained, and he had £12 left. How much had he at first?
47. If a gallon of distilled water weigh 8lb. 5oz, 6.74dr., what is the weight of 17gal. 3qt. 1pt. 1gi.? *Ans.* 149lb. 5oz. 1.19dr.
48. At  $3\frac{1}{2}$  cents per foot, what will be the cost of 12 planks, each measuring 56ft. 9' ? *Ans.* \$23.83 $\frac{1}{2}$ .
49. If, when wheat is worth 6s. 3d. per bushel, a 5-cent loaf weighs 24oz., and allows the baker  $1\frac{1}{2}$  cts. a loaf for his labor, what should it weigh when wheat is 8s. 4d. per bushel, to afford him the same profit on a loaf? *Ans.* 18oz.
50. How much will it cost to carpet a room 21ft. long, 15ft. wide, with carpeting  $\frac{3}{4}$  of yd. wide, at \$1.62 $\frac{1}{2}$  per yd.?
51. What is the value of a pile of wood that is 10rd. long, 4ft. wide, and 1 $\frac{1}{2}$ yd. high, at \$5.75 per cord? *Ans.* \$133.42 —
52. My garden, which is 130ft. long, and 150ft. wide, is surrounded by a tight-board fence  $5\frac{1}{2}$ ft. high. How much will it cost to paint the fence on both sides at 12 cts. per sq. yd.? *Ans.* \$93.86 $\frac{1}{2}$ .
53. A merchant purchased in Manchester 34 bales of cloth for £8 19 5 per bale; he disposed of the cloth at Porto-Rico for 212cwt. of sugar, at £1 5 per cwt. Did he lose or gain? and how much?
54. If a person spends in 6 months what he earns in 4 $\frac{1}{2}$  months; how many dollars can he lay by in a year, supposing he earns \$325 in 2 $\frac{1}{2}$  months? *Ans.* \$390.
55. A man has a piece of land 20 $\frac{1}{2}$  rods long and 41 $\frac{1}{2}$  rods wide, which he wishes to lay out into square lots of the greatest possible size. How many lots will there be? *Ans.* 396.
56. If a man can pay his creditors only 48 cents on a dollar, how much can he pay on a debt of \$52.50?
57. How many bricks, 8in. long, 4in. wide, and 2 $\frac{1}{2}$ in. thick, are required to build the front of a house whose wall is 30ft. long, 24ft. high, and 2ft. thick, allowing the doors and windows to occupy  $\frac{1}{4}$  the surface? *Ans.* 23328 bricks.
58. Paid 3 debts successively, each of which took  $\frac{1}{3}$  the money I had before paying it, and 75 cts. more; after which I had \$25 remaining. How much had I at first? *Ans.* \$210.50.
59. If I buy 120 gallons of rum for \$75, how much water must be added to it that I may sell it at 60 cents per gallon, and gain \$16 in the sale of it? *Ans.* 30gal.

60. Sold 125 equal loads of wood, measuring 115Cd. 3cd. ft. 7cu. ft. for \$492.50. What is the quantity per load, and price per cord?

*Ans.* 118½cu. ft. each load, \$4.26½ per cord.

61. How many francs must a merchant in Paris send to Montreal in payment for a debt of \$15989.862?

62. If a man fill ¼ of a cask with brandy, ¼ with wine, and ½ with water, and if it lack 21½ gallons of being full, how many gallons will that cask contain?

*Ans.* 100gal.

63. If by selling cloth at 10s. 6d., ¼ of the price is gain, what part of the cost would be gained by selling it at 13s.?

64. A ship's chronometer, set at Greenwich, points to 5h. 45min. 24sec. P. M., when the sun is on the meridian. What is the ship's longitude?

*Ans.* 86° 21' E.

65. A grocer bought 15 barrels of salt, of 4 bushels each, at \$1½ a barrel, and retailed it at ¾ of a cent a pint. How much was his whole gain?

*Ans.* \$4.60.

66. James owns ⅓ of a field, and Leo the remainder; ¼ of the difference between their shares is 5A. 3R. 16¼per. What is Leo's share?

*Ans.* 20A. 3R. 9¼per.

67. A gentleman desirous of giving 1s. 6d. apiece to some needy boys, found that he had not money enough in his pocket by 5d.; he therefore gave them 1s. 4d., and had 9d. left. Required the number of boys.

*Ans.* 7.

68. A liquor agent has 50 gallons of wine of superior quality, worth \$7.50 a gallon; he wishes to reduce its quality by the addition of water, so that he may sell it at \$5.25 a gallon. How much water must he add?

*Ans.* 21½gal.

69. A clothier has 960 soldiers' coats to make, each coat containing 2½yd. of cloth 1½yd. wide, and lined with drilling ¾yd. wide. How many yards of lining will be required?

70. A ship captain, sailing from London to Portland, found, on taking an observation, that the sun at noon was 3h. 25min. 40sec. earlier than the London time, as shown by his chronometer. How many degrees west had he sailed?

71. My father's garden is 10½ rods long, and 8½ rods wide, and surrounded by a fence 7½ feet high; he has laid out a walk around it, within the fence, 7½ feet wide on the two sides, and 5½ feet wide on the ends. How much remains for cultivation?

*Ans.* 21296sq. ft.

72. A boy having been sent to a store with 5½ doz. of eggs, was directed to purchase with them equal quantities of sugar, coffee, butter and tea; he disposed of his eggs at the rate of 2 for 5 cents, and paid for the articles purchased 17, 28, 37½ and 137½ cents per pound, respectively. What amount of each did he purchase?

26  
titles,  
of the  
25  
25  
25  
25  
25  
the te  
the n  
ratio  
26  
26  
the co  
26  
the co  
26  
one co  
or ¼ :  
26  
ratios.  
= 1½  
26  
of ratio  
1st.  
conseq  
2nd.  
the ant  
3rd.  
by the

1. 54  
2. 108  
3. 7  
4. 17  
5. 60

## RATIO.

**254.** Ratio is that relation between two numbers or quantities, which is expressed by the *quotient* arising from the division of the one by the other. Thus, the ratio of 12 to 4 is  $12 \div 4 = 3$ .

**255.** The **Terms** of a ratio are the two numbers compared.

**256.** A **Couplet** is the two terms of a ratio taken together.

**257.** The **Antecedent** is the first term, or dividend.

**258.** The **Consequent** is the second term, or divisor.

**259.** A ratio may be expressed either by two dots ( $:$ ) between the terms; or in the form of a fraction, by making the antecedent the numerator and the consequent the denominator. Thus, the ratio of 8 to 4, may be expressed as 8 : 4, or as  $\frac{8}{4}$ .

**260.** A ratio is either *direct* or *inverse*.

**261.** A **Direct Ratio** is the quotient of the antecedent by the consequent. Thus, 8 to 4 is  $\frac{8}{4}$  or 2.

**262.** An **Inverse**, or **Reciprocal Ratio**, is the quotient of the consequent by the antecedent. Thus, 8 to 4 is  $\frac{4}{8}$  or  $\frac{1}{2}$ .

**263.** A **Simple Ratio** is that having but *one antecedent* and *one consequent*; it may be either direct or inverse. Thus, 6 : 3, or  $\frac{6}{3}$ .

**264.** A **Compound Ratio** is the product of two or more ratios. Thus, the ratio compounded of 6 : 3 and 8 : 4 is  $\frac{6}{3} \times \frac{8}{4} = \frac{12}{3} = 4$ , or  $6 \times 8 : 3 \times 4 = 4$ .

**265.** From the foregoing we deduce the following principles of ratio.

1st. *Multiplying the consequent divides the ratio; dividing the consequent multiplies the ratio.*

2nd. *Multiplying the antecedent multiplies the ratio; dividing the antecedent divides the ratio.*

3rd. *Multiplying or dividing both antecedent and consequent by the same number does not alter the ratio.*

## EXAMPLES FOR PRACTICE.

What is the direct ratio of

1. 54 to 6?

Ans. 9.

6. 13 to 52?

Ans.  $\frac{1}{4}$ .

2. 108 to 18?

7. 53 to 212?

3. 7 to 21?

Ans.  $\frac{1}{3}$ .

8. 72yd. to 9yd.?

4. 17 to 68?

9. 60mi. to 4fur.?

Ans. 120.

5. 60 to 12?

10. 3qt. to 20gal.?

Required the inverse ratio of

- |               |         |                   |                      |
|---------------|---------|-------------------|----------------------|
| 11. 27 to 81. | Ans. 3. | 14. 42 to 6.      | Ans. $\frac{1}{3}$ . |
| 12. 72 to 8.  |         | 15. .02 to 2.503. |                      |
| 13. 16 to 48. |         | 16. 256 to 32.    |                      |
17. Which is the greater, the ratio of 86 to 240, or of 45 to 72?  
 18. What is the ratio compounded of 35 to 40, 60 to 75, and 21 to 19?  
 19. If the consequent be 32 and the ratio  $4\frac{1}{7}$ , what is the antecedent?  
 20. If the antecedent be  $7\frac{1}{2}$  and the ratio  $\frac{2}{3}$ , what is the consequent?

## PROPORTION.

**266. Proportion** is the equality of ratios. It is indicated thus,  $6 : 3 :: 8 : 4$ ; or thus,  $6 : 3 = 8 : 4$ , and is read 6 is to 3 as 8 is to 4; or the *ratio* of 6 to 3 = the *ratio* of 8 to 4. Hence every proportion has two couplets and four terms.

**267.** The **Extremes** are the first and fourth terms.

**268.** The **Means** are the second and third terms.

**269.** Since in a proportion, the ratio of the first to the second term is equal to the ratio of the third to the fourth term, the proportion,  $6 : 3 :: 8 : 4$ , becomes  $\frac{6}{3} = \frac{8}{4}$ , multiplying each member by 3 and 4, we have  $6 \times 4 = 8 \times 3$ . Hence,

In every proportion, the product of the means is equal to the product of the extremes.

**270.** From the foregoing principles and illustrations, it follows that, any three terms of a proportion being given, the fourth may readily be found by the following

**271. RULE.—I.** Divide the product of the extremes by one of the means, and the quotient will be the other mean. Or,

**II.** Divide the product of the means by one of the extremes, and the quotient will be the other extreme.

**NOTE.**—We will denote the required term of a proportion by the letter  $x$ .

## EXAMPLES FOR PRACTICE.

1. Find the value of  $x$  in the proportion,

$$9 : 16 :: 36 : x; \quad x = \frac{16 \times 36}{9} = 64, \text{ Ans.}$$

What is the value of  $x$  in each of the following proportions:

- |                          |          |                                            |         |
|--------------------------|----------|--------------------------------------------|---------|
| 1. 24 : 96 :: 14 : $x$ ? | Ans. 56. | 5. $x$ : 15 :: 3 : 9?                      | Ans. 5. |
| 2. 7 : 42 :: $x$ : 96?   | Ans. 16. | 6. \$7\frac{1}{2} \$10 :: 36bu. : $x$ bu.? |         |
| 3. 16 : $x$ :: 10 : 40?  | Ans. 64. | 7. 2yd. : 8yd. :: \$3\frac{1}{2} : $x$ ?   |         |
| 4. 42 : 70 :: 3 : $x$ ?  |          | 8. 7.50 : 18 :: $x$ oz. : 7\frac{1}{2}oz.? |         |

## SIMPLE PROPORTION.

**272. Simple Proportion** is an equality of two simple ratios, and consists of four terms.

**NOTE.**—Simple Proportion is often called the *Rule of Three*, from the circumstance that three terms being given to find a fourth.

*Ex. 1.* If 12 yards of cloth cost \$30, what will 42 yards cost at the same rate?

OPERATION.

$$\begin{array}{r} \text{yd.} \quad \text{yd.} \quad \$ \quad \$ \\ 12 : 42 :: 30 : x \end{array}$$

$$\begin{array}{r} 42 \\ \hline 60 \\ \hline 120 \end{array}$$

$$\begin{array}{r} 12 \ ) \ 1260 \\ \hline \end{array}$$

$$x = \$105, \text{ Ans.}$$

**ELUCIDATION.**—To arrange the given numbers in the order of a proportion, or state the question, we make \$30 the *third* term, because it is of the same kind as the required *fourth* term; and, as from the nature of the question the latter must be greater than the third term, we make the greater of the other two numbers the *second* term, and the less the *first*; and then, the product of the means divided by the given extreme, gives the required extreme.

THE SAME EXAMPLE BY ANALYSIS.

If 12yd. cost \$30, 1 yard will cost  $\frac{1}{12}$  of \$30 = \$2.50; then, if 1yd. cost \$2.50, 42yd. will cost 42 times \$2.50 = \$105, the answer, as before.

*Ex. 2.* If 49 soldiers consume a certain quantity of flour in 28 days, how long will it take 70 soldiers to consume it?

OPERATION.

$$\begin{array}{r} \text{Soldiers.} \quad \text{Soldiers.} \quad \text{days.} \quad \text{days.} \\ 70 : 49 :: 28 : x \end{array}$$

$$\begin{array}{r} 7 \quad 14 \\ 49 \times 28 = 196, \text{ Ans.} \\ \hline 70 \\ \hline 19 \\ \hline 5 \end{array}$$

**ELUCIDATION.**—Since the required answer is days, we make the given days the *third* term. Then, as the flour will not last 70 soldiers so long as it will 49 soldiers, we make 49 soldiers, the smaller of the two terms, the *second* term, and 70 soldiers the *first* term; and proceed as in the first example, except that we shorten the work by cancellation.

THE SAME EXAMPLE BY ANALYSIS.

If 49 soldiers consume the flour in 28 days, it will take 1 soldier 49 times 28 days = 1372 days; then, if 1 soldier consume the flour in 1372 days, 70 soldiers will consume it in  $\frac{1}{70}$  of 1372 days = 19 $\frac{2}{5}$  days.

**273. RULE.**—I. Write the two given numbers, which are of the same name or kind as the required fourth term, or answer, for the third term of the proportion.

II. Of the other two numbers, write the larger for the second term, and the less for the first, when the answer should exceed the third term; but write the less for the second term, and the larger for the first, when the answer should be less than the third term.

Ans. 7.

5 to 72?

76, and 21

Ans. 117.

the ante-

Ans. 7.

the conse-

Ans. 12.

indicated

and 6 is to 3

4. Hence

s.

the second

n, the pro-

member

al to the

ns, it fol-

the fourth

by one of

emes, and

etter x.

ortions:

Ans. 5.

x bu.?

x?

1 $\frac{1}{2}$ oz.?

III. Multiply the second and third terms together, and divide their product by the first; or divide the third term by the ratio of the first term to the second.

NOTES.—1. When the first and second terms are of different denominations, they must be reduced to the same denomination; and when the third term is a compound number, it must be reduced to the lowest denomination mentioned in it. The answer will be of the same denomination as the third term.

2. The pupil should perform these questions by analysis, as well as by proportion, and introduce cancellation when it will abbreviate the operation.

## EXAMPLES FOR PRACTICE.

1. Six laborers earn \$7.68; how much will 10 laborers earn? 36 laborers? *Ans.* \$12.80; \$46.08.
2. If 23yd. of cloth cost £25 8 3; how much will 138yd. cost? 126yd.? 137yd.? *Ans.* £218 15 4½; £139 4 3½; £151 7 4½.
3. One-half a bushel of salt costs 45½ cts.; how much will 16 bushels cost? 34 bushels? 72 bushels? 85½ bushels? 90½ bushels? 105½ bushels? *Ans.* \$14.56; \$30.94; \$65.52; etc.
4. 126lb. of butter cost \$16.38; how many lb. can be had for \$12.61? \$25.74? \$32.37? \$36.40? *Ans.* 97½ lb.; 198lb.; etc.
5. If a cwt. of tobacco is worth \$39.25; what is the value of 1 lb.? 7½ lb.? 56lb.? 98lb.? 107½ lb.? *Ans.* \$0.3925; \$294.37; etc.
6. The ½ of a cwt. of sugar cost \$6.48; what will be the cost of ½ of a cwt.? ½ cwt.? ¼ cwt.? *Ans.* \$6.72; \$7.20; etc.
7. If 403 acres of land are worth \$215.50; what is the value of 6 acres? 70 perchs.? 90 toises? 25½ acres? 10per. 4to. 120? 1103 arpents? *Ans.* \$31.92½; \$3.72½; \$9.52½; etc.
8. The ¼ of an acre produces 16cwt. 1qr. 12lb. of hay; what quantity will 1 acre produce? 8½ acres? 26½ per.
9. At 1s. 8d. per lb., what quantity of coffee can be had for £3 6s. 7 £9 15 6? £11 7 2½? £14 0 10? *Ans.* 39½ lb.; 117½ lb.; etc.
10. If 19 gallons of oil cost \$25.87, how much will 37 gal. cost? 42½ gal.? 50½ gal.? 65½ gal.? ¾ of a gal.? *Ans.* \$71.41; \$82.02; etc.
11. I paid \$78.80 for 11 tons of coal; how much must I pay for 15 tons? 3½ tons? 18½ tons? *Ans.* \$107.45; \$24.67; etc.
12. If 2½ lb. of coffee cost 7½ cts., how much must be paid for 74½ lb.? 96½ lb.? 109½ lb.? 2½ cwt.? *Ans.* \$14.82; \$18.90; etc.
13. Six cwt. 1qr. 1lb. of beef cost £13 7 6, what quantity can be had for £8 12 3? £1 0 8? £17 12 6?
14. For 17½ days' work, \$25.44 were paid; how much will be paid for 1 day's? 45½ days? 89½ days? *Ans.* \$1.44; \$65.52; etc.
15. The rent of a farm containing 12A. 2R. 30per. is \$113.75; what is the rent of another containing 5A. 1R. 7 16¼A. 7 59A. 2R. 20per. 10¼A.? *Ans.* \$47.06; \$145.34; etc.
16. Seven bushels of rice cost \$8.75; how much will 12½ bushels cost? 18½ bushels? 26½ bushels?
17. In paying \$112 for 128 feet of boards, what quantity can be had for \$119? \$230.60? \$370.85? *Ans.* 114½ ft.; etc.
18. I can get 336 pens for 3s. 6d.; how many can I get for 4s. 4d.? £3 10 13? £0 1 10? *Ans.* 324; etc.

19. If 15 yards of cloth are worth 60 yards of linen, how many yards of linen can be procured for 75 yards of cloth? *Ans.* 300.
20. If £100 give £7 interest in 12 months, how much interest will be accumulated in 23 months? in 5½ years? *Ans.* £24 10 0; etc.
21. A workman earned \$140 in 20 days; how much would he have earned, had he worked 6 days more? *Ans.* \$182.
22. If 5 peaches cost as much as 7 apples, how many apples can be had for 35 peaches? 280 peaches? *Ans.* 49 apples; etc.
23. If 3 men can do a piece of work in 51 days, how many men must be added to this number to do it in 17 days? in 9 days?
24. If £1 14s. is paid for the transportation of a weight of 400lb., 97 leagues; to what distance should it be transported for £6 16s.?
25. A merchant having declared bankruptcy, compromised with his creditors to pay \$0.64 on the dollar; how much will one receive on a debt of \$2563.50? *Ans.* \$1640.64.
26. What will be the price of 21A. 3R. 20per. of land, if 36A. 3R. cost £315? *Ans.* £187 10 0.
27. If 10cwt. 2qr. 14lb. of sugar cost \$204, how much should we pay for 3cwt. 1qr. 14lb.?
28. When pomegranates are 40 cts. the hundred, what will a dozen come to? *Ans.* \$0.048.
29. If \$260.70 will purchase 1lb. 4oz. 10gr. of gold, what weight can be had for \$39.50?
30. What is the length of a plane surface of one square foot, of which the breadth is 2½ inches? *Ans.* 57½ inches.
31. A manufacturer having failed owes \$900 to B, \$1200 to C, \$1400 to D, and \$1500 to E. The value of his property is \$2800; how much will each creditor receive? *Ans.* B \$504; C \$672; D \$784; E \$840.
32. What must be paid for 450ft. of split wood, at £4 5s. per M ft. *Ans.* £1 18 3.
33. If a bowl containing 2 cubic yd. is emptied in 12 minutes; how many hours will be required to empty a cistern 4yd. long, 3yd. wide, and 2½yd. deep? *Ans.* 3 hours.
34. One of two pieces of cloth costs \$335, the other, \$390; what is the length of each, knowing that the second is 1½yd. longer than the first? *Ans.* 1st 67yd.; 2nd 78yd.
35. How long will it take to pump 54 barrels of water, if 24 barrels can be pumped in 1h. 45min.?
36. A workman received \$264 for 44 days' labor; how much would he have received by working 14 days more? *Ans.* \$348.
37. What is the value of  $\frac{1}{2}$  of a bushel, knowing that the  $\frac{1}{4}$  cost \$517? *Ans.* £10 18 3.
38. If the moon moves 13° 10' 35" in one day, in what time will it perform its revolution? *Ans.* 27da. 7h. 43 + min.
39. I bought 4950 copies on condition that I should receive 6 per cent more; how many shall I receive? *Ans.* 5247.
40. What is the value of 7lb. 11oz. of gold, knowing that 7oz. are worth \$120? *Ans.* \$1628.57½.
41. The  $\frac{1}{4}$  of a bushel of prunes cost \$½, what part of a bushel can be bought for \$7½?

42. In retailing merchandise for the sale of \$200, I lost \$4.50 on every \$100; how much did I disburse?

43. A pound of cinnamon costs \$1.10; for how much should I retail it to gain at the rate of \$30 on every \$1000? *Ans.* \$1.15 $\frac{1}{2}$ .

44. What metallic pens are 6 $\frac{1}{2}$  cts. a dozen, how much will 10 $\frac{1}{2}$  gross cost? 16 $\frac{1}{2}$  gross? 25 $\frac{1}{2}$  gross? *Ans.* \$8.06 $\frac{1}{2}$ ; etc.

45. When profits are \$50 on every 100 yards of cloth, how many yards must be sold to raise a profit of \$850? *Ans.* 1700yd.

46. What will be the price of 7 chests of tea, each containing 2 $\frac{1}{2}$  cwt., if 51lb. cost £8 10s.?

47. One of two numbers is to the other as 5 : 7 $\frac{1}{2}$ , and the smaller is 164.5; what is the greater?

48. Two pieces of cloth are respectively 41 and 36 yards; the first piece costs \$45 more than the second; required the price of each. *Ans.* 1st. \$369; 2nd. \$324.

49. When wheat is sold at 7s. 6d. the bushel, a loaf of bread weighs 9 ounces; what should be the weight if wheat is but 6s. the bushel? *Ans.* 11 $\frac{1}{2}$ oz.

50. Every soldier in a regiment of 1000 men is to have a watch-coat; each coat will take 3 $\frac{1}{2}$ yd. of cloth which is 1 $\frac{1}{2}$ yd. wide, and is to be lined with flannel, 1 $\frac{1}{2}$ yd. wide; how many yards of flannel will be required to line the whole? *Ans.* 5625yd.

51. To draw success on my business, I propose to give \$5 to the poor every time I gain \$150; how much will I have gained when my gains amount to \$100? *Ans.* \$3000.

52. John can plough a certain field in 5 days, and Maurice in 6 days; what time will both take, working together, to plough the field?

53. A father earns 6s. 5d. per day, his son, 3s. 7 $\frac{1}{2}$ d. in what time will they have economised £1 10s. if they expend but 5s. per day? *Ans.* 5 days.

54. How much must I pay for paving a yard which is 60ft. long and 44ft. wide, if 1425sq. ft. cost \$341?

55. Two gangs composed of 20 and 30 men respectively, did 1500 yards of a certain work in 25 days; how much would they have done had their number been augmented by 15? *Ans.* 1950 yards.

56. One hundred degrees of Centigrade are equivalent to 80 degrees of Reaumur; to how many degrees of Reaumur will 23 $\frac{1}{2}$  degrees of Centigrade equal? *Ans.* 18 $\frac{1}{2}$ ° of Reaumur.

## COMPOUND PROPORTION.

274. Compound Proportion is an expression of equality between a compound and a simple ratio, or between two compound ratios. Thus,

12 : 6 }  
8 : 4 } 6, is a compound proportion:

That is,  $12 \times 8 : 6 \times 4 :: 6 : 6$ ; for,  $12 \times 8 \times 6 = 6 \times 4 \times 24$ .

Now  
would  
called

Es  
how r  
day?

Men.  
6  
9

8  
18  
8

If 6  
time v  
If in 1  
earn y  
5 days  
a day,  
day, t

27  
as the  
II.  
and a  
III  
third,  
quotie

NOTE  
solution

1. T  
horses

2. If  
in 5 m

an aver  
proport

2489 p

4. B  
profit o  
Virginia

NOTE.—Compound proportion embraces that class of questions whose solution would require two or more statements in Simple proportion. It is sometimes called *Double Rule of Three*.

*Ex.* If 6 men can earn \$72 in 10 days, by working 8 hours a day, how many dollars can 9 men earn in 5 days, by working 12 hours a day?

| STATEMENT. |          |     |     |
|------------|----------|-----|-----|
| Men.       | \$.      | Da. | Hr. |
| 6          | 72       | 10  | 8   |
| 9          | <i>x</i> | 5   | 12  |

NOTE.—To aid in remembering the question and in forming the ratios, the pupil should write the conditions upon his slate, or blackboard, as in the margin.

METHOD BY PROPORTION.

|          |                          |                                                                           |
|----------|--------------------------|---------------------------------------------------------------------------|
|          | OPERATION I.             | OPERATION II.                                                             |
| $6 : 9$  | $9$                      | $2$                                                                       |
| $10 : 5$ | $8 : x$                  | $12 : 9$                                                                  |
| $8 : 12$ | $∴ \$72 : x ; x = \$81.$ | $x = \frac{9 \times 8 \times 12 \times 72}{6 \times 10 \times 8} = \$81.$ |

METHOD BY ANALYSIS.

If 6 men in 10 days of 8 hours each earn \$72, 1 man in the same time will earn  $\frac{1}{6}$  of \$72 = \$12; and 9 men will earn  $9 \times \$12 = \$108$ . If in 10 days of 8 hours each, 9 men earn \$108, in 1 day they will earn  $\frac{1}{10}$  of \$108 = \$10.80; and in 5 days,  $5 \times \$10.80 = \$54$ . If in 5 days by working 8 hours a day, 9 men earn \$54, by working 1 hour a day, they will earn  $\frac{1}{8}$  of \$54 = \$6.75; and, by working 12 hours a day, they will earn  $12 \times \$6.75 = \$81$ .

**275. RULE.—I.** Make that number which is of the same kind as the answer required, the third term of a proportion.

**II.** Then take the other numbers in pairs, or two of a kind, and arrange them as in simple proportion.

**III.** Finally, multiply the product of the second terms by the third, and divide the result by the product of the first terms. The quotient will be the fourth term, or answer.

NOTE.—By reference to the above statement, of the question, either method of solution is so plain as to require no rule.

EXAMPLES FOR PRACTICE.

- Twelve horses can plough 11 acres of land in 5 days; how many horses will it require to plough 33 acres in 18 days? *Ans.* 10.
- If \$900 produce \$50 in 9 months, what sum will \$450 produce in 5 months? *Ans.* \$13.88.
- Received \$21 for 15 days' work of 7 horses, each drawing with an average power of 2250 pounds; how much should be received in proportion for 25 days' work of 5 horses, each drawing with a power of 2400 pounds? *Ans.* \$27.
- By selling 75 otter skins, which cost me \$3.60 each, I made a profit of \$24; how much would I have gained in proportion on 45 Virginia silvered fox skins, which cost \$7.70 each? *Ans.* \$30.80.

5. If 144 men, in 6 days, of 12 hours each, build a wall 200ft. long, 3ft. high, and 2ft. thick; in how many days, of 7 hours each, will 30 men build another wall 350ft. long, 6ft. high, and 3ft. thick?

6. If it require 45 tailors to make 300 coats in 36 days, how many will be required to make 200 in 27 days? *Ans.* 40.

7. If 18 men in 24 days, by working 12 hours a day can make 2880 locks; how many men, in 9 days, by working 10 hours a day can make 450 locks? *Ans.* 9.

8. If 6 horses eat 70 bushels of oats in 9 days, how many can be fed with 280 bushels in 27 days? *Ans.* 8.

9. In how many days will 6 persons consume 5 bush. of potatoes, if 3bu. 3pk. suffice for 9 persons during 22 days? *Ans.* 44 days.

10. If 15000 lb. of flour are sufficient to maintain 1500 men during 80 days in a citadel; by how much should this quantity be increased that it may last 2450 men for 232 days? *Ans.* 56050 lb.

11. During 18 days, of 8 hr. each, 14 laborers were employed at a piece of work 136yd. long and 9yd. high; how many yards will 36 laborers do, working 7 hr. per day, during 14 days? *Ans.* 238yd.

12. How many planks 10 $\frac{1}{2}$ ft. long and 1 $\frac{1}{2}$  inches thick, will be necessary to replace 3000 planks, 12ft. 8in. long and 2 $\frac{1}{2}$ in. thick?

13. The  $\frac{1}{2}$  of a wall was constructed by 15 masons in 12 days, after which 7 left; how long did it take the others to finish the work?

14. To perform a piece of work, 46 $\frac{1}{2}$ yd. long, 11 laborers were obliged to work 10 $\frac{1}{2}$  hours a day; how many men would it require to do 41 $\frac{1}{2}$ yd. of the same labor, working 8 $\frac{1}{2}$  hr. per day? *Ans.* 12 men.

15. Paid \$12 for the painting of 5 doors, each measuring 8ft. in height by 3ft. 6in. in breadth; how much should be paid for the painting of 7 windows, each 9ft. high by 4ft. wide, reckoning 2 doors for 3 windows? *Ans.* \$14.40.

16. If 300 bushels of wheat at 6s. 3d., liquidate a certain debt, how many bushels at 4s. 6d. will it require to liquidate a debt 3 times larger? *Ans.* 1250 bushels.

17. If the carriage of 5cwt. 3qr. a distance of 150 miles costs \$24.58, what must we pay for the carriage of 7cwt. 3qr. a distance of 64 miles at the same rate? *Ans.* \$14.135 +.

18. In a fort there are provisions enough for 1520 soldiers for 5 months. If the garrison be augmented by 100 men, what daily ration can be allowed them, if they remain 1 $\frac{1}{2}$  mo. longer?

19. If 4 $\frac{1}{2}$ d. is paid for a loaf, weighing 7 $\frac{1}{2}$ oz., when wheat is 4s. 2d. the bushel; what should a 1s. 2d. loaf weigh when wheat is 5s. 6d. the bushel? *Ans.* 16 $\frac{1}{2}$ oz.

20. Knowing that \$500 give \$10 interest in 3 months, what principal should I place at interest to give me \$200 in 1 year? *A.* \$2500.

21. During how many days, of 8 hr. each, must 49 men work, to do as much work as 7 men did in 28 days, of 10 hr. each? *Ans.* 6 da.

22. A piece of cloth 30yd. long,  $\frac{1}{2}$  of a yard wide, was woven with 26 lb. of thread; what will be the length of a piece  $\frac{1}{2}$  of a yard wide, and which requires 32 lb. of thread? *Ans.* 39 $\frac{1}{2}$  yards.

27  
by th  
signif  
etc.

27  
comp  
27  
the b  
= \$  
27  
of the  
Th  
Th  
Th  
the P  
Th  
the D  
28  
or a c

S

1  
1

28  
Ex

Or,

PERCENTAGE.

**276. Per Cent., or Rate per Cent.,** also written %, signifies *by the hundred*. Thus, 6% means 6 of every hundred, and may signify 6 cents of every 100 cents, 6 dollars of every 100 dollars, etc.

**277. The Base** is the number on which the percentage is computed.

**278. Percentage** is the required number of hundredths of the base. Thus, the percentage of \$200, at 5% is  $\frac{10}{100}$  of \$200 = \$10.

**279. The Amount or Difference** is the sum or difference of the base and percentage. Hence,

*The Amount = the Base + the Percentage.*

*The Difference = the Base - the Percentage.*

*The Base = the Amount - the Percentage, or the Difference + the Percentage.*

*The Percentage = the Amount - the Base, or the Base - the Difference.*

**280.** The rate per cent. may be expressed either by a *decimal* or a *common fraction*, as shown in the following

TABLE.

| Symbols.        |                | Decimals. |         | Common fractions.                  |
|-----------------|----------------|-----------|---------|------------------------------------|
| 1%              | of a number is | .01       | of it = | $\frac{1}{100} = \frac{1}{100}$    |
| 2%              | " " "          | .02       | " " =   | $\frac{2}{100} = \frac{1}{50}$     |
| 4%              | " " "          | .04       | " " =   | $\frac{4}{100} = \frac{1}{25}$     |
| 5%              | " " "          | .05       | " " =   | $\frac{5}{100} = \frac{1}{20}$     |
| 6%              | " " "          | .06       | " " =   | $\frac{6}{100} = \frac{3}{50}$     |
| 8%              | " " "          | .08       | " " =   | $\frac{8}{100} = \frac{2}{25}$     |
| 10%             | " " "          | .10       | " " =   | $\frac{10}{100} = \frac{1}{10}$    |
| 18%             | " " "          | .18       | " " =   | $\frac{18}{100} = \frac{9}{50}$    |
| 75%             | " " "          | .75       | " " =   | $\frac{75}{100} = \frac{3}{4}$     |
| 100%            | " " "          | 1.00      | " " =   | $\frac{100}{100} = 1$              |
| 125%            | " " "          | 1.25      | " " =   | $\frac{125}{100} = \frac{5}{4}$    |
| $\frac{1}{100}$ | " " "          | .005      | " " =   | $\frac{5}{1000} = \frac{1}{200}$   |
| $\frac{3}{100}$ | " " "          | .0075     | " " =   | $\frac{75}{10000} = \frac{3}{400}$ |
| $\frac{7}{100}$ | " " "          | .075      | " " =   | $\frac{75}{1000} = \frac{3}{40}$   |

**281. CASE I.—Given, the base and rate, to find the percentage.**

*Ex.* What is 6% of 512 yards of cloth?

OPERATION.

512  
 .06  
 30.72yd. Ans.

ANALYSIS.—6% = .06. Therefore, 6% of 512yd. is .06 of 512 = 30.72yd.

Or, 6% =  $\frac{6}{100}$ . Therefore, 6% of 512yd. is  $\frac{6}{100}$  of 512 yards = 30.72yd.

Or,  $\frac{6}{100} \times 512 = 30.72$ yd. Ans.

Or,  $100\% = 512\text{yd.}$

$1\% = 5.12\text{yd.}$

$6\% = 30.72\text{yd.}$  Ans.

Or, if  $100\% = 512\text{yd.}$ ,  $1\% = \frac{1}{100}$  of  $512\text{yd.} = 5.12\text{yd.}$ , and  $6\% = 6$  times  $5.12\text{yd.}$  =  $30.72\text{yd.}$  Hence the following

**282. RULE.**—Multiply the base by the rate% expressed decimally, and point off as in decimals. Or,

Find that part of the base which the rate % is of 100.

EXAMPLES FOR PRACTICE.

1. What is  $5\%$  of  $\$462$ ?  $4\%$  of  $1550$ ?  $8\%$  of  $\$630.25$ ?  $7\%$  of  $\$846$ ?  
Ans.  $\$23.10$ ;  $62$ ;  $\$50.42$ ; etc.
2. What is  $9\%$  of  $\$75.37\frac{1}{2}$ ?  $7\%$  of  $580$ ?  $12\frac{1}{2}\%$  of  $1260\text{lb.}$ ?  $11\%$  of  $\$111$ ?  
Ans.  $\$6.78\frac{1}{2}$ ;  $40.6$ ; etc.
3. What is  $32\%$  of  $\$760.60$ ?  $4\frac{1}{2}\%$  of  $487$ ?  $7\frac{1}{2}\%$  of  $345$ ?  $6\frac{1}{2}\%$  of  $\$125$   $12$   $6$ ?  
Ans.  $\$243.392$ , etc.
4. What is  $20\%$  of  $90\text{cwt.}$ ?  $\frac{1}{2}\%$  of  $\$850$ ?  $\frac{1}{4}\%$  of  $\$320$ ?  $87$   $9\frac{1}{2}\%$  of  $550\text{gal.}$ ?  
Ans.  $18\text{cwt.}$ ;  $\$4.25$ ; etc.
5. What is  $15\%$  of  $77$ ?  $1\%$  of  $\$80$ ?  $2\frac{1}{2}\%$  of  $8\frac{1}{2}$ ?  $3\frac{1}{2}\%$  of  $\$20$   $15$   $8$ ?
6. A merchant having  $\$3456$  in the National Bank, wishes to withdraw  $18\%$ ; how much will remain?
7. A man having  $\$3947$ , gave  $15\%$  of it for flour;  $3\%$  for tea;  $45\%$  for cloth and linen; and the remaining  $15\%$ , for sugar. How much did he spend for each?  
Ans. For F.  $\$592.05$ ; T.  $\$986.75$ ; C.  $\$1776.15$ , etc.
8. A merchant bought  $475$  barrels of molasses for  $\$7125$ , and sold  $40\%$  of it at  $\$21$  a barrel;  $30\%$  at  $\$18$  a barrel; and the remainder for what it cost. How much did he gain?  
Ans.  $\$1567.50$ .

**283. CASE II.**—Given, the base and percentage, to find the rate %.

Ex. What per cent. of  $\$450$  is  $27$ ?

OPERATION.

$$\frac{100}{27}$$

$$450 \overline{) 2700} \quad (6\%, \text{ Ans.}$$

$$0.06 \times 100\% = 6\%, \text{ Ans.}$$

$$27 \div 450 = 100\%$$

$$= \frac{1}{15}$$

$$\$ 27 = 6\%, \text{ Ans.}$$

ANALYSIS.— $\$450$  is  $100\%$  of itself.  $\$27$  is  $\frac{27}{450}$  of  $\$450$ ; therefore,  $\$27$  is  $\frac{27}{450}$  of  $100\%$ , or  $\frac{1}{15}$  of  $27$  times  $100\% = 6\%$  of  $\$450$ .

Or,  $\$27$  is  $\frac{27}{450} = \frac{3}{50}$  of  $\$450$ ; therefore, it is  $\frac{3}{50}$  of  $100\% = 6\%$  of  $\$450$ .

Or,  $\$450$  is  $100\%$  of itself; therefore,  $\$1$  is  $\frac{1}{450}$  of  $100\% = \frac{1}{150}$ , and  $\$27$  is  $27$  times  $\frac{1}{150} = 6\%$  of  $\$450$ . Hence the

**284. RULE.**—Multiply  $100\%$  by the percentage and divide by the base. Or,

Find that part of  $100$  per cent. which the percentage is of the base.

EXAMPLES FOR PRACTICE.

1. At what rate per cent. must we place \$20 to have \$2? \$5 to have \$0.25? \$1440 to have \$21.60? £160 5 to have £12 16 4 $\frac{1}{2}$ ? \$4 to have \$0.30? *Ans.* 10%; 5%; etc.
2. What per cent. of 40 is 15? of 480 perches is 24per.? of 3 $\frac{1}{2}$  is 1 $\frac{1}{2}$ ? of  $\frac{1}{4}$  is  $\frac{1}{2}$ ? of 92gal. is 1gal. 2qt.? *Ans.* 37 $\frac{1}{2}$ %; 5%; etc.
3. What per cent. of 148 is 24 $\frac{1}{2}$ ? of 30lb. Avoirdupois is 11lb. 4oz.? of 720lb. is 60lb.? of 620yd. is 46 $\frac{1}{2}$ yd.? of 140lb. is 77lb.? *Ans.* 16 $\frac{1}{2}$ %; 37 $\frac{1}{2}$ %; etc.
4. What per cent. of \$578 is \$26.01? of \$250 is \$80? of  $\frac{3}{4}$  is  $\frac{1}{10}$ ? of £3 15 s. 9d.? *Ans.* 4 $\frac{1}{2}$ %; etc.
5. What per cent. of \$300 will give 25% of \$72? *Ans.* 6%.
6. Bought a horse for \$840, and sold him for \$560; how much did I lose per cent.? *Ans.* 33 $\frac{1}{3}$ %.
7. A number increased by 2 equals 14; required the increase per cent.

