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The Society will not hold itself responsible for any statements or opinions which may be advanced in the following pages.

CONTENTS.

INSTRUCTIONS FOR PREPARING PAPERS, ETC.

In writing papers, or discussions on papers, the use of the first person should be avoided. They should be legibly written on foolscap paper, on one side only, leaving a margin on the left side.

Illustrations, when necessary, should be drawn on the bright side of tracing transfer paper with transfer ink, or on the dull side of tracing linen, to as small a scale as is consistent with distinctness. They should not be more than ten inches in height. Black ink only should be used, and all lines, lettering, etc., must be clear and distinct.

When necessary to illustrate a paper for reading, diagrams must be furnished. These must be bold, distinct and clearly visible in detail for a distance of thirty feet.

Papers which have been read before other Societies, or have been published, cannot be read at meetings of the Society.

All communications must be forwarded to the Secretary of the Society, from whom any further information may be obtained.

The attention of Members is called to By-laws 46 and 47.

ERRATA AND EMENDATIONS.

WICKSTEED ON TRANSITION CURVES, VOL. V, PART 1.

Page.	Line.	
189	37	Write "off" instead of "of."
"	42	Write "199" instead of "417."
190	9	Erase (I. being 0.)
191	..	In Fig. 3 mark the point of curvature "P." instead of "p."
192	5	Write "P I." instead of "P."
"	12	Write "tan $\frac{1}{2}$ I." and "tan. 30°" instead of "cotan $\frac{1}{2}$ I." and "cotan 60°."
"	13	Erase the minus sign before *pi*.
"	18	Put a full stop after "distance," and Ic_1 instead of *Ic*.
"	20	Write "*Ii.*" for *li* and write "*ia.* sec. $aiI = ia\ x$ sec. $\frac{1}{2}$ I = 6.54 *sec.* 30° = 7.55" instead of "= *ia.* sin. $\frac{1}{2}$ P I P_1, &c."
"	33	Between the words "the" and "angles" insert "additional transition."
"	36	For P_1 read P.
"	38	For 417 read 198.
193	34	Append note. "N. B.—When there is an absolute reverse curve it cannot be readjusted without increasing the curvatures to make length for the transition between."
"	35	Erase the words "This, however, matters not, for." Append note: "N.B.—We might have set up at P_5 and run the reverse transition in the ordinary way." "See Fig. 5, page 198."
199	..	Fig. 1, insert "I" at the intersection of the tangents and insert "S" and "C" at the right side at the ends of the tangent and curve respectively.

Page 196 *between lines* 12 *and* 13 *insert.*

Calculate the increased length of the outer over the inner rail of the curve, and cut a 30 foot rail into two parts differing in length by this in creased length. At the beginning of the curve insert the longer of the two segments in the outside rail. This will give broken joints all through the curve. Then at the end of the curve insert the shorter segment in the inside rail which will bring the joints square again for the tangent. Suppose A to be the increased len th, S the shorter segment then S + A will be the longer.

$$2S + A = 30 \text{ ft.}$$

$$S = \frac{30 - A}{2}$$

Widening the guage contributes to ease in rounding curves and - - -

FRIDAY, 14th October.

G. H. Duggan, Member, in the Chair.

Paper No. 66.

THE USE OF SAFE EXPLOSIVES IN MINES.

PART II. THE RESULTS OF EXPERIMENTS.

By E. Gilpin Jun., M. Can. Soc. C. E.

The question of the use of explosives in the provincial coal mines was forced on the attention of the Nova Scotia legislature by the explosion at Springhill. Here apparently no trouble had been spared by the Company to protect its workmen. The locality in which the explosion originated was worked with safety lamps, the shots were fired under the direction of skilled men specially appointed for the purpose, and then at long intervals, not for loosening the coal, but to remove a few inches of stone in part of the roof. Copious watering was resorted to for laying the dust wherever the workings were not naturally damp. In spite of these precautions it appeared upon a careful enquiry that a charge of gunpowder had partially done its work, had flamed out, and the heat acting upon an atmosphere containing dust and gas gave rise to a very serious explosion. This showed that the mining practice of firing charges of gunpowder in places where dust and gas could be present was dangerous in spite of the precautions regarding its use.

After deliberation the Legislature enacted that when gas was found in any mine, in quantity sufficient to show in a safety lamp in three consecutive days, no explosive could be used for two months.

Provision was however made, that the Governor-in-Council could, upon the recommendation of a commission, including the Inspector and persons skilled in the use and composition of explosives, that any explosive was safe, relax the act in respect to explosives in favor of such safe explosive.

In considering the practical application of the use of explosives in coal mines, the first point to be settled is what constitutes a "gassy" mine, that is a mine in which the use of gunpowder becomes unsafe. The theoretical definition is that any mine in which gas is known to be given off may present it in volume sufficient to be ignited either alone or in connection with dust by the explosion of gunpowder. As a matter of fact small percentages are known by special tests to be present almost continuously in mines, although they cannot be detected by the best safety lamps in use. In all parts of the world in well ventilated mines, gunpowder has been continuously used for years with impunity in the presence of these minute percentages of gas. In such mines provided that they are damp, and free from dust, there is little danger even from excessive charges, or blown out shots of powder, so long as the ventilation is adequate.

Such mines when carefully managed and under proper discipline gradually and by imperceptible degrees, pass, as they are worked at an increased depth, into what may be termed the second stage, that of an increased evolution of gas, and usually of a greater degree of dryness. When this stage is reached a deficiency of ventilation in any district, coupled with dryness of the workings, produces a state of affairs highly dangerous in the event of flaming or blown out shots. The third stage is that of deep workings, which add to the dangers of increased exudation of gas, and general dustiness, those of extended fracture of the strata suddenly introducing volumes of gas directly into the workings or pressing it out of the old goaves. Under these conditions prudent management introduces safety lamps and abolishes the use of gunpowder. The second stage is the most dangerous, as when the conditions of safety and danger are balanced, a trifling mishap paves the way for a disaster.

Mining practice has so long sanctioned the use of open lights and high explosives in mines that have reached the second stage, that the reaction now setting in, in favor of their restriction, promises to seriously affect the economic exploitation of some coal beds. The Prussian Commission went so far as to classify as "gassy," mines in which gas had been detected once in two years. It is therefore apparent that in almost every district there are mines varying in their degree of danger. Any hard and fast rule, for example, precluding the use of high explosives whenever gas is found, would not affect the mines of the third stage, but would greatly increase the cost of the coal from the comparatively non-gaseous mines. In the case of many mines giving off little gas, there are serious expenses, off setting the cheapness with which the coal is mined, such as faults, steepness of dip, the presence of stone, weakness of the roof, etc. Such mines would find it difficult to produce coal, if explosives were abolished.

As it was apparent that a mine at any given time fairly classed as not "gassy" might in a few days on cutting a fault, giving off gas, or entering a disturbed section of coal, become decidedly "gassy" it was considered that the limitation imposed in the act would give a fair warning of danger arising in the usual conditions of mining. In the case of mines naturally damp, and decidedly free from gas, permission was also made in the act that any local and temporary detection of gas would not exclude the use of powder until it became evident that the increased proportion of gas was likely to prove permanent.

It must also be remembered that explosives play an important part in mining in addition to their employment for loosening the coal. Faults have to be penetrated, often through stone, tunnels must be driven to connect seams, roofs and pavements have to be removed, etc. These operations when expedited by the use of gunpowder or high explosives, have frequently proved very dangerous, and the source of serious explosions in the presence of gas and dust. So much is this the case that there is evidence tending strongly to show that, in haulage planes containing much dust, and presumably almost entirely free from gas, shots fired to bring down portions of a stone roof have caused disastrous explosions. Still such operations are essential to working coal mines, and their cost would be enormous if they had to be performed by manual labour only.

Upon a careful consideration from every point of view of the difficulties surrounding the problem, it appears that the total prohibition of explosives would be almost impracticable, and result speedily in the closing of mines already compelled to use every economy to make both ends meet in the face of competition.

Under these conditions the importance became evident of ascertaining if there was any explosive that could be safely used in the presence of gas and dust, in order that the exploitation of the Provincial mines might not be injuriously affected. The International and Acadia collieries, in Pictou County, had for some time used Roburite, at first imported from England, but afterwards supplied from a branch factory in Halifax, working under the company controlling the patent in Canada.

The commission appointed to enquire into the subject, under the provisions of the act already referred to, comprised the Inspector and several mining engineers and practical miners familiar with the three principal mining districts of the Province. The Commission met several times at Stellarton, in Pictou County, and experimented in the collieries of the Acadia Company, and appointed a sub-committee to experiment in Cape Breton.

The general value of the explosives tested before the Commission at Stellarton, may be gathered from the following selection of experiments, conducted under the supervision of members of the Commission.

Two parties submitted explosives. The Acadia Powder Company of Halifax, which had been for some time engaged in the investigation of flameless explosives suitable for use in gaseous mines, produced two grades of a dynamite explosive, claimed to be rendered flameless by the addition of certain chemicals. As the explosives were experimental, it was not deemed necessary at that stage to consider their percentage composition. The Roburite Company submitted Roburite as manufactured by them at Halifax, giving its composition as 18 per cent of chloro-dinitro-benzole and 82 per cent of nitrate of ammonia. It may be remarked that the secretary of the English company intimated later that the compound as manufactured there did not contain over 12½ per cent of chloro-dinitro-benzole, and that presumably it was made in Halifax of the same strength. The commission up to the date of its preliminary report, has dealt with the question of exact composition of explosives only in a general manner.

It may be remarked that in these experiments, the shots were fired with detonators, ignited by a victor battery.

1st Experiment. Two 6 oz cartridges of roburite were placed on the ground on the same wire, and covered with a few shovelfulls of dry slack coal. Both shots gave a short bright flash.

2nd Experiment. One 6 oz cartridge of grade "B," and a 3 oz cartridge of grade "C," of the explosives of the Acadia Powder

Company, were connected to the same wire, placed on the ground and covered with slack as before. On firing there was a flash from grade "C" cartridge, but none from grade "B" cartridge.

3rd Experiment.—A 4 oz cartridge of roburite, covered with four inches of slack coal, gave a flash on being fired.

These experiments were made on a dark night, and as far as possible under the same conditions.

The tests were continued in the McGregor pit of the Acadia Coal Company. A number of holes were bored in firm coal, in the high side of a level, in a five foot seem, about half way between the roof and floor. The holes were three feet six inches deep, and from $1\frac{1}{2}$ to $1\frac{3}{4}$ inches in diameter.

1st Experiment.—Charge 7 oz, explosive "B," hole tamped with clay for 25 inches. Shot blew the outside tamping off for a depth of 18 inches. No light visible.

2nd Experiment.—Charge 4 oz. roburite. Hole tamped with clay for 20 inches. Shot blew out tamping. No light visible.

3rd Experiment.—Charge 4 oz explosive "B," hole tamped with clay for 20 inches. Shot blew out tamping. No light visible.

4th Experiment.—Charge 7 oz explosive "B". No tamping. Shot gave bright flash.

5th Experiment.—Detonator of Acadia Powder Company fired outside the hole alone and uncovered, gave flash.

6th Experiment.—A 4 oz cartridge explosive "B" with detonator in rear of cartridge, and pushed in the back of the hole, gave slight flash on being fired.

In the opinion of those witnessing these experiments, the flash observed when the explosives where fired, without tamping, was not greater than that due to the detonator, except perhaps, in the case of the fourth experiment in the McGregor pit. It is probable that the greater or less amount of flash visible in a number of experiments, may be due either to a lack of uniformity of the explosive mixture, or to the detonators not occupying, in each case, the same position in the cartridge. The fact was evident that the explosives, fired unconfined, did not give a flame, but a very brief flash of light. The blown out shots did not flame, nor did they give a light, a very slight tamping being apparently enough to delay the progress of the explosion long enough for the flash to have disappeared, when the rupture of the enclosing matter took place. It may be imagined that the sudden compression of the air in the vicinity of the charge might produce visible heat, in a manner parallel to the ignition of gunpowder by sudden compression of air in a cylinder.

Numerous practical experiments were made in this pit, substituting the new explosives for gunpowder in the ordinary working of the coal. These showed that as soon as the workmen understood the changed methods of apportioning the charge, tamping, etc, they got equally good practical results. As a specimen the following memo gives the particulars of a shot fired in one of the regular working places of the McGregor pit.

Working place 15 feet wide. Bench 6 feet by 7 feet, by 3 feet 9 inches high. (1575 cubic feet.) Hole 5 feet deep, 2 feet 6 inches from the higher side, (the seam dipping about 1 in 3) level, and on bottom of seam. Charge 18 oz. "B" Explosive. First half of hole stemmed with clay, rest with slack coal. The shot was satisfactory. Coal hard and compact, and the bench had a layer of stone on top 9 inches thick.

The committee appointed to experiment in Cape Breton coals, which are softer than those of Pictou, reported that, in spite of their meeting with objections on account of cost, and prejudices in favor of the long established gunpowder, their opinion was that these explosives could readily replace gunpowder in that district.

At the close of the year the Commission submitted a preliminary report to the Governor-in-Council, in which they state that they had selected two of the explosives submitted, as apparently safe and adapted for coal mining, and that they had confined their enquiry solely to the question of safety in blasting, but had not gone into the question of cost, or of safety in manufacture, transportation, or storage, and that no investigation had been carried into the composition of these compounds pending the results of certain changes recommended by the Commission as calculated to render them safer.

The Commission recommended that any of the four explosives, approved of by the French Minister of Public works, August 1st, 1890, be allowed to be used, and any other explosive, not yielding as the product of its detonation any combustible matter, such as carbon, nitrogen, etc, or having its temperature of detonation higher than 1500° C. if employed in coal blasting.

Recommendations are also made as to the proper length of tamping. Arrangements were made for the issue of licenses to manufacturers, testing of samples, firing by electrical fuses by low tension electricity etc.

Since the report was made, samples of ammonite have been received from England. This explosive is put up in thin metallic cartridges to prevent the action of moisture on the nitrate of ammonia. The explosive, which is highly spoken of in England, is likely to prove expensive, where it comes into competition with other explosives, which can be supplied to the miner fresh from the factory, if not as well protected from dampness. Opportunity has hitherto been afforded only of testing this explosive in Cape Breton, and the results were not conclusive, but further tests will be made in Pictou County, where more experience has been gained in handling new explosives.

The Acadia Powder Company have improved their explosive, and are experimenting with an addition which has, it is claimed, the power to effectually waterproof their ammonia nitrate. Their new compound is substituted for the grades "A" & "B" referred to in this paper. The compound contains under 20 per centum of dynamite, and has in addition to the nitrate of ammonia, a chemical, which, stable in itself, is calculated to neutralize any trace of acid that may be present. So soon as the changes in the explosives are finished, the Commission will probably resume its work, and it is hoped will be able to recommend, at least two explosives superior in safety to any yet introduced in England or on the Continent.

NOTE.—The final results, analyses, etc., will be shortly submitted.

CORRESPONDENCE.

Mr. T. C. Brainerd.

Mr. T. C. Brainerd said Mr. Gilpin's paper is a comprehensive and interesting account of recent efforts to obtain improved and safe explosives. Nothing can be more important than to find some method of mining coal that will avoid the terrible catastrophes hitherto incident to that industry. Great progress has been made already, as statistics prove, and the present discussion in all countries shows continued appreciation of its value.

Fortunately many mines are now successfully worked without explosives, but even when the position and character of the vein permit removal of the coal by mechanical means, some blasting is generally necessary for sinking shafts, overcoming faults, and other purposes. The chief objection to the use in the coal of the modern nitro-compounds is that the force developed by the molecular change of their ingredients is not uniform, and is therefore unreliable as compared with the gradual combustion of gunpowder, which can be accurately measured and controlled. The expansion of the former depends on physical conditions, which vary in each case according to the pressure, temperature and other circumstances. Thus an explosive which, from its low specific heat or because the inner gasses are blanketed by a cooler envelope, shows no flame when burned in the open air, may develop brilliant combustion under the pressure produced by heavy stemming or other cause. Most of the "smokeless powders" have very little or no strength if fired in our Canadian winter when the thermometer is much below zero, but the same cartridge, allowed to remain in the chamber of a gun that has been heated by rapid firing, may shatter the barrel itself. The special means used for ignition also have great effect on the character of the explosion, the real discovery of Nobel, which made nitro-glycerine and dynamite useful, being the employment of the "Detonator." One of the "flameless powders" which has been most vaunted requires a detonator charged with no less than 23 grains of fulminate of mercury, an amount quite sufficient to explode even a very sluggish mixture of air and fire damp, and therefore an obvious source of danger. The electrical condition of the atmosphere modifies greatly these chemical actions. The mysterious and much discussed Elmore explosion was accounted for by one witness, an old mine viewer, as the "result of a tremendous lot of electricity in the air that night," but the explanation was not followed up by the Commission, although Mr. Chamberlin was its chairman and Lord Rayleigh one of the members. Yet the manufacturers of gunpowder in America learned long ago to close their mills on the approach of a thunder storm, not from fear of the buildings being "struck by lightning," but because experience proved that the air in the houses, charged with carbonaceous and oxygenated dust, would explode at such a time from a very trifling cause, that would have no effect in other states of the atmosphere. Some so-called safety compounds are as sensitive to shock under certain electric conditions as are Konig's flames to the musical note of the organ. In fact, all explosions may now be considered as rythmic motions, and there are probably few intimate mixtures of suitable ingredients that would not be promptly combined with detonation, if we could ascertain and produce the proper key-note. The object of the present study of the question is to avoid creating this exciting motor cause in coal mines.

Beyond the uncertainty as to what the resulting explosion will really be, there are economic and practical objections to many of the proposed substitutes for gunpowder. Some are liable to internal chemical change, extending even to spontaneous explosion when subjected to the climatic extremes of such a country as Canada. Others, like the compounds containing nitrate of ammonia, are deliquescent and alter rapidly in a damp atmosphere. Some, on burning, yield noxious or even deadly gasses, the coroner's jury having in March of this year given the fumes of roburite as the sole cause of death of a miner at Wigan. All are much more expensive than gunpowder, adding a considerable amount to the cost of each ton of coal, and many break the coal so fine as to create a further loss from the dust and slack. Finally, nearly all require an expert to properly use them, while the ordinary miner is reasonably familiar with the safe handling of gunpowder.

Concerning the latter, one decided source of danger comes from an excessive quantity being employed. Most mine operators sell the powder to their men, at a profit, and therefore think they have no interest in limiting the amount consumed. Hence the drill holes are generally overcharged, causing a volumn of flame at each blast that could be entirely avoided. It should also be remembered that all European experiments and comparisons are from gunpowder made from nitrate of potash, while on this continent nitrate of soda is used almost exclusively for blasting purposes, giving a very different result in temperature of ignition, specific heat and amount of flame.

If any substance can be found that by its combustion or otherwise will develop enough controllable force to promptly and cheaply dislodge coal and other minerals without producing such heat or shock as to inflame fire damp, the world will be a great gainer. It has not been done yet. Many who have investigated the subject thoroughly and without prejudice think that ordinary gunpowder is safer than anything so far produced. In gassy mines the use of any explosive should be very limited, and should be in the hands of a competent blaster, who fires the holes by electricity and between shifts, that is when all the men are out of the workings.

Mr. H. S. Poole.

Mr. H. S. Poole said: As regards the practical application of flameless explosives a little may be said. When compressed lime was so much talked of in England some years ago trials were made with it here, but the results were not considered sufficiently encouraging to continue them. Two years ago in one of the pits under the writer's charge trials were made of Sir F. Abel's water cartridges, but so much time was taken up in the preparation of each shot that it was felt they would not suit for general use in such cases as are here, in which as many as three shots a day may be required for each working place. The shots necessarily being under the supervision of specially appointed shot firers, where a large number are fired they would entail the employment of so many extra officials at a very heavy increased cost. Besides it was evident that a comparatively large number of cartridges might lose their included water so that the search for safety in that direction was abandoned.

Trials would also have been made with such compounds as carbonite could they have been obtained otherwise than by importation from Germany; but not being available, applications were made to the local factory of the Acadia Powder Co., and a compound was prepared which when tried in April, 1890, was, with respect to flamelessness, found satisfactory. Practical difficulties in other directions stood in the way of its general use at that time, and it was not until 1891 that opportunity was had for trial and subsequent use flameless mixtures from the same company and from a branch of the English Roburite Company.

Opposition to the introduction of these explosives was raised by the miners, for a life long experience with black powder had familiarized them with its capabilities, and they objected to the price and the fumes of the modern explosives. The complaint about fumes is no longer heard, many men contending they mind them no more than the smoke of powder, and with the worst of the explosives by keeping away from the seat of the shot for a couple of minutes, no inconvenience is felt.

As to price, the miners who have learnt to proportion quantity used to the work to be done have accepted the gain when the shots are successful, and the loss when misfires occur, and shots are lost.

All these explosives require detonation to make them efficient, and the electric fuses necessary are comparatively costly and doubly so when the shot fails to accompany the detonation of the cap. When this occurs the charge sometimes burns in the hole without exploding, and almost invariably so quietly that no sparks or flame are seen to come through the tamping. But we had one shot out of some twelve thousand that had been then fired with one of these mixtures that burning only showed sparks but no flame, at least so the shot firer and a miner contended, and the writer does not question their veracity. At the same time he has no reason to believe that the sparks so seen were of such a nature that they would have fired gas had it been present in quantity.

The introduction of these "flameless" explosives is to endorse one great element of danger in coal mining. Common powder does the work required of it in coal well, it sends from the working face great blocks little shattered if the shot has been, as the miner say, "properly worked;" while all the high explosives and many of the flameless variety have a local shattering action which largely increases the percentage of slack produced, and thereby seriously affects the profit of the mine; the market value of large and small coal differing very considerably. There has for a long time been recognized the danger attending the use of black powder in pits of a certain character, and the Legislature has imposed restrictions on its use, but the immunity of accident in some mines freely producing gas, and in which millions of powder shots have been fired without disaster, also shows that there are conditions when the use of powder is reasonably safe. The line of safety is a debateable one, but the disastrous side is now generally considered to be where there is fine dust. Not that the dust itself is generally regarded as likely to start an explosion, but that it keeps up an explosion which a cap full of gas may have started, and which alone in a damp pit would have done no damage. To lay the dust, watering has been practiced, but this cannot be done thoroughly in some mines, as for instance where the coal itself is so dry that the working at the face creates a cloud of dust, and where also the roof is broken and timbered, and so offering lodgment for dust in places impossible of access. On account of dust the writer abandoned the use of powder in one pit four years ago.

The writer said that he lately had the honor of reading before the Mining Society of Nova Scotia, a paper[1] on this subject in which the advantages and difficulties attending the introduction and substitution of "flameless explosives" for common black powder in the mines he had to do with were enumerated. He did not consider, therefore, that the present occasion called from him even a partial repetition of the experience therein related; he thought there was, however, one point introduced by Dr. Gilpin which would yet require legislative reconsideration, and that relating to the test defining what was a fiery mine.

The safety lamp test which drew the line on the appearance of a blue cap in the presence of inflammable gas could now hardly be considered a fixed standard, for it not only was subject to "personal error," but now varied with the style of lamp used, some lamps of modern construction being more sensitive than the Davy or even the Mueseler. The spirit lamps of Peeler and Ashworth detecting percentages so low that atmospheres carrying them have been, and even now are by many considered safe to work in.

For his own part he contended that the danger arising in mines from the emission of fire damp was not in proportion to the percentage of gas evolved regardless of other conditions, but that it was largely governed by the hydroscopic state of the air. A free evolution of gas, readily to be both seen in the lamp and heard issuing from the cracks in the coal of a damp mine was

[1] Printed in the Canadian Mining Review, July, 1892.

not to be dreaded to anything like the same extent as a much smaller quantity that imperceptibly escaped into the atmosphere of a dry and dusty pit. He goes further than Dr. Gilpin, and says that "gunpowder has been continuously used for years with impunity in the presence of" not minute percentages, but *large* percentages of gas in damp mines, and therefore he is disposed to contend that as it is a general maxim that legislation should interfere as little as possible with trade, all mines showing gas should not necessarily, therefore be subject to the closest restrictions, *provided* that the hydroscopic condition of the air is at no time disregarded. In thus questioning the necessity and advisability of sweeping enactment to which popular clamour flies for relief when sudden disaster excites attention he ventures to do so as one whose past ought to enable him to speak with experience; he has to do with mines where it was mentioned before a Royal Commission in 1835, "gas issued with the hissing noise of ten thousand snakes," and in his practice was the first to introduce in Nova Scotia, improved forms of safety lamps and to see the advisability of entirely giving up the use of powder in a dry mine. His surprise is not that disasters do sometimes occur, but that they occur so seldom.

He does not consider the clause in the present Mines Regulation Act[1] respecting the use of explosives where gas is occasionally found as sound. It makes the inducement for an imperfect test very great, for example to stop the test six inches of the roof when in the last six inches the probability of finding gas is greatly augmented; or to send ahead of the gas trier a man with a bag to beat and disturb the air at faces where gas is most likely to accumulate. He believes in legislative recognition of the modifying influence of moisture and in the intensifying of danger in dryness, he agrees with Dr. Gilpin in regarding mines of his second class, occasionally producing gas, as proportionally more liable to accident than mines recognized as "gassy." But to avoid misconception he desires to add that not only is he an advocate for the so called flameless explosives as reducers of danger, but that he began their use prior to their consideration by the Legislature. As an illustration of the extent to which gas is given off he would mention that a pair of

[1] "And may order that the use of any explosive is obligatory under this rule only if inflammable gas is found in two consecutive days on any two consecutive weeks." Sec. 24, chap. ix, 1891.

leading places in the deep seam at Stellarton, N. S., produce 180 cubic feet per minute and that the evolution is constant.

Mr. Gilpin. Mr. Gilpin in reply said he would remark with respect to Mr. Brainard's example of the deadly fumes of roburite, that practical trial has shown that in a properly ventilated coal mine its fumes are not objected to by workmen. And the same is the case with the flameless powder of the Acadia Powder Co. If a moment be allowed for the fresh air to mix with the fumes no ill effects can be noticed. The miners, when they are once initiated, as a rule find little difficulty in adopting the new explosives. No gold miner now raises any objection to using dynamite. Whatever may be the weight of opinion of individuals as to the percentage of safety of gunpowder over that of the so-called safety explosives, it is conceded that the thorough and unprejudiced examination of the subject by several European governments has resulted in very emphatic repudiation of gunpowder in the presence of gas or dust. With regard to the definition of a gassy mine it may be said no two mines are alike, and the adoption of a "Plimsol" mark is difficult. The dampening of the air currents is a great safeguard as suggested by Mr. Poole. But in many large and deep mines it would be attended with much expense. It may be remarked on Mr. Poole's statement that the classification of mines as adopted by Provincial legislation makes a great inducement for imperfect examination, is hardly just. No legislation requiring dozens of men daily to do certain things under ground, out of sight, and not before witnesses, can be enforced unless the gas examiners and their employees choose to recognize the fact that the law is one instituted for their safety, and they act as honest men. The writer thinks most mine bosses would hesitate to suggest to a gas examiner that he slur over his work if he reflects that such a course might lead to his being indicted for manslaughter.

FRIDAY, 28th October.

P. A. Peterson, Vice-President, in the Chair.

Paper No. 67.

THE SIMPLIFICATION OF THE QUADRUPLEX, AND THE IMPORTANCE OF ITS ACHIEVEMENT.

By D. H. Keeley, A.M. Can. Soc. C. E.

If there is any one thing more than another in the domain of practical telegraphy that hangs on the consideration of fine points, it is the successful operation of a quadruplex system, and to those who have had opportunity for enquiring into the philosophy of the thing, this fact is more than patent, inasmuch as it is found that a very serious difficulty exists where at the outset none whatever is apprehended.

As a matter of fact, when the quadruplex was a novelty and not yet in extensive use the text books were silent on the one vital but seemingly unimportant point to which the delay in developing the system was referable, and it was sometime before the really difficult part of the problem was appreciated and understood by any excepting those immediately concerned in its conception. However, that time is past, and the student of to-day in consulting his text book will find the whole anatomy of the quadruplex laid bare, and the devices introduced to obviate the difficulty that is due to an *inherent defect* are taken into account as an essential part of the whole. That at any rate is the point of view from which this paper is written; and its object is to show in the first place, that in the established system of quadruplex telegraphy there is an inherent defect whose obviation has encumbered the fundamental principle with complex apparatus—thereby, from a theoretical standpoint at least, demonstrating its inadequacy; and, in the second place, to show how by building upon a different but equally simple principle, the same practical results can be secured with much less machinery.

With this end in view, a little consideration can now be given to a comparison of the different principles involved, and incidentally the whereabouts and character of the inherent defect to which allusion has been made, will be perceived, and the method proposed for its elimination understood.

