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Washington observations, 1865.

APPENDIX I.

DESCRIPTION

OF THE

TRANSIT CIRCLE

OF THE

UNITED STATES NAVAL OBSERVATORY,

WITH AN

INVESTIGATION OF ITS CONSTANTS.

PREPARED BY ORDER OF

REAR ADMIRAL CHARLES HENRY DAVIS, U. S. N.,
SUPERINTENDENT,

BY SIMON NEWCOMB,
PROFESSOR OF MATHEMATICS, U. S. N.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
1867.

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INTRODUCTORY NOTE.

The instrument described in the following pages was procured for the Observatory by the late Captain Gilliss. On his accession to the Superintendency, the want of a suitable Meridian Circle was strongly felt. The military operations of the government temporarily delayed the supply of this want. In October, 1863, however, the necessary authority was granted by the Hon. Secretary of the Navy, and the instrument was ordered from Messrs. Pistor and Martins, of Berlin. Correspondence respecting the size and plan of the instrument occupied the remainder of the year, and terminated by concluding on an object glass of at least eight Paris inches clear aperture, and by leaving the plan of the instrument and its mountings altogether in the hands of the artists.

The parts of the instrument arrived in October, 1865. The work of mounting was commenced on the 16th of that month, and on the 28th the instrument was in position. The next two months were occupied in determining the errors of flexure and division, and in making the necessary preparations for active work. Regular astronomical observations were commenced on January 3, 1866.

APPENDIX I.

DESCRIPTION

OF THE

TRANSIT CIRCLE

OF THE

UNITED STATES NAVAL OBSERVATORY.

PART I.

DESCRIPTION OF THE INSTRUMENT AND ITS ADJUNCTS.

GENERAL DESCRIPTION. (PLATE III.)

(1) The instrument is mounted in the west wing of the Observatory, the room formerly occupied by the Transit Instrument. The interior dimensions of the room are 24.3 feet from east to west, and 18.4 feet from north to south. On the north and south sides of the room are built two recesses, each six feet in length, and six in depth, to make room for the collimators. The slits in the wall and roof for observing are thirty inches in breadth, and closed by four shutters on the roof, and a door on each side.

(2) The piers are solid monoliths of marble. The form of each pier is that of a frustrum of a pyramid, surmounted by a prism. The base at the floor measures 44 inches from north to south, and 38 inches from east to west. The top of the pyramidal portion is 80 inches above the floor, and measures 18.5 inches from north to south, by 24.5 from east to west. The dimensions of the making horizontal sections of the prism are the same, while the height of the prisms is 28 inches, the whole height of the piers 9 feet above the floor. The inside faces of the piers form a vertical continuous plane 19 inches in breadth, and 9 feet in height. On each side of this plane the stone is cut away in the form of a section of a hollow cylinder. The distance of the inside faces is 54 inches. A cylindrical hole 5 inches in diameter is cut from east to west through each prism for the illumination.

Into these openings the inside ends of hollow brass cylinders are firmly set with plaster to the depth of 3 inches. The outside end of each cylinder expands into a disk 8½ inches in diameter and 0.7 of an inch from the face of the pier. The arms which carry the four reading microscopes of each circle are attached to these disks, radiating from the central axis at an angle of 45° from the horizontal and vertical directions. Within each disk, near its centre, is a system of prismatic reflectors for illuminating the divisions of the circle. The illuminating lamps are each at the large end of a conical tube, the small end of which extends through the opening of the pier and fits into the interior of the cylinder carrying the microscope disks. The large end, carrying the lamp, extends three feet from the outer face of the pier. One lamp illuminates the field of the telescope, the other the wires.

The Ys are fastened into semi-cylindrical pieces of brass which extend inward from the head of the disk carrying the microscope arms, the axis of the cylinder being a continuation of that of the openings in the pier, as shown in Plate IV.

(3) The telescope is of 12 feet focal length, and 8.5 inches clear aperture. The eye-piece is furnished with a system of twenty-three fixed vertical wires, (eight of which it is intended to remove,) and two horizontal ones, distant 8″. There is also a horizontal and a vertical micrometer screw, the former carrying one vertical, and the latter four horizontal wires—a central pair, distant 4″.6, and two single ones, 2½′ each side of this pair.

(4) The circles are each 42 inches in diameter, and divided on silver to every 2′. The cylinder on the clamp-end of the axis also has a coarser division to every 10′ for setting. The general character of the arrangement of circles, clamp, counterpoises, &c., may be seen by reference to Plate IV.

Notwithstanding its dimensions, the instrument is reversible, and the operation of reversing can be performed by a single person with great facility. The entire weight of the movable part of the instrument is only about 900 pounds.

(5) The sides of the central tube of the telescope are pierced by openings 2½ inches in diameter, through which the collimators may be set on each other when the instrument is vertical. These are not shown in Plate III.

(6) The instrument is completely spanned from north to south by an arched flight of steps for reflection observations of stars. They are so figured that when the telescope is at any pointing between 120° and 240° of zenith distance, the eye-piece will be in a convenient position to look into. Above the fifth step the arch is bifurcated, so as not to interfere with the line of sight. The highest step is a platform three feet in length, suspended from the roof by iron bars and braces. Hand-rails, not shown in the plate, extend from the bars nearly to the floor.

(7) In the spring of 1867 another mechanical improvement, for convenience and certainty in observing the nadir point, was introduced. On each side of the platform, over the axis of the instrument, a seat is erected. The observer can sit astride of either seat and look into the eye-piece when the telescope points to the nadir. On the inside of each seat, between the observer and the telescope, a board, eight to nine inches wide, rises from the platform nearly to the eye-piece. Each of these boards is furnished with a pair of shutters of the same size, which the observer can turn so that the tube of the telescope shall be completely enclosed in a wooden hexagonal prism, or, more exactly, a frustrum of a pyramid, and thus protected from the heat of the observer's body.

(8) The steps for reading the microscopes need no explanation except that a hand-rail runs along the platform, by which the observer passes from one side of the pier to the other, without descending to the floor.

DETAILED DESCRIPTION, WITH EXPLANATIONS OF THE PLATES.

(9) Plate I is a plan of the observing-room.

Plate II is a section of the walls and masonry below the floor in the plane of the meridian of the instrument, with a view of the room as seen from the west.

E is the entrance from the main building. It is closed by two doors. The floor of the room being thirty inches lower than that of the main building, a platform and flight of steps is erected inside the door for convenience in entering the room.

(10) Below the floor all the masonry is of rough stone set in lime and sand mortar. The base of the masonry rests upon the ground about six feet below the flooring joists.

L L are the collimator piers, the bases being of masonry, three feet square, and the upper parts octagonal monoliths of marble.

S S are piers which support the turn-table, T, and the floor of the room under the instrument.

B, plate II, shows the masonry which supports the circle itself. B is a prism of the masonry already described, eleven feet from east to west, four and a half from north to south, and five in height. Its horizontal dimensions are therefore only sufficient to support the great piers of the instrument. It is covered by a solid cap Q, Plate II, of hard black stone, of the same horizontal dimensions, and about one foot in thickness.

On this cap rest the marble piers P, P'. Their bases are hollowed out so that they each rest on three points, and thus remain secure without cement or other fastening. Their great mass insures perfect steadiness without such aids. C, C, show how the inside corners of the piers are cut away. The concave faces C, C, C, C, are parts of the surfaces of four vertical cylinders, the axes of which are in the plane of the inside faces of the piers, 23½ inches north and south of the middle, and therefore 47 inches apart.

The perforation through each pier, at the end of the axis of the instrument, is shown at *o*, Plate II.

(11) R, R, R, R, R, R, show the railroad on which the reversing carriage runs. Under the instrument the rails rest upon two strong joists, supported by the piers S S. The turn-table T revolves on six cannon balls. By it the reversing carriage, with the circle on it, may be run into either the northeast or the northwest corner of the room. The turn-table is not necessary in reversing, as the Y's of the reversing carriage themselves revolve on a pivot. The whole floor is on the same level, except around the collimator piers, on three sides of which a platform is built.

(12) Plate IV exhibits an end elevation of the instrument from the south, with the hanging level and so much of the reversing carriage as is not concealed by the telescope. The ladders are omitted.

Plate V is an isometric side elevation from the east; the east pier with its appurtenances, the step-ladders, and the hanging level, all being removed.

(13) *a* is a movable arm, for supporting a lamp in observing the nadir point. *b*, *b* are the counterpoises which lighten the weight of the instrument upon the pivots. The levers *c*, *c* which support them have a slight movement around the pivots *c'*, *c'*. The hooks *f*, *f* have friction rollers at the bottom for supporting most of the weight of the instrument. At the top they expand into a strong rectangular frame, through which the end of the lever *c* passes, as shown at *o*, Plate V. Through the top of the frame passes the screw *d*, the lower end of which being rounded off, rests in a socket in the top of the lever, and thus supports the weight. Small pieces of rubber have been placed in these sockets and under the supporting screws, for reasons to be explained hereafter. When the instrument is raised from its pivots the counterpoises are supported by the screws *e*, *e*, which are adjusted so as to allow a small play to the levers.

(14) *Mounting of the Microscopes.*—Enlarged views of the hollow cylinder D with its attachments are shown in Plate VI. The portion P is set into the pier with plaster; grooves are cut through the flanges *p*, *p*, *p* to let the plaster run through. *l* is the disk to which the microscope arms are fastened. It expands in thickness toward its circumference. The portions *j* and *k* of the microscope arms hold the disk firmly between them, being held together by means of clamping screws, the heads of which are shown in Fig. 1. These screws pass through *j* to *k*, just outside the edge of the disk. To keep *j* in position when these screws are loosened, it is fitted with pins which fit corresponding holes in *k*. By loosening the screws the microscope arms can be moved independently around the disk, but will not come nearer to each other than about 43°.

Outside of *k*, the microscope arms are loosely encased in mahogany shields I, to protect them against rapid changes of temperature.

The outer ends of the arms are hollow, holes about ⅜-inch in diameter running into them, to receive the holders which carry the microscope. They are encircled by the circular clamps *i*, which are tightened by means of the screws *h*, partially shown in Fig. 2, but hidden by the clamps in Fig. 1. When *h* is tightened, the microscope is firmly held by the arm. By loosening it the microscope may be drawn out, and three views, as thus removed, are given in Plate VII, Figs. 1, 2, 3, each of which is from a position at right angles to the other two.

(15) *a* (Plate VII) is the solid metallic support which passes into and is held by the microscope arm, as already described. The holders *b* and the circular clamps *c*, enveloping the body of the microscope, form a single piece with *a*. When the screws *d* are loosened, the microscope can be moved longitudinally, and the end *e* carrying the object glass can be slid into or out of the other portion, for adjusting the focus, and the angular value of a revolution of the micrometer. For the accurate adjustment, the independent clamp *y* is used. Loosening *d* and tightening *y* by means of the screw *δ*, a fine motion is given the microscope by turning the screw *ε*. The head of this screw is kept pressed against *b* by the spiral spring *ζ* pressing *y* and *b* apart.

(16) *o* is a perforated reflector for throwing light on the divisions. Its reflecting surface is not polished, but is of a bright white. It can be turned in any direction by the milled head *f*, and can be moved longitudinally by loosening the clamp *g*.

k is a cylinder of white felt, extending very nearly to the face of the circle, at the same time keeping out stray light and assisting the illumination from o.

(17) The micrometer screws of the microscopes revolve with the head. Instead of being attached to the moving frame of the diaphragm, they screw into the latter, and thus give it a slow motion to or from the head. Each revolution of the microscope micrometers is 30″ in angular measure on the circle, and about one hundredth of an inch in linear measure on the screw; 30″ on the circle measuring about $\frac{1}{330}$ inch, the image of the divisions is magnified about 3.3 times in the focus of the microscopes. The face of the circle is about 1.9 inches from the object glass and the micrometer wires about 8.3 inches.

Each microscope micrometer is set upon the image of the division by a pair of parallel spider lines, distant from 10″ to 11″. Microscope VII is also supplied with two extra pair of wires, about 2′ on each side of the original pair, for the convenient measurement of consecutive divisions.

The micrometer head is divided into thirty second-spaces, each of which is again subdivided by a shorter line to half seconds. The entire revolutions are read from the interior by a serrated scale. The divided drum may be turned on the axis of the screw by simple pressure, without turning either the screw or its bearing.

(18) *Supports of the Pivots.*—Referring again to Plate VI, the cylinder D is hollow to the depth of about an inch, the external part in fact consisting of a rim about an inch thick. The cylindrical space forming the interior of this rim contains the metallic plate *e*, Fig. 1, into which screw the antagonistic screws *s*, *s*, the heads of which press against the inside of the rim. The top and bottom of this plate are bevelled toward the pivot, and the bevelled edges are held by the plates *b*, *b′*. When the screws *c*, c are loosened the plate *e* can be moved from right to left by the screws *s*, *s*, but when they are tightened, *e* is firmly held between the plates *b*, *b′*.

The view actually given is that of the west plate, by which the axis is adjusted in azimuth. The east one is similar, except that the plates and screws are turned 90° for adjusting the level.

n is a thin perforated plate of metal which presses against the end of the pivot, to prevent longitudinal motion of the axis. As now arranged for observation, the eastern one is firmly screwed against the V plate, while the western one is pressed against the pivot by a spring behind it.

The V's are carried in the rounded plate *y*, which forms one piece with the plate *e*. They consist simply of small pieces of soft metal *v v*, (Fig. 4,) slightly convex on their upper surfaces, so that only a single point of each pivot at first rests upon them. But a slight hollow is soon worn into each of them by the pivot.

(19) *The Telescope and its External Appendages.*—Returning to Plates IV and V, and passing from the V's, we first have the pivots, which are 2.1 inches in diameter, and 1.8 inch in length, and are apparently of the same diameter throughout their length.

(20) Next to the pivots the axis expands into the frustrum of a cone, the diameters of the bases of which are somewhere about three and a half and four and a half inches, respectively, and the height about three inches. Back of this frustrum the axis again expands perpendicularly to its length. Over these frustrums the circles C, C fit with great nicety; the central perforations in the latter being tightly filled by the former when the circles are pressed against the expansion at their bases. The thickness of the circle is slightly greater than the depth of the frustum, so that the latter does not pass quite through the perforation. The circle is held in its place by the friction of the plates *p*, *p* which are pressed against it by the screws *s*, *s*, *s*, *s*, *s*, (Plate V) passing through them and into the top of the frustrum, without touching the circle. When these screws are loosened, the circle can be turned round so as to take any desired position relatively to the telescope.

(21) The circles each appear to be cast in a single piece. The manner in which they are stiffened can be seen by a comparison of Plates IV and V. The middle of the plane face of each circumference is inlaid with a band of silver on which the divisions are cut to every two minutes. The breadth of each division as seen under the microscope seems to be between two and three seconds, corresponding to a thickness of about $\frac{1}{4000}$ of an inch.

The circle next to the clamp has also a coarse division to every ten minutes, for setting the telescope.

(22) Next to the circles come, on one end of the axis, the clamp *n*, and on the other end a ring to counterbalance it, into which screw four handles *h*, *h* for turning the telescope on its axis. The clamp is tightened by means of the screw *q*, *q*, (Plate V,) and a slow motion may then be given the telescope by another screw, not shown in the figure.

(23) The friction rollers are received by grooves cut around the axis.

On the clamp end of the axis four curved handles *h'*, *h'* (Plate IV) are screwed into the cone for turning the telescope.

Near the base of the cone flat bands are seen, by which the instrument is supported when on the reversing carriage.

(24) The central cube is 16 inches square, and forms a single piece with the axis. Its sides are perforated in a direction perpendicular to the telescope and the axis, by openings 2½ inches in diameter, through which light can pass from one collimator to the other. They are closed by covers which screw into them.

The tubes of the telescope are each fastened to the cube by sixteen screws, as shown in Plate IV.

(25) *The Eye-piece.*—Three views of the eye-piece and its appendages are given in Plate VIII. From Figures 2 and 3 it will be seen that there are six series or strata of plates in the eye-piece. The first or outside stratum, of which a face view can be seen in Fig. 1, carries the ocular *a* and the slide for giving it a horizontal movement by means of the rack work and milled head *b*.

The second stratum consists of the slide and fixed guides for giving a vertical movement to the whole of the first plate, with its guides and slide, by the milled head *c*.

(26) The third stratum consists of the declination micrometer plate with its guides. A portion of its outside surface is seen at *d*, *d*, *d*, *d*, Fig. 1. *e* is a fixed index for reading the approximate position of the plate in micrometer revolutions. *f* is an opening in the plate to admit of the free passage of *e*. An opening about two inches square is cut in the middle of

the plate, and across this opening on the posterior surface of the plate are stretched the horizontal spider lines for zenith distance observations with the micrometer. From Fig. 3 it will be seen that the micrometer plate *d* carries the fulcrum of the milled head *c* by which the ocular is moved vertically. Consequently, when the micrometer is moved, the ocular is carried with it, and the movable spider lines are thus kept in the middle of the field without touching *c*.

(27) The fourth plate is moved in right ascension by the micrometer head R. It has a rectangular opening like Plate III, and across this opening, on the anterior surface of the plate, is stretched a single vertical spider line.

(28) The fifth plate carries the fixed spider lines. These are stretched across the anterior surface of a thin annulus about 1¼ inch in diameter, which projects from the surface of the fifth plate by an amount equal to the thickness of the fourth plate, passing through the rectangular perforation in the latter. Thus, the three sets of spider lines are sensibly in the same plane. The entire plate can be moved horizontally by the three antagonistic screws *g h g*, to adjust the line of collimation. The plate is fastened to the piece through which the screws pass by the projection *i*.

The sixth plate forms the basis for all the rest, and is pierced for the holes in which the micrometer screws turn. It is fastened to the eye-tube E by a cylinder which slides tightly into E, and is fastened by the six screws *k*.

(29) *The Micrometer Movements.*—In all the micrometers of this instrument the screws revolve with the heads. In the microscope micrometers the bearings of the heads are fixed, and the screws passing into the movable plates draw them toward the micrometer heads as the screws turn forward. But in the eye-piece the female screws are bored into the sixth plate, and the bearings of the heads move with the micrometer plate. The projection *d* of the zenith distance micrometer plate is attached to a cross *n*, *n*, *n*, (Fig. 3) through a circular ~~the~~ opening in the centre of which the micrometer screw passes freely, and on which it presses. Thus, as the screw turns forward, the screw, the micrometer head, *n*, and *d* are all slowly moved downward. The middle *n* is the index from which the fractions of a revolution are read, but they can equally be read from the front index. *s* is one of two spiral springs by which the cross is kept pressed against the micrometer head, and made to follow it as it is withdrawn.

(30) *Rogers' Self-registering Micrometer Head.*—The zenith distance micrometer head is furnished with the self-register invented by Mr. Joseph A. Rogers, and described in the Astronomische Nachrichten, No. 1493. The four arms o, o', o'', o''' (Fig. 1) are movable around the same centre with the micrometer, which passes through openings in the middle of them. They are held to the micrometer by the friction of elastic bent plates which surround the centre. Thus, when free, they move with the micrometer head; but when held, the head moves past them. One end of each terminates exactly with the radius of the micrometer head, and serves as an index, as may be seen in Fig. 3. The other end projects a little beyond the cylindrical surface of the head, and may be held by notches in the levers *l l*, (Fig. 2.) When thus held, the index ends are in the same vertical line with the principal fixed index, as shown in Fig. 3.

The mode of operation is as follows: The micrometer being in any position which it is desired to register, one of the levers (Figs. 1 and 2) is moved back with the finger to the position l', and the movable index is thus set free. It now moves round with the micrometer, and thus points continually at the reading indicated when it was set free. Four successive readings may thus be registered in succession without taking the eye from the telescope.

The object of the two extra divided cylinders on the micrometer head is simply to facilitate the reading of the four movable indexes.

(31) *The Oculars.*—The instrument is furnished with five oculars, of which the magnifying powers are, approximately—

No. 1, 130;
2, 150;
3, 180;
4, 260;
5, 360.

No. 3 is that hitherto most used in astronomical observations.

(32) *The Reticule.*—The fixed vertical spider lines are twenty-three in number, arranged as in Fig. 4. Seven of these are at equal intervals of 12½ seconds of time, and are designated by the Roman numerals I–VII. For convenience, a second notation is adopted, as shown at the bottom of the figure, the wires being divided into five groups A to E, and the wires of each group designated by subscript Arabic numerals. Thus the wires II, III, IV, V, and VI are designated indifferently by these numbers or by the symbols A_5, B_1, C_3, D_3, E_1. The wires of set C are 2*s*. apart; those of B and D 1*s*.5 and 2*s*.5 respectively. Sets A and E are rarely used. Stars are usually observed in right ascension over B D and the three middle wires of C. In observations of planets one limb is observed over B and D, the other over C.

The middle of the field is shown by a single pair of fixed horizontal wires about 8″ apart. A single vertical wire is moved by the horizontal micrometer.

(33) The horizontal movable wires consist of a single pair 4″.6 apart, between which objects are usually observed, and a couple of single wires at a distance of ten micrometer revolutions on each side of the central pair. The former were inserted by the machinist of the observatory after the mounting of the instrument, in lieu of a single wire inserted by the makers.

(34) *The Illumination.*—The flames of the two lamps L, L, Plate IV, are nearly surrounded, by parabolic reflectors. The light passes through the supporting tube and the pier into the cylinder D; here it meets a combination of lenses and prisms, shown in Plate VII, Figs. 4 and 5. Fig. 4 gives an end view of the combination, as seen from the lamp. Fig. 5 gives a side view. The nearly cylindrical frustum of a cone C is pierced in the direction of its axis by five holes, *a*, *b*, *b*, *b*, *b*. The light which enters *a* passes through two lenses, and through the perforations in the pivots of the instrument to the centre of the axis. That which passes through the *b's* meets four prisms, *p*, *p*, from the interior surfaces of which it is reflected at an angle of about 42½°, so as to be deflected in all about 85°. C and D fit snugly in a perforation in the centre of the cylinder D, Plate IV; a hole receives the pin *e*, and thus fixes the position of the apparatus. The latter can be removed by unscrewing the lamp-tubes T, and reaching the arm through the pier. When in place, its position is such that the light reflected from the prisms *p* passes down the pipes *k*, (Plate IV,) and into the reflectors of the microscopes, by which it is thrown upon the divided limb.