**285. CASE III.**—Given, the rate per cent. and percentage, to find the base.

*Ex.* I lost \$27, which is 6% of the money I had; how much had I at first?

OPERATION.

$$\$27 \div .06 = \$450, \text{ Ans.}$$

$$\text{Or, } \$27 \div \frac{3}{50} = \$450, \text{ Ans.}$$

$$\text{Or, } 6\% = \$ 27.$$

$$1\% = \frac{3}{2}.$$

$$100\% = \$450, \text{ Ans.}$$

ANALYSIS.—If 6%, or .06 of some number is \$27, that number must be  $\$27 \div .06$ , or  $\frac{3}{50} = \$450$ .

Or, 6% of some number is \$27,  $\frac{1}{6}$  of it is  $\frac{1}{6}$  of \$27 = \$4.50, and 100% of the whole number, is 100 times  $\frac{1}{6}$  = \$450. Hence the

**286. RULE.**—Divide the percentage by the rate % expressed decimally, or in the form of a common fraction. Or, Divide the percentage by the rate  $\frac{1}{100}$ , and multiply by 100.

EXAMPLES FOR PRACTICE.

1. 35 is 10% of what number? 84 is 7% of what number? \$3.60 is 15% of what number? \$55.50 is 4 $\frac{1}{2}$ % of what number? 240 is 12 $\frac{1}{2}$ % of what number? *Ans.* 350; 1200; etc.
2. \$66 is 5 $\frac{1}{2}$ % of what sum? 5 $\frac{1}{2}$  is 1 $\frac{1}{2}$ % of what sum?  $\frac{2}{3}$  is 30% of what sum? *Ans.* \$1200; etc.
3. £32 8 3 is 7 $\frac{1}{2}$ % of how much? 207 is 60% of how much? \$1.32 $\frac{1}{2}$  is 12 $\frac{1}{2}$ % of how much? *Ans.* £432 3 4; etc.
4. \$2.81 $\frac{1}{2}$  is 12 $\frac{1}{2}$ % of how much? 3mi. 1fur. 1per. is 6 $\frac{1}{2}$ % of how much? 16 $\frac{1}{2}$  is 2 $\frac{1}{2}$ % of how much? *Ans.* \$22.50; etc.
5. If the percentage be \$37.50, and the rate 2 $\frac{1}{2}$ %; what is the base? *Ans.* \$1500.
6. A farmer saved annually \$145.50, which was 33 $\frac{1}{3}$ % of his annual income; required his income?

**287. CASE IV.**—Given, the rate per cent. and amount or difference, to find the base.

*Ex.* What number increased by 6% of itself is equal to 477?

## OPERATION.

$$1 + .06 = 1.06$$

$$477 \div 1.06 = 450, \text{ Ans.}$$

$$\text{Or, } \frac{55}{50} = 477$$

$$\frac{10}{50} = 9$$

$$\frac{55}{50} = 450, \text{ Ans.}$$

number, equals  $\frac{55}{50}$  of the number, = 477. If  $\frac{55}{50}$  of the number = 477,  $\frac{1}{50}$  of the number =  $\frac{1}{55}$  of 477 = 9, and  $\frac{55}{50}$  of the number, equals 55 times 9 = 450. Hence the

ANALYSIS.—A number increased by 6% of itself, equals 106%, or 1.06 of itself, which, by the condition of the question, is 477; hence, once the number equals  $477 \div 1.06 = 450$ .

Or, 6% of a number is  $\frac{6}{100} = \frac{3}{50}$  of the number, which being increased by  $\frac{3}{50}$  of the number, = 477. If  $\frac{55}{50}$  of the number = 477,  $\frac{1}{50}$  of the number, equals 50 times 9 = 450.

**288. RULE**—Divide the amount by 1 plus the rate %, expressed decimally, or as a common fraction; and the difference by 1 minus the rate %, expressed decimally, or as a common fraction.

## EXAMPLES FOR PRACTICE.

1. What is that number, which, diminished by 5% of itself, gives 429.40?  
*Ans.* 452.
2. What number increased by 5% of itself, gives £7 1 9?
3. I have \$407.55, or 4% more than my neighbor; what sum does my neighbor possess?  
*Ans.* \$390.
4. The difference is \$9.48 $\frac{1}{2}$ , and the rate, 12%; what is the base?  
*Ans.* \$10.84.
5. Andrew has £189 9 8, which is 7% less than that of Louis; what sum has the latter?  
*Ans.* £203 14 10 $\frac{1}{2}$ .
6. Find the number which, augmented by the  $\frac{1}{5}$  of itself, gives \$52.32 $\frac{1}{2}$ .  
*Ans.* \$52.
7. A teacher spends 45% of his income, and saves \$858; what was his income?
8. After taking 12% of a pile of wheat, there remain 44 bushels; how many bushels were in the pile?  
*Ans.* 50bu.
9. Having increased my capital by 15 $\frac{1}{2}$ % of itself, I find I possess \$5682.60; how much had I at first?
10. A shepherd lost, by disease 12% of his flock; how many sheep composed his primitive flock, knowing that there remain 1100?
11. A clerk spends 20% of 66 $\frac{2}{3}$ % more than  $\frac{1}{2}$  of his income; what is his income, if he saves \$533?
12. A gentleman sold two horses at \$420 each; for one he received 25% more, and for the other 25% less than his value; what is his loss?
13. A man, wishing to sell a horse, asked 25% more than its cost; he finally sold it for 16% less than his asking price, and gained \$7.50. How much did the horse cost him, and what was his asking price?  
*Ans.* cost, \$120; asking price, \$150.

## MISCELLANEOUS EXAMPLES IN PERCENTAGE.

1. Find  $\frac{3}{4}$  of 70wt. lqr. 12lb. *Ans.* 14.074lb.
2.  $\$1\frac{1}{2}$  is  $\frac{1}{2}$  of what number? *Ans.* \$180.
3. Find a number which, diminished by 10% of itself, gives £48.
4. A merchant owes \$4500; his property is valued at \$2295; what rate per cent. can he pay? *Ans.* 51%.
5. A superior officer, having 1500 men under his command, lost 9% of them in a battle, and 40% of the remainder by sickness; how many remain? *Ans.* 819 men.
6. I sold cloth at £1 10 3 a yard, which is but 65% of the cost; how much did it cost a yard? *Ans.* £2 6 6 $\frac{1}{2}$  +
7. A man expends \$18, which is 33 $\frac{1}{3}$ % more than his weekly wages; what are his wages? *Ans.* \$13.50.
8. After paying 42 $\frac{1}{2}$ % of my debt, I find that \$2650 will settle the balance; how much did I owe? *Ans.* \$4608.69 +.
9. What per cent. of £40 will give 20% of £7 15? *Ans.* 3 $\frac{1}{2}$ %.
10. A little boy laid out 40% of his money in play things, 33% in sugar-plums, and has 12 cents remaining; what did his purse contain? *Ans.* 48 cts.
11. What per cent. of a number gives 22 $\frac{1}{2}$ % of the  $\frac{2}{3}$  of this number? *Ans.* 18 $\frac{1}{3}$ %.
12. A cargo of barley having been damaged, the owner was obliged to sell the whole for \$1999.20 which was at a loss of 32%; how much did the cargo cost him? *Ans.* \$2940.
13. A merchant having \$2150 deposited in a bank, desires to draw 15% of it; how much will remain? *Ans.* \$1827.50.
14. There remains 25 $\frac{1}{2}$ yd. of a piece of linen, after having sold 16% of it; what was the length of the piece? *Ans.* 30 yards.
15. The number of deaths in a certain town, during the year, was 1950, which is 3 $\frac{1}{4}$ % of the population; what is the number of its inhabitants? *Ans.* 60000.
16. A fish-monger had 720bbl. of fish, and sold 288bbl.; what per cent. remained unsold? *Ans.* 60%.
17. 18lb. 15oz. is 12 $\frac{1}{2}$ % of how many lb.? *Ans.* 151lb. 8oz.
18. Gave to a Benevolent Society 29bush. of wheat, which was 14 $\frac{1}{2}$ % of my entire crop; how many bushels had I remaining? *Ans.* 171.
19. What per cent. of  $\frac{1}{2}$  of  $\frac{2}{3}$  of  $\frac{3}{4}$  gives  $\frac{1}{4}$ ? *Ans.* 25%.
20. Joseph having received a legacy, deposited 75% of it in a bank. A short time after, he drew forth 30% of his deposit, and there still remained \$1280 17 6; what was the legacy? *Ans.* £2439 15 2 $\frac{1}{2}$ .
21. In a certain coin there are 21 parts copper and 4 parts nickel; what per cent. is the copper and nickel? *A.* copper 84%, nickel 16%.
22. A gentleman has an annual income of \$2700; if he expends 20% for nourishment, 8% for clothing, 3 $\frac{1}{2}$ % in alms, 5% in books, and 14% in casual expenses, what are his annual expenses? *A.* \$1363.50.
23. In an engagement, 5% of the army were killed on the field of battle, and 6% of the remainder died of their wounds in the hospitals. The difference between the number of the dead and the number of the wounded was 154; how many men composed the army? *Ans.* 22000.

24. Edward lives  $3\frac{1}{4}$  miles from the city, which is  $9\frac{3}{8}$  nearer than the residence of Peter; how far from the city does the latter live?
25. An army, having been twice decimated in battle, is reduced to 19440 men; what was the strength of the army before the combat?
26. The sales of a mercantile establishment amount to \$131000 yearly; the  $\frac{2}{3}$  of these sales were made at a profit of 28%; the  $\frac{1}{3}$ , a profit of 40%; and the remainder, a profit of  $17\frac{1}{2}\%$ ; how much did the merchandise cost?
27. The proprietor of the  $\frac{1}{5}$  of a factory, sells 24% of his share to C., and the remainder to D., at the same rate, for \$15800; what is the value of the factory? *Ans.* \$24947.368 +.
28. After cutting a certain number of yards from a piece of cloth, there remains 12yd.  $2\frac{1}{4}$ qr., which is 70% less than the quantity cut; what was the length of the piece? *Ans.* 64yd.  $1\frac{1}{4}$ qr.
29. If £36 17 6 is  $1\frac{1}{4}\%$  of Paul's money, and if  $5\frac{1}{2}\%$  of Paul's money is 12% of Leo's, how much has Paul more than Leo?
30. My crop of potatoes this year is 9% greater than that of last year, and I have gathered in the two years 6479 bushels; of how many bushels did my last crop consist? *Ans.* 3379bu.
31. If the population of the Dominion of Canada, in 1869, was 4260000 inhabitants, what ought it to be in 1879, supposing it to increase at the rate of 27 $\frac{1}{2}\%$ ? *Ans.* 5431500.
32. The net gains of a nursery in two years was £2178; the gains of the second year were 6% greater than those of the first year; what were the gains of each year? *Ans.* £1057 5 7 $\frac{1}{2}$  and £1120 14 4 $\frac{1}{2}$ .
33. I had \$15000 in a bank; I drew out at first 22%, then, 34% of the remainder, and finally deposited 12% of what I had drawn; how much remain in the bank? *Ans.* \$8595.36.
34. A person having a revenue of \$560, makes the following expenses: board, \$130; tailor, \$145; shoemaker, \$28; sundries, \$36; what per cent. of his revenue is each article, and how much remains? *Ans.* 23 $\frac{1}{4}\%$ ; 25 $\frac{3}{4}\%$ ; 5%; 6 $\frac{1}{2}\%$ ; 24 $\frac{1}{2}\%$ .
35. If a number be augmented by 11% of itself, and this amount by 9%, it will become \$7.75; what is the number? *Ans.* \$6.40.
36. A merchant expended the same sum in the purchase of wine, whiskey and coffee. In selling, he gained 8% on the wine and 5% on the whiskey; but he lost 14% on the coffee; he received from his entire sales £630 10; how much did he pay for each sort of merchandise? *Ans.* £216 17 4 $\frac{3}{8}$ .
37. Edmund and Charles have respectively 6% and 4% more money than Maurice, and the three have together \$22320; how much has Maurice? *Ans.* \$7200.
38. A young man commences business on the 1st. of February, with a capital of \$2700. At the end of 10 months, I read in his Ledger as follows: Feb. 2% gain; March, 3 $\frac{1}{2}\%$  gain; April, 1 $\frac{1}{2}\%$  loss; May, 1 $\frac{1}{2}\%$  gain; June, 2 $\frac{1}{2}\%$  gain; July, 1 $\frac{1}{2}\%$  loss; August,  $\frac{1}{2}\%$  gain; Sept. 1 $\frac{1}{2}\%$  loss; Oct. 2 $\frac{1}{2}\%$  gain; Nov. 4 $\frac{1}{2}\%$  gain; what were the net profits of his business during the 10 months?

28  
the le  
29  
29  
of \$10  
rily a  
Notr.

29  
29  
princip  
29

It vari  
Notr.  
country  
intende

29  
Notr.

29

Ex.  
years),

\$5

\$27

1

\$ 28

78

\$106

29

pressed

year.

II. A

as a fr

The

gether.

Notr.

month is

## SIMPLE INTEREST.

**289.** Interest is the compensation made by the borrower to the lender for the use of money.

**290.** The Principal is the sum lent.

**291.** The Rate per cent. is the interest paid for the loan of \$100, £100, etc., during any time whatever, which is ordinarily a year.

*NOTE.*—The rate per cent. is commonly expressed decimally as hundredths.

**292.** The Amount is the sum of the principal and interest.

**293.** Simple Interest is the sum paid for the use of the principal only, during the time of the loan.

**294.** Legal Interest is the rate per cent. established by law. It varies in different countries.

*NOTE.*—When no rate is mentioned, the rate established by the laws of the country in which the transaction takes place, is always understood to be the one intended by the parties.

**295.** Usury is a higher rate % than is allowed by law.

*NOTE.*—The law prohibits usury and makes it subject to a penalty.

**296.** To find the interest on any sum, at any rate %, for any number of years and months.

*Ex.* What is the interest of \$780, for 5 years and 3 months (5½ years), at 7%? What is the amount?

OPERATION.

\$780 Prin.

.07 Rate.

\$54.60 Int. 1yr.

5½

\$273.00 " 5yr.

13.65 " 3mo.

\$ 286.65, " 5½yr.

780.00 Prin. added.

\$1066.65, Amount..

ANALYSIS.—The interest of \$1 for 1 year, at 7% is \$0.07, and of \$780 it is 780 times \$0.07 = \$54.60. If the interest of \$780 for 1 year, at 7% is \$54.60, for 5½ years it is 5½ times \$54.60 = \$286.65.

Or,  $\frac{7}{100}$  of the principal = the interest for 1 year at 7%. The amount is found by adding the principal and interest together.

**297. RULE.**—I. Multiply the principal by the rate % expressed decimally, and the product will give the interest for one year.

II. Multiply this product by the number of years, and the months as a fraction of a year, for the interest required.

The amount is found by adding the principal and interest together.

*NOTE.*—When part of the time for interest is given in months or days, one month is considered as  $\frac{1}{12}$  of a year, and one day as  $\frac{1}{30}$  of a month.

## EXAMPLES FOR PRACTICE.

What is the interest of

- |                                                           |                               |
|-----------------------------------------------------------|-------------------------------|
| 1. \$450 for 3 years, at 4%?                              | Ans. \$54.                    |
| 2. \$16 for 7 years, at 8%?                               | Ans. \$8.96.                  |
| 3. \$656 for 2 years, at 7%?                              |                               |
| 4. \$1728 for 1 year 6 months, at 6%?                     | Ans. \$155.52.                |
| 5. \$1740 for 3 years, at 8 $\frac{1}{2}$ %?              | Ans. \$435.                   |
| 6. \$878.25 for 2 years, at 9 $\frac{1}{2}$ %?            | Ans. \$166.86 $\frac{1}{2}$ . |
| 7. \$118.15 for 2 years 6 months, at 6%?                  |                               |
| 8. \$300 for 3 years 10 months, at 7%?                    |                               |
| 9. \$125.75 for 4 years 6 months, at 8%?                  | Ans. \$45.27.                 |
| 10. \$97.16 for 1 year 5 months, at 5%?                   | Ans. \$8.258.                 |
| 11. \$58.60 for 2 years 9 months, at 8%?                  | Ans. \$12.892.                |
| 12. \$76.50 for 2 years 2 months, at 5%?                  | Ans. \$8.28 +.                |
| 13. \$444.44 for 5 years, at 6 $\frac{3}{4}$ %?           |                               |
| 14. \$960.18 for 1 year 2 months, at 7%?                  | Ans. \$78.414.                |
| 15. \$4501.80 for 2 years 4 months, at 6 $\frac{1}{2}$ %? | Ans. \$656.5125.              |
| 16. \$1671.32 for 14 months, at 6%?                       | Ans. \$116.99.                |

What is the amount of

- |                                                        |                   |
|--------------------------------------------------------|-------------------|
| 17. \$53.68 for 2 years 6 months, at 5%?               | Ans. \$60.39.     |
| 18. \$978.18 for 2 years 3 months, at 6%?              | Ans. \$1110.234.  |
| 19. \$3050 for 4 years 8 months, at 5 $\frac{1}{2}$ %? | Ans. \$3797.25 +. |
| 20. \$31.81 for 8 years 4 months, at 6%?               | Ans. \$122.715.   |
| 21. \$95 for 1 year and 6 months, at 5%?               |                   |
| 22. \$65256 for 4 $\frac{1}{2}$ months, at 7%?         |                   |
| 23. \$894 for 20 months, at 6%?                        |                   |
| 24. \$760 for 5 years 7 months, at 5 $\frac{1}{2}$ %?  |                   |

**298.** To find the interest on any sum, for any time, at any rate %.

## SIX PER CENT. METHOD.

To find the interest of \$1 for any time, at 6%; also, at any other rate %.

ANALYSIS.—At 6% per annum the interest on \$1.

|                                                            |                                                         |
|------------------------------------------------------------|---------------------------------------------------------|
| For 12 months                                              | is \$ .06                                               |
| " 2 months ( $\frac{2}{12}$ = $\frac{1}{6}$ of 12 mo.)     | " .01                                                   |
| " 1 month, or 30 days ( $\frac{1}{12}$ of 12 mo.)          | " .00 $\frac{1}{2}$ = \$.005 ( $\frac{1}{12}$ of \$.06) |
| " 6 days ( $\frac{1}{6}$ of 30 days)                       | " .001                                                  |
| " 1 " ( $\frac{1}{6}$ of 6 da. = $\frac{1}{30}$ of 30 da.) | " .000 $\frac{1}{3}$ . Hence,                           |

1st. The interest on \$1 is \$.005 per month, or \$.01 for every 2 months; 2nd. the interest on \$1 is \$.000 $\frac{1}{3}$  per day, or \$001 for every 6 days. Hence the

**299. RULE.**—I. To find the rate:—Call every year \$.06, every 2 months \$.01, every 6 days \$.001, and any less number of days sixths of 1 mill.

II.

Ex.  
27 dayInt. of  
" "  
" "  
" "

given t

Ex.  
at 7%.ANAL.  
method  
find the  
interest  
rate, \$.30  
—Fin  
preced  
II.  
per cen  
the in

II. To find the interest:—*Multiply the principal by the rate.*

*Ex. 1.* What is the interest of \$660, at 6%, for 3 years 7 months 27 days?

## OPERATION.

|                       |            |
|-----------------------|------------|
| Int. of \$1 for 3yr.  | = \$0.18   |
| " " " 7mo.            | = 0.035    |
| " " " 27 days.        | = 0.004½   |
| " " " 3yr. 7mo. 27da. | = \$0.219½ |
| Principal,            | \$660      |
|                       | .219½      |
|                       | 5940       |
|                       | 660        |
|                       | 1320       |
|                       | 330        |
| Interest,             | \$144.870  |

*ANALYSIS.*—The interest of the given principal is 660 times the interest of \$1 for 3 years 7 months 27 days. As the int. of \$1 for 1yr. is \$.06, for 3yr. it will be \$.18; and since the interest for 2 months is \$.01, for 7 months it will be as many times \$.01, as 2 is contained in 7, or 3½cts. Again, since the interest for 6 days is \$.001, for 27 days, it will be as many times \$.001, as 6 is contained in 27, or 4½ mills. Adding these three results together, we have \$0.219½ which equals the interest of \$1 at 6% for the

given time. Multiplying \$660 by \$0.219½, we obtain \$144.87, the int. required:

*Ex. 2.* Required the interest on \$750 for 8 years 8 months 9 days, at 7%.

## OPERATION.

|             |                        |
|-------------|------------------------|
| \$0.48      | = Int. on \$1 for 8yr. |
| 0.04        | = " " " 8mo.           |
| 0.001½      | = " " " 9 days.        |
| 6) \$0.521½ | = " " " 8yr. 8mo. 9da. |
| \$0.086½    | = " " " " " " " at 1%. |
| 7           |                        |
| \$0.608½    | = " " " " " " " at 7%. |
| \$ 750      |                        |
| \$608½      |                        |
| 6000        |                        |
| 45000       |                        |
| 312½        |                        |
| \$466.312½  | = Int. required.       |

*ANALYSIS.*—After finding the interest of \$1 for the given time, at 6%, by the method laid down in the preceding example, we divide the result by 6, and then find the interest at 1%; we then multiply by the given rate, 7, and obtain the interest on \$1 for the given time, at 7%. Multiplying the principal, \$750, by the rate, \$0.608½, we obtain \$466.31½, which is the interest required. Hence the

**300. RULE.**—I. When the rate is greater or less than 6%:—*Find the interest on \$1, at 6%, for the given time, as in the preceding example.*

II. *Then divide by 6, and multiply the quotient by the given per cent. This result multiplied by the given principal, will give the interest required.*

## SIMPLE INTEREST.

**NOTE.**—The interest of the given principal, at 6%, for the given time, could be obtained at first; then, this result divided by 6 and multiplied by the given per cent. will give the same answer; or, add or subtract from this interest, such a fractional part of itself as the given rate exceeds or falls short of 6% per annum; Thus, if the rate be 9%, the interest at 6% should be increased  $\frac{3}{8}$  or  $\frac{1}{2}$  of itself, because 3, the excess of 9 over 6, is  $\frac{1}{2}$  of 6, and so on;

When the time is short, business men use the following

**301. RULE.**—Multiply the principal by the number of days, divide the product by 6, and remove the decimal point three places toward the left. (The result is the interest at 6%.) Then proceed as in the above rule.

## METHOD BY ALIQUOT PARTS.

*Ex.* What is the interest of \$421.50 for 3yr. 8mo. and 15da. at 9%?

| Principal,                             | \$421.50                        |
|----------------------------------------|---------------------------------|
| Rate %,                                | .09                             |
| Interest for 1 year,                   | \$37.9350                       |
|                                        | 3                               |
| Int. for 3 years,                      | \$113.8050                      |
| Int. for 6mo. = $\frac{1}{2}$ of 1yr.  | 18.9675                         |
| Int. for 2mo. = $\frac{1}{3}$ of 6mo.  | 6.3225                          |
| Int. for 15da. = $\frac{1}{4}$ of 2mo. | 1.5806 $\frac{1}{4}$            |
| Int. for 3yr. 8mo. 15da.               | \$140.6756 $\frac{1}{4}$ , Ans. |

**ANALYSIS.**—Having found the interest for 1yr. and then for 3yr., the int. for 8mo. is obtained by first taking  $\frac{1}{2}$  of 1 year's int., for 6mo., and then  $\frac{1}{3}$  of this last int. for 2mo. And since 15 days are  $\frac{1}{4}$  of 1mo., or  $\frac{1}{4}$  of 2mo., we take  $\frac{1}{4}$  of 2mo.'s int. for 15 days. The int. as found for the several parts of the whole time, added together, gives the interest required.

**NOTE.**—Whenever the number of mills is 5 and upwards, in business transactions, we add 1 cent and drop the mills. Hence, the interest in the above example is called \$140.68.

**302. RULE.**—I. First find the interest for one year by multiplying the principal by the rate %, decimally expressed, and this product by the number of years.

II. Find the interest for the months and days by aliquot parts. The sum of the partial interests will be the interest required.

## METHOD BY MONTHS.

*Ex.* What is the interest of \$24.20 for 4yr. 7mo. and 15da., at 6%?

| OPERATION.                        | OPERATION.                                    |
|-----------------------------------|-----------------------------------------------|
| 6.05                              | 18.5                                          |
| $24.20 \times .06 \times 4$       | $24.20 \times .06$                            |
| $\frac{3}{4} \times 4 = 36.7155.$ | $\frac{7}{12} \times 1.4520 =$ Int. for 1 yr. |

|                                               |
|-----------------------------------------------|
| $\frac{7}{12} \times 1.4520 =$ Int. for 1 yr. |
| $\frac{1}{12} \times 1.4520 =$ Int. for 1 mo. |
| 55.5                                          |
| 6050                                          |
| 6050                                          |
| 6050                                          |
| $36.71550 =$ Int. for 55.5 mo.                |

**NOTES.**—1. 4yr. 7mo. 15da. = 55.5mo.  
2. The above is the product of the principal, rate per cent., decimally expressed in months and decimals of a month, divided by 12 =  $3 \times 4$ .

30  
a mo  
II  
tient  
H  
the p

E3  
and 1

30  
as th

Not  
and fa

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  
9.  
10.  
11.  
12.  
13.  
14.  
15.  
16.  
17.  
18.  
19.  
20.  
21.  
22.  
23.  
28.

**303. RULE.—I.** Reduce the time to months and decimals of a month.

II. Find the interest for 1 year, and divide it by 12; the quotient will be 1 month's interest.

III. Multiply this interest by the time expressed in months, and the product will be the interest required.

## METHOD BY PROPORTION.

*Ex.* What is the interest of \$52.50, at 6%, for 4 years 5 months and 10 days?

Sol.  $100 : 6 \times 4\text{yr. } 5\text{mo. } 10\text{da.} :: \$52.50 : x$ ; whence the

**304. RULE.—**100 is to the per cent. multiplied by the time, as the principal is to the interest.

## EXAMPLES FOR PRACTICE

TO BE SOLVED BY ANY OF THE ABOVE METHODS.

*NOTE.*—If the principal be given in old currency, reduce the shillings, pence and farthings, to the decimal of a £; then proceed as in decimal currency.

What is the interest on

- |                                           |                          |
|-------------------------------------------|--------------------------|
| 1. \$500 for 1yr. 10mo. and 15da., at 6%? | <i>Ans.</i> \$56.25.     |
| 2. \$9862.12½ for 3yr. 5mo., at 4%?       | <i>Ans.</i> \$1347.82 +. |
| 3. £26 10 0 for 2yr. 4mo., at 6%?         | <i>Ans.</i> \$14.84.     |
| 4. \$972.40 for 1yr. 7mo. 18da., at 7%?   | <i>Ans.</i> \$111.177 +. |
| 5. \$143 for 2yr. and 9mo., at 8%?        | <i>Ans.</i> \$31.46.     |
| 6. \$47.25 for 1yr. and 6mo., at 6%?      | <i>Ans.</i> \$4.25½.     |
| 7. £42 18 0 for 3yr. 4mo. 25da., at 6%?   | <i>Ans.</i> £8 15 2½.    |
| 8. \$147.90 for 8mo. 4da., at 5%?         | <i>Ans.</i> \$5.01 +.    |
| 9. \$145.50 for 1yr. 9mo., 24da., at 6%?  | <i>Ans.</i> \$15.85 +.   |
| 10. \$579.75 for 1yr. 3mo. 2da., at 5%?   | <i>Ans.</i> \$36.395 +.  |
| 11. £94 12 6 for 4yr. 6mo. 7da., at 8%?   | <i>Ans.</i> \$136.848 +. |
| 12. \$123.75 for 2yr. 8mo. 12da., at 6%?  | <i>Ans.</i> \$20.04½.    |
| 13. \$50.40 for 1yr. and 10mo., at 7%?    | <i>Ans.</i> \$6.468.     |
| 14. \$475 for 2yr. 7mo. 20da., at 6%?     | <i>Ans.</i> \$75.208½.   |
| 15. £6 11 3 for 2yr. 4mo., at 7%?         | <i>Ans.</i> £1 1 5½.     |
| 16. \$336 for 5mo. 15da., at 6%?          | <i>Ans.</i> \$7.70.      |
| 17. \$1265.60 for 5yr. 2mo. 9da., at 7%?  | <i>Ans.</i> \$459.94 +.  |
| 18. \$72.12½ for 6yr. and 6mo., at 4%?    | <i>Ans.</i> \$18.51 +.   |
| 19. \$497.36 for 1yr. 6mo. 1da., at 5%?   | <i>Ans.</i> \$37.37 +.   |
| 20. £191 5 4 for 2yr. 9mo., at 1%?        | <i>Ans.</i> \$21.039 +.  |
| 21. \$7671.09 for 3yr. 8mo. 5da., at 8%?  | <i>Ans.</i> \$2258.70 +. |
| 22. \$49.80 for 2yr. and 11mo., at 7%?    | <i>Ans.</i> \$10.16½.    |
| 23. \$350.80 for 15mo. and 8da., at 10%?  |                          |

## SIMPLE INTEREST.

24. \$1040 for 5yr. 11mo. 29da., at 7%? *Ans.* \$436.596.  
 25. £24 18 8 for 10mo. and 20da., at 7%? *Ans.* £1 11 0¼+.  
 26. \$51.17 for 10mo. and 29da., at 4%?  
 27. \$548.12 for 6yr. 1mo. 3da., at 7%? *Ans.* \$233.72+.  
 28. \$500 for 2yr. 5mo. 12da., at 6%?  
 29. \$909.50 for 5yr. 5mo. 4da., at 6%? *Ans.* \$296.19+.  
 30. £92 12 0 for 2yr. 10mo., at 6½%?  
 31. \$680 for 4yr. 1mo. 15da., at 6%?  
 32. \$2000 for 1yr. 3mo. 10da., at 9%? *Ans.* \$168.30.  
 33. \$471.11 for 4yr. and 8mo., at 7½%?  
 34. \$190.016 for 3mo. 24da., at 4½%?  
 35. £427 8 8 for 1yr. 5mo., at 5½%? *Ans.* £34 16 4+.  
 36. \$708.20 for 2yr. 2mo. 12da., at 4½%?  
 37. \$640.70 for 8mo. and 26da., at 5½%?  
 38. \$730.50 for 18mo. and 23da., at 6½%?  
 39. \$950 for 4yr. 7mo. 9da., at 8¼%? *Ans.* \$26.037+.  
 40. £81 10 0 for 2yr. and 5mo., at 4½%?  
 41. \$150.80 for 7mo. and 20da., at 7½%?  
 42. \$1072.40 for 5 yr. 10mo. 5da., at 6½%? *Ans.* \$361.178+.  
 43. \$601.20 for 4yr. 2mo. 3da., at 8½%?  
 44. \$1425.20 for 1yr. and 16da., at 4¼%? *Ans.* \$6.98+.  
 45. £319 10 9 for 1yr. 10mo., at 4½%?  
 46. \$742.30 for 4yr. 9mo. 19da., at 6¾%?  
 47. \$1370.40 for 3yr. 4mo. 27da., at 7½%?  
 48. \$160.75 for 2yr. 11mo. 4da., at 5½%?  
 49. \$1463.60 for 7yr. 7mo. 22da., at 6¼%?  
 50. £184 18 8 for 1yr. 9mo. 6da., at 3½%? *Ans.* \$213.35+.  
*Ans.* \$102.518+.  
*Ans.* \$350.30+.  
*Ans.* \$724.24+.

What is the amount of

51. \$0.145 for 9yr. 9mo. and 9da., at 6%? *Ans.* \$0.23+.  
 52. \$1051.50 for 2yr. 10mo., at 7%? *Ans.* \$1260.045+.  
 53. \$168.13 for 8yr. 5mo. 3da., at 6%?  
 54. \$100.25 for 2mo. and 29da., at 4%?  
 55. \$1.011 for 10yr. 10mo. 10da., at 6%?  
 56. \$1000 for 3yr. 3mo. 29da., at 5½%?  
 57. \$168.50 for 1yr. 5mo. and 10da., at 6¼%? *Ans.* \$1183.18.  
 58. \$2000 for 1mo. 5da., at 6½%?  
 59. \$0.05 for 20yr. 10mo. 15da., at 8%? *Ans.* \$2013.12½.  
 60. \$325.25 for 2yr. 9mo. 12da., at 6½%?  
 61. \$495.95 for 5yr. 5mo. 5da., at 6¾%?  
 62. £109 3 9 for 7yr. 9mo. 18da., at 3¾%?  
 63. \$2560.75 for 4yr. 3mo. 25da., at 6½%?  
 64. What is the interest of \$1560 from April 9, to November 10, at 5½%?  
 65. What is the amount of \$175.08 from May 7, 1861, to September 25, 1863, at 7%? *Ans.* \$50.28½.  
 66. What is the interest of \$176.89½ from January 6, 1868, to July 22, 1869, at 6¼%? *Ans.* \$204.289.  
 67. What is the amount of \$1756.75 from June 29, 1860, to February 12, 1863, at 7%?

68.  
7, at  
69.  
Janua  
70.  
Decem  
71.  
1869,  
72.  
July  
73.  
ber 9,  
74.  
uary-  
75.  
March  
76.  
May t

30  
are in  
12 m  
of 36  
not/s

Th  
putin

Nov  
page 1

30  
by th  
prod

1.  
20, 18  
2.  
Novem  
3.  
to No  
4.  
Sept.  
5.  
to Sep  
6.  
1868,

68. What is the interest of £43 2 6 from March 17, to December 7, at  $7\frac{1}{2}\%$ ? *Ans.* £2 5 1 $\frac{1}{2}$  +.
69. What is the interest of \$1530.50 from February 10, 1868, to January 25, 1869, at  $\frac{1}{2}\%$ ? *Ans.* \$7.33 +.
70. What is the amount of \$158.30 from February 17, 1868, to December 30, 1871, at  $7\frac{1}{2}\%$ ?
71. What is the interest of \$1728.19, from May 7, 1868, to July 17, 1869, at  $\frac{1}{2}\%$ ? *Ans.* \$5.16 +.
72. What is the interest of £32 8 9 from September 25, 1867, to July 9, 1869, at  $\frac{3}{4}\%$ ?
73. What is the amount of \$89.96 from June 19, 1870, to December 9, 1871, at  $8\frac{1}{2}\%$ ? *Ans.* \$100.886.
74. What is the interest of \$990.75 from October 5, 1868, to January 15, 1869, at  $1\frac{1}{2}\%$ ?
75. What is the interest of \$1030.10 from November 8, 1867, to March 3, 1869, at  $8\frac{1}{2}\%$ ? *Ans.* \$120.625 +.
76. What is the interest of £45 10 4 from December 10, 1866, to May 5, 1869, at  $\frac{3}{4}\%$ ?

## EXACT METHOD OF COMPUTING INTEREST.

**305.** In the preceding methods of computing interest, which are in general use, we have reckoned 30 days to the month, and 12 months to the year, which allows to each year 360 instead of 365 days. Hence, the results obtained in these calculations are not strictly correct.

The following exact method is used by business men in computing interest when the time is short.

*NOTE.*—The exact time, when it is less than a year, is found by the table on page 124.

**306. RULE.**—*Multiply the interest of the principal for 1 year by the exact number of days it has been on interest, and divide the product by 365, the quotient will be the interest required.*

1. What is the interest of \$345.60, from February 5, 1869, to Aug. 20, 1871, at  $7\%$ ? *Ans.* \$61.374 +.
2. What is the interest, at  $5\frac{1}{2}\%$ , of \$425.50, from January 8th., until November 20th. ? *Ans.* \$20.26 +.
3. What is the interest, at  $6\frac{1}{2}\%$ , of \$140.40, from Aug. 29th., 1870, to Nov. 29th., 1871 ? *Ans.* \$11.426 +.
4. What is the interest, at  $8\%$ , of \$4560, from May 18th., 1868, to Sept. 25th., 1871 ?
5. What is the interest, at  $7\frac{1}{2}\%$ , of \$3790.45, from July 20th., 1869, to Sept. 12th., 1871 ?
6. What is the interest, at  $4\frac{1}{2}\%$  of £48 16 3. from Sept. 12th., 1868, to Aug. 28th., 1871 ?

PARTIAL PAYMENTS.

PARTIAL PAYMENTS.

**307. Partial Payments** are payments of part of a note, bond, or other moneyed obligation, made at different times. The payments are acknowledged by receipts written by the creditor on the back of the note or obligation, which are called **Indorsements**.

**308. RULE.**—I. If the interest be paid by days:—Multiply the principal by the number of days which have elapsed before any payment was made. Subtract the first payment, and multiply the remainder by the number of days which passed between the first and second payments. Subtract the second payment, and multiply this remainder by the number of days which passed between the second and third payments. Subtract the third payment, etc.

II. Add all the products together, and find the interest of their sum for one day.

III. If the interest is to be paid by the week or month:—Substitute weeks or months for days, in the above rule.

*Ex.* 1. How much principal and interest have I to pay on the following note, due Dec. 29, 1871?

\$ 420.

Quebec, Sept. 8, 1868.

For value received, I promise to pay James Carroll, or order, four hundred and twenty dollars, with interest, at 7%? Thomas Brown.

On this note were indorsed the following payments:—

|                         |       |          |
|-------------------------|-------|----------|
| Oct. 1, 1869, received, | ..... | \$22.28. |
| Nov. 20, 1869, " "      | ..... | 50.00.   |
| May 8, 1871, " "        | ..... | 247.87.  |

OPERATION

From Sept. 8, 1868, to Oct. 1, 1869, there are 388 days.

" Oct. 1, 1869, to Nov. 20, 1869, " " 50 "

" Nov. 20, 1869, to May 8, 1871, " " 534 "

" May. 8, 1871, to Dec. 29, 1871, " " 235 "

Whole principal \$420.00 for 388 days = \$162960.00 for 1 day.

First indorsement 22.28

Balance \$397.72 for 50 days = \$ 19886.00 for 1 day.

Second indorsement 50.00

Balance \$347.72 for 534 days = \$185682.48 for 1 day.

Third indorsement 247.87

Balance \$ 99.85 for 235 days = \$ 23464.75 for 1 day.

Whole interest = that of \$991993.23 for 1 day.

## PARTIAL PAYMENTS.

185

Interest on \$391993.23 at 7% for 1 yr. = \$27439.5261.  
 Hence, the int. for 1 day = \$27439.5261 ÷ 365 = \$75.1767 +.  
 Then interest due = \$75.1767 +.  
 Balance on note = 99.8500.

Principal and interest due \$175.0267 +.

\$ 450.

Montreal, January 13, 1869.

2. Nine months after date, I promise to pay Louis Merrill, or order, three hundred and fifty dollars, with interest, at 6%, for value received.  
 A. N. Moreau.

Indorsed as follows: Oct. 7, 1869, \$125.10; Aug. 25, 1870, \$225.35. How much remained due Sept. 19, 1871? *Ans.* \$142.8802 +.

