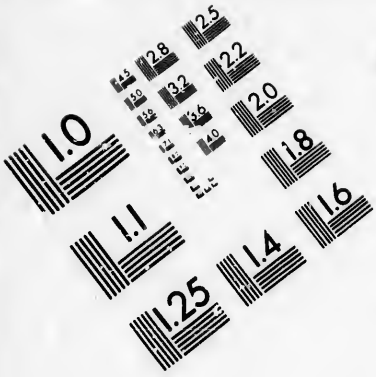

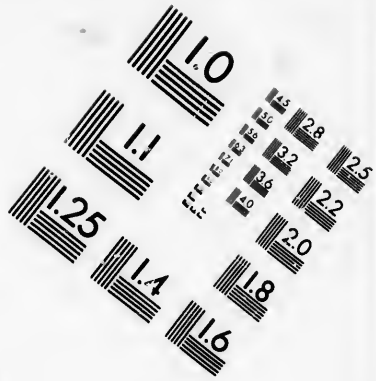

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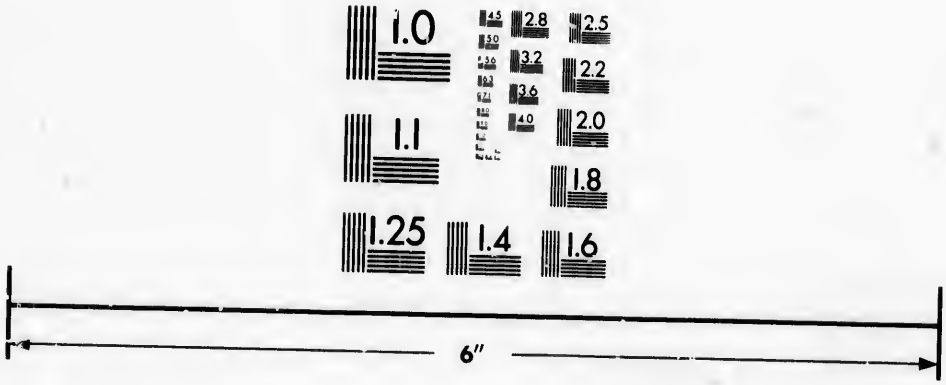

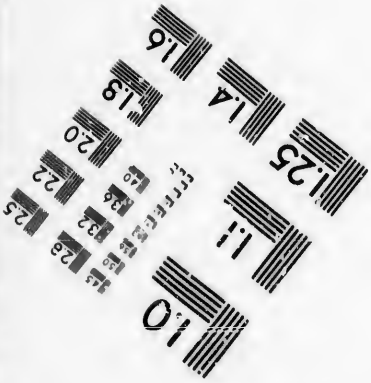

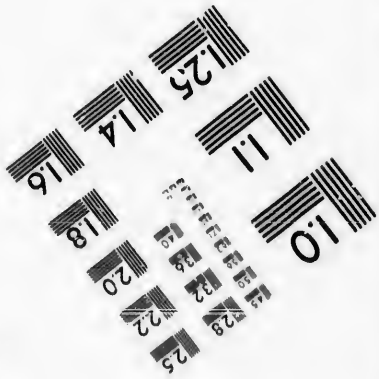

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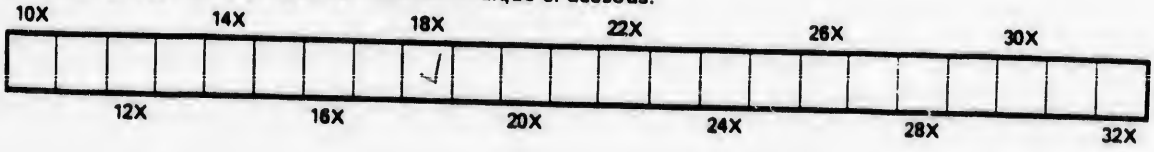

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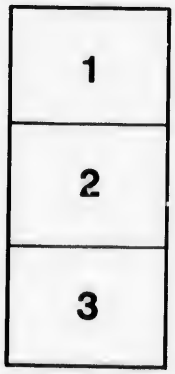
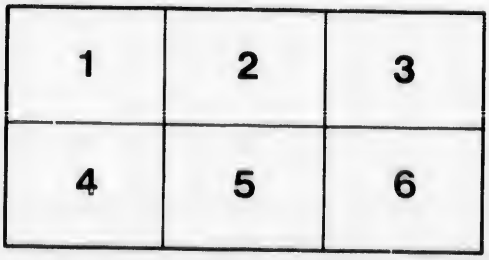




GRAFTON'S

GRADED ARITHMETIC

BOOK I.

TEACHERS' MANUAL

WITH ANSWERS

BY

E. W. ARTHY,

SUPERINTENDENT OF CITY SCHOOLS,
MONTREAL.

MONTREAL:

F. E. GRAFTON & SONS, PUBLISHERS.

1896.

GENERAL SUGGESTIONS.

1. In teaching follow the method and instructions given in this manual. Little time, however, is needed for teaching compared with the time needed for practice. A sufficient number of carefully graded examples will be found in the pupils' book. The best pupils may work them all; slower pupils should work part only. Do not make quick pupils keep pace with slow ones. Both must be taught together, but in busy work the one should do much more than the other. Quick pupils may be asked to prove their work. Avoid *long examples*, which discourage and disgust little children.

2. **Sight Exercises,** when *oral*, should be conducted in a spirited manner. When results are *written*, an exercise should be assigned, and pupils allowed to perform the mental work and record the answers at their own speed.

3. Endeavour first to make pupils *understand the process* of a rule; then *train them to be accurate;* and finally *drill in rapidity.* Never attempt to gain rapidity to the neglect of accuracy.

4. Go slowly, especially at first; do not measure the ability of the child by your ability. Bring yourself down to the level of the child's mind; be patient; repeat everything many times; review daily.

5. **Problems** (*oral and written*).

It should not be forgotten that the number-lesson may be made an excellent language-lesson. It is of the highest importance that the child give his answers in

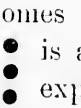
complete sentences, plainly spoken, with clear accent. Explanatory statements made by pupils should be simple but clear. They should represent the pupil's thought, and be clothed in language of his own choice. A *formula* or form of analysis may be given to the class later. A problem is not finished when the answer is found, but when it has been analysed. The language may be taken as a safe test that a pupil has completely mastered a step, though it does not follow from a pupil's inability to make an oral statement that he has failed to understand the process.

Lead children to make original problems.

6. **Slates, scribblers** and **pencils** should be kept in good condition. **Figures** should be large and distinctly made and written in lines parallel to the upper edge of the slate or book. With beginners it is of prime importance that all lines or columns of figures should be *large, even, distinct.*

