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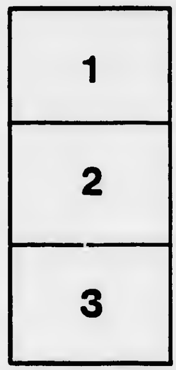
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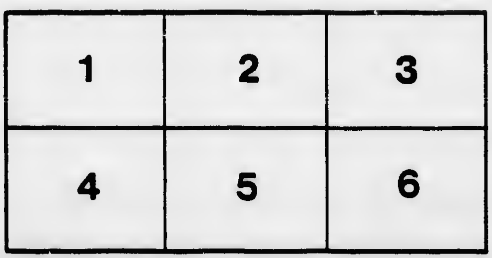
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$a : b = c : d$

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PRI

ARITHMETIC,

For use in Colleges and Schools,

ADAPTED

TO THE DECIMAL SYSTEM OF CURRENCY,

FROM THE ARITHMETIC OF BARNARD SMITH, Esq., M. A.,
Fellow and Tutor of St. Peter's College, Cambridge,

BY THE

REV. A. P. MORRIS, M. A., Oxford,
Principal,
And Mr. JAMES RAVEN, late Mathematical Master,
OF THE GRAMMAR SCHOOL, HAMILTON.

HAMILTON:
PRINTED AND PUBLISHED BY GILLESPY & ROBERTSON.
1860.

If you take $\frac{a}{b} \times \frac{c}{d}$. The definition is In Multiplication ~~[illegible]~~ you perform the same operations on the Multiplicand ~~[illegible]~~ [illegible] did on the unity!

PREFACE.

THE acknowledged superiority of the Treatise on Arithmetic, by MR. BARNARD SMITH, over any that have preceded it from the English press, induced the compilers of the present work to undertake its adaptation to the Decimal System of Currency which is now in use in this Province.

The excellence of the original consisted in its clear explanation of principles and in the numerous examples worked out under each Rule, which enabled the Student to see at a glance the application of the reasoning in each case. In the words of the late learned DR. PEACOCK: "The Rules in this Arithmetic are stated with great clearness. The Examples are well selected, and worked out with just sufficient detail, without being encumbered by too minute explanations; and there prevails throughout it that just proportion of theory and practice which is the crowning excellence of an elementary work."

These characteristics have been retained in the present work; and while its adaptation to the Decimal System has been made complete, care has been taken to retain sufficient examples of the English Currency to enable the Student to become thoroughly familiar with that method of computing money, rendered so necessary by the commercial relations of the two countries.

The present opportunity is taken of thanking MR. MCCALLUM, Principal of the Central School, Hamilton, for a valuable addition to the article on the Double Rule of Three, and for several suggestions which have been adopted in this book.

A. P. MORRIS.

HAMILTON, C.W.
17th January, 1860.

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ARITHMETIC.

DEFINITIONS, NOTATION AND NUMERATION.

ARTICLE 1. By a UNIT is meant a single object or thing, considered as one and undivided.

2. NUMBER is the name by which we signify how many objects or things are considered, whether *one* or *more*. When, for instance, we speak of one horse, two apples, three yards, or four hours, the number of the things referred to will be one, two, three, or four, according to the case; and so one, two, three, four, and the rest, are called numbers.

3. NUMBERS are considered either as ABSTRACT or CONCRETE.

Abstract numbers are those which have no reference to any particular kind of unit; thus, five, as an abstract number, signifies five units only, without any regard to particular objects.

Concrete numbers are those which have reference to some particular kind of unit; thus, when we speak of five hours, six yards, seven horses, the numbers five, six, seven, are said to be concrete numbers, having reference to the particular units one hour, one yard, one horse, respectively.

4. ARITHMETIC is the science of Numbers.

5. All numbers in common Arithmetic are expressed by means of the figure 0, commonly called zero or a cypher, which has no value in itself, and nine significant figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, which denote respectively the numbers one, two, three, four, five, six, seven, eight, nine. These ten figures are sometimes called DIGITS; but this name is often improperly limited to the nine significant figures above mentioned, which are then called the nine digits.

The number one, which is represented by the figure 1, is called UNITY.

6. When any of these figures stands by itself, it expresses its simple or intrinsic value: thus, 9 expresses nine abstract units, or nine particular things: but when it is followed by another figure,

it then expresses ten times its simple value; thus, 94 expresses ten times nine units, together with four units more: when it is followed by two figures, it then expresses one hundred times its simple value; thus, 943 expresses one hundred times nine units, together with ten times four units, and also three units more: and so on by a tenfold increase for each additional figure that follows it.

The value, which thus belongs to a figure in consequence of its position or place, is called its LOCAL VALUE.

Therefore all numbers have a simple or intrinsic value, and also a local value.

7. It appears then, that in common Arithmetic we proceed towards the left from units to tens of units; from tens of units to tens of tens of units, or hundreds of units; from hundreds of units to tens of hundreds of units, or thousands of units; from thousands of units to tens of thousands of units; from tens of thousands of units to tens of tens of thousands of units, that is, to hundreds of thousands of units; thence to tens of hundreds of thousands of units, or millions of units; thence to tens of millions of units, hundreds of millions of units, &c., till we come to millions of millions of units, which are called billions of units, and so on to trillions, quadrillions, &c.

Thus, 10 represents one ten of units, together with no units; or, as it is briefly read, ten. 11 represents one ten of units, together with one unit; or, as it is briefly read, eleven. Similarly 12, 13, 14, 15, 16, 17, 18, 19, respectively represent one ten of units together with two, three, four, five, six, seven, eight, nine units; they are respectively read twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

The next ten numbers are expressed by 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, which respectively represent two tens of units together with no, one, two, three, four, five, six, seven, eight, nine units; they are briefly read twenty, twenty-one, twenty-two, twenty-three, twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine.

The next ten numbers are expressed by 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, which are respectively read thirty, thirty-one, thirty-two, thirty-three, thirty-four, thirty-five, thirty-six, thirty-seven, thirty-eight, thirty-nine: we thus arrive at 40 (forty), 50 (fifty), 60 (sixty), 70 (seventy), 80 (eighty), 90 (ninety.)

99 is the largest number which can be expressed by two figures, since it represents nine tens of units together with nine units; the next number to this is 100, which represents ten tens of units, or

one hundred of units, together with no tens of units, together with no units; or, as it is briefly read, one hundred.

By pursuing the same system in higher numbers the figure occupying the fourth place from the right hand will represent so many tens of hundreds of units, or thousands of units; the figure in the fifth place will represent so many tens of thousands of units; and so on.

205 represents two hundreds of units, together with no tens of units, together with five units; or, as it is briefly read, two hundred and five.

5473 represents five thousands of units, together with four hundreds of units, together with seven tens of units, together with three units; or, as it is briefly read, five thousand, four hundred and seventy-three.

7040730 represents seven millions of units, together with no hundreds of thousands of units, together with four tens of thousands of units, together with no thousands of units, together with seven hundreds of units, together with three tens of units, together with no units; or, as it is briefly read, seven millions, forty thousand, seven hundred and thirty.

107834265 represents one hundred of millions of units, together with no tens of millions of units, together with seven millions of units, together with eight hundreds of thousands of units, together with three tens of thousands of units, together with four thousands of units, together with two hundreds of units, together with six tens of units, together with five units; or, as it is briefly read, one hundred and seven millions, eight hundred and thirty-four thousand, two hundred and sixty-five.

8. Notation (*notare*, to make a sign) is the art of expressing any number of figures which is already given in words. Numeration (*numerare*, to number) is the converse of Notation, being the art of expressing any number in words which is already given in figures.

9. The method above explained of denoting numbers by means of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and combinations of them, was brought into Europe by the Arabs, and it is therefore often called the Arabic Notation. It was derived by the Arabs from the Hindoos. This method of notation is now in common use.

Ex. I.

Exercises in Notation and Numeration.

Express the following numbers in figures:

(1) Sixty-three; eighty-one; ninety-nine; forty; thirteen.

(2) Two hundred; three hundred and three; seven hundred and sixty-four; eight hundred and eighty-eight.

(3) Four thousand; one thousand, four hundred and seventy-one; six thousand, nine hundred and thirty; nine thousand and nine.

(4) Twenty-seven thousand, five hundred and four; thirty-three thousand; nine thousand and sixteen.

(5) One hundred thousand; six hundred and seventy-six thousand and fifty; two hundred and two thousand, five hundred and ninety-three.

(6) Seven millions, three thousand; eleven millions, one hundred and eight thousand, one hundred and six; fifty-four millions, fifty-four thousand and eighty-eight; six hundred and thirteen millions, twenty thousand, three hundred and three.

(7) Two billions; nine billions, three hundred thousand and twenty-one; ninety-four billions, ninety millions, ninety-four thousand, nine hundred and four.

Write down in words at full length the following numbers:

(1) 43; 60; 88; 97; 59; 12; 21; 19.
(2) 256; 401; 500; 999; 365; 578; 837.
(3) 2000; 1724; 3003; 7584; 1075; 4541.
(4) 37003; 47049; 63090; 80008; 341323.
(5) 6850406; 8080808; 7849630; 418254.
(6) 10000001; 20220022; 92568987; 30180070.
(7) 2560530200; 800309560; 9738413208.
(8) 7070000423; 987654321; 5707068080.
(9) 1001987000010090; 487268706341O3264.

ADDITION.

10. ADDITION is the method of finding a number, which is equal to two or more numbers taken together.

The number found by adding two or more numbers together is called the SUM or AMOUNT of the several numbers so added.

11. There are two kinds of Addition, SIMPLE and COMPOUND.

It is Simple Addition, when the numbers to be taken together are all abstract numbers; or when they are all concrete numbers of the same denomination, as *all* pence, *all* days, *all* pints.

It is Compound Addition, when the numbers to be taken together are concrete numbers of the same kind, but of different denominations of that kind; as pounds, shillings, and pence; or years, months, and days; or gallons, quarts, and pints.

12. The sign +, PLUS, placed between two or more numbers signifies that the numbers are to be added together: thus 2+5+7 signifies that 2, 5, and 7 are to be added together, and denotes their sum.

The sign =, EQUAL, placed between two numbers, signifies that the numbers are equal to one another.

The sign $\overline{\quad}$, VINCULUM, placed over numbers, and the sign ( ), or [ ], called a BRACKET, enclosing numbers within it, are used to denote that all numbers under the vinculum, or within the bracket, are equally affected by all numbers not under the vinculum or within the bracket: thus $\overline{2+3}$ or (2+3) or [2+3], each signify, that whatsoever is outside the vinculum or bracket which affects 2 in any way, must also affect 3 in the same way, and conversely.

The sign ∴ signifies 'therefore.'

SIMPLE ADDITION.

13. RULE. Write down the given numbers under each other, so that units may come under units, tens under tens, hundreds under hundreds, and so on; then draw a straight line under the lowest line.

Find the sum of the column of units; if it be under ten, write it down under the column of units, below the line just drawn; if it exceed ten, then write down the last figure of the sum under the column of units, and carry to the next column the remaining figure or figures; treat each succeeding column in the same way, and write down the full sum of the extreme left-hand column. The entire sum so marked down will be the sum or amount of the separate numbers.

14. Add together 5469, 743, and 27.

Proceeding by the Rule given above, we obtain

$$\begin{array}{r} 5469 \\ 743 \\ 27 \\ \hline 6239 \end{array}$$

The reason for the Rule will appear from the following considerations

When we take the sum of 7 units and 3 units and 9 units, we get 10 units and 9 units, or 19 units; we therefore place the 9 units under the column of units and carry on the 1 ten units to the next column, viz. the column of tens.

Now the sum of 1 ten, 2 tens, 4 tens, and 6 tens, is 10 tens and 3 tens, or 13 tens; we therefore place the 3 tens under the column of tens and carry on the 1 hundred units to the next column, viz. the column of hundreds.

Again the sum of 1 hundred, 7 hundreds, and 4 hundreds, is 10 hundreds and 2 hundreds, or 12 hundreds; we therefore place the 2 hundreds under the column of hundreds, and carry on the 1 thousand units to the next column, viz. the column of thousands.

Again, the sum of 1 thousand and 5 thousands, is 6 thousands; we therefore place the 6 under the column of thousands, and the entire sum is 6239.

15. The above example might have been worked thus, putting down at full length the local value of all the figures.

Thus $5469 = 5000 + 400 + 60 + 9$
$+743 = \quad +700 + 40 + 3$
$+27 = \quad +20 + 7.$

Now adding the columns, we get the sum

$$= 5000 + 1100 + 120 + 19$$
$$= 5000 + \overline{1000 + 100} + \overline{100 + 20} + \overline{10 + 9},$$

(since $1100 = 1000 + 100$, $120 = 100 + 20$, and $19 = 10 + 9$)

$$= 6000 + 200 + 30 + 9,$$

(collecting the thousands together, the hundreds together, and so on)

$$= 6239.$$

Note. The truth of all results in Addition may be proved by adding the columns first upwards as in the above example, and then adding them downwards; if the results be the same, the operation in each case will in all probability have been performed correctly.

Ex. II.

Examples in Simple Addition.

(1)	(2)	(3)	(4)
12	57	234	654
35	87	567	321
56	65	753	804
80	43	345	509
183		1899	

(5)	(6)	(7)	(8)
494	1721	750	4789
587	3333	36	2346
656	5046	1843	3857
336	2754	561	5005
		3190	

(9)
9102
479
8776
901

(10)
84670
5437
29
21904

(11)
1790621
206803
353
9003766

(12)
256783
21003
5734
40036
21
100001
423578

(13)
627432
543201
678641
548200
868759
345678

(14)
892764
93687
9482
100
152346
11

(15)
1807353
298743
5987
760003
247
50705

(16)
117064
92973
827569
351
777777
65656

(17) Add together 7384, 326, 6780, and 57; also 6740, 9745, 5769, 8031, 6543, 2002, and 9999; also 89, 4500, 423, 2024, 5408, 60546, and 9401.

(18) Add together 83746, 2478, 692577, 456, and 7; also 935473, 262, 13897, 598453, 25, 3734, 724008, and 649768.

(19) Find the sum of 4738685, 237869513, 148794343978, 865, 4647, and 250; also of 68539582, 78602045, 370489000, 7055591234, 276, 9123456789, and 5000; also of 888929944, 73600, 27978462, 333, 5875396006, 4827532, 486684836, 80032148379, 12345, 1112858673, and 53800000835.

(20) Add together one thousand, four hundred and eighty-three; seven hundred and ninety-six; thirty-nine; forty thousand, seven hundred and forty-four; five thousand, eight hundred and sixty; fifty thousand and seven.

(21) Add together the following numbers: fifteen thousand, seven hundred and ninety-six; four hundred and nine; two hundred and thirty-four thousand and fifty; four millions, three thousand and seventy-six; forty thousand and thirty-six; ten thousand, nine hundred and one.

(22) Add together the following numbers: twenty-two millions, six hundred thousand, five hundred and three; five hundred and sixty-three millions, seventy-six thousand and thirty-four; one hundred and eleven millions, six hundred and fifty thousand and fifty; three hundred and twenty-six millions, seven thousand, nine hundred and ninety-one; one thousand seven hundred and ten millions, one thousand seven hundred and ten; one billion, three hundred thousand and five.

SUBTRACTION.

16. Subtraction is the method of finding what number remains when a smaller number is taken from a greater number.

The number found by subtracting the smaller of two numbers from the greater is called the Remainder.

17. There are two kinds of Subtraction, Simple and Compound, which differ from each other in precisely the same way, in which Simple and Compound Addition differ from each other.

18. The sign —, minus, placed between two numbers, signifies that the second number is to be subtracted from the first number.

SIMPLE SUBTRACTION.

19. Rule. Place the less number under the greater number, so that units may come under units, tens under tens, hundreds under hundreds, and so on; then draw a straight line under the lower line.

Take, if possible, the number of units in each figure of the lower line from the number of units in each figure of the upper line which stands immediately over it, and put the remainder below the line just drawn, units under units, tens under tens, and so on: but if the units in any figure in the lower line exceed the number of units in the figure above it, add ten to the upper figure, and then take the number of units in the lower figure from the number in the upper figure thus increased; put the remainder down as before, and then carry one to the next figure of the lower line. The entire difference or remainder, so marked down, will be the difference or remainder of the given numbers.

20. Ex. Subtract 4938 from 5123.

Proceeding by the Rule given above, we obtain

$$\begin{array}{r} 5123 \\ 4938 \\ \hline 185 \end{array}$$

so that the remainder is one hundred and eighty-five (185).

The reason for the Rule will appear from the following considerations.

We cannot take 8 units from three units, we therefore add 10 units to the 3 units, which are thus increased to 13 units; and taking 8 units from 13 units we have 5 units left; we therefore place 5 under the column of units: but having added 1 ten units to the upper number, we must add the same number of units (1 ten units) to the lower number, so that the difference between the two numbers may not be altered; and adding 1 ten units to the 3 ten units in the lower number, we obtain 4 tens or 40 instead of 3 tens or 30.

Again, we cannot take 4 tens from 2 tens; we therefore add 10 tens or 1 hundred to the 2 tens, which thus become 12 tens or 120; and then taking 4 tens or 40 from 12 tens or 120, we have 8 tens or 80 remaining; we therefore place 8 under the column of tens: but having added 1 hundred to the upper number, we must add 1 hundred to the lower number for the reason given above; and adding 1 hundred to the 9 hundreds in the lower number, we obtain 10 hundreds or 1000 instead of 900.

Again, we cannot take 10 hundreds from 1 hundred, and we therefore add 10 hundreds or 1 thousand to the 1 hundred, which thus becomes 11 hundreds or 1100: and taking 10 hundreds or 1000 from 11 hundreds or 1100, we have 1 hundred or 100 left; we therefore place 1 under the column of hundreds: but having added 10 hundreds or 1 thousand to the upper number, we must add 1 thousand to the lower number for the reason given above; and adding 1 thousand to the 4 thousands in the lower number, we obtain 5 thousands or 5000;

5000 taken from 5000 leaves 0;

therefore the whole difference or remainder is 185.

21. The above Example might have been worked thus, putting down at full length the local values of the figures:

$$\begin{aligned} 5123 &= \underline{5000} + 100 + \underline{20} + 3 \\ &= 4000 + 1000 + 100 + 20 + 3 \\ &= 4000 + 1000 + 100 + 10 + 10 + 3 \\ &= 4000 + 1000 + 110 + 13 \end{aligned}$$

(collecting the first 10 with the 100, and the second 10 with the 3,)

$$4938 = 4000 + 900 + 30 + 8.$$

Therefore, subtracting the columns, thousands from thousands, &c. we get the remainder or difference

$$= 100 + 80 + 5 = 185.$$

Note. The truth of all results in Subtraction may be proved by adding the less number to the difference or remainder; if this sum equals the larger number, the result obtained by subtraction may be presumed to be correct.

Ex. III.

Examples in Simple Subtraction.

(1)	(2)	(3)	(4)
663	976	704	806
580	531	483	720
83			

(5)	(6)	(7)	(8)
4236	80502	46095	555555
3089	38672	28736	123456

(9)	(10)	(11)
1000000	400357261	89437182
100101	99988877	15790293

(12) Find the difference between 6543756 and 412848; 7863927 and 826957; 303233334 and 192001222.

(13) How much greater is 164326289 than 48476798?
" " " 10000001000 than 7077070077?
" " " 7559030640021 than 6990040005679?

(14) Take two thousand and nine, from ten thousand and ninety-six; three thousand and eight, from seven thousand, nine hundred forty-four.

(15) Required the difference between four and four millions; also between one hundred millions and three hundred thousand.

(16) Subtract five hundred and eighty-four thousand and seventy-six, from fifteen millions, one hundred thousand and three.

---

22. The following method of expressing numbers was used by the Romans, and it is still in occasional, though not in common use, among ourselves. They represented the number one by the character I; five by V; ten by X; fifty by L; one hundred by C; five hundred by D or IↃ; one thousand by M or CIↃ.

All other numbers were formed by a combination of the above characters, subject to the following Rules:

First; When a character was *followed* by one of *equal or less* value, the whole expression denoted the *sum* of the values of the single characters; for instance, II stood for 2; III for 3; VI for 6; VIII for 8: LV for 55; LXXVII for 77; CCXI for 211.

Secondly; When a character was *preceded* by one of *less* value, the whole expression denoted the *difference* of the values of the single characters; for instance, IV stood for $5-1$, or 4; IX for $10-1$, or 9; XIX for $10+\overline{10-1}$, or 19; XL for $50-10$, or 40; XC for $100-10$, or 90.

Thirdly; Every Ↄ annexed to IↃ increased the value of the latter tenfold; for instance, IↃↃ stood for 5000; IↃↃↃ for 50000, and so forth. And every C prefixed and Ↄ annexed to CIↃ, increased the value of the latter tenfold; for instance, CCIↃↃ stood 10000; CCCIↃↃↃ for 100000; and so forth.

Fourthly; A line drawn over a character or characters increased the value of the latter a thousandfold; for instance, $\overline{\text{V}}$ stood for 5000; $\overline{\text{C}}$ for 100000; $\overline{\text{IX}}$ for 9000; and so forth.

It follows then that either XXXXVI or XLVI will represent 46: and that either M.DCCC.LIV, or CIↃ.IↃCCCLIV, or $\overline{\text{I}}$.DCCCLIIII will represent 1854.

Ex. IV.

(1) Express in Roman characters, thirty; forty-eight; fifty-nine; 222; 6000; 1843.

(2) Express in words, and also in Arabic figures, the values of XXIII; LXIX; CCXVIII; $\overline{\text{VI}}$; $\overline{\text{CL}}$DCIII; $\overline{\text{MM}}$C.

MULTIPLICATION.

23. Multiplication is a short method of finding the sum of any given number repeated as often as there are units in another given number: thus, when 3 is multiplied by 4, the number produced by the multiplication is the sum of 3 repeated 4 times, which sum is equal to $3+3+3+3$ or 12.

The number to be repeated or added to itself, is called the Multiplicand.

The number which shows how often the multiplicand is to be repeated or added to itself, is called the Multiplier.

The number found by multiplication is called the Product.

The multiplicand and multiplier are sometimes called 'Factors,' because they are factors or makers of the product.

24. Multiplication is of two kinds, Simple and Compound. It is termed Simple Multiplication, when the multiplicand is either an abstract number, or a concrete number of one denomination.

It is termed Compound Multiplication, when the multiplicand contains numbers of more than one denomination, but all of the same kind.

25. The sign ×, placed between two numbers, signifies that the numbers are to be multiplied together.

26. The following Table ought to be learned correctly:

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

In the above Table, the second line from the top shews the product of each of the numbers, 1, 2, 3, 4, &c. 11, 12, in the first line, when multiplied by 2; the several products being placed under the respective numbers of the line above, from the multiplication of which they arise: the third line shews the several products, when the figures in the first line are respectively multiplied by 3, and so on.

Note. One of the factors, namely the multiplier, must necessarily be an 'abstract number'; since it would be absurd to speak of 6 shillings multiplied by 4 shillings. We can multiply 6 shillings by 4, *i. e.* we can find how many shillings there are in four times six shillings; but there is no meaning in 6 shillings multiplied by 4 shillings.

SIMPLE MULTIPLICATION.

27. Rule. Place the multiplier under the multiplicand, units under units, tens under tens, and so on. Multiply each figure of the multiplicand, beginning with the units, by the figure in the units' place of the multiplier (by means of the table given for Multiplication); set down and carry as in Addition. Then multiply each figure of the multiplicand, beginning with the units, by the figure in the tens' place of the multiplier, placing the first figure so obtained under the tens of the line above, the next figure under the hundreds, and so on. Proceed in the same way with each succeeding figure of

the multiplier. Then add up all the results thus obtained, by the rule of Simple Addition.

Note. If the multiplier does not exceed 12, the multiplication can be effected easily in one line, by means of the Table given above.

28. Ex. Multiply 7654 by 397.

Proceeding by the Rule given above, we obtain

```
   7654
    397
  -----
  53578
 68886
22962
-------
3038638
```

The reason for the Rule will appear from the following considerations.

When 7654 is to be multiplied by 7, we first take 4 seven times, which by the Table gives 28, *i. e.*, 8 units and 2 tens; we therefore place down 8 in the units' place and carry on the 2 tens: again, 5 tens taken 7 times give 35 tens, to which add 2 tens, and we obtain 37 tens, or 7 tens and 3 hundreds; we put down 7 in the tens' place, and carry on 3 hundreds: again, 6 hundreds taken 7 times give 42 hundreds, to which add 3 hundreds, and we obtain 45 hundreds, or 4 thousands and 5 hundreds; we put down 5 in the hundreds' place, and carry on the 4 thousands: again, 7 thousands taken 7 times give 49 thousands, to which we add the 4 thousands, thus obtaining 53 thousands, which we write down.

Next, when we multiply 7654 by the 9, we in fact multiply it by 90; and 4 units taken 90 times give 360 units, or 3 hundreds, 6 tens, and 0 units: therefore, omitting the cypher, we place the 6 under the tens' place, and carry on the 3 to the next figure, and proceed with the operation as in the line above.

When we multiply 7654 by the 3, we in fact multiply by 300; and 4 multiplied by 300 gives 1200, or 1 thousand, 2 hundreds, 0 tens, and 0 units; therefore, omitting the cyphers, we place the first figure 2 under the hundreds' place, and proceed as before. Then adding up the three lines of figures which we have just obtained, we obtain the product of 7654 by 397.

29. The above Example might have been worked thus, putting down at full length the local values of the figures:

$$
\begin{array}{r}
7654 = 7\times1000 + 6\times100 + 5\times10 + 4 \\
397 = 3\times100 + 9\times10 + 7 \\
\hline
49\times1000 + 42\times100 + 35\times10 + 28 \\
63\times10000 + 54\times1000 + 45\times100 + 36\times10 \\
21\times100000 + 18\times10000 + 15\times1000 + 12\times100 \\
\hline
21\times100000 + 81\times10000 + 118\times1000 + 99\times100 + 71\times10 + 28
\end{array}
$$

which =

$$
\begin{array}{l}
20\times100000 + 1\times100000 \\
\quad + 8\times100000 + 1\times10000 \\
\quad + 1\times100000 + 1\times10000 + 8\times1000 \\
\qquad + 9\times1000 + 9\times100 \\
\qquad\quad + 7\times100 + 1\times10 \\
\qquad\qquad + 2\times10 + 8 \\
\hline
2000000 + 10\times100000 + 2\times10000 + 17\times1000 + 16\times100 + 3\times10 + 8 \\
= 2000000\times1000000 + 2\times10000 + 10\times1000 + 7\times1000 + 10\times100 + 6\times100 + 3\times10 + 8 \\
= 3000000 + 2\times10000 + 1\times10000 + 7\times1000 + 1\times1000 + 6\times100 + 3\times10 + 8 \\
= 3000000 + 3\times10000 + 8\times1000 + 600 + 30 + 8 \\
= 3000000 + 30000 + 8000 + 600 + 30 + 8 \\
= 3038638
\end{array}
$$

30. If the multiplier or multiplicand, or both, end with cyphers, we may omit them in the working; taking care to affix to the product as many cyphers as we have omitted from the end of the multiplier or multiplicand, or both. Thus, if 263 be multiplied by 6200, and 570 be multiplied by 3200, we have

$$
\begin{array}{r}
263 \\
6200 \\
\hline
526 \\
1578 \\
\hline
1630600
\end{array}
\qquad\qquad
\begin{array}{r}
570 \\
3200 \\
\hline
114 \\
171 \\
\hline
1824000
\end{array}
$$

The reason is clear: for in the first case, when we multiply by the 2, in fact we multiply by 200; and 3 multiplied by 200 gives 600: in the second case, the 7 multiplied by the 2 is the same as 70 multiplied by 200; and 70 multiplied by 200 gives 14000.

31. If the MULTIPLIER contain any cypher in any other place, then, in multiplying by the different figures of the multiplier we may pass over the cypher; taking care, however, when we multiply by the next figure, to place the first figure arising from that multiplication under the third figure of the line above instead of the second figure. The reason of this is clear; for, if we were multiplying by 206, when we multiply by the 6 we take the

multiplicand 6 times, when we multiply by the 2, we really take the multiplicand, not 20 times, but 200 times.

32. When two numbers are to be multiplied together, it is a matter of indifference, so far as the product is concerned, which of them be taken as the multiplicand or multiplier; in other words, the product of the first multiplied by the second, will be the same as the product of the second multiplied by the first.

Thus, $2\times4=2+2+2+2=8,$
$4\times2=4+4 \qquad =8;$

therefore the results are the same, that is, $2\times4=4\times2$.

That the product of one number multiplied by another will be equal to the product of the latter multiplied by the former, may perhaps appear more clearly from the following mode of shewing this equality in the case of the numbers 3 and 5.

$$3=1+1+1;$$

$$\therefore 3\times5=(1+1+1)+(1+1+1)+(1+1+1)+(1+1+1)+(1+1+1)$$
$$\left.\begin{array}{l}=1+1+1\\ +1+1+1\\ +1+1+1\\ +1+1+1\\ +1+1+1\end{array}\right\}=15.$$

Now, if we regard the *ones* from left to right, there are 3 *ones* taken 5 times; if we regard them taken from top to bottom, we have 5 *ones* repeated 3 times; and the number of ones in each case is the same; *i. e.*, $3\times5=5\times3$: and so in the case of any two other numbers multiplied together.

33. The truth of all results in Multiplication may be proved by using the multiplicand as multiplier, and the multiplier as multiplicand: if the product thus obtained be the same as the product found at first, the results are in all probability true.

34. We have hitherto confined our attention to products formed by the multiplication of two factors only. Products may however arise from the multiplication of three or more factors; this is termed CONTINUED MULTIPLICATION: thus, $2\times3\times4$ denotes the continued multiplication of the factors 2, 3, and 4; and means that 2 is to be first multiplied by 3, and the product thus obtained to be then multiplied by 4. The result of such a process would be 24, which is therefore the continued product of 2, 3, and 4: we may express it thus, $2\times3\times4=24$.

Ex. V.

Examples in Simple Multiplication.

(1) 534
4
2136

(2) 673
3

(3) 2867
5

(4) 7492
6

(5) 2057
7

(6) 57409
8

(7) 2745638
9

(8) 5763
11
63393

(9) 35976
11

(10) 91525
12

(11) 257
53
771
1285
13621

(12) 96843
17

(13) 87298
46

(14) 16097
59

(15) 296897
83

(16) 69284
90
6235560

(17) 840607
80

(18) 175
189

(19) 6298
769

(20) 5423
603
16269
32538
3270069

(21) 25607
5004

(22) 78847
8803

(23) Find the product of 234578 by 18, by 29, and also by 53; of 924846 by 67, by 95, and also by 430; 2846067 by 206, by 1008, and also by 907; 8409631 by 21711, by 7009, by 8435, and also by 7980.

(24) Find the product of 1754 and 9306; of 47506 and 4500; of 149570 and 15790; of 554768 and 39314; of 815085 and 20048; 123456789 and 987654321; and of 57298492692 and 700809050321.

(25) Multiply 9487352 by 4731246; 4342760 by 599999; 17376872 by 7399078; 38015732 by 400700065; 574585614865 by 2837154309.

(26) Multiply six hundred and fifty thousand and ninety, by three thousand and eight; also seventy-six millions, eight thousand, seven hundred and sixty-five, by nine millions, nine thousand and nine.

(27) Find the continued product of 12, 17, and 19; of 3781, 3782, and 3783; and of 6565, 6786, and 9898.

(28) Multiply 20470 by 1030, and 2958 by 476, explaining the reason of each step in the process.

DIVISION.

35. Division is the method of finding how often one number, called the Divisor, is contained in another number, called the Dividend. The result is called the Quotient.

36. Division is of two kinds, Simple and Compound. It is called *Simple* Division, when the dividend and divisor are, both of them, either abstract numbers, or concrete numbers of one and the same denomination.

It is called *Compound* Division, when the dividend, or when both divisor and dividend contain numbers of different denominations, but of one and the same kind.

37. The sign ÷, placed between two numbers, signifies that the first is to be divided by the second.

38. In Division, if the dividend be a concrete number, the divisor may be either a concrete number or an abstract number, and the quotient will be an abstract number or a concrete number, according as the divisor is concrete or abstract. For instance, 5 tons taken 6 times give 30 tons, therefore 30 tons divided by 5 tons, give the abstract number 6 as quotient; and 30 tons divided by 6, give the concrete number 5 tons as quotient.

SIMPLE DIVISION.

39. Rule. Place the divisor and dividend thus:

divisor) dividend (quotient.

Take off from the left-hand of the dividend the least number of figures which make a number not less than the divisor; then find by the Multiplication Table, how often the first figure on the left-hand side of the divisor is contained in the first figure, or the first two figures, on the left-hand side of the dividend, and place the figure which denotes this number of times in the quotient: multiply the divisor by this figure, and bring down the product, and subtract it from the number which was taken off at the left of the dividend: then bring down the next figure of the dividend, and place it to the right of the remainder, and proceed as before; if the divisor be greater than this remainder, affix a cypher to the quotient, and bring down the next figure from the dividend to the right of the remainder, and proceed as before. Carry on this operation till all the figures of the dividend have been thus brought down, and the quotient, if there be no remainder, will be thus determined, or if there be a remainder, the quotient and the remainder will be thus determined.

Note 1. If any product be greater than the number which stands above it, the last figure in the quotient must be changed for one of smaller value: but if any remainder be greater than the divisor, or equal to it, the last figure of the quotient must be changed for a greater.

Note 2. If the divisor does not exceed 12, the division can easily be effected in one line, by means of the Multiplication Table.

40. Ex. Divide 2338268 by 6758.

Proceeding by the Rule given above we obtain,

```
6758) 2338268 (346
      20274
      -----
       31086
       27032
       -----
        40548
        40548
        -----
```

Therefore the quotient is 346.

The reason for the Rule will appear from the following considerations:

The divisor represents six thousand, seven hundred and fifty-eight; the first five figures on the left-hand side of the dividend represent two millions, three hundred and thirty-eight thousand, and two hundred.

Now the divisor is contained in this 300 times; and $6758 \times 300 = 2027400$, or omitting the two cyphers at the end for convenience in working, we properly place the 4 under the 2 in the line above; we subtract the product thus found, and we obtain a remainder of 3108, which represents three hundred and ten thousand, and eight hundred. Bring down the 6 by the Rule; this 6 denotes 6 tens or 60, but the cypher is omitted for the reason above stated: the number now represents three hundred and ten thousand, eight hundred and sixty: 6758 is contained 40 times in this, and $6758 \times 40 = 270320$; we omit the cypher at the end as before, and subtract the 27032 from the 31086; and after subtraction the remainder is 4054, which represents forty thousand, five hundred and forty. Bring down the 8 by the Rule, and the number now represents forty thousand, five hundred and forty-eight: 6758 is contained 6 times exactly in this number.

Therefore 346 is the quotient of 2338268 by 6758.

41. The above example worked, without omitting the cyphers, would have stood thus:

```
6758)2338268(300+40+6.
     2027400
     -------
      310868
      270320
      ------
       40548
       40548
       -----
```

hence it appears that the divisor is subtracted from the dividend 300 times, and then 40 times from what remains, and then 6 times from what then remains, and there being now no remainder, 6758 is contained exactly 346 times in 2338268.

The truth of the above method might have been shown as follows:

```
        2338268=2027400+270320+40548
   6758)2027400+270320+40548(300+40+6
        2027400
        -------
               +270320
               +270320
               -------
                       +40548
                       +40548
                       ------
```

42. Ex. Divide 56438971 by 4064.

```
4064) 56438971 (13887
      4064
      -----
      15798
      12192
      -----
       36069
       32512
       -----
        35577
        32512
        -----
         30651
         28448
         -----
          2203
```

therefore 4064 is contained in 56438971, 13887 times, with the remainder 2203.

43. *If the divisor terminate with cyphers, the process can be abridged by the following Rule.*

Rule. Cut off the cyphers from the divisor, and as many figures from the right-hand of the dividend, as there are cyphers so cut off at the right-hand end of the divisor; then proceed with the remaining figures according to the Rule, Art. (39); and to the last remainder annex the figures cut off from the dividend for the total remainder.

Ex. Divide 537523 by 3400.

Proceeding by the Rule,

```
34,00) 5375,23 (158
       34
       ---
       197
       170
       ---
        275
        272
        ---
          3
```

therefore, 3400 is contained in 537523, 158 times, with remainder 323.

The reason for the Rule will appear from the following considerations:

537523 is 5375 hundreds and 23, of which 537500 contains 3400, 158 times, with a remainder 300 over; and as 23 does not contain 3400 at all, the quotient will evidently be 158, with remainder 300+23, or 323.

Note. The same Rule applies when the divisor and dividend both terminate with cyphers.

44. Definitions. A number which cannot be separated into factors, which are respectively greater than unity, is called a prime number. Thus 3, 5, 7, 11, 13, are prime numbers.

A number which can be separated into factors respectively greater than unity, or which, in other words, is produced by multiplying together two or more numbers respectively greater than unity, is called a composite number. Thus 4 which $=2\times 2$, 6 which $=2\times 3$, 8 which $=2\times 2\times 2$, are composite numbers; because they are composed or consist of the product of two or more numbers, each of which is greater than unity.

Numbers which have no common factor greater than unity, are said to be prime to one another. Thus the numbers 3, 5, 8, 11, are prime to each other.

45. When the divisor is a composite number, and made up of two factors, neither of which exceeds 12, the dividend may be divided by one of the factors in the way of Short Division, and then the result by the other factor: if there be a remainder after each of these divisions, the true remainder will be found by multiplying the second remainder by the first divisor, and adding to the product the first remainder.

Ex. Divide 56732 by 45.

$$45\left\{\begin{array}{l|l} 9 & \underline{56732} \\ 5 & \underline{6303}-5 \\ & 1260-3 \end{array}\right.$$

the total remainder is $9\times3+5$, or $27+5=32$.

Therefore the quotient arising from the division of 56732 by 45 is 1260, with a remainder 32 over.

The reason for the above Rule is manifest from the following considerations.

6303 is 5 times 1260 together with 3,
and 56732 is 9 times 6303 together with 5,
or is 9 times (5 times 1260+3,) together with 5,
or is 45 times 1260+27+5,
or is 45 times 1260+32.

46. The accuracy of results in Multiplication is often tested by the following method, which is termed "CASTING OUT THE NINES": add together all the figures in the multiplicand, divide their sum by 9, and set down the remainder; then divide the sum of the figures in the multiplier by 9, and set down the remainder: multiply these remainders together, and divide their product by 9, and set down the remainder: if this remainder be the same as the remainder which results after dividing the product, or the sum of the digits in the product, of the multiplicand and multiplier by 9, the sum is very probably right; but if different, it is sure to be wrong.

This test depends upon the fact that "if any number and the sum of its digits be each divided by 9, the remainders will be the same." The proof of which may be shewn thus:

$$100=99+1,$$

where the remainder must be one, whether 100, or the sum of the digits in 100, viz. 1, be divided by 9, since 99 is divisible by 9 without a remainder.

Similarly,
$$200=2\times 99+2,$$
$$300=3\times 99+3,$$
$$400=4\times 99+4,$$
$$500=5\times 99+5,$$
&c. = &c.

Hence it appears that if 100, 200, 300, 400, 500, &c. be each divided by 9, and the sum of the digits making up the respective numbers be also divided by 9, the two remainders in each case will be the same.

Also the number
$$\begin{aligned}532&=500+30+2\\&=5\times 100+3\times 10+2\\&=\overline{5\times 99+5}+\overline{3\times 9+3}+2;\end{aligned}$$
whence it appears that if the parts 5×100, 3×10, and 2, which make up the entire number, be each divided by 9, the remainders will be 5, 3, 2 respectively; and therefore the remainder, when 532 is divided by 9, will clearly be the same, as when $5+3+2$ is divided by 9.

To explain why the test holds, let us take as an example 533 multiplied by 57.

$$\begin{array}{r}533\\57\\\hline 3731\\2665\\\hline 30381\end{array}$$

Now
$$533=9\times 59+2=531+2$$
$$57=9\times 6+3=54+3$$

It is clear, since 531 contains 9 without a remainder, that 531×57 contains 9 without a remainder; therefore the remainder which is left after dividing the product of 533 and 57 by 9, must be the same as the remainder which is left after dividing the product of 2 and 57 by 9.

Again, since the product of 57 and $2=(54+3)\times 2$, and the product of 54 and 2 when divided by 9 leaves no remainder, therefore the remainder which is left after dividing the product of 533 and 57 by 9, must be the same as the remainder left after dividing the product of 3 and 2 by 9, *i. e.*, after dividing the product of the remainders which are left after the division of the multiplicand and multiplier respectively by 9.

Now, on dividing either 30381, or the sum of its digits, which is 15, by 9, the remainder left is 6, and 3×2 divided by 9, also leaves 6 as a remainder. Therefore we conclude that 30381 is the correct product of 533 and 57.

Note. If an error of 9, or any of its multiples, be committed, the results will nevertheless agree, and so the error in that case remains undetected.

Ex. VI.

Examples in Simple Division.

(1) 456÷2.
(2) 90680÷2.
(3) 261070308÷2.
(4) 6378÷3.
(5) 470850÷3.
(6) 385734÷3.
(7) 372096÷4.
(8) 47392488÷4.
(9) 337625÷5.
(10) 9876540÷5.
(11) 890106÷6.
(12) 3782046÷6.
(13) 623399÷7.
(14) 78432407÷7.
(15) 164864÷8.
(16) 3812312÷8.
(17) 7869231÷9.
(18) 39237840÷9.
(19) 407792÷11.
(20) 91875342÷11.
(21) 211632÷12.
(22) 43600391÷12.
(23) 4045860÷13.
(24) 786543318÷17.
(25) 1234560÷20.
(26) 8224776÷18.
(27) 14683059÷27.
(28) 817286228÷44.
(29) 54906734÷59.
(30) 6848734752÷96.
(31) 70865432÷87.
(32) 649305745÷55.
(33) 28894545÷123.
(34) 433418175÷615.
(35) 1674918÷189.
(36) 31884740÷779.
(37) 536819741÷907.
(38) 1111111111111÷50160.
(39) 8235460800÷1440.
(40) 57380625÷7575.
(41) 353008972662÷5406.
(42) 599961567212÷2468.
(43) 26799534687÷7890000.
(44) 57111104051÷3851.
(45) 10000000000000000÷1111, and also by 11111.
(46) 634394567÷164600.
(47) 67157148372÷90009.
(48) 1220225292÷200563.
(49) 7428927415293÷8496427.
(50) 60435674536845÷79094451.
(51) 65358547823÷5578.
(52) 3968901531620÷687637943.

(53) Divide 152181255 by 3854, and explain the process.

(54) Divide 143255 by 4093. Explain the operation, and shew that it is correct.

(55) Divide 203534191 by 72, first by Long Division, and then by its factors 8 and 9; and show that the results in both cases coincide.

GREATEST COMMON MEASURE.

47. A MEASURE of any given number is a number which will divide the given number exactly, *i. e.* without a remainder.

Thus, 2 is a measure of 6, because 2 is contained 3 times exactly in 6.

When one number is a measure of another, the former is said to measure the latter.

48. A MULTIPLE of any given number is a number which contains it an exact number of times. Thus 6 is a multiple of 2.

49. A COMMON MEASURE of two or more given numbers is a number which will divide each of the given numbers exactly: thus 3 is a common measure of 18, 27, and 36.

The GREATEST COMMON MEASURE of two or more given numbers, is the greatest number which will divide each of the given numbers exactly: thus, 9 is the greatest common measure of 18, 27, and 36.

50. *If a number measure each of two others, it will also measure their sum, or difference; and also, any multiple of either of them.*

Thus, 3 being a common measure of 9 and 15, will measure their sum, their difference, and also any multiple of either 9 or 15.

The sum of 9 and $15 = 9 + 15 = 24 = 3 \times 8$;

therefore 3 measures their sum 24.

The difference of 15 and $9 = 15 - 9 = 6 = 2 \times 3$;

therefore 3 measures their difference 6.

Again, 36 is a multiple of 9, and $36 = 3 \times 12$; therefore 3 measures this multiple of 9; and similarly any other multiple of 9.

Again, 75 is a multiple of 15; and $75 = 3 \times 25$; therefore 3 measures this multiple of 15; and similarly any other multiple of 15.

51. *To find the Greatest Common Measure of two numbers.*

RULE. Divide the greater number by the less; if there be a remainder, divide the first divisor by it; if there be still a remainder, divide the second divisor by this remainder, and so on; always dividing the last preceding divisor by the last remainder, till nothing remains. The last divisor will be the greatest common measure required.

Ex. Required the greatest common measure of 475 and 589.

Proceeding by the Rule given above,

```
475) 589 (1
     475
     ---
     114) 475 (4
          456
          ---
           19) 114 (6
               114
               ---
                 0
```

therefore 19 is the greatest common measure of 475 and 589.

Reason for the above process.

Any number which measures 589 and 475,
also measures their difference, or 589–475, or 114, Art. (50),
also measures any multiple of 114, and therefore 4×114, or 456, Art. (50);
and any number which measures 456 and 475,
also measures their difference, or 475—456, or 19;

and no number greater than 19 can measure the original numbers 589 and 475; for it has just been shown that any number which measures them must also measure 19.

Again, 19 itself will measure 589 and 475.

For 19 measures 114 (since $114 = 6 \times 19$);
therefore 19 measures 4×114, or 456, Art. (50);
therefore 19 measures $456 + 19$, or 475, Art. (50);
therefore 19 measures $475 + 114$, or 589;

therefore since 19 measures them both, and no number greater than 19 can measure them both,

19 is their greatest common measure.

52. *To find the greatest common measure of three or more numbers.*

Rule. Find the greatest common measure of the first two numbers; then the greatest common measure of the common measure so found and the third number; than that of the common measure last found and the fourth number, and so on. The last common measure so found will be the greatest common measure required.

Ex. Find the greatest common measure of 16, 24, and 18.

Proceeding by the Rule given above,

```
16) 24 (1
    16
     8) 16 (2
        16
```

therefore 8 is the greatest common measure of 16 and 24.

Now to find the greatest common measure of 8 and 18,

```
8) 18 (2
   16
    2) 8 (4
       8
       0
```

therefore 2 is the greatest common measure required.

Reason for the above process.

It appears from Art. (50) that every number, which measures 16 and 24, measures 8 also;

therefore every number, which measures 16, 24, and 18, measures 8 and 18;

therefore the greatest common measure of 16, 24, and 18, is the greatest common measure of 8 and 18.

But 2 is the greatest common measure of 8 and 18;

therefore 2 is the greatest common measure of 16, 24, and 18.

Ex. VII.

1. Find the greatest common measure of

(1) 16 and 72. (2) 30 and 75. (3) 63 and 99.
(4) 55 and 121. (5) 128 and 324. (6) 120 and 320.
(7) 272 and 425. (8) 394 and 672. (9) 720 and 860.
(10) 825 and 960. (11) 775 and 1800. (12) 856 and 936.
(13) 176 and 1000. (14) 1236 and 1632. (15) 6409 and 7395.
(16) 689 and 1573. (17) 1729 and 5850. (18) 5210 and 5718.
(19) 2023 and 7581. (20) 468 and 1266. (21) 2484 and 2628.
(22) 3444 and 2268. (23) 5544 and 6552. (24) 4067 and 2573.
(25) 10395 and 16819. (26) 80934 and 110331.
(27) 1242 and 2323. (28) 13536 and 23148.
(29) 42237 and 75582. (30) 285714 and 999999.
(31) 10353 and 14877. (32) 271469 and 30599.

2. Find the greatest common measure of

(1) 14, 18, and 24.
(2) 16, 24, 48, and 74.
(3) 13, 52, 416, and 78.
(4) 837, 1134, and 1347.
(5) 805, 1311, and 1978.
(6) 28, 84, 154, and 343.
(7) 504, 5292, and 1520.
(8) 396, 5184, and 6914.

LEAST COMMON MULTIPLE.

53. A Common Multiple of two or more given numbers is a number which will contain each of the given numbers an exact number of times without a remainder. Thus, 144 is a common multiple of 3, 9, 18, and 24.

The Least Common Multiple of two or more given numbers is the least number which will contain each of the given numbers an exact number of times without a remainder. Thus, 72 is the least common multiple of 3, 9, 18, and 24.

54. *To find the least common multiple of two numbers.*

Rule. Divide their product by their greatest common measure: the quotient will be the least common multiple of the numbers.

Ex. Find the least common multiple of 18 and 30.

Proceeding by the Rule given above.

```
18) 30 (1
    18
    12) 18 (1
        12
         6) 12 (2
            12
             0
```

therefore 6 is the greatest common measure of 18 and 30.

```
   18
   30
6|540
   90
```

therefore 90 is the least common multiple of 18 and 30.

Reason for the above process.

$$18=3\times 6, \text{ and } 30=5\times 6.$$

Since 3 and 5 are prime factors, it is clear that 6 is the greatest common measure of 18 and 30; therefore their least common multiple must contain 3, 6, and 5, as factors.

Now every multiple of 18 must contain 3 and 6 as factors; and every multiple of 30 must contain 5 and 6 as factors; therefore every number, which is a multiple of 18 and 30, must contain 3, 5, and 6 as factors; and the least number which so contains them is $3 \times 5 \times 6$, or 90.

Now, $90 = (3 \times 6) \times (5 \times 6)$, divided by 6,
$= 18 \times 30$, divided by 6,
$= 18 \times 30$, divided by the greatest common measure of 18 and 30.

55. Hence it appears that the least common multiple of two numbers, which are prime to each other, or have no common measure but unity, is their product.

56. *To find the least common multiple of three or more numbers.*

Rule. Find the least common multiple of the first two numbers; then the least common multiple of that multiple and the third number, and so on. The last common multiple so found will be the least common multiple required.

Ex. Find the least common multiple of 9, 18, and 24.

Proceeding by the Rule given above,

Since 9 is the greatest common measure of 18 and 9, their least common multiple is clearly 18.

Now, to find the least common multiple of 18 and 24.

```
18) 24 (1
    18
    --
     6) 18 (3
        18
        --
         0
```

therefore 6 is the greatest common measure of 18 and 24; therefore the least common multiple of 18 and 24 is equal to 18 multiplied by 24, divided by 6,

```
     24
     18
    ---
    192
    24
    ---
6 | 432
    ---
     72
```

therefore, 72 is the least common multiple required.

Reason for the above process.

Every multiple of 9 and 18 is a multiple of their least common multiple 18; therefore every multiple of 9, 18, and 24, is a multiple of 18 and 24; and therefore the least common multiple of 9, 18, and 24, is the least common multiple of 18 and 24; but 72 is the least common multiple of 18 and 24; therefore 72 is the least common multiple of 9, 18, and 24.

57. *When the least common multiple of several numbers is required, the most convenient practical method is that given by the following Rule.*

Rule. Arrange the numbers in a line from left to right, separating them from each other by a comma. Divide those numbers which have a common measure by that common measure, and place the quotients so obtained and the undivided numbers in a line beneath, separated as before. Proceed in the same way with the second line, and so on with those which follow, until a row of numbers is obtained in which there are no two numbers which have any common measure greater than unity. Then the continued product of all the divisors and the numbers in the last line will be the least common multiple required.

Note. It will in general be found advantageous to begin with the lowest prime number 2 as a divisor, and to repeat this as often as can be done; and then to proceed with the prime numbers 3, 5, &c., in the same way.

Ex. Find the least common multiple of 18, 28, 30, and 42.

Proceeding by the Rule given above,

2	18, 28, 30, 42
2	9, 14, 15, 21
3	9, 7, 15, 21
7	3, 7, 5, 7
	3, 1, 5, 1

therefore the least common multiple required,

$$=2\times2\times3\times7\times3\times5=1260.$$

Reason for the above process.

Since $18=2\times3\times3$; $28=2\times2\times7$; $30=2\times3\times5$; $42=2\times3\times7$; it is clear that the last common multiple of 18 and 28 must contain as a factor $2\times2\times3\times3\times7$; and this factor itself is evidently a common multiple of $2 \times 3 \times 3$, or 18, and of $2 \times 2 \times 7$, or 28; now the least number which contains $2 \times 2 \times 3 \times 3 \times 7$ as a factor is the product of these numbers; therefore $2 \times 2 \times 3 \times 3 \times 7$ is the least common multiple of 18 and 28: also it is clear that the least common multiple of 18, 28, and 30, or of $2 \times 2 \times 3 \times 3 \times 7$ and 30, or of $2 \times 2 \times 3 \times 3 \times 7$ and $2 \times 3 \times 5$ must contain as a factor $2 \times 2 \times 3 \times 3 \times 7 \times 5$, and this factor itself is evidently a common multiple of $2 \times 3 \times 3$ or 18, $2 \times 2 \times 7$ or 28, and $2 \times 3 \times 5$ or 30; hence it follows as before that $2 \times 2 \times 3 \times 3 \times 7 \times 5$ is the least common multiple of 18, 28, and 30; again, the least common multiple of $2 \times 2 \times 3 \times 3 \times 7 \times 5$ and 42, or of $2 \times 2 \times 3 \times 3 \times 7 \times 5$ and $2 \times 3 \times 7$ must contain $2 \times 2 \times 3 \times 3 \times 7 \times 5$ as a factor, and this factor, as before, is evidently itself a common multiple of 18, 28, 30, and 42; now the least number which contains $2 \times 2 \times 3 \times 3 \times 7 \times 5$ as a factor, is the product of these numbers.

Therefore this product, or 1260, is the least common multiple required.

Note 2. The above method is sometimes shortened by rejecting in any line, any number, which is exactly contained in any other number in the same line; for instance, if it be required to find the least common multiple of 2, 4, 8, 16, 10 and 48; the numbers 2, 4, 8, 16, since each of them is exactly contained in 48, may be left out of consideration, and 240, the least common multiple of 10 and 48, will evidently be the least common multiple required.

Ex. VIII.

1. Find the least common multiple of

(1) 16 and 24. (2) 36 and 75. (3) 7 and 15.
(4) 28 and 35. (5) 319 and 407. (6) 333 and 504.
(7) 2961 and 799. (8) 7568 and 9504. (9) 4662 and 5476.
(10) 6327 and 23997. (11) 5415 and 30105.
(12) 15863 and 21489.

2. Find the least common multiple of

(1) 12, 8, and 9. (2) 8, 12, and 16.
(3) 6, 10, and 15. (4) 8, 12, and 20.
(5) 27, 24, and 15. (6) 12, 51, and 68.
(7) 19, 29, and 38. (8) 24, 48, 64, and 192.
(9) 63, 12, 84, and 14. (10) 5, 7, 9, 11, and 15.
(11) 6, 15, 24, and 25. (12) 12, 18, 30, 48, and 60.
(13) 15, 35, 63, and 72. (14) 9, 12, 14, and 210.
(15) 54, 81, 63, and 14. (16) 24, 10, 32, 45, and 25.

(17) 1, 2, 3, 4, 5, 6, 7, 8, and 9.
(18) 7, 8, 9, 18, 24, 72, and 144.
(19) 12, 20, 24, 54, 81, 63, and 14.
(20) 225, 255, 289, 1023, and 4095.

Ex. IX.

Miscellaneous Questions and Examples on the foregoing Articles.

I

(1) Explain the principle of the common system of numerical notation. Multiply 603 by 48, and give the reasons for the several steps.

(2) Write at length the meaning of 9090909, and of 90909. Find their sum and difference, and explain fully the processes employed.

(3) Find the difference between the sum of 4715 added to itself 398 times, and the sum of 2017 added to itself 408 times.

(4) A person, whose age is 73, was 37 years old at the birth of his eldest son; what is the son's age?

(5) Explain the meaning of the terms 'vinculum', 'bracket'; and of the signs $+$, $-$, $=$, $\because$, $\times$.

Find the value of the following expression:

$$15 \times 37153 - 73474 - 67152 \div 4 + 40734 \times 2.$$

II.

(1) Define a 'Unit', 'Number', 'Arithmetic'. What is the difference between Abstract and Concrete numbers?

(2) The annual deaths in a town being 1 in 45, and in the country 1 in 50; in how many years will the number of deaths out of 18675 persons living in the town, and 79250 persons living in the country, amount together to 10000?

(3) Define 'Notation', 'Numeration'; express in numbers seven hundred thousand four hundred and nine billions.

(4) Find the value of

$$494871 - 94853 + (45079 - 3177) - (54312 - 3987) - (1763 + 231) + 379 \times 379.$$

(5) What number divided by 528 will give 36 for the quotient, and leave 44 as a remainder?

III.

(1) Define Multiplication, and Division. Shew that the product of two numbers is the same in whatever order the operation is performed.

(2) The Iliad contains 15683 lines, and the Æneid contains 9892 lines; how many days will it take a boy to read through both of them, at the rate of eighty-five lines a day?

(3) Explain what is meant by the greatest common measure, and by the least common multiple of two or more numbers; and shew that the product of two numbers is the product of their least common multiple into their greatest common measure. Find the least common multiple of 12, 16, 21, 52, and 70.

(4) Explain the meaning of the sign $\div$, and find the value of $(7854-4913)\times 3-(20374-12530)\div 53-6+(395456-2364)\div 556$.

(5) At a game of cricket *A*, *B*, and *C* together score 108 runs; *B* and *C* together score 90 runs, and *A* and *C* together score 51 runs; find the number of runs scored by each of them.

IV

(1) Define Addition, and Subtraction. What is meant by a prime number? When are numbers said to be prime to each other? Give examples.

Explain the rule of *carrying* in the addition of numbers; exemplify it in the addition of 3864, 4768, and 15938.

(2) There are two numbers of which the product is 373625; the greater number is 875; find the sum and difference of the numbers.

(3) A father was 21 years old when his eldest son was born; how old will his son be when he is 50 years old, and what will be the father's age when the son is 50 years old?

(4) Write in figures one hundred millions, one hundred thousand, one hundred and one; and in words 1010101010. Express in figures M.DCCC.XL.

(5) When are numbers said to be 'composite'? Find the greatest number which can divide each of the two numbers 849 and 1132; also the least number which can be divided by each of them; explaining the process in each case.

V

(1) Multiply 478 by 146, and test the result by casting out the nines. In what cases does this method of proof fail? Divide 4843 by 99, and prove the correctness of the operation by any test you please.

(2) What number multiplied by 86 will give the same product as 163 by 430?

(3) At a certain Election three Candidates *A*, *B* and *C*, presented themselves; *A* obtained 150 votes more than *B*; and *B* 75 less than *C*; and *C* obtained 515. Find *A's* majority and the whole number of votes polled.

(4) A gentleman dies, and leaves his property thus: 10000 dollars to his widow; 15000 dollars to his eldest son, on the condition of his building a common school at a cost of 350 dollars; 5500 dollars to each of his four younger sons; 3750 dollars to each of his three daughters; 4563 dollars to different societies; and 599 dollars in legacies to his servants. What amount of property did he die possessed of?

(5) The quotient arising from the division of 9281 by a certain number is 17, and the remainder is 373. Find the divisor.

V.

(1) Explain briefly the Roman method of Notation. Express 1563 and 9000 in Roman characters.

(2) Explain the term 'factor', 'product', 'quotient'; shew by an example how the process of Division can be abridged, if the divisor terminate with cyphers.

(3) The remainder of a division is 97, the quotient 665, and the divisor 91 more than the sum of both. What is the dividend?

(4) Express in words the numbers 270130 and 26784; also write down in figures the number ten thousand, two hundred and thirty four; and find the least number which added to the last number will make it divisible by 8.

(5) A gentleman, whose age is 60, has two sons and a daughter; his age equals the sum of the ages of his children; two years since his age was double that of his eldest son; the sum of the ages of the father and the eldest son is seven times as great as that of the youngest son; find the ages of the children.

FRACTIONS.

58. If 1 represent any concrete quantity, as for instance 1 yard, it is divisible into parts: suppose the parts to be equal to each other, and the number of them 3; one of the parts would be denoted by $\frac{1}{3}$ (read *one-third*), two of them by $\frac{2}{3}$ (read *two-thirds*), three of them or the whole yard by $\frac{3}{3}$ or 1; if another equal portion of a second yard divided in the same manner as the first be added, the sum would be denoted by $\frac{4}{3}$; if two such portions were added, by $\frac{5}{3}$; and so on. Such expressions, representing any number of parts of a unit, that is, of the quantity which is denoted by 1, are termed BROKEN NUMBERS or FRACTIONS; we may therefore define a fraction thus:

59. DEF. A FRACTION denotes a part or parts of a unit; it is expressed by two numbers placed one above the other with a line drawn between them; the lower number is called the DENOMINATOR, and shews into how many equal parts the unit is divided; the upper is called the NUMERATOR, and shews how many of such parts are taken to form the fraction.

Thus $\frac{5}{6}$ denotes that the unit is divided into 6 equal parts, and that 5 of these parts are taken to form the fraction: so, if a yard were divided into six equal parts, and 5 of them were taken, then denoting one yard by 1, we should denote the parts taken by the fraction $\frac{5}{6}$. Again, $\frac{7}{6}$ denotes that the unit is divided into 6 equal parts, and that 7 such parts are taken to form the fraction; for instance, in the example before us, one whole yard would be taken, and also one of the equal parts of another yard divided in the same manner as the first.

60. A fraction also represents the quotient of the numerator by the denominator.

Thus, $\frac{5}{6}$ represents $5 \div 6$; for we should obtain the same result, whether we divide *one* unit into 6 equal parts, and take 5 of such parts (which would be represented by $\frac{5}{6}$); or divide *five* units into 6 equal parts, and take 1 of such parts, which would be equivalent to $\frac{1}{6}^{\text{th}}$ part of 5 units, *i.e.* $5 \div 6$: hence $\frac{5}{6}$ and $5 \div 6$ will have the same meaning.

61. When fractions are denoted in the manner above explained they are called VULGAR FRACTIONS.

Fractions, whose denominators are composed of 10, or 10 multiplied by itself, any number of times, are often denoted in a different manner; and when so denoted, they are called DECIMAL FRACTIONS.

VULGAR FRACTIONS.

62. In treating of the subject of Vulgar Fractions, it is usual to make the following distinctions:

(1) A PROPER FRACTION is one whose numerator is less than the denominator; thus, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{6}{7}$ are proper fractions.

(2) AN IMPROPER FRACTION is one whose numerator is equal to or greater than the denominator; thus, $\frac{5}{5}$, $\frac{6}{5}$, $\frac{7}{5}$ are improper fractions.

(3) A SIMPLE FRACTION is one whose numerator and denominator are simple integral numbers; thus, $\frac{1}{3}$, $\frac{4}{3}$ are simple fractions.

(4) A MIXED NUMBER is composed of a whole number and a fraction; thus, $5\frac{1}{6}$, $7\frac{3}{4}$ are mixed numbers, representing respectively 5 units, together with $\frac{1}{6}$th of a unit; and 7 units, together with $\frac{3}{4}$ths of a unit.

(5) A COMPOUND FRACTION is a fraction of a fraction; thus, $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{9}{10}$ are compound fractions.

(6) A COMPLEX FRACTION is one which has either a fraction, or a mixed number in one or both terms of the fraction; thus, $\frac{\frac{2}{9}}{\frac{5}{7}}$, $\frac{2\frac{1}{2}}{3}$, $\frac{3}{4\frac{2}{3}}$, $\frac{2\frac{1}{7}}{5\frac{1}{6}}$, $\frac{\frac{2}{3} \text{ of } \frac{1}{2}}{2\frac{1}{6}}$ are complex fractions.

63. It is clear from what has Leen said, that every integer may be considered as a fraction whose denominator is 1; thus, $5=\frac{5}{1}$, for the unit is divided into 1 part, comprising the whole unit, and 5 of such parts, that is 5 units, are taken.

64. *To multiply a fraction by a whole number, multiply the numerator of the fraction by it.*

Thus $\frac{2}{7} \times 3 = \frac{6}{7}$.

Reason for the above process.

In $\frac{2}{7}$ the unit is divided into 7 equal parts, and two of those parts are taken: whereas in $\frac{6}{7}$ the unit is divided into 7 equal parts, and 6 of those parts are taken; *i. e.* 3 times as many parts are taken in $\frac{6}{7}$ as are taken in $\frac{2}{7}$, the value of each part being the same in each case.

Ex. X.

(1) Multiply $\frac{5}{12}$ separately by 3, 9, 12, 36.
(2) Multiply $\frac{41}{63}$ separately by 7, 15, 21, 45.
(3) Multiply $\frac{7}{9}$ separately by 4, 5, 7.
(4) Multiply $\frac{99}{125}$ separately by 25, 54.

(65) *To divide a fraction by a whole number, multiply the denominator by it.*

Thus, $\frac{2}{7} \div 3 = \frac{2}{7 \times 3} = \frac{2}{21}$.

Reason for the above process.

In the fraction $\frac{2}{7}$, the unit is divided into 7 equal parts, and 2 of those parts are taken; in the fraction $\frac{2}{21}$, the unit is divided into 21 equal parts, and 2 of such parts are taken: but since each part in the latter case is equal to one-third of each part in the former case, and the same number of parts are taken in each case, it is clear that $\frac{2}{21}$ represents one-third part of $\frac{2}{7}$, or $\frac{2}{7}$ divided by 3.

Ex. XI.

(1) Divide $\frac{5}{8}$ separately by 2, 3, 4, 5, 10.
(2) Divide $\frac{15}{587}$ separately by 11, 20, 25, 45.
(3) Divide $\frac{13}{99}$ separately by 6, 9, 12.
(4) Divide $\frac{1}{2}$ separately by 1, 7, 13.

66. *If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction will not be altered.*

Thus, if the numerator and denominator of the fraction $\frac{2}{7}$ be multiplied by 3, the fraction resulting will be $\frac{6}{21}$, which is of the same value as $\frac{2}{7}$.

Reason for the above process.

In the fraction $\frac{2}{7}$ the unit is divided into 7 equal parts, and 2 of those parts are taken; in the fraction $\frac{6}{21}$ the unit is divided into 21 equal parts, and six of such parts are taken. Now there are 3 times as many parts taken in the second fraction as there are in the first fraction; but 3 parts in the second fraction are only equal to 1 part in the first fraction; therefore the 6 parts taken in the second fraction equal the 2 parts taken in the first fraction; therefore $\frac{2}{7} = \frac{6}{21}$.

67. Hence it follows that a whole number may be converted into a vulgar fraction with any denominator, by multiplying the

number by the required denominator for the numerator of the fraction, and placing the required denominator underneath;

$$\text{for } 6=\frac{6}{1}$$

and to convert it into a fraction with a denominator 5 or 14, we have

$$6=\frac{6}{1}=\frac{6\times 5}{1\times 5}=\frac{30}{5},$$

$$6=\frac{6}{1}=\frac{6\times 14}{1\times 14}=\frac{84}{14}.$$

Ex. XII.

(1) Reduce 7, 9, and 11, to fractions with denominators 3, 7, and 22 respectively; and (2) 26, 109, 117, and 125, to fractions with denominators 2, 5, 13, 23, and 35 respectively.

68. *Multiplying the numerator of a fraction by any number, is the same in effect as dividing the denominator by it, and conversely.*

For if the numerator of the fraction $\frac{6}{8}$ be multiplied by 4, the resulting fraction is $\frac{24}{8}$; and if the denominator be divided by 4, the resulting fraction is $\frac{6}{2}$.

Now the fraction $\frac{24}{8}$ signifies that unity is divided into 8 equal parts, and that 24 such parts are taken; these are equivalent to 3 units: also $\frac{6}{2}$ signifies that unity is divided into 2 equal parts, and that 6 such parts are taken; these are equivalent to 3 units: hence $\frac{24}{8}$ and $\frac{6}{2}$ are equal. The proof of their equality may also be put in this form: that since the unit, in the case of the second fraction, is only divided into 2 equal parts, each part in that case is 4 times as great as each part in the case of the first fraction, where the unit is divided into 8 equal parts; and therefore 4 parts in the case of the first fraction are equal to 1 part in the case of the second; or the 24 parts denoted by the first are equal to the 6 denoted by the second; or, in other words, the fractions $\frac{24}{8}$ and $\frac{6}{2}$ are equal.

Again, if we divide the numerator of the fraction $\frac{6}{8}$ by 2, the resulting fraction is $\frac{3}{8}$; and if we multiply the denominator by 2, the resulting fraction is $\frac{6}{16}$.

Now, $\frac{3}{8}$ signifies that the unit is divided into 8 equal parts, and that 3 of such parts are taken; and $\frac{6}{16}$ signifies that the unit is divided into 16 equal parts, and that six of such parts are taken: but each part in $\frac{3}{8}$ is equal to 2 parts in $\frac{6}{16}$: and therefore $\frac{3}{8}$ is of the same value as $\frac{2\times 3}{16}$, or $\frac{6}{16}$.

69. *To represent an improper fraction as a whole or mixed number.*

Rule. Divide the numerator by the denominator: if there be no remainder, the quotient will be a whole number: if there be a remainder put down the quotient as the integral part, and the remainder as the numerator of the fractional part, and the given denominator as the denominator of the fractional part.

Ex. Reduce $\frac{25}{5}$ and $\frac{35}{6}$ to whole or mixed numbers.

By the Rule given above,

$$\frac{25}{5}=5, \text{ a whole number;}$$
$$\frac{35}{6}=5\tfrac{5}{6}.$$

Reason for the above process.

Since $\frac{25}{5}=\frac{5\times 5}{5}=\frac{5}{5}\times 5$, (Art. 64),

and since $\frac{5}{5}$ signifies that the unit is divided into 5 equal parts, and that 5 of those parts are taken, which 5 parts are equal to the whole unit or 1; therefore $\frac{25}{5}=\frac{5}{5}\times 5=1\times 5$, or 5.

Again, $\frac{35}{6}=\frac{30+5}{6}=\frac{6\times 5+5}{6}$

with equals $\frac{6\times 5}{6}$ together with $\frac{5}{6}$, that is, $=5$ together with $\frac{5}{6}$, by what has been said above; or as it is written, $5\frac{5}{6}$.

Ex. XIII.

Express the following improper fractions as mixed or whole numbers:

(1) $\frac{15}{4}$. (2) $\frac{77}{6}$. (3) $\frac{49}{8}$. (4) $\frac{113}{9}$

(5) $\frac{63}{7}$. (6) $\frac{443}{12}$. (7) $\frac{587}{13}$. (8) $\frac{183}{55}$.

(9) $\frac{751}{24}$. (10) $\frac{5801}{63}$. (11) $\frac{648}{108}$. (12) $\frac{5876}{157}$.

(13) $\frac{10000}{111}$. (14) $\frac{231750}{153}$. (15) $\frac{14264}{239}$. (16) $\frac{95950}{999}$.

(17) $\frac{25713}{1168}$. (18) $\frac{632625}{1741}$. (19) $\frac{978911}{36425}$. (20) $\frac{46325}{3724}$.

70. *To reduce a mixed number to an improper fraction.*

Rule. Multiply the integer by the denominator of the fraction, and to the product add the numerator of the fractional part; the result will be the required numerator, and the denominator of the fractional part the required denominator.

Ex. Convert $2\frac{4}{7}$ into an improper fraction.

Proceeding by the Rule given above,

$$2\frac{4}{7}=\frac{2\times7+4}{7}=\frac{18}{7}.$$

Reason for the above process.

$2\frac{4}{7}$ is meant to represent the integer 2 with the fraction $\frac{4}{7}$ added to it.