For convenience of elucidation it will be well to start with the conception that a quadruplex is a combination of two duplexes. A duplex is a system in which the recorder at each end of the line is so arranged as to be unaffected by currents outgoing, while free to respond to currents incoming. In the quadruplex, therefore, the distant recorders respond to the home keys or transmitters, and the home recorders respond to the manipulation of the distant transmitters; the result of which is that a single conductor is made to afford four distinctive circuits. These circuits, however, are not precisely similar, because the constitution of a quadruplex is such that the duplex element cannot be lost sight of, and a system of this kind is therefore necessarily always regarded as having *two sides*. The apparatus on one side (one of the two duplexes) being operated by currents differing in polarity or in degree from those operating on the other side The trouble has been to obviate not only the liability but the *possibility* of an interference of one side with the other.

The several methods of quadruplex telegraphy that have been invented are comprised in two general classes. (1.) The *Polar systems*, or those in which the receivers on the one side are actuated by increase and decrease of a normal current, and on the other side by reversals of that current, and (2.) the *Straight current systems*, or those in which the receivers are actuated by three different strengths of current of a given and unchanging polarity.

We can best consider them separately under distinctive headings, and assume a practicable type as embracing the whole in each instance.

THE POLAR QUADRUPLEX.

It is the polar quadruplex that has found favor in practice, but it presents several undesirable features, and, excepting where dynamic currents are available, is expensive to maintain in consequence of large batteries being required for its exclusive use.

According to this method, the signalling battery is arranged in two sections. One of the transmitters operates to put one or both of the sections to line; and the other transmitter operates to reverse the direction of the current. The receiving apparatus consists of polarized and neutral electro magnets. The armature lever of the polarized instrument is normally held against its limiting stop by the — current of the smaller section of the battery, that traverses the line when both of the transmitting keys are at rest (upraised). and it remains in the same position when the key that puts the second section of the battery in circuit, is depressed; but it passes freely over to its front stop or signalling contact when the + current, from one or both sections of the battery, traverse the circuit, hence the polarized instrument responds to the manipulation of the reversing key. The armature of the neutral instrument is responsive to both — and + currents, but it is held back by a retractile spring exerting a greater force than the magnetic attraction due to the current from the smaller section of the battery. This latter can therefore be reversed repeatedly and continually, thereby producing signals on the polarized receiver, without in any way affecting the neutral instrument. Whenever the increment key is depressed, however, the current from both sections of the battery traverses the circuit, the retractile force is insufficient to withstand the magnetic attraction imparted to the neutral receiver, and its armature passes over to its front stop or signalling contact; the neutral relay is thus responsive to the manipulation of the increment key.

So far the action appears to be smooth and satisfactory, and one might think he could go to work and set up his quadruplex; but he will discover, as did many an early investigator, that his knowledge is as yet incomplete. It will be found that while no effect is produced on the neutral relay by the changes of polarity of the smaller section of the battery, there is a marked interference when the current of both sections together is traversing the circuit. When the increment key is depressed the neutral receiver responds; if now the reversing key is depressed, a short false signal will be produced on the neutral receiver. The reversal of the battery by the reversing key momentarily withdraws the current from the circuit, and in that brief interval the magnetic attraction of the neutral receiver drops sufficiently to allow its armature to fall back from its signalling contact.

Here, then, we have the inherent defect of the system clearly defined. It resides in this lack of continuity of current.

Divers devices—perhaps multitudinous is a better word to describe their number—have been worked out with a view to obviating this ill effect of the current reversals. Out of them all, the clever arrangements due to the ingenuity of Messrs. Gerritt Smith and F. L. Jones have been sifted, and adopted in the standard quadruplex of the telegraph companies. There is not much to choose between these two devices; they are distinctive types, and have gone a long way to make the quadruplex what it is. Smith puts an extra coil on the neutral receiver core and connects it in circuit with a condenser shunting a considerable resistance. The current from both sections of the signalling battery charges the condenser, and the instant the current is withdrawn the condener discharges through the shunted resistance and the extra coil on the neutral receiver, the magnetic attraction of which latter is thereby maintained. Jones substitutes an induction coil for the condenser; the secondary wire is in circuit with the extra coil on the receiver, and the primary is traversed by the current from both sections of the signalling battery; the instant the current is interrupted an induced current traverses the extra coil, and, as in the other case, retains the armature attracted.

Either of these devices necessitates the use of heavier currents than would be required for the mere production of signals on the receivers, because the condenser discharge and the induced wave have to impart nearly the same degree of magnetism as the transmitted current; so it is found that the coils traversed by the transmitted current in the standard quadruplex of to-day are *not* designed to produce a maximum magnetic effect, whereas the coils traversed by the secondary currents *are* designed to that end and wound to suit the high potential of the condenser and induction coil discharges.

It is seen now that the feature of heavy currents has been dragged into and made a necessary part of the standard quadruplex, in consequence of a defect that seemingly could not in any other way be surmounted.

Let us see whether this feature of *heavy currents* is attended with any considerable drawback.

In consequence of electricity being as a general thing now so plentiful and cheap, it might at first sight be considered immaterial, from an economical standpoint, whether heavy or light currents were used in the operation of the quadruplex; but we have to bear in mind that only a small percentage of the tele graph wires of this continent are supplied by dynamos, and that, as regards the vast majority of cases, the customary galvanic battery is in use the world over, and its cost is just as great now as it was 15 or 20 years ago. This fact, dealing with the quadruplex alone, is in itself worth looking at. But we can appreciate the very great importance of this question when we go further and consider the circumstance that in the course of operation of a polar quadruplex, the positions of the transmitting keys are at times such as to throw the entire current + from one end and — from the other into the line, whence it follows that the line becomes inordinately charged and by induction would interfere with every other circuit running parallel with it *were they not also supplied with otherwise unnecessarily large currents in order to obviate the ill effects occasioned in them.*

A few years ago, before the days of the quadruplex, our telegraph circuits were equipped with receivers designed to afford a maximum of magnetic attraction, and the necessity for anything like large currents did not exist. The ordinary run of the old relays measured in the neighborhood of 300 ohms, and we could get good work out of them over long distances. To-day the ordinary run measures 150 ohms, and without going into calculations we can see that on a given line the latter instrument will be less efficient than the former, and to make it answer our purpose an increased current must be supplied. So we have perforce reduced the sensitiveness of our apparatus and increased our currents; and this it appears is the only way we could keep pace with the quadruplex, for had the more sensitive instruments with comparatively weak currents been retained, the induction from the quadruplex circuits would have rendered the ordinary single circuits well nigh inoperative.

We see in all this then that the advances made in telegraphy while admittedly very great from a practical standpoint, have been at the sacrifice of an economic and a scientific principle.

Is it now too late to rectify this error and to return to the old and economical practice that was departed from? Perhaps if an *improvement* in quadruplex apparatus, recently devised with a view to dispensing with the necessity of heavy currents, had been hit upon some twelve or fifteen years ago, this regrettable departure from the old practice might have been averted; and even now it might suggest the practicability of going back to first principles, if the advantages to be gained thereby were considered of sufficient importance to warrant the reversion.

The improvement referred to is in connection with, and might now be taken into consideration under the head of

THE STRAIGHT CURRENT QUADRUPLEX.

In this system the signals are produced, as has already been stated, by three different strengths of current of a given polarity. This method of transmission was for a long time the favorite hunting ground of inventors seeking a solution of the problem of sextuplex telegraphy. It appeared to be an easy matter to form a quadruplex on this plan, and add thereto the reversing key of the polar system and thus obtain as many as six circuits in a single conductor. It, however, came about in the long run that the formation of the quadruplex itself, according to this method, was extremely problematical. As a matter of fact, there never was a straight current quadruplex in practical operation. The difficulties that were presented in the way of its successful adoption have, however, been investigated in many quarters, and the invention as a whole has in the course of time been improved to an extent that warrants its now being characterized *the simplified and perfected system.*

That this simplified quadruplex is fairly calculated to fill the bill already outlined for the practicable operation of the general telegraphs on currents of much less power than are at present employed, will be recognized in view of the fact that the straight current quadruplex, as a system, presents none of the objectionable features of the polar system already pointed out. There are no reversals of the current to occasion disturbance in one of the receivers, so the necessity for extraordinarily large batteries for the operation of devices such as those described is done away with; and there is no need to charge the line beyond the voltage of the maximum current emanating from one end alone, because a system of this description is operative with either like or unlike poles to line, so there are no ill effects due to induction communicated to circuits adjacent to the line upon which this quadruplex may be in operation. The importance and value of the invention need not perhaps, in view of these facts be further emphasized.

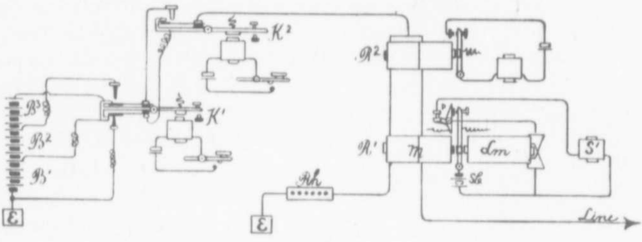
The straight current quadruplex in its simplest and perfected form is shown in the accompanying diagram. It will be noticed that the signalling battery is in one unbroken series, with its — pole to earth. The terminal and two tap wires are connected with the transmitters, which operate to put the main line in connection with the battery at some one of the three points tapped, according to the position of the transmitter levers, and in this way the requisite three strengths of current are presentable for the transmission of signals. The battery being in one unbroken series with one pole constantly to earth, is rendered available for the supply of current to other wires or circuits independent of the quadruplex; just as in the case where several Morse circuits are supplied from a single battery common to them all in one of the regular main offices. This is a feature of great advantage, as it obviates the necessity for separate and exclusive batteries for the operation of the quadruplex circuits. This arrangement of the battery and transmitters was supposed to have originated quite recently, along with a further improvement of the writer's to be described further on. But an examination of the fyles of the United States Patent Office disclosed the fact that its invention was anticipated, as far back as January, 1877, by Benjamin Thompson, of Toledo, to whom it was patented (No. 195,055) in September of that year; and it is perhaps due to the unfortunate circumstance of its being covered in conjunction with an unwieldy and inadequate mass of receiving apparatus, that it never worked its way forward to the recognition that its merit deserves. As will be seen by an examination of the diagram, the transmitters are so connected that when both of the levers are upraised the line is to earth; when K^1 alone is depressed, the line connects with B^1; when K^2 alone is depressed, the line connects with B^3; and when both K^1 and K^2 are depressed, the line connects with B^2. It follows, therefore, that K^1 sends the minimum current to line; K^2 sends the maximum current to line; and when both keys together are in action, an intermediate strength of current goes to line. At the distant end there are two receivers. One of them is an ordinary duplex relay R^2 connected in the usual way; a retractile spring holds its armature with a force superior to the magnetic effect of the minimum current, but it is responsive to the intermediate and maximum currents; R^2 therefore, freely and readily responds to the manipulation of the transmitter K^2. The other receiver comprises the distinctly unique feature that has raised the straight current quadruplex to its present perfection. Its contrivance is such that the sounder or recording instrument, operating in its local circuit, responds promptly and accurately to the manipulation of K^1, under the action of the minimum and intermediate currents, while the maximum current produces upon it absolutely no effect whatever. In this arrangement the sounder is operated directly by the armature lever contacts, and there is consequently no enfeeblement of the signals and no uncertainty whatever about its action. In consequence, the transmissions on both sides are equally rapid and reliable.

For the information of those interested in the solution of the problem involved, a detailed explanation of the receiver R^1 is given in the accompanying note.

A test which the writer was enabled to make, in conjunction with Mr H. Bott of Ottawa, on a line between this place and Toronto, nearly 300 miles long, showed that the apparatus would respond properly to currents (= ·038 amp. max.) derived from batteries of 150 cells at each end of the line. On the self same line wire the standard polar quadruplex is regularly operated with currents (= ·070 amp.) derived from batteries of 275 cells at each end of the line; which, under certain positions of the transmitting keys, produces in the circuit a current of (·070 × 2 =) ·140 amp. max. from the combination of the (275 × 2 =) 550 cells.

The comparison speaks for itself.

We have now arrived at the conclusion of this paper. Its purpose has been to show that the practice of telegraphy to-day, despite the many advances that have been made in the service in one way and another, is not so scientific as we find it to have been so long as twenty years ago. An endeavor has been made to show that the development of the polar quadruplex, now so extensively used, has occasioned this departure from first principles, and how the now perfected straight-current quadruplex would admit of a return thereto. And the question submitted for consideration is, Would it be worth while now to return to the original practice and continue in that good old way?



NOTE.

In the straight-current quadruplex the receiver on one side responds to the stronger currents and the receiver on the other side responds to the weaker currents. It is obviously an easy matter to prevent the former being interfered with by the currents intended for the latter side, as it is only a matter of adjustment of the retractile force on the armature lever. On the other hand, it is equally obvious that an electro magnet cannot be constructed to respond to weak currents and at the same time be unaffected by strong ones. As, however, the signals are, in practice, always taken from a second instrument, operating on a local circuit of the receiver or relay, it is possible to so arrange the intervening mechanism that the local circuit shall be closed only when the currents designed therefor are traversing the magnet coils. In the construction of the instrument, therefore, a supplementary lever is hung in such a way as to hold the armature lever in an intermediate position, between its limiting stops, when attracted by the weaker currents. A strong current will attract the armature with sufficient force to carry it beyond the intermediate position, and when there is no current the armature lever falls back to its rear limiting stop. It then becomes only necessary to connect the local circuit in such a way that it shall be closed in the intermediate position of the lever, and open when the lever is in either of its extreme positions, in order to meet the requirements of the case. That at least was the assumption; but the idea proved impracticable, in consequence of a brief contact, completing the sounder circuit, that was found to obtain in the passage of the armature lever between its extreme limiting stops every time the strong current was applied or withdrawn. This difficulty has never been successfully dealt with until now.

In the receiver R^1 an auxiliary electro magnet Lm, wound to produce a considerable counter e. m. f., is placed directly behind the relay armature so as to act thereupon in opposition to the main circuit coils *m*. In the normal condition, with no current traversing *m*, the armature lever is held against its back stop by a light retractile spring in the usual way. When a weak current, say the minimum, traverses *m*, the armature lever is attracted to the intermediate position, this closes the circuits of both Lm and S^1; the retractile power (that is, what the magnetic attraction in this case becomes) of Lm is delayed by its own counter e. m. f. until the attraction of *m* has grown sufficient to retain the armature in the position to which it was drawn, so the closed circuit of S^1 remains undisturbed. The same action attends the intermediate current; so S^1 responds to the minimum and intermediate currents. When the maximum current traverses *m*, the armature lever is carried from its intermediate position, and S^1 opens, but the circuit through Lm remains uninterrupted. If the current again decreases, the lever returns to its intermediate position, and S^1 closes; but if the maximum current is entirely withdrawn, the armature lever will, in consequence of the steady pull exerted on it by Lm, be drawn sharply back to its rear limiting stop. And if, when the armature is resting in the latter position, the maximum current is applied to *m*, the armature lever will pass directly over to the front limiting stop, in consequence of the counter e. m. f. of Lm robbing it of any retractile power during its passage across the contacts in the intermediate position. There is, therefore, no hindrance to the forward movement of the armature, and there is an acceleration of its movement rearward; hence the maximum current can be applied and withdrawn at pleasure, without in any way affecting the local circuit by which the sounder S^1 is operated.

CORRESPONDENCE.

Mr. A. E. Childs.

Mr. A. E. Childs said: This admirable paper on the Quadruplex by Mr. Keeley must certainly prove interesting to all who read it through carefully aud studiously. It is not, however, until more than half the paper has been covered that one begins to get at the essence of it. Having carefully read through the lengthy description of the Polar Quadruplex, its defects, methods at present used to correct them, and the remarks on the whole by the author, we come to the means proposed for correcting its *inherent defect*, the *clip* of the neutral relay during a reversal of the full strength of the current. This correction is to be achieved by dropping the Polar Quadruplex altogether, and developing the straight current system by an added device in the form of a special local circuit.

He states, and with reason, that the remedies applied to the overcoming of this defect have necessitated the employment of heavy currents, either from batteries or dynamos. This fact is not to be denied, on the contrary it is rather considered by some to be an achievement than otherwise, giving an opportunity, as it does, of doing away with the chemical battery, so troublesome to keep in order and so costly to renew.

Now, as a principle of engineering, anything that tends to do away with devices unmechanical in their nature (even though electrically correct) and to substitute mechanical ones for them, should receive encouragement and consideration. The cost of this substitution may at times be serious, but certainly is not so in the present case. Mr. Keeley himself admits the cheapness and plentifulness of the dynamo.

As a draw-back to the employment of heavy currents, he cites the case of a line acting by induction on other lines, when the full current on it is being reversed. And yet we employ currents of exceedingly high frequency and great power, many times more disturbing than a quadruplex telegraph could be, in the streets of nearly all our towns. Now, the wires carrying these certainly do not extend to great distances as do the telegraph wires, but they are to be found in close proximity to them at the termini. The employment of heavy currents in the Quadruplex acts as a very protection to that system. Apparently the answer to Mr. Keeley's question, "Is it now too late to rectify this error and to return to the old and economical practice that was departed from?" is to be found, in part at least, in the above considerations.

The modified straight current Quadruplex described by the author of the paper is most interesting and is worthy of study, as showing an improvement which is sure to be appreciated and found useful in the future. We are all indebted to Mr. Keeley for his clear and lucid paper, and we cannot but admire the ingenuity that has led him to the solution of the problem set.

Mr. D. H. Keeley.

Mr. D. H. Keeley said in reply to Mr. Childs' remarks: There are one or two points taken in Mr. Childs' thoughtful remarks which, perhaps, should not be allowed to pass without comment, lest there be some misapprehension of the case which has been submitted for consideration.

In the first place, the existence of the Polar Quadruplex is not necessary to the utilization of the dynamo in telegraphy.

The dynamo can be constructed to afford any required voltage; and if the telegraph lines were equipped with suitable apparatus requiring compartively light instead of heavy currents, the dynamo could be advantageously employed wherever the number of circuits is sufficient to warrant the change from ordinary batteries. But, as was shown in the paper, the number of such instances is compartively small.

In the second place, there should be no misconception of the effects producible by induction from local wires carrying alternating currents of high frequency, as compared with the effects produced on telegraph lines by induction from the irregularly reversed currents operating in an adjacent quadruplex circuit.

In the former case there can be no appreciable effect produced, because the current phases of opposite sign follow each other too rapidly to admit of the development of magnetism in the receiver. Whereas, in the other case the induced currents have ample time to exercise their full effect in energizing the electromagnet and producing false signals on the receiver.

The question, therefore, as to whether the use of insensitive apparatus and heavy currents should not be departed from, still remains an open one. There does not appear to be any good ground for persistence in our present practice. The only thing that can perhaps be offered in support of it is that it may be best to "leave well enough alone," but it is difficult to reconcile this conclusion with the real facts of the matter when we become alive to the full significance of the point at issue. The magnetizing power of any electro-magnet is proportional to the number of its ampere-turns, and it is found by calculation that the amount of current energy consumed in the production of a given magnetic effect by a 150 ohm relay, is 141 per cent. greater than that consumed by a 300 ohm relay in producing the same result. In other words, the use of the 150 ohm relay in telegraphy is attended with a dead loss or *sheer waste of nearly* 60 *per cent. of the electrical output*. And this is a consequence that ought not to be overlooked.

Friday, 11th November.

JOHN KENNEDY, President, in the Chair.

Paper No. 68.

"THE TRANSITION CURVE."

By HENRY R. LORDLY.

Stud. Can. Soc. C. E.

Although considerable has been written of late on the subject of "Transition Curves," much of which has been very interesting and of service to the profession, yet the question of deriving and laying out the curve practically has not yet received exhaustive treatment. The necessity of such a curve and the great benefit to be derived by its use has been so thoroughly discussed by different writers, that it will be treated as foreign to this paper. For reference to these points we recommend Mr. Wicksteed's paper, in "Transactions" of this Society, Vol. V, Part I.

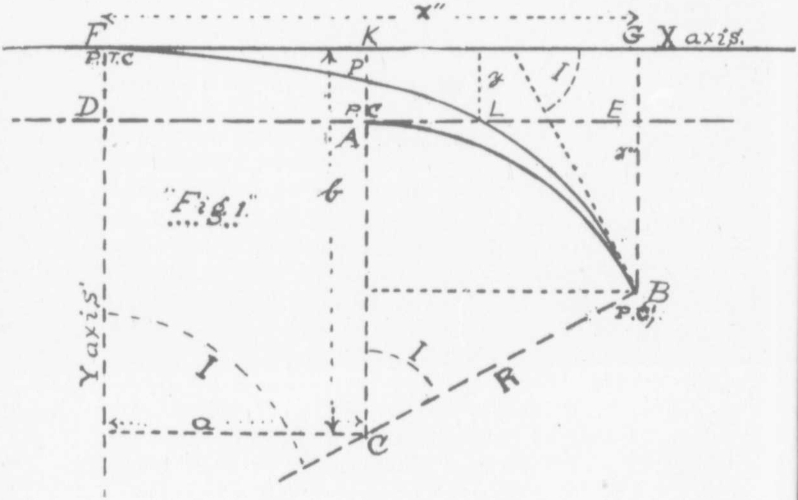
The transition curve to which our attention will be given here, is the curve which has been worked up and developed by Prof. C. L. Crandall, C.E., of Cornell University. (Mr. Ellis Holbrook,[1] it is said, first proposed and used the principles in laying out curves for the Pittsburg, Cincinnati & St. Louis R.R., 1882, and a gentleman at Lehigh University worked up a solution of it a few years ago, but to Prof. Crandall, the honor is due for having put it in its present state of efficiency.)

This curve, as will be seen later, is strictly mathematically correct, and it has now been tested sufficiently in the field to show that besides this, its easy manipulation makes it invaluable to the engineer. In order to discuss it here, we will take it up under the following heading: (1.) Derivation of formulæ. (2.) Tables from the results of (1). (3.) Examples and general conclusions.

[1] R. R. Gaz., Dec. 3, 1880.

(1.) In order to counteract centrifugal force upon a circular curve the outer rail must be elevated, the change from the tangent being gradual to promote easy riding and to prevent twisting the trucks.

Therefore, taking the centrifugal force proportional to the elevation at every point, the curvature, in passing from the circular curve to the tangent, must increase directly with the distance.



Suppose in figure 1, AB to be a circular curve with centre C. Now if we begin at B to reduce the curvature directly with distance, continuing this reduction until curvature is zero, maintaining the same central angle I. as in circular curve, the new curve will pass outside AB, having a tangent FG, parallel to DE, the tangent of AB, and at a certain distance, KA, from it. As is customary A is called the P.C.; F the P.T.C., and B the P.C'. Again let θ be the angle which the curve at any point L, makes with the initial tangent FG, s the length of arc FL. Then since by hypothesis the curvature at F is zero and increases with the distance from P.T.C., therefore the curvature at any point L, distant s from P.T.C., is equal to a constant multiplied by s. For convenience to avoid fractions later, this curvature is expressed by

$2ks$, k being the constant depending on the rate of change of curvature.

The radius of the curvature equals $\frac{ds}{d\theta}$, θ being any angle, and as curvature varies inversely as the radius of curvature, we have

$$\frac{d\theta}{ds} = 2ks \ldots . (1)$$

or, after integrating, $\theta = ks^2$.

If y is the ordinate, we have $dy = ds \sin \theta = ds \sin ks^2$ (a). but sin a (a being any angle) in series is equal to

$$* \quad a - \frac{a^3}{3!} + \frac{a^5}{5!} - \frac{a^7}{7!} \; rc.$$

$$\text{therefore in } (a) \; dy = ds \left(ks^2 - \frac{k^3s^6}{3!} + \frac{k^5s^{10}}{5!} \ldots \right]$$

$$\text{therefore } y = \frac{ks^3}{3} - \frac{k^3s^7}{7.3.2} + \frac{k^5s^{11}}{11.5.4.3.2} \cdot \cdot$$

$$= \frac{ks^3}{3} - \frac{k^3s^7}{42} + \frac{k^5s^{11}}{1320} \; \text{(b)}$$

* 3 ! = 1.2.3; 5 ! = 1.2.3.4.5. and so on.

In the same way, for the abscissa x, we have $dx = ds \cos \theta = ds \cos ks^2$

$$\text{and } \cos a \text{ in series} = 1 - \frac{a^2}{2!} + \frac{a^4}{4!} - \ldots$$

$$\text{therefore } dx = ds\left(1 - \frac{k^2s^4}{2!} + \frac{k^4s^8}{4!}\right) \ldots$$

$$\therefore \quad x = s - \frac{k^2s^5}{10} + \frac{k^4s^9}{216} - \ldots \text{(c)}$$

From these two equations (b) and (c) the length s of the arc FL is expressed in terms of the coördinates of the curve, but before using them the investigation must be carried further.

After passing this point, (P. C.) two methods of procedure are open. First, reducing the curvature from the P. C.[1], in the same ratio as it was increased. Equation (b) will then give the radial ordinates from the circular curve for this part (P to B) of the curve, besides the ordinates—from the tangent—for the first part (F to P). Also the curve and offset will bisect each other at P. If we should now place the second derivative for the first part at P equal to the second derivative of the second part at P, the k will be eliminated, leaving s to be ascertained for any given value of y or AK.

(The work of the above has been omitted here; it is simple application of the calculus, and, if interested, the reader may readily follow the steps through himself.)

Second—By continuing the curve to B, and placing the curve so that it shall be the circle of curvature, we will get for flat curves the same result as in the last case. When the offsets are larger, AK and the curve will not bisect each other, and the ordinates from the circle will differ slightly from the corresponding ones of the tangent. The author claims this to be theoretically the correct method, particularly if the curve is to be run with a transit. It also gives simpler formulæ.

Now from equation (1)

$$\text{(any arc) } \rho=\frac{ds}{d\theta}=\frac{1}{2ks} \text{ and } k=\frac{\theta}{s^2}$$

$$\therefore \quad \rho=\frac{s}{2\theta}.$$

But at its limits $\rho=R$, and at the same time $2\,\theta=2\,I_{(arc)}$; the s then being s'', the distance from the P. T. C. to the P. C.[1]

$$\text{Then } I^\circ=\frac{s''}{2\text{ R} \sin 1^\circ}$$

$$=\frac{s''\,D}{2\times5730\times0.01745}=\frac{s''\,D}{200}$$

(The reciprocal of 5730 being 0.0001745.)

From equations (b) and (c) ,y and x may be found, and from these same equations, remembering that

$$\begin{cases} \text{R}=\dfrac{s}{2\theta}=\dfrac{s''}{2\text{I}} \\ \text{and } k=\dfrac{\theta}{s^2}=\dfrac{\text{I}}{s''^2}=\dfrac{1}{2\text{R}s''}. \end{cases}$$

we get for the point B

$$y''=\frac{s''^2}{6\text{ R}}-\frac{s''^4}{336\text{ R}^3}+\frac{s''^6}{42240\text{ R}^5}$$

$$\text{and } x=s''-\frac{s''^3}{40\text{ R}^2}+\frac{s''^5}{3456\text{ R}^4}.$$

(II.) We have now derived all the formulæ from which to make up our tables. In getting s'' for a given circular curve, the R would be assumed and the x'' and y'' then found. s'' may be more conveniently obtained from given values of F (the offset at the PC from the circular curve to the tangent), for assuming $y''=4$ F; an approximate value of s'' results from using only the first term in the value of y''. This value, slightly increased, if substituted in the second and third terms, will give a value of s'' from the first term which will be sufficiently accurate. Having s'', we are now able to find I, k, F, etc. . . .

The values of F are now compared with the assumed one, and several trials may be necessary before the two values agree. If a few values of F in different parts of the table are ascertained, a certain relation is found to exist between F and y'', so that there is little trouble in getting subsequent values of F. When $x=a$ (fig. 1), the x' of the table is the length of the transition curve from P. C. to P. T. C. The $e=x'-a$, or the excess of transition curve over that of the tangent F K; $e'=s''-x'-(100\,I\div D)$ or the excess of transition curve over that of circular curve from P. C. to P. C[1]; c is the chord length F B; x is tangent length to P. C.=F. K., and y is ordinate from tangent to the curve opposite P. C.=P. K.

Prof. Crandall has worked up a very complete set of tables, the curvature being up to 26°; 2° to 14° inclusive, and from 14° to 26°, taking 14°, 16°, 18°, etc. s'' ranges from 40 ft. to 800 ft.

For intermediate values of F and D we may interpolate. F being very nearly proportional to y'', s'^2 will be proportional to F and therefore to $1\div D$.

s'' is the length of transition curve.

Below are given a few values from the tables, for illustration:

TABLE I.