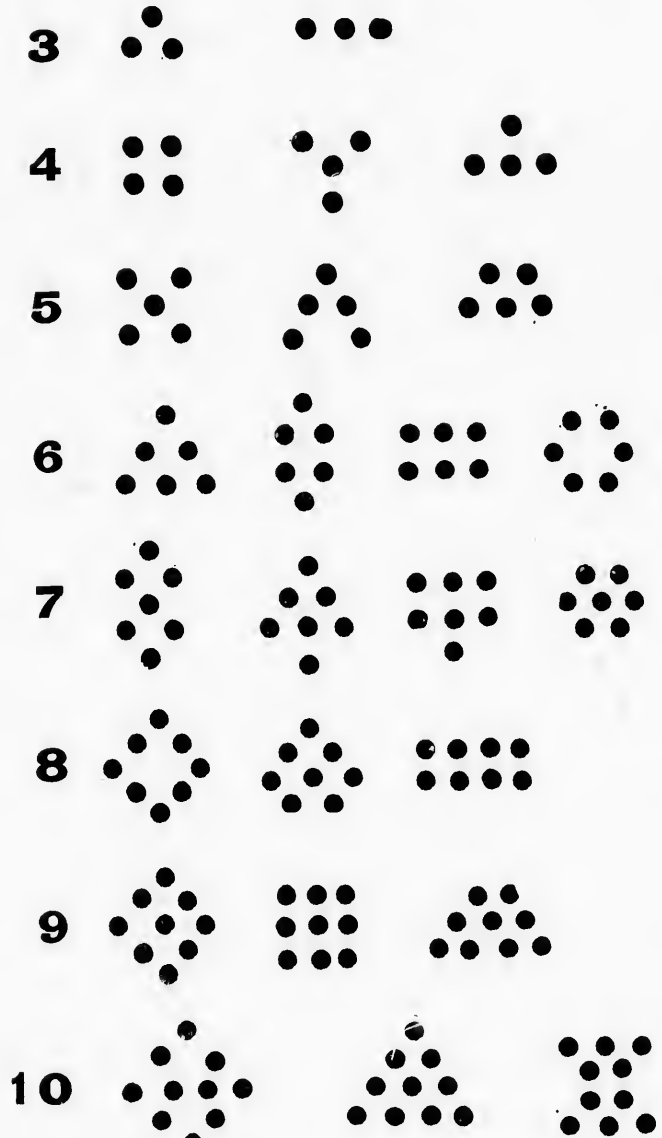
NUMBERS 1 TO 10.

A knowledge of numbers up to 10 is presupposed in the exercises of this book. It is presumed that a child not only knows them, but can use his knowledge. Of what use to the child if he can count to 100, but is unable to separate the number 9 into its elements and use them? By means of the eye and by handling objects (sticks, blocks and other counters) he has mastered the first ten numbers and their combinations. At the 10, if not before, the use of objects should be abandoned. The child should now be able to gain the abstract idea without the help of objects. Objects become a cumbrance as soon as the child can do without them, as they withdraw the attention from the abstract number.

The more thoroughly the numbers from one to ten are known, the surer and more rapid will be all later work in arithmetic. They are the foundation of the whole number system. A right conception of the first ten numbers will be much facilitated by arranging them in **geometrical patterns.** * "With a small number of objects a random grouping is instantaneously recognised; but not with many objects. Careful observation has shown that with most of us the highest number instantly recognised in a promiscuous assemblage of things is five. Higher numbers than five are subdivided by the eye into more easily recognised small groups. If nine pebbles be thrown upon a table before us, most of us will say mentally, here are three and three and three, nine. A few of the more expert will say, here are five and four, nine, on the table. Scarcely one will say at once nine, as we should all say three, if but three were thrown down before us. What is difficult or imposible for us to do, when objects are promiscuously presented, becomes easy in a definite arrangement. This  is at once recognised as nine, and that without explicit breaking up into three and three and three, although that subdivision is implicit in the conception. The formation of such conceptions of the first ten numbers should be regarded as an essential preliminary to arithmetical rules, should be begun at home or in the kindergarten and completed in the first year of the primary school."

The pattern which is presented to the class as the *type-form* of a number should be carefully chosen. It ought

* This quotation is from a treatise on the four simple rules of arithmetic, by Dr. Robins, Principal of the McGill Normal School, which was lent to the author in manuscript, and to which he desires to acknowledge his indebtedness for some valuable suggestions.


3
4
5
6
7
8
9
10

to be (1) well balanced, (2) easily derived from preceding patterns. If the triangle is accepted as the typical *three*, and the square as the typical *four*, good patterns may readily be constructed from these two forms with the aid, perhaps, of some linear arrangement. Special care should be taken in the selection of patterns for numbers above five. The patterns placed first in the series on page 4 can be recommended, and analysis will show how they are related to each other, *e.g.*, the patterns for the seven, the eight, the nine and the ten are all developed from the six, the two triangles of the six being separated by a linear arrangement of 4 to make 10, 3 to make 9, etc.

When a number is being decomposed into its elements for the purpose of comparing and measuring it with other numbers, the remaining patterns will be found useful as suggesting new combinations. If some desired combination is not readily seen, it will be made plain by the use of coloured chalks. Pupils must be trained to make patterns for themselves and to discover in them fresh combinations.

Notes for Book I.

Figures in heavy type at the top of each page indicate corresponding pages in the Pupils' Book. Roman numerals and capital letters on a page indicate corresponding exercises in the Pupils' Book.

I. Numbers 10 to 20.

1. The **Ten.** We have now reached the first number that must be considered as another kind of *one*—the *ten*. We write the figure 1 as before, but to show that this 1 contains ten times as much as the simple 1, we move it *one place to the left*, and say this 1 is a ten. The vacant

place of the simple *one* will be indicated by a cipher, so—10. Teach the *ten* as a group; ten dots joined together, ten sticks bound together, etc.

2. The teaching of each number above ten must precede the working of the exercises in the book. Show that the numbers from ten to twenty are formed by adding the first nine numbers to ten. Their names, from 13 to 19, indicate this, *e.g.*, *fourteen* means four and ten: *fifteen*, five and ten, etc. In writing these numbers the ten is expressed by a 1 in the *second* place and the figure expressing the *ones* is put in the first place.

I. A. This exercise must be worked across the page, not in columns downwards. It involves one new step, viz., *the combination of the ten with the nine digits.*

I. B. This exercise introduces the **key to addition through the ten.** When the sum of two numbers exceeds 10, one (usually the upper in the column) is broken into two parts, the first of which is sufficient to raise the lower number to 10. The remaining part is then added to the 10 thus formed, *e.g.*, the sum of 6 and 5 is found by breaking the 5 into 4 + 1, and the operation becomes 6 and 4 are 10 and 1 more makes 11. This *mental rearrangement* of 6 and 5 into 6 and 4 and 1, for the purpose of addition, is *fundamental*, and must be thoroughly taught. It removes the necessity of committing to memory an addition table, and enables a pupil, who knows the elementary sums up to 10, to add at once more difficult numbers *through* the ten. Practice will soon enable pupils to add by this method rapidly and accurately: when they no longer need the intermediate step, they must be encouraged to do without it, and in most cases they will themselves dispense with it. The process is purely *mental*, and except in explaining it, *oral*

expression of it must not be permitted; in adding 9 and 8, the thought is 9, 10, 17, nothing more, instead of 9, 17, without the intermediate 10.

The fourth line of exercise B must be worked exactly as the fourth column of example A.

I. C. Addition and subtraction should be taught together. The latter process is the opposite of the former, and should be derived from it. As soon as a child sees that 7 and 5 are 12, he is ready to see that 12 less 7 is 5, and 12 less 5 is 7. The difference between 12 and 7 must be inferred from the knowledge that 7 requires 5 to make 12, and not by counting 7 off 12. Examples must be worked *through the ten* as in addition, *e.g.*, 12 − 7 presents itself in this form: "What number must be added to 7 (the lower number) to make 12 (the upper)?" The result (5) is found mentally by raising 7 to 12 in two steps, thus, 7, 10, 12 (7 and 3 are 10 and 2 more are 12).

I. D. Such exercises in computation as are here given must be practised frequently not for any great length at one time, but in a spirited manner at frequent intervals. *Successive results* only must be named as rapidly as they can be given, *e.g.*, adding by *threes* from 1 would require pupils to say, 4, 7, 10, 13, 16, etc. The *spelling process*, 1 and 3 are 4, and 3 are 7, and 3 are 10, etc., cannot be allowed.

II. A. The *teaching* of numbers from 20 to 100 should precede and accompany these exercises. The exercises, if studied by the teacher in advance, will themselves indicate the method of teaching. As before, every ten should be regarded as a group or bundle, and the number of such groups or bundles should be called so many tens, the surplus left over being called ones or units.