(35) The central ray, passing through *a*, if it enters the clamp end of the axis, is conducted to four prisms in the central tube of the axis, from which it is reflected down the side of the tube to four more prisms in the eye-piece, and thence to the wires, in the usual way. Thus the wires may be illuminated or darkened by turning a milled head on the clamp side of the eye-piece.

(36) The light which enters the other end of the axis is reflected from a prism about 4½ inches from the centre of the axis, directly to the eye-piece. It shows the dark wires on the bright field. It may be changed in color from yellow to blue, or cut off entirely by turning a milled head on the side of the eye-piece opposite the clamp.

(37) *The Collimators.*—Fig. 6, Plate VII, gives a general view of each collimator. The object glasses have each 2½ inches clear aperture, and 35 inches focal length. The eye-piece

of one, which is designated collimator A, is furnished with a pair of parallel wires, distant about 9″, with a single wire crossing them at right angles. The wires of the other, called collimator B, form a simple cross. When it is required to set the collimators opposite each other, the coincidence is that of a single wire of B with the mean of the parallel wires of A.

The steel plate which carries the wires is held between four antagonistic screws, but has no binding screws.

The supporting collars of the collimators are only 23 inches apart, so that each end of the collimators projects about six inches from its support.

Each collimator is furnished with a delicate spirit level, which sets on the supporting collars.

(38) The Ys of the collimators, with the way in which they are fastened to the piers, is shown in Figs. 7 and 8. The hemispherical bottoms of the screws *a*, *a*, *a*, rest in three cavities in a Y-shaped block of metal, which is set into the pier with plaster. The outline of the block is shown by the dotted lines. The three screws *b*, *b*, *b*, pass centrally through *a*, *a*, and thus bind the V-plate firmly to the block. The level of either pivot is adjusted by loosening *b*, and turning *a* to the required extent. The azimuth is adjusted by the antagonistic screws *c*, *c*. When the V is in position it is bound by the screws *d*, *d*, *e*, *e*.

PART II.

GENERAL METHOD OF INVESTIGATING THE ERRORS OF A TRANSIT CIRCLE.

(39) The most important and difficult part of this investigation is the determination of the effect of gravity in changing the relative positions of the various parts of the instrument, and the correction of observations for this effect. It is therefore proposed to develop a general method of determining, either rigorously, or with a near approach to rigor, the effect of gravity in changing results of the various observations by which the position of a star is determined.

The necessity for a flexure correction may be regarded as an imperfection in an instrument, since, if it were well balanced, and equally elastic in all its parts, no such correction would ever be necessary. Artists generally attempt to avoid this imperfection by distributing the weight of the instrument so that it shall be perfectly symmetrical with respect to the centre. Thus, in the Washington Transit Circle, the two circles are of equal weight; the clamp is counterpoised by a cylinder at an equal distance on the other end of the axis; and the tubes of the telescope had their outer and inner surfaces turned simultaneously, in order to secure perfect equality of thickness in every rectangular section of the tube. Yet, the effect of irregular flexure cannot, by any means, be neglected, and that of the two circles exhibits a very sensible difference. A similar remark will probably apply to every large instrument ever made. It will not, therefore, do to assume that such an instrument is equally elastic in any of its corresponding parts, however unexceptionable may be its construction.

(40) The only arbitrary hypothesis we shall assume respecting any law of elasticity in the instrument is, that the relative flexure of the various parts is the same as if the instrument, during a revolution on its axis, were always supported by the same particles of its mass. The nature and the results of this hypothesis will be rendered more clear if we suppose the instrument to remain at rest, and gravity, with the forces which oppose it, to act in various directions. The parts of the instrument which sustain these opposing forces are mainly those which press on the friction rollers of the counterpoises. Reflecting, now, that the direction of these forces always passes within two or three inches of the centre of the axis, and that our hypothesis can be incorrect only by a combination of the two following circumstances, viz: 1. An unequal elasticity in the parts of the conical axis on which the friction rollers successively press; 2. Such a relation of elasticities in other parts of the instrument that the unequal elasticity of this narrow band on the conical axis causes the instrument to change its form according to a different law from that which would hold true if the elasticity of the band were uniform—it would seem that the hypothesis must be a safe one.

Experiment shows that the effect on elastic bodies of forces which are very small in proportion to the rupturing force is strictly subject to the law of superposition of small motions; that is, that the combined effect of several such forces is the sum of the effects that each would produce acting separately.

(41) Still supposing the instrument fixed, and the direction of gravity variable, let b represent the amount by which a point of the instrument is moved from its normal position in the direction of an arbitrary fixed axis when gravity acts in the direction of that axis, and let a represent the displacement in the same direction when gravity acts at an angle of 90° with the axis. If gravity acts at an angle W with the axis, it may be resolved into the forces g cos W and g sin W, acting along and perpendicularly to the axis. The first component will, by the law just referred to, produce the displacement b cos W, and the second the displacement a sin W, and the whole effect of gravity will be a sin W+b cos W.

Suppose, then, any system of rectangular co-ordinates fixed in the instrument, and revolving with it; then, as the instrument revolves, the law of displacement of any point in the direction of the movable co-ordinates will be given by the equations

$$\delta x = a \sin W + b \cos W,$$
$$\delta y = a' \sin W + b' \cos W,$$
$$\delta z = a'' \sin W + b'' \cos W.$$

W being the angle of position of the instrument, and a, a', a'', b, b', b'' constants, to be determined by observation.

(42) The method of determining such of these constants as are necessary in the correction of astronomical observations will next be shown. To admit of the application of this method it is necessary that the instrument should be supplied with a pair of collimators, each of which admits of being accurately levelled, in order that it may be used to determine the horizontal point, and an artificial horizon for observing the nadir point by reflection from mercury. A vertical collimator, to be fixed over the centre of the instrument, and set vertical either by reflection of its wires from the mercury bath, or by levelling its axis, would also be valuable as an independent check on some of the results.

(43) Let us now consider the causes which will affect the reading of any microscope. If the divisions are truly cut in direction—that is, if each stroke is sensibly in the same plane with the axis of the circle, the error of each point in any one division may be regarded as the same. We shall therefore regard the divisions as a series of points. When a microscope is read, the position of the point under the microscope is determined by bringing a micrometer wire into coincidence with the image of the point in the microscope. This micrometer wire in the German instruments is an imaginary one bisecting the space between two real ones. The division point is then in the plane which passes through this micrometer wire and the optical centre of the object glass of the microscope. Let us consider, as the zero of the micrometer its position when the plane passing through the object glass and the micrometer wire is at right angles to the face of the circle. The micrometer reading of the division will then be proportional to the tangent of the angle which the plane containing the division makes with this last plane. No appreciable error will result from supposing this adjustment to be perfect.

The subsequent investigation will be precisely the same if we suppose the divisions to radiate from the centre of the circle, and the micrometer wires in the microscope to be a point, which is brought into coincidence with the image of the division. The micrometer reading will then represent the tangent of the angle which the plane, passing through the centre of the object glass and the division, makes with that passing through the centre of the object glass and the axis of the circle.

If the eight microscopes were correctly adjusted in position, and the circles perfect in form and position, and not acted on by gravity, the instrument might be so set that all the microscopes should read zero at the same time. The following include all possible deviations of the circles and the microscopes from this state.

α. *Errors in the Angular Positions of the Microscopes.*—When all the microscope micrometers are set at zero, the planes passing through the micrometer wires and the centre of the object glasses ought to cut the line of divisions at points making angles with the centre of the axis equal to the nominal distance of the microscopes.

β. *Errors in the General Position of the Circle.*—These will be six in number: small motions of translation in the direction of three co-ordinate axes, and small motions of rotation around those axes.

γ. *Errors in the Position of the Division on the Circle.*—These may be three in number. 1. Errors in the angular position of the division. 2. Deviation of the surface on which the divisions are cut from a plane cutting the axis of revolution of the instrument at right angles. 3. Change in the position of the division produced by the effect of gravity on the circle.

The position of a division may be in error by any of the nine causes, β and γ. To determine their effect, let us refer the position of the division to three co-ordinate axes, those of X and Y being in the plane of the circle, and that of Z being the axis of revolution. Let A be the angle which the plane of the micrometer wire and object glass makes with the axis of the circle, B the angle of position of the microscope, counted from the axis of X.

R, the radius of the circle.
c, the distance of the centre of the object glass from the plane of the circle.
r, the reading of a microscope from its own zero.
ρ, the mean reading of a pair of opposite microscopes.
m, the error of position of a microscope.
M, the mean error of a pair of opposite microscopes.
e, the error of division.
ϵ, the mean error of a pair of opposite divisions, or an error in the position of a diameter.
a, b, the flexure coefficients of any point of the circle.
α, β, half the difference of a or b for two opposite points of the circle.
ξ, η, the possible motions of translation of the centre of the circle relatively to X and Y.
ω, the error of pointing of the telescope.

The differential coefficient of tan A, (which is proportional to r) with respect to the co-ordinates of the division, will then be:

$$\frac{d.\tan A}{dx}=\frac{R}{c}\frac{dr}{dx}=-\frac{\sin B}{c};$$

$$\frac{d.\tan A}{dy}=\frac{R}{c}\frac{dr}{dy}=\frac{\cos B}{c};$$

$$\frac{d.\tan A}{dz}=\frac{R}{c}\frac{dr}{dy}=\frac{\tan A}{c}.$$

The effect of small changes δx δy and δz on the reading of a microscope will therefore be

$$\delta r=-\sin B\frac{\delta x}{R}+\cos B\frac{\delta y}{R}+\tan A\frac{\delta z}{R}. \quad (1)$$

Changes in the co-ordinate z can arise only from a motion of translation of the instrument in the direction of its axis; from motions of rotation around the axis of X or of Y, in consequence of irregularity of pivots; from deviation of the circle from a plane cutting the axis at right angles; or from lateral flexure of the circle. If the effect of any of these causes is appreciable, δz must be determined by direct measurement, independently of the readings of the microscopes, and the latter must be corrected accordingly. To judge when δs can be sensible, we remark that in general

$$\tan A = r\frac{R}{c} + \text{constant},$$

the constant being the value of tan A when the micrometer reads zero, and therefore depending on the parallelism of the microscope to the axis of the circle. If the changes in r do not exceed 10″ and $\frac{R}{c}$ does not exceed 12, A will not differ more than 1′ from its mean value in a series of readings. Since, by proper adjustment, this mean value may be made very small, there is no necessity that A should ever exceed 1′ in a series of measures. In this case, s will have to change by $\frac{1}{600}$ the radius of the circle to produce a change of 0″.1 in r. This change being ten times as great as any that a well made instrument need be liable to, the last term in δr may be regarded as insensible, if the microscopes are carefully adjusted. This will dispose of four of the nine causes which may affect the relative position of the division and object glass. The remaining five will be included in the equations of condition.

Every micrometer reading will then, by equation (1), give an equation of the form

$$r = m - \frac{\xi}{R}\sin B + \frac{\eta}{R}\cos B + a\cos B + b\sin B + e + \varpi. \qquad (2)$$

(44) The problem now is to make such mechanical arrangements and such readings, that from a series of equations of this form we may be able to determine the values of the coefficients in the second member, so far as it may be necessary for the correction of observed zenith distances. To obtain a complete correction for the readings of a single microscope world require the solution of equations of extreme complexity; we shall, therefore, hereafter consider the corrections of a pair of opposite microscopes. If, now, another microscope be placed at the angle 180°+B, and its reading, taken simultaneously with (2,) be indicated by an accent, we shall have

$$r' = m' + \frac{\xi}{r}\sin B - \frac{\eta}{R}\cos B - a'\cos B - b'\sin B + e' + \varpi.$$

Adding this equation to (2), and observing that by the rotation already given

$$r + r' = 2\rho,$$
$$m + m' = 2M,$$
$$a - a' = 2\alpha,$$
$$b - b' = 2\beta,$$
$$e + e' = 2\epsilon,$$

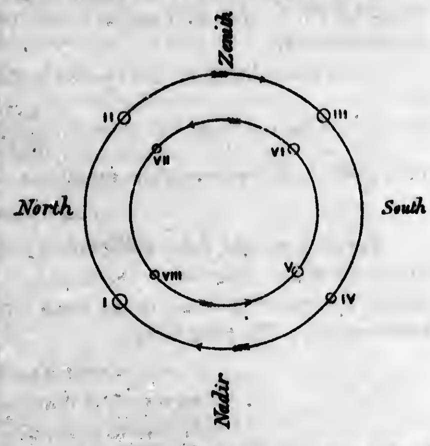
we shall have, by taking the mean reading of a pair of microscopes,

$$\rho = M + \alpha\cos B + \beta\sin B + \epsilon + \varpi, \qquad (3)$$

an equation more simple than (2) and equally complete.

(45) The mechanical arrangements necessary for the proposed method are these: The instrument must be furnished with two finely divided circles, each read by four microscopes. At least one circle must be capable of being turned on the axis of the telescope, and set in any required position relatively to the telescope. In what follows, we shall suppose both circles thus movable. The microscopes must also be capable of being set at different angular positions. The details of the arrangement, and the mode of numbering the microscopes and

counting the degrees, will be supposed to correspond to those of the Washington transit circle. The accompanying diagram shows the arrangements referred to, as seen from the west. The outer circle represents the westernmost one, and shows the positions of microscopes I to IV, which are on the west pier. The inner circle represents the eastern one, and shows the positions of the four microscopes V to VIII, which are on the east pier. The arrows point in the direction in which the degrees, as numbered on each circle, increase. It will be seen that the two circles are divided in opposite directions, so that when the instrument is reversed the circle now east will read in the same direction as the former one, and so for the west. Also, as the instrument is turned in any direction, the sum of the readings of any microscope on the west pier and any microscope on the east pier will be a constant depending on the relative positions of the circles.



We shall suppose the axis of X to pass in the direction of microscopes I–III, and that of Y in the direction of II–IV. Thus, we have for microscopes

I and	VIII,	B=180°;
II and	VII,	B= 90°;
III and	VI,	B= 0°;
IV and	V,	B=270°.

(46) The flexure effect which it is required to determine is the change in the relative direction of two lines, namely, the line joining the object glass and eye-piece micrometer of the telescope, and that joining any pair of opposite divisions which chance to be under a pair of microscopes. This will be determined by finding the change of direction of each line relatively to that part of the central axis to which the circle is fastened. The total flexure will therefore be divided into two parts: 1. The flexure of the central axis relatively to the ends of the telescope. 2. The flexure of the diameters of the circle relatively to the centre. The coefficients of the first flexure we shall represent by f and g for the east circle, and by f' and g' for the west one. Suppose, then, that the line joining the object glass and eye-piece revolves uniformly; that part of the axis which passes through the circle and holds it by pressure may be affected with an inequality of the form

$$f \sin Z + g \cos Z.$$

Again, supposing this part of the axis to revolve uniformly, it is possible that a diameter of the circle may be affected with an inequality of the form

$$\alpha \sin \zeta + \beta \cos \zeta,$$

α and β being coefficients to be determined separately for each pair of divisions, and ζ being the angle of position of the diameter counted from an arbitrary axis. On the east circle we shall for convenience, count ζ from microscope VI through V, VIII, and VII, and on the west circle from microscope II through I, IV, and III. Then, when any division D comes under microscope V, the mean reading of V and VII will, by the effect of gravity, be increased by the quantity

$$\alpha_{\text{D}}.$$

When the same division comes under VI, the mean reading of microscopes VI and VIII will be increased by the quantity

β.D.

The same statement will apply to the west circle by diminishing the number of the microscope by IV, so that α'.D and β'.D will be the flexure when the division D comes under microscopes I and II.

Since changing ζ by 180° is the same as changing the algebraic sign of α and β, we have

$$\alpha.(D+180^\circ)=-\alpha.D,$$
$$\beta.(D+180^\circ)=-\beta.D.$$

In equation (3) we hereafter substitute for $\alpha \cos B+\beta \sin B$ the two flexures

$$f \sin Z+g \cos G+\alpha \sin \zeta+\beta \cos \zeta.$$

Let us now take four divisions on each circle, such that they may all be under the microscopes at once. Represent them by a, b, c, d on the east circle, and a', b', c', d' on the west circle, the lettering being in the same direction with the numbering of the degrees, and of the microscopes. We then have

$\epsilon.a=\epsilon.c=$ error of position of line joining $a-c$,

$\alpha.a=$ error of flexure of $a-c$ when under microscope V,

$\beta.a=$ error of flexure of $a-c$ when under microscope VI.

Suppose, also, that when a is under microscope V, a' is under microscope I. Also put

$$f \sin Z+g \cos Z=f.z.=-f.(z+180^\circ),$$
$$-f' \sin Z+g' \cos Z=f'.z.=-f'.(z+180^\circ),$$

and let

$$Z_0,\quad Z_0+90^\circ,\quad Z_0+180^\circ,\quad Z_0+270^\circ$$

be the four zenith distances of the telescope toward the south when the divisions are brought successively under the four microscopes. Represent by $\rho n.s$ the mean readings of a pair of opposite microscopes. Then we have the following equations of the form (3):

First position of telescope Z_0.

$$\rho5.0=M_5+\epsilon.a+f.z_0+\alpha.a+\varpi.0, \qquad \rho1.0=M_1+\epsilon'.a+f'.z_0+\alpha'.a-\varpi.0,$$
$$\rho6.0=M_6+\epsilon.b+f.z_0+\beta.b+\varpi.0, \qquad \rho2.0=M_2+\epsilon'.b+f'.z_0+\beta'.b-\varpi.0.$$

Second position, Z_0+90°.

$$\rho5.90=M_5+\epsilon.b+f.(z_0+90^\circ)+\alpha.b+\varpi.90, \qquad \rho1.90=M_1+\epsilon'.d+f'.(z_0+90^\circ)+\alpha'.d-\varpi.90,$$
$$\rho6.90=M_6+\epsilon.c+f.(z_0+90^\circ)+\beta.c+\varpi.90, \qquad \rho2.90=M_2+\epsilon'.a+f'.(z_0+90^\circ)+\beta'.a-\varpi.90.$$

Third position, Z_0+180°.

$$\rho5.180=M_5+\epsilon.c+f.(z_0+180^\circ)+\alpha.c+\varpi.180, \qquad \rho1.180=M_1+\epsilon'.c+f'.(z_0+180^\circ)+\alpha'.c-\varpi.180,$$
$$\rho6.180=M_6+\epsilon.d+f.(z_0+180^\circ)+\beta.d+\varpi.180, \qquad \rho2.180=M_2+\epsilon'.d+f'.(z_0+180^\circ)+\beta'.d-\varpi.180.$$

Fourth position, Z_0+270°.

$$\rho5.270=M_5+\epsilon.d+f.(z_0+270^\circ)+\alpha.d+\varpi.270, \qquad \rho1.270=M_1+\epsilon'.b+f'.(z_0+270^\circ)+\alpha'.b-\varpi.270,$$
$$\rho6.270=M_6+\epsilon.a+f.(z_0+270^\circ)+\beta.a+\varpi.270, \qquad \rho2.270=M_2+\epsilon'.c+f'.(z_0+270^\circ)+\beta'.c-\varpi.270.$$

(47) For perspicuity, let us recapitulate the adopted notation, by explaining the meaning of the above equations.

If there were no errors of division or adjustment, and no elasticity of the parts of the instrument, the readings of all the microscopes would be zero. The readings not being zero, and $\rho5.0$ being the mean reading of microscopes V and VII when the telescope points approximately at zenith distance Z_0, the first equation indicates that this value of $\rho5.0$ is due to the five following causes:

1. Error of position of the microscopes V and VII, (M_5.)

2. Errors of the divisions a and c which are under these microscopes, $(\epsilon.a.)$

3. Rotation of the axis of the telescope relatively to its line of sight, in consequence of the irregular effect of gravity, $(f.z_r)$

4. Angular change of line joining divisions a and c relatively to axis of rotation from the same cause, $(a.a.)$

5. Incorrect setting of the telescope, $(\omega.0.)$

In any one of the above four positions take the sum of ρ_1 or ρ_2, and ρ_5 or ρ_6. Take also the corresponding sum in a position of the circle 180° different, then subtract the two sums, remembering that

$$f.z=-f.(z+180^\circ),$$
$$\alpha.a \text{ or } \beta.a=-\alpha.(a+180^\circ) \text{ or } \beta.(a+180^\circ),$$
$$\epsilon.a=\epsilon.(a+180^\circ),$$

and call the difference K, using the notation

$$K.15.i=\rho_1.i+\rho_5.i-\rho_1.(i+180^\circ)-\rho_5.(i+180^\circ).$$

Put also $g.z=f.z+f.'z$, and we have the equations

$$K.15.0=2\alpha.a+2\alpha'.a+2g.z, \qquad K.15.90=\ \ 2\alpha.b-2\alpha'.b+2g.(z+90^\circ),$$
$$K.16.0=2\beta.b+2\alpha'.a+2g.z, \qquad K.16.90=-2\alpha.\beta-2\alpha'.b+2g.(z+90^\circ),$$
$$K.25.0=2\alpha.a+2\beta'.b+2g.z, \qquad K.25.90=\ \ 2\alpha.b+2\beta'.a+2g.(z+90^\circ),$$
$$K.26.0=2\beta.b+2\beta'.b+2g.z, \qquad K.26.90=-2\beta.a+2\beta'.a+2g.(z+90^\circ).$$

Of these eight equations it will be seen that only six are independent, any one of each four being identically derivable from the other three.