But 2 is the same as $\frac{2\times7}{7}$ or $\frac{14}{7}$; and therefore $2\frac{4}{7}$ must be the same as $\frac{14}{7}$ increased by $\frac{4}{7}$ or as $\frac{18}{7}$; for $\frac{18}{7}$ denotes that unity is divided into 7 equal parts, and represents 14 such parts together with 4 such parts.

Ex. XIV.

Reduce the following mixed numbers to improper fractions:

(1) $2\frac{1}{3}$.	(2) $5\frac{3}{7}$.	(3) $4\frac{5}{9}$.	(4) $7\frac{2}{5}$.
(5) $25\frac{11}{12}$.	(6) $43\frac{7}{11}$.	(7) $25\frac{1}{13}$.	(8) $14\frac{13}{15}$.
(9) $2003\frac{1}{7}$.	(10) $857\frac{11}{13}$.	(11) $57\frac{31}{43}$.	(12) $13\frac{83}{84}$.
(13) $3\frac{95}{224}$.	(14) $26\frac{201}{202}$.	(15) $164\frac{118}{441}$.	(16) $106\frac{110}{851}$.
(17) $157\frac{122}{137}$.	(18) $17\frac{2605}{2859}$.	(19) $427\frac{5}{5107}$.	(20) $100\frac{413}{720}$.

71. *To reduce a compound fraction to its equivalent simple fraction.*

Rule. Multiply the several numerators together for the numerator of the simple fraction, and the several denominators together for its denominator.

Ex. Convert $\frac{3}{5}$ of $\frac{7}{8}$ into a simple fraction.

Proceeding by the Rule given above,

$$\frac{3}{5}\text{of }\frac{7}{8}=\frac{3\times7}{5\times8}=\frac{21}{40}.$$

Reason for the above process.

By $\frac{3}{5}$ of $\frac{7}{8}$, we mean $\frac{3}{5}$ths of that part of unity which is denoted by $\frac{7}{8}$; thus if unity be divided into 8 equal parts, and 7 of these be taken, and if each of these be again divided into 5 equal parts, and 3 of each set of parts be taken, then each of the parts will be one-fortieth part of the original unit, and the number of parts taken will be 3×7, or 21; the result therefore is $\frac{21}{40}$, or $\frac{3\times7}{5\times8}$; that is

$$\frac{3}{5}\text{ of }\frac{7}{8}=\frac{3\times 7}{5\times 8}$$

Note. In reducing compound fractions to simple ones, we may strike out factors common to one of the numerators and one of the denominators; for this is in fact simply dividing the numerator and denominator of the fraction by the same number. Art. (66.)

Thus $\frac{3}{5}$ of $2\frac{1}{12}$ of $1\frac{1}{15}=\frac{3}{5}$ of $\frac{25}{12}$ of $\frac{16}{15}$,

$$=\frac{3\times 25\times 16}{5\times 12\times 15}=\frac{3\times 5\times 5\times 4\times 4}{5\times 3\times 4\times 3\times 5}=\frac{4}{3}$$

This result is arrived at by striking out the factors 3, 5, 5, 4, from the numerator and denominator.

Ex. XV.

Reduce the following compound fractions to simple ones:

(1) $\frac{2}{3}$ of $\frac{4}{5}$. (2) $\frac{6}{7}$ of $\frac{9}{10}$. (3) $\frac{2}{5}$ of $\frac{3}{4}$. (4) $\frac{5}{6}$ of $\frac{11}{12}$.

(5) $\frac{7}{8}$ of $\frac{5}{6}$ of 7. (6) $\frac{5}{8}$ of $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{1}{15}$ of 28.

(7) $\frac{5}{11}$ of $2\frac{1}{2}$ of $\frac{6}{7}$ of $10\frac{1}{2}$. (8) $\frac{7}{9}$ of $12\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{8}$ of 9.

(9) $\frac{5}{18}$ of $\frac{7}{3}$ of $\frac{36}{10}$ of $\frac{9}{4}$ of $\frac{3}{10}$ of 2 of $\frac{8}{27}$.

(10) $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{6}{7}$ of $70\frac{2}{9}$ of $\frac{3}{40}$ of $1\frac{7}{11}$ of 147.

72. Def. A Fraction is in its lowest terms, when its numerator and denominator are prime to each other.

Note. When the numerator and denominator of a fraction are not prime to each other, they have (Art. 44) a common factor greater than unity. If we divide each of them by this, there results a fraction *equal* to the former, but of which the terms, that is, the numerator and denominator are less, or *lower* than those of the original fraction; and it may be considered to be the same fraction in *lower* terms. When the numerator and denominator are prime to each other, that is, have no common factor greater than unity, it is clear that its terms cannot be made lower by division of this kind, and on this account the fraction is said to be in its lowest terms.

73. *To reduce a fraction to its lowest terms.*

Rule. Divide the numerator and denominator by their greatest common measure.

Ex. 1. Reduce $\frac{6465}{7335}$ to its lowest terms.

First, find the greatest common measure of 6465 and 7335.

```
6465) 7335 (1
      6465
       870) 6465 (7
            6090
             375) 870 (2
                  750
                  120) 375 (3
                       360
                        15) 120 (8
                            120
```

therefore 15 is the greatest common measure.

```
15) 6465 (431            15) 7335 (489
    60                       60
     46                      133
     45                      120
      15                      135
      15                      135
```

therefore the fraction in its lowest terms $=\frac{431}{489}$.

Reason for the above process.

If the numerator and denominator of a fraction be divided by the same number, the value of the fraction is not altered (Art. 66); and the greatest number which will divide the numerator and denominator is their greatest common measure.

Note. Sometimes it is unnecessary to find the greatest common measure, as it is easier to bring the fraction to its lowest terms by successive divisions of the numerator and denominator by common factors, which are easily determined by inspection.

Ex. 2. Reduce $\frac{540}{750}$ to its lowest terms,

$\frac{540}{750}=\frac{54}{75}$, dividing numerator and denominator by 10,

$=\frac{18}{25}$, dividing numerator and denominator by 3.

Ex. XVI.

Reduce each of the following fractions to its lowest terms:

(1) $\frac{4}{8}$. (2) $\frac{10}{15}$. (3) $\frac{14}{21}$. (4) $\frac{18}{45}$.
(5) $\frac{56}{63}$. (6) $\frac{54}{81}$. (7) $\frac{63}{135}$. (8) $\frac{66}{121}$.

(9) $\frac{2}{3}, \frac{4}{4}, \frac{5}{9}$. (10) $\frac{2}{3}, \frac{4}{2}, \frac{7}{9}$. (11) $\frac{7}{4}, \frac{7}{2}, \frac{3}{5}$. (12) $\frac{3}{7}, \frac{3}{2}, \frac{9}{8}$.
(13) $\frac{9}{3}, \frac{2}{70}, \frac{5}{0}$. (14) $\frac{8}{9}, \frac{3}{3}, \frac{9}{6}$. (15) $\frac{3}{6}, \frac{2}{1}, \frac{4}{2}$. (16) $\frac{9}{23}, \frac{3}{6}, \frac{6}{8}$.
(17) $\frac{5}{6}, \frac{1}{9}, \frac{8}{1}, \frac{4}{2}$. (18) $\frac{3}{3}, \frac{4}{5}, \frac{4}{5}, \frac{4}{6}$. (19) $\frac{7}{90}, \frac{8}{67}, \frac{4}{8}, \frac{5}{0}$. (20) $\frac{2}{3}, \frac{4}{2}, \frac{7}{6}, \frac{2}{4}$.
(21) $\frac{9}{90}, \frac{2}{0}, \frac{5}{00}$. (22) $\frac{8}{46}, \frac{1}{72}$. (23) $\frac{1}{2}, \frac{6}{0}, \frac{3}{7}, \frac{2}{6}$. (24) $\frac{1}{1}, \frac{9}{4}, \frac{2}{3}, \frac{6}{7}, \frac{5}{1}$.
(25) $\frac{4}{95}, \frac{3}{8}, \frac{0}{9}, \frac{1}{7}$. (26) $\frac{4}{7}, \frac{5}{4}, \frac{2}{8}, \frac{4}{4}, \frac{7}{1}$. (27) $\frac{6}{6}, \frac{9}{1}, \frac{9}{7}, \frac{3}{4}$. (28) $\frac{1}{2}, \frac{9}{2}, \frac{8}{8}, \frac{1}{0}, \frac{2}{0}$.
(29) $\frac{2}{8}, \frac{5}{6}, \frac{1}{1}, \frac{9}{7}, \frac{4}{8}$. (30) $\frac{3}{5}, \frac{7}{7}, \frac{4}{5}, \frac{1}{6}, \frac{9}{8}, \frac{2}{0}$. (31) $\frac{1}{2}, \frac{1}{2}, \frac{4}{0}, \frac{1}{6}, \frac{3}{6}, \frac{5}{1}$. (32) $\frac{1}{2}, \frac{2}{3}, \frac{8}{8}, \frac{3}{3}, \frac{6}{6}, \frac{2}{8}$.

74. *To reduce fractions to equivalent ones with a common denominator.*

Rule. Find the least common multiple of the denominators: this will be the common denominator. Then divide the common multiple so found by the denominator of each fraction, and multiply each quotient so found into the numerator of the fraction which belongs to it for the new numerator of that fraction.

Note 1. If the given fractions be in their *lowest* terms, the above rule will reduce them to others having the *least* common denominator; if the *least* common denominator be required, the given fractions should be reduced to their lowest terms before the rule be applied.

Ex. Reduce $\frac{5}{12}, \frac{9}{16}, \frac{11}{24}, \frac{17}{33}$, into equivalent fractions with a common denominator.

Proceeding by the Rule given above,

$$\begin{array}{r|l} 2 & 12,\ 16,\ 24,\ 33 \\ \hline 2 & 6,\ 8,\ 12,\ 33 \\ \hline 2 & 3,\ 4,\ 6,\ 33 \\ \hline 3 & 3,\ 2,\ 3,\ 33 \\ \hline & 1,\ 2,\ 1,\ 11 \end{array}$$

therefore least common multiple $=2\times2\times2\times3\times2\times11$.
$=558$;

therefore the fractions become respectively,

$$\frac{5\times44}{12\times44}=\frac{220}{528}\left(\text{since } \frac{528}{12}=44\right),$$

$$\frac{9\times33}{16\times33}=\frac{297}{528}\left(\text{since } \frac{528}{16}=33\right),$$

$$\frac{11\times22}{24\times22}=\frac{242}{528}\left(\text{since } \frac{528}{24}=22\right),$$

$$\frac{17\times16}{33\times16}=\frac{272}{528}\left(\text{since } \frac{528}{33}=16\right).$$

or the fractions with a common denominator are

$$\frac{220}{528},\ \frac{227}{528},\ \frac{242}{528},\ \frac{273}{528}.$$

Reason for the above process.

The least common multiple of the denominators of the given fractions will evidently contain the denominator of any one of the fractions an exact number of times. If both the numerator and denominator of that fraction be multiplied by that number, the value of the fraction will not be altered (Art. 66); and the denominator will then be equal to the least common multiple of all the denominators. If this be done with all the fractions, they will evidently be, in like manner, reduced to others of the same value, and having the least common multiple of all the denominators for the denominator of each fraction.

Note 2. If the denominators have no common measure, we must then multiply each numerator into all the denominators, except its own, for a new numerator for each fraction, and all the denominators together for the common denominator.

Ex. Reduce $\frac{1}{5}$, $\frac{2}{7}$, $\frac{1}{9}$, to equivalent fractions with a common denominator.

The least common multiple of the denominators

$$=5\times7\times9;$$

therefore the fractions become

$$\frac{1\times7\times9}{5\times7\times9}=\frac{63}{315},\qquad \frac{2\times5\times9}{7\times5\times9}=\frac{90}{315},\qquad \frac{1\times5\times7}{9\times5\times7}=\frac{35}{315}.$$

or the fractions with a common denominator are

$$\frac{63}{315},\ \frac{90}{315},\ \text{and}\ \frac{35}{315}.$$

Ex. XVII.

Reduce the fractions in each of the following sets to equivalent fractions, having the least common denominator:

(1) $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$.

(2) $\frac{2}{5}$, and $\frac{7}{8}$.

(3) $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

(4) $\frac{2}{9}$, and $\frac{5}{27}$.

(5) $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{11}{28}$.

(6) $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{9}$.

(7) $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{17}{18}$.

(8) $\frac{5}{12}$, $\frac{7}{16}$, and $\frac{13}{24}$.

(9) $\frac{5}{6}$, $\frac{9}{10}$, and $\frac{14}{15}$.

(10) $\frac{2}{5}$, $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{7}{10}$.

(11) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

(12) $\frac{1}{7}$, $\frac{4}{11}$, $\frac{7}{13}$, and $\frac{2}{3}$.

(13) $\frac{3}{5}$, $\frac{35}{80}$, and $\frac{14}{2000}$.

(14) $\frac{7}{12}$, $\frac{6}{7}$, $\frac{20}{63}$, and $\frac{13}{84}$.

(15) $\frac{7}{9}$, $\frac{5}{11}$, $\frac{13}{18}$, $\frac{3}{22}$, and $\frac{1}{36}$.

(16) $\frac{1}{3}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{9}{14}$, $\frac{3}{28}$, and $\frac{17}{42}$.

(17) $\frac{2}{3}$, $\frac{4}{9}$, $\frac{7}{27}$, $\frac{8}{81}$, $\frac{16}{243}$, and $\frac{31}{729}$.

(18) $\frac{9}{10}$, $\frac{9}{100}$, $\frac{9}{1000}$, and $\frac{9}{10000}$.

(19) $\frac{31}{60}$, $\frac{17}{90}$, $\frac{13}{25}$, $\frac{1}{105}$, and $\frac{5}{9}$.

(20) $\frac{31}{54}$, $\frac{11}{28}$, $\frac{53}{63}$, and $\frac{3}{12}$.

Note 3. Whenever a comparison has to be made between fractions, in respect of their magnitudes, they must be reduced to equivalent ones with a common denominator; because then we shall have the unit divided, in the case of each fraction so obtained, into the same number of equal parts; and the respective numerators will show us how many of such parts are taken in each case; or which is the greatest fraction, which the next, and so on.

Ex. Compare the values of $\frac{5}{27}$, $\frac{11}{24}$, $\frac{5}{6}$, $\frac{4}{15}$, and $\frac{3}{5}$.

First, to find the least common multiple of the denominators;

$$\begin{array}{r|l} 2 & 27,\ 24,\ 6,\ 15,\ 5 \\ \hline 3 & 27,\ 12,\ 3,\ 15,\ 5 \\ \hline 5 & 9,\ 4,\ 1,\ 5,\ 5 \\ \hline & 9,\ 4,\ 1,\ 1,\ 1 \end{array}$$

therefore the least common denominator

$$=2\times3\times5\times9\times4=1080;$$

therefore the fractions become

$$\frac{5\times40}{27\times40}=\frac{200}{1080},\quad \frac{11\times45}{24\times45}=\frac{495}{1080},\quad \frac{5\times180}{6\times180}=\frac{900}{1080},$$

$$\frac{4\times72}{15\times72}=\frac{288}{1080},\quad \frac{3\times216}{5\times216}=\frac{648}{1080},$$

therefore $\frac{5}{6}$ is the greatest, $\frac{3}{5}$ the next, $\frac{11}{24}$ the next, $\frac{4}{15}$ the next, and $\frac{5}{27}$ the least.

Ex. XVIII.

1. Compare the values of

(1) $\frac{3}{5}$, $\frac{8}{9}$, and $\frac{7}{10}$.

(2) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.

(3) $\frac{1}{5}$ of $\frac{3}{8}$, $\frac{7}{12}$, and $\frac{4}{3}$ of $\frac{6}{7}$.

(4) $\frac{5}{12}$, $\frac{3}{16}$, $\frac{10}{21}$, and $\frac{31}{60}$;

(5) $\frac{3}{7}$, $\frac{7}{13}$, $\frac{9}{22}$, $\frac{8}{11}$, and $\frac{21}{26}$.

(6) $\frac{3}{8}$, $\frac{27}{32}$, $\frac{9}{16}$, $\frac{7}{10}$, and $\frac{27}{40}$.

(7) $\frac{15}{4}$, $3\frac{1}{3}$, and $\frac{2}{7}$ of $9\frac{2}{5}$.

(8) $\frac{6}{7}$, $\frac{13}{28}$, $1\frac{4}{9}$, $\frac{5}{8}$, and $\frac{29}{56}$.

(9) $\frac{8}{9}$, $\frac{3}{11}$, $\frac{7}{18}$, $\frac{9}{22}$, and $\frac{5}{36}$.

(10) $\frac{51}{76}$, $\frac{113}{152}$, $1\frac{1}{38}$, $\frac{401}{448}$, and $\frac{700}{748}$.

(11) $\frac{15}{4}$, $3\frac{1}{3}$, $\frac{2}{7}$ of $9\frac{2}{5}$, and $\frac{2}{7}$ of $\frac{5}{6}$ of $\frac{4}{5}$.

(12) $\frac{3}{7}$ of $\frac{5}{8}$ of 4, $\frac{2}{11}$ of $\frac{3}{5}$ of 5, $\frac{1}{6}$ of $\frac{1}{2}$ of $4\frac{3}{4}$, and $\frac{24}{28}$.

2. Find the greatest and least of the fractions

(1) $\frac{3}{4}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{1}{2}$. (2) $\frac{11}{12}$, $\frac{29}{30}$, $\frac{17}{18}$, $\frac{7}{16}$, and $\frac{47}{48}$.

ADDITION OF VULGAR FRACTIONS.

75. **Rule.** Reduce the fractions to equivalent ones with their least common denominator; add all the new numerators together and under their sum write the common denominator.

Ex. Find the sum of $\frac{7}{15}$, $\frac{10}{21}$, and $\frac{16}{35}$.

Proceeding by the Rule given above,

First, find the least common multiple of the denominators:

$$\begin{array}{r|l} 3 & 15,\ 21,\ 35 \\ \hline 5 & 5,\ 7,\ 35 \\ \hline 7 & 1,\ 7,\ 7 \\ \hline & 1,\ 1,\ 1 \end{array}$$

therefore the least common multiple $= 3 \times 5 \times 7 = 105$; therefore the fractions become

$$\frac{7 \times 7}{15 \times 7} = \frac{49}{105}, \qquad \frac{10 \times 5}{21 \times 5} = \frac{50}{105}, \qquad \frac{16 \times 3}{35 \times 3} = \frac{48}{105},$$

therefore their sum $= \frac{49+50+48}{105} = \frac{147}{105}, = \frac{49}{35}, = 1\frac{14}{35}$.

Reason for the Rule.

In each of the equivalent fractions, we have unity divided into 105 equal parts, and those fractions represent respectively 49, 50, and 48 of such parts; therefore the sum of the fractions must represent $49+50+48$ or 147 such parts, that is, must be $\frac{147}{105}$.

Note 1. If the sum of the fractions be a fraction which is not in its lowest terms, reduce it to its lowest terms; and if the result be an improper fraction, then reduce it to a whole or mixed number: thus $\frac{147}{105} = \frac{49}{35} = 1\frac{14}{35}$: the same remark applies to all results in Vulgar Fractions.

Note 2. Before applying the Rule, reduce all fractions to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones.

Note 3. If any of the given numbers be whole or mixed numbers, the whole numbers may be added together as in simple addition, and the fractional parts by the Rule given above.

Ex. Find the sum of $\frac{3}{8}$, $3\frac{14}{15}$, $10\frac{2}{5}$, and $\frac{9}{22}$.

$$\frac{3}{8}+3\tfrac{14}{15}+10\tfrac{2}{5}+\frac{9}{22} = 3+10+\frac{3}{8}+\frac{14}{15}+\frac{2}{5}+\frac{9}{22}$$

$$=13+\frac{3}{8}+\frac{14}{15}+\frac{2}{5}+\frac{9}{22}.$$

Now to find the sum of $\frac{3}{8}+\frac{14}{15}+\frac{2}{5}+\frac{9}{22}$.

First, find the least common multiple of the denominators;

$$\begin{array}{r|l} 2 & 8,\ 15,\ 5,\ 22 \\ \hline 5 & 4,\ 15,\ 5,\ 11 \\ \hline & 4,\ \ 3,\ 1,\ 11 \end{array}$$

therefore the least common multiple

$$=2\times5\times4\times3\times11=1320;$$

therefore the fractions become

$$\frac{3\times165}{8\times165}=\frac{495}{1320}, \qquad \frac{14\times88}{15\times88}=\frac{1232}{1320},$$

$$\frac{2\times264}{5\times264}=\frac{528}{1320}, \qquad \frac{9\times60}{22\times60}=\frac{540}{1320},$$

therefore the sum of the fractions

$$=\frac{495+1232+528+540}{1320}=\frac{2795}{1320}$$

$$=\frac{559}{264}, \left(\text{dividing numerator and denominator by 5,}\right) =2\tfrac{31}{264}$$

therefore the whole sum $=13+2\frac{31}{264}=15\frac{31}{264}$

Ex. XIX.

1. Add together,

(1) $\frac{2}{3}$ and $\frac{5}{7}$ (2) $\frac{2}{9}$ and $\frac{4}{9}$ (3) $\frac{5}{6}$ and $\frac{7}{8}$

(4) $\frac{1}{7}$ and $\frac{4}{21}$ (5) $\frac{3}{16}$ and $\frac{5}{64}$ (6) $\frac{9}{14}$ and $\frac{13}{21}$

(7) $\frac{5}{12}$ and $\frac{7}{18}$. (8) $\frac{3}{24}$ and $\frac{21}{32}$. (9) $\frac{3}{5}$ and $2\frac{1}{6}$.

(10) $\frac{5}{12}$ and $\frac{7}{13}$. (11) $3\frac{5}{8}$ and $7\frac{2}{5}$. (12) $4\frac{3}{4}$ and $9\frac{11}{12}$.

2. Find the sum of

(1) $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$. (2) $\frac{3}{5}$, $\frac{2}{7}$, and $\frac{1}{3}$.

(3) $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{1}{10}$. (4) $\frac{5}{12}$, $\frac{1}{8}$, and $\frac{7}{24}$.

(5) $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{7}{12}$. (6) $\frac{3}{4}$, $\frac{4}{7}$, and $\frac{7}{9}$.

(7) $\frac{1}{2}$, $\frac{5}{6}$, and $\frac{3}{21}$. (8) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

(9) $\frac{5}{7}$, $\frac{6}{35}$, and $\frac{9}{35}$. (10) $\frac{3}{8}$, $2\frac{1}{7}$, and $13\frac{3}{16}$.

(11) $\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$, and $9\frac{3}{20}$

(12) $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$, $5\frac{1}{2}$, and $\frac{3}{10}$.

(13) $\frac{7}{5}$, $\frac{4}{3}$, $\frac{1}{6}$, and $\frac{11}{30}$.

(14) $\frac{10}{14}$, $\frac{2}{15}$, and $\frac{18}{70}$.

(15) $\frac{4}{5}$, $\frac{7}{10}$, $\frac{4}{7}$, $\frac{2}{21}$.

(16) $\frac{1}{8}$, $\frac{7}{12}$, $\frac{5}{9}$, and $\frac{9}{20}$.

(17) $\frac{1}{5}$, $6\frac{4}{5}$, and $\frac{4}{7}$ of $\frac{1}{3}$.

(18) $100\frac{2}{5}$, $64\frac{5}{9}$, $\frac{3}{5}$ of 701.

(19) $261\frac{1}{3}$, $174\frac{3}{4}$, and $\frac{5}{6}$ of $10\frac{1}{2}$.

(20) $387\frac{1}{2}$, $285\frac{1}{4}$, $394\frac{1}{3}$, and $\frac{2}{5}$ of 3704.

3. Find the value of

(1) $\frac{11}{10}+\frac{11}{100}+\frac{11}{1000}+\frac{11}{10000}$.

(2) $\frac{11}{12}+\frac{17}{18}+\frac{29}{30}+\frac{47}{48}+\frac{59}{60}$.

(3) $\frac{1}{2}+\frac{1}{9}+\frac{3}{17}+\frac{29}{38}+\frac{33}{34}$.

(4) $2\frac{1}{17}+6\frac{21}{34}+\frac{16}{51}+\frac{1}{3}$ of $\frac{16}{17}+\frac{29}{68}+\frac{1}{3}$ of $2\frac{3}{4}$.

(5) $2\frac{2}{3}+3\frac{3}{4}+4\frac{4}{5}+5\frac{5}{6}+6\frac{6}{7}$.

(6) $1\frac{4}{7}+3\frac{1}{10}+\frac{13}{15}+7\frac{5}{16}+\frac{9}{28}+\frac{2}{7}$ of $\frac{1}{5}$.

(7) $5\frac{1}{6}+\frac{8}{9}$ of $\frac{6}{7}$ of $3\frac{1}{2}+9\frac{3}{10}+\frac{3}{7}$ of $\frac{5}{8}$ of 4.

(8) $\frac{5}{8}$ of $12+\frac{3}{5}$ of $\frac{8}{9}+3\frac{3}{8}$ of $1\frac{8}{9}$ of $1\frac{4}{153}+\frac{13}{15}$ of $3\frac{3}{4}$ of $\frac{1}{81}$ of $1\frac{1}{26}$.

(9) $270\frac{3}{4}+650\frac{3}{20}+5000\frac{1}{4}+53\frac{4}{5}+1\frac{1}{20}$.

(10) $\frac{1}{2}$ of $\frac{3}{4}+\frac{7}{31}$ of $(1+\frac{15}{16})+\frac{31}{32}+\frac{63}{64}$ of $\{1+\frac{1}{2}\}$.

SUBTRACTION.

76. Rule. Reduce the fractions to their least common denominator, take the difference of the new numerators, and place the common denominator underneath.

Ex. Subtract $\frac{1}{2}$ from $\frac{7}{8}$.

Proceeding by the Rule given above, since 8 is clearly the least common multiple of the denominators, the equivalent fractions will be $\frac{4}{8}$ and $\frac{7}{8}$.

$$\text{and their difference}=\frac{7-4}{8}=\frac{3}{8}.$$

Reason for the Rule.

The unit in each of the equivalent fractions is divided into 8 equal parts, and there are 7 and 4 parts respectively taken, and therefore the difference must be 3 of such parts, or, in other words, the difference of the two fractions is $\frac{3}{8}$.

Note. 1. Remember always, before applying the above Rule, to reduce fractions to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones.

Note. 2. If either of the given fractions be a whole or mixed number, it is most convenient to take separately the difference of the integral parts and that of the fractional parts, and then add the two results together, as in the following examples.

Ex 1. From $4\frac{3}{8}$ subtract $2\frac{1}{4}$.

Here $4-2=2$, and $\frac{3}{8}-\frac{1}{4}=\frac{3}{8}-\frac{2}{8}=\frac{1}{8}$; therefore the difference of $4\frac{3}{8}$ and $2\frac{1}{4}=2\frac{1}{8}$.

For the process expressed at length is,

$$4+\frac{3}{8}-(2+\frac{1}{4}),$$

which $=4+\frac{3}{8}-2-\frac{1}{4}$, Art. (12.)

or $=4-2+(\frac{3}{8}-\frac{1}{4})=2+\frac{1}{8}=2\frac{1}{8}$.

Ex. 2 Take $2\frac{3}{8}$ from $4\frac{1}{4}$.

Now $\frac{3}{8}$ cannot be taken from $\frac{1}{4}$ since it is the greater of the two; we therefore add 1 to $\frac{1}{4}$, and take $\frac{3}{8}$ from $1+\frac{1}{4}$ or $\frac{5}{4}$; and then, in order that the difference may not be altered, we add 1 to the 2.

Now $\frac{5}{4}-\frac{3}{8}=\frac{10}{8}-\frac{3}{8}=\frac{7}{8}$.

$$4-3=1;$$

therefore the difference of $4\frac{1}{4}$ and $2\frac{3}{8}=1\frac{7}{8}$.

For the process expressed at length is,

$$4+\frac{1}{4}-(2+\frac{3}{8})$$

which $=4+1+\frac{1}{4}-(2+1+\frac{3}{8})$ (adding and subtracting 1),

$=4+\frac{5}{4}-(3+\frac{3}{8})=4-3+\frac{5}{4}-\frac{3}{8}=1+\frac{10}{8}-\frac{3}{8}=1+\frac{7}{8}=1\frac{7}{8}$.

Ex. XX.

1. Find the difference between

(1) $\frac{3}{4}$ and $\frac{1}{2}$. (2) $\frac{5}{8}$ and $\frac{1}{4}$. (3) $\frac{4}{9}$ and $\frac{3}{11}$.
(4) $\frac{7}{12}$ and $\frac{8}{15}$. (5) $\frac{17}{18}$ and $\frac{11}{12}$. (6) $\frac{8}{15}$ and $\frac{9}{20}$.
(7) $2\frac{3}{5}$ and $1\frac{1}{3}$. (8) $37\frac{4}{15}$ and $33\frac{5}{24}$. (9) $6\frac{3}{8}$ and $4\frac{1}{2}$.
(10) $13\frac{5}{12}$ and $9\frac{7}{13}$. (11) $50\frac{1}{16}$ and $47\frac{1}{24}$. (12) 42 and $30\frac{5}{12}$.
(13) $15\frac{32}{729}$ and $1\frac{16}{243}$. (14) $90\frac{10}{111}$ and $25\frac{13}{125}$.
(15) 21 and $1\frac{179}{298}$. (16) 125 and $\frac{2}{7}$ of 14.
(17) $46\frac{5}{8}$ and $15\frac{1}{6}$. (18) $\frac{13}{56}$ and $\frac{5}{28}$ of $1\frac{1}{5}$.
(19) $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{3}{4}$, and $\frac{2}{9}$ of $\frac{3}{4}$.
(20) $\frac{3}{7}$ of $\frac{2}{9}$ of $\frac{4}{5}$ of $8\frac{3}{4}$ and $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{11}{18}$ of $1\frac{7}{11}$.
(21) Find the value of $(\frac{1}{2}+\frac{1}{3})-(\frac{1}{4}+\frac{1}{5}.)$
(22) Find the value of $\{(\frac{3}{4}+\frac{5}{8})-(\frac{7}{12}+\frac{1}{2})\}+\{(\frac{7}{24}+\frac{2}{3})-(\frac{3}{8}-\frac{1}{24})\}$

2. By how much does $\frac{5}{8}$ of $\frac{4}{10}-\frac{7}{9}$ of $\frac{4}{21}$ exceed $\frac{5}{6}$ of $\frac{2}{15}-\frac{3}{8}$ of $\frac{4}{18}$?
3. Add $\frac{3}{15}$ of $\frac{5}{6}$ to $2\frac{7}{8}$, and subtract $\frac{5}{9}$ from the result.
4. From the sum of $11\frac{2}{3}$ and $8\frac{7}{9}$ subtract $9\frac{19}{22}$.

5. By how much does the difference of $5\frac{23}{32}$ and $2\frac{3}{7}$ exceed the sum of $\frac{5}{8}+\frac{5}{14}+\frac{1}{6}$?

6. By how much does the sum of the fractions $1\frac{1}{26}$ and $\frac{9}{13}$ exceed their difference?

MULTIPLICATION.

77. Rule. Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

Ex. Multiply $\frac{3}{7}$ by $\frac{5}{8}$.

Proceeding by the Rule given above, $\frac{3\times 5}{7\times 8}=\frac{15}{56}$

Reason for the Rule.

If $\frac{3}{7}$ be multiplied by 5, the result is $\frac{15}{7}$, Art. (64).

But this result must be 8 times too large, since, instead of multiplying by 5, we have only to multiply by $\frac{5}{8}$, which is 8 times smaller than 5, or, in other words, is $\frac{1}{8}$ part of 5. Consequently the product above, viz. $\frac{15}{7}$ must be divided by 8, and $\frac{15}{7}\div 8=\frac{15}{56}$, Art. (65).

Note. 1. The same reasoning will apply, whatever be the number of fractions which have to be multiplied together.

Note 2. Before applying the above Rule mixed numbers must be reduced to improper fractions.

Note 3. It has been shewn that a fraction is reduced to its lowest terms by dividing its numerator and denominator by their greatest common measure, or, in other words, by the product of those factors which are common to both: hence, in all cases of multiplication of fractions, it will be well to split up the numerators and denominators as much as possible into the factors which compose them; and then, after putting the several fractions under the form of one fraction, the sign of $\times$ being placed between each of the factors in the numerator and denominator, to cancel those factors which are common to both, before carrying into effect the final multiplication. Thus, in the following Examples:

Ex. 1. Multiply $\frac{3}{4}$ and $\frac{4}{5}$ together.

$$\text{Product}=\frac{3\times 4}{4\times 5}.$$

Now cancelling, *i. e.* dividing the numerator and denominator by the common factor 4, we see that

$$\text{Product}=\frac{3}{5}.$$

Ex. 2. Multiply $\frac{1}{2}, \frac{2}{3}$, and $\frac{3}{4}$ together.

$$\text{Product}=\frac{1\times 2\times 3}{2\times 3\times 4},$$

(or cancelling, *i. e.* dividing numerator and denominator by the product of the common factors 2, 3,)

$$=\frac{1}{4}.$$

Ex. 3. Multiply $\frac{8}{9}, \frac{16}{24}, \frac{27}{30}, \frac{45}{60}$ together.

$$\text{Product}=\frac{8\times 16\times 27\times 45}{9\times 24\times 30\times 60}=\frac{8\times 4\times 4\times 3\times 9\times 5\times 9}{9\times 3\times 8\times 5\times 6\times 5\times 12},$$

(or cancelling, *i. e.* dividing the numerator and denominator by the product of the common factors, 8, 3, 9, 5,)

$$\text{product}=\frac{4\times 4\times 9}{6\times 5\times 12},=\frac{4\times 2\times 2\times 3\times 3}{3\times 2\times 5\times 3\times 4},$$

(or cancelling, *i. e.* dividing numerator and denominator by the product of the common factors 4, 2, 3, 3,)

$$\text{product in its lowest terms}=\frac{2}{5},$$

Ex. 4. Multiply $2\frac{1}{2}, 3\frac{3}{8}, 10\frac{1}{8}, 20\frac{4}{9}$, and $5\frac{9}{23}$ together.

$$\text{Product}=2\frac{1}{2}\times 3\frac{3}{8}\times 10\frac{1}{8}\times 20\frac{4}{9}\times 5\frac{9}{23},$$

$$=\frac{5}{2}\times\frac{27}{8}\times\frac{81}{8}\times\frac{184}{9}\times\frac{124}{23},$$

$$=\frac{5\times 9\times 3\times 9\times 9\times 8\times 23\times 4\times 31}{2\times 2\times 4\times 8\times 9\times 23},$$

(or cancelling, *i. e.* dividing numerator and denominator by the product of the common factors 9, 8, 23, 4,)

$$\text{product}=\frac{5\times 3\times 9\times 9\times 31}{2\times 2},=\frac{37665}{4},=9416\tfrac{1}{4},$$

Ex. XXI.

1. Multiply

(1) $\frac{5}{7}$ by $\frac{2}{3}$. (2) $\frac{8}{9}$ by $\frac{13}{15}$. (3) $\frac{3}{4}$ by $\frac{8}{9}$.
(4) $\frac{5}{12}$ by $\frac{8}{15}$. (5) $\frac{51}{60}$ by $\frac{16}{17}$. (6) $7\frac{1}{2}$ by $\frac{1}{3}$.
(7) $3\frac{7}{9}$ by $2\frac{2}{3}$. (8) $7\frac{1}{2}$ by $\frac{1}{5}$ of $\frac{4}{9}$. (9) 12 by $\frac{2}{3}$ of 5.
(10) $\frac{1}{2}$ of $\frac{2}{3}$ by $5\frac{3}{8}$ of 3. (11) $\frac{12}{13}$ of $3\frac{2}{3}$ by $1\frac{1}{33}$ of $\frac{39}{51}$ of $\frac{3}{8}$.
(12) $\frac{11}{15}$ of $1\frac{3}{22}$ of $\frac{1}{70}$ by $\frac{7}{12}$ of $37\frac{14}{35}$ of $3\frac{1}{8}$ of $\frac{1}{11}$.
(13) $\frac{3}{8}$ of $2\frac{1}{16}$ of $1\frac{1}{63}$ of $3\frac{5}{87}$ by $\frac{3}{11}$ of $\frac{10}{19}$.
(14) $5\frac{2}{19}$ of $3\frac{1}{4}$ of $\frac{8}{117}$ of 34 by $\frac{3}{194}$ of $\frac{9}{68}$ of $1\frac{1}{3}$ of 19.

2. Find the continued product of

(1) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$. (2) $\frac{15}{64}$, $\frac{16}{27}$, $\frac{11}{10}$, $\frac{72}{385}$, and $\frac{21}{85}$.
(3) $1\frac{7}{15}$, $2\frac{2}{3}$ of $1\frac{7}{37}$, $\frac{33}{42}$, $\frac{19}{328}$, $5\frac{1}{22}$ of 49, and $\frac{3}{76}$.
(4) $\frac{5}{6}$, $2\frac{3}{4}$, $3\frac{5}{11}$, $5\frac{2}{19}$, and $6\frac{1}{194}$.
(5) $\frac{95}{116}$, $\frac{17}{21}$, $\frac{87}{153}$, $\frac{126}{190}$, and $\frac{4}{7}$ of $1\frac{2}{5}$.
(6) $\frac{147}{196}$, $\frac{1764}{1491}$, $\frac{355}{529}$, $\frac{207}{507}$, and $1\frac{10}{189}$.

DIVISION.

78. Rule. Invert the divisor, *i. e.* take its numerator as a denominator and its denominator as a numerator, and proceed as in Multiplication.

Ex. Divide $\frac{2}{11}$ by $\frac{3}{5}$.

Proceeding by the Rule given above,

$$\frac{2}{11}\div\frac{3}{5}=\frac{2}{11}\times\frac{5}{3}=\frac{10}{33}.$$

Reason for the rule.

If $\frac{2}{11}$ be divided by 3, the result is $\frac{2}{11\times3}$ or $\frac{2}{33}$, (Art. 65).

This result is 5 times too small, or, in other words, is only one fifth part of the required quotient, since, instead of dividing by 3, we have to divide by $\frac{3}{5}$, which is only one-fifth part of 3; and the quotient of $\frac{2}{11}$ divided by $\frac{3}{5}$ must therefore be 5 times greater than if the divisor were 3. Hence the above result $\frac{2}{33}$ must be multiplied by 5 in order to give the true quotient.

Therefore, the quotient $=\frac{2}{33}\times5=\frac{2\times5}{33}=\frac{10}{33}.$

Note 1. Before applying this rule, mixed numbers must be reduced to improper fractions, and compound fractions to simple ones, as in the following Examples:

Ex. 1. Divide $4\frac{1}{3}$ by $2\frac{3}{4}$.

$$4\tfrac{1}{3} \div 2\tfrac{3}{4} = \frac{13}{3} \div \frac{11}{4} = \frac{13}{3} \times \frac{4}{11} = \frac{52}{33} = 1\frac{19}{33}.$$

Ex. 2. Divide $\frac{3}{4}$ of $\frac{7}{8}$ by $\frac{15}{16}$ of 7.

$$\frac{3}{4} \text{ of } \frac{7}{8} \div \frac{15}{16} \text{ of } 7. = \frac{3\times7}{4\times8} \div \frac{15\times7}{16\times1} = \frac{3\times7}{4\times8} \times \frac{16\times1}{15\times7}$$

$$= \frac{3\times7\times16}{4\times8\times15\times7} = \frac{3\times7\times4\times4}{4\times2\times4\times3\times5\times7} = \frac{1}{10}.$$

Note 2. Complex Fractions may by this Rule be reduced to simple ones.

Thus, $$\frac{1\frac{3}{4}}{2\frac{1}{2}} = \frac{\frac{7}{4}}{\frac{5}{2}} = \frac{7}{4} \div \frac{5}{2} \text{ (Art. 60.)}$$

$$= \frac{7}{4} \times \frac{2}{5} = \frac{7}{10}.$$

Or thus, $$\frac{1\frac{3}{4}}{2\frac{1}{2}} = \frac{\frac{7}{4}}{\frac{5}{2}} = \frac{\frac{7}{4}\times4\times2}{\frac{5}{2}\times4\times2},$$

multiplying the numerator and denominator of the complex fraction by the product of the denominators of the simple fractions,

$$= \frac{14}{20} = \frac{7}{10}.$$

Again, $$\frac{4\frac{1}{2}}{30} = \frac{\frac{9}{2}}{30}, = \frac{\frac{9}{2}}{\frac{30}{1}} = \frac{9}{2} \div \frac{30}{1} = \frac{9}{2} \times \frac{1}{30} = \frac{3\times3}{2\times3\times10} = \frac{3}{20}.$$

Or thus, $$\frac{4\frac{1}{2}}{30} = \frac{\frac{9}{2}}{\frac{30}{1}} = \frac{\frac{9}{2}\times2\times1}{\frac{30}{1}\times2\times1} = \frac{9}{60}, = \frac{3}{20}.$$

Again $$\frac{30}{4\frac{1}{2}} = \frac{30}{\frac{9}{2}} = \frac{\frac{30}{1}}{\frac{9}{2}} = \frac{30}{1} \div \frac{9}{2} = \frac{30}{1} \times \frac{2}{9} = \frac{3\times10\times2}{3\times3} = \frac{20}{3} = 6\frac{2}{3},$$

Or thus, $$\frac{30}{4\frac{1}{2}} = \frac{\frac{30}{1}}{\frac{9}{2}} = \frac{\frac{30}{1}\times1\times2}{\frac{9}{2}\times1\times2} = \frac{60}{9} = \frac{20}{3} = 6\frac{2}{3}.$$

Ex. XXII.

1. Divide.

(1) 3 by $\frac{3}{4}$. (2) $\frac{5}{8}$ by $\frac{1}{4}$. (3) $\frac{8}{9}$ by $\frac{13}{18}$.
(4) $\frac{13}{20}$ by $\frac{3}{5}$. (5) $\frac{51}{64}$ by $3\frac{1}{6}$. (6) $\frac{16}{45}$ by $\frac{14}{15}$.

(7) $2\frac{8}{21}$ by $4\frac{1}{6}$. (8) $\frac{69}{80}$ by $\frac{5}{8}$ of $3\frac{2}{7}$. (9) $2\frac{1}{112}$ by $6\frac{2}{7}$ of $2\frac{1}{2}$.
(10) $3\frac{3}{4}$ of $3\frac{1}{3}$ of $\frac{1}{2}$ by 75.
(11) $3\frac{1}{5}$ of $5\frac{5}{8}$ of $3\frac{5}{9}$ by $9\frac{3}{4}$ of $\frac{8}{33}$ of $7\frac{1}{3}$. (12) 119 by $\frac{7}{9}$.
(13) $\frac{3}{8}$ of $\frac{6}{7}$ of $80\frac{1}{9}$ of 9 by $\frac{3}{7}$ of $\frac{2}{9}$ of $\frac{4}{5}$ of $8\frac{3}{4}$.
(14) $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{11}{18}$ of $1\frac{7}{11}$ by $\frac{6}{17}$ of $\frac{95}{114}$ of $\frac{33}{44}$ of $1\frac{6}{11}$.

2. Compare the product and quotient of $2\frac{1}{2}$ by $3\frac{7}{8}$.

3. Reduce to simple fractions the following complex fractions:

(1) $\dfrac{\frac{3}{4}}{1\frac{7}{8}}$. (2) $\dfrac{\frac{14}{27}}{2\frac{1}{3}}$. (3) $\dfrac{2\frac{1}{12}}{\frac{5}{9}}$. (4) $\dfrac{\frac{6}{11}}{\frac{10}{99}}$.

(5) $\dfrac{13\frac{1}{3}}{20}$. (6) $\dfrac{56}{1\frac{5}{9}}$. (7) $\dfrac{13\frac{9}{10}}{1\frac{11}{14}}$.

79. *Miscellaneous Examples in Fractions worked out.*

Ex. 1. What number added to $\frac{7}{8}+\frac{5}{12}$ will give $2\frac{1}{8}$?

This question in other words is the following: "What number will remain after $\frac{7}{8}+\frac{5}{12}$ has been subtracted from $2\frac{1}{8}$?"

Now, $$2\tfrac{1}{8}-(\tfrac{7}{8}+\tfrac{5}{12}) = 2\tfrac{1}{8}-\tfrac{7}{8}-\tfrac{5}{12} = \tfrac{17}{8}-\tfrac{7}{8}-\tfrac{5}{12}$$
$$=\tfrac{10}{8}-\tfrac{5}{12} = \tfrac{30-10}{24}=\tfrac{20}{24}=\tfrac{5}{6}.$$

Therefore the number required $=\frac{5}{6}$.

Note. It will be remembered, that all quantities within a vinculum are equally affected by any sign placed before the vinculum.

Thus in the above expression, $-(\frac{7}{8}+\frac{5}{12})$ means that the sum of $\frac{7}{8}$ and $\frac{5}{12}$ has to be subtracted from $2\frac{1}{8}$; whereas $-\frac{7}{8}+\frac{5}{12}$ would mean that $\frac{7}{8}$ had to be subtracted from $2\frac{1}{8}$, and then $\frac{5}{12}$ had to be added to the result.

Ex. 2. What number subtracted from $14\frac{3}{8}$ will leave $1\frac{3}{4}$ for a remainder?

Number required $$=14\tfrac{3}{8}-1\tfrac{3}{4}=(14+1+\tfrac{3}{8})-(1+1+\tfrac{3}{4})$$
$$=(14+\tfrac{11}{8})-(2+\tfrac{3}{4})=14-2+(\tfrac{11}{8}-\tfrac{3}{4})=12\tfrac{5}{8}.$$

Or thus, $$14\tfrac{3}{8}-1\tfrac{3}{4}=13\tfrac{3}{8}-\tfrac{3}{4}=\tfrac{107}{8}-\tfrac{6}{8}=\tfrac{101}{8}=12\tfrac{5}{8}.$$

Ex. 3. What number multiplied by $1\frac{3}{8}$ will produce $14\frac{3}{4}$?

This question in other words is the following: "If $14\frac{3}{4}$ be divided by $1\frac{3}{8}$, what will the quotient be?"

But $$\frac{14\frac{3}{4}}{1\frac{3}{8}} = \frac{\frac{59}{4}}{\frac{11}{8}} = \frac{59}{4}\times\frac{8}{11} = \frac{59\times 2}{11} = \frac{118}{11} = 10\tfrac{8}{11}$$

Therefore the number required $=10\frac{8}{11}$.

Ex. 4. What number divided by $1\frac{3}{8}$ will produce $10\frac{8}{11}$?

This question in other words is the following: "What is the product of $1\frac{3}{8}$ and $10\frac{8}{11}$?"

The product of $1\frac{3}{8}$ and $10\frac{8}{11} = \frac{11}{8} \times \frac{118}{11} = \frac{118}{8} = \frac{59}{4} = 14\frac{3}{4}$.

Ex. 5. Reduce the expression $\left(\frac{3\frac{1}{3}}{7} + \frac{2}{10\frac{1}{2}} - \frac{5}{18} \text{ of } \frac{4}{7}\right) \times 1\frac{3}{4}$

to its simplest form.

$$\left(\frac{3\frac{1}{3}}{7} + \frac{2}{10\frac{1}{2}} - \frac{5}{18} \text{ of } \frac{4}{7}\right) \times 1\frac{3}{4} = \left(\frac{\frac{10}{3}}{\frac{7}{1}} + \frac{\frac{2}{1}}{\frac{21}{2}} - \frac{5\times 4}{18\times 7}\right) \times \frac{7}{4}$$

$$= \left(\frac{10}{21} + \frac{4}{21} - \frac{10}{63}\right) \times \frac{7}{4} = \left(\frac{14}{21} - \frac{10}{63}\right) \times \frac{7}{4} = \frac{42-10}{63} \times \frac{7}{4} = \frac{32}{63} \times \frac{7}{4} = \frac{8}{9}.$$

Ex. 6. Simplify the expression $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{2}} + \frac{1}{4\frac{1}{2}}}$

$$\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}} + \frac{1}{3\frac{1}{2}} + \frac{1}{4\frac{1}{2}}} = \frac{\frac{6+4+3}{12}}{\frac{1}{\frac{5}{2}} + \frac{1}{\frac{7}{2}} + \frac{1}{\frac{9}{2}}} = \frac{\frac{13}{12}}{\frac{2}{5}+\frac{2}{7}+\frac{2}{9}} = \frac{\frac{13}{12}}{\frac{126+90+70}{315}}$$

$$= \frac{\frac{13}{12}}{\frac{286}{315}} = \frac{13}{12} \times \frac{315}{286} = \frac{13\times 3\times 105}{3\times 4\times 13\times 22} = \frac{105}{88} = 1\frac{17}{88}$$

Ex. 7. Divide $3\frac{1}{4} - \frac{5}{6}$ of $\frac{4}{15}$ by $21\frac{1}{5} + \frac{3}{10} + 4\frac{1}{3}$ of 5.

$$3\frac{1}{4} - \frac{5}{6} \text{ of } \frac{4}{15} = \frac{13}{4} - \frac{2}{9} = \frac{117-8}{36} = \frac{109}{36}.$$

$$21\frac{1}{5} + \frac{3}{10} + 4\frac{1}{3} \text{ of } 5 = 21\frac{1}{5} + \frac{3}{10} + \frac{13\times 5}{3} = 21\frac{2}{10} + \frac{3}{10} + \frac{65}{3}$$

$$= 21\frac{5}{10} + 21\frac{2}{3} = 21+21+\frac{1}{2}+\frac{2}{3} = 21+21+\frac{7}{6} = 43\frac{1}{6} = \frac{259}{6};$$

therefore the quotient required

$$= \frac{109}{36} \div \frac{259}{6} = \frac{109}{36} \times \frac{6}{259} = \frac{109}{6\times 6} \times \frac{6}{259} = \frac{109}{1554}.$$

Ex. 8. Simplify the expression

$$\frac{1}{13} \text{ of } \frac{1}{1+\frac{\frac{1}{3}}{3+\frac{1}{4}}}$$

Now, $\dfrac{1}{1+\frac{\frac{1}{3}}{3+\frac{1}{4}}} = \dfrac{1}{1+\frac{\frac{1}{3}}{\frac{13}{4}}} = \dfrac{1}{1+\frac{4}{39}} = \dfrac{39}{39+4} = \dfrac{39}{43}$;

therefore $\frac{1}{13}$ of $\frac{1}{1+\frac{\frac{1}{3}}{3+\frac{1}{4}}} = \frac{1}{13}$ of $\frac{39}{43} = \frac{3}{43}$.

Ex. 9. Simplify $\left\{2\frac{3}{4}+\frac{5}{2} \text{ of } \frac{7}{3\frac{4}{5}}-\frac{1\frac{2}{3}}{2\frac{1}{2}}\right\} \div 1\frac{77}{228}$.

The expression

$$= \left\{\frac{11}{4}+\frac{5}{2} \text{ of } \frac{\frac{7}{1}}{\frac{19}{5}} - \frac{\frac{5}{3}}{\frac{5}{2}}\right\} \div \frac{305}{228}$$

$$= \left\{\frac{11}{4}+\frac{5}{2}\times\frac{7}{1}\times\frac{5}{19}-\frac{5}{3}\times\frac{2}{5}\right\} \times \frac{228}{305}$$

$$= \left\{\frac{11}{4}+\frac{175}{38}-\frac{2}{3}\right\} \times \frac{228}{305}$$

(the least common multiple of 4, 38, and 3, = $38\times2\times3$)

$$= \left\{\frac{11\times19\times3+175\times2\times3-2\times38\times2}{38\times2\times3}\right\} \times \frac{228}{305}$$

$$= \left\{\frac{627+1050-152}{228}\right\} \times \frac{228}{305}$$

$$= \frac{1677-152}{228} \times \frac{228}{305} = \frac{1525}{305} = 5.$$

Ex. XXIII.

Miscellaneous Questions and Examples on Arts. (58—79).

I.

1. Define a fraction; what is the distinction between a Vulgar and a Decimal fraction? How many different kinds of Vulgar fractions are there? Give an example of each kind.

2. Find the sum and difference of $\frac{2\frac{1}{2}}{5}$ of $7\frac{3}{4}$, and $1\frac{3}{4}$ divided by $2\frac{1}{8}$; and the sum of $5\frac{1}{8}$, $\frac{2}{3}$ of $3\frac{1}{2}$, and $\frac{3}{4}\div\frac{6}{7}$.

3. Simplify

(1) $[\frac{3}{4}+\frac{7}{6} \text{ of } 5\frac{1}{2}]\times[\frac{5}{6}+\frac{2}{3}+3\frac{3}{4}]$. (2) $3\frac{1}{125}$ of $3\frac{4}{7}\div\frac{47}{345}$ of 9.

(3) $\frac{3\frac{3}{4}}{4\frac{2}{7}}-\frac{3\frac{2}{5}}{4\frac{1}{4}}+\frac{\frac{4}{7}}{2\frac{1}{2}}$. (4) $\frac{4\frac{1}{3}\times4\frac{1}{3}\times4\frac{1}{3}-1}{4\frac{1}{3}\times4\frac{1}{3}-1}$. (5) $3+\frac{1}{7+\frac{1}{16}}$.

4. Shew that the fraction $\frac{2+4+6}{3+5+7}$ lies between the greatest and least of the fractions $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{6}{7}$.

5. The difference of two numbers is $15\frac{4}{35}$; the greater number is $20\frac{1}{15}$: find the smaller number.

II.

1. If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction is not altered : prove this by means of an example.

2. What number subtracted from $41\frac{1}{4}$ leaves $19\frac{1}{2}$? and what number multiplied by $2\frac{4}{10}$ of $\frac{4}{13}$ produces $3\frac{1}{2}$ of $\frac{4}{5}$?

3. When is a fraction said to be in its *lowest* terms?

Reduce the fractions $\frac{93208}{137886}$ and $\frac{95469}{359784}$ to their lowest terms.

4. Simplify

(1) $\dfrac{2\frac{1}{2}}{3\frac{1}{4}}+\dfrac{1\frac{1}{2}-\frac{5}{6}}{1\frac{1}{4}+\frac{5}{6}}-1\frac{2}{39}$. (2) $3\frac{2}{5}$ of $5\frac{1}{2}$ of $\frac{7}{9}-\frac{1}{3}$ of $\frac{5}{12}$.

(3) $(\frac{2}{19}+\frac{1}{13})\div(3-\frac{1}{3})\times(\frac{1}{3}+\frac{1}{5})$. (4) $\frac{3}{14}$ of $\dfrac{4\frac{5}{9}}{6\frac{1}{6}}$ of $\dfrac{6\frac{8}{11}}{11\frac{5}{7}}$

5. Divide the product of $2\frac{2}{15}$ and $2\frac{5}{8}$ by the difference of $2\frac{3}{5}$ and $2\frac{1}{2}$. Explain why it is necessary in the addition and subtraction of fractions to reduce the fractions to a common denominator.

III.

1. Shew by an example that multiplying the numerator of a fraction by any number, is the same in effect as dividing the denominator by that number, and conversely.

2. Simplify

(1) $275\frac{1}{3}+62\frac{11}{120}+1031\frac{1}{5}+\frac{7}{8}$ of $4150\frac{1}{7}$. (2) $\frac{32}{51}\div\frac{112}{85}\times\frac{189}{207}\div1\frac{13}{23}$.

(3) $\dfrac{1}{3\frac{1}{5}}-\dfrac{2\frac{1}{4}}{9}+\dfrac{3\frac{5}{8}}{2}+\dfrac{\frac{4}{7}}{4\frac{4}{7}}$. (4) $\dfrac{4\frac{1}{4}-3\frac{3}{8}}{4\frac{1}{4}+3\frac{3}{8}}+\dfrac{3-2\frac{1}{3}}{4-3\frac{1}{4}}$.

3. Which is the greater, $\frac{1}{3}$ of 4 or $\frac{1}{4}$ of 5? and by how much?

4. Divide the sum of the fractions $\frac{3}{7}$ and $\frac{4}{13}$ by the product of $\frac{9}{11}$ and $\frac{13}{14}$; and reduce the result to its lowest terms.

5. What number is that, from which if you deduct $\frac{8}{9}-\frac{3}{7}$ and to the remainder add the quotient of $\frac{2}{15}$ divided by $2\frac{1}{2}$, the sum will be $1\frac{8}{63}$?

IV.

1. Define a vulgar fraction; an improper fraction; and the terms numerator and denominator of a fraction.

Prove by means of an example the rule for the multiplication of fractions; and multiply the sum of $\frac{4}{7}$ of $\frac{1}{2}$ and $1\frac{1}{5}$ by the difference of $\frac{4}{11}$ and $\frac{1}{5}$.

2. Reduce to their most simple forms the following expressions:

(1) $\frac{3}{4}\times\frac{7}{11}\times 8\frac{1}{3}\div\frac{3}{5}$ths of $(7\frac{3}{4}+\frac{4}{2})$. (2) $\frac{1}{6}-\frac{1}{12}+\frac{1}{15}-\frac{1}{20}$.

(3) $\frac{2}{3}\frac{7}{7}\frac{9}{3}\frac{6}{7}\frac{8}{6}$. (4) $\frac{1}{13}$ of $(1+5\frac{1}{2})+\frac{5}{6}$ of $\frac{1}{27}$ of $(7-2\frac{2}{3})-\frac{1}{3}$.

(5) $\dfrac{\frac{10}{3}+\frac{5}{6}-\frac{20}{21}}{\frac{5}{6}-\frac{4}{7}}$

3. What number added to $\frac{3}{4}$ of $(\frac{1}{3}+\frac{1}{5}-\frac{4}{15}+\frac{1}{6})$ makes $3\frac{1}{2}$? and what number divided by $\frac{1}{4}$ of $\frac{1}{6}$ of $\frac{1}{8}$ will give $\frac{64}{315}$?

4. If I pay away $\frac{1}{3}$ of my money, then $\frac{1}{2}$ of what remains, and then $\frac{1}{4}$ of what still remains; what fraction of the whole will be left?

5. Explain the method of 'comparing' fractions.

Compare the product and quotient of the sum and difference of $5\frac{1}{3}$ and $5\frac{1}{4}$.

V.

1. State the rules for multiplying and dividing one fraction by another; and prove them by means of an example.

Divide $\dfrac{2+3}{4+5}$ by $\dfrac{4+3\frac{1}{2}}{5+5\frac{1}{2}}$; and multiply the sum of $\frac{1}{3}$, $1\frac{3}{7}$, and $\frac{5}{6}$ by the difference of $\frac{4}{15}$ and $\frac{3}{20}$, and divide the product by $\frac{11}{15}$ by $1\frac{4}{15}$.

2. Reduce to their simplest forms

(1) $(\frac{4}{2}-\frac{2}{5})\div(\frac{4}{3}-\frac{3}{4})$. (2) $\dfrac{\frac{41}{162}-\frac{9}{40}-\frac{3}{54}}{\frac{4}{9}+\frac{1}{2}-\frac{13}{14}}$.

(3) $\frac{2}{5}$ of $\frac{13}{16}-\dfrac{1\frac{2}{3}}{6\frac{2}{3}}$ of $\frac{19}{20}+\frac{3}{7}$ of $\dfrac{6\frac{5}{12}}{3\frac{2}{5}}$.

(4) $\dfrac{\frac{3}{5}}{\frac{8}{9}}+\dfrac{\frac{5}{8}}{2\frac{1}{2}\times 1\frac{1}{3}}\times\frac{1}{80}$. (5) $2\frac{1}{2}\times\dfrac{1}{3\frac{1}{3}+\dfrac{1}{4\frac{1}{4}}}$.

(6) $\dfrac{\frac{5}{7}\text{ of }\frac{3}{10}+\frac{1}{4}\text{ of }\frac{8}{21}}{\frac{2}{3}\text{ of }\frac{9}{14}-\frac{5}{6}\text{ of }\frac{2}{15}}$. (7) $\dfrac{11\frac{5}{7}-7\frac{5}{11}}{3\frac{1}{2}+5\frac{5}{22}}$.

3. What is meant by the symbol $\frac{2}{5}$?

Find the least fraction which added to the sum of $\frac{3}{4}$, $\frac{7}{6}$, and $\frac{29}{12}$, shall make the result an integer.

4. Find the sum of the greatest and least of the fractions $\frac{3}{8}$, $\frac{5}{12}$, $\frac{4}{9}$ and $\frac{7}{20}$; the sum of the other two; and the difference of these sums.

5. A man has $\frac{3}{8}$ of an estate, he gives his son $\frac{1}{2}$ of his share; what portion of the estate has he then left?

VI.

1. State the rules for addition and subtraction of vulgar fractions; and prove them by means of an example.

2. Simplify

(1) $\frac{4}{5}$ of $\frac{1}{2}-\frac{2}{3}$ of $\frac{9}{17}+\frac{3}{5}$ of $1\frac{10}{17}$. (2) $\dfrac{2\frac{3}{4}+3\frac{2}{5}}{4\frac{1}{5}+5\frac{1}{4}}+\dfrac{3\frac{2}{3}}{10\frac{1}{2}}$.

(3) $\{\frac{5}{7}\times\frac{2}{9}\times 13\frac{1}{2}\}\div\{\frac{1}{9}\times\frac{3}{7}+40\}$. (4) $\dfrac{2\frac{4}{11}}{2\frac{3}{5}}\div\dfrac{2\frac{7}{11}}{8\frac{7}{10}}$.

3. Define a *proper*, *mixed*, and *compound* fraction. Explain the method of reducing a compound fraction to a simple one.

Ex. $\frac{2}{3}$ of $\frac{5}{9}$ of $\frac{81}{141}$ of $1\frac{1}{5}$.

4. Shew by means of an example how a fraction is affected if the same number be added to its numerator and denominator.

5. Multiply $3\frac{1}{9}$ by $3\frac{1}{10}$, and divide $\dfrac{20\frac{3}{4}}{3}$ by $\dfrac{41\frac{1}{2}}{4}$, and find the difference between the sum and difference of these results.

6. What number added to $\frac{31}{35}+\frac{10}{21}$ will produce $3\frac{227}{420}$? and what number divided by $2\frac{1}{16}$ will produce $\frac{4}{15}$?

VII.

1. Shew from the nature of fractions that $\frac{2}{3}+\frac{5}{7}=\frac{29}{21}$; that $\frac{2}{3}$ of $\frac{5}{7}=\frac{10}{21}$; and that $\frac{2}{3}\div\frac{5}{7}=\frac{14}{15}$.

2. Simplify

(1) $\dfrac{\frac{3}{4}}{\frac{5}{4}}+\dfrac{2\frac{1}{4}-\frac{3}{7}}{5\frac{1}{4}+\frac{1}{14}}-\dfrac{2}{3\frac{4}{7}}$. (2) $2\frac{1}{6}+3\frac{3}{8}+\frac{9}{14}+\frac{1}{15}+6\frac{17}{20}$.

(3) $(3\frac{1}{4}$ of $4\frac{1}{3})\div(2\frac{1}{2}-\frac{1}{3})$ of $(3\frac{1}{2}-\frac{1}{4}.)$

(4) $(\frac{1}{26}$ of $3\frac{1}{4})+(\frac{3}{8}\div\frac{27}{64})-\left(\dfrac{1}{1\frac{1}{7}}-\dfrac{1\frac{1}{3}}{3}\right)\div(2-\frac{5}{9})$.

3. Simplify $\dfrac{\frac{2}{3}\text{ of }\frac{5}{7}\text{ of }\frac{35}{9}}{\frac{1}{7}\text{ of }\frac{2}{3}\text{ of }\frac{84}{81}}$, and take the result from the sum of $10\frac{3}{4}$, $3\frac{9}{10}$, $7\frac{24}{25}$.

4. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, subtract the sum from 2, multiply the result by $\frac{2}{3}$ of $\frac{27}{40}$ of 8, and find what fraction this is of 99.

5. In a match of cricket, a side of 11 men made a certain number of runs, one obtained $\frac{1}{8}$th of the number, each of two others $\frac{1}{10}$th, and each of three others $\frac{1}{20}$, the rest made up between them 126; which was the remainder of the score, and 4 of these last scored 5 times as many as the other. What was the whole number of runs, and the score of each man?

DECIMALS.

80. It has been stated that figures in the units' place retain their *intrinsic* values, while those to the *left* of the units' place increase *tenfold* at each step from the units' place; therefore, according to the same notation, as we proceed from the units' place to the *right* every successive figure would decrease *tenfold*. We can thus represent whole numbers or integers and fractions under a uniform notation by means of figures in the units' place and on each side of it; for instance, in the number 5673·2412, the figures on the left of the *dot* · represent *integers*, while those on the right of the dot denote *fractions*. The number written at length would stand thus,

$$5\times1000+6\times100+7\times10+3+\tfrac{2}{10}+\tfrac{4}{100}+\tfrac{1}{1000}+\tfrac{2}{10000}.$$

The dot is termed the decimal point, and all digits to the right of it are called DECIMALS, because they are fractions with either 10, 100, or 10 × 10, 1000, or 10 × 10 × 10, &c, as their respective denominators.

81. It may here be observed, that, when a number is multiplied by itself any number of times, the product is called a POWER of that number; being called the *second*, *third*, *fourth*, &c. power, according as the number is multiplied *once*, *twice*, *three* times, &c. by itself, that is, according as it is employed *twice*, *three* times, &c. as a factor.

82. It will be seen from what has been said, that DECIMALS are in fact fractions having either 10 or some power of 10, for their denominators. For this reason also they are called DECIMAL (*Decem—ten*) FRACTIONS, in contradistinction to VULGAR FRACTIONS, which, as we have seen, are represented by a different notation, and not limited in their denominators to 10, or powers of 10.

83. From the preceding observations, it appears that

First, $$\cdot2345=\frac{2}{10}+\frac{3}{100}+\frac{4}{1000}+\frac{5}{10000}.$$

Now the least common multiple of the denominators of the fractions is 10000: therefore, reducing the several fractions to equivalent ones with their least common denominator, we get

$$\cdot2345=\frac{2}{10}\times\frac{1000}{1000}+\frac{3}{100}\times\frac{100}{100}+\frac{4}{1000}\times\frac{10}{10}+\frac{5}{10000}$$

$$=\frac{2000+300+40+5}{10000}=\frac{2345}{10000}.$$

Secondly, $\cdot00324=\frac{0}{10}+\frac{0}{100}+\frac{3}{1000}+\frac{2}{10000}+\frac{4}{100000}$

(the least common multiple of the denominators is 100000)

$$=\frac{0}{10}\times\frac{10000}{10000}+\frac{0}{100}\times\frac{1000}{1000}+\frac{3}{1000}\times\frac{100}{100}+\frac{2}{10000}\times\frac{10}{10}+\frac{4}{100000}$$

$$=\frac{300+20+4}{100000}=\frac{324}{100000}.$$

Thirdly, $56.816=5\times10+6+\frac{8}{10}+\frac{1}{100}+\frac{6}{1000}$

(the least common multiple of the denominators is 1000)

$$=\frac{5\times10}{1}\times\frac{1000}{1000}+\frac{6}{1}\times\frac{1000}{1000}+\frac{8}{10}\times\frac{100}{100}+\frac{1}{100}\times\frac{10}{10}+\frac{6}{1000}$$

$$=\frac{50000+6000+800+10+6}{1000}=\frac{56816}{1000}.$$

Hence, we infer that every decimal, and every number composed of integers and decimals, can be put down in the form of a vulgar fraction, with the figures comprising the decimal or those composing the integer and decimal part (the dot being in either case omitted) as a numerator, and with 1 followed by as many zeros as there are decimal places in the given number for the denominator.

84. Conversely, any fraction having 10 or any power of 10 for its denominator, as $\frac{56816}{1000}$, may be represented in the form 56·816.

For $\frac{56816}{1000}=\frac{5\times10000+6\times1000+8\times100+1\times10+6}{1000}$

$$=\frac{5\times10000}{1000}+\frac{6\times1000}{1000}+\frac{8\times100}{1000}+\frac{1\times10}{1000}+\frac{6}{1000}$$

$$=5\times10+6+\frac{8}{10}+\frac{1}{100}+\frac{6}{1000}$$

$=56·816$ (by the notation we have assumed).

85. Again, by what has been said above, it appears that

$$\cdot327=\frac{327}{1000}$$

$$\cdot0327=\frac{327}{10000}$$

$$\cdot3270=\frac{3270}{10000}=\frac{327}{1000}.$$

We see that ·327, ·0327, and ·3270 are respectively equivalent to fractions which have the same numerator, and the first and third of which have also the same denominator, while the denominator of the second is greater.

Consequently, ·327 is equal to ·3270, but ·0327 is less than either.

The value of a decimal is therefore not affected by *affixing* cyphers to the right of it; but its value is decreased by *prefixing* cyphers: which effect is exactly opposite to that which is produced by affixing and prefixing cyphers to integers.

86. Hence it appears that a decimal is *multiplied* by 10, if the decimal point be removed *one* place towards the *right* hand; by 100, if *two* places; by 1000, if *three* places; and so on: and conversely, a decimal is *divided* by 10, if the point be removed *one* place to the *left* hand; by 100, if *two* places; by 1000, if *three* places; and so on.

Thus

$$5·6 \times 10 = \tfrac{56}{10} \times 10 = 56.$$

$$5·6 \times 1000 = \tfrac{56}{10} \times 1000 = 5600.$$

$$5·6 \div 10 = \tfrac{56}{10} \times \tfrac{1}{10} = \tfrac{56}{100} = ·56.$$

$$5·6 \div 1000 = \tfrac{56}{10} \times \tfrac{1}{1000} = \tfrac{56}{10000} = ·0056.$$

87. The advantage arising from the use of decimals consists in this, viz: that the addition, subtraction, multiplication, and division of *decimal* fractions, are much more easily performed than those of *vulgar* fractions; and although all vulgar fractions cannot be reduced to finite decimals, yet we can find decimals so near their true value, that the error arising from using the *decimal* instead of the *vulgar* fraction is not perceptible.

Ex. XXIV.

1. Convert the following decimals into vulgar fractions:

·1; ·3; ·31; ·311; ·31111; ·31111111.

2. Convert the following decimals into vulgar fractions in their lowest terms:

·5; ·25; ·35; ·05; ·005; ·0256; ·000256; ·00008125.

3. Express as vulgar fractions in their lowest terms:

·075; ·848; 3·02; 3·434; 343·4; ·03434; ·050005; 230·409; 2·30409; 2137·2; 91300·0008; 24·000625; 8213·7169125; ·00083276; 1·0000009; ·000000001.

4. Express as decimals,

$\frac{1}{10}$; $\frac{3}{10}$; $\frac{7}{10}$; $\frac{53}{100}$; $\frac{7}{100}$; $\frac{3}{1000}$; $\frac{9178}{10000}$; $\frac{9178}{100}$; $\frac{9178}{1000000}$; $\frac{91}{10000}$; $\frac{9}{1000000}$; $\frac{5203}{10}$; $\frac{90}{100}$; $\frac{30142}{10000}$; $\frac{672819}{1000000}$; $\frac{672819}{10000000000}$; $\frac{67281900}{10000}$.

5. Multiply

·7 separately by 10, 100, 1000, and by 100000;
·006 separately by 100, 10000 and by 10000000;
·0431 separately by 100, and by 1000000;
16·201 separately by 10, 1000, and by a million;
9·0016 by ten hundred thousand, and by 100.