\$ 325. <sup>25</sup>/<sub>100</sub>.

Kingston, July 26, 1866.

3. Four years after date, we promise to pay Lawrence Boyce, or order, three hundred twenty-five and <sup>25</sup>/<sub>100</sub> dollars, with interest, at 7%. Value received.  
 L. R. Whelan & Co.

Indorsed as follows: Jan. 20, 1867, \$121.18; March 14, 1868, \$72.45; July 26, 1869, \$133.65. How much remained due Sept. 3, 1870? *Ans.* \$41.01 +.

\$1737. <sup>50</sup>/<sub>100</sub>.

Toronto, March 6, 1868.

4. On demand, we promise to pay Fisher & Howe, or order, one thousand seven hundred thirty-seven and <sup>50</sup>/<sub>100</sub> dollars, for value received, with interest, at 6%.  
 T. Johnson & Bro.

Indorsed as follows: June 1, 1868, \$623.80; Sept. 10, 1868, \$700. How much was due Jan. 31, 1869? *Ans.* \$466.763 +.

\$1240.

Ottawa, Aug. 18, 1869.

5. For value received, I promise to pay R. N. Kelly, or order, two hundred and forty dollars, on demand, with interest, at 6%.  
 Joseph Rogers.

Indorsed as follows: Sept. 25, 1869, \$95; Oct. 28, 1869, \$217.86; Dec. 12, 1869, \$432.36; April 6, 1870, \$120.20; July 3, 1870, \$366.50. How much remained due Sept. 10, 1870? *Ans.* \$43.758 +.

£ 304 6 6.

Halifax, June 2, 1868.

6. For value received, I promise to pay N. J. Webster, or order, on demand, three hundred and four pounds six shillings and six pence, with interest, at 6%.  
 A. G. Murphy.

Indorsed as follows: July 17, 1868, £51 19 0; Oct. 6, 1868, £52 8 0; Dec. 11, 1868, £80 4 6; March 29, 1869, £105 9 14. How much was due Oct. 7, 1869? *Ans.* £24 13 0 <sup>21</sup>/<sub>100</sub>.

\$ 14696.50.

St. John, June 17, 1866.

7. For value received, we jointly and severally promise to pay Edward Hammond, or order, on demand, fourteen thousand six hundred and ninety-six  $\frac{50}{100}$  dollars, with interest, at 8%. J. P. Rooney.  
S. E. Hamilton.

Indorsed as follows: Sept. 5, 1866, \$4927.60; Dec. 7, 1866, \$784.40; June 11, 1867, \$1964.40; Feb. 7, 1868, \$5685.80; Dec. 19, 1868, \$634.46. How much remained due May 1, 1869?  
*Ans.* \$2006.266+

8. A farmer gave a mortgage on his farm for \$4875, dated June 1, 1867, to be paid in 4 years, with 7  $\frac{1}{2}$ % interest. Six months from date he paid \$223.25; Oct. 20, 1869, \$1250; July 3, 1870, \$750; Jan. 1, 1871, \$250; how much was due at the expiration of the given time?  
*Ans.* \$3595.31+

## PROBLEMS IN INTEREST.

**309.** It will be observed that there are *five parts or terms* connected with each of the preceding questions in interest, viz.: the Principal, the Rate %, the Time, the Interest, and the Amount. The investigation of these involves five cases: I. To find the interest; II. To find the amount; III. To find the principal; IV. To find the rate %; V. To find the time.  
The Cases I. and II. have already been solved (296, 298).

**310. CASE III.**—*The interest, time, and rate %, being given, to find the PRINCIPAL.*

*Ex.* What principal in 3 years, at 6%, will gain \$47.70 interest?

OPERATION.

.06 int. of \$1 for 1yr.

3

.18) \$47.70 (*\$265, Ans.**By proportion.*

\$100 : x :: \$6 × 3 : \$47.70.

*ANALYSIS.*—We find the interest of \$1 for 3 years. Since it requires 3 years from a principal of \$1 to gain 18 cents, it will require a principal of as many dollars to gain \$47.70 as \$0.18 is contained times in \$47.70; dividing, we obtain \$265, the required principal. Hence

**311. RULE.**—*Divide the given interest or amount by the interest or amount of \$1 for the given time and rate, and the quotient will be the principal.*

## EXAMPLES FOR PRACTICE.

What principal will in

1. 6yr. 3mo., at 6%, give \$56.25?
2. 1yr. 6mo., at 6%, give \$1.2924 int.?

*Ans.* \$150.  
*Ans.* \$14.36.

3. 4  
4. 1  
5. 3  
6. 4  
7. 8  
8. 1  
9. I  
days,  
10.  
to pro

y 31

Ex.

\$  
\$1  
31  
princi  
rate %

1. \$  
2. \$  
3. \$  
4. £  
5. \$  
6. \$  
7. \$  
8. \$  
9. A  
double  
10.  
semi-a

31

3. 4mo. 18da., at 4%, give \$27.60 int. ? Ans. \$1800.
4. 1yr. 4mo., at 8½%, give \$13.20 int. ? Ans. \$120.
5. 3yr. 8mo. 15da., at 6%, give \$76.095 int. ? Ans. \$342.
6. 4yr. 9mo. 18da., at 9%, give \$65.016 interest ?
7. 8yr. 8mo. 12da., at 5%, gain \$147.9435 ? Ans. \$340.10.
8. 10yr. 10mo. 20da., at 6½%, gain \$1.7770 ?
9. If the interest on a sum borrowed at 2% a month, is \$24 for 90 days, what is the sum ? Ans. \$400.
10. What sum of money loaned for 183 days; at 7½%, is sufficient to produce \$619.15 ?

**312. CASE IV.**—*The principal, time and interest being given, to find the RATE %.*

*Ex.* The interest of \$750 for 4 years is \$180, what is the rate % ?

|            |                      |
|------------|----------------------|
| OPERATION. |                      |
| \$750      |                      |
| .04        |                      |
| \$30.00    | ) \$180.00 (6%, Ans. |
|            | 180 00               |

*By proportion.*

$$\$100 : \$750 :: x \times 4 : \$180.$$

*ANALYSIS.*—We find the interest on the principal for 4 years at 1%. Since the interest of \$1 at 1% for 4 years is 4 cts., the interest of \$750 will be 750 times as much, or \$30. Now, if \$30 is 1%, \$180 will be as many % as \$30 is contained times in \$180; dividing, we obtain 6, the required rate %. Hence the

**313. RULE.**—*Divide the given interest by the interest of the principal for the given time, at 1%, and the quotient will be the rate % required.*

EXAMPLES FOR PRACTICE.

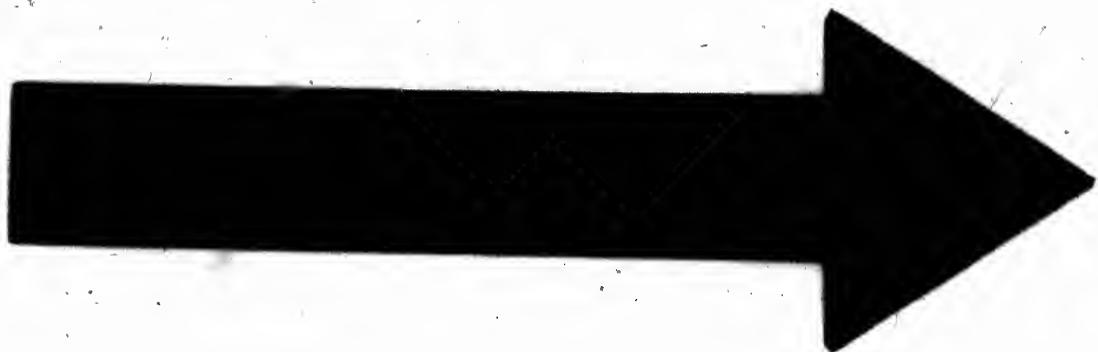
Required the rate per cent. if the interest of

1. \$500 for 1yr. 3mo. is \$56.25. Ans. 9%.
2. \$40 for 2yr. 9mo. 12da. is \$13.36. Ans. 12%.
3. \$540 for 1yr. 2mo. 6da. is \$38.34. Ans. 6%.
4. £37 15 0 for 1yr. 4mo. is £3 10 5½. Ans. 7%.
5. \$125 for 3yr. 6mo. is \$32.37½. Ans. 7½%.
6. \$1500 for 3yr. 3mo. 29da. is \$274.77.
7. \$124 for 4yr. 3mo. 10da. is \$29.17½. Ans. 5½%.
8. \$36 for 3yr. 8mo. 19da. is \$8.034.
9. At what rate % must \$1, or any other sum, be on interest, to double itself in 14½ years ? Ans. 7%.
10. A man invested \$4500 in the Montreal Bank, and received a semi-annual dividend of \$167.50; what % was the dividend ?

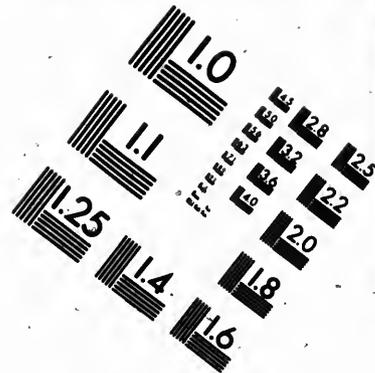
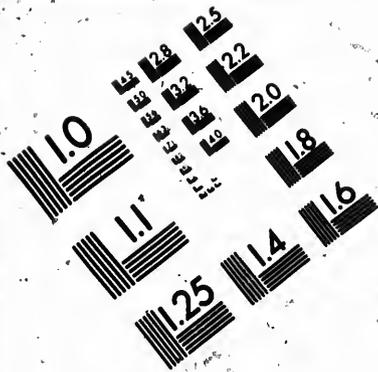
**314. CASE V.**—*The principal, interest, and rate % being given, to find the TIME.*



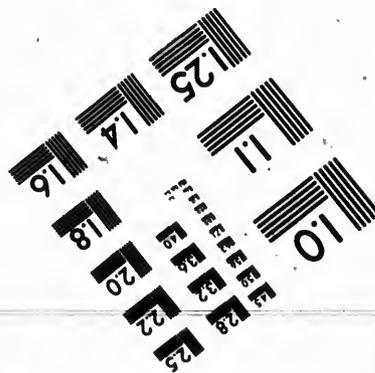
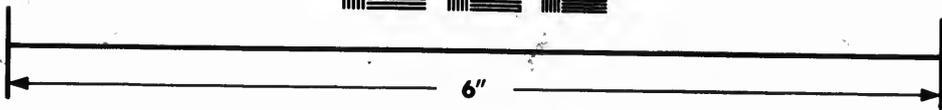








**IMAGE EVALUATION  
TEST TARGET (MT-3)**



**Photographic  
Sciences  
Corporation**

23 WEST MAIN STREET  
WEBSTER, N.Y. 14580  
(716) 872-4503

18  
20  
22  
25  
28  
32  
36

10



3?  
 and the interest  
 for 1 year.  
 \$1 for 1 year is  
 of \$450 will be  
 or \$27. Now, if  
 the given prin-  
 will require as  
 \$64 as \$27 is  
 54; dividing, we  
 required time.

interest on the  
 e required in

uced to months

1 yr. 3mo.  
 5 years.  
 Ans. 8mo.

0mo. 10da.

11 mo.

Ans. 20yr.

INTEREST.

Ans. \$9600.  
 \$5237 10.  
 Ans. \$900.  
 \$9 13 9 +  
 \$1582.42.  
 Ans. \$9000.  
 Ans. \$675.  
 \$7287.71.

Ans. 7yr.  
 Ans. 9yr.  
 3yr. 4mo.

- 12. \$242, at 4½%, give \$55 int. ? *Ans. 4yr. 9mo. 12da.*
- 13. £460, at 5½%, give £50 int. ? *Ans. 2yr. 25da.*
- 14. \$2178, at 4½%, give \$635.25 int. ? *Ans. 7yr.*
- 15. £405, at 6%, give £151 17 6 int. ? *Ans. 6yr. 3mo.*
- 16. \$481.25, at 5%, give \$192.50 int. ? *Ans. 8yr.*

Required the rate %, if the interest of

- 17. \$978.20 for 1yr. is \$48.91. *Ans. 5%.*
- 18. £110 12 6 for 50da. is £1 16 10½. *Ans. 12%.*
- 19. \$1290 for 124da. is \$19.99½. *Ans. 4½%.*
- 20. \$4340 for 3yr. is \$585.90. *Ans. 4½%.*
- 21. \$675 for 44mo. is \$142.31½. *Ans. 5½%.*
- 22. \$7500 for 48da. is \$60. *Ans. 6%.*
- 23. \$11004.75 for 1yr. is \$550.23¾. *Ans. 5%.*
- 24. £120 for 6mo. 15da. is £32 10 0. *Ans. 50%.*
- 25. The annual sales of a starch manufacturer amount to £2737 10; supposing that his profits are 5% per year, in how many years will they reach £323 18 9? *Ans. 2yr. 4mo. 12da.*
- 26. An individual disposed of the ¼ of his funds at 4% and ¼ at 5%; every year he draws as much as will pay the harnessing of a horse which harness is worth \$117.60; what is the amount of his funds? *Ans. \$2800.*
- 27. What is the interest of \$17.18, from July 29th., 1864, to Sept. 1st., 1863, at 6%? *Ans. \$4.214 +.*
- 28. What will be the amount of £19 15 9, at 7½%, from Feb. 17th., 1864, to Dec. 30th., 1867? *Ans. £25 10 7 +.*
- 29. If \$1756.75 is placed on interest, June 29th., 1866, what will it amount to Feb. 12th., 1869, at 7%? *Ans. \$2078.869 +.*
- 30. What principal, at 5%, during 1yr. 8mo. 12da. will amount to £231 12 11¾? *Ans. £213 10 0.*
- 31. On Aug. 15th., 1860, I lent \$5259, at 6%; what amount will be due me on May 1st., 1868? *Ans. \$7692.164.*
- 32. An individual buys 65¼ acres of land at the rate of \$509.72 per 100 acres; if he pays only at the end of 3yr. 1mo. 15da., the int. will equal to ¼ of the principal; what is the rate? *Ans. 4%.*
- 33. A person placed a certain sum on interest at 4%, which produced £427 10, in 3 years; what is the sum? *Ans. £3562 10.*
- 34. What is the interest on a bill of \$257.81, dated March 1st., 1865, and payable July 16th., 1867, at 7%? *Ans. \$42.86 +.*
- 35. Find the amount of \$17041.20, at 4½%, for 1yr. 7mo. 28da. *Ans. \$2000.*
- 36. What sum is that which will give an interest of \$900, in 10yr., at 4½%? *Ans. \$2000.*
- 37. A principal of £112 10 was put on interest, and at the end of 8yr. amounted to £144; at what rate was the principal placed? *Ans. \$2800.*
- 38. A boy has accumulated a sum of money by his savings, and wishes to obtain an annual revenue of \$140; if the rate is 5%, what principal must he have? *Ans. \$2800.*
- 39. A merchant borrows the sum of £938 12 3, which is owned by a minor aged 15yr. 3mo. 20da. He keeps it until the owner is 21 years old; what sum will be then due, at 6% simple interest?

40. What will be the interest of \$325, from June 5th., 1866, to July 4th., 1868, at  $7\frac{1}{2}\%$ ? *Ans.* \$49.02 +.
41. A merchant says that his gain, during the nine years he carried on business, equals the price of 3659 yards of cloth at \$2.08 a yard; what was his annual revenue, supposing he placed his capital on interest at  $5\%$ ? *Ans.* \$380.536.
42. From 1857 to 1867, the population of Syracuse augmented  $24\frac{1}{2}\%$ ; knowing the last year's number of inhabitants to be 102295, tell us what was the population in 1857? *Ans.* 82000 inhab.
43. What sum must be placed on interest, at  $4\%$ , to amount to £627 18 6 in 2yr. 10mo 16da.? *Ans.* £563 2 1 $\frac{1}{2}$ .
44. A man assures me that if he places on interest a sum equivalent to 968 yd. of cloth at \$3.18 a yard, he will secure an annual revenue of \$153.91 $\frac{1}{2}$ ; what must be the rate? *Ans.*  $5\%$ .
45. From an investment of \$35680 in commercial concerns, I withdraw a gain of \$223 per month; what is the annual rate of the interest? *Ans.*  $7\frac{1}{2}\%$ .
46. A property was sold for £2830; the conditions were £800 in cash, £875 in 6 months, £625 in 10 months, and the remainder in 1yr. 3mo., with interest at  $7\%$ ; what was the amount paid?
47. A merchant having raised, during the 6 years of his business, a capital of \$2965.10, desires to know in what time he will receive \$889.53 as interest at  $5\%$ ? *Ans.* 6yr.
48. An individual borrowed £3750 at  $7\%$ , and then lent it at  $6\%$ ; what will he lose in 146 days, if the year, for the first transaction, consists of 360 days, and that of the second, 365 days?
49. During what time must a certain sum be on interest at  $4\frac{1}{2}\%$  to produce  $\frac{1}{3}$  of it? *Ans.* 9mo. 10da.
50. In selling merchandise at 12s. the yard, I made a profit of  $6\frac{1}{2}\%$ ; what is the price per yard? *Ans.* 11s. 3 $\frac{1}{2}$  + d.
51. The  $\frac{1}{2}$  of a sum of money is lent at  $4\%$ ; and the  $\frac{1}{2}$ , at  $5\%$ ; what is the sum, knowing that the annual interest is \$28.82? *Ans.* \$655.
52. An apparatus for astronomical purposes cost £49; but, as this sum could not be paid before 3yr. 9mo., the price was augmented  $\frac{1}{5}$  of its primitive value; what was the rate? *Ans.*  $4\%$ .
53. A man placed on interest, at  $4\%$ , a certain sum of money which produced in 5 years the funds requisite for the purchase of 368 lbs. of preserved tamarinds, at 46 $\frac{1}{2}$  cts. a lb.; what was the sum?
54. A merchant has invested in business a capital of \$21840 which produces him  $12\frac{1}{2}\%$  annually; but, for sanitary reasons, he retires from mercantile affairs, and loans his money at  $7\frac{1}{2}\%$ ; how much will he lose in 2yr. 5mo. 10da. by the change? *Ans.* \$2535.86 $\frac{1}{2}$ .
55. What is that principal the  $\frac{1}{2}$  of which at  $6\%$ , and the remainder at  $7\%$ , will give \$4340 interest? *Ans.* \$70000.00.
56. A speculator desires to purchase a tract of land, containing 450 acres, at £6 17 6 per acre, and, for this purpose, borrows money at  $5\frac{1}{2}\%$ . At the expiration of 4yr. 11mo. 20da., he sells the  $\frac{1}{2}$  of the land at £8 10 an acre, and the remainder, at £8 2 9 the acre; how much does he lose by the transaction?

31  
interc

Norm  
nally,  
compu  
deman

Ex

31  
the gi  
ond ye

II.  
princi  
numbe

III  
the ren

Norm  
for the  
it to the  
2. W  
of the g  
ond inte

1. W  
24 day  
2. W  
3. W  
at 6 %  
4. W  
payabl  
5. W  
semi-a

COMPOUND INTEREST.

**316. Compound Interest** is interest on both principal and interest, when the latter is not paid when due.

*NOTE.*—The simple interest may be added to the principal annually, semi-annually, quarterly, or monthly, according to agreement. A creditor may receive compound interest without being liable to the charge of usury, but cannot legally demand it.

*Ex.* What is the compound interest of \$390 for 3 years, at 5%?

OPERATION.

|                   |             |                          |
|-------------------|-------------|--------------------------|
|                   | \$390.00    | Principal for 1st. year. |
| \$390.00 × .05 =  | 19.50       | Interest for 1st. year.  |
|                   | <hr/>       |                          |
|                   | \$409.50    | Principal for 2nd. year. |
| \$409.50 × .05 =  | 20.475      | Interest for 2nd. year.  |
|                   | <hr/>       |                          |
|                   | \$429.975   | Principal for 3rd. year. |
| \$429.975 × .05 = | 21.49875    | Interest for 3rd. year.  |
|                   | <hr/>       |                          |
|                   | \$451.47375 | Amount for 3 years.      |
|                   | <hr/>       |                          |
|                   | \$390.00000 | Given principal.         |
|                   | <hr/>       |                          |
|                   | \$ 61.47375 | Compound interest.       |

**317. RULE.—I.** Find the amount of the given principal at the given rate for one year, and make it the principal for the second year.

**II.** Find the amount of this new principal, and make it the principal for the third year, and so continue to do for the given number of years.

**III.** Subtract the given principal from the last amount, and the remainder will be the compound interest.

*NOTES.*—1. When the time contains years, months, and days, find the amount for the years, upon which compute the interest for the months and days, and add it to the last amount, before subtracting.

2. When the interest is payable semi-annually or quarterly, find the amount of the given principal for the first interval, and make it the principal for the second interval, proceeding in all respects as when the interest is payable yearly.

EXAMPLES FOR PRACTICE.

1. What is the compound interest of \$970 for 2 years 9 months and 24 days, at 6%?  
*Ans.* \$173.295.
2. What is the compound interest of \$520 for 3 years, at 5%?
3. What is the amount of \$128 for 3 years 5 months and 18 days, at 6%, compound interest?  
*Ans.* \$156.717.
4. What is the compound interest of \$340 for 2 yr., interest being payable semi-annually, at 6%?  
*Ans.* \$42.57 +.
5. What is the compound interest of \$737.75 for 2½ years, payable semi-annually, at 7%?

6. What will \$900 amount to in 1 year, at 7%, compound interest, payable quarterly?

Ans. \$964.67 +

7. What is the amount of \$500 for 1 yr., interest payable every 3 months, compound interest, at 8%?

8. Find the compound interest of \$948 for 3 years 4 months and 18 days, at 6%?

Ans. \$207.051.

**318.** Compound interest may be computed more expeditiously by the use of the following

TABLE

Showing the amount of \$1, or £1, at 3, 4, 5, 6, 7, and 8 per cent., compound interest, for any number of years from 1 to 34.

| Years | 3 per cent. | 4 per cent. | 5 per cent. | 6 per cent. | 7 per cent. | 8 per cent. |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|
| 1     | 1.030000    | 1.040000    | 1.050000    | 1.060000    | 1.070000    | 1.080000    |
| 2     | 1.060900    | 1.081600    | 1.102500    | 1.123600    | 1.144900    | 1.166400    |
| 3     | 1.092727    | 1.124864    | 1.157625    | 1.191016    | 1.225043    | 1.259712    |
| 4     | 1.125509    | 1.169859    | 1.215506    | 1.262477    | 1.310796    | 1.360489    |
| 5     | 1.159274    | 1.216653    | 1.276282    | 1.338226    | 1.402552    | 1.469328    |
| 6     | 1.194052    | 1.265319    | 1.340096    | 1.418519    | 1.500730    | 1.586374    |
| 7     | 1.229874    | 1.316932    | 1.407100    | 1.503630    | 1.605782    | 1.713824    |
| 8     | 1.266770    | 1.368569    | 1.477455    | 1.593848    | 1.718186    | 1.850930    |
| 9     | 1.304773    | 1.423312    | 1.551328    | 1.689479    | 1.838459    | 1.999006    |
| 10    | 1.343916    | 1.480244    | 1.628895    | 1.790848    | 1.967151    | 2.158925    |
| 11    | 1.384234    | 1.539454    | 1.710339    | 1.898299    | 2.104852    | 2.331639    |
| 12    | 1.425761    | 1.601032    | 1.795856    | 2.012197    | 2.252192    | 2.518170    |
| 13    | 1.468534    | 1.665074    | 1.885649    | 2.132928    | 2.409845    | 2.719624    |
| 14    | 1.512590    | 1.731676    | 1.979932    | 2.260904    | 2.578534    | 2.937194    |
| 15    | 1.557967    | 1.800944    | 2.078928    | 2.396558    | 2.759032    | 3.172169    |
| 16    | 1.604706    | 1.872981    | 2.182875    | 2.540352    | 2.952164    | 3.425943    |
| 17    | 1.652848    | 1.947901    | 2.292018    | 2.692773    | 3.168815    | 3.700018    |
| 18    | 1.702433    | 2.025817    | 2.406619    | 2.854339    | 3.379932    | 3.996020    |
| 19    | 1.753506    | 2.106849    | 2.526950    | 3.026600    | 3.616528    | 4.315071    |
| 20    | 1.806111    | 2.191123    | 2.653298    | 3.207136    | 3.869685    | 4.660957    |
| 21    | 1.860295    | 2.278768    | 2.785693    | 3.399564    | 4.140562    | 5.033834    |
| 22    | 1.916103    | 2.369919    | 2.925261    | 3.603537    | 4.430402    | 5.436540    |
| 23    | 1.973587    | 2.464716    | 3.071524    | 3.819750    | 4.740530    | 5.871464    |
| 24    | 2.032794    | 2.563304    | 3.225100    | 4.048935    | 5.072367    | 6.341181    |
| 25    | 2.093778    | 2.665836    | 3.386355    | 4.291871    | 5.427433    | 6.848475    |
| 26    | 2.156591    | 2.772470    | 3.555673    | 4.549383    | 5.807353    | 7.396353    |
| 27    | 2.221289    | 2.883369    | 3.733456    | 4.822346    | 6.213868    | 7.988062    |
| 28    | 2.287928    | 2.998703    | 3.920129    | 5.111687    | 6.648838    | 8.627106    |
| 29    | 2.356566    | 3.118651    | 4.116136    | 5.418388    | 7.114257    | 9.317275    |
| 30    | 2.427262    | 3.243398    | 4.321942    | 5.743491    | 7.612255    | 10.062657   |
| 31    | 2.500080    | 3.373133    | 4.538040    | 6.088101    | 8.145113    | 10.867669   |
| 32    | 2.575083    | 3.508059    | 4.764942    | 6.453387    | 8.715271    | 11.737083   |
| 33    | 2.652335    | 3.648381    | 5.003189    | 6.840590    | 9.325340    | 12.676050   |
| 34    | 2.731905    | 3.794316    | 5.253348    | 7.251025    | 9.978114    | 13.690134   |

NOTE.—The compound interest of \$1, or £1, is \$1, or £1, less than the amounts in the above table.

*Ex.* What is the compound interest of \$90 for 7 years and 6 months, at 7%?

OPERATION.

|                           |                  |
|---------------------------|------------------|
| Amt. of \$1 for 7yr.,     | \$1.605782       |
| Principal,                | 90               |
| Amt. \$90 for 7yr.,       | 144.520380       |
| Interest of \$1 for 6mo., | .031             |
|                           | 4.3356114        |
|                           | .7226019         |
| Int. of amt. for 6mo.,    | 5.0582133        |
| Amt. added,               | 144.520380       |
| Amt. for 7yr. 6mo.,       | 149.5785933      |
| Principal subtracted,     | 90.              |
| Comp. int. for giv. time, | \$59.57 + , Ans. |

ANALYSIS. — We find the amount of \$1 for 7 years in the table, and multiplying it by the given principal, obtain the amount of the \$90 for 7 years. We then find on this amount the interest for the 6 months, and add it to its principal. From the last amount subtracting the original principal, we have left the compound interest required. Hence the

ound interest,  
s. \$964.67 +  
able every 3  
months and 18  
\$207.051.  
xpedientiously  
8 per cent.,  
1 to 34.

|             |
|-------------|
| 8 per cent. |
| 1.080000    |
| 1.166400    |
| 1.259712    |
| 1.360489    |
| 1.469328    |
| 1.586874    |
| 1.713824    |
| 1.850930    |
| 1.999005    |
| 2.158925    |
| 2.331639    |
| 2.518170    |
| 2.719624    |
| 2.937194    |
| 3.172169    |
| 3.425943    |
| 3.700018    |
| 3.996020    |
| 4.315701    |
| 4.660957    |
| 5.033834    |
| 5.436540    |
| 5.871464    |
| 6.341181    |
| 6.848475    |
| 7.396353    |
| 7.988062    |
| 8.627106    |
| 9.317275    |
| 0.062657    |
| 0.867669    |
| 1.737083    |
| 2.676050    |
| 3.690134    |

the amounts

**319. RULE.**—Multiply the amount of \$1 for the given rate and time, as found in the table, by the given principal, and the product will be the amount. Subtract the principal from the amount, and the remainder will be the compound interest.

EXAMPLES FOR PRACTICE.

1. What is the compound interest of \$60 for 8 years and 6 months, at 7%?  
*Ans.* \$46.69 +.
2. What is the amount of \$25.50 for 20 years 2 months and 12 days, at 7%, compound interest?  
*Ans.* \$100.058.
3. What is the compound int. of \$3000 for 2yr. 6mo. 18da., at 6%?  
*Ans.* \$12,364 +.
4. What is the amount of \$12 for 6 months, the interest to be compounded monthly, at 6%?  
*Ans.* \$12.364 +.
5. What is the compound interest of \$600 for 10 years 7 months and 15 days, at 6%?
6. To what sum will \$75, deposited in a savings bank, amount, at compound interest, for 18 years, at 3%, payable semi-annually?

PROMISSORY NOTES.

**320. A Promissory Note** is a written or printed engagement to pay a certain sum either on demand or at a specified time.

**321. The Maker or Drawer** of a note is the person who signs it and thus becomes responsible for its payment when due.

**322. The Payee** of a note is the person to whom or to whose order it is made payable.

**323. The Indorser** of a note is the person who signs his name on the back of it, and by so doing guarantees its payment, unless he writes "Without Recourse" over his name at the time.

**324. A Negotiable Note** is a promissory note which is made payable to bearer or the order of some person (*see Notes, Forms, 2; 3, 4*).

**NOTES.**—1. If a note is payable to the bearer, it may be negotiated without indorsement.

2. A note should contain the words "value received", and the sum for which it is given should be written out in words.

**325.** A note may be made payable on demand, as in *Form No. 1*, or at the expiration of a certain time after its date, as in *Forms No. 2, 3, and 4*. A note may be made payable to a particular person, as in *Form No. 1*; or to any person who is the bearer or holder of it, as in *Form No. 2*; or to the order of a person named in it, as in *Form No. 3*; and may be made payable at a particular place, as in *Form No. 4*.

The Note, *Form No. 1*, is due when the payee demands its payment from the maker of it.

**REMARK.**—If no time is fixed, in a note, for payment, it is payable on demand.

The Note, *Form No. 2*, is payable to the holder of it at the expiration of six calendar months from its date.

The Note, *Form No. 3*, is due at the time specified in it, to the payee who indorses it. Jos. A. Walter may indorse this note in blank, that is to say, only write his name, and thus make any person lawfully holding the note, the payee; or, he may indorse it payable to the order of a particular person, in which case such person can make another person the payee, as Jos. A. Walter could, by indorsing the note in blank or otherwise.

The Note, *Form No. 4*, is payable only at the Bank named in it.

**326. The Face** of a note is the sum named in it.

**327. Days of Grace** are the three days usually allowed by law for the payment of a note after the expiration of the time specified in the note.

**328. The Maturity** of a note is the expiration of the days of grace; a note is *due* at maturity.

**REMARK.**—1. When a note promises interest, as *Forms 1, 2, and 3*, the interest begins at the date of the note, and continues until the note is paid. If the time expressed in a note for its maturity be stated in months, calendar months are understood; and if a note promises interest without stating the rate %, it bears the legal interest of the country in which it is dated; also, a note which does not promise interest, if not paid when due, bears the same legal rate % of interest from the time it matures until paid.

2. If a note be not paid by its maker when it matures, it may on the same day be protested for non-payment, and the indorsers may be required to pay it if they are at once notified of the protest.

3. If a note matures on Sunday or a legal holiday, it must be paid on the day previous.

3:  
sider  
payee

3:  
uable  
payee  
been

Not  
money

3:  
who  
Note  
either

3:  
to a

3:  
mone

3:  
to see  
non-p

3:  
propo  
dition  
the n

\$ 64

sixty  
Wa

(STAN)

**329. A Business Note** is a note given for a valuable consideration. It renders the maker liable for the amount to the payee, or to any subsequent *bona fide* holder.

**330. An Accommodation Note** is a note given for no valuable consideration. It does not render the maker liable to the payee, but makes him liable to any *bona fide* holder after it has been negotiated for value.

**NOTE.**—Accommodation notes are usually given to enable the payee to borrow money on the credit of the makers of the notes.

**331. A Joint Note** is a note signed by two or more persons, who are jointly liable for its payment. A *Joint and Several Note* is a note signed by two or more persons, who may be held, either jointly or singly, for its payment.

**332. A Produce Note** is a written promise to deliver goods to a specified amount.

**333. A Due Bill** is an acknowledgment of a debt due in money, or its equivalent in goods.

**334. A Bond** is a written obligation, authenticated by a seal, to secure the payment of a sum of money of the performance or non-performance of certain acts.

**335. A Mortgage or Mortgage Deed** is a conveyance of property given to secure the payment of a bond or debt, on condition that when payment is made, the conveyance is void and the mortgage is discharged.

## FORMS OF NOTES.

*Form No. 1.*—DEMAND NOTE.

\$ 64  $\frac{00}{100}$

*Quebec, January 15th., 1870.*

*On demand, I promise to pay R. N. Harris,  
sixty-four and  $\frac{00}{100}$  dollars, with interest, at 5%.*

*Value Received.*

(STAMP.)

*Louis Garneau.*

*Form No. 2.—NOTE PAYABLE TO BEARER (NEGOTIABLE.)*\$ 205 <sup>17</sup>/<sub>100</sub>.*Montreal, February 3, 1870.*

*Six months after date, I promise to pay  
W. S. Colton, or bearer, two hundred sixty-five  
and <sup>17</sup>/<sub>100</sub> dollars, with interest at 6%. Value received.*

(STAMP.)

*J. W. Power.**Form No. 3.—NOTE PAYABLE TO ORDER (NEGOTIABLE.)*\$ 99 <sup>17</sup>/<sub>100</sub>.*Hingham, March 10, 1870.*

*One year after date, I promise to pay to the  
order of J. H. Samson, ninety-nine and <sup>17</sup>/<sub>100</sub> dol-  
lars, with interest, at 7%. Value received.*

(STAMP.)

*E. J. Byrne.**Form No. 4.—NOTE PAYABLE AT A BANK (NEGOTIABLE.)*\$ 87 <sup>20</sup>/<sub>100</sub>.*Toronto, April 12, 1870.*

*Forty days after date, I promise to pay to  
the order of C. D. Nichols, at the Ontario  
Bank, eighty-seven and <sup>20</sup>/<sub>100</sub> dollars. Value received.*

(STAMP.)

*John Douglas.*

Form of Produce Note.

\$ 58<sup>35</sup>/<sub>100</sub> Halifax, N. S., May 14, 1870.

For value received, we promise to pay to  
J. H. Martel, on demand, fifty-eight and <sup>35</sup>/<sub>100</sub>  
dollars in goods, at our store.

(STAMP.)

*S. Richard & Co.*

Form of Due Bill.

\$103.

Ottawa, June 9, 1870.

Due Edward G. Larkin, for value received,  
one hundred and three dollars, with interest.

(STAMP.)

*Philip Alloyn.*

PROFIT AND LOSS.

**336.** Profit and Loss are commercial terms, used to express the gain or loss in business transactions.

**337.** There are four terms or quantities to be considered in Profit and Loss, viz. :—

- 1st. The *Cost*, or original number, which is the **Base**.
- 2nd. The *Rate %* of gain or loss, which is the **Rate %**.
- 3rd. The *Gain*, or *Loss*, which is the **Percentage**.
- 4th. The *Selling Price*, which is the **Amount**, or **Difference**.

The questions follow the same rules as in Percentage.

*Selling Price* = *Cost* + *Gain*, or *Cost* — *Loss*.

*Cost* = *Selling Price* — *Gain*, or *Selling Price* + *Loss*.

*Gain* = *Selling Price* — *Cost*.

*Loss* = *Cost* — *Selling Price*.

## EXAMPLES FOR PRACTICE.

1. I bought cloth, at \$2.50 per yard, and sold it so as to gain 25 %; for how much did I sell it a yd. ? *Ans.* \$3.12½.

To solve this Example, see Case I., 282, RULE.

2. A farm was bought for \$4600, and sold so as to gain \$900; how much was the gain % ? *Ans.* 20 %.

To solve this Example, see Case II., 284, RULE.

3. By selling a building lot, a man gained \$175, which was 12 % of the cost; what was the cost ? *Ans.* \$1458.33½.

To solve this Example, see Case III., 286, RULE.

4. A gentleman sold a horse for \$180, and thereby gained 20 %; what was the cost of the horse ? *Ans.* \$150.

To solve this Example, see Case IV., 288, RULE.

5. A merchant lost 15 % on his old stock of goods; how much did he lose on those that cost 12½ cts., \$6½, 38½ cts., 33½ cts., and \$18½ ? *Ans.* 1½ cts.; \$1; 5½ cts.; etc.

6. Bought sugar, at 12 cts. a pound, and sold it so as to gain 1½ cts. a pound; required the gain %.

7. Sold butter at ¼ of a dollar a pound, which was at a gain of 25 %; required the cost per pound. *Ans.* 66½ cts.

8. A market woman sold oranges so as to gain ¾ of a cent on each orange, which was at a gain of 33½ %; what was the cost of an orange ? *Ans.* 2 cts.

9. Sold a horse at 33¼ % gain, and with the money bought another horse, which I sold for \$120, and lost 25 %. Did I gain or lose by my trading? and how much?

10. If I make a profit of 15½ % by selling paper for \$0.85 above the cost per ream, how much must I advance on the price to realize a profit of 32½ % ? *Ans.* 93½ cts.

11. What should I sell a barrel of flour for, that cost me £1 2 6, to gain 16½ % ? *Ans.* £1 6 3.

12. A neighbor offers his house, which cost him \$6900, for 20 % less than cost; what is his price ? *Ans.* \$5520.

13. A merchant sells cloth for \$5 a yard, which cost him \$3.75 a yard; what is his gain per cent. ? *Ans.* 33½ %.

14. I bought 640 yards calico at 15 cts. per yd., and sold it at a reduced price of 24 %; what did I lose ? *Ans.* \$2.40.

15. A grocer sells coffee at 7½d. a lb. which cost him 9d.; what is his loss per cent. ? *Ans.* 17½ %.

16. A merchant buys at auction \$9562.50 worth of goods; if he sell them at an advance of 20 % on the cost, what will be his net profits, deducting \$600 for expenses ? *Ans.* \$1312.50.

17. How much should I sell different qualities of sugar which cost me £1 15, £2 1 3, and £2 12 6 the cwt., to gain 12½ % ?

18.  
railroad  
freight  
selling  
19.  
5 wheel  
20.  
was m  
21.  
which  
was h  
22.  
drayag  
the bu  
23.  
tity at  
24.  
been lo  
found  
lose by  
25.  
cargo  
26.  
I have  
27.  
28.  
\$4.25  
29.  
would  
30.  
per cen  
31.  
should  
32.  
making  
33.  
or loss  
34.  
and ga  
the wo  
35.  
my cur  
what is  
36.  
cost; y  
37.  
bought  
horse c  
38.  
and res  
bought

18. Bought 45 bbl. of apples at \$3.50 per bbl., and sent them by railroad, to be sold on commission at 5%; knowing that I paid for freight and other expenses \$5.38, what will be my total loss if the selling price is 10% below the buying price? *Ans.* \$28.2175.

19. Bought a horse for \$130, paid \$6 for his nourishment during 5 weeks, and then sold him for \$120; what was my loss per cent.?