Now, let the position of the east circle on the axis of the telescope be changed by 180°, and let the same divisions be again read. Treating the equations derived from the readings in the same manner, and distinguishing the new K's by an accent, we shall have

$$K'.15.0=-2\alpha.a+2\alpha'.a+2g.z, \qquad K'.15.90=-2\alpha.b-2\alpha'.b+2g.(z+90^\circ),$$
$$K'.16.0=-2\beta.b+2\alpha'.a+2g.z, \qquad K'.16.90=\ \ 2\beta.a-2\alpha'.b+2g.(z+90^\circ),$$
$$K'.25.0=-2\alpha.a+2\beta'.b+2g.z, \qquad K'.25.90=-2\alpha.b+2\beta'.a+2g.(z+90^\circ),$$
$$K'.26.0=-2\beta.b+2\beta'.b+2g.z, \qquad K'.26.90=\ \ 2\beta.a+2\beta'.a+2g.(z+90^\circ).$$

Now, turn the west circle 180° on its centre and again repeat, distinguishing the K's by two accents, and we have

$$K''.15.0=-2\alpha.a-2\alpha'.a+2g.z, \qquad K''.15.90=-2\alpha.b+2\alpha'.b+2g.(z+90^\circ),$$
$$K''.16.0=-2\beta.b-2\alpha'.a+2g.z, \qquad K''.16.90=\ \ 2\beta.a+2\alpha'.b+2g.(z+90^\circ),$$
$$K''.25.0=-2\alpha.a-2\beta'.b+2g.z, \qquad K''.25.90=-2\alpha.b-2\beta'.a+2g.(z+90^\circ),$$
$$K''.26.0=-2\beta.b-2\beta'.b+2g.z, \qquad K''.26.90=\ \ 2\beta.a-2\beta'.a+2g.(z+90^\circ).$$

These equations suffice to give two values of each of the required quantities. But, to have as many independent determinations as possible, a fourth series of readings are taken with the east circle restored to its original position. We then have

$$K'''.15.0=2\alpha.a-2\alpha'.a+2g.z, \qquad K'''.15.90=\ \ 2\alpha.b+2\alpha'.b+2g.(+z90^\circ),$$
$$K'''.16.0=2\beta.b-2\alpha'.a+2g.z, \qquad K'''.16.90=-2\beta.a+2\alpha'.b+2g.(z+90^\circ),$$
$$K'''.25.0=2\alpha.a-2\beta'.b+2g.z, \qquad K'''.25.90=\ \ 2\alpha.b-2\beta'.a+2g.(z+90^\circ),$$
$$K'''.26.0=2\beta.b-2\beta'.b+2g.z, \qquad K'''.26.90=-2\beta.a-2\beta'.a+2g.(z+90^\circ).$$

Adding together the corresponding equations of the first and third series, and also those of the second and fourth, we shall have

$$4gz=K.15.0+K''.15.0=K.16.0+K''.16.0=K.25.0+K''.25.0=K.26.0+K''.26.0$$
$$=K'.15.0+K'''.15.0=K.'16.0+K'''.16.0=K'.25.0+K.''25.0=K'.26.0+K'''.26.0.$$

Of these eight values only six are really independent, being subject to the condition that the sum of the extreme values is equal to the sum of the means, which furnishes a check on the accuracy of the computation.

Again, by simple subtraction of the corresponding equations in successive series, we find the following four distinct values of each of the eight quantities $\alpha.a$, $\beta.a$, $\alpha.b$, $\beta.b$, $\alpha'.a$, $\beta'.a$, $\alpha'.b$, $\beta'.b$, which give the circle flexure.

$4\alpha.a=$	$4\beta.a=$	$4\alpha.b=$	$4\beta.b=$
K .15.0—K′ .15.0,	K′ .16.90—K .16.90,	K .15.90—K′ .15.90,	K .16.0—K′ .16.0,
K .25.0—K′ .25.0,	K′ .26.90—K .26.90,	K .25.90—K′ .25.90,	K .26.0—K′ .26.0,
K‴.15.0—K″ .15.0,	K″.16.90—K‴.16.90,	K‴ 15.90—K″.15.90,	K‴.16.0—K″ .16.0,
K‴.25.0—K″ .25.0.	K″.26.90—K‴.26.90.	K‴.25.90—K″.25.90.	K‴.26.0—K″ .26.0.

$4\alpha'.a=$	$4\beta'.a=$	$4\alpha'.b=$	$4\beta'.b=$
K .15.0—K‴.15.0,	K .25.90—K‴.25.90,	K‴.15.90—K .15.90,	K .25.0—K‴.25.0,
K .16.0—K‴.16.0,	K .25.90—K‴.26.90,	K‴ 16.90—K .16.90,	K .26.0—K‴.26.0,
K′ .15.0—K″ .15.0,	K′ .25.90—K″ .25.90,	K″ .15.90—K′ .15.90,	K′ .25.0—K″ .25.0,
K′ .16.0—K″ .16.0.	K′ .26.90—K″ .26.90.	K″ .16.90—K′ .16.90.	K′ .26.0—K″ .26.0.

(48) Thus, the flexure of the circles, in so far as it affects the mean reading of any pair of opposite microscopes, and the relative flexure of the two ends of the axis, are completely and rigorously determined. It remains to determine the flexure of the line joining the eye and object end of the telescope relatively to the axis itself. We have supposed, for brevity,

$$g.z=(f-f')\sin Z+(g+g')\cos Z,$$

and have shown how to find $g.z$, and therefore $f-f'$, and $g+g'$. The coefficients f g, f' g', may now be found separately and independently by observations of the nadir point and collimators. An observation of a levelled collimator, corrected for inequality of collimator pivots, difference of latitude of circle and collimator, aerial refraction, and collimation error of the collimator, shows the circle reading when the line of sight of the telescope is truly horizontal. The coincidence of the direct and reflected images of the zenith-distance wires shows the circle reading when the telescope is truly vertical. When these readings are corrected for circle flexure, the horizontal and vertical circle readings will be as follows:

	East circle.	West circle.
South horizontal reading,	C_0+f	C'_0-f'
North horizontal reading,	C_0-f	C'_0+f'
Nadir reading,	C_0-g	C'_0-g'.

C_0 being the true reading, independent of flexure. From these equations the values of C_0, f, and g, C'_0, f', and g', may all be determined.

The values of f and f' may also be determined independently by viewing the wires of one collimator through the other, and making the horizontal wires of the latter coincide with the images of those of the former. If, then, the micrometer wires of the telescope be set successively upon the images of the two collimator wires, the line joining its object glass and micrometer wires will have moved accurately through the space of 180°, while the circle will indicate a motion of $180°\pm2f$. The agreement of the two values of f will afford a check upon the accuracy of the collimator determinations.

(49) One precaution is, however, indispensable to an accurate result. The different strata of air in the room must be as nearly as possible of the same temperature. For, if there be any admixture of warm and cool air, the former will ascend, and we shall thus have a temperature increasing with the height. If, now, this increase amount to one degree Fahrenheit in nine feet, the curvature of the ray will be equal to that of the water level, so that it would never leave the earth.

The course of the ray being curved, the value of f obtained from the opposing collimators will be vitiated. On the other hand, if we adopt the other method, there is always a possibility of error arising from irregularities or uncertain differences of diameter of the collimator pivots, or possible flexure of some part of the collimator itself. However f is determined, the observations should be made with the shutters open, and under atmospheric conditions, as nearly as practicable, like those under which the astronomical observations are made. Considering the difficulty of rigorously fulfilling this condition, it would, perhaps, be better to determine f by reflection observations of stars. g also admits of being determined by a comparison of North Polar Distances of stars near the zenith in reversed positions of the instrument.

ERRORS OF DIVISION.

(50) The method of determining the errors of division is, in principle, that usually adopted, the operation being so conducted as to eliminate all irregular flexure of the circles.

Let us suppose the two pairs of opposite microscopes set at the distance B, the angle position of the first pair being 0°, and that of the second pair, B. Take a series of pairs of divisions at the distance B, and let these divisions be successively brought under two pairs of microscopes. Then, using the same notation as in § (43),

$$\begin{aligned}
\text{Mic. I.}\quad & r_1=m_1+\frac{\eta}{R}+a_1+e_1+\omega_1,\\
\text{II.}\quad & r_2=m_2+\frac{\eta}{R}\cos B-\frac{\xi}{r}\sin B+a_2\cos B+b_2\sin B+e_2+\omega_1,\\
\text{III.}\quad & r_3=m-{}_3\frac{\eta}{R}-a_3+e_3+\omega_1, \qquad\qquad (1)\\
\text{IV.}\quad & r_4=m_4-\frac{\eta}{R}\cos B+\frac{\xi}{R}\sin B-a_4\cos B-b_4\sin B+e_4+\omega_1.
\end{aligned}$$

Whence, adding the odd and even equations

$$\rho_1^{(1)}=M_1+a_1+e_1+\omega_1,$$

$$\rho_2^{(1)}=M_2+a_2\cos B+\beta_2\sin B+e_2+\omega_1.$$

Turning the circles through the distance B, we shall have

$$\rho_1^{(2)}=M_1+a_2+e_2+\omega_2,$$

$$\rho_2^{(2)}=M_2+a_3\cos B+\beta_3\sin B+e_3+\omega_2. \qquad (2)$$

Turning again

$$\rho_1^{(3)}=M_1+a_3+e_3+\omega_3,$$

$$\rho_2^{(3)}=M_2+a_4\cos B+\beta_4\sin B+e_4+\omega_4.$$

Let $2i$ be the number of motions necessary to bring the circle back to its first position. Then, the $(i+1)$th reading will have brought it round 180°, so that the same divisions as at first will be under the microscopes. The readings after the ith will then give

$$\rho_1^{(i+1)}=M_1-a_1+e_1+\omega(i+1),$$

$$\rho_2^{(i+1)}=M_2-a_2\cos B-\beta_2\sin B+e_2+\omega(i+1),$$

$$\rho_1^{(i+2)}=M_1-a_2+e_2+\omega(i+2), \qquad (3)$$

$$\rho_2^{(i+2)}=M_2-a_3\cos B-\beta_3\sin B+e_3+\omega(i+2),$$

&c., &c., &c.

Taking the sums of readings in opposite positions of the circle, we have

$$\rho_1^{(1)}+\rho_1^{(i+1)}=2M_1+2\epsilon_1+\omega_1+\omega.(i+1),$$
$$\rho_2^{(1)}+\rho_2^{(i+1)}=2M_2+2\epsilon_2+\omega_1+\omega.(i+1).$$

Using the notation

$$2I.1.2=\rho_2^{(1)}-\rho_1^{(1)}+\rho_2^{(i+1)}-\rho_1^{(i+1)}, \qquad (4)$$

we have

$$\begin{aligned} M_2-M_1+\epsilon_2-\epsilon_1&=I.1.2,\\ M_2-M_1+\epsilon_3-\epsilon_2&=I.2.3,\\ M_2-M_1+\epsilon_4-\epsilon_3&=I.3.4,\\ \&c., \quad \&c., \quad &\&c. \end{aligned} \qquad (5)$$

Taking the sum of the entire series of values of I, and dividing by their number, the ϵ's will destroy each other, and we shall have

$$M_2-M_1=\frac{\Sigma I}{i}. \qquad (6)$$

Putting

$$\Delta.1.2=I.1.2-(M_2-M_1), \qquad (7)$$

we shall have

$$\begin{aligned} \epsilon_2-\epsilon_1&=\Delta.1.2,\\ \epsilon_3-\epsilon_2&=\Delta.2.3,\\ \epsilon_4-\epsilon_3&=\Delta.3.4,\\ \&c., \quad &\&c. \end{aligned} \qquad (8)$$

Thus, where one ϵ is known, all the others may be found by successive addition of the differences $\triangle$. If, however, the number i is considerable, the accidental errors of the various $\triangle$s may in the additions become considerable; it is therefore necessary that at least every fifth ϵ in each series be corrected by the results of another series in which the value of i shall be less than in the series in question. The correction will be really applied to M'_2-M_1, which will thus be determined independently for each sub-series. Suppose, for example, that in a first series the microscopes are at the distance 45°, so that $i=4$. The preceding formulæ will then give

$$\begin{aligned} \epsilon.45\ -\epsilon.0\ \ &=\Delta\ .0.45,\\ \epsilon.90\ -\epsilon.45\ &=\Delta.45.90,\\ \epsilon.135-\epsilon.90\ &=\Delta.90.135,\\ \epsilon.0\ \ -\epsilon.135&=\Delta.135.0. \end{aligned} \qquad (9)$$

In a second series, let the microscopes be placed at the distance 15°. The formulæ will then give

$$\begin{aligned} &\epsilon.15-\epsilon.0\ =\Delta.0.15, && \epsilon.60-\epsilon.45=\Delta.45.60, && \&c.;\\ &\epsilon.30-\epsilon.15=\Delta.15.30, && \epsilon.75-\epsilon.60=\Delta.60.75, && \&c.;\\ &\epsilon.45-\epsilon.30=\Delta.30.45, && \epsilon.90-\epsilon.75=\Delta.75.90, && \&c. \end{aligned} \qquad (10)$$

Adding each of (10) and comparing with (9), we have

$$\begin{aligned} \Delta\ .0.15+\Delta.15.30+\Delta.30.45&=\Delta\ .0.45,\\ \Delta.45.60+\Delta.60.75+\Delta.75.90&=\Delta.45.90,\\ \&c., \quad \&c., \quad &\&c. \end{aligned}$$

If the readings were perfect, these equations ought to be exactly satisfied. Not being satisfied, the difference is due to the accidental errors of the readings. Let ϵ' be the probable error of each of the left-hand Δs, ϵ that of the right-hand ones, and e the amount of the discrepancy. The most probable distribution of the discrepancy will then be,

to each left-hand Δ,

$$\frac{e\epsilon'^2}{\epsilon^2+3\epsilon'^2};$$

and to each right hand Δ,

$$\frac{e\epsilon^2}{\epsilon^2+3\epsilon'^2}.$$

The corrected values of $\triangle$ being substituted in (10), we shall have a series from which the most probable values of ϵ_0, ϵ_{15}, &c., may be found by successive addition from any arbitrarily assumed value of ϵ_0.

DETERMINATION OF THE ERRORS OF SPECIAL DIVISIONS.—PROBABLE ACCIDENTAL ERROR OF ISOLATED DIVISIONS.

(51) The determination of the error of each separate division being impracticable, this question arises: What is the probable error of any isolated division relatively to any considerable number of the divisions near it? Measuring a series of contiguous spaces on any part of the circle, we have the data for answering this question. These measures can also be made available for the termination of the special divisions used in observing the nadir and horizontal points, or any special star the position of which is required with a high degree of accuracy. The method of treating these measures is as follows:

The space between two adjacent divisions being very nearly four revolutions of the micrometer, let the measure of the first, second, third, &c., spaces be

$$4 \text{ rev.} + d_1, \quad 4 \text{ rev.} + d_2, \quad 4 \text{ rev.} + d_3, \quad \text{\&c.}$$

Represent also by x the error of runs of the microscope in four revolutions, and by e_0, e_1, &c., the errors of division. Then the measures give the series of equations

$$\begin{aligned} x+a_1&=e_1-e_0,\\ x+a_2&=e_2-e_1,\\ x+a_3&=e_3-e_2,\\ &\cdots\\ &\cdots\\ x+a.n&=e.n-e.(n-1.) \end{aligned} \qquad (1)$$

Here we have n equations for the determination of $n+2$ unknown quantities. The remaining two equations are given by the condition that the sum of the squares of the errors of division must be a minimum. We thus have

$$e_0\frac{de_0}{dx}+e_1\frac{de_1}{dx}+ \text{\&c.}, +e.n\frac{de.n}{dx}=0, \qquad (2)$$

$$e_0\frac{de_0}{de}+e_1\frac{de_1}{de}+ \text{\&c.}, +e.n\frac{de.n}{de}=0, \qquad (3)$$

e in the last equation representing any error of division taken at pleasure. Regarding x and e as independent variables, any change in e (x being given) will cause an equal change in all the other errors of division. We have, therefore,

$$\frac{de_0}{de}=\frac{de_1}{de}= \text{\&c.} \ldots =\frac{de.n}{de}=1,$$

and therefore,

$$e_0+e_1+ \text{\&c.} \ldots +e.n=0. \qquad (4)$$

This equation, united with the equations (1,) will enable us to express each error of division in terms of x. Multiplying the first of equations (1) by n, the second by $n-1$, the third by $n-2$, &c., and adding, remembering that

$$\Sigma e=0,$$

we have an equation which gives e_0 in terms of x. By using the proper coefficients of elimination, we have each of the other errors in terms of x. These coefficients are indicated in the following table, in which each column is to be read downward, in connection with the words on the left,

To express		e_0	e_1	e_2		$en-1$	en
the coefficient of a_1	is	n	-1	-1	. . .	-1	-1
" " " a_2	is	$n-1$	$n-1$	-2	. . .	-2	-2
" " " a_3	is	$n-2$	$n-2$	$n-2$	. . .	-3	-3
.		.	.	.		.	.
" " " $a.(n-1)$	is	2	2	2	. . .	$-(n-1)$	$-(n-1)$
" " " $a.n$	is	1	1	1	. . .	1	$-n$

The equations thus obtained are as follows:

$$\frac{n(n+1)}{2}x+na_1+(n-1)a_2+ \dots +a.n+(n+1)e_0=0,$$

$$\frac{(n-2)(n+1)}{2}x- a_1+(n-1)a_2+ \dots +a.n+(n+1)e_1=0,$$

$$\frac{(n-4)(n+1)}{2}x- a_1-2a_2+(n-2)a_3+ \dots +a\,n+(n+1)e_2=0, \qquad (5)$$

$$\vdots$$

$$-\frac{n(n+1)}{2}x- a_1-2a_2-3a_3- \dots -na.n+(n+1)e.n=0.$$

We have now only to suppose the last members of these equations to be accidental errors, and solve by least squares. This solution gives for the equation in x,

$$\frac{n(n+1)\,(n+2)}{6}x+na_1+2(n-1)a_2+3(n-2)a_3+ \dots +na.n=0.$$

Having thus obtained the value of x, the quantities e_1-e_0, e_2-e_1, &c., are immediately formed from equations (1.) Thus, when we have e_0, the other e's are formed in succession by the adding on of successive differences. The value of e_0 may be obtained from the equation

$$(n+1)e_0+n(e_1-e_0)+(n-1)(e_2-e_1) \dots +e.n-e.n_1=0,$$

formed by multiplying the first member of (1) by n, the second by $n-1$, &c., and adding with reference to (4.) But the most ready way of obtaining all the e's is perhaps by forming them from $e_0=0$, and then subtracting from each of them the mean value of the e's thus obtained.

If one of the e's is known by the methods already explained, the values of the differences e_0-e_1, &c., will not be changed, and the values of all the other errors may be found by addition of the differences to this known value.

PART III.

DETERMINATION OF THE CONSTANTS OF THE TRANSIT CIRCLE AND ITS SUBSIDIARY APPARATUS.

(52) *Value of one division of the Spirit Levels.*—It will be remembered that there are three levels—one striding level for each collimator, and a hanging level for the axis of the instrument. The former are distinguished by the letters A and B.

1865, October 18. The old mural circle was turned until the telescope was horizontal, and the collimator levels were set astride of the telescope. Microscopes A and B of the circle, and the two ends of the bubble of the level, were then read in different positions of the telescope, as follows:

	Mic. A.	Mic. B.	Level A.		Level B.	
			S. end.	N. end.	S. end.	N. end.
	′ ″	″	d.	d.	d.	d.
	57 67.9	70.8	18.4	42.3	18.5	38.3
	54.2	56.4	1.1	25.2	1.6	21.5
	75.2	77.9	27.6	51.3	27.5	47.3
	54.4	57.1	2.3	26.2	3.0	22.8
	75.1	77.5	26.6	50.5	26.4	46.2
	65.2	67.8	15.0	39.1	16.4	36.2
Levels reversed	54.1	56.8	44.5	25.5	41.0	21.2
	60.9	63.4	36.5	12.5	34.1	14.2
	49.7	52.8	50.0	26.1	46.0	26.2
	71.0	73.8	23.2	−0.6	21.4	1.6
	50.2	52.8	49.6	25.5	45.8	26.0
	68.8	71.2	26.7	2.5	24.3	4.6
	59.9	62.7	37.2	13.4	34.0	14.2

Temperature 57°.

From the observations is concluded—

One division of level A=0″.826.

One division of level B=0″.850.

The level error of the collimators being kept quite small, rarely so great as 3″, the value of one division,

0″.84,

has been adopted for both levels.

1865, November 21. The hanging level was suspended from the telescope of the Mural Circle, and the following readings taken:

Mic. A.	Mic. B.	Level.	
		N. end.	S. end.
″	″	d.	d.
16.8	21.1	32.7	84.5
26.9	31.2	20.5	72.3
40.3	45.1	5.2	57.2
14.9	19.4	34.4	86.3
41.5	46.1	4.0	55.9
35.7	40.6	10.2	61.1 62.1(?)
30.2	35.2	16.4	68.3
24.2	29.0	22.8	74.9
18.6	23.8	29.2	81.2
13.2	17.8	36.2	88.2
42.9	47.7	2.0	54.1

Temperature 53°. From which is concluded

One division=0″.872=0s.058.

The three readings preceding the last seem to indicate a diminution of the value as we approach the end of the scale, but the diminution is no greater than what may be due to errors of the microscopes.

(53) *Difference of collars of collimators.*—It is requisite that we know the difference between the true level of the axis of the collimator and the readings of the spirit level set upon the collars on which the collimators turn. This is effected by reading the level when the collimator is in its regular position, and when reversed, end for end, around a vertical axis, the operation being repeated a number of times in succession to eliminate the possible changes of level during the operation.

The following are the level indications of the two collimators in different angles of position. The angle of position of the collimator indicates its position as it is turned upon its Ys, without being raised from them. They are used only in those four positions in which one of the wires is horizontal; and, in practice, the positions are almost entirely confined to two, which are designated as 0° and 180°. The angle is measured by the position of the clamp which binds the eye-piece, being called 0° when this clamp points horizontally to the right of an observer looking into the eye end of the collimator, and 90° when it points upward.

The collimator is said to be direct when the collimator points toward the Transit Circle; reverse when it points from it. The level readings are positive when the pivot farthest from the Transit Circle is too high.

COLLIMATOR A.

First series.—Position changed from 0° to 180°, and the collimator reversed alternately one pair level readings between each change.

