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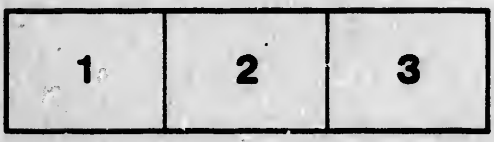
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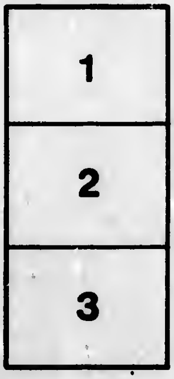
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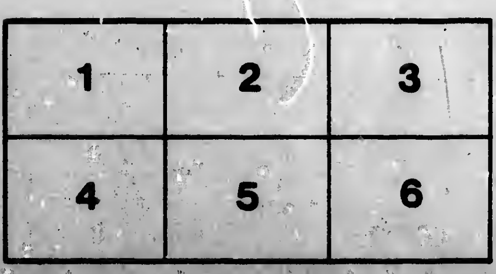
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ON THE DYNAMICS OF THE EARTH'S ROTATION, WITH RESPECT TO THE PERIODIC VARIATIONS OF LATITUDE.

BY

SIMON NEWCOMB.

On the Dynamics of the Earth's Rotation, with respect to the Periodic Variations of Latitude. By Simon Newcomb.

The recent remarkable discovery of Mr. S. C. Chandler, that the axis of rotation of the Earth revolves around the axis of maximum moment of inertia in a period of about 427 days, is worthy of special attention.* At first sight it seems in complete contradiction to the principles of dynamics, which show that the ratio of the time of such a rotation to that of the Earth's revolution should be equal to the ratio of the polar moment of inertia of the Earth to the difference between the equatorial and the polar moments. Representing these moments by A and C, it is well known that the theory of rotation of a rigid body gives the equation

$$\tau = \frac{A}{C-A},$$

τ being the period of rotation of the pole in sidereal days.

Now the ratio in question is given with an error not exceeding a few hundredths of its total amount by the magnitude of the precession and nutation. The value found by Oppolzer is $\frac{1}{305}$, giving the time of rotation as 305 days.

This result has long been known, and several attempts have been made to determine the distance between the two axes, especially at Poulkova and Washington. A series of observations was made with the Washington Prime Vertical Transit during the years 1862–1867, including six complete periods of the inequality. Thus the determination of the coefficient and zero of the argument is completely independent of all sources of error having an annual or diurnal period. Such errors are

* *Astronomical Journal*, Numbers 248, 249.

liable to affect the determination unless it is continued over this period.

A preliminary discussion of the observations, which was made at the request of Sir William Thomson, and published by him, gave a coefficient of $0''.05$ for the inequality. A more complete discussion, undertaken quite recently, reduces the coefficient to $0''.03$, corresponding to a distance of three feet between the two axes. This result was quite within the limits of errors of observation, and seemed to show that there was no appreciable difference between the two axes. This result was in complete accordance with the conclusions reached from the Poulkova observations, and seemed to show, beyond doubt, that there could be no inequality of the kind looked for.

Mr. Chandler's discovery gives rise to the question whether there can be any defect in the theory which assigns 306 days as the time of rotation. The object of this paper is to point out that there is such a defect—namely, the failure to take account of the elasticity of the Earth itself, and of the mobility of the ocean.

The mathematical theory of the rotation of a solid body, on which the conclusions hitherto received have been based, presupposes that the body is absolutely rigid. As the Earth and ocean combined are not absolutely rigid, we have to inquire whether their flexibility appreciably affects the conclusions. That it does can be shown very simply from the following consideration:—

Imagine the Earth to be a homogeneous spheroid, entirely covered by an ocean of the same density with itself. It is then evident that, if the whole mass be set in uniform rotation around any axis whatever, the ocean will assume the form of an oblate ellipsoid of revolution, whose smaller axis coincides with that of rotation. Hence, the axes of rotation and of figure will be in perfect coincidence under all circumstances.

