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1	2	3
4	5	6

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AND A CHAPTER ON

THE INSTITUTE OF CHARTERED ACCOUNTANTS

REVISED BY HARRY VIGEON, ESQ., F.C.A., SEC'Y TO THE INSTITUTE

WITH THEIR

EXAMINATION QUESTIONS IN MERCANTILE ARITHMETIC

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TEXT BOOK AND READY-REFERENCE MANUAL FOR THE COMMER-
CIAL STUDENT, MERCHANT, ACCOUNTANT, LUMBERMAN,
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COMPILED AND EDITED BY

CLARKE MOSES, | **R. C. CHESWRIGHT,**
PUBLIC SCH. INSP., HALDIMAND CO. | MATH. MASTER, SEAFORTH H.S.

PUBLISHERS' PREFACE

THIS Arithmetic has been so well received by Business Colleges, Schools, Accountants, and the General Public, and has been found so thoroughly practical and helpful, that another edition has been called for. Advantage has been taken of this fact to add several new features, among them a full and complete Index, for the careful compilation of which we are indebted to MR. ELVEN J. BENOOFOH. By means of this Index, which has been arranged Topically as well as Alphabetically, the complete contents of the work can be rapidly reviewed. A glance over this Index will illustrate more forcibly than any words of ours the comprehensive character of the CANADIAN COMMERCIAL ARITHMETIC.

THE METRIC SYSTEM having been legalized in Cana   a chapter has been added deal:. g with this important subject. This chapter is the mos complete and practical to oe found in any work. It has been compiled with special refer. ce to commercial usage, and avoids Physics on the one hand, and Higher Mathematics on the other. The MS. was submitted for revision to SIR HENRI JOLY, Minister of Inland Revenue for Canada, who is the highest authority on the subject in the Dominion, having made a special study of the Metric System, and having gone to the trouble and expense of getting diagrams, etc., from France, recently. Sir Henri has examined our MS. with care, and offered valuable suggestions, and he considers that "the chapter will be very useful in "teaching the Metrical System, and that the comparison between that system, which "appears so logical, with the prese t syst ms of measurement, will certainly be a most "useful and intellectual exercise for students."

The new chapter on the INSTITUTE OF CHARTERED ACCOUNTANTS will be of value to all students who aspire to become expert Accountants and members of the Institute; while the EXAMINATION QUESTIONS IN MERCANTILE ARITHMETIC, now for the first time made generally public, will give a correct idea as to the scope of the Institute, and will furnish valuable material for exercise by students. This chapter has been revised by the Secretary of the Institute, HARRY VIOZON, ESQ., F.C.A., who, with the President, GEORGE EDWARDS, ESQ., F.C.A., and the Treasurer, W. B. TINDALL, ESQ., F.C.A., have shown practical interest in the CANADIAN COMMERCIAL ARITHMETIC by valuable suggestions.

A thoroughly practical and scientific arithmetical education can be obtained from this W ·L. which embraces a treatment of all the subjects necessary therefor.

Attention is directed to the following features :—

1. To the clearness and conciseness of the definitions, solutions, and rules, the latter of which are logically deduced from preceding solutions.

2. To the many short methods in addition, multiplication, division etc.

3. To the numerous solutions, and the large number of exercises, the practical character of which, will doubtless commend them to all teachers. In this importa t particular the authors believe a long-felt want has been supplied.

4. To the thoro h tr atment of Percentage, and its applications in Interest, discount, Partial Payments, Equation of Accounts, etc.

5. To the clear statement of the Commercial Law relating to Interest, Discount, etc.

Believing that this volume in it improved form will find an ever-increasing number of favorites, it is confidently sent forth.

Toronto, November, 1900.

IN EX TO TABLES.

INDEX TO ILLUSTRATIONS.

INDEX TO CONTENTS.

ARRANGED ALPHABETICALLY AND TOPICALLY.

Black-faced figures indicate page numbers.
Light-faced figures indicate numbers of sections or paragraphs, of which there are 755 in the Book.
The titles in SMALL CAPITALS indicate general headings of sections, of which there are 64.

CHARACTERS AND ABBREVIATIONS

USED IN BUSINESS.

@   At

a/c   Account.

₵   Cents.

%   Per cent.

№   Number.

1^1 One and one-quarter.

1^2 One and one-half.

     One and three-quarters.

√   Check mark.

×   By, as 14 × 18 inches.

$ Dollars.

£   Pound sterling.

6/3.   English shillings and pence are frequently written in this manner, the shillings on the left of the sloping line, and the pence on the right, the above meaning, 6 shillings and 3 pence.

May 18 21. The day of maturity, as expressed in a note, and the last day of grace are indicated by writing the first on the left and the second on the right of the sloping line.

15 doz. $\$^5_{12}$, $\$^5_{15}$, $\$^5_{18}$.   Fifteen doz., 5 of which are $12 per doz., 5 doz. at $15, and 5 doz. at $18 per doz.

^{1} hhd. Sugar.   .1100 pounds gross
1100   weight, 155 lbs. tare,
155 945 lbs.,   or weight of hhd., 945 lbs., net weight.

$4 ps. \left.\begin{matrix} 42 \\ 36 \\ 28 \\ 32 \end{matrix}\right\} 138$ yds.   The numbers in the bracket are the number of yards in each piece respectively.

10 doz. $\frac{4}{5}$ @ 2'. $\frac{6}{4}$ @ 3/6.   4 doz. No. 5 @ 2 shillings per doz. ; 6 doz. No. 8 @ 3s. 6d. per doz.

W. W. and similar characters and letters are placed on packages to designate a particular lot or shipment.

(Goods are numbered and marked that they may be distinguished without minute description.)

7 × 9, or 7 by 9 in.   7 in. wide, 9 in. long.

A 1 .... First class.

Acct. .... Account.

Adv .... Adventure.

Agt. .... Agent.

Amt. .... Amount.

Ass'd .... Assorted.

B. B. .... Bill-book.

Bal. .... Balance.

Bbl. .... Barrel.

Blls .... Bundles.

Bgs .... Bags.

Bkts. .... Baskets.

Blk .... Black.

Bls. .... Bales.

Bot .... Bought.

B. L. or B. of L. Bill of Lading.

Bills Pay ..... Bills payable.

Bills Rec ..... Bills receivable.

Bnk .... Bank.

Brot .... Brought.

Bquo .... Barque.

Br .... Brig.

Bus .... Bushels.

Bxs .... Boxes.

C. .... Cents.

C or centum .. Hundred.

C. B. .... Cash-book.

Ck .... Check.

Cap. .... Capital.

Co. .... Company.

C.O.D .... Collect on delivery.

Col'd .... Colored.

Cr .... Creditor.

Com .... Commission.

Cons't .... Consignment.

Cs .... Cases.

Cwt .... Hundredweight.

C/o .... Care of.

d .... Pence.

Dft. .... Draft.

Div .... Dividend.

Disct. .... Discount.

Do. or Ditto. . . The same.
Doz. . . . . Dozen.
Dr. . . . . Debtor.
Ds. . . . . Days.
Ea. . . . . Each.
E. E. . . . . Errors expected.
E. & O. E. . . . . Errors and omissions
                excepted.
Eng . . . . English.
Ent'd . . . . Entered.
Ex. . . . . Without, as ex-divi-
                dend.
Exch. . . . . Exchange.
Exps. . . . . Expenses.
Emb'd . . . . Embroidered.

Fig'd. . . . . Figured.
Fir. . . . . Firkin.
F. o. b . . . . Free on board.
Fol . . . . Folio.
F'wd or for'wd. Forward.
Fr . . . . From or French.
Fc . . . . Franc.
Fr't . . . . Freight.
Ft . . . . Feet.

Gal . . . . Gallon.
Gro. . . . . Gross.
Guar. . . . . Guarantee.

Hdkf. . . . . Handkerchief.
Hhd . . . . Hogshead.
Hund . . . . Hundred.

I. B. . . . . Invoice-book.
In. or " . . . . . . Inches.
Ins. . . . . Insurance.
Insol. . . . . Insolvency.
Inst. (Instant). This month.
Int. . . . . Interest.
Inv . . . . Invoice.
Invt'y . . . . Inventory.
I. O. U. . . . . . . I owe you.

Lbs . . . . Pounds.

M . . . . Thousand.
Mdse. . . . . Merchandise.
Mo. . . . . Month.
Mols. . . . . Molasses.
M.'t . . . . Empty.

Net . . . . Without deduction.

No. . . . . Number.
N. P. . . . . Notary public.

O. I. B. . . . . . . Outward invoice-book
Oz . . . . Ounces.

Paym't . . . . . . Payment.
P'd. . . . . Paid.
Pkgs. . . . . Packages.
Pr. or Per . . . . By.
Per cent. . . . . . By the hundred.
Pp. . . . . Pages.
Pr . . . . Pair.
Prem. . . . . Premium.
Prox (Proximo) The next month.
Ps . . . . Pieces.
Pts. . . . . Pints.

Qr. . . . . Quarter.
Qts . . . . Quarts.
Qtls . . . . Quintals.

Rec'd . . . . Received.
Recpt . . . . Receipt.
R. R. . . . . Railroad.
Rs. or Rls. . . . . Reals.
R. W. . . . . Regular way.

s . . . . Shilling.
Shipt . . . . Shipment.
Shs . . . . Shares.
Schr . . . . Schooner.
S.S. . . . . Steamship.
Sq. . . . . Square.
Stor . . . . Storage.
Stb't . . . . Steamboat.
Sunds. . . . . Sundries.
Super . . . . Superfine.
Str. . . . . Steamer.

Tes . . . . Tierces.

Ult. (Ultimo) . . The last month.

Ves . . . . Vessels.
Vs . . . . Against.
Viz . . . . Namely.

Wt . . . . Weight.
W. I. . . . . West Indies.

Yds. . . . . Yards.
Yr. . . . . Year.

ADDITION.

1. Rapidity and accuracy in addition are of the first importance to the commercial student.

These can be acquired only by a thorough familiarity with the simple combinations of numbers, and a proper practice with these combinations.

The following Tables exhibit all the combinations of numbers, and the attention of the student is especially directed to the endings, that is the right-hand figure in such combinations :

Combinations ending with 0.

1	2	3	4	5
9	8	7	6	5
10	10	10	10	10

Combinations ending with 1.

1	2	3	4	5
0	9	8	7	6
1	11	11	11	11

Combinations ending with 2.

1	2	3	4	5	6
1	0	9	8	7	6
2	2	12	12	12	12

Combinations ending with 3.

2	3	4	5	6
1	0	9	8	7
3	3	13	13	13

Combinations ending with 4.

2	3	4	5	6	7
2	1	0	9	8	7
4	4	4	14	14	14

Combinations ending with 5.

3	4	5	6	7
2	1	0	9	8
5	5	5	15	15

Combinations ending with 6.

3	4	5	6	7	8
3	2	1	0	9	8
6	6	6	6	16	16

Combinations ending with 7.

4	5	6	7	8
3	2	1	0	9
7	7	7	7	17

Combinations ending with 8.

4	5	6	7	8	9
4	3	2	1	0	9
8	8	8	8	8	18

Combinations ending with 9.

5	6	7	8	9
4	3	2	1	0
9	9	9	9	9

After the student becomes familiar with the foregoing combinations his attention is directed to the use of the endings. For example:

7 & 6 = 13,　17 & 6 = 23,　27 & 6 = 33,　37 & 6 = 43, &c.
6 & 7 = 13,　16 & 7 = 23,　26 & 7 = 33,　36 & 7 = 43, &c.

i.e., the sum of any two numbers, one of which ends with 6 and the other with 7, produces a number ending with 3. A thorough drill of this kind should be given with all the combinations.

2. An effective drill may be given to the student by the use of the following diagram :
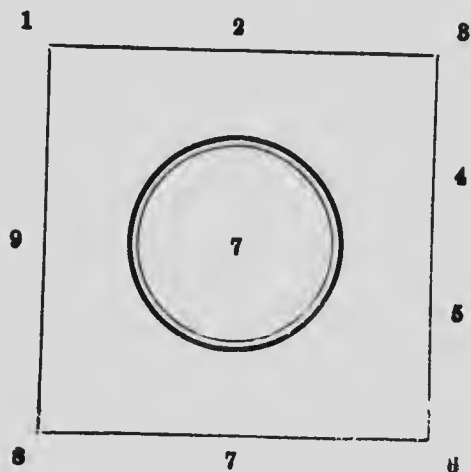
The teacher places any number within the circle and requires the pupils to add to it any number or succession of numbers to which he may point.

Rapidity and accuracy in addition can be gained only by adding columns of figures.

3. In adding ledger columns, accountants frequently use the following devices :

EXAMPLE 1.—

$$\begin{array}{r} \$926.42 \\ 49.98 \\ 67.84 \\ 876.55 \\ 4867.89 \\ 916.74 \\ 6487.45 \\ \hline \$14222.87 \\ 4454\,3 \end{array}$$

The figure to be carried is placed under the column to which it belongs so that in case of interruption or mistake it may be used for reference.

4.   EXAMPLE 2.—

```
93746
 2385
91642
28735
82614
79186
───────  328308
25738
87264
19285
63127
58432
───────  203846
82691
35417
63529
48763
21734   252134
───────
        784288
```

The column to be added is divided into several parts. These parts are added and the sum of the results then taken.

5. Addition of two or more columns at the same time.

EXAMPLE 3—

```
35
89
76
47
───
247
```

METHOD OF ADDITION—

     47 & 6 make 53,   53 & 70 make 123,   123 & 9 make 132,

   132 & 80 make 212,   212 & 5 make 217,   217 & 30 make 247.

Columns of three or four figures may be added in the same way, or by adding two columns at a time.

The methods employed in Examples 2 and 3 are excellent tests of the correctness of addition performed in the ordinary way.

6. To find the sum of any series of numbers which have a common difference.

<center>RULE.</center>

Multiply the sum of the first and last terms by the number of terms and divide the result by 2.

Example 1.—Add, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,

OPERATION.   Common difference is 1.

```
    16   first term.
    27   last term.
   ───  ─────────
    43
    12   number of terms.
   ───
2 ) 516
   ───
    258
```

Example 2.—Add, 48, 56, 64, 72, 80, 88, 96, 104, 112.

```
     48
    112        Common difference is 8.
   ───
    160
      9
   ─────
9 ) 1440
   ─────
     720
```

MULTIFLICATION.

SHORT METHODS IN MULTIPLICATION.

7. To multiply by any of the numbers from **11** to 19 inclusive.

Multiply 4625 by 14.

FIRST METHOD.

```
 4625        5 × 4                    = 20 carry 2
   14        2 × 4 + 2 (carried)  =  15   "   1
 ─────
64750        6 × 4 + 1 (   "   ) + 2 = 27   "   2
             4 × 4 + 2 (   "   ) + 6 = 24   "   2
                     2 (   "   ) + 4 =  6
```

The student will observe that we multiply by 4 in the ordinary way, but in addition to the ordinary number to be carried we also carry the figure to the *right* of the figure multiplied.

SECOND METHOD.

```
4625 × 14
18500
─────
64750
```

Multiply by 4, placing the product one place to the right and add.

NOTE.—This method may be applied when the multiplier has one or more ciphers between the two figures, by writing the product two or more places to the right, and adding.

EXERCISE I.

Multiply—

1. 79526 by 11, 12, 13, 14, 15, 16, 17, 18, 19.
2. 37295 by 102, 104, 105, 107.
3. 49273 by 1003, 1006, 1008, 1009.

8. To multiply by any number of two figures ending with 1.

Multiply 846 by 41.

FIRST METHOD.

```
  846          6 × 1
   41          6 × 4                        =  6
34686          4 × 4 + 2 (carried)    + 4 = 28   carry 2
               8 × 4 + 2     "        + 2 = 26    "    2
                                            = 34
```

The student will observe that we place the units figure of the multiplicand as the units figure of the product. Then multiply by 4, and in addition to the ordinary number to be carried, we carry the figure to the left of the figure multiplied.

SECOND METHOD.

```
  846 × 41
8384

34686
```

Multiply by 4, placing the product one place to the left and add.

NOTE.—This method may be applied when the multiplier has one or more ciphers between the two figures by writing the product two or more places to the left.

EXERCISE 2.

Multiply—

1. 64278 by 21, 31, 41, 51, 61, 71, 81, 91.
2. 87396 by 301, 501, 601, 801.
3. 93254 by 2001, 3001, 7001, 9001.

9. To multiply two numbers in which the units figures added make 10, the other figures being the same in each.

EXAMPLE 1.—Multiply 74 by 76.

```
   74              METHOD.
   76           4 × 6 = 24
 5624        (7 + 1) × 7 = 56
```

EXAMPLE 2.—   Multiply 123 by 127.

```
  123              METHOD.
  127            3 × 7 =  21
15621        (12 + 1) × 12 = 156
```

EXERCISE 3.

1.	34 × 36.	7.	92 × 98.	13.	112 × 118.	19.	153 × 157.
2.	55 × 55.	8.	64 × 66.	14.	123 × 127.	20.	491 × 499.
3.	72 × 78.	9.	95 × 85	15.	104 × 106.	21.	694 × 696.
4.	65 × 65.	10.	97 × 93.	16.	105 × 105.	22.	225 × 225.
5.	63 × 67.	11.	78 × 72.	17.	101 × 109.	23.	392 × 398.
6.	31 × 39.	12.	91 × 99.	18.	292 × 298.	24.	173 × 177.

10. To multiply two numbers in which the units figures are the same.

Multiply 46 by 66.

```
    46                      METHOD.
    66              6 × 6                    = 36 carry 3
   ----
   3036            (4 + 6) × 6 + 3 (carried) = 63 carry 6
                    4 × 6 + 6 (   "    ) = 30
```

EXERCISE 4.

1.	21 × 51.	7.	64 × 54.	13.	19 × 29.	19.	105 × 125.
2.	53 × 53.	8.	86 × 36.	14.	27 × 47.	20.	113 × 133.
3.	45 × 25.	9.	47 × 87.	15.	36 × 56.	21.	114 × 144.
4.	67 × 57.	10.	58 × 48.	16.	84 × 34.	22.	136 × 126.
5.	28 × 38	11.	81 × 91.	17.	83 × 73.	23.	125 × 135.
6.	92 × 72.	12.	42 × 72.	18.	116 × 146.	24	117 × 197.

11. To multiply two numbers in which the units figures are unlike, the remaining figures being alike.

EXAMPLE 1.—Multiply 78 by 72.

```
    78                      METHOD.
    72              8 × 2                    = 16 carry 1
   ----            (8 + 2) × 7 + 1 (carried) = 71 carry 7
   5616             7 × 7 + 7 (carried) = 56
```

EXAMPLE 2.—Multiply 126 by 122.

```
   126                      METHOD.
   122              6 × 2                    =  12 carry 1
   ----            (6 + 2) × 12 + 1 (carried) =  97 carry 9
  15372            12 × 12 + 9 (carried) = 153
```

EXERCISE 5.

1.	37 × 35.	7.	68 × 61.	13.	48 × 43.	19.	116 × 113.
2.	54 × 52.	8.	74 × 78.	14.	26 × 27.	20.	124 × 125.
3.	75 × 76.	9.	85 × 84.	15.	57 × 59.	21.	136 × 134.
4.	83 × 82.	10.	91 × 92.	16.	38 × 37.	22.	117 × 141.
5.	27 × 29.	11.	74 × 72.	17.	61 × 69.	23.	157 × 159.
6.	46 × 45.	12.	63 × 65.	18.	78 × 74.	24.	323 × 322.

12. To multiply by means of cross multiplication.

EXAMPLE 1.—Multiply 56 by 63.

```
     56                    METHOD.
     63        6 × 3                          = 18, carry 1
    ——         5 × 3 + 1 (carried) + 6 × 6 = 52,   "    5
   3528        5 × 6 + 5 (    "    )            35
```

EXAMPLE 2.—Multiply 346 by 23.

```
     346                   METHOD.
      23        6 × 3                          = 18, carry 1
    - —         4 × 3 + 1 (carried) + 6 × 2 = 25,   "    2
   7958         3 × 3 + 2 (    "    ) + 4 × 2 = 19,  "    1
                3 × 2 + 1 (    "    )          =  7
```

EXERCISE 6.

1. 86 × 32.	6. 45 × 62.	11. 346 × 43.
2. 51 × 24.	7. 39 × 74.	12. 608 × 37.
3. 79 × 45.	8. 82 × 51.	13. 543 × 23.
4. 87 × 53.	9. 37 × 22.	14. 760 × 48.
5. 28 × 51.	10. 46 × 25.	15. 3268 × 79.

13. To multiply by a number ending with 9.

Multiply by 1 more than the given multiplier and substract the multiplicand.

Multiply 263 by 69.

```
              OPERATION.
        18410 (product by 70)
          263 (    "      "    1)
        ————
        18147 (    "      "   69)
```

EXERCISE 7.

Multiply—

1. 3764 by 79, 49.	5. 13256 by 119, 399, 109.
2. 46251 by 39, 59.	6. 47305 by 299, 999, 799.
3. 37284 by 99, 69.	7. 27634 by 119, 249, 189.
4. 29655 by 89, 29.	8. 17825 by 9999, 499, 139.

14. To multiply by a number which is a little less than 100, 200, 300, 400, etc.

Multiply the multiplicand by the difference between the multiplier and 100, 200, 300, or etc., and substract the product from the product of the multiplicand by 100, 200, 300, or etc.

Multiply 675 by 97.

OPERATION.

67500 (product by 100)
  2025 ("   "   3)
————
65475 ('   " 97)

EXERCISE 8.

Multiply—

1.   8684 by   95, 96, 94.
2. 41523 by   93, 95, 97
3. 27136 by 296, 495.
4.   8124 by 794, 897.

5. 36092 by 993, 795, 990.
6. 14613 by 988, 691, 791.
7. 80257 by 989, 995, 9993.
8. 17824 by 992, 6999, 996

15. To multiply two numbers, one of which is more and the other less than 100, 1000, etc.

The complement of a number is the difference between that number and the unit of the next higher order.

RULE.

Multiply the sum of the numbers less the unit of comparison by the unit of comparison, and from the product substract the product of the excess and the complement.

Multiply 108 by 94.

Unit of comparison is 100.

108 .. .. 8 excess.

94 .. .. 6 complement.

————

$\left. \begin{array}{l} 10200 \\ 48 \end{array} \right\}$ product of the excess and complement.

————

10152

METHOD.

$$108 - 6 = 102$$
$$\text{or } 94 + 8 = 102$$
$$\text{or } 108 + 94 - 100 = 102$$
$$102 \times 100 = 10200$$
$$8 \times 6 = 48$$
$$————$$
$$10152$$

EXERCISE 9.

1. 107×97.
2. 105×95.
3. 113×88.
4. 103×94.
5. 106×92.

6. 112×91.
7. 115×93.
8. 108×96.
9. 114×95.
10. 104×87.

11. 1012×994.
12. 1015×988.
13. 1032×998.
14. 1064×993.
15. 1025×989.

16. To multiply two numbers of the same number of figures over and near 100, 1000, etc.

RULE.

From the sum of the numbers substract the unit of comparison, and to the right of the result write the product of the excesses.

NOTES.

1. When there are fewer figures in the product of the excesses than ciphers in the unit of comparison, write ciphers in the result to supply the deficiency.

2. When there are figures in the product of the excesses than ciphers in the unit of comparison, add excess on the left hand to the first part of the result.

3. After practice, ting of the complements or the excesses in examples where the may be omitted.

Multiply 112 6

METHOD.

$$112 \ldots \ldots 12 \qquad 112 + 6 = 118$$
$$106 \ldots \ldots 6 \qquad or\ 100 + 12 = 118$$

11872

$$or\ 112 + 106 - 100 = 118$$
$$12 \times 6 = 72$$

Unit of comparison is 100.

EXERCISE 10.

1. 112×108	6. 1×10	11. 1006×1003.
2. 108×103.	7. 107×115.	12. 1017×1008.
3. 115×105.	8. $\times 1 7$.	13. 1125×1009.
4. 113×4.	9. $\times 108$.	14. 1034×1005.
5. 105×6.	10. $\times 1$	15. 1075×1012.

17. To multiply by means of complements.

RULE.

From either number subtract the complement of the other, and to the right of the result write the product of the complements.

NOTE.—The notes of Art. 16 apply in these problems if we substitute the word "complements" in place of "excesses."

EXAMPLE 1.—Multiply 94 by 98.

94 .. .. 6 complement.
98 .. .. 2 "

9212

METHOD.
$$94 - 2 = 92$$
$$or\ 98 - 6 = 92$$
$$or\ 94 + 98 = 192,\ omit\ the\ 1$$

Example 2—
    997 3
    992 8
    ———.
    989024 (see Note 1.)

Example 8—
    685 815
    996 4
    ———.
    682260 (see Note 2.)

EXERCISE 11.

1. 97×96.	6. 88×93.	11. 993×995.
2. 95×93.	7. 87×88.	12. 997×992.
8. 94×95.	8. 84×92.	13. 995×993.
4. 99×94.	9. 75×96.	14. $98 \quad 788$.
5. 98×92.	10. 93×85.	15. 991×885.

18. To multiply by means of factors.

The factors of a number are the numbers whose product is equal to that number.

Multiply 865 by 35.

$$35 = 7 \times 5$$

    865
      7
    ————
    6055 produced by 7
      5
    ————
    30275   "   " 35 (5 times 7)

METHOD OMITTING MULTIPLIERS.
    865
    ————
    6055 product by 7
    ————
    30275   "   " 35

EXERCISE.

1. 626×36.	5. 296×99.	9. 1351×42.	13. 30687×105.
2. 327×54.	6. 343×72.	10. 4164×35.	14 20956×121.
3. 495×48.	7. 764×56.	11. 8127×126	15. 41378×154
4. 378×77.	8. 827×45.	12. 3174×64.	16. 36254×225.

19. To multiply when one part of the multiplier is a factor of the other.

RULE.

Multiply by the part of the multiplier which is a factor of another part, placing the first figure of each partial product under the right hand figure of the multiplier which produced it.

Example 1—Multiply 467 by 248.

    467
    248
    ————
    3736 product by 8.
    11208   "   " 24 (3 times the product by 8).
    ————
    115816

Example 2—Multiply 643 by 436.

$$
\begin{array}{l}
\quad 643 \\
\quad 436 \\
\hline
2572 \quad \text{product by 4.} \\
23148 \quad \text{``} \quad \text{`` 36 (9 times the product by 4).} \\
\hline
280348
\end{array}
$$

Example 3—Multiply 3247 by 842.

$$
\begin{array}{l}
\quad 3247 \\
\quad 842 \\
\hline
6494 \quad \text{product by 2} \\
12988 \quad \text{``} \quad \text{`` 4 (2 times the product by 2).} \\
25976 \quad \text{``} \quad \text{`` 8 (2 ``} \qquad \text{``} \qquad 4). \\
\hline
2733974
\end{array}
$$

EXERCISE 12.

1. 864×126.	8. 3164×427.	15. 37281×832.
2. 475×279.	9. 4275×246.	16. 41325×776.
8. 896×142.	10. 8137×189.	17. 63587×618.
4. 857×357.	11. 2956×284.	18. 49126×428.
5. 943×426.	12. 4765×927.	19. 64273×535.
6. 854×369.	13. 8259×936.	20. 47821×1682.
7. 375×632.	14. 4371×183.	21. 45314×2468.

20. To multiply by a mixed number.

Example 1—Multiply 363 by $6\frac{1}{4}$.

$$
\begin{array}{l}
\quad 363 \\
\quad 6\frac{1}{4} \\
\hline
90\frac{3}{4} \quad \text{product by } \tfrac{1}{4} = 363 + 4 = 90 \\
2178 \quad \text{``} \quad \text{`` 6.} \\
\hline
2268\frac{3}{4}
\end{array}
$$

Example 2—Multiply 3426 by $5\frac{2}{3}$.

$$
\begin{array}{l}
\quad 3426 \\
\quad 5\frac{2}{3} \\
\hline
1370\frac{2}{3} \quad \text{product by } \tfrac{2}{3} = 3426 \times 2 + b \\
17130 \quad \text{``} \quad \text{`` 5.} \\
\hline
18500\frac{2}{3}
\end{array}
$$

EXERCISE 13.

Multiply—

1. 3126 by $3\frac{1}{4}$, $5\frac{1}{3}$, $8\frac{1}{2}$, $17\frac{3}{5}$, $13\frac{3}{8}$, $11\frac{1}{8}$, $10\frac{1}{8}$.
2. 4371 by $15\frac{7}{8}$, $14\frac{2}{3}$, $25\frac{1}{4}$, $33\frac{2}{3}$.
3. 2137 by $41\frac{3}{4}$, $21\frac{1}{4}$, $36\frac{1}{10}$, $47\frac{1}{2}$.
4. 4645 by $22\frac{3}{11}$, $35\frac{2}{3}$, $42\frac{3}{4}$, $52\frac{2}{3}$.
5. 1316 by $161\frac{1}{13}$, $241\frac{3}{13}$, $110\frac{4}{17}$, $95\frac{4}{21}$.

21. In multiplying by a mixed number, it is often a shorter method to reduce the mixed number to an improper fraction and to multiply as in fractions. The following exercise contains multipliers of this kind.

Multiply 689 by $33\frac{1}{3}$.

$$689 \times 33\frac{1}{3}$$
$$= 689 \times \frac{100}{3}$$
$$= \frac{68900}{3} = 22966\frac{2}{3}$$

EXERCISE 14.

Multiply—

1. 3964 by $1\frac{1}{3}$, $1\frac{3}{4}$, $3\frac{1}{8}$, $9\frac{1}{11}$, $11\frac{1}{8}$.
2. 1375 by $14\frac{2}{3}$, $33\frac{1}{3}$, $18\frac{2}{11}$, $22\frac{2}{9}$, $28\frac{4}{7}$.
3. 4137 by $66\frac{2}{3}$, $42\frac{2}{3}$, $133\frac{1}{3}$, $57\frac{1}{4}$.
4. 3164 by $44\frac{1}{8}$, $36\frac{4}{11}$, $71\frac{3}{4}$, $55\frac{5}{8}$, $45\frac{5}{11}$.
5.   227 by $85\frac{7}{8}$, $54\frac{6}{11}$, $233\frac{1}{3}$, $77\frac{7}{8}$, $63\frac{7}{11}$.
6.   383 by $266\frac{2}{3}$, $114\frac{2}{7}$, $88\frac{6}{8}$, $72\frac{6}{11}$, $128\frac{4}{7}$.
7.   752 by $81\frac{9}{11}$, $333\frac{1}{3}$, $142\frac{6}{7}$, $111\frac{1}{8}$, $99\frac{1}{11}$.

22. To multiply by a number which is a convenient aliquot part of 10, 100, 200, 300, etc.

Multiply 638 by $2\frac{1}{2}$.

Since $10 \div 4 = 2\frac{1}{2}$, therefore to multiply by $2\frac{1}{2}$ we multiply by 10 and divide the result by 4. $638 \times 10 \div 4 = 1595$.

The following list comprises some of the multipliers that may be used in this way. All the multipliers used in the preceding exercise are examples of this class.

1.	$1\frac{1}{4}$	$=$	$10 \div$	8.
2.	$1\frac{2}{3}$	$=$	$10 \div$	6.
S.	$2\frac{1}{2}$	$=$	$10 \div$	4.
4.	5	$=$	$10 \div$	2.
5.	$8\frac{1}{3}$	$=$	$100 \div$	12.
6.	$12\frac{1}{2}$	$=$	$100 \div$	8.
7.	$16\frac{2}{3}$	$=$	$100 \div$	6.
8.	25	$=$	$100 \div$	4.
9.	$37\frac{1}{2}$	$=$	$300 \div$	8.
10.	75	$=$	$300 \div$	4.
11.	$41\frac{2}{3}$	$=$	$500 \div$	12.

12.	$62\frac{1}{2}$	$=$	$500 \div$	8.
13.	$58\frac{1}{3}$	$=$	$700 \div$	12
14.	$87\frac{1}{2}$	$=$	$700 \div$	8.
15.	$116\frac{2}{3}$	$=$	$700 \div$	6
16.	175	$=$	$700 \div$	4.
17.	$112\frac{1}{2}$	$=$	$900 \div$	8.
18.	225	$=$	$900 \div$	4.
19.	$83\frac{1}{3}$	$=$	$1000 \div$	12.
20.	125	$=$	$1000 \div$	8.
21.	$166\frac{2}{3}$	$=$	$1000 \div$	6.
22.	$333\frac{1}{3}$	$=$	$1000 \div$	3.

EXERCISE 15.

Multiply—

1. 346 by $1\frac{1}{4}$, $1\frac{2}{3}$, $2\frac{1}{2}$,   5, $8\frac{1}{3}$.
2. 258 by $12\frac{1}{2}$, $16\frac{2}{3}$, 25, $37\frac{1}{2}$, 75.
3. 512 by $41\frac{2}{3}$, $62\frac{1}{2}$, $58\frac{1}{3}$, $87\frac{1}{2}$.
4. 545 by $116\frac{2}{3}$, 175, $112\frac{1}{2}$, 225.
5. 357 by $83\frac{1}{3}$, 125, $166\frac{2}{3}$, $333\frac{1}{3}$.

23. To multiply by 75.

Multiply by 100 and subtract one quarter of the product.

EXAMPLE—Multiply 863427 by 75.

$$75 = 100 - 25 \text{ (one-fourth of 100)}$$

OPERATION—86342700 – product by 100.
21585675 – one-fourth of the product.

64757025

24. To multiply by 125.

$$125 = 100 + 25 \text{ (one-fourth of 100)}$$

Multiply by 100 and add one-fourth of the product.

EXAMPLE—Multiply 1234769 by 125.

OPERATION—123476900 – product by 100.
31094225 – one-fourth of the product by 100.

155471125

EXERCISE 16.

1. $367258 \times 66\frac{2}{3}$.
2. 43729×95.
3. 27864×975.
4. 376298×950.

5. 36254×105.
6. $27936 \times 133\frac{1}{2}$.
7. 478256×150.
8. 236471×1025.

DIVISION.

___

25. A number is said to be divisible by another number when the latter will divide the former without a remainder.

26. An **even number** is a number of which 2 is an exact divisor.

27. An **odd number** is a number of which 2 is not an exact divisor.

28. Any number is divisible—

1. By 2, if it is an even number as 2, 4, 8, 26.

2. By 3, if the sum of its digits is divisible by 3, as 744, 7 + 4 + 4 = 15, 15 is divisible by 3.

3. By 4, if the two right hand figures are ciphers, or express a number divisible by 4, as 1500, 7328.

4. By 5, if the right hand figure is 0 or 5, as 60, 95.

5. By 6, if it is an even number and has the sum of its digits divisible by 3, as 348.

6. By 8, if the three right hand figures are ciphers, or express a number divisible by 8, as 4000, 9218.

7. By 9, if the sum of its digits is divisible by 9, as 45387.

8. By 11, if the difference of the sum of the digits in the even places, and the sum of the digits in the odd places is 0, or is divisible by 11, as 43263, 459173.

9. By 25, if the two right hand figures are ciphers or express a number divisible by 25, as 4700, 3675.

10. By 75, the same as for 25, providing also that the sum of the digits is divisible by **3, as** 3900, 41475.

29. To divide one number by another leaving out the products.

RULE.

Subtract the several products from the next number greater ending with the corresponding figure in the dividend, and carry each time the left hand figure of the minuend to the next product.

Divide 42343014 by 973.

ORDINARY METHOD.

973) 42343014 (43518
3892
———
3423
2919
———
5040
4865
———
1751
973
———
7784
7784
———
0000

LEAVING OUT THE PRODUCTS.

42343014   973
——— |———
3423 | 43518
———
5040
———
1751
———
7784
——
ι)0

METHOD.

The first quotient figure is 4, by which we multiply. 4 times 3 are 12, which, subtracted from 14 (the next number greater ending with 4), leaves 2. Write 2 in the remainder and carry 1. 4 times 7 are 28, and 1 carried makes 29, which, subtracted from 33 (the next number greater

ending with 3), leaves 4.   Write 4 in the remainder and carry 3.   4 times 9 are 36 and 3 carried makes 39, which, subtracted from 42 (the next number greater ending with 2), leaves 4.   4 subtracted from 4 leaves 0. Bring down 3 the next figure in the dividend.   So proceed until the division is completed.

EXERCISE 17.

1. $743297 \div 527.$
2. $14839 \div 869.$
3. $87654 \div 743.$
4. $36287 \div 567.$
5. $64925 \div 784.$
6. $34681 \div 429.$

7.   Divide 3642789 by 625, 436, 8173, 2106.

30. To divide by a mixed number.

PRINCIPLE.

Multiplying both divisor and dividend by the same number does not alter the quotient.

Divide 786 by $5\frac{2}{3}$.

```
      5⅔ ) 786 (
      3      3
      ──    ──
      17 ) 2208 ( 129 15/17
           17
           ──
           50
           34
           ──
           168
           153
           ──
           15
```

EXERCISE 18.

Divide—
1.   475 by $3\frac{1}{2}$, $4\frac{2}{3}$, $7\frac{1}{3}$, $3\frac{1}{4}$, $4\frac{4}{7}$, $5\frac{6}{8}$.
2.   2624 by $13\frac{1}{3}$, $4\frac{3}{5}$, $9\frac{1}{11}$, $31\frac{1}{4}$, $4\frac{7}{9}$.
3.   6712 by $7\frac{3}{8}$, $11\frac{7}{9}$, $2\frac{1}{13}$, $6\frac{1}{13}$, $10\frac{5}{8}$.

31.   To divide when all the figures in the divisor except the first on the left hand can be changed to ciphers by using a convenient multiplier.

EXAMPLE 1—Divide 624395 by 35.

```
      35 ) 624395 (
      2        2
      ──     ───────
      70 ) 1248790
           17839 - 70/70
```

EXAMPLE 2—Divide 13476 by 16⅔.

$$16\tfrac{2}{3} \,)\, 13476 \,($$
$$\begin{array}{cc} 6 & 6 \\ \hline 100 \,)\, 8085\cancel{6} \\ 808\tfrac{53}{100} \end{array}$$

NOTE.—If the true remainder is required it may be obtained by dividing the remainder found by the number by which we multiply the divisor.

EXERCISE 19.

1.	4826 ÷ 5.	7.	32068 ÷ 12½.	13.	21396 ÷ 41⅔.
2.	3827 ÷ 25.	8.	68934 ÷ 8⅓.	14.	9201 ÷ 36₁₁⁴.
3.	9109 ÷ 75.	9.	32165 ÷ 1⅔.	15.	7345 ÷ 57¼.
4.	4863 ÷ 175.	10.	8327½ ÷ 83⅓.	16.	6287 ÷ 125.
5.	3798 ÷ 225.	11.	49323 ÷ 33½.	17.	31264 ÷ 87½.
6.	8306 ÷ 45.	12.	9306 ÷ 62½.	18.	31907 ÷ 142⅔.

32. To divide by any number that can be changed to a convenient divisor by increasing or diminishing it by an aliquot part of itself.

RULE.

After dividing by the divisor so increased or diminished, increase or diminish the quotient in the same proportion.

Divide 1920 by 24.

OPERATION.

$$\begin{array}{r} 30 \,)\, 1920 \\ 4 \,)\, 64 \\ 16 \\ \hline 80 \quad \text{the quotient.} \end{array}$$

EXPLANATION.

¼ of 24 = 6   24 + 6 = 30

$$1920 \div 30 = 64$$
$$\tfrac{1}{4} \text{ of } 64 = 16$$
$$\overline{}$$
$$80 \quad \text{the quotient.}$$

EXERCISE 20.

1.	1845 ÷ 45.	5.	7704 ÷ 24.	9.	24300 ÷ 18¾.
2.	3640 ÷ 35.	6.	8343 ÷ 27.	10.	24500 ÷ 87½.
3.	2322 ÷ 54.	7.	41472 ÷ 81.	11.	337500 ÷ 75.
4.	15216 ÷ 48.	8.	121120 ÷ 180	12.	425100 ÷ 125.

33. To divide by means of factors of the divisor.

EXAMPLE 1.—Divide 25380 by 108.

$$108 = 9 \times 4 \times 3 \text{ or } 6 \times 6 \times 3 \text{ or } 9 \times 6 \times 2$$

```
108 ) 25380 ( 235
      216
      ────         3 ) 25380      8 ) 25380      9 ) 25380
      378          4 ) 8460       6 ) 8460       6 ) 2820
      324          9 ) 2115       6 ) 1410       2 ) 470
      ────            ────          ────           ────
      540             235           235            235
      540
```

EXAMPLE 2.—Divide 6326 by 75.

$$75 = 3 \times 5 \times 5.$$

```
3 ) 6326
5 ) 2108 .. 2 |                2        1 × 5 + 3 =  8
5 ) 421 .. 3 |     3 × 3 =     9  or  8 × 3 + 2 = 26
      84 .. 1 |  1 × 5 × 3 =  15
                               ──
                               26   true remainder.
```

NOTE 1.—To find the true remainder, take the product of each remainder by all the divisors preceding the one that produced it. The sum of these products with the first remainder will be the true remainder.

2. Take the product of the last remainder by the divisor preceding the one that produced it. To this product add the preceding remainder. Multiply this result by the next divisor and add the next remainder. Continue this process until the first divisor has been used as a multiplier.

EXERCISE 21.

1.	25380 ÷ 36.	5. 31279 ÷ 72.	9. 43716 ÷ 168.
2.	178584 ÷ 48.	6. 43827 ÷ 81.	10. 29373 ÷ 81.
3.	23741 ÷ 42.	7. 19375 ÷ 125.	11. 41658 ÷ 45.
4.	43165 ÷ 64.	8. 41643 ÷ 165.	12. 23725 ÷ 96.

34. To divide by cancellation.

35. Cancellation is the process of shortening operations by rejecting equal factors from both dividend and divisor.

36. The sign of cancellation is an oblique mark (/) drawn through the number from which the factor is rejected.

Divide 18 × 16 × 28 by 12 × 7 × 14

$$\frac{\overset{3}{\cancel{18}} \times 16 \times \overset{2}{\cancel{28}}}{\underset{2}{\cancel{12}} \times 7 \times \cancel{14}} = \frac{3 \times 16}{7} = 6\frac{4}{7} \quad \text{or}$$

2	3	
12	18	
7	16	
14	28	2
7	48	
	6⁴⁄₇	

RULE.

Cancel the factors common to the divisor and dividend, and divide the product of those remaining in the dividend by the product of those remaining in the divisor.

EXERCISE 22.

Divide—

1. 5 × 9 × 7 × 11 by 7 × 5 × 3 × 11.
2. 80 × 56 × 18 by 2 × 3 × 4 × 6.
3. 70 × 39 × 13 by 26 × 21 × 7.
4. 28 × 49 × 75 by 7 × 15 × 84.
5. 3 × 6 × 8 × 72 by 2 × 3 × 4 × 18.
6. 74 × 12 × 14 × 16 by 28 × 72 × 24.
7. 112 × 27 × 178 by 54 × 63 × 89.
8. 128 × 16 × 72 by 44 × 32 × 18.
9. 135 × 12 × 29 by 27 × 18 × 154.
10. 45 × 63 × 144 by 72 × 24 × 2.

FACTORING.

37. A **Factor,** a **Measure,** or an **Exact Divisor** of a given number is an integral number that will divide the given number without a remainder.

38. A **Prime Number** is a number that has no factors except itself and 1, as 3, 7, 13, 19.

39. A **Prime Factor** is a prime number used as a factor.

10. A **Composite Number** is a number that has other factors besides itself and 1, as 24, 32, 70.

41. **Factoring** is the process of finding the factors of a composite number.

42. To resolve a number into its prime factors.

RULE.

Divide the number by the least prime number which will divide it exactly. In like manner divide the resulting quotient. Continue this process until a quotient which is a prime number is reached. The several divisors and the last quotient are the prime factors.

Find the prime factors of 420.

```
2 ) 420          420 = 2 × 2 × 3 × 5 × 7
2 ) 210
3 ) 105
5 ) 35           2, 3, 5 and 7 are the prime factors.
    7
```

EXERCISE 23.

Find the prime factors of—

1. 1050.	6. 4985.	11 8140.	16. 1906.	21. 2526.
2. 2625.	7. 4620.	12. 8712.	17. 1858.	22. 2978.
3. 1820.	8. 4802.	13. 1320.	18. 1478.	23. 2992.
4. 1485.	9. 5132.	14. 1768.	19. 2956.	24. 3936.
5. 1155.	10. 7000.	15. 1848.	20 2406.	25. 8430.

HIGHEST COMMON FACTOR.

43. A Common Factor of two or more numbers is a number that will exactly divide each of them ; thus 2, 4, 6, or 12 is a common factor of 24 and 36.

44. The Highest Common Factor, also called the Greatest Common Divisor or Greatest Common Measure, of two or more numbers, is the greatest number that will exactly divide each of them, thus 12 is the H. C. F. of 24 and 36.

45. To find the H. C. F. of two or more numbers :

RULE.

Divide the greater number by the less, and the less number by the remainder, if any, and so continue to divide the last divisor by the last remainder until there is no remainder. The last divisor will be the H. C. F.

If more than two numbers are given, find the H. C. F. of two of them, then of this factor and the third number and so on.

Find the H. C. F. of 1386 and 2268.

FIRST METHOD.

```
1386 ) 2268 ( 1
       1386

       882 ) 1386 ( 1
             882

             504 ) 882 ( 1
                   504

                   378 ) 504 ( 1
                         378

H. C. F.       126 ) 378 ( 3
                     378
```

SECOND METHOD.

	QUOTIENTS	2268
1386	1	1386
882	1	882
50:	1	504
378	1	378
H. C. F. 126	3	378

NOTE.—Observe that the second method is the same as the first, the work being arranged so as not to necessitate the writing of the divisor more than once.

The column for quotients may be omitted.

THIRD METHOD.

		MULTIPLIERS	2268	
subtract	1386	**2**	2772	subtract downwards.
downwards.	1512	**3**	504	
H. C. F.	126	**4**	504	

METHOD.

In this method we use such a multiplier for 1386 as will give a product nearest to 2268, that is 2. From the product 2772 take 2268, which leaves a remainder 504. Next take as a multiplier of 504 such a number as will give a product nearest to 1386, that is 3, etc.

FOURTH METHOD.

By means of prime factors.

PRIME FACTORS FOUND.

2	1386		2	2268
3	693		2	1134
3	231		3	567
7	77		3	189
	11		3	63
			3	21

PRIME FACTORS ARRANGED.

$1386 = 2 \quad \times 3 \times 3 \qquad \times 7 \times 11$

$2268 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 7$

Common prime factors multiplied.

$2 \times 3 \times 3 \times 7 = 126 = $ H. C. F.

RULE.

Resolve the given numbers into their prime factors; the product of all the prime factors common to them is the H. C. F.

FIFTH METHOD.

By means of common prime factors.

2	1386	2268
3	693	1134
3	231	378
7	77	126
	11	18

$2 \times 3 \times 3 \times 7 = 126$ H. C. F.

RULE.

Divide the given numbers by the prime factors common to each: the product of these prime factors will be the H. C. F.

EXERCISE 24.

Find the H. C. F. of

1.	323,	425.	8.	961,	1178.	15.	45,	57,	81.
2.	228,	399.	9.	5355,	6545.	16.	63,	99,	90.
3.	615,	735.	10.	4155,	4720.	17.	72,	84,	96.
4.	819,	945.	11.	7865,	5784.	18.	306,	408,	510.
5.	949,	871.	12.	3876,	1983.	19.	420,	462,	84.
6.	825,	960.	13.	7956,	7668.	20.	546,	462,	882.
7.	689,	1575.	14.	9861,	9523.	21.	900,	936,	2520.

LEAST COMMON MULTIPLE.

46. A Multiple of a number is one that is exactly divisible by that number, thus 36 is a multiple of 6.

47. A Common Multiple of two or more numbers is a number which is exactly divisible by each of them, thus 18, **36, 72,** are common multiples of 2, 3, 6 and 9.

48. The Least Common Multiple of two or more numbers is the least number which is exactly divisible by each of them, thus 18 is the least common multiple of 2, 3, 6, and 9.

Find the L. C. M. of 18, 28, **42.**

FIRST METHOD

By means of prime factors.

$$18 = 2 \times 3 \times 3$$
$$28 = 2 \times 2 \times 7$$
$$42 = 2 \times 3 \times 7$$

L. C. M. $= 2 \times 2 \times 3 \times 3 \times 7 = 252$

RULE.

Resolve the given numbers into their prime factors; the product of the different prime factors taking each the greatest number of times it appears in any of the numbers will be the L. C. M.

SECOND METHOD.

Find the L. C. M. of 9, 15, 18, 16, 12, 30, 45.

$$
\begin{array}{r|l}
2 & 9, \ 15, \ 18, \ 16, \ 12, \ 30, \ 45 \\
2 & \quad\quad 9, \ 8, \ 6, \ 15, \ 45 \\
\hline
 & \quad\quad\quad 4, \ 3, \quad\ 45
\end{array}
$$

$2 \times 2 \times 4 \times 45 = 720$ L. C. M.

or

$$
\begin{array}{r|l}
2, 2, 3 & 9, \ 15, \ 18, \ 16, \ 12, \ 30, \ 45 \\
\hline
 & 3, \ 4, \ 1, \ 3, \ 15
\end{array}
$$

$2 \times 2 \times 3 \times 4 \times 45 = 720$ L. C. M.

RULE.

Write the numbers in a horizontal line, cancelling such of the smaller numbers as are factors of the larger, and divide by any prime factor or prime factors that will exactly divide two or more of the given numbers. Write the quotients and the undivided numbers, if any, in a line beneath.

Continue this process until the results are prime to each other.

The product of all the divisors and the numbers in the last line will be the L. C. M.

EXERCISE 25.

Find the L. C. M. of

1.	5, 6, 15.	11.	27, 24, 15.
2.	7, 14, 21, 28.	12.	63, 27, 84.
3.	4, 8, 12, 16.	13.	12, 51, 68.
4.	5, 7, 15, 21.	14.	35, 63, 72.
5.	3, 14, 21, 28.	15.	9, 12, 14, 210.
6.	9, 2, 6, 18, 24.	16.	60, 15, 24, 25.
7.	8, 7, 12, 21, 24.	17.	54, 81, 63, 14.
8.	5, 2, 15, 7, 35.	18.	18, 27, 36, 63.
9.	3, 6, 9, 54.	19.	22, 27, 54, 108.
10.	7, 9, 12, 14, 36.	20.	9546, 6364, 14319.

FRACTIONS.

49. A **Fraction** is one or more of the equal parts of a unit, or anything regarded as a whole ; thus, one-half, two-thirds, three-fourths, are fractions.

50. The unit of the fraction is the unit which is divided. One of the equal parts is the **fractional unit.**

51. Fractions obtained by the division of the unit into tenths, hundredths, thousandths, etc., are called **Decimal Fractions.** All other fractions are called **Common Fractions.**

52. A Common Fraction is expressed by two numbers, called the **Numerator** and the **Denominator,** the former written over the latter, with a line between th

One-third	is written ⅓	Five-sixths	is wri
Three-fourths	" ¾	Seven thirteenths	"
Three-eighths	" ⅜	Eleven-twentieths	"

53. The numerator and the denominator are called the **terms** of a fraction.

54. The **Denominator** of a fraction, written below the line, shows the **number of equal parts** into which the unit is divided and also **names the unit :** thus in ⅞. 8 is the denominator and shows that the unit is divided into **eight** equal parts, named **eighths.**

55. The **Numerator** of a fraction, written above the line, shows the number of equal parts **taken** to form the fraction ; thus in ⅞, 7 is the numerator, and shows that **seven** of the eight equal parts are taken or expressed by the fraction.

56. Since the denominator of a fraction shows how many fractional units in the numerator are equal to one integral unit, it follows

That a fraction is an expression of unperformed division. The numerator is the dividend, the denominator is the divisor, and the value of the fraction is the quotient.

57. GENERAL PRINCIPLES OF FRACTIONS.

I. *Multiply by the numerator or dividing the denominator by any number multiplies the value of the fraction by that number.*

If we multiply the numerator of the fraction $\frac{1}{2}$ by 3, the result is $\frac{3}{2}$, which is three times as great as $\frac{1}{2}$. If we divide the denominator of $\frac{1}{4}$ by 2, the result is $\frac{1}{2}$, which is twice as great as $\frac{1}{4}$.

II. *Dividing the numerator or multiplying the denominator by any number divides the fraction by that number.*

If we divide the numerator of the fraction $\frac{2}{4}$ by 2, the result is $\frac{1}{4}$, which is $\frac{1}{2}$ as great as $\frac{2}{4}$. If the denominator of $\frac{1}{2}$ is multiplied by 2, the result is $\frac{1}{4}$, which is $\frac{1}{2}$ as great as $\frac{1}{2}$.

III. *Multiplying or dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction.*

If we multiply both the numerator and the denominator of $\frac{1}{2}$ by 2, the result is $\frac{2}{4}$, which has the same value as $\frac{1}{2}$. If we divide both numerator and denominator of $\frac{2}{4}$ by 2, the result is $\frac{1}{2}$, which has the same value as $\frac{2}{4}$.

58. A **Simple Fraction** is one whose terms are both integers, as $\frac{7}{9}$, $\frac{11}{11}$.

59. A **Proper Fraction** is one whose numerator is less than its denominator ; hence its value is less than 1, as $\frac{3}{4}$, $\frac{7}{8}$.

60. An **Improper Fraction** is one whose numerator equals or exceeds its denominator, as $\frac{5}{5}$, $\frac{10}{9}$, $\frac{24}{4}$.

61. A **Mixed Number** is a number composed of an integer and a fraction, as $3\frac{1}{2}$, $5\frac{3}{7}$.

EXERCISE 26.

1. Read the following fractions, and tell what each numerator and each denominator shows:

$\frac{3}{10}$, $\frac{7}{12}$, $\frac{15}{33}$, $\frac{45}{100}$, $\frac{185}{1002}$, $1\frac{15}{25}$, $12\frac{12}{13}$, $\frac{5}{7}$ of $\frac{3}{8}$.

2. Express the following in figures:

one *third ;*

ten *twentieths ;*

thirty *one hundred-and-eighths ;*

twelve hundred *ninety-thousandths ;*

three *sevenths* of nineteen *forty-fifths.*

four *ninths ;*

seventeen *twenty-thirds ;*

three *five-thousandths ;*

3. Write:

three and a half;

sixty-five and twenty-three *forty-eighths* ·

eighteen and eleven *eighty-fourths.*

fourteen and a quarter;

REDUCTION.

62. Reduction of Fractions is the changing of their form without changing their value.

63. To reduce integers or mixed numbers to improper fractions.

EXAMPLE 1.—In 18 units how many fifths?

SOLUTION.

In 1 unit there are 5 fifths

" 18 units " 18 times 5 fifths

or 90 fifths $(\frac{90}{5})$

Hence 18 = $\frac{90}{5}$

EXAMPLE 2.—Reduce $18\frac{3}{5}$ to an improper fraction.

SOLUTION.		EXPLANATION.
$18\frac{3}{5}$	(Example 1)	18 = 90 fifths
$\dfrac{93}{5}$		$\frac{3}{5} = $ 3 fifths
		$18\frac{3}{5} = $ 93 fifths $(\frac{93}{5})$

RULE.

Multiply the whole number by the denominator of the fraction, to the product add the numerator, and set their sum over the denominator.

EXERCISE 27.

Reduce to improper fractions—

I.	II.	III.	IV.	V.
$7\frac{1}{2}$.	$2\frac{1}{6}$.	$19\frac{2}{3}$.	$27\frac{7}{12}$.	$5\frac{2}{3}$.
$2\frac{3}{4}$.	$3\frac{7}{11}$.	$18\frac{1}{17}$.	$5\frac{7}{15}$.	$5\frac{3}{4}$.
$3\frac{1}{7}$.	$8\frac{9}{10}$.	$37\frac{9}{13}$.	$95\frac{11}{35}$.	$19\frac{8}{18}$.
$4\frac{1}{3}$.	$7\frac{9}{13}$.	$33\frac{5}{17}$.	$41\frac{12}{37}$.	$112\frac{12}{47}$.

64. To reduce an improper fraction to an integer or a mixed number.

EXAMPLE.—Reduce $\frac{48}{5}$ to a mixed number.

SOLUTION.

$$5\,)\,48$$
$$\overline{9\frac{3}{5}}$$

EXPLANATION.

Since $\frac{48}{5}$ expresses an unperformed division (Art. 56), therefore by performing the division we obtain $9\frac{3}{5}$ for quotient.

EXERCISE 28.

Reduce to mixed numbers—

I.	II.	III.	IV.
$\frac{48}{5}$	$\frac{112}{7}$	$\frac{540}{37}$	$\frac{216}{17}$
$\frac{45}{4}$	$\frac{1234}{18}$	$\frac{569}{7}$	$\frac{378}{8}$
$\frac{32}{7}$	$\frac{328}{11}$	$\frac{1828}{14}$	$\frac{974}{85}$
$\frac{134}{9}$	$\frac{3213}{17}$	$\frac{1837}{13}$	$\frac{2140}{75}$
$\frac{4264}{11}$	$\frac{104}{13}$	$\frac{478}{35}$	$\frac{784}{27}$

65. To reduce a fraction to higher terms.

EXAMPLE.—Reduce $\frac{3}{4}$ to sixteenths.

SOLUTION.

$$\frac{3}{4}=\frac{3\times4}{4\times4}=\frac{3}{4}\times\frac{4}{4}=\frac{12}{16}$$

EXPLANATION.

Since it is required to change $\frac{3}{4}$ to sixteenths, (*i.e*) a fraction whose denominator is 16, we must multiply the denominator 4 by 4; then by Art. 57, III., so as not to change the value of the fraction, we must multiply the numerator 3 by 4.

RULE.

To reduce a fraction to higher terms, divide the required denominator by the denominator of the given fraction and multiply both terms by the quotient.

EXERCISE 29.

Reduce—

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$ to twelfths.
2. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{2}{9}$ to eighteenths.
3. 1, $\frac{1}{4}$, $\frac{3}{2}$, $\frac{1}{2}$ to eighths.
4. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, 3 to twenty-fourths.
5. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{7}{9}$, $\frac{5}{18}$, $\frac{2}{3}$ to seventy-seconds.
6. 5, $\frac{3}{8}$, $\frac{5}{16}$, $\frac{5}{4}$, $\frac{9}{8}$, 1 to sixteenths.
7. $\frac{5}{6}$, $\frac{1}{9}$, $1\frac{1}{2}$, $\frac{5}{18}$, $\frac{1}{2}$ to fifty-fourths.
8. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{7}{?}$, $\frac{8}{15}$, $\frac{2}{45}$ to forty-fifths.
9. $\frac{5}{8}$, $\frac{1}{4}$, $\frac{5}{16}$, $1\frac{7}{6}$, $\frac{5}{3}$ to forty-eighths.
10. $\frac{7}{6}$, $\frac{3}{4}$, $\frac{7}{12}$, $\frac{1}{3}$, $1\frac{7}{9}$ to thirty-sixths.

66. To reduce a fraction to its lowest terms.

SOLUTION.

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$
or
$$4 \left| \frac{12}{16} = \frac{3}{4} \right.$$

EXPLANATION.

By Art. 57, III., we may divide both numerator and denominator by 4 without changing the value of the fraction.

RULE.

Divide both terms of the fraction successively by all the prime factors common to the two, or by the continued product of all the prime factors, (i.e) their highest common factor. (H. C. F.)

NOTE.—A fraction is in its lowest terms when the numerator and denominator have no common factor.

EXERCISE 30.

Reduce to lowest terms—

I.	II.	III.	IV.	V.
$\frac{4}{11}$	$\frac{66}{140}$	$\frac{7}{8}$	$\frac{158}{100}$	$\frac{1844}{1855}$
$\frac{13}{8}$	$\frac{72}{144}$	$\frac{84}{63}$	$\frac{148}{216}$	$\frac{1584}{2772}$
$\frac{84}{74}$	$\frac{10}{85}$	$\frac{64}{144}$	$\frac{192}{340}$	$\frac{1563}{1736}$
$\frac{7}{16}$	$\frac{15}{45}$	$\frac{41}{164}$	$\frac{324}{463}$	$\frac{6003}{10061}$
$\frac{4}{50}$	$\frac{48}{72}$	$\frac{63}{207}$	$\frac{288}{744}$	$\frac{1917}{3357}$
$\frac{13}{33}$	$\frac{34}{85}$	$\frac{188}{444}$	$\frac{396}{728}$	$\frac{4823}{2733}$

67. To reduce two or more fractions which have different denominators to equivalent fractions having a least common denominator.

NOTE.—Since the common denominator must be the same for each fraction, and fractions can be changed to equivalent fractions having a

different denominator (Art. 65), therefore the common denominator must contain each of the denominators of the given fractions exactly. The least number that will contain each of the given denominators is their L. C. M. Therefore the least common denominator of the fractions must be the L. C. M. of their denominators.

EXAMPLE.—Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{8}$ to equivalent fractions having a least common denominator.

SOLUTION.

The least common denominator
= L. C. M. of 2. 3 ,8 = 24.

$\frac{1}{2} = \frac{12}{24}$

$\frac{2}{3} = \frac{16}{24}$

$\frac{5}{8} = \frac{15}{24}$ (Art. 65)

EXPLANATION.

We first find the L. C. M. of the given denominators which is 24. This must be the least common denominator to which the given fractions can be reduced (Note Art. 67.) Reducing each fraction to the denominator 24 (Art. 65), we obtain $\frac{12}{24}$, $\frac{16}{24}$, $\frac{15}{24}$, as results.

RULE.

I. Find the L. C. M. of the given denominators for the least common denominator.

II. Divide the common denominator by each of the given denominators, and multiply the numerator and denominator of each fraction by the corresponding quotient.

EXERCISE 31.

Reduce to their least common denominator.

1. $\frac{3}{4}$, $\frac{2}{3}$, $\frac{4}{9}$.
2. $\frac{2}{5}$, $\frac{4}{11}$, $\frac{9}{10}$.
3. $\frac{1}{6}$, $\frac{3}{8}$, $\frac{7}{12}$.
4. $\frac{3}{8}$, $\frac{4}{15}$, $\frac{9}{10}$.
5. 3, $\frac{1}{3}$, $\frac{5}{8}$.
6. $\frac{3}{7}$ of $\frac{2}{3}$, $\frac{13}{21}$, 42.
7. $\frac{1}{8}$, $2\frac{3}{4}$, $\frac{7}{11}$.

8. $\frac{3}{5}$, $\frac{4}{9}$, $\frac{2}{3}$, $\frac{1}{7}$, $\frac{3}{14}$.
9. $\frac{2}{3}$, $\frac{4}{13}$, $2\frac{5}{28}$, $\frac{9}{50}$, 7.
10. $2\frac{3}{5}$, $\frac{4}{15}$, $\frac{5}{24}$, $\frac{37}{80}$, $\frac{9}{17}$.
11. $3\frac{1}{4}$, $2\frac{1}{3}$, $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{2}{3}$ of $\frac{1}{8}$.
12. $\frac{101}{333}$, $\frac{232}{333}$, $\frac{113}{333}$.
13. $\frac{9.15}{1825}$, $\frac{3127}{5723}$, $\frac{8123}{6673}$.
14. $\frac{5}{7}$, $1\frac{3}{4}$, $\frac{3}{14}$, $\frac{3}{17}$, $\frac{3}{18}$, $\frac{11}{21}$.