	Position 0°.		Position 180°.	
	Direct.	Reverse.	Direct.	Reverse.
	d.	d.	d.	d.
Level A . .	+ 3.84	+ 1.35	+ 3.71	+ 1.26
	+ 2.91		+ 2.56	
Level B . .	− 0.45	− 0.73	+ 1.02	− 0.90
	+ 0.62	− 0.98	+ 0.55	− 1.06

Mean, D−R. +1.32 d. +1.80 d.

Second series.—Position 0°.

Direct.	Reverse.
d.	*d.*
+0.81	−1.24
+0.32	−0.94
−0.24	−1.42
0.00	
	d.
Mean, D−R.	+1.42.

Third series.—Position 180°.

d.	*d.*
−0.18	−0.95
+0.36	−1.65
−0.39	−0.89
	d.
Mean, D−R.	+1.09

The concluded correction for difference of collars, when in the position 0° or 180°, is one-fourth D—R, or 0″.29, the eye-pivot being too large.

Two similar, but more accordant, series gave for the correction to position 90°—270° 0″.09, so that the collars are not perfectly cylindrical.

COLLIMATOR B.

The determinations have been made only for the positions 0° and 180°. The results are

		d.
D−R for	0°,	+1.06
for	180°,	+0.55

Concluded correction, 0″.17, the eye collar being too large, as in the other collimator.

The collars are decidedly conical, diminishing toward the ends of the telescope, so that entire dependance cannot be placed on the absolute horizontal point obtained from a single collimator. But, by interchanging the collimators, this error is completely eliminated from the zenith point.

(54) *Periodic Inequalities of the Micrometer Screws.*—In the case of the eight microscope micrometers the inequalities were determined by measuring the intervals between the parallel wires of each pair with different portions of the screw. The circle being clamped and properly set, one of the micrometer wires was brought to a distance from the edge of a division approximately equal to the thickness of the wire. The observer retained a quite accurate idea of the intervening space, though the idea could not be defined in language. The micrometer was then read. The other wire was then brought into the same position, and the micrometer was again read. The operation was twice repeated, making three measures in all. The circle was then moved forward 5″ by the tangent screw, and a similar series of measures again taken. The operation was continued through the two revolutions of the screw most used.

Repeated trials showed that the wires could be set more accurately in position by this method than by making them coincide with the circle division, the probable error of a single setting being about 0″.10, scarcely greater than that in putting the division midway between the wires.

The results of a determination made in November, 1865, were:

For microscopes I, II, III, inequality insensible;

IV. Ineq. $= -0''.53 \cos u + 0''.57 \sin u + 0''.13 \cos 2u - 0''.11 \sin 2u$;

V. $-0\ .09 \cos u + 0\ .02 \sin u$;

VI. $+0\ .24 \cos u + 0\ .28 \sin u$;

VII. $-0\ .06 \cos u + 0\ .06 \sin u$;

VIII. $+0\ .06 \cos u + 0\ .27 \sin u$;

u being the angle of the reading of the head.

Microscopes V–VIII being alone employed in astronomical operations, their inequalities were redetermined in March, 1866, with the following result:

$$\begin{aligned} \text{V.}\quad & -0''.05 \cos u + 0''.05 \sin u; \\ \text{VI.}\quad & +0\ .24 \cos u + 0\ .25 \sin u; \\ \text{VII.}\quad & -0\ .07 \cos u + 0\ .08 \sin u; \\ \text{VIII.}\quad & +0\ .11 \cos u + 0\ .25 \sin u. \end{aligned}$$

The mean of the two results is adopted as the correction for the year 1866.

(55) A rough check upon the general accuracy of the screws is given by the mean distance of the wires as measured in different revolutions of the screw. The following are the distances given by the same measures which determine the inequalities:

	1865.		1866.		Difference.	
	29 rev.	30 rev.	29 rev.	30 rev.	1865.	1866.
	"	"	"	"	"	"
Mic. V .	10.98	11.05	10.80	10.91	+ 0.07	+ 0.11
Mic. VI .	10.88	10.82	10.80	10.95	− 0.06	+ 0.15
Mic. VII .	10.53	10.47	11.85	11.75	− 0.06	− 0.10
Mic. VIII .	10.88	10.87	10.74	10.92	− 0.01	+ 0.18

The method of observing is such that the microscope micrometers are seldom moved through more than a fraction of a revolution. No very accurate investigations have therefore been made to find whether the value of the revolution of any one of them changes progressively; but the measures occasionally made for runs show that the change, if it exists, is entirely inappreciable.

There are, however, outstanding discrepancies, amounting sometimes to three or four tenths of a second, which I have not been able to refer to any law.

(56) *The Declination Micrometer of Telescope.*—Neither the collimators nor the telescope micrometer were originally furnished with double wires; the usual method could not, therefore, be used to determine the inequality of the latter. The plan was, therefore, adopted of measuring successive half revolutions of the telescope micrometer with the microscope micrometers, by setting the wire of the former upon the collimator. The following are the value of eight successive half revolutions, as given by two microscopes of each circle, in one series, and the four microscopes V–VIII in the other:

	First series.				
	Mic. I.	Mic. III.	Mic. VI.	Mic. VIII.	Mean.
r. r.	"	"	"	"	"
24.0 — 24.5	7.70	7.61	7.91	8.39	7.90
24.5 — 25.0	7.77	7.76	7.85	7.49	7.72
25.0 — 25.5	7.56	7.37	7.66	7.66	7.56
25.5 — 26.0	7.81	7.41	7.45	7.41	7.52
26.0 — 26.5	7.75	7.85	8.00	8.25	7.96
26.5 — 27.0	7.86	7.51	7.63	7.38	7.60
27.0 — 27.5	7.55	7.65	8.03	7.85	7.77
27.5 — 28.0	7.83	7.63	7.61	7.76	7.71

The mean of the odd measures is 7″.80, and of the even ones, 7″.64.

	Second series.				
	Mic. I.	Mic. III.	Mic. VI.	Mic. VIII.	Mean.
r. r.	"	"	"	"	"
31.25 — 31.75	7.54	7.69	7.51	7.36	7.52
31.75 — 32.25	7.54	7.98	7.73	8.02	7.82
32.25 — 32.75	7.75	7.65	7.86	7.86	7.78
32.75 — 33.25	7.51	7.49	7.24	7.22	7.36
33.25 — 33.75	7.37	7.50	7.64	7.71	7.56
33.75 — 34.25	7.57	7.76	7.66	7.61	7.65
34.25 — 34.75	7.94	7.64	7.75	7.85	7.79
34.75 — 35.25	7.49	7.57	7.62	7.60	7.57

The mean of the odd measure is 7".66, and of the even ones, 7".60.

From the differences 0".16 and 0".06 of the half revolutions the screw would seem to be affected with the error

$$0''.04 \cos u + 0''.02 \sin u\,;$$

an error so small that it has been neglected, especially as the screw is used about equally through several revolutions for every class of observations, by which the periodic errors will disappear from the final result of the observations. After double wires were placed in the collimators, measurements of successive half revolutions of the micrometer were made, which gave for the periodic correction,

$$0''.07 \cos u - 0''.05 \sin u.$$

(57) *Value of a Revolution of the Micrometer Screws.*—1866, June 12.—The telescope was set on collimator B. The record does not state whether it was north or south. The microscopes, however, indicate that it was south.

The following readings of the microscopes and zenith distance micrometer were taken successively in different positions of the telescope. Each micrometer reading is the mean of nine; three for coincidence of collimator image with each wire, and three for a position midway between wires:

Minute of circle division.	Readings of microscopes—				Mean.	Corr. for—		Corrected circle reading.	Microm. reading.
	V.	VI.	VII.	VIII.		Ineq.	Div.		
'	r. "	"	"	"	"			' "	r.
56	9 19.5	20.3	21.2	23.7	21.18	−.15	−.04	55 50.99	28.408
54	16.5	18.0	19.2	21.8	18.89	−.14	−.12	53 48.63	20.392
58	21.4	22.8	23.2	26.0	23.35	−.12	+.16	57 53.39	36.377
54	15.8	17.9	18.9	21.2	18.45	−.14	−.12	53 48.19	20.373
58	21.6	22.8	23.4	25.9	23.43	−.12	+.16	57 53.47	36.393
56	19.0	20.6	21.4	23.7	21.18	−.15	−.04	55 50.99	28.395

These readings give from 20 r. to 28 r., 1 rev. = 15".286,
28 r. to 36 r., 1 rev. = 15 .337.

An increase in the value of a revolution as the turns increase seems to be indicated.

In the great majority of observations the micrometer is used between 26 and 34 revolutions; a determination of the value between those limits was therefore made which gave 15".296. The value actually adopted was the mean of the two determinations, or 15".303.

A third determination, made on September 3, 1866, gave

From 26 r. to 34 r., 1 rev. = 15".311;
34 r. to 45 r., 1 rev. = 15 .375;

which seems to confirm the suspicion of an increase in the value of a revolution as the screw is advanced.

(58) *Irregularity of the Screw.*—It is desirable to know whether this change in the value of revolution is regularly progressive, or whether it is subject to sudden changes. To learn this, advantage was taken of the fact that the distance of the pair of wires in collimator A, and of the close pair of wires moved by the micrometer, are together nearly equal to a revolution of the latter. Consequently, turning the telescope on that collimator when its double wires are horizontal, we have a measure of a constant space by setting first the upper micrometer wire on the lower image of the collimator wire, and then the lower micrometer wire on the upper image. The measures were commenced, as nearly as practicable, at an even revolution, and therefore ended nearly at the beginning of the next revolution. The measures were made on three different dates, and three measures of each revolution made on each date. But the measures were not always continued through the entire range of the screw. About 30 revolutions they were prevented by the interference of the fixed horizontal wires of the reticule.

Commencement of measure.	End of measure.	Measure.			Residuals of cor. arc.	No. of dates.
		Rev.	Arc.	Cor. arc.		
r.	r.	r.	"	"	"	
17.7	18.7	0.984	15.06	14.97	+ 0.12	1
18.7	19.7	.984	15.06	14.97	+ 0.12	1
19.6	20.6	.972	14.88	14.80	− 0.05	1
20.0	21.0	.986	15.09	15.02	+ 0.17	3
20.5	21.5	.980	15.00	14.93	+ 0.08	4
21.0	22.0	.971	14.86	14.79	− 0.06	4
22.0	23.0	.976	14.94	14.88	+ 0.03	4
23.0	24.0	.972	14.88	14.83	− 0.02	3
24.0	25.0	.973	14.89	14.85	0.00	3
25.0	26.0	.965	14.77	14.73	− 0.12	3
26.0	27.0	.972	14.88	14.85	0.00	3
27.0	28.0	.965	14.77	14.75	− 0.10	3
28.0	20.0	.964	14.76	14.75	− 0.10	3
31.0	32.0	.970	14.85	14.86	+ 0.01	2
32.0	33.0	.968	14.82	14.83	− 0.02	2
33.0	34.0	.975	14.92	14.94	+ 0.09	1
34.0	35.0	.961	14.71	14.74	− 0.11	1
35.0	36.0	.965	14 77	14.81	− 0.04	1
36.0	37.0	.966	14.79	14.83	− 0.02	1

Of the three columns headed "Measure," the first gives the measures in revolutions of a micrometer; in the second these revolutions are turned into arc, using 1 rev.=15″.303; in the third they are corrected for progressive change in the value of the revolution. The residuals show the excesses of the individual corrected measures over the mean value 14″.85. I conceive that they proceed mainly from the accidental errors of reading and temporary derangements of the motion of the screw by dust, displacement of the oil, and other causes, and that the screw itself may be regarded as sensibly regular.

(59) *R. A. Micrometer.*—The value of a revolution of this micrometer seems to be exactly the same as that of the other. Wide measures give 15″.300. Owing to its limited use, no special investigation of its movement has been entered upon.

(60) *Flexure of the Circles.*—The work of determining separately the flexure of the different parts of each circle was commenced in 1866, January 19. But after taking one series of readings, it was found that the axes of several of the microscopes deviated quite sensibly from the perpendicular to the face of the circle; some deviating as much as 40′. To adjust them with entire accuracy appeared to be a difficult and troublesome operation; but a kind of T-square was made by which they could be set without an error exceeding 5′, and they were adjusted by it on April 7. The set of readings previously made were not used.

The following are the details of the operations by which the definitive values of the flexure coefficients were obtained:

In the first series of readings the position of the circles was such that when the telescope pointed to the zenith, the divisions of circle A, which were under microscopes V–VIII, were:

Mic. V, 135°; VI, 225°; VII, 315°; VIII, 45°.

The divisions of circle B, under microscopes I–IV, were:

Mic. I, 315°; II, 45°; III, 135°; IV, 225°.

When the telescope pointed at zenith distance z, the above readings of microscopes V–VIII would be increased, and those of I–IV diminished by z.

The operation was begun on divisions 0°, 90°, 180°, 270°. The division 0° of circle B was brought under microscope I, and the eight microscopes were read, one observer reading each circle. The telescope was then turned 90°, and the microscopes were again read. The telescope was again turned 90°, and the operation of turning and reading continued until the microscopes had been read three times in each of the four positions of the telescope. The mean of the three readings was taken as that corresponding to each position.

The telescope was then set 15° backward from its first position, and the same operation was performed on divisions 75°, 165°, &c., of circle A, and 15°, 105°, &c., of circle B. The operation was repeated through every 15° of the quadrant. The first series was now complete.

In the second series circle A was loosened and turned 180° on its axis, so that when the telescope pointed to the zenith, the division 315° was under microscope V. A series of readings exactly like the first was then made on the two circles.

Circle B was then turned on its axis 180°, and a third series of readings were made.

Finally, circle A was returned to its original position, and a fourth series of readings were made.

The mean of the three readings for each position of the instrument is given in the following table:

SERIES I.

Zenith distance of telescope.	Division under microscope I.	Division under microscope V.	Readings of microscopes.							
			I.	II.	III.	IV.	V.	VI.	VII.	VIII.
°	°	°	"	"	"	"	"	"	"	"
45	2.0	180	27.90	25.63	4.97	0.67	4.97	7.07	2.87	2.90
135	180	270	27.40	25.60	4.77	29.13	5.33	6.27	0.67	3.37
225	90	0	23.43	23.70	3.73	27.23	5.43	4.63	2.17	5.03
315	0	90	24.37	21.90	3.67	28.50	4.50	6.73	4.57	5.43
30	285	165	26.57	23.67	3.40	28.93	6.23	8.60	4.77	4.37
120	195	255	26.43	25.10	3.83	28.27	6.20	8.00	1.83	4.30
210	105	345	23.47	23.70	4.40	27.00	5.60	4.47	1.63	4.53
300	15	75	22.27	20.13	1.80	27.17	6.10	8.60	5.57	7.13
15	300	150	26.87	24.77	4.57	29.70	4.83	6.83	3.67	3.43
105	210	240	27.33	25.63	4.77	29.40	5.27	7.43	1.67	3.10
195	120	330	24.03	23.63	3.30	27.07	6.00	5.50	1.43	3.80
285	30	60	23.17	21.50	2.47	27.03	5.17	6.77	4.63	5.63
0	315	135	26.57	23.90	4.53	29.27	4.67	7.30	3.93	4.13
90	225	225	27.73	26.90	5.33	0.40	4.67	6.67	1.50	2.37
180	135	315	25.47	24.60	4.70	27.77	4.93	5.07	0.63	3.27
270	45	45	23.30	22.35	2.50	27.75	5.00	6.20	5.20	5.15
345	330	120	25.03	22.43	3.30	28.13	5.23	8.13	4.17	5.60
75	240	210	27.13	26.07	4.70	0.23	5.17	7.00	2.87	2.93
165	150	300	25.37	24.17	4.03	27.47	5.23	6.43	1.03	3.73
255	60	30	22.85	22.60	2.60	27.35	5.45	5.35	2.95	4.80
330	345	105	24.77	21.97	3.10	28.57	5.20	7.70	5.03	5.90
60	255	195	27.43	25.87	4.60	0.03	5.27	7.77	3.30	3.77
150	165	285	26.30	25.57	4.93	28.67	5.40	6.30	0.93	3.50
240	75	15	22.70	22.37	3.13	27.57	5.33	5.77	2.83	5.63

SERIES II.

Zenith distance of telescope.	Division under microscope I.	Division under microscope V.	Readings of microscopes.							
			I.	II.	III.	IV.	V.	VI.	VII.	VIII.
°	°	°	"	"	"	"	"	"	"	"
315	0	270	29.60	28.30	6.70	0.45	4.70	8.40	0.40	29.70
45	270	0	1.70	0.90	7.30	1.50	5.10	5.20	28.75	30.30
135	180	90	26.90	26.65	2.95	26.55	6.05	8.35	3.80	4.55
225	90	180	25.35	26.25	3.45	25.85	5.80	9.55	4.60	2.55
300	15	255	26.70	26.00	4.60	28.73	5.67	9.90	1.83	1.50
30	285	345	0.80	29.23	5.87	0.20	5.73	5.80	29.57	0.87
120	195	75	27.40	27.63	3.00	26.30	5.37	7.60	2.77	4.23
210	105	165	24.73	25.57	3.20	24.77	5.40	9.27	4.53	2.37
285	30	240	24.80	24.23	2.33	25.63	6.17	11.10	3.60	2.13
15	300	330	28.97	27.90	4.67	28.53	6.33	7.53	0.10	0.77
105	210	60	26.70	26.17	2.17	25.63	6.57	7.90	2.60	4.43
195	120	150	22.80	23.37	0.67	22.83	6.57	9.77	5.67	4.20
270	45	225	23.67	23.77	0.66	24.57	7.17	11.93	4.70	2.57
0	315	315	27.77	25.90	3.57	27.03	7.33	9.33	1.47	2.07
90	225	45	26.47	26.73	2.07	25.97	7.20	8.27	2.77	4.07
180	135	135	23.30	23.33	0.63	22.73	7.13	10.53	5.80	5.07
255	60	210	22.73	23.60	0.83	23.83	7.00	10.80	5.60	3.00
345	330	305	26.90	25.13	3.13	26.77	7.10	10.47	1.83	2.03
75	240	30	27.03	27.10	2.97	27.30	6.87	7.20	1.50	2.90
165	150	120	23.20	22.97	0.10	22.40	7.00	10.23	5.37	5.70
240	75	195	22.50	23.10	0.80	23.23	7.23	11.70	6.03	3.93
330	345	285	26.80	25.37	3.22	27.20	7.07	10.43	2.37	1.93
60	255	15	27.47	27.13	3.47	27.33	6.97	7.87	1.33	3.07
150	165	105	24.87	24.10	0.77	23.23	7.27	9.83	5.20	5.83

SERIES III.

135	0	90	26.30	24.80	6.40	27.97	6.57	9.40	4.47	4.93
225	270	180	29.57	27.80	5.77	28.93	6.77	10.37	5.17	3.50
315	180	270	28.77	28.63	5.87	26.50	6.90	10.87	3.37	2.90
45	90	0	25.17	27.80	7.17	25.63	7.20	8.03	1.97	3.20
120	15	75	23.70	23.17	4.97	26.40	7.37	9.97	4.87	5.77
210	285	165	28.40	25.87	4.30	27.70	7.17	10.60	5.73	3.93
300	195	255	28.07	23.03	4.40	25.20	7.10	11.47	3.70	2.97
30	105	345	25.53	27.87	6.97	24.93	7.00	7.47	1.23	2.33
105	30	60	24.23	24.40	5.47	26.10	6.70	8.37	3.10	4.63
195	300	150	27.60	24.97	4.47	27.27	6.83	9.97	5.57	4.27
285	210	240	28.40	27.30	4.40	25.70	6.87	11.67	4.17	2.87
15	120	330	26.17	27.93	6.17	24.80	6.93	8.27	0.93	1.50
90	45	45	23.63	24.80	5.27	26.00	7.53	8.67	3.23	4.60
180	315	135	26.67	23.97	4.53	26.47	7.37	10.87	5.77	5.13
270	225	225	27.63	26.70	3.60	25.87	7.67	12.40	5.20	3.47
0	135	315	25.80	26.87	6.00	24.17	7.70	9.60	1.80	2.63
75	60	30	23.90	26.83	6.00	25.80	7.37	7.40	1.20	3.43
165	330	120	25.80	23.27	5.77	26.23	7.20	10.40	5.30	5.70
255	240	210	27.27	26.27	3.90	26.97	7.30	11.30	5.50	3.30
345	150	300	26.77	26.93	5.70	24.20	7.23	10.67	2.10	2.40
60	75	15	24.37	26.70	6.90	25.90	7.23	7.77	1.47	3.27
150	345	105	25.77	23.60	5.53	27.30	7.23	9.67	5.13	5.27
240	255	195	28.17	26.60	4.53	27.30	7.13	11.40	5.77	4.03
330	165	285	27.50	26.00	5.90	25.27	7.23	10.53	2.63	2.13

SERIES IV.

Zenith distance of telescope.	Division under microscope I.	Division under microscope V.	Readings of the microscopes. I.	II.	III.	IV.	V.	VI.	VII.	VIII.
°	°	°	"	"	'	"	"	"	"	"
135	0	270	22.87	20.97	3.60	25.50	5.20	8.80	1.10	0.33
225	270	0	26.33	23.63	2.47	26.17	5.87	6.43	20.93	1.87
315	180	90	23.20	22.33	29.87	20.77	5.67	7.97	3.33	4.73
45	90	180	19.40	21.57	1.73	20.17	5.37	8.80	3.83	3.17
120	15	255	21.67	20.90	3.47	25.03	5.13	8.80	1.20	1.70
210	285	345	27.10	23.53	3.17	26.77	5.13	5.33	20.20	0.73
300	195	75	23.80	23.13	0.17	21.23	5.07	7.00	2.17	3.93
30	105	165	20.33	22.37	1.93	19.70	5.20	8.00	3.10	2.07
105	30	240	20.70	20.33	2.40	23.07	5.13	9.37	2.23	2.00
195	300	330	25.80	23.10	2.63	25.80	5.17	6.13	20.33	0.20
285	210	60	24.47	22.93	0.23	21.93	5.20	6.70	1.13	3.20
15	120	150	20.47	22.10	1.30	19.23	5.23	7.37	3.10	2.80
90	45	225	20.43	21.47	1.97	22.63	5.17	8.83	2.17	1.53
180	315	315	25.40	22.50	3.60	25.80	5.30	6.57	29.47	0.67
270	225	45	24.70	23.33	0.60	23.23	5.07	6.50	0.83	2.57
0	135	135	20.53	21.20	0.37	18.60	5.33	8.20	3.33	3.43
75	60	210	19.67	21.30	1.70	21.63	5.20	8.23	2.73	1.80
165	330	300	23.97	21.00	2.80	24.97	5.30	8.03	0.07	1.03
255	240	30	25.13	23.63	1.40	24.73	5.27	5.37	29.90	1.47
345	150	120	21.47	21.37	0.10	18.47	5.13	7.83	2.63	3.90
60	75	195	19.30	21.10	1.97	20.90	5.63	9.17	3.67	2.73
150	345	285	23.60	21.43	2.93	25.60	5.70	8.53	0.63	0.97
240	255	15	25.77	23.63	1.67	24.93	5.60	6.23	0.10	2.07
330	165	105	22.20	22.17	0.17	19.77	5.47	7.80	3.27	4.20

One of the most obvious conclusions, from the above table, is that the two circles do not give the same result for the distance the telescope has moved. If they did, the sum of the eight microscope readings would be constant for each series and each set of circle divisions. In the first set of the first series, for instance, there is a difference of 1".45 after the circle has been turned 180°. One or both circles are therefore affected with a quite sensible flexure.