20. Bought codfish at \$4.25 the cwt., and sold it at \$4.93; what was my gain per cent.?

*Ans.* 16%.

21. A grocer sold tea which cost 3s. 1½d. for 3s. 9d. per lb.; sugar which cost 5½d. for 7½d.; flour which cost £1 6 0 for £1 8 3; what was his gain per cent. on each article? *Ans.* 20% on the tea; etc.

22. Bought 9cwt. 72lb. of sugar for \$65; paid \$5.15 for freight and drayage; at how much per pound should it be sold to gain 25% on the buying price?

*Ans.* \$ 83.

23. A dealer in furs made a profit of \$156 in selling a certain quantity at 12% advance; what was the amount sold? *Ans.* \$1300.

24. A merchant bought a hogshead of wine for \$189; a part having been lost by leakage, he sold the remainder at \$3.99 per gallon, and found that his loss was 5% on the cost; how many gallons did he lose by leakage?

*Ans.* 18 gallons.

25. Sold a cargo of corn for £4000, at 25% profit; what did the cargo cost?

*Ans.* £3200.

26. In selling tea at 90 cts. a lb., I gained 20%; how much would I have gained had I sold it at \$1 a pound?

*Ans.* 33½%.

27. By selling cloth at \$4 the yard, I lose 20%; what was the cost?

28. What will I gain per cent. by selling silks at \$5 which cost \$4.25?

*Ans.* 17½%.

29. By selling lard at £1 15s. per cwt. I gain 75%; how much % would I gain or lose by selling it for 18s.?

*Ans.* Lose 10%.

30. Sold wheat at \$1.25 the bushel thereby losing 15%; how much per cent. would I have gained had I sold it at \$1.647, the bushel?

31. Lost 15% by selling a lot of paper for \$480; for how much should I have sold it to gain 12%?

*Ans.* \$632.4705 +.

32. Sold a field containing 106A. 3R. 30rd., at \$120 an acre, thereby making a profit of 18% on the cost; what did the field cost?

33. Tea, sold at 25% loss, is \$1.25 a lb.; what would be the gain or loss per cent. in selling it at \$1.60 a lb.?

*Ans.* 4% loss.

34. A lumber merchant sold 36840 feet of wood at £5 5 7½ per M., and gained 28%; how much would he have gained or lost by selling the wood at £4 5 per M.?

35. The retail prices of my goods are 40% above the cost. I supply my customers wholesale at a reduction of 12% on the retail price; what is my profit on the goods sold by wholesale? *Ans.* 23½%.

36. An engineer sold an engine for \$8812.50 and lost 6% on the cost; what should it have been sold for to gain 12%?

37. I sold a horse at an advance of 30%, and with this money bought another which I sold for £45 10, losing 12%; what did each horse cost me?

*Ans.* 1st. horse £40; 2nd. horse £52.

38. A speculator sold the goods of a store at a reduction of 7½%, and realized a profit of 5%; at what rate of reduction were the goods bought?

*Ans.* 12%.

39. My retail price for grey cloth is \$4.75 per yd., by which I make a profit of 33 $\frac{1}{2}$ %. I sell 100yd. by wholesale at 30% reduction on the retail price. What is my gain or loss per cent., and how much do I receive a yard?

*Ans.* 6 $\frac{1}{2}$ % loss; \$3.32 $\frac{1}{2}$  a yd.

40. A merchant sells linen 2 $\frac{1}{2}$  cts. more than the cost and realizes a profit of 8%; what is the cost of a yard?

*Ans.* 31 $\frac{1}{2}$  cts.

41. A grocer demanded for a certain quantity of prunes a price 22% above the cost; but being a little musty, he sold them at 10% less than his first demand, and thus gained \$98 by the sale; what was his first demand?

*Ans.* \$1220.

42. At what price should I sell codfish which costs 16s. 6 $\frac{1}{2}$ d. per cwt. to realize a profit of 12 $\frac{1}{2}$ % on the cost, after deducting 12 $\frac{1}{2}$ % of the price?

*Ans.* £1 1 2 $\frac{1}{2}$ .

43. Bought a quantity of cheese at 12 cts. a pound. Supposing the weight to be 5% less than that calculated, and 10% of the sales to be in bad debts, for how much must it be sold a pound to make a net profit of 14% on the cost?

*Ans.* 16 cents a pound.

44. J. Moran & Co. bought dry goods for the amount of \$6840; they sold  $\frac{1}{2}$  at 15% profit;  $\frac{1}{4}$  at 18 $\frac{1}{2}$ %,  $\frac{1}{4}$  at 20%, and the remainder at 33 $\frac{1}{2}$ % profit; what was their total profit?

*Ans.* \$1482.00.

## COMMISSION AND BROKERAGE.

**338. Commission and Brokerage** are the percentages paid an agent, or broker, for the transaction of business, and is estimated at a certain rate per cent. on the amount of the sale, purchase, collection, etc., effected.

**339. An Agent, Factor, Broker, Collector, or Commission Merchant**, is a person who transacts business for another.

**NOTE.**—1. An agent may be a *Special Agent*,—that is, authorized to transact only such business as is specified,—or a *General Agent*, who, as such, can transact any business of the person who employs him.

2. Merchandise and Produce sent to a person for sale or superintendence are said to be *consigned*. The person sending them is termed a *Consignor*; the person to whom they are sent, is termed a *Consignee*.

3. A consignee whose business office is remote from a consignor is sometimes termed a *Correspondent*, and usually acts as agent of the firm who consigns to him the goods.

4. Brokers are classified according to the nature of the sales and contracts they effect. Thus, a *Bill-broker* is one who negotiates the discount on bills of exchange, etc.; a *Real-estate broker* is one who negotiates the sale of houses and land; *Insurance-broker*, *Ship-broker*, *Stock-broker*, *Pawnbroker*, etc.

5. A collector may have the business of settling accounts between individuals, or he may be an officer of the government, as a *Collector of the Port*, whose business is to collect duties; a *Collector of Taxes*, etc.

**340. The Net Proceeds** is the amount received from a sale or collection, less the commission and other charges.

Questions in Commission and Brokerage follow the same rules as those in Percentage.

EXAMPLES FOR PRACTICE.

1. A broker sold \$15800 worth of stock for C; required his brokerage at  $\frac{1}{4}\%$ ?  
*Ans.* \$39.50.

To solve this Example, see Case I., 282, RULE.

2. An agent received \$1600 for selling a house and lot for \$25600; what was his rate of commission?  
*Ans.*  $6\frac{1}{4}\%$ .

To solve this Example, see Case II., 284, RULE.

3. A commission merchant receives \$84 for selling wood, at  $5\frac{1}{2}\%$ ; what is the amount sold?  
*Ans.* \$1575.

To solve this Example, see Case III., 286, RULE.

4. An agent receives \$3105 to be invested in dry goods; after retaining his commission,  $3\frac{1}{2}\%$ , how much was invested? *Ans.* \$3000.

To solve this Example, see Case IV., 288, RULE.

5. What is the commission on \$374, at  $2\frac{1}{2}\%$ ? on \$71.50, at  $3\frac{1}{2}\%$ ? on \$1580.70, at  $4\frac{1}{2}\%$ ? on \$309.10, at  $5\frac{1}{2}\%$ ? on \$4705.20, at  $6\%$ ?  
*Ans.* \$19.66 $\frac{1}{2}$ ; \$2.50 $\frac{1}{2}$ ; etc.

6. What is the commission on £15 9 10, at  $3\%$ ? on £170 10 6, at  $4\frac{1}{2}\%$ ? on £630 9 0, at  $6\frac{1}{2}\%$ ? on £96 12 3, at  $5\frac{1}{2}\%$ ? on £918 7 0, at  $6\frac{1}{2}\%$ ?  
*Ans.* £0 9 3 $\frac{1}{2}$ +; £7 13 5 $\frac{1}{2}$ +; etc.

7. How much will I pay for the brokerage of \$750, at  $\frac{1}{2}\%$ ? of \$1540.40, at  $\frac{1}{4}\%$ ? of \$3610.80, at  $1\frac{1}{2}\%$ ? of \$823.50, at  $\frac{1}{2}\%$ ? of \$1560.70, at  $1\frac{1}{2}\%$ ?  
*Ans.* \$1.87 $\frac{1}{2}$ ; \$7.702; etc.

8. Sold merchandise as follows: 1st. for £942 16 0, at  $4\frac{1}{2}\%$  commission; 2nd. for £15 11 6, at  $5\%$ ; 3rd. for £310 5 7, at  $6\%$ ; 4th. for £530 0 5, at  $3\frac{1}{2}\%$ ; what is the total com.? *Ans.* £80 7 6 $\frac{1}{2}$ +.

9. What amount of brokerage must I pay for exchanging greenbacks, as follows: \$590, at  $26\%$ ; \$745.30, at  $28\%$ ; \$1615.72, at  $30\%$ ; \$4532.09, at  $32\%$ ; \$87.30, at  $29\%$ ? *Ans.* \$2322.385+.

10. A farmer paid a broker  $\frac{7}{8}\%$  to invest \$11730, in Ontario bonds; what is the brokerage?  
*Ans.* \$102.637+.

11. A broker received \$465 for buying stocks, at  $\frac{3}{4}\%$  brokerage; how much stock did he buy?  
*Ans.* \$74400.

12. A flour merchant remits to his agent in Toronto \$4740 for the purchase of grain, after deducting the commission at  $2\%$ ; how much will the agent expend for his employer, and what will be his commission? *Ans.* \$4647.06—, for grain; \$92.94+ for commis.

13. An agent sold real estate on  $4\%$  commission, and remitted \$10095.36 to the owner as the net proceeds; for what price did he sell the property, and what was his commission?

14. An agent receives \$4920 to expend in purchasing cows at \$32 a head; after reserving his commission,  $2\frac{1}{2}\%$ , how many cows did he purchase?  
*Ans.* 150.

15. A merchant having on hand 4700 barrels of sugar, gave an agent  $3\frac{1}{2}\%$  for selling it; what are the net proceeds, if sold at \$16 a bbl.?

16. I purchased 6000 bushels of wheat in Buffalo, at \$1.37½, and shipped the same to my agent in Kingston, who sold it at \$1.62½. How much did I make, after paying \$543 for expenses and a commission of 2½%?

*Ans.* \$723.

17. A broker charged me 5s. 3d. % for the exchange of £681 4 10, in greenbacks; what was his brokerage?

*Ans.* £35 15 3¼.

18. A commission merchant sold a consignment of oats for \$12686. He charged \$66 for storage, and 6¼% commission; what were the net proceeds?

*Ans.* \$11827.12½.

19. An architect charges ¼% for his plan and survey of a building which cost \$24000, and 1¼% for superintending the work; how much did he receive?

*Ans.* \$450.

20. I sent to my correspondent in Bordeaux £2097 10, with advice to invest in the purchase of wines, after deducting his commission of 3¼%; what was the sum invested and what was his commission?

*Ans.* £2026 11 4½, wines; £70 18 7½, commission.

21. An agent having a debt of \$1570 to collect, compromises for 90%; what was his commission at 5¼%?

*Ans.* \$77.71¼.

22. Paid Folger Brothers \$5.46 for exchanging \$364 in United States' money; what was the rate of brokerage?

*Ans.* 1¼%.

23. A consignee in Glasgow informs his constituent of the purchase of Dry Goods to the amount of £395 15 5; what is his commission at 2¼%?

*Ans.* £8 18 1+.

24. Bought at Halifax a cargo of wheat, 9500 bushels, at \$1.20 per bushel, and sent it to my agent in Portland who sold it at \$1.50 per bushel; what did I realize on the whole after paying \$320 for expenses, and commission at 3¼%?

*Ans.* \$2031.25.

25. My correspondent at Bordeaux charges \$74.20 for purchasing 264 cwt. of honey at \$10.50 per cwt.; what was the rate of commission?

*Ans.* 2¾%.

26. A broker receives £2085 7 6 comprising the sum to be invested in Railroad stock at £20 15 a share, and his brokerage at ¼%; how many shares can he buy, and what is his brokerage?

27. A certain piece of land was sold for \$3925, but the owner received \$3866.12½ as the net proceeds; what was the rate of commission?

*Ans.* 1¼%.

28. I remitted \$5500 to my broker with advice to invest in Bank stock, after deducting his brokerage at 7%; what was the investment?

29. The net proceeds of a sale were £1408 15, and the commission, £28 15; what was the rate of commission?

*Ans.* 2%.

30. In charging 1¼% for the investment of a certain sum, a broker realized \$285; what was the amount of the investment?

*A.* \$19000.

31. My agent in Cincinnati gives me information of the purchase of 4000 bushels of Indian meal at 80 cts. per bushel, and desires me to remit a check on New York which he can sell to a broker at ¼% premium; what should the amount of the check be, his commission being 3%?

*Ans.* \$3271.464.

32. A factor received £5 12 for the sale of grain at 4% commission; what was the amount sold?

*Ans.* £140.

33. Received from A. \$700 in specie; paid 3¼% for changing it to

gold;  
ance  
was t  
34.  
of cal  
many  
35.  
allow  
36.  
and in  
10 6,  
amou  
37.  
freigh  
purch  
his co  
per lb  
38.  
150 b  
10 %  
whole  
must

34  
party  
risk o  
34  
on lif  
34  
risk;  
34  
34  
reckon  
sured,  
sumed

34  
and M  
34  
loss ca

(1) I

gold; and, after deducting the commission at 2%, employed the balance in the purchase of fruit; what was paid for the fruit; and what was the commission? *Ans.* \$661.99, fruit; \$13.51 commission.

34. Remitted to my correspondent at Rouen £255, for the purchase of calico at 9d. per yard, after deducting his commission at 2%; how many yards will I receive? *Ans.* 6666 $\frac{1}{2}$  yd.

35. A speculator receives \$4113.50 as the net proceeds of a sale, allowing 5% commission; what was the value of the property?

36. A commission merchant who charges 5% commission on sales and investments, receives 260 cwt. of cheese, at 6d. per lb., and £748 10 6, in cash, with advice to purchase a cargo of cotton for the whole amount; what will be his total commission? *Ans.* £97 10 11 $\frac{1}{2}$

37. A Halifax agent buys 34 boxes of chocolate; he pays \$7.50 for freight and cartage, and his commission is 1 $\frac{1}{2}$ % on the amount of the purchase. He sends me a bill of \$740.83 $\frac{1}{2}$  for the whole; what was his commission; and, allowing 250lb. per box, how much did I pay per lb. for the chocolate? *Ans.* \$10.83 $\frac{1}{2}$  com.; \$0.08 $\frac{1}{2}$  per lb.

38. A commission merchant receives 125 barrels of flour from A, 150 bbl. from B, 225 bbl. from C; he finds on inspection that A's is 10% better than B's, and C's is 5 $\frac{1}{2}$ % better than A's; he sells the whole lot at \$7 per barrel, and charges 4% commission. How much must be remitted to each? *Ans.* A, \$842.30; B, \$918.87; C, \$1598.83.

## INSURANCE.

**341. Insurance** is a contract of indemnity, by which one party engages, for a stipulated sum, to insure another against a risk or loss to which he is exposed.

**342.** It is of two kinds: insurance on property, and insurance on life (1).

**343.** The **Insurer** or **Underwriter** is the party taking the risk; and the **Insured** or **Assured**, the party protected.

**344.** The **Policy** is the written obligation or contract.

**345. Premium** is the sum paid for insurance. It is always reckoned at a certain per cent. on the value of the property insured, varying according to the degree or nature of the risk assumed.

## FIRE AND MARINE INSURANCE.

**346.** Insurance on property is of two kinds: *Fire Insurance*, and *Marine Insurance*.

**347. Fire Insurance** is an indemnification of damage and loss caused by *fire* or *lightning*.

(1) Life insurance will be treated of later.

**348. Marine Insurance** is an indemnification of damage and loss caused by the perils peculiar to navigation.

**349.** In insurance, the calculations are based on the following principles:

- I. Premium is *percentage*. (278)
- II. The sum insured is the *base* of premium.
- III. The sum covered by insurance is *difference*.

EXAMPLES FOR PRACTICE.

1. What premium must be paid for insuring goods to the amount of \$4500, at  $2\frac{1}{2}\%$ ? *Ans.* \$112.50.

To solve this Example, see Case I., 282, RULE.

2. A man paid \$64.80 for the insurance of \$8640 on his house; what was the rate of insurance? *Ans.*  $\frac{3}{4}\%$ .

To solve this Example, see Case II., 284, RULE.

3. The premium for insuring a tannery for  $\frac{2}{3}$  of its value, at  $1\frac{1}{2}\%$ , was \$145.60; required the value of the tannery. *Ans.* \$11648.

To solve this Example, see Case III., 286, RULE.

4. What must be paid for an insurance of \$5728 at  $1\frac{1}{2}\%$ ?

5. What premium must be paid for the insurance of a vessel and cargo valued at £3649 8, at  $3\frac{1}{4}\%$ ? *Ans.* £118 12 11 $\frac{1}{4}$  +.

6. A schooner, insured for \$5000, at  $2\frac{1}{4}\%$ , was completely wrecked; how much of the loss was covered by the insurance? *A.* \$4887.50.

7. A hotel valued at £3750 is insured for  $\frac{2}{3}$  of its value, at  $\frac{3}{4}\%$ . The policy and survey of the premises are charged 7s. 6d.; what is the insurance? *Ans.* £9 15 0.

8. A store and its stock are worth \$6370; what sum must be insured, at 2%, to cover both property and premium? *Ans.* \$6500.

9. What is the premium of insuring £695 11 8, at £5 13 9%?

10. I pay annually \$45 insurance for my library, and this sum is 3% of the amount for which I am insured; what is the amount?

11. What is the premium for an insurance of £1486 13 9, at £3 16 8%? *Ans.* £56 19 9 $\frac{1}{4}$  +.

12. A business man, having \$12000 worth of goods, gets them insured for  $\frac{1}{4}$  of their value, at  $\frac{3}{4}\%$ ; if, in a conflagration, he saves but \$2000 of the stock, what real loss will he sustain? *Ans.* \$472.

13. For what sum must a house, valued at \$8274, be insured, at  $1\frac{1}{2}\%$ , to cover the entire loss, in case it is destroyed by fire? *A.* \$8400.

14. My goods are worth £1563 12. For what sum must I insure them to cover, in case of loss, both premium and property, at £2 5 6%? *Ans.* £1600.

15. The premium of a school-house, insured at  $1\frac{1}{2}\%$ , is \$50; for what sum was it insured? *Ans.* \$4000.

16. An Insurance Company, after having insured a block of buildings for \$36000, at  $2\frac{1}{2}\%$ , re-insure the  $\frac{1}{2}$  at 3%; what is the balance of premium? *Ans.* \$360.

17. For what sum must goods worth £1938 12 6 be insured to cover both premium and goods in case of loss, the rate being  $5\frac{1}{2}\%$ ?

18. A brig estimated at \$40000 is insured for  $\frac{1}{2}$  of its value at  $1\frac{1}{2}\%$ , and its cargo, worth \$36000, at  $\frac{1}{2}\%$ ; what is the insurance?

19. A merchant paid \$1450 for premium of insurance on a cargo of cotton coming from Havana, the rate of insurance being  $2\frac{1}{2}\%$ ; what was the value of the cargo? *Ans.* \$58000.

20. I paid \$18 for an insurance of \$1200; what is the rate of the premium? *Ans.*  $1\frac{1}{2}\%$ .

21. To £579 16 10, add  $7\frac{1}{2}\%$  commission, and find the insurance of the sum, at  $4\frac{1}{2}\%$ ? *Ans.* £27 5 4 $\frac{1}{2}$  +.

22. A merchant, having a cargo of 500 bbl. flour, has it insured for 80% of its value at  $3\frac{1}{2}\%$ , and paid \$107.25 for premium; what was the price per bbl.? *Ans.* \$8.25.

23. A ship-owner has two of his vessels insured for \$30000 in the Royal Insurance Co., at  $\frac{1}{2}\%$ , and for \$45000 in the Colonial Insurance Co., at  $\frac{1}{2}\%$ ; what is the rate of premium for the whole insurance?

24. A house estimated at £300 was insured for  $\frac{2}{3}$  of its value, during 3 years, at 1% per annum. Towards the end of the third year, it was destroyed by fire; what is the actual loss of the proprietor without any allowance of interest? *Ans.* £106.

25. My house was insured for \$45000 during 5 years. The first year I paid \$1.50 for the policy and plans, and  $\frac{1}{2}\%$  premium; every succeeding year, I paid  $\frac{1}{2}\%$  premium. The house having been destroyed the fifth year, what was the loss of the insurance, no interest having been allowed? *Ans.* \$43817.25.

26. I paid \$46.75 for insuring a store for the  $\frac{1}{2}$  of its value, at  $1\frac{1}{2}\%$ ; what is the store worth? *Ans.* \$6800.

27. I took a policy of £3011 5 for the value of both property and premium; what is the worth of the insured property, the rate being  $\frac{1}{2}\%$ ? *Ans.* £3000.

28. A shipment of wheat was insured at  $2\frac{1}{2}\%$ , to cover  $\frac{2}{3}$  of its value; the premium paid was \$44.07; the wheat being worth 80 cts. per bushel, how many bushels were shipped? *Ans.* 2825 bu.

## ASSESSMENT OF TAXES.

**350.** A **Tax** is a sum of money assessed on the person or property of an individual, for public purposes.

**351.** When a **tax** is assessed on *property*, it is apportioned at a certain *per cent.* on the estimated value. When assessed on the *person*, it is apportioned *equally* among the male citizens liable to assessment, and is called a *poll tax*.

**352.** Property is of two kinds, viz.: *real estate*, and *personal property*.

**353.** **Real Estate** is *fixed* or *immovable* property, such as lands, houses, etc.

**354. Personal Property** is *movable* property, such as money, stocks, furniture, cattle, etc.

**355. An Inventory** is a written list of articles of property, with their value.

**356. A Schedule** is a list of taxable property with its owners' names and its value as estimated by assessors.

**357. Assessors** are officers appointed to make out a schedule of taxable property, and apportion taxes thereon.

*Ex.* A tax of \$840.75 is to be raised in a town containing 65 polls; the taxable property of the town amounts to \$48000, and each poll tax is 75 cts.; what will be the tax on a dollar, and how much will be C's tax, whose property is valued at \$5600, and who pays for 2 polls?

## OPERATION.

$\$0.75 \times 65 = \$48.75$ , amount assessed on the polls.

$\$840.75 - \$48.75 = \$792$ , amt. to be assessed on the property.

$\$792 \div \$48000 = \$0.0165$ , tax on \$1.

$\$5600 \times \$0.0165 = \$92.40$ , C's tax on property.

$\$0.75 \times 2 = \$1.50$ , C's tax on 2 polls.

$\$92.40 + \$1.50 = \$93.90$ , amount of C's tax. Hence the

**358. RULE.—I.** Find the amount of poll tax, if any, and subtract it from the whole tax to be raised; the remainder will be the property tax.

II. Divide the property tax by the whole amount of taxable property; the quotient will be the per cent., or the tax on \$1.

III. Multiply each man's taxable property by the tax on \$1, and to the product add his poll tax, if any; the result will be the whole amount of his tax.

## EXAMPLES FOR PRACTICE.

1. The tax assessed on a certain town is \$1485; its property, both personal and real, is valued at \$42000, and it contains 300 polls, which are assessed 75 cts. a piece. What per cent. is the tax; that is, how much is the tax on a dollar; and how much is A's tax who pays for 3 polls, and whose property is valued at \$2250?

*Ans.* 3 cts. on \$1; \$69.75, A's tax.

2. What is the tax of a non-resident, having property in the same town, worth \$7900?

*Ans.* \$

3. How much will B's tax be, in the same town, who pays for 3 polls, and whose real estate is valued at \$32000, and his personal property, at \$18880?

*Ans.* \$1628.65.

4. What sum must be assessed in order to raise a net amount of \$11123, and pay the commission for collecting at 2½%?

5. The expense for repairs of a public building was \$2521.06, which was defrayed by a tax upon the property of the town. The rate of taxation was 3½ mills on one dollar, and the collector's commission was 3¼%; what was the valuation of the property? *Ans.* \$303843.69 +

**35**  
goods,  
indust

**36**  
Foreign  
places  
a Cust

**36**  
ment t  
cers a  
is to i  
to insp

NOTE  
merce a  
etc.; th  
2. To  
by law,

**36**  
**36**

of goo  
**36**  
measu  
ances

**36**  
the bu  
article

**36**  
lowan  
duties  
the ta  
boxes,  
cask.

NOTE  
specific

**36**

*Ex.*  
which

\$256.5

## CUSTOM-HOUSE BUSINESS.

**359. Duties, or Customs,** are taxes levied on imported goods, for the support of government and the protection of home industry.

**360.** All goods coming into the Dominion of Canada from Foreign countries are required by law to be landed at certain places or ports called *Ports of Entry*. Every Port of Entry has a *Custom-House*.

**361.** A *Custom-House* is an office established by government for the transaction of business relating to duties. The officers attached to it are called *Custom-House Officers*. Their business is to inspect the cargoes of all vessels entering at any of these ports; to inspect the invoice of goods, collect the duties, etc.

**NOTES.**—1.—Besides the duties on merchandise, all vessels engaged in commerce are required to pay certain charges for the privilege of entering the port, etc.; these charges are called harbor dues.

2. To carry on foreign commerce secretly, without paying the duties imposed by law, is *smuggling*.

**362.** Duties are of two kinds — *Ad Valorem* and *Specific*.

**363. Ad Valorem Duty** is a certain per cent. on the cost of goods, as stated in the invoice.

**364. Specific Duty** is a tax computed on the weight or measure of the goods, without regard to their cost; hence, allowances are made before computing the duty.

**365.** An *Invoice* is a statement of goods, from the seller to the buyer, or importer, showing the quantity and prices of the articles.

**366.** In the United States Custom-Houses, certain legal allowances are made for draft, tare, leakage, etc., before specific duties are imposed. In Canada, however, these are not known, the tare being found by actually weighing one or more of the boxes, etc., containing the goods, and the leakage by gauging the cask.

**NOTE.**—At present, the various kinds of spirits are the only articles upon which specific duties are charged by the Canadian Tariff.

**367.**—To compute *ad valorem* duties.

*Ex.* What is the ad valorem duty, at 18 %, on an invoice of merino which cost \$256.50 ?

## OPERATION.

$$\$256.50 \times .18 = \$46.17, \text{ Ans.}$$

**ANALYSIS.**—According to Case I, (282), we multiply the invoice, \$256.50, which is the *base* of the duty, by the given *rate*, and obtain the duty, \$46.17. Hence the

**368. RULE.**—*Find the percentage on the invoiced value of the goods, at the given rate of tariff, and the result will be the ad valorem duty.*

**369.** To compute *specific duties*.

*Ex.* What is the duty on 4 hogsheads of sugar, each weighing 1280 lb., gross weight, at  $2\frac{3}{4}$  cts. a pound; tare 14%?

## OPERATION.

1280 × 4 = 5120lb., gross weight.  
 5120 × .14 = 716.8lb., tare.  
 5120 - 716.8 = 4403.2lb., net value.  
 4403.2 × .02 $\frac{3}{4}$  = \$121.088, duty.

ANALYSIS.—We first find the whole weight of the invoice which is 5120lb. From this amount we deduct the allowance for tare, 716.8lb., and compute the duty on the remainder: Hence the following

**370. RULE.**—*Deduct allowances, if necessary, and compute the duty, at the given rate, on the net value.*

## EXAMPLES FOR PRACTICE.

1. What is the ad valorem duty, at 19%, on 15780 lb. of cordage, invoiced at 15 cts. per lb.?  
*Ans.* \$449.73.
2. At 7 cts. a pound, what is the specific duty on 346 kegs of tobacco, each weighing 130 lb., allowing 6 $\frac{1}{2}$  lb. per keg for tare?
3. At 30 cts. per gallon, what is the specific duty on 40 hhd. of wine, each gauging 58 $\frac{1}{2}$  gallons?
4. What is the duty at 33%, on a bale of Holland linens which cost \$1593.50?  
*Ans.* \$525.85 $\frac{1}{2}$ .
5. What is the duty, at 20%, on an invoice of broadcloth which cost in Liverpool £657 1 0, the pound sterling being valued at \$4.86 $\frac{1}{2}$ ?
6. What is the specific duty, at 10 cts. per lb., on 25 chests of tea, each weighing 120 lb.; tare 10%?
7. What was the rate % of duty on whose invoice value was \$2250, and for which \$337.50 duty was paid?  
*Ans.* 15%.
8. A merchant imported 64 casks of wine, each containing 42 gal. net, the duty at 30% amounting to \$1036.80; at what price per gal. was the wine invoiced?
9. A merchant in Montreal makes an importation of goods invoiced at \$16448. On goods invoiced at \$2400, the duties were at the rate of 4%; on goods invoiced at \$3360, the duties were at the rate of 15%; goods invoiced at \$4800, were free of duty; and on the remainder, the duties were at the rate of 30%; what was the whole amount of the duties?  
*Ans.* \$2366.40.
10. What is the duty at 18% on 60 kegs of prunes, each weighing 1 cwt., invoiced at 7 $\frac{1}{2}$  cts. per lb.; tare at 3 $\frac{1}{2}$ %?
11. A. Hamel & Bro., of Quebec, import from Manchester 15 pieces of Belgian carpeting, 40 yd. each, purchased at 5s. per yd., duty 24%; 300 yd. of merino, at 4s. per yd., duty 19%; 150 yd. Irish linen, at

2s. 6d.  
 is the  
 to be \$  
 12.  
 48 pie  
 at 24  
 was th  
 charg

37  
 payme

37  
 future  
 legal i

Ex  
 payab

\$ 1.00  
 25.44  
 25.44  
 24.00

\$ 1.44

37  
 for th  
 WORT  
 II.  
 remain

I.  
 100 -  
 l

One  
 hundr

II.  
 100 +  
 fo

2s. 6d., duty 15%; and leather to the cost of £90, duty 4%. What is the whole amount of duty, allowing the value of the pound sterling to be \$4.86 $\frac{2}{3}$ ? *Ans.* \$261.88 +.

12. S. R. Wilson & Co., of Toronto, imported from Amsterdam 48 pieces of linen of 32 yd. each, on which they paid for the duties, at 24%, \$184.32, and other charges to the amount of \$61.44. What was the invoice value per yd., and the cost per yd. after duties and charges were paid?

## DISCOUNT AND PRESENT WORTH.

**371. Discount** is an allowance or deduction made for the payment of a debt before it is due.

**372. The Present Worth** of a note or debt, payable at a future time, without interest, is such a sum as, being placed at legal interest, will amount to the given debt when it becomes due.

*Ex.* What is the present worth and discount of \$25.44, at 6%, payable in 1 year?

## OPERATION.

\$ 1.06, amount of \$1.

25.44 ÷ 1.06 = \$24.

25.44, given sum.

24.00, present worth.

\$ 1.44, discount.

*ANALYSIS.*—Since \$1 is the present worth of \$1.06, it is evident that the present worth of \$25.44 will be as many dollars as 1.06 is contained in 25.44, or \$24. We find \$24 to be the present worth which, subtracted from the given sum, gives \$1.44 discount. Hence the following

**373. RULE.**—I. *Divide the given sum by the amount of \$1 for the given time and rate, and the quotient will be the PRESENT WORTH.*

II. *Subtract the present worth from the given sum, and the remainder will be the DISCOUNT.*

*By proportion.*

I. To determine the present-worth:—

$100 + (6 \times 1) : 100 :: 25.44 : x = \$24$ ; whence the following formula:

*One hundred plus the rate multiplied by the time, is to one hundred as the given sum is to x, or the present worth of this sum.*

II. To determine the discount:—

$100 + (6 \times 1) : 6 \times 1 :: 25.44 : x = \$1.44$ ; whence the following formula:

*One hundred plus the rate multiplied by the time, is to the rate multiplied by the time, as the given sum is to X, or the discount of this sum.*

NOTES.—1. The terms *present worth*, *discount*, and *debt*, are equivalent to *principal*, *interest*, and *amount*. Hence, when the time, rate, % and amount are given, the principal may be found by Case III., (311); and the interest by subtracting the principal from the amount.

2. When payments are to be made at different times without interest, find the present worth of each payment separately. Their sum will be the present worth of the several payments, and this sum subtracted from the sum of the several payments will leave the total discount.

## EXAMPLES FOR PRACTICE.

What is the present worth of the following notes: (1)

1. Dated Feb. 3rd., amounting to \$104.60, on 5 months' credit, discounted June 6th., at 5%?  
*Ans.* \$104.20 +.
2. Dated March 4th., amounting to £58 10 5, on 7 months' credit, discounted Aug. 10th., at 4%?  
*Ans.* £58 3 5 +.
3. Dated April 2nd., amounting to \$206.15, on 4 months' credit, discounted May 30th., at  $4\frac{1}{2}$ %?  
*Ans.* \$204.564 +.
4. Dated May 15th., amounting to £135 9 0, on 8 months' credit, discounted Nov. 15th., at 6%?  
*Ans.* £134 2 2 +.
5. Dated Aug. 7th., amounting to \$8000.00, on 6 months' credit, discounted Dec. 5th., at 5%?  
*Ans.* \$7931.699 +.
6. Dated Jan. 3rd. amounting to £90 3 6, on 9 months' credit, discounted Sept. 20th., at 7%?  
*Ans.* £89 18 11 $\frac{1}{2}$ .
7. Dated June 14th., amounting to \$1560.90, on 3 months' credit, discounted Aug. 2nd., at 6%?  
*Ans.* \$1550.049 +.
8. Dated Sept. 8th., amounting to \$795.10, on 10 months' credit, discounted Feb. 12th., at 5%?  
*Ans.* \$779.297 +.
9. Dated Nov. 25th., amounting to £875 6 8, on 7 months' credit, discounted May 11th., at 6%?  
*Ans.* £868 19 2 $\frac{1}{2}$  +.
10. Dated Dec. 6th., amounting to \$630.50, on 11 months' credit, discounted Sept. 18th., at 5%?  
*Ans.* \$626.324 +.
11. Dated Oct. 9th., amounting to £95 15 0, on 9 months' credit, discounted June 7th., at  $6\frac{1}{2}$ %?  
*Ans.* £95 4 5 +.
12. Dated July 15th., amounting to \$208.95, on 5 months' credit, discounted Oct. 12th., at  $4\frac{1}{2}$ %?  
*Ans.* \$207.20 +.
13. Dated March 2nd., amounting to £140 16 4, on 8 mos.' credit, discounted Sept. 28th., at  $6\frac{1}{2}$ %?  
*Ans.* £139 19 1 $\frac{1}{2}$ .
14. Dated Jan. 7th., amounting to \$780.50, on 11 months' credit, discounted Nov. 3rd., at  $7\frac{1}{2}$ %?  
*Ans.* \$775.19 +.
15. Dated April 10th., amounting to £780 5 3, on 10 mos.' credit, discounted Dec. 4th., at  $4\frac{1}{2}$ %?  
*Ans.* £773 10 6 $\frac{1}{2}$ .
16. Dated May 17th., amounting to \$436.75, on 3 months' credit, discounted June 22nd., at  $5\frac{1}{2}$ %?  
*Ans.* \$433.110 +.
17. Dated March 14th., amounting to \$600.00, on 7 months' credit, discounted Sept. 7th., at 7%?  
*Ans.* \$595.714.

(1) We reckon only 30 days to the month for all the notes in true discount.

18. Dated Feb. 9th., amounting to £850 18 0, on 5 months' credit, discounted April 13th., at  $7\frac{1}{2}\%$ ? *Ans.* £835 18 5 $\frac{1}{2}$  +.
19. Dated Nov. 11th., amounting to \$175.30, on 7 months' credit, discounted May 4th., at  $6\%$ ? *Ans.* \$174.225 +.
20. Dated March 6th., amounting to £701 9 6, on 4 mos.' credit, discounted June 9th., at  $7\frac{1}{2}\%$ ? *Ans.* £697 11 0 $\frac{1}{2}$  +.
21. What is the present worth of \$117.60, payable in 1 year, at  $12\%$ ? *Ans.* \$105.
22. What is the present worth of a debt of £96 6 6 $\frac{3}{4}$ , due 5mo. 15da. hence, at  $6\%$ ? *Ans.* £93 15 0.
23. What should be the discount on \$373.75, paid 11mo. before the term of maturity, at  $6\frac{1}{2}\%$ ? *Ans.* \$21.01 +.
24. What is the discount on £200 12 6, at  $7\frac{1}{2}\%$ , payable in 1yr.? *Ans.* £13.881 +.
25. A note of \$139.94 is payable in 9 months; what is the present worth, discount being  $5\%$ ? *Ans.* \$134.881 +.
26. Discounted a note of £75, payable in 4 years, at  $5\frac{1}{2}\%$ ; what sum shall I receive? *Ans.* £61 9 6 $\frac{3}{4}$ .
27. What is the actual discount of a note of \$429.98 $\frac{1}{2}$ , due in 1yr. 6mo. 1da., at  $5\frac{1}{2}\%$ ? *Ans.* \$32.82 +.
28. The sum of \$195.10 is payable in 13 months; what will be the discount, at  $4\%$ , by immediate payment? *Ans.* \$8.10 +.
29. What is the present worth of £169 13 9, payable in 3yr. and 7mo., at  $7\frac{1}{2}\%$  discount? *Ans.* £129 3 7 $\frac{1}{2}$ .
30. Bought cloth, on 21 months' credit, for £140 0 7 $\frac{1}{2}$ ; how much ready money will acquit me of the debt, if  $\frac{2}{3}\%$  discount per month, is allowed? *Ans.* £129 3 7 $\frac{1}{2}$ .
31. I sold a house, which cost me \$2964.12 ready money, for \$3665.20 payable in 1yr. 6mo.; what will be my gain, in ready money, by discounting at  $8\%$ ? *Ans.* \$308.38.
32. I bought silks for \$43713.60, on 15 months' credit; but, by paying before the time due, I will obtain  $5\%$  discount; at what epoch should I pay the debt, so as to disburse but \$41632? *Ans.* In 3mo.
33. A flour-mill was offered for \$25000 cash, or for \$12000 payable in 6mo., and \$15000 payable in 15mo. Accepting the latter condition, I would like to know whether I gained or lost, and how much, money being worth  $10\%$ ? *Ans.* Lost \$238.09 +.
34. Louis bought goods to the amount of £82 0 6 $\frac{3}{4}$ , on 20 mos.' credit; at what time did he pay, knowing that he obtained  $\frac{2}{3}\%$  discount per month, and that he disbursed but £75 19? *Ans.* 8mo.
35. A merchant gave out two notes: the first, of \$243.36, payable May 6th., 1867; the second, of \$178.64, payable Sept. 25th. 1867; what sum is required to pay the two notes Oct. 11th., 1866, discount at  $7\%$ ? *Ans.* \$390 8.
36. What quantity of produce must be bought at 5s. per lb., on 22 months' credit, in order to pay but £50 19 10 $\frac{1}{2}$ , after deducting the discount at  $7\%$ ? *Ans.* £390 8.
37. On 9 months' credit, I bought 120 bales of cotton, each bale weighing 488lb., at 5 $\frac{1}{2}$ d. the lb. Selling it immediately for £1610 8 cash, I paid my own debt, and received  $8\%$  discount; how much did I gain? *Ans.* £390 8.

to the rate  
of discount

equivalent to  
of amount are  
interest by sub-

interest, find the  
present worth  
of the several

credit, dis-  
04.20 +.  
ths' credit,  
3 5 +.  
ths' credit,  
4.564 +.  
ths' credit,  
4 2 2 +.  
ths' credit,  
1.699 +.  
ths' credit,  
18 11 $\frac{1}{2}$  +.  
ths' credit,  
0.049 +.  
ths' credit,  
9.297 +.  
ths' credit,  
9 2 $\frac{1}{2}$  +.  
ths' credit,  
3.324 +.  
ths' credit,  
4 5 +.  
ths' credit,  
7.20 +.  
os.' credit,  
19 1 $\frac{1}{2}$  +.  
ths' credit,  
5.19 +.  
os.' credit,  
10 6 $\frac{1}{2}$  +.  
ths' credit,  
110 +.  
ths' credit,  
95.714.

discount.