6. Divide

·51 separately by 10, 1000, and by 100000;
·008 separately by 100, and by a million;
5·016 separately by 1000, and by 100000;
378·0186 separately by 1000, and by a million.

7. Express according to the decimal notation, five tenths; seven-tenths; nineteen hundredths; twenty-eight hundredths; five thousandths; ninety-seven tenths; one millionth; fourteen and four-tenths; two hundred and eighty, and four ten-thousandths; seven and seven-thousandths; one hundred and one hundred-thousandths; one one-thousandth and one ten-millionth; five billionths.

8. Express the following decimals in words:

·4; ·25; ·75; ·745; ·1; ·001; ·00001; 23·75; 2·375; ·2375; ·00002375; 1.000001; ·1000001; ·00000001.

ADDITION OF DECIMALS.

88. Rule. Place the numbers under each other, units under units, tens under tens, &c., one-tenths under one tenths, &c.; so that the decimals be all under each other: add as in whole numbers, and place the decimal point in the sum under the decimal point above.

Ex. Add together 27·5037, ·042, 342, and 2·1.

Proceeding by the Rule given above,

```
 27·5037
   ·042
342·
  2.1
--------
371·6457
```

Note. The same method of explanation holds for the fundamental rules of decimals, which has been given at length in explaining the Rules for Simple Addition, Simple Subtraction, and the other fundamental rules in whole numbers.

Reason for the above process.

If we convert the decimals into fractions, and add them together as such, we obtain

$$27{\cdot}5037 + {\cdot}042 + 342 + 2{\cdot}1,$$

$$= \frac{275037}{10000} + \frac{42}{1000} + \frac{342}{1} + \frac{21}{10};$$

(or reducing the fractions to a common denominator),

$$= \frac{275037}{10000} + \frac{420}{10000} + \frac{3420000}{10000} + \frac{21000}{10000}$$

$$= \frac{3716457}{10000} = 371{\cdot}6457, \text{ (Art. 84).}$$

Ex. XXV.

1. Add together:
   (1) ·234, 14·3812, ·01, 32·47, and ·00075.
   (2) 232·15, 3·225, 21, ·0001, 34·005, and ·001304.
   (3) 14·94, ·00857, 1.5, 5607·25, 530, and ·0057.
2. Express in one sum:
   (1) ·08 + 165 + 1·327 + ·0003 + 2760·1 + 9.
   (2) 346 + ·0027 + ·25 + ·186 + 72·505 + ·0014 + ·00004.
   (3) 6·3084 + ·006 + 36·207 + ·0001 + 364 + 008022.
   (4) 725·1201 + 34·00076 + ·04 + 50·9 + 143·713.
   (5) 67·8125 + 27.105 + 17·5 + ·000375 + 255 + 3·0125.
3. Add together:

(1) 2·0068, ·04137, ·987641, 1·0000009, 57, and 1·5; and prove the result.

(2) ·0003025, 29·99987, 143·2, 5·000025, 9000, and 3·4073; and verify the result.

(3) 21·74, ·075, 103·00375, ·0005495, and 4957·5; and verify the result.

(4) Five hundred, and nine-hundredths; three hundred and seventy-five; twenty thousand and eighty-four, and seventy-eight hundred thousandths; eleven millions, two thousand, and two hundred and nine millionths; eleven thousand millionths; one billion, and one billionth.

SUBTRACTION OF DECIMALS.

89. RULE. Place the less number under the greater, units under units, tens under tens, &c., one-tenths under one-tenths, &c.; suppose cyphers to be supplied if necessary in the upper line to the right of the decimal: then proceed as in Simple Subtraction of whole numbers, and place the decimal point under the decimal point above.

Ex. Subtract 5·473 from 6·23.

Proceeding by the rule given above,

$$\begin{array}{r} 6{\cdot}23 \\ 5{\cdot}473 \\ \hline {\cdot}757 \end{array}$$

Reason for the above process.

If we convert the decimals into fractions, and subtract the one from the other as such, we obtain

$$6{\cdot}23 - 5{\cdot}473 = \frac{623}{100} - \frac{5473}{1000}$$
$$= \frac{6230}{1000} - \frac{5473}{1000}$$
$$= \frac{757}{1000}$$
$$= {\cdot}757, \text{ Art. (84.)}$$

Ex. XXVI.

1. Find the difference between 2·1354 and 1·0436; 7·835 and 2·0005; 15·67 and 156·7; ·001 and ·0009; ·305 and ·000683.

2. Find the value of

(1) 213·5 — 1·8125. (2) ·0516 — ·0094187.
(3) 603 — ·6584003. (4) 17·5 — 13·0046.
(5) ·582 — ·09347. (6) 9·233 — ·0536.

3. Take ·01 from ·1; 57·704 from 713·00683; 35·009876 from 56·078; 27·148 from 9816; and prove the truth of each result.

4. Required the difference between seven and seven tenths; also between seven tenths and seven millionths; also between seventy-four + three hundred and four thousandths and one hundred and seventy-four + one hundredths; and verify each result.

MULTIPLICATION OF DECIMALS.

90. RULE. Multiply the numbers together as if they were whole numbers, and point off in the product as many decimal places in both the multiplicand and the multiplier; if there are not figures enough, supply the deficiency by prefixing cyphers.

Ex. 1. Multiply 5·34 by ·21.

Proceeding by the Rule given above,

$$\begin{array}{r} 5{\cdot}34 \\ {\cdot}21 \\ \hline 534 \\ 1068 \\ \hline 11214 \end{array}$$

Now the number of decimal places in the multiplicand + the number of those in the multiplier $=2+2=4$;

therefore product $=1{\cdot}1214$.

Ex. 2. Multiply 5·34 by ·0021.

$$\begin{array}{r} 5{\cdot}34 \\ {\cdot}0021 \\ \hline 534 \\ 1068 \\ \hline 11214 \end{array}$$

We must have 6 decimal places in the product; but there are only 5 figures: and therefore we must prefix one zero, and place a point before it thus ·011214.

Reason for the above process.

$$5{\cdot}34\times{\cdot}21=\frac{534}{100}\times\frac{21}{100}$$

$$=\frac{11214}{10000}=1{\cdot}1214.$$

Again $$5{\cdot}34\times{\cdot}0021=\frac{534}{100}\times\frac{21}{10000}$$

$$=\frac{11214}{1000000}$$

$$={\cdot}011214.$$

Ex. XXVII.

1. Multiply together:

(1) 3·8 and 42; ·38 and ·42; 3·8 and 4·2; ·038 and ·0042.
(2) 417 and ·417; ·417 and ·417; 71956 and ·000025.
(3) 2·052 and ·0031; 4·07 and ·916; 476 and ·00026.

2. Multiply (proving the truth of the result in each case.)

(1) 81·4632 by ·0378. (2) 27·35 by 7·70071.
(3) ·04375 by ·0754.

3. Find the product of

(1) ·0046 by 7·85. (2) ·00846 by ·00324. (3) ·314 by ·0021.
(4) ·009 by ·00846. (5) ·009207 by 6·056. (6) ·00948 by 29;
proving the truth of each result.

4. Find the continued product of 1, ·01, ·001, and 100; also of ·12, 1·2, ·012, and 120; and prove the truth of the results.

5. Find the value of

(1) $7{\cdot}6 \times {\cdot}071 \times 2{\cdot}1 \times 29$.
(2) ${\cdot}007 \times 700 \times 760{\cdot}3 \times {\cdot}00416 \times 100000$.

DIVISION OF DECIMALS.

91. *First. When the number of decimal places in the dividend exceeds the number of decimal places in the divisor.*

Rule. Divide as in whole numbers, and mark off in the quotient a number of decimal places equal to the excess of the number of decimal places in the dividend over the number of decimal places in the divisor; if there are not figures sufficient, prefix cyphers as in Multiplication.

Ex. 1. Divide 1·1214 by 5·34.

Proceeding by the rule given above,

```
5·34) ·1·1214 (21
       1068
        534
        534
```

Now the number of decimal places in the dividend — the number of decimal places in the divisor=4—2=2;

therefore the quotient =·21.

Ex. 2. Divide ·011214 by 53·4.

```
53·4) ·011214 (21
        1068
        ----
         534
         534
         ---
```

Now the number of decimal places in the dividend — the number of decimal places in the divisor

$$=6-1=5;$$

therefore we prefix three cyphers, and the quotient is ·00021.

Reason for the above process.

$$1{\cdot}1214 \div 5{\cdot}34$$

$$=\frac{11214}{10000} \div \frac{534}{100} = \frac{11214}{10000} \times \frac{100}{534} = \frac{11214}{534} \times \frac{100}{10000} = \frac{21}{1} \times \frac{1}{100}$$

$$\left(\text{since } \frac{11214}{534} = 21 \text{ and } \frac{100}{10000} = \frac{1}{100}\right)$$

$$= \frac{21}{100} = {\cdot}21$$

Again, $\cdot 011214 \div 53{\cdot}4$

$$=\frac{11214}{1000000} \div \frac{534}{10} = \frac{11214}{1000000} \times \frac{10}{534} = \frac{11214}{534} \times \frac{10}{1000000}$$

$$=\frac{21}{1} \times \frac{1}{100000} = \frac{21}{100000} = {\cdot}00021.$$

92. *Secondly. When the number of decimal places in the dividend is less than the number of decimal places in the divisor.*

RULE. Affix cyphers to the dividend until the number of decimal places in the dividend equals the number of decimal places in the divisor; the quotient up to this point of the division will be a whole number; if there be a remainder, and the division be carried on further, the figures in the quotient after this point will be decimals.

Ex. Divide 1121·4 by ·534.

Proceeding by the Rule given above,

```
·534) 1121·400 (2100
      1068
      ----
        534
        534
        ---
         00
```

Reason for the above process.

$$1121{\cdot}4 \div {\cdot}534 = \frac{11214}{10} \div \frac{534}{1000}$$

$$= \frac{11214}{10} \times \frac{1000}{534} = \frac{11214}{534} \times \frac{1000}{10} = 21 \times 100 = 2100$$

Note. In order to prevent mistakes in the proof of examples in Division of Decimals, always contrive in the process to separate 10, 100, &c. in the two fractions from the other figures, as in the above examples; and be sure never to effect the multiplication if there be tens left in the denominator; nor, if there be tens left in the numerator, to effect it until the last step of the operation.

Ex. Divide 172·9 by ·142 to three places of decimals.

```
·142) 172·900000 (1217·605
      142
      ---
       309
       284
       ---
        250
        142
        ----
        1080
         994
         ---
          860
          852
          ---
           800
           710
           ---
            90
```

Here we must affix 5 cyphers to 172·9; for if we affix two according to the rule, the division up to that point will give the integral part of the quotient only, and therefore as the quotient is to be obtained to three places of decimals, we must affix three cyphers more, that is, we must affix five altogether.

Reason for the above process.

$$172{\cdot}9 \div {\cdot}142$$

$$= \frac{1729}{10} \div \frac{142}{1000} = \frac{1729}{142} \times \frac{1000}{10} = \frac{1729}{142} \times \frac{100000}{1000}$$

$$= \frac{172900000}{142} \times \frac{1}{1000}$$

Now $\frac{172900000}{142} = 1217605\ldots$ from above;

therefore the result $= \frac{1217605\ldots}{1000} = 1217{\cdot}605.$

Ex. XXVIII.

1. Divide, (proving the truth of each result by Fractions :)

(1) 10·836 by 5·16, and 34·96818 by ·381.
(2) ·02[illegible]7[illegible] by 1·003, and ·02916 by ·0012.
(3) ·00081 by 27, and 1·77089 by 4·735.
(4) 1 by ·1, by ·01, and by ·0001.
(5) 31·5 by ·126, and 5·2 by ·32.
(6) 3217 [illegible] ·0625, and ·03217 by 6250.
(7) 4·63638 by 31·34, and 15·4546 by ·019.
(8) ·429408 by 59·64, and 2147·04 by ·036.
(9) 12·6 by ·0012, and ·065341 by ·000475.
(10) 3·012 by ·0006, and 293916·669 by 541·283.
(11) 130·4 by ·0004 and by 4, and 46·634205 by 480[illegible]65.
(12) 1·69 by 1·3, by ·13, by 13, and also by ·013.
(13) ·00281 by 1·405, by 1405, and by ·001405.
(14) 72·36 by 36 and by ·0036, and ·003 by 1·6.
(15) 6725402·3544 by 7089, and by ·7089.
(16) 10363284·75 by 396·25, and ·09844 by ·0046.
(17) 816 by ·0004, and ·001961065287[illegible] by 2·38645.
(18) 18368830·5 by 2315, by 231·5, and by ·2315.
(19) ·00005 by 2·5, by 25, and by ·00000[illegible]5.
(20) 684·1197 by 1200·21, and also by ·0120021.

2. Divide to four places of decimals each of the following, and prove the truth of the results by Fractions:

(1) 32·5 by 8·7; ·02 by 1·7; 1 by ·013.

(2) ·009384 by ·0063; 51846·734 by 1·02.

(3) 7380·964 by ·023; 6·5 by 3·42; 25 by 19.

(4) 176432·76 by ·01257; 7457·1345 by 6535496·2.

(5) 37·24 by 2·9; ·0719 by 27·53.

3. Find the quotient (verifying each result) of

(1) ·0029202 by 157, and by 1·57.

(2) 5005 by 1953125; of 50·05 by 195·3125; of ·05005 by ·0001953125.

(3) ($7\frac{1}{2}$ of $\frac{1}{5}+\frac{17}{25}$) by ·0005; of 31·008 by $\frac{428}{515}$ of $1\frac{1}{4}$ of $\frac{5253}{18190}$; ·7575 by $16\frac{2}{3}$.

93. *Certain Vulgar Fractions can be expressed accurately as Decimals.*

Rule. Reduce the fraction to its lowest terms; then place a dot after the numerator and affix cyphers for decimals; divide by the denominator, as in division of decimals, and the quotient will be the decimal required.

Ex. 1. Convert $\frac{3}{5}$ into a decimal.

$$5 \,|\, \underline{3·0}$$
$$·6$$

There is one decimal place in the dividend and none in the divisor; therefore there is one decimal place in the quotient.

Note. In reducing any such fraction as $\frac{3}{50}$ or $\frac{3}{500}$ to a decimal we may proceed in the same way as if we were reducing $\frac{3}{5}$; taking care however in the result to move the decimal point one place further to the left for each cypher cut off.

Thus
$$\frac{3}{5}=·6,$$
$$\frac{3}{50}=·06,$$
$$\frac{3}{500}=·006,$$

for in fact, we divide by 5, and then by 10, 100, &c., according as the divisor is 50, 500, &c.

Ex. 2. Reduce $\frac{5}{16}$ to a decimal.

$$\begin{array}{l} 16)\ 5\cdot0000\ (\cdot3125 \\ \quad 48 \\ \quad \overline{\ \ 20} \\ \quad\ \ 16 \\ \quad\ \ \overline{\ \ 40} \\ \quad\ \ \ \ 32 \\ \quad\ \ \ \ \overline{\ \ 80} \\ \quad\ \ \ \ \ \ 80 \end{array}$$

or thus,
$$16\left\{\begin{array}{r|l} 4 & 5\cdot00 \\ 4 & \overline{1\cdot2500} \\ & \overline{\ \ \cdot3125} \end{array}\right.$$

$\because \frac{5}{16} = \cdot3125$

Ex. 3. Convert $\frac{3}{512}$ and $\frac{3}{51200}$ into decimals.

Now $512 = 8 \times 64 = 8 \times 8 \times 8$

$$\begin{array}{r|l} 8 & 3\cdot000 \\ 8 & \overline{\ \cdot375000} \\ 8 & \overline{\ \cdot046875000} \\ & \overline{\ \cdot005859375} \end{array}$$

or $\frac{3}{512}$ is equivalent to $\cdot005859375$, and $\frac{3}{51200}$ is equivalent to $\cdot00005859375$.

Ex. 4. Convert $\frac{3}{5} + 3\frac{1}{8} + 2\frac{9}{40} + 6\frac{11}{125}$ into a decimal.

$$\tfrac{3}{5} + 3\tfrac{1}{8} + 2\tfrac{9}{40} + 6\tfrac{11}{125} = 11 + \tfrac{3}{5} + \tfrac{1}{8} + \tfrac{9}{40} + \tfrac{11}{125}.$$

$$\begin{array}{r|l} 5 & 3.0 \\ & \overline{\ \ \cdot6} \end{array} \qquad \begin{array}{r|l} 8 & 1\cdot000 \\ & \overline{\ \cdot125} \end{array} \qquad \begin{array}{r|l} 4 & 9\cdot00 \\ & \overline{\ 2\cdot25} \end{array}$$

$$\begin{array}{r|l} 5 & 11\cdot0 \\ 5 & \overline{\ 2\cdot20} \\ 5 & \overline{\ \ \cdot440} \\ & \overline{\ \ \cdot088} \end{array} \qquad \therefore \tfrac{9}{40} = \cdot225$$

therefore $\frac{3}{5} = \cdot6,\ \frac{1}{8} = \cdot125,\ \frac{9}{40} = \cdot225,\ \frac{11}{125} = \cdot088$;
therefore the whole expression

$$= 11 + \cdot6 + \cdot125 + \cdot225 + \cdot088$$
$$= 12\cdot038.$$

Ex. XXIX.

1. Reduce to decimals :

(1) $\frac{1}{4}$; $\frac{3}{4}$; $\frac{5}{8}$; $\frac{9}{25}$; $\frac{5}{16}$; $\frac{19}{20}$.

(2) $\frac{66}{128}$; $\frac{54}{125}$; $\frac{570}{200}$; $\frac{170}{125}$; $\frac{1}{160}$.

(3) $6\frac{11}{64}$; $\frac{57}{240}$; $\frac{13}{256}$; $\frac{3}{512}$; $15\frac{588}{78125}$.

2. Reduce to decimals :

(1) $3\frac{5}{8}$ of $\frac{1}{512}$. (2) $\frac{1}{2}+\frac{1}{4}+\frac{1}{16}+\frac{1}{32}$ (3) $\frac{1}{64}\times \cdot 0064$.

(4) $\frac{3}{5}+\cdot 061$. (5) $\frac{1}{2}+\frac{1}{5}-\frac{1}{8}$. (6) $\frac{47\frac{5}{8}}{94}$ of $\frac{11\frac{3}{4}}{7\cdot 5}$.

(7) $\frac{7\cdot 75}{9}$ of $\frac{2\frac{1}{2}}{2\frac{7}{9}}$ of $\frac{20}{31}$. (8) $5\frac{5}{640}+\cdot 75$ of $\frac{6}{5}$ of $7\frac{1}{2}$.

(9) $3\frac{4}{25}+\frac{33}{110}+81\frac{37}{1000}+\frac{7\frac{3}{4}}{3\frac{7}{8}}$.

(10) $\frac{247}{\frac{5}{7}}+\frac{1512}{108}+\frac{17}{7\frac{5}{9}}+200\frac{7}{10}+\frac{11}{62\cdot 5}$.

Note. 10 is sometimes called the *first power* of 10.
10 × 10 .... *second power* of 10.
10 × 10 × 10 .... *third power* of 10.
10 × 10 × 10 × 10 × 10 ..... *fifth power* of 10.

and so on, similarly of other numbers.

94. We have seen that, in order to convert a vulgar fraction into a decimal, after reducing the fraction to its lowest terms and affixing cyphers to the numerator, we have in fact to divide 10, or some multiple of 10 or of its powers, by the denominator of the fraction: now $10=2\times 5$, and these are the only factors into which 10 can be broken up; therefore, when the fraction is in its lowest terms, if the denominator be not composed solely of the factors 2 and 5, or one of them, or of powers of 2 and 5, or one of them, then the division of the numerator by the denominator will never terminate. Decimals of this kind, that is, which never terminate, are called indeterminate decimals, and they are also called Circulating, Repeating, or Recurring Decimals, from the fact that when a decimal does not terminate, the same figures must come round again, or recur, or be repeated: for since we always affix the same figure to the dividend, namely a cypher, whenever any former remainder recurs, the quotient will also recur. Now when we divide by any number, the remainder must

always be less than that number, and therefore some remainder must recur before we have obtained a number of remainders equal to the number of units in the divisor.

95. PURE CIRCULATING DECIMALS are those which recur from the beginning; thus ·3333..., ·272727..., are pure circulating decimals.

MIXED CIRCULATING DECIMALS are those which do not begin to recur, till after a certain number of figures.

Thus ·128888......, ·0113636......, are mixed circulating decimals.

The circulating part, or the part which is repeated is called the PERIOD OR REPETEND.

Pure and mixed circulating decimals are generally written down only to the end of the first period, a dot being placed over the first and last figures of that period.

Thus $\cdot\dot{3}$	represents the pure circulating decimal	333...
$\cdot\dot{3}\dot{6}$	----------------------------	·3636...
$\cdot\dot{6}3\dot{9}$	----------------------------	·639639...
$\cdot 13\dot{8}$	------------mixed------------	·1388...
$\cdot 011\dot{3}\dot{6}$	----------------------------	·0113636...

96. *Pure Circulating Decimals may be converted into their equivalent Vulgar Fractions by the following Rule.*

RULE. Make the period or repetend the numerator of the fraction, and for the denominator put down as many *nines* as there are figures in the period or repetend; this fraction, reduced to its lowest terms, will be the fraction required.

Note. The fraction is only reduced to its lowest terms for the sake of exhibiting it in its simplest form. It is not of course actually necessary so to reduce it.

Exs. Reduce the following pure circulating decimals, $\cdot\dot{3}$ $\cdot\dot{2}\dot{7}$, $\cdot\dot{8}5714\dot{2}$, to their respective equivalent vulgar fractions.

Proceeding by the rule given above,

$$\cdot\dot{3}=\frac{3}{9}=\frac{1}{3}.$$

$$\cdot\dot{2}\dot{7}=\frac{27}{99}=\frac{3}{11}.$$

$$\cdot\dot{8}5714\dot{2}=\frac{857142}{999999}=\frac{95238}{111111}=\frac{6\times 15873}{7\times 15873}=\frac{6}{7}.$$

The truth of these results will appear from the following considerations.

Let the circulating decimal ·3333... be represented by a symbol x; then

$$x = \cdot 3333...$$

therefore

$$10 \text{ times } x = 10 \text{ times } \cdot 3333...$$
$$= 3\cdot 3333... \text{ (Art. 86).}$$

Now 10 times x, diminished by 1 time x, will leave 9 times x, and

$$3\cdot 3333... - \cdot 3333 = 3.3333...$$
$$- \cdot 3333...$$
$$= 3$$

$$\text{or } 9 \text{ times } x = 3$$

therefore 1 time x, that is x or $\cdot 3333... = \frac{3}{9} = \frac{1}{3}$.

Next, let the circulating decimal ·2727··· be represented by x. Then,

$$x = \cdot 272727...$$

here, since there are two figures in each period, we multiply by 100, and we have

$$100 \text{ times } x = 100 \text{ times } \cdot 2727...$$
$$= 27\cdot 2727... \text{ (Art. 86).}$$

Therefore 100 times x, diminished by 1 time x, will be equal to

$$27\cdot 2727... - \cdot 2727...$$
$$\text{or } 99 \text{ times } x = 27;$$

therefore x or $\cdot 2727... = \frac{27}{99} = \frac{3}{11}$,

Next, let the recurring decimal $\cdot\dot{8}5714\dot{2}$ be represented by x. Then,

$$x = \cdot 857142857142...$$

here since there are six figures in each period, we multiply 1000000, and we have

$$1000000 \text{ times } x = 1000000 \text{ times } \cdot 875142...,$$
$$= 875142\cdot 875142...;$$

therefore

$$999999\, x = 857142,$$

or

$$x = \frac{857142}{999999},$$

which fraction, reduced to its lowest terms, $= \frac{6}{7}$.

Note 1. The object in each case is to multiply the recurring decimal by such a power of 10, as will bring out the period a whole number.

Note 2. The *powers* of numbers are often expressed by placing *a small figure* (equivalent to the number of factors and called the INDEX OR EXPONENT of the power) at the right hand of the number, a little above the line.

Thus 10×10, or the *second* power of 10 is expressed by 10^2,
$10 \times 10 \times 10$, or the *third* power of 10............. 10^3,
$10 \times 10 \times 10 \times 10 \times 10$, or the *fifth* power of 10.......... 10^5,
and so on.

97. *Mixed Circulating Decimals may be converted into their equivalent Vulgar Fractions by the following Rule.*

RULE. Subtract the figures which do not circulate from the figures taken to the end of the first period, as if both were whole numbers; make the result the numerator; and write down as many *nines* as there are figures in the circulating part, followed by as many *zeros* as there are figures in the non-circulating part for the denominator.

Exs. Reduce the following mixed circulating decimals, $\cdot 1\dot{4}$, $\cdot 013\dot{8}$, $\cdot 2\dot{4}1\dot{8}$, to their respective equivalent vulgar fractions.

Proceeding by the Rule given above,

$$\cdot 1\dot{4} = \frac{14-1}{90} = \frac{13}{90}.$$

$$\cdot 013\dot{8} = \frac{138-13}{9000} = \frac{125}{9000}$$

$$= \frac{1}{72}, \text{ in its lowest terms,}$$

$$\cdot 2\dot{4}1\dot{8} = \frac{2418-2}{9990} = \frac{2416}{9990}$$

$$= \frac{1208}{4995}.$$

The truth of these results will appear from the following considerations.

Let the mixed circulating decimal be represented by x in each of the above cases.

First, let $x = \cdot 1444...$

If, by multiplication, we change the decimal in such a manner that the non-circulating part is rendered a whole number, and also change it so that the non-circulating and circulating parts to the end of the first period are rendered a whole number, and then subtract the first result from the second, we shall get rid of the circulating part. Thus, multiplying first by 10 to get the 1 out as a whole number, and then by 100 to get the 14 out as a whole number, we have

$$10 \text{ times } x = 10 \text{ times } \cdot 1444\ldots$$
$$= 1\cdot 444\ldots$$
$$100 \text{ times } x = 14\cdot 444\ldots;$$

therefore

$$100 \text{ times } x - 10 \text{ times } x$$
$$= 14\cdot 444\ldots - 1.444\ldots$$

$$\text{or } 90 \text{ times } x = 14.444\ldots$$
$$-1.444\ldots$$
$$= 13$$

therefore $$x = \frac{13}{90}.$$

Next, let $$x = \cdot 013888\ldots$$

Here there are three places in the non-recurring part, and one in the recurring part; therefore multiplying first by 1000, and then by 10000, we have

$$1000 \text{ times } x = 1000 \times \cdot 013888\ldots = 13\cdot 8888\ldots;$$

and $$10000 \text{ times } x = 138\cdot 8888\ldots;$$

therefore subtracting, as before,

$$9000 \text{ times } x = 138 - 13 = 125;$$

therefore $$x = \frac{125}{9000} = \frac{1}{72}.$$

Next, let $$x = \cdot 2418418\ldots$$

Now we have one place in the non-recurring part, and three places in the recurring part; therefore multiplying first by 10, and then by 10000 we have

$$10 \text{ times } x = 2\cdot 418418\ldots$$

$$10000 \text{ times } x = 2418\cdot 418418\ldots;$$

therefore $$9990 \text{ times } x = 2418 - 2 = 2416;$$

therefore $$x = \frac{2416}{9990} = \frac{1208}{4995}$$

Ex. XXX.

1. Reduce the following vulgar fractions and mixed numbers to circulating decimals:

(1) $\frac{5}{9}$; $\frac{2}{11}$; $\frac{1}{37}$; $\frac{3}{7}$. (2) $\frac{17}{30}$; $\frac{368}{495}$; $\frac{16}{81}$; $15\frac{52}{333}$.

(3) $\frac{32231}{35200}$; $7\frac{962}{3367}$; $\frac{17}{99000}$. (4) $24\frac{83}{9768}$; $17\frac{13}{700}$; $2\frac{139868}{833325}$.

2. Find the vulgar fractions equivalent to the recurring decimals;

(1) $\cdot\dot{7}$; $\cdot 0\dot{7}$; $\cdot 2\dot{2}\dot{7}$. (2) $\cdot 5\dot{8}\dot{3}$; $\cdot\dot{1}3\dot{5}$; $\cdot 26\dot{3}$;

(3) $\cdot 00\dot{1}8\dot{5}$; $3\cdot\dot{0}2\dot{4}$; $\cdot 012\dot{3}\dot{6}$. (4) $\cdot\dot{1}4285\dot{7}$; $\cdot 39791\dot{6}$; $\cdot 382\dot{1}4285\dot{7}$

(5) $\cdot\dot{3}0769\dot{2}$; $\cdot 6\dot{3}0769\dot{2}$; $2\cdot 7\dot{8}5714\dot{2}$.

(6) $\cdot 342\dot{7}5\dot{3}$; $\cdot 03132\dot{1}3\dot{2}$; $8\cdot 0208\dot{3}$.

(7) $85\cdot 6080\dot{6}$; $3\cdot 6\dot{4}2857\dot{1}$; $127\cdot 00022\dot{0}9\dot{5}$.

98. The value of the circulating decimal ·999... is found by Art. (96) to be $\frac{9}{9}$ or 1; but since the difference between 1 and ·9=·1, between 1 and ·99=·01, between 1 and ·999=·001, &c., it appears that however far we continue the recurring decimal, it can never at any stage be *actually*=1. But the recurring decimal is considered =1, because the difference between 1 and ·99... becomes less and less, the more figures we take in the decimal, which thus, in fact, approaches nearer to 1 than by any difference that can be assigned.

In like manner, it is in this sense that any vulgar fraction can be said to be the value of a circulating decimal; because there is no assignable difference between their values.

99. In arithmetical operations, where circulating decimals are concerned, and the result is only required to be true to a certain number of decimal places, it will be sufficient to carry on the circulating part to two or three decimal places more than the number required: taking care that the last figure retained be increased by 1, if the succeeding figure be 5, or greater than 5; because, for instance, if we have the mixed decimal ·6288, and and stop at ·628, it is clear that ·628 is less, and ·629 is greater than the true value of the decimal: but ·628 is less than the true value by ·000888,... and ·629 is greater than the true value by ·000111...

Now ·000111... is less than ·000888...

Therefore ·629 is nearer the true value than ·628.

Ex. 1. Add together $\cdot 3\dot{3}$, $\cdot 0\dot{4}3\dot{2}$, $2\cdot 3\dot{4}\dot{5}$, so as to be correct to 5 places of decimals.

$$\begin{array}{r} \cdot 3333333 \\ \cdot 0432432 \\ 2\cdot 3454546 \\ \hline 2\cdot 7220311 \end{array} \qquad \textit{Ans.}\ 2\cdot 72203.$$

Ex. 2. Subtract $\cdot 291\dot{6}$ from $\cdot 98958\dot{3}$, so as to be correct to 5 places of decimals.

$$\begin{array}{r} \cdot 9895833 \\ \cdot 2916667 \\ \hline \cdot 6979166 \end{array} \qquad \textit{Ans.}\ \cdot 69791.$$

Note. This method may be advantageously applied in the Addition and Subtraction of circulating decimals. In the Multiplication and Division, however, of circulating decimals, it is always preferable to reduce the circulating decimals to Vulgar Fractions, and having found the product or quotient as a Vulgar Fraction, then, if necessary, to reduce the result to a decimal.

Ex. XXXI.

(1) Find the value (correct to 6 places of decimals) of

1. $2\cdot\dot{4}1\dot{8} + 1\cdot 1\dot{6} + 3\cdot\dot{0}0\dot{9} + \cdot 735\dot{4} + 24\cdot 042$.
2. $234\cdot\dot{6} + 9\cdot 92\dot{8} + \cdot\dot{0}12345678\dot{9} + \cdot\dot{0}04\dot{4} + 456$.
3. $6\cdot 45 - \cdot\dot{3}$; and $7\cdot\dot{7}\dot{2} - 6\cdot 0\dot{4}\dot{5}$; and $309 - \cdot 94\dot{7}2\dot{4}$.

(2) Express the sum of $\frac{49}{52}$, $\frac{278}{390}$, and $\frac{7}{12}$, and the difference of $18\frac{1}{12}$ and $4\frac{5}{14}$, as recurring decimals.

(3) Multiply

1. $2\cdot\dot{3}$ by $3\cdot\dot{6}$; $\cdot 7\dot{5}7\dot{5}$ by $36\dot{6}$. 2. $\cdot 40\dot{6}$ by 62; 825 by $\cdot\dot{3}\dot{6}$.
3. $7\cdot 5\dot{2}$ by $48\cdot\dot{3}$; 368 by $\cdot\dot{6}$. 4. $3\cdot\dot{1}4\dot{5}$ by $\cdot\dot{4}29\dot{7}$; $20\frac{4}{7}$ by $\cdot\dot{8}\dot{4}$.

(4) Divide

1. $195\cdot 0\dot{2}$ by 4; $\cdot 37\dot{5}9\dot{2}$ by $\cdot 0\dot{5}$. 2. 54 by $\cdot 1\dot{7}$; $13\cdot\dot{2}$ by $5\cdot\dot{6}$.
3. $411\cdot 3\dot{5}1\dot{9}$ by $58\cdot 7\dot{6}4\dot{5}$; $2\cdot 1659\dot{5}$ by $\cdot 0\dot{4}$; $\cdot 6559\dot{9}0\dot{3}$ by $48\cdot 76$.

Ex. XXXII.

Miscellaneous Questions and Examples on Arts. (80—99).

I.

(1) Define a Decimal; and shew how its value is affected by

affixing and prefixing cyphers. Reduce ·0625, and 3·14159 to fractions; and express the difference between $20\frac{5}{14}$ and $17\frac{1}{12}$ as a decimal.

(2) Find the value of $10\frac{3}{8}+1\frac{5}{40}+\frac{7}{10}+\frac{13}{16}$ both by vulgar fractions and by decimals; and shew that the results coincide.

(3) Find the sum, difference, product, and quotient of 573·005 and ·000754; and of 1·015 and ·01015, and prove the truth of each result.

(4) If a vulgar fraction, being converted into a decimal, do not terminate, prove that it must recur. What must be the limit to the number of figures in the recurring part? Is $\frac{3}{6144}$ convertible into a terminating decimal?

(5) Simplify 1. $2\frac{1}{3}+72\frac{5}{8}+316\frac{1}{6}+2\cdot875$. 2. $\cdot026649\div2\frac{15}{16}$.
3. $\dfrac{1-\cdot05}{5+\cdot5}\times\dfrac{3-\cdot8}{3\cdot8}\div\dfrac{1}{10}$. 4. $\{\cdot18+\cdot009\}\div\cdot016$.

(6) Divide $\dfrac{48\frac{4}{7}}{1085\frac{7}{10}}$ by $\dfrac{7\frac{3}{11}}{174\frac{3}{17}}$; reduce the quotient to the form $1\cdot\dot{0}71428\dot{5}$. Divide $91\cdot\dot{8}6\dot{3}$ by $87\cdot5\dot{6}$.

II.

(1) Write down in a decimal form seven hundred thousand four hundred and nine billionths. Express 12·1345 as a fraction, and $\frac{32546}{1000000000}$ as a decimal.

(2) State the effect as regards the decimal point of multiplying and dividing a decimal by any given power of 10. Write down in words the meaning of 397008·405009; multiply it by 1000, and also divide it by 1000; and write down the meaning of each result in words.

(3) What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375 and 2·46?

(4) Multiply 1·05 by 10·5; and reduce the result to a fraction in its lowest terms. Divide ·8727588 by 1620; find the value of $\dfrac{\cdot0003\times\cdot004}{\cdot006}$; reduce $\frac{1}{16}+\frac{9}{400}-\frac{13}{625}$ to a decimal.

(5) Simplify, expressing each result in a decimal form,

1. $\frac{1}{10000}$ of $\frac{21}{23}$. 2. $(2\frac{1}{2}+6)\div(3\frac{1}{2}-\frac{1}{2})$.

3. $\dfrac{4\cdot4+\frac{3}{5}}{7\cdot375+\frac{3}{4}-\frac{1}{8}}$ 4. $2\frac{1}{3000}+1\frac{5}{1600}+5\frac{1}{6000}+2\cdot000875$.

(6) Find a number which multiplied into 3132·458 will give a product which differs only in the 7th decimal place from 7823·6572.

III.

(1) Divide 684·1197 by 1200·21, and also by ·0120021; and 594·27 by ·047 to three places of decimals, and explain fully how the position of the decimal point is determined in each of the quotients.

(2) Simplify, expressing each result in a fractional and decimal form,

1. $\dfrac{\cdot 015 \times 2\cdot 1}{\cdot 035}$. 2. $\dfrac{3\frac{1}{2} - \cdot 04}{5 - \cdot 0625}$.

3. $\frac{3}{5} + \cdot 14 + \frac{3}{4}$ of 1·0784. 4. $(\frac{1}{4} - \frac{1}{9}) \times (\frac{3}{5} + 1\frac{1}{2})$.

(3) What is meant by a 'Recurring Decimal'? What kind of vulgar fractions produce such decimals? State the rules for reducing any recurring decimal to a vulgar fraction. Multiply $5\cdot\dot{8}\dot{1}$ by $\cdot 458\dot{3}$, and divide $1\cdot 1\dot{3}$ by $\cdot 000\dot{1}3\dot{2}$. Is $\frac{91}{560}$ reducible to a recurring decimal?

(4) Shew that if $1\frac{1}{12}$, $2\frac{2}{15}$, $3\frac{3}{20}$, $4\frac{4}{27}$ be added together, (1) as fractions, and (2) as decimals, the results coincide.

(5) A man walked in 4 days 60 miles; in each of the three first days he walked an equal distance, in the fourth day he walked 13·95 miles; find the amount of his daily walking.

(6) A person has ·1875 part of a mine, he sells $\cdot 1\dot{7}$ part of his share; what fractional part of the mine has he still left?

IV.

(1) State the Rules for the Addition and Subtraction of Decimals. Add together 1·23, ·123, ·0123, ·00123, and 123; and find the vulgar fraction corresponding to the result. Find the fraction equivalent to $31\cdot 457\dot{4}5\dot{7}$, and subtract it from the fraction $\frac{494}{15}$.

(2) Write down in figures the number, three millions six thousand and five. Also write down in words the signification of the same figures when the last is marked off as a decimal.

(3) Compare the values of 5 × ·05, 1·5 × ·75, and 2·625 ÷ 5.

(4) Find the product of $\cdot 014\dot{7}14\dot{7}$ by $\cdot 33\dot{3}$; and the quotients of $\cdot 1269\dot{3}$ by $19\cdot\dot{3}\dot{9}$; of 132790 by ·245; of ·014904 by $3\frac{6}{25}$; of 61061 by 3·05; and of 6106·1 by 305000.

(5) Shew that the decimal ·90437532 is more nearly represented by ·90438 than by ·90437; and find the value of

$$16 \times \left\{ \frac{1}{5} - \frac{1}{3 \times 5^3} + \frac{1}{5 \times 5^5} - \frac{1}{7 \times 5^7} + \&c. \right\} - \frac{4}{239}$$

accurately to 5 places of decimals.

(6) A person sold ·15 of an estate to one person, and then $\frac{6}{17}$ of the remainder to another person. What part of the estate did he still retain?

V.

(1) Express $\frac{1}{2}(6\frac{1}{2}+2\frac{2}{3}-3)$, $\frac{8978}{3125}$, and also the product of $3\frac{5}{8}$ and $(3\frac{1}{4}-\frac{2}{3})$ of $\frac{6}{7}$ as decimals.

(2) Simplify

1. $\dfrac{4{\cdot}255 \times {\cdot}032}{{\cdot}00016}$ 2. $(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}) \div (\frac{1}{3}+\frac{1}{5}+\frac{1}{9}+\frac{1}{15})$.

3. $(\frac{9}{11}$ of $35\frac{1}{5}-3\frac{1}{8})+(2{\cdot}5625+7\frac{1}{4})$.

4. $59\dot{3} \div 1{\cdot}78 \times {\cdot}\dot{3}\dot{6} \div {\cdot}072$.

(3) State at length the advantages which decimals possess over vulgar fractions; what disadvantages have they?

Shew whether $\frac{22}{7}$ or $\frac{333}{106}$ is nearer to the number 3·14159.

(4) Find the value of $1+\frac{1}{1}+\frac{1}{1 \times 2}+\frac{1}{1 \times 2 \times 3}+\&c.$, to 7 places of decimals; and also of

$$\frac{1}{10^3} \times \left(1 - \frac{3}{10^2} + \frac{3 \times 4}{1 \times 2} \times \frac{1}{10^4} + \frac{3 \times 4 \times 5}{1 \times 2 \times 3} \times \frac{1}{10^6} \right)$$

expressing it (1) as a decimal, and (2) as a fraction.

(5) Find the Earth's equatorial diameter in miles, supposing the Sun's diameter, which is 111·454 times as great as the equatorial diameter of the Earth, to be 883345 miles.

(6) In what sense is a vulgar fraction said to be the value of a recurring decimal? Explain how a sufficient degree of accuracy may be obtained in the addition and subtraction of circulating decimals to any given number of decimal places, without converting the decimals into fractions.

Ex. Find the sum of $\cdot 12\dot{5}$, $4{\cdot}16\dot{3}$, and $9{\cdot}45\dot{7}$, correct to 5 places of decimals.

VI.

(1) Prove the Rule for Multiplication of decimals by means of the example 404·04 multiplied by ·030303. Multiply $\cdot\dot{3}4\dot{5}$ by $\dfrac{\cdot 111}{4\cdot 3}$; and divide ·04813480963 by ·6593, and by ·006593.

(2) Explain the meaning of 7^2, and 7^3; and find what vulgar fraction is equivalent to the sum of 20·5 and 2·05 divided by the difference.

(3) Reduce to their lowest terms $\dfrac{123\cdot 48}{1033\cdot 2}$, and $\dfrac{36\cdot 595}{5\cdot 7980}$.

(4) Shew that $\dfrac{\cdot 375\times\cdot 375-\cdot 025\times\cdot 025}{\cdot 375-\cdot 025}=\dfrac{2}{5}$, and that

$$3+\frac{1}{7}+\frac{1}{16}=3\cdot 14159 \text{ nearly.}$$

Reduce $\cdot 12931\dot{3}\dot{1}$ to its equivalent vulgar fraction.

(5) What decimal added to the sum of $1\frac{7}{24}$, $\frac{5}{8}$, and $\frac{13}{36}$ will make the sum total equal to 3?

(6) The quotient being $2\frac{1}{12}$ and the divisor ·15, find the dividend.

DECIMAL COINAGE.

100. *Table of Canadian and United States Currency.*

10 mils, ...	make 1 cent	abbreviated	1c.
10 cents, or 100 mils, ...	make 1 dime	"	1d.
10 dimes, 100 cents, or 1000 mils,	make 1 dollar	"	\$1.

It will be seen from the preceding table that this system of currency has 10 for its radix or base, that is, it is so arranged, that it takes 10 coins of a lower denomination, to make one coin of the next higher denomination; thus it takes 10 mils to make one cent, 10 cents or 100 mils to make one dime, and ten dimes, or 100 cents, or 1000 mils, to make a dollar. The student will observe that under this arrangement, all monetary operations can be readily calculated by means of the rules that apply to *abstract* numbers; for the base on which we proceed is exactly the same, since it takes 10 dimes to make one dollar, just as it takes 10 tenths to make one unit; and it takes 10 cents to make one dime or one tenth of a dollar, just as it takes 10 hundreths to make 1 tenth of a unit, &c. Hence, regarding the dollar as the unit of money, it is plain that dimes, cents and mils, can be expressed as the decimal of a dollar, just as $\frac{1}{10}$ths, $\frac{1}{100}$ths, $\frac{1}{1000}$ths can be expressed as the decimal of a unit.

For example, take 2 dimes 7 cents.

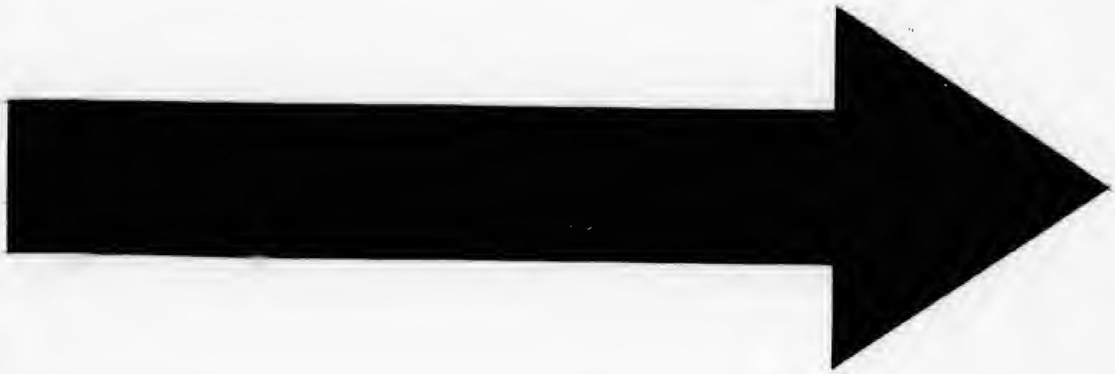
Now since 10 dimes $=$ \$1, and 10 cents $=$ 1 dime,

And consequently 1 dime $= \$\frac{1}{10}$, and 1 cent $= \frac{1}{10}$ dime or $\$\frac{1}{100}$.

$$\therefore \text{2 dimes, 7 cents.}$$
$$= \$\tfrac{2}{10} + \$\tfrac{7}{100}.$$
$$= \$(\tfrac{20}{100} + \tfrac{7}{100}.)$$
$$= \$\tfrac{27}{100} = \$\cdot 27 \text{ Art. (84); usually written } \$0\cdot 27.$$

Again take \$191, 7 cents, 3 mils.

$$\$191, \text{ 7 cents, 3 mils.}$$
$$= \$(191 + \tfrac{0}{10} + \tfrac{7}{100} + \tfrac{3}{1000}).$$
$$= \$(191 + \tfrac{0}{1000} + \tfrac{70}{1000} + \tfrac{3}{1000}).$$
$$= \$(191 + \tfrac{73}{1000}).$$
$$= \$191 + \tfrac{73}{1000} = \$191\ \cdot 073. \quad \text{Art. (84.)}$$



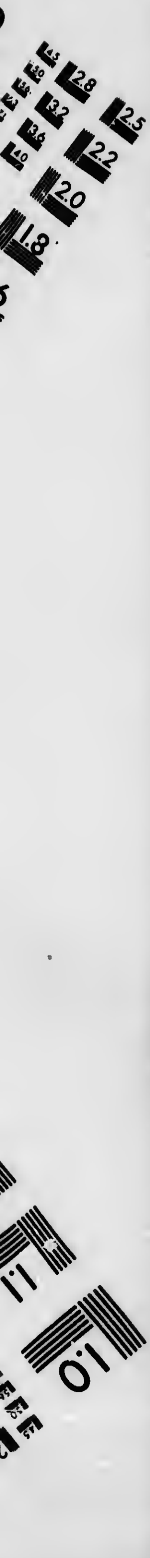


TABLE OF CANADIAN AND UNITED STATES' COINS.

CANADIAN COINS.	UNITED STATES' COINS.
GOLD.	GOLD.
There are no gold coins at present current in Canada.	Double Eagle, or............. \$20.
	Eagle, or.................... \$10.
	Half Eagle, or............... \$5.
	Quarter Eagle, or............ \$2½.
	Dollar..
SILVER.	SILVER.
	Dollar.
	Half Dollar.
20 cent piece	Quarter Dollar.
10 cent piece answers to	Dime.
5 cent piece answers to	Half Dime.
	3 cent piece.
1 cent answers to................	1 cent, (copper.)
mil, not coined.	mil, not coined.

The decimal system of currency has been so recently introduced into these Provinces, that the full complement of coins to be used is not, as yet, completed.

Note 1. As the dollar is the unit of money, the eagle is always expressed in dollars; thus, 2 eagles 3 dollars would be written 23 dollars; also, the dime is expressed in cents.

Note 2. The gold coinage of the United States consists of $\frac{9}{10}$ pure metal and $\frac{1}{10}$ alloy.

The English gold coinage consists of $\frac{11}{12}$ pure metal and of $\frac{1}{12}$ alloy. Should a gold coinage eventually become current in Canada, the standard of purity will no doubt be the same as that adopted in England.

The *standard* of silver coin in Canada and England is $\frac{37}{40}$ pure metal and $\frac{3}{40}$ copper.

The *standard* of silver coin in the United States is $\frac{1489}{1668}$ pure metal and $\frac{179}{1668}$ copper.

Note 3. The student must be careful to remember that one cent is the hundredth part of a dollar, and consequently that any number of cents will be equal to so many hundredths of a dollar. For instance: 2 cents, 5 cents or 13 cents, equal $\frac{2}{100}$ of a dollar, $\frac{5}{100}$ of a dollar, or $\frac{13}{100}$ of a dollar, which equal \$0·02, \$0·05, or \$0·13, as the case may be. It would manifestly be wrong to express 1 cent as \$0·1, since \$0·1=\$$\frac{1}{10}$, and 1 cent is only equal to \$$\frac{1}{100}$.

The half cent may be expressed either as ½ ct., or \$0·005.

101. *The rules which have been given in the chapter on Decimals are applicable to all the examples in Decimal coinage.*

Ex. 1. Read $19·923.

$$\$19·923.$$
$$=\$(19+\tfrac{9}{10}+\tfrac{2}{100}+\tfrac{3}{1000}). \quad \text{Art. (80).}$$
$$=\$19+9d+2c+3m.$$
$$=\$19+92c+3m.$$
$$=\$19 \text{ “ } 92c \text{ “ } 3m.$$

Ex. XXXIII.

(1.) Read the following :

1. $9·34 ; $7·560 ; £95·636 ; $731·236 ; $816·001.
2. $9·10 ; $6·01 ; $10·001 ; $7·036 ; $0·007.

(2.) Express in figures,

1. Fifty dollars ; seven dollars, thirteen cents ; nine dollars, nineteen cents, seven mils ; eighteen dollars, one cent, seven mils ; ten cents ; ninety dollars, nine mils ; one cent ; one mil ; one million dollars, one cent, one mil ; three hundred thousand dollars and three cents ; five hundred thousand and one dollars, five mils.

ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF DECIMAL CURRENCY.

Ex. 1. Find the sum of $19·408 ; 117·18 ; $3·2.

Proceeding as in Art. (88),

$$\begin{array}{r} \$\ 19·408 \\ 117·18 \\ 3·2 \\ \hline \$139·788 \end{array}$$

Ex. 2. From $119·05 take $34·687.

Proceeding as in Art. (89)

$$\begin{array}{r} \$119·050 \\ 34·687 \\ \hline \$\ 84·363 \end{array}$$

Multiply \$18·053 by 17.

Proceding as in Art. (90)

```
 $18·053
      17
 -------
 126371
 18053
 -------
$306·901
```

Ex. 4. Divide \$368·736 by 23.

Proceeding as in Art. (91)

```
23)$368·736($16·032
   23
   ---
   138
   138
   ---
     73
     69
     --
      46
      46
      --
```

Ex. XXXIV.

(1) Add together

1. \$76·853; \$27·909; \$84·01; \$56·362; \$19·001.
2. \$252·25; \$300·025; \$·45; \$·052.

Add together fractionally and decimally, and show that the results coincide,

3. \$$19\frac{1}{2}$; \$$319\frac{5}{8}$; \$$7\frac{3}{16}$; \$$11\frac{7}{10}$.
4. \$$\frac{3}{4}$; \$$\frac{59}{64}$; \$$\frac{121}{128}$; \$$\frac{19}{32}$; \$156.

(2) Find the difference between

1. \$19·50 and \$16·39.
2. \$20 and \$19·999.
3. \$5·55 and \$4·45.

4. Take \$$79\frac{24}{25}$ from \$$98\frac{1}{1000}$; also \$$18\frac{1}{2}$ from \$$50\frac{1}{5}$; work them both fractionally and decimally, and show that the results coincide.

(3.) Multiply

1. \$76803 separately by 5 and 63.
2. \$0·925 separately by 18 and 1000.
3. \$150·005 separately by 2095 and 18576.

Multiply fractionally and decimally, and shew that the results coincide:

4. $\$\frac{1}{10}$ separately by 106 and 795.
5. $\$25\frac{3}{8}$ separately by 56 and 73.

Divide

1. \$194·575 separately by 5 and 15.
2. \$10764·284 separately by 11 and 33.
3. \$342136·80 separately by 7380 and 1845.

Divide both fractionally and decimally, and show that the results coincide:

4. $\$4\frac{2}{5}$ separately by 15 and 125.
5. $\$117\frac{7}{16}$ separately by 64 and 128.

NOTE.—Since 100 cents make 1 dollar and 10 mils make 1 cent, it follows that any number of dollars may be expressed in the denomination of cents or mils by multiplying the dollars by 100 or by 100 × 10, as the case may be.

Ex. 1. $\$10 = 10 \times 100c = 1000c.$

Ex. 2. $\$15 = 15 \times 100c = 15 \times 100 \times 10m = 15000m.$

Again, conversely, cents or mills may be expressed as dollars by dividing the number of cents or mils by 100 or 100 × 10, as the case may be;—thus,

Ex. 1. 150 cents $= \$\frac{150}{100} = \$1·5.$ Art. (84.)

Ex. 2. 150 mils $= \frac{150}{10}c. = \$\frac{150}{10 \times 100} = \frac{150}{1000} = \$·150.$ Art. (84.)

Ex. XXXV.

Miscellaneous Questions and Examples in Decimal Currency.

I.

(1.) What coin is regarded as the unit of money in Canada and the United States? Mention the gold coins current in the United States, and those of silver, current in Canada.

(2.) What is the *standard* of purity as regards gold and silver in American and English coins?

(3.) Read \$9·001, also \$177·101; express in figures nine dollars one cent and one mil; fifteen dollars one mil.

(4.) Bought a case of preserved fruit at \$3·50; a bushel of pears at \$75; a side of bacon for \$6·35; a cooking stove at \$31. Find the sum of money expended, and express the answer in a fractional form.

(5.) A farm of 125 acres is sold for \$5324. How much is that per acre? Express the answer as a fraction, and find the difference between it and $\$25\frac{3}{4}$.

(6.) A man borrowed \$606·75, and has repaid \$216·36, what remains of his debt? Express the result in mils.

II.

(1.) Find the value, in dollars, of

1. $\dfrac{\$5\frac{2}{5}+5\frac{2}{5}c+5\frac{2}{3}m}{5\frac{2}{5}}$.

2. $\dfrac{\$(7{\cdot}01+\frac{3}{4}+{\cdot}07)}{{\cdot}006}$.

3. $\dfrac{80016c+\frac{3}{8}m}{1{\cdot}25}$

4. $\dfrac{\$7\frac{1}{2}}{{\cdot}05}\times\dfrac{3\frac{1}{2}}{{\cdot}5}$

(2) A bankrupt tradesman realized \$1536 from the sale of some land; \$1856·15 from his stock in trade; \$395·35 from his horse and carriage; and \$59.63 from other effects. What did he possess after paying \$2563·758 to his creditors and \$125·50 for the expenses of the auction?

(3) Find the difference between

$$\frac{\$({\cdot}1001+3{\cdot}572)}{{\cdot}0001} \text{ and } \frac{\$(7{\cdot}356+{\cdot}101)}{{\cdot}101}$$

and divide the remainder by $1\frac{3}{4}$.

(4) A merchant possesses a fortune of \$206,060 and wishes to start his three sons in life; to the eldest he gives $\frac{3}{16}$ of his fortune, to the second $\frac{5}{32}$, and to the third $\frac{9}{64}$, find the value of each one's share; also what remains of the father's fortune.

(5) Find the value of

$$\frac{\$2{\cdot}8 \text{ of } 2{\cdot}\dot{2}\dot{7}}{1{\cdot}1\dot{3}\dot{6}} + \frac{\$(4{\cdot}\dot{4} - 2{\cdot}8\dot{3})}{1{\cdot}\dot{6} + 2{\cdot}\dot{6}2\dot{9}} \text{ of } \frac{6{\cdot}8 \text{ of } \dot{3}}{2{\cdot}25}$$

(6) A gentleman, in furnishing his house, had to buy 95 yards of carpet for his drawing room at \$2·75 a yard; 87 yards for his dining room at \$1·625; 165 yards for his study and bed-rooms at an average price of \$0·875 per yard; find the whole cost of carpeting the house, and the average price of the carpet per yard.

III.

(1.) The estimated cost of the Great Eastern steamship was \$5,000,000; what would be the value of $\frac{\frac{1}{2}+\frac{7}{9}}{\frac{1}{3}}$ of $\frac{1}{23}$ of the vessel? And supposing her shares rose in the market from \$5 to \$7·25 per share, what would be the increase on 51·2 original shares?

(2.) A farmer goes into town with \$15·75 in his pocket to pay his grocer's bill, which consisted of the following items: 7 lbs. of tea at 75c per lb.; 13 lbs. of sugar at $9\frac{1}{2}$c per lb.; $\frac{1}{2}$lb. of arrow root at 48c per lb.; 5 ounces of spices at $7\frac{1}{4}$c per ounce; what money will he have remaining after paying the debt?

(3) Find the difference between \$1·60 of 3·40 of 1·125, and \$$\frac{7}{128}$ of 3·60 of 9·1125.

(4) A speculator bought 500 acres of land for \$$987\frac{4}{5}$; and 250 acres for \$$647\frac{5}{6}$. He sold $487\frac{1}{2}$ acres for \$$1245\frac{1}{2}$. How much land has he remaining, and for what must he sell it per acre, so as neither to gain nor lose by the transaction?

(5) What is the value of 20344 feet of oak planking at \$12·75 per 1000 feet?

(6) Find the value of $6{\cdot}8\dot{3}$ of $\$3{\cdot}867708\dot{3} + 5{\cdot}8$ of $\$2{\cdot}411458\dot{3} - 4{\cdot}375$ of \$1·30.

IV.

(1) Read \$6·00101; and express in figures seven hundred and three thousand and nine dollars, seven mils; also, express 759 dimes in cents and mils, and 706854 mils in dollars.

(2) What sum of money should be paid for 37·875 lbs. of coffee at \$0·224 ℔ lb.; 14 lbs of tea at \$0·6875 ℔ lb.; 30 lbs. of raisins at \$0·1257 ℔ lb.; and 8·635 gallons of molasses at \$0·653 ℔ gallon?

(3) Find what decimal of a dollar multiplied by 175 will give $\$(\frac{1}{4}+\frac{16}{25}+\frac{43}{50}+3\frac{1}{2}.)$

(4) If 625 bushels of peaches cost \$1306·8713, find the cost of 1 bushel; also of ·56 of a bushel.

(5) What is the value of sugar a quarter when ·75 quarters cost \$6·375, and what must be paid for $16\frac{3}{4}$ quarters of sugar at the same rate?

(6) A person bought 750 shares in a coal mine, for which he paid \$250·70 per share. In a short time the shares rose in the market to the value of \$351·16 per share; he then sold out. What did he make by the transaction? and find the value of $\dfrac{\frac{2}{5}+\frac{7}{10}}{\frac{2}{3}\text{ of }\frac{9}{10}}$ of a share at the advanced price?

102. It has been for some years in contemplation in England to introduce a system of decimal coinage, but the practical difficulties in the way render it necessary that its introduction should be gradual. The florin, which has now been in use for some years, was coined with the view of its ultimately occupying a place in a decimal scale of currency. The scale which has been most favorably received for the proposed change, takes the pound sterling for its unit of value, and is as follows:

10 mils	make 1 cent,	abbreviated	1 c.
10 cents	1 florin		1 fl.
10 florins	1 pound		£1.

The remarks in Art. 100 on the decimal currency of Canada and the United States, will be equally applicable here; the only difference being that some of the coins have different names; thus,

$$£71·526=£(71+\tfrac{5}{10}+\tfrac{2}{100}+\tfrac{6}{1000})=£71\ 5\text{fl. } 2\text{c. } 6\text{m.}$$

Corresponding in arrangement to

$$\$71·526=\$(71+\tfrac{5}{10}+\tfrac{2}{100}+\tfrac{6}{1000})=\$71\ 5\text{d. } 2\text{c. } 6\text{m.}$$

It must not be thought, however, that £71·526 is equal to \$71·526; for the pound, the unit of money in the one case, being greater in value than the dollar, the unit of money in the other, the several parts will differ in a corresponding degree.

CONCRETE NUMBERS.

TABLES.

103. Our operations hitherto have been carried on with regard only to abstract numbers, or concrete numbers of one denomination. It is evident that if concrete numbers were all of one denomination; if, for instance pounds were the only units of weight, yards of length, years of time, and so on, such numbers would be subject to the common rules for abstract numbers. Again, if the concrete numbers were of different denominations, and those denominations differed from each other by 10 or multiples of 10, then all operations with such concrete numbers could be carried on by the rules which have been given for Decimals. But generally with concrete numbers such a relation does not hold between the different denominations, and therefore it is necessary to commit to memory tables, which connect the different units of weight together, the different units of length together, the different units of time together, and so on.

We shall now put down some of the most useful of these tables, with a few brief remarks on each.

TABLE OF ENGLISH MONEY.

2 Farthings	make	1 Half-penny.
2 Half-pence	....	1 Penny.
12 Pence	....	1 Shilling.
20 Shillings	......	1 Pound.

Pounds, shillings, pence, and farthings were formerly denoted by £, *s*, *d*, and *q*, respectively, these letters being the first letters of the Latin words, *libra*, *solidus*, *denarius*, and *quadrans*, the Latin names of certain Roman coins or sums of money. £ *s*, *d*, are still the abbreviated forms for pounds, shillings, and pence respectively; but $\frac{1}{4}$ annexed to pence denotes 1 farthing, $\frac{1}{2}$ denotes a half-penny, $\frac{3}{4}$ denotes three farthings; shewing that one farthing, two farthings, and three farthings are respectively $\frac{1}{4}$th, $\frac{2}{4}$ths, or $\frac{1}{2}$, and $\frac{3}{4}$ths of the concrete unit, one penny.

The following coins are in common use in England:

COPPER COINS.

A Farthing, the coin of least value.
A Half-penny=2 Farthings.
A Penny....=4 Farthings.

SILVER COINS.

Threepenny piece ... } =3 Pence.
Fourpenny-piece ... } =4 Pence.
A Sixpence ..=6 Pence.
A Shilling ...=12 Pence.
A Florin ....=2 Shillings.
A Half-Crown=2 Shillings and 6 pence.
A Crown ....=5 Shillings.

GOLD COINS.

A Half-Sovereign=10 Shillings.
A Sovereign ....=20 Shillings.

The following coins have been in use at various periods in England, but with the exception of the first two, which are used under different names, they are now obsolete:

SILVER COINS.

A Groat.... = 4 Pence.
A Tester ... = 6 Pence.

GOLD COINS.

	£	s.	d.
A Noble........	=0	6	8
An Angel.......	=0	10	0
A Half-Guinea...	=0	10	6
A Mark or Merk.	=0	13	4
A Guinea.......	=1	1	0
A Carolus	=1	3	0
A Jacobus......	=1	5	0
A Moidore......	=1	7	0

Note. The office at which coin is made and stamped, so as to pass or become current for legal money, is called *the Mint.*

MEASURES OF WEIGHT.

TABLE OF TROY WEIGHT.

104. This table derives its name probably from *Troyes* in France, the first city in Europe where it was adopted. It seems to have been brought thither from Egypt. It has also been derived from *Troy-novant*, the monkish name for London. It is used in weighing gold, silver, diamonds, and other articles of a costly nature; also in determining specific gravities; and generally in philosophical investigations.

The different units are grains (written grs.), pennyweights (dwts), ounces (oz.), and pounds (lbs. or ℔s.), and they are connected thus:

24 Grains........ make 1 Pennyweight .. 1 dwt.
20 Pennyweights ...... 1 Ounce.......... 1 oz.
12 Ounces .... 1 Pound .... 1 lb. or ℔.

Note 1. As the origin of weights, a grain of wheat was taken from the middle of the ear, and being well dried, was used as a weight, and called a '*grain*'.

Note 2. Diamonds and other precious stones are weighed by

'*Carats*,' each carat weighing about $3\frac{1}{6}$ grains. The term 'carat' applied to gold has a relative meaning only; any quantity of pure gold, or of gold alloyed with some other metal, being supposed to be divided into 24 equal parts (carats); if the gold be pure, it is said to be 24 carats fine; if 22 parts be pure gold and 2 parts alloy, it is said to be 22 carats fine.

Standard gold is 22 carats fine: jewellers' gold is 18 carats fine.

TABLE OF APOTHECARIES' WEIGHT.

105. Apothecaries weight only differs from Troy weight in the subdivisions of the pound, which is the same in both. This table is used in mixing medicines. The different units are grains (grs.) scruples (℈.), drams (ʒ.), ounces (℥.), pounds (lbs. or ℔s.), and they are connected thus:

20 Grains	make	1 Scruple	1 sc. or 1 ℈.
3 Scruples		1 Dram	1 dr. or 1 ʒ.
8 Drams		1 Ounce	1 oz. or 1 ℥.
12 Ounces		1 Pound	1 lb. or 1 ℔.

TABLE OF AVOIRDUPOIS WEIGHT.

106. Avoirdupois weight derives its name from *Avoirs* (goods or chattels,) and *Poids* (weight). It is used in weighing all heavy articles, which are coarse and drossy, or subject to waste, as butter, meat, and the like, and all objects of commerce, with the exception of medicines, gold, silver, and some precious stones. The different units are drams (drs.) ounces (oz.) pounds (lbs.) quarters (qrs.) hundredweights (cwts.) tons (tons.) and they are connected thus:

16 Drams	make	1 Ounce	1 oz.
16 Ounces		1 Pound	1 lb.
25 Pounds		1 Quarter	1 qr.
4 Quarters		1 Hundredweight	1 cwt.
20 Hundredweights		1 Ton	1 ton.

1 lb. Avoirdupois weighs 7000 grains Troy;
1 lb. Troy weighs 5760 grains Troy;
therefore 1 lb. Avoirdupois $=\frac{7000}{5760}$ of lb. Troy
$=\frac{700}{576}$ of 1 lb. Troy
$=\frac{175}{144}$ of 1 lb. Troy
=14 oz. 11 dwt. 16 grs. Troy
=1 lb. 2 oz. 11 dwt. 16 grs. Troy.

Note. In England, 28 lbs. =1 qr. or 112 =1 cwt. In Liverpool, however, a new weight has lately been introduced, called the CENTAL, which corresponds with the hundredweight of 100 lbs.

MEASURES OF LENGTH.

TABLE OF LINEAL MEASURE.

107. In this measure, which is used to measure distances, lengths, breadths, heights, depths, and the like, of places or things :

3 Barley Corns (in length)	make 1	Inch, which is written	1 in.
12 Inches	1	Foot,	1 ft.
3 Feet	1	Yard,	1 yd.
6 Feet	1	Fathom	1 fth.
$5\frac{1}{2}$ Yards	1	Rod, Pole or Perch	1 po.
40 Poles	1	Furlong	1 fur.
8 Furlongs	1	Mile,	1 m.
3 Miles	1	League	1 lea.
$69\frac{1}{6}$ Miles	1	Degree	1 deg. or 1°

Note. A grain of Barley, or a Barley-corn, is supposed to have been the original element of Lineal Measure.

The following measurements may be added, as useful in certain cases :

4 Inches make 1 Hand (used in measuring horses),
22 Yards make 1 Chain }
100 Links make 1 Chain } used in measuring land,
a Palm=3 inches, a Span=9 inches, a Cubit=18 inches,
a Pace=5 feet, 1 Geographical Mile=$\frac{1}{60}$th of a degree,
a Line=$\frac{1}{12}$th of an inch.

TABLE OF CLOTH MEASURE.

108. In this measure which is used by linen and woollen drapers :

$2\frac{1}{4}$ Inches	make 1 Nail.	
4 Nails	1 Quarter	1 qr.
4 Quarters	1 Yard	1 yd.
5 Quarters	1 English Ell.	
6 Quarters	1 French Ell.	
3 Quarters	1 Flemish Ell.	

MEASURES OF SURFACE.

TABLE OF SQUARE MEASURE.

109. This measure is used to measure all kinds of superficies, such as land, paving, flooring, in fact everything in which length and breadth are to be taken into account.

Def. A **square** is a four sided figure, whose sides are equal, each side being perpendicular to the adjacent sides.

A square inch is a square, each of whose sides is an inch in length; a square yard is a square, each of whose sides is a yard in length.

144 Square Inches make	1 Square Foot. .	1 sq. ft. or 1 ft.
9 Square Feet.	1 Square Yard. .	1 sq. yd. or 1 yd.
30¼ Square Yards	1 Square Pole. .	1 sq. po. or 1 po.
40 Square Poles.	1 Square Rood	1 ro.
4 Roods	1 Acre.	1 ac.
640 Acres.	1 Square mile.	

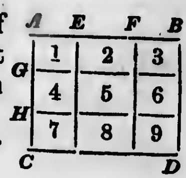
25000 Square Links=1 Rood.
100000 . . . .=1 Acre.
10 Square Chains=1 Acre.

Note. This table is formed from the table for lineal measure, by multiplying each lineal dimension by itself.

The truth of the above table will appear from the following considerations.

Suppose AB and AC to be lineal yards placed perpendicular to each other.

Then by definition $ABCD$ is a square yard. If AE, EF, FB, AG, GH, HC=1 lineal foot each, it appears from the figure that there are 9 squares in the yard, and that each square is one square foot.



The same explanation holds good of the other dimensions.

The following measurements may be added:

A Rod of Brickwork . . . . =272¼ Square Feet.

(*The work is supposed to be* 14 *in., or rather more than a brick-and-a half, thick.*)

A Square of Flooring. . . . =100 Square Feet.
A Yard of Land . . . . =30 Acres.
A Hide of Land . . . . =100 Acres.

MEASURES OF SOLIDITY.

TABLE OF SOLID OR CUBIC MEASURE.

110. This measure is used to measure all kinds of solids, or figures which consist of three dimensions, length, breadth, and depth or thickness.

DEF. A CUBE is a solid figure contained by six equal squares; for instance, a die is a cube.

A cubic inch is a cube whose side is a square inch.
A cubic yard .... square yard.
$12 \times 12 \times 12$ or 1728 cubic inches make 1 cubic foot.
$3 \times 3 \times 3$ or 27 cubic feet............ 1 cubic yard.

Note. This table is formed from the table for lineal measure by multiplying each lineal dimension by itself twice.

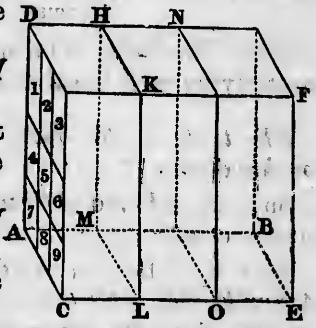
The truth of the above table will appear from the following considerations.

If *AB*, *AC*, and *AD* be perpendicular to each other, and each of them a lineal yard in length, then the figure *DE* is a cubic yard.

Suppose *DH* a lineal foot, and *HKLM* a plane drawn parallel to side *DC*.

By last table there are 9 square feet in side *DC*. There will therefore be 9 cubic feet in the solid figure *DL*.

Similarly if another lineal foot *HN* were taken, and a plane *NO* were drawn parallel to *HL*, there would be 9 cubic feet contained in the solid figure *HO*.