The above readings were corrected for inequality of screw, and the flexure coefficients were then computed from the formulæ already given. We give an example of the form of computation adopted, by presenting in full so much of the computation as relates to the four cardinal divisions.

SERIES I.

Z	$Z_0=315°$; $a=90°$ $a'=0°$ 2ρ.1	2ρ.2	2ρ.5	2ρ.6	2ρ.(1+5)	2ρ.(1+6)	2ρ.(2+5)	2ρ.(2+6)	2K.15	2K.16	2K.25	2K.26	8α.a	8β.a	8g.z_0
°	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
315	28.0	19.9	9.1	12.7	7.1	10.7	29.0	2.6	−1.0	−1.7	−1.2	−1.9	−2.7	−2.2	+ 0.2
45	32.9	26.0	7.8	10.5	10.7	13.4	33.8	6.5	+5.9	+5.9	+5.9	+5.9	−2.3	−1.7	− 0.5
135	32.2	24.3	5.9	10.2	8.1	12.4	30.2	4.5	. .	. .	. .	. .	−1.9	−1.4	+ 0.4
225	27.2	20.3	7.6	10.3	4.8	7.5	27.9	0.6	. .	. .	. .	. .	−2.1	−1.9	− 0.3
SERIES II.															
°									2K'.15, etc.				8α.b	8β.b	"
315	36.3	28.4	5.0	8.4	11.3	14.7	3.4	6.8	+1.7	+1.5	+1.1	+0.9	+1.3	−3.2	0.0
45	39.0	32.1	4.8	6.0	13.8	15.0	6.9	8.1	+4.6	+3.7	+5.1	+4.2	+0.8	−2.8	+ 0.9
135	29.8	22.5	9.8	13.4	9.6	13.2	2.3	5.9	. .	. .	. .	. .	+1.7	−0.8	+ 0.6
225	28.8	21.4	10.4	12.5	9.2	11.3	1.8	3.9	. .	. .	. .	. .	+2.2	−1.0	+ 1.5

SERIES III.

Z	$Z_0 = 315°$; $a = 90°$ $a' = 0°$								2K''.15, etc.				8α'.a	8β'.a	8g.(Z₀+90°)
	2ρ.1	2ρ.2	2ρ.5	2ρ.6	2ρ.(1+5)	2ρ.(1+6)	2ρ.(2+5)	2ρ.(2+6)							
°	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
315	4.6	24.5	10.3	14.0	14.9	18.6	4.8	8.5	+1.2	+1.2	+1.6	+1.6	−0.3	+10.2	0.0
45	2.3	22.7	9.1	11.7	11.4	14.0	1.8	4.4	−5.9	−5.5	−6.5	−6.1	−2.1	+10.1	+ 0.4
135	2.7	22.2	11.0	14.7	13.7	17.4	3.2	6.9	. .	. .	. .	. .	+0.5	+11.6	− 0.6
225	5.3	26.3	12.0	14.2	17.3	19.5	8.3	10.5	. .	. .	. .	. .	+0.3	+10.3	− 0.2
SERIES IV.															
°													8α'.b	8β'.b	
315	23.1	12.6	9.0	13.2	32.1	6.3	21.6	25.8	−0.7	+0.4	−0.5	+0.6	−10.1	−0.7	− 0.6
45	21.1	11.3	9.2	12.4	30.3	3.5	20.5	23.7	−4.2	−4.1	−4.3	−4.2	−10.0	−2.5	− 1.4
135	26.5	15.8	6.3	9.4	32.8	5.9	22.1	25.2	. .	. .	. .	. .	−10.5	−0.5	+ 0.6
225	28.8	19.1	5.7	8.8	34.5	7.6	24.8	27.9	. .	. .	. .	. .	− 9.2	−0.7	0.0

The computation for the other divisions was performed in the same way. The complete results for every 15° are given in the following table, which is so arranged that the two coefficients which correspond to the same position of the circle are under each other. For this purpose some of the angles are changed by 180°, and the signs of the coefficients are changed to correspond.

CIRCLE A.

a =	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
	"	"	"	"	"	"	"	"	"	"	"	"
Values of 8α.a	−1.3	−4.0	−3.8	−0.9	−2.4	−2.7	−2.7	−0.7	−1.2	+0.7	+0.9	+1.5
	−0.8	−3.2	−3.3	−0.8	−2.5	−2.8	−2.3	−2.1	0.0	+1.1	+0.5	+0.8
	−1.7	−3.1	−2.4	−2.9	−3.8	−2.6	−1.9	−1.5	−0.5	−1.2	+1.3	+0.9
	−2.2	−2.8	−2.6	−3.1	−3.3	−2.7	−2.1	−2.1	−1.4	−0.9	+0.8	+1.7
α.a =	−0.19	−0.41	−0.38	−0.24	−0.38	−0.34	−0.28	−0.20	−0.10	−0.01	+0.11	+0.15
a =	90°	105°	120°	135°	150°	165°	180°	195°	210°	225°	240°	255°
	"	"	"	"	"	"	"	"	"	"	"	"
Values of 8β.a	−2.2	−3.1	−3.1	−2.6	−3.3	−3.0	−3.2	−0.5	−0.7	+0.8	+0.6	+2.0
	−1.7	−2.3	−2.6	−2.5	−3.4	−3.1	−2.8	−1.9	+0.5	+1.2	+0.2	+1.3
	−1.4	−3.0	−2.9	−2.8	−4.1	−2.9	−0.8	−1.1	0.0	−0.6	+1.3	+0.6
	−1.9	−2.7	−3.1	−2.9	−3.6	−3.0	−1.0	−1.7	−0.9	−0.3	+0.8	+1.4
β.a =	−0.22	−0.35	−0.37	−0.34	−0.[illegible]	−0.38	−0.24	−0.16	−0.03	+0.03	+0.09	+0.17

CIRCLE B.

a =	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°
	"	"	"	"	"	"	"	"	"	"	"	"
Values of 8α'.a	−0.3	−2.2	−4.1	−4.7	−8.4	−9.1	−10.1	−9.7	−8.2	−8.2	−4.8	−2.5
	−2.1	−2.2	−4.8	−6.7	−8.0	−8.8	−10.0	−9.9	−7.9	−7.7	−4.8	−1.8
	+0.5	−2.1	−5.8	−6.7	−7.0	−8.2	−10.5	−9.1	−8.6	−6.3	−5.5	−1.7
	+0.3	−2.1	−5.6	−6.9	−7.8	−8.7	− 9.2	−8.5	−8.6	−6.3	−6.5	−1.2
α'.a =	−0.05	−0.27	−0.63	−0.77	−0 98	−1.09	−1.24	−1.16	−1.04	−0.89	−0.64	−0.22

CIRCLE B.—Continued.

$a=$	90°	105°	120°	135°	150°	165°	180°	195°	210°	225°	240°	255°
	″	″	″	″	″	″	″	″	″	″	″	″
Values of $\delta\beta'.a$	−0.7 −2.5 −0.5 −0.7	−1.9 −1.9 −1.8 −1.8	−4.2 −4.9 −5.0 −5.1	−4.1 −6.1 −6.4 −6.6	−8.8 −8.4 −8.1 −8.9	−9.3 −9.0 −8.9 −9.4	−10.2 −10.1 −11.6 −10.3	− 9.5 − 9.7 −10.4 − 9.8	−8.4 −8.1 −8.7 −8.7	−8.0 −7.5 −6.0 −6.0	−5.1 −5.1 −3.8 −3.7	−2.7 −2.0 −2.7 −2.2
$\beta'.a=$	−0.14	−0.23	−0.60	−0.72	−1.07	−1.14	− 1.32	− 1.23	−1.06	−0.86	−0.55	−0.30

(61) It will be remembered that α represents the excess of reading of microscopes V–VII, when a division of circle A is brought under microscope V, and β the excess of VI–VIII, when a division is brought under microscope VI. Also, when the division a is under microscope V, the division $a+90°$ is under microscope VI. The same remarks hold true for circle B, by diminishing the number of the microscope by IV. Now, comparing the values of α with those of β immediately under them, it will be seen that there is generally a quite close agreement, the difference amounting to one-tenth of a second in only one case out of the twenty-four, and the mean difference being less than 0″.05. If we suppose, as seems probable, that these differences are no greater than the unavoidable errors of the determinations, we arrive at the conclusion:

The geometrical form of the circle relative to any system of fixed axes remains invariable as it revolves.

The large values of α and β show that *if the central part of the circle revolves uniformly, the circumference does not revolve uniformly, but is affected with a periodic inequality.*

I am disposed to attribute this singular phenomena to a slight deviation of the centre of gravity of the circle from its centre of figure. The circles weigh about 80 pounds each, and a weight of a few ounces on their circumference is sufficient to produce a flexure of 1″. But to whatever cause we attribute it, the circumstance of invariability of form of the circle involves the law that the flexure shall be of the form of $a \sin z + b \cos z$. If, then, we suppose

$$\alpha = A \sin (D + 45°) + B \cos (D + 45°),$$
$$\beta = A \sin (D - 45°) + B \cos (D - 45°),$$
$$\alpha' = A' \sin (D' + 45°) + B' \cos (D' + 45°),$$
$$\beta' = A' \sin (D' - 45°) + B' \cos (D' - 45°).$$

D being the circle division, we find by equating the preceding values of α and β, and solving by least squares,

$$A = -0''.37; \qquad B = +0''.01;$$
$$A' = -0''.84; \qquad B' = +0''.86.$$

(62) The outstanding apparent errors are seen in the following table, which includes the combined errors of the two hypothesis; first, that the geometrical form of the circle remains invariable, in other words, that

$$\alpha.a = \beta.(a + 90°).$$

Second, that α and β are each of the form,

$$A \sin D + B \cos D.$$

The first column of the table gives the reading of the finding microscope, which, for circle A, is midway between V and VI, and for circle B, midway between I and II.

The second gives the flexure for that position of the circle as computed by the formulæ, which is assumed to be the same for each pair of microscopes.

The third gives the observed flexure of the mean of microscopes VI–VII, or I–III, for that position of the circle, in other words, the value of $\alpha.(R-45^{\circ}.)$

The fourth gives the observed flexure of the mean of microscopes VI–VIII, or II–IV, for the same position of the circle, or the value of $\beta.(R+45^{\circ}.)$

The fifth and sixth give the outstanding errors.

R	Flexure by formula.	Observed flexure.		Errors.		R′	Flexure by formula.	Observed flexure.		Errors.	
		V–VII.	VI–VIII.	V–VII.	VI–VIII.			I–III.	II–IV.	I–III.	II–IV.
°	″	″	″	″	″	°	″	″	″	″	″
45	−0.25	−0.19	−0.22	+0.06	+0.03	45	+0.02	−0.05	−0.14	−0.07	−0.16
60	−0.31	−0.41	−0.35	−0.10	−0.04	60	−0.30	−0.27	−0.23	+0.03	+0.07
75	−0.36	−0.38	−0.37	−0.02	−0.01	75	−0.59	−0.63	−0.60	−0.04	−0.01
90	−0.37	−0.24	−0.34	+0.13	+0.03	90	−0.84	−0.77	−0.72	+0.07	+0.12
105	−0.36	−0.38	−0.45	−0.02	−0.09	105	−1.03	−0.98	−0.07	+0.05	+0.04
120	−0.32	−0.34	−0.38	−0.02	−0.06	120	−1.16	−1.09	−1.14	+0.07	+0.02
135	−0.27	−0.28	−0.24	−0.01	+0.03	135	−1.20	−1.24	−1.32	−0.04	−0.12
150	−0.19	−0.20	−0.16	−0.01	+0.03	150	−1.16	−1.16	−1.23	0.00	−0.07
165	−0.11	−0.10	−0.03	+0.01	+0.03	165	−1.05	−1.04	−1.06	+0.01	−0.01
180	−0.01	−0.01	+0.03	0.00	+0.04	180	−0.86	−0.89	−0.86	−0.03	0.00
195	+0.09	+0.11	+0.09	+0.02	0.00	195	−0.61	−0.64	−0.55	−0.03	+0.06
210	+0.18	+0.15	+0.17	−0.03	−0.01	210	−0.32	−0.22	−0.30	+0.10	+0.02
225	+0.25	+0.16	+0.19	−0.09	−0.06	225	−0.02	−0.01	+0.14	+0.01	+0.15

The lower line of the table is, it will be seen, only a repetition of the upper with the sign changed. We always have

$$f.R+f.(R+180^{\circ})=0$$

by the fundamental hypothesis of the investigation.

(63) *Values of g z.*—Taking the differences of the K's, we have eight distinct values of 8 *g.z*, the sum of which gives the value of 64 *g.z*, derived from the observation. The mean is as follows:

Z	gz
	″
0	+.01
15	+0.14
30	+0.07
45	−0.02
60	+0.08
75	+0.10
90	+0.06
105	+0.06
120	+0.02
135	−0.04
150	+0.02
165	−0.02

Though these values are quite well marked, indicating a twisting flexure coefficient of 0″.06, I am not at all satisfied of their reality, and have therefore preferred to dispense with their use, and derive the telescope flexure for each end of the axis directly from the observations.

(64) *Flexure of the Telescope.*—The preceding investigation gives the flexure of the circle divisions relatively to the central nucleus of the circle. We next wish to know the flexure of the line joining the micrometer wire, and the optical centre of the object glass relatively to the same nucleus. During the early part of the year 1866, I was greatly troubled by finding a constant difference of a large fraction of a second between the horizontal flexure determined from the opposing and that from the levelled collimators. It seemed to follow from this, that if the axes of the collimators were set optically in the same line, the difference of their level

errors would not be equal to their difference of latitude, as it should be. On April 17, 1866, this was tested directly, in the following way: The error both of level and collimation of each collimator was reduced as much as possible. The telescope was set vertically, the cube opened, and collimator A turned till its double wires were horizontal. The shutters were closed, as usual, in observing the collimators. Three observers were employed; one to read the level of each collimator, and one to make the images of the horizontal wires coincide. The latter looking into collimator B, set the wires opposite in each of the four combinations of position of the collimators, A 90°, B 180°; A 90°, B 0°; A 270°, B 0°; B 180°, and in each combination a set of level readings of each collimator was taken. To eliminate any possible personal error in setting, the observer then went to collimator A, and the operation was repeated. The result was as follows:

Mean level of A, (North,)	0″.87,	(S. end high.)
" " B, (South.)	0 .29,	" "
Difference - - -	+0″.58 S.	
Corr. for diff. of collars,	+0 .26	
Corr. for diff. of latitude,	+0 .25	
Sum - - - - -	1″.09	

This sum ought to be zero, so that there is a seeming discrepancy of 1″.09. That this is due to refraction, I entertain no doubt, for the following reasons: (1.) *A priori;* an increase of temperature amounting to 1° Fahrenheit in two feet, will entirely account for it; and the actual increase from the floor to the roof is found to exceed this on a sunny day. (2.) A few days afterward the observations were partly repeated with the shutters open, and a cold wind blowing through the room. The discrepancy was 0″.61 in the opposite direction. The images were quite unsteady, and the wind troublesome.

(65) The flexure by the opposing collimators was determined by setting the telescope on one collimator, and reading the telescope and microscope micrometers. The telescope was then pointed upward, and the horizontal wires of the other collimator set on those of the first. The circle reading was then determined for the other collimator, and the telescope again pointed to the zenith. The first collimator was then set independently on the second, and the two collimators were thus alternately set and read as often as was deemed advisable. The following are the separate results obtained on different dates:

1865. Dec.	16,	$f = +0''.15$;	
1866. Mar.	29,	+0 .83,	$f' = 0''.77$;
April	16,	+1 .42,	1 .30;
	26,	+0 .71, wt. =1;	
May	31,	+0 .89, wt. =2;	
June	9,	+0 .49, wt. =1.	

After the first determination the screws of the object end of the telescope tube were tightened. The two next were made without suspecting that the results might be vitiated by refraction, and therefore without attention to the equality of temperature in the different strata of air. They are therefore rejected. The last three were made with the shutters open, at times when the internal and external temperatures were nearly equal. That of May 31 was particularly satisfactory, and depends on four readings of one collimator, and three of the other, the separate readings being

N.	S.
48″.39	50″.77
47 .81	50 .87
48 .12	50 .09
47 .87	

The value of f, concluded from observations of the opposing collimators is

$$0''.75.$$

In the beginning of 1867 the object glass was taken out and cleaned. Conceiving a change in the elasticity of its bearings possible, a careful determination of f' was made on September 9, 1867. The circumstance taken advantage of to secure equality of temperature was a cold rain. Two thermometers were fastened to the stairway below the line from the object glass of the telescope to each collimator, and two more were suspended just under the roof. The upper pair indicated a higher temperature of 1°.2 before the observations, and 2°.2 afterward. The separate readings uncorrected for circle flexure were:

South coll.	North coll.
R′=90°	R′=270°
14″.69	15″.13
15. 10	15. 18
14. 97	15. 25
15. 43	15. 57
15. 51	

The readings were commenced on the south collimator. On looking into it to set it on the north one, preparatory to its second reading, the mean of wires were seen to differ quite sensibly from coincidence with the wire of the other collimator. The first reading is therefore regarded as doubtful. The result of these readings is

$$f'=+0''.78.$$

During the autumn of 1866 the levelled collimators were regularly observed at night with the shutters open, so that the mean result ought to be free from refraction. The result, from observations made by Messrs. Hall and Rogers and myself, was:

Mean excess of reading for S. collimator,	1″.86;
Uncorrected flexure coefficient - - - -	0 .93;
Correction for difference of latitude - -	−0 .12;
difference of pivots - - -	−0 .23;
circle of flexure - - - -	−0 .37;
Resulting coefficient - - - - - - - -	+0 .21.

o there is a discrepancy of more than half a second between the flexure coefficients found by the two methods. The error is probably in that determined from the levelled collimators, the conical character of their shoulders rendering their results uncertain.

The discrepancy is so great that I think it best to try also the method of comparison of direct and reflection observations.

(66) *Vertical Flexure.*—Thus far, the coefficient of cos Z has been found only by the method already set forth, namely, by comparison of the nadir reading obtained from observations of the collimators, and that obtained directly by coincidence of the wires with their images reflected from mercury. The observations were so conducted as to completely eliminate every constant error of the collimator itself, the following being the usual order:

(1) Nadir;
(2) Collimator B (north);
(3) Collimator A (south);
(4) Collimator B (south);
(5) Collimator A (north);
(6) Nadir.

If, now, either collimator be affected with any constant cause of error when on one side of the instrument, that cause will act in the opposite direction on the circle reading when the collimator is carried to the other side. By interchanging the collimators we therefore eliminate all constant errors peculiar to them.

The following are the separate results obtained in this way:

	C_0	C_0-g	g	Wt.	C'_0	C'_0-g'	g'.	Wt.
1865.	"	"	"		"	"	"	
Dec. 29	13.22	13.76	−0.54	2	16.35	15.96	+0.30	2
30	28.34	28.44	−0.10	2	23.61	23.61	0.00	2
1866.								
Jan. 30	22.34	22.48	−0.14	3	52.82	52.78	+0.04	3
Apr. 7	24.87	24.96	−0.09	3	14.32	14.28	+0.04	3
16	19.80	20.00	−0.20	3	12.43	12.32	+0.11	3
17	17.62	17.40	+0.22	3	16.50	16.66	−0.16	3
18	11.62	11.88	−0.26	3	16.44	16.17	+0.27	3

From which results

$$g=-0''.14,$$
$$g'=+0''.09.$$

There must always be a possibility of the nadir determinations being affected with undiscoverable sources of error, depending either upon the habits of the observer, or the disturbing conditions to which the instrument may be subjected, as, for example, the heat of the observer's body. I think it best, therefore, to depend for the final value of g upon the comparison of observations made in reversed positions of the instrument, the effect of the cosine flexure being reversed with the instrument. For the present, therefore, the quantity $-0''.14$ is regarded simply as the reduction of an observed nadir reading of circle A to the mean of the horizontal readings.

(67) In the flexure of the telescope is included the effect of gravity in changing the position of the declination micrometer slide relatively to the fixed plates of the eye-piece. As the telescope turns, the reading of the micrometer for coincidence of the fixed and movable wires is affected with the inequality

$$-0r.0376 \sin Z - 0r.0197 \cos Z,$$

Z being the zenith distance of the telescope counted in such a direction that sin Z is positive when the micrometer head is above the screw, and negative when below it.

The flexures already found being corrected for this inequality, the value of the sine coefficient would be quite small, while that of the cosine coefficient would be increased to $0''.44$.

(68) In observing the sun, the aperture of the telescope is diminished to about three inches by means of a cap weighing 5.3 ounces. It is found, by experiment, that this weight causes a flexure of $-0''.10 \sin Z$. A further flexure correction of

$$+0''.10 \sin Z$$

is therefore required in reducing observations of the sun.

ERRORS OF DIVISION.