38. I paid \$320 for a sum I owed; what was this sum, knowing that  $5\frac{1}{2}\%$  discount was allowed? *Ans.* \$336.80.

39. Paid £23 15 for 50yd. of cloth; having received  $5\%$  discount, how much did it cost me per yard? *Ans.* 9s. 11 $\frac{1}{2}$ d.

40. Is it more advantageous to purchase flour at \$6.25 per bbl. on 6 months' credit, or at \$6.50 on 9 months' credit, discount being  $8\%$ ? *Ans.* Flour at \$6.25 is the more advantageous.

## BANK DISCOUNT.

**374.** A **Bank** is a corporation, legally established for the purpose of receiving and loaning money, and of furnishing a paper circulation.

**375.** **Bank Notes**, or **Bank Bills**, are the notes made and issued by banks to circulate as money. They are payable in specie at the banks.

*Obs.*—A bank which issues notes to circulate as money, is called a *bank of issue*; one which lends money, a *bank of discount*; and one which takes charge of money belonging to other parties, a *bank of deposit*. Some banks perform two and some all these duties.

**376.** The **Capital** of a bank is the money paid in by its stockholders, as the basis of business.

**377.** The affairs of a bank are usually managed by a *board of directors* chosen by the stockholders, and the *principal officers* are a *president*, a *cashier*, and one or more *tellers*.

*Obs.*—The president and cashier sign the notes issued; the cashier superintends the bank accounts; and the tellers receive and pay out money. A *bank check* is an order, payable to bearer and drawn on the cashier for money.

**378.** **Bank Discount** is the simple interest of a note, draft, or bill of exchange, deducted from it in advance, or before it becomes due. Thus, the *bank discount* on a note of \$106, payable in 1 year, at  $6\%$  is \$6.36; while the *true discount* is but \$6.

The interest is computed not only for the specified time, but for three days additional called *days of grace*.

*Obs.*—1. The difference between *bank discount* and *true discount* is the same as the difference between interest and true discount.

2. The legal rate of discount is ordinarily the same as the legal rate of interest.

**379.** The **Proceeds**, **Avails**, or **Cash Value** of a note is its face or amount minus the discount.

**380.** *Obs.* I.—The face of a note being given, to find the discount and proceeds.

*Ex.* V.  
note of \$

Sum disc  
Int for  
Bank dis  
Proceeds

**381**  
for three  
discount  
II. S  
mainden

*NOTE.*—  
bank disc  
instead of  
greater ac  
rule, must  
page 183,

1. Wh  
\$1000, d  
2. Wh  
and disc  
3. Des  
8%, I ga  
I add to  
4. A m  
acre, and  
Being in  
how muc  
5. Fin  
of the fol

£40 2.

Six mo  
Lee & Co  
Bank of  
Disc  
*Ans.* Du



\$1066.75.  
100.

Montreal, April 19th., 1869.

Ninety days after date, we promise to pay C. Simson, one thousand sixty-six and  $\frac{75}{100}$  dollars, at the Union Bank, for value received.

RAPPE, WEBBER, &amp; Co.

Discounted May 8th., at 7%.

Ans. Due July 18 | 21; term of disc., 74da.; proceeds, \$1051.40 +

6. What is the difference between the true discount and bank discount of \$950, for 3mo., at 7%?

7. What is the difference between the true discount and the bank discount of £2000 0 9, for 6 months, at 3%?

**382. CASE II.**—*The proceeds of a note being given, to find the face.*

*Ex.* What is the amount of a bill, payable in 60 days, which discounted at a bank, at 6%, gives \$989.50 for the proceeds?

OPERATION.

|                                |                 |
|--------------------------------|-----------------|
|                                | \$1.0000        |
| Int. of \$1 for 63 days        | .0105           |
| Proceeds of \$1                | <u>\$0.9895</u> |
| 989.50 ÷ 0.9895 = \$1000, Ans. |                 |

*ANALYSIS.*—Since \$0.9895 is the proceeds of \$1, the note of which \$989.50 is the proceeds, must be as many dollars as \$0.9895 is contained in \$989.50. Hence the

**383. RULE.**—*Divide the proceeds of the note, by the proceeds of \$1, for the time and at the rate mentioned; the quotient will be the face of the note.*

*By proportion.*

$$100 - (6 \times \frac{60}{360}) : 989.50 :: 100 : x = \text{the face.}$$

## EXAMPLES FOR PRACTICE.

1. What sum, payable in 90 days, and discounted at 7%, at a bank, will give £170?  
*Ans.* £173 2 7 $\frac{1}{2}$ .
2. A merchant desires to draw \$5000 from a bank, and for this purpose discounts his bill, payable in 90 days, at 6%; what should be the amount of it?  
*Ans.* \$5078.72 +.
3. The proceeds of a note, due in 4 months, and discounted at the bank, at 6%, are £407 18; what is the face of the note?
4. Bought goods at Toronto for the sum of \$1486.90, and gave in payment my note at 4 months, at 7 $\frac{1}{2}$ % discount; what should be the amount of the note?  
*Ans.* \$1526 +.
5. A merchant wishes to borrow \$750 in a bank; what should be the face of his note, payable in 30da., allowing 1% discount per mo.?
6. I gave my note at 60 days for a debt of £163 16; if discount is 1 $\frac{1}{2}$ % monthly, what was the face of the note?

38

*Ex.*  
discou

\$0.06

interest  
as man

38  
imally  
time a

1. W  
discoun  
2. A  
at what  
3. A  
to days  
corresp  
4. W  
at what  
5. W  
discoun  
6. W  
discoun

386

*Ex.*  
that his

90 days  
Base,  
Int. for  
Amt.  
\$6.20 ÷

**384. CASE III.**—*The rate of bank discount being given, to find the corresponding rate of interest.*

*Ex.* What is the rate of interest of a note payable in 90 days and discounted at 6%?

OPERATION.

$$\$0.06 \div 0.9845 = 0.06\frac{188}{1985}, \text{ Ans.}$$

ANALYSIS.—Every \$1 discounted for the given time and rate yields as its proceeds \$0.9845. Then, if \$1 in the given time yield a certain interest at 6 per cent., \$0.9845 in the same time will yield the same interest, as many per cent. as the given rate, .06, contains .9845.

**385. RULE.**—*Divide the given rate per cent., expressed decimally, by the number denoting the proceeds of \$1 for the given time and rate. The quotient will be the rate of interest required.*

*By proportion.*

$$100 - (6 \times \frac{90}{360}) : 100 :: 6 : x = 6\frac{188}{1985} \%$$

EXAMPLES FOR PRACTICE.

1. What rate of interest is paid when a note payable in 30 days is discounted at 6%? *Ans.*  $6\frac{22}{83} \%$ .
2. A note payable in 2 months was discounted at 2% per month; at what rate was the interest? *Ans.*  $25\frac{2}{5} \%$  annually.
3. A note, payable in 1 year, was discounted at 6%, without regard to days of grace; to what rate % of interest does the bank discount correspond? *Ans.*  $6\frac{1}{3} \%$ .
4. When a note, payable in 90 days, is discounted at  $1\frac{1}{2} \%$  per mo., at what rate was the interest paid? *Ans.*  $18\frac{1}{3}\frac{1}{4} \%$ .
5. What was the rate per cent. of a note payable in 60 days, and discounted at  $\frac{1}{2}, 1, 2, 2\frac{1}{2}, 3 \%$  monthly? *Ans.*  $9\frac{55}{57} \%, 12\frac{1}{2} \%, \text{ etc.}$
6. What is the rate of interest corresponding to 5, 6, 7, 10, 12% discount on a bill due in 10 months, without days of grace? *Ans.*  $5\frac{2}{3} \%, 6\frac{2}{3} \%, \text{ etc.}$

**386. CASE IV.**—*The rate of interest being given, to find the corresponding rate of bank discount.*

*Ex.* A man buys notes payable in 90 days, at a discount such that his money brings him 2% per month; what is the rate of discount?

OPERATION.

|                          |                                   |
|--------------------------|-----------------------------------|
| 90 days + 3 days =       | 93 days.                          |
| Base,                    | \$100.00                          |
| Int. for 93 days,        | 6.20                              |
| Amt. " "                 | \$106.20                          |
| $\$6.20 \div 0.278775 =$ | $22\frac{1}{99} \%, \text{ Ans.}$ |

ANALYSIS.—If we assume \$100 for the proceeds of a note, the int. for 93 days, at 2%, will be \$6.20, and the face of the note \$106.20. We have then, the face of the note, \$106.20, the interest, \$6.20, and the time, 93 days, to find the rate per cent., which is done according to the preceding case. Hence the

**387. RULE.—I.** Find the interest and the amount of \$1 or \$100 for the time the note has to run.

**II.** Divide the interest by the interest of the amount at 1% for the same time.

By proportion.

$$100 + (24 \times \frac{93}{360}) : 100 :: 24 : x = 22\frac{1}{4}\% \text{, Ans.}$$

EXAMPLES FOR PRACTICE.

1. At what rate of bank discount must a note, payable in 60 days, be discounted to obtain 6% interest? *Ans. 5\frac{1}{4}\%*
2. At what rate must a note, due in 30 days, be discounted to obtain 6% interest? *Ans. 5\frac{3}{4}\%*
3. At what rate must a note, payable in 120 days, be discounted to obtain 8% interest? *Ans. 7\frac{5}{8}\%*
4. What rates of bank discount, of notes payable in 30 days, correspond to 5, 6, 7, 10% interest? *Ans. 4\frac{1}{4}\%, 5\frac{1}{4}\%, etc.*
5. What will be the rate of bank discount, on a note payable 8yr. and 4mo. hence, without grace, corresponding to 5% interest?
6. At what rates must notes, payable at 60 days, be discounted, to pay a broker 1, 1\frac{1}{2}, 2, 2\frac{1}{2}% per month? *Ans. 11\frac{1}{4}\%, etc.*

PROMISCUOUS EXAMPLES IN DISCOUNT.

What was the present worth, at true discount, of the following notes, when discounted:—

1. Dated Feb. 3rd., discounted June 6th., amounting to \$313.80, payable in 5 months, at 5%? *Ans. \$312.62 +.*
2. Dated March 4th., discounted Aug. 10th., amt'g to £175 11 3, payable in 7 mo., at 4%? *Ans. £174 10 3 +.*
3. Dated April 2nd., discounted May 30th., amounting to \$618.45, payable in 4 mo., at 4\frac{1}{2}%? *Ans. \$613.55 +.*
4. Dated May 15th., discounted Nov. 15th., amt'g to £406 7 0, payable in 8 mo., at 6%? *Ans. £402 6 6\frac{1}{2} +.*
5. Dated Aug. 7th., discounted Dec. 5th., amounting to \$8000.00, payable in 6 mo., at 5%? *Ans. \$7931.69 +.*
6. Dated Jan. 3rd., discounted Sept. 20th., amt'g to £270 10 6, payable in 9 mo., at 7%? *Ans. £269 16 10\frac{1}{2} +.*
7. Dated June 14th., discounted Aug. 2nd., amounting to \$4682.70, payable in 3 mo., at 6%? *Ans. \$4650.14 +.*
8. Dated Sept. 8th., discounted Feb. 18th., amounting to \$2385.30, payable in 10 mo., at 5%? *Ans. \$2337.89 +.*
9. Dated Nov. 25th., discounted May 11th., amt'g to £2626 5 3, payable in 7 mo., at 6%? *Ans. £2607 2 10\frac{1}{2} +.*
10. Dated Dec. 6th., discounted Sept. 18th., amounting to \$1891.50, payable in 11 mo., at 5%? *Ans. \$1878.97 +.*

What were the proceeds, at bank discount, of the following notes, when discounted:—

11. Dated Oct. 9th., discounted June 7th., amounting to £287 5 0, payable in 9 mo., at  $6\frac{1}{4}\%$ ? *Ans.* £285 10 1+.

12. Dated July 16th., discounted Oct. 12th., amt'g to \$626.85, payable in 5 mo., at  $4\frac{3}{4}\%$ ? *Ans.* \$621.225+.

13. Dated March 2nd., discounted Sept. 28th., amt'g to £422 9 0, payable in 8 mo., at  $6\frac{1}{2}\%$ ? *Ans.* £419 11 0 $\frac{1}{4}$ +.

14. Dated Jan. 7th., discounted Nov. 3rd., amounting to \$2341.50, payable in 11 mo., at  $7\frac{1}{2}\%$ ? *Ans.* \$2324.052+.

15. Dated April 10th., discounted Dec. 4th., amt'g to £2340 15 6, payable in 10 mo., at  $4\frac{3}{4}\%$ ? *Ans.* £2318 16 11+.

16. Dated May 17th., discounted June 22nd., amt'g to \$1310.25, payable in 3 mo., at  $5\frac{1}{2}\%$ ? *Ans.* \$1298.439+.

17. Dated March 14th., discounted Sept. 7th., amounting to \$1800, payable in 7 mo., at  $7\%$ ? *Ans.* \$1786.

18. Dated Feb. 9th., discounted April 13th., amt'g to £2552 14 0, payable in 5 mo., at  $7\frac{1}{2}\%$ ? *Ans.* £2504 16 8 $\frac{1}{2}$ +.

19. Dated Nov. 11th., discounted May 4th., amounting to \$525.90, payable in 7 mo., at  $6\%$ ? *Ans.* \$522.306+.

20. Dated March 6th., discounted June 9th., amt'g to £2104 8 6, payable in 4 mo., at  $7\frac{1}{2}\%$ ? *Ans.* £2091 5 5 $\frac{1}{2}$ +.

21. On March 12th., discounted at a bank, at  $6\%$ , a note of \$705.60, payable June 28th.; what sum did I receive? *Ans.* \$692.546+.

22. A bill on 4 months' credit having been discounted at  $5\%$ , bank discount, was reduced to £37 5 4 $\frac{1}{2}$ ; what was the amt. of the bill?

23. The contract for a public school was given to a builder on the deduction of  $12\%$  of his tender. The building being finished, he was ordered to do extra work for \$1529. Required the amount of the extras, so that the contractor may receive the \$1529, after deducting the  $12\%$ ? *Ans.* \$1737.50.

24. The proceeds of a note, payable Aug. 2nd., and discounted May 9th., at the bank, are £39 0 9 $\frac{3}{4}$ ; what is the face of the note, discount being  $6\%$  yearly? *Ans.* £39 12 4 $\frac{1}{2}$ +.

25. I owe the sum of \$514.22 as follows: \$208.32 payable in 10mo., \$123.20 in 18 mo., and the remainder in 22 mo.; if I can obtain true discount at  $4\%$ , how much must I pay? *Ans.* \$488.043+.

26. A bill amounts to £300 7, and the discount allowed is  $2\frac{1}{2}\%$ ; to what sum is the amount of the bill reduced? *Ans.* £292 16 9 $\frac{1}{2}$ .

27. What is the present worth of \$769.60, due 3 years and 5 months hence, at  $6\%$ ? *Ans.* \$638.67+.

28. Paul invested in business the sum of £1441 10 payable in 3 years, and is at liberty to advance the payment at the rate of  $\frac{1}{2}\%$ , bank discount, per month, without days of grace. At the end of 15 months he gave £716 2 6; in what time did he balance the remainder, knowing that he disbursed but £532 7? *Ans.* After 22 mo. 20 da.

29. The sum of \$1720 is payable in 1 year, and \$10900, in 18 months; but by paying immediately,  $5\%$  true discount, on the first sum, and  $5\frac{1}{2}\%$  on the second, can be obtained; what is the diminution?

30. For what sum must a note, to run 4mo. 15da., at 6%, be given that the bank proceeds may be \$1954? *Ans.* \$2000.

31. A person owes £2250 0 4 $\frac{1}{2}$ , payable in 6 months; if he pays ready money at 2% discount for the 6 months, how much will he pay? *Ans.* £2205 0 4 $\frac{1}{2}$ .

32. Had I bought goods for £875, I would have obtained £120 discount; but as I bought them for £620, the discount amounted to only £98; did I obtain more diminution in proportion to my purchases, and at what % does the surplus amount to? *Ans.* 2 $\frac{2}{3}$  17%.

33. A merchant bought \$4612.80 worth of oil, on 3 years' credit, and has the liberty of advancing the payment, at a discount of  $\frac{1}{4}$ %. After 15 months he gave \$2291.60; at what time did he settle the remainder, knowing that he disbursed but \$1703.52? *Ans.* 22mo. 20da. after the purchase.

34. What sum discounted for 7mo. 9da., at 6 $\frac{1}{2}$ % per annum, can produce a discount with which may be purchased the makings of 8 covered benches, using 1 $\frac{1}{2}$ yd. for each, at \$1.80 per yd.? *A.* \$662.79 +

35. Having bought two clocks for \$505, on 16mo.'s credit, and having paid them before the term of maturity, I obtained \$18.05 discount, at 5% per annum; at what epoch did I acquit the debt? *A.* 7mo. 3da. aft.

36. In a new building, two iron floors were laid, each floor being 15.36yd. long and 8.25yd. wide. The weight of the iron is 70lb. per yard of superficic, and after being laid costs \$5 per 100lb. I ask, 1st. the total price of the two floors; 2nd. the discount that can be obtained by paying 68 days before the time, at  $\frac{1}{4}$ % discount per month.

## STOCKS.

**388.** **Stocks** is a general name given to government bonds, and to money capital invested in corporations.

**389.** A **Corporation** is a body formed and authorized by law to act as a single person.

**390.** The legal act of incorporation which defines the rights and powers of the corporation is called a **Charter**.

**391.** The **Capital Stock** of a corporation is the money contributed and employed to carry on the business of the company.

**Notes.**—1. When the capital stock has been all paid in; money may be raised, if necessary, by loans, secured by mortgage upon the property. The bonds issued for these loans entitle the holders to a fixed rate of interest. Thus, bonds drawing 6% annually are called 6 per cent. stock, or 6 $\frac{1}{2}$ ; &c.

2. To the bonds are attached what are called *coupons*, each of which is a due bill for the interest on the bond to which it is attached, representing the amount of the periodical dividend or interest, and the time of payment, which coupons are severally cut off and presented for payment as they become due.

3. *Consols* is a term abbreviated from the expression "consolidated annuities." The British government having at various times borrowed money at different

rat  
the  
sen  
pra  
dec  
pre  
bec

titl  
sto  
of  
rai

val  
for

tha  
qui  
3  
mee

3  
prof  
4  
or a  
jobb

E  
stock  
No  
unlos

\$270  
\$270  
Or,  
\$1.04

rates of interest, and payable at different times, consolidated the stock or bonds thus issued, by issuing new stock drawing interest at 3% per annum, payable semi-annually, and redeemable only at the option of the government, becoming practically *perpetual annuities*. With the proceeds of this the old stock was redeemed. The quotations of these 3% perpetual annuities or *consols*, indicate pretty accurately the state of the money market, as they form a staple credit and become a standard for reference.

**392. Stockholders** are the owners of stock, either by original title or by subsequent purchase.

**393. A Share** is one of the equal parts into which capital stock is divided. The value of a share in the original contribution of capital varies in different companies; in bank, insurance, and railroad companies of recent organization, it is usually \$100.

**394. Stocks are At Par** when they sell for their original value.

**395. Above Par**, at a premium or advance, when they sell for more than their original value.

**396. Below Par**, or at a discount, when they sell for less than their original value.

**397. An Installment** is a portion of the capital stock required of the stockholders, as a payment on their subscription.

**398. An Assessment** is a sum required of stockholders, to meet the losses or the business expenses of the company.

**399. A Dividend** is a sum paid to the stockholders from the profits of the business.

**400. A person** who buys and sells stocks, either for himself, or as the agent of another, is called a **Stock Broker** or **Stock-jobber**.

#### EXAMPLES FOR PRACTICE.

*Ex. 1.* What is the cost of 27 shares of Grand Trunk Railroad stock at  $4\frac{1}{2}\%$  premium?

NOTE.—In all these examples, \$100 will be considered as the value of a share, unless otherwise mentioned.

#### OPERATION.

$$\$2700 \times .045 = \$121.50, \text{ premium.}$$

$$\$2700 + \$121.50 = \$2821.50, \text{ Ans.}$$

$$\text{Or, } \$2700 \times \$1.045 = \$2821.50, \text{ Ans.}$$

$$\$1.045, \$2700 \text{ will cost } \$2700 \times \$1.045 = \$2821.50, \text{ Ans.}$$

ANALYSIS.—We calculate firstly the premium on the par value, which we find to be \$121.50; we add this to \$2700, and obtain \$2821.50 which is the cost. Or, since \$1 of the stock costs \$1 plus the premium, or

$$\text{By proportion, } 100 : 100 + 4.5 :: 27 \times 100 : x.$$

*Ex. 2.* Bought from an agent 64 shares of the Ocean Steamers Co. stock, at 15% discount, for which he charged me  $\frac{1}{4}$ % brokerage; how much did I pay?

## OPERATION.

$$\begin{aligned} \$0.15 + .0025 &= 0.1525. \\ \$1.00 - \$0.1525 &= \$0.8475 \text{ proceeds} \\ &\text{of \$1 of stock.} \\ 6400 \times \$0.8475 &= \$5424, \text{ Ans.} \end{aligned}$$

ANALYSIS.—Adding the rate of brokerage to the rate of discount, we have .1525; hence \$1 will bring \$1 — \$0.1525 = \$0.8475, and 64 shares or \$6400 will bring  $6400 \times .8475 = \$5424$ .

*By proportion.*  $100 : 100 - (15 + 0.25) :: 64 \times 100 : x$ .

*Ex. 3.* I put \$17700 into the hands of a broker to be invested in Ontario Province Bonds when their market value is 12% below par; how many shares will I receive if the broker charges  $\frac{1}{4}$ % for his services?

## OPERATION.

$$\begin{aligned} \$1.00 - \$0.12 &= \$0.88, \text{ market value of \$1.} \\ \$0.88 + \$0.00\frac{1}{4} &= \$0.885, \text{ cost of \$1.} \\ \$17700 \div \$0.885 &= \$20000 = 200 \text{ shares, Ans.} \end{aligned}$$

ANALYSIS.—Since the stock is 12% below par, the market value of \$1 will be \$0.88; adding the rate of brokerage, we find that every dollar of the

stock will cost \$0.885. Hence, for \$17700, the broker can purchase  $\$17700 \div .885 = \$20000$  or 200 shares.

*By proportion.*  $100 - (12 + .5) : 100 :: 17700 : x \div 100$ .

*Ex. 4.* The Richelieu Company declares a dividend of 15 $\frac{1}{2}$ %; what will I receive for 24 shares?

## OPERATION.

$$\$2400 \times .15\frac{1}{2} = \$372.$$

ANALYSIS.—According to 282, we multiply the base, \$2400, by the rate, 15 $\frac{1}{2}$ , and obtain the dividend, \$372.

*By proportion.*  $100 : 15\frac{1}{2} :: 24 \times 100 : x$ .

*Ex. 5.* What income can we obtain by investing \$10260 in Quebec Province 6% bonds, purchased at 95%?

## OPERATION.

$$\begin{aligned} \$10260 \div .95 &= \$10800, \text{ stock purchased.} \\ \$10800 \times .06 &= \$648, \text{ annual income.} \end{aligned}$$

And since the stock bears 6% interest, we have  $\$10800 \times .06 = \$648$ , the annual income.

ANALYSIS.—We divide the investment, \$10260, by the cost of \$1, and obtain \$10800, the stock which the investment will purchase, (288).

*By proportion.*  $95 : 100 :: 10260 : x \times .06$ .

*Ex. 6.* A person desires to secure \$450 annual revenue; what capital must he invest in 5% bonds, when stock is purchased at 80%?

OPERATION.

$\$450 \div .05 = \$9000$ , stock required.  
 $\$9000 \times .80 = \$7200$ , cost, or investment.

ANALYSIS.—Since \$1 of the stock will secure \$0.05 income, to obtain \$450 will require  $\$450 \div .05 = \$9000$ , (Ex. 5). Multiplying the par

value of the stock by the market price of \$1, we have  $\$9000 \times .80 = \$7200$ , the cost of the required stock, or the sum to be invested.

*By proportion.*  $5 : 100 :: 450 : x \times .80$ .

*Ex. 7.* What per cent. of my investment shall I secure, by purchasing Montreal 7 per cents., at 105%?

OPERATION.

$.07 \div 1.05 = 6\frac{2}{3}\%$ .

ANALYSIS.—Since \$1 of stock will cost \$1.05, and pay .07, the income is  $\frac{7}{105} = 6\frac{2}{3}\%$  of the investment.

*By proportion.*  $105 : 100 :: 7 : x$ .

*Ex. 8.* A man invested in a Steamboat Company, and received a dividend of 9%, which was  $8\frac{1}{2}\%$  on his investment; at what price did he purchase?

OPERATION.

$\$0.09 \div \$0.08\frac{1}{2} = \$108$ , Ans.

ANALYSIS.—Since \$0.09, the income of \$1 of the stock, is  $8\frac{1}{2}\%$  of the sum paid for it, we have,  $\$0.09 \div \$0.08\frac{1}{2} = \$108$ , the purchase price.

*By proportion.*  $8\frac{1}{2} : 100 :: 9 : x$ .

9. A person buys 25 shares of the Marine Bank, of \$100 each, at 12% discount; how much must he pay? *Ans.* \$2200.

10. What will I receive for 20 shares of the Central Railroad stock, at 135%, brokerage being  $1\frac{1}{2}\%$ ? *Ans.* \$2665.

11. At  $7\frac{1}{2}\%$  premium, and  $\frac{1}{4}\%$  brokerage, what will be the cost of 36 shares of the Bank of Commerce? *Ans.* \$3879.

12. A canal cost £400000; all expenses defrayed it brings in £15000 annually. Suppose it to have been constructed by means of shares of £50 each, and that an individual took 25 shares, what dividend will he receive annually? *Ans.* £45 17 6.

13. If 300 shares of the Ottawa Bank sell for \$30112.50, what is the premium, each share being \$100? *Ans.*  $\frac{1}{2}\%$  premium.

14. When the nominal value of stock is £12 10, and the discount  $3\frac{1}{2}\%$ , how much must I pay for 30 shares? *Ans.* £361 17 6.

15. The steamboat company of the Saguenay declares a dividend of 15%; what shall I receive for 65 shares the nominal value of which is \$100 per share? *Ans.* \$975.

16. Bought stock at par, and sold it at 3% premium, gaining £187 10 0; how many shares did I purchase? *Ans.*  $62\frac{1}{2}$  shares.

17. An individual bought, at the rate of \$168.75, a number of shares in the Pictou coal-mine company, the annual income of which is \$10 per share. With the income he purchases \$260 worth of goods; what was his investment, the brokerage being  $\frac{1}{4}\%$ ? *Ans.* \$4398.46 $\frac{1}{2}$ .

18. A merchant retires from business with a sum of \$34520.50, and buys with this capital government 6's, at the rate of \$70.45; what will be his annual income? *Ans.* \$2940.

19. Ontario 4 $\frac{1}{2}$ 's are sold at the rate of £94 17; what income will I obtain for £3794? *Ans.* £180.

20. Sold \$16400 worth of North Bank Stock at 13% premium; what shall I receive? *Ans.* \$18532.

21. A person, having £2250, invests this sum in Ocean Telegraph Company Stock which sells at 17% discount; what amount of capital does he purchase? *Ans.* £2710 16 10 $\frac{1}{2}$  +.

22. Bought 36 shares of the Western Copper Mine Company, the par value of each being \$500, at 2% premium, and sold it at 28% discount; what is my loss? *Ans.* \$5400.

23. I have an investment of \$15000 in a transatlantic steamship company; how many shares shall I own after a dividend of 8% is declared and payable in capital stock? *Ans.* 162 shares of \$100 each.

24. What should be the rent of a farm, which cost \$16992.10, in order that the purchase capital may produce the same revenue as would be produced by the same sum, employed in the purchase of 6 $\frac{1}{2}\%$  bonds, at 91 $\frac{1}{2}\%$ ? *Ans.* \$1203.80.

25. A farmer invests £36, the price of three oxen, in the purchase of 5% bonds sold at the rate of £78 10; at what real rate was his money placed? *Ans.* 6 $\frac{5}{8}\%$ .

26. An exchange agent having \$45000 invested in bonds of the Canadian Transatlantic Steamship Company, exchanged them at 88% for capital stock in the same company valued at 62 $\frac{1}{2}\%$ . The bonds brought 7% annually, while the shareholders received two dividends during the year, the first of 3%, and the second of 3 $\frac{1}{2}\%$ ; how much did the agent gain annually by the exchange? *Ans.* \$968.40.

27. An agent receives \$25000, with instructions to deduct his brokerage at 1 $\frac{1}{2}\%$ , and then purchase bank stock for the balance; if the stock is selling at 3% discount, what will be the amount of his capital stock? *Ans.* \$25329.92 +

28. An individual desires to invest \$11158 in 5% bonds. The market value being but \$67.35, he waits a few days, when it rises to \$69.10. Find, now, what income did he lose, and what income he would have gained had the market value lowered to \$66.25, brokerage being  $\frac{1}{4}\%$ ? *Ans.* Lost \$20.95 + income, would have gained \$13.73 +

29. I have \$60500 to invest in bonds. I can purchase 4 $\frac{1}{2}\%$  bonds at the rate of \$95.30, and 3% bonds at the rate of \$69.25; which would be the more profitable of the two? *Ans.* The 4 $\frac{1}{2}\%$  bonds.

30. How much more advantageous is it to invest \$1128 in 4 $\frac{1}{2}\%$  bonds, at 91 $\frac{1}{2}\%$ , than \$1128 in 3% bonds, at 69 $\frac{1}{2}\%$ , brokerage being at  $\frac{1}{4}\%$ ? *Ans.* \$6.923 +.

31. A banker owns 150 shares in the Quebec Insurance Company.

I ow  
how  
char  
32  
he b  
90 $\frac{1}{2}$   
the  
wha  
price  
33  
\$104  
a fa  
of th

34  
£37  
annu  
35  
bein  
95 $\frac{1}{2}$   
tain  
case  
36  
desir  
nix l  
inter  
Prov  
teres  
the r  
he se  
3 sha

4  
sons  
socia  
Not  
signif

4

Es  
A put  
of the

I order my agent to buy them when they will rate at  $5\frac{1}{2}\%$  premium; how much will the 150 shares cost me, knowing that the agent will charge me  $\frac{1}{2}\%$  brokerage? *Ans.* \$15956.25.

32. A farmer sold corn for the amount of \$4134.40. With this sum he buys three  $4\frac{1}{2}\%$  bonds which produce an annual income of \$18, at  $90\frac{3}{8}\%$ , and one  $3\%$  bonds, producing annually \$20, at  $64\frac{7}{16}\%$ . With the remainder diminished by \$1.95, he buys  $3\%$  bonds at  $68\frac{1}{2}\%$ ; at what average rate should he purchase  $4\frac{1}{2}\%$  bonds, to have, for the price of the corn sold, the same quantity of revenue? *Ans.* \$98.43 +

33. In buying stock in the Labrador Company for the value of \$10425, at 500 per share, and producing \$36 for interest and dividend, a farmer secured a revenue of \$540. Required the market value of the stock per share, and at what rate he let out his money?

*Ans.* 1st. \$695; 2nd.  $55\frac{25}{100}\%$ .

34. In January 1848, the total amount of British consols was £378019855; what was the amount of interest paid on them semi-annually?

*Ans.* £5670297 $\frac{3}{8}$ .

35. A person desires to sell \$3500 of Montreal 7's; the market value being at  $95\frac{1}{2}\%$ , he waits a few days longer when stock rises to  $95\frac{3}{8}\%$ ; what profits did he realize? What loss would he have sustained had the market value lowered to  $94\frac{7}{16}\%$ , brokerage, in both cases, being  $\frac{1}{2}\%$ ?

*Ans.* \$22.75 gain, and \$17.50 loss.

36. A mason built 965 sq. yd. of a wall at \$21.80 per sq. yd. He desires to invest this sum in insurance company stock. In the Phoenix Insurance Co., the shares are \$5000 each; they produce \$200 as interest and dividend, and are negotiated at 40% premium. In the Providence Co. the shares are \$2500 each; they produce \$50 as interest and dividend, and are negotiated at 45% premium. Which are the most advantageous, and by how much %? How many shares can he purchase in taking the most advantageous, and what revenue could he secure? *Ans.* The first are the more advantageous by \$1.478%; 3 shares; and \$600 of revenue.

## PARTNERSHIP.

**401.** A Partnership is an association of two, or more persons in business, each of whom is called a *Partner*. Such an association is called a *Company*, *Firm*, or *House*.

*NOTE.*—The terms *Capital* or *Stock*, *Dividend*, and *Assessment*, have the same signification in Partnership as in Stocks.

**402.** CASE I.—To find each partner's share of the profit or loss, when there is no regard to time.

*Ex.* Three merchants, A, B, and C, associate together in business; A puts in \$275, B \$475, and C \$500. They gained \$150; what part of the profit must be given to each?

|       |                               | OPERATION.                               |
|-------|-------------------------------|------------------------------------------|
| A's   | stock, \$275                  | $\$275 \times 0.12 = \$33$ , A's profit. |
| B's   | " 475                         | $475 \times 0.12 = 57$ , B's profit.     |
| C's   | " 500                         | $500 \times 0.12 = 60$ , C's profit.     |
| Whole | " <u>\$1250</u>               | Proof \$150, whole profit.               |
|       | $\$150.00 \div 1250 = \$0.12$ | profit on \$1.                           |

**ANALYSIS.**—Since the whole stock is \$1250, and the whole profit, \$150, the profit on every \$1 of stock will be as many dollars as 150 contains times 1250, or \$0.12 on every \$1 of stock. Then, each merchant's stock multiplied by .12 gives his part of the whole profit. The same result also may be obtained, as follows:—

*By proportion.*

$$\begin{array}{l}
 275 \\
 + 475 \\
 + 500
 \end{array}
 \left. \vphantom{\begin{array}{l} 275 \\ + 475 \\ + 500 \end{array}} \right\} = 1250 : 150 :: \left\{ \begin{array}{l} 275 \\ 475 \\ 500 \end{array} \right\} : x = \text{Ans. } \left\{ \begin{array}{l} \$33, \text{ A's profit.} \\ 57, \text{ B's profit.} \\ 60, \text{ C's profit.} \end{array} \right.$$

Proof, \$150, whole profit.

**403. RULE.**—*The whole profit or loss, divided by the number denoting the whole stock, will give the profit or loss on each dollar of stock; and each partner's stock, multiplied by the number denoting the profit on \$1, will give his share of the whole profit or loss.*

Or,

*As the whole stock is to each partner's stock, so is the whole profit or loss to each partner's profit or loss.*

#### EXAMPLES FOR PRACTICE.

1. With £200, two men gained £50; the first man contributed £125, the second, £75; what part of the gain is each entitled to?

*Ans.* The first, £31 5; the second, £18 15.

2. Four merchants associated and raised a capital of \$45000, to which each man contributed equally. At the expiration of the partnership, the capital was found to be augmented by \$26877. What shall be the part of each man, knowing that the 1st. ought to have 13 parts; the 2nd., 11; the 3rd., 8; and the 4th., 7?

*Ans.* 1st., \$23959; 2nd., \$20273; 3rd., \$14744; 4th., \$12901.

3. Three men associating together, gained £287 10; the 1st., put in 400 yd. of velvet at £1 per yard; the 2nd., 350 yd. of cloth at £2; the 3rd., 450 yd. of cassimere at 15s.; what part of the gain should each have?

*Ans.* £80, £140, and £67 10.

4. Four persons having joined in partnership agree that the 1st. put in £1250; the 2nd.,  $\frac{1}{4}$  more than the first; the 3rd., as much as the two others together; and the 4th., his industry during the year, which was estimated at £2000; what share of the profits, £1525, shall each receive?

*Ans.* £250, £312, £562, and £400.

5. Four associates made a profit of \$1500. The first is to have 3 parts; the 2nd., 4; the 3rd., 5; and the 4th., 6. How much will each receive?  
*Ans.* \$250, \$333 $\frac{1}{3}$ , \$416 $\frac{2}{3}$ , and \$500.

6. The first of five men, associated in partnership, put in \$800; the 2nd., \$100 more than the first; the 3rd., 100 more than the second; and so on, with the others, always augmenting by \$100. If the gain is \$1800, what ought to be the part of each?  
*Ans.* \$288, \$324, \$360, \$396, \$432.

7. Three speculators have together a capital of \$4928, which brings them a profit of \$616; the 1st. received \$150 for his share of the gain; the 2nd., \$206; and the 3rd., \$260. What was each one's investment?  
*Ans.* \$1200, \$1648, \$2080.

8. Two speculators shipped 6000 tons of corn to Cuba. During the voyage 650 tons were thrown overboard on account of a storm which arose. If 250 tons were spoiled, how much did each man lose, knowing that 3500 tons belonged to the first?  
*Ans.* 525 and 375 tons.

9. Three farmers bought 148 sheep at \$4.12 $\frac{1}{2}$  per head, for the payment of which the 1st. furnished \$218.85, the 2nd., \$236.32 $\frac{1}{2}$ , and the 3rd. the remainder. They sold the sheep, after having nurtured them during 6 months, at a profit of \$1.60 per head; how much did each receive of the profits?  
*Ans.* \$84.88 $\frac{1}{2}$ , \$91.66 $\frac{1}{2}$ , \$60.24 $\frac{1}{2}$ .

10. Three lumber merchants bought 76500 saplings, on which they realized a profit of £296 8 9. The first man contributed £460 15 7 $\frac{1}{2}$ ; the second, £527 6 10 $\frac{1}{2}$ ; the third man's part is not known, but he received, however, £98 16 3 as his share of the profits. Tell us the contribution of the third merchant, the profits of the two others, and the price of the saplings per hundred?  
*Ans.* Third merchant's share £494 1 3. The profits of 1st., £92 3 1 $\frac{1}{2}$ ; 2nd., £105 9 4 $\frac{1}{2}$ ; £1 18 9 per hundred.

11. Two dealers in furs made a joint purchase of 268 assorted fox and beaver skins, at £112 10 per hundred; the first dealer advanced £48 10 more than the second, and, together they realize a profit of 18% on the buying price. Required what is due to each, and at what price they sold the skins a piece?  
*Ans.* £149 5 4 $\frac{1}{2}$  due to the 2nd.; £206 10, to the first. The skins cost £1 6 6 $\frac{3}{4}$  a piece.

12. Three students in Astronomy join in raising \$698.50 for the purchase of a telescope. The second furnished  $\frac{1}{3}$  of what the first gave, and the third furnished  $\frac{1}{4}$  of what the two others had advanced; what was the contribution of each?  
*Ans.* \$277.81 $\frac{1}{2}$ , \$166.68 $\frac{2}{3}$ , \$254.