ADDITION.

68. EXAMPLE 1.—Find the sum of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{12}$.

SOLUTION.

	24ths.
$\frac{3}{4}$	18
$\frac{7}{8}$	21
$\frac{5}{12}$	10
	$\frac{49}{24} = 2\frac{1}{24}$ Ans.

EXPLANATION.

In order that fractions may be added they must have like denominators and be parts of like units.

$\frac{3}{4}$ = 18 twenty-fourths.

$\frac{7}{8}$ = 21 twenty-fourths.

$\frac{5}{12}$ = 10 twenty-fourths.

49 twenty-fourths = $\frac{49}{24}$ = $2\frac{1}{24}$ Ans.

EXAMPLE 2.—Find the sum of $2\frac{3}{4}$, $13\frac{7}{8}$, $4\frac{5}{12}$.

SOLUTION.

	24ths.
$2\frac{3}{4}$	18
$13\frac{7}{8}$	21
$4\frac{5}{12}$	10
$21\frac{1}{24}$ Ans. $\frac{49}{24} = 2\frac{1}{24}$.	

EXPLANATION.

The sum of the integers, 2, 13 & 4 $= 19$

The sum of the fractions $= \frac{49}{24} = 2\frac{1}{24}$

$21\frac{1}{24}$ Ans.

RULE.

I. *To add Fractions.—When necessary reduce the fractions to their least common denominator ; then add the numerators and place the sum over the common denominator.*

II. *To add Mixed Fractions.—Add the integers and fractions separately, and then add their sums.*

NOTE.—All fractions should be reduced to their lowest terms, and if improper, to whole or mixed numbers.

EXERCISE 32.

Find the sum of—

1. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$.
2. $\frac{5}{6}$, $\frac{5}{12}$, $\frac{3}{15}$.
3. $\frac{2}{5}$, $\frac{3}{7}$, $\frac{3}{8}$.
4. $\frac{1}{3}$, $\frac{5}{7}$, $\frac{17}{21}$, $\frac{7}{15}$.
5. $\frac{7}{8}$, $\frac{5}{12}$, $\frac{3}{15}$, $\frac{3}{24}$, $\frac{2}{27}$.
6. $\frac{1}{4}$, $\frac{5}{6}$, $\frac{3}{11}$, $\frac{7}{13}$, $\frac{2}{3}$.
7. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{3}{8}$, $\frac{8}{9}$, $\frac{9}{10}$.
8. $3\frac{1}{2}$, $1\frac{2}{3}$, $2\frac{5}{15}$.
9. $1\frac{1}{2}$, $2\frac{3}{4}$, $3\frac{3}{7}$, $4\frac{1}{5}$, $5\frac{1}{6}$, $6\frac{4}{9}$.
10. $7\frac{3}{8}$, $10\frac{1}{3}$, $4\frac{1}{7}$, $7\frac{7}{16}$.
11. $1\frac{7}{13}$, $8\frac{5}{21}$, $2\frac{2}{23}$.
12. $4\frac{1}{4}$, $2\frac{1}{4}$, $1\frac{5}{13}$, $2\frac{6}{24}$, $5\frac{7}{8}$.
13. $21\frac{2}{3}$, $13\frac{1}{4}$, $4\frac{5}{6}$, 70.
14. 21, $13\frac{2}{3}$, $5\frac{3}{4}$, $4\frac{8}{27}$, $6\frac{3}{8}$.

SUBTRACTION.

69. EXAMPLE 1.—Find the difference between $\frac{7}{12}$ and $\frac{3}{8}$.

SOLUTION.

	24ths.
$\frac{7}{12}$	14
$\frac{3}{8}$	9
	$\frac{5}{24}$ Ans.

EXPLANATION.

In order that fractions may be subtracted, they must have like denominators and be parts of the same unit.

$\frac{7}{12} = 14$ twenty-fourths (Art. 65.)

$\frac{3}{8} = 9$ twenty-fourths.

$\frac{7}{12} - \frac{3}{8} = 5$ twenty-fourths $= \frac{5}{24}$ Ans.

EXAMPLE 2.—Find the difference between $83\frac{7}{12}$ and $45\frac{3}{8}$.

SOLUTION.

	24ths.
$83\frac{7}{12}$	14
$45\frac{3}{8}$	9
	$\frac{5}{24}$
$38\frac{5}{24}$ Ans.	

EXPLANATION.

The difference between the integers $= 38$

The difference between the fractions $= \frac{5}{24}$

The result $= 38\frac{5}{24}$ Ans.

EXAMPLE 3.—Find the difference between $36\frac{7}{9}$ and $19\frac{5}{6}$.

EXPLANATION.

SOLUTION.

	18ths
$36\frac{7}{9}$	4
$19\frac{5}{6}$	15
$16\frac{7}{9}$ Ans.	$\frac{7}{18}$

You can't take $\frac{15}{18}$ from $\frac{4}{18}$. Borrow unity from 36. Reduce it to eighteenths, and then add result to $\frac{4}{18}$ which makes $\frac{22}{18}$ $\frac{15}{18}$ from $\frac{22}{18}$ leaves $\frac{7}{18}$. 19 from 35 leaves 16. Result, $16\frac{7}{18}$.

RULE I.

To subtract fractions.—*When necessary, reduce the fractions to their least common denominator. Subtract the numerator of the subtrahend from the numerator of the minuend, and place the difference over the common denominator.*

RULE II.

To subtract mixed numbers.—*Reduce the fractions, if necessary, to a common denominator, and if the fraction in the subtrahend is smaller than that in the minuend, subtract one fraction from the other, and the smaller whole number from the larger whole number. But if the fraction in the subtrahend is larger than that in the minuend, borrow 1 from the whole number. After changing it to the same denominator as the fraction, add it to the fraction in the minuend. Then subtract as before.*

EXERCISE 33.

Find the difference between—

1. $\frac{4}{7}$ and $\frac{3}{8}$.	8. $\frac{5}{21}$ and $\frac{1}{15}$.	15. $8\frac{1}{7}$ and $5\frac{1}{4}$.
2. $\frac{7}{8}$ and $\frac{3}{4}$.	9. $\frac{7}{16}$ and $\frac{13}{170}$.	16. $3\frac{5}{17}$ and $1\frac{3}{4}$.
3. $\frac{3}{8}$ and $1\frac{1}{2}$.	10 $\frac{9}{25}$ and $\frac{9}{30}$.	17. $5\frac{1}{3}$ and $31\frac{11}{12}$.
4. $1\frac{7}{15}$ and $\frac{3}{8}$.	11. $16\frac{3}{8}$ and $7\frac{1}{4}$.	18. 19 and $4\frac{7}{9}$.
5. $2\frac{11}{14}$ and $\frac{13}{14}$.	12. $3\frac{4}{7}$ and $1\frac{4}{8}$.	19. $11\frac{3}{8}$ and $7\frac{1}{3}$.
6. $\frac{9}{14}$ and $\frac{13}{3}$.	13. $2\frac{1}{2}$ and $1\frac{1}{16}$.	20. $38\frac{11}{3}$ and $23\frac{3}{14}$.
7. $1\frac{4}{3}$ and $\frac{13}{8}$.	14. $6\frac{1}{2}$ and $2\frac{11}{4}$.	21. $18\frac{3}{8}$ and $5\frac{4}{5}$.

MULTIPLICATION.

70. EXAMPLE 1.—Multiply $\frac{3}{4}$ by $\frac{2}{3}$.

SOLUTION.

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$$

EXPLANATION.

The numerators are multiplied for a new numerator and the denominators for a new denominator.

EXAMPLE 2.—Multiply $\frac{1}{2}$ by $\frac{2}{3}$ by $\frac{6}{8}$ by $\frac{8}{9}$.

SOLUTION.

$$\frac{1}{2} \times \frac{2}{3} \times \frac{\overset{2}{\cancel{6}}}{8} \times \frac{8}{9} = \frac{2}{9}$$

EXPLANATION.
See Art. 36.

RULE.

Reduce integers and mixed numbers to improper fractions. Multiply the numerators together for a new numerator, and the denominators for a new denominator. Reduce the result to its simplest form.

NOTE.—Cancellation often shortens the operation.

EXERCISE 34.

Find the product of—

1. $\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}$.   8. $\frac{2}{3} \times 12 \quad \times \quad \frac{3}{4} \times 16 \quad \times \quad \frac{4}{5}$ of 20.

2. $\frac{1}{4} \times \frac{1}{4} \times \frac{3}{4}$.   9. $\frac{2}{3} \times 15 \quad \times \quad \frac{3}{4} \times 18 \quad \times \quad \frac{2}{7} \times 21$.

3. $\frac{1}{4} \times \frac{3}{8} \times \frac{3}{4}$.   10. $\frac{1}{5} \times 18 \quad \times \quad \frac{3}{4} \times 20 \quad \times \quad \frac{4}{9} \times 27$.

4. $\frac{1}{3} \times \frac{3}{7} \times \frac{3}{4}$.   11. $2\frac{1}{3} \times \quad 3\frac{1}{3} \quad \times \quad 1\frac{1}{3} \quad \times \quad \frac{3}{4} \times 1\frac{1}{4} \times \quad \frac{1}{4}$.

5. $\frac{3}{4} \times \frac{1}{4} \times 1\frac{5}{17}$.   12. $3\frac{1}{4} \times \quad 1\frac{3}{10} \quad \times \quad 1\frac{2}{3} \quad \times \quad 1\frac{3}{4} \times 2\frac{3}{7} \times 22$.

6. $1\frac{5}{13} \times \frac{13}{25} \times \frac{1}{4}$.   13. $8\frac{1}{4} \times \quad 1\frac{3}{4} \quad \times \quad 3\frac{1}{4} \quad \times \quad 1\frac{3}{4} \times 2\frac{1}{4} \times \quad \frac{1}{4}$.

7. $\frac{3}{8} \times 1\frac{1}{4} \times \frac{1}{17}$.   14. $\frac{3}{4} \times \quad 1\frac{3}{11} \quad \times \quad 1\frac{9}{13} \times 77 \quad \times \quad \frac{3}{7} \times \quad \frac{1}{11} \times 91$.

DIVISION.

71. To divide a fraction by an integer.

EXAMPLE 1.—Divide $\frac{21}{25}$ by 3.

SOLUTION.

$$\frac{21}{25} \div 3 = \frac{21 \div 3}{25} = \frac{7}{25}$$

EXPLANATION.
Art. 57, 3.

EXAMPLE 2.—Divide $\frac{3}{4}$ by 2.

SOLUTION.

$$\frac{3}{4} \div 2 = \frac{3}{4 \times 2} = \frac{3}{8}$$

EXPLANATION
Art. 57, 2.

EXAMPLE 3.—Divide $34\frac{2}{3}$ by 11.

SOLUTION.

$34 \div 11 = 3, \text{ rem. } 1$

$1\frac{2}{3} = \frac{5}{3}$

$\frac{5}{3} \div 11 = \frac{5}{33}$

$3\frac{5}{33}$ Ans.

EXPLANATION.
Divide the integer by 11, quotient 3, rem. 1. This rem. prefixed to the fraction makes $1\frac{2}{3}$, or $\frac{5}{3}$, yet to be divided. Divide this improper fraction and combine the results.

or

$$34\frac{2}{3} \div 11$$

$$= \frac{104}{3} \div 11 = \frac{104}{3 \times 11}$$

$$= \frac{104}{33} = 3\frac{5}{33} \text{ Ans.}$$

EXPLANATION.

Reduce the mixed number to an improper fraction and proceed as in example **2.**

EXERCISE 35.

Divide--

1.	$1\frac{2}{3}$ by 4.	6.	$1\frac{1}{3}$ by 11.	11.	$67\frac{1}{4}$ by 6.
2.	$\frac{2}{5}$ by 6.	7.	$2\frac{3}{5}$ by 6.	12.	$19\frac{1}{3}$ by 8.
3.	$1\frac{5}{11}$ by 8.	8.	$1\frac{3}{4}$ by 5.	13.	$16\frac{1}{5}$ by 7.
4.	$\frac{5}{6}$ by 7.	9.	$16\frac{1}{4}$ by 7.	14.	$11\frac{1}{11}$ by 11.
5.	$\frac{3}{11}$ by 3.	10.	$42\frac{1}{2}$ by 3.	15.	$24\frac{2}{3}$ by 6.

72. To divide a fraction by a fraction.

EXAMPLE.--Divide $\frac{3}{5}$ by $\frac{2}{3}$.

EXPLANATION.

$\frac{3}{5} \div \frac{2}{3} =$

5 fifths ÷ 2 thirds

= 9 fifteenths ÷ 10 fifteenths Art. 65

SOLUTION.

$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$

$= \frac{9}{10} = \frac{3 \times 3}{5 \times 2} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$ Ans

(*i.e*) $\frac{3}{5}$ multiplied by $\frac{3}{2}$, (the divisor inverted).

RULE.

Invert the divisor and proceed as in multiplication of fractions.

EXERCISE 36.

Divide --

1.	$\frac{3}{5}$ by $\frac{1}{4}$.	14.	$2\frac{2}{3}$ by $1\frac{1}{3}$.
2.	$\frac{9}{7}$ by $\frac{3}{11}$.	15.	$18\frac{2}{3}$ by $4\frac{1}{2}$.
3.	$\frac{2}{17}$ by $\frac{8}{15}$.	16.	$6\frac{1}{3}$ by $6\frac{3}{17}$.
4.	$\frac{7}{12}$ by $\frac{4}{5}$.	17.	$2\frac{3}{11}$ by $\frac{1}{3} \times \frac{3}{8} \times \frac{3}{4}$
5.	$\frac{9}{15}$ by $\frac{3}{8}$.	18.	$2\frac{1}{4}$ by $\frac{3}{4}$.
6.	$\frac{3}{4} \div \frac{7}{4}$.	19.	$1\frac{7}{8}$ by $1\frac{1}{4}$.
7.	$\frac{2}{3}$ of $\frac{5}{11}$ by $\frac{3}{11}$ of $\frac{6}{15}$.	20.	73 by $8\frac{1}{3}$.
8.	$2\frac{1}{2} \times 7\frac{1}{3}$ by $3\frac{1}{3} \times 3\frac{3}{17}$.	21.	$\frac{4}{5}$ of $\frac{6}{11}$ by $\frac{9}{11} \times \frac{5}{13}$.
9.	11 by $\frac{3}{4} \times 5\frac{1}{3} \times 7$.	22.	$\frac{7}{12} \times \frac{8}{13}$ by $\frac{7}{8}$ of $\frac{5}{11}$.
10.	16 by $\frac{1}{4}$.	23.	$\frac{2}{3}$ of $13\frac{3}{14}$ by $\frac{1}{2}$ of $12\frac{9}{13}$.
11.	49 by $\frac{7}{5}$.	24.	$4\frac{1}{2}$ by $\frac{1}{3}$ of $\frac{3}{15}$.
12.	73 by $8\frac{1}{3}$.	25.	$\frac{6}{13}$ of $\frac{3}{8}$ by $\frac{7}{15}$.
13.	$6\frac{3}{4}$ by $\frac{1}{3}$.	26.	$18\frac{4}{7}$ by $4\frac{12}{17}$.
		27.	$1301\frac{3}{4}$ by $35\frac{1}{2}$.

GREATEST COMMON MEASURE.

73. A Measure of a fraction is any number that is contained in the fraction an exact integral number of times; thus $\frac{1}{12}$ is a measure of $\frac{1}{4}$, being contained in it 3 times.

Hence,

74. A fraction is a measure of a given fraction when its numerator is a measure of the given numerator, and its denominator is a multiple of the given denominator.

75. A Common Measure of two or more fractions is any number that is contained in each an exact integral number of times; thus, $\frac{1}{24}$ is a common measure of $\frac{1}{3}$ and $\frac{1}{4}$, being contained in $\frac{1}{3}$ 8 times, and in $\frac{1}{4}$ 6 times.

Hence,

76. A fraction is a common measure of two or more given fractions when its numerator is a common measure of the given numerators, and its denominator is a common multiple of the given denominators.

77. The Greatest Common Measure of two or more given fractions is the greatest number that is contained in each an exact integral number of times; thus, $\frac{1}{12}$ is the greatest common measure of $\frac{1}{3}$ and $\frac{1}{4}$.

Hence,

78. A fraction is the greatest common measure of two or more given fractions when its numerator is the greatest common measure of the given numerators and its denominator is the least common multiple of the given denominators.

EXAMPLE.—Find the greatest common measure of $\frac{5}{6}$, $\frac{5}{12}$ and $\frac{15}{16}$

SOLUTION.

The G. C. M. of 5, 5 and 15 = 5

The L. C. M. of 6, 12 and 16 = 48

Therefore the G. C. M. of the given fractions is $\frac{5}{48}$ Ans.

PROOF.

$$\tfrac{5}{6} \div \tfrac{5}{48} = 8$$
$$\tfrac{5}{12} \div \tfrac{5}{48} = 4$$
$$\tfrac{15}{16} \div \tfrac{5}{48} = 9$$

The quotients 8, 4 and 9 are prime to each other.

From these principles and illustrations we derive the following rule :

I. *Reduce whole and mixed numbers to improper fractions and all fractions to their lowest terms.*

II. *Find the greatest common measure of the given numerators for a new numerator, and the least common multiple of the given denominators for a new denominator. This fraction will be the greatest common measure sought.*

EXERCIS: 37.

Find the greatest common measure of—

1.   ½,   ⅞,   ⅜.

2.   ⅝,   4⅓,   ⅞

3.   4⅞,   3⅛,   4⅚.

4.   2⅔,   1⅞, $\frac{63}{100}$.

5. $1\frac{4}{10}$,   2$\frac{4}{16}$,   3$\frac{2}{25}$, $\frac{88}{105}$.

6. $1\frac{83}{121}$, $1\frac{57}{77}$, $4\frac{2}{52}$, $5\frac{2}{3}$, 12$\frac{5}{11}$.

7.   ½,   ⅔,   ⅞,   ⅘,   ⅜,   ⅞,   ⅓,   ⅘, $\frac{1}{16}$.

8.   3,   ⅞,   9,   4½, $\frac{9}{25}$, 1⅞.

LEAST COMMON MULTIPLE.

79. A Multiple of a fraction is any number that contains the fraction an exact integral number of times ; thus, ¼ is a multiple of $\frac{1}{12}$, since ¼ contains $\frac{1}{12}$ 3 times.

Hence,

80. A fraction is a multiple of a given fraction when its numerator is a multiple of the given numerator, and its denominator a measure of the given denominator.

81. A Common Multiple of two or more given fractions is any number that contains each an exact integral number of times ; thus, ⅔ is a common multiple of $\frac{1}{12}$ and ⅑, containing $\frac{1}{12}$ 8 times, and ⅑ 6 times.

Hence,

82. A fraction is a common multiple of two or more given fractions when its numerator is a common multiple of the given numerators, and its denominator is a common measure of the given denominators.

83. The **Least Common Multiple** of two or more given fractions is the least number that contains each an exact integral number of times; thus, $\frac{1}{3}$ is the least common multiple of $\frac{1}{15}$ and $\frac{1}{4}$.

Hence,

84. A fraction is the least common multiple of two or more given fractions when its numerator is the least common multiple of the given numerators and its denominator the greatest common measure of the given denominators.

EXAMPLE.—Find the least common multiple of $\frac{3}{4}$, $\frac{5}{12}$, and $\frac{15}{16}$.

SOLUTION.

L. C. M. of 3, 5 and 15 = 15
G. C. M of 4, 12 and 16 = 4

Therefore the L. C. M. of the given fractions = $\frac{15}{4}$

PROOF.

$\frac{15}{4} \div \frac{3}{4} = 5$
$\frac{15}{4} \div \frac{5}{12} = 9$
$\frac{15}{4} \div \frac{15}{16} = 4$

The quotients 5, 9 and 4 are prime to each other.

From these principles and illustrations we derive the following rule:

RULE.

I. *Reduce whole and mixed numbers to improper fractions and all fractions to their lowest terms.*

II. *Find the least common multiple of the given numerators for a new numerator, and the greatest common measure of the given denominators for a new denominator. This fraction will be the least common multiple sought.*

EXERCISE 38.

Find the least common multiple of—

1   $\frac{2}{5}$, $\frac{7}{10}$, $\frac{14}{15}$, $\frac{8}{15}$.
2. $\frac{7}{24}$, $\frac{35}{88}$, $\frac{49}{55}$, $\frac{24}{120}$.
3. $\frac{8}{15}$, $\frac{24}{25}$, $\frac{18}{105}$, $\frac{24}{35}$.
4. $2\frac{12}{13}$, $1\frac{17}{15}$, $\frac{63}{100}$.
5. $\frac{18}{55}$, $\frac{110}{121}$, $1\frac{23}{140}$, $\frac{82}{81}$, $1\frac{7}{55}$.
6   $1\frac{1}{4}$, $2\frac{1}{3}$, $3\frac{1}{2}$, $4\frac{1}{5}$, $5\frac{1}{4}$.
7.   4, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{27}$, $\frac{6}{81}$, $\frac{7}{18}$, $1\frac{1}{4}$.

DECIMALS.

85. A **Decimal Fraction**, commonly called **Decimal**, is one whose denominator is 1 followed by one or more ciphers:

As $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, $\frac{3}{10000}$.

86. Since the denominators of decimal fractions increase or decrease by the uniform scale of 10 (the same scale as that used in expressing integers), a system of notation similar to that of integers is employed to express them, thus saving the trouble of writing the denominators.

87. The Decimal Sign (.) or decimal point determines, by its position, the denominator of the fraction, and, in a number composed of an integer and a decimal, it shows where the decimal part begins.

88. This system of notation will be best explained by the following examples:

$\frac{3}{10}$	is written	.3, and is read	3 tenths.
$\frac{3}{100}$	"	.03, " "	3 hundredths.
$\frac{3}{1000}$	"	.003, " "	3 thousandths.

The numerator alone is written, and there must be as many figures to the right of the decimal point as there are ciphers in the denominator of the fraction. The vacant orders, if any, must be filled with ciphers.

89. The relation of decimals and integers to each other is clearly shown in the following table:

TABLE.

Names.	etc.	Hundred millions.	Ten millions.	Millions.	Hundred-thousands.	Ten thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	etc.
Units.		2	2	2	2	2	2	2	2	2.	2	2	2	2	2	2	2	2	
Orders.		9th.	8th.	7th.	6th.	5th.	4th.	2nd.	3rd.	1st.	2nd.	3rd.	4th.	5th.	6th.	7th.	8th.	9th.	

From this it appears that

$$2222.222 = 2000 + 200 + 20 + 2 + \tfrac{2}{10} + \tfrac{2}{100} + \tfrac{2}{1000}.$$

90. The method of representing decimal fractions is merely an extension of the method by which integers are represented, since the local value of each digit increases tenfold as we advance from right to left, and also decreases in the same proportion as we advance from left to right.

From the foregoing we derive the following principles:

PRINCIPLES.

91. 1. *Decimals are governed by the same law of local value that governs the notation of integers.*

2. *The different orders of decimal units decrease from left to right and increase from right to left in a tenfold ratio.*

3. *The value of any decimal figure depends upon the place it occupies at the right of the decimal point.*

4. *Each removal of a decimal order one place to the left increases its value tenfold.*

5. *Each removal of a decimal order one place to the right decreases its value tenfold.*

6. *Prefixing a cipher to a decimal diminishes its value tenfold, since it removes every decimal figure one place to the right.*

7. *Annexing a cipher to a decimal does not alter its value, since it does not change the place of any figure in the decimal.*

EXERCISE 39.

Express in decimal form and read—

I.	II.	III.	IV.	V.
$\tfrac{8}{10}$	$\tfrac{125}{1000}$	$\tfrac{13}{100}$	$\tfrac{215}{100}$	$\tfrac{300}{1000}$
$\tfrac{4}{10}$	$\tfrac{6}{1000}$	$\tfrac{4}{1000}$	$\tfrac{274}{10000}$	$\tfrac{38}{10000}$
$\tfrac{80}{100}$	$\tfrac{17}{100}$	$\tfrac{350}{1000}$	$\tfrac{958}{100}$	$\tfrac{34}{100000}$
$\tfrac{9}{100}$	$\tfrac{9}{100}$	$\tfrac{355}{100}$	$\tfrac{937}{10000}$	$\tfrac{137}{1000}$

Express in the form of a fraction and read—

VI.	VII.	VIII.	IX.	X.
.9	.27	8.7	.0005	.0304
.06	.006	4.05	.31600	.00001
.25	.450	.005	.0404	.15000
	.003			
.007	.010	.136	.912	35.003

Express as decimals—

XI.   Five-*tenths*,   ninety-seven-*hundredths*.
               eleven *ten-thousandths*.

XII.   Thirty, and *seven-tenths*,   fourteen, and *nine-hundredths*

XIII.   Seventy-four *ten-millionths*,   thirty-six *ten-thousandths*.

92. A **Complex Decimal** has a fraction in its right hand place, as .12⅓ which is read 12⅓ hundredths, the fraction not being counted as a decimal place.

Express as common fractions in their lowest terms—

XIV.   .75,   .72,   .625,   .024,   .0032,   .12,

XV.   .13⅓,   .16⅔,   .57½,   .66⅔,   .444⅘,   .024⅘.

93. To reduce a common fraction to a decimal.

Reduce ⅝ to its equivalent decimal.

$$\text{OPERATION.} \qquad\qquad \text{REASON.}$$
$$8\,\overline{)\,5.000} \qquad \frac{5}{8} = \frac{5000}{8000} = \frac{\dfrac{5000}{8}}{1000} = \frac{625}{1000} = .625$$
$$.625$$

94. From this and similar examples we derive the following rule :

RULE.

Annex ciphers to the numerator and divide by the denominator.

Point off as many decimal places in the quotient as there are ciphers annexed.

NOTE 1.—If the division is not exact, when a sufficient number of decimal places has been obtained, the sign + may be annexed to show that the division is not complete, or it may be expressed as a complex decimal.

2. A fraction in its lowest terms can be reduced to a pure decimal only when its denominator contains no prime factors but 2 and 5. If the denominator contain any other prime factor the division will not end. The decimals thus produced are called *Repeating* Decimals, and the figures repeated, *Repetends*.

EXERCISE 40.

Reduce to equivalent decimals—

1. ¾	5. 1¼	9. ½	13. 12½
2. 1⁄16	6. 1¾	10. ⅘	14. 16¾
3. ½	7. ⅜	11. ⅝	15. 25 1⁄16
4. ⅞	8. ⅔	12. 7⁄11	16. 31¼

ADDITION.

95. Since integers and decimals increase and decrease uniformly by the scale of 10, it is evident that decimals may be added, subtracted, multiplied and divided in the same manner as integers.

Add 13.6, 5.034, .3172, 14.52.

13.6
5.034
.3172
14.52
33.4712

EXPLANATION.

If the decimal points are in the same vertical line it will necessarily bring tenths under tenths, hundredths under hundredths, etc., and the numbers may therefore be added as in integers.

RULE.

96. *Write the numbers so that their decimal points are in the same vertical line. Add as in integers, and place the decimal point in the result directly under the points in the numbers added.*

EXERCISE 41.

Add—

1. .3642, 26.035, .0037, 3.4, .017.
2. 41.234, 17.015, 3.3, 400.2, .0045.
3. .0126, 40.371, .7251, .0021, 311.5.
4. .06⅜, 3.805¾, 40.036, .0031⅜.
5. .0044⅖, 36.02⅔, 7.34, .37⅓.
6. Ninety-seven *hundredths*, four hundred and three *thousandths*, thirteen *ten-thousandths*, sixteen, and fifteen *hundredths*, forty-seven, three hundred and twelve, and sixty-four *thousandths*.

SUBTRACTION.

97. From 13.65 take 9.3652.

13.6500
9 3652
4.2848

NOTE.—The affixing of ciphers to right of the decimal does not alter its value. In practice we omit the decimals, and merely conceive them to be annexed, subtracting as otherwise.

RULE.

98. *Write the numbers so that the decimal places shall stand directly under each other. Subtract as in whole numbers, and place the decimal point in the result directly under the points in the given numbers.*

EXERCISE 42.

Find the difference between—

1. 17.205 and 13.6.
2. 4 037 and .2735.
3. 37.001 and 16.39253.
4. 400.7 and .00362.

5. From .03165 take .0127.
6. From 3074 take .29653.
7. Subtract 30.365 and 30.3782.
8. Subtract .0034715 and .0126.

MULTIPLICATION.

99. In multiplication of decimals, the position of the decimal point in the product depends upon the following principles :

1. The number of ciphers in the denominator of a decimal is equal to the number of decimal places.

2. If two decimals in the fractional form be multiplied together, the denominator of the product must contain as many ciphers as there are decimal places in both factors. Therefore,

3. The product of two decimals expressed in the decimal form must contain as many decimal places as there are decimals in both factors.

Multiply .314 by .23.

$$
\begin{array}{r}
.314 \\
.23 \\
\hline
942 \\
628 \\
\hline
.07222
\end{array}
$$

NOTE —The number of decimal places in both factors is 5. The number of figures in the product is only 4, and therefore a cipher must be prefixed.

EXERCISE 43.

Multiply—

1. .75 by .4.
2. .410 by .32.
3. 5 75 by .38.
4. 741 by .025.
5. 3.26 by 40.4.

6. 015 by .003.
7. 2.371 by .018.
8. .435 by 1.203.
9. .03 × .05 × .016 × .54
10. .301 × .2 × .03 × .25

RULE.

100. *Multiply as in whole numbers and from the right hand of the product point off as many figures for decimals as there are decimal places in both factors.*

CONTRACTIONS IN MULTIPLICATION.

101. Multiply 62.87416 by 2.84169 so as to retain only 4 places of decimals.

ORDINARY METHOD.

$$
\begin{array}{r}
62.37416 \\
2.34169 \\
\hline
56\,136714 \\
371\,24496 \\
623\,7416 \\
24949\,664 \\
187122\,43 \\
1247183\,2 \\
\hline
146\,0609\,467304
\end{array}
$$

CONTRACTED METHOD.

$$
\begin{array}{rlcl}
62.37416 & & & \\
96113.2 & & & \\
\hline
1247183 & = & 62374 \times 2 & + 1 \\
187122 & = & 62371 \times 3 & \\
24950 & = & 6237 \times 4 & + 2 \\
624 & = & 623 \times 1 & + 1 \\
374 & = & 62 \times 6 & + 2 \\
56 & = & 6 \times 9 & + 2 \\
\hline
146\,0609 & & &
\end{array}
$$

102. It frequently happens in multiplication that a greater number of decimal figures is obtained in the product than is necessary for practical accuracy. This may be avoided by contracting each partial product to the required number of decimal places.

103. From this principle and illustrations similar to the foregoing example we derive the following:

RULE.

Write the multiplier with the order of the figures reversed, and with the units place under that figure of the multiplicand which is the lowest decimal to be retained in the product.

Find the product of each figure of the multiplier by the figures above and to the left of it in the multiplicand, increasing each partial product by as many units as would have been carried from the rejected part of the multiplicand, and one more when the highest figure in the rejected part of any product is 5 or greater than 5; and write these partial products with the lowest figure of each in the same column.

Add the partial products, and from the right hand point off the required number of decimal figures.

NOTE 1 —In obtaining the number to be carried it is generally necessary to multiply (mentally) only one figure at the right of the figure above the

multiplying figure; but when the figures are large the multiplication should commence at least two places to the right.

2. There is always a liability to an error of one or two units in the last place.

3. When the number of places in the multiplicand is less than the number to be retained in the product, supply the deficiency by annexing ciphers.

EXERCISE 44.

1.	36.275	×	4.3678	retaining 2 decimal places.		
2.	41.3075	×	467.52	"	3	"
3.	17.0036	×	.08245	"	4	"
4.	.43261	×	.73153	"	5	"
5.	.003647	×	.12759	"	4	"
6.	700.375	×	.02736	"	3	"
7.	.371325	×	.693847	"	5	"

DIVISION.

PRINCIPLE.

104. *Multiplying both divisor and dividend by the same number does not alter the quotient.*

105. Multiplying a decimal expression by 10, moves the decimal point one place to the right; by 100, two places to the right; by 1000, three places to the right, etc. Therefore, moving the decimal point in divisor and dividend the same number of places to the right, multiplies each of them by the same number.

EXAMPLE 1.—Divide 16.578 by 5.4.

$$5.4) 16.578 ($$

EXPLANATION.

Multiply the divisor and dividend by 10 and we obtain 54 as divisor and 165.78. Now 54 will divide into 165, 3 times, and therefore 3 is the integral part of the quotient.

```
54 ) 165.78 ( 3.07
     162
     ───
     378
     378
```

EXAMPLE 2.—Divide .736644 by 234.6

$$234.6) .736644 ($$

```
2346 ) 7.36644 ( .00314
       7038
       ────
       3284
       2346
       ────
       9384
       9384
       ────
```

Here in dividing we use as the first partial dividend 7.366 or 7366 thousandths, and hence our first quotient figure 3 thousandths which expressed as a decimal is .003.

106. *Move the decimal point to the right of the divisor, and the same number of places to the right in the dividend. Divide as in simple division, placing the decimal point in the quotient as soon as the tenths figure is used or brought down.*

Note.—If the dividend does not contain as many decimal places as the divisor, annex ciphers to the right of the decimal before removing the points.

EXERCISE 45.

1. 48.591 $+$ .96.   5.   .0774 $\div$ 480.   9.   10 66 $\div$ 1.3.
2. 2.56 $+$ .0032.   6. 21 3 $\div$ 37.5.   10. 15.77 $+$ .19.
3. 3 1 $+$ .025.   7. 202 $\div$ .01.   11. 134.25 $+$ 7 5
4. .0012 $\div$ 1.6.   8. 406.8 $\div$ .018. 12.   .73326 $\div$ 33.
  13. Divide 1.21 by 11, 1.1, .11, .011, .0011, .00011.
  14. Divide .036 by 1800, 180, 18, .18, .018.

CONTRACTED DIVISION.

107. Divide 763.14163 by 21.3642, correct to four places of decimals.

ORDINARY METHOD.

213642) 7631416.3 (35.7205
           640926
         122215 6
         106821 0
          15394 63
          14954 94
            439 690
            427 284
             12 40600
             10 68210
              1 72390

CONTRACTED METHOD.

213642) 76314163 (**35.7205**
          640926
          122215
          106821
           15394
           14955
             439
             427
              12
              11
               1

108 *Compare the highest or left hand figure of the divisor with the units of like order in the dividend, and determine how many figures will be required in the quotient.*

For the first contracted divisor take as many significant figures from the left of the given divisor, as there are places

required in the quotient, and, at each subsequent division reject one place from the right of the last preceding divisor.

In multiplying by the several quotient figures, carry from the rejected figures of the divisor as in contracted multiplication.

NOTE.—Before commencing the work, supply ciphers at the right of either divisor or dividend, when necessary.

EXERCISE 46.

Divide—

1.	27.3782	by	4.3267	correct to	3	decimal	places.
2.	487.24	by	1.003675	"	2	"	"
3.	8.47326	by	75.43	"	5	"	"
4.	.8487564	by	.075637	"	3	"	"
5.	478.325	by	1.433	"	3	"	"
6.	8972.436	by	756.3152	"	4	"	"
7.	1	by	1.007633	"	6	"	"
8.	.953728	by	44.73654	"	3	"	"

REPEATING, CIRCULATING OR INTERMINATE DECIMALS.

109. In reducing common fractions to equivalent decimals, reference was made in Article 94, Note 1, to the methods of expressing the decimals in cases where the division does not terminate. But if the division were carried far enough (never to number of places in the quotient greater than the number represented by the divisor) a remainder would be obtained which had occurred before, and hence a figure or set of figures in the quotient would be repeated in the same order in a never-ending succession. A decimal of this kind is called a **repeating** or **circulating** decimal, or simply a **repetend**.

110. When a repetend consists of a single figure it is indicated by a point placed over it; when it consists of more than one figure a point is placed over the first and one over the last figure repeated. Thus the circulating

decimals .4444 + and .324324324 + are written .$\dot{4}$ and .$\dot{3}2\dot{4}$.

111. If we take any fractions whose denominator consists of any number of 9's, as $\frac{4}{9}$, $\frac{24}{99}$, $\frac{324}{999}$, and we reduce each of them to decimals, we obtain

$\frac{4}{9}$ = .444 + = .$\dot{4}$; $\frac{24}{99}$ = .2424 + = .$\dot{2}\dot{4}$; $\frac{324}{999}$ = .324324 + = .$\dot{3}2\dot{4}$.

From these and similar examples we infer that **all possible repetends** *can thus be derived from fractions whose numerators are the repeating figures, and whose denominators are as many 9's as there are repeating figures.*

EXAMPLE 1.—Express $\frac{5}{7}$ as a repeating decimal.

7) 5 (.714285714285 + = .$\dot{7}1428\dot{5}$

EXAMPLE 2.—Express $\frac{13}{28}$ as a repeating decimal.

28) 13 (.46428571428571 + = 46$\dot{4}$2857$\dot{1}$.

112. Decimals in which all the figures do not repeat are called **Mixed Circulating Decimals.**

EXAMPLE 3.—Express .$\dot{2}\dot{5}$ as a common fraction.

$$.\dot{2}\dot{5} = \frac{25}{99}$$

EXPLANATION.

.$\dot{2}\dot{5}$ = .252525 +

∵ 100 times .$\dot{2}\dot{5}$ = 25.252525 +

Add 1 times .$\dot{2}\dot{5}$ = .252525 +

Subtracting 99 times .$\dot{2}\dot{5}$ = 25

∴ .$\dot{2}\dot{5}$ = $\frac{25}{99}$

From similar examples to this we derive the following rule :

Omit the points and decimal sign and write the figures of the repetend for a numerator and as many 9's as there are places in the repetend for a denominator.

EXAMPLE 4.—Express .24$\dot{5}\dot{6}$ as a common fraction.

SOLUTION.

2456

24

————

2432 $\frac{2432}{9900}$ Ans

EXPLANATION.

$$.2 4 5 \overset{\cdot}{5} \overset{\cdot}{6} = \qquad .2456565656 \; +$$
$$\because \; 10000 \text{ times } .245\overset{\cdot}{5}\overset{\cdot}{6} = 2456.565656 \quad +$$
$$\text{And} \quad 100 \text{ times } .245\overset{\cdot}{5}\overset{\cdot}{6} = \quad 24.565656 \quad +$$
$$\text{Subtracting} \quad 9900 \text{ times } .245\overset{\cdot}{5}\overset{\cdot}{6} = 2432 = 2456 - 24$$
$$\therefore \; .245\overset{\cdot}{5}\overset{\cdot}{6} = \frac{2456 - 24}{9900}$$

From examples similar to the preceding, we derive the following rule for reducing mixed circulating decimals (those in which only a portion of the figures in the decimal repeat) to common fractions.

RULE.

Subtract the part of the decimal which does not repeat from the whole decimal as if each were whole numbers, and place the remainder as a numerator, and for a denominator as many 9's as there are figures repeating, followed by as many 0's as there are figures in the part which does not repeat.

EXERCISE 47.

Express as circulating decimals—

1. ⅓, ⅔, ⅙, 7/12, 1/15, 2/7, 4/9, 5/7, 3/16, 5/11.
2. 13/18, 2/15, 11/13, 15/23, 13/21, 7/15, 31/41, 13/33.

Express as fractions in their lowest terms—

3. $.\overset{\cdot}{7}$, $.\overset{\cdot}{5}\overset{\cdot}{7}$, $.3\overset{\cdot}{0}\overset{\cdot}{6}$, $.\overset{\cdot}{4}\overset{\cdot}{5}$, $.3\overset{\cdot}{6}\overset{\cdot}{9}$, $.1\overset{\cdot}{6}\overset{\cdot}{2}$, $.\overset{\cdot}{2}63\overset{\cdot}{5}$.

4. $.2\overset{\cdot}{7}$, $.4\overset{\cdot}{7}$, $.3\overset{\cdot}{1}$, $.2\overset{\cdot}{3}\overset{\cdot}{5}$, $.24\overset{\cdot}{5}$, $.3\overset{\cdot}{4}73\overset{\cdot}{4}$, $.7127\overset{\cdot}{1}$.

5. $.03\overset{\cdot}{6}$, $.0024\overset{\cdot}{7}$, $.03\overset{\cdot}{5}\overset{\cdot}{6}$, $.251\overset{\cdot}{6}$, $.035\overset{\cdot}{7}$, $.71\overset{\cdot}{9}\overset{\cdot}{4}$.

WEIGHTS AND MEASURES.

CANADIAN CURRENCY.

113. Money is the measure of value.

114. Currency is the money employed in trade.

115. Coins or **Specie** are species of metal of known purity and weight, stamped at the Mint, and authorized by the Government to be used as money at fixed value.

116. Bullion is uncoined gold or silver, and includes bars, gold-dust, etc.

117. Paper Money is a substitute for metallic currency. It consists of Dominion Notes issued by the Government and Bank Notes issued by Chartered Banks.

118. Canada money is the legal currency of the Dominion of Canada. It is founded on the Decimal Notation, and its denominations are, Dollars, Cents and Mills.

119. The Silver coins are the fifty-cent piece, the twenty-five-cent piece, the twenty-cent piece, the ten-cent piece and the five-cent piece.

The Copper coin is the cent.

There are no Canadian gold coins ; those of England and the United States are a legal tender.

TABLE.

120.

10 Mills	= 1 Cent	.	*ct.* or ¢.
100 Cents	= 1 Dollar	.	*dol.* or $.

UNITED STATES MONEY.

121. U. S. Money is the legal currency of the United States, and is often called Federal Money. Its denominations are Eagles, Dollars, Dimes, Cents and Mills.

122. The Gold coins are the double eagle, eagle, half-eagle, quarter-eagle, three-dollar piece, and dollar.

123. The Silver coins are the dollar, half-dollar, quarter-dollar, and dime.

The Nickel coins are the one-cent and three-cent pieces.

The Bronze coin is the one-cent piece.

TABLE.

124.	10 Mills	.	.	= 1 Cent	.	*ct.*
	10 Cents	.	.	= 1 Dime	.	*d.*
	10 Dimes or 100 Cents		= 1 Dollar	.	*dol.* or **$**
	10 Dollars	.	.	= 1 Eagle	.	E.

ENGLISH MONEY.

125. English or Sterling money is the currency of Great Britain.

126. The unit is the **Pound Sterling**, which is represented by a gold sovereign, is equal in value to $4.8665.

TABLE.

127.	4 Farthings (*qr.* or *far.*)	= 1 Penny	.	*d.*
	12 Pence	.	.	= 1 Shilling	.	*s.*
	20 Shillings .	.	= 1 Pound or Sovereign	*£.*
	21 Shillings .	.	= 1 Guinea.

128. The **gold coins** are the sovereign, and the half-sovereign.

129. The **silver coins** are the crown (= 5*s.*), the half-crown (2*s.* 6*d.*), the shilling, and the sixpenny piece.

130. The **copper coins** are the penny, half-penny, and farthing.

131. The standard purity of the gold coins of Great Britain is 22 carats fine; that is $\frac{11}{12}$ pure gold and $\frac{1}{12}$ alloy. That of the silver coins is $\frac{37}{40}$ pure silver and $\frac{3}{40}$ alloy.

TROY WEIGHT.

132. Troy Weight is used in weighing gold, silver and jewels; in philosophical experiments.

The **measuring unit** is the pound.

133.

24 Grains (*gr.*)	= 1 Pennyweight	*dwt.*
20 Pennyweights	= 1 Ounce	*oz.*
12 Ounces	= 1 Pound	*lb.*

134. The value of diamonds and other jewels is estimated by **carats.**

A carat is the weight of four grains.

APOTHECARIES WEIGHT.

135. Apothecaries Weight is used by druggists and physicians in compounding medicines, but drugs and medicines are bought and sold by avoirdupois weight.

The **measuring** unit is the pound.

The pound, ounce, and grain are the same as in troy weight.

136.

TABLE.

20 Grains	= 1 Scruple	*sc.* or Ͻ
8 Scruples	= 1 Dram	*dr.* or 3
8 Drams	= 1 Ounce	*oz.* or ℥
12 Ounces	= 1 Pound	*lb.*

APOTHECARIES' FLUID MEASURE.

137. Apothecaries' Fluid Measure is used in mixing liquid medicines.

138.

TABLE.

6? Minims, or Drops (*m.*)	= 1 Fluid Drachm	*f3*
8 Fluid Drachms	= 1 Fluid Ounce	*f℥*
20 Fluid Ounces	= 1 Pint	*O*
8 Pints	= 1 Gallon	*Cong.*

AVOIRDUPOIS WEIGHT.

139. Avoirdupois Weight is used for all the ordinary purposes of weighing.

The measuring unit is the pound.

140.

TABLE.

16 Ounces (*oz.*)	= 1 Pound	*lb.*
100 Pounds	= 1 Hundredweight	*cwt.*
2000 Pounds, or 20 *cwt.*	= 1 Ton	*T.*

141.

16 Ounces (oz.)	= 1 Pound	. .	lb
112 Pounds	= 1 Hundredweight	cwt.
2240 Pounds	= 1 Ton	. . .	T.

SPECIAL AVOIRDUPOIS WEIGHTS.

142.

100 *lbs.* Nails . = 1 Keg.
100 *lbs.* Dry Fish = 1 Quintal.
196 *lbs.* Flour . . = 1 Barrel.
200 *lbs.* Beef or Pork = 1 Barrel.

COMPARATIVE TABLE OF WEIGHTS

143.

	TROY.	AVOIRDUPOIS.	APOTHECARIES
1 Pound =	5760 Grains =	7000 Grains	= 5760 Grains
1 Ounce =	480 " =	437½ "	= 480 "
	175 Pounds =	144 Pounds =	175 Pounds

GRAIN MEASURE.

TABLE.

144.

14 *lbs.* Blue Grass Seed	= 1 Bushel.
34 *lbs.* Oats	= "
36 *lbs.* Malt	= "
40 *lbs.* Castor Beans	= "
44 *lbs.* Hemp Seed	= "
48 *lbs.* Barley	= "
48 *lbs.* Buckwheat	= "
48 *lbs.* Timothy Seed	= "
50 *lbs.* Flax Seed	= "
56 *lbs.* Indian Corn	= "
56 *lbs.* Rye	= "
60 *lbs* Wheat	= "
60 *lbs.* Beans	= "
60 *lbs.* Red Clover Seed	= "
60 *lbs.* Potatoes	= "
60 *lbs.* Turnips	= "
60 *lbs.* Carrots	= "
60 *lbs.* Parsnips	= "
60 *lbs.* Beets	= "
60 *lbs.* Onions	= "
70 *lbs.* Bituminous Coal	= "

DRY MEASURE.

145. Dry measure is used in measuring substances not liquid, as grain, fruit, salt, roots, etc.

TABLE.

146.		
2 Pints (*pt.*)	= 1 Quart	. *qt.*
4 Quarts	= 1 Gallon	. *gal.*
2 Gallons	= 1 Peck	. *pk.*
4 Pecks	= 1 Bushel	. *bush.*

147. The Imperial Standard Gallon, for liquids and all dry substances, is a measure that will contain 10 pounds avoirdupois of distilled water, weighed in air at 62° Fahrenheit, the barometer at 30 inches

148. The Imperial Gallon contains 277 274 cubic inches.

149. The Imperial Standard Bushel is equal to 8 gallons or 80 pounds of distilled water, weighed in a manner above described.

150. The Standard Bushel contains 2218.192 cubic inches.

LIQUID MEASURE.

151. Liquid Measure is used in measuring liquids; as liquors, molasses, water, etc.

TABLE.

152.				
4 Gills (*gi.*)	. .	= 1 Pint	. .	*pt.*
2 Pints	. . .	= 1 Quart	. .	*qt.*
4 Quarts	. . .	= 1 Gallon	.	*gal.*
31½ Gallons	. . .	= 1 Barrel	.	*bbl.*
2 Barrels, or 63 gallons		= 1 Hogshead	.	*hhd.*

153. The following denominations are also in use :

42 Gallons	. . .	= 1 Tierce.
2 Hogsheads, or 126 Gallons		= 1 Pipe, or Butt.
2 Pipes, or 4 Hogsheads		= 1 Tun.

NOTE.—The tierce, hogshead, pipe, butt, and tun, are the names of casks, and do not express any fixed definite measures. They are usually gauged, and have their capacities in gallons marked on them.

154. A Measure is a standard unit established by law or custom, by which extent, dimension, capacity, amount, or value is estimated.

MEASURES OF EXTENSION.

155. Measures of Extension are those used to ascertain how long a line is, or in calculating the size (extent) of a surface or solid.

A **line** has only one dimension—length.

LINEAR OR LINE MEASURE.

In measuring length, linear or line measure is used.

TABLE.

156.

12	Inches (*in.*) .	.	= 1 Foot .	*ft.*
3	Feet .	.	= 1 Yard .	*yd.*
5½	Yards, or 16½ *ft.*	= 1 Rod .	*rd.*
320	Rods .	.	= 1 Mile .	*mi.*

EQUIVALENTS.

1 Mile = **320** Rods = 1760 Yards = 5280 Feet = 63360 Inches.

SURVEYORS' MEASURE.

157. Gunter's Chain, used by land surveyors, is 4 rods, or 66 feet long, and consists of 100 links, each 7.92 inches long.

TABLE.

158.

7.92	Inches .	.	= 1 Link .	*lk.*
25	Links .	.	= 1 Rod .	*rd.*
4	Rods, or 66 Feet	= 1 Chain .	*ch.*
80	Chains .	.	= 1 Mile .	*mi.*

SQUARE MEASURE.

159. Square Measure is used in measuring surfaces : as of land, boards, painting, plastering, etc.

160. Area or Surface has **length** and **breadth** only, and is the space or surface included within any given lines.

161. A square inch, square foot or square yard, is a square, each side of which is respectively, 1 inch, 1 foot, or 1 yard in length.

TABLE.

162.

144	Square Inches (*sq. in.*)	= 1 Square Foot	*sq. ft.*
9	Square Feet . .	= 1 Square Yard	*sq. yd.*
30¼	Square Yards . .	= 1 Square Rod	*sq. rd.*
160	Square Rods . .	= 1 Acre . . .	*A.*
640	Acres	= 1 Square Mile	*sq. mi.*

Artificers estimate their work as follows :

By the square foot : glazing and stone-cutting.

By the square yard : painting, plastering, paving, ceiling, and paper-hanging.

By the square of 100 square feet : flooring, partitioning, roofing, slating, and tiling.

Bricklaying is estimated by the thousand bricks, by the square yard, and by the square of 100 square feet.

NOTES 1.—In estimating the painting of moldings, cornices, etc., the measuring-line is carried into all the moldings and cornices.

2. In estimating brick-laying by either the square yard or the square of 100 feet, the work is understood to be 12 inches or 1½ bricks thick.

3. A thousand shingles are estimated to cover 1 square, being laid 5 inches to the weather.

SURVEYORS' SQUARE MEASURE.

163. This measure is used by surveyors in computing the area of land.

TABLE.

164.

625	Square Links	= 1 Pole . . .	*P.*
16	Poles . .	= 1 Square Chain	*sq. ch.*
10	Square Chains	= 1 Acre . . .	*A.*
640	Acres . .	= 1 Square Mile	*sq. mi.*

CUBIC MEASURE.

165. Cubic Measure is used in measuring solids or volume.

166. A **solid** is that which has **length**, **breadth**, and **thickness**.

167. A **Cube** is a regular solid bounded by six equal squares called faces. Hence length, breadth, and thickness are equal to each other.

TABLE.

168.

1728 Cubic Inches (*cu. in.*) · ·	= 1 Cubic Foot - *cu. ft.*
27 Cubic Feet · · · ·	= 1 Cubic Yard - *cu. yd.*
40 Cubic Feet of Round Timber, or }	= 1 Ton · · *T.*
50 Cubic Feet of Hewn " }	
16 Cubic Feet · · · ·	= 1 Cord Foot - *cd. ft.*
8 Cord Feet, or 128 Cubic Feet	= 1 Cord of Wood *Cd.*
24¾ Cubic Feet · · · · ·	= 1 Perch of Stone } *Pch.*
	or Masonry }

NOTES.—1. A cubic yard of earth is called a load.

2. Railroad and transportation companies estimate light freight by the space it occupies in cubic feet, and heavy freight by weight.

3. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 cord ; and a cord foot is 1 foot in length of such a pile.

4. A perch of stone or of masonry is 16½ feet long, 1½ feet wide, and 1 foot high.

5. Joiners, bricklayers, and masons, make an allowance for windows, doors, etc., of one half the openings or vacant spaces. Bricklayers and masons, in estimating their work by cubic measure, make no allowance for the corners of the walls of houses, cellars, etc., but estimate their work by the *girt*, that is, the entire length of the wall on the *outside.*

MEASURE OF TIME.

169. Time is the measure of duration. The measuring unit is the day.

170. Time is naturally divided into **days** and **years.** The former are measured by the revolution of the earth on its axis ; the latter by its revolution around the sun.

TABLE.

171.

60 Seconds (*sec.*) · ·	= 1 Minute · · *min.*
60 Minutes · · ·	= 1 Hour · · *hr.*
24 Hours · · · ·	= 1 Day · · *da.*
7 Days · · · ·	= 1 Week · · *wk.*
365 Days · · · ·	= 1 Common Year *c. yr.*
366 Days · · · ·	= 1 Leap Year · *l. yr.*
12 Calendar Months · ·	= 1 Civil Year · *yr.*
100 Years · · · ·	= 1 Century · *C.*

172. The Civil Year includes both **common** and **leap** years, and is divided into 12 Calendar Months, viz. :

January (*Jan.*) . . . 31 Days.	July (*July*) . . 31 Days.
February (*Feby*) . . . 28 "	August (*Aug.*) . 31 "
In Leap Year 29 "	September (*Sept.*) 30 "
March (*Mar.*) . . . 31 "	October (*Oct.*) . 31 "
April (*Apr.*) . . . . 30 "	November (*Nov.*) 30 "
May (*May*) . . . , 31 "	December (*Dec.*) 31 "
June (*June*) . . . . 80 "	

173. The numbers of days in each month may be easily remembered from the following lines :

"Thirty days h.. ᐧ September,
April, June and November;
February, twenty-eight alone,
All the rest have thirty-one,
But in leap year, then is the time
When February has twenty-nine."

LEAP YEAR.

174. The period of time required by the sun to pass from one vernal equinox to another, called the vernal or tropical year, is exactly 365 *da.* 5 *hr.* 48 *min.* 49.7 *sec.*

175. If 365 days be reckoned as one year, the time lo~ in the calendar will be,

In 1 Year - - 5 *hr.* 48 *min.* 49.7 *sec.*
In 4 " - - 23 *hr.* 15 *min.* 18.8 *sec.*

The time thus lost in 4 years will 'ack only 44 *min.* 41.2 *sec.* of 1 entire day. Hence,

If every fourth year be reckoned as leap year, the time gained in the calendar will be,

In 4 Years - - - 44 *min.* 41.2 *sec.*
In 100 " (= 25 × 4) 18 *hr.* 37 *min.* 10 *sec.*

The time thus gained in 100 years will lack only 5 *hr.* 22 *min* 50 *sec.* of 1 day. Hence,

If every fourth year be reckoned as leap year, the centennial years excepted, the time lost in the calendar will be,

In 100 Years - - 5 *hr.* 22 *min.* 50 *sec.*
In 400 " - - 21 *hr.* 31 *min.* 20 *sec.*

The time thus lost in 400 years lacks only 2 *hr.* 28 *min.* 40 *sec.* of 1 day. Hence,

If every fourth year be reckoned as leap year, 3 of every 4 centennial years excepted, the time gained in the calendar will be,

In 400 Years - - 2 hr. 28 min. 40 sec.
In 4000 " - - 24 hr. 46 min. 40 sec.

176. The following rule for leap year will therefore render the calendar correct to within 1 day for a period of 4000 years.

RULE.

I. *Every year that is exactly divisible by 4 is a leap year, the centennial years excepted; the other years are common years.*

II. *Every centennial year that is exactly divisible by 400 is a leap year; the other centennial years are common years.*

177. Circular Measure is used principally in surveying, navigation, astronomy, and geography, for reckoning latitude and longitude, determining locations of places and of vessels, and in computing difference of time.

178. Every circle, great or small, is divided into the same number of equal parts; as quarters, called quadrants; twelfths, called signs; three hundred and sixtieths, called degrees, etc. Consequently the parts of different circles, although having the same names, are of different lengths.

The unit is the degree, which is $\frac{1}{360}$ part of the circumference of any circle.

TABLE.

179.

60 Seconds (")	= 1 Minute	.	'
60 Minutes	= 1 Degree	.	°
30 Degrees	= 1 Sign -	.	S.
12 Signs, or 360°	= 1 Circle	.	C.

MISCELLANEOUS TABLES.

COUNTING.

180.

12 Things	=	1 Dozen.
12 Dozen	=	1 Gross.
12 Gross	=	1 Great Gross.
20 Things	=	1 Score.

PAPER.

181.

24 Sheets	=	1 Quire.
20 Quires	=	1 Ream.
2 Reams	=	1 Bundle.
5 Bundles	=	1 Bale.

BOOKS.

182.

2 Leaves	=	1 Folio.
4 Leaves	=	1 Quarto, or 4to.
8 Leaves	=	1 Octavo, or 8vo.
12 Leaves	=	1 Duodecimo, or 12mo.

The terms folio, quarto, octavo, denote the number of leaves into which a sheet of paper is folded in making books.

LONGITUDE AND TIME.

STANDARD TIME.

183. During the year 1883 the principal railroads of Canada and the United States adopted what is known as the "**Standard Time System.**" This system divides Canada and the United States into four sections or time-belts, each covering 15 degrees of longitude, $7\frac{1}{2}°$ of which are east and $7\frac{1}{2}°$ are west of the governing or standard meridian, and the time throughout each belt is the same as the astronomical or local time of the governing meridian of that belt.

The governing meridians are the **75th**, the **90th**, the **105th**, and the **120th**, west of the Greenwich Observatory, London, England, and as these meridians are just 15• apart, there is a difference in time of **exactly** one hour between any one of them and the one next on the east, or the one next on the west; the standard meridian next on the east being one hour faster, and the one next on the west one hour slower. Hence, the 60° of longitude is four hours, the 75° five hours, the 90° six hours, the 105° seven hours, and the 120° eight hours slower than Greenwich time, making five different standards of time between the Atlantic and the Pacific Oceans, viz.: Intercolonial, Eastern, Central, Mountain, and Pacific.

184. Since every circle may be divided into 360 equal parts called degrees, and since the time in which the earth makes one revolution on its axis may be divided into 24

equal parts called hours, it follows that the earth on revolving on its axis passes through $\frac{1}{24}$ of 360° or 15° of longitude in one hour; through 1° of longitude in $\frac{1}{15}$ of an hour, or 4 minutes, and through 1' of longitude in $\frac{1}{60}$ of 4 minutes or 4 seconds.

185.

TABLE.

360° of Longitude	=	24 Hours or 1 Day of time	.	*da.*
15°	"	=	1 Hour of time	. . .	*hr.*
1°	"	=	4 Minutes of time	. .	*min.*
1'	"	=	4 Seconds of time	. .	*sec.*

186. To find the difference in time between two places or meridians when the difference of longitude is known.

EXAMPLE—

If the difference in longitude of two places be 7° 18', what must be their difference in time?

EXPLANATION.

SOLUTION.

7°   18'

      4

29 *min.* 12 *sec.*

Since each minute of distance equals 4 seconds of time, 18 minutes of distance will equal 72 seconds, or 1 minute 12 seconds of time.

Since each degree of distance equal 4 minutes of time, 7 degrees will equal 28 minutes, plus 1 minute, gives 29 minutes.

RULE.

Multiply the distance between the two places expressed in degrees and minutes by 4, and the result is the difference in time expressed in minutes and seconds.

NOTES.— 1. If one place be in east and the other in west longitude, the difference of longitude is found by **adding** their longitudes, and if the sum be greater than 180 degrees, it must be subtracted from 360°.

2. Since the sun appears to move from east to west, when it is exactly 12 o'clock at one place, it will be **past** 12 o'clock at all places east, and **before** 12 at all places west. Hence, if the difference of time between two places be **subtracted** from the time at the easterly place, the result will be the time at the westerly place; and if the difference be **added** to the time at the westerly place the result will be the time at the easterly place.

187. To find the difference of longitude between two places or meridians, when the difference of time is known.

EXAMPLE—

If the difference of time between two places be 28 minutes, 20 seconds, find the difference in longitude.

EXPLANATION.

SOLUTION.

$4 \overline{)\ 28\ min.\ \ 20\ sec.}$
      $7°$ $5'$

Since 4 minutes of time equal 1 degree of longitude, 28 minutes of time equal 7° of longitude.

Since 4 seconds of time equal 1 minute of longitude, 20 seconds of time equal 5' of longitude.

RULE.

Divide the difference in time between the places expressed in minutes and seconds by 4 and the quotient is the difference in longitude expressed in degrees and minutes.

TABLE OF LONGITUDES.

188.

Toronto,	.	79° 21′ 15″ W.	Belleville, .	77° 26′ 12″ W.
Kingston,	.	76° 28′ 26″ W.	Quebec, . .	71° 31′ 35″ W.
Ottawa,	.	75° 40′ 35″ W.	Berlin, . .	13° 23′ 45″ E.
Winnipeg,	.	97° 30′ 42″ W.	Philadelphia,	75° 10′ W.
Chicago,	.	87° 37′ 45″ W.	Victoria, .	123° 12′ 15″ W.
Calcutta,	.	88° 19′ 2″ E.	Hamilton, .	79° 52′ 30″ W.
Montreal,	.	75° 28′ 15″ W.	London (Eng.)	0° 5′ 38″ W.
London(Can.)		81° 15′ 5″ W.	Regina, . .	105° 2′ 26″ W.
New York,	.	74° 0′ 3″ W.	Brantford, .	80° 28′ 38″ W.
Paris, .	.	2° 20′ 22″ E.	Halifax, .	63° 36′ 42″ W.

EXERCISE 48.

Find the difference in longitude between—

1. Toronto   and London (Eng.)
2. Quebec   and Calcutta.
3. Ottawa   and Victoria.
4. Hamilton and Berlin.
5. Brantford and Winnipeg.
6. Kingston   and Paris.

Find the difference in solar time between—

7. Toronto   and Greenwich.
8. Kingston   and Winnipeg.
9. Ottawa   and Victoria.
10. Montreal   and Regina.
11. London (Can.) and London (Eng.)
12. Philadelphia   and Calcutta.

