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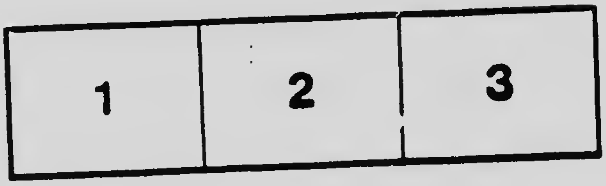

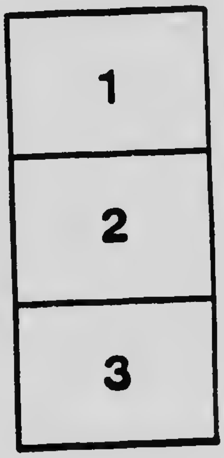

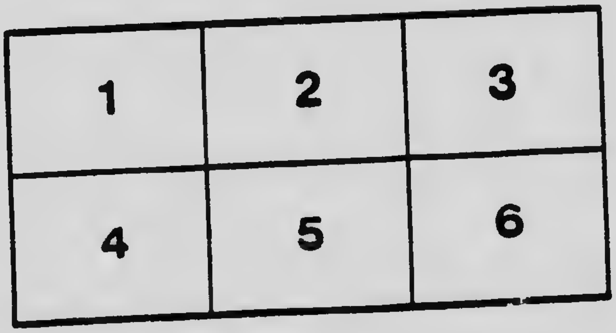

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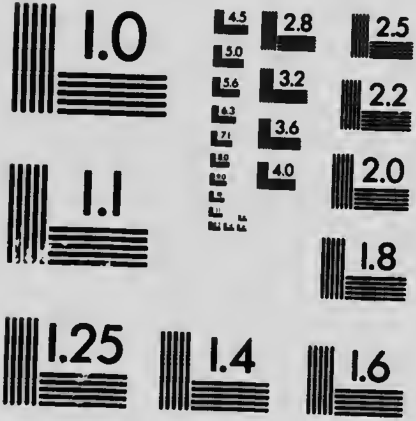

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In compliance with the Program of Public Instruction

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It was the aim of the Author to furnish the pupil a thorough training in both oral *and written Arithmetic by well-classified exercises;*

To treat Mental Arithmetic *with as much care as becomes its importance as an everyday necessity, and a well-recognized mind driller;*

To supply a large number of simple, graded, practical problems dealing with trading, farming, manufacturing features as well as with up-to-date questions, like money saving, hygiene, anticlcoholism;

To give theory *(unhappily slighted nowadays) a pretty extensive development, but based upon observation and induction.*

We hope that this new book will appeal to the teachers as both pithy and practical, and that they will amically greet it.

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SCHOOL ARITHMETIC

Intermediate Course

INTRODUCTION

1. A *quantity* is anything that can be counted or measured, increased or decreased.

EXAMPLES.—The number of pupils in a class, the height of a steeple, the surface of a field, the volume of a load of brick, the weight of a bag of flour.

2. A *unit* is a single thing or a certain quantity used to measure other quantities of the same kind.

EXAMPLES.—When I count the trees of an orchard, the unit is *one tree;* when I measure cloth, the unit may be *one yard;* when I find the capacity of a barrel, the unit may be *one gallon*, etc.

3. A *number* expresses how many *units* there are in a *quantity*.

EXAMPLE.—Let us measure the length of a fence with a yardstick, and suppose we lay down our stick ninety-nine times, ninety-nine is the *number*, the yardstick is the *unit*, and the length of the fence is the *quantity*.

4. There are three classes of numbers: an integer, a fraction, and a mixed number.

5. An *integer* is a number that contains one or several whole units.

EXAMPLE.—The length of the fence measures ninety-nine yards exactly. *Ninety-nine* is an integer.

6. A *fraction* expresses a quantity less than the unit.

EXAMPLE.—The length of my desk is three quarters of a yard. *Three quarters* is a fraction.

7. A *mixed number* contains both an integer and a fraction.

EXAMPLES.—One yard and a quarter; three yards and three quarters.

8. A number of any of these three classes may be either *concrete* or *abstract.* It is concrete when the nature of its unit is given: five *yards.* It is abstract when the nature of its unit is not given: five.

9. *Arithmetic* is the science of numbers; and it teaches how to form them, name them, write and combine them.

10. A *principle* is a truth that is used as a stepping-stone to other truths of the same order.

11. A *problem* is a question requiring some unknown result when certain quantities are given.

12. A *solution* comprises the different operations that lead to the answer of a problem.

13. A *rule* points out the method to be followed in a particular class of problems.

14. *Arithmetical language* is the method of expressing numbers.

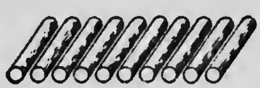
15. Arithmetical language is of two kinds, *oral* and *written.* The former is called *numeration,* and the latter *notation.*

NUMERATION

16. Numeration teaches how to name the numbers; it is the oral expression of numbers.

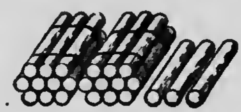
17. Units.—A single unit is named *one;* it is the first number. One and one more are named *two;* two and one more, *three;* three and one more, *four;* we obtain likewise the other simple numbers, *five, six, seven, eight, nine.* The nine first numbers are called *simple units* or units of the *first order.*

18. Tens.—Nine and one are *ten.* Regarding the collection *ten* as a single thing we have a unit of the *second order;* and we might count *one and ten, two and ten,* etc., as far as *ten and ten,* or *two tens,* which modified by use have given *eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty.*

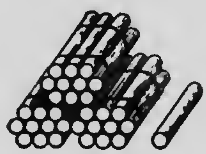


ten units

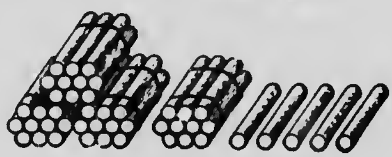
one ten



two tens and three units



three tens and one unit

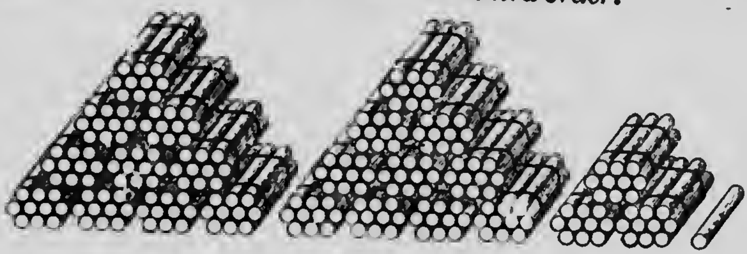


four tens and five units

We count by tens as we have counted by units: two tens or *twenty;* three tens or *thirty;* four tens or *forty;* five tens or *fifty;* six tens or *sixty;* seven tens or *seventy;* eight tens or *eighty;* nine tens or *ninety.*

To name the numbers comprised between the different sets of tens, we add the nine first numbers to twenty, thirty, forty, etc.: twenty-one, twenty-two, ... twenty-nine; thirty-one, thirty-two, thirty-three, ... thirty-nine; ...... ninety-nine.

19. Hundreds.—Ninety-nine and one, or ten tens, are named one *hundred*, or a unit of the *third order.*



ten tens or one hundred — ten tens one hundred — three tens — one unit

Two hundreds, three tens and one unit; or two hundreds and thirty-one units.

We count by hundreds as we have counted by units and tens: *one hundred, two hundreds, three hundreds,...... nine hundreds.*

To name the numbers ranging between the different sets of hundreds, we add the ninety-nine first numbers to one hundred, two hundred, eight hundred: one hundred and one, one hundred and two, one hundred and three, ... two hundred and one, ... three hundred and forty-six, ... nine hundred and ninety-nine.

The first three orders constitute the *first* or *units' period.*

20. Thousands.—Nine hundred and ninety-nine and one, or ten hundreds, are named one *thousand.*

We count by thousands, by ten-thousands and by hundred-thousands as we have counted by units, by tens and by hundreds. So, we say: one thousand, two thousands, fifty thousands, ninety-nine thousands, one hundred thousands, five hundred thousands, etc.; we thus reach the number nine hundred and ninety-nine thousand nine hundred and ninety-nine.

The thousands form the *fourth order;* the ten-thousands, the *fifth order;* the hundred-thousands, the *sixth order.*

These three orders constitute the *second* or *thousands' period.*

21. Millions.—Nine hundred and ninety-nine thousand nine hundred and ninety-nine and one, or one thousand thousand, are named one *million*, or the *seventh order*.

We count by millions as we have counted by thousands, and thus reach the *eighth order* (ten-millions), and the *ninth order* (hundred-millions).

These three orders constitute the *third* or *millions' period*.

Billions, trillions and quadrillions are formed in like manner, etc. There is no limit to the series of integers that may be thus formed.

22. Basic Principle.—Numeration is based on the following principle: *Ten units of a given order are worth one unit of the next higher order.*

Numeration Table.

5th PERIOD			4th PERIOD			3rd PERIOD			2nd PERIOD			1st PERIOD		
Trillions			**Billions**			**Millions**			**Thousands**			**Units**		
15th order	14th order	13th order	12th order	11th order	10th order	9th order	8th order	7th order	6th order	5th order	4th order	3rd order	2nd order	1st order
Hundred-trillions	Ten-trillions	Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Simple units

23. Note.—You see that the different orders are divided into periods: the *units' period*, the *thousands' period*, the *millions' period*, etc., and that each period has three orders: *units, tens, hundreds*. But in the first period, we have units, tens, and hundreds of the *simple units;* in the second period, we have units, tens, and hundreds of the *thousands' units*, and so on.

NOTATION.

24. **Notation** is the method of writing numbers by characters called *figures*.

25. Ten **figures** are used, and they form the *Alphabet* of the Arabic notation. They are:

Figures:	1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
Names:	one	two	three	four	five	six	seven	eight	nine	zero

The first nine figures indicate respectively the first nine numbers. They are called *significant figures*.

The tenth character, 0, (called *zero, naught* or *cipher*) indicates the negation, or absence of number.

26. **Basic Principle.**—*Any figure placed on the left of another represents units of the next higher order, that is, units ten times greater.*

By this *device of place*, all numbers may be expressed.

A figure standing alone or in the first place at the right of other figures, expresses *simple units*.

A figure standing in the second place, counting from the right, expresses *tens;* in the third place, *hundreds;* in the fourth place, *thousands*, etc.

Thus in the number 2 635, the figure 5, standing in the first place, represents five simple units.

The figure 3, standing in the second place, represents three tens.

The figure 6, standing in the third place, represents six hundred.

The figure 2, standing in the fourth place, represents two thousand.

27. It follows that figures have two values: an *absolute* value and a *relative* value.

28. The **absolute value** of a figure is the number of units it expresses when it stands alone or in the units' place.

The **relative value** of a figure is the number it expresses when in any other than units' place.

Thus in the number 4 852, the absolute value of the third figure is 8 simple units, and its relative value is 8 hundred or 800 simple units.

Exercises in Numeration.

29. Rule.—*To read an integer 1° begin at the right hand and separate the number into periods of three figures each; 2° then begin at the left and read each period in succession, giving the name of each period.*

Examples.—

87 is read: eighty-seven units.
245 is read: two hundred and forty-five units.
3 778 is read: three thousand seven hundred and seventy-eight units.
29 106 is read: twenty-nine thousand one hundred and six units.
705 349 is read: seven hundred and five thousand three hundred and forty-nine units.
4 037 402 is read: four million thirty-seven thousand four hundred and two units.

30. Note I.—The name of the last period (*units*) is generally omitted, it being understood.

Note II.—The highest period may contain only *one* or *two* figures.

Note III.—When a period or order is missing, its name is not called.

Read the following numerical expressions:

47	55 010	1 043 285
96	93 567	2 887 339
104	10 025	12 876 372
789	45 103	92 065 478
589	600 275	325 000 532
7 654	876 535	6 240 432 700
5 045	810 003	85 325 431 112
5 701	1 237 875	12 852 421 808

Exercises in Notation.

31. Rule.—*To write an integer 1° begin at the left and write the hundreds, tens and units of each period, as if it were a period of units; 2° when there are vacant places, fill them with ciphers.*

EXAMPLES.—The number five hundred and four is written thus: 504; five hundred and two thousand and forty-two, 502 042; five hundred million three hundred and eight, 500 000 308.

Express the following numbers in figures:

1. Forty-two.

2. Ninety-nine.

3. Five hundred and forty-nine.

4. Nine hundred and ninety-two.

5. Seven thousand two hundred and three.

6. Fifty-eight thousand and three.

7. Ninety-nine thousand five hundred.

8. Thirty-three thousand six hundred and four.

9. Two hundred and thirty-nine thousand two hundred and two.

10. Four hundred and thirty thousand and one.

11. Three million three hundred thousand two hundred.

12. Fifty-four million sixty thousand and four.

13. Seventy-eight million one hundred and eight thousand and sixteen.

14. Three million and nine.

15. Nine hundred and forty-seven thousand and two.

16. Four hundred and three million nine hundred and four thousand and eight.

17. Nine hundred and nine million two thousand and thirty-eight.

18. Fifty-seven thousand and twenty-three.

19. One billion two hundred million and three thousand.

20. Thirty-six billion and twenty-two thousand.

Analysis of Integers.

32. The units of a number of two figures equal 10 times the tens' figure plus the units' figure; the units of a number of three figures equal 100 times the hundreds' figure plus 10 times the tens' figure plus the units' figure; the units of a number of four figures equal 1000 times the thousands' figure plus 100 times the hundreds' figure plus 10 times the tens' figure plus the units' figure.

Examples.—1° Let us analyze 45.

The right-hand figure represents 5 units; the left-hand figure represents 4 tens, or 40 units. Hence, the number 45 expresses 40 units plus 5 units or 45 units.

2° Let us analyze 231.

From right to left, the first figure represents 1 unit, the second figure represents 3 tens or 30 units, the third figure represents 2 hundreds or 20 tens or 200 units. Hence, 231 expresses 200 units plus 30 units plus 1 unit or 231 units.

3° Let us analyze 7 283

From right to left, the first figure represents 3 units; the second figure represents 8 tens or 80 units; the third figure represents 2 hundreds or twenty tens or 200 units; the fourth figure represents 7 thousands or 70 hundreds or 700 tens or 7 000 units. Hence, 7 283 expresses 7 000 units plus 200 units plus 80 units plus 3 units or 7 283 units.

7 000 units
200 units
80 units
3 units
———
7 283 units.

Analyze the following numbers:

18	104	1 234	21 641
21	267	4 525	30 102
29	348	3 600	45 678
33	491	7 254	35 042
38	567	6 301	56 378
47	401	2 568	90 194
78	669	8 742	62 708
85	704	5 003	52 350
98	819	7 654	50 042
99	901	3 010	68 790

ROMAN NOTATION AND NUMERATION.

33. The **Roman notation** expresses numbers by letters.

34. Seven capital letters are used; they express the following values:

I	V	X	L	C	D	M
1,	5,	10,	50,	100,	500,	1000.

35. Note.—This notation was used by the Romans, hence its name. It is still used to denote the chapters and sections of books, pages of preface and introduction, dates on monuments, hours on clock dials, etc.

36. Principles.—Writing numbers by the Roman system is based on the following principles:

1° When a letter is followed by the same letter or by one of less value, the values of the letters are to be added.

Ex.—XXX represents 30; VI represents 6.

2° When a letter is followed by one of greater value, the value of the smaller is to be subtracted from that of the greater.

Ex.—IV represents 4; XL represents 40.

3° When a letter is placed between two letters of greater value, the value of the smaller is to be subtracted from the sum of the other two.

Ex.—XIV represents 14; XIX represents 19.

4° A bar placed over a letter multiplies its value by 1 000.

Ex.—$\overline{\text{V}}$ represents 5 000.

Roman Table.

I	1	XI	11	XXX	30	CD	400
II	2	XII	12	XL	40	D	500
III	3	XIII	13	L	50	DC	600
IV	4	XIV	14	LX	60	CM	900
V	5	XV	15	LXX	70	M	1 000
VI	6	XVI	16	LXXX	80	MD	1 500
VII	7	XVII	17	XC	90	MCM	1 900
VIII	8	XVIII	18	XCIX	99	MCMIX	1 909
IX	9	XIX	19	C	100	MM	2 000
X	10	XX	20	CC	200	MMIV	2 004

37— Note I.—The following rule will prove helpful in expressing numbers by the Roman notation: *1° analyze the number; 2° write the highest order, then the next highest order, and so continue until the lowest order is written.*

Example.—Write 2 689 by the Roman method.

2 689 = 2 000 + 600 + 80 + 9.
2 000 = MM.
600 = DC.
80 = LXXX.
9 = IX.
2 689 = MMDCLXXXIX.

38. Note II.—It is worthy of remark that the Roman numerals are not all to be subtracted from greater ones.

Thus, I is to be subtracted from V and X only; X is to be subtracted from L and C only; C is to be subtracted from D and M only. V and L are never to be subtracted from another higher numeral.

Express the following numbers by the Arabic notation:

1. IX	7. XCIII	13. CXLI
2. XVII	8. CXXIX	14. CCXCIX
3. XXIX	9. CCCXCIV	15. DLVII
4. LXXXIX	10. DLV	16. CMX
5. XCIX	11. DCLXXXI	17. CMXLVI
6. XLVI	12. LXXVII	18. DCLXVII

19. DLVIII	**25.** CMXCII	**31.** CLXXII
20. MDXXXIV	**26.** MDCVIII	**32.** CDII
21. MCMXVIII	**27.** MDCCCXL	**33.** DCCCLIV
22. MDCCXCI	**28.** DCVI	**34.** MDCCLXXIV
23. MDCCCXX	**29.** MDLXV	**35.** MMXVIII
24. DXXIII	**30.** LXXIV	**36.** MMLXVII

Express the following numbers by the Roman notation:

1. 17.	**7.** 235.	**13.** 779.	**19.** 999.	**25.** 1 791.
2. 75.	**8.** 285.	**14.** 800.	**20.** 1 492.	**26.** 1 837.
3. 83.	**9.** 400.	**15.** 299.	**21.** 1 534.	**27.** 1 917.
4. 92.	**10.** 440.	**16.** 563.	**22.** 1 642.	**28.** 2 000.
5. 111.	**11.** 505.	**17.** 716.	**23.** 1 663.	**29.** 2 110.
6. 184.	**12.** 621.	**18.** 614.	**24.** 1 840.	**30.** 1 910.

Questions on Theory.

(*The figures following the questions refer to the rules*).

1. What is 1° a quantity? 2° a unit? 3° a number? (1, 2, 3).

2. What is 1° an integer? 2° a fraction? 3° a mixed number? (5, 6, 7).

3. When is a number concrete? abstract? (8).

4. What is Arithmetic? (9).

5. What is 1° a principle? 2° a problem? 3° a solution? 4° a rule? (10, 11, 12, 13).

6. What is arithmetical language? (14).

7. Name the two kinds of arithmetical language. (15).

8. What is numeration? (16).

9. How are numbers formed? (17).

10. Name the first nine numbers.

11. How are the tens formed? How do we count by tens? (18).

12. How do we name the numbers ranging between the different sets of tens? Are there exceptions? (18).

13. How are the hundreds formed? How do we count by hundreds? (19).

14. How do we name the numbers ranging between the different sets of hundreds? (19).

15. What are the 4th, 5th and 6th orders? How are they formed? (20).

16. What are the units of the 7th, 8th and 9th orders? (21).
17. What is the basic principle of numeration? (22).
18. Name the first five periods. (23).
19. How many orders in a period? (23).
20. What is notation? (24).
21. How many figures are there? (25).
22. What do the first nine figures represent? Name them. (25).
23. What does zero express? (25).
24. What is the basic principle of notation? (26).
25. How may all numbers be expressed by means of the basic principle? (26).
26. Figures have how many values? (27).
27. What is the absolute and the relative value of a figure? (28).
28. How is an integer read? (29).
29. How is an integer written? (31).
30. What is Roman notation? (33).
31. Name the Roman numerals. (34).
32. Are Roman numerals still used? (35).
33. What are the four principles of the Roman notation? (36).
34. What rule is employed in expressing numbers by the Roman notation? (37).
35. Are all Roman numerals to be subtracted from greater ones? (38).

FUNDAMENTAL OPERATIONS

39. *Operations,* in arithmetic, are the modifications undergone by numbers.

40. There are four fundamental operations: *addition, subtraction, multiplication* and *division.*

41. The *proof* of an operation is a second operation testing the accurateness of the first.

ADDITION

PROBLEM.—Henry had two apples and his brother gave him three apples; how many apples had he then? To know we may unite all these apples and count them. This operation is an addition.

2 apples and 3 apples are 5 apples

42. Addition is the process of uniting into one number several numbers of the same kind or considered as such; it is a short method of counting.

EXAMPLES.—Two apples and three apples are five apples; 7 apple trees and 5 pear trees are 12 fruit trees.

43. The numbers to be added are called *addends,* and the result of an addition is called the *sum.*

44. Principles.—1° The addends must be *like numbers,* that is numbers of the same kind.

2° The sum expresses units of the *same kind* as its addends.

45. Addition is indicated by the sign called *plus* (+); it is placed between the numbers to be added.

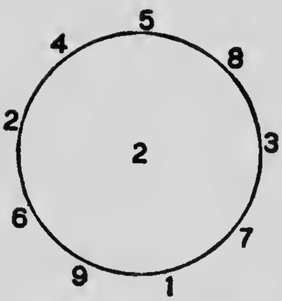
The sign of equality is =, and it reads *equals.*

Thus, $4+5=9$ is read: 4 plus 5 equals 9.

MENTAL ADDITION.

Oral Exercises.

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2
	2	3	4	5	6	7	8	9
		3	3	3	3	3	3	3
		3	4	5	6	7	8	9
			4	4	4	4	4	4
			4	5	6	7	8	9
				5	5	5	5	5
				5	6	7	8	9
					6	6	6	6
					6	7	8	9
						7	7	7
						7	8	9
							8	8
							8	9
								9
								9



By changing the middle number these 45 simple combinations may be thoroughly dealt with.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
11	11	11	12	13	15	15	16	14	16
2	4	6	3	4	5	7	6	8	5

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
16	17	15	19	17	18	19	18	19	15
7	4	8	6	7	8	8	9	9	4

(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
32	26	43	27	45	53	76	47	98	54
9	7	8	8	7	9	8	6	3	7

These exercises and similar ones must be thoroughly drilled on.

Announce the sums at sight:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
9	8	7	6	9	4	5	3	7	9
1	8	4	2	1	3	5	6	1	2
2	1	1	3	3	3	5	2	4	2
7	1	5	5	6	4	4	2	5	6

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
17	18	24	39	43	67	76	43	61	55
2	3	5	2	8	4	2	2	1	3
3	3	4	6	1	4	7	4	7	5
5	6	1	2	1	2	1	4	2	2

(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)
56	67	83	47	82	69	43	67	79	88
2	3	6	3	8	1	4	2	3	4
4	2	3	4	1	4	4	6	6	1
4	5	1	3	1	5	2	2	1	5

(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)
81	47	74	82	71	89	74	90	27	44
6	3	4	4	6	2	6	8	3	4
2	4	5	3	1	2	1	1	4	4
2	3	1	3	3	6	3	1	3	2

1. Count by 2's, 1° from 0 to 100; 2° from 1 to 101.
2. " " 3's, 1° from 0 to 99; 2° from 1 to 100.
3. " " 4's, 1° from 0 to 100; 2° from 3 to 99.
4. " " 5's, 1° from 0 to 100; 2° from 2 to 102.

5. Count by 6's, 1° from 0 to 100; 2° from 5 to 101.
6. " " 7's, 1° from 0 to 100; 2° from 2 to 100.
7. " " 8's, 1° from 0 to 100; 2° from 3 to 99.
8. " " 9's, 1° from 0 to 100; 2° from 8 to 98.

Add horizontally:

1. 16+3+5+4+7+6+5+5+8+4.
2. 13+9+5+1+8+4+6+1+5+2.
3. 15+7+4+5+2+3+8+4+3+4.
4. 25+4+9+6+3+9+3+7+2+6.
5. 48+4+6+9+8+7+2+9+5+6.
6. 36+6+4+2+2+2+6+7+3+7.
7. 82+7+5+4+8+5+8+6+7+3.
8. 79+4+7+7+9+8+6+5+5+7.
9. 56+7+7+4+2+3+8+6+9+8.
10. 83+8+3+5+2+9+6+7+6+8.

46. Short Method.—Analyzing Numbers.—When the addends slightly exceed an exact number of tens or hundreds, analyze them to shorten the operation.

Examples.—1° Add 32 and 63. Say 3 tens and 6 tens are 9 tens or 90; 2 and 3 are 5; 90 and 5 are 95.

2° Add 331 and 413. Say 3 hundreds and 4 hundreds are 7 hundreds or 700; 31 and 13 are 44; 700 and 44 are 744.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(9)	(8)	(10)
21	23	34	43	51	63	74	83	93	85
24	42	51	62	23	22	52	71	64	74

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
303	531	326	402	519	709	916	741	827	318
509	406	711	336	312	413	209	319	622	924

47. Short Method. — Rounding Numbers. — A round number is an exact number of tens without units, an axact number of hundreds, etc. 80 is a round number; 77 is not.

A number may be rounded by adding to it a few units, that are afterwards subtracted.

Example —1° Add 35 and 9. Say $35+10$ are 45; $45-1=44$. 2° Add 129 and 97. Say $129+100=229$; $229-3=226$.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
9	19	29	39	49	99	65	39	289	399
164	154	136	149	217	316	198	595	97	637

The following table may afford many varied useful exercises.

Examples.—1° Add 6 to each number of line A, B, C, etc.; 2° Add 49 to each number of column I, III, IX, etc.

	I	II	III	IV	V	VI	VII	VIII	IX	X
A	7	16	30	34	49	52	61	73	85	98
B	5	14	27	31	48	56	69	72	87	93
C	2	19	23	38	47	51	70	74	86	91
D	4	20	26	37	43	58	65	79	81	100
E	9	15	21	33	46	57	62	80	88	96
F	1	17	24	35	42	60	63	78	89	92
G	10	12	28	36	41	53	64	75	83	97
H	8	13	29	32	44	55	66	77	90	95
I	3	11	25	39	50	54	68	76	82	94
J	6	18	22	40	45	59	67	71	84	99

48. Principles of Analysis relating to Addition.

1. I add because I wish to find the *sum* of......

2. I add because ...... (what is asked) is *more than* ...... (what is given).

3. I add because I wish to find how many there are *in all* or *together*.

NOTE I.—The pupil should learn these principles by heart, as they will greatly help him in analyzing his problems. Let the principles remain on the blackboard until they are well mastered.

NOTE II.—When a problem involves the application of several principles, the pupil will take them up in their logical order, first giving the number of the principle, next its statement.

NOTE III.—Let it be the aim of much drill work to familiarize the pupil with the following key words or their synonyms: *the sum of, more than, in all, together.*

Oral Problems.

FIRST PRINCIPLE OF ANALYSIS.

1. Find the *sum* of 4 apples and 5 apples.

2. I earned 23 cents yesterday, and 20 to-day; what *sum* have I now?

3. A hen has 12 chickens, and another, 13 chickens. Find the *sum* of the chickens.

4. Henry has learned 60 words this week, and 40 last week. Find the *sum* of the words learned.

5. Mary deposited 25 cents in the School Savings Bank, and Helen, 22 cents. Can you find the *sum* of their deposits?

SECOND PRINCIPLE OF ANALYSIS.

1. Patrick wrote 24 words on his tablet; John wrote 6 *more* on his tablet; how many words did John write?

2. The Beauharnois Canal is 11 miles long; the Welland Canal is 15 miles *longer* (*more*). How long is the Welland Canal?

3. Albert saved 27 cents last week, and 11 cents *more* this week. How much has he saved this week?

4. Frontenac lived 78 years, and Bishop de Laval 8 years *longer* (*more*). How old was Bishop de Laval at his death?

5. Michael had 66 notes for his report last month, and 5 *more* this month. How many notes has he obtained this month?

6. Mount Belœil is 450 feet *higher* (*more*) than Mount Royal. How high is Mount Belœil if Mount Royal is 750 feet high?

7. A hare lives 7 years, and a fox 7 years *longer* (*more*). How long does a fox live?

8. A boy sold 30 newspapers on Friday, and 15 *more* on Saturday. How many did he sell on Saturday?

9. The number of German-speaking people in the world is 75 million; find the number of English-speaking people if it is 50 million *greater* (*more*).

10. The manufacturing industries of Great Britain use 150 million tons of coal per year. Find that country's annual coal consumption if it is 30 million *greater* (*more*).

THIRD PRINCIPLE OF ANALYSIS.—REVIEW.

1. Father paid \$2 for my cap, \$5 for my coat, \$2 for my shoes. How much did he pay *in all?*

2. There were 16 robins in a tree, 24 on the barn, 30 in the meadow. How many robins were there *in all?*

3. A farmer has 6 horses, 13 cows, and 21 sheep. How many animals has he *in all?*

4. Roy has 44 stamps in his collection, and Robert has 33. How many stamps have they *in all?*

5. A club played 24 games at home, and 2 *more* abroad. How many games were played *in all?*

6. Wilfrid spent 25 cents for a ball, and 15 cents *more* for a glove. How much did he spend *in all?*

7. There are 30 days in September, and one *more* in October. That makes how many days *in all?*

8. In 1915, Canada published 1 500 newspapers and periodicals, and England 7 500 *more* than Canada. How many does that make *in all?*

9. 2 thirds and 3 thirds and 4 thirds make how many thirds *together?*

10. Amy won 7 good notes more than Irene. If Irene won 27, how many good notes did they win *together?*

WRITTEN ADDITION.

49. Find the sum of 3 937, 614 and 546.

OPERATION.		ANALYSIS.
3 937	=	3 thousands + 9 hundreds + 3 tens + 7 units.
614	=	6 hundreds + 1 ten + 4 units.
546	=	5 hundreds + 4 tens + 6 units.
5 097	=	3 thousands + 20 hundreds + 8 tens + 17 units.
5 097	=	5 thousands + 0 hundreds + 9 tens + 7 units.

EXPLANATION.—1° Write the numbers so that the like units may fall in the same column;

2° Adding the units' column, think: 6, 10, 17 units. State results only; 17 units equal 1 ten and 7 units; write 7 units under the units' column, and carry 1 ten to the tens' column;

3° Adding the tens' column, think: 1 (the carrying number), 5, 6, 9 tens. Write the 9 under the tens' column;

4° Adding the hundreds' column, think: 5, 11, 20 hundreds. 20 hundreds equal 2 thousands; write 0 under the hundreds' column and carry 2 thousands to the thousands' column;

5° Adding the thousands' column, think: 2 (the carrying number), 5 thousands. Write the 5 under the thousands' column.

50. Rule.—*1° Write the addends so that terms of the same order stand in the same column, and draw a line beneath;*

2° Begin at the units, add the terms of each column separately and write the sum underneath, if less than ten;

3° When the sum of any column is ten or more than ten, write the units only, and add the tens to the next column;

4° Write the entire sum of the last column.

51. PROOF OF ADDITION.—Add each column in the reverse order. If the results agree, the work is supposed to be correct.

52. NOTE I.—The one matter of prime importance in addition, is *accuracy;* without it rapidity is of no advantage. To avoid errors make legible figures, dispose them in straight columns, and keep your mind concentrated on your work.

NOTE II.—In adding dollars and cents, dispose the numbers so that the decimal points may form a vertical line; then like units fall in the same column.

NOTE III.—The sum is never correct if the *decimal point* is missing or out of place.

NOTE IV.—In short additions the *carrying number* should be added mentally; but in long additions it may be advisable to write it under the column to which it belongs, that is, the next column at the left.

```
247
362
228
438...1275
128
326
121... 575
321
316
405...1042
———    ————
       2892
```

NOTE V.—When the addends are numerous, the operation may be divided into groups of three of four numbers each; the partial results are written down and footed as in the illustration. This method may also be used for checking one's work.

NOTE VI.—Never say: 5 and 8 are 13 and 9 are 22, etc. Simply announce the results: 13, 22, etc.

53. Short Method.—GROUPING NUMBERS. — This method consists in combining two or three numbers that form exact tens or hundreds. The method involves *looking ahead* with a view of forming 10's, 20's, 100's, etc.

EXAMPLES.—1° Add 6, 15 and 4. We combine 6 and 4 and add 10 to 15: 25.

2° Add 7, 76, 13 and 24. Combine 76 and 24: 100; 13 and 7: 20; sum: 120.

The following table should be learned by heart as it facilitates the forming of *couples* and *terns* whose sum is 10 or 20:

10

(*couples*)		(*terns*)	
1+9	4+6	1+1+8	2+2+6
2+8	5+5	1+2+7	2+3+5
3+7		1+3+6	2+4+4
		1+4+5	3+3+4

20

(couples)		(terns)	
1+19	6+14	2+9+9	5+6+9
2+18	7+13	3+8+9	5+7+8
3+17	8+12	4+7+9	6+6+8
4+16	9+11	4+8+8	6+7+7

Add the following by forming couples and terns aggregating 10 or 20:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
4	7	4	6	3	9	7	4	5	6
6	3	3	3	4	1	3	4	3	2
8	9	3	1	3	4	6	2	2	2
2	1	7	4	5	2	2	5	4	3
8	5	2	4	2	3	2	4	2	5
7	5	1	2	3	1	8	1	4	2

11. 4, 8, 7, 6, 3, 2, 7, 9 = ?
12. 7, 9, 7, 4, 5, 8, 9, 1 = ?
13. 6, 4, 7, 9, 4, 7, 9, 4 = ?
14. 2, 4, 6, 8, 6, 4, 2, 0 = ?
15. 1, 3, 7, 5, 9, 3, 7, 5 = ?
16. 7, 8, 9, 3, 2, 1, 6, 4, 3, 7 = ?
17. 9, 9, 9, 8, 8, 1, 1, 1, 2, 2 = ?
18. 9, 2, 9, 8, 4, 8, 6, 5, 9, 0 = ?
19. 8, 2, 3, 7, 6, 5, 9, 2, 3, 1 = ?
20. 6, 5, 4, 9, 8, 1, 2, 8, 2, 8 = ?

Add the following by grouping the numbers that form exact 10's or 100's:

21. 56+17+44+23 = ?
22. 64+ 9+11+36 = ?
23. 71+ 6+29+64 = ?
24. 19+31+26+14 = ?
25. 27+19+23+71 = ?
26. 37+62+18+43 = ?
27. 91+44+ 9+36 = ?
28. 82+19+31+18 = ?
29. 74+26+37+23 = ?
30. 92+72+28+ 8 = ?

Add the following, and check the results by adding in reverse order. Time yourself.

(1)	(2)	(3)	(4)	(5)	(6)
364	327	643	842	634	3 426
742	645	864	375	784	3 548
436	327	963	842	643	7 263
743	846	327	963	426	4 986
375	364	842	643	962	5 432
20					
24					
24					
2 660					

NOTE.—The sum of the columns may be recorded partially as illustrated.

(7)	(8)	(9)	(10)	(11)
4	4 983 649	9	21 524	34 253
26	837 463	84	13 065	25 321
342	54 289	356	43 753	12 579
5 463	7 436	8 974	10 476	40 055
79 834	842	765	44 342	54 204
378 426	97	98	13 526	13 652
5 897 634	8	6	26 437	31 148

(12)	(13)	(14)	(15)	(16)
22 736	13 125	$ 987.65	$ 847.56	$ 369.25
16 853	24 143	683.75	728.39	378.74
40 632	13 560	482.96	548.64	964.76
12 345	30 234	953.87	675.39	658.38
43 210	45 746	846.75	935.74	725.47
25 607	25 612	839.64	492.86	696.86
34 051	14 200	968.57	786.98	787.95
15 779	17 488	248.35	563.76	876.54

(17)	(18)	(19)	(20)
$ 426.37	$ 764.85	$ 937.89	$ 649.58
946.85	386.29	586.27	963.85
382.75	857.46	947.83	268.47
679.68	469.37	462.38	854.99
538.47	943.75	838.47	246.78
572.66	635.86	673.54	839.64
896.47	468.57	769.28	468.57
456.83	539.86	849.96	972.53
746.59	467.23	485.38	394.89
538.62	583.96	968.73	678.56
576.48	745.87	294.68	695.93
839.98	693.96	899.88	854.76

Add vertically and horizontally. Prove the work by adding the vertical and horizontal results.

	A	B	C	D	E	F
1.	$ 169.60	$ 12.60	$ 2.20	$ .40	$ 6.10	$ 148.76
2.	75.80	3.50	1.75	.25	7.20	66.96
3.	314.20	1.95	1.50	.70	3.20	302.03
4.	172.10	7.20	2.00	.18	96.22	156.05
5.	96.43	1.85	2.50	.11	18.37	88.23
6.	19.96	1.40	1.82	.27	173.26	14.07
7.	181.34	1.75	14.00	1.16	126.42	159.10
8.	48.60	2.25	1.93	.96	.38	41.44
9.	113.65	4.60	7.25	2.10	1.52	96.93
10.	94.75	11.75	4.60	.34	836.26	75.21
11.	178.30	2.93	11.75	.17	32.06	157.98
12.	93.75	3.74	1.92	2.05	8.16	73.51
13.	196.13	1.69	2.41	1.21	3.29	185.05
14.	205.60	4.75	19.65	.38	306.92	174.66

1.	To 36 342 622	add	22 222 222	5 times	successively.
2.	" 45 793 868	"	33 333 333	" "	"
3.	" 72 567 392	"	44 444 444	" "	"
4.	" 78 310 631	"	55 555 555	" "	"
5.	" 32 423 690	"	66 666 666	" "	"
6.	" 31 296 396	"	77 777 777	" "	"
7.	" 41 679 627	"	88 888 888	" "	"
8.	" 56 389 458	"	99 999 999	" "	"
9.	" 39 470 269	"	12 345 678	" "	"
10.	" 32 561 370	"	90 843 639	" "	"

Many similar exercises may be afforded by thus adding any number five times successively to a given number. The work should be done in one minute. Keep on drilling till you can add with accuracy and facility.

54. The two-column method.—The method consists in adding first the tens and then the units of each new number.

Example.—54
36
43
58
67
———
258

Explanation.—Beginning at the bottom of the column, 67 and 50 make 117 and 8 make 125, and *thinking* the remaining results without naming each process, 165, 168, 198, 204, 254, 258.

Add the following naming only the results:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
64	94	87	98	64	$ .38	$ .59	$ .67	$ .83	$ .92
85	83	69	75	85	.58	.98	.85	.58	.76
93	78	54	86	99	.92	.37	.69	.67	.39
78	69	83	45	88	.67	.54	.57	.99	.85
69	38	95	93	76	.85	.86	.48	.84	.47
35	54	89	25	89	.49	.75	.28	.58	.62

Checking the Work.

55. The accountant's proof.—The best method of checking addition is to add the columns in the reverse order; many accountants use the following method:

OPERATION.

$$\left.\begin{array}{l}1864=19=10=1\\1953=18=\ 9\\1764=18=\ 9\\1025=\ 8\\3789=27=\ 9\end{array}\right\}36=9,\ \textit{the final unitate of the addends.}$$

$$\overline{10395}=18=9,\ \textit{the final unitate of the sum.}$$

EXPLANATION.—1° Add the figures in each line *horizontally;* the first line $(1+8+6+4)$ adds to 19, and the sum of these two figures is 10, and the sum of these is 1.

2° The second line adds to 18, and the sum of these two figures is 9. The third line adds to 18, and $8+1$ is 9. The fourth line adds to 8, and the fifth adds to 27, and $7+2$ is 9. In each case keep on adding until you have reduced the line to *one figure*, or *unitate*.

3° Now, find the sum of all these unitates, which is 36, and $3+6$ is 9, the final unitate.

4° Then add the figures in the answer $1+0+3+9+5=18$; $8+1=9$, the final unitate.

5° If the addition is correct, the final unitates will be the same.

NOTE.—This method is a modification of the plan called *casting out the nines.*

	A	B	C	D	E
1.	34 568	93 876	69 784	38 976	94 768
2.	72 487	47 876	72 864	72 487	48 694
3.	95 438	83 297	14 897	92 764	38 972
4.	64 876	45 872	32 571	12 476	66 666
5.	45 987	76 978	89 724	53 897	89 764
6.	98 321	32 589	89 724	69 748	92 787
7.	87 643	13 489	97 247	48 724	77 777
8.	97 987	87 943	83 971	93 697	43 938
9.	34 786	76 897	72 481	54 328	55 555
10.	49 876	37 643	92 482	97 648	48 743
11.	24 689	28 971	76 893	63 872	33 333
12.	39 872	59 761	64 762	59 763	98 697
13.	97 246	97 246	59 769	97 687	88 888
14.	72 489	38 972	43 872	53 792	46 479
15.	93 247	89 726	74 672	47 867	99 999
16.	59 876	97 642	94 897	83 897	72 687
17.	72 489	57 921	58 932	46 876	22 222
18.	72 979	38 761	72 468	93 872	48 769

The preceding table can form 60 drills. Cover the useless addends with clips of paper.

Add the columns:

1.	A,	B,	C,	D,	E,	vertically	from	1	to	7,	inclusive.
2.	"	"	"	"	"	"	"	2	"	10	"
3.	"	"	"	"	"	"	"	3	"	12	"
4.	"	"	"	"	"	"	"	4	"	13	"
5.	"	"	"	"	"	"	"	5	"	14	"
6.	"	"	"	"	"	"	"	6	"	15	"
7.	"	"	"	"	"	"	"	7	"	16	"
8.	"	"	"	"	"	"	"	8	"	18	"
9.	"	"	"	"	"	"	"	2	"	16	"
10.	"	"	"	"	"	"	"	3	"	17	"
11.	"	"	"	"	"	"	"	4	"	18	"
12.	"	"	"	"	"	"	"	1	"	18	"

Questions on Theory.

1. What is an operation in arithmetic? (39).

2. Name the fundamental operations. (40).

3. What is the proof of an operation? (41).

4. What is an addition? (42).

5. What are addends? What is the sum? (43).

6. What is the sign of addition? How is it called? Where is it placed? (45).

7. Must the addends be of the same kind? (44).

8. The sum represents units of what kind? (44).

9. When is it advisable to analyze numbers before adding them? (46).

10. What is meant by *rounding* a number? (47).

11. What are the three principles of analysis relating to addition? (48).

12. Recite the rule for adding. (50).

13. In how many ways can you prove your addition? (51, 55).

14. In adding dollars and cents, how are the numbers laid out? (52, II).

15. Must the carrying number be written down? (52, IV).

16. What is meant by *grouping* numbers? (53).

17. Explain the two-column method. (54).

Written Problems.

1. Add three hundred and ninety-seven, four hundred and eighteen thousand and seven, one million thirty-six thousand and four, twenty-eight thousand and fifteen, and forty-seven.

2. Express by the Arabic notation and add: XCVII+MM+DXLIII+CMXXXVIII+XIX.

3. The distance from Montreal to Smith's Falls, by the Canadian Pacific R. R., is 129 miles; from Smith's Falls to Toronto, 209 miles; from Toronto to Detroit, 231 miles. Find the distance from Montreal to Detroit.

4. A man leaves Montreal to settle at Bell River, Abitibi. From Montreal to Cochrane he travels 600 miles; from Cochrane to La Reine, 80; from La Reine to Privat, 48; from Privat to Amos, 37; from Amos to Bell River, 40. Find the total distance.

5. By the Intercolonial R. R., it is 162 miles from Montreal to Levis; 115 miles from Levis to Rivière-du-Loup; 188 miles from Rivière-du-Loup to Campbellton; 185 miles from Campbellton to Moncton; 186 miles from Moncton to Halifax. It is how many miles from Montreal to Halifax?

6. From Montreal to Ottawa, by the C.P.R., it is 111 miles; from Ottawa to Port Arthur, 877 miles; from Port Arthur to Winnipeg, 424 miles; from Winnipeg to Vancouver, 1 473 miles. How far is it from Montreal to Vancouver?

7. In 1666, there were 625 white persons in the district of Montreal; 455 in that of Three Rivers; 2 135 in that of Quebec; moreover there were 1 200 men of the King's troops. Find the white population of New France in 1666.

8. After the Capitulation of Montreal in 1760, the English transported to France the following regiments of French regulars: Berri, 772 men; La Reine, 417; La Sarre, 276; Royal-Roussillon, 329; Languedoc, 382; Guyenne, 297; Béarn, 408; and 1 013 Marines. How many men were there in all?

9. To withstand Montgomery's attack, in 1775, Quebec had 63 officers, 186 recruits, 176 Royal Riflers, 352 sailors, and 585 militiamen. Quebec had how many defenders?

10. In 1775, to hold out against the Americans, the Canadian forces at St. Johns stood as follows: 7th Regiment, 228; 26th Regiment, 189; McLean's Corps, 37; sailors, 19; carpenters, 22; volunteers, 78; and 38 men of the Royal Artillery. How many men were there in all to defend St. Johns?

11. According to the census of 1911, the rural population of Canada amounted to 3 924 394, and the urban population, to 3 280 444. Required the total population of Canada in 1911.

12. In a recent year the greatest wheat yielders stood as follows: Russia, 962 587 000 bu.; the United States, 763 380 000 bu.; India, 358 388 000 bu.; France, 321 571 000 bu.; Canada, 231 717 000 bu. Required the total.

13. The Shawinigan Falls can develop 237 000 horse power; the Montmorency Falls, 10 000 H. P.; the Grand'Mère Falls, 57 000 H. P.; The Ouiatchuan Falls, 18 000 H. P.; the falls of the Chicoutimi River, 23 000 H. P.; the falls of Aux Outardes River, 59 000 H. P. Find the total horse power of these falls.

14. The buildings erected in Montreal were worth $5 062 226 in 1908; $7 783 621 in 1909, and $15 815 859 in 1910. Find the total value.

15. In 1910, Canada received 208 794 immigrants; in 1911, 311 084; in 1912, 354 237; in 1913, 402 432; in 1914, 384 878. How many immigrants were received in five years?

16. In 1913, Canada's exports of forest produce were thus distributed: $10 103 469 to the United Kingdom; $29 951 880 to the United States; $3 199 711 to other countries. Find the sum.

17. Find the total railway mileage of the world in 1912 with the following statistics: America, 327 070 miles; Europe, 207 432 miles; Asia, 63 320 miles; Africa, 22 892 miles; the other parts of the world, 19 267 miles.

18. In 1911, there were 1 605 339 French-speaking people in Quebec; 202 442 in Ontario; 163 474 in the Maritime Provinces, and 83 635 in the Western Provinces. Find the total French-speaking population in Canada.

19. In 1911, the population of Ontario was 2 523 274; of Quebec, 2 003 232; of Saskatchewan, 492 432; of Nova Scotia, 492 338; of Manitoba, 455 614; of British Columbia, 392 480; of Alberta, 374 663; of New Brunswick, 351 889; of Prince Edward Island, 93 728; of the Northwest Territories, 18 481; and of Yukon, 8 512. Required the total population of Canada.

20. The area of Prince Edward Island is 2 184 sq. mi.; of Nova Scotia, 21 428 sq. mi.; of New Brunswick, 27 985 sq. mi.; of Quebec, 706 834 sq. mi.; of Ontario, 407 262 sq. mi.; of Manitoba, 251 832 sq. mi.; of Saskatchewan, 251 700 sq. mi.; of Alberta, 255 285 sq. mi.; of British Columbia, 355 855 sq. mi.; of Yukon, 207 076 sq. mi.; and of Northwest Territories, 1 242 224 sq. mi. Find the total area of Canada.

21. A farmer has sheep in three pastures; in the first 157 sheep; in the second 48 more than in the first; in the third as many as in the two others together. How many sheep has he in all?

22. Find the number of muscles in the human body, if the head contains 63; the upper limbs, 35 more than the head; the trunk, 29 more than the head and the upper limbs together; and the lower limbs, 6 more than the upper limbs.

23. A driver lays six loads of gravel along a road, each 12 feet apart, the first being 450 feet from the pit. How far will he travel for each load, out and in? How far in all?

24. How many Indians were there in the District of Quebec in 1783? (Add vertically and horizontally).

Reservations.	Chiefs.	Messengers.	Warriors.	Women.	Children.	
Caughnawaga.....	20	8	139	202	243	..
Lake of the Two Mountains...	37	24	169	266	258	..
Saint Regis........	23	9	100	134	114	..
Saint François....	9	2	103	149	79	..
Lorette............	4	0	28	42	29	..
Oswegatchie.......	8	2	26	39	26	..
Carleton Isle.....	8	3	184	230	157	..
	..	..	..	..	..	..

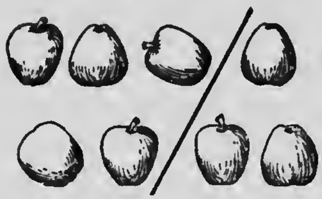
25. Find the population of the Province of Quebec in 1784.

DISTRICTS.	MARRIED MEN.	MARRIED WOMEN.	BOYS.	GIRLS.	SERVANTS.	
Montreal.....	10 140	9 727	15 994	14 612	5 161	..
Three Rivers	2 080	2 247	3 786	3 603	902	..
Quebec.....	7 911	7 380	14 153	14 985	2 126	..

Add vertically and horizontally.

SUBTRACTION

PROBLEM.—Eveline had eight apples; she gave away three; how many has she left? After giving one away, she had seven left; after giving another away, she had six left; and then after giving another away, she had five left. This operation is a subtraction.



56. **Subtraction** is the operation of taking one number from another of the same kind.

57. The result of a subtraction is called the *remainder* or *difference*.

58. The greater number is called the *minuend;* and the lesser number, the *subtrahend*.

59. The *sign* of subtraction is —, and is read *minus*. It denotes that the number after the sign is to be subtracted from the one before it.

Thus, 18—7 = 11 is read: 18 minus 7 equals 11.

60. When a parenthesis ( ) is used, begin by performing the operation indicated within the parenthesis.

Thus, 18—(5+7); 5 plus 7 = 12, and 18 minus 12 = 6.

MENTAL SUBTRACTION.

Oral Exercises.

1. Count backwards by 2's beginning with 100; 99.

2. Count backwards by 3's beginning with 100; 99; 98.

3. Count backwards by 4's beginning with 100; 99; 98; 97.

4. Count backwar 's by 5's beginning with 100; 99; 98; 97; 96.

5. Count backwards by 6's beginning with 100; 99; 98; 97; 96; 95.

6. Count backwards by 7's beginning with 100; 99; 98; 97; 96; 95; 94.

7. Count backwards by 8's beginning with 100; 99; 98; 97; 96; 95; 94; 93.

8. Count backwards by 9's beginning with 100; 99; 98; 97; 96; 95; 94; 93; 92.

Find the missing term:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
7	2	8	5	6	8	5	7	1	9
20	14	30	13	18	28	19	11	16	11

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
16	15	12	14	32	12	17	15	39	18
20	18	19	24	38	100	30	35	49	30

Give the differences at sight:

1. 3—2 = ?	**6.** 12—7 = ?	**11.** 11—9 = ?	**16.** 10—4 = ?
2. 4—1 = ?	**7.** 9—4 = ?	**12.** 7—4 = ?	**17.** 19—3 = ?
3. 5—4 = ?	**8.** 9—8 = ?	**13.** 9—5 = ?	**18.** 17—9 = ?
4. 7—3 = ?	**9.** 11—4 = ?	**14.** 17—8 = ?	**19.** 11—3 = ?
5. 8—4 = ?	**10.** 9—7 = ?	**15.** 12—6 = ?	**20.** 12—9 = ?

Find the value of:

1. 60—7+6+9—2—7+5 —8+3+5—9.
2. 18—7+8+1—8—4+7+8—6—2+9.
3. 88—8+6—5—6—9+6+6—9+2—6.
4. 47—7+8—3+4+6—8+3—9+3+4.
5. 35—9+7—4+5—8+7+7—9—2—8.

61. The *round number* method may be used in subtracting as well as in adding.

EXAMPLE.—Subtract 97 from 126. Say: 126 minus 100 = 26; 26 plus 3 = 29.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
92	92	77	96	95	127	235	346	354	633
49	38	29	79	39	98	198	299	297	597

SUGGESTION.—The drill table on page 24 may supply much extra work.

62. Principles of Analysis relating to Subtraction.

4. I subtract because I wish to find the *difference between* ...... and ......

5. I subtract because I wish to find *what is left* or *what remains.*

6. I subtract because ...... (what is asked) is *less than* ...... (what is given).

Oral Problems.

FOURTH PRINCIPLE OF ANALYSIS.

1. Emma has 25 roses; Eva has 20. Find the *difference.*
2. Duncan won 65 good notes; Donald won 75. What is the *difference?*
3. In Quebec there are 24 Legislative Councilors and 81 Deputies. Find the *difference* and the sum.
4. Louis Hippolyte Lafontaine was born in 1807 and he died in 1864. At what age did he die?
5. Bishop Bourget was born in 1799; he died in 1885. How old was he at his death?

FIFTH PRINCIPLE OF ANALYSIS.

1. Add 20 and 30; then take away 10 and say *what is left.*

2. Viator had 40 cents at the School Savings Bank; he withdrew 10 cents to give alms. Find *what is left.*

3. Edmund had 74 marbles; he lost 20; how *many* has he *left?*

4. Guy said he counted 37 crows in his father's cornfield; he threw a stone and scared 13 away. *How many* then *remained?*

5. Evangeline received 18 cents from her uncle and 12 cents from her aunt. If she spent 15 cents, *how many* cents *remained?*

SIXTH PRINCIPLE OF ANALYSIS.

1. Cora has 4 cents *less than* Clara. Clara has 16 cents; how many has Cora?

2. Harold's book contains 92 pictures; Hubert's book contains 16 *less.* Required the number of pictures in Hubert's book.

3. James has $30 in the School Savings Bank; Hugh has $14 *less.* How much money has Hugh?

4. Norman has 20 rabbits; Owen has 25; Matthew has 10 *less than* Norman and Owen together. How many has Matthew?

5. Beatrice has 50 cents; Bertha has 10 cents *less than* Beatrice. How many cents have they together?

NOTE.—The *difference* is how much a small number is *less than* a great one; and also how much a great number is *more than* a small one.

1.	20 is how much *less than*	25?	(*difference*).
2.	30 is how much *less than*	45?	"
3.	60 is how much *less than*	70?	"
4.	80 is how much *less than*	105?	"
5.	105 is how much *less than*	120?	"
6.	120 is how much *more than*	100?	"
7.	140 is how much *more than*	125?	"
8.	180 is how much *more than*	160?	"
9.	200 is how much *more than*	175?	"
10.	1000 is how much *more than*	200?	"

NOTE.—The words *more than, less than,* are sometimes understood or replaced by synonyms.

This kind of drill work should be carried on till the pupil becomes perfectly familiar with the *key words.*

1. If I had $2 more, I should have $5; how much money have I?

2. If I had $3 less, I should have $15; how much have I?

3. If I had $10 more, I should have $11; how much have I?

4. If I were 10 years older, I should be 22 years old. Find my age.

5. If Alice were 2 years older, she would be as old as Etta who is 11 years old; how old is Alice?

6. Give me 3 cents and I shall have 12. How many cents have I?

7. A workman spent $6 at the saloon; if he had spent $3 more, how much would he have spent? If he had spent $3 less?

8. If I had $30 more, I should have $59. How much money have I?

9. Give me 3 apples and I shall have 12. How many have I?

10. If I had $60 less, I should have $60. How many dollars have I?

WRITTEN SUBTRACTION.

63. From 8 740 take 3 904.

Operation.	Analysis.
8 740	= 8 thousands+7 hundreds+4 tens+0 units.
3 904	= 3 thousands+9 hundreds+0 tens+4 units.
4 836	= 4 thousands+8 hundreds+3 tens+6 units.

Explanation.—1° Place the subtrahend under the minuend so that units of the same order shall fall in the same column.

2° As you cannot take 4 units from 0 units, borrow 1 ten from 4 tens; this makes 10 units. Take 4 units from 10 units. This leaves 6 units.

3° Three tens remain in the minuend. Taking 0 tens from 3 tens leaves 3 tens.

4° You cannot take 9 hundreds from 7 hundreds. Borrow 1 thousand from 8 thousands; this with the 7 hundreds makes 17 hundreds. Taking 9 hundreds from 17 hundreds leaves 8 hundreds.

5° Seven thousands remain in the minuend. Taking 3 from 7 leaves 4 thousands.

64. Rule.—*1° Write the subtrahend under the minuend so that units of the same order shall stand in the same vertical line;*

2° Begin at units, and subtract each term of the subtrahend from the corresponding term of the minuend, writing the remainder beneath;

3° If any term of the subtrahend is greater than the corresponding term of the minuend, add 10 to the latter and then subtract; then subtract 1 from the next term of the minuend and proceed as before.

65. Proof of subtraction.—Add the difference to the subtrahend, and if the work is correct the sum will equal the minuend.

66. Principles.—I. The difference of two numbers added to the lesser, gives the greater; and subtracted from the greater, gives the lesser.

II. *Like units* alone can be taken from each other.

III. The difference is the same if both numbers be equally increased or decreased.

IV. The difference is a number *similar* to the minuend and subtrahend.

Written Exercises.

1. From	867 495 312	take	22 222 222	five times	successively.
2. "	934 817 526	"	33 333 333	" "	"
3. "	945 128 376	"	44 444 444	" "	"
4. "	987 135 642	"	55 555 555	" "	"
5. "	927 683 514	"	66 666 666	" "	"
6. "	993 872 645	"	77 777 777	" "	"
7. "	983 764 152	"	88 888 888	" "	"
8. "	951 472 863	"	99 999 999	" "	"
9. "	848 369 786	"	23 864 795	" "	"
10. "	965 417 328	"	38 924 765	" "	"

Similar exercises may be formed by thus subtracting a lesser number from a greater one a given number of times.

67. Addition and Subtraction.—When several numbers are to be subtracted from a given number, it is preferable to do so at a single operation:

From 5 243 take 624, 778 and 893.

Operation.	Explanation.
5 243	$3+8+4=15$, and since 8 more will make 23, we write 8 for the first term of the remainder.
624	$2+9+7+2=20$, and since 4 more makes 24, we write 4 in the remainder.
778	$2+8+7+6=23$, and since 9 more makes 32, we write 9 in the remainder.
893	3 and 2 more makes 5, so we write 2 in the remainder.
2 948	Note.—Observe the same rule in carrying as in addition.

1. From 4 332 take 346, 876, 896.
2. " 9 636 " 236, 399, 677.
3. " 8 342 " 249, 927, 639.
4. " 6 339 " 942, 837, 934.
5. " 3 634 " 336, 836, 922.

Find the balances of the following accounts.

NOTE.—The business man says *balance* instead of *remainder*.

(1)

Dr. Merchandise.	Cr.
$ 926.50	$1 536.24
536.24	850.70
543.27	926.50
1 000.00	1 143.27
100.00	2 152.31
152.31	
200.00	
250.00	

(2)

Dr. Cash.	Cr.
$ 1.25	$23.40
.75	14.10
15.40	5.50
117.25	2.11
81.32	
11.18	
12.21	
5.36	
175.50	

(3)

Dr. S. A. Hart.	Cr.
$4 632.40	$ 892.40
19.64	436.51
1 275.38	1 475.73
932.14	1 162.41
1 762.41	523.42
3 827.50	1 574.51
5 963.14	1 963.75
	2 012.31

(4)

Dr. A. Bates.	Cr.
$3 562.75	$ 164.25
137.40	110.29
1 141.38	58.37
964.25	1 073.26
59.61	176.38
1 263.75	840.50
1 964.76	95.24
2 075.38	382.40
941.37	3 426.50
	827.41

(5)

Dr.	W. Durnin. Cr.
$ 637.20	$ 832.50
110.72	58.75
340.21	161.74
17.50	327.41
1 240.00	110.10
930.16	73.41
1 145.73	568.20

(6)

Dr.	J. Daly. Cr.
$ 413.50	$ 943.70
217.71	2 974.58
511.30	6 275.81
1 263.73	3 963.53
814.60	4 764.12
1 511.26	6 372.90
5 963.40	5 251.01

68. Making change.—*Austrian Method.*

You go to the grocer's and buy for $1.38 worth of goods, and you tender in payment a two-dollar bill. How much change shall you receive?

The clerk will return to you 2 pennies, a ten-cent piece, and a half-dollar, saying as he does so, "$1.38, 40, 50, $2, thank you!"

Note.—This method is used by many in ordinary subtraction. It is practical, since conducive to both rapidity and accuracy.

What change should be given in the following cases?

a) For a $2 bill.

1.	$1.56.	**6.**	$1.17.
2.	1.27.	**7.**	.96.
3.	.87.	**8.**	.88.
4.	.58.	**9.**	1.49.
5.	1.33.	**10.**	1.68.

b) For a $5 bill.

11.	$1.72.	**16.**	$4.82.
12.	2.49.	**17.**	2.54.
13.	1.27.	**18.**	3.43.
14.	2.67.	**19.**	1.40.
15.	3.76.	**20.**	2.32.

69. Checking the work.—The method of checking given on page 33 may be employed here as well by simply subtracting the unitate of the subtrahend from the unitate of the minuend. The final unitate must be the same.

$$\left.\begin{array}{l}864\ 535=31=4\\ 254\ 892=30=3\end{array}\right\}4-3=1,\text{ final unitate.}$$

$$609\ 643=28=10=1,\text{ final unitate.}$$

Questions on Theory.

1. What is subtraction? (56).

2. What is the remainder, or difference? (57).

3. What is the minuend? the subtrahend? (58).

4. What is the sign of subtraction? What does it denote? (59).

5. How is a parenthesis dealt with? (60).

6. May the round number method be used in subtraction? (61).

7. What are the three principles of analysis relating to subtraction? (62).

8. Recite the rule for subtracting. (64).

9. Why do you add 10, and no other number, to the minuend figure, when it is less than the corresponding subtrahend figure?

10. In how many ways can you prove subtraction? (65, 69).

11. Is the difference changed if the minuend and subtrahend are equally increased or decreased? (66).

12. How do you make change? (68).

Written Problems.

1. Express by the Arabic notation and subtract CXVII from CDIX.

2. The area of Lake Winnipegosis is 2 086 sq. mi.; the area of Lake Manitoba is 1 900 sq. mi. What is the difference in their areas?

3. Eiffel Tower, at Paris, is 984 feet high; Washington's obelisk is 554 feet high. Find the difference in height.

4. Queen Victoria died in 1901, after having reigned 64 years. In what year did she ascend the throne?

5. The distance from Havre to Halifax is 2 705 miles; the distance from Havre to Montreal is 3 041 miles. What is the difference?

6. What number taken from 65 831 will leave 39 653?

7. In 1914, the world's Mercantile Navy aggregated 24 444 vessels; of this number 10 123 belonged to Great Britain. How many did the other countries of the world possess?

8. The altitude of Mount Everest, in Asia, is 29 002 feet; Mount Logan, Canada's highest peak, is 19 519 feet. Mount Logan is how many feet less high than Mount Everest?

9. The area of New Brunswick is 27 985 sq. mi.; the area of Nova Scotia is 21 428 sq. mi. New Brunswick is how many miles larger than Nova Scotia?

10. In 1912, Canada's railway mileage was 24 725; in 1914, it was 30 795. Required the increase.

11. In 1908, Manitoba, Alberta and Saskatchewan cultivated 11 664 943 acres of land; in 1913, they cultivated 18 963 200. Find the increase.

12. The Columbia River is 1 150 miles long; it flows 465 miles in British Columbia and then runs into the United States. How long is the American course of that river?

13. The votes registered at an election numbered 5 473. If the losing candidate got 1 423 votes, what was the majority of the winning candidate?

14. At the battle of the Monongahela, Braddock's army was 2 200 strong, while Beaujeu commanded 146 regulars, 72 Marines and 637 Indians. How many more men had Braddock than Beaujeu?

15. At the battle of Carillon, Abercromby had 6 367 regulars and 9 034 volunteers; Montcalm had only 3 506 men. Montcalm had how many men less than Abercromby?

16. At Carillon, in 1758, Abercromby had 1 944 men killed or wounded, and Montcalm 1 592 less. Find the latter number.

17. Canada was a French colony from 1608 till 1760. During how many years?

18. The population of the Province of Quebec, in 1861, was 567 865; in 1911, it was 2 003 232. Find the increase in half a century.

19. A charitab'e person on his death bequeathed $49 785 as follows: $24 575 to a technical school; $16 935 to a hospital; $2 400 to a public library; and the rest to the poor. What sum did the poor receive?

20. A bank clerk had $4 875.75 in his cash box; what was left after cashing four checks amounting respectively to $354, $548.50, $674.80, $857.85?

21. Our School Savings Bank registered the following deposits and withdrawals in the four first months of the year:

	DEPOSITS.	WITHDRAWALS.
September....................	$48.76	$15.98
October........................	$55.63	$10.12
November....................	$42.75	$ 8.15
December....................	$66.09	$45.48

What was the balance on hand at the end of December?

22. In 1897, Canada's exports and imports amounted to $249 244 274; in 1911, to $759 094 389. Find the increase.

23. The distance from the earth to the moon is 240 000 miles, and from the earth to the sun, 95 000 000 miles. The sun is how many miles farther from us than the moon?

24. A father left $7 300 to his son and two daughters. Andrew received $3 000; Bridget, $600 less than Andrew; Mabel had the remainder. How much did each girl receive?

25. A man bought 128 750 bricks. He sold 56 845 to B, and 11 830 less to C. How many has he left?

MULTIPLICATION

Problem.—Agnes had three pears; three others were given to her. How many has she now? I set the pears in groups, add them, and say: "2 times three are six". I see that I may also say: "3 times two are six". When I act thus, *without adding*, I multiply.

70. Multiplication is an operation by which one number is repeated as many times as there are units in another number: it is therefore a short method of adding equal numbers.

To multiply 8 by 4 means to take 8 four times, as 8+8+8+8, or 32. If I remember that four 8's equal 32, the next time I shall have a similar case, I shall not need to add, but shall say simply: 4 times 8 are 32.

71. The number to be multiplied is the *multiplicand;* the number by which we multiply is the *multiplier.*

72. The result obtained by multiplying is the *product.* The multiplicand and the multiplier are called the *factors* of the product.

73. The sign of multiplication is ×, and is read *multiplied by,* when the multiplicand precedes it, and *times,* when the multiplier precedes it.

Thus, 8×4 is read 8 multiplied by 4, when 8 is the multiplicand, but it is read 8 times 4, when 8 is the multiplier.

MENTAL MULTIPLICATION.

Multiplication Table.

The combinations already given are not repeated.

$2\times 2=4.$
$2\times 3=6.$
$2\times 4=8.$
$2\times 5=10.$
$2\times 6=12.$
$2\times 7=14.$
$2\times 8=16.$
$2\times 9=18.$
$2\times 10=20.$
$2\times 11=22.$
$2\times 12=24.$
$2\times 13=26.$
$2\times 14=28.$
$2\times 15=30.$
$2\times 16=32.$
$2\times 17=34.$
$2\times 18=36.$
$2\times 19=38.$
$2\times 20=40.$
$2\times 21=42.$
$2\times 22=44.$
$2\times 23=46.$
$2\times 24=48.$
$2\times 25=50.$
$2\times 26=52.$
$2\times 27=54.$
$2\times 28=56.$
$2\times 29=58.$
$2\times 30=60.$
$2\times 31=62.$
$2\times 32=64.$
$2\times 33=66.$
$2\times 34=68.$
$2\times 35=70.$
$2\times 36=72.$
$2\times 37=74.$
$2\times 38=76.$
$2\times 39=78.$

$2\times 40=80.$
$2\times 41=82.$
$2\times 42=84.$
$2\times 43=86.$
$2\times 44=88.$
$2\times 45=90.$
$2\times 46=92.$
$2\times 47=94.$
$2\times 48=96.$
$2\times 49=98.$
$2\times 50=100.$

$3\times 3=9.$
$3\times 4=12.$
$3\times 5=15.$
$3\times 6=18.$
$3\times 7=21.$
$3\times 8=24.$
$3\times 9=27.$
$3\times 10=30.$
$3\times 11=33.$
$3\times 12=36.$
$3\times 13=39.$
$3\times 14=42.$
$3\times 15=45.$
$3\times 16=48.$
$3\times 17=51.$
$3\times 18=54.$
$3\times 19=57.$
$3\times 20=60.$
$3\times 21=63.$
$3\times 22=66.$
$3\times 23=69.$
$3\times 24=72.$
$3\times 25=75.$
$3\times 26=78.$
$3\times 27=81.$
$3\times 28=84.$

$3\times 29=87.$
$3\times 30=90.$
$3\times 31=93.$
$3\times 32=96.$
$3\times 33=99.$

$4\times 4=16.$
$4\times 5=20.$
$4\times 6=24.$
$4\times 7=28.$
$4\times 8=32.$
$4\times 9=36.$
$4\times 10=40.$
$4\times 11=44.$
$4\times 12=48.$
$4\times 13=52.$
$4\times 14=56.$
$4\times 15=60.$
$4\times 16=64.$
$4\times 17=68.$
$4\times 18=72.$
$4\times 19=76.$
$4\times 20=80.$
$4\times 21=84.$
$4\times 22=88.$
$4\times 23=92.$
$4\times 24=96.$
$4\times 25=100.$

$5\times 5=25.$
$5\times 6=30.$
$5\times 7=35.$
$5\times 8=40.$
$5\times 9=45.$
$5\times 10=50.$
$5\times 11=55.$
$5\times 12=60.$
$5\times 13=65.$

$5\times 14=70.$
$5\times 15=75.$
$5\times 16=80.$
$5\times 17=85.$
$5\times 18=90.$
$5\times 19=95.$
$5\times 20=100.$

$6\times 6=36.$
$6\times 7=42.$
$6\times 8=48.$
$6\times 9=54.$
$6\times 10=60.$
$6\times 11=66.$
$6\times 12=72.$
$6\times 13=78.$
$6\times 14=84.$
$6\times 15=90.$
$6\times 16=96.$

$7\times 7=49.$
$7\times 8=56.$
$7\times 9=63.$
$7\times 10=70.$
$7\times 11=77.$
$7\times 12=84.$
$7\times 13=91.$
$7\times 14=98.$

$8\times 8=64.$
$8\times 9=72.$
$8\times 10=80.$
$8\times 11=88.$
$8\times 12=96.$

$9\times 9=81.$
$9\times 10=90.$
$9\times 11=99.$

$10\times 10=100.$

Oral Exercises.

NOTE.—If either factor contains ciphers at the right, multiply regardless of the ciphers, and annex them to the product.

1. 13 × 3 = ?
2. 15 × 6 = ?
3. 19 × 2 = ?
4. 16 × 6 = ?
5. 14 × 5 = ?
6. 13 × 6 = ?
7. 29 × 2 = ?
8. 24 × 3 = ?
9. 17 × 4 = ?
10. 46 × 2 = ?
11. 18 × 4 = ?
12. 23 × 3 = ?
13. 19 × 5 = ?
14. 15 × 5 = ?
15. 36 × 2 = ?
16. 28 × 3 = ?
17. 14 × 6 = ?
18. 18 × 5 = ?
19. 13 × 7 = ?
20. 23 × 4 = ?
21. 350 × 2 = ?
22. 1 900 × 4 = ?
23. 18 × 30 = ?
24. 260 × 2 = ?
25. 1 700 × 5 = ?
26. 160 × 30 = ?
27. 300 × 48 = ?
28. 130 × 60 = ?
29. 2 800 × 3 = ?
30. 470 × 2 = ?
31. 1 300 × 40 = ?
32. 380 × 20 = ?
33. 1 700 × 30 = ?
34. 140 × 300 = ?
35. 180 × 40 = ?
36. 120 × 10 = ?
37. 160 × 50 = ?
38. 500 × 19 = ?
39. 700 × 130 = ?
40. 29 × 30 = ?
41. 1 314 × 3 = ?
42. 1 912 × 5 = ?
43. 1 713 × 4 = ?
44. 2 648 × 2 = ?
45. 1 815 × 5 = ?
46. 918 × 3 = ?
47. 514 × 6 = ?
48. 1 918 × 4 = ?
49. 2 946 × 2 = ?
50. 1 525 × 3 = ?
51. 1 327 × 3 = ?
52. 2 319 × 4 = ?
53. 1 131 × 3 = ?
54. 1 618 × 5 = ?
55. 2 114 × 3 = ?
56. 1 729 × 3 = ?
57. 3 213 × 3 = ?
58. 1 319 × 5 = ?
59. 4 728 × 2 = ?
60. 16 140 × 6 = ?
61. $1.90 × 5 = ?
62. $18.25 × 3 = ?
63. $16.40 × 2 = ?
64. $13.18 × 4 = ?
65. $12.29 × 3 = ?
66. $6.48 × 2 = ?
67. $7.15 × 5 = ?
68. $28.17 × 3 = ?
69. $47.25 × 2 = ?
70. $15.17 × 5 = ?
71. $14.18 × 4 = ?
72. $26.45 × 2 = ?
73. $18.32 × 3 = ?
74. $12.16 × 6 = ?
75. $ 8.19 × 5 = ?
76. $27.16 × 3 = ?
77. $19.25 × 2 = ?
78. $47.28 × 2 = ?
79. $13.25 × 3 = ?
80. $15.18 × 5 = ?
81. 504 × 19 = ?
82. 1 700 × 40 = ?
83. $1 316 × 50 = ?
84. 204 × 23 = ?
85. 320 × 30 = ?
86. 17 019 × 3 = ?
87. 141 517 × 5 = ?
88. 12 018 × 4 = ?
89. $190.29 × 3 = ?
90. $240.45 × 2 = ?
91. $180.15 × 5 = ?
92. 305 × 16 = ?
93. $507.03 × 13 = ?
94. $306.04 × 14 = ?
95. $1 825 × 30 = ?
96. $1 315 × 60 = ?
97. $25 016 × 30 = ?
98. $140.15 × 5 = ?
99. $1 700 × 40 = ?
100. 1 800 × 50 = ?

74. Principles of Analysis relating to Multiplication.

7. I multiply because I wish to find the *product* of ...... by ......

8. I multiply because having the *value of one object* and the *number of objects*, I wish to find the *total value.*

9. I multiply because I wish to find ...... *times as much as* what I have.

Oral Problems.

SEVENTH PRINCIPLE OF ANALYSIS.

1. What is the *product* of 5 by 4?
2. If I multiply 10 by 5, what will the *product* be?
3. One pencil costs 4 cents; I buy 5; what is the *product?*
4. 22 times 4 makes what *product?*
5. A man earns $5 a day; what is the *product* of 6 days' work?

EIGHTH PRINCIPLE OF ANALYSIS.

NOTE.—Point out, each time, the value of the object and the number of objects.

1. A penholder costs 5 cents (*value of one*); how much will 4 penholders cost? (*number of objects*).

2. A yard of cloth costs 40 cents; find the cost of 3 yards.

3. If one pound of butter sells for 37 cents, find the cost of 3 pounds.

4. Joseph writes 15 words in a minute; how many does he write in 6 minutes?

5. Henry takes 40 steps in a minute; how many steps will he take in 9 minutes?

6. If a car runs 25 miles in an hour, how far will it run in 6 hours?

NOTE.—A *total value* may represent a total cost price, a total selling price, a total expense, a total distance, a total salary, a total production, etc.

7. There are four quarts in a gallon; how many quarts in 20 gallons? 30 gallons? 40 gallons? 50 gallons? 45 gallons?

8. How many days are there in 8 weeks? 12 weeks? 20 weeks? 30 weeks? 50 weeks?

9. Twelve inches make one foot. How many inches in 6 feet? 8 feet? 40 feet? 60 feet? 80 feet?

10. An apprentice spends 10 cents a day on cigarettes. How much more does he waste in 3 days? 8 days? 15 days? in a month? in 7 weeks?

11. An automobile runs 15 miles in an hour. How far will it run in 3 hours? 5 hours? 6 hours? 7 hours? 10 hours?

12. Find the cost of 18 pair of shoes at \$2 a pair; at \$3; at \$4; at \$5; at \$6.

13. If a man breathes 19 times in a minute, how often does he breathe in 3 minutes? 5 minutes? 10 minutes? 30 minutes? 1 hour?

14. A healthy man's pulse beats 70 times in a minute. How often does it beat in 30 minutes? 40 minutes? 100 minutes? 1 hour? 2 hours?

15. A man earns 18 cents per hour. How much money does he earn in 6 hours? 7 hours? 10 hours? in 2 days of 10 hours? in 6 days of 10 hours?

16. Sound travels 1 100 feet in one second. How far off is the storm, if you hear a clap of thunder 3 seconds after seeing the lightning that caused it? 4 seconds after? 5 seconds after? 6 seconds after? 7 seconds after?

17. If it costs \$4.25 a week to run a small automobile, what will it cost in 2 weeks? 3 weeks? 4 weeks? 5 weeks? 50 weeks?

18. With improved machinery, a pin maker can turn out 1 500 000 pins a day. How many pins can be turned out in 2, 3, 4, 5, 6 days?

19. Before the war of 1914, the value of the toys made in Germany was 22 million dollars a year. At that rate, find the value of the toys manufactured in 2 years; 4 years; 8 years; 10 years; 12 years.

20. A boy smokes 5 cigarettes a day. How many does he smoke in 200 days? 365 days? 400 days? 10 years? 20 years?

Ninth principle of analysis.

1. Harold has 8 cents; Paul has 3 *times as many;* how many has he?

2. Peter has 90 hens; Bernard has 3 *times as many.* How many hens has Bernard?

3. Maurice has \$40; Ralph has 4 *times as much* money as he. Find how much.

4. Walter has 15 *times as much* money as his sister; how much has he if his sister has \$3?

5. In 1911, typhoid fever killed 700 persons in the Province of Quebec, and tuberculosis 5 times as many. How many victims did tuberculosis make?

WRITTEN MULTIPLICATION.

75. First case.—*When one of the factors consists of only one figure.*

Multiply 3 457 by 6.

OPERATION.

$$\left.\begin{array}{r} 3\ 457 = \text{multiplicand} \\ 6 = \text{multiplier} \end{array}\right\} = \text{factors.}$$

$$\begin{array}{r} \underline{} \\ 20\ 742 = \text{product.} \end{array}$$

EXPLANATION.

Multiplying 3 457 by 6 is finding 6 times the number 3 457, or 6 times the simple units, 6 times the tens, 6 times the hundreds, and 6 times the thousands of that number.

Write the multiplier under the multiplicand, and draw a line beneath. Proceed mentally: 6 times 7 units are 42 units, or 4 tens and 2 units; write the 2 units and carry the 4 tens. 6 times 5 tens are 30 tens, plus 4 tens, are 34 tens, or 3 hundreds and 4 tens; write the 4 tens and carry the 3 hundreds. 6 times 4 hundreds are 24 hundreds, plus 3 hundreds, are 27 hundreds, or 2 thousands and 7 hundreds; write the 7 hundreds and carry the 2 thousands. 6 times 3 thousands are 18 thousands, plus 2 thousands, are 20 thousands; write the 20 thousands; hence the product 20 742.

76. Rule.—*1° Place the factor having one figure under the other factor and draw a line beneath;*

2° Begin at the right, and multiply each order of the upper factor by the lower factor, carrying as in addition.

77. Second case.—*When both factors consist of two or more figures.*

Multiply 78 645 by 793.

OPERATION.

$$\left.\begin{array}{r} 78645 = \text{multiplicand} \\ 793 = \text{multiplier} \end{array}\right\} \text{factors}$$

$$\begin{array}{rl} 235935 & = \text{product of 78 645 by 3 units} = 235\ 935 \text{ units.} \\ 707805 & = \text{product of 78 645 by 9 tens} = 707\ 805 \text{ tens.} \\ 550515 & = \text{product of 78 645 by 7 hundreds} = 550\ 515 \text{ hundreds.} \\ \hline 62365485 & = \text{total product.} \end{array}$$

EXPLANATION.

Multiplying 78 645 by 793 is finding 793 times the number 78 645, or 3 times 78 645, plus 9 tens times 78 645, plus 7 hundred times 78 645.

The product of 78 645 by 3 units gives 235 935 units; the product of 78 645 by 9 tens times gives 707 805 tens; the product of 78 645 by 7 hundreds gives 550 515 hundreds. The several products are first written so that figures of the same order may stand in the same column; the several products are then added.

78. Rule.—*Begin at the right, and multiply the multiplicand by each figure of the multiplier, writing the first figure of each product under the figure of the multiplier which produces it. Add the partial products.*

79. NOTE.—When there are ciphers between some others figures of the multiplier, pass over them in multiplying, taking care to place the first figure of the next product under the figure of the multiplier which produced it.

80. PROOF OF MULTIPLICATION.—Multiply the multiplier by the multiplicand; the results must agree.

81. Principles.—I. The multiplier is always an abstract number and shows how many times the multiplicand is taken.

II. The multiplicand and product are like numbers.

III. The factors of a product may be interchanged without affecting the value of the product.

Thus, the product of $2\times3\times4$ is the same as the product of $4\times3\times2$, and that of $3\times2\times4$.

IV. To multiply a given number by the *product of several factors*, find the continued product of the given number by each factor in turn.

Thus, to multiply 35 by 15, that is by 3 times 5, multiply 35 by 3, and the result by 5.

82. REMARKS ON PARENTHESIS.

I. Sums, differences, and products may be placed within parentheses.

EXAMPLES.—$(4+5)\times6$ means that the *sum* 9 is to be multiplied by 6.

$(8-5)\times4$ means that the *difference* 3 is to be multiplied by 4.
$(6\times4)\times5$ means that the *product* 24 is to be multiplied by 5.

II. The sign × may be suppressed before or after a parenthesis, as well as between two parentheses.

EXAMPLES. 4(8—5) =4×3, or 12.
(8—5)4 =3×4, or 12.
(3+4) (4+7)=7×11, or 77.

III. A parenthesis preceded by a plus sign may be removed without changing the signs of terms in the parenthesis.

EXAMPLE. 8+(5+6—4) =8+5+6—4, or 15.

IV. A parenthesis preceded by a minus sign may be removed if the signs of all the terms in the parentheses are changed.

EXAMPLE. 6—(3—9+8) =6—3+9—8, or 4.

V. In an expression involving a parenthesis, begin by performing the operations indicated within the parenthesis.

Written Exercises.

Multiply 5 times running:

1. 65 387 by 2.
2. 73 869 " 3.
3. 73 969 " 4.
4. 49 835 by 5.
5. 67 839 " 6.
6. 84 937 " 7.
7. 85 497 by 8.
8. 96 547 " 9.

(9)

I	243	×	*a*) 921.
II	376		*b*) 330.
III	405		*c*) 407.
IV	221		*d*) 920.
V	392		*e*) [illegible].

Multiply I, II, III, IV, V, successively by *a*, *b*, *c*, *d*, *e*. Thus, 243, 376, 405, 221, 392, must be multiplied by 921; then by 330, and so forth.

(10)

I	827	×	*a*) [illegible]48.
II	906		*b*) 229.
III	432		*c*) 226.
IV	507		*d*) 642.
V	581		*e*) 379.

(11)

I	18 360	×	*a*) 2 467.
II	32 342		*b*) 3 459.
III	45 546		*c*) 7 692.
IV	65 473		*d*) 1 918.
V	69 398		*e*) 2 992.

83. When there are ciphers at the right of one or both factors, multiply regardless of the ciphers, and annex them to the product.

Ex.—

```
   562          234000        243000
    2400          16            2500
 -------       -------       --------
   2248         1404          1215
  1124          234           486
 -------       -------       --------
 1348800       3744000       607500000
```

Multiply:

1. 768 by 1 000.
2. 346 " 4 600.
3. 1 234 000 " 75.
4. 346 700 " 9 000.
5. 106 000 " 30 400.
6. 740 000 by 228 000.
7. 82 400 " 5 000.
8. 89 600 " 4 000.
9. 94 800 " 800.
10. 9 800 " 3 000.

84. When a number is to be multiplied several times *by itself*, thus $3 \times 3 \times 3 \times 3 \times 3$, it is customary to indicate this operation by means of a little figure written above and at the right of the number: 3^5.

$4 \times 4 = 4^2$; $5 \times 5 \times 5 = 5^3$; $6 \times 6 \times 6 \times 6 = 6^4$.

Find the products in the following exercises:

1. 10^2.
2. 12^2.
3. 4^3.
4. 8^4.
5. 9^5.
6. 100^2.
7. 144^2.
8. 36^3.
9. 49^2.
10. 26^4.
11. $1\,424^2$.
12. $2\,632^2$.
13. 240^3.
14. 180^3.
15. 60^4.
16. 10^4.
17. 19^6.
18. 20^5.
19. 25^6.
20. 300^3.

The little figure written beside the factor is called the *exponent*; it indicates how many times the number is used as a factor, or the degree of its power.

$25^3 = 25 \times 25 \times 25$, or the 3rd power of 25.

SHORT METHODS.

CROSS MULTIPLICATION.—By inspection we notice that 1° units × units = *units;* 2° (units × tens) and (tens × units) = *tens;* 3° (units × hundreds) and (hundreds × units) and (tens × tens) = *hundreds;* 4° (tens × hundreds) and (hundreds × tens) = thousands; 5° hundreds × hundreds = ten-thousands.

These are the principles of *cross multiplication.* Time is gained by using them, as they permit us to add mentally the several products that contribute to make up *each order* of the total product.

85. *When both factors contain two figures.*

Multiply 78 by 36.

OPERATION.	EXPLANATION.
78 36 —— 2 808	1° $8 \times 6 = 48$ units; write the 8 units, carry the 4 tens; 2° $7 \times 6 = 42$ tens; $8 \times 3 = 24$ tens; $42 + 24 + 4$ (carrying number) $= 70$ tens; write the 0 tens and carry the 7 hundreds;

3° $7 \times 3 = 21$ hundreds; $21 + 7$ (carrying number) $= 28$ hundreds. Write the 28 hundreds.

Multiply:

1. $22 \times 26 =$	**6.** $33 \times 18 =$	**11.** $36 \times 24 =$	**16.** $29 \times 39 =$
2. $41 \times 33 =$	**7.** $27 \times 14 =$	**12.** $22 \times 27 =$	**17.** $49 \times 21 =$
3. $16 \times 18 =$	**8.** $18 \times 22 =$	**13.** $36 \times 28 =$	**18.** $62 \times 36 =$
4. $24 \times 31 =$	**9.** $43 \times 26 =$	**14.** $42 \times 19 =$	**19.** $32 \times 82 =$
5. $27 \times 13 =$	**10.** $17 \times 24 =$	**15.** $61 \times 34 =$	**20.** $37 \times 95 =$

86. *When one factor contains three figures.*

Multiply 378 by 54.

OPERATION.	EXPLANATION.
378 54 —— 20 412	1° $8 \times 4 = 32$ units; write the 2 units, carry the 3 tens; 2° $(7 \times 4) + (8 \times 5) + 3 = 71$ tens; write 1 ten, carry the 7 hundreds;

3° $(3 \times 4) + (7 \times 5) + 7 = 54$ hundreds; write the 4 hundreds, carry the 5 thousands;

4° $(3 \times 5) + 5 = 20$ thousands. Write the 20 thousands.

Multiply:

1. $234 \times 54 =$	**6.** $436 \times 76 =$	**11.** $664 \times 67 =$
2. $262 \times 32 =$	**7.** $538 \times 39 =$	**12.** $837 \times 84 =$
3. $164 \times 44 =$	**8.** $456 \times 37 =$	**13.** $406 \times 26 =$
4. $286 \times 62 =$	**9.** $831 \times 47 =$	**14.** $509 \times 57 =$
5. $346 \times 74 =$	**10.** $289 \times 38 =$	**15.** $704 \times 89 =$

87. *When both factors contain three figures.*

Multiply 524 by 436.

OPERATION.	EXPLANATION.
524 436 —— 228 464	1° $4 \times 6 = 24$ units; write the 4 units, carry the 2 tens; 2° $(2 \times 6) + (4 \times 3) + 2 = 26$ tens; write the 6 tens, carry the 2 hundreds;

3° $(5 \times 6) + (2 \times 3) + (4 \times 4) + 2 = 54$ hundreds; write the 4 hundreds, carry the 5 thousands;

4° (5×3)+(2×4)+5=28 thousands; write the 8 thousands, carry the 2 ten-thousands;

5° (5×4)+2=22 ten-thousands, or 2 hundred-thousands and 2 ten-thousands. Write 22.

Multiply:

1. 234×432=	6. 707×632=	11. 836×638=
2. 267×262=	7. 909×393=	12. 393×938=
3. 432×342=	8. 436×637=	13. 242×422=
4. 336×434=	9. 535×936=	14. 636×363=
5. 668×206=	10. 724×427=	15. 825×527=

88. Rounding one factor.

Multiply 342 by 99.

Operation.	Explanation.
342×100=34 200; 34 200—342=33 858.	99 times 342=(100—1) times 342.

Multiply:

1. 876× 99=	5. 236×998=	9. 324× 997=
2. 54× 999=	6. 365× 98=	10. 452× 996=
3. 763×9 999=	7. 99×677=	11. 29×9 998=
4. 84× 98=	8. 367× 97=	12. 57× 997=

89. Resolving the multiplier into factors.

The factors of 48 being 6 and 8; of 42, 6 and 7; of 49, 7 and 7; of 56, 7 and 8, replace the multiplier by its factors in the following exercises.

Thus, 326×42=(326×6=1 956; 1 956×7=) 13 692.

(1)

I 309	×	a) 48.	Multiply I, II, III, IV, successively by *a*, *b*, *c*, *d*.
II 836		b) 42.	
III 387		c) 49.	
IV 532		d) 56.	

Multiply using 6 and 9 for 54, 7 and 9 for 63, 8 and 8 for 64, 9 and 9 for 81, 8 and 9 for 72.

(2)

Multiply I, II, III, IV, V, successively by *a*, *b*, *c*, *d*, *e*.	I 1 234	×	a) 54.
	II 3 432		b) 63.
	III 6 382		c) 64.
	IV 9 238		d) 81.
	V 8 367		e) 72.

90. Exercises involving parentheses.

Note.—First perform the operations indicated within the parentheses.

1. $6\times(4+2)=$
2. $7\times(7+3)=$
3. $(8-5)+4=$
4. $(8+2)\times7=$
5. $(3\times3)\times8=$
6. $(6\times5)\times4=$
7. $7\times(8-6)=$
8. $10-(3\times2)=$
9. $6(6+4)=$
10. $5(7-5)=$
11. $6(10+2)=$
12. $(6+4)7=$
13. $8+(3+4+5)=$
14. $10-(3+2+4)=$
15. $37-(8-9+2)=$
16. $(2+3)(4+2)=$
17. $(8-5)(3+2)=$
18. $16-(7-8+9)=$
19. $8+(2\times3)=$
20. $(8\times5)-(8\times2)=$
21. $(6+4)-(6-5)=$
22. $(6\times3)+(8+4)=$
23. $(10\times3)+6(3+2)=$
24. $18-(6+7-9)=$
25. $18+(7+2-3)=$
26. $(2+3)-(5-4)=$
27. $600-(50+40)=$
28. $800+(60-40)=$
29. $900(4+3)=$
30. $1\ 800-(50\times8)=$

91. Checking the work.—The method used in addition may be used in multiplication. Find the *product* of the unitates of the factors. The final results must agree.

$$\begin{array}{l} \left.\begin{array}{r} 326 = 11 = 2 \\ 241 = \ 7 \end{array}\right\} 7\times2 = 14 = 5. \\ \hline 78\ 566 = 32 = 5. \end{array}$$

92. Multiply, and check the results:

1. 287×232.
2. 284×237.
3. 345×357.
4. 689×416.
5. 284×238.
6. 932×427.
7. 967×488.
8. 366×358.
9. 976×459.
10. 875×756.
11. $1\ 249\times648$.
12. $3\ 147\times639$.
13. $2\ 489\times763$.
14. $3\ 271\times654$.
15. $5\ 298\times324$.

Questions on Theory.

1. What is multiplication? (70).
2. What is the multiplicand? the multiplier? (71).
3. What is the result of multiplication called? (72).
4. What are the factors of the product? (72).
5. What is the sign of multiplication? How is it read? (73).
6. Recite the table of *threes, fours, fives, sixes*, etc.
7. What are the three principles of analysis relating to multiplication? (74).
8. What is the first case of multiplication? (75).
9. Give the rule of multiplication when one of the factors consists of only one figure. (76).
10. What is the second case of multiplication? (77).
11. Give the rule of multiplication when both factors consist of two or more figures. How do you treat intervening ciphers? (78, 79).
12. How can you prove your multiplication? (80).
13. Is the multiplier always an abstract number? (81, I).
14. Are the multiplicand and product like numbers? (81, II).
15. Does interchanging the factors affect the product? Example. (81, III).
16. What may be done to multiply a number by the product of several factors? Example. (81, IV).
17. What operations may a parenthesis contain? Examples. (82, I).
18. Is the sign × necessary before, after or between parentheses? Examples. (82, II).
19. What must be done to remove a parenthesis preceded by a plus? Example. (82, III).
20. What must be done to remove a parenthesis preceded by a minus? Example. (82, IV).
21. How must parentheses be dealt with? (82, V).
22. How do you multiply when there are ciphers at the right of one or both factors? (83).
23. How can you indicate that a number is to be multiplied several times by itself? (84).
24. What is cross multiplication? On what is it based?
25. How can you check your work in multiplication? (91).

Written Problems.

What is the product of:

1. 302 days of work at $2.75 each?
2. 245 days of work at $3.25 each?
3. 367 days of work at $3.50 each?
4. 836 days of work at $3.75 each?
5. 938 days of work at $4.25 each?

6. If an ocean steamer sails 246 miles in one day, how far will she sail in 26 days?

7. In one barrel of flour there are 196 pounds; how many pounds are there in 25 barrels?

8. In an orchard there are 32 rows of trees with 46 trees in a row. Required the total number of trees.

9. Find the cost of 125 yards of cloth at $3.25 per yard.

10. 1 760 yards make one mile; how many yards in 18 miles?

11. 5 760 grains make one pound Troy· how many grains in 137 pounds?

12. At 54 cents each, what will 3 686 grammars cost?

13. At 6 cents a quart, what will be paid for 3 678 quarts of milk?

14. There are 144 pins in one gross; find how many pins there are in 256 gross.

15. There are 277 cubic inches in one gallon of water; how many cubic inches in 48 gallons?

16. There being 4 840 square yards in one acre, how many square yards are there in 365 acres?

17. There being 5 280 feet in one mile, how many feet are there in 156 miles?

18. 63 360 inches make one mile; how many inches are there in 640 miles?

19. 198 inches make one rod; how many inches are there in 76 rods?

20. If a scholar learned 14 words of English every day, how many would he know in 198 days?

21. There are 128 cubic feet in one cord of wood; how many cubic feet in 67 cords?

22. At $6 each, what is the cost of 18 cords of wood?

23. At $245 each, what will 24 horses cost?

24. Find the value of 3 200 francs at 19 cents each.

25. Find the value of 4 800 marks at 24 cents each.

26. The Canadian bushel contains 2 218 cubic inches; how many cubic inches are there in 282 bushels?

27. If a pound sterling sells for $4.84 at the bank, find the value of 378 pounds.

28. There are 5 280 feet in one mile; how many feet are there in the diameter of the earth, if it is 7 912 miles?

29. Find the annual laundry expenses of a family, at the rate of $2.40 a week.

30. A stagecoach conveys an average number of 65 travelers a week; find the driver's yearly income, if the fare is 40 cents.

31. If one soldier consumes 2 pounds of wheat per day, what will be the yearly consumption of a regiment 5 464 strong?

32. By the Canadian Northern R. R., it is 176 miles from Montreal to Quebec. Find the price of a first-class ticket at 3 cents a mile; of a second-class ticket at 2 cents a mile.

33. What number is 39 times 24? 36 times as great as 122? 37 times as big as 49?

34. What number is 49 times 639? 23 times as great as 63? 39 times as big as 79?

35. What number is 153 times 69? 152 times as great as 63? 89 times as big as 98?

36. What number is 79 times 79? 63 times as great as 57? 39 times as big as 39?

37. Adelaide has $14.68; Rachel has 15 times as much money. How much have they together?

38. I have $59.63; what sum is $12 less than 15 times my money?

39. John has $6 more than 6 times $36. How much has he?

40. Clarence has $10 less than 13 times $49.50. How much has he?

41. If Daniel had $13 more, he would have 13 times $13.13. How much has he?

42. If Anthony had $3 more, he would have 39 times $39. What sum has he?

43. If Alexander had $16 less, he would have 13 times as much money as Arthur, who has $18. How much have they together?

44. If Caleb had $249 more, he would have 6 times $86.16. How much money has he?

45. Leander has $18 less than 49 times $239.16. What is his fortune?

46. Philip has $249 more than 3 times as much money as Eugene. How much have they together if Eugene has $1 142?

47. Amelia has 123 good marks; Theresa has 15 less than 3 times that number. How many good marks have they together?

48. If Blanche had $200 more, she would have 19 times $12. How much has she?

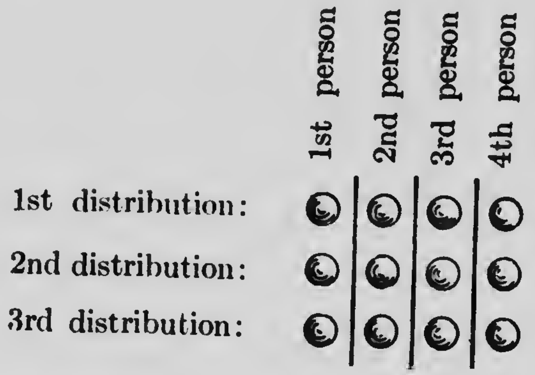
49. If Benedict had $239 less, he would have 3 times as much money as Albert, who has $121. How much money have they together?

50. A farmer sold 37 hens and 5 dozen chickens, the chickens 40 cents each, and the hens 3 times as much. Required the total selling price.