(69) The readings for errors of division of every 5° were made during November and December, 1865, before the commencement of astronomical observations with the instrument. The observers, beside myself, were Professors Hall and Eastman, and aides Rogers and Thirion. Each observer read two microscopes. Any personal error in reading will appear only in the distance of the microscopes.

A few readings were first taken on the 0° and 90° divisions, with the microscopes 90° apart, to determine their angle. This, however, is of little importance, since any error, in its value, is eliminated in the mean of four microscopes, the number always read in astronomical observation.

To determine the error of every 45°, the microscopes were set 45° apart, and both circles were read thirteen times in each of the eight positions. Assuming the error of 0° to be zero, the following are the resulting values of 4ϵ, or the negative of four times the correction for error of division for every 45°:

	Circle A.	Circle B.
°	"	"
0	0.00	0.00
45	−1.19	−2.87
90	−1.12	−0.80
135	−0.67	−0.59

To find the errors of every 15°, two determinations were made: the first being made with the microscopes 60° apart, the second with the microscopes 75° apart. The circle was read five times in each position in each series. For circle A another and more exact series was made with microscopes 75° apart.

The first series could not give an independent determination of the 45° spaces, but the latter did, and small corrections were applied to them accordingly; not, however, with exact reference to the formulæ for weights already given. The following are the results:

	Circle A.				Circle B.		
	Mic. 60.	75° (1st.)	75° (2d.)	Concluded.	60°.	75°.	Concluded.
°	"	"	"	"	"	"	"
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	− 0.02	− 0.21	− 0.12	− 0.12	− 1.73	− 2.52	− 2.13
30	+ 0.14	+ 0.38	+ 0.36	+ 0.31	− 1.07	− 1.42	− 1.25
45	− 1.19	− 1.37	− 1.11	− 1.19	− 2.93	− 2.93	− 2.93
60	− 2.29	− 2.21	− 2.28	− 2.26	− 3.91	− 3.88	− 3.90
75	− 1.26	− 1.79	− 2.05	− 1.79	− 1.51	− 0.99	− 1.25
90	− 1.12	− 1.13	− 1.12	− 1.12	− 0.80	− 0.80	− 0.80
105	− 0.52	+ 0.03	− 0.02	− 0.13	− 1.11	− 1.07	− 1.09
120	− 1.14	− 2.19	− 1.11	− 1.39	− 1.75	− 2.32	− 2.03
135	− 0.67	− 0.76	− 0.62	− 0.67	− 0.43	− 0.43	− 0.43
150	− 0.60	− 1.43	− 2.22	− 1.62	− 0.94	− 1.43	− 1.18
165	+ 1.15	+ 1.15	+ 0.90	+ 1.02	− 0.13	− 0.25	− 0.19

For the errors of every 5° two series of readings were made; one with a distance of 50°, the other with one of 55°. Three readings were made in each position of the circle in each series, except the second series of circle B, when only two readings were made. The following are the separate results. The last column in each table gives one-fourth the negative of the mean by weights of the two preceding columns, and is the correction to be applied to the mean of opposite microscopes on account of errors of division:

Division.	Circle A.			Circle B		
	50°.	55°.	Concluded — ε.	50°.	55°.	Concluded — ε.
0°	0″.00	0″.00	0″.00	0″.00	0″.00	0″.00
5	+ 1.44	+ 0.20	− 0.22	− 0.86	− 1.06	+ 0.24
10	+ 1.55	+ 1.39	+ 0.37	− 0.99	− 0.76	+ 0.22
15	− 0.17	− 0.12	+ 0.04	− 2.06	− 2.13	+ 0.52
20	− 0.93	− 1.10	+ 0.26	− 1.96	− 1.91	+ 0.48
25	− 1.55	− 2.05	+ 0.45	− 1.88	− 2.02	+ 0.48
30	+ 0.34	+ 0.31	− 0.08	− 1.22	− 1.33	+ 0.32
35	− 0.64	− 0.96	+ 0.20	− 2.77	− 3.07	+ 0.72
40	− 1.41	− 0.83	+ 0.28	− 3.19	− 2.62	+ 0.74
45	− 1.19	− 1.19	+ 0.30	− 2.93	− 2.93	+ 0.73
50	− 1.09	− 0.02	+ 0.14	− 3.80	− 3.33	+ 0.90
55	− 1.08	− 2.29	+ 0.42	− 3.35	− 3.44	+ 0.85
60	− 2.22	− 2.18	+ 0.55	− 3.92	− 3.84	+ 0.97
65	− 2.20	− 2.32	+ 0.56	− 3.47	− 4.06	+ 0.93
70	− 0.88	− 0.36	+ 0.16	− 3.22	− 3.93	+ 0.88
75	− 1.79	− 1.70	+ 0.44	− 1.27	− 1.31	+ 0.32
80	+ 1.79	+ 2.25	− 0.51	− 0.18	− 0.96	+ 0.12
85	− 0.66	− 0.38	+ 0.13	+ 0.24	− 0.91	+ 0.06
90	− 1.12	− 1.12	+ 0.28	− 0.80	− 0.80	+ 0.20
95	− 2.05	− 1.78	+ 0.48	− 1.34	− 1.46	+ 0.35
100	− 2.24	− 1.36	+ 0.45	− 1.81	− 0.88	+ 0.36
105	− 0.22	− 0.13	+ 0.04	− 1.06	− 1.04	+ 0.26
110	+ 0.71	+ 0.09	− 0.10	+ 0.28	− 0.29	− 0.01
115	− 1.09	− 1.04	+ 0.27	− 1.32	− 1.30	+ 0.33
120	− 1.51	− 1.43	+ 0.36	− 2.07	− 1.96	+ 0.51
125	− 2.26	− 0.90	+ 0.40	− 1.10	− 1.81	+ 0.34
130	− 1.79	− 2.07	+ 0.48	− 0.84	− 0.78	+ 0.20
135	− 0.63	− 0.67	+ 0.16	− 0.43	− 0.43	+ 0.11
140	+ 1.48	+ 1.51	− 0.37	+ 0.56	− 0.03	− 0.08
145	− 0.27	− 0.08	+ 0.04	− 0.16	+ 0.10	+ 0.02
150	− 1.50	− 1.53	+ 0.38	− 1.30	− 1.21	+ 0.32
155	− 0.40	− 0.44	+ 0.10	− 1.19	− 1.41	+ 0.32
160	+ 1.61	+ 0.97	− 0.32	− 0.22	− 0.44	+ 0.08
165	+ 0.97	+ 1.09	− 0.26	− 0.18	− 0.20	+ 0.05
170	+ 0.89	+ 0.65	− 0.19	− 0.38	− 0.11	+ 0.07
175	+ 0.87	+ 0.51	− 0.17	− 0.28	− 0.78	+ 0.12

We have here two entirely independent determinations of each division error, except those which are multiples of 15°. The mean difference between the two values of 4ϵ is 0″.52 for circle A, and 0″.41 for circle B. The difference of accuracy between the circles arises from differences in the eye-sight of the observers. It appears then that the probable value of a concluded ϵ is about 0″.065 for circle A, and 0″.051 for circle B. The probable error of the mean reading of four microscopes, when these divisions are under them, will therefore be about 0″.046 for circle A, and 0″.036 for circle B, a quantity smaller than the probable accidental error of the isolated divisions. Little advantage would therefore be gained by making the determination more exact.

(70) When the mean of divisions, 90° apart, is taken, the progression of the errors in the case of circle B are so regular that a determination of the intermediate divisions was supposed to be hardly necessary. But, as circle A was used for all the observations in 1866, it was thought desirable to determine at least some intermediate points on that circle. Accordingly, the microscopes of that circle were placed at the distance 43°20′, and one complete series of readings were made, which gave the error of every 10°40′. *The result showed that the errors of the intermediate divisions were systematically greater than those of the 5° divisions, the mean difference being +0″.39.*

This was supposed to indicate a cyclical inequality in the error, the period of which was 5°. In order to determine it accurately, it was necessary to determine the error of every degree of both circles. This was done on September 17–19, 1866. The microscopes of each circle were placed at the distance 48°, and the circles were both read once in each position Again the result was anomalous, and the cyclic hypothesis had to be modified or abandoned. The systematic mean difference between the 5° divisions and the intermediate even degrees was only 0″.14 for circle A, and 0″.08 for circle B.

A last attempt to discover the general law of the inequality without the laborious operation of determining the error of every 10′ was made in January, 1867. The 5° interval was divided into six parts by setting the microscopes of circle B 44° 10′ apart, and this circle was read once in each position. The cycle now seemed to be reduced to 30′.

To show the nature of the law the correction for the mean of four microscopes, as it resulted from the above determination, is shown for each circle in the following table:

CIRCLE A.

A	Correction for error of division of—						
	A	A+ 1°	A+ 1° 40′	A+ 2°	A+ 3°	A+ 3° 20′	A+ 4°
°	″	″	″	″	″	″	″
0	+0.14	+0.08	−0.34	−0.10	−0.05	−0.24	+0.07
5	+0.13	+0.10	−0.29	−0.33	−0.40	−0.12	−0.22
10	+0.04	−0.24	−0.56	+0.25	−0.22	−0.23	−0.41
15	+0.04	+0.15	+0.02	+0.06	−0.18	−0.12	+0.24
20	+0.08	−0.05	−0.33	+0.14	+0.17	+0.15	+0.15
25	+0.35	−0.17	−0.64	−0.08	−0.16	−0.18	+0.08
30	+0.14	+0.18	−0.14	+0.07	−0.25	−0.08	+0.10
35	+0.30	+0.18	−0.14	+0.24	+0.18	−0.08	+0.10
40	+0.38	+0.26	−0.02	+0.15	+0.35	+0.02	+0.22
45	+0.23	+0.06	−0.44	+0.10	−0.04	−0.24	−0.25
50	−0.12	−0.13	−0.45	+0.02	+0.02	−0.30	+0.18
55	+0.23	[illegible].04	−0.34	+0.15	+0.12	−0.18	+0.30
60	−0.46	+0.52	−0.07	+0.18	+0.20	+0.10	+0.08
65	+0.33	+0.15	−0.12	+0.26	0.00	+0.04	−0.05
70	−0.08	−0.08	−0.76	−0.18	−0.10	−0.78	−0.16
75	+0.02	−0.20	−0.42	−0.29	−0.44	−0.46	−0.18
80	−0.35	0.28	−0.58	−0.40	−0.43	−0.42	−0.29
85	−0.02	−0.12	−0.39	−0.28	−0.07	−0.25	+0.35
Mean . .	+0.13	+0.02	−0.33	0.00	−0.08	−0.19	+0.02
M.−0.″13	0.00	−0.15	−0.46	−0.13	−0.21	−0.32	−0.11

CIRCLE B.

A	Correction for error of division of—									
	A	A+ 0° 50′	A+ 1°	A+ 1° 40′	A+ 2°	A+ 2° 30′	A+ 3°	A+ 3° 20′	A+ 4°	A+ 4° 10′
°	″	″	″	″	″	″	″	″	″	″
0	+0.10	−0.29	0.10	−0.05	0.12	−0.09	0.11	−0.05	0.06	−0.28
5	.30	.04	− .01	.21	− .06	.08	.25	− .06	.18	− .01
10	.29	− .09	.14	.00	.16	.26	.37	.24	.04	− .05
15	.39	− .01	.32	.34	.31	.40	.10	.14	.39	.14
20	.24	.22	.22	.10	.20	.31	.28	.34	− .09	.01
25	.40	.20	.24	.06	.22	.25	.52	.34	.15	.04
30	.42	.16	.59	.36	.33	.41	.26	.54	.58	.21
35	.53	.24	.37	.46	.46	.31	.38	.39	.04	.16
40	.47	.20	.47	.30	.40	.61	.45	.52	.43	.11
45	.42	.08	.28	.01	.03	.19	.10	.20	.10	.20
50	.41	.30	.25	.16	.22	.25	.21	.19	.22	.15
55	.44	.18	.39	− .04	.35	.24	.15	.18	.45	.29
60	.64	.68	.82	.51	.56	.64	.42	.54	.30	.26
65	.62	.40	.44	.42	.42	.60	.52	.61	.64	.29
70	.48	.25	.53	.36	.28	.39	.58	.26	.27	.44
75	.18	.20	.42	.05	.22	.09	.26	.11	.25	− .04
80	.10	− .01	− .04	− .08	.40	.20	.15	.22	.22	− .04
85	0.09	−0.42	0.02	−0.28	−0.02	−0.29	−0.05	0.01	−0.16	−0.28
Mean . .	0.″36	0.″13	0.″31	0.″16	0.″26	0.″27	0.″28	0.″26	0.″23	0.″09
M.−0.″36	0.00	−0.23	−0.05	−0.20	−0.10	−0.09	−0.08	−0.10	−0.13	−0.27

The systematic irregularity is well marked in this table, and seems to follow the same law in the two circles, except that it is nearly twice as great for A as for B. Two important questions now present themselves respecting the nature of the law.

1. Do the intermediate errors really depend upon those of the 5° spaces on each side of them more than on any other part of the circle? If not, the attempt to determine the errors with precision might as well be abandoned. Inspection of the above tables, however, shows that for circle B, at least, the question is to be answered in the affirmative; the corrections of the intermediate divisions increase and diminish with those of the 5° ones, and to about the same extent with the latter.

2. Do the systematic errors of the 5° spaces peculiar to them affect them alone, or is the law of error continuous? For example, are the systematic errors of the divisions 4° 58′ and 5° 2′ the same as those of 5°, or are they the same as those of 4° 10′ and 5° 50′? If the former, the spaces on each side of the 5° divisions will be equal; if the latter, they will differ by half a second. Twenty pairs of intervals of circle B—those adjacent to every 15° from 0° to 135°, and from 180° to 315°—were measured and compared, and the mean difference found to be 0″.07. The law of periodic error, whatever it may be, is therefore continuous. It seems result, in great part at least, from two cycles in the errors of division; the one having a period of 5°, the other a period of 30′.

(71) The course which it seems best to adopt is this: instead of attempting to determine the errors of division with the last degree of precision, we shall seek to eliminate them by changing the position of the circles from year to year, so that the position of any one star will depend on different divisions in different years. If the periodic errors be entirely neglected, the effect of their probable amount, at least in the case of circle B, will be less than 0″.1, and the total probable effect of the difference between the actual and the adopted error of any isolated error of division will not much exceed that amount. The uncertainty of the mean of four divisions will, therefore, scarcely exceed the probable error of the declinations of fundamental stars derived from all the observations hitherto made.

(72) To obtain a general table of the corrections of the divisions, we have first corrected the determinations given in the preceding tables for periodic error, so as to take the mean of the entire degree divisions, those which are a multiple of 5 excepted, as the standard. This has been effected by applying the following corrections to the different vertical columns:

Column.	Circle A.	Circle B.
° ′	″	″
A	−0.14	−0.19
A+0 50	.....	+ .14
A+1	.00	.00
A+1 40	+ .32	+ .11
A+2	.00	.00
A+2 30	.....	.00
A+3	.00	.00
A+3 20	+ .16	.00
A+4	.00	.00
A+4 10	.....	+ .18

The corrections are thus reduced to what they would have been had there been no periodic error, and arranged consecutively in a table. The mean difference between consecutive numbers was now found to be 0″.150 for circle A, and 0″.135 for circle B, indicating a probable error of each individual determination, combined with the accidental error of division, of less than 0″.1.

Table of corrections to mean of four microscopes for errors of division. Argument, reading of horizontal microscope.

Arg.	Circle A.	Circle B.	Arg.	Circle A.	Circle B.
°	"	"	°	"	"
0	+0.12	+0.30	45	+0.02	−0.01
1	+ .06	.23	46	.00	.00
2	.00	.18	47	− .02	.00
3	− .05	.18	48	− .04	.00
4	− .11	.22	49	− .04	+ .06
5	− .17	.27	50	− .04	.12
6	− .10	.27	51	− .08	.12
7	− .02	.26	52	− .22	.12
8	.00	.26	53	− .20	.13
9	+ .04	.27	54	− .16	.14
10	.07	.29	55	− .12	.15
11	.08	.20	56	− .12	.16
12	.10	.29	57	− .12	.17
13	.13	.36	58	− .12	.19
14	.20	.44	59	− .12	.20
15	.28	.52	60	− .05	.22
16	.27	.56	61	+ .05	.26
17	.24	.57	62	.05	.28
18	.23	.57	63	.05	.27
19	.20	.52	64	.02	.25
20	.18	.48	65	.00	.22
21	.17	.49	66	.02	.22
22	.16	.52	67	.09	.22
23	+ .10	.53	68	.15	.22
24	− .02	.50	69	.14	.23
25	− .14	.46	70	+ .08	.24
26	− .18	.44	71	− .03	.25
27	− .22	.42	72	− .12	.27
28	− .24	.40	73	− .12	.28
29	− .20	.34	74	− .05	.31
30	− .16	.27	75	+ .04	.35
31	− .23	.24	76	.04	.40
32	− .29	.22	77	.02	.45
33	− .32	.19	78	.00	.46
34	− .35	.15	79	.01	.45
35	− .37	.11	80	.12	.44
36	− .36	.12	81	.16	.41
37	− .36	.15	82	.17	.38
38	− .36	.14	83	.18	.36
39	− .29	.08	84	.19	.36
40	− .22	+ .02	85	.20	.36
41	− .17	− .07	86	.23	.40
42	− .12	− .12	87	.25	.44
43	− .06	− .13	88	.25	.44
44	− .02	− .07	89	.19	.37
45	+0.02	−0.01	90	+0.12	+0.30

The corrections thus obtained were now made continuous, and the above table was formed in the following way. Represent the correction for y° by (Y). Then, for circle A was taken

$$[1\tfrac{2}{3}]=\tfrac{1}{3}\{(1)+(1\tfrac{2}{3})+(2)\},$$
$$[3\tfrac{1}{3}]=\tfrac{1}{3}\{(3)+(3\tfrac{1}{3})+(4)\},$$
&c., &c.
$$[0]=\tfrac{1}{3}\{[88\tfrac{1}{3}]+(0)+[1\tfrac{2}{3}]\},$$
$$[5]=\tfrac{1}{3}\{[3\tfrac{1}{3}]+(5)+[6\tfrac{2}{3}]\},$$
&c., &c.
$$[2\tfrac{1}{2}]=\tfrac{1}{2}\{(2)+(3)\},$$
$$[7\tfrac{1}{2}]=\tfrac{1}{2}\{(7)+(8)\},$$
&c., &c.
$$[[2\tfrac{1}{2}]]=\tfrac{1}{3}\{[1\tfrac{2}{3}]+[2\tfrac{1}{2}]+[3\tfrac{1}{3}]\},$$
$$[[7\tfrac{1}{2}]]=\tfrac{1}{3}\{[6\tfrac{2}{3}]+[7\tfrac{1}{2}]+[8\tfrac{1}{3}]\},$$
&c. &c.

The concluded corrections were then interpolated between [0], [[2½]], [5], [[7½]], &c.

For circle B was taken

$$[1\tfrac{1}{4}]=\tfrac{1}{4}\{ (\tfrac{1}{2})+(1)+(1\tfrac{1}{2})+(2) \},$$
$$[3\tfrac{3}{4}]=\tfrac{1}{4}\{ (3)+(3\tfrac{1}{2})+(4)+(4\tfrac{1}{2}) \},$$
$$\text{\&c.,} \qquad \text{\&c.}$$

$$[0]=\tfrac{1}{3}\{ [88\tfrac{3}{4}]+(0)+[1\tfrac{1}{4}] \},$$
$$[5]=\tfrac{1}{3}\{ [3\tfrac{3}{4}]+(5)+[6\tfrac{1}{4}] \},$$
$$\text{\&c.,} \qquad \text{\&c.}$$

$$[2\tfrac{1}{2}]=\tfrac{1}{9}\{ 4[1\tfrac{1}{4}]+(2\tfrac{1}{2})+4[3\tfrac{3}{4}] \},$$
$$[7\tfrac{1}{2}]=\tfrac{1}{9}\{ 4[6\tfrac{1}{4}]+(7\tfrac{1}{2})+4[8\tfrac{3}{4}] \},$$
$$\text{\&c.,} \qquad \text{\&c.}$$

The concluded corrections were then interpolated between [0], [2½], [5], [7½], &c.

In the table the argument is changed 45°, so as to correspond to the reading of the finding microscope.

ERRORS OF CERTAIN ISOLATED DIVISIONS.

(73) During the year 1866 the circle was so set that when the telescope pointed toward the nadir, the reading of the finding microscope was 359° 56′. It therefore becomes necessary to determine the error of the particular divisions then under the microscopes, relatively to the others. For this purpose microscope VII was furnished by the machinist of the Observatory with two extra pair of spider lines at a distance of, as nearly as possible, 2′ on each side of the central pair. Each division, from 44° 42′ to 45° 8′, was then brought in succession under the middle pair of wires, and at each setting the three pairs were placed in succession over their corresponding divisions. Thus, two measures of each space were obtained. These measures, being treated in the way already set forth, gave the following corrections for each division relatively to the mean of the fourteen divisions from 44° 42′ to 45° 8′, 134° 42′ to 135° 8′, &c.:

Div.		Cor.	Div.		Cor.	Div.		Cor.	Div.		Cor.	Mean.
°	′	″	°	′	″	°	′	″	°	′	″	″
44	42	+0.25	134	42	+0.27	224	42	−0.01	314	42	+0.12	+0.16
	44	+ .19		44	+ .07		44	− .09		44	− .09	+0.02
	46	− .09		46	− .31		46	− .23		46	− .11	−0.18
	48	− .05		48	− .01		48	+ .03		48	+ .01	0.00
	50	− .05		50	+ .03		50	− .07		50	− .23	−0.08
	52	+ .04		52	− .04		52	+ .10		52	+ .22	+0.08
	54	− .12		54	− .09		54	.00		54	− .26	−0.12
	56	− .05		56	− .13		56	+ .09		56	− .08	−0.04
	58	+ .35		58	− .21		58	+ .40		58	+ .09	+0.16
45	0	− .03	135	0	+ .21	225	0	+ .04	315	0	+ .10	+0.08
	2	− .02		2	+ .43		2	− .01		2	− .08	+0.08
	4	− .25		4	− .03		4	− .11		4	− .14	−0.13
	6	− .14		6	− .19		6	+ .04		6	+ .22	−0.02
	8	+ .19		8	− .01		8	− .20		8	+0.16	+0.04

TESTS FOR OTHER POSSIBLE ERRORS.

