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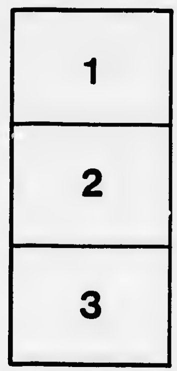
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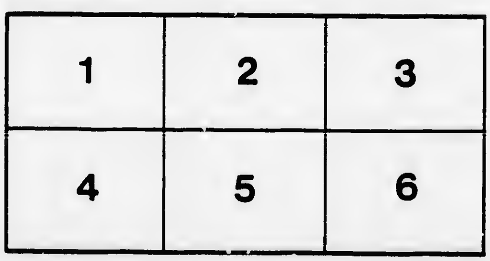
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# KEY,

TO THE

NATIONAL

# MENSURATION;

CONTAINING

SOLUTIONS

TO ALL THE QUESTIONS

LEFT UNSOLVED IN THAT WORK.

BY D. McKAY.

TORONTO:
PRINTED FOR THE AUTHOR.
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# PREFACE.

The following Key, being designed not only to assist those who are pursuing their studies without an instructer, but to abridge the labour of teachers, by furnishing them with the means of reference, care has been taken to illustrate the solutions, by referring, when necessary, to appropriate rules and observations in the Mensuration; and so to exhibit the results of the different operations, as to make it easy to detect errors in the work of a pupil.

In the solution of some of the questions, reference is made to some of the figures in the Mensuration.

It will be observed, that quantities are frequently used in the form of vulgar fractions, instead of decimal, as being shorter and more accurate.

Reference is also made, in the solution of some of the Miscellaneous Problems, to the Remarks in the beginning of the Key.

D. McK.

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# REMARKS.

1st. An angle in a semicircle is a right angle.

2nd. All angles in the same segment of a circle are equal.

3rd. Triangles that have all the three angles of the one respectively equal to the three angles of the other, are callad equiangular triangles, or similar triangles.

4th. In similar triangles the like sides, or the sides opposite to the equal angles, are proportional.

5th. The areas of similar triangles are to each other as the squares of their like sides.

6th. The areas of circles are to each other as the squares of their diameters, radii, or circumferences.

7th. The areas of similar figures are to each other as the squares of their like sides.

8th. Similar solids are to each other as the cubes of their like sides.

9th. The solidities of cylinders, prisms, parallelopepedons, &c., which have their altitudes equal, are to each other as the squares of their diameters or like sides. The same remark is applicable to frustrums of a cone or pyramid when the altitude is the same, and the ends proportional.

10th. In finding the content of the hoof of a cylinder, if the cutting plane passes through the centre of the base, the content of the hoof becomes $\frac{2}{3} r^2 h$; where $h =$ the height and $r =$ the radius.

# MENSURATION OF SUPERFICIES.

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## PROBLEM I.

Ex. 2. $15 \times 15 = 225$ chains, the area.

Ex. 3. $7\frac{3}{4}$ ft. $\times$ $7\frac{3}{4}$ ft. $= 60\frac{1}{16}$ ft., the area.

Ex. 4. $4769 \times 4769 = 22{,}743{,}361$ links, the area.

## PROBLEM II.

Ex. 2. 5ft. 6 inches $\times$ 7ft. 8 inches $=$ 42ft. 2 inches.

Ex. 3. $176 \times 154 = 27{,}104$ ; then $27{,}104 \div 100{,}000$ $= .27{,}104$ of an acre.

Ex. 4. 40ft. 6 inches $\times$ 28ft. 9 inches $=$ 1,164 feet, 4 inches, 6 parts.

## PROBLEM III.

Ex. 2. $7.6 \times 5.7 = 43.32$ chains, the area.

Ex. 3. 7ft. 6 inches $\times$ 3ft. 4 inches $=$ 25 feet.

Ex. 4. 3 yards $=$ 9 feet ; then, 9 feet $\times$ 2 feet 3 inches $=$ 20 feet 3 inches.

## PROBLEM IV.

Ex. 2. 72.7 × 36.5 ÷ 2 = 1,326.775 yards, the area.

Ex. 3. (1276 × 976 ÷ 2) ÷ 100,000 = 6 acres, 36.3008 perches.

Ex. 4. 15 feet 6 inches × 12 feet 7 inches ÷ 2 = 97 feet 6¼ inches.

## PROBLEM V.

Ex. 2. (50 + 40 + 30) ÷ 2 = 60 ; then, 60 — 50 = 10, 60 — 40 = 20, and 60 less 30 = 30 ; then, $\sqrt{(60 \times 30 \times 20 \times 10)} = \sqrt{360000} = 600$.

Ex. 3. ( 4900 + 5025 + 2569 ) ÷ 2 = 6247 ; then, 6247 — 4900 = 1347, 6247 — 5025 = 1222, and 6247 — 2569 = 3678 ; then, $\sqrt{(6247 \times 3678 \times 1347 \times 1222)} = \sqrt{37820044225844} = 6149800 \div 100000$ = 61 acres, 1 rood, 39.68 perches.

Ex. 4. $(20 + 15 + 15) \div 2 = 25$; then, $25 - 20 = 5$, $25 - 15 = 10$, and $25 - 15 = 10$; $\sqrt{(25 \times 5 \times 10 \times 10)} = \sqrt{12500} = 111.803$.

Ex. 5. $(380 + 420 + 765) \div 2 = 782.5$; then, $782.5 - 380 = 402.5$, $782.5 - 420 = 362.5$, and $782.5 - 765 = 17.5$; then, $\sqrt{(782.5 \times 402.5 \times 362.5 \times 17.5)} = \sqrt{1998003710.9375} = 44699.04$ yards $\div 4840 = 9$ acres, 0 roods, 38 poles, nearly.

Ex. 6. $(13 + 14 + 15) \div 2 = 21$; then, $21 - 13 = 8$, $21 - 14 = 7$, and $21 - 15 = 6$; then, $\sqrt{(21 \times 8 \times 7 \times 6)} = \sqrt{7056} = 84 \div 9 = 9\frac{1}{3}$ square yards.

Ex. 7. $(49 + 50.25 + 25.69) \div 2 = 62.47$; then, $62.47 - 49 = 13.47$, $62.47 - 50.25 = 12.22$, and $62.47 - 25.69 = 36.78$; then, $\sqrt{(62.47 \times 36.78 \times 13.47 \times 12.22)} = \sqrt{378200.44225844} = 614.98$ chains $\div 10 = 61$ acres, 1 rood, 39.68 perches.

## PROBLEM VI.

Ex. 2. $70^2 = 4900 \div 4 = 1225$ ; then, $1225 \times \sqrt{3}$ or $1.732 = 2121.7 \div 160 =$ 13 acres, 1 rood, 1 perch.

Ex. 3. The perimeter of any figure is the sum of its sides. $27 \div 3 = 9$, the side of an equilateral triangle whose perimeter is 27. $9^2 = 81 \div 4 = 20.25 \times \sqrt{3} = 35.074$.

## PROBLEM VII.

Ex. 2. 14 acres $\times 4840 = 67760$ yards ; then, $(67760 \div 7) \times 2 = 19360$ yards.

## PROBLEM IX.

Ex. 4. $\sqrt{(120^2 + 210^2)} = \sqrt{58500} = 241.86$ feet.

Ex. 5. $\sqrt{(60^2 - 40^2)} = \sqrt{2000} = 44.7213$ ; and $\sqrt{(60^2 - 50^2)} = \sqrt{1100} = 33.1662$ ; then, $44.7213 + 33.1662 = 77.8875$ feet. *Ans.*

Ex. 6. $\sqrt{(10^2 + 14^2)} = \sqrt{296} = 17.204$. *Ans.*

## PROBLEM X.

Ex. 2. $\sqrt{(10^2 + 12^2)} = \sqrt{244} = 15.6204$ feet, the breadth of the building. $12^2 = 144 \div 15.6204 = 9.2186$ feet, the greater segment; then, $15.6204 - 9.2186 = 6.4018$ feet, the lesser segment. Again, $\sqrt{(9.2186 \times 6.4018)} = \sqrt{59.015633} = 7.68$; and $30 + 7.68 = 37.68$ feet, the length of the prop.

## PROBLEM XI.

Ex. 2. $(4105 + 3755 + 4835) \div 2 = 6347.5$; then, $6347.5 - 3755 = 2592.5$, $6347.5 - 4835 = 1512.5$, and $6347.5 - 4105 = 2242.5$; then, $6347.5 \times 2592.5 = 16455893.75 \times 2242.5 = 36902341734.375 \times 1512.5 = 55814791873242.1875$; $\sqrt{55814791873242.1875} = 7470929.786$ links, area of one triangle.

$(4835 + 3575 + 2740) \div 2 = 5575$;

then, 5575 — 4835 = 740, 5575 — 3575 = 2000, and 5575 — 2740 = 2835 ; then, 5575 × 2835 = 15805125 × 740 = 11695792500 × 2000 = 23391585000000 ; then, √23391585000000 = 4835484.777 links, the area of the other triangle.

Again, ( 7470929.786 + 4836484.777 ) ÷ 100,000 = 123 acres, 0 roods, 11.8633 perches. *Ans.*

Ex. 3. ( 15 + 13 + 16 ) ÷ 2 = 22 ; then, 22 — 15 = 7, 22 — 13 = 9, and 22 — 16 = 6 ; then, √( 22 × 9 × 7 × 6 ) = √8316 = 91.19, area of the triangle A C D.

( 14 + 12 + 16 ) ÷ 2 = 21 ; then, 21 — 14 = 7, 21 — 12 = 9, and 21 — 16 = 5 ; then, √( 21 × 9 × 7 × 5 ) √6615 = 81.33, area of the triangle A B C.

Then, 81.33 + 91.19 = 172.52.

Ex. 4. $\sqrt{(220^2 - 100^2)} = \sqrt{38400} \doteq 195.959$; and $(265^2 - 70^2) = 65325$; then, $\sqrt{65325} = 255.5875$;

Then, $(195.959 + 255.5875) \times 378 = 170684.577 \div 2 = 85342.2885$ yards;

Then, $85342.2885 \div 4840 = 17$ acres, 2 roods, 21 perches.

Ex. 5. $\sqrt{(220^2 - 195.959^2)} = \sqrt{10000} = 100$; and $\sqrt{(265^2 - 255.5875^2)} \doteq \sqrt{4900} = 70$; then, $100 + 70 + 208 = 378$, the diagonal;

Then, $(195.959 + 255.5875) \times 378 \div 2 = 85342.2885$.

Ex. 6. $\sqrt{(26^2 - 25^2)} = \sqrt{51} = 7.1414$; and $\sqrt{(22^2 - 19^2)} = \sqrt{123} = 11.09$; and $\sqrt{(32^2 - 25^2)} = \sqrt{399} = 19.9754$;

Then, $7.1414 + 11.09 + 19.9754 = 38.207$, the diagonal; and $(19+25) \div 2 = 22 \times 38.207 = 840.554$ square yards.

## PROBLEM XII.

Ex. 2. $(25 + 34 + 35 + 16) \div 2 = 55$;

Then, 55 — 25 = 30, 55 — 34 = 21,
55 — 35 = 20, 55 — 16 = 39;
Then, 39 × 21 × 30 × 20 = 491400;
and $\sqrt{491400}$ = 700.99.

## PROBLEM XIII.

Ex. 2. ( 750 + 1225 ) ÷ 2 = 987.5 × 1540 = 1520750 ÷ 100,000 = 15 acres, 0 roods, 33.2 perches.

Ex. 3. ( 4 feet 6 inches + 8 feet 3 inches ) ÷ 2 = 6 feet 4½ inches × 5 feet 8 inches = 36 feet 1½ inches.

Ex. 4. ( 1476 + 2073 ) ÷ 2 = 1774.5 × 976 = 1731912 ÷ 7840 (the number of square yards in an Irish acre) = 220 acres, 3 roods, 25 perches, 7 yards, Irish.

## PROBLEM XIV.

Ex. 2. 28 × 14 ÷ 2 = 196, and ( 13 + 7 ) ÷ 2 = 10 × 35 = 350;
Then, 350 + 196 = 546 ÷ 160 = 3 acres, 1 rood, 26 perches.

Ex. 3. $15 \times 5 \div 2 = 37.5$, $(10 + 5) \div 2 \times 8 = 60$, $(10 + 12) \div 2 \times 14 = 154$, $12 \times 6 \div 2 = 36$, $12 \times 14 \div 2 = 84$, $(14 + 20) \div 2 \times 14 = 238$, $17 \times 20 \div 2 = 170$;

Then, $37.5 + 60 + 154 + 36 + 84 + 238 + 170 = 779.5 \div 160 = 4$ acres, 3 roods, $19\frac{1}{2}$ perches.

## PROBLEM XV.

Ex. 2. $14 \times 6 \div 2 \times 12.1243556 = 509.2229352$.

Ex. 3. $5.7^2 \times 4.8284271 = 156.875596479$.

Ex. 4. $19.38^2 \times 3.6339124 = 1364.84$.

Ex. 5. $10^2 = 100 \times 4.8284271 = 482.84271$.

Ex. 6. $50^2 = 2500 \times 6.1818242 = 15454.5605$.

Ex. 7. $20^2 \times 9.3656404 = 3746.25616$.

Ex. 8. $40^2 \times 11.1961524 = 17913.84384$.

## PROBLEM XVI.

Ex. 3. As 7 : 22 : : 10 : 31.4285.

Ex. 4. As 22 : 7 : : 50 : 15.909.

Ex. 5. As 1 : 3.1416 : : 7958 : 25000.8528.

Ex. 6. As 3.1416 : 1 : : 25000.8528 : 7958.

## PROBLEM XVII.

Ex. 3. $3\frac{1}{2} \times 8 = 28$; then, $(28 - 6) \div 3 = 7\frac{1}{3}$.

Ex. 4. $40 \div 2 = 20$; then, $\sqrt{(20^2 + 15^2)} = \sqrt{625} = 25$, the chord of half the arc. Then, $25 \times 8 = 200 - 40 = 160 \div 3 = 53\frac{1}{3}$.

Ex. 5. $(30.5 \times 8) - 48.7 = 193.26 \div 3 = 64.42$.

Ex. 6. $\sqrt{(15^2 + 8^2)} = \sqrt{289} = 17$, the chord of half the arc; then, $(17 \times 8) - 30 = 106 \div 3 = 35\frac{1}{3}$.

## PROBLEM XVIII.

Ex. 2. As 7 : 22 : : 7 : 22, the circumference ; then, $7 \div 2 = 3\frac{1}{2}$, and, $22 \div 2 = 11$ ; then, $11 \times 3\frac{1}{2} = 38\frac{1}{2}$.

Ex. 3. As 7 : 22 : : $1\frac{1}{6}$ : $3\frac{2}{3}$, the circumference ; then, $3\frac{2}{3} \div 2 = 1\frac{5}{6}$, and, $1\frac{1}{6} \div 2 = \frac{7}{12}$ ; then, $1\frac{5}{6} \times \frac{7}{12} = 1\frac{5}{72} = 1.069$.

Ex. 4. $200 \div 2 = 100 \times 16\frac{1}{2} = 1650$ feet ; then, $1650 \div 3 = 550 \div 5\frac{1}{2}$ yards $= 100$ poles ; then, $100^2 \times .07958 \div 160 =$ 4 acres, 3 roods, 35.8 perches.

Ex. 5. $56^2 \times .07958 = 249.56288$.

Ex. 6. $100 \times 2 = 200$, diameter ; then, $200^2 = 40000 \div 4 = 10000 \times .7854 = 7854$.