DIVISION

PROBLEM.—I wanted to distribute twelve marbles equally among four persons. How many marbles did each person receive?

I first gave one marble to each person, that is four marbles in all, and then had eight left. I did the same a second time; I had then given eight marbles away and had four left. At last, I gave each person another marble. I had none left after having given away twelve. What have I done? I have taken four marbles from twelve as often as I could, that is, I have found out how many times four marbles were contained in twelve marbles. So, four is contained in twelve (or may be taken from twelve) three times. When I act as above, *without subtracting*, I divide.



93. Division is an operation by which we find how many times one number is contained in another; it is therefore a short method of subtracting several times the same number.

94. The number to be divided is the *dividend;* the number by which we divide is the *divisor*.

95. The result of the division is the *quotient;* it shows *how many times* the divisor is contained in the dividend; it is also called *ratio*.

96. The *sign* of division is ÷ or :, and is read *divided by*. It shows that the number on the left is to be divided by the one on the right.

Thus, $27 \div 9$ is read 27 divided by 9.

Division is sometimes indicated by writing the dividend above the divisor.

Thus, $\frac{36}{9}$ is read 36 divided by 9, or 36 over 9.

97. The part of the dividend remaining when the division is not exact, is called the *remainder*.

Thus, $28 \div 9 = 3$, with a remainder of 1.

MENTAL DIVISION.

Oral Exercises.

In multiplying you have said: $6 \times 5 = 30$; here you must reverse the operation and say: $30 \div 6 = 5$; $30 \div 5 = 6$. Hence, recalling the products of the multiplication table, find the missing factor in the following exercises:

	A	B	C	D	E	F	G	H
1.	2)26	2)52	2)72	13)26	4)84	13)39	31)62	15)75.
2.	3)45	2)34	7)91	14)42	5)75	13)78	14)70	15)90.
3.	2)38	3)54	5)85	29)58	2)74	17)68	24)72	31)93.
4.	3)39	2)56	3)87	32)64	2)78	23)69	29)87	19)95.
5.	2)54	4)60	6)84	18)36	4)76	18)90	27)81	14)98.
6.	2)28	3)57	5)90	17)51	5)80	18)54	16)64	47)94.
7.	3)42	5)65	2)92	13)65	3)81	19)57	6)90	16)80.
8.	2)48	2)64	3)93	17)34	2)82	18)72	23)92	22)88.
9.	4)52	4)68	2)94	16)48	2)84	26)52	14)42	36)72.
10.	2)32	3)66	4)96	14)56	4)72	21)63	13)91	32)96.
11.	2)58	3)63	6)96	19)38	5)95	17)85	26)78	39)78.
12.	3)51	5)70	7)98	21)84	6)78	15)60	24)96	13)52.

Give the quotients and remainders of the following:

EXAMPLE.— 7) 22 = 3 times, and 1 over 7, or $3\frac{1}{7}$.

	A	B	C	D	E	F	G	H
1.	7)24	8)44	8)41	6)71	7)55	26) 53	13) 15	15)100.
2.	3)56	7)62	7)60	12)59	12)82	15) 76	14) 79	21)100.
3.	8)42	6)43	6)51	11)60	19)97	13) 68	18) 91	12)100.
4.	9)56	5)52	12)62	9)89	3)53	14) 58	14)100	18)100.
5.	5)38	9)49	8)20	6)70	18)59	24)100	16)100	17)100.

Give the quotients in the following indicated divisions. $\frac{20}{5}$ is read 20 over 5, and indicates that 20 must be divided by 5.

1. $\frac{92}{46}$.	5. $\frac{68}{17}$.	9. $\frac{64}{16}$.	13. $\frac{57}{19}$.	17. $\frac{96}{16}$.
2. $\frac{96}{12}$.	6. $\frac{91}{13}$.	10. $\frac{36}{18}$.	14. $\frac{63}{21}$.	18. $\frac{72}{24}$.
3. $\frac{121}{11}$.	7. $\frac{54}{27}$.	11. $\frac{78}{26}$.	15. $\frac{90}{18}$.	19. $\frac{92}{23}$.
4. $\frac{144}{12}$.	8. $\frac{85}{17}$.	12. $\frac{81}{27}$.	16. $\frac{45}{15}$.	20. $\frac{90}{15}$.

Find the quotients:

1. 91 : 7.	4. 51 : 3.	7. 64 : 21.	10. 95 : 13.	13. 5 : 2.
2. 87 : 3.	5. 100 : 10.	8. 39 : 8.	11. 12 : 30.	14. 17 : 5.
3. 51 : 17.	6. 50 : 14.	9. 30 : 12.	12. 5 : 17.	15. 210 : 100.

16. 180 ÷ 10.	19. 180 ÷ 5.	22. 800 ÷ 160.	25. 800 ÷ 400.
17. 1 800 ÷ 10.	20. 640 ÷ 32.	23. 400 ÷ 200.	26. 160 ÷ 16.
18. 200 ÷ 20.	21. 900 ÷ 180.	24. 800 ÷ 40.	27. 160 ÷ 10.

98. Principles of Analysis relating to Division.

10. I divide because I wish to find the *quotient* of ...... by ......

11. I divide because having the *total value* and the *number of objects*, I wish to find the *value of one object.*

12. I divide because having the *total value* and the *value of one object*, I wish to find the *number of objects.*

13. I divide because I wish to find ...... *times less than* what I have.

Oral Problems.

TENTH PRINCIPLE OF ANALYSIS.

1. Find the *quotient* of 20 by 4.
2. Find the *quotient* of 100 by 20.
3. What is the *quotient* of 200 divided by 40?
4. What is the *quotient* of 144 divided by 12?
5. What is the *quotient* of 90 divided by 18?

ELEVENTH PRINCIPLE OF ANALYSIS.

1. Six pencils cost 12 cents; what is the cost of *one* pencil?

2. Three pounds of butter cost $1.11; what is the cost of *one* pound?

3. George takes 320 steps in 8 minutes; how many does he take in *one* minute?

4. A car runs 125 miles in 5 hours; how many miles does it run in *one* hour?

5. A workman earned $36 in 6 days; what did he earn in *one* day?

6. There are 96 ounces in 6 pounds; how many ounces in *one* pound?

7. A lady divided $1.08 equally among 9 beggars; how many cents did each (*one*) beggar receive?

8. A man spends $8 in 4 weeks at the saloon; how much does he spend in *one* week?

9. Ten men drank 30 quarts of beer in 3 days; what did *each* man drink in *one* day?

10. In 1832, the 10 independent schools of Montreal received $3 975.50 from the Government of Lower Canada. How much did *each* school receive on the average?

11. Six pencils cost 24 cents; find the cost of *one* pencil; 2, 4, 8, 15 pencils.

12. Eleven acres of land cost $1 100; find the cost of 3, 8, 9, 15, 16 acres.

13. A man pays $60 for three months' rent; what would he pay for 8, 12, 24, 36 months?

14. Six suits cost $90; find the cost of 3, 7, 10, 20 suits.

15. Four wagons cost $480; find the cost of 3, 5, 6, 10 wagons.

TWELFTH PRINCIPLE OF ANALYSIS.

1. At 5 cents each, how many pencils can I buy for 30 cents?

2. There are 16 ounces in one pound; how many pounds in 192 ounces?

3. There are 32 quarts in one bushel; how many bushels in 224 quarts?

4. A car runs 180 miles at the rate of 30 miles an hour. Required the number of hours.

5. How long will it take an automobile to run 105 miles, at the rate of 15 miles an hour?

6. The circumference of the earth is 25 000 miles; at the rate of 50 miles a day, how long will it take a person to walk it?

7. I have $1 500 to pay in installments of $100 each. Required the number of installments.

8. A man divided $1 600 equally among a certain number of persons, each one receiving $40. Required the number of persons.

9. An orchard contains 360 trees arranged in 12 rows. How many tress are there in each row?

10. It is 450 miles from Winnipeg to Fort William by the Grand Trunk Pacific R. R. At 30 miles per hour, how long will it take a train to run that distance?

THIRTEENTH PRINCIPLE OF ANALYSIS.

1. Alice has 60 cents; Emma has 3 times *less*. How much has Emma? How much have they together?

2. Christopher has 90 hens; Lewis has 6 times *less;* how many has he? How many have they together?

3. Susan has $2.50; Kate has 5 times *less*. How much money has Kate? How much have they together?

4. What number is 16 times *less* than 80? Find the product of both, their sum, their difference.

5. Clara's age is 3 times *less* than Cora's; if Cora is 9 years old, how old is Clara?

6. 64 is how many times 16? 81, how many times 9? 132, how many times 12?

7. Lionel has $6; Lorenzo, $24; Lorenzo has how many times as much money as Lionel? Lionel has how many times less money than Lorenzo?

8. Eva is 4 years old; Mary is 12 years old. Mary is how many times as old as Eva? Eva is how many times less old than Mary?

9. 12 is how many times less than 48? 48 is how many times 12? 48 is how many times as much as 12?

10. 108 is how many times 12? how many times as much as 12? 12 is how many times less than 108?

WRITTEN DIVISION.

99. There are two cases in written division, namely, *short division* and *long division*.

100. First case.—SHORT DIVISION.—*When the divisor is one figure.*

Divide 42 364 by 7.

OPERATION.	EXPLANATION.
7)42 364 ――― 6 052	7 is not contained in 4; in 42 it is contained 6 times; 7 is not contained in 3, write 0 under the 3; 7 is contained 5 times in 36, with a remainder of 1; in 14, 2 times. Say mentally: 7 in 42, 6; in 3, 0; in 35, 5; in 14, 2.

101. Second case.—LONG DIVISION.—*When the divisor is two or more figures.*

Divide 26 765 by 463.

OPERATION.

Divisor	Dividend	Quotient
463)	26 76,5	$(57\frac{374}{463}$
	23 15	
	3 615	
	3 241	
	374	

EXPLANATION.

$463 \times 10 = 4\ 630$, a number smaller than the dividend; $463 \times 100 = 46\ 300$, a number greater than the dividend; the quotient must therefore be greater than 10 and smaller than 100, that is, be composed of two figures, a tens' figure and a units' figure.

Now, the tens of the quotient, multiplied by the divisor, give tens, that are all contained in the 2 676 tens of the dividend. It follows that 2 676 divided by 463 will give the tens of the quotient. The greatest number that can be taken from 2 676 is the product of 463 by 5, 2 315; hence, 5 is the tens' figure of the quotient, with 361 tens remaining.

361 tens and 5 units are 3 615 units. This partial dividend contains the product of the divisor by the units' figure of the quotient, plus the remainder, if any. The greatest number that can be taken from 3 615 is the product of 463 by 7, 3 241 units; hence, 7 is the units' figure of the quotient, with a remainder of 374 units.

SIMPLIFIED OPERATION.

463)	26 765	(57
	3 615	
	374	

Take enough of the left-hand digits of the dividend to contain the divisor at least once, at most 9 times, and say mentally:

1° 463 is contained how many times in 2 676? 5 times; 5 times 3, $15+1=16$; I write the remainder 1 and carry 1 ten; 5 times 6, $30+1=31$; $31+6=37$; I write the remainder 6 and carry the 3 hundreds; 5 times 4, $20+3=23$; $23+3=26$; I write the remainder 3.

2° 361 tens+5 units = 3 615 units; 463 is contained how many times in 3 615? 7 times; 7 times 3, $21+4=25$; I write the remainder 4 and carry the 2 tens; 7 times 6, $42+2=44$; $44+7=51$; I write the remainder 7 and carry the 5 tens; 7 times 4, $28+5=33$; $33+3=36$; I write the remainder 3.

102. General rule.—*1° Draw curved lines at both sides of the dividend, and place the divisor at the left;*

2° With a comma, mark out at the left of the dividend such a number as will contain the divisor at least once, at most 9 times; this number is the first partial dividend;

3° See how many times the first left-hand figure of the divisor is contained in the first figures of the partial dividend. Write the quotient figure thus obtained at the right of the dividend; multiply the divisor by this quotient figure and subtract the product from the partial dividend used, and to the remainder annex the next figure of the dividend;

4° Divide as before, until all the figures have been annexed to the remainder;

5° If any partial dividend will not contain the divisor, write a cipher in the quotient, then annex the next figure of the dividend, and proceed as before;

6° If there is a final remainder, annex it, with the divisor beneath, to the integral part of the quotient.

103. Proof of division.—Multiply the quotient by the divisor, and add the remainder, if any, to the product. The result should equal the dividend.

104. Note.—When the divisor and the dividend have ciphers on the right:

1° Cut off an equal number of ciphers from the divisor and the dividend;

2° Divide the remaining numbers one by the other.

105. Principles.—I. To divide a number by the product of several factors, it is sufficient to find the continued quotient of the given number by each factor in turn.

Thus, to divide 840 by 105, which is the product of $3 \times 5 \times 7$, we may say: $840 \div 3 = 280$; $280 \div 5 = 56$; $56 \div 7 = 8$. When the successive divisions give remainders, the case is dealt with in rule N° 199.

II. Division is the converse of multiplication: the dividend is a product, and the divisor and quotient are the factors of the product.

III. If the dividend and divisor be both multiplied or both divided by a same number, the quotient is not changed.

EXAMPLE.—$24 \div 6 = 4$.

$$\left.\begin{array}{l}24 \times 3 = 72 \\ 6 \times 3 = 18\end{array}\right\} 72 \div 18 = 4. \qquad \left.\begin{array}{l}24 \div 3 = 8 \\ 6 \div 3 = 2\end{array}\right\} 8 \div 2 = 4.$$

Written Exercises.

Divide 5 times successively:

1. 98 848 768 by 2.
2. 3 813 266 322 by 3.
3. 67 519 905 792 by 4.
4. 428 964 843 750 by 5.
5. 1 774 440 400 896 by 6.
6. 17 777 297 320 566 by 7.
7. 50 302 851 283 968 by 8.
8. 304 173 124 005 636 by 9.

Divide I, II, III, IV, V, successively: by a, then successively by b, and so on.

(1)

$$\left.\begin{array}{rr}\text{I} & 2\,420 \\ \text{II} & 3\,236 \\ \text{III} & 2\,636 \\ \text{IV} & 3\,569 \\ \text{V} & 3\,199\end{array}\right\} \div \left\{\begin{array}{lr}a) & 364. \\ b) & 396. \\ c) & 339. \\ d) & 675. \\ e) & 839.\end{array}\right.$$

(2)

$$\left.\begin{array}{rr}\text{I} & 6\,464\,341 \\ \text{II} & 7\,846\,760 \\ \text{III} & 5\,864\,548 \\ \text{IV} & 8\,645\,341 \\ \text{V} & 9\,624\,872\end{array}\right\} \div \left\{\begin{array}{lr}a) & 268. \\ b) & 354. \\ c) & 676. \\ d) & 758. \\ e) & 865.\end{array}\right.$$

Find the value of:

1. $4 \times 12 - 16 \div 4$.
2. $7 + 8 \times 7 - 26$.
3. $(14 + 8 - 6) \times 9$.

NOTE.—First perform the operations indicated within the parentheses.

4. $(87 - 65 + 96) \times 24$.
5. $(240 + 98) \times (688 - 425)$.
6. $(56 - 18) \times (11 + 4) - 6 \times 4$.
7. $(84 - 7 \times 6 + 9 \times 4 + 6) \div 9$.
8. $(56 + 7) \times 12 + 97 - 7 \times 9$.
9. $6 + 10 \times 5 + 8 \div 2 - 4 - 2 + 8$.
10. $7 \times (5 + 4) + (8 \times 6) \div 2 - 3 \div 4$.

106. SHORT METHODS.

I. To divide by 10, 100, 1000.

Divide 4 370 by 10.

OPERATION.	EXPLANATION.
10)4 370. 437.0	The operation shows that to divide by 10, it is only necessary to remove the point one place to the left. Likewise, to divide by 100, 1000, move the point 2 places, 3 places to the left.

Divide each of the following numbers 1° by 10; 2° by 100; 3° by 1 000.

1.	130 000.	**6.**	24 750 000.	**11.**	3 489.
2.	1 250 000.	**7.**	400 000.	**12.**	7 248.
3.	40 000.	**8.**	1 800 000.	**13.**	5 673.
4.	750 000.	**9.**	2 000 000.	**14.**	8 470.
5.	1 160 000.	**10.**	14 500 000.	**15.**	19 872.

II. To divide or multiply by 25, 50, 125 and 250.

$$\text{To divide by}\left\{\begin{array}{r}25\\50\\125\\250\end{array}\right\}\text{divide by}\left\{\begin{array}{r}100\\100\\1\,000\\1\,000\end{array}\right\}\text{and multiply by}\left\{\begin{array}{l}4.\\2.\\8.\\4.\end{array}\right.$$

$$\text{To multiply by}\left\{\begin{array}{r}25\\50\\125\\250\end{array}\right\}\text{multiply by}\left\{\begin{array}{r}100\\100\\1\,000\\1\,000\end{array}\right\}\text{and divide by}\left\{\begin{array}{l}4.\\2.\\8.\\4.\end{array}\right.$$

Divide by 25, 50, 125 and 250:	Multiply by 25, 50, 125 and 250:
1. 153 000.	**1.** 480.
2. 72 000.	**2.** 560.
3. 16 000.	**3.** 640.
4. 99 000.	**4.** 720.
5. 1 450.	**5.** 1 240.

I 216 000	÷	*a*) 2 400.	
II 648 000		*b*) 3 600.	
III 504 000		*c*) 4 800.	
IV 792 000		*d*) 6 000.	
V 576 000		*e*) 7 200.	

III. In these exercises cut off an equal number of ciphers from both terms, and divide successively I, II, III, IV, V, by *a*, then by *b*, and so on.

IV. To divide using factors.

EXAMPLE.—1 176 ÷ 42 = 6)1 176.
7)196.
28.

Divide I, II, III, IV, successively by *a*, then by *b*, and so on.

(1)

I 2 352	÷	*a*) 48.
II 18 816		*b*) 42.
III 25 872		*c*) 49.
IV 11 760		*d*) 56.

(2)

I 5 184	÷	*a*) 54.
II 15 552		*b*) 36.
III 20 736		*c*) 64.
IV 46 656		*d*) 81.

107. Division by continued subtraction.

Divide 784 by 149.

OPERATIONS.

I	II	III	IV	V
784	635	486	337	188
149	149	149	149	149
635	486	337	188	39

EXPLANATION.

Subtract the divisor from the dividend and from the successive remainders until the last remainder is less than the divisor. The number of operations represents the quotient.

PROOF.—(5×149)+39 = 784; or 784÷149 = 5 times and 39 over.

Divide by continued subtraction and prove your work:

1. 489 764÷42 875.
2. 447 859÷48 727.
3. 128 976÷54 738.
4. 249 768÷55 987.
5. 473 964÷75 428.

108. Averaging.

An average is found by adding like quantities, and dividing their sum by the number of addends.

1. Find the average of 6, 7, 8, 9, 10.

2. Find the average of 40, 50, 60, 70, 80.

3. In 7 days my hens laid 10, 12, 14, 18, 10, 14, and 13 eggs. Find the daily average.

4. Find the average of the notes I got in four weeks: 50%, 60%, 65%, and 73%.

5. My notes for 10 months were as follows: 65%, 64%, 63%, 72%, 50%, 62%, 65%, 70%, 61%, and 78%. Required the average note per cent.

109. Checking the work.

Division.

17 759	(494
2 939	35
469	

Proof.

1° $494 = 17 = 8 \\ 35 = 8 \Big\} 8 \times 8 = 64 = 10 = 1.$

$469 = 19 = 10 = 1;\ 1 + 1 = 2.$

2° $17\,759 = 29 = 11 = 2.$

The method is similar to that already given in addition. Proceed as follows: *1° Multiply the unitate of the divisor by the unitate of the quotient, and add the unitate of the remainder; 2° find the unitate of the dividend. If the final unitates agree, the work is deemed correct.*

Divide and check the results as illustrated above:

1. 7 349 ÷ 36.
2. 4 872 ÷ 42.
3. 7 879 ÷ 56.
4. 5 987 ÷ 72.
5. 9 478 ÷ 96.
6. 9 764 ÷ 84.
7. 10 632 ÷ 99.
8. 42 367 ÷ 108.
9. 27 945 ÷ 54.
10. 87 497 ÷ 88

Questions on Theory.

1. What is division? (93).
2. What is the dividend? What is the divisor? (94).
3. What is the quotient? (95).
4. How many signs of division are there? Make and name them. (96).

5. What is the remainder? (97).

6. Recite the four principles of analysis relating to division. (98).

7. What is short division? (100).

8. When must short division be used? (100).

9. What is long division? (101).

10. Recite the general rule for dividing. (102).

11. How can you prove your division? (103).

12. How do you proceed in division when both terms contain ciphers? (104).

13. What may be done instead of dividing a number by the product of several factors? (105, I).

14. Is division the converse of multiplication? (105, II).

15. Explain averaging. (108).

16. Can you check a division by the Accountant's method? (109).

Written Problems.

1. Find the quotient of $62 377 by 49 days.

2. If 6 horses cost $1 212, what does one horse cost?

3. I paid $1 905 for 127 saddles; find the price of each.

4. A man earned $2 639 in 13 weeks; what were his weekly wages?

5. Find the contents of one barrel, if 56 barrels contain 3 192 gallons of oil.

6. Sound moves 37 060 feet in 34 seconds; how far does it move in one second?

7. A man travels 1 728 miles in 36 days; how far does he travel in one day?

8. A ship sails 6 120 miles in 90 days; how far does she sail in one day?

9. If 217 acres of land cost $12 152, what does one acre cost?

10. If 69 cows are worth $3 450, what is the average value of each?

11. There are 7 756 cubic inches in 28 gallons; how many cubic inches in one gallon?

12. If 41 barrels contain 2 583 gallons, find the contents of one gallon.

13. 345 hats cost $696.90; find the cost of one hat.

14. Lawrence has $54.18 in the School Savings Bank. If that sum represents 43 deposits, find the average value of each deposit.

15. A printing press turns out 27 000 newspapers in 45 minutes. How many in one minute?

16. In 1907, the estimated yield of potatoes in Quebec was 22 929 885 bushels from 138 969 acres. Find the average yield per acre.

NOTE.—It is advisable to approximately estimate an answer before writing it down; many absurd answers may be thus avoided. In N° 16, the estimate would give a number of three figures, as 100 000 is contained 220 times in 22 000 000.

17. A field contains 41 595 hills of potatoes. If there were 177 rows, how many hills were there in each row?

18. In 1909, 414 301 280 letters were mailed in Canada. Find the daily average.

19. In 1911, Canada's population was 7 205 000 inhabitants. The total area of Canada being 2 298 385 000 acres, find the average area per head.

20. If 11 acres of land cost $1 485, find the cost of one acre, 2 acres, 3 acres, 8 acres, 10 acres.

21. If a dozen suits of clothes are worth $336, what is the price of 1, 2, 3, 7, 9 suits?

22. If I pay $1 276 for 11 months' rent, find the outlay for 1, 2, 4, 8, 12 months.

23. If 9 carriages cost $1 215, find the cost of 1, 2, 7, 8, 18 carriages.

24. If 8 acres of land sell for $784, find the selling price of 5, 24, 40, 200 acres.

25. At $5 each, how many sheep can I buy with $675?

26. At 11 cents a quart, how many quarts of cherries will 1 243 cents buy?

27. There being 60 seconds in one minute, how many minutes are there in 12 900 seconds?

28. In one bushel there are 32 quarts; how many bushels are there in 16 192 quarts?

29. The diameter of the earth is 7 912 miles; at the rate of 46 miles a day, how many days will it take a person to walk that distance?

30. The distance from the earth to the moon is 240 000 miles; at the rate of 75 miles an hour, how long would it take an airship to reach it?

31. The sun is 95 000 000 miles from the earth; a cannon ball moving at the rate of 50 miles a minute would take how many minutes to reach it?

32. At the rate of 260 miles a day, how many days would a steamer take to go from Vancouver to the Fiji Islands, a distance of 5 200 miles?

33. The circumference of a wheel is 15 feet; how many times will it turn in going one mile (5 280 feet)?

34. How many sheep were sold for $869.50, at the rate of $4.70 each?

35. At 325 miles a day, how long will a steamer take to go from Quebec to Liverpool, 2 600 miles?

36. How long will it take an automobile to run from Montreal to Quebec, 176 miles, at the rate of 22 miles an hour?

37. It is 1 170 miles from Chicago to Boston, via Montreal. At 39 miles an hour, how long will a train take to cover that distance?

38. At $3.75 a day, how long will it take a workman to earn $45?

39. How long will a supply pipe take to fill up a cistern containing 1 215 gallons, if 27 gallons run in every minute?

40. A tract of land containing 1 728 acres was divided into lots of 24 acres each. Required the number of lots.

41. What number is 12 times less than 468? Find the sum of the two numbers.

42. What number is 3 times less than 12 120? Find their difference also.

43. What number is 16 times less than 784? Find their product also.

44. A horse cost $240; a cow cost 4 times less. How much did they cost together?

45. I paid $300 for a horse, and 5 times less for a cow. What would 2 horses and 3 cows have cost?

46. Henry has $450; Patrick has 5 times less. How much have they together?

47. Mary has 420 good notes; Ann has 3 times less. Mary has how many more than Ann?

48. Leo is 13 times less old than his grandfather who is 78. Required Leo's age, the sum and the difference of their ages.

SUGGESTION.—Remark that the sum is 14 times Leo's age, and the difference 12 times Leo's age.

49. 1 728 is how many times as great as 144? 144 is how many times less than 1 728?

50. 4 356 is how many times as great as 121? 121 is how many times less than 4 356?

GENERAL PROPERTIES OF NUMBERS

FACTORS.

110. The **factors** of a number are the numbers which, when multiplied together, will produce it.

111. The factors of a number are also its *exact divisors:* 6 and 7 are the factors or exact divisors of 42; 2, 3 and 5 are the factors or exact divisors of 30.

112. An exact divisor will divide a number without having a remainder. The number is then said to be *divisible.*

113. A number is *divisible by* 2, that is, contains the factor 2, when its units' figure is 0, 2, 4, 6 or 8.

114. An *even number* is that divisible by 2. An *odd number* is that not divisible by 2.

115. A number is *divisible by* 4, when its two right-hand figures are ciphers, or when the number they express is divisible by 4.

Ex.—**1 116, 2 508, 1 396, 1 900.**

116. A number is *divisible by* 8, when its three right-hand figures are ciphers, or when the number they express is divisible by 8.

Ex.—**5 128, 6 008, 7 000, 5 240.**

117. A number is *divisible by* 5, when its units' figure is either 0 or 5.

118. A number is *divisible by* 10, when its units' figure is 0.

119. A number is *divisible by* 3, when the sum of its figures is divisible by 3. A number is *divisible by* 9, when the sum of its figures is divisible by 9.

Ex.—2 343 is *divisible by* 3, because $(2+3+4+3)$ or 12 is divisible by 3. 56 178 is divisible by 9, because $(5+6+1+7+8)$ or 27 is divisible by 9.

120. A number is *divisible by* 6, when it is even, and the sum of its figures is divisible by 3.

Ex.—56 178 is divisible by 6.

Oral Exercises.

Point out the numbers divisible by 2, by 4, by 8, by 5, by 10, by 3, by 6, by 9.

1. 86.
2. 94.
3. 96.
4. 72.
5. 123.
6. 221.
7. 918.
8. 819.
9. 515.
10. 450.
11. 660.
12. 279.
13. 849.
14. 639.
15. 300.
16. 700.
17. 4 907.
18. 6 255.
19. 3 027.
20. 1 356.
21. 7 268.
22. 4 867.
23. 7 075.
24. 9 620.
25. 8 124.
26. 3 672.
27. 1 235.
28. 5 778.
29. 9 100.
30. 8 328.

31. Find the factors of 39, 51, 58, 69, 78, 87.

32. Find the leap years between 1906 and 1931.

NOTE.—Leap years are divisible by 4; centennial years must be divisible by 400 to be leap years.

33. Find one factor of 395, 123, 777, 692, 1 275, 1 263.

34. In how many ways can you factor 60?

35. Find the missing factor:

$2 \times ? \times 5 \times 7 = 210.$ $3 \times 2 \times ? \times 5 = 270.$

$2 \times 2 \times 6 \times 2 \times ? = 96.$ $2 \times 8 \times 3 \times ? = 144.$

PRIME FACTORS.

121. A **prime number** is one that has no exact divisor except itself and 1.

Thus, 2, 3, 5, 11, 19, 23 are prime numbers.

122. A *prime factor* is a prime number used as a factor.

Thus, 3×5 are the prime factors of 15, since 3 and 5 are prime numbers; 4×8 are factors of 32, but are not the prime factors of 32; $2 \times 2 \times 2 \times 2 \times 2$ are the prime factors of 32.

Table of prime numbers, from 1 to 200:

1	13	37	61	89	113	151	181
2	17	41	67	97	127	157	191
3	19	43	71	101	131	163	193
5	23	47	73	103	137	167	197
7	29	53	79	107	139	173	199
11	31	59	83	109	149	179	

123. A number not prime is called *composite.* It is composed of other factors than itself and 1, and may be divided by simple prime numbers like 2, 3, 5, 7, 11, etc.

EXAMPLE.—What are the prime factors of 462?

OPERATION.

$$\begin{array}{r} 2)\underline{462} \\ 3)\underline{231} \\ 7)\underline{77} \\ 11 \end{array}$$

EXPLANATION.

Since 462 is an even number, it is divisible by the prime number 2; 231 $(2+3+1=6)$ is divisible by the prime number 3; 77 is divisible by the prime number 7, and the quotient obtained is the prime number 11. Hence, 462 is composed of the *prime factors* 2, 3, 7, 11.

124. Rule.—*Divide the given number by its smallest prime factor; divide the quotient, if composite, in the same manner; and thus continue until the quotient is a prime number. The divisors and the last quotient will be the prime factors required.*

PROOF.—The continued product of all the prime factors should equal the given number.

Oral Exercises.

1. Name the prime numbers from 200 to 300; from 300 to 400; from 400 to 500.
2. Close your book and recite the prime numbers from 1 to 200.
3. What are the prime factors of 4? 6? 8? 9? 10?
4. What are the prime factors of 20? 21? 22? 24? 25?
5. What are the prime factors of 33? 34? 35? 36? 38?

Written Exercises.

Find the prime factors of the following:

1. 120.	**6.** 225.	**11.** 375.	**16.** 2 430.
2. 136.	**7.** 775.	**12.** 8 910.	**17.** 5 390.
3. 216.	**8.** 891.	**13.** 2 970.	**18.** 1 280.
4. 270.	**9.** 130.	**14.** 1 250.	**19.** 1 538.
5. 390.	**10.** 420.	**15.** 1 375.	**20.** 1 024.

125. Resolving numbers into their prime factors enables us to find:

1° all the divisors of a number;
2° the greatest common divisor of two or more numbers;
3° the least common multiple of two or more numbers.

COMMON FACTORS OR COMMON DIVISORS.

The word *common* implies *belonging to* or *pertaining to several.* Thus, two or three persons may own a property *in common;* and anger may be defined: *a passion common to man and beast.*

126. The **common factor** or *common divisor* of two or more numbers is any number that will exactly divide each of them.

Thus, 2 is a common factor of 24, 60 and 84.

127. When numbers have no common factor or divisor, they are said to be *prime to each other.*

Thus, 16 and 25 are prime to each other.

128. The *greatest common divisor* (G. C. D.) of two or more numbers is the largest factor common to the numbers.

Thus, 12 is the greatest common divisor of 24, 60 and 84.

All the exact divisors of 24 are 2, 3, 4, 6, 8, 12; of 60 are 2, 3, 4, 5, 6, 10, 12, 15, 20, 30; of 84 are 2, 3, 4, 6, 7, 12, 14, 21, 28, 42.

The divisors *common* to the three numbers are 2, 3, 4, 6 and 12; and the *greatest common divisor* of the three numbers is therefore 12.

Oral Exercises.

1. Find the greatest number that will exactly divide 96 and 72.
2. Find the greatest number that will exactly divide 60 and 90.
3. What is the greatest common divisor of 90 and 108?
4. What is the greatest number that will exactly divide 30, 60, 54 and 48?
5. Find a divisor common to 34, 51 and 85.
6. Find all the numbers less than 100 of which 19 is a common factor.
7. Name 5 numbers of which 12 is the greatest common divisor.
8. Name 3 composite numbers that are prime to each other.
9. Find the greatest common divisor of 48, 72, 96 and 120,

10. Find the greatest common divisor of 12, 40, 52, 28, 480 and 4.

NOTE.—52 is the sum of 40 and 12; 28, their difference; 480, their product, and 4, their remainder in a division.

129. Principles.—I. **A number is the exact divisor of only such numbers as contain all its prime factors.**

Thus, 4 is an exact divisor of 16 and 24.

	Prime factors	
of 4:	of 16:	of 24:
2×2.	$2\times2\times2\times2$.	$2\times2\times2\times3$.

II. **The greatest common divisor of two or more numbers is composed exclusively of the prime factors common to those numbers.**

Thus, 12 is the greatest common divisor of 48 and 60.
Prime factors of $48 = 2\times2\times2\times2\times3$.
Prime factors of $60 = 2\times2 \qquad \times3\times5$.

The only prime factors common to both numbers are $2\times2\times3$; they form 12, the greatest common divisor.

III. **The greatest common divisor of two numbers is also an exact *divisor* of their sum, of their difference, of their product, and of their *remainder after division*.**

Thus, 4, the greatest common divisor of 12 and 40, is also an *exact divisor*

$$\text{of their sum}\begin{cases}12 = 3 \text{ times } 4;\\ 40 = 10 \text{ times } 4;\\ \hline 52 = 13 \text{ times } 4;\end{cases} \qquad \text{of their difference}\begin{cases}40 = 10 \text{ times } 4;\\ 12 = 3 \text{ times } 4;\\ \hline 28 = 7 \text{ times } 4;\end{cases}$$

$$\text{of their product}\begin{cases}40 = 10 \text{ times } 4;\\ 12 = 3 \text{ times } 4;\\ \hline 480 = 30 \text{ times } 4\times4;\end{cases} \qquad \text{and of their remainder after division}\begin{cases}12)\ 40\ (3\\ \quad\ \ 36\\ \hline \quad\ \ \ 4\end{cases}$$

From 10 times 4, or 40, were subtracted 9 times 4; the remainder is therefore 1 time 4.

130. Finding the greatest common divisor.

I. Factoring Method.—What is the greatest common divisor of 54, 66 and 90?

Operation.

$54 = 2 \times 3 \times 3 \times 3.$
$66 = 2 \times 3 \times 11.$
$90 = 2 \times 3 \times 3 \times 5.$

$$\begin{array}{r|rrr} 2) & 54 & 66 & 90 \\ \hline 3) & 27 & 33 & 45 \\ \hline & 9 & 11 & 15. \end{array}$$

$2 \times 3 = 6$, G. C. D.

Explanation.

As the prime number 2 is common to each of these numbers, it is therefore a factor of their greatest common divisor; as the prime number 3 is common to each of the resulting quotients, it is also a factor of the greatest common divisor. There being no other common factor, the product of the common factors obtained $(2 \times 3 = 6)$ is the greatest common divisor.

131. Rule I.—*Write the numbers one beside another and divide by any prime number common to all the numbers; divide the quotients in the same manner, and thus continue until the quotients have no common factor; the product of the several divisors will be the greatest common divisor.*

Written Exercises.

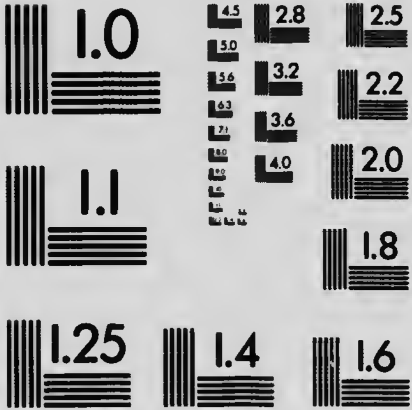
Find the greatest common divisor by the *Factoring Method:*

1. 9, 18, 45, 72.	**6.** 60, 72, 84, 108.	**11.** 58, 87, 116, 145.
2. 16, 32, 40, 56.	**7.** 28, 42, 56, 70.	**12.** 56, 84, 112, 140.
3. 36, 84, 66, 60.	**8.** 63, 72, 81, 90.	**13.** 64, 96, 128, 160.
4. 33, 44, 55, 66.	**9.** 36, 54, 72, 90.	**14.** 375, 500, 625, 750.
5. 26, 39, 52, 65.	**10.** 42, 63, 84, 147.	**15.** 1 024, 1 280, 1 792.

Note.—When a smaller number exactly divides a greater one, it is the greatest common divisor of both. For example, 12 and 48.

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



II. Division Method.—What is the greatest common divisor of 2 592 and 384?

Operation.

		2 592
384	6	2 304
288	1	288
96	3	288.

Explanation.

According to Principle III, the greatest common divisor of two numbers is *also an exact divisor of their remainder after division.* 384 is contained 6 times in 2 592 and the *remainder* is 288. This remainder must therefore contain the greatest common divisor of 2 592 and 384. But 288 is not their *greatest common divisor*, as it does not exactly divide 384. But, repeating the principle, we may say that the greatest common divisor of 384 and 288 is also an exact divisor of their remainder; this remainder is 96. 96 therefore *contains* the greatest common divisor, and *it is* the greatest common divisor, as it exactly divides 288. So does it also divide 384 and 2 592.

132. Rule II.—*Divide the greater number by the less, and then divide the preceding divisor by the remainder, and so on, until there is no remainder. The last divisor will be the greatest common divisor.*

Note I.—This method should be used when the numbers are large and cannot be readily factored.

Note II.—When there are three or more numbers, find the greatest common divisor of two of them, and then the greatest divisor of this common divisor and of the third number, and so on.

Note III.—If the last remainder is 1, the numbers have no common divisor

Written Exercises.

Find the greatest common divisor by the *Division Method:*

1. 869 and 1 580.
2. 810 and 1 215.
3. 615 and 820.
4. 468 and 585.
5. 1 538 and 2 307.
6. 7 610 and 9 132.
7. 7 976 and 8 973.
8. 2 030 and 2 900.
9. 1 390, 1 529 and 1 807.
10. 5 870, 18 197 and 24 067.

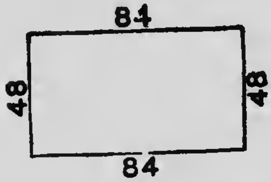
Written Problems.

NOTE.—Try to solve the five first mentally.

1. What is the length of the longest pole with which you can exactly measure 18, 24 and 36 feet?

2. A, B, C and D have $21, $28, $35 and $42 respectively; they want to buy calves at such a price as will exactly use each man's money. What is the highest price per head they can give, and how many calves will each buy at that price?

3. Three pieces of carpet of 24, 32 and 40 yards were cut into equal strips the longest possible. How long was each strip?

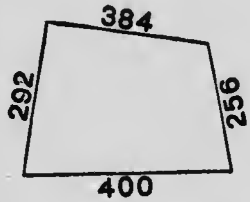


4. Find the length of the longest string that will exactly measure the length and the width of a hall 84 feet by 48 feet.

5. Three boards of 15, 20 and 25 feet, were cut into equal pieces the longest possible. How long was each piece?

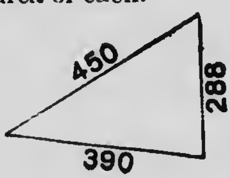
6. Find the length of the longest string that will exactly measure the length and the width of a field 126 yards by 156.

7. My field measures 292 by 384 by 256 by 400 feet. I want to fence it with equal rails the longest possible. Find the length of each. Find the number of rails, if the fence is 5 rails high.



8. A grocer wishes to put 272 quarts of strawberries and 304 quarts of plums into cans of uniform size without mixing the fruits. What is the capacity of the largest can that may be used, all being filled?

9. A farmer has three tracts of land, containing respectively 870, 1 479 and 1 740 acres. He wishes to divide them into equal lots the largest possible. Required the number of lots and the area of each.



10. A triangular field measures 288 by 390 by 450 feet; it is to be fenced with equal rails the longest possible. Find the length of each. Find the number of rails, if the fence is 5 rails high.

COMMON MULTIPLES.

133. The **multiple** of a number is one or more times that number.

Thus, 20 is a multiple of 5, since it is *four* times 5.

134. The *common multiple* of two or more numbers is the number which is a multiple of each of them.

Thus, 24 is a common multiple of 4 and 6, since it is several times each of them.

135. The *least common multiple* (L. C. M.) of two or more numbers is the smallest number which is a multiple of each of them.

Thus, 12 is the L. C. M. of 4 and 6, since it is the smallest number that is several times each of them. In other words, 12 is the smallest number exactly divisible by 4 and 6.

Oral Exercises.

1. Name a multiple of 3; of 4; of 6; of 7; of 8.
2. Name two multiples of 8; of 10; of 15; of 21; of 40.
3. Name three multiples of 9; of 7; of 11; of 12; of 13.
4. Name a common multiple of 4 and 6; of 5 and 6; of 6 and 8.
5. Find a common multiple of 3 and 4; of 6 and 9; of 8 and 12; of 9 and 12.
6. Is 70 the least common multiple of 5 and 7?
7. Is 90 the least common multiple of 9 and 10?
8. Find the least common multiple of 213 and 4; of 3, 4 and 5.
9. What is the least common multiple of 4, 5 and 8? of 5, 6 and 10?
10. What is the least common multiple of 10, 6 and 12? of 9, 6 and 3?

136. Principle.—The least common multiple of two or more numbers must contain all the prime factors of each number, and no other factors.

Thus, the least common multiple of 6 and 15 is 30.

OPERATION.

$6 = 2 \times 3$

$15 = 3 \times 5$

$L.C.M. = 2 \times 3 \times 5 = 30.$

EXPLANATION.

The prime factors of 6 are 2 and 3; hence the multiple must contain 2 and 3. The prime factors of 15 are 3 and 5. As the *three* is already taken as a factor of the multiple, there only remains to add 5 to 2 and 3. The least common multiple, therefore, is $2 \times 3 \times 5$, or 30.

137. Finding the least common multiple.

EXAMPLE.—Find the least common multiple of 120, 150 and 180.

OPERATION.

2)	120	150	180
2)	60	75	90
3)	30	75	45
5)	10	25	15
	2	5	3

EXPLANATION.

Each of these numbers contains the prime factor 2; hence, the L. C. M. must contain the factor 2 at least once. But 2 is also a prime factor of 60 and 90; the L. C. M. must therefore contain the factor 2 at least one more time. Since 3 is a prime factor of 30, 75 and 45, the L. C. M. must also contain the factor 3 at least once. Since 5 is a prime factor of 10, 25 and 15, the L. C. M. must contain the factor 5 at least once.

In addition to the factors already obtained $(2 \times 2 \times 3 \times 5)$, the L. C. M. must contain the remaining quotients 2, 5 and 3, which are also prime factors of the three given numbers. Hence, the L. C. M. must be $2 \times 2 \times 3 \times 5 \times 2 \times 5 \times 3$, or 1 800.

138. Rule.—*1° Write the numbers in a horizontal line; then divide by any prime number that will exactly divide two or more of them, and set down, in a line below, the quotients and the undivided numbers;*

2° Divide as before, until all the numbers in the lower line are all prime to each other;

3° Then multiply together the divisors and final quotients, and their product will be the least common multiple.

NOTE.—If no two of the given numbers have a common prime factor, the least common multiple will be the product of all the given numbers.

Find the L. C. M. of:

1. 16, 30, 33, 55.
2. 28, 36, 50, 75.
3. 60, 120, 240, 360.
4. 21, 24, 72, 30.
5. 25, 75, 125, 225.
6. 180, 216, 120, 720.
7. 100, 110, 440, 500.
8. 225, 400, 725, 925.
9. 302, 401, 121, 363.
10. 450, 700, 900, 960.

Written Problems.

NOTE.—Try to solve the five first mentally.

1. I have three poles respectively 3, 6 and 8 feet long. What is the shortest distance that can be exactly measured with each of these poles?

2. If three steamers start together from Montreal, the first will be there again in 15 days, the second in 20 days, and the third in 30 days. In what time will they all be together again at the starting place?

3. John can walk round a pond in 8 minutes; Paul, in 10 minutes; Leo, in 12 minutes. If they start together, how many minutes will elapse before they are all three together at the starting point?

4. What is the least number that can be divided by 2, 3, 4, 5 and 6, and leave a remainder of 1 each time?

5. What is the least sum which I could pay either with one-dollar, two-dollar, five-dollar, or ten-dollar bank notes?

6. A lady wishes to purchase the shortest possible piece of silk that can be cut without waste into parts either 12, 15, 20, or 30 yards long. Required the length of the piece of silk.

7. Four hoops are respectively 36, 40, 42 and 48 inches in circumference. What is the shortest distance in which each hoop can make an exact number of revolutions?

8. The G. C. D. of 64 and 96 is contained how many times in their L. C. M.?

9. The G. C. D. of 32, 72 and 192 is contained how many times in their L. C. M.?

10. The G. C. D. of 36, 324 and 162 is contained how many times in their L. C. M.?

CANCELLATION.

139. $144 \div 36 = 4$. If we separate 144 into the factors 9 and 16, and 36 into the factors 9 and 4, we may write $144 \div 36$ as follows: $(9 \times 16) \div (9 \times 4) = 4$. If we strike out the common factor 9 in both cases, the proposition becomes $16 \div 4 = 4$. (See Principle III, N° 105). The factors of the dividend are usually written above a line, and the factors of the divisor below.

$$\frac{\cancel{9} \times 16}{\cancel{9} \times 4} = 4.$$

140. Principle.—Striking out a common factor from both dividend and divisor does not change the quotient.

141. This method of shortening division is called *cancellation.*

EXAMPLE.—Divide $(12 \times 31 \times 40 \times 36)$ by $(14 \times 48 \times 9 \times 5)$.

OPERATION.

$$\frac{\cancel{12} \times 31 \times \overset{\overset{4}{\cancel{8}}}{\cancel{40}} \times \cancel{36}}{\underset{7}{\cancel{14}} \times \underset{\cancel{4}}{\cancel{48}} \times \cancel{9} \times \cancel{5}} = \frac{124}{7} = 17\tfrac{5}{7}.$$

EXPLANATION.

Cancel 12 in dividend and 48 in divisor, dividing each by 12, and write the quotient 4 below 48; when the quotient is 1, it is not written; 4 and 9 in the divisor will cancel 36 in the dividend; 5 in the divisor will cancel 40 in the dividend, leaving 8; 2 will cancel 8 in the dividend and 14 in the divisor, leaving 4 and 7. Two factors remain in the dividend, 31×4, making a product of 124, which being divided by 7, the remaining factor in the divisor, gives a quotient of $17\frac{5}{7}$.

142. Rule.—*Strike out all factors common to both dividend and divisor; then divide the product of the uncancelled* *factors of the dividend by the product of the uncancelled factors of the divisor.*

NOTE.—The expression $\frac{3+4+5}{6+8+10}$ does not admit of cancellation, its terms not being factors. But $\frac{3\times4\times5}{6\times8\times10}$ may be cancelled.

Written Exercises.

Divide, using cancellation:

1. $(4\times5\times6)\div(3\times4\times5)$.
2. $(4\times6\times14)\div(3\times5\times7)$.
3. $(7\times9\times10)\div(3\times5\times7)$.
4. $(8\times10\times12)\div(4\times5\times16)$.
5. $(27\times12\times14)\div(9\times4\times7)$.
6. $(72\times45\times140)\div(18\times24\times35)$.
7. $(27\times56\times38)\div(19\times35\times40)$.
8. $(100\times33\times250)\div(125\times150)$.
9. $(225\times65\times320)\div(26\times150\times16)$.
10. $(16\times40\times60\times28)\div(80\times24\times7)$.

Written Problems.

1. How many pounds of butter, worth 36 cents a pound, must be given in exchange for 48 pounds of coffee, worth 42 cents a pound?

2. How many pounds of sugar, at 7 cents a pound, can be had for 15 dozen of eggs, at 35 cents a dozen?

3. How many bushels of potatoes, worth 75 cents a bushel, must be given for 3 chests of tea, containing 30 pounds each, at 40 cents a pound?

4. By selling 70 pieces of bunting, each piece containing 52 yards, at 18 cents a yard, a lady receives enough money to buy how many pieces of calico averaging 42 yards per piece, and worth 5 cents a yard?

5. Find the cost of 15 680 pounds of coal, if 2 240 pounds cost $6.

6. How many acres of land, at $35 an acre, will pay for 84 tons of hay at $20 a ton?

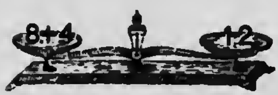
7. How many boys working during 15 days, and 8 hours a day, at 10 cents per hour, will earn as much money as 4 men in 18 days of 10 hours each and at 25 cents per hour?

8. The members of a Cooperative Farming Society buy 20 barrels of sugar, each containing 288 pounds, at 6 cents a pound; how many loads of potatoes, each containing 24 barrels, at $1.60 a barrel, will they give in exchange?

9. How many times is the area of a field 24×36 feet contained in the area of another field 108×144 feet?

10. A cellar $72 \times 96 \times 6$ feet is how many times as large as a cellar $24 \times 36 \times 4$ feet?

THE EQUATION.



The solution of many problems may often be made easier by giving it the form of an *equation*.

$8+4=12$ is an equation, that is, the expression of equality between two quantities.

$$(1)\quad 8+4=12.$$
$$(2)\quad 8 \quad =12-4.$$

[illegible]e these two equations; you will see that in (2) the 4 [illegible] taken from both sides to keep up the equality. Thus,

$$\begin{array}{r} 8+4= \quad 12. \\ -4=-\ 4. \\ \hline 8 \quad =12-4. \end{array}$$

Study $8=6+2$, and $8-2=6$, and you will understand that *a number may be moved from one side of an equation to the other side by changing its sign.*

What is done to one side of an equation must be done to the other side to preserve the equality:

$$(a)\quad \begin{array}{r} 10= \quad 10 \\ +\ 5=+\ 5 \\ \hline 15= \quad 15. \end{array} \qquad (b)\quad \begin{array}{r} 10= \quad 10 \\ -\ 5=-\ 5 \\ \hline 5= \quad 5. \end{array}$$

Change the following equations so that the first number will remain alone at the left of the equality sign:

1. $20-10=10$.	4. $100+75=175$.	7. $90-10+5=85$.
2. $40-15=25$.	5. $75-20=55$.	8. $100+10-20=90$.
3. $80+15=95$.	6. $85-5-10=70$.	9. $60-2+8=66$.

Examples.

1. Multiply my money by 5 and I shall have \$35. How much money have I?

ANALYSIS.—

Let 1 time = my money.
5 times my money = \$35.
1 time = \$7, my money.

2. There are 444 sheep in two fields. If one field contains 3 times as many as the other, how many sheep are there in each field?

ANALYSIS.—

Let 1 time = the number of sheep in one field.
Then *3 times = the number of sheep in the other field.*
And 4 times = the number of sheep in both fields.
Therefore 4 times = 444 sheep.
Hence 1 time = 111 sheep, or the number in one field.
And 3 times = 333 sheep, or the number in the other field.

3. Mr. Boyd and Mr. O'Hagan together have 240 acres of land. If Mr. Boyd has 40 acres more than Mr. O'Hagan, how many has each?

ANALYSIS.—

Let 1 time = Mr. O'Hagan's acres.
Then 1 *time + 40 = Mr. Boyd's acres.*
And 2 times + 40 = the acres of both.
Therefore 2 times + 40 = 240 acres.
Subtracting 40 from both sides of the equation, we have:
2 times = 200.
1 time = 100, Mr. O'Hagan's acres.
And 1 time + 40 = 140, Mr. Boyd's acres.

NOTE.—Instead of writing 1 *time*, 2 *times*, 3 *times*, you may simply write 1 *t*, 2 *t*, 3 *t*, or any other letter. For example, *n* may stand for *number*, *m* may stand for *money*, etc. It is even customary to represent 1 *time an unknown quantity* by the letter *x*, 2 *times* by 2 *x*, etc.

4. Multiply my money by 4, then add \$6 and I shall have \$50. How much money have I?

ANALYSIS.—

$$\begin{aligned} \text{Let } 1\,t &= \text{my money.} \\ 4\,t + \$6 &= \$50. \\ 4\,t &= \$50 - \$6. \\ 4\,t &= \$44. \\ 1\,t &= \$11, \text{ my money.} \end{aligned}$$

5. The sum of two numbers is 8, and their difference is 2. Required the two numbers.

ANALYSIS.—

$$\begin{aligned} \text{Let } 1\,t &= \text{the smaller number.} \\ 1\,t+2 &= \textit{the greater number.} \\ 2\,t+2 &= 8, \text{ the sum of both numbers.} \\ 2\,t &= 8-2. \\ 2\,t &= 6. \\ 1\,t &= 3, \text{ the smaller number.} \\ 1\,t+2 &= 5, \text{ the greater number.} \end{aligned}$$

Written Problems.

1. Twice the cost of my bicycle and \$5 more equals \$75. Find its cost.

2. J[illegible]'s age plus twice his age, plus 6 years, equals 42 years. How old is he?

3. Robert and Roy together have \$3 000, and Robert has 2 times as much money as Roy. How much money has each?

4. The sum of two numbers is 75, and the larger is 4 times the smaller. What are the numbers?

5. A says: "My age is twice B's." B says: "My age is twice C's;" and C says: "The sum of our ages is 140 years." Find their ages.

6. Divide \$840 among 3 men so that the first shall have twice as much as the third, and the second shall have as much as the first and the third together.

7. My money plus \$20 and minus \$14 equals \$19. How much have I?

8. Subtract 64 from 10 times a certain number and you will have twice that number. What is the number?

9. There are 3 400 antelopes and deer in Yellowstone Park. The number of antelopes is 200 less than twice the number of deer. Required the number of deer.

10. A and B had equal sums of money. But A gave B $4, and then B had twice as much as A had left. How much had each at first?

Questions on Theory.

NOTE.—Give an example in each case.

1. What are the factors of a number? (110).

2. Is there any difference between the factors and the exact divisors of a number? (111).

3. When is a number said to be divisible? (112).

4. When is a number divisible by 2? (113).

5. What is an even number? an odd number? (114).

6. When is a number divisible by 4? by 8? (115, 116).

7. When is a number divisible by 5? by 10? (117, 118).

8. When is a number divisible by 3? by 9? by 6? (119, 120).

9. What is a prime number? (121).

10. What is a prime factor? (122).

11. Give the rule for finding the prime factors of a number. (124).

12. Resolving numbers into their prime factors enables us to find what? (125).

13. What is the common factor of two or more numbers? the common divisor? (126).

14. When are numbers said to be prime to each other? (127).

15. What is the greatest common divisor of two or more numbers? (128).

16. Must a number contain all the prime factors of its exact divisor? (129, I).

17. The greatest common divisor of two or more numbers is composed of what factors? (129, II).

18. Is the greatest common divisor of two numbers contained exactly in their sum? in their difference? in their product? in their remainder after division? (129, III).

19. How do you find the greatest common divisor by the *Factoring Method*? by the *Division Method*? (131, 132).

20. What is the multiple of a number? (133).

21. What is the commun multiple of two or more numbers? (134).

22. What is the least common multiple of two or more numbers? (135).

23. What prime factors must the least common multiple contain? (136).

24. How do you find the least common multiple? (138).

25. Is the quotient changed if you reject a common factor from both dividend and divisor? (140).

26. What is cancellation? (140, 141).

27. What is the rule for cancellation? (142).

REVIEW OF INTEGERS.

Questions on Theory.

1. What must you do in adding when the sum of a column exceeds 10? Why?

2. What must you do in subtracting when the units' figure of the subtrahend is greater in value than the units' figure of the minuend?

3. How do you multiply when the multiplier contains an intervening cipher?

4. How would you manage to find the average age of your classmates?

5. If you knew the cost of a certain number of horses, how would you find the average cost per horse?

6. The product of two numbers being given, and one of the two numbers, how would you find the other number?

7. How do you find the dividend when the divisor, quotient and remainder are given?

8. Having the dividend, remainder and quotient, how could you find the divisor?

9. I know a workman's wages per hour, and the number of hours' work; how shall I find his salary?

10. You know how many hours a car has been running and its average rate per hour in miles; how can you find the distance?

RAPID DRILL TABLE.

Each of the following exercises comprises 20 operations.

Ex.—For N° 1, $(A \times B = ?)$ multiply 4×7; 5×4; 6×3; 7×8, and so on.

A slip of paper or a ruler will help you avoid misreading the numbers.

	A	B	C	D	E	F
1.	4	7	42	294	882	4 410
2.	5	4	36	144	576	3 456
3.	6	3	24	72	360	3 240
4.	7	8	40	320	960	7 680
5.	5	7	56	392	1 568	15 680
6.	2	8	72	576	1 152	4 608
7.	3	3	27	81	729	5 103
8.	8	7	28	196	1 960	9 800
9.	4	8	64	512	1 536	3 072
10.	3	5	40	200	1 800	5 400
11.	6	6	30	180	7 200	28 800
12.	7	9	27	243	1 701	8 505
13.	9	8	48	384	768	2 304
14.	3	6	36	216	864	3 456
15.	8	8	56	448	2 240	4 480
16.	3	7	63	441	3 528	7 056
17.	9	7	49	343	3 773	15 092
18.	4	6	42	252	1 008	5 040
19.	8	9	54	486	2 430	7 [illegible]90
20.	4	5	40	200	1 000	9 000

Oral Exercises.

1. $A \times B = ?$
2. $C \div B = ?$
3. $C - B = ?$
4. $C \times 10 = ?$
5. $100 - C = ?$
6. $D \times 2 = ?$
7. $A + B + C = ?$
8. $D \times 100 = ?$
9. $C \times 5 = ?$

Written Exercises.

10. $C + D + E + F = ?$
11. $E - D = ?$
12. $F - E = ?$
13. $D \times B = ?$
14. $D \times C = ?$
15. $D \times 1\,400 = ?$
16. $D \times 203 = ?$
17. $E \div B = ?$
18. $E \div C = ?$
19. $F \div C = ?$
20. $F \div D = ?$
21. $A \times B \times C = ?$

Written Problems.

WORKMEN'S WAGES AND SAVINGS.

1. A father earns $2.50 a day and his son $1.75. How much will they both have earned in a year if they work 302 days?

2. A mason's wages per day are \$3.75; he works 23 days a month and 8 months a year. If the daily household expenses amount to \$1.75, what can he save in a year?

3. An artisan is paid 275 cents a day. He works 300 days a year and saves 14 975 cents. Find his daily expenses in cents; in dollars.

4. A laborer earns \$2.50 a day, and works 24 days a month. What is left of his annual income if indulging in spirituous liquors costs him on an average 35 cents a day?

5. A mechanic who is paid \$2.60 a day, spends \$688.40 and saves \$122.80 a year. How many days a month does he work?

Household finances.

1. "Can you figure up the family's cost of living?" said Mr. Hancock on New Year's eve to his oldest children Mary and Charles. "I paid for rent, \$240; for heat, \$52.75; for light, \$17.60; for food, \$406.80; for clothing, \$160.50; for tram and car fare, \$64; for church and alms, \$35; for toys and sweets, \$30.60; for doctor fees, \$16; for books, newspapers, tobacco and sundries, \$44.75. How much money remains of my income which amounts to \$1 230?

2. We are six; find the cost of living *per capita* (*by the head*).

3. What is the monthly cost of food *per capita*?

4. Find the average cost of clothing *per capita*.

5. Now suppose we should next year reduce our expenses as follows: clothing, \$4 a month; tram and car fare, \$2 a month; toys and sweets, \$1 a month; books, newspapers, tobacco and sundries, \$2 a month. What would our annual savings amount to?

6. If, like our unfortunate neighbor X, I had this year spent \$3 a week on spirituous liquors, what would now remain of my salary?

7. Ten years ago, continued Mr. Hancock, I used to earn \$840 a year; rent cost, \$14 a month; light and heat, \$44 a year; food, \$240; all other expenses, \$298. What were then the annual savings?

8. If the family was then composed of four members, what was the cost of living *per capita*? Compare it with the actual cost of living. Find the difference.

9. If during the last 10 years we had saved $45 more than we did save, according to N° 7, what sum would our total savings amount to?

10. If my actual salary were only $840, what would be this year's shortage?"

Helping mother.

1. Mrs. Strong, a charwoman, earns $1.25 a day. If she spends 50 cents a week for car fare, and works 6 days a week, what is her net income weekly? yearly, if she works 51 weeks?

2. Her late husband left a life-insurance policy, the amount of which, $1 000, was invested so as to yield $50 a year. What sum must she add to the $50 to pay off her year's rent, if one month's rent is $9?

3. Guy earns money for his mother before and after school. Every week he gets $1 for delivering newspapers, and 25 cents for running errands. On Saturday evenings he makes 75 cents in a grocery. How much does he bring in every month?

4. Kate, Guy's sister, does her bit too. A neighbor pays her 5 cents an hour for wheeling the baby. If she works 30 hours a month, how much does she earn in a year?

5. Andrew, their brother, grows vegetables in a vacant lot. Guy and Kate each lend him $1.50 to pay for seeds and tools. If he sells $4.35 worth of cabbages, $3.75 worth of tomatoes, $6.25 worth of potatoes, and $3.25 worth of onions, and pays back the money borrowed, how much is he also able to give his mother?

Wastefulness.

1. If a man earning $1.75 a day idles away 72 working days in a year, how much money does he lose?

2. The cost price of a farm wagon is $56; it will last only 7 years if left out in the yard exposed to the weather; but it will last twice as long if carefully sheltered. Leaving it out represents what annual loss?

3. Two neighbors, the same year, each bought for $300 worth of improved farming implements. One of them took such care of his, that after 15 years they were still in good working order,

while the other carelessly left his exposed to the weather and had to spend \$315 for repairing and renewing them. What was the annual cost of negligence?

4. A ton of straw contains 12 lb. of nitrogen, 10 lb. of potash, and 2 lb. of phosphoric acid. Nitrogen is worth 20 cents a pound; potash and phosphoric acid each 5 cents a pound. What sum does that farmer waste who instead of using his straw as a fertilizer, burns 20 tons of it?

5. A man addicted to drinking spent 25 cents a day on spirituous liquors and was idle for 50 working days worth \$2.50 each. What total sum of money did he lose in a year?

FINDING THE SELLING PRICE, THE PROFIT, THE LOSS.

Cost price + Profit = Selling price.
Cost price — Loss = Selling price.

1. A merchant buys 372 yd. of cloth at 80 cents a yard. To realize a profit of \$75, he must sell for what total price?

2. A dealer bought 14 pieces of cloth, each containing 85 yd., at \$22.10 apiece. What must be the total selling price, if he wants to gain 8 cents a yard?

3. I bought 25 yd. of cloth at 64 cents a yard. If I lost 5 cents a yard, what was the total selling price?

4. I bought 10 \$2.25 hats and sold them so as to gain \$7.50 on the whole. How much did one hat sell for?

5. A hatter received 150 caps billed at 32 cents each. In consideration of immediate cash payment, he was allowed a rebate of \$1.20 on the whole. He sells 38 caps at 45 cents, and 56 at 28 cents. At what price apiece must he sell the remaining caps to net a profit of \$13.98?

Selling price — Cost price = Profit.
Cost price — Selling price = Loss.

6. A drover buys 85 sheep at \$7.35 each, and sells them all for \$599.98. Does he gain or lose? How much?

7. A butcher buys 65 rabbits at 28 cents, and sells them at 45 cents a head. Find his net profit, if freight and other charges amount to \$2.36.

8. I bought 126 sheep at \$5.12. Three months after I sold 62 for \$586, and the rest at \$9.50 each. Find my net profit, if they average \$2.10 a head for pasturage.

9. An agent buys for me 125 sheep at \$6.35 each. He sells 18 at \$7.35, and 45 at \$8.25; 5 die, and the others are sold at \$6.90 each. Find my gain, sundry expenses amounting to \$43.25.

10. An agent buys for me 50 cows at the average price of \$71.50, and charges \$15 for his trouble. He then sells the cows at \$92 each, retains \$25 for his services and \$35 for extra charges. Find my net gain.

Gross cost = (price of one) × (n. of cows) + buying charges.
Net sales = (price of one) × (n. of cows) — selling charges.
The difference between the net sales and the gross cost = the gain or the loss.

11. A speculator buys 2 500 bushels of wheat at 89 cents a bushel; he pays \$175 for freight, \$23 for drayage and \$15 for insurance. Find his loss or gain if the wheat sells for 99 cents a bushel.

12. A merchant sold 600 bushels of oats at 48 cents a bushel and 750 bushels of rye at 69 cents a bushel. If the selling charges aggregated \$85, find his net gain, the cost price being \$626.50.

13. A man bought a house for \$5 600, spent \$900 for improvements, and then sold it for \$7 500. If the selling charges were \$50, find the net gain.

14. I bought 60 barrels of apples at \$2.25 each. Sorting them gave 35 barrels of first quality and 21 barrels of second quality; the others were unsalable. I got \$3.50 a barrel for the first choice and \$2.15 for the second. Find my gain or loss if the sorting cost \$6.

15. My annual purchases amount to \$47 500 and my sales to \$62 500; buying charges aggregate \$700 and selling charges \$800. Find my net gain.

Profit apiece.

1. 32 yards of cloth cost \$107.20 and sold for \$163.20. What is the profit per yard?

2. I bought 25 yards of cloth at \$2.50 a yard and sold the whole for \$80. Find the profit per yard.

3. I paid $29.50 for a piece of cloth of 118 yd. Find the gain per yard, if the selling price was 42 cents a yard.

4. A tailor bought 6 yd. of cloth at $2.40 a yard, and trimmings for $1.35. He made 7 coats which he sold at $3 each. Find his profit per coat.

5. A merchant bought 6 pieces of linen, each containing 48 yd., for $144. Find his loss or gain per yard, if each yard sold for 39 cents.

TOTAL COST PRICE.

Cost price = Selling price — Profit.
Cost price = Selling price + Loss.

1. 875 yd. of linen sold for 39 cents each. Find the total cost if a profit of $87.50 was realized.

2. I lost 30 cents per yard by selling 275 yd. of cloth for $500. What was the total cost?

3. A merchant bought 12 pieces of merino, each containing 104 yd. He realized a profit of $13.25 per piece by selling the merino at 95 cents a yard. What was the total cost?

4. I bought a piece of cloth measuring 66 yd. Find its cost, if I gained 35 cents per yard by selling 15 yd. for $18.75.

5. A merchant buys 208 yd. of cloth. He sells 45 yd. at $1.95 a yard, 85 yd. at $2.05, and what remains at $1.75 a yard. If his total gain is $135.75, find the cost of all the cloth.

COST PRICE APIECE.

1. 85 hats sold for $80.75, which was an advance of $10.20 on the total cost. Required the cost per hat.

2. A milliner buys 36 hats and sells them at $9 apiece, thus making a total profit of $90. Find the cost per hat.

3. A milliner bought 65 hats at a factory. She gained $1.25 per hat by selling the whole for $211.25. Find the cost apiece.

4. A hatter buys 3 dozen of hats which he sells for $432, thereby losing 75 cents per hat. What was the cost apiece?

5. A hatter one season sold 432 hats. 324 hats brought in $[illegible]44; the others sold for $5 apiece. If his average gain per hat was $1.25, find the cost price of one hat.

SPEED AND DISTANCE.

(N. of miles an hour) × (n. of hours) = distance.
Distance ÷ (n. of miles an hour) = n. of hours.
Distance ÷ (n. of hours) = n. of miles an hour.

1. Two couriers 684 miles apart, started towards each other traveling at the rate of 15 and 13 mi. an hour respectively. How far were they from each other after 19 hours?

2. A and B started together from the same point and traveled in the same direction, A at the rate of 22 mi. a day, and B at the rate of 18 mi. After 6 days, B retraced his steps and reduced his walking rate to 14 mi. a day. How far apart were they 9 days after starting off? In what time could A walk 286 mi.?

3. Two steamers, bound for Montreal, left Havre at the same time, one going 14 mi. an hour, the other, 20. How far apart were they after 4 days? In how many hours did each cover the distance, 3 080 mi.?

4. Two steamers start towards each other at the same time, one from Yokohama (Japan), the other from Victoria (British Columbia), cities 4 283 mi. distant. How far apart will the steamers be after 5 days, if the first goes 15 mi. an hour, and the second 17? In how many hours will they meet?

5. It is 2 450 mi. from Liverpool to Halifax, 3 600 mi. from Halifax to Vancouver, 6 766 mi. from Vancouver to Sydney (Australia). Allowing 2 days for delays, how long will it take to go from Liverpool to Sydney, via Halifax and Vancouver, at the rate of 16 mi. an hour by boat and 30 mi. an hour by rail?

UNEQUAL SHARING.

1. I paid $37.40 for a suit of clothes and an overcoat. Find the cost of each, if the suit of clothes cost $5.20 more than the overcoat.

2. I paid $73.50 for two pieces of cloth of like quality invoiced at $1.05 a yard. Find the length of each piece, if one was 14 yards longer than the other.

3. Divide $5 200 among 3 persons, giving $900 more to the first than to the second, and $800 more to the second than to the third.

4. $6 490 were divided among 4 partners as follows: the first had $160 more than the second; the second had $240 more than the third; and the third had $350 more than the fourth. How many dollars did each get?

5. I bought two pieces of cloth, each containing 30 yards, and paid $114 for both, thereby giving $18 more for one than for the other. What was the price of one yard in each case?

PROPERTIES OF NUMBERS.

1. If I counted the trees of my orchard 2 by 2, 3 by 3, 4 by 4, 5 by 5, 6 by 6, or 7 by 7, the result would always be an exact number of trees. Find the minimum number of trees that the orchard can contain.

2. Find the prime factors of 1 380 and 276; then find their greatest common divisor and least common multiple.

3. Three steamboats started from Montreal together on the 2nd of June. Find the next date at which they started together again from Montreal, if the first leaves Montreal every four days; the second, every nine days; the third, every fifteen days.

4. Find the greatest common divisor of 930, 620, and their difference.

5. Find the smallest number which when divided by 5, by 15, by 12, or by 30, always gives a remainder of 4.

POPULATION OF CANADA AT DIFFERENT PERIODS.

1. The population of New France was 340 souls in 1640; from 1640 to 1663, Canada received 1 264 immigrants; during the same period, the excess of births over deaths was 891. Find the population in 1663.

2. According to Talon's census, the population of 1667 was 3 918 souls. Find the population of 1685 if it represents an increase of 6 807 souls. Find the increase between 1663 and 1667.

3. There were 3 090 more inhabitants in 1698 than in 1685. Required the population of 1698.

4. The population of 1714 exhibited an increase of 5 149 souls over that of 1698, and the population of 1739 an increase of

23 737 souls over that of 1714. Required the population of 1714 and that of 1739.

5. From 1739 to 1754, the excess of births over deaths was 12 308; from 1754 to 1765, the increase was 14 801. What was the population in 1754? in 1765?

6. In 1784, the districts of Quebec, Montreal and Three Rivers numbered 113 012 Canadians, and Upper Canada 10 000 United Empire Loyalists. The population of Newfoundland was 10 244; that of Nova Scotia 32 000; the Acadians were close upon 11 000. What was the total population of British North America?

7. In 1806, the population of New Brunswick was 35 000; of Prince Edward Island 9 676; of Nova Scotia 65 000; of Upper Canada 70 718; of Lower Canada 250 000; of Newfoundland 26 505. Required the population of 1806. What was the increase over that of 1784?

8. In 1827, Upper Canada numbered 177 174 souls, and Lower Canada 473 475; in 1832, Upper Canada numbered 263 554, and Lower Canada 553 134; in 1848, Upper Canada numbered 725 879, and Lower Canada 786 693; in 1860, Upper Canada numbered 1 396 091, and Lower Canada 1 111 566. Find the successive increases for Upper Canada, then for Lower Canada.

9. The population of the Dominion of Canada was 3 689 257 in 1871; of 4 324 810 in 1881; and of 4 833 239 in 1891. Find the successive increases.

10. In 1901, there was an increase of 538 076 over 1891; and in 1911, an increase of 1 835 328 over 1901. Find the population of 1901, and that of 1911.

POPULATION OF QUEBEC.

Groups.	1901	1911
French	1 322 155	1 605 339
English	114 710	153 295
Irish	114 842	103 147
Scotch	60 068	58 555
Indians	9 166	9 993
Hebrews	7 607	30 648
Other races (14)	20 390	42 255

1. There were how many more French people in 1911 than in 1901?

2. Add the English, Irish and Scotch groups for 1911 and 1901, and then find the difference between the two sums; then find the increase or decrease of each of these groups separately.

3. How many more Hebrews were there in 1911 than in 1901? Find the increase of the Indian population.

4. Find the total population of Quebec for 1901 and 1911, and then the increase.

5. From the French population of 1901 subtract the sum of all the other groups. Do the same with the 1911 column.

ANTIALCOHOLISM.

Province of Quebec, 1915.—*Licenses for the sale of intoxicating liquors.*

CITIES.	POPULATION.	BARS.	GROCERIES (*retail and wholesale*).	BOTTLERS.
Hull................	22 000	14	5	4
Maisonneuve.......	39 770	19	28	—
Montreal..........	650 000	400	565	18
Quebec..........	90 000	50	117	7
St. Hyacinthe......	11 670	10	10	3
Sorel............	8 715	9	8	1
Sherbrooke.......	19 305	15	8	3
Three Rivers.....	19 000	12	15	6
Valleyfield........	9 487	8	3	2

1. Find the total population of these nine cities.

2. Required the number of bars retailing spirits.

3. Find the number of groceries carrying on the liquor business.

4. How many bottlers were there?

5. How many inhabitants were there for one bar in each of the above cities?

NOTE.—*Overlook remainders in Problems* 5, 6 *and* 10.

6. If you divided the total population of these nine cities by their total number of bars, what average would you get?

NOTE.—Of course the answer would contain the abstemious as well as the non-abstemious. Divide the answer by 5 and you will know the average number of *clients* to a bar.

7. In the 1 221 other municipalities of the Province, there were 176 bars less than in the nine given municipalities. How many bars were there outside of the nine given municipalities? How many in all the Province?

8. In 1915, out of 1 230 municipalities, 853 had prohibited the sale of alcoholic liquors. How many were still carrying on that pernicious trade?

9. In 1910, the Province had 1 282 bars. How many were barred out in 5 years?

10. In Montreal, in 1915, there were 400 bars and 565 licensed groceries. Find the sum; then the average number of persons to one liquor counter.

Propagation of Faith.

Statement of the alms contributed the world over to the Propagation of Faith in 1913.

Countries.	Francs.	Countries.	Francs.
France	3 333 860.	United States	2 196 053.
Germany	626 883.	**Canada**	38 763.
Belgium	363 383.	Mexico	23 496.
Italy	296 818.	Chili	84 719.
Great Britain	234 709.	Brazil	45 885.
Spain	165 710.	Uruguay	37 585.
Switzerland	98 261.	Peru	3 000.
Austria-Hungary	77 405.	Bolivia	2 463.
Netherlands	61 672.	Rest of South America	3 063.
Luxemburg	26 435.	Central America	5 978.
Portugal	20 978.	Oceanica	23 787.
Levant	56 962.	Africa	23 029.
Rest of Europe	2 404.	Asia	9 082.

1. How much more than Germany did France give?

2. How much more than Germany and Austria-Hungary did France give?

3. At 20 cents to the franc, what is the value in our money of France's contribution?

4. France gave how many more francs than all the other countries of Europe together?

5. Switzerland gave how many more francs than Canada?

6. How much less than France did the United States contribute?

7. Find the total contribution of the United States, Canada and Mexico.

8. Find how many dollars Canada gave, counting 5 francs to the dollar.

9. Find the total sum given in 1913 for the Propagation of Faith 1° in francs, 2° in dollars.

10. From 1822 to 1893, France contributed 255 million francs to that excellent work, thereby giving 93 million francs more than all the other countries of the world together. How much money did the latter give 1° in francs, 2° in dollars?

THE WAR OF 1914.

On the first of January 1916, after 17 months of war, the approximative standings of the belligerents were as follows:

ALLIES.	ENLISTED.	ENLISTABLE.	KILLED.	WOUNDED.	CAPTURED.
Great Britain.	1 253 000	4 463 000	115 000	351 000	71 000
France.........	3 000 000	1 590 000	270 000	840 000	180 000
Russia.......	5 000 000	11 050 000	450 000	1 400 000	375 000
Italy........	800 000	3 150 000	72 000	224 000	48 000
Belgium......	300 000	550 000	27 000	84 000	18 000
Serbia..........	300 000	100 000	27 000	84 000	18 000
Montenegro...	50 000	30 000	4 500	14 000	3 000
TEUTONS.					
Germany......	5 330 000	2 500 000	485 375	1 510 040	323 580
Austria-Hungary...	3 546 000	2 290 000	319 140	992 880	265 000
Turkey.........	500 000	2 225 000	45 000	140 000	30 000
Bulgaria.......	400 000	350 000	36 000	112 000	24 000

1. Find the number of men enlisted *a*) among the Allies; *b*) among the Teutons; *c*) in all; *d*) find the difference in favor of the Allies.

2. Find the number of men killed *a*) among the Allies; *b*) among the Teutons; *c*) in all; *d*) the difference between the Allies and the Teutons.

3. How many men were made prisoners *a*) among the Allies; *b*) among the Teutons; *c*) in all; *d*) what difference?

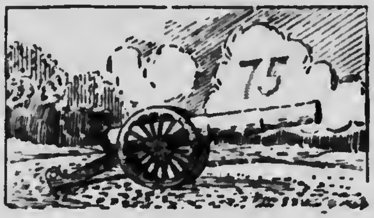
4. Find the number of wounded *a*) among the Allies; *b*) among the Teutons; *c*) in all; *d*) the difference.

5. Find the total losses (men killed, wounded, captured) *a*) among the Allies; *b*) among the Teutons; *c*) on both sides; *d*) find the difference.

6. How many men were enlistable *a*) among the Allies; *b*) among the Teutons; *c*) in all; *d*) what difference?

7. War expenses are so great that each man killed represents an outlay of $15 000; at that rate what did it cost the Allies to kill 885 515 Teutons?

8. If Canada's contribution to the war averaged 30 million dollars a month, how much was it for 45 months? Taking the population as 7 500 000, how much does this mean that every man, woman and child must, on an average, pay or be responsible for paying? What did it cost Canada to back each of her brave 450 000 soldiers?



9. A 75 fieldgun costs $3 600, shots about 400 shells in one day, and at that rate, lasts only 15 days; a shell costs $6 and weighs 16 lb. In 15 days, *a*) how many shells are shot by one gun? *b*) how much money is spent for one gun and the shells used? how much for the 120 guns of an army corps? *c*) how many pounds of metal are hurled against the enemy?

10. At the great battle of Verdun, during 100 days, 16 million tons of metal were daily sent whizzing in the air by both sides together. At $4 per 100 lb., what is the value of Verdun's new *iron mine*? (1 *ton* = 2 000 *lb.*)

MISCELLANEOUS PROBLEMS.

1. A young girl knits 15 pair of stockings in 50 evenings using 7 lb. of yarn purchased at 50 cents a pound. If she sells her stockings at 60 cents a pair, how much has she earned per evening?

2. Five dozen of shirts were made out of 180 yd. of linen bought at 37 cents a yard. A needlegirl, paid 49 cents a day, did the work in 48 days. Buttons and thread cost $1.08. Find the cost of one shirt.

3. I paid $14.40 for 80 yd. of calico, out of which 2 dozen shirts were made. Find 1° the price of one yard of calico; 2° the gross cost of one shirt if its making cost 25 cents.

4. A merchant bought 80 dozen of plates at $1.25 a dozen. Twenty were broken, and he sold the rest at 15 cents apiece. Find his profit.

5. A lady buys 12 yd. of silk at $1.25 a yard and 9 yd. of lace at 54 cents a yard for a dress. Find its gross cost, allowing $7.30 for the making.

6. I bought cloth at the rate of 5 yd. for $12.50 and sold it at the rate of 8 yd. for $22.40 thereby gaining $12. How many yards did I buy?

7. Three pipes supply a fountain with water; the first pours out 650 gallons an hour; the second, 80 gallons more than the first; the third, 120 gallons more than the second. How much water runs into the fountain in 48 hours?

8. A young man smoked 10 cigarettes a day, that is, 15 cents worth, during 12 years. How many cigarettes did he smoke? How much money did he waste?

9. A drunken workman, struck by an auto, was laid up for 3 weeks in a hospital. He had to pay $2.25 a day for his room, $0.75 a day for the doctor, and $8.25 for drugs. Find his total loss, including 18 days' salary, at $4 a day.

10. Two trains leave Montreal at the same time and take opposite directions, the first running 25 mi. an hour, the second, 43. How far apart will they be after 15 hours?

11. Two couriers separated, one north bound and the other south bound, and traveled 9 and 13 mi. an hour respectively. Find the distance between them after 8 hours, and the difference between their respective mileages.

12. 471 mi. of model road cost $2 325 798; find the average cost per mile.

13. 287 mi. of railroad cost $5 236 602; find the average cost per mile.

14. In 1830, the population of Lower Canada was 432 105. Divide that number by 15, add 20 100 to the quotient, and the sum will give the number of Protestants there were at that time in Lower Canada. Required the number of Protestants and Catholics.

15. Montreal and its suburbs had a population of 602 437 souls in 1912. Multiply that number by 4, then subtract 407 036 and you will have the population of the Province of Quebec at that time. What was the population of the Province of Quebec in 1912?

16. Abercromby, at Carillon, had 15 401 soldiers. Subtract 1 377 from that number, then divide by 4 and you will have Montcalm's fighting forces. How many soldiers did Montcalm have?

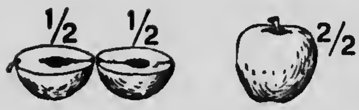
17. My barn cost $2 430; my house, 3 times as much, and my farm twice as much as my barn and house together. Find the cost of my farm.

18. At Queen Elizabeth's death, England had 43 battleships. At King George the Second's death, England had 10 times as many, less 18. Find the latter number.

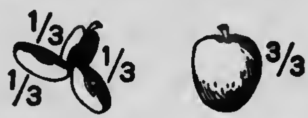
19. The course of the St. Lawrence River is 2 200 mi. To that length add 50, multiply the sum by 2, then subtract 200, and you will have the entire length of the Mississippi River. How long is the Mississippi?

20. In 1911, 583 605 000 cigarettes were smoked in Canada. Find the consumption per head, if the population of Canada was then about 7 205 000.

COMMON FRACTIONS



I cut an apple into *two* equal parts. These equal parts are fractions. Each fraction is named *one half* and is written $\frac{1}{2}$. Two halves make one whole.



I cut an apple into *three* equal parts. These equal parts are fractions. Each fraction is named *one third* and is written $\frac{1}{3}$. Three thirds make one whole. If I give away two of these equal parts, I give away two thirds, $\frac{2}{3}$. One third is smaller than one half.

143. One or more of the equal parts of a unit is called **a fraction.**

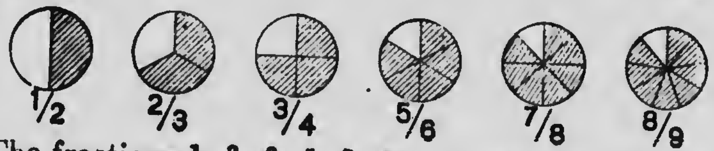
If I divide a unit into *four* equal parts, each part is named *one quarter*, or *one fourth*, and is written $\frac{1}{4}$. Three of these parts are named *three fourths* and are written $\frac{3}{4}$.

144. In the fraction $\frac{3}{4}$, the 4 denominates or names the parts as *fourths*. It is called the *denominator*. The denominator shows into how many parts a unit has been divided. The 3 numerates or counts the parts taken. It is called the *numerator*. The numerator shows how many parts the fraction contains.

The *terms* of a fraction are its numerator and denominator. A fraction which has both terms expressed is called a **common fraction.**

145. To express a fraction, first name the numerator, then the denominator, adding *th* to it.

The denominators 2, 3 read *half* and *third*, 4 either *quarter* or *fourth.*

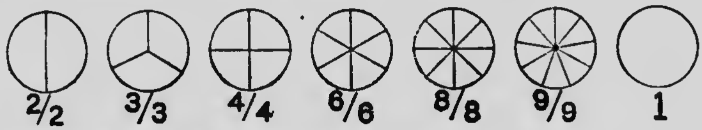


The fractions $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{8}{9}$ read *one half, two thirds, three fourths, five sixths, seven eighths, eight ninths.*

146. When the numerator is less than the denominator, the value of the fraction is *less* than 1. Such a fraction is called a *proper fraction*.

Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{8}{9}$ are all *smaller than* 1.

147. When the numerator is equal to the denominator, the value of the fraction is *equal to* 1.



Thus, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{6}{6}$, $\frac{8}{8}$, $\frac{9}{9}$ are all *equal to* 1.

148. When the numerator is greater than the denominator, the value of the fraction is *greater than* 1.

Thus, $\frac{8}{3}$ is *greater than* 1.

149. A fraction whose numerator equals or exceeds its denominator is called an *improper fraction*.

150. A *mixed number* consists of an integer and a fraction.

Thus, $2\frac{2}{3}$ is a mixed number; we see it is equal to $\frac{8}{3}$.

Oral Exercises.

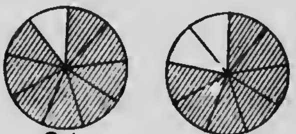
1. What is one half? one third? one quarter? one fifth? one seventh of an apple?

2. What is one eighth? one ninth? one tenth? one twentieth? one hundredth of a bushel?

3. How would you take 2 thirds of an apple? 2 fifths? 3 elevenths? 4 thirteenths? 9 twelfths?

4. Read aloud the following: $\frac{11}{12}$, $\frac{7}{33}$, $\frac{19}{31}$, $\frac{7}{160}$, $\frac{7}{143}$, $\frac{6}{95}$, 21/[illegible], $3\frac{6}{7}$, $9\frac{1}{3}$, $8\frac{3}{17}$, $33\frac{1}{3}$, $12\frac{1}{2}$, $37\frac{1}{2}$, $66\frac{2}{3}$.

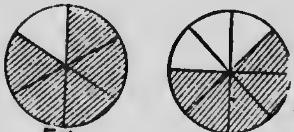
5. Read aloud the following: $\frac{19}{7}$, $\frac{18}{7}$, $\frac{16}{11}$, $\frac{17}{11}$, $\frac{9}{3}$, $\frac{151}{49}$, $\frac{1}{1728}$ cu. ft., $\frac{1}{5280}$ mi., $\frac{1}{63360}$ mi.



8/9 7/9

151. Comparing two fractions.—I. The greater of two fractions having similar denominators, is that whose numerator is greater.

Thus, $\frac{8}{9}$ is greater than $\frac{7}{9}$.



5/6 5/8

152. —II. The greater of two fractions having similar numerators, is that whose denominator is smaller.

Thus, $\frac{5}{6}$ is greater than $\frac{5}{8}$.

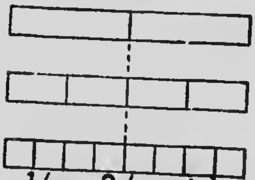
153. The following exercises will give a concrete idea of the different cases to be later taken up.

One inch here represents one unit.

1. How many thirds are there in 2 inches and $\frac{1}{3}$?

We see that one unit contains 3 thirds, that 2 units contain 6 thirds, and that in all there are 6 plus 1, or 7 thirds.

2. How many thirds in $3\frac{1}{3}$? $5\frac{1}{3}$? How many quarters in $1\frac{3}{4}$? How many fifths in $3\frac{3}{5}$?



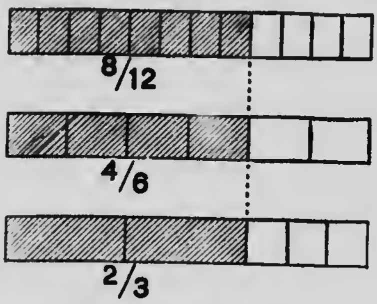
$1/2 = 2/4 = 4/8$

3. In $\frac{1}{2}$, how many fourths? eighths?

We see that the unit is successively cut into two, four, eight parts and that 1 half equals 2 fourths, and 4 eighths.

4. In $\frac{4}{8}$, how many fourths? halves? In $\frac{3}{4}$, how many eighths?

5. In $\frac{6}{8}$, how many fourths? In $\frac{2}{1}$, how many fourths? In $\frac{3}{1}$, how many eighths?



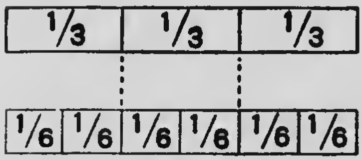
6. In $\frac{8}{12}$, how many sixths? thirds?

We see that $\frac{8}{12}=\frac{4}{6}=\frac{2}{3}$.

7. In $\frac{2}{3}$, how many sixths? In $\frac{1}{6}$, how many twelfths?

8. In $\frac{3}{6}$, how many halves? twelfths? In $\frac{1}{3}$, how many ninths?

9. In $\frac{2}{3}$, how many ninths? Does one third equal $\frac{4}{12}$, $\frac{2}{6}$, $\frac{3}{9}$?

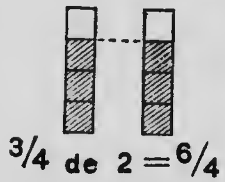


10. Find $\frac{1}{2}$ of $\frac{1}{3}$.

To find one half of $\frac{1}{3}$, we divide $\frac{1}{3}$ into 2 equal parts; if we thus divided the three thirds of one unit, we would have six equal parts, and each part would be 1 sixth.

Hence, $\frac{1}{2}$ of $\frac{1}{3}=\frac{1}{6}$.

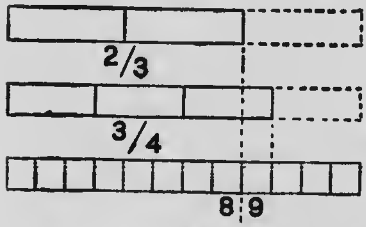
11. What is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{4}$?



12. Find $\frac{3}{4}$ of [illegible].

Let us divid[illegible]h units into fourths, and take 3 four[illegible]rom each unit. In all we have 6 fourths.

13. Find $\frac{2}{3}$ of 2; $\frac{2}{3}$ of 3; $\frac{3}{4}$ of 3; $\frac{3}{[illegible]}$ of 4.

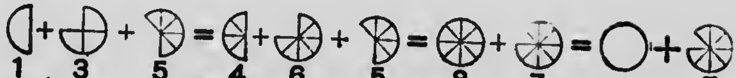


14. Divide $\frac{2}{3}$ by $\frac{3}{4}$.

We see that $\frac{2}{3}$ equal 8 twelfths; that $\frac{3}{4}$ equal 9 twelfths; it therefore only remains to divide 8 things by 9 things of the same value; $8 \div 9 = \frac{8}{9}$.

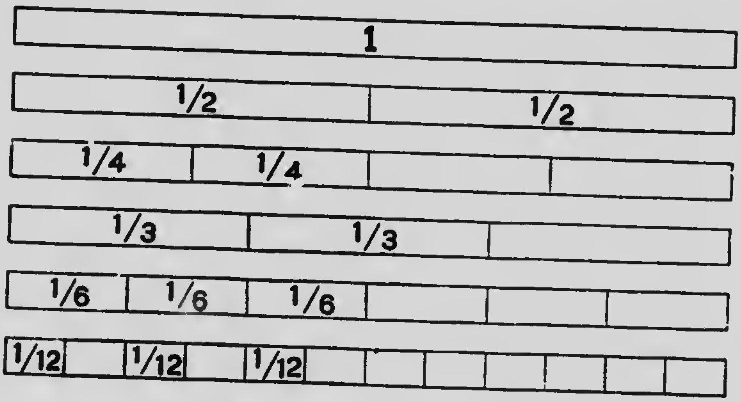
15. [illegible]ide $\frac{3}{4}$ by $\frac{2}{3}$; $\frac{1}{2}$ by $\frac{2}{3}$; $\frac{2}{3}$ by $\frac{5}{6}$; $\frac{1}{2}$ by $\frac{3}{4}$.

16. Find the sum.



$$\frac{1}{2} + \frac{3}{4} + \frac{5}{8} = \frac{4}{8} + \frac{6}{8} + \frac{5}{8} = \frac{8}{8} + \frac{7}{8} = 1 + \frac{7}{8}$$

17. The relations which exist between $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$ are indicated in the following diagram, which may supply many practical drills.



154. Principles relating to fractions.

I. Multiplying the numerator of a fraction by a number, *multiplies* the fraction by that number.

Thus, $\frac{3}{15} \times 4 = \frac{3\times 4}{15} = \frac{12}{15}$.

II. Dividing the numerator of a fraction by a number, *divides* the fraction by that number.

Thus, $\frac{15}{16} \div 5 = \frac{15 \div 5}{16} = \frac{3}{16}$.

III. Multiplying the denominator of a fraction by a number, *divides* the fraction by that number.

Thus, $\frac{3}{7} \div 4 = \frac{3}{7 \times 4} = \frac{3}{28}$.

IV. Dividing the denominator of a fraction by a number, *multiplies* the fraction by that number.

Thus, $\frac{5}{32} \times 4 = \frac{5}{32 \times 4} = \frac{5}{8}$.

V. Multiplying or dividing both terms of a fraction by the same number, does not change the value of the fraction.

Thus, multiplying both terms of $\frac{3}{5}$ by $4 = \frac{3\times4}{5\times4} = \frac{12}{20}$, which is equivalent to $\frac{3}{5}$; and dividing both terms of $\frac{6}{15}$ by $3 = \frac{6\div3}{15\div3} = \frac{2}{5}$, which is equivalent to $\frac{6}{15}$.

155. A fraction represents an uneffected division, its numerator being the dividend, and its denominator the divisor. The sign of division itself has the form of a fraction, a point over a point, $\div$.

Oral Exercises.

1. (*Principle I*) Multiply $\frac{5}{19}$, $\frac{6}{19}$, $\frac{3}{25}$, $\frac{15}{100}$, $\frac{18}{90}$, $\frac{13}{85}$, $\frac{18}{97}$, $\frac{20}{31}$, $\frac{25}{100}$, $\frac{33}{90}$ by 2; by 3; by 4.

2. (*Principle II*) Divide $\frac{6}{19}$, $\frac{12}{31}$, $\frac{18}{3}$, $\frac{24}{100}$, $\frac{30}{4}$, $\frac{120}{100}$, $\frac{90}{11}$, $\frac{240}{19}$, $\frac{18}{100}$, $\frac{12}{99}$ by 2; by 3; by 6.

3. (*Principle III*) Divide $\frac{5}{19}$, $\frac{11}{31}$, $\frac{17}{3}$, $\frac{23}{100}$, $\frac{29}{4}$, $\frac{119}{100}$, $\frac{89}{11}$, $\frac{239}{19}$, $\frac{17}{100}$, $\frac{11}{99}$ by 2; by 3; by 6.

4. (*Principle IV*) Multiply $\frac{5}{12}$, $\frac{6}{24}$, $\frac{3}{36}$, $\frac{15}{48}$, $\frac{18}{60}$, $\frac{13}{72}$, $\frac{18}{84}$, $\frac{20}{96}$, $\frac{25}{108}$, $\frac{33}{144}$ by 2; by 3; by 6.

5. (*Principle V*) Are $\frac{6}{12}$, $\frac{6}{24}$, $\frac{24}{60}$, $\frac{12}{30}$, $\frac{120}{30}$, $\frac{90}{12}$, $\frac{240}{90}$, $\frac{6}{90}$, $\frac{18}{120}$, $\frac{12}{600}$ changed if you multiply both terms of each by 2? by 3? by 4? If you divide both terms of each by 2? by 3? by 6?

REDUCTION OF FRACTIONS.

156. The process of changing the forms of fractions without changing their values is called **reduction of fractions.** There are five cases of reduction.

157. An integer may be made to assume the form of a fraction by writing the denominator 1 under it; thus, $7 = \frac{7}{1}$.

158. First case.—To reduce whole or mixed numbers to improper fractions.

EXAMPLE I.—How many fifths in 7?

EXPLANATION.—*a*) In one unit there are 5 fifths, and in 7 units there are 7 times 5 fifths, or 35 fifths, or $\frac{35}{5}$.

EXPLANATION.—*b*) $7=\frac{7}{1}$; multiplying both terms by 5 we have $\frac{7\times5}{1\times5}=\frac{35}{5}$ (*Principle V*).

EXAMPLE II.—How many ninths in $8\frac{5}{9}$?

EXPLANATION.—*a*) In one unit there are 9 ninths, and in 8 units there are 8 times 9 ninths, or 72 ninths, which added to 5 ninths give 77 ninths, or $\frac{77}{9}$.

EXPLANATION.—*b*) $8=\frac{8}{1}$; $\frac{8\times9}{1\times9}=\frac{72}{9}$; $\frac{72}{9}+\frac{5}{9}=\frac{77}{9}$.

159. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and write the denominator under the result.*

Oral Exercises.

Reduce:

1. 2, 3, 4, 5, 6 to halves.
2. 3, 5, 8, 9, 11 to thirds.
3. $1\frac{1}{4}$, $2\frac{3}{4}$, 3, 4, 5 to fourths.
4. $1\frac{2}{5}$, $2\frac{3}{5}$, $3\frac{3}{5}$, $4\frac{1}{5}$, $5\frac{4}{5}$ to fifths.
5. $1\frac{1}{6}$, $2\frac{3}{6}$, $3\frac{4}{6}$, $4\frac{5}{6}$, 6 to sixths.
6. $1\frac{2}{7}$, $2\frac{3}{7}$, $4\frac{4}{7}$, $5\frac{5}{7}$, $7\frac{6}{7}$ to sevenths.
7. $1\frac{3}{8}$, $2\frac{4}{8}$, $4\frac{5}{8}$, $5\frac{6}{8}$, $7\frac{7}{8}$ to eighths.
8. $1\frac{4}{9}$, $2\frac{2}{9}$, $4\frac{7}{9}$, $5\frac{5}{9}$, $7\frac{8}{9}$ to ninths.
9. $1\frac{3}{10}$, $2\frac{7}{10}$, $3\frac{5}{10}$, $4\frac{8}{10}$, $5\frac{7}{10}$ to tenths.
10. $1\frac{1}{12}$, $2\frac{3}{12}$, $3\frac{4}{12}$, $4\frac{5}{12}$, $9\frac{3}{12}$ to twelfths.