6° CURVE.

s''	I°	x''	y''	c	e	e'	F	x	y
60	1.80	60	0.63	60		—.01	0.16	30	0.08
100	3.00	100	1.74	100		.01	0.44	50	0.22
200	6.00	199.8	6.98	199.9	.01	—.01	1.75	100	0.87
300	9.00	299.3	15.68	299.7	.02	.04	3.92	149.9	1.96
400	12.00	398.2	27.83	399.2	.05	.18	6.96	199.7	3.48
600	18.00	594.1	62.38	597.3	.20	.80	15.67	299.0	7.79

s'' is taken for every 20 ft. In computing s''. I. F. and x' only, are really required.

(III.) The curve may be laid out by deflection angles or by offsets. The distance s'' is divided into 20 equal parts, y and x being found from the formula, for each point, then $y \div x$ gives the tangent for the respective deflection angles, the transit being at the P. T. C. These angles are tabulated in parts of I, and are almost proportional to I as D and F vary. The greatest error is really very small. Also from the values of θ in the formula, the central angles, beginning at the P. T. C. are proportional to I, the same being true of the central angles subtended by the short chords. Below we give the notes for a transition curve, by deflections, just as it appears in the transit book. From this we believe the reader will see the method of operation without much further explanation.

Station.	Point.	Bearing.	Vernier.	Curve Data.
52+40.5	P. T. C.[1]		3° 00′=⅓ I	
+80.5			3° 43′	
51+20.5			4° 41′	
+60.5			5° 53′	
50+00.5			7° 19′	
● +40.5	P. C.[1] ⊙		4° 44′	
49			3° 31′	
48			0° 31′	Vertex=48+67
* +82.7	P. C. ⊙	6° Left.	3° 00′=⅓ I	△=27° 28′
47+22.7			1° 55′.2	D= 6°
+62.7			1° 04′.8	I= 9°
46+02.7			0° 28′.8	T=234.44
45+42.7			0° 07′.2	s''=300
44+82.7	Offset 3.92 P. T. C. ⊙	N. 20 W.		

* Set up transit and set to 6° for backsight.
● " " " 4° 44′ + I3 = 4° 44′ for backsight, etc.

Here we have taken $s''=300$. $\therefore$ F=3.92, and s or $x = 149.9$, T=234.44. We divide 300 by 5, which is 60 ft. for chord length, which is reasonable length. Then as central angles are as square of distance :

$(\frac{1}{5})^2 \times 9° = 0° \ 21.6'$ ∴ $07'.2$ = deflection.

$(\frac{2}{5})^2 \times 9° = 1° \ 26.4'$ ∴ $0° \ 28'.8$ = "

$(\frac{3}{5})^2 \times 9° = 3° \ 14.4'$ ∴ $1° \ 04'.8$ = "

$(\frac{4}{5})^2 \times 9° = 5° \ 45.6$ ∴ $1° \ 55'.2$ = "

$(\frac{5}{5}) \times 9° = 9° \ 00'$ ∴ $3° \ 00'$ = " $= \frac{1}{3}$ I.

Since $\triangle = 27° \ 28'$ and 18° is used up for T. curve ∴ 9° 28′ remains for simple curve, which is (9° 28÷6°) long=1+57.8. (The deflections from P. C.[1] to P. T. C.[1] are taken from Table No. 2, a portion of which is given below. It is constructed from the the tangent, in series and an equation, $d = \frac{1}{3} \cdot \frac{1}{s'^2} (s^2 + s'^2 + ss^1) - B$.

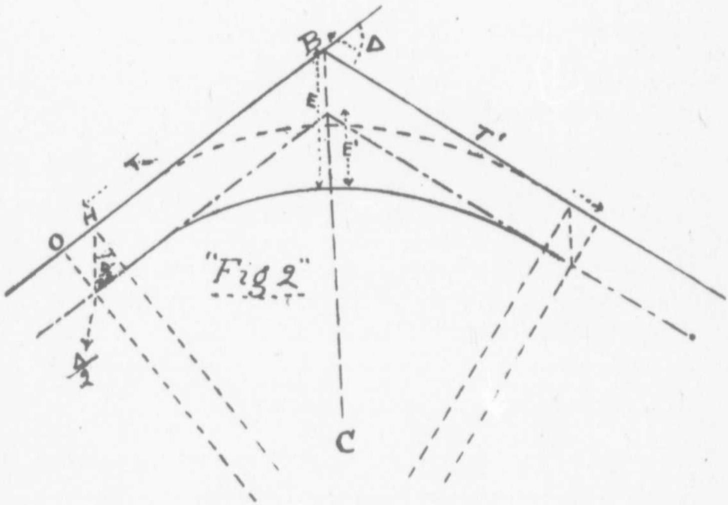
The first term of right member being the $\frac{1}{3}$ A of table mentioned. This table should be very valuable to the engineer. Without it we begin from the P. T. C.[1] and run to the P. C.[1] with the deflections first found.

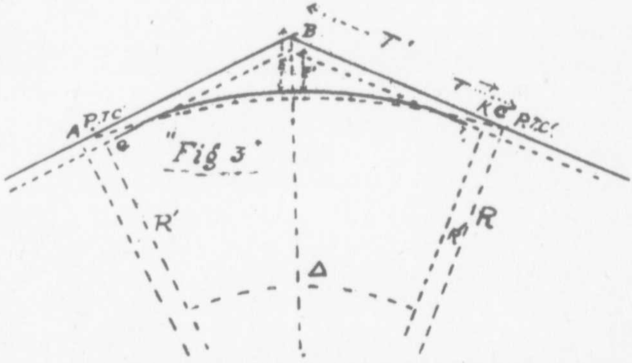
If we wish to simply put in the offsets and run the curve later, we place the stakes as follows :

P. I. (point of intersection)	=48+67
	T^1= 2+34.4
∴ P. C.	=46+32.6
	+ 4+57.8
P. T.	50+90.4

Then at Sts. 46+32.6 and 50+90.4 offsets are placed (4+57.8)= $\frac{\triangle}{D} = \frac{27° \ 28'}{6°}$.

As will be seen, it is better to work forward instead of from the P. T. C. and P. T. C.[1] to the circular curve.





If the front tangent has not been located, in beginning the circular curve proceed as follows. Set up transit at offset distance inside the tangent, or at the P. C.; backsight to a point similarly offset; then run the curve as usual. At the P. T. the operation will have to be reversed. If, on the contrary, the front tangent has been fixed, T^1 and not T, must be measured from the vertex to locate a point from which to lay off F from the P. C., $T^1=(R+F)\tan\frac{\triangle}{2}$, and is found, from fig. 2, as follows:

$$T^1=HB+HO$$
$$=R\tan\frac{\triangle}{2}+F\tan\frac{\triangle}{2}$$
$$=(R+F)\tan\frac{\triangle}{2}$$
$$\text{or } =T+F\tan\frac{\triangle}{2}.$$

Here we have $\triangle$, D and F given.

As the figure indicates, the circular curve is moved parallel to itself to a distance F, from its former position, in order to make room for the transition curve. The new curve then has an external distance, with reference to the old tangent, equal to or slightly less than the old, the offset being small. Thus $E^1 \overline{\overline{<}} E-\frac{F}{\cos\frac{\triangle}{2}}$. From Searles' Table VI we may take the E for a 1° curve; divide this value by E^1 for D^1, and then change the latter value enough to avoid fractional minutes, before finding the length of the curve and T^1.

In case the new curve should fit the roadbed better by extending as far outside the old curve at centre as inside at the P. C. we would have:

$$E^1=E-\frac{F}{\cos\frac{\triangle}{2}}-\frac{F}{2}.$$

Another important case arises where a transition curve is to be put in on old track, the new track being same length as the old. This is to prevent cutting the rails. In fig. 3 let

$$BC=T=R\tan\frac{\triangle}{2}$$
$$\text{and } BK=T^1=(R^1+F)\tan\frac{\triangle}{2}.$$

The arc AC=length of old track

$$=R\triangle^\circ \text{ arc } 1^\circ$$
$$\text{and arc } GL=R^1\triangle^\circ \text{ arc } 1^\circ.$$

Now, the length of new track from A to C, the transition curve being put in, is equal to GL+2 (BC—BK)+2 $(e+e^1)$, therefore by substitution we get $R\triangle°$ arc $1°=R^1\triangle°$ arc $1°+2$ R tan $\frac{\triangle°}{2}$ -2 (R^1+F) tan $\frac{\triangle}{2}+2$ $(e+e^1)$, therefore

$$R^1=\frac{R\triangle° \text{ arc } 1°-2\ (R-F) \tan\frac{\triangle}{2}-2\ (e+e^1)}{\triangle° \text{ arc } 1°-2 \tan\frac{\triangle}{2}}.$$

The following will show the use of the above equation. Find the data for a transition curve where the track is already laid on a 6° curve, 800 ft. long.

Taking 25 ft. of transition curve per degree we have $c=2\times150=300$. Then from the tables we get F=3.92 ft.; $(e+e^1)=(.02+.04)=0.06$; $\triangle=48°$; $R=\frac{5730}{6}=955$. Substituting in the equation

$$R^1=\frac{799.90-846.69-0.12}{0.83760-0.89046}$$

$$=\frac{-46.91}{-0.05286}=887.43.$$

therefore from Searles' Table VI we get

$$D^1=6°28'=6°.466.$$

$$L^1=48\div6.466=742.35 \text{ ft.}$$

$$T^1=(887.43+3.92)\tan\frac{\triangle}{2}=396.85 \text{ ft.},$$

which is the data required. Other problems might be taken, but we believe enough has been given to show the working of the curve.

Some engineers, it might be remarked, seem to think that using what they call "elaborate transition curves" is a waste of time. No reason is offered, however, to show why it should take more time to do it right than wrong. At any rate, present railroad practice demands the best, and a properly qualified engineer is one able to respond to these demands. Surely, if a thing is worth doing at all, it is worth doing well.

In conclusion the writer wishes to extend thanks to Prof. Crandall for his kindness in allowing the use of his notes and tables.

TABLE No. 2.

DEFLECTION ANGLES FOR TRANSITION CURVE.

Instrument at Station n″=0. Instrument at Station n″=1.

n	Θ for $\frac{I}{3}=1$	A	B for $\frac{I}{3}=$ 4°	6°	8°	10°	12°	14°	16°	18°	20°	A	B for $\frac{I}{3}=$ 4°	6°	8°	10°	12°	14°	16°	18°	20°
.0	0000	.0										1.	1	5	12	23	41	64	97	138	191
.05	.0075	.0025										1.0525	1	4	11	20	36	58	86	124	171
.1	.03	.01										1.11	1	4	9	18	32	51	76	109	150
.15	.0675	.0225										1.1725	1	3	8	16	28	44	66	95	130
.2	.12	.04										**1.24**	1	3	7	13	23	37	56	80	110
.25	.1875	.0625										1.325	1	2	6	11	20	31	47	68	93
.3	.27	.09										1.39	1	2	5	9	16	26	39	56	77
.35	.3675	.1225										1.475		2	4	7	13	21	31	44	61
.4	.48	.16									1	**1.56**		1	3	6	10	16	24	34	47
.45	.6075	.2025						1	1	1	1	1.6525		1	2	4	8	12	18	36	36
.5	.75	.25					1	1	1	2	3	1.75		1	2	3	6	9	14	19	26
.55	.9075	.3025				1	1	2	3	4	5	1.8525			1	2	4	6	9	13	19
.6	1 08	.36			1	1	2	3	4	5	9	**1.96**			1	1	3	4	6	9	13
65	1.2675	.4225			1	2	3	5	7	10	14	2.0725				1	2	3	4	6	8
.7	1.47	.49		1	1	3	5	7	11	16	22	2.19					1	1	2	3	5
.75	1.6875	.5625		1	2	4	7	11	17	24	33	2.3125						1	1	2	3
.8	1 92	.64		1	3	6	11	17	25	36	49	**2.44**								1	1
.85	2.1675	.7225	1	2	4	9	15	24	36	52	71	2.5725									1
.9	2.43	.81	1	3	6	12	21	34	51	73	100	2.71									
.95	2.7075	.9025	1	4	9	17	30	47	71	102	139	2.8525									
1.0	3.000	1.0000	1	5	12	23	41	64	97	138	191	3.									

B is in thousandths of a degree.

The complete table has I up to 60°, and has three intermediate transit points.

The values of A in heavy type, multiplied by 3, give the values of deflections found in example, from the P.C.[1] Thus 2°.44 × 3 = 7°.19′ and so on.

Friday, 25th November.

HENRY T. BOVEY, Member of Council, in the Chair.

Paper No. 69.

DISCUSSION ON TRANSITION CURVES.

By M. W. HOPKINS, B.A.Sc., A.M.Can.Soc.C.E.

The much vexed question of transition curves is getting pretty nearly settled when such excellent methods as that of Mr. Lordly's are given us and good tables made out which he tells us is being done.

His method is very accurate and not so difficult to use. I have employed a method which is not quite so accurate without corrections, as the one in Mr. Lordly's paper, but it is much more simple to apply, and is, to my mind, sufficiently accurate for 999 cases out of 1000, and with corrections given below can be made as accurate as we please.

It is composed of two separate curves, and is in fact two separate cubic parabolas, one of which is measured along the tangent from the *P. T. C.* to the middle of the offset opposite to the *P. C.* marked *P* in Mr. Lordly's fig. 1, which I will refer to as simply fig. 1 in my following remarks, and the other is measured along the circular arc beginning at the $P.\ C.^1$ and continuing back to the point *P*, where it meets the first cubic parabola.

The following is the formula employed for the first cubic parabola :—

Let the curvature increase in proportion to x measured along the tangent and not along the curve as in Mr. Lordly's paper.

Then we have

$$\frac{d^2y}{dx^2} = 2\,kx = \frac{1}{r} \qquad (1)$$

$$\frac{dy}{dx} = kx^2 \qquad (2)$$

$$y = \frac{kx^3}{3} \qquad (3)$$

From fig. 1 it will be seen

$$\text{when } x = FK,\ r = 2\text{ R and } x = a$$

$$\text{" } x = FK,\ y = \frac{o}{2} \text{ and } x{=}a$$

where o = the offset

$$\therefore\ 2\,ka = \frac{1}{2\,R} = \frac{D}{2 \times 5730}$$

$$\therefore\ ka = \frac{D}{22920} \qquad (4)$$

$$\text{and } \frac{o}{2} = \frac{ka^3}{3} = \frac{Da}{3\ (22920)}$$

$$\therefore\ a = 185{\cdot}4\sqrt{\frac{o}{D}} \qquad (5)$$

Now, this part of the transition is only used from the *P. T. C.* up to the point *P* opposite to the *P. C.* in the circular arc in fig. 1, and the difference of length between '*a*' in this formula and the length of the curve is smaller than is usually measured in the field.

To show how to put in this part of the curve, let us take an example. Suppose we desire to put a transition on a 4° curve and we choose an offset of 2 ft. These numbers are taken as a simple case in order to be better understood. From (5) we see that this gives us "*a*"=131 ft. Now, the ordinates from the tangent *F R* (fig. 1) are prop. to x^3, as will be seen from (3), and the ordinate at *P* will be $\frac{2}{2} = 1$ ft. The ordinates then will be, beginning at the *P. T. C.*,

1 ft. from the *P. T. C.* $\left(\frac{1}{131}\right)^3 \times 1 = 00$

2 " " " $\left(\frac{2}{131}\right)^3 \times 1 = 00$

10 " " " $\left(\frac{10}{131}\right)^3 \times 1 = 00$

20 " " " $\left(\frac{20}{131}\right)^3 \times 1 = 00$

30 " " " $\left(\frac{30}{131}\right)^3 \times 1 = {\cdot}01$

$$35 \text{ ft. from the } P.\ T.\ C. \left(\frac{35}{131}\right)^3 \times 1 = \cdot 02$$

$$40 \text{ " " " } \left(\frac{40}{131}\right)^3 \times 1 = \cdot 03$$

$$45 \text{ " " " } \left(\frac{45}{131}\right)^3 \times 1 = \cdot 04$$

$$50 \text{ " " " } \left(\frac{50}{131}\right)^3 \times 1 = \cdot 06, \text{ etc.}$$

$$125 \text{ " " " } \left(\frac{125}{131}\right)^3 \times 1 = \cdot 87$$

$$130 \text{ " " " } \left(\frac{130}{131}\right)^3 \times 1 = \cdot 97$$

This is a very convenient length, 131 ft., of transition, which being doubled gives us 262 ft. of transition curve. If we always make the offset half the degree of curvature, that is $\frac{o}{D} = \frac{1}{2}$, it will be seen from (5) that we will always get this length, and indeed it can be nearly always used to good advantage. But as we will sometimes require other lengths or other offsets, we can prepare, say, half a dozen such little tables as the above, and put them in the back of the field-book.

We can have, perhaps,

$$\frac{o}{D} = 1 \quad \therefore \quad a = 186 \text{ ft.}$$

$$\frac{o}{D} = \frac{1}{2} \quad \therefore \quad a = 131 \text{ ft.}$$

$$\frac{o}{D} = \frac{1}{4} \quad \therefore \quad a = 93 \text{ ft.}$$

$$\frac{o}{D} = 2 \quad \therefore \quad a = 262 \text{ ft.}$$

$$\frac{o}{D} = 4 \quad \therefore \quad a = 372 \text{ ft.,}$$

and tabulate the ordinates as above for the transition curve for every 5 ft., say, from the *P. T. C.* to the *P. C.* One table does for each length of '*a*.' Make it out for an offset of 2 ft., and then, if your offset is any other multiple of 2, multiply the ordinates so tabulated by this multiple mentally when you are putting in the transition curve to get the proper ordinates.

So far we have only given the curve for the first half of the total, that is from the *P. T. C.* to the *P. C.*

I intend to show that an exactly similar formula can be used for the second part of the transition curve, only it will be measured from the *P. C.*[1] backwards from the point *B* in fig. 1 to the *P. C.* at the point *P*. But the curvature will be supposed to vary as the distance from *B* measured *along the circular arc towards A* and up to the *P. C.*

Now, since the curvature of the straight line *F K* is constant (and always zero), its curvature can properly be represented by a horizontal straight line as a base of no curvature. Let *A B* (fig. 4) represent this.

Fig. 4.

Again, since the curvature of the transition varies as the distance from the *P. T. C.*, its curvature can be properly represented by a straight line beginning at *A* and making an angle with *A B* depending on the constant $2K$ in (1). Let such a line be *A C* and let $A B = a$ in fig. 1. Then the ordinate from *A B* to *A C* at any point x from *A* will represent the curvature at that point, and of course will represent the difference of curvature between the tangent and the transition curve at that point.

Now, a similar diagram will apply to the second part of the transition curve. For since the curvature of the circular arc is constant, its curvature can properly be represented by a horizontal straight line at a distance of $\frac{1}{R}$ above the base line *A B* of no curvature, since the curvature of the circle is $\frac{1}{R}$. Let this be represented by the straight line *D E*. Now make *D E* equal to the length of the circular arc between the *P. C.* and the *P. C.*[1]

and also equal to the length of tangent from the *P. T. C.* to the *P. C.*, and also, of course, equal to *A B* in fig. 4.

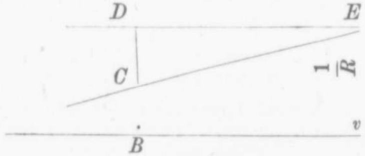


FIG. 5.

Then, since at the *P. C.*[1] the curvature of the transition curve is equal to that of the circular arc, its curvature at this point can properly be represented by the ordinate from *E* to the base line of no curvature, the same as that of the circular arc. And since the decrease in the curvature of the transition curve is proportionate to the distance from the *P. C.*[1] measured backwards towards the *P. C.*, this decrease of curvature can properly be represented by a straight line drawn from the point *E* in (fig. 5) below *D E*, and making an angle with it depending on the constant *K* in (1), and this angle will, of course, be equal to the angle *B A C* in (fig. 4) and the curvature of the transition at any point x from the *P. C.*[1] will be represented by the ordinate between this inclined line and the base line of no curvature. Then let the curvature of the transition curve be represented by the strait line *C E*. Then the difference of curvature between the circle and transition at any point x from *E* will be represented by the ordinate from *D E* to *C E* in fig. 5.

Now, on examining figs. 4 and 5, it will at once be seen that they nicely fit into one another, and that one is not complete without the other. Of course *B C* in fig. 4 is equal to *B C* in fig. 5; that is, the curvature of the two parts of the total transition is the same at the *P. C.* or at point *P* in fig. 1. And as the second part of the transition curve recedes from the circular arc at any point with the same angle as the first part of the transition recedes from the tangent at the corresponding point, both depending equally on the constant k in (2), and as the length of the circular arc is equal to the length of the tangent from the *P. T. C.* to the *P. C.*, it follows that the inclination of the first part at *P* to the tangent *F K* is equal to the inclination of the second part at *P* to the tangent of the circular arc at *P. C.*, and consequently the first and second parts of the transition have a common tangent at the point *P*, and from (5) it is evident that the point *P* must bisect the offset at the *P. C.*

It will be seen from the explanation above that the same equations will apply to the second part of the transition curve as for the first part, with this distinction: the formulæ for the second part represent not the total curvature, or tangent, or ordinate, as in the formulæ for the first part, but represent the *difference* between these functions of the circular arc and the transition curve. Then the above equations are identical for the two parts.

Now, since the length of the tangent from the *P. T. C.* is so very nearly equal to that of the first part of the transition, and the length of the circular arc is so very nearly the same length as the second part of the transition curve, I am pretty safe in asserting that a passenger riding in the train passing over this part of the track would not be able to say whether the variation in curvature was calculated according to the length of the transition curve or of the combined lengths of the tangent and the circular arc.

The second part of the transition is put in exactly as the first part excepting that you commence at the *P. C.*[1] and measure *backwards* towards the *P. C. along the circular arc*, and measure the ordinates outward from the circular arc in fig. 1. But in practice the circular arc will be the distance of the offset "*o*" outside of the circular arc (which we will call the inner circular arc), and will be a continuation of the tangent *F K* instead of the line *D A*, and will have its *P. C.* at the point *K* instead of at *A*. Consequently it will be necessary to measure the ordinates for the transition curve *inwards the distance of the offset, minus the tabulated ordinates*, or, if "*y*" is the tabulated ordinate from the inner circular arc to the transition curve, it will be necessary to measure the distance (*o—y*) *inwards* instead of the distance "*y*" *outwards*. I always run in the regular circular curve in location just as if there were to be no transition, and then, just before construction, go along the line and, with an assistant, pull up the stakes in the circular curve and move them in the distance (*o—y*) for the second part of the transition. But for the first part of the transition the stakes are moved in only the distance "*y*."

Two men can do this as fast as they can walk along, and one pull up the stakes and move them in the proper distance, while the other gives that distance from the little tables. If the stakes are set 50 ft. apart in a transition of 262 ft., only five stakes will have to be moved on each end of the circular curve, and of course all the stakes on the remaining part of the circular curve must be moved in the whole distance of the offset "*o*," which will give an idea of the rapidity with which this can be done. A number of miles can be done in a few hours even where there is considerable curvature, as in a hilly country.

It must be remembered that this transition is composed of two distinct parts which happen to fit in together nicely. But one is measured *along a circular arc backwards*, while the first is measured *along the tangent forwards.*

I always keep the same hubs as were used for the ordinary circular curve, and when the transition is put in no other hubs are required, but the old ones must be preserved, and if it is necessary to run in the line again the circular arc is run in first as if no transition were going to be put in, and at any time convenient the stakes moved as directed above. In the transit book it is only necessary to note down the offset chosen or "*a*," as the degree of curvature will have been already noted for the ordinary circular curve. Then the transition curve can be put in any time afterwards.

And then we have to consider that by this method we can lay down the transition curve from lines already established, and without any calculation more than can easily be done mentally by any one who can do ordinary multiplication.

As said above, this method, without using the corrections, is only an approximation, but a very close approximation, so close, in fact, that as far as making the trains ride easily over it, it is as good as if the variations of curvature were calculated to vary with the distance from the *P. C.* along the transition curve itself. But as there may be times when it will be neeessary to know the exact difference between the length of the total transition and "2 *a*", and as this is a very easy matter to find, and the resulting formula is very simple, the solution is given in what follows.

For the first part of the transition let

s = distance of any point from the *P. T. C.* measured along the transition curve, and let

x = the corresponding distance measured along the tangent.

Then we have

$$ds = \sqrt{dx^2 + dy^2} = \sqrt{1 + k^2 x} \; dx \text{ from } (2)$$

$$= \left(1 + \frac{k^2 x^4}{2} - \frac{k^4 x^8}{8} + \frac{k^6 x^{12}}{16} - \&c.\right) dx$$

$$\therefore\; s = \left(x + \frac{k^2 x^5}{10} - \frac{k^4 x^9}{72} + \frac{k^6 x^{13}}{208} - \&c.\right)$$

When $x = a$, $s = s_1'' =$ length of first half of the transition curve

$$\therefore\; s_1'' = a\left(1 + \frac{k^2 a^4}{10} - \frac{k^4 a^8}{72} + \frac{k^6 a^{12}}{208} - \&c.\right)$$

$$s_1'' = a\left(1 + \frac{D^2 a^2}{10\,(22920)^2} - \frac{D^4 a^4}{72\,(22920)^4} + \frac{D^6 a^6}{208\,(22920)^6} - \&c.\right)$$

$$= a\left\{1 + \frac{1}{10}\left(\frac{D\,a}{22920}\right)^2 - \frac{1}{72}\left(\frac{D\,a}{22920}\right)^4 + \frac{1}{208}\left(\frac{D\,a}{22920}\right)^6 - \&c.\right\}$$

Writing b for $\left(\frac{D\,a}{22920}\right)$ we have

$$s_1'' = a\left\{1 + \frac{b^2}{10} - \frac{b^4}{72} + \frac{b^6}{208} - \&c.\right\}$$

$$\therefore\; \frac{s_1''}{a} = 1 + \frac{b^2}{10} - \frac{b^4}{72} + \frac{b^6}{208} - \&c.$$

$$\text{and } s_1'' - a = a\left\{\frac{b^2}{10} - \frac{b^4}{72} + \&c.\right\} \;\ldots\ldots\; (6)$$

Thus we see that the ratio $\frac{s_1''}{a}$ depends only on the product $D\,a$. If we make this product 1310,

then $b = \cdot 057155$

$$\therefore\; \frac{s_1''}{a} = 1 + \cdot 000326669 - \cdot 0000001482 + \&c.$$

$$= 1 \cdot 00032652.$$

Now if $a = 131$ ft. and $D = 10°$

$$s_1'' - a = .043 \text{ ft.}$$

$$= \text{half an inch.}$$

It will be seen that it is a very simple matter to find the value of the ratio $\frac{s_1''}{a}$ from the above formula, as with a table of logarithms one logarithm will do for all the terms, and when this logarithm is multiplied by the respective exponents of "b" all the values of b^2, b^4, &c., can be found at once. But it is evident that b^2 and b^4 are all the terms that need to be considered to secure the greatest accuracy ever required.

It is also easy to find the difference in length between the second part of the transition curve and that of the circular arc, from which it is measured off, in the following manner:

Let s be the length of any point from the *P. C.*[1] measured along the transition curve, and x the length of the corresponding point measured along the inner circular arc. As before, let y = ordinate measured along the inner circular arc, and let R_1 = radius of curvature of the inner circular arc, and D_1 its degree of curvature, and a_1 its length measured along the inner circular arc. Then,

$$y = \frac{k\,x^3}{3}$$

$$\therefore\ \frac{ds}{dx} = \frac{R_1 + y}{R_1} = \frac{R_1 + \frac{k_1\,x^3}{3}}{R_1} = 1 + \frac{k\,x^3}{3\,R_1}$$

$$\therefore\ s = x + \frac{k\,x^4}{12\,R_1}$$

when $x = a_1\ s = s_2''$

$$\therefore\ s_2'' = a_1\left(1 + \frac{k\,a_1^3}{12\,R_1}\right) = a_1\left(1 + \frac{D_1^2\,a_1^2}{3\,(22920)^2}\right)$$

$$\therefore\ \frac{s_2''}{a_1} = 1 + \frac{D_1^2\,a_1^2}{3\,(22920)^2}\quad \therefore\ s_2'' - a_1 = a\,\frac{b^2}{3}\ \ldots (6\tfrac{1}{2})$$

But as in practice the last half length of the transition curve is measured along the outer circular arc, which lies just the distance of the offset "o" radially outward from the inner circular arc, what we really want is not the ratio $\frac{s_2''}{a_1}$ but the ratio $\frac{s_2''}{a}$. If we let D, a and R stand for similar functions of this outer circular arc, as follows, we can easily transform the above equation to give us this requirement, for $D_1\,a_1 = D\,a$ since the factors in each vary inversely as one another,

$$\text{and } a_1 = \frac{R - o}{R}\,a = \frac{5730 - o\,D}{5730}\,a$$

$$\therefore\ \frac{s_2''}{a} = \frac{5730 - o\,D}{5730}\left(1 + \frac{D^2\,a^2}{3\,(22920)^2}\right)$$

or using "b" to stand for $\frac{D\,a}{22920}$ we have

$$\frac{s_2''}{a} = \frac{5730 - o\,D}{5730}\left(1 + \frac{b^2}{3}\right)$$

$$\therefore\ a - s_2'' = a\left\{\frac{o\,D}{5730}\left(1 + \frac{b^2}{3}\right) - \frac{b^2}{3}\right\}\ \ldots . (7)$$

It will be noticed that in formula (5) the D is there the degree of curvature of the inner circular arc instead of that of the outer circular arc which is the one used in practice. So the correction for 'a' in (5) will take the form

$$a_0 = \sqrt{\frac{5730 - o\,D}{5730}}\,.\,a \;.\;.\;.\;.\;.\; (8)$$

consequently $\sqrt{\frac{5730 - o\,D}{5730}}\,.\,a$ must be substituted for a in all the formulæ.