Ex. 7. 4840, the number of square yards in an acre, $\div .7854 = 6162.48$ ; $\sqrt{6162.48} = 78\frac{1}{2}$, the diameter, $\div 2 = 39\frac{1}{4}$.

Ex. 8. $91^2 = 8281 \times .07958 = 659.00198$.

Ex. 9. $15^2 = 225 \times .7854 = 176.715$.

Ex. 10. $20 \times 2 = 40$, the diameter; then, $40^2 \times .7854 = 1256.64$, area of the whole circle, $\div 2 = 628.34$.

## PROBLEM XIX.

Ex. 1. $100 \times .8862269 = 88.62269$.

Ex. 2. $200 \times .8862269 = 177.24538$.

## PROBLEM XX.

Ex. 1. $100 \times .2820948 = 28.20948$. *Ans.*

Ex. 2. $200 \times .2820948 = 56.41896$. *Ans.*

## PROBLEM XXI.

Ex. 1. $100 \times .7071068 = 70.71068$.

Ex. 2. $200 \times .7071068 = 141.42136$.

## PROBLEM XXII.

Ex. 1. $100 \times .6366197 = 63.66197$; then, $\sqrt{63.66197} = 7.97884$.

Ex. 2. 200 × .6366197 = 127.32394 ; then, $\sqrt{127.32394} = 11.2837$.

## PROBLEM XXIII.

Ex. 1. 10 × 1.4142136 = 14.142136.

Ex. 2. 20 × 1.4142136 = 28.284272.

## PROBLEM XXIV.

Ex. 1. 100 × 4.4428934 = 444.28934.

Ex. 2. 30 × 4.4428934 = 133.286802.

## PROBLEM XXV.

Ex. 1. 100 × 1.1283791 = 112.83791.

Ex. 2. 200 × 1.1283791 = 225.67582.

## PROBLEM XXVI.

Ex. 1. 100 × 3.5449076 = 354.49076.

Ex. 2. 300 × 3.5449076 = 1063.47228.

## PROBLEM XXVII.

Ex. 4. 25 × 2 = 50, the diameter ; then, $50^2$ ×

$.7854 = 3475002.3$. As $360 : 147\frac{20}{60} :: 3475002.3 : 804.3986$.

Ex. 5. $3 \times 2 = 6$, the diameter; then, $6^2 \times .7854 = 28.2744$. As $360 : 18 :: 28.2744 : 1.41372$.

## PROBLEM XXVIII.

Ex. 5. $12 \div 2 = 6$; then, $6^2 = 36 \div 18 = 2$; and, $18 + 2 = 20$, the diameter of the circle. Then, $2 \div 20 = .100$, area in table corresponding to $.100$ is $.040875 \times 20^2 = 16.35$, area of the lesser segment. And, $20^2 \times .7854 = 314.16$, area of the whole circle; then, $314.16 - 16.35 = 297.81$, the area of the greater segment.

## PROBLEM XXIX.

Ex. 2. $(48 - 30) \div 2 = 9 = XB$; then, $\sqrt{(DB^2 + XB^2)} = \sqrt{(15.8114^2 + 9^2)} = 18$ nearly. Then, $AB - XB = 48 - 9 = 39 =$

AX; then, $39 \times 9 \div 13 = 27 + 13 = 40 = DF$, and, $\sqrt{(40^2 + 30^2)} = \sqrt{(2500)} = 50$, the diameter. Then, $15.8114 \times 39 \div (13 \times 2) = 23.7171$. And, $50 \div 2 = 25$, radius; then, $25 - 23.7171 = 1.2829 = YZ \div 50 = .0256$, the tabular segment answering to which is .005434. See note 2nd, page 51, in Mensuration. Then, $.005434 \times 50^2 = 13.585 \times 2 = 27.17$, areas of segments AZC and DB. Then, $(30 + 48) \div 2 \times 13 = 507$; and $507 + 27.17 = 534.17$. *Ans.*

Ex. 3. $(20 - 15) \div 2 = 2.5$; then, $(20 - 2.5) \times 2.5 \div 17.5 = 2.5$, and $17.5 + 2.5 = 20 = DF$; then, $\sqrt{(20^2 + 15^2)} = 25$, diameter; then, $2.5 \div 25 = .100$, the tabular area corresponding to which is $.040875 \times 25^2 = 25.546875$. Again, $20 - 15 = 5 \div 25 = .200$, the tabular area corresponding to which is $.111823 \times 25^2 =$

69.889375; then, 69.889375 + 25.546875 = 95.43625; then, $25^2 \times .7854$ = 490.875; and, 490.875 − 95.43625 = 395.43875. *Ans.*

Ex. 4. (96 − 60) ÷ 2 = 18 = XB; then, $\sqrt{(18^2 + 26^2)}$ = 31.6227 = DB. Again, (96 − 18) = 78 = AX; then, 78 × 18 ÷ 26 = 54 = FX, 54 + 26 = 80 = DF; then, $\sqrt{(60^2 + 80^2)}$ = 100, and 100 ÷ 2 = 50 = GF. Then, 31.62277 × 78 ÷ (26 × 2) = 47.434105 = GZ; then, 50 − 47.434105 = 2.565895 = YZ, and 2.5658 ÷ 100 = $.025\frac{658}{1000}$, the tabular area corresponding to which is .005437928 × $100^2$ = 54.37928 × 2 = 108.75856; And (96 + 60) × 26 ÷ 2 = 2028; then, 2028 + 108.75856 = 2136.75856. *Answer.*

## PROBLEM XXX.

Ex. 2. 30 + 40 = 70; and, 40 − 30 = 10;

then, $70 \times 10 = 700 \times .7854 = 549.78$.

Ex. 3. $50 + 45 = 95$; and $50 - 45 = 5$; then, $95 \times 5 = 475 = .7854 = 373.065$.

## PROBLEM XXXI.

Ex. 2. $(60 + 40) \div 2 = 50$; then, $50 \times 2 = 100$.

Ex. 3. $(25 + 15) \div 2 = 20 \times 6 = 120$.

## PROBLEM XXXII.

Ex. 2. $(20^2 \div 15) + 15 = 41\frac{2}{3}$; then, $15 \div 41\frac{2}{3} = .36$, the tabular area corresponding to which is $.25455 \times 41\frac{2}{3}^2 = 441.9$; $(20^2 \div 2) + 2 = 202$; then, $2 \div 202 = .009$, the tabular area corresponding to which is $.001329 \times 202^2 = 53.4$; then, $441.9 - 53.4 = 388.5$.

## PROBLEM XXXIII.

Ex. 2. $(4.8 + 7.2) \div 2 = 6$; then, $(6 + 5.2 + 4.1 + 7.3) \div 4 = 5.65$; $5.65 \times 39 = 220.35$.

Ex. 3. $(5.5 + 8.8) \div 2 = 7.15$; then, $(7.15 + 6.2 + 7.3 + 6 + 7.5 + 7) \div 6 = 6.8583 \times 50 = 342.915$.

Ex. 4. $(0 + 6 \cdot 1) \div 2 = 3.05$; then, $(3.05 + 4.4 + 6.5 + 7.6 + 5.4 + 8 + 5.2 + 6.5) \div 8 = 5.83125 \times 37.6 = 219.255$.

## EXERCISES.

Ex. 1. $35.25 \times 35.25 = 1242.5625 \div 10 = 124$ acres, 1 rood, 1 perch.

Ex. 2. $12\frac{1}{2} \times \frac{3}{4} = 9\frac{3}{8}$.

Ex. 3. $(4^2 + 5^2 + 6^2) = 77$; then, $\sqrt{77} = 8.7749$.

Ex. 4. $10.51 \times 4.28 = 44.9828 \div 10 = 4$ acres, 1 rood, 39 perches.

Ex. 5. $12.6 \times 6.4 \div 2 = 40.32$.

Ex. 6. The same as question 2nd, Problem V.

Ex. 7. $(150 + 200 + 250) \div 2 = 300$; then, $300 - 150 = 150$, $300 - 200 = 100$, and $300 - 250 = 50$; then, $\sqrt{(300 \times 150 \times 50 \times 100)} = \sqrt{225000000} = 15000 \div 4840 = 3$ acres, 15 perches $= 3\frac{3}{32}$ acres at 9s. 6d. = £1 9s. 5d.

Ex. 8. $36 \div 9 = 4 \times 2 = 8$.

Ex. 9. $36 \div 12 = 3 \times 2 = 6$.

Ex. 10. $\sqrt{(320^2 - 103^2)} = \sqrt{91791} = 302.97$.

Ex. 11. 6 inches $= \frac{1}{2}$ foot; then $12\frac{1}{2} - \frac{1}{2} = 12$; and $\sqrt{(12\frac{1}{2}^2 - 12^2)} = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3\frac{1}{2}$ feet.

Ex. 12. $\sqrt{(3^2 + 8^2)} = \sqrt{73} = 8.544 = BC$; then, as $3 : 8 :: 8.544 : 22.78$; and $\sqrt{(22.78^2 - 8^2)} = \sqrt{454.9284} = 21\frac{1}{3}$.

Ex. 13. $(26.5 + 30) \div 2 = 28.35 \times 70.5 = 1998.675$.

Ex. 14. (56 feet 3 inches + 60 feet 9 inches) ÷ 2 = 58 feet 6 inches × 108 feet 6 inches = 6347 square feet 36 square inches.

Ex. 15. (75 + 122) ÷ 2 = 98.5 × 154 = 15169.

Ex. 16. (6340 + 4380) ÷ 2 = 5360 × 121 ÷ 4840 = 134 acres; then, £207 14s. ÷ 134 = £1 11s.

Ex. 17. (24 + 26 + 28 + 30) ÷ 2 = 54; then, 54 — 24 = 30, 54 — 26 = 28, 54 — 28 = 26, 54 — 30 = 24; then, $\sqrt{(30 \times 28 \times 26 \times 24)} = \sqrt{524160} =$ 723.99.

Ex. 18. Solve the same as the first question under Problem XIV, Mensuration, having double the dimensions there given.

Ex. 19. $10^2 = 100 \times 4 = 400$; then, $400 \div \sqrt{3} = 400 \div 1.732 = 230.99$; then, $\sqrt{230.99} = 15.196$.

Ex. 20. $8 \times 9 \div 2 = 36$; then, $36 \times 10.99 = 395.64$.

Ex. 21. $20.5^2 = 420.25$; then, $420.25 \times 7.6942088 = 3233.4912482$.

Ex. 22. 5280, the number of feet in a mile $\times 10 \div 4400 = 12$ feet $\div 3.1416 = 3.819708$ feet.

Ex. 23. As $7 : 22 :: 9 : 28\frac{2}{7}$.

Ex. 24. $14 \times 60 = 840 \times .01745329 = 14.6607$ feet.

Ex. 25. $30 \div 2 = 15$; then, $\sqrt{(15^2 + 8^2)} = 17$; then, $17 \times 8 = 136 - 30 = 106 \div 3 = 35\frac{1}{3}$.

Ex. 26. $200^2 \div 4 = 10000 \times .7854 = 7854$.

Ex. 27. $15 + 10 = 25$, and $15 - 10 = 5$; then, $25 \times 5 \times .7854 = 98.175$.

Ex. 28. $628.32 \div 3.1416 = 200$, the diameter, $\div 2 = 100$, and $628.32 \div 2 = 314.16 \times$

100 = 31416 ÷ 2 = 15708 ÷ .7854 = 141.42, diameter, ÷ 2 = 70.71.

Ex. 29. 3 × .8862269 = 2.6586807.

See figure, Problem XXIX.

Ex. 30. (16 — 12) ÷ 2 = 2 = XB;

then, $\sqrt{(XD^2 + XB^2)} = \sqrt{(2^2 + 2^2)}$ = 2.828 = DB; AB — BX = 16 — 2 = 14 = AX, and AX × BX ÷ DX = 14 × 2 ÷ 2 = 14 = YX; then, 14 + 2 = 16 = DF, and $\sqrt{(12^2 + 16^2)}$ = 20, the diameter.

Then, 20 ÷ 2 = 10 = GZ.

And, DB × AX ÷ 2DX = 2.828 × 14 ÷ 4 = 9.898 = GY; then, GZ — GY = 10 — 9.898 = .102 = ZY.

Then, .102 ÷ 20 = .005, the tabular area segment answering to which is .00047; then .00047 × $20^2$ = .188, and .188 × 2 =

.376. Then, $(16 + 12) \times 2 \div 2 = 28$; then $28 + .376 = 28.376$.

See figure, Problem XXII.

Ex. 31. $15 \div 2 = 7\frac{1}{2} = AE$; then, $(AE^2 \div EC) + EC = (7\frac{1}{2}^2 \div 7) + 7 = 15.035$; then, $7 \div 15.035 = .465$, the tabular area answering to which is $.357727 \times 15.035^2 = 80.8626$.

Again, $(AE^2 \div ED) + ED = (7\frac{1}{2}^2 \div 4) + 4 = 18.0625$; then, $4 \div 18.0625 = .221$, the tabular area segment answering to which is $.128942 \times 18.0625^2 = 42.19607$.

Then, $80.8626 - 42.19607 = 38.7$. *Answer.*

# CONIC SECTIONS.

Ex. 32. $\sqrt{(1^2 + 1^2)} = \sqrt{2}$, the hypothemuse; which is to be the diameter of the circle. Then, $\sqrt{2}$ squared $= 2 \times .7854 = 1.5708$.

Ex. 33. $100 \times .6366197 = 63.66197$.

## PROBLEM I.

Ex. 1. $35 \times 25 \times .7854 = 687.225$. *Ans.*

Ex. 2. $70 \times 50 \times .7854 = 2748.9$.

Ex. 3. $80 \times 60 \times .7854 = 3769.92$.

Ex. 4. $50 \times 45 \times .7854 = 1767.5$.

## PROBLEM II.

Ex. 2. $(60 \times 40) - (30 \times 10) = 2100$; then, $2100 \times .7854 = 1649.34$.

Ex. 3. $(30 \times 24) - (19 \times 13) = 473$; then, $473 \times .7854 = 371.4942$.

## PROBLEM III.

Ex. 2. $10 \div 35 = .285\frac{5}{7}$, the tabular versed sine, the area segment answering to which is $.185166 \times 35 \times 25 = 162.0202$.

Ex. 3. $10 \div 30 = .333\frac{1}{3}$; the area segment answering to which is $.229172 \times 40 \times 30 = 275.0064$.

Ex. 4. $10 \div 70 = .146\frac{6}{7}$; the area segment answer ing to which is $.068824 \times 70 \times 50 = 240.884$.

## PROBLEM IV.

Ex. 2. $30 + 20 = 50$; then, $50 \times 1.5708 = 78.54$.

Ex. 3. $60 + 40 = 100$; then, $100 \times 1.5708 = 157.08$.

Ex. 4. $6 + 4 = 10$; then, $10 \times 1.5708 = 15.708$.

Ex. 5. $3 + 2 = 5$; then, $5 \times 1.5708 = 7.854$.

## PROBLEM V.

Ex. 2. As $15^2 : 10^2 :: (20 \times 10) : (800 \div 9) = TB^2$; then, $\sqrt{\frac{800}{9}} = 9.42278 = TB$, and $\sqrt{(9.42278^2 + 5^2)} = \sqrt{113.8} = 10.67187 = OG$ or $OB$; then, $(10.67187 + 10) \div 2 = 10.335 = OY$;

$(10.67187 - 9.42278) = 1.25$ nearly, the versed sine;

then, $\sqrt{(1.25^2 + 5^2)} = 5.153$, half the arc.

Then, by Rule II., Problem XVII.