Written Exercises.

Reduce to improper fractions:

1. $15\frac{2}{3}$.	6. $21\frac{6}{7}$.	11. $87\frac{1}{2}$.	16. $18\frac{9}{10}$.
2. $18\frac{7}{8}$.	7. $24\frac{5}{8}$.	12. $100\frac{1}{16}$.	17. $19\frac{3}{16}$.
3. $28\frac{5}{8}$.	8. $19\frac{7}{8}$.	13. $200\frac{1}{4}$.	18. $17\frac{5}{6}$.
4. $16\frac{2}{3}$.	9. $12\frac{1}{2}$.	14. $13\frac{7}{8}$.	19. $66\frac{2}{3}$.
5. $33\frac{1}{3}$.	10. $37\frac{1}{2}$.	15. $11\frac{10}{11}$.	20. $38\frac{4}{5}$.

160. Second case.—To reduce improper fractions to whole or mixed numbers.

EXAMPLE.—How many units in $\frac{25}{7}$?

EXPLANATION.—*a*) In 7 sevenths there is one unit, and in 25 sevenths there are as many units as 7 is contained times in 25, or 3 units and 4 sevenths remaining, $3\frac{4}{7}$.

EXPLANATION.—*b*) Dividing both terms by 7 does not change the value of the fraction (*Principle V*); so $\frac{25 \div 7}{7 \div 7} = 3\frac{4}{7}$.

161. Rule.—*Divide the numerator by the denominator.*

Oral Exercises.

Reduce to whole or mixed numbers:

1. $\frac{12}{6}$, $\frac{15}{3}$, $\frac{16}{2}$, $\frac{81}{9}$, $\frac{25}{5}$, $\frac{125}{25}$, $\frac{64}{4}$, $\frac{48}{12}$, $\frac{36}{4}$, $\frac{85}{17}$.
2. $\frac{95}{19}$, $\frac{78}{13}$, $\frac{75}{15}$, $\frac{65}{5}$, $\frac{87}{29}$, $\frac{99}{33}$, $\frac{121}{11}$, $\frac{98}{49}$, $\frac{94}{47}$, $\frac{84}{21}$.
3. $\frac{15}{7}$, $\frac{27}{18}$, $\frac{19}{3}$, $\frac{31}{10}$, $\frac{36}{7}$, $\frac{74}{12}$, $\frac{41}{5}$, $\frac{63}{8}$, $\frac{33}{4}$, $\frac{19}{5}$.
4. $\frac{52}{25}$, $\frac{19}{4}$, $\frac{47}{13}$, $\frac{88}{16}$, $\frac{97}{13}$, $\frac{75}{12}$, $\frac{90}{16}$, $\frac{80}{18}$, $\frac{72}{17}$, $\frac{53}{13}$.
5. $\frac{47}{6}$, $\frac{92}{11}$, $\frac{75}{13}$, $\frac{31}{15}$, $\frac{50}{7}$, $\frac{80}{9}$, $\frac{121}{5}$, $\frac{53}{6}$, $\frac{51}{7}$, $\frac{65}{8}$.

Written Exercises.

1. $\frac{125}{24}$.
2. $\frac{263}{15}$.
3. $\frac{398}{16}$.
4. $\frac{633}{32}$.
5. $\frac{191}{48}$.
6. $\frac{177}{128}$.
7. $\frac{624}{48}$.
8. $\frac{423}{48}$.
9. $\frac{347}{32}$.
10. $\frac{426}{32}$.
11. $\frac{163}{10}$.
12. $\frac{248}{100}$.
13. $\frac{387}{100}$.
14. $\frac{489}{100}$.
15. $\frac{243}{100}$.
16. $\frac{631}{200}$.
17. $\frac{842}{300}$.
18. $\frac{976}{400}$.
19. $\frac{842}{500}$.
20. $\frac{625}{125}$.

162. Third case.—To reduce fractions to higher terms.

EXAMPLE.—Reduce $\frac{4}{5}$ to twentieths.

EXPLANATION.—*a*) In one unit there are 20 twentieths; in $\frac{1}{5}$ of one unit there are 4 twentieths, and in $\frac{4}{5}$ of one unit there are 4 times 4 twentieths, or 16 twentieths, $\frac{16}{20}$.

EXPLANATION.—*b*) Multiplying both terms by 4 does not change the value of the fraction (*Principle V*); so $\frac{4 \times 4}{5 \times 4} = \frac{16}{20}$.

163. Rule.—*Multiply both numerator and denominator by the number that will give the required denominator.*

Oral Exercises.

Reduce:

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{2}{3}$ to twelfths.
2. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$ to sixteenths.
3. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{12}$, $\frac{3}{8}$, $\frac{2}{3}$ to twenty-fourths.
4. $\frac{1}{10}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{1}{20}$, $\frac{2}{5}$ to fortieths.
5. $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ to twenty-eighths.

Written Exercises.

Reduce:

1. $\frac{9}{13}$ to 78ths.
2. $\frac{15}{22}$ to 132ds.
3. $\frac{17}{18}$ to 72ds.
4. $\frac{7}{15}$ to 135ths.
5. $\frac{11}{23}$ to 276ths.
6. $\frac{17}{25}$ to 275ths.
7. $\frac{18}{31}$ to 372ds.
8. $\frac{21}{38}$ to 494ths.
9. $\frac{41}{45}$ to 765ths.
10. $\frac{51}{83}$ to 415ths.

164. Fourth case.—To reduce fractions to lower terms.

EXAMPLE.—Reduce $\frac{24}{72}$ to ninths.

EXPLANATION.—*a*) 72 seventy-seconds equal one unit, or 9 ninths; so 8 seventy-seconds equal 1 ninth, and 24 seventy-seconds equal as many ninths as there are times 8 in 24, or 3 ninths, $\frac{3}{9}$.

EXPLANATION.—*b*) Dividing both terms by 8 does not change the value of the fraction (*Principle V*); so $\frac{24 \div 8}{72 \div 8} = \frac{3}{9}$.

165. Rule.—*Divide both numerator and denominator by the number that will give the required denominator.*

Oral Exercises.

Reduce:

1. $\frac{10}{25}$, $\frac{18}{30}$, $\frac{21}{35}$, $\frac{24}{40}$, $\frac{20}{50}$ to fifths.
2. $\frac{24}{36}$, $\frac{12}{24}$, $\frac{30}{36}$, $\frac{24}{48}$, $\frac{50}{60}$ to sixths.
3. $\frac{15}{40}$, $\frac{40}{80}$, $\frac{20}{32}$, $\frac{24}{64}$, $\frac{18}{72}$ to eighths.
4. $\frac{28}{40}$, $\frac{45}{50}$, $\frac{36}{60}$, $\frac{49}{70}$, $\frac{64}{80}$ to tenths.
5. $\frac{18}{24}$, $\frac{18}{36}$, $\frac{28}{48}$, $\frac{45}{60}$, $\frac{42}{72}$ to twelfths.

Written Exercises.

Reduce:

1. $\frac{294}{336}$ to 8ths.
2. $\frac{336}{378}$ to 9ths.
3. $\frac{630}{735}$ to 7ths.
4. $\frac{392}{588}$ to 6ths.
5. $\frac{366}{915}$ to 15ths.
6. $\frac{105}{126}$ to 6ths.
7. $\frac{205}{328}$ to 8ths.
8. $\frac{498}{830}$ to 5ths.
9. $\frac{485}{679}$ to 7ths.
10. $\frac{549}{671}$ to 11ths.

166. Fifth case.—To reduce fractions to their lowest terms.

A fraction is in its lowest terms when the numerator and denominator are prime to each other.

Example I.—Reduce $\frac{42}{54}$ to its lowest terms.

Explanation.—As the fraction is not changed when its terms are divided by the same number (*Principle V*), we shall divide both terms by their common factors; so $\frac{42 \div 2}{54 \div 2} = \frac{21}{27}$; and $\frac{21 \div 3}{27 \div 3} = \frac{7}{9}$; and since 7 and 9 are prime to each other, the fraction is in its lowest terms.

Example II.—Reduce $\frac{314}{471}$ to its lowest terms.

Explanation.—The greatest common divisor of two numbers is the greatest number that exactly divides them; so if we divide both terms by their greatest common divisor, we shall obtain numbers prime to each other. The greatest common divisor of 314 and 471 is 157; hence, $\frac{314 \div 157}{471 \div 157} = \frac{2}{3}$, or the lowest terms of $\frac{314}{471}$.

167. Rule I.—*Divide both terms successively by their common factors.*

168. Rule II.—*Divide both terms by their greatest common divisor.*

169. Note.—Fractions must always be reduced to their lowest terms before being used in any operation whatever, or when given as answers.

Oral Exercises.

At sight, reduce to their lowest terms:

1. $\frac{2}{4}$, $\frac{3}{6}$, $\frac{8}{16}$, $\frac{9}{18}$, $\frac{10}{20}$, $\frac{20}{40}$, $\frac{14}{28}$, $\frac{15}{30}$, $\frac{16}{32}$, $\frac{17}{34}$.
2. $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, $\frac{5}{15}$, $\frac{6}{18}$, $\frac{7}{21}$, $\frac{8}{24}$, $\frac{9}{27}$, $\frac{10}{30}$, $\frac{12}{36}$.
3. $\frac{4}{6}$, $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{12}{18}$, $\frac{14}{21}$, $\frac{16}{24}$, $\frac{18}{27}$, $\frac{20}{30}$, $\frac{24}{36}$.
4. $\frac{2}{8}$, $\frac{3}{12}$, $\frac{4}{16}$, $\frac{5}{20}$, $\frac{6}{24}$, $\frac{6}{8}$, $\frac{9}{12}$, $\frac{12}{16}$, $\frac{15}{20}$, $\frac{18}{24}$.
5. $\frac{7}{28}$, $\frac{8}{32}$, $\frac{9}{36}$, $\frac{10}{40}$, $\frac{12}{48}$, $\frac{21}{28}$, $\frac{24}{32}$, $\frac{27}{36}$, $\frac{30}{40}$, $\frac{36}{48}$.

Written Exercises.

Reduce to their lowest terms:

1. $\frac{72}{81}$.
2. $\frac{84}{96}$.
3. $\frac{54}{72}$.
4. $\frac{58}{74}$.
5. $\frac{81}{96}$.
6. $\frac{121}{132}$.
7. $\frac{128}{176}$.
8. $\frac{125}{325}$.
9. $\frac{480}{660}$.
10. $\frac{182}{196}$.
11. $\frac{264}{333}$.
12. $\frac{315}{345}$.
13. $\frac{200}{450}$.
14. $\frac{528}{624}$.
15. $\frac{288}{444}$.
16. $\frac{147}{196}$.
17. $\frac{435}{630}$.
18. $\frac{126}{189}$.
19. $\frac{414}{999}$.
20. $\frac{322}{504}$.
21. $\frac{672}{936}$.
22. $\frac{756}{924}$.
23. $\frac{567}{621}$.
24. $\frac{294}{476}$.
25. $\frac{840}{960}$.

COMMON DENOMINATOR.

170. A **common denominator** of two or more fractions is any denominator to which all can be reduced.

171. Principle.—The common denominator of several fractions is a common multiple of their denominators.

EXAMPLE.—Reduce $\frac{3}{5}$, $\frac{6}{7}$, and $\frac{7}{8}$ to a common denominator.

OPERATION.

$5\times7\times8=280$

$280\div5=7\times8 \qquad \frac{3\times7\times8}{5\times7\times8}=\frac{168}{280}$

$280\div7=5\times8 \qquad \frac{6\times5\times8}{7\times5\times8}=\frac{240}{280}$

$280\div8=5\times7 \qquad \frac{7\times5\times7}{8\times5\times7}=\frac{245}{280}$

EXPLANATION.

$5\times7\times8$, or 280 = the common multiple (that is, the common denominator); dividing the common denominator by any one of the denominators, gives for quotient the *product* of the two other denominators; and multiplying both terms of each fraction by the quotient thus obtained, reduces each fraction to 280ths. The fractions will preserve the same value (*Principle V*), and they will have a common denominator.

172. Rule.—*Multiply both terms of each fraction by the denominators of the other fractions.*

Oral Exercises.

Reduce to a common denominator:

1. $\frac{1}{2}$, $\frac{1}{3}$.	**6.** $\frac{1}{7}$, $\frac{1}{2}$.	**11.** $\frac{1}{6}$, $\frac{1}{7}$.	**16.** $\frac{3}{4}$, $\frac{1}{5}$.
[illegible] $\frac{1}{2}$, $\frac{1}{5}$.	**7.** $\frac{1}{7}$, $\frac{1}{3}$.	**12.** $\frac{1}{5}$, $\frac{1}{4}$.	**17.** $\frac{3}{4}$, [illegible].
[illegible], $\frac{1}{5}$.	**8.** $\frac{1}{7}$, $\frac{1}{4}$.	**13.** $\frac{2}{3}$, $\frac{1}{2}$.	**18.** $\frac{3}{4}$, $\frac{1}{7}$.
[illegible] $\frac{1}{4}$.	**9.** $\frac{1}{7}$, $\frac{1}{5}$.	**14.** $\frac{2}{3}$, $\frac{1}{4}$.	**29.** $\frac{3}{4}$, $\frac{1}{8}$.
[illegible] $\frac{[illegible]}{4}$, $\frac{1}{8}$.	**10.** $\frac{1}{7}$, $\frac{1}{8}$.	**15.** $\frac{2}{3}$, $\frac{1}{5}$.	**20.** $\frac{1}{9}$, $\frac{1}{2}$.

Written Exercises.

Reduce to a common denominator:

1. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.	**6.** $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{9}$.	**11.** $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.	**16.** $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{8}$.
2. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$.	**7.** $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$.	**12.** $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.	**17.** $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{9}$.
3. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$.	**8.** $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$.	**13.** $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{5}$.	**18.** $\frac{3}{8}$, $\frac{3}{7}$, $\frac{2}{9}$.
4. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{7}$.	**9.** $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{8}$.	**14.** $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$.	**19.** $\frac{3}{8}$, $\frac{4}{7}$, $\frac{2}{5}$.
5. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$.	**10.** $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{9}$.	**15.** $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{7}$.	**20.** $\frac{5}{8}$, $\frac{5}{7}$, $\frac{2}{3}$.

LEAST COMMON DENOMINATOR.

173. The **least common denominator** of two or more fractions is the *smallest* denominator to which all can be reduced.

174. Principle.—The least common denominator (L. C. D.) of two or more fractions is the least common multiple of their several denominators.

EXAMPLE.—Reduce $\frac{5}{6}$, $\frac{5}{8}$, and $\frac{8}{9}$ to their least common denominator.

OPERATION.

$$\begin{array}{r|rrr} 2) & 6 & 8 & 9 \\ \hline 3) & 3 & 4 & 9 \\ \hline & 1 & 4 & 3 \end{array}$$

$2 \times 3 \times 4 \times 3 = 72$, l.c.m.

$72 \div 6 = 12 \quad \frac{5 \times 12}{6 \times 12} = \frac{60}{72}$

$72 \div 8 = 9 \quad \frac{5 \times 9}{8 \times 9} = \frac{45}{72}$

$72 \div 9 = 8 \quad \frac{8 \times 8}{9 \times 8} = \frac{64}{72}$

EXPLANATION.

The least common multiple of 6, 8, and 9 is 72; so 72 is the L. C. D. To reduce 5 sixths, 5 eighths, and 8 ninths to seventy-seconds, multiply both terms of $\frac{5}{6}$, of $\frac{5}{8}$, and of $\frac{8}{9}$ respectively by 12, 9, and 8. This is done without changing the value of the fractions. (*Principle V*).

175. Rule.—*1° Find the least common multiple of the denominators: this will be the least common denominator of the fractions;*

2° Divide the least common denominator by the denominator of each fraction, separately, and multiply both terms by the quotient.

NOTE.—Before applying the above rule, reduce all fractions to their lowest terms.

Oral Exercises.

Reduce to fractions having the least common denominator:

1. $\frac{1}{2}$, $\frac{1}{4}$.	**6.** $\frac{5}{8}$, $\frac{3}{4}$.	**11.** $\frac{1}{3}$, $\frac{3}{6}$.	**16.** $\frac{7}{9}$, $\frac{5}{6}$.	**21.** $\frac{5}{6}$, $\frac{7}{12}$.
2. $\frac{1}{2}$, $\frac{3}{4}$.	**7.** $\frac{7}{8}$, $\frac{1}{4}$.	**12.** $\frac{1}{3}$, $\frac{5}{6}$.	**17.** $\frac{6}{9}$, $\frac{5}{6}$.	**22.** $\frac{5}{12}$, $\frac{1}{4}$.
3. $\frac{1}{2}$, $\frac{3}{4}$.	**8.** $\frac{1}{3}$, $\frac{1}{2}$.	**13.** $\frac{2}{3}$, $\frac{4}{9}$.	**18.** $\frac{8}{9}$, $\frac{5}{6}$.	**23.** $\frac{10}{12}$, $\frac{5}{6}$.
4. $\frac{1}{2}$, $\frac{1}{8}$.	**9.** $\frac{1}{3}$, $\frac{2}{4}$.	**14.** $\frac{2}{3}$, $\frac{3}{6}$.	**19.** $\frac{5}{9}$, $\frac{1}{3}$.	**24.** $\frac{1}{5}$, $\frac{1}{10}$.
5. $\frac{1}{2}$, $\frac{3}{8}$.	**10.** $\frac{1}{3}$, $\frac{3}{8}$.	**15.** $\frac{1}{6}$, $\frac{1}{8}$.	**20.** $\frac{1}{12}$, $\frac{1}{2}$.	**25.** $\frac{3}{5}$, $\frac{2}{15}$.

Written Exercises.

Reduce to fractions having the least common denominator:

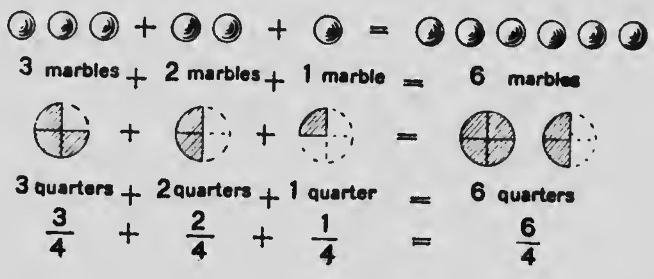
1. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$.
2. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$.
3. $\frac{4}{5}, \frac{9}{10}, \frac{1}{2}$.
4. $\frac{5}{8}, \frac{2}{3}, \frac{1}{12}$.
5. $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}$.
6. $\frac{8}{9}, \frac{2}{3}, \frac{5}{18}$.
7. $\frac{7}{8}, \frac{9}{16}, \frac{5}{12}$.
8. $\frac{3}{7}, \frac{5}{21}, \frac{9}{28}$.
9. $\frac{3}{11}, \frac{5}{9}, \frac{7}{33}$.
10. $\frac{7}{8}, \frac{8}{9}, \frac{13}{15}$.
11. $\frac{15}{16}, \frac{16}{18}, \frac{24}{26}$.
12. $\frac{17}{14}, \frac{19}{21}, \frac{27}{36}$.
13. $\frac{11}{26}, \frac{17}{39}, \frac{41}{75}$.
14. $\frac{16}{51}, \frac{25}{68}, \frac{28}{85}$.
15. $\frac{24}{63}, \frac{21}{45}, \frac{11}{27}$.

NOTE.—Reduce mixed numbers to improper fractions, before reducing to the L. C. D.

16. $\frac{1}{18}, \frac{5}{21}, \frac{9}{28}, \frac{31}{54}$.
17. $\frac{11}{18}, 3\frac{1}{2}, \frac{22}{24}, 1\frac{5}{6}$.
18. $\frac{3}{14}, \frac{9}{16}, 3\frac{3}{8}, 2\frac{17}{21}$.
19. $\frac{8}{9}, \frac{14}{25}, 3\frac{7}{15}, \frac{3}{5}$.
20. $\frac{9}{22}, \frac{16}{33}, \frac{19}{66}, 1\frac{2}{3}$.
21. $\frac{587}{1761}, \frac{593}{2965}, 1\frac{1}{3}$.
22. $\frac{613}{2452}, \frac{619}{2476}, 1\frac{1}{4}$.
23. $1\frac{857}{2571}, 2\frac{863}{3452}$.
24. $1\frac{1514}{2271}, 1\frac{2253}{3755}$.
25. $2\frac{1982}{2973}, 1\frac{2991}{3988}, \frac{997}{11964}$.

ADDITION OF FRACTIONS

Only like units can be added.



176. To be added, fractions must express like fractional units, that is, have a common denominator. The numerators are the fraction units to be added; the like denominators indicate the value of each of these units.

MENTAL ADDITION.

Oral Exercises.

EXAMPLE I.—Find the sum of $\frac{1}{3}$ and $\frac{1}{4}$.

OPERATION.

$\frac{1}{3}+\frac{1}{4}$

$\frac{4+3}{3\times 4}=\frac{7}{12}$

EXPLANATION.

When two factors have 1 for a numerator, take *the sum of the denominators* for the required numerator, and *the product of the denominators* for the required denominator.

Add the following:

1. $\frac{1}{2}+\frac{1}{3}$.
2. $\frac{1}{2}+\frac{1}{5}$.
3. $\frac{1}{2}+\frac{1}{7}$.
4. $\frac{1}{2}+\frac{1}{9}$.
5. $\frac{1}{2}+\frac{1}{11}$.
6. $\frac{1}{2}+\frac{1}{13}$.
7. $\frac{1}{2}+\frac{1}{15}$.
8. $\frac{1}{3}+\frac{1}{4}$.
9. $\frac{1}{3}+\frac{1}{5}$.
10. $\frac{1}{3}+\frac{1}{7}$.
11. $\frac{1}{3}+\frac{1}{8}$.
12. $\frac{1}{3}+\frac{1}{10}$.
13. $\frac{1}{3}+\frac{1}{11}$.
14. $\frac{1}{3}+\frac{1}{13}$.
15. $\frac{1}{4}+\frac{1}{5}$.
16. $\frac{1}{4}+\frac{1}{7}$.
17. $\frac{1}{4}+\frac{1}{9}$.
18. $\frac{1}{4}+\frac{1}{11}$.
19. $\frac{1}{4}+\frac{1}{13}$.
20. $\frac{1}{4}+\frac{1}{15}$.

EXAMPLE II.—Find the sum of $\frac{2}{3}$, $\frac{1}{12}$, and $\frac{3}{4}$.

EXPLANATION.—*Think* the operation, naming only the results: "The L. C. D. is 12; the first numerator is 8; the second is 1, 9; the third is 9, 18; $\frac{18}{12}=\frac{9}{6}=\frac{3}{2}=1\frac{1}{2}$."

Oral Exercises.

Add the following:

21. $\frac{2}{5}+\frac{2}{6}$.
22. $\frac{2}{5}+\frac{2}{8}$.
23. $\frac{2}{3}+\frac{2}{7}$.
24. $\frac{2}{8}+\frac{2}{9}$.
25. $\frac{2}{7}+\frac{2}{9}$.
26. $\frac{2}{5}+\frac{3}{6}$.
27. $\frac{3}{5}+\frac{5}{6}$.
28. $\frac{2}{7}+\frac{2}{11}$.
29. $\frac{2}{5}+\frac{2}{11}$.
30. $\frac{3}{4}+\frac{3}{5}$.
31. $\frac{3}{6}+\frac{3}{8}$.
32. $\frac{3}{7}+\frac{3}{8}$.
33. $\frac{2}{5}+\frac{3}{7}$.
34. $\frac{4}{5}+\frac{3}{4}$.
35. $\frac{5}{8}+\frac{5}{9}$.

Principles of Analysis relating to Addition.

1. I add because I wish to find the *sum* of ......

2. I add because ...... (what is asked) is *more than* ...... (what is given).

3. I add because I wish to find how many there are *in all*, or *together*.

Oral Problems.

First principle of analysis.

1. The three sides of a triangle measure $4\frac{1}{2}$, $5\frac{1}{2}$, and $6\frac{3}{4}$ in.; find the *sum* of these lengths.

2. Joseph has $\$\frac{3}{4}$ and Eva $\$1\frac{1}{2}$. Find the *sum* of their money.

3. Henry is $4\frac{1}{2}$ ft. tall and Paul $4\frac{3}{4}$; find the *sum* of their statures.

Second principle of analysis.

4. I paid $\$\frac{3}{4}$ for my geography and $\$\frac{1}{4}$ *more* for my dictionary. Find the cost of my dictionary.

5. I had $66\frac{1}{2}$ notes this week, and Lewis, $\frac{3}{4}$ of a note *more* than I. How many notes did Lewis have?

6. James has two apple trees; one yields $7\frac{1}{2}$ bu. of apples, the other $1\frac{1}{2}$ bu. *more*. How many bushels does the second tree yield?

7. My brother is $10\frac{1}{3}$ yr. old; my sister, $2\frac{1}{3}$ yr. *older*. How old is my sister?

8. The class is $14\frac{1}{2}$ ft. wide; find the length if it measures $1\frac{1}{3}$ ft. *more*.

9. It is $14\frac{3}{10}$ miles from Montreal to Laprairie; and $1\frac{3}{10}$ miles *more* from Laprairie to St. Remi. How far is it from Laprairie to St. Remi?

10. Mabel has $\$\frac{3}{4}$; Maud, $\$\frac{1}{2}$ *more*. How much has Maud?

11. If Harold lives $\frac{1}{8}$ mile from school, and Cuthbert $\frac{1}{4}$ mile *farther*, how far from school does Cuthbert live?

12. Anselm owns $\$\frac{5}{10}$, and Conrad $\$\frac{3}{10}$ *more*. Find the sum of their money.

Third principle of analysis.

13. If a cyclist goes $7\frac{1}{2}$ mi. the first hour, and $6\frac{1}{2}$ mi. the second, how many miles does he make *in all*?

14. A can do $\frac{1}{3}$ of a work in 1 day, and B can do $\frac{1}{4}$ of the same work in 1 day; what can they do *together* in 1 day?

15. Ambrose can do $\frac{1}{8}$ of a work in 1 day, and Edmund can do $\frac{1}{3}$ of the same work in 1 day. What part of the work can they do *together* in 1 day?

16. A boy paid $\$1\frac{1}{2}$ for a ball, $\$\frac{3}{4}$ for a bat, and $\$2\frac{1}{2}$ for a mitten; how much was that *in all*?

17. A family consumed $\frac{1}{3}$ of a barrel of flower one week, and $\frac{1}{15}$ of a barrel *more* the next week; how much was that *in all*?

18. Albert has \$$7\frac{3}{4}$, and Bertha \$$1\frac{1}{2}$ *more* than Albert. How much have they *together*?

19. I have two pieces of silk; one measures $4\frac{1}{4}$ yards, the other, $2\frac{1}{2}$ yards *more*. Find the *total* length.

20. Edwin deposited \$$3\frac{4}{5}$ in the School Savings Bank, and Walter, \$$2\frac{1}{5}$ *more*. How much did both deposit *together*?

	A	B	C	D	E
1.	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{2}{7}$
2.	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{5}{8}$
3.	$\frac{1}{6}$	1	$\frac{5}{6}$	$\frac{2}{5}$	$\frac{4}{9}$
4.	$\frac{1}{8}$		$\frac{5}{9}$	$\frac{3}{8}$	$\frac{7}{9}$
5.	$\frac{1}{10}$	$\frac{1}{12}$	$\frac{7}{8}$	$\frac{3}{7}$	$\frac{4}{5}$

This table can afford many oral exercises; for instance, by adding A and B; B and C; A, B, and C; 1 and 2; 2 and 4; 1, 2, and 5; etc.

WRITTEN ADDITION.

EXAMPLE I.—Find the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$, $\frac{11}{24}$.

OPERATION (*a*).

$$\frac{3}{4}+\frac{7}{8}+\frac{9}{16}+\frac{11}{24}=?$$

$$\frac{36+42+27+22}{48}=\frac{127}{48}$$

$$\frac{127}{48}=2\tfrac{31}{48}.$$

OPERATION (*b*).

$$\left.\begin{array}{l} \phantom{\frac{3}{4}=}\ 48 \\ \frac{3}{4}=36 \\ \frac{7}{8}=42 \\ \frac{9}{16}=27 \\ \frac{11}{24}=22 \end{array}\right\}=\frac{127}{48}=2\tfrac{31}{48}.$$

EXPLANATION.—Before these fractions can be added they must be reduced to a common denominator; they will then express *like fractional units.* Their L. C. D. is 48. The numerators 36, 42, 27, and 22 give 127 equal parts, called forty-eighths, $\frac{127}{48}$, or $2\frac{31}{48}$.

EXAMPLE II.—Find the sum of $14\frac{1}{2}$, $25\frac{3}{8}$, $9\frac{5}{6}$, and $11\frac{5}{12}$.

OPERATION (*a*).

$$14\tfrac{1}{2}+25\tfrac{3}{8}+9\tfrac{5}{6}+11\tfrac{5}{12}=?$$

$$\begin{array}{lllll} 14 & +25 & +9 & +11 & =59 \\ \frac{1}{2}+ & \frac{3}{8}+ & \frac{5}{6}+ & \frac{5}{12} & \end{array}$$

or $\frac{12+9+20+10}{24}=\frac{51}{24}$

$\frac{51}{24}=2\frac{3}{24}=2\frac{1}{8}$

$59+2\frac{1}{8}=61\frac{1}{8}$.

OPERATION (*b*).

$$\left.\begin{array}{r} 24 \\ 14\tfrac{1}{2}=12 \\ 25\tfrac{3}{8}=\ 9 \\ 9\tfrac{5}{6}=20 \\ 11\tfrac{5}{12}=10 \end{array}\right\}=\frac{51}{24}=2\tfrac{1}{8}.$$

$$\begin{array}{l} 59 \\ \ \ 2\tfrac{1}{8} \\ \hline 61\tfrac{1}{8}. \end{array}$$

177. Rule.—*Reduce the fractions to their least common denominator; add the numerators, and place the sum over the least common denominator.*

178. NOTE I.—Reduce each fraction to its lowest terms before adding; after the addition is performed, reduce the result to its simplest form; that is, an improper fraction to a mixed number, and the fractional part to its lowest terms.

179. NOTE II.—To add mixed numbers, add the integers and fractions separately, and then unite the two results,

Written Exercises.

Add:

1. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{5}{12}$.
2. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{5}{24}$.
3. $\frac{5}{7}$, $\frac{3}{14}$, $\frac{4}{21}$, $\frac{5}{14}$.
4. $\frac{3}{8}$, $\frac{5}{9}$, $\frac{7}{18}$, $\frac{3}{4}$.
5. $\frac{5}{4}$, $\frac{6}{7}$, $\frac{3}{14}$, $\frac{1}{2}$.
6. $\frac{5}{8}$, $\frac{5}{7}$, $\frac{5}{4}$, $\frac{5}{14}$.
7. $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{18}$, $\frac{3}{4}$.
8. $\frac{7}{8}$, $\frac{5}{12}$, $\frac{7}{24}$, $\frac{2}{3}$.
9. $\frac{6}{7}$, $\frac{5}{28}$, $\frac{3}{14}$, $\frac{3}{4}$.
10. $\frac{2}{5}$, $\frac{7}{10}$, $\frac{5}{30}$, $\frac{2}{3}$.
11. $\frac{5}{8}$, $\frac{5}{6}$, $\frac{3}{9}$, $\frac{5}{24}$.
12. $\frac{4}{7}$, $\frac{4}{8}$, $\frac{4}{14}$, $\frac{6}{28}$.
13. $\frac{3}{10}$, $\frac{7}{15}$, $\frac{7}{30}$, $\frac{7}{20}$.
14. $\frac{7}{12}$, $\frac{9}{16}$, $\frac{13}{24}$, $\frac{11}{18}$.
15. $\frac{5}{9}$, $\frac{7}{18}$, $\frac{7}{36}$, $\frac{7}{12}$.
16. $\frac{5}{8}$, $\frac{4}{9}$, $\frac{3}{4}$, $\frac{5}{6}$.
17. $\frac{7}{12}$, $\frac{8}{15}$, $\frac{15}{36}$, $\frac{8}{24}$.
18. $\frac{8}{9}$, $\frac{7}{18}$, $\frac{7}{16}$, $\frac{5}{24}$.
19. $\frac{3}{10}$, $\frac{8}{9}$, $\frac{3}{8}$, $\frac{13}{15}$.
20. $\frac{2}{3}$, $\frac{5}{12}$, $\frac{17}{24}$, $\frac{11}{14}$.
21. $\frac{14}{15}$, $\frac{17}{30}$, $\frac{23}{28}$, $\frac{19}{20}$.
22. $\frac{7}{12}$, $\frac{19}{24}$, $\frac{31}{36}$, $\frac{9}{10}$
23. $\frac{3}{5}$, $\frac{2}{15}$, $\frac{1}{3}$, $\frac{1}{9}$.
24. $\frac{1}{4}$, $\frac{6}{7}$, $\frac{9}{10}$, $\frac{2}{35}$.
25. $\frac{1}{10}$, $\frac{2}{7}$, $\frac{5}{6}$, $\frac{2}{3}$.
26. $\frac{4}{5}$, $\frac{3}{10}$, $\frac{4}{15}$, $\frac{7}{25}$.
27. $\frac{1}{7}$, $\frac{2}{9}$, $\frac{5}{6}$, $\frac{2}{3}$.
28. $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{24}$, $\frac{7}{12}$.
29. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{7}$, $\frac{2}{9}$.
30. $\frac{1}{8}$, $\frac{3}{10}$, $\frac{1}{3}$, $\frac{1}{15}$.
31. $1\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.
32. $2\frac{1}{3} + \frac{1}{6} + \frac{5}{9}$.
33. $2\frac{3}{4} + 1\frac{7}{8} + 4\frac{1}{16}$.
34. $2\frac{1}{2} + 1\frac{3}{4} + \frac{1}{8}$.
35. $3\frac{2}{3} + \frac{5}{6} + \frac{7}{9}$.
36. $3\frac{3}{4} + 2\frac{5}{8} + 3\frac{3}{16}$.
37. $3\frac{1}{2} + 2\frac{3}{4} + 1\frac{5}{8}$.
38. $1\frac{2}{3} + 2\frac{5}{6} + \frac{8}{9}$.
39. $4\frac{2}{3} + 3\frac{3}{4} + \frac{5}{12}$.
40. $4\frac{3}{4} + 8\frac{5}{8} + 7\frac{2}{3}$.

41.	42.	43.	44.	45.	46.
$364\frac{5}{8}$	$327\frac{4}{5}$	$634\frac{3}{16}$	$842\frac{3}{25}$	$634\frac{5}{12}$	$3\,426\frac{3}{4}$
$742\frac{7}{12}$	$645\frac{5}{6}$	$864\frac{7}{12}$	$375\frac{7}{10}$	$784\frac{4}{15}$	$3\,548\frac{5}{8}$
$436\frac{5}{6}$	$327\frac{2}{3}$	$963\frac{11}{24}$	$842\frac{4}{5}$	$643\frac{17}{20}$	$7\,263\frac{1}{2}$
$743\frac{3}{4}$	$846\frac{7}{10}$	$327\frac{5}{6}$	$963\frac{1}{2}$	$426\frac{7}{10}$	$4\,986\frac{1}{4}$
$375\frac{2}{3}$	$364\frac{8}{15}$	$842\frac{2}{3}$	$643\frac{3}{25}$	$962\frac{11}{30}$	$5\,432\frac{7}{8}$

47.	48.	49.	50.
$4\frac{118}{177}$	$7\frac{479}{1437}$	$8\frac{303}{808}$	$3\frac{751}{3755}$
$5\frac{194}{291}$	$9\frac{797}{2391}$	$5\frac{309}{824}$	$2\frac{1514}{3785}$
$6\frac{278}{417}$	$12\frac{977}{2931}$	$3\frac{107}{428}$	$7\frac{1522}{3805}$

Written Problems.

1. Josephine is $11\frac{11}{12}$ years old; Rose is $1\frac{5}{12}$ years older. How old is Rose?

2. Joseph and Napoleon are working together; Joseph can cut $\frac{9}{21}$ of a cord of wood in 1 day, and Napoleon, $\frac{1}{6}$ of a cord more than Joseph in 1 day. What can Napoleon do in 1 day?

3. In time of war, a German soldier's knapsack weighs $58\frac{3}{4}$ lb.; a French soldier's, $1\frac{1}{4}$ lb. more. How heavy is a French soldier's knapsack?

4. A farmer sold his oats for \$$17\frac{3}{4}$, and his corn for \$$3\frac{1}{8}$ more. What did he get for the corn?

5. I buy two horses, paying \$$76\frac{3}{4}$ for one, and \$$23\frac{4}{5}$ more for the other. Find the cost of the second horse.

6. The distance from Montreal to Toronto is $334\frac{1}{20}$ mi. Find the distance from Toronto to Chicago, if it is $225\frac{11}{20}$ mi. greater.

7. A farmer harvested $1\,250\frac{4}{15}$ bu. of wheat; find his neighbor's harvest, if it was $399\frac{1}{3}$ bu. greater.

8. In 1910, the minerals produced in Ontario were worth $43\frac{1}{2}$ million dollars; in 1912, $8\frac{2}{9}$ million dollars more. What was the production worth in 1912?

9. In 1886, Canada's mineral production was worth $10\frac{1}{5}$ million dollars; in 1905, it was worth $59\frac{17}{20}$ million dollars more. What was its value in 1905?

10. In 1900, Canada exported $29\frac{3}{5}$ million dollars' worth of forest products; in 1913, $13\frac{9}{20}$ millions more. Find the value for 1913.

11. The imports of the Province of Quebec aggregated $128\frac{7}{10}$ million dollars in 1910; $141\frac{3}{10}$ millions in 1911; $164\frac{1}{5}$ millions in 1912; $187\frac{3}{10}$ millions in 1913; and $185\frac{3}{10}$ millions in 1914. Find the sum.

12. The exports of the Province of Quebec aggregated $126\frac{3}{10}$ million dollars in 1910; $123\frac{7}{10}$ millions in 1911; $123\frac{1}{10}$ millions in 1912; $147\frac{7}{10}$ millions in 1913; and $177\frac{1}{2}$ millions in 1914. Find the sum.

13. The exchanges of the Montreal Clearing House amounted to $2\,088\frac{1}{2}$ million dollars in 1910; to $2\,368\frac{4}{10}$ millions in 1911;

to 2 845$\frac{4}{10}$ millions in 1912; to 2 879$\frac{1}{10}$ millions in 1913; and to 2 631$\frac{3}{10}$ millions in 1914. Find the sum.

14. The value of Canada's mineral production was 106$\frac{8}{10}$ million dollars in 1910; 103$\frac{1}{5}$ millions in 1911; 135$\frac{1}{20}$ millions in 1912. Find the sum.

15. In 1912, Canada produced 36$\frac{1}{20}$ million dollars' worth of coal, 13$\frac{2}{5}$ millions of nickel, 19$\frac{2}{5}$ millions of silver, 12$\frac{2}{5}$ millions of gold, 12$\frac{7}{10}$ millions of copper. Find the total value of these productions.

16. At Quebec, in 1914, fresh fish cost 8$\frac{1}{3}$ cents a pound; fresh eggs, 34$\frac{3}{4}$ cents a dozen; milk, 10$\frac{1}{3}$ cents a quart; butter, 31$\frac{1}{2}$ cents a pound. Find the total cost of 1 pound of fish, 1 dozen of eggs, 1 quart of milk, and 1 pound of butter.

17. A man walked 10$\frac{2}{3}$ mi. on Monday; 1$\frac{1}{3}$ mi. more on Tuesday than on Monday; and 2$\frac{7}{8}$ mi. more on Wednesday than on Tuesday. Find the distance covered in the 3 days.

18. Find the total yield of my 3 wheat fields. I had 947$\frac{3}{4}$ bu. from the first; 135$\frac{7}{8}$ bu. more from the second than from the first; and as many bushels from the third as from the two first together.

19. A and B start together from the same place and travel in opposite directions, A 20$\frac{7}{18}$ mi., and B 22$\frac{5}{16}$ mi. How far apart are they then?

20. How many feet of fencing will be needed for a field 194$\frac{7}{12}$ ft. long and 84$\frac{1}{6}$ ft. wide? (*This implies* 2 *lengths and* 2 *widths*).

21. Your class is 32$\frac{2}{3}$ ft. long and 29$\frac{7}{8}$ ft. wide. How long a string would it take to measure around it?

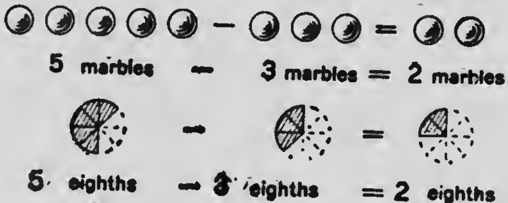
22. The titanic iron ore of the Eastern Townships contains 40$\frac{7}{10}$ parts of peroxide, 48$\frac{3}{5}$ parts of titanic acid, 2$\frac{11}{25}$ parts of magnesia, and 4$\frac{1}{10}$ parts of water. Find the sum of these mixed numbers.

23. Find the distance by water from Montreal to Ottawa with the following statements: Lachine Canal, 8$\frac{1}{2}$ mi.; Lake St. Louis, 15 mi.; St. Anne's Lock and outlet of Ottawa River, $\frac{1}{8}$ mi.; Lake of Two Mountains and Ottawa River, 27 mi.; Carillon Canal, $\frac{3}{4}$ mi.; Ottawa River, 6$\frac{1}{4}$ mi.; Grenville Canal, 5$\frac{3}{4}$ mi.; Ottawa River to Ottawa, 56 mi.

24. Find the length of the waterway from Montreal to the International boundary, near Lake Champlain, with the following intermediate distances: St. Lawrence River to Sorel, $45\frac{9}{10}$ mi.; Richelieu River, $14\frac{1}{8}$ mi.; St. Ours' Lock, $\frac{1}{8}$ mi.; Richelieu River, $32\frac{1}{6}$ mi.; Chambly Canal, 12 mi.; from St. Johns to boundary line, $22\frac{7}{8}$ mi.

25. Find the length of the through route between Montreal and Port Arthur with the following navigation sections: Lachine Canal, $8\frac{1}{2}$ mi.; Lake St. Louis and River St. Lawrence, $15\frac{7}{8}$ mi.; Soulanges Canal, 14 mi.; Lake St. Francis and River St. Lawrence, 33 mi.; Cornwall Canal, $10\frac{9}{10}$ mi.; River St. Lawrence, 5 mi.; Farran's Point Canal, $1\frac{1}{4}$ mi.; River St. Lawrence, $9\frac{7}{8}$ mi.; Rapide Plat Canal, $3\frac{2}{3}$ mi.; River St. Lawrence, $4\frac{1}{10}$ mi.; Galops Canal, $7\frac{1}{3}$ mi.; River St. Lawrence and Lake Ontario, $235\frac{7}{8}$ mi.; Welland Canal, $26\frac{3}{4}$ mi.; Lake Erie, Detroit River, Lake St. Clair, Lake Huron, $579\frac{9}{10}$ mi.; Sault St. Marie Canal, $1\frac{1}{4}$ mi.; Lake Superior to Port Arthur, $272\frac{9}{10}$ mi.

SUBTRACTION OF FRACTIONS



$$\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$$

180. The fraction of the subtrahend and the fraction of the minuend must express like fractional units, that is, have a common denominator.

MENTAL SUBTRACTION.

Oral Exercises.

Example I.—Subtract $\frac{1}{3}$ from $\frac{1}{2}$.

OPERATION.

$\frac{1}{2} - \frac{1}{3}$

$\frac{3-2}{2\times 3} = \frac{1}{6}$

EXPLANATION.

When two fractions have 1 for a numerator, take *the difference between the denominators* for the required numerator, and *the product of the denominators* for the required denominator.

Subtract mentally:

1. $\frac{1}{3}-\frac{1}{4}$. 6. $\frac{1}{4}-\frac{1}{7}$. 11. $\frac{1}{5}-\frac{1}{11}$. 16. $\frac{1}{7}-\frac{1}{11}$. 21. $\frac{1}{10}-\frac{1}{11}$.
2. $\frac{1}{3}-\frac{1}{5}$. 7. $\frac{1}{4}-\frac{1}{9}$. 12. $\frac{1}{5}-\frac{1}{12}$. 17. $\frac{1}{7}-\frac{1}{12}$. 22. $\frac{1}{10}-\frac{1}{17}$.
3. $\frac{1}{3}-\frac{1}{7}$. 8. $\frac{1}{4}-\frac{1}{11}$. 13. $\frac{1}{6}-\frac{1}{7}$. 18. $\frac{1}{8}-\frac{1}{9}$. 23. $\frac{1}{2}-\frac{1}{9}$.
4. $\frac{1}{3}-\frac{1}{8}$. 9. $\frac{1}{5}-\frac{1}{6}$. 14. $\frac{1}{6}-\frac{1}{11}$. 19. $\frac{1}{8}-\frac{1}{11}$. 24. $\frac{1}{2}-\frac{1}{15}$.
5. $\frac{1}{3}-\frac{1}{9}$. 10. $\frac{1}{6}-\frac{1}{7}$. 15. $\frac{1}{7}-\frac{1}{8}$. 20. $\frac{1}{8}-\frac{1}{15}$. 25. $\frac{1}{2}-\frac{1}{17}$.

EXAMPLE II.—Subtract $\frac{2}{5}$ from $\frac{3}{4}$.

EXPLANATION.—*Think* the operation, naming only the results: "The L. C. D. is 20; the first numerator is 15; the second is 8, 7; $\frac{7}{20}$."

Oral Exercises.

Subtract mentally:

26. $\frac{2}{7}-\frac{2}{9}$. 31. $\frac{2}{5}-\frac{1}{4}$. 36. $\frac{4}{5}-\frac{3}{7}$. 41. $\frac{3}{8}-\frac{1}{3}$.
27. $\frac{2}{5}-\frac{2}{7}$. 32. $\frac{5}{6}-\frac{2}{9}$. 37. $\frac{4}{5}-\frac{2}{7}$. 42. $\frac{3}{8}-\frac{1}{9}$.
28. $\frac{2}{3}-\frac{2}{7}$. 33. $\frac{2}{5}-\frac{2}{10}$. 38. $\frac{4}{3}-\frac{1}{12}$. 43. $\frac{3}{8}-\frac{1}{7}$.
29. $\frac{2}{3}-\frac{2}{5}$. 34. $\frac{2}{5}-\frac{2}{12}$. 39. $\frac{2}{3}-\frac{2}{9}$. 44. $\frac{3}{8}-\frac{2}{11}$.
30. $\frac{3}{5}-\frac{3}{7}$. 35. $\frac{3}{5}-\frac{2}{9}$. 40. $\frac{2}{3}-\frac{2}{11}$. 45. $\frac{5}{8}-\frac{1}{3}$.

Complete the following:

46. $\frac{3}{4}+?=\frac{9}{8}$. 51. $\frac{5}{9}-?=\frac{1}{4}$. 56. $\frac{1}{2}+\frac{1}{3}-\frac{1}{6}=?$
47. $?+\frac{2}{3}=\frac{5}{2}$. 52. $\frac{7}{36}+?=\frac{1}{2}$. 57. $\frac{1}{3}+\frac{1}{4}-\frac{5}{12}=?$
48. $\frac{5}{6}-?=\frac{1}{9}$. 53. $\frac{5}{9}+?=\frac{3}{2}$. 58. $\frac{4}{3}-\frac{3}{4}+\frac{1}{12}=?$
49. $\frac{3}{4}-?=\frac{1}{5}$. 54. $\frac{7}{5}-?=\frac{3}{4}$. 59. $\frac{4}{3}+\frac{2}{3}-\frac{1}{9}=?$
50. $?+\frac{1}{9}=\frac{3}{8}$. 55. $\frac{1}{8}+?=\frac{8}{7}$. 60. $\frac{6}{5}-\frac{1}{2}+\frac{3}{10}=?$

NOTE.—Subtract the fractions from the fractions, and the integers from the integers. If the fraction of the minuend is smaller than the fraction of the subtrahend, *borrow* 1 unit from the minuend.

61. $4\frac{2}{3}-2\frac{2}{3}$. 66. $10\frac{5}{6}-5\frac{1}{6}$. 71. $9\frac{7}{8}-6\frac{1}{4}$.
62. $10\frac{2}{3}-7\frac{2}{3}$. 67. $6\frac{7}{8}-3\frac{3}{8}$. 72. $15\frac{1}{3}-10\frac{1}{2}$.
63. $9\frac{3}{4}-4\frac{3}{4}$. 68. $8\frac{1}{2}-6\frac{1}{4}$. 73. $17\frac{1}{6}-6\frac{1}{3}$.
64. $1\frac{2}{3}-1\frac{1}{3}$. 69. $15\frac{5}{6}-10\frac{2}{3}$. 74. $19\frac{1}{4}-8\frac{3}{8}$.
65. $9\frac{3}{4}-6\frac{1}{4}$. 70. $20\frac{3}{5}-10\frac{1}{10}$. 75. $9\frac{2}{3}-8\frac{3}{4}$.

Principles of Analysis relating to Subtraction.

4. I subtract because I wish to find the *difference* between ...... and ......

5. I subtract because I wish to find ***what is left*** or *what remains.*

6. I subtract because ...... (what is asked) is *less than* ...... (what is given).

FOURTH PRINCIPLE OF ANALYSIS.

1. George has \$$\frac{3}{4}$; Catharine has \$$\frac{1}{4}$. Find the *difference.*

2. John is $12\frac{1}{4}$ years old; Edmund is 14 years old. Find the *difference* between their ages.

3. Wilfrid won $65\frac{3}{10}$ goods marks, and Henry, $61\frac{1}{5}$. Find the *difference.*

Oral Problems.

FIFTH PRINCIPLE OF ANALYSIS.

4. What *remains* if I subtract $3\frac{1}{4}$ from the sum of $2\frac{1}{2}$ and $4\frac{1}{2}$?

5. Viator had \$$4\frac{3}{4}$ in the School Savings Bank; he withdrew \$$1\frac{1}{2}$. What was *left* in the Bank?

6. Irene received \$$1\frac{1}{2}$ from her brother and \$$\frac{3}{4}$ from her sister; she then spent \$$1\frac{1}{4}$; how much money had she *left*?

7. A lady had $14\frac{1}{4}$ yards of cloth. How many yards *remained* after she had employed $8\frac{3}{4}$?

SIXTH PRINCIPLE OF ANALYSIS.

8. Rose has \$$\frac{1}{2}$ *less than* Gertrude. If Gertrude has \$$3\frac{1}{2}$, how much has Rose?

9. I have \$$5\frac{1}{2}$ and my brother has \$$1\frac{1}{4}$ *less than* I. How much has he?

10. At Quebec, in 1910, green tea sold for \$$\frac{3}{10}$ a pound, and black tea sold for \$$\frac{1}{15}$ *less* a pound; what did it sell for?

11. In Montreal, 1913, a carpenter's salary was \$$3\frac{3}{5}$ a day. In Quebec, \$$1\frac{1}{10}$ *less;* what was it in Quebec?

12. In Sherbrooke, 1909, a bricklayer's salary was \$$4\frac{1}{2}$ a day; at St. Hyacinthe, \$$1\frac{1}{2}$ *less*; what was it at St. Hyacinthe?

13. In Hull, 1909, carpenters were paid \$$10\frac{4}{5}$ a week, and laborers, \$$1\frac{3}{5}$ *less*. Find the weekly salary of the latter.

14. In Three Rivers, 1909, plumbers earned $24 a week, and plasterers, $$2\frac{1}{2}$ *less*. How much did the latter earn?

15. In Quebec, 1909, stonecutters were paid $$19\frac{1}{2}$ a week; in 1899, they were paid $$4\frac{1}{2}$ *less*. What was their salary in 1899?

WRITTEN SUBTRACTION.

EXAMPLE I.—Subtract $\frac{11}{24}$ from $\frac{9}{16}$.

OPERATION (*a*).

$$\frac{9}{16}-\frac{11}{24}=\ ?$$
$$\frac{27-22}{48}=\frac{5}{48}.$$

OPERATION (*b*).

$$\begin{array}{c} 48 \\ \hline \left.\begin{array}{l} \frac{9}{16}=27 \\ \frac{11}{24}=22 \end{array}\right\}=\frac{5}{48}. \end{array}$$

EXPLANATION.—After these fractions are reduced to a common denominator, they express like fractional units. Their L. C. D. is 48.

NOTE.—If small mixed numbers are given, reduce to improper fractions before applying the above method. Thus, $1\frac{1}{2}-\frac{1}{3}=\frac{3}{2}-\frac{1}{3}=\frac{9}{6}-\frac{2}{6}=\frac{7}{6}$, or $1\frac{1}{6}$.

EXAMPLE II.—Subtract $12\frac{5}{6}$ from $17\frac{3}{4}$.

OPERATION (*a*).

$$17\frac{3}{4}-12\frac{5}{6}=\ ?$$
$$17\frac{9}{12}-12\frac{10}{12}=\ ?$$
$$16\frac{21}{12}-12\frac{10}{12}=4\frac{11}{12}.$$

OPERATION (*b*).

$$\begin{array}{c} 12 \\ \hline \left.\begin{array}{l} 16\frac{7}{4}=21 \\ 12\frac{5}{6}=10 \end{array}\right\}=\frac{11}{12}. \\ \hline 4\frac{11}{12}. \end{array}$$

EXPLANATION.—$\frac{5}{6}$ is larger than $\frac{3}{4}$; so, borrow 1 from the 17 and add it to the $\frac{3}{4}$ to have $\frac{7}{4}$, or add it to the $\frac{9}{12}$ to have $\frac{21}{12}$. The L. C. D. is 12.

181. Rule.—*Reduce the fractions to their least common denominator, take the difference of the numerators and write it over the common denominator.*

182. NOTE I.—Reduce each fraction, and also the difference, to its lowest terms.

183. NOTE II.—When there are mixed numbers, subtract the fractions and integers separately and combine the results.

Written Exercises.

Find the value of:

1. $\frac{7}{8}-\frac{3}{4}$.
2. $\frac{9}{10}-\frac{11}{15}$.
3. $\frac{7}{12}-\frac{7}{18}$.
4. $\frac{25}{27}-\frac{11}{18}$.
5. $\frac{55}{96}-\frac{11}{24}$.
6. $\frac{13}{15}-\frac{17}{21}$.
7. $\frac{23}{24}-\frac{11}{18}$.
8. $\frac{91}{108}-\frac{13}{36}$.
9. $\frac{11}{12}-\frac{10}{13}$.
10. $\frac{23}{36}-\frac{13}{28}$.
11. $\frac{28}{63}-\frac{13}{35}$.
12. $\frac{25}{56}-\frac{19}{42}$.
13. $1-\frac{19}{31}$.
14. $2\frac{1}{2}-\frac{19}{37}$.
15. $3\frac{1}{3}-\frac{17}{20}$.
16. $7\frac{3}{4}-3\frac{5}{12}$.
17. $9\frac{5}{8}-2\frac{4}{7}$.
18. $18\frac{17}{21}-8\frac{4}{9}$.
19. $30\frac{1}{4}-20\frac{19}{30}$.
20. $45\frac{1}{6}-24\frac{5}{9}$.
21. $50\frac{5}{13}-11\frac{5}{6}$.
22. $63\frac{11}{18}-24\frac{5}{36}$.
23. $71\frac{7}{22}-19\frac{5}{44}$.
24. $18\frac{1}{4}-15\frac{7}{8}$.
25. $120\frac{3}{4}-69$.
26. $5\frac{2}{3}-2\frac{1}{6}$.
27. $12\frac{1}{7}-5\frac{2}{3}$.
28. $7\frac{1}{5}-2\frac{1}{9}$.
29. $2\frac{1}{9}-\frac{1}{11}$.
30. $6-\frac{4}{99}$.
31. $2\frac{3}{16}+3\frac{7}{32}+4\frac{5}{64}=?$
32. $2\frac{3}{16}+9\frac{3}{32}-\frac{3}{64}=?$
33. $3\frac{1}{16}+5\frac{3}{32}-3\frac{7}{64}=?$
34. $4\frac{3}{16}+7\frac{7}{32}-3\frac{3}{64}=?$
35. $8\frac{7}{16}+5\frac{11}{32}-2\frac{9}{64}=?$
36. $46-(18\frac{7}{12}+6\frac{1}{3})=?$
37. $17\frac{5}{6}-(2\frac{1}{12}+3\frac{1}{24})=?$
38. $23\frac{5}{6}-(3\frac{5}{12}-2\frac{5}{24})=?$
39. $33\frac{1}{6}-(2\frac{7}{12}+3\frac{1}{48})=?$
40. $41\frac{1}{6}-(3\frac{11}{12}+2\frac{5}{48})=?$

Written Problems.

1. In 1912, Canada exported $40\frac{9}{10}$ million dollars' worth of forest products; and the exports to Great Britain alone aggregated $10\frac{13}{20}$ million dollars. Find the difference.

2. In 1913, Canada exported $43\frac{1}{4}$ million dollars' worth of forest products; in 1900, $29\frac{13}{20}$ millions. Find the difference.

3 In 1910, the value of Canada's dairy production was $39\frac{1}{25}$ million dollars; in 1900, $29\frac{7}{10}$ millions. Find the difference.

4. In 1910, Canada's cheese production aggregated $199\frac{9}{10}$ million lb.; Ontario's, $13\frac{9}{100}$ million lb. Find the difference.

5. In 1910, Canada's butter production amounted to $64\frac{7}{10}$ million lb.; Quebec's, to $41\frac{3}{4}$ million lb. Find the difference.

6. From a barrel containing $63\frac{1}{2}$ gal. of syrup, there were drawn $45\frac{3}{7}$ gal. How many gallons remained?

7. Lambert owes $129\frac{7}{20}$; he pays \$$56\frac{13}{25}$. How much does there remain to pay?

8. From a lot containing $\frac{13}{20}$ of an acre of land, I sold $\frac{3}{25}$ of an acre to a man, and $\frac{3}{8}$ of an acre to another. How much land had I left?

9. A merchant had $219\frac{3}{4}$ yd. of linen. He sold $98\frac{7}{8}$ yd. at \$0.50, and the remainder at \$0.65 a yard. How many yards were sold at \$0.65?

10. In 1914, the weekly cost of living for a family of 5 members was $\$13\frac{3}{8}$ in Montreal, $\$\frac{1}{2}$ less in Quebec than in Montreal, and $\$2\frac{1}{8}$ less in Three Rivers than in Montreal. What was it in Quebec and Three Rivers?

11. In 1914, the weekly cost of staple foods for a family of 5 persons was $\$7\frac{4}{5}$ in Montreal, and $\$\frac{3}{5}$ less in Sherbrooke. What was it in Sherbrooke?

12. The average wholesale prices of commodities in Canada are represented by 100 points for 1899, by $102\frac{1}{2}$ for 1893, by $108\frac{1}{5}$ for 1900, by $126\frac{1}{5}$ for 1907, by $127\frac{3}{5}$ for 1911, by $134\frac{3}{5}$ for 1912, by $135\frac{1}{2}$ for 1913, and by $136\frac{1}{5}$ for 1914. Find the rise in each case over the initial price 100.

13. In the preceding problem, 1907 was how many points less than 1914? 1913 was how many points more than 1900?

14. A earns $\$\frac{3}{10}$ an hour, and B, $\$\frac{1}{12}$ less than A. How much do they earn together in an hour?

15. In an hour Joseph earns $\$\frac{9}{17}$, and Charles, $\$\frac{1}{10}$ less. How much do they earn together in an hour?

16. The value of the building permits given out in Montreal, 1913, was $27\frac{1}{30}$ million dollars; in 1914, it was $13\frac{7}{30}$ million dollars less. What was it in 1914?

17. In Maisonneuve, 1913, the value of the building permits issued was $2\frac{9}{20}$ million dollars; in 1914, $2\frac{14}{25}$ million dollars. Find the difference.

18. In Quebec, 1913, the value of the building permits granted was $2\frac{3}{4}$ million dollars, that is, $\frac{17}{20}$ million dollars less than in 1914. Find the sum for these two years.

19. The People's Bank failed in 1895, paying out $75\frac{1}{4}$ cents on a dollar to depositors. The Ville Marie Bank failed in 1899, paying out $57\frac{3}{4}$ cents less on a dollar. How much was received in the latter case?

20. The Lachine Canal cost $11\frac{1}{6}$ million dollars for enlargement, that is, $8\frac{37}{60}$ million dollars more than for construction. Find the total cost.

21. The Beauharnois Canal cost $1\frac{3}{5}$ million dollars, and the Chambly Canal cost $\frac{87}{100}$ million dollars less. Find the cost of the two together.

22. The Lachine Canal and the Soulanges Canal together cost $21\frac{11}{20}$ million dollars, that is, $13\frac{7}{10}$ million dollars more than *all the other* canals of the Province of Quebec. What did *all* the canals cost?

23. The eight canals of the Province of Quebec cost $12\frac{19}{20}$ million dollars for construction, and $16\frac{9}{20}$ million dollars for enlargement. How much less was paid for construction than for enlargement?

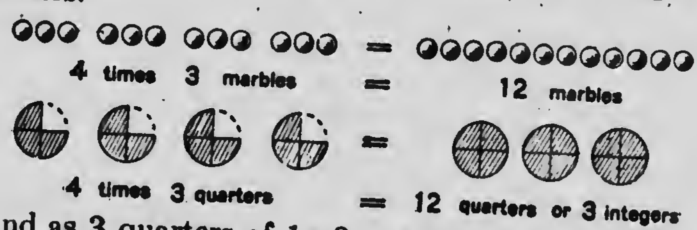
24. The total rise of the water in the Beauharnois Canal is $82\frac{1}{2}$ ft.; in the Grenville Canal it is $38\frac{3}{4}$ ft. less. Find the lockage of the Grenville Canal.

25. The lockage of the Soulanges Canal is 84 ft.; the lockage of the Chambly Canal is $8\frac{1}{2}$ ft. less. How high does the water rise in the Chambly Canal?

MULTIPLICATION OF FRACTIONS

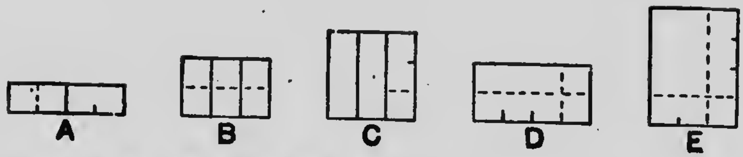
MENTAL MULTIPLICATION.

184. Multiplication is a short process of adding equal numbers.



And as 3 quarters of $1 = 3$ quarters, 3 quarters of $4 =$ 4 times 3 quarters, or 12 quarters; so $\frac{3}{4}\times 4 = 4\times\frac{3}{4}$, or $\frac{12}{4}$. But 4 may be written $\frac{4}{1}$; so $\frac{3}{4}\times\frac{4}{1} = \frac{12}{4}$, and $\frac{4}{1}\times\frac{3}{4} = \frac{12}{4}$. 12 is the product of the numerators, and 4 is the product of the denominators.

NOTE.—The word *of* written between fractions, or between fractions and integers, indicates that they are to be multiplied.



What is $\frac{1}{2}$ of $\frac{1}{2}$ of A? $\frac{1}{2}$ of $\frac{1}{3}$ of B?

Is $\frac{1}{2}$ of $\frac{1}{3}$ of B the same as $\frac{1}{3}$ of $\frac{1}{2}$ of B?

What is $\frac{1}{3}$ of $\frac{1}{3}$ of C?

Point to $\frac{1}{2}$ of D; to $\frac{1}{4}$ of $\frac{1}{2}$ of D; to $\frac{1}{2}$ of $\frac{1}{4}$ of D; to $\frac{1}{2}$ of $\frac{3}{4}$ of D; to $\frac{3}{4}$ of $\frac{1}{2}$ of D.

Point to $\frac{1}{3}$ of E; to $\frac{1}{4}$ of $\frac{1}{3}$ of E; to $\frac{1}{4}$ of E; to $\frac{1}{3}$ of $\frac{1}{4}$ of E.

Draw rectangles on your copy books and divide them so as to show:

1. $\frac{1}{5}$ of $\frac{1}{3}$; $\frac{1}{3}$ of $\frac{1}{5}$.
2. $\frac{2}{3}$ of $\frac{1}{5}$; $\frac{1}{5}$ of $\frac{2}{3}$.
3. $\frac{3}{5}$ of $\frac{2}{3}$; $\frac{2}{3}$ of $\frac{3}{5}$.
4. $\frac{1}{8}$ of $\frac{1}{2}$; $\frac{1}{2}$ of $\frac{1}{8}$.
5. $\frac{1}{2}$ of $\frac{1}{6}$; $\frac{1}{6}$ of $\frac{1}{2}$.
6. $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{2}{3}$.
7. $\frac{2}{5}$ of $\frac{1}{2}$; $\frac{1}{2}$ of $\frac{2}{5}$.
8. $\frac{3}{10}$ of $\frac{1}{2}$; $\frac{1}{2}$ of $\frac{3}{10}$.

185. Multiplying a fraction by $\frac{2}{3}$ is the same as multiplying by 2 (*Principle I, page* 00) and dividing it by 3 (*Principle III, page* 00); it is therefore finding the product of the numerators and the product of the denominators.

EXAMPLE.—$\frac{9}{8}\times\frac{2}{3}=\frac{9\times2}{8\times3}=\frac{18}{24}=\frac{3}{4}$.

Oral Exercises.

Multiply mentally using cancellation when possible:

1. $\frac{5}{9}\times 9$. **2.** $\frac{3}{4}\times 2$. **3.** $\frac{3}{8}\times 16$. **4.** $\frac{5}{8}\times 16$. **5.** $\frac{2}{3}\times 12$.
6. $12\times\frac{3}{4}$. **7.** $6\times\frac{5}{6}$. **8.** $8\times\frac{7}{8}$. **9.** $25\times\frac{2}{5}$. **10.** $25\times\frac{3}{5}$.
11. $20\times\frac{1}{10}$. **12.** $20\times\frac{3}{10}$. **13.** $12\times\frac{5}{6}$. **14.** $24\times\frac{5}{6}$. **15.** $90\times\frac{1}{9}$.
16. $\frac{1}{2}\times\frac{3}{8}$. **17.** $\frac{1}{3}\times\frac{7}{8}$. **18.** $\frac{1}{4}\times\frac{4}{7}$. **19.** $\frac{3}{4}\times\frac{7}{8}$. **20.** $\frac{3}{4}\times\frac{4}{7}$.

NOTE.—When the multiplier is a proper fraction, multiplication decreases the multiplicand.

Since the *value of one object* and the *number of times it is repeated* may be fractions, the following principles apply to fractions as well as to integers.

Principles of Analysis relating to Multiplication.

7. I multiply because I wish to find the *product* of ...... by ......

8. I multiply because having the *value of one object* and the *number of objects,* I wish to find the *total value.*

9. I multiply because I wish to find ...... *times as much as* what I have.

Oral Problems.

Seventh principle of analysis.

1. What is the *product* of 30 by $1\frac{1}{2}$? of 20 by $2\frac{1}{2}$? of 60 by $3\frac{1}{2}$?
2. What is the *product* of 40 by $4\frac{1}{2}$? of 50 by $5\frac{1}{2}$? of 10 by $6\frac{1}{2}$?
3. What is the *product* of 15 by $2\frac{1}{3}$? of 21 by $3\frac{1}{3}$? of 12 by $4\frac{1}{3}$?

Eighth principle of analysis.

Note.—Total value = total cost, total distance, total salary, total production, total output, etc.

4. One penholder is worth $\$\frac{1}{20}$; find the *value* of 6 penholders.
5. One orange costs $\$\frac{1}{30}$; find the *cost* of 9 oranges.
6. A train goes $\frac{2}{3}$ mi. in 1 minute; find the *distance* passed over in 15 minutes.
7. A man walks $\frac{1}{15}$ mi. in 1 minute; what *distance* can he go over in 12 minutes?
8. An automobile runs $\frac{3}{4}$ mi. in one minute; what *distance* does it cover in 9 minutes?
9. Albert can do $\frac{1}{4}$ of a work in 1 day; what can he do in 3 days?
10. Francis can do $\frac{2}{15}$ of a work in a day; what can he do in 4 days?
11. One ton of coal costs $\$6\frac{3}{4}$; find the cost of 4 tons.
12. My artesian well is $233\frac{1}{3}$ ft. deep; what did the boring cost at \$3 a foot?
13. A machine weaves $4\frac{2}{3}$ yd. of cloth in an hour; how many yards in 3 hours? 6 hours? 12 hours?
14. At $\$\frac{3}{4}$ a yard, what is the cost of $\frac{2}{3}$ yd.?
15. At $\$\frac{2}{3}$ a yard, what is the cost of $\frac{3}{4}$ yd.?

NINTH PRINCIPLE OF ANALYSIS.

16. In Quebec, 1909, a laborer earned $\$1\frac{1}{4}$ a day, and a plasterer earned *twice as much.* Find a plasterer's daily salary.

17. In Quebec, 1899, a plumber earned $\$1\frac{1}{2}$ a day, and a stonecutter, 2 *times as much.* Find the daily salary of a stonecutter.

18. In St. Hyacinthe, 1909, a laborer earned $\$1\frac{1}{4}$ a day, and a stonecutter, 3 *times as much.* Find the daily salary of a stonecutter.

19. In Sherbrooke, 1909, a carpenter earned $12 a week, and a locomotive engineer, $2\frac{1}{2}$ *times as much.* Find the daily salary of an engineer.

20. In St. Johns, 1913, Moka coffee cost 30 cents a pound; in St. Hyacinthe, it cost $1\frac{1}{3}$ *times as much.* How much was that?

21. In Sorel, 1913, green tea cost 30 cents a pound; in St. Hyacinthe, it cost $1\frac{1}{2}$ *times as much.* How much was that?

22. Andrew can do $\frac{3}{8}$ of a work in 1 day, and George can do *twice as much.* What can they do together in 1 day?

23. In St. Hyacinthe, 1913, milk cost $6\frac{1}{4}$ cents a quart; in Quebec, it cost $1\frac{1}{2}$ *times as much.* What was the price in Quebec?

24. There are 24 members in the Legislative Council of Quebec, and $3\frac{5}{8}$ *times as many* members in the Senate of Ottawa. Find the number of senators.

25. In 1901, the Province of Quebec spent 6 thousand dollars for good roads, and in 1911, $15\frac{5}{8}$ *times as much* money. Find the latter expenditure.

WRITTEN MULTIPLICATION.

186. CANCELLATION METHOD.

EXAMPLE I.—Multiply 1° $\frac{13}{18}$ by 36; 2° 36 by $\frac{13}{18}$; 3° $\frac{17}{18}$ by $\frac{6}{11}$; 4° $\frac{6}{11}$ by $\frac{17}{18}$; 5° $\frac{3}{4}$ by $\frac{8}{15}$ by $\frac{3}{7}$.

OPERATIONS.

1. $\dfrac{13\times\overset{2}{\cancel{36}}}{\cancel{18}\times 1}=26.$ 2. $\dfrac{\overset{2}{\cancel{36}}\times 13}{1\times\cancel{18}}=26.$ 3. $\dfrac{17\times\cancel{6}}{\underset{3}{\cancel{18}}\times 11}=\frac{17}{33}.$

4. $\dfrac{\overset{}{\cancel{6}}\times 17}{11\times \underset{3}{\cancel{18}}}=\frac{17}{33}$ 5. $\dfrac{\cancel{3}\times \overset{2}{\cancel{8}}\times 3}{\cancel{4}\times \underset{5}{\cancel{15}}\times 7}=\frac{6}{35}$.

NOTE.—In practice the denominator 1 is suppressed.

EXAMPLE II.—Multiply 1° 5 by $8\frac{3}{11}$; 2° $5\frac{1}{7}$ by 8; 3° $5\frac{1}{7}$ by $8\frac{3}{11}$.

OPERATIONS.

1. $\dfrac{5\times 91}{1\times 11}=\dfrac{455}{11}=41\frac{4}{11}$. 2. $\dfrac{36\times 8}{7\times 1}=\dfrac{288}{7}=41\frac{1}{7}$.

3. $\dfrac{36\times \overset{13}{\cancel{91}}}{\cancel{7}\times 11}=\dfrac{468}{11}=42\frac{6}{11}$.

187. Rule.—*Multiply the numerators together and the denominators together, cancelling common factors.*

188. NOTE.—If one or both factors are *small* mixed numbers, reduce to improper fractions and employ the above rule.

Written Exercises.

Multiply:

1. $\frac{5}{16}\times 38$.
2. $\frac{11}{12}\times 66$.
3. $\frac{7}{8}\times 84$.
4. $\frac{15}{16}\times 5$.
5. $\frac{19}{20}\times 50$.
6. $\frac{21}{22}\times 66$.
7. $\frac{12}{25}\times 75$.
8. $\frac{13}{20}\times 48$.
9. $\frac{15}{19}\times 57$.
10. $\frac{19}{21}\times 84$.
11. $24\times\frac{5}{7}$.
12. $81\times\frac{8}{9}$.
13. $96\times\frac{15}{16}$.
14. $273\times\frac{80}{91}$.
15. $286\times\frac{25}{26}$.
16. $6\frac{2}{5}\times 2\frac{3}{11}\times 7\frac{8}{9}$.
17. $8\frac{7}{10}\times 2\frac{7}{9}\times 2\frac{2}{3}$.
18. $10\frac{1}{2}\times 1\frac{1}{3}\times 7\frac{3}{4}$.
19. $14\frac{1}{7}\times 17\frac{3}{5}\times 8\frac{2}{5}$.
20. $27\frac{15}{16}\times 42\frac{2}{3}\times 6\frac{1}{4}$.
21. $2\frac{1}{2}\times 3\frac{3}{4}\times 4\frac{4}{5}\times 5\frac{5}{6}$.
22. $6\frac{3}{7}\times 5\frac{4}{5}\times 7\frac{7}{8}\times 2\frac{3}{10}$.
23. $20\frac{2}{3}\times 20\frac{4}{7}\times 20\frac{11}{18}\times 10$.
24. $7\frac{31}{32}\times 15\times 2\frac{18}{55}\times 12\frac{9}{17}$.
25. $172\frac{1}{2}\times 2\frac{7}{25}\times 3\frac{7}{8}\times 2\frac{1}{4}$.

189. Direct method.—With *large* mixed numbers, direct multiplication is resorted to.

Examples.—Multiply 1° $179\frac{3}{4}$ by 7; 2° 118 by $99\frac{3}{4}$; 3° $79\frac{3}{11}$ by $15\frac{2}{5}$.

Operations.

1.
$$\begin{array}{rl} 179\frac{3}{4} & \\ 7 & \\ \hline 5\frac{1}{4} & (=\frac{3}{4}\times 7) \\ 1\ 253 & (=179\times 7) \\ \hline 1\ 258\frac{1}{4} & (=179\frac{3}{4}\times 7) \end{array}$$

2.
$$\begin{array}{rl} 118 & \\ 99\frac{3}{4} & \\ \hline 88\frac{1}{2} & (=118\times\frac{3}{4}) \\ \left.\begin{array}{r} 1062\ \\ 1062\ \ \end{array}\right\} & (=118\times 99) \\ \hline 11770\frac{1}{2} & (=118\times 99\frac{3}{4}) \end{array}$$

3.
$$\begin{array}{rrl} & 79\frac{3}{11} & \\ & 15\frac{2}{5} & \\ \cline{2-2} & & \underline{55} \\ (\frac{3}{11}\times\frac{2}{5}=) & \frac{6}{55} & \left.\begin{array}{r} =\ 6 \\ =33 \\ =\ 5 \end{array}\right\}=\frac{44}{55}=\frac{4}{5}. \\ (79\times\frac{2}{5}=) & 31\frac{3}{5} & \\ (\frac{3}{11}\times 15=) & 4\frac{1}{11} & \\ (79\times 15=) & 395 & \\ & 79 & \\ \hline (79\frac{3}{11}\times 15\frac{2}{5}=) & 1\ 220\frac{4}{5}. & \end{array}$$

190. Rule.—*Multiply the multiplicand* (fraction and integers) *by the fraction of the multiplier, then by the integers of the multiplier; unite the results.*

Note.—Reduce fractions to lowest terms before and after multiplying.

Written Exercises.

Multiply:

1. $137\frac{1}{6}\times 15$.
2. $248\frac{5}{6}\times\ 8$.
3. $630\frac{1}{9}\times 12$.
4. $560\frac{1}{4}\times 16$.
5. $730\frac{1}{5}\times 20$.
6. $481\frac{13}{18}\times 36$.
7. $119\frac{5}{8}\times\ 64$.
8. $827\frac{1}{16}\times\ 8$.
9. $275\ \times 413\frac{3}{8}$.
10. $897\ \times\ 37\frac{5}{8}$.
11. $495\ \times\ 31\frac{5}{6}$.
12. $171\ \times\ 35\frac{2}{7}$.
13. $490\times 98\frac{3}{7}$.
14. $360\times 97\frac{3}{8}$.
15. $320\times 89\frac{1}{16}$.
16. $820\times 87\frac{1}{11}$.
17. $45\frac{4}{9}\times 18\frac{1}{6}$.

18.

$$\left.\begin{array}{rl} \text{I} & 27\tfrac{3}{4} \\ \text{II} & 31\tfrac{1}{4} \\ \text{III} & 45\tfrac{1}{2} \\ \text{IV} & 56\tfrac{3}{4} \\ \text{V} & 38\tfrac{1}{2} \end{array}\right\} \times \left\{\begin{array}{ll} a) & 7\tfrac{1}{2}. \\ b) & 8\tfrac{3}{4}. \\ c) & 9\tfrac{1}{4}. \\ d) & 6\tfrac{3}{4}. \\ e) & 7\tfrac{3}{4}. \end{array}\right.$$

19.

$$\left.\begin{array}{rl} \text{I} & 72\tfrac{3}{4} \\ \text{II} & 42\tfrac{1}{2} \\ \text{III} & 63\tfrac{1}{4} \\ \text{IV} & 16\tfrac{3}{4} \\ \text{V} & 22\tfrac{1}{2} \end{array}\right\} \times \left\{\begin{array}{ll} a) & 10\tfrac{1}{4}. \\ b) & 9\tfrac{1}{2}. \\ c) & 7\tfrac{1}{2}. \\ d) & 15\tfrac{1}{2}. \\ e) & 18\tfrac{3}{4}. \end{array}\right.$$

20.

$$\left.\begin{array}{rl} \text{I} & 26\tfrac{1}{4} \\ \text{II} & 32\tfrac{3}{4} \\ \text{III} & 49\tfrac{1}{2} \\ \text{IV} & 38\tfrac{1}{2} \\ \text{V} & 59\tfrac{3}{4} \end{array}\right\} \times \left\{\begin{array}{ll} a) & 26\tfrac{1}{4}. \\ b) & 31\tfrac{1}{2}. \\ c) & 27\tfrac{3}{4}. \\ d) & 25\tfrac{1}{2}. \\ e) & 19\tfrac{1}{4}. \end{array}\right.$$

Successively multiply I, II, III, IV, V by *a*; then successively multiply I, II, etc., by *b*; and so on by *c*, by *d*, by *e*.

191. Short methods in multiplication.

$4\times2\frac{1}{2}=10$; to multiply by $2\frac{1}{2}$, add 0 and divide by 4.

$3\times3\frac{1}{3}=10$; to multiply by $3\frac{1}{3}$, add 0 and divide by 3.

$16\times6\frac{1}{4}=100$; to multiply by $6\frac{1}{4}$, add 00 and divide by 16 $(=4\times4)$.

$15\times6\frac{2}{3}=100$; to multiply by $6\frac{2}{3}$, add 00 and divide by 15 $(=5\times3)$.

$12\times8\frac{1}{3}=100$; to multiply by $8\frac{1}{3}$, add 00 and divide by 12 $(=3\times4)$.

$8\times12\frac{1}{2}=100$; to multiply by $12\frac{1}{2}$, add 00 and divide by 8.

$6\times16\frac{2}{3}=100$; to multiply by $16\frac{2}{3}$, add 00 and divide by 6.

$3\times33\frac{1}{3}=100$; to multiply by $33\frac{1}{3}$, add 00 and divide by 3.

1.

$$\left.\begin{array}{rl} \text{I} & 720 \\ \text{II} & 840 \\ \text{III} & 900 \\ \text{IV} & 960 \\ \text{V} & 108 \\ \text{VI} & 288 \\ \text{VII} & 312 \\ \text{VIII} & 576 \end{array}\right\} \times \left\{\begin{array}{ll} a) & 2\tfrac{1}{2}. \\ b) & 3\tfrac{1}{3}. \\ c) & 6\tfrac{1}{4}. \\ d) & 6\tfrac{2}{3}. \\ e) & 8\tfrac{1}{3}. \\ f) & 12\tfrac{1}{2}. \\ g) & 16\tfrac{2}{3}. \\ h) & 33\tfrac{1}{3}. \end{array}\right.$$

2.

$$\left.\begin{array}{rl} \text{I} & 252 \\ \text{II} & 276 \\ \text{III} & 348 \\ \text{IV} & 372 \\ \text{V} & 492 \\ \text{VI} & 588 \\ \text{VII} & 612 \\ \text{VIII} & 708 \end{array}\right\} \times \left\{\begin{array}{ll} a) & 2\tfrac{1}{2}. \\ b) & 3\tfrac{1}{3}. \\ c) & 6\tfrac{1}{4}. \\ d) & 6\tfrac{2}{3}. \\ e) & 8\tfrac{1}{3}. \\ f) & 12\tfrac{1}{2}. \\ g) & 16\tfrac{2}{3}. \\ h) & 33\tfrac{1}{3}. \end{array}\right.$$

Multiply I, II, III, IV, V, VI, VII, VIII successively by *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, employing the abridged method.

192. *To multiply by itself a mixed number ending in* $\frac{1}{2}$.

EXAMPLE.

$3\frac{1}{2}$	$(3\times4)=12.$	Multiply the integer by itself plus 1, and add $\frac{1}{4}$.
$3\frac{1}{2}$	$(\frac{1}{2}\times\frac{1}{2})=\frac{1}{4}.$	
$12\frac{1}{4}$		

Multiply by itself:

1. $4\frac{1}{2}$.	**4.** $7\frac{1}{2}$.	**7.** $10\frac{1}{2}$.	**10.** $13\frac{1}{2}$.	**13.** $20\frac{1}{2}$.
2. $5\frac{1}{2}$.	**5.** $8\frac{1}{2}$.	**8.** $11\frac{1}{2}$.	**11.** $14\frac{1}{2}$.	**14.** $30\frac{1}{2}$.
3. $6\frac{1}{2}$.	**6.** $9\frac{1}{2}$.	**9.** $12\frac{1}{2}$.	**12.** $15\frac{1}{2}$.	**15.** $40\frac{1}{2}$.

193. *To multiply together any two mixed numbers ending in* $\frac{1}{2}$.

EXAMPLE I.

$13\frac{1}{2}$	$(13\times7)=91$
$7\frac{1}{2}$	$(13+7)\div2=10$
$101\frac{1}{4}$	$(\frac{1}{2}\times\frac{1}{2})=\frac{1}{4}$

EXAMPLE II.

$13\frac{1}{2}$	$(13\times8)=104$
$8\frac{1}{2}$	$\left.\begin{array}{r}(13+8)\div2=10\frac{1}{2}\\(\frac{1}{2}\times\frac{1}{2})=\frac{1}{4}\end{array}\right\}=\frac{3}{4}$
$114\frac{3}{4}$	

To the product of the integers add half their sum plus the fraction $\frac{1}{4}$.

Remark that when the sum of the integers is an uneven number, you have $\frac{3}{4}$ instead of $\frac{1}{4}$ in the answer.

In the following multiply I, II, III, IV, V successively by *a*, *b*, *c*, *d*, *e*, employing the above method.

1.

$$\left.\begin{array}{rl}\text{I} & 12\frac{1}{2}\\ \text{II} & 14\frac{1}{2}\\ \text{III} & 16\frac{1}{2}\\ \text{IV} & 18\frac{1}{2}\\ \text{V} & 20\frac{1}{2}\end{array}\right\}\times\left\{\begin{array}{lr}a) & 4\frac{1}{2}.\\ b) & 6\frac{1}{2}.\\ c) & 8\frac{1}{2}.\\ d) & 10\frac{1}{2}.\\ e) & 2\frac{1}{2}.\end{array}\right.$$

2.

$$\left.\begin{array}{rl}\text{I} & 10\frac{1}{2}\\ \text{II} & 20\frac{1}{2}\\ \text{III} & 30\frac{1}{2}\\ \text{IV} & 40\frac{1}{2}\\ \text{V} & 50\frac{1}{2}\end{array}\right\}\times\left\{\begin{array}{lr}a) & 3\frac{1}{2}.\\ b) & 5\frac{1}{2}.\\ c) & 7\frac{1}{2}.\\ d) & 9\frac{1}{2}.\\ e) & 11\frac{1}{2}.\end{array}\right.$$

Written Problems.

1. Find the cost of $18\frac{3}{4}$ yd. of ribbon at $31\frac{1}{4}$ cents a yard.

2. Find the cost of $27\frac{2}{3}$ acres of land at \$$45\frac{1}{5}$ an acre.

3. If a boat sails 246 mi. a day, how far will she sail in $26\frac{1}{3}$ days?

4. If an auto goes $15\frac{2}{3}$ mi. an hour, how far will it go in $12\frac{1}{3}$ hours?

5. If a train runs $31\frac{1}{4}$ mi. an hour, how far will it run in $8\frac{1}{4}$ hours?

6. If a hurricane moves on at the rate of 90 mi. an hour, what distance will it cover in $3\frac{1}{3}$ hours?

7. Henry earns \$$14\frac{3}{4}$ a week. What does he earn in a year if he loses $6\frac{1}{2}$ weeks through sickness?

8. If a railway passenger car cost \$$7\ 034\frac{3}{4}$, what will 386 cars cost?

9. The area of the Ottawa Experimental Farm is 466 acres. What is its value at \$$134\frac{1}{2}$ an acre?

10. At \$$6\frac{1}{20}$ each, find the value of the 24 025 tons of copper ore produced by the Province of Quebec in 1910.

11. In 1911, the operating expenses of the Grand Trunk R. R. Co. were \$$946\frac{1}{2}$ per mile of line. Find the expenditure for the 3 545 mi. in operation.

12. A boy buys his newspapers at $1\frac{1}{2}$ cents and sells them at 2 cents apiece. Find his net earnings if he sells 80 a day.

13. A clerk earns \$75 a month. What does he save in 7 months if his monthly expenses are \$$27\frac{2}{5}$ for board and lodging, \$$3\frac{3}{4}$ fow washing, and \$$15\frac{3}{4}$ for traveling, newspapers, etc.?

14. For $3\frac{3}{4}$ years, Henry has spent 10 cents a day on cigarettes. If he had deposited that money in the School Savings Bank, how much would he have, exclusive of interest?

15. A steamer has 2 750 passengers aboard. How many pounds of meat are required for a voyage of $6\frac{1}{2}$ days, if each passenger gets $\frac{7}{8}$ lb. a day?

16. What sum was received for $100\frac{7}{8}$ tons of iron ore, if $40\frac{3}{4}$ tons were sold at \$$42\frac{1}{2}$ each, and the rest at \$$40\frac{1}{4}$?

17. In France, 1880, alcohol killed 5 000 persons; in 1898 $2\frac{9}{10}$ times as many. Find the number of victims for 1898.

18. In Quebec City, 1900, the Bell Telephone Co. had 1 650 subscribers; in 1910, $2\frac{5}{11}$ times as many. Find how many.

19. In a recent year, the average cost of tilling on the Ottawa Experimental Farm was $\$11\frac{3}{4}$ per acre; if the harvest was worth $3\frac{6}{7}$ times as much, find the average yield per acre.

20. In 1906, the Quebec Government was paid $66 000 for fishing and shooting licenses; and in 1909, $1\frac{5}{22}$ times as much. Find the amount for 1909.

21. The falls of the Chaudière River are 114 ft. high; the falls of the Hamilton River, $2\frac{14}{19}$ times as high. Find the height in the latter case.

22. In 1909, Canada exported $27 035 worth of mica; in 1910 $2\frac{1}{6}$ times as much. Find the value of the 1910 exports.

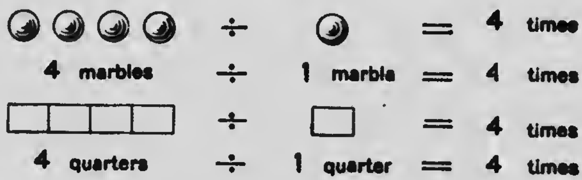
23. In 1909, the United States produced 2 835 tons of graphite, and imported $7\frac{1}{7}$ times as much. How many tons were imported?

24. Russia annually produces close upon 12 000 tons of asbestos; and Quebec, $6\frac{2}{3}$ times as much. Find the Quebec production of asbestos.

25. The distance by rail from Montreal to Vaudreuil is $24\frac{1}{2}$ mi.; from Montreal to Brockville, it is $5\frac{1}{6}$ times as great. How far is it from Montreal to Brockville?

DIVISION OF FRACTIONS

MENTAL DIVISION.



$\frac{4}{4}\div\frac{1}{4}=4$ times; but $\frac{4}{4}=1$; so, $1\div\frac{1}{4}=4$ or $\frac{4}{1}$. If $1\div\frac{1}{4}=\frac{4}{1}$, $2\div\frac{1}{4}=2$ times $\frac{4}{1}$ or $\frac{4\times2}{1\times1}=\frac{8}{1}=8$.

If $1\div\frac{1}{4}=\frac{4}{1}$, $\frac{2}{3}\div\frac{1}{4}=\frac{2}{3}$ times $\frac{4}{1}$, or $\frac{4\times2}{1\times3}=\frac{8}{3}=2\frac{2}{3}$.

 = 4 over 3 or $^4/_3$

4 quarters + 3 quarters = 4 over 3 or $^4/_3$

$\frac{4}{4} \div \frac{3}{4} = \frac{4}{3}$; but $\frac{4}{4} = 1$; so $1 \div \frac{3}{4} = \frac{4}{3}$.

If $1 \div \frac{3}{4} = \frac{4}{3}$, $3 \div \frac{3}{4} = 3$ times $\frac{4}{3}$ or $\frac{4 \times 3}{3 \times 1} = \frac{12}{3} = 4$.

If $1 \div \frac{3}{4} = \frac{4}{3}$, $\frac{3}{5} \div \frac{3}{4} = \frac{3}{5}$ times $\frac{4}{3}$ or $\frac{4 \times 3}{3 \times 5} = \frac{12}{15} = \frac{4}{5}$.

194. Dividing by a fraction is multiplying by that fraction inverted.

Dividing by an integer is dividing by a fraction whose denominator is 1.

Thus, $\frac{2}{3} \div 4 = \frac{2}{3} \div \frac{4}{1} = \frac{2 \times 1}{3 \times 4} = \frac{2}{12} = \frac{1}{6}$.

195. When the divisor is a proper fraction, division increases the dividend.

Oral Exercises.

Divide:

1. $1 \div \frac{1}{2}$.	**11.** $3 \div \frac{1}{3}$.	**21.** $\frac{2}{3} \div \frac{1}{2}$.	**31.** $\frac{1}{2} \div 2$.
2. $1 \div \frac{1}{3}$.	**12.** $3 \div \frac{1}{5}$.	**22.** $\frac{2}{3} \div \frac{3}{4}$.	**32.** $\frac{1}{2} \div 3$.
3. $1 \div \frac{1}{4}$.	**13.** $3 \div \frac{1}{6}$.	**23.** $\frac{2}{3} \div \frac{3}{5}$.	**33.** $\frac{1}{2} \div 4$.
4. $1 \div \frac{1}{5}$.	**14.** $3 \div \frac{1}{7}$.	**24.** $\frac{2}{3} \div \frac{6}{7}$.	**34.** $\frac{1}{2} \div 5$.
5. $1 \div \frac{1}{6}$.	**15.** $3 \div \frac{1}{8}$.	**25.** $\frac{2}{3} \div \frac{9}{5}$.	**35.** $\frac{1}{2} \div 6$.
6. $2 \div \frac{1}{3}$.	**16.** $\frac{1}{2} \div \frac{1}{2}$.	**26.** $\frac{3}{4} \div \frac{1}{2}$.	**36.** $1\frac{1}{2} \div 1\frac{1}{2}$.
7. $2 \div \frac{1}{4}$.	**17.** $\frac{1}{2} \div \frac{1}{4}$.	**27.** $\frac{3}{4} \div \frac{1}{3}$.	**37.** $1\frac{1}{3} \div 1\frac{1}{3}$.
8. $2 \div \frac{1}{5}$.	**18.** $\frac{1}{2} \div \frac{1}{3}$.	**28.** $\frac{3}{4} \div \frac{2}{3}$.	**38.** $1\frac{3}{4} \div 1\frac{3}{4}$.
9. $2 \div \frac{1}{6}$.	**19.** $\frac{1}{2} \div \frac{1}{5}$.	**29.** $\frac{4}{5} \div \frac{5}{4}$.	**39.** $1\frac{9}{11} \div 1\frac{9}{11}$.
10. $2 \div \frac{1}{7}$.	**20.** $\frac{1}{2} \div \frac{1}{6}$.	**30.** $\frac{7}{8} \div \frac{5}{8}$.	**40.** $2\frac{1}{2} \div 2\frac{1}{2}$.

Principles of Analysis relating to Division.

10. I divide because I wish to find the *quotient* of ...... by .....

11. I divide because having the *total value* and the *number of objects*, I wish to find the *value of one object*.

12. I divide because having the *total value* and the *value of one object*, I wish to find the *number of objects*.

13. I divide because I wish to find ...... *times less than* what I have.

NOTE.—The *number of objects*, or the *value of one*, or the *total value*, may be fractions; the application of these principles is always the same.

Oral Problems.

TENTH PRINCIPLE OF ANALYSIS.

1. Find the *quotient* of $\frac{4}{15}$ by 4.
2. Find the *quotient* of $\frac{4}{7}$ by 5
3. Find the *quotient* of $1\frac{1}{2}$ by 7.

ELEVENTH PRINCIPLE OF ANALYSIS.

4. A workman earns $\$\frac{3}{4}$ in 3 hours; what does he earn in *one* hour?

5. A train goes $4\frac{1}{2}$ mi. in 10 minutes; what distance does it cover in *one* minute?

6. I paid $\$\frac{3}{4}$ for 2 lb. of coffee. Find the price of *one* pound.

7. Find the cost of *one* pencil, if 6 were paid $\$\frac{1}{2}$.

8. If 5 quarts of milk cost $\$\frac{5}{8}$, what will *one* quart cost?

9. If I can do $\frac{1}{4}$ of a work in 2 hours, what can I do in *one* hour?

10. If a train runs 100 mi. in $2\frac{1}{2}$ hours, what is its rate *per hour*?

11. I paid \$15 for $7\frac{1}{2}$ yd. of silk. Find the price of *one* yard.

12. What is the price of *one* cord of wood when $12\frac{1}{2}$ cords cost \$50?

13. A girl can knit a stocking in three days, what can she do in *one* day?

14. If a man can saw a cord of wood in 4 days, what can he do in *one* day?

15. A mason did a work in 6 days, what part of the work did he do in *one* day?

16. If $\frac{2}{3}$ yd. of cotton cost $\$\frac{1}{3}$, what is the price of *one* yard?

17. $\frac{1}{2}$ yd. of cloth costs $\$\frac{1}{3}$; find the price of *one* yard.

18. I paid $\$\frac{1}{4}$ for $\frac{1}{8}$ lb. of medicine; find the price of *one* pound.

19. I paid $\$\frac{1}{20}$ for $\frac{3}{4}$ lb. of sugar; find the price *per pound*.

TWELFTH PRINCIPLE OF ANALYSIS.

20. A laborer earns $\$2\frac{1}{2}$ a day; in *how many days* will he earn \$10? \$5? $\$\frac{1}{2}$?

21. A penholder costs $\$\frac{1}{30}$; *how many* can I buy for $\$\frac{1}{2}$? $\$\frac{1}{10}$? $\$\frac{1}{4}$?

22. A pedestrian goes $\frac{1}{5}$ mi. in one minute; in *how many minutes* will he go 1 mi.?

23. A farmer mows $\frac{1}{4}$ of a field in 1 day; *how many days* will he take to mow $\frac{3}{4}$ of the field?

24. If a pound of brown sugar costs $4\frac{1}{2}$ cents, *how many pounds* can be bought for 18 cents? 9 cents?

25. A laborer earns $\$1\frac{1}{2}$ a day; in *how many days* will he earn $\$3$? $\$6$? $\$\frac{1}{2}$?

26. At $\$1\frac{1}{3}$ each, how *many yards* of silk can be bought for $\$4$? $\$2\frac{2}{3}$? $\$\frac{1}{3}$?

27. At $\$7\frac{1}{2}$ each, *how many tons* of coal can be bought for $\$15$? $\$1\frac{1}{2}$? $\$\frac{1}{2}$?

28. At $\$\frac{1}{3}$ a pound, *how much* butter can be bought for $\$1$? $\$10$? $\$\frac{1}{2}$?

29. If a sheet of zinc is $\frac{1}{16}$ in. thick, *how many sheets* are there in a thickness of 1 in.? $1\frac{1}{4}$ in.? $\frac{1}{2}$ in.?