A child must be taught to give clearly and exactly an analysis of numbers, written or spoken. Upon his ability to do this rapidly will depend his power to compute. *E.g.*, in 49 he must see at once 4 tens and an added 9. Concerning such a number he must be able to tell (*a*) that it consists of 4 tens and 9 units, (*b*) that it requires 1 unit more to make it 5 tens.

In counting, the following device may be tried with advantage :—

Count by ones (say) from 30 to 50: 31, 32, 33, 34, 35, 36—— "Stop!" the teacher says, "Where are we?" *Ans.*—"We have passed the third ten by 6; we require 4 more to make 4 tens, and still another ten to make 50."

II. B. All these exercises are to be worked in *tens, e.g.*:

90 + 50 = 140 (9 tens and 5 tens are 14 tens).

100 − 60 = 40 (6 tens requires 4 tens to make 10 tens).

79 + 20 = 99 (7 tens and 2 tens are 9 tens; 9 tens and 9 units are 99).

93 − 40 = 53 (4 tens needs 5 tens to make 9 tens or 90; and 3 units more to make 93.

III. The practice of giving to the **unit figure** great prominence in elementary instruction in number is to be commended. Even in grades where the *last figure* is not spoken of as the *unit figure*, the *same method* of instruction should be pursued. In adding digits to decades, *e.g.*, 7 successively to 14, 24, 34, etc., pupils must be made to observe that 7 and 4, or 4 and 7, added together will always give 1 as the unit figure. Results must still be obtained by computation through the ten; 34+7=34+6 +1=40+1=41, or *in words*, 34 requires 6 to make 40, and 1 more makes 41. But the memory soon comes in, so that pupils will instantaneously remember 1 as the unit figure resulting from an addition of 7 and 4 or 4 and 7. So conversely in subtraction, where 1 and 7 are the unit figures, a resulting 4 (or from 1 and 4, a resulting 7) will be instantaneously remembered.

The book exercises, pp. 10 to 13, on digits and decades, are to be all worked mentally. Children may copy them or not before working, as the teacher thinks wise. Examples which involve *several additions*, or *additions and subtractions combined*, may be omitted at this stage by duller pupils.

IV. A. These exercises, *like all others written in columns*, should not be copied, but added from the book, results only being recorded. Pupils should here be taught to check each addition by adding from the *top down*, as well as from the *bottom up*.

IV. B. **Practical Questions.**

1. The first of the *three* exercises here given is intended as an exercise in adding concrete numbers silently. The teacher should dictate the numbers with sufficient slowness to give the pupils time to *add*, but with sufficient rapidity to *prevent counting*. Answers are to be written simultaneously on slates.

The examples here given are models only, and their number should be increased. Digits and decades should be added in the same way, *e.g.*, 25 pencils and 8 pencils? 17 dollars and 6 dollars?

2. The second exercise will form a similar model for the silent subtraction of concrete numbers.

3. The third exercise has quite a different purpose, and may be employed and added to with much benefit. It is intended to train the pupil to *think* and to *talk*. The teacher says, "There are 8 girls and 11 boys in the class." One pupil may say, "There are 19 children in the class." Another may say, "There are 3 more boys than girls in the class." Another may say, "The number of girls is 3 less than the number of boys." This exercise is incidentally a *language lesson*. The teacher should see that pupils always answer with a *sentence*.

By way of preparation for this exercise it will be well to give some simple concrete questions in addition and subtraction, to which the pupils must reply orally in sentences, first finding the sum and then the difference; *e.g.*, the teacher says, "7 counters and 3 counters?" The pupils say, "7 counters and 3 counters are 10 counters, and 7 counters less 3 counters are 4 counters." The words "sum" and "difference" may now be required from pupils, *e.g.*, the above question may be answered, "The sum is 10 counters; the difference is 4 counters."

Before passing to multiplication and division let the teacher write on the board two columns of figures as indicated in the margin. These are for review work in addition and subtraction, and ought not to be erased, being kept for constant practice whenever a few minutes can be spared. The teacher with a pointer will indicate the successive numbers, the operation being indicated by the sign at top of the column. When a figure in the first column is pointed to, the number which it represents will be added; but when a figure in the second column is pointed to, the number indicated will be subtracted from the result which the pupils have previously obtained.

+	−
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9

V. Multiplication Table.

1. See that pupils have a clear idea of "*times*," and then of a number taken several *times*. The successive ticks of a watch, sounds of a bell or strokes of a clock will illustrate *times*.

2. Multiplication is to be taught as a *short method of adding a number to itself*. Thus $4 \times 6 = 24$ is a short method of finding $6 + 6 + 6 + 6 = 24$. Children must construct for themselves each table. In teaching a table observe carefully the following steps:—

(*a*) Let the results that are to be memorized in the table first be proved by rows of dots as shown in the pupils' book, p. 18.

(*b*) Let pupils make the table by counting (adding) by *equal increments*, *e.g.*, if the table of 6's is to be learnt, this step calls for adding by 6's up to 60:—6, 12, 18, 24, 30, 36, 42, 48, 54, 60.

The rate at which these successive numbers are named will at first be slow, but will become gradually quicker as facility is attained.

(*c*) The table must then be read and recited as follows:—

1 six, 6; 2 sixes, 12; 3 sixes, 18; 4 sixes, 24; etc.

No other words should be spoken or thought of.

(*d*) Study, recitation and frequent repetition are still required to fix the table in the memory.

Division Table.

The division table, though given in full, must not be committed to memory. Division must be taught as the *reverse of multiplication*; and the elementary *quotients* must be derived from the elementary *products*. Division and multiplication at this stage must be taught together. As soon as a pupil has learned that 6 *fives* are 30, he is prepared to see that 30 contains 5 six times. The question, reversing the process, should be first put thus: How many 5's in 30? or, In 30 how many fives? The form of question may afterwards be varied as follows: How many 5's make 30? 30 contains how many 5's? 30 contains 5 how many times? 5 is contained in 30 how many times? etc.

The book exercises, pp. 18–26, on each table should accompany the learning of the table. The problems at this stage both in multiplication and division ought to be illustrated by drawings, rows of dots, etc.

E.g., How many cents must I pay for 4 three-cent stamps?

This problem may be illustrated by drawing four oblongs representing the stamps, and placing 3 dots above each to represent the cents.

Halves, *thirds*, *fourths*, etc., should be taught from **fraction disks**, and problems involving their use illustrated afterwards, *e.g.*, 16 ounces in a lb. How many ounces in ¼ lb.? 16 dots (4 rows of 4

each) may be drawn, representing 16 ounces, and one-quarter of them then marked off.

The task of committing to memory the multiplication table may be shortened by several devices which are educationally sound. The following are recommended:—

1. The table of *twos* should be derived from addition as something already known; and that of *tens* from numeration and notation. These tables may be taken first.

2. Prove (but do not enunciate) the principle that the product of two factors is the same in whichever order they are taken, *e.g.*, $3 \times 5 = 5 \times 3$. This may be shown by rows of dots which, read horizontally, make 3 fives; read vertically, make 5 threes; or by adding a column of 3 fives and then one of 5 threes. This principle, which proves that 3 fives and 5 threes give the same result, and need not be memorised as independent facts, reduces the number of products to be memorised in each table progressively by one. In this way, when *nine times* is reached, 9 nines is the only result not already learned.

3. *Five times* is easy to remember, but, for the sake of the training involved, is best taught in connection with *ten times*, *every two fives making a ten*. *2 fives* = 1 ten; 3 fives = 1 ten and five or 15; 4 fives = 2 tens or 20; 5 fives = 2 tens and five or 25, etc.

5. The *nines*, which are difficult to memorise, may be thus simplified: *Nine* is one less than *ten*; therefore, 2 nines are 2 less than 2 tens or 18; 3 nines are 3 less than 3 tens or 27; etc.