$(5.153 \times 8) - 10 = 31.224 \div 3 = 10.408 = 2BG$.

As $10.67187 : 10.335 :: 10.408 : 10.079 = 2XY$;

then, $10.079 \div 2 = 5.039$.

Ex. 3. As $20^2 : 15^2 :: 25 \times 15 : 211$; then, $\sqrt{211} = 14.5 = TB$, and $\sqrt{(14.5^2 + 5^2)} = \sqrt{236} = 15.36 = OB$ or $OG$, and $(15.36 + 15) \div 2 = 15.18 = OY$; then, $15.36 - 14.5 = .86$, and $\sqrt{(5^2 + .86^2)} = \sqrt{25.76} = 5.075$, half the arc; then, by Rule II. Problem XVII.,

$(5.075 \times 8) - 10 = 30.6 \div 3 = 10.2 = 2BG$.

As $15.36 : 15.18 :: 10.2 : 10.08 = 2XY = 2BC$;

then, $10.08 \div 2 = 5.04$.

## PROBLEM VI.

Ex. 2. $35 - 7 = 28$; then, as $35 : 25 :: \sqrt{(28 \times 7)} : 10$.

Ex. 3. $70 - 10 = 60$; then, as $70 : 60 :: \sqrt{(60 \times 10)} : 20.9956$.

## PROBLEM VII.

Ex. 2. $40 \div 2 = 20$; then, $\sqrt{(20^2 - 16^2)} = \sqrt{144} = 12$.

As $40 : 120 :: 12 : 36$, the distance between the ordinate and the centre;

then, $120 \div 2 = 60$, and $60 - 36 = 24$, lesser abscissa, and $36 + 60 = 96$, greater abscissa.

## PROBLEM VIII.

Ex. 2. $25 \div 2 = 12.5$; then, $\sqrt{(12.5^2 - 10^2)} = \sqrt{56.25} = 7.5$, and $12.5 - 7.5 = 5$; then, as $10^2 : 25 \times 28 :: 5 : 35$.

## PROBLEM IX.

Ex. 2. $96 \times 24 = 2306$; then, $\sqrt{2306} = 48$.

As $48 : 16 :: 120 : 40$.

## PROBLEM X.

Ex. 2. $24 \times 4 \times \frac{2}{3} = 64$.

Ex. 3. $12 \times 2 \times \frac{2}{3} = 16$.

## PROBLEM XI.

Ex. 3. $30^2 = 900$, $25^2 = 625$, and $25 \times 30 = 750$;

$900 + 625 + 750 = 2275$.

Again, $30 + 25 = 55$;

then, $2275 \div 55 = 41\frac{4}{11} \times 6 \times \frac{2}{3} = 165\frac{5}{11}$.

## PROBLEM XII.

Ex. 2. $8^2 = 64$; and $3^2 = 9 \times \frac{4}{3} = 12$;

then, $\sqrt{(64 + 12)} = \sqrt{76} = 8.7177$, the length of the single curve;

and $8.7177 \times 2 = 17.4354$.

## PROBLEM XIII.

Ex. 2. As $9 : 16 :: 6^2 : 64$; then, $\sqrt{64} = 8$.

## PROBLEM XIV.

Ex. 2. $8^2 : 9^2 :: 10 : 12.656$.

## PROBLEM XV.

Ex. 2. $4^2 = 16$, $3^2 = 9$, then, $16 - 9 = 7$;

then, as $7 : 2 :: 16 : 4\frac{4}{7}$.

As $7 : 2 :: 9 : 2\frac{4}{7}$.

## PROBLEM XVI.

Ex. 2. $120 + 40 = 160$, the greater abscissa;

then, $\sqrt{(160 \times 40)} = \sqrt{6400} = 80$.

As $120 : 72 :: 80 : 48$.

Ex. 3. $60 + 20 = 80$, the greater abscissa;

$\sqrt{(80 \times 20)} = \sqrt{1600} = 40$.

As $60 : 36 :: 40 : 24$.

## PROBLEM XVII.

Ex. 2. $72 \div 2 = 36$; and $\sqrt{(36^2 + 48^2)} =$

$\sqrt{3600} = 60$;

then, as $72 : 120 :: 60 : 100$, half the sum of the abscissa;

Then $120 \div 2 = 60 + 100 = 160$, the greater abscissa ;

And $100 - 60 = 40$, the lesser abscissa.

## PROBLEM XVIII.

Ex. 2. $72 \div 2 = 36$, and $\sqrt{(36^2 + 48^2)} = \sqrt{3600} = 60$ ; then, $60 + 36 = 96$.

As $48^2 : 72 \times 40 :: 96 : 120$.

Ex. 3. $36 \div 2 = 18$, and $\sqrt{(18^2 + 24^2)} = \sqrt{900} = 30 + 18 = 48$ ;

then, as $24^2 : 36 \times 20 :: 48 : 60$.

## PROBLEM XIX.

Ex. 2. $60 + 20 = 80$, greater abscissa ;

and $\sqrt{(80 \times 20)} = \sqrt{1600} = 40$ ;

then, as $40 : 24 :: 60 : 36$.

## PROBLEM XXI.

Ex. 1. $14 + 25 = 39$, and $15 + 17 + 20 + 23 = 75$, and $16 + 18 + 22 = 56$ ;

then, $75 \times 4 = 300$, and $56 \times 2 = 112$;

then, $39 + 300 + 112 = 451 \times 2 \div 3 = 300\frac{2}{3}$.

Ex. 2. $5 + 8 = 13$, and $7 \times 4 = 28$;

then, $28 + 13 = 41$.

$AE = 10$; then, $AC = 10 \div 2 = 5$;

then, $41 \times 5 \div 3 = 68\frac{1}{3}$.

Ex. 3. $\frac{10}{10} + \frac{10}{20} = 1\frac{1}{2}$; then, $\frac{10}{11} + \frac{10}{13} + \frac{10}{15} + \frac{10}{17} + \frac{10}{19} = \frac{2396890}{692835} \times 4 = \frac{9587560}{692835}$;

$\frac{10}{12} + \frac{10}{14} + \frac{10}{16} + \frac{10}{18} = \frac{1375}{504} \times 2 = \frac{2750}{504}$;

then, $\frac{9587560}{692835} + \frac{2750}{504} + 1\frac{1}{2} = \frac{2420403250}{116396280} \times \frac{1}{3} \times \frac{1}{10} = \frac{2420403250}{3491888400}$, which reduced to a decimal $= .693150$.

## PROBLEM XXII.

Ex. 2. $80^2 = 6400 \times 19 = 121600$, and $60^2 = 3600 \times 21 = 75600$;

then, $75600 + 121600 = 197200 \times 2.1637 = 426681.64$.

Again, $80^2 = 6400 \times 9 = 57600$,

and $57600 + 75600 = 133200 \times 2.1637 = 288204.84$;

then, $60^2 = 3600 \times 80 \times 15 = 4320000$,

and $288204.84 + 4320000 = 4608204.84$,

and $426681.64 + 4320000 = 4746681.64$;

then, as $4608204 : 4746681.64 :: 10 : 10.3005$.

## PROBLEM XXIII.

Ex. 2. $25 \times 50 = 1250$, and $25^2 = 625 \times \frac{5}{7} = 446.42857$; then, $\sqrt{(1250 + 446.42857)} = \sqrt{1696.42857} = 41.1878 \times 21 = 864.9438$.

Again, $\sqrt{1250} = 35.3552 \times 4 = 141.4208 + 864.9438 = 1006.3646 \div 75 = 13.4181817$ nearly; then, $30 \times 25 \times 4 \div 50 = 60 \times 13.4181817 = 805.0909$.

Ex. 3. $50 \times 100 = 5000$, and $50^2 \times \frac{5}{7} = 1785.7142857$;

then, $\sqrt{(50000 + 1785.71428571)} = \sqrt{6785.7142857} = 82.3756$ nearly $\times 21 = 1729.8876$.

Again, $\sqrt{5000} = 70.7104 \times 4 = 282.8416 + 1729.8876 = 2012.7292 \div 75 = 26.8363634$ nearly;

then, $60 \times 50 \times 4 \div 100 = 120 \times 26.8363634 = 3220.3636$.

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# MENSURATION OF SOLIDS.

## PROBLEM I.

Ex. 2. 5 feet 7 inches $\times$ 5 feet 7 inches $\times$ 5 feet 7 inches $=$ 174 feet.

Ex. 3. 12 feet 3 inches $\times$ 12 feet 3 inches $\times$ 12 feet 3 inches $=$ 1838 feet 3 inches.

## PROBLEM II.

Ex. 2. 10 inches $= \frac{5}{6}$ of a foot, and 8 inches $= \frac{2}{3}$; then, $26 \times \frac{5}{6} \times \frac{2}{3} = 14\frac{4}{9}$.

Ex. 3. 10 inches $= \frac{5}{6}$; then, $40 \times \frac{5}{6} \times \frac{5}{6} = 27\frac{7}{9}$.

Ex. 4. 15 inches $=$ 1.25 ft.; then, $18 \times 1.25 \times 1.25 = 28.125$, and $1.25 \times 1.25 = 1.5625$; then, 12 inches $\div 1.5625 = 7.68$.

Ex. 5. 2 feet 9 inches $= 2\frac{3}{4}$ feet, and 1 foot 7 inches $= \frac{19}{12}$ ft.; then, $2\frac{3}{4} \times \frac{19}{12} = \frac{209}{48}$; then, $12 \div \frac{209}{48} = 12 \times 48 \div 209 = 2.756$.

## PROBLEM III.

Ex. 2. $1\frac{1}{3} \times 1\frac{1}{3} \times 5\frac{1}{2} = 9\frac{7}{9}$.

Ex. 3. $4^2 \div 4 = 4$; then, $4 \times \sqrt{3} = 6.928 \times 10 = 69.28$.

Ex. 4. $3^2 = 9 \times 7 = 63$.

## PROBLEM IV.

Ex. 2. $96 \div 12 = 8$ feet ; then, $8^2 = 64 \times .07958 = 91.67616$.

Ex. 3. $1728 \times 3 = 5184$ ; then, $44^2 = 1936 \times .07958 = 154.0688$,

and $5184 \div 154.0688 = 33.64$.

## PROBLEM V.

Ex. 2. $50 \div 2 = 25$, and $25 - 21.75 = 3.25$ ; then, $3.25 \div 50 = .065$, the tabular versed sine ; the area segment answering to which is .021659 ;

then, $.021659 \times 50^2 = 54.1475 \times 20 = 1082.95$.

## PROBLEM VI.

Ex. 2. $3^2 = 9 \div 4 = 2.25 \times \sqrt{3} = 3.8971125$
$\times 30 \div 3 = 38.971125.$

Ex. 3. 5 feet 10 inches $= \frac{70}{12}$ of a foot, and 4 feet 11 inches $= \frac{59}{12}$ of a foot;
then, $\frac{70}{12}^2 = \frac{4900}{144} \times 4.8284271 = 164.300643,$
and $\frac{59}{12}^2 = \frac{3481}{144} \times 4.8284271 = 116.720519 \times 41 \div 3 = 1595.180426$;
then, $164.300643 \times 45 \div 3 = 2464.509645,$
and $2464.509645 - 1595.180426 = 869.32\text{-}9219 \div 27 = 32.19738$ solid yards, nearly.

Ex. 4. $10^2 = 100 \times 2.5980762 = 259.80762 \times 45 \div 3 = 3897.1143.$

## PROBLEM VII.

Ex. 2. $12^2 = 144 \times .7854 = 113.0976 \times 100 \div 3 = 3769.92.$

Ex. 3. $37.6992 \div 3.1416 = 12$, the diameter ;

then, $12^2 = 144 \times .7854 = 113.0976 \times 100 \div 3 = 3769.92$.

Ex. 4. $70 \div 3.1416 = 22.28164$, the diameter, $\div 2 = 11.1408$ ;

then, $\sqrt{(30^2 - 11.1408^2)} = \sqrt{(900 - 124.117)} = \sqrt{775.883} = 27.855$ nearly, the perpendicular.

Again, $22.28164^2 = 496.4714810896 \times .7854 = 389.9283284 \times 27.855 \div 3 = 3620.4818 \div 27 = 134.09$, solid yards.

Ex. 5. $20^2 - (19^2 + 8^2) \div (8 \times 2) = 400 - 320 \div 16 = 5$, the distance between the perpendicular and obtuse angle ;

then, $\sqrt{(16^2 - 5^2)} = \sqrt{231} = 15.19868$, the perpendicular.

Again, $8^2 = 64 \times .7854 = 50.2656 \times 15.19868 \div 3 = 254.656588$.

## PROBLEM VIII.

Ex. 2. 1 foot 6 inches $= 1\frac{1}{2}$ ft.; and $1\frac{1}{2}^2 = \frac{9}{4} \times 1.7204774 = 3.8711$;
then, 6 inches $= \frac{1}{2}$ foot, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \times 1.7204774 = .43012$ nearly;
then, $\sqrt{(3.8711 \times .43012)} = 1.2904$,
and $1.2904 + .43012 + 3.8711 = 5.5915 \times 5 \div 3 = 9.3192$.

Ex. 3. 18 inches $= 1\frac{1}{2}$ foot, and $1\frac{1}{2} \times 1\frac{1}{2} = \frac{9}{4} \times 2.5980762 = 5.8456714$;
Again, 12 inches $= 1$ foot, and $1^2 \times 2.5980762 = 2.5980762$;
Then, $\sqrt{(5.8456714 \times 2.5980762)} = \sqrt{15.188499735362} = 3.89724$, and $3.89724 + 5.84567 + 2.5980762 = 12.3409862 \times 6 \div 3 = 24.6819724$.

Ex. 4. $504 \div 144 = 3.5$ feet, and $372 \div 144 = 2.583$; then, $\sqrt{(2.583 \times 3.5)} = \sqrt{9.0416} = 3.0069$, and $3.0069 + 2.583 + 3.5 = 9.0902 \times 31.5 \div 3 = 95.447$.

Ex. 5. 18 inches $= 1\frac{1}{2}$ ft., and $1\frac{1}{2} \times 1\frac{1}{2} = 2\frac{1}{4}$.

Again, $1^2 = 1$;

Then, $\sqrt{(2\frac{1}{4} \times 1)} = 1\frac{1}{2}$, and $2\frac{1}{4} + 1\frac{1}{2} + 1 = 4\frac{3}{4} \times 18 \div 3 = 28.5$.

## PROBLEM IX.

Ex. 2. 18 inches $= 1.5$ foot, and $1.5^2 = 2.25 \times .7854 = 1.76715$; and 9 inches $= .75$ ft.; then, $.75^2 \times .7854 = .44178$; then, $\sqrt{(1.76715 \times .44178)} = \sqrt{.780691527} = .883567$, and $1.76715 + .44178 + .883567 = 3.092504 \times 14.25 \div 3 = 14.689394$.

Ex. 3. 2 feet 4 inches $= \frac{7}{3}$ feet, and $\frac{7}{3}^2 = \frac{49}{9} \times .7854 = 4.276$;

Again, 1 foot 8 inches $= \frac{5}{3}$ ft., and $\frac{5}{3}^2 = \frac{25}{9} \times .7854 = 2.1816\cdot$;

Then, $\sqrt{(2.1816\cdot \times 4.276)} = \sqrt{9.32877816} = 3.0543$, and $2.181 + 4.276 + 3.054 = 9.511$;

Then, $9.511 \times 1\frac{2}{3} \div 3 = 5.284$. *Ans.*

## PROBLEM X.

Ex. 2. $(70 \times 2) + 110 = 250 \times 24.8 = 186000 \div 6 = 31000$.