30. A pad is $\frac{1}{16}$ in. thick. *How many pads* are there in a pile 5 in. thick?

31. *How many boards* each $\frac{3}{4}$ in. thick are there in a pile 12 in. high?

32. *How many bricks* each $2\frac{1}{2}$ in. thick are there in a pile 15 in. high?

33. A machine can weave 4 yd. of cloth in an hour; in *what time* can it weave 10 yd.? 5 yd.? $\frac{1}{2}$ yd.?

34. A boy gets $\$\frac{1}{20}$ for each errand he runs. *How many errands* entitle him to $\$\frac{1}{5}$?

THIRTEENTH PRINCIPLE OF ANALYSIS.

35. Arthur's salary is $\$6\frac{2}{3}$ a week; find John's salary if it is 2 times *less*.

36. I have $\$4\frac{1}{2}$, and my brother, 9 times *less* than I. How much has he?

37. An orange costs $\$\frac{1}{40}$, and an apple, 2 times *less*. How much is that?

38. My dictionary is $\frac{3}{16}$ ft. thick, and my arithmetic is 3 times *less* thick. How much is that?

39. A train runs $\frac{1}{2}$ mi. in 1 minute; another goes 2 times *less* fast. Find its speed per minute.

40. I have $\$5$. If my sister has $2\frac{1}{2}$ times *less* money than I, how much has she?

WRITTEN DIVISION.

196. CANCELLATION METHOD.

EXAMPLES.—Divide 1° $\frac{13}{14}$ by 5; 2° $\frac{13}{14}$ by $\frac{1}{7}$; 3° $\frac{13}{14}$ by $\frac{5}{7}$.

OPERATIONS.

$$1.\ \frac{13}{14\times5}=\frac{13}{70}.\qquad 2.\ \frac{13\times\cancel{7}}{\underset{2}{\cancel{14}}}=\frac{13}{2}=6\tfrac{1}{2}.\qquad 3.\ \frac{13\times\cancel{7}}{\underset{2}{\cancel{14}}\times5}=\frac{13}{10}=1\tfrac{3}{10}.$$

EXPLANATIONS.—1. Dividing $\frac{13}{14}$ by 5 is multiplying the denominator by 5 (*Principle III, page* 118).

2. Dividing $\frac{13}{14}$ by $\frac{1}{7}$ is dividing by 7 *times less than* 1; it is therefore obtaining a quotient 7 *times as great as* $\frac{13}{14}$ divided by 1, or $\frac{13\times7}{14\times1}$, or $6\frac{1}{2}$.

3. Dividing $\frac{13}{14}$ by $\frac{5}{7}$ is dividing by 7 *times less than* 5; it is therefore obtaining a quotient 7 *times as great as* $\frac{13}{14}\div5$, or $\frac{13\times7}{14\times5}$ or $1\frac{3}{10}$.

197. Rule.—*Multiply by the fraction inverted.*

NOTE I.—If the dividend or the divisor is a whole number, it may be expressed fractionally by writing 1 under it as a denominator.

NOTE II.—Reduce mixed numbers to improper fractions.

Written Exercises.

Divide:

1. $\frac{13}{14}\div\frac{6}{7}$.
2. $\frac{15}{16}\div\frac{11}{24}$.
3. $\frac{25}{36}\div\frac{15}{16}$.
4. $\frac{65}{72}\div\frac{5}{8}$.
5. $\frac{49}{108}\div\frac{91}{132}$.
6. $18\div\frac{6}{7}$.
7. $25\div\frac{8}{9}$.
8. $40\div\frac{25}{36}$.
9. $48\div\frac{15}{16}$.
10. $56\div\frac{16}{3}$.
11. $31\div\frac{22}{9}$.
12. $50\div\frac{40}{3}$.
13. $21\div\frac{7}{8}$.
14. $60\div\frac{8}{9}$.
15. $18\div\frac{2}{9}$.
16. $\frac{4}{7}\div21$.
17. $\frac{9}{10}\div27$.
18. $\frac{1}{15}\div21$.
19. $\frac{7}{15}\div35$.
20. $\frac{15}{7}\div75$.
21. $\frac{19}{3}\div95$.
22. $\frac{21}{4}\div84$.
23. $22\frac{3}{4}\div2\frac{5}{8}$.
24. $6\frac{3}{16}\div8\frac{5}{8}$.
25. $60\div1\frac{15}{32}$.
26. $10\frac{1}{8}\div6\frac{3}{4}$.
27. $3\frac{1}{2}\div2\frac{1}{5}$.
28. $(2\frac{1}{3}\times\frac{1}{4})\div(1$ [illegible] $\times\frac{1}{5})$.
29. $(28\frac{1}{2}\times\frac{3}{19})\div($ [illegible] $)$.
30. $($ [illegible] $\frac{11}{19}\times\frac{38}{44})\div($ [illegible] $\times\frac{36}{21})$.

198. Direct Method.—It is resorted to when large mixed numbers are dealt with.

First case.—Short division.—*To divide a mixed number by a small integer.*

Examples.—Divide 1° 5 $184\frac{18}{23}$ by 9; 2° 4 $853\frac{1}{4}$ by 3.

Operations.

1. $9)\ \underline{5\,184\frac{18}{23}}$
   $576\frac{2}{23}$.

2. $3)\ \underline{4\,853\frac{1}{4}}$
   $1\,617\frac{3}{4}$.

Explanations.

1.—9 exactly divides 5 184; $\frac{18}{23}$ divided by 9 $= \frac{18 \div 9}{23} = \frac{2}{23}$ (*Principle II, page 118*), or $\frac{18}{23 \times 9} = \frac{2}{23}$ (*Principle III, page 118*).

2.—Dividing by 3 leaves a remainder of $2\frac{1}{4}$; $2\frac{1}{4} = \frac{9}{4}$; $\frac{9}{4} \div 3 = \frac{3}{4}$.

199. Rule.—*Separately divide the integers and the fractional part by the divisor, and combine the results.*

Written Exercises.

Divide, employing short division:

1. $6\,532\frac{11}{12} \div 8$.	**6.** $3\,936\frac{1}{4} \div 3$.	**11.** $7\,632\frac{1}{2} \div 8$.
2. $25\,681\frac{2}{11} \div 7$.	**7.** $10\,935\frac{3}{5} \div 4$.	**12.** $5\,937\frac{1}{4} \div 9$.
3. $1\,286\frac{6}{7} \div 9$.	**8.** $5\,961\frac{1}{3} \div 10$.	**13.** $8\,396\frac{1}{5} \div 7$.
4. $6\,875\frac{9}{10} \div 5$.	**9.** $4\,428\frac{13}{15} \div 12$.	**14.** $9\,360\frac{1}{9} \div 5$.
5. $9\,532\frac{11}{12} \div 6$.	**10.** $9\,720\frac{2}{3} \div 11$.	**15.** $8\,398\frac{1}{11} \div 7$.

Divide, making successive short divisions.

Thus, to divide a number by 81, divide it by 9, and then divide the quotient by 9.

(1)

$$\left.\begin{array}{r} \text{I}\ \ 180\frac{1}{4} \\ \text{II}\ \ 360\frac{1}{2} \\ \text{III}\ \ 720\frac{1}{3} \\ \text{IV}\ \ 270\frac{1}{5} \\ \text{V}\ \ 540\frac{1}{3} \end{array}\right\} \div \left\{\begin{array}{ll} a) & 18. \\ b) & 27. \\ c) & 36. \\ d) & 45. \\ e) & 54. \end{array}\right.$$

(2)

$$\left.\begin{array}{r} \text{I}\ \ 144\frac{1}{2} \\ \text{II}\ \ 288\frac{1}{3} \\ \text{III}\ \ 216\frac{1}{2} \\ \text{IV}\ \ 432\frac{1}{3} \\ \text{V}\ \ 504\frac{1}{4} \end{array}\right\} \div \left\{\begin{array}{ll} a) & 63. \\ b) & 72. \\ c) & 81. \\ d) & 64. \\ e) & 56. \end{array}\right.$$

Divide I, II, III, IV, V, successively by *a*, *b*, *c*, *d*, *e*, using the factors of the divisor.

Second case.—LONG DIVISION.—*To divide a mixed number by a mixed number.*

EXAMPLE.—Divide $683\frac{2}{3}$ by $44\frac{3}{5}$.

OPERATION.	
$44\frac{3}{5}$)	$683\frac{2}{3}$ (
15	15
9	10
220	3415
44	683

669) 10255 ($15\frac{220}{669}$.

EXPLANATION.

$683\frac{2}{3} \div 44\frac{3}{5}$ may be written fractionally thus: $\frac{683\frac{2}{3}}{44\frac{3}{5}}$; if I multiply the numerator and the denominator by 15 (15 is the L. C. D. of $\frac{2}{3}$ and $\frac{3}{5}$), I obtain two integral terms: $\frac{683\frac{2}{3} \times 15}{44\frac{3}{5} \times 15} = \frac{10255}{669}$, and then I divide as with other integers.

200. Rule.—*1° Multiply both dividend and divisor by the least common denominator of their terminal fractions; 2° Divide as with other integers.*

Written Exercises.

Divide:

1. $75\frac{2}{3} \div 14\frac{1}{2}$.
2. $465\frac{1}{2} \div 18\frac{3}{4}$.
3. $219\frac{2}{5} \div 12\frac{3}{5}$.
4. $926\frac{1}{4} \div 19\frac{5}{6}$.
5. $783\frac{2}{9} \div 28\frac{1}{3}$.
6. $516\frac{1}{12} \div 35\frac{1}{3}$.
7. $791\frac{5}{8} \div 16\frac{2}{5}$.
8. $415\frac{1}{2} \div 70\frac{1}{3}$.
9. $517\frac{2}{15} \div 92\frac{1}{15}$.
10. $782\frac{1}{9} \div 38\frac{1}{3}$.
11. $836\frac{1}{2} \div 13$.
12. $937\frac{1}{4} \div 17$.
13. $833\frac{1}{5} \div 19$.
14. $237\frac{1}{16} \div 23$.
15. $340\frac{1}{15} \div 29$.
16. $341\frac{1}{9} \div 31$.
17. $450\frac{1}{7} \div 37$.
18. $300\frac{1}{2} \div 41$.
19. $400\frac{1}{3} \div 43$.
20. $500\frac{1}{4} \div 47$.
21. $880\frac{1}{6} \div 53$.
22. $395\frac{1}{10} \div 59$.
23. $387\frac{1}{8} \div 61$.
24. $210\frac{1}{2} \div 67$.
25. $110\frac{1}{9} \div 71$.
26. $230 \div 18\frac{1}{2}$.
27. $436 \div 17\frac{1}{3}$.
28. $377 \div 16\frac{1}{4}$.
29. $830 \div 20\frac{1}{5}$.
30. $943 \div 19\frac{1}{6}$.

COMPLEX FRACTIONS.

201. A *complex fraction* is one whose numerator or denominator or both are fractional.

Thus, $\frac{\frac{1}{2}}{3}$, $\frac{4}{\frac{2}{3}}$, $\frac{\frac{3}{4}}{\frac{1}{5}}$, and $\frac{3\frac{1}{2}}{4\frac{1}{4}}$ are complex fractions. These fractions mean that $\frac{1}{2}$ is to be divided by 3, that 4 is to be divided by $\frac{2}{3}$, that $\frac{3}{4}$ is to be divided by $\frac{1}{5}$, and that $3\frac{1}{2}$ is to be divided by $4\frac{1}{4}$.

This is only another form of expression for the division of fractions.

202. Rule.—*Multiply the numerator of the complex fraction by its denominator inverted.*

Written Exercises.

Reduce to simpler forms:

1. $\dfrac{\frac{2}{3}}{4}$.

2. $\dfrac{\frac{5}{8}}{\frac{7}{8}}$.

3. $\dfrac{4}{\frac{3}{22}}$.

4. $\dfrac{\frac{5}{9}}{\frac{13}{18}}$.

5. $\dfrac{\frac{3}{5}}{\frac{2}{3}}$.

6. $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$.

7. $\dfrac{3\frac{4}{5}}{6\frac{7}{8}}$.

8. $\dfrac{8\frac{2}{3}}{16\frac{1}{3}}$.

9. $\dfrac{4\frac{1}{6}}{4\frac{2}{3}}$.

10. $\dfrac{3\frac{1}{3}}{9\frac{1}{4}}$.

11. $\dfrac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{5}+\frac{1}{3}}$.

12. $\dfrac{2\frac{1}{2}+\frac{2}{3}}{3\frac{1}{2}-\frac{1}{3}}$.

13. $\dfrac{4\frac{1}{2}+\frac{2}{3}}{4\frac{1}{2}-\frac{2}{3}}$.

14. $\dfrac{2\frac{1}{2}+3\frac{1}{3}}{2}$.

15. $\dfrac{4\frac{1}{3}-2\frac{1}{6}}{9}$.

Written Problems.

1. A laborer earned $\$67\frac{1}{2}$ in 45 days; find his daily salary.

2. A mechanic earned $\$113\frac{3}{4}$ in $45\frac{1}{2}$ days; find his daily wages.

3. An artisan received $\$2\frac{5}{8}$ for $1\frac{7}{8}$ days' work. How much was that for one day?

4. Find the daily earnings o. a man who gets $\$\frac{3}{4}$ for $\frac{7}{8}$ of a day's work.

5. If a train runs $877\frac{1}{2}$ mi. in 30 hours, what is its running rate per hour?

6. A train runs $457\frac{1}{4}$ mi. in $15\frac{1}{2}$ hours; find its running rate per hour.

7. A train runs 55 mi. in $1\frac{5}{6}$ hours; find the running rate per hour.

8. If a train runs $25\frac{5}{6}$ mi. in $\frac{5}{6}$ of an hour, how many miles does it run in 1 hour?

9. I paid $181\frac{1}{3}$ cents for 32 lb. of sugar. Find the cost per pound.

10. I paid $109\frac{2}{3}$ cents for $15\frac{2}{3}$ lb. of sugar. What is the price of one pound?

11. If $1\frac{7}{8}$ lb. of sugar cost $12\frac{3}{16}$ cents, what does one pound cost?

12. If $\frac{7}{8}$ lb. of sugar cost $6\frac{9}{16}$ cents, what does one pound cost?

13. A steamer sails 1 $620\frac{1}{2}$ mi. in 7 days. What is her rate per day?

14. A steamer sails 1 485 mi. in $6\frac{3}{4}$ days. Find the number of miles a day.

15. A steamer sails $386\frac{3}{4}$ mi. in $1\frac{3}{4}$ days. Find the daily rate in miles.

16. A steamer sails $171\frac{3}{4}$ mi. in $\frac{3}{4}$ of a day. How many miles a day does she sail?

17. If a man earns $\$2\frac{1}{2}$ a day, how long will it take him to earn $\$287\frac{1}{2}$?

18. If a man earns $\$3\frac{1}{2}$ a day, how long will it take him to earn $\$2\frac{1}{2}$?

19. If a man earns \$4 a day, how long will it take him to earn $\$\frac{15}{16}$?

20. At the rate of $30\frac{1}{2}$ mi. an hour, how long will a train take to run $198\frac{1}{4}$ mi.?

21. At the rate of $30\frac{3}{4}$ mi. an hour, how long will a train take to run $15\frac{3}{8}$ mi.?

22. At the rate of $29\frac{1}{2}$ mi. an hour, how long will a train take to run $\frac{1}{2}$ mi.?

23. How many pounds of brown sugar can be bought for 90 cents, at $4\frac{1}{2}$ cents a pound?

24. How many pounds of brown sugar can be bought for $54\frac{5}{8}$ cents, at $4\frac{3}{4}$ cents a pound?

25. How many pounds of brown sugar can be bought for $\$2\frac{1}{24}$, at $\$\frac{1}{24}$ a pound.

26. How many pounds of brown sugar can be bought for $\$\frac{1}{15}$, at $\$\frac{1}{25}$ a pound?

27. What quantity of honey can be bought for $25\frac{3}{4}$ cents, at $30\frac{1}{2}$ cents a pound?

28. In Quebec, 1910, 557 thousand quintals of codfish were caught; in 1914, $1\frac{192}{365}$ times less. Find the latter number.

29. In Quebec, 1914, $12\frac{2}{3}$ thousand quintals of salmon were caught; find the number for 1910, if it was $1\frac{11}{27}$ times less.

30. In Quebec, 1912, 23 thousand quintals of herring were caught; in 1913, $2\frac{1}{11}$ times less. Find the latter number.

31. In 1908, 6 100 vessels were used in the sea and inland fisheries of Quebec; in 1914, $1\frac{8}{53}$ times less. Find the latter number.

32. In 1914, Quebec used 636 thousand cords of wood in the manufacture of pulp; in 1910, $1\frac{49}{57}$ times less. Find the latter number.

33. In 1913, Quebec exported 802 thousand cords of wood pulp: in 1914, $1\frac{1}{6}$ times less. Find the latter number.

34. In 1914, Canada manufactured 935 thousand tons of pulp; in 1910, $1\frac{92}{95}$ times less. Find the latter number.

35. In 1914, Canada employed $8\frac{1}{10}$ million dollars' worth of wood in the manufacture of pulp; in 1910, $2\frac{1}{4}$ times less. Find the latter amount.

FRACTIONAL RELATIONS.

When I say: $\frac{1}{2}$ of *an apple*, $\frac{3}{4}$ of *a dollar*, the unit is *a single object*.

When I say: $\frac{1}{2}$ of *my fortune*, $\frac{3}{4}$ of 20 *dollars*, $\frac{1}{5}$ of $\$\frac{3}{4}$, the unit is *a quantity*, great or small, considered as *a single unit*. In the latter case, the unit is generally called the *base*, and we represent it by 1, or $\frac{2}{2}$, or $\frac{3}{3}$, or $\frac{4}{4}$, etc.

203. The *base* generally follows the expressions *part of*, *fraction of*, *$\frac{3}{4}$ of*, *$\frac{3}{4}$ as much as*, *$\frac{1}{4}$ more than*, *$\frac{1}{5}$ less than*.

204. The **base** is the quantity used to measure another quantity which is greater or less than it; the base is therefore the *unit of comparison*, or the quantity considered as *a whole*.

205. The product is the quantity compared with the base; it may be *a part of the base*, or a quantity *greater* or *less* than *the base*.

206. The fraction expresses the ratio, or *relation* of *the part to the base.*

Thus, in the expression: \$8 is $\frac{2}{3}$ of \$12, \$12 is the *base,* \$8 is the *product* or *part,* and $\frac{2}{3}$ is the *fraction* or *relation.*

207. Principles.—I. *The Base multiplied by the Fraction equals the Product.*

II. *The Product divided by the Base equals the Fraction.*

III. *The Product divided by the Fraction equals the Base.*

NOTE.—Problems may be solved either by *Analysis* or by *Formulas* drawn from the above principles, full illustrations of which are given. However, teachers are invited to exact analysis in the oral work, and occasionally in the written work.

208. First case.—To find the Product when the Base and the Fraction are given.

EXAMPLE I.—What is $\frac{3}{4}$ *of* \$60?

ANALYSIS.—\$60, the base, is the quantity considered as a whole, or 1, or $\frac{4}{4}$.

4 fourths of $\$60 = \$60$;
1 fourth of $\$60 = \frac{60}{4}$;
3 fourths of $\$60 = \frac{60 \times 3}{4}$, or \$45.

FORMULA.—Base × Fraction = Product (*Principle I*).
PRACTICE.—\$60 × $\frac{3}{4}$ = \$45, Ans.

EXAMPLE II.—I have \$300; my neighbor has $\frac{1}{3}$ *less than* I; how many dollars has he?

ANALYSIS.—\$300, the base, is the quantity considered as 1, or $\frac{3}{3}$. My neighbor has $\frac{1}{3}$ less than I, that is, a sum equal to $\frac{2}{3}$ of my money.

3 thirds of my money = \$300;
1 third of my money = $\frac{300}{3}$;
2 thirds of my money = $\frac{300 \times 2}{3}$, or \$200.

FORMULA.—Base × Fraction = Product.
PRACTICE.—\$300 × $\frac{2}{3}$ = \$200, Ans.

EXAMPLE III.—A lot cost \$800; I sold it for $\frac{1}{4}$ *more than* it cost. Find the selling price.

ANALYSIS.—\$800, the base, is the quantity considered as 1, or $\frac{4}{4}$. I sold the lot for $\frac{1}{4}$ more than the cost, that is, for $\frac{5}{4}$ of the cost.

$$\begin{aligned} &\text{4 fourths of the cost} = \$800; \\ &\text{1 fourth of the cost} = \$\tfrac{800}{4}; \\ &\text{5 fourths of the cost} = \$\tfrac{800 \times 5}{4}, \text{ or } \$1\,000. \end{aligned}$$

FORMULA.—Base × Fraction = Product.
PRACTICE.—\$800 × $\frac{5}{4}$ = \$1 000, Ans.

NOTE I.—$\frac{1}{3}$ *less than* a number, implies $\frac{2}{3}$ of that number; also $\frac{1}{4}$ *more than* a number, implies $\frac{5}{4}$ of that number.

NOTE II.—Remember that the *Base* and the *Fraction* are factors of the *Product*.

Oral Exercises.

1. $\frac{1}{2}$ of \$ 4 = ?
2. $\frac{1}{3}$ of \$ 6 = ?
3. $\frac{1}{4}$ of \$ 8 = ?
4. $\frac{1}{5}$ of \$10 = ?
5. $\frac{1}{6}$ of \$12 = ?
6. $\frac{2}{3}$ of \$ 6 = ?
7. $\frac{3}{4}$ of \$ 8 = ?
8. $\frac{3}{5}$ of \$10 = ?
9. $\frac{5}{6}$ of \$12 = ?
10. $\frac{4}{3}$ of \$ 6 = ?
11. $\frac{1}{7}$ of \$$\frac{3}{4}$ = ?
12. $\frac{1}{6}$ of \$$\frac{2}{3}$ = ?
13. $\frac{2}{3}$ of \$$\frac{3}{4}$ = ?
14. $\frac{5}{6}$ of \$$\frac{1}{3}$ = ?
15. $\frac{3}{4}$ of \$$\frac{1}{2}$ = ?
16. $\frac{1}{5}$ less than \$10 = ?
17. $\frac{1}{3}$ less than \$ 9 = ?
18. $\frac{1}{4}$ less than \$ 8 = ?
19. $\frac{2}{3}$ less than \$12 = ?
20. $\frac{2}{5}$ less than \$15 = ?
21. $\frac{1}{3}$ less than \$ $\frac{1}{2}$ = ?
22. $\frac{1}{2}$ less than \$ $\frac{1}{4}$ = ?
23. $\frac{2}{3}$ less than \$ $\frac{3}{4}$ = ?
24. $\frac{3}{4}$ less than \$ $\frac{2}{3}$ = ?
25. $\frac{1}{5}$ less than \$ $\frac{1}{3}$ = ?
26. $\frac{2}{3}$ more than \$ 6 = ?
27. $\frac{3}{4}$ more than \$12 = ?
28. $\frac{2}{5}$ more than \$10 = ?
29. $\frac{3}{5}$ more than \$15 = ?
30. $\frac{4}{5}$ more than \$20 = ?
31. $\frac{1}{3}$ more than \$ $\frac{1}{2}$ = ?
32. $\frac{1}{4}$ more than \$ $\frac{1}{4}$ = ?
33. $\frac{1}{5}$ more than \$ $\frac{3}{4}$ = ?
34. $\frac{2}{3}$ more than \$ $\frac{1}{2}$ = ?
35. $\frac{3}{4}$ more than \$ $\frac{2}{3}$ = ?

Oral Problems.

1. A shepherd had 60 sheep; he sold $\frac{1}{3}$ of them; how many did he sell?

2. I had \$12 in the School Savings Bank; I withdrew $\frac{2}{3}$ of my money. How much did I withdraw?

3. I bought \$100 worth of sugar, and the merchant granted me a rebate of $\frac{1}{10}$ of the price. Find the amount of the rebate.

4. A house was sold for \$1 000, $\frac{1}{10}$ of this price being paid cash. Find the cash payment.

5. A merchant failed in business and paid only $\frac{3}{4}$ of his debts amounting to \$800. How much did he pay?

6. A bicycle was sold for $\frac{1}{4}$ less than its cost. Find the selling price if the cost price was \$40.

NOTE.—It was sold for $\frac{3}{4}$ of its cost.

7. I have \$40; my brother has $\frac{1}{5}$ less than I. How much money has he?

8. A and B are partners; A invests \$1 000; B invests $\frac{1}{10}$ less than A. Find B's investment.

9. John earns 60 cents an hour; Thomas earns $\frac{1}{3}$ less than John; how much does he earn in an hour?

10. A horse cost \$80; it was sold for $\frac{1}{4}$ less than it cost. Find the selling price.

11. I paid \$100 for a horse, and $\frac{1}{4}$ more for a carriage. Find the cost of the carriage.

NOTE.—It cost $\frac{5}{4}$ of \$100.

12. I sold two pianos, one for \$200, and the other for $\frac{1}{8}$ more than the first. What did the second piano sell for?

13. I have 300 bushels of wheat; how many bushels of oats have I, if I have $\frac{1}{3}$ more than of wheat?

14. There are 400 boys in a school; how many girls are there, if the number of girls is $\frac{1}{10}$ greater than the number of boys?

15. Henry earns \$20 a week; how much does Charles earn if his salary is $\frac{1}{10}$ greater than Henry's?

16. Luke has \$$\frac{7}{8}$; Patrick has $\frac{1}{4}$ more money than Luke; how much has he?

17. I had \$1 000; I spent $\frac{3}{8}$ of this sum to buy an automobile. How much had I left?

18. A child had $\frac{7}{8}$ of a quart of milk; he spilt $\frac{1}{4}$ of what he had. How much milk was spilt? How much remained?

19. A, B, and C own \$100; A has $\frac{1}{4}$ of the sum; B, $\frac{1}{5}$ of the sum, and C, the remainder. How much has each?

20. William had a farm of 100 acres; he sold $\frac{1}{5}$ of it, and then $\frac{1}{2}$ of the remainder. How many acres had he left?

21. A drunkard received a \$15 salary; he gave $\frac{1}{3}$ of it to pay for the drinks he had bought on credit; then he spent $\frac{1}{2}$ of the remainder drinking. What sum did he bring home?

22. My father lost $\frac{1}{3}$ of his fortune, and $\frac{1}{4}$ of the remainder. What did he lose the second time? What remained?

23. I sold $\frac{1}{3}$ of my sheep; then $\frac{2}{5}$ of the remainder died. What had I left?

24. I gave $\frac{1}{5}$ of my flowers to Irene, and $\frac{3}{4}$ of the remainder to Rose. What did Rose receive? Find Clara's share if she got what was left.

25. Wilfrid gave $\frac{1}{3}$ of his prizes to Henry, and $\frac{1}{4}$ of the remainder to Charles. What did Charles get? What did Wilfrid then have?

Written Exercises.

Find the product:

1. $\frac{3}{4}$ of \$288 = ?	6. $\frac{21}{100}$ of \$ 200 = ?
2. $\frac{5}{4}$ of \$624 = ?	7. $\frac{78}{100}$ of \$ 900 = ?
3. $\frac{3}{8}$ of \$960 = ?	8. $\frac{17}{100}$ of \$ 400 = ?
4. $\frac{7}{8}$ of \$880 = ?	9. $\frac{93}{100}$ of \$1000 = ?
5. $\frac{3}{16}$ of \$128 = ?	10. $\frac{150}{100}$ of \$ 300 = ?

11. $\frac{1}{4}$ less than \$624 = ?	16. $\frac{1}{15}$ less than \$195 = ?
12. $\frac{1}{5}$ " \$125 = ?	17. $\frac{2}{15}$ " \$135 = ?
13. $\frac{2}{5}$ " \$250 = ?	18. $\frac{1}{12}$ " \$108 = ?
14. $\frac{3}{8}$ " \$136 = ?	19. $\frac{1}{16}$ " \$128 = ?
15. $\frac{2}{9}$ " \$117 = ?	20. $\frac{4}{100}$ " \$400 = ?

21. $\frac{1}{4}$ more than \$400 = ?	26. $\frac{1}{10}$ more than \$240 = ?
22. $\frac{1}{3}$ " \$138 = ?	27. $\frac{1}{9}$ " \$117 = ?
23. $\frac{1}{5}$ " \$195 = ?	28. $\frac{1}{12}$ " \$144 = ?
24. $\frac{1}{6}$ " \$480 = ?	29. $\frac{1}{25}$ " \$625 = ?
25. $\frac{2}{7}$ " \$147 = ?	30. $\frac{1}{16}$ " \$224 = ?

Written Problems.

1. A farmer harvested 2 100 bushels of potatoes and sold $\frac{7}{10}$ of them. How many bushels did he sell?

2. In Montreal, 1906, 8 650 persons died, $\frac{17}{173}$ with tuberculosis. How many victims did the *white plague* make?

3. The $\frac{10}{23}$ of 1 978 dunkards were found to be consumptives. Find the number of consumptives.

4. The eruption of a volcano in Tokio, 1703, caused the death of 210 000 persons. In St. Pierre (Martinique), 1902, $\frac{4}{21}$ of that number perished. Find how many.

5. I have $\frac{3}{4}$ of $\frac{5}{8}$ of \$288; how much money have I?

6. A farmer carried to the mill 239 bushels of wheat, weighing 60 lb. each. How many pounds of flour did he bring back, if wheat gives $\frac{3}{4}$ of its weight in flour?

7. A man retired from business with a fortune of \$17 280; he invested $\frac{1}{4}$ of it in real estate, $\frac{3}{8}$ of it in stocks, and the remainder in bonds. Find the amount of each investment.

8. I bought 80 acres of land, $\frac{1}{4}$ at \$45 each, $\frac{2}{5}$ at \$37.50 each, and the remainder at \$30 each. Find the total cost.

9. In 1911, Canada's immigrant arrivals were 310 thousand; and in 1912, $\frac{22}{155}$ greater. Find the latter number.

10. In 1913, Canada's immigrant arrivals were 402 thousand; and in 1914, $\frac{3}{201}$ less. Find the latter number.

11. In 1915, Canada's immigrant arrivals were 144 thousand; and in 1916, $\frac{2}{3}$ less. Find the latter number.

12. A, B, and C own a factory worth \$75 000. A owns $\frac{1}{3}$ of it, and B, $\frac{1}{5}$ more than A. Find C's share, if he owns the remainder.

13. A, B, and C form a partnership; A invests \$5 200; B, $\frac{1}{4}$ more than A; C, $\frac{3}{20}$ less than B. Find the capital of the firm.

14. A music dealer bought 3 pianos at \$500 each. He sold the first for $\frac{3}{10}$ more than it cost; the second for $\frac{4}{10}$ more than it cost, and the last for $\frac{1}{4}$ less than it cost. What was the total selling price?

15. The sales of a merchant, in 1913, amounted to \$50 000; in 1914, his sales were $\frac{1}{5}$ less than in 1913; in 1915, $\frac{1}{10}$ greater than in 1914; in 1916, $\frac{3}{11}$ greater than in 1915. Required the amount of his sales in 1916.

209. Second case.—To find the Fraction when the Base and the Product are given.

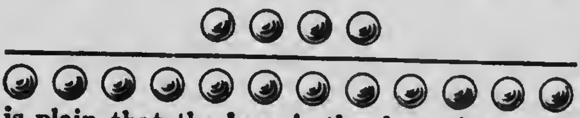
EXAMPLE I.—\$4 is what *part of* \$12?

ANALYSIS.—\$12, the base, is the quantity considered as a whole, or 1.

\$12 = the whole, or 1;
\$ 1 = 1 twelfth of the whole, or $\frac{1}{12}$;
\$ 4 = 4 twelfths of the whole, or $\frac{1}{3}$.

Let us suppose the question reads as follows: "I had twelve marbles; I lost 4 of them; what part of all the marbles did I lose?"

Then, the whole is composed of 12 equal parts; I lose 4 of them, or 4 parts over 12, or $\frac{4}{12}$, or $\frac{1}{3}$.



$= \frac{4}{12} = \frac{1}{3}$

It is plain that the base is the denominator of the fraction, and the product is the numerator of the fraction.

{FORMULA.—Product ÷ Base = Fraction (*Principle II*).
{PRACTICE.— \$4 ÷ \$12 = $\frac{1}{3}$, Ans.

EXAMPLE II.—My sales amounted to \$1 000 in January, and to \$900 in February. The sales of February are *what fraction less than* the sales of January?

ANALYSIS.—The sales of February are \$100 less than the sales of January; this is the *decrease* expressed in dollars;

\$1 000, the base, is the quantity considered as a whole, or 1;

\$1 000 = the whole, or 1;
\$1 = 1 thousandth of the whole, or $\frac{1}{1000}$;
\$100 = 100 thousandths of the whole, or $\frac{1}{10}$;
this is the *decrease* expressed fractionally.

{FORMULA.—Product ÷ Base = Fraction.
{PRACTICE.—\$100 ÷ \$1 000 = $\frac{1}{10}$, Ans.

EXAMPLE III.—My sales amounted to \$1 500 in November, and to \$2 000 in December. The sales of December are *what fraction greater than* the sales of November?

ANALYSIS.—The sales of December are \$500 more than the sales of November; this is the *increase* expressed in dollars.

\$1 500, the base, is the quantity considered as a whole, or 1;

\$1 500 = the whole, or 1;
\$1 = 1 fifteen-hundredth of the whole, or $\frac{1}{1500}$;
\$500 = 500 fifteen-hundredths of the whole, or $\frac{1}{3}$; this is the *increase* expressed fractionally.

{Formula.—Product ÷ Base = Fraction.
{Practice.—\$500 ÷ \$1 500 = $\frac{1}{3}$, Ans.

Note.—*Fraction less than* implies a decrease; *fraction greater than* implies an increase.

Oral Exercises.

What part of:

1. \$12 is \$ 6?
2. \$ 9 " \$ 3?
3. \$ 8 " \$ 2?
4. \$10 " \$ 2?
5. \$20 " \$10?
6. \$ 15 is \$ 5?
7. \$ 12 " \$ 3?
8. \$ 60 " \$30?
9. \$100 " \$50?
10. \$100 " \$60?

Find the fraction:

11. \$ 4 × ? = \$ 2.
12. \$10 × ? = \$ 5.
13. \$20 × ? = \$10.
14. \$ 8 × ? = \$ 4.
15. \$ 3 × ? = \$ 1.
16. \$ 6 × ? = \$2.
17. \$ 8 × ? = \$1.
18. \$12 × ? = \$3.
19. \$12 × ? = \$4.
20. \$12 × ? = \$6.
21. \$ 50 × ? = \$25.
22. \$ 60 × ? = \$30.
23. \$ 80 × ? = \$40.
24. \$100 × ? = \$25.
25. \$100 × ? = \$10.

What fraction of:

26. \$ 8 is \$4?
27. \$ 6 is \$3?
28. \$20 is \$5?
29. \$12 is \$1?
30. \$20 is \$3?
31. \$ 9 is \$8?
32. \$ 7 is \$6?
33. \$ 5 is \$2?
34. \$ 3 is \$2?
35. \$100 is \$1?

What part of:

36. \$$\frac{3}{6}$ is \$$\frac{1}{6}$?
37. \$$\frac{5}{6}$ is \$$\frac{2}{6}$?
38. \$$\frac{3}{8}$ is \$$\frac{1}{8}$?
39. \$$\frac{7}{8}$ is \$$\frac{1}{8}$?
40. \$$\frac{4}{8}$ is \$$\frac{3}{8}$?
41. \$$\frac{5}{12}$ is \$$\frac{2}{12}$?
42. \$$\frac{9}{12}$ is \$$\frac{1}{12}$?
43. \$$\frac{5}{100}$ is \$$\frac{1}{100}$?
44. \$$\frac{25}{100}$ is \$$\frac{1}{100}$?
45. \$$\frac{50}{100}$ is \$$\frac{25}{100}$?

What fraction less than:

46. \$ 6 is \$5?
47. \$ 7 is \$6?
48. \$ 5 is \$4?
49. \$ 9 is \$8?
50. \$10 is \$5?
51. \$5 is \$3?
52. \$5 is \$2?
53. \$6 is \$3?
54. \$6 is \$2?
55. \$9 is \$7?
56. \$100 is \$80?
57. \$100 is \$60?
58. \$100 is \$90?
59. \$100 is \$75?
60. \$100 is \$50?

What fraction more than:

61. \$ 6 is \$ 7?
62. \$ 5 is \$ 6?
63. \$ 8 is \$ 9?
64. \$ 5 is \$ 7?
65. \$10 is \$15?
66. \$5 is \$ 8?
67. \$5 is \$ 9?
68. \$6 is \$ 9?
69. \$6 is \$10?
70. \$9 is \$15?
71. \$100 is \$105?
72. \$100 is \$120?
73. \$100 is \$150?
74. \$100 is \$180?
75. \$100 is \$175?

Oral Problems.

1. A young man earns \$10 a week and spends \$3 on superfluities. What fraction of his salary does he waste?

2. George is 26 years old, and Joseph, 32. George's age is what part of Joseph's?

3. An agent gets \$100 for selling \$1 000 worth of goods. What fraction of his sales is that?

4. A laborer earned \$800 in a year, and saved \$300. What part of his salary did he save?

5. A and B formed a partnership, A investing \$500 and B \$400. What part of the capital (\$900) did each invest?

6. A grocer sold 400 lb. of coffee, and had 600 lb. remaining. What part of what he had did he sell?

7. If 10 gallons of water are mixed with 30 gallons of vinegar, what part of the mixture is water? is vinegar?

8. A, B, and C are partners; A put in \$500; B, \$700; and C, \$800. What part of the capital did each invest?

NOTE.—When there is *increase* or *decrease*, the quantity increased or decreased is the normal base.

9. The population of St. Johns was 3 thousand in 1871, and 4 thousand in 1901. What fraction represents the increase?

10. The population of Belœil was 13 hundred in 1914, and 15 hundred in 1911. What fraction represents the decrease?

11. The population of Acton Vale was 14 hundred in 1911, and 15 hundred in 1914. What fraction represents the increase?

12. The population of Cartierville was 15 hundred in 1914, and 9 hundred in 1911. Find the fraction representing the increase.

13. The population of Longueuil was 4 thousand in 1911, and 6 thousand in 1914. Find the fraction representing the increase.

14. The population of Three Rivers was 14 thousand in 1911, and 19 thousand in 1914. Find the fraction representing the increase.

15. The Indian population of the Province of Quebec was 12 thousand in 1886, and 13 thousand in 1914. Find the fraction representing the increase.

16. A mechanic can do a work in 6 days. What part of the work can he do in 1 day? 2 days? 8 days?

17. A woman can do a work in 9 days. What part of the work can she do in 1 day? 2 days? 8 days?

18. A pipe can fill a reservoir in 16 hours. What part of the reservoir can it fill in 1 hour? 3 hours? 10 hours?

19. A pipe can fill a tank in 20 hours. What fraction of the tank can it fill in 1 hour? 2 hours? 10 hours?

20. A can do a work in 6 days, and B can do it in 7 days. What part of the work can each do in 1 day? What part of the work can they do together in 1 day?

Written Exercises.

What part of:

1. \$ 125 is \$25 ?	**5.** \$150 is \$$37\frac{1}{2}$?	**9.** \$ $\frac{7}{8}$ is \$ $\frac{3}{4}$?
2. \$ 160 is \$40 ?	**6.** \$100 is \$$33\frac{1}{3}$?	**10.** \$ $1\frac{1}{2}$ is \$ $\frac{5}{6}$?
3. \$ 100 is \$$12\frac{1}{2}$?	**7.** \$$\frac{3}{4}$ is \$ $\frac{3}{8}$?	**11.** \$ $3\frac{1}{3}$ is \$ $2\frac{1}{3}$?
4. \$1 000 is \$125 ?	**8.** \$$\frac{3}{8}$ is \$ $\frac{1}{3}$?	**12.** \$$16\frac{1}{2}$ is \$$12\frac{1}{3}$?

Find the relation of:

13. \$610 to \$1 220.	**17.** \$ 90 to \$80.
14. \$336 to \$ 672.	**18.** \$110 to \$90.
15. \$ 75 to \$ 625.	**19.** \$$\frac{2}{3}$ to \$$\frac{3}{8}$.
16. \$ 75 to \$ 60.	**20.** \$$\frac{3}{4}$ to \$$\frac{1}{3}$.

What fraction less than:

21. \$125 is \$ 95 ?	**24.** \$875 is \$125 ?	**27.** \$165 is \$ 55 ?
22. \$625 is \$600 ?	**25.** \$936 is \$312 ?	**28.** \$185 is \$ 95 ?
23. \$770 is \$660 ?	**26.** \$888 is \$333 ?	**29.** \$205 is \$195 ?

What fraction more than:

30. \$125 is \$150 ?	**33.** \$825 is \$925 ?	**36.** \$300 is \$400 ?
31. \$250 is \$300 ?	**34.** \$325 is \$350 ?	**37.** \$800 is \$950 ?
32. \$525 is \$675 ?	**35.** \$675 is \$700 ?	**38.** \$600 is \$750 ?

39. \$121 is \$132 ? **40.** \$702 is \$729 ?

Written Problems.

1. A dunner charged $29 for collecting a bill of $435. What part of the sum collected did he charge for his services?

2. A and B engage in partnership, A investing $5 700, and B, $5 400. What fraction of the capital of the firm does each invest?

3. A farm contained 75 acres in wheat, 110 acres in oats, 65 acres in grass, and 125 acres in woodland. What fraction of the farm was in wheat? in oats? in grass? in woodland?

4. An importer paid $1 250 for an invoice of goods, and $250 for customhouse charges. What part of the invoice price were the charges?

5. A tank contained 175 gallons of water after 50 had leaked out. What fraction of the original contents was lost by leakage?

6. In a certain school 1 150 pupils are enlisted. By what fraction would you represent a daily attendance of 1 100 pupils?

7. A grazier owned 1 184 sheep, and sold 296. He sold what part of his flock?

8. A man walked 24 mi. on Monday, 18 mi. on Tuesday, and 30 mi. on Wednesday. He walked what part of the entire distance each day?

9. I pay $345 for taxes on a property assessed at $46 000. What fraction does that represent?

10. An army corps numbered 9 600 men after losing 2 400 men. What fraction of the army corps was lost?

11. 1 050 deaths were registered in a city whose population was 25 000. Find the fractional death rate.

12. In Quebec, 1911, there were 390 thousand men 18 to 45 years of age; 23 thousand were British-born, 26 thousand foreign-born, and the remainder Canadian-born. What part of the 390 thousand did each group represent?

13. In Ontario, 1911, there were 582 thousand men 18 to 45 years old; 106 thousand were British-born, 64 thousand foreign-born, and the remainder Canadian-born. What part of the 582 thousand did each group represent?

14. In 1881, Canada had 662 thousand farmers; in 1911, 933 thousand. Express the increase fractionally.

15. In 1901, Canada had 45 thousand professionals; in 1911, 63 thousand. Express the increase fractionally.

16. In 1901, Canada's manufactures employed 274 thousand laborers; in 1911, 491 thousand. Express the increase fractionally.

17. In 1901, Canada had 11 600 barkeepers; in 1911, 10 800. Express the decrease fractionally.

18. In 1901, Canada had 39 000 teachers; in 1911, 42 000. Express the increase fractionally.

19. In the Province of Quebec, 1911, there were 74 000 births; in 1914, 80 000. Express the increase fractionally.

20. In the Province of Quebec, 1760, the birth rate per thousand inhabitants, was 65; in 1900, it was 35. The birth rate for 1900 was what fraction less than the birth rate for 1760?

210. Third case.—To find the Base when the Product and the Fraction are given.

EXAMPLE I.—How much money have I, if $\frac{3}{5}$ *of* my money equals $30?

ANALYSIS.—My money is the base, or the quantity to be considered as a whole, or 1, or $\frac{5}{5}$.

3 fifths of my money = $30;
1 fifth of my money = $10;
5 fifths of my money = $50.

FORMULA.—Product ÷ Fraction = Base (*Principle III*).
PRACTICE.— $30 ÷ $\frac{3}{5}$ = $50, Ans.

EXAMPLE II.—A has $30, which is $\frac{1}{4}$ *less than* B has. How much has B?

ANALYSIS.—B's money is the base, or the quantity to be considered as a whole, or 1, or $\frac{4}{4}$. A has $\frac{1}{4}$ less than B, that is, a sum equal to $\frac{3}{4}$ of B's money.

3 fourths of B's money = $30;
1 fourth of B's money = $10;
4 fourths of B's money = $40.

FORMULA.—Product ÷ Fraction = Base.
PRACTICE.— $30 ÷ $\frac{3}{4}$ = $40, Ans.

EXAMPLE III.—A has \$20, which is $\frac{1}{4}$ *more than* B has. How much has B?

ANALYSIS.—B's money is the base, or the quantity considered as a whole, or 1, or $\frac{4}{4}$. A has $\frac{1}{4}$ more than B, that is, a sum equal to $\frac{5}{4}$ of B's money.

5 fourths of B's money = \$20;
1 fourth of B's money = \$4;
4 fourths of B's money = \$16.

FORMULA.—Product ÷ Fraction = Base.
PRACTICE.— \$20 ÷ $\frac{5}{4}$ = \$16, Ans.

NOTE.—When an equation can be made between a fraction and any quantity whatever, divide the quantity by the fraction to find the *Base.*

Oral Exercises.

Find the missing factor:

1. $\frac{1}{2}$ of ? = \$4.
2. $\frac{1}{3}$ of ? = \$3.
3. $\frac{1}{4}$ of ? = \$2.
4. $\frac{1}{5}$ of ? = \$3.
5. $\frac{1}{6}$ of ? = \$2.
6. $\frac{2}{3}$ of ? = \$ 4.
7. $\frac{3}{4}$ of ? = \$ 6.
8. $\frac{3}{5}$ of ? = \$ 9.
9. $\frac{4}{5}$ of ? = \$ 8.
10. $\frac{5}{6}$ of ? = \$10.
11. ? × $\frac{1}{2}$ = \$5.
12. ? × $\frac{1}{3}$ = \$4.
13. ? × $\frac{1}{4}$ = \$3.
14. ? × $\frac{1}{5}$ = \$2.
15. ? × $\frac{1}{6}$ = \$3.

Find the sum which decreased by:

16. $\frac{1}{4}$ of itself = \$3.
17. $\frac{1}{3}$ of itself = \$2.
18. $\frac{1}{5}$ of itself = \$4.
19. $\frac{1}{6}$ of itself = \$10.
20. $\frac{1}{7}$ of itself = \$6.
21. $\frac{3}{10}$ of itself = \$14.
22. $\frac{3}{4}$ of itself = \$4.
23. $\frac{2}{9}$ of itself = \$14.
24. $\frac{2}{3}$ of itself = \$ 5.
25. $\frac{1}{20}$ of itself = \$19.
26. $\frac{1}{4}$ of itself = \$$\frac{6}{5}$.
27. $\frac{1}{3}$ of itself = \$$\frac{4}{5}$.
28. $\frac{1}{5}$ of itself = \$$\frac{8}{7}$.
29. $\frac{1}{6}$ of itself = \$$\frac{10}{3}$.
30. $\frac{1}{8}$ of itself = \$$\frac{14}{15}$.

Find the sum which increased by:

31. $\frac{1}{4}$ of itself = \$10.
32. $\frac{1}{3}$ of itself = \$ 8.
33. $\frac{1}{2}$ of itself = \$ 6.
34. $\frac{1}{5}$ of itself = \$12.
35. $\frac{1}{7}$ of itself = \$16.
36. $\frac{2}{5}$ of itself = \$14.
37. $\frac{2}{3}$ of itself = \$10.
38. $\frac{4}{5}$ of itself = \$18.
39. $\frac{3}{5}$ of itself = \$16.
40. $\frac{2}{5}$ of itself = \$21.
41. $\frac{1}{4}$ of itself = \$$\frac{10}{3}$.
42. $\frac{1}{3}$ of itself = \$$\frac{8}{9}$.
43. $\frac{1}{8}$ of itself = \$$\frac{18}{5}$.
44. $\frac{1}{6}$ of itself = \$$\frac{14}{9}$.
45. $\frac{1}{8}$ of itself = \$$\frac{18}{5}$.

Oral Problems.

1. I drew from the bank $20, or $\frac{2}{5}$ of my deposit. What was my deposit?

2. I sold a horse for $100, which was $\frac{4}{5}$ of the cost. Find the cost.

3. If I take 21 days to do $\frac{3}{4}$ of a work, how many days will the whole require?

4. If $25 is $\frac{5}{8}$ of Leo's salary, find his salary.

5. I lost 20 marbles, which is $\frac{2}{5}$ of all I had. How many had I?

6. I paid $\frac{5}{8}$ of a bill and still owed $21. How much did I owe at first?

7. A, B, and C are partners; A invested $\frac{1}{4}$ of the capital; B, $\frac{1}{3}$ of the capital; and C, the remainder. If C's share is 15 thousand dollars, find the shares of A and B.

8. A farmer first sold $\frac{1}{3}$ and $\frac{1}{4}$ of all his sheep and had 10 remaining. How many had he?

9. A young man gives $\frac{1}{3}$ of his salary for board, $\frac{1}{4}$ of his salary for lodging, and has $15 left. What is his salary?

10. How much money had I, if after spending $\frac{1}{4}$ and $\frac{1}{5}$ of my money, I have $22 left?

11. What was the amount of a bill, if after obtaining a deduction of $\frac{1}{20}$ of it, I paid $38 to settle it?

12. The difference between $\frac{3}{4}$ and $\frac{2}{3}$ of a number equals $12. What is that number?

13. If 27 equals $\frac{3}{4}$ of a number, find $\frac{1}{3}$ of that number.

14. John has $30, which is $\frac{1}{6}$ less than his brother has. How much has the latter?

15. I spent $10, which was $\frac{1}{3}$ less than the sum I had left. How much money had I left?

16. A and B together have $21, but A has $\frac{1}{4}$ less than B. How much money has B?

17. Laura and Rachel together have 40 pinks, but Laura has $\frac{2}{3}$ less than Rachel. How many pinks has Rachel?

18. Alice receives a weekly salary of $20, which is $\frac{1}{3}$ more than Emma receives. Find Emma's salary.

19. Ralph and Omer together have 210 acres of land, but Ralph has $\frac{1}{3}$ more than Omer. How many acres has Omer?

20. Hugh has 50 apple trees, which is $\frac{1}{4}$ more than Charles has. How many has Charles?

21. Peter bought 10 cows, which was $\frac{2}{3}$ more than the number he had at first. How many cows has he now?

22. Stephen spent $\frac{1}{3}$ of his money at the saloon; he then lost $\frac{1}{2}$ of what remained, and still had \$3. How much had he at first?

23. I paid \$25 in settlement of an account, and afterwards discovered that I had paid $\frac{1}{4}$ more than the correct sum. What was the amount of the overcharge?

24. I sold a watch for \$12, which was $\frac{1}{3}$ more than its cost. Find the cost.

25. I lost $\frac{3}{7}$ of my sheep, and had 28 remaining. How many had I at first?

Written Exercises.

Find the *Base* in the following equations:

1. $\frac{3}{4}$ of ? = \$627.
2. $\frac{5}{6}$ of ? = \$475.
3. $\frac{7}{8}$ of ? = \$343.
4. $\frac{5}{8}$ of ? = \$265.
5. $\frac{3}{15}$ of ? = \$912.
6. $\frac{3}{100}$ of ? = \$135.
7. $\frac{25}{100}$ of ? = \$625.
8. $\frac{45}{100}$ of ? = \$405.
9. $\frac{70}{100}$ of ? = \$490.
10. $\frac{90}{100}$ of ? = \$630.
11. $\frac{2}{3}$ of ? = \$ $\frac{1}{2}$.
12. $\frac{3}{4}$ of ? = \$ $\frac{1}{5}$.
13. $\frac{5}{6}$ of ? = \$ $\frac{1}{9}$.
14. $\frac{7}{4}$ of ? = \$3$\frac{1}{2}$.
15. $\frac{3}{7}$ of ? = \$7$\frac{1}{2}$.

Find the sum which decreased by:

16. $\frac{2}{9}$ of itself = \$357.
17. $\frac{3}{11}$ of itself = \$408.
18. $\frac{4}{15}$ of itself = \$319.
19. $\frac{3}{16}$ of itself = \$247.
20. $\frac{20}{45}$ of itself = \$625.
21. $\frac{3}{100}$ of itself = \$291.
22. $\frac{4}{100}$ of itself = \$480.
23. $\frac{25}{100}$ of itself = \$675.
24. $\frac{35}{100}$ of itself = \$715.
25. $\frac{45}{100}$ of itself = \$605.
26. $\frac{1}{3}$ of itself = \$ $\frac{1}{9}$.
27. $\frac{1}{4}$ of itself = \$ $\frac{3}{7}$.
28. $\frac{1}{5}$ of itself = \$ $\frac{2}{3}$.
29. $\frac{1}{6}$ of itself = \$2$\frac{1}{2}$.
30. $\frac{1}{7}$ of itself = \$6$\frac{1}{3}$.

Find the sum which increased by:

31. $\frac{2}{9}$ of itself = \$319.
32. $\frac{3}{11}$ of itself = \$126.
33. $\frac{4}{15}$ of itself = \$475.
34. $\frac{3}{16}$ of itself = \$399.
35. $\frac{20}{45}$ of itself = \$715.
36. $\frac{3}{100}$ of itself = \$927.
37. $\frac{4}{100}$ of itself = \$728.
38. $\frac{25}{100}$ of itself = \$625.
39. $\frac{35}{100}$ of itself = \$405.
40. $\frac{45}{100}$ of itself = \$725.
41. $\frac{1}{3}$ of itself = \$ $\frac{1}{7}$.
42. $\frac{1}{4}$ of itself = \$ $\frac{1}{8}$.
43. $\frac{1}{5}$ of itself = \$ $\frac{1}{6}$.
44. $\frac{1}{6}$ of itself = \$2$\frac{1}{3}$.
45. $\frac{1}{7}$ of itself = \$7$\frac{1}{4}$.

Written Problems.

1. A merchant sold 1 260 bushels of grain, which was $\frac{14}{100}$ of his supply. How many bushels had he at first?

2. A trader withdrew \$4 116 from bank, which was $\frac{28}{100}$ of his deposit. How much had he remaining in bank?

3. A owns $\frac{15}{100}$ of a ship; B, $\frac{25}{100}$; C, $\frac{28}{100}$; and D, the remainder. Find the value of A's share, if D's share is worth \$34 464.

4. I receive a commission of \$250. If this sum represents $\frac{5}{200}$ of my sales, what are my sales?

5. I pay \$21 for municipal taxes. If this sum represents $\frac{3}{400}$ of the valuation of my property, find the valuation of my property.

6. A man having a two-third interest in a piece of property, sells $\frac{3}{5}$ of his share for \$1 600. What is the value of the property?

7. If $\frac{5}{8}$ of the length of Victoria Bridge equals 4 190 feet, find the total length.

8. A farmer left $\frac{1}{3}$ of his fortune to his wife, $\frac{1}{4}$ to his daughter, and the remainder, \$5 125, to his son. Find the share of his wife, and that of his daughter.

9. After spending $\frac{1}{4}$ and $\frac{1}{5}$ of my money, I have \$1 551 left. How much had I at first?

10. Instead of taking $\frac{3}{4}$ of a sum, I took $\frac{3}{5}$ of it. How much should I have taken, if the error amounted to \$513?

11. Shawinigan Falls are 120 ft. high. If $\frac{5}{8}$ of this height equals $\frac{3}{10}$ of the height of Montmorency Falls, how high are the latter?

12. A man spends $\frac{1}{7}$ of his yearly income for food, $\frac{4}{21}$ of it for clothes, $\frac{3}{35}$ of it for rent, and has \$793 remaining. Find his income.

13. A merchant owned $\frac{3}{8}$ of a store, and sold $\frac{1}{4}$ of his share for \$2 718. Find the value of the whole store.

14. Three brothers inherited a fortune; the first received $\frac{1}{3}$ of it; the second, $\frac{5}{8}$ of the remainder. If the third received \$3 631, find the value of the inheritance.

15. If $\frac{2}{3}$ of a farm consists in oats, $\frac{3}{4}$ of the remainder in wheat, what is the area of the farm, the 19 acres left being in barley?

16. Half of a stock of goods was destroyed by fire; half of the remainder was destroyed by water; and the rest was sold at cost for \$6 730. What was the cost of the entire stock?

17. This year Patrick harvested 728 bushels of potatoes, which was $\frac{1}{7}$ more than last year. What was the yield for both years?

18. In Quebec, 1913, there died 13 300 children under one year, which was $\frac{4}{137}$ less than in 1911. How many died in 1911?

19. In Quebec, 1912, there died 12 300 children under one year, which was $\frac{2}{43}$ less than in 1910. How many died in 1910?

20. In Quebec, 1913, there died 3 800 children, 1 to 5 years of age, which was $\frac{9}{29}$ more than in 1912. How many died in 1912?

21. In Quebec, 1913, there died 4 500 persons, 70 years of age and over, which was $\frac{1}{10}$ less than in 1911. How many died in 1911?

22. In Quebec, 1912, 33 000 deaths were registered, which was $\frac{4}{29}$ more than in 1907. How many deaths were registered in 1907?

23. In Quebec, 1913, tuberculosis caused 3 250 deaths, which was $\frac{7}{58}$ more than in 1906. How many victims were there in 1906?

24. In Quebec, 1912, tuberculosis caused 3 228 deaths, which was $\frac{18}{287}$ less than in 1911. How many victims were there in 1911?

25. In Quebec, 1913, typhoid fever, rubeola, scarlet fever, and diphtheria caused 2 040 deaths, which was $\frac{1}{2}$ more than in 1912. How many deaths were caused by these four diseases in 1912?

THE EQUATION.

The following examples will illustrate the use of the equation when fractions are involved.

EXAMPLE I.—Add \$40 to $\frac{3}{5}$ of my monthly salary and you will have $\frac{2}{3}$ of my salary. Find my salary.

ANALYSIS.—My salary is the base, or 1.

$\frac{2}{5}$ of my salary + \$40 = $\frac{2}{3}$ of my salary.

Reducing the fractions to a common denominator gives:

$\frac{6}{15}$ of my salary + \$40 = $\frac{10}{15}$ of my salary;

\$40 = $\frac{10}{15}$ — $\frac{6}{15}$ of my salary;

$\frac{4}{15}$ of my salary = \$40;

$\frac{1}{15}$ of my salary = \$10;

$\frac{15}{15}$, or the whole of my salary = \$150.

EXAMPLE II.—The difference between $\frac{1}{4}$ and $\frac{1}{8}$ of a number equals 90 less than $\frac{1}{5}$ of that number. Find the number.

ANALYSIS.—The number is the base, or 1.

$(\frac{1}{4}$ of $n)$—$(\frac{1}{8}$ of $n) = (\frac{1}{5}$ of $n)$—90.

Reducing fractions to a common denominator gives:

$\frac{10}{40}$—$\frac{5}{40}$ of $n = \frac{8}{40}$ of n—90;

$\frac{5}{40}$ of $n = \frac{8}{40}$ of n—90;

Reversing the equation gives:

$\frac{8}{40}$ of n—90 $= \frac{5}{40}$ of n;

$\frac{8}{40}$—$\frac{5}{40}$ of $n = 90$;

$\frac{3}{40}$ of $n = 90$;

$\frac{1}{40}$ of $n = 30$;

$\frac{40}{40}$ of n, or the number itself = 1 200.

Written Problems.

1. $\frac{5}{8}$ of my age equals 2 years less than $\frac{2}{3}$ of my age. Find my age.

2. $\frac{1}{3}$ of my money equals \$6 less than $\frac{1}{2}$ of my money. How much money have I?

3. $\frac{1}{2}$ minus $\frac{1}{4}$ of a number equals 90 more than $\frac{1}{7}$ of that number. What is it?

4. The sum of $\frac{1}{3}$ and $\frac{1}{2}$ of my money equals \$10 less than $\frac{7}{8}$ of my money. How much have I?

5. $\frac{2}{3}$ of $\frac{1}{2}$ of my money equals \$10 less than $\frac{3}{4}$ of $\frac{2}{3}$ of my money. How much have I?

6. $\frac{3}{5}$ of $\frac{3}{4}$ of the length of the Gatineau River equals 25 mi. more than $\frac{4}{5}$ of $\frac{5}{8}$ of the length of the Chaudière River. How long is the Gatineau if the Chaudière is 80 mi. long?

7. $\frac{3}{4}$ of my trees are elms, $\frac{1}{10}$ ash trees, and the remainder maple trees. Required the number of trees, if the maple trees equal 20 more than $\frac{1}{8}$ of all the trees.

8. John spent $\frac{2}{5}+\frac{1}{3}+\frac{1}{6}$ of his money; how much had he at first, if the remainder equals $24 more than $\frac{1}{15}$ of all his money?

9. I sold $\frac{4}{7}$ of $\frac{1}{8}$ of my harvest; how many bushels had I at first, if the remainder equals 42 bushels more than $\frac{5}{28}$ of all the harvest?

10. A boy spent $\$4\frac{3}{10}$ less than $\frac{3}{4}$ of his money, and had $\$10\frac{9}{20}$ left. How much money had he at first?

REVIEW OF COMMON FRACTIONS.

FIRST SERIES.

Written Exercises.

1.

	A	B	C	D	E
I	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$
II	$1\frac{1}{8}$	$1\frac{3}{4}$	$1\frac{7}{8}$	$2\frac{1}{16}$	$2\frac{1}{4}$
III	$2\frac{1}{3}$	$2\frac{1}{2}$	$2\frac{2}{3}$	$2\frac{5}{6}$	$3\frac{1}{3}$
IV	$3\frac{1}{32}$	$4\frac{1}{16}$	$4\frac{3}{32}$	$5\frac{1}{8}$	$5\frac{5}{32}$
V	$4\frac{9}{32}$	$5\frac{5}{16}$	$5\frac{11}{32}$	$6\frac{3}{8}$	$7\frac{1}{2}$

Perform the following combinations in each of the above numbers (I, II, III, IV, V):

1. A+B+C.
2. (B+C)—A.
3. B+C+D.
4. (C+D)—E.
5. A+B+D+E.
6. (C+E)—B.
7. (A+B+C+D) × 3.
8. (B+C+D+E) ÷ 4.
9. (A+B+E) × $\frac{3}{2}$.
10. (B+D) × $201\frac{1}{2}$.

2.

	A	B	C	D	E	F	G
I	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{8}{9}$	$\frac{4}{9}$	$4\frac{1}{2}$	2	$\frac{23}{24}$
II	$\frac{7}{8}$	$\frac{1}{3}$	$\frac{7}{10}$	$\frac{1}{12}$	$3\frac{1}{4}$	3	$\frac{19}{24}$
III	$\frac{5}{6}$	$\frac{1}{4}$	$\frac{9}{10}$	$\frac{5}{12}$	$3\frac{3}{4}$	$1\frac{1}{8}$	$\frac{17}{48}$
IV	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{16}$	$4\frac{1}{8}$	$1\frac{3}{8}$	$\frac{39}{48}$
V	$\frac{3}{5}$	$\frac{1}{6}$	$\frac{7}{12}$	$\frac{3}{16}$	$5\frac{3}{8}$	$2\frac{5}{8}$	$5\frac{1}{4}$

Perform the following combinations in each of the above numbers (I, II, III, IV, V):

1. A×B. **6.** E÷F. **11.** (E−D)÷B.
2. A÷B. **7.** (E+G)÷2. **12.** (F+G)×A.
3. C×D. **8.** (E+F)×2. **13.** (E+F+G)×B.
4. C÷D. **9.** (E−G)×3. **14.** (A−B)×C.
5. E×F. **10.** (C+D)×B. **15.** (C+G)÷D.

3.

PAY ROLL

For the week ending..............................19....

No	NAME	No. of hours per day.						Total time	Wages per hour	Total Wages
		M.	T.	W.	T.	F.	S.			
1	W. Hubert...	8	8	8	$6\frac{1}{2}$	8	4	$42\frac{1}{2}$	50c	\$21.25
1	C. Hancock..	8	$7\frac{1}{2}$	8	8	4	4	*	50c	* *
3	J. Prairie.....	7	$7\frac{1}{4}$	8	$7\frac{1}{2}$	8	4	*	50c	* *
4	S. Martin.....	8	7	6	8	$7\frac{1}{2}$	3	*	55c	* *
5	O. Mead.....	8	8	7	$7\frac{3}{4}$	8	$3\frac{1}{2}$	*	55c	* *
6	A. Monet.....	8	6	8	0	7	$3\frac{3}{4}$	*	$27\frac{1}{2}$c	* *
		*	*	*	*	*	*	*		* *

Fill out the spaces marked with an asterisk (*).

Written Problems.

1. A pedestrian goes $12\frac{2}{3}$ mi. in $2\frac{1}{3}$ hours, and $17\frac{1}{2}$ mi. in $3\frac{3}{4}$ hours. How far does he go, and how long does he walk?

2. A tailor uses $\frac{3}{4}$ yd. of cloth to make a pair of pantaloons. At \$2 a yard, what will the cloth cost for two dozen pair?

3. A milkman daily supplies a family with $1\frac{3}{4}$ qt. of milk at $9\frac{1}{3}$ cents a quart. What will the bill amount to in 30 days?

4. A lamp is kept burning $4\frac{1}{3}$ hours a day, and it consumes $\frac{1}{100}$ gal. of kerosene an hour. Find the expense per week, if kerosene sells for $\$\frac{1}{4}$ a gallon.

5. Find the total cost of the following articles: $2\frac{1}{4}$ lb. of beef at $21\frac{2}{3}$ cents a pound, $2\frac{1}{4}$ lb. of veal at $14\frac{2}{3}$ cents a pound, $1\frac{1}{2}$ lb. of pork at $18\frac{1}{3}$ cents a pound.

6. A merchant buys $37\frac{3}{4}$, $41\frac{1}{4}$, and $52\frac{1}{2}$ yd. of linen at $\$\frac{1}{4}$ a yard. Find the total cost.

7. From a piece of silk $48\frac{3}{4}$ yd. long, I sell $10\frac{3}{4}$, $7\frac{1}{2}$ and $17\frac{1}{4}$ yd. What is the remainder worth at $\$2\frac{1}{4}$ a yard?

8. A basin receives $4\frac{3}{4}$ gal. and loses $1\frac{1}{4}$ gal. of water in $\frac{1}{2}$ hour. How much water will it contain after $3\frac{3}{4}$ hours?

9. Every stroke of a pump gives out $\frac{1}{3}$ gal. of water. How many strokes will be necessary for 200 gal.?

10. In going 418 yd., how many revolutions will be made by a wheel $4\frac{2}{3}$ yd. in circumference?

11. A lady pays $\$121\frac{1}{4}$ for 5 pieces of cloth each $12\frac{3}{4}$ yd. long. Find the cost of one yard.

12. A spout gives out $2\frac{1}{3}$ gal. of water in a minute. In what time will it give out $60\frac{2}{3}$ gal.?

13. I added 6 to each term of the fraction $\frac{3}{4}$; how much was the increase or decrease?

14. If $5\frac{1}{2}$ tons of coal cost $\$28\frac{3}{4}$, find the cost of $12\frac{3}{8}$ tons.

15. When the dividend is 165 and the quotient $6\frac{7}{8}$, what is the divisor?

16. I had 200 lb. of butter; I sold $27\frac{1}{2}$, $30\frac{1}{4}$, $24\frac{1}{3}$, $32\frac{7}{8}$, and $34\frac{3}{4}$ lb. How many pounds remained? How much is the remainder worth at $29\frac{1}{2}$ cents a pound?

17. Multiply the sum of $\frac{5}{8}$ and $\frac{3}{4}$ by their difference.

18. A square rod contains 272$\frac{1}{4}$ sq. ft. How many square rods in 43 560 sq. ft.?

19. Find the cost of 18 000 envelops at \$21$\frac{1}{4}$ a thousand.

20. The product of two fractions is $\frac{4}{5}$; one of the fractions is $\frac{11}{15}$; find the other.

21. A and B start from the same point in opposite directions, A at the rate of 3$\frac{3}{5}$ mi. an hour, and B at the rate of 2$\frac{4}{5}$ mi. an hour. How far apart will they be in 4$\frac{1}{2}$ hours?

22. Two men start from Beauharnois in opposite directions, one at the rate of 18$\frac{5}{8}$ mi. a day, the other at the rate of 14$\frac{3}{4}$ mi. a day. How far apart will they be in 3 days?

23. An automobile runs 1 mi. in 3$\frac{1}{3}$ minutes; a horse trots 1 mi. in 6$\frac{2}{3}$ minutes. How long will each take to go from Montreal to Terrebonne, a distance of 18 mi.?

24. Two trains 87$\frac{1}{2}$ mi. apart move toward each other, one at the rate of 50$\frac{3}{4}$ mi. an hour, the other at the rate of 20$\frac{5}{8}$ mi. an hour. How far apart will they be in half an hour?

25. A man walks 2$\frac{1}{3}$ mi. in $\frac{4}{5}$ hours; how far can he go in 2$\frac{1}{3}$ hours?

26. By the Canadian Pacific R. R., it is 173 mi. from Montreal to Quebec. At what distance from Quebec will two trains meet, if they respectively leave Quebec and Montreal at the same time, the first going 35$\frac{1}{3}$ mi. an hour, and the other 33$\frac{13}{15}$?

27. A grocer mixes 8$\frac{5}{8}$ lb. of one kind of tea with 7$\frac{1}{4}$ lb. of another kind, and sells the mixture at \$$\frac{4}{5}$ a pound. How much does he receive?

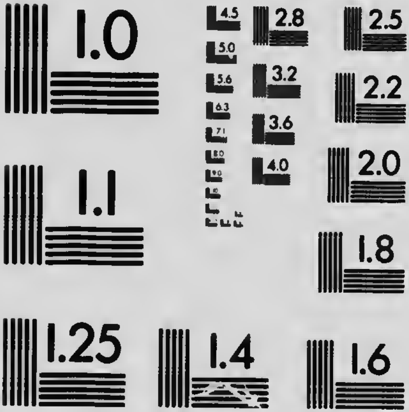
28. A bookkeeper earns \$4$\frac{1}{4}$ a day, his son \$2$\frac{1}{2}$, and his daughter \$1$\frac{5}{8}$. Suppose there are 300 working days in a year, that the family expenses average \$3$\frac{1}{2}$ a day, and find their yearly savings.

29. Alfred multiplied by $\frac{5}{8}$ instead of multiplying by $\frac{5}{16}$, and obtained $\frac{5}{8}$ for product. What should the product have been?

30. An engraver whose wages are \$3$\frac{1}{4}$ a day, began to drink and was discharged. Now he earns \$1$\frac{1}{2}$ a day. How much money does he lose in 5 years of 300 working days?

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



31. Each of 4 cows gives $3\frac{3}{4}$ gal. of milk a day; $\frac{1}{3}$ lb. of butter can be made from 1 gal. of milk. Find the quantity of butter made in 30 days.

32. From a piece of cloth $42\frac{1}{8}$ yd. long, $3\frac{1}{4}$, $5\frac{2}{3}$, $3\frac{5}{8}$, and $\frac{3}{8}$ yd. were sold. What is the remnant worth at $\$\frac{3}{4}$ a yard?

33. From a barrel of vinegar containing $31\frac{1}{2}$ gal., $6\frac{3}{4}$ and $12\frac{1}{8}$ gal. were sold. Find the value of the remainder at $54\frac{1}{2}$ cents a gallon.

34. A farmer's yield of oats averaged $27\frac{1}{2}$ bushels an acre, and of buckwheat $28\frac{1}{12}$ bushels an acre. If $9\frac{1}{3}$ acres were planted in oats, and $12\frac{3}{8}$ acres in buckwheat, what was the total number of bushels harvested?

35. I bought $\frac{5}{6}$ yd. of linen at $\$\frac{5}{16}$ a yard, in return for which I gave $\frac{1}{8}$ yd. of cloth worth $\$3\frac{1}{2}$ a yard. What is the difference? Is it in my favor?

36. In Quebec, 1909, 46 400 acres were planted in peas. The average yield was $16\frac{1}{5}$ bushels an acre, and the average price $\$1\frac{1}{4}$ a bushel. Find the value of the harvest.

37. A thoughtless lad tore down 5 chickadees' nests in which were 40 young ones. If a chickadee destroys on an average 50 caterpillars a day, how many caterpillars would these 40 birds have destroyed in $98\frac{1}{2}$ days?

38. The average yield of peas is $16\frac{1}{5}$ bushels an acre, of corn $32\frac{1}{2}$ bushels an acre. If peas sell for $\$1\frac{1}{4}$ a bushel, and corn for $\$\frac{9}{10}$ a bushel, would it be more advantageous to plant $74\frac{1}{2}$ acres in corn than in peas?

39. A fruit vender buys pears 4 for 7 cents, and sells them 6 for 15 cents. How many dozen pears must he sell to gain 90 cents?

40. C buys 48 lemons 6 for 11 cents, and twice as many 4 for 7 cents. He sells the whole lot 2 for 5 cents. How much does he gain or lose?

41. A grocer bought $54\frac{1}{3}$ gal. of maple syrup at $\$\frac{4}{5}$ a gallon. $6\frac{3}{4}$ gal. leaked out, and he sold the remainder at $\$1\frac{1}{8}$ a gallon. Find his gain.

42. In 1901–1902, Quebec spent \$6 000 for good roads; in 1910–1911, $15\frac{5}{6}$ times as much. Find the latter expenditure.

43. In 1836, Canada had 16 mi. of railway; in 1909, the total mileage was 1 506½ times as great. Find the mileage for 1909.

Doing the work together.

44. A can do a piece of work in 4 days; so can B. What part of the work will be done in one day by A and B together?

45. A can do a piece of work in 4 days; B can also do it in 4 days. In how many days can they together perform the work?

46. A can do a piece of work in 4 days, and B can do it in 5 days. In how many days can they together perform the work?

47. A, B, and C can each alone perform a piece of work in 8 days. In how many days can all together do the work?

48. A can plow a field in 2 days, B in 3 days, and C in 4 days. In what time can all together do the work?

49. A can cut a cord of wood in 3 days, B in 4 days, and C in 6 days. In what time will the cord be cut if all work together?

50. A and B can do a piece of work in 2 days. If A alone can do the work in 5 days, how long will it take B?

51. A, B, and C could dig a trench in 2 days; A alone would take 6 days, B 8 days. In what time could C alone dig the trench?

52. A can do a piece of work in 12 days, and B in 15 days. In what time can they together do half the work?

53. A can do a piece of work in 8 days, and B in 13 days. In what time can they together do three fourths of the work?

54. A can mow a field in 6 days, B in 8 days, and C in 10 days. In what time can they together do two thirds of the work?

55. Mary can make a dress in $3\frac{2}{3}$ days, Agnes in $4\frac{1}{3}$ days. How long will it take them together?

Supply and discharge pipes.

56. One pipe can fill a cistern in 5 hours; a second pipe can fill it in 3 hours, and a third pipe can fill it in 6 hours. What part of the cistern will the three pipes together fill in 1 hour?

57. A supply pipe can fill a cistern in 6 hours, and a discharge pipe can empty it in 9 hours. The cistern being empty, both pipes are opened; in what time will it be filled?

58. A supply pipe can fill a cistern in 7 hours, and a discharge pipe can empty it in 10 hours. The cistern being empty, both pipes are opened; in what time will three fourths of the cistern be filled?

SECOND SERIES.

Oral Exercises on fractional relations.

	A	B	C	D	E	F
I	6	8	5	1	$\frac{1}{6}$	$\frac{3}{2}$
II	8	9	6	2	$\frac{1}{4}$	$\frac{4}{3}$
III	10	12	8	4	$\frac{1}{5}$	$\frac{5}{4}$
IV	12	14	10	3	$\frac{1}{6}$	$\frac{3}{2}$
V	16	20	12	4	$\frac{1}{4}$	$\frac{8}{5}$
VI	20	25	15	10	$\frac{1}{5}$	$\frac{4}{5}$
VII	25	30	20	6	$\frac{2}{5}$	$\frac{5}{6}$
VIII	30	32	28	12	$\frac{3}{5}$	$\frac{5}{4}$
IX	40	45	35	15	$\frac{3}{4}$	$\frac{8}{5}$
X	50	60	40	8	$\frac{4}{5}$	$\frac{25}{20}$

NOTE.—Answer the following questions with the numbers in line I, then do the same for II, III, IV, V, VI, VII, VIII, IX, X.

1. The number in column B is *what fraction greater than* the number in column A?

2. The number in column C is *what fraction less than* the number in column B?

3. The number in column D is *what fraction of* the number in column C?

4. What is the *product*, if column A is the *base*, and column E the *fraction*?

5. What is the *product*, if column B is the *base*, and column F the *fraction*?

6. What is the *base*, if column D is the *product*, and column E the *fraction*?

7. What is the *base*, if column A is the *product*, and column F the *fraction*?

Questions on Theory.

NOTE.—Give an example with each answer.

1. What is a fraction? (143).
2. What does the denominator show? (144).
3. What does the numerator show? (144).
4. When is the value of a fraction less than 1? (146).
5. When is the value of a fraction equal to 1? (147).
6. Define an improper fraction. (149).
7. Define a mixed number. (150).
8. Which is the greater of two fractions having similar denominators? (151).
9. Which is the greater of two fractions having similar numerators? (152).
10. Does multiplying the numerator of a fraction by an integer increase the fraction? (154, I).
11. Does dividing the numerator of a fraction by an integer decrease the fraction? (154, II).
12. Does multiplying the denominator of a fraction by an integer decrease the fraction? (154, III).
13. Does dividing the denominator of a fraction by an integer increase the fraction? (154, IV).
14. Does multiplying both terms of a fraction by the same number change the value of the fraction? (154, V).
15. Does dividing both terms of a fraction by the same number change the value of the fraction? (154, V).
16. Does a fraction represent an uneffected division? (155).
17. How may an integer be made to assume the form of a fraction? (157).
18. Give the rule for reducing whole or mixed numbers to improper fractions. (159).

19. Give the rule for reducing improper fractions to whole or mixed numbers. (161).

20. How are fractions reduced to higher terms? (163).

21. How are fractions reduced to lower terms? (165).

22. How are fractions reduced to their lowest terms? (167, 168).

23. What is a common denominator of two or more fractions? (170).

24. How are fractions reduced to a common denominator? (172).

25. What is the least common denominator of two or more fractions? (173).

26. Compare a least common denominator with a least common multiple. (174).

27. When must a fraction be reduced to its lowest terms?

28. How are fractions reduced to their least common denominator? (175).

29. To be added must fractions express similar fractional units? What does that mean? (176).

30. Give the rules for adding fractions, mixed numbers. (177, 179).

31. Must both fractions have a common denominator in subtraction? Why? (180).

32. Give the rule for finding the difference between two fractions. (181).

33. How is the multiplication of fractions performed by the cancellation process? By the direct process? (187, 190).

34. How is the division of fractions performed by the cancellation process? By the direct process? (197, 200).

35. Is a number increased or decreased if you multiply it by a proper fraction?

36. Is a number increased or decreased if you divide it by a proper fraction?

37. Do the principles of analysis apply when the price of an object, the quantity, the total value are proper fractions?

38. What is a complex fraction? (201).

39. How is a complex fraction simplified? (202).

40. What is the Base? Is $\frac{3}{5}$ of the base the same as $\frac{3}{5}$ of 1 time the base? (204).

41. By what expressions is the Base generally introduced? (203).

42. How do you find the Product when the Base and the Fraction are given? (207).

43. How is the Base found when the Product and the Fraction are given? (207).

44. How is the Fraction found when the Product and the Base are given? (207).

45. Does adding the same number to each term of a fraction change the value of this fraction?

Written Problems.

1. The Ottawa River is 800 mi. long. How long is the St. Maurice River if it is $\frac{7}{16}$ as long?

2. D owns $\frac{7}{12}$ of an estate valued at \$3 600, and sells $\frac{2}{3}$ of his share. How much money will he receive?

3. A remnant of velvet was $\frac{8}{9}$ yd. long; I sold $\frac{3}{5}$ of it at \$$1\frac{4}{5}$ a yard. Find the length of the part sold and its price.

4. A piece of linen was $68\frac{8}{9}$ yd. long; I bought $\frac{3}{4}$ of it at \$$\frac{2}{5}$ a yard. How much did I pay?

5. A farmer paid \$150 for a horse, $\frac{2}{5}$ as much for a cow, and $\frac{1}{10}$ as much for a sheep. How much did he pay in all?

6. C paid $\frac{3}{5}$ of \$20 for $\frac{2}{5}$ of $5\frac{1}{3}$ cords of wood. Find the price of one cord.

7. Donald inherited \$6 500. He spent $\frac{3}{5}$ of that sum for a lot, and $\frac{5}{7}$ of the remainder for a house. How much has he left?

8. A man willed $\frac{2}{7}$ of his fortune to his relatives, $\frac{2}{5}$ of it to a school, and $\frac{5}{11}$ of the remainder to an orphan asylum. If his fortune amounted to \$35 000, what sum was willed to the orphan asylum?

9. A lady's yearly income is \$6 000. She sets apart $\frac{2}{5}$ of it for her children's schooling, $\frac{1}{6}$ for charitable works, $\frac{1}{12}$ for traveling, and the remainder for the household. Find the greater expenditure.

10. The potato gives $\frac{4}{25}$ of its weight in starch; starch gives $\frac{3}{2}$ of its weight in syrup, and syrup gives $\frac{9}{20}$ of its weight in alcohol. 125 lb. of potatoes will make how many pounds of alcohol?

11. Mary bought 14 lb. of currents to make jelly. The currents gave $\frac{4}{7}$ of their weight in juice; to the juice she added an equal quantity of sugar. What was the cost of one pound

of jelly, if the currents cost $7\frac{1}{2}$ cents a pound, and the sugar $5\frac{1}{2}$ cents a pound?

12. A owns $\frac{3}{5}$ of a 200-acre farm. He sells $\frac{2}{3}$ of his share to B, and the latter sells $\frac{1}{4}$ of his to C. How many acres has each now?

13. In 1908–1909, the Quebec Government granted \$210 000 to the Public Schools, $\frac{1}{15}$ of this sum to the poor municipalities, and $6\frac{11}{28}$ times as much to the Normal Schools as to the poor municipalities. How much money was granted in the last two instances?

14. In 1914, there were 435 300 pupils enrolled in the Quebec schools; and in 1915, $\frac{1}{10}$ more. Find the latter number.

15. In Quebec, 1913, strikes and lockouts occasioned a loss of 85 748 working days; in 1915, $\frac{14}{17}$ less. Find the latter number.

16. In Quebec, 1914, the total value of the field crops was \$99 279 000; in 1915, it was $\frac{1}{20}$ greater. Find the value for 1915.

17. In 1914, Canada's total receipts amounted to $162\frac{5}{8}$ millions of dollars; in 1915, they were $\frac{2}{11}$ less. Find the latter amount.

18. In France, 1865, the consumption of alcoholic liquors was $19\frac{1}{5}$ million gallons; in 1893, it was $\frac{95}{96}$ greater. In France, 1865, 438 suicides were registered as being due to alcohol; in 1893, $\frac{205}{146}$ as many. Find the consumption of alcohol and the number of suicides for 1893.

19. In 1915, there were 2 064 convicts in the penitentiaries of Canada; of this number only 360 were teetotalers. By what fraction can you represent the non-abstainers?

20. According to French statistics, out of 225 epileptic children, 162 were born of alcoholic parents. By what fraction can you represent the epileptics born of alcoholic parents?

21. In 1915, Canada's debt was 700 million dollars; of this amount, 338 million dollars were owed to England. What part of the national debt was owed to England?

22. In 1915, Canada's excise and other revenues amounted to 22 millions of dollars; of this amount, spirits contributed $8\frac{3}{4}$ million dollars. What fraction was furnished by spirits?

23. From a farm of 120 acres, 35$\frac{1}{4}$ acres were sold. What part of it was sold?

24. If A can perform a work in 20 days, what part of it can he perform in 7$\frac{2}{5}$ days?

25. A lady paid \$56$\frac{1}{4}$ for a dress, and \$6$\frac{1}{4}$ for a hat. The cost of the hat is what part of the cost of the dress?

26. Three ounces of silver were fused with 5 ounces of copper. What quantity of silver is there in $\frac{2}{3}$ ounces of this alloy?

27. The bronze of a bell is composed of 25 parts of copper to 7 parts of tin. If copper is worth 53$\frac{1}{2}$ cents a pound, and tin 55$\frac{8}{10}$ cents a pound, what will a 600-pound bell cost?

28. In Canada, 1915, out of 2 064 convicts, 276 were illiterate. By what fraction can you represent the literate?

29. In Montreal, 1914, 20 thousand births and 11 thousand deaths were registered. The births were what fraction greater than the deaths?

30. In Quebec, 1914, there were 80 thousand births; in Ontario, 66 thousand. The births were what fraction greater in Quebec than in Ontario?

31. English statistics prove that for a given period of time, there died of different sicknesses, 576 farmers, 653 workmen, 895 clerks, 1 470 saloon keepers, and 2 142 bartenders. 1° Mortality was what fraction greater among bartenders than among saloon keepe[illegible]hat fraction of all the deaths did the saloon keepers a[illegible]ders together represent?

32. A Parisian do[illegible]ssor Lancereaux, having followed 2 190 cases of tube[illegible], attributed 46 to contagion, 92 to probable heredity, 822 to poverty, and 1 230 to alcoholism. 1° The cases due to misery were what fraction less than the cases due to alcoholism? 2° Alcoholism was responsible for what part of the 2 190 cases?

33. In Quebec, when 162 country people die of tuberculosis, 297 city people die of it. 1° Country mortality is what fraction less than city mortality? 2° City mortality is what fraction greater than country mortality?

34. In Canada, 1913, the consumption of spirits was 5 million gallons, and in 1915, 4$\frac{1}{2}$ million gallons. Express the decrease fractionally.

35. I sold lightly spoiled hay at $\$7\frac{1}{4}$ a ton, which was $\frac{7}{8}$ of the ordinary price. Find the ordinary price.

36. In 1910, Canada's production of copper ore was 24 000 tons, which was $\frac{3}{4}$ of what it was in 1902. Find the latter production.

37. $\frac{2}{3}$ of $\frac{4}{5}$ of the length of Lake Ontario equals $37\frac{1}{3}$ mi. Required the total length.

38. If $\frac{2}{3}$ of $\frac{3}{4}$ of a field sold for $\$1\,326\frac{1}{4}$, what would the whole field have sold for?

39. I have paid $\frac{2}{3}$ of $\frac{3}{8}$ of the purchase price of my property, and still owe \$8 340. Required the purchase price.

40. A sum of money was divided among 4 persons; the first got $\frac{3}{10}$ of it; the second, $\frac{1}{4}$; the third, $\frac{1}{6}$; and the fourth, the remainder, \$5 270. What did the sum amount to?

41. James worked $25\frac{3}{4}$ days. Find his daily salary, if he has $\$12\frac{9}{25}$ left after giving $\frac{2}{5}$ of his salary for board.

42. The difference between $\frac{1}{4}$ and $\frac{1}{8}$ of a number is 39. Find the number.

43. A sum of money was divided among 4 persons; the first received $\frac{1}{2}$ of it; the second, $\frac{1}{5}$; the third, $\frac{1}{6}$; and the fourth, the remainder, which was \$48 less than the share of the third. What did the sum amount to?

44. A owns $\frac{7}{12}$ of a mill, and B the remainder. What is the value of the mill, if A's share is worth \$2 000 more than B's?

45. An estate was divided between two brothers and their sister. The older brother received $\frac{3}{7}$ of the estate; the younger received $\frac{4}{7}$ of the remainder; and their sister received the second remainder, which was \$1 720 less than the share of the younger brother. Find the value of the estate.

46. Subtract 65 feet from the height of Niagara Falls, and you will have $\frac{10}{11}$ of $\frac{2}{3}$ of their height. How high are they?

47. $\frac{2}{5}$ of a farm consists in oats, $\frac{3}{8}$ in barley, and the remainder in potatoes. What is the area of the farm if the second patch is $9\frac{3}{5}$ acres greater than the third?

48. I lost $\frac{1}{2}$ of my money, and $\frac{1}{4}$ of the remainder. How much had I at first, if I still possess \$96?

49. How much money had Mary, if after spending $\frac{1}{3}$ of her money, and $\frac{2}{5}$ of the remainder, she still has $\$3\frac{3}{5}$?

50. A man did $\frac{1}{3}$ of a work in one day; the next day he did $\frac{1}{4}$ of the remainder. What part of the work remained to be done? How much money will he get for the whole work, if he earned $4 the second day?

51. My father spent $\frac{2}{5}$ of his money, then $\frac{1}{2}$ of the remainder, and at last $\frac{3}{4}$ of the second remainder. How much had he at first if he still has $15?

52. Divide $630 between two persons so that the share of the second may equal $\frac{3}{4}$ of the share of the first.

53. In 10 alcoholic families there were 57 children. If the number of healthy children was $\frac{37}{47}$ less than the number of unhealthy children, how many were healthy? (*Dr Demme*, Berne).

54. In 10 abstemious families there were 61 children. If the number of unhealthy children was $\frac{39}{50}$ less than the number of healthy children, how many were unhealthy? (*Dr Demme*, Berne).

55. In 1915, the population of the United States was 91 000 000. The number of prohibitionists was $\frac{1}{45}$ greater than the number of antiprohibitionists. Find the number of prohibitionists.

56. A merchant bought a piece of linen 98 yd. long. He sold $\frac{3}{4}$ of it at $\$\frac{1}{3}$ a yard, $\frac{2}{3}$ of the remainder at $\$\frac{9}{25}$ a yard, and the rest at $\$\frac{12}{25}$ a yard, gaining $\$10\frac{3}{10}$ on the whole. Find the cost of one yard of linen.

57. I bought 45 yd. of cloth at $\$1\frac{1}{5}$ a yard. Find my profit if I sold $\frac{1}{2}$ of the cloth at $\$1\frac{7}{10}$ a yard, $\frac{1}{3}$ of it at $\$1\frac{2}{3}$ a yard, and the remainder at $\$1\frac{1}{5}$ a yard.

58. A merchant bought 544 yd. of cloth at $\$1\frac{1}{2}$ a yard, and sold $\frac{3}{4}$ of it at $\$1\frac{3}{4}$ a yard. What should he ask per yard for the remainder so as to gain $153 on the total cost?

59. I paid $\$38\frac{2}{5}$ for a piece of linen. I sold $\frac{1}{4}$ of it at cost, and the remainder at a profit of $\$\frac{1}{4}$ per yard. I required the number of yards and the cost price of each, the total selling price being $\$44\frac{2}{5}$.

60. I sold $\frac{3}{5}$ of a piece of linen at $\$\frac{11}{10}$ a yard, and the remaining 20 yd. at $\$\frac{1}{2}$ each. Find the number of yards and the cost price of each, if my total gain was $5.

61. Quantity of nitrogenous (*body-building*) substance contained in one pound of each of the following foods:

CHEESE	$\frac{13}{40}$ lb.	EGGS	$\frac{1}{8}$ lb.	POTATOES	$\frac{1}{50}$ lb.
BEANS	$\frac{1}{4}$ lb.	PORK	$\frac{1}{11}$ lb.	MILK	$\frac{1}{22}$ lb.
BEEF	$\frac{1}{5}$ lb.	BREAD	$\frac{1}{7}$ lb.	BEER	$\frac{1}{250}$ lb.
FISH	$\frac{1}{6}$ lb.	BUTTER	$\frac{1}{12}$ lb.	WINE	$\frac{1}{1000}$ lb.
		Alcohol	**=0.**		

(*Dr Dujardin-Beaumetz*).

1. 20 lb. of cheese contain how many more pounds of nitrogen than 1 000 lb. of beer?
2. 5 lb. of beans contain how many more pounds of nitrogen than 1 000 lb. of wine?
3. How many pounds of milk will supply as much nitrogen as 5 000 lb. of beer and 1 000 lb. of wine together?
4. What quantity of nitrogenous food is there in 1 000 lb. of beer, 1 000 lb. of wine, and 1 000 lb. of alcohol?
5. How many pounds of body-building substance in 200 lb. of beef, 300 lb. of fish, and 400 lb. of eggs?
6. How many pounds of bread will furnish as much nitrogen as 3 000 lb. of beer?
7. What quantity of beer is necessary to provide as much nitrogen as 22 lb. of pork and 100 lb. of potatoes?
8. Milk is how many times as nutritious as beer?

Alcohol is not an aliment.

62. CONSUMPTION OF BEER IN CANADA.

1911,	$41\frac{3}{4}$ million gal.
1912,	$47\frac{1}{2}$ million gal.
1913,	$52\frac{1}{3}$ million gal.
1914,	56 million gal.
1915,	48 million gal.

1. Find the total consumption for these five years.
2. The consumption for 1915 was what fraction less than the consumption for 1914?
3. The consumption for 1914 is what fraction greater than the consumption for 1911?

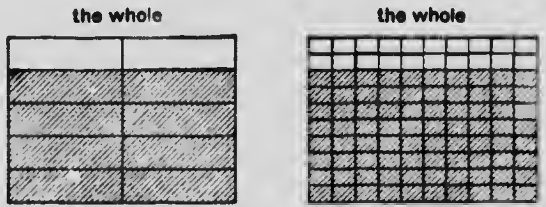
4. What fraction greater than the consumption for 1912 is the consumption for 1913?

5. A gallon of beer contains, on the average, $1\frac{1}{2}$ lb. of alcohol. How many pounds of alcohol in the 1915 consumption?

6. Find the consumption per head in 1915, the population being 7 500 000.

7. In Canada, 1914, the consumption of beer per head was $7\frac{1}{5}$ gal.; in 1870, $2\frac{1}{5}$ gal. The former equals how many times the latter?

DECIMAL FRACTIONS



$$\frac{8}{10} = 0.8 \quad = \quad \frac{80}{100} = 0.80$$

8 tenths, 80 hundredths, 800 thousandths are written $\frac{8}{10}$, $\frac{80}{100}$, $\frac{800}{1000}$. They may also be written, without denominator, 0.8, 0.80, 0.800: these are called decimal fractions.

$15\frac{8}{10}$ and $25\frac{80}{100}$ may also be written 15.8, 25.80: these are called mixed decimals.

Note.—Strictly speaking, $\frac{8}{10}$, $\frac{80}{100}$, $\frac{800}{1000}$ are decimal fractions; but the name is generally indicative of the following: 0.8, 0.80, 0.800.

211. A common fraction whose denominator is 10, 100, 1 000, etc., is a **decimal fraction**; it is usually expressed, without its denominator, by means of an extension of the notation method.

212. An integer followed by a decimal fraction is a *mixed decimal*. For example, 33.3.

213. The period used to separate an integer and decimal (33.3) is called the *decimal point*.

The expression 0.3 means the same as .3; the 0 may be written or not when there are no integers.

In the example 33.3, the value of the units' figure 3 is ten times smaller than the value of the tens' figure 3; so is the value of .3 ($\frac{3}{10}$) ten times smaller than the value of 3 units, etc.

This is simply an extension of the notation method, in which *the value of any order is ten times smaller than the value of the next order at the left, and ten times greater than the value of the next order at the right.*

Numeration Table.

billions.	hundred-millions.	ten-millions.	millions.	hundred-thousands.	ten-thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.	billionths.
3	3	3	3	3	3	3	3	3	3	.	3	3	3	3	3	3	3	3	3

The first figure at the left of the units' figure represents *tens*, and the first figure at the right of the units' figure represents *tenths*; the second figure at the left of the units' figure represents *hundreds*, and the second figure at the right of the units' figure represents *hundredths*; the third figure at the left of the units' figure represents *thousands*, and the third figure at the right of the units' figure represents *thousandths*, etc.

NUMERATION OF DECIMALS.

214. Rule.—*To read a mixed decimal: 1° Read the whole number; 2° read the decimal as a whole number, and give it the denominator of the last term at the right.*

NOTE.—Use *and* only between the units and tenths, and before common fractions.

EXAMPLES.— 33.3 is read 33 *and* 3 tenths.
633.333 is read 633 *and* 333 thousandths.
.3333 is read 3333 ten-thousandths.
.07958 is read 7958 hundred-thousandths.
$.007\frac{2}{3}$ is read 7 *and* $\frac{2}{3}$ thousandths.
$.000\frac{1}{3}$ is read one-third thousandths.

NOTE.—The last two examples are sometimes called *complex decimals.*

Read the following:

6.25.	58.675.	$191.075\frac{3}{4}$.	16.0005.
4.78.	0.375.	$301.009\frac{2}{3}$.	3.14165.
2.95.	75.005.	$123.116\frac{1}{4}$	9.0101058.
7.34.	132.375.	$0.256\frac{1}{2}$.	5.000075.
0.125.	100.025.	$0.679\frac{1}{3}$.	1.700625.

NOTATION OF DECIMALS.

215. Rule.—*To write a mixed decimal: 1° Write the whole number and the decimal point; 2° at the right of the point write the tenths, the hundredths, the thousandths, etc. Replace the missing orders by ciphers.*

EXAMPLES.—48 *and* 4 tenths = 48.4.
9 *and* 56 hundredths = 9.56.
27 *and* 3 ten-thousandths = 27.0003.
12 hundred-thousandths = .00012.
$6\frac{1}{2}$ millionths = $.000006\frac{1}{2}$.

Express decimally:

1. 6 *and* 8 tenths.
2. 4 *and* 3 thousandths.
3. 375 thousandths.
4. 12 ten-thousandths.
5. 60 *and* 40 ten-thousandths.
6. 1 578 hundred-thousandths.
7. 3 *and* 39 thousandths.
8. 112 *and* 56 millionths.
9. 352 millionths.
10. 3 hundred-thousandths.