Find the difference in standard time between—

13. Quebec   and Ottawa.
14. Montreal and Victoria.
15. Toronto   and Winnipeg.
16. Kingston and Regina.
17. Montreal and Winnipeg.
18. Halifax   and Victoria.

Find the difference between the standard time and the solar time in the following cities :

19. Toronto,   Ottawa.
20. Montreal,   Victoria.
21. Winnipeg,   Halifax.

22. A navigator finds that when it is noon at his place of observation it is 16 *min.* 34 *sec.* past 10 p.m. by his chronometer, Greenwich time; what is his longitude?

23. When it is 6:40 a.m. at Halifax, what is the time at Victoria?

24. If the difference of solar time between two places is 1 *hr.* 18 *min.* 4 *sec.*, what is the difference of longitude?

25. When it is Monday 10 p m., solar time, at Toronto, what day and time is it in London (Eng.) (Greenwich time)?

REDUCTION.

189. Reduction is the process of changing the denomination of a quantity without changing its value. It is of two kinds, Descending and Ascending.

190. Reduction Descending is changing a number of one denomination to another denomination of less unit value

191. Reduction Ascending is changing a number of one denomination to another denomination of greater unit value.

192. To reduce Higher denominations to Lower.

EXAMPLE.—Reduce 26 *bbl.* 8 *gal.* 3 *qt.* to quarts.

SOLUTION.	EXPLANATION.
26 *bbl.* 8 *gal.* 3 *qt.*.	Since $31\frac{1}{2}$ *gal.* make 1 *bbl.*, there are
$31\frac{1}{2}$	$31\frac{1}{2}$ times as many gallons as barrels,
827	and $819 + 8 = 827$ gallons. Like-
4	wise, there are 4 times as many quarts
3311 *qts.* Ans.	as gallons, and $(827 \times 4) + 3 = 3311$
	quarts.

RULE.

Multiply the highest denomination by the number required of the next lower to make a unit of the higher, and to the product add the lower denomination.

Proceed in this manner with the successive denominations, till the one required is reached.

EXERCISE 49.

1. In 17 *dys.* 18 *hrs.* 27 *min.*, how many seconds?
2. Reduce 12 *mi.* 8 *rd.* 3 *yd.* 2 *ft.* to inches.
3. Reduce 243 *lb.* 8 *oz.* 6 *dwt.* to grains.
4. In 83 *c. yds.* how many cubic inches?
5. £133 6*s.* 8*d.*, how many farthings?
6. How many pence are there in £164 8*s.* 0½*d.*?
7. In 481 sovereigns how many pence?
8. In 4 *mi.* 120 *rd.* 2 *yd.* 1 *ft.* 6 *in.*, how many rods? yards? feet? inches?
9. Reduce 16 *T.* 8 *cwt.* 86 *lb.* to pounds.
10. Reduce 18 *A.* 22 *sq. rd.* 25 *sq. yd.* to square feet.
11. How many grains in 16 *lb.* Avoirdupois?
12. In 2 *mi.*, in 3½ *mi.*, in ¼ *mi.*, how many rods? yards? feet? inches?
13. In 47 guineas how many pounds and shillings?
14. In 12 *lb.*, Troy, how many drams, Apothecaries?
15. Find the cost of 2 *bl.* 3 *bun.* 1 *rm.* 4 *qr.* 21 sheets of paper, at $3.87½ a ream.

193. To reduce Lower denominations to Higher.

EXAMPLE.—Reduce 157540 minutes to weeks.

EXPLANATION.

Dividing the given number of minutes by 60, because there are $\frac{1}{60}$ as many hours as minutes, we obtain 2625 hours plus a remainder of 40 minutes.

SOLUTION.

60) 157540 *min.*
24) 2625 *hr.* + 40 *min.*
7) 109 *da.* + 9 *hr.*
15 *wk.* + 4 *da.*

15 *wk.* 4 *da.* 9 *hr.* 40 *min.* Ans.

We next divide the 2625 hours by 24, because there are $\frac{1}{24}$ as many days as hours, and we find that 2625 hours = 109 days plus a remainder of 9 hours. Lastly, we divide the 109 days by 7, because there are $\frac{1}{7}$ as many weeks as days, and we find that 109 days = 15 weeks plus a remainder of 4 days. The last quotient and the several remainders arranged in the order of the succeeding denominations form the answer.

EXERCISE 50.

Reduce—

1.	1913551	ounces	to tons.
2.	97920	grains	to lbs.
3.	43769	inches	to miles.
4.	27150	pounds	to long tons.
5.	3276	pints	to gallons.
6.	184760	seconds	to days.
7.	278648	cubic inches	to cubic yards.
8.	32459	farthings	to £
9.	478960	cubic feet	to cords.
10.	283516	sheets of paper	to reams.
11.	2468	pence	to half-crowns.
12.	23750	grains, Troy,	to lbs.
13.	15630	mills	to dollars.
14.	1800356	links	to miles.
15.	4562	pints	to bushels.
16.	20436	rods	to miles.
17.	1020300	"	to S.
18.	70	lbs. Avoirdupois	to lbs. Troy.
19.	350	oz. Troy	to oz. Avoirdupois.
20.	46030	grains, Apoth.	to lbs. Avoirdupois.

21. Find the value of 921640 *lbs.* of coal at $4.75 per long ton.

22. Find the price of 462 *bush.* 23 *lbs.* of wheat at 95c. a bushel.

23. How many bushels are there in 5160 *lbs.* of timothy seed ?

24. What is the freight on 528 bushels of corn at 32c. a cwt.?

25. What is the freight on 16 *T.* 17 *cwt.* 20 *lb.* of coal at $1.20 per ton of 2240 *lbs.* ?

26. Find the amount of the following bill of grain:

      1360 *lbs.* of oats   @   45c. a bushel.
      1216 *lbs.* of barley @   68c.   "
      5160 *lbs.* of beans   @ $1 00   "
      2130 *lbs.* of rye   @   56c.   "
      9468 *lbs.* of wheat @   98c.   "

DENOMINATE NUMBERS.

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194. The process of adding, subtracting, multiplying and dividing denominate numbers is based on the same principles that govern similar operations in simple numbers; the principal difference being that denominate numbers have an **irregular** scale of increase and decrease, while simple numbers have a uniform decimal scale.

ADDITION.

Find the sum of 3 *lb.* 7 *oz.* 10 *dwt.* 12 *gr.* ; 17 *lb.* 5 *os.* 18 *dwt.* 4 *gr.* ; and 12 *lb.* 11 *oz.* 9 *dwt.* 15 *gr.*

SOLUTION.				EXPLANATION.
lb.	*os.*	*dwt.*	*gr.*	Write the numbers of the same unit
3	7	10	12	value in the same column. Beginning
17	5	18	4	with the lowest denomination, add as in
12	11	9	15	simple numbers, and reduce to higher
34 *lb.*	0 *os.*	18 *dwt.*	7 *gr.*	denominations according to the scale.

EXERCISE 51.

Add—

(1)				(2)			(3)			
bush.	*pk.*	*qt.*	*pt.*	£	*s.*	*d.*	*hhd.*	*gal.*	*qt.*	*pt*
91	3	7	1	145	0	9½	79	62	3	1
14	3	5	1	169	17	8	3	59	2	0
17	2	3	0	175	14	7¼	61	13	2	1
68	3	1	1	166	15	8¾	159	4	1	1
9	1	6	1	1199	5	10	66	27	0	0

Add—

(4)				(5)				(6)		
gal.	qt.	pt.	gi.	mi.	A.	sq. p.	sq. yd	T.	cwt.	lb.
49	2	1	3	50	75	30	15	55	16	17
71	3	0	2	791	11	15	11	14	11	5
6	1	i	1	87	315	31	16	63	19	24
16	3	1	3	75	473	29	30	919	6	20
68	3	1	2	15	29	18	26	89	1	4

7. Add 236 *lb.* 4 *oz.* 15 *dwt.*, 83 *lb.* 11 *oz.* 21 *gr.*, 46 *lb.* 16 *dwt.*, 105 *lb.* 9 *oz.* 11 *gr.*

8. Add 7 *T.* 14 *cwt.* 25 *lb.*, 14 *T.* 9 *cwt.* 16 *lb.* 8 *oz.*, 36 *cwt.* 17 *lb.*, 14 *T.* 12 *cwt.*, and 5 *cwt.* 10 *lb.* 14 *oz.*

9. Find the sum of 12 *wk.* 3 *da.* 5 *hr.* 20 *min.* 42 *sec.* 4 *da.* 12 *hr.* 30 *min.*, 3 *wk.* 1 *da.* 10 *hr.* 40 *min.*, and 16 *hr.* 36 *min.* 30 *sec.*

10. Add 6 *cd.* 5 *cd. ft.*, 3 *cd.* 6 *cd. ft.* 9 *cu. ft.*, 4 *cd. ft.* 14 *cu. ft.*, and 5 *cd.* 24 *cu. ft.*

11. Off of one field of wheat were raised 37 *bush.* 1 *pk.* 3 *qt.*; of a second field, 41 *bush.* 2 *pk.* 5 *qt.*; of a third, 35 *bush.* 1 *pk.* 3 *qt.*; of a fourth, 43 *bush.* 3 *pk.* 1 *qt.* How much was the whole?

12. A grocer received an invoice of 7 *hhd.* of sugar; the first weighed 11 *cwt.* 15 *lb.*; the second, 12 *cwt.* 15 *lb.*; the third, 9 *cwt.* 16 *lb.*; the fourth, 12 *cwt.*; the fifth, 11 *cwt.* 24 *lb.*; the sixth, 9 *cwt.* 24 *lb.*; the seventh, 13 *cwt.* How much did the seven hogsheads contain?

13. A person has 5 pieces of ground; the first contains 16 *A.* 3 rd.; the second, 17 *A.* 1 *sq* p. 45 *sq. ft.*; the third, 11 *A.* 14 *sq. p.* 62 *sq. ft.*; the fourth, 2 *A.* 120 *sq. ft.*; and the fifth, 41 *A.* 7 *sq. p.* What is the amount of the whole?

14. A person owes several sums of money; to one 17*s.* 6*d.*; to another, £3 5*s.* 8*d.*; to another, £25 11*s.* 10¹⁄₂*d.*; to another, £12 8*d.*; to another, 16*s.* 3¾*d.*; to another, £126 4*s.*; to another, £31 6*s.* 10¼*d.*; to another, £50 4*s.* 1¾*d.* What is the whole amount?

15. A person travelling goes 26 *mi.* 12 *rd.*, the first day; 28 *mi.* 5 *fur.* 9 *rd.* 9 *ft.*, the second day; 31 *mi.* 15 *rd.* 14 *ft.*, the third day; 26 *mi.* 12 *ft.*, the fourth day; and 33 *mi.* 16 *rd.* 11 *ft.*, the fifth day. How far does he go during the five days?

16. A jeweller receives on one day 11 *lb.* 6 *oz.* of gold; on another day, 10 *lb* 5 *oz.* 20 *gr.*; on another, 6 *oz.* 3 *dwt.* 16 *gr.*; on another, 5 *lb.* 17 *dwt.* 1 *gr.*; on another, 16 *lb.* 4 *oz.* 15 *dwt.* 15 *gr.* How much does he receive in all?

SUBTRACTION.

EXAMPLE.—Subtract 12 *lb*. 9 *oz*. 11 *dwt*. 15 *gr*. from 27 *lb*. 5 *oz*. 16 *dwt*. 12 *gr*.

EXPLANATION.

Write the numbers as for simple subtraction; take each subtrahend term from its corresponding minuend term. In case any subtrahend term be greater than the minuend term, borrow 1 as in simple subtraction, and reduce it to the denomination required, etc.

SOLUTION.

lb.	os.	dwt.	gr.
27	5	16	12
12	9	11	15
14 *lb*.	8 *os*.	4 *dwt*.	21 *gr*.

EXERCISE 52.

(1)			(2)				(3)			
£	s.	d.	lb.	oz.	dwt.	gr.	cwt.	lb.	os.	dr
186	4	0¼	114	3	16	12	58	16	2	5
98	11	2¾	91	4	12	19	27	20	1	6

(4)			(5)					(6)			
A.	sq.p.	sq.yd.	lb.	℥	ℨ	℈	gr.	rd.	yd.	ft.	in.
75	14	11	68	1	7	2	12	16	5	1	11
73	10	16	15	0	7	2	15	14	5	2	9

7. A person owes £78 3*s*. 2¼*d*.; he pays £17 17*s*. 1¾*d*.; how much does he still owe?

8. A. owes B. for 2 invoices of merchandise; one worth £17 16*s*. 8½*d*., the other £11 2*s*. 9*d*.; he pays £25 16*s*. 4*d*.; how much does he still owe?

9. A farmer has a farm consisting of 2000 acres. He gave his eldest son 109 *A*. 3 *rd*. 20 *sq.p*.; to his second son. 48 *A* 1 *rd*.; the remainder he gave to his third son. What was the remainder?

10. How long is it from June 21st, 1886, to December 14th, 1888?

11. The latitude of Hamilton is 43° 12′ 40″, of Quebec, 46° 50′ 10″; how many degrees is Quebec north of Hamilton?

12. The latitude of Brantford is 42° 21′ 22″; how far is Brantford from the North Pole?

13. A merchant bought 3 pieces of cloth; the first measured 47 *yd*. 3 *qr*.; the second, 43 *yd*.; the third, 41 *yd*. 3 *qr*.; when he came to examine it, he found 13 *yd*. worthless; how much good cloth was there?

MULTIPLICATION.

EXAMPLE.—Each of seven bars of silver weighs 17 *lb.* 5 *os.* 18 *dwt.* 16 *gr.* Find the total weight?

SOLUTION.

lb.	os.	dwt.	gr.
17	5	13	16
			7

122 *lb.* 8 *oz.* 15 *dwt.* 16 *gr.*

EXPLANATION.

Write the multiplier under the lowest denomination of the multiplicand, and multiply as in simple numbers, thus:

16 *gr.* × 7 = 112 *gr.* = 4 *dwt.* 16 *gr.*   Put down 16 under *gr.*   Carry 4 to *dwt.*

13 *dwt.* × 7 + (4 *dwt.* carried) = 95 *dwt.* = 4 *oz.* 15 *dwt.*   Put down 15 under *dwt.*   Carry 4 to *oz.*

5 *os.* × 7 + (4 *oz.* carried) = 39 *os.* = 3 *lb.* 3 *os.*   Put down 3 under *os.*   Carry 3 to *lb.*

17 *lb.* × 7 + (3 *lb.* carried) = 122 *lb.*   Put down 122 under *lb.*

EXERCISE 53.

1. Multiply   88 *lb.* 6 *os.* 17 *dwt.* by 17.
2. Multiply   19 *T.* 13 *cwt.* 18 *lb.* by 19.
3. Multiply   3 *lb.* 4 ℥ 2 ʒ 1 Ɵ 17 *gr.* by 11.
4. Multiply   13 *yd.* 1 *ft.* 11 *in.* by 21.
5. Multiply   17 *mi.* 2 *rd.* 16 *ft.* by 23.
6. Multiply   15 *rd.* 2 *yd.* 1 *ft.* by 29.
7. Multiply 144 *A.* 17 *sq. p.* 19 *sq. yd.* by 5.
8. Multiply   17 *C.* 59 *cu. ft.* 718 *cu. in.* by 13.
9. Multiply   73 *hhd.* 61 *gal.* 3 *qt.* 1 *pt.* by 26.
10. If one cord of wood cost £1 16*s.* 9½*d.*, what will 25 cords cost?
11. If you can buy 3 *bu.* 3 *pk.* 3 *qt.* for $1, how many bushels can be bought for $79?
12. Bought 17 yards of lace, at £3 17*s.* 1*d.* per yard; 14 yards of crape, at £2 10*s.* per yard. What is the value of both purchases?
13. If you can exchange one acre of wheat for 17 *A.* 7 *sq. p.* of pasture, how many acres of pasture can you get for 41 acres of wheat?
14. Bought 16 pieces of lace, each containing 62 yards, at £1 11*s.* 2*d.* per yard, and sold 7 pieces for £1 15*s.* per yard, and the rest at £1 13*s.* 10*d.* per yard; how much was gained?

DIVISION.

Example.—If 122 *lb.* 3 *oz.* 15 *dwt.* 16 *gr.* of silver be made into 7 bars of equal weight, what will be the weight of one bar?

SOLUTION.   EXPLANATION.

	lb.	*os.*	*dwt.*	*gr.*
7)	122	3	15	16
	17 *lb.*	5 *oz.*	13 *dwt.*	16 *gr.*

Write the dividend and divisor as in short division, and divide as in simple numbers, thus:

⅐ of 122 *lb.* = 17 *lb.* and an undivided remainder of 3 *lb.*

Reduce this remainder to *oz.*; add the 3 *oz.* of dividend = 39 *oz.*

⅐ of 39 *oz.* = 5 *oz.* and an undivided remainder of 4 *oz.*

Reduce this remainder to *dwt.*; add the 15 *dwt.* of dividend = 95 *dwt.*

⅐ of 95 *dwt.* = 13 *dwt.* and an undivided remainder of 4 *dwt.*

Reduce this remainder to *gr.*; add the 16 *gr.* of dividend = 112 *gr.*

⅐ of 112 *gr.* = 16 *gr.*

EXERCISE 54.

1. Divide £91 12*s.* 6*d.* by 6.

2. Divide 386 *lb.* 0 *oz.* 16 *dwt.* 23 *gr.* by 29.

3. Divide 9 *T.* 16 *cwt.* 16 *lb.* 3 *oz.* 4 *dr.* by 17.

4. Divide 61 ℔ 10 ℥ 43 ℨ 0 ℈ 16 *gr.* by 36.

5. Divide 78 *mi.* 14 *r.* by 31.

6. Divide 4 *yd.* 1 *ft.* 11 *in.* by 15.

7. Divide 19861 *sq. mi.* 179 *A.* 20 *sq. p.* 11 *sq. yd.* by 61.

8. Divide 738 *cu. yd.* 20 *cu. ft.* 1100 *cu. in.* by 399.

9. Divide 20 *hhd.* 16 *gal.* 3 *qt.* 1 *gi.* by 147.

10. Divide 175 *bush.* 3 *pk.* 1 *qt.* 1 *pt.* by 67.

11. Divide 1 circle by 128.

12. Divide 365 *da.* 6 *hr.* by 240.

13. If 16 bushels of oysters cost £75 17*s.* 4*d.*, what will one bushel cost?

14. If one yard cost 2*s.* 6*d.*, how many yards can be bought for £180?

15. If you can buy 15 square rods of land for £1, for how many pounds can you buy one acre?

16. Divide a square mile into 15 equal parts.

17. A man travelled 1249 *mi.* 36 *rd.* in 61 days; how far did he travel in a day?

18. A cartman carried 117 *cd* 110 *cu. ft.* in 100 loads; how much did he average a load?

DENOMINATE FRACTIONS.

195. A Denom'nate **Fraction** is a fraction whose integral unit is a denominate number.

NOTE.—The principles, analyses, and rules of denominate fractions are essentially the same as those of denominate integers; therefore, no special rules are necessary for their reduction. A sufficient number of examples are given to fully explain the different cases that may arise.

196. To reduce a denominate fraction or decimal to integers of lower denominations.

EXAMPLE.—Reduce $£\frac{7}{16}$ (.4375) to integers of lower denominations.

SOLUTION.

$£\frac{7}{16} \times \frac{20}{1} \times \frac{12}{1} = 105d.$
$105d. = 8s.\ 9d.$
     or

$£\frac{7}{16} \times \frac{20}{1} = \frac{35}{4} = 8\frac{3}{4}s.$

$\frac{3}{4}s. \times \frac{12}{1} = 9d.$

$\therefore £\frac{7}{16} = 8s.\ 9d.$
     or   7
         20
   16) 140 (**8s.**
      128
       12
       12
   16) 144 (**9d.**
      144

$£.4375 \times 20 \times 12 = 105d.$
$105d. = 8s.\ 9d.$
     or

£.4375
   20
s. 8.7500
   12
d. 9.0000
$\therefore £.4375 = 8s.\ 9d.$

197. To change a fraction or decimal of one denomination to a higher or lower denomination.

EXAMPLE 1.—Reduce $\frac{1}{40}$ (.025) of a yard to the fraction of an inch.

OPERATION.

$\frac{1}{40}yd. \times \frac{3}{1} \times \frac{12}{1} = \frac{9}{10}in.$ Ans.

OPERATION.

$.025\ yd. \times 3 \times 12 = 9 = \frac{9}{10}in.$ Ans.

EXAMPLE 2.—Change $\frac{9}{10}$ (.9) of an inch to the fraction of a yard.

OPERATION.

$\frac{9}{10}in. \times \frac{1}{12} \times \frac{1}{3} = \frac{1}{40}yd.$ Ans.

OPERATION.

12) .9 in.
3) .075
   .025 or $\frac{1}{40}yd.$

198. To change one denominate number to the fraction or to the decimal of another.

EXAMPLE 1.—Reduce 3 *qt* 1 *pt.* to (1) the fraction of a gallon (2). to the decimal of a gallon.

SOLUTION.

TO A COMMON FRACTION.

$$3\,qt.\ 1\,pt. = 7\,pt.$$
$$1\,gal. = 8\,pt.$$
$$\therefore 7\,pt. = \tfrac{7}{8}\text{ of a gal.}$$

SOLUTION.

TO A DECIMAL.

$$2\ )\ 1.0\ pt.$$
$$4\ )\ 3\,5\ qt.$$
$$= .875\ gal.\quad \textbf{Ans.}$$

EXAMPLE 2.—Reduce 15*s.* 6*d.* 3*far.* **(1)** to the fraction of a **£ (2)**, to the decimal of a **£.**

SOLUTION.

TO A COMMON FRACTION.

$$15s.\ 6\tfrac{3}{4}d. = 747\,far.$$
$$£1 = 960\,far.$$
$$\therefore 747\,far. = £\tfrac{747}{960}$$

SOLUTION.

TO A DECIMAL.

$$4\ )\ 3\,far.$$
$$12\ )\ 6.75\,d.$$
$$20\ )\ 15.5625\,s.$$
$$.778125\text{ of a £.}$$

EXAMPLE 3.—Reduce £1 3*s.* 4*d.* (1) to the fraction (2), to the decimal of £1 17*s.* 4*d.*

SOLUTION

$$£1\quad 3s.\ 4d. = 380d.$$
$$£1\quad 17s.\ 4d. = 448d.$$
$$\therefore £1\quad 3s.\ 4d. = \tfrac{380}{448}\text{ of £1 17s. 4d.}$$
$$= \tfrac{5}{8}\text{ of }\quad\text{``}$$
$$= .625\text{ of }\quad\text{``}$$

EXERCISE 55.

1. Reduce $\frac{1}{2112}$ of a mile to the fraction of a yard.
2. What is the value of .8525 of a £ ?
3. Reduce $\frac{5}{7}$ of a pennyweight to the fraction of a pound, **Troy.**
4. What part of 3 weeks is 4 *da.* 16 *hr.* 30 *min.* ?
5. What part of 1¼ bushels is .45 of a peck ?
6. Reduce .425 of a foot to the fraction of a mile.
7. Reduce £617 1*s.* 1*d.* to the decimal of a £.
8. What is the value of $\frac{1}{8}$ of a mile ?
9. What part of an inch is $\frac{5}{78}$ of a yard ?
10. What part of a *lb.* Troy is .75 of a grain ?
11. Reduce 3 *bush.* 1 *pk.* 3 *qt.* to the decimal of a bushel.
12. Reduce 2.333⅓ years to integers of lower denominations
13. Reduce £14 15*s.* 9*d.* to the decimal of a £.
14. Reduce $\frac{7}{8}$ of a hundredweight to the fraction of an ounce.
15. Reduce $\frac{4}{5}$ of a mile to the fraction of $\frac{3}{4}$ of a rod.
16. Reduce £2 10*s.* 0¼*d.* to the decimal of £2 17*s.* 2*d.*

ALIQUOT PARTS.

199. An aliquot part of a number or quantity is an exact divisor of that number or quantity. Thus 5 is an aliquot part of 20; $33\frac{1}{3}$ of 100.

Many business calculations may be shortened by combining the values of convenient aliquot parts.

EXAMPLE 1.—What will 576 yards of cloth cost at $1.87\frac{1}{2}$ a yard?

SOLUTION.

At $1.00 per yard,			the price would be $5.76
" .50 "	$\frac{1}{2}$ of $1.00	"	" 2.88
" .25 "	$\frac{1}{2}$ of .50	"	" 1.44
" .12$\frac{1}{2}$ "	$\frac{1}{2}$ of .25	"	" .72
" 1.87\frac{1}{2}$ "		"	" $10.80. Ans.

EXAMPLE 2.—What will 7 *bush.* 3 *pk.* 6 *qt.* of wheat cost at $1.60 a bushel?

SOLUTION.

7 bushels		@ $1.60	=	$11.20
2 pecks	= $\frac{1}{2}$ bushel		=	.80
1 peck	= $\frac{1}{4}$ bushel		=	.40
4 quarts	= $\frac{1}{2}$ peck		=	.20
2 quarts	= $\frac{1}{2}$ of $\frac{1}{2}$ peck		=	.10
7 *bush.* 3 *pk.* 6 *qt.*				$12.70 Ans.

EXAMPLE 3.—Find the cost of 972 *oz.* of gold dust at £3 14s. 8$\frac{1}{4}$d. per *oz.*

SOLUTION.

At £3		per *oz.* the price would be	£2916	0s.	0d.
" 10s.	= $\frac{1}{6}$ of £3	" " "	486	0	0
" 3s. 4d.	= $\frac{1}{3}$ of 10s.	" " "	162	0	0
" 10d.	= $\frac{1}{4}$ of 3s. 4d.	" " "	40	10	0
" 5d.	= $\frac{1}{2}$ of 10d.	" " "	20	5	0
" 1$\frac{1}{4}$d.	= $\frac{1}{4}$ of 5d.	" " "	5	1	3
" £3 14s. 8$\frac{1}{4}$d.	" " "	£3629	16s.	3d. Ans.

EXAMPLE 4.—What will 34 *bush.* 3 *pk.* 4 *qt.* of clover-seed cost at $4.50 per bushel?

SOLUTION.

½ of $4.50 =	$153 00	= Cost of 34 *bush.*	
½ of 2.25 =	2.25	= "	2 *pk*
½ of 1.12½ =	1.12½	= "	1 *pt*
	56¼	= "	4 *qt.*
	$156.93¾	"	34 *bush.* 3 *pk.* 4 *qt.*

EXERCISE 56.

What is the cost of—

1. 75 *lbs.* of coffee at 33⅓c. a *lb.*
2. 120 *lbs.* of sugar at 12½c. a *lb.* ?
3. 84 yards of carpet at $1.33⅓ a yard?
4. 144 bushels of wheat at $1.16⅔ a bushel ?
5. 5386 boxes of oranges at 15*s.* 9½*d.* a box ?
6. 886 pieces of silk at £9 6*s.* 7¼*d.* a piece ?
7. 26 *T.* 18 *cwt.* 47 *lbs.* of copper at $245.20 a ton ?
8. 615 *A.* 152 *pr.* of land at $164.80 an acre ?
9. 43 *bush.* 2 *pk.* 7 *qt.* of corn at 58c. a bushel ?
10. 12 *lb.* 10 *oz.* 14 *dwt.* of gold at £63 12*s.* a pound ?
11. 270 *yds.* silk at £1 5*s.* 6*d.* per *yd* ?
12. 326 *bbls.* flour at $7.87½ per *bbl.* ?
13. 15 *A.* 3 *r.* 20 *rd.* land at $60 per acre ?
14. 12 *T.* 17 *cwt.* freight at $4 per ton ?
15. 7 *cwt.* 3 *qr.* 12 *lb.* at $61.50 per long ton r
16. 27 *yds.* of cloth at 3*s.* 9¾*d.* per *yd.* ?
17. 84 *cu. yds.* 24 *cu. ft* at $2.50 per *cu yd.* ?
18. 13 *gal.* 1 *qt.* 1 *pt.* wine at $3 per *gal.* ?
19. 17 *cwt.* 2 *qr.* at $7.50 per ton ?
20. ¾ *doz.* elbows at $2.75 per *doz.* ?

MISCELLANEOUS PROBLEMS.

EXERCISE 57.

I.

1. Find the total distance around a rectangular field at 1,728 feet long and 1,683 feet wide.

2. A manufacturer sells 236½ barrels of flour on Monday, 3,124 barrels on Tuesday, 46¾ barrels on Wednesday, 3632 barrels on Thursday, 25.6 barrels on Friday, and 33¼ barrels on Saturday How many barrels did he sell during the week?

3. A certain building contains 74 windows, each window containing 8 panes of glass. Find the cost of the glass at 14 cents per pane.

4 How many pounds of wire will it require to fence a field 304 feet square, the fence being 6 wires high, if 10 feet of the wire weigh one pound?

5. A man deposited in a bank $8,752; he drew out at one time $4,234, at another, $1,700, at another, $962, at another, $49. How much had he remaining in the bank?

6 A man invests in trade at one time $680, at another time, $820, at a third time, $1,580, and on a fourth occasion, $420. How much must he add to the sum of these that the amount may be $5,000?

7. A merchant bought 240 barrels of flour for $1,920, and sold it at $10.50 a barrel. What did he gain?

8. A farmer exchanged 754 bushels of wheat, at $1.25 a bushel, for 78 barrels of flour, at $2 per barrel, and received the balance in money. How much money did he receive?

9. A man bought 15 acres of land at $38 an acre, and 76 acres at $47 an acre, and sold the whole at $45 an acre. Did he gain or lose, and how much?

10. The cost of the Atlantic Telegraph Cable, as originally made, was as follows: 2,500 miles at $485 per mile, 10 miles deep sea cable at $1,450 per mile, and 25 miles shore ends at $1,250 per mile. What was its total cost?

II.

1. How many bags will be required to hold 108 bushels, if 4 bags hold 9 bushels?

2. If 5 barrels of flour cost $60, how many cords of wood at $4 a cord will pay for 3 barrels of flour?

3. If 12 yards of cloth cost $60, for how much a yard must it be sold to gain $20?

4. A man received $50 for 5 barrels of pears, and paid all but $14 for 4 chairs. What did each chair cost?

5. If a man received 16 pounds of sugar in exchange for 20 pounds of cheese at 8 cents a pound What is the price of the sugar a pound?

6. If a woman pay 60 cents for some lemons at the rate of 10 cents for 6, and sell them at the rate of 9 for 20 cents, how many cents will she gain?

7. What is the smallest sum of money with which I can purchase either sheep at $3.50 a head, calves at $10.50, cows at $35, oxen at $70, or horses at $105?

8. A coal dealer sold 5 tons of coal for $57.50, which was $\frac{3}{8}$ as much as he received for all he had left at 7.66\frac{2}{3}$ per ton. How many tons did he sell?

9. How many times is the G. C. M. of $4\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{2}{3}$ and $7\frac{7}{11}$ contained in the L. C. M. of the same numbers?

10. If $3\frac{3}{11}$ tons of coal will last as long as $4\frac{1}{2}$ cords of wood, how many tons of coal will last as long as $13\frac{7}{11}$ cords of wood?

III.

1. What will 45 *bush.* 3 *pk.* 1 *qt.* of wheat cost at $1.75 a bushel?

2. Wishing to travel in Great Britain, I exchanged $1,500 for English money. How many pounds did I receive?

3. What will 25 *T.* 6 *cwt.* 94 *lbs.* of coal cost at $6.40 a long ton?

4. From a pile of wood containing 960 *cu. ft.*, was sold at one time 3$\frac{1}{4}$ *cd.*, at another, 2$\frac{3}{5}$ *cd.* What was the remainder worth at 4\frac{1}{4}$ a cord?

5. How many acres of land can be bought for $25,000, if a square foot cost 25..?

6. A carriage wheel 12 *ft.* 3 *in.* in circumference will make how many revolutions in a distance of 65.5 miles?

7. If 5$\frac{1}{2}$ *lbs.* of coffee cost 1\frac{7}{18}$, what will 27$\frac{1}{3}$ *lbs.* cost?

8. How many times can a vessel containing $\frac{1}{4}$ of a gallon be filled from $\frac{2}{3}$ of a barrel containing 31$\frac{1}{2}$ gallons?

9. $\frac{A}{T}$ of a certain number exceeds $\frac{4}{9}$ of the same number by 156. What is the number?

10. A certain number multiplied by 2.5 and divided by 5.2 produces 1. What is the number?

IV.

1. Divide the sum of .075 and .0075 by the difference of 7.5 and .75.

2. Find the least common multiple of $\frac{2}{3}$, $\frac{7}{10}$, $\frac{18}{18}$ and $\frac{A}{18}$.

3. Divide $2,000 between two persons so that one should have $\frac{7}{9}$ as much as the other.

4. Bought a cord of wood for $4.625, a cheese for 7.56\frac{1}{2}$, and $14\frac{A}{11}$ *lbs.* of butter at 25c. per *lb.* What was the cost of the whole?

5. At 1\frac{1}{4}$ a bushel, how many bushels of wheat can be bought for 37.68\frac{3}{4}$?

6. If a *lb.* of tea be worth $.62$\frac{1}{2}$, what is .8 of a *lb.* worth?

7. What is the value of 720 pounds of hay at $12.75 a ton, and 912 pounds of shorts at 15\frac{1}{2}$ a ton?

\ 8. Bought 12 *yds.* cloth at 37\frac{1}{2}$ per *yd.*, and agreed to pay $\frac{1}{2}$ the cost in butter, at $.16$\frac{3}{4}$ per *lb.*; $\frac{1}{4}$ in money and the remainder in eggs, at $.12$\frac{1}{2}$ a dozen. How many pounds of butter and dozens of eggs were required?

\ 9. What is the value of 1,046 pounds of beef at 4\frac{5}{8}$ per *cwt.*?

10. How many pairs of pants can be made from 48.6 *yds.* of cloth, allowing 1.8 *yd.* per pair?

V.

1. Sold 125 equal loads of wood, measuring 115 *cd.* 3 *cd. ft.* 7 *cu. ft.*, for $492.50. What is the quantity per load, and price per cord?

\ 2. If I buy 120 gallons of rum for $75, how much water must be added to it that I may sell it at 60 cents a gallon, and gain $15 on the sale of it?

3. What part of a short ton is $\frac{5}{9}$ of a long ton?

\ 4. I have a field 96 rods long and 50 rods wide. How much will it cost to build a fence around it at $.12$\frac{1}{2}$ per foot?

5. A. owns $\frac{5}{13}$ of a field, and B. the remainder; $\frac{3}{4}$ of the difference between their shares is 5 *A.* 3 *rds.* 16$\frac{1}{2}$ *pr.* What is B.'s share?

6. What part of a cord of wood is a load 7$\frac{1}{2}$ *ft.* long, 2$\frac{1}{4}$ *ft.* high, 3$\frac{1}{3}$ *ft.* wide?

7. Reduce $\frac{31}{35}$ of a long ton to the decimal of a short ton.

\ 8. A farmer sold 8 loads of potatoes, averaging 27 *bush.* each, for $.45 a bushel. How much did he receive?

9. A merchant in selling groceries sells $14\frac{9}{15}$ oz. for a *lb.*; how much does he cheat a customer who buys of him to the amount of $38.40?

10. If the longitude of Belleville is 77° 26′ 12″ W., what will be the time in that place when it is 3 *hr.* 35 *min.* a.m. in London, Eng.?

VI.

1. How many bricks, each containing $121\frac{1}{2}$ cubic inches, can be packed in 3 cubic yards?

2. Telegraph posts are placed 66 yards apart; a train passes one every 3″. Find at what rate per hour the train is travelling?

3. What is the cost per hour of lighting a room with 3 burners, each consuming 5 cubic inches of gas per second, the price of the gas being $2 for 1000 cubic feet?

4. A man bought 35 bushels of barley, and sold the whole for $30. He made $5.50 in the trade. What did he give per bushel?

5. A tailor has $67\frac{3}{4}$ yards of cloth, from which he wishes to cut an equal number of coats, pants and vests. What number of each can he cut if they contain respectively $3\frac{7}{8}$, $2\frac{3}{4}$, $1\frac{1}{2}$ yards?

6. Bought 12 *T.* 3 *cwt.* 70 *lbs.* of sugar at $8.25 per *cwt.* What was the cost?

7. How many bales of cotton, of 400 *lb.* each, at 36 cents per *lb.*, are equal in value to 18 *hhd.* of sugar, of 1,500 *lb.* each, at 8 cents per *lb.*?

8. What part of 5 *da.* 28 *hr.* 58 *min.* is 4 *da.* 6 *hr.* 50 *min.*?

9. Thirty-two men agree to build 14 *mi.* 234 *rd.* 6 *ft.* of road. When the work is $\frac{1}{3}$ done, they employ 8 more men. What distance does each man construct?

10. I wish to put 111 *bush.* 2 *pk.* 4 *qt.* of grain into bags that should contain 2 *bush.* 1 *pk.* 4 *qt.* each. How many bags will be required?

VII.

1. If a man travel at the rate of a minute of distance in 10 minutes of time, how long will he be in travelling around the world?

2. St. Thomas is 81° 15′, and Halifax 63° 36′ West Longitude. When it is 12 o'clock noon at St. Thomas, what is the time at Halifax?

3. The ice on a pond, whose area is $\frac{1}{2}$ an acre, is 10 inches thick. How many tons of ice may be taken from the pond, supposing a cubic foot of ice to weigh 56 pounds?

MICROCOPY RESOLUTION TEST CHART
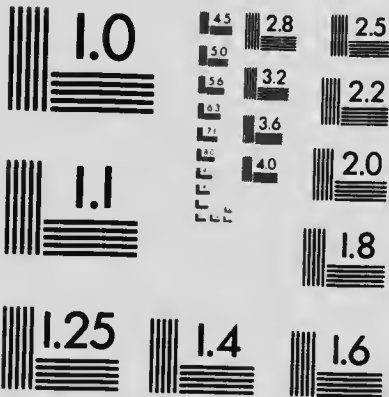
(ANSI and ISO TEST CHART No. 2)

APPLIED IMAGE Inc

4. If the regular fare on a railway is 3 cents a mile, but $\frac{1}{4}$ is allowed off full fare when return tickets are bought, find the distance between two places if a return ticket costs $1.80.

5. 450 leaves of a certain kind of paper make an inch of thickness. Find the thickness of a book 6 inches by 4 inches, in which 10 square yards of the paper are used.

6. It costs $23.10 to fence a square field at 3½ cents per yard. How many acres in the field?

7. From 10 acres take 8 A. 159 pr. 30 yd. 6 ft. 108 in.

8. What is the result, when 500 is divided by .25, the quotient by .025, the second quotient by 50?

9. Express 3.74976 minutes as the decimal of a week.

10. What is the least number from which 1,224 and 1,656 may each be taken an exact number of times?

VIII.

1. If water in freezing expands $\frac{1}{10}$, find the weight of a cubic foot of ice, a cubic foot of water weighing 1,000 ounces.

2. Find the difference between 9 A. 159 pr. 30 yd. 2 ft. 36 in. and 10 A. ?

3. Divide $760 among A. B. and C., so that B. may have $160 more than A., but $50 less than C.

4. How far may a person ride in a carriage going at the rate of 8 miles per hour, so that if he walked back at the rate of 3 miles per hour he may be gone 5½ hours?

5. What will it cost to dig a ditch on each side of a road 4 miles 80 chains long at 40 cents a rod?

6. Walking 4¼ miles an hour, I start after a friend whose pace is 3 miles an hour; how long shall I be in overtaking him?

7. How many square rods are there in 100 square chains?

8. A man owns .1875 of a mine; he sells .17 of his share. What fractional part has he left?

9. Reduce $\frac{2}{3}$ of an hour to the decimal of $\frac{3}{4}$ of 48 minutes.

10. What will it cost to fence a square 10 acre field at 80 cents a rod?

IX.

1. At $2.40 per rod, what will it cost to fence a piece of land 84.5 rods long by 24.75 rods wide?

2. A ship with its cargo is worth $340,000, $\frac{2}{3}$ of the value of the cargo is worth $\frac{3}{4}$ the value of the ship. Find the value of each?

3. Divide 6 *dy.* 17 *hr.* 11 *min.* by $\frac{5}{11}$.

4. How many reams of paper will be reqnired to supply 7,500 subscribers with a weekly newspaper for a year, allowing a sheet for one copy?

5. Telegraph poles are placed 8 rods apart, and a train passes one every $4\frac{1}{2}$ seconds. How many miles an hour is the train travelling?

6. A man charged me 15 cents for a scuttle of coal, when coal was selling at $7 per ton. How many pounds ought the scuttle to hold?

7. Divide $82.60 among 27 men and 37 boys, so that each man may have three times as much as each boy.

8. By selling my cloth at $1.26 a yard, I gain 11 cents more than I lose by selling it at $1.05 a yard. What would I gain by selling 800 yards at $1.40 a yard?

9. If $\frac{3}{8}$ of an estate be worth £220, find the value of $\frac{3}{11}$ of the estate.

10. If a railway train goes 45 miles an honr, how many yards will it go in a second?

X.

1. How many times will the seconds hand of a watch go around in 12 *wk.* 2 *hr.* 15 *min.*?

2. Divide $600 between two persons, so that one shall have $\frac{7}{8}$ as much as the other.

3. A regiment marching $3\frac{1}{2}$ miles an hour makes 110 steps in a minnte. What is the length of the step?

4. I bought 20 pounds of opium by Avoirdupois weight, at 55 cents an ounce, and sold by Troy weight at 60 cents an ounce. Did I gain or lose, and how mnch?

5. The G. C. M. of two nnmbers is 12; their L. C. M. is 72; one of the numbers is 24; find the other?

6. Divide $345 among A. B. and C., so that B. will receive $5 for A.'s $4, while C. receives $6 for A.'s $5.

7. Which is the greater .0025 of a mile or ·79 of a rod?

8. How long will it take a train 20 rods long, and going at the rate of 15 miles an honr, to cross a bridge 15 rods long?

9. When an onnce of gold is worth $19.45, what is the value of .04 of a pound?

10. A coal dealer bought a qnantity of coal at $6 a ton, and sold it for 48 cents a hundredweight, gaining thereby $43 20. How many tons did he buy?

PERCENTAGE.

200. Percentage is the method of calculating by hundredths, or it is the term applied to such computations as involve the number 100 as the basis or unit of measure.

201. Per cent. is an abbreviation of the Latin phrase *per centum*, and signifies on or by the hundred. Thus 4 per cent. means 4 of every hundred and may signify 4 cents of every 100 cents, $4 of every $100, 4 *lbs.* of every 100 *lbs.*, etc.

202. The sign % stands for the phrase per cent. ; thus 8 per cent. is written 8%.

203. To express any per cent. as a decimal or as a common fraction.

Since any per cent. is some number of hundredths, it is properly expressed by a decimal fraction, or by a common fraction.

Since 6 % means six-hundredths, therefore 6 % = .06 = $\frac{6}{100}$.

TABLE.

SYMBOLS.		DECIMALS.		COMMON FRACTIONS.
1%	=	.01	=	$\frac{1}{100} = \frac{1}{100}$
2%	=	.02	=	$\frac{2}{100} = \frac{1}{50}$
4%	=	.04	=	$\frac{4}{100} = \frac{1}{25}$
5%	=	.05	=	$\frac{5}{100} = \frac{1}{20}$
10%	=	.10	=	$\frac{10}{100} = \frac{1}{10}$
25%	=	.25	=	$\frac{25}{100} = \frac{1}{4}$
40%	=	.40	=	$\frac{40}{100} = \frac{2}{5}$
100%	=	1.00	=	$\frac{100}{100} = 1$
125%	=	1.25	=	$\frac{125}{100} = \frac{5}{4}$
$\frac{1}{2}$%	=	$.00\frac{1}{2} = .005$	=	$\frac{\frac{1}{2}}{100} = \frac{1}{200}$
$\frac{3}{4}$%	=	$.00\frac{3}{4} = .0075$	=	$\frac{\frac{3}{4}}{100} = \frac{3}{400}$
$12\frac{1}{2}$%	=	$.12\frac{1}{2} = .125$	=	$\frac{12\frac{1}{2}}{100} = \frac{1}{8}$
$33\frac{1}{3}$%	=	$.33\frac{1}{3}$	=	$\frac{33\frac{1}{3}}{100} = \frac{1}{3}$
$14\frac{2}{7}$%	=	$.14\frac{2}{7}$	=	$\frac{14\frac{2}{7}}{100} = \frac{1}{7}$

The student will observe that any per cent is expressed as a decimal by removing the decimal point two places to the left in the number expressing the rate per cent., that is, dividing the rate by 100.

EXERCISE 57.

What decimals and what common fractions are equivalent to—

1. 3%, 7%, 17%, 56%, 225%.
2. $\frac{1}{4}$%, $\frac{2}{3}$%, $\frac{1}{3}$%, $\frac{1}{18}$%, $\frac{3}{4}$%.
3. $2\frac{1}{2}$%, $18\frac{2}{3}$%, $37\frac{1}{2}$%, $31\frac{1}{4}$%.
4. $43\frac{3}{4}$%, $56\frac{1}{4}$%, $62\frac{1}{2}$%, $68\frac{3}{4}$%.
5. $81\frac{1}{4}$%, $8\frac{1}{3}$%, $16\frac{2}{3}$%, $14\frac{7}{9}$%, $22\frac{3}{4}$%.
6. $41\frac{2}{3}$%, $58\frac{1}{4}$%, $66\frac{2}{3}$%, $83\frac{1}{3}$%.
7. $28\frac{1}{7}$%, 60%, $55\frac{5}{9}$%, 75%
8. $85\frac{5}{7}$%, $87\frac{1}{2}$%, $91\frac{3}{8}$%, $6\frac{1}{4}$%.
9. $10\frac{5}{7}$%, $3\frac{1}{5}$%, $3\frac{1}{4}$%, $23\frac{1}{4}$%.
10. $9\frac{2}{3}$%, 175%, 70%, $93\frac{3}{4}$%.
11. $77\frac{5}{7}$%, $57\frac{1}{7}$%, $15\frac{5}{9}$%, 80%.
12. 135%, $1\frac{3}{4}$%, 90%, $42\frac{4}{7}$%.

204. To change a decimal or a common fraction to an equivalent per cent.

205. Since any per cent. is changed to an equivalent decimal or common fraction by expressing it as so many **hundredths**, that is by **dividing** it by 100, it follows that any decimal or common fraction can be changed to an equivalent per cent. by **multiplying** such decimal or fraction by 100.

EXAMPLE 1.—What per cent. is equivalent to .06 ?

SOLUTION

$$.06 = (.06 \times 100) \% = 6\%.$$

EXAMPLE 2.—What per cent. is equivalent to the fraction $\frac{3}{4}$?

SOLUTION.

$$\tfrac{3}{4} = (\tfrac{3}{4} \times 100) \% = 75\%$$

NOTE.—A decimal is multiplied by 100 by removing the decimal point two places to the left.

EXERCISE 58.

What per cents. are equivalent to the following fractions ?

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
2. $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{1}{6}$, $\frac{1}{6}$.
3. $\frac{5}{6}$, $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$.
4. $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{9}$, $\frac{5}{9}$, $\frac{7}{9}$, $\frac{8}{9}$, $\frac{1}{10}$.
5. $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{3}{11}$, $\frac{7}{11}$, $\frac{8}{11}$, $\frac{9}{11}$.
6. $\frac{1}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{4}{13}$, $\frac{1}{13}$, $\frac{3}{13}$, $\frac{5}{13}$, $\frac{11}{13}$.
7. $\frac{5}{18}$, $\frac{7}{18}$, $\frac{3}{20}$, $\frac{1}{20}$, $\frac{7}{20}$, $\frac{9}{20}$, $\frac{1}{8}$, $\frac{13}{20}$.
8. $\frac{3}{75}$, $\frac{7}{75}$, $\frac{11}{45}$, $\frac{11}{40}$, $\frac{15}{55}$, $\frac{23}{30}$, $\frac{1}{7}$.
9. $\frac{4}{75}$, $\frac{11}{75}$, $\frac{38}{75}$, $\frac{52}{75}$, $\frac{17}{125}$, $\frac{47}{225}$, $3\frac{4}{75}$.
10. $\frac{13}{750}$, $\frac{7}{800}$, $\frac{11}{2000}$, $\frac{1}{1800}$, $\frac{1}{900}$, $\frac{7}{2500}$.

What per cents. are equivalent to the following decimals ?
11. . .7. .5, .03, .07, .05, .003, .007, .005, .75, .055, 006
12. 6.5, .064, .09, .01, .001, .3875, .0625, .03125, .0025.
13. .03½, .028¼, .00⅛, .00½, .06¾, .000¼, .33½, .011¼.

206. To find the value of any per cent. of a number or quantity.

EXAMPLE.—Find 8% of 625.

SOLUTION 1.   OPERATION.   EXPLANATION.
                6.25   =   1% (₁₀₀) of 625.
                 8
              ─────
              50.00   =   8% "   "

SOLUTION 2.   OPERATION.   EXPLANATION.
                625   8% of 625 = .08 of (or times) 625 = 50.00.
                .08
              ─────
              50.00

SOLUTION 3.   OPERATION.   EXPLANATION.
            ₁₀₀ × 625 = 50   8% of 625 = ₁₀₀ of 625 = 50.

The student should use whichever of the preceding methods gives the shortest solution.

Find—   **EXERCISE 59.**

1. 20% of 5,   25,   45,   75, 125,   95.
2. 25% of 4,   36,   76,   96, 128, 240.
3.   4% of 25,   75, 125, 250, 300, 1000.
4. 12½% of 64,   96, 160, 320, 480, 500.
5. 16⅔% of 6,   36,   72,   84, 132, 324.
6.   8⅓% of 12,   72,   60, 240, 252, 372.
7. 37½% of 80,   32,   48,   75, 90, 724.
8. 66⅔% of 9,   27,   75, 335, 47, 520.
9.   6¼% of 32,   64, 256,   90, 750.
10. 31¼% of 48,   80, 144,   75, 380.
11. 87½% of 16,   72, 108, 356, 968.
12. 22⅔% of 27,   45,   63, 567, 656.
13. 28⁴% of 21,   35,   56, 987, 770.
14. 7⁸⁄₁₃% of 26, 39,   78, 117, 273.
15. 75% of 24,   32,   28, 264, 760.
16. 90% of 70, 110,   40, 350, 660.
17. 31% of 86, 475, 373, 254.
18. 44% of 374, 228, 937, 8321.
19. 50% of ½, ¼, ⅔, ₁₁, ₇⁄₁₈.
20. 125% of $7.50, $375, 436 bushels, 328 tons.
21. 8% of 37½, 62½, 87½, 6¼ 4½, 33⅓.
22. 6% of 350 yds., 450 men, 375 lbs., 580 oz.

207. Given the value of any per cent. of a number, to find the number.

EXAMPLE.—24 is 8% of what number?

SOLUTION 1.

OPERATION.

.08) 24 (300.

EXPLANATION.

The question is $08 \times$ what number $= 24$. If 24 is the product of two factors, one of which is .08, the other factor may be found by dividing 24 by .08.

SOLUTION 2.

OPERATION.

$\frac{24}{8} \times 100 = 300.$

EXPLANATION.

If 8% of the number $= 24$

then   1%   "   " $= \frac{1}{8}$ of 24 $=$ 3

"   100%   "   " $= 100 \times 3 = 300$

SOLUTION 3.

OPERATION

$24 \times \frac{100}{8} = 300.$

EXPLANATION.

The question is $\frac{8}{100}$ of what number $= 24$. If 24 is composed of two factors, one of which is $\frac{8}{100}$, the other factor may be found by dividing 24 by $\frac{8}{100}$.

EXERCISE 60.

Find the numbers of which—

1.	60 is	4%,	3%,	2%,	5%,	6%.	
2.	96 is	20%,	25%,	50%,	75%,	90%.	
3.	640 is	125%,	150%,	225%,	160%,	80%.	
4.	32 is	$8\frac{1}{3}$%,	$9\frac{1}{11}$%,	$7\frac{9}{13}$%,	$7\frac{1}{7}$%,	$6\frac{1}{3}$%,	$5\frac{5}{8}$%.
5.	320 is	$16\frac{2}{3}$%,	35%,	$44\frac{4}{9}$%,	$14\frac{2}{7}$%,	$12\frac{1}{8}$%.	
6.	252 is	30%,	40%,	60%,	90%,	70%.	
7.	105 is	$12\frac{1}{4}$%,	$37\frac{1}{2}$%,	$62\frac{1}{4}$%,	$87\frac{1}{2}$%,	$3\frac{1}{8}$%.	
8.	84 is	$41\frac{3}{3}$%,	$58\frac{1}{3}$%,	$66\frac{2}{3}$%,	$42\frac{6}{7}$%,	$15\frac{5}{9}$%.	
9.	350 is	15%,	35%,	45%,	55%,	65%.	85%.
10.	220 is	$77\frac{7}{8}$%,	$3\frac{1}{8}$%,	$9\frac{7}{8}$%,	$23\frac{1}{3}$%,	$83\frac{1}{3}$%.	
11.	48 is	24%,	16%,	12%,	10%,	24%,	36%.

208. To find what per cent. one number is of another.

EXAMPLE.—What per cent. of 60 is 15?

SOLUTION 1.

$\frac{15}{60} = (\frac{15}{60} \times 100)\% = 25\%.$

EXPLANATION.

As 15 is $\frac{15}{60}$ of 60, and as the fraction $\frac{15}{60}$ expressed as % is $(\frac{15}{60} \times 100)\%$ $= 25\%$ Art. 205, it follows that 15 is 25% of 60.

SOLUTION 2.

.6) 15 (25

1 % × 25 =   5 %.

EXPLANATION.

1 % of 60 = .6,
15 is 25 times .6 and
therefore 25 times 1 % of 60

EXPLANATION.

SOLUTION 3.

60) 15 (.25

.25 = 25 %.

The question is 60 × what % ?
— . = 15. If 15 is the product of two
factors, one of which is 60, the other
factor can be found by dividing 15 by
60.   15 ÷ 60 = .25, and .25 = 25 %.

EXERCISE 61.

1. What % is 30 of 60 ? 12 of 48 ? 15 of 45 ? 7 of 35 ? 9 of 63 ?
2. What % of 12 is 2 ? 36 is 16 ? 35 is 28 ? 49 is 21 ? 75 is 50 ?
3. What % of 10 is 1 ? 5 ? 10 ? 20 ? 30 ? 40 ? 50 ? 60 ? 70 ? 80 ?
4. What % of 50 is 9 ? 12 ? 15 ? 18 ? 30 ? 45 ? 55 ? 100 ? 125 ? 300 ?
5. What % of 200 is 25 ? 75 ? 125 ? 250 ? $12\frac{1}{2}$? $87\frac{1}{2}$? $16\frac{2}{3}$? $62\frac{1}{2}$?
6. What % is $26\frac{1}{4}$, $29\frac{3}{4}$, $33\frac{1}{2}$, $36\frac{3}{4}$, $17\frac{7}{9}$, $29\frac{2}{3}$ of 175 ?
7. What % is 49.5, 56.25, 58.50, 63, 14.3 of 225 ?
8. What % is $.024\frac{1}{5}$, $.4\frac{3}{5}$, $0\frac{1}{5}$, $.00\frac{1}{4}$, .25 of $2\frac{1}{4}$?
9. What % of 1 is $\frac{1}{80}$? $\frac{2}{25}$? $\frac{1}{16}$? $\frac{1}{3}$? $\frac{3}{4}$? $\frac{7}{8}$? $1\frac{1}{4}$?
10. What % of 18.79, $187\frac{9}{10}$, 281.85, 319.43, 394.59 of .1879 ?

209. To find a number, which, if increased or
diminished by a certain per cent. of itself, will be equal
to a given number.

EXAMPLE 1.— What number increased by 25 % of itself will equal
300 ?

EXPLANATION.

SOLUTION 1.

125 % of required number = 300

Therefore the   "   "   = $\frac{100}{125}$ × 300 = 240.

See Art. 207.

If any number be in-
creased by 25 % of itself
the result will be (100 %
+ 25 %) = 125 % of the
original number.

EXPLANATION.

SOLUTION 2.

$\frac{5}{4}$ of required number = 300

Therefore the   "   "   = $\frac{4}{5}$ × 300 = 240.

25 % = $\frac{1}{4}$.   A number
increased by $\frac{1}{4}$ of itself
will be equal to $\frac{5}{4}$ of it-
self, that is ($\frac{4}{4}$ + $\frac{1}{4}$) of
itself.

EXAMPLE 2.—What number decreased by 20 % of itself will equal 360 ?

SOLUTION 1.

80 % of required number = 360

Therefore the "   "   = $\frac{100}{80}$ × 360 = 450.

EXPLANATION.

If any number be decreased by 20 % of itself the result will be (100 % − 20 %) = 80 % of the original number.

SOLUTION 2.

$\frac{4}{5}$ of required number = 360

Therefore the "   "   = $\frac{5}{4}$ of 360 = 450.

EXPLANATION

20 % = $\frac{1}{5}$. A number decreased by $\frac{1}{5}$ of itself will be ($\frac{5}{5}$ − $\frac{1}{5}$) = $\frac{4}{5}$ of itself.

EXERCISE 62.

What number increased by—

1. 10 %   of itself equals 110 ?
2. 75 %   of itself equals $420?
3. 62½ % of itself equals $89.37½ ?
4. 21½ % of itself equals $32.56½ ?
5. 83⅓ % of itself equals $87.12 ?
6. 15 %   of itself equals 345 ?
7. 36 %   of itself equals 238 A. ?
8. 100 %   of itself equals 84.6 cwt. ?
9.   6 %   of itself equals 1272 ?
10. 22 %   of itself equals $549 ?
11.   ⅔ % of itself equals $9.06 ?
12.   ⅜ % of itself equals $81.72 ?

What number diminished by—

13. 65 %   of itself equals $2,590 ?
14. 50 %   of itself equals 28.5 feet ?
15. 16⅔ % of itself equals 1,035 miles ?
16.   4 %   of itself equals $465.60 ?
17. 37½ % of itself equals $203.37½ ?
18.   5 %   of itself equals $6.65 ?
19. 20 %   of itself equals 80 ?
20.   9 %   of itself equals 9$\frac{1}{10}$?
21. 87½ % of itself equals 10 ?
22. 5⅜ % of itself equals 95$\frac{5}{12}$?
23.   ¾ % of itself equals 67.95 ?
24.   ⅔ % of itself equals 216.38 ?

PROFIT AND LOSS.

210. Profit and Loss are commercial terms used to express gain or loss in business transactions.

211. *Gains and losses are usually estimated at some rate per cent. of the cost of the goods including the expenses.*

212. To find the Gain, Loss, or Selling Price, the cost and the rate per cent. of gain or loss being given.

EXAMPLE 1.—A merchant sold cloth which cost $1.75 per yard, so as to gain 8% in selling. What was the gain and selling price?

SOLUTION.

$$
\begin{array}{lcl}
\text{Cost} & = & \$1.75 \\
\text{Gain} = 8\% \text{ of } \$1.75 & = & \underline{.14} \quad \text{Art. 206.} \\
\text{Selling price} & = & \$1.89
\end{array}
$$

EXAMPLE 2 —Goods which cost $2.40 are sold at a loss of 5%. Find the loss and the selling price.

SOLUTION.

$$
\begin{array}{lcl}
\text{Cost} & = & \$2.40 \\
\text{Loss} = 5\% \text{ of } \$2.40 & = & \underline{12} \quad \text{Art. 206.} \\
\text{Selling price} & = & \$2.28
\end{array}
$$

213. To find the Cost Price, the Selling Price, and the rate per cent. of gain or loss being given.

EXAMPLE 1.—By selling goods for $132, I gain 10%. What is the cost price?

SOLUTION.

$$
\begin{array}{ll}
100\% \text{ Cost price} = \text{Cost price} \\
\underline{10\% \quad \text{"} \quad \text{"}} = \text{Gain} \\
\therefore 110\% \text{ Cost price} = \text{Selling price} \\
\therefore 110\% \text{ Cost price} = \$132 \\
\therefore \qquad \text{Cost price} = \tfrac{100}{110} \text{ of } \$132 = \$120. \quad \text{Art. 207.}
\end{array}
$$

EXAMPLE 2.—I find that by sellin · an article for $1 80 I lose 10 . What is the cost price ?

SOLUTION.

100 % Cost price = Cost price
10 % " " = Loss
∴ 90 % Cost price = Selling price
∴ 90 % Cost price = $1.80
∴ Cost price = $\frac{100}{90}$ of $1.80 = $2.00. Art. 207.

214. To find the Cost Price. the Gain or Loss and the rate per cent. of gain or loss being given.

EXAMPLE 1.—By selling a farm at a gain of 20 %, I realized a profit of $850. Find the cost of farm.

SOLUTION.

20 % Cost of farm = $850
∴ Cost of farm = $\frac{100}{20}$ × $850 = $4,250.

EXAMPLE 2.—A yacht was sold for $1,200 less than it cost, its owner thereby losing 12½ % of the cost. What was the cost ?

SOLUTION.

12½ % of the cost = $1200
∴ the cost = $\frac{100}{12\frac{1}{2}}$ × 1200 = $9600. Art 207.

215. To find the rate per cent. of gain or loss, the selling price and the cost price being given.

EXAMPLE 1.—Goods which cost $5 are sold for $7. What is the gain % ?

EXPLANATION.

$7 − $5 = $2 gain Since the gain % is computed on the cost, the question becomes, $2 is what % of $5.

By Art. 20 $2 is ($\frac{2}{5}$ of 100) % = 40 % of $5 (the cost).

SOLUTION.
($\frac{2}{5}$ of 100) % = 40 %. Ans.

EXAMPLE 2.—Goods which cost $7 are sold for $5. What is the loss % ?

EXPLANATION.

$7 − $5 = $2 loss. Since the loss % is computed on the cost, the question becomes, $2 is what % of $7.

By Art. 208 $2 is ($\frac{2}{7}$ of 100) % = 28$\frac{4}{7}$ % of the cost ($7).

SOLUTION.
($\frac{2}{7}$ × 100) % = 28$\frac{4}{7}$ %. Ans.

216. To find the Selling Price, the Cost Price and the gain or loss per cent. of the selling price being given.

EXAMPLE 1.—For what must I sell an article which cost $2.25 so as to gain 25 % of the selling price?

SOLUTION.

Selling price   =   100 % Selling price.
Gain   =   25 %   "   "
Cost price   =   75 % Selling price.

∴ **75** % Selling price = $2.25
∴   Selling price = $\frac{100}{75}$ × 2.25 = $3.00.   Art. 207

EXAMPLE 2.—I sold goods which cost $2.50, so that I lost 25 % of the selling price. Find the selling price.

SOLUTION.

Selling price   =   100 % Selling price.
Loss   =   25%   "   "
Cost   =   125 % Selling price.

∴ **125** % of the selling price = $2.50
∴   the selling price = $\frac{100}{125}$ of 2.50 = $2.00.   Art. 207.

EXERCISE 63.