To apply a similar reasoning to the case of the Earth, imagine that the axis of rotation is displaced by $0''.20$ from that of greatest moment of inertia, which I shall call the axis of figure. Then, with an ocean of the same density as the Earth, its equator would be displaced by the same amount. The ocean level would change in middle latitudes by about one inch at the maximum. But this change would have for its effect a corresponding change in the axis of figure. As the ocean covers only three-fourths of the Earth, the axis would be displaced by three-fourths of the distance between the two axes, were ocean and Earth of equal density. But, as the density of the Earth is some five times as great, the actual change would be only one-fifth of this. It would even be less than one-fifth, because the displacement of the ocean equator would be resisted by the attraction of the Earth itself. The exact amount of this resistance cannot be accurately given, but I think the displacement would thereby be reduced to one-half. I therefore think that one-fourteenth would be an approximate estimate of the displacement of the axis of figure, in consequence of the movement of the ocean. As Mr. Chandler's period requires a displacement of two-sevenths, the ocean displacement only accounts for one-fourth of the difference.

The remainder is to be attributed to the elasticity of the Earth itself. It is evident that the flexure caused by the non-coincidence of the two axes tends to distort the Earth into a spheroid of the same form as that which the ocean assumes, and thus to bring the two axes together.

We have now to show how this deformation of the Earth changes the time of revolution. Let us imagine ourselves to be looking down upon the North Pole, and let P be the actual mean pole of the Earth when the two axes are in coincidence, and R the end of the axis of rotation. Then, in consequence of the rotation around R, the actual pole will be displaced to a certain point, P′. Now, the law of rotation of R is such that it constantly moves around the instantaneous position of P′ in a period of 305 days, irrespective of the instantaneous motion of P′ itself. In other words, the angular motion of R at each moment is that which it would have if P′ had remained at rest. Hence, the angular motion as seen from P is less than that from P′, in the ratio of P′ R : P R.

But, as R rotates, P′ continually changes its position and rotates also, remaining on the straight line PR. Thus the time of revolution of R around P is increased in the same ratio.

We may next inquire what degree of rigidity the Earth must have in order that the total displacement of the axis of figure produced by the change in the centrifugal force may be two-sevenths that of the displacement of the axis of rotation; in other words, that the ratio P′R : PR may be 5 : 7. A rigorous treatment of the problem is scarcely possible, as the rigidity probably varies from the surface inward; I shall therefore only attempt a rough estimate, founded on certain conclusions as to the deformation of a rotating spheroid reached in Thomson and Tait's *Natural Philosophy*. To proceed in the simplest way, I shall assume the earth to have the rigidity of steel, and inquire to what displacement the axis of figure would be subject, in consequence of the centrifugal force arising from a rotation around an axis differing from the normal axis of figure.

Conceive a solid sphere, of the same size and general constitution as the Earth, to be set in rotation like the Earth. Let ϵ' be the ellipticity induced in it by the rotation, and let ϵ be the actual ellipticity of the Earth. We shall then have a superposition of two ellipticities, the one ϵ, such that P is the pole of figure; the other, ϵ' such that R is the pole of figure. P′ being the pole arising from the combined ellipticities, I assume that we have the proportion

$$PP' : \epsilon' = P'R : \epsilon.$$

To find the value of ϵ' I start from the conclusion of Thomson and Tait (§837), that a ball of steel of any radius rotating with an equatorial velocity of 10,000 centimètres per second will be flattened to an ellipticity of $\frac{1}{7220}$. The Earth's equatorial velocity is 4·65 times this. Its density is less than that of steel: the density which we should assume is not the actual mean density but a mean in which greater weight is given to the superficial portions, because these have the greatest centrifugal force. Probably the actual mean to be adopted is 0·6 of the density of steel. We have, therefore, neglecting the effect of gravitation,

$$\epsilon'_0 = \frac{0{\cdot}6 \times 4{\cdot}65^2}{7220} = \frac{1}{557}.$$

But the deformation of the Earth is resisted by the gravitation of its parts. By a theorem given by Thomson and Tait, we should have, taking this effect into account—

$$\frac{1}{\epsilon'} = \frac{1}{\epsilon'_0} + \frac{1}{\epsilon} = 557 + 292 = 849.$$

Hence we have

$$\epsilon' = \frac{1}{849}.$$

Hence, considering only the solid Earth,

$$PP' : P'R = 292 : 849.$$

We have already concluded that the motion of the ocean will shift P′ one-fourteenth of the way from P′ to R. Hence, finally,

$$PP' : P'R = 353 : 788$$
$$PR : P'R = 1142 : 788.$$

Time of revolution of pole	= 443 days
Period for a rigid earth	= 306 „
Computed increase of period	= 137 „
Observed increase of period	= 121 „

The conclusion is that the Earth yields slightly less to the centrifugal force than it would if it had the rigidity of steel, and that it is consequently slightly more rigid than steel.