11. $\frac{7}{10}$.
12. $\frac{85}{100}$.
13. $\frac{72}{1000}$.
14. $\frac{45}{10000}$.
15. $\frac{1256}{1000000}$.
16. $3\frac{16}{100}$.
17. $201\frac{101}{1000}$.
18. $267\frac{342}{1000}$.
19. $3632\frac{10012}{1000000}$.
20. $36370\frac{1001}{10000}$.

216. Principles.—I. The denominator of a decimal is 1 with as many ciphers annexed as there are places at the right of the decimal point.

Thus, $.0015 = \frac{15}{10000}$.

II. Annexing ciphers to the right of a decimal or dropping them from it, does not affect the value of the decimal.

Thus, $.50 = .500 = .5$.

III. Moving the decimal point 1, 2, 3 ... places to the right, multiplies the decimal by 10, 100, 1 000 ...; ciphers are suffixed if necessary.

Thus, 36.33 multiplied by 10 = 363.3; by 100 = 3 633; by 1 000 = 36 330.

IV. Moving the decimal point 1, 2, 3 ... places to the left, divides the decimal by 10, 100, 1 000 ...; ciphers are prefixed if necessary.

Thus, 19.625 divided by 10 = 1.9625; by 100 = .19625; by 1 000 = .019625.

Reduction of common fractions to decimals.

EXAMPLES.—Reduce to decimals: 1° $\frac{3}{4}$; 2° $\frac{3}{8}$; 3° $\frac{7}{11}$.

OPERATIONS.

1° 4) 3.00
.75

2° 8) 3.000
.375

3° 11) 7.0000
.6363
or $.6363\frac{7}{11}$

EXPLANATIONS.

1° $\frac{3}{4}$ = 3 units ÷ 4; 3 units = 30 tenths = 300 hundredths; 300 hundredths ÷ 4 = 75 hundredths, or .75.

2° $\frac{3}{8}$ = 3 units ÷ 8; 3 units = 30 tenths = 300 hundredths = 3 000 thousandths; 3 000 thousandths ÷ 8 = 375 thousandths, or .375.

3° $\frac{7}{11}$ = 7 units ÷ 11; 7 units = 70 tenths = 700 hundredths = 7 000 thousandths = 70 000 ten-thousandths; 70 ten-thousandths ÷ 11 = 6 $363\frac{7}{11}$ ten-thousandths, or $.6363\frac{7}{11}$.

217. NOTE.—In the first two examples, the decimals are said to be *terminate;* in the last example, the decimal is said to be *interminate*, as it can not be exactly expressed by a decimal.

218. Rule.—*1° Annex ciphers to the numerator and divide by the denominator; 2° point off as many decimal places in the quotient as there have been ciphers annexed.*

Written Exercises.

Reduce the following to decimals:

1. $\frac{5}{100}$.	**6.** $\frac{1}{2}$.	**11.** $\frac{5}{8}$.	**16.** $\frac{7}{20}$.	**21.** $1\frac{1}{25}$.
2. $\frac{17}{100}$.	**7.** $\frac{1}{4}$.	**12.** $\frac{7}{8}$.	**17.** $\frac{24}{25}$.	**22.** $5\frac{3}{20}$.
3. $\frac{41}{100}$.	**8.** $\frac{2}{5}$.	**13.** $\frac{1}{16}$.	**18.** $\frac{7}{125}$.	**23.** $13\frac{4}{5}$.
4. $\frac{37}{100}$.	**9.** $\frac{3}{5}$.	**14.** $\frac{3}{16}$.	**19.** $\frac{5}{64}$.	**24.** $19\frac{1}{32}$.
5. $\frac{51}{100}$.	**10.** $\frac{4}{5}$.	**15.** $\frac{5}{32}$.	**20.** $\frac{13}{80}$.	**25.** $26\frac{21}{25}$.

NOTE.—When 3 decimal terms are required, find *four;* if the fourth term is less than 5, replace it by the sign +; if the fourth term is 5 or more, increase the third term by 1, and replace the fourth term by —.

Ex.—$\frac{7}{11}=.6363=.636+$; $\frac{1}{16}=.0625=.063-$; $\frac{2}{3}=.6666=667-$.

Reduce the following to decimals (3 *decimal places*):

26. $6\frac{2}{3}$.	**28.** $3\frac{5}{12}$.	**30.** $36\frac{1}{64}$.	**32.** $30\frac{9}{11}$.
27. $9\frac{7}{8}$.	**29.** $8\frac{7}{32}$.	**31.** $31\frac{7}{9}$.	**33.** $37\frac{1}{128}$.

NOTE.—In the following, reduce the common fraction to a decimal, and annex this decimal to the others.

Reduce the following to decimals (5 *dec. places*):

34. $18.278\frac{5}{8}$.	**38.** $95.41\frac{7}{12}$.	**42.** $0.00\frac{1}{7}$.
35. $31.13\frac{15}{16}$.	**39.** $1.0005\frac{1}{2}$.	**43.** $0.00\frac{1}{8}$.
36. $8.04\frac{5}{6}$.	**40.** $6.008\frac{2}{3}$.	**44.** $0.00\frac{1}{16}$.
37. $17.51\frac{3}{7}$.	**41.** $3.5001\frac{3}{11}$.	**45.** $0.000\frac{1}{6}$.

Reduction of decimals to common fractions.

EXAMPLES.—Reduce to common fractions: 1° .375; 2° $.06\frac{2}{3}$.

OPERATIONS.

$$1°\ .375=\frac{375\div125}{1000\div125}=\frac{3}{8}.$$

EXPLANATIONS.

1° According to *Principle I*, the denominator must be 1 followed by 3 ciphers (= 1 000); the common fraction equals $\frac{375}{1000}$, or $\frac{3}{8}$.

$$2° \quad .06\tfrac{2}{3} = \frac{6\frac{2}{3} \times 3}{100 \times 3} = \frac{20}{300} = \tfrac{1}{15}.$$

2° According to *Principle I*, the denominator must be 1 followed by 2 ciphers (=100); the common fraction equals $\frac{6\frac{2}{3}}{100}$, or $\frac{1}{15}$.

219. Rule.—*1° Omit the decimal point and prefix ciphers; 2° supply the proper denominator; 3° reduce the fraction to its lowest terms.*

Written Exercises.

Reduce to common fractions or to mixed numbers:

1. 0.875.	**6.** 8.0375.	**11.** $1.06\frac{1}{4}$.	**16.** $0.00\frac{1}{8}$.
2. 0.625.	**7.** 3.1625.	**12.** 0.31125.	**17.** $1.00\frac{1}{2}$.
3. 0.075.	**8.** $3.08\frac{1}{3}$.	**13.** $6.03\frac{2}{9}$.	**18.** $0.31\frac{2}{5}$.
4. 3.005.	**9.** $1.28\frac{8}{9}$.	**14.** $0.18\frac{1}{3}$.	**19.** 0.10875.
5. 0.0025.	**10.** $8.037\frac{1}{2}$.	**15.** $0.00\frac{1}{3}$.	**20.** 0.34375.

ADDITION OF DECIMALS

Example I.—What is the sum of .7, 2.43, .865, 11.5, 113.2075, 200.00165?

Operation.

```
      .7
     2.43
      .865
    11.5
   113.2075
   200.00165
   ---------
   328.70415
```

Explanation.

Only like orders can be added, therefore like decimal orders must be written under one another; if the decimal points are in a column, units will come under units, tenths under tenths, and so on. Then we add as with whole numbers.

Example II.—What is the sum of $.82\frac{3}{8}$, $14.1\frac{3}{4}$, $12.065\frac{5}{7}$, and $190.004\frac{1}{3}$, to three decimal places.

Operation.

```
      .8238—
    14.175
    12.0657+
   190.0043+
   ---------
   217.0688
or 217.069—
```

Explanation.

Expand the complex decimals to 4 decimal places to insure accuracy; the first decimal would be .82375, write .8238—; the third decimal would be $.0657\frac{1}{7}$, write .0657+; the fourth decimal would be $.0043\frac{1}{3}$, write .0043+; arrange the decimals and add as in Ex I.

220. Rule.—*1° Write the numbers so that the decimal points stand in a vertical column; 2° proceed as in the addition of whole numbers; 3° place the point in the result directly beneath the points of the addends.*

Written Exercises.

Add the following:

1. 5.18, 54.375, 28.32, 1.0048, 2006.75, 0.0001.
2. 3.04, 25.001, 0.67, 0.2146, 819.2562.
3. 30.1257, 605.2146, 1000.864532, 16.25694.
4. 5.4203, 29,000111, 8.005, 0.00608, 1200.12000014.
5. 8.0012, 250.000001, 311.00555, 89.0071004.
6. 101.404, 72.000004, 5000.5005, 5.50053004.
7. 14.0000864, 0.0096, 250.4, 700.0007, 1.001001.
8. 625.475, 107.35, 9.0056, 1.08, 42.8, 0.8.
9. 423.82, 25.3875, 17.0095, 326.3245, 100.01.
10. 10001.0001, 0.5, 3.001, 96.00101, 60.8901, 39.5.

Find the sum of the following to 4 decimal places:

NOTE.—Expand the complex decimals of the addends to one more decimal place than is required in the sum.

(11)	(12)	(13)	(14)
325.85	$43.08\frac{1}{4}$	4.25	0.0043
$25.38\frac{3}{4}$	$690.8\frac{5}{8}$.	$.00\frac{3}{8}$	$1.25\frac{1}{4}$
$25.009\frac{1}{2}$	$16.13\frac{5}{6}$	463.2504	$72.00\frac{11}{25}$
$116.01\frac{1}{7}$	$0.65\frac{7}{8}$	$5.0\frac{16}{25}$	$16.0\frac{4}{5}$
$0.003\frac{1}{3}$	$17.00\frac{1}{3}$	0.0436	$4.325\frac{1}{2}$

Find the exact sum of the following:

NOTE.—Extend the complex decimals until the remaining fractions are of the same order. Then proceed as in adding ordinary mixed numbers.

(15)	(16)	(17)	(18)
0.4532	$60.\frac{13}{25}$	$0.22\frac{2}{9}$	$136.0\frac{1}{5}$
$7.00\frac{3}{4}$	50.305	$0.3333\frac{1}{3}$	$72.126\frac{2}{3}$
$9.700\frac{14}{25}$	$13.275\frac{1}{7}$	$0.44444\frac{4}{9}$	$13.001\frac{1}{6}$
$6.0\frac{5}{8}$	$1.\frac{16}{25}$	$0.66\frac{2}{3}$	$18.0\frac{1}{16}$
$0.000\frac{1}{5}$	$0.0000\frac{4}{2}$	$16.05\frac{1}{3}$	$3.0001\frac{1}{3}$

Add the following:

19. Seven hundred eight *and* twenty-six *ten-thousandths*, fifty-eight *and* three *millionths*, six hundred ten *and* fifty-four *hundred-thousandths*, eight *hundred-thousandths*, thirty-nine *hundredths*.

20. Two hundred eighteen *and* seven *hundred-millionths*, nineteen *thousandths*, ten *and* eighty-six *hundred-thousandths*, eight hundred *and* eighty-nine thousand *millionths*.

SUBTRACTION OF DECIMALS

Example I.—From 37.682 subtract 24.125.

Operation.	Explanation.
37.682	
24.125	Write like orders under one another and subtract as with whole numbers.
13.557	

Example II.—Find the difference between 836.0$\frac{5}{9}$ and 575.13$\frac{3}{8}$ to 3 decimal places.

Operation.	Explanation.
836.0556—	
575.1338—	Expand the complex decimals to one more decimal place than is required in the remainder; 0.$\frac{5}{9}$ = .0555$\frac{5}{9}$, write .0556—; .13$\frac{3}{8}$ = .13375, write .1338—.
260.9218	
or 260.922 —.	

221. Rule.—*1° Write the numbers so that the decimal points stand in a vertical column; 2° proceed as in the subtraction of whole numbers; 3° place the point in the remainder directly beneath the other points.*

Note.—If the minuend has not so many places as the subtrahend, annex ciphers until both have the same number of places; but the ciphers may be supposed to be there even though they are not written.

Written Exercises.

Find the difference between:

1. 0.985 and 0.573.	**7.** 10000.1 and 0.0001.
2. 0.668 and 0.1382.	**8.** 25 and 24.60852.
3. 1.549 and 0.8627.	**9.** 1.000625 and 0.11001.
4. 2.3 and 0.7543.	**10.** 0.9 and 0.0009.
5. 100.52 and 42.2906.	**11.** 6.98754 and 3.0276.
6. 4246.5 and 345.005.	**12.** 0.375 and 0.00462.

Subtract to 4 decimal places:

(13)	(14)	(15)	(16)
$25.\frac{7}{8}$	$18.0\frac{4}{5}$	$42.0982\frac{5}{16}$	$150.03\frac{5}{64}$
$18.72\frac{3}{4}$	$1.003\frac{3}{8}$	$32.728\frac{3}{32}$	$92.48\frac{1}{6}$

Subtract, giving exact answer:

(17)	(18)	(19)	(20)
$.03\frac{1}{3}$	$26.$	$6.\frac{7}{32}$	$50.0081\frac{5}{16}$
$.0036\frac{2}{3}$	$15.99\frac{1}{9}$	3.003125	$2.728\frac{3}{32}$

(21)	(22)	(23)
$127.\frac{3}{4}$	$3867.1278\frac{7}{13}$	$630.005\frac{1}{11}$
$68.513\frac{1}{11}$	$319.21864\frac{1}{3}$	$301.00001\frac{1}{7}$

24. From nineteen thousand *units* subtract nineteen *ten-millionths.*

25. From one hundred million *units* subtract one *hundred-millionth.*

MULTIPLICATION OF DECIMALS

EXAMPLE I.—Multiply 7.23 by .46.

OPERATION.

```
 7.23
  .46
 ----
 4338
2892
------
3.3258
```

EXPLANATION (*a*).

$7.23 = 723 \div 100$; $.46 = 46 \div 100$; $723 \times 46 = 33\,258$; $33\,258 \div 100 = 332.58$; $332.58 \div 100 = 3.3258$ (*Principle IV*).

EXPLANATION (*b*).

$7.23 \times .46 = \frac{723}{100}\ \frac{46}{100} = \frac{33258}{10000} = 33\,258 \div 10000 = 3.3258$ (*Principle IV*).

EXAMPLE II.—Multiply .205 by .024.

OPERATION.

$$\begin{array}{r} .205 \\ .024 \\ \hline 820 \\ 410 \\ \hline .004920 \end{array}$$

EXPLANATION.

205 thousandths multiplied by 24 thousandths = 4 920 millionths, or .004920; hence the product has 6 decimal places, or as many as there are in both factors.

222. Rule.—*1° Multiply as if the factors were whole numbers; 2° from the right of the product point off as many decimal places as there are in both factors, prefixing ciphers when necessary.*

NOTE I.—Multiplying $38.4\frac{1}{4}$ by $.05\frac{3}{4}$ is the same as multiplying $384\frac{1}{4}$ by $5\frac{3}{4}$, and then pointing off three decimal places in the product.

NOTE II.—To multiply by 10, 100, 1 000, move the decimal point 1, 2, 3 places to the right (*Principle III*).

NOTE III.—Multiplying by .1, .01, .001, etc., is the same as dividing by 10, 100, 1 000, etc.

Written Exercises.

Multiply:

1. 1.649×11.
2. 59.247×0.25.
3. 24.0123×0.0031.
4. 0.369×0.0045.
5. 7.412×0.26.
6. 35.025×0.009.
7. 0.485×16.
8. 0.0024×0.0021.
9. 44.65×0.05.
10. 31×0.0005.
11. 43.2×0.0017.
12. 0.000875×2.75.
13. 0.0009×0.003.
14. 18.73×0.6.
15. 17×0.001.

Multiply, leaving factors as they stand:

16. $0.234\frac{1}{2} \times 0.25$.
17. $38.4\frac{1}{4} \times 0.5\frac{1}{4}$.
18. $1.345\frac{1}{2} \times 2.5\frac{1}{3}$.
19. $1342 \times 3.2\frac{1}{3}$.
20. $1624 \times 4.1\frac{1}{4}$.
21. $3792 \times 6.23\frac{1}{3}$.
22. $178.32 \times 1.24\frac{2}{3}$.
23. $0.396\frac{2}{3} \times 66\frac{2}{3}$.
24. $18.35\frac{6}{7} \times 2.41\frac{1}{3}$.
25. $5.20\frac{3}{4} \times 0.042\frac{4}{9}$.
26. $63.11\frac{1}{4} \times 4.44\frac{4}{9}$.
27. $72.6\frac{7}{20} \times 4800$.

28. $14.45\frac{3}{8} \times 0.085$.
29. $6.0081\frac{1}{3} \times 0.000\frac{2}{3}$.
30. $45.783\frac{2}{3} \times 0.0025$.
31. 78.7×100.
32. 0.434×100.
33. 0.3857×100.
34. 0.3067×1000.
35. 1.001×1000.
36. 36.02×1000.
37. $7000 \times 0.007\frac{1}{2}$.
38. $25000 \times 0.00025\frac{1}{2}$.
39. $6300 \times 0.005\frac{1}{2}$.
40. $10000 \times 0.00001\frac{1}{10}$.

DIVISION OF DECIMALS

EXAMPLE I.—Divide 230.67486 by 3.042.

OPERATION.

```
3.04200) 230.67486 (
 100000    100000
-------  ---------
 304200) 23067486 ( 75.83
          1773486
          2524860
           912600
           000000
```

EXPLANATION.

If the dividend and divisor be both multiplied by the same number, the quotient is not changed; so, multiplying both terms by 100 000, we have whole numbers. To continue the division, add a cipher to the remainder 252 486, and you will find the quotient 8 tenths; add a cipher to the remainder 91 260, and you will find the quotient 3 hundredths.

EXAMPLE II.—Divide $3.181\frac{5}{9}$ by $1.70\frac{2}{3}$, carrying quotient to 3 decimal places.

OPERATION.

```
1.70⅔)     3.181 5/9(
   9           9
-------  -----------
15.36       28.634
15360 ) 28634 (1.864+
        132740
         98600
          64400
           2960
```

EXPLANATION.

Multiply both complex decimals by the L. C. D. of their terminal fractions, obtaining 15.36 and 28.634; then divide as in the preceding case.

223. Rule.—*1° Give both terms the same number of places by annexing ciphers; 2° discard the decimal points; 3° proceed as with whole numbers.*

NOTE I.—Immediately before annexing the tenths' figure to the integral remainder, place the decimal point in the quotient.

NOTE II.—For each decimal order wanted in the quotient, add a cipher to the integral remainder.

224. NOTE III. *Another Method.*—Multiply both divisor and dividend by 10 enough times to make the divisor an integer, annexing ciphers to the dividend if necessary. Divide as with whole numbers, placing the decimal point in the quotient immediately before carrying down the tenths' figure.

NOTE IV.—Always try to determine beforehand what position the decimal point will occupy in the quotient.

Thus, in dividing 61.678 by 12.2, we see readily that 12 is contained 5 times in 61; the decimal point must therefore be placed immediately after the first figure.

NOTE V.—To divide by 10, 100, 1 000, move the decimal point 1, 2, 3 places to the left.

NOTE VI.—Dividing by .1, .01, .001, etc., is the same as multiplying by 10, 100, 1 000, etc.

Written Exercises.

1. $24.65 \div 0.005$.
2. $62 \div 0.0004$.
3. $864 \div 0.0016$.
4. $0.000875 \div 1.25$.
5. $0.0009 \div 0.003$.
6. $250 \div 0.0625$.
7. $36.5 \div 0.073$.
8. $0.95 \div 0.019$.
9. $9.5 \div 190$.
10. $9.5 \div 1.9$.
11. $1100 \div 4.4$.
12. $15.25 \div 0.005$.
13. $0.625 \div 2.5$.
14. $3.6 \div 1800$.
15. $0.005 \div 200$.
16. $27.465 \div 0.0015$
17. $1396.875 \div 250$.
18. $131300 \div 0.025$.
19. $62.5 \div 1.25$.
20. $0.00875 \div 125$.
21. $17.5 \div 1750$.
22. $0.44 \div 0.00011$.
23. $10000 \div 0.0001$
24. $0.001 \div 1000$.
25. $1.6 \div 0.064$.
26. $6400 \div 0.000016$.
27. $0.0081 \div 0.054$.
28. $1860 \div 0.000031$.
29. $195.36 \div 2.22$.
30. $8.118 \div 0.615$
31. $1 \div 0.1$.
32. $1 \div 0.01$.
33. $10 \div 0.1$.
34. $10 \div 0.01$.
35. $0.1 \div 1$.
36. $0.1 \div 0.1$.
37. $0.1 \div 0.01$.
38. $0.1 \div 0.001$.
39. $0.1 \div 10$.
40. $1 \div 10$.
41. $1 \div 100$.
42. $.01 \div 1000$.
43. $0.001 \div 100$.
44. $0.0001 \div 0.1$.
45. $100 \div 0.00001$.

46. $1000 \div 0.001$.
47. $0.00001 \div 1000$.
48. $10 \div 10000$.
49. $10000 \div 0.00001$.
50. $0.001 \div 100$.
51. $0.22 \div 11$.
52. $2.2 \div 0.011$.
53. $220 \div 11000$.
54. $0.022 \div 110$.
55. $0.00022 \div 11000$.
56. $2.2 \div 0.000011$.
57. $2200 \div 0.00011$.
58. $0.022 \div 11000$.
59. $0.000022 \div 110000$.
60. $22000 \div 0.00022$.
61. $3.6 \div 2.5$.
62. $360 \div 0.25$.
63. $0.0036 \div 250$.
64. $36 \div 0.0025$.
65. $360 \div 25000$.
66. $3600 \div 0.00025$.
67. $0.0036 \div 0.0025$.
68. $0.000036 \div 25000$.
69. $3600 \div 0.25$.
70. $36000 \div 0.000025$.
71. $0.875 \div 1250$.
72. $875 \div 0.0125$.
73. $8.75 \div 0.000125$.
74. $0.00875 \div 12500$.
75. $875000 \div 0.00125$.

Note.—In the following, carry the quotients to 3 decimal places.

76. $49 \div 0.06$.
77. $672.51 \div 17$.
78. $35.44 \div 7.1835$.
79. $100 \div 1758$.
80. $58.1351 \div 2.9$.
81. $41\frac{1}{9} \div 0.05$.
82. $74.28\frac{3}{16} \div 12.75$.
83. $5.004\frac{1}{7} \div 38\frac{1}{2}$.
84. $0.0\frac{2}{3} \div 9.5$.
85. $15\frac{3}{4} \div 18\frac{5}{7}$.
86. $5.1837\frac{2}{3} \div 0.08\frac{1}{7}$.
87. $9.180\frac{2}{9} \div 2.74\frac{2}{3}$.
88. $13.701\frac{1}{3} \div 0.17\frac{5}{6}$.
89. $457.62\frac{1}{5} \div 0.059\frac{1}{2}$.
90. $\frac{5}{7} \div 0.00\frac{1}{3}$.

91. $\dfrac{3.9 \times 0.84}{15.4} = ?$

92. $\dfrac{1.8 \times 5.05}{9.9 \times 0.1} = ?$

93. $\dfrac{0.21 \times 0.06}{0.0126} = ?$

94. $\dfrac{0.08 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10} = ?$

95. $\dfrac{1.2 \times 0.36 \times 1.2}{2.4 \times 0.42 \times 2.} = ?$

96. $\dfrac{0.056 \times 0.3}{0.63 \times 0.8} = ?$

97. $\dfrac{0.022 \times 0.24}{4 \times 0.77} = ?$

98. $\dfrac{0.0055 \times 0.42}{0.077} = ?$

99. $\dfrac{0.2 \times 3.2 \times 1.44}{9.6 \times 0.4 \times 2.4} = ?$

100. $\dfrac{6.3 \times 4.2 \times 1.8}{2.7 \times 3.6 \times 0.45} = ?$

Questions on Theory.

1. What is a decimal fraction? (211).
2. What is a mixed decimal? (212).
3. For what purpose is the decimal point used? (213).
4. Give the rule for reading a mixed decimal. (214).
5. Give the rule for writing a mixed decimal. (215).
6. Of what is the denominator of a decimal composed? (216, I).
7. Does annexing ciphers to the right of a decimal, or dropping them from it, affect the value of the decimal? (216, II).
8. Give the effect of moving the decimal point to the right; to the left. (216, III, IV).
9. How are common fractions reduced to decimals? (218).
10. How are decimals reduced to common fractions? (219).
11. Give the rule for adding decimals. (220).
12. Give the rule for the subtraction of decimals. (221).
13. How are decimals multiplied? (222).
14. Give the rule for dividing decimals. (223).
15. Give the other rule, which consists in reducing the divisor alone to a whole number. (224).

REVIEW OF DECIMALS.

First Series.

In the *first series*, will be applied to decimals the 13 principles of analysis. If you do not know them by heart, keep them close at hand.

Give exact answers, except when otherwise directed.

1. In 1916, the mileage in Quebec of the Canadian Pacific Railway was 560.16; of the Grand Trunk Railway, 450.74; of the Intercolonial Railway, 328.75; of the Quebec and Lake St. John Railway, 298.26; of the Quebec Central Railway, 252.85. Find the total length of these railways.

2. In 1900, the railway mileage of Quebec was 3 387.11; in 1915, 4 353.69. Find the increase.

3. In 1898, the railway mileage of Quebec was 3 377.32; in 1914, 961.21 greater. What was the mileage in 1914?

4. In 1867, the railway mileage of Quebec was 575.25; in 1907, 3 366.79 greater. What was the mileage in 1907?

5. Find the total mileage of the Quebec electric railways for 1916: Montreal Tramways, 124.32; Quebec Railway, 17.22; Hull Electric, 14.50; Sherbrooke Street, 7; Montreal Park & Island, 37.99; Levis County, 10.25; Montreal Terminal, 18.34.

6. In 1915, the mileage in Canada of the Canadian Pacific Railway was 12 823.49, and the mileage of all the other railways was 9 935.46 greater. Find the mileage of the other railways, and the total railway mileage of Canada.

7. In 1915, the mileage of the Grand Trunk Railway was 3 551.64; of the Grand Trunk Pacific, 1 322.73 less. Find the mileage of the Grand Trunk Pacific.

8. In 1915, the operating expenses of the railways of Canada were $4 151.57 per mile of line. Find the earnings per mile of line, if they were $1 464.84 greater.

9. In 1915, the operating expenses of the railways of Canada were $1 595 per train mile. Find the earnings per train mile if they were $0.559 greater.

10. The total earnings of our Canadian railways were 199.843 million dollars in 1915; the total expenses were 52.112 million dollars less. Find the total expenses.

11. In 1915, the total earnings of the Canadian Pacific Railway amounted to 90.83 million dollars, which was 30.618 million dollars more than the operating expenses. Find the expenses.

12. In 1915, the construction of Government Railways cost 21.865 million dollars, which was 7.549 million dollars less than in 1909. How much was it in 1909?

13. In 1915, the Government aid to independent railway companies amounted to 183.479 million dollars, which was 4.645 million dollars more than in 1914. Find the amount for 1914.

14. In 1914, the wages paid by the railway companies of Canada amounted to 111.762 million dollars, which was 3.987 million dollars less than in 1913. Find the amount for 1913.

15. In 1915, our Canadian railways hauled 87.204 million tons of freight, which was 19.788 million tons less than in 1913. Find the number of tons for 1913.

16. In 1915, the gross earnings of our Canadian electric railways amounted to $26.922\frac{9}{10}$ million dollars, and this amount was $8.791\frac{1}{10}$ million dollars greater than the operating expenses. Find the operating expenses.

17. The total tonnage of the seagoing vessels entered and cleared at Canadian ports in 1915 was $25.402\frac{1}{2}$ million, which was $4.165\frac{9}{10}$ million less than in 1914. Find the tonnage for 1914.

18. In 1915, Canada importations amounted to $587.439\frac{1}{3}$ million dollars, which was $46.253\frac{1}{15}$ million dollars less than in 1914. Find the amount for 1914.

19. Find the sum of the following agricultural produce imported by Canada in 1915 (in million dollars): rice $1.571\frac{1}{5}$, corn $6.734\frac{1}{5}$, wheat $1.803\frac{1}{3}$, cotton $6.533\frac{2}{3}$, dried fruits $4.935\frac{3}{5}$, bananas $2.296\frac{1}{5}$, oranges $4.246\frac{3}{5}$, tobacco $4.718\frac{3}{5}$, vegetables $3.039\frac{1}{3}$.

20. In 1915, Canada's agricultural imports amounted to $52.449\frac{1}{3}$ million dollars, which was $2.941\frac{2}{3}$ million dollars less than in 1913. Find the amount for 1913.

Note.—The 10 following problems give the lumber production of Quebec in 1915. In each problem, find the sum of the values. (*2 decimals*).

21. Spruce, 599 810 754 ft. (*board measure*) at \$15.41 per M. ft.

Note.—Multiply 599 810 754 by \$15.41, then divide by 1 000.

22. Fir, 170 793 935 ft. at \$14.32 per M. ft.; white pine, 157-255 605 ft. at \$22.68 per M. ft.

23. Red pine, 17 895 439 ft. at \$17.15 per M. ft.; gray pine, 12 005 561 ft. at \$16.48 per M. ft.

24. Hemlock, 38 064 229 ft. at \$13.91 per M. ft.; black birch, 41 991 860 ft. at \$17.98 per M. ft.

25. Cedar, 4 492 545 ft. at \$16.33 per M. ft.; tamarack, 2 791 077 ft. at \$17.01 per M. ft.

26. Maple, 6 404 675 ft. at \$18.03 per M. ft.; elm, 3 490 177 ft. at \$16.35 per M. ft.

27. Ash, 6 155 738 ft. at \$17.56 per M. ft.; white birch, 2 987 758 ft. at \$14.76 per M. ft.

28. Poplar, 1 086 225 ft. at \$14.37 per M. ft.; bass, 11 889 508 ft. at \$20.35 per M. ft.

29. Oak, 459 429 ft. at $27.37 per M. ft.; aspen, 907 225 ft. at $13.93 per M. ft.

30. Walnut, 245 728 ft. at $38.58 per M. ft.; cherry, 57 700 ft. at $30 per M. ft.

31. The value of the pulpwood produced in Quebec, 1915, was $8 327 891. Find how many cords at an average price of $6.50.

32. How many shingles were made in Quebec, 1915, if their total value was $1 264 553.40, and the market price was $2.20 per thousand?

33. In 1915, Quebec had 71 000 acres sown with wheat; in 1850, 410 000 acres. In 1915, how many times less than in 1850? (2 *decimals*).

34. In 1915, 71 000 acres produced 1 411 000 bushels of wheat. What was the average yield per acre? (2 *decimals*).

35. In Quebec, 1915, 1 400 000 acres produced 42 182 000 bushels of oats. Find the average yield per acre. (2 *decimals*).

36. In 1915, Quebec produced 42 182 000 bushels of oats; in 1890, 17 819 000 bushels; in 1890, how many times less than in 1915? (2 *decimals*).

37. In 1914, Quebec produced $24 429 020 worth of oats; how many bushels at $0.58 each?

38. How many bushels of barley were harvested in Quebec, 1915? Total value, $1 939 300; average price per bushel, $0.86.

39. In Quebec, 1915, 85 000 acres were sown with barley; with the number of bushels harvested according to the preceding problem, find the average yield per acre. (2 *decimals*).

40. In Quebec, 1915, 8 700 acres produced 145 000 bushels of rye. Find the average yield per acre. (2 *decimals*).

41. If the 145 000 bushels of rye harvested in 1915 were worth $162 000, find the average price per bushel. (2 *decimals*).

42. In Quebec, 1890, 25 939 acres were sown with rye; in 1915, 8 700 acres. In 1915, how many times less than in 1890? (2 *decimals*).

43. In Quebec, 1890, 155 000 acres were sown with peas; in 1915, only 24 400. In 1915, how many times less than in 1890? (2 *decimals*).

44. In 1915, Quebec harvested 404 000 bushels of peas from 24 400 acres. Find the average yield per acre. (2 *decimals*).

45. The harvest mentioned in problem 44 was worth $998 000. Find the average price of one bushel of peas. (2 *decimals*).

46. In Quebec, 1915, 4 700 acres produced 103 000 bushels of beans. Find the average yield per acre. (2 *decimals*).

47. If the harvest of beans mentioned in problem 46 was worth $327 000, what was the average price per bushel? (2 *decimals*).

48. In 1907, Quebec produced 330 000 bushels of beans. In 1907, the crop was how many times as great as in 1915? (2 *decimals*).

49. How many bushels of buckwheat were harvested in Quebec, 1915, if the total crop was worth $2 157 120, and 1 bushel, $0.84? (2 *decimals*).

50. With the preceding answer, find the number of bushels produced by one acre, if 104 000 acres were sown with buckwheat. (2 *decimals*).

51. In 1850, Quebec produced 532 000 bushels of buckwheat. How many times less than in 1915? (2 *decimals*).

52. In 1915, Quebec sowed 600 acres with flax; in 1890, 2 878 acres. The latter number is how many times as great as the former? (2 *decimals*).

53. Find the value of the 7 000 bushels of linseed harvested in Quebec, 1915, at $2.18 a bushel.

54. In Quebec, 1915, 117 000 acres produced 17 510 000 bushels of potatoes. Find the average yield per acre (2 *decimals*), and the value of one acre's yield at $0.55 a bushel.

55. Quebec's potato yield, in 1850, was 4.4 million bushels; in 1914, 21.8 million. The latter was how many times as great as the former? (2 *decimals*).

56. The hay yielded in Quebec, 1915, was worth $58.50\frac{2}{3}$ million dollars; in 1908, $38.19\frac{1}{3}$ million. In 1908, the yield was worth how many times less than in 1915? (2 *decimals*).

57. In Quebec, 1915, the yield of fodder corn was worth $1 872 000; that of corn for husking was worth $569 000. The former was worth how many times the latter? (2 *decimals*).

58. The diameter of a wheel is 33 inches; find the circumference, if it is 3.1416 times as great.

59. A cistern contains 162.125 cubic feet of water. If 1 cubic foot weighs 62.5 lb., find the total weight of the water.

60. The Canadian bushel contains 2 218.192 cubic inches; the Canadian gallon, 277.274 cubic inches. The bushel is worth how many times the gallon?

Second Series.

The following problems deal with the three cases of fractional relations. The *Base* is the quantity considered as a *whole*, or 1. The expressions *part of*, *decimal fraction of*, *fraction less than*, *fraction greater than*, 5 *hundredths of*, 3 *tenths more than*, 2 *tenths less than*, 6 *hundredths as many as*, are always followed by the *Base*.

FIRST CASE.—Base × Fraction (*common or decimal*) = Product.

SECOND CASE.—Product ÷ Base = Fraction (*common or decimal*).

THIRD CASE.—Product ÷ Fraction (*common or decimal*) = Base.

Written Exercises.

	A	B	C	D	E
	$	$	$		
1	20	200	180	.2	.25
2	60	300	160	.3	.20
3	40	600	450	.1	.30
4	30	200	160	.5	.50
5	50	300	275	.6	.60
6	70	800	700	.8	.75
7	80	600	500	.4	.80
8	40	300	100	.7	$1.33\frac{1}{3}$
9	30	300	175	.9	$1.66\frac{2}{3}$
10	180	240	200	$.2\frac{1}{2}$	$1.37\frac{1}{2}$

Answer the following questions with the numbers in line 1 of the table; then do the same for 2, 3, 4, 5, 6, 7, 8, 9, 10.

1. What is the *product*, if column A is the *base*, and column D, the *fraction*?

2. What is the *product*, if column B is the *base*, and column E, the *fraction*?

3. Column B is the *base*; column A, the *product*; what is the *decimal fraction*? (*2 decimals*).

4. Column A is *what fraction less* than column B? (*2 decimals*).

5. Column B is *what fraction greater* than column C? (*2 decimals*).

6. What is the *base*, if column A is the *product*, and column D, the *fraction*?

7. What is the *base*, if column B is the *product*, and column E, the *fraction*?

8. What is the *product*, if column B is the *base*, and column D, the *fraction*?

9. What is the *base*, if column B is the *product*, and column D, the *fraction*?

10. What is the *base*, if column A is the *product*, and column E, the *fraction*?

Written Problems.

List of equivalent common and decimal fractions.

$\frac{1}{2} = .50$	$\frac{1}{3} = .33\frac{1}{3}$	$\frac{1}{5} = .20$
$\frac{1}{4} = .25$	$\frac{2}{3} = .66\frac{2}{3}$	$\frac{2}{5} = .40$
$\frac{3}{4} = .75$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{3}{5} = .60$
$\frac{1}{8} = .12\frac{1}{2}$	$\frac{5}{6} = .83\frac{1}{3}$	$\frac{4}{5} = .80$
$\frac{3}{8} = .37\frac{1}{2}$	$\frac{1}{12} = .08\frac{1}{3}$	$\frac{1}{15} = .06\frac{2}{3}$
$\frac{5}{8} = .62\frac{1}{2}$	$\frac{1}{9} = .11\frac{1}{9}$	$\frac{1}{20} = .05$
$\frac{7}{8} = .87\frac{1}{2}$	$\frac{1}{10} = .10$	$\frac{1}{25} = .04$
$\frac{1}{16} = .06\frac{1}{4}$	$\frac{1}{100} = .01$	$\frac{1}{40} = .02\frac{1}{2}$

NOTE.—Commit the above to memory, and in the following problems shorten your work by replacing a decimal fraction by a more convenient common fraction.

1. My annual income is $1 500; I spend .2 of it for board, .05 of it for clothes, and .04 of it for books. Find the amount of each outlay.

2. I had a deposit of \$6 000 in bank and withdrew .25 of it. What sum did I withdraw?

3. A hawk captured $.12\frac{1}{2}$ of 152 chickens. How many were captured? How many were left?

4. A farmer sold $.06\frac{1}{4}$ of 960 sheep. How many were sold? How many remained?

5. My salary, \$1 200, was increased $.33\frac{1}{3}$; find the increase, and my actual income.

6. I sold .6 of 350 cows. How many cows remained?

7. A farmer had 376 acres of land, and sold .50 of it. How many acres remained?

8. A man's income is \$675 a year, and he spends $.67\frac{1}{2}$ of it. How much does he save?

9. A farm cost \$4 750; but since it was purchased the farm has increased $.16\frac{1}{2}$ in value. What is the present value of the farm?

10. A man having a fortune of \$24 000, invested $.06\frac{2}{3}$ of it in bank stock, $.08\frac{1}{3}$ in bonds, $.06\frac{1}{4}$ in mortgages, and the rest in the pulpwood industry. Find the latter investment.

11. Find the difference between $.87\frac{1}{2}$ of \$1 600 and $.83\frac{1}{3}$ of \$1 200.

12. A farmer had 300 acres of land and sowed .40 of it with wheat, and .50 of the remainder with oats. How many acres were there of each kind of grain?

13. Three men earned \$1 200; the first received .25 of the sum; the second, $.33\frac{1}{3}$ of the remainder, and the third, the rest. Find the share of each.

14. I sold \$2 400 worth of goods in 3 days; the first day brought in $.37\frac{1}{2}$ of this sum; the second day, .20 of the remainder. Find the sales of the third day.

15. A man bequeathed \$12 000 as follows: .50 to his family; .25 of the remainder to a charitable institution, and what remained to a Catholic newspaper enterprise. Find the latter amount.

16. I paid \$200 for a horse, and sold it for .25 more than it cost. Find the selling price.

17. I paid \$4 000 for a farm, and sold it for $.08\frac{1}{3}$ less than it cost. Find the selling price.

18. At what price must I sell a house of \$5 000 to receive $.08\frac{1}{2}$ more than it cost?

19. I owned .24 of a ship valued at \$375 000. How much did I receive for my share, if I sold it for $.37\frac{1}{2}$ more than it cost?

20. I bought 37 yd. of cloth at \$1.50 each, and 48 yd. of calico at \$0.14 each. If I sold the whole for .20 more than it cost, find the total selling price. (2 *decimals*).

NOTE.—Give the 20 following answers in complex decimals of two places.

21. The product of two numbers is 360; if one of the numbers is 30 000, what is the other?

22. What decimal fraction of 1 728 is 144?

23. What part of 375 is 75?

24. \$12.50 is what part of \$84?

25. An insolvent debtor owed me \$2 092; I accepted \$1 150.62 in full payment of his debt. What part of the debt did he pay?

26. My house, worth \$16 500, was entirely destroyed by fire; if I received \$7 260 from the underwriters, what part did I lose?

27. I bought a house for \$24 000, and sold it \$27 840. The selling price was what fraction greater than the cost?

28. My father's yearly income is \$1 600; if the household expenses amount to \$1 300, what fraction of his salary does he save?

29. I bought a house for \$3 720, and sold it for \$232.50 more than it cost. The profit represents what fraction of the cost?

30. I paid a premium of \$63.75 for insuring a house for \$8 500. The premium represents what part of the amount of insurance?

NOTE.—Food is not only a body-builder, it is also a heat-giver. Our bodies are human machines, and like all other machines, require fuel for their maintenance. The fuel value of food is measured by *calories*. A *calorie* is the amount of heat necessary to raise the temperature of 1 000 grams of water one degree centigrade.

31. Leg of lean mutton furnishes 790 calories per pound, and rib of beef, 1 150 per pound. The latter furnishes what fraction more than the former?

32. Shad furnishes 380 calories per pound, and chicken, 505 per pound. The latter furnishes what fraction more than the former?

33. Apples furnish 290 calories per pound, and bananas, 460 per pound. Apples furnish what fraction less than bananas?

34. Prunes furnish 370 calories per pound, and watermelons, 140 per pound. Watermelons furnish what fraction less than prunes?

35. Carrots furnish 210 calories per pound, and Lima beans, 570 per pound. Carrots give how many hundredths less than beans?

36. Lettuce furnishes 90 calories per pound, and cucumber, 80 per pound. 1° Cucumber gives what fraction less than lettuce? 2° Lettuce gives what fraction more than cucumber?

37. Macaroni furnishes 1 665 calories per pound, and peanuts furnish 2 560 per pound. 1° Peanuts give what fraction more than macaroni? 2° Macaroni furnishes what fraction less than peanuts?

38. Fifteen cents' worth of milk furnishes 1 000 calories; 15 cents' worth of bread furnishes 2 500 calories; and 15 cents' worth of potatoes furnishes 3 000 calories. 1° Bread gives what fraction more than milk? 2° Potatoes give what fraction less than milk and bread together?

39. Fifteen cents' worth of beef furnishes 800 calories; 15 cents' worth of rice furnishes 3 000 calories; and 15 cents' worth of pork furnishes 1 000 calories. Beef and pork together give what fraction less than rice?

40. Ten cents' worth of cornmeal gives 6 500 calories, and 10 cents' worth of oatmeal gives 4 000 calories. The former furnishes how many hundredths more than the latter?

NOTE.—The full value of foods does not depend on their price. Bread, cornmeal, oatmeal, beans, potatoes, rice, can take the place of meat in this respect, and at less than a quarter of its cost.

Science proves that alcohol, far from increasing the heat of the body, decreases it on the contrary.

41. What sum must be multiplied by 25 hundredths to produce $400?

42. 6 hundredths of what number equals 324?

43. The product of two numbers is 75; one of the factors is .03; find the other.

44. A farmer spent .42 of his money and had $53 070 remaining. How much had he at first?

45. .35 of my fortune is invested in bank stock, .10 of it in mortgages, .20 of it in real estate, and the remainder in bonds. If $7 000 are in bonds, what is the amount of my fortune?

46. I withdrew $2 058 from bank, which was .28 of my bank account. How much money have I remaining in bank?

47. A owns .15 of a ship, B .25 of it, C .30 of it, and D the remainder. Find the value of A's share, if B's share is worth $12 000.

48. I paid .35 of a debt and still owe $6 750. How much did I give?

49. A grocer sold .20 of his stock of sugar, then .50 of the remainder. How much sugar had he at first, if 800 lb. remained?

50. I own .40 of a mill, and sell .25 of my share for $1 000. How much is the mill worth?

51. I sold a horse for $600, which was .20 more than it cost. Find the cost.

52. I sell my house for $5 000, and this sum equals .16$\frac{2}{3}$ more than the cost of the house. Find the cost.

53. What number increased by .25 of itself equals $500?

54. I paid $1 701 to settle an account, and afterwards found that I had given .26 more than the amount due. What was the amount due?

55. A quantity of soap lost .06 of its weight by desiccation, and then weighed 8 460 lb. Find the original weight.

56. A quantity of coffee lost .08 of its weight in roasting, and then weighed 736 lb. Find the original weight.

57. A and B together have $4 500, and B's share equals .25 more than A's share. How much has each?

58. A and B together have $3 360, and A's share equals .25 less than B's share. How much has each?

59. A herder has charge of 1 140 sheep and goats, and the number of sheep is .40 greater than the number of goats. Find the number of sheep.

60. A merchant found that his sales in 1914 were .20 greater than his sales in 1913; in 1915, .30 greater than in 1914; and in 1916, .25 greater than in 1915. If his sales in 1916 were $1 900 greater than his sales in 1913, what were his sales in 1915?

CARRYING OF GOODS.

A BAD ROAD.

(*Give exact answers*).

1. Bad roads are expensive. Here is a farmer that had 3 tons (1 *ton* = 2 000 *lb.*) of barbed wire to haul over 25 miles of impassable roads. If each load weighed .75 of a ton and required 1.5 days of painful efforts, how many days did he take

in all? What sum did the cartage represent, if the time of a man and team is worth $4 a day?

2. In these conditions, what was the cost of conveying 1 ton 1 mile?

3. Over the same road macadamized, the farmer can easily in one day convey a load weighing 1.5 tons. What will it cost to convey the 3 tons of wire? What will the cartage cost per ton-mile?

4. The cost per ton-mile is what fraction less in the second case than in the first? (2 *decimals*).

5. What would it cost to convey 32 tons of goods 15 miles: 1° over bad roads? 2° over good roads?

A GOOD ROAD.

6. A farmer lives 10 miles from the railway station. Before the roads were macadamized, hauling 40 bushels of wheat to the station took him a complete day. What did the cartage of one bushel of wheat come to at $4 a day for man and team?

7. The municipality borrowed money to make good roads, and the farmer can now, without any fatigue for his horses, haul 75 bushels of wheat to the station in one day. What does the cartage of one bushel now come to?

8. The cartage of one bushel of wheat now costs what fraction less than it did before? (2 *decimals*).

9. The cartage per ton-mile costs how many cents less in N° 7 than in N° 6? (1 *bushel of wheat* = 60 *lb.*). At that rate, how much would be saved in conveying 50 bushels of wheat 15 miles?

10. If an acre of land yields on an average 20 bushels of wheat, what fraction of a ton does it yield? (2 *decimals*).

11. Let us suppose that every farmer in this country must convey his wheat 5 miles, and that good roads reduce the cost of hauling 5 cents per ton-mile; then, what sum would be economized by every farmer conveying an acre's yield over good roads?

12. In N° 11, what sum would be economized per square mile (640 *acres*)?

13. In Quebec, 1915, 71 000 acres were sown with wheat; with the preceding data, figure out what sum would have been economized in conveying the wheat over good roads.

14. In Quebec, 1915, 1 400 000 acres were sown with oats, yielding on an average 30 bushels each. A bushel of oats weighs 34 lb. Five cents being economized on the cartage per ton-mile, what sum would have been saved in conveying the oats 5 miles over good roads?

15. In 1915, the Quebec Government spent $6 140 273.13 for good roads; in 1914, $4 069 307.68. Find the sum.

From 1894 to 1914, only $4 375 100.31 were spent. Find the total expenditure, from 1894 to 1916.

Per cent.—[illegible] *per cent* is a commonly used expression equivalent [illegible] *ths*. For example, 25 hundredths may be expressed 2[illegible]

There is a special sign [illegible] *per cent:* %.

Thus, .15 = 15%; .20 = 20%; .01 = 1%.

A decimal like .3 may be written .30, or 30%; $\frac{1}{2}$ may be written $\frac{50}{100}$, or .50, or 50%.

So, $\frac{1}{2}$ = .50 = 50%.	$\frac{1}{5}$ = .20 = 20%.
$\frac{1}{4}$ = .25 = 25%.	$\frac{3}{5}$ = .60 = 60%.
$\frac{3}{4}$ = .75 = 75%.	$\frac{1}{10}$ = .10 = 10%.
$\frac{1}{8}$ = .12$\frac{1}{2}$ = 12$\frac{1}{2}$%.	$\frac{1}{20}$ = .05 = 5%.

Written Exercises.

Add, subtract, multiply, and divide, using each number of the first column with each lettered number of the second column. Carry quotients to 3 decimal places.

I	182.716		*a*)	0.4.
II	3790.52		*b*)	0.7.
III	5.69182		*c*)	1.5.
IV	75236.3	+	*d*)	3.4.
V	92.824	−	*e*)	1.2.
VI	111.8162	×	*f*)	2.6.
VII	13.18092	÷	*g*)	0.58.
VIII	1517.162		*h*)	0.11.
IX	17.05338		*i*)	$1.20\frac{1}{2}$.
X	18.9162		*j*)	$0.77\frac{1}{4}$.

COUNTINGHOUSE PRACTICE

Simplify your work by changing the *decimal* price to its equivalent *common fraction* of a dollar.

Fractions of a dollar. (*To be thoroughly memorized*).

1 c = $\frac{1}{100}$ of \$1.	25 c = $\frac{1}{4}$ of \$1.
2 c = $\frac{1}{50}$ of \$1.	$33\frac{1}{3}$c = $\frac{1}{3}$ of \$1.
$2\frac{1}{2}$c = $\frac{1}{40}$ of \$1.	$37\frac{1}{2}$c = $\frac{3}{8}$ of \$1.
4 c = $\frac{1}{25}$ of \$1.	40 c = $\frac{2}{5}$ of \$1.
5 c = $\frac{1}{20}$ of \$1.	50 c = $\frac{1}{2}$ of \$1.
$6\frac{1}{4}$c = $\frac{1}{16}$ of \$1.	$62\frac{1}{2}$c = $\frac{5}{8}$ of \$1.
$8\frac{1}{3}$c = $\frac{1}{12}$ of \$1.	$66\frac{2}{3}$c = $\frac{2}{3}$ of \$1.
10 c = $\frac{1}{10}$ of \$1.	75 c = $\frac{3}{4}$ of \$1.
$12\frac{1}{2}$c = $\frac{1}{8}$ of \$1.	80 c = $\frac{4}{5}$ of \$1.
$16\frac{2}{3}$c = $\frac{1}{6}$ of \$1.	$83\frac{1}{3}$c = $\frac{5}{6}$ of \$1.
20 c = $\frac{1}{5}$ of \$1.	$87\frac{1}{2}$c = $\frac{7}{8}$ of \$1.

First case.—Finding the cost.

Example.—What is the cost of 800 bushels of wheat at $1.25 each?

Operation (*a*).

$\$1 \times 800 = \$800;$
$\$\frac{1}{4} \times 800 = \$200;$
$\$800 + \$200 = \$1\,000.$

Explanation (*a*).

800 bushels at $1 = $800; 800 bushels at $\$\frac{1}{4} = \$200$; $800 + $200 = $1 000.

Operation (*b*).

$\$\frac{5 \times 800}{4} = \$1\,000.$

Explanation (*b*).

$\$1.25 = \$1\frac{1}{4}$, or $\$\frac{5}{4}$; $\$\frac{5}{4} \times 800 = \$1\,000.$

Written Exercises.

Find the total cost of:

1.	2.
125 lb. b[illegible] 20c;	960 lb. rice, at $6\frac{1}{4}$c;
363 doz. [illegible] at $33\frac{1}{3}$c;	400 lb. beans, at 5c;
144 qt. m[illegible], at $8\frac{1}{3}$c;	360 lb. dried apples, at $12\frac{1}{2}$c;
720 lb. butter, at $37\frac{1}{2}$c;	300 lb. prunes, at 10c;
336 lb. cheese, at 25c;	288 lb. sugar, at $8\frac{1}{3}$c;
125 lb. bread, at 4c;	560 bags of potatoes, at $87\frac{1}{2}$c;
310 lb. oatmeal, at 5c.	540 qt. of vinegar, at $16\frac{2}{3}$c.

3.	4.
123 bu. wheat, at $\$1.33\frac{1}{3}$;	180 bu. linseed, at $\$2.16\frac{2}{3}$;
632 bu. oats, at $0.50;	320 bu. shelled corn, at $\$1.12\frac{1}{2}$;
540 bu. barley, at $\$0.83\frac{1}{3}$;	615 bu. turnips, at $\$0.33\frac{1}{3}$;
240 bu. rye, at $\$1.12\frac{1}{2}$;	640 lb. tobacco, at $\$0.37\frac{1}{2}$;
250 bu. peas, at $2.40;	120 cords of wood, at $\$4.83\frac{1}{3}$;
120 bu. beans, at $\$3.16\frac{2}{3}$;	336 gal. kerosene, at $0.25;
420 bu. buckwh. at $\$0.87\frac{1}{2}$.	80 tons of coal, at $8.50.

Second case.—Finding the number of objects.

Example.—At 25c each, how many yards can be bought for $75?

Operation.

$75 \times \frac{4}{1} = 300.$

Explanation.

$25c = \$\frac{1}{4}$; for $1, I can buy 4 yd., and for $75, 75 times 4, or 300 yd.
Also, $\$75 \div \$\frac{1}{4} = 300$ yd.

Written Exercises.

Find the number of yards.

	Cost.	Price per yard.
1.	$61.	$0.12½.
2.	$79.	.25.
3.	$89.	.16⅔.
4.	$91.	.33⅓.
5.	$52.	.06¼.
6.	$93.	.75.
7.	$69½.	.12½.
8.	$73⅕.	.20.
9.	$42.	.37½.
10.	$84.	.62½.

Find the number of gallons.

	Cost.	Price per gallon
11.	$70.	$0.87½.
12.	$95.	.83⅓.
13.	$54.	.66⅔.
14.	$72.	1.12½.
15.	$55.	1.37½.
16.	$78.	1.62½.
17.	$75.	1.87½.
18.	$42.	1.75.
19.	$54.	2.25.
20.	$125.	1.66⅔.

RECEIPTS, BILLS AND ACCOUNTS

RECEIPT.

Arthur Roy hired Joseph Gorman's house for a year at $25 a month. The rent is payable in advance on the first of each month. This is the *receipt* that Joseph Gorman gave Arthur Roy on the first of September 1917, for one month's rent.

(b) *$25.00* Montreal, (a) *September 1, 1917.*

Received of Mr. (c) *Arthur Roy*

the sum of (d) *twenty-five* $\frac{00}{100}$ *Dollars,*

for (e) *the rent of September.*

(f) *Joseph Gorman.*

EXPLANATIONS.—(*a*) Place and date; (*b*) amount in figures; (*c*) name of payer; (*d*) amount in words; (*e*) what the money was given for; (*f*) signature of receiver.

A **receipt** is a writing acknowledging the taking of money. A receipt may be *in part* or *in full* payment of a debt, and it operates as a discharge of the debt either in part or in full.

Make out receipts to suit the following cases:

1. You are a milkman, and Donald Bates hands you $6.75 in part payment of his account.

2. You are a grocer, and Owen Kennedy hands you $13.22 in full of account to date.

3. You are a baker, and Henry Clark hands you all he owes, $12.50.

4. You are a shoemaker, and William Hancock hands you $7.40 to apply on account.

5. You are a butcher, and Leo Brooks hands you $18.10 in full of account to date.

Make out the receipts that would be handed to you in the following cases:

1. You pay $5.90 to Paul Wilson, your milkman, in full of account to date.

2. You pay $16.85 to your grocer, Daniel Walsh, to apply on account.

3. You pay $11.25 to your landlord, Albert Dalphond, for the rent of March.

4. You pay $3.75 to your shoemaker, Patrick Murphy, in full of account to date.

5. You pay $4.75 to your baker, Arthur Forest, to apply on account.

ORDER FOR GOODS.

(a) *Rawdon, Que., September 17, 1917.*

(b) *Messrs. Laporte, Martin, Limited,*
584, St. Paul Street West,
Montreal.

(c) *Gentlemen:*

(d) *Please ship me by the Ca[illegible]n Northern Railway:*

280 lb. dried peaches;
48 cans cocoa;
120 lb. rice.

(e) *Yours truly,*

(f) *Francis Slavin.*

EXPLANATIONS.—(*a*) Place and date; (*b*) name and address of the merchant; (*c*) salutation; (*d*) body of the letter; (*e*) complimentary closing; (*f*) signature.

1. Write a letter ordering goods from the different editors of your schoolbooks.

2. Order goods from your grocers, Messrs. Sanche & Leblanc, 1355, St. Lawrence Boulevard, Montreal.

RECEIPTED BILL.

(a) MONTREAL, *Sept. 18,* 1917.

(b) *Mr. Francis Slavin,*
Rawdon, Que.

(c) Bought of **LAPORTE, MARTIN, LIMITED,**
584, ST. PAUL STREET WEST.

(d) TERMS: *Cash.*

(e)	(f)	(g)		
280	*lb. Dried Peaches*	*.22*	(h) *$61*	*60*
48	*cans Cocoa*	*.25*	*12*	
120	*lb. Rice*	*.08*	*9*	*60*
			(i) *$83*	*20*

(j) *Received payment,*
(k) *Sept. 21, 1917.*
(l) *Laporte, Martin, Limited,*
per J. R.

Explanations.—(a) Place and date; (b) name and address of buyer; (c) name of seller; (d) terms: *cash* or *on account*—30 days, 60 days, 90 days; (e, f, g) quantity, name, and price of each item; (h) amount of each item; (i) total amount; (j) the words "Received payment" mean that the bill was paid; (k) date of payment; (l) name of firm, and initials of clerk.

A **bill** or **invoice** is a detailed list of merchandise sold. It is forwarded to the buyer, who sends it back to have it receipted when he pays, and it is then again forwarded to the buyer.

Make out in proper form and receipt the following bills:

1. On March 7, 1917, James Fallon, Rigaud, Que., bought of Sanche & Leblanc, wholesale grocers, Montreal: 25 lb. tea @ 55c; 40 lb. butter @ 35c; 12 lb. maple sugar @ 15c. Terms: 30 days.

2. On May 30, 1917, A. S. Hart, Joliette, Que., bought of Monet & Prairie, Montreal: 60 pr. rubbers @ 85c; 24 pr. shoes @ $1.95. Terms: cash.

3. On July 31, 1917, Sylvio Martin, Terrebonne, Que., bought of Lessard & Bernard, Montreal: 30 yd. linen @ 45c; 35 yd. carpet @ 55c; 6 doz. handkerchiefs @ $2; 40 yd. satin @ $1. Terms: 60 days.

4. On Sept. 18, 1917, Arthur Robert, Cartierville, Que., bought of Roy & Hubert, Montreal: 6 chairs @ $2.25; 2 tables @ $9.75; 4 armchairs @ $5.25; 2 sofas @ $16.25. Terms: cash.

5. On Oct. 22, 1917, J. W. Maguire, St. Lambert, Que., bought of Blain & Rochon, Montreal: 500 lb. granulated sugar @ $7\frac{1}{2}$c; 300 lb. prunes @ 12c; 150 lb. coffee @ 45c. Terms: 30 days.

ACCOUNT OF A CUSTOMER.

(b) *Dr.* (a) *William Hancock.* (c) *Cr.*

(d) 1918	(f)	(g)	(h)		(d) 1918	(f)	(i)	(j)	
May	*4*	*Mdse.*	*796*	*45*	*May*	*10*	*Cash*	*500*	
(e)	*15*	*Mdse.*	*347*	*63*	(e)	*18*	*Cash*	*125*	*68*
	19	*Mdse.*	*467*	*38*		*22*	*Cash*	*900*	
	26	*Mdse.*	*531*	*09*					

Explanations.—(a) William Hancock has bought on credit of J. A. Prairie, his supplier, May 4th, 15th, 19th, 26th, 1918; he has made cash payments May 10th, 18th, 22d, 1918. The record of these sums due, and paid, is an *account;* (b) Dr. = the debit or left side of an account; (c) Cr. = the credit or right side of an account; (d) year; (e) month; (f) day of the month; (g) *Mdse*, abbreviation of *Merchandise*, stands for *merchandise bought;* (h) the aggregate amount of each purchase; (i) *Cash* stands for the money paid in; (j) the amount of each cash payment.

This **account** is kept in the ledger of the merchant selling goods on credit. At the end of each month, the merchant sends each customer a written exhibit of all purchases and payments made during the month; this is called a *statement of account.*

STATEMENT OF ACCOUNT.

(a) MONTREAL, *May 31,* 19*18.*

(b) *Mr. William Hancock,*

Montreal.

(c) IN ACCOUNT WITH **J. A. PRAIRIE.**

(e)	(f)	(d) *Dr.*	(k)			
May	*4*	*Mdse., a. p. b. r.*	*$796*	*45*		
	15	" "	*347*	*63*		
	19	" "	*467*	*38*		
	26	" "	*531*	*09*	(l)	
		(g) (h)			*$2142*	*55*
(e)	(f)	(i) *Cr.*	(m)			
May	*10*	*Cash*	*500*			
	18	"	*125*	*68*	(n)	
	22	"	*900*		*$1525*	*68*
		(j)				
		(o) *Balance due*			*$616*	*87*
					(p)	

EXPLANATIONS.—(*a*) Place and date; (*b*) name and address of customer; (*c*) supplying merchant; (*d*) Dr. = debit side; (*e*) month; (*f*) day of the month; (*g*) what was bought on credit; (*h*) a. p. b. r. = *as per bill rendered;* this means that the details of each purchase have been previously rendered; (*i*) Cr. = credit side; (*j*) *Cash* = payments made; (*k*) the aggregate amount of each bill; (*l*) total amount of the purchases; (*m*) separate amounts paid in; (*n*) total amount given in cash; (*o*) balance due; the payments were subtracted from the purchases and amounted to (*p*) $616.87.

A **statement of account** is not an itemized bill; it contains only the aggregate of each purchase, and each cash payment made.

If, in the preceding example, William Hancock had had any older debts than those mentioned, we should have written their amount on the first line, as follows: April 30, *as per statement rendered*, $. . . . ; and this amount would have been footed with the debits of May.

DENOMINATE NUMBERS

When you say "*My watch lost* 8 *hours and* 58 *minutes in a month*", you employ a *compound denominate number.*

225. A compound denominate number is a number composed of units of two or more denominations that are related to each other.

A denominate number which is composed of units of one denomination only is called a *simple denominate number.*

Denominate numbers are used to measure time, weight, extension, capacity, value, etc.

MEASURES OF TIME.

226. The standard unit for measuring time is a *solar day* or the average time required by the earth to make one revolution on its axis.

TIME.

60 seconds (*sec.*)	= 1 minute,	(*min.*)
60 minutes	= 1 hour,	(*hr.*)
24 hours	= 1 day,	(*da.*)
7 days	= 1 week,	(*wk.*)
12 months (*mo.*)	= 1 year,	(*yr.*)

A century is 100 years. The common year has 365 days, or 52 weeks and 1 day; the leap year has 366 days.

Thirty days has *September*,
April, *June*, and *November*.

The other months have 31 days, except *February*, which has 28 days in common years and 29 days in leap years.

Years whose numbers are divisible by 4 are *leap years:* 1904, 1908, 1912, 1916, 1920; centennial years though must be divisible by 400 to be leap years: 1600, 2000.

A solar year is the time in which the earth makes one revolution around the sun, or 365 da. 5 hr. 48 min. and 49.7 sec.

Oral Exercises.

1. The school year lasts 10 mo.; find how many weeks it lasts, counting 4 wk. to the month.

2. How many minutes are there in the $5\frac{1}{2}$ hr. of a school day?

3. Name the months having 30 da., 31 da.

4. How many hours in 180 min.? 240 min.? 300 min.?

5. How many hours in 4 da.? 5 da.? 1 wk.?

6. How many days in 2 yr.?

7. In 1 hr., how many seconds?

8. In 12 hr., how many minutes?

9. How many days in 48 hr.? 96 hr.?

10. Name the leap years between 1890 and 1920.

Note.—The 17th century began with 1601, the 18th with 1701, the 19th with 1801, the 20th with 1901.

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

227. *Avoirdupois Weight* is used for weighing goods.

TABLE.

16 ounces (*oz.*)	= 1 pound,	(*lb.*)
100 pounds	= 1 hundredweight,	(*cwt.*)
20 hundredweight	= 2000 lb. = 1 ton,	(*T.*)

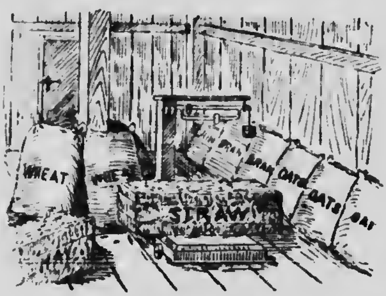
In Great Britain, 112 lb. equal 1 cwt., and 2 240 lb. equal 1 ton; the latter is called the *long ton*; it is used here only in coal and iron mines. The ton of 2 000 lb. is sometimes called the *short ton.*

An 8-ounce weight maintains the box in equilibrium. Wilfrid is holding the 4-ounce weight.

Near Irene are the 1-pound, 2-ounce, and 1-ounce weights.

A PAIR OF SCALES.

Oral Exercises.



BEAM SCALE.

1. How many ounces in $\frac{1}{2}$ lb.? $\frac{1}{4}$ lb.? $\frac{3}{4}$ lb.? $1\frac{1}{2}$ lb.?

2. How many pounds in $\frac{1}{2}$ cwt.? $\frac{1}{4}$ cwt.? $\frac{3}{4}$ cwt.? $1\frac{1}{2}$ cwt.? $2\frac{1}{2}$ cwt.?

3. How many hundredweight in 100 lb.? 200 lb.? 300 lb.? 400 lb.? 500 lb.?

4. How many pounds in $\frac{1}{2}$ T.? $\frac{1}{4}$ T.? $\frac{3}{4}$ T.? $1\frac{1}{2}$ T.? 2 T.?

5. In the first picture, with the 5 weights represented, could 1 oz. of sugar be weighed? 3 oz.? 5 oz.? 6 oz.? 7 oz.? $\frac{3}{4}$ lb.? 10 oz.? 15 oz.? $1\frac{1}{4}$ lb.? $1\frac{1}{2}$ lb.? $1\frac{3}{4}$ lb.? 2 lb.?

6. A boy weighs 90 lb. How many ounces does he weigh?

7. How many hundredweight in 600 lb.? 700 lb.? 750 lb.? 875 lb.?

8. How many tons in 4 000 lb.? 8 000 lb.? 5 000 lb.? 1 500 lb.?

9. How many ounces in 2 lb. 4 oz.?

10. What fraction of a hundredweight is an object weighing 10 lb.? 20 lb.? 50 lb.? 75 lb.? 90 lb.?

Other Weights.

228. Goldsmiths still use the old table of *Troy Weight*, and druggists use the table of *Apothecaries' Weight* in compounding dry medicines; but neither is important enough to be long dwelt on.

TROY WEIGHT.

24 grains (*gr.*)	=	1 pennyweight,	(*pwt.*)
20 pennyweights	=	1 ounce,	(*oz.*)
12 ounces	=	1 pound,	(*lb.*)
5 760 grains	=	1 pound.	

APOTHECARIES' WEIGHT.

20 grains (*gr.*)	=	1 scruple,	(℈).
3 scruples	=	1 dram,	(ʒ).
8 drams	=	1 ounce,	(℥).
12 ounces	=	1 pound,	(*lb.*)
5 760 grains	=	1 pound.	

The pound, ounce, and grain weights are identical in both cases.

The Avoirdupois pound contains 7 000 grains.

Oral Exercises.

Troy weight:

1. How many grains in 2 pwt.? 3 pwt.? 4 pwt.?
2. How many grains in 1 oz.?
3. How many pennyweights in 8 oz.? 9 oz.?
4. How many pennyweights in 1 lb.? 2 lb.?
5. How many ounces in $\frac{1}{2}$ lb.? $\frac{3}{4}$ lb.? $1\frac{1}{2}$ lb.?

Apothecaries' weight:

1. How many grains in 2℈? 3℈? 4℈?
2. How many scruples in 60 gr.? 80 gr.?
3. How many grains in 2ʒ?
4. How many ounces in 40ʒ?
5. How many ounces in $\frac{1}{2}$ lb.? $\frac{3}{4}$ lb.? $1\frac{1}{2}$ lb.?

NOTE I.—The word *carat* is used to denote the fineness of gold and means $\frac{1}{24}$; 15 *carats fine* means $\frac{15}{24}$ pure gold.

NOTE II.—Physicians use the Roman notation in writing prescriptions, employing the small letters, preceded by the symbols. Thus, 6 ounces, 7 drams, are written ℥ vj, ʒ vij.

MEASURES OF EXTENSION

MEN MEASURING A DISTANCE WITH A TAPELINE.

LINEAR MEASURE.

229. *Linear* or *long measure* is used in measuring length, breadth, height, distance, etc.

The standard unit is the *yard*.

Table.

12 inches (*in.*)	= 1 foot,	(*ft.*)
3 feet	= 1 yard,	(*yd.*)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ ft.	= 1 rod,	(*rd.*)
320 rods	= 1 mile,	(*mi.*)
1 760 yd. or 5 280 ft.	= 1 mile.	

Carpenters and architects often write 6″ for 6 inches, and 2′ for 2 feet.

The following denominations are sometimes used: 1 cubit = $1\frac{1}{2}$ ft.; 1 hand = 4 in.; 1 span = 9 in.; 1 pace = 3 ft.; 1 fathom = 6 ft.; 1 knot = 6 086 ft., or 1.15 mi.; 1 furlong = $\frac{1}{8}$ mi., or 40 rods; 1 league = 3 mi.; 1 *arpent* = 192 ft.

Oral Exercises.

1. How many inches in 2 ft.? 3 ft.? 4 ft.? $1\frac{1}{2}$ ft.? $\frac{3}{4}$ ft.? $\frac{1}{3}$ ft.? 5 ft.?

2. How many inches in 1 yd.? $\frac{1}{2}$ yd.? $1\frac{1}{2}$ yd.? 2 yd.? $\frac{1}{3}$ yd.?

3. How many feet in 2 yd.? 3 yd.? 10 yd.? $\frac{1}{3}$ yd.? $5\frac{1}{2}$ yd.?

4. How many feet in 1 rd.? 10 rd.?

5. In $\frac{1}{4}$ yd., how many inches?

6. What part of 1 ft. is 1 in.?

7. What part of 1 yd. is 1 ft.?

8. What part of 1 yd. is 1 in.?

9. What part of 1 mi. is 1 rd.?

10. What part of 1 mi. is 1 ft.?

11. How many yards in 36 in.? 72 in.? 144 in.?

12. How many yards in 9 ft.? 12 ft.? 15 ft.?

13. How many yards in 1 rd.? 2 rd.? 4 rd.?

14. Find the cost of 72 in. of velvet at $6 a yard.

15. How many miles in 1 league? 2? 3? $4\frac{1}{2}$?

16. In the diagram of page 233, 1 in. = 64 ft. Use your ruler to answer the following questions:

1. If 64 ft. are represented by 1 in., how many feet are represented by $\frac{1}{8}$ in.? $\frac{1}{2}$ in.? $\frac{3}{8}$ in.? $\frac{5}{8}$ in.?

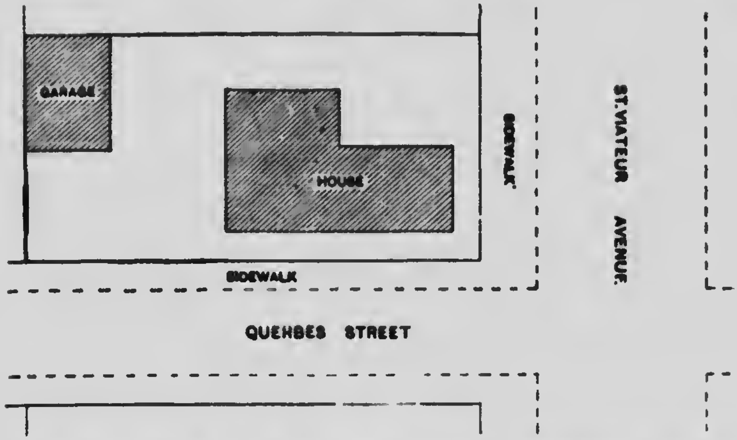
2. Find how many feet this lot measures on St. Viateur Avenue.

3. Find how many feet it measures on Querbes Street.

4. Find the front and rear measures of the house fronting St. Viateur Avenue.

5. Find the length and width of the garage.

6. Find the breadth of St. Viateur Avenue, between both sidewalks.



7. Find the breadth of Querbes Street, between both sidewalks.

8. Find the breadth of the sidewalk on St. Viateur Avenue, on Querbes Street.

9. Find the space between the house and the sidewalk.

10. How long a string would measure around the garage? the house? the lot?

Surveyors' Linear Measure.

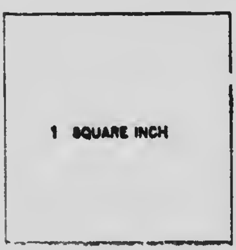
TABLE.

7.92	inches	= 1 link,	(*l.*)
100	links	= 1 chain,	(*ch.*)
80	chains	= 1 mile,	(*mi.*)

This table is used by surveyors in measuring boundaries of land, etc. 1 chain = 66 ft.

For railroad and other purposes, engineers now use a chain 100 ft. long, the feet being divided into tenths.

MEASURES OF SURFACE.



230. The *surface* or *square measure* is used in measuring surfaces, such as land, boards, plastering, painting, etc.

231. A surface is the *exterior* of a solid or body. A unit of square measure is a square bounded by lines of some known length. Thus, a square inch is a square whose sides are one inch long.

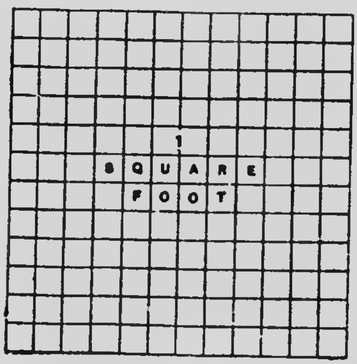
TABLE.

144 square inches (*sq. in.*)	= 1 square foot,	(*sq. ft.*)
9 square feet	= 1 square yard,	(*sq. yd.*)
$30\frac{1}{4}$ square yards	= 1 square rod,	(*sq. rd.*)
160 square rods	= 1 acre,	(*A.*)
640 acres	= 1 square mile,	(*sq. mi.*)

All the above, except the *acre*, are derived from the corresponding units of linear measure. The acre is the common unit of land measure, and is equivalent to a square whose side is 209 ft. long. A square arpent = 0.84 A.; a rood = 40 sq. rd.

Oral Exercises.

1. How many square inches in the top of a table whose area is 3 sq. ft.?



(*Scale* $\frac{1}{8}$.)

2. How many square feet in a school platform whose area is 3 sq. yd.?

3. How many square rods in a campus whose area is $\frac{1}{2}$ A.?

4. How many square inches in $\frac{1}{2}$ sq. ft.? 10 sq. ft.?

5. How many square feet in 432 sq. in.? 72 sq. in.?

6. What part of a square foot is an area of 12 sq. in.?

7. How many acres in $\frac{1}{4}$ sq. mi.?

8. How many square yards in 2 sq. rd.?

9. What part of a mile is an area of 32 acres?

10. How many lots each 100 sq. ft. in 5 A.?

Surveyors' Square Measure.

TABLE.

10 000 square links (*sq. l.*)	= 1 square chain,	(*sq. ch.*)
10 square chains	= 1 acre,	(*A.*)
640 acres	= 1 *sq. mi.*, or section,	*sq. mi.*
36 sq. mi., or sections	= 1 township.	

This table is used in measuring land. 1 square chain = 16 square rods.

MEASURES OF VOLUME.

232. *Solid* or *cubic measure* is used in measuring things which have length, breadth, and thickness.

233. A unit of cubic measure is a cube, each of whose edges is a unit of some known length. Thus, a cubic inch is a cube, each of whose edges is one inch.

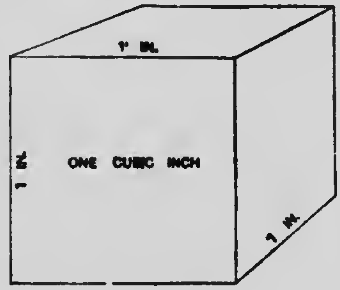
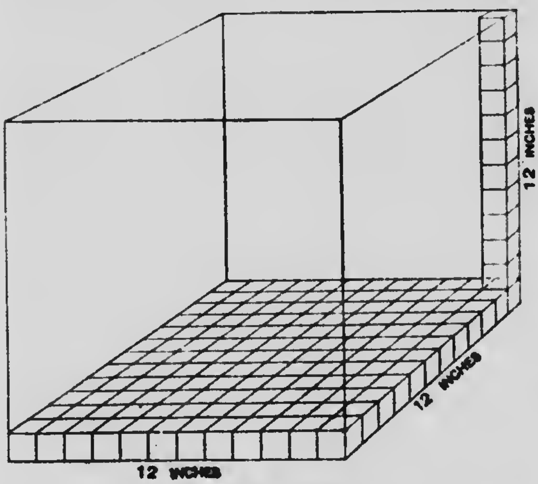


TABLE.

1 728 cubic inches (*cu. in.*)	= 1 cubic foot,	(*cu. ft.*)
27 cubic feet	= 1 cubic yard,	(*cu. yd.*)

Also, in measuring wood:

16 cubic feet	= 1 cord foot,	(*cd. ft.*)
8 cd. ft., or 128 cu. ft.	= 1 cord,	(*cd.*)



1 cubic foot (*Scale* $\frac{1}{8}$.)

A perch of masonry = $24\frac{3}{4}$ cu. ft.

A cubic yard of earth is called a *load.*

In computing the tonnage of vessels, a *registered ton* is 100 cubic feet of their internal capacity.

Oral Exercises.

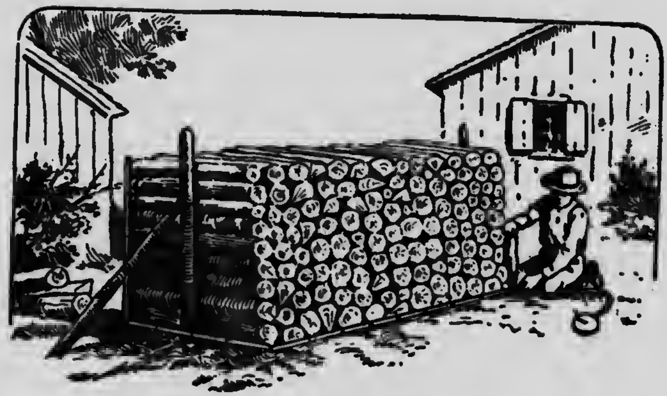
1. How many cubic feet in 1 cu. yd.? 2 cu. yd.?

2. How many cubic yards in 54 cu. ft.? 270 cu. ft.?

3. A volume of 9 cu. ft. is what fraction of 1 cu. yd.?

4. How many cubic inches in $\frac{1}{2}$ cu. ft.?

5. A volume of 144 cu. in. equals what part of 1 cu. ft.?



A cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

MEASURES OF CAPACITY

Rose is pouring the contents of a pint measure into a quart measure; she will have to pour in another pint of water to fill the quart measure. Four quarts of water would be necessary to fill the other vessel on the table: it is a gallon measure.

LIQUID MEASURE.

234. *Liquid measure* is used in measuring liquids.

TABLE.

4	gills (*gi.*)	= 1 pint,	(*pt.*)
2	pints	= 1 quart,	(*qt.*)
4	quarts	= 1 gallon,	(*gal.*)
$31\frac{1}{2}$	gallons	= 1 barrel,	(*bbl.*)
63	gal., or 2 bbl.	= 1 hogshead,	(*hhd.*)

The Imperial gallon contains 277.274 cu. in.; the American gallon contains 231 cu. in.

Barrels and hogsheads are of variable capacity. The above values are used in estimating the capacity of cisterns and tanks.

In compounding liquid medicines, druggists use a special table: 60 minims = 1 fluid dram; 8 fluid drams = 1 fluid ounce; 16 fluid ounces = 1 pint; 8 pints = 1 gallon.

Oral Exercises.

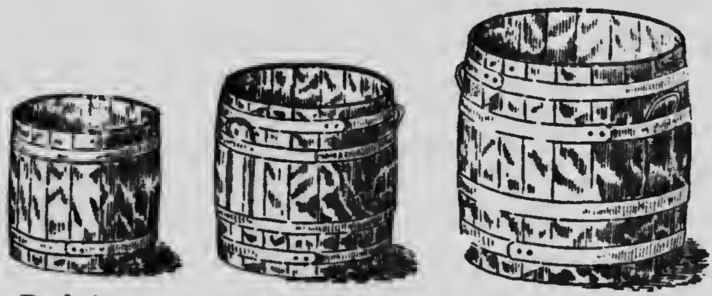
1. How many gills in $\frac{1}{2}$ pt.?
2. How many quarts in 3 pt.?
3. What part of a gallon is 1 qt.? 3 pt.?
4. How many gills in 1 pt.? 1 qt.? 1 gal.?
5. How many quarts in 2 gal.? 3 gal.? 4 gal. and 1 qt.?
6. What part of a gallon is 2 qt.? $2\frac{1}{2}$ qt.?
7. A milkman buys his milk at 16c a gallon, and sells it at 5c a pint. Find his profit per gallon.
8. A lamp contains 1 pt. of kerosene; how often can I fill it, if I have $1\frac{1}{2}$ gal. of kerosene?
9. How many quarts in 24 gills? 40 gills?
10. In the preceding picture, the bucket can hold $2\frac{1}{2}$ gal., and the small fruit jar, 1 qt. The capacity of the bucket is how many times as great as that of the jar?

DRY MEASURE.

235. *Dry measure* is used in measuring grain, fruit, vegetables, etc.

MEASURES USED IN TRADE.

Pint. Quart. Small meas. (*2 qt.*) Half-peck (*4 qt.*)



Peck (*8 qt.*) Half-bushel (*16 qt.*) Bushel (*32 qt.*)

TABLE.

2 pints (*pt.*)	= 1 quart,	(*qt.*)
8 quarts	= 1 peck,	(*pk.*)
4 pecks	= 1 bushel,	(*bu.*)

The half-peck is sometimes called a gallon. The Imperial bushel contains 2 218.192 cu. in. The American bushel contains 2 150.42 cu. in.

Oral Exercises.

1. How many quarts in 1 pk.? $\frac{3}{4}$ bu.? $\frac{1}{2}$ bu.? $\frac{1}{8}$ bu.? $\frac{1}{16}$ bu.?
2. How many gallons in $1\frac{1}{2}$ bu.?
3. How many half-bushels in $3\frac{1}{2}$ bu.? $5\frac{1}{2}$ bu.?
4. How many quarts in $1\frac{1}{2}$ bu.?
5. How many quarts in $1\frac{1}{4}$ bu.? $3\frac{1}{4}$ bu.?

6. A quart is what fraction of a bushel? of a peck?
7. What part of a bushel is 24 qt.?
8. How many gallons in 7 bu.?
9. How many quarts in 3 half-bushels?
10. At 10c a peck, how many bushels of apples will $8 buy?

Legal weight of a bushel of grain, and other farm produce.

Wheat	60 lb.	Clover seed	60 lb.	Onions	50 lb.
Beans	60 lb.	Carrots	60 lb.	Barley	48 lb.
Peas	60 lb.	Beets	60 lb.	Buckwheat	48 lb.
Potatoes	60 lb.	Rye	56 lb.	Timothy seed	48 lb.
Turnips	60 lb.	Corn, shelled	56 lb.	Oats	34 lb.

MEASURES OF VALUE

236. The *value* of any thing is its worth, or what it will produce when sold. Money is the measure of value.

237. *Coin*, or *specie*, is made chiefly of gold, silver, and bronze.

Paper money consists of printed promises emitted by the Government to circulate as money. Banks are also authorized to issue paper money.

Canada Money.

238. The standard unit is the *dollar*. It is issued by the Ottawa Government in paper currency.

The Government also issues 2-dollar, 4-dollar, 5-dollar, 50-dollar, 100-dollar, 500-dollar, 1 000-dollar, and 5 000-dollar notes.

Banks emit 5-dollar, 10-dollar, 20-dollar, 50-dollar, and 100-dollar notes.

239. Our specie currency, coined in Ottawa, consists of the 5-dollar, 10-dollar, and 20-dollar *gold pieces;* of the 5-cent, 10-cent, 25-cent, and 50-cent *silver pieces;* and of the 1-cent *bronze piece.*

TABLE.

10 mills	= 1 cent,	(*c*).
100 cents	= 1 dollar,	($).

The 1-dollar silver piece and the $2\frac{1}{2}$-dollar gold piece have not yet been coined.

The *mill* is only a convenient designation for the tenth part of a cent; it is never coined.

United States Money.

240. The standard unit is the *dollar.* The denominations of United States money are the mill, the cent, and the dollar; they have the same value as the corresponding denominations of Canada money.

English or Sterling Money.

241. The standard unit is the *pound,* or *sovereign;* its value is $\$4.86\frac{2}{3}$.

TABLE.

4 farthings (*far.*)	= 1 penny,	(*d.*)
12 pence	= 1 shilling,	(*s.*)
20 shillings	= 1 pound,	(£).

The guinea = 21 shillings; a crown = 5 shillings; a half-crown = $2\frac{1}{2}$ shillings.

Oral Exercises.

1. How many shillings in £3? £5?
2. How many shillings in 48 d.?

3. How many pence in 1½ s.?
4. 1 s. is what part of £1?
5. How many shillings in £1 and 12 s.?

UNITS OF CURRENCY OF DIFFERENT COUNTRIES.

Country.	*Unit.*	*Value.*
Austria	crown	$0.203.
Belgium	franc	$0.193.
Brazil	milreis	$0.546.
France	franc	$0.193.
Germany	mark	$0.238.
Italy	lira	$0.193.
Japan	yen	$0.498.
Netherlands	fiorin	$0.402.
Russia	rouble	$0.515.
Spain	peseta	$0.193.
Switzerland	franc	$0.193.

Paper Measure.

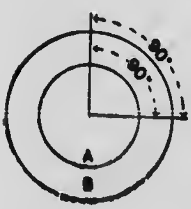
242. In small quantities, sheets of paper are counted thus: 24 sheets = 1 quire (*qr.*); 20 quires, or 480 sheets = 1 ream (*rm.*).

In great quantities, paper is sold by the pound.

Counting.

20 units = 1 score; 12 units = 1 dozen (*doz.*); 12 dozen = 1 gross (*gro.*); 12 gross = 1 great gross (*gr. gro.*).

Circular Measure.



243. The circumference of a circle (its bounding line) is measured in *degrees*.

There are 360 degrees in a circumference, whether great or small. One fourth of a circle measures 90 degrees; ¼ of circle A, and ¼ of circle B both measure 90 degrees.

Oral Exercises.

Table.

60 seconds (″)	= 1 minute,	(′)
60 minutes	= 1 degree,	(°)
360 degrees	= 1 circle,	(C.)

1. How many degrees in one half of a circumference?

2. How many minutes in 5°? 7°? 10°?

3. Find how many seconds in 3′? 5′? 20′? 1°?

4. What part of a circumference is 1°?

5. One degree of the circumference of the earth equals $69\frac{1}{6}$ mi.; how many miles are there in 2 degrees?

LONGITUDE AND TIME

244. The circumference of the earth equals 360°; the earth turns on its axis once every 24 hours; therefore 15° equal 1 hour.

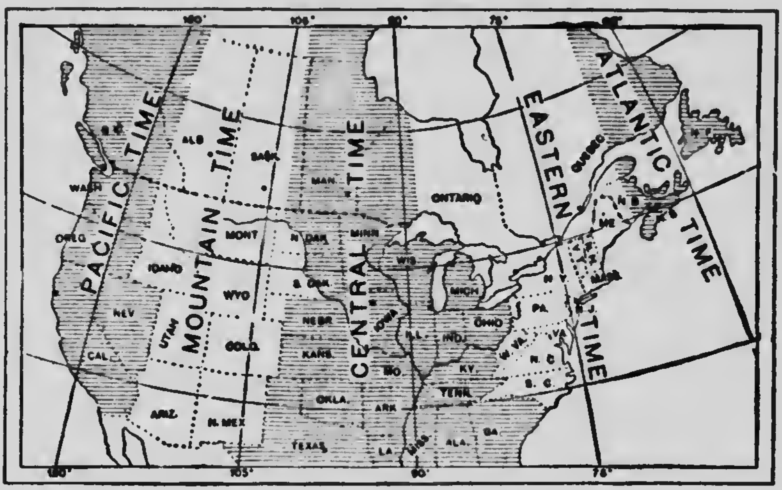
245. The earth moves from west to east; that is why the sun *appears* to rise in the east and to pass through 15° an hour.

From Halifax to Vancouver there are 60°; the sun, therefore, rises in Halifax 4 hours before rising in Vancouver; so when it is noon in Halifax it is only 8 A. M. in Vancouver. A person traveling from Halifax to Vancouver would have to set his watch back an hour every 15 degrees.

246. Geographers place the prime meridian in Greenwich, England; the other meridians are reckoned to 180 east and west of the prime meridian.

247. The **longitude** of a place is its distance in degrees from the prime meridian.

This map represents Canada and the United States divided into 5 time sections. All places in the same section have the same time. *Atlantic time* is that of the 60th meridian; *Eastern time* is that of the 75th meridian; *Central time* is that of the 90th meridian; *Mountain time* is that of the 105th meridian; and *Pacific time* is that of the 120th meridian.



It follows that the time throughout each time section will be one hour later than the time of the section west of it.

When it is 11.12 A. M. by Atlantic time, it is 10.12 A. M. by Eastern time, 9.12 A. M. by Central time, 8.12 A. M. by Mountain time, and 7.12 A M.. by Pacific time.

Oral Exercises (*Use the Map*).

1. Montreal is in the Eastern time section, and Winnipeg in the Central time section; when it is 3 P. M. in Montreal, what time is it in Winnipeg?

2. When it is 10 A. M. in Quebec City, what time is it in New York City? in Ottawa? in Halifax?

3. What time is it now in your academy? in Regina? in Chicago? in San Francisco?

4. When it is 5.30 P. M. in Montreal, what time is it in Louisiana? in Montana? in Alberta?

5. When it is noon in Ottawa, what time is it in Halifax? in Vancouver? in Regina? in Calgary? in Winnipeg? in Montreal? in Toronto? in St. John, N. B.?

Oral Exercises.

To be thoroughly dealt with.

How many:

1. gallons in 1 bu.?
2. rods in 1 mi.?
3. degrees in 1 C.?
4. grains in 1 lb. (*Troy*)?
5. days in 1 leap year?
6. feet in 1 rd.?
7. ounces in 1 lb. (*Avoir.*)?
8. feet in 1 fathom?
9. mills in $1?
10. dollars in £1?
11. feet in 1 mi.?
12. sheets in 1 qr.?
13. cu. inches in 1 cu. ft.?
14. grains in 1 lb. (*Avoir.*)?
15. yards in 1 rd.?
16. sq. rods in 1 A.?
17. units in 1 gro.?
18. sq. yards in 1 sq. rd.?
19. quarts in 1 gal.?
20. gallons in 1 bbl.?
21. acres in 1 sq. mi.?
22. ounces in 1 lb. (*Troy*)?
23. sq. feet in 1 sq. yd.?
24. pounds in 1 ton?
25. grains in 1 pwt.?
26. cu. feet in 1 cu. yd.?
27. cu. inches in 1 gal.?
28. shillings in £1?
29. yards in 1 mi.?
30. gills in 1 pt.?
31. inches in 1 yd.?
32. pence in 1 s.?
33. cu. feet in 1 cord?
34. pints in 1 gal.?
35. quarts in 1 half-bushel?
36. pounds in 1 *long* T.?
37. pennyweights in 1 oz.?
38. grains in 1℈?
39. sq. inches in 1 sq. ft.?
40. cu. inches in 1 bu.?
41. feet in 1 knot?
42. inches in 1 span?
43. feet in 1 cubit?
44. scruples in 1ʒ?
45. drams in 1℥?
46. cents in 1 franc?
47. feet in 1 arpent?
48. quarts in 1 pk.?
49. gallons in 1 pk.?
50. gallons in 1 hhd.?

REDUCTION OF DENOMINATE NUMBERS

DESCENDING REDUCTION.

Example.—Reduce 3 hr. 30 min. 12 sec. to seconds.

Operation.	Explanation.
$60 \times 3 = 180$ min.	1 hr. = 60 min.;
180 min. + 30 min. = 210 min.;	3 hr. = 180 min.;
$60 \times 210 = 12\,600$ sec.;	180 min. + 30 min.
12 600 sec. + 12 sec. = 12 612 sec.	= 210 min.; 1 min. = 60 sec.; 210 min.

= 12 600 sec.; 12 600 sec. + 12 s. = 12 612 sec.

248. Rule.—*A denominate number is reduced to lower denominations by successive multiplications.*

Written Exercises.

Reduce:

1. 20 da. 10 hr. 30 min. to minutes.
2. 3 da. 5 hr. 25 min. 6 sec. to seconds.
3. 7 cwt. 25 lb. 12 oz. to ounces.
4. 3 T. 3 cwt. 21 lb. 5 oz. to ounces.
5. 12 lb. 10 oz. 12 pwt. 7 gr. to grains.
6. 8 lb. 3 oz. 3 pwt. 19 gr. to grains.
7. 7 drams 2 scruples 18 gr. to grains.
8. 2 oz. 2 drams 3 gr. to grains.
9. 2 mi. 16 rd. 3 yd. to feet.
10. 2 mi. 4 rd. 4 yd. 3 in. to inches.
11. 12 A. 24 sq. rd. to square feet.
12. 1 A. 16 sq. yd. to square feet.
13. 3 cu. yd. 2 cu. ft. to cu. inches.
14. 8 cu. yd. 7 cu. in. to cu. inches.
15. 3 gal. 2 qt. 1 gi. to gills.
16. 8 gal. 1 qt. 1 pt. to gills.
17. 4 bu. 8 qt. to pints.
18. 3 bu. 5 pt. to pints.
19. £4 3 s. 2 d. to pence.
20. £3 9 s. to farthings.

NOTE.—A denominate fraction, like £$\frac{5}{6}$, is reduced to lower denominations by successive multiplications.

EXAMPLE.—$20 \times \frac{5}{6} = \frac{20 \times 5}{6}$, or $16\frac{2}{3}$ s.; $12 \times \frac{2}{3} = 8$ d.; £$\frac{5}{6}$ = 16 s. 8 d.

Likewise, £.825 = 20 × .825 or 16.5 s.; 12 × .5 = 6 d.; £.825 = 16 s. 6 d.

Reduce to lower denominations:

21. $\frac{3}{5}$ da.
22. $\frac{9}{10}$ gal.
23. $\frac{3}{7}$ lb. (*Troy.*)
24. $\frac{3}{8}$ mi.
25. £ $\frac{2}{3}$.
26. $\frac{5}{12}$ mi.
27. $\frac{2}{3}$ A.
28. $\frac{7}{8}$ cu. yd.
29. $\frac{3}{16}$ mi.
30. $\frac{2}{3}$ bu.
31. £ .625.
32. .375 yd.
33. .875 mi.
34. .92 lb. (*Avoir.*).
35. .1375 gal.
36. .375 lb. (*Troy*).
37. .88 da.
38. .125 cu. yd.
39. .0675 cu. yd.
40. £ .0015.

ASCENDING REDUCTION.

EXAMPLE.—Reduce 113 628 sq. in. to higher denominations.

OPERATIONS.

1° 144) $\underline{113628}$

789 sq. ft., with a remainder of 12 sq. in.

2° 9) $\underline{789}$

87 sq. yd., with a remainder of 6 sq. ft.

3° $30\frac{1}{4}$) $\underline{87}$

2 sq. rd., with a remainder of $26\frac{1}{2}$ sq. yd.

4° 2 sq. rd. 26 sq. yd. 6 sq. ft. 12 sq. in.

$\frac{1}{2}$ sq. yd. = 4 sq. ft. 72 sq. in.

2 sq. rd. 27 sq. yd. 1 sq. ft. 84 sq. in.

NOTE I.—$\frac{1}{2}$ sq. yd. reduced to lower denominations has been added to 6 sq. ft. 12 sq. in.

NOTE II.—If in dividing by $30\frac{1}{4}$ sq. yd. both terms were *multiplied* by 4, it is then necessary to *divide* the remainder by 4 to have it express square yards.

249. Rule.—*A denominate number is reduced to higher denominations by successive divisions.*

Written Exercises.

Reduce to higher denominations:

1. 3 610 132 minutes.
2. 73 251 600 seconds.
3. 151 670 ounces (*Avoir.*).
4. 56 170 ounces (*Avoir.*).
5. 53 120 grains (*Troy*).
6. 5 216 pennyweights.
7. 7 190 drams.
8. 26 375 grains (*Apoth.*).
9. 181 612 inches.
10. 139 287 inches.
11. 48 729 square inches.
12. 360 192 square feet.
13. 1 500 162 cubic inches.
14. 426 790 cubic inches.
15. 31 213 gills.
16. 8 274 pints (*liq.*).
17. 6 178 pints (*dry*).
18. 816 quarts (*dry*).
19. 17 260 farthings.
20. 17 534 pence.

NOTE.—A denominate number or fraction is reduced to fractional higher denominations by successive divisions.

EXAMPLE I.—Reduce 2 qt. to a fraction of a gallon.
2 quarts $\div 4 = \frac{2}{4}$, or $\frac{1}{2}$ gal.