These processes, once understood, must be worked mentally, not repeated aloud. Dull pupils will still use their unreasoning memory.

VI. Practical Questions.

A contains easy mental questions in multiplication. B and C contain similar questions in division. In both cases the questions should be considered as samples, and their number increased.

This exercise is intended to develop *oral expression* and *analysis*. The analysis given in the Arithmetic must be followed. Each example should be analysed *clearly* and *concisely*, and recited with *distinctness* and *promptitude*, first by some one pupil, then by the class. Practise this exercise until you have attained the desired result.

Before passing to written work, let the teacher write four columns of figures on the blackboard, as represented in the margin. This is a practical exercise to make pupils rapid and accurate in the mechanical processes of adding, subtracting, multiplying and dividing. For the method of conducting this exercise see directions on p. 10 of this Manual.

+	−	÷	×
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

VII. Numbers to 1000.

1. Teach the *place-value* of a figure, *i.e.*, that the value of a figure in the *first* place is so many units; in the *second* place, so many tens; in the *third* place, so many hundreds.

2. Require pupils to write numbers from dictation, their component parts being given in order and out of order, *e.g.*, 1 unit, 5 tens, 6 hundreds; and to name at sight the component parts of any number of 3 figures.

3. The addition and multiplication of *hundreds* will result in giving a figure in the *fourth* place, viz., *units of thousands*. Give the name *thousands* to figures in the fourth place, but do not teach the *thousand period*.

4. In writing numbers from dictation give special drill on numbers that present difficulties, *e.g.*, 101, 106, 413, 219. Such numbers as 756 are seldom read or written incorrectly.

VIII. The result found by addition is called the **sum**. The numbers to be added are called **addends**. These terms must be now explained, and afterwards used constantly. No definitions need be memorised.

Addition.—Two cases, (*a*) *without carrying*, (*b*) *with carrying*.

A. 341 + 415 + 122 (no carrying):—

341 = 3 hundreds + 4 tens + 1 unit.
415 = 4 hundreds + 1 ten + 5 units.
122 = 1 hundred + 2 tens + 2 units.

878 = 8 hundreds + 7 tens + 8 units.

Like is added to *like*, *units* to *units*, *tens* to *tens*, *hundreds* to *hundreds*. Impress and illustrate this principle.

B. 594 + 687 + 555 (carrying):—

594 = 5 hundreds + 9 tens + 4 units.
687 = 6 hundreds + 8 tens + 7 units.
555 = 5 hundreds + 5 tens + 5 units.

1836 = 18 hundreds + 3 tens + 6 units.

The sum of the units is 16 units; but 16 units make 1 ten with 6 units over. We write down the 6 units and carry the 1 ten to the column of tens (*like to like*).

The sum of the tens 1 (carried), 5, 8, 9, is 23 tens; but 23 tens make 2 hundreds with 3 tens over. Write down the 3 tens and carry the 2 hundreds to the column of hundreds.

The sum of the hundreds, 2 (carried), 5, 6, 5 is 18 hundreds.

The answer may be read, *eighteen hundred and thirty-six*.

NOTE 1.—Teach the extended method and the short method together.

NOTE 2.—The extended method must be reviewed again and again till pupils thoroughly grasp it, and are able to reproduce it. This accomplished, the showing of the extended work should only be called for occasionally.

NOTE 3.—Where examples in addition are arranged in columns, pupils should add from the book without copying, results only being recorded on their slates. They will get plenty of practice in copying other examples not arranged in columns.

NOTE 4.—Book exercises must be commenced as soon as pupils *understand the process of the rule.* Do not put off the mechanical drill until they are able to reproduce the extended work. In teaching the four simple rules observe the following steps:—(1) *Make pupils understand the process of the rule.* (2) *Train them to be accurate by mechanical practice.* (3) *Drill them in rapidity.* (4) *Make them show the process by reproducing the extended work.*

IX. The method of finding the *part that remains,* when a smaller number is taken from a larger, is called **subtraction.** The number that remains after subtracting is called the **difference.** Subtraction is very closely connected with addition (being its *opposite*), and must be taught by means of addition. The larger number is the *sum,* the smaller number is *one of two addends.* The remainder is the *other addend.* To show this relation introduce subtraction as a variation of addition in the following way:—

432	Suppose this example in addition has just been worked.
546	Erase one of the addends, say 546, and then ask the class to
978	discover what it was. Lead them by judicious questioning

to reproduce the missing addend, *e.g.*, the sum is 8 and one of the two addends is 2; therefore, the other must be 6. The sum is 7 and one of the two is 3; therefore, the other must be 4, and so on. When the missing addend is thus reproduced, prove its correctness by addition. The other addend may next be erased and reproduced in the same way.

The next step will be to rearrange the numbers after the manner usual in subtraction, and again find the missing addends.

978	978
546	432
—	—

This method has the following advantages to recommend it:—

(*a*) It shows clearly the relation between addition and subtraction.

(*b*) We easily derive from it a rule for subtraction, viz., *to raise the lower line so as to equal the top line.*

(*c*) We easily derive the proof of subtraction, viz., *that the remainder and lower line* (the two addends) *will give the top line* (the sum).

(*d*) The *borrowing* becomes the *carrying* of addition.

NOTE.—Do not mention or teach the terms *minuend* or *subtrahend*. They will come in later.

Pupils must be able to show the work extended as before :—

623 = 6 hundreds + 2 tens + 3 units.
237 = 2 hundreds + 3 tens + 7 units.

386 = 3 hundreds + 8 tens + 6 units.

7 units require 6 units to make 13 units. Write down the 6 units and carry 1 ten. 4 tens (3+1) require 8 tens to make 12 tens. Write down 8 tens and carry 1 hundred. 3 hundreds (2+1) require 3 hundreds to make 6 hundreds.

NOTE 1.—Carrying 1 ten means that having raised 7 up to 13 (instead of 3), we are 1 *ten to the good* in raising the next figure of the lower line up to the next figure of the upper line. Do not waste time in explaining this. It will be evident when the subtraction is proved by adding 386 and 237.

NOTE 2.—Though the exercises in the text-book give 4 figures (thousands), extended work need not go beyond 3 figures (hundreds).

X. Multiplication is a *short* method of *adding a number to itself*. Prove this by working a few examples (as 3 times 413) both by addition and multiplication. Pupils should occasionally be required to prove an example by addition.

The number to be multiplied is called the **multiplicand.** The number by which we multiply is called the **multiplier.** The result is called the **product.** Explain and afterwards constantly employ these terms.

A complete analysis of the steps used in multiplication is too difficult for pupils at this stage; but they must be taught to show extended work, as before, up to hundreds.

495 = 4 hundreds + 9 tens + 5 units.
4 4

1980 = 19 hundreds + 8 tens + 0 units.

4 times 5 units (4 fives) are 20 units. 20 units are 2 tens and 0 units over. Write down 0 and carry 2 tens. 4 times 9 tens are 36 tens, which with 2 tens carried make 38 tens. 38 tens are 3 hundreds and 8 tens over. Write down 8 tens and carry 3 hundreds. 4 times 4 hundreds are 16 hundreds, which with 3 hundreds carried make 19 hundreds.

XI. Division is finding *how many times* one number is contained in another (*e.g.*, how often 5 days are contained in 20 days), or separating a number into *equal parts* (*e.g.*, distributing 20 apples equally among 5 boys).

In division there are two numbers, called **dividend** and **divisor.** The *dividend* is the number to be divided. The *divisor* is the number by which we divide. The result or answer found by division is called the **quotient.** The part of the dividend left after dividing is called the **remainder.** Explain and afterwards employ these terms.

A complete analysis of division need not be attempted, but pupils must be taught to show extended work up to hundreds.