Similarly, there would be 9 cubic feet in the solid figure *NE*.

Therefore, there are 27 cubic feet in the solid figure *DE*, or in 1 cubic yard.

The following measurements may be added:

A Load of rough Timber = 40 cubic feet.
A Load of squared Timber = 50 cubic feet.
A Ton of Shipping = 42 cubic feet.
A Cord of Wood = 128 cubic feet.

Note. A pile of wood 4 feet wide, 4 feet high and 8 feet long makes a cord,—1 foot in length, of such a pile, is sometimes called a *cord foot*. It contains 16 solid feet; consequently 8 cord feet make 1 cord.

MEASURES OF CAPACITY.

TABLE OF WINE MEASURE.

111. In this measure, by which wines and all liquids, with the exception of malt liquors and water, are measured:

4 Gills make	1 Pint	1 pt.
2 Pints	1 Quart	1 qt.
4 Quarts	1 Gallon	1 gal.
10 Gallons	1 Anker	1 ank.
18 Gallons	1 Runlet	1 run.
42 Gallons	1 Tierce	1 tier.
2 Tierces	1 Puncheon	1 pun.
63 Gallons	1 Hogshead	1 hhd.
2 Hogsheads	1 Pipe	1 pipe.
2 Pipes	1 Tun	1 tun.

TABLE OF ALE AND BEER MEASURE.

112. In this measure, by which all malt liquors and water are measured:

2 Pints make	1 Quart	1 qt.
4 Quarts	1 Gallon	1 gal.
9 Gallons	1 Firkin	1 fir.
18 Gallons	1 Kilderkin	1 kil.
36 Gallons	1 Barrel	1 bar.
1½ Barrels or 54 Gallons	1 Hogshead	1 hhd.
2 Hogsheads	1 Butt	1 butt.
2 Butts	1 Tun	1 tun.

TABLE OF CORN OR DRY MEASURE.

113. In this measure, by which all dry commodities, as corn, and the like, which are not usually heaped above the measure, are measured:

2 Quarts make	1 Pottle	1 pot.
2 Pottles	1 Gallon	1 gal.
2 Gallons	1 Peck	1 pk.
4 Pecks	1 Bushel	1 bush.
2 Bushels	1 Strike	1 str.
4 Bushels	1 Coomb	1 coomb.
2 Coombs or 8 Bushels	1 Quarter	1 qr.
5 Quarters	1 Load	1 load.
2 Loads or 10 Quarters	1 Last	1 last.

TABLE OF COAL MEASURE.

114. In this measure, which is not much used now, as coals are sold by weight:

4 Pecks make	1 Bushel.
3 Bushels....	1 Sack.
36 Bushels....	1 Chaldron.

MEASURES OF NUMBER.

TABLE OF NUMBER.

115.

12 Units....make	1 Dozen.
12 Dozen........	1 Gross.
20 Units........	1 Score.
120 Units........	1 Long Hundred.
24 Sheets of Paper	1 Quire.
20 Quires........	1 Ream.
10 Reams.......	1 Bale.

MEASURES OF TIME.

TABLE OF TIME.

116.

* 1 Second is written thus 1″

60 Seconds make	1 Minute..	1′
60 Minutes.....	1 Hour ...	1 hr.
24 Hours......	1 Day	1 day.
7 Days.......	1 Week...	1 wk.

* A year is divided into 12 months, called Calendar months, the number of days in each of which are easily remembered by means of the following lines:

Thirty days hath September,
April, June, and November:
February has twenty-eight alone,
And all the rest have thirty-one:
But leap-year coming once in four,
February then has one day more.

A day, or rather a *mean solar day*, which is divided into 24 equal portions called *mean solar hours*, is the standard unit for the measurement of time, and it is the mean or average time which elapses between two successive transits of the Sun across the meridian of any place.

The time between the Sun's leaving a certain point in the *Ecliptic* and its return to that point consists of 365·242218 mean *solar days*, or 365 days, 5 hours, 48 minutes, 47½ seconds, very nearly, and is called a *solar year*. Therefore the *civil* or *common* year, which contains 365 days, is about ¼th of a day less than the *solar* year; and this error would of course in time be very considerable, and cause great confusion.

Julius Cæsar, in order to correct this error, enacted that every 4th year should consist of 366 days; this was called *Leap or Bissextile year*. In that year February had 29 days, the extra day being called 'the *Intercalary*' day.

But the solar year contains 365·242218 days, and the Julian year contains 365·25 or 365¼ days.

Now 365·25 − 365·242118 = ·007782.

Therefore in one year, taken according to the Julian calculation, the Sun would have returned to the same place in the Ecliptic ·007782 of a day before the end of the Julian year.

Therefore in 400 years the sun would have come to the same place in the Ecliptic ·007782 × 400 or 3·1128 days before the end of the Julian year; and in 1257 years would have come to the same place, ·007782 × 1257 or 9·7819, or about ten days before the end of the Julian year. Accordingly, the vernal equinox which, in the year 325 at the council of Nice, fell on the 21st of March, in the year 1582 (that is 1257 years later) happened on the 11th of March; therefore Pope Gregory caused 10 days to be omitted in that year, making the 15th of October immediately succeed the 4th, so that in the next year the vernal equinox again fell on the 21st of March; and to prevent the recurrence of the error, ordered that for the future in every 400 years, 3 of the leap years should be omitted, viz. those which complete a century, the numbers expressing which century, are *not* divisible by 4; thus 1600 and 2000 are leap years, because 16 and 20 are exactly divisible by 4; but 1700, 1800, and 1900 are not leap years, because 17, 18, and 19 are not exactly divisible by 4.

This Gregorian style, which is called the *new style*, was adopted in England on the 2nd of September 1752, when the error amounted to 11 days.

The Julian calculation is called the *old style*: thus Old Michaelmas and old Christmas take place 12 days after New Michaelmas and New Christmas.

In Russia, they still calculate according to the *old style*, but in the other countries of Europe the new style is used. Sir Harris Nicolas in his Chronology gives the dates at which the new style was adopted in different countries. Of course it was almost immediately adopted by most of the Roman Catholic courts of Europe.

TABLE OF ANGULAR MEASURE.

117.

1 Second is written 1 sec. or 1″.
60 Seconds make 1 Minute......1 min. or 1′
60 Minutes..... 1 Degree......1 deg. or 1°.
90 Degrees..... 1 Right Angle .1 rt. ang. or 90°.

The circumference of every circle is considered to be divided into 360 equal parts, each of which is often called a degree, as it subtends an *angle* of 1° at the centre of the circle.

118. An Act of the Imperial Parliament "FOR ASCERTAINING AND ESTABLISHING UNIFORMITY OF WEIGHTS AND MEASURES," in England, came into operation on the first of January, 1826.

It is thereby enacted,

First; that the *brass Standard Yard of* 1760, then in custody of the Clerk of the House of Commons, shall be the *Imperial Standard Yard*, (the brass being at the temperature of 62° by Fahrenheit's thermometer); and that this Imperial Standard Yard shall be the unit or only standard measure of extension, wherefrom or whereby all other measures of extension whatsoever, whether the same be lineal, superficial, or solid, shall be divided, computed, and ascertained; and that the *thirty-sixth* part of this yard shall be *an Inch*.

Now the length of a *Pendulum* vibrating *seconds* in the latitude of *London*, in a vacuum, and at the level of the sea, is found to be 39·1393 such inches, *i.e.* 39 such inches, and 1393 ten-thousandths of another such inch.

This affords the means of recovering the Imperial Standard Yard, should it be lost. In fact, the brass Standard Yard of 1760 was destroyed or rendered useless by the fire at the House of Commons in 1834.

Secondly; That the *brass weight of one Pound Troy of the year* 1758, then in the custody of the same officer, shall continue the unit or *Standard Measure of Weight*, from which all other weights shall be derived, computed and ascertained; that 5760 grains shall be contained in the Imperial Standard Troy Pound, and 7000 such grains in the Avoirdupois Pound.

Now the weight of a *cubic inch* of distilled water is 262·458 grains Troy, the barometer being at 30 inches, and the thermometer at 62°. This affords the means of recovering the Imperial Standard Pound should it be lost. In fact, the brass weight of 1758 was destroyed or lost at the above-mentioned fire.

Thirdly; That the *Standard Measure of Capacity* for Liquids and Dry Goods shall be "the *Imperial Standard Gallon*," containing 10 Pounds Avoirdupois weight of distilled water, weighed in air at a temperature of 62° Fahrenheit's thermometer, and the barometer being at 30 inches.

Now this weight fills 277·274 cubic inches, therefore the Imperial Standard Gallon contains 277·274 cubic inches.

The *Imperial Bushel* consisting of *eight* gallons, will consequently be 2218·192 cubic inches.

REDUCTION.

119. REDUCTION is the method of expressing numbers of a superior denomination in units of a lower denomination, and conversely. Thus £1 is of the same value as 240*d.*, and £21 as 5040*d.*, and conversely; and the process, by which we ascertain this to be so, is termed *Reduction.*

First. *To express a number of a higher denomination in units of a lower denomination.*

RULE. "Multiply the number of the highest denomination in the proposed quantity by the number of units of the next lower denomination contained in one unit of the highest, and to the product add the number of that lower denomination, if there be any in the proposed quantity; repeat this process for each succeeding denomination till the required one is arrived at."

Ex. 1. How many pence are there in £23. 15*s.*?

Proceeding by the Rule given above,

£23 . 15*s.*
20
460+15 or 475*s.*
12
5700*d.*

or £23. 15*s.*=5700*d.*

Reason for the process.

There are 20 shillings in £1.

Therefore there are (23×20)*s.* or 460*s.* in £23, and so there are 460*s.*+15*s.*, or 475*s.* in £23. 15*s.*

Again, since there are 12 pence in 1*s.*; therefore there are (475×12)*d.*, or 5700*d.* in 475*s.* *i. e.* in £23. 15*s.*

Ex. 2. Reduce 2 tons, 7 cwt., 3 qrs., 24 lbs. into lbs.

```
tons  cwt.  qrs.  lbs.
 2  .  7  .  3  . 24
20
--
40+7 or 47 cwt.
         4
       ---
       188+3 or 191 qrs.
                 25
                ---
                955
               382
               ----
               4775+24
                    or 4799 lbs.
```

Ex. 3. How many inches are there in 106 miles, 6 furlongs, 25 perches, and $2\frac{1}{2}$ yards?

```
miles  fur.  per.  yds.
 106 .  6  . 25 .  2½
   8
 ---
 848+6=854 fur.
        40
     -----
     34160+25 per.
     =34185
          5½
     ------
     170925
      17092½
     -------
     188017½+2½ yds.
    =188020
         36
    -------
    1128120
     564060
    -------
    6768720 in
```

Secondly. *To express a number of inferior denomination in units of a higher denomination.*

Rule. "Divide the given number by the number of units which connect that denomination with the next higher, and the remainder, if any, will be the number of surplus units of the lower denomination. Carry on this process, till you arrive at the denomination required."

Ex. 1. How many pounds and shillings are there in 5700 pence?

Proceeding by the Rule given above,

12 | 5700
2,0 | 47,5
£23. 15*s*.

In dividing 475 by 20 we cut off the 0 and 5 by Art. (43.)

Reason for the above process.

Since 12 pence=1 shilling; therefore in any given number of pence, for every 12 pence there is 1 shilling, so that in 5700*d*. or $(12\times475)d$. there are 475*s*.

Again, since 20*s*=£1; therefore in any given number of shillings, for every 20 shillings there is £1.

Hence, in 475*s*., or $(20\times23+15)s$. there are £23, and 15*s*. over.

Note. Since each of the above Rules is the converse of the other, the accuracy of any result obtained by either of them may be tested by working the result back again by the other rule.

Ex. 2. In 272668 inches how many miles, &c. are there? Verify the result.

In this Example it will be convenient to bring the inches to half-yards, and the half-yards to poles. In a half-yard there are 18 or 3×6 inches, and in a pole there are $5\frac{1}{2}$ yards or eleven half-yards.

18 { 3 | 272668—1 } 4 in.
18 { 6 | 90889—1 }
11 | 15148—1 half-yard or 18 inches.
4,0 | 137,7—17 po.
8 | 34—2 fur.
4

therefore the answer is 4 miles, 2 fur., 17 po., 22 in.

```
                miles  fur.  poles   in.
Proof             4  .  2  .  17  .  22
                  8
                 ---
                 34 furlongs.
                  40
                 ----
                1360+17
                   =1377 poles.
                     1377
                       11
                    -----
                    15147 half-yards.
                       18
                   ------
                   121176
                   15147
                       22
                   ------
                   272668 inches.
```

Ex. 3. How many grains of gold are contained in 9 lbs., 11 oz., 13 dwts., 20 grs.? Prove the result.

```
lbs.  oz.  dwts.  grs.
 9  . 11  . 13  .  20
12
---
108+11=119 oz. in 9 lbs., 11 oz.
        20
      ----
      2380+13 or 2393 dwts in 9 lbs., 11 oz., 13 dwts.
                 24
               ----
               9572
              4786
              -----
              57432+20
or          57452 grs. in 9 lbs., 11 oz., 13 dwts., 20 grs.

Proof       { 4 | 57452—0 }
            { 6 | 14363—5 } 20 grs.
             2,0 |  239,3
             12  |   119—13 dwts.
                       9—11 oz.
```

therefore in 57452 grs., there are 9 lbs., 11 oz., 13 dwts., 20 grs.

Ex. XXXVI.

(1) Reduce (verifying each result);

1. £57 to pence; and 613 guineas to farthings.
2. £15 12*s.* to pence; and 5000 guineas to pence.
3. £83 15*s.* $6\frac{1}{4}d.$ to farthings; and £393 0*s.* $11\frac{1}{2}d.$ to half-pence.

(2) Find the number of pounds in 5673542 farthings, and prove the truth of the result.

(3) Reduce the following, verifying the result in each case:

1. 59 lbs., 7 oz., 14 dwts., 19 grs., to grains; and 37400157 grs. to lbs.
2. 56332005 scrs. to lbs. Troy; and 536 lbs. to drams and scruples.
3. 7 tons, 15 cwt., 2 qrs., 16 lbs. to ounces; and 7593241 drs. to tons.
4. 5838297 oz. to tons; and 33 tons, 17 cwt., 3 qrs., 23 lbs., 15 drs. to drams.
5. 17lbs., 2 ℥, 2 ℈ to grains; and 34678 grs. Apoth. to oz. Troy.
6. 3 m., 7 fur., 8 po. to yards; and 573 miles to inches.
7. 1364428 in. to leagues; and 74 m., 3 fur., 4 yds. to inches.
8. 4 lea., 2 m., 2 in. to barleycorns; and 50 m., 3 po. to yards.
9. 7 fur., 200 yds. to chains; and 6 cubits, 1 span to feet.
10. 84 yds., 1 qr. to nails; and 56 Eng. ells, 1 qr. to nails.
11. 83 Fr. ells, 3 qrs. to nails; and 73 Fl. ells, 1 qr. to nails.
12. 35 ac., 2 ro. to poles; and 56 ac., 2 ro. to yards.
13. 3 ro., 37 po., 26 yds. to inches; and 3 ac., 30 po. to feet.
14. 15 ac., 3 ro. to links; and 50000 po. to acres.
15. 29 cub. yds. to feet; and 158279 cub. in. to yards.
16. 17 cub. yds., 1001 cub. in. to inches; and 26 cub. yds., 19 cub. ft. to inches.
17. 563 gals. to pints; and 365843 gills to gallons.
18. 5 pipes, 1 hhd., 35 gals. to pints; and 487634 gills to tierces.
19. 760 bus., 3 pks. to quarts; and 2 qrs., 1 coomb, 3 pks. to gallons.
20. 3659712 pints to loads; and 7 lds., 1 qr., 2 bus. to pecks.
21. 56 reams, 19 quires to sheets; and 52073 sheets of paper to reams.
22. 36 wks., 5d., 17 hrs. to seconds; and 1 mo. of 30 days, 23 hrs., 59 sec. to seconds.

(4.) How many barrels, gallons, quarts, and pints are there in 1376381 half-pints?

(5) One year being equivalent to 365 days, 6 hours, find how many seconds there are in 27 years, 245 days.

(6) From 9 o'clock, P.M., Aug. 5, 1852, to 6 o'clock, A.M., March 3, 1853, how many hours are there, and how many seconds?

(7) In 5972 cords of wood, how many cubic feet, and cord feet; and in 76267 cubic feet, how many cords.

(8) In the United States there are 3,250,000 square miles, in British America there are 2,450,000 square miles, and in the British Islands there are 115,000 square miles; how many acres do they collectively contain?

COMPOUND ADDITION.

120. Compound Addition is the method of collecting several numbers of the same kind, but containing different denominations of that kind, into one sum.

Rule. "Arrange the numbers, so that those of the same denomination may be under each other in the same column, and draw a line below them. Add the numbers of the lowest denomination together, and find by reduction how many units of the next higher denomination are contained in this sum. Set down the remainder, if any, under the column just added, and carry the quotient to the next column: proceed thus with all the columns."

Ex. 1. Add together £2. 4*s*. $7\frac{1}{2}$*d*., £3. 5*s*. $10\frac{1}{4}$*d*., £15. 15s., and £33. 12*s*. $11\frac{1}{2}$*d*.

Proceeding by the Rule given above,

£	*s*.	*d*.
2 .	4 .	$7\frac{1}{2}$
3 .	5 .	$10\frac{1}{4}$
15 .	15 .	0
33 .	12 .	$11\frac{1}{2}$
£54 .	18 .	$5\frac{1}{4}$

Reason for the above process.

The sum of 2 farthings, 1 farthing, and 2 farthings, =5 farthings, = one penny, and 1 farthing; we therefore put down $\frac{1}{4}$, that is, one farthing, and carry 1 penny to the column of pence. Then

$$(1+11+10+7)d.=29d.=(12\times 2+5)d.$$

or 2 shillings, and 5 pence; we therefore put down 5*d.*, and carry on the 2 to the column of shillings.

Then $(2+12+15+5+4)s.=38s.=(20\times 1+18)s.=£1.$ and 18*s.*; we therefore put down 18*s.*, and carry on the 1 pound to the column of pounds. Then $(1+33+15+3+2)$ pounds equal £54.

Therefore the result is £54. 18*s.* $5\frac{1}{4}d.$

Note. The method of proof is the same as that in Simple Addition.

Ex. 2. Add together 34 tons, 15 cwt, 1 qr., $14\frac{1}{2}$ lbs.; 42 tons, 3 cwt., $18\frac{1}{8}$ lbs.; 18 tons, 19 cwt., 3 qrs.; 7 cwt., $6\frac{3}{4}$ lbs; 2 qrs., 19 lbs.; and 3 tons, $7\frac{5}{16}$ lbs.

	tons	cwts.	qrs.	lbs.
	34 ·	15 ·	1 ·	$14\frac{1}{2}$
	42 ·	3 ·	0 ·	$18\frac{1}{8}$
	18 ·	19 ·	3 ·	0
	0 ·	7 ·	0 ·	$6\frac{3}{4}$
	0 ·	0 ·	2 ·	19
	3 ·	0 ·	0 ·	$7\frac{5}{16}$
Ans.	99 ·	6 ·	0 ·	$15\frac{11}{16}$

Proceeding in this case as in ordinary fractions we have

$$(\tfrac{5}{16}+\tfrac{3}{4}+\tfrac{1}{8}+\tfrac{1}{2})\text{ lbs.}$$
$$=(\tfrac{5}{16}+\tfrac{12}{16}+\tfrac{2}{16}+\tfrac{8}{16})\text{ lbs.}$$
$$=\tfrac{27}{16}\text{ lbs.}\ =1\tfrac{11}{16}\text{ lbs.}$$

we therefore put down $\frac{11}{16}$ lb. and carry one to the column of lbs. and then proceed as in the former example.

Ex. XXXVII.

(1)

£.	s.	d.
1	7	6
6	0	3
5	11	4
8	8	8
2	1	11

(2)

£.	s.	d.
33	16	$3\frac{3}{4}$
67	0	$7\frac{1}{2}$
73	19	$10\frac{3}{4}$
29	9	$9\frac{1}{4}$
47	16	$8\frac{1}{2}$

(3)

£.	s.	d.
528	14	$11\frac{7}{16}$
854	19	4
578	18	$9\frac{1}{8}$
507	0	$0\frac{3}{4}$
859	14	$11\frac{1}{2}$

(4)

tons.	cwt.	qrs.	lbs.
16	17	2	23
13	10	0	20
17	15	2	19
84	0	3	22
11	11	1	11

(5)

oz.	drs.	sc.	grs.
22	3	2	$19\frac{1}{4}$
56	0	1	$10\frac{2}{9}$
3	2	2	11
15	6	1	$9\frac{1}{3}$
79	4	1	10

(6)

ac.	ro.	po.
82	2	$24\frac{7}{8}$
18	3	$14\frac{9}{16}$
20	1	27
56	0	$0\frac{7}{32}$
45	3	30

(7) Find the sum £28. 14*s.* $6\frac{3}{4}d.$, £27. 18*s.* $4\frac{1}{4}d.$, £79. 12*s.* 6*d.*, £19. 18*s.* $10\frac{1}{2}d.$, and £85. 14s. $3\frac{3}{4}d.$; also of £678. 10*s.* 2d., £325.

6*s*. 5*d*., £487. 18*s*. 9*d*., £507. 0*s*. 11*d*., and £779. 10*s*. 8*d*.; also of £4. 14*s*. $8\frac{3}{4}$*d*. £2. 0*s*. $7\frac{1}{3}$d.; £12. 16*s*. $0\frac{3}{4}$*d*.; £10. 0*s*. $0\frac{1}{2}$*d*.; £1. 7*s*. $5\frac{3}{8}$*d*., and £14. 15*s*. 7[illegible]*d*.; also of £20. 16*s*. 5*d*$\frac{1}{8}$., £14. 15*s*. $0\frac{3}{7}$*d*., £5. 13*s*. $8\frac{11}{12}$*d*., £33. 19*s*. $1\frac{1}{3}$*d*. and £18. 3*s*. $4\frac{4}{7}$*d*, and prove the result in each case.

(8) Add together 2 lbs., 9 oz., 1 dwt., $23\frac{1}{4}$ grs.; 8 lbs., 6 oz., 4 dwts., 20 grs.; 1 lb., 10 oz. 5 dwts., $12\frac{3}{8}$ grs.; 14 lbs., 11 oz., 14 dwts., 19 grs.; and 21 lbs., 8 oz., 13 dwts. $11\frac{1}{2}$ grs.: verify the result.

(9) Add together 3 drs., 2 scr., 19 grs.; 2 drs., 2 scr., 11 grs.; 7 drs. 17 grs.; 6 drs. 1 scr., 9 grs.; and 5 drs., 1 scr., $13\frac{7}{12}$ grs.: explain the process.

(10) Find the aggregate of 18 lbs., 14 oz., 6 drs.; 9 lbs., 6 oz., 15 drs.; 45 lbs., 9 oz., 8 drs.; 9 lbs., 15 oz., 4 drs.; and 14 lbs., 12 oz., 12 drs.; also of 1 cwt., 2 qrs., 24 lbs., $10\frac{2}{3}$ oz.; 11 cwt., 18 lbs., $9\frac{1}{4}$ oz.; 13 cwt., 3 qrs., 17 lbs., $14\frac{1}{2}$ oz.; 7 cwt., 1 qr., 20 lbs., 9 oz.; and 19 cwt., 2 qrs., 19 lbs., 14 oz.

(11) Find the sum of 11 yds., 2 ft., 9 in.; 27 yds., 1 ft., $3\frac{2}{3}$ in.; 36 yds., 2 ft., $10\frac{1}{2}$ in.; 48 yds., 2 ft., 11 in.; and 51 yds., 1 ft., $8\frac{1}{3}$ in.; also of 26 m., 7 fur., 23 po., 3 yds.; 22 m., 5 fur., 27 po., $5\frac{1}{2}$ yds.; 37 m., 4 fur., $3\frac{2}{3}$ yds.; 86 m., 6 fur., 38 po., $3\frac{1}{10}$ yds.; and 25 m., 1 fur., 29 po., $2\frac{1}{2}$ yds.

(12) Find the sum of 43 yds., 2 qrs., 3 na.; 37 yds., 2 qrs., $1\frac{1}{2}$ na; 23 yds., 3 qrs., 2 na.; 41 yds., 2 qrs., $2\frac{1}{6}$ na.; and 38 yds. 2 qrs. 3 na.; and of 11 Eng. ells, 2 qrs., 3 na.; 13 Eng. ells, 2 qrs., $1\frac{1}{7}$ na.; 39 Eng. ells, 4 qrs., 2 na.; 37 Eng. ells, 4 qrs., $3\frac{8}{21}$ na.; and 79 Eng. ells, 3 na.: and prove each result.

(13) Find the sum of 25 ac., 2 ro., 16 po.; 30 ac., 2 ro., 25 po.; 26 ac., 2 ro., 35 po.; 63 ac., 1 ro., 31 po.; and 34 ac., 2 ro., 29 po.: also of 5 ac., 2 ro., 15 po., $25\frac{1}{4}$ sq. yds., 101 sq. in.; 9 ac. 1 ro., 35 po., $12\frac{1}{2}$ sq. yds., 87 sq. in.; 42 ac., 3 ro., 24 po., $23\frac{3}{4}$ sq. yds., 57 sq. in.; 12 ac., 2 ro., 5 po., $13\frac{3}{4}$ sq. yds., 23 sq. in.; and 17 ac., 24 po., 30 sq. yds., 113 sq. in.: explain each process.

(14) Find the sum of 3 c. yds.. 23 c. ft., 171 c. in.; 17 c. yds., 17 c. ft., 31 c. in.; 28 c. yds., 26 c. ft., 1000 c. in.; and 34 c. yds., 23 c. ft., 1101 c. in.: also of 12 po., 18 sq. yds., 7 sq. ft., 35 sq., in.; 13 po., $24\frac{3}{4}$ sq. yds., 8 sq. ft., 63 sq. in.; 14 po., $29\frac{1}{4}$ sq. yds., 5 sq. ft., 131 sq. in.; 15 po., 19 sq. yds., 3 sq. ft., 126 sq. in.; and 16 po., $28\frac{1}{4}$ sq. yds., 130 sq. in.

(15) Add together 39 gals., 3 qts., $1\frac{1}{3}$ pt.; 48 gals., 2 qts., $1\frac{1}{2}$ pt.; 56 gals., $1\frac{3}{4}$ pt.; 74 gals., 3 qts.; and 84 gals., 3 qts., $1\frac{5}{6}$ pt.;

also 2 pipes, 42 gals., 3 qts.; 36 gals., 1 qt.; 5 pipes, 48 gals.; 12 pipes, 53 gals., $3\frac{1}{3}$ qts., and 27 pipes, $2\frac{4}{5}$ qts., of wine.

(16) Add together 4 mo., 3 w., 5 d., 23 h., 46 m.; 5 mo., 1 d., 17 h., 57 m.; 6 mo., 2 w., 1 h.; 1 w., 6 d., 23 h., 59 m.; and 11 mo., 1 w., 58 m.; also, 7 yrs., 28 w., 3 s.; 26 yrs., 5 w., 5 d.; 58 yrs., 6 d., 23 h., 59 s.; 43 w., 23 h., 50 m., 12 s.; and 124 yrs., 14 w., 19 h., 37s.

(17) When *B* was born, *A*'s age was 2 yrs., 9 mo., 3w., 4 d.; when *C* was born, *B*'s age was 13 yrs., and 3 d.; when *D* was born, *C*'s age was 9 mo., 2 w., 3 d., 23 h.; when *E* was born, *D*'s age was 6 yrs., 11 mo., 23 hrs.; when *F* was born, *E*'s age was 7 yrs., 3 w., 5 d., 15 h. What was *A*'s age on *F*'s 5th birth-day?

COMPOUND SUBTRACTION.

121. Compound Subtraction is the method of finding the difference between two numbers of the same kind, but containing different denominations of that kind.

Rule. "Place the less number below the greater, so that the numbers of the same denomination may be under each other in the same column, and draw a line below them. Begin at the right hand, and subtract if possible each number of the lower line from that which stands above it, and set the remainder underneath.

But when any number in the lower line is greater than the number above it, add to the upper one as many units of the same denomination as make one unit of the next higher denomination; subtract as before, and carry one to the number of the next higher denomination in the lower line; proceed thus throughout the columns."

Ex. 1. Subtract £88. 18*s.* $8\frac{1}{2}$*d.* from £146. 19*s.* $6\frac{1}{4}$*d.*

Proceeding by the Rule given above,

£	*s.*	*d.*
146 ·	19 ·	$6\frac{1}{4}$
88 ·	18 ·	$8\frac{1}{2}$
£58 ·	0 ·	$9\frac{3}{4}$

Reason for the above process.

Since $\frac{1}{2}d.$ is greater than $\frac{1}{4}d.$, we add to $\frac{1}{4}d.$ 4 farthings or 1 penny, thus raising it to 5 farthings: and when 2 farthings are subtracted from 5 farthings, we have three farthings left; we therefore place down $\frac{3}{4}d.$: and in order to increase the lower number equally with the upper number, we add one penny to the 8 pence.

Now 9 pence cannot be taken from 6 pence; we therefore add 12 pence or 1*s.* to 6 pence, thus raising the latter to 18*d.*: we take the 9*d.* from 18., and put down the remainder 9*d.*; then adding 1*s.* to 18*s.*, the latter becomes 19*s.*: 19*s.* taken from 19*s.* leave no remainder: we then subtract £88. from £146., as though they were abstract numbers. It is manifest that in this process, whenever we add to the upper line, we also add a number of the same value to the lower line, so that the final difference is not altered.

Ex. 2. Subtract 106 lbs., 11 oz., $16\frac{2}{3}$ dwts., from 144 lbs., 8 oz., $14\frac{1}{9}$ dwts.

lb.	oz.	dwts.
144	8	$14\frac{1}{9}$
106	11	$16\frac{2}{3}$
37	8	$17\frac{4}{9}$

$\frac{2}{3}$ is greater than $\frac{1}{9}$, therefore we add 1 or $\frac{9}{9}$ to $\frac{1}{9}$, which makes it $\frac{10}{9}$. Now $\frac{10}{9}-\frac{2}{3}=\frac{10}{9}-\frac{6}{9}=\frac{4}{9}$. We must repay the dwt. by adding 1 dwt. to the 16 dwts. Art. (76.)

Ex. XXXVIII.

	£	s.	d.
(1)	149	4	$6\frac{3}{4}$
	86	13	$2\frac{3}{4}$

	£	s.	d.
(2)	309	13	$11\frac{1}{4}$
	119	19	$10\frac{3}{4}$

	cwt.	qr.	lb.	oz.
(3)	63	0	18	1
	58	1	12	10

	fu.	po.	yds.
(4)	14	34	5
	1	38	4

	ac.	ro.	po.
(5)	63	1	$29\frac{7}{9}$
	57	2	$38\frac{2}{3}$

	qrs.	bus.	pk.	gal.
(6)	64	3	1	0
	8	5	3	1

(7) Subtract £456. 15*s.* $11\frac{3}{8}d.$ from £534. 13*s.* $10\frac{1}{2}d.$; and prove the result.

(8) Find the difference between the following numbers, and verify the results:

1. 426 lbs., 8 oz., 1 dwt., 7 grs., and 385 lbs., 3 oz., 11 dwts., 21 grs.
2. 5836 lbs., and 4976 lbs., 7 oz., 13 dwts., 19 grs.
3. 26 tons, 2 qrs., $23\frac{4}{5}$ lbs., and 19 tons, 3 cwt., 3 qrs., 18 lbs.
4. 806 tons, 14 cwt., 7 lbs., and 789 tons, $16\frac{5}{16}$ lbs.
5. 144 lbs., 9 oz., 4 drs., 1 scr.. and 129 lbs., 7 dr., 3 scr.

6. 418 yds., 1 qr., 1 na., and 387 yds., 3 qrs., 3 na.
7. 15 yds., 1 ft., 5 in., and 13 yds., 2 ft., 7 in.
8. 99 yds., and 87 yds., 1 ft., 11 in.
9. 13 m., 6 fur., 35 po., $3\frac{1}{2}$ yds., and 12 m., 38 po., 4 yds.
10. 35 lea., 4 fur., 23 po., 4 yds., 1 ft., and 28 lea., 5 fur., 39 po., $4\frac{1}{2}$ yds., 2 ft.
11. 56 ac., 2 ro., 34 po., and 48 ac., 3 ro., 38 po.
12. 3 ro., 28 po., 27 sq. yds., 7 sq. ft., and 1 ro., 39 po. $28\frac{1}{4}$ sq. yds., 8 sq. ft.
13. 37 cub. yds., 18 cub. ft., 857 cub. in., and 35 cub. yds., 24 cub. ft., 1280 cub. in.
14. 203 tuns, 19 gals., 3 qts., 1 pt., of wine, and 187 tuns, 1 hhd., 29 gals., 2 qts.
15. 83 bar., 2 fir., 7 gals., of beer, and 77 bar., 2 fir., 8 gals., $29\frac{3}{8}$ qts.
16. 23 lds., 2 qrs., 5 bus., $3\frac{7}{11}$ pks., and 18 lds., 2 qrs., 6 bus.
17. 216 yrs., 9 mo., 2 w., 4 d., and 217 yrs.
18. The latitude of St. Peter's at Rome is 41°, 53′, 54″ north, and that of St. Paul's at London is 51°, 30′, 49″ north. Find the difference of their latitude.
19. What sum added to £947. 19*s*. $7\frac{3}{4}$*d*. will make £1000?
20. A furnished house is worth £4759. 10*s*. $9\frac{1}{4}$*d*.; unfurnished, it is worth £1494. 11*s*. $9\frac{3}{4}$*d*. By how much does the value of the furniture exceed the value of the house?

COMPOUND MULTIPLICATION.

122. Compound Multiplication is the method of finding the amount of any proposed compound number, that is, of any number composed of different denominations, but all of the same kind, when it is repeated a given number of times.

Rule. "Place the multiplier under the lowest denomination of the multiplicand; multiply the number of the lowest denomination by the multiplier, and find the number of units of the next denomination contained in this first product; if there be a remainder, place it down, adding on the number of units just found to the second product; for this second product multiply the number of the next denomination in the multiplicand by the multiplier, and after carrying on to it the above-mentioned number of units, proceed with the result as with the first product; carry this operation through with all the different denominations of the multiplicand."

Ex. Multiply £56. 4*s*. $6\frac{1}{2}$*d*. by 5.

Proceeding by the Rule given above,

£.	s.	d.
56	4	$6\frac{1}{2}$
		5
£281	2	$8\frac{1}{2}$

Reason for the above process.

$\frac{1}{2}d.$ multiplied by 5 is the same as $(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2})d.=5$ half-pence$=2\frac{1}{2}d.$; we therefore put down $\frac{1}{2}d.$, and carry on 2*d*. to the denomination of pence:

6*d*. multiplied by $5=30d.$; therefore $(2+6\times5)d.=32d=(2\times12+8)d.=2s.+8d.$; we therefore put down 8*d*., and carry on 2*s*. to the denomination of shillings:

4*s*. multiplied by $5=20s.$; therefore $(2+4\times5)s.=22s.=(20+2)s.=£1+2s.$; we therefore put down 2*s*., and carry £1 to the denomination of pounds:

Now by Simple Multiplication $£56\times5=£280$; therefore $£(1+56\times5)=£(1+280)=£281$.

Therefore the total amount is £281. 2*s*. $8\frac{1}{2}$*d*.

123. When the multiplier exceeds 12 it will be the easiest method to split the multiplier into factors, or into factors and parts: thus $15=3\times5$; $17=3\times5+2$; $23=4\times5+3$; $240=4\times6\times10$: and so on.

Ex. Multiply 1 ton, 3 cwt., 2 qrs. $7\frac{5}{8}$ lbs., by 23.

ton.	cwt.	qrs.	lbs.	
1	3	2	$7\frac{5}{8}$	by 23.
			4	
4	14	1	$5\frac{1}{2}$	= value of 1 ton, 3 cwt., 2 qrs., $7\frac{5}{8}$ lbs., multiplied by 4.
			5	
23	11	2	$2\frac{1}{2}$	= value of 4 tons, 14 cwt., 1 qr., $5\frac{1}{2}$ lbs., multiplied by 5, or of 1 ton, 3 cwt., 2 qrs., $7\frac{5}{8}$ lbs., multiplied by $(4\times5$, or) 20.
3	10	2	$22\frac{7}{8}$	= value of 1 ton, 3 cwt., 2 qrs., $7\frac{5}{8}$ lbs., multiplied by 3.
27	2	1	$0\frac{3}{8}$	= value of 1 ton, 3 cwt., 2 qrs., $7\frac{5}{8}$ lbs., multiplied by $(20+3)$ or 23.

In the above example $\frac{5}{8}$ lb. $\times4=\frac{5}{2}$ lb. $=2\frac{1}{2}$ lbs, consequently we put down the $\frac{1}{2}$ lb. and carry on the 2 lbs. to the lbs. The fractions in the other multiplicands are treated in a similar manner.

NOTE.—When the multiplier is a high number, the simplest method is to break it up into factors in the following manner:

$$456 = 400+50+6.$$
$$= (10\times10\times4)+(10\times5)+6.$$

Ex. XXXIX.

Multiply

(1) £11. 13*s*. 6*d*. separately by 2 and 5.
(2) £709. 17*s*. $11\frac{1}{12}$*d*. separately by 6, 26, and 120.
(3) £2579. 0*s*. $0\frac{3}{4}$*d*. separately by 147, 155, 474, and 2331.
(4) 86 lbs., 7 oz., 16 dwts., 11 grs. separately by 8 and 36.
(5) 3 tons, 22 lbs., 13 oz. separately by 11 and 76.
(6) 45 lbs., 7 oz., 3 drs., 2 sc. separately by 12 and 68.
(7) 67 yds., 1 qr., 2 na. separately by 9 and 53.
(8) 70 yds., 2 ft., 10 in. separately by 7 and 29.
(9) 67 ro., 38 po., 27 yds., 2 ft. separately by 11 and 112.
(10) 380 ac., 3 ro., 32 po. separately by 12 and 106.
(11) 57 gals., 3 qts. separately by 10 and 257.
(12) 76 qrs., 5 bus., 2 pks. separately by 13 and 240.
(13) 5 wks., 6 d., 18 h., 14 m. separately by 11 and 339.
(14) 84 hhds., 43 gals., 1 pt. of wine separately by 27 and 364.
(15) 43 bar., 13 gals., 1qt., 1 pt. of beer separately by 39 and 764.
(16) A person buys 67 lambs at £1. 0*s*. $9\frac{1}{2}$*d*. each; 73 sheep at £2. 2*s*. $11\frac{1}{4}$*d*. each; 12 cows at the average of £37. 0*s*. $2\frac{3}{4}$*d*. for every 3 of them; and 17 horses at 37 guineas each: the expenses of getting them all home amount to $17\frac{1}{2}$ guineas. What money must he draw from his bankers to pay for the whole outlay?
(17) There are 7 chests of drawers: in each chest there are 18 drawers; and in each drawer 8 divisions; and in each division there is placed £16. 6*s*. 8*d*. How much money is deposited in the chests?

COMPOUND DIVISION.

124. COMPOUND DIVISION is the method of dividing a compound number, that is, a number composed of several denominations, but all of the same kind, into as many equal parts as the divisor contains units; and also of finding how often one compound number is contained in another of the same kind.

When the divisor is an abstract number.

RULE. "Place the numbers as in Simple Division: then find how often the divisor is contained in the highest denomination of the dividend; put this number down in the quotient; multiply as in Simple Division and subtract; if there be a remainder, reduce that remainder to the next inferior denomination, adding to it the number of that denomination in the dividend, and repeat the division: carry on this process through the whole dividend."

Ex. Divide £199. 6*s.* 8*d.* by 130.

Proceeding by the Rule given above,

```
         £.   s.  d.
130) 199 . 6 . 8 (1£
     130
     ---
      69
      20
     ----
130) 1386 (10s.
     130
     ----
       86
       12
     ----
130) 1040 (8d.
     1040
     ----
```

Therefore the answer is £1. 10*s.* 8*d.*

Reason for the above process.

We first subtract £1 taken 130 times, from £199. 6*s.* 8*d.*, and there remains £69. 6*s.* 8*d.*

Now £69. 6*s.* 8*d.*=1386*s.* 8*d.*; from this amount we subtract 10*s.* taken 130 times, and there remains 86*s.* 8*d.*

Again, 86*s.* 8*d.*=1040*d.*; from this amount we subtract 8*d.* taken 130 times, and nothing remains.

Therefore £1. 10*s.* 8*d.* is contained 130 times in £199. 6*s.* 8*d.*

Note. When the divisor is not greater than 12, the division can be easily performed in one line: thus for example, divide £8. 18*s.* 6*d.* by 12.

```
     £.    s.    d.
12 | 8  .  18  .  6
   ----------------
          14  . 10½
```

Since we cannot divide 8 by 12, we reduce the £8 to shillings, and adding in the term 18*s.*, we have to divide 178*s.* by 12; we obtain 14s., with remainder 10*s.*; and since 10*s.*=120*d.*; therefore, adding in the term 6*d.*, we have to divide 126*d.* by 12; we obtain 10*d.*, with remainder 6*d.*; and since 6*d.*=24*q.*, we divide 24*q.* by 12, and thus we obtain 2*q.*, or $\frac{1}{2}d.$

125. It may sometimes be found convenient to break up the divisor into factors: thus,

Ex. Divide £37. 14*s.* by 24.

24=4×6.

		£.	s.	d.
24	4	37	14	0
	6	9	8	6
		£1	11	5

Ex. XL.

(1) £245. 14*s.* 8*d.*÷4.
(2) £435. 17*s.* $2\frac{1}{4}d.$÷7.
(3) £33. 18*s.* 6*d*÷23.
(4) £605. 0*s.* $1\frac{1}{2}d.$÷9.
(5) £62. 1*s.* $7\frac{1}{2}d.$÷198.
(6) £162. 3*s.* 6*d.*÷156.
(7) £492710. 1*s.* 8*d.*÷6352.
(8) 178 cwt., 3 qrs., 14 lbs÷53.
(9) 1283 cwt., 4 lbs.÷75.
(10) 684 d., 8 h., 9 m.÷47.
(11) 206 mo. of 28 days, 4 d.÷26.
(12) 76 cwt.÷963.
(13) 15 cwt., 2 lb., 11 oz.÷456.
(14) 13 ac., 1 ro.÷147.
(15) 75 ac., 3 ro., 39 po.÷26.
(16) 97 qrs., 3 bus., 3 pks.÷107.
(17) 91 yds., 2 qrs., 1 na.÷903.
(18) £12÷·000625; and £36÷·001875.

126. If the Divisor be 10, 100, 1000, &c., the operation of Division is usually performed, by pointing off as decimals, one, two, three, &c., figures accordingly at the right hand of the dividend.

Thus: Divide Tons 5362. 10 cwts. by 100.

Ex.

Tons.	Cwts.
53·62 ·	10
20	
12.40+10 cwts.	
=12·50 cwts.	
4	
2.00 qrs.	

Therefore the quotient is Tons 53. 12 cwts. 2 qrs.

Reason for the above process.

$$5362 \text{ tons, } 10 \text{ cwts.} \div 100 = \frac{5362}{100} \text{Tons,} + \frac{10}{100} \text{cwts.}$$

$$= \text{Tons } 53.62 + \frac{10}{100}\text{cwts.} = 53 \text{ Tons} + \frac{62}{100}\text{Tons} + \frac{10}{100}\text{cwts.}$$

$$= \text{Tons } 53 + \frac{(62 \times 20)}{100} \text{ Tons} + \frac{10}{100}\text{cwts.}$$

$$= \text{Tons } 53 + \frac{(1240+10)}{100} \text{ cwts.}$$

$$= \text{Tons } 53 + \frac{1250}{100} \text{ cwts.}$$

$$= \text{Tons } 53 + 12{\cdot}50 \text{ cwts.}$$

$$= \text{Tons } 53 + 12 \text{ cwts.} + \frac{50}{100}\text{cwts.}$$

$$= \text{Tons } 53 + 12 \text{ cwts.} + \frac{(50 \times 4)}{100} \text{qrs.}$$

$$= \text{Tons } 53 + 12 \text{ cwts.} + 2 \text{ qrs.}$$

$$= \text{Tons } 53.\ 12 \text{ cwts. } 2 \text{ qrs.}$$

Ex. 2. Divide 1668 tons. 15 cwts. by 1500.

$$1500 = 3 \times 5 \times 100;$$

first divide by the factors 3 and 5, and then by 100: it will be found best in all cases of this kind to do so.

	tons.	cwts.
3	1668 ·	15
5	556 ·	5
Ton	1·11 ·	5
		20
	2·25 cwts.	
		4
	1·00 qrs.	

Therefore the quotient is 1 ton 2 cwts. 1 qr.

Ex. XLI.

(1) £396. 9*s*. 2*d*. ÷ 10. (2) £2025 ÷ 1000.
(3) 1447 lbs. 10 oz. 4 dwts. ÷ 1000. (4) 262 tons, 10 cwt. ÷ 2400.

(5) 26380 mo. 3 w. 5 d. ÷ 25000.
(6) 21 ac., 3 ro., 17 perches × ·02; and £375. 3*s.* × ·0507.
(7) 24 ac., 3 ro., 10 perches × 112, and × 11·2.

127. *When the divisor and dividend are both compound numbers of the same kind.*

Rule. "Reduce both numbers to the same denomination: divide as in Simple Division, and the result will be the answer required."

Ex. How often is 5*s.* $3\frac{3}{4}$*d.* contained in £15. 8*s.* 9*d.*?
Proceeding by the above Rule,

s.	*d.*		£	*s.*	*d.*
5 ·	$3\frac{3}{4}$		15 ·	18 ·	9
12			20		
63			318		
4			12		
255			3825		
			4		
			15300		

255) 15300 (60
        1530

Therefore 60 is the answer.

Reason for the above process.

5*s.* $3\frac{3}{4}$*d.* = 255 farthings,
£15. 18*s.* 9*d.* = 15300 farthings;

and 255 farthings subtracted 60 times from 15300 farthings leave no remainder.

Ex. XLII.

(1) £160. 4*s.* $8\frac{1}{4}$*d.* ÷ £1. 10*s.* $6\frac{1}{4}$*d.*
(2) £401 4*s.* 3*d.* ÷ £2. 11s. $5\frac{1}{4}$*d.*
(3) 44 cwt., 2 qrs., 11 lbs. ÷ 1 cwt., 2 qrs., 17 lbs.
(4) 272 yds., 1 qr. ÷ 7 yds., 2 qrs., 1 na.
(5) 9487 bus., 2 pks. ÷ 143 bus. 3 pks.
(6) 1416 ac., 2 ro., 16 po. ÷ 4 ac., 3 ro., 27 po.
(7) 57 lea., 1 mi., 956 yds. ÷ 7 fur., 87 yds., 1 ft., 5 in.
(8) 617 lds., 1 qr. ÷ 12 qrs., 1 pk.

128. We shall now add some examples of the Multiplication and Division of numbers, comprising different denominations, but of the same kind, by mixed numbers.

In the case of Multiplying by a mixed number, it will generally be found advantageous, first to multiply by the integral part, and then to add to the result thus obtained the result given by multiplying by the fractional part.

Thus, for example: Multiply £2. 6*s*. 8*d*. by $3\frac{7}{10}$.

$$(£2.\ 6s.\ 8d.) \times 3 = £7.$$

$$\frac{(£2.\ 6s.\ 8d.) \times 7}{10} = \frac{£16.\ 6s.\ 8d.}{10} = £1.\ 12s.\ 8d$$

Therefore $(£2.\ 6s.\ 8d.) \times 3\frac{7}{10} = £7 + £1.\ 12s.\ 8d. = £8.\ 12s.\ 8d.$

In Division it will be found advantageous to reduce the mixed number to an improper fraction.

Thus, for example: Divide £89. 17*s*. $6\frac{3}{4}d$. by $19\frac{3}{4}$.

$$19\frac{3}{4} = \frac{79}{4}.$$

$$\text{Now } £89.\ 17s.\ 6\tfrac{3}{4}d. \div \frac{79}{4} = \frac{(£89.\ 17s.\ 6\frac{3}{4}d.) \times 4}{79} = £4.\ 11s.\ 0\tfrac{15}{79}d.$$

Ex. XLIII.

(1) £40. 11*s*. $6\frac{3}{8}d. \times 57\frac{1}{20}$.
(2) £20. 18*s*. $2\frac{157}{310}d \div 12\frac{10}{12}$.
(3) 3 ro., 35 po., $27\frac{1}{2}$ yds. $\times 81\frac{1}{2}$.
(4) 84 tons, 13 cwt., 3 lbs. $\times 23\frac{5}{9}$.
(5) 597 cwt., 2 qrs., 8 lbs. $\div 13\frac{1}{3}$.
(6) 6491 yrs., 8 mo. $\div 375\frac{8}{11}$.
(7) 571 yds., 2 qrs., 1 na. $\div 23\frac{5}{7}$.
(8) 4 mi., 3 fur., 37 po., $4\frac{1}{5}$ yds $\times 5\frac{5}{7}$.

REDUCTION OF FRACTIONS.

129. *To find the value of a fractional part of a number of one denomination in terms of the same or lower denominations.*

RULE. Multiply the given number by the numerator of the fraction, and divide the product (if possible) by the denominator; if there be a remainder, multiply the numerator of the fraction which remains by the number of units connecting the given denomination with the next lower denomination, and divide the product by the denominator; if there still be a remainder, proceed with it in the same way as with the last remainder, and so on, till you come to the lowest denomination. The compound number formed of the integral parts reserved from the successive quotients, and of the result of the last reduction, will be the value required.

Note. If the given number comprise different denominations of the same kind: reduce the different denominations to the lowest denomination involved, and the above rule may be then applied; or the value may be found by the method shewn in Art. (128).

Ex. 1. Find the value of $\frac{7}{8}$ of £1.

Proceeding by the Rule given above,

$$\frac{7}{8} \text{ of } £1 = \frac{7 \times 20}{8} s. = \frac{7 \times 5}{2} s.$$

$$= \frac{35}{2} s. = 17\tfrac{1}{2} s.,$$

$$\text{and } \frac{1}{2} \text{ of } 1s. = \frac{1 \times 12}{2} d. = 6d.;$$

therefore the value required $= 17s.\ 6d.$

Reason for the above process.

$$\frac{7}{8} \text{ of } £1 \text{ is the same as 7 times } \frac{1}{8} \text{ of } £1.,$$

$$\text{and } \frac{1}{8} \text{ of } £1 = \frac{20s.}{8} = \frac{5s.}{2};$$

therefore 7 times $\frac{1}{8}$ of £1 $= 7$ times $\frac{5s}{2} = \frac{35s}{2} = 17\tfrac{1}{2}s. = 17s.\ 6d.$

Ex. 2. Find the value of $\frac{2}{7}$ of £15 $+ 3\frac{3}{7}$ of £1 $+ \frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of £1 $+ \frac{2}{3}$ of $\frac{3}{7}$ of 1s.

$$\frac{2}{7} \text{ of } £15 = £\frac{2 \times 15}{7} = £\frac{30}{7} = £4\tfrac{2}{7},$$

$$£\frac{2}{7} = \frac{2 \times 20}{7} s. = \frac{40}{7} s. = 5\tfrac{5}{7} s.,$$

$$\frac{5}{7} s. = \frac{5 \times 12}{7} d. = \frac{60}{7} d. = 8\tfrac{4}{7} d.$$

therefore $\frac{2}{7}$ of £15 $= £4.\ 5s.\ 8\frac{4}{7}d.$

$$£3\tfrac{3}{7} = £3 + £\tfrac{3}{7}.$$

$$£\frac{3}{7} = \frac{3 \times 20}{7} s. = \frac{60}{7} s. = 8\tfrac{4}{7} s.,$$

$$\frac{4}{7}s.=\frac{4\times 12}{7}d.=\frac{48}{7}d.=6\tfrac{6}{7}d.$$

therefore £$3\frac{3}{7}$ = £3. 8s. $6\frac{6}{7}$d.

$$\tfrac{1}{3}\text{ of }\tfrac{5}{7}\text{ of }\tfrac{3}{5}\text{ of }£1=\tfrac{1}{7}\text{ of }£1.$$

$$=\frac{20}{7}s.$$

$$=2\tfrac{6}{7}s.$$

$$\frac{6}{7}s.=\frac{6\times 12}{7}d.=\frac{72}{7}d.=10\tfrac{2}{7}d.$$

therefore $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of £1 = 2s. $10\frac{2}{7}$d.

$$\frac{2}{3}\text{ of }\frac{3}{7}\text{ of }1s.=\frac{2}{7}\text{ of }1s.=\frac{2\times 12}{7}d.$$

$$=\frac{24}{7}d.=3\tfrac{3}{7}d.$$

therefore required value = £4. 5s. $8\frac{4}{7}$d. + £3. 8s. $6\frac{6}{7}$d. + 2s. $10\frac{2}{7}$d. + $3\frac{3}{7}$d. = £7. 17s. $5\frac{1}{7}$d.

Ex. 3. Find the value of $\frac{5}{9}$ of a bushel—$\frac{5}{7}$ of a peck.

$$\frac{5}{9}\text{ of a bus.}=\frac{5\times 4}{9}\text{ pks.}=\frac{20}{9}\text{pks.}=2\tfrac{2}{9}\text{ pks.},$$

$$\frac{2}{9}\text{ pk.}=\frac{2\times 8}{9}\text{ qts.}=\frac{16}{9}\text{ qts.}=1\tfrac{7}{9}\text{ qts.};$$

therefore $\frac{5}{9}$ of a bus. = 2 pks., $1\frac{7}{9}$ qts.,

$$\frac{5}{7}\text{ of a pk.}=\frac{5\times 8}{7}\text{ qts.}=\frac{40}{7}\text{ qts.}=5\tfrac{5}{7}\text{ qts.};$$

therefore required value = 2 pks., $1\frac{7}{9}$ qts. — $5\frac{5}{7}$ qts.
= 1 pk., $4\frac{4}{63}$ qts.

Ex. XLIV.

(1) Find the respective values of

1. $\frac{2}{5}$ of 4s. 7d.; $1\frac{3}{5}$ of £1. 2s. 9d.; $\frac{4}{14}\times\frac{11}{18}$ of 21s.; $\frac{1}{3}$ of $\frac{3}{5}$ of 9s. $10\frac{1}{2}$d.
2. $3\frac{1}{12}$ of 2s. 6d.; $\frac{9}{11}$ of £4. 14s. 5d.; $\frac{2}{3}$ of $\frac{4}{21}$ of 10s. 6d.; $\frac{1}{50}$ of 100 guineas.

3. $\frac{3}{5}$ of $\frac{7}{9}$ of $\frac{20}{21}$ of 5 guineas; $\frac{3}{5}$ of £16. 16*s.* $3\frac{1}{4}$*d.*; [illegible] of £44 12*s.* 6*d.*
4. $\frac{5}{16}$ of a cwt.; $\frac{3}{4}$ of a lb. Avoird.; $\frac{1}{5}$ of a mile; $\frac{5}{8}$ of an acre.
5. $\frac{5}{11}$ of a mile; $\frac{3}{10}$ of a day; $\frac{2}{3}$ of a yard; $\frac{2}{7}$ of 3 cwt., 1 qr., 14 lb.
6. $7\frac{3}{5}$ of a lb. Avoird.; $1\frac{3}{4}$ of a lb. Troy; $2\frac{2}{5}$ of a gal.; $4\frac{5}{16}$ of an acre.
7. $3\frac{5}{12}$ of a hhd. of beer; $2\frac{3}{8}$ of a tun of wine; $6\frac{23}{25}$ of a bus.
8. $2\frac{1}{6}$ of a load; $3\frac{109}{270}$ of a cub. yd.; $9\frac{5}{12}$ guineas.
9. $\frac{7}{8}$ of $\frac{3}{5}$ of $10\frac{2}{3}$ hrs.; £$\frac{15\frac{3}{5}}{7\frac{4}{5}}$; $\frac{7\frac{1}{8}}{8\frac{1}{7}}$ of $\frac{5\frac{1}{7}}{7\frac{1}{5}}$ of a moidore.
10. $\frac{\frac{4}{45}}{\frac{5}{14} \text{ of } 1\frac{1}{3}}$ of £16. 8*s.* $1\frac{1}{2}$*d.*; $\frac{3}{7}$ of $1\frac{2}{5}$ of $12\frac{1}{2}$ of $\frac{3}{44}$ of £2 $\times \frac{3}{44}$.
11. $\frac{5}{7}$ of 1 cwt. $\times 5\frac{3}{5}$; $\frac{3}{8}$ of $\frac{2}{9}$ of 1lb. $\div \frac{1}{4}$.
12. $19\frac{3}{4}$ of 5ac. 1ro. $1\frac{1}{4}$po.; $2\frac{2}{5}$ of 8 yds. 2ft. $2\frac{7}{12}$in. $\div 8\frac{4}{5}$.

(2) Find the values of

1. $\frac{3}{5}$ of £1 + $\frac{2}{7}$ of a guinea + 3*s.* 2*d.*
2. $\frac{3}{8}$ of £1 + $\frac{3}{8}$ of 2*s.* 6*d.* + $\frac{3}{8}$ of 1*s.*
3. $\frac{1}{7}$ of £1 + $\frac{2}{9}$ of 1*s.* + $\frac{5}{12}$*d.*
4. $\frac{2}{3}$ of 1lb. + $\frac{4}{5}$ of 1oz. + $\frac{5}{6}$ of 1dwt.
5. $\frac{3}{8}$ of 5 acres + $\frac{2}{5}$ of 3ro. + 7 po.
6. $\frac{\frac{3}{4}+\frac{2}{3}}{1\frac{7}{3}}$ of 6tons + $\frac{7}{16}$ of 4cwts. + $\frac{4}{5}$ of 1qr.
7. $\frac{7}{8}$ of 3 cords + $\frac{1}{2}$ of 3 cord feet.
8. $\frac{2}{9}$ of 21*s.* + $\frac{1}{2}$ of $\frac{3}{4}$ of 1£. — $\frac{1}{6}$ of $\frac{3}{4}$ of 5*s.* + $\frac{1}{3}$ of $\frac{5}{8}$ of 1*s.*
9. $\frac{2}{3\frac{1}{2}}$ of $\frac{4}{\frac{5}{7}-\frac{1}{9}}$ of 2 guineas + $\frac{1\frac{7}{8}+\frac{3}{4}}{3\frac{4}{5}-\frac{2}{3}}$ of £5.
10. $\frac{3}{5}$ of a ton + $\frac{5}{8}$ of a cwt. + $\frac{2}{3}$ lb.
11. $\frac{3}{8}$ lb. Troy + $\frac{5}{6}$ lb. Troy — $\frac{8}{9}$ oz. Troy.
12. $\frac{7}{10}$ of a mile — $\frac{5}{8}$ of a fur. + $\frac{4}{11}$po.
13. $\frac{9}{111}$ cub. yds. + $2\frac{7}{8}$ cub. ft.
14. $\frac{2}{3}$ of a qr. + $\frac{3}{4}$ of a bus. — $\frac{1}{6}$ of a qr.
15. $\frac{3}{5}$ of 7 fur., 29 po., $3\frac{1}{4}$yds. + $\frac{5}{7}$ of 5 mi., 3 fur., 37 po., $4\frac{1}{5}$ yds.
16. $7\frac{2}{5}$ of $365\frac{1}{4}$ d. $\times 3\frac{9}{10}$ of $\frac{5}{6}$ wks. + $\frac{3}{4}$ of $5\frac{5}{9}$ hrs.
17. $\frac{1}{11}$ of 91 ac., 3 ro. 36 po., $2\frac{3}{4}$ yds. — $\frac{3}{5}$ of 6 ac., 2 ro., 17 po. $25\frac{1}{3}$ yds.

130. *To reduce a number, or a fraction, of any denomination, to a fraction of another denomination.*

Rule. "Reduce the given number, or fraction, and also the number or fraction to the fraction of which it is to be reduced, to their respective equivalent values in terms of some one and the same denomination: then the fraction of which the former is made the numerator, and the latter the denominator, will be the fraction required."

Ex. 1. Reduce 3*s*. 5*d*. to the fraction of £1.

Proceeding by the above Rule,

3*s*. 5*d*. = 41 pence,
£1 = 240 pence;

therefore fraction required $=\frac{41}{240}$.

Reason for the above process.

£1, or unity, is here divided into 240 equal parts; and 41 of such parts being taken, the part of unity, or £1, which they make up, is represented by $\frac{41}{240}$.

Ex. 2. Reduce $\frac{5}{8}$ of £1 to the fraction of 27*s*.

$$\tfrac{5}{8} \text{ of } £1 = 20 \text{ times } \tfrac{5}{8} \text{ of } 1s.$$
$$=\frac{5\times 20}{8}s.$$
$$=\frac{5\times 5}{2}s.$$
$$27s. = 27s.$$

therefore fraction required $=\dfrac{\frac{5\times 5}{2}}{27}=\dfrac{5\times 5}{2}\div\dfrac{27}{1}$

$$=\frac{5\times 5}{2}\times\frac{1}{27}=\frac{25}{54}.$$

For 27*s*. is divided into 27 equal parts; and $\frac{5}{8}$ of £1 is divided into $\frac{25}{2}$ of such parts; therefore the part of unity, or 27*s*., which the latter represents, is $\dfrac{\frac{25}{2}}{27}=\dfrac{25}{54}$.

Ex. 3. What part of $\frac{1}{3}$ of a ton is $2\frac{2}{3}$ of $1\frac{1}{3}$ of $\frac{1}{7}$ of a cwt.?

$$2\tfrac{2}{3} \text{ of } 1\tfrac{1}{3} \text{ of } \tfrac{1}{7} \text{ of a cwt.} = \tfrac{8}{3} \text{ of } \tfrac{4}{3} \text{ of } \tfrac{1}{7} \text{ of a cwt.}$$
$$=\tfrac{8}{3}\times\tfrac{4}{3}\times\tfrac{1}{7} \text{ cwt.}$$
$$\tfrac{1}{3} \text{ of a ton} = \tfrac{20}{3} \text{ cwt.}$$

Therefore fraction required $=\dfrac{\frac{8}{3}\times\frac{4}{3}\times\frac{1}{7}}{\frac{20}{3}}$

$$=\tfrac{8}{3}\times\tfrac{4}{3}\times\tfrac{1}{7}\times\tfrac{3}{20}$$
$$=\tfrac{8}{105}.$$

Ex. XLV.

(1) Reduce

1. 5*d.* to the fraction of 1*s.*; and 3*s.* 4$\frac{1}{2}$*d.* to the fraction of £1.

2. £1. 3*s.* 4*d.* to the fraction of £9. 6*s.* 8*d.*; and 2*s.* 0$\frac{1}{4}$*d.* to the fraction of 10*s.* 6*d.*

3. £18. 7*s.* 6*d.* to the fraction of £2; and 6*s.* 7$\frac{3}{4}$*d.* to the fraction of 7*s.* 9*d.*

4. 3 qrs., 19 lbs. to the fraction of a ton; and 61$\frac{7}{8}$ lbs. to the fraction of 4 oz.

5. 3 qrs., 4 lbs. to the fraction of 2 cwt.; and 5 oz., 2$\frac{3}{5}$ drs. to the fraction of a grain.

6. 3 ro., 27$\frac{1}{2}$ po. to the fraction of an acre; and 26$\frac{3}{4}$ sq. yds. to the fraction of 2 acres.

7. 126 yds., 2 ft., 6 in. to the fraction of a mile; and 6 cub. ft., 100 cub. in. to the fraction of a cubic yard.

8. 2 qrs., 2$\frac{2}{3}$ na. to the fraction of an Eng. ell; and 8 h., 3 m. to the fraction of a day.

9. 2 ac., 1 ro. to the fraction of 9 ac., 2 ro.; and 1540 yds., 2 ft., 9 in. to the fraction of 2 miles.

10. 5$\frac{1}{2}$ lbs. to the fraction of a cental; and 8$\frac{2}{3}$ feet of wood to fraction of a cord.

11. 1 ft., $\frac{7}{8}$ in. to the fraction of a sq. yd.; and 2 qts., 1$\frac{1}{2}$ pt. to the fraction of a barrel..

12. 2 wks., 5 days, 7 h., 27 m. to the fraction of a day; and 1 ro. 20 po. to the fraction of an acre.

13. 4 bush., 2$\frac{2}{5}$ qts. to the fraction of a load; and 3 quires, 7 sheets to the fraction of a ream.

14. 2$\frac{1}{2}$ guineas to the fraction of £2$\frac{1}{2}$; and 2$\frac{1}{2}$ cwt. to the fraction of 2 tons, 12 lbs.

15. 10$\frac{5}{6}$ months to the fraction of 13 months; and 100$\frac{1}{2}$ guineas to the fraction of 4*d.*

16. 6 ft., 3$\frac{5}{8}$ in. to the fraction of 13 ft., 8$\frac{9}{10}$ in. aud 1$\frac{1}{2}$ yds. to the fraction of 1$\frac{1}{2}$ in.

(2) Reduce

1. $\frac{3}{4}$ of 5*s* to the fraction of £1; and $\frac{4}{5}$ of a farthing to the fraction of 1*s.*

2. $\frac{3}{5}$ of a dwt., to the fraction of 1 lb.; and $\frac{5}{6}$ of 2 lbs. to the fraction of 2$\frac{1}{2}$ tons.

3. $\frac{3}{8}$ of a lb. to the fraction of a cwt.; and $\frac{5}{8}$ of a yd. to the fraction of a mile.

4. $\frac{1}{300}$ of £1 to the fraction of a penny ; and $\frac{3}{9680}$ of a mile to the fraction of a yard.

5. $\frac{2}{3}$ of $\frac{4}{7}$ of 10*s.* 6*d.* to the fraction of 2*s.* 6*d.* ; and 1 oz. Troy to the fraction of 1 oz. Avoirdupois.

6. $\frac{3}{7}$ of a pole to the fraction of a league; and $3\frac{7}{8}$ furlongs to the fraction of $2\frac{7}{8}$ miles.

7. $\frac{3}{5}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards to the fraction of a furlong; and $\frac{1}{7}$ of $\frac{2}{11}$ of a guinea to the fraction of 2*s.* 6*d.*

8. $\frac{7}{9}$ of 16*s.* $0\frac{1}{2}$*d.* to the fraction of 17*s* 6*d.*; and $\frac{7}{310}$ of a lb. Troy to the fraction of a pennyweight.

9. $\frac{7}{8}$ of a lb. Avoird, to the fraction of 2 lbs. Troy; and $\frac{7}{9}$ of 2*s.* 6*d.* to the fraction of £1. 11*s.* 6*d.*

10. $\frac{5}{9}$ of French ell to the fraction of a yd. ; and $\frac{3}{8}$ of a crown to the fraction of $\frac{1}{3}$ of 7*s.* 6*d.*

11. $\frac{16}{23}$ of a sq. in. to the fraction of a sq. yd. ; and $\frac{1}{5}$ of a yd. to the fraction of an English ell.

12. $\frac{21}{15000}$ of a year to the fraction of a day; £$\frac{\cdot036}{1875}$ to the fraction of a farthing.

(3)

1. What part of £9 is $\frac{1}{2}$ of $\frac{9}{10}$ of 2*s.* 6*d.*?
2. What part of a second is $\frac{1}{1000000}$ of a day?
3. What part of $\frac{3}{4}$ of a league is $\frac{7}{9}$ of a mile?
4. What part of $4\frac{1}{2}$ guineas is $5\frac{5}{8}$ of $\frac{4}{15}$ of £7?
5. What part of 3 weeks, 4 days, is $\frac{1}{2}$ of $5\frac{3}{5}$ sec.?
6. What part of $\frac{1}{3}$ of an acre is $25\frac{9}{11}$ po.?
7. What part of $\frac{1}{30}$ of a min. is $\frac{7}{183}$ of a month of 28 days?
8. What part of $\frac{1}{3}$ of 4 tuns of wine is $3\frac{1}{4}$ hhds.?
9. What part of 3 fathoms is $\frac{3}{14}$ of $\frac{7}{3}$ of a pole?
10. Which is the greater, $\frac{1}{25}$ of a day, or $\frac{6}{7}$ of an hour and by how much?

REDUCTION OF DECIMALS.

131. *To reduce a decimal of any denomination to its proper value.*

Rule. " Multiply the decimal by the number of units connecting the next lower denomination with the given one, and point off for decimals as many figures in the product, beginning from the right hand, as there are figures in the given decimal. The figures on the left of the decimal point will represent the whole numbers in the next denomination. Proceed in the same way with the decimal part for that denomination, and so on."

Ex. 1. Find the value of ·0484 of £1.

Proceeding by the Rule given above,

$$\begin{array}{r} \pounds \\ \cdot 0484 \\ 20 \\ \hline \cdot 9680s. \\ 12 \\ \hline 11\cdot 6160d. \\ 4 \\ \hline 2\cdot 4640q. \end{array}$$

therefore the value of ·0484 of £1 $= 11d.\,2\cdot 4640q.$

$= 11d.\,2\frac{464}{1000}q.$

$= 11\frac{1}{2}\frac{29}{250}d.$

Reason for the above process.

$$\cdot 0484 \text{ of } \pounds 1 = \frac{484}{10000} \text{ of } \pounds 1.$$

$$= \frac{9680}{10000}s. = \frac{116160}{10000}d.$$

$$= 11\frac{616}{1000}d. = 11d. + \frac{616 \times 4}{1000}q.$$

$$= 11d. + \frac{2464}{1000}q.$$

$$= 11d. + 2\frac{464}{1000}q.$$

$$= 11\frac{1}{2}\frac{29}{250}d.$$

Ex. 2. Find the value of 13·3375 acres.

$$\begin{array}{r} \text{Acres.} \\ 13\cdot 3375 \\ 4 \\ \hline 1\cdot 3500 \text{ ro.} \\ 40 \\ \hline 14.0000 \text{ po.} \end{array}$$

therefore the value is 13 ac., 1 ro., 14 po.

Ex. 3. Find the value of $\cdot 97291\dot{6}$ of £1.

1st method.

$$\begin{array}{r} £. \\ \cdot 972917 \\ 20 \\ \hline 19\cdot 458340s. \\ 12 \\ \hline 5\cdot 500080d. \\ 4 \\ \hline 2\cdot 000320q. \end{array}$$

therefore the value is 19*s* $5\frac{1}{2}$*d.* nearly.

2nd method.

$$\cdot 97291\dot{6} \text{ of } £1 = \frac{972916-97291}{900000} \text{ of } £1, \text{ Art. (97)},$$

$$= \frac{875625}{900000} \text{ of } £1 = \left(\frac{467}{480} \times 20\right) s.$$

$$= \frac{467}{24} s. = 19s.\ 5\tfrac{1}{2}d.$$

Note. The latter is generally the better course to adopt.