EXAMPLE II.—Reduce $\frac{1}{2}$ gi. to a fraction of a gallon.
$\frac{1\times1\times1\times1}{2\times4\times2\times4} = \frac{1}{64}$ gal.

EXAMPLE III.—Reduce .25 gi. to a decimal fraction of a gallon.
$.25 \div 4 = .0625$; $.0625 \div 2 = .03125$; $.03125 \div 4 = .0078125$ gal.

Reduce:

21. 3 hr. to a common fraction of a day.

22. 2 gi. to a common fraction of a quart.

23. $\frac{1}{4}$ qt. to a common fraction of a gallon

24. $\frac{7}{8}$ ft. to a common fraction of a mile.

25. 3 s. to a common fraction of a pound sterling.

26. .75 gi. to a decimal of a quart.

27. .27 in. to a decimal of a yard.

28. 3 d. to a decimal of a shilling.

29. 16 s. to a decimal of a pound sterling.

30. 13 s. 9 d. to a decimal of a pound sterling.

NOTE.—Divide 9 d. by 12 to have a decimal of a shilling; add this fraction to 13 s., and divide by 20.

31. 3 qt. 1 pt. to a common fraction of a gallon.

NOTE.—Reduce 3 qt. 1 pt. to pints, and divide by the number of pints in a gallon.

32. 15 s. 6 d. to a common fraction of a pound sterling.

33. 2 ft. 6 in. to a common fraction of a yard.

34. 15 min. 20 sec. to a common fraction of an hour.

35. 16 s. 3 d. to a decimal of a pound sterling.

36. 1 s. 3 d. to a decimal of a pound sterling.

37. 2 ft. 3 in. to a decimal of a yard.

38. 2 ft. 9 in. to a decimal of a yard.

39. 1 qt. 1 pt. to a decimal of a gallon.

40. 1 cwt. 50 lb. to a decimal of a ton.

ADDITION OF COMPOUND NUMBERS

250. The addition, subtraction, multiplication, and division of compound numbers are performed as with simple numbers, when the denominations are the same; but to pass from one denomination to the other, it is necessary to use the different tables.

COMPARATIVE EXAMPLE.

Simple numbers.

hundreds	10 tens	10 units
4	2	8
6	4	2
3	5	4
14	2	4

A scale of 10.

Compound numbers.

hr.	60 min.	60 sec.
2	38	42
3	25	36
5	17	58
11 hr.	22 min.	16 sec.

A varying scale.

EXPLANATION.—The sum of the seconds is 136 sec., or 2 min. 16 sec. Write 16 under the seconds' column and carry 2 min. The sum of the minutes is 82 min., or 1 hr. 22 min. Write 22 under the minutes' column and carry 1 hr. The sum of the hours is 11.

Written Exercises.

Add:

1.

ft.	in.
5	4
9	10
22	6

2.

sq. ft.	sq. in.
94	20
82	85
71	63

3.

cu. ft.	cu. in.
45	820
96	524
75	993

4.

lb. (*Avoir.*)	oz.
148	8
862	12
421	6

5.

ft.	in.
28	11
17	10
35	6

6.

sq. ft.	sq. in.
36	42
56	84
93	57

7.

cu. ft.	cu. in.
38	792
47	874
75	680

8.

lb. (*Avoir.*)	oz.
382	15
421	6
632	9

9.

gal.	qt.
34	2
34	1
32	1
31	2
18	3

10.

gal.	qt.
46	6
27	1
44	2
72	3
81	4

11.

ft.	in.
25	6
36	7
35	8
44	3
28	9

12.

hr.	min.	sec.
3	25	12
2	38	45
4	16	10
2	54	44
6	51	32

13.

£.	s.	d.
16	17	2
26	2	3
45	13	8
28	9	3
15	18	4

14.

£.	s.	d.
12	18	1
13	11	5
14	16	6
13	9	8
45	6	5

15.

£.	s.	d.
8	10	2
30	12	6
41	14	5
36	12	5
41	23	3

16. Add $\frac{1}{4}$ gal. and 3 gi. (*Ans. in gills*).

17. Add $\frac{3}{4}$ yd. and 6 in. (*Ans. in inches*).

18. Add £.75 and 12 s. (*Ans. in shillings*).

19. Add $2\frac{1}{2}$ gal., 3 qt., 1 pt. (*Ans. in pints*).

20. Add 3 A. 57 sq. rd. 22 sq. yd. 7 sq. ft. 68 sq. in., 5 A. 100 sq. rd. 17 sq. yd. 5 sq. ft. 120 sq. in., 7 A. 72 sq. rd. 28 sq. yd. 8 sq. ft. 64 sq. in.

NOTE.—Reduce the fraction of a square yard to lower denominations.

SUBTRACTION OF COMPOUND NUMBERS

COMPARATIVE EXAMPLE.

Simple numbers.

hundreds	10 tens	10 units
7	4	3
4	2	7
3	1	6

Compound numbers.

cwt.	100 lb.	16 oz.
14	67	6
12	50	8
2 cwt.	16 lb.	14 oz

NOTE.—With simple numbers, *borrowing* 1 ten is *borrowing* 10 units; with compound numbers, *borrowing* 1 lb. is *borrowing* 16 oz.

Written Exercises.

Subtract:

1.

ft.	in.
62	8
15	10

2.

yd.	ft.
35	1
18	2

3.

cu. ft.	cu. in.
225	450
128	1236

4.

s.	d.
32	1
18	9

5.

ft.	in.
91	$4\frac{1}{2}$
38	$8\frac{1}{4}$

6.

bu.	qt.
31	13
17	16

7.

cu. ft.	cu. in.
325	47
278	698

8.

£.	s.
71	0
39	6

9.

sq. ft.	sq. in.
96	72
38	107

10.

A.	sq. rd.
64	75
43	119

11.

£.	s.
350	9
275	14

12.

gal.	qt.
9	2
5	3

13.

sq. ft.	sq. in.
81	$35\frac{1}{4}$
38	$16\frac{1}{2}$

14.

hr.	min.	sec.
20	13	40
12	40	39

15.

£.	s.	d.
274	0	4
39	15	9

16. From $\frac{3}{4}$ yd. subtract $\frac{1}{2}$ ft. (*Ans. in inches*).
17 From $\frac{1}{2}$ gal. subtract $\frac{1}{2}$ pt. (*Ans. in gills*).
18 From $\frac{1}{2}$ hr. subtract 3 min. 20 sec. (*Ans. in seconds*).
19. From $\frac{3}{4}$ mi. subtract 15 ft. (*Ans. in feet.*)

20. From 16 rd. 4 yd. 2 ft. 4 in. subtract 12 rd. 5 yd. 2 ft. 8 in.

21. Find the interval between Jan. 17, 1885, and Jan. 1, 1901.

NOTE.—Consider the borrowed month as equal to 30 da.

22. Find the interval between May 31, 1839, and Dec. 17, 1916.

23. Montcalm was born Feb. 28, 1712, and died Sept. 13, 1759. How old was he at his death?

24. Bishop Bourget was born Oct. 30, 1799, and died June 8, 1885. What was his age at his death?

25. The battle of Carillon took place July 8, 1758. Find the interval between that date and July 1, 1867.

26. Find the exact number of days between Feb. 28, 1916, and Sept. 3, 1916.

NOTE.—In finding the exact number of days between two dates, omit the first day and count the last. In leap years, give February 29 days.

27. A note was discounted Aug. 3, 1917; if it was due Sept. 4, 1917, find the term of discount in exact days.

28. What was the exact number of days from June 3, 1917, to May 2, 1918?

29. Find the exact time from Oct. 21, 1917, to Dec. 25, 1917.

30. Find the exact time from Mar. 19, 1917, to Sept. 8. 1917.

MULTIPLICATION OF COMPOUND NUMBERS

COMPARATIVE EXAMPLE.

Simple numbers.			*Compound numbers.*		
	10	10		20	12
hun-dreds	tens	units	£	s.	d.
6	3	4	4	10	4
		7			7
44	3	8	£31	12 s.	4 d.

EXPLANATION.—4 d. × 7 = 28 d., or 2 s. 4 d.; write 4 d. and carry 2 s.; 10 s. × 7 = 70 s.; 70 s. + 2 s. = 72 s., or £3 12 s.; write 12 s. and carry £3; £4 × 7 = £28; £28 + £3 = £31.

Written Exercises.

Multiply:

1. 47 ft. 6 in. by 20.
2. 45 s. 2 d. by 15.
3. 39 ft. 3 in. by 18.
4. 61 yd. 3 in. by 68.
5. 47 lb. 15 oz. (*Avoir.*) by 5.
6. 58 lb. 12 oz. (*Avoir.*) by 7.
7. 59 ft. $6\frac{1}{2}$ in. by 84.
8. £ 54 2 s. by 28.
9. £ 17 18 s. by 84.
10. 44 mi. 3 ft. by 89.
11. 48 gal. 2 qt. by 29.
12. 56 gal. 3 pt. by 27.
13. 4 mi. 113 rd. by 21.
14. 5 mi. 1 250 ft. by 13.
15. 2 sq. ft. 40 sq. in. by 4.
16. 3 sq. yd. 15 sq. ft. by 7.
17. 5 cu. ft. 50 cu. in. by 75.
18. 8 cu. ft. 75 cu. in. by 96.
19. 3 bu. 8 qt. by 32.
20. 8 bu. 3. pt. by 16.

21. One cubic foot of water weighs 62 lb. 8 oz.; find the weight of 48 cu. ft. of water.

22. An auto consumes 8 gal. 2 qt. of gasoline a day; how much in 7 days?

23. Multiply $\frac{2}{3}$ mi. by 7. (*Reduce the fraction*).

24. Multiply $\frac{5}{16}$ gal. by 31. (*Reduce the fraction*).

25. Multiply .1835 T. by 2.5. (*Reduce the fraction*).

DIVISION OF COMPOUND NUMBERS

Comparative Example.

Simple numbers.

		10	10	10
	thou-sands	hun-dreds	tens	units
7)	4	4	3	8
		6	3	4

Compound numbers.

		20	100	16
	T.	cwt.	lb.	oz.
8)	26	17	48	12
	3 T.	7 cwt.	18 lb.	$9\frac{1}{2}$ oz.

Explanation.—$26 \div 8 = 3$ T., with a remainder of 2 T., or 40 cwt; 40 cwt. + 17 cwt. = 57 cwt.; 57 cwt. $\div 8 = 7$ cwt., with a remainder of 1 cwt., or 100 lb.; 100 lb. + 48 lb. = 148 lb.; 148 lb. ÷ 8 = 18 lb., with a remainder of 4 lb., or 64 oz.; 64 oz. + 12 oz. = 76 oz.; 76 oz. ÷ 8 = 9½ oz.

Written Exercises.

Divide:

1. 72 ft. 6 in. by 3.
2. 81 ft. 9 in. by 7.
3. 73 ft. 8 in. by 9.
4. 7 lb. 8 oz. (*Avoir.*) by 10.
5. 27 lb. 12 oz. (*Avoir.*) by 12.
6. 2 mi. 120 rd. by 38.
7. 3 mi. 40 rd. by 125.
8. 26 hr. 26 min. by 13.
9. 2 yr. 5 mo. by $14\frac{1}{2}$.
10. £ 36 8 s. by 14.
11. 8 T. 5 cwt. 8 lb. by 7.
12. £ 44 17 s. 6 d. by 6.
13. 17 cu. yd. 1 264 cu. in. by 16.
14. 76 gal. 3 qt. 1 pt. by 3.
15. 5 hr. 1 min. 57 sec. by 9.
16. 15 gal. 3 qt. by 18.
17. 54 yd. 1 ft. 4 in. by 20.
18. 188 mi. 12 rd. 2 yd. by 6.
19. 1 629 yd. 1 ft. by 96.
20. 863 gal. 2 qt. 1 pt. by 47.

21. A cubic foot of gold weighs 1 187 lb. 4 oz. (*Avoir.*); if water weighs 19 times less than gold, find the weight of one cubic foot of water.

22. Divide $3\frac{1}{8}$ gal. by 15. (*Reduce the fraction*).

23. Divide .3 T. by .625. (*Reduce the fraction*).

24. Divide $\frac{5}{11}$ yd. by $\frac{5}{7}$. (*Reduce the fraction*).

25. Divide 48 sq. rd. 22 sq. yd. 7 sq. ft. 108 sq. in. by 9.

26. Divide £ 226 9 s. 5 d. by £ 17 8 s. 5 d.

Note.—First reduce both terms to pence.

27. Divide 41 ft. 3 in. by 2 ft. 9 in.

28. Divide 5 bu. 15 qt. 1 pt. by 19 qt. 1 pt.

29. Divide $6\frac{1}{4}$ gal. by $2\frac{1}{2}$ pt.

30. Divide 31 gal. 2 qt. $1\frac{3}{4}$ pt. by $1\frac{3}{4}$ pt.

REVIEW OF DENOMINATE NUMBERS.

First Series.

NOTE FOR BOTH SERIES.—Reduce denominate fractions to lower terms.

1. A man travels in an auto 7 days a week for 3 weeks; the first week his auto consumes 8 gal. 2 qt. of gasoline a day; the second week, 7 gal. 3 qt. a day; the third week, 8 gal. 3 qt. a day. Find the sum.

2. A building has a basement 15 ft. 9 in. high, and above it, 9 stories each 11 ft. 10 in. high. How high is the building?

3. It took me 8 hr. 31 min. 24 sec. to walk a certain distance; how long did it take me to return in an auto going 12 times as fast?

4. I bought 12 bu. of chestnuts at $2.50 a bushel, and sold them at 5c a pint. Find my gain.

5. The circumference of a wheel is 10 ft. 8 in.; how often will the wheel turn in going 6 mi.?

6. A cyclist traveled 63 mi. 150 rd. 2 yd. one day, and 10 mi. 56 rd. the next day. What distance did he travel in all?

7. A man had 1 000 A. of land; he sold 96 A. 150 sq. rd. to John, and 4 times as much to Paul. What area was left?

8. A digs 3 rd. 4 yd. 2 ft. 8 in. in a trench every day; and B, 3 ft. 6 in. less. What length of trench will both dig in 1 day? in 5 days?

9. At the rate of 25 mi. 220 rd. an hour, how long will it take a train to run 2 500$\frac{1}{4}$ mi.?

10. At 10c each, how many pints of nuts can I buy for $12.80?

11. If 3 lb. of coal cost 1c, find the cost of a short ton.

12. How many bottles, each holding 1 qt., will be required to empty a cask containing 50 gal. of wine?

13. How many cords in 3 840 cu. ft. of wood?

14. How many pills, each containing 3 grains, can be made from $\frac{1}{2}$ oz. of pepsin?

15. At $1.15 a pennyweight what is the value of a gold chain weighing 384 gr.?

16. How many square feet in 1 acre of land?

17. A cask was emptied in 8½ hr. by a faucet giving 1 pt. a second. Find the capacity of the cask.

18. Find the cost of 15 T. 7 cwt. 52 lb. of coal at $10 a ton.

19. Find the cost of 32 eggs at $2.52 a gross.

20. I bought twelve 8-ounce cakes of maple sugar for 66c. At that rate, what would 100 lb. of sugar cost?

21. How many miles can a horse run in 45 minutes, at the rate of 15 ft. a second?

22. At the rate of ½ gi. of alcohol thrice a day, what quantity will a man drink in a year?

23. A dealer bought 25 long tons of coal at $5.60 per ton, and sold it at $7.75 per short ton. What was his gain?

24. Two square miles of land were divided equally among 24 settlers. How much land did each receive?

25. I sold three loads of wheat, containing respectively 1 566, 1 642, and 1 592 lb., at $1.32 a bushel. How much did I receive?

26. What is the cost of 5 340 lb. of beans at $3.17 a bushel?

27. I have 250 bu. 10 qt. of potatoes to ship; how many barrels, each containing 2 bu. 25 qt., will be required?

28. What is the cost of 2 T. of oats at 55c a bushel, and of 3 T. of barley at 86c a bushel?

29. A milkman adulterates his milk in such a way that every gallon he sells contains 2 gi. of water. Every week he sells 224 gal. of this milk at 10c a quart. What sum does he dishonestly obtain?

30. A horse is given 8 qt. of oats every day; at 55c a bushel, what sum was spent on oats from Sept. 20, 1917, to March 21, 1918?

Second Series.

31. Find $\frac{1}{8}$ of 275 ft. 4 in.

32. Find $\frac{3}{4}$ of 11 lb. 8 oz. (*Avoir.*).

33. A milkman sold milk at 8c a quart as follows: 115 gal. 3 qt. 1 pt. the first week; 105 gal. 3 qt. 1 pt. the second week; 115 gal. 1 qt. the third week; 103 gal. 1 qt. 1 pt. the fourth week. What did he receive if he collected only $\frac{7}{8}$ of the amount due him?

34. An Englishman bequeathed £8 000 13 s. 4 d. as follows: $\frac{1}{4}$ to his wife; the remainder to be divided equally among his 5 children. Find each child's share.

35. In N° 34, what is the value in Canada money of each child's share?

36. If 1 ounce (*Troy*) of drugs is worth $3, what is 1 lb. (*Avoir.*) worth?

37. An American bushel contains 2 150$\frac{2}{5}$ cu. in.; how many cubic feet in 3 450 American bushels? (2 *decimals*).

38. A Canadian bushel contains 2 218.192 cu. in.; how many cubic feet in 350 Canadian bushels? (2 *decimals*).

39. A car runs 63$\frac{9}{10}$ mi. in 1$\frac{1}{2}$ hr. Find its rate per hour.

40. If $\frac{1}{2}$ gi. of water is added to 1 pt. of alcohol, what part of the mixture is the water?

41. A watch loses $\frac{1}{2}$ sec. in 5 min.; what time will it lose in 4 da. 2 hr. 18 min.?

42. Find the cost of 3 qt. 1 pt of molasses at 64c a gallon.

43. A lake is 155 fathoms deep; what fraction of a mile is that? (3 *decimals*).

44. How many times is $\frac{3}{4}$ yd. contained in $\frac{3}{4}$ mi.?

45. A pedestrian walks 15$\frac{3}{4}$ mi. in 4$\frac{1}{5}$ hr. How far can he walk in 1$\frac{2}{3}$ hr.?

46. At 4.86\frac{2}{3}$ to the pound sterling, what is the value in Canada money of £1 867 17 s. 6 d.?

47. At \$0.193 to the franc, what is the value in Canada money of 437.52 francs?

48. How many marks can be bought for \$197.54 at \$0.238 per mark?

49. Find the cost of 47.0625 gal. of milk at 10c a quart.

50. At \$1.28 a bushel, what is the cost of 2 qt. 1 pt. of wheat?

51. What part of a mile is 200 ft.?

52. Reduce $\frac{1}{4}$ in. to a fraction of a rod.

53. What fraction of a ton is 3 cwt. 8 lb. 4 oz.? (3 *decimals*).

54. Reduce $\frac{7}{11}$ mi. to lower denominations.

55. What part of a day is 3 min. 3 sec.?

56. What is the cost of 3 cwt. 15 lb. of iron at \$140 a ton?

57. How many knots in 300 miles? (2 *decimals*).

58. What is the area of a field if $\frac{3}{4}$ of it is 13 A. 2 sq. rd. 20 sq. yd.?

59. I had 30 bu. 10 qt. 2 pt. of potatoes; I sold $\frac{1}{4}$ of them, and then $\frac{2}{3}$ of the remainder. What quantity had I left?

60. An Englishman spent $\frac{1}{4}$ of his money one day, and $\frac{2}{3}$ of the remainder the next day. How much had he at first, if he still possessed £30 12 s. 8 d.?

ABOUT AN ELECTRIC CAR.

1. This car runs between 2 cities 7 mi. distant, and takes 20 min. for a single trip. How many miles will it run in 1 hr.?

2. Each rail is 30 ft. long and weighs 65 lb. per yard. Find its weight.

3. Find the cost of *all* the rails at \$35 a ton.

4. The cross ties are 10 in. wide and 14 in. apart. How many are there in 1 mile of line?

5. Find the cost of *all* the cross ties at \$34.59 per hundred.

6. Find the space between the poles, if there are 44 in 1 mile. (*Ans. in feet*).

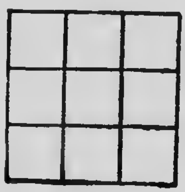
7. Find the cost of *all* the poles at \$1.78 each.

8. There are fences on both sides of the line. At 7c each, find the cost of *all* the posts, if they are 1 rd. apart.

9. How many pounds of wire are necessary for both fences, if 15 ft. of wire weigh 1 lb.? Find the cost of the wire at \$2.30 per hundredweight.

10. The electric cable weighs 2 128 lb. per mile of length; what is it worth at 21c a pound?

SQUARE ROOT.



251. The *square* of a number is the product of that number by itself.

The square of 3 is 3×3, or 9.

The square of 3 is written 3^2; $3^2 = 9$.

The squares of the first ten numbers are:

$1^2=1$	$3^2=9$	$5^2=25$	$7^2=49$	$9^2=81$
$2^2=4$	$4^2=16$	$6^2=36$	$8^2=64$	$10^2=100.$

252. The *square root* of a number is one of the two equal factors of this number.

Thus, the square root of 16 is 4, since $4 \times 4 = 16$. The square root of 16 is written $\sqrt{16}$; $\sqrt{16} = 4$.

The symbol $\sqrt{\ }$ is called the *radical sign*.

Since $\sqrt{9} = 3$, and $\sqrt{16} = 4$, it is plain that $\sqrt{12}$ will be 3 plus a fraction.

253. Squares of the smallest and greatest numbers of one, two, and three figures:

	$1^2 = 1$	$10^2 = 100$	$100^2 = 10\,000$
	$9^2 = 81$	$99^2 = 9801$	$999^2 = 998\,001.$
Then	$\sqrt{1} = 1$	$\sqrt{100} = 10$	$\sqrt{10000} = 100$
	$\sqrt{81} = 9$	$\sqrt{9801} = 99$	$\sqrt{998001} = 999.$

It follows that the square root of a number expressed by one or two figures is a number of *one figure;* the square root of a number expressed by three of four figures is a number of *two figures;* and the square root of a number expressed by five or six figures is a number of *three figures.*

254. Principle.—*The square root of any number contains as many figures as the number itself contains periods of two figures, beginning at units. The left-hand period may contain only one figure.*

How many figures in the square root of 576? 4 225? 12 544? 42 875? 133 225? 810 000?

255. THE SQUARE OF A NUMBER OF TWO FIGURES.

What is the square of 25?

We know that 25 = 2 tens and 5 units, or 20 units plus 5 units, 20+5; we may square 25 in two ways:

$$\begin{array}{rl} 25 = & \qquad\qquad\qquad\qquad 20+5 \\ 25 = & \qquad\qquad\qquad\qquad 20+5 \\ \hline 125 = & \qquad\qquad (20\times5) + (5\times5) \\ 500 = & (20\times20) + (20\times5) \\ \hline 625 = & 20^2 + 2 \text{ times } (20\times5) + 5^2 \end{array}$$

We have multiplied (20+5) by 5; then (20+5) by 20. It follows that:

$$25^2 = \left\{\begin{array}{lr} 1^\circ & 20^2 = 400 \\ 2^\circ \text{ 2 times } & (20\times5) = 200 \\ 3^\circ & 5^2 = 25 \end{array}\right\} = 625.$$

256. Principle.—*The square of a number of two figures equals the square of the tens plus twice the product of the tens by the units plus the square of the units.*

Finding the Square Root.

257. Example.—What is the square root of 625?

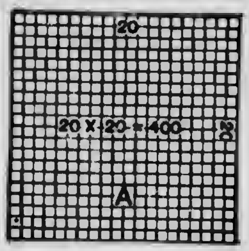
Operations.

```
                     6,25 | 2 tens, or 20 units.
               20² = 4 00 |
trial divisor         225 |
(2 times 20) = 40         | 5 units.
                5         | 25 = square root.
true divisor = 45     225
```

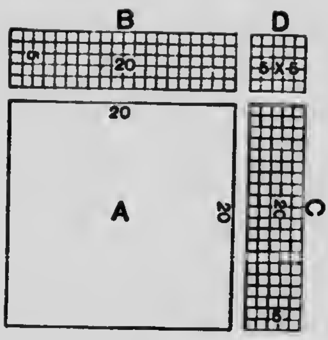
In practice:

```
6,25 | 25 = square root.   2 tens × 2 = 4 tens;
4    |                     22 tens ÷ 4 tens = 5 times;
---- | 45                  4 tens + 5 units = 45;
22.5 |                     45 × 5 = 225.
22 5 |
```

Explanation.—As 625 forms two periods, its root is composed of 2 figures, tens and units. Since the square of tens is hundreds,



6 hundreds must be the square of at least 2 tens. 2 tens, or 20 units, is 400 when squared (*see A*); and 400 subtracted from 625 leaves 225; therefore the root, 20, must be increased by such an amount as will exhaust the remainder, 225.



These 225 units are composed of (2 times 20 × the units' figure) + (the square of the units' figure).

B, C, and D must be added to A to exhaust these 225 units and still keep the figure a square.

225 divided by 2 times 20 (40) must give approximately the units' figure, which is 5. And we see in fact that 2 times 20 × 5, plus 5 × 5 = 225; we simplify by saying $(40+5) \times 5 = 225$. The complete root is 2 tens, or 20 units, + 5 units, or 25.

258. Rule.—*To find the square root:*

1° Separate the number into periods of two figures each, beginning at the decimal point;

2° Find the greatest number whose square is contained in the left-hand period, place it at the right as a quotient, subtract its square from the left-hand period, and annex the next period to the remainder for a dividend;

3° Double the root already found for a trial divisor, *and by it divide the dividend, disregarding the right-hand figure; the quotient will be the second term of the root;*

4° Annex the second term of the root to the trial divisor for the true divisor; *multiply this divisor by the second term of the root; subtract the product from the dividend, and bring down the next period for the next dividend;*

5° Double the root now found for a second trial divisor; *find the third term of the root as before, and thus proceed until all the periods have been used.*

259. Note I.—When the product of the true divisor by a term of the root exceeds the dividend, diminish the term by one unit.

When a cipher occurs in the root, annex a cipher to the trial divisor, add the next period to the dividend, and proceed as before.

260. Note II.—When an integer is not a perfect square, annex periods of decimal ciphers and continue the process. Decimals are pointed off into periods of two places each, by beginning at the decimal point and passing to the right.

The square root of a common fraction is naturally the square root of both numerator and denominator separately. When the terms are not perfect squares, reduce the fraction to a decimal, and extract the root.

Written Exercises.

Extract the square root of the following:

1. 289.	**4.** 729.	**7.** 2 025.	**10.** 1 225.	**13.** 3 721.
2. 961.	**5.** 576.	**8.** 1 296.	**11.** 3 249.	**14.** 2 209.
3. 484.	**6.** 324.	**9.** 1 089.	**12.** 1 681.	**15.** 3 969.

16. 98 596.	**21.** 1 900.96.	**26.** 2 (4 *dec.*)
17. 65 536.	**22.** .514089.	**27.** 3 (4 *dec.*)
18. 11 449.	**23.** 97.8121.	**28.** 5 (4 *dec.*)
19. 41 616.	**24.** .001225.	**29.** 6 (5 *dec.*)
20. 52 441.	**25.** .009216.	**30.** 7 (5 *dec.*)

Questions on Theory.

1. What is the square of a number? (251).

2. What is the square root of a number? (252).

3. How many figures in the square root of a number of 5 figures? (253).

4. How do you extract the square root of an integer? (258).

5. How do you extract the square root of a decimal? of a common fraction? (260).

PRACTICAL MEASUREMENTS

261. This block of wood, like all solids, occupies a certain space. This space is its **volume.** Volume has three dimensions: *length*, *width*, and *height* or *thickness.*

262. This solid is limited; each of the limits of a solid is a **surface.** Here there are six; each of these surfaces has two dimensions: *length* and *width.*

263. A surface is also limited; each of the limits of a surface is a **line.** A line has only one dimension: *length.*

264. Each end of a line is a **point.** A point has neither length, breadth, nor thickness.

265. The *volume* of a solid is the *space* it occupies.

A *surface* is one of the *limits* of a solid.

A *line* is one of the *limits* of a surface.

A *point* is the *end* of a line.

Lines.

Straight line. Curved line. Broken line.

266. There are two kinds of lines: the straight line and the curved line.

267. A *straight* line has all its points in the same direction; it is the shortest than can be traced from one point to another.

A tightly stretched string represents a straight line.

268. A *curved* line changes its direction at every point.

A string incompletely stretched represents a curved line.

269. NOTE.—A *broken* line is formed of straight lines going in different directions.

270. Two straight lines are *parallel* when they can never meet, however far they may be extended.

In this staff, lines 1, 2, 3, 4 and 5 are parallel to one another; so are lines 6, 7, 8 and 9.

271. A straight line is *horizontal* when it is parallel to the horizon, or water level.

These floating logs are in a horizontal position.

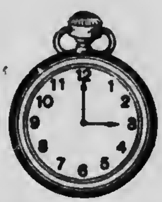
272. A straight line is *vertical* when it is standing according to a plumb line.

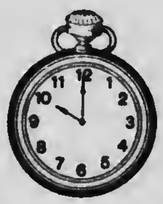
These poplar trees are in a vertical position.

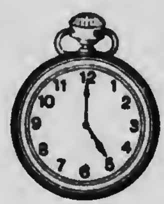
273. A *perpendicular* line is a straight line that meets another straight line without any inclination.

An *oblique* line is a straight line that meets another straight line with inclination.

Angles.


Right angle.


Acute angle.


Obtuse angle.

274. An *angle* is the opening between two lines diverging from a common point. The two lines forming an angle are its *sides*. The point from which the sides diverge is the *vertex* of the angle.

A *right* angle is formed by two perpendicular sides.

The quarter of a circle is a right angle. At 3 o'clock the hands of a watch form a right angle.

An *acute* angle is smaller than a right angle.

The hands of a watch form an acute angle at 10 o'clock.

An *obtuse* angle is greater than a right angle.

The hands of a watch form an obtuse angle at 5 o'clock.

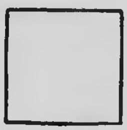
RECTILINEAR SURFACES

Polygons (*several angles*).

275. A figure bounded by straight lines is a *polygon*. The sum of all these straight lines is the *perimeter* of the polygon.

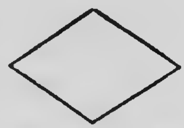
276. The principal polygons are: the *quadrilateral* (4 sides); the *triangle* (3 angles); the *pentagon* (5 angles); the *hexagon* (6 angles); and the *octagon* (8 angles).

QUADRILATERALS.



1. Square.

2. Rectangle.



3. Rhombus.

277. A *square* is a quadrilateral having 4 right angles and 4 equal sides. (*Fig.* 1).

278. A *rectangle* is a quadrilateral having 4 right angles. (*Fig.* 2).

279. A *rhombus* is a quadrilateral having 4 equal sides, but no right angles. (*Fig.* 3).

4. Parallelogram.

5. Trapezoid.

6. Trapezium.

280. A *parallelogram* is a quadrilateral having its two pair of opposite sides parallel. (*Fig.* 4).

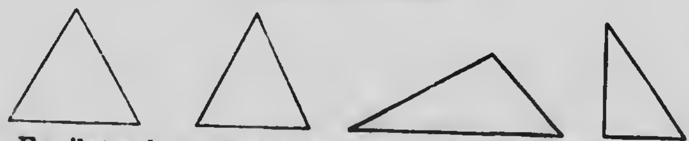
A square, a rectangle, and a rhombus are also parallelograms.

A rhomboid is a parallelogram having no right angles. Figure 4 may also be called a rhomboid.

281. A *trapezoid* is a quadrilateral having only one pair of opposite sides parallel. (*Fig.* 5).

282. A *trapezium* is a quadrilateral having no two of its sides parallel. (*Fig.* 6).

TRIANGLES.



7. Equilateral triangle. 8. Isosceles triangle. 9. Scalene triangle. 10. Right-angled triangle.

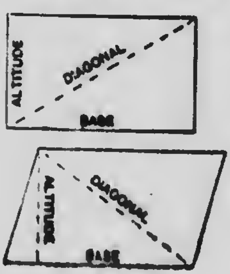
283. An *equilateral* triangle has three equal sides. (*Fig.* 7).
An *isosceles* triangle has two equal sides. (*Fig.* 8).
A *scalene* triangle has all its sides unequal. (*Fig.* 9).
A *right-angled* triangle has one right angle. (*Fig.* 10).

OTHER POLYGONS.

11. Pentagon. 12. Hexagon. 13. Octagon. 14. Irregular polygon.

284. A polygon is *regular* when all its angles and sides are equal.

Figures 1, 7, 11, 12, 13 are regular polygons; figures 2, 3, 4, 5, 6, 8, 9, 10, 14 are irregular polygons.

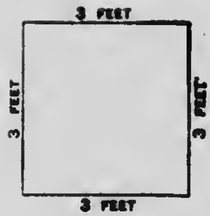


285. The *base* of a polygon is the side on which it stands.

The *altitude* of a polygon is a line perpendicular to the base, drawn from the angle opposite.

The *diagonal* of a polygon is a straight line which joins two opposite vertices.

THE SQUARE.



286. To find the perimeter of a square.

EXAMPLE.—Find the perimeter of a square whose side is 3 ft. long.

EXPLANATION.—The perimeter of a square is equal to the sum of its four sides, or to 4 times the length of the side.

NOTE.—Draw figures of the correct shape in all your problems, and write the proper formula underneath.

Per. = S + S + S + S.
Per. = S × 4.

Oral Problems.

1. Find the perimeter of a carpet 4 ft. square.

NOTE.—A carpet 4 ft. square is a square whose side is 4 ft. long.

2. At 50c a yard, find the cost of the golden braid necessary to border a banner 2 yd. square.

3. How many posts, 3 ft. apart, will be needed in fencing a garden 30 yd. square?

4. Around a grass plot 45 ft. square, flowers were planted 1½ ft. apart. Find the cost of the flowers at 2c apiece.

5. A garden 35 ft. square was fenced with a triple row of wire. How long was the wire?

Written Problems.

1. Around a garden 23½ yd. square a lattice fence was built at the rate of 75c per running yard. Find its cost.

2. I fenced a garden 35.75 yd. square with a wirework at the rate of 65c per running yard. Find the total expenditure if the making cost $3.75.

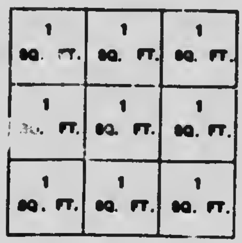
3. I set up posts, 2.5 yd. apart, around a lot 277.75 ft. square. Find their cost at 9c each.

4. At 12c each for posts and 75c a running yard for wirework, what is the cost of fencing a garden 73.5 yd. square, the posts being 3.5 yd. apart?

5. How many yards of barbed wire must I buy to surround a lot 125 rd. 3 yd. 2 ft. square, with 5 rows of wire?

287. To find the area of a square.

NOTE.—An area is always the product of two dimensions.



EXAMPLE.—What is the area of a square whose side is 3 ft. long?

EXPLANATION.—This square may be divided into 3 rows of 3 square feet; its area will therefore be 3 times 3 sq. ft., or 9 sq. ft.

$$\boxed{\begin{array}{l}\text{Area} = \text{Side} \times \text{Side.} \\ \quad A = S^2.\end{array}}$$

Oral Problems.

1. Find the area of a square whose side is 7 ft. long.
2. At $2 a sq. yd., find the cost of a rug 4 ft. square.
3. Find the area of a lot 20 yd. square.
4. A linoleum 5 yd. square costs $50. Find the cost of 1 sq. yd.
5. What is the value of a lot 40 yd. square at 33$\frac{1}{3}$c a sq. ft.?

Written Problems.

1. A lot 55.5 yd. square cost $11 088.90. Find the cost per square foot.

2. If 3 A. 40 sq. rd. of land cost $1 415.70, what would be the price of a lot 120 ft. square?

3. I have two lots, one 65 yd. square, the other 128 yd. square. If the first is worth $1 056.25, what is the second worth?

4. A person has two lots of equal quality; the area of the first is 152 sq. rd. 27 sq. yd.; the second is 75 yd. square. Find the value of each, if the second is worth $200 more than the first.

To find the area of a square when the perimeter is given.

5. The perimeter of a square lot is 240 ft. Find the length of its side and its surface.

6. There are 81 posts 4 yd. apart around a square garden. Find 1° the perimeter of the garden; 2° the length of its side; 3° its area.

7. The perimeter of a square garden is 136 yd. Find 1° its area; 2° its value at 4c a square foot.

8. A square garden, 146 ft. in perimeter, was sold for $159.87. Find 1° its surface; 2° the price of one acre.

9. The fence around a square lot cost $127.96 at 70c per running yard. Find the perimeter of the lot, and its surface.

10. A square garden is surrounded with 92 posts, each 4 yd. apart. Find the value of the garden at $48.40 an acre.

To find the side of a square when the area is given.

288. Since the area of a square is the product of the two equal factors which represent its sides, it is obvious that the length of a side is found by extracting the square root of the area.

11. The area of a square field is 2 809 sq. yd.; how long is its side?

12. Express in rods the length of a square whose area is 10 acres.

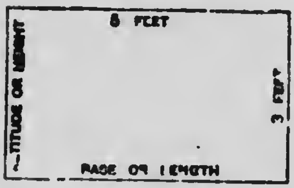
13. How many yards of steel wire are needed to fence a 7-acre square field with 4 rows of wire?

14. At \$1.33$\frac{1}{3}$ a running rod, what will it cost to fence a square field whose area is 9 025 sq. ft.?

15. Two lots are respectively 30 and 40 ft. square. Find the side of a square lot which is as large as both.

THE RECTANGLE.

289. To find the perimeter of a rectangle.



EXAMPLE.—Find the perimeter of a rectangle 5 ft. long and 3 ft. wide.

EXPLANATION.—The perimeter is equal to the sum of the 4 sides of the rectangle: $5+5+3+3=16$ ft.

Oral Problems.

Per. = B+B+H+H.
Per. = 2 times (B+H).

1. Find the perimeter of two rectangles each 10 ft. long and 5 ft. wide.

2. Find the perimeter of a class 30 ft. long and 20 ft. wide.

3. Find the perimeter of a frame 28 in. by 22 in.

4. Find the perimeter of a house 75 ft. by 50 ft.

5. Find the perimeter of your desk.

Written Problems.

1. Find the perimeter of a class 15 ft. 10 in. wide and 21 ft. 8 in. long.

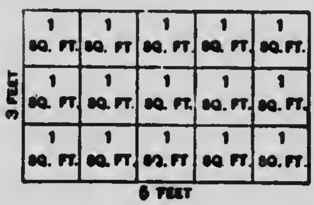
2. The width of a mat is $\frac{2}{3}$ its length. Find the cost of fringing it at 30c a running yard, if its length is 1.8 yd.

3. Mary bordered 4 curtains, 4.15 yd. by 0.85 yd., with fringe purchased at 16c a running yard; she bought 20c worth of thread, and valued her 4$\frac{1}{2}$ days' work at \$1.40 each. What did all this amount to?

4. A rectangular field, 30.25 yd. by 20.5 yd., was surrounded with a hedge worth 25c a running yard. Find the total expense.

5. A man planted trees 3.5 yd. apart around a rectangular garden 91 yd. by 63 yd., and charged $8 for his trouble. Find the total expense, if the trees cost $50 per hundred.

290. To find the area of a rectangle.



EXAMPLE.—Find the area of a rectangle 5 ft. long and 3 ft. wide.

EXPLANATION.—This rectangle may be divided into 3 rows of 5 sq. ft.; its area will therefore be 3 times 5 sq. ft., or 15 sq. ft.

Oral Problems.

Area = Base × Altitude.
A = B × Al.

1. How many square inches in a rectangle 1 ft. long and 10 in. wide?

2. Find the areas of the following rectangles: 1° 5 ft. by 9 ft.; 2° 8 ft. by 7 ft.

3. Find the areas of the following rectangles: 1° 4 yd. by 1½ yd.; 2° 6 ft. by 4½ ft.

4. Find the areas of the following rectangles: 1° 9 in. by 7½ in.; 2° 1 ft. by 8 in.

5. Find the number of square feet in the top of a desk 2½ ft. by 2 ft.

Written Problems.

1. Find the area of a classroom floor 30 ft. long and 20 ft. wide. What should be the area of the classroom windows, School Laws requiring it to be $\frac{1}{6}$ of the floor area?

2. A rectangular lot 138.6 ft. long and 120.5 ft. wide was sold at 50c a square foot. Find the selling price.

3. Find the cost of a rectangular lot 200 yd. by 150 yd., at $48.40 an acre.

4. A man dug 16 garden beds each 5 yd. by 3 yd., at the rate of $24.20 per acre. How much money will he receive?

5. A carriage road is 500 yd. long and 12 yd. wide. Each side is bordered with a walk 1.75 yd. wide. Find the cost of paving the road and sidewalks at 80c a square yard in the first case, and $1.75 a square yard in the second case.

6. From a garden whose area was 1 232 sq. yd., I sold a rectangular strip of land 32 yd. by 1.5 yd. Find the number of square yards remaining.

7. A rectangular meadow, 190.5 yd. by 90 yd., is crossed lengthwise by a lane 2.5 yd. wide. What area remains?

8. I bought a rectangular field, 62 yd. by 38 yd., at 30c a square yard and had it fenced at 50c a running foot. Find the total expenditure.

9. A rectangular prairie, 120 ft. by 60 ft., is surrounded by a fence 5 ft. high. Find 1° the length of the fence; 2° the surface of the fence.

NOTE.—To find the area of the fence multiply the perimeter by the height.

10. I bought a rectangular lot, 720 ft. by 242 ft., at $55 an acre, and had it surrounded with wirework $4\frac{1}{2}$ ft. high and weighing $7\frac{1}{2}$ lb. per square yard. Find the total cost at $3.00 per hundredweight.

291. Plastering and painting.—The unit of measure is the *square yard.*

Allowances for openings are not made unless so specified. In practice, painters rarely make allowances. The extra care required in painting sills, sashes, etc., is considered a sufficient offset.

11. How many square yards of plastering are there in the walls and ceiling of a hall 60 ft. by 40 ft., and 27 ft. high, deducting 36 sq. ft. for openings and baseboard?

NOTE.—Area of the walls = Perimeter × Height.

Area of the ceiling = Length × Width.

Use a cardboard box to have a clear idea of the work.

12. Find the cost of plastering the walls and ceiling of a classroom 32 ft. by 18 ft., and 12 ft. high, at 32c a square yard, deducting $\frac{1}{6}$ of the area of the walls on account of blackboards, baseboard and openings.

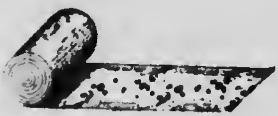
13. What will it cost to paint both sides of the fence surrounding a rectangular field, 150 ft. long by 100 ft. wide, if the fence is 8 ft. high, and the painter is paid at the rate of 18c a square yard?

14. A contractor filed a bid to plaster the walls and ceiling of a hall 54 ft. by 28 ft., and 18 ft. high, at 35c a square yard, deducting $\frac{1}{2}$ of the area of 8 windows each $6\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft., and of 2 doors each 7 ft. by $4\frac{1}{2}$ ft. Another bidder offered to do the work at 33c per square yard, not allowing for any openings. How much would be saved in choosing the lower bid?

15. What will it cost to paint the outside walls of a freight warehouse 96 ft. by 56 ft., and 24 ft. high, at 27c a square yard?

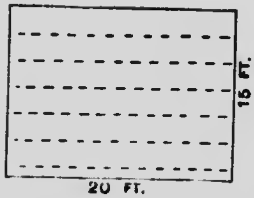
16. Assuming that 100 laths can cover a surface of 5 sq. yd., how many bundles of 100 laths each will be necessary to cover the walls and ceiling of a store 75 ft. by 20 ft., and 15 ft. high, deducting 480 sq. ft. for openings? Find the cost of the laths at 35c a bundle.

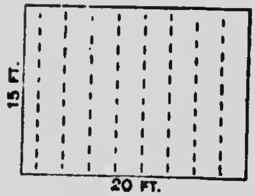
292. Carpeting.—Carpets are sold by the *running yard.*



Some carpets are one yard in width; others are $\frac{3}{4}$ yd., or 27 in. in width. Any length of carpet can be bought, but the width is necessarily entire; therefore, in computing the cost of carpet the number of *strips* must be found.

EXAMPLE.—How many yards of carpet are required to cover the floor of a room 20 ft. long and 15 ft. wide, if the width of the carpet is 27 in., 1° if the strips are laid lengthwise; 2° if the strips are laid crosswise?





EXPLANATION.—1° Strips laid lengthwise: $\frac{15 \text{ ft.} \times 12}{27} = 6\frac{2}{3}$ strips. But 7 will have to be bought. Each strip is 20 ft. long; 20 ft. $\times$ 7 = 140 running ft.; 140 ft. $\div$ 3 = $46\frac{2}{3}$ running yd.

2° Strips laid crosswise: $\frac{20 \text{ ft.} \times 12}{27} = 8\frac{8}{9}$ strips. But 9 will have to be bought. Each strip is 15 ft. long; $15 \times 9 = 135$ running ft.; 135 ft. $\div 3 = 45$ running yd.

17. How many yards of 27-inch carpet will be required for the following rooms, strips running lengthwise: 1° 20 ft. by 18 ft.; 2° 25 ft. by 22 ft.; 3° 28 ft. 6 in. by 22 ft. 6 in.?

18. How many yards of 36-inch carpet will be needed for the following rooms, strips running crosswise: 1° 26 ft. 9 in. by 15 ft. 8 in.; 2° 23 ft. by 17 ft. 6 in.; 3° 20 ft. 9 in. by 15 ft.?

19. How many yards of 27-inch carpet are needed for a room 20 ft. by 17 ft. 8 in., if the strips run lengthwise, and if there is a waste of 6 in. on each strip (*except the first*) for matching the pattern?

20. Find the cost of a carpet 27 in. wide, at $1.45 a yard, for a parlor 18 ft. by 15 ft. 8 in., if the strips are laid lengthwise, allowing 9 in. on each strip (*except the first*) for matching the pattern.

293. Paper hanging.—Wall paper is sold and hung by the roll. A single roll is 18 in. wide and 24 ft. long; a double roll is 18 in. wide and 48 ft. long. Fractional parts of rolls cannot be bought.

294. Rule. –*To find the number of rolls needed: 1° Find the perimeter of the room; 2° Subtract the width of all the openings; 3° Divide the net perimeter by 18 in., to find the* *number of strips required; 4° Find how many whole strips can be cut from a roll; 5° Divide the number of strips required by the number of whole strips that can be cut from a roll; 6° If there is a fraction in dividing, add 1.*

Note.—The ends of rolls are used for the surface above the doors and above and below the windows.

For the ceiling, divide the number of strips required by the number of whole strips that can be cut from a roll.

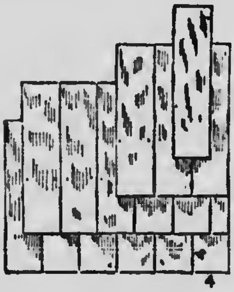
21. The net perimeter of a room is 72 ft. and the height of the walls above the baseboard is $8\frac{1}{2}$ ft. How many double rolls of paper will be required for the walls only?

Note.—72 ft. ÷ width of a roll (18 in.) = the number of strips required. Each strip is $8\frac{1}{2}$ ft. long; divide 48 ft. by $8\frac{1}{2}$ ft. to find the number of whole strips that can be cut from a roll, which is 5. Then divide the number of strips required by 5 to find the number of rolls.

22. How many double rolls of paper will it take to cover the walls and ceiling of a dining room 22 ft. by 15 ft., and 11 ft. high above the baseboard, allowing for 4 openings 7 ft. by $3\frac{1}{2}$ ft.? The strips of the ceiling are to be laid lengthwise.

23. How many double rolls of paper will be required to cover the walls and ceiling of a room 23 ft. by 18 ft., and $7\frac{1}{2}$ ft. above the baseboard, allowing for 4 openings 8 ft. by 4 ft.? The strips of the ceiling are to be laid lengthwise. What will the paper cost at 30c a roll?

295. Roofing.—Shingles, slates, and tiles are used in roofing. The unit of measurement is the *square* (= 100 sq. ft.).



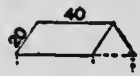
4

Ordinary shingles are 16 in. long by 4 in. wide, and are usually laid 4 in. to the weather, thus covering an area of 16 sq. in. each. This would mean 9 shingles per square foot (144 sq. in. ÷ 16 sq. in. = 9), and 900 per *square*. But allowing for waste, 1 000 shingles are computed per square.

Shingles are put in bundles of 250 each. Parts of a bundle are not sold.

Count 900 shingles per square, if the exposed portion of each is 4 in. by $4\frac{1}{2}$

in.; 800 shingles per square, if the exposed portion of each is 4 in. by 5 in.; and 700 shingles per square if the exposed portion of each is 4 in. by $5\frac{1}{2}$ in.



24. How many shingles, exposed portion 4 in. by 4 in., would be required for a roof 40 ft. long, and measuring 20 ft. from ridge to eaves? How many bundles?

25. At $3.75 per M shingles, exposed portion 4 in. by 4 in., and $1.15 per square for labor, what is the cost of the two sides of a roof, each 25 ft. by 10 ft.?

26. The roof of a barn is 80 ft. long; its width on both sides of the ridge consists of two parts each 20 and 15 ft. Find how many bundles of shingles are needed for roofing, the exposed portion of each shingle being 4 in. by 4 in.

27. How many shingles would be needed in N° 26, if the upper portion of the roof were covered with shingles laid $4\frac{1}{2}$ in. to the weather, and the lower portion with shingles laid 5 in. to the weather?

28. How many slates 16 in. by 10 in. would be required per square, the exposed portion of each being 10 in. by 6 in.?

296. Paving.

29. How many bricks 8 in. long and 4 in. wide are necessary to pave a street 80 rd. long and 36 ft. wide? Find the total cost at $4 per square yard.

30. How many paving stones 9 in. by 5 in. are necessary to cover a street 1 mi. long and 40 ft. wide?

297. To find one dimension of a rectangle when the area and the other dimension are given.

Area ÷ Altitude = Base.	Area ÷ Base = Altitude.
$\frac{A}{Al.} = B.$	$\frac{A}{B} = Al.$

Oral Problems.

1. The area of a rectangle is 20 sq. ft., and its width is 4 ft. Find its length.

2. Find the breadth of a desk whose area is 12 sq. ft. and whose length is 4 ft.

3. If the area of a floor is 600 sq. ft., and its length 30 ft., what is its breadth?

4. The area of a window is 32 sq. ft., and its height 8 ft. Find its breadth.

5. If a rectangular field which contains 1 A. is 20 rd. long, what is its width expressed in feet?

Written Problems.

1. Find the length of a rectangular field which contains 1 102 sq. yd., if its width is 29 yd.

2. Find the width of a rectangular field whose length is 65 yd., if it cost $1 287 at 45c a square yard.

3. Find the perimeter of a rectangular lot whose area is 1 904 sq. yd. and length 56 yd.

4. A rectangular piece of land 36 yd. long cost $3 564 at 45c a square yard. What would the fencing come to at 70c a running foot?

5. A piece of land is 180 ft. wide and contains 1 A. 38 sq. rd. 10 sq. yd. 4 sq. ft. 72 sq. in. How many trees could be planted around, placing them 10 yd. apart?

298. Increasing or decreasing areas.

Written Problems.

1. What must be the area of a cloth that will reach over .25 yd. on every side of a rectangular table 2 yd. long and 1.25 yd. wide?

2. A rectangular sheet of paper is 14 in. by $8\frac{1}{2}$ in. If you were to cut off a slip $\frac{1}{2}$ in. wide from every side, what area would you thus take away?

3. A portrait is 20 in. long and 16 in. wide. The frame around it is 3 in. wide. Find 1° the area of the portrait; 2° the outside perimeter of the frame in yards.

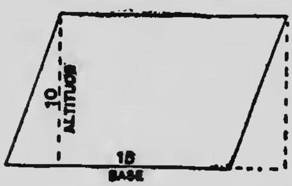
NOTE.—Picture-frame makers always use the outside perimeter in their calculations; it is found by adding twice of the frame to each dimension of the picture. This is because one half of each corner is sawed off in making.

4. An artist painted a picture 8 ft. by 6 ft. Find the cost of framing it at $1 a running foot, if the frame is 6 in. wide.

5. How long a fence is needed to surround a house 12 yd. front and 15 yd. deep, if the fence is to be $5\frac{1}{2}$ yd. from the house on every side? Find the area of the space between the house and the fence.

THE PARALLELOGRAM.

To find the area of a parallelogram.



EXAMPLE.—Find the area of a parallelogram whose base is 15 ft. and altitude 10 ft.

EXPLANATION.—If the left-hand triangle were cut from this parallelogram and placed at the right, the figure would become a rectangle 15 ft. long and 10 ft. high, the area of which would be 150 sq. ft.

Oral Problems.

$$\text{Area} = \text{Base} \times \text{Altitude}.$$
$$A = B \times Al.$$

1. The base of a parallelogram is 6 ft., and its altitude is 4 ft. Find its area.

2. Find the area of a parallelogram whose base is $3\frac{1}{2}$ ft. and altitude 2 ft.

3. What is the area of a parallelogram whose base is 3 ft. 8 in., and altitude 3 ft.?

4. The base of a parallelogram is $\frac{2}{3}$ yd., and its altitude is 24 in. What is its area?

5. The base of a parallelogram is 16 ft., and its altitude 8 ft. Find its area.

Written Problems.

1. What is the area of a parallelogram whose base is 40 ft. and altitude 30 ft.?

2. A crossbarred pattern contains 42 parallelograms each 1.5 in. long and .8 in. wide. Find the area of the parallelograms.

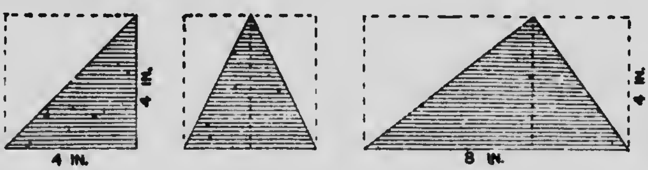
3. How many parallelograms are there in a piece of check silk containing 108.864 sq. in., if the base of each parallelogram is .18 ft. and its altitude .12 ft.?

4. The area of a parallelogram is 30.25 A.; its altitude is $60\frac{1}{2}$ rd. Find its base.

5. A tessellated work contains 35 squares and 30 parallelograms. Find its area, if the side of each square is 1.1 in. long, and if the base and altitude of each parallelogram are respectively 1.4 in. and .7 in.

THE TRIANGLE.

To find the area of a triangle when its base and altitude are given.



299. You see that the area of a triangle equals one half of the area of a rectangular figure having its base and altitude.

The first two triangles contain $\frac{4\times4}{2}=8$ sq. in.; the other, $\frac{8\times4}{2}=16$ sq. in.

$$\text{Area} = \frac{\text{Base} \times \text{Altitude.}}{2}$$

$$A = \frac{B \times Al.}{2}; \text{ or } A = B \times \frac{Al.}{2}; \text{ or } A = \frac{B}{2} \times Al.$$

Oral Problems.

1. A triangle is what part of a rectangle of the same base and altitude?

2. What is the area of a triangle whose base is 8 in., and altitude 6 in.?

3. The base of a triangle is 10 in., and its altitude 8 in. Find its area.

4. The base of a triangle is 15 in., and its altitude 10 in. Required the area of the triangle.

5. Find the area of a triangle whose base is 120 in. and altitude $16\frac{2}{3}$ in.

Written Problems.

1. Find the areas of the triangles whose bases and altitudes are as follows: 1° 40 ft., 30 ft.; 2° 1.35 yd., .86 yd.; 3° 158.8 ft., 108.6 ft.

2. Find the value of a triangular lot whose base is 12.8 yd. and altitude 11.6 yd., at 25c a square yard.

3. How many square feet are there in the gables of a house that is 20 ft. wide, the ridge of the roof of the house being 10 ft. higher than the foot of the rafters?

4. The surface of a church steeple is made up of 6 triangles. Find the total area, if the base of each triangle is 20 ft., and its altitude 65 ft.

5. The base of a triangular field is 30 rd., and the altitude is 40 rd. How many acres does it contain?

To find one dimension of a triangle when the area and the other dimension are given.

300. Rule.—*Multiply the area by 2, and divide the product by the given dimension; or divide the area by one half of the given dimension.*

Oral Problems.

1. The area of a triangle is 50 sq. ft., and its altitude 10 ft. What is its base?

2. What is the altitude of a triangle whose area is 100 sq. in., and base 1 ft. 8 in.?

3. What is the base of a triangle whose area is 16 sq. yd., and altitude 12 ft.?

4. The area of a triangle is $3\frac{1}{3}$ sq. yd., and its base 10 ft. Find its altitude.

5. The area of a triangle is $\frac{1}{2}$ A., and its base 40 rd. What is its altitude in feet?

Written Problems.

1. Find the altitude of a triangle whose area is 1 064 sq. ft., and base 56 ft.

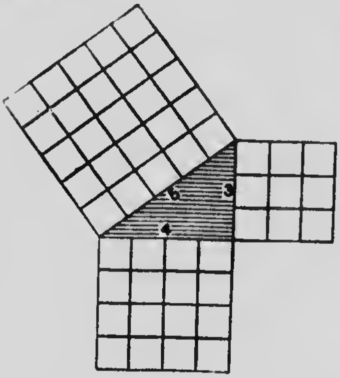
2. The area of a triangle is 6 960 sq. ft., and its altitude 96 ft. What is its base?

[illegible] paid $257.60 for a triangular piece of land bought at 1 [illegible] yard. If its altitude is 35 yd., what is its base?

[illegible] changed a rectangular field 30 yd. by 24 yd. for a triangular field of an equal area. Find the altitude of this triangle, if its base is 40 yd.

5. A triangle whose base is 5 in., and altitude 4.8 in., is equal in size to another triangle whose base is 8 in. Find the altitude of the latter triangle.

To find the hypotenuse of a right-angled triangle.



301. The *hypotenuse* is the side opposite the right angle.

Observe the squares formed on the sides of the annexed right-angled triangle. You will see that the square on the hypotenuse contains as many units as the square on the base and the square on the altitude together. Hence the principle:

The square of the hypotenuse equals the sum of the squares of the other two sides.

$$\begin{aligned} Hy.^2 &= B^2 + Al.^2 \\ Hy.^2 - Al.^2 &= B.^2 \\ Hy.^2 - B^2 &= Al.^2 \end{aligned}$$

And, the square of the hypotenuse minus the square of the altitude equals the square of the base; and, the square of the hypotenuse minus the square of the base equals the square of the altitude.

$$\sqrt{Hy.^2} = Hy.; \quad \sqrt{Al.^2} = Al.; \quad \sqrt{B^2} = B.$$

Written Problems.

1. Find the length of the hypotenuse when the sides are: 1° 30 ft. and 40 ft.; 2° 45 yd. and 60 yd.

2. Find the base of each of the right-angled triangles whose other sides are here given: 1° hypotenuse 25 ft., altitude 15 ft.; 2° hypotenuse 12.5 yd., altitude 7.5 yd.

3. Find the altitude of each of the right-angled triangles whose other sides are here given: 1° hypotenuse 1 ft., base .8 ft.; 2° hypotenuse 35 yd., base 28 yd.

4. How far apart are the opposite corners of a rectangular lot 68 ft. long and 51 ft. wide?

5. How long is the diagonal of a blackboard 4 ft. square? (2 *decimals*).

6. Find the perimeter of a right-angled triangle whose base is 240 ft., and altitude 160 ft. (2 *decimals*).

7. Two yachts sailed from the same port; one went 40 mi. north, and the other 30 mi. east. How far apart were they then?

8. A ladder leaning against a wall reaches 36 ft., its foot being 27 ft. from the wall. How long is the ladder?

9. What is the distance from a lower corner to the opposite upper corner of a room 16 ft. by 12 ft., and 10 ft. high? (2 *decimals*).

10. A 60-foot ladder is so placed in a street as to reach on one side to a window 23 ft. high, and on the other side to a window 37 ft. high. Can you find the width of the street? (2 *decimals*).

To find the area of a triangle when its three sides are given.

302. Rule.—*1° Find half the sum of the three sides; 2° separately subtract each side from half the sum of the three sides; 3° find the continued product of the three remainders and half the sum of the three sides: 4° extract the square root of the continued product.*

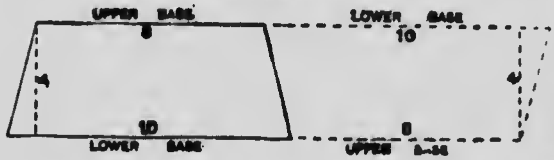
Written Exercises.

Find the areas of the triangles whose three sides are:

1. 16, 58, and 70 ft.
2. 38, 40, and 74 ft.
3. 45, 60, and 75 ft.
4. 270, 170, and 150 ft. (2 *decimals*).
5. 200, 300, and 400 ft. (2 *decimals*).
6. 80, 90, and 120 yd. (2 *decimals*).
7. 20, 20, and 30 yd. (2 *decimals*).
8. 11.5, 11.5, and 20 yd. (2 *decimals*).
9. 8.75, 8.75, and 15 yd. (2 *decimals*).
10. 60, 20, and 50 yd. (2 *decimals*).

THE TRAPEZOID.

To find the area of a trapezoid.



303. Example.—Find the area of a trapezoid whose altitude is 4 in., and parallel sides are respectively 10 in. and 8 in.

Explanation.—If a trapezoid has its duplicate turned over and fitted to it as shown in the diagram, the two together form a large parallelogram. In this example, the parallelogram would be 18 in. long and 4 in wide; its area would be 72 sq. in.; $72 \div 2 = 36$, or ½ the area of the parallelogram, or the area of the trapezoid.

$$\text{Area} = \frac{(\text{Longer base} + \text{shorter base}) \times \text{Altitude}}{2}$$

$$A = \frac{(B + b) \times Al.}{2}$$

Oral Problems.

Find the areas of trapezoids with altitudes first given below, followed by the two parallel sides:

1. 6 in., 7 in., and 13 in.
2. 10 in., 8 in., and 12 in.
3. 3.5 in., 1 in., and 3 in.
4. $6\frac{1}{4}$ in., 9 in., and 11 in.
5. 12 in., 1[illegible] in., and 19 in.

Written Problems.

1. Find the areas of trapezoids whose dimensions are: 1° longer base, 1.20 yd.; shorter base, .60 yd.; altitude, 1.10 yd.; 2° longer base, 118 ft.; shorter base, 56 ft.; altitude, 65 ft.

2. The parallel sides of a platform are 30 and 20 ft.; and its breadth is 15 ft. At $1.05 per square yard, what will it cost to cover this platform with carpet?

3. There is a field in the form of a trapezoid which has an altitude of 968 ft. The parallel sides are 550 ft. and 350 ft. long respectively. What is the field worth at $40 an acre?

4. A lot in the form of a trapezoid has its parallel sides 534 ft. and 366 ft. in length. The perpendicular distance between them is 484 ft. Find the price of 1 A., if the whole lot is worth $250.

5. A garden was given a second tillage by 3 men, at the rate of $9 an acre. How much should each man receive, if the parallel sides of this garden are 312 and 172 ft., and its altitude is 600 ft.?

To find one of the dimensions of a trapezoid.

304. Rule.—*To find the altitude of a trapezoid, divide twice its area by the sum of its parallel sides.*

To find the sum of the parallel sides of a trapezoid, divide twice its area by its altitude.

Oral Problems.

1. Find the altitude of a trapezoid whose area is 48 sq. in. and parallel sides are 5 and 7 in.

2. Find the shorter base of a trapezoid whose area is 60 sq. in., altitude 6 in., and longer base 12 in.

3. Find the longer base of a trapezoid whose area is 100 sq. in., altitude 5 in., and shorter base 5 in.

4. Find both bases of a trapezoid whose area is 15 sq. in., and altitude 2 in., if the longer base is twice the shorter base.

5. Find both bases of a trapezoid whose area is 75 sq. in., altitude 5 in., and shorter base ½ of the longer.

Written Problems.

1. The area of a trapezoid is 48 sq. in.; if its parallel bases are 7 and 5 in., find its altitude.

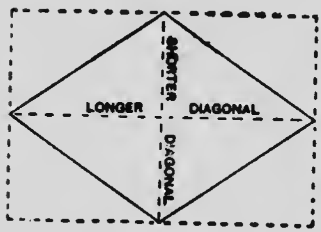
2. If the area of a trapezoid is 1 963.5 sq. ft., the altitude 38.5 ft., and the longer base 68 ft., what is the shorter base?

3. Find the longer base of a trapezoid whose area is 2 122.8 sq. yd., altitude 36.6 yd., and shorter base 47.5 yd.

4. Find both bases of a trapezoid whose area is 1 925.70 sq. yd., altitude 49 yd., if the longer base is double the shorter base.

5. Find both bases of a trapezoid which has the same area as a square whose side is 56 ft. long. The altitude of the trapezoid is 112 ft., and its longer base equals 3 times its shorter base.

THE RHOMBUS.



To find the area of a rhombus.

305. The area of a rhombus equals one half of the area of a rectangle whose base and altitude are the same as the longer and shorter diagonals of the rhombus.

$$\text{Area} = \frac{\text{Longer Diagonal} \times \text{shorter diagonal}}{2}$$

$$A = \frac{D \times d}{2}$$

Oral Problems.

Find the area of each rhombus whose diagonals are:

1. 8 ft. and 6 ft.
2. 10 ft. and 8 ft.
3. $1\frac{1}{2}$ ft. and 6 in.
4. $\frac{2}{3}$ yd. and 1 ft.
5. 18 in. and $\frac{1}{6}$ yd.

Written Exercises.

1. Find the area of each rhombus whose diagonals are: 1° 85 and 50 ft.; 2° 14.50 and 11.60 yd.

2. Find the area of each rhombus whose diagonals are: 1° 32 and 25 ft.; 2° 89.90 and 66.7 yd.

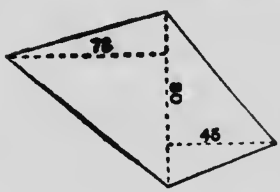
3. How many acres are there in a rhombus whose diagonals are 80 and 40 rd.?

4. Find the area of a glazed window composed of 3 050 lozenge-shaped panes whose diagonals are .18 yd. and .12 yd.

5. A homemade carpet is composed of 56 lozenges whose diagonals are .16 ft. and .12 ft., and of 34 triangles whose bases and altitudes are .16 ft. and .06 ft. Find the area of the carpet.

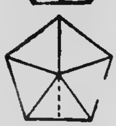
MISCELLANEOUS POLYGONS.

Written Problems.

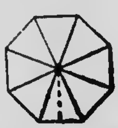


1. Find the area of a trapezium whose diagonal is 80 ft., if the two triangles formed by the diagonal are respectively 75 and 45 ft. high.

2. What is the area of a regular hexagon composed of equilateral triangles whose bases and altitudes are respectively 10 in. and 8.66 in.?



3. What is the area of a regular pentagon composed of isosceles triangles whose bases and altitudes are respectively 10 in. and 6.882 in.?



4. What is the area of a regular octagon composed of isosceles triangles whose bases and altitudes are respectively 10 in. and 10.383 in.?

5. What is the area of a regular hexagon each of whose sides is 8 ft., if the perpendicular distance from the center to each side is 6.928 ft.?

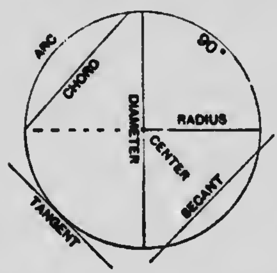
CURVILINEAR SURFACES

THE CIRCLE.

306. A *circle* is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *center*.

The curved line that bounds a circle is called its *circumference*.

The bottom of a bucket represents a circle; the rim of a bucket represents a circumference.



307. The *radius* of a circle is the distance from the center to the circumference.

Any number of radii may be traced in a circle. All the radii of a circle are equal.

308. The *diameter* of a circle is a straight line passing through the center and terminating in the circumference.

Any number of diameters may be traced in a circle. All the diameters of a circle are equal.

The diameter equals two radii.

309. Any part of a circumference is an *arc*.

One fourth of the circumference is an arc of 90 degrees.

A straight line uniting the extremities of an arc is a *chord*.

A straight line cutting a circumference is a *secant;* it is a prolonged chord.

A straight line which touches a circumference without cutting it is a *tangent*.

310. To find the circumference of a circle.

An unrolled circumference forms a straight line which can easily be measured. With a string you can find the circumference of a hoop, bucket, or round table; the length of the string in each instance would be a little more than 3 times the length o the diameter, exactly 3.1416 times.

Oral Problems.

$$\boxed{\begin{gathered}\text{Circumference} = \text{Diameter} \times 3.1416. \\ C = D \times 3.1416.\end{gathered}}$$

1. Find the diameter when the radius is 10 feet.; 8½ ft.

2. Find the radius when the diameter is 14 in.; 21 ft.

3. Find the circumference when the diameter is 10 ft.

4. Find the circumference when the radius is 5 ft.

5. Find the circumference when the diameter is 100 in.

Written Problems.

1. Find the circumference, given the diameter: 1° 40 in.; 2° 7.5 ft.; 3° .75 yd.

2. What is the circumference of a circle whose radius is 1° 3 in.; 2° 3.5 ft.; 3° .9 yd.?

3. The hind wheel of a carriage made 1 000 revolutions in going a certain distance. Required the distance, if the diameter of the wheel is 1.5 yd.

4. The hind wheel of a carriage makes 50 revolutions a minute. At this rate, how many miles will the carriage run in 8 hr. 20 min., if the radius of the wheel is .65 yd.? (2 *decimals*).

5. What is the circumference of a wire whose diameter is .63 in.?

To find the diameter of a circle.

311. Rule.—*Divide the circumference by 3.1416.*

$$\boxed{D = \frac{C}{3.1416}}$$

Written Problems.

1. Find the diameter, given the circumference: 1° 314.16 in.; 2° 471.24 ft.; 3° 376.992 yd.

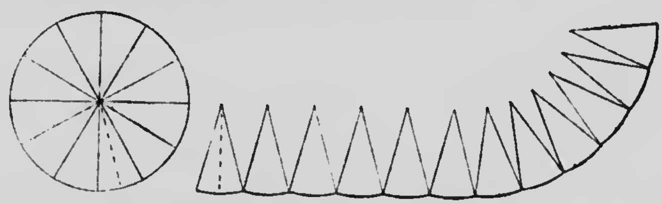
2. Find the radius, given the circumference: 1° 62.832 in.; 2° 14.1372 ft.; 3° 188.496 yd.

3. The fence around a circular pond cost $157.08 at $5 a running yard. What is the diameter of the pond?

4. What is the diameter of a tree that is 125.664 in. in circumference?

5. The circumference of the earth is 24 856.3392 miles; what is its diameter?

312. To find the area of a circle.



A close observation of the accompanying figures will make it evident for you that a circle may be regarded as composed of a large number of small triangles, the sum of whose bases forms the circumference of the circle, and whose altitude is the radius of the circle. Hence, *the area of the circle* must be the same as that of all these small triangles, or *the product of the circumference by half the radius.*

$$\boxed{\begin{array}{l}\text{Area} = \text{Circumference} \times \dfrac{\text{Radius.}}{2} \\ A = C \times \dfrac{R}{2}\end{array}}$$

When the radius alone is given, first find the circumference by multiplying twice the radius by 3.1416; then, multiply by half the radius.

ILLUSTRATION.—Area $= \dfrac{\text{Radius} \times 2 \times 3.1416 \times \text{Radius}}{2}$, which simplified gives: Area = Square of the radius $\times 3.1416$; or $A = R^2 \times 3.1416$.

When the diameter alone is given, first multiply it by 3.1416: this gives the circumference of the circle; then multiply by half the diameter (*radius*), and divide by 2.

ILLUSTRATION.—Area $= \dfrac{\text{Diameter} \times 3.1416 \times \text{Diameter}}{2 \times 2}$, which simplified gives: Area = Square of the diameter $\times .7854$; or $A = D^2 \times .7854$.

Written Problems.

1. Find the area, given the radius as follows: 1° 10 in.; 2° .4 ft.; 3° 12.75 yd.

2. Find the area, given the diameter as follows: 1° 10 in.; 2° 2.7 ft.; 3° 12.9 yd.

3. Find the area, given the circumference as follows: 1° 62.832 in.; 2° 141.372 ft.; 3° 18.8496 yd.

4. Find the cost of a circular oilcloth 2.6 yd. in diameter, at 50c a square yard.

5. The distance around a circular park is 1½ mi. What is its area in acres? (2 *decimals*).

6. A cow is fastened to a stake by a halter 22 ft. long. Over what area can the cow graze?

7. A circular pond is 251.328 ft. in circumference. What would the cementing of this pond cost at $1.05 a square yard?

8. In the middle of a lot 25 yd. square is a circular pond 20 yd. in diameter. Find the area without the pond.

9. One circle has a diameter 10 in. long; another circle has a diameter twice as long. The second circle is how many times larger than the first?

10. Within a figure 20 ft. square is a circle 20 ft. in diameter. The circle is what part of the square? (4 *decimals*).

To find the radius of a circle when the area is given.

313. Rule.—*Divide the area by 3.1416, and extract the square root of the quotient.*

NOTE I.—Multiply the radius by 2 to find the diameter.

NOTE II.—Multiply the radius by 2 and by 3.1416 to find the circumference.

Written Problems.

1. Find the radius of a circle whose area is 314.16 sq. ft.

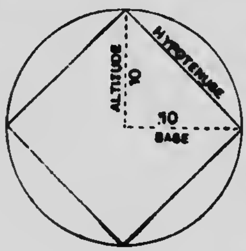
2. Find the diameter of a circle whose area is 157.08 sq. ft.

3. Find the circumference of a circle whose area is 785.4 sq. yd.

4. The area of a circular court is 38.4846 sq. rd. Find the diameter.

5. The area of a circle is 286.488 sq. ft. Find the radius, diameter, and circumference.

To find the sides of a square inscribed in a given circle.



EXAMPLE.—What is each side of the square which can be inscribed in a circle whose radius is 10 in.?

EXPLANATION.—The side of the square is equal to the hypotenuse of a right-angled triangle having for base and altitude the radius of the circle: $\sqrt{10^2+10^2} =$ 14.142 in.

Written Problems.

1. What is each side of the square which can be inscribed in a circle whose radius is 5 in.? (3 *decimals*).

2. A circle is 18 in. in diameter; what is the side of the largest square that can be inscribed in this circle? (3 *decimals*).

3. Find the area of the square inscribed in a circle whose radius is 20 in.

4. Find the side of the largest square log that can be hewn from a round piece of timber whose smaller end is $2\frac{2}{3}$ ft. in diameter. (3 *decimals*).

5. Find the side of the largest square log that can be hewn from a round piece of timber whose smaller end is 4 ft. in circumference. (1 *decimal*).

THE RING.

To find the area of a ring.

314. A *ring* is the surface included between two concentric circles.

315. Rule.—*Subtract the area of the smaller circle from the area of the larger circle.*

NOTE.—Or else, subtract the square of the shorter radius from the square of the longer radius, and multiply by 3.1416: $A = (R^2 - r^2) \times 3.1416$.

Written Problems.

1. The radius of the larger circle being 75 in., and that of the smaller circle 60 in., find the area of the ring.

2. A circle is 2.3 yd. in diameter, and within it is another circle 1.95 yd. in diameter. What area of the larger circle is outside the smaller?

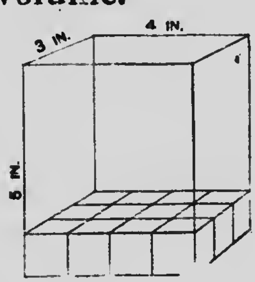
3. Find the area of the ring formed by two concentric circles whose respective diameters are .78 in. and .67 in.

4. A walk 6 ft. wide extends around a circular pond whose radius is 6 yd. What is the area of the walk?

5. A concrete sidewalk 4 ft. wide extends around a circular pond 20 ft. in diameter. Find the cost of the walk at $1.50 a square yard.

RECTANGULAR VOLUMES

316. To find the contents of a rectangular volume.



NOTE.—A volume is the product of three dimensions.

EXAMPLE.—What is the volume in cubic feet of a rectangular block 4 in. long, 3 in. wide, and 5 in. high?

EXPLANATION.—Each small cube represents 1 cu. in.; there are 12 such cubes (3×4) in the bottom row; and as there are 5 rows in all, we have $3 \times 4 \times 5 = 60$ cu. in.

Volume = Length × Width × Height.

Oral Problems.

Find the volumes of boxes having the following dimensions:

1. 4″×3″×2″.
2. 4″×5″×6″.
3. 3″×3″×7″.
4. 2″×3″×12″.
5. 9″×9″×10″.

Written Problems.

1. How many cubic feet in a pile of wood 8 ft. long, 8 ft. wide, and 12 ft. high?

2. How many cubic feet of air in a room 18 ft. long, 16 ft. wide, and 10 ft. high?

3. How much will it cost to excavate a cellar 30 ft. long, 18 ft. wide, and 8 ft. deep, at 35c a cubic yard?

4. Find the cost of a pile of wood 32 ft. by 4 ft. by 6 ft., at $7.50 a cord.

5. Find the weight of a squared piece of timber 24 ft. long, 18 in. wide, and 18 in. thick, at 48 lb. per cubic foot.

6. How many pupils can be placed in a schoolroom 30 ft. by 20 ft. by 12 ft., at the rate of 150 cu. ft. per pupil?

7. A schoolroom 30 ft. by 25 ft. by 12 ft., is occupied by 29 pupils and the teacher. Find the number of cubic feet of air allowed to each.

8. The interior dimensions of a freight car are 34 ft., 8 ft., and 7 ft. Find its capacity in cubic feet. A farmer loads it with wheat to a height of 6 ft.; find the number of bushels of wheat, allowing $1\frac{1}{4}$ cu. ft. to the bushel.

9. How many cubic yards of earth were removed in digging a tunnel 495 ft. long, 40 ft. wide, and 20 ft. high?

10. A dirt car is 27 ft. long, 6 ft. wide, and $3\frac{1}{4}$ ft. deep. What is its capacity in cubic yards?

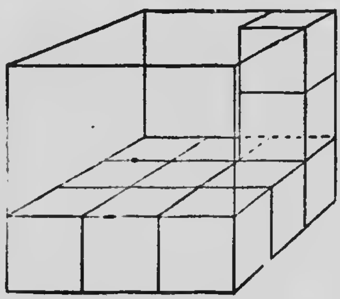
11. A cistern is 12 ft. long, 6 ft. wide, and 4 ft. deep. How many gallons can it contain at the rate of $6\frac{1}{4}$ gal. per cubic foot?

12. If the volume of a ton of hay is 500 cu. ft., how many tons of hay can be placed in a barn 48 ft. by 15 ft. 6 in. by 10 ft.?

13. If the volume of a ton of coal is 35 cu. ft., how many tons of coal can be placed in a bin 22 ft. by 17.5 ft. by 8 ft.?

14. A cubic foot of water weighs 1 000 ounces, and a gallon of water weighs 10 lb. How many gallons of water in a tank 20 ft. by 5 ft. by 5 ft.?

15. To drain a farm, I had a trench dug at the rate of 9c a cubic yard. How much did the digging come to if the trench was 1 mi. long, 16 ft. wide, and 3 ft. deep?



Cubic yard. (SCALE $\frac{1}{36}$.)

317. Concrete, stone or brick work.

CONCRETE AND STONE. — The unit of measurement is the *cubic yard*.

BRICKS.—The unit of measurement is the *cubic foot* of brickwork, which generally contains 22 bricks.

In labor estimates, the volume equals the outside perimeter of the walls × the height × the thickness of the walls. No allowance is made for openings.

Written Problems.

1. How many cubic yards of stone in the walls of a cellar 28 ft. long, $21\frac{1}{2}$ ft. wide, and $6\frac{1}{2}$ ft. deep, if the walls are $1\frac{1}{2}$ ft. thick?

2. At the rate of $1.15 a cubic yard for concrete, what did I pay for the foundation of a house 30 ft by 20 ft., if the foundation is 4½ ft. high and 2 ft. thick?

3. How many cubic yards of concrete in the foundation of a house 120 ft. long and 110 ft. wide, the foundation being 9 ft. high and 4 ft. thick?

4. If in the preceding problem, ⅙ of the concrete was cement, ⅓ sand, and the remainder gravel, how many cubic yards of each of these materials were used?

5. In problem N° 2, what would be the cost of brick walls 21 ft. high and 1 ft. thick, at $8.50 per M bricks?

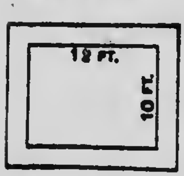
6. At $8 per M bricks, find the cost of the walls of a house 36 ft. long, 32 ft. wide, and 22½ ft. high, if the walls are 1¼ ft. thick.

7. A house is 72 ft. long, 50 ft. wide, and 42 ft. high. What did the brickwork cost at $17.50 per M bricks, if the walls are 14 in. thick?

8. A contractor made a bid on the following work: 1° To dig a cellar 36 ft. by 24 ft. by 6 ft., at 30c a cubic yard; 2° to construct a stone foundation 1½ ft. thick, projecting 2 ft. above ground, at $1.50 a cubic yard; 3° to furnish the stone and lime, at $6 a cubic yard. Find the amount of his bid.

9. An ordinary brick is 8 in. long, 4 in. wide, and 2 in. thick. To each dimension add ¼ in. for mortar, and find the volume of a brick. How many times is this volume contained in a cubic foot?

10. A farmer constructed a concrete silo with inside measurements 12 ft. long, 10 ft. wide, and 28 ft. high. The walls are 1½ ft. thick, and the concrete floor upon which the silo stands is 2 ft. thick. Find the cost of this silo, at $1.15 per cubic yard for concrete, taking the outside perimeter to find the volume of the walls. How many tons of ensilage can this silo contain, at the rate of 40 cu. ft. per ton?



To find any dimension of a rectangular solid.

318. Rule.—*Divide the volume by the product of any two dimensions.*

Written Problems.

1. A box 20 in. long and 13 in. wide contains 3 900 cu. in. What is its depth?

2. A hall 41.3 ft. long and 36.4 ft. wide contains 41 341.3 cu. ft. What is its height?

3. A ditch is 3.5 ft. wide and 8 ft. deep. Find its length, if its volume is 1 244$\frac{4}{9}$ cu. yd.

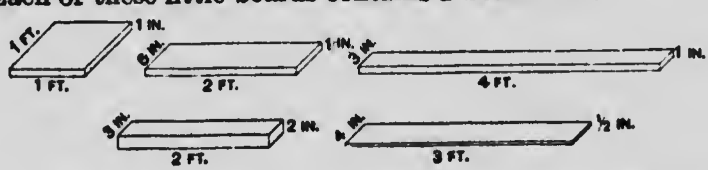
4. A grain store contains 15 552 bushels of grain. How high is it, if it is 36 ft. long and 18 ft. wide, allowing 1$\frac{1}{4}$ cu. ft. to the bushel?

5. The volume of a rectangular room is 4 221.36 cu. ft., and the surface of the floor is 469.04 sq. ft. Find the height and breadth of the room, if its length is 32.8 ft.

Board Measure.

319. The unit of measure is the *board foot* (*bd. ft.*); it is represented by a board that is 1 square foot on one surface, and 1 inch or less in thickness.

Each of these little boards contains 1 board foot.



EXAMPLE I.—How many board feet in a board 16 ft. long, 9 in. wide, and 1 in. thick?

EXPLANATION.—A board 1 in. thick contains as many *board feet* as *square feet*, that is, $16 \times \frac{9}{12}$, or $16 \times \frac{3}{4}$, or 12 bd. ft.

EXAMPLE II.—How many board feet in a joist 16 ft. long, 3 in. wide, and 2 in. thick?

EXPLANATION.—The area in square feet equals $16 \times \frac{3}{12}$ ($\frac{1}{4}$), or 4 sq. ft.; 4 square feet 1 in. thick = 4 bd. ft., and 2 in. thick = 8 bd. ft.

320. Rule.—*To find the number of board feet in a piece of lumber, 1° find its area in square feet; 2° multiply by its thickness in inches.*

Lumber less than one inch thick is counted the same as lumber one inch thick.

Oral Problems.

How many board feet in:

1. A small board 3 ft. long, 1 ft. wide, and 1 in. thick? 10 similar boards?

2. A joist 16 ft. long, 3 in. wide, and 2 in. thick? 10 similar joists?

3. A beam 18 ft. long, 10 in. wide, and 8 in. thick? 10 similar beams?

4. A board 10 ft. long, 18 in. wide, and 1 in. thick? 10 similar boards?

5. A plank 12 ft. long, 18 in. wide, and 2 in. thick? 10 similar planks?

Written Problems.

Find the number of feet, board measure, in:

1. 1° 20 pieces 1″×12″×16′; 2° 40 pieces 4″×4″×18′; 3° 60 pieces 10″×12″×18′.

2. 1° 80 pieces 6″×8″×12′; 2° 40 pieces 10″×12″×18′; 3° 40 pieces 6″×12″×18′.

3. 1° 540 pieces 1″×10″×16′; 2° 1 200 pieces 1″×5″×16′; 3° 960 pieces 1″×8″×14′.

4. 1° 30 pieces 2″×9″×16′; 2° 18 pieces 2″×8″×12′; 3° 27 pieces 2″×8″×6′.

5. 1° 300 pieces $2\frac{1}{2}$″×12″×16′; 2° 400 pieces $1\frac{1}{4}$″×9″×12′; 3° 100 pieces $\frac{3}{4}$″×6″×16′.

6. At $22 per M bd. ft., find the cost of 12 planks 2″×12″×16′, and of 10 planks 3″×12″×14′.

7. At $25 per M bd. ft., find the cost of 694 pieces 2″×6″×16′.

8. At \$30 per M bd. ft., find the cost of 160 pieces 2″ × 8″ × 14′.

9. At \$28 per M bd. ft., find the cost of 188 pieces 6″ × 6″ × 18′.

10. At \$24 per M bd. ft., find the cost of 628 pieces 6″ × 8″ × 24′.

Boardwork.

The area in square feet (boards being 1 inch thick or less) represents the number of board feet of lumber in fences, floors, sidewalks, partitions, etc.

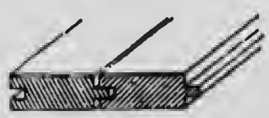
1. A lot is 792 ft. long and 528 ft. wide, How many board feet of lumber do I need to enclose it with a 5-feet fence, if the boards are to be 1 in. thick?

2. In N° 1, how many boards will the lumber dealer deliver, if the boards are 1″ × 10″ × 16′?

3. A sidewalk 300 ft. long and 5 ft. wide is made of planks 2 in. thick. How many board feet of lumber does it contain?

4. In N° 3, find the number of planks, if each is 2″ × 15″ × 12′.

5. A floor is 18 ft. 6 in. long, 14 ft. wide, and 1 in. thick. How many board feet does it contain?



Matched lumber.—If the boards are tongued and grooved, matching them causes a loss. Carpenters find the number of board feet required by adding to the floor space an allowance of $\frac{1}{4}$, as lumber dealers charge for the full width of the lumber.

6. What will be the cost of a matched-wood floor 1 in. thick, for a dining room 30 ft. by 20 ft., the lumber costing \$28 per M, and the laying 65c per 100 sq. ft. of floor space?

7. A contractor undertook to floor a hall 100 ft. by 75 ft., with matched pine 1 in. thick, at the rate of \$3.75 per 100 sq. ft. of floor space. If the lumber cost him \$19.50 per M, and the laying 70c per 100 sq. ft. of floor space, what was the contractor's profit on the job?

REVIEW OF PRACTICAL MEASUREMENTS.

1. A cloth covers a table 1.25 yd. square and laps over .3 yd. on every side. Find its area.

2. A rectangular lot is 630 yd. long, and its area is equal to that of a lot 210 yd. square. How wide is the rectangular lot?

3. The floor of a hall 5 yd. long and 4 yd. wide was covered with tiles .2 yd. square, at $9 per hundred. Find the total expenditure if the laying cost $4.50.

4. Find the length of the fence surrounding a rectangular pavilion 12.5 yd. by 10.25 yd., if the fence is 4.6 yd. from the pavilion on every side.

5. How many slates .3 yd. by .2 yd., $\frac{1}{3}$ exposed to the weather, will be required to cover a roof 14.5 yd. long and 10.3 yd. from eave to ridge, on both sides?

6. A garden is 16.5 yd. square. What area would be left if I surrounded it with a wall .3 yd. thick?

7. Both sides of two doors 2.4 yd. by 1.3 yd. were painted at the rate of 10c a square yard. How much did the work cost?

8. The area of a trapezoid is 962.85 sq. yd., and its altitude 24.5 yd. Find the length of its parallel sides, if one is twice as long as the other.

9. A man put a double sloping roof on a house 12.5 yd. square. How many squares (100 *sq. ft.*) does the roof contain, if the altitude of each gable is 7.4 yd.?

10. A young girl may choose between two kinds of silk for a dress. The first kind is .78 yd. wide and costs 49c a running yard; the other kind is 1.2 yd. wide and costs 65c a running yard. If 12.54 running yards of the first are needed for the dress, how many of the other? How much do the prices differ?

11. Find the cost of plastering a hall 6.44 yd. long, 5.28 yd. wide, and 3.4 yd. high, at 25c a square yard for the walls, and 50c a square yard for the ceiling, allowing for one door 1.95 yd. by 1.05 yd., and 2 windows each 2 yd. by 1.5 yd.

12. Find the cost of whitewashing the walls and ceiling of a room whose length is 7.15 yd., height 4 yd., and width $\frac{3}{5}$ of its length, at 12c a square yard, deducting 19 sq. yd. for openings.

13. There is a yard in the form of a triangle whose altitude is 12.4 yd., and base 15.6 yd. Find the cost of paving it with flagstones at the rate of $1.50 a square yard.

14. What is the area of a triangular field whose sides are respectively 72, 64 and 58 yd.?

15. The roof of a pavilion forms 4 equal triangles whose bases are 8.4 yd. and altitudes 7.6 yd. Find the cost of covering it with zinc at $2.25 a square yard.

16. The area of a triangle equals 3 times that of a square whose side is 45.6 yd. Find its altitude, if its base is 82 yd.

17. Find the cost of plastering the walls and ceiling of a room 16 ft. long, 14 ft. wide, and 8.5 ft. high, at 50c a square yard, allowing for 3 windows each 3 ft. by 6 ft., and 1 door 3 ft. by 7 ft.

18. Find the cost of carpeting the room described in N° 17 with $\frac{3}{4}$ yd. wide American Brussels at $1.75 a running yard, if the strips run lengthwise and there is a waste of 6 in. in each strip (*except the first*) in matching the pattern.

19. Find the number of double rolls of paper (48 ft. by 18 in.) needed to cover the walls and ceiling (*lengthwise*) of the room described in N° 17. Find the cost at $1.60 a double roll.

20. Before being carpeted, the room described in N° 17 was floored with matched lumber. How much lumber was needed, and what did it cost at $55 per M bd. ft.?

21. Find the cost of 48 joists 18 ft. long, 8 in. wide, and 2 in. thick, at $32.50 per M bd. ft.

22. A man puts a sloping A roof on his house, each slope being 34 ft. by 18 ft. How much will the shingles cost at $1.50 a bundle (250), allowing 1 000 shingles per square (100 sq. ft.)?

23. I enclose a garden 15 rd. by 12 rd. with a stone wall 4 ft. high and $2\frac{1}{2}$ ft. thick. Find the cost of the wall at $3.25 a cubic yard, outside dimensions being used in calculating the volume.

24. My lot is 48 yd. by 28 yd. Along its inside borders I made an alley 1.5 yd. wide, which I covered with a layer of sand .04 yd. thick. Find the cost of the sand at $1.05 a cubic yard.

25. Every stroke of a pump furnishes 3 pt. of water. How many will be required to fill up a rectangular cistern 2.7 yd. by 2.4. yd., and 1.5 yd. high, allowing 6.25 gal. to the cubic foot? How long will it take at the rate of 50 strokes a minute?

26. I dug a trench 6.8 yd. long, .8 yd. wide, and .45 yd. deep. The earth removed was evenly spread over a field whose area is 248 sq. yd. How thick is the layer?

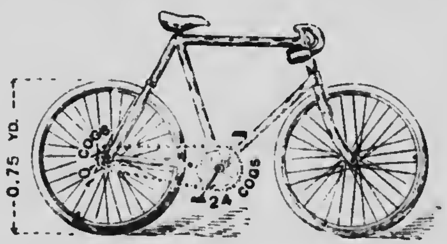
27. A cubic foot of water weighs 62.5 lb., and a cubic foot of ice, $\frac{1}{11}$ less. How many pounds of ice can I place in a box 5 ft. by 4 ft. by 3 ft.?

28. Find the cost of cementing a circular basin 21.6 yd. in diameter, at $1.25 a square yard.

29. All around a circular pond 15.8 yd. in diameter, is a grass border 8.4 yd. wide. Find the area of the pond and that of the border.

30. Find the radius of a circle whose area is equal to the united areas of the following polygons: a rectangle 40 yd. long and 30 yd. wide; a triangle whose altitude is 3 times its base, which is equal to that of the rectangle; a parallelogram whose base and altitude are respectively 50 and 30 yds.

31. The diagonals of a rhombus are respectively 15 and 8.7 yd. Find the radius of a circle of an equal area.

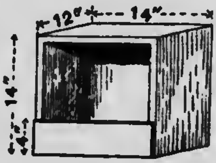


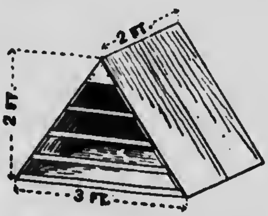
32. A cyclist starts at 9 A.M. for a town 20 mi. off. The hind wheel of his bicycle is .75 yd. in diameter; the rear sprocket has 10 cogs; the front sprocket has 24 cogs, and is operated so as to make 50 revolutions a minute. At what time will the cyclist reach his destination?

NOTE.—The space covered when the front sprocket makes one revolution, is equal to the circumference of the hind wheel multiplied by the *quotient* of the front cogs by the rear cogs.

33. The hind wheel of a boy's bicycle has a radius of 10 in.; the front sprocket has 20 cogs, and the rear sprocket, 9. How many revolutions does the front sprocket make when the boy rides 3 miles?

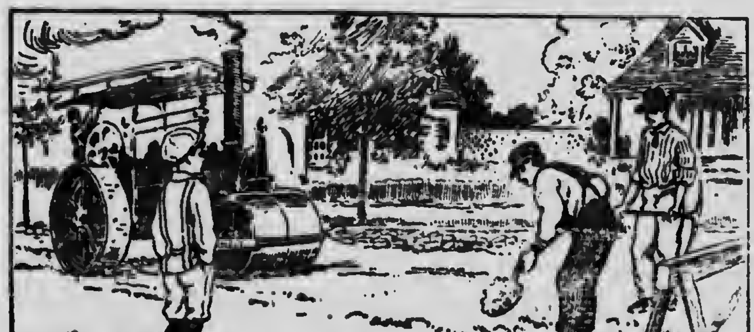
34. Peter raises chickens. This figure represents one of the nests he made for his sitting hens. How many feet of lumber did he employ for one nest? Find the cost of 10 similar nests at \$40 per M bd. ft. (*Use outside dimensions*).





35. Peter made 10 small coops like this one for his hens and chickens. Find their cost at \$40 per M bd. ft. Add \$1.50 for extra expenses.

36. STREET PAVING.—1. A street $\frac{1}{2}$ mi. long and 50 ft. wide is to be paved. First, 14 in. of earth will have to be removed. How many cubic yards? What will this work cost, at 50c a cubic yard?



2. Find the cost of an 8-in. concrete foundation at $1.50 a square yard.

3. It will cost how much more to cover the concrete with asphalt than to cover it with brick, at $3.40 a square yard for asphalt, and $2.50 a square yard for brick?

4. Find the cost of the curbstone for both sides of this street at 82c a running foot, deducting 1 000 ft. from the total length for street crossings.

5. A man has a frontage of 44 ft. on this street. The municipality requires him to pay half the cost of the curbing in front of his house, and half the cost of the paving in front of his house to the middle of the street. How much will he pay if asphalt is used?

Questions on Theory.

Elucidate each answer by examples or drawings.

1. What is the volume of a body? (265).
2. What is a surface? a line? a point? (266).
3. Define a straight line; a curved line; a broken line. (267, 268, 269).
4. When are straight lines parallel? (270).
5. When is a straight line horizontal? vertical? (271, 272).
6. What is a perpendicular line? an oblique line? (273).
7. What is an angle? a right angle? an acute angle? an obtuse angle? (274).
8. What is a polygon? the perimeter of a polygon? (275).
9. What is a 4-sided polygon? What are 3-angled, 5-angled, 6-angled, 8-angled polygons? (276).
10. Name the different 4-sided polygons.
11. What is a square? a rectangle? (277, 278).
12. What is a rhombus? a parallelogram? (279, 280).
13. What is a trapezoid? (281).
14. What is an equilateral triangle? an isosceles triangle? a scalene triangle? a right-angled triangle? (283).
15. When is a polygon said to be regular? (284).
16. Define the base, the altitude, the diagonal of a polygon. (285).
17. How do you find the perimeter of a square? the area? (286, 287).