5) 732	= 5) 7 hundreds + 3 tens + 2 units.	
146 *with* 2 *rem.*	1 hundred + 4 tens + 6 units *with* 2 *rem.*	

5 is contained in the hundreds, which are 7, once with 2 remainder. Write down 1 under the hundreds. The 2 hundreds remaining are equal to 20 tens, to which are added the 3 tens in the dividend, making 23 tens. 5 is contained in 23 4 times with 3 remainder. Write 4 under the tens. The 3 tens remaining are equal to 30 units, to which are added the 2 units of the dividend, making 32 units. 5 is contained in 32 6 times with 2 remainder. Write the 6 under the units and the 2 as a remainder.

4) 136	= 4) 1 hundred + 3 tens + 6 units.	
34	= 0 hundred + 3 tens + 4 units.	

4 is not contained in the hundreds, which are 1. We show this (in the extended form) by placing 0 under the hundreds. We then write the 1 hundred (= 10 tens), with the 3 tens of the dividend, and proceed as before. The absence of a remainder is not recorded.

XII. Time.—Teach time at this stage by the clock, begining with the *hours, half-hours* and *quarter-hours,* and ending with the *minutes.* The following units of time should be taken: *minute, hour; hour, day; day, week; week, month; month, year.*

The Roman numerals to XII must be taught from the clock-dial. I *one,* V *five* and X *ten* should be taught first. The others are combinations of these three. When I *precedes* V or X, it must be subtracted from it; when I follows V or X, it must be added.

XIII. Capacity.—By way of introducing a table, have a talk with the children to find out what they know about it. Even little children will know something about buying milk, coal-oil, etc., by the *pint, quart* and *gallon.* Have these measures in your class-room, and prove the reality of your table, using water or dry sand. The table must then be copied and learnt, and drill given upon it.

XIV. Length.—The measuring strip here suggested should be of thick paper or cardboard. It may be prepared by the pupil at home, if it cannot be conveniently done in school. No work is to be done by means of this measure: it is to be used as a *test of correctness only.* The length of all lines or objects must be *guessed,* then *measured,* the difference between the estimate and true measure being found and recorded.

XV. Mental Examples.

In these examples, when the problems are given in concrete form, an oral statement should be called for;

E.g., 40 roses; 9 faded; how many left? *Ans.* 40 roses less 9, or 31 roses are left; or 31 roses are left, the difference between 40 and 9.

Mary is 5 years older than Jane, who is 8. What is Mary's age? *Ans.* Mary is 13 years old, because 5 and 8 make 13 (or because the sum of 5 and 8 is 13).

For oral statements of examples in multiplication and division see pp. 27 and 28 Pupils' Book.

XVI. When problems involving concrete numbers are given in the Test Exercises, the following plan is recommended :—

1. Let pupils first endeavour to solve such examples without aid and without giving any written statement of the process.

2. Let the teacher draw from the class by questioning an oral analysis of these examples.

3. Let pupils now work the examples a second time, giving a written statement sufficient to indicate the process.

E.g., If 4 fishermen catch 920 fish and divide them equally, how many fish will each have ?

(*a*) What is given in this example ? *Ans.* (drawn from several pupils). That there are 4 fishermen ; that they catch 920 fish ; that they divide them equally.

(*b*) What is required ? *Ans.* To find how many fish each fisherman will have.

(*c*) Which of the four methods or rules shall we use to find this ? *Ans.* Division.

(*d*) Employing division, what shall we do ? *Ans.* Divide 920 by 4, or 920 fish into four equal parts.

(*e*) Why ? *Ans.* Because each fisherman ought to get $\frac{1}{4}$ of the whole.

The pupils, after this oral analysis, proceed to work the example a second time, and are now expected to give some written statement of the process. Something like the following will be sufficient :—

Each fisherman will have $\frac{1}{4}$ of 920 fish. 4) 920 fish.
$\frac{1}{4}$ of 920 fish is 230 fish. 230 fish.

XVII. Before teaching numbers to 1,000,000, study the questions in this exercise and the *test questions* that

follow. The division of numbers into periods of 3 figures each must now be taught. Review the *place-value* of a figure. The three places of the units' period have already been taught, viz., the units' place, the *first;* the tens' place, the *second;* the hundreds' place, the *third.* On commencing instruction on the *thousands' period,* teach the names of the 3 places, *thousands* (or units of thousands), *tens of thousands, hundreds of thousands;* also the corresponding numbers of the places, *fourth, fifth* and *sixth.*

Notation, numeration, decimal scale must be explained and the definitions given in the book committed to memory.

XVIII. The object of this exercise is not to teach Canadian money (which is dealt with in Book II., Ch. I.), but only how to read and write it. Attention must be drawn to the following points:—

(*a*) Dollars and cents are separated by a *point.*

(*b*) The cents occupy *two places,* the first being single cents (units), the second ten-cent pieces (tens).

(*c*) When the number of cents is less than 10, a *nought* must be put in the *second* place to indicate the absence of tens.

XIX. The definitions relating to addition here given (and all definitions hereafter given in the pupils' book) are to be committed to memory. Before this is done, not only the terms used, but the arithmetical rule to which they apply, should be fully understood. *"The question, 'How shall pupils learn to express each rule and its reasons?' does not so much concern arithmetic as com-

* This extract is from Dr. Robins' treatise.

position. It is not a question of calculation, it is a question of the right use of language. Nothing must be done that the child does not understand as he does it. It is not good training to say to a child, 'Do or learn this, that or the other thing. You will understand it later.' He may understand it better later, but even as he does it or learns it, he should have within himself a sufficient reason for the doing or learning. Therefore, not so much in the interest of the pupils' arithmetical instruction and training, as of his general mental culture, let him attempt the enunciation of the rule, and let him argue the correctness and convenience of the rule. Let this be done at first in answer to questions given by the teacher, 'What do you do first in adding several numbers (or in subtracting, multiplying or dividing one number from or by another)? What then? And then?' The answers will very likely be imperfect. If erroneous, the error will probably be an error of omission, and the imperfection or omission may be brought to light by further questioning, or, better still, by the teacher's attempting to do exactly what the pupil has said should be first done. So step by step all the elements of the rule are prominently and distinctly presented."

XIX. A. Drill in *adding by equal increments* must be given at frequent intervals. (For method see p. 7, **I.** D.)

XIX. C. To the first number add the *units* of the second number, and to the result add the *tens* of the second number; *e.g.*, 65+36. 65+6=71; 71+30=101.

The following method may be used, if preferred. Add the tens first; then the units. Combine the two, *e.g.*, 65+36. 6 *tens*+3 *tens* =9 tens or 90; 5+6=11; 90+11=101.

In adding dollars and cents (involving four figures) pupils may be allowed at first trial to record the sum of the cents before adding the dollars. The exercise may be repeated later without such indulgence.

XIX. D and E. The suggestions, including the notes, made for the teaching of written addition on p. 14, still hold good. Read and follow them. Extended methods need not be taught beyond four figures.

Such examples as *9 times 987* (see E. 13), when given under addition, must be worked *by adding* 987 *to itself nine times.* They may be proved by multiplication.

NOTE.—A figure in the *seventh* place will sometimes occur in the answers, as the result of adding a column of figures occupying the *sixth* place. Give the name *millions* to a figure in the *seventh* place, but do not teach the millions' *period.*

XIX. F. All problems ought to be analysed orally by asking (*a*) what is given, (*b*) what is required, (*c*) what steps and methods must be employed. For method of analysing see p. 19, **XVI.**

The *denomination* must be marked in all answers involving concrete quantities.

XX. A and B. Raise the smaller number so as to equal the greater. This may conveniently be done in two steps: (1) raise the *units* of the smaller to the units of the larger; (2) raise the *tens.*

E.g., 79-19. 19 requires 60 to make 79. (Second step only, the units being already the same.)