Ex. 3. $(32 \times 2) + 21 = 85 \times 4\frac{1}{2} = 382.5 \times 14 \div 6 = 892.5$.

## PROBLEM XI.

Ex. 2. 1 foot 2 inches $= 1\frac{1}{6}$ ft., the length; and 1 foot the breadth; also, 6 inches $= \frac{1}{2}$ foot, and 4 inches $= \frac{1}{3}$ foot;

Then, $(1\frac{1}{6} \times 2) + \frac{1}{2} = 2\frac{5}{6} \times 30\frac{1}{2} \times \frac{1}{6} = 14.402$;

Again, $(\frac{1}{2} \times 2) + \frac{7}{6} = 2\frac{1}{6} \times \frac{1}{3} \times 30\frac{1}{2} \times \frac{1}{6} = 3.672$;

Then, $3.672 + 14.402 = 18.074$.

## PROBLEM XII.

Ex. 2. $10 \div 2 = 5 + 13 = 18$; then $18 \times 8 = 144$, and $13 \div 2 = 6.5 + 10 = 16.5$;

Then, $16.5 \times 5.2 = 85.8$, and $144 + 85.8 = 229.8 \times .2618 = 60.16164 \times 12 = 721.93968$.

Ex. 3. 12 inches = 1 foot, and 7 inches = $\frac{7}{12}$ ft., 14 inches = $\frac{7}{6}$ ft., 12 inches = 1 foot; Then, $1 \div 2 = \frac{1}{2} + \frac{7}{6} = \frac{5}{3}$, and $\frac{7}{6} \div 2 = \frac{7}{12} + 1 = \frac{19}{12} \times \frac{7}{12} = \frac{133}{144}$; Then, $\frac{5}{3} + \frac{133}{144} = \frac{373}{144} \times 10 \times .2618 =$ 6.78 feet.

## PROBLEM XIII.

Ex. 2. $24^3 = 13824 \times .5236 = 7238.2464$.

Ex. 3. $25000 \div 3.1416 = 7957.75$, the diameter; then, $(7957.75 \div 2) \times (25000 \div 2) = 263858149120$.

Ex. 4. $30^3 = 27000 \times .5236 = 14137.2$.

## PROBLEM XIV.

Ex. 2. $16 \div 2 = 8$, the radius; then $(8^2 \times 3) + 4^2 = 208 \times 4 = 832 \times .5236 = 435.6352$.

Ex. 3. $20 \times 3 = 60$, and $5 \times 2 = 10$;

Then, $60 - 10 = 50$, and $5^2 = 25 \times 50 = 1250 \times .5236 = 654.5$.

## PROBLEM XV.

Ex. 4. $3 \div 2 = 1\frac{1}{2}$, and $1\frac{1}{2}^2 = 2.25 + 2.25 = 4.5$, and $4^2 \times \frac{1}{3} = 5\frac{1}{3}$;

Then, $5\frac{1}{3} + 4\frac{1}{2} = 9\frac{5}{6} \times 4 = 39\frac{1}{3} \times 1.5708 = 61.7848$.

Ex. 5. $20 \div 2 = 10$, and $15 \div 2 = 7.5$;

Then, $(10^2 + 7.5^2) = 156.25 + (10^2 \div 3) \times 1.5708 \times 10 = 2977.92264$.

Ex. 6. $12 \div 2 = 6$, and $6^2 = 36$; also, $10 \div 2 = 5$, and $5^2 = 25$.

$2^2 \times \frac{1}{3} = 1\frac{1}{3}$;

Then, $(36 + 25 + 1\frac{1}{3}) \times 2 \times 1.5708 = 195.8264$.

## PROBLEM XVI.

Ex. 2. $48 \div 2 = 24$, and $24^2 = 576 \div$

$(36 \div 2) = 32, + 18 = 50$, the diameter.

Also, $(50 - 36) \div 2 = 7$, the central distance.

Now, $18 \div 50 = .36$, the area segment corresponding to which is .25455 ;

Then, $50^2 = 2500, \times .25455 = 636.575$, the area of the generating segment ABC, the half of which is $318.1875, \times 7 = 2227.3125$ ;

Then, $24^3 \div 3 = 4608, - 2227.3125 = 2380.6875 \times 12.5664 = 29916.6714$.

## PROBLEM XVII.

Ex. 2. First, $40 \div 2 = 20$, and $20^2 = 400 \div 10 = 40$, $40 + 10 = 50$, the diameter ;

Again, $25 - 18 = 7$, the central distance.

Also $(36 - 16) \div 2 = 10$, and $10 \div 50 = .2$, the area segment corresponding to which is $.111823 \times 50^2 = 279.5575$, area of PLQ ;

Then, $16 \div 2 = 8 \times 40 = 320$, and $320 + 279.5575 = 599.5575$, the area of the generating surface PDLE.

Again, $50 \div 2 = 25$;

Then, $25^2 - 7^2 = 625 - 49 = 576$, the square of half the length of the spindle;

Then, $(576 - \frac{400}{3}) \times 20 = 8853\frac{1}{3}$;

And, $599.5575 \times 7 = 4196.9025$;

Then, $8853.3\dot{} - 4196.9025 = 4656.4315 \times 6.2832 = 29257.2904$.

## PROBLEM XVIII.

Ex. 2. $100^2 \times 6 \times .5236 = 31416$, cubic feet.

Ex. 3. $40^2 \times 50 \times .5236 = 41888$.

Ex. 4. $20^2 \times 10 \times .5236 = 2094.4$.

## PROBLEM XIX.

Ex. 3. $100 \times 3 = 300$, and $10 \times 2 = 20$;

Then, $300 - 20 = 280$, and $280 \times 10^2 \times .5236 = 14660.8$;

Then, $100^2 : 60^2 :: 14660.8 : 5277.888$.

Ex. 4. $10 \times 3 = 30$, and $1 \times 2 = 2$;

Then, $30 - 2 = 28 \times 1^2 \times .5236 = 14.6608$;

Then, as $10^2 : 6^2 :: 14.6608 : 5.277888$.

## PROBLEM XX.

Ex. 2. $30^2 = 900 \times 2 = 1800 + 18^2 = 2124 \times 40 \times .2618 = 22242.528$.

## PROBLEM XXI.

Ex. 3. $12^2 = 144 \times .3927 \times 40 = 2261.952$.

Ex. 4. $8^2 = 64 \times .3927 \times 30 = 753.948$.

## PROBLEM XXII.

Ex. 2. $(20^2 + 10^2) = 500 \times 12 \times .3927 = 2356.2$.

Ex. 3. $(30^2 + 10^2) = 1000 \times 50 \times .3927 = 19635$.

Ex. 4. $(15^2 + 12^2) = 369 \times 8 \times .3927 = 1159.2504$.

## PROBLEM XXIII.

Ex. 2. $12^2 = 144 \times .7854 = 113.0976 \times 30 \times \frac{8}{15} = 1809.5616$.

Ex. 3. $3^2 = 9 \times .7854 = 7.0686 \times 9 \times \frac{8}{15} = 33.92928$.

Ex. 4. $6^2 \times .7854 = 28.2744 \times 10 \times \frac{8}{15} = 150.7968$.

Ex. 5. $30^2 \times .7854 = 706.86 \times 50 \times \frac{8}{15} = 18849.6$.

## PROBLEM XXIV.

Ex. 2. $(30^2 \times 2 + 20^2) = 2200$, and $(30 - 20) = 10$, and $10^2 = 100 \times \frac{4}{10} = 40$;

Then, $2200 - 40 = 2160 \times 40 \times .2618 = 22619.52 \div 282 = 80.211$.

Ex. 3. $(40^2 \times 2 + 30^2) = 4100$, and $40 - 30 = 10$;

Then, $10^2 = 100 \times \frac{4}{10} = 40$;

Then, $4100 - 40 = 4060 \times 60 \times .2618 \div 231 = 276.08$,

## PROBLEM XXV.

Ex. 2. $10 \times 2 = 20$, $30 \times 3 = 90$;

Then, $90 + 20 = 110$;

And, $12^2 = 144 \times 110 \times 10 \times .5236 \div (30 + 10) = 2073.456$.

## PROBLEM XXVI.

Ex. 2. $\frac{8}{5} \sqrt{310} = \sqrt{\frac{64}{25} \times 310} = \sqrt{\frac{19840}{25}}$,

squared $= \frac{19840}{25}$;

Then, $\frac{19840}{25} \times 4 = 3174.4 + 24^2 + 32^2 = 4774.4 \times 40 = 190976 \times .1309 = 24998.7584$.

## PROBLEM XXVII.

Ex. 2. $10^2 + 5^2 = 125$, and $8 \times 2 = 16$;

Then, $16^2 = 256 + 125 = 381 \times 20 \times .1309 = 997.458$.

### PROBLEM XXVIII.

Ex. 2. $(2 + 18) = 20 \times 2^2 \times 2.4674 = 197.392.$

Ex. 3. $(7 + 20) = 27 \times 7^2 \times 2.4674 = 3264.3702.$

Ex. 4. $(2 + 12) = 14 \times 2^2 \times 2.4674 = 138.1744.$

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# REGULAR BODIES.

### PROBLEM I.

Ex. 2. $12^3 = 1728 \times .1178511 = 203.6467.$

### PROBLEM II.

Ex. 1. $3 \times 3 \times 3 = 27.$

### PROBLEM III.

Ex. 2. $2^3 = 8 \times .4714045 = 3.77126949.$

## PROBLEM IV.

Ex. 2. $12^3 = 1728 \times 7.6631189 = 13241.8675.$

## PROBLEM V.

Ex. 2. $12^3 = 1728 \times 2.181695 = 3769.9689.$

## PROBLEM VI.

Ex. 2. $12^2 = 144 \times 1.7320508 = 249.4153152.$

## PROBLEM VII.

Ex. 2. $4^2 = 16 \times 6 = 96.$

## PROBLEM VIII.

Ex. 2. $12^2 = 144 \times 3.4641016 = 498.8306304.$

Ex. 3. $4^2 = 16 \times 3.4641016 = 55.4256256.$

## PROBLEM IX.

Ex. 2. $2^2 = 4 \times 20.6457289 = 82.58292.$

## PROBLEM X.

Ex. 2. $2^2 = 4 \times 8.660254 = 34.641.$

Ex. 3. $3^2 = 9 \times 8.660254 = 77.9423.$

# SURFACES OF SOLIDS.

## PROBLEM I.

Ex. 2. 3 feet 6 inches × 4 × 3 feet 6 inches = 49 feet upright surface, and (3 feet 6 inches × 3 feet 6 inches) × 2 = 24 feet 6 inches;

Then, 49 feet + 24 feet 6 inches = 73 feet 6 inches.

Ex. 3. (4 feet 8 inches + 2 feet 3 inches) × 2 = 13 feet 10 inches, the perimeter;

Then, 13 feet 10 inches × 12 feet 9 inches = 176 feet $4\frac{1}{2}$ inches, and 4 feet 8 inches × 2 feet 3 inches × 2 = 21 feet, the area of both ends;

Then, 176 feet $4\frac{1}{2}$ inches + 21 feet = 197 feet $4\frac{1}{2}$ inches.

## PROBLEM II.

Ex. 2. 15 inches ÷ 12 = 1.25 foot, and 1.25 ×

1.0382617 (see Table, Mensuration, page 40) $= 1.297827125$, the perpendicular;

Then, $\sqrt{(1.297827125^2 + 13.5^2)} = \sqrt{(1.684355246385765625 + 182.25)} = \sqrt{183.934355246385765625} = 13.562235$, the slant height;

Then, $(15 \times 7 \div 2) \div 12 = 4.375$ feet, and $13.562235 \times 4.375 = 59.334778075$.

Also, $1.297827125 \times 4.375 = 5.677993$;

Then, $5.677993 + 59.334778075 = 65.012761$.

## PROBLEM III.

Ex. 2. $9 \div 3.1416 = 2.8648$, the diameter, $\div 2 = 1.4324$;

Then, $\sqrt{(1.4324^2+10.5^2)}=\sqrt{112.30176976} = 10.59725$, the slant height;

Then, $(10.59725 \times 9) \div 2 = 47.68762$, and $9^2 \times .07958 = 6.44598$, area of the base;

Then, $47.68762 + 6.44598 = 54.1336$.

## PROBLEM IV.

See Table, Mensuration, page 40.

Ex. 2. $1.2071068 \times 9 \div 12 = .9053301$, and $1.2071068 \times 5 \div 12 = .51029611$ ft.;

Then, $.9053301 - .51029611 = .40237$ nearly;

And $\sqrt{(10.5^2 + .40237^2)} = \sqrt{(110.4119016169)} = 10.512$, nearly, the slant height;

Again, 9 inches $\times 8 \div 12 = 6$ feet, and 5 inches $\times 8 \div 12 = 3\frac{1}{3}$ feet,

And $6 + 3\frac{1}{3} = 9\frac{1}{3}$, the sum of the perimeters of the ends, $\div 2 = 4\frac{2}{3} \times 10.512 = 49.05$;

Also, 9 inches $= \frac{3}{4}$ of a foot, and $\frac{3}{4}^2 = \frac{9}{16} \times 4.8281271 = 2.71$,

And 5 inches $= \frac{5}{12}$ of a foot,

And $\frac{5}{12}^2 = \frac{25}{144} \times 4.8281271 = .83$;

Then, $49.05 + 2.71 + .83 = 52.59$.

## PROBLEM V.

Ex. 2. $(18 + 9) \div 2 \times 3.1416 = 42.4116 \times$

171.0592 = 7254.894, and $18^2 \times .7854 =$ 254.4696, also, $9^2 \times .7854 = 63.6174$; Then, $7254.894 + 254.4696 + 63.6174 =$ 7572.981.

## PROBLEM VI.

Ex. 2. $20 \times 2 = 40$, the area of the back, and $(20 + 20) \div 2 \times 10 = 200$, area of one face $\times 2 = 400$, area of both faces, and $\sqrt{(10^2 - 1^2)} = 9.949 \times 2 =$ 19.898, area of both ends;

Then, $400 + 40 + 19.898 = 459.898$.

## PROBLEM VII.

Ex. 2. $10 \times 4 = 40$, area of the back,

$5 \times 2 = 10$, area of the upper section,

and $(4 + 2) \div 2 \times 20 = 60$, area of one end $\times 2 = 120$, area of both ends,

and $(10 + 5) \div 2 \times 20 = 150$, area of one of the faces × 2 = 300, area of both faces.

Then, (40 + 10 + 120 + 300) = 470.

## PROBLEM VIII.

Ex. 3. 7957.75 × 25000 = 198943750.

## PROBLEM IX.

Ex. 2. 7970 × 3.1416 = 25038.552 × 2143.623 5535 = 53673229.812734532.

Ex. 3. 7970 × 3.1416 × 3178.030327 = 79573277.600166504.

Ex. 4. 3 × 3.1416 × 1 = 9.4248.

Ex. 5. 33 × 4 = 132.

## PROBLEM X.

Ex. 2. 21.5 ÷ 12 = 1.7925 feet, diameter × 3.1416 = 5.631318, circumference, × 16 = 90.101088, convex surface, and $1.7925^2 \times 2 \times .7854 = 5.046$, area of both ends;

Then, $90.101 + 5.046 = 95.147$.

Ex. 3. $20.75 \times 3.1416 = 65.1882 \times 55 = 3585.351$, convex surface,

and $20.75^2 \times 2 \times .7854 = 676.327575$, area of both ends;

Then, $(3585.351 + 676.327575) \div 144 = 29.595$ nearly.