18. An area is the product of how many dimensions? (287).

19. How do you find the length of the side of a square whose area is given? (288).

20. How do you find the area of the walls of a room? (290).

21. Give the rule for carpeting a room lengthwise, crosswise. (292).

22. How is wall paper sold? (293).

23. If you wanted to paper your class walls, how would you find the number of rolls needed? (294).

24. What is the unit of measurement for roofing? (295).

25. How do you find the area of a triangle whose base and altitude are given? (299).

26. How do you find the altitude of a triangle whose area and base are given? (300).

27. Name the third side of a right-angled triangle. How can it be found when the base and altitude are given? (301).

28. Give the rule for finding the area of a triangle whose three sides are given.

29. How do you find the area of a trapezoid? of a rhombus? (303, 305).

30. What is a circle? a circumference? a radius? a diameter? (306, 307, 308).

31. Is a diameter worth two radii? (308).

32. What is an arc? the chord of an arc? (309).

33. What is a secant? a tangent? (309).

34. How do you find the circumference when the diameter is given? (310).

35. How do you find the area of a circle? (312).

36. Given the area of a circle, how do you find the radius? the diameter? the circumference? (313).

37. How do you find the area of a ring? (315).

39. What is the unit of measurement for stone and concrete work? (317).

40. What is the unit of measurement for brickwork? How many bricks per unit? (317).

41. By what would you divide a volume to find its thickness? (318).

42. What is a board foot? (319).

43. How do you find the number of board feet of lumber in a floor? (320).

44. Given the length and width of a board, can you find the number of board feet it contains? What is missing?

45. A board foot equals what part of a cubic foot?

PERCENTAGE

321. You have already learnt, in decimal fractions, that *per cent* stands for *hundredths*, and that % is the symbol for *per cent.*

Thus, 1 per cent $=1\% = .01 = \frac{1}{100}$; and

$$10\% = 0.10 = \frac{10}{100} = \frac{1}{10}.$$
$$25\% = 0.25 = \frac{25}{100} = \frac{1}{4}.$$
$$100\% = 1.00 = \frac{100}{100} = 1.$$
$$125\% = 1.25 = \frac{125}{100} = \frac{5}{4}.$$
$$2\tfrac{1}{2}\% = 0.02\tfrac{1}{2} = \frac{2\frac{1}{2}}{100} = \frac{1}{40}.$$

The symbol % takes the place of two orders at the right of the point in decimal fractions, and of the denominator 100 in common fractions.

Blackboard Exercises.

Express as decimal fractions:

1. 15%.	6. 35%.	11. $6\frac{2}{3}$%.	16. 120%.	21. $112\frac{1}{2}$%.
2. 18%.	7. 40%.	12. $8\frac{1}{3}$%.	17. 250%.	22. 600 %.
3. 25%.	8. 4%.	13. $12\frac{1}{2}$%.	18. $\frac{1}{4}$%.	23. 101.5%.
4. 30%.	9. 5%.	14. $37\frac{1}{2}$%.	19. $\frac{3}{4}$%.	24. $105\frac{1}{2}$ %.
5. 32%.	10. 8%.	15. $33\frac{1}{3}$%.	20. $\frac{1}{8}$%.	25. .25 %.

NOTE.—Do not confound 25% and .25%.

Express as simplified common or improper fractions:

1. 5%.	6. 60%.	11. 150%.	16. $8\frac{1}{3}$%.
2. 10%.	7. 80%.	12. 175%.	17. $37\frac{1}{2}$%.
3. 20%.	8. 75%.	13. 250%.	18. $87\frac{1}{2}$%.
4. 25%.	9. 100%.	14. 300%.	19. $2\frac{1}{2}$%.
5. 50%.	10. 125%.	15. $12\frac{1}{2}$%.	20. $\frac{1}{8}$%.

Written Exercises.

Express 1° as decimal fractions; 2° as simplified common or improper fractions:

1. 45%	**6.** 105%	**11.** 600%.	**16.** $112\frac{1}{2}$%.
2. 55%.	**7.** 180%.	**12.** 800%.	**17.** $137\frac{1}{2}$%.
3. 65%.	**8.** 275%.	**13.** 350%.	**18.** .25%.
4. 95%.	**9.** $62\frac{1}{2}$%.	**14.** 750%.	**19.** $\frac{1}{4}$%.
5. 39%.	**10.** $1\frac{1}{2}$%.	**15.** 900%.	**20.** $100\frac{1}{2}$%.

Blackboard Exercises.

Express as per cents (%):

1. 0.01.	**6.** 0.35.	**11.** 1.25.	**16.** $0.62\frac{1}{2}$.
2. 0.02.	**7.** 0.625.	**12.** 0.065.	**17.** $0.56\frac{1}{4}$.
3. 0.10.	**8.** 0.875.	**13.** 0.005.	**18.** $0.12\frac{1}{2}$.
4. 0.15.	**9.** 0.375.	**14.** 0.025.	**19.** $0.00\frac{1}{4}$.
5. 0.25.	**10.** 0.625.	**15.** 1.375.	**20.** $0.00\frac{1}{8}$.

NOTE.—First reduce the common fractions to hundredths.

21. $\frac{1}{2}$.	**26.** $\frac{3}{5}$.	**31.** $\frac{1}{3}$.	**36.** $\frac{1}{8}$.
22. $\frac{1}{4}$.	**27.** $\frac{4}{5}$.	**32.** $\frac{2}{3}$.	**37.** $\frac{7}{8}$.
23. $\frac{1}{5}$.	**28.** $\frac{1}{50}$.	**33.** $\frac{1}{6}$.	**38.** $\frac{3}{8}$.
24. $\frac{3}{4}$.	**29.** $\frac{1}{20}$.	**34.** $\frac{1}{40}$.	**39.** $\frac{3}{2}$.
25. $\frac{2}{5}$.	**30.** $\frac{1}{25}$.	**35.** $\frac{1}{16}$.	**40.** $\frac{5}{4}$.

Written Exercises.

Express as per cents (%):

1. $0.33\frac{1}{3}$.	**6.** 0.0025.	**11.** $\frac{1}{15}$.	**16.** $\frac{1}{80}$.
2. $0.66\frac{2}{3}$.	**7.** 0.003.	**12.** $\frac{1}{7}$.	**17.** $\frac{1}{60}$.
3. $0.06\frac{2}{3}$.	**8.** 0.0875.	**13.** $\frac{1}{12}$.	**18.** $\frac{1}{30}$.
4. $0.08\frac{1}{3}$.	**9.** 1.0025.	**14.** $\frac{5}{6}$.	**19.** $\frac{1}{11}$.
5. $0.83\frac{1}{3}$.	**10.** 1.005.	**15.** $\frac{7}{6}$.	**20.** $\frac{5}{12}$.

322. Problems in percentage are simply problems involving fractional relations, either common or decimal. The wording differs a little. What was called the *Base* or the *whole*, is always termed the *Base*.

The *Base* follows the expressions *per cent of*, *per cent as much as*, *per cent more than*, *per cent less than.*

The *Base* is considered as a whole, or 1, or 1.00, or 100%.

What was called the *Fraction*, common or decimal, is here named the *Rate.*

What was called the *Product*, is here named the *Percentage.*

The *Product* obtained by multiplying the Base by 1 less the fraction, is here named the *Difference.*

The *Product* obtained by multiplying the Base by 1 plus the fraction, is here named the *Amount.*

But, *Percentage*, *Difference*, and *Amount* are nothing but the products of the *Base* by a fraction.

After the *Rate* has been changed into a common or decimal fraction, we stand in presence of a case already dealt with in fractional relations.

Some per cent rates are best treated as common fractions. Follows a table of these, which should be thoroughly committed to memory.

$50\% = \frac{1}{2}$	$10\% = \frac{1}{10}$	$33\frac{1}{3}\% = \frac{1}{3}$	$4\% = \frac{1}{25}$
$25\% = \frac{1}{4}$	$20\% = \frac{1}{5}$	$66\frac{2}{3}\% = \frac{2}{3}$	$5\% = \frac{1}{20}$
$75\% = \frac{3}{4}$	$30\% = \frac{3}{10}$	$16\frac{2}{3}\% = \frac{1}{6}$	$2\frac{1}{2}\% = \frac{1}{40}$
$12\frac{1}{2}\% = \frac{1}{8}$	$40\% = \frac{2}{5}$	$83\frac{1}{3}\% = \frac{5}{6}$	$1\frac{1}{4}\% = \frac{1}{80}$
$37\frac{1}{2}\% = \frac{3}{8}$	$60\% = \frac{3}{5}$	$8\frac{1}{3}\% = \frac{1}{12}$	$7\frac{1}{7}\% = \frac{1}{14}$
$62\frac{1}{2}\% = \frac{5}{8}$	$70\% = \frac{7}{10}$	$3\frac{1}{3}\% = \frac{1}{30}$	$14\frac{2}{7}\% = \frac{1}{7}$
$87\frac{1}{2}\% = \frac{7}{8}$	$80\% = \frac{4}{5}$	$6\frac{2}{3}\% = \frac{1}{15}$	$11\frac{1}{9}\% = \frac{1}{9}$
$6\frac{1}{4}\% = \frac{1}{16}$	$90\% = \frac{9}{10}$	$1\frac{2}{3}\% = \frac{1}{60}$	$9\frac{1}{11}\% = \frac{1}{11}$

FIRST CASE.

To find the Percentage when the Base and Rate are given.

323. Rule.—

Base × Rate = Percentage.
Base × (1 — Rate) = Difference.
Base × (1 + Rate) = Amount.

Example I.—What is 25% of $400?

Analysis.—$400, the base, is the quantity considered as a whole, or 100%.

$$
\begin{aligned}
100\%\ (\tfrac{4}{4}) \text{ of } \$400 &= \$400;\\
1\% \quad\ \text{ of } \$400 &= \$4;\\
25\%\ (\tfrac{1}{4}) \text{ of } \$400 &= \$100.
\end{aligned}
$$

Formula.—Base × Rate = Percentage.
Practice.—$400 × 25% = $100, Ans.

Example II.—I have $80; my neighbor has 40% less than I; how many dollars has he?

Analysis.—$80, the base, is the quantity considered as a whole, or 100%. My neighbor has 40% less than I, that is, a sum equal to 60% of my money.

$$
\begin{aligned}
100\%\ (\tfrac{5}{5}) \text{ of my money} &= \$80;\\
1\% \quad\ \text{ of my money} &= \$0.80;\\
60\%\ (\tfrac{3}{5}) \text{ of my money} &= \$48.
\end{aligned}
$$

Formula.—Base × (1 — Rate) = Difference.
Practice.—$80 × 60% = $48, Ans.

Example III.—A man paid $1 000 for a lot, and 25% more than this sum for a house. How much did he pay for the house?

Analysis.—$1 000, the base, is the quantity considered as a whole, or 100%. He paid 25% more for the house than for the lot, that is, a sum equal to 125% of the cost of the lot.

$$
\begin{aligned}
100\%\ (\tfrac{4}{4}) \text{ of the cost of the lot} &= \$1\,000;\\
1\% \quad\ \text{ of the cost of the lot} &= \$10;\\
125\%\ (\tfrac{5}{4}) \text{ of the cost of the lot} &= \$1\,250.
\end{aligned}
$$

Formula.—Base × (1 + Rate) = Amount.
Practice.—$1 000 × 125% = $1 250, Ans.

Note.—Orally: $1 000 plus $\tfrac{1}{4}$ of $1 000 = $1 000 + $250 = $1 250.

Oral Exercises.

Find:

1. 50% of $100.
2. 25% of $400.
3. 75% of $300.
4. 10% of $ 50.
5. 20% of $500.
6. 40 % of $ 100.
7. 60 % of $ 200.
8. 40 % of $1000.
9. $12\frac{1}{2}$% of $ 400.
10. $37\frac{1}{2}$% of $ 800.
11. $62\frac{1}{2}$% of $80.
12. $87\frac{1}{2}$% of $ 8.
13. $16\frac{2}{3}$% of $60.
14. $33\frac{1}{3}$% of $24.
15. $66\frac{2}{3}$% of $30.

What is:

1. 1 % less than $100?
2. 4 % " $ 50?
3. $6\frac{2}{3}$% " $ 15?
4. $3\frac{1}{3}$% " $ 30?
5. $16\frac{2}{3}$% " $ 60?
6. 25% more than $ 4?
7. 50% " $ 10?
8. 10% " $100?
9. 5% " $ 20?
10. 20% " $ 10?

Oral Problems.

1. A workman spends drinking 20% of his annual salary, which is $600. What sum does he annually waste?

2. My property is valued at $50 000. How much do I pay for taxes at the rate of 1% of this valuation?

3. 50% of 1 000 pupils enrolled in a school have a Savings Bank account. Find how many.

4. Of 1 000 convicts 75% were addicted to drinking. Find how many.

5. Of 200 suicides 35% were caused by alcohol. Find how many.

6. John paid $40 for a bicycle, and sold it for 5% less than it cost. Find the selling price.

7. A and B are partners. A has invested $1 000, and B 20% less than A. Find B's investment.

8. One workman earns 42c an hour; find the hourly salary of another workman earning $16\frac{2}{3}$% less than the first.

9. One field contains 40 acres of land, and another $37\frac{1}{2}$% less. Find the area of the latter.

10. A watch bought for $16 was sold for $12\frac{1}{2}$% less than it cost. Find the selling price.

11. I have 450 bu. of wheat, and 20% more oats than wheat. How many bushels of oats have I?

12. My goods cost $800; find the selling price, if I sold the goods for 10% more than they cost.

13. Find t[illegible]mber of my pupils, if it is $16\frac{2}{3}$% greater than 24.

14. Henry earns $16 a week, and Charles, $37\frac{1}{2}$% more than Henry. How much does Charles earn a week?

15. From 1896 to 1906, scarlet fever, measles, typhoid fever, and diphtheria caused 24 000 deaths in Quebec. Find the number of deaths due to tuberculosis, it being $33\frac{1}{3}$% greater.

16. A, B, and C together have $1 000; A has 25% of this sum; B, 20% of it; and C, the remainder. How much has each?

17. A, B, and C are partners, A having invested $2 000; B, 25% more than A; and C, 20% less than A. How much did B and C invest?

18. I own $66\frac{2}{3}$% of a lot valued at $3 000; and I sell 50% of my share. What sum shall I receive?

19. I had 200 sheep; I first sold 25% of them, and then $33\frac{1}{3}$% of the remainder. How many have I left?

20. I paid $20 for a suit of clothes, and 40% more than this sum for an overcoat. How much did I pay for both?

Written Exercises:

Find:

1. $1\frac{1}{4}$% of $720.

2. $1\frac{2}{3}$% of $540.

3. $2\frac{1}{2}$% of $440.

4. $3\frac{1}{3}$% of $570.

5. 4 % of $625.

6. 5 % of $640.

7. $6\frac{2}{3}$% of $225.

8. $7\frac{1}{7}$% of $182.

9. $9\frac{1}{11}$% of $209.

10. $6\frac{1}{4}$% of $208.

11. $18\frac{3}{4}$% of $147.20.

12. $31\frac{1}{4}$% of $137.60.

13. $16\frac{2}{3}$% of $109.50.

14. $8\frac{1}{3}$% of $242.40.

15. $83\frac{1}{3}$% of $307.50.

What is:

16. 1% less than	$146 ?	21. 37½%	more than	$19.20 ?
17. ½%	"	$238 ?	22. 66⅔%	"	$66.30 ?
18. ¼%	"	$548 ?	23. 75 %	"	$19.24 ?
19. ⅛%	"	$312 ?	24. 87½%	"	$16.80 ?
[illegible]%	"	$639 ?	25. 1¼%	"	$12.80 ?

Written Problems.

1. In a school of 400 pupils, 4¼% were in the Sixth Year. How many pupils were there in the Sixth Year?

2. In a class of 48 pupils, 6¼% were absent on the Inspector's visit. How many pupils were absent?

3. Of 1 256 pupils enrolled in a school, 37½% received a prize for perfect assiduity. How many pupils received prizes?

4. Of 560 pupils enrolled in a school, 11¼% received a prize for good manners. How many pupils received prizes?

5. The tax rate being 1½% on the valuation of property, what shall I pay if my house and lot are assessed at $58 312?

6. A man had $15 824 in bank, and withdrew $\frac{1}{16}$ % of this sum to buy a watch. How much did the watch cost?

7. In the United States, 1896, 495 persons were sun-stricken; [illegible]ese, 80% were habitual drinkers. Alcohol may be held [illegible]ble for how many cases?

[illegible]00 children born of alcoholic parents, 21⅞% died in their [illegible]ancy. Find how many.

9. A man died at 30, after having drunk 5 gills of whisky a day for 10 years. If the whisky contained 48% of alcohol, how many gallons of alcohol did this man drink? (*Count two leap years*).

10. A bankrupt owed $25 200; if he paid only 25% of his indebtedness, how much did his creditors lose?

11. A house cost $3 600; $62\frac{1}{2}\%$ of the cost was paid in cash, and a mortgage given for the remainder. What was the amount of the mortgage?

12. A man invested $53 408 in railroad stock. If he lost $6\frac{1}{4}\%$ of his investment, how much money remained?

13. The number of immigrant arrivals in Canada, 1906, was 190 000; in 1907, $34\frac{4}{19}\%$ less. Find the latter number.

14. In 1915, there were 109 000 girls in the Catholic Elementary Schools of Quebec. Find the number of boys, if it was $6\frac{46}{109}\%$ less.

15. In 1915, there were 54 000 boys in the Catholic Model Schools of Quebec. Find the number of girls, if it was $5\frac{5}{9}\%$ less.

16. In 1915, there were 42 000 girls in the Catholic Academic Schools of Quebec. Find the number of boys, if it was $21\frac{3}{7}\%$ less.

17. In all the Quebec schools, Catholic or Protestant, 1915, there were 233 000 girls. Find the number of boys, if it was $2\frac{34}{233}\%$ greater.

18. In Quebec, 1914, the expenditure for Public Education amounted to $8 900 000; what did it amount to in 1915, if it was $29\frac{18}{89}\%$ greater?

19. In 1915, the Quebec hospitals received 18 500 women. Find the number of men received, if it was $18\frac{14}{37}\%$ greater.

20. The sum collected in the Province of Quebec by the St. Vincent de Paul Society, in 1914, amounted to $178 000; what did it amount to in 1915, if it was $57\frac{25}{89}\%$ greater?

21. In 1913 and 1914, the St. Vincent de Paul Society assisted in all 20 500 persons; in 1915, $37\frac{3}{41}\%$ more. Find the latter number.

22. A, B, and C engaged in partnership, B investing $14 848; C, $12\frac{1}{2}\%$ more than B; and A, $37\frac{1}{2}\%$ more than B and C together. How much did C and A invest?

23. A merchant bought 330 yd. of goods, sold 25% of the purchase at one sale, and 20% of the remainder at a second sale. How many yards remained unsold?

24. A, B, C, and D started in business with a joint capital of \$15 000, A investing 20% of it; B, $16\frac{2}{3}\%$ more than A; C, 20% less than B; and D, the remainder. How much did each invest?

25. In 1913, Paul's sales amounted to \$100 000; in 1914, they were 20% less than in 1913; in 1915, 10% greater than in 1914; and in 1916, $27\frac{3}{11}\%$ greater than in 1915. Find the amount of his sales for 1916.

SECOND CASE.

To find the Rate when the Base and Percentage are given.

324. Rule.— $\boxed{\text{Percentage} \div \text{Base} = \text{Rate.}}$

The product and one factor are given; divide the product by the given factor to find the other factor.

EXAMPLE I.—What per cent of 24 is 6?

ANALYSIS.—24 = the base, or 100%;

$1 = \frac{1}{24}$ of the base, or $\frac{1}{24}$ of 100%;
$6 = \frac{6}{24}$ of the base, or $\frac{1}{4}$ of 100%, or 25%.

FORMULA.—Percentage ÷ Base = Rate.
PRACTICE.— 6 ÷ 24 = 25%, Ans.

EXAMPLE II.—What per cent less than 80 is 60?

ANALYSIS.—60 is 20 units less than 80.

80 = the base, or 100%;
$1 = \frac{1}{80}$ of the base, or $\frac{1}{80}$ of 100%;
$20 = \frac{20}{80}$ of the base, or $\frac{1}{4}$ of 100%, or 25%.

FORMULA.—Percentage ÷ Base = Rate.
PRACTICE.— 20 ÷ 80 = 25%, Ans.

EXAMPLE III.—What per cent greater than 40 is 50?

ANALYSIS.—50 is 10 units greater than 40.

40 = the base, or 100%;
1 = $\frac{1}{40}$ of the base, or $\frac{1}{40}$ of 100%;
10 = $\frac{10}{40}$ of the base, or $\frac{1}{4}$ of 100%, or 25%.

FORMULA.—Percentage ÷ Base = Rate.
PRACTICE.— 10 ÷ 40 = 25%, Ans.

Oral Exercises.

What per cent of:

1. 2 is 1?
2. 4 is 1?
3. 4 is 3?
4. 8 is 6?
5. 10 is 7?
6. 80 is 40?
7. 70 is 35?
8. 90 is 45?
9. 60 is 20?
10. 32 is 8?
11. 36 is 27?
12. 64 is 48?
13. 81 is 27?
14. 25 is $12\frac{1}{2}$?
15. 300 is 15?

16. 4 is what per cent of 12? 16? 24? 40? 64?
17. 5 " " of 10? 20? 15? 60? 100?
18. 3 " " of 6? 8? 9? 15? 20?
19. 12 " " of 24? 48? 36? 72? 96?
20. 20 " " of 40? 80? 30? 50? 120?

What per cent less than:

21. 8 is 6?
22. 12 is 9?
23. 20 is 15?
24. 25 is 20?
25. 80 is 60?
26. 20 is 18?
27. 30 is 20?
28. 50 is 40?
29. 75 is 50?
30. 80 is 50?
31. 120 is 100?
32. 150 is 100?
33. 200 is 100?
34. 300 is 100?
35. 5 is $2\frac{1}{2}$?

What per cent more than:

36. 8 is 12?
37. 12 is 15?
38. 20 is 25?
39. 60 is 80?
40. 16 is 24?
41. 80 is 100?
42. 60 is 100?
43. 75 is 100?
44. 100 is 120?
45. 100 is 150?
46. 500 is 600?
47. 600 is 700?
48. 500 is 750?
49. 25 is $37\frac{1}{2}$?
50. $\frac{1}{3}$ is $\frac{1}{2}$?

Oral Problems.

1. A club wins 6 games out of 10; what per cent does it win?

2. My income is $1 200, and I save $200. What per cent do I save?

3. A pupil made 10 mistakes in writing a dictation of 80 words. What per cent of the dictation was incorrectly spelt?

4. A school worth $12 500 is insured for $7 500. What per cent of the valuation does the insurance represent?

5. A collector charged $20 for collecting a bill of $400. What per cent did he charge for collection?

6. I obtained a rebate of $10 on a suit of clothes marked $40. What per cent was the rebate?

7. The assets of a bank are $200 000, and its liabilities, $400 000. What per cent of its indebtedness can it pay?

8. A grocer sold 600 lb. of sugar, and had 400 lb. remaining. What per cent of his stock did he sell?

9. If 20 gal. of water are added to 40 gal. of vinegar, what per cent of the mixture is water? What per cent is vinegar?

10. I had 50 gal. of gasoline. What per cent had I left after spending 10 gal.?

11. Forming a partnership, A invests $5 000; B, $7 000; and C, $8 000. What per cent of the firm's capital does each invest?

12. A battalion numbered 600 men before an engagement, and 500 men after. What was the loss per cent?

13. A merchant sold 5 000 lb. of sugar in 1915, and 4 000 lb. in 1916. What per cent less did he sell in 1916 than in 1915?

14. A and B engage in partnership, A investing $10 000, and B, $15 000. A invests what per cent less than B?

15. John has 15 cows, and Paul has 25. John has what per cent less than Paul?

16. In Quebec, 1904, 30 000 deaths were registered; in 1914, 36 000 deaths. Find the per cent of increase.

17. In 1911, the population of Lachine was 10 000; in 1915, 15 000. Find the per cent of increase.

18. A merchant started in business with a capital of $6 000, and at the expiration of the first year, he found that his capital was $7 000. What was the per cent of his increase?

19. The population of Joliette was 6 000 souls in 1911, and 8 000 in 1915. Find the per cent of increase.

20. A farmer sowed 50 bu. of wheat, and harvested 500 bu. What was the per cent of yield? What was the per cent of increase?

NOTE.—A quotient of 1 = 100%; a quotient of 10 = 1 000%.

Written Exercises.

What per cent of:

1. 160 is 30?	**6.** 800 is 650?	**11.** 480 is 280?
2. 320 is 100?	**7.** 1600 is 1500?	**12.** 720 is 660?
3. 480 is 210?	**8.** 1280 is 80?	**13.** 220 is 20?
4. 640 is 360?	**9.** 240 is 20?	**14.** 630 is 180?
5. 160 is 110?	**10.** 360 is 150?	**15.** 560 is 240?

What per cent less than:

16. 125 is 95?	**21.** 165 is 55?	**26.** 540 is 90?
17. 625 is 600?	**22.** 800 is 790?	**27.** 210 is 30?
18. 770 is 660	**23.** 600 is 590?	**28.** 360 is 210?
19. 875 is 125	**24.** 800 is 780?	**29.** $\frac{3}{4}$ is $\frac{2}{3}$?
20. 936 is 312?	**25.** 240 is 232?	**30.** $\frac{8}{9}$ is $\frac{7}{8}$?

What per cent more than:

31. 480 is 680?	**36.** 720 is 990?	**41.** 480 is 900?
32. 240 is 243?	**37.** 240 is 390?	**42.** 420 is 770?
33. 300 is 305?	**38.** 800 is 1550?	**43.** 900 is 1300?
34. 160 is 190?	**39.** 360 is 630?	**44.** $\frac{2}{3}$ is $\frac{3}{4}$?
35. 320 is 500?	**40.** 640 is 1040?	**45.** $\frac{7}{8}$ is $\frac{8}{9}$?

Written Problems.

1. My annual income is $1 250. What per cent of my income do I save if I lay up $262.50?

2. You have read 78 pages in a book of 300 pages; what per cent of the book have you read?

3. Of 15 problems a pupil figured out 8 correctly. What per cent will he be marked?

4. From a cask containing 25 gal. 2 qt., there escaped 9 gal. $2\frac{1}{4}$ qt. What per cent leaked out?

5. A farm contained 50 A. in wheat, 65 A. in corn, 45 A. in oats, 75 A. in hay, 125 A. in woodland, and 15 A. in orchard. What per cent of the farm was in hay? in orchard? in wheat?

6. In 1910, Canada's copper ore production was 56 million lb., of which British Columbia furnished 36 million lb. What per cent of Canada's production was furnished by British Columbia?

7. If 1 200 lb. of wheat make 960 lb. of flour, what per cent of the wheat does the flour represent?

8. With 960 lb. of flour a baker makes 1 $267\frac{1}{5}$ lb. of bread. The weight of the bread represents what per cent of the weight of the flour?

9. The director of a Paris jail on a record of 3 000 convicts counted 2 115 drinkers. What per cent did the drinkers represent?

10. A man having \$5 000, invests \$2 750 in a farm, \$1 875 in town property, and the remainder in bank stock. What per cent is in bank stock?

11. My house is worth \$3 500 and rents for \$240 a year. If I pay \$30 for taxes and insurance, what per cent of the valuation of my house does the net income represent?

12. In Holland, 1882, there were 43 000 saloons. In 1910, owing to the salutary steps taken by the government, there were only 10 000. Find the per cent of decrease.

13. The rapids of Lachine represent 400 000 horse power; those of Coteau, Cèdres and Cascades represent 960 000 horse power. What per cent less in the first than in the second case?

14. The hydraulic forces developed in Ontario, 1910, amounted to 400 000 horse power; in Quebec, 1910, only 190 000 horse power. What per cent less in Quebec than in Ontario?

15. In 1915, Canada produced 520 million bu. of oats, and the United States 1 540 million bu. Canada produced what per cent less than the United States?

16. In 1913, Canada produced 230 million bu. of wheat; in 1915, 380 million bu. Find the per cent of increase.

17. In 1913, the United States produced 760 million bu. of wheat; in 1915, 1 000 million bu. Find the per cent of increase.

18. In 1914, Russia produced 575 million bu. of wheat, and 765 million bu. in 1915. Find the per cent of increase.

19. In 1915, Quebec produced 4 000 000 lb. tobacco, and Ontario, 4 950 000 lb. Ontario produced what per cent more than Quebec?

20. Quebec's cheese and butter production in 1900 was worth 20 million dollars; in 1910, 31 million dollars. Find the per cent of increase.

21. In 1901, the population of Montreal and its suburbs was 285 000; in 1912, 600 000. Find the per cent of increase.

22. In Canada, 1914, 290 million cigars were smoked; in 1915, 240 million. Find the per cent of decrease.

23. In Canada, 1910, 450 million cigarettes were smoked; in 1911, 585 million. Find the per cent of increase.

24. In Canada, 1913, the consumption of spirits was 5 000 000 gal.; in 1914, 4 750 000 gal. Find the per cent of decrease.

25. In Canada, 1914, the consumption of beer was 56 million gal.; in 1915, 48 million gal. Find the per cent of decrease.

THIRD CASE.

To find the Base when the Percentage and Rate are given.

325. Rule.—

Percentage	÷	Rate	=	Base.
Difference	÷	(1 — Rate)	=	Base.
Amount	÷	(1 + Rate)	=	Base.

The Percentage, the Difference, and the Amount are products; divide them by the given factor (*the Rate*) to find the Base.

EXAMPLE I.—If 25% of my money is \$100, how much money have I?

ANALYSIS.—My money is the base, or 100%.

25% ($\frac{1}{4}$) of my money = \$100;
1% of my money = \$4;
100% ($\frac{4}{4}$) of my money = \$400.

{FORMULA.—Percentage ÷ Rate = Base.
{PRACTICE.— \$100 ÷ 25% = \$400, Ans.

EXAMPLE II.—A has \$40, which is 20% less than B has. How much has B?

ANALYSIS.—B's money is the base, or 100%. A has 20% less than B, that is, a sum equal to 80% of B's money.

80% ($\frac{4}{5}$) of B's money = \$40;
1% of B's money = \$0.50;
100% ($\frac{5}{5}$) of B's money = \$50.

{FORMULA.—Difference ÷ (1—Rate) = Base.
{PRACTICE.— \$40 ÷ 80% = \$50, Ans.

EXAMPLE III.—A has \$30, which is 25% more than B has. How much has B?

ANALYSIS.—B's money is the base, or 100%. A has 25% more than B, that is, a sum equal to 125% of B's money.

$$125\%\ (\tfrac{5}{4}) \text{ of B's money} = \$30;$$
$$1\% \text{ of B's money} = \$0.24;$$
$$100\%\ (\tfrac{4}{4}) \text{ of B's money} = \$24.$$

$\begin{cases} \text{FORMULA.—Amount} \div (1+\text{Rate}) = \text{Base.} \\ \text{PRACTICE.—}\ \$30 \div 125\% = \$24, \text{ Ans.} \end{cases}$

NOTE.—When you can make an equation between a rate and a quantity, always divide the quantity by the rate.

Oral Exercises.

Find the number of which:

1. 20% = 10.	**6.** 5% = 7.	**11.** 12½% = 8.
2. 25% = 15.	**7.** 75% = 6.	**12.** 37½% = 12.
3. 50% = 30.	**8.** 40% = 10.	**13.** 62½% = 20.
4. 10% = 9.	**9.** 60% = 9.	**14.** 87½% = 28.
5. 4% = 3.	**10.** 80% = 16.	**15.** 2½% = 4.

What number decreased:

16. 25% = 6?	**21.** 40% = 15?	**26.** 12½% = 21?
17. 50% = 5?	**22.** 60% = 18?	**27.** 33⅓% = 30?
18. 10% = 18?	**23.** 80% = 21?	**28.** 16⅔% = 45?
19. 20% = 12?	**24.** 4% = 24?	**29.** 75 % = 22?
20. 30% = 14?	**25.** 5% = 38?	**30.** 37½% = 25?

What number increased:

31. 25% = 25?	**36.** 40 % = 28?	**41.** 16⅔% = 28?
32. 20% = 24?	**37.** 33⅓% = 48?	**42.** 12½% = 27?
33. 10% = 22?	**38.** 66⅔% = 50?	**43.** 8⅓% = 26?
34. 50% = 21?	**39.** 37½% = 33?	**44.** 75 % = 21?
35. 60% = 40?	**40.** 62½% = 39?	**45.** 87½% = 30?

Oral Problems.

1. Find Paul's salary, if \$14 equal 25% of it.

2. How much money have I, if \$15 equal 37½% of my money?

3. I sold a horse for \$100, which was 80% of what it cost. Find the cost.

4. I sold a horse for 80% of its value, and thus received \$120. What was the horse worth?

5. I save \$6 a week, which is 37½% of my wages. Find my wages.

6. I sold 41 sheep, which was 33⅓% of my flock. What was the whole flock worth at \$5 a head?

7. I lost 16⅔% of my money and have \$15 left. How much had I at first?

8. A, B, and C are partners; A owns 25% of the capital; B, 37½% of it; and C, the remainder. How much have A and B if C has \$3 000?

9. After having spent 37½% and 12½% of my money, I have \$400 remaining. How much had I at first?

10. After obtaining a rebate of 5%, I settle a bill with \$95. What was the amount of the bill?

11. Viator has \$30, which is 16⅔% less than Vincent has. How much has Vincent?

12. Agnes has \$40, which is 33⅓% less than Gertrude has. How much has Gertrude?

13. I sold my house for \$4 000, which was 20% less than its value. Find its value.

14. I sold cloth at \$1 a yard, which was 33⅓% more than it cost. Find its cost.

15. I burned 15 tons of coal this year, which was 25% more than last year's consumption. Find last year's consumption.

16. A farmer bought 14 cows, which was 40% more than the number he already had. How many cows had the farmer then?

17. I settled a bill with \$25, which was 25% more than the sum due. Find the amount of overcharge.

18. Edmund and Edward together have 90 books; how many has each, if Edmund has 25% more than Edward?

19. A man spent \$10, which was $33\frac{1}{3}\%$ less than what he had left. How much had he at first?

20. Patrick purchased 10 cows, which was $16\frac{2}{3}\%$ less than the number he already had. How many cows had he then?

Written Exercises.

Find the sum of which:

1. 8% = \$ 2.40.	**6.** 43% = \$860.	**11.** 250 % = \$755.
2. 12% = \$ 3.60.	**7.** $\frac{1}{4}$ % = \$ 48.	**12.** $128\frac{4}{7}\%$ = \$900.
3. 6% = \$72.	**8.** $\frac{1}{2}$ % = \$321.	**13.** $116\frac{2}{3}\%$ = \$833.
4. 32% = \$360.	**9.** $\frac{8}{9}$ % = \$424.	**14.** $266\frac{2}{3}\%$ = \$848.
5. 45% = \$72.	**10.** $\frac{3}{8}$ % = \$ $4\frac{1}{2}$.	**15.** $56\frac{1}{4}\%$ = \$378.

What number decreased:

16. 16 % = 168?	**21.** $18\frac{3}{4}\%$ = 325?	**26.** $8\frac{1}{3}\%$ = 352?
17. 45 % = 55?	**22.** 23 % = 308?	**27.** 17% = 249?
18. 18 % = 246?	**23.** 95 % = 25?	**28.** $8\frac{1}{3}\%$ = 990?
19. $62\frac{1}{2}\%$ = 27?	**24.** 10 % = $4\frac{1}{2}$?	**29.** $\frac{1}{2}\%$ = 1 990?
20. 50 % = $22\frac{1}{2}$?	**25.** $12\frac{1}{2}\%$ = 350?	**30.** $\frac{3}{4}\%$ = 3 970?

What number increased:

31. 17 % = 585?	**36.** 50 % = 690?	**41.** 15 % = \$4 025?
32. 8 % = 324?	**37.** 250% = 105?	**42.** 27 % = \$5 969?
33. 30 % = 260?	**38.** $16\frac{2}{3}\%$ = 1050?	**43.** 500% = \$ 60?
34. $37\frac{1}{2}\%$ = 550?	**39.** 70 % = 510?	**44.** 1% = \$90.90?
35. $\frac{3}{4}\%$ = $201\frac{1}{2}$?	**40.** 35 % = 405?	**45.** $\frac{1}{16}\%$ = \$16.01?

Written Problems.

1. I withdrew \$1 550 from bank, which was $83\frac{1}{3}$ % of my deposit. What was my deposit?

2. I rent a house for \$752 per annum, which is 16% of its value. Find the value of the house.

3. I annually save $31\frac{1}{4}$% of my salary, and this permits me to acquit a debt of \$1 875 in 6 years. What is my yearly income?

4. A house is rented for \$900 per annum; when the annual expenses are \$250, the net income represents $6\frac{1}{4}$% of the value of the house. Find the value of the house.

5. If 24 % of a contract can be executed in 252 days, in what time can the whole contract be executed?

6. I was granted a rebate of \$25.20, which was $3\frac{1}{3}$% of the amount of the bill. What was the amount of the bill?

7. Find the cost of goods sold at a profit of \$75, if the profit represents $6\frac{1}{4}$% of the cost.

8. I paid a tax of \$25, which was $\frac{1}{4}$% of the valuation of my property. Find the valuation of my property.

9. I sold a lot for \$2 750, which was $33\frac{1}{3}$% less than it cost. How much did the lot cost?

10. I sold my watch for 10% less than it cost, thereby receiving \$27.36. Find the cost of the watch.

11. In 1913, the United States produced 330 million bu. of potatoes, which was $21\frac{3}{7}$% less than in 1912. Find the yield for 1912.

12. In 1915, Canada's imports of merchandise for home consumption amounted to 470 million dollars, which was $27\frac{9}{13}$% less than in 1913. Find the amount for 1913.

13. In 1915, Canada's exports of home produce amounted to 670 million dollars, which was $52\frac{3}{11}$% more than in 1913. Find the amount of exports for 1913.

14. According to the *London Economist*, the first two years of the war of 1914 cost to the Allies 2 700 million dollars, which was $58\frac{14}{17}$% more than they cost the Teutons. Find the expenses of the Teutons.

15. In England, 1812, a bushel of wheat sold for \$3.85, which was 40% more than in 1805. What was the price of a bushel of wheat in 1805.

16. In England, 1912, a bushel of wheat sold for \$1.05, which was $16\frac{2}{3}\%$ more than in 1905. What was the price of a bushel of wheat in 1905?

17. In England, 1915, a bushel of wheat sold for \$1.60, which was $52\frac{8}{21}\%$ more than in 1914. What was the price of a bushel of wheat in 1914?

18. After spending 25% and 20% of my money, I have \$31.02 left. How much had I at first?

19. A man willed $33\frac{1}{3}\%$ of his fortune to his wife, 25% of it to his children, and the remainder, \$10 250, to charitable institutions. What was the amount of his fortune?

20. After spending 50% of my money, and 25% of the remainder, I have \$168.75 left. How much had I at first?

21. A merchant owning $66\frac{2}{3}\%$ of the capital of a firm, sells 40% of his share for \$8 000. Find the value of what he has left.

22. I bought two houses for \$18 816, paying 24% more for one than for the other. Find the price of each.

23. A merchant's sales for 1915 and 1916 were \$17 600, the sales for 1916 being $16\frac{2}{3}\%$ less than the sales for 1915. Find the sales for 1916.

24. I withdraw 60% of my bank deposit, and spend $16\frac{2}{3}\%$ of the money thus withdrawn in the purchase of a bill of goods amounting to \$54. How much had I at first in bank?

25. A merchant increased his bank account 25%; a month after he again increased his bank account 20%; then he withdrew 8% of the whole. How much did he have in bank at first if he now has \$651.36?

REVIEW OF PERCENTAGE.

Oral Exercises.

	A	B	C	D	E	F	G
1.	100	120	80	20 %	25 %	20	10
2.	50	60	45	25 %	20 %	15	20
3.	60	75	50	50 %	$16\frac{2}{3}$%	20	18
4.	20	24	15	60 %	40 %	6	7
5.	40	45	36	$12\frac{1}{2}$%	10 %	12	15
6.	50	56	42	60 %	$12\frac{1}{2}$%	8	5
7.	54	72	48	$12\frac{1}{2}$%	$11\frac{1}{9}$%	9	6
8.	44	55	33	10 %	50 %	4	11
9.	30	42	28	40 %	30 %	15	9
10.	80	90	75	$12\frac{1}{2}$%	$16\frac{2}{3}$%	2	6

Written Exercises.

	A	B	C	D	E	F	G
1.	160	192	144	20 %	$12\frac{1}{2}$%	60	16
2.	144	180	126	$12\frac{1}{2}$%	10 %	36	24
3.	324	360	288	$16\frac{2}{3}$%	20 %	27	108
4.	300	360	270	$37\frac{1}{2}$%	50 %	50	30
5.	180	210	140	25 %	$14\frac{2}{7}$%	60	48
6.	540	630	450	$66\frac{2}{3}$%	$12\frac{1}{2}$%	120	180
7.	400	450	350	$12\frac{1}{2}$%	25 %	100	50
8.	120	144	96	20 %	25 %	10	40
9.	144	168	120	$33\frac{1}{3}$%	$16\frac{2}{3}$%	36	18
10.	360	432	288	20 %	$16\frac{2}{3}$%	60	36

NOTE.—Apply the following questions to each horizontal line of the preceding tables.

1. The number in column B is what per cent greater than the number in column A?

2. The number in column C is what per cent less than the number in column A?

3. If column A is the Base, and column D the Rate, what is the Amount?

4. If column A is the Base, and column E the Rate, what is the Difference?

5. If column A is the Base, and column F the Percentage, what is the Rate?

6. The number in column G is what per cent of the number in column A?

7. If column B is the Base, and column C the Difference, what is the Rate?

8. If column C is the Base, and column B the Amount, what is the Rate?

9. If column B is the Amount, and column D the Rate, what is the Base?

10. If column B is the Difference, and column E the Rate, what is the Base?

Written Problems.

THE HOLY CHILDHOOD IN 1913.

1. The contributions to this genuine work of charity, whose directors are in Paris, amounted to $824 000 in 1913. Find France's offering, it being $21\frac{1}{8}\%$ of the total amount.

2. Italy and Belgium together furnished $178 000. Find the offering of each country, if Belgium's share was $22\frac{1}{2}\%$ greater than Italy's.

3. Germany contributed $321 360. What per cent of the whole amount was that?

4. Austria-Hungary poured in $40 000, and Holland, $32 000. What per cent less than Austria-Hungary did Holland contribute?

5. The United States gave $9\frac{3}{8}\%$ less than Holland. What amount was that?

6. The foregoing countries together gave what per cent of the total amount? What per cent of the total amount did each of these countries furnish? (*Extend rates to one-decimal place only*).

7. In 1913, through its missionaries, the Holy Childhood baptized 425 000 heathen babies, and this number was $19\frac{1}{21}\%$ less than the number of children evangelized and instructed. Find the latter number.

8. In 1913, there were 185 000 boys in the Quebec Catholic Public Schools, and this number was $7\frac{1}{2}\%$ less than the number of girls. How many pupils were there in all?

9. If each one of these pupils had contributed 1 cent a month to the Holy Childhood, what would have been Quebec's offering in 1913?

10. The Chinese babies bought and baptized by the good missionaries represent an average outlay of 10c each. How many heathen children might the Catholic boys and girls of Quebec have saved in 1913? (1).

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(1) "Small children with their small monthly offering and daily prayer may conquer the greatest empire in the world and submit it to Christ Jesus." (*Lacordaire*).

MISCELLANEOUS PROBLEMS.

11. The United States Civil War destroyed 205 000 lives; every year, in the same country, tuberculosis destroys $78\frac{2}{41}\%$ as many lives. How many victims does tuberculosis annually make?

12. In 1909, a certain county of the Province of Quebec spent \$120 000 for religious, municipal, and educational purposes, and 145% more than that amount for alcoholic liquors. Find the amount wasted on alcohol.

13. From 1900 to 1910, the number of our immigrant arrivals from the United Kingdom was 560 000. Find the number of immigrant arrivals from France, if it was $97\frac{1}{2}\%$ less.

14. In 1755, the number of Acadians was 18 000, and in 1910, $816\frac{2}{3}\%$ greater. How many Acadians were there in 1910?

15. In 1902, Canada exported 12 million dozen of eggs; in 1909, 600 000 dozen. The latter number is what per cent of the former?

16. In Canada, 1901, there were 2 400 000 milch cows; in 1909, 2 850 000. Find the per cent of increase.

17. In 1900, Canada exported 25 million pounds of butter; in 1911, 3 million pounds. Find the per cent of decrease.

18. In 1901, the cheese and butter production of Canada amounted to 66 million dollars; in 1910, 98 million dollars. What was the per cent of increase?

19. In 1912, the mines of Canada produced 110 000 tons of asbestos; in 1910, 75 000 tons. Find the per cent of increase.

20. The value of the annual consumption of alcohol, the world over, is 2 450 million dollars; of bread, 350 million dollars. What per cent more is spent for alcohol than for bread?

21. In 1913, Canada imported from France and the United States 465 million dollars' worth of merchandise. If our imports from France were $3\frac{1}{8}\%$ of our imports from the United States, find the amount of the latter.

22. In 1913, our imports from Italy amounted to \$1 800 000, which was $12\frac{1}{2}\%$ more than our imports from Austria-Hungary. Find the amount of the latter.

23. In 1913, our imports from the United Kingdom were 140 million dollars, being an increase of $27\frac{3}{11}\%$ over 1911. Find the amount for 1911.

24. In 1913, our imports from Cuba aggregated \$2 800 000, which was $12\frac{1}{2}\%$ less than our imports from Japan. Find the amount of our imports from Japan.

25. In 1913, our imports from Argentine Republic amounted to \$4 200 000, which was $31\frac{1}{4}\%$ more than our imports from Mexico. Find the amount of the latter.

26. In 1912 and 1913, Canada imported from Germany 25 million dollars' worth of merchandise, and the amount for 1913 shows an increase of $27\frac{3}{11}\%$ over the amount for 1912. Find the amount for 1913.

27. In 1913, our imports from Spain and Brazil aggregated \$2 520 000. If the imports from Spain were $6\frac{2}{13}\%$ less than those from Brazil, find the amount of the latter.

28. In 1913, Canada imported from Japan \$1 115 000 worth of tea and rice. Find the value of the rice, if the tea was worth $97\frac{1}{3}\%$ more than the rice.

29. In 1913, Canada imported \$45 000 000 worth of wool and cotton. If the cotton was worth $39\frac{2}{7}\%$ less than the wool, find the value of each.

30. In 1913, Canada imported \$1 170 000 worth of goods from Switzerland, Belgium and Holland. If the goods imported from Switzerland and Belgium were respectively worth $28\frac{1}{8}\%$ and $37\frac{1}{2}\%$ more than those imported from Holland, find the value of the goods imported from each of these countries.

Questions on Theory.

1. Is there a difference between 25% and $\frac{1}{4}$ of a sum of money?

2. Is there a difference between 84% and .84 of a number?

3. How do you find the Percentage when the Base and Rate are given? (323).

4. How do you find the Base when the Percentage and Rate are given? (325).

5. How do you find the Rate when the Base and Percentage are given? (324).

6. How do you find the Amount when the Base and Rate are given? (323).

7. How do you find the Difference when the Base and Rate are given? (323).

8. How do you find the Base when the Amount and Percentage are given?

9. How do you find the Base when the Difference and Percentage are given?

10. How do you find the Rate when the Amount and Percentage are given?

11. Given the Difference and Percentage, ho do you find the Rate?

12. Given the Amount and the Rate, how do you find the Base? (325).

13. Given the Difference and the Rate, how do you find the Base? (325).

14. Given the Amount and the Rate, how do you find the Percentage?

15. Given the Difference and the Rate, how do you find the Percentage?

APPLICATIONS OF PERCENTAGE

Percentage has many applications in business transactions. The following will be dealt with: *Profit and Loss, Commercial Discount, Commission, Interest, Bank Discount, Taxes, Insurance, Stocks and Bonds.*

In the different cases, care should be taken to see clearly the *base* upon which the percentage is reckoned. *Analysis* should be used in the oral work, and occasionally in the written work. When the subject has been thoroughly mastered, *formulas* may be used to advantage.

PROFIT AND LOSS

326. The profits or losses resulting from purchases and sales are usually expressed as per cents of the cost.

Let us apply the principles of percentage to purchases and sales.

The Base is the *cost price.*

The Rate is the *per cent of profit* or *loss.*

The Percentage is the *profit* or *loss.*

The Amount is the cost price plus the profit, or the *selling price.*

The Difference is the cost price minus the loss, or the *selling price.*

FIRST CASE.

Given the cost price and the rate of profit or loss, to find the profit or loss, or the selling price.

327. Rule.—

$$\text{Cost Price} \times \begin{cases} \text{Rate of profit} = \text{Profit.} \\ \text{Rate of loss} = \text{Loss.} \end{cases}$$

When there is a *Profit:*

Cost Price + Profit = Selling Price.

Cost Price × (1 + Rate) = Selling Price.

When there is a *Loss:*

Cost Price — Loss = Selling Price.

Cost Price × (1 — Rate) = Selling Price.

Example I.—(*a*) A book costs \$5. I sell it at 20% profit. Find the profit. (*b*) A book costs \$5. I sell it at 20% loss. Find the loss.

Operation (*a*).

$0.20 × 5 = $1, profit.

Operation (*b*).

$5 × ⅕ = $1, loss.

Explanation (*a*).—At 20%, the profit on \$1 is \$0.20, and the profit on \$5 is 5 times \$0.20, or \$1.

Explanation (*b*).—At 20%, the loss equals $\frac{20}{100}$, or ⅕ of the cost price (\$5), or \$1.

EXAMPLE II.—A book costs \$5. I sell it at 20% profit. Find the selling price.

OPERATION (*a*).
\$1.20×5 = \$6, selling price.

EXPLANATION (*a*). — At 20% profit, the selling price of \$1 is \$1.20, and the selling price of \$5 is 5 times \$1.20, or \$6.

OPERATION (*b*).
\$5 × $\frac{6}{5}$ = \$6, selling price.

EXPLANATION (*b*). — At 20% profit, the selling price is 100% plus 20%, or 120% of the cost price; 120% of the cost price = $\frac{6}{5}$ of \$5, or \$6.

EXAMPLE III.—A book costs \$5. I sell it at 20% loss. Find the selling price.

OPERATION (*a*).
\$0.80×5 = \$4, selling price.

EXPLANATION (*a*). — At 20% loss, the selling price of \$1 is \$0.80, and the selling price of \$5 is 5 times \$0.80, or \$4.

OPERATION (*b*).
\$5 × $\frac{4}{5}$ = \$4, selling price.

EXPLANATION (*b*). — At 20% loss, the selling price is 100% minus 20%, or 80% of the cost price; 80% of the cost price = $\frac{4}{5}$ of \$5, or \$4.

Oral Exercises.

Find 1° the profit or loss; 2° the selling price when there is a profit; 3° the selling price when there is a loss:

	COST.	RATE.		COST.	RATE.		COST.	RATE.
1.	\$30	20 %	11.	\$16	$18\frac{3}{4}$%	21.	\$ 80	$1\frac{1}{4}$%
2.	\$50	10 %	12.	\$50	40 %	22.	\$ 60	$1\frac{2}{3}$%
3.	\$75	$6\frac{2}{3}$%	13.	\$60	60 %	23.	\$ 80	$2\frac{1}{2}$%
4.	\$60	50 %	14.	\$40	80 %	24.	\$ 90	$3\frac{1}{3}$%
5.	\$36	75 %	15.	\$75	$33\frac{1}{3}$%	25.	\$ 75	4 %
6.	\$48	25 %	16.	\$28	$14\frac{2}{7}$%	26.	\$100	$\frac{1}{2}$%
7.	\$56	$12\frac{1}{2}$%	17.	\$45	$66\frac{2}{3}$%	27.	\$200	$\frac{1}{4}$%
8.	\$24	$37\frac{1}{2}$%	18.	\$24	$16\frac{2}{3}$%	28.	\$800	$\frac{1}{4}$%
9.	\$16	$87\frac{1}{2}$%	19.	\$60	$83\frac{1}{3}$%	29.	\$3.20	$\frac{1}{10}$%
10.	\$32	$6\frac{1}{4}$%	20.	\$48	$8\frac{1}{3}$%	30.	\$ 10	1 %

Oral Problems.

1. An article cost \$20. I sold it at 10% profit. What was the profit?

2. A hat cost \$5. I sold it at 20% profit. What was the profit?

3. If I pay \$4 000 for a house, and sell it gaining $12\frac{1}{2}$%, what is my profit?

4. I bought a book for \$1.50, and sold it at an advance of $33\frac{1}{3}$%. What was the gain?

5. I purchased a horse for \$150, and sold it at a loss of 10%. Find the loss.

6. A watch which cost \$50 was sold at a loss of 5%. What was the loss?

7. A lot purchased for \$1 200 was sold at a loss of $66\frac{2}{3}$%. Find the loss.

8. I paid \$4 000 for a farm, and sold it at $12\frac{1}{2}$% loss. What was my loss?

9. Cloth is bought at \$3 a yard, and sold at a profit of $16\frac{2}{3}$%. What is the selling price?

10. A horse that cost \$200 was sold at an advance of 25%. What was the selling price?

11. A gross of penholders cost \$4.32; it was sold at a profit of $12\frac{1}{2}$%. Find the selling price.

12. At what price must a house be sold to bring in a profit of 20%, if it cost \$3 500?

13. A farm was bought for \$4 200, and sold at a gain of $16\frac{2}{3}$%. What was the selling price?

14. A bag of potatoes cost \$2.40, and was sold at a loss of $12\frac{1}{2}$%. Find the selling price.

15. Old-fashioned hats were sold at a loss of 40%. Find the selling price if the cost price was \$125.

Written Exercises.

Find 1° the profit or loss; 2° the selling price when there is a profit; 3° the selling price when there is a loss:

	Cost.	Rate.		Cost.	Rate.		Cost.	Rate.
1.	\$ 75	20 %	11.	\$ 189	$11\frac{1}{9}$%	21.	\$ 38	88 %
2.	\$ 96	$33\frac{1}{3}$%	12.	\$16.80	$62\frac{1}{2}$%	22.	\$ 24	$22\frac{1}{2}$%
3.	\$ 115	15 %	13.	\$ 42	39 %	23.	\$1.76	$31\frac{1}{4}$%
4.	\$ 227	19 %	14.	\$ 37	36 %	24.	\$5.28	$43\frac{3}{4}$%
5.	\$ 356	21 %	15.	\$ 1.24	75 %	25.	\$7.04	$56\frac{1}{4}$%
6.	\$ 132	28 %	16.	\$ 9.60	$87\frac{1}{2}$%	26.	\$1.44	$68\frac{3}{4}$%
7.	\$6.20	5 %	17.	\$ 352	$\frac{1}{16}$%	27.	\$1.28	$81\frac{1}{4}$%
8.	\$8.80	$12\frac{1}{2}$%	18.	\$ 69	3 %	28.	\$2.24	$93\frac{3}{4}$%
9.	\$3 25	8 %	19.	\$ 43	27 %	29.	\$1.26	$7\frac{1}{7}$%
10.	\$ 16	$\frac{1}{4}$%	20.	\$ 6.50	90 %	30.	\$7.70	$9\frac{1}{11}$%

Written Problems.

1. How much did I gain by selling at a profit of 8% a house that had cost \$3 250?

2. I bought 3 lots for \$1 200, \$1 500, and \$1 800 respectively. I sold the first at 8% profit, the second at $16\frac{2}{3}$% profit, and the third at 12% profit. Find my total gain.

3. A farmer bought 70 acres of land for \$3 570, and sold the whole at 20% profit. What was his profit per acre?

4. I sold at 26% profit a lot of goods which had cost \$2 205. Find my gain.

5. What is the loss on goods which cost \$2 948.13 and are sold at $33\frac{1}{3}$% loss?

6. I bought 1 280 yd. of calico at 15c a yard, and sold the entire lot at $2\frac{1}{3}$% loss. Find my loss.

7. I had \$715.28 worth of old-fashioned goods, and sold them at $12\frac{1}{3}$% loss. Find my loss.

8. I bought 500 yd. of muslin at 9c a yard. Find my loss, if I sold the entire quantity at a loss of $1\frac{1}{3}$%.

9. A merchant buys cloth at $1.50 a yard. At what price must he sell it to gain $16\frac{2}{3}$%?

10. A drover buys 45 sheep at $6 each; he sells 20 at $6.50 each; at what price each must he sell the remainder so as to gain 20% on the whole?

11. A merchant bought 200 yd. of cloth at $3.25 a yard. How much did he receive per yard if he sold the whole at 8% profit?

12. A dealer buys 160 yd. of lawn at 15c a yard, and sells it at a loss of 5%. Find the total selling price.

13. A lot that cost $1 275 was sold at a loss of $33\frac{1}{3}$%. How much was received for it?

14. A grocer bought 16 bbl. of sugar, each containing 315 lb., at $6\frac{1}{4}$c a pound. Find the total selling price, if the sugar was sold at $6\frac{2}{3}$% loss.

15. Three houses were bought for $4 000, $5 500, and $7 500 respectively. If the first was sold at a gain of $12\frac{1}{2}$%, the second at a loss of $9\frac{1}{11}$%, and the third at a loss of $2\frac{1}{2}$%, what were the net loss, and the total amount received for the houses?

NOTE I.—Buying expenses, such as commission. freight, etc., must be added to the amount of a purchase to form the *entire cost*, which is then taken as the *base*.

NOTE II.—Selling expenses must be deducted from the amount of a sale to form the *net proceeds*.

NOTE III.—The difference between the net proceeds and the entire cost is the *profit* or the *loss*.

16. I bought 160 bu. of wheat at $1.50 a bushel and paid $10 for freight. At what price must I sell the entire quantity to gain $37\frac{1}{2}$%?

17. I bought a house for $5 600, paid $400 additional for repairs, and then sold it at a profit of $16\frac{2}{3}$%. Find the selling price.

18. A merchant bought 2 400 plates at $4 a hundred, and paid $4 for freight. What must be the selling price per dozen,

if 48 plates were broken, and if the merchant wishes to realize a profit of $27\frac{3}{8}\%$?

19. A farmer's wife bought 5 dozen of chickens at 30c a chicken and spent $12 raising them. If 16 chickens died, at what price must she sell each of the remaining chickens to realize a profit of $46\frac{2}{3}\%$ on the whole?

20. A merchant bought 200 bags of potatoes at $1.75 each, and paid $25 for incidental expenses. What price must he ask per bag to realize a profit of $33\frac{1}{3}\%$ on the transaction?

SECOND CASE.

Given the cost price and the profit or loss, or the selling price, to find the rate.

328. Rule.—

Profit ÷ Cost Price = Rate of profit.
Loss ÷ Cost Price = Rate of loss.

or else:

(Selling Price — Cost Price) ÷ Cost Price = Rate of profit.

(Cost Price — Selling Price) ÷ Cost Price = Rate of loss.

EXAMPLE I.—(*a*) A horse cost $200. I sold it at a profit of $80. Find the rate of profit. (*b*) A horse cost $200. I sold it at a loss of $80. Find the rate of loss.

OPERATION (*a*).

$80 ÷ 200 = $0.40, or 40% profit.

EXPLANATION (*a*).—On $200, the profit is $80; on $1, it is 200 times less, or $0.40, or 40%.

OPERATION (*b*).

$\frac{80}{200} = \frac{2}{5} = 40\%$ loss.

EXPLANATION (*b*).—A loss of $80 on the cost price ($200) $= \frac{80}{200}$, or $\frac{2}{5}$, or 40% of the cost price.

Example II.—A horse cost $200. I sold it for $280. Find the rate of profit.

Operation.

$280 — $200 = $80, profit;
$80 ÷ 200 = $0.40, or 40% profit.

Explanation. — The selling price ($280) minus the cost price ($200) = the profit ($80); $80 ÷ 200 = $0.40, or 40%.

Example III.—A horse cost $200. I sold it for $120. Find the rate of loss.

Operation.

$200 — $120 = $80, loss;
$80 ÷ 200 = $0.40, or 40% loss.

Explanation. — The cost price ($200) minus the selling price ($120) = the loss ($80); $80 ÷ 200 = $0.40, or 40%.

Oral Exercises.

Find the rate of profit or loss:

	Cost.	Profit.		Cost.	Loss.		Cost.	Sales.		Cost.	Sales.
1.	$10	$ 5	11.	$10	$ 1	21.	$ 6	$ 9	31.	$10	$ 5
2.	$ 8	$ 2	12.	$ 7	$ 1	22.	$16	$20	32.	$16	$12
3.	$12	$ 9	13.	$ 9	$ 1	23.	$12	$21	33.	$24	$21
4.	$16	$ 2	14.	$12	$ 1	24.	$24	$27	34.	$40	$25
5.	$ 8	$ 3	15.	$20	$ 4	25.	$16	$2[illegible]	35.	$32	$12
6.	$16	$10	16.	$15	$ 6	26.	$40	$65	36.	$16	$ 2
7.	$24	$21	17.	$25	$15	27.	$80	$82	37.	$16	$15
8.	$12	$ 8	18.	$20	$16	28.	$15	$20	38.	$30	$50
9.	$24	$16	19.	$32	$ 2	29.	$21	$35	39.	$15	$14
10.	$12	$ 2	20.	$80	$ 2	30.	$18	$21	40.	[illegible]	$80

Oral Problems.

1. A sewing machine cost $40, and was sold at a profit of $8. Find the rate of profit.

2. Find the per cent of gain on coffee which cost 24c a pound and was sold at a profit of 6c per pound.

3. What is the per cent of gain on goods which cost $750 and are sold at a profit of $125?

4. I bought a house for $12 000, and sold it for $2 000 more than it cost. Find the rate of profit.

5. I bought a piece of cloth for $75, and sold it for $15 less than it cost. Find the rate of loss.

6. A hat cost $3.75, and was sold at a loss of $1.25. Find the per cent of loss.

7. Goods which cost $620 are sold at a loss of $310. What is the rate of loss?

8. A lot which cost $1 500 was sold at a loss of $250. Find the per cent of loss.

9. A house which cost $2 000 was sold for $2 500. What was the per cent of gain?

10. A hat dealer buys his hats at $48 a dozen and sells them at $6 apiece. Find his per cent of gain.

11. Soap bought at 75c a dozen cakes, and retailed at 3 cakes for 25c, will produce what per cent of profit?

12. A lot which cost $1 200 was sold for $600. Find the per cent of loss.

13. I bought a horse for $180 and sold it for $150. What was my per cent of loss?

14. A set of furniture which had cost $500 was sold by au tion for $300. Find the per cent of loss.

15. I bought $15 worth of strawberries. Find my loss per cent, if I sold them for $12.

Written Exercises.

Find the rate of profit or loss:

Cost	Profit or loss.	Cost.	Sales.	Cost.	Sales.
1. $150	$ 12	11. $100	$105	21. $ 8.50	$ 7.65
2. $350	$ 14	12. $175	$210	22. $ 2.16	$ 1.89
3. $750	$165	13. $576	$624	23. $ 6.00	$ 5.97
4. $850	$221	14. $800	$810	24. $ 8.00	$ 7.98
5. $720	$108	15. $600	$610	25. $ 7.36	$ 0.92
6. $650	$117	16. $200	$205	26. $192	$ 72
7. $189	$ 21	17. $154	$168	27. $160	$150
8. $209	$ 19	18. $640	$760	28. $320	$ 20
9. $424	$371	19. $ 1.44	$ 1.89	29. $120	$ 70
10. $848	$ 0.53	20. $ 1.28	$ 1.84	30. $217	$155

Written Problems.

1. A house which cost \$7 490 was sold at a profit of \$1 498. Find the profit per cent.

2. A ship cost \$100 000, and was sold at a gain of \$11 500. What was the gain per cent?

3. I bought a farm at \$4 an acre, and sold it at a profit of \$10.50 an acre. What was the per cent of profit?

4. I bought a bicycle for \$50, and sold it for \$7.50 less than it cost me. Find the per cent of loss.

5. Hats that cost \$54 a dozen were sold at a rebate of 50c each. What was the loss per cent?

6. If I sell an article for $2\frac{1}{2}$ times its cost, what is my gain per cent?

7. A grocer bought 300 bags of salt, each weighing 56 lb., for \$126. He sold $66\frac{2}{3}$% of the invoice at 1c a pound, and the remainder at $\frac{1}{2}$c a pound. Find his per cent of gain.

8. A dealer buys penholders at \$4 a gross, and retails them at 5c each. What per cent does he gain?

9. A merchant bought 300 bu. of wheat at \$1.20 a bushel, and 400 bu. at \$1.10 a bushel. Find his gain per cent, if he sells the whole quantity at \$1.40 a bushel.

10. What is the per cent of loss if $\frac{4}{5}$ of a farm be sold at $\frac{3}{4}$ of the cost of the whole farm?

11. A house worth \$3 072 was sold for \$2 560. Find the loss per cent.

12. A violin worth \$51 was sold for \$35.70. Find the loss per cent.

NOTE.—Selling price—Profit = Cost price.
Selling price + Loss = Cost price.

13. I sold a horse for \$200, thereby gaining \$50. Find the per cent of profit.

14. I lose \$0.005 by selling pencils \$0.045 apiece. What do I lose per cent?

15. I gain $2\frac{1}{2}$c a quart by selling milk at 30c a gallon. What do I gain per cent?

16. I bought a 160-acre farm at \$15 an acre, spent \$354 for repairs and \$246 for draining. What per cent of gain did I realize, if I sold the farm for \$3 800?

17. I imported \$2 232.18 worth of French goods, and paid \$267.82 for freight and custom dues. If I sold the goods for \$3 000, what was my gain per cent?

18. I bought a house for \$3 500, and paid \$250 additional for repairs and other charges. An agent sold the house for \$5 000 and claimed \$50 for his services. Find my gain per cent.

19. One thousand copies of a book were printed for \$300. The book sold for 50c a copy, and the selling charges amounted to \$100. Find the per cent of gain.

20. I bought 7 200 bu. of wheat at \$1.35 a bushel, paid \$120 for drayage, \$108 for storage, and \$52 for other charges. Then I sold the whole quantity at \$1.99 a bushel. Find my gain per cent, if the selling charges amounted to \$328.

THIRD CASE.

Given the rate and the profit or loss, or the selling price, to find the cost.

329. Rule.—

$$\left.\begin{matrix}\text{Profit}\\ \text{or}\\ \text{Loss}\end{matrix}\right\} \div \text{Rate} = \text{Cost Price.}$$

When there is a *Profit:*

Selling Price ÷ (1 + Rate) = Cost Price.

When there is a *Loss:*

Selling Price ÷ (1 — Rate) = Cost Price.

EXAMPLE I.—(*a*) A man sold a house at a profit of \$200, thereby gaining 25%; required the cost. (*b*) A man sold a house at a loss of \$200, thereby losing 25%; required the cost.

OPERATION (*a*).

\$200 ÷ \$0.25 = 800 times, or \$800, cost price.

EXPLANATION (*a*).—At 25%, a profit of \$0.25 derives from a cost price of \$1, and a profit of \$200 derives from a cost price of as many dollars as \$0.25 is contained times in \$200, or \$800.

OPERATION (*b*).

$\$200 \div \frac{1}{4} = \$200 \times \frac{4}{1} = \$800$, cost price.

EXPLANATION (*b*).—25% or $\frac{1}{4}$ of the cost price = \$200; $\frac{4}{4}$(100%) of the cost price = 4 times \$200, or \$800.

EXAMPLE II.—A man sold a house for \$1 000, thereby gaining 25%; required the cost.

OPERATION (*a*).

\$1 000 ÷ \$1.25 = 800 times, or \$800, cost price.

EXPLANATION (*a*).—At 25% profit, a selling price of \$1.25 derives from a cost price of \$1, and a selling price of \$1 000 derives from a cost price of as many dollars as \$1.25 is contained times in \$1 000, or \$800.

OPERATION (*b*).

$\$1\,000 \div \frac{5}{4} = \$1000 \times \frac{4}{5} = \$800$, cost price.

EXPLANATION (*b*).—100% + 25%, or 125%, or $\frac{5}{4}$ of the cost price = \$1 000; $\frac{4}{4}$ (100%) of the cost price = $\$1\,000 \div \frac{5}{4}$, or \$800.

EXAMPLE III.—A man sold a house for \$600, thereby losing 25%; required the cost.

OPERATION (*a*).

\$600 ÷ \$0.75 = 800 times, or \$800, cost price.

EXPLANATION (*a*).—At 25% loss, a selling price of \$0.75 derives from a cost price of \$1, and a selling price of \$600 derives from a cost price of as many dollars as \$0.75 is contained times in \$600, or \$800.

OPERATION (*b*).

$\$600 \div \frac{3}{4} = \$600 \times \frac{4}{3} = \$800$, cost price.

EXPLANATION (*b*).—100% − 25%, or 75%, or $\frac{3}{4}$ of the cost price = \$600; $\frac{4}{4}$ (100%) of the cost price = $\$600 \div \frac{3}{4}$, or \$800.

Oral Exercises.

Find the cost price:

Profit or Loss.	Rate.	Sales.	Rate of Profit.	Sales.	Rate of Loss.
1. \$ 6	50 %	11. \$34	$6\frac{1}{4}$%	21. \$18	10 %
2. \$ 7	25 %	12. \$18	20 %	22. \$24	$14\frac{2}{7}$%
3. \$ 9	75 %	13. \$21	40 %	23. \$48	4 %
4. \$ 4	$12\frac{1}{2}$%	14. \$24	60 %	24. \$28	$6\frac{2}{3}$%
5. \$ 6	$37\frac{1}{2}$%	15. \$27	80 %	25. \$20	$9\frac{1}{11}$%
6. \$10	$62\frac{1}{2}$%	16. \$28	$16\frac{2}{3}$%	26. \$16	$11\frac{1}{9}$%
7. \$21	$87\frac{1}{2}$%	17. \$22	$83\frac{1}{3}$%	27. \$18	$14\frac{2}{7}$%
8. \$15	$33\frac{1}{3}$%	18. \$41	$2\frac{1}{2}$%	28. \$21	30 %
9. \$30	$66\frac{2}{3}$%	19. \$62	$3\frac{1}{3}$%	29. \$18	70 %
10. \$ 7	$8\frac{1}{3}$%	20. \$42	5 %	30. \$30	90 %

Oral Problems.

1. I sold a book for 25c above cost, and gained 25%. What was the cost of the book?

2. If by selling calico at a profit of 8c a yard, I gain 50%, find the cost of the calico per yard.

3. I sold a lot for \$400 more than it cost, thereby gaining $12\frac{1}{2}$%. What did it cost?

4. What is the cost of a watch which is sold at \$6 profit, if the rate of gain is $16\frac{2}{3}$%?

5. Find the cost of a pound of sugar, if I lose 1c a pound by selling it at $14\frac{2}{7}$% loss.

6. Required the cost of old-fashioned hats which were sold for 50c less than they cost, if the rate of loss was 20%.

7. What is the cost of a dozen of oranges sold at a loss of 10c a dozen, if the rate of loss is $33\frac{1}{3}$%?

8. I sold damaged fruit at a loss of 40%. Find the cost if my loss amounts to \$8.

9. I sold a carriage for \$90, thereby gaining 20%; required the cost.

10. If by selling a farm for \$4 500 I gained $12\frac{1}{2}$%, what was the cost of the farm?

11. If I make 60% profit by selling a pencil for 8c, what did it cost me?

12. I sold a dozen of pencils for 48c, thereby gaining $33\frac{1}{3}\%$. What did the pencils cost a dozen?

13. I lost 20% by selling a horse for \$200. How much had I paid for the horse?

14. I sold a bicycle for \$25, losing $16\frac{2}{3}\%$. How much had it cost?

15. How much was paid for a mine sold for \$12 000, at 50% below cost?

Written Exercises.

Find the cost price:

Profit or Loss.	Rate	Sales	Rate of Profit	Sales.	Rate of Loss.
1. \$171	18 %	11. \$220	10 %	21. \$492	18 %
2. \$900	25 %	12. \$900	$12\frac{1}{2}\%$	22. \$115	8 %
3. \$110	44 %	13. \$119	$16\frac{2}{3}\%$	23. \$8.55	$6\frac{1}{4}\%$
4. \$492	15 %	14. \$143	$8\frac{1}{3}\%$	24. \$6.92	$6\frac{2}{3}\%$
5. \$990	$12\frac{1}{2}\%$	15. \$2.34	$62\frac{1}{2}\%$	25. \$6.63	$7\frac{1}{7}\%$
6. \$330	$\frac{1}{4}\%$	16. \$4.18	$37\frac{1}{2}\%$	26. \$9.80	$9\frac{1}{11}\%$
7. \$267	$\frac{1}{3}\%$	17. \$2.79	$12\frac{1}{2}\%$	27. \$5.52	$14\frac{2}{7}\%$
8. \$2.88	48 %	18. \$9.75	25 %	28. \$8.59	$83\frac{1}{3}\%$
9. \$6.48	72 %	19. \$7.02	30 %	29. \$8.99	$3\frac{1}{3}\%$
10. \$5.72	52 %	20. \$8.04	$\frac{1}{2}\%$	30. \$9.36	$2\frac{1}{2}\%$

Written Problems.

1. I sold a house for \$874.08 above cost, and gained 24%; required the cost.

2. I sold a lot for \$470.82 more than it cost, thereby making 38% profit. What was the cost of the lot?

3. What is the cost of an automobile which is sold at a gain of \$341, if the rate of gain is 22%?

4. By selling a lot for \$825 less than it cost, I lost 66%. Find the cost.

5. I sold out my stock at a loss of 37%. Find the cost price and selling price, if my loss was \$5 309.50.

6. If I make $16\frac{2}{3}\%$ profit by selling a house for \$5 075, what was the cost of that house?

7. How much was paid for a house sold for \$4 500, at 125% above cost?

8. A druggist sells a remedy at 49c a bottle, and gains 75%. How much did he pay for a dozen bottles?

9. I sold a farm for \$4 800, and gained 20%. What was the cost of the farm?

10. I sold a farm for \$4 800, and lost 20%. What was the cost of the farm?

11. A merchant sold a bill of goods for \$178.60, thereby losing 6%. What did the goods cost him?

12. A stock of goods was sold for \$3 850, which was at a loss of 12%. What did the goods cost?

13. A farmer sold two horses for \$472.50 each. On one he gained 35%, and on the other he lost 10%. Find the cost of each horse, and the net gain.

14. A speculator sells two lots for \$3 600 each, gaining 25% on one, and losing 25% on the other. How much does he gain or lose by the transaction?

15. I sold two lots for \$1 599 each. On one I gained $2\frac{1}{2}\%$, and on the other I lost $2\frac{1}{2}\%$. What was the total cost of both lots?

16. I sold a horse for \$265, and paid \$15 for sundry selling expenses. How much had the horse cost, if I gained 25%?

17. I sold a house through an agent for \$3 550. Find the cost of the house, if the agent charged \$50 for his services, and if I realized a profit of $16\frac{2}{3}\%$.

18. I sold an automobile for \$1 260, gaining 25%. Find the cost price, if the selling charges amounted to \$10.

19. An agent bought a house for me, and received \$25 for his services. At what price did the agent buy the house, if I sold it for \$11 000, thus realizing a profit of $37\frac{1}{2}\%$?

20. Find the cost price of a house which was sold for \$8 550, at a profit of $13\frac{1}{3}\%$, if the buying expenses aggregated \$25, and the selling expenses were \$50.

REVIEW OF PROFIT AND LOSS.

Oral Exercises.

	A	B	C	D	E	F	G
1.	\$ 50	\$15	\$20	25 %	20 %	\$ 60	\$45
2.	100	20	10	20 %	25 %	120	80
3.	20	6	7	60 %	40 %	24	15
4.	60	20	18	50 %	$16\frac{2}{3}\%$	75	50
5.	50	8	5	60 %	$12\frac{1}{2}\%$	56	42
6.	40	12	15	$12\frac{1}{2}\%$	10 %	45	36
7.	44	4	11	10 %	50 %	55	33
8.	54	9	6	$12\frac{1}{2}\%$	$11\frac{1}{9}\%$	72	48
9.	80	2	6	$12\frac{1}{2}\%$	$16\frac{2}{3}\%$	90	75
10.	30	15	9	40 %	30 %	42	28

Written Exercises.

	A	B	C	D	E	F	G
1.	\$144	\$ 36	\$ 24	$12\frac{1}{2}\%$	$8\frac{1}{3}\%$	\$180	\$126
2.	160	60	16	20 %	$12\frac{1}{2}\%$	192	140
3.	300	50	30	$37\frac{1}{2}\%$	50 %	360	250
4.	324	27	108	$16\frac{2}{3}\%$	5 %	360	288
5.	540	120	180	$66\frac{2}{3}\%$	$12\frac{1}{2}\%$	630	480
6.	180	60	48	25 %	$14\frac{2}{7}\%$	210	144
7.	120	10	40	20 %	25 %	144	100
8.	400	100	50	$12\frac{1}{2}\%$	25 %	450	350
9.	360	60	36	20 %	$16\frac{2}{3}\%$	432	288
10.	144	36	18	$33\frac{1}{3}\%$	$16\frac{2}{3}\%$	168	128

Apply the following questions to each horizontal line of the preceding tables:

1. A = the cost price; B = the profit. What is the profit per cent?

2. A = the cost price; C = the loss. What is the per cent of loss?

3. A = the cost price; D = the rate of profit. What is the selling price?

4. A = the cost price; E = the rate of loss. What is the selling price?

5. A = the cost price; F = the selling price. What is the profit per cent?

6. A = the cost price; G = the selling price. What is the loss per cent?

7. B = the profit; E = the rate of profit. What is the cost price?

8. G = the loss; D = the rate of loss. What is the cost price?

9. F = the selling price; D = the rate of profit. What is the cost price?

10. F = the selling price; E = the rate of loss. What is the cost price?

Written Problems.

1. I bought \$7 224 worth of goods, and paid \$108 for freight and drayage. Find the selling price, if my profit was 20%.

2. A farmer sold 375 bu. of corn at 80c a bushel. At what price must the buyer sell the corn to realize a profit of 20%?

3. I bought two horses for \$150.25 each; I sold one at a profit of 40%, and the other at a loss of 28%. What was my net gain?

4. Find the per cent of gain on a carriage which cost \$348.50, and was sold for \$425.17.

5. A merchant bought 45 T. 16 cwt. 40 lb. of iron at \$75 a ton, and sold the whole quantity at \$78.50 a ton. Find his gain, and the per cent of his gain.

6. I bought \$7 200 worth of lumber. I sold 50% for \$4 000, then 50% of the remainder for \$2 500, and the second remainder for \$844. What was my gain per cent?

7. I bought \$1 280 worth of lumber. If 35% was sold at $12\frac{1}{2}$% profit, 35% at 10% profit, and the remainder at 20% profit, what was the per cent of profit realized on the whole transaction?

8. I bought 1 060 bu. of barley at 95c a bushel. I sold 300 bu. at 20% profit, 150 bu. at 12% profit, 100 bu. at cost, 400 bu. at 5% loss, 50 bu. at 50% loss. If the remainder was unsalable, what per cent of the total cost did I lose?

9. A merchant sold 30% of an invoice of wheat at 20% profit, and the remainder at 10% loss. Find the cost price and the selling price of the invoice, if the net loss was \$94.50.

10. A farmer sold a cow for \$37.50 above cost, and gained 30%. What would have been the rate of profit, if the cow had been sold for \$175?

11. A speculator sold a lot at 20% profit, and with the proceeds he purchased another lot which he sold at 30% profit, realizing a total gain of \$940.80 on both. What did he pay for each?

12. A farmer sold a farm for \$8 400, gaining 5%. If the farmer had sold the farm \$8 800, what would have been his per cent of gain?

13. I sold a harness for \$48, losing 20%. What selling price would have yielded 20% profit?

14. A man lost 20% by selling an automobile for \$640. What would he have lost per cent by selling it for \$780?

15. A sold a farm to B, and lost 30%. B sold it to C, and gained 10%. How much had A paid for the farm, if C bought it for \$6 930?

16. A manufacturer sold a bicycle at 20% profit; the wholesaler by selling it made $16\frac{2}{3}$% profit; and the retailer sold it for \$42, gaining 25%. What did the bicycle cost the manufacturer?

17. A sold a house to B at a gain of 20%; B sold it to C at a profit of 15%; and C sold it to D for $3 036, thereby making 20% profit. How much had A paid for the house?

18. A speculator bought a house for $16 800, paid $2 700 for improvements, and then sold it at a profit of 20%. Find the selling price.

19. In the preceding problem, what would the speculator have gained per cent by selling the house for $22 425?

20. What would have been the gain per cent, in N° 19, if the selling expenses had amounted to $195?

Questions on Theory.

1. What is the entire cost of a purchase?

2. What are the net proceeds of a sale?

3. When is the difference between the entire cost and the net proceeds a gain? When is it a loss?

4. How do you find the profit when the cost price and the rate of profit are given? (327).

5. How do you find the selling price when the cost price and the rate of profit are given? (327).

6. How do you find the cost price when the profit and rate of profit are given? (329).

7. How do you find the cost price when the selling price and the rate of profit are given? (329).

8. How do you find the profit when the selling price and the rate of profit are given?

9. Given the selling price and the cost price, how do you find the rate of profit? (328).

10. Given the selling price and profit, how do you find the rate of profit?

11. Given the cost price and the profit, how do you find the rate of profit? (328).

12. Given the cost price and the rate of loss, how do you find the loss? (327).

13. Given the cost price and the rate of loss, how do you find the selling price? (327).

14. How do you find the cost price when the loss and rate of loss are given? (329).

15. How do you find the cost price when the selling price and rate of loss are given? (329).

16. How do you find the loss when the selling price and rate of loss are given?

17. How do you find the selling price when the loss and rate of loss are given?

18. How do you find the rate of loss when the cost price and selling price are given?

19. How do you find the rate of loss when the loss and selling price are given?

20. How do you find the rate of loss when the cost price and loss are given?

21. Given the cost price, the selling price, the buying and selling expenses, how do you find the rate of profit?

22. Given the cost price, the selling price, the buying and selling expenses, how do you find the rate of loss?

23. Given the selling price, the rate of profit, the buying and selling expenses, how do you find the cost price?

24. Given the selling price, the rate of loss, the buying and selling expenses, how do you find the cost price?

25. Given the cost price, the rate of profit, the buying and selling expenses, how do you find the selling price?

COMMERCIAL DISCOUNT

330. Manufacturers and dealers offer their merchandise at a certain catalogue or list price, which is also called the *marked price*, or *gross price.*

331. A *commercial discount* is a percentage allowance, or deduction made from the catalogue or list price.

332. The *net price* is the gross price minus the discount.

The seller may offer a discount to his customer 1° to induce him to make a greater purchase; 2° to prompt him to pay cash, or to pay within a specified period; 3° to adjust prices to a falling market, etc.

The buyer may therefore be entitled to several discounts; in such cases, the discounts are not added, but calculated consecutively.

333. Let us apply the principles of percentage to commercial discount.

The Base is the *gross price.*
The Rate is the *per cent of discount.*
The Percentage is the *discount.*
The Difference is the *net price.*

FIRST CASE.

Given the gross price and the rate of discount, to find the discount, or net price.

334. Rule.—

Gross price × Rate of discount = Discount.

Gross price — Discount = Net price.

Gross price × (1 — Rate of discount) = Net price.

Example I.—The gross price of a sewing machine is $60, and the rate of dicount 20%. Find the discount.

Operation (*a*).

$0.20×60=$12, discount.

Explanation (*a*).— At 20%, the discount on $1 is $0.20; and the discount on $60 is 60 times $0.20, or $12.

OPERATION (*b*).

$\$60 \times \frac{1}{5} = \$12$, discount.

EXPLANATION (*b*).— At 20%, the discount is $\frac{20}{100}$, or $\frac{1}{5}$ of the gross price ($60), or $12.

EXAMPLE II.—The gross price of a sewing machine is $60, and the rate of discount 20%. Find the net price.

OPERATION (*a*).

$\$0.80 \times 60 = \$48$, net price.

EXPLANATION (*a*).—At 20%, the discount on $1 is $0.20, and the net price of $1 is $0.80; the net price of $60 is 60 times $0.80, or $48.

OPERATION (*b*).

$\$60 \times \frac{4}{5} = \$48$, net price.

EXPLANATION (*b*). — At 20% discount, the net price is 100%—20%, or 80% of the gross price. 80% of the gross price $= \frac{4}{5}$ of the gross price ($60), or $48.

Oral Exercises.

Find 1° the discount; 2° the net price:

GROSS PRICE.	RATE OF DISCOUNT.	GROSS PRICE.	RATE OF DISCOUNT.	GROSS PRICE.	RATE OF DISCOUNT.
1. $1.55	20 %	6. $0.80	30 %	11. $8.00	15 %
2. $6.00	10 %	7. $0.60	75 %	12. $5.50	40 %
3. $2.40	5 %	8. $8.00	$2\frac{1}{2}$%	13. $9.60	$16\frac{2}{3}$%
4. $3.60	$33\frac{1}{3}$%	9. $ 500	2 %	14. $8.40	$12\frac{1}{2}$%
5. $5.00	25 %	10. $ 700	1 %	15. $8.30	50 %

Oral Problems.

1. The gross price of a coat is $20, and the rate of discount 20%. Find the discount.

2. Find the discount at 25% on a bill of $60.

3. Find the discount on an invoice of $150, at 2% for cash.

4. At 4% discount find the net cost of an invoice of $250.

5. I bought $350 worth of goods; what sum did I pay if I was allowed a discount of 3%?

Written Exercises.

Find 1° the discount; 2° the net price:

	Gross price.	Rate of discount.		Gross price.	Rate of discount.		Gross price.	Rate of discount.
1.	$792	25 %	6.	$9.95	20%	11.	$936	$2\frac{1}{2}$%
2.	$5.58	$16\frac{2}{3}$%	7.	$6.70	30%	12.	$838	15%
3.	$261	$33\frac{1}{3}$%	8.	$9.30	10%	13.	$324	60%
4.	$6.16	$37\frac{1}{2}$%	9.	$6.25	4%	14.	$310	17%
5.	$875	40 %	10.	$8.60	5%	15.	$3.52	$6\frac{1}{4}$%

Written Problems.

1. A person bought 28 yd. of cloth at $1.75. What was the net price, the rate of discount being 2% ?

2. A lady bought 24 yd. of linen at $1.45 a yard. She was allowed a discount of $2\frac{1}{2}$% for cash. Find the net amount of the bill.

3. What was the net value of the following bill, discount off 3%: 10 doz. bottles of ink at $3; 300 lb. of paper at 30c; 12 gross of pencils at $2.75 ?

4. A discount of $16\frac{2}{3}$% was allowed on the following invoice: 150 bags of potatoes at $2.50; 200 bu. of wheat at $1.75; 360 bags of beans at $3. Find the net amount.

5. Find the net cost of the following bill, $12\frac{1}{2}$% off: 420 yd. of silk at $1.50; 320 yd. of velvet at $1.75; 200 yd. of ribbon at 23c.

335. Note.—When several discounts are allowed, the first discount is to be computed upon the gross price and deducted; then the second is to be computed upon the remainder and deducted; and so on for each successive discount.

The result is not affected by the order in which the discounts are taken.

EXAMPLE.—What is the discount and net price of hardware invoiced at $280, and subject to a discount of 25% and 20%?

OPERATION.

4) $280
$ 70, 1st discount;
5) $210
$ 42, 2nd discount;
$168, net price;
$280—$168 = $112, total discount.

EXPLANATION.—The 1st discount is ¼ of $280, or $70; $280 — $70 = $210, proceeds of the 1st discount. The 2nd discount is ⅕ of $210, or $42; $210 — $42 = $168, proceeds of the 2nd discount, or the required net price; $280 — $168 = $112, the total discount.

The following method is sometimes used: $\frac{\$280 \times 3 \times 4}{4 \times 5} = \$168$, net price. And $280—$168 = $112, total discount.

Written Problems.

1. Find the net value of a bill of goods amounting to $120, discount off 33⅓% and 5%.

2. I was allowed 20%, 10%, and 10% discount on an invoice of furniture amounting to $375. What was the net price?

3. The list price of an auto was $1 630; what was the net price, discounts of 20%, 10%, and 5% being allowed?

4. I bought 675 yd. of carpet at $2.50 a yard, less 20% and 5%. What was the net price?

5. The gross amount of a bill of goods is $1 440, the rates of discount being 20%, 5%, and 2½%. Find the net price and discount.

6. Find the net amount of a bill of $2 000, the series of discounts being 25%, 20%, and 5%.

7. I bought a piano at a list price of $575, with discounts of 33⅓% 25%, and 20%. How much did I pay?

8. What is the net amount of a bill of $3 000, the discounts being 10%, 10%, and 10%?

9. I pay two bills of $100 each, the first less 10% and 5%, the second less 15%. Find the difference between the net prices.

10. A bookseller's list price of a set of works is $90 less 20% and 5%; another bookseller's list price of the same set of works is $100 less 20% and 15%. How much would be saved by choosing the better offer?

336. Note.—A discount series may be replaced by a single discount.

Example.—What single discount is equivalent to a discount series of 25%, 20%, and 5%?

Operation.

100% = gross price;
25% = 1st discount;
75% = 1st proceeds;
($\frac{1}{5}$ of 75) 15% = 2nd discount;
60% = 2nd proceeds;
($\frac{1}{20}$ of 60) 3% = 3rd discount;
57% = net price;
100%—57% = 43%, single discount.

Explanation.—Represent the gross price by 100%. The 1st discount is $\frac{1}{4}$ of 100%, or 25%; deduct it, obtaining 75%. The 2nd discount is $\frac{1}{5}$ of 75%, or 15%; deduct it, obtaining 60%. The 3rd discount is $\frac{1}{20}$ of 60%, or 3%; deduct it, obtaining 57%, the rate of the net price. 100% — 57% = 43%, the rate of the single discount.

Written Exercises.

Reduce the following series rates to single rates:

1. 20% and 10%.
2. 10% and 10%.
3. 25% and 5%.
4. 20% and $2\frac{1}{2}$%.
5. 25% and 15%.
6. 25%, 20%, and 10%.
7. 20%, 10%, and 5%.
8. 10%, 10%, and 10%.
9. 30%, 10%, and 5%.
10. 20%, 5%, and $2\frac{1}{2}$%.
11. $12\frac{1}{2}$%, 20%, and $2\frac{1}{2}$%.
12. $33\frac{1}{3}$%, 25%, and 5%.
13. 40%, 25%, and $12\frac{1}{2}$%.
14. 5%, $2\frac{1}{2}$%, and 20%.
15. 40%, 20%, and 10%.

SECOND CASE.

Given the gross price and the net price, to find the rate of discount.

337. Rule.—(Gross price — Net price) ÷ Gross price = the Rate of discount.

EXAMPLE.—The gross price of a hat is \$6, and its net price is \$5; required the rate of discount.

OPERATION.

\$6—\$5=\$1;
\$1÷ 6=\$0.16⅔, or 16⅔%.

EXPLANATION.—The gross price (\$6) minus the net price (\$5) = the discount, \$1. On \$6 the discount is \$1; on \$1, it is 6 times less, or \$0.16⅔, or 16⅔%.

Oral Exercises.

Find the rate of discount:

	GROSS PRICE.	NET PRICE.		GROSS PRICE.	NET PRICE.		GROSS PRICE.	NET PRICE.
1.	\$7.50	\$6.00	6.	\$0.75	\$0.60	11.	\$50	\$35
2.	\$8 00	\$7.00	7.	\$0.15	\$0.14	12.	\$60	\$55
3.	\$100	\$75	8.	\$600	\$500	13.	\$70	\$60
4.	\$900	\$600	9.	\$0.05	\$0.04	14.	\$90	\$85
5.	\$5.00	\$4.50	10.	\$0.50	\$0.45	15.	\$75	\$50

Oral Problems.

1. The list price of a gross of copy books is \$4.50; the net price, \$4.05. Find the rate of discount.

2. The catalogue price of a gross of penholders is \$3.20; the net price, \$3.04. What is the per cent of discount?

3. The gross price of a dozen bottles of ink is \$3, and the net price, \$2.40. Find the rate of discount.

4. The marked price of a gross of pencils is \$2.50; the net price is \$2. Find the per cent of discount.

5. I offered my pens at 75c a gross, but sold them at 70c a gross. What rate of discount did I allow?

Written Exercises.

Find the rate of discount:

Gross price.	Net price.	Gross price.	Net price.	Gross price.	Net price.
1. $600	$550	6. $810	$720	11. $7.17	$2.39
2. $4.05	$3.24	7. $576	$528	12. $6.24	$3.90
3. $3.96	$3.30	8. $630	$504	13. $1.95	$1.82
4. $4.90	$4.20	9. $8.40	$8.19	14. $6.24	$5.85
5. $1.28	$1.12	10. $5.19	$3.46	15. $960	$630

Written Problems.

1. The list price of an automobile was $3 500; I purchased it for $3 080. What per cent of discount was I allowed?

2. I asked $5 250 for a house, but I obtained only $5 040 for it. What rate of discount did I allow?

3. I settle a bill of $950 by paying $900 cash. Find the rate of discount.

4. The net price of a bill is $950, and the discount, $50. Find the rate of discount.

5. I gave a $1 080 cheque to obtain a rebate of $120 on a bill of goods. Find the rate of discount.

NOTE.—For the seller, the net price represents the cost price plus the profit, or the cost price minus the loss.

EXAMPLE.—A hat which costs $6 is marked $10. What per cent discount may the seller allow and yet gain $33\frac{1}{3}\%$?

OPERATION.

$ $6 \times \frac{4}{3} = \$8$, net price;
$ $10 - \$8 = \$2$, discount;
$ $2 \div 10 = \$0.20$, or 20%.

EXPLANATION.— Reckon the profit upon the cost price ($6); $\$6 \times \frac{4}{3} = \$8$, net selling price which will yield $33\frac{1}{3}\%$ profit. $\$10 - \$8 = \$2$, the discount that may be allowed on the list price ($10); $\$2 \div 10 = \$0.20$, or 20%.

Written Problems.

1. I ask $10 000 for a house that cost me $7 500. What per cent discount may I offer and yet gain 20%?

2. A piano which cost \$320 was listed at \$500. What per cent discount did the seller allow, if he sold the piano at 25% profit?

3. A sewing machine which costs \$30 is marked \$50; what rate of discount may the seller allow to lose $16\frac{2}{3}$%?

4. I paid \$7.50 for a table, and I wish to sell it at 20% profit. What rate of discount may I offer on the list price, \$12?

5. The gross price of a bicycle is \$40, and its cost price \$20. What per cent discount may the seller offer and yet gain 50%?

THIRD CASE.

Given the net price and the rate of discount, to find the gross price.

338. Rule.— Net price ÷ (1—Rate of discount) = Gross price.

EXAMPLE I.—The net price of a hat is \$6; find the gross price, if 25% discount was allowed.

OPERATION.

\$6 ÷ \$0.75 = 8 times, or \$8, gross price.

EXPLANATION.—At 25% discount, a net price of \$0.75 derives from a gross cost of \$1, and a net price of \$6 derives from a gross cost of as many dollars as \$0.75 is contained times in \$6, or \$8.

EXAMPLE II.—The net amount of a bill is \$360; find the gross amount, the discount series being 25%, 20%, and 20%.

OPERATION.

100% — 25% = 75%;
75% — 15% = 60%;
60% — 12% = 48%,
rate of net price;
\$360 ÷ \$0.48 = 750 times, or \$750, gross price.

EXPLANATION.—When there are several discounts, first find the rate of the net price. Here it is 48%. Then divide the net price by its rate.

Oral Exercises.

Find the gross price:

NET PRICE.	RATE OF DISC'T.	NET PRICE.	RATE OF DISC'T.	NET PRICE.	RATE OF DISC'T.
1. $9	10%	6. $14	$12\frac{1}{2}$%	11. $17	15 %
2. $6	25%	7. $10	$37\frac{1}{2}$%	12. $10	$16\frac{2}{3}$%
3. $5	50%	8. $12	40 %	13. $ 7	50 %
4. $8	20%	9. $10	$33\frac{1}{3}$%	14. $39	$2\frac{1}{2}$%
5. $7	30%	10. $15	$6\frac{1}{4}$%	15. $38	5 %

Oral Problems.

1. Find the gross price of a watch sold at 20% discount, the net price being $16.

2. What was the list price of an auto that I bought for $600 after obtaining a discount of 25%?

3. The net price of an article is $3.50, and the rate of discount $16\frac{2}{3}$%. Find the gross price.

4. I sold a lot for $750 net. What price had I first asked for the lot, if I allowed the buyer a discount of 40%?

5. The net amount of a bill is $78. What was the gross amount, the rates of discount being 20% and $2\frac{1}{2}$%?

Written Exercises.

Find the gross cost:

NET PRICE.	RATE OF DISC'T.	NET PRICE.	RATE OF DISC'T.	NET PRICE.	RATES OF DISC'T.
1. $2.72	15%	6. $634	$33\frac{1}{3}$%	11. $312	20% $2\frac{1}{2}$%
2. $6.70	20%	7. $875	$16\frac{2}{3}$%	12. $567	10% 10%
3. $3.75	25%	8. $849	$62\frac{1}{2}$%	13. $675	25% 10%
4. $8.19	10%	9. $695	$37\frac{1}{2}$%	14. $765	15% 10%
5. $2.64	12%	10. $819	$2\frac{1}{2}$%	15. $696	25% 20%

Written Problems.

1. The net cost of a bill of goods was $51.87 with discounts of 25%, 20%, and 5% off. What was the list price?

2. At what price was a piano marked, if I paid \$218.88 for it, discount off 20%, 10%, and 5%?

3. The net amount of a bill is \$145.80. Find the gross amount, the rates of discount being 40%, 10%, and 10%.

4. I bought goods with discounts of 20%, $2\frac{1}{2}$%, and $2\frac{1}{2}$%. Find the gross amount of the bill, if the net amount was \$380.25.

5. I gave a \$179.55 cheque to settle a bill upon which I was granted a discount of 30%, 10%, and 5%. Find the gross amount of the bill.