Ex. 4. Find the value of $\frac{133}{400}$ of $3\frac{3}{4}$ tons—$\cdot \dot{3}40\dot{5}$ of $1\frac{2}{3}$ qrs.

$$\frac{133}{400} \text{ of } 3\tfrac{3}{4} \text{ tons} = \left(\frac{133}{400} \times \frac{15}{4}\right) \text{ tons} = \frac{133 \times 3}{80 \times 4} \text{ tons},$$

$$= \left(\frac{133 \times 3}{80 \times 4} \times 20\right) \text{cwt.} = \frac{399}{16} \text{ cwt.}$$

$$= 24 \text{ cwt., } 3 \text{ qrs., } 18\tfrac{3}{4} \text{ lbs.}$$

$$\cdot \dot{3}40\dot{5} \text{ of } 1\tfrac{2}{3} \text{ qrs.} = \left(\frac{3405-3}{9990} \text{ of } \frac{5}{3}\right) \text{qrs.},$$

$$= \left(\frac{3402}{9990} \times \frac{5}{3} \times 25\right) \text{lbs.},$$

$$= \left(\frac{21 \times 25}{37}\right) \text{lbs.} = 14\tfrac{7}{37} \text{ lbs.}$$

therefore the value of the expression

$$= 24 \text{ cwt., } 3 \text{ qrs., } 18\tfrac{3}{4} \text{ lbs.} - 14\tfrac{7}{37} \text{lbs.}$$

$$= 24 \text{ cwt., } 3 \text{ qrs. } 4\tfrac{83}{148} \text{lbs.}$$

$$= 1 \text{ ton, } 4 \text{ cwt., } 3 \text{ qrs., } 4\tfrac{83}{148} \text{lbs.}$$

Ex. XLVI.

(1) Find the respective values of

1. ·45 of £1; ·16875 of £1; ·87708 of £1.
2. ·875 of a lea.; 2·5384375 of a day; ·6 of 1 lb. Troy.
3. 6·156510416 of £4; ·046875 of 3 qrs., 12 lbs.
4. ·85076 of a cwt.; ·07325 of a cwt.; ·045 of a mile.
5. 4·16525 of a ton; 3·625 of a cwt.; ·05 of an acre.
6. 3·8343 of a lb. Troy; 2·46875 of a qr.; 4·106 of 3 cwt., 1 qr., 21 lbs.
7. 3·8375 of an acre; 3·5 of 18 gallons.
8. ·925 of a furlong; ·34375 of a lunar month.
9. 5·06325 of £100; 3·8 of an Eng. ell.
10. 2·25 of $3\frac{1}{2}$ acres; 2·465 of 25 shillings.
11. 1·605 of £3. 2*s.* 6*d.*; 2·0396 of 1 m., 530 yds.
12. 4·751 of 2 sq. yds., 7 sq., ft.; 2·0005 of £63. 0*s.* $3\frac{1}{4}$*d.*
13. 2·009943 of 2 miles; 1·005 of 15 centals.

(2) Find the respective values of

1. $\cdot 38\dot{3}$ of £1; $\cdot 4708\dot{3}$ of £1; $\cdot 469\dot{4}$ of 1lb. Troy.
2. $\cdot 5\dot{7}4\dot{0}$ of 27*s.*; $\cdot 13\dot{8}$ of 10*s.* 6*d.*; $2\cdot\dot{6}$ of 5oz. Troy.
3. $\cdot\dot{1}4285\dot{7}$ of 6 ounces; $3\cdot\dot{2}09532\dot{8}$ of 3 drams.
4. $4\cdot 0\dot{5}$ of $1\frac{1}{2}$ sq. yds.; $\cdot 16\dot{3}$ of $2\frac{1}{2}$ miles; $4\cdot\dot{9}\dot{0}$ of 4 d., 3 hrs.
5. $3\cdot 24\dot{2}$ of $2\frac{1}{2}$ acres; $\frac{\cdot 093\dot{1}\dot{8}}{\cdot 56\dot{8}\dot{1}}$ of $2\frac{1}{12}$ of 2·5 days.

(3) Find the difference between ·77777 of a pound and 8*s.* 6·6648*d.*; and between ·70323 of a pound and 3·5646 of a shilling.

(4) Find the respective values of the following expressions:

1. ·68125 of £1 + ·375 of 13*s.* 4*d.* + $\cdot\dot{6}05$ of £3. 2*s.* 6*d.*
2. $3\frac{7}{8}$ of 5 cwts — $\cdot 4097\dot{2}$ of 5 lbs + 2·75 of 5 oz.
3. 18·731 of a mile + $17\cdot 50\dot{5}$ of a mile + $\frac{7}{8}$ of a yard.
4. $2\cdot\dot{8}\dot{1}$ of $365\frac{1}{4}$ days + 5·75 of a week — $\frac{3}{4}$ of $5\frac{5}{9}$ hours.
5. $\frac{7}{9}$ of $\frac{3}{14}$ of 3 acres — 2·00875 square yards + $\cdot 02\dot{2}\dot{7}$ of $3\frac{1}{2}$ square feet.

(5). Which is the greater, ·0231 of a ton, or ·19 of a cwt?

132. *To reduce a number or fraction of any denomination, to the decimal of another denomination.*

RULE. "Reduce the given number or fraction, to a fraction of the proposed denomination; and then reduce this fraction to its equivalent decimal."

Ex. 1. Reduce 13*s*. 6¼*d*. to the decimal of £1.

$$13s.\ 6\tfrac{1}{4}d. = 162\tfrac{1}{4}d. = \tfrac{649}{4}d.$$
$$£1 = 240d.;$$

therefore the fraction $= \dfrac{\frac{649}{4}}{240} = \dfrac{649}{960}.$

$$\begin{array}{r} 960)\ 649{\cdot}00\ ({\cdot}67 \\ 5760 \\ \hline 7300 \\ 6720 \\ \hline 580 \end{array}$$

We may work such an example as the above more expeditiously by first reducing ¼*d*. to the decimal of a penny, which decimal will be ·25, and then reducing 6·25*d*. to the decimal of a shilling by dividing by 12, which decimal will be $\cdot 52083\dot{3}$, and then reducing $13\cdot 52083\dot{3}s.$ to the decimal of £1 by dividing by 20, which process gives $\cdot 6760416\dot{6}$ as the required decimal of £1.

The mode of operation may be shewn thus:

$$\begin{array}{r|l} 4 & 1{\cdot}00 \\ 12 & 6{\cdot}25 \\ 2,0 & 13{\cdot}52084\dot{3} \\ & {\cdot}6760416 \end{array}$$

Ex. 2. Reduce 3 bush., 1 pk. to the decimal of a load: and verify the result.

$$\begin{array}{r|l} 4 & 1{\cdot}00 \\ 8 & 3{\cdot}25 \\ 5 & {\cdot}40625 \\ & {\cdot}08125 \end{array}$$

therefore ·08125 is the decimal required.

$$\begin{array}{r l} \cdot08125 & \text{ld.} \\ 5 & \\ \hline \cdot40625 & \text{qrs.} \\ 8 & \\ \hline 3\cdot25000 & \text{bush.} \\ 4 & \\ \hline 1\cdot00000 & \text{pk.} \end{array}$$

therefore ·08125 of a load=3 bus., 1 pk.

Ex. 3. Add together ⅖ of a rood and ¾ of 5 perches. Reduce the result to the decimal of an acre:

$$\tfrac{2}{5} \text{ of } 1 \text{ ro.} = \frac{2\times40}{5}\text{perch.} = 2\times8 \text{ perch.} = 16 \text{ perches};$$

$$\tfrac{3}{4} \text{ of } 5 \text{ perches}, = \frac{15}{4}\text{perches}, = 3\tfrac{3}{4} \text{ perches};$$

therefore the sum=19¾ perches.

Now to reduce 19¾ perches to the decimal of an acre:

$$\begin{array}{r|l} 4 & 3\cdot \\ \hline 4{,}0 & 19\cdot75 \\ \hline & \cdot49375 \end{array}$$

therefore the decimal required=·49375 acre.

Or it may be worked thus:

$$19\tfrac{3}{4} \text{ perches}, = \frac{79}{4} \text{ perches}.$$

$$= \frac{79}{4} \div 40 \text{ ac.}$$

$$= \frac{79}{160} \text{ ac.}$$

$$= \cdot49375 \text{ ac.}$$

Ex. 4. Express the sum of $\cdot\dot{4}2857\dot{1}$ of £15, $\frac{1}{4}$ of $\frac{1}{2\frac{1}{4}}$ of $\frac{1}{3}$ of £1. 12*s.* and $\frac{4}{7}$ of 3*d.*, as the decimal of £10.

$$\cdot\dot{4}2857\dot{1} \text{ of } £15 = \tfrac{428571}{999999} \text{ of } £15.$$

$$= \tfrac{3}{7} \text{ of } £15 = £\tfrac{45}{7}$$

$$= £6.\ 8s.\ 6\tfrac{6}{7}d.;$$

$\frac{1}{4}$ of $\frac{1}{2\frac{1}{3}}$ of $\frac{2}{3}$ of £1. 12*s*. $=\frac{1}{4}$ of $\frac{6}{14}$ of $\frac{2}{3}$ of 32*s*.

$=\frac{16}{7}$*s*. $=$ 2*s*. $3\frac{3}{7}$*d*.;

$\frac{4}{7}$ of 3*d*. $=\frac{12}{7}$*d*. $=1\frac{5}{7}$*d*;

therefore the sum $=$ £6. 8*s*. $6\frac{6}{7}$*d*. $+$ 2*s*. $3\frac{3}{7}$*d*. $+1\frac{5}{7}$*d*.

$=$ £6. 11*s*.

2,0	11
10	6·55
	·655

therefore the decimal required $=$ ·655.

Ex. XLVII.

(1) Reduce

1. 6*s*. 4*d*. to the decimal of £1; and 8*s*. $8\frac{1}{2}$*d*. to the decimal of £1.
2. 4*s*. $7\frac{1}{2}$*d*. to the decimal of £1; and 15*s*. $11\frac{1}{4}$*d*. to the decimal of £1.
3. 10*s*. $0\frac{3}{4}$*d*. to the decimal of £1; and 5*s*. $8\frac{3}{4}$*d*. to the decimal of £5.
4. £3. 11*s*. $9\frac{3}{4}$*d*. to the decimal of £1; and also to the decimal of £2. 10*s*.
5. 2 oz., 13 dwts. to the decimal of 1 lb.; and 4 lbs., 2 sc. to the decimal of 1 oz.
6. 2 qrs., 21 lbs. to the decimal of 1 ton; and 3 cwt., 3 oz. to the decimal of 10 cwt.
7. 2 fur., 41 yds. to the decimal of a mile; 1 fur. 30 po. to the decimal of a league.
8. 2 sq. ft., 73 sq. in. to the decimal of a square yard; and 3 ro., 20 po. to the decimal of an acre.
9. 14 gals., 2 qts. to the decimal of a barrel; and 3 qrs., 3 pks. to the decimal of a load.
10. 4 days, 18 hrs. to the decimal of a week; and 11 sec. to the decimal of 5 days.
11. $1\frac{1}{3}$ guineas to the decimal of £$1\frac{1}{2}$; and 1 lb. Troy to the decimal of 1 lb. Avoirdupois.
12. $2\frac{1}{8}$ inches to the decimal of $2\frac{1}{2}$ miles; and $20\frac{1}{2}$ lbs. to the decimal of $3\frac{1}{2}$ lbs.
13. $3\frac{3}{4}$ pks. to the decimal of $3\frac{1}{2}$ qrs.; and $27\frac{1}{2}$ gals. to the decimal of $1\frac{1}{3}$ qts.

14. $5\frac{3}{4}$ yds. to the decimal of 2 Fr. ells; and 1 ton, $2\frac{1}{4}$ cwt. to the decimal of 1 cwt., $2\frac{1}{4}$ qrs.

15. 3 wks., $5\frac{1}{4}$ d. to the decimal of $5\frac{1}{2}$ hrs.; and 1 min., $2\frac{1}{4}$ sec. to the decimal of $\frac{1}{25}$ of a lunar month.

16. 3 reams to the decimal of 19 sheets; and $3\frac{1}{2}$ acres to the decimal of $3\frac{1}{4}$ sq. yards.

17. 33 yds. to the decimal of a mile; and 7*s*. $8\frac{12432}{100000}$*d*. to a the decimal of 10*s*. 6*d*.

(2) Reduce

1. $\frac{4}{5}$ of 5*s*. to the decimal of 21*s*.; and $6\frac{2}{3}$ cwt. to the decimal of a ton.

2. $\frac{7}{8}$ of a pk. to the decimal of 2 qrs.; and $\frac{21}{15000}$ of a year to the decimal of a day.

3. $\frac{3}{7}$ of $\frac{1}{10}$ of 40 yds. to the decimal of $\frac{1}{5}$ of 2 miles; and $\frac{1}{4}$ of $3\frac{1}{2}$ sq. yds. to the decimal of 2 acres, 1 ro.

4. $\frac{3}{5}$ of $4\frac{4}{5}$ hrs. to the decimal of $365\frac{1}{4}$ days; and $9\frac{6}{11}$ of $\frac{1}{8}$ pecks to the decimal of $3\frac{1}{3}$ qrs.

5. 3 lbs., 6 oz. Troy to the decimal of 10 lbs. Avoird.; and $\frac{1}{5}$ oz. Avoird. to the decimal of $\frac{1}{3}$ oz. Troy.

(3) Express $\frac{3}{8}$ of a crown + $\frac{4}{5}$ of a shilling as a decimal of 7*s*.

(4) Express $\frac{3}{5}$ of half-a-crown + $\cdot 4$ of a shilling as a decimal of £2.

(5) Add together $\frac{3}{5}$ of a day, $\frac{2}{3}$ of an hour and $\frac{4}{5}$ of 6 hours; and express the result as the decimal of a week.

(6) Express the difference of $\frac{2}{5}$ of a rood and $\frac{3}{4}$ of a perch as the decimal of an acre.

(7) Express the value of $\cdot 8\dot{3}$ of 8*s*. + $\cdot\dot{0}\dot{5}$ of 2 guineas + $1\cdot 8$ of 5*s*. as the decimal of half-a-guinea.

(8) Add $5\frac{1}{2}$ cwt. to $3\cdot 125$ qrs.; and reduce the sum to the decimal of a ton.

REDUCTION OF CURRENCIES.

133. Pounds, shillings and pence, the denominations of the English monetary system, are still in use in various parts of this Continent. The value of the pound, however, in America, has depreciated in comparison with the pound sterling, owing to the great amount of

paper circulation. This depreciation in value has been greater in some districts than in others, and consequently the pound here has various values. The respective equivalents of the pound in dollars and cents will be seen in the following table:

HALIFAX OR CANADA CURRENCY.

£1 in Canada, and Nova Scotia.......................=\$4.00,
and \$1=5s=£$\frac{1}{4}$.

NEW YORK CURRENCY.

£1 in New York, Ohio, N. Carolina.......................=\$2$\frac{1}{2}$,
and \$1=8s.=£$\frac{2}{5}$.

NEW ENGLAND CURRENCY.

£1 in New England States, Kentucky, Virginia, Tennessee=\$3$\frac{1}{3}$,
and \$1=6s.=£$\frac{3}{10}$.

PENNSYLVANIA CURRENCY.

£1 in New Jersey, Pennsylvania, Delaware, Maryland ..=\$2$\frac{2}{3}$,
and \$1=7s. 6d.=£$\frac{3}{8}$.

GEORGIA CURRENCY.

£1 in South Carolina and Georgia.......................=\$4$\frac{2}{7}$,
and \$1=4s. 8d.=£$\frac{7}{30}$.

The value of the pound sterling in Canada is \$4.87 or in Canada currency, £1. 4s. 4d. Its value in the United States is fixed by the Senate at \$4.84, but its real value as compared with the gold coins of that country is \$4·866.

134. The following table shows the value of the principal foreign coins in dollars and cents, as fixed by the Senate of the United States:

1 Pound Sterling or Sovereign	\$ 4.84
1 Guinea, English	5.00
1 Crown	1.06
1 Shilling piece	0.23
1 Franc, French	0.186
1 Doubloon, Mexico	15.60
1 Specie Dollar of Sweden and Norway	1.06
1 Specie Dollar of Denmark	1.05
1 Thaler of Prussia, and N. States of Germany	0.69
1 Florin of Austrian Empire, and City of Augsburg	0.485
1 Ducat of Naples	0.80
1 Ounce of Sicily	2.40

135. *To reduce any Currency to dollars and cents.*

Rule. Express the sum given, decimally, in the principal unit of the currency, and multiply by the dollars and cents equivalent to that unit.

Ex. 1. Reduce £5. 12*s*. 6*d*. sterling to dollars and cents.

Proceeding by the above rule:

£5. 12*s* 6*d*. = £5·625 (Art. 132)

```
     5·625
      4·87
   -------
     39375
    45000
   22500
   -------
  $27·39375
```

therefore £5. 12*s*. 6*d*. = $27·393+

Reason for the above process.

Since £1 = $4·87,

therefore £5·625 = $(5·625 × 4·87) = $27·39375.

Note.—To reduce dollars and cents to any other currency, it will of course simply be necessary to divide by the number of dollars and cents contained in the principal unit of that currency.

Ex. 2. Reduce $75679·80 into sterling money; also into French Francs.

75679·80 ÷ 4·87
= £15540

therefore $75679·80 = £15540.

again 1 Franc = ·186

therefore $75679·80 ÷ ·186 = No. of Francs in $75679·80
= 406880 $\frac{2}{31}$ Francs.

Ex. XLVIII.

(1) Reduce the following Sterling money to dollars and cents; first taking the pound sterling at $4·87, and secondly at $4·84.

1. £17 $\frac{3}{5}$, and £715. 10*s*.
2. £75. 15*s*. 6*d*., and £189. 14*s*. 3*d*.
3. £618. 15 $\frac{5}{16}$ *s*., and £459. 13*s*. 6 $\frac{7}{8}$ *d*.

(2) Reduce to sterling money at \$4·87, to the pound sterling.

1. \$84.738 and \$3484.485
2. \$369·02425, \$923·890 and \$796076$\frac{1}{4}$.
3. £18. 11*s.* 3*d.*; and £196. 15*s.* 10$\frac{3}{4}$*d.* Canada currency.

(3) Reduce:

1. \$7928·04 to Canada currency.
2. \$8037·06 to New York currency.
3. \$684·125 to New England currency.

(4) Reduce to dollars and cents.

1. £85. 17*s.* 10*d.*, Canada Currency.
2. £7. 19*s.* 3*d.*, New York Currency.
3. £15. 7*s.* 6*d.*, Georgia Currency.

(5) Reduce,

1. 715 francs to dollars and cents.
2. 18·6 Austrian Florins to Canada Currency.
3. 189·5 Mexican Doubloons to dollars and cents, and also to sterling money.

(6) Find the value, in dollars and cents, of

1. £75 sterling + £16. 17*s.* 3*d.*, Canada currency + £18. 5*s.* Georgian currency + £1087. 7*s.* 6*d.*, New York Currency.
2. 71 guineas + 15·5 Prussian Thalers—2·8 Crowns.
3. \$71687·03 + £17$\frac{2}{3}$ Canada Currency + 18$\frac{1}{2}$ Danish dollars—£4$\frac{1}{2}$ sterling + $\frac{1}{3}$ of $\frac{5}{16}$ of $\frac{3}{2}$ of \$0·05.

(7) What would be gained by changing £198. 17$\frac{1}{2}$*s.* sterling in Canada, instead of the United States?

(8) Which is the greater 987$\frac{1}{2}$ Mexican Doubloons, or \$16692·50, and by how much?

(9) Find the value in dollars of $\frac{7}{16}$ of £1 sterling + $\frac{7}{16}$ of £1 Canada Currency + $\frac{7}{16}$ of £1 New York Currency + £$\frac{7}{16}$ of 1 Danish Thaler + £$\frac{7}{16}$ of 1 Ducat of Naples.

(10) Find the difference between $\frac{4}{5}$ of £25 Georgia Currency, and $\frac{4}{5}$ of £25 Canada Currency.

PRACTICE.

136. **Def.** An Aliquot Part of a number is such a part as, when taken a certain number of times, will exactly make up that number. Thus, 4 is an aliquot part of 12, 6 an aliquot part of 18, &c.

Practice is a compendious mode of finding the value of any number of articles by means of Aliquot Parts, when the value of *an unit* of any denomination is given.

The rule for Practice will be easily shown by the following example.

Ex. Find the value of 84 cwts., 3 qrs., 5 lbs. of Sugar at $56.75 per cwt.

The method of working such an example is the following:

1 cwt. costs $56.75

$\therefore$ 84 cwts. cost $ $(56\cdot75\times84)$ $=\$4767\cdot00$.

2 qrs. $=\frac{1}{2}$ of 1 cwt. $\therefore$ the cost of 2 qrs. $=\frac{1}{2}$ the cost of 1 cwt. $=\$\ 28\cdot375$.

1 qr. $=\frac{1}{2}$ of 2 qrs. $\therefore$ the cost of 1 qr. $=\frac{1}{2}$ the cost of 2 qrs. $=\$\ 14\cdot1875$.

5 lbs. $=\frac{1}{5}$ of 1 qr. $\therefore$ the cost of 5 lbs. $=\frac{1}{5}$ the cost of 1 qr. $=\$\ 2\cdot8375$,

therefore by adding up the vertical columns,

the cost of 84 cwts. 3 qrs. 5 lbs. $=\$4812\cdot39$.

The operation is usually written thus:

2 qrs. $=\frac{1}{2}$ of 1 cwt.	$56·75 = value of 1 cwt.
	84
	22700
	45400
	4767·00 = value of 84 cwt.
1 qr. $=\frac{1}{2}$ of 2 qrs.	28·375 = value of 2 qrs.
5 lbs. $=\frac{1}{5}$ of 1 qr.	14·1875 = value of 1 qr.
	2·8375 = value of 5 lbs.
	$4812·3900 = value of 84 cwts. 3 qrs. 5 lbs.

Note.—It will generally be found more convenient to express qrs. and lbs. in the denomination of lbs. and take aliquot parts of the cwts., thus:

2 qrs. 15 lbs. = 65 lbs.

of which 50lbs. = ½ of a cwt., 10 lbs. = ⅕ of 50lbs., and 5lbs. = ½ of 10lbs.

137. The following are examples of Practice in Canada Currency:

Ex. 1. Find the value of 3825 things at £2. 17*s*. 4½*d*. each.

The operation is as follows:

10*s*. = ½ of £1	£3825. 0*s*. 0*d*.	=value at £1 each.
	2	
	7650. 0 . 0	=value at £2 each.
5*s*. = ½ of 10*s*.	1912.10 . 0	=value at 10*s*. each.
2*s*. = ⅕ of 10*s*.	956. 5 . 0	=value at 5*s*. each.
(∴ take ⅕ of £1912. 10*s*.)	382.10 . 0	=value at 2*s*. each.
4*d*. = ⅙ of 2*s*.	63.15 . 0	=value at 4*d*. each.
½*d*. = ⅛ of 4*d*.	7.19 . 4½	=value at ½*d*. each.
	£10972.19 . 4½	=value at £2. 17*s*. 4½*d*. each.

Note.—In the above Example the given number is expressed in the same denomination as the unit, whose value is given. This case is called *Simple* Practice.

Ex. 2. Find the value of 37 yds., 2 ft.; 7 in. of silk, at 5*s*. 3¼*d*. a yard.

The operation is as follows:

1 ft. = ⅓ of 1 yd.	£0. 5*s*. 3¼d.	= value of 1 yd.
	4	
	1 . 1 . 1	= value of 4 yds.
	9	
	9 . 9 . 9	= value of 36 yds.
	0 . 5 . 3¼	= value of 1 yd.
	9 .15 . 0¼	= value of 37 yds.
1 ft. = ⅓ of 1 yd.	0 . 1 . 9$\frac{1}{12}$	= value of 1 ft.
6 in. = ½ of 1 ft.	0 . 1 . 9$\frac{1}{12}$	= value of 1 ft.
1 in. = ⅙ of 6 in.	0 . 0 .10$\frac{13}{24}$	= value of 6 in.
	0 . 0 . 1$\frac{109}{144}$	= value of 1 in.
	£9. 19*s*. 6½*d*. $\frac{31}{36}$	=value of 37 yds. 2 ft. 7 in

Note.—In the above example the given number is not wholly expressed in the same denomination as the unit whose value is given. This case is called *Compound* Practice.

Ex. XLIX.

1. Find the value of 5 cwts., 2 qrs., 14 lbs., at \$75·50 per cwt.
2. Find the value of 33 cwt., 3 qrs., 7 lbs., at \$35·05 per cwt.
3. Find the value of 72 cwts., 3 qrs., 7 lbs., of sugar, at \$35·60 per cwt.
4. Find the value of 9 yds., 2 ft., 10 in., at \$1·75 per yard.
5. Find the value of 15 oz., 6 dwts., 17 grs., at \$1·25 per oz.
6. Find the value of 7 cwts., 1 qr., 15½ lbs., at \$12·37 per cwt.
7. What will, the rent of a farm, consisting of 196 acres, 3 roods, 30 poles, amount to at \$4·84 per acre?
8. What would be the cost of putting up 230 rods, 2¾ yards of fencing at \$0·75 per rod?
9. Find the cost of 18 tons, 16 cwt., 3 qrs., 16 lbs., at \$378·63 per ton.
10. Suppose a steamship require 45 tons, 15 cwts., 2 qrs. of coal per day; what would she cost a year for fuel, the coal being \$4·50 per ton?
11. Find the value of 48 sq. yds., 8 sq. ft., 114 sq. in., at \$3·33½ per sq. yd. and of 139 cords 25 feet of wood at \$3·25 per cord.
12. Find the cost of 157 things at £1. 18*s.* 6½*d.* each; and of 30 cwt., 2 qrs., 14 lbs., at £1. 17*s.* 8¼*d.*

SQUARE AND CUBIC MEASURE.
CROSS MULTIPLICATION, DUODECIMALS.

138. DEFINITIONS:

(1) A PARALLELOGRAM is a four-sided figure, of which the opposite sides are parallel.

(2) A RECTANGLE is a right-angled parallelogram.

(3) The AREA of a figure is the quantity of surface contained in it; and is estimated numerically by the number of times or parts of a time it contains a certain fixed area, which is assumed for its measuring unit.

(4) A SOLID is that which hath length, breadth, and thickness.

(5) The CAPACITY, or VOLUME of a solid, is the quantity of space, comprehending length, breadth, and thickness, which it contains or takes up.

(6) The word *Content* is also frequently used to denote length, area and capacity or volume; the length of a line being called its *linear content;* the area of a figure, its *superficial content;* and the capacity or volume of a body, or of a portion of space, comprehending length, breadth and thickness, its *solid content.*

(7) A PARALLELOPIPED is a solid contained by six quadrilateral figures, whereof every opposite two are parallel.

(8) A RECTANGULAR PARALLELOPIPED is one in which the several angles of the quadrilateral figures, which contain it, are right angles.

139. By reference to the Tables, Arts. (109, 110), and the observations upon them, we see that, in the sense there indicated, length multiplied by length, produces area, and area multiplied by length produces capacity; the units in the products in these cases differing in kind from the units in the factors; thus, a rectangular area, whose adjacent sides are 4 and 3 feet respectively, is divisible into (4 × 3) or 12 equal squares, as shewn by the accompanying figure, the length of a side of each square being one linear foot. The rectangular area in this case is said to be the product of the two adjacent sides, represented respectively by numbers, the units in the numerical product being no longer linear feet, but square feet.

	1	2	3	4
1	1	2	3	4
2	5	6	7	8
3	9	10	11	12

Similarly, if the adjacent edges of a rectangular parallelopiped be 3, 4, and 2 feet, respectively, the capacity of the solid is equivalent to 24 cubes, each containing one cubic foot; and thus the capacity of the parallelopiped is correctly expressed by the product of the three adjacent edges represented respectively by numbers, the units in the numerical product being no longer linear feet, as in the factors, but cubic feet.

Perhaps the readiest way of working examples in square and cubic measure is that of reducing all the different denominations to the same denomination; and proceeding as in the examples subjoined.

Ex. 1. Find the area of a rectangular court-yard, 17 ft. 6 in. long, and 13 ft. 4 in. broad.

$$\text{The area}=(17 \text{ ft. } 6 \text{ in.})\times(13 \text{ ft. } 4 \text{ in.})$$
$$=17\tfrac{1}{2} \text{ ft.}\times 13\tfrac{1}{3} \text{ ft.}$$
$$=\left(\frac{35}{2}\times\frac{40}{3}\right)\text{sq.ft.}$$
$$=\frac{700}{3}\text{sq.ft.}$$
$$=233\tfrac{1}{3} \text{ sq. ft.}$$
$$=25 \text{ sq. yds., } 8 \text{ sq. ft., } 48 \text{ sq. in.}$$

Ex. 2. Find the expense of paving a floor, whose length is 33 ft. 2 in., and breadth 18 ft., at \$1·50 per square yard.

$$\text{Area of floor}=(33 \text{ ft. } 2 \text{ in.})\times 18 \text{ ft.}$$
$$=33\tfrac{1}{6} \text{ ft.}\times 18 \text{ ft.}$$
$$=\left(\frac{199}{6}\times 18\right)\text{sq.ft.}$$
$$=\left(\frac{199}{6}\times\frac{18}{9}\right)\text{sq. yds.}$$
$$=\frac{199\times 2}{6}\text{sq. yds.}$$

Therefore cost, which=cost per yard × number of yards

$$\text{is}=\$\left(1\cdot 50\times\frac{199\times 2}{6}\right)$$
$$=\$99.50.$$

Ex. 3. How many square yards, feet and inches will remain out of 400 square feet of carpeting, after covering the floor of a room 21 ft. 9 in. long and 16 ft. 11 in. broad?

$$\text{Area of the floor}=(21\tfrac{3}{4}\times 16\tfrac{11}{12})\text{sq. ft.}$$
$$=\left(\frac{87}{4}\times\frac{203}{12}\right)\text{sq. ft.}$$
$$=\frac{29\times 203}{4\times 4}\text{ sq. ft.}=\frac{5887}{16}\text{sq. ft.}$$

Therefore the number of square feet of carpet remaining after covering the floor $=400-\dfrac{5887}{16}$

$$=\frac{6400-5887}{16}$$

$$=\frac{513}{16}$$

=32 sq. ft., 9 sq. in.

=3 sq. yds., 5 sq. ft., 9 sq. in.

Ex. 4. Find the capacity of a cube, of which each edge is 1 ft. 8 in.

Capacity = length × breadth × height,

$$=(1\tfrac{2}{3}\times 1\tfrac{2}{3}\times 1\tfrac{2}{3}) \text{ cubic ft.}$$

$$=\left(\frac{5}{3}\times\frac{5}{3}\times\frac{5}{3}\right) \text{ cub. ft.}$$

$$=\frac{125}{27} \text{cub. ft.}$$

=4 cub. ft., 1088 cub. in.

Ex. 5. A block of marble is 2 yds. 1 ft. 3 in. long, 1 ft. 7 in. broad, and 2 ft. thick; find its solid content, and its value at \$2·25 per cub. ft.

Its content = (2 yds. 1 ft. 3 in.) × (1 ft. 7 in.) × 2 ft.

$$=7\tfrac{1}{4}\text{ft.}\times 1\tfrac{7}{12}\text{ft.}\times 2 \text{ ft.}$$

$$=(7\tfrac{1}{4}\times 1\tfrac{7}{12}\times 2) \text{ cub. ft.}$$

$$=\left(\frac{29}{4}\times\frac{19}{12}\times 2\right) \text{ cub. ft.}$$

$$=\frac{29\times 19}{2\times 12}\text{cub. ft.}=22 \text{ cub. ft.}, 1656 \text{ cub. in.}$$

And \$2·25 = \$$2\tfrac{1}{4}$ = \$$\tfrac{9}{4}$

$$\text{Therefore its value}=\frac{9}{4}\left(2\tfrac{9}{4}\times\frac{29\times 19}{2\times 12}\right)=\frac{1653}{32}=\$51{\cdot}656\tfrac{1}{4}$$

Note 1. Since linear feet multiplied by linear feet give square feet, it follows that square feet divided by linear feet give linear feet. Similarly, square yards divided by linear yards give linear yards, and so on. Again, since linear feet multiplied by square feet give cubic feet, it follows that cubic feet divided by linear feet give square feet, and cubic feet divided by square feet give linear feet.

Note 2. When surface alone is concerned, length and breadth, or length and height, or breadth and height, have to be multiplied together; but never length and breadth and height to be multiplied together; the latter only takes place where solid content is required.

Ex. 6. Find the expense of carpeting a room 15 ft. 9 in. long, and 12 ft. 5 in. broad, with carpet $\frac{3}{4}$ yd. wide, at \$1·00 per yard.

$$\text{Area of floor} = (15\tfrac{3}{4} \times 12\tfrac{5}{12})\text{sq. ft.}$$

$$= \left(\frac{63}{4} \times \frac{149}{12} \times \frac{1}{9}\right)\text{sq. yds.}$$

$$= \left(\frac{7}{4} \times \frac{149}{12}\right)\text{sq. yds.}$$

It is clear that the required length of carpet in yards × given width of carpet in yards must give the whole area of floor in square yards;

$$\therefore \text{required length of carpet in linear yds} \times \frac{3}{4}\text{linear yd.} = \left(\frac{7}{4} \times \frac{149}{12}\right)\text{sq. yds.}$$

$$\therefore \frac{\text{required length of carpet in linear yds.} \times \frac{3}{4}\text{linear yd.}}{\frac{3}{4}\text{linear yd.}} = \frac{\left(\frac{7}{4} \times \frac{149}{12}\right)\text{sq. yds.}}{\frac{3}{4}\text{linear yd.}}$$

$$\text{or, required length of carpet in linear yds.} = \frac{7}{4} \times \frac{149}{12} \times \frac{4}{3} = \frac{7 \times 149}{12 \times 3}.$$

$$\therefore \text{cost of carpet} = \$\left(1 \times \frac{7 \times 149}{12 \times 3}\right) = \$\frac{1043}{36}.$$

$$= \$28\tfrac{35}{36}$$

Ex. 7. What must be the length of a beam, the end of which is 18 sq. in., in order that its solid content may be the same as that of another beam, whose width, depth, and length are respectively 4 ft. 6 in., 3 ft. 9 in., and 12 ft. 10 in.?

Content of 1st beam=(length of beam in linear ft. $\times 1\frac{1}{2} \times 1\frac{1}{2}$) cub. ft.

..........2nd....=$(4\frac{1}{2} \times 3\frac{3}{4} \times 12\frac{5}{6})$ cub. ft.

By the question,

∴ (length of beam in linear ft. $\times 1\frac{1}{2} \times 1\frac{1}{2}$) cub. ft.=$(4\frac{1}{2} \times 3\frac{3}{4} + 12\frac{5}{6})$ cub. ft. ;

$$\therefore \frac{\left(\text{length of beam in linear ft.} \times \frac{3}{2} \times \frac{3}{2}\right)\text{cub. ft.}}{\left(\frac{3}{2} \times \frac{3}{2}\right)\text{sq. ft.}} = \frac{\left(\frac{9}{2} \times \frac{15}{4} \times \frac{77}{6}\right)\text{cub. ft.}}{\left(\frac{3}{2} \times \frac{3}{2}\right)\text{sq. ft.}}$$

$$\text{or length of beam in linear ft.} = \frac{9}{2} \times \frac{15}{4} \times \frac{77}{6} \times \frac{2}{3} \times \frac{2}{3}$$

$$= \frac{385}{4} = 96\tfrac{1}{4} = 96 \text{ ft. } 3 \text{ in.}$$

32 yds. 0 ft. 3 in.

Ex. 8. Find the expense of painting the walls and ceiling of a room, whose height, length, and breadth are 17 ft. 6 in., 35 ft. 4 in., and 20 ft. respectively, at \$·30 per sq. yd.

The area of 2 of the walls=2 length × height,
..........of the other 2..=2 breadth × height,
..........of the ceiling =length × breadth.

Therefore whole area to be painted

=2 height × length+2 height × breadth+length × breadth
=2 height × (length+breadth)+length × breadth
=$2 \times 17\frac{1}{2}$ ft. $\times (35\frac{1}{3}+20)$ft.$+(35\frac{1}{3} \times 20)$sq. ft.

$$= \left(2 \times \frac{35}{2} \times 55\tfrac{1}{3} + \frac{106}{3} \times 20\right) \text{sq. ft.}$$

$$= \frac{7930}{3} \text{sq. ft.} = \left(\frac{7930}{3} \times \frac{1}{9}\right) \text{sq. yds.}$$

$$\text{therefore cost} = \$\left(\frac{3}{10} \times \frac{7930}{3} \times \frac{1}{6}\right) = \$\frac{793}{9}$$

$$= \$88\tfrac{1}{9}.$$

Ex. 9. Let it be required to find the expense of papering the walls of the above room with paper $\frac{5}{8}$yd. wide, at \$·50 per yard, there being three doorways in it, which each measure 7 ft. by $4\frac{1}{2}$ ft.

Now area of walls to be papered

$$=\left(2\times 17\tfrac{1}{2}\times 55\tfrac{1}{3}-3\times 7\times\frac{9}{2}\right)\text{ sq. ft.}$$

$$=\frac{11053}{6}\text{ sq. ft.}=\left(\frac{11053}{6}\times\frac{1}{9}\right)\text{ sq. yds.}$$

$$\therefore \text{ no. of linear yds. of paper required}\times\tfrac{5}{8}\text{ linear yd.}=\left(\frac{11053}{6}\times\frac{1}{9}\right)\text{ sq. yd.}$$

$$\therefore \text{ no. of linear yds. of paper required}=\frac{11053}{6}\times\frac{1}{9}\times\frac{8}{5}.$$

$$\text{Therefore cost}=\$\left(\frac{1}{2}\times\frac{11053}{6}\times\frac{1}{9}\times\frac{8}{5}\right)=\$\frac{22106}{135}$$

$$=\$163.74\tfrac{22}{27}.$$

140. It may be well to note that each of the foregoing Examples in square and cubic measure might have been worked by bringing all the denominations into the *lowest* denomination, or by reducing the lower denominations into decimals of the highest involved.

There is, however, a method of working examples in square and cubic measure without reducing the different denominations to the same denomination. This method is styled CROSS MULTIPLICATION OR DUODECIMALS, and it is generally employed by painters, bricklayers, &c., in measuring work. They take the dimensions of their work in feet, inches, parts, &c., decreasing from the left to the right in a twelve-fold proportion ; thus, 12 inches=1 foot, 12 parts =1 inch, &c. : the inches, parts, &c., are termed primes, seconds, thirds, &c., and are distinguished by the accents $'$, $''$, $'''$, &c., placed a little to the right above the numbers to which they belong.

The Rule for performing Cross Multiplication is the following :

Write the terms of the multiplier under the corresponding terms of the multiplicand. Multiply every term in the multiplicand, beginning at the lowest, by each term of the multiplier successively, beginning with the highest ; divide each product which is not of the denomination of feet by 12, add the quotient to the next product, and place the remainder under the term of the multiplicand just used, when the denomination of the multiplier is feet, one place removed to the right when it is primes, two places when it is seconds, three when it is thirds, &c. Add the products together, carrying 1 for every 12, and the sum will be the answer.

Ex. 1. Multiply 4 ft. 7 in. by 9 ft. 6 in.

Proceeding by the rule given above,

$$\begin{array}{r}
\text{ft.}\quad\quad\;\; \\
4 \cdot 7'\quad\;\; \\
9 \cdot 6'\quad\;\; \\
\hline
41 \cdot 3\quad\;\;\; \\
2 \cdot 3 \cdot 6'' \\
\hline
43 \cdot 6 \cdot 6''
\end{array}$$

which is the required product, and is $=43$ square feet $+\frac{6}{12}$ths of a square foot (or 6 *superficial primes*, as they are called) $+\frac{6}{12}$ths of a superficial prime, *i. e.* $\frac{6}{144}$ths of a square foot (or 6 *superficial* seconds as they are called).

We may express this product in square feet and inches, thus:

$$\left(43+\frac{6}{12}+\frac{6}{144}\right)\text{sq. ft.}=43\text{ sq. ft.}+\left(\frac{6\times12+6}{144}\right)\text{sq. ft.}$$

$$=43\text{ sq. ft.}+\frac{78}{144}\text{sq. ft.}$$

$$=43\text{ sq. ft. }78\text{ sq. in.}$$

Reason for the above process.

$$9\text{ ft.}\times4\text{ ft.}=36\text{ sq. ft.}$$

$$9\text{ ft.}\times7'=\left(9\times\frac{7}{12}\right)\text{sq.ft.}=\frac{63}{12}\text{sq.ft.}$$

$$=\left(\frac{60+3}{12}\right)\text{sq.ft.}$$

$$=5\text{ sq. ft.}+\frac{3}{12}\text{sq.ft.}$$

$$=5\text{ sq. ft.}+3\text{ superficial primes.}$$

$$6'\times4\text{ ft.}=\left(\frac{6}{12}\times4\right)\text{sq.ft.}=\frac{24}{12}\text{sq. ft.}=2\text{ sq. ft.};$$

$$6'\times7'=\left(\frac{6}{12}\times\frac{7}{12}\right)\text{sq.ft.}=\frac{42}{144}\text{sq.ft.}=\left(\frac{36+6}{144}\right)\text{sq.ft.}=\left(\frac{3}{12}+\frac{6}{144}\right)\text{sq.ft.}$$

$$=3\text{ superficial primes}+6\text{ superficial seconds.}$$

Now 36 sq. ft. + 5 sq. ft. + 3 superficial primes + 2 sq. ft. + 3 superficial primes + 6 superficial seconds

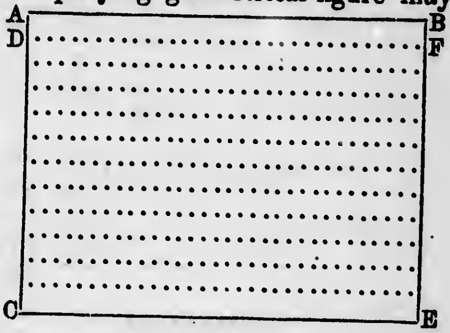
$$=43 \text{ sq. ft.} + 6 \text{ superficial primes} + 6 \text{ superficial seconds.}$$

$$=\left(43+\frac{6}{12}+\frac{6}{144}\right)\text{sq.ft.}$$

$$=\left(43+\frac{12\times6+6}{144}\right)\text{sq.ft.}$$

$$=43 \text{ sq. ft. } 78 \text{ sq. in.}$$

Note. Attention to the accompanying geometrical figure may perhaps explain more clearly the result obtained by multiplying 9 ft. by 7 primes.



Take $AB=9$ ft.,
$AC=7$ ft.,
$AD=7$ in.

Then 9 ft. × 7 ft., or rect. $AB, AC=$ rectangular figure $ACEB$, which contains 63 sq. ft., and 9 ft. × 7 primes, or rect. $AB, AD=$ rectangular figure $ADFB$, which is $\frac{1}{12}$th part of 63 sq. ft.

For since there are 12 lines in AC, each $=AD$, it follows that there are 12 rectangular figures, each $=ADFB$ in rectangular figure $ACEB$.

Ex. 2. Multiply 17 ft. 3 in. 6 pts. by 12 ft. 6 in. 3 pts.

$$\begin{array}{r}
\text{ft.} \\
17\ .\ 3'\ .\ 6'' \\
12\ .\ 6\ .\ 3 \\
\hline
207\ .\ 6\ .\ 0 \\
8\ .\ 7\ .\ 9'''\ .\ 0'''' \\
4\ .\ 3\ .\ 10'''\ .\ 6'''' \\
\hline
216\ .\ 6\ .\ 0\ .\ 10'''\ .\ 6''''
\end{array}$$

$$=216 \text{ sq. ft.} + \left(\frac{6}{12}+\frac{0}{144}+\frac{10}{12\times144}+\frac{6}{12\times12\times144}\right)\text{sq.ft.}$$

$$=216 \text{ sq. ft.} + \left(\frac{72}{144}+\frac{10}{12\times144}+\frac{6}{12\times12\times144}\right)\text{sq.ft.}$$

$$=216 \text{ sq. ft.} + 72 \text{ sq. in.} + \frac{10}{12} \text{ sq. in.} + \frac{6}{144} \text{sq. in.}$$

Ex. 3. Find by cross multiplication the capacity of a cube, whose edge is 2 ft. 8 in.; and prove the truth of the result by vulgar fractions.

```
 ft.
  2 . 8'
  2 . 8
 ---------
  5 . 4
  1 . 9 . 4"
 ---------
  7 . 1 . 4"
  2 . 8
 ---------
 14 . 2 . 8
  4 . 8 .10 . 8'"
 ---------------
 18 .11 . 6 . 8'"
```

$$=18 \text{ cub. ft.}+\left(\frac{11}{12}+\frac{6}{144}+\frac{8}{1728}\right)\text{cub. ft.}$$

$$=18 \text{ cub. ft.}+\frac{1584+72+8}{1728}\text{ cub. ft.}$$

$$=18\tfrac{1664}{1728} \text{ cub. ft.}$$

$$=18 \text{ cub. ft. } 1664 \text{ cub. in.}$$

Proof by vulgar fractions.

$$\text{Content}=(2\tfrac{2}{3}\times 2\tfrac{2}{3}\times 2\tfrac{2}{3}) \text{ cub. ft.}$$

$$=\left(\frac{8}{3}\times\frac{8}{3}\times\frac{8}{3}\right) \text{ cub. ft.}=\frac{512}{27}\text{cub. ft.}$$

$$=18 \text{ cub. ft. } 1664 \text{ cub. in.}$$

141. In the examples of Cross Multiplication we see that a decimal and duodecimal scale of notation is employed, the figures of the feet being expressed and multiplied in the ordinary way; whereas in other places the number 12 is always used instead of 10: Cross Multiplication is not, therefore, properly termed Duodecimal Multiplication or Duodecimals; because, although the different denominations are connected with each other by the number 12, still the different digits of those denominations are connected with each other by the number 10.

Ex. L.

1. Find the area of a rectangular board, whose sides are 2 ft. 9 in. and 10 ft. 4 in. respectively.

2. A room is 7 ft. 3 in. long, and 13 ft. 10 in. broad; find the area of the floor in feet and inches.

3. Find the number of square feet and inches in a rectangular piece of ground 9 ft. 3 in. by 3 ft. 5 in.

4. The floor of a room, which is $15\frac{1}{2}$ ft. wide, contains 91 sq. yards; find the length of the room.

5. A rectangular plot of ground 26 ft. broad, contains 92 sq. yds. 4 sq. ft.; find its length.

6. Find the breadth of a room, whose length is $22\frac{1}{2}$ ft. and whose area is $397\frac{1}{2}$ ft.

7. How many planks 12 ft. 6 in. long, and $8\frac{1}{2}$ in. wide, will floor a room 50 ft. by 16 ft.?

8. Find the area of a square building, whose side is 26 yds. 5 in.

9. An area, measuring 30 ft. 6 in. by 8 ft. 9 in., is to be paved; what will it cost at the rate of \$1·13 per square foot?

10. Find the cost of a slab 5 ft. 7 in. long, and 3 ft. 8 in. broad, at \$0·75 per sq. ft.

11. Find the area of a floor which measures 18 ft. 6 in. by 12 ft. 3 in., and the expense of carpeting it at \$0·80 per sq. yd.

12. What will be the expense of painting the surfaces, which measure respectively as follows?

(1) 23 ft. 6 in. by 20 ft., at \$1·25 per sq. yd.
(2) 14 ft. 3 in. by 11 ft. 11 in., at \$1·10 per sq. ft.
(3) 13 ft. 6 in. by 8 ft. 9 in., at \$2·05 per sq. yd.

13. Work by Cross Multiplication each of the following examples, and prove the truth of each result by Vulgar Fractions:

(1) 18 ft. 9 in. × by 14 ft. 7 in.
(2) 23 ft. 8 in. × by 16 ft. 9 in.
(3) 27 ft. 6′. 9″. × by 5 ft. 3′.
(4) 22 ft. $8\frac{3}{4}$ in. × 16 ft. $7\frac{1}{2}$ in.
(5) 4 ft. 6′. 5″. × by 9 ft. 4′. 7″.
(6) 75 ft. $7\frac{1}{3}$ in. × by 38 ft. $3\frac{1}{4}$ in.
(7) 5 yds. 2 ft. 2 in. 3 pts. × by 5 yds. 11 in. 7 pts.

14. How many yards of carpet $\frac{1}{2}$ yd. wide will cover a room 40 ft. 8 in. by 24 ft. 6 in.

15. What length of paper $\frac{3}{4}$ of a yard wide will be required to cover a wall 15 ft. 8 in. long by 11 ft. 3 in. high ?

16. Find the cost of a carpet $\frac{3}{4}$ yard wide at \$0·83$\frac{1}{2}$ a yard, for a room 20 feet by 18.

17. Find the expense of carpeting the following rooms :

(1) 12 ft. 4 in. long, and 12 ft. 6 in. broad, with carpet $\frac{3}{4}$ yd. wide, at \$2·75 a yard.
(2) 29$\frac{1}{2}$ ft. long, and 14$\frac{1}{4}$ ft. broad, with carpet $\frac{5}{8}$ yd. wide, at \$1·50 a yard.
(3) 15 ft. 6 in. long, and 12 ft. 9 in. broad, with carpet 24 in. wide, at \$1·85 a yard.
(4) 26$\frac{1}{2}$ ft. long, and 18 ft. broad, with carpet $\frac{3}{4}$ yd. broad, at \$0·87$\frac{1}{2}$ a yard.
(5) 19 ft. 7 in. long, and 18 ft. 11 in. broad, with carpet 25 in. broad, at \$1·25 a yard.

18. Find the content, and (when required) the cost, of the following :

(1) A piece of timber, whose length, breadth and thickness are respectively 54$\frac{1}{2}$ ft., 5 ft., and 2 ft. 5 inches, at \$0.03 a solid foot.
(2) A cube, whose edge is 1 ft. 8 in., at \$0·12 a solid inch.
(3) Digging a cubical cellar, whose length is 12 ft., at \$0·25 a solid yard.
(4) A cistern 6 ft. deep, having a square bottom, of which each side is 2$\frac{1}{2}$ ft.
(5) A wall 1000 ft. long, 10$\frac{1}{2}$ ft. high, and 2 ft. 1$\frac{1}{2}$ in. thick.
(6) A cube, whose edge is 13 ft. 7'. 7''.

19. Find the number of feet and inches in the floor, and the number of cubic feet and inches in the volume of a room 23 ft. 10 in. long, 18 ft. 4 in. broad, and 11 ft. 3 in. high.

20. Find the length of paper, $\frac{7}{8}$ths of a yard wide, required to cover the walls of a room, whose length is 27 ft. 5 in., breadth 14 ft. 7 in., and height 12 ft. 10 in.

21. What would be the cost of painting the four walls of a room, whose length is 24 ft. 3 in., breadth 15 ft. 8 in., and height 11 ft. 6 in., at \$1·10 a square foot ?

22. Find the expense of painting the walls and ceilings of each of the first two, and the walls of each of the last two of the following rooms:

(1) A room whose length is 16 ft. 8 in., breadth 15 ft. 9 in., and height 14 ft., at $0·30 a sq. yd.
(2) One whose length is 15 ft., breadth 10 ft., and height 9 ft. 9 in., at $0·37$\frac{1}{2}$ a sq. yd.
(3) One whose circuit is 41$\frac{1}{2}$ ft., and height 8 ft. 5 in., at $0·35 a sq. yd.
(4) One whose circuit is 72 ft., and height 10$\frac{1}{4}$ ft., at $0·25 a sq. yd.

And find also the expense of papering the walls of the first two of the above rooms with paper 1 ft. 9 in. wide, at the following prices—the first at $1.00 a yard, and the second at $0·37$\frac{1}{2}$ a yard.

23. The length, breadth and height of a room are 7 yds. 1 ft. 3 in., 5 yds, 2 ft. 9 in., and 4 yds. 6 in., respectively. What length of paper two feet broad will be required to cover the walls, and what will it cost at $0·05 per yard?

24. Supposing the cost of a carpet in a room 25 feet long, at $1.50 a square yard, to be $37·50, determine the breadth of the room.

25. In a rectangular court, which measures 96 ft. by 84 ft., there are four rectangular grass plots, measuring each 22$\frac{1}{2}$ ft. by 18 ft.; find the cost of paving the remaining part of the court at $0·17 per square yard.

26. If a piece of cloth be 94$\frac{1}{2}$ yds. long, and 1$\frac{1}{8}$ yds. broad, how broad is a piece of the same content whose length is 74$\frac{2}{3}$ yds.?

27. How many square feet and square inches remain out of 313 sq. ft. of carpeting, after covering a room 16 ft. 9 in. by 12 ft. 11 in? What is the price of the requisite carpeting at $0·75 a yard?

28. On laying down a bowling-green with sods 2 ft. 6 in. long by 9 in. wide, it is found that it requires 75 sods to form one strip extending the whole length of the green, and that a man can lay down one strip and a quarter each day: find the space laid down in 8 days.

29. A piece of land, whose length is 151 yds. 1$\frac{1}{4}$ ft., and breadth 35 yds., is to be exchanged for part of a strip of land of the same

quality, whose breadth is 15 yds. $2\frac{1}{2}$ ft. Find the length of the equivalent strip.

30. Find the difference between the content of a floor 80 ft. 9 in. long and 65 ft. 6 in. broad, and the sum of the contents of three others, the dimensions of each of which are exactly one-third of those of the other.

31. A reservoir is 24 ft. 8 in. long by 12 ft. 9 in. wide; how many cubic feet of water must be drawn off to make the surface sink 1 foot.

32. Divide 1532 ft. $9\frac{8}{12}'$ in. by 81 ft. 9 in.: and find the breadth of a room, the length of which is $17\frac{1}{2}$ ft., and the area $250\frac{5}{6}$ ft.

33. How many sq. ft. of glazing are contained in the windows of a house of 4 stories, each story containing 12 windows, the breadth of each window being 3 ft. 6 in.; the height of the windows on the ground and first floors being $7\frac{1}{2}$ ft., on the second floor 6 ft. 10 in., and on the third floor 6 ft.? What will the cost be at $0·20 a sq. ft.?

34. How many bricks will be required to build a wall 20 yds. long, $7\frac{1}{2}$ ft. high, and 14 in. deep; supposing a brick to be 9 in. long, $3\frac{1}{2}$ in. broad, and $2\frac{1}{2}$ in. deep?

35. How many tons of water are there in a cistern 18 ft. 8 in. long, 18 ft. 4 in. broad, and 6 ft. 9 in. deep, supposing a cubic foot of water to weigh 1000 oz.?

36. How many rods of brickwork are there in a wall 77 ft. long, 16 ft. high, and 1 ft. $10\frac{1}{2}$ in. thick?

37. Find the expense of painting the outside of a cubical iron chest, whose edge is 2 ft. 5 in. at $0·33 per sq. yd.

38. What will the painting of a room cost which is $20\frac{1}{4}$ ft. long, $18\frac{1}{2}$ ft. broad, and 10 ft. high, containing 2 windows whose dimensions are 7 ft. by 4 ft. each, at the rate of $0·50 per sq. yd.?

39. A piece of cloth five times as long as broad cost $19·50: supposing the price of cloth to be $1.00 a square yard, find the dimensions of the piece.

40. What length must be cut off a straight plank $1\frac{1}{3}$ ft. broad, and $\frac{3}{4}$ ft. deep, in order that it may contain $11\frac{1}{4}$ cubic ft.

41. A Turkey carpet, measuring 11 ft. 6 in. by 9 ft. 8 in., is laid down on the floor of a room measuring 14 ft., by 12 ft. 6 in.;

determine the quantity of Brussels carpet, $\frac{3}{4}$ yd. wide, which will be required to complete the covering of the area; what will be the cost of it at $1·37$\frac{1}{2}$ a yard?

42. Shew by Cross Multiplication and by Vulgar Fractions how many cubic feet are contained in a beam 20 ft. 4 in. long, 1 ft. 5 in. broad, and 10 in. thick.

43. If 69 yds. of carpet, $\frac{3}{4}$ yd. wide, cover a room which is $10\frac{1}{2}$ yds. long, find the width of the room.

44. If a postage stamp be an inch long and $\frac{7}{8}$ths of an inch broad, how many stamps will be required for papering a room 16 ft. 10 in. long, 15 ft. 9 in. broad, and 12 ft. 6 in. high?

45. The length, width, and height of a room are respectively 36 ft., 24 ft., and 20 ft.; how many yards of painting are there in the walls of it, deducting for a fire-place 6 ft. by $5\frac{1}{2}$ ft., and two windows, each $7\frac{1}{2}$ ft. by $3\frac{3}{4}$ ft.?

What would it cost to paper the above room with paper $2\frac{1}{4}$ ft. wide, at $0·28 a yard?

46. How many bricks, each 9 in. long, $4\frac{1}{2}$ in. wide, and 3 in. thick, will be required for a wall 100 yds. long, 15 ft. high, and 1 ft. $10\frac{1}{2}$ in. thick?

47. A gentleman has a garden 200 ft. long and 180 ft. broad, and a gravel walk is to be made to run lengthways across it; how wide must the path be so as to take up $\frac{1}{8}$th of the garden?

48. A wall is to be built 15 yds. long, 7 ft. high and 13 in. thick, containing a doorway 6 ft. high and 4 ft. wide. How many bricks will it require, the solid content of a brick being 108 cubic inches?

49. What would be the cost of paving a road of a uniform breadth of 4 yards, extending round a rectangular piece of ground, the length of which is 85 yds., and breadth 56 yds., the cost of paving a square yard being $0·45?

50. How many paving stones, each of them a foot long, and $\frac{4}{15}$ of a foot wide, will be required for paving a street 45 feet wide, surrounding a square, the side of which is 225 ft.?

51. What will be the expense of paving a rectangular court-yard, whose length is 126 ft. and breadth 98 ft., with pebbles, at $0·20 per sq. yd.; and by how much will the expense be increased if a granite path, $5\frac{1}{2}$ ft. wide, at $3·75 per sq. yd., be laid down all round between the outside walls and the pebbles?

52. A gentleman wishes to raise his lawn (which is 1902 feet long, and 1020 feet broad) 2 ft., and for that purpose digs a moat round it 17 yds. broad in every part; supposing the depth of the moat to be uniform, how deep must it be in order that he may have soil sufficient for his purpose?

53. Find the expense of lining a cistern, 10 ft. 3 in. long, 6 ft. 6 in. broad, and 5 ft. $4\frac{1}{2}$ in. deep, with lead, at \$11.50 a cwt., which weighs 8 lbs. per sq. ft.

54. How many imperial gallons will a cistern contain whose length, depth and breadth are 7 ft. 3 in., 3 ft. 8 in., and 2 ft. 10 in. respectively?

142. Examples which are usually classed under particular Rules, such as the Rule of Three, &c., can nevertheless be readily solved independently, by means of the foregoing principles.

The following examples, which are worked out, are intended to exemplify various methods of reasoning. In the examples for practice which follow them, questions will be found the solution of which may be easily arrived at in a similar way; the number of such questions in this place must necessarily be very limited, and therefore the student is strongly recommended to apply to all questions which are hereafter classed under particular Rules, an independent method of solution, as well as the one denoted by the Rule to which they are respectively affixed.

Ex. 1. Express a degree ($69\frac{1}{2}$m.) in metres, 32 metres being = 35 yds.

$$35 \text{ yards} = 32 \text{ metres}.$$

$$\therefore 1 \text{ yard} = \frac{32}{35} \text{ metres};$$

$$\therefore 1 \text{ degree} = (69\tfrac{1}{2} \times 1760) \text{ yards} = (139 \times 880) \text{ yards},$$

$$= \left(\frac{139 \times 880 \times 32}{35}\right) \text{ metres} = 111835\tfrac{3}{7} \text{ metres}.$$

Ex. 2. If $\frac{2}{3}$rds of a lottery ticket be worth \$220, what is the value of $\frac{3}{11}$ths of the same?

$$\therefore \tfrac{2}{3}\text{rds of the ticket} = \$220.$$

$$\therefore \tfrac{1}{3}\text{rd of the ticket} = \$110.$$

$$\therefore \text{whole ticket} = \$110 \times 3 = \$330.$$

$$\therefore \tfrac{3}{11}\text{ths of the ticket} = \tfrac{3}{11}\text{ths of } \$330 = \$\frac{330 \times 3}{11} = \$90.$$

Ex. 3. A person has $\frac{3}{7}$ths of an estate of 4000 acres left him; he sells $\frac{2}{3}$rds of his share: how many acres has he remaining, and what fraction of the whole estate will they be?

$$\text{He sells } \frac{2}{3} \text{ of } \frac{3}{7} \text{ of } 4000 \text{ acres, or } \frac{2}{7} \text{ of } 4000 \text{ acres,}$$

$$\therefore \text{ he has remaining } \left(\frac{3}{7} \text{ of } 4000 - \frac{2}{7} \text{ of } 4000\right) \text{ acres}$$

$$= \frac{1}{7} \text{ of } 4000 \text{ acres} = 571\tfrac{3}{7} \text{ acres.}$$

Ex. 4. The sum of \$463·80 is to be raised in a township, the assessment of which is \$6184; what is the rate in the \$?

$$\$6184 \text{ produce } \$463\tfrac{4}{5} \text{ or } \$\frac{2319}{5},$$

$$\because \$1 \text{ produces } \$\left(\frac{2319}{5} \times \frac{1}{6184}\right), \text{ or } \$\cdot075.$$

Ex. 5. A met two beggars, B and C; and having $\frac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of \$$3\frac{6}{7}$ in his pocket, gave B $\frac{1}{7}$ of $\frac{3}{4}$ of that sum, and C $\frac{3}{5}$ of the remainder; what did each receive?

$$A \text{ had at first } \frac{\frac{40}{11}}{\frac{30}{7}} \text{ of } \frac{\frac{75}{7}}{\frac{15}{2}} \text{ of } \frac{77}{540} \text{ of } \$\frac{27}{7}, \text{ or } \$\frac{2}{3}$$

$$B \text{ received } \frac{1}{7} \text{ of } \frac{3}{4} \text{ of } \$\frac{2}{3}, \text{ or } \$1\frac{1}{4} \text{ or } \$\cdot07\tfrac{1}{7}$$

$$A \text{ had left afterwards } \$\left(\frac{2}{3} - \frac{1}{14}\right) = \$\frac{25}{42} = \$\cdot595\tfrac{5}{21}$$

$$\therefore C \text{ received } \frac{3}{5} \text{ of } \$\frac{25}{42}, \text{ or } \$\frac{15}{42} \text{ or } \$\cdot35\tfrac{5}{7},$$

Ex. 6. If \$4·87 be worth 25 francs, 60 centimes; and also worth 6 thalers, 20 silber groschen; how many francs and centimes is a thaler worth? (One thaler=30 silber groschen, 1 franc=100 centimes.)

6 thalers, 20 silber groschen=25 francs, 60 centimes,

$$\text{or } 6\tfrac{20}{30} \text{ thalers}=25\tfrac{60}{100} \text{ francs,}$$

$$1 \text{ thaler}=(25\tfrac{3}{5}\div 6\tfrac{2}{3}) \text{ francs}$$

$$=\frac{384}{100}\text{francs}=3 \text{ francs, } 84 \text{ centimes.}$$

Ex. 7. Standard gold contains 12 parts of pure gold to one part of copper, and 20lbs. Troy are coined into 934 sovereigns and a half-sovereign; find the weight of pure gold in a sovereign.

Number of parts=12+1=13, of which $\frac{12}{13}$ is pure gold.

By the question,

$$934\tfrac{1}{2} \text{ sovereigns weigh } 20 \text{ lbs. Troy,}$$

$$\therefore 1 \text{ sov. weighs} \frac{20\times 2}{1869} \text{lbs. Troy,}$$

$$\therefore \text{weight of pure gold in a sov.}=\left(\frac{12}{13}\times\frac{20\times 2}{1869}\right) \text{lb. Troy}$$

$$=113\tfrac{6413}{8099} \text{ grs.}$$

Ex. 8. If a person, travelling $13\frac{3}{4}$ hours a day, perform a journey in $27\frac{5}{8}$ days, in what length of time will he perform the same if he travel $10\frac{2}{7}$ hours a day?

If he travel $13\frac{3}{4}$ hrs. a day, he does the journey in $27\frac{5}{8}$ days,

.............1 hr. ......, ...$(27\frac{5}{8}\times 13\frac{3}{4})$days,

.............$10\frac{2}{7}$ hrs. ....., ... $\dfrac{27\frac{5}{8}\times 13\frac{3}{4}}{10\frac{2}{7}}$ days,

which worked out, gives $36\frac{2141}{2304}$ days.

Ex. 9. If 858 men in 6 months consume 234 bus. of wheat, how many bushels will be required for the consumption of 979 men for three months and a half?

858 men in 6 months consume 234 bushels,

$\therefore$ 1 man in 1 month consumes $\dfrac{234}{858 \times 6}$ bus.,

$\therefore$ 979 men in 1 month consume $\dfrac{979 \times 234}{858 \times 6}$ bus.,

$\therefore$ 979 men in $3\frac{1}{2}$ months consume $\left(\dfrac{979 \times 234}{858 \times 6} \times \dfrac{7}{2}\right)$ bus. or $155\frac{3}{4}$ bus.

Ex. 10. If 5 men or 7 women can do a piece of work in 37 days; in what time will 7 men and 5 women do a piece of work twice as great?

5 men = 7 women,

$\therefore$ 1 man $= \dfrac{7}{5}$ woman,

$\therefore$ 7 men $= \dfrac{49}{5}$ women,

$\therefore$ 7 men and 5 women $= \left(\dfrac{49}{5} + 5\right)$ women $= \dfrac{74}{5}$ women.

Now by the question,

7 women in 37 days do the piece of work,

$\therefore$ 1 woman in (37×7) days does ....

$\therefore$ 74 women in $\dfrac{37 \times 7}{74}$ days do ....

$\therefore$ $\dfrac{74}{5}$ women in $\dfrac{37 \times 7 \times 5}{74}$ days do ....

$\therefore$ $\dfrac{74}{5}$ women in $\dfrac{37 \times 7 \times 5 \times 2}{74}$ or in 35 days do twice as much.

Ex. 11. A bankrupt owes three creditors *A*, *B*, and *C* \$250, \$330, and \$420 respectively, and his property is worth \$125; how much will each creditor receive, and how many cents in the dollar?

Debts amount to \$(250+330+420), or \$1000.

If the bankrupt has \$1, he pays $\dfrac{1}{1000}$ part of his debt,

..................\$125...... $\dfrac{125}{1000}$ part of debt.

...... $\dfrac{1}{8}$ part of debt.

∴ A gets \$31.25., B gets \$41.25., and C gets \$52.50. He pays $\frac{1}{8}$ of \$1., or \$·125., in the dollar.

Ex. 12. Gunpowder being composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts; find how much of each is required for 16 cwt. of powder.

The whole number of parts $=(15+3+2)=20$.

Of every 20 parts,

$$\frac{15}{20} \text{ or } \frac{3}{4} \text{ is nitre, } \frac{3}{20} \text{ is charcoal, } \frac{2}{20} \text{ or } \frac{1}{10} \text{ is sulphur.}$$

$\therefore \frac{3}{4}$ of 16 cwt., or 12 cwt. = quantity of nitre required.

$\frac{3}{20}$ of 16 cwt., or $2\frac{2}{5}$ cwt. = ... charcoal ......

$\frac{1}{10}$ of 16 cwt., or $1\frac{3}{5}$ cwt. = ... sulphur .....

Ex. 13. Of a certain dynasty, $\frac{1}{3}$ of the kings are of the same name, $\frac{1}{4}$ of another, $\frac{1}{8}$ of a third, and $\frac{1}{12}$ of a fourth, and there are 5 besides: how many are there of each name?

Representing the whole dynasty by unity, or 1.

$\frac{1}{3}$ = number of kings of one name,

$\frac{1}{4}$ = ... of a second ....

$\frac{1}{8}$ = ... of a third ....

$\frac{1}{12}$ = ... of a fourth ....

$$\text{Now } \frac{1}{3}+\frac{1}{4}+\frac{1}{8}+\frac{1}{12}=\frac{19}{24},$$

$\therefore$ whole dynasty $-\frac{19}{24}$, or $1-\frac{19}{24}$, or $\frac{5}{24}$ = no. of remaining kings in it.

But by the question,

$$\frac{5}{24} \text{ of unity, or } \frac{5}{24} \text{ of the whole dynasty} = 5\,;$$

$$\therefore 1, \text{ or the whole dynasty}, = 5 \times \frac{24}{5} = 24\,;$$

$\therefore$ there are 8 kings of the 1st name, 6 of the 2nd, 3 of the 3rd, and 2 of the 4th.

Ex. 14. *A* can do a piece of work in 5 days, *B* can do it in 6 days, and *C* can do it in 7 days; in what time will *A*, *B*, and *C*, all working at it, finish the work? Find also in what time *A* and *B* working together, *A* and *C* together, and *B* and *C* together, could respectively finish it.

Representing the work by unity or 1.

In one day *A* does $\frac{1}{5}$ part of the work,

.... *B* does $\frac{1}{6}$ .... ;

.... *C* does $\frac{1}{7}$ .... ;

$\therefore$ .... $A+B+C$ do $\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)$, or $\frac{107}{210}$ part;

$\therefore$ time in which $A+B+C$ would finish the work

$$= \frac{1}{\frac{107}{210}} \text{ days} = \frac{210}{107} \text{ days} = 1\tfrac{103}{107} \text{ days}.$$

Again, in one day, $A+B$ do $\left(\frac{1}{5}+\frac{1}{6}\right)$, or $\frac{11}{30}$ of the work;

therefore time in which they would finish it $= \frac{1}{\frac{11}{30}}$ or $2\frac{8}{11}$ days.

In like manner, it may be shewn that *A* and *C* would finish the work in $2\frac{11}{12}$ days; and *B* and *C* in $3\frac{3}{13}$ days.

Ex. 15. It being given that A and B can do a piece of work in $2\frac{8}{11}$ days; that A and C can do the same in $2\frac{11}{12}$ days; and that B and C can do it in $3\frac{3}{13}$ days: find the time in which A, B and C would do the work: working, first, all together, secondly, separately.

In one day A and B do $\frac{11}{30}$ of the work,

.... A and C do $\frac{12}{35}$ ....

.... B and C do $\frac{13}{42}$ ....

$\therefore$ by addition,

In one day $2A+2B+2C$ would do $\left(\frac{11}{30}+\frac{12}{35}+\frac{13}{42}\right)$, or $\frac{214}{210}$ of the work,

$\therefore$ in one day $A+B+C$ do $\frac{107}{210}$ ....

$$\therefore \text{ time required} = \frac{1}{\frac{107}{210}} = \frac{210}{107} \text{ days} = 1\tfrac{103}{107} \text{ days.}$$

Again,

work done by $A+B+C$ in one day — work done by $B+C$ in one day,

or, work done by A in one day $= \frac{107}{210} - \frac{13}{42} = \frac{1}{5}$;

therefore time required, in which A would do the work $=5$ days.

In like manner it may be shewn, that B would do the work in 6 days, and that C would do it in 7 days.

Ex. 16. A cistern is fed by a spout which can fill it in 2 hours, how long would it take to fill it if the cistern has a leak which would empty it in 10 hours?

In one hour spout fills $\frac{1}{2}$ of the cistern,

.. ...leak empties $\frac{1}{10}$ ....