## PROBLEM XI.

Ex. 1. $(2 + 5) \times 2 = 14$; then, $14 \times 9.8696 = 138.1744$.

## PROBLEM XII.

Ex. 2. $5 \times 4 \times 2 = 40$, the area of both faces;

$5 \times 3 \times 2 = 30$, the area of both faces;

$3 \times 4 \times 2 = 24$, the area of both ends;

Then, $40 + 30 + 24 = 94$.

# TIMBER MEASURE.

## PROBLEM I.

Ex. 2. 8 feet 6 inches $\times$ 1 foot 3 inches $= 10\frac{5}{8}$ ft. at 5d. $=$ 4s. 5d.

Ex. 3. 12 feet 9 inches $\times$ 1 foot 3 inches $= 15\frac{15}{16}$ ft. at 5d. $=$ 6s. $7\frac{1}{2}$d.

Ex. 4. $(4 + 2) \div 2 = 3 \times 12 = 36$ feet, at 6d. $=$ 18 shillings.

Ex. 5. $(15 + 17) \div 2 = 16$ inches $= 1\frac{1}{3}$ foot $\times$ $6 = 8$ feet.

Ex. 6. 3 feet 3 inches $\times 20 = 65$ feet.

## PROBLEM II.

Ex. 2. $(15 \times 15) \div 144 = 1\frac{9}{16}$ foot $\times 18 =$ $28\frac{1}{8}$ feet.

Ex. 3. $16 \times 12 \div 144 = 1\frac{1}{3} \times 12 = 16$ feet.

Ex. 6. $81\frac{1}{2} + 41 = 122\frac{1}{2}$, and $55 + 29\frac{1}{2} = 84\frac{1}{2}$;

Then, $122\frac{1}{2} \times 84\frac{1}{2} = 10351\frac{1}{4}$, and $81\frac{1}{2} \times 55 = 4482\frac{1}{2}$, and $41 \times 29\frac{1}{2} = 1209\frac{1}{2}$;

Then, $(10351\frac{1}{4} + 4482\frac{1}{2} + 1209\frac{1}{2}) \div 6 = 2673.25 \times 47.25 = 126340.59375$.

## PROBLEM III.

Ex. 1. $144 \div 6 = 24$ inches $\div 12 = 2$ feet.

Ex. 2. $144 \div 8 = 18$ inches $\div 12 = 1\frac{1}{2}$ foot $\times 4 = 6$ feet.

Ex. 3. $144 \div 16 = 9$ inches $\div 12 = \frac{3}{4}$ of a foot $\times 7 = 5\frac{1}{4}$ feet.

## PROBLEM IV.

Ex. 2. $1728 \div (20 \times 10) = 8.64 = 8\frac{16}{25}$ inches.

Ex. 3. $1728 \div (9 \times 6) = 32$ inches $= 2\frac{2}{3}$ feet $\times 3 = 8$ feet.

## PROBLEM V.

The following questions are solved by the Table in the Mensuration, page 140.

Ex. 3. $1.41 \times 20 = 28.2$.

Ex. 4. $.562 \times 40 = 22.48$.

Ex. 5. $.444 \times 32 = 14.208$.

Ex. 6. $.39 \times 8.5 = 3.315$.

Ex. 7. $5.252 \times 40 = 210.08$.

Ex. 8. $5.252 \times 30 = 160.186$.

Ex. 9. $1.129 \times 25\frac{3}{4} = 29.071$.

Ex. 10. $1.265 \times 12 = 15.18$.

Ex. 11. $1.511 \times 38 = 57.418$.

# CARPENTERS' AND JOINERS' WORK.

Ex. 2. $51.5 \times 40.75 \div 100 = 20.98625$.

Ex. 3. $36.25 \times 16.5 \div 100 = 5.98125 =$ 5 squares $98\frac{1}{8}$ feet.

Ex. 4. 86 feet 11 inches $\times$ 21 feet 2 inches $=$ 1839 feet $8\frac{5}{6}$ inches $= 1839.73 \div 100 = 18.3973$.

Ex. 5. 1 foot 2 inches $= 1\frac{1}{6} \times 1 \times 22 = 25\frac{2}{3}$, content of the girder; 3 inches $= \frac{1}{4}$ foot, and 6 inches $= \frac{1}{2}$ foot; then, $\frac{1}{4} \times \frac{1}{2} \times 22 \times 9 = 24\frac{3}{4}$, content of the bridgings;
Again, 8 inches $= \frac{2}{3}$ foot, and 4 inches $= \frac{1}{3}$ foot; then, $\frac{2}{3} \times \frac{1}{3} \times 10 \times 9 = 20$ feet, content of the binding joists;
Again, 4 inches $= \frac{1}{3}$, and 3 inches $= \frac{1}{4}$; then, $\frac{1}{3} \times \frac{1}{4} \times 7 \times 25 = 14\frac{7}{12}$, content of

the ceiling joists ; then, $25\frac{2}{3} + 24\frac{3}{4} + 20 + 14\frac{7}{12} = 85$ feet.

Ex. 6. $30 \times 20 \times 3 = 1800$ ; and $6 \times 4 \times 2 = 48$, and 5 feet 6 inches $\times 6 \times 2$ inches = 66 feet, and 5 feet 6 inches $\times 4 \times 2 = 44$ feet, and 5 feet $\times$ 4 feet = 20 feet ;

Also, $10 \times 8 \times 3 = 240$ ;

Then, $1800 - (240 + 48 + 66 + 44 + 20) = 1382$ feet $\div 100 = 13.82$ squares, at £5 = £69 2s.

## OF PARTITIONING.

Ex. 1. 173 feet 10 inches $\times$ 10 feet 7 inches = 1839 feet $8\frac{5}{6}$ inches = 1839.73 feet $\div 100 = 18.3973$.

Ex. 2. 80 feet $\times$ 50 feet 6 inches = 4040 feet $\div 100 = 40\frac{2}{5}$ squares.

Ex. 3. 10 feet 6 inches $\times$ 10 feet 9 inches = 112 feet $10\frac{1}{2}$ inches = 112.875 ÷ 100 = 1 square $12\frac{7}{8}$ feet.

Ex. 4. 50 feet 6 inches × 12 feet 9 inches = 643 feet $10\frac{1}{2}$ inches ÷ 100 = 6 squares 43 feet $10\frac{1}{2}$ inches.

## OF ROOFING.

Ex. 2. 50 feet 9 inches × 30 feet = 1522 feet × $1\frac{1}{2}$ = $2283\frac{3}{4}$ feet, at 11 shillings ÷ 100 = £12 11s. $2\frac{11}{50}$d.

Ex. 3. 40 × 18 = 720 × $1\frac{1}{2}$ = 1080 ÷ 100 = 10 squares 80 feet.

Ex. 4. 14 feet 6 inches × 2 = 29 feet × 60 ÷ 100 = 17 squares 40 feet.

Ex. 5. 15 feet × 2 = 30 × 50 = 1500 ÷ 100 = 15 squares.

Ex. 6. 13 feet × 2 = 26 × 37 = 962 ÷ 100 = 9 squares 62 feet.

Ex. 7. 14 feet 6 inches × 2 = 29 feet × 70 feet 6 inches = $2044\frac{1}{2}$ feet ÷ 100 = 20 squares $44\frac{1}{2}$ feet.

Ex. 8. 50 × 30 = 1500 ÷ 100 = 15 squares.

# OF WAINSCOTTING.

Ex. 2. 142 feet 6 inches × 15 feet 6 inches = $2208\frac{3}{4}$ feet ÷ 9 = $245\frac{5}{12}$ yards.

Ex. 3. 60 feet 6 inches × 6 feet 4 inches = 383 feet 2 inches × $1\frac{1}{2}$ = $574\frac{3}{4}$ ÷ 9 = $63\frac{31}{36}$ yards.

Ex. 4. 129 feet 6 inches × 16 feet 3 inches = 2104 feet $4\frac{1}{2}$ inches, the superficial content of the room;

And 3 feet 9 inches × 7 feet = 26 feet 3 inches ÷ 2 = 13 feet $1\frac{1}{2}$ inches, half work of door.

And 7 feet 3 inches × 4 feet 6 inches = 32 feet $7\frac{1}{2}$ inches × 2 = 65 feet 3 inches, the superficial content of the shutters $\div$ 2 $=$ 32 feet $7\frac{1}{2}$ inches, for half work of shutters;

And 15 inches $=$ 1 foot 3 inches $\times$ 2 $=$ 2 feet 6 inches $+$ 7 feet 3 inches $=$ 9 feet 9 inches $\times$ 1 foot 2 inches $=$ 11 feet $4\frac{1}{2}$ inches $\times$ 4 $=$ 45 feet 6 inches, superficial content of the cheek-boards of the windows;

And 4 feet 6 inches $\times$ 1 foot 2 inches $\times$ 2 $=$ 10 feet 6 inches, superficial content of the boards over the top of the windows;

And 16 inches $=$ 1 foot 4 inches $\times$ 7 $\times$ 2 $=$ 18 feet 8 inches;

And 3 feet 9 inches $\times$ 1 foot 4 inches $\times$ 2 $=$ 10 feet;

Then, 10 feet $+$ 18 feet 8 inches $=$ 28 feet 8 inches, superficial content of the lining-boards round the door;

Then, 3 feet 9 inches $\times$ 3 feet $=$ 11 feet 3 inches, the chimney;

And 2104 feet $4\frac{1}{2}$ inches $+$ 13 feet $1\frac{1}{2}$ in.

+ 32 feet 7½ inches + 28 feet 8 inches + 45 feet 6 inches + 10 feet 6 inches = 2234 feet 9½ inches — 11 feet 3 inches = 2223 feet 6½ inches ÷ 9 = 247$\frac{13}{24}$ feet, at 3s. 6d., = £43 4s. 8d.

Ex. 5. 7 feet 8 inches × 12 feet 6 inches = 1045 feet 10 inches, superficial content of the room;

And 7 feet 8 inches × 3 feet 9 inches × 3 = 80 feet 6 inches, the superficial content of the shutters, ÷ 2 = 40 feet 3 inches, for half work;

And 3 feet 6 inches × 7 = 24 feet 6 in., ÷ 2 = 12 feet 3 inches, half work of door;

Then, 1045 feet 10 inches + 40 feet 3 in. + 12 feet 3 inches = 1098⅓ feet ÷ 9 = 122$\frac{1}{27}$ feet, at 6 shillings per foot, = £36 12s. 2½d.

# BRICKLAYERS' WORK.

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## OF TILING OR SLATING.

Ex. 2. 40 feet 9 inches × 47 feet 6 inches = 1935 feet $7\frac{1}{2}$ inches ÷ 100 = 19 squares $35\frac{5}{8}$ feet.

Ex. 3. 43 feet 9 inches × 27 feet 5 inches × $1\frac{1}{2}$ = 1802 feet $7\frac{3}{4}$ inches, and 43 feet 10 inches × 1 foot 4 inches × 2 = 116 feet $10\frac{2}{3}$ inches;

Then, 1802 feet $7\frac{3}{4}$ inches + 116 feet $10\frac{2}{3}$ inches = 1919 feet $6\frac{5}{12}$ inches = $1919\frac{77}{144}$ feet;

As 100 : $1919\frac{77}{144}$ : : £1 5s. 6d. : £24 9s. $5\frac{3}{4}$d.

Ex. 4. 45 feet 9 inches × 34 feet 3 inches = 1566 feet $11\frac{1}{4}$ inches ÷ 9 = 174.104 yards.

# OF WALLING.

Ex. 2. 57 feet 3 inches $\times$ 24 feet 6 inches $\times$ 5 half bricks $\div$ 3 $=$ 2337.7083 $\div$ 272.25 $=$ 8.5866 rods.

Ex. 3. 28 feet 10 inches $\times$ 20 feet $\times$ 5 half-bricks $\div$ 3 $= 961\frac{1}{9}$;

And 28 feet 10 inches $\times$ 20 feet $\times$ 4 half-bricks $\div$ 3 $= 768\frac{8}{9}$;

And $42 \div 4 = 10\frac{1}{2}$ feet, the height of the gable-end;

Then, 28 feet 10 inches $\times 10\frac{1}{2} \div 2 =$ $100\frac{11}{12}$;

Then, $(961\frac{1}{9} + 768\frac{8}{9} + 100\frac{11}{12}) \div 9 = 253\frac{1277}{2052}$ $= 253.62$.

# MASONS' WORK.

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Ex. 2. 120 feet 4 inches × 30 feet 8 inches = 3390 feet $2\frac{2}{3}$ inches = $3390\frac{2}{9}$ feet.

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Ex. 3. 112 feet 3 inches × 16 feet 6 inches = 1852 feet $1\frac{1}{2}$ inches ÷ 63 = 29 rods, 25 feet.

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Ex. 4. 5 feet 7 inches × 1 foot 10 inches = $10\frac{17}{72}$ feet, at 8 shillings per foot, = £4 1s. $10\frac{1}{2}$d.

# PLASTERERS' WORK.

Ex. 2. 14 feet 5 inches + 13 feet 2 inches × 2 = 55 feet 2 inches × 9 feet 3 inches = 510 feet $3\frac{1}{2}$ inches ÷ 9 = 53 yards 5 feet, of rendering; 5 inches × 2 = 10 inches;
Then, 14 feet 5 inches — 10 inches = 13 feet 7 inches — 10 inches = 12 feet 4 in. × 13 feet 7 inches = 167 feet ÷ 9 = 18 yards 5 feet of ceiling.

Ex. 3. (275 feet 5 inches × 105 feet 6 inches) ÷ 9 = $3228\frac{83}{216}$ yards, at 1s., = £161 8s. $5\frac{3}{4}$d.

Ex. 4. (18 feet 6 inches + 12 feet 3 inches) × 2 = 61 feet 6 inches × 10 feet 6 inches = 645 feet 9 inches;
And 3 feet 8 inches × 7 = 25 feet 8 inches;
And 5 feet × 5 = 25 feet;
Then, 25 feet 8 inches + 25 feet = 50 feet 8 inches;

And 645 feet 9 inches — 50 feet 8 inches = 595 feet 1 inch ÷ 9 = $66\frac{13}{108}$ yards, at 3 pence, = 16s. $4\frac{13}{36}$d ;

Again, (18 feet 6 inches × 12 feet 3 inches) ÷ 9 = $25\frac{13}{72}$ yards, at 8 pence, = 16s. $9\frac{4}{9}$d ;

Then, 16s. $4\frac{13}{36}$d. + 16s. $9\frac{4}{9}$d. = £1 13s. 2d.

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## PLUMBERS' WORK.

Ex. 2. 15 feet 6 inches × 10 feet 3 inches = $158\frac{7}{8}$ feet × 6 lbs. = $953\frac{1}{4}$ lbs. = 8 cwt. 2 qrs. $1\frac{1}{4}$ lbs.

Ex. 3. 43 feet × 32 feet = 1376 feet, and 57 × 2 = 114 ;

Then, 1376 + 114 = 1490 feet, at $8\frac{3}{4}$ lbs. per square foot = $13037\frac{1}{2}$ lbs. ;

Then, as 112 : $13037\frac{1}{2}$ : : 18 shillings : £104 15s. $3\frac{3}{4}$d.

Ex. 4. 130 yards × 18 lbs. = 2340, at 4d. = £39.