NOTE.—For the seller, the net price represents the cost price plus the profit, or the cost price minus the loss.

EXAMPLE.—What must be the marked price of hats costing \$3, that I may deduct 20% from it, and yet gain $33\frac{1}{3}$% on the cost?

OPERATION.

$\$3 \times \frac{4}{3} = \$4$, net price;
$\$4 \div \frac{4}{5} = \$5$, gross price.

EXPLANATION.— Reckon the profit upon the cost price (\$3); $\$3 \times \frac{4}{3} = \$4$, net selling price which will yield $33\frac{1}{3}$% profit; divide the net price by its rate, to find the gross price.

Written Problems.

1. What must I ask for a horse costing \$216, in order to abate 40%, and yet make a profit of 25%?

2. Find the list price of a lawn mower that cost \$15 and is sold at a discount of 25% and 20%, and at a profit of 20%.

3. A merchant sells an overcoat at a discount of 20%, 10%, and $2\frac{1}{2}$%, and thereby gains 25%. What is the list price, if the cost is \$28.08?

4. What was the list price of velvet costing \$5 a yard, if a loss of 10% was sustained by selling it at a discount of 25% and 20%?

5. A gross of buttons costs 36c. What list price will yield 50% profit, the rates of discount being 25% and 10%?

REVIEW OF COMMERCIAL DISCOUNT.

Oral Exercises.

	A	B	C	D	E	F
1.	16	$12\frac{1}{2}$%	4	21	12	20
2.	25	20 %	8	24	12	15
3.	28	25 %	10	15	20	25
4.	10	50 %	6	18	30	32
5.	30	$16\frac{2}{3}$%	8	15	40	45
6.	21	$33\frac{1}{3}$%	9	20	36	42
7.	16	$37\frac{1}{2}$%	12	25	18	24
8.	20	5 %	3	38	15	16
9.	40	10 %	5	27	30	45
10.	35	40 %	16	30	64	72

Written Exercises.

	A	B	C	D	E	F
1.	120	50 %	72	216	360	384
2.	240	10 %	30	162	180	270
3.	875	20 %	280	840	420	525
4.	220	5 %	33	418	165	176
5.	455	40 %	208	390	832	936
6.	378	$33\frac{1}{3}$%	162	360	648	756
7.	812	25 %	290	435	580	600
8.	240	$16\frac{2}{3}$%	64	120	320	360
9.	656	$12\frac{1}{2}$%	164	861	492	840
10.	480	$37\frac{1}{2}$%	360	750	540	720

Apply the following questions to each horizontal line of the preceding tables.

1. A = the gross price; B = the rate; what is the discount?
2. A = the gross price; B = the rate; what is the net price?
3. B = the rate; C = the discount; what is the gross price?
4. B = the rate; D = the net price; what is the gross price?
5. C = the discount; E = the net price; what is the rate?

6. B = the rate; C = the discount; what is the net price?
7. C = the discount; E = the gross price; what is the rate?
8. E = the net price; F = the gross price; what is the rate?
9. B = the rate; D = the net price; what is the discount?
10. A = the net price; C = the discount; what is the gross price?

Written Problems.

1. What is the net amount of a bill of \$360, the discounts being $12\frac{1}{2}\%$ and 8%?

2. What must I ask for a carriage costing \$80, in order to abate 25% and yet make a profit of 25%?

3. I paid \$92.12 net after being allowed a discount of 2% on a bill of goods. What was the gross amount of the bill, and what had the merchant himself paid for the goods, if he realized a profit of 12% on the transaction?

4. I bought 325 books at a discount of 40% on the list price, which was \$1.20 each. I sold them at a discount of 10% on the same list price. Find my net gain.

5. A bookseller granted a discount of \$1.70 on a work listed at \$10.20. What was the rate of discount? What should the bookseller have asked for the work, in order to abate $37\frac{1}{2}\%$, and yet receive the same net price?

6. I bought a bill of goods amounting to \$1 500, at a discount of 20% and 15%. If I sold the goods for \$1 500, at a discount of 15%, 10%, and 5%, what sum did I gain?

7. I buy goods at 20%, 20%, and 10% off. What is the gross price, and the net price, the total discount being \$9.54?

8. What is the difference on a bill of \$100 between a discount of 40% and 10%, and a discount of 30% and 20%?

9. What must I ask for a horse that cost \$100, in order to abate $12\frac{1}{2}\%$, and yet make a profit of 40%?

10. Three merchants offer me the same quality of hats at the same list price; the first with discounts of 20%, 10%, and 5%; the second with discounts of 5%, 20%, and 10%; the third with discounts of 10%, 5%, and 20%. Which is the best offer?

NOTE.—Merchants sometimes mark their goods by means of letters. The prices thus marked can only be read by those who know the key.

With the following price key, $\begin{cases}\text{n o w b e s m a r t,}\\ \text{1 2 3 4 5 6 7 8 9 0}\end{cases}$ 75 would be written: *me*; 90, *rt*; $\frac{me}{rt}$ would indicate that an object costs 75c, and sells for 90c.

Questions on Theory.

1. What is the gross cost of an article? (330).

2. What is the discount? the net price? (331, 332).

3. What reasons may a seller have to allow a discount? (332).

4. May the same buyer be entitled to several discounts? (332).

5. Given the gross cost and the rate of discount, how do you find the discount? (334).

6. Given the gross cost, and a series of three discounts, how do you find the net price? (335).

7. Given the discount and the rate of discount, how do you find the gross price?

8. Given the net price and the rate of discount, how do you find the gross price? (337).

9. How do you find the rate of discount when the net price and discount are given?

10. How do you find the net price when the discount and rate of discount are given?

11. How do you find the rate of discount when the gross price and discount are given?

12. How do you find the discount when the net price and rate of discount are given?

13. Give a few synonyms of *gross price*. (330).

14. How can a discount series be replaced by a single discount? (336).

15. Given the cost price of an article, the rate of profit to be realized, the rate of discount to be allowed, how can you find the gross price?

COMMISSION

339. The sum charged by one person for buying or selling for another is often computed at a certain per cent of the price, and is called *commission* or *brokerage*.

340. A *Commission Merchant, Agent,* or *Broker* is a person who transacts business for another person called the *Principal.*

It is very important not to mistake a purchasing agent for a selling agent, and *vice versa*.

341. Let us apply the principles of percentage to commission:

In a purchase:	The Base is the cost price or *market price.* The Rate is the *per cent of commission.* The Percentage is the *commission.* The Amount is the market price plus the commission, or *entire cost.*
In a sale:	The Base is the selling price, or *market price.* The Rate is the *per cent of commission.* The Percentage is the *commission.* The Difference is the market price minus the commission, or *net proceeds.*

By *market price* is meant the value of a purchase before the expenses are added, or the value of a sale before the expenses are deducted.

FIRST CASE.

Given the market price and the rate of commission, to find the commission, or the entire cost, or the net proceeds.

342. Rule.—

Market price × Rate = Commission.

In a purchase:

Market price + Commission = Entire cost.

Market price × (1 + Rate of commission) = Entire cost.

In a sale:

Market price — Commission = Net proceeds.

Market price × (1 — Rate of commission) = Net proceeds.

EXAMPLE I.—An agent buys (*or* sells) $1 000 worth of goods at 5% commission. Find the commission.

OPERATION (*a*).

$0.05 × 1 000 = $50, com.

EXPLANATION (*a*).—At 5% the commission on $1 is $0.05, and the commission on $1 000 is 1 000 times $0.05, or $50.

OPERATION (*b*).

$1 000 × $\frac{1}{20}$ = $50, com.

EXPLANATION (*b*).—At 5% the commission is $\frac{5}{100}$ or $\frac{1}{20}$ of the market price ($1 000), or $50.

EXAMPLE II.—An agent buys $1 000 worth of goods at 5% commission. Find the entire cost.

OPERATION (*a*).

$1.05 × 1 000 = $1 050, entire cost.

EXPLANATION (*a*).—At 5% the commission on $1 is $0.05, and the entire cost of $1 is $1.05; the entire cost of $1 000 is 1 000 times $1.05, or $1 050.

OPERATION (*b*).

$1 000 × $\frac{21}{20}$ = $1 050, entire cost.

EXPLANATION (*b*).—At 5% the entire cost is 100% plus 5%, or 105% of the market price; 105% of the market price is $\frac{21}{20}$ of the market price ($1 000), or $1 050.

EXAMPLE III.—An agent sells $1 000 worth of goods at 5% commission. Find the net proceeds.

OPERATION (*a*).

$0.95 × 1 000 = $950, net proceeds.

EXPLANATION (*a*). At 5%, the commission on $1 is $0.05, and the net proceeds of $1 are $0.95; the net proceeds of $1 000 are 1 000 times $0.95, or $950.

OPERATION (*b*).

$1 000 × $\frac{19}{20}$ = $950, net proceeds.

EXPLANATION (*b*).—At 5% commission, the net proceeds are 100% minus 5%, or 95% of the market price; 95% of the market price is $\frac{19}{20}$ of the market price ($1000), or $950.

Oral Exercises.

Find 1° the commission; 2° the entire cost; 3° the net proceeds.

MARKET PRICE.	RATE OF COM.	MARKET PRICE.	RATE OF COM.	MARKET PRICE.	RATE OF COM.
1. $200	5 %	6. $150	20 %	11. $100	1%
2. $600	16⅔%	7. $800	12½%	12. $300	2%
3. $450	6⅔%	8. $400	2½%	13. $500	4%
4. $480	6¼%	9. $700	10 %	14. $700	3%
5. $900	3⅓%	10. $600	8⅓%	15. $900	5%

Oral Problems.

1. An agent bought $5 000 worth of wheat at 5% commission. Find his commission and the entire cost of the wheat.

2. What is an agent's commission on a purchase of potatoes amounting to $2 000 at 2½%? Find the entire cost.

3. A drover had $6 000 worth of cattle bought at 2% commission. Find the agent's commission, and the entire cost of the transaction.

4. A real estate dealer sells $2 000 land at 1¼% commission. Find his commission and the net proceeds of the sale.

5. An agent sells $5 000 of furniture. How much is his commission at 5%? What sum does the agent remit to his principal?

Written Exercises.

Market Price.	Rate of Com.	Market Price.	Rate of Com.	Market Price.	Rate of Com.
1. $9 660	$3\frac{1}{5}$%	6. $9 040	$12\frac{1}{2}$%	11. $8 480	$\frac{1}{8}$%
2. $6 400	$6\frac{1}{4}$%	7. $7 080	$16\frac{2}{3}$%	12. $6 528	$\frac{1}{16}$%
3. $7 695	$6\frac{2}{3}$%	8. $5 445	20 %	13. $3 124	$\frac{1}{4}$%
4. $4 920	$2\frac{1}{2}$%	9. $3 692	10 %	14. $3 690	$\frac{1}{10}$%
5. $7 356	$8\frac{1}{3}$%	10. $8 696	$\frac{1}{2}$%	15. $3 048	5%

Written Problems.

1. An agent buys 2 340 bu. of wheat at $1.75 a bushel. What is his commission at $2\frac{2}{5}$%, and what is the entire cost of the wheat?

2. I bought $18 400 worth of property through an agent who charges $\frac{3}{8}$% commission. Find the agent's commission and the entire cost of the property.

Note.—When there are other charges than the commission, the *entire cost* equals the market price plus the commission and other charges.

3. An agent purchased for me 7 320 bags of potatoes at $1.50 each. What was the entire cost of the potatoes, if the agent charged $2\frac{1}{4}$% commission, and if the other charges amounted to $150.50?

4. An agent buys for me 3 600 bu. of wheat at $1.60 a bushel. What is his commission at $\frac{3}{16}$%? Find the entire cost of the wheat, the other expenses amounting to $205.

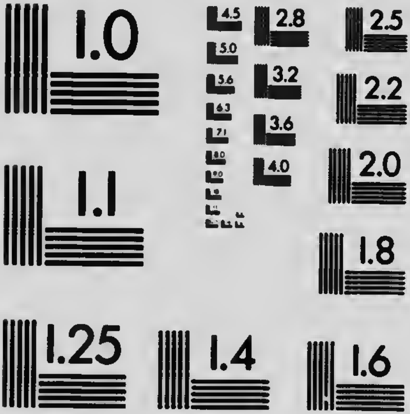
5. An agent buys for me 380 bbl. of apples at $2.25 each, charges $3\frac{1}{3}$% for commission, and pays $150 for freight and drayage. Find the entire cost of the apples.

6. An agent sold $2 560 worth of goods at $2\frac{1}{4}$% commission. Find his commission, and the net proceeds of the transaction.

7. A commission merchant sold 540 bbl. of flour at $8.25 each, and charged 5% commission. Find his commission, and the sum he remitted to his principal.

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)



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8. An agent sold 3 476 lb. of cheese at $12\frac{1}{2}$c a pound. If his commission was $3\frac{1}{5}$%, what sum did he remit to his principal?

NOTE.—When there are other charges than the commission, the *net proceeds* equal the market price minus the commission and other charges.

9. An agent sold \$750 worth of merchandise at 4% commission. What were the net proceeds due the principal, the other charges amounting to \$8.75?

10. My agent sold 180 bbl. of sugar, each containing 275 lb., at $6\frac{1}{2}$c a pound. What were the net proceeds, if he charged 2% for commission, and \$72.34 for sundry other expenses?

SECOND CASE.

Given the market price and the commission, or the entire cost or the net proceeds, to find the rate of commission.

343. Rule.—

Commission ÷ Market price = Rate of commission.

In a purchase:

(Entire cost — Market price) ÷ Market price = Rate of com.

In a sale:

(Market price — Net proceeds) ÷ Market price = Rate of com.

EXAMPLE I.—An agent buys (*or* sells) \$1 000 worth of goods, and receives a commission of \$50. Find the rate of commission.

OPERATION (*a*).

\$50 ÷ 1 000 = \$0.05, or 5% com.

OPERATION (*b*).

$\frac{50}{1000}$ = 5% com.

EXPLANATION (*a*). — On \$1 000, the commission is \$50; on \$1, it is 1 000 times less, or \$0.05, or 5%.

EXPLANATION (*b*).— A commission of \$50 on the market price (\$1 000) = $\frac{50}{1000}$, or $\frac{1}{20}$, or 5% of the market price.

Example II.—What is a purchasing agent's rate of commission when the market price is \$1 000, and the entire cost, \$1 050?

OPERATION.

\$1 050—\$1 000 = \$50, com.;
\$50 ÷ 1 000 = \$0.05, or 5% com.

EXPLANATION.—The entire cost (\$1 050) minus the market price (\$1 000) = the commission (\$50). \$50 ÷ 1 000 = \$0.05, or 5%.

EXAMPLE III.—What is a selling agent's rate of commission when the market price is \$1 000, and the net proceeds are \$950?

OPERATION.

\$1 000—\$950 = \$50, com.;
\$50 ÷ 1000 = \$0.05, or 5% com.

EXPLANATION.—The market price (\$1 000) minus the net proceeds (\$950) = the commission (\$50). \$50 ÷ 1000 = \$0.05, or 5%.

Oral Exercises.

Find the rate of commission :

	MARKET PRICE.	COM.		MARKET PRICE.	ENTIRE COST.		MARKET PRICE.	NET PROCEEDS.
1.	\$100	\$20	6.	\$200	\$250	11.	\$100	\$ 95
2.	\$200	\$10	7.	\$750	\$900	12.	\$200	\$175
3.	\$300	\$20	8.	\$250	\$275	13.	\$350	\$300
4.	\$400	\$10	9.	\$800	\$900	14.	\$250	\$225
5.	\$500	\$50	10.	\$150	\$200	15.	\$150	\$140

Oral Problems.

1. An agent bought a farm for \$12 000, and received a commission of \$120. Find the rate of his commission.

2. A commission merchant collected \$8 000, and his commission was \$200. What was the rate of commission?

3. An agent charged \$300 for selling 40 horses at \$150 each. What was the rate of commission?

4. If the market price was \$4 000, and the entire cost, \$4200, what was the rate of commission?

5. If the sales were \$800, and the net proceeds, \$750, what was the rate of commission?

Written Exercises.

Find the rate of commission:

Market price.	Com.	Market price.	Entire cost.	Market price.	Net proceeds.
1. $380	$19	6. $580	$609	11. $780	$741
2. $640	$16	7. $810	$837	12. $960	$936
3. $990	$33	8. $920	$943	13. $588	$539
4. $732	$61	9. $708	$767	14. $840	$819
5. $496	$31	10. $656	$697	15. $672	$630

Written Problems.

1. An agent bought a property for $12 500, and received a commission of $187.50. What was the rate of his commission?

2. An agent charged $1 539 for collecting a sum of $7 695. Find the rate of commission.

3. A commission merchant sold 40 bbl. of apples at $3.25 each, and retained $7.80 for his services. What was the rate of his commission?

4. My agent bought 350 bu. of oats at 60c a bushel. If the entire cost amounted to $231, what was the rate of commission?

5. An agent bought 4 000 bu. of barley at 95c a bushel. Find the rate of commission, if the entire cost of the barley was $4 275.

6. The entire cost of 3 500 bu. of peas was $9 625. If my agent charged $875 for buying the peas, what was his rate of commission? What was the market price per bushel?

7. The net proceeds of the sale of 500 tons of alfalfa was $5 600, and the commission $400. Find the market price per ton, and the rate of commission.

Note.—The entire cost minus the commission and other charges equals the buying market price. The net proceeds plus the commission and other charges equal the selling market price.

8. What is the rate of commission, if the entire cost of a purchase is $250, the commission $40, and the sum of the other charges $10?

9. A commission merchant remitted $355 as the net proceeds of a sale, after deducting $25 for his commission, and $20 for freight and storage. What was the rate of commission?

10. An agent sold 1 860 bu. of wheat at $1.75 a bushel; having deducted $35 for freight, $25 for storage, and a certain sum for his commission, he remitted $2 544 to his principal. Find the commission and the per cent of commission.

THIRD CASE.

Given the rate and the commission, or the entire cost or the net proceeds, to find the market price.

344. Rule.—

> Commission ÷ Rate of commission = Market price.
>
> In a purchase:
>
> Entire cost ÷ (1 + Rate of commission) = Market price.
>
> In a sale:
>
> Net proceeds ÷ (1 — Rate of commission) = Market price.

Example I.—What is the market price of a purchase (*or* a sale), if the commission is $50, and the rate of commission, 5%?

Operation (*a*).

$50 ÷ $0.05 = 1 000 times, or $1 000, market price.

Explanation (*a*).—At 5%, a commission of $0.05 derives from a market price of $1, and a commission of $50 derives from a market price of as many dollars as $0.05 is contained times in $50, or $1 000.

Operation (*b*).

$\$50 \div \frac{1}{20} = \$50 \times \frac{20}{1} = \$1\ 000$, market price.

Explanation (*b*).—5%, or $\frac{1}{20}$ of the market price = $50; $\frac{20}{20}$ (100%) of the market price = 20 times $50, or $1 000.

Example II.—What is the market price of a purchase, if the entire cost is \$1 050, and the rate of commission, 5%?

Operation (a).

\$1 050 ÷ \$1.05 = 1 000 times, or \$1 000, market price.

Explanation (a).—At 5% commission, an entire cost of \$1.05 derives from a market price of \$1, and an entire cost of \$1 050 derives from a market price of as many dollars as \$1.05 is contained times in \$1 050, or \$1 000.

Operation (b).

\$1 050 ÷ $\frac{21}{20}$ = \$1 050 × $\frac{20}{21}$ = \$1 000, market price.

Explanation (b).—100% + 5%, or 105%, or $\frac{21}{20}$ of the market price = \$1 050; $\frac{20}{20}$ (100%) of the market price = \$1 050 ÷ $\frac{21}{20}$, or \$1 000.

Example III.—What is the market price of a sale, if the net proceeds are \$950, and the rate of commission is 5%?

Operation (a).

\$950 ÷ \$0.95 = 1 000 times, or \$1 000, market price.

Explanation (a).—At 5% commission, a net proceeds of \$0.95 derives from a market price of \$1, and a net proceeds of \$950 derives from a market price of as many dollars as \$0.95 is contained times in \$950, or \$1 000.

Operation (b).

\$950 ÷ $\frac{19}{20}$ = \$950 × $\frac{20}{19}$ = \$1 000, market price.

Explanation (b).—100% − 5%, or 95%, or $\frac{19}{20}$ of the market price = \$950; $\frac{20}{20}$ (100%) of the market price = \$950 + $\frac{19}{20}$, or \$1000.

Oral Exercises.

Find the market price:

Com.	Rate of com.	Entire cost.	Rate of com.	Net proceeds.	Rate of com.
1. \$4	2 %	6. \$240	20 %	11. \$150	$6\frac{1}{4}$ %
2. \$3	$2\frac{1}{2}$ %	7. \$220	10 %	12. \$140	$6\frac{2}{3}$ %
3. \$6	3 %	8. \$210	$16\frac{2}{3}$ %	13. \$190	5 %
4. \$5	4 %	9. \$900	$33\frac{1}{3}$ %	14. \$110	$8\frac{1}{3}$ %
5. \$7	5 %	10. \$450	$12\frac{1}{2}$ %	15. \$990	1 %

Oral Problems.

1. If the commission was \$100, and the rate of commission, 2%, what was the market price of a purchase?

2. What was the amount of my agent's sales, if his commission was \$1 000, at 5%?

3. I remitted to my agent \$1 020 to invest in wheat, at 2% commission. What was the market price of the wheat?

4. The entire cost of a purchase effected at $2\frac{1}{2}$% commission was \$410. Find the market price.

5. The net proceeds of the sale of 100 bbl. of apples amounted to \$360. Find the market price of one barrel, if the rate of commission was 10%.

Written Exercises.

Find the market price:

Com.	Rate of com.	Entire cost.	Rate of com	Net proceeds	Rate of com.
1. \$6.85	2%	6. \$477	6 %	11. \$615	$6\frac{1}{4}$%
2. \$9.36	3%	7. \$327	9 %	12. \$630	$6\frac{2}{3}$%
3. \$7.32	4%	8. \$517	10 %	13. \$803	$8\frac{1}{3}$%
4. \$8.12	5%	9. \$943	$2\frac{1}{2}$%	14. \$398	$\frac{1}{2}$%
5. \$6.41	7%	10. \$887	$3\frac{1}{3}$%	15. \$798	$\frac{1}{4}$%

Written Problems.

1. An agent received a commission of \$175 for buying a property. If the rate of commission was $2\frac{1}{2}$%, at what price did the agent buy the property?

2. An agent charged \$226.80 for selling a consignment of potatoes. Find the market value of the sale, if the rate of commission was $3\frac{1}{8}\%$.

3. A commission merchant received \$5 392.05 to invest in sugar after deducting his commission of 3%. How much was invested in sugar?

4. At 28c a pound, how many pounds of wool will my agent buy with \$2 870, if his rate of commission is $2\frac{1}{2}\%$?

5. A commission merchant remitted to his principal \$15 029 as the net proceeds of a consignment sold on a commission of 5%. Find the market value of the transaction.

NOTE.—(Entire cost — charges other than commission) ÷ (1 + Rate of com.) = market price.

(Net proceeds + charges other than commission) ÷ (1 — Rate of com.) = market price.

6. Find the market price of a purchase, if the entire cost is \$6 700, the rate of commission, $3\frac{1}{8}\%$, and the sum of the remaining charges, \$100.

7. An agent received \$768.45 to invest in cotton after deducting all expenses. How much did he pay for the cotton, if his rate of commission was $1\frac{7}{8}\%$, and the sum of the other charges, \$10.50?

8. Find the market price of a sale effected at $2\frac{1}{2}\%$ commission if the net proceeds were \$2 322.75, and the charges other than commission, \$17.25.

9. An agent remitted to his principal \$298.30 as the net proceeds of a sale. His charges were \$11.70 for drayage, and $3\frac{1}{8}\%$ for commission. Find the market value of the transaction.

10. I received \$2 550.50 as the net proceeds of a consignment of 350 bbl. of flour. If my agent's charges were 2% for commission, and \$22 for drayage, what was the selling price of one barrel?

REVIEW OF COMMISSION.

Written Exercises on Purchases.

	A	B	C	D	E	F
1.	$672	5 %	$21	$42	$231	$294
2.	$375	4 %	$26	$52	$442	$520
3.	$636	$6\frac{2}{3}$%	$16	$32	$288	$336
4.	$124	$6\frac{1}{4}$%	$17	$34	$459	$510
5.	$915	$3\frac{1}{3}$%	$31	$62	$248	$341

Apply the following questions to each horizontal line of the preceding table.

1. A = the market price; B = the rate of commission; what is the entire cost?

2. E = the entire cost; B = the rate of commission; what is the market price?

3. E = the entire cost; D = the commission; what is the rate of commission?

4. D [illegible] commission; B = the rate of commission; what is the [illegible]?

5. [illegible] entire cost; E = the market price; what is the rate of [illegible]sion?

6. A = the market price; B = the rate of commission; C = the charges other than commission; what is the entire cost?

7. E = the entire cost; B = the rate of commission; C = the charges other than commission; what is the market price?

8. E = the entire cost; D = the commission; C = the charges other than commission; what is the rate of commission?

9. F = the entire cost; E = the market price; C = the charges other than commission; what is the rate of commission?

10. D = the commission; B = the rate of commission; C = the charges other than commission; what is the entire cost?

Written Exercises on Sales.

	A	B	C	D	E	F
1.	$600	10%	$ 9	$18	$198	$171
2.	$840	$2\frac{1}{2}$%	$39	$78	$273	$156
3.	$144	$8\frac{1}{3}$%	$11	$22	$143	$110
4.	$480	2 %	$49	$98	$294	$147
5.	$836	5 %	$19	$38	$171	$114

Apply the following questions to each horizontal line of the preceding table.

1. A = the market price; B = the rate of commission; what are the net proceeds?

2. E = the net proceeds; B = the rate of commission; what is the market price?

3. E = the net proceeds; D = the commission; what is the rate of commission?

4. D = the commission; B = the rate of commission; what are the net proceeds?

5. F = the net proceeds; E = the market price; what is the rate of commission?

6. A = the market price; B = the rate of commission; C = the charges other than commission; what are the net proceeds?

7. E = the net proceeds; B = the rate of commission; C = the charges other than commission; what is the market price?

8. E = the net proceeds; D = the commission; C = the charges other than commission; what is the rate of commission?

9. F = the net proceeds; E = the market price; C = the charges other than commission; what is the rate of commission?

10. D = the commission; B = the rate of commission; C = the charges other than commission; what are the net proceeds?

Written Problems.

1. A clerk receives $6 a week and 2% commission on goods sold. If he sells $1 500 worth of goods a month, what is his monthly income?

2. A commercial traveler has a fixed salary of \$18 a week; besides he is entitled to $2\frac{1}{2}\%$ commission on his sales. If he sells \$40 000 worth of goods in a year, what is his annual income?

3. A collecting agent working on a commission of 4% turns over to his principal \$1 286.40 as the net amount due him. What was the sum collected, and the agent's commission?

4. An agent received \$4 526.25 to invest in potatoes. How many bushels did he buy at 88c a bushel, if his charges were \$16.25 for drayage, and $2\frac{1}{2}$ % for commission?

5. I paid an agent \$200 for buying 5 000 bu. of wheat at a commission of $2\frac{1}{2}\%$. Find the entire cost of a bushel, the other expenses being \$50.

6. An agent sold a consignment of 3 000 bu. of wheat, charged $2\frac{1}{2}\%$ for commission, \$22.50 for storage, and remitted \$5 242.50 as the balance due his principal. Find the market price of a bushel of wheat.

7. The entire cost of a purchase is \$1 729.10, which includes the commission \$50.10, and other charges aggregating \$9. Find the rate of commission.

8. The net proceeds of a consignment were \$2 425, the commission, \$50, and sundry other expenses, \$25. What was the rate of commission?

NOTE.—The rate of profit is to be computed upon the entire cost.

9. An agent buys for me 200 bbl. of sugar, each containing 275 lb., at $5\frac{1}{4}$c a pound, charges 2% for commission, and pays \$64.50 for freight. Find the entire cost of the sugar. What shall I gain per cent, if I sell the sugar at 7c a pound?

10. What should I gain per cent in the preceding problem, if the sales were effected by an agent at 3% commission, the other charges aggregating \$59.50?

Questions on Theory.

1. What is commission? brokerage? (339).

2. What person is called a commission merchant? an agent? a broker? (340).

3. What person is called the principal? (340).

4. What is the market price? (341).

5. What is the base of commission, 1° in a purchase? 2° in a sale? (341).

6. How do you find the commission when the market price of a sale and the rate of commission are given? (342).

7. How do you find the net proceeds of a sale when the market price and the rate of commission are given? (342).

8. How do you find the net proceeds of a sale when the market price, the rate of commission, and the freight charges are given?

9. How do you find the market price of a sale when the commission and rate of commission are given? (344).

10. How do you find the market price when the net proceeds of a sale, and the commission are given?

11. How do you find the rate of commission when the net proceeds of a sale, and the commission are given?

12. Given the net proceeds of a sale, the commission and other charges, how do you find the market price?

13. Given the net proceeds of a sale, the commission and other charges, how do you find the rate of commission?

14. Given the net proceeds of a sale, and the rate of commission, how do you find the market price? (344).

15. Given the net proceeds of a sale, the freight charges, and the rate of commission, how do you find the market price?

16. Given the net proceeds of a sale, and the market price, how do you find the commission?

17. Given the net proceeds of a sale, and the market price, how do you find the rate of commission? (343).

18. Given the net proceeds of a sale, the market price, and the freight charges, how do you find the rate of commission?

19. Given the commission and rate of commission, how do you find the net proceeds of a sale?

20. Given the rate of commission, the commission, and freight charges, how do you find the net proceeds of a sale?

21. How do you find the commission when the market price of a purchase, and the rate of commission are given? (342).

22. How do you find the entire cost of a purchase when the market price and rate of commission are given? (342).

23. How do you find the entire cost of a purchase when the market price, the freight charges, and the rate of commission are given?

24. How do you find the market price of a purchase when the commission and rate of commission are given? (344).

25. How do you find the rate of commission when the market price of a purchase, and the commission are given? (343).

26. How do you find the market price of a purchase when the entire cost and commission are given?

27. How do you find the rate of commission when the commission and entire cost of a purchase are given?

28. How do you find the market price of a purchase, when the entire cost, commission, and freight charges are given?

29. How do you find the rate of commission when the commission, freight charges, and entire cost are given?

30. How do you find the market price of a purchase when the entire cost and rate of commission are given? (344).

31. Given the entire cost of a purchase, the freight charges and rate of commission, how do you find the market price?

32. Given the entire cost and market price of a purchase, how do you find the rate of commission? (343).

33. Given the market price of a purchase, the entire cost, and freight charges, how do you find the rate of commission?

34. Given the commission and rate of commission, how do you find the entire cost of a purchase?

35. Given the commission, freight charges, and rate of commission, how do you find the entire cost of a purchase?

INTEREST

A man who has money may either invest his money in goods that will sell at a profit, in a property that will yield rent, or *loan his money*, that is to say, lend it, to have it bring in an income called *interest*. Money produces money, and the *use* of another man's money must be paid for.

345. *Interest* is an allowance for the use of money.

346. The *principal* is the sum loaned or invested.

The sum of the principal and its accrued interest is called the *amount*.

347. The *rate* of interest is the interest on $1 for 1 year.

In Canada, the legal rate of interest is 5%.

If the use of $1 for 1 year costs 5c (5%), the use (interest) of $100 for 1 year will cost 100 times 5c, or $5; and the use of $100 for 2 years will cost 2 times $5, or $10; and the use of $100 for 6 months will cost ½ of $5, or $2.50.

348. Percentage now involves a new element: *time*, or the period during which the money is on interest.

To find the interest for one year, we apply the principles of percentage:

The Base is the *principal.*

The Rate is the *interest on* $1 *for* 1 *year.*

The Percentage is the *interest.*

Then, when we have the interest for one year, we find the interest for several years, or for fractional parts of a year, by applying the 8th principle of analysis.

FIRST CASE.

Given the principal, the rate per annum, and the time, to find the interest or the amount.

349. Rule.—

> Principal × Rate per annum = Annual interest.
>
> Annual interest × Time = Total interest.

350. Interest for years and months.

EXAMPLE.—(*a*) What is the interest of \$500 for 3 years at 5% per annum? (*b*) What is the interest of \$500 for 3 years and 4 months at 5% per annum?

OPERATION (*a*).

\$0.05 × 500 = \$25, int. for 1 year;

\$25 × 3 = \$75, int. for 3 years.

EXPLANATION (*a*).—At 5%, the interest of \$1 for 1 year is \$0.05, and the interest of \$500 for 1 year is 500 times \$0.05, or \$25; and the interest for 3 years is 3 times \$25, or \$75.

OPERATION (*b*).

\$0.05 × 500 = \$25, int. for 1 year;

\$25 × $3\frac{1}{3}$ ($\frac{10}{3}$) = \$$83\frac{1}{3}$, int. for 3 yr. 4 mo.

EXPLANATION (*b*).—The interest for 3 years and 4 months, or $3\frac{1}{3}$ years, is $3\frac{1}{3}$ times \$25, or \$$83\frac{1}{3}$.

351. Interest for years and days.

EXAMPLE.—What is the interest of \$600 for 3 years and 132 days at 6% per annum?

OPERATION.

\$0.06 × 600 × 3 = \$108, int. for 3 years;

\$0.06 × 600 × $\frac{132}{365}$ = \$13.02 —;

or

\$600 × $\frac{6}{100}$ × $\frac{132}{365}$ = \$13.02, int. for 132 days;

\$108 + \$13.02 = \$121.02, int. for 3 yr. 132 da.

EXPLANATION.—At 6%, the interest of \$1 for 1 year is \$0.06, and the interest of \$600 for 1 year is 600 times \$0.06, or \$36; and the interest for 3 years is 3 times \$36, or \$108.

The interest for 1 day is 365 times less than \$36, and the interest for 132 days is 132 times as much, or $\frac{132}{365}$ of \$36, or \$13.02.

\$108, int. for 3 years, plus \$13.02, int. for 132 days = \$121.02, int. for 3 yr. 132 da.

352. NOTE.—When the time is given in years and days, compute the interest separately for the years and days, and add the results.

When the time is given in months and days, reduce the months to days, considering one month 30 days, and then proceed as in the preceding example.

Oral Exercises.

Find the interest 1° for 1 year; 2° for 2 years:

Principal.	Rate.	Principal.	Rate.	Principal.	Rate.
1. $100	6%	6. $600	3 %	11. $1 000	4%
2. $300	4%	7. $750	4 %	12. $2 000	5%
3. $500	5%	8. $800	$4\frac{1}{2}$%	13. $3 000	6%
4. $120	5%	9. $900	5 %	14. $5 000	7%
5. $300	3%	10. $400	5 %	15. $9 000	3%

Oral Problems.

1. What fraction of a year is: 3 mo.? 6 mo.? 7 mo.? 9 mo.? 73 da.? 146 da.? 219 da.? 292 da.?

2. If you had $40 in the Montreal Bank, how much interest would you receive at 3% for one year? for 6 months?

3. At 6%, what is the interest of $400 for 6 mo.? 3 mo.? 2 yr.?

4. I borrowed $350 at 6%; what is the interest for 6 mo.? 2 yr. 6 mo.?

5. Find the interest of $8 000 at 5% for 2 yr. 6 mo.

Written Exercises.

Find the interest:

Principal.	Rate.	Time.	Principal.	Rate.	Time.
1. $270	5%	73 da.	11. $1 095	$5\frac{1}{2}$%	1 yr. 73 da.
2. $450	6%	146 da.	12. $1 460	$6\frac{1}{2}$%	2 yr. 146 da.
3. $380	7%	219 da.	13. $1 825	$7\frac{1}{2}$%	3 yr. 219 da.
4. $620	4%	292 da.	14. $2 555	$4\frac{1}{2}$%	4 yr. 292 da.
5. $730	3%	63 da.	15. $3 650	5 %	2 yr. 100 da.
6. $950	5%	33 da.	16. $3 470	6 %	4 yr. 200 da.
7. $375	7%	93 da.	17. $5 140	7 %	3 yr. 100 da.
8. $835	5%	200 da.	18. $9 205	5 %	1 yr. 300 da.
9. $745	6%	180 da.	19. $3 705	4 %	5 yr. 40 da.
10. $430	4%	175 da.	20. $4 015	6 %	4 yr. 80 da.

Find the interest of $1:

(*Give exact answers*).

21. for 100 da. at 6%.
22. for 300 da. at 5%.
23. for 120 da. at 7%.
24. for 93 d. at 6%.
25. for 63 da. at 5%.
26. for 183 da. at 6%.
27. for 123 da. at 5%.
28. for 72 da. at 4%.
29. for 34 da. at 7%.
30. for 90 da. at 5%.
31. for 1 yr. 73 da. at 5%.
32. for 2 yr. 40 da. at 6%.
33. for 3 yr. 72 da. at 5%.
34. for 2 yr. 183 da. at 7%.
35. for 1 yr. 200 da. at 6%.

To find the amount.

353. Rule.—

Principal + Interest = Amount.

Principal × (1 + int. of $1 for given time at given rate) = Amount.

Find the amount of:

36. $300 for 100 da. at 6%.
37. $400 for 300 da. at 5%.
38. $120 for 120 da. at 7%.
39. $200 for 93 da. at 6%.
40. $500 for 63 da. at 5%.
41. $4 000 for 1 yr. 73 da. at 5%.
42. $3 650 for 2 yr. 40 da. at 6%.
43. $4 540 for 3 yr. 72 da. at 5%.
44. $9 500 for 2 yr. 183 da. at 7%.
45. $5 000 for 1 yr. 200 da. at 6%.

354. For less than one year, find the time by counting the *exact* number of days in each month.

Find 1° the interest; 2° the amount of:

46. $ 730 from Apr. 4, 1917, to Dec. 21, 1917, at 6%.
47. $3 650 from Jan. 14, 1918, to July 29, 1918, at 7%.
48. $1 825 from Apr. 15, 1918, to Nov. 25, 1918, at 5%.
49. $1 095 from Jan. 17, 1919, to Sept. 30, 1919, at 6%.
50. $7 665 from Feb. 11, 1920, to Sept. 23, 1920, at 7%.
51. $4 015 from Feb. 25, 1917, to Dec. 30, 1918, at 6%.
52. $2 920 from Apr. 22, 1918, to Mar. 4, 1920, at 5%.
53. $9 125 from May 6, 1919, to Mar. 18, 1921, at 7%.
54. $6 205 from May 13, 1918, to Jan. 7, 1920, at 6%.
55. $6 935 from Dec. 30, 1918, to Jan. 7, 1921, at 7%.

Written Problems.

1. A man loaned \$2 900 at 5%, and \$5 200 at $6\frac{1}{2}$%. Find his annual income.

2. A lady having a sum of \$9 000, loaned one half of it at $6\frac{1}{2}$%, and with the remainder bought a house that rents for \$30 a month. Find her yearly income.

3. On Feb. 24, 1917, a farmer bought 50 head of cattle at \$45 a head, promising to pay for them later with interest at 7%. What amount did he pay on Apr. 26, 1917?

4. I borrowed \$750 on May 5, at 5%, and \$800 on July 4, at 6%. I paid the amount of both debts on December 15 of the same year. How much did I pay?

5. A man borrowed \$2 750, at $6\frac{1}{2}$%. What amount should he pay 93 days later?

SECOND CASE.

PART I.

Given the principal, the time, and the interest, or the amount, to find the rate per annum.

355. Rule.—*1° Find the interest of the principal, for the given time, at 1%; 2° divide the given interest by the interest of \$1. (12th principle of analysis).*

NOTE.—If the amount is given, diminish it by the principal to find the interest, and proceed as before.

EXAMPLE I.—At what rate per annum will \$800 in 3 years give \$144 interest?

OPERATION.

\$0.01 × 800 × 3 = \$24, int. for 3 yr. at 1%;
\$144 ÷ \$24 = 6 times, or 6%.

EXPLANATION.— At 1% the interest of \$800 for 3 yr. = \$24; \$24 is the interest at 1%, \$144 is the interest at such a per cent as \$24 is contained times in \$144, or 6%.

EXAMPLE II.—At what rate per annum must $730 be loaned for 3 yr. 200 da. to produce $129.50 interest?

OPERATION.

$0.01 \times 730 \times 3 = \$21.90$, int. for 3 yr. at 1%;

$0.01 \times 730 \times \frac{200}{365} = \$4$, int. for 200 da. at 1%;

$21.90 + $4 = $25.90;

$129.50 ÷ $25.90 = 5 times, or 5%.

EXPLANATION.—At 1%, the interest of $730 for 3 yr. 200 da. is $25.90; $25.90 is the interest at 1%, $129.50 is the interest at such a per cent as $25.90 is contained times in $129.50, or 5%.

Oral Exercises.

Find the rate per annum:

	PRINCIPAL.	TIME.	INTEREST.		PRINCIPAL.	TIME.	INTEREST.
1.	$100	1 yr.	$ 5	6.	$700	6 mo.	$14
2.	$200	2 yr.	$24	7.	$800	3 mo.	$12
3.	$300	3 yr.	$45	8.	$400	1 yr. 6 mo.	$36
4.	$400	4 yr.	$48	9.	$800	2 yr. 6 mo.	$60
5.	$500	5 yr.	$75	10.	$600	3 yr. 6 mo.	$84

Oral Problems.

1. At what rate will $200 in 1 yr. give an interest of $8? $6? $10? $15?

2. At what rate must $300 be loaned for 4 yr. to produce an interest of $12? $18? $36? $60?

3. At what rate will $400 in 6 mo. yield an interest of $10? $12? $9? $11?

4. At what rate will $800 in 3 mo. produce an interest of $6? $9? $11? $15?

5. At what rate must $200 be loaned for 2 yr. 6 mo. to yield an interest of $15? $25? $30? $35?

Written Exercises.

Find the rate per annum:

Principal.	Time.	Interest.	Principal.	Time.	Amount.
1. $ 705	73 da.	$ 8.46	11. $ 511	155 da.	$ 524.02
2. $ 730	208 da.	$20.80	12. $1 022	1 yr. 70 da.	$1 076.81
3. $ 875	146 da.	$24.50	13. $1 752	1 yr. 200 da.	$1 887.60
4. $1 095	183 da.	$32.94	14. $1 250	2 yr. 146 da.	$1 370.
5. $ 735	219 da.	$17.64	15. $1 314	1 yr. 65 da.	$1 391.40
6. $1 825	93 da.	$32.55	16. $ 990	2 yr. 219 da.	$1 118.70
7. $3 475	292 da.	$83.40	17. $ 876	1 yr. 35 da.	$ 914.40
8. $4 015	33 da.	$25.41	18. $ 500	3 yr. 292 da.	$ 623.50
9. $1 460	300 da.	$90.	19. $ 803	2 yr. 100 da.	$ 894.30
10. $2 285	63 da.	$28.35	20. $ 438	1 yr. 155 da.	$ 472.32

Written Problems.

1. At what per cent per annum will $900 yield $9 interest from Dec. 1, 1917, to Feb. 12, 1918?

2. At what rate per annum will $310 produce $26.04 interest from Oct. 21, 1918, to Mar. 15, 1920?

3. At what rate per annum must $1 387 be loaned to produce $123.50 interest from Jan. 2, 1918, to May 17, 1919?

4. A debt of $876 with interest from Feb. 17, 1917, to May 28, 1917, amounts to $888. Find the rate of interest.

5. At what rate will $2 044 amount to $2 075.92 from Mar. 3, 1919, to June 6, 1919?

Part II.

Given the principal, the rate per annum, and the interest or the amount, to find the time.

356. Rule.—*1° Find the interest of the principal at the given rate for one year; 2° divide the given interest by the interest for one year.* (*12th principle of analysis*).

Note.—When the amount is given, subtract the principal from the amount to find the interest, and then proceed as before.

EXAMPLE I.—In what time will \$600, at 5% per annum, give \$90 interest?

OPERATION.

\$0.05 × 600 = \$30, int. for 1 yr.;
\$90 ÷ \$30 = 3 times, or 3 years.

EXPLANATION.— For 1 yr., the interest of \$600 at 5% is \$30; \$30 is the interest for 1 yr., \$90 is the interest for as many years as \$30 is contained times in \$90, or 3 years.

EXAMPLE II.—In what time will \$600 produce \$12 interest, if loaned at 5% per annum?

OPERATION.

\$0.05 × 600 = \$30, int. for 1 yr.;
\$12 ÷ \$30 = $\frac{12}{30}$, or $\frac{2}{5}$ times, or $\frac{2}{5}$ of a year (365 da.), or 146 days.

EXPLANATION. — \$30 is the interest for 1 year; \$12 is the interest for such a part of one year as \$30 is contained times in \$12, or $\frac{2}{5}$ times, or $\frac{2}{5}$ of 1 yr. (365) da., or 146 days.

We may also find the interest of \$600 at 5% for 1 day; \$0.05 × 600 × $\frac{1}{365}$ = \$$\frac{6}{73}$, int. for 1 day; \$12 ÷ \$$\frac{6}{73}$ = 146 days.

Oral Exercises.

Find the time:

	PRINCIPAL.	RATE.	INTEREST.		PRINCIPAL.	RATE.	INTEREST.
1.	\$200	4%	\$ 16	6.	\$150	6%	\$45
2.	\$300	5%	\$ 45	7.	\$200	7%	\$42
3.	\$600	6%	\$ 72	8.	\$500	3%	\$75
4.	\$500	7%	\$105	9.	\$250	4%	\$25
5.	\$800	5%	\$140	10.	\$400	5%	\$50

Oral Problems.

1. In what time will \$100, at 4%, produce an interest of \$4? \$12? \$20? \$2?

2. In what time will \$400, at 3%, give an interest of \$36? \$18? \$72? \$84?

3. In what time will \$600, at 2½%, yield an interest of \$15? \$30? \$45? \$60?

4. In what time will \$1 000, at 5%, produce an interest of \$100? \$200? \$300? \$25?

5. In what time will \$10 000, at 6%, give an interest of \$1 200? \$3 000? \$900? \$300?

Written Exercises.

Find the time:

	Principal.	Rate.	Interest.		Principal.	Rate.	Amount.
1.	\$ 695	3 %	\$ 16.68	11.	\$ 949	4%	\$ 993.20
2.	\$ 584	7½%	\$ 36	12.	\$1 679	6%	\$1 879.10
3.	\$1 314	5 %	\$ 11.34	13.	\$2 044	7%	\$2 057.72
4.	\$8 030	7 %	\$ 50.82	14.	\$2 409	3%	\$2 588.19
5.	\$1 168	6 %	\$ 32.64	15.	\$2 993	5%	\$3 157
6.	\$1 533	4½%	\$ 37.80	16.	\$3 139	6%	\$3 397
7.	\$1 898	5½%	\$114.40	17.	\$2 701	7%	\$3 063.60
8.	\$6 570	4 %	\$ 45.36	18.	\$3 431	4%	\$3 769.40
9.	\$1 241	6½%	\$110.50	19.	\$3 723	5%	\$3 755.13
10.	\$1 971	5 %	\$ 80.73	20.	\$4 088	6%	\$4 155.20

Written Problems.

1. In what time will \$4 800 loaned at 5% per annum amount to \$4 896?

2. In what time will \$630 amount to \$655.20, if loaned at 5% per annum?

3. I borrowed \$500 on May 2, 1917, at 6%, and on the date of maturity I paid \$512 for the principal and interest. Find the date of payment.

4. I borrowed \$850, at 7%, and on May 1, 1918, paid an amount of \$885.70. When was the money borrowed?

5. On Dec. 31, 1918, I loaned \$350, at 3½%, and when the debt became due I received \$352.45 in full for principal and interest. At what time was the debt paid?

THIRD CASE.

PART I.

Given the rate per annum, the time, and the interest, to find the principal.

357. Rule.—*1° Find the interest of \$1 for the given time and at the given rate; 2° divide the given interest by the interest of \$1. (12th principle of analysis).*

EXAMPLE I.—What principal will yield \$40 interest in 4 years at 5%?

OPERATION.

$0.05 × 4 = $0.20, int. of $1;
$40 ÷ $0.20 = 200 times, or $200, principal.

EXPLANATION.— The interest of \$1 for 4 yr. at 5% is \$0.20; \$0.20 is the interest of \$1, \$40 is the interest of as many dollars as \$0.20 is contained times in \$40, or \$200.

EXAMPLE II.—What sum of money loaned for 1 yr. 183 da. at 5% will yield \$32.88 interest?

OPERATION.

\$0.05 = int. of \$1 for 1 yr.;
\$0.05 × $\frac{183}{365}$ = \$0.02$\frac{37}{73}$, int. for 183 da.
\$0.05 + \$0.02$\frac{37}{73}$ = \$0.07$\frac{37}{73}$, int. for 1 yr. 183 da.;
\$32.88 ÷ \$0.07$\frac{37}{73}$ = 438 times, or \$438, principal.

EXPLANATION.— The interest of \$1 for 1 yr. 183 da. at 5% is \$0.07$\frac{37}{73}$; \$32.88 is the interest of as many dollars as \$0.07$\frac{37}{73}$ is contained times in \$32.88, or \$438.

Oral Exercises.

Find the principal:

	RATE.	TIME.	INTEREST.		RATE.	TIME.	INTEREST.
1.	4½%	2 yr.	\$ 9	6.	4 %	2½ yr.	\$30
2.	5 %	3 yr.	\$15	7.	5½%	4 yr.	\$44
3.	6½%	4 yr.	\$52	8.	6 %	2½ yr.	\$30
4.	7 %	5 yr.	\$70	9.	7½%	2 yr.	\$45
5.	3 %	3 yr.	\$27	10.	3½%	4 yr.	\$42

Oral Problems.

1. What principal in 6 mo. at 4% will yield an interest of $10? $12? $20?

2. What principal in 3 mo. at 8% will produce an interest of $6? $8? $14?

3. What principal in 1 yr. 6 mo. at 6% will give an interest of $18? $45? $36?

4. What principal in 2 yr. 6 mo. at 8% will yield an interest of $40? $60? $10?

5. What principal in 18 mo. at 4% will produce an interest of $12? $24? $36?

Written Exercises.

Find the principal:

	RATE.	TIME.	INTEREST.		RATE.	TIME.	INTEREST
1.	4 %	146 da.	$ 4.80	**11.**	5 %	1 yr. 100 da.	$ 23.25
2.	5 %	200 da.	$12	**12.**	6 %	1 yr. 73 da.	$ 17.28
3.	6 %	219 da.	$ 4.14	**13.**	7 %	1 yr. 40 da.	$ 51.03
4.	7 %	100 da.	$15.40	**14.**	6½%	1 yr. 146 da.	$ 36.40
5.	7½%	63 da.	$15.12	**15.**	5½%	2 yr. 20 da.	$181.50
6.	6½%	292 da.	$26	**16.**	4½%	2 yr. 219 da.	$ 46.80
7.	5½%	35 da.	$21.56	**17.**	3½%	1 yr. 165 da.	$ 51.94
8.	3 %	73 da.	$ 6.	**18.**	3 %	2 yr. 292 da.	$ 42.
9.	3½%	93 da.	$ 6.51	**19.**	4 %	1 yr. 201 da.	$ 45.28
10.	4½%	183 da.	$32.94	**20.**	5 %	2 yr. 300 da.	$257.50

Written Problems.

1. What sum loaned from Mar. 12, 1918, to June 22, 1918, at 4% per annum, will produce $8.16 interest?

2. What sum of money will yield $19.53 interest in 93 da. at 3½%?

3. On Jan. 2, 1919, a merchant borrowed a certain sum at 5%; on April 1, 1919, he paid the principal and interest. What was the principal, if the interest was $15.13?

4. What principal at 6% will yield $79.20 interest from Oct. 21, 1919, to Mar. 1, 1920?

5. A lender received \$20.16 interest on a sum loaned at 7% on June 5, 1918, and paid Jan. 2, 1920. What was the sum loaned?

Part II.

Given the rate per annum, the time, and the amount, to find the principal.

358. Rule.—*1° Find the amount of \$1 for the given time and at the given rate; 2° divide the given amount by the amount of \$1. (12th principle of analysis).*

Example I.—What principal will amount to \$240 in 4 yr. at 5%?

Operation.

$0.05 × 4 = $0.20, int. of \$1 for 4 yr.;
\$1 + \$0.20 = \$1.20, amount of \$1 for 4 yr.;
\$240 ÷ \$1.20 = 200 times, or \$200, principal.

Explanation.—The interest of \$1 for 4 yr. at 5% is \$0.20; the amount of \$1 is \$1 + \$0.20, or \$1.20; \$240 is the amount of as many dollars as \$1.20 is contained times in \$240, or \$200.

Example II.—What principal will amount to \$470.88 in 1 yr. 183 da., if loaned at 5%?

Operation.

\$0.05 + (\$0.05 × $\frac{183}{365}$) = \$0.07$\frac{37}{73}$, int. for 1 yr. 183 da.;
\$1 + \$0.07$\frac{37}{73}$ = \$1.07$\frac{37}{73}$, amount of \$1;
\$470.88 ÷ \$1.07$\frac{37}{73}$ = 438 times, or \$438, principal.

Explanation. — The interest of \$1 for 1 yr. 183 da. at 5% is \$0.07$\frac{37}{73}$, and the amount of \$1 is \$1.07$\frac{37}{73}$; \$470.88 is the amount of as many dollars as \$1.07$\frac{37}{73}$ is contained times in \$470.88, or \$438.

Oral Exercises.

Find the principal:

Rate.	Time.	Amount.	Rate.	Time.	Amount.
1. 4%	1 yr.	\$104	6. 4½%	2 yr.	\$327
2. 3%	2 yr.	\$106	7. 5½%	4 yr.	\$336
3. 5%	3 yr.	\$230	8. 6%	6 mo.	\$412
4. 6%	1½ yr.	\$218	9. 7½%	2 yr.	\$345
5. 7%	2 yr.	\$228	10. 3%	3 yr.	\$436

Oral Problems.

1. What principal will amount to $102 in 6 mo. at 4%?
2. What principal will amount to $530 in 1 yr. 6 mo. at 4%?
3. What principal will amount to $888 in 2 yr. at 5½%?
4. What principal will amount to $600 in 2 yr. 6 mo. at 8%?
5. What principal will amount to $460 in 3 yr. at 5%?

Written Exercises.

Find the principal:

	Rate.	Time.	Amount.		Rate.	Time.	Amount.
1.	6 %	219 da.	$ 238.28	11.	3½ %	1 yr. 40 da.	$1 365.03
2.	5 %	100 da.	$ 888	12.	6½ %	2 yr. 146 da.	$ 462.40
3.	5 %	146 da.	$ 408	13.	5½ %	1 yr. 20 da.	$1 699.17
4.	7 %	200 da.	$ 833 80	14.	4½ %	1 yr. 219 da.	$ 423.44
5.	7½ %	126 da.	$1 198.24	15.	5 %	1 yr. 150 da.	$ 390.75
6.	5½ %	70 da.	$2 065 56	16.	4 %	1 yr. 165 da.	$1 081.36
7.	5 %	73 da.	$ 203.01	17.	5 %	1 yr. 300 da.	$1 991.25
8.	6 %	292 da.	$1 241.88	18.	5 %	1 yr. 73 da.	$ 508.80
9.	7 %	93 da.	$ 371.51	19.	6 %	2 yr. 292 da.	$1 168
10.	4½ %	200 da.	$1 570.80	20.	4 %	1 yr. 101 da.	$ 767.28

Written Problems.

1. What principal will amount to $670.80 in 292 da., if loaned at 4%?

2. On Apr. 1, 1918, I borrowed a sum of money at 5% per annum, and on Dec. 2, 1918, I paid $3 470.70 in full of interest and principal. What was the sum borrowed?

3. What principal loaned at 7% per annum will amount to $1 486.04 in 93 days?

4. What principal loaned May 31, 1917, at 6% per annum will amount to $2 652.02 on Jan. 17, 1918?

5. What sum of money loaned from May 31, 1917, to Dec. 2, 1918, at 7% per annum, will amount to $8 150.70?

REVIEW OF INTEREST.

1. A man borrows $10 000 at 5% per annum. After how many days will the principal amount to $10 600?

2. What principal loaned for 183 da. at 6% per annum, will amount to $1 127.04?

3. An investment of $17 885 amounted to $19 600 after 1 yr. 135 da. What was the annual rate of interest on the investment?

4. In what time will $657 loaned at $4\frac{1}{2}\%$ amount to $678.06?

5. A principal of $1 387 yielded $98.80 interest from July 2, 1918, to Aug. 6, 1919. What was the rate of interest?

6. How much money must be put at interest at 5%, Jan. 7, 1917, so that on Mar. 11, 1920, $150.67 interest may be due?

7. I may settle an account by paying $950 on Mar. 5, 1918, or $1 000 on June 5, 1918. If I borrow $950 at 7% in order to pay on Mar. 5, 1918, what sum shall I have gained on June 5, 1918?

8. I borrowed $650 on May 1, 1918, at 5%, and $1 000 on July 2, 1918, at $4\frac{1}{2}\%$. I paid the amount of both debts on Dec. 16, 1918. How much did I pay?

INCOME ON REAL ESTATE.

9. The entire cost of a house is $3 600. What annual rent will represent $6\frac{1}{2}\%$ interest on the investment?

10. A block worth $36 000 is occupied by 12 tenants. How much rent must each tenant pay annually, so that the landlord may realize 8% on his investment?

11. My father bought a house for $6 000. The annual expenses for taxes, insurance, and repairs being $120, what monthly rent must he receive to net 6% interest on his investment?

12. A house worth $5 000 costs every year $40 for repairs, $90 for taxes, and $12 for insurance. What monthly rent will represent a net income of 7% per annum on the investment?

13. A house worth $4 000 is occupied by 2 tenants. Put down $104 for yearly expenses, and figure out what monthly rent each tenant must pay so that the landlord may net 7% interest on his investment.

14. A house which cost $14 750 is rented for $1 464 per annum. What is the net rate of interest on the investment, if the average annual expenses are $357.75?

15. A landlord sets apart 10% of his rental for good works, 75% of it for his personal expenses, and has $675 left. What is the value of his property, if it yields $4\frac{1}{2}$% annual interest?

16. TENANT OF LANDLORD?

1. I suppose your father pays a monthly rent of $20 to the owner of the house you live in. Then find what the annual rent amounts to. If your father has $3 000 savings which yield 5% annually, how much more than the yearly interest does your father pay for rent?

2. If the house you inhabit cost $5 500, and besides yourself contains a tenant whose monthly rent is $22, what net rate of interest does it yield, the annual expenses averaging $119?

3. Your father buys this house for $5 500, paying $3 000 cash, and binding himself to pay 5% interest on the balance, $2 500. Now your father has no rent to pay, and he receives $22 a month from his tenant; but he loses the interest on his $3 000, and he pays on an average $119 every year for taxes, insurance and repairs. Does he gain or lose, and how much a year?

4. What net rate of interest on its value ($5 500) does the house yield now?

5. If, after two years, your father sold the house for $6 000, what net rate per annum on its value ($5 500) would the house have yielded during those two years?

COMPOUND INTEREST.

Mr. Murphy loans $200 to Mr. Fallon, at 6% per annum. At the end of the year, Mr. Fallon says to Mr. Murphy: "I am not able to pay the $12 interest which I owe you; but you may add them to the principal $200, and next year I shall pay you interest on $212."

359. When the interest as it becomes due is added to the principal, and the total bears interest, the lender is said to receive *compound interest.*

Written Problems.

1. What is the amount of $200 for 2 years at 6% compound interest?

2. Find the difference between the compound interest and the simple interest of $300 for 3 years at 3%.

3. Required the compound interest of $4 000 for 3 years at 5%.

4. What sum must be paid after 3 years to discharge a debt of $500 bearing interest at 4%, if each year the interest was added to the principal?

5. Find the amount of $500 for 5 years at 5% compound interest.

A MORTGAGE.

360. A *mortgage* is a conditional conveyance of property as security for the payment of a debt.

361. Mr. Quinlan wishes to begin some business for which he needs $3 000 in cash; he has no money; he only possesses a house worth $10 000, but he does not want to sell it. So he borrows $3 000 from Mr. Hubert who agrees to the loan, at a certain rate of interest, and in consideration of a mortgage on Mr. Quinlan's house.

Mr. Quinlan and Mr. Hubert meet at a notary's where the *mortgage deed* is drawn up; it is then recorded and kept at the registry of the county town.

When the obligation falls due, if Mr. Quinlan pays back the $3 000, the mortgage premises are redeemed, that is, the mortgage is canceled. Should the interest and debt not be properly paid, the mortgage may be *foreclosed*, and the property is then sold by the sheriff to the highest bidder, and the mortgage paid off from the proceeds.

362. It is very important to loan money only on a *first mortgage*, as the second mortgage can not be paid before the first is fully paid, and hence may not be a very good security.

Questions on Theory.

1. What is interest? (345).

2. What is the legal rate of interest in Canada? (347).

3. What is principal? (346).

4. What distinguishes interest problems from ordinary percentage problems? (348).

5. How do you find the interest when you have the principal, the rate per annum, and the time 1° in years? 2° in days? 3° in years and days? (349).

6. How do you find the amount when you have the principal, the rate per annum, and the time 1° in years? 2° in days? 3° in years and days? (353).

7. Given the principal, the interest, and the rate per annum, how do you find the time 1° in years? 2° in days? (356).

8. How do you find the rate per annum, given the principal, the interest, and the time 1° in years? 2° in days? 3° in years and days? (355).

9. How do you find the principal when you have the rate per annum, the interest, and the time 1° in years? 2° in days? 3° in years and days? (357).

10. How do you find the principal when you have the amount, the rate per annum, and the time 1° in years? 2° in days? 3° in years and days? (358).

11. Given the principal and the interest, how do you find the amount? (353).

12. The amount and the interest being given, how do you find the principal?

13. How do you find the rate per annum when you have the principal, the amount, and the time 1° in years? 2° in days? 3° in years and days?

14. Given the principal, the amount, and the rate per annum, how do you find the time?

15. Given the amount, the time, and the rate per annum, how do you find the interest?

16. When does money bear compound interest? (359).

17. Is it possible to borrow large sums of money without giving some kind of security?

18. What is a mortgage? (360).

19. Give an example of how a mortgage is given, and canceled. (361).

20. Is a second mortgage a very good security? (362).

COMMERCIAL PAPER AND BANKING

363. Commercial paper embraces written instruments that circulate as money. The *cheque* and the *promissory note* are the two most important.

THE CHEQUE.

Let me suppose that you begin business in Montreal to-day. You have $3 000 in bank notes which you wish to deposit in the Provincial Bank of Canada. You will first obtain the manager's consent to your having an account with the bank he represents. This done, wou will make out a deposit slip, and present it with your money to the receiving teller. The latter

DEPOSIT SLIP.

PROVINCIAL BANK
OF CANADA

Date........(*a*)............

Credit M............(*b*)............

Residence............... (*c*).......

Cheques.

(On which bank)

... . (*d*)

...........(*e*)..........

....

Total of cheques (*f*)

Bills.

× 1 =
× 2 =
× 4 =
× 5 =
(*g*) × 10 = (*h*)
× 20 =
× 25 =
× 50 =
× 100 =
× 500 =

Total of bills (*i*)

Specie, etc........... (*j*)

Gold................ (*k*)

Total (*l*)

..........(*m*)............*Dollars*

Signature.........(*n*) ....

will make you sign your name in a special book wherein the signatures of all depositors are kept for identification. Then he will give you a pass book in which the amount of your deposit is written, and a cheque book containing blank cheques and stubs. Now you are entitled to make your payments by drawing cheques on the bank.

364. A *deposit slip* is a detailed record of the paper or specie deposited.

EXPLANATIONS.—(*a*) The date; (*b*) your name; (*c*) your address; (*d*) the amount of each cheque deposited, and (*e*) the name of the bank on which it is drawn; (*f*) the total amount of the cheques deposited; (*g*) number of bank notes of \$1, \$2, \$4, etc; (*h*) value in notes of \$1, \$2, \$4, etc.; (*i*) total amount of all the bank notes; (*j*) amount in specie, etc.; (*k*) amount in gold; (*l*) the total amount of your deposit in figures; (*m*) the total amount of your deposit in words; (*n*) your signature; it must always be like that you left in the signature book.

365. A *cheque* is an order on a bank for money drawn by one who has funds on deposit.

No. ......(*f*)......	$ ....(*b*).... Montreal, ...(*a*)...... 19
......(*a*)...... 19	**Provincial Bank of Canada**
(*c*)	Pay to ......(*c*)...... or order,
(*e*)	the sum of ......(*d*)...... $\frac{}{100}$ Dollars,
(*b*)	......(*e*)......
....	No. ....(*f*).... ......(*g*)......

EXPLANATIONS.—(*a*) The date; (*b*) the amount in figures; (*c*) the name of the *payee;* (*d*) the amount in words; (*e*) what the cheque was given for: *the rent of May, on account,* etc.; (*f*) the number of the cheque; (*g*) the signature of the *drawer;* it must always be identical to that left in the signature book.

A cheque is usually drawn to the *order* of the payee, who to collect it must *endorse* it, that is, write his name across the back. The payee of a cheque must, moreover, be known of the banker to whom he applies for payment. If the depositor wishes to draw out money for himself, he will make the cheque payable to "Self". A cheque is always payable on demand. It is important so to carefully fill out the stub, that it may contain the material facts of the cheque for reference.

Written Exercises.

1. Draw a cheque for $100 to the order of John Mackay in payment of May's rent.

2. Draw a cheque for $200 in favor of yourself.

3. Make out a deposit slip for the following: 1 cheque on the Hochelaga Bank, $200; Bank notes: 5 of $1; 3 of $10; 4 of $25; Silver: $10.

4. Make out a deposit slip for the following: 1 cheque on the Merchants Bank, $400; Bank notes: 6 of $1; 3 of $2; 7 of $5; 4 of $50; Silver: $7.50.

5. Draw a cheque for $15 in favor of some Catholic enterprise that you wish to patronize.

THE PROMISSORY NOTE.

You buy $500 worth of goods of Sanche & Leblanc on 60 days' credit, and you remit them a written promise to pay the sum due at maturity; you have given them a promissory note.

366. A *promissory note* is a written promise to pay to a person named, at a specified time, a certain sum of money

> *No.* ......(h)...... *$* ......(e)...... *Due* ......(i)......
>
> *Montreal,* ....(a).................. *19*
>
> *Sixty days* (b) *after date, for value received, I promise to pay to the order of* *Sanche & Leblanc* (c) *at the office of*
>
> THE PROVINCIAL BANK OF CANADA
> 7 PLACE D'ARMES
>
> *Five Hundred* (d)———————— $\frac{xx}{100}$ *Dollars,*
>
> ......(j)...................... .... (f) ....
>
> .... (g) ....

EXPLANATIONS.—(*a*) The date, (*b*) the time; (*c*) the name of the payees; (*d*) the amount in words (the *par value* or *face* of the note); (*e*) the amount in figures; (*f*) your signature; (*g*) your address; (*h*) the number of the note; (*i*) the maturity, that is, the date at which the note falls due; in this case, it will be 63 days after the date of issue. Three *days of grace* are allowed by law; they are added to the 60 days specified in the note. (*j*) If the words *with interest at....%* are omitted, the note will not draw interest until it is due. After it is due, it will draw interest at the legal rate.

367. When the merchant allows a term of credit, as Sanche & Leblanc did, he does not exact an interest-bearing note. But, outside of this case, merchants and loaners accept none but interest-bearing notes. Sometimes the interest is computed beforehand and added to the sum due to form the amount of the note, which then, of course, does not draw interest.

A note is generally made payable to the *order* of the payee; if he wishes to sell it, he must endorse it.

368. The place of payment of a note is usually a bank. If the above note had been dated May 3, 1917, it would have been due 63 days later, on July 5. If it had been payable 2 months after date, it would have matured 3 days after July 3, on July 6.

Written Exercises.

1. Find the date of maturity on: 1° a note dated May 7, 1917, and given for 90 da.; 2° a note dated Sept. 30, 1917, and given for 2 mo.; 3° a note dated Dec. 30, 1917, and given for 90 da.; 4° a note dated Sept. 3, 1917, and given for 60 da.; 5° a note dated Feb. 20, 1917, and given for 3 mo.

2. Make out a 90-day note for $150, dated to-day, payable to C. N. Prairie's order at some bank.

3. Make out a 60-day note for $300, dated to-day, payable to Roy Robert's order at some bank.

4. Make out a 7% interest-bearing note for $225.50, dated to-day, payable in 6 mo. to O. Kennedy's order at some bank.

5. Make out a 7% interest-bearing note for $301.60, dated to-day, payable in 3 mo. to J. Tarrant's order at some bank.

6. What is the amount due June 4, 1917, on a note of $500, dated Apr. 2, 1917, and drawing interest at 7% per annum?

7. Find the amount due at maturity on a note of $150, dated Feb. 25, 1918, bearing interest at 6%, and payable 90 days after date.

8. Find the amount due at maturity on a 30-day note of $600, dated June 3, 1918, and bearing interest at 7%.

9. Find the amount due at maturity on a note of $800, dated Dec. 2, 1918, bearing interest at 6%, and payable 6 mo. after date.

NOTE.—Compute the interest for 6 mo. plus 3 da., or 183 days.

10. Find the amount due at maturity on a note of $750, dated Feb. 5, 1918, and payable 3 mo. after date with interest at 7%.

NOTE.—Compute the interest for 3 mo. plus 3 da., or 93 days.

BANK DISCOUNT

369. Suppose to-day is the 14th of March 1918, and that you have the following note in your hands.

500\frac{XX}{100}$.	Montreal, Mar. 14, 1918.
Ninety days after date, I promise to pay to the order of (*your name is written here*), the sum of Five Hundred Dollars $\frac{XX}{100}$. Value received.	
Due June 15, 1918.	S. A. Hart, 721, Bloomfield Avenue.

This note is in your favor, but it will be paid by S. A. Hart only on June 15, 1918. You need money at once: then you will apply to your banker and ask him to buy the note, that is, to *discount* it. If the banker is willing to do so, he will not pay you $500 for your note, but $500 minus the interest of $500 for 93 days at 7%, which is $8.92. This interest deducted in advance is called *bank discount*. You will receive $500 minus $8.92, or $491.08. This is the price for which you sold the note; it is the *net proceeds* of the note.

370. Bank discount is computed on the *face value* of a note which does not bear interest, and on the *face value plus its accrued interest at maturity* of a note which bears interest: the *base* of bank discount is always the sum due at maturity. So, bank discount is the simple interest of the sum due at maturity, computed for the number of days between the date of discount and the date of maturity.

Written Exercises.

Find the *date of maturity, term of discount, bank discount,* and *net proceeds* of the following:

	Face of note.	Date of note. 1918	Time.	Date of discount. 1918	Rate of discount.
1.	$1 800	Feb. 2	30 da.	Feb. 18	6 %
2.	$2 500	Mar. 4	60 da.	Apr. 16	6 %
3.	$1 630	Apr. 15	90 da.	June 19	7 %
4.	$1 820	May 3	4 mo.	July 23	5 %
5.	$1 240	June 15	6 mo.	Sept. 4	6½%
6.	$3 545	July 30	2 mo.	Aug. 7	7 %
7.	$6 000	Aug. 31	3 mo.	Sept. 25	7½%
8.	$1 475	Sept. 3	5 mo.	Sept. 3	6 %
9.	$3 240	Oct. 21	30 da.	Oct. 31	7 %
10.	$3 100	Nov. 30	60 da.	Dec. 24	5 %

Find the *interest, amount due at maturity, discount,* and *net proceeds* of the following:

	Face of note.	Date of note. 1919	Time.	Rate of int.	Date of discount. 1919	Rate of disc't.
1.	$1 200	Jan. 17	90 da.	6 %	Jan. 31	7 %
2.	$1 500	Feb. 8	60 da.	5 %	Feb. 28	6 %
3.	$1 620	Mar. 31	4 mo.	7 %	Apr. 4	7 %
4.	$1 800	Apr. 22	6 mo.	5½%	May 20	7½%
5.	$2 100	June 10	30 da.	6 %	June 17	7 %
6.	$3 500	July 29	90 da.	7 %	Aug. 5	7½%
7.	$1 750	Sept. 9	60 da.	7 %	Sept. 30	7½%
8.	$1 150	Oct. 7	3 mo.	5 %	Oct. 21	7 %
9.	$ 412.50	Nov. 18	2 mo.	6 %	Nov. 25	6½%
10.	$ 875.50	Dec. 30	90 da.	5 %	Dec. 31	7 %

Questions on Theory.

1. What is meant by commercial paper? Name two kinds. (363).

2. Say how you would proceed to open an account with a bank.

3. What is a deposit slip? What does it contain? (364).

4. What is a cheque? What entities a person to draw cheques on a bank? (365).

5. What does a cheque contain? the stub?

6. What is a promissory note? (366).

7. Do all notes draw interest? (367).

8. How do you find the maturity of a note given for 60 days? 2 months?

9. What are *days of grace*?

10. Is interest computed on the 3 days of grace?

11. What is discounting a note? (369).

12. What is bank discount? (369).

13. How is discount computed on a note that does not draw interest? (370).

14. How is discount computed on an interest-bearing note? (370).

TAXES

Much money is needed to administer a city, a municipality: the streets must be paved, repaired, cleaned, lighted; the police and fire departments must be kept up; public buildings must be constructed and cared for; a salary must be paid to public officers, etc.

371. To defray all their expenses, municipalities annually levy a *certain per cent* on the real property belonging to each citizen: this is the *municipal tax.*

For educational purposes, a *school tax* is levied on real property.

Let us apply the principles of percentage to property taxes.

The Base is the *assessed value of the property.*

The Rate is the *per cent of tax.*

The Percentage is the *tax.*

372. Rule.—

Assessed value × Rate = Tax.
Tax ÷ Assessed value = Rate.
Tax ÷ Rate = Assessed value.

Written Problems.

1. The municipal tax in Montreal is 1%; moreover, Catholics pay a school tax of $\frac{4}{10}$%. What must a Catholic citizen pay on $20 000 worth of property?

2. Using the data in the preceding problem, what is the entire tax of a Catholic citizen whose property is assessed at $45 000?

3. What is the entire tax of a Protestant citizen whose property in Montreal is assessed at $50 000, the school tax for Protestants being $\frac{1}{2}$%?

4. A town has an assessed valuation of $3 200 000, and it has to raise $16 000 by taxation. What is the rate?

5. The taxable property in a certain town is valued at $4 500 000, and a tax of $9 000 is voted for school purposes. What is the rate of taxation?

6. My school taxes amount to $30; the tax rate being $\frac{4}{10}$%, find the assessed valuation of my property.

7. In Quebec City, 1915, the school taxes paid by the Catholics amounted to $235 600. Find the assessed valuation of their property, if the rate of taxation was .38%.

8. In 1915, the taxes levied for the Catholic schools of Outremont amounted to $22 325. What was the rate of taxation on $8 930 000 worth of property?

9. The taxable property in Joliette is assessed at $2 600 000. What do the school taxes amount to at $\frac{3}{8}$%?

10. Your father's property is assessed at $9 000 and has a frontage of 40 ft. on the street. The municipal tax is 1%, the school tax $\frac{3}{8}$%, the special tax .02$\frac{1}{2}$%; moreover 5c per foot of frontage must be paid for the clearing off of snow. How much does your father annually pay for taxes?

INSURANCE

It would be a great misfortune to your family if fire burned down the house you live in; but this misfortune would be less hard to bear, if your parents received from some insurance company a sum of money nearly equal to the loss sustained.

373. By annually paying a small sum of money called a *premium*, any person may protect his property against fire or other perils.

374. A *policy* is the written agreement of an insurance company to pay a certain amount in case of loss.

375. The premium is usually estimated at a certain per cent of the amount insured. Rates vary according to the risk involved.

Let us apply the principles of percentage to insurance.

The Base is the amount insured, or *face of the policy.*

The Rate is the *per cent of premium.*

The Percentage is the *premium.*

376. Rule.—

Face of policy × Rate = Premium.
Premium ÷ Face of policy = Rate.
Premium ÷ Rate = Face of policy.

Written Problems.

1. A house was insured for $12 000 at a premium of 2%. What was the cost of insurance?

2. A factory worth $140 000 is insured for $\frac{4}{5}$ of its value at $1\frac{3}{10}$%. What is the premium?

3. A stock of goods valued at $45 000 is insured for $\frac{4}{5}$ of its value at $\frac{13}{20}$%. How much is the premium? In case of total loss, how much would the owner receive?

4. If a premium of \$12 is paid for an insurance of \$3 000 on a house, what is the rate of insurance?

5. If a three-year policy for \$5 500 costs \$41.25, what is the rate of premium per annum?

6. I insure a house worth \$4 800 for $\frac{3}{4}$ of its value, paying as premium \$21. What is the rate of premium?

7. [illegible] id \$192 premium on a house insured for $\frac{4}{5}$ of its value a[illegible] 1%. Find the face of the policy.

8. A schooner is insured for $\frac{3}{4}$ of its value at $3\frac{1}{4}$%. The premium is \$585. What is the face of the policy? What is the value of the schooner?

9. I may insure my house for \$6 000 at $\frac{3}{8}$% a year, or at $1\frac{1}{2}$% for 3 years. In the latter case, how much would be saved in 3 years?

10. A farmer insures his house for \$2 800 at $1\frac{1}{4}$%, his barns for \$1 600 at $\frac{7}{8}$%, and his furniture for \$800 at 1%. What is his annual premium?

NOTE.—There are also life insurance policies. A prud[illegible]nt man must be careful to insure his life, and thereby provide for the future welfare of those who may be dependent u[illegible]n him. This is not only provident care of futurity, it is econom[illegible].

Premiums in life insurance are paid on \$1 000 and vary with the age of the person insured. The younger one is, the lower is the premium.

In the "Sauvegarde", the annual premium payable for life on \$1 000 is \$19.40 at the age of 21; \$24.25 at the age of 30; \$3[illegible] the age of 40.

[illegible] ates for the following problems.

11. [illegible] n of money should be laid aside monthly by a person ag[illegible], to pay the annual premium on a life policy of \$6 000?

12. Suppose this person died after paying his 20th annual premium; then what sum did his heirs realize above the cost of insurance?

13. Answer the preceding question, should 40 annual premiums have been paid.

14. A man aged 30 took out a life policy of $5 000. How much premium did he pay in 20 years?

15. A man aged 40 took out a $4 000 life policy. What was the total cost of his insurance, if he died after making 15 annual payments?

STOCKS AND BONDS

STOCKS.

377. Twelve pupils of the Sixth Year organize a baseball club. They figure out that uniforms will cost $32, and bats and balls, $16. All parts being equal, each member should contribute $\frac{1}{12}$ of $48, or $4. As the members can not all afford to pay this sum, they divide the $48 into 48 shares of $1 each, and sell to each as many shares as he may be willing to buy.

Francis, the captain, buys 12 shares, and pays in $12. Elmer takes 8 shares, and pays in $8. The other members take 5, 4, 3, or 2 shares, paying $1 for each.

The boys play several games that bring in nice receipts; they pay off some sundry expenses, and divide the net receipts among themselves, not equally, but according to the number of shares of each.

To insure greater profits for the future, Elmer tries to buy some of the shares that Francis has. The latter refuses to sell any for less than $1.05; he says that they are now *above par*.

378. These pupils have formed a real *joint stock company*. The *capital stock* is $48; there are 48 *shares of stock*, and the *par value* of each is $1. The net earnings that were distributed to the stockholders proportionally to the number of shares held by them, are the *dividends*. And the playing business is so profitable that the *market value* of the shares of stock is *above par*, at a quotation of 105.

Banks, insurance and railroad companies, commercial and industrial corporations are formed in this way, for real business purposes.

379. The par value of stock is ordinarily $100. The market value of a share (*its quotation*) is the sum for which it can be bought or sold. If the corporation is doing excellent business and pays a good rate of dividend, a share may be sold for more than its par value, that is, above par, or *at a premium*. On the other hand, stock in an unprosperous corporation that pays a low rate of dividend, may have a market value less than its par value; it is then said to be below par, or *at a discount*.

380. Stock operations are carried on through *brokers*, at a place called *Stock Exchange*. The broker's *commission* is usually $\frac{1}{8}$% of the par value, or $12\frac{1}{2}$c per share of \$100.

Besides, in Quebec, the seller must pay the Provincial Government a tax of 2c on every \$100 share sold. Thus, a share bought at a quotation of 118 (*the par value being \$100*) will cost \$118 + \$0.125, or \$118.125; and a share sold at a quotation of 120 (*the par value being \$100*) will net \$120 — \$0.145 (\$0.125 + \$0.02), or \$119.855.

But the dividend will be paid at the office of the corporation on the par value of each share, whatever may be the market value of the stock.

In the following problems, the par value of each share is \$100.

EXAMPLE I.—Find the cost of 20 shares of stock bought at $137\frac{1}{8}$, brokerage $\frac{1}{8}$%.

OPERATION.

\$137.125 + \$0.125 = \$137.25;
\$137.25 × 20 = \$2 745.

EXPLANATION.— One share costs \$$137\frac{1}{8}$ + \$$\frac{1}{8}$, or \$$137\frac{1}{4}$; 20 shares cost 20 times \$$137\frac{1}{4}$, or \$2 745.

EXAMPLE II.—I sold 40 shares of stock at $96\frac{5}{8}$, brokerage $\frac{1}{8}$%. Find the net proceeds of the sale.

OPERATION.

\$96.625 – (\$0.125 + \$0.02) = \$96.48; \$96.48 × 40 = \$3 859.20.

EXPLANATION. — One share brings \$$96\frac{5}{8}$ minus \$$\frac{1}{8}$ brokerage and 2c tax, or \$96.48; 40 shares bring 40 times \$96.48, or \$3 859.20.

EXAMPLE III.—A company declares a dividend of 6%. What is my income on 75 shares of stock?

OPERATION.

\$6 × 75 = \$450.

EXPLANATION.—The dividend is computed on the par value (\$100); 6% of \$100 = \$6, dividend of 1 share; 75 shares yield 75 times \$6, or \$450.

Written Problems.

1. Find the cost of 10 shares of Canadian Pacific R. R. stock quoted at 161, brokerage $\frac{1}{8}$%.

2. I buy 20 shares of New York Central stock quoted at $85\frac{1}{8}$, brokerage $\frac{1}{8}$%. What does the stock cost?

3. A speculator sold 40 shares of Hochelaga Bank stock at 147, brokerage $\frac{1}{8}\%$, and tax 2c. Find the net proceeds of the sale.

4. I sell 30 shares of Montreal Bank stock at 224, brokerage $\frac{1}{8}\%$, tax 2c. Find the net proceeds.

5. I bought 15 shares of Quebec Bank stock when it was quoted at 135, brokerage $\frac{1}{8}\%$, and sold it at 142, brokerage $\frac{1}{8}\%$, tax 2c. What was my net gain?

NOTE.—Fractions of a share cannot be bought.

6. How many shares of Hochelaga Bank stock, at 146, can be bought for $5 000, brokerage $\frac{1}{8}\%$?

7. I bought 30 shares of Crown Reserve at 172, brokerage $\frac{1}{8}\%$, and sold them at 175, brokerage $\frac{1}{8}\%$, tax 2c. Find my gain.

8. The Bank of Montreal pays semiannual dividends of 5%. What is my yearly income on 15 shares of stock?

9. The Canadian Pacific R. R. issues quarterly dividends of $2\frac{1}{2}\%$. What is my yearly income on 25 shares of stock?

10. I bought 10 shares of Hochelaga Bank stock at 145, brokerage $\frac{1}{8}\%$, received thereon a dividend of 9%, and then sold out at 147 brokerage $\frac{1}{8}\%$, tax 2c. What was my total gain?

BONDS.

381. *Bonds* are written promises, made under seal, to pay a certain sum of money at a specified time. When the Government, or a municipality, or any incorporated company wishes to borrow a considerable sum of money for some enterprise, bonds are prepared and sold. These bonds are simply *promissory notes* issued under seal. They bear a fixed rate of interest payable annually, semiannually, or quarterly. Some bonds are called *debentures*.

Government and municipal bonds rest upon the honor of the country or city that issues them. Commercial bonds are secured by mortgage on the property of the corporation issuing them; this means that the holders of the bonds may sell the property mortgaged if the bonds and the interest are not paid.

382. DIFFERENCE BETWEEN STOCKHOLDERS AND BONDHOLDERS.—The stockholders are the *owners* of a corporation; the bondholders are its *creditors*, that is, its money lenders. The

rate of interest on bonds is always the same. The dividend of stocks depends on the net profits of the corporation after the interest on the bonds has been paid.

Computations in both cases are the same.

11. Find the cost of 48 bonds quoted at 92, brokerage $\frac{1}{8}\%$.

12. I sold 25 bonds at 95, brokerage $\frac{1}{8}\%$, tax 2c. Find the net proceeds.

13. I own 150 bonds bearing 5% interest. Find my annual income.

14. I invested $5 000 in 5% bonds, quoted at 96, brokerage $\frac{1}{8}\%$. Find the unexpended balance, and my yearly income.

15. I invested $15 000 in $6\frac{3}{4}\%$ bonds quoted at 92, brokerage $\frac{1}{8}\%$. Find the unexpended balance, and my yearly income.

Questions on Theory.

(*Taxes, insurance, stocks and bonds*).

1. What are municipal taxes? (371).
2. How are taxes computed? (372).
3. What is an insurance policy? (374).
4. What is the premium? (373).
5. How is the premium computed? (375, 376).
6. Give an example to show how corporations are formed. (377).
7. Name the place where stock operations are carried on. (380)
8. What is a dividend? (378).
9. When is stock *at a premium? at a discount?* (379).
10. What difference is there between a stockholder and a bondholder? (382).

RATIO AND PROPORTION

The quotient of 14 ÷ 7 is 2; comparing 14 with 7, we say that 14 is 2 times 7; 2 *times* is the expression of a ratio.

383. The **ratio** or *relation* of two numbers is the quotient of the first by the second.

The ratio of 12 to 3 is 12 ÷ 3, or $\frac{12}{3}$, or $\frac{4}{1}$. The ratio of 3 to 12 is 3 ÷ 12, or $\frac{3}{12}$, or $\frac{1}{4}$. Notice that reversing the order of the numbers *inverts* the ratio.

384. The two numbers of a ratio are called *terms.* The first is the *numerator* or *antecedent;* the second is the *denominator* or *consequent.*

385. Ratios are common fractions: all the rules and principles relative to common fractions may be applied to ratios.

The ratio of 9 to 17 may be written $\frac{9}{17}$, or 9 : 17. We shall employ the latter form.

386. An equality of ratios is a **proportion.**

Thus, $\frac{1}{3} = \frac{5}{15}$ is a proportion; it may be read: 1 is to 3 as 5 is to 15, and be written: 1 : 3 :: 5 : 15. The terms 1 and 15 are called *extremes;* 3 and 5, *means.*

387. You see that the product of the extremes is equal to the product of the means (*fundamental principle*).

To find an omitted term of a proportion.

NOTE.—The missing term is replaced by x.

EXAMPLE.—36 : 6 :: 24 : x.

OPERATION.

$$36 \times x = 24 \times 6$$
$$x = \frac{24 \times 6}{36}$$
$$x = 4.$$

EXPLANATION.—Since the product of the extremes is equal to the product of the means, 36 times x equals 24 times 6. One time x equals 36 times less than 24 × 6, or $\frac{24 \times 6}{36}$. Solving, $x = 4$.

388. Rule.—*Divide the product of the means by the given extreme; or divide the product of the extremes by the given mean.*

Written Exercises.

Find the value of x in the following proportions:

1. 4 : 26 :: 10 : x.
2. 36 : 18 :: 12 : x.
3. 48 : 20 :: 120 : x.
4. x : 16 :: 18 : 9.
5. x : 30 :: 8 : 48.
6. x : 18 :: 30 : 20.
7. 100 : x :: 50 : 75.
8. 65 : x :: 45 : 9.
9. 30 : x :: 10 : 9.
10. 18 : 9 :: x : 27.
11. 40 : 5 :: x : 9.
12. 16 : 32 :: x : 4.
13. 6 : 15 :: x : 75.
14. 18 : 4 :: 24 : x.
15. 16 : 28 :: $\frac{4}{7}$: x.
16. $\frac{3}{7}$: 5 :: x : 40.
17. 6 : 15 :: $3\frac{3}{5}$: x.
18. 6 : 10 :: x : $\frac{1}{2}$.
19. $\frac{5}{8}$: x :: 25 : 8.
20. $\frac{3}{5}$: $\frac{2}{3}$ :: 9 : x.

CAUSE AND EFFECT.

389. The application of proportion to problems consists in comparing two causes and two effects.

Example.—If 5 horses (*1st cause*) eat 10 tons of hay (*1st effect*), 10 horses (*2d cause*) will eat 20 tons of hay (*2d effect*).

390. A *cause* is that which produces a result; it is all that is needed to do a work: men, time, animals, land, capital, material, etc.

391. An *effect* is that which is produced; it is the work done, the money yielded, etc.

392. The cause and the effect are *simple* when they contain only one element; they are *compound* when they contain several elements.

When I say: "10 men cut 100 cords of wood", the cause and the effect are simple. When I say: "10 men in 4 days cut 100 cords of wood", the cause (10 *men in 4 days*) is compound; the effect (100 *cords of wood*) is simple.

393. Causes and effects expressing quantities may be represented by numbers; these numbers will be related to each other as the things they represent; again, like causes should produce like effects, and the effects should be in proportion to their causes, namely:

1st cause : 2d cause : : 1st effect : 2d effect.

SIMPLE PROPORTION.

394. A proportion is simple when its four terms consist of single numbers. Thus, 8 : 12 :: 16 : 24 is a simple proportion.

Example I.—If 30 acres of land yield 1 650 bushels of corn, how many bushels of corn will 60 acres yield?

Operation.

acres		acres		bushels		bushels
30	:	60	::	1650	:	x
1st cause		2d cause		1st effect		2d effect

$$30 \times x = 60 \times 1650$$

$$x = \frac{60 \times 1650}{30}$$

$$x = 3\,300 \text{ bushels.}$$

Explanation. — The acres are the causes; the bushels are the effects; and the 1st cause is to the 2d cause as the 1st effect is to the 2d effect. The product of the extremes is equal to the product of the means. Solving, $x = 3\,300$ bushels.

EXAMPLE II.—If 3 horses can draw 10 tons, how many horses will be required to draw 30 tons?

OPERATION.

horses		horses		tons		tons
3	:	x	::	10	:	30
1st cause		2d cause		1st effect		2d effect

$$10x = 3 \times 30$$

$$x = \frac{3 \times 30}{10}$$

$$x = 9 \text{ horses.}$$

EXPLANATION.—The horses are the causes; the tons are the effects; and the 1st cause is to the 2d cause as the 1st effect is to the 2d effect. The product of the means is equal to the product of the extremes. Solving, $x = 9$ horses.

Written Problems.

1. If 8 volumes of a work cost $36, find the cost per dozen.

2. If 100 bu. of wheat cost $96, how much must be paid for 75 bushels of the same kind of wheat?

3. If 35 acres of a field will produce 875 bu. of wheat, how many bushels of wheat will 87 acres of the same field produce?

4. If 4 bbl. of flour can be made from 18 bu. of wheat, how many barrels of flour can be made from 207 bu. of wheat?

5. If a locomotive vaporizes 1 200 gal. of water in running 90 mi., how many gallons of water will be required for a 300-mile trip?

6. If 20 lb. of cream make 4 lb. of butter, how much cream will be required for 180 lb. of butter?

7. A stick 5 ft. high casts a shadow of 2 ft. How high is a tree which, at the same time, casts a shadow of 36 ft.?

8. If for cattle, 55 lb. of potatoes have the same nutritive value as 60 lbs. of beets, how many pounds of beets should I take to replace 8 800 lb. of potatoes?

9. A farm boy was to receive $50 for 60 days' work. What is due him if he leaves after 9 days?

10. A fountain gives 30 gal. of water in 45 minutes. In what time will it fill a 2 500-gallon tank?

11. A farmer plows 9 furrows in 63 minutes. How many furrows will he plow in 2 hours and 48 minutes?

12. If 220 lb. of wood produce 50 lb. of charcoal, how many pounds of wood will be required for 2 500 lb. of charcoal?

13. How many pounds of bread can be made from 196 lb. of flour, if 27 lb. of bread are made from 20 lb. of flour?

14. If 17 lb. of paper are made from 25 lb. of rags, how many pounds of rags would be required to make 8 500 lb. of paper?

15. What would $2\frac{1}{2}$ yd. of cloth cost, if I paid \$1.50 for $\frac{5}{8}$ of a yard?

16. If $2\frac{3}{4}$ bbl. of beef cost \$20.75, how much shall I pay for $7\frac{1}{2}$ bbl.?

17. A farmer sowed 6 bu. of wheat on $4\frac{4}{5}$ acres of land. How many bushels of wheat would he need for $13\frac{1}{2}$ acres of land?

18. How many bushels of potatoes can be raised on $3\frac{1}{4}$ acres of land, if 165 bu. are raised on $1\frac{1}{32}$ acres of land?

19. If a pole 5 ft. high casts a shadow 6 ft. long, how high is a church spire whose shadow, at the same time, is 162 ft. long?

20. If a boy can skate 75 yd. in $8\frac{1}{3}$ seconds, in what time will he skate 200 yd., at the same rate of speed?

COMPOUND PROPORTION.

395. A proportion is compound when two of its terms, or all of them, are composed of two or more numbers.

EXAMPLE I.—If 4 men in 7 days of 10 hours each can earn \$84, how much can 6 men earn in 10 days of 9 hours each?

Operation.

$$\begin{array}{ccccccc} \text{1st cause} & & \text{2d cause} & & \text{1st effect} & & \text{2d effect} \\ 4 & & 6 & & & & \\ 7 & : & 10 & :: & 84 & : & x \\ 10 & & 9 & & & & \end{array}$$

$$4 \times 7 \times 10 \times x = 6 \times 10 \times 9 \times 84$$

$$x = \frac{6 \times 10 \times 9 \times 84}{4 \times 7 \times 10}$$

$$x = \$162.$$

Explanation.—4 men in 7 da. of 10 hr. is the 1st cause; \$84 is the 1st effect. And 6 men in 10 da. of 9 hr. is the 2d cause; x is the 2d effect. And the product of all the terms in the extremes equals the product of all the terms in the means. Solving, x = \$162.

Note.—The first cause might be expressed as 280 hours of work (4 × 7 × 10); and the second cause, as 540 hours of work (6 × 10 × 9).

Example II.—If 15 men can plow a farm in 8 days of 9 hours each, in how many days of 12 hours each can 2 men perform the same work?

Operation.

$$\begin{array}{ccccccc} 15 & & 2 & & & & \\ 8 & : & x & :: & 1 & : & 1 \\ 9 & & 12 & & & & \end{array}$$

$$2 \times 12 \times x = 15 \times 8 \times 9$$

$$x = \frac{15 \times 8 \times 9}{2 \times 12}$$

$$x = 45 \text{ days.}$$

Explanation.—When the same work is to be done, the effects are in the ratio of 1 to 1. The causes are: men working for so many days and so many hours a day. We replace by x the number of days in the 2d cause.

Written Problems.

1. If 35 men earn \$2 030 in 29 days, how many dollars can 43 men earn in 92 days?

2. A garrison of 1 200 men has provisions for 45 days. How long will these provisions last if the garrison is reinforced by 300 men?

Note.—The effects are in the ratio of 1 to 1, since in both cases the same quantity of provisions is involved.

3. In how many days can 6 men do the same work that 24 men can do in 8 days?

4. By laboring 9 hours a day, an artisan could finish a work in 12 days. How many hours a day should he labor to finish it in 10 days?

5. In 20 days, 15 men performed one half of a certain work. In how many days will the work be finished if 3 men quit?

6. If 6 persons consume 700 lb. of bread in 73 days, how many pounds of bread will 15 persons consume in 365 days?

7. If 20 men can dig out 5 000 cu. yd. of dirt in 40 days of 10 hours each, how many hours a day will 18 men have to work to dig out 4 140 cu. yd. in 46 days?

8. If 4 horses consume 1 650 lb. of fodder in 15 days, in how many days will 11 horses consume 8 470 lb. of fodder?

9. Three men took 12 days to dig a ditch 75 yards long; how many yards could 8 men dig in 6 days?

10. If 9 spinning jennies, of 24 spindles each, can spin 2 000 lb. of wool in 8 days of 10 hours, in how many days of 12 hours each will 4 jennies, of 30 spindles each, spin 5 000 lb. of wool?

11. Fifteen men could perform a certain work in 24 days of 10 hours each. How many men should I employ to have the same work done in 20 days of 9 hours each?

12. Five men in 3 days of 8 hours each wheelbarrowed 60 cu. yd. of sand from a pit to a workyard. In how many days of 9 hours each could 6 men have conveyed 108 cu. yd. of sand?

13. If 7 men can dig a ditch 60 ft. long, 8 ft. wide, and 6 ft. deep, in 12 days, how many men will be required to dig a ditch 80 ft. long, 3 ft. wide, and 8 ft. deep, in $2\frac{2}{3}$ days?

14. If 15 men can perform 144 yd. of work in 8 days, how many men could perform one half of the same work in 6 days?

15. If 3 pipes of equal size take 3 hours to fill a cistern 5 ft. long, 4 ft. wide, and 9 ft. deep, in what time will 5 such pipes fill a cistern 3 ft. long, 2 ft. wide, and 5 ft. deep?

Questions on Theory.

1. What is the ratio of two numbers? (383).

2. Name the two numbers of a ratio. (384).

3. What difference is there between a ratio and a common fraction? (385).

4. What is a proportion? (386).

5. What is the fundamental principle of a proportion? (387).

6. Apply this principle to problems involving cause and effect. (389, 390, 391, 392).