And in the formulæ for the first part of the transition curve wherever 'D' occurs it is, of course, the degree of curvature of the inner circular arc or 'D.' So the form of correction which must be applied to it is

$$D = \frac{5730}{5730 - o\,D}\,D \;.\;.\;.\;.\;.\;.\; (8\tfrac{1}{2})$$

which expression must be substituted for 'D' everywhere in the formulæ for the first part of the transition but not in the second part.

This is only a correction of a correction, it will be noticed, and a small one at that if '$o\,D$' is kept small. Of course this correction would seldem need to be applied to more than the second term to give the greatest accuracy required. For instance, if we take the example given of the application of that formula where $D = 10$ and $a = 131$, and consequently 'o' $= 5$, the correction is only

$$\begin{aligned} &\cdot 0000287\,a \\ = &\cdot 0037 \text{ ft.} \end{aligned}$$

when applied to the second term, and the correction from the succeeding terms is practically infinitely small. But for larger values of '$o\,D$,' it will be seen from equation (8) the correction increases very rapidly.

Hence the total difference between '$2\,a$' and the total length of the two parts of the transition curve without considering the last correction can now be put in one single formula, which will be very simple.

For the first part,

$$s_1'' - a = a\left(\frac{b^2}{10} - \frac{b^4}{72}\right)$$

For the second part,

$$a - s_2'' = a\left\{\frac{o\,D}{5730}\left(1+\frac{b^2}{3}\right) - \frac{b^2}{3}\right\}$$

$$\therefore \text{ total difference} = d_2 - d_1$$

$$= d = 2a - s_1'' - s_2'' = a\left\{\frac{o\,D}{5730}\left(1+\frac{b^2}{3}\right) - \frac{b^2}{3}\right\} - a\left(\frac{b^2}{10} - \frac{b^4}{72}\right)$$

As the term $\frac{b^4}{72}$ will only have to be used when 'b' is exceptionally large indeed, it can be neglected for railroad work, and still the equation will give the correct result to a small fraction of an inch when small offset is used.

$$\therefore\ d = a\left\{\frac{o\,D}{5730}\left(1+\frac{b^2}{3}\right) - \frac{b^2}{3}\right\} - a\,\frac{b^2}{10}$$

$$= a\left\{\frac{o\,D}{5730}\left(1+\frac{b}{3}\right) - \frac{b^2}{3} - \frac{b^2}{10}\right\} \ldots\ldots (8\tfrac{3}{4})$$

If the maximum degree of curvature used be 10° then the last 'd' will be correct to less than half an inch, even when 'a' is as great as 262 ft.

It will be evident from the above that it would be waste of time to make out a set of tables for these corrections, as they will not have to be applied in more than one case in a thousand, and for that isolated case it would be better to calculate the correction than to carry a needless load around so long.

As for a set of tables for the ordinates along 'a' for the transition curve, a dozen such as the one given above would be enough and to spare. In every case where it is not desirable to choose a particular offset so as to fit the ground, it is likely that the one where 'a' $= 131$ and $\frac{o}{D} = \frac{1}{2}$ will almost always be employed. This method has this in its favor, among other things, that it is so simple and easy to put in on the ground that, no matter how indolent or ill-informed an engineer might be, he would in all probability put it in. And then, after it is in, it is as good as the best. I can't help but think that it is the one that will always be used when all engineers come to understand the importance of transition curves, and to see how easy it is to put this one in on the ground. It is simplicity itself, is as good as if it were better, and can be accurately calculated as to length, &c., from the formulæ given above, when such calculation is necessary, which will not be once in a thousand times. Enough is as good as a feast and much more wholesome. And last but not least, it can be put in at any time after location by simply moving the stakes as directed above.

If, as might happen sometimes, it is necessary to have a result infinitely correct, the combination of (6), (7)(8) and ($8\frac{1}{2}$) will give it, and then we have

Total length of transition curve:

$$s_2'' + s_1'' = \sqrt{\frac{5730 - o\,D}{5730}}\,.\,a \left[\left\{ \frac{5730 - o\,D}{5730} \left(1 + \frac{\left(\sqrt{\frac{5730 - o\,D}{5730}}\,.\,D\,a \right)^2}{3\,(22920)^2} \right\} + 1 + \frac{\left(\sqrt{\frac{5730}{5730 - o\,D}}\,.\,D\,a \right)^2}{10\,(22920)^2} \right. \right.$$

$$\left. \left. - \frac{\left(\sqrt{\frac{5730}{5730 - o\,D}}\,.\,D\,a \right)^4}{72\,(22920)^4} + \&c. \right\} \right] \quad \ldots\ldots (9)$$

In which the first line on right of equation is the length of the second part of the transition curve and the second line is the length of the first part of the transition curve. Whenever the product '$o\,D$' is large this last equation (9) should be used, and for accuracy it should be run out the same number of terms as is done above. If '$o\,D$' is small formula ($8\frac{3}{4}$) can be used. If '$o\,D$' is very small no correction is required.

With this correction the method is practically correct, and when it is required to find the difference in length between the total length of the transition curve and '$2\,a$' this last formula (9) should always be used, unless the offset 'o' is small. For accuracy it should be run out the same number of terms as is done above.

If it should be desired to run in new tangents parallel to the old ones from an already established circular curve, this circular curve would then become the inner circular arc, and the second part of the transition would be measured along it. If it should be required to know the difference in length between the whole transition and '2 a,' this can be found by combining (6) and ($6\frac{1}{2}$) and we get

$$d = a\left\{\frac{b^2}{10} - \frac{b^4}{72} + \&c.\right\} + a\frac{b^2}{3}$$

$$= a\left\{\frac{13\,b^2}{30} - \frac{b^4}{72} + \&c.\right\}$$

$$= a\,\frac{13\,b^2}{30} \text{ very approximately.}$$

The transition should be put in as in the other case, of course, except the ordinate 'y' for the transition curve is measured *outward from the circular arc*, and (o—y) *outward from the old* tangent or 'y' *inward from the new* tangent in all cases in same direction as the other case.

DISCUSSION.

Mr. H. Irwin said it is evident that engineers, both in this country and in the United States, have lately been turning their attention to the use of transition curves, as there are now published three or four books of tables on their use, and doubtless others are in preparation. There have also been three methods for laying out such curves lately brought before this Society. Mr. H. Irwin

The principal reason why transition curves should be used, viz.: that the elevation of the outer rail and the degree of curvature should simultaneously increase from zero till the main curve is reached, has been fully discussed by Mr. Wicksteed in his paper published in part 1, volume 5, of the Transactions of this Society. (See page 188).

When the conditions above mentioned are fulfilled and the transition curve is sufficiently long (say at least 200 feet), trains will ride much more smoothly over curves than they do when transition curves are not used. All roadmasters and section men know very well that curves must be eased off at the ends and invariably put in transition curves of their own, with the inevitable result that there is a hump where their transition curve joins the main curve. Were transition curves properly laid out at first there would be no hump. Any one who has made a carefull survey of a piece of old track knows that it is impossible to tell where the curves begin or end, and engineers who have plotted such a survey on a large scale are well aware that the ends of curves are extremely irregular.

A member remarked to the speaker a few days ago that it would be useless to go to the trouble of putting in transition curves, since on an old piece of track originally laid out as (say for example) a 5-degree curve will generally be found to be somewhat like a short piece of a 1-degree, another piece of a 2-degree and the rest about a 6-degree curve. This line of argument is rather in favor of, than against, transition curves, since it shows that the section men, who seem to understand the proper working of curves better than many of those who are their superiors in office, have done their best to ease the curve, but could not do so without sharpening the main curve, which would not have been necessary had transition curves been put in at the ends at first. The speaker, indeed, is quite at a loss to understand the reason why railway managers and superintendents do not insist on transition curves being used on all new roads, since by using a good system very little time would be lost in comparison with the benefit to be derived, and in many cases grades might be considerably reduced and heavier trains run, were sharper curves used with proper transition curves at their ends. As the data necessary for laying out the curves are obtained on entirely different lines by the authors of the three methods above referred to, and as the calculations to be made in the field would be different for each, the speaker has thought that it might be of some practical use to compare the actual working of these methods by means of an example as nearly as possible similar in each case.

The first paper brought before the Society is that by Mr. Wicksteed, above referred to. It was read on the 7th May, 1891, but the speaker understands from Mr. Wicksteed's paper that he had been using the method brought forward at that date for some time previously. This method, and the curve laid out by it, is identical with that referred to in Mr. Lordly's paper which was read on the 28th October last. The length of the sub-tangent and the angles to be laid off to fix points on the curve are calculated in precisely the same manner in both cases. The great difference in the two papers above referred to is that Mr. Wicksteed's deals principally with the practical side of the question, and with regard to the theoretical side gives only the proof of the simpler parts, and in mentioning the more difficult part, simply states that "*it can be demonstrated.*" Mr. Wicksteed's paper, therefore, appears as it really is, very concise and simple, and for those who are satisfied with the term "*it can be demonstrated,*" is much more satisfactory than if the complete demonstration were given.

Mr. Lordly's paper, on the other hand, gives the method of determining the equation of the curve and of working out various quantities necessary to make a set of tables for facilitating the calculation of the length of the sub-tangent, of the angles to be laid off to points on the curve, of the abscissæ and ordinates to the curve and of other values mentioned on page 3. It is unfortunate, however, that Mr. Lordly did not explain more clearly his method of calculating the deflections from the tangent to the points on the curve, and that he condensed parts of the wo.king out of his formulæ so much as to make it rather difficult in places, for those who do not keep their mathematics quite at their fingers' ends, to follow his line of reasoning. The writer thinks that in a theoretical paper clearness should never be sacrificed to brevity. Mr. Lordly's paper is, however, more satisfactory than Mr. Wicksteed's in so far as that any one, who wishes to thoroughly examine it, and to work out for himself the entire proof of the system, can satisfy himself of its correctness. It appears, from Mr. Lordly's paper, that the system he has brought forward was proposed and used by Mr. Ellis Holbrook in laying out curves for the P. C. and St. L. Ry. in 1882, or nine years before Mr. Wicksteed's paper was read, and that Professor Crandall, of Cornell University, has lately worked up a set of tables to facilitate its use.

Mr. Lordly states that the *curve* is strictly mathematically correct. This is, no doubt, true, but certain *approximations* have to be used in order to make it workable. It must also be remembered that rails cannot readily be bent to suit the curve. The best that can be done is to bend the rails to the mean curvature which they *should* have at the ends, since section men or track-layers could never be got to bend a rail to different curvatures. The result of this is that the rails, when laid, will not have a common tangent at the joints, so that the theoretical advantage of this curve over one made up of rail lengths of circular curves vanishes, so far as this point is concerned, because in the latter case each rail can be bent to suit its own curve, and the rails, when laid, have a common tangent at the joint. This is, of course, a point of no great practical importance, but, so far as it goes, the rail lengths of circular curves have the advantage. The curve proposed by Messrs. Holbrook, Wicksteed and Lordly seems to the speaker to be extremely neat and simple to lay out, and, provided no tables were furnished, to be easier to lay out than any he has seen.

It is probable, however, that, on account of certain approximations used in calculating the length of the subtangent and of the angles to be turned off to the various points on the transition curve, the curve would not close very well on the P. T. T. when the main curve is sharper than six degrees, especially where the transition curve is over two hundred feet long. The speaker, however, hopes to be able to lay out a few curves shortly by each method, and to see how they work on the ground under various conditions.

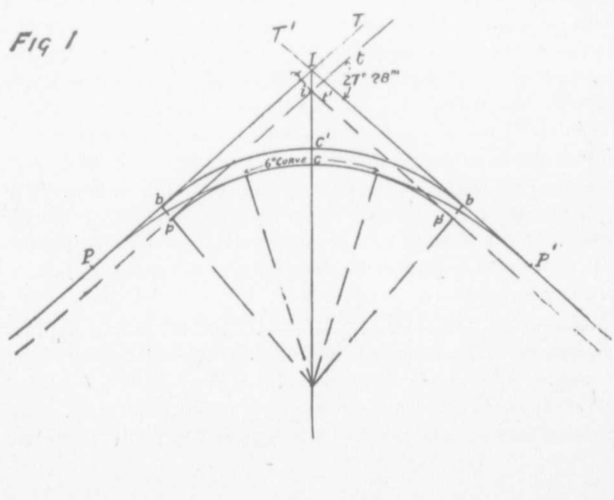
In the case of the method proposed by the speaker at page 200, part I, volume 5, of the Transactions of this Society, which he first worked out in the spring of 1888, and in the use of which tables are necessary, and have been calculated to suit almost every possible case, no approximations whatever are used.

The speaker next proposes to give the actual working required in the field according to the systems above referred to, in so far as he understands them, no working figures being omitted.

As Mr. Lordly chose for example a 6-degree main curve with a transition curve 300 feet long on each side, the curvature increasing gradually at the rate of 1° 12′ for each 60 feet, the speaker will use the same example to illustrate Mr. Wicksteed's method, no tables being used, except such as are found in "Shunk" or similar works. The intersection angle is taken as being 27° 28′. The speaker proposes to give first, in consecutive order, a brief demonstration of the method used by Messrs. Wicksteed and Lordly, giving the working figures by themselves.

The first quantity to be calculated is the subtangent P. I. (See fig. 1.)

Fig 1



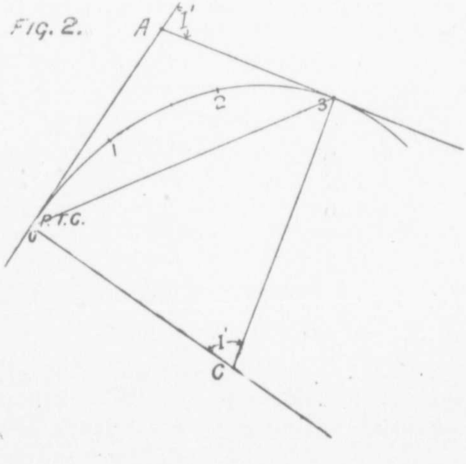
This is found as follows:

P I = P b + p i + α I. P b is assumed as being = 150′, p i = the subtangent of on ordinary 6° curve, having the intersection angle = 27° 28′, which can be found in "Shunk," and α I = i α $\tan\frac{\alpha i t}{2} = i\,\alpha \tan\frac{27^\circ\,28'}{2} = i\,\alpha \tan 13^\circ\,44'$. Now $i\,\alpha = b\,p$, $b\,p$ = double the offset to the curve at b, and the offset to the curve at b is = the offset to the curve at the end of the first 60 feet multiplied by $\left(\frac{5}{2}\right)^3$, since the offsets vary as the cube of the distance from the P. T. C. and $150' = \frac{5}{2} \times 60'$.

The angles to be turned off to each point on the transition curve are calculated on the assumption that they are equal to one-third of the total central or deflection angle taken up by the curve from the P. T. C. to each point. The speaker will show further on that this assumption is practically correct, as Mr. Wicksteed simply says that it can be demonstrated, and Mr. Lordly does not give any proof either. It may fairly be assumed that it would be necessary to move up the transit to an intermediate point on one of the two transition curves, say at point 3.

Since the tangential angles are one-third of the total central angles from the P. T. C. up to any point, it follows that on ar-

FIG. 2.



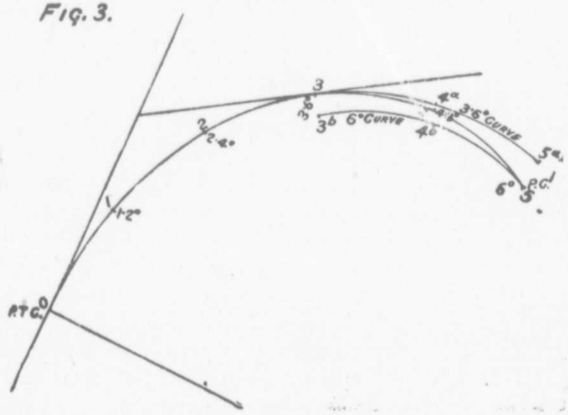
riving at that point on the transition curve, the tangent can be laid out by sighting at the P. T. C. and turning off double the tangential angle from the P. T. C.

This is readily proved. (See fig. 2.) Let the P. T. C. be at O, let 1, 2 and 3 be points on the transition curve. Let I′ be the intersection angle which the tangent at 3 makes with the tangent at the P. T. C. Let C be the point of intersection of the normals to the curves at O and 3.

Then $O C 3 = I' = A O 3 + A 3 O$, but $A O 3 = \frac{O C 3}{3} = \frac{I}{3}$ $\therefore 3 \times A O 3 = A O 3 + A 3 O$, or $2 \times A O 3 = A 3 O$. Q. E. D.

Having set up at 3 and laid off the tangent at that point, the tangential angles to points 4 and 5 must be found. The speaker has not been able to find in either Mr. Wicksteed's or Mr. Lordly's paper any demonstration of how this is to be done, but the theory made use of by Mr. Wicksteed for running the curve from the P. T. to the P. T. T. (see page 190, part 1, volume 5) would seem to suit for this purpose. This theory is, that in continuing from the P. T. to the P. T. T. the transition curve would leave the main curve, if it were produced, at the same rate that the transition curve at the P. T. C. leaves the tangent at that point. (See fig. 3.) The speaker has not had time to verify the correctness of this theory, which at first sight appears to be an

FIG. 3.



approximation which would be practically correct for such flat curves as are used on railways. Certainly, in running from the P. T. to the P. T. T. the rate at which the transition curve leaves the main curve produced is the same as the rate at which the transition curve leaves the tangent at the P. T. C. (which is 1·2° per 60 feet in the example used herein), but it does not follow that the *angles* to be turned off from the tangent at the P. T. to the transition curve will be *exactly* equal to the difference between the angles to points on the main curve produced, and the angles to similar points on the transition curve from the tangent at the P. T. C.; indeed, for sharp curves, it seems as if it could not be true, as the long chords for similar points on the two curves would soon differ too much in length. However, as the angles used by Mr. Lordly to run from the P. T. to the P. T. T. are identical with Mr. Wicksteed's, and are, Mr. Lordly says, calculated in a different manner, the theory may be assumed to be practically correct for flat curves.

Now, being at said point 3, and having set on to the tangent at that point, it is necessary to set out a curve increasing in curvature from a 3·6° to a 6° curve in 120 feet, putting in a peg also at the end of the first 60′, at point 4, where the curvature is 4·8°.

At point 3 sight at point O, or P. T. C., and set off (1°04′·8) × 2 = 2°09′.6 to get on the tangent. (In the case of the transition curve of the speaker's system, which most nearly coincides with that under discussion, viz, five lengths of 60 feet each of a 1°, 2°, 3°, 4° and 5° curve respectively, the corresponding angle from the tables is 2° 12′.)

Now, supposing the 3°·6 curve at point 3 to be produced, the transition curve keeps inside of the 3°·6 curve at the rate of 1°·2 for every 60 feet, or at the same rate that the transition curve beginning at the P. T. C. leaves the tangent at that point. Therefore, supposing the above theory to hold good at intermediate points, we should set off from the tangent at point 3 an angle =

$$\frac{3°\cdot 6 \times .6}{2} = 1°\cdot 08 = 1°04'.8.$$

to reach the 3°·6 curve produced, and 7′·2. (equal to the angle from the P. T. C. to point No. 1) to reach point 4, or a total angle of 1° 12′ from tangent to point 4. (In the speaker's system the tables give 1° 12′ for the corresponding angle.) Similarly, to get point 5 we set off $\frac{3{\cdot}6^\circ \times 1{\cdot}2}{2} = 2^\circ{\cdot}9'{\cdot}6$ to reach the $3^\circ{\cdot}6$ curve produced, and $28'.8$ more to reach point 5, or a total angle of $2^\circ{\cdot}38'{\cdot}4$ from the tangent at No. 3 to reach point No. 5 (in the speaker's system the tables give $2^\circ\ 33'$ for the corresponding angle).

As the transition curve mentioned above, which in the speaker's system would correspond most nearly to the present example, the speaker thinks that, as the angles just given differ so little, the last mentioned theory will apply to intermediate points. Having arrived at the P. C'., or point 5, the angle to be turned off to get on the tangent, when sighting back to point No. 3 (an intermediate point) instead of to point O, or the P. T. C., must be found. Messrs. Wicksteed and Lordly do not give definite information on this point, so far as the speaker can see, but it is evident they would turn off the same angle to get on the tangent as they would use to set point 3 from point 5 in running from P. T. to P. T. T. Therefore, applying the last mentioned theory to this case also, we see that were the main 6° curve produced from point No. 5 towards No. 3, it would leave the transition curve at the same rate as the transition curve leaves the tangent in going from P. T. C. (See fig. 3.) Therefore, sighting at point No. 3 and turning off an angle equal to $\left(\frac{6^\circ}{2} \times 1{\cdot}2 - 28'{\cdot}8\right) = 3^\circ\ 07'{\cdot}2$, the tangent is reached at point No. 5. (In the speaker's system the tables give the corresponding angle as $= 2^\circ\ 51'$.) Having laid off the tangent at 5, the main curve is put in as usual. Neither Mr. Lordly or Mr. Wicksteed deal with the case where it would be necessary to set the transit at (say) points Nos. 2 and 4; but the last mentioned theory would meet these also; additional figuring would, however, be required. In the speaker's system the tables give all the angles necessary to lay out the tangent at any point when sighting at any other, as well as to lay out any point from any other. Having reached the P. T., the tangent at that point is laid off from the main curve in the usual manner.

The theory above mentioned, that the transition curve would leave the main curve, if produced, at the same rate as the transition curve leaves the tangent at the P. T. C., is then used te run in the transition curve backwards, in Mr. Wicksteed's system. The working out need not be again explained.

Mr. Lordly would use Prof. Crandall's tables instead of figuring out the angles, and very wisely so, since figuring done in the field wastes time, and is always liable to error.

The actual figuring necessary to calculate the various functions of the curve above referred to, without the help of any tables except a "Shunk," as proposed by Mr. Wicksteed, can now be collected, no further explanation being necessary.

Sin 0° 8′ 0·0023271
Sin 0° 7′ 0·0020362

Diff. for 1′ 0·0002909

Diff. for 0′.2 0·0000582
Sin 0° 7′ 0·0020362

Sin 0° 7′.2 0·0020944 Log. 125 = 2·09691
60

0·1256640 and Log. = $\bar{1}$·09921

1·19612

P b = 150·00 Log. 4 = 0·60206
I α = 0·96 Log. $i\ \alpha$ = 0·59406 and $i\ \alpha$ = 3·927
$p\ i$ = 233.38 Log. tan 13° 44′ = 9·38808

384.34=subtan. 10+Log. I α = 9·98214 I α = 0·96

Subtangent for 27° 28′ = 1400·3, by Shunk:

1-6th = 233·38 = $p\ i$.

Angles from P. T. C.:

To No. 1 = $\frac{3'.6}{3} \times 6 = 7'{\cdot}2$

To No. 2 = 7·2 × 4 = 28′·8

To No. 3 = 7·2 × 9 = 64′·8 = 1° 04′·8

Angle to get on tangent at point 3:

(1° 04′·8) × 2 = 2° 09′·6

Angles from No. 3 to Nos. 4 and 5:

3°·6	1° 04′·8
× ·3	× ·2
1°·08 = 1° 04′·8	2° 09′·6
7′·2	28′·8
From 3 to 4 = 1° 12′	2° 38′·4 from 3 to 5.

Angle from 5 to 3 to get on tangent:

$$6° \times 6 = 3{\cdot}6° = 3°\ 36'$$
$$0°\ 28'{\cdot}8$$
$$\overline{3°\ 07'{\cdot}2}$$

Angles from P. T. to P. T. T.:

$$3° \times {\cdot}6 = 1{\cdot}8° = 1°\ 48' - 7'{\cdot}2 = 1°\ 40'{\cdot}8$$
$$1°\ 48' \times 2 = 3°\ 36' - 29' = 3°\ 07'$$
$$1°\ 48' \times 3 = 5°\ 24' - 1°\ 05' = 4°\ 19'$$
$$7'{\cdot}2 \times 16 = 115'{\cdot}2 \qquad 1°\ 48' \times 4 = 7°\ 12' - 1°\ 55'{\cdot}2 = 5°\ 16'{\cdot}8$$
$$7'{\cdot}2 \times 25 = 180' \qquad 1°\ 48' \times 5 = 9°\ 00' - 3°\ 00' = 6°\ 00'$$

The last angle being 6°, or double the angle from the P. T. T. to the P. T., as it should be, which verifies the correctness of the last angle.

The figures just given are necessary in laying out the transition curves straight round all in one direction, on the supposition that the transit has only to be moved up once on the transition curve at one end. As it might easily have to be moved up twice at the other end, still further calculations might be necessary.

It will be noticed that in Mr. Lordly's paper on page 176, the angles from P. T. to P. T. T. do not appear to correspond with those given. This is due to the fact that Mr. Lordly commences to lay out the transition curve from the P. T. with his vernier reading 9° 00, and subtracts the angles to be turned off from the 9°, thus:

$$9° - (1°\ 40'{\cdot}8) = 7°\ 19'{\cdot}2$$
$$9° - (3°\ 07') = 5°\ 53'$$
$$9° - (4°\ 19') = 4°\ 41'$$
$$9° - (5°\ 16'{\cdot}8) = 3°\ 43'{\cdot}2$$
$$9° - (6°\ 00') = 3°\ 00'$$

Mr. Lordly has not given a full explanation of what Prof. Crandall's tables contain, but, so far as one can judge, they give only the angles to be laid off from the P. T. C. to the P. C'. and from the P. T. to the P. T. T., as well as the data mentioned on page 3 of Mr. Lordly's paper. Therefore, with the help of Prof. Crandall's tables, as proposed by Mr. Lordly, one could omit from the calculations just given, as necessary under the terms of Mr. Wicksteed's paper, the calculations of the angles from the P. T. C. and from the P. T. to the P. T. T., and also the calculations of $i\ \alpha$, which would reduce the figuring just given by about one-half, leaving it as follows, viz:

Subtangent for 27° 28′	=	1400·3
$p\ i$ = 1-6th	=	233·38
Log. 3·92 (= F)	=	0·59329
Log. tan. 13° 44′	=	9·38808
Log. I x	=	9·98137
I x	=	0·96
$p\ i$	=	233·38
$p\ b$	=	149·90 (tables)
Subtangent	=	384·24

Angle to get on tangent at point 3:

1° 04′·8 × 2 = 2° 09′·6

Angle from 3 to 4:

3°·6 × 3 = 1°·08 1° 04′·8
7·2

From 3 to 4, 1° 12′·0

Angle from 3 to 5:

1° 04′·8
2

2° 09′·6
28′·8

From 3 to 5, 2° 38′·4

Angle from 5 to 3 to set off tangent:

6° × ·6 = 3·6° = 3° 36′
0° 28′·8

3° 07′·2

It will be seen that the calculations given above are very simple, though they are quite as long as those required under the speaker's system, above referred to, which was adopted for two reasons, viz: (1) that no approximations are required, so that should the curve not close on the tangent at the P. T. T. the error would be due to mistakes made on the ground, and (2) that no knowledge of mathematics is necessary to understand the method employed beyond that necessary for running the ordinary curves. According to this system the subtangent is the only quantity to be calculated. When the speaker first began to work out a set of tables, the method by which he proposed to find the subtangent was that given at page 200, vol. 5, part 1, of the Transactions of this Society. It is as follows (see fig. 6):

Rad. of 5°—6° = 190·98 (tables)

b_4 = 766·58 (tables) — $I\frac{1}{2}$ = 13° 44′

Log. 957·56 = 2·98116 — b_8 = 9° 00′ (tables)

Log. sin. 4° 44′ = 8·91655 — 4° 44′

11·89771

Log. sin. 13° 44′ = 9·37549

2·52222 = Log. 332·83

Rad. of 1° = 5729·58 — A_4 = 3823·36

4156·19 — 4156·19

L G = 1573·39 and Log = 3·19684

Log. tan. 13° 44′ = 9·38808

10 × Log. subtan. = 12 58492

Subtangent = 384·52

It seems to the speaker that the calculations given above, which are all connected, could be worked out just as quickly as the calculations of the various quantities required under Mr.