We have next to consider the effect of viscosity of the earth. Those geologists who have given special attention to the subject regard it as well established that the Earth yields under the weight of deposits as if it were a thin crust floating upon a liquid interior, and must therefore be a viscous solid, if a solid at all. The effect of viscosity is that the normal pole P of the Earth would be in slow but continuous motion towards the revolving pole R. Both P and R would then describe logarithmic spirals, so related that the tangent to the inner spiral at the position of P at any moment would pass through the position of R at that moment, and cut the R spiral normally. Thus the line PR would diminish from century to century by equal fractions of its amount in equal times. Thus the poles would eventually appear to meet, unless separated from time to time by the action of causes changing one or both of them.

Since the position of the pole of figure of the Earth may be supposed to have been originally determined by the rotation itself, and continually to approach the pole of rotation if it were very slightly separated from it, the presumption would appear to be that the two poles would now be in apparent coincidence, in the absence of disturbing causes. Moreover, the evidence of the most accurate observations hitherto made with Prime Vertical Transits seems to show that the separation of the two poles at the epochs 1842 and 1864 could scarcely have exceeded the tenth of a second. But observations made with probably equal exactness at the present time seem to show, according to Mr. Chandler, a separation of $0''\cdot3$. It would seem, therefore, accepting these provisional numerical results, that some disturbing cause has acted. A *vera causa* was pointed out some years ago by Sir William Thomson, in the motions of the winds and oceans, and especially in changes in the polar ice-cap. In order to have its greatest effect such a movement of matter must occur in the middle latitudes; a change in the polar ice-cap would be the less appreciable in its effect the nearer it occurred to the pole. A heavy snow-fall over the whole of Northern Asia, unaccompanied by a corresponding fall on the American continent, would undoubtedly cause a slight displacement; but I doubt whether the greatest effect of this kind could amount to $0''\cdot05$.

But we have also to consider the effect of an annually repeated disturbance of this kind. Mr. Chandler's period is such that the pole of rotation makes six revolutions in seven years. Hence, during one-half the period of seven years, the effect of an annually repeated cause will be cumulative. In a recent volume of the *Bulletin Astronomique*, Mr. Radau has investigated the effect of an annual periodic change in the position of the Earth's axis of figure, and shown that it will be multiplied three times, in consequence of this cumulative effect. But his analysis rests on the hypothesis of a 306-day period. It is worth while to show how such an annual cause would act when we adopt Mr. Chandler's period.

[redacted]

Let Q be the mean position of the pole of figure of the Earth, and let us assume that the actual pole P revolves around it in a radius a, and in a period of one year. Let R be the position of the pole of rotation at any time. Then, at each moment, R is revolving around the fixed position P with a uniform motion, which, if continued, would cause it to complete a revolution in 427 days. Let us put

n, the mean motion of the radius PQ;

μ, the mean motion of R around the position of P;

x, y, the rectangular co-ordinates of R referred to Q as an origin.

The law of rotation then gives the equations

$$\frac{dx}{dt} = -\mu y + a\mu \sin(nt + c)$$

$$\frac{dy}{dt} = \mu x - a\mu \cos(nt + c).$$

The integration of these equations gives

$$x = \alpha \cos \mu t - \beta \sin \mu t - \frac{a\mu}{n-\mu} \cos(nt + c)$$

$$y = \alpha \sin \mu t + \beta \cos \mu t - \frac{a\mu}{n-\mu} \sin(nt + c),$$

α and β being arbitrary constants.

Substituting for μ and n their numerical values, we have, approximately,

$$x = \alpha \cos \mu t - \beta \sin \mu t + 6a \cos(nt + c)$$

$$y = \alpha \sin \mu t + \beta \cos \mu t + 6a \sin(nt + c).$$

Such a rotation as we have supposed, around a circle of $0''{\cdot}05$ in radius, would suffice to produce anomalies as large as those actually observed.

If the winters in Siberia and in North America occurred at opposite seasons, we should have no difficulty in accepting the sufficiency of annual falls of snow to account for the anomaly. But, under the actual circumstances, we must await the results of further investigations into the whole subject.

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