78-34. 34 requires 4 to make 38, and 40 more to make 78. *Ans.* 44.

86-29. 29 requires 7 to make 36, and 50 more to make 86. *Ans.* 58.

The first step gives the *units*; the second the *tens.*

XX. C. Draw attention to the different ways of *wording* a question in subtraction. The method of working remains the same.

XXI. C. Do not forget the oral analysis of the problems.

XXII. This exercise takes up examples for the solution of which both *addition* and *subtraction* are required. See that each step is fully understood. The examples

increase in difficulty so that some of those given under D and E may require explanation. Problems must be analysed orally. The short method of adding by multiplication must not be permitted.

XXII. B. **Brackets,** inclosing numbers, indicate that whatever is contained within the brackets is to be treated as a single number.

Therefore, if within brackets there are several numbers connected by signs, the operations denoted by these signs must be performed, before any operation denoted by a sign outside the brackets. *E.g.*,

$31-(8+6)$. The numbers $8+6$ inside the brackets are to be treated as a single number, and our first step is to find their sum. $31-(8+6) = 31-14$ (first step) $= 17$ (second step). This example should be read "*From* 31 *take the sum of* 8 *and* 6."

XXIII. The *millions' period* is to be taught like the thousands' period. It occupies the *seventh, eighth* and *ninth* places, consisting respectively of *millions* (units of millions), *tens of millions* and *hundreds of millions.* Dictate numbers as follows: 801 million, 607 thousand, 760; 19 million and 1; etc.

XXIV. The tables of *elevens* and *twelves* are best learned from *tens.* Eleven is 1 ten and 1; 2 elevens are 2 tens and 2 or 22; 3 elevens are 3 tens and 3 or 33, etc. Twelve is 1 ten and 2: 2 twelves are 2 tens and 4 or 24; 3 twelves are 3 tens and 6 or 36, etc.

11×11 and 11×12 must be memorised and should receive special attention on account of their difficulty.

XXV. A. In multiplying at sight begin with the *tens.* *E.g.*, $40 \times 9 = 4$ *tens* $\times 9 = 36$ *tens* $= 360$.

84×7: 7 times $80 = 560$; 7 times $4 = 28$; 560 and 28 $= 588$.

XXV. B. 22. Division by *factors*. *How to find the remainder* when one exists, *e.g.*, $670408 \div 35$ (5×7).

5) 670408

7) 134081 — 3 *units* over

19154 — 3 *fives* over } 3 fives + 3 units = 18 rem.

The first division by 5 distributes the units into *groups of five each*, with 3 units over. When we again divide these groups of five by 7, we get 3 over; but these are 3 *fives*, not 3 units. The true remainder is then found by adding the two. From this derive the rule, "*Multiply the second remainder by the first divisor and add the first remainder.*"

XXVI. A. Multiplying a number by 10 is to raise it from *units* up to *tens*, *e.g.*, 23 × 10 = 23 tens = 230. Examining the result we find that the figures composing the number remain unaltered, but each has been moved to the next higher place, and a cipher has been inserted in the units' place. Hence a number is multiplied by 10 by annexing a cipher.

For the same reason annexing *two* ciphers increases the value of a number 100 times (raising each digit two places), and therefore multiplies the number by 100.

To multiply a number by 20, multiply by 10 and 2, *i.e.*, annex a cipher and multiply by 2; to multiply by 700, multiply by 100 and 7, *i.e.*, annex two ciphers and multiply by 7, etc.

In the same way and for the same reason, a number is *divided* by 10 by moving each figure to the *next lower* place; or by 100 by moving each figure *two places down*. The last figure or figures of the number, which are thus cut off, become the remainder.

XXVI. B. 13. You are now ready to explain multiplication where the multiplier consists of more than one

figure. Suppose the product of 571 and 23 is required. The multiplier $23 = 20$ (2 tens) $+ 3$, and the product is found by multiplying 571 first by the 3 units and then by the 2 tens. The partial products are then added.

571						571
23						23
1713	=	571	×	3	or	1713
11420	=	571	×	20	more briefly	1142
13133	=	571	×	23		13133

The cipher at the right of the second partial product does not affect the result of the addition, and may be omitted (as in shorter method), if care is taken in writing down the partial results, so that the *first figure* of each shall be *directly under that figure of the multiplier* which was used to obtain this result.

NOTE 1.—Do not wait for a complete understanding of full method before giving practice by shorter method.

NOTE 2.—It is essential to good work in multiplication that figures should be large and plain, and columns and lines even and distinct.

XXVI. D. Review the principles taught in **XXVI.** A, and now derive and establish the following rule for multiplying when either or both factors have ciphers on the right of their significant figures.

Find the product of the significant figures, and to the result annex as many ciphers as are on the right of both factors.

Give test questions like the following: "*How many ciphers will there be in the product of* 670 *and* 800 *?*"

The above rule is not to be memorised.

If a multiplier, *e.g.*, 4007, contains ciphers, not to the right of, but between, its significant figures, *two* points

need careful attention. (1) *The products that correspond to these ciphers will consist of ciphers and need not be written.*

(2) *The first figure of each partial result must be written under the figure used as a multiplier.*

2341
2007
16387
4682
4698387

XXVII. C. For method see Manual, **XV.** p. 18.

XXVII. D. For method see Manual, **XVI.** p. 19.

XXVIII. Long Division. The difficulty of teaching long division will be much lessened by a careful grading of the exercises. The book exercises have been, therefore, carefully graded, and teachers are advised not to diminish or increase them without due precaution, lest some step may be unwittingly omitted or prematurely introduced.

A. **Short Method.** Examples in A, 1 to 17.

13	
31) 403	31 is not contained in the first figure of the dividend.
31	31 is contained in 40 (the first two figures) 1 time with 9 over, (or, if preferred, 3 is contained in 4, taking the first figure only of the divisor).
93	
93	

Write the 1 over the second figure of the dividend (the last figure taken).

Annex 3, the next figure of the dividend, to the remainder 9. 31 is contained in 93 3 times with no remainder.

Write the 3 over the third figure of the dividend.

NOTE 1.—Pupils must be made to observe that there are *four steps* in each partial operation: (1) *Divide*, (2) *Multiply*, (3) *Subtract*, (4) *Bring down.*

NOTE 2.—If, when we multiply, the product is greater than the partial dividend, the quotient figure is too large and must be diminished.

NOTE 3.—If, when we subtract, the remainder is equal to, or greater than, the divisor, the quotient figure is too small and must be increased.

NOTE 4.—The method of writing the figures of the quotient *above* the *corresponding* figures of the dividend is recommended for two reasons; (*a*) it closely resembles the method of short division, the quotient figures being now written immediately above instead of below; (*b*) it prevents the omission of figures (especially 0's) from the quotient, for (the place of the first quotient figure being determined) there will be a quotient figure over each succeeding figure of the dividend.

C. **Extended Method.**

$403 \div 31$

$403 = 40 \text{ tens} + 3 \text{ units}.$

As 31 is not contained in the hundreds, which are 4, we arrange the dividend into 40 tens and 3 units.

31 is contained in 40 (*tens*) 1 (*ten*) with 9 (*tens*) over. Write the 1 *ten* above the *tens* of the dividend. Add the 3 units of the dividend to the 90 (9 tens) of the remainder.

31 is contained in 93 (units) 3 (units) with no remainder. Write the 3 units above the units of the dividend. Quotient = 1 ten + 3 units = 13.

1 ten + 3 units = 13 *Ans.*

$31 \overline{)\ 40 \text{ tens} + 3 \text{ units}}$

31

9 tens = 90 units

93

93

NOTE.—Teach the short method first and give practice upon it. Introduce the extended method later.

XXVIII. A. Examples 18 to 51.

$2494 \div 29$

How to find the quotient figures.