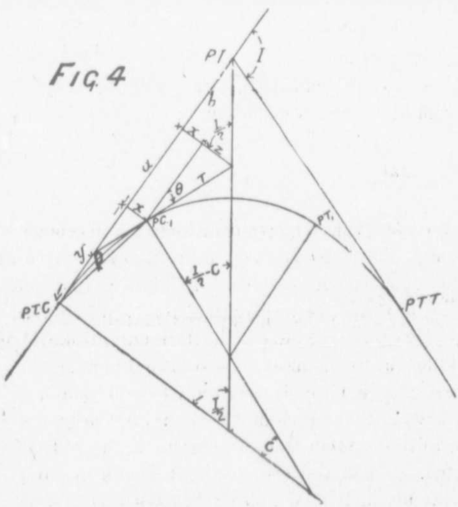


FIG. 4

Lordly's method, though the latter do not require so many logarithms to be looked for. After having worked out a few tables, however, the speaker thought it advisable to work out the ordinates and abscissæ to the various points on the transition curves, and these values supply another method of finding the length of the subtangent, which appears to be slightly shorter than the above and also more direct and more easily remembered.

The second method is as follows, viz (see fig. 4):

$$\text{Subtangent} = y + a + b$$

$$\text{do} \quad = y + \text{T} \cos \text{D} + (x + \text{T} \sin. \text{O}) \tan. \frac{\text{I}}{2}$$

where T equals the subtangent of a curve having the same curvature as the main curve and whose intersection or central angle is equal to I — 2 C.

Using the same example as before, we have therefore I = 27° 28′ and D = C = 9°, and the calculations would be as follows, viz:

I=27° 28′	Subtan. for 9° 28′ (Shunk)
2 C=18° 00′	=474·43
(I—2 C)= 9° 28′	T=1–6th= 79·071
Log. T= 1·89807	- - - - 1·89807
Log. cos 9°= 9·99462	Log. sin= 9·19433
10+Log. a= 11·89269	10+Log. T sin D= 11·09240
a= 78·101	T×sin D= 12·37
b= 7·239	x= 17·25 (tables)
y=299·206 (tables)	Log. 29·62=1·47159
Subtan.=384·546	Log. tan. 13° 44′ =9·38808
	10+Log. b=10·85967
	b=7·239

The calculations given above are all that are required under the speaker's system. The tables supply all other information necessary to lay out both transition curves and the main curve all in one direction, which is very *often* necessary in locating on rough or sidehill ground, where a few feet of lateral deviation will make a very great difference in the profile. The tables give the angles required to set off any point from any other, or to lay out a tangent at any point when sighting at any other, either

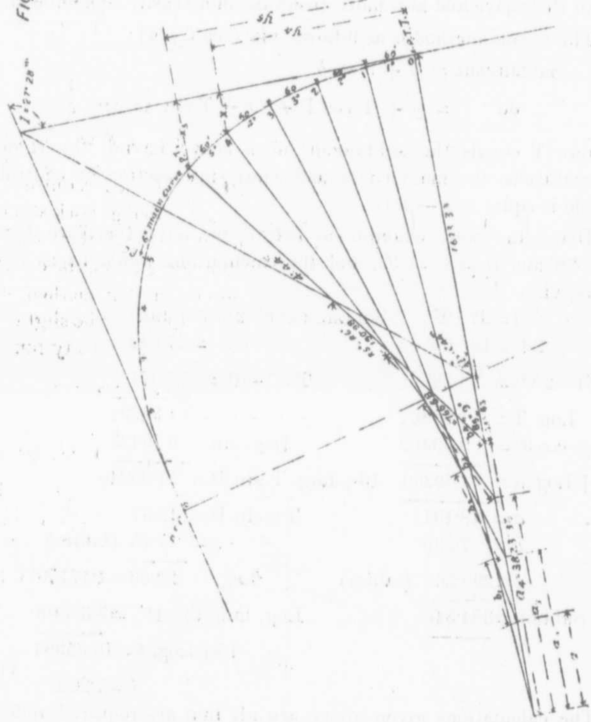


FIG. 6.

backwards or forwards. One set of tables is given herewith for a transition curve which is very nearly identical with that discussed above. (See fig. 6.) The curve is made up of 60-foot lengths of 1°, 2°, 3°, 4°, 5° and 6° curves respectively. Of course it is not necessary to use all the lengths. In the present example only five lengths are used, the main curve being 6°. In the accompanying set of tables, which are marked Series 21, there are, as in each of the series, three tables. In the table 1, column 1 gives radii of the various curves used in the transition, column 2 gives the differences of the radii, and columns 3 and 4 give the values of "*a*" and "*b*". (See fig. 6.) These four first columns are only used for finding the length of the subtangent under the first system above referred to, and they might be omitted only that they might be used to check the length of the subtangent as calculated by the second method above mentioned. Column 5 is used in finding the values in column 6. These latter are used in calculating the length of the subtangent by the second method above referred to, and correspond to the angle C of fig. 4. Columns 7 and 8 give respectively the abscissæ along the tangent, and the ordinates therefrom, to the various points on the transition curve, and the values in these columns would be used to lay out the curve by offsets, when that is necessary, and they are also used, as already explained, to calculate the length of the subtangent.

Table 2 gives first the angles (D) to be turned off from the tangent at any point to fix any other point, the various points being numbered from the P. T. C. or P. T. T. as 0, 1, 2, 3, etc. Thus, when at point 0 (the P. T. C. or P. T. T.), to set point 1 look in column 1 for D_{0-1}, which is found to be 0° 18′; to set point 3 look in same column for D_{0-3}, which is found to be 1° 24′. When at point 3, to set point 5 look column 4 for D_{3-5}, which is found to be 2° 33′, and so on, the numbers affixed to D indicating the points from which and to which the angle is to be turned off, always from the tangent at the point from which the angle is to be laid out.

It will be noticed that D_{0-1} is put down as also equal to F. $T._{0-1}$. "F. T." stands for the angle to be laid off when sighting *forward* to get on the *tangent*. These angles, as, for example, F. $T._{0-1}$, F. $T._{3-6}$, etc., would be used in running *backwards* from the P. T. to the P. T. T. (see fig. 6), the term *forwards* being

Transition Curves – Series 21 – 60 Feet each of 1°, 2°, 3°, 4°, 5° and 6° Curves respectively.

H. Irwin, Montreal, December, 1892.

Table 1.

1	2	3	4	5	6	7	8
Radii	Differences of Radii.	Values of "a"	Values of "b"	Values of "C" or Central Angle of each portion	Values of "B" or sums of Central angles to each point.	Values of "y" or Abscissæ from B.T.C. along Tangent.	Values of "x" or Ordinates from tangent at each point
1° = 5729·58 2° = 2864·79 3° = 1909·86 4° = 1432·39 5° = 1145·91 6° = 954·93 7° = 818·51	1°-2° = 2864·79 2°-3° = 954·93 3°-4° = 477·46 4°-5° = 286·48 5°-6° = 190·98 6°-7° = 126°·42	a_1 = 1910·04 a_2 = 2865·50 a_3 = 3439·55 a_4 = 3823·36 a_5 = 4098·98	b_1 = 955·07 b_2 = 955·46 b_3 = 860·76 b_4 = 766·58 b_5 686·68	C_1 = 0° 36m C_2 = 1° 12m C_3 = 1° 48m C_4 = 2° 24m C_5 = 3°·00m C_6 = 3° 36m	B_1 = 0°·36m B_2 = 1° 48m B_3 = 3° 36m B_4 = 6° 00m B_5 = 9° 00m B_6 = 12°·36m	y_1 = 59·999 y_2 = 119·996 y_3 = 179·929 y_4 = 239·719 y_5 = 299·206 y_6 = 358·143	x_1 = 0·314 x_2 = 1·511 x_3 = 4·396 x_4 = 9·418 x_5 = 17·249 x_6 = 28·492

Table 2.

Angles to be turned off from the tangent at any point to set any given point ahead and also the angles to be turned off to lay out the tangent at any point when set up at the latter point and sighting to any point forward.

1	2	3	4	5	6
D_{0-1} = 0°·18m = $F.T._{0-1}$. D_{0-2} = 0°·45m = $F.T._{0-2}$ D_{0-3} = 1°·24m = $F.T._{0-3}$ D_{0-4} = 2°·15m = $F.T._{0-4}$ D_{0-5} = 3°·18m = $F.T._{0-5}$ D_{0-6} = 4°·33m = $F.T._{0-6}$	D_{1-2} = 0°·36m = $F.T._{1-2}$ D_{1-3} = 1°·21m = $F.T._{1-3}$ D_{1-4} = 2°·18m = $F.T._{1-4}$ D_{1-5} = 3°·27m = $F.T._{1-5}$ D_{1-6} = 4°·48m = $F.T._{1-6}$	D_{2-3} = 0°·54m = $F.T._{2-3}$ D_{2-4} = 1°·57m = $F.T._{2-4}$ D_{2-5} = 3°·12m = $F.T._{2-5}$ D_{2-6} = 4°·39m = $F.T._{2-6}$	D_{3-4} = 1°·12m = $F.T._{3-4}$ D_{3-5} = 2°·33m = $F.T._{3-5}$ D_{3-6} = 4°·06m = $F.T._{3-6}$	D_{4-5} = 1°·30m = $F.T._{4-5}$ D_{4-6} = 3°·09m = $F.T._{4-6}$	D_{5-6} = 1°·48m = $F.T._{5-6}$.

Table 3.

Angles to be turned off to lay out a tangent at any point when set up at said point and sighting to any point behind, and also the angles to be turned off from the tangent at any point to set any given point behind.

1	2	3	4	5	6
$B.T._{1-0}$ = 0°·18m = D_{1-0}	$B.T._{2-0}$ = 1°·03m = D_{2-0} $B.T._{2-1}$ = 0°·36m = D_{2-1}	$B.T._{3-0}$ = 2°·12m = D_{3-0} $B.T._{3-1}$ = 1°·39m = D_{3-1} $B.T._{3-2}$ = 0°·54m = D_{3-2}	$B.T._{4-0}$ = 3°·45m = D_{4-0} $B.T._{4-1}$ = 3°·06m = D_{4-1} $B.T._{4-2}$ = 2°·15m = D_{4-2} $B.T._{4-3}$ = 1°·12m = D_{4-3}	$B.T._{5-0}$ = 5°·42m = D_{5-0} $B.T._{5-1}$ = 4°·57m = D_{5-1} $B.T._{5-2}$ = 4°·00m = D_{5-2} $B.T._{5-3}$ = 2°·51m = D_{5-3} $B.T._{5-4}$ = 1°·30m = D_{5-4}	$B.T._{6-0}$ = 8°·03m = D_{6-0} $B.T._{6-1}$ = 7°·12m = D_{6-1} $B.T._{6-2}$ = 6°·09m = D_{6-2} $B.T._{6-3}$ = 4°·54m = D_{6-3} $B.T._{6-4}$ = 3°·27m = D_{6-4} $B.T._{6-5}$ = 1°·48m = D_{6-5}

applied to working from the P. T. C. or P. T. T. towards the P. C. or P. T., respectively, or from a point of lower number to a point of higher number; and the term *backwards* being applied to running the curve, or sighting, from the P. C. or P. T. towards the P. T. C. or P. T. T., or from a point of higher to a point of lower number.

For example, in laying out the curve from the P. T. at point 5 up to point 3, and setting transit at the latter point, to find the angle to be set off at point 3 to get on the tangent, look in column 4 for F. $T._{3-5}$, which is found to be 2° 33′; therefore sighting *forwards* (or really backwards with reference to the direction in which the curve is being run) from point 3 to point 5, and setting off an angle of 2° 33′, the tangent at point 3 can be laid out.

Table 3 similarly gives the angles (B. T.) to be used in sighting *back* at any point to lay out the *tangent*. For example, in running from P. T. C. towards P. C., on arriving at point 3, and setting transit at that point, to find the angle to set out to get on the tangent, look in column 3 for B. $T._{3-0}$, which is found to be 2° 12′. The angles "B. T." are, of course, equal to the angles "D" of corresponding numbers; for example, B. $T._{2-0} = D_{2-0}$. These latter angles, as D_{2-0}, D_{4-2}, etc., are used in running the transition curve *backwards* from P. T. towards P. T. T., or from any point of higher number to a point of lower number. For example, being at point 5, to set point 4, having first set off on the tangent at point 5, look in column 5 for D_{5-4}, which is found to be 1° 30; to set point 3, D_{5-3} is found in the same column to be 2° 51′, and so on. Having transit at point 3, and having set off the tangent at that point by laying off F. $T._{3-5}$ = 2° 33′, to lay out points 2, 1 and 0 (or P. T. T.), look in column 3 for the angles D_{3-2}, D_{3-1} and D_{3-0}, which are found to be respectively 0° 54′, 1° 39′ and 2° 12′. It will be seen therefore that, having calculated the subtangent, it is a very simple matter to lay out the entire curve straight round with the help of the tables.

On referring to Mr. Lordly's paper, on page 176, it will be noticed that the subtangent, which is equal to x + T′, is given as being 149·9 + 234·44. This value of T′, however, is not quite correct, as T′ is really 234·34 or 0·96 + 233·38, as already given, making the subtangent equal to 384·24 (the correct value), while as calculated by Mr. Wicksteed's method it is equal to 384·33, the difference being due to taking n as 150′ instead of

149′·9. This difference is, of course, of very little consequence, and is only mentioned in order to explain how it arose.

As certain approximations are used in calculating the subtangent by Mr. Wicksteed's and Mr. Lordly's methods, the speaker has calculated the exact length of the subtangent by the second method used to calculate the subtangent in his system, using Mr. Lordly's values of x and y, which are, of course, slightly different from their corresponding values under the speaker's system. (See fig. 4.)

We have $T = R \times \tan \frac{1}{2}$ (I—D) and R = radius of main curve. In the example under discussion I = 27° 28′, D = C = 9°, ∴ $\frac{1}{2}$ I = 13° 44′, and $\frac{1}{2}$ (I—D) = 4° 44′; by Mr. Lordly's tables — y = 299·3 and x = 15′·68, R = 955′.

From these values the subtangent can be found to be 384.24, or exactly the value found by Mr. Lordly as corrected. It would appear therefore that the approximations used do not affect the value of the subtangent in the present case by any appreciable amount, provided that the values of x and y can be relied on.

It has been already mentioned that neither Mr. Wicksteed nor Mr. Lordly give any proof of the statement that the angle to be laid off from the tangent at the P. T. C., to set any point, is one-third of the total deflection angle, or total central angle, taken up by the transition curve from the P. T. C. up to said point. That the statement is practically correct may be shown as follows, viz:

Let the angle which the tangent to the curve at any point makes with the initial tangent be D, then D is evidently equal to the total central angle from P. T. C. up to said point.

From page 173 of Mr. Lordly's paper we have

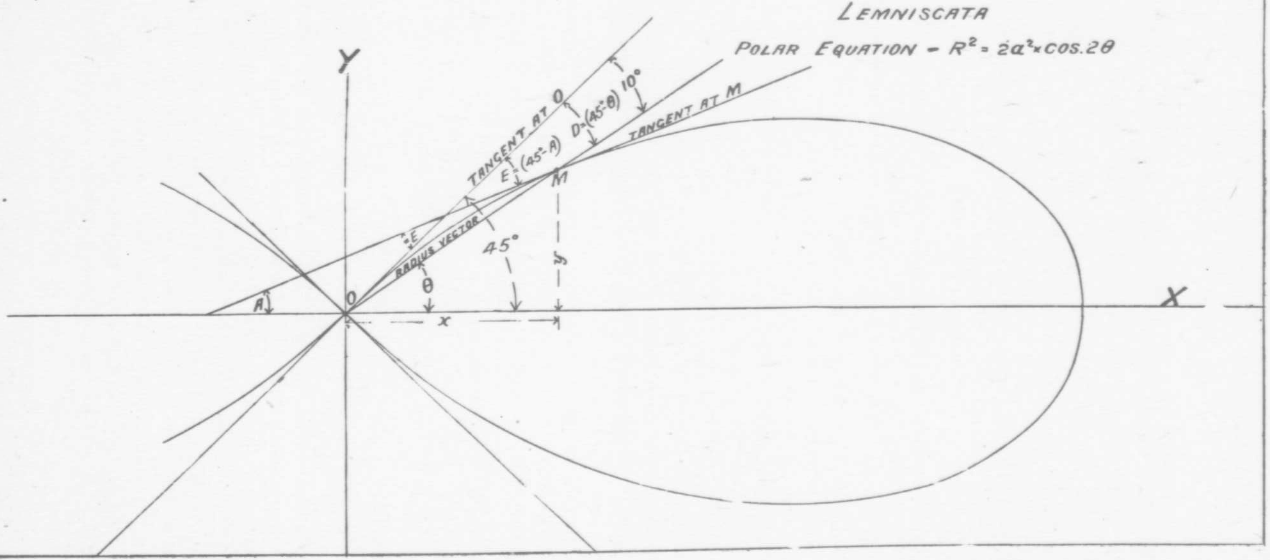
$$\tan. A = \frac{y}{x} = \frac{\frac{k.s^3}{3} - \frac{k^3.s^7}{42} + \frac{k^5.s^{11}}{1320} - \&c.}{s - \frac{k^2.s^5}{10} + \frac{k^4.s^9}{216} - \&c.}$$

Now neglecting all terms after the third and dividing out, we find that $\tan. A = \frac{k.s^2}{3} + \frac{(k.s^2)^3}{3 \times 5 \times 7}$, &c. Now $k.s^2$ equals the circular measure of D, and D would seldom exceed 12°, therefore $k.s^2$ would seldom exceed 0·20943, and therefore $\frac{k.s^3}{3 \times 5 \times 7}$ would seldom exceed 0·0000875; therefore when D does not exceed about 12°, the tangent of the angle (A) to be set off to any point, x, y, may be taken as equal to $\frac{k.\,s^2}{3}$, for, supposing A to be 4°, a difference of 0·0000875 in the value of the tangent would only amount to an error of about 20 seconds; we may assume therefore that $\tan A = \frac{k.\,s^2}{3}$. Now D in circular measure equals $k.\,s^2$, $\therefore \tan A = \frac{D}{3}$, and supposing D not to exceed 12°, we have to find what error might be introduced by assuming that since $\tan A = \frac{D}{3}$, therefore $A = \frac{D}{3}$. Now since D is not likely to exceed 12°, and the circular measure of 12° divided by 3 = 0.0698132 = tan 3° 59′ 36″ instead of 4°, we find that the assumption that $A = \frac{D}{3}$ makes the value of A about 24″ too large in an extreme case, but the assumption that $\tan A = \frac{k.\,s^2}{3}$ leaves A about twenty seconds too small; therefore the two approximations seem to about balance each other; however, any small errors in the angles to be turned off would be cumulative and would increase in proportion to the length of the curve.

Mr. Wicksteed has referred to the curve under discussion as identical with the cubic parabola for short arcs. The equation of the cubic parabola is $a^2 y = x^3$. Giving D and A the same values as above, we have $\tan A = \frac{y}{x}$, but $y = \frac{x^3}{a^3}$, $\therefore \tan A = \frac{x^2}{a^2}$, also $\tan D = \frac{d\,y}{d\,x} = \frac{3\,x^2}{a^2}$, $\therefore \tan A = \frac{1}{3} \tan D$. Now if we assume D to be 12°, we have tan A = 0·0708522, ∴ A = 4° 03′ 12″, but assuming $A = \frac{D}{3}$ we would have A = 4°, involving an error of 03′ 12″ in an extreme case, and indeed the cubic parabola cannot be very readily used for the purpose under discussion, except in some modified form as that proposed by Mr. Wellington of the "Engineering News."

Prof. McLeod has mentioned to the speaker that the curve under discussion is almost identical with the Leniniscata. This can be readily shown. The speaker has therefore plotted one-quarter of this curve as an example of the form it takes. (See fig, 5.) It will be noticed that the curve reverses at 0, as does

FIG. 5.



LEMNISCATA
POLAR EQUATION - R² = 2a² × COS. 2θ
TANGENT AT O
TANGENT AT M
D=(45°-θ)
10°
E=(45°-A)
E
RADIUS VECTOR
45°
θ
A
M
O
X
Y
x
y

also the cubic parabola, therefore the radius of curvature is infinite at 0. At 0, when r (the radius vector) becomes zero, cos 2 D = 0 ∴ D = 45°, D being the angle which the radius vector makes with the axis of x; also the angle which the tangent to the curve at 0 makes with the axis of x is $= \frac{d\,y}{d\,x}$, and since the equation to the curve is $(x^2 + y^2)^2 = 2\,a^2\,(x^2 - y^2)$ we have

$$\frac{d\,y}{d\,x} = \frac{x}{y}\left(\frac{a^2 - (x^2 + y^2)}{a^2 + (x^2 + y^2)}\right) = \tan A.$$

$$\therefore \tan A = \frac{x}{y}\,\frac{(a^2 - r^2)}{(a^2 + r^2)}$$

Now, when $r = 0$, $\tan A = \frac{a^2\,x}{a^2\,y} = \frac{x}{y} = \tan D$, ∴ at O tan A = tan D = 45°; *i.e.*, the tangent to the curve at 0 makes an angle of 45° with the axis of x.

It will be seen that only a small portion of the curve would be used for a transition curve in railway work, since the line 0 m, which makes an angle of 10° with the tangent at 0, does not take up one-third of the part of the curve which lies above the axis of x and to the right of the axis of y.

Were the leniniscata to be used as a transition curve, it is evident that the angles to be laid off to different points on the curve would be laid off from the tangent at 0.

The polar equation to the curve is $r^2 = 2\,a^2 \cos 2\,D$, but D = (45° — D), ∴ 2 D = (90° — 2 D), ∴ $r^2 = 2\,a^2 \cos (90 - 2\,D)$, $\therefore \frac{r^2}{2\,a^2} = \sin 2\,D$, therefore sin 2 D varies as the square of the radius vector. To show that D itself varies practically as r^2, the speaker has worked out two curves for a length of 180 feet, one being nearly the same as that already used, namely, a curve with $r = 60'$ and A = 7·2$^{m.}$, and the other where T = 60′ and A = 1°. It is assumed that the length of the curve is the same as r.

In the first case we find A to be 655·8′. Then taking $r = 120'$ $A_1 = 0°\ 28{\cdot}79^{m.}$, and when $r = 180'$ $A_2 = 1°\ 04{\cdot}73^{m.}$, as compared with 0° 28·8$^{m.}$ and 1° 04·8$^{m.}$ for the curve as proposed by Messrs. Wicksteed and Lordly. In the second case, with $r = 60'$ and A = 1°, which would be a very extreme case, corresponding to a curve increasing from 0° to about 15° in 180 feet, A is found to 227·1′.

From this value of A and giving r the values of 120′ and 180′,

A_1 is found to be 4° 00$^{m.}$ 30$^{s.}$ and A_2 to be 9° 09$^{m.}$, therefore in this case the angles increase rather more quickly than the squares of the distance along the curve, but in the first case they increase practically as the squares of the distances.

In the case of the leniniscata, the central angle taken up by any portion of the curve from 0 to any point *m* (see fig. 5), which angle is evidently equal to the angle E between the tangent at 0 and the tangent at *m*, is exactly equal to three times the angle D to be turned off from the tangent at D to set the point *m*. This can be proved as follows:

The equation to the curve referred to rectangular ordinates is $(x^2+y^2)^2=2a^2(x^2-y^2)$ (1), and the polar equation is $r^2=2a^2\cos 2D$ (2), from (1) $x^4+2x^2y^2+y^4=2a^2x^2-a^2y^2$

$$\therefore\ 4x^3\,dx+4xy^2\,dx+4x^2y\,dy+4y^3\,dy=4a^2x\,dx-4a^2y\,dy.$$

$$\therefore\ (y^3+x^2y+a^2y)\,dy=(a^2x-xy^2-x^3)\,dx$$

$$\therefore\ \frac{dy}{dx}=\tan A=\frac{x}{y}\times\frac{a^2-(x^2+y^2)}{a^2+(x^2+y^2)}=\frac{x\,(a^2-r^2)}{y\,(a^2+r^2)}$$

$$\therefore\quad \tan A=\frac{x}{y}\times\frac{(a^2-2a^2\cos 2D)}{(a^2+2a^2\cos 2D)}=\frac{x}{y}\times\frac{(1-2\cos 2D)}{(1+2\cos 2D)}$$

but $A=(45°-E)$, $D=(45°-D)$ and $\frac{x}{y}=\cot D=\cot(45°-D)$.

$$\therefore\ \tan(45°-E)=\cot(45°-D)\times\left(\frac{1-2\cos(90°-2D)}{1+2\cos(90°-2D)}\right)$$

$$\therefore\ \frac{\tan 45°-\tan E}{1+\tan 45°\times\tan E}=\frac{(\cot 45°\times\cot D+1)}{(\cot D-\cot 45°\quad)}\times\frac{(1-2\sin 2D)}{(1+2\sin 2D)}$$

$$\therefore\ \frac{1-\tan E}{1+\tan E}=\frac{(\cot D+1)\ (1-4\sin D\cos D)}{(\cot D-1)\ (1+4\sin D\cos D)}$$

$$\therefore\ \frac{1-\tan E}{1+\tan E}=\frac{\cot D-4\cos^2 D+1-4\sin D\cos D}{\cot D+4\cos^2 D-1+4\sin D\cos D}$$

Now add and subtract nominator and denominator,

$$\text{then}\ \frac{2\tan E}{2}=\frac{8\cos^2 D-2}{2\cot D-8\sin D\cos D}=\frac{4\sin D\cos^2 D-\sin D}{\cos D-4\sin^2 D\cos D}$$

$$\therefore\ \tan\ E=\frac{4\sin D\,(1\sin^2 D)-\sin D}{\cos D-4\cos D\,(1\cos^2 D)}=\frac{3\sin D-4\sin^3 D}{4\cos^3 D-3\cos D}$$

$$\therefore\ \tan\ E=\frac{\sin 3D}{\cos 3D}=\tan 3D$$

$$\therefore\ E=3D.$$

It appears therefore that the leniniscata, for ordinary cases of flat curves, practically agrees with the curve proposed by Messrs. Wicksteed and Lordly, but from its equations it does not seem to be very well adapted for use in laying out transition curves, and it would be necessary to determine the radius of curvature at the end of any portion used in order to see that it would suit the main curve which it was to merge into.

With regard to Mr. Hopkins' paper, the theoretical part is fully and clearly explained, and the method by which he proposes to lay out transition curves, viz, by offsets, would suit very well on fairly flat and clear ground. The mode of finding the values of the offsets turns out to be very simple and easy of application under the above conditions. However, Mr. Hopkins' method of first laying out the ordinary main curve and afterwards offsetting from it and from the tangent would not work at all in location on rough ground with steep side slopes, where the tangents are not run out to intersection and where it is desirable to run the actual curves to be used at once and entirely in one direction, in order to get the profile of the actual line as closely as possible.

The curve proposed by Messrs. Wicksteed and Lordly will suit all cases, and, with the aid of Prof. Crandall's tables, is just as easily laid out as Mr. Hopkins', and, in the speaker's opinion, it could be laid out more expeditiously.

The speaker also thinks that his own method, with the help of his tables, is just as general in its application and as easy to use as either of the above, and he quite agrees with Mr. R. F. Tate that "a combined transition and circular curve can be run in by transit from its P. T. C. to the P. T. T. on the other tangent as easily as the simple curve."

Mr. Wicksteed remarks that *possibly* Mr. Lordly and Mr. Hopkins may have frightened some members of the profession by the number of complicated formulæ they indulge in. It was principally with a view of keeping clear of any mathematics, beyond what is necessary in order to understand how to lay out a simple curve, that the speaker worked up the method already referred to and the tables, one set of which is given herewith.

The speaker fully agrees with Mr. Wicksteed that "in the case of sharp curves with long transition ends the method of offsets becomes cumberous; that the exact mathematical form of the curve is of no consequence; that the method which will come into general use is the one which can be most readily calculated and manipulated in the field with the least use of figures, and that the proposition to keep the transition curve length constant is radically wrong in principal."

With regard to the latter point, it is not only wrong in principle, but it could not be carried out in practice. In the common case of reverse curves with a short piece of tangent between, the length of the transition portion will too often depend on the nature of the ground. The speaker cannot, however, agree with Mr. Wicksteed as to the use of tables. Such tables, relating to curves, as are to be found in Shunk, Henck and Trautwine are of material assistance to engineers, and the speaker fails to see why any exception should be made in the case of transition curves. Nor can the speaker agree with the idea that the curvature of a transition curve should change by one degree in any fixed distance. The nature of the ground will in many cases determine the length of the curve.

As to the use of curves as sharp as 10°, the speaker thinks that a train would run as safely and as easily over a 10° curve with say 300 feet of transition curve at each end as it would over a 6° curve without transition ends.