Therefore in one hour, when the spout and leak are both open, the part of the cistern filled by what runs in—what runs out,

$$=\left(\frac{1}{2}-\frac{1}{10}\right)=\frac{2}{5},$$

$\therefore$ time required for filling the cistern $=\dfrac{1}{\frac{2}{5}}$ hrs. $=\dfrac{5}{2}$ hrs. $=2\frac{1}{2}$ hrs.

Ex. 17. *A* can perform a certain quantity of work in 5 days, *B* twice as much in 6 days, and *C* four times as much in 9 days; in what time can *A*, *B* and *C*, working together, perform a piece of work 11 times as great?

In one day *A* does $\frac{1}{5}$ of the work,

.... *B* does $\frac{2}{6}$ or $\frac{1}{3}$ ....

.... *C* does $\frac{4}{9}$ ....

$\therefore$ in one day $A+B+C$ do $\left(\frac{1}{5}+\frac{1}{3}+\frac{4}{9}\right)$ or $\frac{44}{45}$ of the work,

$\therefore$ they would finish this piece of work in $\frac{45}{44}$ days,

$\therefore$ they would finish required piece of work in $\left(\frac{45}{44}\times 11\right)$ or $11\frac{1}{4}$ dys.

Ex. 18. *A* and *B* can do a piece of work in 15 and 18 days respectively; they work together at it for 3 days, when *B* leaves, but *A* continues, and after 3 days is joined by *C*, and they finish it together in 4 days; in what time would *C* do the piece of work by himself?

Representing the work by unity, or 1.

In one day $A+B$ do $\left(\frac{1}{15}+\frac{1}{18}\right)$ of the work,

in 3 days they do $\left(\frac{1}{15}+\frac{1}{18}\right)\times 3$ ....

or $\frac{11}{30}$ ....

$$\therefore \frac{19}{30} \text{ of the work remains to be done.}$$

In 3 days more A does $\frac{3}{15}$ or $\frac{1}{5}$ of the work;

$\therefore$ when A is joined by C,

$$\frac{19}{30}-\frac{1}{5}, \text{ or } \frac{13}{30} \text{ of the work remains to be done.}$$

In 4 days more A does $\frac{4}{15}$ of the work;

$\therefore$ work which has to be done by C in 4 days

$$=\frac{13}{30}-\frac{4}{15}=\frac{5}{30}=\frac{1}{6};$$

$$\therefore \text{part of work to be done by } C \text{ in one day} = \frac{1}{24},$$

$\therefore$ time in which C would do the whole work $=24$ days.

Ex. LI.

Miscellaneous Questions and Examples on preceding Articles.

I.

1. State the rules for the multiplication and division of decimals, and divide 34·17 by $3\frac{1}{4}$.

2. What is the value in English money of 1556·85 francs when the exchange is at 24·25 francs per £?

3. Reduce $\frac{1}{3}+\frac{1}{6}+\frac{1}{14}+\frac{3}{56}$ to a decimal fraction. What decimal of a cwt. is 1 qr. 7 lbs.?

4. Explain the principle of the Rule of Practice. Find (by Practice) the cost of 365 tons 13 cwt. 3 qrs., of coal at $6·75 a ton; and the rent of 315 ac., 3 ro., 7 po, at £1. 16*s.* 8*d.* Canada currency, an acre.

5. If $\frac{4}{7}$ of an estate be worth $11564, what is the value of $\frac{3}{4}$ of it?

6. If a bankrupt pay $0·17 in the dollar, what will be received on a debt of $$7265\frac{3}{4}$?

7. A person possessing $\frac{3}{14}$ of an estate, sold $\frac{2}{7}$ of $\frac{1}{3\frac{1}{5}}$ of his share for \$120$\frac{5}{8}$; what would $\frac{1}{5}$ of $\frac{3}{16}$ of the estate sell for at the same rate ?

8. A man, his wife, and 3 children earn \$19·25 a week; the wife earns twice as much as each child, and the man three times as much as his wife; required the man's weekly earnings.

9. If \$5 be worth 12 florins, and also worth 25 francs, 56 centimes; how many francs and centimes is one florin worth ? (100 centimes=1 franc.)

10. The wages of 5 men for 6 weeks being \$172·50, how many weeks will 4 men work for \$276.

11. Define a square, a cube ; shew clearly by a figure, how many cubic feet there are in a cubic yard. Reduce 4203239040 cnb. in. to cub. yds.; and find how many grains of wheat there are in a load, if a pint contains 7000 grains.

12. A merchant bought 125 hhds. 30·5 gals. 3 qts. of sugar for \$1585·12$\frac{1}{2}$; and sold $\frac{3}{5}$ of it for \$21·75 a hogshead; and the remainder at \$28·93$\frac{3}{4}$ a hogshead. How much did he gain by the operation ?

II

1. What is meant by saying that one sum is a certain fraction (for example $\frac{2}{5}$) of another ? If 26 francs are equivalent to a pound sterling, what fraction of a shilling is a franc ? Give the reasons for the process which you adopt in answering the question.

2. Express $\frac{2}{3}$ of 1$\frac{5}{9}$ of a mile in terms of a metre, supposing 32 metres=35 yards.

3. *A*, *B*, and *C* rent a pasture for \$200. *A* puts in 8 cattle, *B*, 9, and *C*, 11 : how much should each pay for his share ?

4. Reduce 3$\frac{3}{4}$ lbs. to the decimal of 10 cwts. and divide the result by 12·5. Explain the process employed.

5. Find the value of 45 ac., 3 ro., 20 po., at \$111·75 per acre, by Practice, and prove the result decimally.

6. If the property in a town be assessed at \$60000, what must be the rate in the \$ in order that \$2500 may be raised ?

7. If the circumference of a circle=Diameter × 3·14159; find the number of revolutions passed over by a carriage-wheel, 5 ft. in diameter, in 10 miles.

8. A farmer has to pay yearly to his landlord the price of $7\frac{1}{8}$ bushels of wheat at \$1·10 per bushel, and $9\frac{1}{4}$ of malt at \$1·75, and $6\frac{2}{5}$ of oats at \$·38. What is the whole amount of his rent?

Reduce the answer to sterling money.

9. *A* alone can do a piece of work in 10 hours, and *B* can do it in 12 hours, find the time in which both working together can do it.

10. Ten excavators dig 12 loads of earth in 16 hours, whilst 12 others can dig only 9 loads in 15 hours; in what time will they jointly dig 100 loads?

11. Reduce \$100·20 to Canada, and to the different currencies of the United States; also, £75 15*s.* 6*d.* of the respective currencies to dollars and cents.

12. Find the value of 5 acres of land, at $100\frac{1}{2}$ guineas per acre; of 97 bushels of wheat at 5*s.* $9\frac{1}{2}$*d.* per bush.; of 365 lbs. of spices at 14*s.* $8\frac{3}{4}$*d.* per lb. Work out each sum by Compound Multiplication, and prove the result by Practice. Also, find the sum of the answers, and express it in decimal currency, allowing \$4·87 to the £. sterling.

III.

1. Divide 28 tons, 4 cwt., 3 qrs. into 36 equal portions; and find the value of one of them at \$35·$37\frac{1}{2}$ per cwt.

2. Reduce 186 yds., 2 ft., $8\frac{1}{25}$ in. to the decimal of a chain. If one chain=10 chainlets=100 links=1000 linklets; express the above in chains, chainlets, links, linklets.

3. If I bottle off two-thirds of 2 pipes of wine into quarts, and the rest into pints, how many dozens of each shall I have?

4. If the rents of a township amount to \$2514, and a school rate is granted of \$$83\frac{1}{2}$, how much is that in the \$?

5. If a tradesman, with a capital of \$1000 gains \$90 in 7 months, in what time will he gain \$$20\frac{1}{4}$ with a capital of \$315?

6. What is the difference between simple and compound Practice?

Required the price of [illegible] cwt., 3 qrs. 16 lbs. at £4. 6*s.* $0\frac{1}{2}$*d.* per quarter, by practice.

7. Determine the expense of papering a room 12 ft. high, measuring 20 ft. by 15 ft., at the rate of \$·05 per square yard.

8. In the civil year, 97 days are intercalated into 400 years; what is the average length of the year?

9. If 15 horses, and 148 sheep can be kept 9 days for \$75, what sum will keep 10 horses and 132 sheep for 8 days, supposing 5 horses to eat as much as 84 sheep?

10. *A*, *B*, and *C* are three workmen: *A* can do half a piece of work in 3 hours, being twice as much as *B* can do; and *A*, *B*, and *C* can together do the whole in $2\frac{1}{2}$ hours. Shew that *C* can do in 5 hours as much as *B* can do in 9 hours.

11. Find the sum and difference of 7 tons, 13 cwts. 2 qrs. 15 lbs. 8oz., and 9 tons, 3 cwts. and divide the sum by 95.

12. What are the different uses to which Troy weight and Avoirdupois weight are respectively applied? Express 56 lbs. Avoirdupois in lbs. &c., Troy.

IV.

1. Explain how whole numbers are represented in the decimal or common system of notation. Multiply 729 by 37, and explain the process.

2. Add together the fifth of a shilling, two-sevenths of a crown, and four-ninths of a guinea; and reduce the result to the decimal of £25.

3. Two persons gained in trade \$375; one having put in \$500 and the other \$850; what part of the profit ought each person to receive?

4. Taking the circumference of a circle at $3\frac{1}{7}$ times its diameter, find the cost of a marble column of two feet breadth and five yards height, marble being $\$5\frac{1}{2}$ per cub. ft. (Area of circle = $\frac{1}{2}$ circumference × semi-diameter.)

5. If a certain number of men can throw up an entrenchment in 12 days, when the day is 6 hours long, in what time will they do it when the day is 8 hours long?

6. Bought 3 loads of wood: the first was 8 ft. long, 4 ft. wide, and 3 ft. high; the second 7 ft. long, 4 ft. wide, and 2 ft. high; the third was 9 ft. long, 3 ft. wide, and 3 ft. high. How many solid ft. in the whole? How many cord ft., and how many cords?

7. Reduce $\$2375\frac{3}{4}$ Swedish dollars to dollars and cents, the exchange being at \$1·06 per dollar. And find the value of 1,000,000 rupees at $\$53\frac{1}{2}$ each.

8. The roller used for rolling a bowling-green, being 6 ft. 6 in. in circumference, by 2 ft. 3 in. wide, is observed to make 12

revolutions as it rolls from one extremity of the green to the other; find the area rolled when the roller has passed 10 times the whole length of it.

9. Divide \$1400 among *A*, *B* and *C*, in such a manner that as often as *A* gets \$5, *B* shall get \$4, and as often as *B* gets \$3, *C* shall get \$2.

10. A fraudulent wine-merchant sells as brandy a mixture of brandy and rum at \$10½ a gallon, which is the proper price of his brandy, that of his rum being \$5¼ a gallon. Supposing one-third of the whole mixture to be rum, ascertain how much a gallon he gains by his dishonesty.

11. How many revolutions will a wheel, which is 4 yards in circumference, make in 3 miles?

12. Find the difference between £1000 Canada currency, and £1000 Georgia currency; also between £596½ sterling and £596½ Canada currency. Express the answers in dollars and cents.

V.

1. Divide 550974 by 1472; find the quotient and remainder. Explain the operation, and prove the result.

2. Shew that the value of a fraction is not altered by multiplying the numerator and denominator by the same number.

3. Express the fractions $\frac{6}{45}$, $\frac{5}{7}$, and $\frac{9}{35}$ by corresponding fractions having the same denominator, and find the sum.

4. Eight bushels of wheat are consumed annually by each person in Canada; if wheat be at \$1·25 a bushel, and the population 2,500,000, what is the value of a quarter of a year's consumption?

5. A certain number of men mow 4 acres of grass in 3 hours; and a certain number of others mow 8 acres in 5 hours: how long will they be mowing 11 acres, if all work together?

6. The depth of water in a cistern whose base contains 1344 sq. in. is 3 ft. 9 in. Find (1) the number of cub. ft. of water in it, and (2) the depth of the same quantity of water in another cistern whose base contains 1088 square inches.

7. If a man can do a piece of work in 8½ days by working 6 hours a day, how many hours a day must he work to finish it in 5 days?

8. If 7 men or 11 women can finish a piece of work in 17 days, how many days will it take 11 men and 7 women to finish it?

9. A room is 20 ft. 6 in. long by 15 ft. 6 in. wide, and 15 ft. high; it has two doors, each 8 ft. high by 3 ft. 9 in. wide, and 3 windows, one 5 ft. by 7 ft., the other two 5 ft. by 4 ft. each. What will it cost to paper the room with paper a yard wide at 10 cents a yard?

10. A loaded truck weighs 4 tons, 3 qrs. 1 lb. the truck itself weighs a ton and a half, and it contains 758 equal packages; find the weight of each package.

11. William Shakspear was born April 23rd, 1564. How long is it since his birth to January 1st, 1860.?

12. How much cloth is there in 3 pieces, measuring as follows: first piece 37 yds. 3 qrs. 1 nail; second piece 41 yds. $1\frac{1}{2}$ Flemish ells; third piece 43 yds. $1\frac{1}{2}$ English ells.

VI.

1. Multiply £10. 17*s.* $6\frac{3}{4}$*d.* Canada currency by 764; and find by Practice the value of 8764 things at £10. 17*s.* $6\frac{3}{4}$*d* each.

2. A bankrupt's assets amounted to \$542. and his creditors received $12\frac{1}{2}$ cts. in the dollar: find the amount of his debts.

3. A piece of cloth, when measured with a yard measure which is two-thirds of an inch too short, appears to be $10\frac{1}{2}$ yards long, what is its true length?

4. A merchant purchased 120 yards of cloth for \$780, and sold $\frac{5}{8}$ of it at a profit of $\$1\frac{3}{8}$ a yard, and the remainder at a loss of $\$\frac{5}{8}$ a yard. How much did he gain by the operation?

5. Estimate the cost of a dish of almonds and raisins consisting of six ounces of almonds and three quarters of a pound of raisins: supposing almonds to be $37\frac{1}{2}$ cts., and raisins 17 cts. a pound.

6. If 5 cwt. 3 qrs. 14 lbs. cost \$30 per cwt., what will be the cost per pound when the cost of the whole has been reduced by $\$25{\cdot}62\frac{1}{2}$?

7. A grocer buys 10 cwt. 3 qrs. 21 lbs. of sugar \$109·60, and pays \$5·48 for expenses; at what rate must he sell it per pound to clear \$21·92 by his bargain?

8. Explain the difference between Cross Multiplication and Duodecimals.

Find the cost of papering a room 20 ft. long, $16\frac{1}{2}$ ft. broad, and 12 ft. high, the price of a piece of paper 12 yds long, and 3 qrs. broad, being \$1·50.

9. If a snail, on the average, creep 2 ft. 7 in. up a pole during 12 hours in the night, and slip down 16 in. during the 12 hrs. in the day, how many hours will he be in getting to the top of a pole 35 ft. high?

10. The profits of a tradesman average $135·75 per week, out of which he pays 3 foremen, 10 clerks, and 5 assistants, at the rate of $10, $5, and $4, per week respectively; His yearly outgoings for rent, &c., amount to $3640·62½. Find his net annual profit.

11. A traveller walks 22 miles a day, and after he has gone 84 miles another follows him at the rate of 34 miles a day; in what time will the second traveller overtake the first?

12. Divide 100 acres, 3 roods, 8 rods, of land, between four persons, *A*, *B*, *C*, *D*, so that *A* shall have $\frac{1}{6}$ of the whole, *B* $\frac{1}{4}$ of the remainder, *C* $\frac{1}{3}$ of what then remains, and *D* the rest. How much will each one have?

VII.

1. What is meant by a fraction? Find the value of $\frac{2}{3}$ of $\frac{1}{4}$ of 13 tons, and then express the result as the fraction and decimal of 237 tons, 10 cwts.

2. By what number must £5. 6*s*. 3¼*d*., be multiplied, in order to give as product £85. 0*s*. 4*d*.? Divide £34. 13*s*. into 3 parts, one of which shall be twice, and the other 4 times as great as the third.

3. If a year consists of 365·242264 days, in how many years will its defect from the civil year of 365¼ days amount to 1 day?

4. If 15 men take 17 days to mow 300 arres of grass, how long will 27 men take to mow 167 acres?

5. If 20 men can perform a piece of work in 12 days, how many men will accomplish another piece of work, which is six times as great, in a tenth part of the time?

6. I am owner of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{1}{3}$ of a ship worth $6549 and sell $\frac{1}{6}$th of the ship; what part of her will then belong to me, and what will it be worth?

7. A bankrupt owes $900 to his three creditors; and his whole property amounts to $675; the claims of two of his creditors are $125 and $375 respectively; what sum will the remaining creditor receive for his dividend?

8. Shew that a cistern 13 ft. 4 in. long, 8 ft. broad, and 5 ft. 3 in deep holds just twice as much water as another which is 7 ft. long, 6 ft. 8 in. broad, and 6 ft. deep.

9. Explain the calender as now in use. On June 21, of 1851, the Duke of Wellington had lived 30,000 days. Find the day and year of his birth.

10. What will be the cost of $10\frac{3}{4}$ bales of cotton, each bale weighing 3 cwt., 2 qrs., at \$10·62½ per cwt.? Also, find how many solid feet there are in a stick of timber 40 ft. 9 in. long, 1 ft. 3 in. wide, and 1 ft. 9 in. deep. What would be its value at \$0·25 a ft.?

11. A person counts on the average \$7000 in an hour; what sum will he count in 67 days, if he work 9 hours a day?

12. Reduce £539. 9*s*. 10½*d*. sterling, to Canada currency; and £55. 12*s*. 6*d*., Canada currency, to sterling money.

VIII.

1. Find the value, at \$16 per oz., of 13 lbs. 9 ozs. 3 dwts. of gold.

2. How many times will a pendulum vibrate in 24 hours, which vibrates 5 times in 2 seconds.

3. A crystal palace has a glazed area of 28,000 sq. yds.; find how many square inches of glass there are, and what it would cost at \$0·10 a sq. ft.?

4. A creditor receives, on a debt of \$950, a dividend of \$0·63 in the dollar, and he receives a further dividend, upon the deficiency, of \$0·25 in the dollar; what does the creditor receive on the whole?

5. Reduce 12 ft., 4½ in., to the fraction of a mile, and find the corresponding decimal.

6. If 29,040 copies of a paper be printed, each copy consisting of 3 sheets, and each sheet being 3½ ft. long by 2 ft. broad; how many acres will one edition cover?

7. A man has an income of \$800 a year; an extra tax is established of 14 cents in the dollar, while a duty of 3 cents per lb. is taken off sugar; what must be his yearly consumption of sugar that he may just save the extra tax?

8. If *A* can do as much work in 5 hours as *B* can do in 6 hours, or as *C* can do in 9 hours, how long will it take *C* to complete a piece of work, one-half of which has been done by *A* working 12 hours and B working 24 hours?

9. Find the number of shillings and pence which are equivalent to the recurring decimal ·3333.... of a pound.

10. Explain how the Statute defines "a yard," with reference to a natural standard of length. Find the corresponding linear unit, when an acre is one hundred thousand square units.

11. A Liverpool merchant has bills falling due in New York and Montreal: what number of sovereigns will he have to transmit, supposing the New York bill to amount to $7260, and the Montreal one to $9740?

12. A merchant has 10 pieces of cloth, of equal length, and together containing 575 yards, 2 qr. 2 na. What is the length of each piece?

IX.

1. Explain the process of Long Division.

Reduce $\frac{9275}{371}+\frac{1522842}{719}$ to its equivalent whole number.

2. Shew how to convert any proper fraction into a decimal.

Reduce $\frac{2}{5}$ and $\frac{75}{1875}$ to the decimal form.

3. State what kind of vulgar fractions can be expressed in finite decimals? Can the quantity $\frac{1}{3}-\frac{1}{6}-\frac{1}{24}$ be so expressed?

How many cents should be given in exchange for $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{2}+\frac{1}{3}}$ of a dollar.

4. If two-thirds of an academic term exceed one-half of it by $13\frac{1}{3}$ days, how many days are there in the whole term?

5. How many barleycorns will reach round the earth, supposing the circumference of it to be 25000 miles?

6. The length of a rectangular field, which contains 7 ac. 1 ro. 15 po., is 453 yds. 2 ft. 3 in.; find its breadth.

7. A butler concocts a bowl of punch, of which the following are the ingredients: milk $2\frac{1}{6}$ qts., the rind of one lemon, 2 eggs, 1 pint of rum, and half-a-pint of brandy. Compute the value of the punch, reckoning milk at $·05 a quart, lemons at $·60 a dozen, eggs at $·02 each, rum at $3·50 per gallon, and brandy at $5·00 per gallon.

8. A Cochin China hen eats a pint of barley and lays a dozen eggs, while a Canadian hen eats half-a-pint of barley and lays five eggs. Supposing the eggs of the Canadian hen to be half as large again as those of the Cochin China, which is the more economical layer?

9. If 72 men dig a trench 20 yds. long, 1 ft. 6 in. broad, 4 feet deep, in 3 days of 10 hours each, how many men would be required to dig a trench 30 yds. long, 2 ft. 3 in. broad, and 5 feet deep, in 15 days of 9 hours each?

10. A person engages to build 100 rods and 10 ft., of stone fence; at one time he builds 17 rods, 5 ft., at another 37 rods, 15 ft. How much still remains to be built?

11. Five Chinamen divide 100 lbs of rice as follows; the first takes $\frac{1}{7}$ of $\frac{3}{4}$ of the whole; the second takes $\frac{1}{6}$ of $\frac{3}{4}$ of the remainder; the third takes $\frac{1}{5}$ of $\frac{3}{4}$ of the second remainder; the fourth takes $\frac{1}{4}$ of $\frac{3}{4}$ of the third remainder; and the fifth had what was left. How much did each receive?

12. Add together $\frac{1}{7}$ of 4 degrees of $69\frac{1}{6}$ miles each, and $\frac{1}{3}$ of 4 furlongs.

X.

1. Explain the rule for the addition of decimals; add together $\frac{3}{5}$ and ·061; subtract ·003 from ·02; and divide ·0672 by ·006.

2. Subtract $\frac{1}{2}$ of $\frac{5}{6}$ from $\frac{2}{3}$ of $\frac{7}{11}$, and multiply the result by $\frac{3}{7}$ of $\frac{5}{8}$.

3. The fore wheel of a carriage is 10 feet in circumference, and the hind wheel is 16 feet; how many revolutions will one make more than the other in 100 miles?

4. If 6 men earn \$36·75 in $7\frac{1}{2}$ days, how much will 10 men earn in $11\frac{3}{4}$ days?

5. A person expends \$72 in the purchase of cloth, how much can he buy at the rate of \$0·64 a yard?

6. What is the cost per hour of lighting a room with ten burners, each consuming 4 cub. in. of gas per second; the price of gas being \$1·50 for a thousand cubic feet?

7. What is the value of 8 qrs. 5 bushels, 3 pecks of wheat, at \$1·$37\frac{1}{2}$ a bushel?

If 70 bushels, 2 pecks of malt cost \$100, what is the price per bushel?

8. What length of paper, $\frac{3}{4}$ of a yard wide, will be required to cover a wall 15 ft. 8 in. long by 11 ft. 3 in. high?

9. "Define a Rectangular Parallelopiped."

A block of wood, in the form of a rectangular parallelopiped, measures along its edges 18½ feet, 5¼ feet, and 3 feet, respectively; determine its value on the supposition that a cubical block, measuring 11 inches along the edge, is worth $0·50.

10. If 36 men, working 8 hours a day for 16 days, can dig a trench 72 yards long, 18 wide, and 12 deep, in how many days will 32 men, working 12 hours a day dig a trench 64 yards long, 27 wide, and 18 deep?

11. *A* received $\frac{1}{6}$ of a legacy, *B* $\frac{1}{10}$, and *C* the remainder. Now it is found that *A* had $80 more than *B*. How much did each receive?

12. Suppose a man consumes $\frac{3}{8}$ of a day in sleep, $\frac{1}{8}$ of a day in eating, $\frac{2}{3}$ hr. each day in amusement, $\frac{3}{4}$ hr. each day in idleness. How many days, of 10 hours each, has he for work in the course of the year?

XI.

1. Express $\frac{11}{133}$ as a decimal; and thence find its value when unity represents $300.

2. A Township containing 2456 acres is rated on a rental of $10700; a rate of $0·5 in the dollar being levied, what on the average is the charge per acre?

3. Find the price of 2 tons, 16 cwt., 17 lbs. of sugar at $1·25 a Ton.

4. If 1 cwt. of an article cost $70, at what price per lb. must it be sold to gain $\frac{1}{10}$ of the outlay?

5. Find in inches and fractions of an inch the value of $\cdot 000035511\dot{3}\dot{6}$ of a mile. Explain the process employed.

6. Express £17. 18*s*. 3$\frac{1}{5}$*d*. sterling money, and £19. 13*s*. 4*d*. Canada currency, in dollars and cents; also, convert their sum into Georgia currency.

7. Sound travels at the rate of 1142 feet a second; if a gun be discharged at the distance of 4½ miles, how long will it be after seeing the flash before I hear the report?

8. *A* and *B* can do a piece of work in 6 days, *B* and *C* in 7 days, and *A*, *B*, and *C* can do it in 4 days; how long would *A* and *C* take to do it?

9. If a sheet of paper $5\frac{1}{2}$ feet long by $2\frac{1}{2}$ feet broad be cut into strips an inch broad; how many sheets would be required to form a strip that would reach round the earth (25,000 miles?)

10. Find the cost of 76 cords, 16 feet of wood at $3·50 a cord; also of 17 yds. 2 ft. 8 in. of cloth at $4·50 a yard.

11. Two boys run a race of 1 mile; one of them gains 5 feet in every 110 yards: how far will the other be left behind at the end of the race?

12. From a piece of cloth containing 20 yds. 2 qrs. 2 na., three suits, each requiring $4\frac{1}{2}$ yds., were taken; and $\frac{1}{3}$ of the remainder was sold for $10·68$\frac{3}{4}$. How much was that per yard?

RULE OF THREE.

143. We may compare one number with another, or ascertain the relation which one bears to the other in respect of magnitude, in two different ways; either by considering how much one is greater or less than the other; or by considering what multiple, part, or parts, one is of the other, that is, how many times or parts of a time, or both, one number is contained in the other. Thus if we compare the number 12 with the number 3, we observe, adopting the first mode of comparison, that 12 is greater than 3 by the number 9; or, adopting the second mode of comparison, that 12 contains 3 four times, and is thus $\frac{12}{3}$ or four times as great as 3. Again if we compare the number 7 with the number 13, we observe, according to the first mode of comparison, that 7 is less than 13 by the number 6; and, according to the second, that as 1 is one thirteenth part of 13, so 7 is seven thirteenth parts of 13, or $\frac{7}{13}$ths of 13.

144. The relation of one number to another, in respect of magnitude, is called RATIO; and when the relation is considered in the first of the above methods, that is, when it is estimated by the difference between the two numbers, it is called ARITHMETICAL RATIO; but when it is considered according to the second method, that is, when it is estimated by considering what multiple, part, or parts, one number is of the other, or, which is seen from above to be the same thing, by the fraction which the first number is of the second, it is called GEOMETRICAL RATIO. Thus, for instance, the arithmetical ratio of the numbers 12 and 3 is 9; while their geometrical ratio is $\frac{12}{3}$ or 4. In like manner the arithmetical ratio of 7 and 13 is 6, while their geometrical ratio is $\frac{7}{13}$.

145. It is more common, however, in comparing one number with another to estimate their relation to one another in respect of magnitude according to the second method, and to call that relation so estimated by the name of RATIO. According to this mode of treatment, which we shall adopt in what follows, "RATIO is the relation which one number has to another in respect of magnitude, the comparison being made by considering what multiple, part, or parts, the first number is of the second, or how many times or parts of a time, or both, the second is contained in the first."

146. It is plain that, for any two numbers, the fraction in which the first is numerator and the second denominator, will correctly express the multiple or part, or both which the first number is of the second, or the number of times or parts of a time, or both, of a time the second is contained in the first. Thus, if we take the numbers 12 and 3, the fraction $\frac{12}{3}$, which is equivalent to the whole number 4, shows the multiple which 12 is of 3, or the number of times 3 is contained in 12. And again if we take the numbers 7 and 13, the fraction $\frac{7}{13}$ will express the part or parts which the number 7 is of 13, or will express the part or parts of a time that 13 is contained in 7: for 1 is one thirteenth part of 13, so that 7 must be 7 thirteenth parts of 13, that is, $\frac{7}{13}$ths of it; and 1 is contained 7 times in 7, so that 13 must be contained only $\frac{7}{13}$ths of a time in 7. We conclude therefore that the ratio of one number to another may be estimated and expressed by the fraction in which the former number is the numerator and the latter the denominator.

147. The ratio of one number to another is often denoted by placing a colon between them. Thus the ratio of 7 to 13 is denoted by 7 : 13. As we have shown that the ratio of one number to another may be expressed by the fraction in which the former is the numerator, and the latter the denominator, we see that 7 : 13 is $=\frac{7}{13}$. The two numbers which form a ratio are called its *terms;* the first number, or the number compared, being called the first term, or THE ANTECEDENT, and the second number or that with which the former is compared, the second term, or THE CONSEQUENT of the ratio.

148. If the two numbers to be compared together be concrete, they must be of the *same kind.* We cannot compare together 7 days and 13 miles in respect of magnitude; but we can compare 7 days with 13 days; and it is clear that 7 days will have the same relation to 13 days in respect of magnitude, which the

number 7 has to the number 13, so that the ratio of 7 days to 13 days will be the same as the ratio of the abstract number 7 to the abstract number 13, and may be expressed by the fraction $\frac{7}{13}$. If however the concrete numbers, though of the same kind, be not in the same denomination of that kind, it will be convenient to reduce them to one and the same denomination in order to find their ratio. Thus, if one of the numbers be 7 days and the other be 13 hours, the ratio of the former to the latter will not be that of 7 to 13, but that of 7 *days* to 13 *hours*, that is, 168 hours to 13 hours, which will clearly be the same as that of the abstract number 168 to the abstract number 13, and so will be expressed not by $\frac{7}{13}$, but by $\frac{168}{13}$. We see, then, that 7 days : 13 hours is the same as 168 : 13, and that each is $=\frac{168}{13}$. Thus it is plain that when the numbers are concrete, we may reduce them to one and the same denomination, and then, in considering their ratio, treat them as abstract numbers.

149. Proportion is the equality of two ratios; so that, when the ratio of one number to a second is equal to the ratio of a third number to a fourth, proportion is said to exist among the numbers, and the numbers are called Proportionals. Thus, the ratio of 8 to 9 is equal to that of 24 to 27, for the former ratio is $\frac{8}{9}$, and the latter ratio is $\frac{24}{27}$, which is also equal to $\frac{8}{9}$. The ratios being equal, proportion exists among the numbers 8, 9, 24, 27; and thus those numbers are proportionals.

150. When proportion exists among four numbers, that is, when the ratio of the first to the second is equal to that of the third to the fourth, this proportion or equality is often denoted by writing down the two ratios in the manner mentioned in (Art. 147) in one line, and placing a double colon (::) between them. Thus the existence of proportion among the numbers 3, 4, 9, 12, is indicated as follows,

$$3 : 4 :: 9 : 12,$$

which is commonly read thus, "three are to four as nine to twelve," or "as three to four so nine to twelve." It will appear from what has preceded, that by the expression 3 : 4 :: 9 : 12, it is meant in fact that $\frac{3}{4}=\frac{9}{12}$.

151. In order to form a proportion four numbers are required. It may indeed happen that the second and third are the same, in which particular case it might be said that only three numbers are required; thus 9 : 6 :: 6 : 4; but even in such a case it is better to consider the second and third as distinct numbers, and to regard

the proportion as consisting of four numbers, of which indeed two are equal. The four numbers required to form a proportion are called its *terms*. In the proportion 3 : 4 :: 9 : 12, we have 3 for the first term, 4 for the second, 9 for the third, and 12 for the fourth term, of the proportion.

152. It has been stated that proportion is the equality of two ratios, and we have explained that the two numbers constituting a ratio must either be both abstract, or (if concrete) both of the same kind. In a proportion if one of the ratios be formed by two abstract numbers, the other may arise from two concrete numbers. For it has been explained (Art. 148) that if a ratio consist of two concrete numbers, we may reduce them both to the same denomination, and then treat the resulting numbers as abstract, the ratio of those abstract numbers being the same as that of the two concrete numbers from which they have arisen. For the same reason, one of the two ratios constituting a proportion may be formed from concrete numbers of one kind, while the other is formed from concrete numbers of a different kind; for 7 days : 13 days :: 7 miles : 13 miles, each ratio being in fact that of 7 to 13. Indeed, it appears by (Art. 148) that the ratio of two concrete numbers may always be expressed by a ratio of two abstract numbers. If both or either of the ratios in a proportion be formed from concrete numbers, we may thus replace each such ratio by one arising from abstract numbers, and in this way every term of the proportion will become an abstract number; so that, notwithstanding the remark in note (Art. 26), any one of the terms may then be multiplied or divided by any other.

153. It is readily seen that if proportion exist among four numbers taken in a certain order, it will exist also among the same numbers taken in the contrary order. Thus the numbers 8, 9, 24, 27, being proportionals in the order in which they stand, the numbers 27, 24, 9, 8, will also be proportionals. For,

$$\frac{8}{9}=\frac{24}{27};$$

$$\therefore 1\div\frac{8}{9}=1\div\frac{24}{27},$$

$$\text{or } 1\times\frac{9}{8}=1\times\frac{27}{24},$$

$$\text{or } \frac{9}{8}=\frac{27}{24};$$

$$\therefore \frac{27}{24} = \frac{9}{8},$$

or 27 : 24 :: 9 : 8.

It is apparent also from (Art. 66) that $\frac{27}{24} = \frac{9}{8}$.

154. If only three of the numbers in a proportion be given, we can by means of them find the fourth, and the method or Rule by which it may be found, is one of great importance in Arithmetic. We have seen that proportion exists among the numbers 8, 9, 24, 27. If the first three numbers only were given, and we were required, by means of these, to find the fourth, the method or rule to be adopted ought to determine a number to which 24 would have the same ratio, as 8 to 9; or, which is seen from the last article to be the same thing, it ought to determine a number which will have the same ratio to 24, which 9 has to 8; this number being of course 27. Almost all questions which arise in the common concerns of life, so far as they require calculation by numbers, might be brought within the scope of the Rule of Three, which enables us to find the fourth term in a proportion, and which, on account of its great use and extensive application, is often called the Golden Rule.

155. The Rule of Three, then, is a method by which we are enabled, from three numbers which are given, to find a fourth which shall bear the same ratio to the third as the second to the first, that is, shall be the same multiple, part, or parts of the third, as the second is of the first; in other words, it is a Rule by which, when three terms of a proportion are given, we can determine the fourth.

As most of the practical cases in which this rule is made use of relate to concrete numbers, we shall express the Rule with especial reference to such cases, adding, however, a short direction for cases in which abstract numbers only are concerned.

156. Rule. "Leaving out of consideration superfluous quantities, find, out of the three quantities which are given, that which is of the same kind as the fourth or required quantity; or that which is distinguished from the other terms by the nature of the question: place this quantity as the third term of the proportion.

"Now consider whether, from the nature of the question, the fourth term will be greater or less than the third; if it be greater, then put the larger of the other two quantities in the second term, and the smaller in the first term; but if less, put the smaller in the second term, and the larger in the first term.

"Take care to reduce the first and second terms to one and the same demomination, and also to reduce the third so that it may be wholly in one denomination; remembering, however, that if the quantities involved be all of the same kind, it is unnecessary to reduce all the three terms to the same denomination, but only the first and second terms to one and the same denomination, and the third to a single denomination, which will not necessarily be the same as the former. When the terms have been properly reduced, multiply the second and third together, and divide by the first, treating all three as abstract numbers. The quotient will be the answer to the question, in the denomination to which the third term was reduced."

If the case be one in which abstract numbers only are concerned, the question itself will show at once which of the numbers will form the third term of the proportion: the second and first will be determined as above explained; and then the answer to the question will be found by such multiplication and division as are directed in the Rule.

The arrangement of the given terms in the manner mentioned at the beginning of the Rule, is commonly called *stating the question.* Sometimes a word or two, or a letter, or a symbol, will be added to represent the fourth or required term.

Note 1. The process denoted by the above Rule may often be much abbreviated by dividing the first and second, or the first and third terms, (but never the second and third) by any number which will divide each of them without a remainder, and using the quotients instead of the numbers themselves.

For, 9 : 12 : : 21 : 28 is the same as $\frac{9}{12}=\frac{21}{28}$, which is the same as $\frac{3}{4}=\frac{21}{28}$, which is the same as 3 : 4 : : 21 : 28, which represents the first proportion after its first and second terms have each been divided by the same number 3.

Again, 9 : 12 : : 21 : 28 is the same as $\frac{9}{12}=\frac{21}{28}$, which is the same as $\frac{3}{12}=\frac{7}{28}$, which is the same as 3 : 12 : : 7 : 28, which represents the first proportion after the first and third terms have each been divided by 3.

Again, 9 : 12 : : 21 : 28 is the same as $\frac{9}{12}=\frac{21}{28}$, but this is *not* the same as $\frac{9}{4}=\frac{7}{28}$, which is the same as 9 : 4 : : 7 : 28, which represents the first proportion after the second and third terms have each been divided by 3. Moreover $\frac{9}{4}$ is *not* equal to $\frac{7}{28}$, and of course 9 : 4 : : 7 : 28 is not a true proportion.

Note 2. Although we have said in the Rule, multiply the second and third terms together and then divide their product by the first; it will be found in most cases advisable not to perform the actual multiplication until we have discovered, by putting the expression in the form of a fraction, whether there be any factor or factors common to the numerator and denominator, and if so, have rejected such factor or factors.

157. It may be proper to observe that the Rule of Three is applicable in two different kinds of cases, according to which it is called the Rule of Three Direct or the Rule of Three Inverse. The method just stated (Art. 156) is applicable to both kinds of cases; but as the distinction between the two is commonly noticed by writers on Arithmetic, it will be right to show in what it consists.

The Rule of Three Direct is that in which more requires more, or less requires less; or, in other words, in which a greater number requires a greater answer, or a less number a less answer. Thus in the question, "If 4 acres of land cost $550, find the cost of 15 acres, after the same rate." The 15 acres being more than the four acres, will require a larger sum than $550 for their purchase, and so, in this case, more requires more. Again in the question, "If 15 acres of land cost $1160 find the cost of 4 acres, after the same rate," the four acres being less than the 15 acres, will require a less sum than $1160 for their purchase, and, therefore, in this case, less requires less. Such cases belong to the Rule of Three Direct.

The Rule of Three Inverse is that in which more requires less, or less requires more: or, in other words, in which a greater number reqnires a less answer, or a less number a greater answer. Thus in the question, "If 4 men can mow a certain meadow in 3 days, find the time in which 6 men ought to mow it," the six men being more than the four, should perform the work in less time, and so, in this case, more requires less. Again, in the question, "If 6 men can mow a certain meadow in 2 days, find the time in which 4 men ought to mow it," the 4 men being fewer than the 6, will require a longer time for performing the work, and therefore, in this case, less require more. Such cases belong to the Rule of Three Inverse.

Rule of Three Direct.

Ex. 1. Find the value of 37 yards of silk, when 25 yards cost $150.

There are here three given quantities, 25 yards, 37 yards, and $150, and we have to find a fourth which will be the price of 37 yards. It is manifest that three given quantities, 25 yards, 37 yards, $150, and the required sum, must form a proportion, because the 25 yards must have the same relation in respect of magnitude to the 37 yards, which the $150 (cost of 25 yards) has to the required sum (cost of 37 yards). Proceeding then by Rule (Art 157) we observe that the $150 is of the same kind as the required term, viz. money; we make that the third term of the proportion; and since the required sum (cost of 37 yards) must necessarily be greater than $150 (cost of 25 yards), we make 37 the second term, and 25 the first. We have thus the first three terms arranged as follows:

25 yds : 37 yds. : : $150.

And the entire proportion will be as follows:

25 yds. : 37 yds. : : $150 : required cost.

The first and second terms are in one and the same denomination, and require no reduction; therefore the proportion is,

25 yds. : 37 yds. : : $150 : no. of dollars in required sum.

And by our rule we must now treat the numbers as abstract, multiply the second and third together, and divide by the first.

```
            150
             37
           ----
           1050
            450
     ( 5 | 5550
  25 {   |-----
     ( 5 | 1110
         |-----
            222
```

The quotient 222 gives the number of dollars required.

The above process is usually written down as follows:

```
  yds.  yds.   $
  25 : 35 : : 150
               37
             ----
             1050
             450
     ( 5 | 5550
  25 {   |-----
     ( 5 | 1110
         |-----
          $222
```

Reason for the above process.

We have the cost of 25 yards given, viz. $150, in order to enable us to find the cost of 37 yards.

It is manifest that the required sum must have the same relation in respect of magnitude to $150 which 37 yards have to 25 yards; that is, the ratio of the required sum must be equal to that of 37 yards to 25 pards.

Now the ratio of the number of dollars in the required sum to $150, is the same as that of the *abstract* number which indicates how many dollars the required sum contains to the abstract number 150, and may (if the former number be called the ***required number***) be expressed by the fraction $\frac{\text{required number}}{150}$.

$$\therefore \frac{\text{required number}}{150} = \frac{37}{25};$$

$$\therefore \frac{\text{required number}}{150} \times 150 = \frac{37}{25} \times 150,$$

$$\text{or } \frac{\text{required number} \times 150}{150} = \frac{37 \times 150}{25},$$

$$\text{or required number} = \frac{37 \times 150}{25}, \text{ (Art. 66.)}$$

$$\text{or} = \frac{150 \times 37}{25}.$$

This result shows that if we arrange the three given terms, 25 yards, 37 yards, and $150 in the following manner:

$$\overset{\text{yds.}}{25} : \overset{\text{yds.}}{37} :: \$150,$$

and then consider the numbers to be abstract, as if they had been written

$$25 : 37 :: 150,$$

we shall obtain the abstract number which will show us how many dollars there are in the required sum by multiplying the second and third terms together and dividing the product by the first; and then by treating this number as concrete, that is, as so many dollars, we have the required answer in dollars.

The reason for the process may also be shown as follows:

The cost of 25 yards is \$150;

$$\therefore \text{ the cost of 1 yard is } \frac{150}{25} \text{ dollars;}$$

$$\therefore \text{ the cost of 37 yards is } \left(\frac{150}{25} \times 37\right) \text{ dollars} = \frac{150 \times 37}{25} \text{ dollars;}$$

or if we arrange the numbers in the form

$$\overset{\text{yds.}}{25} : \overset{\text{yds.}}{37} :: \$150,$$

and then treat them as abstract numbers, multiply the second and third together, and divide the product by the first, the quotient will give the number of dollars in the required sum of money.

Ex. 2. If the tax on \$565 be \$35·60, what will be the tax on \$3570?

The \$35·60, being of the same nature with the sum required, must be placed in the third term in the proportion; and as the required tax must clearly be greater than \$35·60, we must place \$3570 as the second, and \$565 as the first term.

\$565 : \$3570 :: \$35·60 : the required tax.

```
              3570
            ------
            249200
           17800
          10680
          --------
565)      127092·00 (224·94 18/113
          1130
          ----
           1409
           1130
           -----
            2792
            2260
            -----
             5320
             5085
             -----
              2350
              2260
              -----
                90
```

$\therefore$ the required tax is $\$224\cdot94\frac{18}{113}$.

Note 1. The student who is expert in the use and reduction of fractions will very often find it convenient, after reducing the terms of his proportion in the manner mentioned in the Rule, to defer the *actual* multiplication and division, and express the required result in a fractional form; to reduce the fraction as much as possible by the method indicated in Art. 77, note 3; and to effect the requisite multiplication or division, or both, after the fraction has been so simplified.

Ex. 3. If I can travel 198 miles by railway for \$4·94, how far, at the same rate of charge, ought I to be carried for \$29·70?

$$\$4{\cdot}94 : \$29{\cdot}70 :: 198 \text{ m.} : \text{required distance.}$$

$$\therefore \text{Required distance} = \frac{29{\cdot}70 \times 198}{5{\cdot}94} \text{ miles} = \frac{5 \times 198}{1} \text{ miles}$$

$$= 990 \text{ miles.}$$

Note 2. There are certain examples in which, at first sight, more than three terms appear to be given, but they nevertheless in certain cases come under this rule, as in the following instance:

Ex. 4. If the carriage of 5 cwt. 5 lbs. for 84 miles cost me \$5, what will it cost me to have 30 cwt. 1 qr. 5 lbs. carried the same distance?

The 84 miles may evidently be left out of consideration, since the distance in both cases is the same.

Proceeding then according to our Rule,

5 cwt. 5 lbs. : 30 cwt. 1 qr. 5 lbs. :: \$5 : required cost;

whence it will be found that

Required cost = \$30.

Rule of Three Inverse.

Ex. 5. If a piece of cloth is 20 yards in length and $\frac{3}{4}$ yard in breadth, how broad is another piece which is 12 yards long, and which contains as much cloth as the other?

As the length of the second piece is less than that of the first, its breadth must necessarily be greater, in order that the content may be the same. Therefore in this case a less length requires a greater breadth, and so the example belongs to the Rule of Three Inverse.

We have the *breadth* of the second piece to find. That of the first piece is $\frac{3}{4}$ yard: place this therefore as the third term. Now the required breadth is to be greater than this; therefore place the 20 yards as the second term, and the 12 yards as the first.

12 yds. : 20 yds. : : 3 qrs. of a yd. : required breadth in qrs. of a yd.

$$\begin{array}{r|l} & 3 \\ \hline 12 & 60 \\ \hline & 5 \end{array}$$

5 qrs. of a yard $= 1\frac{1}{4}$ yd.

Or thus;

12 yds. : 20 yds. : : $\frac{3}{4}$ yd. : required breadth in yds.

$$\therefore \text{required breadth} = \frac{20 \times \frac{3}{4}}{12} \text{yds.}$$

$$= \frac{5 \times 3}{12} \text{yds.} = \frac{5}{4} \text{ yds.} = 1\frac{1}{4} \text{ yds.}$$

The required breadth is therefore a yard and a quarter.

Ex. 6. If 12 men can reap a field in 4 days, in what time can the same work be performed by 32 men?

It is clear that 32 men can perform the work in a less time than 12 men, and so the time required will be less than 4 days, the third term in our proportion. We must therefore place the 12 as the second term and the 32 as the first.

32 : 12 : : 4 : required time in days.

$$\begin{array}{r l} & 4 \\ \hline 32) & 48 \; (1\frac{16}{32} \text{ days;} \\ & 32 \\ \hline & 16 \end{array}$$

$\therefore$ the required time is $1\frac{16}{32}$ days $= 1\frac{1}{2}$ days.

Or thus;

$$\text{Required time} = \frac{12 \times 4}{32} \text{days} = \frac{12}{8} \text{days} = \frac{3}{2} \text{ days} = 1\frac{1}{2} \text{ days.}$$

Ex. 7. What was the price of wheat per bushel when the 5 cent loaf weighed 8 ounces; the statute being that it must weigh 10 oz. when wheat is \$1·50 a bushel?

Here are two numbers, viz. 1 bushel and 5 cents, which can evidently have no effect on the answer, for if any other measure had been named instead of the bushel, and any other loaf in place of the 5 cent loaf, the answer would be the same.

Now as wheat is dearer, or as the price is more, the weight of any given loaf is less, and conversely, as the weight of a given loaf is less, the price of wheat is greater; so that the price required must clearly be greater than \$1·50, which, according to our Rule, must be the third term of the proportion. Therefore the 10 oz. must be the second term, and the 8 oz. the first.

$$\begin{array}{rl} 8 \text{ oz.} : 10 \text{ oz.} :: & \$1{\cdot}50 \\ & \underline{\quad 10} \\ 8 \,| & \underline{15{\cdot}00} \\ & \$1{\cdot}875, \text{ the required price per bush.} \end{array}$$

Or thus:

$$\text{Required price} = \$\frac{10 \times 1{\cdot}50}{8} = \$\frac{5 \times 1{\cdot}50}{4} = \$1{\cdot}875.$$

Note 3. Examples, such as the following, are easily worked out by the Rule of Three.

Ex. 1. A clock, which is 4 min. $8\frac{9}{32}$ sec. too fast at half-past nine, A. M. on Tuesday, loses 2 min. 45 sec. daily; what will be the time indicated by the clock at a quarter past five P. M. on the following Friday?

From $9\frac{1}{2}$ A. M. on Tuesday, till $5\frac{1}{4}$ P. M. on Friday, there are $79\frac{3}{4}$ hours.

$$\therefore 24 \text{ hrs.} : 79\tfrac{3}{4} \text{ hrs.} :: 2'.45'' : \text{time lost by clock,}$$

$$\text{whence, time lost by clock} = 9'.8\tfrac{9}{32}'';$$

$\therefore$ time by the clock at $5\frac{1}{4}$ P. M. on Friday

$$= 4'.8\tfrac{9}{32}'' + 5 \text{ hrs. } 15' - 9'.8\tfrac{9}{32}'' = 5 \text{ hrs. } 10 \text{ min.}$$

Ex. 2. A hare, pursued by a greyhound, was 130 yards before him at starting; whilst the hare ran 5 yards the dog ran 7 yards: how far had the hare gone when she was caught by the greyhound?

For every 5 yards the hare runs, the dog gains 2 yards, and when he has gained 130 yards he will have caught her.

$\therefore$ 2 yds. : 130 yds. :: 5 yds. : required number of yards;

whence, required number of yards $=325$.

Ex. 3. A gentleman spends on the average \$150 a fortnight; what must be his daily income in order that with his savings at the end of $3\frac{1}{2}$ years he may buy a farm worth \$6725? (supposing a year to consist of 52 weeks).

His expenditure in $3\frac{1}{2}$ years is $(150 \times 26 \times 3\frac{1}{2})$ dollars;

$\therefore$ his income in $3\frac{1}{2}$ years must be $(150 \times 26 \times 3\frac{1}{2})$ dollars $+$ \$6725 $=$ \$20375.

$\therefore$ $(3\frac{1}{2} \times 364)$ days : 1 day :: \$20375 : daily income,

whence, daily income $=$ \$$15{\cdot}99\frac{187}{637}$.

Ex. 4. Two places, *A* and *B*, are distant from each other 324 miles by railway. A train leaves *A* for *B* at the same time that a train leaves *B* for *A*; the trains meet at the end of 6 hours, the train from *A* to *B* having travelled 16 miles an hour more than the other. How many miles did each travel an hour?

Each train is supposed to run with uniform speed: when the trains meet, the whole distance must have been passed over by them.

$\therefore$ 6 hrs. : 1 hr. :: 324 miles : miles passed over by both trains in 1 hr.

whence, miles passed over by both trains in 1 hr. $=54$;

therefore by question, $(54-16)\div 2$, or $19=$ miles travelled per hour by one train, and therefore $54-19$, or $35=$ miles travelled per hour by the other.

Ex. 5. An English gentleman, after paying an income-tax of 7*d.* in the £, has £248. 10*s.* 8*d.* left; what was his gross annual income?

For every 19*s.* 5*d.* which he now has, he had £1. before he paid his income-tax;

$\therefore$ 19*s.* 5*d.* : £248. 10*s.* 8*d.* :: £1. : required income,

Ex. LII.

1. If 4 yards of cloth cost \$12, what will 96 yards of the same cloth cost?

2. If 9 yards of cloth cost \$45, how many yards can be bought for \$225?

3. If 7 bushels of wheat be worth \$7·70, what will be the value of 3 bushels of the same quality?

4. The rent of 42 acres of land is \$63, how many acres of the same quality of land ought to be rented for \$273?

5. If the cost of 72 tons of coals be \$540, what will be the cost of 42 tons?

6. How much must be given for 13 articles at the rate of £3. 16*s.* 6*d.*, Canada currency, for 6 articles?

7. How long will a person be saving \$194·26, if he put by \$1·75 per week?

8. Find a number which shall bear the same ratio to 9, which 20 does to 15.

9. If 2 cwt., 3 qrs., 14 lbs. of sugar cost \$28·90 what quantity of the same quality of sugar can be bought for \$86·70?

10. If 3 cwt., 3 qrs. cost \$33·75, what will be the price of 2 cwt., 2 qrs.?

11. Find the value of 23 yds., 1 ft. of cloth, supposing 4 yds., 31 in. of the same quality to cost \$17·50.

12. What will be the school tax, at 3 cents in the dollar, on \$257·10?

13. 5 gallons, 2 pints of parafine oil cost \$5·88, what will 75¼ gallons cost?

14. What is the tax upon \$872·25, when \$1170 is rated at \$3·15?

15. If one bushel of malt cost \$1·23, how much can I buy for \$897·62½?

16. Find the price of 2 tons, 3 cwt., 14 lbs. at \$15·11 per quarter.

17. If 9 acres of land sell for \$230·62½, what should 5 acres bring at the same rate?

18. Find the amount of a servant's wages for 215 days at \$1·25 a day.

19. A bankrupt's debts amount to \$10550, and his assets to \$1055; how much in the dollar can he pay?

20. A cistern is filled with water, by 2 pipes, in 3 hours 25 minutes; in what time would it be filled by 5 pipes of like size?

21. A bankrupt pays \$·075 in the dollar, and his assets amount to \$950; find the amount of his debts.

22. The velocity of a locomotive on a railroad is 35 miles an hour, how far does it move in 30 seconds?

23. If $\frac{3}{5}$ of a bushel of wheat cost \$$1\frac{5}{6}$, what will \$$\frac{2}{3}$ purchase?

24. If $1\frac{1}{2}$lbs. of indigo cost \$3·84, what will 49·2 lbs. cost?

25. Find a fourth proportional to the numbers 3, 3·75, and 40.

26. If 10 men can mow a field in 12 days, in how many days will 15 men mow it?

27. If a man walk 62 miles in 3 days, in how many days will he walk 80 miles?

28. How many yards worth \$3·25 a yard must be given in exchange for $935\frac{1}{2}$ yards worth \$4·60 per yard?

29. If 7 times $\frac{1}{3}$ of $\frac{7}{8}$ of an estate be worth \$15000, what is $\frac{3}{5}$ of $\frac{3}{4}$ of it worth?

30. Find the price of 2 tons, 16 cwt., 17 lbs. of sugar at \$0·25 for $2\frac{1}{2}$ lbs.

31. If a person travelling 19 hours a day perform a journey in 24 days, in what length of time will he perform the same journey if he travel 16 hours a day?

32. If $3\frac{5}{8}$ oz. Avoir. cost \$$2{\cdot}87\frac{1}{2}$, what will $30\frac{3}{7}$ lbs. cost?

33. How many men must be employed to finish a piece of work in 15 days, which 5 men can do in 24 days?

34. If 356 ac., 3 ro., $39\frac{1}{2}$ po. be rented at \$1713·585, what is the rent of 2 acres?

35. The governor of a besieged place has provisions for 54 days, at the rate of $1\frac{1}{2}$ lbs. of bread to each man per day, but is desirous to prolong the siege to 80 days, in expectation of succour; in that case, what must the ration of bread be?

36. If 27 bush. 2 pks., cost \$65, what is the price of $16\frac{1}{2}$ bush.?

37. How many yards of drugget, an ell wide, will cover 40 yds. of carpet $\frac{3}{4}$ yd wide?

38. A merchant, owning $\frac{2}{3}$ of a vessel, sells $\frac{2}{3}$ of his share for \$500; what is the whole vessel worth?

39. A field is 121 yds. long and 86 yds. broad; what will be its value at \$80 an acre?

40. If the price of 1 lb. of sugar be \$0·0625, what is the value of ·75 of a cwt.?

41. If $3\frac{1}{2}$ shares in a mine cost \$22·645, what will $28\frac{3}{4}$ shares cost?

42. If $34\frac{1}{2}$ yards of cloth cost $\$53\frac{5}{8}$, how many yards can be bought for $\$35\frac{5}{16}$?

43. Find the rent, at \$3 an acre, of a rectangular field, whose sides are respectively 50 chains 40 links, and 56 chains 25 links.

44. In what time will 25 men do a piece of work which 12 men can do in 3 days?

45. If $\dot{3}$ of 4·5 cwt. cost \$11·55, what is the price per lb.?

46. When $\$\frac{1}{8}$ will buy $\frac{1}{7}$ dwt. of gold, how much gold can be bought for \$8000?

47. If a piece of building land, 375 ft. 6 in. by 75 ft. 6 in., cost \$118, what will be the price of a piece of similar land, 278 ft. 9 in. by 151 ft.?

48. A servant enters on a situation at 12 o'clock at noon, on Jan. 1, 1859, at a yearly salary of \$175, he leaves it at noon on the 27th of May following; what ought he to receive for his services?

49. *A* was owner of $\frac{4}{17}$ of a vessel, and sold $\frac{3}{11}$ of $\frac{2}{5}$ of his share of his share for $\$\frac{400}{3}$; what was the value of $\frac{1\frac{2}{3}}{4\frac{1}{4}}$ of $\frac{2}{5}$ of the vessel?

50. *A* exchanged with *B* 60 yards of silk, worth \$1·44 a yard, for 48 yards of velvet; what was the price of the velvet a yard?

51. If it require 30 yds. of carpeting, which is $\frac{3}{4}$ of a yard wide, to cover a floor, how many yards of carpeting, which is $1\frac{1}{4}$ yds. wide, will be necessary to cover the same floor?

52. If by a leak of a ship $\frac{3}{5}$ enough water run in to sink her in 4 hours, how long will she float?

53. A watch is 10 minutes too fast at 12 o'clock (noon) on Monday, and it gains 3′.10″ a day; what will be the time by the watch at a quarter past 10 o'clock A. M. on the following Saturday?

54. The circumference of a circle is to its diameter as 3·1416 : 1; find (in feet and inches) the circumference of a circle whose diameter is $22\frac{1}{2}$ feet.

55. If \$100, in 12 months, bring an interest of \$7, what will be the interest of \$75 for the same time?

56. If the carriage of 3 cwt. cost $\$2\frac{1}{2}$ for 40 miles, how much ought to be carried for the same price for $25\frac{3}{4}$ miles?

57. If I spend 20 dollars in a fortnight, what must my income be that I may lay by \$200 in a year?

58. If $\frac{3}{16}$ of a city lot be sold for \$500, what would $\frac{7}{16}$ of the same lot sell for at the same rate?

59. A silver tankard, which weighs 1 lb. 10 oz. 10 dwts., cost \$30·55; what is the value of the silver per ounce?

60. A man, working $7\frac{1}{2}$ hours a day, does a piece of work in 9 days; how many hours a day must he work to finish it in $4\frac{1}{2}$ days?

61. Suppose sound to move 1100 feet in a second, how many miles distant is a cloud, in which lightning is observed 16 seconds before the thunder is heard, no allowance being made for the motion of light?

62. How much did a person spend in 64 days, who, with an annual income of \$8180, is 900 dollars in debt at the end of a year?

63. If 15 men, 12 women, and 9 boys, can complete a piece of work in 50 days, what time would 9 men, 15 women, and 18 boys take to do four times as much, the parts done by each in the same time being as numbers 3, 2, and 1?

64. A person possesses \$800 a year; how much may he spend per day in order to save \$48·25, after paying a tax of \$5 on every \$100 of income?

65. If 3 cows or 7 horses can eat the produce of a field in 29 days, in how many days will 7 cows and 3 horses eat it up?

66. How many yards of carpet, $\frac{3}{4}$ yd. wide, will cover a room whose width is 16 feet and length $27\frac{1}{2}$ feet?

67. When $\frac{1}{2}$ of $\frac{3}{4}$ of a gallon of wine costs $\$\frac{4}{5}$, what will $5\frac{1}{2}$ gallons cost?

68. A church-clock is set at 12 o'clock on Saturday night; at noon on Tuesday it is 3 minutes too fast: supposing its rate regular, what will be the true time when the clock strikes four on Thursday afternoon?

69. A person after paying a rate of 10 cents in the dollar has $7284 remaining; what had he at the first?

70. If a piece of work can be done in 50 days by 35 men working at it together, and if after working together for 12 days, 16 of the men were to leave the work; find the number of days in which the remaining men could finish the work.

71. A regiment of 1000 men are to have new coats; each coat is to contain $2\frac{1}{2}$ yards of cloth $1\frac{1}{4}$ yards wide; and it is to be lined with shalloon of $\frac{3}{4}$ yard wide; how many yards of shalloon will be required?

72. If 5 ounces of silk can be spun into a thread two furlongs and a half long, what weight of silk would supply a thread sufficient to reach to the Moon, a distance of 240,000 miles?

73. How many revolutions will a carriage-wheel, whose diameter is 3 feet, make in 4 miles? (See Ex. 54.)

74. If 8 oz. of sugar be worth $0·0625, what is the value of ·75 of a ton?

75. The price of ·0625 lbs. of tea is $0·4583; what quantity can be bought for $205?

76. Two watches, one of which gains as much as the other loses, viz. 2′. 5″ daily, are set right at 9 o'clock, A.M. on Monday; when will there be a difference of one hour in the times denoted by them?

77. How many yards of matting, 2·5 feet broad, will cover a room 9 yards long, and 20 feet broad?

78. A person bought 1008 gallons of spirits for $3200; 48 gallons leaked out: at what rate must he sell the remainder per gallon so as not to lose by his bargain?

79. If a soldier be allowed 12 lbs. of bread in 8 days, how much will serve a regiment of 850 men for the year 1860?

80. If 2000 men have provisions for 95 days, and if after 15 days 400 men go away; find how long the remaining provisions will serve the number left.

81. A gentleman has 10000 acres; what is his yearly rental, if his weekly rental for 20 square poles be 5 cents? (1 year=52 weeks.)

82. If an ounce of gold be worth $20·189583, what is the value of ·36822916 lbs?

83. If 1000 men have provision for 85 days, and if after 17 days 150 of the men go away; find how long the remaining provisions will serve the number left.

84. What is the quarter's rent of $182\cdot\dot{3}$ acres of land, at \$4·65 per acre for a year?

85. A grocer bought 2 tons, 3 cwt., 3 qrs. of goods for \$120, and paid 50 cents for expenses; what must he sell the goods at per cwt. in order to clear \$62 on the outlay?

86. What must be the breadth of a piece of ground whose length is $40\frac{1}{2}$ yards, in order that it may be twice as great as another piece of ground whose length is $14\frac{4}{5}$ yards, and whose breadth is $13\frac{2}{13}$ yards?

87. If 3·75 yards of cloth cost \$3·825, what will 38 yds. 2 qrs. 3 nails cost?

88. Four horses and 6 cows together find sufficient grass on a certain field; and 7 cows eat as much as 9 horses; what must be the size of a field relatively to the former, which will support 18 horses and 9 cows?

89. *A* alone can reap a field in 5 days, and *B* in 6 days, working 11 hours a day; find in what time *A* and *B* can reap it together, working 10 hours a day.

DOUBLE RULE OF THREE.

158. There are many questions, which are of the same nature with those belonging to the Rule of Three, but which if worked out by means of that Rule as before given, would require two or more distinct applications of it. Every such question, in fact, may be considered to contain two or more distinct questions belonging to the Rule of Three, and when each of those questions has been worked out by means of the Rule, the answer obtained for the last of them will be the answer to the original question.

159. The following example may serve to illustrate the preceding observations. "If the carriage of 15 cwt. for 17 miles cost me \$20 what would the carriage of 21 cwt. for 16 miles cost me?"

We observe that this question, though of a like nature with those which engaged our attention under the Rule of Three, is nevertheless of a more complicated description; and the student, without further explanation, would find some difficulty in obtaining an answer to it by means of a single application of the Rule. For we observe, that instead of three given quantities, we have five, every one of which must necessarily have a bearing on the answer, so that none of them can be superfluous. If however the question be divided into two distinct questions, each of these, when superfluous terms are rejected, will be found to comprise only three given terms of a proportion, from which three terms the fourth is to be ascertained; and the student would have no difficulty in working out each of these two questions by means of a single application of the Rule, so that in this way he will obtain the correct answer by applying the Rule of Three twice over.

The first question may be this; "If the carriage of 15 cwt. for 17 miles cost me \$20, what would the carriage of 21 cwt. for 17 miles cost me? In this question the 17 miles would have no effect upon the answer, because the distance is the same in both parts of the question, and the answer would clearly remain unaltered, if any other number of miles, or if the words "a certain distance," had been used instead of the 17 miles. This number may therefore be neglected as superfluous, and we have then three terms of a proportion remaining, and the fourth is to be found. Solving the question by the Rule of Three, we find that the answer will be \$28.

The second question may be this, "If the carriage of 21 cwt. for 17 miles cost me \$28, what will the carriage of 21 cwt. for 16 miles cost me?" In this question, for reasons similar to those before given, the 21 cwt., will be a superfluous quantity. Applying the Rule of Three to the question, we find the answer to be $\$26.35\frac{5}{17}$.

From the connection of the two questions with that originally proposed, we observe that $\$25\cdot35\frac{5}{17}$, thus obtained through two distinct applications of the Rule of Three, must be the answer to the original question.

160. We might give still more complicated instances, in which more than two distinct applications of the Rule of Three would be needed, in order to obtain the required answer; but the practical questions which most commonly occur, of the kind we have been treating of, would require only a double application of the Rule of Three, and, like the question which has been used by way of illustration, would comprise only five given quantities for the determination of a sixth, which is not given.

161. The Double Rule of Three is a shorter or more com-

pendious method of working out such questions as would require two or more applications of the Rule of Three; and it is sometimes called the Rule of Five, from the circumstance, that in the practical questions to which it is applied, there are commonly five quantities given to find a sixth.

162. For the sake of convenience, we may divide each question into two parts, the *supposition* and the *demand:* the former being the part which expresses the conditions of the question, and the latter the part which mentions the thing demanded or sought. In the question, "If the carriage of 15 cwt. for 17 miles cost me $20, what would the carriage of 21 cwt. for 16 miles cost me?" the words "if the carriage of 15 cwt. for 17 miles cost me $20," from the supposition; and the words, "what would the carriage of 21 cwt. for 16 miles cost me?" form the demand. Adopting this distinction we may give the following rule for working out examples in the Double Rule of Three.

163.* Rule. "Take from the supposition that quantity which corresponds to the quantity sought in the demand; and write it down as a third term. Then take one of the other quantities in the supposition and the corresponding quantity in the demand, and consider them with reference to the third term *only*, (regarding each other qantity in the supposition and its corresponding quantity in the demand as being equal to each other); when the two quantities are so considered, it from the nature of the case, the fourth term would be greater than the third, then, as in the Rule of Three, put the larger of the two quantities in the second term, and the smaller in the first term; but if less, put the smaller in the second term, and the larger in the first term.

"Again, take another of the quantities given in the supposition, and the corresponding quantity in the demand; and retaining the same third term, proceed in the same way to make one of those quantities a first term and the other a second term.

"If there be other quantities in the supposition and demand, proceed in like manner with them.

"In each of these statings reduce the first and second terms to the same denomination. Let the common third term be also reduced to a single denomination, if it be not already in that state. The terms may then be treated as abstract numbers."

Note. In dealing with the final statement obtained by our Rule, the two notes on Art. 156 will often be found useful.

"Multiply all the first terms together for a final first term, and all the second terms together for a final second term, and retain the former third term. In this final stating multiply the second and third terms together and divide the product by the first. The quotient will be the answer to the question in the denomination to which the third term was reduced."

Ex. 1. If a tradesman, with a capital of $2000, gain $50 in 3 months, how long will it take him, with a capital of $3000, to gain $175?

The 3 months in the supposition correspond with the quantity sought in the demand. We make the 3 months therefore the third term. Then taking the capital of $2000 in the supposition, and that of $3000 in the demand, and considering them with reference to the time in the third term, we see that if the amount of capital be increased, the time in which a given gain would be produced would be diminished, so that a fourth term would be less than the third; therefore we place the $3000 as a first term and $2000 as a second. Again, taking the gain of $50 from the supposition, and that of $175 from the demand, and considering them in like manner with reference to the time in the third term, we see that if the amount of gain be increased, the time in which a given capital would produce it must be increased also, so that here the fourth term would be greater than the third; and therefore we place the $50 as a first term, and the $175 as a second term; thus we have the following statements:

$$\left.\begin{matrix} \$3000 : \$2000 \\ \$50 : \$175 \end{matrix}\right\} :: 3\ m.$$

Proceeding according to our Rule, we have the following statement:

$$3000 \times 50 : 2000 \times 175 :: 3,$$

and the required number of months $= \dfrac{2000 \times 175 \times 3}{3000 \times 50}$

$$= \frac{2 \times 175}{50}$$

$$= \frac{175}{25} = 7.$$

The required answer is therefore 7 months.

Reason for the above process.

The tradesman, with a capital of \$2000, gains \$50 in 3 months. Let us first find, by the Rule of Three, how long he would be in gaining \$175 with the *same* capital. Thus,

$$\$50 : \$175 :: 3 \text{ m.} : \text{required time.}$$

$$\text{Required time} = \left(\frac{175 \times 3}{50}\right) \text{months.}$$

Since then the tradesman, with a capital of \$2000, would gain \$175 in $\left(\frac{175 \times 3}{50}\right)$ months, let us next find, by the Rule of Three, how long it would take him to gain the same sum with a capital of \$3000, and we must have the answer to the original question. Thus,

$$\$3000 : \$2000 :: \frac{175 \times 3}{50} \text{ months} : \text{required time.}$$

$$\text{Required time in months} = \left(\frac{175 \times 3}{50} \times 2000\right) \div 3000$$

$$= \frac{175 \times 3 \times 2000}{50} \div 3000$$

$$= \frac{175 \times 3 \times 2000}{50} \times \frac{1}{3000}$$

$$= \frac{175 \times 3 \times 2000}{50 \times 3000}$$

$$= \frac{2000 \times 3 \times 175}{3000 \times 50};$$

whence it appears that if we arrange the quantities given by the question as follows:

$$\left.\begin{matrix} \$3000 : \$2000 \\ \$50 : \$175 \end{matrix}\right\} :: 3\ m,$$

and treat the numbers as abstract; and then multiply the two first terms together for a single first term, and the two second terms together for a single second term; and then divide the product of the second and third terms by the first, we shall obtain the answer in that denomination to which the third term was reduced.

Or thus:

A capital of \$2000 gains \$50 in 3 months,

.... \$1 .... \$50 in (3×2000) months,

.... \$1 .... \$1 in $\left(\frac{3 \times 2000}{50}\right)$ months,

.... \$3000 .... \$1 in $\left(\frac{3 \times 2000}{50 \times 3000}\right)$ months,

.... \$3000 ... \$175 in $\left(\frac{3 \times 2000 \times 175}{50 \times 3000}\right)$ months,

or $\left(\frac{2000 \times 175 \times 3}{3000 \times 50}\right)$ months,

that is, if we arrange the given quantities as follows,

$$\left.\begin{matrix}\$3000 : \$2000 \\ \$50 : \$175\end{matrix}\right\} :: 3\ m,$$

we obtain the required time in months by multiplying the two first terms together for a final first term, the two second terms together for a final second term; and then dividing the product of the second and third terms by the first term.

Ex. 2. If a tradesman, with a capital of \$2000, gain \$50 in 3 months, what sum will he gain, with a capital of \$3000, in 7 months?