13. Four farmers associated in furnishing a quantity of straw which they sold at \$7 per hundred bundles; what did each receive, knowing that the 1st. furnished  $\frac{1}{4}$  of it; that the 2nd. furnished a quantity not mentioned, and that the 3rd. furnished 600 bundles, which quantity equalled the delivery of the 1st. and 4th., who furnished 240 bundles?  
*Ans.* \$25.20, \$8.40, \$42.00, \$16.80.

14. Two clockmakers joined in the purchase of 120 clock works at the average price of \$7.37 $\frac{1}{2}$ ; in the speculation, they lost \$135. The loss of the 1st. surpassed that of the 2nd. by \$33.50; what were the loss and investment of each?  
*Ans.* 1st. Inv. \$552.30 $\frac{1}{2}$ ; loss \$84.25. 2nd. Inv. \$332.69 $\frac{1}{2}$ ; loss \$50.75.

15. Several persons agreed to conduct, during one year, a paper manufactory. The first put in  $\frac{1}{3}$  of the stock; the second, \$4000 less than the first; the third, \$4000 less than the second, and so on until the last. If the investments had been in sums equal to the highest, the capital stock would be augmented by  $\frac{1}{3}$ . The merchandise sold produced a sum equal to the  $\frac{1}{3}$  of what was put in, which was employed in buying rags. In admitting that the  $\frac{1}{3}$  of the sum proceeding from sales serve to cover the expenses of fabrication and investment, it is required to ascertain how many persons there were, how much each one put in, and what part of the gain each is entitled to?

**404. CASE II.**—*To find each partner's share of the profit or loss, when the stock is employed for different periods of time.*

*Ex.* A and B entered into partnership; A furnished \$240 for 8 months, and B \$560 for 5 months. They lost \$118; what was each man's share of the loss?

## OPERATION.

$$\begin{array}{r} \$240 \times 8 = \$1920. \\ 560 \times 5 = 2800. \\ \hline \$4720. \end{array} \quad \begin{array}{r} \$1920 \times 0.025 = \$48, \text{ A's loss.} \\ 2800 \times 0.025 = 70, \text{ B's loss.} \\ \hline \text{Proof, } \$118, \text{ entire loss.} \end{array}$$

$$\$118.00 \div 4720 = \$0.025, \text{ loss on } \$1.$$

**ANALYSIS.**—It is evident that \$240 for 8 mo. is the same as  $240 \times 8 = \$1920$  for 1 mo., since \$1920 would lose as much in 1 mo. as \$240 in 8 mo.; and \$560 for 5 mo. is the same as  $560 \times 5 = \$2800$  for 1 month. The question then is the same as if A had furnished \$1920, and B \$2800, for equal times. Then, if  $\$1920 + \$2800 = \$4720$  lose \$118, \$1 will lose  $\frac{1}{4720}$  of \$118 = \$0.025, and  $\$1920 \times .025 = \$48$ , A's loss;  $\$2800 \times .025 = 70$ , B's loss. The same results may be obtained as follows:—

*By proportion.*

$$\left. \begin{array}{l} \$240 \times 8 = 1920 \\ 560 \times 5 = 2800 \end{array} \right\} = 4720 : \left\{ \begin{array}{l} 1920 \\ 2800 \end{array} \right\} :: 118 : x = \text{Ans. } \left\{ \begin{array}{l} \$48, \text{ A's loss.} \\ 70, \text{ B's loss.} \end{array} \right.$$

Proof, \$118.

**405. RULE.**—*Multiply each partner's stock by the time it was in trade, and divide the whole profit or loss by the sum of the several products; by the quotient, multiply the product of each partner's stock and time, and the result will be his share of the profit or loss.*

Or,

*Multiply each partner's stock by the time it was in trade; then, as the sum of these products is to each product, so is the whole profit or loss to each partner's profit or loss.*

## EXAMPLES FOR PRACTICE.

1. Two persons contribute unequal sums towards a capital: the first puts in \$2300 for 2 years; the second, \$1500 for 18 months. What part of the gain, \$1400, should each person receive?

*Ans.* \$940.15, \$459.85.

2. Three individuals raised a capital sum with which they gained £1137 10: the first contributed £200 for 2½ years; the second, £125 for 25 months; and the third, £248 15 for 35 months. What part of the gain should each have?

*Ans.* 1st. £382 15 1½; 2nd. £199 7 0½; 3rd. £555 7 10½.

3. A porter associated with a pedler and raised a capital of \$16000. After two years they divided the gain, and the pedler, who had contributed \$9000, received \$1800; what did his companion receive, knowing that the latter left his share in the business but during 20 months?

*Ans.* \$1166.66½.

4. Four persons agree to form a partnership for 3 years. The first puts in at the beginning \$350, and 5 months after \$240 more; the second puts in \$8000 at first, and at the end of 20 months withdraws the half of his share, and 5 months after withdraws \$2400 more; the third puts in \$1500 in the beginning, and \$5000 at the end of 2 years; the fourth puts in at first \$600, and every six months augments his portion by a like amount; the gain being \$80000, what part did each receive?

*Ans.* \$14677.35 +, \$33336.15 -, \$19232.39 +, \$12754.11 +.

5. Three merchants joined in business. The first put in £1001 12 for 10 months; the second, £1751 12 6 for 15½ months; and the third, £2000 3 9 for 17 mo. and 20 days. Required each merchant's share of the profits which amount to £350 3?

*Ans.* £48 7 5½ -; £131 2 4½ +; £170 13 1½ -.

6. Two clothiers associate together; one of them contributed a sum with which could be bought 90 yd. of Broadcloth at \$6 per yard, the other put in a sum with which 60 yd. could be purchased at the same rate. In supposing the 1st. to have had \$6 of the profits more than the 2nd., to how much did the profits amount?

*Ans.* \$30.

7. Four farmers rent a pasture for \$975. The first put 5 beeves on it during 54 days; the second, 7 cows during 63 days; the third, 8 heifers during 75 days; and the fourth, 6 horses during 50 days. It was calculated that 1 beef consumed 1½ times as much as a cow, or twice as much as a heifer, or 1½ times as much as a horse; how much must each farmer pay?

*Ans.* \$238.45 +; \$259.65 -; \$264.94 +; \$211.96 -.

8. In the working of a mine during 6 years, three partners gain £21750. The first partner had put in £13437 10 in the beginning, but after 2½ years, he withdrew £3275. The second put in his share, which was £41000, only 1½ years after the commencement of the work. Finally, the third made his contribution of £53750, but 3 years after the installment of the first. What part of the profits should each receive? *Ans.* £3566 16 0½ +; £9867 6 4½ -; £8315 17 7 +.

## EXCHANGE.

**406. Exchange** is the process of remitting money from one place to another by **Drafts** and **Bills of Exchange**.

*NOTE.*—For a full treatment of this and of the following subjects, see the *Commercial Arithmetic*.

*Form of a Draft.*

\$400. [STAMP.] *Quebec, P. O., March 1, 1871.*

*Thirty days after sight, pay to Henry Simms,  
or order, Four Hundred Dollars, and charge the  
same to my account.*

*To James Benton. Louis O'Neil.  
No. 12, Richard Street, Toronto, P. O.*

**407.** The **Drawer**, or **Maker**, is the person who signs the draft.

**408.** The **Drawee** is the person on whom the draft is made.

**409.** The **Payee** is the person to whom the draft is made payable.

**410.** An **Acceptance** is the promise of the **Drawee**, to pay the draft at maturity, and is usually acknowledged by writing the word "Accepted" with his signature, across the face of the draft.

**411.** An **Indorsement** of a draft, by the payee, is made in the same manner as the indorsement of a note.

**412.** A **Sight Draft** is an order to pay at sight.

**413.** A **Time Draft** is an order requiring payment at a specified time.

**414.** A **Draft** or **Bill of Exchange** is at a *Premium*, when the price paid is greater than its face; and at a *Discount*, when the price paid is less than its face.

**415.** **Domestic**, or **Inland Exchange**, is when both the drawer and drawee reside in the same country.

4  
E.  
exch.

\$640

E.  
Halif

\$1.00  
.00

\$ .99  
.02

\$1.01

\$3500

41

draft  
1 mi

II.

discou  
is pur

or sub  
draft

I. A  
exchan

2. A  
and ex

3. V  
\$800,

2% ?

4. A  
\$840,

rate of  
5. V  
exchan

**416. CASE I.**—Given the face of a draft, the rate per cent. of exchange, and the time, to find its cost.

*Ex. 1.* What must I pay in Ottawa for a draft of \$640 on Quebec, exchange being  $1\frac{1}{2}\%$  premium?

OPERATION.

$$\$640 \times 1.015 = \$649.60, \text{ Ans.}$$

ANALYSIS.—The cost of exchange of \$1 is \$1 + \$0.015 = \$1.015, and of \$640,  $640 \times \$1.015 = \$649.60$ .

*Ex. 2.* What must be paid in Montreal for a draft of \$3500 on Halifax, at 33 days, exchange  $2\frac{1}{2}\%$  premium.

OPERATION.

$$\begin{aligned} \$1.000 & \\ .006 & = \text{disc. for 36da. at } 6\% : \\ \hline \$.994 & = \text{cost at par of } \$1. \\ .022 & = \text{rate of exchange.} \\ \hline \$1.016 & = \text{cost of } \$1 \text{ of the draft.} \\ \$3500 \times 1.016 & = \$3556, \text{ Ans.} \end{aligned}$$

ANALYSIS.—The discount of \$1 at 6% for 36 days is \$0.006, which being subtracted from \$1 leaves \$0.994, the cost of \$1 of the draft, if the exchange was at par. To this add the premium of \$1, \$0.022, and we have \$1.016, the cost of \$1 of the draft. Hence the cost of \$3500, the draft, is  $\$3500 \times 1.016 = \$3556$ .

**417. RULE.**—I. For sight drafts.—Multiply the face of the draft by 1 plus the rate when exchange is at a premium, and by 1 minus the rate when exchange is at a discount.

II. For drafts payable after sight.—Find the cost of \$1 at bank discount for the specified time, at the legal rate where the draft is purchased; then add the rate of exchange when at a premium, or subtract it when at a discount, and multiply the face of the draft by this result.

#### EXAMPLES FOR PRACTICE.

1. A merchant in Toronto wishes to pay in Montreal \$7930, and exchange is  $\frac{1}{2}\%$  premium; what will be the cost of the draft?

*Ans.* \$7989.47 $\frac{1}{2}$ .

2. A merchant in St. John, N. B., wishes to pay in Ottawa, \$980, and exchange is  $1\frac{1}{2}\%$  discount; required the cost of the draft?

*Ans.* \$962.85.

3. What will be the cost, in Kingston, of a draft on Halifax for \$800, payable 60 days after sight, exchange being at a premium of 2%?

4. A merchant in Kingston purchased a draft on Fredericton for \$840, payable 30 days after sight, at 6%; what did it cost him, the rate of exchange being  $1\frac{1}{2}\%$  discount?

*Ans.* \$822.78.

5. What will be the cost of a draft of \$4250, for 60 days, at 6%, exchange being  $1\frac{1}{2}\%$  premium?

*Ans.* \$4285.06 $\frac{1}{2}$ .

6. A merchant in Quebec receives from his agent 1200 bushels red wheat, purchased in Toronto at 65 cts. per bushel; in payment for which he remits a draft on Toronto, at  $\frac{3}{4}\%$  discount. The transportation of his wheat cost \$98. What must he sell it for per bushel to gain \$225?  
*Ans.* \$0.91 $\frac{1}{2}$ .

**418. CASE II.**—Given the cost of a draft, the rate per cent. of exchange, and the time, to find its face.

*Ex.* A merchant in Three Rivers paid \$6856.10 for a 60 days' draft on Toronto, exchange being  $1\frac{1}{2}\%$  premium, and interest  $6\%$ ; required the face of the draft.

## OPERATION.

\$1.0000  
 .0105 = the discount for 63 days.  
 \$ .9895 = the cost of \$1 at par.  
 .01875 = the rate of exchange.  
 \$1.00825 = the cost of \$1 of the draft.  
 \$6856.10 ÷ \$1.00825 = \$6800, *Ans.*

**ANALYSIS.**—By 416, Case I., *Ex.* 2, we find the cost of \$1 of the draft to be \$1.00825. Hence, \$6856.10 ÷ \$1.00825 = \$6800, is the face of the draft.

**419. RULE.**—Divide the given cost by the cost of a draft for \$1, at the given rate of exchange; the quotient will be the face of the required draft.

## EXAMPLES FOR PRACTICE.

1. What draft may be purchased for \$16415.10, exchange being at  $3\frac{1}{2}\%$  premium? *Ans.* \$15860.
2. Required the face of a draft for \$158.40, exchange being at  $1\%$  discount? *Ans.* \$160.
3. An agent in Kingston is directed to make the remittance by draft, of \$565.32, to his employer in Quebec, drawn at 60 days. What will be the face of the draft, exchange being at  $1\frac{1}{2}\%$  premium?
4. What will be the face of a draft for \$962.85, exchange being at  $\frac{3}{4}\%$  discount?
5. A man in Halifax, has \$4800 due him in Montreal; how much more will he realize by making a draft for this sum on Montreal and selling it at  $\frac{1}{4}\%$  discount, than by having a draft on Halifax remitted to him, purchased in Montreal for this sum, at  $\frac{1}{2}\%$  premium?  
*Ans.* \$11.73 +.

## FOREIGN EXCHANGE.

**420. A Foreign Bill of Exchange** is a draft in which the drawer and drawee live in different countries,

Form of a Foreign Bill of Exchange.

£ 300. [STAMP.] *Montreal, P. Q., April 7, 1871.*

*At sight of this, our first Bill of Exchange (second and third of same tenor and date unpaid), pay to the order of William C. Maguire, London, Three hundred pounds sterling, value received, and charge the same to my account.*

*To F. J. Moore & Co., R. N. Wallace.*

*36, Wellington Street, London.*

NOTE.—In foreign exchange, to prevent loss or delay, two or three drafts of the same tenor and date are drawn up and sent by different conveyances, or at different times; on the payment of one, the other two are worthless. Each draft must have a stamp attached.

**421.** Foreign exchange is computed as inland exchange, except that the currency of one country must be reduced to that of another.

**422.** Rates of exchange between the Dominion of Canada and Great Britain are commonly reckoned, at a certain per cent. on the old par of exchange, instead of on the new par.

NOTE.—By an old act of Provincial Parliament, it was enacted that £1 sterling = \$4.444 Canadian money. But by a recent act the value of the pound sterling was fixed at \$4.866. Now, the new par is equal to the old par plus 9 1/4 % of the old par, that is, \$4.444 + 9 1/4 % of \$4.444, which is .422, equal to \$4.866, the new par. Consequently, the rate of exchange between the Dominion of Canada and Great Britain, must reach the nominal premium, or 9 1/4 % before it is at par, according to the new standard.

*Ex. 1.* A merchant in Quebec wishes to remit to London £560 3 6 sterling; exchange being at 11 % premium. How much must he pay for the bill of exchange?

OPERATION.

$$\begin{aligned} \$49 \times 1.11 &= \$4.93\frac{1}{2}; \\ £560\ 3\ 6 &= £560.175; \\ £560.175 \times 4.93\frac{1}{2} &= \$2763.53, \text{ Ans.} \end{aligned}$$

ANALYSIS.—Since the old par

of £1 sterling = \$4.444, or \$49, we multiply \$49 by 11 %, or \$1.11, the given rate, decimally expressed, and we obtain \$4.93 1/2, the cost of £1 at that

rate; multiplying the face of the bill, £560 3 6, decimally expressed by the cost of exchange of £1, we obtain \$2763.53, the required cost of the bill.

*Ex. 2.* What will be the face of a bill of exchange on Liverpool, purchased in Montreal for \$5537.40, exchange being at 10 % premium ?

OPERATION.

$$\begin{aligned} \$40 \times 1.10 &= \$4.88\frac{2}{3}; \\ \$5537.40 \div 4.88\frac{2}{3} &= \text{£}1132 \text{ } 13 \text{ } 0. \end{aligned}$$

ANALYSIS.—We find, as in the preceding example, the cost of £1, at the given rate of exchange; then we divide \$5537.40, the given cost, by the cost of exchange for £1, and obtain £1132 13 0, the face.

*Ex. 3.* What is the cost in Toronto of a bill on Paris, for 1780 francs, exchange being at 2½ % discount ?

OPERATION.

$$\begin{aligned} \text{Commercial value of the franc,} &= \$0.186 \\ \text{Deduct } 2\frac{1}{2} \% \text{ discount,} &\dots\dots\dots 0.00465 \end{aligned}$$

$$\text{Value of 1 franc,} \dots\dots\dots \$0.18135$$

$$\$0.18135 \times 1780 = \$322.803, \text{ Ans.}$$

**423.** From these illustrations we derive the following

**RULE.—I.** To find the cost of a bill, the face being given.—*Multiply the face by the cost of a unit of the currency in which the bill is expressed.*

**II.** To find the face of a bill, the cost being given.—*Divide the given cost by the cost of a unit of the currency in which the bill is to be expressed.*

### REDUCTION OF THE STERLING MONEY TO THE OLD OR TO THE NEW CANADIAN CURRENCY, NEW PAR.

*Ex.* Reduce £560 3 4 sterling, to Old Canadian Currency.

OPERATION.

$$\begin{aligned} &\text{£}560 \text{ } 3 \text{ } 4 \\ + \frac{1}{4} \text{ of } \text{£}560 \text{ } 3 \text{ } 4 &= 112 \text{ } 0 \text{ } 8 \\ + \frac{1}{12} \text{ of } 112 \text{ } 0 \text{ } 8 &= 9 \text{ } 6 \text{ } 8\frac{2}{3} \\ \hline &\text{£}681 \text{ } 10 \text{ } 8\frac{2}{3}, \text{ Ans.} \end{aligned}$$

$$\begin{aligned} \text{And in Decimal Currency,} \\ \text{£}681 \text{ } 10 \text{ } 8\frac{2}{3} (233) &= \$2726.13\frac{2}{3}. \end{aligned}$$

ANALYSIS.—The pound sterling = \$4.86⅔, and the Old currency pound = \$4; diff., \$0.86⅓. Then £1 ster. = £1 +  $\frac{86\frac{1}{3}}{400}$  = £1⅓ old

currency. Now,  $\frac{1}{12}$  of a number =  $\frac{1}{4}$  plus  $\frac{1}{12}$  of  $\frac{1}{4}$  of that number. Hence the

**424. RULE.—**To reduce sterling money to Old Canadian Currency, new par,—*Add to the given sum its fifth plus one twelfth of the fifth.*

1  
chan  
2.  
the c  
3.  
for 3  
4.  
at ½ %  
5.  
cost i  
35 ct  
6.  
sterlin  
7.  
franc  
8.  
Mont  
9.  
on Ly  
excha  
10.  
£381  
mal c

42  
mean  
ferent  
42  
becom  
42  
before  
once, v  
42  
debts

42

*Ex.*  
the first  
60 days  
erage to

## EXAMPLES FOR PRACTICE.

1. What will be the commercial value in Ottawa, of a bill of exchange on London for £390 10 ster., at 9% prem.? *Ans.* \$1891.75 $\frac{3}{4}$ .
2. What will cost, in Amsterdam, a bill on Montreal for \$681.34, the course of exchange being at \$0.38 per guilder? *Ans.* \$171.20.
3. What must be paid in Kingston for a bill of exchange on Paris for 3000 francs, exchange being at 2% above par?
4. What will be the cost in Montreal of a bill on Boston, for \$2000, at  $\frac{1}{2}$ % premium? *Ans.* \$2010.
5. What will a bill of exchange on Hamburg, for 5000 marcs banco, cost in Quebec, at 1% above par, the marc banco being equal to 35 cts.?
6. Bought in Toronto a bill of exchange on Glasgow for £675 2 6 sterling; what did it cost me at 8 $\frac{1}{2}$ % premium? *Ans.* \$3255.60 +.
7. What will cost in Halifax a bill of exchange on Rouen for 56245 francs, the course of exchange being 5 fr. 54 centimes per dollar? *Ans.* \$10152.53 +.
8. What will be the face of a bill on Dublin, that may be bought in Montreal for \$7125.50, exchange being at 9 $\frac{1}{2}$ % premium?
9. Paid in Quebec £2170 15 7, old Canadian currency, for a bill on Lyons amounting to 49335 fr. 20 centimes; what was the rate of exchange below par? *Ans.* \$0.053 +.
10. Received from L. Nelson & Co., London, a bill of exchange for £381 5 0, on J. Chalmers & Co., Quebec; what is its value in decimal currency of Canada, at 9% premium? *Ans.* \$1846.94 +.

## EQUATION OF PAYMENTS.

**425. Equation of Payments** is the process of finding the mean or equitable time of payment of several sums, due at different times without interest.

**426. The Term of Credit** is the time to elapse before a debt becomes due.

**427. The Average Term of Credit** is the time to elapse before several dates, due at different times, may all be paid at once, without loss to debtor or creditor.

**428. The Equated Time** is the date at which the several debts may be cancelled by one payment.

**429. CASE I.**—To find the average or equitable time of paying several debts due at different times.

*Ex. 1.* On the first of March 1870, a man gave notes, as follows: the first for \$250 payable in 30 days; the second for \$200 payable in 60 days; the third for \$300 payable in 90 days. What was the average term of credit, and what the equated time of payment?

## EQUATION OF PAYMENTS.

## OPERATION.

$$\$250 \times 30 = 7500$$

$$200 \times 60 = 12000$$

$$300 \times 90 = 27000$$

$$\underline{\$7500} \quad \underline{46500}$$

$$46500 \div 750 = 62 \text{ da. average credit.}$$

March 1 + 62 da. = May 3, Ans.

the interest of \$1 for 7500 da. + 12000 da. + 27000 da. = 46500 days. Now, if \$1 require 46500 days to gain a certain sum, \$250 + \$200 + \$300 = \$750 will require  $\frac{1}{750}$  of 46500 days; 46500 da.  $\div$  750 = 62 days, the average term of credit; and, March 1, the date at which the credits begin, + 62 da. = May 3, the equated time of payment.

ANALYSIS.—The interest of \$250 for 30 days is the same as the interest of \$1 for 7500 days; and of \$200 for 60 days, the same as of \$1 for 12000 days; and of \$300 for 90 days, the same as of \$1 for 27000 days. Hence, the interest of all the sums to the time of payment is the same as

\$250 + \$200 + \$300 = \$750 will require 46500 days. Now, if \$1 require 46500 days to gain a certain sum, \$250 + \$200 + \$300 = \$750 will require  $\frac{1}{750}$  of 46500 days; 46500 da.  $\div$  750 = 62 days, the average term of credit; and, March 1, the date at which the credits begin, + 62 da. = May 3, the equated time of payment.

Ex. 2. Bought of D. I. Lyons several bills of goods, at different times, and on various terms of credit, as by the following statement. What is the equated time for the payment of the whole?

|           |                            |              |
|-----------|----------------------------|--------------|
| Jan. 1,   | a bill amounting to \$300, | on 4 months. |
| Feb. 7,   | " " " " 185,               | on 5 months. |
| March 15, | " " " " 280,               | on 4 months. |
| April 20, | " " " " 210,               | on 6 months. |

## OPERATION.

|     |          |              |              |                    |
|-----|----------|--------------|--------------|--------------------|
| Due | May 1,   | \$300        |              |                    |
| "   | July 7,  | 185          | $\times$ 67  | = 12395            |
| "   | July 15, | 280          | $\times$ 75  | = 21000            |
| "   | Oct. 20, | 210          | $\times$ 172 | = 36120            |
|     |          | <u>\$975</u> |              | <u>69515</u> days. |

$$69515 \div 975 = 71 \frac{55}{105} \text{ days.}$$

May 1 + 71 days = July 11, Ans.

ANALYSIS.—We first find the time when each of the bills will become due. Then, since it will shorten the operation and not change the result, we take the first time when any bill becomes due, instead of its date, or the point from which to compute the average time. Now, since May 1 is the period from which the average time is computed, no time will be reckoned on the first bill, but the time for the payment of the second bill extends 67 days beyond May 1, and we multiply its amount by 67. Proceeding in the same manner with the remaining bills, we find the average term of credit to be 71 days, and July 11, the equated time of payment.

**430. RULE.**—Multiply each payment by its own time of credit, and divide the sum of the products by the sum of the payments.

NOTE.—If the date of the average time of payment is required, as in Ex. 2, find the time when each of the sums becomes due. Multiply each sum by the number of days intervening between the date of its becoming due and the earliest date on which any sum becomes due. Then proceed as in the rule, and the quotient will be the average time required, in days forward from the date of the earliest sum becoming due.

1.  
pay  
what  
2.  
and th  
3.  
6 mo.,  
as to r  
4. I  
follow  
but on

5. C  
first fo  
60 day  
time of  
6. A  
of merc  
and the  
single p

431  
debt, w

Ex.  
4 mont  
the exp

\$18  
200  $\div$  1

main anp

432  
before i  
balance  
time.

1. A  
in 12 mo  
the equa

## EXAMPLES FOR PRACTICE.

1. A merchant purchased £4750 worth of cloth, and agreed to pay  $\frac{1}{3}$  of the sum every month until the cancellation of his debt; what will be the amount of each payment? *Ans.* £950.

2. A man owes \$15960 payable as follows:  $\frac{1}{2}$  in cash,  $\frac{1}{3}$  in 6 mo., and the remainder in 1 year; required the amount of each payment?

3. The sum of \$1710 is to be paid in two installments, viz.:  $\frac{1}{2}$  in 6 mo., and the other  $\frac{1}{2}$  in 10 mo. At what time should it be paid so as to make but one payment? *Ans.* In 8 months.

4. Bought 25 casks of wine for \$1125 which I agreed to pay as follows: \$525 in 6 mo., and the balance in 9 mo. Wishing to make but one payment, how long should this payment be deferred? *Ans.* 7mo. 18da.

5. On the 1st. of January, 1868, a merchant gave three notes: the first for \$500 payable in 30 days; the second for \$400 payable in 60 days; the third for \$600 payable in 90 days. Required the equated time of payment? *Ans.* March 3, 1868.

6. A merchant bought, on the 15th. of May, 1868, \$8000 worth of merchandise and agreed to pay  $\frac{1}{3}$  of the price in 6 mo.,  $\frac{1}{3}$  in 8 mo., and the remainder in 10 mo. But wishing to cancel his debt by a single payment, how long should this payment be deferred? *Ans.* 8mo. 24da.

**431. CASE II.**—To find the time of paying the balance of a debt, when partial payments have been made before the debt is due.

*Ex.* Bought \$180 worth of goods, at 8 months' credit. At the end of 4 months, I paid \$30, and 2 months later, \$40; when, in equity, after the expiration of 8 months, shall I owe the balance?

## OPERATION.

$$30 \times 4 = 120$$

$$40 \times 2 = 80$$

$$\frac{70}{200}$$

$$\$180 - \$70 = \$110;$$

$$200 \div 110 = \text{Info. } 25\text{da.}, \text{ Ans.}$$

**ANALYSIS.**—The interest on the \$30 for 4 months is equal to the interest of \$1 for 120 months, and the int. of \$40 for 2 months is equal to that of \$1 for 80 months; and thus the int. on both partial payments, at the expiration of the 8 months, is equal to the int. of \$1 for 120 + 80 = 200 months. To equal this credit of int., the bal. of the debt, which we find to be \$110, should remain unpaid, after the 8 months,  $\frac{110}{200}$  of 200 months, or 1mo. 25da.

**432. RULE.**—Multiply each payment by the time it was made before it becomes due, and divide the sum of the products by the balance remaining unpaid; and the quotient will be the required time.

## EXAMPLES FOR PRACTICE.

1. A vintner agreed to pay \$1895.20 for 2369 gal. of cognac brandy in 12 mo.; but at the end of 10 mo. he paid for 633 gal. Required the equated time of the balance? *Ans.* 12 mo. 22 da.

2. Bought of C. Lyons, at 6 mo. £432 worth of goods; at the end of 1 mo. I paid him £75, and 4 mo. after £200 more. How long after the expiration of the 6 mo. should I pay the balance?

Ans. 3 mo. 20 da.

3. A grocer bought \$2829.69 worth of coffee which he desires to pay in three different payments: the first is to the second as 4 is to 5, and the third is equal to half the second. The first payment should be made in 4 mo.; the second in 7 mo.; and the third in 1 year. But at the end of 6 mo. he paid \$975, how long can he keep the balance?

Ans. 7 mo. 18 da.

4. An undertaker built a house for £6035 payable in 15 mo.; but being in want of some money, the proprietor pays him £2847 10 eight months before the time. How long, in equity, can the proprietor keep the balance to compensate the advance he made the undertaker?

Ans. 22 mo. 4 da.

5. Andrew having sold \$8400 worth of linen, at 12 mo. credit, received the  $\frac{1}{2}$  of the price only 15 mo. after. When did he receive the  $\frac{3}{4}$ ?

Ans. In 10 mo. 15 da.

6. I owed \$600 at 13 months; I paid  $\frac{2}{3}$  of this sum before it was due, so that I can keep the remainder 2 years without injuring my creditor. Required the time when the  $\frac{2}{3}$  were paid? A. 7 mo. 15 da.

7. A trader owes \$3000 payable in 6 mo.; \$4500 payable in 8 mo., and \$9500 payable in 10 mo. At the end of 5 mo. he pays \$12000. How long can he keep the balance? Ans. 17 mo. 24 da.

### ALLIGATION.

**433.** Alligation treats of mixing or compounding articles or ingredients of different qualities or values. It is of two kinds—*Alligation Medial*, and *Alligation Alternate*.

#### ALLIGATION MEDIAL.

**434.** *Alligation Medial* is the process of finding the mean or average rate of a mixture composed of articles of different qualities or values, the quantity and rate of each being given.

**435.** To find the average value of several articles mixed, the quantity and rate of each being given.

*Ex.* A grocer mixed 2cwt. of sugar worth \$9 per cwt. with 1cwt. worth \$7 per cwt. and 2cwt. worth \$10 per cwt.; what is 1cwt. of the mixture worth?

OPERATION.

$$\begin{array}{r} \$ 9 \times 2 = \$ 18 \\ 7 \times 1 = 7 \\ 10 \times 2 = 20 \\ \hline 5) \quad \$ 45 \\ \hline \end{array}$$

\$9, Ans.

ANALYSIS.—Since 2cwt. at \$9 per cwt. is worth \$18, 1cwt. at \$7 per cwt. is worth \$7, and 2cwt. at \$10 per cwt. is worth \$20; 2cwt. + 1cwt. + 2cwt. = 5cwt. is worth \$18 + \$7 + \$20 = \$45; and 1cwt. is worth as many dollars as 45 contains times 5, or \$9.

43  
divide  
the art

1. A  
bu. of  
what is  
2. If  
60 cts.  
value o  
3. A  
wine at  
gallon o  
4. A  
10½ cts.  
doz. I  
he rece  
5. A  
fine wit  
alloy; v

437  
proport  
whose p  
rate or

438  
ingredi  
*Ex.*  
and clov  
worth \$

5 {

he a loss,  
quantities  
one bushel  
\$1 would  
bushel of  
require  
In ever  
per bushel  
seed, we t  
and we st

**436. RULE.**—Find the value of each of the articles, and divide the sum of their values by the number denoting the sum of the articles. The quotient will be the average value of the mixture.

EXAMPLES FOR PRACTICE.

1. A farmer mixes together 10 bush. of oats at 40 cts. per bu., 15 bu. of corn at 50 cts. per bu., and 25 bu. of rye at 70 cts. per bu.; what is the value of a bushel of the mixture? *Ans.* 58 cts.

2. If I mix 20 pounds of tea at 70 cts. per pound with 15 pounds at 60 cts. per pound, and 80 pounds at 40 cts. per pound; what is the value of 1 lb. of this mixture? *Ans.* 47 $\frac{1}{3}$  cts.

3. A dealer in liquors would mix 14 gal. of water with 12 gal. of wine at \$.75, 24 gal. at \$.90, and 16 gal. at \$1.10; how much is a gallon of the mixture worth? *Ans.* \$.73 $\frac{1}{3}$ .

4. A man bought 3 $\frac{1}{2}$  dozen of eggs at 12 cts. a dozen, 4 dozen at 10 $\frac{1}{2}$  cts. a dozen, 4 $\frac{1}{2}$  dozen at 11 cts. a doz., and 5 $\frac{1}{2}$  doz. at 10 cts. a doz. He sells them so as to make 50% on the cost; how much did he receive per dozen? *Ans.* 16 $\frac{1}{2}$  cts.

5. A goldsmith wishes to compound 3 lb. 6 oz. of gold 23 carats fine with 4 lb. 8 oz. 21 carats, 3 lb. 9 oz. 20 carats, and 2 lb. 2 oz. of alloy; what will be the fineness of the composition? *Ans.* 18 carats.

ALLIGATION ALTERNATE.

**437. Alligation Alternate** is the process of finding the proportional quantities to be taken of several articles or ingredients, whose prices or qualities are known to form a mixture of any given rate or quality.

**438.** To find the proportional quantity to be used of each ingredient, when the mean price or quality of the mixture is given.

*Ex.* 1. What relative quantities of timothy seed worth \$2 a bushel, and clover seed worth \$7 a bushel, must be used to form a mixture worth \$5 a bushel?

OPERATION.

$$5 \left\{ \begin{array}{l|l|l} 2 & \frac{1}{3} & 2 \\ 7 & \frac{1}{3} & 3 \end{array} \right\} \text{Ans.}$$

ANALYSIS.—Since on every ingredient used whose price or quality is less than the mean rate there will be a gain, and on every ingredient whose price or quality is greater than the mean rate there will

be a loss, and since the gains and losses must be exactly equal, the relative quantities used of each should be such as represent the unit of value. By selling one bushel of timothy seed worth \$2, for \$5, there is a gain of \$3; and to gain \$1 would require  $\frac{1}{3}$  of a bushel, which we place opposite the 2. By selling one bushel of clover seed worth \$7, for \$5, there is a loss of \$2; and to lose \$1 would require  $\frac{1}{2}$  of a bushel, which we place opposite the 7.

In every case, to find the unit of value we must divide \$1 by the gain or loss per bushel or pound, &c. Hence, if, every time we take  $\frac{1}{3}$  of a bushel of timothy seed, we take  $\frac{1}{2}$  of a bushel of clover seed, the gain and loss will be exactly equal, and we shall have  $\frac{1}{3}$  and  $\frac{1}{2}$  for the proportional quantities.

ALLIGATION.

*Ex. 2.* What proportions of coffees worth respectively 3, 4, 7 and 10 shillings a pound, must be taken to form a mixture worth 6 shillings a pound?

OPERATION.

|   |               |               |               |               |   |   |   |   |   |
|---|---------------|---------------|---------------|---------------|---|---|---|---|---|
| 6 | 3             | 4             | 7             | 10            | 1 | 2 | 3 | 4 | 5 |
|   | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ | 4 | 1 | 2 | 3 | 4 |

**ANALYSIS.**—To preserve the equality of gains and losses, we must always compare two prices or simples, one *greater* and one *less* than the mean rate, and treat each pair or couplet as a separate example. In the given example we form two couplets, and may compare either 3 and 10, 4 and 7, or 3 and 7, 4 and 10.

We find that  $\frac{1}{2}$  of a lb. at 3s. must be taken to gain 1 shilling, and  $\frac{1}{3}$  of a lb. at 4s. to gain 1 shilling, and  $\frac{1}{4}$  of a lb. at 7s. to lose 1 shilling. These proportional numbers, obtained by comparing the two couplets, are placed in columns 1 and 2. If, now, we reduce the numbers in columns 1 and 2 to a common denominator, and use their numerators, we obtain the integral numbers in columns 3 and 4, which, being arranged in column 5, give the proportional quantities to be taken of each.

It will be seen that in comparing the simples of any pair or couplet, one of which is greater, and the other less than the mean rate, the proportional number finally obtained for either term is the difference between the mean rate and the other term. Thus, in comparing 3 and 10, the proportional number of the former is 4, which is the difference between 10 and the mean rate 6; and the proportional number of the latter is 3, which is the difference between 3 and the mean rate. The same is true of every other couplet. Hence, when the simples and the mean rate are integers, the intermediate steps taken to obtain the final proportional numbers as in columns 1, 2, 3, and 4, may be omitted, and the same results readily found by taking the difference between each simple and the mean rate, and placing it opposite the one with which it is compared.

From the foregoing examples and analyses we derive the following

**439. RULE.**—I. Write the several prices or qualities in a column and the mean price or quality of the mixture at the left.

II. Form couplets by comparing any price or quality less, with one that is greater than the mean rate, placing the part which must be used to gain 1 of the mean rate opposite the less simple, and the part that must be used to lose 1 opposite the greater simple, and do the same for each simple in every couplet.

III. If the proportional numbers are fractional, they may be reduced to integers, and if two or more stand in the same horizontal line, they must be added; the final results will be the proportional quantities required.

**NOTES.** 1. If the numbers in any couplet or column have a common factor, it may be rejected.

2. We may also multiply the numbers in any couplet or column by any multiplier we choose, without affecting the equality of the gains and losses, and thus obtain an indefinite number of results, any one of which being taken will give a correct final result.

1. A pound; 12 cts. p  
2. W a gallon lon ?

3. A number head ? and 3 of  
4. W cent. str  
Ans. of the 5t

**440**  
ingredient

*Ex. A*  
barley w  
60 cents  
how man

{ 30  
60 { 45  
84

40 bushels the equali each of th bustels of

**441.**  
Divide t same in quantities

1. A m how many 75 cents, Ans. 2 fourth.

2. A fa buy at \$3 price of \$4

EXAMPLES FOR PRACTICE.

1. A grocer has sugars worth 10 cents, 11 cents, and 14 cents per pound; in what proportions may he mix them to form a mixture worth 12 cts. per lb. ? *Ans.* 1 lb. at 10 cts., and 2 lb. at 11- and 14 cts.

2. What proportions of water at no value, and wine worth \$1.20 a gallon, must be used to form a mixture worth 90 cents a gallon ? *Ans.* 1 gal. of water to 3 gal. of wine.

3. A farmer had sheep worth \$2, \$2½, \$3, and \$4 per head; what number could he sell of each, and realize an average price of \$2½ per head ? *Ans.* 5 of the 1st. kind, and 1 each of the 2nd. and 3rd., and 3 of the 4th. kind.

4. What relative quantities of alcohol 80, 84, 87, 94, and 96 per cent. strong must be used to form a mixture 90 per cent. strong ?

*Ans.* 6 of the first two kinds, four of the 3rd., 3 of the 4th. and 16 of the 5th.

**440.** To find the proportional quantity to be used of each ingredient, when the quantity of one of the simples is limited.

*Ex.* A miller has oats worth 30 cents, corn worth 45 cents, and barley worth 84 cents per bushel; he desires to form a mixture worth 60 cents per bushel, and which shall contain 40 bushels of corn; how many bushels of oats and barley must he take ?

OPERATION.

$$60 \left\{ \begin{array}{l} 30 \\ 45 \\ 84 \end{array} \right. \left\{ \begin{array}{l} \frac{1}{3} \\ \frac{1}{5} \\ \frac{1}{84} \end{array} \right. \left\| \begin{array}{l} 4 \\ 8 \\ 5 \end{array} \right. \left\| \begin{array}{l} 4 \\ 8 \\ 10 \end{array} \right. \left\{ \begin{array}{l} 20 \\ 40 \\ 50 \end{array} \right\} \text{ Ans.}$$

**ANALYSIS.** By the same process as in Case I we find the proportional quantities of each to be 4 bushels of oats, 8 of corn, and 10 of barley. But we wish to use

40 bushels of corn, which is 5 times the proportional number 8, and to preserve the equality of gain and loss we must take 5 times the proportional quantity of each of the other simples, or  $5 \times 4 = 20$  bushels of oats, and  $5 \times 10 = 50$  bushels of barley. Hence the following

**441. RULE.**—Find the proportional quantities as in 438. Divide the given quantity by the proportional quantity of the same ingredient, and multiply each of the other proportional quantities by the quotient thus obtained.