Find gain or loss and selling price—

	COST.	GAIN %.		COST.	LOSS %.
1.	$8.00,	20 %.	6.	$15.60,	37½ %.
2.	$3.60,	10 %.	7.	$14.75,	4 %.
3.	$4.20,	15 %.	8.	$13.60,	62½ %.
4.	$5 60,	12½ %.	9.	$10 80,	16⅔ %.
5.	$13.20,	40 %.	10.	$4.50,	33⅓ %.

Find cost price—

	SELLING PRICE.	GAIN %.		SELLING PRICE.	LOSS %.
11.	$7.50,	50 %.	16.	$4.75,	5 %.
12.	$3.90,	30 %.	17.	$5.64,	60 %.
13.	$4.59,	28¼ %.	18.	$12.60,	42⅞ %.
14.	$5.50,	22¾ %.	19.	$24.30,	85 %.
15.	$4.56,	14 %.	20.	$5.61,	8¼ %.

Find gain or loss %

SELLING PRICE.	COST.		SELLING PRICE.	COST.
21. $10.00,	$8.00.		26. $10.40,	$8.00.
22. $7.00,	$5.00.		27. $6.50,	$7.50.
23. $4.00,	$4.00.		28. $12.50,	$15.00.
24. $3.60,	$1.00.		29. $10.60,	$12.00.
25. $7.50.	$5.50.			

Find cost—

	GAIN.	GAIN %.		LOSS.	LOSS %
30.	$3.00,	10 %.	34.	$2.50,	30 %.
31.	60c.,	12½ %.	35.	$4.80,	25 %.
32.	37½c.	16⅔ %.	36.	$1.20,	8½ %.
33.	$5.60.	40 %.	37.	$3.00.	6¼ %.

88. Goods which cost $2.40 were sold so as to gain 25 % of the selling price. Find the selling price

39. An article which cost $3.50 was sold so that 12½ % of the proceeds were lost. Find the selling price of the article.

40. What is the selling price of a horse which cost $120, and which was sold so as to gain ？ % of the proceeds.

TRADE DISCOUNT.

217. It is customary for merchants and manufacturers to have fixed price lists of their goods, and when the market varies, instead of changing the fixed price they change the rate of discount.

218. Trade Discount is a percentage deducted from the face of bills, the list prices of goods, or from the amount of a debt without regard to time, and is expressed by the term **per cent. off.**

219. Thus 20 % off, means a deduction of 20 % from the nominal or asking price. 20 and 5 % off, means a discount of 20 %, and then 5 % from the remainder, etc.

The result is not affected by the order in which the discounts are taken.

220. Dealers usually announce their terms upon their bill heads thus, Terms 3 months, or 30 days less 5 %, meaning that a credit of 3 months is given, but if the bill be paid within 30 days a discount of 5 % will be allowed.

221. Goods are marked by wholesale dealers or jobbers at a rate % above, which will allow a certain per cent. of discount from the list or marked price, and still realize a margin of gain.

222. The net price of goods is the list price less the trade discount.

223. To find the net price, the list price and discounts being given.

EXAMPLE.—Goods are invoiced at $600, with discounts of 25, 10, and 5 % off. Find cost of goods?

SOLUTION.

$640
160 = 25 % of $640
―――
$480

48 = 10 % of $480
―――
$432

21.60 = 5 % of $432
―――
$410.40 = Net price.

224. To find the single discount equivalent to two or more discounts.

EXAMPLE.—Find the direct discount equal to two successive discounts of 20 % and 10 %.

SOLUTION.

Set list price = $100
1st Discount = 20 = 20 % of $100
―――
80
2nd Discount = 8 = 10 % of $80
Net price = $72.
Total discount on $100 = $100 − $72 = $28
∴ discount = 28 %.

225. From similar examples we derive the following rule to find a single discount equal to two successive discounts.

RULE.

From the sum of the discounts subtract $\frac{1}{100}$ of their product.

226. Then in the above example the discount = 20 + 10 − $\frac{20 \times 10}{100}$ = 28 %.

When a third discount is given, combine it with the result obtained from the other two.

Thus, if discounts of 20, 10 and 5 % off are given.

From the preceding illustration, 20 % and 10 % are equal to a single discount of 28 %, combining 28 % and 5 % we get a discount of 28 + 5 − $\frac{5 \times 28}{100}$ = 31$\frac{2}{5}$ %, the single discount equal to the discounts of 20, 10 and 5 % off.

227. To mark goods so that a given per cent. may be deducted and leave a given per cent. profit.

EXAMPLE.—At what price must I mark an article which cost $4.00 so that, after deducting 20 %, I may still have a profit of 25 ?

SOLUTION.

Selling price $= \$4.00 + 25\%$ of $\$4.00 = \$5.00$,
     and 20 % less than the marked price $=$ Selling price $\$5.00$.
     ∴ 80 % of marked price $= 5.00$.
     ∴   marked price $= \frac{100}{80} \times 5.00 = \$6.25$.

EXERCISE 64.

Find cash price of—

LIST PRICE.	TRADE DISCOUNT.	LIST PRICE.	TRADE DISCOUNT.
1. $360,	5 and 20 % off.	9. $360.60,	10, 5, and 3 % off.
2. $475,	30 and 5 % off.	10. $2142.45,	5, 2½, and ⅛ % off.
3. $800,	20 and 10 % off.	11. $402.18,	20, 5, and 2½ % off.
4. $750,	10 and 8 % off.	12. $675.36,	10, 8½, and ¼ % off.
5. $1600,	40 and 20 % off.	13. $471.25,	40, 10, and 5 % off.
6. $1750,	25 and 10 % off.	14. $396.60,	50, 30, and 1 % off.
7. $1840,	30 and ¼ % off.	15. $4362.50,	20, 10, and 3⅓ % off
8. $3200,	40 and ⅛ % off.	16. $3169.20,	33⅓, 20, and 10 % off.

What direct discounts are equal to discounts—

17.   5 % and 20 % ; 30 % and 5 % ; 20 % and 10 % ; 10 % and 5 %.
18. 40 % and 20 % ; 25 % and 10 % ; 30 % and ¼ % ; 40 % and ⅛ %.
19. 10 %, 5 % and 3 % ; 50 %, 10 % and 5 % ; 40 %, 20 % and 10 %.
20. 10 %, 10 % and 10 % ; 20 %, 10 % and 5 % ; 10 %, 5 % and 5 %.
21. 10 %, 8⅓ % and ¼ % ; 33⅓ %, 20 % and 8½ % ; 5 %, 2½ % and ⅛ %.

22. What is the difference on a bill of $425 between a discount of 50 % and a discount of 30 % and 20 % ?

23. A bookseller wishes to mark a book which cost $2.00 that he may allow a discount of 25 % and still make a profit of 20 %. What must be the marked price ?

24. If the list price of certain goods is $12 per gross, what will I gain or lose by buying of Mr. A., whose discounts are 25 % and 10 %, instead of from Mr. B., whose discounts are 20, 10 and 5 % off ?

25. For what must I mark goods which cost $3.60, so that I may allow discounts of 20 and 10% off, and still have a profit of 25%?

26. A bookseller wishes to mark up the price of a book which he now sells for $1.70, so that he can deduct 15% and yet receive the present price. What must be the marked price.

MISCELLANEOUS EXERCISE 65.

I.

1. A man having 1,000 bushels of apples, sold 5% of them at $1.25 per bushel; 8% of the remainder at $1 per bushel; 50% of what was then left at 75c. per bushel, and the rest at 60c. per bushel, thus receiving 10% more than he paid; how much did he pay for the whole quantity?

2. Mr. Brooks bought a farm, which was in very poor condition, for $1,586: and, after two years of careful cultivation, which paid for itself with some improvements, he sold it for 65% more than he paid for it. What did he sell it for?

3. The number of inmates in a workhouse 5 years ago was 110; this number has since increased 180%. How many inmates are there now?

4. A merchant bought goods for $297.70, and paid an additional sum equal to 7% of the purchase price for cartage, freight, etc. What must he sell them for to gain 40% on the whole cost?

5. In a mixture of alcohol and water 85% is alcohol. How many gills of alcohol in 3 gallons of the mixture, and how many gills of water?

6. 560 bushels of wheat, bought at $1.10 per bushel, were sold at a profit of 10%. What did the wheat sell for?

7. Bought a bill of goods amounting to $875.50, from which was deducted 5 %. What was the percentage allowed, and the amount paid ?

8. Having $10,720, I invested 25 % of it in land, and 12½ % of the remainder in fencing it. What remained ?

9. Two men engaged in trade, each with $3,540. One of them gained 33⅓ of his capital, and the other gained 60 %. How much more did the one gain than the other ?

10. A little boy who has 8 apples gives 25 % of them to his brother, 12½ % to his sister, and 50 % to his mother. What per cent. and how many has he left ?

11. Charles sold his sled, which had cost him $1.75, at 20 % below cost. How much did he get for it ?

12. A lot of damaged calicoes are to be sold at 75 % below the marked price. What prices must be asked for those that are marked 8c., 10c., 12½c., 16c., 20c., 30c. ?

13. A grain dealer bought wheat for $9,384, and sold it at a gain of 4½ %. What did he receive for it ?

14. If a man owes $2,500, and agrees to pay it in 4 instalments, the first to be 50 % of the whole, the second 25 %, the third 15 %, the fourth 10 %. What will each instalment be ?

II.

1. A merchant owes $6,500, and his property is worth only $5,425. What per cent. of his debt can he pay ?

2. A man shipped 3,800 barrels of flour to England, and during a storm 19 barrels were thrown overboard. What per cent. was lost ?

3. If I have $374.50 in currency, how much gold can I buy when it sells at a premium of 7 %?

4. The population of a certain village increased in 5 years from 6,000 to 7,800. What was the average rate of increase per year?

5. A man bought 350 acres of land, at $40 an acre, and sold part of it for $2,240 at the same rate. What per cent. of the land did he sell?

6. An agent received $67.50 for collecting $4,500. What per cent. was his commission?

7. Bought sugar for $150 and sold it for $167.50. What per cent. was the gain?

8. A merchant owes $8,250, his assets are $3,240. What per cent. of his debts can he pay?

9. Sold ⅓ acres of land for what the whole cost. What was the per cent. gain?

10. What per cent. of 365 days are 30 days?

11. Bought a number of eggs, and sold 11 for the money paid for 18. What per cent. was the gain?

12. A regiment went into battle with 600 men, and came out with 320. What per cent. were lost?

13. Of 4,000 acres of land, I sell 140 acres. What per cent. do I retain?

14. A grocer sold from a hogshead containing 600 pounds of sugar, ¼ of it at one time, and ⅓ of the remainder at another time. What per cent. of the whole remained?

III.

1. A merchant owes $15,120, and his assets are $9,828. What per cent. of his debts can he pay?

2. If $52.50 is paid for the use of $750, 1 year, what is the rate per cent. if $56.70 is paid for the use of $1,260?

3. A man shipped 2,600 bushels of grain from Chicago, and 455 bushels were thrown overboard during a gale. What was the rate per cent. of his loss?

4. One number is 6% of another. What per cent. is the latter number of the former?

5. My furniture is worth $7,200, which is 90% of the value of my lot; and the value of the lot is $33\frac{1}{3}$% of that of my house. How much are lot, house, and furniture together worth?

6. A gentleman who had a yearly income of $2,000 for four years, spent $1,800 the first year, $1,500 the second, $1,200 the third, and $2,260 the fourth. What per cent. of his income did he save during the four years?

7. A person expended 16% of all he was worth in buying 20% of the stock of a mining company. If the entire stock of the company sold for $100,000, how much was the person worth?

8. A merchant, embarking in two speculations, in the first made £37 9s. 3d., which was 4% of his investment; in the second he lost £27 16s. 8d., which was 5% of his investment. How much had he invested in both enterprises?

9. A.'s yearly income, which is 7% of $27,000, is 150% of B.'s income. If B. receives an income of 10% annually from his property, how much is he worth?

10. A leap year is what per cent. of a common year?

11. C. from an income of $5,340, spends $4,966.20; D. from an income of $2,790.40, spends $2,650.88; E. on an income of $1,559.50, saves as much per cent. as the rate per cent. that C. saves, exceeds the rate per cent. that D. saves. How much does E. save?

12. What is the cost of a house which sells at a loss of $7\frac{1}{2}$%, the selling price being $11,500?

13. A merchant owes $12,575, and his assets are $7,500. What per cent. can he pay?

14. Sold two city lots at $1,500 each; on one I made 15 %, on the other I lost 15 %. What did I gain or lose?

IV.

1. Adding to a certain number 11 % of itself, we have 109.835. What would we get, if we subtracted from the same number 11 % of itself?

2. In a certain nursery, 15 % of the trees are pear trees, 1 % cherry trees, 4 % plum trees, and the rest, numbering 480, are apple trees. How many trees in all, and how many pear, cherry, and plum trees does the nursery contain?

3. P. having lost 20% of his capital, was worth exactly as much as Q., who had just gained 12 % on his capital. Q.'s capital was originally $15,000. How much was P.'s?

4. A railway company sold 12 % of its land, and then mortgaged 5 % of what was left. It then had 50,800 acres unencumbered. How many acres had it originally?

5. What number, increased by $2\frac{1}{2}$ % of itself, equals $12\frac{1}{2}$ diminished by $33\frac{1}{2}$ % of itself?

6. What fraction, increased by 21 % of itself, equals $\frac{111}{165}$?

7. 240 is $33\frac{1}{3}$ % more than what number?

8. A collector who has 8 % commission, pays $534.75 for a bill of $775. What amount of the bill does he collect?

9. What is $\frac{7}{8}$ % of $1,728?

10. What is $9\frac{1}{4}$ % of 275 miles?

11. What is the difference between $5\frac{1}{2}$ % of $800, and $6\frac{1}{2}$ % of $1,050?

12. 25 % of 800 bushels is 2½ % of how many bushels?

13. Sold 105 barrels of potatoes, which was 35 % of all I raised. How many did I raise?

14. A farmer sold 7.5 acres of land, which was 15 % of all he owned. How many acres did he own?

V.

1. What per cent. of a number is 25 % of ¾ of it?

2. ¾ % of 1,258 is ¼ % of what number?

3. What per cent. of a number is 20 % of ⅘ of it?

4. A man spends $325.60. which is 33⅓ % of his salary. How much is his salary?

5. A man drew out 9 % of his bank deposit to pay a debt of $243.72. How much had he in bank?

6. If a man owning 40 % of an iron foundry, sells 25 % of his share for $1,246.50, what is the value of the whole foundry?

7. A farmer sold 3,150 bushels of grain and had 30 % of his entire crop left. What was his entire crop?

8. If a man owning 45 % of a steamboat sells 16⅔ % of his share for $5,860, what is the value of the whole boat?

9. The assets of a business man are $135,700, which sum is 43 % of his debts. What is his indebtedness?

10. A fruit dealer sold a lot of oranges for $337.50, which allowed him a profit of 12½ %. What did he pay for them?

11. A city lot was sold for $25,500, the gain on the cost being 325%. What was the cost?

12. A grocer sold 300 bushels of potatoes for $285, which was 16⅔% less than he had paid for them. How much did they cost him per bushel?

13. A. sold goods at a gain of 18%. His profit was $29.70. How much did he sell them for ?

14. By selling a lot of goods for $380, I gain 3 times the per cent. that would be gained by selling them for $340. What per cent. is gained in the latter case ? ($380 − $340 = 2 times the gain.)

15. In the schools of a village yesterday there were 1,235 pupils present, which was 95% of the whole number belonging. How many belonged to the schools ?

VI.

1. Sold a horse for $340, which was 15% less than his value. What was his value ?

2. A man having increased his bank deposit 40%, it amounted to $840. How much had he at first ?

3. My income this year is $2,232, which is 7% less than it was last year. How much was it last year ?

4. A man sold 160 acres from his farm, which was $12\frac{1}{2}$% less than the number of acres he retained. How many acres in his farm ?

5. The price of a single ticket from Princeton to Woodstock is 30c., but 20 coupon tickets can be bought for $5. What per cent. is saved by buying coupon tickets ? What per cent. is lost by buying single tickets ?

6. 10% of a flock of sheep were killed by dogs; $6\frac{2}{3}$% of the rest were lost; $33\frac{1}{3}$% of the remaining number were sold, and 28 then remained. What was the original number ?

7. At harvest time a farmer sold 60 bushels of wheat, which was 25% of the quantity he sent to mill, and what he sent to mill was 40% of what he kept over till the next spring. How many bushels had he at first ?

8. When a merchant sold his goods for $261, he gained twice as much as he would have lost had he sold them for $207. What was his gain per cent. ? (How many times the loss is the difference between $261 and $207?)

9. A grocer sold butter at 12 % profit. Had he sold it for 2c. more per pound, he would have gained 20 %. What did 50 pounds cost him ?

10. A boy buys an old pair of skates for 50c. and sells them for 25c. He then buys a pair for 25c. which he sells for 50c. What per cent. did he lose on the first pair, what per cent. did he gain on the second ?

11. If a dealer buys a hat for $3, and sells it for $4, what per cent. does he gain ? If he buys it for $4 and sells it for $3, what per cent. does he lose ?

12. One hundred pounds of beef were sold for $6, having been bought at 4c. a lb. What per cent. profit ?

13. A retail dealer in boots and shoes sold 50 pairs of boots for $300, they cost him $5 a pair. What rate per cent. did he gain ?

14. A merchant bought goods for $500. What per cent. would he gain by selling them for $530 ? For $525 ? For $550 ? For $540 ? For $560 ? For $575 ? For $600 ? For $1,500?

VII.

1. William buys a penknife for 20c. and sells it to James for 25c. What per cent. does William gain, and what per cent does James lose ?

2. If the 25 minutes of school time given to recesses are $8\frac{1}{3}$ % of the daily session, how many hours . 1 the session ?

3. If a book is marked to be sold at 25 % above cost, but it is sold at 20% below the marked price, what was the gain or loss per cent. ?

4. If 90 pounds of coffee are exchanged for 120 pounds of sugar, what per cent. is the coffee worth per pound more than the sugar?

5. What per cent. do I gain by selling an article for $3 for which I paid $2.25? What per cent. do I lose by buying an article for $3 and selling it for $2.25?

6. A drover sold a horse for $226, and thus gained 25 %. What did he pay for him?

7. Bought 300 long tons coal at $3.75 a ton and sold it at $4 60 a short ton. What is the per cent. profit?

8. Bought a barrel of syrup for $20. What must I charge a gallon in order to gain 20 % on the whole?

9. Sold 25 tons of coal at $5.64 per ton, and made $62. What did the coal cost, and what per cent. was the profit?

10. A quarter section of land was sold for $4,563, which was 8 % less than cost. What was the cost per acre?

11. If 15 % of what is received for goods is gain, what is the gain per cent.?

12. Sold goods for $29,900 and made 15 % after deducting 5 % for cash. What was the cost and the market price?

13. A dealer sold 1,600 bbls. beef for $24,000, which was a loss of 25 %. What did the whole cost, and what did he get a barrel?

14. A builder sold a house for $8,250, which was 12 % more than it cost him. What was the cost?

VIII.

1. A merch. sold cloth at $3 per yard, and thereby gained 20 %. What per cent. would he have gained if he had sold the cloth at $3.75 per yard?

2. A person at two auction sales bought 1,170 books, buying at the second 80 % of the number purchased at the first. How many did he buy at the second?

3. What number, diminished by 25 % of half of itself, equals 12,600?

4. Mr. A. paid three times as much for his horse as for his gig. If he had paid 15 % more for his gig, and 8⅓ % less for his horse, they would together have cost $468. How much did he give for each?

5. A merchant in 1872 made 8 % on his capital, and in 1873, 3⅓ % on his capital thus increased. Capital and profit then equaled $22,351. What was his original capital? What was his profit in 1873?

6. A. offered B. $6,015 for a farm; which B. declined, as it was 2½ % less than it cost him. B. afterward sold it for $6,355. Did he gain or lose on the farm, and what per cent?

7. A statue was sold for $753.75, which was ½ of 1 % more than it cost. Had it been sold for $700, what per cent. would have been gained or lost?

8. Sold goods for $4,026.75, at a loss of 3¼ %. What would they have had to sell for to yield a profit of 3¼ %?

9. B. bought a horse for $200, and sold it at 20 % advance to C., who sold it to D. at a loss of 10%, and D. sold it to E. for 5 % more than it cost him. If E. had paid $21.60 less for the horse, would D. have lost or gained, and what per cent.?

10. K. sold X. some goods for $394, at a loss of 1½%. X. sold them to Y., at a profit of 1½ %. Did they cost Y. more or less than K., and how much?

11. Sold 40 bush. 1 pk. of apples for $31.39½, clearing 4 %. What would they have had to sell for per bushel, to yield a profit of 9 %?

12. A drover laid out equal sums for sheep, cows, and hogs. On the hogs he lost 7%, on the sheep he made 15%, and on the cows he lost 1%. he received for the whole $1,535, and bought 25 hogs, what did each hog cost him? What did all the sheep cost him?

13. Jones offered his house for 15% more than it cost him, but afterward sold it for $15,525, which was 10% less than his original offer. How much did his house cost him?

14. The population of a certain city in 1871 increased 4% on that of 1870; in 1872 it increased 5% on that of 1871; in 1873 it increased 6% on that of 1872, and amounted to 1,389,024. What was its population in 1870?

15. If a certain number be increased by $16\frac{2}{3}$% of itself, and the sum is diminished by 50% of itself, 10% of the remainder is 14. Required, the number.

IX.

1. If a merchant who buys goods on 6 months' credit is allowed a deduction of 5% for paying his bill within 30 days, what can he save on a bill of $560? How much on $3,650?

2. If a man fails to pay his water rent until he is charged 12% for delay, how much will he lose if his water rate is $18.75?

3. If 1% per month, counting from the time of payment, is allowed on all taxes paid before July 1st, and 1% per month charged on all taxes remaining unpaid thereafter, how much more does A. pay than B., if B. pays his taxes February 1st, and A. pays his taxes November 1st, their tax-bills each being $180?

4. What is the net amount of a bill of goods, the list price of which is $435, sold 5% off for cash, trade discount 8%?

5. Sold books on 8 mo. amounting to $854.75 at a discount of $12\frac{1}{2}\%$ from retail price, and 10% off for cash.   What is the net value of the bill ?

6. The gross amount of a bill is $236.37 ; the rates of discount are 15 % and 8 %.   What is the net amount ?

7. Find a direct discount equal to a discount of $12\frac{1}{2}\%$ and 8%.

8. What direct discount is equal to a discount of 25 % and 17 %?

∧9. On a bill of $625, what is the difference between a discount of 30 % and a discount of 25 % and 5 % ?

10. Bought books at a discount of 20 % on the retail price, and sold them at the retail price.   What per cent. did I gain ?

11. What per cent. would I gain at a discount of $33\frac{1}{3}\%$?

12. With a trade discount of 8 % and 5 % for cash, goods were sold for $825 at a profit of 15 %.   What was the cost ?

13. A bookseller wishes to mark up the price of a book which he now sells for $2, so that he can deduct 15 % and yet receive the present price.   What must be the marked price ?

14. A merchant sells cloths for $268 by which he gains 23 %.   How must he mark them so that he may deduct 4% and make the same profit ?

15. Bought diamonds at $920.   How must I mark the price that after abating 5 % the profit may be 25 % ?

16. What must be the price of an article from which you deduct 20 % and leave 20 cents ?

MARKING GOODS.

228. It is customary in mercantile houses to use a private mark, which is placed on the goods to denote their cost and selling price. A word or phrase containing ten different letters is taken, the letters of which are used to indicate the ten digits. For example, the word " Sutherland " is selected ; then the letters represent the figures as follows :

$$S\ u\ t\ h\ e\ r\ l\ a\ n\ d$$
$$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 0$$

If it is required to mark $1.75, it is done thus, **Sle**; 47 **hl**; 90 **nd**.

229. The following are among the words and phrases that may be used : Haliburton, Chelmsford, Cumberland, Blacksmith, Now be smart, Strike hard, Cash profit, Black horse, etc.

230. It sometimes happens that the selling price contains three figures, while the cost price contains but two. To prevent this difference from being noticed, the letter denoting the cipher is prefixed to the cost price. For instance, the cost price was 85 cents, it would be marked **dae**; and the selling price, **sue**; thus each price would be indicated by **three** letters.

231. An extra letter, called a "**Repeater,**" is used to prevent the repetition of a figure. Instead of writing **see** for 1.55, which would show that the two right hand figures were alike, and thus aid in giving a clue to the key-word, some additional letter is selected for a repeater,—y, for instance—and then the price would be written **sey**; 337 would be written **tyl**.

232. Arbitrary characters are frequently used instead of letters, thus :

$$⌐\ Z\ T\ ⊐\ ⊿\ ◻\ ×\ ⊥\ ⊓\ +$$
$$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 0$$

233. Fractions may be designated by additional letters or characters; thus **g** may represent ½; **f,** ⅓, etc.

EXERCISE 66.

1. What is the profit and what is the selling price of the following :

Cost $1.10,	Freight 10 %,	Gain 20 %.	Selling price.
" 1.80,	" 8 %,	" 16 %.	"
" 4.50,	" 10 %,	" 25 %.	"
" 1.75,		' 20 %.	"
" 2.50,	" 10 %,	" 30 %.	"

Mark the selling price of the above, using the word "Chelmsford."

2. Knowing a merchant's profit on cloth to be 25 % and his key-word Haliburton, what letters would be used in indicating the cost price per yard, the selling price being **hrb** ?

3. What letters would be used in marking the selling price of single articles which were bought at $3.50 per dozen, and sold at a profit of 20 %, using the word "Cumberland."

4. A publisher marks each copy of a work **skd.** What mark should he put on each so as to be able to allow the trade 30 % discount ? (Key-word "Strike hard.")

5. What would be the selling price of imported articles bought at $4.60, on which the charges were 50 % of the purchase price, if they were sold at 40 % profit on total cost ? How would the selling price be marked if the phrase "Cash profit" be used, with **y** as a repeater ?

6. A merchant using as his key-word "Chelmsford," indicates the cost per yard of a piece of silk, thus **cod.** What mark will indicate the selling price so that he may sell it at 10 % less than the marked selling price and still make 20 % profit ?

7. A man wishing to sell a web of silk asks 40 % per yard more than it cost him, but he finally sold it at 10 % less than his asking price, and made a profit of 52 cents a yard. Using the phrase "Now be smart" indicate the cost price, the asking price and the selling price.

COMMISSION AND BROKERAGE.

234. Commission is an allowance made to agents or commission merchants for transacting business. It is usually calculated at so much per cent. on the amount of money received for sales or expended in purchase.

235. A Commission Merchant or Agent is a person engaged in the buying and selling of goods for another, as the purchase or sale of merchandise or real estate, collecting or investing money, etc.

236. An Agent's Commission for sale is computed on the gross proceeds, and for purchase on the prime cost.

237. A Broker is one who effects purchases or sales in the interest of buyer or seller.

A broker does not generally take possession of the article bought or sold. He usually contracts in the name of the party from whom he receives his compensation.

238. Brokerage is the compensation paid to a Broker.

239. The Principal is the person for whom the business is transacted.

240. A Consignment is property received to be sold on commission.

241. The Consignor or Shipper is the person who ships the goods to be sold.

242. The Consignee is the person to whom the goods are sent to be sold.

243. A Guarantee is the charge made for assuming the risk of loss from non-payment by the purchaser.

244. The **Gross Proceeds** of a sale or collection is the total amount received by the agent before deducting commission or other charges.

245. The **Net Proceeds** is what remains after all charges have been deducted.

246. An **Account Sales** is a statement in detail rendered by the Consignee to the Consignor, showing the sales of the consignment, all charges or expenses attending the same, and the net proceeds.

247. An **Account Purchase** is a detailed statement made by the purchasing agent to his principal, showing the quantity, grade and price of goods bought on his account, all expenses incident to the purchase, and the gross amount of the purchase.

248. To find the **Commission** on a sale of goods, the gross proceeds, and per cent. of commission being given.

EXAMPLE.—How much commission will be due an agent who sold a house and lot for $6,000, and charged 3% for his services?

SOLUTION.

$6,000 × .03 = $180. Ans.

249. To find the **Commission** on the purchase of goods when the prime cost and the per cent. of commission are given.

EXAMPLE.—My agent in London bought for me 350 yards of silk, which cost $2.50 a yard, at a commission of 20%. Find the amount of his commission.

SOLUTION.

$2.50 × 350 = $875.10 = Cost of silk.
$875.00 × .02 = $17.50. Ans.

250. To find the amount of a **Sale** when the amount of commission and the per cent. of commission are given.

EXAMPLE.—Received $245 for selling a shipment of goods at a commission of 5%. How much did I receive for the goods?

SOLUTION.

5% of amount received = $245

1% " " = $\dfrac{\$245}{5}$

100% " " = $\dfrac{\$245 \times 100}{5}$

∴ Amount received for goods = $4,900. Ans.

251. To find the Commission on an investment when the amount sent the agent includes both the amount to be invested and the agent's commission.

EXAMPLE 1.—A commission merchant received a check for $5,150, to be invested in tea after deducting his commission of 3%. How much money did he invest, and what was the amount of his commission?

SOLUTION.

The amount to be invested is 100% of itself, the commission is 3% of amount invested.

∴ 103% of amount to be invested = $5,150

1% " " = $\dfrac{\$5,150}{103}$

100% " " = $\dfrac{\$5,150 \times 100}{103}$

∴ The amount to be invested = $5,000.
Commission, $5,150 − $5,000 = $150.

EXAMPLE 2.—Having sold a consignment of cotton on 3% commission, I am instructed to invest the proceeds in city property, receiving a commission of 2% on the price paid for the property. My whole commission is $200. Find the amount for which the cotton sold.

SOLUTION 1.

Take the amount for which the cotton sold as a unit

then $\frac{3}{100}$ of the amount of sales = *first commission.*
$\frac{97}{100}$ " " = what is left after deducting 1st Com.

On every $102 of amount left after deducting 1st Com., the agent receives $2 for his second commission.

∴ The agent's commission = $\frac{2}{102}$ of the amount to be invested.

Hence $\frac{2}{102}$ of $\frac{97}{100}$ = $\frac{2}{5100}$ of sales = *second commission.*

∴ $(\frac{3}{100} + \frac{2}{5100})$ of sales = Agent's total commission.

$\frac{8}{51}$ of sales = $200.

Sales = $4,080. Ans.

Solution 2.

$$3\% + 2\% = 5\%.$$

If the 5% commission had been charged on the whole amount of sales, the commission would have been 2% of $200 = $4 more, *i.e.*, the entire commission would have been $200 + $4 = $204 = 5% of sales.

∴ 5% of sales = $204.

Sales = $4,080. Ans.

Again: If the 5% commission had been taken on the amount of **purchase** money, the entire commission would have been 3% of $200 = $6 less than it was. *i.e.*, the entire commission would have been $200 - $6 = $194 = 5% of **purchase** money.

∴ 5% of purchase money = $194.

Purchase money = $3,880.

Solution 3.

It will be found that on every $102 from *sale* there is $5 entire commission. Suppose we allow for commission for *selling*, $2 of the $102, leaving $100. For commission for *purchasing*, $3 of the $100, leaving $97. The entire commission would be $5.

In the former case we have charged 2% of $3 = 6 cents *too much.* But in the latter case we have charged 3% of $2 = 6 cents too little, *i.e.*, the *excess* equals the deficit, and we have still $5 entire commission.

Then, $\frac{8}{51}$ of sales = $200.

Sales = $4,080

SOLUTION 4.

$$
\begin{aligned}
\text{Let } 100\% &= \text{Sale.} \\
3\% \text{ of sale} &= \text{First Commission.} \\
\tfrac{3}{2} \text{ of } 97\% = 1\tfrac{41}{100}\% \text{ of sale} &= \text{Second} \quad \text{``} \\
3\% \text{ of sale } + 1\tfrac{41}{100}\% \text{ of sale} &= \text{Total} \quad \text{``} \\
4\tfrac{41}{100}\% \text{ of sale} &= \$200. \\
100\% \text{ of sale} &= \$4,080.
\end{aligned}
$$

From the foregoing solutions we obtain the following

If commission on sale is 4% and on purchase 3%, the entire commission $= \dfrac{4+3}{100+3}$, *i.e.*, $\dfrac{7}{103}$ of **sale** money, and $\dfrac{4+3}{100-4}$, *i.e.*, $\dfrac{7}{96}$ of **purchase** money.

And generally if we have m per cent. on **sales**, and n per cent. on **purchase,** the entire commission $= \dfrac{m+n}{100+n}$ of **sale** money, and $\dfrac{m+n}{100-m}$ of **purchase** money.

EXERCISE 67.

Find the commission—

1. On the sale of merchandise for $3,150, at $2\tfrac{1}{2}\%$.
2. On the sale of a mill for $8,450, at $2\tfrac{3}{4}\%$.
3. On the sale of 375 bbl. of flour, at $6.25 a bbl., at $3\tfrac{1}{4}\%$.
4. On the purchase of a farm for $12.370, at $2\tfrac{1}{4}\%$.
5. On the sale of 255 bales of cotton, each weighing 520 lb., at $14\tfrac{3}{4}$ cents a lb., at $1\tfrac{1}{2}\%$.

Find the rate of commission—

6. When $78 is paid for selling goods for $5,200.
7. When $84 is paid for collecting a debt of $4,800.
8. When $189 is paid for selling a farm for $7,560.

Find the amount of sales—

9. When a commission of $360 is charged, at $2\tfrac{1}{2}\%$.
10. When the brokerage charged is $48, at $\tfrac{1}{4}\%$.
11. When the agent charges $59.60 commission at $1\tfrac{3}{4}\%$.
12. When a commission of $57.82\tfrac{1}{2}$ is charged, at $4\tfrac{1}{2}\%$.
13. When the net proceeds are $38.70, commission $3\tfrac{1}{4}\%$.
14. When the net proceeds are $2,444.55, brokerage $\tfrac{3}{4}\%$.

Find the amount to be invested and commission—

15. If $4,455 is remitted, deducting $1\frac{1}{4}$ % commission.

16. If $9,909.40 is remitted, deducting $3\frac{1}{2}$ % commission.

17. If $6,500 is received, and $1\frac{1}{4}$ % brokerage deducted.

18. If $2,846.25 is remitted, deducting $3\frac{1}{2}$ % commission.

19. What weight of wool, at 52 cents a lb., can be bought for $1,109.60, after deducting a commission of 4%.

20. Sent to my agent in Hamilton $1,508.80, to invest in flour at $5.75 a bbl., after deducting his commission at $2\frac{1}{2}$ %. How many bbls. can he buy?

21. An agent sold a house and lot for $8,500, and charged 3 % for his services. How much was his commission?

22. If an agent's charges are 2 %, how much commission will he earn by selling property valued at $10,500 ?

23. A real estate agent sold a farm of 75 acres at $85 an acre, on a commission of 2 %; and the stock and implements on the farm for $3,250, on a commission of 3 %. Find the total amount of his commission.

24. An agent received $612.50 for selling grain, on a commission of $1\frac{1}{4}$ %. What was the amount of his sales?

25. A collector's charges for collecting a note amounted to $14.10, at a commission of 5 %. What sum was collected?

26. An agent receives $12,501 20 to invest in wheat, on a commission of 3 %. Find the amount of money invested in wheat.

27. How many lbs. of wool at 27c. a lb., can be bought for $8,424, if the agent is allowed 4 % for purchasing ?

28. Paid an agent a commission of $133.12½, at $2\frac{1}{2}$ %, to purchase wheat at $1.87½ a bushel. How many bushels did he buy, and what was the amount of his bill ?

29. Paid a broker $38.10 for buying 120 shares of railroad stock, at 95½ % a share. What was the rate of his brokerage ?

30. An agent in Montreal remitted $3,795.66 on a sale of 540 barrels of flour, at $7.25 a barrel. What was his rate of commission ?

31. A real estate broker charges $182.34 for investing $12,156 in a factory. What was his rate of brokerage ?

32. I sell through my broker 7 tons of Brazil nuts at $7.50 per cwt. How much do I receive if the broker charges 1 % for selling ?

33. Sent $414 to an agent in Toronto to be invested in prints, at 12½ cents a yard, after taking out his commission of 3½ %. How many yards can he purchase ?

34. My attorney collected 80 % of a note for $1,200, and charged 5½ % commission. What amount should he pay me ?

35. An agent sells a consignment of flour for $7,532.80 and then purchases 1,840 bushels of wheat, at $1 40 a bushel, his commission being 2¼ %. What sum must he remit to the consignor ?

36. An auctioneer, who charged 2 % for selling, found his commission for the sale of a certain house just sufficient to pay for a Cyclopædia in 16 volumes, worth $5.50 a volume. What did the house sell for ?

37. A commission merchant received a remittance of $1,000 to be invested in sugar, after deducting his commission of 2 %. The sugar costing 8⅝c. a lb., how many pounds could he buy ?

38. How much does a house bring, for which the owner receives $24,255, 1 % of the purchase money having been first deducted for the agent who sold it ?

39. How many barrels of flour, at $5.60, can be bought for $2,545.20, a commission of 1% for purchasing having also to be paid out of this sum?

40. A commission merchant sold 500 lbs. of butter at 18c. per lb., and invested the proceeds in oats at 42c. a bushel. He charged ·½ % for selling and 1½ % for buying. What was his total commission, and how many bushels of oats did he buy?

41. A fruit broker sold $680 worth of apples, and after deducting 5 % commission and 20 % for freight and other charges, invested the balance in oranges. How much did he invest in oranges if he charged 2 % for buying?

42. My agent in Brantford sells for me a quantity of dry goods on commission at 6 %. How much must be sold that my agent can buy flour with the proceeds to the value of $5,400, after retaining his commission, for buying, of 2½ ?

43. Sold goods at 2½ % commission, which I invested in sugars, and sold them at a profit of 15 %, realizing a gain of $240. How much commission did I receive, and how much did the goods sell for?

44. A merchant purchased an invoice of grain, which, including a commission of 1½ %, cost $5,050.65. The freight charges were $15.35. He sold the grain at a profit of 15 % on the entire cost, and invested the proceeds in sugar, which he sold at a profit of 5 %. What was the amount paid for commission? What the entire cost of the grain, and how much were his profits?

45. A commission merchant bought goods for which he received 5 % commission for buying and $63.25 for charges. The total cost of goods, commission, and charges was $3,250. What was paid for the goods?

46. An agent bought coffee at $\frac{3}{4}$% brokerage, and received $350. He afterwards sold the coffee at a profit to his principal of $5,160, after deducting $1\frac{1}{2}$% commission on the amount for which it was sold. How much was his commission?

47. I received from Day & Son, of Chicago, a ship load of corn, which I sold for 60c. per bushel, on a commission of 4%; and, by the shipper's instructions, invested the net proceeds in barley, at 75c. per bushel, charging 5% for buying; my total commission was $1,350. How many bushels of corn did Day & Son ship, and how many bushels of barley should they receive?

48. A Buffalo brewer remitted $21,500 to a Toronto commission merchant, with instructions to invest 40% of it in barley, and the remainder, less all charges, in hops. The agent paid 60c. per bushel for barley, and 20c. per pound for hops, charging 2% for buying the barley, 3% for buying the hops, and 5% for guaranteeing the quality of each purchase. If his incidental charges were $187.50, what quantity of each product did he buy, and what was the amount of his commission?

49. A Toronto factor received from Cincinatti a consignment of corn, which he sold at 75c. per bushel, on a commission of 5%; and by instructions of the consignor invested the net proceeds in wool, at 20c. per pound, charging 2% for buying, and 3% additional for guaranty of quality. If the total amount of the agent's commission and guaranty was $1,640, now many bushels of corn were received?

50. My Memphis agent sends me an account purchase of 350 bales of cotton, averaging 480 lbs. each, bought at 15c. per lb., on a commission of $2\frac{1}{2}$%. His charges, other than for commission, were: freight advanced, $126.50,

cartage, $53.25, and insurance, $13.75. What sum should I remit to pay the account?

51. An agent sells a consignment of goods for $2.100. He pays $33.50 for freight, and, reserving his commission remits $2,024.77. Find the rate of his commission

52. An agent sells 1,100 barrels of flour, at $4.50 a barrel, and charges 2½% commission. He invests the proceeds in steel, at 1½c. a lb., charging 1½% commission. What is his entire commission, and how many tons of steel (2,240 lbs. to a ton) does he buy?

53. A commission merchant has consigned to him 5,000 lbs. of cotton, which he sells at 14c. a lb., and charges 2% commission. With the net proceeds he buys cotton cloth, at 10c. a yard, charging 1½% commission for buying. How many yards of cloth does he buy?

54. A commission merchant has consigned to him 5,000 barrels of flour, which he sells at $5.50 a barrel, and charges 2½% commission; the expenses for freight, etc., amounted to $250. With the net proceeds he buys sugar, at 6¼c. a lb., charging 2½% commission for buying. How much sugar does he buy, and what is the amount of his commissions?

CUSTOM HOUSE BUSINESS.

252. **Duties** or **Customs** are taxes levied by the Dominion Government on imported goods, for revenue purposes and for the protection of home industry.

253. Duties are of two kinds, **ad valorem** and **specific.**

254. An **Ad Valorem Duty** is a certain per cent. assessed or levied on the actual cost of the goods in the country from which they are imported, as shown by the income.

255. A **Specific Duty** is a tax assessed at a certain sum per ton, foot, yard, gallon, or other weight or measure, without reference to the value.

Note.—Upon certain goods both specific and ad valorem duties are levied.

256. A **Custom House** is an office established by the Dominion Government for the transaction of business relating to duties, and for the entrance and clearance of vessels.

257. **Ports of Entry** are places at which custom houses are established ; and it is lawful to introduce merchandise into a country only at these places.

258. A **Clearance** is a certificate given by the Collector of a Port after the requirements of law have been complied with, that the vessel has been properly entered.

259. An **Invoice** or **Manifest** is a statement made by the seller or shipper, giving a description of the same, showing actual cost, or value of such merchandise; showing also, marks, numbers, quantity, charges, and other details.

260. All invoices are made out in the weights and measures of the country from which the importation is made.

261. All invoices of merchandise subject to an ad valorem duty, are made out in the currency of the country from which the importation is made.

262. When the value of the foreign currency is fixed by law the value is to be taken in estimating the duties ; when the value is not fixed by law, the invoice must be acompanied by a consular certificate showing its value.

263. A **Tariff** is a schedule of goods, and the rates of import duties imposed by law on the same.

264. The **Free List** includes classes of goods that are exempt from duty.

265. **Tonnage** is a tax levied upon a vessel independent of its cargo, for the privilege of coming into a port of entry.

266. **Allowances** are deductions made in estimating Specific Duties, and are distinguished as *Leakage, Breakage, Draft, Tare,* etc.

267. **Leakage**, determined by gauging, is an allowance for the waste of liquids imported in barrels or casks.

268. **Breakage** is an allowance made for loss of liquids imported in bottles.

269. **Draft** is an allowance made for waste or impurities.

270. **Tare** is an allowance made for the box, bag, crate, or other covering of the goods.

271. **Gross weight** is the weight before any allowances are made.

272. **Net weight** is the weight after all allowances are made.

273. Drawback.—When distilled spirits, fermented liquors, and tobacco upon which an excise duty has been paid, and foreign merchandise upon which an import duty has been paid, are exported, the tax or duty upon the same is refunded. Such return of the tax or duty is called a **Drawback.**

274. An **Appraiser** is an officer of the customs who examines imported merchandise and determines the dutiable value and the rate of duty of the same.

275. A **Bonded Warehouse** is a place for the storage of merchandise on which the duties have not been paid.

NOTES 1.—The law requires an entry for goods to be made within three days after arrival. If no entry is made the goods may be conveyed to the Queen's Warehouse, and may be sold after thirty days for duties.

2. In case goods are warehoused, that is, claimed by the importer and transferred by proper entry to some bonded warehouse, they cannot be sold within two years from the date of such transfer.

3. When goods arrive at a Port of Entry and are unclaimed, they are taken to the Queen's Warehouse, and are subject to sale by auction within thirty days. The proceeds of the sale, after paying all expenses, are paid over to the Receiver General, and may be recovered by proving ownership.

276. A **Custom House Broker** is a person who makes entries, secures permits, and transacts other business at Custom Houses for merchants. He is familiar with the tariff laws, and the details and regulations of Custom House business. He usually acts under the power of an attorney.

277. To find Specific Duty.

EXAMPLE.—What is the specific duty on 150 casks of alcohol, of 60 gallons each, at 20c. per gallon; leakage, 5 %?

SOLUTION.

$$60 \text{ gal.} \times 150 = 9000 \text{ gal.} = \text{Gross quantity.}$$
$$\text{Less } 5\% \text{ for leakage} = 450 \text{ gal.}$$
$$8550 \text{ gal.} = \text{Net quantity.}$$
$$20c. \times 8550 = \$1710.00 = \text{Specific duty.}$$

278. To find Ad Valorem Duty.

EXAMPLE.—What is the ad valorem duty, at 40% on 120 boxes of brass rivets, at 50 lb. per box, invoiced at 9c. a lb., tare 8 lb. per box ?

SOLUTION.

$$50 \text{ lb.} \times 120 = 6000 \text{ lb.} = \text{Gross weight.}$$
$$8 \text{ lb.} \times 120 = 960 \text{ lb.} = \text{Tare.}$$
$$5040 \text{ lb.} = \text{Net weight.}$$
$$9c. \times 5040 = \$453.60 = \text{Net value.}$$
$$\$453.60 \times .40 = \$181.44 = \text{Duty.}$$

EXERCISE 68.

Find the specific duty—

1. On 50 hhd. of sugar, each weighing 480 lb., at $1\frac{1}{4}$c a lb., tare 78 lb. per hhd.

2. On 360 doz. bottles of porter, duty 50c. a doz., breakage 10%.

3. On 250 chests of tea, each 75 lb., invoiced at 15c. a lb., duty $3\frac{1}{2}$c. a lb.

4. On 120 bags of coffee, gross weight 148 lb. each, allowing 3% tare, at $3\frac{1}{4}$c. a lb.

5. On 60 packages of figs, each 16 lb. weight, at $2\frac{1}{2}$c. per lb., tare 5%.

6. On 897,120 lb. of bituminous coal at 75c. per ton.

7. On an importation of 200 boxes of plate glass, each box containing 20 plates 24 × 48 in. in size, at 25c. per sq. ft.

8. On 40 doz. bottles of wine, at $2 per doz., on allowance of 10% for breakage.

9. On 1,500 doz. empty bottles, breakage 4%, and rate of duty 10c. per doz.

10. On 6 blocks of marble, each 10 ft. long, 3 ft. wide, 2 ft. high. at 65c. per cu. ft.

Find the ad valorem duty—

11. On 16 tons of steel, invoiced at 18c. per lb., at 25 %.

12. On 175 boxes of raisins, 18 lb. per box, at 17 %.

13. On 650 doz. kid gloves, invoiced at $6.50 a doz., at 52 %.

14. On 600 gal. sperm oil, of 42 gal. each, invoiced at 45c. a gal., at 20 % ; $3\frac{1}{2}$ % being allowed for leakage.

15. What is the duty at 40 % on an invoice of French jewellery, amounting to 8,560 francs ?

16. What is the duty on an invoice of books from Vienna the value of which was 6,429 florins, at 38 %.

17. What is the duty on an invoice of linens amounting to £3,256 sterling at 27 %, allowing $4.866\frac{1}{2}$ to a pound ?

18. Find the duty on an invoice of woollen cloths from Germany valued at 8,437 Reichmarks, at 45 %.

19. What is the duty on 1,000 yd. of brussels carpet, 27 in. wide, invoiced at 6s. 9d. per yd. ; duty 44c. per sq. yd. specific, and 35 % ad valorem ?

20. An invoice of woollen cloth, imported from England, was valued at £956 6s. If its weight was 684 lb., how much was the duty, at 50c. per lb. specific, at 35 % ad valorem ?

21. I imported from the United States 7,240 bush. of corn and $17\frac{1}{2}$ tons of hay, invoiced at $9.50 per ton. What amount of duties had I to pay, at 15c. per bush. on the corn and 20 % on the hay ?

22. The duty, at 19 %, on an importation of satin, is $309.70. What is the invoice of the goods ?

23. How much duty must be paid on an importation of 27,640 lb. of wool, invoiced at £1,497 10s. 4d., if the rate of duty is 10c. per lb. specific, and 11 % ad valorem ?

24. What is the duty and total cost of 2,500 pieces bleached calico, 33 yd. each in length, and $1\frac{1}{2}$ yd. wide; price 6d. per yd., duty 4c. per sq. yd., and expenses at Liverpool £65 10s.? What is the amount of a bill of exchange at $4.87 to the £ to cover the cost?

25. Find the duty on 50 cases of tobacco, each weighing 60 lb., and 50,000 Havanna cigars weighing 55 lb, invoiced at $75 per M., the duty being 50c. per lb. specific on the tobacco and $2.50 per lb. specific on the cigars, and 25 % ad valorem on both.

26. Paid $22.40 duty on 100 bbl. of sugar, each weighing 220 lb., invoiced at 8c. a lb., tare 4%. What was the rate?

27. Required the duty and total cost of 1 case of French silks, value 3,500 francs, duty 50 % ad valorem; 1 case velvets, value 28,000 francs, duty 50%, expenses, cartage, shipping, etc., 625 francs, and commission $2\frac{1}{2}$ %.

28. A merchant imported 80 pieces three-ply carpet, 75 sq. yd. in a piece, and paid $2,591.84 duty, at 16c. per sq. yd., and 30 % ad valorem. What was the invoice price per yd., in sterling money?

29. A merchant imported 300 pieces of three-ply carpet, each piece containing 75 sq. yd., invoiced at 3s. 6d. per sq. yd., upon which he paid a duty of 17c. per sq. yd. specific, and 35 % ad valorem. What was the total amount of duty paid?

30. On 40 cases of tobacco, each weighing 65 lb., and 20,000 Havana cigars, weighing 200 lb., invoiced at $45 per M., the duty on tobacco being $.90 per lb., and on cigars $2½ per lb. specific, and 40 % ad valorem.

31. Find the duty at 33 % ad valorem, on 1 case of shawls valued at £42 5s., 1 case of linens at £37 10s., duty 40 % ;

1 case prints at £8 5s., duty 20 %; incidental expenses £1 5s., commission 2½ % · consul's fees 15s. What is the total cost in Canadian money ?

32. W. A. Murray & Co. imported 10 cases of shawls, averaging 216 lb. a case, invoiced at 24884.10 francs, the duty being $.50 a lb., and 35 % ad valorem. The invoice was paid with a bill of exchange, bought at 5.16 francs to the dollar. What was the duty, and what did the shawls cost, after paying other charges to the amount of $75.80 ?

INSURANCE.

279. Insurance is a contract by which one party engages for a stipulated consideration to make up a loss which another may sustain. It is distinguished as *Property Insurance, Life Insurance, Accident Insurance,* and *Health Insurance.*

280. An Insurance Company is a company or corporation which insures against loss or damage.

281. Insurance companies may be classified according to principles of organization as follows :—1. *Stock;* 2. *Mutual;* 3. *Mixed,* or *Stock and Mutual.*

282. A Stock Insurance Company is one in which the capital stock is owned by the members of the company called stockholders. They alone share the profits and are liable for the losses.

The business of a stock company is managed by directors chosen by the stockholders.

283. A Mutual Insurance Company is one in which the persons insured receive a share or division of the profits.

284. Non-participating policies, the holders of which do not share in the profits or losses, are issued by certain mutual and mixed companies.

285. A Mixed Insurance Company is one which is conducted upon a combination of the stock and mutual plan.

286. The Insurer or Underwriter is the party who assumes the risk, or agrees to indemnify against loss.

287. The Policy is the name applied to the written agreement of contract between the Insurance Company (the Insurer or Underwriter) and the party insured.

288. A Valued or Closed Policy is one in which the amount insured is definitely determined at the time the insurance is effected. Houses, furniture, and goods in a store are insured in policies of this kind.

289. An Open Policy is one upon which additional insurances may be entered at any time from port to port, at rates and under conditions agreed upon.

290. The Premium is the amount paid for the insurance.

291. An Insurance Agent is a person who represents one or more Insurance Companies, and acts for them in soliciting business, collecting premiums, adjusting loses, etc.

292. An Insurance Broker is a person who effects insurance for a compensation called brokerage or commission.

FIRE INSURANCE.

293. Fire Insurance refers to insurance against loss or damage by fire.   Losses may be total or partial.

294. Fire Insurance Losses are usually adjusted by the insurance company paying the full amount of the loss, provided that such loss does not exceed the sum insured ; if the policy, however, contains the "average clause," the payment made is such proportion of the loss as the amount of insurance bears to the total value of the property.

295. The Term of Insurance is the period of time for which the risk is taken, or the property insured.

296. Short Rates are certain rates of premium charged by the companies when the term of insurance is less than a year.

297. In case a policy is terminated at the request of the insured, he is charged the "short rate" premium ; if, however, it be terminated at the option of the company, the lower long rate will be charged, and the company refund the premium for the unexpired time of the policy.

298. To guard against fraud, property is not usually insured for its full value, and no more can be recovered than the amount of actual loss.   The party insured must also have an interest in the property insured.

299. Dwelling-houses and permanent property, about the value of which opinions differ, and which deteriorate in time, may generally be insured for from one-half to three-fourths their estimated value ; goods in store, at their cash value.

Insurance companies usually reserve the privilege of rebuilding, replacing, or repairing damaged property.

MARINE INSURANCE.

300. **Marine Insurance** refers to insurance of vessels and their cargoes against the dangers of navigation.

301. **Inland or Transit Insurance** refers to insurance of merchandise while being transported from place to place either by rail or water routes, or both.

302. **Marine Insurance losses** are adjusted by the insurance company paying only such a proportion of the loss as the sum insured is to the entire value of the vessel.

303. **Policies on Cargoes** are issued for a certain voyage, and on vessels, for a voyage, or for a specified time.

304. **Salvage** is an allowance made to those rendering voluntary aid in saving vessels or cargoes from marine casualties.

305. When the insured ships goods, or receives information of goods shipped to him, he must notify the insurance company as soon as he is in receipt of bill of lading or other advice of shipment, that it may be entered on the open policy.

306. Goods at sea may generally be insured from 5 % to 25 % more than their cost or invoice price, in order to cover the expenses of freight, insurance, and a share of the profits.

307. To find the cost of insurance, the amount insured, and per cent. of premium being given.

EXAMPLE.—A house and its contents are insured for $8,500. What is the cost of insurance for one year at 1½% premium?

SOLUTION.

$8,500 × .015 = $127.50.

RULE.

Multiply the amount of insurance by the rate per cent. of premium, and the product will be the cost of insurance.

308. To find the amount insured, the premium, and the per cent. of premium being given.

EXAMPLE.—I paid $170 to insure a stock of goods for one year at a premium of 2%. For what amount was the policy insured?

SOLUTION.

$$2\% \text{ of amount of policy } = \$170$$

$$1\% \quad " \quad " \quad = \frac{170}{2}$$

$$100\% \quad " \quad " \quad = \frac{170 \times 100}{2}$$

$$\therefore \text{ Amount of policy } = \$8,500. \text{ Ans.}$$

or

$$\$170 \div .02 = \$8,500. \text{ Ans.}$$

RULE.

Divide the premium by the rate per cent. of premium, and the quotient will be the amount insured.

309. To find the rate per cent. of premium, the premium and the amount of insurance being given.

EXAMPLE.—I paid $85 premium on a house insured for $6,800 What was the rate per cent. of insurance?

SOLUTION.

$$\text{Cost of insuring } \$6,800 \text{ is } \$85$$

$$" \quad " \quad \$1 \quad " \quad \frac{85}{6800}$$

$$" \quad " \quad \$100 \quad " \quad \frac{85 \times 100}{6800}$$

$$\therefore \text{ Rate } = 1\tfrac{1}{4}\%. \text{ Ans.}$$

or

$$\$85 \div \$6,800 = .0125, \text{ or } 1\tfrac{1}{4}\%. \text{ Ans.}$$

RULE.

Divide the premium by the sum insured, and the quotient will be the rate.

310. To find the sum to be insured that will cover both premium and insurance, in case of loss, the value of the property and the rate being given.

EXAMPLE.—For what amount must property worth $7,600 be insured, at 5%, so that, in case of loss, both the premium and the value of the goods may be recovered?

SOLUTION.

To realize $95 we must insure $100 (Since 5 is paid in premium)

" $1   " $\dfrac{100}{95}$

" $7,600   " $\dfrac{100 \times 7600}{95}$ = $8,000. **Ans.**

or

$100\% - 5\% = 95\%$

$7,600 \div .95 = 8,000.$ Ans.

RULE.

Divide the value of property by 100%, minus the rate of insurance, and the quotient will be the sum insured.

311. To estimate proportionate losses.

EXAMPLE.—A merchant insured $2,500 in the Ontario Mutual, $1,500 in the Phœnix, and $3,500 in the Western. A loss by fire of $6,000 occurred. How much should each company pay?

SOLUTION.

$2,500 Ontario Mutual.
 1,500 Phœnix.
 3,500 Western.

$7,500 = Sum insured.

$6,000 \div 7,500 = .80 = $ Rate of loss on $1. **Ex.**
 2,500 $\times$ .80 $= $2,000 = $ Share of Ontario Mutual.
 1,500 $\times$ .80 $= 1,200 =$ "   Phœnix.
 3,500 $\times$ .80 $= 2,800 =$ "   Western.

RULE.

Divide the loss by the total insurance, the quotient will be the per cent. which each must pay.

EXERCISE 69.

1. What will it cost to insure a factory worth $26,000 at $\frac{1}{4}$%, and machinery worth $16,800 at $\frac{5}{8}$%, with $1.50 for policy?

2. What premium must be paid for insuring $6,500 on a store for 3 years at $2\frac{1}{2}$%?

3. My house cost me $8,400. I insured it for $\frac{3}{4}$ of its value, at $\frac{3}{4}$% per year. My books and furniture were insured for $8,000 at the same rate. What did I pay annually for insurance on both ?

4. If $125 are paid annually for insuring $24,000, what is the rate per cent ?

5. Paid $350 on a shipment of goods to insure $\frac{3}{4}$ the value, at $3\frac{1}{2}$%. What was the whole value ?

6. A house is insured at $\frac{3}{8}$%, and the premium is $93.60. For how much is it insured ?

7. The cargo of steamer Gallion, bound for Liverpool, is insured at $\frac{1}{2}$%. For what sum is it insured, the premium being $1,500 ?

8. A manufacturing company paid $214.80 premium for insurance on $\frac{3}{4}$ of the cost of its building and machinery, at 60c per $100. What was their cost ?

9. A company had $125 premium for insuring property worth $18,000. If similar property worth $45,000 were insured at the same rate in another company, what would be the premium ?

10. A merchant sent a cargo of goods worth $25,275 to Canton. What sum must he get insured at 3%, that he may suffer no loss, if the ship is wrecked ?

11. What sum must be insured, at 3%, on a consignment of tea worth $4,200, to cover property and premium ?

12. A shipowner insures a ship and cargo for $89,325, at $4\frac{1}{2}$%, the policy covering both property and premium. What is the value of the property ?

13. If a warehouse is worth $266,250, what sum must be insured, at 2%, to cover the property and premium ?

14. The premiums paid for insuring two stores are $98.24 and $146.50 ; the rate is $1\frac{3}{4}$%. What sum must be insured to cover the property and premium ?

15. The loss by fire on a store and contents was $4,525; the property was insured $2,500 in Western. $1,000 in British American, $2,000 in Provincial, and $3,000 in Royal Canadian. How much should each pay?

16. The loss by fire on a piece of property was $8,000, of which $2,000 was insured in the Ottawa Agricultural, $3,000 in the London Mutual, and $3,000 in the Citizen. How much did each company contribute?

17. A block of stores and contents was insured for $220,000, and became damaged by fire and water to the amount of $150,000. Of the risk, $40,000 was taken by the Quebec Co , $65,000 by the British American, $35,000 by the Western, and the remainder was divided equally between the Royal Canadian and the London Mutual. What was the net loss of each company, if the premium paid was $1\frac{7}{8}$%?

18. A man owing $\frac{2}{5}$ of a ship, insured $\frac{2}{3}$ of his interest at $1\frac{1}{2}$%, and paid $91.50 for premium, and a policy charge of $1.50. If the ship becomes damaged to the extent of $12,000, how much can be recovered on the policy?

19. For how much must a house worth $6,000, and furniture worth $2,000, be insured, at $1\frac{1}{2}$%, to cover the cost of the policy, which was $2, the amount of premium paid, and $\frac{3}{4}$ of the value of the property?

20. A schooner is valued at $10,500, and has a cargo of 8,500 barrels of apples, worth $2.10 per barrel. What amount of insurance must be obtained, at $2\frac{1}{2}$%, to provide, in case of loss, for the value of the property, the premium, and $5 additional which the owner paid for survey and policy?

21. The furniture in my house is estimated at one-half the value of the house. I get both insured for $7,687.50 for 5 years, at $2\frac{1}{2}$%, and find that in case of total destruc-

tion the face of the policy will be full indemnity for both the property and premium. Find the value of the house.

22. A factory worth $45,000 is insured, with its contents, for $62,500; $30,000 of the insurance is on the building, $12,500 on machinery worth $20,000, and $20,000 on stock worth $35,000. A fire occurs by which the building and the machinery are both damaged, each to the amount of $15,000, and the stock is entirely destroyed. How much is the claim against the company, if the risk is covered by an "ordinary" policy? How much if the policy contains the "average clause?"

23. A merchant, owning a store worth $12,000, and goods to the same amount, insures them both for two-thirds of their value, at the rate of 50c. on $100, through a broker, who allows him a discount of 10% on the premium and retains 5% himself. How much does the insurance cost the merchant, what does the broker get, and what is the net premium received by the company?

24. Three companies insure, at $\frac{3}{4}$ of its value, a building worth $16,000. The first company takes $\frac{1}{3}$ the risk, at $\frac{3}{4}$ of 1%; the second, $\frac{2}{5}$ of it, at $\frac{7}{8}$ of 1%; and the third, the remainder, at $\frac{3}{4}$ of 1%. Find the total premium?

TAXES.

312. **A Tax** is the sum assessed on the person, property or income of an individual for local improvement, payment of officers, support of schools, and other general purposes.

313. **A Poll Tax** is a certain sum required of each male citizen liable to taxation, without regard to his property.

314. A **Property Tax** is a tax assessed on real or personal estate, and is assessed at a given rate per cent. of the valuation, but usually at so many cents on $100, or so many mills on $1.

315. Property is of two kinds,—**Real** and **Personal**.

316. **Real Property** or **Real Estate** is immovable property, such as lands and houses.

317. **Personal Property** is **movable** property, such as merchandise, furniture, ships, cattle, money, mortgages, etc.

318. An **Assessor** is an officer appointed to estimate the value of property.

319. An **Assessment Roll** is a list or schedule containing the names of all the persons liable to taxation in the municipality, and the valuation of each person's taxable property.