The speaker cannot agree with Mr. Butler that it is not practicable to lay out a transition curve with a transit on account of the small angles and short chords. It is just as easy to lay off 7 minutes with a transit as to lay off 5°.07$^{m.}$, and as to the short chords, a tape can be used, and in any case a curve can be laid out more accurately with 25 foot chords than with 100 foot chords, especially when the curve is sharp, since the short chords give the length of the curve more closely, and in fact errors in length would mount up much more quickly in setting off the curves by offsets unless great care were taken in getting them in square.

The speaker has recently been shown by Mr. H. E. Vautelet an article on transition curves and on the distance which should be left between two reverse curves, published in the "Revue Generale des Chemins de Fer," Vol. 2, July to December, 1879, pp. 410 to 426, and thinks that a few notes from the article would be of interest, as these matters are subject to Government regulation in France.

The article referred to deals firstly with the method of joining main circular curves with the tangents, and secondly with the length of tangent which should be left between two reverse curves.

It states that the question of the best means of joining two tangents might have been supposed to have been settled, theoretically at least, some years previously, by the use of parabolic instead of circular curves, but that this theory had not been fully carried into practice.

The author, M. Jules Michel, therefore proposed to study the question as completely as possible, to see if it were really necessary, in the case of several new railways to be built in rough and difficult districts, to follow the theoretical rule then in force of substituting for a simple circular curve a concentric circular curve of rather sharper radius joined to the tangents by a parabolic curve of the form $y = m x^3$.

The author also proposed to show that, in practice, an arc of a circle differs very little from the theoretical parabola, especially if the latter be altered to take into account the position of the centre of gravity of the rolling stock; and, further, that experience teaches that a simple circular curve is all that is required to join the main curve with the tangent, in order to secure safety in building lines under the most varied conditions.

With regard to the second part of the paper, it appears that strategic railways in 1878 were obliged to have a tangent of at least 100 metres (328 feet) between reverse curves of less than 500 metres radius (3·5° curves), though experience had proved that a length of from 35 to 40 metres (115 to 131 feet) was sufficient.

In the case of local railways, their charters required that 50 to 60 metres (164 to 197 feet) of tangent should be left between reverse curves.

Returning to the question of the means of joining tangents by curves, the author goes on to state that, from the first, circular curves have been used on railways without any endeavour to find, as has sometimes been done in the case of roads, a suitable curve by using a parabola of the second degree. Also, that for flat curves circular arcs sufficed, but that for sharp curves it became necessary to see if they should be used, and also to determine how the super-elevation of the outer rail should be arranged so as to ensure safety and easy riding.

The author also states that Mr. Chavis, in 1865, was the first who discussed the question of determining a curve so that, at each point, its radius of curvature should agree with the superelevation of the outer rail, which latter increased uniformly, and who showed that the curve was a parabola of the third degree, osculating to the tangent and to the main circular curve; but thought that such a curve could not be carried out in practice; that, in 1867, M. Nordling took up the question and carried the theory into practice; and that, in 1869, M. Combier proposed to substitute for part of the main curve a circular curve of shorter radius, and to join this latter with the tangent by means of a parabola of the third degree.

The author then takes up the question somewhat as follows, the speaker having condensed M. Michel's paper where possible: The elevation of the outer rail is given by the equation $u_o = \frac{e \cdot v^2}{g \cdot R}$ (1) where "e" is the gauge.

The Mediterranean Railway Company, however, considered that the value given by this equation was not sufficient, and substituted a simpler formula, viz: $u_1 = \frac{A}{R}$ (2), where A is the speed in kilometres per hour.

Formula (2) gives the same elevation as (1) for a speed of 84 kilometres per hour (53 miles per hour), and higher values in proportion as lower speeds are dealt with. This is proved as follows, viz: In (1) "e" $= 1'\cdot5$, $g = 9\cdot8$, and to substitute "A" for "v" (1) must be multiplied by $\frac{1000^2}{3600^2}$ $\therefore u_o = \frac{1\cdot5}{9\cdot8} \times \frac{1000^2}{3600^2} \times \frac{A}{R} = \frac{A^2}{84\ R}$, and if this be equated to u_1 we have $\frac{A^2}{84\ R} = \frac{A}{R}$ or $A = 84$.

If A be taken as $= 30$, the value of the superelevation is the same as would be given for 50 kilometres per hour by formula (1).

In the following calculations "u" will be taken as $= \frac{A}{R} = u$ (2^a).

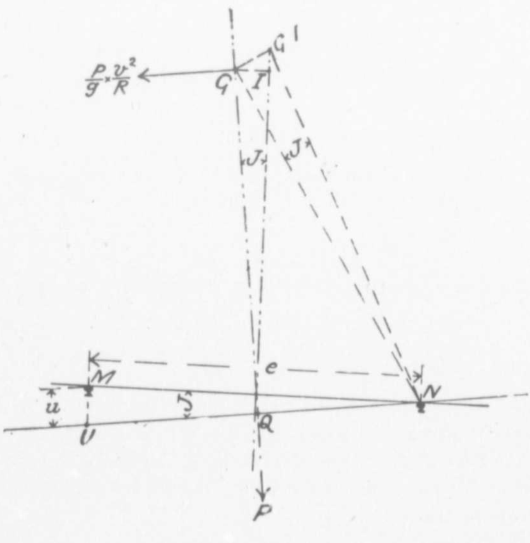
If the superelevation increase uniformly in a given distance "s," from zero to its full value, and if the grade of the outer rail be $\frac{1}{i}$, then $\frac{u}{s} = \frac{1}{i}$. $\therefore u = \frac{s}{i}$ (3). Equating (2^a) and (3), we have $\frac{s}{i} = \frac{A}{R}$ or $\frac{1}{R} = \frac{s}{i.\,A}$, $\therefore \frac{d^2\,y}{d\,x^2} = \frac{s}{i.\,A}$.

If only the part of the curve near the tangent be condered we may put $s = x$ and we have $\frac{d^2 y}{d x^2} = \frac{x}{i \cdot A}$ (4), $\therefore \frac{d y}{d x} = \frac{x^2}{2 i \cdot A}$ (5) and $y = \frac{x^3}{6 i A}$ (6), from which it can be proved that, when the superelevation increases uniformly, the parabola of the third degree, put in between a tangent and a main circular curve, is bisected by the ordinate from the original P. C. of the main curve. The manner of proving this is given a little further on in connection with curve (8).

If the centre of gravity of the rolling stock were at the level of the track, equation (6) would give the curve of the centre line of the track; and assuming, as above, that only portions of the curve close to the tangent are dealt with, equation (6) would also represent the curve of the outer rail.

Now, the centre of gravity of the rolling stock, situated from $1{\cdot}25^{m.}$ to $1{\cdot}50^{m.}$ above the rail, does not remain fixed with refer-

FIG 7



ence to the centre line of the track when the rails are canted, but it describes an arc of a circle G G^1 round the top of the inner rail (fig. 7), and its horizontal projection no longer agrees with the centre line of the track.

In fig. 7 G Q is the vertical let fall from G, and it bisects N U at Q. The perpendicular from G^1 on M N also passes practically through Q. Considering the triangles M N U and G I Q as similar, we have $\frac{\text{G I}}{\text{G Q}} = \frac{\text{M U}}{\text{N U}} = \frac{u}{e}$.

G I is the displacement of the centre of gravity in a direction normal to the centre line, and G^1 I is also approximately equal to its vertical displacement. G Q is the height of the centre of gravity of the vehicle above the track. Calling "*n*" the ratio of this height to the gauge, we have G Q $= n \times e$, $\therefore$ since $\frac{\text{G I}}{\text{G Q}} = \frac{u}{e}$ we have G I $= n \times u$ (7).

If we suppose that the centre of gravity describes in plan the same curve as before, the curve of the centre of the track or of the rail would be found by *adding* the value of G I with its proper sign, at each point to the ordinate of the curve (6).

The equation to the curve of the centre line would then be $y = \frac{x^3}{6\,i\,\text{A}} - n \cdot u$, or since $i = \frac{s}{u}$, and since by the assumption already made $s = x$, $y = \frac{x^3}{6\,i\,\text{A}} - n\frac{x}{i}$ (8).

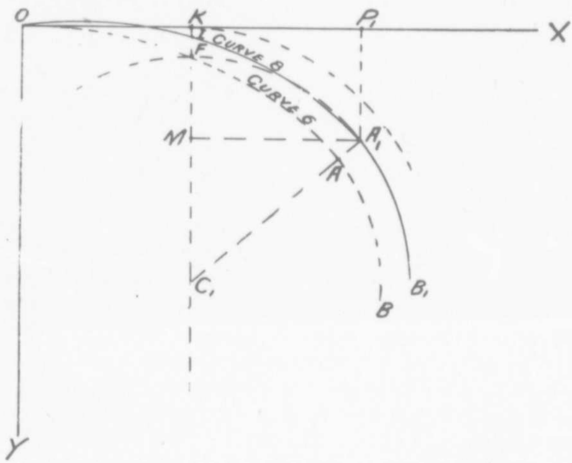
This would be the true theoretical transition curve to be substituted for the curve (6) proposed by M. Combier.

From (8) we obtain $\frac{d\,y}{d\,x} = \frac{x^2}{2\,i\,\text{A}} - \frac{n}{i}$ (9), which gives the inclination of the tangent at any point to the axis of X, and $\frac{d\,y^2}{d\,x^2} = \frac{x}{i\,\text{A}}$ (10), the approximate value of the reciprocal of the radius of M. Combier. See Equation (4).

It is evident then that, at points corresponding to the same abscissæ, the curves (6) and (8) have approximately the same curvature, and that, even on M. Nordling's hypothesis, radius of the curve (8) would be of as convenient form as a parabola of the third degree.

On examination of equation (8) it is readily seen (see fig. 8) that—

FIG. 8



Firstly—That the curve (8) passes through the origin of curve (6), since when $x = o$, $y = o$, and the curve also shows an inflexion at the origin and cuts the axis of X at a distance from the origin of $x = \sqrt{6 \text{ A} \times n}$, since when $x = \sqrt{6 \text{ A } n}$, $y = o$; and

Secondly—That the tangent of the inflexion makes with the axis of X an angle whose tangent $= \frac{n}{i}$, since the tangent $= \frac{x^2}{2\, i \text{ A}} - \frac{n}{i}$ and when $x = o$ this $= \frac{n}{i}$.

The greatest divergence of this curve (8) beyond the axis of X takes place between the origin and the second point of intersection with the axis of X, and the maximum value of this divergence will be at the point α (see fig. 9), where the tangent is horizontal; that is when $\frac{d\, y}{d\, x} = o$, or when $\frac{x^2}{2\, i \text{ A}} = \frac{n}{i}$, or when $x^2 = \sqrt{2\, n + \text{A}}$, the value of the offset is then $y_2 =$

$\frac{(2\,n \times A)^{\frac{3}{2}}}{6\,i\,A} - \frac{n}{i}\sqrt{2\,n \times A} = \sqrt{2\,n \times A} \times \left(\frac{2\,n\,A}{6\,i\,A} - \frac{n}{i}\right) =$ $\sqrt{2\,n \times A}\left(-\frac{2\,n}{3\,i}\right)$ or calling $\triangle^d_m$ the maximum distance that the transition curve passes beyond the tangent we have $\triangle^d_m = -\frac{2\,n}{3\,i}\sqrt{2\,n\,A}$ (11). Note $\triangle^d_m = \alpha\,\alpha_1$ of fig. 9.

On comparing the positions of the curve (8) of the centre of the track, and of the curve (6) of the centre of gravity, we see that they begin at the same origin O, and that they end in two parallel axes of circles A B and A^1 B^1, the points of contact being on the same radius C_1 A A_1 (fig. 8).

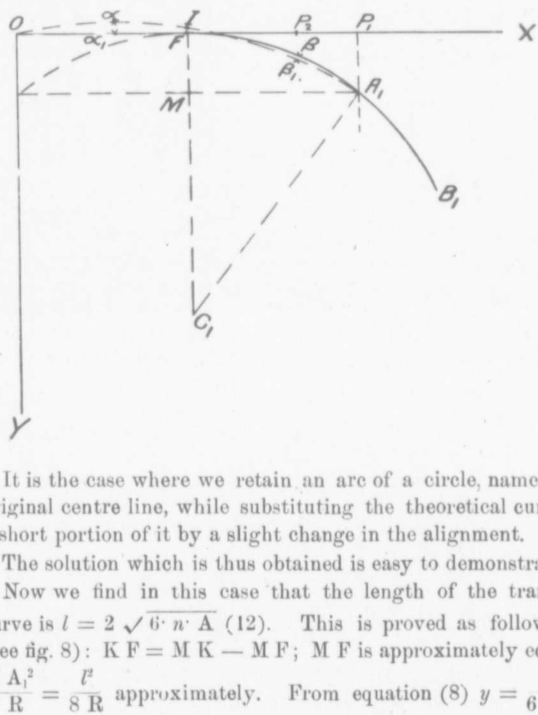
In fact, when the centre of gravity begins to describe the arc A B, the cant of the track has reached its maximum, and from thence it remains constant, the centre of gravity has no longer any further displacement with respect to the centre line (or to the rails), and the latter takes the shape of an arc of a circle parallel to and concentric with the former. It is also readily seen, as M. Combier has remarked in reference to the parabolic curve, that the transition curve is bisected by the perpendicular let fall from the centre to the tangent.

This is proved as follows (fig. 8): Let α = the angle which the tangent at A or A_1 makes with the axis of X, then tan $\alpha = \frac{d\,y}{d\,x} = \frac{x^2}{2\,i\,a} - \frac{n}{i}$, but as $\frac{n}{i}$ is very small it may be neglected; therefore, assuming also that the arc is equal to its projection on the tangent, as has already been done, we have tan $\alpha = \frac{l^2}{2\,i\,A}$. Also, if "$h$" be the elevation of the outer rail corresponding to a radius "R," we have $\frac{h}{l} = \frac{1}{i}$, and $h = \frac{A}{R}$ as already assumed; $\therefore \frac{A}{R \times l} = \frac{1}{i}$ or $i \times A = l \times R$, $\therefore$ tan $\alpha \frac{l^2}{2\,l\,R} = \frac{l}{2\,R}$, but the angle α being small it may be assumed that tan $\alpha = \sin\alpha = \frac{M\,A_1}{R}$, $\therefore \frac{M\,A_1}{R} = \frac{l}{2\,R}$, $\therefore l = 2\,M\,A_1$ or $O\,K = \frac{l}{2}$; and

Thirdly—That the radius of the arc A_1 B_1, which joins the two tangents, is not fixed; we can therefore vary the offset K F, so that the distance between the curve and the arc of the circle A_1 F shall be as small as possible.

We shall only examine the case where K F = O (see fig. 9).

FIG. 9.



It is the case where we retain an arc of a circle, namely, the original centre line, while substituting the theoretical curve for a short portion of it by a slight change in the alignment.

The solution which is thus obtained is easy to demonstrate.

Now we find in this case that the length of the transition curve is $l = 2\sqrt{6 \cdot n \cdot A}$ (12). This is proved as follows, viz (see fig. 8): K F = M K — M F; M F is approximately equal to $\frac{M A_1^2}{2 R} = \frac{l^2}{8 R}$ approximately. From equation (8) $y = \frac{x^3}{6 i A} - \frac{n}{i} \times x$, taking $x = l$ and $i A = l R$ as before, and since $y = P_1 A_1 = M K$ when $x = l$, we have $M K = \frac{l^2}{6 R} - \frac{n A}{R}$, $\therefore K F = \frac{l^2}{6 R} - \frac{n A}{R} - \frac{l^2}{8 R} = \frac{l^2}{24 R} - \frac{n A}{R}$. Now when K F = O, $\frac{l^2}{24 R} = \frac{n A}{R}$ or $l^2 = 24 n A$, $\therefore l = 2\sqrt{6 n A_1}$ (12). And we see that, in this case when K F=O, the length of the arc, over which the cant of the track is increased from zero to its maximum, depends only on the height of the centre of gravity of the rolling stock above the rail level, and on the speed of the trains.

Let us now investigate the maximum distance between the curve O I A_1 and the axis of X (fig. 9), as well as between the latter curve and the arc of the circle F A_1 B_1.

We have already found that the maximum distance $\alpha\,\alpha_1$ is given by the general equation $\alpha\,\alpha_1 = \frac{-2\,n}{3\,i}\sqrt{2\,n\,A}$, but $A = \frac{l\,R}{i}$ and $l = 2\sqrt{6\,n\,A}$ when K F = O, $\therefore A = \frac{2\sqrt{6\,n\,A} \times R}{i}$, $\therefore A^2 = \frac{4\,R^2 \times 6\,n\,A}{i^2}$ or $A = \frac{4\,R^2 \times 6\,n}{i^2}$ $\therefore \alpha\,\alpha_1 = \frac{-2\,n}{3\,i}\sqrt{\frac{4\,R^2 \times 12\,n^2}{i^2}}$, $\therefore \alpha\,\alpha_1 = \frac{-8\,n^2}{3\,i^2} \times R\sqrt{3}$. (13.)

With regard to the ordinate from the axis of X to the arc of the circle opposite P_2, the centre of F P_1, if O $P_2 = x$, since O P_1 has been assumed to be $= l$, this ordinate is approximately $= \frac{\left(x - \frac{l}{2}\right)^2}{2\,R} = \frac{(x - \sqrt{6nA})^2}{2\,R}$, and the ordinate to the curve at $P_2 = \frac{x^3}{6\,i\,A} - \frac{n}{i} \times x$ (8). $\therefore$ the difference, $\beta\,\beta_1$, between the ordinate to the curve and the ordinate to the circle has for its value $\Delta_o = \frac{x^3}{6\,i\,A} - \frac{n}{i} \times x - \frac{(x - \sqrt{6nA})^2}{2\,R}$ (14), but A $= \frac{4\,R^2 \times 6\,n}{i^2}$, $\therefore \Delta_o = \frac{x^3}{6\,i \times \frac{4\,R^2 \times 6\,n}{i^2}} - \frac{n}{i} \times x -$

$$\frac{\left(x - \sqrt{\frac{(6\,n)^2 \times 4\,R^2}{i^2}}\right)^2}{2\,R} = \frac{x^3 \times i}{36\,n \times 4\,R^2} - \frac{n}{i} \times x - \frac{\left(x - \frac{12\,n\,R}{i}\right)^2}{2\,R}$$

$$\therefore \Delta_o = x^3\left(\frac{i}{36\,n \times 4\,R^2}\right) - \frac{x^2}{2\,R} - x\left\{\frac{n}{i} - \frac{12\,n}{i}\right\} - \frac{72\,n^2 R}{i^2} \quad (15),$$

which is a maximum when its differential equals O, or when $x^2\left(\frac{i}{48\,n\,R^2}\right) - \frac{x}{R} = -11 \times \frac{n}{i}$, or when $x^2 - \frac{48\,n\,R}{i} \times x =$

$\frac{-11 \times 48 \times R^2 n^2}{i^2}$, or when $x = \frac{24\,n\,R}{i} \pm \sqrt{3} \times \frac{4\,n\,R}{i} = \frac{4\,n\,R}{i}(6 \pm \sqrt{3})$. Substitute this value of x in (14) and we have

$$\left\{4(6 \pm \sqrt{3})\right\}^3 \times \frac{n^3 R^3}{i^3} \times \frac{i}{36\,n \times 4\,R^2} - \left\{4(6 \pm \sqrt{3})\right\}^2 \times \frac{n^2 R^2}{i^2 \times 2\,R} - \frac{4\,n\,R}{i}(6 \pm \sqrt{3})\left(\frac{-11\,n}{i}\right) - \frac{72\,n^2\,R}{i^2} = \Delta^c_{max.}$$

$$= \frac{n^2}{i^2} \times R\left\{\frac{64(6 \pm \sqrt{3})^3}{144} - \frac{16(6 \pm \sqrt{3})^2}{2} \times 44(6 \pm \sqrt{3}) - 72\right) = \frac{n^2}{i^2} \times R\left\{4\sqrt{3} - \frac{4}{\sqrt{3}}\right\} \therefore \Delta^c_{max.} = \frac{n^2}{i^2} \times R \times \frac{8}{\sqrt{3}}$$

(13[a]); from which we see that the maximum difference ($\Delta^c_{max.}$) between the main circle and the second half of the transition curve is the same as the maximum ordinate ($\alpha\ \alpha_1$ max.) from the main tangent to the first half of the transition curve.

The equation (13[a]) can be put in another form by obtaining the value of i in terms of R, A and n, thus: $\frac{h}{l} = \frac{1}{i}$ and $h = \frac{A}{R}$, $\therefore i = \frac{l \times R}{A}$; but in the case under consideration (viz, where K F $=$O), $l = 2\sqrt{6\,n\,A}$, $\therefore i = \frac{2\ R\ \sqrt{6\,n\,A}}{A}$ or $i^2 = \frac{4\,R^2 \times 6\,n}{A}$ (16), and $\therefore$ substituting this value of i in (13) or (13[a]) we obtain

$$\Delta^c_{max.} = \frac{8}{\sqrt{3}} \times \frac{n^2\,R \times A}{4\,R^2 \times 6\,n} = \frac{n}{3\sqrt{3}} \times \frac{A}{R}\ (17).$$

The latter equation shows that, if we take A (the speed) as constant, the divergence of the transition curve from the main circular curve increases with the height of the centre of gravity of the vehicles above the level of the rails, and that this divergence varies inversely as the radius of the curve.

If, on the contrary, we regard "n" as constant, which is practically true, we find that the maximum divergence varies as the cant of the track, which is equal to $\frac{A}{R}$.

We may now apply the preceding investigation to two extreme cases where the cant is found by assuming A = 70 and

R = 500^m (equivalent to a speed of 43·5 miles per hour and a 3° 30^m curve; and A = 30, with a curve of 250^m radius (corresponding to a speed of 18·6 miles per hour and a 7° curve.)

The height of the centre of gravity is, on the average, equal to the gauge, $\therefore n = \frac{h}{e} = 1$.

If the locomotive alone be considered, the value of "h" will be very little greater than 1·25^m, so that by taking $n = 1$ we shall obtain the greatest value of the divergence of the theoretic transition curve from the main circle.

In the first case (A = 70), we have $\Delta^c_{max.} = \frac{70}{500} \times \frac{1}{3\sqrt{5}} =$ 0^m·0268 (1·05 inches); and in the second case $\Delta^c_{max.} = \frac{30}{250} \times \frac{1}{3\sqrt{3}}$. = 0^m·0224 (= 0·88 inches).

We see then that the greatest difference is less than 0·03^m (= 1·18 inches), which is not much more than the deviations from correct alignment which might be due to want of accuracy in laying out the centres or in keeping the track in line.

There is still an important remark to be made with reference to the theoretic curve (8), $\left(y = \frac{x\,3}{6\,i\,A} - n\frac{x}{i}\right)$.

It has been shown that the tangent at the origin of this curve is directed towards the exterior of the curve (see fig. 8), and that the angle which it makes with the main tangent has for its value $\frac{n}{i}$.

This sudden change of direction is not permissible in practice, unless, however, it does not exceed the angles which two consecutive straight rail lengths make with each other on a curve. Now what is the value of $\frac{n}{i}$ under the preceding conditions?

When A = 70 (43·5 miles per hour), R = 500^m (3° 30′ curve), and $n = 1$, we have from (16) $(i^2) = \frac{4 \times 250{,}000 \times 6}{70}$, $\therefore i =$ 292 and $\frac{n}{i} = \frac{1}{292} = 0{\cdot}0034$; therefore the tangent of the angular divergence between the final direction to be given to the centre lieu and the main tangent is less than 0·004, which is so small that it need not be considered. It is about the same as the angle between two 6^m (19′ 8″) rails on a curve of 1500^m radius (1° 10′ curve).

When A = 30 (18·6 miles per hour) and R = 250^m (a 7° curve), then $i = 224$ and $\frac{1}{i} = 0{\cdot}0044$, which is very little more than the amount 0 004 above mentioned.

Thus the greatest difference between the arc of a circle joining two tangents and the theoretical curve for connecting a tangent with the main circular curve, without an abrupt change of curvature, is, at the most, $0^m{\cdot}03$ (1·18 inches), and also the tangent of the angular deviation of the tangent at the origin is always less than $0^m{\cdot}005$ (0·2 inches).

Are we not, therefore, justified in stating that, in practice, the defects in the lining up of the track, oscillations of the train, oscillations due to the rails being out of gauge and those due to the lateral bending of the rails, all produce deviations from the true alignment exactly of the same amount as the differences pointed out above.

In other words, in undertaking to lay out the theoretical transition curve, one is in no sense sure of realizing in practice the theoretical conditions that are aimed at, and these conditions can be approximately fulfilled by simply laying out an arc of a circle to connect the two main tangents.

It is scarcely necessary to remark that the theoretical curve only suits a single rate of speed.

Another very important conclusion follows from the above, viz, that since the circular curve can be considered as practically identical with the parabolic curve, we can obtain the gradual superelevation of the outer rail by carrying out the same rule in the one case as in the other.

Now we have seen that, with a parabolic curve, it is sufficient to take a length of from 15^m to 18^m (50 to 59 feet) on each side of the B. C. of the original main curve, and that the superelevation easily reaches its maximum in a length of from 30^m to 36^m (99 to 118 feet), and that it is thus distributed half on the tangent and half on the curve.

If in formula (12), where $l = 2\sqrt{6\,n\,A}$, we take $n = \frac{5}{4}$ an $A = 70$, we have $l = 2\sqrt{\frac{2100}{4}} = 46^{m}$ (= 151 feet), which is a maximum value.

We can conclude from this that, in the case where we simply use an arc of a circle for connecting the tangents, it is useless to undertake, as is commonly done, to bring the cant of the track up to its full value at the commencement of the curve by raising the superelevation up to its full value entirely on the tangent.

Once we admit that the arc of the circle and the parabolic curve can be substituted the one for the other, the superelevation should be carried out in the same manner in both cases and according to the same law.

We insist on this point, since it must serve us to solve the second question which we have proposed, viz: What is the length of the tangent which should be left between two reverse curves?

After reviewing the various orders on this question, M. Michel concludes that, supposing the superelevation to increase with a grade of 0·25 per cent., it is sufficient to leave 45^{m} (148 feet) between two reverse curves, and goes on to state that even with a cant of $0{\cdot}15^{m}$ (6 inches), which may be considered as the maximum, and which corresponds to an effective speed of 58^{k} per hour (36 miles per hour) on a curve of 250^{m} radius (a 7° curve), a length of 35^{m} (115 feet) is sufficient between two reverse curves, and that of this 15^{m} (49·5 feet) at each end should be taken up in getting in half the superelevation of the outer rail, and that the track should be level transversely for the central 5^{m} (16 feet). M. Michel also considers that it should be remembered that several examples could be given of tangents of from 42^{m} to 60^{m} (138 to 197 feet) in length between reverse curves of from 600^{m} to 1000^{m} radius (2° 55′ to 1° 44′ curves) on railways where there are trains whose regular speed is 70^{k} per hour (43·5 miles per hour), which is sometimes increased to 80^{k} (50 miles) per hour, and which run quite smoothly over the portions of the track where these reverse curves exist. He also states that, when we examine on the ground how the problem has been solved, we see that the section men have arranged the cant half on the tangent and half on the curve; and that they have also altered by the eye the point of commencement of the curve to ease the change from the tangent to the curve, and have thus instinctively arrived at the solution which we should like to see carried out.

M. Michel concludes by stating that, in his opinion, after twenty-five years of practice, it is sufficient to leave a tangent of from 40^m to 50^m (131 to 164 feet) in length between two reverse curves.

The speaker would remark that on this side of the Atlantic the curves are much sharper than those referred to by M. Michel, and that speed has increased considerably since his paper was written; therefore the question of transition curves is now of still more importance than when M. Michel's paper was written. Also that M. Michel's final conclusions as to transition curves are based on the assumption that the quantity K F = O, and that the maximum length of the portion of the curve on which the super-elevation is to attain to its full height is 151 feet, the maximum speed in the latter case being 70 kilometers, or 43·5 miles, per hour. And further, that M. Michel did not consider the effect of the oblique impact of the truck wheels against the outer rail when they have run as far as possible along the curve, while at the same time following the direction of the tangent, an impact the effect of which would certainly be aggravated by the use of Curve No. 8.

Mr. Sproule said Mr. Lordly's paper is valuable in giving a method of obtaining a curve of such a form that, at every point in passing from the tangent, the outer rail can be elevated so as to exactly counterbalance the centrifugal force of the train for any given speed. If this kind of curve were used in railways and kept in perfect order the wear of rolling stock and danger of derailment, as well as the discomfort to passengers, would be reduced to a minimum. In running location lines, however, on difficult ground, where repeated trials are necessary in order to put the line in its proper position, where the curve must be shortened, lengthened, flattened, sharpened, thrown a foot to left or right, etc., etc., this curve must be more difficult to manipulate than an ordinary circular curve, and until the importance of exact engineering work is better appreciated the use of such curves will not likely be encouraged by Boards of management. Desirable as proper transition curves are, railways are more in need of scientific superintendence in keeping the curves they use at present in proper line. Probably few of the trackmen who flatten a curve at its junction with the tangent Mr. W. J. Sproule.

realize that in so doing they must sharpen the curve elsewhere, making the curve irregular, and the elevation of the outer rail incorrect, thus often giving a rough if not dangerous road to ride on.