(74) The errors of an instrument may be divided into two classes; those which we expect to find, determine, and allow for in the reduction of observations, and those we expect the

artist to avoid entirely, or at least render insensible. The classification is somewhat arbitrary, depending, as it does, upon the degree of precision sought by the astronomer, and the degree of excellence attained by the artist; yet the custom of astronomers has rendered it quite definite. The errors we have investigated are generally recognized as of the first class; we shall now consider those of the second.

(75) *Irregularity of Pivots.*—No apparatus for determining directly the influence of possible irregularity of pivots upon the axis of rotation was furnished with the instrument, but the artists did furnish an extremely delicate instrument for determining any difference of diameters of the same pivot. It consists of a pair of calipers, which are screwed upon one of the Y bearers, and grasp the horizontal diameter of the pivot. The telescope being turned, any change in the distance of the calipers amounting to the two hundred thousandth of an inch will be rendered sensible by a pair of multiplying levers, the end of the last of which moves over a divided scale. The telescope being turned through an entire revolution with the calipers on one pivot, no difference of diameters so great as this was detected. Only one pivot was thus tested.

As an additional test, the hanging level was placed upon the pivots, and read at every 20° of zenith distance of telescope, from 20° to 160°; the telescope being moved by one of the west handles. It was then returned by the same handle, and the readings repeated. The effect of the pressure of the handle in changing the level of the pivot was quite sensible, amounting to 0″.25; but this effect was reversed by the backward motion, and the extreme range of the mean level reading for different positions of the telescope was 0″.05. It was concluded that the form of the pivots might be regarded as perfect.

(76) Another possible source of irregularities in the motion of the optical axis of the telescope at first caused me considerable solicitude. It has been seen in the description that the fulcrum of the levers of the great counterpoises are not, as I conceive they should be in so large an instrument, knife edges, but pivots. Owing to the unavoidable friction of these pivots, and also to the friction of the end springs, the instrument, when balanced by the counterpoises, will allow changes of weight of perhaps eight or ten pounds without causing motion of the lever. Consequently, the division of the weight between the friction rollers and the pivots will be uncertain to this amount. If, now, there be any irregularity in the grooves of the axis by which the friction rollers act on the instrument, this division may vary in different positions of the instrument, the friction roller acting more powerfully on points more distant from the centre of rotation. And such a change of pressure will produce a vertical flexure of the axis, which will change the direction of the optical axis of the telescope. To avoid any possible error from this source, small pieces of rubber cloth were inserted in the sustaining sockets on the levers, the elasticity of which would take up any minute irregularities of the kind referred to, and make the pressure nearly constant.

To test the effectiveness of this contrivance, a piece of thin paper, probably $\frac{1}{400}$ of an inch thick, was drawn under the friction rollers, and the effect upon the level of the axis and the verticality of the telescope was noted. The former was not changed at all. The latter, which was determined by comparing the position of the vertical middle wire, and its image reflected from mercury, did not admit of exact measurement, as the image was nearly hidden by the wire. Certainly, however, there was no change as great as 0″.3. The paper being many times as thick as any probable irregularity in turning the axis, there is little danger of error from the source in question.

(77) Part of the same general investigation was the determination of the effect on the level of the axis when the action of the counterpoise was changed from its maximum to its minimum amount. One pivot being raised from its Y, by pressing on the counterpoise, was gently let down again, and the level carefully noted. It was then pressed downward by pressing the

counterpoise upward, and the level again noted. The changes of reading varied from 0″.3 to 1″.0. They are, I conceive, almost entirely due to flexure of the Y's under the weights of the pivots.

(78) *Coincidence of the Divided Faces of the Circles with Planes perpendicular to the Axis of Rotation.*—There is no error in this respect which affects the definition of the divisions in the fields of the microscope, and I have made such measures as to satisfy myself that no appreciable error arises from the product tan A δx, p. 11–12.

(79) *General Remark.*—Beside the above systematic examinations, the instrument is from time to time in an irregular way examined for every cause which I can think of, as liable to vitiate the results of observations. Nothing serious has yet been detected.

PART IV.

REMARKS ON THE PERFORMANCE AND USE OF THE TRANSIT CIRCLE.

(80) The general design of the instrument is entirely that of the makers. Specific directions were sent them only on a few minor points, such as the arrangement of the micrometers, the self-registering micrometer head, and the wires of the reticule. It was the opinion of Captain Gilliss, that, considering the reputation and experience of the artists, he would be more likely to secure a good instrument by allowing them to carry out their own views, and holding them responsible for the performance of the instrument, than by designing it himself.

As a general remark, it may be said that the mechanical execution of every part of the instrument is of the first order of excellence. I cannot speak with certainty of the object glass, as it has not been severely tested. Certainly, however, it has no defect which interferes with the performance of the instrument. After being transported by land and water a fourth of the way round the globe, all the delicate and complicated parts of the instrument were put together without impediment or delay, and immediately went into successful operation.

As a knowledge of the defects in design and performance, which we have thus far succeeded in discovering, may be valuable to astronomers, I shall set them forth.

The only defects of design which can yet be pronounced upon with certainty have already been alluded to. They are:

1. Making the zenith distance micrometer carry the slides of the ocular, thus causing the screw to carry too much weight when the head points downwards.
2. The form of the supporting fulcrums of the counterpoise levers.
3. The instability of the collimators and their levels, owing to the small distance (22 inches) between the supporting shoulders. The only inconvenience which results from this construction is the increased labor of levelling the collimator.

(81) *Stability of the Instrument.*—Invariableness of instrumental constants is generally considered one of the most desirable qualities in a meridian instrument. A deficiency in this respect, however, need not vitiate the results of observations, if only the astronomer can determine and apply the constants with a frequency proportioned to the instability of his instrument.

The most precise way to measure and indicate the variableness or uncertainty of the instrumental constants is to take the mean difference between consecutive determinations of the constants. This I have done for the latter part of the year 1866, and the results are given in the following table. During the summer of 1867 the stability of the zenith point has decidedly improved, the mean difference being reduced to 0''.60.

Collimation, October, 1866, to July, 1867	interval 1 week	0''.22
Level, July to November, 1866	interval 1 to 4 days	0''.57
Level on consecutive days	interval 1 day	0''.29
Mean difference of consecutive transit of Polaris	interval 1 day	3s. 2
Resulting mean change of azimuth	interval 1 day	1''. 5
Zenith point	interval 3 to 12 hours	0''. 7
Inclination of E. pier	interval 1 day	0''.65

If ϵ be the mean error of the determination itself, we may expect a mean difference of $\sqrt{2}\epsilon$ owing to that error alone, supposing the instrument to remain invariable. The stability of the line of collimation may therefore be regarded as perfect, and that of the level error practically so, if determined for each day of observation.

It is far otherwise with the zenith point and the azimuth. Not only are they variable to an annoying degree, but the causes of the variations are not definitely determined. Some light may, however, be thrown upon them.

(82) *Zenith Point.*—Previous to any trial of the instrument, its most objectionable feature seemed to be the mode of mounting the microscopes, and many astronomers would have predicted instability from this cause. But the relative positions of the microscopes have proved unexpectedly steady. The following table shows the amount by which the line through the zero of V and VII was in excess of 90° from that through VI and VIII, at various dates between June 29 and September 17, 1866, the longest interval as yet in which the microscopes have been subjected to no disturbance. They are formed by subtracting the mean reading of V and VII for the two collimators, from that of VI and VIII, and are therefore the difference between the nadir points of the circle as given by the two pairs. The dates are taken at random, except with reference to the observer:

Date.	Δ.
1866. June 29,	+0.05″;
29 9,	+0.02;
July 2,	−0.12;
..	−0.08;
9,	−0.05:
12,	−0.32;
26,	−0.10;
30,	+0.30;
Aug. 6,	+0.32;
15,	−0.20;
20,	−0 28;
29,	+0.15;
Sept. 3,	−0.15;
10,	+0.12;
14,	−0.15.

I am persuaded that this degree of steadiness has never been exceeded, so that if the central core has been set in the pier in such a way as to secure immobility, the positions of the microscopes relative to the pier will be as invariable in this mounting as in any other.

Passing from the microscopes to the circle—the nicety of fit and firmness of connection of every part, from the divisions of the circle to the ends of the tube of the telescope, is beyond reasonable doubt.

The constancy of position of the optical axis of the tube is rendered highly probable by the steadiness of the error of coiiimation. Moreover, the object end of the telescope has been subjected to shocks several times greater than it ever receives in ordinary use, without any effect upon the nadir point.

The constancy of the reading of the zenith-distance micrometer head for a given position of the wires is all that could be desired.

(83) Supposing, from these considerations, that the changes observed must be due to movements of the pier itself, a horizontal cylinder was fastened to it in July, admitting of being levelled by one of the collimator levels and the changes in the inclination of the pier thus determined. But the changes of nadir point were still only partially accounted for, and the correct nadir reading sometimes exhibits a progressive change, continuing through a period of several days.

(84) Apparently, the only untested link in the chain is the stability of the setting of the microscope holder (D, plate IV, D–P, plate VI, Fig. 2) into the pier. To insure the solidity of this setting, a hole was cut from the top of the pier to the perforation which received the core of the holder, and the plaster poured in until the hole was full, when it oozed out on all sides of the core. The setting is, therefore, as solid as it can be with plaster. It will, indeed, yield, and allow the microscopes to turn by a strong pressure of the hand; but under pressures several times as great as they are ever subject to in observing, the microscopes are not disturbed at all. Still, considering the known hygrometric qualities of plaster; considering, also, that the relative readings of the microscopes on the two piers are subject to changes of the same general character and magnitude with the zenith point, I decidedly think that the greater part of the instability of the zenith point is due to the want of firmness of the plaster setting.

(85) *The Azimuthal Error.*—Of the cause of the variation of azimuth I entertain little doubt. The extreme breadth of the masonry on which the piers are supported is only four and a half feet from north to south. With so narrow a base injurious changes of inclination of the piers, from motion of the ground and consequent tipping of the masonry, seem to me unavoidable. And that such changes do take place is shown conclusively by the levelling apparatus attached to the piers. If every part of the masonry tipped equally, the nadir point alone, and not the azimuth, would be affected. But since the masonry is not perfectly rigid, this condition would be fulfilled only by the ground giving way equally at each end of the pier, which we have no reason to suppose the case. The piers tipping unequally, we may look for changes of azimuth as well as of nadir point. The height of the axis above the centre of the pier being three times the distance of pivots, the change of azimuth from the cause in question will be three times the change of relative inclination of the piers.

(86) In the spring of 1867 a levelling cylinder was attached to the west pier also, and the difference of tipping of the piers compared with the changes of azimuth. The latter were not accounted for, a fact which may be attributable to the imperfections of the apparatus itself.

(87) *Dependence of the Collimation Error on Temperature.*—Toward the end of the year 1866 the amount of this error was found to be dependent on the temperature, varying 0″.05 for every degree of Fahrenheit. The cause was discovered when the object glass was taken out at the end of the year. It was then found that the glass was held in its cell by the pressure of three chucks, 120° apart, two of them being fixed, and the third pressed in by a strong spring. The direction of action of the spring was horizontal. Hence, owing to the different expansibilities of the brass and glass, the centre of the latter would take different positions relative to the centre of the former at different temperatures. The calculated change of collimation on the hypothesis of perfect rigidity of both object glass and cell, is 0″.07. The difference between this and the observed change is probably due to the fact that the spring is not perfectly flexible, nor the glass and brass perfectly rigid.

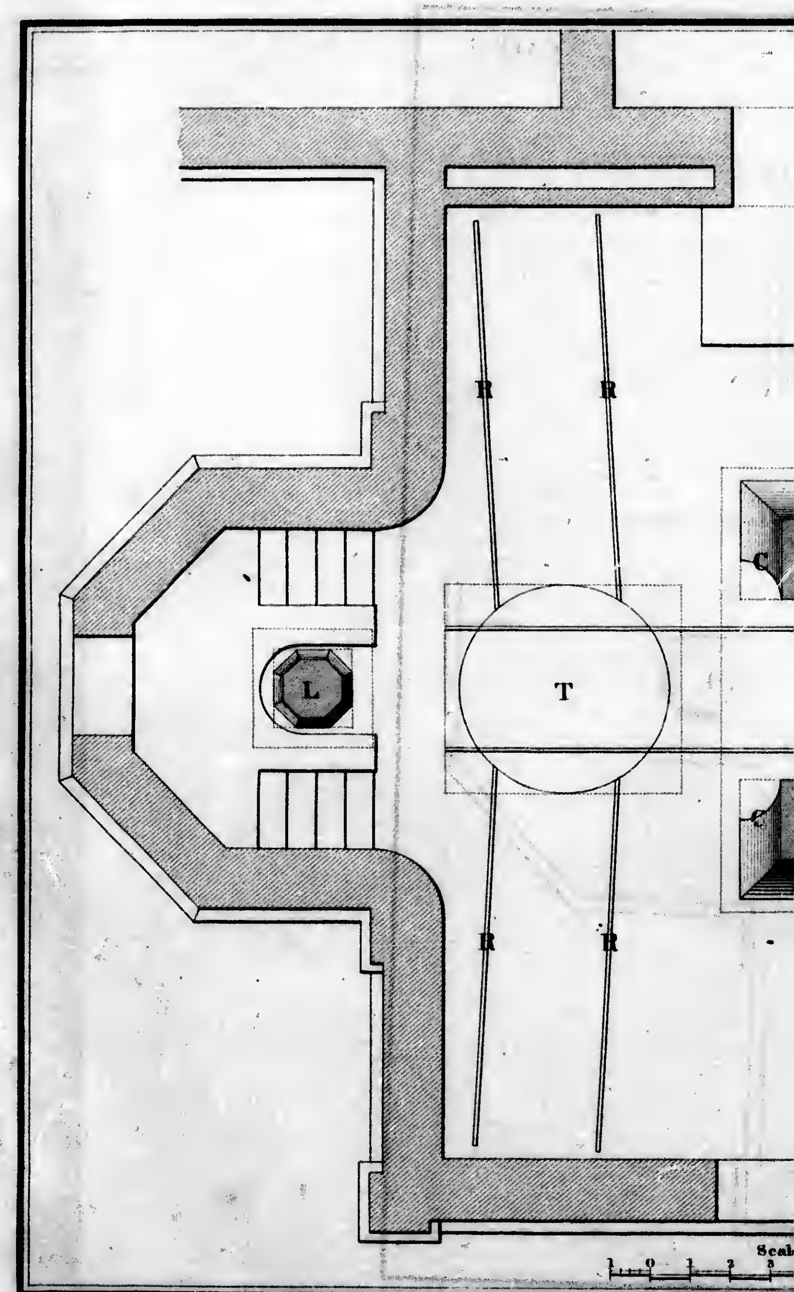
(88) It is probable that such changes will ultimately be made in the mounting of the object glass, the microscope holders, and the great piers as may seem sufficient and necessary to secure greater immobility of the collimation, azimuth, and zenith point, the instability of the two latter arising, as has been seen from defects of mounting rather than of construction.

[illegible]

ALPHABETICAL INDEX.

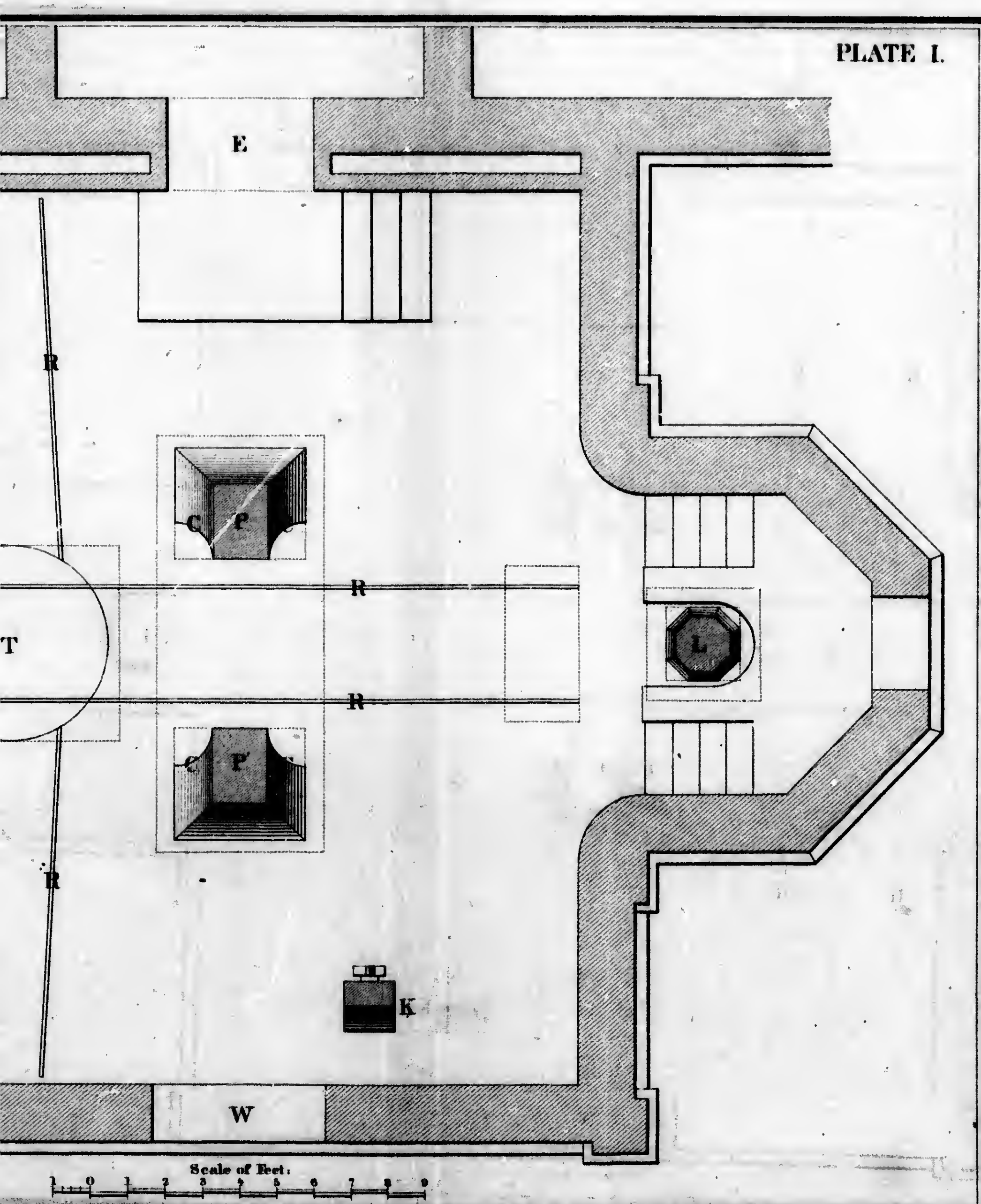
(THE NUMBERS REFER TO THE PARAGRAPHS.)

7

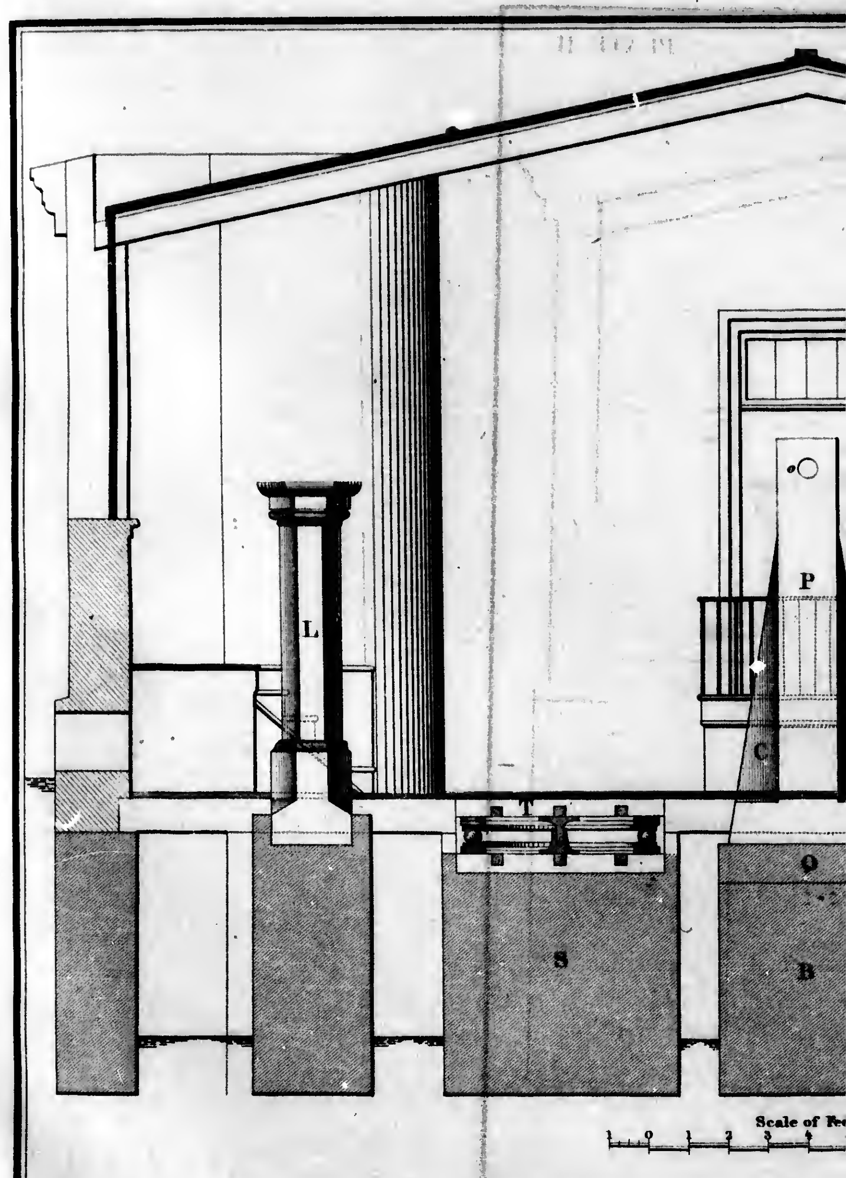


W. Baumann del.

PLATE I.