EXAMPLES FOR PRACTICE.

1. A merchant has teas worth 40, 60, 75, and 90 cents per pound; how many pounds of each must he use with 20 pounds of that worth 75 cents, to form a mixture at 80 cents ?

*Ans.* 20 lbs. each of the first three kinds, and 130 lbs. of the fourth.

2. A farmer bought 24 sheep at \$2 a head; how many must he buy at \$3 and \$5 a head, that he may sell the whole at an average price of \$4 a head, without loss ? *Ans.* 24 at \$3, and 72 at \$5.

3. How much alcohol worth 60 cents a gallon, and how much water, must be mixed with 180 gallons of rum worth \$1.40 a gallon, that the mixture may be worth 90 cents a gallon?

*Ans.* 60 gallons each of alcohol and water.

5. How many acres of land worth 35 dollars an acre must be added to a farm of 75 acres, worth \$50 an acre, that the average value may be \$40 an acre?

*Ans.* 150 acres.

6. A merchant mixed 80 pounds of sugar worth 6½ cents per pound with some worth 8½ cents and 10 cents per pound, so that the mixture was worth 7½ cents per pound; how much of each kind did he use?

**442.** To find the proportional quantity to be used of each ingredient, when the quantity of the whole compound is limited.

*Ex.* A grocer has sugars worth 6 cents, 7 cents, 12 cents, and 13 cents per pound. He wishes to make a mixture of 120 pounds worth 10 cents a pound; how many pounds of each kind must he use?

OPERATION.

|    |   |    |  |   |  |   |  |   |  |   |  |    |
|----|---|----|--|---|--|---|--|---|--|---|--|----|
| 10 | { | 6  |  | ¼ |  | 3 |  | 2 |  | 3 |  | 30 |
|    |   | 7  |  | ½ |  | 3 |  | 2 |  | 2 |  | 20 |
|    |   | 12 |  | 1 |  | 4 |  | 3 |  | 3 |  | 30 |
|    |   | 13 |  | 1 |  | 4 |  | 4 |  | 4 |  | 40 |
|    |   |    |  |   |  |   |  |   |  |   |  |    |

**ANALYSIS.** By Case 1 we find the proportional quantities of each to be 3 lbs. at 6 cts., 2 lbs. at 7 cts., 3 lbs. at 12 cts., and 4 lbs. at 13 cts. By adding the proportional quantities, we find that the mixture would be but 12 lbs. while the required mixture is 120, or 10 times 12. If the whole mixture is

to be 10 times as much as the sum of the proportional quantities, then the quantity of each simple used must be 10 times as much as its respective proportional, which would require 30 lbs. at 6 cts., 20 lbs. at 7 cts., 30 lbs. at 12 cts., and 40 lbs. at 13 cts. Hence we deduce the following

**443. RULE.**—Find the proportional numbers as in 438. Divide the given quantity by the sum of the proportional quantities, and multiply each of the proportional quantities by the quotient thus obtained.

EXAMPLES FOR PRACTICE.

1. A farmer sold 170 sheep at an average price of 14 shillings a head; for some he received 9s., for some 12s., for some 18s., and for others 20s.; how many of each did he sell?

*Ans.* 60 at 9s., 40 at 12s., 20 at 18s., and 50 at 20s.

2. A jeweler melted together gold 16, 18, 21, and 24 carats fine, so as to make a compound of 51 ounces 22 carats fine; how much of each sort did he take?

*Ans.* 6 ounces each of the first three, and 33 ounces of the last.

3. A man bought 210 bushels of oats, corn, and wheat, and paid for the whole \$178.50; for the oats he paid \$¾, for the corn, \$1, and for the wheat \$1½ per bushel; how many bushels of each kind did he buy?

*Ans.* 78 bu. each of oats and corn, and 54 bu. of wheat.

4.  
of co  
and  
that  
5.  
recei  
man,  
boys

44  
given  
44  
num  
44  
place  
many

Thus,

Henc

44  
times,  
power.

Norm  
fraction

1. Squ
2. Squ
3. Cub
4. Cnb
5. 24<sup>4</sup>
6. (1.2)

448  
number  
449  
450

how much  
40 a gallon,

and water.  
ere must be  
the average  
50 acres.  
s per pound  
at the mixt-  
kind did he

4. A, B, and C are under a joint contract to furnish 6000 bushels of corn, at 48 cts. a bushel; A's corn is worth 45 cts., B's 51 cts., and C's 54 cts.; how many bushels must each put into the mixture that the contract may be fulfilled?

5. One man and 3 boys received \$84 for 56 days' labor; the man received \$3 per day, and the boys \$ $\frac{1}{2}$ , \$ $\frac{2}{3}$ , and \$ $\frac{1}{4}$  respectively; how many days did each labor?  
*Ans.* The man 16 days, and the boys 24, 4, and 12 days respectively.

## INVOLUTION.

**444.** **Involution** is the process of raising a number to a given power.

**445.** A **Power** is the product arising from multiplying a number by itself, or repeating it several times as a factor.

**446.** The **Index** or **Exponent** of a power is a small figure placed at the right and a little above the number, to show how many times it is used to produce the power:—

$$\text{Thus, } \left\{ \begin{array}{l} 3^1 = \text{the first power of 3, or the root.} \\ 3^2 = 3 \times 3 = 9, \text{ the second power, or square of 3.} \\ 3^3 = 3 \times 3 \times 3 = 27, \text{ the third power, or cube of 3.} \\ 3^4 = 3 \times 3 \times 3 \times 3 = 81, \text{ the fourth power of 3.} \\ (3)^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243, \text{ the fifth power of 3.} \end{array} \right.$$

Hence, from these several powers of 3, we derive the following

**447. RULE.**—Multiply the given number by itself as many times, less 1, as there are units in the exponent of the required power.

**NOTE.**—A mixed number may be either reduced to an improper fraction, or the fractional part reduced to a decimal, before involution.

## EXAMPLES FOR PRACTICE.

|                  |                      |                            |                               |
|------------------|----------------------|----------------------------|-------------------------------|
| 1. Square 25.    | <i>Ans.</i> 225.     | 7. $(1.06)^4 = ?$          | <i>Ans.</i> 1.262476.         |
| 2. Square 79.    | <i>Ans.</i> 6241.    | 8. $(\frac{3}{4})^3 = ?$   | <i>Ans.</i> $\frac{27}{64}$ . |
| 3. Cube 47.      | <i>Ans.</i> 103823.  | 9. $(\frac{1}{2})^3 = ?$   | <i>Ans.</i> $\frac{1}{8}$ .   |
| 4. Cube 39.      | <i>Ans.</i> 59319.   | 10. $(2\frac{1}{2})^4 = ?$ | <i>Ans.</i> 5044.             |
| 5. $24^4 = ?$    | <i>Ans.</i> 331776.  | 11. $(1\frac{1}{2})^6 = ?$ | <i>Ans.</i> 16125.            |
| 6. $(1.2)^5 = ?$ | <i>Ans.</i> 2.48832. | 12. $(2\frac{1}{2})^5 = ?$ | <i>Ans.</i> 157.243.          |

## EVOLUTION.

**448.** **Evolution** is the process of extracting the root of a number considered as a power; it is the reverse of Involution.

**449.** The **Root** of a number is one of its equal factors.

**450.** The **First Root** of a number is the number itself.

**451.** The **Second Root**, or **Square Root**, of a number, is one of its two equal factors. Thus, 4 is the square root of  $16 = 4 \times 4$ .

**452.** The **Third Root**, or **Cube Root**, of a number, is one of its three equal factors. Thus, 4 is the cube root of  $64 = 4 \times 4 \times 4$ .

**453.** The **Radical Sign** is the character,  $\sqrt{\quad}$ , which, placed before a number, indicates that its root is to be extracted.

**454.** The **Index** of the root is the figure placed above the radical sign, to denote what root is to be taken. When no index is written, the index, 2, is always understood.

**455.** The names of roots are derived from the corresponding powers, and are denoted by the indices of the radical sign. Thus,  $\sqrt{36}$  denotes the *square root* of 36;  $\sqrt[3]{36}$  denotes the *cube root* of 36;  $\sqrt[4]{36}$  denotes the *fourth root* of 36; etc.

**456.** A **Rational Root** is a root which can be exactly obtained.

**457.** A **Surd** is one which cannot be exactly obtained.

## SQUARE ROOT.

The roots of the first ten integers and their squares are :

1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

**NOTES.**—1. It will be observed that the second power or square of each of the numbers contains twice as many figures as the root, or twice as many wanting one. Hence, to ascertain the number of figures in the square root of a given number, — *Beginning at the right, point it off into as many periods as possible, of two figures each; and there will be as many figures in the root as there are periods.*

2. When the given number contains an *odd* number of figures, the period at the left can contain but one figure.

*Ex.* What is the square root of 4096 ?

**OPERATION.**

$$\begin{array}{r|l} 4096 & 64, \text{ Ans.} \\ 36 & \\ \hline 124 & 496 \\ & 496 \\ \hline & 0 \end{array}$$

**ANALYSIS.**—Beginning at the right, we separate the number into periods of two figures each, by placing a point (.) over the right-hand figure of each period. Now, the greatest square of 40, the left-hand period, is 36, the root of which is 6. Placing the 6 on the right of the number, we subtract its square from the period 40, and to the right of the remainder bring down the next period. We then double the 6, the part of the root already found, and, placing it on the left of the dividend for a partial divisor, we perceive

it is contained in the dividend, (omitting its right-hand figure), 4 times. Placing the 4 on the right of the root, also on the right of the partial divisor, we multiply the divisor thus completed by 4, and subtract the product from the dividend. The root or answer is 64.

4  
figu  
I  
writ  
I  
peri  
I  
and  
of t  
of t  
V  
last  
V  
iden  
cont

No  
cont  
the d  
used  
2.  
period  
be de  
3. I  
root is  
off the  
a poin  
last p  
4.  
suar  
are pe  
5. I  
root;  
fractio

1.

3 ×  
3 ×  
66 ×  
36 ×  
725 ×

2. V  
of 324

**458. RULE.—I.** Point off the given number into periods of two figures each, counting from units' place toward the left and right.

**II.** Find the greatest perfect square in the left-hand period and write its root on the right for the first figure in the root.

**III.** Subtract the square of the root figure from the left-hand period, and to the remainder annex the next period for a dividend.

**IV.** Double the part of the root already found for a trial divisor, and see how many times it is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next divisor of the root, and also at the right of the trial divisor.

**V.** Multiply the divisor thus formed by the figure of the root last found, and subtract the product from the dividend.

**VI.** To this remainder annex the next period for a new dividend, and divide the same by twice the root already found, and continue in this manner until all the periods are used.

**NOTES.—1.** When any dividend, exclusive of its right-hand figure, does not contain the divisor, a cipher must be placed in the root, and also at the right of the divisor; then, after bringing down the next period, this last divisor must be used as the divisor of the new dividend.

**2.** When there is a remainder after all the periods have been brought down, periods of ciphers may be annexed, and the figures of the root thus obtained will be decimals.

**3.** If the given number is a decimal, or a whole number and a decimal, the root is extracted in the same manner as in whole numbers, except, in pointing off the decimals, either alone or in connection with the whole number, we place a point over every second figure toward the right, from the separatrix, filling the last period, if incomplete, with a cipher.

**4.** The square root of a common fraction may be obtained by extracting the square roots of the numerator and denominator separately, provided the terms are perfect squares; otherwise, the fraction may be reduced to a decimal.

**5.** Mixed numbers may be reduced to the decimal form before extracting the root; or, if the denominator of the fraction is a perfect square, to an improper fraction.

EXAMPLES FOR PRACTICE.

1. What is the square root of 133225? of 62.8?

| OPERATION. |                   | OPERATION.    |                 |
|------------|-------------------|---------------|-----------------|
|            | 133225 (365, Ans. |               | 62.80(7.92+, A. |
| 3 × 3 =    | 9                 | 7 × 7 =       | 49              |
| 3 × 2 =    | 6 6 ) 432         | 7 × 2 =       | 14.9 ) 13.80    |
| 66 × 6 =   | 396               | 14.9 × .9 =   | 13.41           |
| 36 × 2 =   | 72 5 ) 3625       | 7.9 × .2 =    | 15.82 ) .3900   |
| 725 × 5 =  | 3625              | 15.82 × .02 = | .3164           |
|            | 0                 |               | .0736           |

2. What is the square root of 169? of 576? of 1225? of 2401? of 3249? of 4096? of 5329? of 6724? of 9801? of 10816?

Ans. 13, 24, 35, 49, 57, 64, etc.

3. What is the square root of 61009 ? of 454276 ? of 505521 ? of 637821 ? of 648132 ? of 738417 ? of 809215 ? of 927748 ? of 977137 ? of 999999 ? *Ans.* 247, 674, 711, 798, 805, 859, 899, etc.
4. What is the square root of 234.09 ? of 5.4756 ? of 17.3056 ? of 256.6404 ? of 0.1024 ? of 0.120409 ? of 0.00008836 ? of 609151.76100 ? *Ans.* 15.3, 2.34, 4.16, 16.02, 0.32, 0.347, 0.0094, 780.481.
5. What is the square root of  $\frac{3}{4}$  ? of  $4\frac{27}{4}$  ? of  $\frac{49}{25}$  ? of  $60\frac{1}{16}$  ? of  $1\frac{3}{4}$  ? of  $\frac{3}{8}$  ? of  $28\frac{3}{4}$  ? of  $\frac{4}{9}$  ? of  $19\frac{3}{8}$  ? of  $95\frac{1}{16}$  ? *Ans.* 0.86602 +,  $2\frac{7}{12}$ ,  $\frac{7}{3}$ ,  $7\frac{3}{8}$ ,  $\frac{7}{2}$ , 0.7745 +,  $5\frac{3}{8}$ , 0.858 +,  $1\frac{3}{8}$ ,  $9\frac{3}{4}$ .

## APPLICATIONS OF THE SQUARE ROOT.

1. What is the length of one side of a square farm containing 90 acres ? *Ans.* 120 rods.
2. A certain general has an army of 141376 men ; how many must he place in rank and file to form them into a square ? *Ans.* 376.
3. A company of persons spent \$75.69 ; each spending as many cents as there were persons in the company. How much did each expend ? *Ans.* \$0.87.
4. Bought 200 yards of carpeting  $1\frac{1}{4}$  yards wide ; what is the length of one side of the square room which this carpet will cover ? *A.* 45ft.
5. A man owns three pieces of land ; the first is 125 rods long, and 53 wide ; the second is  $62\frac{1}{4}$  rods long, and 34 wide ; and the third contains 37 acres : what will be the length of the side of a square field whose area will be equal to the three pieces ? *Ans.* 121.11 + rods.
6. Purchased 2 house-lots ; the first is 242 feet square, and the second contains 9 times the area of the first ; how many feet square in the second ? *Ans.* 726 feet.
7. Required the sides of a rectangular court-yard having an area of 432 rods, and whose breadth is only the  $\frac{3}{4}$  of the length ?
8. A certain field contains 48020 square rods ; the length exceeds the breadth by 49 rods : what are the sides ? *Ans.* 245 rods long ; 196 rods wide.
9. A school-master says that the number of his pupils multiplied by  $\frac{1}{2}$  of itself is 2523 ; how many pupils has he ? *Ans.* 87.
10. How much will it cost to roughcast the walls of a garden, having a surface of 8100 yards, at  $87\frac{1}{2}$  cts. per yard, the walls being  $2\frac{3}{10}$  yd. high ? *Ans.* \$1449.
11. The greater of two numbers is 40, and the sum of their squares 1625 ; what is the smaller number ? *Ans.* 5.
12. A clock-maker sold three watches whose prices are as 5 is to 6, and as 6 is to 9 ; the sum of the squares of the prices is \$3550. What is the price of each watch ? *Ans.* \$25, \$30, \$45.
13. What is the price of a raking machine, knowing that the price added to its square gives \$186 for result ? *Ans.* \$13.13 $\frac{1}{2}$ .
14. In dividing the square of the number of dollars that I have by  $\frac{1}{4}$  of the number itself I obtain \$96 for result. How many barrels of codfish, at \$4 per barrel, can I buy with the money I possess ? *Ans.* 6 barrels.

N  
bers  
wan  
cube  
peri  
root

E

tria

plus  
The o  
the n  
sands  
125 th  
32 the  
32404  
the te  
three  
the sq  
three  
somev  
the re  
units,  
units,  
hunder  
der, d  
the sq  
units i  
comple  
we fin  
one le  
root, w  
root in  
divisor  
from th  
54 lbs

## CUBE ROOT.

The roots of the first ten integers and their cubes are:—

|    |    |     |     |      |      |      |      |      |       |
|----|----|-----|-----|------|------|------|------|------|-------|
| 1, | 2, | 3,  | 4,  | 5,   | 6,   | 7,   | 8,   | 9,   | 10.   |
| 1, | 8, | 27, | 64, | 125, | 216, | 343, | 512, | 729, | 1000. |

**NOTE.**—It will be observed that the cube or third power of each of the numbers contains three times as many figures as the root, or three times as many wanting one, or two at most. Hence, to determine the number of figures in the cube root of a given number,—beginning at the right, point it off into as many periods as possible of three figures each, and there will be as many figures in the root as there are periods.

*Ex.* What is the cube root of 157464 ?

OPERATION.

|               |                              |             |
|---------------|------------------------------|-------------|
|               |                              | 157464 ( 54 |
|               |                              | 125         |
| trial div.,   | $3 \times 50^2 = 7500$       | 32464       |
|               | $3 \times 50 \times 4 = 600$ | 8116        |
|               | $4^2 = 16$                   | 8116        |
| True divisor, | $8116 \times 4 =$            | 32464       |
|               |                              | 0           |

PROOF.

$$54 \times 54 \times 54 = 157464.$$

**ANALYSIS.**—Beginning at the right, we separate the given number into periods, by placing a point over the units' figure, then over thousands. Since the number of periods is two, the root will consist of two figures, tens and units. Then 157464 = the cube of tens, plus three times the square of the tens into the units,

plus three times the tens into the square of the units, plus the cube of the units. The cube of tens is thousands, and must therefore be found in the thousands of the number. The greatest number of tens whose cube does not exceed 157 thousands is 5, which we write as the tens figure of the root. We then subtract the 125 thousands, the cube of the 5 tens, from the 157 thousands, and there remain 32 thousands; and, annexing the next period, we have as the entire remainder, 32464, equal three times the square of the tens into the units, plus three times the tens into the square of the units, plus the cube of the units, or the product of three times the square of the tens, plus three times the tens into the units, plus the square of the units, multiplied by the units. By dividing this remainder by three times the square of the tens of the root, we obtain the units, or a number somewhat too large. Although it may be too large, it cannot be too small, since the remainder 32464 contains not only three times the square of the tens into the units, but three times the tens into the square of the units, plus the cube of the units. We therefore make three times the square of the tens of the root, = 75 hundreds, a trial divisor, with which we divide the 324 hundreds of the remainder, disregarding the 64 units, since they cannot form any part of the product of the square of the tens by the units. The quotient figure obtained, 5, must be the units figure of the root, or a number somewhat larger. But on undertaking to complete the divisor on the supposition that 5 is the true units figure of the root, we find a divisor too large for the remainder. We therefore take 4, a number one less, and to determine whether it expresses the real number of units in the root, we add to the 75 hundreds of the trial divisor three times the 3 tens of the root into the 4 units, plus the square of the 4 units; and multiplying the true divisor, 8116, thus formed, by the units, and subtracting the product, 32464, from the remainder, there is nothing left. Hence, 157464 is a perfect cube, and 54 its cube root.

**459. RULE.—I.** Point off the given number into periods of three figures each, counting from units place toward the left and right.

**II.** Find the greatest cube that does not exceed the left-hand period, and write its root for the first figure in the required root; subtract the cube from the left-hand period, and to the remainder bring down the next period for a dividend.

**III.** At the left of the dividend write three times the square of the first figure of the root, and annex two ciphers, for a trial divisor; divide the dividend by the trial divisor, and write the quotient for a trial figure in the root.

**IV.** Add to the trial divisor three times the product of the tens figure of the root by the units figure with a cipher annexed, and the square of the last figure, for a true divisor.

**V.** Multiply the complete divisor by the trial figure; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.

**VI.** Multiply the square of the root figures already found, by 3, and to the product annex two ciphers for a new trial divisor; and proceed as before until all the periods are brought down.

**NOTE.**—The observations made in Notes 1, 2, 3, 4, and 5, under the rule for the extraction of the square root (458), are equally applicable to the extraction of the cube root, except that two ciphers must be placed at the right of a true divisor when it is not contained in its corresponding dividend; and, in pointing off decimals, each period must contain three figures.

#### EXAMPLES FOR PRACTICE.

1. What is the cube root of 12326391 ?

OPERATION.

|                                          |                               |                         |
|------------------------------------------|-------------------------------|-------------------------|
|                                          |                               | 1 2 3 2 6 3 9 1   2 3 1 |
|                                          |                               | 8                       |
| Trial divisor, $3 \times 20^2 = 1200$    | $3 \times 20 \times 3 = 180$  | 4 3 2 6                 |
|                                          | $3^2 = 9$                     |                         |
| True divisor, $1389 \times 3 =$          |                               | 4 1 6 7                 |
| Trial divisor, $3 \times 230^2 = 158700$ | $3 \times 230 \times 1 = 690$ | 1 5 9 3 9 1             |
|                                          | $1^2 = 1$                     |                         |
| True divisor, $159391 \times 1 =$        |                               | 1 5 9 3 9 1             |

2. What is the cube root of 1331 ? of 3375 ? of 12167 ? of 32768 ? of 110592 ?  
*Ans.* 11, 15, 23, 32, etc.

3. What is the cube root of 185193 ? of 272144 ? of 456533 ? of 704969 ? of 970299 ?  
*Ans.* 57, 64, 77, 89, etc.

4. What is the cube root of 1367631? of 9938375? of 41781923? of 96071912? of 184220009? of 300763000? of 476379541? of 709732288? of 736314327? of 997002999?

*Ans.* 111, 215, 347, 458, 569, 670, 781, 892, 903, etc.

5. What is the cube root of 9.15? of 51662.1837824? of 11.03? of 0.518? of 0.12965? *Ans.* 2.091, 37.244, 2.22, 0.803, 0.2349.

6. What is the cube root of  $34\frac{1}{2}$ ? of  $\frac{1}{8}$ ? of  $39\frac{2}{3}$ ? of  $\frac{1}{27}$ ? of  $81\frac{1}{11}$ ? of  $166\frac{2}{3}$ ? of  $11\frac{2}{3}$ ? of  $\frac{1}{27}$ ?

*Ans.*  $3\frac{1}{2}$ ,  $.87+$ ,  $3\frac{2}{3}$ ,  $\frac{1}{3}$ ,  $4.334+$ ,  $5\frac{1}{2}$ ,  $2\frac{1}{2}$ ,  $\frac{1}{3}$ .

## APPLICATIONS IN CUBE ROOT.

1. A mason wishes to make a cubical cistern that shall contain 2744 cubic feet of water; what must be the length of one of its sides?

*Ans.* 14 feet.

2. A miller has a cubical box that will hold 400 bushels of wheat; what is the depth of the box?

3. What quantity of paper will be required to make a cubical box which shall contain  $\frac{3}{4}$  of a solid foot? *Ans.*  $\frac{3}{4}$  of a yd.

4. A carpenter has a plank 1 foot wide,  $22\frac{1}{2}$  feet long, and 2 $\frac{1}{2}$  inches thick; and wishes to make a box whose width shall be twice its height, and whose length shall be twice its width. Required the contents of the box.

*Ans.* 5719 cub. in.

5. How much must be paid for a certain number of pounds of li-seed, bought at 35 cents per lb., knowing that the  $\frac{2}{3}$  of the cube of the number equal 26509168?

6. A mattress-maker purchased 84 lb. of hair, for which he gave a sum such that the  $\frac{1}{3}$  of the cube of the price, diminished by  $\frac{1}{4}$  of the same cube equal \$0.6591. How much did the 84 lb. cost him?

*Ans.* \$163.80.

7. Required the value of the articles contained in 25 boxes, each containing as many articles, which cost as many cents as there are boxes?

*Ans.* \$156.25.

8. What is that number, whose  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{4}$  multiplied together, give 9 for product?

*Ans.* 6.

9. Bought \$164.64 worth of oranges packed up in a certain number of boxes, each containing three times as many oranges as there are boxes; and each orange costs twice as many cents as there are boxes. Required the number of boxes and oranges.

*Ans.* 14 boxes; 588 oranges.

10. In dividing the cube of a certain number by the  $\frac{2}{3}$  of the square of the same number, we obtain  $13\frac{1}{3}$  for quotient; what is this number?

*Ans.* 9.

11. A reservoir, whose length is to its breadth as 13 is to 5, and depth as 13 is to 3, contains 99840 cubic feet of water; what are the dimensions of the reservoir?

*Ans.* length 104 ft., breadth 40 ft., depth 24 ft.

12. Some merchants formed a partnership in which each partner invested 1000 as many dollars as there were associates. Having made

a profit of \$2560, they find that they have gained the half as much per cent. as there are associates. How many partners were there in the company? *Ans.* 8.

13. An inlayer bought a certain quantity of pearl-shells; by paying \$1.35 per lb., and multiplying the square of the sum he laid out by the  $\frac{3}{4}$  of itself, it gives a product of 59049. Required the number of lbs. he bought? *Ans.*  $35\frac{5}{7}$  lb.

14. How much must a merchant pay, at 55 cents per lb., for a certain number of bales of wool, each bale containing 145 lb., the number of bales being such that in multiplying together its  $\frac{3}{4}$ ,  $\frac{2}{3}$ , and  $\frac{1}{2}$ , the product will be 8640? *Ans.* \$3828.

## PROGRESSIONS.

## ARITHMETICAL PROGRESSION.

**460.** An **Arithmetical Progression** is a series of numbers increasing or decreasing by a constant difference.

**461.** The **Terms** of a series are the numbers of which it is formed.

**462.** The **Extremes** are the first and last terms.

**463.** The **Means** are the intermediate terms.

**464.** The **Common Difference** is the number added or subtracted, in order to form each successive term.

**465.** An **Ascending Series** is produced by adding the common difference to each term successively; as, 1, 3, 5, 7, 9, 11, 13, 15, and 17.

**466.** A **Descending Series** is produced by subtracting the common difference from each term successively; as, 17, 15, 13, 11, 9, 7, 5, 3, and 1.

**467.** The sum of the extremes is equal to the sum of any two terms equally distant from them, or to double the middle term. Thus,

|    |    |    |    |    |
|----|----|----|----|----|
| 1  | 3  | 5  | 7  | 9  |
| 17 | 15 | 13 | 11 | 9  |
| 18 | 18 | 18 | 18 | 18 |

**468.** The following are the *five quantities* considered, *three* of which being given, the other *two* may be found:—

- |                              |            |    |
|------------------------------|------------|----|
| 1. The first term,           | denoted by | a. |
| 2. The last term,            | " "        | l. |
| 3. The common difference,    | " "        | d. |
| 4. The number of terms,      | " "        | n. |
| 5. The sum of all the terms, | " "        | s. |

**NOTE.**—Half the sum of any two numbers is called their *Arithmetical Mean*.

**469. CASE I.**—Given the first term, the common difference, and the number of terms, to find the last term.

*Ex.* The first term of an ascending series is 4, the common difference 3, and the number of terms 19; what is the last term?

$$18 = 19 - 1$$

$$\frac{3}{54}$$

$$\frac{4}{4}$$

58, the last term.

*ANALYSIS.*—The first term is 4, the second term = 4 + once the common difference, the third term = 4 + twice the common difference, etc. Therefore the last term = 4 + 18 times the common difference. Hence the Formula,  $a + (n - 1)c = l$ , or the

**470. RULE.**—To the first term add the product of the common difference and the number of terms less one.

*NOTE.*—If the series is descending, subtract the product from the first term.

EXAMPLES FOR PRACTICE.

1. The first term of an ascending series is 8, the common difference 5, and the number of terms 40; what is the last term? *Ans.* 203.
2. I bought 16 acres of land, giving \$1 for the first acre, \$9 for the second, \$17 for the third, and so on; what did the last acre cost at this rate? *Ans.* \$121.
3. The first term of a descending series is 75, and the common difference 5; what is the 13th. term? *Ans.* 15.
4. A board,  $2\frac{1}{2}$  inches wide at the narrow end, and 10 feet long, increases in width  $1\frac{1}{4}$  inches for every foot in length. What is the width of the wide end? *Ans.*  $17\frac{1}{4}$  in.
5. If the first term of an ascending series be  $\frac{1}{3}$ , the common difference  $\frac{1}{3}$ , and the number of terms 20, what is the last term? *Ans.*  $7\frac{1}{3}$ .

**471. CASE II.**—Given the extremes and number of terms, to find the common difference.

*ANALYSIS.*—Since  $a + (n - 1)c = l$ ,  $c = \frac{l - a}{n - 1}$  Hence, the

**472. RULE.**—Divide the difference of the extremes by the number of terms less one.

EXAMPLES FOR PRACTICE.

1. The first term is 3, the last term is 15, and the number of terms is 7; what is the common difference? *Ans.* 2.
2. The extremes are 2 and 17, and the number of terms is 6; what is the common difference? *Ans.* 3.
3. A man has 10 sons; the youngest is 8, and the eldest 44 years old; their ages increase in arithmetical progression. Required the difference of their ages? *Ans.* 4 years.

4. If the extremes are 0 and  $2\frac{1}{2}$ , and the number of terms is 18, what is the common difference? *Ans.*  $\frac{1}{36}$ .

**473. CASE III.**—Given the extremes, and the common difference, to find the number of terms.

*ANALYSIS.*—Since,  $a + (n - 1)c = l$ ,  $n = \frac{l - a}{c} + 1$ . Hence, the

**474. RULE.**—Divide the difference of the extremes by the common difference, and increase the quotient by 1.

**EXAMPLES.**

1. The first term is 8, the last term 203, and the common difference 5; what is the number of terms? *Ans.* 40.
2. A man going a journey travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles; how many days did he travel? *Ans.* 12.
3. The extremes are  $2\frac{1}{2}$  and 40, and the common difference is  $7\frac{1}{2}$ ; what is the number of terms? *Ans.* 6.
4. In what time can a debt be discharged, supposing the first week's payment to be \$1, and the payment of every succeeding week to increase by \$2, till the last payment shall be \$103? *Ans.* 52 weeks.

**475. CASE IV.**—Given the extremes, and the number of terms, to find the sum of all the terms.

*ANALYSIS.*—Since, the sum of the extremes of an arithmetical progression is equal to the sum of any two terms equally distant from them, it follows that the terms must average half the sum of the extremes. Hence,  $s = \frac{1}{2}(a + l)n$ .

**476. RULE.**—Multiply half of the sum of the extremes by the number of terms.

**EXAMPLES.**

1. The extremes of an arithmetical series are 3 and 19, and the number of terms 9; what is the sum of the series? *Ans.* 99.
2. A man bought 16 acres of land, giving \$1 for the first acre, and \$121 for the last acre; the prices of the successive acres form an arithmetical progression. How much did the 16 acres cost? *Ans.* \$976.
3. A gentleman wishes to discharge a debt in 11 annual payments such that the last payment shall be \$220, and each payment greater than the preceding by \$17; what is the amount of the debt, and the first payment? *Ans.* 1st. payment, \$50.
4. A merchant bought 20 pieces of cloth, giving for the first, \$2, and for the last \$40; the prices of the pieces form an arithmetical series; how much did the cloth cost? *Ans.* \$420.
5. If 100 oranges are placed in a line, exactly 2 yards from each other, and the first 2 yards from a basket; what distance must a boy travel, starting from the basket, to gather them up singly, and return with each to the basket?

GEOMETRICAL PROGRESSION.

**477.** A **Geometrical Progression** is a series of numbers increasing or decreasing by a constant ratio.

**478.** The **Ratio** is the constant multiplier or divisor.

**479.** An **Ascending Series** is produced by any ratio greater than 1, as 2, 4, 8, 16, 32, 64, etc.

**480.** A **Descending Series** is produced when the ratio is less than 1; as 1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$ ,  $\frac{1}{32}$ ,  $\frac{1}{64}$ , etc.

**481.** The following are the *five quantities* considered, *three* of which being given, the other *two* may be found:—

- |                              |            |           |
|------------------------------|------------|-----------|
| 1. The first term,           | denoted by | <b>a.</b> |
| 2. The last term,            | “ “        | <b>l.</b> |
| 3. The ratio,                | “ “        | <b>r.</b> |
| 4. The number of terms,      | “ “        | <b>n.</b> |
| 5. The sum of all the terms, | “ “        | <b>s.</b> |

**NOTE.**—The *Geometrical Mean* between two numbers is the square root of their product.

**482. CASE I.**—Given the first term, the ratio, and the number of terms, to find the last term.

*Ex.* The first term of a geometrical series is 4, and the ratio is 3; what is the 9th. term?

**ANALYSIS.**—The first term = 4, and from the nature of the series,  
 The second term =  $4 \times 3^1$   
 The third term =  $4 \times 3^2$   
 The fourth term =  $4 \times 3^3$   
 and so on. Hence, the last term,  $l = a \times r^{n-1}$ .

**OPERATION.**

$4 \times 3^8 = 26244, \text{ Ans.}$

**483. RULE.**—Multiply the first term by that power of the ratio denoted by the number of terms, less one.

**NOTE.**—If the series is decreasing, consider the first term as the last, and the last as the first, the ratio will then be greater than 1.

**EXAMPLES.**

1. The first term of a geometrical series is 2, and the ratio 3; what is the 10th. term?  
*Ans.* 39366.
2. The first term of a series is 1458, the number of terms 7, and the ratio  $\frac{1}{3}$ ; what is the last term?  
*Ans.* 2.
3. A woman bought 9 eggs, agreeing to pay 1 mill for the first egg, 2 mills for the second, and so on; what did the last egg cost her?  
*Ans.* \$0.256.

4. If the first term of a series is 30, the ratio 1.06, and the number of terms 6; what is the last term? *Ans.* 40.146767328.

5. A person traveling goes 2 miles the first, 4 miles the second, 8 miles the third day, and so on, increasing in geometrical progression for 10 days. How far did he travel the last day? *Ans.* 1024 miles

6. Bought a lot of land containing 15 acres, agreeing to pay for the whole what the last acre would come to, reckoning 5 cts. for the first acre, 15 cts. for the second, and so on, in a threefold ratio. What did the lot cost me? *Ans.* \$239148.45.

**484. CASE II.**—Given the extremes and ratio, to find the sum of all the terms.

*Ex.* The first term is 2, the last term is 128, and the ratio 4; required the sum of all the terms.

## OPERATION.

$$\begin{array}{r} 8 + 32 + 128 + 512 = 4 \times \text{sum of the series.} \\ 2 + 8 + 32 + 128 = 1 \times \text{sum of the series.} \\ \hline 512 - 2 = 3 \times \text{sum of the series.} \\ \text{Hence } \frac{512 - 2}{3} = 170, \text{ the sum of the series.} \end{array}$$

**ANALYSIS.**—Since  $512 = 1r$ ,  $2 = a$ , and  $3 = r - 1$ ,  $s = \frac{1r - a}{r - 1}$ . Hence, the

**485. RULE.**—Multiply the last term by the ratio, subtract the first term from the product, and divide the remainder by the ratio less one.

**NOTES.**—1. If the ratio is less than 1, the product of the last term, multiplied by the ratio, must be subtracted from the first term; and, to obtain the divisor, the ratio must be subtracted from the unity, or 1.

2. When a descending series is continued to infinity, it becomes what is called an INFINITE SERIES, whose last term must be regarded as 0, and its ratio as a fraction.

To find the sum of an Infinite Series,—Divide the first term by a unit diminished by the fraction denoting the ratio.

## EXAMPLES.

1. The first term of a series is 4, the last term is 62500, and the ratio 5; what is the sum of all the terms? *Ans.* 78124.

2. If the first term of a series is 12, the ratio 3, and the number of terms 8; what is the sum of the series? *Ans.* 39360.

3. The first term of a decreasing series is 102, the last term 4, and the ratio  $\frac{1}{2}$ ; what is the sum of the series? *Ans.* 151.

4. If the first term of a series is 5, the ratio  $\frac{2}{3}$ , and the number of terms 6; required the sum of the series. *Ans.* 131 $\frac{1}{3}$ .

5. The first term of a decreasing series is 106, the last term 10, and the ratio  $\frac{1}{2}$ ; required the sum of the terms. *Ans.* 130.

6.  
men  
and  
7.  
first  
were  
wha  
8.  
10 s  
thele  
for t  
cost

4  
boar

4  
boar

R  
boar

4

R  
width

M  
uct a

No  
the w

E  
wide

2.  
wide

3.  
ends

4

R  
in in  
produ

Not  
width  
thickn  
half the

144.

6. In what time will a certain debt be discharged by monthly payments in geometrical progression, if the first and last payments are \$1 and \$2048, and the ratio 2? *Ans.* In 12 months.

7. A young man agreed to serve in a store for 6 months. For the first month he was to receive \$3, and each succeeding month's wages were to be increased by  $\frac{1}{3}$  of his wages for the month next preceding; what sum did he receive for the 6 months? *Ans.* \$91.95 +.

8. A gentleman wishing to purchase a piece of ground, measuring 10 square rods, thought \$1 per sq. rod too high a price; he, nevertheless, agreed to give 1 cent for the first sq. rod, 4 for the second, 16 for the third, and so on, in a fourfold ratio; how much did that ground cost him? *Ans.* \$3495.25.

## MEASUREMENT OF LUMBER.

**486.** Boards are usually measured by the square foot. The board is considered to be 1 inch in thickness.

**487.** Planks, Beams, Joists, etc., are usually measured by board measure.

Round timber is sometimes measured by the ton, and sometimes by board measure.

**488.** To find the contents of a board.

*RULE.*—Multiply the length of the board, taken in feet, by its width in feet, and the product will be the contents in sq. feet. Or, Multiply its length in feet by its width in inches, and the product divided by 12 will give the contents in square feet.

*NOTE.*—If the board is tapering, take half the sum of the width of its ends for the width.

*Ex.* 1. What are the contents of a board 36 feet long, and  $1\frac{1}{2}$  feet wide? *Ans.* 54 sq. feet.

2. What are the contents of a board 24 feet long, and 15 inches wide? *Ans.* 30 sq. feet.

3. What are the contents of a tapering board, 20 feet long, whose ends are, the one 24 inches, and the other 13 inches wide?

**489.** To find the contents of planks, beams, joists, etc.

*RULE.*—Multiply the width, taken in inches, by the thickness, in inches, and this product by the length, in feet; and the last product divided by 12 will give the contents in feet, board measure.

*NOTE.*—If the plank, beam, etc. is tapering in width, take half the sum of the width of the ends for the width; and if the taper be both of the width and the thickness, the common rule of obtaining the contents in cubic feet is, to multiply half the sum of the areas of the two ends by the length, and divide the product by 144.