The \$50 in the supposition corresponds to the quantity sought in the demand. Make this \$50 the third term. Then taking the capital of \$2000 in the supposition, and that of \$3000 in the demand, and considering them with reference to the gain in the third term, we observe that if the amount of capital be increased, so also will be the gain in a given time, and thus the fourth term would be greater than the third; therefore we place the \$2000 as the first term, and the \$3000 as the second. Again, taking the 3 months in the supposition, and the 7 months in the demand, and considering them in like manner with reference to the gain in the third term, we observe, that as the time is increased so also will be the gain from a given capital, and thus the fourth term would be greater than the third; therefore we place the three months as a first term and the 7 months as a second.

We thus obtain the following statements:

$$\left.\begin{matrix} \$2000 : \$3000 \\ 3m : 7\,m \end{matrix}\right\} :: \$50.$$

Proceeding according to our Rule, we obtain the following statement:

$$2000 \times 3 \ : \ 3000 \times 7 :: 50,$$

$$\text{and the required sum in dollars} = \frac{3000 \times 7 \times 50}{2000 \times 3}$$

$$= \frac{3 \times 7 \times 50}{2 \times 3} = 7 \times 25 = 175.$$

The answer is therefore \$175.

Ex. 3. If 7 horses be kept 20 days for \$14, how many will be kept 7 days for \$28?

The 7 horses in the supposition correspond to the required quantity (number of horses) in the demand. Make this the third term. Then, taking the 20 days in the supposition, and the 7 days in the demand, and considering them with reference to our third term, we observe that if the number of days be diminished, the number of horses which can be kept in them for a given sum of money will be increased, and thus a fourth term would be greater than the third; we therefore place the 7 days in a first term, and the 20 days in a second. Again, taking the \$14 in the supposition, and the \$28 in the demand, and considering them with reference to the third term, we observe that if the sum be increased, the number of horses which can be kept by it in a given time will be increased also; so that here also a fourth term would be greater than the third; we therefore place the \$14 in a first term, and the \$28 in a second. We thus obtain the following statements:

$$\left.\begin{matrix} 7 \text{ days} : 20 \text{ days} \\ \$14 : \$28 \end{matrix}\right\} :: 7 \text{ horses},$$

which, by our Rule, will give the following single statement:

$$7 \times 14 \ : \ 20 \times 28 :: 7,$$

$$\text{and thus, the required number of horses} = \frac{20 \times 28 \times 7}{7 \times 14}$$

$$= 40.$$

The answer is therefore 40 horses.

Ex. 4. If I get 8 oz. weight of bread for 6*d.* when wheat is 15*s.* a bushel, what ought a bushel of wheat to be when I get 12 oz. of bread for 4*d.*?

The price of a bushel of wheat is required; to this the 15*s.* in the supposition corresponds. Place this as the third term. Then taking the 8 oz. in the supposition and the 12 oz. in the demand, and considering them with reference to the price in the third term, we observe that the greater the weight of bread we obtain for a given sum the less will be the price of a bushel of wheat, and so a fourth term would be less than the third; we therefore place the 12 oz. as a first term, and the 8 oz. as a second term. Again, taking the 6*d.* in the supposition and the 4*d.* in the demand, we consider that the less we pay for a given weight of bread, the less will be the price of a bushel of wheat, so that here also a fourth term would be less than the third; therefore we place the 6*d.* as a first term, and the 4*d.* as a second. Thus we have the following statements:

$$\left.\begin{array}{r} 12 \text{ oz.} : 8 \text{ oz.} \\ 6d. : 4d. \end{array}\right\} :: 15s.$$

which, by our Rule will give the following single statement:

$$12\times6 : 8\times4 :: 15,$$

and thus, the required price will be

$$\frac{8\times4\times15}{12\times6}s. = \frac{8\times15}{3\times6}s. = \frac{4\times5}{3}s. = \frac{20}{3}s. = 6s.\ 8d.$$

Ex. 5. If 20 men can perform a piece of work in 12 days, find the number of men who could perform another piece of work, 3 times as great, in $\frac{1}{5}$th of the time.

The first piece of work being reckoned as 1, the second must be reckoned as 3.

The 20 men in the supposition must be taken as the third term. Then, taking the piece of work (represented by 1) in the supposition, and the piece of work (represented by 3) in the demand, we observe that if the work be increased, the number of men to perform it in a given time must be increased, and we therefore place the 1 as a first term, and the 3 as a second. Again, taking the 12 days in the supposition and the $\frac{12}{5}$ days in the demand, we observe that if the number of days be diminished, the number of men required to perform any given work will be increased, and therefore we place the $\frac{12}{5}$ days as a first term, and the 12 days as a second term. Thus we have the following statements:

$$\left.\begin{matrix} 1 & : & 3 \\ \frac{12}{5}\text{ days} & : & 12\text{ days} \end{matrix}\right\} :: 20\text{ men},$$

which, by our Rule, will give the following single statement:

$$\tfrac{12}{5} : 3 \times 12 :: 20,$$

and thus the required number of men will be

$$\frac{3 \times 12 \times 20}{\frac{12}{5}} = \frac{3 \times 12 \times 20 \times 5}{12} = 300.$$

Ex. 6. If 252 men can dig a trench 210 yards long, 3 wide, and 2 deep, in 5 days of 11 hours each; in how many days of 9 hours each will 22 men dig a trench of 420 yds. long, 5 wide, and 3 deep?

The first trench contains $(210 \times 3 \times 2)$ cubic yds.
$=1260$ cubic yds.

The second.................$(420 \times 5 \times 3)$ cubic yds.
$=6300$ cubic yds.

On the supposition therefore that 252 men can remove 1260 cubic yds. of earth in 55 hours, we have to find in how many hours 22 men can remove 6300 cubic yds.

The 55 hours correspond to the quantity sought. Make this the third term. Then, taking the 252 men in the supposition, and the 22 men in the demand, we observe that if the number of men be diminished, the number of working hours in which a given work can be performed will be increased, and we therefore place the 22 men as a first term, and the 252 men as a second. Again, taking the 1260 cubic yds. in the supposition and the 6300 cub. yds. in the demand, we consider that if the number of cubic yds. be increased, the number of working hours in which a given number of men can perform the work will be increased also, and therefore we place the 1260 cubic yds. as a first term, and the 6300 cubic yds. as a second.

Then we have the following statements:

$$\left.\begin{matrix} 22\text{ men} & : & 252\text{ men} \\ 1260\text{ cub. yds.} & : & 6300\text{ cub. yds.} \end{matrix}\right\} :: 55\text{ hours},$$

which, by our Rnle, will give the following single statement:

$$22 \times 1260 : 253 \times 6300 :: 55,$$

and thus the required time

$$=\frac{252\times 6300\times 55}{22\times 1260}\text{ working hours}$$

$$=\frac{252\times 5\times 55}{22}\text{ working hours}$$

$$=3150\text{ working hours}$$

$$=\frac{3150}{9}\text{ days of 9 working hours}$$

$$=350\text{ such days.}$$

Ex. 7. If 560 flag stones, each $1\frac{1}{2}$ feet square, will pave a court-yard, how many will be required for a yard twice the size, each flag-stone being 14 in. by 9 in.?

Superficial content of each of former flag-stones

$$=(1\tfrac{1}{2}\times 1\tfrac{1}{2})\text{ sq. ft.}=(\tfrac{3}{2}\times\tfrac{3}{2})\text{ sq. ft.}=\tfrac{9}{4}\text{ sq. ft.}$$

Superficial content of each of the latter flag-stones

$$=(\tfrac{14}{12}\times\tfrac{9}{12})\text{ sq. ft.}=(\tfrac{7}{6}\times\tfrac{3}{4})\text{ sq. ft.}=\tfrac{7}{8}\text{ sq. ft.}$$

Considering the first court-yard as 1, and therefore the second as 2, our statements will be

$$\left.\begin{array}{rcl}\tfrac{7}{8}\text{ sq. ft.} & : & \tfrac{9}{4}\text{ sq. ft.}\\ 1 & : & 2\end{array}\right\} :: 560\text{ flag-stones,}$$

which, by our Rule, will give us the following single statement:

$$\tfrac{7}{8} : \tfrac{9}{4}\times 2 :: 560,$$

and thus the required number of flag-stones

$$=(\tfrac{9}{4}\times 2\times 560)\div\tfrac{7}{8}$$

$$=(\tfrac{9}{2}\times 560\times\tfrac{8}{7})$$

$$=\frac{9\times 560\times 8}{2\times 7}=2880.$$

Ex. 8. If 10 cannon, which fire 3 rounds in 5 minutes, kill 270 men in an hour and a half, how many cannon, which fire 5 rounds in 6 minutes, will kill 500 men in one hour?

The first 10 cannon, firing $\frac{3}{5}$ of a round in a minute, kill 270 men in $\frac{3}{2}$ hours. It is required to find how many cannon, firing $\frac{5}{6}$ of a round in a minute will kill 500 men in one hour.

The 10 cannon in the supposition correspond to the quantity sought in the demand. We make this the third term. Then, taking the $\frac{3}{5}$ of a round in the supposition and the $\frac{5}{6}$ of a round in the demand, we observe that if the part of a round which is fired in a minute be increased, the number of cannon for effecting a certain slaughter would be diminished; and therefore we place the $\frac{5}{6}$ of a round as a first term, and the $\frac{3}{5}$ of a round as the second. Again, taking the 270 men in the supposition and the 500 men in the demand, we observe that an increase in the number of men killed would require an increase in the number of cannon; and therefore we place the 270 men as a first term, and the 500 men as a second. Again, taking the $\frac{3}{2}$ hours in the supposition and the 1 hour in the demand, we consider that if the time in which a certain number of men are killed be diminished, the number of cannon would be increased; and therefore we place the 1 hour as a first term and the $\frac{3}{2}$ hours as a second. Our statements will therefore be,

$$\left.\begin{array}{rcl} \frac{5}{6} \text{ round} & : & \frac{3}{5} \text{ round} \\ 270 \text{ men} & : & 500 \text{ men} \\ 1 \text{ hour} & : & \frac{3}{2} \text{ hours} \end{array}\right\} :: 10 \text{ cannon},$$

which, by our Rule, will give us the following single statement:

$$\tfrac{5}{6}\times 270\times 1 : \tfrac{3}{5}\times 500\times \tfrac{3}{2} :: 10,$$

$$\text{or } 5\times 45 : 3\times 50\times 3 :: 10,$$

$$\therefore \text{ required number of cannon} = \frac{3\times 50\times 3\times 10}{5\times 45} = 20.$$

Ex. 9. A town which is defended by 1200 men, with provisions enough to sustain them 42 days, supposing each man to receive 18 oz. a day, obtains an increase of 200 men to its garrison; what must now be the allowance to each man, in order that the provisions may serve the whole garrison for 54 days?

The 1400 men will belong to the demand: for the question is, what must be the allowance to each man, when the garrison is increased to 1400 men, in order that the provisions may last 54 days.

The 18 oz. must clearly, according to our Rule, be the third term. Taking the 1200 men from the supposition, and the 1400 men from the demand, we consider that if the number of men be increased, the allowance to each must be diminished, in order that the provisions may last a given time; and we therefore place the 1400 men as a first term, and the 1200 men as a second. Again, taking the 42 days in the supposition and the 54 days in the demand, we consider that if the number of days during which a garrison must be sustained be increased, the allowance to each man must be diminished; and we therefore place the 54 days as a first term and the 42 days as a second term. Our statements will therefore be,

$$\left.\begin{array}{r} 1400 \text{ men} : 1200 \text{ men} \\ 54 \text{ days} : 42 \text{ days} \end{array}\right\} :: 18 \text{ oz.}$$

which, by our Rule, will give us the following single statement:

$$1400 \times 54 : 1200 \times 42 :: 18,$$

$$\therefore \text{ required allowance} = \frac{1200 \times 42 \times 18}{1400 \times 54} \text{ oz.} = 12 \text{ oz.}$$

so that 12 oz. will be the answer.

164. (1.) All questions in Proportion, simple and compound, may be solved by simple ratios, without the lengthened processes usually pursued in such cases; and when cancelling is employed, the operation is very much shortened. As the various terms, kinds of ratios, antecedents and consequents, perfect and imperfect, rising and falling, have already heen explained, to allude to them further is unnecessary. The problem under consideration should be carefully examined by the pupil, and as all such questions consist of perfect ratios, except one, and as the terms of each of these ratios consist of the same kinds of quantities, all we have to do is to place them together in vertical columns. All questions about the ratios should be asked with reference to the term sought, and the ratios put down *rising* or *falling*, as the terms affect the answer, in accordance with the requirements of the problem. An example will make this plain.

If 8 men mow a field in 12 days, how many men would do the same work in 4 days?

$$\begin{array}{r} x : 8 \\ 4 : 12 \end{array}$$

Here we first put down the imperfect ratio $x : 8$ using x (or any other mark) for the term in which it is deficient, next we ask the question, would 4 days require more or less men than 12 days? the answer is affirmative, and we put down the other as a rising ratio $4 : 12$.

which, after cancellation, will stand as follows:

$$\frac{12 \times 2}{1} = 24 \text{ men.}$$

(2.) Next let us add another ratio to the question :—If 8 men in 12 days, working 10 hours a day, mow a field, how many men could do it in 4 days, working 8 hours a day? Ans. 30. The work stands thus,

x : 8 (Imperfect ratio, men required).
4 : 12
8 : 10

The imperfect ratio x : 8 as before, we ask, using the term of demand with reference to the term wanting in the imperfect ratio, would 4 days require more or less men than 12 days in which to perform the same amount of labor? the answer is, 4 days would require more men (it is evident the shorter the time the more must be the men employed) then the ratio is a rising one, and put down 4 : 12; likewise we ask would 8 hours per day require more or less men than 10 hours a day? answer affirmative a rising ratio 8 : 10. Then we cancel, and as all the antecedents or factors of our divisor are reduced to unity we multiply the consequents remaining $3 \times 10 = 30$ for the answer.

(3.) Again let us introduce a fourth ratio, thus :—If 8 men mow a field of 20 acres in 12 days working 10 hours each day how many men could mow 16 acres in 4 days working 8 hours a day? Ans. 24 men.

x : 8 (Imperfect ratio, men required).
20 : 16
4 : 12
8 : 10

which after cancellation will stand as follows: $6 \times 4 = 24$ Ans.

This is the same as before, except the second ratio, would 16 acres require more or less men than 20 acres? less, a falling ratio 20 : 16.

(4.) The following problem having no fewer than seven ratios is just as easily solved as any of the preceeding, and so of any question so far as the statement is concerned.

If 5 compositors in 16 days of 14 hours long can compose 20 sheets of 24 pages in each sheet, 50 lines in a page and 40 letters in a line; in how many days of 7 hours long can 10 compositors compose a volume to be printed in the same letter, containing 40 sheets, 16 pages in a sheet, 60 lines in a page and 50 letters in a line? Ans. 32 days.

$$
\begin{array}{r@{\;:\;}l}
x & 16 \\
7 & 14 \\
10 & 5 \\
20 & 40 \\
24 & 16 \\
50 & 60 \\
40 & 50
\end{array}
$$

which after cancellation will stand as follows: $16 \times 2 = 32$ Ans.

We find by inspection the imperfect ratio and put it down x : 16; then we ask, would 7 hours require more or less days than 14? more, a rising ratio 7 : 14; would 10 compositors require more or less time than 5 compositors? less, a falling ratio 10 : 5; would 40 sheets require more or less than 20 sheets? more, a rising ratio 20 : 40; would 16 pages require more or less than 24 pages? less, a falling ratio 24 : 16; would 60 lines require more or less time than 50 lines? more, a rising ratio 50 : 60; lastly, would 50 letters require more or less than 40 letters? they would require more, a rising ratio, 40 : 50.

It is evident that if the value of x be substituted for it in these examples, both sides will just cancel; this must always be so when the ratios are put down correctly, as the product of all the antecedents equals the product of all the consequents.

Ex. LIII.

1. If 7 men can reap 6 acres in 12 hours, how many men will reap 15 acres in 14 hours?
2. If 3 men earn £15 in 20 days, how many men will earn 15 guineas in 9 days, at the same rate?
3. If 16 horses eat 96 bushels of corn in 42 days, in how many days will 7 horses eat 66 bushels?
4. If 800 soldiers consume 5 sacks of flour in 6 days, how many will consume 15 sacks in 2 days?
5. If 17 bushels be consumed by 6 horses in 13 days, what quantity will 8 horses eat in 11 days, at the same rate?
6. 16 horses can plough 1280 acres in 8 days, how many acres will 12 horses plough in 5 days?
7. If 11 cwt. can be carried 12 miles for \$1·50, how far can 36 cwt. 23 lbs. be carried for \$5·25?

8. If the carriage of 8 cwt. of goods for 124 miles be $30, what weight ought to be carried 53 miles for half the money?

9. If 5 men on a tour of 11 months, spend £641. 13*s.* 4*d.*, how much at the same rate would it cost a party of 7 men for 4 months?

10. If with a capital of $1000 a tradesman gain $100 in 5 months, in what time will he gain $49·50 with a capital of $225?

11. If it cost $184·30 to keep 3 horses for 7 months, what will it cost to keep 2 horses for 11 months?

12. The carriage of 4 cwt., 3 qrs., for 160 miles cost $3·85; what weight ought to be carried 100 miles for $30·05.

13. If 1 man can reap $345\frac{4}{7}$ sq. yds. in an hour, how long will 7 such men take to reap 6 acres?

14. If 20 men in 3 weeks earn $900, in what time will 12 men earn $1500?

15. If the carriage of 1 cwt., 3 qrs., 21 lbs. for $52\frac{1}{4}$ miles come to $3·25, what will be charged for $2\frac{1}{2}$ tons for $46\frac{1}{2}$ miles?

16. If 10 men can reap a field of $7\frac{1}{2}$ acres in 3 days of 12 hours each, how long will it take 8 men to reap 9 acres, working 16 hours a day?

17. If 25 men can do a piece of work in 24 days, working 8 hours a day, how many hours a day would 30 men have to work in order to do the same piece of work in 16 days?

18 If the rent of a farm of 17 ac., 3 ro., 2 po., be $390, what would be the rent of another farm, containing 26 ac., 2 ro., 23 po., if 6 acres of the former be worth 7 acres of the latter?

19. If 1500 copies of a book of 11 sheets require 55 reams of paper, how much paper will be required for 5000 copies of a book of 25 sheets, of the same size as the former?

20. If 5 men can reap a rectangular field whose length is 800 feet and breadth 700 ft. in $3\frac{1}{2}$ days of 14 hours each; in how many days of 12 hours each can 7 men reap a field whose length is 1800 ft. and breadth 960 ft.?

21. If a thousand men besieged in a town with provisions for 5 weeks, allowing each man 16 oz. a day, be reinforced with 500 men more, and have their daily allowance reduced $6\frac{2}{3}$ oz.; how long will the provisions last them?

22. If 20 masons build a wall 50 feet long, 2 feet thick, and 14 feet high, in 12 days of 7 hrs. each, in how many days of 10 hrs. each will 60 masons build a wall 500 feet long, 4 thick, and 16 high?

23. If 10 men can perform a piece of work in 24 days, how many men will perform another piece of work 7 times as great, in one-fifth of the time?

24. If 125 men can make an embankment 100 yards long, 20 feet wide, and 4 feet high, in 4 days, working 12 hours a day, how many men must be employed to make an embankment 1000 yards long, 16 feet wide, and 6 feet high, in 3 days, working 10 hours a day?

25. What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. broad, and 8 ft. 3 in. deep, when a block of the same stone 5 ft. long, 3 ft. 9 in. broad, and 2 ft. 6 in. deep, weighs 7500 lbs.?

26. If 100 men drink $200 worth of wine at $1·10 per bottle, how many men will drink $720 worth at $1·20. per bottle, in the same time, at the same rate of drinking?

27. If 5 horses require as much corn as 8 ponies, and 15 quarters last 12 ponies for 64 days, how long may 25 horses be kept for $205 when corn is $2·75 a bus.

28. If $42\frac{1}{2}$ yds. of cloth, which is 18 in. wide, cost $236·50, what will $118\frac{1}{4}$ yds. of yard-wide cloth of the same quality cost?

29. 124 men dig a trench 110 yds. long, 3 ft. wide, and 4 ft. deep, in 5 days of 11 hours each; another trench is dug by half the number of men in 7 days of 9 hours each; how many feet of water is it capable of holding?

30. If the 5 cent loaf weigh 3·35 lbs. when wheat is at $1·75 a bus., what ought to be paid for $47\frac{1}{2}$ lbs. of bread when wheat is at $2·40 a bus.?

31. A pit 24 ft. deep, 14 sq. ft. horizontal section cost $30 to dig out; how deep will a pit be of horizontal section 7 ft. by 9 ft. which costs $40·50?

32. The value of the paper required for papering a room, supposing it $\frac{3}{4}$ yard wide, and 6 cents a yard, is $10.75; what would it come to if it were 2 feet wide and 5 cents a yard?

33. 7 men working 16 days can mow a field of corn 1320 yards long and 880 wide; what will be the length of the side of a field 1320 yards broad, which 4 men can mow in 42 days?

34. A beam 16 feet long, $2\frac{1}{4}$ feet broad, and 8 inches thick, weighs 1280 lbs; what must be the length of another beam of the same material, whose breadth is $3\frac{1}{4}$ feet, thickness $7\frac{1}{2}$ inches, and weight 2028 lbs?

35. If 12 oxen and 35 sheep eat 12 tons, 12 cwt. of hay in 8 days, how much will it cost per month (of 28 days) to feed 9 oxen and 12 sheep, the price of hay being \$40 a ton, and 3 oxen being supposed to eat as much as 7 sheep?

36. If 1 man and 2 women do a piece of work in 10 days, find in how long a time 2 men and 1 woman will do a piece of work 4 times as great, the rates of working of a man and a woman being as 3 to 2?

37. A person is able to perform a journey of 142·2 miles in $4\frac{1}{2}$ days, when the day is 10·164 hours long; how many days will he be in travelling 505·6 miles, when the days are 8·4 hours long?

38. If the sixpenny loaf weigh 4·35 lbs. when wheat is at 5·75*s.* per bushel, what weight of bread, when wheat is 18·4*s.* per bushel, ought to be purchased for 18·13*s.*?

39. If a family of 9 people can live comfortably in England for 1560 guineas a year, what will it cost a family of 8 to live in Canada in the same style for seven months, prices being supposed to be $\frac{3}{5}$ of what they would be in England?

INTEREST.

165. Def. Interest is the sum of money paid for the loan or use of some other sum of money, lent for a certain time at a fixed rate; generally at so much for each \$100 for one year.

The money lent is called the Principal.

The interest of \$100 for a year is called the Rate per Cent.

The principal + the interest is called the Amount.

Interest is divided into Simple and Compound. When interest is reckoned only on the original principal, it is called Simple Interest.

When the interest at the end of the first period, instead of being paid by the borrower, is retained by him and added on as principal to the former principal, interest being calculated on the new principal for the next period, and this interest again, instead of being paid, is retained and added on to the last principal for a new principal, and so on; it is called Compound Interest.

SIMPLE INTEREST.

166. *To find the interest of a given sum of money at a given rate per cent. for a year.*

Rule. "Multiply the principal by the rate per cent., and divide the product by 100, as in (Art. 126)"

Note 1. The interest for any given number of years will of course be found by multiplying the interest for one year by the number of years; and the interest for any parts of a year may be found from the interest for one year, by Practice, or by the Rule of Three.

Note 2. If the interest has to be calculated from one given day to another, as for instance from the 30th of January to the 7th of February, the 30th of January must be left out in the calculation, and the 7th of February must be taken into account, for the borrower will not have had the use of the money for one day till the 31st of January.

Note 3. If the amount be required, the interest has first to be found for the given time, and the principal has then to be added to it.

Ex. Find the simple interest of $250 for one year at 5 per cent. per annum.

Proceeding according to the Rule given above

$$\begin{array}{r} \$250 \\ 5 \\ \hline \$12{\cdot}50 \end{array}$$

therefore the interest is $12·50

Reason for the Process.

The sum of $100 must have the same relation in respect of magnitude to $250 as the simple interest of $100 for a year has to the simple interest of $250 for a year; and thus the $100, $250, $5, and the required interest must form a proportion. (Art. 149).

We have then

$$\$100 : \$250 :: \$5 : \text{required interest},$$

$$\text{whence, required interest} = \frac{250 \times 5}{100} \text{(Art. 156)}.$$

which agrees with the Rule given above.

Examples worked out.

Ex. 1. Find the simple interest and amount of \$460.14 for 1 year, 10 months, at $3\frac{1}{4}$ per cent.

$$\begin{array}{r} \$460{\cdot}14 \\ 3\frac{1}{4} \\ \hline 1380{\cdot}42 \\ 115{\cdot}035 \\ \hline 1495{\cdot}455 \end{array} \qquad 4\,)\,\underline{\$460{\cdot}14} \\ \quad 115{\cdot}035$$

$$\begin{array}{lr} \text{Int. for 1 year} & =\$14{\cdot}95455 \\ \text{Int. for 6 mos., or } \frac{1}{2} \text{ of 1 year} = & 7{\cdot}477275 \\ \text{Int. for 4 mos., or } \frac{1}{3} \text{ of 1 year} = & 4{\cdot}98485 \\ \hline \therefore \text{Int. for 1 year, 10 months} & =\$27{\cdot}416675 \end{array}$$

$$\therefore \text{Amount} = \$460{\cdot}14 + \$27{\cdot}416675$$
$$= \$487{\cdot}556675.$$

Note. In examples like the above we may reckon 12 months to the year; but if calendar months are given, the interest will then be best found by the Rule of Three; as for instance in the following example:

Ex. 2. Find the simple interest and the amount of \$500, from June 15, 1843, to Aug. 27, 1843, at $4\frac{1}{2}$ per cent. ?

$$\begin{array}{r} 500 \\ 4{\cdot}5 \\ \hline 2500 \\ 2000 \\ \hline \$2250{\cdot}0 \end{array}$$

\$22·50=interest for 1 year.

The number of days from June 15 to Aug. 27

$$=15+31+27 = 73,$$

Hence, 365 days : 73 :: \$22·50,

whence it may be found that interest required=\$4·50;

$$\therefore \text{amount}=\$500+\$4{\cdot}50=\$504{\cdot}50.$$

Questions on Commission, Brokerage and Insurance, these charges being usually made at so much per cent., amount to the same thing as finding the interest on a given amount at a given rate for one year, and may therefore be worked by the Rule given above for Simple Interest.

There is, however, one case of Insurance which it may be well to notice by an example worked out.

Ex. If goods worth $1200 be insured at $16 per cent., to what amount must they be insured, so that in case of loss the party insured may recover the value of the goods and the premium?

If they be insured at their actual worth the premium paid will be lost, since the insurer will get $1200 only.

But if every ($100—16.), or $84, be insured for $100, then, in case of loss, the value of the goods $84 + $16, (the premium paid) will be recovered.

Thus we have

$84 : $1200 :: $100 : sum which is required to be insured; whence, sum required to be insured = $1428·571$\frac{3}{7}$.

Note. If it be required to find the interest on a sum of money in *£. s. d.* it will be necessary to reduce the given sum to the decimal of a pound, and then to proceed under the ordinary rule.

Ex. LIV.

1. Find the simple Interest

(1) On $85 for 1 year at 5 per cent.
(2) On $310 for 1 year at 4 per cent.
(3) On $1000 for 1 year at 4$\frac{1}{2}$ per cent.
(4) On $475 for 3 years at 5 per cent.
(5) On $936·75 for 2 years at 4 per cent.
(6) On 556·50 for 6 years at 5 per cent.
(7) On $945 for 2 years at 4 per cent.
(8) On $198 for 1 year at 3$\frac{1}{2}$ per cent.
(9) On 236\frac{1}{5}$ for 2$\frac{1}{2}$ years at 3 per cent.
(10) On $98 for $\frac{1}{2}$ year at 2$\frac{1}{2}$ per cent.

2. Find the amount

(1) Of $1000 for 2 years at 4$\frac{1}{4}$ per cent.
(2) Of 2833\frac{3}{4}$ for 4$\frac{1}{2}$ years at 3 per cent.
(3) Of $1050·625 for 6 years at 4$\frac{1}{4}$ per cent.
(4) Of 139\frac{5}{8}$ for 3$\frac{1}{2}$ years at 5$\frac{1}{2}$ per cent.
(5) Of $1895·35 for 4$\frac{3}{4}$ years at 2$\frac{3}{4}$ per cent.

(6) Of £1534. 6*s*. 3*d*. for $1\frac{5}{8}$ years at $3\frac{1}{4}$ per cent.
(7) Of £411. 10*s*. for $\frac{1}{4}$ year at $4\frac{7}{8}$ per cent.
(8) Of $1595·125 for $5\frac{3}{4}$ years at $3\frac{3}{4}$ per cent.

3. Find the simple Interest and Amonnt

(1) Of $375 for 3 years, 8 months, at $3\frac{1}{2}$ per cent.
(2) Of $$446\frac{1}{2}$ for 3 years, 3 months, at 5 per cent.
(3) Of $220 for 7 months at $3\frac{3}{4}$ per cent.
(4) Of £243. 10*s*. for 2 years, 5 months, at $4\frac{3}{4}$ per cent.
(5) Of $10 for 117 days at $3\frac{1}{2}$ per cent.
(6) Of $684 for 1 year 11 months, at $1\frac{1}{8}$ per cent.
(7) Of $500 from March 16, 1850, to January 23, 1851, at $5\frac{1}{8}$ per cent.
(8) Of $7290·56 for 2 years, 35 days, at $7\frac{3}{4}$ per cent.
(9) Of £34. 10*s*. from August 10 to October 21, at $6\frac{1}{2}$ per cent.

4. What is the annual cost of insuring $40000 worth of property at $\frac{1}{3}$ per cent. ?

5. What must be the sum insured at $4\frac{1}{2}$ per cent. on goods worth $19100 so that in case of loss the worth of the goods and the premium may be recovered ?

6. At $7\frac{1}{2}$ per cent. what will be the cost of insuring property worth $500, so that in the event of loss the worth of the goods and the premium of insurance may be recovered ?

167. *In all questions of Interest, if any three of the four (principal, rate per cent., time, amount) be given, the fourth may be found; as, for instance, in the following examples.*

Ex. 1. Find the amount of $225 in 4 years at $3\frac{1}{2}$ per cent., simple interest.

$$\text{Interest for 1 year} = \$\frac{225 \times \frac{7}{2}}{100} = \$\frac{9 \times \frac{7}{2}}{4}$$

$$= \$\frac{63}{8} = \$7{\cdot}875$$

$$\therefore \text{Interest for 4 years} = \$(7{\cdot}875 \times 4) = \$31{\cdot}50.$$

$$\text{Amount} = \text{Principal} + \text{Interest.}$$

$$= \$225 + \$31{\cdot}50$$

$$= \$256{\cdot}50$$

Ex. 2. In what time will $225 amount to $256.50 at $3\frac{1}{2}$ per cent?

$256·50—$225=$31·50 which is the interest to be obtained on $225 in order that it may amount to $256·50.

But Int. of $225 for 1 year=$7·875; which must have the same relation in respect of magnitude to the $31·50, as the one year has to the required time.

∴ $7·876 : $31·50 :: 1 year : required number of years.

whence required number of years=4.

Ex. 3. At what rate per cent., simple interest, will $225 amount to $256·50 in 4 years? In other words, at what rate per cent. will $225 give $31·50 for interest in 4 years, or $\frac{\$31\cdot50}{4}$, or $7·875 in one year?

Then $225 : $100 :: $7·875 : required rate per cent.,

whence required rate per cent.=$3\frac{1}{2}$.

Ex. 4. What sum of money will amount to $256·50 in 4 years at $3\frac{1}{2}$ per cent., simple interest?

$100 in 4 yrs. at $3\frac{1}{2}$ per cent., amounts to $100+$($3\frac{1}{2}\times4$) or $114; and this $114 must be to the $256·50 as the $100 is to the required sum of money;

∴ $114 : $256·50 :: $100 : required number of dollars,

whence required number of dollars=$225.

168. The student who has made sufficient progress in Algebra to understand the following equations, will find it advantageous to remember the formula $M=P+Pnr$, by which all examples in Interest may be easily worked, and which may be explained as follows:

Let P represent the principal (in dollars).
n .... number of years.
r .... interest of $1 for 1 year.
M .... amount of $$P$ in n years.

Then $1 : $$P$ :: r : interest of $$P$ for 1 year

or interest of $$P$ for 1 year=Pr.

.... n years=Prn.

(1) $\therefore m = P + Prn.$

(2) $\quad = P(1+rn.)$

and $m-P=Prn.$

(3) $\therefore \frac{m-P}{Pn} = r$

(4) also $\frac{m-P}{Pr} = n$

and since $m=P(1+rn.)$

(5) $\therefore \frac{m}{1+rn} = P.$

Wherefore having any three of the quantities $P.\ m.\ r.\ n.$ given, the fourth may easily be found.

Note.—Care must be taken not to misunderstand the meaning of r which is the interest of \$1, not of \$100.

169. The following formula, deduced from the rule under Art. 166, will be found applicable to all cases of interest when three of the four quantities (time, rate, interest, principal) are given, and from its simplicity will be easily understood and remembered.

(1) $\frac{prt.}{100} = i$

$\therefore prt. = i \times 100$

(2) $\therefore p = \frac{i \times 100}{rt}$

(3) and $r = \frac{i \times 100}{pt}$

(4) and $t = \frac{i \times 100}{pr}$

The following examples will illustrate this formula:

What is the interest of \$100 for 16 yrs. 8 mo., at 6 per cent?

(1) $i = \$\frac{100 \times 6 \times 16\frac{2}{3}}{100} = \$\frac{300}{3} = \$100.$

What is that sum, the interest of which at 7 per cent for 5 yrs. 4 mo. is \$728?

(2) $p = \$\frac{728 \times 100}{7 \times 5\frac{1}{3}} = \$\frac{104 \times 100}{\frac{16}{3}} = \$10400 \times \frac{3}{16} = \$1950.$

At what rate per cent. will \$600 gain \$300 in 5 years ?

(3) $r = \$\dfrac{300 \times 100}{600 \times 5} = \$10.$

How long will \$1800 be in gaining \$702 at 6 per cent ?

(4) $t = \dfrac{702 \times 100}{1800 \times 6} = \dfrac{117}{18} = 6\tfrac{1}{2} = 6$ yrs. 6 mo.

Ex. LV.

1. What sum will bring \$250 in 4 years at 5 per cent., simple interest ?

2. At what rate per cent. will \$540 amount to \$734 in 9 years, at simple interest ?

3. In what time will \$350 amount to \$402·50 at 3 per cent. simple interest ?

4. At what rate per cent. will \$752·60 amount to \$9970 in $3\tfrac{1}{2}$ years, at simple interest ?

5. In what time will \$729 amount to \$1250·85 at $7\tfrac{1}{2}$ per cent., simple interest ?

6. At what rate will \$729·37½ amount to \$5695 in 25 years at simple interest ?

7. What sum will produce for interest \$75 in $2\tfrac{1}{4}$ years at $6\tfrac{1}{2}$ per cent. simple interest ?

8. What sum will amount to \$529·30 in $3\tfrac{1}{2}$ years at $6\tfrac{1}{4}$ per cent. simple interest ?

9. What sum will amount to \$38·5 in 3 years at 4 per cent., simple interest ?

10. In what time will \$1275 amount to to \$1549·25 at $3\tfrac{4}{5}$ per cent. simple interest ?

11. At what rate per cent., simple interest, will \$936 amount to \$1157·75, in $4\tfrac{7}{8}$ years ?

12. In what time will \$125 double itself at 5 per cent. simple interest ?

13. What sum will amount to £425. 19*s*. $4\tfrac{4}{5}$*d*. in 10 years at $3\tfrac{1}{2}$ per cent. simple interest, and in how many more years will it amount to £453. 11*s*. 7*d*. ?

14. What sum of principal money, lent out at 5 per cent. per annum, simple interest, will produce in 4 years the same amount of interest as £250, lent out at 3 per cent. per annum, will produce in 6 years?

15. What time will be required for $2,500 to gain $470, at 8 per cent?

16. In what time will $284·75, at 5¾ per cent. give $18.75 interest?

17. What principal will in 7 years and 6 months, at 8 per cent. amount to $2600?

18. The interest of $120 for 2 years 9 months and 12 days is $13·36, what is the rate per cent?

19. What is the interest at 8⅔ per cent. of $384·25 from January 12th, 1859, to April 4th, 1860?

20. What is the amount of $87·95 for 2 years 3 months and 20 days, at 7 per cent?

21. What principal will, in 4 years and 9 months, at 8 per cent., give $19·38 interest?

22. The interest of $248 for 2 years 1 month and 20 days is $29·194, what is the rate per cent?

COMPOUND INTEREST.

170. *To find the Compound Interest of a given sum of money at a given rate per cent. for any number of years.*

Rule. "At the end of each year add the interest of that year, found by Art. (166), to the principal at the beginning of it; this will be the principal for the next year; proceed in the same way as far as may be required by the question. Add together the interest so arising in the several years, and the result will be the compound interest for the given period."

The reason for the above Rule is clear from what has been stated in Arts. (165 and 166).

Ex. Required the compound interest and the amount of $720 for 3 years at 5 per cent.

Proceeding as in Simple Interest for the 1st year;

$720
5
$36·00
$720=1st principal,
36=1st interest,
by addition, $756=2nd principal, of which find in. at 5 per cent.
5
$37·80=2nd interest,
$756=2nd principal,
37·80=2nd interest,
by addition, $793·80=3rd principal of which find in. as above.
5
$39·6900=3rd interest,
$793·80= principal for 3rd year,
39·69= interest for 3rd year,
by addition, ∴ $833.49= amount of $720 in 3 years at 5 per cent.

The compound interest for that time
=sum of interests for each year,
=$36+$37·80+$39·69=$113·49.

If the rate per cent. be a factor of a 100, the following method, by practice, will be more concise than the preceding. Thus:

$5=\frac{1}{20}$ of 100	$720	=1st principal,
	36	=1st interest,
$5=\frac{1}{20}$ of 100	756	=2nd principal,
	37·8	=2nd interest,
$5=\frac{1}{20}$ of 100	793·8	=3rd principal,
	39·69	=3rd interest,
	833 49	=amount in 3 years.

Note 1. It is customary, if the compound interest be required for any number of entire years and a part of a year, (for instance for $5\frac{3}{4}$ years), to find the compound interest for the 6th year, and then take $\frac{3}{4}$ths of the last interest for the $\frac{3}{4}$ths of the 6th year.

Note 2. If the interest be payable half-yearly, or quarterly, it is clear that the compound interest of a given sum for a given time will be greater as the length of each given period is less; the simple interest will not be affected by the length of each period.

Note 3. In working sums in compound interest it will generally be found advisable to reduce to decimals any vulgar fractions that may occur in the time given, or the rate per cent.

171. Any example in Compound Interest may be worked by remembering the formula, $M=P(1+r)n$.

Let P denote the principal (in dollars),
n .... number of years,
r .... interest of \$1 for 1 year,
M .... amount of \$$P$ in n years.

Since at the end of the first year, \$1 amounts to (\$1+r), and since the same proportion holds for each successive year, we obtain

$$\$1 : (\$1+r) :: \$P : \$P(1+r),$$

or the amount of \$$P$ in 1 year is \$$P(1+r)$.

Similarly,

$$\$1 : (\$1+r) :: \$P(1+r) : \$P(1+r)(1+r), \text{ or } \$P(1+r)^2,$$

or the amount \$$P$ in 2 years is \$$P(1+r)^2$.

Similarly, the amount of \$$P$ in n years is \$$P(1+r)^n$,

$$\text{or } M=P(1+r)^n;$$

In which equation any three of the quantities M, P, r, n being given, the fourth may be found.

Ex. Find the amount of \$200 in 2 years, at 4 per cent., Compound Interest.

$$\text{Here } P=\$200,\ n=2,\ r=\tfrac{4}{100},\ n=\cdot04;$$
$$\therefore M=P(1+r)^n=\$200\,(1\cdot04)^2$$
$$=\$216\cdot32$$

Ex. LVI.

1. Find the compound interest of \$2000 in 2 years at 4 per cent. per annum.

2. Find the amount of \$600 in 3 years at $3\frac{3}{4}$ per cent., allowing compound interest.

3. Find the compound interest of \$270 in 2 years, at 3 per cent.

4. Find the amount of \$690 for 3 years at $4\frac{1}{2}$ per cent., compound interest.

5. Find the amount of \$230·75. for 3 years, at 5 per cent., compound interest.

6. Find the difference in the amount of \$415·50, put out for 4 years at $2\frac{1}{2}$ per cent., 1st at simple, 2nd at compound interest.

7. Find the compound interest of \$130 in 3 years at 4 per cent. (interest being payable half-yearly).

8. What will \$1760·75 amount to in $2\frac{1}{2}$ years, allowing 4 per cent. compound interest?

9. A person lays by \$230 at the end of each year, and employs the money at $6\frac{1}{2}$ per cent., compound interest; what will he be worth at the end of 3 years?

10. Find the difference between the simple and compound interest of \$$416\frac{2}{3}$ for 2 years at $6\frac{1}{4}$ per cent.

11. What is the difference between the simple and the compound interest of £13,333. 6*s*. 8*d*., for 5 years, at 5 per cent.?

12. Find the amount of \$180 in 3 years at $4\frac{1}{2}$ per cent. compound interest.

13. What sum of money, put out at compound interest for 2 years at 5 per cent., will amount to \$100?

14. What sum at 10 per cent., compound interest, will amount in 2 years to \$264·60?

15. *A* and *B* each lend £256 for 3 years, at $4\frac{1}{2}$ per cent. per annum, one at simple interest, the other at compound interest; find the difference in the amount of interest they respectively receive.

PRESENT WORTH AND DISCOUNT.

172. *A* owes *B* \$600, which is to be paid at the end of 6 months from the present time; it is clear that, if the debt be discharged at once, (interest being reckoned, we will suppose, at 4 per cent. per annum,) *B* ought to receive a less sum of money than \$600; in fact, such a sum of money as will, being now put out at 4 per cent. interest, amount to \$600 at the end of 6 months. The sum which *B* ought to receive *now* is called the Present Worth of the \$600

due 6 months hence, and the sum to be deducted from the \$600, in consequence of immediate payment, which is in fact the interest of the Present Worth, is called the Discount of the \$600 discharged 6 months before it is due.

DEF. We may therefore define PRESENT WORTH to be the actual worth, at the present time, of a sum of money due some time hence, at a given rate of interest; and we may define the Discount of a sum of money to be the interest of the Present Worth of that sum, calculated from the present time to the time when the sum would be properly payable.

PRESENT WORTH.

173. RULE. Find the interest of \$100 for the given time at the given rate per cent., and state thus:

"\$100 + its interest for the given time at the given rate per cent. : given sum :: \$100 : present worth required."

Ex. 1. Find the present worth of \$600, due 6 months hence, at 4 per cent. per annum.

Proceeding according to the above Rule,

Interest of \$100 for 6 months at 4 per cent. is \$2.

$\therefore$ \$102 : \$600 :: \$100 : required present worth,

whence, required present worth = $\$588{\cdot}23\frac{9}{17}$.

The Reason for the above process is clear from the consideration, that \$100 in 6 months at 4 per cent. interest would amount to \$102, and therefore \$100 is the present value of \$102 due 6 months hence: and consequently we have

1st debt : 2nd debt :: 1st present worth : 2nd present worth.

Ex. 2. Find the present worth of \$838, due 19 months hence, at 6 per cent. simple interest.

Since the interest of \$100 for 19 months, at 6 per cent.

$$=\$(\tfrac{19}{12}\times 6)=\$\tfrac{19}{2}=\$9\tfrac{1}{2}$$

$\therefore$ $\$109\frac{1}{2}$: \$838 :: \$100 : required present worth,

whence, required present worth = \$765·29+

Ex. 3. What is the value, at 16 years of age, of a legacy of \$1000 payable at 21 years of age, allowing simple interest at 4 per cent.?

Since \$100 at 4 per cent. simple interest will in 5 years amount to \$120, therefore the present worth of \$120 due 5 years hence will at that rate be \$100.

Hence $\$120 : \$1000 :: \$100 :$ required value,

whence, required value $=\$833\frac{1}{3}$.

DISCOUNT.

174. Rule. Find the interest of \$100 for the given time at the given rate per cent., and state thus:

\$100+its interest for the given time at the given rate per cent.: given sum :: interest of \$100 for the given time at the given rate per cent. : discount required."

Ex. 1. Find the discount of \$600, due 6 months hence, at 4 per cent. per annum.

Proceeding according to the above Rule,

The interest of \$100 for 6 months at 4 per cent.=\$2; therefore proceeding according to the Rule,

$\$102 : \$600 :: \$2$ required discount,

whence, required discount $=\$11{\cdot}76\frac{8}{17}$.

The reason for the above process is clear from the consideration, that \$2 is the interest for 6 months, at 4 per cent., of \$100, the present worth of \$102 due at the end of that time; and consequently we have

1st debt : 2nd debt :: discount on 1st debt : discount on 2nd debt.

Ex. 2. Find the discount on \$1000, due 15 months hence, at 5 per cent. per annum.

The interest of \$100 for 15 months at 5 per cent.=\$6·25.

$\therefore \$106{\cdot}25 : \$1000 :: \$6{\cdot}25 :$ required discount,

whence required discount $=\$58{\cdot}82\frac{1}{3}$ nearly.

Ex. 3. Find the discount on £127. 2*s* for half-a-year at 5 per cent.

$£100\frac{5}{2} : £127\frac{1}{10} :: £\frac{5}{2} :$ required discount;

whence required discount=£3. 2*s*.

Note 1. Discount=given sum *less* Present Worth; Present Worth=given sum *less* Discount.

Note 2. Bankers and Merchants in discounting bills calculate interest, instead of discount, on the sum drawn for in the bill, from the time of their discounting it to the time when it becomes due, adding THREE DAYS OF GRACE, which are usually allowed after the time a bill is NOMINALLY due, before it is LEGALLY due; which is of course an additional advantage. When a bill is payable on demand, the days of grace are not allowed.

Note 3. If a bill, without the days of grace, should appear to be due on the 31st of any month which contains only 30 days, the last of that month, and not the first day of the next, is considered as the day on which the bill is due. Thus a bill drawn on the 31st of October, at 4 months, would be really due, adding in the days of grace, on the 3rd of March. Also bills which fall due on Sunday, are paid in England on the previous Saturday.

Ex. A bill of \$1000 is drawn on Feb. 16th, 1851, at 7 months' date; it is discounted on the 8th of July at 5 per cent. What does the banker gain by the transaction?

The bill is legally due on Sept. 19; and from July 8 to Sept. 19 are 73 days.

The interest of \$1000 for the time = \$10.
The true discount .... = $9{\cdot}90\frac{90}{101}$
∴ the banker's gain .... $\${\cdot}09\frac{11}{101}$.

Ex. LVII.

1. Find the Present worth of

(1)	\$233½	due 1 year hence,	at 5	pr. ct. pr. ann., simple interest.
(2)	\$252.9675	....	6½	....
(3)	\$676⅔	..6 months	3	....
(4)	\$284·90	..6	3½	....
(5)	\$460·50	..7	4	....
(6)	\$390	..7	3¼	....
(7)	\$572	..8	7½	....
(8)	\$1261·05	..1	1	....
(9)	\$35	..4	4½	....
(10)	\$1250	..3	6	....
(11)	\$2110	.11	5	....
(12)	\$275⅓	.15	4	....
(13)	\$918	..4 years	5	....
(14)	\$500	.19 months	5½	....
(15)	\$800	.20 years.	5½	....
(16)	\$2197	..3 years.	4	........compound interest

2. Find the Discount on

(1) $\$63\frac{1}{3}$ due 4 months hence, at 4 per cent. per annum, [simple interest.

(2)	\$1380·375	..9	3 ....
(3)	\$107·25	..6	5 ....
(4)	\$125·50	..3	$3\frac{1}{2}$ ....
(5)	\$487	..5	$3\frac{1}{2}$ ....
(6)	\$340	..5	4 ....
(7)	£3640	.10	$4\frac{4}{5}$ ....
(8)	£813 9*s.*	..$1\frac{1}{3}$	$4\frac{3}{4}$ ....
(9)	£250 15*s.*	.17 months ...	5 ....
(10)	\$55	146 days......	$4\frac{3}{4}$ ....

(11) A bill of \$649 is dated on June 23, 1853, at 6 months, and is discounted on July 8, at $3\frac{1}{4}$ per cent.; what does the banker gain thereby?

(12) Find the true discount on a bill drawn March 17, 1859, at 3 months, and discounted May 2, at $5\frac{3}{4}$ per cent.

(13) Find the simple interest on \$545 in 2 years, at $3\frac{1}{2}$ per cent. per annum; and the discount on \$583·15, due 2 years hence, at the same rate of interest. Explain clearly why these two sums are identical.

(14) Explain the difference between Discount and Interest.

Five volumes of a work can be bought for a certain sum, payable at the end of a year; and six volumes of the same work can be bought for the same sum in ready money: what is the rate of discount?

(15) A tradesman marks his goods with two prices, one for ready money, and the other for one year's credit allowing discount, at 5 per cent.; if the credit price be marked £2 9*s.*, what ought to be the cash price?

STOCKS, BROKERAGE AND COMMISSION.

175. Stocks are Government funds, the Capital of Banks and other Joint Stock Companies.

Brokerage is the allowance paid to a Broker, or dealer in stocks or bills of exchange for transacting business. This allowance in

England is £$\frac{1}{8}$ or 2*s.* 6*d.* per cent. The rate of allowance varies, however, on this continent.

Commission is an allowance made to an agent for the sale, purchase, or care of property, on the money employed.

The *capital* of a company, or *money paid in*, is divided into *shares*, which are owned by *Stockholders.* The original cost of a share is its *par value.* If it sell in the market for more than its original, it is said to be *above par*, or at an advance; if it sell for less, it is *below par*, or at a discount.

The original cost of a share is usually $100, though it is sometimes $25, $50, $500, &c.

The rise and fall in stocks is a per cent. on the par value. Thus a share, whose par value is $\$\frac{100}{100}$, at 16 per cent. advance, will bring $\frac{116}{100}$ of its original cost; at 16 per cent. discount it will bring but $\frac{84}{100}$ of its original cost.

The profits of these companies are every year, or every half-year, divided among the Stockholders. The amount so paid out is called a *dividend.*

The actual cost of $100 share in any company, when, for instance, the $100 share is worth in the market 94\frac{1}{4}$, will be 94\frac{1}{4}$ + the brokerage. The actual sum received on such share where the brokerage is $\frac{1}{4}$ per cent. will therefore be (94\frac{1}{4}$—$\frac{1}{4}$) $94.

All examples in Stocks depend on the principles of Proportion; those of most frequent occurrence will be now explained.

Ex. 1. Required the sum which will purchase $1500 worth of mining shares, at $82; the $100 share.

In this case $100 stock cost $82 in money;

∴ $100 : $1500 :: $82 money ; required sum of money.

whence, required sum of money=$1200.

Ex. 2. What amount of Railway stock will $4050 purchase, the market value of the $100 share being $90?

In this case $90 money will purchase $100 stock;

∴ $90 : $4050 :: $100 stock ; required amount of stock.

whence, required amount of stock=$4500.

Ex. 3. If I invest $1520 in a mining company paying 3 per cent., the stock being 93\frac{1}{4}$ per $100 share, and pay $$\frac{1}{4}$ for brokerage, what does it cost me?

Every \$100 stock costs me (\$$93\frac{1}{4}+\frac{1}{8}$), or \$$93\frac{3}{8}$;

$\therefore$ \$100 stock : \$1520 stock :: \$$93\frac{3}{8}$: required sum of money;

whence, required sum of money=\$1419·30.

Ex. 4. What sterling money shall I receive for £1920 13*s*. 4*d*. in the $3\frac{1}{2}$ per cent. government stock at $98\frac{7}{8}$, brokerage being £$\frac{1}{8}$ per cent.?

£100 stock realizes £$(98\frac{7}{8}-\frac{1}{8})=98\frac{3}{4}$;

$\therefore$ £100 stock : £$1920\frac{2}{3}$ stock :: £$98\frac{3}{4}$: required sterling money;

whence, required sterling money=£1896. 13*s*. 2*d*.

Ex. 5. If I invest £7927 10*s*. in the 3 per cent. government stock at $94\frac{3}{8}$, what annual income shall I receive from the investment?

For every £$94\frac{3}{8}$ I get £100 stock, and the interest on £100 stock is £3; therefore for every £$94\frac{3}{8}$ of money I get £3 interest;

$\therefore$ £$94\frac{3}{8}$: £7927 10*s*. :: £3 : required annual income;

whence, required annual income=£252.

Note 1. If it be required to find the income arising from a certain quantity of stock, it is merely a question of simple interest.

Note 2. It may be noticed in the above examples, that when the question was simply to find the amount of stock, or money realized by sale of stock, the 3, 4, or other rate per cent. never entered into the *statement*; and when the question was simply to find income arising from any sum invested in the funds, then the \$100 never entered into the *statement*.

Note 3. All questions of the transfer of stock from one kind to another, belong to the Rule of Three inverse.

Note 4. The par value of all English Government stock is £100 per share.

Ex. LVIII.

1. Find the quantity of Railway stock, the par value being \$100 per share, purchased by investing:

(1) \$2850 paying 3 per cent. at 75.
(2) \$712 paying $3\frac{1}{2}$ per cent. at 89.
(3) \$504 paying 6 per cent. at 96.
(4) \$883·50 paying 4 per cent. at 93.
(5) \$3741 paying $3\frac{3}{4}$ per cent. at 87.
(6) \$500 paying 3 per cent. at $83\frac{3}{4}$.

2. Find the yearly income arising from the investment in government securities, par value the same as preceding.

(1) \$1008 paying 6 per cent. at 84.
(2) \$5580 paying 6 per cent. at 93.
(3) \$1138·50 paying $4\frac{1}{2}$ per cent. at 92.
(4) \$1638 paying $4\frac{1}{2}$ per cent. at $93\frac{3}{5}$.
(5) \$2000 paying 6 per cent. at $88\frac{1}{2}$.
(6) £3500 in the 3 per cent. consols at $94\frac{1}{8}$, brokerage $\frac{1}{8}$ per cent.

3. Find the quantity of Bank stock purchased by investing

(1) \$800 paying 4 per cent. at $75\frac{1}{2}$.
(2) \$4311 paying $3\frac{1}{2}$ per cent. at $85\frac{3}{8}$.
(3) \$2000 paying $3\frac{1}{2}$ per cent. at 94.
(4) \$2353 paying 3 per cent. at $90\frac{3}{8}$, brokerage $\frac{1}{8}$ per cent.
(5) \$3277 paying 4 per cent. at $105\frac{7}{8}$, brokerage $\frac{1}{8}$ per cent.
(6) \$10000 paying $3\frac{1}{4}$ per cent. at $99\frac{1}{4}$, brokerage $\frac{1}{8}$ per cent.

4. (1) I bought 27 shares of Railway Stock, at 13 per cent. discount and sold them again at an advance of 2 per cent. How much did I gain by the operation? The par value was \$100.

(2) A gentleman paid a broker $\frac{3}{4}$ per cent. to invest \$19278 in government funds. How much was the brokerage?

(3) A person in England transfers £1000 stock from the 4 per cents at 90, to the 3 per cents at 72; find the alteration in his income.

(4) Which is the better investment, \$1896 in city debentures, paying 6 per cent. at 87, or in Railway shares, paying $6\frac{1}{3}$ at \$89?

(5) A lady has a bequest of \$10500; she paid an agent $2\frac{1}{2}$ per cent. commission per annum to take care of the money for her. How much did the commission amount to?

(6) A factor sells 43 bales of cotton at \$375 per bale, and charges 2 per cent. commission. How much money must he pay to his principal?

(7) What is the value of 50 shares of Bank stock, 12 per cent. below par; the par value of which was \$200 per share?

(8) Find the income produced by \$12600, and invested in a blanket factory, paying $7\frac{1}{2}$ per cent., the stock being at the time of purchase \$85 per \$100 share.

PARTIAL PAYMENTS—FORMS OF NOTES.

176. When notes, bonds or obligations receive only partial payments or endorsements, the following Rule is that usually adopted:

RULE. The rule for casting interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceed the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance, as aforesaid.

Ex.

$620 HAMILTON, Nov. 1, 1857.

For value received, I promise to pay SAMUEL THOMPSON, or order, the sum of six hundred and twenty dollars on demand, with interest.

THOMAS JONES.

The following endorsements were made on this note:—1858, Oct. 6, received $61·07; March 4th, 1859, $89·03; Dec. 11, 1859, $107·77; July 20th, 1860, $200·50.

What is the balance due, Oct. 15, 1860, allowing 7 per cent. interest?

The amount of note, or principal, is	$620·000
Int. on the same to Oct. 6, 1858, at 7 per cent., is	40·386
Amount due on note, Oct. 6, 1858, is	660·386
The first endorsement is	61·070
	599·316
Interest from Oct. 6, 1858, to March 4, 1859, is	17·247
Amount due March 4, 1859, is	616·563
The second endorsement is	89·030
	527·533
Interest from March 4, 1859, to Dec. 11, 1859, is	28·414
	555·947
The third endorsement is	107·770
	448·177
Interest from Dec. 11, 1859, to July 20, 1860, is	19·085
	467·262
The fourth endorsement is	200·500
	266·762
Interest from July 20, 1860, to Oct. 15, 1860, is	4·409
Ans.	271·171

Ex. LIX.

$350 TORONTO, May 1, 1856.

1. For value received, I promise to pay TIMOTHY SNOOKS, or order, three hundred and fifty dollars, with interest, at 6 per cent.

M. BROWN.

Dec. 25, 1856, there was endorsed $50; June 30th 1857, $5; Aug. 22, 1858, $15; June 4, 1859, $100. How much was due April 5, 1860?

143\frac{50}{100}$ LONDON, C. W. Aug. 1, 1857.

2. For value received, I promise to pay HENRY TOMS, or bearer, one hundred and forty-three dollars, and fifty cents, on demand, with interest.

S. JONES.

Dec. 17, 1857, there was endorsed $37·40; July 1st, 1858, $7·09; Dec. 22, 1859, $13·13; Sept. 9, 1860, $50·50. How much remains due Dec. 28, 1860, the interest being 7 per cent.?

PROFIT AND LOSS.

177. Def. All questions in Arithmetic which relate to gain or loss in mercantile transactions, fall under the head of Profit and Loss.

Examples in Profit and Loss are worked by the principle of Proportion: various examples will now be worked out by way of illustration.

Ex. 1. If a cask of wine containing 84 gallons cost \$412.50, what is gained by selling it at \$6 per gallon?

The gain = selling price *less* first cost;
the selling price$=\$(6\times 84)=\$504$;
therefore the gain$=\$504-\$412\cdot50=\$91\cdot50$.

Ex. 2. A ream of paper cost me \$5·25 what must I sell it at, so as to realize 20 per cent.?

The reasoning in this case is, If \$100 gain \$20, or produce \$120, what will \$5·25 produce?

$\therefore \$100 : \$5\cdot25 :: \$120 :$ required amount in dollars,
whence, required amount$=\$6\cdot30$.

Ex. 3. A man buys 33 geese for \$30·50; at how much per head must he sell them to gain 10 per cent. on his outlay?

In this case,

$\$100 : \$30\cdot50 :: \$110 :$ selling price of the geese in dollars,
whence, selling price $=\$33\cdot55$.
$\therefore$ selling price of each goose $=\$\frac{33\cdot55}{33}=\$1\cdot01\frac{2}{3}$.

Ex. 4. A person buys shares in a railway when they are at £19½, £15 having been paid, and sells them at £32. 9*s*. when £25 has been paid: how much per cent. does he gain?

He buys each share at £19½, and he afterwards pays upon it £(25—15), or £10; therefore at the time he sells, he has paid on each share £29. 10*s*.; therefore by selling at £32. 9*s*. he gains on each £29. 10*s*.; therefore by selling at £32. 9*s*. he gains on each £29. 10*s*. which he has paid (£32. 9*s*.—£29. 10*s*.)=£2. 19*s*.;

$\therefore £29\frac{1}{2} : £100 :: £2\frac{19}{20} :$ gain per cent. in pounds;
whence, gain per cent.=£10, or gain is 10 per cent.

Ex. 5. What was the prime cost of an article, which when sold for \$12, realized a profit of 20 per cent.?

Here what cost £100 would be sold for \$120;

$\therefore$ \$120 : \$12 :: \$100 : prime cost in dollars,

whence, prime cost = \$10.

If the above example had been, "What was the prime cost of an article, which when sold for \$12, entails a loss of 20 per cent.?"

then \$80 : \$12 :: \$100 : prime cost in dollars,

whence, prime cost = \$15.

Ex. LX.

1. Bought 5 cwt. 3 qrs. 14 lbs. of cheese at \$9·50 per cwt., and sold it again for \$11·20 per cwt. What was the gain upon the whole?

2. If 5 cwt. 3 qrs. 14 lbs. be bought for \$47 and sold for \$65·80 what is the rate of gain per cwt.?

3. Find the total value of 43 articles at £4. 6*s*. 8*d*. each, 57 at £11. 8*s*. 4*d*. each. and 4 at £13. 15*s*. 4*d*. each. What is gained or lost by selling them at the rate of 3 for £28?

4. A person buys 400 yards of silk at \$500 and sells 300 yards at \$1·50 a yard, and the rest, which is damaged, at \$·75 a yard; find how much per cent. he gains or loses.

5. A grocer buys 2 cwt. of sugar at 10 cents per pound, and 4 cwt. at 9 cents; he sells 3 cwt. at $9\frac{1}{2}$ cents per pound; at what rate per pound will he be able to sell the remainder so as neither to gain nor lose by the bargain?

6. If a commodity be bought for \$6.50 a cwt. and sold for 9 cents a lb., find the rate of profit per cent.

7. Bought goods at 10 cents per pound, and sold them at \$7.50 per cwt.; what is the gain or loss per cent.?

8. An article which cost 3*s*. 6*d*. is sold for 3*s*. $10\frac{1}{2}$*d*.; find the gain per cent.

9. Goods were sold at \$12, at a profit of $22\frac{38}{41}$ per cent.; what was the prime cost?

10. If a tradesman gain \$5·50 on an article which he sells for \$22, what is his gain per cent.?

11. A man sells a cow for \$24·60, and loses \$18. per cent. on what the cow cost him; what was the original cost?

12. By selling an article for $5. a person loses 5 per cent. ; what was the prime cost, and what must he sell it at to gain $4\frac{1}{2}$ per cent. ?

13. The cost price of a book is $$\frac{1}{3}$; the expense of sale 5 per cent. upon the cost price ; and the profit 25 per cent. upon the whole outlay ; find the selling price of the book.

14. If by selling an article for $25·50, 8 per cent. be lost, what per cent. is gained or lost if it be sold at $38 ?

15. I bought 500 sheep at $2·10 a-head ; their food cost me $27\frac{1}{2}$ cents. a-head : I then sold them at $2·425 a-head. Find my whole gain, and also my gain per cent.

16. A person having bought goods for $40 sells half of them at a gain of 5 per cent. ; for how much must he sell the remainder so as to gain 20 per cent. on the whole ?

17. A person has goods worth $30 ; he sells one-third of them so as to lose 10 per cent. ; what must he sell the remainder at so as to gain 20 per cent. on the whole ?

18. I buy a house for $5000, and sell it immediately at a profit of 30 per cent. ; what do I receive, supposing the expenses of the sale to be 5 per cent. ?

19. The prime cost of a 76-gallon cask is $23·625, but 13 gallons are lost by leakage ; 9 gallons of water is then mixed with the remainder, and it is sold at $·375 a gallon. Find the whole gain, and also the gain per cent.

20. A stationer sold quills at 11*s.* sterling a thousand, by which he cleared $\frac{3}{8}$ of the money ; he raises the price to 13*s.* 6*d.* What does he clear per cent. by the latter price ?

21. A person sold 72 yards of cloth for $43·50 ; his profit being the cost of 11·52 yards ; how much did he gain per cent. ?

26. A person expends $3000 in railway shares at $15\frac{1}{2}$ per cent. discount, and sells them at par ; what does he gain by the transaction, and what per cent. ?

27. A wine-merchant bought $14\frac{1}{2}$ pipes of wine, which having received damage, he sold for $1120 $\frac{1}{2}\frac{4}{5}$, thereby losing 20 per cent. ; find the cost of the wine per pipe, and the selling price of it per gallon.

28. A farm is let for £96 and the value of a certain number of quarters of wheat. When wheat is 38*s.* a quarter, the whole

rent is 15 per cent. lower than when it is 56s. a quarter. Find the number of quarters of wheat which are paid as part of the rent.

29. A man having bought a lot of goods for $150, sells $\frac{1}{3}$rd at a loss of 4 per cent.; by what increase per cent. must he raise that selling price, in order that by selling the rest at the increased rate, he may gain 4 per cent. on the whole transaction?

30. A person bought a French watch, bearing a duty of 25 per cent., and sold it at a loss of 5 per cent.; had he sold it for $3 more, he would have cleared 1 per cent. on his bargain. What had the French maker for it?

DIVISION INTO PROPORTIONAL PARTS.

178. *To divide a given number into parts which shall be proportional to certain other given numbers.*

This is merely an application of the Rule of Three; still it may be well to state a general Rule, by which examples which come under the above head may be worked.

RULE. State thus: "As the sum of the given parts : any one of them :: the entire quantity to be divided : the corresponding part of it."

This statement must be repeated for each of the parts, or at all events for all but the last part, which of course may either be found by the Rule, or by subtracting the sum of the values of the other parts from the entire quantity to be divided.

Ex. 1. Divide $40 among *A*, *B*, and *C*, so that their proportions may be as 7, 11, and 14, respectively.

Proceeding according to the Rule given above,

32 : 7 :: 40 dollars : *A*'s share,
32 : 11 :: 40 dollars : *B*'s share,

whence *A*'s share=$8·75, and *B*'s share= $13·75.

C's share may be found from the proportion

32 : 14 :: 40 dollars : *C*'s share;

whence *C*'s share=$17·50;

or by subtracting $8·75+$13·75, or $22·50, from $40, which leaves $17·50, as above.

The reason for the above process is clear from the consideration, that 40 dollars is to be divided into 32 equal parts, of which A is to have 7 parts, B 11, and C 14.

Ex. 2. Divide \$11000 among 4 persons, A, B, C, D, in the proportions of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

$$\text{Sum of shares} = \tfrac{77}{60};$$

$$\therefore \tfrac{77}{60} : \tfrac{1}{2} :: \$11000 : A\text{'s share in dollars},$$

$$\text{whence } A\text{'s share} = \$4285\tfrac{5}{7}$$

Similarly,

$$B\text{'s share} = \$2857\tfrac{1}{7},\ C\text{'s share} = \$2142\tfrac{6}{7}.$$

$$D\text{'s share} = \$1714\tfrac{2}{7}.$$

Ex. 3. Divide \$45000 among A, B, C, and D, so that A's share : B's share :: 1 : 2, B's : C's :: 3 : 4, and C's : D's :: 4 : 5.

In this case,

$$B\text{'s share} = 2\ A\text{'s share},\ 3\ C\text{'s share} = 4\ B\text{'s share},$$

$$4\ D\text{'s share} = 5\ C\text{'s share};$$

$$\therefore \text{ we have } C\text{'s share} = \tfrac{4}{3}\ B\text{'s share} = \tfrac{8}{3}\ A\text{'s share},$$

$$\text{and } D\text{'s share} = \tfrac{5}{4}\ C\text{'s share} = \tfrac{10}{3}\ A\text{'s share};$$

$$\therefore A\text{'s share} + B\text{'s share} + C\text{'s share} + D\text{'s share}$$

$$= A\text{'s share} \times (1 + 2 + \tfrac{8}{3} + \tfrac{10}{3}),$$

$$= 9\ A\text{'s share};$$

$$\therefore A\text{'s share} = \$5000,\ B\text{'s} = 10000,\ C\text{'s} = \$13333\tfrac{1}{3},$$

$$D\text{'s} = \$16666\tfrac{2}{3}.$$

FELLOWSHIP OR PARTNERSHIP.

179. Def. Fellowship or Partnership is a method by which the respective gains or losses of partners in any mercantile transactions are determined.

Fellowship is divided into Simple and Compound Fellowship; in the former, the sums of money put in by the several partners continue in the business for the same time; in the latter, for different periods of time.

SIMPLE FELLOWSHIP.

180. Examples in this Rule are merely particular applications of the Rule in Art. (178), and that Rule therefore applies.

Ex. 1. Two merchants, *A* and *B*, form a joint capital; *A* puts in \$240, and *B* \$360: they gain \$80. How ought the gain to be divided between them?

\$(240+360) : \$240 :: \$80 : *A's* share in dollars,
whence, *A's* share=\$32, and ∴ *B's* share=\$48.

Note. The estate of a Bankrupt may be divided among his creditors by the same method.

Ex. 2. A bankrupt owes three creditors, *A*, *B*, and *C*, \$175, \$210, and \$265, respectively; his property is worth \$422·50: what ought they each to receive?

\$650 : \$175 :: \$422½ : *A's* share,
\$650 : \$210 :: \$422½ : *B's* share,
whence *A's* share=\$113¾ *B's* share=£136½;
∴ *C's* share=\$172¼.

COMPOUND FELLOWSHIP.

181. Rule. "Reduce all the times into the same denomination, and multiply each man's stock by the time of its continuance, and then state thus:

As the sum of all the products : each particular product :: the whole quantity to be divided : the corresponding share."

Ex. 1. *A* and *B* enter into partnership; *A* contributes \$3000 for 9 months, and *B* \$2400 for 6 months, they gain \$1150: find each man's share of the gain.

Proceeding by the Rule given above,

\$(3000 × 9 + 2400 × 6) : \$(3000 × 9) :: \$1150 : *A's* share of gain,
or \$41400 : \$27000 :: 1150 : *A's* share of gain,
and \$41400 : \$14400 :: \$1150 : *B's* share of gain;
whence, *A's* share=\$750, and *B's* share=\$400.

The reason for the above process, is evident from the consideration, that a stock of \$3000 for 9 months would be equivalent to a stock of 9 times \$3000 for one month; and one of \$2400 for 6 months, to one of 6 times \$2400 for 1 month: hence, the increased stocks being considered, the question then becomes one of Simple Fellowship.

EQUATION OF PAYMENTS.

182. Def. When a person owes another several sums of money, due at different times, the Rule by which we determine the just time when the whole debt may be discharged at one payment, is called the Equation of Payments.

Note. It is assumed in this Rule that the sum of the interests of the several debts for their respective times equals the interest of the sum of the debts for the equated time.

Rule. "Multiply each debt into the time which will elapse before it becomes due, and then divide the sum of the products by the sum of the debts; the quotient will be the equated time required."

Ex. 1. *A* owes *B* \$100, whereof \$40 is to be paid in 3 months and \$60 in 5 months: find the equated time.

Proceeding according to the Rule given above,

then $(40 \times 3 + 60 \times 5) = 40 + 60) \times$ equated time in months,

whence, equated time $= 4\frac{1}{5}$ months.