# PAINTERS' WORK.

Ex. 2. (24 feet 6 inches + 16 feet 3 inches) × 2 = 81 feet 6 inches × 12 feet 9 inches = 1039 feet $1\frac{1}{2}$ inches,

3 feet 6 inches × 7 = 24 feet 6 inches,

and (7 feet 9 inches × 3 feet 6 inches) × 2 = 54 feet 3 inches,

and (8 feet 6 inches × 1 foot 3 inches) × 4 = 42 feet 6 inches ;

and 3 feet 6 inches × 1 foot 3 inches × 4 = 17 feet 6 inches ;

Also, 5 feet 6 inches × 5 feet = 27 feet 1 inch ;

Then, 1039 feet $1\frac{1}{2}$ inch + 24 feet 6 inches + 54 feet 3 inches + 42 feet 6 inches + 17 feet 6 inches — 27 feet 1 inch = 1150 feet ÷ 9 = $127\frac{7}{9}$ yards ;

Then, $127\frac{7}{9}$ × 3 coats, at 2d. = £3 3s. $10\frac{1}{2}$d.

Ex. 3. (20 feet + 14 feet 6 inches) × 2 × 10 feet 4 inches = 713 feet;

And 4 feet 4 inches × 4 feet = 17 feet 4 inches;

Also, 3 feet 2 inches × 6 feet × 2 = 38 feet + 17 feet 4 inches = 55 feet 4 inches;

Then, 713 — $55\frac{1}{3}$ feet = $657\frac{2}{3}$ feet ÷ 9 = $73\frac{2}{27}$ yards.

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## GLAZIERS' WORK.

Ex. 2. 4 feet 8 inches 9 parts × 1 foot 4 inches 3 parts = 6 feet 4 inches $10\frac{3}{16}$ parts × 10 = 64 feet 0 inches $5\frac{7}{8}$ parts = $64\frac{846}{20736}$ = 64.0407.

Ex. 3. 3 feet 6 inches 9 parts × 1 foot 3 inches 3 parts × 20 = 90 feet, 6 inches, $6\frac{3}{4}$ parts = 90.546875.

Ex. 4. 7 feet 6 inches × 3 feet 4 inches = 25 feet.

Ex. 5. 14 feet 6 inches × 4 feet 9 inches = 68 feet, 10 inches.

Ex. 6. (12 feet 6 inches × 6 feet 9 inches) ÷ 2 = 42 feet 2 inches 3 parts = $42\frac{3}{16}$ feet, at 1s. 8d. = £3 10s. $3\frac{3}{4}$d.

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## PAVERS' WORK.

Ex. 2. 62 feet 7 inches × 44 feet 5 inches = 2779 feet $8\frac{11}{12}$ inches,

and 62 feet 7 inches × 5 feet 6 inches = 344 feet $2\frac{1}{2}$ inches ÷ 9 = $38\frac{53}{216}$ yards, at 3 shillings = £5 14s. $8\frac{3}{4}$ $\frac{1}{3}$d.;

Then, 2779 feet $8\frac{11}{12}$ inches − 344 feet 2 inches 6 parts = $2435\frac{77}{144}$ feet ÷ 9 = $270\frac{797}{1296}$ yards, at 2s. 6d. = £33 16s. $6\frac{1}{4}$ $\frac{43}{54}$d. + £5 14s. $8\frac{3}{4}$ $\frac{1}{3}$d. = £39 11s. $3\frac{1}{4}$d.

Ex. 3. 27 feet 10 inches $\times$ 14 feet 9 inches $=$ 410 feet $6\frac{1}{2}$ inches $\div$ 9 $=$ $45\frac{133}{216}$ yards, at 3s. 2d. $=$ £7 4s. $5\frac{1}{4}$d.

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Ex. 4. $5 \times 2 = 10 + 40 = 50$, greater diameter;

and $40 + 50 = 90$;

also, $50 - 40 = 10$;

Then, $90 \times 10 = 900 \times .7854 \div 9 =$ 78.54, at 2s. 4d. $=$ 183.26 shillings $=$ £9 3s. $3\frac{3}{25}$d.

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Ex. 5. $4 \times 2 = 8 + 60 = 68$, and $50 + 8 =$ 58;

Then, $68 \times 58 = 3944$, and $60 \times 50$ $= 3000$;

Then, $3944 - 3000 = 944 \times .7854 \div$ $= 82.3797$ yards.

# VAULTED AND ARCHED ROOFS.

## PROBLEM I.

Ex. 3. $15 \times 30 \times .7854 = 353.43 \times 90 = 31808.7$.

## PROBLEM II.

Ex. 2. 40 feet 6 inches $\times 100 = 4050$ feet.

Ex. 3. $40.5 \times 60 = 2430$ feet.

## PROBLEM III.

Ex. 2. $20^2 = 400$; then, (by Table, Mensuration, page 40,) $4.8284271 \times 400 = 27039.19176$.

Ex. 3. $4 \times 2 = 8 + 40 = 48$, and $30 + 8 = 38$, the diameters from the outer extremity of the wall;

Also, $17.32 + 4 = 21.32$, the height.

Then, $48 \times 38 = 1824 \times .7854 \times 21.32 \times \frac{2}{3} = 20361.589248$;

Again, $40 \times 30 \times .7854 = 942.48 \times 17.32 \times \frac{2}{3} = 10882.5024$;

Then, $20361.58924 - 10882.5024 = 9479.086848$.

## PROBLEM IV.

Ex. 2. $40 \times 30 \times 1.5708 = 1884.96$.

Ex. 3. $10^2 = 100 \times 2 \times 2.5980762$ (see Table, Mensuration, page 40) $= 519.61524$.

## PROBLEM V.

Ex. 2. D E $= 3.2$ feet, D F $= 4.5$ feet, 9 inches $= .75$ of a foot;

Then, $4.5 \div 2 = 2.25$, and $\sqrt{(2.25^2 + .75^2)} = \sqrt{5.635} = 2.37$, half the arc;

Again, $\sqrt{(4.5^2 - 3.2^2)} = \sqrt{10.01} = 3.1638 =$ F E or C D;

Again, $3.1638 \times 3.2 \times \frac{1}{2} = 5.06208$;

And $2.37 \times \frac{4}{3} = 3.16 + 4.5 = 7.66 \times \frac{2}{5} \times .75 = 2.298$;

Then, $5.06208 - 2.298 = 2.76408 \times 50 = 138.204$.

### PROBLEM VI.

Ex. 2. $98 \times 12 = 1176$ feet.

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## SPECIFIC GRAVITY.

### PROBLEM I.

Ex. 2. $10 - 6\frac{3}{4} = 3\frac{1}{4}$;

Then, as $3\frac{1}{4} : 10 :: 1000 : 3077$ nearly.

Ex. 4. $120 - 80 = 40$; and $20 + 120 = 140 - 16\frac{2}{3} = 123\frac{1}{3} - 40 = 83\frac{1}{3}$;

Then, as $83\frac{1}{3} : 20 :: 1000 : 240$.

Ex. 6. $166\frac{5}{8} - 42\frac{3}{4} = 123\frac{7}{8}$;

Then, as $166\frac{5}{6} : 123\frac{7}{8} :: 1333 : 991$.

## PROBLEM II.

Ex. 2. 112 lbs. = 1792 ounces ; one cubic foot = 1728 inches ;

Then, as 2520 : 1792 : : 1728 : $1228\frac{84}{105}$.

Ex. 3. 1 lb. = 16 ounces ;

Then, as 1745 : 16 : : 1728 : $15\frac{1473}{1745}$.

Ex. 4. One ton = 35840 ounces ;

Then, as 925 : 35840 : : 1728 : $38\frac{138}{185}$.

## PROBLEM III.

Ex. 2. $10 \times 3 \times 2\frac{1}{2} = 75$ ;

Then, as 1 : 75 : : 925 : 69375 ounces ÷ 16 = $4335\frac{15}{16}$ lbs.

Ex. 3. $12 \times 12 \times 63 = 9072$, content ; by the table, the specific gravity of marble is 2742, and one ton = 35840 ounces ;

Then, as 1 : 9072 : : 2742 : 24875424 ounces ÷ 35840 = $694\frac{11}{160}$ tons.

## PROBLEM IV.

Ex. 2. 6 — 5 = 1, and 9 — 6 = 3 ;

Then, 3 + 1 = 4 ;

As 4 : 3 : : 100 : 75, ounces of gold.

As 4 : 1 : : 100 : 25, ounces of silver.

## PROBLEM V.

Ex. 2. As 1 : 1 : : 970 : 970, ounces;

Then, as 1000 : 1728 : : 970 : 1676.16, cubic inches immersed $\div$ 144 = 11.64.

## PROBLEM VI.

Ex. 2. $9 \times 6 \times 3 = 162 \div 1728 = \frac{162}{1728}$ of a solid foot;

and $1000 - 970 = 30$;

Then, $30 \times \frac{162}{1728} = 2\frac{13}{16}$ ounces.

Ex. 3. An anker contains 10 gallons; then the cask contained 5 gallons; and a gallon contains 231 cubic inches $\times\ 5 = 1155 + 216 =$ 1371 cubic inches contained in the cask of brandy;

As 1728 : 1371 : : 1030 : 817.20486 ounces, the weight of an equal bulk of sea-water;

As 1728 : 216 : : 932 : 116.5 ounces, weight of the cask ; As 1728 : 1155 : : 927 : 619.609375 ounces, the weight of the brandy ; 619.609375 + 116.5 = 736.19375 ounces, the weight of both, and 817.20486 — 736.19375 = 81.01111, difference ;

The specific gravity of lead ts 11325, and of sea-water 1030 ;

Then, 11325 -- 1030 = 10295 ;

As 10295 : 81.01111 : : 11325 : 89.09495 ounces ÷ 16 = 5 lbs. 9 ounces.

## PROBLEM VII.

Ex. 2. $49\frac{3}{8}$ lbs. =790 onnces ;

As 11325 : 790 : : 1728 : 120 cubic inches of lead ;

As 1728 : 120 : : 1030 : $71\frac{19}{36}$ ounces ;

1030 — 240 = 790, and 790 ounces — $71\frac{19}{36}$ = $718\frac{17}{36}$ ;

As 790 : $718\frac{17}{36}$ : : 1728 : 1571.54.

# TONNAGE OF SHIPS.

## PROBLEM VIII.

Ex. 2. $20 \div 2 = 10$;

Then, $50.5 \times 20 \times 10 = 10100 \div 94 = 107.4$.

Ex. 3. $30 \div 2 = 15$;

Then $100 \times 30 \times 15 \div 94 = 478$.

# WEIGHT AND DIMENSIONS OF BALLS AND SHELLS.

## PROBLEM I.

Ex. 2. $3^3 = 27 \times 9 \div 64 = 3.8$ lbs.

Ex. 3. $5.54^3 = 170.030464 \times 9 \div 64 =$ 24 lbs.

## PROBLEM II.

Ex. 2. $6.6^3 = 287.496 \times 2 \div 9 = 63.888$ lbs.

Ex. 3. $3.5^3 = 42.875 \times 2 \div 9 = 9.53$ lbs.

Ex. 4. $6^3 = 216 \times 2 \div 9 = 48$ lbs.

## PROBLEM III.

Ex. 2. $24 \times 7\frac{1}{9} = 170.030464$ ;

Then, $\sqrt[3]{170.030464} = 5.54$.

Ex. 3. $3.8 \times 7\frac{1}{9} = 27$;
Then, $\sqrt[3]{27} = 3$.

## PROBLEM IV.

Ex. 2. $27\frac{7}{9} \times 9 \div 2 = 125$;
Then, $\sqrt[3]{125} = 5$.

Ex. 3. $63.888 \times 9 \div 2 = 287.496$;
Then, $\sqrt[3]{287.496} = 6.6$.

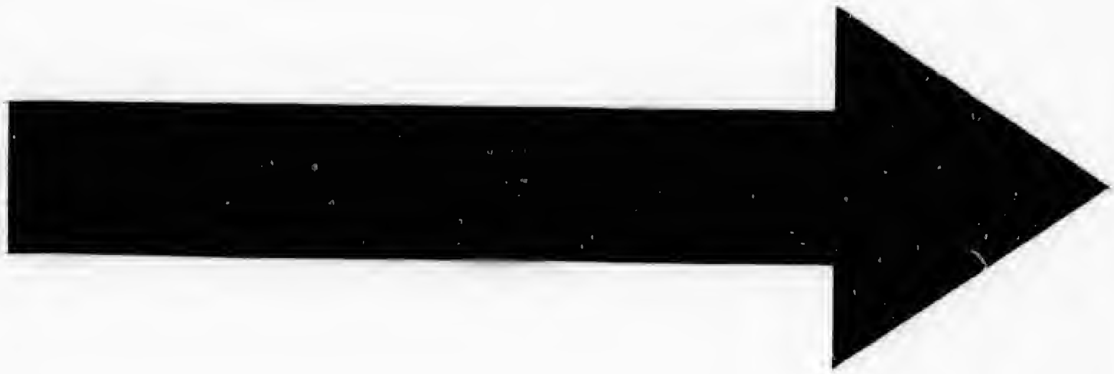
## PROBLEM V.

Ex. 2. $1\frac{1}{2} \times 2 = 3$; then, $9 - 3 = 6$, internal diameter;
$9^3 = 729$, cube of external diameter,
and $6^3 = 216$, cube of internal diameter;
Then, $729 - 216 = 513 \times 9 \div 64 = 72.14$ lbs.

Ex. 3. $9.8^3 = 941.192$, and $7^3 = 343$;
Then, $941.192 - 343 = 598.192 \times 9 \div 64 = 84.12$ lbs.

## PROBLEM VI.

Ex. 2. $13^3 = 2197 \div 59.32 = 37$ lbs.



Ex. 3. $7^3 = 343 \div 57.5 = 6$ lbs.

PROBLEM VII.

Ex. 2. $10 \times 5 \times 2 = 100 \times .0322 = 3.22$ lbs.

Ex. 3. $5 \times 2 \times 10 = 100 \times .0322 = 3.22$ lbs.

PROBLEM VIII.

Ex. 2. $5^2 = 25 \times 40 \div 40 = 25$ lbs.

Ex. 3. $5^2 = 25 \times 12 \div 40 = 7.5$ lbs.

PROBLEM IX.

Ex. 2. $14^2 = 196$;

And, $10 \times 40 = 400$;

Then, $400 \div 196 = 2.05$.

Ex. 3. $12^2 = 144 \times 40 \div 144 = 40$.

# PILING OF BALLS AND SHELLS.

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### PROBLEM I.

Ex. 2. 15 × 16 × 17 = 4080 ÷ 6 = 680 balls.

Ex. 3. 30 × 31 × 32 = 29760 ÷ 6 = 4960 balls.

### PROBLEM II.

Ex. 2. (19 + 1) = 20, and (19 × 2) + 1 = 39;

Then, 19 × 20 × 39 = 14820 ÷ 6 = 2470.

Ex, 3. 21 + 1 = 22, and (21 × 2) + 1 = 43;
Then, 21 × 22 × 43 = 19866 ÷ 6 = 3311.

### PROBLEM III.

Ex. 2. (46 × 3) + 1 = 139 − 15 = 124,

and $15 + 1 = 16$;

Then, $124 \times 15 \times 16 \div 6 = 4960$.

## PROBLEM IV.

Ex. 2. $(24 \times 3) + 1 = 73 - 24 = 49$, and

$24 + 1 = 25 \times 49 = 1225 \times 24 \div 6$

$= 4900$, the number of balls in a complete pile;

Again, $8 - 1 = 7$;

Then $(7 \times 3) + 1 = 22$, and $22 - 7$

$= 15 \times 8 \times 7 \div 6 = 140$.

Then, $4900 - 140 = 4760$.