When the second figure of the divisor is greater than 5, use the first figure only as a *trial* divisor, and increase it by 1. Also increase the *trial* dividend by

86 *Ans.*

$29 \overline{)\ 2494}$

232

174

174

1. Thus, instead of saying how often is 29 contained in 249, say how often is 3 contained in 25. A *trial* quotient figure 8 is thus obtained.

XXVIII. B. *Ciphers* are now introduced into the *quotients*. The introduction of 0's in the quotient involves no new principle. Care must be taken that they are not omitted. Their omission will be best prevented by close attention to Note 4, p. 27.

XXIX. B. Division is the *reverse* of multiplication. The former separates a number into equal parts; the latter unites equal parts into one number. The *dividend* corresponds to the *product*, and the *divisor* and *quotient* to the *multiplier* and *multiplicand*. Explain and illustrate this.

XXX. A. See instructions, p. 18. **XV.**

XXX. B. See instructions, p. 19. **XVI.**

Time —Review the units previously taught (see pupils' book, p. 41). Now teach *seconds*, using the small dial and second-hand of the clock-face. Show how to write the time of day, explaining A.M., M., and P.M.

Teach also the number of days in a year, the months and the *seasons* with their months. Teach the number of days in each month. February 28 or 29; those with 30 days easily fixed in the memory by means of the familiar couplet, "Thirty days hath September," etc; *all* the others 31.

Capacity.—Review previous work p. 42 pupils' book, and see suggestions p. 18 of manual.

Measurements.—Review previous work p. 43 pupils' book, and see suggestions p. 18 of this manual.

Weight.—For this lesson you need scales and the following weights: 1 lb., 8 oz. ($\frac{1}{2}$ lb.), 4 oz. ($\frac{1}{4}$ lb.), 1 oz. Introduce by a talk about these weights.

Roman Notation.—The following *key* to Roman notation must be explained:—

1. When letters representing equal values are placed side by side, their values are to be added, *e.g.*, XX.

2. When a smaller number (*i.e.*, a letter representing that number) is placed on the right of a larger number, it must be added to the larger, *e.g.*, VI, LX, XV.

3. When a smaller number is placed at the left of a larger, it must be subtracted from the larger, *e.g.*, IV, XL.

Mental Problems, p. 84.—An oral analysis of these problems is required. The answer may be recorded by all pupils and the oral analysis given by one.

Review Examples.—For suggestions see p. 19, **XVI.** of manual.

ANSWERS.

VIII. Pages 31, 32, 33.

A. 1. 1036. 2. 775. 3. 889. 4. 898. 5. 965. 6. 878.
7. 798. 8. 889. 9. 797. 10. 768. 11. 789. 12. 988.

1. 781. 2. 1117. 3. 1331. 4. 1714. 5. 2058. 6. 2301.
7. 2568. 8. 2939. 9. 2643. 10. 2143. 11. 2080. 12. 2094.
13. 2436. 14. 2720. 15. 2106. 16. 2055. 17. 2164. 18. 1596.
19. 1872. 20. 2285. 21. 2082. 22. 2791. 23. 2359. 24. 2740.

C. 1. 3286. 2. 3012. 3. 3681. 4. 2474. 5. 2696. 6. 2236.
7. 2301. 8. 1705. 9. 3715. 10. 3543. 11. 3540. 12. 2197.
13. 2197. 14. 2301. 15. 2204. 16. 1705. 17. 3393. 18. 3347.

D. 1. 913. 2. 919. 3. 565. 4. 867. 5. 805. 6. 1055.
7. 1693. 8. 1747. 9. 1092. 10. 1578. 11. 1397. 12. 1625.

E. 1. 230. 2. 1207. 3. 378. 4. 958. 5. 965. 6. 768.
7. 683. 8. 331. 9. 697. 10. 1639. 11. 601. 12. 231.

F. 1. 825. 2. 658. 3. 678. 4. 908. 5. 864. 6. 998.
7. 953. 8. 1498. 9. 1604. 10. 341. 11. 1723. 12. 1681.

G. 1. 711. 2. 103. 3. 210. 4. 407. 5. 408. 6. 1275.
7. 1887. 8. 1442. 9. 1564. 10. 2168. 11. 2650.

IX. Pages 34, 35, 36.

A. 1. 3243. 2. 3232. 3. 3244. 4. 3424. 5. 3525. 6. 3213.
7. 5022. 8. 5024. 9. 6257. 10. 1361. 11. 2001. 12. 2170.

B. 1. 2728. 2. 2710. 3. 2780. 4. 2419. 5. 1449. 6. 2324.
7. 1825. 8. 3085. 9. 1306. 10. 4093. 11. 111. 12. 453.
13. 2462. 14. 1250. 15. 643. 16. 6332. 17. 3545. 18. 3553.
19. 1881. 20. 5760. 21. 5365. 22. 3016. 23. 6154. 24. 1844.

C. 1. 4623. 2. 2780. 3. 4790. 4. 1147. 5. 4572. 6. 3144.
7. 3754. 8. 3553. 9. 2432. 10. 737. 11. 138. 12. 2901.
13. 2281. 14. 2288. 15. 3258. 16. 3154. 17. 2184. 18. 2017.
19. 6432. 20. 3653. 21. 4499. 22. 6335. 23. 4930. 24. 7899.

D. 1. 6737. 2. 4927. 3. 6834. 4. 2586. 5. 5462. 6. 5027.
7. 9397. 8. 8539. 9. 944. 10. 4606. 11. 3387. 12. 5493.

E. 1. 1546. 2. 2717. 3. 4429. 4. 4622. 5. 4797. 6. 3726.
7. 6382. 8. 2861. 9. 6896. 10. 3552. 11. 7744. 12. 5879.
13. 3212. 14. 7179. 15. 5026. 16. 8383. 17. 8778. 18. 619.

F. 1. 7560. 2. 4587. 3. 4887. 4. 5841. 5. 4763. 6. 8677.
7. 4217. 8. 5170. 9. 3231. 10. 3055. 11. 907. 12. 4979.
13. 11. 14. 228. 15. 813. 16. 408. 17. 124. 18. 233.

G. 1. 236. 2. 253. 3. 36. 4. 27. 5. 461. 6. 578.
7. 4299. 8. 431. 9. 81. 10. 4135.

X. Pages 37, 38.

A. 1. 46. 2. 52. 3. 123. 4. 171. 5. 208. 6. 300.
7. 110. 8. 340. 9. 192. 10. 372. 11. 329. 12. 371.
13. 696. 14. 768. 15. 450. 16. 666. 17. 658. 18. 430.
19. 371. 20. 702. 21. 688. 22. 470. 23. 332. 24. 864.
25. 291. 26. 159. 27. 329. 28. 316. 29. 280. 30. 172.
31. 592. 32. 504. 33. 135. 34 222. 35. 276. 36. 290.
37. 510. 38. 469. 39. 616. 40. 891.

B. 1. 942. 2. 1146. 3. 2764. 4. 2860. 5. 3822. 6. 5887.
7. 4656. 8. 6849. 9. 2748. 10. 3220. 11. 1435. 12. 3822.
13. 4550. 14. 6264. 15. 3411. 16. 412. 17. 1461. 18. 3744.
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4. 53424, 61056, 68688
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6. 17806, 26709, 35612, 44515, 53418, 62321, 71224, 80127.
7. 12374, 18561, 24748, 30935, 37122, 43309, 49496, 55683.

D. 1. 976. 2. 679. 3. 1134. 4. 445, 534, 623, 712, 801.
5. $776. 6. 978 lbs. 7. 243 miles. 8. 784 lbs. 9. 168 hrs.
10. 2456 lbs. 11. $600. 12. $475. 13. 522 cents.