320. A **Collector** is a person appointed to collect the taxes.

321. To find the tax, the sum assessed and the rate of taxation being given.

EXAMPLE.—The rate of taxation in a certain city was $11\frac{1}{4}$ mills on the dollar. What tax was paid by a person whose property was assessed for $12,000?

SOLUTION.

On $1 　　　the tax is .001125.

∴　 " $12,000　　" 　　　.001125 × 12000 = $135.　Ans

RULE.

Multiply the sum assessed by the rate of taxation, and the product will be the tax.

322. To find the rate of taxation, the sum assessed and the tax being given.

EXAMPLE 1.—In a certain village a school-house is to be built at a cost of $5,725, to be paid by a tax upon the assessed property valued at $229,000. What rate of taxation will cover the cost?

SOLUTION.

On $229,000 there is a tax of $5,725.

∴　　" $1　　　　"　　"　　$\frac{5,725}{229,000}$ = $2\frac{1}{2}$c.　Ans.

RULE.

Divide the property tax by the sum assessed, and the quotient is the rate of taxation.

EXAMPLE 2.—A tax of $16,230 is to be assessed upon the village of Caledonia; the valuation of the taxable property is $800,000, and there are 115 polls, to be assessed at $2 each. What will be the tax on the dollar, and how much will be the tax of Mr. Scott, whose property is valued at $12,500, and who pays for 2 polls?

SOLUTION.

$2　　　× 115　　　= $230.　.　Amount of poll tax.

$16,230 − $230　　= $16,000.　　"　　property tax.

$16,000 ÷ $800,000 = .02.　.　Rate of taxation.

$12,500 × .02　　= $250.　.　Mr. Scott's property tax.

$250　+ $4 (2 polls) = $254.　.　　"　　total tax.

323. To find the sum assessed, the rate of taxation and the tax being given.

EXAMPLE.—The tax on a certain property was $96.10, and the rate of taxation $7\frac{3}{4}$ mills on the dollar. For how much was the property assessed?

SOLUTION.

$0.00775 is the tax on $1

$1   "   " $\dfrac{1}{.00775}$

$96.10   "   " $\dfrac{96.10}{.00775} = \$12,400.$ **Ans.**

RULE.

Divide the tax by the rate of taxation, and the quotient will be the sum assessed.

324. To find what sum must be levied on the assessed valuation to raise a given net amount.

EXAMPLE.—What sum must be levied to raise $38,800 net, allowing 3 % for collection?

SOLUTION.

To raise $97   net, $100   .   must be levied.

" $1   " $\dfrac{100}{97}$.   "   "

" $38,800   " $\dfrac{100 \times 38,800}{97}$ "   " $= \$40,000.$ **Ans.**

or

$1.00   − .03 = .97

$38,800 ÷ .97 = $40.000.

RULE.

Subtract the rate allowed from $1, and divide the net amount to be raised by the remainder; the quotient will be the sum to be levied.

325. When the rate of taxation is ascertained, for convenience a Tax Table is usually prepared on that basis. The following is based on the rate of 3 mills on the dollar. By its use much labor and time may be saved.

TAX TABLE AT THREE MILLS PER DOLLAR.

$		$	$		$	$		$	$		$
1	pays	.003	10	pay	.03	100	pay	.30	1000	pay	3.00
2	"	.006	20	"	.06	200	"	.60	2000	"	6.00
3	"	.009	30	"	.09	300	"	.90	3000	"	9.00
4	"	.012	40	"	.12	400	"	1.20	4000	"	12.00
5	"	.015	50	"	.15	500	"	1.50	5000	"	15.00
6	"	.018	60	"	.18	600	"	1.80	6000	"	18.00
7	"	.021	70	"	.21	700	"	2.10	7000	"	21.00
8	"	.024	70	"	.24	800	"	2.40	8000	"	24.00
9	"	.027	90	"	.27	900	"	2.70	9000	"	27.00
10	"	.030	100	"	.30	1000	"	3.00	10000	"	30.00

EXAMPLE —Find, from the tax-table. the amount of taxes Mr. A. has to pay on a property assessed at $2,475.

SOLUTION.

Tax on $2,000 = $6.00
"   400 =   1.20
"   70 =   .21
"   5 =   .01½
" $2,475 = $7.42½.

EXERCISE 70.

1. **My** property is assessed at $6,400. At the rate of 3½ mills on the dollar, how much tax will I be required to pay?

2. What amount of tax must a man pay who is assessed $12,000 for real estate and $4,500 for personal property, if he pays a rate of 2½%?

3. At what rate must property, valued at $1,250,000, be assessed to raise a tax of $15,000?

4. What is the assessed value of a property that pays a tax of $182, at the rate of 3¼ mills on the dollar?

5. A tax of $7,380 was levied upon the taxable property of a county, valued at $2,460,000. What was the rate, and what was the tax on a farm assessed at $4,000?

6. My entire property is worth $8,000, and is assessed at ¾ of its value, at the rate of 7¾ mills on the dollar. I pay an additional tax for 2 polls, at $2 each. What is my total tax?

7. What sum must be assessed on a school-section to build a school-house, at a cost of $2,730, and pay 2½ % for collection?

8. What amount of tax must a man pay who is assessed $10,500 for real estate, and $5,000 for personal property, if he pays 1½ % City tax, ½ % County tax?

9. A Town-hall, costing $12,250, was built by a tax assessed upon the property of the town. The tax rate was 5 mills on the dollar, and the cost of collection 2 %. What was the valuation ?

10. If my property is valued at $2,500, and the rate of taxation for school purposes is 5 mills on the dollar, what does the tuition of each one of my three children cost me if all of them attend the public schools ?

From the table find out how much—

11. Mr. W. H. Hull pays on $ 6,000
12. Mr. M. Howard   "   " $ 5,583
13. Mr. H. Brierly   "   " $ 5,354
14. Mr. E. Munroe   "   " $10,000
15. Mr. W. Galer   "   " $ 7,534
16. Mr. D. Turnbull   "   " $ 5,821

17. Make out a tax table, rate 15 mills on the dollar.

18. Allowing 5 % for taxes uncollectable, and 2 % for collection, what sum must be levied that $50,000 may be realized for the building of a school-house ?

19. The Council of the Village of Dunville wish to levy a tax which will net them $18,979, after paying the expense of collection, which will be 3 %. The assessed value of real and personal property is $1,260.000, and there are 828 polls, each taxed $2. How much will $1 be assessed ?

20. For the purpose of building a town-hall, a tax of $15,961.60 is to be levied on property valued at $1,856,000. What will be the tax on Dr. Burns' property, which is valued at $8,650 ?

21. A bridge costing $18,185 was built by the proceeds of a tax levied upon the property of a town, the rate of taxation being 50c. on $100 (5 mills on $1), the cost of collection being $2\frac{1}{2}$ %. What was the assessed valuation of the property ?

22. If the assessed value of the real and personal property of a city is $80,000,000, and a special tax is desired for the construction of sewers, what must be the rate of levy to realize $188,160 for the purpose, if 2 % be allowed for collection and 4% of the levy be uncollectable ?

23. If a tax of $240 is assessed upon a cotton mill valued at $48,000, what is the valuation of a piece of property that pays a tax of $35.50, at the same rate ?

24. The assessed valuation of the real estate of a county is $1,910,887, of the personal property, $921,073. The year's estimated expenditure is :—For schools, $8,400; interest, $6,850; highways, $7,560; salaries, $5,150; and contingent expenses, $13,675. What tax must be levied on a dollar to meet expenses and provide a sinking fund of $7,000 ?

MISCELLANEOUS.

EXERCISE 7I.

I.

1. A commission merchant, whose rate both for selling and investing is 5 %, receives 24,000 lb. of pork, worth 6c. a lb., and $3,000 in cash, with instructions to invest in a shipment of cotton. What will be his entire commission ?

2. Having sold a consignment of cotton on 3% commission I am instructed to invest the proceeds in town lots, after deducting my purchase commission of 2%. My total commission is $265. How much money did I invest in town lots ?

3. I send a quantity of goods into the country to be sold by auction, on a commission of 9 %. What amount of goods must be sold, that my agent may buy produce with the avails, to the value of $3,500, after retaining his purchase commission of 4 %?

4. A commission merchant sells a consignment of wheat for $7,240. He pays $40 for freight and storage, and charges a commission of $2\frac{1}{4}$%. What are the net proceeds ?

5. A merchant buys, through an agent, 480 yds. of carpet at 80c. per yd., and pays the agent $\frac{3}{4}$ % commission. The freight amounted to $1.92. At what price per yard must the carpet be sold to realize a profit of $33\frac{1}{3}$ % ?

6. I purchased 6,000 bushels of wheat in Winnipeg at 85c. a bushel, and shipped the same to my agent at Ottawa, who sold it at $1.10 per bushel. How much did I make, after paying $543 for expenses and a commission of 2%?

7. I remit to my agent at Chicago $84,650, to purchase flour. After deducting his commission of 1½ % and $20.25 for other expenses, how many barrels of flour at $5 a barrel will the money purchase ?

8. A flour merchant in Montreal remitted to his correspondent in Toronto the proceeds of a consignment amounting to $2,453.75 per draft, which he purchased at the expense of the consignor, at 1½ % premium. What was the amount of the consignment, his commission being 2¼ % ?

9. Sold 2,978 bushels of wheat at $1.05 a bushel; invested the proceeds in sugar, as per order, reserving my commission of 5 % for selling and 1½ % for buying, and the expenses of shipping, $53.37. How much did I invest in sugar ?

10. Sold goods to a certain amount on a commission of 5 %, and having remitted the net proceeds to the owner, received for prompt payment ¼ %, which amounted to $16.15. What was the amount of commission ?

II.

1. My agent bought tea at ⅝ % brokerage, and was paid $450. He afterwards sold the tea at a profit to me of $6,150, deducting 1½ % commission on the sale. How much was his commission ?

2. 11,500 bushels of wheat were bought through an agent, who charged ⅛ % for buying. If the agent paid 85c. per bushel for the wheat, $762.50 freight, and $12.50 insurance, what sum should be remitted to him in full settlement ?

3. From a consignment of 3,160 lbs. of tea, sold by an agent at 30c. per lb., the consignor received as net proceeds $853.74. What was the per cent. of commission charged for selling, if the charges for storage and insurance amounted to $51.60 ?

4. A man wishes to draw on New York for an amount sufficient to cover expenses of 2% exchange and $2\frac{1}{2}$% commission, and leave him the sum of $5,242.50. For how much must he draw?

5. A farmer received from his city agent $190 as the net proceeds of a shipment of butter. If the agent's commission is 3%, delivery charges $6.80, and 5% charge is made for guaranty of quality to purchasers, how many pounds, at 27c. per lb., must have been sold, and how much commission was allowed?

6. Sold by consignee 16,000 bushels of wheat, at 95c.; 5,760 bushels of corn, at $86\frac{1}{2}$c.; 9,245 bushels of oats, at 63c.; and 1,120 bushels of barley, at $1.73. Required, the gross proceeds; also the net proceeds, the charges amounting to $515.20, and the commission being $2\frac{1}{2}$% for selling, and $2\frac{1}{2}$% for guaranteeing payment?

7. A broker sold ⁓ ⁓ ⁓ les of cotton, averaging 395 lbs. to the bale, at $16\frac{1}{4}$c., his commission being $2\frac{3}{4}$%, and the charges $179. He invested 25% of the net proceeds in flour for the consignor, charging a commission of $1\frac{1}{4}$%. How much was still due the consignor?

8. An agent bought butter on a commission of 10%, cheese on a commission of 6%, and eggs on a commission of 5%. If his commission for buying the butter was $21, for buying the cheese $21.60, and for buying the eggs $22, and he charges 25% additional for guaranteeing the freshness of the eggs, what sum should the principal remit to pay for purchases and charges?

9. A merchant sent to his agent in New Orleans a consignment, the gross proceeds of which were $7,689, the charges being $323.50, and the commission $3\frac{1}{2}$%. He directed the agent to buy sugar with the net proceeds, and pay himself his commission for buying ($2\frac{1}{2}$%) out of the same. What was the amount invested, and the agent's commission for both transactions?

10. An agent sold 2,000 bushels Alsike clover seed, at $7.85 per bushel, on a commission of 5%; and 1,200 bushels medium red, at $5.20 per bushel, on a commission of 2½%: taking the purchaser's 3 month's note for the amount of the sales. If the agent charges 4% for his guaranty of the notes, what amount does he earn by the transaction?

III.

1. A consignment of butter was sold for $1,570, of which $1,546.45 were the net proceeds. What was the rate per cent. of commission?

2. An Australian buyer shipped 33,000 lbs. of coarse wool to a London agent to be sold on commission, and gave instructions for the net proceeds to be invested in leather. If the agent sold the wool at 18c. per ' on a commission of 2%, and charged 10% for the pur se and guaranty of grade of the leather, what was the amount of his commissions?

3. What are the net proceeds from the sale of 2,250 bbls. of flour. at $6.25 a bbl., if the charges for freight and storage be 50c. a bbl., commission for selling 2%, for guaranteeing paying 1½%?

An agent sold, on commission, 1,750 bbls. of mess-pork, at $16.50 per bbl., and 508 bbls. of short-ribs, at $18 per bbl., charging $112.50 for cartage, and $5.55 for advertising. He then remitted to his principal $36,000, the net proceeds. Find the rate of commission.

5. A commission merchant received $1,640 with which to buy corn, after deducting a commission of 2½%. What is the amount of his commission, and how many bushels of corn, at 62½c. a bushel, can he buy?

6. The holder of a doubtful claim of $850, handed it to an agent for collection, agreeing that, for every dollar sent him by the agent, the agent might keep for himself 20c. The agent succeeded in collecting but 80% of the debt. How much did the agent remit, how much commission did he receive, and what was his per cent. of commission?

7. A merchant buys, through an agent, 730 yds. of carpeting, at $1.25 a yd., and pays the agent ¼ of 1% commission; the freight amounted to $7.37. At what price per yard must the carpeting be sold to realize a profit of 20%?

8. I remitted $10,500 to a Duluth agent to be invested in wheat, allowing him a commission of 3% for investing. The agent paid 95c. per bushel for the wheat, and charged me 1½ cts. a bushel per month for storage. At the end of 4 months the agent sold the wheat at $1.10 per bushel, on a commission of 5%. If I paid $350 for the use of the money, did I gain or lose by the operation, and how much?

9. A commission merchant sells a consignment of cotton for $5,216. He pays $51 for freight and storage, and charges a commission of 2¼%. What are the net proceeds?

10. The net proceeds of a consignment of wheat was 96½% of the net proceeds of a consignment of oats, and the rate of commission on each was 4½%. The sum of the net proceeds on both consignments was $5,895, and the sum of the charges, other than commission, was $330, of which $175.00 was charged to the consignment of wheat. How much was the commission on the consignment of oats?

IV.

1. Find the duty on 3 dozen clocks, invoiced at $21.50 each, and 6 dozen watches, invoiced at $35 each, if the ad valorem duty was 35% on the clocks, and 25% on the watches.

2. A ⟨...⟩ merchant imported 6 casks of wine, and paid $⟨...⟩ ⟨...⟩, at $2 per gallon, leakage 10% allowed. How many gallons to each cask, had no leakage been allowed?

3. Paid $⟨...⟩25 d⟨...⟩ goods which had been damaged; allowance ⟨...⟩ age is 24%, and the duty was 24%. W⟨...⟩ was ⟨...⟩ price of the goods?

4. An import⟨...⟩ ⟨...⟩aid $825 duty on an invoice of silks, the duty o⟨...⟩ing 24%. But damages of 15% were allowed at the custom-house. What was the entire cost of the goods?

5. A sugar refiner imports 50 hhds. of sugar weighing 480 lbs. each, and 120 hhds. of molasses containing 63 gals. each. What is the amount of the duties, if the sugar pay 3c. a lb. and the molasses 8c. a gal., an allowance being made on the sugar of 10%, and 2% on the molasses?

6. A liquor dealer receives an invoice of 120 dozen bottles of porter, rated at $1.25 per dozen. If 2% of the bottles are found broken, what will be the duty at 24%?

7. A merchant imported 56 casks of wine, each containing 36 gals. net, the duty at 30% amounting to $907.20. At what price per gallon was the wine invoiced?

8. The duty on an invoice of French lace goods at 24% was $132, an allowance of 12% having been made at the custom-house for damage received since the goods were shipped. What was the cost or invoice of the goods?

9. A quantity of Valencias, invoiced at $1,654, cost me $1,980.50 in store, after paying the duties and $12.24 for freight. What was the rate of duty?

10. A merchant imported 50 casks of port wine, each containing originally 36 gals., invoiced at $2.50 per gal. He paid freight at $1.30 per cask, and duty at 30%, 1½% leakage being allowed at the custom-house, and $8.50 for cartage. What did the wine cost him in store?

V.

1. The duty at **19**% on an importation of Denmark satin was $619.40. What was the invoice of the goods?

2. The duty on 600 drums of figs. each containing 14 lbs., invoiced at **5¼**c. per lb., was $35.28. Required, the rate of duty?

3. The duty on an importation of Bay rum, after allowing 2 % for breakage, was $823.20, and the invoice price of the rum was $.25 per bottle. How many dozen bottles did the importer receive, duty at 24 %?

4. A merchant in New York imports from Havana 200 hhds. of W. I. molasses, each containing 63 gals, invoiced at $.30 per gal.; 150 hhds. of B. coffee sugar, each containing 500 lbs., invoiced at $.05 per lb.; 80 boxes of lemons, invoiced at $2.50 per box; and 75 boxes of sweet oranges, invoiced at $3.00 per box. What was the whole amount of duty, estimated at 24 % on molasses and sugar, and at 8 % on lemons and oranges?

5. The duty on an invoice of 300 dozen Buffalo porter, at 30 %, was $190,512; breaka . 2 %. Required, the invoiced price per dozen?

6. Imported 12 casks of wine. each containing 42 gals., invoiced at $3.25 per gal.; paid $96 for freight and a duty of 40 %. How much shall I gain % in selling the whole for $2,747.58?

7. Paid $63.90 duties, at the rate of 9 %, on 50 casks of raisins, tare, 15 lbs. per cask; allowing the gross weight of each cask to have been 115 lbs., what was the invoiced value per lb.?

VI.

1. A man paid $175 for insuring his dwelling, at ⅞ %, and $100 for insuring the furniture, at 1¼ %. If both are destroyed by fire, how much is he entitled to receive?

2. A canal-boat load of 840 bushels of wheat, worth 90c. per bushel, is insured for three-fourths of its value, at $1\frac{3}{8}$% premium. In case of the total destruction of the wheat, how much will the owner lose?

✗ 3. A company took a risk at $2\frac{1}{4}$%, and re-insured $\frac{3}{4}$ of it in another company at $2\frac{1}{2}$%. The premium received exceeded the premium paid by $72. What was the amount of the risk? ⟩ ⁄⁄⁄⁄

✗4. I insured my grocery store, valued at $13,500, and its contents, valued at $33,000, and paid $350 for premium and policy. If the policy cost $1.25, what was the rate per cent. of premium? ⁄⁄⁄⁄

5. A merchant shipped a cargo to London, and to cover both the cargo and the premium, he took out a policy of $100,800, at $3\frac{1}{2}$%. What was the value of the cargo?

6. The steamer Cibola, valued at $90,000, is insured for $75,000, at $2\frac{1}{2}$%. What will be the actual loss to the insurance company, in case the steamer is damaged to the amount of $20,000?

7. Insured for their full value 200 barrels of flour, worth $5.75 a barrel, and 400 barrels worth $6.25, at $\frac{2}{10}$ of 1%. 125 barrels of the first lot and 250 of the second were burned. What was the actual loss to the company?

✗8. A speculator bought 2,000 barrels of flour, and had it insured for 80% of its cost, at $3\frac{1}{2}$%, paying a premium of $429. At what price must he sell the flour, to make a net profit of 10%?

✗ 9. A vessel is so insured that if lost the owner may receive both the value of the vessel and the premium. The value of the vessel is $96,084, and the rate of insurance $1\frac{7}{8}$%. Find the premium.

10. An underwriter agreed to insure some property for enough more than its value to cover the premium. A policy was issued for $25,087.81. The rate being 35c. on $100, what was the property worth?

✗ 11. For what sum must a policy be issued, to insure a vessel for $36,000 and cover also the premium, the rate being 1½ % ?

— 12. A speculator bought 1,000 bbls. of flour, and had it insured for 80 % of its cost, at 3¼ %, paying a premium of $214.50. At what price must he sell the flour to realize a profit of 20 % ?

13. Four companies join in insuring a ship and cargo for $60,000. One company takes ⅓, at ⅜ of 1 %; a second takes $10,000, at ¼ of 1 %; a third, $15,000, at ⅝ of 1 %; a fourth, the remainder, at ½ of 1 %. How much is paid for insurance ?

VII.

1 A town containing $541,250 taxable real estate and $15,620 personal property, levies a tax of .009 %. If 2 % is paid for collecting, what is the net amount realized from the tax ?

✗ 2. In a school section the valuation of the taxable property is $752,400, and it is proposed to repair the school-house and ornament the grounds at an expense of $5,000. If old material sells for $673.70, what will be the rate per cent. of taxation, and what will be B's tax, whose property was valued at $9,400 ?

✗ 3. A tax of $11,466, besides the cost of collecting at 2½ %, is to be raised in a certain town. The polls, 560 in number, are taxed $1 each. The real estate is assessed at $1,270,000, and the personal property at $130,000. Determine the rate, make an assessors' table for that rate, and find A's tax for 2 polls, $2,300 real estate, and $1,400 personal property ?

4. The cost of maintaining the public schools of a city during the year 1888, was $112.000, and the taxable property of the city was $44,800,000. How many mills on a dollar must be assessed for school purposes? If 10 % of the tax assessed cannot be collected, how many mills on a dollar must then be assessed?

5. The total assessed value of a town, real and personal, is $630,000, and the town expenses are $3,913.95. How much tax must be collected to provide for town expenses and allow 3 % for collecting? If the same town contains 310 polls, taxed $1.50 each, what will be the rate of taxation, and how much will be the tax of a man who pays for two polls and owns property assessed at $14,500?

6. A tax of $13.943.20 is assessed upon a town containing 860 taxable polls; the real estate is valued at $2,708,000, and the personal property at $151,600. If the polls be taxed $1.25 each, what will be the rate of property taxation, and what will be the tax of Peter Parley, who pays for three polls, and has real and personal estate valued at $23,750?

7. The assessed value of a town is, on real estate, $1,197,500, and on personal property, $432,500. A poll tax of $.50 per head is assessed on each of 1,870 persons. The town votes to raise $8,000 for schools, $1,500 for highways, $1,500 for salaries, $1,000 for support of poor, and $310 for contingent expenses. How much tax will a milling company have to pay on a mill valued at $46,500, and stock at $19,750?

INTEREST.

326. Interest is money paid for the use of money.

327. The **Principal** is the money for the use of which interest is paid.

328. The **Amount** is the sum of the principal and interest.

329. The **Rate** is the per cent. of the principal paid for its use for 1 year, or a specified time.

NOTE.— When the rate is given, it is to be understood in this work to mean rate per annum, unless otherwise specified.

330. **Legal Interest** is the rate fixed by law for cases in which no rate is specified in the agreement between the parties interested.

In all the Provinces of Canada the legal rate is 6 %.

331. **Usury** is a higher rate than the legal rate.

332. In computing interest, a legal year is 12 months or 365 days.

333. **Simple Interest** is the interest on the principal only.

ACCURATE INTEREST.

(12 months or 365 days to a year).

334. To find the interest on a sum of money for a given number of years, or fraction of a year, at a given rate.

EXAMPLE 1.—Find the interest on $650 for 2 years at 4 %.

SOLUTION 1.

650 Principal
.04
—————
$26.00 Int. for 1 **yr.**
2
—————
$52.00   "   2 yrs.

SOLUTION 2.

$6.50 is int. for 1 yr. at 1 %.
4
—————
$26.00   "   "   "   " 4 %.
2
—————
$52.00   "   "   2 yrs. " 4 %.

EXPLANATION.

Interest for 1 year is 4 % of the principal $650 = $650 × .04 = $26.00, and the interest for 2 years is twice the interest for 1 year, or $26.00 × 2 = $52.00.

SOLUTION 3.
$6.50
8 = 4 × 2
—————
$52.00

EXAMPLE 2.—Find the interest on $960 for 3 yrs. 4 mos., at 6 %.

SOLUTION 1.

$960
.06
—————
$57.60 Int. for 1 yr.
3⅓
—————
$192.00   "   " 3⅓ yrs.(3 yrs. 4 mos.) $192.00

SOLUTION 2.
$9.60
6
—————
$57.60
3⅓

SOLUTION 3.
$9.60
20 = 6 × 3⅓
—————
$192.00

NOTE 1.—1 % of a number is found by removing the decimal point in the number, 2 places to the left.

2 The result will be the same in Ex. 1, whether we multiply by 4 and then by 2, as in Solution 2, or by 8 (4 × 2), as in Solution 3.

EXERCISE 72.

Find the interest for one year of—

1. $450 at 4½ %.
2. $680 at 3½ %.
3. $960 at 7½ %.
4. $840 at 5½ %.
5. $1,720 at 6¼ %.
6. $2,630 at 4½ %.
7. $4,920 at 5 %.
8. $5,000 at 3¾ %.
9. $3,720 at 3½ %.
10. $4,680 at 4½ %.
11. $7,428 at 5½ %.
12. $9,654 at 6 %.
13. $7,851 at 6¼ %.
14. $9,643 at 7 %.
15. $5,430 at 5 %.

Find the interest and amount—

	PRINCIPAL.	RATE.	TIME.
16.	$600.00,	5%,	2 yr .
17.	$700.00,	6%,	2 yr . 6 mos.
18.	$500.00,	7%,	5 yrs
19.	$950.00,	8 %,	3 yrs. 3 mos.
20.	$800.00,	9 %,	6 yrs
21.	$740.00,	8½ %,	7 yrs.
22.	$1,320.00,	10 %,	2 yrs 10 mos.
23	$960.50,	12 %,	3 yrs. 9 mos.
24.	$475.80,	6¼ %,	4 yrs. 6 mos
25.	$363.20,	2½ %,	8 yrs 8 mos.
26.	$1,020.00,	3¾ %,	1 yr. 7 mos.
27.	$4,075.00,	6 %,	2 yrs. 4 mos.
28.	$4,028.75,	4 %,	5 yrs.
29.	$4,026.00,	8 %,	3 yrs. 2 mos.
30.	$270.36,	3½ %,	1 yr. 11 mos.
31.	$840.00,	9 %,	1 yr. 9 mos.
32.	$100.00,	6 %,	2 yrs 7 mos.
33.	$900.00,	5 %,	3 yrs. 6 mos.
34.	$360 00,	7 %,	5 yrs. 4 mos.
35.	$750.80,	4 %,	2 yrs. 7 mos.
36.	$475 30,	3 %,	6 yrs. 3 mos.
37.	$328.00,	6¼ %,	2 yrs. 5 mos.
38.	$474.90,	8½ %,	4 yrs 6 mos.
39.	$640.80,	5⅝ %,	1 yr. 3 mos.
40.	$143 33,	3⅓ %,	6 yrs
41.	$360.96,	12 %,	2 yrs
42.	$796.00,	11 %,	3⅓ yrs.
43.	$1,800.00,	13 %,	4¼ yrs.
44.	$1,080.00,	10 %,	2⅙ yrs.
45.	$894.00,	4½ %,	3½ yrs.

335. To find the interest on a sum of money, for a given number of days, at a given rate.

EXAMPLE 1.—Find the interest on $850 for 62 days at 5 %.

SOLUTION 1.

$8.50
  5
$42.50 Int. for 1 yr.
  62
365) 2635.00 ($7 21 +
    or
    $7 72.

SOLUTION 2.

CANCELLATION METHOD.

$$\frac{8.50 \times 5 \times 62}{365} = \frac{52700}{73} = \$7.22.$$

EXPLANATION.

Sixty-two days is $\frac{62}{365}$ of 1 year. The interest for 62 days is therefore $\frac{62}{365}$ of the interest for 1 year, and this may be found by multiplying the interest for 1 year ($42.50) by 62 and dividing the result by 365, as in Solution 1, or by cancellation, as in Solution 2.

EXAMPLE 2.—Find the interest on $3,250 from April 16th, 1889, to June 18th, 1891, at 6 % per annum.

(From April 16th, '89, to June 18th, '91, is 2 years and 63 days.)

SOLUTION 1.

$32.50

6
――――
$195.00

$2\frac{63}{365}$
――――
$423.66.

SOLUTION 2.

$32\ 50 \times 6 \times \frac{63}{365} = 33.66$ Int. for 63 ds.

$32.50 \times 6 \times 2 = 390.00$ " 2 yrs.
――――
$423.66 " 2 yrs. 63 da

336. It is the custom with banks when the time is given in months, to consider them calendar months in reference to the maturity of the paper, but even then they compute the discount by days.

Time table, showing the number of days:

FROM ANY DAY OF	To the Corresponding Day of											
	1 Jan.	2 Feb.	3 Mar.	4 Apr.	5 May	6 June	7 July	8 Aug.	9 Sept.	10 Oct.	11 Nov.	12 Dec.
January ...	365	31	59	90	120	151	181	212	243	273	304	334
February...	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September.	122	153	181	212	242	273	303	334	365	30	61	91
October ..	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December.	31	62	90	121	151	182	212	243	274	304	335	365

1. How many days from May 13th to August 23rd?

EXPLANATION.

Find "May in the column of months at the left; and on the same line under "August" find 92, which is the number of days from any day in May to the same day in August. But August 23 is 10 days more than August 13, and 92 + 10 = 102 days. Ans.

NOTE 1.—If the required date be earlier in the month than the date from which time is counted, subtract the difference from the tabular number.

2. If in Leap Year, and the month of February be included in the time reckoned, add 1 day to the number of days found by the table.

EXERCISE 73.

Find interest on—

PRINCIPAL.	TIME.	RATE.	PRINCIPAL.	TIME.	RATE.
1. $3,600,	65 da.,	5%.	7. $340.80,	130 da.,	$5\frac{1}{2}$ %.
2. $4,500,	80 da.,	7%.	8. $424.40,	67 da.,	6 %
3. $800,	90 da.,	8%.	9. $625.30,	48 da.,	$3\frac{1}{2}$ %.
4. $750,	45 da.,	$4\frac{1}{2}$ %.	10. $426.50,	292 da.,	4 %.
5. $9,360,	135 da.,	6 %.	11. $370.75,	73 da.,	7 %
6. $4,850,	219 da.,	$3\frac{1}{2}$ %.	12. $420.80,	60 da.,	8 %

Find the amount—

	PRINCIPAL.	RATE.	TIME.
13.	$542.00,	7%,	From 1888, Oct. 27, to 1890, May 12.
14.	$684.00,	8%,	" 1887, Sept. 19, to 1889, June 1.
15.	$960 00,	9%,	" 1882, Dec. 31, to 1892, Oct 1.
16.	$1,100.00,	10 %,	" 1889, Jan. 1, to 1892, Dec. 20.
17.	$1,186.20,	11%,	" 1885, April 1, to 1886, July 28.
18.	$1,260.48,	12%,	" 1858, Aug. 31, to 1893, Nov. 1.
19.	$1,040.25,	8%,	" 1890, Feb. 20, to 1891, May 10.
20.	$1,097.76,	6%,	" 1885, Mar. 15, to 1885, Jan. 15.
21.	$976.80,	7%,	" 1886, June 19, to 1889, April 7.
22.	$896.84,	9%,	" 1887, Nov. 24, to 1887, Nov. 30.
23.	$1,272.24,	10%,	" 1891, Sept. 27, to 1892, Dec. 9.
24.	$1,284.96,	12%,	" 1890, Dec 8, to 1891, May 1.
25.	$1,200.00,	11%,	" 1888, Dec. 25, to 1890, May 28.
26.	$989.00,	12%,	" 1889, Mar. 21, to 1890, June 30.

27. A note for $560.60, dated May 5th, 1881, was paid Dec. 31st, 1882, with interest at 7 %. What was the amount?

28. If I have the use of $275 for 4 years 10 months from Jan. 12th, 1883, what amount must I return to the owner, allowing 6 % interest, and what will be the date of maturity?

29. Required the amount of $108.60 from Aug. 20th to Dec. 18th, 1886, at 10 % ?

30. What is the interest on a note for $515.62, dated March 1st, 1883, and payable July 16th, 1885, at 7 % ?

31. What is the value of a note of $65.75, due with interest for 1 year 2 months, at $6\frac{1}{2}$ % ?

32. If a person borrow $375 at 5 %, what will be due the lender at the end of 2 years 6 months ?

33. A man sold his house and lot for $12,500 ; the terms were, $4,000 in cash on delivery, $3,500 in 9 months, $2,600 in 1 year 6 months, and the balance in 2 years 4 months, with 6 % interest.   What was the whole amount paid ?

SIX PER CENT. METHOD.

337. The **Six Per Cent. Method** is formed on a basis of 360 days to the year and 30 days to the month.

338. At 6% per annum the interest of $1.

For 1 yr. 12 mo., or 360 da., is 6c. = .06 of the principal.
For ⅙ yr. 2 mo., or 60 da., is 1c. = .01 of the principal.
For 1/12 yr. 1 mo., or 30 da., is 5m. = .005 of the principal.
For   ⅕ mo., or 6 da., is 1m. = .001 of the principal.
For   1/30 mo., or 1 da., is ½m. = .000½ of the principal.

Hence the following—

PRINCIPLES.

339. *1. The interest of $1 at 6% is half as many cents as there are months in the given time.*

2. The interest of $1 at 6% is one-sixth as many mills as there are days in the given time.

3. The interest for 60 days at 6% is found by removing the decimal point two places to the left in the principal.

4. The interest for 30 days at 6% is found by removing the decimal point two places to the left in the principal and dividing the result by 2.

5. The interest for 6 days at 6% is found by removing the decimal point 3 places to left in the principal.

6 The interest for 1 day at 6% is found by removing the decimal point 3 places to right in the principal and dividing the result by 6.

340. To find the interest for any number of years, months and days at 6%.

EXAMPLE 1.—What is the interest on $450.75 for 1 yr. 3 mos. 21 da. at 6 %?

SOLUTION 1.

Int. on $1 for 15 mos.   = $.075.   (Principle 1)

" " $1 " 21 "   = .0035.   (Principle 2)

Int. on $1 for 1 yr. 3 mos. 21 da.   = $.0785.

∴ Int. on $450.75 for 1 yr. 3 mos. 21 da   = $450.75 × .0785 = $35. 383875

SOLUTION 2.

1 yr. 3 mos. 21 da. = 471 da				SHORTER PROCESS.
$4.5075	= Int. for 60 da.	(Principle 3)		$4.503
$31.5525	= " 420 "	(60 × 7)		$31.556
2.25375	= " 30 "	(60 ÷ 2)		2 254
1.126875	= " 15 "	(30 ÷ 2)		1.127
.45075	= " 6 "	(Principle 5)		.451
$35.383875	= Int. for 471 da.			$35.388

NOTE 1.—For business purposes it is sufficiently exact to carry the work to mills, as in the shorter process.

2. In this process when the decimal in the fourth places is less than 5 it is rejected; when 5 or greater than 5, the figure in the third decimal place is increased by one, and the decimals to the right of the third decimal place are rejected.

341. *To find the interest at any other rate than 6% by this method, first find the interest at 6%, and then increase or diminish the result by as many sixths as the given rate is units greater or less than 6%. Thus, for 7% add $\frac{1}{6}$, for 8% add $\frac{2}{6}$ or $\frac{1}{3}$, for 4% subtract $\frac{2}{6}$ or $\frac{1}{3}$, etc.*

EXERCISE 74.

Find the interest at 6 % of—

1. $267.27 for 6 mo. 24 da.
2. $146.18 for 1 yr. 21 da.
3. $256.84 for 2 yr. 4 mo. 12 da.
4. $597.25 for 7 mo. 18 da.
5. $418.75 for 1 mo. 25 da.
6. $309.18 for 2 yr. 24 da.
7. $38.90 for 1 yr. 1 mo. 6 da.
8. $146.48 for 9 mo. 10 da.
9. $275.50 for 11 mo. 13 da.
10. $1,298 for 3 yr. 1 mo. 27 da.
11. $2,000 for 2 yr. 7 mo. 24 da.
12. $1,010 for 1 yr. 1 mo. 13 da.
13. $680 for 2 yr. 6 mo. 10 da.
14. $1,895 for 1 yr 7 mo. 7 da.
15. $468 for 5 yr. 5 mo. 1 da.
16. $1,000 for 11 yr. 1 mo. 20 da.
17. $645 for 4 yr. 4 mo. 5 da.
18. $500 for 3 yr. 1 mo. 27 da.
19. $895 for 5 yr. 11 mo. 11 da.
20. $1650 for 1 yr. 10 mo. 23 da.
21. $1,463 for 9 yr. 1 mo. 9 da.
22. $365 for 4 yr. 1 mo. 25 da.

Find the interest and amount—

	PRINCIPAL.	RATE.	TIME.		PRINCIPAL	RATE.	TIME.
23.	$1,080.50,	7%,	1 yr. 9 mo.	35.	$1,218.00,	9%,	9 mo. 25 da.
24.	$420.25,	8%,	2 yr 9 mo.	36.	740 00,	6%,	1 yr 9 mo 15 da.
25.	$960 00,	9%,	3 yr. 4 mo.	37.	$960 00,	7%,	1 yr. 9 mo. 24 da
26.	$576.48,	10%,	3 yr 6 mo.	38.	$1,296 00,	8%,	2 yr. 3 mo. 9 da.
27.	$645.00,	12%,	5 yr. 10 mo.	39.	$1,080.00,	9%,	2 yr. 9 mo. 21 da.
28.	$1,200.00,	5%,	6 yr. 3 mo.	40.	$1,800 00,	10%,	3 yr. 6 mo. 15 da.
29.	$1,200.00,	10%,	12 yr. 6 mo.	41.	$600.00,	11%,	4 yr. 7 mo. 18 da.
30.	$828.00,	6%,	8 mo. 16 da.	42.	$796.00,	12%,	5 yr. 10 mo. 6 da.
31.	$972.36,	8%,	17 mo. 18 da.	43.	$976.28,	7%,	7 yr. 9 mo. 27 da.
32.	$600.60,	10%,	23 mo. 14 da.	44.	$869.44,	9%,	8 yr. 4 mo. 17 da.
33.	$1,155.17,	12%,	40 mo. 6 da.	45.	$1,126.56,	11%,	10 yr. 5 mo. 1 da.
34.	$891.00,	7%,	14 mo. 17 da.	46.	$1,295.28,	8%,	13 yr. 4 mo. 29 da.

342. To find the interest for any number of days at 6%

EXAMPLE 1.—Find the interest on $672 for 216 days at 6%.

SOLUTION 1.

$6.72	=	Int. for 60 da.	(Principle 3)
$20 16	=	" 180 "	(60 × 3)
3.36	=	" 30 "	(60 ÷ 2)
.672	=	" 6 "	(Principle 5)
$24.192	=	Int. for 216 da.	

SOLUTION 2.

$672
.036
4032

2016
$24.192.

EXPLANATION.

By Principle 2, the interest on $1 for 216 days = 36 mills = $.036. ∴ Interest on $672 for 216 days = $672 × .036 = $24.192.

SOLUTION 3.

$.672 + 6 = $.112 = Int. for 1 da. (Principle 6)

∴ $.112 × 216 = $24.192 = " 216 da.

EXAMPLE 2.—Find the interest on $760.48 for 174 days at 6%.

SOLUTION.

$7.6048	=	Int. for 60 da.	(Principle 3)
$22.8144	=	" 180 "	(60 × 3)
.76048	=	" 6 "	(Principle 5)
$22.05392	=	Int. for 174 da.	

SHORTER PROCESS.

7.605

22.815

.760

$22.055.

EXERCISE 75.

Find the interest on—

1. $1,750.00, for 15 days, at 6 %.	18.	$5,178.00, for 9 days, at 9 %
2. $1,125.00, for 21 days, at 7 %.	19.	$732.00, for 11 days, at 6 %
3. $742.50, for 30 days, at 6 %.	20.	$1,171.51, for 12 days, at 8 %.
4. $900.00, for 95 days, at 7½ %.	21.	$340.00, for 70 days, at 10 %
5. $660.00, for 63 days, at 8 %.	22.	$1,478.00, for 80 days, at 6 %.
6. $136.42, for 33 days, at 9 %.	23.	$2,150.00, for 96 days, at 4½ %.
7. $1,000.00, for 21 days, at 10 %.	24.	$1,200.00, for 53 days, at 6 %.
8. $2,000.00, for 12 days, at 5 %.	25.	$1,500.00, for 87 days, at 7 %.
9. $351.00, for 10 days, at 4½ %.	26.	$120.00, for 41 days, at 5 %
10. $1,368.00, for 50 days, at 3 %.	27.	$360.00, for 81 days, at 6 %
11. $93.00, for 150 days, at 6 %	28.	$2,347.50, for 18 days, at 7 %
12. $550.00, for 75 days, at 7 %.	29.	$1,112.49, for 25 days, at 8 %
13. $812.50, for 45 days, at 6 %.	30.	$1,300.00, for 13 days, at 6 %.
14. $800.00, for 27 days, at 5 %.	31.	$17,000.00, for 3 days, at 5½ %.
15. $1,725.00, for 57 days, at 9 %.	32.	$195.50, for 33 days, at 10 %.
16. $125.00, for 55 days, at 6 %.	33.	$1,050.00, for 43 days, at 7 %.
17. $3,741.85, for 6 days, at 7 %.	34.	$1,560.00, for 44 days, at 7½ %

Find the interest in—

	PRINCIPAL	FROM		TO		RATE.
35.	$35.61,	Nov.	11, 1891,	Dec.	15, 1893,	6 %
36.	$50.00,	Sept.	4, 1890,	Jan.	1, 1892,	3½ %
37.	$97.86,	May	17, 1886,	Dec.	20, 1893,	7 %
38.	$325.28,	June	20, 1882,	Sept.	4, 1884,	8 %.
39.	$154.75,	April	10, 1888.	Nov.	24, 1888,	6 %.
40.	$861.50,	June	3, 1889,	March 25, 1890,		5 %.

Find the amount of—

41.	$450.80,	March 6, 1893,	Dec.	20, 1893,	6 %.
42.	$1,500.00,	May 5, 1894,	Jan.	20, 1895,	4 %.
43.	$127.36,	Dec. 12, 1889,	July	3, 1891,	4⅛ %

ACCURATE INTEREST.

(12 months or 365 days to a year.)

343. Since interest in Canada is reckoned upon a basis of 365 days to a year, the interest found by the " Six Per Cent. Method," which is based upon the supposition that 360 days make a year and 30 days a month, is not strictly accurate.

344. *Since the year contains 365 days, the interest, found by the Six Per Cent Method for 360 days to the year, is $\frac{5}{3\,7\,3}$ or $\frac{1}{73}$ part of itself too large.*

345. In many States of the American Union interest is reckoned on the basis of 360 days to the year, and many people in Canada still reckon the interest on their accounts on this basis.

346. On account of the shortness of the Six Per Cent Method, many accountants prefer to reckon by this method, and to then make the necessary correction of $\frac{1}{73}$ of itself.

EXAMPLE.—Find the accurate interest on $750 for 96 days at 8 %.

SOLUTION.

$7.50 = Int. for 60 da. at 6 %.
3.75 =   "   30   "   "
  75 =   "   6   "   "
──────
$12.00 =   "   96   "   6 %.
 4 00
──────
$16.00 =   "   96   "   8 %.   Art. 341.
$16.00 − $\frac{1}{73}$ of $16.00 = $15.78.   Accurate interest.

EXERCISE 76.

Find the interest at 6 % on—

1. $2,500 for 75 days.
2. $750 for 48 days.
3. $6,253 for 96 days.
4. $4,525 for 47 days.
5. $8,360 for 78 days.
6. $4,780 for 51 days.
7. $3,651 for 43 days.
8. $9,875 for 153 days.

Find the interest and amount of—

9. $850 00 for 63 days at 6 %.
10. $945.50 for 33 days at 6 %.
11. $378.68 for 75 days at 6 %.
12. $354.75 for 130 days at 6 %.
13. $510.00 for 63 days at 7 %.
14. $615.00 for 93 days at 6 %.
15. $450.00 for 78 days at 5 %.
16. $120.00 for 96 days at 7½ %.
17. $353.00 for 80 days at 10 %.

18. $670.00 for 78 days at 5 %.
19. $785.00 for 45 days at 7 %.
20. $1,200.00 for 68 days at 5 %.
21. $2,500.00 for 93 days at 8 %.
22. $1,935.50 for 75 days at 5 %.
23. $2,136.88 for 70 days at 4 %.
24. $1,000.00 for 73 days at 6 %.
25. $2,000.00 for 146 days at 9 %.
26. $1,500.00 for 219 days at 4½ %.

Find the interest of—

	PRINCIPAL.	TIME.	RATE.
27.	$450,	From Aug. 10 to Nov. 8, 1885,	6 %.
28.	$720,	" Jan. 25 to April 7, 1885,	7 %.
29.	$960,	" Feb. 3 to Mar. 19, 1884,	8 %.
30.	$540,	" April 8 to May 18, 1890,	9 %.
31.	$100,	" Jan. 30 to Mar. 6, 1892,	4 %.
32.	$900,	" Feb. 12 to Mar. 4, 1893,	7½ %.
33.	$240,	" May 31 to Nov. 27, 1895,	10 %.
34.	$333,	" Oct. 1 to Nov. 29, 1886,	5 %.
35.	$672,	" Feb. 28 to Oct. 25, 1880,	4½ %.
36.	$60,	" June 19 to Nov. 10, 1881,	12 %.
37.	$600,	" July 4 to Oct. 20, 1889,	8 %.
38.	$630,	" Feb. 1 to Aug. 20, 1889,	5¼ %.
39.	$480,	" Jan. 21 to Dec. 2, 1891,	5 %.
40.	$270,	" May 10 to July 29, 1894,	6 %.
41.	$386,	" Oct. 13 to Dec. 12, 1895,	9 %.

42. A person borrows $3,754.45, being the property of a minor who is 15 years 3 months old. He retains it until the owner is 21 years old. How much money will then be due at 6 % ?

43. A note for $710.50, with interest after 3 months at 7 %, was given Jan. 1st, 1884, and paid Aug. 12th, 1886. What was the amount due ?

44. A speculator borrowed $9,675, at 6 %, April 15th, 1884, with which he purchased flour at $6.25 a barrel. May 10th, 1885, he sold the flour at $7⅝ a barrel, cash. What did he gain by the transaction ?

45 A man, engaged in business with a capital of $21,840, is making $12\frac{1}{2}$% per annum on his capital ; but on account of ill health he quits his business, and loans his money at $\frac{3}{4}$% a month. How much does he lose in 2 years 5 months by the change ?

46. Bought 4,500 bushels of wheat at 1.12\frac{1}{2}$ a bushel, payable in 6 months ; I immediately realized for it $1.06 a bushel, cash, and put the money at interest at 10%. At the end of the 6 months I paid for the wheat. Did I gain or lose by the transaction, and how much ?

347. To find the principal, the rate, time, and interest being given.

EXAMPLE 1.—What principal will yield $44.80 interest in 2 yrs. 4 mos. at 4%?

SOLUTION 1.

$1.00

.04

.04

$2\frac{1}{3}$

.$09\frac{1}{3}$

$.09\frac{1}{3}$) 44.80

3   3

.28) $\overline{13440}$ ($480.

EXPLANATION.

The interest on $1 for 2 yrs 4 mos. at 4% is $.09$\frac{1}{3}$, therefore $44.80 must be the interest on as many dollars as $.09$\frac{1}{3}$ is contained in $44.80 or $480. Ans

SOLUTION 2.

$4\% \times 2\frac{1}{3} = 9\frac{1}{3}\%$

$9\frac{1}{3}$% of the principal = $44.80

∴ the principal $= 44.80 \times \frac{100}{9\frac{1}{3}}$

$= $480.

EXPLANATION

The interest each year = 4% of the principal, and for $2\frac{1}{3}$ years $= 4\% \times 2\frac{1}{3} = 9\frac{1}{3}$% of the principal, and therefore $9\frac{1}{3}$% of the principal = $44.80.

EXAMPLE 2.— On what sum of money is $45.60 the interest for 76 days at 5%.

SOLUTION.

$5\% \times \frac{76}{365} = 1\frac{3}{73}\%$

$1\frac{3}{73}$% of the principal = $45.60

∴ the principal $= $45.60 \times \frac{100}{1\frac{3}{73}}$

$= $4,380.

EXPLANATION.

Interest for each year = 5% of the principal, and for 76 days $= 5\% \times \frac{76}{365} = 1\frac{3}{73}$ of the principal and therefore $1\frac{3}{73}$% of the principal = $45.60.

RULE.

Divide the given interest by the interest on $1 for the given time and rate.

EXERCISE 77.

Find the principal—

	RATE.	TIME.	INTEREST.		RATE.	TIME.	INTEREST.
1.	3½%,	1 yr.,	$45½.	7.	5%,	7 yrs.,	$29.75.
2.	5½%,	1 "	$41¼.	8.	3½%,	1½ "	$91 50.
3.	4½%,	⅓ "	$25½.	9.	4%,	1¾ "	$68.25.
4.	3¾%,	½ "	$3⅜.	10.	4½%,	1¼ "	$47.25.
5.	8%,	⅝ "	$18.	11.	6%,	5¾ "	$170.00.
6.	2½%.	6 "	$52½.	12.	3⅓%,	4¼ "	$136.00.

Find the principal—

	INTEREST.	RATE.	TIME.
13	$42.70	7%,	From Jan. 1, 1886, to Sept. 1, 1887.
14.	$197.80.	8%,	" Jan. 1, 1887, to July 12, 1889.
15.	$26 08,	6%,	" Jan 1, 1888, to Sept. 9, 1890.
16.	$60.75.	5%,	" Jan. 1, 1890, to Oct. 10, 1891.
17.	$987.75,	9%,	" Jan 1, 1890, to July 1, 1891.
18.	$366.32,	10%,	" Jan. 1, 1888, to Oct. 18, 1890.
19.	$90.06 +	11%,	" Jan. 1, 1892, to July 1, 1894
20.	$561.56,	12%,	" Jan. 1, 1889, to Oct. 1, 1893.
21.	$445.19,	7%,	" Jan. 1, 1888, to July 24, 1893.
22.	$277.76,	8%,	" Jan. 1, 1892, to Nov. 15, 1895.
23.	$315.64 +	5%,	" Jan. 1, 1887, to Aug. 6, 1892
24	$95.97,	6%,	" Jan. 1, 1891, to Nov 1, 1893.
25.	$700.70,	9%,	" Jan. 1, 1890, to Oct. 10, 1899.
26.	$1,150.86,	12%,	" Jan. 1, 1880, to July 20, 1887

348. To find the principal, the amount, time and rate being given.

EXAMPLE 1.—What principal will amount to $760.20 in 2 yrs. 7 mos. at 8%?

SOLUTION 1.

$1.00
.08
―――
.08 $1.20⅔) $760 20 (
2.⁷₁₂   3   3
―――   ―――― ―――――
.20⅔   3.62) 2280.60 ($630.
1.00
――――
$1.20⅔

EXPLANATION.

The amount of $1 for 2 yrs. 7 mos. at 8% is $1.20⅔, therefore the principal will be as many dollars as $1.20⅔ is contained times in $760.20 or $630. Ans.

Solution 2.

$100\% + 8\% \times 2\frac{7}{12} = 120\frac{2}{3}\%$

$120\frac{2}{3}\%$ of the principal = \$760.20

$\therefore$ the principal = $\$760.20 \times \dfrac{100}{120\frac{2}{3}}$

= \$630. Ans.

Explanation.

Interest for 1 year = 8 % of the principal and for 2 yrs. 7 mos. = $8\% \times 2\frac{7}{12} = 20\frac{2}{3}\%$ of the principal, hence the amount = 100 % of the principal + $20\frac{2}{3}\%$ of the principal = $120\frac{2}{3}\%$ of the principal, therefore $120\frac{2}{3}\%$ of the principal = \$760.20.

EXAMPLE 2.—What principal will amount to \$2,235.60 in 152 days at 5 % ?

Solution.

$100\% + 5 \times \frac{152}{365} = 102\frac{6}{73}\%$

$102\frac{6}{73}\%$ of the principal = \$2,235.60

$\therefore$ the principal = $\$2,235.60 \times \dfrac{100}{102\frac{6}{73}}$ = \$2,190. Ans.

Explanation.

Interest for each year = 5 % of the principal, and for 152 days = $5\% \times \frac{152}{365} = 2\frac{6}{73}\%$ of the principal, and therefore $102\frac{6}{73}\%$ of the principal = \$2,235.60, the amount.

RULE.

Divide the given amount by the amount on $1 for the given time and rate.

EXERCISE 78.

What sum must be put out at interest for—

1.	2 years at	4%	to amount to	\$540.
2.	4 "	6 %	"	\$2,180.00.
3.	6 "	2½ %	"	\$2,760.00.
4.	3 "	3 %	"	\$87.20.
5.	10 "	7 %	"	\$342.00.
6.	8 "	5 %	"	\$616.00.
7.	2½ "	2 %	"	\$53.00.
8.	3⅓ "	6 %	"	\$120.00.
9.	7½ "	8 %	"	\$960.00.
10.	1½ "	3 %	"	\$1,353.00.
11.	9¾ "	1 %	"	\$175.60.
12.	6⅔ "	5 %	"	\$360.00.
13.	3 yr. 1 mo. at 4%		"	\$1,011 00.
14.	2 yr. 5 mo. " 6 %		"	\$114 50.
15.	3 yr. 7 mo " 8 %		"	\$386 00.
16.	1 yr. 8 mo " 3 %		"	\$945.00.
17	2 yr 2 mo " 6⅔ %		"	\$1,630 00.

18.	1 yr. 6 mo.	at 3½ %	to amount to	$840.00.
19.	2 yr. 8 mo.	" 8 %	"	$1,092.00.
20.	1 yr. 9 mo.	" 10 %	"	$940.00.
21.	5 yr. 2 mo.	" 12 "	"	$972.00.
22.	3 yr. 1 mo.	" 6 %	"	$1,185.00.
23.	45 da.	" 6 %	"	$1,470.80.
24.	16 da	" 7½ %	"	$1,098.60.
25.	12 da.	" 3½ %	"	$2,923.36.
26.	87 da.	" 4 %	"	$1,812.40.
27.	104 da.	" 5 %	"	$1,141.20.
28.	318 da.	" 3 %	"	$2,246.70.
29.	75 da.	" 6 %	"	$2,586.50.
30.	150 da.	" 2½ %	"	$3,318.75.
31	200 da.	" 6¼ %	"	$755.00.
32	80 da	" 6 %	"	$516.95.
33.	174 da	" 3½ %	"	$594.28.
34.	312 da	" 6¾ %	"	$462.96.

349. Find the time, the principal, interest and rate being given.

EXAMPLE 1.   In what time will $607.50 produce $125.55 interest at 8 %?

SOLUTION.

6 075

   8

48 60 = Int. for 1 yr. at 8 %.

48.60) 125.55 (2 $\frac{7}{13}$

1 yr. × 2 $\frac{7}{13}$ = 2 yrs. 7 mos. Ans.

EXPLANATION.

The interest for 1 year at 8 % is $48.60 but the interest is 2 $\frac{7}{13}$ times $48.60   ∴ the time = 2 $\frac{7}{13}$ times 1 year = 2 yrs. 7 mos.

EXAMPLE 2.   In what time will $584 produce $6.72 interest at 4 %?

SOLUTION.

5.84

  4

23.36 = Int. for 1 yr. at 4 %.

$\frac{6.72}{23.36}$ × 365 = 105 days. Ans.

EXPLANATION.

The interest for 1 year at 4 % is $23.36, but the interest is only $\frac{6.72}{23.36}$ of this sum, and ∴ the time is $\frac{6.72}{23.36}$ of 1 year = $\frac{6.72}{23.36}$ of 365 days = 105 days.

RULE.

Divide the given interest by the interest of the principal for 1 year at the given rate.

NOTES 1.—If the quotient consists of a fraction, or of a whole number and a fraction, reduce the fractional parts to days by multiplying the fraction by 365

2. If the amount is given instead of the interest, find the part omitted and proceed as above.

3. At 100%, any sum of money will *double* itself in 1 year, therefore any per cent. will require *as many years* to double the principal as the given per cent. is contained times in 100%.

EXERCISE 79.

Find the time—

	PRINCIPAL.	RATE.	INTEREST.		PRINCIPAL	RATE.	INTEREST.
1.	$810.00,	3½%,	$70.	12.	$615.75,	9%,	$206.64.
2.	$1,050.00,	4%,	$136¼.	13.	$727.35,	12%,	$118.954.
3.	320 mark,	4½%,	72 m.	14.	$865.10,	11%,	$317.065.
4.	180 lire,	5%,	4½ l.	15.	$978.60,	10%,	$518.658.
5.	$1,000.00,	5%,	$50.	16.	$998.52,	5%,	$185.115.
6.	$650.00,	1%,	$78	17.	$1,002.24,	7%,	$358.958.
7.	$820.00,	5%,	$215¼	18.	$1,192.32,	9%,	$584.729.
8.	$150.00,	4%,	5¢.	19.	$1,192.30,	8%,	$751.161.
9.	$895.00,	5%,	$80.64.	20.	$1,200.00,	6%,	$1,200.00.
10.	$768.00,	7%,	$111.853.	21.	$1,268.40,	12%,	$1,268.40.
11.	$984.00,	8%,	$284.64.	22.	$1,288.83,	10%,	$1,261.142.

	PRINCIPAL.	RATE.	AMOUNT.		PRINCIPAL.	RATE.	AMOUNT.
23.	$1,460.00,	6%,	$1,150.80.	27.	$4,350,	5%	$1,111.20.
24.	$1,095.00,	7½%,	$1,092.60.	28.	$2,190,	3½%	$2,216.70
25.	$2,920.00,	3½%,	$2,923.36.	29.	$2,555,	6%,	$2,586.50.
26.	$1,825.00,	4%,	$1,842.10.	30.	$3,285,	2½%,	$3,318.75

27. B. loaned $1,600 at 6% until it amounted to $2,000. What was the time?

28. Mr. Roper paid $48 interest. For what period did he pay it, the principal being $640, and the rate 5%?

29. Borrowed Jan. 1st, 1889, $60 at 6%, to be paid as soon as the interest amounted to one-half the principal. When is it due?

80. May 18th a speculator bought 1,606 bushels of wheat at $1.00 a bushel. He afterwards sold the whole for $1,658.80 cash, his profit being equivalent to 6% per annum on the amount invested. What was the date of the sale?

81. In what time will any sum of money double itself at 4 %, 5 %, 6 %, 8 % and 10 % per annum?

350. To find rate, when principal, interest, and time are given.

EXAMPLE.—At what rate will $1,248 in 2 years 5 months produce $135.72 interest?

SOLUTION.

12.48 = Int. for 1 yr. at 1 %

$2\frac{5}{12}$

$30.16 = Int. for $2\frac{5}{12}$ yrs. at 1 %.

$30.16) $135.72 ($4\frac{1}{2}$

1 % × $4\frac{1}{2}$ = $4\frac{1}{2}$ %. Ans.

EXPLANATION.

The interest on $1,248 for 2 yrs. 5 mos. at 1 % = $30.16, but the interest is $4\frac{1}{2}$ times as great as $30.16. ∴ the rate per cent. is $4\frac{1}{2}$ times 1 % = $4\frac{1}{2}$%.

EXPLANATION.

$\frac{$135.72}{$1,248.00}$ expresses what frac-tion the interest is of the princi-pal for $2\frac{5}{12}$ years; this fraction divided by $2\frac{5}{12}$ expresses what fraction of the principal the interest is for 1 year; this latter fraction is expressed as per cent. by multiplying by 100.

SOLUTION 2.

$$\frac{$135.72}{$1,248.00} \times \frac{1}{2\frac{5}{12}} \times 100 \% = 4\frac{1}{2}\%$$

EXAMPLE 2.—At what rate will $4,380 in 76 days. produce $45.60 interest?

SOLUTION 1.

$43.80 = Int. for 1 yr. (365 da.) at 1 %

$9.12 = " 76 da. at 1 %

$9.12) $45.60 (5

1 % × 5 = 5 %. Ans.

EXPLANATION.

Interest on $4,380 for 76 days at 1 % = $9.12, but the interest is 5 times as great as 9.12. ∴ the rate is 5 times 1 % = 5 %.

Explanation.

$\dfrac{\$15.60}{\$4,380.00}$ expresses what fraction of the principal the interest is for $\frac{74}{365}$ year; this fraction divided by $\frac{74}{365}$ expresses what fraction of the principal is for 1 year; this latter fraction is expressed as per cent. by multiplying by 100.

Solution 2.

$$\frac{\$45.60}{\$4,380.00} \times \frac{1}{\frac{74}{365}} \times 100\% = 5\%. \text{ Ans.}$$

RULE.

Divide the given interest by the interest of the principal at 1 % for the given time.

NOTE.—If the amount be given instead of the interest, find the part omitted and proceed as above.

EXERCISE 80.

Find the rate—

	PRINCIPAL.	INTEREST.	TIME.		PRINCIPAL.	INTEREST.	TIME.
1.	$260 00,	$18.00,	1 yr.	21.	$1,231.36,	$923 52,	8 yr. 4 mo.
2.	$150.00,	$90.00,	10 yr.	22.	$800.00,	$18 00,	9 mo.
3.	$180.00,	$76 00,	5 yr.	23.	$8,450.00,	$118 00,	3 mos.
4.	$300.00,	$50.00,	3½ yr.	24.	$311.50,	$26·92,	1 yr. 4 m.
5.	$500.00,	$60.00,	2 yr.	25.	.$750.00,	$93.73,	1 yr. 3 m.
6.	$450.00,	$81 00,	3 yr.	26.	$900 00,	$186 00,	2 yr. 7 m.
7.	$600.00,	$39.00,	1½ yr.	27.	$720 00,	$40 80,	1 yr 5 m.
8.	$120.00,	$18.00,	2½ yr.	28.	$369 00,	$70 50,	3 yr 11 m.
9.	$2,000.00,	$90.00,	1 yr.	29.	$438 00,	$24 96,	312 da.
10	$1,000.00,	$340.00,	2 yr.	30.	$584 00,	$9 28,	174 d.
11.	$2,500.00,	$225.00,	2¼ yr.	31.	$511.00,	$5 95,	87 da.
12	$3,600 00,	$306.00,	1 5⁄12 yr.	32	$1,400.00,	$10 80,	45 da.
13.	$4,850.00,	$582.00,	3 yr.	33.	$1,095 00,	$3 60,	16 da.
14.	$3,500.00,	$315.00,	1½.	34.	$2,920.00,	$3 36,	12 da.
15.	$900.00,	$88.80,	1 yr. 6 m.	35	$1,825.00,	$17 10,	87 da.
16.	$796.20,	$171.98,	2 yr. 8 m	36.	$1,380 00,	$61.20,	102 da
17	$897.50,	$251.30,	3 yr. 6 m	37	$2,190.00,	$56.70,	318 la
18.	$1,224.72,	$481 04,	5 yr. 7 m.	38	$2,555 00,	$31 50,	75 da
19.	$1,152.00,	$403.20,	3 yr. 10 m.	39.	$3 285.00,	$33 75,	150 da
20.	$867 40,	$320 94,	7 yr 4 m	40.	$730 00,	$25 00,	200 da.