CORRESPONDENCE.

Mr. R. F. Tate.

Mr. R. F. Tate remarked that he has been more inclined to the "taper curve theory" of transition from tangent to constant circular arc, viz: a series of circular arcs varying in degree in arithmetical progression, based on the assumption that "such transition at point of constant curve produced to parallel tangency with the main tangent, bisects the vertical distance between such tangents, and also that at the same bisection, the length of the transition curvature used is also bisected (see Wellington's Economic Location, page 869), the claims for which are the ample fulfilment of the needs of the subject under discussion and practically identical geometric results with the parabola within these restrictions; and it seems to the writer to be further enhanced by its simple conception, easily memorized and figured without the use of further tables than generally used for the simple curve. Take the author's example of a constant 6° curve with five transition chords of 60 feet each; his deflection and tangential angles are equivalent to a taper curved series of 1°.12′—2°24′—3°36′—4°48′—6°00, which give identically the same angles as shown by him in this case of a 9° transition deflection angle, which on an intersection of 90° permits 50% of the entire curvature to be of the transition character; the taper curve angles per author's example being:—

0°.36′	×	0.6	=	0°.21′.6	÷ 3	=	0°.07′.2
1°.12′	×	1.2	=	1°.26′.2	÷ 3	=	0°.28′.8
1°.46′	×	1.8	=	3°.14′.4	÷ 3	=	1°.04′ 8
2°.24′	×	2.4	=	5°.45′.6	÷ 3	=	1°.55′.2
3°.00′	×	3.0	=	9°.00′	÷ 3	=	3°.00′

Of course such precise mathematical agreement will not hold good with that of the author's, if extended beyond certain limits, but the 'range' within which it does is considerable, and though enlarged beyond strict mathematical identity, produces a curve so closely identical with that of the author's that there is practically no difference so far as its fitness for the purpose is concerned. The field instrumental work of the one will be as precise as that of the other.

Take the case of a transition deflection angle of 15° upon an intersection of 90°; the transition curvature will be equal in length to the constant curve used, or with a 20° curve, 150 feet of transition at each end and 300 feet of constant curve. In street railway practice transitions have been introduced to form union with a 143.1/4° curve (radius 40 feet) and similarly as before about 21 feet of transition at each end could be used with about 42 feet of constant curve, so that even with this moderate limitation, the relation of length between the two classes of curve in turning a right angle is well maintained, as well as the parabolic figure.

A very neat formula for determining the offset "O" between the parallel tangents of the constant and transition curve was used by Mr. Geo. P. Snyder, Assistant Engineer P. & R. R. Rd. in the revision of existing alignment on a portion of that system (Engineering News, Jan. 9, 1892), as follows:—

$O = \frac{L^2}{24\,R}$; where L = length of the transition curve used at either end, and R = the radius of the constant curve, both in feet. This was in connection with the same transition theory as the writer describes. While there are disciples of each treatment of the subject, the usage of either will evidently afford all the requisite efficiency the very important *desiderata* covers.

Mr. Hopkin's parabolas will apply correctly if he places them properly, and entirely *outside* his permanent circular curve. He cannot run in his circular curve from his permanent tangents as if there were to be no transition, and then apply any *correct* transition lying *inside* his permanent circular curve without changing the position of *all* that circular curve. If he does so, it appears to the writer that the effect is not only to *sharpen* the curvature about his P. C., in excess of his circular curve, but that his parabolas would consume but half "O" at P. C., and all "O" entirely ignored at P. C'., if the original position of his circular curve is adhered to. The writer may not follow Mr. Hopkins properly in this, but contends that "*O*" *is an indispensable relative transition function* outside and in addition to the circumference of the permanent circular arc; and that in dealing with an existing circular curve for transition approaches without disturbing the position of its tangents *a new circular curve should be laid out*, and if placed upon the *inside* of the existing circular curve, at the distance "O" at P.C. and P.C'. then in this position will it lie parallel to the existing circular curve throughout, and the radius of this new permanent circular curve R', = (R—O). Taking R' and its tangents as a basis, if it be desired to place any permanent circular curve in any other position nearer the apex, at any known desired distance measured in that direction along its middle radius, then R' will diminish and vary in direct ratio with the external secant.

If in the first case it is desired to retain R then the new permanent circular curve will lie *inside* the original circular curve at its middle radius by $\frac{O}{\cos \frac{1}{2} I}$.

Take Mr. Hopkins' case of a 4° curve, R, = 1432.7 feet, O = 2 feet, and say I = 60°, then R, = 1430.7 feet. If it be desired to place a new curve for transition approaches, so that it will lie coincident with the existing curve at its middle radius, then its radius will be 1417.8 feet or 14.9 feet less than the existing curve, and its P. C. 7.4 feet *forward* of the original P. C. Where O and R are fixed the length of the transition will vary with each change of R, I being constant.

It may be said that when O and D are small and relatively also the length of the transition, that the sharpening of a circular curve about P.C'. by moving the track *in*, may be insignificant in effect, but the writer sees no reason to do even in a small way, what trackmen frequently do in a very considerable one, and considers such practice should be altogether abandoned relative to a transition curve.

A combined transition and circular curve can be run in by transit from its P. T. C. to the P. T. C'. on the other tangent as easily as the simple curve; moreover running in location through wooded districts, this would generally be desired, in order to avoid the extra chopping for a view of a back sight from which to deflect when over an "offsetted" P. C.

Mr. Hopkins' geometric demonstration that the transition and offset are bisected at a point coincident with the P. C. of the circular curve accords with the writer's assumption in his remarks upon the taper curve transition in connection with Mr. Lordly's paper, and he still sees no reason to change his inclination to that theory, in which, if offsetting is desired, it may be done quite as simply as Mr. Hopkin's method without restrictions to *any* arbitrary lengths, but *always* entirely *outside* the permanent circular curve.

Cecil B. Smith—The writer has read with interest the various papers read before the society on the subject, and while believing in them theoretically, sees difficulties in bringing them into extended use in steam railroad construction. Doubtless, as engineers become more generally possessed of a technical education the introduction of this improvement will be facilitated, but at present it will be difficult to find a chief engineer and his assistants who are one in mind on the subject, and until such is the case little progress can be hoped for. Another difficulty confronting the innovator is description of Right of-way, such as to make it intelligible to the ordinary lawyer and to continue descriptions from the circular curve and place the track elsewhere is to provide litigation for the railroad company. In the Southern States the title of land is vested still in the landowner and the railroad company gets merely an easement. When such is the case it matters little where the track is, but in the Northern States and Canada land is bought outright by the railroad company and fences laid out from the track. Thus it will be seen that, where land is valuable, to have one piece of land described in the registrar's office and another fenced in, is a consummation hardly to be wished for. This, in the opinion of the most experienced engineers, is a serious obstacle yet to be provided for. With reference to the particular curve mentioned by Mr. Hopkins the writer can vouch for its simplicity, as he has often put it into practice and used the tables referred to by making insets from the circular curve. The originator of this modified cubic parabola for transition curves is Mr. A. M. Wellington, a member of the Society, and in the issue of "Engineering News," February 8th, 1890, he has fully developed its leading qualities without finding it necessary to envelop it in a haze of calculus, which obscures its worth, spoils its simplicity, and gives it that nameless odor which disgusts practical engineers with much that otherwise would have its merits sooner recognized. The question naturally presents itself:—Can this particular curve be put in by transit at once in location without delay to the locating corps, and if so, Mr. C. B. Smith.

can its length be easily varied to suit the degree of curve, for evidently if it can be done, then it is much better to do so, for the locating engineer is supposed to place his curves on the best ground and, except by an improbable preconcerted action, will never have them waiting to be moved inward to find their best position. A short study of Mr. *Wellington's* curve will show:—

(1) That by having transit at P. T. C. the deflection necessary to reach the full length of transition is $\frac{1}{3}$ that of the total angle from the ordinary P. C. to end of transition instead of $\frac{1}{2}$ as in the circular curve, the length of transition being twice as long as from P. C. to end of transition.

(2) Any intermediate transit deflections vary as the square of the distance instead of directly as the distance in circular curves.

(3) The transition at end of curve can be put in by reversing the process.

(4) The transit being at the end of the first transition the vernier is brought forward $\frac{2}{3}$ the total angle before referred to and the transit is ready to run in the ordinary circular curve. Thus the sum of $(\frac{1}{3} + \frac{2}{3})$ is all that is necessary to remember and that the P. T. C. is placed back of the P. C. by $\frac{1}{2}$ the length intended for the transition.

Example:—10° curve—P. C. Sta. 10. Length of transition 300 feet. Place transit on hub at Sta. 8 + 50.

Deflection	⊙ Sta. 8+50	Vernier	$= 0° \ 0'$	300 feet transition curve. Deflection forward 5° 00′ Backsight deflection 10° 00′
"	Sta. 9	$(\frac{1}{6})^2 \times 5° \ 00'$	$= 0° \ 8\frac{1}{3}'$	
"	Sta. 9+50	$(\frac{1}{3})^2 \times 5° \ 00'$	$= 0° \ 33\frac{1}{3}'$	
"	Sta. 10	$(\frac{1}{2})^2 \times 5° \ 00'$	$= 1° \ 15'$	
"	Sta. 10+50	$(\frac{2}{3})^2 \times 5° \ 00'$	$= 2° \ 13\frac{1}{2}'$	
"	Sta. 11	$(\frac{5}{6})^2 \times 5° \ 00'$	$= 3° \ 28\frac{1}{3}'$	
"	⊙ Sta. 11+50		$= 5° \ 00'$	

Then placing transit at Sta. 11 + 50, take backsight on Sta. 8 + 50 with vernier at — 2° 30′, and bringing it forward to + 7° 30′, you are ready to run in circular curve with vernier in the ordinary position of $\frac{1}{2}$ total deflection.

Mr. H. K. Wicksteed.

Mr. H. K. Wicksteed said Mr. Lordly and Mr. Hopkins have certainly given us two very interesting papers on the transition curve, but the writer thinks they may possibly have *frightened* some of the members of the profession by the number of complicated formulas they indulge in, into the belief that no one but an accomplished mathematician is capable of understanding them. He thinks too that both of these gentlemen have laid too much stress upon the properties of the cubic parabola properly so called and too little upon the kindred curve which he has for want of a better name termed the "quadratic" curve. As the circle is to the parabola, so is the "quadratic" to the cubic parabola. In former days before the modern method of locating circular curves by tangential angles was introduced, ordinates and offsets from the tangent were the means used to lay the curve out on the ground. Parabolic curves on account of the simplicity of the formulas connected with them and the ease with which the offsets could be calculated were therefore, common, and for all practical purposes were equally useful with the circular with which in fact in the case of *moderate radii* and central angles they were to all intents and purposes identical.

When the angular method came into vogue it was so much simpler and more expeditious that the old method of offsets is now only used in peculiar and exceptional cases. So with the transition curve. As long ago as 1870 or thereabouts I remember a couple of paragraphs in Rankin's engineering devoted to a description of what he calls a "curve of adjustment" which is identical in principle with the cubic parabola. That it has not come into general use is, the writer thinks, undoubtedly due to the roundabout method of obtaining it by offsets from other lines already run. Mr. Hopkins describes a precisely similar method which, while it is as he claims rapid, is by no means precise enough for use in centring track although quite sufficiently accurate for grading. He has probably dealt only with curves of moderate radius in which the offsets are only from a few inches to a couple of feet in length. Where, in the case of sharp curves with long transition ends they run up to several feet the method of offsets becomes crude and cumbrous, independent of the loss of time. In the "quadratic" curve we have a means of running the correct centre line in the first place with practically no loss of time and with the transit instrument. The exact mathematical form of the curve is of no consequence, any one of the forms in use is as good as the other. That which will come into general use is the one which can be most readily calculated and manipulated in the field with the least use of figures and tables.

Mr. Hopkins has again lost sight of the fact that in offsetting the central portion of the 'curve inwards he is shortening the radius In the ordinary range of curves from 2° to 6° the difference would be scarcely appreciable but it becomes rapidly more so as the curve gets sharper, and in the case of a 20° curve offsetted by his method to conform to a transition with a pitch of 1° to every 30 feet the central curve would be sharpened to more than 24°. The writer thinks too that the proposition to keep the transition curve length constant is radically wrong in principle The curvature should he thinks change by 1° in a fixed distance in every curve on the same road or piece of road. In other words, the transition curve is always the same in all its elements except the length and deflection, which vary exactly as the degree of curvature of the central curve. This he thinks is not only correct in principle but simplifies the calculations in each case so greatly that no tables are necessary, except such as one of those given in his paper on "Railway Curves" published in the Transactions of the Society, and even these may be readily committed to memory or worked out for any particular case in the field. Tables are good, but a method quite independent of tables is surely better. The writer's favorite transition is one in which the curvature changes by 1° in every 30 feet. There are reasons for the choice which have been partially explained in the former paper and which it is unnecessary to enumerate here. Suffice it to say that this makes a curve amply easy in any case. Having the chord length settled, the total length for any central curve of D° curvature is simply 30 D, and the curvature at any distance S, from the P. T. C. is simply $\frac{S}{30}$. The total deflection at any such distance S from the same point is (in degrees) $\frac{S^2}{6000}$ and the tangential angle one-third of this or $\frac{S^2}{18000}$. These are in a majority of cases all the formulas which are needed, and these can be readily committed to memory by any one. Suppose for instance we have placed on the location plan an 8° curve for 56° of central angle. The offset for the tangent for this degree of curvature is 3.3 feet which may be allowed for in the plotting, or where both tangents are absolutely fixed the subtangent may be worked out from the P. I., as in the example given in the paper referred to. Working from the plotted plan we should merely take the B. C. of the offsetted simple curve and deduct from the chainage the half length of the transition curvature. By our formula, the total length of transition as above will be $8 \times 30 = 240$ feet, and the half length 120 feet, the total angle for each transition curve will be $\frac{240^2}{6000} = 9° 36'$; doubling this we get 19° 12′, and deducting from 56° there remains 36° 48′ for the central constant 8° curve. Supposing the P. T. C. to fall at 51 + 20, we are now in a position to make up our transit book, which will look something like this.

					Tangt'l Angle.	Deflection.
	60+60	E. T. C.	Total Angle 56°	o Hub.	6° 24′	3° 12′
T. C. 1° to 30 ft. ∠9° 36′	60				5° 24′	
	9				2° 51′	
	8+20	E. C. C.		o Hub.	0= 18° 24′	24° 48
Constant curve 8° to Right Angle 36° 48′					17° 36′	
	7				13° 36′	
	6				9° 36′	
	5				5° 36′	
	4				1° 36′	
T. C. 1° in 30 ft. ∠9° 36′	3+60	B. C. C.		o Hub.	3° 12′	24° 48
					1° 48′	
	2				0° 21′	
	51+20	P. T. C.		o Hub.	0° 00′	3° 12
						56° 00

The tangential angles are obtained from the formula $\frac{S^2}{18000}$ for the first transition curve, for the constant curve they are got in the usual way and in the final transition by subtracting the values obtained by the formula $\frac{S^2}{18000}$ from the corresponding values for a continuous 8° curve. The last column of deflection is kept as a check on the work and represents the total deflection at each point, first 3° 12′ from the tangent to chord of transition, then 6° 24′ + 18° 24′ = 24° 48′ from chord of transition to chord of constant, and finally the same two repeated in reverse order.

Now, it will be seldom in actual field practice that the 700 feet of 8° curve can be run in from B. C. to E. C. without at least an intermediate point or hub, so that in a majority of cases the extra labour involved in running the curve with transition ends direct with the instrument is merely an extra setting up of the transit over those required for the simple curve and the five minutes figuring to obtain the elements of the transition ends. The writer thinks the postulate already advanced will be readily granted, that what we want is a curve which may be readily run by angular measurements rather than by offsets for much the same reasons as an accurate survey is made in the field by angular measurements and laid down on the plan by linear measurements calculated from them. The parabola plain or cubic is a very taking curve for the mathematician because of the simplicity of the equation by which it is represented and the co-ordinates of any point obtained, but in field practice any method involving co-ordinates is, at best, cumbrous and unhandy. We have not only to measure the offsets very carefully and exactly (not always an easy thing to do), but we must be careful to lay them off exactly at right angles to our base line. When the latter, as in Mr. Hopkin's method, is a curve for part of the distance this is not a thing which can be rapidly done, even with an optical square or cross-staff.

The writer has been rather taken to task for suggesting the use of curves as sharp as 10° and over. The question of their use has so much to do with necessity for and the best form of transition curves that he may, perhaps, be pardoned for recurring to the subject of extremely sharp curves. He instanced a 21° curve, built under his own supervision, and he had occasion within the last few days to revisit the locality. It had been in constant use by all trains on the road for nearly three years. There has never been a wheel off the track and a large portion of it built on pile work has literally never been touched with maul or claw-bar, but stands (without guard rail) as true as when first laid. The care with which it was centred and the rails bent has no doubt much to do with its success. The superelevation of outer rail is only three inches, but it is essentially a low speed curve from its position on the road. The question of superelevation is another which is closely correlated to the form and "pitch" of transition curves, and the writer advances with some diffidence the opinion that, instead of an amount of elevation varying directly as the degree of curvature, we should, on account of the higher velocities indulged in on the flatter curves and the paramount influence on centrifugal force of speed, employ some formula involving the square root of the degree of curvature.

The simple formula $E = \sqrt{D}$ * answers in his experience very well in practice without any "trimmings" or constants. The elevations would be:

For a 1° curve	0·10	feet.
" 4° "	0·20	"
" 9° "	0·30	"
" 16° "	0·40	"

Which he thinks are not far from what the best practice demands. The writer begs to apologize for the shortcomings of this hurriedly written critique. The subject is so fascinating a one for him that even the short time at his disposal did not deter him from putting in his oar.

Mr. M.J. Butler—For ten years past it has been the writer's custom to use some form of transition curve on all railway work. At first use was made of a table that was published in the *Railway Gazette*, afterwards the "Spiral Curve" by W. H. Searle. All along, however, the lack of flexibility in the published curves was felt as a drawback to the convenient use of this most valuable adjunct to railway location. On February 8th, 1890, Mr. A. M. Wellington, M. Inst. C. E., published in *Engineering News* a formula that seems to the writer to best fill the requirements. Mr. M. J. Butler.

It is as follows:

$$n = 1.86 \sqrt{\frac{O}{D}}$$

$$O' = \left(\frac{m}{n}\right)^{\frac{3-O}{2}}$$

Where n = half length of transition curve O = total offset in from tangent at B. C. of regular curve, D = degree of main curve, O' = offset in from tangent, out from curve, at distance m

*E in tenths of feet, D in degrees.

from beginning of transition curve. The offsets O' are given for values of $m = 30$ feet, (a rail length) as being on the whole the most convenient in use.

On location work the tangents are produced to intersection as usual and the data for the regular curve calculated as usual. Then an offset in towards the centre of the curve is taken "O," as great as the topography will permit of. The regular curve is then located in the usual way from the offsetted hubs, the stake for the transition curve at the B. C., being half the amount of "O." Then look in tables and move nearest stake on curve *out*, on tang. *in*, the amount shown for the distance given.

The accompanying tables are calculated for curves of 1° 2° 3° 4° 5° 6° and are extended too far for ordinary work; it was thought, however, best to show the actual values of the various "O'" offset and half lengths "n."

See diagram :

The writer does not consider any method practicable that calls for deflection with a transit instrument, the small angles and short chords leading to very unsatisfactory results. The accompanying tables have been calculated for the writer's own use and are given as a practical easy way to put in transition curves.

The subject theoretically treated is one calling for a considerable amount of mathematics and is hence viewed almost with horror by all old engineers who have long since forgotten the little higher mathematics they may have once known. Practically a transition curve facilitates work on difficult location, and is in every respect a desirable improvement.

VALUES OF O′.

Values of O.......		.5	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0	12.0
Values of *n*.......		52	74	96	130	152	169	186	201	212	227	238	251	262
6° Curve Values of *m*.	30	.04	.03	.03	.018	.014	.013	.012	.009	.009	.008	.008	.008	.008
	60	....	.26	.24	.15	.108	.10	.096	.095	.094	.08	.08	.07	.06
	90	....	....	....	.49	.39	.37	.324	.29	.28	.27	.25	.23	.215
	120	....	....	....	1.16	1.02	.89	.77	.71	.68	.64	.63	.57	.51
	150	....	....	....	....	....	1.70	1.50	1.41	1.35	1.29	1.25	1.12	1.00
	180	....	....	....	....	....	....	2.65	2.46	2.34	2.22	2.11	1.97	1.72
	210	....	....	....	....	....	....	....	....	....	3.50	3.41	3.14	3.07
	240	....	....	....	....	....	....	....	....	....	....	....	....	4 52

VALUES OF O'.

Values of O..		.5	1.0	2.0	3.0	4.0	5.0	6.	7.	8.	9.	10.
Values of *n*.......		58	82	117	143	166	186	203	219	234	249	262
5° Curve. Values of *m*.	30	.03	.02	.01	.01	.01	.01	.009	.009	.008	.007	.006
	60	....	.19	.12	.11	.08	.08	.08	.07	.07	.06	.05
	90	....	....	.45	.37	.31	.27	.25	0.24	0.22	.20	.18
	120	....	....	.....	.89	.74	.64	.61	0.58	.53	.49	.43
	150	....	....	.....	.....	.....	1.25	1.17	1.10	1.05	.99	.92
	180	....	....	.....	.....	.....	2.26	2.04	1.93	1.83	1.70	1.57
	210	....	....	.....	.....	.....	.....	...	3.00	2.82	2.70	2.56
	240	....	....	.....	.....	.....	.....	.....	.....	.....	.....	3.77

TABLES OF VALUES OF OFFSETS IN FROM TANGENT OUT FROM CURVE FOR VARIOUS VALUES OF O.

	Offsets in feet O =	0.5′	1′.0	2′.0
	Values of n =			
	Half length transition curve	130′	186′	262
	Distance.			
1° Curve. Values of m.	30	0.003	0.002	0.001
	60	0.024	0.016	0.010
	90	0.082	0.054	0.036
	120	0.192	0.128	0.085
	150	....	0.25	0.17
	180	....	0.43	0.29
	210	....	....	0.51
	240	....	....	0.75

VALUES OF O′.

	Values of O =	0.5.	1.0.	2.0	3.0	4.0
	Values of n	93	130	186	227	262
	Distance.					
2° Curve. Values of m.	30	0.008	0.006	0.004	.003	0.002
	60	0.065	0.048	0.032	0.026	0.020
	90	....	0.164	0.108	0.081	0.071
	120	....	0.384	0.256	0.192	0.170
	150	....	....	0.500	0.375	0.33
	180	....	....	0.80	0.74	0.57
	210	....	....	....	1.17	1.02
	240	....	....	....	....	1.51

VALUES OF O′.

Values of O =		0.5	1.0	2.0	3.0	4.0	5′.0	6′.0′
Values of n =		74	107	152′	186′	214′	238	262
° Curve. Values of m.	*Distance.*							
	30′	.016	.01	.007	.006	.005	.004	.004
	60	.13	.087	.054	.048	.043	.039	.030
	90		.296	.185	.162	.148	.127	.100
	120			0.51	.384	.351	.312	.26
	150				.75	.68	.62	.50
	186				1.29	1.18	1.05	.86
	210					1.88	1.70	1.54
	240							2.26

Values of O =		.5.	1.0.	2.0.	3.0.	4.0.	5.0.	6.0.	7.0.	8.0
Values of n		65′	93′	130′	160′	186′	206′	227′	245′	262
4 ° Curve. Values of m.	*Distance.*									
	30	.02	.016	.012	,009	.008	.007	.006	.006	.005
	60		.13	.097	.079	.064	.06	.053	.048	.04
	90			.328	.263	.216	.198	.162	.163	.14
	120			.779	.633	.512	.487	.384	.386	.34
	150				1.206	1.0	.933	.86	.794	.66
	180					1.75	1.65	1.48	1.36	1.15
	210							2.34	2.15	2.05
	240									3.01

The transition Curve bisects the central offset O and is bisected by it.

Mr. H. R. Lordly.

Mr. H. R. Lordly in reply said:—

Mr. Hopkins offers a method using two separate cubic parabolas and gives formulæ which he has worked up for the same. His method is not so accurate as the one under discussion and according to many engineers has one serious objection, viz., that it is laid out by offsets. Deflection angles are considered better, particularly if there is any length of transition curve. The remark that "no matter how indolent or ill-informed an engineer is, etc," is not a very strong argument in favour of its adoption. In all probability if a railroad company deem it necessary to put in transition curves at all, they will require the work to be properly done.

Mr. Sproule's remarks regarding wear of rolling stock, etc., should have considerable weight in relation to the use of transition curves. He magnifies a good deal the difficulty of laying out the curve, and it is to be hoped that this idea, which is quite common among engineers, will soon die out. It is quite erroneous. Perhaps scientific superintendence, of which he speaks, might do away with much of the customary "track tampering" and the result be a benefit all around.

Mr. Butler does not consider any method of using deflections practicable. In this he differs from most of the engineers the writer has consulted. The method he uses, while perhaps sufficient under certain circumstances, is quite approximate.

Mr. Wicksteed's remarks show that he is well acquainted with both the practical and theoretical working of the transition curve. As will be seen by Mr. Irwin's comparison, the results obtained by Mr. Wicksteed's method do not differ materially from those obtained by the writer. As to the calculation of the curve his method is not as short, thus giving an advantage to the curve under discussion. It may be a good thing to be able to get along without the use of tables, but their adoption simply means convenience and saving of time. The writer would refer him to Mr. Irwin's discussion regarding the relation of the original curve to the cubic parabola.

Mr. Wicksteed thinks that Mr. Hopkins and the writer may have frightened some of the members by the complicated? formulæ. This, of course, if it is the case, is to be regretted, but in the writer's case it was a necessary evil as the paper contributed was largely theoretical. It was expected that the example worked out would interest the more practical reader, and it is hoped that it has to some extent fulfilled its mission.

Mr. Tate prefers the "taper curve," but the writer does not believe he would continue to do so should he become quite familiar with the method under discussion. It is not as accurate as the latter nor as easily handled as regards computation.

Replying to Mr. Irwin, the author regrets that his paper should be considered lacking in clearness and conciseness.

Brevity was an object; first on account of space, and second, because in technical papers it is considered desirable. As to whether the paper is obscure depends upon the kind of spectacles through which it is seen. It was intended to be a theoretical paper, and the author begs to remark that the fact of its being a student's paper should not be lost sight of.

Mr. Irwin says "the curve proposed is easier to lay out than any he has seen, provided no tables were furnished," but later on he agrees with the writer in using tables, saying that "figuring done in the field wastes time and is always liable to error." This statement may be regarded as a point in favor of the original method, and explains why Professor Crandall worked up the tables.

The method of calculating the deflection angles is illustrated under the example given and the first paragraph of section 3 gives the principles.

Mr. Irwin makes a very good comparison of his own method with that of Mr. Wicksteed and the method under discussion. The latter appears to be shorter than either of the others.

Mr. Irwin's method of getting the tangent and subtangent is longer than the writer's method. The latter simply requires the value of X from the table to be added to the tangent to get the subtangent. The tangent$=(R+F)\tan\frac{\Delta}{2.}$ the natural function being used.

The slight discrepancy mentioned in the computed values of tangent, is caused by the writer using 955.36 as the value of R. It is really correct.

Mr. Irwin's curve, as worked up so far, is only for a maximum of 6°, and has a constant chord length of 60 feet. Professor Crandall's tables provide for any probable curvature and the length of chord may be selected to suit the case.

The proof that deflection at the tangent = $\frac{1}{3}$., was omitted it is not a new statement and has been used elsewhere.

Mr. Irwin's proof covers Professor Crandall's table No. 2 and gives corrections to be applied, but they are very small till after 10 or 12 degrees are reached.

Mr. Irwin shows how the Lemniscate might be used for a transition curve.

While the results obtained may not vary much from those obtained by the methods under discussion, for flat curves, it seems doubtful if it is adapted to practical use.

The author has gone rapidly over the principal points brought up. There remain one or two statements of a theoretical nature which might be interesting to look into but time will not permit.

He wished to show in his original paper that there was such a thing as a " correct transition curve," and that it was in actual use. It is the method now taught at the College of Civil Enineering, Cornell University.

To those who took an interest in his paper the writer extends thanks, and feels particularly indebted to Mr. Irwin, whose able discussion clearly shows the advantage the transition curve possesses.

Mr. M. W. Hopkins.