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B. 1. 214. 2. 132. 3. 212. 4. 488. 5. 244. 6. 163.
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7. 839. 8. 389. 9. 738. 10. 647. 11. 583. 12. 739.
13. 837. 14. 485 15. 537. 16. 863. 17. 749. 18. 837.
19. 579. 20. 496. 21. 371. 22. 695. 23. 738. 24. 597.
25. 836. 26. 948. 27. 379. 28. 957. 29. 657. 30. 598.

D. 1. 1260, 840, 630, 504, 420, 360, 315, 280.
2. 1980, 1320, 990, 792, 660, 565-5, 495, 440.
3. 2376, 1584, 1188, 950-2, 792, 678-6, 594, 528.
4. 385. 5. 324. 6. 81 plums. 7. 894 times. 8. 106 slates.
9. 31 children. 10. 97 yds. 11. 126 qts. 12. 1123. 13. 873.

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A. 1. 736. 2. 219. 3. 47 counters. 4. 12627 people 5 230 fish.
B. 1. 1313. 2. 899. 3. 8830 yds. 4. 54 cows. 5. 3600 times.

C. 1. 881. 2. 11 marbles. 3. 53. 4. 725-5. 5. 2920 days.
D. 1. 151 days. 2. 137 children. 3. 141. 4. 41909. 5. 107 cents.
E. 1. 850. 2. 88 sheep. 3. 574 pts. 4. 133. 5. 2384.
F. 1. 299 pupils. 2. 99. 3. 181. 4. 6867. 5. 90.
G. 1. 2147 bushels. 2. 343 kittens. 3. 1188 panes. 4. 52 yds. 5. 317 eggs.

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D. 1. 22998. 2. 19983. 3. 28310. 4. 242724.
5. 122904. 6. 267974. 7. 390370. 8. 262412.
9. 244372. 10. 146506. 11. 409448. 12. 205782.
13. 1124838. 14. 1070465. 15. 2250524. 16. 271305.
17. 401968. 18. 323049. 19. 5133357. 20. 1469297.

E. 1. 202821. 2. 154810. 3. 3140069. 4. 779053.
5. 2009637. 6. 1333435. 7. 1034683. 8. 1126047.
9. 2498457. 10. 931108. 11. 72084. 12. 391385.
13. 8883. 14. 68331. 15. 62378. 16. 1406361.
17. 20797. 18. 8704. 19. 4956. 20. 578312.

F. 1. 71 panes. 2. 5665 pupils. 3. 2192 apples. 4. 7642 trees.
5. 1984 pages. 6. 1008 pens. 7. 2309 sheep. 8. 365 days.
9. 78 strokes. 10. 41015. 11. 1773 potatoes. 12. 171979.

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6. 1618. 7. 13031. 8. 15708. 9. 17368. 10. 32131.
11. 161376. 12. 411389. 13. 492064. 14. 370504. 15. 524433.
16. 237797. 17. 168299. 18. 162798. 19. 11999. 20. 253676.

B. 1. 39672. 2. 730926. 3. 80857. 4. 170595. 5. 599071.
6. 92514. 7. 96444. 8. 662733. 9. 54322. 10. 779044.
11. 877507. 12. 291111. 13. 111109. 14. 260679. 15. 299999.
16. 20000. 17. 6778. 18. 1. 19. 39672. 20. 699731.
21. 730926. 22. 78552. 23. 80857. 24. 163896. 25. 22468.
26 9946. 27. 63642. 28. 594044. 29. 212503. 30. 91002.

C. 1. 67805. 2. 103875. 3. $242.19. 4. $126.17. 5. 423570.
6. 42 yrs. 7. 1815. 8. 969370. 9. 18394. 10. $319595.
11. 749 yrs. 12. 152367 males. 13. 586 trees. 14. 339752.
15. 44769 grains. 16. 13269 ft.

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6. 676. **7**. 10388. **8**. 426. **9**. 90408. **10**. 92.
11. 1. **12**. 28177. **13**. 605483. **14**. 4098.

E. **1**. 988. **2**. 177. **3**. 200. **4**. 10. **5**. 19013. **6**. 827.
7. 65454. **8**. 710. **9**. 2397. **10**. 5329. **11**. $5072.43.
12. 7065. **13**. 985. **14**. 498 pages. **15**. $4405.
16. 382 pupils. **17**. 44 yds. **18**. Mary 786, John 1069.

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3. 664279, 724668; 779845, 850740; 701899, 765708; 963952, 1051584; 9999099, 10908108.

4. 66555-2, 61008-11; 1089905-4, 999079-11; 1000999, 917582-5; 6910100-1, 6334258-5; 9829199-9, 9010099-10.

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13. 854904. **14**. 2823408. **15**. $46530.56. **16**. $175179.24.
17. $200408.88. **18**. $201150.60. **19**. $145952.96.
20. $571069.80. **21**. $275096.88. **22**. 4419-4. **23**. 18119-8.
24. 19154-18. **25**. $3.24. **26**. $5.63.
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6. 6084. **7**. 14450. **8**. 15215. **9**. 11414. **10**. 4250.
11. 68026. **12**. 34466. **13**. 110768. **14**. 254736. **15**. 486726.
16. 199617. **17**. 7278538. **18**. 3592212. **19**. 3131672.
20. 82052571. **21**. 760852. **22**. 939015. **23**. 2408217.
24. 6754755. **25**. 68604840. **26**. 54949721. **27**. 9320556.
28. 76661002. **29**. 6424019S. **30**. 16978476.

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21. 2075994000. 22. 3603200. 23. 5544700. 24. 40593600.
25. 809600. 26. 95060000. 27. 3860040. 28. 8271960.
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37. 407166647. 38. 361277560. 39. 421001350. 40. 280507995.
41. 367229800. 42. 389710881. 43. 46293952. 44. 76370766.
45. 9164688. 46. 15087224. 47. 19764888. 48. 1057904889.

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D. 1. 82280 yds. 2. 360 miles. 3. 317 tons. 4. $5475.
5. 2700 feet. 6. $12.15. 7. $567. 8. 27,000,000 seeds.
9. 360,000,000 eggs. 10. 149176 people. 11. 122896 fish.
12. 174 marbles. 13. 2,895,588 ounces. 14. 817200 pages.
15. 462000 bricks.

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23. $4.70. 24. $30.05. 25. $56.20. 26. $70.03. 27. $70.03.
28. 36000-56. 29. 10010-6. 30. 2010-23.

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B. 1. 31 yrs. 28 weeks over. 2. 75 dresses. 3. $50. 4. 27 days.
5. 126 boxes 6. 288 eggs. 7. 75 buns. 8. 17 cars.
9. 106 daily, 742 weekly. 10. 49 tons. 11. 7610 boxes.
12. $634. 13. 63182 acres, $28 over. 14. 9080. 15. $804.
16. 73298 dozen. 17. 1908.

Review Examples. Pages 86, 87, 88.

1 1. 5623 soldiers. 2. 10063 plums
3. 39 payments, 3 yrs. 3 mos. 4. 141816.
2 1. 2553. 2. 880 counters. 3. 8 wks. 6 days. 4. 95049.
3 1. 346 apples. 2. 776. 3. 261-8. 4. 40299.
4 1. 83149. 2. 77654. 3. 4470. 4. 156 strokes
5 1. 245 days. 2. 10119 men. 3. 85. 4. 36 marbles.
6 1. 20700. 2. $3.80. 3. $227. 4. $108.
7 1. 82. 2. 17430. 3. 80 counters each, 16 over. 4. 4840 trees.
8 1. 40 apples each, 11 apples over. 2. 40 boys, 11 apples over.
3. 209-584. 4. 1259.
9 1. $234. 2. $1.60. 4. 3 ft. 6 in. 5. 360 minutes
6. $600, $900, $2100.

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