41. A house bought for $12,500 paid $1,000 rent. If $200 were paid for taxes and repairs what rate of interest did the purchase money yield ?

COMPOUND INTEREST.

351. Compound Interest is the interest of the principal and of the unpaid interest after it becomes due.

NOTES 1.—The simple interest may be added to principal annually semi-annually, or quarterly, as the parties may agree.

2. Compound interest can not be collected by law, except as per written agreement, but a creditor may receive it without incurring the penalty of usury.

3. In the Post Office Savings Banks, interest is calculated to the thirtieth day of June in every year, and is then added to and becomes part of the principal money, unless withdrawn.

4. Some Savings and Loan Companies compound interest semi-annually.

352. To compute compound interest when the principal, rate and time are given.

EXAMPLE 1.—Find the compound interest on $2,000 for 3 years at 5 %.

SOLUTION 1.

Principal	$2,000 00
Int. for 1st yr. ($2,000 × .05)	100.00
Amt. for 1 yr., or 2nd Principal	$2,100.00
Int. for 2nd yr. (2,100 × .05)	105 00
Amt. for 2 yrs., or 3rd Principal	$2,205.00
Int. for 3rd. yr. ($2,205 × .05)	110.25
Amt. for 3rd yr.	$2,315.25
Original Principal to be subtracted	2,000.00
Compound Interest for 3 yrs.	$315 25

SOLUTION 2.   SOLUTION 3.

$2,000   1.05
    1.05   Amt. of $1 for 1 yr.   1.05
$2,100   Amt. of $2,000 for 1 yr.   1.1025
    1.05   1.05
$2,205   Amt. of $2,100 for 1 yr.   1 157625
    1.05   2000
$2,315.25 Amt of $2,205 for 1 yr. $2,315.25
  2,000   Principal.   2,000
$315.25 Compound Interest. $315.25

EXAMPLE 2.—Find the compound interest on $1,000 for 2 years
3 months at 8 %.

SOLUTION 1.   SOLUTION 2.

$1,000   Principal. $1,000
    80   Int. 1st yr.   1.08   Amt. of $1 for 1 yr.
$1,080   Amt. 1st yr. $1,080
    86.40   Int. 2nd yr.   1.08
$1,166.40   Amt. 2nd yr. $1,166.40
    23.328   Int. for 3 mos.   1.02   Amt. of $1 for 3 mos.
$1,189.728   Int. for 3 yrs. 3 mos. $1,189.728
$1,000   Principal.   1 000
$189.728   Compound Interest. $189.728

SOLUTION 3.

1.08
1.08
1.1664
1.02
1.189728
    1000
$1189.728
$1000
$189.728

353. The use of the following table will greatly shorten
calculations in compound interest.

TABLE.

Showing the amount of $1 or £1, at different rates for any number of years from 1 to 40.

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2¼ per ct.	3 per ct.
1	1.0100 000	1 0150 000	1.0200 000	1.0250 000	1.0300 000
2	1.0201 000	1.0302 250	1.0404 000	1.0506 250	1.0609 000
3	1.0303 010	1.0456 784	1.0612 080	1.0768 906	1.0927 270
4	1.0406 040	1.0613 636	1.0824 321	1.1038 128	1.1255 088
5	1.0510 101	1.0772 840	1.1040 808	1.1314 082	1.1592 740
6	1 0615 202	1.0934 433	1.1261 624	1.1596 934	1.1910 523
7	1.0721 354	1.1098 450	1.1486 856	1.1886 857	1.2298 738
8	1.0828 567	1.1264 926	1.1716 593	1.2184 029	1.2667 700
9	1.0936 853	1.1433 900	1.1950 925	1.2488 629	1.3047 731
10	1.1046 221	1.1605 408	1.2189 944	1.2800 845	1.3439 163
11	1.1156 683	1.1779 489	1.2433 743	1.3120 866	1 3842 338
12	1.1268 250	1.1956 182	1 2682 417	1 3448 888	1.4257 608
13	1.1380 933	1.2135 524	1.2936 066	1.3785 110	1.4685 337
14	1.1494 742	1.2317 557	1.3194 787	1.4129 738	1 5125 897
15	1.1609 690	1.2502 321	1.3458 683	1.4482 981	1.5579 674
16	1.1725 786	1.2689 855	1.3727 857	1.4845 056	1 6047 064
17	1.1843 044	1.2880 203	1.4002 414	1.5216 182	1.6528 476
18	1.1961 475	1.3073 406	1.4282 462	1.5596 587	1.7024 380
19	1.2081 090	1.3269 507	1.4568 111	1.5986 501	1 7535 060
20	1.2201 900	1.3168 550	1.4859 474	1.6386 164	1.8061 112
21	1.2323 919	1.3670 578	1.5156 663	1.6795 818	1.8602 945
22	1.2447 159	1 3875 637	1.5459 796	1.7215 714	1.9161 034
23	1.2571 630	1.4083 772	1.5768 992	1.7646 106	1 9735 865
24	1.2697 346	1.4295 028	1.6084 372	1.8087 259	2.0327 941
25	1.2824 320	1.4509 454	1.6406 059	1 8539 441	2.0937 779
26	1.2925 563	1.4727 095	1.6734 181	1.9002 927	2.1565 912
27	1.3082 089	1.4948 002	1.7068 864	1.9478 000	2.2212 890
28	1.3212 910	1.5172 222	1.7410 242	1.9964 950	2.2879 276
29	1.3345 039	1.5399 805	1.7758 446	2.0464 073	2.3565 655
30	1.3478 490	1.5630 802	1.8113 615	2.0975 675	2.4272 624
31	1.3613 274	1.5865 264	1.8475 888	2.1500 067	2.5000 803
32	1.3749 407	1.6103 243	1.8845 405	2.2037 569	2 5750 827
33	1.3886 901	1.6344 792	1.9222 314	2.2588 508	2.6523 352
34	1.4025 770	1.6589 964	1.9606 760	2.3153 221	2.7319 053
35	1 4166 028	1.6838 813	1.9998 895	2.3732 051	2.8138 624
36	1.4307 688	1.7091 395	2.0398 873	2.4325 353	2.8982 783
37	1.4450 765	1.7347 766	2.0806 850	2.4933 487	2.9852 266
38	1.4595 272	1.7607 983	2.1222 987	2.5556 824	3.0747 834
39	1.4741 225	1.7872 103	2.1647 447	2.6195 744	3.1670 269
40	1.4838 637	1.8140 184	2.2080 396	2.6850 638	3.2620 377

TABLE.

Yrs.	3½ per ct.	4 per ct.	4½ per ct.	5 per ct.	5½ per ct.
1	1.0350 000	1.0400 000	1.0450 000	1 0500 000	1.055 •
2	1.0712 250	1.0816 000	1.0920 250	1.1025 000	1.113
3	1.1087 178	1 1248 640	1.1411 661	1 1576 250	1.174
4	1.1475 230	1.1698 585	1.1925 186	1 2155 063	1.239
5	1.1876 863	1.2166 529	1 2461 819	1.2762 816	1 307
6	1.2292 553	1.2653 190	1.3022 601	1.3400 956	1.379
7	1.2722 792	1.3159 317	1.3608 618	1.4071 004	1.455
8	1 3168 090	1.3685 690	1.4221 006	1.4774 554	1.535
9	1.3628 973	1.4233 118	1.48 0 951	1.5513 282	1.619
10	1.4105 987	1.4802 442	1.5529 694	1.6288 946	1.708
11	1.4599 697	1.5391 540	1.6228 530	1.7103 391	1.802
12	1.5110 686	1.6010 322	1.6958 814	1.7958 563	1.901
13	1.5639 560	1.6650 735	1.7721 9 1	1.8856 4.1	2.006
14	1.6186 945	1.7316 764	1.8519 449	1.9799 316	2.116
15	1.6753 488	1.8009 435	1 9352 824	2.0789 282	2.232
16	1.7339 860	1.8729 812	2.0223 701	2.1828 746	2.355
17	1.7946 755	1.9479 005	2.1133 768	2.2920 183	2.485
18	1.8574 892	2.0258 165	2.2084 787	2.4066 192	2.621
19	1.9225 013	2.1068 491	2.3078 603	2.5269 502	2.766
20	1.9897 888	2.1911 231	2.4117 140	2.6532 977	2.918
21	2.0594 314	2.2787 680	2.5202 411	2.7859 626	3.078
22	2.1315 115	2.3699 187	2.6336 520	2.9252 607	3.248
23	2.2061 144	2.4647 155	2.7521 663	3.0715 238	3.426
24	2 2833 284	2 5633 041	2.8760 138	3.2250 999	3.615
25	2.3632 449	2 6658 363	3.0051 314	3.3863 549	3.813
26	2.4459 585	2.7724 697	3 1406 790	3.5565 727	4.023
27	2.5315 671	2.8833 685	3.2820 095	3.7344 563	4.244
28	2.6201 719	2.9987 033	3.4296 999	3.9201 291	4.478
29	2.7118 779	3 1186 514	3.5840 364	4.1161 356	4.724
30	2.8067 937	3.2433 975	3.7453 181	4.3219 424	4.984
31	2.9050 314	3.3731 334	3.9138 574	4.5380 395	5.258
32	3.0067 075	3.5080 587	4 0899 810	4.7649 415	5.517
33	3.1119 423	3.6183 811	4.2740 301	5.0031 885	5.832
34	3.2208 603	3.7943 163	4.4663 615	5.2533 480	6.174
35	3 3335 904	3.9460 889	4.6673 478	5.5160 154	6.514
36	3.4502 661	4.1039 325	4.8773 784	5.7918 161	6.872
37	3.5710 254	4.2680 898	5.0968 604	6.0811 069	7.250
38	3.6960 113	4.4388 134	5.3262 192	6.3854 773	7.649
39	3.8253 717	4.6163 659	5.5658 990	6.7017 512	8.069
40	3.9592 597	4.8010 206	5.8163 645	7.0399 887	8.513

•NOTE.—As the 5½ table is seldom used in business, we only extend it three figures.

MICROCOPY RESOLUTION TEST CHART
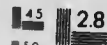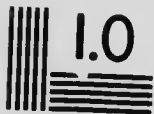
(ANSI and ISO TEST CHART No. 2)

TABLE.

Yrs.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.
1	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000
2	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000
3	1.1910 160	1·2250 430	1.2597 120	1.2950 290	1.3310 000
4	1.2624 770	1.3107 960	1 3604 890	1.4115 816	1.4641 000
5	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100
6	1.4185 191	1.5007 304	1.5668 743	1.6771 001	1.7715 610
7	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171
8	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888
9	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477
10	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425
11	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167
12	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284
13	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712
14	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983
15	2.3965 582	2.7590 315	8.1721 691	3.6424 825	4.1772 482
16	2.5403 517	2.9521 638	3.4259 426	8.9703 059	4.5949 730
17	2.6927 728	3.1588 152	3.7000 181	4.3276 834	5.0544 703
18	2.8543 392	3.3799 323	8.9960 195	4.7171 204	5.5599 173
19	3.0255 995	3.6165 275	4.8157 011	5.1416 613	6.1159 390
20	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000
21	3.3995 636	4.1405 624	5.0338 337	6.1038 077	7.4002 499
22	8.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749
23	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024
24	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327
25	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059
26	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765
27	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942
28	5.1116 867	6.6488 384	8.6271 064	11.1671 895	14.4209 936
29	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930
30	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023
31	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425
32	6.4533 867	8.7152 708	11.7870 830	15.7633 288	21.1137 768
33	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544
34	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699
35	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 869
36	8.1472 520	11.4239 422	15.9681 718	22.2512 250	80.9126 805
37	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486
38	9.1542 524	13.0792 714	18.6252 756	26.4366 805	87.4043 434
39	9.7035 075	13.9948 204	20.1152 977	28·8159 817	41.1447 778
40	10.2857 179	14.9744 578	21.7245 215	81.4094 200	45.2592 556

NOTES 1.—If each of the numbers in the table be diminished by 1, the remainder will denote the compound interest of $1, instead of its amount.

2. If interest is compounded semi-annually, take ½ the given rate and twice the number of years; if compounded quarterly, take ¼ the given rate for 4 times the number of years, etc.

3. The amount for any number of years not given in the table may be computed by finding the products of the amounts for any two numbers of years whose sum equals the given time.

4. To find the amount of any given principal at compound interest, multiply the principal by the amount of $1 for the time and rate.

5. If the time contains parts of a period, as months or days, find the amount due for the full periods, and to this add its interest for the months or days.

EXERCISE 81.

Find the amount and the compound interest of—

1. $312 for 3 years at 6%; $800 for 4 years at 4%.

2. $640 for 4 years at 5%; $376 for 3 years 8 months and 15 days at 6%.

3. $1,200 for two years 4 months at 4½%; for 3 years 8 months at 7%.

4. $400 for 1 year 6 months at 7%, payable semi-annually.

5. $2,000 for 1 year at 8%, payable quarterly.

6. $1,000 for 23 years at 7%.

7. $750 for 12 years at 3%.

8. $920 for 8 years at 5%.

9. $2,600 at 6%, from Jan. 1st, 1870, to Jan. 1st, 1894.

10. $1,410 at 8%, from March 30th, 1889, to August 15th, 1894.

11. What is the amount of $3,500 for 5 years at 5% compound interest?

12. What is the amount of $1,350 for 12 years at 7%?

13. What is the compound interest of $1,469 for **15** years at at 3 %.

14. What is the compound interest of $2,500 for **24** years at 6 %.

15. What is the compound interest of $1,650 for **30** years at $3\frac{1}{2}$ %.

16. What is the amount of $1,800 for 3 years at 6 % compound interest, payable semi-annually ?

17. What is the amount of 1,500 for 2 years, at 12 % compound interest, payable quarterly ?

18. What is the compound interest of $5,000 for 2 years, at 6 %, if the interest is due annually ?   If the interest is payable half-yearly ?   If the interest is payable quarterly ?

19. By how much does interest compounded semi-annually exceed simple interest, on $400, for 2 years 6 months at 7 % ?

20. What is the amount of $2,400 from May 1st, 1887, to Jan. 14th, 1890, interest compounded half-yearly, at 5 % ? What is the amount, if the interest is compounded yearly ? What is the amount, at simple interest ?

21. What is the compound interest on $7,325 for 2 years 2 months at 7 % ?

22. Find the compound interest on $3,333 at $3\frac{1}{3}$ % semi-annually for 1 year 7 months.

23. What amount was due March 25th, 1886, on $1,512 borrowed Jan. 25th, 1885, with compound interest at $1\frac{1}{2}$ % quarterly ?

24. What is the amount of $4,615 at compound interest for 2 years 5 months at 8 % ?

25. Find the amount of $3,500 at compound interest from Oct. 29th, 1888, to Nov. 15th, 1889, at 2 % quarterly.

26. How much greater, at compound than at simple interest, would be the amount of $1,568 in 3 years 8 months at 6 %?

27. Find the amount due Sept. 18th, 1889, on $450, loaned Sept. 18th, 1886. Interest compounded annually at 4½ %.

28. What is the interest, compounded every six months, of $600 from July 1st, 1890, to July 1st, 1894, at 8 %?

29. What will $16,000 invested Jan. 14th, 1883, amount to Sept. 16th, 1893, at 10 %, interest payable half-yearly?

30. How much must a lady invest when her son is 12 years old, that, on arriving at 21, he may have $25,000, the rate being 6 % and the interest compounded semi-annually?

To find the principal or present worth of an amount at compound interest, divide the given amount by the amount of $1 for the given time and rate at compound interest, (i. e., the same method as finding the present worth as in simple interest.)

31. What is the present worth of $6,036.25 due in 8 years, at 6 % compound interest?

32. What principal at compound interest will amount to $2,375.92 at 5 % in 14 years?

33. What is the present worth of $2,521.81 due in 14 years, at 6 % compound interest?

What principal at 10 % will amount to $265.83 in 10 years, interest payable semi-annually?

35. What sum at compound interest at 4 %, interest due annually, will amount to $1,000 in three years?

36. What sum would have to be put out at 6 %, interest payable every six months, to produce $543.3456 compound interest in 3 years?

37. At what rate would $500 have to be loaned, to amount to $1079.46 in 10 years, the interest being compounded annually?

DISCOUNT.

354. Discount is an abatement or allowance made from the amount of a debt, a note or other obligation.

355. The term discount is often used without reference to *time* to imply an abatement at a certain rate per cent. on a price asked.

356. When Time enters in as an element, two kinds of Discount are distinguished, viz.: **True Discount** and **Bank Discount.**

TRUE DISCOUNT.

357. The Present Worth of a debt, note or other obligation, payable at a future time without interest, is such a sum as, being placed at interest at a legal rate, will amount to the given sum when it becomes due.

358. True Discount is the difference between any sum of money payable at a future time and its present worth, and is equal to the interest on the present worth.

ILLUSTRATION.—Suppose A. owes B. $106 payable a year hence without interest. The current rate of interest being 6 %, the present worth of the debt is $100, because that sum would amount to $106 in 1 year at 6 %.

The true discount is $106 – $100 or $6, which is evidently the interest on the present worth $100, for 1 year at 6 %.

359. To find the present worth and true discount, the face of the debt, rate per cent. per annum, and time being given.

Example.—Find the present worth and true discount of a debt of $1,360 for 6 years at 6 %.

SOLUTION.

Interest on $1.00 for 6 years at 6 % = $.36

∴ $1.36 has for its present worth $1.00

$1. " " $\dfrac{1.00}{1.36}$

$1,360 " " $\dfrac{100 \times 1,360}{136}$

= $1,000, present worth.

$1,360 − $1,000 = $360, true discount.

RULE.

1. Divide the face of the debt by the amount of $1 for the given time, and the quotient will be the present worth.

2. Subtract the present worth from the face of the debt, and the remainder will be the true discount.

360. To find the true discount of a debt.

Example.—Find the discount on $1,781.40, due 4 years hence, interest at 5 %.

SOLUTION.

Interest on $1.00 for 4 years at 5 % = $.20.

∴ $1.20 has for its discount $.20

$1 " " $\dfrac{.20}{1.20}$

$1,781.40 " " $\dfrac{1,781.40 \times 20}{1.20}$

= $296.90, discount.

RULE.

Divide the interest of the debt for the given time and rate by the amount of $1, and the quotient will be the discount.

EXERCISE 82.

Find the present worth and true discount—

1. Of $475.50 at 7 % due in 2 yr. 9 mo.
2. Of $661.50 at 7 % due in 3 yr. 9 mo.
3. Of $500 at 5 % due in 11 mo.
4. Of $4,261.33 at 4½ % due in 1 yr. 6 mo.
5. Of $1,575 at 7 % due in 1 yr. 3 mo 15 da.
6. Of $860 at 6½ % due in 90 da.
7. Of $678.40 at 4½ % due in 16 mo.
8. Of $715.20 at 3¼ % due in 1 yr. 4 mo.
9. Of $990.75 at 10 % due in 73 da.
10. Of $1,315.45 at 8 % due in 219 da.

Find the true discount on—

> 11. $1,500 due in 3 yr. 6 mo. at 6 %.
> 12. $3.550 due in 90 da. at 7 %.
> 13. $4,960.75 due in 18 mo. at 6½ %.
> 14. $960.40 due in 73 da. at 10 %.
> 15. $625.13 due in 8 mo. at 7 3/10 %.

16. Which is the better, to buy flour at $8 a barrel, on 6 months' credit, or $7.50 cash, money being worth 8 %?

17. What is the difference between the interest and true discount of $1,650, at 6 %, due in 8 months?

18. Which is worth the most, $610 in 12 months, $620 in 6 months, or $600 cash, money being worth 8 %?

19. Bought a farm for $2,964.12 ready money, and sold it again for $3,665.20, payable in 1 year, 6 months. How much would be gained in ready money, reckoning true discount at 8 %?

20. Having bought a house for $5,048 cash, I at once sold it for $7,000, to be paid in 18 months without interest. If money is worth 8 % per annum, did I gain or lose, and how much?

21. A man bought a flouring mill for $10,000 cash, or for $12,000 payable in 6 months, or $15,000 payable in 1 year 3 months. He accepted the latter offer; did he gain or lose, and how much, money being worth to him 10 %.

22. Goods to the amount of $510 were sold on 6 months' credit. If the selling price was $30 less than the goods cost, and money is worth 6 % per annum, how much was the loss and the per cent. of loss?

23. A speculator bought 120 bales of cotton, each bale containing 488 pounds, at 9 cents a pound, on a credit of 9 months for the amount. He immediately sold the cotton for $6,441.60 cash, and paid the debt at 8 % discount; how much did he gain?

24. How much must be discounted for the present payment of a debt of $8,741.50, $2,000 of which is on credit for 5 months; $3,000 for 8 months, and the remainder for 15 months, money being worth 10% per annum?

25. A merchant bought a bill of goods for $2,150, on 6 months' credit, and the seller offered to discount the bill at 5%, for cash. If money is worth $7\frac{1}{2}$% per. annum, how much would the merchant gain by accepting the seller's offer.

26. A merchant bought a bill of goods on 6 months' credit amounting to $1,450. What will he gain by present payment of the bill, if allowed 5% off, money being worth $\frac{3}{4}$% a month?

27. A dealer bought grain to the amount of $2,700, on 4 months' credit, and immediate y sold it at an advance of 10%. If from the proceeds of the sale he paid the present worth of his debt at a rate of discount of 8% per annum, how much did he gain?

28. After carrying a stock of silk for 4 months, I sold it at an advance of 80% on first cost, extending to the purchaser a credit of one year without interest. If money is worth 5% per annum, what was my per cent. of profit or loss?

29. Bought a house for $3,786 ready money, and sold it for $5,250, payable in 1 year 6 months. How much would be gained in ready money, discounting at the rate of 8%? How much, discounting at the rate of 6%?

80. The asking price of a hardware stock is $5,460, on which a trade discount of 25%, 15%, and 10% is offered, and a credit of 90 days on the selling price. If money is worth $5\frac{1}{2}$%, what sum should be discounted for the payment of the bill ten days after its purchase?

BANK DISCOUNT.

361. **Bank Discount** is a deduction usually made by banks for paying a note before it is due. This deduction is the interest on the face of the note for the time it has to run, including three additional days, called *Days of Grace.*

362. **Days of Grace** are three days usually allowed for the payment of a note, after the expiration of the time specified in the note.

363. The **Proceeds** of a note is the amount received by the holder from the bank when the note is discounted. It is the amount of the note at maturity less the interest on that amount for the term of discount.

ILLUSTRATION.—A person holds a note for $1,000 payable in 73 days, including the days of grace. Wishing to use the money immediately he indorses the note and offers it to his bank for discount. If both maker and indorser are considered responsible, the bank retains the note, and if the legal rate is 6 %, deducting $12 (the interest of $1,000 for 73 days) pays over the balance $988 to the holder. The note is thus discounted; the bank discount is $12; the proceeds are $988.

364. **Negotiable paper** commonly includes all orders and promises for the payment of money, the property interest in which may be negotiated or transferred by indorsement.

365. A **Promissory Note** is a written, or partly written and partly printed, agreement to pay a certain sum of money, either on demand or at a specified time.

366. The **Face of a Note** is the sum for which it is given.

367. The **Maturity** of a note is the expiration of the time including days of grace.

368. The **Time** in bank discount is always the number of days from the date of discounting to the date of maturity.

369. The **Term of Discount** is the time the note has to run after being discounted.

370. **Value of a note** at its maturity is its face, if it does not bear interest; if the note is given with interest, its value at maturity is its face plus the interest for the time, including days of grace.

NOTES 1. In Ontario and Manitoba promissory notes and drafts are governed by commercial law as to days of grace, protest and notice. In these provinces notes do not bear interest, unless expressed on the face, until after maturity, from which period however, the legal rate—six per cent.—can be recovered. If a rate of interest is named on face of note, and in addition to the words, " with interest at the rate of —," the words " until paid " are added, the *named* rate can be collected till date of payment of note, otherwise, without these words (or others of similar import), the expressed rate only during period of currency, and legal rate from maturity till payment can be collected.

2. The person who promises to pay is called the *maker;* the person in whose favour the promise is made is called the *payee,* and the person who writes his name on the back of the note is called an *endorser* and is responsible for the payment of the note.

3. A note is *non-negotiable* when it is made payable only to the person whose name is mentioned in it. Such a note cannot be transferred ; it must be held by the payee until it falls due.

4. A note is *transferable* only when it is made payable to the *order* of the payee or to *bearer.*

5. The maker of a note is the one *primarily* liable, but each endorser is liable to succeeding holders in default of payment by the maker. A subsequent endorser is not liable to a prior endorser.

6. When the payee writes only his name it is termed a *blank* endorsement, and it has the effect of rendering the note payable afterwards to any *bona fide* holder. The payee thus becomes the endorser and as such is responsible for payment.

7. If the payee writes above his signature, " Pay to the order of A. B." it is called a *full endorsement.* In this case A. B. will have to endorse it before he can negotiate it.

8. If the payee writes above his signature, " Pay to A. B only," it is termed a *restrictive* endorsement.

9. If the endorser does not wish to render himself liable for payment he should write, " Without recourse to me," above his name. This is called a *qualified* endorsement.

10. When a note is made payable to bearer it is negotiable without endorsement, delivery being all that is necessary.

11. In calculating the date of maturity of a note, the three days grace must be allowed after the time expressed, that is, it falls due on the third day after its term has expired. The day on which the note is dated is not counted in computing the date of maturity.

12. When a note becomes due which happens on the third day after the time expressed, it must be presented for payment during business hours at the place mentioned in it. If no place is stated it should be presented at the maker's place of business or at his residence.

13. Should the maker refuse to pay it, the proper demand being made, it is the duty of the holder to *give due notice* to all the parties to it. He may have it protested if he chooses.

371. A Protest is a declaration in writing by a Notary Public, giving legal notice to the maker and endorsers of a note of its non-payment. In Ontario a note must be protested on the day of its maturity, otherwise the endorsers are released from all obligation to pay the note.

NOTES 1. When a note becomes due on Sunday or a legal holiday, it must be paid on the day following.

2. The person paying a note has a right to a receipt, which is usually written on the back of the note.

3 The person who pays a note has a right to it as his voucher, if it is negotiable, but not otherwise.

4. When a note is made payable with interest it bears interest from the date of it, and not merely from its maturity. In such a case the interest is part of the debt.

5. When a note bears interest, the discount is computed on the face of the note with the interest added.

6. When the term of a note is given in months, calendar months are meant and no allowance is made for a deficiency in the number of days in any month. This being the case the student will see that four notes drawn at 2 months and bearing dates, Dec. 28, Dec. 29, Dec 30, Dec. 31, respectively, will become due on the same day, viz.: March 3rd, of next year.

7. When the time is expressed in *days*, the day of maturity is found by counting forward from the date of the note the *number of days* named in the note, and the three days of grace. When the time is in months, the day of maturity is found by counting the number of calendar months, and the three days of grace.

372. Banks in discounting notes always reckon discount for an exact number of days from the time of discounting to date of maturity. Thus on a note maturing July 5th, and discounted May 25th, the term of discount would be reckoned as follows: 6 days in May, + 30 days in June + 5 days in July = 41 days.

373. To find the bank discount and proceeds of a note.

EXAMPLE 1.—Find the bank discount and proceeds of a note for $681, due 90 days hence, at 7%.

SOLUTION.

The term of discount is 93 days.

Interest of $681 for 93 days at 7% = $12 20 = Bank disco
$$\$681 - \$12\ 20 = \$671\ 80 = \text{Proceeds.}$$

EXAMPLE 2.—A note of $375 dated October 23rd, payable in 30 days, with interest at 7%, is discounted at a bank November 12th at 8%. Find the proceeds.

SOLUTION.

The date of maturity is November 26th.

The note bears interest for 34 days.

$3.75	=	Int. for 60 da. at 6%.	
1.875	=	" 30 "	
.1875	=	" 3 "	
.0625	=	" 1 "	
2.125	=	" 31 "	
.354	=	" "	1
$2.479	=	Int. for 34 da. at 7%.	

$2.479 (360 da. int.) less $\frac{1}{73}$ of $2.479 = $2.45 (actual int.)

The amount of note at maturity is $375 + $2.45 = $377.45.

The note is held by the bank from November 12th until November 26th. or 14 days.

$$\$3.7745 \;\;= \text{ Int. for } 60 \text{ da. at } 6\%.$$
$$\frac{.7549}{} \;\;= \;\;\text{''}\quad \overline{12}\quad \text{''}$$
$$\frac{.1258 \;+ \;=}{} \;\;\text{''}\quad 2\quad \text{''}$$
$$\frac{.8807}{} \;\;= \;\;\text{''}\quad \overline{14}\quad \text{''}$$
$$\frac{.2935 \;+ \;=}{} \;\;\text{''}\quad \text{''}\quad 2$$
$$\$1.1742 \;\;= \text{ Int. for } 14 \text{ da. at } \overline{8}\%.$$

$\$1.174$ (360 da. int.) less $\frac{1}{3}$ of $\$1.174 = \$1.15$ (actual int.)

$$\$377.45 = \text{ Amt. of note at maturity.}$$
$$\frac{1.15}{} = \text{ Disct. for time held by bank.}$$
$$\$376.30 = \text{ Proceeds.}$$

EXAMPLE 3.—A note of $750 dated August 4th, 1888, payable in 6 months with interest at 6%, is discounted at a bank October 20th, at 7%. Find the proceeds.

SOLUTION.

The date of maturity is February 7th, 1889.

The note bears interest from August 4th, 1888, to February 7th, 1889, or 187 days.

$$\$7.50 = \text{ Int. for } 60 \text{ da. at } 6\%$$
$$\frac{22.50}{} = \;\;\text{''}\quad 180\quad \text{''}$$
$$.75 = \;\;\text{''}\quad 6\quad \text{''}$$
$$\frac{.125}{} = \;\;\text{''}\quad 1\quad \text{''}$$
$$\$23.375 = \;\;\text{''}\quad 187\quad \text{''}$$

$\$23.375$ (360 da. int.) less $\frac{1}{3}$ of $\$23.375 = \$23.06$ (actual int.)

The amount due at maturity is $750 + \$23.06 = \$773.06$.

The note is held by the bank from October 20th, '88, to February 7th, '89, or 110 days.

$$\$7.7306 = \text{ Int. for } 60 \text{ da. at } 6\%$$
$$3.8653 = \;\;\text{''}\quad 30\quad \text{''}$$
$$\frac{2.5763 \;+ \;=}{} \;\;\text{''}\quad 20\quad \text{''}$$
$$\$14.1727 = \;\;\text{''}\quad 110\quad \text{''}$$
$$\frac{2.3621 \;+ \;=}{} \;\;\text{''}\quad \text{''}\quad 1$$
$$\$16.5348 = \text{ Int. for } 110 \text{ da. at } \overline{7}\%.$$

$\$16.53$ (360 da. int.) less $\frac{1}{3}$ of $\$16 53 = \$16.31$ (actual int.)

$$\$773.06 = \text{ Amt of note at maturity.}$$
$$\frac{16.31}{} = \text{ Disct for time held by bank.}$$
$$\$756.75 = \text{ Proceeds.}$$

EXAMPLE 4.—Find *day of maturity*, the *time to run*, the discount, and proceeds of the following note:

$1,800.   OTTAWA, February 3rd, 1889.

　　　Five months after date, I promise to pay John Craig, or order, the sum of One Thousand Eight Hundred Dollars, value received, with interest at 6%.

　　　　　　　　　　　　　　　　　　　　　　THOMAS COWAN.

Discounted May 22nd, 1889, at 7%.

SOLUTION.

Date of maturity will be 5 months and 3 days from February 3rd, 1889,—or July 6th, 1889.

The *time to run* will be the interval between the date of discount, May 22nd, and July 6th,—or 45 days.

As the note bears interest, the discount must be computed on the *amount* of $1,800, from February 3rd to July 6th, or 153 days.

Interest on $1,800 for 153 days at 6 % = $45.27 +

The amount of note at maturity = $1,800 + $45.27 = $1,845.27.

The note is held by the bank from May 22nd, to July 6th, or 45 days.

Interest on $1,845.27 for 45 days at 7 % = $15.92 = *discount.*

Proceeds = $1,845.27 − $15.92 = $1,829.35.

RULE.

1. For the bank discount, find the interest on the face of the note (or, if the note bears interest, on the amount due at maturity), at the given rate, from the date of discount to the date of maturity.

2. For the proceeds, subtract the bank discount from the face of the note (or, if the note bears interest, from its amount).

EXERCISE 83.

Find the bank discount and proceeds of a note for—

1.	$440.00, payable in 90 da.,	discounted at 6 % on the day drawn
2.	$500.00,	"	60	"	9 %	"
3.	$1,000.00,	"	45	"	9 %	"
4	$140.25,	"	80	"	5 %	"
				4½ %	"

Find the date of maturity and proceeds of the following notes:

	DATE OF NOTE.	TIME.	FACE.	DATE OF DISCOUNT.	RATE OF DISCOUNT.
5.	January 20 ..	90 da. ...	$2,500 ..	January 20..	6 %.
6.	May 7	60 " ..	$1,200 ..	May 31	7 %.
7.	June 4	4 mos..	$3,600 ..	July 18	8 %.
8.	July 27	60 da ..	$8,200 ..	September 2.	6 %.
9.	November 1.	90 " ..	$6,000 ..	November 28	8 %.
10.	May 27	6 mos..	$4,880 ..	August 15 ..	5 %.

Find the proceeds and date of maturity of the following notes discounted through a broker, his commission being $\frac{1}{4}$ % of the face of the notes :

	DATE OF NOTE.	TIME.	FACE.	DATE OF DISCOUNT.	RATE OF DISCOUNT
11.	February 18 .	4 mos..	$2,000 ..	February 18.	5 %.
12.	June 1	90 da...	$6,000 ..	June 12	6 %.
13.	January 10..	120 " ..	$5,500 ..	January 10..	7 %.
14	March 3.. ..	6 mos .	$8 700 ..	April 30.. ..	8 %.
15.	May 18	3 " .	$5,280 ..	May 18	4 %.
16	January 3 ..	60 da...	$9,000 ..	February 28.	6 %.

17. Find the proceeds of a note of $850, due in 3 months, at 6 % ?

18. Find the proceeds of a draft of $885, on 60 days, at 6 % ?

19. Find the maturity, the term of discount and the proceeds of a note of $5,250, on 60 days, dated July 1st, 1889, and discounted August 21st, 1889, at 5 %.

20. Find the difference between the true and bank discount on $6,000 for 1 year, allowing each 3 days grace, at 7 % ?

21. A merchant bought $6,800 worth of goods for cash, sold them on 4 months, at 15 % advance, and got the note discounted at 6 % to pay the bill.   How much did he make ?

22. $652.45.   OTTAWA, Jan. 25th, 1889.

Five months after date I promise to pay to the order of Charles Barrett six hundred and fifty-two and $\frac{45}{100}$ dollars. value received, with interest at six per cent.

Discounted at $4\frac{1}{2}$ %, Mar. 15.   WILLIAM KIMBALL.

23. $215.   PETERBOROUGH, Jan. 28th, 1889.

Thirty days after date, I promise to pay to the order of James Fogg two hundred and fifteen dollars, value received.

Discounted at 6 %, Feb. 3rd.   JOHN ROGERS.

24. $2,017.85.   GALT, Jan. 14th, 1889.

Three months after date I promise to pay to the order of
John Brown two thousand and seventeen and $\frac{85}{100}$ dollars,
value received.

Discounted at 10%, Mar. 1st.   TIMOTHY BRUCE.

25. $4,760   GUELPH, Jan. 1st, 1889.

Ninety days after date I promise to pay to the order of
James Pike four thousand seven hundred and sixty dollars,
value received.

Discounted at $7\frac{1}{2}$%, Feb. 15th.   WILLIAM CLEMENT.

26. $5,000.   BRANTFORD, Oct. 4th, 1889.

Six months after date I promise to pay to John Adams
or order five thousand dollars, value received, with interest
at seven per cent.

Discounted at 8%, Dec. 31st.   WILLIAM DUNN.

27. $9,040.   LONDON, Jan. 19th, 1889.

Sixty days from date I promise to pay to the order of
Charles Carroll nine thousand and forty dollars, value
received.

Discounted $5\frac{1}{2}$%, Feb. 16th.   JAMES MONROE.

28. $650.   BERLIN, Nov. 3rd, 1888.

Six months from date we jointly and severally promise to
pay to the order of Charles Full six hundred and fifty dol-
lars, value received, with interest at six per cent.

Discounted at 7%, Jan. 3rd, 1889.

                              JOHN HENDERSON.
                              JAMES HENDRICKS.

29. A note for $3,600 with interest, dated Jan. 15th,
1889, and payable 3 months after date, was discounted at
a bank Feb. 15th, the legal rate being 7%; with the pro-
ceeds was paid on account 40% of a bill due that day. How
much remained due on the bill ?

30. A merchant sold some goods that cost him $840. at a profit of 12%, and took in payment a four-month note dated May 15th, which after 52 days he got discounted at a bank for 7 %.   How much did he receive from the bank?

81. A merchant, having sold 200 barrels of flour at $6.80 a barrel, and having taken in pay ment a 30-day note, found, on getting the note discounted at a bank the day of its date for 7 %, that he had realized on the transaction a cash profit equal to 300 % on the bank discount.   What had the flour cost him per barrel ?

32. A person owing for 117 A. 5 sq. rd, of land, which he had bought for $32 an acre, paid on account the proceeds of a sixty-day note for $2,000, which he got discounted at a bank, for 7 %, on the day it was drawn.   How much remained due ?

33. I paid in cash $950 for an engine, and sold it the same day for $975, taking a 60-day note, which I discounted at a bank at 8 %.   What was my gain or loss ?

34. Perkins, Ince & Co's bank account is overdrawn $11,516.19 ; they now discount, at 6 %, a 90-day note for $3,975.21 ; a 60-day note for $5,514.25 ; a 30-day note for $1,546.19 ; a 20-day note for $2,546.85 ; proceeds of all to their credit at the bank.   What is the condition of their bank account after they receive credit as above ?

35. W. Darling & Co.'s bank account is overdrawn $12,915.47 ; they now discount, at 6 %, a 90-day note for $2,428.40 ; a 60-day note for $6,311.25 ; a 30-day note for $1,120.50 ; a 20-day note for $4,500; a 10-day note for $1,550.50 ; Proceeds of all to their credit at the bank. What is the condition of their bank account after they receive the above credits ?

374. To find the face value of a note that shall produce a given sum when discounted at bank.

EXAMPLE.—For how much must a note be drawn, payable in 70 days, that, when discounted at a bank at 8%, it may yield $1,968.

SOLUTION.

Bank discount of $1 for 73 days at 8% $.016.

$$\$1 - \$.016 = \$.984 \quad \text{proceeds of } \$1.$$

$$\$.984 = \text{proceeds of } \$1$$

$$\$1 = \text{"} \quad \frac{1}{.984}$$

$$\$1,968 = \text{"} \quad \frac{1,968}{.984} = \$2,000. \quad \text{Ans.}$$

RULE.

Divide the given sum by the proceeds of $1 for the given rate and time, and the quotient will be the face value of the note.

EXERCISE 84.

Find the face of note or draft—

PROCEEDS.		TERM OF DISCOUNT.	RATE OF INTEREST.
1.	$361.56	90 days	8 %.
2.	$394.40	70 "	7 %.
3.	$118.20	60 "	9 %.
4.	$595.20	48 "	6 %.
5.	$717.80	108 "	5 %
6.	$796.20	30 "	10 %.

BANK DISCOUNT.		TERM OF DISCOUNT.	RATE OF INTEREST.
7.	$2.80	84 days	5 %
8.	$36.00	135 "	8 %.
9.	$9.80	36 "	7 %.
10.	$4.90	73 "	6 %
11.	$12.60	146 "	10 %.
12.	$5.94	66 "	6 %.

13. What sum, due 73 days hence, at 7 %, should be discounted, so that the present payment may be $900 ?

14. What is the face of a note at 60 days, the proceeds of which, when discounted at bank at 6%, are $1,275 ?

15. If a merchant wishes to draw $5,000 at bank, for what sum must he give his note at 90 days, discounting a. 6 % ?

16. The avails of a note having 3 months to run, discounted at a bank at 7 %, were $276.84. What was the face of the note ?

17. For what sum must a note be drawn at 30 days, to ret $1,200 when discounted at 5 % ?

18. Find the face of a 6 months' note, the proceeds of which, discounted at 2 % a month, are $496.

19. Owing a man $575, I give him a 60 day note. What should be the face of the note, to pay him the exact debt. if discounted at $1\frac{1}{2}$ % a month ?

20. James T. Fisher buys a bill of merchandise in Montreal at cash price, to the amount of $1,486.90, and gives in payment his note at 4 months at $7\frac{1}{2}$ %. What must be the face of the note ?

375. Given, the rate of interest to find the corresponding rate of bank discount.

EXAMPLE.—A broker buys a 70 day note at such a discount that his money earns him 10 %. What is his rate per cent. of discount ?

SOLUTION.

70 day note = 73 days' time.

Interest on $100 for 73 days at 10 % = $2.

∴ Amount of $100 = $102.

$102 in 73 days gives $2 interest.

∴ 100 " 365 " $9\frac{41}{51}$ "

∴ Rate of discount = $9\frac{41}{51}$ %. Ans.

376. Given, the rate of bank discount, to find the corresponding rate of interest.

EXAMPLE.—What rate of interest is paid, when a note payable in 70 days is discounted at 10 % ?

SOLUTION.

10 day note = 73 days' time.

Interest on $100 for 73 days at 10 % = **$2.**

∴ Proceeds of $100 = $98.

$98 in 73 days gives $2 interest.

∴ 100 " 365 " 10\frac{13}{49}$ "

∴ Rate of interest = 10$\frac{13}{49}$ %. Ans.

EXERCISE 85.

1. What rate of interest is paid, when a note payable in 30 days is discounted at 6 %?

2. A speculator discounted a note due in 90 days, at 12 % per annum, what was the actual rate of interest received on the sum invested ?

8. If a note payable in 8 months without grace be discounted at 10% per annum, what will be the rate of interest ?

4. If a note for $500, maturing in 96 days, without grace, can be purchased for $12 less than its face, what is the rate of interest.

5. A broker discounted a note due in 4 months, without grace, at the rate of 6 % per annum, what was the actual rate of interest realized on the sum advanced ?

6. At what rate should a 3 month's note be discounted to produce 8 % interest ?

7. What rates of bank discount on 30 day notes correspond to 5, 6, 7, and 10 per cent. interest.

377. The bank discount exceeds the true discount by the simple interest on the true discount.

Bank discount = Interest on principal.

True discount = Interest on present worth of principal.

" " = Interest on (principal—true discount).

" " = (Interest on principal)—(interest on true discount).

" " = (Bank discount)—(interest on true discount).

or,

Let P = Principal; t = time; r = rate.

∴ P t r = Interest, or bank discount.

$$\frac{P t r}{1 + t r} = \text{True discount.}$$

$$P t r - \frac{P t r}{1 + t r} = \text{Difference B. D. and T. D.}$$

$$= \left(P - \frac{P}{1 + t r} \right) t r.$$

$$= \left(\frac{P t r}{1 + t r} \right) t r.$$

= Simple interest on the true discount.

B. D. on $100 for 1 yr. at 6% = $6

T. D.   "   "   " = $\dfrac{6}{1.06}$

Difference = $6 − $\dfrac{6}{1.06}$ = $\dfrac{36}{106}$

But $\dfrac{36}{106}$ is the simple interest on $\dfrac{6}{1.06}$ for 1 year at 6%.

= Simple interest on the true discount.

378. If the bank discount or simple interest on a sum of money for a given time and rate is $\frac{a}{b}$ of that sum, then the true discount will be $\frac{a}{a + b}$ of the sum.

If interest = $\frac{a}{b}$ of principal, then $a is interest on $b.

∴ $b (*i.e.* principal) + $a (*i.e.* interest) = $ (a + b) = Amount.

∴ $b is present worth of $ (a + b), and $a is the true discount of $ (a + b).

∴ True discount is $\dfrac{a}{a + b}$ of principal.

Thus:

Simple interest on $100 for 1 yr. at 6 % = $6.

i.e., the interest is $\frac{6}{100}$ of principal.

Then $6 is interest on $100.

∴ $100 of principal + $6 of interest = $106. Amt.

∴ $100 is present worth of $106,

and $6 is true discount of $106.

∴ True discount = $\dfrac{6}{106}$ of principal, *i.e.*, $\dfrac{6}{100 + 6}$ of principal.

EXERCISE 86.

1. The interest is $\frac{2}{7}$ of the principal, and the difference between the interest and discount is $8. Find the principal.

2. The interest is $5, the discount for the same time and rate is $4. Find the principal.

3. The interest is $2, and the difference between the interest and discount is $16\frac{2}{3}$ cents. Find the principal.

4. If the interest is $\frac{1}{4}$ of the principal, what fraction of the principal is the true discount?

5. The interest of a certain sum is $\frac{1}{7}$ of the principal. The amount is $640. Find the principal.

6. The difference between the interest and the discount on a sum of money for $1\frac{1}{2}$ years at 8% is $18. Find the sum of money.

7. Reckoning bank discount at 5%, a person would receive $21 less than the nominal value of a note which has a year to run. What would he receive for the note if true discount were deducted.

8. I have two notes (1 year to run) amounting to $38; both are discounted at 20% one at bank discount, the other at true discount, the entire discount being $7. Find the face of the note on which bank discount was allowed.

9. The interest on a certain sum for 6 years is $261, and the discount for the same time is $180. Find the sum and rate per cent.

10. If $4 is allowed as 12 months' discount off a bill for $76, and at the same rate $7 be allowed off a bill for $91, for how long was the latter sum discounted?

11. The interest on a sum of money for 5 years is $140, and the discount for the same time and rate $100. Find the sum and rate per cent.

12. The interest on a certain sum of money is $180, and the discount on the same sum for the same time and same rate is $150. Find the sum.

PARTIAL PAYMENTS.

379. **Partial Payments** are part payments made at different times of notes, acceptances, bonds, mortgages or other written and interest-bearing instruments which the debtor is obliged to pay.

380. **Indorsements** are the acknowledgments of the payments written on the back of the note, acceptance, etc., stating the amount and date of the payment.

Special receipts are sometimes given for partial payments made, instead of writing the acknowledgment on the back of the obligation.

381. The method of computing interest when partial payments have been made is based on the following principles :

1. Payments must be applied first to discharge accrued interest, and then the remainder, if any, towards the discharge of the principal.

2. Only unpaid principal can draw interest.

EXAMPLE 1.—A note the face of which was $3,600, bearing interest at 6%, was given October 17th, 1884, and settled February 14th, 1889. Find the balance due, the following indorsements having been made : March 3rd, 1885, $600; October 25th, 1886, $1,000; December 6th, 1888, $2,400.

SOLUTION.

Face of note	$3,600.00
Interest to date of first payment (137 da.)	81.07
Amount of principal and interest at time of first payment ..	$3,681.07
First payment (Mar. 3rd, 1885)	600.00
Remainder after deducting first payment	$3,081.07
Interest to date of second payment (1 yr. 236 da.)..	304.39
Amount due at time of second payment	$3,385.46
Second payment (Oct. 25th, 1886)	1,000 00
Remainder after deducting second payment	$2,385.46
Interest to date of third payment (2 yr. 42 da.)	302.72
Amount due at time of third payment	$2,688.18
Third payment (Dec. 6th, 1888)	2,400 00
Remainder after deducting third payment	$288.18
Interest to time of settlement (70 da.)	3.31
Balance due at time of settlement (Feb. 14th, 1889)	$291 49

EXAMPLE 2.—

$1,000.   *Toronto, May 15th, 1881.*

     Two years after date, for value received, I promise to pay to Elwin J. Mills, or order,   thousand dollars, with interest at 7%.

           James H. Ross.

On this note were indorsed the following payments :

September 20th, 1882	$150.60
October 25th, 1884	200.90
July 11th, 1886	75.20
September 20th, 1887	112.10
December 5th, 1888	105.00

What remained due May 20th, 1889 ?

SOLUTION.

Face of note	$1,000.00
Interest to Sept. 20th, 1882 (1 yr. 128 da.)	94.55
Amount of principal and interest at time of first payment ..	$1,094.55
First payment (Sept. 20th, 1882)	150.60
Remainder after deducting first payment	$943.95
Interest from first payment to Oct. 25th, 1884 (2 yrs. 35 da.)	138 49
Amount due at time of second payment	$1,082.44
Second payment (Oct. 25th, 1884)	200.90
Remainder after deducting second payment	$881.54
Interest from second payment to Dec. 5th, 1888 (4 yrs. 41 da.)	253.76
Amount due at time of fifth payment	$1,135.80
Third payment, less than interest due $75.20	
Fourth " 112.10	
Sum of third and fourth payments, less than interest due $187.30	
Fifth payment 105.00	
Sum of third, fourth, and fifth payments	$292.30
Remainder after deducting third, fourth, and fifth payments..	$843.00
Interest from fifth payment to May 20th, 1889 (166 da.) ..	26.84
Balance due at time of settlement (May 20th, 1889)	$869.84

RULE.

382. *1. Compute the interest on the given principal from the date of the note to the time of the first payment. If this payment equals or exceeds the interest due, subtract the payment from the amount, and treat the remair   as a new principal.*

2. If any payment is less than the accrued interest, compute the interest on the same principal, to a date when the sum of the payments equals or exceeds th interest then due, and subtract the sum of the payments from the amount, and regard the remainder as a new principal.

3. Proceed in the same manner with the remaining payments, until the date of settlement.

EXERCISE 87.

1. A note of $4,560, dated Jan. 22nd, 1887, and drawing interest at 7%, had payments endorsed upon it as follows: Jan. 10th, 1888, $2,000; Aug. 31st, 1888, $500; Jan. 15th, 1889, $1,200; Mar. 4th, 1889, $860. Find the balance due June 15th, 1889.

2. On a claim for $3,000, dated Aug. 12th, 1885, and bearing interest at 7%, payments were made as follows: Dec. 15th, 1885, $30; April 1st, 1887, $550; Jan. 20th, 1888, $85; June 12th, 1888, $1,651.50. How much was due May 30th, 1889?

3. I held a bond against Ira Fox, dated May 1st, 1885, for $4,000, on interest at 6%. The following payments were endorsed on this bond: May 21st, 1886, $800; June 10th, 1887, $1,200; Aug. 10th, 1888, $1,500. What was due May 1st, 1889?

4. On a mortgage for $5,500, dated Aug. 13th, 1882, and bearing 6% interest, the following payments were made: Jan. 1st, 1883, $100; Mar. 2nd, 1883, $25; Aug. 13th, 1885, $2,500; Dec. 19th, 1887, $2,500; Mar. 1st, 1889, $500. How much was required for full settlement, Mar. 11th, 1889?

5. Required, the balance due May 1st, 1889, on a note for $4,119.82, at 6%, dated June 25th, 1888, on which a payment of $150.25 was made Aug. 1st, 1888, and a payment of $21.19 on the 15th of each subsequent month.

6. On a loan of $2,000, made Mar. 19th, 1885, and bearing 6% interest, payments were made as follows: Nov. 1st, 1886, $500; May 3rd, 1888, $700; Feb. 1st, 1889, $1,000. How much will be required for settlement in full, Mar. 2nd, 1889?

7. I gave a mortgage for $10,000, May 9th, 1882, bearing 6% interest, and made thereon the following payments: Sept. 19th, 1882, $500; Jan. 1st, 1883, $500; April 25th, 1884, $4,000; Oct. 15th, 1884, $4,000; May 1st, 1889, $3,525. How much was due at final settlement, June 2nd, 1889?

8. A bond was given Mar. 3rd, 1883, for $8,650, with interest at 8%. The following payments were made on account: April 17th, 1884, $1,000; May 10th, 1885, $550; June 23rd, 1885, $540; Dec. 22nd, 1886, $803; Feb. 15th, 1887, $25; Mar. 18th, 1887, $25; April 19th, 1887, $115; April 25th, 1888, $146. How much remained due, May 7th, 1889?

9. A note of $1,520, dated May 20th, 1888, and drawing interest at 6%, had payments endorsed upon it as follows: Oct. 2nd, 1888, $300; Feb. 26th, 1889, $25; April 2nd, 1889, $570; Aug. 8th, 1889, $600. Find the amount due Dec. 6th, 1889.

10. A note of $2,000, dated Jan. 22nd, 1889, and drawing interest at 6%, had payments endorsed upon it as follows: May 20th, 1889, $100; July 20th, 1889, $325; Nov. 2nd, 1889, $20; Dec. 23rd, 1889, $125. Find the balance due Mar. 1st, 1890.

11. A note of $1,662.50, dated Jan. 15th, 1888, and drawing interest at $6\frac{1}{2}$ %, had payments endorsed upon it as follows : April 30th, 1888, $25 ; June 24th, 1888, $25 ; Sept. 2nd, 1888, $625 ; Jan. 31st, 1889, $700. Find the balance due May 12th, 1889.

12. Oct. 1st, 1885, a note for $1,000 was given, payable in 4 years, with 6 % interest. A payment of $50 was made 1 year from date ; a payment of $250 was made 1 year 6 months from date ; a payment of $224 was made 2 years from date ; a payment of $20 was made 2 years 8 months from date ; a payment of $110 was made 2 years 10 months from date.   How much remained due at the maturity of the note ?

13. A mortgage for $5,400 was dated Strathroy, Jan. 1st, 1886, and endorsed as follows : May 22nd, 1887, $1,200 ; Feb. 9th, 1888, $150 ; Oct. 28th, 1888, $1,500. What was due Mar. 1st, 1889, interest 5 % ?

14. A note of $302.25, dated Aug. 4th, 1887, and drawing interest at $6\frac{1}{2}$ %, had payments endorsed upon it as follows : Oct. 14th, 1887, $100 ; July 21st, 1888, $100 ; Oct. 11th, 1888, $50 ; Jan. 18th, 1889, $50.   Find the amount due July 22nd, 1889.

15.   On the following note, payments were endorsed as follows ; Nov. 3rd, 1887, $50 ; Mar. 16th, 1888, $50 ; Oct. 1st, 1888, $50 ; Dec. 30th, 1888, $1,000 ; April 1st, 1889, $625.   How much was due, if paid in full, May 8th, 1889, money being worth 6 % ?

$1,600.00.   BRANTFORD, April 1st, 1887.
Three years after date, I promise to pay to the order of Silas Hopkins, one thousand six hundred dollars, value received.   JAS. MURRAY.

16. On the following note endorsements were made as follows : Aug. 1st, 1883, $350 ; Nov. 3rd, 1883, $1,000 ;

Mar. 20th, 1885, $600; Mar. 31st, 1885, $2,500; Dec. 11th, 1888, $2,000. What was the balance due Jan. 30th, 1889?

$6,500.00.   BROCKVILLE, Mar. 19th, 1882.

On demand, I promise to pay to the order of T. Gilmour, six thousand five hundred dollars, with interest at 6 %.

                         W. HINDSON.

17. The following note was settled Oct. 13th, 1888; a payment of $25 having been made Jan. 15th, 1887; one of $300, July 12th, 1887; and one of $200, April 1st, 1888. If money be worth 8 %, how much was due at final settlement?

$585.50.   GALT, Aug. 1st. 1886.

Six months after date, I promise to pay to Alex. Buchanan, or order, five hundred eighty-five and $\frac{50}{100}$ dollars, value received.   F. McHARDY.

18. $500.   ST. THOMAS, Feb. 1, 1888.

For value received, I promise to pay D. E. Broderick, or order, five hundred dollars three months after date, with interest at 7 %.   ·   JAMES MONROE.

Endorsed as follows, May 1, 1888, $40.

      "   Nov. 14, 1888, $8.

      "   April 1, 1889, $12.

      "   May 1. 1889. $30.

How much was due Sept. 16. 1889?

19. $5,000.   STRATFORD, May 1st, 1887.

Six months after date I promise to pay G. T. Smith, or order, five thousand dollars, with interest at 5 per cent., value received.   JOHN ADAMS.

Endorsed, Oct. 1st, 1887, $700.

      "   Feb. 7th, 1888, $45.

      "   Sept. 13th, 1888, $480.

What was the balance due Jan. 1st, 1889?

20. **$2,460.** Trenton, April 10th, 1887.

Four months after date I promise to pay W. H. Austin, or order, two thousand four hundred sixty dollars, with interest at 6 per cent., value received.

George G. Williams.

Endorsed, Aug. 20th, 1888, $840.
" Dec. 26th, 1888, $400.
" May 2d, 1889, $1,000.

How much was due Aug. 20th, 1889 ?

21. **$650.** Guelph, Jan. 1st, 1887.

For value received, I promise to pay Alexander Mo-Kenzie, or order, six hundred fifty dollars on demand, with interest at 6 per cent. George Law.

Endorsed, Aug. 13th, 1887, $100.
" April 13th, 1888, $120.

What was due on the note, Jan. 20th, 1889 ?

EQUATION OF ACCOUNTS.

383. Equation of Accounts, also called **Equation of Payments**, and **Averaging Accounts**, is the process of finding the time at which several debts due at different times may be paid in one sum without loss of interest to either party. It is also the process of finding the time when the balance of an account having both debits and credits, may be paid without loss of interest to either party.

384. The **Equated Time** is the date at which the several debts due at different times may be equitably paid in one sum.

385. The **Term of Credit** is the time between the contraction of a debt and its maturity.

386. The **Average Term of Credit** is the time to elapse before several debts due at different times may all be paid at once without loss to debtor or creditor.

387. The Focal Date is any assumed date of settlement, with which the dates of the several accounts are compared for the purpose of finding the equated time.

NOTES 1.—Any conceivable date may be taken as the focal date; the most common dates used being, the earliest due date, the latest due date, the first day of the month of the earliest due date, and the last day of the month preceding the month of the earliest due date.

2. In Equation Tables, Dec. 31st, or Jan. 1st, is taken for all examples.

3. Interest may be calculated at any rate per cent., and either on a 360 day basis, or a 365 day basis, without varying the result, providing only that a uniformity in rate and manner of computing interest be observed throughout.

4. The student is recommended to choose one method of equating accounts, that method being uniform regarding choice of focal date, rate, and form of solution.

388. Equation of accounts depends upon the following principles :

1. The rate and time remaining the same. Double the principal produces twice the interest. Half the principal produces half the interest, etc.

2. The rate and principal remaining the same. Double the time produces twice the interest. Half the time produces half the interest, etc.

3. Hence, the interest on any given principal for 1 year, 1 month, or 1 day, is the same as the interest of $1 for as many years, months, or days, as there are dollars in the given principal.

4. Hence, the interest on any given principal for any number of years, months, or days, is the same as the interest for 1 year, 1 month, or 1 day, on as many dollars as is expressed by the product of the given principal multiplied by the given number of years, months, or days.

389. The several rules in equation of accounts are based upon the principle of bank discount, for they imply that the discount of a sum paid before it is due equals the interest of the same amount paid after it is due.

390. To find the average time when the items are all debits or all credits, having the same date and different terms of credit.

EXAMPLE.—A. bought a farm June 24th and was to pay $500 down, $300 in 2 months, $400 in 6 months, and $600 in 8 months. Find the average term of credit and the equated time.

SOLUTION 1.

By the interest method.

Interest on $500 for 0 mo. at 6 % = $0.00.
     " $300 for 2 "   "   = 3.00.
     " $400 for 6 "   "   = 12.00.
     " $600 for 8 "   "   = 24.00.

Amount of payments = $1,800   Interest = 39.00.

Interest on $1,800 for 1 month at 6 % = $9. $39 ÷ $9 = 4⅓.

1 mo. × 4⅓ = 4⅓ mo. the average term of credit.

June 24th + 4⅓ mo. = Nov. 3rd, the equated time.

If we take June 24th as the time for payment of all the items, A. would lose the interest of $300 for 2 months, $400 for 6 months, and $600 for 8 months, in all $39 interest. He is therefore entitled to the use of $1,800, the amount of the debt, for such a time as the interest on it would be equal to $39, and which is shown above to be 1½ months, and 4½ months, from June 24th, gives the equated time Nov. 3rd. A. could therefore pay the amount of the debt, $1,800, on Nov. 3rd, without loss of interest either to himself or his creditor.

RULE FOR INTEREST METHOD.

Find the interest on each item for its term of credit, and divide the sum of these interests by the interest of the sum of the items for 1 day, 1 month or 1 year as the case may be.

The quotient will be the number of months or days from the focal date to the equated time of payment.

Add this number to the focal date and the result will be the equitable date of payment.

NOTES 1.—In computing by the interest method the *rate* forms no element of the calculation, hence *any* rate may be used. The most convenient rates are 6 % and 12 %.

2. The result will be the same whether we reckon 365 days to the year or 360 days to the year.

SOLUTION 2.

By the product method.

ITEMS.		TIME.		PRODUCT.
500	×	0 mo.	=	00 mo.
300	×	2 mo.	=	600 mo.
400	×	6 mo.	=	2,400 mo.
600	×	8 mo.	=	4,800 mo.
1,800				7,800 mo.

7,800 ÷ 1,800 = 4½ mo.

EXPLANATION.

This method is the same in principle as the interest method. The interest on $300 for 2 months is the same as the interest on $1 for 600 months; the interest on $400 for 6 months equals the interest on $1 for 2,400 months; and the interest on $600 for 8 months equals the interest on $1 for 4,800 months. A. would therefore lose the interest on $1 for 7,800 months. He would therefore be entitled to the use of $1,800 for such a time as the interest on it would equal the interest on $1 for 7,800 months, or 4½ months.

RULE FOR PRODUCT METHOD.

Multiply each item by its term of credit, and divide the sum of the products by the sum of the items; the quotient will be the aver 'erm of credit.

EXERCISE 88.

1. On a certain day A. bought a horse for $175 on 30 days, 3 cows for $120 on 45 days, 80 sheep for $250 on 60 days, and 5 tons of hay for $130 on 90 days. What is the average term of credit?

2. Bought a ship for $30.000; the payments were $5,000 cash, $8,000 in 4 months, $7,500 in 6 months, $4,500 in 8 months, and the balance in a year. What is the average term of credit?

3. Sept. 1st, 1891, I bought goods, as follows: $200 on 2 months' time, $400 on 3 months, and $450 on 4 months. What was the average term of credit, and the average date of maturity?

4. On the first day of December, 1890, a man gave 3 notes, the first for $500, payable in 3 months; the second for $750, payable in 6 months; and the third for $1,200, payable in 9 months. What was the average term of credit, and the equated time of payment?