Ex. 3. $(40 \times 3) + 1 = 121 - 20 = 101$;

and $20 + 1 = 21$;

Then, $101 \times 21 \times 20 \div 6 = 7070$;

Again, $40 - 12 = 28$, and $20 - 12 =$

$8$, and $8 + 1 = 9$;

Then, $(28 \times 3) + 1 = 85 - 8 = 77 \times$

$9 \times 8 \div 6 = 924$;

Then, $7070 - 924 = 6146$.

## DETERMINING DISTANCES BY SOUND.

Ex. 2. 1142 × 8 = 9136 ÷ 5280, the number of feet in a mile = $1\frac{7}{10}$ mile.

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# GAUGING.

## PROBLEM II.

Ex. 2. 40 × 40 = 1600 ÷ 277.274 = 5.77 gallons.

Ex. 3. 37 × 40 = 1480 × .0036065 = 5.33762.

Ex. 4. 144 × 144 = 7776 × .0036065 = 74.785.

Ex. 5. 64 × 6 ÷ 2 = 192 × 55.42 = 10640.64 ÷ 277.274 = 38.38 gallons; and as there are 8 gallons in a bushel, 38.38 ÷ 8 = 4.8 bushels nearly.

## PROBLEM III.

Ex. 2. $10^2 = 100 \times .002832 = .2832.$

Ex. 3. $30^2 = 900 \times .002832 = 2.548.$

Ex. 4. $24^2 = 556 \times .002832 = 1.631.$

## PROBLEM IV.

Ex. 2. $10 \times 20 = 200 \times .002832 = .566$, of a gallon.

Ex. 3. $70 \times 50 \times .002832 = 9.912$ gallons $\div 8 = 1.24$ bushels, nearly.

Ex. 4. $24 \times 18 = 432 \times .002832 = 1.2234$ gallons.

## PROBLEM V.

Ex. 2. $20 \times 20 \times 10 = 4000 \div 282 = 14.2$ gallons.

Ex. 3. $15 \times 20 \times 10 = 3000 \div 282 = 10.638$ gallons.

Ex. 4. $20 \times 8 \times 10 = 1600 \div 231 = 6.92$ gallons.

## PROBLEM VI.

Ex. 2. $100 + 80 = 180$, sum of the lengths ;
and $70 + 56 = 126$, sum of the breadths ;
Then, $180 \times 126 = 22680$, the product,
and $100 \times 70 = 7000$, area of the bottom ;
Also, $56 \times 80 = 4480$, area of the top ;
Then, $22680 + 7000 + 4480 = 34160 \times 42 \div 6 = 239120 \div 277.274 = 862.39$.

## PROBLEM VII.

Ex. 2. $20.2 \times 38 = 767.6 \times 3 = 2302.8$,
and $38 - 20.2 = 17.8$ ;
and $17.8^2 = 316.64$ ;
Then, $2302.8 + 316.64 = 2619.44 \times 21 \div 3 = 18336.08$ ;
And $282 \div 7854 = 359.05$, the circular divisor of old ale gallons, found the same as

the other divisors by Problem I., Gauging; Then, $18336.08 \div 359.05 = 51.07$ gallons.

## PROBLEM IX.

Ex. 2. $20 \times 20 \times 20 = 8000 \times .5236 = 4188.8 \div 282 = 14.85$, old ale gallons.

And $4188.8 \div 231 = 18.133$.

Ex. 3. $100^3 \times .5236 = 523600 \div 277.274 = 1888\frac{1}{3}$ gallons.

## PROBLEM XIV.

Ex. 2. Here $(12^2 + 16^2 \times 2) = 656 \times 20 = 13120$;

And, $13120 \times .0009\frac{1}{4} = 12.136$, old ale gallons.

Also, $13120 \times .0011\frac{1}{3} = 14.869$, wine gallons.

# LAND SURVEYING.

## PROBLEM III.

Ex. 2. $(57 + 91) \div 2 = 74$, and $785 - 634 = 151$;

Then, $151 \times 74 = 11174$, area of BNMG.

$(57 + 88) \div 2 = 72.5$, and $634 - 510 = 124$,

Then, $124 \times 72.5 = 8990$, the area of GMLF;

$(88 + 70) \div 2 = 79$, and $510 - 340 = 170$;

Then $79 \times 170 = 13430$, area of EFLK;

$(84 + 70) \div 2 = 77$, and $340 - 220 = 120$;

Then, $120 \times 77 = 9240$, area of DEKI;

$(84 + 62) \div 2 = 73$, and $220 - 40 = 175$;

Then, 175 × 73 = 12775, area of CDIH; 62 ÷ 2 × 45 = 1395, area of ACH; Then, 11174 + 8990 + 13430 + 9240 + 12775 + 1395 = 57004 links ÷ 100,000 = .57004 of an acre = 0 acre, 2 roods, 12 perches, nearly.

## PROBLEM IV.

Ex. 2. (12.25 × 8.5) ÷ 2 = 52.0625 ÷ 10 chains = 5.20625 = 5 acres, 0 rood, 33 perches.

## PROBLEM V.

Ex. 2. DF = 342, and CE = 625; Then, (625 + 342) ÷ 2 = 483.5 × 1360 = 65756 links ÷ 100,000 = 6.5756 = 6 acres, 2 roods, 12 perches.

Ex. 3. (2.25 + 3.6) ÷ 2 = 2.925 × 4.75 = 13.89375 ÷ 10 = 1.389375 acres = 1 acre, 1 rood, 22.5 perches.

## PROBLEM VI.

Ex. 2. EG = 900, and FS = 268;

Then, 268 × 900 ÷ 2 = 120600, area of the triangle GFE;

HE = 1100, DV = 280, and GW = 410;

Then, (410 + 280) ÷ 2 = 345 × 1100 = 379500, area of HGED;

CH = 1180, DN = 140, and IO = 280;

Then, (280 + 140) ÷ 2 = 210 × 1180 = 247800, area of CDHI;

AC = 900, BM = 200, and IA = 450,

Then, (450 + 200) ÷ 2 = 325 × 900 = 292500, area of AICB;

Then, 379500 + 292500 + 247800 + 120600 = 1040400 links ÷ 100,000 = 10.404 acres = 10 acres, 1 rood, 24.64 perches.

# MISCELLANEOUS PROBLEMS.

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Ex. 1. $(12 + 20 + 28) \div 2 = 30$;

Then, $30 - 12 = 18$, $30 - 20 = 10$, and $30 - 28 = 2$;

Then, $\sqrt{(30 \times 18 \times 10 \times 2)} = \sqrt{10800}$ $= \sqrt{(3600 \times 3)} = 60\sqrt{3}$.

Ex. 2. $(3 + 4 + 5) \div 2 = 6$;

Then, $6 - 3 = 3$. $6 - 4 = 2$, and $6 - 5 = 1$, and $\sqrt{(6 \times 3 \times 2 \times 1)}$ $= \sqrt{36} = 6$, area of the triangle;

Again, $(3 + 4 + 5) \div 3 = 4$, the side of the equilateral triangle of equal perimeter;

Then, $4^2 = 16 \div 4 = 4 \times \sqrt{3} = 6.928$, area of the equilateral triangle;

Then, $6.928 - 6 = .928$ of a square foot.

Ex. 3. $24 \div 2 = 12$; then, $12^2 = 144 \times 3 = 432$, and $10^2 = 100 + 432 = 532 \times 10 \times .5236 = 2785.552$.

Ex. 4. $18\frac{1}{2} = 18.5$, and $18.5^2 = 342.25$;
Then, as $7\frac{1}{2} : 8 :: 342.25 : 365.06$;
Then, $\sqrt{365.06} = 19.107$. See Remark 9 in Key.

Ex. 5. $\sqrt{(40^2 - 33^2)} = \sqrt{511} = 22.605$,
and $\sqrt{(40^2 = 21^2)} = \sqrt{1159} = 34.044$;
Then, $34.044 + 22.605 = 56.649 =$ 56 feet $7\frac{3}{4}$ inches.

Ex. 6. The circumference and diameter of the circle described by the outer wheel, is to that described by the inner, as 2 is to 1; and the radius of the circle described by the outer wheel is 5 feet greater than that described by the inner; therefore the diameter is 10 feet greater, and consequently the diameter of the circle described by the outer wheel is 20 feet;
Then, as $7 : 22 :: 20 : 62\frac{6}{7}$, or 63 feet, nearly.

Ex. 7. Here $9^2 \times .07958 \times 3 = 19.33794 =$ the content of the smaller rope;
and $12^2 \times .07958 \times 6 = 68.75712$, content of the larger rope;
Then, as $19.33794 : 68.75712 :: 22$ lbs. : $78\frac{2}{9}$ lbs.
See Remark 9.

Ex. 8. $8^3 = 512$, and $4^3 = 64$;
Then, $512 \div 64 = 8$.

Ex. 9. The perimeter of a figure is the sum of its sides;
Then, $40 \div 3 = 13.333$, side of the equilateral triangle, and $13.333^2 \div 4 \times \sqrt{3} = 76.980035 =$ area of the triangle;
Again, $40 \div 4 = 10$;
Then, $10^2 = 100$, area of the square;
Also, $40 \div 6 = 6.666 \times .8660254$ (see Table, Mensuration, page 40) $= 5.772953$, and $40 \div 2 = 20 \times 5.772953 = 115.47$ $=$ area of the hexagon.

Ex. 10. See Remark 9th. Here $3\frac{1}{2} = 3.5$, and $3.5^2 = 12.25$;

Then, as $\left\{\begin{matrix} 3 : 5 \\ 1 : 2 \end{matrix}\right. :: 12.25 : 40.8333$;

Then, $\sqrt{40.8333} = 6.39$ feet.

Ex. 11. See Remark 8th. By similar cones we have $3 : 1 :: 20^3 : 2666.666$;

Then, $\sqrt[3]{2666.666} = 13.867$, altitude of the upper part.

Also, $3 : 2 :: 20^3 : 5333.333$;

Then, $\sqrt[3]{5333.333} = 17.472$, the altitude of the middle and upper part;

Then, $17.472 - 13.867 = 3.604$, and $20 - 17.472 = 2.528$, altitude of the lower part.

Ex. 12. 21 inches $\div$ 12 = 1.75 feet;

Then, $1.75^2 = 3.0625 \times 4.8284271$ (see Table, Mensuration, page 40) $= 14.788$, nearly;

Again, 9 inches $\div$ 12 = .75 foot, and $.75^2 = .5625 \times 4.8284271 = 2.716$, nearly.

Also, $\sqrt{(14.788 \times 2.716)} = \sqrt{40.164208} = 6.337$, the mean proportional between the areas of the ends;

Then, $6.337 + 14.788 + 2.716 = 23.84 \times 15 \div 3 = 119.2$.

Ex. 13 As 4.1 : 65 : : 5 : 79.26.

Ex 14. Since there are 160 square poles in an acre, $160 \times 2 \div 40 = 8$, perpendicular of the triangle;

And $\sqrt{(20^2 - 8^2)} = \sqrt{336} = 18.3303$, the distance between the perpendicular and obtuse angle;

And $40 - 18.3303 = 21.67$;

Then, $\sqrt{(21.67^2 + 8^2)} = \sqrt{533.5889} = 23.099$;

Again, $40 + 18.3303 = 58.3303$, and the $\sqrt{(58.3303^2 + 8^2)} = \sqrt{3466.388999} = 58.876$.

Ex. 15. Here half-an-acre = 2420 square yards, and the area of a circle whose diameter is 1 = .7854;

Then, as $.7854 : 2420 :: 1^2 : \frac{2420}{.7854}$ = the square of the diameter = 3081.2325;

Then, $\sqrt{3081.2325} = 55.5$ diameter $\div 2 = 27.75$, radius.

Ex. 16. $\sqrt{(100^2 + 80^2)} = 128.06248$, and $100 - 80 = 20$; then, $128.06248 - 20 = 108.06248 \div 2 = 54.0312$, and 80 —

54.0312 = 25.9688 = twice the breadth of the walk ÷ 2 = 12.9844 feet.

Ex. 17. 4 ÷ 2 = 2, the height of the base of the hoof, and 2 ÷ 4 = .5, the area segment corresponding to which is .392699 × $4^3$ = 25.132736, first content;

Again, 4 — 3 = 1, and 2 — 1 = 1;

Then, 1 ÷ 3 = .333⅓, the area corresponding to which is .229172, which is found thus:—Area segment answering to .333 = .228858. and to .334 is .229801;

Then, .229801 — .228858 = .000943 × ⅓ = .000314 + .228858 = .229172, as above;

Then, $3^3$ = 27 × .229172 = 6.187644, and 4 — 3 = 1;

Then, 2 ÷ 1 = 2, and $\sqrt{2}$ = 1.4142;

Then, 6.187644 × 2 × 1.4142 = 17.5011323, and 25.132736 — 17.5011323 = 7.63 ÷ (4 — 3) = 7.63 × 6 ÷ 3 = 15.2 cubic inches, the quantity of liquor left in the cup;

Again, $3^2$ = 9 × .7854 = 7.0686, and $4^2$ = 16 × .7854 = 12.5664;

Then, $\sqrt{(12.5664 \times 7.0686)} = \sqrt{88.82686504}$ $= 9.425$ nearly, the mean proportional between the areas of the ends;

Then, $(12.5664 + 7.0686 + 9.425) = 29.05 \times 6 \div 3 = 58.1$ cubic inches, the content of the frustrum;

Then, $58.1 - 15.2 = 42.9$ cubic inches, the quantity of liquor the person drank, $\div 282 = .152127$ ale gallons.

Ex. 18. $5^2 \times .7853 = 19.635 \times 8 \div 3 = 52.36$, the solidity of a cone whose base is 5 and altitude 8;

Again, £5 13s. 7d. $\div$ 10s. $= \frac{1363}{120}$,

And by Remark 8th,

As $52.36 : \frac{1363}{120} :: 8^3 : 111.067$, the $\sqrt[3]{111.067} = 4.8068$, the altitude of the cone;

As $8 : 5 :: 4.8068 : 3.005$, nearly, the base;

Ex. 19.

Again, $3.005 \div 2 = 1.5025$, and

$\sqrt{(1.5025^2 + 4.8068^2)} = \sqrt{25.36232624}$

$= 5.0361$, the slant height of the whole cone;

Also, $8 + 5 = 13$;

Then, by Remark 9th,

As $13 : 8 :: 25.36232624 : 15.6077$, nearly;

Then, $\sqrt{15.6077} = 3.9506$, the slant height of the upper part,

And $5.0361 - 3.9506 = 1.0854$, the slant height of the under part.

Ex. 19. A general formula for such questions deduced from a simple and easy investigation, founded on the principle involved in the calculation of this question, may be expressed thus :—

$h = \dfrac{d}{n-2}$, in which $h$ = the height above the earth's surface; $d$ the earth's diameter;

$n$ the part of the earth's surface required to be visible from the altitude $h$; and, consequently $\dfrac{d}{n}$ = the height of the segment of the sphere containing the given altitude;

Then, $400 \div 5280 = \frac{5}{66}$ of a mile.

As $22 : 7 :: 25000 : 7954.5$, the earth's diameter.

Hence, $\frac{5}{66} = \dfrac{7954.5}{n - 2} = 5n - 10 =$ $524997$, and $5n - 525007$;

Then, $n = 525007 \div 5 = 105001.4$.

Again, $7954.5 \times 25000 = 198862500$ square miles $\times$ 640, the number of acres in a square mile, $= 127272000000$ acres $\div$ $105001.4 = 1212098.12436$.

Ex. 20. By Remark 10th. $4 \div 2 = 2$, radius, and $2^2 = 4 \times \frac{2}{3} \times 6 = 16$, the quantity of milk drank by the second boy.