- Ex.* 1. What are the contents of a plank 40 feet long, 2 feet wide, and 3 inches thick? *Ans.* 240 sq. ft.  
 2. How many feet are there in 9 joists, which are 15 feet long, 5 inches wide, and 3 inches thick? *Ans.* 168½ feet.  
 3. How many feet in 3 beams 24 feet long, 10 inches thick, whose width tapers from 18 to 16 inches? *Ans.* 858 feet.

**490.** To find the contents of round timber.

**RULE.**—Multiply the length, taken in feet, by the square of one fourth of the mean girth, taken in inches; and, this product, divided by 144, will give the contents in cubic feet.

**NOTES.**—1. The girth of tapering timber is usually taken about one third the distance from the larger to the smaller end.  
 2. This rule is that in common use, though very far from giving the actual number of cubic feet in round lumber measured by it. 40 cubic feet, as given by the rule, are in fact equal to 50.82 true cubic feet. The following rule gives results more nearly accurate, requiring to be diminished by only one foot in 190, to give exact contents. Multiply the square of one fifth of the mean girth, taken in inches, by twice the length, in feet; and divide by 144.

- Ex.* 1. How many cubic feet in a stick of timber which is 50 feet long, and whose girth is 60 inches? *Ans.* 78½ cub. ft.  
 2. What are the contents of a stick of timber whose length is 30 feet, and girth 40 inches? *Ans.* 20½ feet.  
 3. How many cubic feet in a log 90 feet long, and whose circumference is 120 inches? *Ans.* 562½ cub. feet.

### MISCELLANEOUS EXAMPLES.

1. A man sleeps  $7\frac{1}{2}$  hours each day. What per cent. of his time does he sleep? *Ans.*  $31\frac{1}{2}\%$ .
2. What number is that to which, if you add  $\frac{2}{3}$  of  $\frac{1}{11}$  of itself, the sum will be 61? *Ans.* 55.
3. A gentleman bought 95 yards of cloth,  $\frac{2}{3}$  of a yard wide, for \$100, and gave the same and \$25 for cloth of the same quality, 1 yard wide. How many yards did he buy? *Ans.*  $89\frac{1}{3}$  yd.
4. A father devised  $\frac{1}{11}$  of his estate to one of his sons, and  $\frac{1}{11}$  of the residue to the other, and the remainder to his wife. The difference of his sons' legacies was found to be £257 3 4. What money did he leave for his widow? *Ans.* £635 0 10½.
5. How many bricks 8 inches long, 4 inches wide, and 2 inches thick, will it take to build a wall 40 feet long, 20 feet high, and 2 feet thick? *Ans.* 43200 bricks.
6. If a man can paint 4 square yards in one hour, and is 31 h. 6 min. 40 sec. in painting two sides of a wall 7 feet high; how long is the wall? *Ans.* 80 ft.

7. By selling wheat at 12 s. 6 d. per bushel, I gain £30 on the quantity purchased; but if I sell it for 13 s. 6 d. a bushel, I shall gain £42 on the same quantity. How many bushels were bought? *Ans.* 240.

8. A grocer bought a hogshead of wine for \$28.35; how much water must be added to reduce the first to 35 cts. per gal.? *Ans.* 18 gal.

9. A father, dying, left his son a legacy,  $\frac{1}{4}$  of which he spent in 8 months;  $\frac{3}{4}$  of the remainder lasted him 12 months longer, after which he had only \$410 left. What amount did his father bequeath him? *Ans.* \$956.66 $\frac{2}{3}$ .

10. A man had  $\frac{1}{4}$  of a yard of broad cloth, for which he paid at the rate of \$8 $\frac{1}{4}$  per yard; he gave the broad cloth and 50 cents for 1 $\frac{1}{4}$  yards of cassimere. What did the cassimere cost him per yd.? *A.* \$2.66 $\frac{2}{3}$ .

11. How many dollars, Canadian currency, are equal to \$160 United States currency?

12. A grocer wishes to mix together brandy at 80 cts. a gal., wine at 70 cts., cider at 10 cts., and water, in such proportions that the mixture may be worth 50 cts. a gal.; what quantity of each must be used? *Ans.* 3 gal. of water, 2 of cider, 4 of wine, and 5 of brandy.

13. If the longitude of Boston is 70° 4' west, what will be the time in that place when it is 3 h. 35 min. A. M. in London?

*Ans.* 10 h. 54 min. 44 sec. P. M. of the previous day.

14. A merchant sold goods to a certain amount, on a commission of 4%; and, having remitted the net proceeds to the owner, received  $\frac{1}{4}$ % for prompt payment, which amounted to \$15.60. What was his commission? *Ans.* \$260.

15. I purchased railroad stock to the amount of \$2356.80, and found that the sum invested was 40% of what I had left; what sum had I at first? *Ans.* \$3248.80.

16. If 13 $\frac{1}{2}$  bushels of wheat make 3 barrels of flour, how many bushels of wheat will be required to make 40 barrels of flour? *Ans.* 180.

17. The capital of an insurance company is \$250000; its receipts for one year are \$58760; its losses and expenses are \$4910. What rate of dividend can it declare? *Ans.* 7 $\frac{1}{2}$ %.

18. By selling a lot of books for \$438, a bookseller loses 10%; how much should the books have been sold for, to gain 12 $\frac{1}{2}$ %?

19. What is the difference between the interest and the discount of \$540 at 6%, for 6 years 10 months?

20. I own 25 shares of \$50 each in the Gosford Railroad Co., which has declared a semi-annual dividend of 3 $\frac{1}{4}$ %. How much do I receive? *Ans.* \$43.75.

21. If 12 boarders eat \$25 worth of bread in 2 mo., when flour is \$9.50 per bbl.; in how many months will 15 boarders eat \$60 worth of bread, when flour is \$12 per bbl.? *Ans.* 3 $\frac{1}{2}$  mo.

22. B hired a house for one year for \$300; at the end of 4 months he takes in C as a partner, and at the end of 8 months he takes in D. At the end of the year, what rent must each pay? *Ans.* B \$183 $\frac{1}{3}$ ; C \$83 $\frac{1}{3}$ ; D \$33 $\frac{1}{3}$ .

23. A person mixed 12 cwt. of sugar at \$10, with 3 cwt. at \$8 $\frac{1}{2}$ , and 8 cwt. at \$7 $\frac{1}{2}$ ; how much was 1 cwt. of the mixture worth? *Ans.* \$8 $\frac{2}{3}$ .

24. A shipment of wheat was insured at  $2\frac{3}{8}\%$ , to cover  $\frac{3}{4}$  of its value; the premium paid was \$44.07; the wheat being worth 80 cts. per bushel, how many bushels were shipped? *Ans.* 2825 bush.

25. A stack of hay will keep 24 cows or 18 horses one week. How many days will it keep 5 cows and 5 horses? *Ans.*  $14\frac{1}{2}$  da.

26. C, of Montreal, remits to D, of Quebec, a bill of exchange on Liverpool, the avails of which he wishes to be invested in goods on his account. D, having disposed of the bill at  $7\frac{1}{4}\%$  advance, received \$9675; and, having reserved for himself  $\frac{1}{4}\%$  on the sale of the bill, and 2% for commission, he invests the remainder. What is the amount invested, and for how much was the bill drawn?

*Ans.* Investment, \$9461.58 $\frac{3}{4}$ ; the bill was £2025.

27. What per cent. is gained by buying oil at 80 cents a gallon, and selling it at 12 cents a pint? *Ans.* 20%.

28. A merchant pays \$10050 for a stock of goods; he sells them at an advance of  $33\frac{1}{3}\%$ ; the expenses connected with the business are \$1750. How much does he gain? *Ans.* \$1600.

29. What o'clock is it when the time from noon is  $\frac{1}{11}$  of the time to midnight? *Ans.* 5 o'clock. 24 min. P. M.

30. A merchant receives on commission three kinds of flour; from C he receives 20 bbl., from D 25 bbl., and from E 40 bbl. He finds that C's flour is 10% better than D's, and that D's is 20% better than E's. He sells the whole at \$6 per bbl. What in justice should each man receive? *Ans.* C receives \$1391 $\frac{1}{11}$ ; D, \$1581 $\frac{1}{11}$ ; E, \$2111 $\frac{1}{11}$ .

31. For what sum must a note be drawn at 4 mo., that the proceeds of it, when discounted at bank, at 7%, shall be \$875.50?

32. If  $2\frac{1}{2}$  yards of merino  $1\frac{1}{2}$  yards wide cost \$3.37 $\frac{1}{2}$ , what will be the cost of  $36\frac{1}{2}$  yards  $1\frac{1}{4}$  yards wide? *Ans.* \$52.779.

33. What must be the face of a note at 60 days, the proceeds of which, when discounted at Bank, at 6%, are \$100? *Ans.* \$101.06 +

34. A merchant sold a piece of cloth for \$24, and thereby lost 25%; what would he have gained had he sold it for \$34? *Ans.*  $6\frac{1}{4}\%$ .

35. A bankrupt compromises with his creditors for  $37\frac{1}{2}\%$ ; how much will he pay on a claim of \$3656? *Ans.* \$1371.

36. A man, dying, left \$3565 to be placed at interest for his son, who was 16 yr. 5 mo. 15 da. old; how much will he receive when he is 21 years old, allowing 7% interest? *Ans.* \$4698.37 +.

37. A garrison, consisting of 360 men, was provisioned for 6 months; but at the end of 5 months they dismissed so many of the men that the remaining provision lasted 5 months longer; how many men were sent away? *Ans.* 288.

38. What sum must I invest in the New Brunswick 6% stock, selling at  $2\frac{1}{2}\%$  premium, to secure an annual income of \$840? *Ans.* \$14350.

39. A grocer divided a barrel of flour into two parts, so that the smaller contained  $\frac{2}{3}$  as much as the other; how many pounds were there in each? *Ans.* 78 $\frac{1}{2}$  lb., 117 $\frac{1}{2}$  lb.

40. A sportsman spends  $\frac{1}{3}$  of his time in smoking,  $\frac{1}{4}$  in gunning, 2 ho. per day in loafing, and 6 ho. in eating, drinking, and sleeping; how much remains for useful purposes? *Ans.* 2 ho.

41. Exchanged 250 shares of 6% stock, at 70%, for stock bearing 8%, at 120%; what is the difference in my income? *Ans.* \$333.33 $\frac{1}{2}$ .

42  
sold  
pay,  
mon  
recei  
43  
8 in.  
dime  
44  
at th  
a pro  
45  
flour  
many  
46  
\$100  
sum  
47  
bills  
becar  
\$520  
48  
at 7 $\frac{1}{2}$   
worth  
sugar  
49  
is sur  
walk  
rema  
50  
will a  
to h  
51  
6 $\frac{1}{4}$   
and t  
ment  
52  
goods  
credi  
he m  
mone  
53  
plante  
and n  
54  
with c  
if it w  
55  
mainc  
betwe  
his wi

42. Purchased 100 barrels herrings, at \$5 per bbl. and immediately sold them on a credit of six months. The note which I received for pay, I got discounted at the Union Bank; and, on examining my money, I found that I had gained 20% on my purchase. What did I receive per bbl. for the herrings? *Ans.* \$6.18 +.

43. How many bricks are required to build the front of a house 50 ft. 8 in. in length, 15 ft. 8 in. in height, and 1 ft. 6 in. in thickness, the dimensions of a brick being 8, 4 and 2 inches? *Ans.* 32148 bricks.

44. A woman buys apples at the rate of 5 for 2 cts., and sells them at the rate of 4 for 3 cts.; how many must she buy and sell to make a profit of \$4.20? *Ans.* 1200.

45. Sent \$12300 to my agent in Toronto, with which to purchase flour at \$10 per bbl., after deducting his commission of 2½%. How many barrels of flour did I receive? *Ans.* 1200.

46. Borrowed of A \$150 for six months; afterwards I lent him \$100; how long shall he keep it to compensate him for the use of the sum he lent me? *Ans.* 9 mo.

47. A broker charges me 1½% for purchasing some uncurrent bank bills at 25% discount; of these bills, three of \$10 each, and one of \$50 became worthless; I dispose of the remainder at par, and thus make \$520. What was the amount of bills purchased? *Ans.* \$2500.

48. A grocer mixed 5 lbs. of sugar, at 8½ cts. per lb., with 80 lbs., at 7½ cts. per lb., and 60 lbs. at such a price that the mixture was worth 9¾ cts. per lb. Required the price per lb. of the last kind of sugar. *Ans.* 12½ cts.

49. A gentleman's garden is 23¼ rods long, and 13¼ rods wide, and is surrounded by a good fence 7½ ft. high. Now, if he shall make a walk around his garden within the fence 7½ ft. wide; how much will remain for cultivation? *Ans.* 1 A. 3 R. 7 p. 85½ ¾ ft.

50. A certain principal, at compound interest for 5 years, at 6%, will amount to \$669.113; in what time will the same principal amount to the same sum, at 6% simple interest? *Ans.* 5 yr. 7 mo. 19 + da.

51. I invested ¾ of my money in R. R. stock, which depreciated 6½%; the remainder I invested in real estate, which advanced 15%, and thereby I gained \$1500. How much did I gain in both investments? *Ans.* \$250.

52. What % in advance of the cost must a merchant mark his goods, so that, after allowing 5% of his sales for bad debts, an average credit of 6 months, and 7% of the cost of the goods for his expenses, he may make a clear gain of 12½% on the first cost of the goods, money being worth 6%? *Ans.* 29.56 + %.

53. What is the greatest possible number of hills of rye that can be planted on a square acre, the hills to occupy only a mathematical point, and no two hills to be nearer than 3¼ feet? *Ans.* 4165.

54. I wish to line the carpet of a room, 6¼ yd. long and 5¼ yd. wide, with duck, ¾ yd. wide. How many yards of lining must I purchase, if it will shrink 4% in length and 5% in width? *Ans.* 43¾ yd.

55. A man bequeathed ¼ of his estate to his son, and ⅓ of the remainder to his daughter, and the residue to his wife; the difference between his son and daughter's portion was \$100; what did he give his wife? *Ans.* \$600.

56. Eight workmen, laboring 7 hours a day for 15 days, were able to execute  $\frac{1}{2}$  of a job; in how many days can they complete the residue, by working 9 hours a day, if 4 workmen be added to their number?

*Ans.*  $15\frac{1}{2}$  days.

57. Exchanged 60 Ontario bonds of \$1000 each, at  $8\frac{1}{2}\%$  premium, for Nova Scotia bonds of \$200 each, at  $5\%$  premium. How many of the latter did I receive?

*Ans.* 310.

58. I lent a friend \$700, which he kept 20 months. Some years after I borrowed of him \$300; how long should I keep it to balance the favor?

*Ans.*  $46\frac{1}{2}$  months.

59. Bought merchandise as follows: July 3, \$35.26; July 4, \$48.65, on 30 da.; Aug. 17, \$6.48; Sept. 12, \$50. What is due on the account Oct. 12, interest at  $9\%$ ?

*Ans.* \$142.60.

60. Lent a certain sum of money to A, and at the end of 3 yr. 7 mo. 20 da., I received for interest and principal \$1000; what sum did I lend?

*Ans.* \$820.79 +.

61. If 12 oz. of wool make  $2\frac{1}{2}$  yd. of cloth,  $1\frac{1}{2}$  wide, how many pounds of wool are required to make 115 yd. of cloth 1 yd. wide?

*Ans.* 24.

62. Bought goods for \$1500,  $\frac{1}{3}$  payable in 3 months,  $\frac{1}{3}$  in 6 months, and the remainder in 9 months. How much ready cash ought I to pay for the goods, money being worth  $6\%$ ?

*Ans.* \$1456.52 -.

63. Purchased a quantity of oats, April 1; May 1 its value had increased  $25\%$ ; June 1 its value was  $30\%$  more than May 1; July 1 I sold it for  $15\%$  less than its value June 1, receiving in payment a 6-months' note, which I got discounted at a bank, at  $7\%$ , receiving \$2950 on it. How much was my profit on the oats?

*Ans.* \$3238.52.

64. If 24.4 cubic inches of lead weigh 16 lb., required the number of feet of lead pipe that can be made from 80 lb. of lead, the caliber of the pipe to be 1 inch, and the thickness of it  $\frac{1}{4}$  of an inch.

*Ans.* 10.35 + feet.

65. One-third of a quantity of goods was sold to gain a certain  $\%$ , one-fourth to gain  $1\frac{1}{2}$  times as much  $\%$ , and the remainder to gain  $2\frac{1}{2}$  times as much  $\%$ . What is the gain  $\%$  on each part, the gain upon the whole being  $21\%$ ?

*Ans.* 1st.,  $12\%$ ; 2nd.,  $18\%$ ; 3rd.,  $30\%$ .

66. A merchant in Kingston has 5000 francs due him on account in Paris. He can draw on Paris for this amount, and negotiate the bill at  $19\frac{1}{2}$  cts. per franc; or he can advise his correspondent in Paris to remit a draft on Canada, purchased with the sum due him, exchange on Canada being at the rate of 5 fr. 20 centimes per dollar. What sum will the merchant receive by each method?

*Ans.* By draft on Paris, \$970; by remittance from Paris, \$961.63.

67. A miller is required to grind 160 bushels of provender, worth \$1 a bushel, from oats worth \$.40, corn worth \$.80, barley worth \$.90, and rye worth \$1.10, and wheat worth \$1.30 per bushel. How many bushels of each kind may he take?

*Ans.* 20, 20, 60, and 40, respectively.

68. How much coffee at \$.37 $\frac{1}{2}$  a lb., must be given for 12 gal. 3 qt. of sirup, at \$.75 a gallon?

*Ans.*  $25\frac{1}{2}$  lb.

69. A servant draws off a gallon on each day, for 20 days, from a cask containing 10 gallons of wine, each time supplying the deficiency by the addition of a gallon of water; and then, to escape detection, he

again draws off 20 gal., supplying the deficiency each time by a gallon of wine. How much water still remains in the cask?

*Ans.* 1.0679577 gal., or more than a gallon and half a pint.

70. A merchant has \$216 due him, to be paid in 7 months; but the debtor agrees to pay one-half ready money, and  $\frac{1}{4}$  of the remainder in 6 months. What time should he be allowed for paying the balance?

*Ans.* 3 yr. 2 mo.

71. A house that cost £3931 5 0, rents for £369 10 9; the insurance is  $\frac{1}{4}$ %, and the repairs  $\frac{3}{4}$ % each year. What rate of interest does it pay?

*Ans.* 8%.

72. I owe a man the following notes: one of \$500, due April 1; one of \$750, due July 15; and one of \$1750, due Sept. 10. The holder wishes to exchange them for two notes of \$1500 each, and wants one to fall due May 10; when should the other be payable?

*Ans.* Oct. 20.

73. A trader bought merchandise as follows: April 8, \$150.22; May 23, \$55.64, on 30 da.; June 2, \$82.60, on 2 mo., and July 14, \$90. What was due on the account Sept. 26, money being worth 7%?

*Ans.* \$386.67.

74. By working 9 hours a day, for 155 days, 12 men were able to execute  $\frac{1}{2}$  of a job, how many men may be withdrawn, and the residue be finished in 15 days more, if the laborers are employed only 7 hours a day?

*Ans.* 4 men.

75. At a certain time between 2 and 3 o'clock, the minute-hand was between 3 and 4. Within an hour after, the hour-hand and minute-hand had exactly changed places with each other. What was the precise time when the hands were in the first position?

*Ans.* 2 hr. 15 min. 56  $\frac{2}{3}$  sec.

76. D and E traded together; D put in £100 for 512 days, and received  $\frac{1}{4}$  of the gain; the number of dollars which E put in was equal to the number of days it was employed in trade. What was E's capital?

*Ans.* £160.

77. If stock bought at 8% discount will pay 7% on the investment, at what rate should it be bought to pay 10%?

*Ans.* 35.6% disct.

78. An importer sold cloth to a wholesale dealer at 10% advance; the wholesale dealer sold it to a clothier at  $1\frac{1}{2}$ % advance; the clothier sold it at a farther advance of 25%, and received \$1452. How much did it cost the importer?

*Ans.* \$938.66  $\frac{2}{3}$ .

79. What is the difference between the interest and discount of \$730, for 5 yr. 9 mo., at 8%?

*Ans.* \$105.80.

80. A merchant sold  $\frac{1}{4}$  of his goods at an advance of 25%;  $\frac{2}{5}$  of them at a loss of 8%;  $\frac{1}{10}$  of them at a profit of 30%, and  $\frac{1}{10}$  of them at a discount of 20%. For what % of the cost must the remainder be sold in order to lose 5% on the whole?

*Ans.* 68  $\frac{1}{2}$ %.

81. I received an 8% dividend on Montreal city railroad stock, and invested the money in the same stock at 80%. My stock having increased to \$13750, what was the amount of my dividend? *A.* \$1000.

82. A tailor bought 40 yards of broadcloth,  $2\frac{1}{4}$  yd. wide; but on sponging it, it shrunk in length upon every 4 yd. half a quarter, and in width, one nail and a half upon every  $1\frac{1}{4}$  yd. To line this cloth, he bought flannel 5 quarters wide, which, being wet, shrunk the whole

width on every 20 yards in length, and in width it shrunk half a nail. Required the number of yards of flannel used in lining the cloth.

*Ans.*  $71\frac{1}{2}$  yards.

83. Stock purchased at 5% premium pays 6% on the investment, what % will it pay if purchased at 15% discount? *Ans.*  $7\frac{1}{2}$ %.

84. A merchant failing in business can pay 76 cts. on a dollar. He offers, to pay his whole indebtedness without interest in 5 years if his creditors will allow him to go on with his business; his offer being accepted, how much will his creditors lose in the 5 years, money being worth 7%? *Ans.* \$.026 on a dollar.

85. Purchased a quantity of wine for \$675.32 $\frac{1}{2}$ , at 85 cents per gallon; but a part having leaked out, the remainder was sold at 40% advance, and the original cost was realized. What quantity leaked out? *Ans.* 227 gal.

86. A owes B \$600 due in 4 months, and \$840 due in 6 months; B owes A \$1600 due in 7 months. If A should make present payment of his debts, when ought B in justice to pay A? *Ans.* In 2mo. 10 $\frac{1}{2}$  da.

87. How many pounds of sugar at 8, 13, and 14 cts. per pound, may be mixed with 3 lb. at 9 $\frac{1}{2}$  cts., 2 lb. at 8 $\frac{1}{2}$  cts., and 4 lb. at 14 cts. a lb., so as to gain 16% by selling the mixture at 14 $\frac{1}{2}$  cts. per lb.? *Ans.* 1 lb. at 8; 8 $\frac{1}{2}$  lb. at 13; 8 lb. at 14.

88. What is the difference between the true and bank discount of \$3000, payable in 120 days, at 8 $\frac{1}{2}$ %? *Ans.* \$4.467-

89. A general, forming his army into a square, had 284 men remaining; but increasing each side by one man, he wanted 25 men to complete the square. How many men had he? *Ans.* 24000.

90. C bought a house of D, and gave him his bond for \$6000, dated April 1, 1866, payable in 5 equal annual installments of \$1200; the first to be paid April 1, 1867; C took up his bond April 1, 1869, semi-annual discount at the rate of 7% per annum on the payments due after April 1, 1869, being deducted. What sum cancelled the bond? *Ans.* \$3365.94 +.

91. I have a plank 42 $\frac{1}{2}$  feet in length, 24 inches wide, and 3 inches thick; required the side of a cubical box that can be made from it? *Ans.* 48 inches.

92. If B owes \$500 due in 6 months, \$400 due in 4 months, and \$300 due in 7 months, and pays  $\frac{1}{2}$  of the whole in 3 months, when ought the remainder to be paid? *Ans.* In 10 $\frac{1}{2}$  mo.

93. A wholesale merchant sent a quantity of goods into the country to be sold at auction, on a commission of 4 $\frac{1}{2}$ %. What amount of goods must be sold, that his agent may buy produce with the avails to the amount of \$1910, after retaining a commission of 2%? *Ans.* \$2040.

94. If the annual rent of 23 A. 1 R. 27 per. of land be \$187.35, how much will be the rent of 71 A. 20 per.? *Ans.* \$569.

95. A Halifax merchant shipped 1000 barrels of salmon to his agent in New Orleans, directing him to sell it, and invest the proceeds in cotton; his agent sold the salmon at \$14 per bbl., paid \$274 charges, and bought cotton at \$.65 per lb., charging 3% commission for selling the salmon and 5% for buying the cotton. How many pounds of cotton did he buy? *Ans.* 19494.5 + lb.

96.  
12, a  
he fin  
owe?  
97.  
E hac  
mone;

98.  
of \$10  
99.  
and B  
quired

100.  
only \$  
101.  
selling  
by inv  
money  
much

102.  
8 more  
did I b  
103.  
in how  
104.  
retailed  
gain?

105.  
head +  
togethe

106.  
chant,  
The co  
\$8.40 p  
draft;

107.  
every 3  
every 2  
108.  
\$22.50,  
gain on

109.  
factory

110.  
for ban  
I receiv

111.  
brokera  
1 $\frac{1}{2}$ %.

96. A man owes a debt to be paid in 4 equal installments at 4, 9, 12, and 20 months, respectively; discount being allowed at 5%, he finds that \$750 ready money will pay the debt; how much did he owe?  
*Ans.* \$784.74 +.

97. D's money was to E's as 2 to 3; when D had spent \$40, and E had spent 40% more than D, D's money, minus \$20, was to E's money, plus \$2, as 4 to 9. How much had each at first?  
*Ans.* D, \$108; E, \$162.

98. What is the cost of a 90 days' bill on Montreal, to the amount of \$1000, at  $\frac{1}{2}$ % premium, and int. off at 6%?  
*Ans.* \$997.75.

99. Three men engaged in the lumber trade; A furnished \$4000, and B \$6000; they gained \$1680, of which C's share was \$840. Required C's stock and A's and B's gain.  
*Ans.* C's stock, \$10000; A's gain, \$336; B's \$504.

100. A man having lost  $\frac{2}{3}$  of his money, found he had remaining only \$672; how much had he at first?  
*Ans.* \$1792.

101. A speculator invested a certain amount in railroad stocks, by selling these stocks at a deteriorated price he lost  $\frac{1}{3}$  of his investment; by investing the remainder he cleared \$240, and afterward lost  $\frac{1}{3}$  of the money he had remaining, which left him possessed of \$480; how much did he invest?  
*Ans.* \$3600.

102. Bought a certain number of horses for \$2600; had I bought 8 more at \$10 less each, all would have cost \$3560; how many horses did I buy?  
*Ans.* 20.

103. Louis can do a piece of work in 8 days, and John in 12 days; in how many days can both do it?  
*Ans.*  $4\frac{2}{3}$  days.

104. A grocer bought 11 bushels of chestnuts at \$3 a bushel, and retailed them at 3 cents a half pint. What per cent. profit was his gain?  
*Ans.* 28%.

105. The head of a fish is 12 inches long, the tail is as long as the head +  $\frac{1}{3}$  of the body, and the body is as long as the head and tail together; what is the length of the fish?  
*Ans.* 96 inches.

106. A consignor sends 500 barrels of flour to a commission merchant, with instructions to sell it and remit the net proceeds by draft. The consignee pays \$120.40 for freight and expenses, sells the flour at \$8.40 per bbl., charges  $2\frac{1}{2}$ % commission, and pays  $\frac{1}{2}$ % premium for draft; how much does the consignor receive?  
*Ans.* \$4008.31 +.

107. How many horses could be kept on 25 acres of land, if for every 3 horses there is of the 25 acres, 1 acre of plowed land, and for every 2 horses, 1 acre of pasture?  
*Ans.* 30 horses.

108. Purchased 240 bushels of oats at the rate of 18 bushels for \$22.50, and sold it at the rate of 22 $\frac{1}{2}$  bu. for \$33 $\frac{1}{2}$ ; how much did I gain on the whole?  
*Ans.* \$60.

109. I paid \$93 15 0, at the rate of  $2\frac{1}{2}$ %, for insurance on a shoe factory; for what amount was the policy given?

110. Exchanged 75 railroad bonds of \$500 each, at 36% below par, for bank stock at 5% premium, how many shares of \$100 each did I receive?  
*Ans.* 228 $\frac{1}{2}$ .

111. Invested £858 in Government bonds at 106%, paying  $1\frac{1}{2}$ % brokerage, and afterward sold the stock at 12% premium, brokerage  $1\frac{1}{2}$ %. What was my gain?  
*Ans.* £26.

112. The longitude of Paris is  $2^{\circ} 20' 22''$  E., and of Constantinople,  $8^{\circ} 59'$  E. When it is 1 A. M. at the latter place, what time is it at the former? *Ans.* 33 min. 25  $\frac{1}{2}$  sec. past midnight.

113. Having placed a bill of \$775 in the hands of a collector, who succeeded in obtaining 75% of it, and charged 8% commission, how much did I receive?

114. Suppose that the earnings of the Grand Trunk R. R. for December 1870 were \$472240, which was an increase of 11% over the earnings for the same month in 1869. How much was the increase? *Ans.* \$47224.

115. In a cask containing brandy and water,  $\frac{3}{4}$  of the whole + 3 gal. is brandy, and  $\frac{1}{4}$  of the whole + 2 gal. is water; required the number of gal. of each. *Ans.* 43 gal. brandy, 17 gal. water.

116. Hamel, Perry, Lane, and Garneau are partners; Hamel takes  $\frac{1}{4}$  of the gains or losses; Perry  $\frac{1}{4}$ , Lane  $\frac{1}{4}$ , and Garneau the remainder. At the close of the year, the resources of the firm are: Cash \$10312.50, Merchandise \$13447.50, Bonds and Mortgages \$11475, Bank Stock \$4500; Hamel has drawn from the business \$900, Perry \$525, and Lane \$285; the liabilities are: Notes outstanding \$460; Balance in favor of Ross & Co., \$1120; Balance in favor of J. L. Murphy, \$3967.50; Hamel invested \$9547.50, Perry \$7905, Lane \$6270, and Garneau \$3480. What is each partner's interest in the business at the close of the year? *Ans.* Hamel, \$9877.50; Perry, \$8302.50; Lane, \$6723; Garneau, \$4279.50.

117. What is the difference in cost between a draft on Toronto of \$17302.80, at 1  $\frac{1}{2}$ % premium, and one on St. John, N. B., for the same amount, at  $\frac{1}{2}$ % discount? *Ans.* \$302.80.

118. A mechanic received \$3.75 a day for his labor, and paid \$1.25 a day for his board; at the expiration of 100 days he had saved \$200; how many days did he work? *Ans.* 65 days.

119. For two successive years, a merchant annually contributed \$100 for charitable purposes, and added yearly to that part of his capital not thus expended, a sum equal to its half; at the end of the second year his capital was doubled. Required his capital. *Ans.* \$1500.

120. A merchant in Halifax purchased 350 bales of cotton, each containing 450 pounds, at \$.80 a lb., and shipped them to Liverpool at a cost of 16% for freight and duties. How much in Canada currency did he gain by selling them at 2s. 10d. sterling per lb., rate of exchange 171%? *Ans.* \$23415.

121. A piece of merino cost \$.80 per yard; at what price shall it be marked, that the merchant may sell it at 10% less than the marked price, and still make 20% profit? *Ans.* \$1.06  $\frac{2}{3}$ .

122. A merchant in Quebec gave \$2000 for a bill of exchange of £400 to remit to London; what was the rate in London of England?

123. What yearly debt can be discharged by monthly payments, the first being \$2, the second \$6, and the third \$10, and so on, in geometrical progression? *Ans.* \$531440.

124. A farmer sold one hog, weighing 250 lb., at 4 cts. per lb.; a second, weighing 300 lb., at 4  $\frac{1}{2}$  cts.; and a third, weighing 369 lb., at 5 cts.; what was the average price per lb. for the whole? *Ans.* 4  $\frac{1}{2}$  cts.

125. be on

126. amou  
wine s127.  $\frac{1}{4}$  of th  
and t  
goods.128. \$2660.  
exchian129. his ol  
what t  
among  
ber of

130. how m

131. was th

132. its seas  
price, s133. able in  
present

134. than by

135. each da  
each co

the grai

136. 6, a pro  
counted

ceive?

137. bank de  
price of138. sends to  
the cotto

pays for

less his

bushel, s  
obtained139. the short  
piece?

125. John's age is 4 times Mary's, but in 12 years John's age will be only  $2\frac{1}{2}$  times Mary's; required the age of each.

*Ans.* Mary's age is 12 yr.; John's is 48.

126. A company of 50 men drank wine at 2s. 6d. per bottle, to the amount of £10. How many men at the same rate will £18 worth of wine supply, when wine is worth 2s. 3d. per bottle? *Ans.* 100 men.

127. The sales of a clothing house amount to \$100000 a year;  $\frac{1}{4}$  of the sales are made at a profit of 25%,  $\frac{2}{5}$  at a profit of 20%, and the remainder at a loss of 4%. Required the cost of the goods.

*Ans.* \$88750.

128. A merchant in Toronto purchased a draft on Quebec for \$2660, drawn at 60 days, paying \$2570.89. What was the course of exchange?

*Ans.*  $2\frac{1}{2}\%$  discount.

129. A man gave  $\frac{1}{4}$  of his estate to his wife,  $\frac{1}{5}$  of the remainder to his oldest son,  $\frac{1}{4}$  of the residue to his oldest daughter, and  $\frac{1}{4}$  of what then remained, which was \$1500, was to be equally distributed among his other children, who received \$150 each; required the number of his children, and the value of his estate.

*Ans.* 12 children; \$10000.

130. A merchant by selling a lot of goods for \$438, loses 10%; how much should the goods have been sold for, to gain  $12\frac{1}{2}\%$ ?

131. An agent received \$65 for collecting a debt of \$1300. What was the rate of his commission?

132. A merchant marked a piece of goods 25% above the cost, but its season passing, he determined to sell it 20% below the marked price, supposing he should make 5%. Did he make or lose?

133. A man can sell his farm for \$4000 cash, or for \$5000, payable in 2 years. If he accept the last offer, and received instead its present worth, at 10% discount; how much more would he receive than by the former?

*Ans.* \$166.66 $\frac{2}{3}$ .

134. A laborer worked 3 months, 25 days each month, 10 hours each day, for \$.08 an hour, and received in payment 2 loads of grain, each containing 15 bags of  $2\frac{1}{2}$  bushels each. What was the price of the grain per bushel?

*Ans.* \$0.80.

135. Sold goods to the amount \$348.25, taking in payment, April 6, a promissory note for sixty days, which I indorsed and had discounted at the bank, April 20, at 7%; how much cash did I receive?

*Ans.* \$344.93 +.

136. Suppose bank stock purchased at 28% premium, and the bank declares a dividend of 9% per share, what % is that on the cost price of the stock?

*Ans.*  $7\frac{1}{2}\%$ .

137. A person, wishing to buy wheat with the proceeds of cotton, sends to his agent 32 bales, each weighing 380 lb. The agent sells the cotton at 26 cts. per lb., for which he charges  $2\frac{1}{2}\%$  commission; pays for freight and charges, \$34.60; and expends the remainder, less his commissions for selling and buying, in wheat at 85 cts. per bushel, for which he charges  $1\frac{1}{2}\%$  commission; how much wheat is obtained through this factor?

*Ans.* 3642 + bu.

138. A pole 63 feet long, in falling, was broken into two pieces; the shorter piece being  $\frac{2}{3}$  of the longer; what is the length of each piece?

*Ans.* 16 and 45 ft.

139. A farmer had a dairy of 48 cows, each furnishing 18 qt. of milk a day, from which he made 40 tubs of butter of 60 lb. each in 30 days. He made a contract to deliver 100 tubs of 96 pounds each in 80 days. How many cows must he add to his dairy provided each additional cow furnish 4 gallons of milk daily? *Ans.* 33.
140. In what time will \$3045.20 gain \$190.32 $\frac{1}{2}$  if the gain of \$2494.75 for 1 yr. 13 da., is \$258.48, and what is the rate per annum? *Ans.* 7 mo. 15 da.; rate 10%.
141. Andrews, Baker, and Childs entered into partnership. Andrews put in £3000, Baker £2000, and Childs £1750. At the end of the first year Andrews drew out £500, Baker £250, and Childs put in £750. At the close of the second year, Andrews and Baker each drew out £250, and Childs put in £500 more. At the end of the third year they dissolved partnership, and found that their joint property was £7125. What was each partner's share? *Ans.* Andrews', £2393 10 4 $\frac{1}{2}$ ; Baker's, £1597 4 5 $\frac{1}{2}$ ; Childs', £3134 5 2 $\frac{1}{2}$ .
142. If I buy 50 shares Grand Trunk railroad stock at 141%, and 50 shares Canada Central railroad stock at 139%, the former paying a semi-annual dividend of 4 $\frac{1}{10}$ %, the latter of 5%; what rate of interest shall I realize on my investment? *Ans.* 6 $\frac{1}{2}$ %.
143. What is the cost of a bill on London for £800 17 6 sterling, when the rate of exchange is 9 $\frac{1}{2}$ % premium?
144. J. Sheridan bought of L. H. Miles & Co., the following bills of goods; Nov. 1, 1870, a bill of \$750, on 6 mo. credit; Dec. 15, 1870, a bill of \$300, on 5 mo.; Jan. 1, 1871, a bill of \$425, on 4 mo.; Feb. 5, 1871, a bill of \$275, on 2 mo. What sum would settle the account, May 29, 1871, interest at 7%? *Ans.* \$1760.10.
145. When exchange on England is at 10% premium, and freight at 1s. 3d. sterling per Winchester bushel, how much can be paid in Montreal for wheat per bushel, in answering an order from London limited to £3 10 per Imperial quarter?
146. The duty on an invoice of 300 dozen London porter, at 30%, was \$190.512; breakage, 2%. Required the invoiced price per dozen. *Ans.* \$2.16.
147. Three merchants have an interest in a steam vessel; A puts in \$960 for 6 months; B, a sum unknown, for 12 months; C, \$640 for a time not known when the accounts were settled; A received \$1200 for his share, stock and profit; B, \$2400 for his, and C, \$1040 for his. What was B's stock, and C's time? *Ans.* B's stock, \$1600; C's time, 15 mo.
148. Merrill, Wells and Roche were partners in the grain business; Merrill had invested  $\frac{1}{3}$ , Wells  $\frac{1}{4}$ , and Roche  $\frac{1}{6}$  of the capital. They were to share equally the gains or losses. The business not being successful, the partnership was dissolved at the close of the year, when the resources of the firm were found to be: Cash, \$1785; barley on hand, \$2500; corn, \$1752; rye, \$350; oats, \$1650; wheat, \$5000. The liabilities were: Notes outstanding, \$1562; they owed S. Myler, \$1200, and P. Riley, \$1875. The net losses were \$750. What was the net capital of the firm at commencing, and what was each partner's net capital?

ng 18 qt. of  
0 lb. each in  
pounds each  
rovided each

Ans. 33.  
the gain of  
rate per an-  
rate 10 %.

p. Andrews  
end of the  
hilds put in  
Baker each  
end of the  
r joint prop-  
Andrews',

24.  
141 %, and  
rmer paying  
rate of in-  
ns.-6 1/2 %.  
6 sterling,

owing bills  
t; Dec. 15,  
, on 4 mo.;  
l settle the  
\$1760.10.  
and freight  
n be paid in  
om London

er, at 30 %,  
price per  
s. \$2.16.  
el; A puts  
; C, \$640  
A received  
d C, \$1040

, 15 mo.  
n business;  
tal. They  
not being  
the year,  
1785; bar-  
0; wheat,  
they owed  
were \$750.  
what was