5. Bought merchandise Jan. 1st, 1893, as follows: $350 on 2 months, $500 on 3 months, $700 on 6 months. What is the equated time of payment?

6. Jan. 15th, I bought a bill of goods amounting to $900, $275 of which was on 30 days' credit, $300 on 60 days, and $325 on 90 days. What was the equated time of payment?

7. James Hudson, June 12th, owes $317.75 due in 4 months, $216.38 due in 5 months, and $170 due in 6 months. Find the average time of payment and date of maturity.

8. Dec. 1st. 1894, bought goods to the amount of $1,200, on terms as follows: 25 % in cash, 30 % in 3 months, 20 % in 4 months, and the balance in 6 months. Find the equated time of payment.

9. May 1st, 1890, I purchased property for $8,500, paid cash $1,500, and gave notes, one for $3,000, payable in 2 years, and another for $4,000, payable in 4 years. Find the average term of credit on the notes.

10. Bought a bill of goods April 20th amounting to $6,000, on the following terms: $\frac{1}{8}$ cash, $\frac{1}{4}$ in 4 months, and the balance in 6 months. At what date may the whole be justly paid?

11. A stock of groceries was purchased Jan. 1st, 1889, the purchase price payable as follows: $\frac{1}{4}$ in 1 month, $\frac{1}{4}$ in 3 months, $\frac{1}{8}$ in 4 months, $\frac{1}{3}$ in 5 months. When may the whole amount be equitably paid in one sum?

12. William Owens bought a farm of 320 acres at $68 per acre, $\frac{1}{4}$ payable in cash, $\frac{1}{4}$ in 1 year, $\frac{1}{3}$ in 3 years, and the remainder in 5 years. What was the average term of credit?

391. To find the average time when the items have different dates and different terms of credit, all the items being on the same side of the account.

EXAMPLE.—L. O. Hill bought goods of Wm. Grant as follows: June 1st, 1890, amounting to $350, on 2 months' credit; July 15th, 1890, $400 on 3 months; Aug. 10th, 1890, $450, on 4 months; Sept. 12th, 1890, $600 on 6 months. What is the equated time?

SOLUTION 1.

Interest method.

DUE.	ITEMS.	DAYS.	INTEREST AT 6%.
Aug. 1,	$350	0	$00.00.
Oct. 15,	400	75	5.00.
Dec. 10,	450	131	9.82$\frac{1}{2}$.
Mar. 12,	600	223	22.30.

Amount = $1,800 37.12\frac{1}{2}$ Interest.

Interest on 1,800 for 1 day at 6 % = $.30.

37.12$\frac{1}{2}$ ÷ 30 = 123$\frac{3}{4}$ days.

Aug. 1 + 124 days = Dec. 3.

EXPLANATION.

If we take Aug 1st as the time for payment of all the items, L. O. Hill would lose the interest on $400 for 75 days, on $450 for 131 days, and on $600 for 223 days, in all 37.12\frac{1}{2}$. In justice he should be allowed the use

of $1,800 for such time as the interest will amount to $37.12½. or as shown above for 124 days.

Hence the equated time is 124 days, after Aug. 1st or Dec. 3rd.

Rule for Interest Method.

Take as the focal date the earliest due date. Find the interest on each item from the standard date to the date of its maturity and divide the sum of the interests by the interest of the sum of the items for 1 day.

The quotient will be the number of days from the standard date to the average date of payment. Add this number to the standard date and the result will be the equated time of payment.

Notes 1.—If the earliest or latest due date is the focal date, its *item* has no interest, but such item must be included in the sum of the debts.

2. If the fraction in the quotient is ½ day or more, 1 day is added; if less than ½ day it is rejected.

3. *Any* date may be assumed as the focal date, the most preferable being the earliest or latest due date.

4. In business practice, odd cents and even odd dollars are rejected from the items in the interest calculations.

5. In the solution given above the gain of interest to the payee on the first two bills, which are to be paid after they are due, equals the loss of interest on the last two which are to be paid before they are due.

6. In regard to the foregoing problem, it may be urged that a debt can not be paid before it is contracted, but, it must be remembered, that the object of the solution is really to find at what date a note, given in settlement of the account, should be dated, in order that neither party would lose interest.

7. When terms of credit are given in months, calendar months are meant.

Solution 2.

By the product method.

Assume August 1st as the focal date.

DUE.	ITEMS.		TIME.		PRODUCTS.
Aug. 1,	$350	×	0 da.	=	00.
Oct. 15,	400	×	75 "	=	300.00.
Dec. 10,	450	×	131 "	=	589.50.
Mar. 12,	600	×	223 "	=	1,338.00.
	$1,800				$2,227.50.

1800) 222750 (123⅞.

Aug. 1 + 124 days = Dec. 3.

EXPLANATION.

This method of solution may be explained in a manner similar to that given to Solution 2, Art. 390.

RULE FOR PRODUCT METHOD.

1. Find the date at which each item matures, and find the number of days between the focal date and the date of maturity of each item.

2. Multiply each item by its number of days, and divide the sum of the products by the sum of the items. The quotient will be the average term of credit.

3. Add this quotient to the focal date, and the result will be the equitable date of payment.

SOLUTION 3.

Interest method.

Assume the latest date, March 12th, 1889, as the **focal** date.

DUE.	ITEMS.	DAYS.	INTEREST AT 6 %.
Aug. 1,	$350	223	13.00\frac{5}{6}$.
Oct. 15,	400	148	9.86$\frac{2}{3}$.
Dec. 10,	450	92	6.90.
Mar. 12,	600	0	00.

Amount = $1,800

29.77\frac{1}{2}$ = Interest.

Interest on $1,800 for 1 day at 6 % = $ 30.

29.77$\frac{1}{2}$ ÷ 30 = 99$\frac{1}{4}$ days.

Mar. 12 − 99 da. = Dec. 3.

EXPLANATION.

Reckoning the days from the due dates Aug. 1st, Oct. 15th, Dec. 10th, Mar. 12th, to the focal date Mar. 12th, we find the number of days to be 223, 148, 92, and 0 days respectively. If the debt were not paid until Mar. 12th, 1891, William Grant would lose the interest, on $350 for 223 days, on $400 for 148 days, $450 for 92 days, or a total interest of 29.77\frac{1}{2}$ on Mar. 12th, 1891. The problem then becomes, "For what length of time should Wm. Grant be allowed interest on the debt $1,800 so as to receive 29.77\frac{1}{2}$ interest?" and which is shown above to be 99 days. The time at which the debt should be paid so that neither party would lose interest, would therefore be 99 days before Mar. 12th, 1891, or Dec. 3rd, 1890.

SOLUTION 4.

By product method.

Assume March 12th as the focal date.

DUE.	ITEMS.	DAYS.	PRODUCT
Aug. 1.	$350 ×	223 =	$78,050.
Oct. 15.	400 ×	148 =	59,200.
Dec. 10.	150 ×	92 =	41,100.
Mar. 12.	600 ×	0 =	00.

Amount $1,800 $1,800) $178 650.
                         99¼ days.

Mar. 12, 1891 — 99 days = Dec. 3, 1890.

EXPLANATION.

The number of days is found as in Solution 3.

If the debt is settled on Mar. 12th, 1891, William Grant will lose the interest on $350 for 223 days, or the interest on $78,050 for 1 day; on $400 for 148 days, or the interest on $59,200 for 1 day; and on $450 for 92 days, or the interest on $41,400 for 1 day. The total loss of interest is therefore the interest on $178,650 for 1 day. We have then to determine for how many days the interest on $1,800 will equal the interest on $178,650 for 1 day, which is found to be 99¼ days. Therefore the debt is due 99 days before Mar. 12th, 1891, or Dec. 3rd, 1890.

EXERCISE 89.

1. A merchant bought goods as follows :

Sept. 5, 1890, a bill of $250 on a credit of 6 mos.
Oct. 10,   "   "   500   "   8 "
Nov. 11,   "   "   350   "   60 days.
Dec. 5,   "   "   425 for cash.

What is the average date for the payment of the whole ?

2. John E. Lewis purchased goods of Isaac S. Smyth & Co. to the amount of $5,000, $1,250 to be paid June 2nd, 1889, $1,000 to be paid July 5th, $2,000 to be paid Aug. 15th; the balance, $750, will become due Aug. 30th. At what date must a single note for the whole amount be drawn, payable in 3 months, that it may become due at the average date ?

3. Bought goods as follows:

    Jan. 8, 1889, $250 @ 3 mos. credit.
    Feb. 18, "   360 " 4 "   "
    Mar. 6, "   125 " 60 days "

What is the average date of payment?

4. When shall a note to settle the following account be made payable?

Henry Field.

                                         To James L. Edwards, Dr.

1892.				
Mar. 3	To Mdse. @ 3 mos., as per bill rendered.			$250 00
Apr. 4	" " 30 days. " "			100 00
" 16	" " 60 " " "			300 00
May 1	" " 60 " " "			420 00
				$1070 00

5. Average the following statement of account:

    Mar. 6, To Mdse. @ 30 days $315.00
    "   18,   "   " 60 "   420.00
    "   25,   "   " 3 mos.   612.50
    Apr. 4,   "   " 60 days 210.25
    "   12,   "   " 30 "   400.00

6. The following items were sold on a credit of 30 days each. What is the average time for the payment of the whole amount?

    Apr. 1, 20 bbls. ex. fam. flour @ $8.50
    "   11, 500 bush. Manitoba wheat " 1.25
    "   21, 30 bbls. Ontario flour " 6.75
    "   26, 100 bush. oats " .45

7. Find the average of the following:

    June 3, Mdse. @ 3 mos. $1,275.00
    "   15,   "   " 60 days   500.00
    July 12,   "   " 3 mos.   450.50
    Aug. 18,   "   " 90 days   320.87
    Sept. 25,   "   " 3 mos.   145.62

                                 $2,692.00

8.	May 5, Mdse. @ 60 days	$600.00
	" 16, " " 30 "	896.40
	June 10, Cash	250.00
	July 7, Mdse. (net)	420.00
	Aug. 14, " @ 60 days	538.28
		$2,204.68

9. A young man, having money advanced to help him pay his way through college, received :

Sept. 1, 1888, $75.	Feb. 15, 1890, $86.
Feb. 15, 1889, $80.	Sept. 20, 1890, $128.
Aug. 31, 1889, $95.	Aug. 30, 1891, $175.

What was the equated time at which he should date a single interest bearing note for the whole sum ?

10. Five years from the date of the first loan, the above mentioned note was paid, with interest at 4 %. What was the amount ?

11. What is the average time at which the following bills become due ?   Feb. 10th, 1892, $400 on 2 months' credit; May 10th, 1892, $300 on 4 months' credit; June 16th, 1892, $350; Aug. 6th, 1892, $150.

12. Find the equitable date for a single note given on the following bills for merchandise: June 1st, 1895, $20, July 1st, $30, Aug. 1st, $30, Sept. 1st, $20, each on 2 months' credit.

13. Bought goods of Messrs Holt & Co., as follows: Mar. 11th, $35, on 30 days' credit; July 20th, $95, on 2 months' credit; Sept. 8th, $215, on 3 months' credit. What was the average term of credit ?

392. To find the extension of credit to which the balance of a debt is entitled when partial payments have been made before they are due.

A. sold B. a bill of goods Mar. 12th on 6 months' credit amounting to $1,740; July 10th, B. paid him $500; Aug. 6th B. paid $700. To what additional credit is B. entitled on the balance?

<div align="center">SOLUTION 1.</div>

Interest method.

<div align="center">Bill is due Mar. 12 + 6 months = Sept. 12.</div>

INTEREST AT 6% ON	FROM	TO	DAYS.	
$500,	July 10,	Sept. 12,	64	= $5.33⅓.
$700,	Aug. 6,	Sept. 12,	37	= 4.31¾.
$1,200.				$9.65 interest.

<div align="center">Balance = $1,740 − $1,200 = $540.</div>
<div align="center">Interest on $540 for 1 day, at 6% = $0.09.</div>
<div align="center">$9.65 ÷ .09 = 107⅔ days</div>

Sept. 12 + 107 days = Dec. 28, the equitable time of payment.

<div align="center">EXPLANATION.</div>

If a partial payment is made before a debt is due, equity requires that the *debtor* should have an extension of credit on the balance, equivalent to the interest of the pre-payment.

B., by paying a portion of his debt before it is due, loses the interest on $500 for 64 days, and the interest on $700 for 37 days, in all $9.65 interest. A. should therefore allow B. the use of the balance, $540, until the interest on it amounts to $9.65, and which is shown above to be 107 days.

NOTE.—Equity requires an extension of credit, but the creditor is not always willing to allow this and is not required to do so by law.

<div align="center">SOLUTION 2.</div>

By the product method.

ITEMS.			DAYS.
$500	×	64 =	32000
$700	×	37 =	25900
$1,200			57900

<div align="center">$1,740 − $1,200 = $540</div>
<div align="center">540) 57900 (107⅔ days.</div>
<div align="center">Sept. 12 + 107 days = Dec. 28.</div>

<div align="center">EXPLANATION.</div>

A similar explanation to that, given in Solution 4, Art. 391, may be given.

EXERCISE 90.

1. P. owed me $1,300 due in 1 year. At the end of 4 months he paid me $500, and at the end of 7 months $300, on condition that I would let the balance stand an equitable time in consideration of these pre-payments. What was the balance, and when should it be paid?

2. A man bought a bill of goods on 90 days, amounting to $2,340.75; if he pays $1,000 down, what extension ought he to have on the balance?

3. A man owes $1,569.75, payable in 90 days; 60 days before it is due he pays $350.86, and 30 days later $211.89 more; what extension ought he to have on the balance?

4. A person owes a debt of $1,680 due in 8 months, of which he pays $\frac{1}{3}$ in 3 months, $\frac{1}{4}$ in 5 months, $\frac{1}{6}$ 6 months, and $\frac{1}{8}$ in 7 months. When is the remainder due?

5. Bought a bill of goods, amounting to $1,500 on 6 months' credit. At the end of 2 months, I paid $300 on account, and 2 months afterward, paid $400 on account, giving my note for the balance. For what time was the note drawn?

6. The following sums are due from E. to F. :—$500, at the present time; $600, in 30 days; $400, in 40 days; and $900, in 60 days. If E. pays F. $500 to-day, and $1,000 in 10 days, how long from the present time should the rest stand, to balance the pre-payments?

7. A debt of $2,000 is due in 1 year from Jan. 1st, 1890. In consideration of the payment of $400 March 2nd, and $800 April 1st, till what date should the balance be allowed to stand?

8. $\frac{1}{3}$ of a certain debt is paid 78 days before it is due; $\frac{1}{4}$, 60 days; $\frac{1}{5}$, 27 days. What extension should the debtor be allowed for the payment of the balance?

9. A. sold B. a bill of goods March 12th, on 6 months, amounting to $1,740; July 10th, B. paid him $500; Aug. 6th he paid $700 more, to what additional credit is B. entitled on the balance?

10. On a debt of $2,500 due in 8 months from Feb. 1st, the following payments were made: May 1st, $250; July 1st, $300; and Sept. 1st, $500. When is the balance due?

11. Find the average term of credit, and the equated time of payment from Dec. 15th, of $225 due in 35 days, $350 due in 60 days, and $750 due in 90 days.

393. To find the equated time for the payment of the balance of an account having both debit and credit items.

EXAMPLE.—What is the equated time and date of paying the following account:

Dr.	H. BRIERLEY in acct. with MURRAY & Co.			*Cr.*

1890.				1890.		
May 21	To Mdse. 3 mos.	$500		May 24	By Cash	$300
" 28	" 3 mos.	$250		June 8	" Sundries, 60 da.	$400
June 9	" 30 da.	$160		July 21	" Cash	$100

SOLUTION 1.

Interest method.

DUE.	ITEMS.	TIME.	INTEREST.	DUE.	ITEMS.	TIME.	INTEREST.
Aug. 21	$500	112 da.	$9.33⅓	May 24	$300	23	$1.15
" 28	$250	119 da.	$4.95⅚	Aug. 7	$400	98	$6.53⅓
July 9	$160	69 da.	$1.84	July 21	$100	81	$1.35
	$910		$16.13⅓		$800		$9.03⅓
	$800		9.03⅓				
	$110		$7.09⅚				

Int. on $110 for 1 day at 6% = $.018⅓

$7.09⅚ ÷ $.018⅓ = 387²⁄₁₁ days.

May 1, 1890 + 387 days = May 23, 1891.

NOTES 1.—May 1st is chosen as the focal date. Any date may be chosen however.

2. In this example the balance of interest on May 1st is in favor of H. Brierley, hence he is entitled to the interest on the balance of the account for 387 days after May 1st.

Had the balance of interest been on the credit side of the account, we should then have subtracted the equated time from the focal date.

Rule for Interest Method.

1. Find the interest on each item for the time from the focal date to the maturity of the respective items, and divide the balance of the interests by the interest of the balance of the items for 1 day or 1 month; the quotient will be the number of days or months, as the case may be, between the standard date and the time of settlement.

2. When the balance of an account and the balance of interest are both on the same side, add the quotient to the focal date; if on opposite sides, subtract it: the result will be the date of settlement.

Notes.—1. In finding the maturity of notes and drafts 3 days of grace should be added to the specified time of payment.

2. When no time of credit is mentioned the transaction is understood to be for cash, and the payment due at once.

Solution 2.

By the product method.

Dr.   *Cr.*

DUE.	ITEMS.	DAYS.	PRODUCTS.	DUE.	ITEMS.	DAYS.	PRODUCTS.
Aug. 21	$500	112	56000	May 24	$300	23	6900
" 28	$250	119	29750	Aug. 7	$400	98	39200
July 9	$160	69	11040	July 21	$100	81	8100
	$910		96790		$800		54200
	800		54200				
	110		)42590(387$\frac{2}{11}$ days.				

May 1, 1890 + 387 days = May 23, 1891.

Rule for the Product Method.

1. Find the number of days from the focal date to the maturity of each item.

2. Multiply each item by its number of days, and divide the difference between the sums of products by the difference between the sums of items ; the quotient will be the equated time.

3. If the greater sum of items and the greater sum of products are both on the same side of the account, add the equated time to the focal date ; if on opposite sides subtract it ; the result will be the date when the balance of the account will be equitably due.

EXERCISE 91.

1. When did a note given in settlement of the following account begin to bear interest ?

Dr. **L. R. CLEM.** *Cr.*

1890.			1890		
July 2	To mds.	$5-0	Aug. 14	By cash,	$150

2. When did interest begin on the following account, and what was due on settlement, Jan 1st, 1892, interest 5 % ?

Dr. **C. L. HOOSACK.** *Cr.*

1891.				1891.		
June 17	To mdse., 2 mos.	$270		June 30	By mdse.	$250
Sept. 20	" 3 mos.	$650		Oct. 1	" cash,	$500
Oct. 1	" 1 mo.	$100		Nov. 30	" mdse.	$150

3. When is the balance of the following account due by equation ?

Dr. **FRANK H. BARNARD.** *Cr.*

1889.			1889.		
Jan. 15	To mdse.	$600	1	By cash,	$300
Feb. 28	"	$300	Mar. 31	"	$300

4. What is the balance of the following account, and when due by equation?

Dr.	BENJ. F. HAWKINS.				Cr.
1889. Jan. 11 " 28 Feb. 3 " 15	To mdse. " " "	$600 $300 $5 0 $600	1889. Jan. 20 Feb. 10	By cash, "	$1,000 $700

5. Balance the following account by two methods:

Dr.	J. H. STRONG & Co. in acct. with SMITH & CRANE.				Cr.
1893. Mar. 25 Apr. 7 May 2	To mdse., 60 da. " " " "	$560 $830 $730	1893. Apr. 30 July 13 Oct. 31	By sundries, 80 da. " cash, " draft, 30 da.	$450 $500 $260

NOTE.—In this example the balance of items and excess of products being on opposite sides, the average time is *subtracted* from the standard date.

6. What is the balance of the following account and when due?

Dr.	H. MORGAN in acct. with LOCKWOOD & Co.				Cr.
1890. July 20 Aug. 10 Sept. 15	To sundries. " "	$760 $540 $850	1890. Aug. 26 Sept. 12 Oct. 1	By flour, " stocks, 30 da. " cash,	$520 $300 $385

7. Find the average time of paying the following account:

Dr.	GEORGE JENKINS.				Cr.
1891. Mar. 1 Apr. 5 May 20	To mdse., 30 da. " 3 mos. " 4 mos	$500 $700 $850	1891. Apr. 12 May 10 June 4	By draft, 20 da. " cash, " "	$400 $540 $600

8. Find the equated time for the payment of the balance due on the following account :

Dr. W. T. Dawes. *Cr.*

1892.				1892.		
Mar. 1	To mdse., 60 da.		$200	Mar. 6	By mdse.	$200
May 10	" "		$900	May 16	" cash,	$150
June 20	"	90 da.	$400	June 26	" "	$350
July 30	"	30 da.	$700	July 1	" "	$500
Aug. 14	"	60 da.	$100	Aug. 28	" mdse.	$245

9. Average the following account :

Dr. James Green & Co. *Cr.*

1892.				1892.		
Jan. 10	To mdse., 3 mos.		$450	June 1	By bal. of acct.	$485
" 25	"	30 da.	$265	Feb. 10	" note, 3 mos.	$2,500
Apr. 20	"	3 mos.	$850	Mar. 1	" draft, 30 da.	$360

10. Balance the following account :

Dr. C. J. Hamilton. *Cr.*

1890.				1890.		
Jan. 20	To sundries, 30 da.		$500	Jan. 20	By real estate, 60 da.	$400
Feb. 12	"	60 da.	$340	Mar. 1	" draft, 60 da.	$200
Mar. 1	"	30 da.	$800	" 20	" cash,	$500

11. Find the balance of the following account and when due :

Dr. A. B. in acct. with C. D. *Cr.*

1890.			1890.		
Aug. 11	For mdse.	$160	Sept. 2	By sundries,	$75
Sept. 5	"	$210	Oct. 10	" note, 30 da.	$100
Oct. 20	For 1 horse,	$175	Nov. 1	" cash,	$110

12. Find the balance of the following account and when due :

Dr. Wm. Gorman in acct. with John Hendrie. *Cr.*

1890.				1890.		
Feb. 10	For mdse., 4 mos.		$450	Mar. 20	By sundries, 3 mos.	$325
May 11	" 5 "		$500	July 9	" draft, 60 da.	$150
July 26	" 2 "		$360	Sept. 15	" cash,	$400

13. When is the balance of the following account due by equation ?

Dr.   SAMUEL PECK & SON.   *Cr.*

1889.			1889.		
Mar. 3	To mdse.	$60	Apr. 1	By cash,	$150
Apr. 24	"	$100	June 1	"	$150
May 1	"	$150	Aug. 1	"	$150
" 30	"	$90	Oct. 1	"	$90
Aug. 17	"	$200			

14. Find 1st, the balance of the following account, 2nd, when due by equation :

Dr.   WALTER L. PARKER.   *Cr.*

1889.				1889.		
May 11	To mdse., 2 mos.		$108.40	June 1	By cash,	$124.27
July 1	"	30 da.	$225.00	Oct. 31	" 4 mos. note	
Aug 31	"		$280.80		(no interest),	$167 91
Oct. 1	"		$137.50	Dec. 1	" cash,	$305.05

15. Find when the following account is due by equation :

Dr.   JOHN MONTGOMERY & Co.   *Cr.*

1889.				1889.		
Dec. 15	To mdse.		$200	Jan. 2	By cash,	$300
" 28	"	2 mos.	$300	Mar. 1	" 60 da. note (no	
1888.					interest),	$150
Jan 14	"	30 da.	$300			

AVERAGING ACCOUNT SALES.

393. An account sales is an account rendered by a commission agent, of goods sold on account of a consignor, and contains a statement of the sales, attendant charges, and the net proceeds due the owner.

NOTES.—1. The charges include freight, cartage, storage, advertising, insurance, commission, guaranty, etc.

2. The *sales* form the *credit* side of the account and the *charges* and *advances* the *debit* side.

394. Guaranty is a charge made in addition to the commission, for insuring the owner against the risk of non-payment in case of goods sold on credit.

395. The charges for transportation, cartage, advertising, storage and insurance are considered due at the time of payment of the same.

396. The commission, guaranty, and other after charges of the commission merchant are considered *due* by some at the *average date of sales ;* by others at the average *due date of sales ;* while some merchants enter the commission at the date the account sales is rendered.

NOTES.—1. When the commission is small compared with the gross sales, either of these methods produce a result, which is practically sufficiently accurate.

2. In this work they will be considered due at the average due date of the sales.

3. Of course the due date of the commission must be a matter of agreement between the parties concerned.

397. The method of averaging an account sales is the same as that for averaging an account having both debits and credits, except in the matter of adjusting the date for the commission and other charges.

398. To average an account sales, and find when the net proceeds are due.

EXAMPLE.—Average the following, and find the due date of the net proceeds:

Received on consignment 1,000 barrels of flour from Scott. Bros. Caledonia.

SALES.

July 11	200 bbls. of flour, sold on 30 da.....	5.50	$1,100 00	
Aug 5	350 " " " 10 da.....	6 20	2,170 00	
" 20	250 " " " 30 da.....	6.0	1,500 00	
Sept. 2	200 " " " 60 da.....	5.75	1,150.00	$5,920.00

CHARGES.

July 1	Freight		450 25	
" 1	Cartage		30 75	
" 3	Storage		150 00	
	Commission 2½ % on $5,920.. ..		148 00	$779 00
	Commercial balance			$5,141 00

SOLUTION.

1. Find average date of sales—Focal date, July 1st.

DUE.	ITEMS.	DAYS.	INTEREST AT 6%.
Aug 10.	$1,100.	40.	$7.33⅓.
" 15.	2,170.	45.	16.27½.
Sept. 19.	1,500.	80.	20.00.
Nov. 1.	1,150.	123.	23.57½.
	$5,920.		$67·18⅓.

Int. on $5,920 for 1 day at 6% = $.98⅔.

$67 18⅓ ÷ .98⅔ = 68 days.

Sales due July 1st + 68 days = Sept. 7th.

2. Find average date of charges, focal date July 1st.

DUE.	ITEMS.	DAYS.	INTEREST AT 6%.
July 1.	$450.25.	0.	$ .00.
July 1.	30 75.	0.	.00.
July 4.	150.00.	2.	.05.
Sept. 7.	148.00.	68.	1.67¹⅓.
	$779.00.		$1.72¹⅓.

Int. on $779 for 1 day at 6% = $.12⅞⅔.

$1.72¹⅓ ÷ $.12⅞⅔ = 13 days.

Charges due July 1 + 13 days = July 14.

3. Averaging sales and expenses, they now stand as follows : Focal date July 1st.

DUE.	ITEMS.	DAYS.	PRODUCT.	DUE.	ITEMS.	DAYS.	PRODUCT
July 14	$779	13	10,127.	Sept. 7	$5,920	68	402,560.
					779		10,127
					$5,141		) 392,433
						Average time	76 days.

Net proceeds $5,141 due July 1 + 76 days = Sept. 15.

RULE.

1. Find the amount and the average date of sales. The date of the sales will be the date of the commission and guaranty.

2. Find the amount and the average date of the charges.

3. Make the charges the debits and the sales the credits, and find the average date for paying the balance.

EXERCISE 92.

1. Put the following items into the form of an account sales, find the net proceeds and date of payment :

A. B. Harrison, of Montreal, sold a consignment of goods from Chase & Co., Toronto, as follows : Nov. 15th, 1889, 135 chests tea at $45, on 30 days ; Nov. 20th, 75 sacks coffee at $28, on 2 months ; Dec. 1st, 256 kegs lard at $4.50, 30 days ; same date 285 tubs butter at $18.37 on 2 months. Paid freight Dec. 1st, $23.75 ; cartage, $5.40 ; storage, Dec. 10th, $7.80 ; commission, $2\frac{1}{2}$%.

2. Same parties sold Sept. 1st, on 3 months, 3,520 lb. sugar, at $.12\frac{1}{2}$; Sept. 15th, 25 chests tea, each 85 lbs., at $.98, on 2 months ; October 2nd, 28 half-chests Oolong tea, 42 lbs. each, at $1.05, on 2 months. The charges were paid October 15th, freight and cartage $85, commission and guaranty 5%.

3. Average the four following account sales :

1888.		SALES.		
Sept.	23	95 barrels to Hudson & Son,	@ $5.60, cash,	
Oct.	1	200 " " Chas. H. Knapp,	@ $5.75, 1 mo.,	
"	18	65 " " " "	@ $5.80, 60 da ,	
Nov.	3	110 " " Wm. Clark & Bro.,	@ $5.80, 30 ds.,	
"	25	130 " " Clinton McPherson,	@ $5.75, cash.	
		CHARGES.		
Sept.	24	Freight		62 50
"	26	Cartage		30 00
Oct.	28	Cash advanced on consignment		2,000 00
Nov.	15	Cooperage		5 00
"	25	Commission, 4 %		137.78

1892.		SALES.		
July	15	500 barrels, 30 da @ $6.50		$3,250 00
"	30	300 " " " 7.00		2,100 00
Aug.	10	600 " " " 6.75		4,050 00
				$9,400 00
		CHARGES.		
Aug.	10	Storage, labor and cooperage,	$71.25	
July	1	Insurance on $9,000 @ ⅛ %,	11.25	
		Commission on $9,400 @ 2½ %,	235.00	317 50
		Net proceeds due per average,		$9,082 50

1892.		SALES.		
April	9	Sold Leonard Barker & Co., @ 6 mos. :		
		15 hhds. Cuba sugar, 25,422 lbs. @ 16c.		$4,067 52
		32 half-chests Oolong tea, 1,805 lbs., tare 480 =		
		1,325, @ 1.10,		1,457.50
				$5,525.02
		CHARGES.		
"	5	Fire ins. on $6,000 @ 1½ %,	$90.00	
"	7	Cooperage, weighing, labor, etc.,	17.37	
		Com. and guar. on $5,525.02 @ 5 %,	276.25	383.62
		Net proceeds due per average,		$5,141 40

1892.		COSTS.		
May	10	1,000 kilogrammes prunes, 60 days @ 70c.,		$700.00
June	12	2 bbls. currants, 30 days,		85.05
"	25	1 case figs, 60 days.		69.60
July	17	100 bags peanuts, 30 days,		757.50
				$1,612.15
		CHARGES.		
May	10	Duties on $950 @ 20 %,	$190.00	
June	6	Freight, storage and labor.	225.50	
		Commission on $1,612.15 at 2½ %,	40.30	455.80
		Net proceeds due.		$1,156.35

4. Average the following account of sales:

Account sales of 500 barrels of pork received from Conover & Drowne, of Cincinnati, to be sold on their account and risk.

1874.	Sold to	Description.	Bar.	ℬ	
July 6	Fox & Son ...	New Mess...	100	$7.60, 30 da.	$
" 14	A. Rohr	Prime Mess.	130	16.35, cash	
" 15	H. Queen ..	New Mess...	240	17.50, 1 mo.	
" 18	Clay & Co...	Extra Prime.	30	15 25, 2 mo.	
					$

Charges.

July 3, Freight on 500 bbls., at 75c., ... $ ——
" 3, Cartage " 43.50
" 28, Storage and insurance, ... 12.50
——, Commission on $——, at 2½ .. ..

Total charges .. ..

Net proceeds, due as per average,——, ... ●

ACCOUNTS CURRENT.

399. An Account Current is an itemized record of the mercantile transactions between two parties, showing the cash balance due at a certain date.

NOTES.—1. An account current is a transcript of the ledger account, with the addition of certain details taken from the books of original entry, and is arranged in a different form.

2. Whether the items bear interest or not depends on custom or agreement between the parties.

3. It is customary for merchants, bankers, and brokers to render their accounts at stated times, as monthly, quarterly, semi-annually, or annually.

4. Among retail dealers, mechanics, farmers, etc., the items seldom bear interest; hence, in settling such accounts it is necessary to find only the merchandise balance.

5. In the illustrative example interest is calculated on the 360 days' basis, the necessary change to 365 days' basis being afterwards made.

6. In Ontario and Manitoba, interest may be recovered on open accounts from and after demand of payment and notice that interest will be charged.

400. The Commercial or Merchandise Balance is the difference between the debit and credit items.

401. The Cash Balance is the sum required to settle an account at a given date.

402. To find the cash balance of an account at a given date.

EXAMPLE.—Find the cash balance of the following account, due on July 15th, 1890, at 6 % interest:

Dr.   J. M. DOYLE in acct. with R. HISCOX.   *Cr.*

1890.			1890.		
Mar. 10	To mdse., 30 da.	$650	Apr. 20	By bal. acct.	$500
Apr. 1	" cash,	1000	May 13	" draft on 90 da.	910
May 26	" note, 60 da.	1260	June 1	" bank stock,	1000

SOLUTION.

DUE.	DAYS.	ITEMS	INTEREST.	DUE.	DAYS.	ITEMS.	INTEREST.
Apr. 9	97	$650	$10.51	Apr 20	86	$500	$7.17
" 1	105	1000	17.50	Aug 14	-30	940†	†
July 28	-13	1260*	*	June 1	44	1000	7 33
		$2910	4.70†			$2440	2.73 *
		2440	$32 71				$17.23
Bal. of items		$470	17.23				
			$15.48				

Interest. 360 days to year.

$15.48 − $\frac{1}{72}$ of $15.48 = $15.27. Actual interest.

$470 + $15.27 = $485.27. Cash balance.

EXPLANATION.

The third item on the Dr. side is not due until 13 days (indicated by − 13) after the date of settlement, and therefore J. M. Doyle is entitled to the discount on $1,260 for 13 days. This amount may either be deducted from the interest on the Dr. side or added to the interest on the Cr. side as in the problem. Similar remarks apply to the second item on the Cr. side.

NOTES.—1. The *reason* for placing the interest of an item on its own side, when it becomes due before the time of settlement, is because it is entitled to interest for the intervening time.

2. In like manner, if a credit extends beyond the settlement, equity requires that interest should be allowed on that item. Hence, its interest for that time must either be subtracted from its own side or be added to the opposite. The latter is the more convenient, and therefore adopted.

3. Interest tables are much used in making out accounts current.

4. If the account has been averaged, the amount due at a given date may be found by calculating the interest on the balance of the account from the time it is due to the date of settlement. If the date of settlement is earlier than the average date, subtract the interest from the balance of the account; if later than the average date, add the interest.

5. The interest method of finding a Cash Balance is recommended because it gives the interest or discount on each item, it is readily understood, it is more satisfactory to those to whom accounts current are sent than the product method, and when interest tables are used it is shorter than any other method.

RULE FOR INTEREST METHOD.

1. Find the due date of each item of the account. Then find the interest on each item from the date it becomes due to the day of settlement. The difference between the sums of the debit and the credit interests will be the balance of interest.

2. To find the cash balance due, when the balance of inter-
est and the balance of items are on the same side, take their
sum; when on opposite sides, take their difference.

EXERCISE 93.

1. Find the cash balance of the following account, Aug.
5th, 1892, at 6 %:

Dr.   H. MEADOWS in acct. with J. P. HUME.   *Cr.*

1892.			1892		
June 10	To mdse.	$200	June 15	By cash,	$100
" 30	"	300	" 30	"	150
July 11	"	120	July 6	"	200
" 24	"	250	" 30	"	300

2. Find the cash balance of the following account, Oct.
30th, 1892, at 6 %:

Dr.   J. S. CARSON in acct. with JAMES FERGUSON.   *Cr.*

1892.				1892.		
Jan. 5	To mdse., 60 da.	$182	Feb. 1	By bal. of acct,	$300
Feb 12	" 30 da.	270	Mar. 30	" cash,	250
Mar. 7	" "	430	Apr. 20	" "	200
Apr. 15	" 60 da.	640	June 15	" note, 30 da.	300
May 9	" "	530	Aug. 1	" cash,	400

3. Find the cash balance of the same account at 8 %.

4. Find the balance due Aug. 1st, 1892, at 8 %.

5. Find the balance of the same account due Jan. 1st,
1893, at 6 %.

6. Find 1st, the balance of the following account; 2nd,
when due by equation; 3rd, cash balance due Jan. 1st, 1888,
if money be worth 6 % per annum. Prove the result.

Dr.   JOHN McMILLAN & Co.   *Cr.*

1887.			1887.		
May 14	To mdse., 1 mo.	$300	May 31	By 2 mo. note (no interest),	$240
June 3	" 60 da.	200	July 15	" 30 da. note (no interest),	150
July 31	" 2 mos.	400	1888.		
			Jan. 1	" cash,	100

7. Find the cash balance due on the following account on the latest day of maturity, interest 6 % :

Dr.   W. NICKLE.   Cr.

1882.				1882.		
Mar. 30	To mdse.,	60 da.	$300	Mar. 10	By mdse.	$180
Apr. 2	"	90 da.	700	June 20	"	980
July 16	"	60 da.	150	July 27	" draft,	200

8. What sum in cash will settle the following account on Jan 1st, 1893, interest at 6 % ?

Dr.   GEO. MILLS & Co.   Cr.

1892.				1892.		
Sept 14	To mdse.,	3 mos.	$125.00	Sept.30	By mdse. 30 da.	$250
Oct. 4	"	60 da.	416.50	Nov. 15	" note, 3 mos.	300
Nov. 11	"	30 da.	217.45	" 25	" mdse. (net),	650
Dec. 12	" cash,		300.0			

9. Find cash balance of the following account due July 21st, 1892, interest 8 % :

Dr.   THOS. McKAY.   Cr.

1892.				1892.		
May 22	To mdse.,	3 mos.	$500	May 25	By cash,	$300
" 29	"	"	250	June 9	" sundries,	400
June 10	"	30 da.	150	July 2	" cash,	100

10. Find 1st, the balance of the following account ; 2nd, when due by equation ; 3rd, the cash balance due March 1st, 1889, if money be worth 5 % per annum. Prove the result.

Dr.   S. S. COOK.   Cr.

1888.				1888.		
Aug. 31	By mdse.,	1 mo.	$150	Oct. 2	By 30 da. note (no interest),	$100
Sept. 5	"	60 da.	200	" 30	" cash,	200
Oct. 31	"	4 mo.	600	Dec. 1	" 60 da. note (no interest),	300
Dec. 19	"	30 da.	150	1889		
1889.				Jan. 25		
Jan. 1	"	1 mo.	100		" 1 mo. accept. (no interest),	500

11. Find cash balance due Jan. 1st, 1893, interest 6 %.

Dr.   J. BRADFIELD & Co.   *Cr.*

1892.			1892.		
Oct. 10	To mdse., 60 da.	$150	Aug. 25	By mdse. 30 da.	$500
" 1	" cash,	350	Sept. 20	" "	350
" 18	" draft, 30 da.	250			

12. Find the cash balance of the following account, due Nov. 3rd, 1893, interest 8 % :

A. B. bought of C. D., July 16th, 1993, merchandise $350; Aug. 11th, $465; Sept. 9th, $570; Sept. 14th, $850; Oct. 18th, $780. The former paid August 1st, $360; Sept. 30th, in grain $340; Oct. 5th, cash $500; Oct. 21st, $625.

13. Reduce the following memoranda to the form of an account, and find the cash balance due Jan. 1st, 1889 :

Aug. 1st, 1888, A. bought goods of B. amounting to $560; Aug. 26th, $840; Sept. 21st, $1,000; Oct. 12th, $1,370; and Nov. 1st, $600. A. sold B. Sept. 11th, 1888, wheat amounting to $350; Oct. 1st, wool amounting to $760; Oct. 31st, $400 worth of butter; Nov. 16th, paid him $1,000 cash.

14. What is the cash balance of the following account, Dec. 31st, 1889, at 7 % ?

Dr.   S. MORGAN in acct with J. D. BISSONNETTE.   *Cr.*

1889.			1889.		
Sept.10	To mdse., 30 da.	$1,250.15	Sept.25	By mdse., 60 da.	$1,560 50
Oct. 1	" 60 da.	1,051.60	Oct. 10	" 90 da.	948.30
" 23	" 46 da.	1,500.85	" 30	" 40 da.	1,430 65
Nov. 15	" 60 da.	1,743.44	Dec 15	" 30 da.	1,365.42

15. What is the cash balance on the following account, Jan. 10th, 1892 ?

Dr.   W. R. Telford in acct with A. T. Stewart.   *Cr.*

1891.				1891.		
Aug. 4	To sundries, 3 mos.	$1,400		July 5	By mdse., 3 mos.	$685
" 20	" "	1,050		" 18	" "	840
Sept 10	" "	780		Aug. 11	" "	900
" 24	" "	1,300		" 18	" draft, 30 da.	800

16. Reduce the following transactions to the form of an account bearing interest at 6 %, and find the cash balance :

Feb. 11th, 1890, C. bought goods of D. amounting to $1,250; March 14th, a bill of $2,160; Apr. 10th, a bill of $1,700; Apr. 30th, a bill of $1,070; May 6th, a bill of $2,000. March 1st, 1890, C. sold a bill to D. of $1,640, March 20th, a bill of $1,160; Apr. 15th, a bill of $1,600; May 1st, a bill of $1,340; May 21st, a bill of $1,000. What was the cash balance June 10th, 1890 ?

17. What was the cash balance due July 20th, 1889, on the following account, at 7 % interest ?

Dr.   C. W. Harrison in acct with L. Congdon.   *Cr.*

1889.				1889.		
Mar. 1	For mdse., 3 mos.	$500		Apr. 5	By mdse., 3 mos.	$350
" 20	" 2 mos.	750		" 20	" 2 mos.	900
Apr. 10	" 5 mos.	410		May 1	" 4 mos.	620
May 21	" 1 mo.	600		" 22	" cash,	200

STORAGE.

403. Storage is a provision made for keeping goods in a warehouse for a time agreed upon, or for an indefinite time, subject to accepted conditions.

The t— storage is used also to designate the charges for keeping the goods stored.

Rates of storage may be fixed by agreement of the parties to the contract, but are often regulated by Boards of Trade, Chambers of Commerce, or Warehouse Companies, and are estimated at a certain price per barrel, bale, bag, bushel, etc., per *storage term*.

404. A storage term is the number of days for which the storage is charged. The storage term is usually one week, 10 days, 20 days or 30 days. The rates of storage often vary for grains, or goods of different grades or values, and also on account of different modes of shipment.

405. Cash storage is a term applied to cases in which the payment of charges is made on each withdrawal or shipment, at the time of such withdrawal or shipment, notwithstanding the fact that the owner may still have goods of the same kind in store at the warehouse.

406. Credit storage is a term applied to cases in which sundry deposits or consignments are received, from which sundry withdrawals or shipments are made, and all charges adjusted at the time of final withdrawal.

407. A grain elevator is a building erected for the convenience of storing and shipping grain.

408. Storage receipts, especially of grains, are frequently bought and sold under the name of " warehouse receipts " or " elevator receipts," as representing so much value by current market reports.

NOTE. —When deposits or consignments, and withdrawals or shipments, are made at different times, debit is to be given for the amount of each

deposit or consignment, from date to its final withdrawal or shipment, and credit given to the owner or consignor for each withdrawal or shipment, from date up to the time of settlement.

409. To find the average storage when goods have been received at different dates, but none delivered.

EXAMPLE.—There was received at a storage warehouse: Oct. 15th, 500 bbls. flour; Oct. 21th, 120 bbls. apples; Nov. 5th. 125 bbls. potatoes; Nov. 20th, 200 bbls. quinces; Nov. 24th, 340 bbls. apples. The merchandise was all delivered Dec 12th. If the storage charge was 1c. per bbl. for a period of 30 days average storage, what was the storage bill?

SOLUTION.

The storage of 500 bbls. for 58 days = 29,000 bbls. stored for 1 day
"   120   "   49   "   =   5,880   "   "   "   "
"   125   "   37   "   =   4,625   "   "   "   "
"   200   "   22   "   =   4,400   "   "   "   "
"   340   "   18   "   =   6,120   "   "   "   "

50,025 bbls. stored for 1 day.

50,025 bbls. for 1 day = $\frac{50,025}{30}$ = 1,667½ bbls. for 30 days
1,667½ bbls. @ 4c. a bbl. = $66.70, storage bill.

RULE.

Multiply the number of articles of each receipt by the number of days between the time of its deposit and withdrawal and divide the sum of these products by the number of days in the storage term. The quotient will be the average storage for that term.

EXERCISE 94.

1. There was received at a warehouse: May 15th, 2,500 bush. wheat; June 8th, 2,500 bush. oats; July 24th, 3,500 bush. barley; July 30th, 5,000 bush. corn. If all of this was shipped August 20th, what was the storage bill, the charge being 1½c. per bushel per term of 30 days average storage?

2. A farmer received for pasture: April 30th, 12 head of cattle; May 15th, 14 head of cattle; May 23rd, 27 head of cattle; June 9th, 5 head of cattle; June 30th, 8 head of cattle; July 16th, 40 head of cattle. All were delivered

July 25th, and the charges were 75c. per head for each week of 7 days' average pasturage. How much was his bill ?

3. The following produce was received at a warehouse : Oct. 19th, 250 bbls. flour ; Oct. 27th, 160 bbls. potatoes ; Nov. 2nd, 240 bbls. apples ; Nov. 24th, 60 bbls. onions ; Dec. 6th, 180 bbls. flour. The merchandise was all delivered Dec. 8th. What was the storage bill, the charge being $2\frac{1}{2}$c. per bbl. per term of 30 days ?

410. To find the average storage when goods have been received and delivered at different times.

EXAMPLE.—A warehouseman received and delivered the following :

RECEIVED.	DELIVERED.
Jan. 19, 300 bbls.	Feb. 9, 150 bbls.
Feb. 24, 200 "	Mar. 18, 200 "
Mar. 8, 150 "	Apr. 4, 150 "
Apr. 21, 400 "	May 7, 550 "

What was paid for storage at 2c. a bbl., for a period of 30 days average storage, a settlement having been made May 7th ?

First Method.

SOLUTION.

From Jan. 19 to Feb. 9 = 21 da.; 300 bbl. stored for 21 da. = 6,300 for 1 da.
Feb. 9 .. .. 150 bbl. delivered.

From Feb. 9 to Feb. 24 = 15 da.; 150 bbl. rem'g for 15 da. = 2,250 "
Feb. 24.. .. .. .. 200 received.

From Feb. 24 to Mar. 8 = 12 da.; 350 bbl. stored for 12 da. = 4,200 "
Mar. 8.. .. .. .. 150 bbl. received.

From Mar. 8 to Mar. 18 = 10 da.; 500 bbl. stored for 10 da. = 5,000 "
Mar. 18 .. .. .. 200 bbl. delivered.

From Mar. 18 to Apr. 4 = 17 da; 300 bbl. stored for 17 da = 5,100 "
Apr. 4 .. .. .. 150 bbl. delivered.

From Apr. 4 to Apr. 21 = 17 da.; 150 bbl. rem'g for 17 da = 2,550 "
Apr. 21 .. .. .. 400 bbl. received.

From Apr. 21 to May 7 = 16 da; 550 bbl. stored for 16 da = 8,800 "
May 7 .. .. .. 550 bbl. delivered.

Total .. .. .. 34,200 "

34,200 bbl. for 1 day = $\frac{34,200}{30}$ = 1,140 bbl. for 30 da.
1,140 bbl. @ 2c. a bbl. = $22.80 Cost of storage.

RULE.

1. Multiply the number of barrels, bales, etc., by the number of days between the date of their receipt and the date of the next receipt or delivery ; add the number of articles of such next receipt, or subtract the number of such delivery, as the case may be, and so proceed to the time of the final delivery.

2. Divide the sum of the products thus found by the number of days in the storage term, and the quotient will be the Average Storage for that term.

Second method.

SOLUTION.

RECEIVED.					DELIVERED.				
Jan. 19, 800 bbl.	×	108	=	32,400	Feb. 9, 150 bbl.	×	87	=	13,050
Feb. 24, 200 bbl.	×	72	=	14,400	Mar. 18, 200 bbl.	×	50	=	10,000
Mar. 8, 150 bbl.	×	60	=	9,000	Apl. 4, 150 bbl.	×	33	=	4,950
Apl. 21, 400 bbl.	×	16	=	6,400	May 7, 550 bbl.	×	0	=	0,000
				62,200					28,000
				28,000					
				34,200					

84,200 ÷ 30 = 1,140.

1,140 bbl. @ 2c. per bbl. = $22.80. Cost of storage.

EXERCISE 95.

1. What will be the storage charge, at 4c. per bbl., for a term of thirty days average, on the following transaction ?

RECEIVED.				DELIVERED.			
1889.—June 12,	200 bbls.,	potatoes.		1889.—June 17,	75 bbls.	potatoes.	
" " 20,	150 "	apples.		" " 25,	125 "	"	
" July 18,	60 "	turnips.		" " 30,	90 "	apples.	
" Aug. 2,	90 "	onions.		" July 5,	60 "	"	
				" " 25,	40 "	turnips.	
				" Aug. 9,	20 "	"	
				" " 15,	90 "	onions.	

2. What will be the storage charge, at $4\frac{1}{2}$c. per bbl., for a term of thirty days average, in the following transaction ?

RECEIVED.				DELIVERED.			
1889.—Feb. 8,	180 bbls.	flour.		1889.—Mar. 1,	100 bbls.	apples.	
" " 27,	100 "	apples.		" " 28,	190 "	flour.	
" Mar. 8,	60 "	potatoes.		" Apr. 15,	60 "	potatoes	
" " 13,	300 "	flour.		" " " 60	"	flour.	
				" " 29,	230 "	"	

8. What is the storage on the following account to **Dec.** 31st, 1889, at $2\frac{1}{2}$c. per bbl., for 30 days ?

RECEIVED.		DELIVERED.	
1889.—Aug. 17, 250 bbls. mdse.		1889.--Aug. 23, 200 bbls. mdse.	
" " 25, 90 " "		" Sept. 25, 240 " "	
" Sept. 19, 200 " "		" Oct. 13, 300 " "	
" Oct. 12, 300 " "		" Nov. 20, 150 " "	
" Nov. 18, 200 " "		" Dec. 25, 550 " "	
" Dec. 17, 400 " "			

411. To find the Cash Storage on goods received and delivered at different dates, when charges vary.

EXAMPLE.--At a warehouse there was received and delivered merchandise as follows :

RECEIVED.	DELIVERED.
Jan. 3, 150 bbl.	Jan. 23, 250 bbl.
Jan. 20, 200 bbl.	Mar. 1, 400 bbl.
Feb. 1, 300 bbl.	

How much must be paid for storage on the above, at the rate of 5c. per bbl. for the first 10 days, or part thereof, and 3c. per bbl. for each subsequent 10 days, or part thereof ?

SOLUTION.

Date.   Receipts and Deliveries.

Jan   3, received 150 bbl.
"   20,   "   200 "
                     350 bbl. in store.

Jan. 23, delivered 250 bbl. $\begin{cases} 150 \text{ bbl. stored 20 da. or 2 terms, 8c.} = \$12.00 \\ 100 \text{ "   "   3 da. or 1 term, 5c.} = \quad 5.00 \end{cases}$
                     100 bbl remaining.

Feb.   1, received 300 bbl.
                     400 bbl. in store.

Mar. 1, delivered 400 bbl. $\begin{cases} 100 \text{ bbl. stored 40 da. or 4 terms, 14c.} = \$14.00 \\ 300 \text{ "   "   28 \quad " \quad 3 \quad " \quad 11c.} = \quad 33.00 \end{cases}$

                            Total cost of storage. $64.00

EXERCISE 96.

1. How much must be paid for storage on the following account at the rate of 5 cents per bbl. for the first 10 days, or part thereof, and 3 cents per bbl. for each subsequent 10 days, or part thereof?

RECEIVED.	DELIVERED.
1889.—May 7, 350 bbl. flour.	1889.—May 26, 250 bbl. flour.
" " 26, 150 " "	" June 1, 100 " "
" June 15, 200 " "	" " 9, 100 " "
	" " 30, 250 " "

2. The receipts and deliveries at a certain warehouse on the following account were as follows :

RECEIVED.	DELIVERED.
1889.—June 20, 350 bbl. pork.	1889.—July 10, 90 bbl. pork.
" Aug. 1, 250 " "	" Aug. 15, 100 " "
" " 25, 100 " "	" " 25, 250 " "
" Sept. 12, 90 " "	" Sept. 10, 50 " "
	" " 20, 30

What was the total storage paid, the rate being 5 cents per bbl. for the first 10 days, and 3 cents for each subsequent 10 days, or part thereof?

3. Find the cash storage on the following storage account :

RECEIVED.	DELIVERED.
1889.—Sept. 2, 100 bbl.	1889.—Sept. 20, 100 bbl.
" " 25, 200 "	" " 30, 100 "
" Oct. 19, 350 "	" Oct. 10, 100 "
" " 31, 150 "	" " 20, 100 "
" Nov. 7, 200 "	" " 30, 100 "
	" Nov. 20, the remainder.

The contract required the payment of 6c. per bbl. for the present term of 30 days or fraction thereof, and 3c. per bbl. for each subsequent term of 30 days or fraction thereof.

MISCELLANEOUS.

EXERCISE 97.

1. The interest on $1,805, loaned on May 13th, at $5\frac{1}{4}$ % per annum is $37.905; on what day was the money returned ?

2. A sum of money at simple interest has in four and one-half years amounted to $735, the rate of interest being 5 per cent. per annum ; what was the sum at first, and in how many years *more* will it amount to $1,140 ?

3. I am offered a house that rents for $27 per month, at such a price that, after paying $67.20 taxes, and other yearly expenses amounting to $24.85, my net income will be $8\frac{1}{2}$ % on my investment. What is the price asked for the house ?

4. In order to engage in business, I borrowed $3,750 at 6 %, and kept it until it amounted to $4,571.25. How long did I keep the money ?

5. October 12th, 1889, I purchased 2,700 bushels of wheat, at $1.05 per bushel, and afterwards sold it at a profit of 6 % On what date was the wheat sold, if my gain was equivalent to 10 % interest on my investment ?

6. December 11th, 1888, a lumber dealer borrowed money and bought shingles at $4.50 per M.; September 17th, 1889, he sold the shingles and paid his debt, and 8 % interest, amounting to $3,462.60. How many thousand shingles did he buy ?

7. I loaned a bridge builder $17,500 for seven years, at 10 % per annum, compound interest payable quarterly, and took a bond and mortgage to secure the debt and its interest. Nothing having been paid until the end of the seven years, how much was required in full settlement ?

8. **Harry** is ten, and Fred seven years old. If 7 % compound interest investments can be secured by their father, for what amounts must such investments be made in order that at the age of twenty-one the boys may each have $12,500 ?

9. The day Charles was six years old, his father deposited for him in a savings bank such a sum of money that, at 4% interest, compounded quarterly, there will be $7,500 to his credit on the day he attains his majority. What sum was deposited ?

10. Having purchased July 15th 1,150 barrels of pork, at $16 per barrel, on four months' credit, the dealer, thirty days later, sold it at $17.50 per barrel, receiving therefor a six months' note without interest. When the purchase money became due, he discounted the note on a basis of 7 %, and paid his debt. How much was gained ?

11. I loaned a friend a sum of money for nine months, at 6 % per annum, and w. the loan was due he paid $851.50 in cash, which was 75 % of the amount due me ; the remainder was paid six months, fifteen days later, with interest at the rate of 10 %. Find the amount paid at final settlement.

12. Having bought a mill for $12,000, I paid cash $4,000 on delivery, and gave a bond and mortgage for eight years without interest to secure the balance ; to secure the interest, which was to be paid semi-annually, at the rate of 7 % per annum, I gave sixteen non-interest bearing notes, without grace, for $280 each, one maturing at the end of each six months for the eight years. If the four of the notes first maturing were paid when due, and no other payment was made until the mortgage became due, how much was required for full settlement ?

13. The discount on $56?.50 for nine months is $16.50 : find the rate of interest.

14. Bought 5,000 bushel ... at at $1.25 a bushel, payable in six months; I ... realized for it at $1.20 cash, and put the mone. ... at interest at 10 %. At the appointed time I paid for the wheat; did I gain or lose by the transaction, and how much ?

15. Jones loaned $2,400 ... interest, until it amounted to $3,000. F. ... the loan made ?

16. A man invested $16,000 in busi... and at the end of three years, three mor'us. wi'h'v $22,880, which sum included investment and gains. What yearly per cent. of interest did his investment pay ?

17. Sold an invoice of crockery on two months' credit; the bill w.. paid three months, eighteen days, after the date of purchase, with interest at 8 %, by a check for $1,963.45. How much was the interest ?

18. A bond, bearing interest at 8 %, and dated May 1st, 1881, was settled in full November 16th, 1889, by the payment of $17,685. For what face amount was the bond given ?

19. What sum will be due January 18th, 1892, on a debt of $5,100, dated March 17th, 1885, bearing interest at 7 % per annum, payable semi-annually, if the first five payments were made when due and no subsequent payments were made ?

20. A merchant sold a stock of glassware on one month's credit; the bill was not paid until three months, twenty-one days after it became due, at which time the seller received a draft for $4,716.21 for the bill, and interest thereon at the rate of 5 %. Find the selling price of the goods.

21. A tradesman who is ready to allow 5 % per annum, compound interest, for ready money, is asked to give credit for two years. If he charged $110.25 in his bill, what ought the ready money price to have been?

22. A speculator borrowed $6,250, at 7½ % interest, and with the money bought a note, the face of which was $7,500, maturing in nine months without interest, but which was not paid until two years from the date of its purchase. If the note drew 6 % interest after maturity, did its purchaser gain or lose, and how much?

23. A jobber bought 6,000 yards of Axminster carpet, at $2.80 per yard, payable in six months, and immediately sold it at $3.15 per yard, giving a credit of two months; at the expiration of the two months he anticipated the payment of his own paper, getting a discount off of 10 % per annum. How much did he gain by the transaction?

24. On the 20th of March, 1889, I borrowed $13,500, at 5 % interest; on April 5th, I loaned $5,000 of the money until December 20th, 1889, at 8 %; April 15th, I purchased with the remainder a claim for $10,000, due August 1st. but which, not being paid at maturity, was extended until the $5,000 became due, at the rate of 6 %. How much did I gain, both claims having been paid on the day the loan of $5,000 became due?

25. Find the present worth of $842.70 for two years at 6 %, compound interest.

26. If $20 be allowed off a bill of $120 due in six months, how much should be allowed off the same bill due in twelve months, reckoning true discount?

27. If $15 be the interest on $115 for a given time, what should be the true discount off $115 for the same time?

28. If $10 be allowed off a bill of $110 due eight months hence, what should be the bill from which the same sum is allowed as four months' discount?

29. How much may be gained by hiring money at 5 % to pay a debt of $6,400, due in eight months, allowing the present worth of this debt to be reckoned by deducting 5 % per annum discount?

30. The discount on a certain sum due nine months hence is $20, and the interest on the same sum for the same time is $20.75. Find the sum and the rate of interest.

31. Having bought goods to the amount of $2,431.80 cash, I gave my 60-day note in settlement. If discount be at 7 %, what should have been the face of the note?

32. A note dated September 1st, 1889, payable in 90 days, with interest at $7\frac{1}{2}$ %, was discounted twenty-one days after date, at 10 %. If the proceeds were $690.52, what must have been the face?

33. If, on a note made for $700, bearing interest at 6 %, and dated January 1st, 1889, $50 is paid on the first of every month, commencing February 1st, following the date, what is due January 1st, 1890?

34. F. J. Ramsay & Co. bought goods of John Hope & Co. as follows: July 1st, $150, at three months; July 20th, $200, at four months; August 16th, $300, at two months; and October 4th, $250 at four months. Find the equated time of payment, and what would be due on the account March 15th following, at 6% interest.

35. I owe $480 payable in ninety days, and $320 payable in sixty days. My creditor consents to an extension of time to one year, and offers to take my note for the

whole amount on interest at 6 % from the equated time, or a note for the true present worth of both debts, on interest from date. How much will I gain if I choose the latter condition ?

36. I sell goods to A. at different times, and for different terms of credit, as follows :

Sept. 12, 1859, a bill on thirty days' credit, for $180
Oct. 7, "   "   thirty   "   300
Nov. 16, "   "   sixty   "   150
Dec. 20, "   "   ninety   "   350
Jan. 25, 1860,   "   thirty   "   130
Feb. 24, "   "   thirty   "   140

If I take his note in settlement; at what time should interest commence ?

37. A person owes $350, due in three months, and $750, due in six months ; but at the end of two months he pays $200, and three months afterwards, $500. When is the remainder due ?

38. A note for $1,000, dated April 1st, 1889, payable on demand, with interest at 7%, bears the following endorsements : May 6th, $200 ; July 5th, $225.37 ; October 18th, $322. What is due January 1st, 1889 ?

39. Bought goods to the amount of $10,000, of which $2,000 was to be paid in one month ; $2,000 in two months ; $4,000 in three months, and the balance in six months. If a note is given for the whole amount, how long should it run ?

40. Four notes, made by J. Simpson, and payable as follows : $560, due September 10th, 1888 ; $800, due October 15th, 1888 ; $1,100, due December 1st, 1888 ; $900, due February 1st, 1889, were exchanged for a single note. When will it fall due ?

41. Asa May has given three notes; one for $300, due May 1st; one for $350, due June 15th; and one for $550, due August 1st. Desiring to exchange them for two notes of $600 each, he makes one payable June 15th; when should the other fall due?

42. Bought a bill of goods amounting to $1,200, on six months' credit. Paid cash on account $100; at the end of three months paid $300 more; and two months afterwards paid $400, giving a note for the balance. For what time was the note drawn?

43. A note for $835.25, dated July 1st, 1888, payable on demand, with interest at $6\frac{1}{2}\%$, bears the following endorsements: August 20th, $157.50; September 21st, $180.25; October 5th, $200; December 1st, $80. What is due January 1st, 1889?

44. On a bill of goods bought March 1st, amounting to $1,500, on eight months' credit, the following payments were made: May 1st, $350; August 1st, $500; September 1st, $150. What is the equated time for the payment of the balance?

45. A note for $618.75, dated April 17th, 1888, payable on demand, bears the following endorsements: June 5th, $126.50; August 20th, $127.25; November 17th, $210. What is due January 1st, 1889, reckoning interest at 6%?

46. Bought of A. T. Stewart & Co., the following bills of goods on five months' credit: February 10th, 1888, $900; March 15th, 1888, $2,000; May 10th, 1888, $750; June 12th, 1888, $2,000. Find the present worth of a note drawn July 1st, in payment of the whole, discounted at 6%.

47. Bought goods at different dates, as follows :

Aug. 15, amounting to $475, on 6 months' credit.
Sept. 10,   "   600, " 5   "
Oct. 5,   "   750, " 4   "
Nov. 1,   "   450, " 8   "

What sum will equitably discharge the whole debt November 10th, allowing true discount at 7 % ?

48. Purchased merchandise of W. Duncan & Co., as follows :

Jan. 1, a bill amounting to $375.50, on 4 months' credit.
Jan. 20,   "   168.75,   5   "
Feb. 4,   "   386.25,   4   "
Mar. 11,   "   144.60,   5   "
Apr. 7.   "   386.90,   3   "

What is the present worth of a note made May 1st, in payment of the whole, discounted at 6 %?