Mr. Hopkins in reply said:—The great majority of railroad engineers do not put in transition curves, and if it is not made simple they will not. The method used by the writer is so simple that all would use it, rather than none, until they became desirous of having a more accurate method. The errors in this are so small as not to be noticeable in a train running over the road. It gives the track man an opportunity of easing off the curves even if they should be put in very far from the theoretically correct position. This, however, is far from being the case, and when the offset used is small the error is very small indeed. Track men always use a small offset and only use it for the purpose of having a transition curve to make the train ride more easily around the curves and to admit of the necessary elevation of the outer rail being gradually attained. This is the chief object to be sought. All further niceties are unnecessary, and by asking too much we shall have no transition curve at all. And it seems to the writer that the advantage of being able to put in the transition curve at any time after the main circular curve is located is a material one and will very often be the advantage which will decide an engineer to use it.

As to the objection that this is not accurate enough to run the centre line, the writer does not understand why. A transition curve put in this way will give smooth running and will entirely satisfy any engineman.

As to the objection that the offset would often throw the line off a high embankment, if the offset is not more than four or five feet this objection has no effect.

What is wanted is a transition to aid us in easing off the circular curves. All extra refinements are serious obstacles to the general use of transition curves.

Friday, 9th December.

JOHN KENNEDY, President, in the Chair.

Paper No. 70.

PLUMBAGO, AND SOME OF ITS USES.

By JOHN FRASER TORRANCE, M. Can. Soc. C. E.

Plumbago, graphite or black lead is well known to all of us in its various forms of application as stove polish, foundry facings, lubricating powder, pencil leads, graphite grease, graphite packing, graphite paint, etc., etc.

It sometimes occurs in nature in flat hexagonal crystals, but generally it occurs massive or more or less radiated, foliated, scaly or compact. It is of a grayish-black color with an almost metallic lustre and a black shining streak. It is too soft to strike fire with a steel and it is a splendid conductor of electricity. Its specific gravity ranges from 1·8 to 2·24. It is composed chiefly of carbon; but usually it contains more or less alumina, silica, lime, iron, etc., apparently in mechanical mixture rather than in chemical combination. Next to the diamond it is the most incombustible form of carbon. For this reason it is used in combination with fire-clay for the manufacture of crucibles to resist the highest temperatures.

As far as the author can learn, all the known deposits of plumbago of any economic value occur in rocks of Laurentian age. The only mine working on a large scale in the United States is operated by the Jos. Dixon Crucible Co. at Ticonderoga, N. Y. In a report by Albert Williams, Jr., of the United States Geological Survey on the Mineral Resources of the United States, it is stated that the deposit now being exploited is a bed of graphitic schists fifteen feet thick, carrying from 8 to 15 per cent. of graphite. This is treated by a wet process, wherein the ordinary practice is reversed; the "heads" being the refuse and the "tails" being the valuable graphite. The average output is placed at 500,000 lbs. valued at an average of 8 cents per lb. at the works. Apart from this Company's output the North American trade is supplied almost entirely from the amorphous earthy deposit near Passau in Bavaria, and the large veins of graphite in the Laurentian gneiss near Travancore in Ceylon. But the finest pencil lead comes from the mines in Irkutsk, Siberia.

Some interesting notes on the geology of Ceylon were published as far back as 1818 by Dr. John Davy, who says: "Graphite is pretty commonly disseminated in minute scales through gneiss. It is worthy of remark that graphite is generally found in company with gems. I have had so often occasion to make the observation that now I never see the former without suspecting the presence of the latter."

The Ceylon graphite is extracted from large fissure veins in the gneiss, which are completely filled by the graphite. It requires to be merely cobbed and sized before going to market. It is known as "dust," "chip" and "lump." Dust sells at 2½ cents per lb. in New York, chip at 3½ cents and lump at 5½ cents. It is used for all purposes except pencil-making.

The German black lead is far more impure, containing only 35 to 40 per cent. of carbon, the balance being of the composition of clay. But it is suitable for use in pencil-making. Very refractory crucibles are made by mixing 2 to 3 parts of this impure plumbago with one part of clay. Such crucibles will undergo 70 to 80 meltings in brass foundries, about 50 meltings with bronze or 8 to 10 meltings with steel.

The American and Canadian graphite is used for all purposes of the trade and excels all others as a lubricant. Many tests have proved this conclusively.

As graphite or plumbago is found almost exclusively in rocks of Laurentian age and there is a greater development of these rocks in Canada than anywhere else, we should naturally expect to find many deposits of graphite in this country. It is nearly half a century since two of Canada's most distinguished citizens called attention to the large deposits of graphite in the Laurentian formation both north and south of the Ottawa River, and pointed out the possibility of their profitable exploitation for shipment to the British market. Since the date of this first report by Sir Wm. Logan and Dr. T. Sterry Hunt, the marvellous growth of the American nation has developed a demand for this material on our own continent such as they hardly anticipated. And it is not creditable to the enterprise and skill of our Canadian capitalists and miners that this market is supplied from similar deposits in Bavaria and far-distant Ceylon, while our own lie idle and almost unknown.

In Canada the graphite is usually found in close relation to some of the large bands of massive crystalline Laurentian limestone that can be traced through Burgess and Elmsley (near the Rideau) and re-appear in Hull, Templeton, Buckingham, Lochaber and Grenville. This mineral generally occurs disseminated in scales in beds of limestone, sandstone or pyroxenite; or else in veins from a few inches to several feet thick. These beds are often interrupted, producing lenticular masses which are sometimes pure and sometimes mixed with carbonate of lime, pyroxene and other minerals. At times it is so finely disseminated through the limestone as to give it a blueish-grey color, which serves to mark the stratification of the rock. In one locality at Grenville sphene, zircon, pyroxene and tabular spar have been found associated with it, reminding us of Dr. Davy's observations about the precious stones in Ceylon. No veins of graphite, however, have been found yet in Canada of sufficient size and extent for profitable exploitation. All attempts at plumbago mining in Canada have been confined to the beds of disseminated graphite in Buckingham, Lochaber and Grenville townships as well as some rich masses where a number of small veins are seen to intersect; and to a sandstone bed near Oliver's Ferry on the Rideau Lake, which is richly impregnated with graphite.

Some ten years ago it was the author's duty in connection with the Geological Survey of Canada to examine some of the principal deposits of plumbago in Ottawa County of this Province, and enquire the reasons why they lay idle. His report was published in the volume for 1882–83–84. But these reports are so little seen by our members that he would give a few quotations :—

An American manufacturer wrote the author that his company tried a great deal of the Canadian graphite some years previously, but were obliged to give it up *because it did not run uniform.* Some of the crucibles made from it were as good as any, but others would crack or melt. They gave it a thorough test and used a great many barrels of it. It contained sulphur and other impurities.

Surely this statement is an ample explanation of the idleness and decay of the Canadian plumbago works. But the author did not hesitate to assert then (and he repeats most emphatically here) that "as long as the price of dressed plumbago does not fall below forty dollars per ton, many of our Canadian deposits could be profitably worked, always provided that they are worked by competent mining engineers. But no mining company need hope to succeed in Canada or any other place unless the manager has had a careful technical training or the ore is of phenomenal richness."

The relative value of our Canadian deposits depends not only upon the freedom from lime and iron, as well as their richness and extent, but also upon their situation with regard to easy exploitation, cheap labour and supplies, low rates of freight, etc. On nearly all these counts the author is inclined to rank the deposit once worked near Oliver's Ferry on the Rideau Lake as far superior to any in this Province. This deposit occurs in a five-foot bed of sandstone which has been opened on the crown of a flat anticlinal. The actual character of this deposit is not very clear. The overlying beds are sandstone and the gneiss lies immediately below. It may prove by farther work to be a bedded vein. But the author holds it to be a regular bed. A sample lot shipped from here this autumn and dressed by S. R. Krom's pneumatic jigs yielded ten per cent. of dressed graphite. Men with some experience in phosphate mining can be hired here for $1.25 per day: plenty of dry tamarac can be bought for $1.50 to $1.60 per cord; the wharf at Oliver's Ferry is only one mile distant, and the main line of the Canadian Pacific Railway comes within three miles at "Elmsley," a way station between Perth and Smith's Falls.

The sudden closing of this mine in 1874 is attributed partly to the wide-spread ruin among the iron masters of the United States, who were its chief customers, and partly to an agreement between its owner and some of the large operators in graphite across the boundary. Many facts connected with its closing are not readily explained in any other way. The proprietor Mr. Eaton, of Rochester, N. Y., himself stated to Mr. Morris of Perth, that it took only six months' profits to pay for the property, plant and all.

The quantity of plumbago, graphite or black lead imported into the United States in the year ending 30th June, 1891, amounted to 10,136 tons, valued at $509,809; while it increased in the following year to 13,511 tons, valued at $726,648. This shows conclusively that there is no danger of any properly conducted Canadian plumbago mine failing to market its wares, if it is worked economically and the output is thoroughly uniform and up to standard.

From the valuable report on the Mineral Resources of the United States published in 1884, the author quotes the proportions of the output, etc., devoted to various uses, as follows:—

Making crucibles and refractory wares	30%
Stove polish	30%
Lubricating graphite	13%
Foundry facings	10%
Graphite greases	8%
Pencil leads	3%
Graphite packing	3%
Polishing shot and powder	2%
Paint ½; electrotyping and miscellaneous ½	1%

But it is altogether likely that this comparative statement will be radically modified, as a result of the general introduction of the wonderful new lubricating composition invented by Mr. Philip H. Holmes of Gardiner, Maine, known as fibre-graphite. The author's attention was first called to it about twelve months ago by a letter from Mr. J. T. Taylor, M.E., the Mechanical Superintendent for Messrs. W. & J. Sloane, who had been professionally investigating its merits. His claims for this material were almost too strong to be readily believed. But subsequent investigations by Prof. Henry T. Bovey, Messrs. Frank Redpath, R. F. Ogilvy and the author have fully confirmed the immense value of this new anti-friction material. Prof. Bovey was permitted to bring back two boxes bushed with this material, which he set up in the Workman workshops at McGill University last June. They have been running ever since and giving perfect satisfaction without any oiling or special attention.

This fibre-graphite is simply an intimate mechanical mixture of finely divided plumbago and mechanical wood pulp in varying proportions according to the purposes of the special bearing. These materials are mixed in water and pumped with a hand-pump into the moulds, which are made of brass with grooves on the outside and small holes possibly $\frac{1}{16}$ inch in diameter spaced about $\frac{5}{8}$ inch apart. Each mould is inclosed in a heavy case made of a steel casting. The mass is compressed by hydraulic pressure to about $\frac{1}{3}$ its original bulk, while the water escapes through the holes and along the grooves. (The inventor considers this system of drainage very important, as the flow of water tends to arrange the wood fibres radially from the centre of the bearing.) After sufficient pressure the piece is removed from the mould and dried. It is then immersed in a bath of hot linseed oil and finally subjected to a slow baking in a gas oven.

This product can be cut and tooled with ease. But it takes and retains the form of the mould so perfectly that any tooling is unnecessary.

In August, 1891, a committee of the Franklin Institute composed of H. R. Hoyl, Chairman, and Messrs. J. Sellers Bancroft, Thos. P. Conard, Philip H. Fowler, Luther L. Cheney, Stockton Bates, H. W. Spangler and S. H. Vauclain made a very exhaustive investigation of the merits of this invention, which resulted in the award of the Elliot Cresson gold medal to Mr. Holmes. In the award they state that there is nothing in the Holmes compound that can be injured by any degree of heat that can arise from frictional causes, or can cause surrounding objects to ignite, therefore the fire risk due to the over-heating of journals will disappear in direct proportion to the use of these graphite bearings." They say :—" We find further that the ratio of friction with the Holmes' bearings is much less than with well oiled metal bearings, and greatly superior to the results obtained in common practice, thereby effecting a great saving in power even in comparison with the most carefully attended metal bearings. The co-efficient of friction of the Holmes' bearing is practically constant, being no greater at starting than when running at any speed. * * *

" The remarkable qualities of this bearing material are strikingly exhibited in its application to the spinning frame, when the spindles are run at very high velocities. In this direction your committee has taken special pains to verify, by personal tests, the excellent results vouched for by others. Spindles running with unusually tight belts constantly for ten hours a day for three weeks, at a speed of 8,400 revolutions per minute, did not heat or show any perceptible wear either of the spindles or the graphite bearings. Thus, through Mr. Holmes' invention, it has become practicable to run an entire spinning plant without using a single oiled bearing, which means in economy, cleanliness and freedom from fire risk, conditions of inestimable value. Your committee have also practically tested the brushes made of this material for use upon dynamos, and found them practically indestructible, that they do not wear the commutators, and give most satisfactory results in every way."

The history of this invention was told to the author very simply by the inventor himself, who is also the inventor and maker of one of the most successful turbine water wheels in the New England States. Mr. Holmes was experimenting with wooden steps for these wheels, when he reflected that plumbago was the best known lubricant for wooden cog wheels and that it would probably be the best for his wooden steps also, if he could devise some means of fixing it there. Suddenly the idea occurred to him that he might incorporate the plumbago in the step. A series of careful experiments with plumbago and wood pulp gradually led him to a more complete success than he could have hoped for.

One of the facts cited in a paper by Mr. John H. Cooper, M.E., on this subject read at a recent meeting of the American Society of Mechanical Engineers, shows how perfectly Mr. Holmes solved the problem of his water-wheel. Mr. Cooper says:—

" When used as a step bearing in water upon which a loaded vertical shaft runs, it is proven that 300 lbs may be supported in revolution of 470 turns per minute upon an obtuse angled cone 2 inches in diameter, which is the equivalent of 100 lbs. per square inch of surface covered, in which case the tool marks were not all worn out nor any detrimental effect of submersion or abrasion observed after three months of running."

In conclusion the author would add that, if his lengthy presentation of this subject has not exhausted your interest in the matter, he hopes next session to be able to lay before you the record of some practical experience in mining and preparing our Canadian graphite for market, and also some observations on actual work done by the Holmes' fibre-graphite bearings in Canadian industrial establishments.

DISCUSSION.

Mr. Ogilvy said, in regard to the use of fibre-graphite for bearings, he might mention two cases which came under his notice some months ago and which are rather interesting. One was in the workshops attached to a paper pulp mill in Yarmouth, Maine. All the shafting in the shop was running with only fibre-graphite as a bearing, and had been working very satisfactorily for six months. The master mechanic stated that as far as he could judge by external examination, the graphite bearings were as good as when put in and would outlast babbit. The other case was in the works of a manufacturing company in Waltham, Mass., where some bearings had been in use for over two years. In this case also the graphite was in shaft bearings and had given perfect satisfaction, the guarantee having been more than fulfilled. Mr. Ogilvy.

In reply to a question as to whether these bearings were for sale in Canada yet, the author said:

No. In order to protect the Canadian patents on this new material and its applications, the bearings cannot be offered for sale here until they are manufactured by a Canadian company, such as we are now taking steps to organize. The material is protected by patents. They will probably charge every cent that the consumer is able to pay.

Mr. Kennedy asked has the journal that is running this box been tested in any way since it began to rnn? Mr. Kennedy.

Mr. Torrance said he was not aware of any tests yet made of the box before the meeting, which has been running very nearly six months on the line shafting in the Workman workshops at McGill University. During that long period it had received no oiling or any special care. Mr. Torrance.

Mr. Kennedy said it must be known to some one here why it is that although stated in the paper that the friction is not less than ordinary, the box runs all the time so, that one can with difficulty bear the hand on it. The speaker said he did not know where the heat came from, nor just what temperature it was, but it was a temperature that if you put your hand on the box would make you take it off very quickly. He would suppose it was about 150 or so. Mr. Kennedy.

The author said in his opinion the unusual heating of this box is caused by its being about 1-16′ too small for the shaft running Mr. Torrance.

in it. The rough chiselling on the back of this box shows plainly enough that it was intended for quite a different style of hanger. From the author's own observations in a number of factories in various parts of the United States, he can testify that under proper conditions boxes bushed with this material run cooler than well oiled metal bearings. This is borne out by the tests given by Prof. Bovey as made on a Thurston testing machine, which established the co-efficient of friction of well oiled metal bearings as 0.16, while the co-efficient for fibre-graphite bearings running without oil was 0.10, and when these patent bearings were themselves lubricated with oil in the ordinary way this co-efficient fell to 0.033.

In reply to a question as to whether he had any particulars of tests of Canadian and American plumbago, the author said:

Mr. Torrance. He believed that a valuable series of tests of the lubricating values of American and Canadian graphites were made by Professor Thurston, at the Stevents' Institute about eight years ago, but the author has not been able to consult the published report. He understands that they clearly established the superiority of our North American plumbago over the Ceylon article for purposes of lubrication.

It was asked that if oil had to be used, as is sometimes done, would it do away with the breaking or reduce the cost?

Mr. Torrance. Mr. Torrance said customers may use oil on these bearings in the usual way, if they are so inclined. The oil can do the bearings no injury and it reduces the friction still lower. But the author is not aware of any person using oil upon them. On the contrary, our largest mill owners and manufacturers are eager to banish oil, with its dirt and danger of fire, from their establishments. The largest mill owner in Canada told him that he would gladly change all the bushings in all his mills as soon as tests in his own works showed that our claims were justified. Another manufacturer here in Montreal told him that it was not the cost of the oil that troubled him or the wages of the men employed as oilers so much as the fact that if they neglected a single metal box, and it heated he was obliged to shut down that whole department until it cooled again.

One rather serious difficulty in the way of the rapid introduction of this material into general use in Canada, is the fact that the machinists of Canada have no standards at all in regard to shafting, or boxes or hangers. This will necessitate the use of a much larger number of patterns for our business than would be required for a business of equal size in the United States. Of course the customers will have to pay this extra cost in some way.

It was asked had anything been done to induce mining men to take an interest in this industry?

Mr. Torrance said the attention of capitalists in England and France was called to the deposits of plumbago near Buckingham many years ago, and he believes that a large sum of British money was lost there. A friend told him that a company was almost floated in Paris some years ago with a very large capital to work the Walker property there. He was not familiar with all the facts, but it seems likely that the very high price demanded for a comparatively undeveloped property was the main cause of its collapse. American capital is now working one of the properties in that region on a small scale. Mr. Torrance.

In reply to a question as to what was the annual output from mines of this mineral?

The author said he could not give the annual output of all the known mines of plumbago. The figures given in his paper for the output of the mine at Ticonderoga cannot be depended on as anything better than a rough approximation. Mr. Torrance.

Mr. Kennedy said that he thought that something must be wrong with the deposits in this province, as large amounts of capital and plenty of skill had been employed in the attempt to develop them. Mr. Kennedy.

Mr. Torrance said he granted that sufficient British and local capital was sunk at Buckingham and Lochaber to establish plumbago mining on a permanent basis, but he must emphatically deny that the proper skill was ever displayed there. The fact that customers complained of serious quantities of lime and iron being left in the finished product, and finally abandoned its use on that account, is the plainest possible evidence that the proper technical skill was utterly lacking. Mr. Torrance.

In reply to a question as to whether there was a great deal of difference between the Canadian and Ceylon plumbago,

Mr. Torrance said the difference between Canadian and Ceylon plumbago is exactly what we might expect from their different modes of deposit as described in his paper. The Ceylon veinstone when properly cobbed and selected will run up to 99 per cent carbon, while the Canadian disseminated plumbago runs only 66 per cent or thereabouts. But the Canadian ore can be Mr. Torrance.

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dressed by currents of water or air, or else by fusion with caustic soda up to the same standard of purity as the Ceylon. For lubricating purposes the properly cleaned Canadian or American plumbago is superior to that from Ceylon.

In closing this discussion the author wished to call attention to another successful oilless bearing recently introduced in England. It is called the Carboid Oilless Bearing, and it is described at some length in *Engineering* for 2nd December, 1892. Some months ago the author noticed in technical journals a report of a paper about these bearings read by Prof. Unwin at the Edinburgh meeting of the British Association. No reference to them was embodied in his paper, because the first report stated them to be a mixture of carbon and steatite. But this recent article in *Engineering* describes them as being a mixture of plumbago and Steatite. It is probable that when they began to manufacture these bearings on a commercial scale they found difficulty in obtaining a large supply of carbon of anything approaching uniform lubricating value. Naturally they would then turn to plumbago as the most valuable and convenient lubricating material. This article states that the steatite is mixed with the plumbago to harden the mass and thus enable it better to resist violent shocks. Apparently, this mixture is treated with hot oil and then baked in very much the same way as the Holmes' fibre-graphite bearings. These bearings have very successfully endured the shocks which they underwent in long continued use as brasses on train cars in Edinburgh. And it has successfully undergone lengthened trials in boxes on line shafting. Dust and grit do not seem to injure it any more than fibre-graphite. *But* the introducers do not claim that the carboid boxes diminish friction as compared with well-oiled metal boxes. This point is well worth special notice in any comparison of the respective merits of carboid and fibre-graphite bearings.

The invention and commercial introduction of these carboid bearings in England is of interest to us in Canada chiefly, perhaps, on account of its stimulating the demand for our Canadian plumbago.

When we find *The Engineer*, published in London, quoting the price of No. 1 Ceylon plumbago at £25 to £26 sterling per ton, our capitalists should look very favorably on any well matured proposals for developing any of our well-known deposits of this valuable mineral.

Thursday, 28th January.

P. ALEX. PETERSON, Vice-President, in the chair.

The following candidates having been balloted for, were declared duly elected as:

MEMBERS,

JOHN LANGTON.

ASSOCIATE MEMBERS,

DAMIEL ISAAC VERNON EATON, THOMAS HENRY JONES.

STUDENT,

M. J. E. LENOBLET DU PLESSIS.

The following were transferred from the class of Associate Members to that of Members:

SAMUEL FORTIER, HARTLEY GISBORNE.

CHARLES HODGSON OSTLER.

The discussion on the papers on "Transition Curves" occupied the evening.

OBITUARY.

Richard Plunkett Cooke was born at Parsonstown, Ireland, on the 12th October, 1824. He was a graduate of Trinity College, Dublin, and studied Engineering under Sir John McNeil. In 1852 he came to Canada and obtained a position as assistant Engineer on the Toronto and Guelph Railway and subsequently, on the road becoming part of the Grand Trunk Railway, held the position of Divisional Engineer in charge of the line west of Toronto. In 1858 he was transferred to the central division and had charge of the Grand Trunk line between Montreal and Toronto. This position he resigned in 1861, to take charge of the Brockville and Ottawa Railway as General Manager and Chief Engineer. About the year 1869 he undertook and carried to a successful completion the building of the Boston, Barre & Gardner R. R. in Massachusetts. In later years he executed many important works, including the new canal at Carillon, and at the time of his death was engaged in mechanical work at Vancouver, B.C. Mr. Cooke was elected a member of this Society on the 19th April, 1888.

Walter W. Gilbert was born in 1849. He patented a steam-moved valve for steam pumps in 1868. He was three years in the Columbia School of Mines, New York, from 1868 to 1871, and two years with Hicks, Hargreaves & Co., Bolton, Eng. He was Vice-President of the Gilbert Bros. Engineering Co. and member of the firm of E. E. Gilbert & Sons. He took an active part in designing most of the machinery used in submarine rock drilling and dredging at Galops Rapids, River St. Lawrence and St. Anne's Channel and River Ottawa. Mr. Gilbert died March 8th, 1892. He was elected a member of this Society on the 3rd February, 1887.

(Written by D. H. Keeley, A.M. Can. Soc. C.E.

Frederic Newton Gisborne.—The name of F. N. Gisborne, the late General Superintendent of Government Telegraphs, has been for a long period prominently before the public, and especially in the engineering world, in connection with the subject of transatlantic telegraphy. In several publications the story is told at length of the pioneer's work performed by him in Newfoundland in the early days of telegraphy, and of his claim to recognition as the original projector of the first Atlantic cable. Whether it can ever be determined that the idea of establishing telegraphic communication by means of a submarine cable between Great Britain and America was the conception of a single mind, is an open question. There can, however, be no doubt about the fact of the late Mr. Gisborne's sincere belief in that the idea was his and his alone. He always maintained that ground; and it is very oddly coincident that, after a lapse of 30 years, the very first counter-claim (apart from the popular assumption that the whole thing originated with Mr. Field, to whose indomitable energy and marvellous enterprise the successful issue of the project is commonly ascribed) made its appearance in one of the electrical journals (*The Electrician*, Vol. XXIX, No. 745) just at the time when the man whom it should above all others most concern was past beyond knowledge of its existence![1] However, putting the question of the conception of the idea aside, the substantial fact remains—established as it is by the circumstance of his name being one of those specified for the provisional directors in the original charter granted to the New York, Newfoundland and London Telegraph Company, by the Legislature of Newfoundland in 1854—that Mr. Gisborne was one of the prime movers in that stupendous enterprise. How it came about that he was subsequently ignored by his fellow-directors, when the Atlantic cable became an accomplished fact, is not clear; but by those familiar with his disposition and temperament it would most likely be attributed to his simply attaching a too broad significance to the circumstance of an enthusiastic demonstration in his honour by the people of Newfoundland, on the occasion when he was presented with a magnificent testimonial, a massive silver statuette, that is still in the possession of his family. The local event appealed perhaps too strongly to his imagination, and so unhappily was regarded by him as a world-wide mark of recognition of his unquestioned merit. If that were the case, it is hardly to be

[1] Mr. Gisborne passed away on the morning of the 30th August, and only a few days later the issue of the journal referred to, bearing date 26th August, came to hand by the mail from Great Britain.

wondered at that he should, in the single-mindedness of his purpose, neglect to look to his laurels in a studied way. At any rate it appears that when he awoke to the real state of affairs, his opportunity for sharing in the honours and acknowledgments showered upon the participants in one of the greatest achievements of modern times, was past and gone; and no amount of subsequent effort on the part of the deceased gentleman could get that wrong thing righted. It was, and with good reason, a sore spot in his life's experience, often occasioning him pain of no common kind; and those who knew him most intimately will revere his memory for the manfulness that enabled him to ward off, through it all, the slightest taint of an embittered spirit. One by one all of the eminent men who were associated together some 30 years ago in the work of telegraphically connecting the old world and the new, have passed away. The death of the illustrious Cyrus Field preceded that of Mr. Gisborne by only a few weeks. Henceforth, therefore, the histories written of that great work should come from entirely disinterested hands. Let us hope and trust that when all is said and done, the meed of honour shall be given wheresoe'er 'tis justly due. The originality of the subject of this brief memoir is attested by several inventions for which he obtained patents in Great Britain and in the United States and Canada. He was a member of the Council of the Can. Soc. C.E., a fellow of the Royal Society of Canada, and a member of the Institute of Electrical Engineers of England; and his regretted death has removed from amongst us a striking example of the type of man whose prestige, while no doubt enhanced by such memberships, imparts lustre to the composition of any society in which he may happen to have the honour to be enrolled. The Cyclopædia of Canadian Biography: Rose Publishing Co., Toronto, 1886, contains in a lengthy sketch a very full and interesting account of his eventful career.

MR. HAMILTON H. KILLALY was engaged during his earlier professional career on the construction of the Grand Trunk, Northern and other Canadian railways. He was afterwards employed on the Victoria and International bridges. He then accepted an engagement, under Mr. T. C. Clarke, on the staff of the Quincy Bridge Company, during the years 1867 and 1868. There he served as assistant engineer and chief draughtsman, the duties being to prepare maps from the notes of surveys, and to make all the mechanical drawings of both the structure itself and the apparatus used in its construction. The position was the more important inasmuch as all the masonry and foundations of the bridge were built by day work, the company having to provide its own plant for the purpose. This involved preparing plans, not only of plant peculiar to Canadian constructions, but also of many new and special contrivances intended to facilitate the progress of the work. In this Mr. Clarke was greatly aided by Mr. Killaly's skill as a mechanical as well as a civil engineer, and he entertained the highest opinion of his abilities. His next important work was in connection with the construction of a bridge over the Missouri River at St. Joseph, under Col. Mason.

He was a very modest man, and his retiring manners probably prevented his taking the prominent position which his talents would have enabled him to fill. During the last seventeen years of his life he was employed by the Dominion Government in the construction and enlargement of canals on the St. Lawrence River. He was a gentleman in every sense of the word, and was much loved by his associates. Mr. Killaly died on the 10th of September, 1892. He was elected a member of this Society on the 20th of January, 1887.

Mr. George Mitchell was born in New Glasgow, N.S., May 10th, 1870. Having completed his school course, he matriculated as a mechanical engineering student, Faculty of Applied Science, McGill University, in the autumn of 1888, but did not, however, complete his course in the University, and died in the autumn of 1892. He was elected a student of this Society on the 14th March, 1889.

George Reaves was born at Rahway, New Jersey, 26th March, 1847. He was well known throughout Canada as representing important Engineering firms in Europe and America as an importer of materials of construction. He was elected an associate of this Society on the 20th of January, 1887, and died in September 1892.

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