Again, $4^2 = 16 \times .7854 = 12.5664 \times$

6 = 75.3984, the content of the cylinder ;

Then, 75.3984 — 16 = 59.3984, the quantity of milk drank by the first boy ;

And $4\frac{1}{2}$d. = 18 farthings ;

As 75.3984 : 16 : : 18 : 3.81971 farthings, for the second ;

Then, 18 — 3.81971 = 14.18029 farthings, for the first.

Ex. 21. Suppose 30 ; then, 30 × 30 × 6 = 5400 square inches, the surface of a cube whose side is 30 inches.

Again, 30 + 1 = 31 ;

Then, 31 × 31 × 6 = 5766 square inches ;

And 5766 — 5400 = 366 — 246 = 120 plus of an error.

Again, suppose 11 ; then, 11 × 11 × 6 = 726 square inches ;

Also, 11 + 1 = 12, the side increased one inch ;

Then, $12 \times 12 \times 6 = 864$ square inches,

and $864 - 726 = 138$,

and $246 - 138 = 108$ minus of an error ;

30 ╲╱ 120 + = 1320

11 ╱╲ 108 — 3240

$$\frac{228}{} = \frac{3240}{4560} = 20.$$

---

Ex. 22. As 30 : 20 : : 36 : 24 inches, the length of the arc of the sector whose radius is 20 inches.

Since the sides of the frustrum fold over each other $\frac{1}{8}$th of an inch ;

the circumference of the base and top of the frustrum will be $\frac{1}{4}$th of an inch less than the length of the arcs.

Then, $\frac{1}{4} = .25$, and $36 - .25 = 35.75$, the circumference of the bottom ;

And $24 - .25 = 23.75$, circumference of the top.

Also, $35.75 \div 3.1416 = 11.41$, diameter of the bottom,

and $23.75 \div 3.1416 = 7.56$, diameter of the top.

Again, $10 - \frac{1}{8} = 9.875$, the slant height; and $(11.41 - 7.56) \div 2 = 1.925$, the distance between the perpendicular and slant height;

Then, $\sqrt{(9.875^2 - 1.925^2)} = \sqrt{(97.515625 - 3.705625)} = \sqrt{94.81} = 9.6$, perpendicular height.

Also, $35.75^2 = 1278.0625 \times .07958 = 101.70821375$, area of the base of the frustrum,

And $23.75^2 \times .07958 = 44.88809375$, area of the top.

Then, $\sqrt{(101.70821375 \times 44.88809375)} = \sqrt{4565.4878339550359625} = 67.5683$, the mean proportional between the areas of the ends;

And $(101.7082 + 44.888 + 67.5683) =$ $214.1645 \times 9.6 \div 3 = 685.3264$.

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Ex. 23. $40 \div 2 = 20$, radius ;

Then, as $20 : 5 :: 20^2 : 100$,

and $\sqrt{100} = 10$, the part of the radius the third man must grind down ;

$6 + 5 = 11$ shillings ;

Then, as 20s. : $11 :: 20^2 : 220$,

and $\sqrt{220} = 14.832397$ ;

Then, $14.832397 - 10 = 4.832397$, the part of the radius the second must grind ;

And $20 - 14.832397 = 5.167603$, the part the first must grind.

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Ex. 24, $5^2 = 25 \times .7854 = 19.635$, area of the base, and $2^2 = 4 \times .7854 = 3.1416$, area of tne top ;

Then, $\sqrt{(19.635 \times 3.1416)} = \sqrt{61.685316}$ $= 7.8539$, the mean proportional;

And $7.8539 + 3.1416 + 19.635 = 30.6305$; then, $30.6305 \times 12 \div 3 = 122.522$, the solidity.

Then, by Remark 9th,

As $122.522 : 20 :: 5^2 : 4.0809$;

Then, $\sqrt{4.0809} = 2.02012$, greater diameter.

Then, as $5 : 2 :: 2.02012 : .80804$ feet.

Ex. 25. $6^3 = 216$ cubic feet; and $4^3 = 64 \times 2 = 128$;

Then, $216 - 128 = 88$ cubic feet still due.

Ex. 26. Let $x =$ the depth of the bowl,

$a =$ half the transverse diameter,

$b =$ half the conjugate diameter,

$y =$ half the diameter of the top of the bowl;

Then, $y^2 = \dfrac{b^2}{a^2} \times (2ax - x^2)$;

Hence, $y^2 = \frac{3b^2}{4}$, but $y^2 = 100$;

Then, $\frac{3b^2}{4} = 100$, and $b^2 = 133.\dot{3}$;

Then, $\sqrt{133.\dot{3}} = 11.547$, half the conjugate, $\times\ 2 = 23.094$, the conjugate diameter,

And, as $3 : 4 :: 23.094 : 30.792$, the transverse diameter;

Then, $30.792 \div 4 = 7.698$, the depth of the bowl;

Again, $30.792 \times 3 = 92.376$, and $7.698 \times 2 = 15.396$;

Then, $92.376 - 15.396 = 76.98$;

Also, $3^2 \div 4^2 = .5625$;

Then, $76.98 \times .5625 = 43.30125$;

And $7.689^2 = 59.25929 \times 43.30125 = 2566.00012 \times .5236 = 1343.55822832$, cubic inches, content of the bowl;

Again, $1.5^2 = 2.25 \times .7854 = 1.76715$

$\times\ 2 \div 3 = 1.1781$, cubic inches, content of the glass;

Then, $1343.55822832 \div 1.1781 = 1140.4$ $4497 \div 10 = 114.044497$.

Ex. 27. Here, 1 cubic foot = 1728 cubic inches;

And $\left(\frac{1}{40}\right)^2 \times .7854 = \frac{.7854}{1600} = \frac{.3927}{800}$ = the area of one end of the wire;

Hence, $1728 \div \frac{.3927}{800} = \frac{1382400}{.3927} =$ 3520244.4614 inches, = 97784.5684 yards, $= 55.\dot{5} = 55\frac{5}{9}$ miles.

Ex. 28. $48 \times 3 = 144 + 1 = 145 - 30 =$ 115, and $30 + 1 = 31$;

Then, $115 \times 31 = 3565 \times 30 = 106950$ $\div\ 6 = 17825$ balls, the number in a complete pile whose base is 48 by 30;

Again, 24 × 3 = 72 + 1 = 73 − 6 = 67, and 6 + 1 = 7;

Then, 67 × 7 × 6 ÷ 6 = 469 balls in a pile whose base is 24 by 6;

And 17825 — 469 = 17356 balls, the answer in the Mensuration, which is incorrect.

The correct answer is found thus:—

24 — 1 = 23, and 6 — 1 = 5;

Then, (23 × 3) + 1 = 70 — 5 = 65 × 6 × 5 ÷ 6 = 325 balls in a pile whose base is 23 by 5;

Then, 17825 — 325 = 17500 balls; the correct answer.

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Ex. 29. 12 — 1 = 11, and 40 + 11 = 51 balls in the length of the base;

And 10 + 11 = 21 balls in the breadth of the base;

Then, (51 × 3) + 1 = 154 — 21 = 133, and 21 + 1 = 22;

Then, (133 × 22 × 21) ÷ 6 = 10241

balls, the number in a pile whose base is 51 by 21.

Again, 40 — 1 = 39, and 10 — 1 = 9;

Then, (39 × 3) + 1 = 118 — 9 = 109 × 10 × 9 ÷ 6 = 1635 balls in a pile whose base is 39 by 9;

Then, 10241 — 1635 = 8606 balls.

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Ex. 30. See question 2nd, Problem III., in Weight and Dimensions of Balls and Shells. Since $\frac{1}{10}$th of an inch = .1; then 5.54 + .1 = 5.64.

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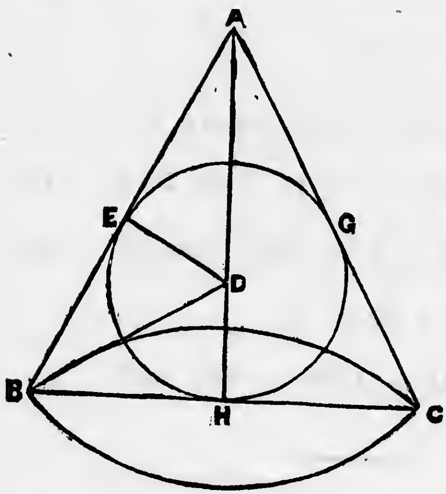
Ex. 31. Here, (9.43 + 7.92 + 6.15 + 3.16) = 31.4 ÷ 5 = 6.28, mean girt;

Then, 6.28 ÷ 4 = 1.57, and $1.57^2$ = 2.4649 × 17¼ = 42.5195.

Ex. 32.




Let ABC be the equilateral cone circumscribing the sphere EHG;

Then, since EH $= \frac{1}{3}$ of the circumference, the angle EDH = 120 degrees;

Then, the angle EBH = 60 degrees, and the angle EBD = 30 degrees, and DB or DA = 2DE;

Hence, AH = 3DE;

Let $r$ = radius of the sphere,

and Z = 3.1416;

Then, $2Zr^3$ = content of the cylinder circumscribing the sphere EHG;

And $2Zr^3 \times \frac{2}{3} = \frac{4Zr^3}{3}$ = the content of the sphere;

Also, $BH^2 = BD^2 - DH^2 = 3r^2$;

Then, $Z \times BH^2 \times \frac{AH}{3} = Z3r^3$

= content of the circumscribing equilateral cone;

Then the sphere, cylinder, and circumscribing cone are as $\frac{4}{3}$, 2, 3, or as the numbers 4, 6, and 9.

Ex. 33. $10^2 = 100 \div 4 = 25 \times \sqrt{3} = 43.30127$, area of the equilateral triangle;

Then, $43.30127 \div 3 = 14.43376$, area of each of the triangles into which the equilateral triangle is divided by bisecting the angles and producing the lines until they meet;

Again, $10 \div 2 = 5$;

Then, $14.43376 \div 5 = 2.8868$, nearly, the perpendicular of each of the triangles into which the equilateral triangle is divided, or the radius of the inscribed circle;

But the radius of the circumscribing circle is double the radius of the inscribed ;

Then, $2.8868 \times 2 = 5.7736$.

Ex. 34. Let the perpendicular $= p$, the sum of the sides $= s$, and the difference of the segments of the base $= d$ ;

Also, let $y + \frac{1}{2}d$ = greater segment, and $y - \frac{1}{2}d$ = lesser segment ;

Then, $\sqrt{\{(y + \frac{1}{2}d)^2 + p^2\}}$ = the greater side ;

And $\sqrt{\{(y - \frac{1}{2}d)^2 + p^2\}}$ = the lesser side ;

And, by the question, $\sqrt{\{(y + \frac{1}{2}d)^2 + p^2\}} + \sqrt{\{(y - \frac{1}{2}d)^2 + p^2\}} = s$ ;

Squaring both sides, and transposing,

$$2\sqrt{\{(y+\tfrac{1}{2}d)^2 + p^2\}} \times \sqrt{\{(y-\tfrac{1}{2}d)^2 + p^2\}}$$
$$= s^2 - 2y^2 - \tfrac{1}{2}d^2 - 2p^2 ;$$

Or, in order to simplify the expression,

Let, $s^2 - \frac{1}{2}d^2 - 2p^2 = 2m$;

Then, $\sqrt{\left\{(y + \frac{1}{2}d)^2 + p^2\right\}} \times \sqrt{\left\{(y - \frac{1}{2}d)^2 + p^2\right\}} = m - y^2$;

Squaring again both sides, and actually performing the multiplication of the first two factors, we have

$(y^2 - \frac{1}{4}d^2)^2 + 2p^2(y^2 + \frac{1}{4}d^2) + p^4 = m^2 - 2my^2 + y^4$;

And, by involving and collecting the terms,

$\frac{1}{16}d^4 - \frac{1}{2}d^2y^2 + 2p^2y^2 + \frac{1}{2}p^2d^2 + p^4 = m^2 - 2my^2$;

$(2m + 2p^2 - \frac{1}{2}d^2)y^2 = m^2 - \frac{1}{2}p^2d^2 - \frac{1}{16}d^4 - p^4$;

Whence, $y = \sqrt{\left(\frac{m^2 - \frac{1}{2}p^2d^2 - \frac{1}{16}d^4 - p^4}{2m + 2p^2 - \frac{1}{2}d^2}\right)}$

In which formula, substituting the values of $p$, $d$, and $m$, we have $y = 472\frac{1}{2}$;

Then, $y + \frac{1}{2}d = 720$, greater segment, and $y - \frac{1}{2}d = 225$, lesser segment;

And $\sqrt{(720^2 + 300^2)} = 780$, greater side;

Also, $\sqrt{(225^2 + 300^2)} = 375$, lesser side;

And $720 + 225 = 945$, the base.

Ex. 35. Here, $(12^2 + 10^2) = 244$, the square of the distance from the top of the higher wall, to that of the lower; and $40 - 30 = 10$;

Then, $10^2 = 100$; and $\sqrt{(244 - 100)} = 12$, breadth of the building;

Again, $\sqrt{244} = 15.6204$, and $12^2 = 144 \div 15.6204 = 9.2186$ feet, greater segment;

Then, $15.6204 - 9.2186 = 6.4018$, lesser segment; and $9.2186 - 6.4018 = 2.8168$;

Then, by Remark 4th, $15.6204 : 2.8168 :: 12 : 2.163$; and $\sqrt{(12^2 - 2.163^2)} = \sqrt{139.321431} = 11.803$; then, $11.803 + 30 = 41.803$, the length of the upright.

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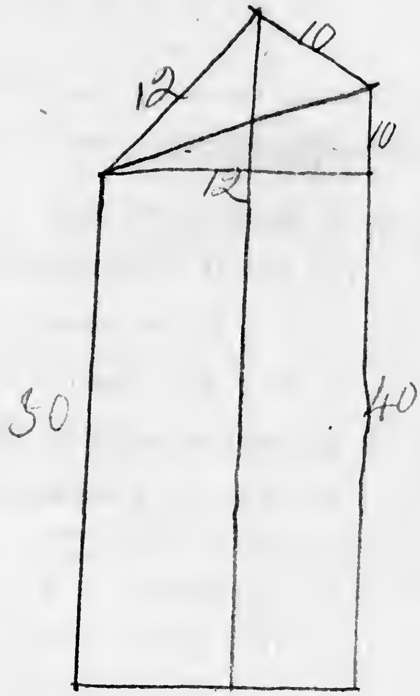

# ERRATA.

The attention of the Teacher is respectfully directed to the following typographical errors in the Key:—

| | | | |
|---|---|---|---|
| Page 12, | Ex. 2, | 4835484.777 should be **4836484.777** |
| " 18, | " 10, | 628.34 should be 628.32 |
| " 67, | " 6, | 6 × 2 inches, should be 6 × 2 |
| " 72, | " 3, | 43 ft. 9 in. should be 43 ft 10 inches. |
| " 74, | " 2, | 3390 feet should be 3690 feet. |
| " 75, | " 2, | after 13 ft. 7 in. should be and **13 ft.** 2 in. — 10 inches. |
| " 80, | " 5, | ÷ should be ÷ 9 = |
| " 83, | " 6, | $166\frac{5}{6}$ should be $166\frac{5}{8}$ |
| " 95, | " 2, | 7854 should be .7854. |
| " 101, | " 5, | $\sqrt{(40^2=21^2)}$ should be $\sqrt{(40^2-21^2)}$ |
| " 106, | " 18, | .7853 should be .7854 |
| " 108, | " 19, | $5n - 325007$ should be $5n = 325007$ |

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