The reason for the above process, in accordance with our assumption, is clear from the consideration that the sum of the interests of \$40 for 3 months, and \$60 for 5 months, is the same as the interest of \$(120+300), or \$420 for 1 month; if therefore *A* has to pay \$100 in one sum, the question is, how long ought he to hold it so that the interest on it may be the same as the interest on \$420 for one month. The statement therefore will be thus:

\$100 : \$420 :: 1 month : required number of months;

whence, required number of months $= 4\frac{1}{5}$ months;

which is evidently the equated time of payment, and agrees with the result obtained by the Rule given above.

Ex. 2. *A* owed *B* \$1000, to be paid at the end of 9 months:

he pays however \$200 at the end of 3 months, and \$300 at the end of 8 months: when was the remaider due?

In this case,

$$(200\times 3+300\times 8+500\times \text{number of months required})=1000\times 9,$$
$$\text{or } 500\times \text{number of months required}=6000;$$
$$\text{whence number of months required}=12.$$

Ex. LXI.

1. A company of militia consisting of 72 men is to be raised from 3 towns, which contain respectively 1500, 7000, and 9500 men. How many must each town provide?

2. Divide £17·5875 into two parts which shall be to each other as 5 : 16.

3. Divide 4472 into parts which shall be to each other in the ratio of 3, 5, 7, 11; and also \$500 into parts which shall be in the ratio of $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{4}{7}$.

4. A bankrupt owes *A* \$$256\frac{1}{3}$, *B* \$$203\frac{1}{2}$, and *C* \$$141\frac{2}{3}$; his estate is worth \$$421\frac{1}{20}$; how much will *A*, *B*, and *C* receive respectively?

5. A mass of counterfeit metal is composed of fine gold 15 parts, silver 4 parts, and copper 3 parts: find how much of each is required in making 18 cwt. of the composition.

6. Two persons have gained in trade \$720; the one put in \$2200 and the other \$1800; what is each person's share of the profits?

7. In a certain substance there are 11 parts tin to 100 of copper. Find the weight of tin in a piece weighing 24 cwt.?

8. A man leaves his property amounting to \$13,000 to be divided amongst his children, consisting of 4 sons and 3 daughters; the three younger sons are each to have twice the share of each of the daughters, and the eldest son as much as a younger son and a daughter together; find the share of each.

9. Two persons, *A* and *B*, are partners in a mercantile concern, and contribute \$12000 and \$20000 capital respectively; *A* is to have 10 per cent. of the profits for managing the business, and the remaining profits to be divided in proportion to the capital

contributed by each; the entire profit at the year's end is $8000; how much of it must each receive?

10. Divide $100 among *A*, *B*, *C* and *D*, so that *B* may receive as much as *A*; *C* as much as *A* and *B* together; and *D* as much as much as *A*, *B*, and *C* together.

11. Divide $11,875 among *A*, *B*, and *C*, so that as often as *A* gets $4, *B* shall get $3, and as often as *B* gets $6, *C* shall get $5.

12. *A* commences business with a capital of $1000, two years afterwards he takes *B* into partnership with a capital of $15,000, and in 3 years more they divide a profit $1500; required *B's* share.

13. $700 is due in 3 months, $800 in 5 months, and $500 in 10 months; find the equated time of payment.

14. Find the equated time of payment of $750, one half of which is due in 4 months, $\frac{3}{8}$ in 5 months, and the rest in 6 months.

15. *A* owes *B* a debt payable in $7\frac{1}{30}$ months, but he pays $\frac{1}{3}$ in 4 months, [illegible] in 6 months, $\frac{1}{5}$ in 8 months; when ought the remainder to be paid?

16. *A*, *B*, and *C* invest capital to the amount of $700, $500, and $300 respectively; *A* was to have 25 per cent. of the profits, which amount to $450; what share of the profits ought *C* to have?

17. *A* and *B* enter into a speculation: *A* puts in $500 and *B* puts in $450; at the end of 4 months *A* withdraws $\frac{1}{2}$ his capital, and at the end of 6 months *B* withdraws $\frac{1}{3}$ of his; *C* then enters with a capital of $700; at the end of 12 months their profits are $2540; how ought this to be divided amongst them?

APPLICATIONS OF THE TERM PER CENT.

183. In Art. 165, and those which follow, wherever the term "Per Cent." occurred it referred to $100 money, or $100 stock; there are however many cases in which the term Per Cent. occurs, where the reference is neither to the one nor the other, but to the number 100, where the unit is an abstract number, or a concrete number of a different kind from the above mentioned.

All such examples depend on the principles of proportion: some examples will now be worked by way of illustration, and others subjoined for practice.

Ex. 1. Find how much per cent. 7 is of 16?

In other words the question is; find what number bears the same ratio to 100, that 7 bears to 16.

By Rule, Art. 156,

$$16 : 100 :: 7 : \text{number required};$$

$$\therefore \text{number required} = \frac{700}{16} = 43{\cdot}75.$$

Ex. 2. In a parish school of 153 children, 125 learn to write. What is the percentage?

In other words, what number bears the same ratio to 100, which 125 bears to 153?

$$\therefore 153 : 100 :: 125 : \text{percentage};$$

$$\therefore \text{percentage} = \frac{12500}{153} = 81\frac{107}{153}.$$

Ex. 3. 23 per cent. of the population of a city containing 30000 people died of cholera; find the number of deaths.

If 23 died out of every 100, how many died out of 30000?

$$100 : 30000 :: 23 : \text{number of deaths};$$

$$\therefore \text{number of deaths} = \frac{690000}{100} = 6900.$$

Ex. 4. If of a regiment of 750 men, 26 per cent. are in hospital, 32 per cent. in trenches, and the rest in camp, how many are in hospital, trenches and camp respectively?

$$100 : 750 :: 26 : \text{number in hospital};$$

$$\therefore \text{number in hospital} = \frac{750 \times 26}{100} = 195.$$

$$100 : 750 :: 32 : \text{number in trenches};$$

$$\therefore \text{number in trenches} = \frac{750 \times 32}{100} = 240;$$

$$\therefore \text{number in camp} = 750 - (195 + 240) = 315.$$

Ex. 5. The percentage of children who are learning to write is 65 in a school of 60 children, and 78 in another school of 70, what is the percentage in the two schools together?

In the first school,

$$100 : 60 :: 65 : \text{number who learn to write};$$

$$\therefore \text{ number who learn to write} = \frac{60 \times 65}{100} = 39.$$

In the second school,

$100 : 70 :: 78 :$ number who learn to write;

$$\therefore \text{ number who learn to write} = \frac{70 \times 78}{100} = 54\tfrac{3}{5};$$

$\therefore$ in a school of $(60+70)$ or of 130, there are $93\frac{3}{5}$ who learn to write;

$\therefore 130 : 100 :: 93\frac{3}{5} :$ percentage required;

$$\therefore \text{ percentage required} = \frac{100 \times 93\frac{3}{5}}{130} = 72.$$

Ex. 6. In standard gold 11 parts out of 12 are pure gold; how much per cent. is dross?

In every 12 parts 1 part is dross,

$\therefore 12 : 100 :: 1 :$ percentage of dross;

$$\therefore \text{ percentage of dross} = \frac{100}{12} = 8\tfrac{1}{3}.$$

Ex. 7. Archimedes discovered that the crown made for King Hiero consisted of gold and silver in the ratio of $2 : 1$. How much per cent. was gold, and how much per cent. silver?

Out of every 3 parts, 2 were gold, and 1 silver;

$\therefore 3 : 100 :: 2 :$ percentage of gold;

$$\therefore \text{ percentage of gold } \frac{100 \times 2}{3} = 66\tfrac{2}{3};$$

and percentage of silver $= 33\frac{1}{3}$.

Ex. 8. The numbers of male and female criminals are 1235 and 988 respectively; while the decrease in the former is 4·6 per cent., the increase in the latter is 9·8 per cent.; find the increase or decrease per cent. in the whole number of criminals.

1st. $100 : 1235 :: 4{\cdot}6 :$ whole decrease of male criminals;

$$\therefore \text{ whole decrease of male criminals} = \frac{1235 \times 4{\cdot}6}{100} = 56{\cdot}81.$$

2nd. $100 : 988 :: 9{\cdot}8 :$ whole increase of female criminals;

$$\therefore \text{ whole increase of female criminals} = \frac{988 \times 9{\cdot}8}{100} = 96{\cdot}824;$$

$\therefore$ in (1235+988) or 2223 persons there is an increase of (96·824—56·81) or 40·014 persons.

$\therefore$ 2223 : 100 :: 40·014 : percentage required;

$$\therefore \text{percentage required} = \frac{4001\cdot 4}{2223} = 1\cdot 8.$$

Ex. LXII.

1. What is the percentage on 56394 at $\frac{1}{3}$; $\frac{5}{8}$; 4; $7\frac{3}{8}$; 10; $150\frac{1}{2}$?

2. How much per cent. is 15 of 96; 19 of 81; 23 of 256; $185\frac{1}{4}$ of 7321·75; $5\cdot\dot{3}$ of 11080·5?

3. Write in a decimal form $\frac{1}{2}$; $2\frac{3}{4}$; $4\frac{1}{3}$; $5\frac{5}{8}$; $26\frac{1}{3}$; 230·05; $500\cdot013\dot{8}$ per cent.

4. A cask, which contained 2005 gallons, leaked 27 per cent., how much remained in the cask?

5. A malster malts 7500 bushels of barley, which in the process increases $12\frac{1}{2}$ per cent., how many bushels of malt has he?

6. A grocer uses for a 1 lb. weight one which only weighs 15·75 oz., what does he gain per cent. by his dishonesty?

7. Out of 14804 cases of Small-Pox 1588 persons died, and out of 2422 cases of Scarlet Fever 211 persons died: find the rate per cent. of mortality in each case, also the rate per cent. of mortality in the whole number of sick people.

8. The population of Ireland was 7767401 in 1831, 8175124 in 1841, 6515794 in 1851. Find the increase per cent. in the first ten years, the decrease per cent in the second ten years, and the decrease per cent. in the 20 years, from 1831 to 1851.

9. The population of a city is a million; it rises $1\frac{1}{2}$ per cent. for 3 years successively; find the population at the end of 3 years.

10. A school contains 383 scholars, 3 are of the age of 18 years; 5 per cent. of the remainder are between the ages of 15 years and 18 years; 10 per cent. between 12 and 15; 35 per

cent. between 10 and 12, and the remainder under that age; find the number of each class.

11. Sugar being composed of 49·856 per cent. of oxygen, 43·265 per cent. of carbon, and the remainder hydrogen; find how many pounds of each of these materials there are in one ton of sugar.

12. If the increase in the number of male and female criminals be 1·8 per cent., while the decrease in the number of males alone is 4·6 per cent., and the increase in the number of females is 9·8. Compare the number of male and female criminals respectively.

184. Questions are often given, in which the term "Average" occurs; a few examples of such a kind will now be worked by way of illustration, and others subjoined for practice.

Ex. 1. In a school of 27 boys, 1 of the boys is of the age of 17 years, 2 others of 16, 4 others of $15\frac{1}{2}$, 1 of $14\frac{3}{4}$, 2 of $14\frac{1}{2}$, 5 of $13\frac{3}{4}$ 10 of $12\frac{1}{4}$, and 2 of 10; find the average age of the boys.

The object is to find, what must be the age of each boy supposing all to be the same age, that the sum of their ages may = the sum of the ages in the question.

sum of ages in question

$$=17+32+62+14\tfrac{3}{4}+29+68\tfrac{3}{4}+122\tfrac{1}{2}+20=366;$$

$$\therefore \text{ average age}=\frac{366}{27}=13\tfrac{5}{9} \text{ years.}$$

Ex. 2. In a class of 25 children, 19 have attended during the week. Days attended by children : 5 for 5 days, 6 for $4\frac{1}{2}$, 3 for 4, 2 for $3\frac{1}{2}$, 1 for 3, 1 for 2, 1 for $\frac{1}{2}$ day. Find the average number of days attended by each child.

The whole number of days attended by class

$$=(5\times5+6\times4\tfrac{1}{2}+3\times4+2\times3\tfrac{1}{2}+1\times3+1\times2+1\times\tfrac{1}{2})$$

$$=25+27+12+7+3+2+\tfrac{1}{2}=76\tfrac{1}{2} \text{ days;}$$

$$\therefore \text{ average attendance}=\frac{76\frac{1}{2}}{25}=\frac{153}{50}=\frac{306}{100}=3\cdot06 \text{ days.}$$

Ex. 3. In a school the numbers for the week were :—Monday morning 67, Tuesday morn. 60, Wednesday morn. 65, Thursday morn. 68, Friday morn. 62, Monday afternoon ·5 more than the average of Monday and Tuesday mornings, Tuesday aft. 59, Wednesday aft. ·5 less than the average of Tuesday, Thursday the average of Monday morn. and Tuesday aft., Friday aft. 60. Find the average attendance for the week.

Number of children who attended on

$$\begin{aligned} \text{Monday} &= 67+64; \\ \text{Tuesday} &= 60+59; \\ \text{Wednesday} &= 65+59; \\ \text{Thursday} &= 68+63; \\ \text{Friday} &= 62+60; \end{aligned}$$

$\therefore$ the total number of children who attended on the 10 occasions $=627$;

$$\therefore \text{ average attendance} = \frac{627}{10} = 62.7.$$

Ex. 4. A farm of 500 acres is let at a corn-rent equally apportioned between wheat and barley; it is valued at \$930 a year when the average price of wheat is \$1·10 a bushel, and that of barley \$·75 a bushel; find the rent when wheat rises to the average price of \$1·50 per bushel, and barley to that of \$1·00 per bushel.

First we must find the number of bushels of wheat and barley at the given rent of \$930.

$$\frac{\$930}{2} = \$465 \text{ the sum to be raised by each kind of grain;}$$

$$\therefore \frac{465}{1\cdot 10} = 422\tfrac{8}{11} \text{ bushels of wheat;}$$

$$\therefore \frac{465}{\cdot 75} = 620 \text{ bushels of barley;}$$

$$\therefore \text{ rent in latter case} = (422\tfrac{8}{11} \times 1\tfrac{1}{2} + 620 \times 1)$$

$$= \$1254\tfrac{1}{11}.$$

Ex. 5. A person's average annual income from 1830 to 1850 was \$1550. In 1830 his income was \$1680, and in 1851 his income was \$1625, what was his average annual income from 1831 to 1851 (inclusive)?

His total income from 1831 to 1851 inclusive

$=\$1550\times21+\$1625-\$1680.$

$=\$32495$

$\therefore$ his average income $=\frac{\$32495}{21}=\$1547\frac{8}{21}$.

Ex. LXIII.

1. In 1845 the rental of an estate amounted to $18697, in 1846 to $17292, in 1847 to $20135·50, in 1848 to $20078·75, in 1849 to $18582, in 1850 to $24048·25, in 1851 to $21631; find the average rental of the 7 years.

2. The number of bushels of grain exported from a country in 11 successive years were 2679438, 2958272, 3030293, 3474302, 2243151, 2327782, 2855525, 2538234, 3206482, 2801204, 3251901; find the average exportation during that period.

3. In a class of 23 children, 8 are boys, 15 girls. The age of the boys—4 of 8, 2 of 11, 2 of 12. Of the girls—5 the average age of the boys, 4 of 9, 2 of 10, 4 of 13. Find the average age of (*a*) the boys, (*b*) the girls, (*d*) the whole class.

4. There are 25 children on the register of one class in a school. 19 have been present at one time or other during the week. The sum of days on which the children have attended is $84\frac{1}{2}$. What is the average number of days per week attended by each child ever present during the week, there being no school on Saturday or Sunday? Give the answer in decimals.

5. In a school of 7 classes, the average number of days attended by each child in Class I. is 4·5; Class II., 4; Class III., 3·9; Class IV., 4·1; Class V., 3·6; Class VI., 4·2; Class VII., 3·3. Find the average number of days attended by each child in the school.

6. Divide $960 among three persons in such a manner that their shares shall be to each other as 5, 4, and 3, respectively.

SQUARE ROOT.

184. The SQUARE of a given number is the product of that number multiplied by itself. Thus 36 is the square of 6.

The square of a number is frequently denoted by placing the figure 2 above the number, a little to the right. Thus 6^2 denotes the square of 6, so that $6^2=36$.

185. The Square Root of a given number is a number, which when multiplied by itself, will produce the given number.

The square root of a number is sometimes denoted by placing the sign $\sqrt{}$ before the number, or by placing the fraction $\frac{1}{2}$ above the number, a little to the right. Thus $\sqrt{36}$ or $(36)^{\frac{1}{2}}$ denotes the square root of 36; so that $\sqrt{36}$ or $(36)^{\frac{1}{2}}=6$.

186. The number of figures in the integral part of the Square Root of any whole number may readily be known from the following considerations:

The square root of	1	is	1
	100	is	10
	10000	is	100
	1000000	is	1000
	&c.	is	&c.

Hence it follows that the square root of any number between 1 and 100 must lie between 1 and 10, that is, will have one figure in its integral part; of any number between 100 and 10000, must lie between 10 and 100, that is, will have two figures in its integral part; of any number between 10000 and 1000000, must lie between 100 and 1000, that is, must have three figures in its integral part; and so on. Wherefore, if a point be placed over the units' place of the number, and thence over every second figure to the left of that place, the points will shew the number of figures in the integral part of the root. Thus the square root of $9\dot{9}$ consists, so far as it is integral, of *one* figure; that of $\dot{1}9\dot{8}$ of *two* figures; that of $1\dot{7}6\dot{4}3\dot{2}$ of *three* figures; that of $\dot{1}7\dot{6}4\dot{3}2\dot{1}$ of *four* figures; and so on.

187. The following Rule may be laid down for extracting the square root of a whole number:

Rule. "Place a point or dot over the units' place of the given number, and thence over every second figure to the left of that place, thus dividing the whole number into several periods. The number of points will shew the number of figures in the required root. (Art. 186.)

Find the greatest number whose square is contained in the first period at the left; this is the first figure in the root, which place in the form of a quotient to the right of the given number. Subtract its square from the first period, and to the remainder bring down the second period. Divide the number thus formed, omitting the last figure, by twice the part of the root already obtained, and annex the result to the root and also to the divisor. Then multiply the divisor, as it now stands, by the part of the root last obtained, and subtract the product from the number formed, as above mentioned, by the first remainder and second period. If there be more periods to be brought down, the operation must be repeated."

Ex. 1. Find the square root of 1369.

```
      1369 (37
      9
67 | 469
   | 469
```

After pointing, according to the Rule, we take the first period, or 13, and find the greatest number whose square is contained in it. Since the square of 3 is 9, and that of 4 is 16, it is clear that 3 is the greatest number whose square is contained in 13; therefore place 3 in the form of a quotient to the right of the given number. Square this number, and put down the square under the 13; subtract it from the 13, and to the remainder 4 affix the next period 69, thus forming the number 469. Take 2×3, or 6, for a divisor; divide the 469, omitting the last figure, that is, divide the 46 by the 6, and we obtain 7. Annex the 7 to the 3 before obtained and to the divisor 6; then multiplying the 67 by the 7 we obtain 469, which being subtracted from the 469 before formed, leaves no remainder; therefore 37 is the square root of 1369.

Reason for the above process.

Since $(37)^2=1369$, and therefore 37 is the square root of 1369; we have to investigate the proper Rule by which the 37, or $30+7$, may be obtained from the 1369.

$$\begin{aligned}\text{Now } 1369 &= 900+469=900+49+420\\ &=(30)^2+7^2+2\times 30\times 7\\ &=(30)^2+2\times 30\times 7+7^2\end{aligned}$$

where we see that the 1369 is separated into parts in which the 30 and the 7, together constituting the square root, or 37, are made distinctly apparent. Treating then the number 1369 in the following form, viz.

$$(30)^2+2\times30\times7+7^2$$

we observe that the square root of the first part or of $(30)^2$, is 30; which is one part of the required root. Subtract the square of the 30 from the whole quantity $(30)^2+2\times30\times7+7^2$, and we have $2\times30\times7+7^2$ remaining. Multiply the 30 before obtained by 2, and we see that the product is contained 7 times in the first part of the remainder, or in $2\times30\times7$; and adding the 7 to the 2×30, thus making $2\times30+7$ or 67, this latter quantity is contained 7 times exactly in the remainder $2\times30\times7+7$ or 469; so that by this division we shall gain the 7, the remaining part of the root. If we had found that the $2\times30+7$ or 67, when multiplied by the 7, had produced a larger number than the 469, the 7 would have been too large, and we should have had to try a smaller number, as 6, in its place.

The process will be shewn as follows:

$$\begin{array}{r|l}
 & (30)^2+2\times30\times7+7^2\ (30+7 \\
 & (30)^2 \\
\hline
2\times30+7 & 2\times30\times7+7^2 \\
 & 2\times30\times7+7^2 \\
\hline
\end{array}$$

This operation is clearly equivalent to the following:

$$\begin{array}{r|l}
 & 900+420+49\ (\ 30+7 \\
 & 900 \\
\hline
60+7 & 420+49 \\
 & 420+49 \\
\hline
\end{array}$$

This again is equivalent to the following:

$$\begin{array}{r|l}
 & 1\dot{3}6\dot{9}\ (\ 37 \\
 & 9 \\
\hline
67 & 469 \\
 & 469 \\
\hline
\end{array}$$

which is the mode of operation pointed out in the Rule.

Note 1. The reasoning will be better understood when the student has made some progress in Algebra.

Note 2. The divisor obtained by doubling the part of the root already obtained, is often called a *trial divisor*, because the quotient first obtained from it by the Rule in (Art. 187), will sometimes be too large. It will be readily found, in the process, whether this is the case or not, for when, according to our Rule, we have annexed the quotient to the trial divisor, and multiplied the divisor as it then stands by that quotient, the resulting number should not be greater than the number from which it ought to be subtracted. If it be, the quotient is too large, and the number next smaller should be tried in its place.

Note 3. If at any point of the operation, the number to be divided by the trial divisor be less than it, we then affix a cypher to the root, and also to the trial divisor, bring down the next period, and proceed according to the Rule.

Ex. 2. Find the square root of 74684164.

		74684164 (8642
		64
$\{2\times 8=16\}$	166	1068
		996
$\{2\times 86=172\}$	1724	7241
		6896
$\{2\times 864=1728\}$	17282	34564
		34564

Therefore 8642 is the square root of 74684164.

Ex. 3. Find the square root of 71690512350625.

		71690512350625 (8467025
		64
$\{2\times 8=16\}$	164	769
		656
$\{2\times 84=168\}$	1686	11305
		10116
$\{2\times 846=1692\}$	16927	118912
		118489
$\{(2\times 8467=16934)$, $(2\times 84670=169340)\}$	1693402	4233506
		3386804
	16934045	84670225
		84670225

$\therefore$ 8467025 is the required square root.

188. Again, since the square root of ·01 is ·1
·0001 is ·01
·000001 is ·001
·00000001 is ·0001
&c. &c.

it appears, that in extracting the square root of decimals, the decimal places must first of all be made even in number, by affixing a cypher to the right, if this be necessary; and then if points be placed over every second figure to the right, beginning as before from the units' place of whole numbers, the number of such points will shew the number of decimal places in the root.

189. If there be no whole number, or integral part in the given number, we must, in pointing, begin with the second figure from that which would be the unit's place, if there were a whole number, and mark successively over every second figure to the right. If there be a whole number as well as a decimal, it will be the safest method to begin at the units' place, and point over every second figure to the right and left of it. The number of points over the whole numbers and decimals will shew respectively the numbers of figures in the integral and decimal parts of the root. Thus if the given number were 6115·23, place the first point over the 5, and mark from it to the right and left, thus $6\dot{1}1\dot{5}\cdot2\dot{3}$. If the given number were 58·432, first make the decimal places even in number thus, 58·4320, and then point thus $5\dot{8}\cdot4\dot{3}2\dot{0}$.

190. With the above explanation (Arts. 186 and 188) on the subject of pointing, the rule for extracting the square root of a decimal, or of a number consisting partly of a whole number and partly of a decimal, will be the same as that before given (Art. 187) for finding the square root of a whole number. As the decimal notation is only an extension or continuance of the ordinary integral notation, and quite in agreement with it, the reason before given for the process, will in fact apply also here.

191. To extract the square root of a vulgar fraction, if the numerator and denominator of the fraction be perfect squares, we may find the square root of each separately, and the answer will thus be obtained as a vulgar fraction; if not we can first reduce the fraction to a decimal, or to a whole number and decimal, and then find the root of the resulting number. The answer will thus be obtained either as a decimal, or as a whole number and decimal, according to the case. Also a mixed number may be reduced to an improper fraction, and its root extractedin the same way.

Ex. 4. Extract the square root of ·4 to four places of decimals.

```
        ·40000000 (·6324
         36
  123  | 400
       | 369
  1262   | 3100
         | 2524
  12644    | 57600
           | 50576
               7024
```

Ex. 5. Extract the square root of ·0006 to four places of decimals.

```
      ·00060000 (·0244
           4
   44  | 200
       | 176
   484   | 2400
         | 1936
             464
```

Ex. 6. Extract the square root of ·0365 to five places of decimals.

```
        ·0365000000 (·19104
          1
  29    | 265
        | 261
  381     | 400
          | 381
  38204     | 190000
            | 152816
               37184
```

Ex. 7. Extract the square root of 53111·8116.

```
         5̇31̇1̇1·8̇11̇6̇ (230·46
         4
   43 | 131
      | 129
 4604 | 21181
      | 18416
46086 | 276516
      | 276516
```

Ex. 8. Find the square root of $\frac{529}{2401}$.

```
    5̇29̇ (23              2̇40̇1 (49
    4                     16
43 | 129              89 | 801
   | 129                 | 801
```

therefore square root required $=\frac{23}{49}$.

Ex. 9. Find the square root of $\frac{5}{7}$.

This may be done by first reducing $\frac{5}{7}$ to a decimal, and then by extracting the square root of the decimal, thus $\frac{5}{7}=\cdot714285\ldots$

```
      ·7̇14̇28̇5 (·845...
       64
 164 | 742
     | 656
1685 | 8685
     | 8425
        260
```

or thus $\sqrt{\frac{5}{7}}=\frac{5\times 7}{7\times 7}=\frac{\sqrt{35}}{7}$.

```
        3̇5·0̇00̇00̇0 (5·916
        25
  109 | 1000
      |  981
 1181 | 1900
      | 1181
11826 | 71900
      | 70956
          944
```

therefore $\sqrt{\frac{5}{7}}=\frac{5\cdot916}{7}=\cdot845\ldots$

Ex. LXIV.

1. Find the square roots of
   (1) 289; 576; 1444; 4096. (2) 6561; 21025; 173056.
   (3) 98596; 37249; 11664. (4) 998001; 978121; 824464
   (5) 29506624; 14356521; 5345344.
   (6) 236144689; 282429536481; 282475249.
   (7) 295066240000; 4160580062500.

2. Find the square roots of
   (1) 167·9616; 28·8369; 57648·01.
   (2) ·3486784401; 39·15380[illegible]29.
   (3) ·042849; ·00139876; ·00203401.
   (4) 5774409; 5·774409.
   (5) 120888·68379025; 240398·012416.

3. Extract the square roots of
   (1) 16; 1·6; ·16; ·016. (2) 235·6; ·1; ·01; 5; ·5.
   (3) ·0004; ·00081; 379·864. (4) $20\frac{1}{4}$; $153\frac{7}{9}$; $\frac{1}{3}$; $\frac{2209}{9801}$.
   (5) $\frac{3}{5}$; $\frac{1}{17}$; $2\frac{1}{6}$; $\frac{3\frac{1}{2}}{4\frac{1}{2}}$. (6) $\frac{5·04}{·021}$; $1\frac{56}{169}$; 23·1; 42;
   to four places of decimals in each case where the root does not terminate.

4. Extract the square root of ·0019140625 and reduce the result to the corresponding equivalent fraction in its lowest terms.

5 Find the side of a square field equal in area to a rectangular field 700 yards wide and 2800 yards long.

6. A rectangular field measures 223 yards in length, and 120 yards in breadth; what will be the length of a diagonal path across it?

7. Find the length of the side of a square enclosure, the paving of which cost £27. 1*s.* 6*d.* at 8*d.* per sq. yard.

8. The hypothenuse of a right-angled triangle is 51 yards, and the perpendicular is 24 yards, find the base.

9. A ladder, whose length is 91 feet, reaches from the extremity of a path 35 feet wide, to a point in a building on the other side, which is within 9 inches of the top of it; find the height of the building.

10. Extract the square root of ·0050722884, and find within an inch the length of a side of a square field, the area of which is 2 acres.

11. Two persons start from a certain point at the same time, the one goes due east at the rate of 12 miles an hour, and the other due north at the rate of 9 miles an hour; how far are they distant from each other at the end of six hours?

12. A ladder 36 feet long will reach to a window 28 feet from the ground, on one side of a street; and if the foot of the ladder be retained in the same position, will reach to a window 26 feet high on the other side. Find the breadth of the street.

13. A society collected among themselves for certain purposes a fund of $45·9375: each person paid as many cents as there were members in the whole society. Find the number of members.

14. The area of a circular lake is 295066·24 square yards, how many yards are contained in the side of a square of equal superficies?

CUBE ROOT.

192. The CUBE of a given number is the product which arises from multiplying that number by itself, and then multiplying the result again by the same number. Thus $6\times6\times6$ or 216 is the cube of 6.

The cube of a number is frequently denoted by placing the figure 3 above the number, a little to the right. Thus 6^3 denotes the cube of 6, so that $6^3=6\times6\times6$ or 216.

193. The CUBE ROOT of a given number is a number, which, when multiplied into itself, and the result again multiplied by it, will produce the given number. Thus 6 is the cube root of 216; for $6\times6\times6$ is $=216$.

The cube root of a number is sometimes denoted by placing the sign $\sqrt[3]{\ }$ before the number, or placing the fraction $\frac{1}{3}$ above the number, a little to the right. Thus $\sqrt[3]{216}$ or $(216)^{\frac{1}{3}}$ denotes the cube root of 216; so that $\sqrt[3]{216}$ or $(216)^{\frac{1}{3}}=6$.

194. The number of figures in the integral part of the cube root of any whole number may readily be known from the following considerations:

The cube root of	1	is	1
	1000	is	10
	1000000	is	100
	1000000000	is	1000
	&c.	is	&c.

Hence it follows that the cube root of any number between 1 and 1000 must lie between 1 and 10, that is, will have one figure in its integral part; of any number between 1000 and 1000000, must lie between 10 and 100, that is, will have two figures in its integral part; of any number between 1000000 and 1000000000, must lie between 100 and 1000, that is, must have three figures in its integral part; and so on. Wherefore, if a point be placed over the units' place of the number, and thence over every third figure to the left of that place, the points will shew the number of figures in the integral part of the root. Thus the cube root of 677 consists, so far as it is integral, of *one* figure; that of 198999 of *two* figures; that of 134198999 of *three* figures; and so on.

195 The following Rule may be laid down for extracting the Cube Root of a whole number:

Rule. "Place a point or dot over the units' place of the given number, and thence over every third figure to the left of that place, thus dividing the whole number into several periods. The number of points will shew the number of figures in the required root. (Art. 194.)

"Find the greatest number whose cube is contained in the first period at the left; this is the first figure in the root, which place in the form of a quotient to the right of the given number.

"Subtract its cube from the first period, and to the remainder bring down the second period.

"Divide the number thus formed, omitting the last two figures, by 3 times the square of the part of the root already obtained, and annex the result to the root.

"Now calculate the value of 3 times the square of the first figure in the root (which of course has the value of so many tens) + 3 times the product of the two figures in the root + the square of the last figure in the root. Multiply the value thus found by the second figure in the root, and subtract the result from the number formed, as above mentioned, by the first remainder and the second period. If there be more periods to be brought down, the operation must be repeated."

Ex. 1. Find the cube root of 15625.

		$\dot{1}5\dot{6}25$ (25
	$2^3 = 8$	8
$3 \times 2^2 = 12$		7625
$3 \times (20)^2 = 3 \times 400 = 1200$		
$3 \times 20 \times 5 = 300$		
$5^2 = 25$		
	1525	
Multiply by	5	
	7625	7625

After pointing, according to the Rule, we take the first period, or 15, and find the greatest number whose cube is contained in it. Since the cube of 2 is 8, and that of 3 is 27, it is clear that 2 is the greatest number whose cube is contained in 15; therefore place 2 in the form of a quotient to the right of the given number.

Cube 2, and put down its cube, viz. 8, under the 15; subtract it from the 15, and to the remainder 7 affix the next period, 625, thus forming the number 7625. Take 3×2^2, or 12, for a divisor; divide 76 by 12, 12 is contained 6 times in 76; but when the other terms of the divisor are brought down, 6 would be found too great, therefore take 5. Annex the 5 to the 2 before obtained, and calculate the value of $3 \times (20)^2 + 3 \times 20 \times 5 + 5^2$, which is 1525; multiplying 1525 by 5 we obtain 7625, which being subtracted from 7625 before formed leaves no remainder,

therefore 25 is the cube root required.

Reason for the above process.

Since $(25)^3 = 15625$, and therefore 25 is the cube root of 15625; we have to investigate the proper Rule by which the 25, or $20+5$, may be obtained from 15626.

$$\begin{aligned}\text{Now } 15625 &= 8000 + 7500 + 125 \\ &= 8000 + 6000 + 1500 + 125 \\ &= (20)^3 + 3 \times (20)^2 \times 5 + 3 \times 20 \times 5^2 + 5^3,\end{aligned}$$

where we see that the 15625 is separated into parts in which the 20 and the 5, together constituting the cube root, or 25, are made distinctly apparent. Treating then the number 15625 in the following form, viz.

$$(20)^3 + 3 \times (20)^2 \times 5 + 3 \times 20 \times 5^2 + 5^3,$$

we observe that the cube root of the first part or of $(20)^3$ is 20; which is one part of the required root. Subtract the cube of the 20 from the whole quantity, and we have $3\times(20)^2\times 5+3\times 20\times 5^2+5^3$ remaining. Multiply the square of the 20 before obtained by 3, and we see that the prodnct is contained 5 times in the first part of the remainder, or in $3\times(20)^2\times 5$; and adding 3 times the product of the two terms of the root + the square of the last term of the root, thus making $3\times(20)^2+3\times 20\times 5+5^2$, we see that this latter quantity is contained 5 times exactly in the remainder $3\times(20)^2\times 5+3\times 20\times 5^2+5^3$, so that by this division we shall obtain the 5, the remaining part of the root.

The process will be shewn as follows:

$$\begin{array}{l|l}
 & (20)^3+3\times(20)^2\times 5+3\times(20)\times 5^2+5^3\ (20+5 \\
 & (20)^3 \\
\hline
\text{divisor}=3\times(20)^2, & 3\times(20)^2\times 5+3\times 20\times 5^2+5^3 \\
\text{and}\dfrac{3\times(20)^2\times 5}{3\times(20)^2}=5; & \\
\therefore\{3\times(20)^2+3\times 20\times 5+5^2\}\times 5= & 3\times(20)^2\times 5+3\times 20\times 5^2+5^3 \\
\hline
\end{array}$$

This operation is clearly equivalent to the following:

$$\begin{array}{r|l}
 & 8000+6000+1500+122\ (20+5 \\
 & 8000 \\
\hline
3\times(20)^2=1200,\ \text{and}\ \frac{6000}{1200}=5 & 6000+1500+125 \\
(1200+300+25)\times 5= & 6000+1500+125 \\
\hline
\end{array}$$

This again is equivalent to the following:

$$\begin{array}{r|l}
 & 15625\ (25 \\
 & 8 \\
\hline
3\times 2^2=3\times 4=12,\ \text{and}\ \frac{76}{12}=5 & 7625 \\
3\times(20)^2=1200 & \\
+3\times 20\times 5=\ \ 300 & \\
+5^2=\ \ \ 25 & \\
\overline{1525} & \\
5 & \\
\overline{7625} & 7625 \\
\hline
\end{array}$$

which is the mode of operation pointed out in the Rule.

Note 1. The reasoning will be better understood when the student has made some progress in Algebra.

Note 2. The divisor which is obtained according to the Rule given in (Art. 195) is sometimes called a *trial* divisor, because the number from that division may be too large, as was the case in the above Example, in which case we must try a smaller number. We shall readily ascertain whether the number obtained from the division is too large or not, because if it be too large, the quantity which we ought to subtract from the number formed by a remainder and a period will turn out in that case to be larger than that number, which of course it ought not to be, and so we must try a smaller number.

Note 3. If at any point of the operation, the number to be divided by the trial divisor be less than it; we affix a cypher to the root, two cyphers to the trial divisor, bring down the next period, and proceed according to the Rule.

Ex. 2. Extract the cube root of 95443993.

$$\dot{9}5443\dot{9}93\dot{3}\ (457$$

	$4^3=64$	
trial divisor $=3\times 4^2=48$	31443	
$3\times(40)^2=4800$		$\frac{314}{48}$ goes 6 times, but 6 will be found too large; try 5.
$3\times 40\times 5=\ \ 600$		
$5^2=\ \ \ 25$		
5425		
5		
27125	27125	
trial divisor $=3\times(45)^2=6075$	4318993	
Now 45 has the value of 450;		$\frac{43189}{6075}$ goes 7 times, and we are led to conclude that 7 is the figure, because $7^3=343$, and 3 is the final figure in the remainder.
$\therefore 3\times(450)^2=607500$		
$3\times 450\times 7=\ \ \ 9450$		
$7^2=\ \ \ \ 49$		
616999		
7		
4318993	4318993	

Therefore 457 is the cube root required.

Ex. 3. Find the cube root of 223648543.

$$223\dot{6}48\dot{5}4\dot{3}\ (607$$
$$6^3 = 216$$

trial divisor $= 3 \times 6^2 = 108$	7648	76 is not divisible by 108;
trial divisor $= 3 \times (60)^2 = 10800$	7648543	bring down the next period and affix 0 to the root; $\frac{76485}{10800}$ goes 7 times, and 7 seems likely to be the figure required; since $7^3 = 343$ and 3 is the final figure in the remainder.
$3 \times (600)^2 = 1080000$		
$3 \times 600 \times 7 = 12600$		
$7^2 = 49$		
1092649		
7		
7648543	7648543	

Therefore 607 is the cube root required.

196. Again, since the cube root of ·001 is ·1,
the cube root of ·000001 is ·01,
the cube root of ·000000001 is ·001,
&c. is &c.,

it appears, that in extracting the cube root of decimals, the decimal places must first of all be made three, or some multiple of three in number, by affixing cyphers to the right, if this be necessary; and then if points be placed over every *third* figure to the right, beginning as before from the units' place of *whole numbers*, the number of such points will shew the number of decimal places in the cube root.

197. If there be no whole number or integral part in the given number, we must in pointing begin with the *third* figure from that which would be the units' place, if there were a whole number, and mark successively every *third* figure to the right. If there be a whole number as well as a decimal, it will be the safest method to begin at the units' place, and point over every third figure to the right and left of it; the number of points over the whole numbers and decimals will shew respectively the numbers of figures in the integral and decimal parts of the root. Thus, if the given number were 5623·453134, place the first point over the 3, and mark from it to the right and left, thus, $\dot{5}62\dot{3}{\cdot}45\dot{3}13\dot{4}$. If the given number were 5·23, make the number of decimal places equal to 3, by affixing a cypher thus, 5·230; place the first point over the 5, and

the second over the 0 : if the root be required to more decimals than one, more cyphers must be affixed.

198. With the above explanation (Arts. 194, 196) on the subject of pointing, the rule for extracting the cube root of a decimal, or of a number consisting partly of a whole number and partly of a decimal, will be the same as that before given (Art. 195) for finding the cube root of a whole number. As the decimal notation is only an extension or continuance of the ordinary integral notation and quite in agreement with it, the reason before given for the process, will in fact apply also here.

199. To extract the cube root of a vulgar fraction, if the numerator and denominator of the fraction be perfect cubes we may find the cube root of each separately ; and the answer will thus be obtained as a vulgar fraction ; if not, we can first reduce the fraction to a decimal, or to a whole number and decimal, and then find the root of the resulting number. The answer will thus be obtained either as a decimal, or as a whole number and decimal, according to the case. Also a mixed number may be reduced to an improper fraction, and its root extracted in the same way.

Ex. 4. Find the cube root of 48228·544.

		48228·544 (36·4
	$3^3=27$	
	$3\times 3^2=27$	21228
$3\times(30)^2=2700$		
$3\times 30\times 6=\ 540$		
$6^2=\ 36$		
3276		
6		
19656		19656
	$3\times(36)^2=3888$	1572544
$3\times(360)^2=388800$		
$3\times 360\times 4=\ 4320$		
$4^2=\ 16$		
393136		
4		
1572544		1572544

Therefore 36·4 is the cube root required.

Ex. 5. Find the cube root of ·000007 to three places of decimals.

$$
\begin{array}{rr|l}
 & & \cdot 00\dot{0}00\dot{7}00\dot{0}\ (\cdot 019 \\
 & & 1 \\
 & 3 \times 1^2 = 3 & 600 \\
3 \times (10)^2 = 300 & & \\
3 \times 10 \times 9 = 270 & & \\
9^2 = 81 & & \\
651 & & \\
9 & & \\
5859 & & 5859 \\
 & & 141
\end{array}
$$

Ex. 6. Find the cube root of $\frac{5}{9}$ to three places of decimals.

$$\tfrac{5}{9} = \cdot 555555555\ldots$$

$$
\begin{array}{rr|l}
 & & \cdot 55\dot{5}55\dot{5}55\dot{5}\ (\cdot 822 \\
 & 8^3 = 512 & \\
 & 3 \times 8^2 = 192 & 43555 \\
3 \times (80)^2 = 19200 & & \\
3 \times 80 \times 2 = 480 & & \\
2^2 = 4 & & \\
19684 & & \\
2 & & \\
39368 & & 39368 \\
 & 3 \times (82)^2 = 20172 & 4187555 \\
3 \times (820)^2 = 2017200 & & \\
3 \times 820 \times 2 = 4920 & & \\
2^2 = 4 & & \\
2022124 & & \\
2 & & \\
4044248 & & 4044248 \\
 & & 143307
\end{array}
$$

200. Higher roots than the square and cube can sometimes be extracted by means of the Rules for square and cube root; thus the 4th root is found by taking the square root of the square root; the 6th root by taking the square root of the cube root, and so on.

Ex. LXV.

1. Find the cube roots of

(1) 1728; 3375; 29791. (2) 54872; 110592; 300763.
(3) 681472; 804357; 941192.
(4) 2406104; 69426531; 8365427.
(5) 251239591; 28372625; 48228544.
(6) 17173512; 259694072; 926859375.
(7) 27054036008; 219365327791.

2. Find the cube roots of
·389017; 32·461759; 95443·993; ·000912673; ·001906624; ·000024389.

3. Find the cube roots of

(1) 3, ·3, ·03. (2) $\frac{8}{27}$; $\frac{250}{686}$; 44·6.

(3) 405$\frac{28}{125}$; 7$\frac{1}{5}$; 3·00415. (4) ·0001; $\frac{1257\cdot728}{16384}$

to three places of decimals, in those cases where the root does not terminate.

4. Find the cube root of 233·744896, and also the cube root of the last-mentioned number multiplied by ·008.

5. The cost of a cubic mass of metal is £10481. 1*s.* 4*d.* at 10*s.* 5*d.* a cubic inch. What are the dimensions of the mass?

6. A cubical block of stone contains 50653 solid feet, what is the area of its side?

7. A cube contains 56 solid feet, 568 solid inches; find its edge.

8. Find the cost of carpeting a cubical room, whose content is 21717·639 solid feet, with carpet 21 inches broad, at $1·25 a yard.

9. A cubical box contains 941192 solid inches; find the cost of painting its outside surface at 6 cents a square foot.

10. If the solid content of a cube be 37 ft. 64 in., shew that its surface will be 66 ft. 96 in.

11. The edge of a cubical vessel is 2 feet long: what is the length of the edge of another cubical vessel containing 3 times as much?

12. Find the 4th root of 43046721; and the 6th root of ·000000004096.

PROGRESSION.

ARITHMETICAL PROGRESSION.

201. A series of numbers that increase or decrease by a common difference is said to be an Arithmetical Progression. When the terms are constantly increasing, the series is an *Arithmetical Progression Ascending*; when constantly decreasing, the series is an *Arithmetical Progression Descending*. Thus, 1, 3, 5, 7, 9, &c., is an Ascending Arithmetical Progression; 10, 8, 6, 4, 2, is a Descending Arithmetical Progression.

The first and last terms of a Progression are called the *extremes*; the intermediate terms are called the *means*.

The terms of an Arithmetical Progression may be fractional, thus:

$\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, &c., having a common difference of $\frac{1}{2}$;

$\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, $2\frac{2}{3}$, 3, &c., having a common difference of $\frac{1}{3}$.

From the nature of an Arithmetical Progression it follows that the sum of the extremes is equal to the sum of any other two terms equally distant from them, or to *twice* the middle term, if the number of terms be unequal; this will be evident from inspecting the following Progression: 1, 3, 5, 7, 9, 11, 13.

1	3	5	7	9	11	13
13	11	9	7	5	3	1
14	14	14	14	14	14	14

In Arithmetical Progression there are five terms to be considered. 1, *the first term*; 2, *the last term*; 3, *the common difference*; 4, *the number of terms*; 5, *the sum of all the terms*.

These quantities bear such a relation to each other that any three of them being given, the remaining two can be found. Since there are five terms and only three of them necessary to be known, it follows that there are twenty distinct cases in Arithmetical Progression. We shall, however, notice but two of the most important, and refer the student to Algebra for the others.

THE LAST TERM.

202. To find the last term, when the first term, the common difference and the number of terms are given.

Rule. To the first term add the product of the common difference into the number of terms, less 1.

Ex. What is the last term of an Arithmetical Progression, whose first term is 4, the common difference 3, and the number of terms 5?

Proceeding by the rule given above,

$$4+(3\times4)=16.$$

Reason for the process.

It is clear that the second term of an Ascending Progression is equal to the first, increased by the common difference; the third is equal to the first, increased by twice the common difference; the fourth is equal to the first, increased by three times the common difference; and so on, for the succeeding terms.

In a decreasing Progression the last term is equal to the first term, *less* the same product. thus:

What is the last term of a Progression, whose first term is 16, the common difference 3, and the number of terms 5?

$$16-(4\times3)=4.$$

THE SUM OF ALL THE TERMS.

203. To find the sum of all the terms, when the first term, the last term and the number of terms are given.

Rule. Multiply half the sum of the extremes by the number of terms.

Ex. The first term of an Arithmetical Progression is 2, the last term is 50, and the number of terms 17; what is the sum of all the terms?

Proceeding by the rule given above,

$$\frac{2+50}{2}\times17=442.$$

Reason for the process.

The sum of the extremes of an Arithmetical Progression being equal to the sum of any two terms equally distant from them, it follows that the terms must average half the sum of the extremes.

LXVI.

1. What is the 100th term of an Arithmetical Progression, whose first term is 2, and common difference 3?

2. What is the 50th term of an Arithmetical Progression, whose first term is 1, the common difference $\frac{1}{2}$?

3. The first term of a *decreasing* Arithmetical Progression is 4680, the common difference is 3, and the number of terms 120. What is the last term? and the sum of all the terms?

4. A tapering board, 6 inches wide at the narrow end, and 12 feet long, is found to increase $\frac{1}{2}$ an inch for every foot in length. What is the width of the wide end?

5. A person travels 25 days, going 11 miles the first day, 135 miles the last day: the miles which he travelled in the successive days form an Arithmetical Progression. How far did he go in the 25 days?

6. A person makes 12 monthly deposits in a Savings Bank; the first deposit consisted of $25, the second of $30, the third of $35, and so on, in Arithmetical Progression. How much did he deposit in 12 months?

7. What is the sum of an Arithmetical Progression whose first term is 7, and last term 1113?

8. A man has in his orchard 34 rows of trees; in the first row there are 20 trees, in the second 24, in the third 28, increasing in Arithmetical Progression. How many trees are there in the last row?

9. A merchant bought a certain number of pieces of cloth, the prices of which increased by $2; the first piece cost $3 and the last $43. How many pieces did he buy?

GEOMETRICAL PROGRESSION.

204. A series of numbers which succeed each other by a constant multiplier is called a *Geometrical Progression*.

The constant factor by which the successive terms are multiplied is called the *ratio*.

When the ratio is greater than a unit, the series is called an *Ascending Geometrical Progression*; when less than a unit, the series is called a *Descending Geometrical Progression*. Thus, 1, 3, 9, 27, is an Ascending Geometrical Progression, whose ratio is 3; and 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, is a Descending Geometrical Progression, whose ratio is $\frac{1}{4}$.

In Geometrical, as in Arithmetical Progression, there are five terms to be considered: 1, *the first term*; 2, *the last term*; 3, *the common ratio*; 4, *the number of terms*; 5, *the sum of all the terms.*

These quantities are so related to each other that any three being given the remaining two can be found.

We will demonstrate two of the most important, leaving the student to pursue the remainder in Algebra.

THE LAST TERM.

205. To find the last term, when the first term, the ratio and the number of terms are given.

Rule. Multiply the first term by that power of the ratio which is expressed by the number of terms *less one.*

Ex. Find the last term of a Geometrical Progression, whose first term is 2, the ratio 3, and the number of terms 8.

Proceeding by the rule given above,

$$2 \times 3^7 = 4374.$$

The reason of the process is rendered evident by the following consideration:

If the first term be 3 and the ratio of the progression 5, the series will be

$$3,\ 3 \times 5,\ 3 \times 5 \times 5,\ 3 \times 5 \times 5 \times 5 \text{ \&c.}$$
$$\text{or } 3,\ 3 \times 5,\ 3 \times 5^2\quad 3 \times 5^3 \text{ \&c.}$$

in which it is plain that the last term (3×5^3) will always be equal to the first term (3) multiplied by that power of the ratio (5) which is expressed by the number of terms *less one.*

THE SUM OF ALL THE TERMS.

206. To find the sum of the terms of a Geometrical Progession when the first term, the last term, and the ratio are given.

Rule. Divide the difference between the first term and the last term multiplied by the ratio, by the difference between the ratio and a unit.

The following examples will explain the process :

Take the progression of which the ratio is 3,

$$4,\ 4\times3,\ 4\times3^2,\ 4\times3^3,\ 4\times3^4$$

multiplying each term by the ratio, we obtain the series

$$4+3,\ 4+3^2,\ 4+3^3,\ 4\times3^4,\ 4\times3^5$$

The sum of which is 3 times the sum of the first series.

If the first series be subtracted from the second, the remainder will be

4×3^5—4 which must be twice the given series.

But this remainder is the difference between the first term (4,) and the product of the last term (4×3^4), multiplied by the ratio (3) while 2 is the difference between the ratio and a unit. The sum of the series will therefore be found according to the Rule.

207. If the number of terms of a *decreasing* Geometrical Progression were infinite, that is increased without limit, their sum would be equal to the first term divided by the difference between the ratio and a unit.

Thus, if the series $9, 3, \frac{1}{3}, \frac{1}{9}$, &c., in which the ratio $\frac{1}{3}$ were continued to an infinite number of terms; the last term would be diminished without limit, that is it would be 0, and by the preceding proposition the sum of all the terms would be

$$(9-0\times\tfrac{1}{3})\div(1-\tfrac{1}{3})=9\div\tfrac{2}{3}=13\tfrac{1}{2}.$$

On this principle the value of a Repeating Decimal may be computed. (Art. 96.)

For example, the Repetend $\cdot444\dot{4}$ is equal to $\frac{4}{10}+\frac{4}{100}+\frac{4}{1000}+\frac{4}{10000}$ and so on without limit.

Now this is a decreasing Geometrical Progression, in which the ratio is $\frac{1}{10}$, the sum of all the terms is therefore $\frac{4}{10}\div\frac{9}{10}=\frac{4}{9}=\frac{2}{3}$.

Ex. LXVII.

1. The first term of a Geometrical Progression is 100, the ratio

of the progression is $\frac{1}{3}$, and the number of a terms 7; what is the last term? and the sum of all the terms?

2. The first term of a Geometrical Progression is 5, the ratio is 4 and the number of term 9; what is the last term?

3. A person travels 5 miles the first day, 10 miles the second day, 20 miles the third day, and so on, increasing in Geometrical Progression: if he continues to travel in this way for 7 days, how far will he go the last day?

4. A ball falling from the height of 10 feet, by its elasticity bounds 5 feet, and again falling, bounds $2\frac{1}{2}$ feet, and so on continuing to bound $\frac{1}{2}$ as high as it fell. What will be the whole distance made in the successive falls before coming to a state of rest? and what the whole distance made by its successive bounds?

5. The first term of a Geometrical Progression is 4, the last term is 78732, and the ratio is 3. What is the sum of all the terms?

6. What is the sum of an infinite number of terms of a Geometrical Progression whose first term is 1000 and ratio $\frac{1}{2}$?

7. A man commences business with a capital of $20 and doubled it in every 2 years. What was his capital at the end of 25 years?

8. What is the sum of the infinite series 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c.?

9. What is the sum of the infinite series $\frac{1}{10}$, $\frac{3}{100}$, $\frac{9}{1000}$, $\frac{27}{10000}$, &c.

10. If 11 yards of cloth were sold at 1 cent for the first yard, 3 for the second, 9 for the third, and so on, what would be the price of the last yard? and what would the whole amount to?

11. What is the sum of the infinite series 1, $\frac{1}{3}$, $\frac{1}{9}$, &c., also of 1, $\frac{2}{3}$, $\frac{4}{9}$, &c.

Miscellaneous Questions and Examples on preceding Arts.

I

1. Find the length of the interior edge of a cubical bin which contains 310 bushels of wheat. (A bushel fills 2218·192 cubic inches.)

2. What is the whole value of $6\frac{3}{4}$ yds. of cloth at \$.$3\frac{1}{2}$ a yard, $10\frac{3}{4}$ lbs. of tea at \$·75 a lb., and 43 bush. of corn at \$·70 a bushel? Divide the sum among 4 people in the proportions 1, 2, 3, 4.

3. Assuming only the definition of a vulgar fraction, prove that the numerator and denominator of any vulgar fraction may be multiplied or divided by the same integer without altering its value.

(*a*) What fraction of a pound sterling is $4\frac{14}{21}-10\frac{44}{55}+9\frac{3}{45}-\frac{52}{117}$ of a penny?

(*b*) Find the value of $\frac{1}{3}\times\frac{17}{3}$ of $\frac{27}{85}+\left(2\frac{1}{3}+\frac{1}{3+\frac{1}{4}}\right)\times\frac{39}{103}$

4. A debt is to be discharged at the expiration of $6\frac{1}{2}$ months, $\frac{1}{4}$ is paid immediately, and $\frac{1}{4}$ at the expiration of 3 months: when ought the remainder to be paid?

5. If 10 men or 15 boys can reap 20 acres of corn in 6 days working 14 hours a day, how many boys must be employed to assist 3 men to reap 6 acres in $1\frac{3}{4}$ days of 8 hours a day?

6. What is the height of a closet $8\frac{1}{3}$ ft. by $6\frac{2}{3}$ ft. which will exactly contain 12 boxes $4\frac{1}{6}$ ft. long, $3\frac{1}{3}$ ft. wide, $2\frac{1}{2}$ ft deep?

7. Two lines are 41·06328 and ·0438 of an inch long respectively. How many lines as long as the latter can be cut off from the former? What will be the length of the remaining line?

8. Explain the method of extracting the cube root of a number. Find the area of the surface of a cube which contains 733626753859 cubic inches.

9. Shares in a certain Railway pay \$3·25 dividend per annum. How much must I give for them to get 5 per cent. for my money?

A person having bought 20 shares at this price sells them when they have risen \$7 each, and buys mining stock at 90, paying $3\frac{1}{4}$ per cent. Find the change in his income.

10. What sum of money will amount to $845 in 2 years at 4 per cent. compound interest, and what will it amount to in 2 more years?

11. A merchant sells 72 bushels of corn at a profit of 8 per cent., and 37 bushels at a profit of 12 per cent.; if he had sold the whole at a uniform profit of 10 per cent. he would have received $2·7125 more than he actually did; what was the price he paid for the corn?

12. The gross receipts of a railway company in a certain year are apportioned as follows; 41 per cent. to pay the working expences, 56 per cent. to give the shareholders a dividend at the rate of $3\frac{1}{2}$ per cent. on their shares; and the remainder $15000, is reserved; find the paid-up capital of the company.

II.

1. Express in figures one billion, three hundred thousand millions, five hundred and seven thousands, three hundred and sixty four; and in writing 236045978213428.

2. When the pound sterling was worth 24 francs, 75 centimes, a traveller at Dover received 15*s.* for a Napoleon (20 francs). Of how much was he cheated?

3. Shew how by first principles to calculate values by Practice. Find by Practice the value 750 articles at £5. 8*s.* 4*d.* each; and the price of 3 cwt. 2 qrs. $18\frac{1}{2}$ lbs. at £3. 7*s.* 6*d.* per cwt. (Canada currency.)

4. Explain the difference between a Vulgar and a Decimal Fraction. Simplify

(*a*) $\dfrac{2\times\sqrt{1+\frac{1}{3}}\div\sqrt{1-\frac{1}{5}}}{5\times\sqrt{1+\frac{1}{3}}\times\sqrt{1-\frac{1}{5}}}$.

(*b*) $\dfrac{2\frac{4}{11}}{2\cdot 6}\div\dfrac{2\frac{7}{11}}{8\cdot 7}$.

(*c*) $\frac{2}{5}\left(6\frac{2}{3}+2\frac{1}{2}\right)£+\dfrac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\text{ of }3\frac{1}{3}+\frac{13}{36}}\times\cdot 95\text{ of }5s.+\dfrac{16\cdot 8}{\cdot 024}d.$

(*d*) $\cdot 0576\times 1\cdot 97+\cdot 142857\div 2\frac{1}{7}+\cdot 0454864$.

If the latter result represent a square in yards, find the length of its side in inches.

5. *A* and *B* can finish a piece of work in $1\frac{1}{2}$ days, *A* and *C* in

2 days, and B and C in 3 days. If \$6 be paid for the piece of work, what are a day's wages of each workman?

6. A tax of \$530 is to be raised from 3 towns, the numbers of inhabitants of which are respectively 2500, 3000, and 4200. How much should each town pay, and each person in it?

7. If 15 men or 40 boys do a piece of work in 12 days, how many days would 10 men and 20 boys take to do a piece of work 7 times as great?

8. Define Interest and Discount. Shew that the Interest and Discount on \$64·50 for 8 months at $4\frac{1}{2}$ per cent. per annum, differ by \$·5625 nearly.

9. The breadth of a room is 14 ft.; the cost of papering the walls at \$·05 a square yard is \$4; and that of carpeting the room at \$·225 a square yard is \$5.60. Determine the height and length of the room.

10. It is observed that 20 men, all of equal strength, build a wall 15 feet high, 30 feet long, in 60 days, and 35 others, also of equal strength, build a wall 20 feet high, and 40 feet long, in 64 days; what is the ratio of the strength of the men of the two classes?

11. A person has 200 shares in a certain Railway, for which he gives \$100 per share. When they are paying \$2 per cent. he sells them all at \$46 per share, and invests the proceeds in City Debentures at 92, paying 6 per cent. Find the alteration in his income.

12. Explain the method of pointing in the Extraction of the cube root of decimals.

Find the square root of $\frac{\cdot 00123}{\cdot 18}$ and the cube root of 423564·751.

III.

1. Shew from first principles how to divide one fraction by another.

Prove that the fraction $\frac{6+7}{7+8}$ is greater than $\frac{6}{7}$ and less than $\frac{7}{8}$.

Simplify

$$\frac{1\frac{1}{4}-\frac{5}{12}}{1\frac{1}{4}+\frac{5}{12}}+\frac{7}{6} \text{ of } \frac{9\times 5}{14\times 3}-\frac{11\frac{1}{4}}{15}$$

2. Express

(*a*) $(\frac{1}{2}+\frac{3}{4})$£$+(\frac{1}{3}+\frac{3}{5})s.+(\frac{1}{4}+\frac{4}{5})d.$ as the decimal of £1.

(*b*) $5\frac{1}{2}$ cwts. as the decimal of a ton, and $2\frac{1}{2}$ qrs. as the decimal of a cwt.

3. A man contracts to perform a piece of work in 60 days, and immediately employs upon it 30 men; at the end of 48 days the work is only half done; required the additional number of men necessary to fulfil the contract.

4. A person increased his capital annually $\frac{1}{3}$rd part, and at the end of 4 years, one year's interest thereon at $4\frac{1}{2}$ per cent. amounted to $270. What capital did he start with?

5. *A* can do a piece of work in 12 hours, *B* in four hours, and *C* in 3 hours. *A*, *B* and *C* all work together for half an hour, when *A* leaves off. How long will it take *B* and *C* to finish the piece of work?

6. Explain the method of pointing in extracting the square root of a whole number, and also of a decimal.

(*a*) The surface of a cube is 86·94 square feet, find the length of its edge.

(*b*) Given that the square of 10129 is 102596641, find the square of 101293 without going through the operation of squaring.

(*c*) Given that the square root of 105625 is 325, find that of 10582009.

7. If 144 men can dig a trench 40 yds long, 1 ft. 6 in. broad, and 48 ft. deep, in 3 days of 10 hours each; how long must another trench 5 ft. deep, and 2 ft. 3 in. broad be, in order that 51 men may dig it in 15 days of 9 hours each?

8. If a cubic foot of marble weigh 2·716 times as much as a cubic foot of water, find the weight of a block of marble 9 ft. 6 in. long, 2 ft. 3 in. broad, 2 ft. thick, supposing a cubic foot of water to weigh 1000 oz.

9. Find the equated time of payment of $200 due 14 months' hence, and of $300 due 19 months' hence; and determine the present value of the whole sum, (supposed to be due at the equated time) allowing $3\frac{3}{4}$ per cent. simple interest.

10. A publisher wishes to net $·70 for each copy of a work; what price should he put upon it so as to be able to allow the trade 30 per cent. discount?

11. What is the present worth of $840, due 3 years and 6 months hence, at 6 per cent.?

12. Which is the greater $\sqrt{2}$ or $\sqrt[3]{3}$? Find the cube root of $\frac{5030 \cdot 912}{65536}$

IV.

1. A stationer bought 40 reams of paper at $·625 a ream, and 60 reams at $·775 a ream; find the whole cost, and the average price per ream, and if the whole be sold at $·75 a ream, find the profit.

2. Express in figures thirty-four and two thousandths, and by it divide 2825565·2. What alteration must be made in the quotient if the decimal point in the dividend be moved eight places to the left?

3. Three men, working 9 hours a day, take 16 days to pave a road 315 yds. long and 30 ft broad; how many days will four men, two of whom work 8 hours, and two 10 hours a day, take to pave a road 1575 yds. long, and 35 ft. 6 in. broad?

4. The areas of two cubes are respectively 5359·375 and 5·359375 cubic feet; find the difference of the lengths of their edges in inches.

5. *A* and *B* agree to divide their travelling expenses in the proportion of the numbers 7 and 5. *A* pays in the whole $25·80, and *B* pays in the whole $15·85. What is the one to pay and the other to receive in order to settle the account?

6. When are four quantities said to be in proportion? Shew by means of your definition that $191·625 : $31·50 : : 365 days : 60 days and deduce the method of working the following question:

If 3 workmen earn between them $191·625 in a year, in what time will they earn $31·50?

7. The Discount on a sum due one year hence at 5 per cent. per annum interest is $15. What is the sum?

8. If 8 variegated silk scarfs, measuring each 3 cubits in breadth and 8 in length cost 100 nishcas; what will a like scarf $3\frac{1}{2}$ cubits long and $\frac{1}{2}$ a cubit wide cost in terms of drammas, panas, cacinio, and cowry shells?

1 nishca=16 drammas, 1 dramma=16 panas, 1 pana=4 cacinis, 1 cacini=20 cowry shells.

9. A person invests a sum of money in 50 casks of sugar each containing 11 cwt. 3 qrs. 2 lbs. at $3·50 per cwt of 28 lbs. what price must he sell them at after 6 months to realize the same interest as he might have had for his money at $4\frac{1}{2}$ per cent.?

10. It is agreed that the rent of a farm shall consist of a fixed sum together with the value of a certain number of bushels of wheat; when wheat is \$2 a bushel the rent is \$250, when wheat is \$2·25 a bushel the rent is \$260, what will the rent be when wheat is \$2·50 a bushel?

11. *A* and *B* can do a piece of work in 10 days; *B* and *C* in 15 days and *A* and *C* in 25 days; they all work at it for 4 days; *A* then leaves, and *B* and *C* go on for 5 days; *B* then leaves: In how many days will *C* finish the work?

12. A ship's hold is 99 ft. long, 40 ft. broad, and 5 ft. deep, how many bales can be stowed in it each 3 ft. 6 in. long, 2 ft. 8 in broad, and 2 ft. 6 in deep, leaving a gangway of 4 ft. broad?

V.

1. (*a*) The French metre being 39·37 in., how many yards are there in 3600 metres?

(*b*) 3 versts being = 2 miles, in what time will a man travel over 2500 versts at the rate of 10 miles an hour?

2. State what fractions produce terminating decimals, and what produce recurring decimals. Explain the reason.

Reduce to decimals the vulgar fractions $\frac{3}{5}$, $\frac{30}{57}$, $\frac{600}{576}$, and add them; and divide their sum by [illegible]3741 to two decimal places.

3. A silversmith purchases a large dish weighing 80 oz., and forms it into 2 dozen of dessert spoons, and one dozen of table-spoons. If the latter weigh 28 oz., what is the weight of each dessert-spoon, and what is its value at $\frac{3}{16}$ of \$·02 per grain?

4. Add together $\frac{3}{4}$ of $\frac{5}{9}$ of £2. 5*s*., $\frac{2}{7}$ of 3 guineas, $\dot{2}\dot{7}$ of £1. 18*s*. 6*d*., and $2\dot{1}5\dot{4}$ of £2. 15*s*., and reduce the result to the fraction of 25 guineas.

5. How much may a person who has an annual income of \$840·50 spend per day, in order to save \$63·966$\frac{2}{3}$ m. after paying a Tax of \$·16 in the dollar?

6. Find the square root of $\frac{100.10001}{1000}$, and the cube root of ·03.

7. If a piece of work can be finished in 45 days by 35 men, and if the men drop off by 7 at a time at the end of every 15 days, how long will it be before the work is finished?

8. Divide £16984 among *A*, *B*, *C* and *D*, so that *A*'s share : *B*'s share :: 6 : 5; *B*'s share : *C*'s share :: 2 : 3; and *C*'s share : *D*'s share :: 4 : 3.

9. What is the cost of paper for the walls of a room 30 ft. long, 15 ft. broad, and 15 ft. high, the paper be $1\frac{1}{2}$ yds. wide, and its price $4\frac{1}{2}$ cents per yard? What would be the cost for a room twice as long, twice as broad, and twice as high, the paper twice as wide, and costing twice as much per yard as before?

10. If when 25 per cent. is lost in grinding wheat, a country has to import ten million quarters, but can maintain itself on its own produce if only 5 per cent. be lost, find the quantity of wheat grown in the country.

11. How many flag-stones, each 5·76 ft. long and 4·15 ft. wide are required for paving a cloister which encloses a rectangular court 45·77 yds. long and 41·93 yds. wide; the cloister being 12·45 ft. wide?

12. A man wishing to invest $1000 in Government bonds bearing interest at 6 per cent, inquires the price of the stock, and finds it to be 86 per cent.; he delays the investment however, until bonds have risen to 87. What effect has the delay on his income?

VI.

1. Explain our decimal system of arithmetic, and how it is that we are enabled with digits to express any number, however great.

2. If 12 men or 18 boys can do $\frac{3}{4}$ of a piece of work in $6\frac{1}{2}$ hours, in what time will 11 men and 9 boys do the rest?

3. Express ·0025 by a simple fraction, and $\frac{3}{125}+6\frac{3}{40}-\frac{17}{25}$ by a decimal.

4. Explain what is meant by Compound Interest. What is the difference between the Simple and Compound Interest of $345·50 for 2 years at 3·5 per cent.?

5. Define discount. If the discount on $226·33, due at the end of a year and a half be $12·80 what is the rate of interest?

6. Show how to divide 4 things of the same size and material among 3 children, *A*, *B*, and *C*, by merely breaking *one* of the four, and so that *A*'s share shall be $\frac{2}{5}+\frac{5}{6}+\frac{1}{2}$ of a whole one; *B*'s share $\frac{3}{8}+\frac{7}{10}+\frac{3}{20}$ of a whole one; and *C*'s share the remainder.

7. A grain of pure gold can be drawn out into a wire 550 feet long; find the cost of a wire of the same thickness which would extend round the earth, assuming the circumference of the earth to be 25,000 miles, and the value of gold to be $21·25 per oz. troy.

8. (*a*) If $A = 1\frac{1}{6}$ of B, and $C = 2\frac{1}{6}$ of B, find the ratio of A to C.

**IMAGE EVALUATION
TEST TARGET (MT-3)**

6"


(*b*) Simplify $\frac{\frac{3}{5}}{\frac{5}{9}} + \frac{\frac{5}{7}}{7\frac{1}{2} \text{ of } 1\frac{1}{2}} + \left(\frac{3\frac{3}{4}}{4\frac{4}{7}} - \frac{3\frac{3}{5}}{4\frac{1}{4}} + \frac{\frac{2}{7}}{2\frac{1}{2}}\right) \div 4\frac{4}{7}$.

(*c*) Divide 19·836 by 51·6; and 1083·6 by 5·16, and also by ·00516, and prove each result by vulgar fractions.

9. A shopkeeper buys $\frac{1}{2}$ cwt. of tea at 4*s*. 2*d*. per lb., and mixes it with tea which cost him 2*s*. 11*d*. per lb. How much of the latter must he add to the former that he may sell the mixture at 3*s*. [illegible] per lb., and gain 20 per cent. on his outlay?

10. 4 ft. 4 in. being the area of a map which $\frac{1}{4}$ is laid down on the scale of an inch to a mile, required the number of acres represented.

11. A person's taxes amount to 10 per cent. on his income, and after paying them he has $1250 a year; find his gross income.

12. In an election of a member of Parliament, $\frac{1}{16}$th of the constituency refused to vote, and of two candidates the one who is supported by $\frac{19}{40}$th of the whole constituency is returned by a majority of five; find the number of votes for each.

VII.

1. Prove that 29 multiplied by 15=15 multiplied by 29. What is the difference between abstract and concrete numbers?

2. On the roof of a public building there was a tank holding 18 tons of water. Supposing it cubical, what would have been its dimensions? One cubic foot of water weighs 1000 oz.

3. If it take 3′ to read over two pages of a book containing 30 lines in each page, with an average of 10 words in a line, how many pages of another book can be read in 20′ where there are 50 lines in a page and 12 words in a line?

4. Required the expense of painting the outside of a cubical box, whose edge is 3·5 ft., at $1·30 per sq. yd.

5. The wages of 25 men amount to £76. 13*s*. 4*d*. in 16 days, how many boys must work 24 days to receive £103. 10*s*., the daily wages of the latter being one half those of the former?

6. If a bookseller gain $\frac{2}{7}$th of the prime cost of a book by selling it at $1·50, what would be his gain per cent. if he sold it at $1·75?

"SPECTATOR" STEAM PRESS, COURT HOUSE SQUARE, HAMILTON, C. W.