J.C. Kondrup Eng.[d]



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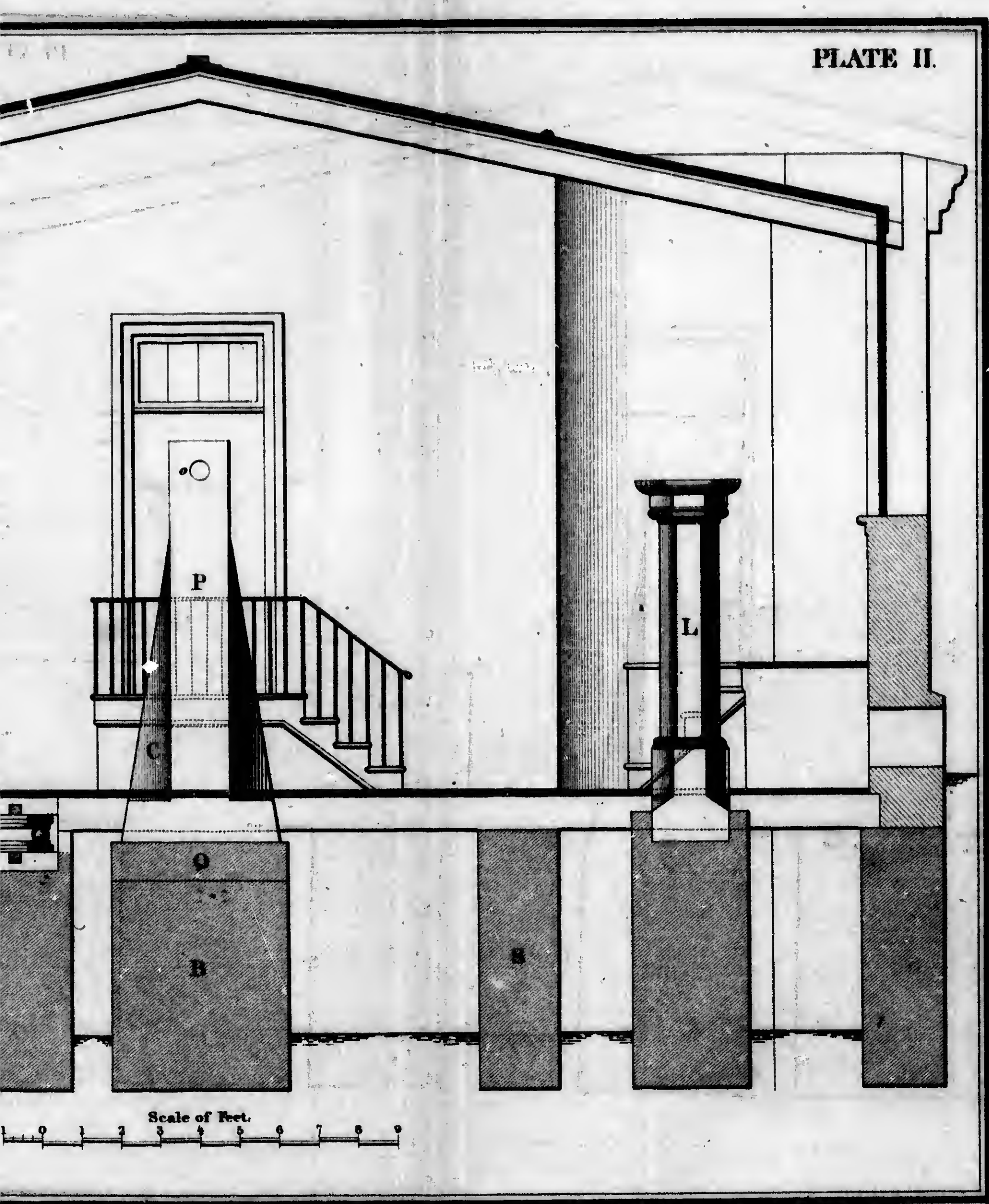
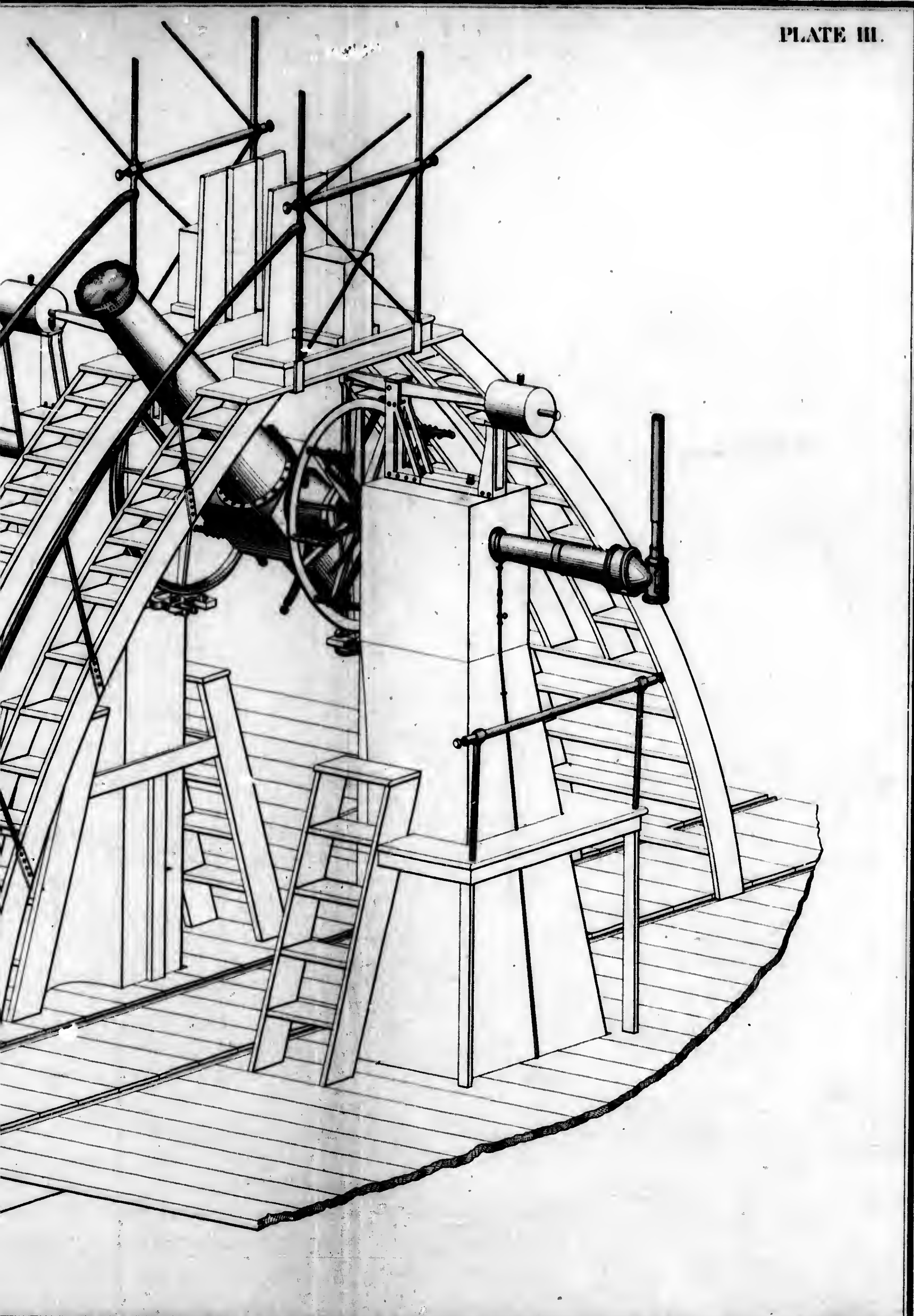


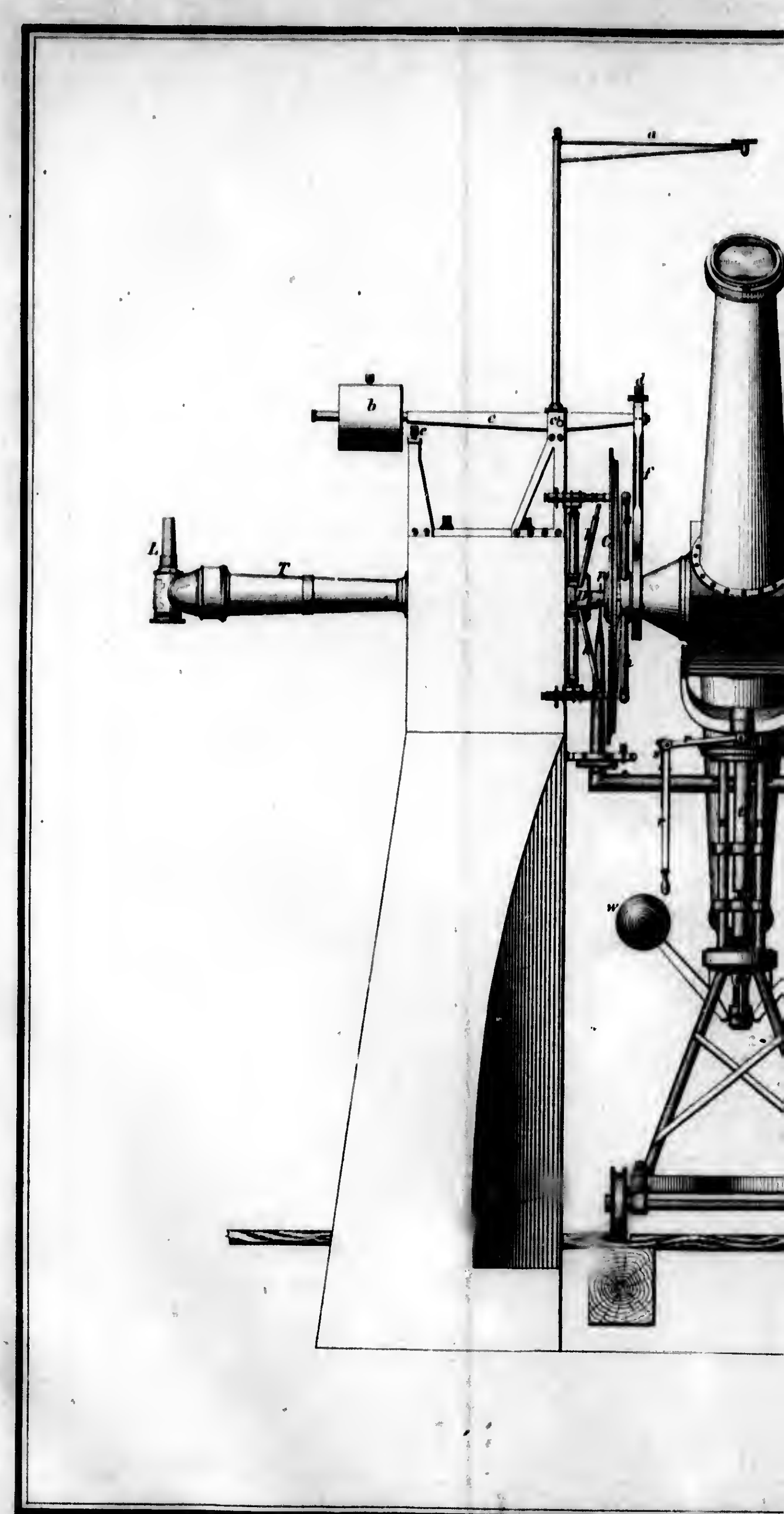
PLATE II.
P
L
C
O
B
S
Scale of Feet.
0 1 2 3 4 5 6 7 8 9
J.C.Kondrup Engr.

W.Baumann del.

PLATE III.

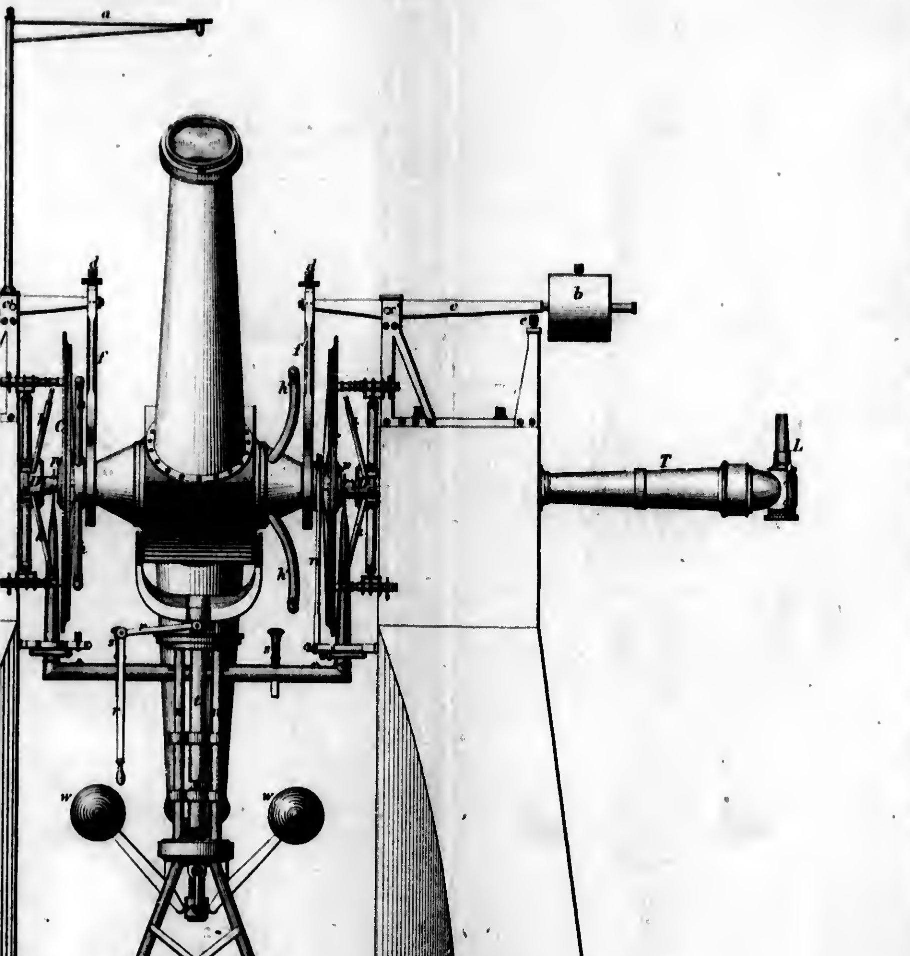


J.C.Kondrup Engr.

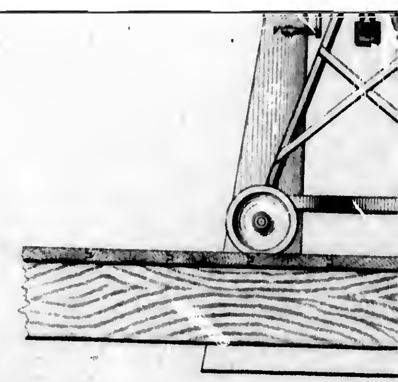


W. Baumann del.

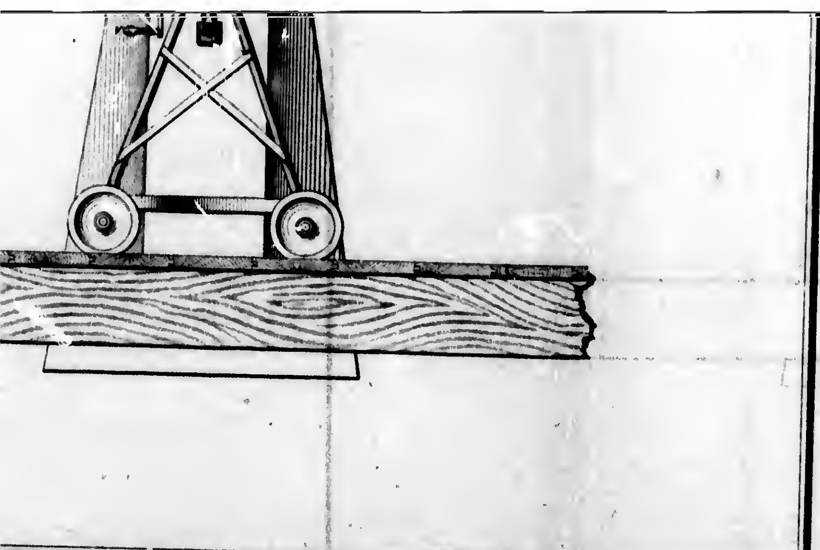
PLATE IV.



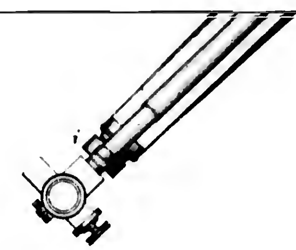
J.C.Kondrup Eng.r

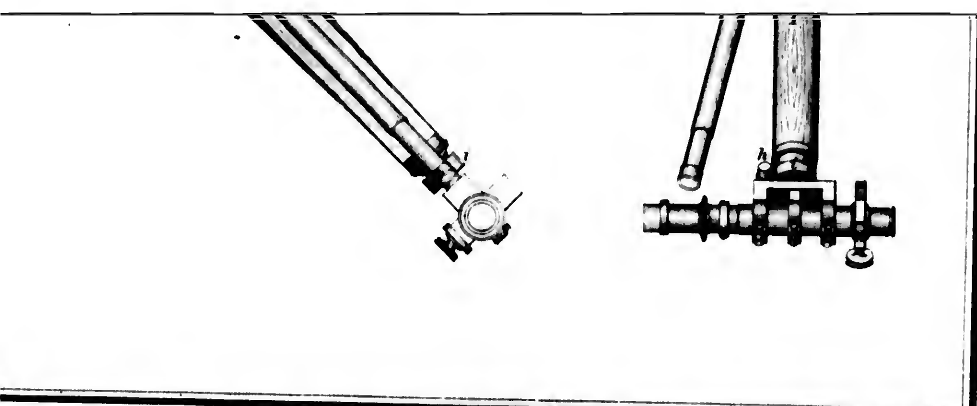


W.Baumann del.



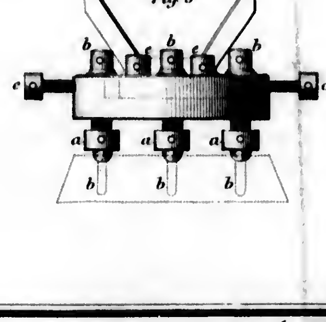
J.C.Koudrup Eng.r





Bowen & C° lith. Philada.

Fig 8



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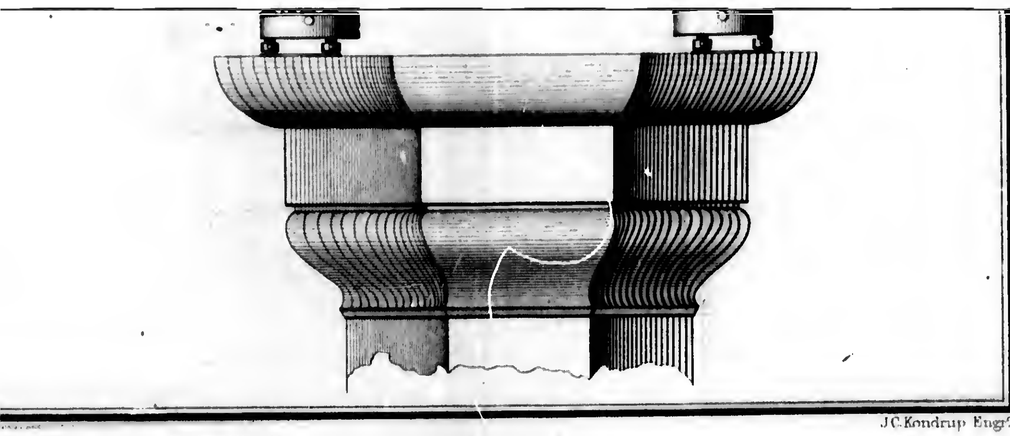

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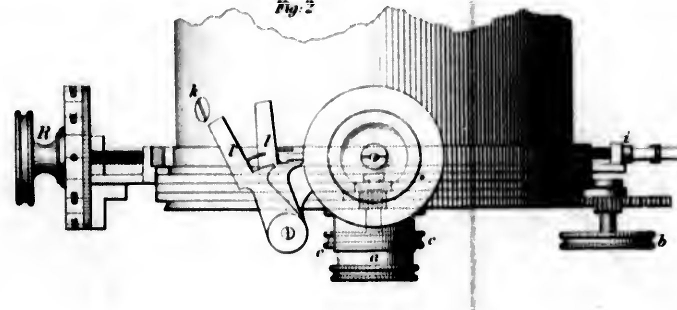
Fig: 2



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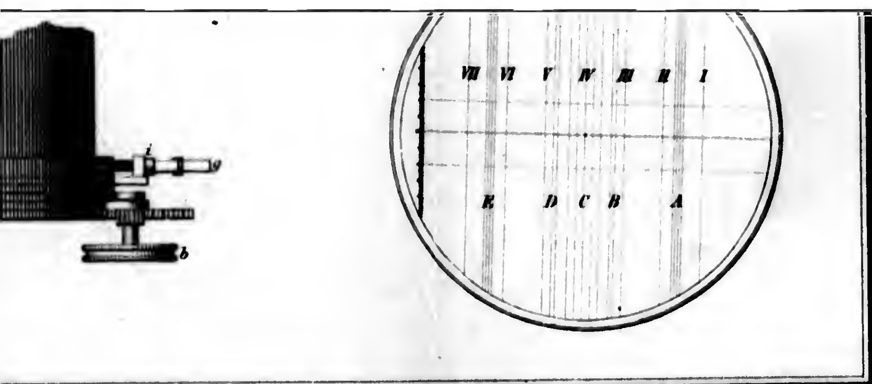

J.C.Kondrup Eng^d