PERCENTAGE.

STOCKS.

412. **Stocks** represent the capital or property of **incorporated** companies.

413. An **Incorporated Company** is an association authorized by law to transact business, and having the same *rights* and *obligations* as a single individual.

414. A **Share** is *one* of the equal parts into which the capital stock of a corporation is divided.

NOTE.—The par value of a share varies in different companies. It is usually $100. and will be so regarded in this work unless otherwise stated. Shares of $50 and $25 are called half-stock and quarter-stock respectively.

415. A **Certificate of Stock** is a paper issued by a corporation specifying the number of shares to which the holder is entitled, and the par value of each share.

416. The **par value** of a stock is the sum named in the certificate.

417. The **Market Value** of stock is the sum for which it can be sold.

NOTE.—When shares sell for their *nominal* value, they are at *par*; when they sell for *more*, they are above *par*, at a *premium*, or at an *advance*; when they sell for less, they are *below par*, or at a *discount*.

When stocks sell at par they are quoted at 100; when at 5 % above par they are quoted at 105; when at 10 % discount they are quoted at 90.

418. A **Dividend** is a sum divided among the stockholders from the net profits of the company, and is a certain percentage computed on the par value of the stock.

NOTE.—Companies sometimes declare a Scrip Dividend, entitling the holder to the sum named payable in stock at par value.

419. A **Preferred Stock** is one which is entitled annually to a stated per cent. dividend out of the net profits before the *common* stock dividend is declared.

420. A **Stock Broker** is one who buys and sells stocks for others, on a commission called brokerage which is always a certain percentage computed on the par value of the stock purchased or sold.

421. A **Stock Jobber** is one who buys and sells stocks on his own account.

422. An **Instalment** is a payment of part of the capital.

423. An **Assessment** is a sum required of stockholders to meet the losses or the business expenses of the company.

424. The **Gross Earnings** of a company are its entire receipts from its ordinary business.

425. The **Net Earnings** is the remainder after all expenses are deducted.

426. A **Bond or Debenture** is a written agreement to pay a sum of money, with a fixed rate of interest, at or before a specified time. The term is applied to the Dominion, Provincial, County, Township, City, Town, Village, Railroad Bonds, etc.

NOTE.—Bonds or Debentures are named from the corporations who issue them, the rate of interest they bear, the date at which they are payable or from a combination of any of these.

Bonds are also known, First Mortgage, Second Mortgage, etc., Income Bonds, Consols, Sinking Fund, etc.

427. **Coupon Bonds** are those having small certificates attached representing the different instalments of interest payable at the times specified, and which are to be cut off when paid, as a receipt.

NOTE.—1. Bonds are also issued without coupons, in what is known as the registered form. In t...s case the bond is only payable to the registered owner, or his assignee, and the interest is paid by cheque or in cash to the owner or to his attorney.

2. Bonds are sometimes issued with coupons attached payable to bearer, but the principal of which may or may not be registered at the choice of the owner.

428. The principal United States government bonds are the $4\frac{1}{2}$'s of 91, redeemable at the option of the government after Sept. 1st, 1891 ; 4's of 1907, redeemable at the option of the Government after July 1st, 1907 ; Refunding Certificates of the denomination of $10, bearing interest at 4 %, and convertible at any time with accrued interest, into 4 % bonds ; Currency 6's, issued to aid in the construction of Pacific railroads, payable in thirty years after date, and maturing at different dates from 1895 to 1899.

Consols are the leading funded securities of the English Government, bearing 3 % interest, payable half-yearly, and redeemable only at the pleasure of the Government.

The funded debt of France bears the title of **Rentes,** bearing usually, interest at the rate of 5 %.

The German Empire has a funded debt bearing 4 % interest, known as 4 %, **Imperial bonds.**

The funded debt of Austria is known as the **Austrian Consols,** the largest part of which bears 5 % interest.

Russia has a debt which bears a nominal interest of 5 %, or $5\frac{1}{2}$ %. The bonds are known as **Oriental Loans,** and are below par.

The bonds in Italy are called **Rentes,** and bear interest of 8 %, or 5 %.

STOCK EXCHANGE.

429. **Stock Exchanges** are associations organized for buying and selling stocks, bonds, and other similar securities.

430. **Quotations** are usually made at so much per cent. on the basis of a par value of $100 per share.

431. **Stocks** are usually bought or sold either "cash," "regular way," "seller three," "buyer three."

NOTE.—1. A stock sold "*cash*" is deliverable the day sold, a stock sold "*regular way*" is deliverable next day, or if bought "*regular way*" is to be paid for the next day. "*Seller three*" means deliverable on either of three days at the option of the seller. "*Buyer three*" means the buyer can demand delivery within three days, but must take and pay for it the third day.

2. Quotations are termed "*flat*" when the accrued interest is included in the price named.

3. Transactions on any of the above terms carry no interest.

4. If the option is over three days, interest on the selling value of the stock is paid by the buyer to the seller.

5. One day's notice is required of intention to terminate an option of a longer period than three days.

6. Should the stock pay a dividend during the pendency of a contract, the dividend belongs to the purchaser of the stock, unless otherwise previously agreed.

432. **Margin** is cash or other security deposited with a broker on account of either the purchase or sale of securities, and to protect the broker against loss, in case the market price of the securities, bought or sold, varies so as to be against the interests of the customer. It is usually 10% of the par value of the stock.

NOTE.—1. Brokers charge interest on the amount furnished by them for "carrying the stock."

2. The margin deposited with the broker is simply to protect the broker against losing any money should the stock move in the wrong direction. In case of the stock so doing, the margin must be made good by the deposit of an additional amount, otherwise the broker will sell the stock to protect himself from losing any of the money he has advanced It is usually 10% of the par value of the stock.

433. 1. A Bear is an operator who is " short " of stock. He wishes to buy at a lower rate, and tries to depress the price of the stock of which he is "short."

2. A Bull is an operator who is holding stock for an advance. He is said to be "long" of stock. Bulls try to advance the price of the stock of which they are "long."

3. **Collaterals.** Stocks, bonds, notes, or other value given in pledge as security, when money is borrowed.

4. **Hyphothecating Stocks** and bonds, is depositing them as collaterals.

5. **B.C.** "between calls." The sale not taking place on the call of the stock but after the first call and before the second call.

6. **Short.** When one has sold stock which he does not own hoping to realize a profit by buying it at lower prices, he is said to be "short."

7. A "Put" is a contract which secures to the holder the privilege of delivering to the person named therein a number of shares of stock at a specified price per share, within a limited time (usually thirty days), without the obligation to deliver it. The holder of a "put" is not required to pay interest.

8. A "Call" is a contract which secures to the holder the privilege of buying a number of shares of stock at a specified price, within a limited time without the obligation to purchase it. The holder of the "call" must pay interest on the purchase price of the stocks to the day of delivery.

9. A "Spread" is a contract which secures to the holder the privilege of either buying or selling within a limited time, a number of shares of stock, at a specified price. without the obligations of taking or delivering it.

10. A "Straddle" is a contract which secures to the holder the privilege of either buying or selling, within a limited time a number of shares of stock, not only at the price mentioned in the contract, but, also at the market price of the stocks at the date the privilege was purchased.

11. Puts, Calls, Spreads and Straddles, are privileges not recognized by the Stock Exchange.

12. **Cover, to "cover one's shorts."** Where stock has been sold short and the seller buys it in to realize his profit or to protect himself from loss, or to make his delivery, he is said to be " covering short sales."

13. **Ex.-Div. or Ex.-Dividend.** When the price of stock does not include, and the stock does not carry to the buyer a recently declared dividend.

14. **Difference.** When the price at which a stock is bargained and the price of the stock on the day of delivery are not the same, the broker against whom the variation exists, frequently pays the " difference " in money, instead of furnishing or receiving the stock.

15. **Watering Stock** is increasing the number of shares of an incorporated company without a corresponding increase of their value. This is usually done in the re-organization of a railroad or in the consolidation of two or more railroads.

16. A "Corner" is produced when one or more operators *owning* or *controlling all* the stock of a company are able to purchase still more for either immediate or future delivery, from one who is " short." When they demand the stock, the sellers are unable to find it in the market.

17. Brokerage. The usual brokerage for buying and selling stocks is $\frac{1}{8}$ %, and is calculated on the par value of the stock.

434. Given number of shares, the par value of a share. To find the stock, or vice versa.

EXAMPLE 1.—What amount of stock is represented by 40 shares of Bank of Montreal stock, par value $200 per share ?

SOLUTION.

40 shares at $200 each $=$ $200 $\times$ 40 $=$ $8,000 stock.

EXAMPLE 2.—How many shares, par value $200 each, are represented by $8,000 Bank of Montreal stock ?

SOLUTION.

$200 $=$ value of 1 share.

$\therefore$ $8,000 $=$ " " $\frac{8000}{200}$ $=$ 40 shares.

EXAMPLE 3.—What is the par value of a share, when 40 shares of Bank of Montreal stock represent $8,000 stock ?

SOLUTION.

40 shares represent $8,000 stock

$\therefore$ 1 share represents $\frac{8000}{40}$ $=$ $200 stock.

EXERCISE 98.

What amount of stock is represented by—

1. 120 shares Western Assurance, par value $40 per share?
2. 60 " Bank of Montreal, " $200 "
3. 200 " " Toronto, " $200 "
4. 150 " " Commerce, " $50 "
5. 175 " " Hamilton, " $100 "
6. 240 " Imperial Bank, " $100 "
7. 98 " Dominion Bank, " $50 "
. 75 " Standard Bank, " $50 "

Find the par value of a share when—

9. 40 shares Imperial Bank represent $4,000 stock?
10. 75 " Merchants' Bank " $7,500 "
11. 90 " Ontario Bank " $9,000 "
12. 120 " Standard Bank " $6,000 "
13. 300 " Western Assurance Co. $12,000 "
14. 70 " Imp. S. & Invest. " $7,000 "
15. 80 " B. & L. Association " $2,000 "
16. 110 " Dominion Telegraph " $5,500 "

How many shares are represented by--

17. $8,500 stock Merchants' Bank, par value $100.
18. $9,600 " Bank of Montreal, " $200
19. $7,525 " Lon. & Can. L. & A., " $50.
20. $2,640 " Western Assurance Co., " $40.
21. $3,150 " Bank of Toronto, " $200.
22. $8,175 " B. & L Association, " $25.
23. £475 " North-West Land Co., " £5.
24. $6,400 " Imperial Bank, " $100.

435. To find the cost price or selling price of any number of shares, the market value of the shares being given, and vice versa.

EXAMPLE 1.—What is the cost of 60 shares of Bank of Commerce Stock at 121 %, brokerage $\frac{1}{4}$ % ?

SOLUTION.

Cost of 1 share $= \$121 + \$\frac{1}{4} = \$121\frac{1}{4}$

" 60 shares $= \$121\frac{1}{4} \times 60 = \$7,275.$

EXAMPLE 2.—What will be received as proceeds of a sale of 60 shares of Bank of Commerce Stock at 121 %, brokerage $\frac{1}{4}$ % ?

SOLUTION.

Selling price 1 share $= \$121 - \$\frac{1}{4} = \$120\frac{3}{4}$

" 60 shares $= \$120\frac{3}{4} \times 60 = \$7,245.$

EXAMPLE 3.—If 60 shares of Bank of Commerce Stock cost $7,275, find the market value, brokerage $\frac{1}{4}$ %.

SOLUTION.

60 shares cost $7,275

$\therefore$ 1 share costs $\frac{7,275}{60} = \$121\frac{1}{4}$

$\$121\frac{1}{4} - \$\frac{1}{4}$ brokerage $= \$121 =$ market value.

EXAMPLE 4.—If 60 shares Bank of Commerce Stock sold for $7,245, find the market value of the stock, brokerage $\frac{1}{4}$ %.

SOLUTION.

60 shares sold for $7,245

$\therefore$ 1 share sold for $\frac{\$7,245}{60} = \$120\frac{3}{4}$

$\$120\frac{3}{4} + \$\frac{1}{4}$ brokerage $= \$121 =$ market value.

EXAMPLE 5.—How many shares Bank of Commerce Stock at 121 can be bought for $7,275, brokerage $\frac{1}{4}$ % ?

SOLUTION.

Cost of 1 share $= \$121 + \$\frac{1}{4} = \$121\frac{1}{4}$

$\$7,274 \div \$121\frac{1}{4} = 60$ shares. Ans.

EXAMPLE 6.—How many shares Bank of Commerce Stock at 121 must I sell to realize $7,245, brokerage ¼%?

SOLUTION.

Receipts from sale 1 share = $121 − $¼ = $120¾

$7,245 ÷ 120¾ = 60 shares. Ans.

EXERCISE 99.

Find the amount of cash required to purchase—

	SHARES.	MAR. VAL.	BROK.		SHARES.	MAR. VAL.	BROK.
1.	70	110	¼ %.	9.	135	11 ½	¼ %.
2.	60	75	¼ %.	10.	46	87¼	⅛ %.
3.	120	35	⅛ %.	11.	130	93¾	¼ %.
4.	300	140	⅛ %.	12.	200	75¼	¼ %.
5.	45	220	½ %.	13.	75	86¼	⅜ %.
6.	90	206	¼ %.	14.	170	122¾	¼ %.
7.	110	105	⅛ %.	15.	800	261¼	½ %.
8.	36	80	¼ %.	16.	360	87¾	⅛ %.

Find the cash received from the sale of—

	SHARES.	MAR. VAL.	BROK.		SHARES.	MAR. VAL.	BROK.
17.	160	96	¼ %.	25.	200	110½	¼ %.
18.	70	47	1/16 %.	26.	48	223¼	⅛ %.
19.	200	135	¼ %.	27.	120	260⅝	½ %.
20.	96	120	½ %.	28.	36	150¼	½ %.
21.	118	110	½ %.	29.	45	75¼	¼ %.
22.	250	80	⅓ %.	30.	160	37⅝	⅛ %.
23.	36	84	⅛ %.	31.	240	145⅛	¼ %.
24.	87	120	⅛ %.	32.	60	75¾	½ %.

Find the market value of the stock when—

	SHARES.			BROK.		SHARES.			BROK.
33.	30	cost	$3,615	¼ %.	41.	70	sold for	$5,600	¼ %.
34.	40	"	2,405	⅛ %.	42.	84	"	6,720	⅛ %.
35.	50	"	3,795	¼ %.	43.	100	"	7,525	¼ %.
36.	60	"	7,215	¼ %.	44.	60	"	4,890	½ %.
37.	80	"	6,410	½ %.	45.	48	"	3,858	⅛ %.
38.	120	"	14,520	⅛ %.	46.	56	"	3,962	¼ %.
39.	360	"	25,245	⅛ %.	47.	75	"	4,500	⅛ %.
40.	90	"	6,750	¼ %.	48.	80	"	7,270	⅜ %.

How many shares may be bought for—

	COST.	MAR. VAL.	BROK.		COST.	MAR. VAL.	BROK.
49.	$13,155	225	½ %.	53.	$1,923	80	⅜ %.
50.	$9,760	121⅞	¼ %.	54.	$3,850	96	½ %.
51.	$5,610	140	⅜ %.	55.	$12,025	210	¼ %.
52.	$13,620	85	⅜ %.	56.	$4,134	86	⅜ %.

How many shares must be sold to realize—

	S. P.	MAR. VAL.	BROK.		S. P.	MAR. VAL.	BROK.
57.	$8,505	121⅞	¼ %.	61.	$10,755	220	½ %.
58.	$10,245	85½	⅜ %.	62.	$2,400	96¼	¼ %.
59.	$4,314	90	⅜ %.	63.	$4,336	130⅜	½ %.
60.	$4,350	87⅜	⅜ %.	64.	$10,548	110	⅜ %.

436. Given the number of shares or amount of stock held and rate per cent of dividend, to find income, or vice versa.

EXAMPLE 1.—What income will be derived from 60 shares G. T. R Stock paying 6 % dividends?

SOLUTION.

Income from 1 share is $6
"   60 shares is $6 × 60 = $360.

EXAMPLE 2.—What would a stockholder, who owns $4,000 Bank of Commerce Stock, receive from a 5 % dividend?

SOLUTION.

$4,000 stock = 40 shares
40 shares at $5 income per share = $200.

EXAMPLE 3.—What number of shares does a person hold who receives $300 income, from a 6 % dividend?

SOLUTION.

$6 income is derived from 1 share
∴ $300   "   "   300 ÷ 6 = 50 shares.

EXAMPLE 4.—What amount of stock must be held to obtain $200 Income from a 4 % dividend?

SOLUTION.

$4 income is derived from 1 share
∴ $200   "   "   200 ÷ 4 = 50 shares.
50 shares = 50 × 100 = $5,000 stock.

EXAMPLE 5.—What is the rate per cent. dividend when 40 shares yield an income of $240 ?

SOLUTION.

40 shares yield an income of $240

∴ 1 share yields an income of $6

∴ rate per cent, dividend is 6 %.

EXAMPLE 6.—$300 income is derived from $3,750 stock ; find the rate per cent. of dividend.

SOLUTION.

$3,750 stock = 37½ shares

37½ shares yield an income of $300

∴ 1 share yields an income of $\frac{300}{37\frac{1}{2}} = \$8$

∴ rate per cent. dividend = 8 %.

EXERCISE 100.

What income will be derived from—

SHARES.	DIV.		SHARES.	DIV.		SHARES.	DIV.
1. 70	6 %.	5.	120	3 %.	9.	130	5½ %.
2. 120	5½ %.	6.	110	3½ %.	10.	145	6 %.
3. 150	4½ %.	7.	75	9 %.	11.	64	7 %.
4. 65	8 %.	8.	126	8½ %.	12.	87	8¼ %.

What income will be derived from—

STOCK.	DIV.		STOCK.	DIV.		STOCK.	DIV.
13. $5,000	7 %.	17.	$3,600	6 %.	21.	$4,100	5½ %.
14. $8,750	3 %.	18.	$4,500	9¼ %.	22.	$2,225	8 %.
15. $4,400	4 %.	19.	$9,150	5½ %.	23.	$4,520	8½ %.
16. $3,620	5 %.	20	$4,375	8 %.	24.	$3,200	6 %.

What number of shares and what stock must be held to obtain—

INCOME.	DIV.		INCOME.	DIV.		INCOME.	DIV.
25. $300	6 %.	29.	$64	4 %	33.	$150	4 %.
26. $420	5 %.	30.	$240	6 %	34.	$450	5½ %.
27. $600	2¼ %.	31.	$520	3¼ %	35.	$160	8¼ %.
28. $500	4 %.	32.	$360	4½ %	36.	$340	3½ %.

What is the rate per cent. of dividend when—

	SHARES.		INCOME.			SHARES.		INCOME.
37.	50	yield	$275.		42.	36	yield	$196.
38.	60	"	$300.		43.	42	"	$189.
39.	90	"	$390.		44.	80	"	$500.
40.	75	"	$450.		45.	54	"	$351.
41.	34	"	$170.		46.	120	"	$900.

What is the rate per cent. of dividend when—

	STOCK.		INCOME.			STOCK.		INCOME.
47.	$3,500	yields	$245.		52.	$4,500	yields	$135.
48.	$3,640	"	$182.		53.	$7,550	"	$453.
49.	$2,250	"	$225.		54.	$8,600	"	$301.
50.	$4,000	"	$380.		55.	$3,275	"	$131.
51.	$2,300	"	$115.		56.	$4,125	"	$330.

437. Given cash invested, market value of stock and rate per cent. dividend to find income, or vice versa.

EXAMPLE 1.—What income will be derived from investing $6,315 in the 6 per cents at 105, brokerage $\frac{1}{4}$%?

SOLUTION.

$\frac{6315}{105\frac{1}{4}}$ = Number of shares bought. Art. 435.

$\frac{6315}{105\frac{1}{4}} \times 6 = $360. Income. Art. 436.

EXAMPLE 2.—What sum must be invested to secure an income of $360 from the 6 per cents at 105, brokerage $\frac{1}{4}$%?

SOLUTION.

$\frac{360}{6}$ = 60, Number of shares held. Art. 436.

$105\frac{1}{4} \times 60 = $6,315, Cash invested. 435.

EXERCISE 101.

1. What income is derived from investing—

	CASH.	RATE.	MAR. VAL.	BROK.		CASH.	RATE.	MAR. VAL.	BROK
1.	$4,210	5 %	105	$\frac{1}{4}$ %.	9.	$8,510	4 %	106$\frac{1}{4}$	$\frac{1}{8}$ %.
2.	$5,715	4$\frac{1}{2}$ %	95	$\frac{1}{4}$ %.	10.	$5,311	5$\frac{1}{2}$ %	110$\frac{1}{2}$	$\frac{1}{4}$ %.
3.	$1,683	3 %	70	$\frac{1}{8}$ %.	11.	$23,070	6$\frac{1}{4}$ %	96	$\frac{1}{2}$ %.
4.	$3,524	6 %	110	$\frac{1}{2}$ %.	12.	$27,320	3$\frac{1}{2}$ %	85$\frac{1}{4}$	$\frac{1}{8}$ %.
5.	$15,025	7 %	150	$\frac{1}{4}$ %.	13.	$5,949	8 %	140	$\frac{1}{4}$ %.
6.	$7,938	8 %	220	$\frac{1}{2}$ %.	14.	$13,025	7 %	130	$\frac{1}{4}$ %.
7.	$24,050	9 %	240	$\frac{1}{2}$ %.	15.	$15,785	10 %	225	$\frac{1}{8}$ %.
8.	$10,189	7$\frac{1}{2}$ %	140	$\frac{1}{4}$ %.	16.	$6,099	4$\frac{1}{2}$ %	76	$\frac{1}{4}$ %.

2. What amount of cash must be invested in order to derive an—

	INCOME.	RATE.	MAR. VAL.	BROK.		INCOME.	RATE.	MAR. VAL.	BROK.
1.	$200	5 %	105	$\frac{1}{4}$ %.	9	$320	4 %	106$\frac{1}{2}$	$\frac{1}{8}$ %.
2.	$270	4$\frac{1}{2}$ %	95	$\frac{1}{2}$ %.	10.	$264	5$\frac{1}{2}$ %	110$\frac{1}{2}$	$\frac{1}{2}$ %
3.	$72	3$\frac{1}{2}$ %	70	$\frac{5}{8}$ %.	11.	$1.500	6$\frac{1}{4}$ %	96	$\frac{1}{8}$ %
4.	$192	6 %	110	$\frac{1}{8}$ %.	12.	$112	3$\frac{1}{2}$ %	85$\frac{1}{4}$	$\frac{1}{8}$ %.
5.	$700	7 %	150	$\frac{1}{4}$ %.	13.	$288	8 %	140	$\frac{1}{4}$ %
6.	$288	8 %	220	$\frac{1}{2}$ %.	14.	$700	7 %	130	$\frac{1}{4}$ %.
7.	$900	9 %	240	$\frac{1}{2}$ %.	15.	$700	10 %	225	$\frac{1}{2}$ %.
8.	$540	7$\frac{1}{2}$ %	140	$\frac{5}{8}$ %.	16.	$360	4$\frac{1}{2}$ %	76	$\frac{5}{8}$ %.

438. To find the per cent. of income from a given investment without regard to its maturity.

EXAMPLE.—What per cent. of my investment shall I secure by purchasing Ontario Bank stock at 105, paying 7 % dividends?

SOLUTION.

On $105 investment, $7 income is derived.

" $1   " $\frac{7}{105}$ "   "

" $100   " $100 \times \frac{7}{105}$ = 6\frac{2}{3}$ income is derived.

∴ rate per cent = 6$\frac{2}{3}$ %.

439. To find how stock must be bought, which pays a given per cent. dividend, to realize a specified per cent. on the investment.

EXAMPLE.—At what price must I buy stock which pays 6 % dividend to realize 8 % on my investment?

SOLUTION.

Since the income derived from 1 share is $6, $6 must therefore be 8 % of my investment for 1 share.

8 % of purchase price of 1 share = $6

∴ 100 %   "   "   "   = $\frac{100}{8} \times 6$ = $75. Ans.

EXERCISE 102.

What per cent. of my investment will be derived from investing in the—

1. 4 per cents at 120.	5. 8 per cents at 125.	9. 3$\frac{1}{2}$ per cents at 70.
2. 5 " 80.	6. 9 " 175.	10. 4$\frac{1}{2}$ " 76.
3. 6 " 110.	7. 10 " 225.	11. 5$\frac{1}{2}$ " 110.
4. 3$\frac{1}{2}$ " 90.	8. 12 " 240.	12. 6 " 90.

At what price must I buy stock which pays—

13.	6 %	dividends	to realize	9 %	on my investment?
14.	4 %	"	"	5 %	"	"
15.	5 %	"	"	6 %	"	"
16.	8 %	"	"	4½ %	"	"
17.	3½ %	"	"	5 %	"	"
18.	4½ %	"	"	3½ %	"	"
19.	7 %	"	"	4 %	"	"
20.	9 %	"	"	16 %	"	"

440. To find the per cent. income derived from investing in bonds or debentures payable in a given time.

Example.—What per cent. income will be received if I buy Dominion 6's at 120, payable at par in 16 years?

Solution 1.

Cost price of $100 of bonds = $120
Selling   "   "   " = $100 = par value.

Loss in 16 years = $20
"   1 year   = $1¼

Income each year from $100 of bonds = $6

∴ Gain each year on $100 of bonds = $6 − $1¼ = $4¾
On $120 invested, the income cleared = $4¾

∴ On $100   "   " = $\frac{4¾}{120}$ × 100 = $3\frac{31}{32}$

∴ $3\frac{31}{32}$ is derived from the investment.

Solution 2.

Receipts of $100 of bonds = $100 par value at end of 16 years
Income   "   " = 96, $6 per year for 16 years

Total receipts   "   " = $196 at end of 16 years
Cost   "   " = 120

∴ Gain on $120 investment = $76 for 16 years
∴   "   100   " = $3\frac{31}{32}$ for 1 year
∴ $3\frac{31}{32}$ % of interest is derived from the investment.

441. To find how bonds must be bought, which have several years to run, and which pay a given per cent. dividend, to realize a specified per cent. on the investment.

EXAMPLE.—At what price must 6 % bonds, payable in 10 years, be bought so as to realize 5 % on the investment ?

SOLUTION 1.

By simple interest.

Amount of $100 of bonds in 10 yrs. at 6 % = $160.

In order to realize 5 % on the investment we can afford to pay the present worth of $160 due in 10 years, reckoning interest at 5 %.

Present worth of $160 for 10 yrs. at 5 % = $\frac{100}{150}$ × 160 = 106\frac{2}{3}$.

We can therefore afford to pay 106\frac{2}{3}$ for $100 of bonds.

SOLUTION 2.

By compound interest.

If $6 income be invested at compound interest as soon as received each year at 5 %, the income at the end of 10 years will amount to $75.467 (see Table of Annuities).

∴ Amount of $100 of bonds at end of 10 years = $175.467, and the present worth of this amount for 10 years at 5 %, compound interest = $175.467 ÷ $1.6289 + = $107.72 +   Ans.

EXERCISE 103.

1. What per cent. of the investment is received as income by purchasing C. P. R. 5's at 105, payable at par in twenty years ?

2. What per cent. income will be received if I buy Dominion 4's at 112, payable at par in sixteen years ?

3. Bought Intercolonial Railway bonds at 90, bearing 4 % interest, having twenty-five years to run.   What per cent. will be realized if they are paid at par at maturity ?

4. What per cent. income will be gained from 8 % bonds, bought at 80, and payable at par in twenty years ?

5. In 1882, Intercolonial 6's, due at par in 1930, were bought for 108.   What interest will this pay ?

6. If I pay 108 for Dominion 4's, having fifteen years to run, what per cent. will I receive if I keep them till they mature, and they are paid at par?

7. At what price must 6 % debentures, payable at par in eight years, be brought to realize 4 % on the investment?

8. Bought railroad bonds payable in five years, and expect to realize 7 % on the investment. What did I pay?

9. What must I pay for 5 % debentures, which mature in fifteen years, that my investment may yield 4 %? (Both simple and compound interest).

10. What shall I pay for a bond of $500 having twelve years to run, with interest at 6 %, in order to make it an 8 % investment? (Both methods).

11. What must be paid for a $600 debenture, due in five years, with interest annually at 4 %, so as to realize 5 % on the investment?

EXERCISE 104.

1. What income will $19,650 invested in Dominion 3½'s at 97¾ yield, brokerage ½ %?

2. If $48,000 is invested, ½ in 5 % stock, at 95½, and ½ in 6 % stock at 112, brokerage ½ % in each case, what annual income is secured?

3. A farm which rents for $411.45 per annum, is sold for $8,229, and the proceeds invested in 5 % bonds at 105, brokerage ½ %. Is the yearly income increased or diminished, and how much?

4. How much must a gentleman invest for his daughter in 7 % bonds, selling at 95, to secure to her a semi-annual income of $315?

5. Bought 800 shares of Michigan Central at 101 ; held them twenty days, paying interest at 7 % on the purchase-money, and sold them at 102⅞. Deducting interest, and brokerage ¼ %, for purchase and sale, what was the net profit ?

6. A man bought 100 shares Canadian Pacific at 79¼, and sold the same at 82⅜. What was the gain, less ⅛ % brokerage ?

7. Governments yielding $240 income a year at 4 % interest, were sold at 108, and the proceeds invested in land at $75 an acre. How many acres were bought ?

8. Which is the ·r investment, R. R. stock at 25 % discount, and paying a semi-annual dividend of 4 %, or money loaned at 10 %, interest payable annually ? What per cent. better ?

9. What per cent. of his money will a man obtain by investing in 6 % stock at 108, at a discount of 16 % ?

10. If stock paying 10 % dividends is at a premium of 12½ %, what per cent. of income will be realized on an investment in it ?

11. Which will yield the better income, 8 % bonds at 110, or 5's at 75 ; 5's at 70, or 6's at 80 ?

12. Which is the more profitable, and how much, to buy B. & L. H. 7's at 105, or 6 % bonds, at 84 ?

13. If a man buys stock at 17 % above par, what per cent does he receive on his investment, if the stock pays a dividend of 8½ % on its par value ($100) ?

14. A man bought 8 shares of stock at 108¾, and after keeping it eleven months received a dividend of $7 a share, and sold the stock then at 109½. What per cent. did he receive on his investment ?

15. How many shares of Dominion Telegraph stock at 84⅜, can be bought for $12,000, brokerage ⅛ % ?

16. Bought Oct. 12th, 400 G. W. R. at 42¼, and 200 Michigan Central at 92½; Nov. 10th, sold the former at 42⅞, and the latter at 93¾. What was my gain, money being worth 5 % ?

17. Which would be the better investment, $12,120 in Michigan Central at 84, paying 3 % annual dividends, or the same invested in Canada Bank stock at 2,020, paying 15 % every two months ?

18. On 84 shares of stock two semi-annual dividends were declared, one at 5 %, the other at 4 %, the investment paid 10 %. What did the stock cost ?

19. A man's income from $2,000 worth of stock is $75 semi-annually. What is the per cent. per annum ?

20. At what per cent. discount must 6 % stock be bought, that the investment may pay 9 % ?

21. If a stock yields 15 % per annum, what is its value when money is worth 8 % ?

22. Which is the more profitable investment, a stock at 120, paying 8 % annually, or a 20-year bond at 90, paying 6 % annually ?

23. At what price must 6 % bonds, payable in eight years, be bought to realize 4 % on the investment.

24. How many shares of a half stock, standing at 5 % above par, should be given in exchange for 700 shares of the stock of an express company, at 25 % below par ?

25. A man subscribed for 300 shares of stock in a manu-facturing company, the par value of which was placed at $50 per share; but, after paying three instalments, amounting to 75 % of the par value, a dividend of 3 % was declared. How much will he receive, and at what rate per cent. on the actual cost ?

26. The gross earnings of a stock company with a capital of $3,500,000 are $420,000; their expenses are 60 % of their gross earnings. What per cent. dividend can they declare, after putting aside $28,000 as a surplus?

27. The receipts of a mining company in one year are $170,000, clear of all expenses. The company has a capital of $500,000, divided into shares of $10 each, reserving $50,000 as a contingent fund. What rate of dividend can it declare for the year? what per month? and how much can be paid on each share of stock?

28. March 4th, deposited with my broker $500 margin, for purchasing 50 shares Canada Pacific R. R. stock at $92\frac{1}{4}$. The stock was sold March 28th at $96\frac{3}{4}$. Allowing 6 % interest on the deposit, and charging 6 % interest on the purchase, and $\frac{1}{8}$ % brokerage, what was the net profit on the transaction?

29. Sold " short " through my broker 200 shares Michigan Central at 90, and "covered" my " short " at $86\frac{5}{8}$. Allowing $\frac{1}{8}$ % commission for buying and selling, what was my net profit?

30. May 6th, I bought through my broker 300 shares of a certain stock at $93\frac{1}{4}$, depositing with him $3,000 as " margin," for his security against loss by a fall of price. On the first of the following month, he sold them for my account at 95. How much does he owe me besides the $3,000, if he charges $\frac{1}{8}$ % brokerage for each transaction, interest at 6 % (for the exact number of days) on the money used in excess of my deposit?

31. Three companies, A, B, and C, are to be consolidated on the basis of the relative market values of their stock.

Thus, A's capital $1,000,000, Market value 100 %;

   B's " $1,500,000,   "   50 %;

   C's " $625,000,   "   40 %.

The capital of the consolidated company is to be
$2,000,000, in 20,000 shares of $100 each. What propor-
tion and what amount of the capital should be allotted to
each of the old companies ; and how much stock in the
new company should the holder of 1 share of the stock
of each of the old companies be entitled to ?

32. A customer deposited $500 margin with a broker
November 23rd, who purchased for him 50 shares Michigan
Central at 80. He sold the same stock November 30th, at
98. What was the gain, brokerage $\frac{1}{8}$ % ?

33. Aug. 30th, a broker purchased for the account of a
customer 300 shares of Railroad Stock at 78. He deposited
as a margin $3,000. On Sept. 22nd , the stock was sold at
74$\frac{3}{4}$. What was the loss ? Interest 6 %, and commission
$\frac{1}{8}$ %.

34. May 10th, a speculator deposited with his broker
$5,000 as a margin, and directed him to purchase for his
account 500 shares Dominion Saving & Loan, preferred at
90$\frac{3}{8}$. May 20th, the stock was sold at 94$\frac{1}{8}$. What was the
gain ? Interest 6 %, brokerage $\frac{1}{8}$ %.

. 35. Sept. 10th, I deposited with my broker $5,000 as
margin, and he purchased for me 200 shares, C. P. R. at
90$\frac{1}{2}$, 200 shares, Lon. & Can. L. & A. (half stock) at 122$\frac{1}{4}$,
and 200 shares Intercolonial Railway Stock at 49$\frac{3}{4}$. The
stocks on Sept. 30th were quoted as follows : C. P. R. 80$\frac{3}{4}$,
Lon. & Can. L. & A., 120$\frac{1}{8}$, Intercolonial Railway 41$\frac{5}{8}$.
How much should I have deposited with my broker to make
my margin of 10 % good, and to cover commission of $\frac{1}{8}$ %
for buying and selling, and interest at 6 % ? If I had been
unable to have made an additional deposit, and the broker
had " sold me out," what would have been my loss ?

EXCHANGE.

—

442. **Exchange** is the system by which merchants in distant places discharge their debts to each other without the transmission of money.

Suppose for example that A. of Toronto owes B. of Halifax $2,000 for grain, and C. of Halifax owes D. of Toronto $2,000 for dry goods. The two debts may be discharged by means of one draft or bill of exchange without the transmission of money. Thus B. of Halifax draws on A. of Toronto for $2,000 and sells the draft to C. of Halifax, who remits it to D. of Toronto, D. of Toronto presents the draft to A. of Toronto for acceptance or payment, and thus both debts are cancelled. There is in effect a setting off or exchange of one debt for the other.

443. A **Bill of Exchange** is a written order, drawn by one party on another, to pay a specified sum of money to a party named therein, or to his order, or to bearer.

444. Bills of Exchange are of two kinds, viz. : **Inland** or **Domestic, and Foreign.**

445. An **Inland Bill of Exchange** is one which is drawn and made payable in the same country.

446. A **Foreign Bill of Exchange** is one which is drawn in one country and made payable in another country.

447. Inland Bills of Exchange are usually called **Drafts,** and are distinguished as **Time Drafts** and **Sight Drafts.**

448. A **Sight Draft** is one which is made payable upon presentation or on demand.

449. A **Time Draft** is one which is made payable at a certain specified time after date or after time of presentation for acceptance.

450. A Bill of Exchange is negotiable when it may be transferred from one person to another by endorsement or assignment.

451. The Rate of Exchange is the rate per cent. which is computed on the Bill of Exchange.

452. The Course of Exchange is the current price paid in one place for bills of exchange on another place. This price varies, according to the relative conditions of trade and commercial credit at the two places, between which exchange is made.

The course of exchange between two countries, depends on their relative amount of indebtedness to each other; and these, in turn, are largely dependent on "the balance of trade," or comparative amount of exports and imports. Thus, if the United States owes Great Britain more than Great Britain owes the United States, which is likely to be the case if it has imported from Great Britain more than it has exported thither, exchange on that country will be in demand, and will consequently command a premium. If, on the other hand, the balance of trade is in favor of the United States—that is, if the exports exceed the imports,— Great Britain will be indebted to the United States, the supply of bills on Great Britain will more than meet the demand, and exchange will fall below par.

The premium for exchange on any country can not long exceed the cost of shipping specie thither; for merchants will transmit coin to pay their indebtedness abroad, if it is cheaper so to do than to buy exchange.

453. The Par of Exchange is the estimated value of the coins of one country as compared with those of another, and is either intrinsic or commercial.

454. The Intrinsic Par of Exchange is the comparative value of the coins of different countries, as determined by their weight and purity.

Thus, according to the mint regulations of Great Britain and France, £1 sterling is equal to 25 fr. 20 cent., which is said to be the par between London and Paris. Exchange between the two countries is said to be at par when bills are negotiated at this rate; that is, when a bill for £100 drawn in London is worth 2,520 francs in Paris, and conversely. When

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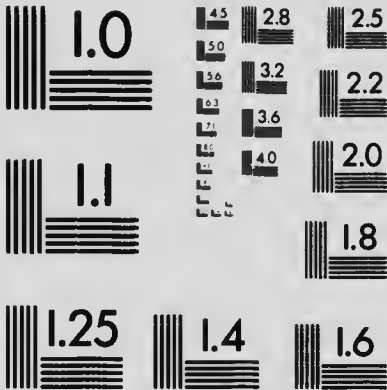
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£1 in London buys a bill on Paris for more than 25 fr. 20 cent., the exchange is said to be in favor of London and against Paris; when £1 in London will not buy a bill on Paris for 25 fr. 20 cent., exchange is against London and in favor of Paris.

Exchange is made to diverge from par by any discrepancy between the actual weight or fineness of the coins and the mint standard, and by the variations in the demand and supply of bills of exchange.

455. The Commercial Par of Exchange is the comparative value of the coins of different countries, as determined by their nominal or market price.

NOTE.—The intrinsic par is always the same while the coins remain unchanged; but the commercial par, being determined by commercial usage, fluctuates.

456. When exchange sells for more than the face of the draft, it is above par, or at a premium, and below par, or at a discount, when sold for less than its face.

———

INLAND OR DOMESTIC EXCHANGE.

457. To find the cost of a draft at sight.

EXAMPLE 1.—How much must be paid for a sight draft of $1,000, on the Bank of Montreal, at a premium of $1\frac{1}{2}$ %?

SOLUTION.

$1 + $.015 = $1.015, course of exchange

∴ $1 costs $1.015

∴ $1,000 cost $1.015 × 1,000 = $1,015. Ans.

EXAMPLE 2.—How much must be paid for a sight draft of $600, on the Bank of Ottawa, at a discount of 1 %?

SOLUTION.

$1 − $.01 = $.99, course of exchange

∴ $1 costs $.99

∴ $600 cost $.99 × 600 = $594. Ans.

458. To find the cost of a time draft.

EXAMPLE 1.—What will be the cost of the following draft, exchange on Hamilton being in Toronto at $2\frac{1}{4}$ % premium?

$600.   TORONTO, July 18th, 1889.

Seventy days after sight, pay to J. S. Carson, or order, six hundred dollars, value received, and charge the same to my account.

JAMES FERGUSON.

To Bank of Montreal, Hamilton.

SOLUTION.

$1 + $.0225 = $1.0225, course of exchange

.012,   bank discount of $1 for 73 da. at 6 % (legal rate)

$1.0105, cost of exchange of $1

$1 cost $1.0105

$600 " $1.0105 × 600 = $606.30.

EXAMPLE 2.—Find the cost of a 60 days' draft on the Bank of Quebec, Toronto, for $900, at a discount of $2\frac{1}{2}$ %.

SOLUTION.

$1 − $.025 = $.975, course of exchange

          .0104 +, bank discount of $1 (63 da.), at 6 % (legal rate)

          $.9646, cost of exchange of $1

      $1 cost $.9646

∴ $900 " $.9646 × 900 = $868.14.

EXERCISE 105.

1. Find the cost of a draft on Montreal for $1,100, at $\frac{1}{4}$ of 1 % premium.

2. Find the cost of a draft on Winnipeg for $1,350, at $\frac{1}{4}$ of 1 % discount.

3. What is the cost of a draft on Chatham for $1,800, at $1\frac{3}{4}$ % premium ?

4. Exchanged $600 in bank notes for gold at 5 % premium. How m .. lid I receive ?

5. Sold $375 uncurrent money at $2\frac{1}{4}$ % discount. How much did I receive ? How much did I lose ?

6. What was the cost of a bill for $240 on Belleville, purchased at $1\frac{1}{4}$ % premium ?

7. Required the amount to pay for a draft to be remitted to Hart & Denton, Kingston, for $1,250, exchange at $\frac{3}{4}$ % discount.

8. Shipped goods to Winnipeg, and received a draft for $2,500, which gave me a profit of 20 % ; sold the draft at $4\frac{1}{2}$ % premium. How much did I gain by both transactions ?

9. Bought goods for $1,250, and sold them at a profit of 25 % ; purchased a draft on Fredricton with the proceeds, at a discount of $\frac{3}{4}$ %. What was the amount of the draft ?

10. A commission merchant sold goods, the net proceeds of which were $2,750. How large a draft can he buy to remit to his consign, if he pays $2\frac{1}{2}$% premium for the draft? How large a draft if he purchases at $2\frac{1}{2}$% discount?

11. Find the cost of a draft for $1,600, payable 30 days after sight, when exchange is $\frac{1}{4}$ of 1 % premium, and interest 6%.

12. Find the cost of a draft for $950, payable in 30 days, when exchange is at par and interest $4\frac{1}{2}$%.

13. Find the cost of a draft for $500, payable 60 days after sight, when exchange is $\frac{1}{2}$ of 1 % discount, and interest 7 %.

14. Find the cost of a draft for $1,200, payable in 90 days after sight, when exchange is $\frac{1}{2}$ of 1 % premium, and interest 7 %.

15. Find the cost of a draft for $810, payable in 90 days, when exchange is at $\frac{1}{4}$ of 1 % premium, and interest $5\frac{1}{2}$%.

16. Find the cost of a draft for $725, payable in 60 days, when exchange is at $\frac{1}{4}$ of 1 % discount, and interest 5 %.

17. What must be paid in Toronto for a draft on Victoria at 90 days, for $4,800, the course of exchange being $101\frac{3}{8}$%?

18. A firm in Toronto bought a 60 days' draft on Montreal for $2,500, at $\frac{3}{8}$% premium, 6 % interest. What did the draft cost?

19. A broker in Montreal bought a 90 days' draft on Halifax for $1,299 at $\frac{1}{2}$% discount. He paid $\frac{1}{8}$% additional for brokerage. How much did he pay for the draft?

20. A commission merchant in Winnipeg sold for a firm in Hamilton a consignment of cotton. The sales amounted to $12.240, and his commission was 5 % on the sales. He bought and remitted a 30 days' draft at $\frac{5}{8}$% discount for the proceeds due the firm. How much did the draft cost?

459. To find the face of a draft at sight.

EXAMPLE 1.—I paid $652.86 for a sight draft on the Bank of Commerce, Winnipeg, at a premium of $\frac{3}{4}$ %. What was the amount of its face ?

SOLUTION.

$1 + $.0075 = $1.0075, course of exchange

$1.0075 is paid for $1   face

$1   "   " $\dfrac{1}{1.0075}$ "

$652.86   "   " $\dfrac{652.86}{1.0075}$ "

∴ Face of draft = $648.

EXAMPLE 2.—A commission merchant in Belleville wishes to remit to his employer at Halifax a sight draft purchased with $7,202.70. What is the face of the draft, exchange at $\frac{5}{8}$ % discount ?

SOLUTION.

$1 − $.00625 = $.99375, course of exchange

$.99375 is paid for $1   face

$1   "   " $\dfrac{1}{.99375}$ "

$7,202.70 "   " $\dfrac{7,202.70}{.99375}$ "

∴ Face of draft = $7.248.

460. To find the face of a time draft.

EXAMPLE 1.—The cost in London of a 70 days' draft on Ottawa, exchange $\frac{7}{8}$ % premium, was $797.40. What was the face of the draft ?

SOLUTION.

$1 + $.00875 = $1.00875, course of exchange

         .012, bank discount of $1 for 73 da. at 6 %

     $.99675 = cost of $1

$.99675 is paid for $1   face

$1   "   " $\dfrac{1}{.99675}$ "

$799.40   "   " $\dfrac{797.40}{.99675}$ "

∴ Face of draft = $800.

EXAMPLE 2.—A commission merchant in Stratford wishes to remit to his employer in Montreal $987.10 by a draft at 30 days. What is the face of the draft which he can purchase with this sum, exchange being at a discount of $\frac{3}{4}$ % ?

SOLUTION.

$1 - $.0075 = $.9925, course of exchange

.0054 +, bank discount for 33 da. at 6 %

$.9871 = cost o' $1

$.9871 is paid for $1   face

$1   "   " $\dfrac{1}{.9871}$ "

$987.10   "   " $\dfrac{987.10}{.9871}$ "

∴ Face of draft = $1,000.

461. To find the rate of exchange on a sight draft.

EXAMPLE 1.—The cost of a sight draft on Winnipeg for $1,200 was $1,213.50. Find the rate of exchange.

SOLUTION.

Cost   = $1,213.50

Face   = $1,200.00

Premium = $13.50

$1,200 was purchased at a premium of $13.50

$1   "   "   " $\dfrac{13.50}{1,200}$

$100   "   "   " $\dfrac{13.50 \times 100}{1,200} = \$1\frac{1}{8}$

∴ Rate of exchange = $1\frac{1}{8}$% premium.

EXAMPLE 2.—The cost of a sight draft on Victoria for $600 was $594.75. What was the rate of exchange ?

SOLUTION.

Face   = $600.00

Cost   = $594.75

Discount = $5.25

$600 was purchased at a discount of $5.25

$1   "   "   " $\dfrac{5.25}{600}$

$100   "   "   " $\dfrac{5.25 \times 100}{600} = \$\frac{7}{8}$

∴ Rate of exchange = $\frac{7}{8}$% discount.

462. To find the rate of exchange on a time draft.

EXAMPLE 1.—The cost in Collingwood of a 70 days' draft for $1,000 is $1,020. Interest being 6 %, what was the rate of exchange?

SOLUTION.

Cost	$1,020
Face	$,1000
Premium, less interest	$20
Interest for 73 da. at 6 %	$12
Full premium	$32

$1,000 was purchased at a premium of $32

$1 　　 " 　　 " 　　 " 　　 $\dfrac{32}{1,000}$

$100 　　 " 　　 " 　　 " 　　 $\dfrac{32 \times 100}{1,000} = $3\frac{1}{5}$

∴ Rate of exchange = $3\frac{1}{5}$ % premium.

EXAMPLE 2.—The cost in Quebec of a 70 days' draft for $6,000 is $5,910. Interest being 6 %, what is the rate of exchange?

SOLUTION.

Cost	$6,000
Face	$5,910
Discount, plus interest	$90
Interest for 73 da ..	$72
Full discount.. ..	$18

$6,000 was purchased at a discount of $18

$1 　　 " 　　 " 　　 " 　　 $\dfrac{18}{6,000}$

$100 　　 " 　　 " 　　 " 　　 $\dfrac{18 \times 100}{6,000} = $\frac{3}{10}$

∴ Rate of exchange = $\frac{3}{10}$ % discount.

EXERCISE 106.

1. A sight draft was purchased for $550.62, exchange being at a premium of $3\frac{1}{2}$ %; what was the face?

2. What is the face of a sight draft bought for $7,500 at a premium of $2.50 ? ($2.50 on $1,000 = $\frac{1}{4}$ %.)

3. Find the largest draft payable 30 days after date that can be bought for $4,985.00, exchange being at a premium of $\frac{1}{4}$%.

4. What per cent. of its face is the cost of a 90 days' draft, if exchange is 1% premium, and interest is allowed at 4%?

5. Find the face of a 60 days' draft, bought for $620.75, if exchange is $2.50 discount, and interest 6%.

6. Find the face of a draft, payable 60 days after date, that can be bought for $1,125, when exchange is at $\frac{1}{4}$ of 1% discount, and interest $5\frac{1}{2}$%.

7. Find the face of a draft, payable 30 days after date, that can be bought for $520, when exchange is at $\frac{1}{2}$ of 1% premium, and interest 4%.

8. Find the face of a draft, payable 60 days after sight, that can be bought for $1,250, when exchange is at $\frac{1}{4}$ of 1% premium, and interest 7%.

9. Find the face of a draft, payable 30 days after sight, that can be bought for $274, when exchange is at par, and interest 6%.

10. Find the face of a draft, payable 90 days after date, that can be bought for $10,000, when exchange is at par, and interest $4\frac{1}{2}$%.

11. A commission merchant in Detroit wishes to remit to his employer in St. Louis, $512.36 by draft at 60 days; what is the face of the draft which he can purchase with this sum, exchange being at $2\frac{1}{2}$% discount?

12. An agent in Halifax having $1,324.74 due his employer, is instructed to purchase with the same a draft drawn at 30 days; what will be the face of the draft, exchange being at $1\frac{3}{4}$% premium?

13. My agent in Winnipeg sells a house and lot for $7,500, on commission of $1\frac{1}{2}$%, and remits to me the proceeds in a draft purchased at $\frac{1}{2}$% premium; what sum do I receive from the sale of my property?

14. The Merchants' Bank of New York having declared a dividend of $6\frac{1}{4}$%, a stockholder in Toronto drew on the bank for the sum due him, and sold the draft at a premium of $1\frac{3}{4}$%, thus realizing $509.75 from his dividend; how many shares did he own?

15. A man in Owen Sound has $4,800 due him in Quebec; how much more will he realize by making a draft for this sum on Quebec and selling it at $\frac{1}{2}$% discount, than by having a draft on Owen Sound remitted to him, purchased in Quebec for this sum at $\frac{3}{4}$% premium?

16. A man in Brantford purchased a draft on Montreal for $5,320, drawn at 60 days, paying $5,141.78; what was the course of exchange?

17. An agent owing his principal $5,059.20, was directed to buy a draft with this amount, and remit it. The principal received $4,960; what was the rate of exchange?

18. Sight exchange on Toronto for $5,000 cost $5.075; what was the course of exchange?

FOREIGN EXCHANGE.

463. Foreign Exchange is the exchange which is carried on between different countries, and is distinguished as direct and circuitous.

Exchange with Europe is effected mainly through the great financial centres, London, Paris, Antwe p, Berlin, Hamburg, Frankfort, and Amsterdam.

464. Direct Exchange is confined to the two places between which the money is to be remitted.

465. There are always two methods of transmitting money between two places. Thus, if A. is to receive money from B.,

1st. A. may draw on B. and sell the draft;

2nd B. may remit a draft made in favor of A.

No   One person is said to *draw on* another person when he is the make   a draft addressed to that person.

466. A set of exchange is a bill usually drawn in triplicate   bearing the same date, payable to the same party, and   essed that when one of the bills is paid the oth   me void.

The   oject of drawing Bills of Exchange in sets of three is   provide against loss in transmitting from one country   another. The bills are sometimes sent by different r   tes or by the same route at different dates. Some merc   ts send only the first and second and preserve the third

SET OF EXCHANGE.

(1.)

£1,000.　　　　　　　　　　　　　Toronto, July 23, 1889.

Sixty days after sight of this First of Exchange (Second and Third of the same tenor and date unpaid), pay to the order of H. E. Clarke One Thousand Pounds Sterling, value received, and charge the same to account of

　　　　　　　　　　　　　　　JOHN McDONALD & Co.

To Brown, Shipley & Co.,
　　　　　　　London, England.

No. 179.

(2.)

£1,000.　　　　　　　　　　　　　Toronto, July 23, 1889.

Sixty days after sight of this Second of Exchange (First and Third of the same tenor and date unpaid), pay to the order of H. E. Clarke, One Thousand Pounds Sterling, value received, and charge the same to account of

　　　　　　　　　　JOHN　　DONALD & Co.

To Brown, Shipley & Co.,
　　　　　　　London, England.

No. 179.

(8.)

£1,000.　　　　　　　　　　　　　Toronto, July 23, 1889.

Sixty days after sight of this Third of Exchange (First and Second of the same tenor and date unpaid), pay to the order of H. E. Clarke, One Thousand Pounds Sterling, value received, and charge the same to account of

　　　　　　　　　　　　　　　JOHN McDONALD & Co.

To Brown, Shipley & Co.,
　　　　　　　London, England.

No. 179.

467. Foreign Bills of Exchange are usually drawn in the currency of the country in which they are paid. Thus drafts on England are usually drawn in pounds, shillings, and pence; on France, Belgium, and Switzerland, in francs; on Germany in marks, etc.

468. Foreign Bills of Exchange are usually drawn at sight (3 days), or at sixty (63 days) days' sight.

469. Quotations for £ days refer to sight exchange, on the theory that 3 days' grace are allowed on sight drafts, though custom varies in this respect.

470. Sight drafts are frequently called "short" exchange, and 60 day drafts, "long" exchange.

71. "Long" exchange is sold at a rate below that for "short" exchange, sufficient to equalize the difference in interest between the dates of maturity of the two classes of bills, the banker having the use of the money from the time the draft is drawn till it is paid.

472. A Letter of Credit is a draft made by a banker in one country, addressed to *foreign* bankers, by which the holder may draw funds at different places to any amount not exceeding the limits of the letter of credit.

473. Exchange on England (sterling exchange) is quoted by giving the value of £1 in dollars and cents.

Thus, when exchange is 4.84, a draft of £1 will cost $4.84; of £100, $484.

474. By Act of Parliament the value of the pound sterling was fixed at $4⅔ (9£=$40). This is much below its intrinsic value, which is now fixed at $4.86⅔. The rates of exchange usually quoted in commercial papers are

calculated at a certain per cent. on the old par of exchange.

Exchange is at par between Great Britain and Canada when the old par of exchange is at a premium of $9\frac{1}{2}$ per cent., for $\$4\frac{4}{9}$ increased by $9\frac{1}{2}$ per cent., equals $\$4.86\frac{2}{3}$.

475. Sterling quotations usually range between 4.80 and 4.91 (*i. e.* $\$4.80$ to $\$4.91$ to the £ sterling). Two quotations are mentioned for each kind of exchange, and indicate the highest and lowest price paid on the same day. Thus 60 days' sterling 4.86 @ 4.87, means that the lowest quotation to the £ was $\$4.86$, and the highest $\$4.87$.

Quotations are frequently given with reference to the old par of exchange. Thus 60 days' sterling $9\frac{1}{4}$ to $9\frac{1}{2}$ means that the old par of exchange (£1 = $\$4\frac{4}{9}$) ranges from $9\frac{1}{4}$ % to $9\frac{1}{2}$ % premium, *i. e.* the lowest course of exchange is $\$4\frac{4}{9} \times 1.09\frac{1}{4}$; the highest, $\$4\frac{4}{9} \times 1.09\frac{1}{2}$.

476. Exchange on France, Belgium, and Switzerland, is quoted by giving the value of $1 in francs and centimes. Thus, when exchange is $5.27\frac{1}{2}$, $1 will buy 5 francs and $27\frac{1}{2}$ centimes.

477. Exchange on Amsterdam, (Netherlands), is quoted by giving the value of one guilder or florin in Canadian currency.

The intrinsic par value of one guilder is $40\frac{2}{10}$ cents.

478. Exchange on Germany is quoted by giving the value of 4 marks (reichsmarks) in cents.

The intrinsic par value of 1 mark is $23\frac{8}{10}$ cents.

VALUES OF FOREIGN MONEY IN CANADIAN CURRENCY.

COUNTRY.	MONETARY UNIT.	STANDARD.	Value in Canadian Money.	STANDARD COIN.
Argentine Republic	Peso-fuerte	Gold and Silver	$.96.5	
Australia				(See Great Britain.)
Austria	Florin	Silver	.40.7	
Belgium	Franc	Gold and Silver	.19.3	5, 10 and 20 francs.
Bolivia	Bolviano	Silver	.82.9	Boliviano.
Brazil	Milrels of 1,000 reis	Gold	.54.6	
Bogota	Peso	Silver	.96.5	
Central America	Dollar	Silver	.93.5	
Chili	Peso	Gold and Silver	.91.9	Condor, doubloon and escudo.
China	Taol	Silver	1.36.0	
Cuba	Peso	Gold and Silver	.92.2	1-16, ¼, ½, ¾, and 1 doubloon.
Denmark	Crown	Gold	.26.8	10 and 20 crowns.
Ecuador	Peso	Silver	.82.3	Peso.
Egypt	Piaster	Gold	.04.9	5, 10, 25, 50 and 100 piasters.
France	Franc	Gold and Silver	.19.3	5, 10 and 20 francs.
Great Britain	Pound Sterling	Gold	4.86.6½	½ sovereign and sovereign.
Greece	Drachma	Gold and Silver	.19.3	5, 10, 20, 50 and 100 drachmas.
German Empire	Mark	Gold	.23.8	5, 10 and 20 marks.
India	Rupee of 16 annas	Silver	.39.0	
Italy	Lira	Gold and Silver	.19.3	5, 10, 20, 50 and 100 lire.
Japan	Yen	Silver	.88.8	1, 2, 5, 10 and 20 yen, gold and silver or yen.
Liberia	Dollar	Gold	1.00.0	
Mexico	Dollar	Silver	.89.4	Peso or dollar, 5, 10, 25 and 50 centavo.
Netherlands	Florin	Gold and Silver	.40.2	10 an 1 90 crowns.
Norway	Crown	Gold	.24.8	10 and 1 90 crowns.
Peru	Sol	Silver	.82.3	Sol.
Portugal	Milrels of 1,000 rels	Gold	1.08.0	2, 5 and 10 milrees.
Russia	Konbles of 100 copecks	Silver	.65.8	¼, ½ and one rouble.
Sandwich Islands	Dollar	Gold	1.00.0	
Spain	Peseta of 100 centimes	Gold and Silver	.19.3	5, 10, 20, 50 and 100 pesetas.
Sweden	Crown	Gold	.96.8	10 and 20 crowns.
Switzerland	Franc	Gold and Silver	.19.3	5, 10 and 20 francs.
Tripoli	Mahbub of 20 piastors	Silver	.94.3	
Turkey	Piaster	Gold	.04.4	25, 50, 100, 250 and 500 plasters.
United States	Dollar	Gold and Silver	1.00.0	
U. S. of Columbia	Peso	Silver	.82.3	Peso.
Uruguay	Patacon	Silver	.94.9	
Venezuela	Bolivars	Gold and Silver	.19.2	5, 10, 20, 50 and 100 bolivars.

480. To find the cost of a foreign bill of exchange.

EXAMPLE 1.—How much must be paid in Toronto for a bill of exchange on Liverpool for £1,200, exchange being quoted at $4.86¾ to the £ sterling?

SOLUTION.

Cost of £1 = $4.86¾

∴ " £1,200 = $4.86¾ × 1,200 = $5,841. Ans.

EXAMPLE 2.—How much must be paid in Hamilton for a draft on Paris for 2,072 francs, exchange being quoted at 5.18?

SOLUTION.

5.18 francs = $1

1 franc = $$\frac{1}{5.18}$$

2,072 francs = $$\frac{2,072}{5.18} = \$400. \quad Ans.$$

EXAMPLE 3.—What will be the cost in Montreal of the following bill of exchange on Liverpool, at 9½ % premium?

£432. MONTREAL, July 22nd, 1889.

At sight of this first of exchange (second and third of same tenor and date unpaid), pay to the order of W. R. Telford, Montreal, four hundred and thirty-two pounds, value received, and charge the same to the account of,

J. P. HUME & Co.

To Alex. Grant & Son.,
    Liverpool, England.

SOLUTION.

£9 = $40 × 1.095

£1 = $$\frac{40 \times 1.095}{9}$$

£432 = $$\frac{40 \times 1.095 \times 432}{9} = \$2,102.40 \quad Ans.$$

EXPLANATION.

Since exchange on Liverpool is at 9½% premium, £9 will cost $40 × 1.095. Art. 475.

EXERCISE 107.

1. Sold to a broker 480 English sovereigns at 4.86. I was paid in currency when gold was quoted at 1.05¼. How much did I receive?

2. An importer purchased a bill of exchange on London, at 3 days' sight, for £488 16s. 6d., at 4.85½. What was the cost?

3. Find the cost of a bill of exchange on Manchester, for £485 12s. 6d. at the par value.

4. An exporter sold a draft for £540 3s. on Liverpool, payable in London, at 4.84, brokerage ⅛%. What were the proceeds?

5. What is the cost in Kingston of a bill on London, Eng., for £425 6s. 8d., at 9¾% premium?

6. How much will a draft on Berlin for 2,400 marks cost, exchange being quoted at 94½?

7. Bought a bill of exchange on Paris for 3,760.20 francs, when exchange was 5.22¼. What did the bill cost?

8. What is the cost in Toronto of a bill of exchange on St. Petersburg for 3000 roubles at 1¼% premium, the par of exchange being $.754 for 1 rouble?

9. What is the cost of a bill of exchange on New York for $7,200, at ⅝% premium?

10. Bought at par, 260 rupees of India, 560 Austrian florins, and 480 crowns of Denmark. How much did I pay for all?

11. Sold a bill of exchange on Amsterdam for 1,440 guilders. Exchange 39⅜. What was the sum obtained?

12. Sold exchange on Geneva, through a broker, for 8,000 francs at 60 days' sight. What were the proceeds of the draft, exchange being 5.20⅝, brokerage ⅛%?

13. What will it cost to remit 8,750 francs to Antwerp at par value?

14. What were the proceeds of a draft, sold through a broker, for 8,748 marks (Reichsmarks), at 94⅜, brokerage ⅛%?

15. What are the proceeds of a draft on Paris for 12,420 francs, at 5.19¾, brokerage on exchange ⅛%?

481. To find the course of exchange.

EXAMPLE 1.—The cost of a bill of exchange on Liverpool for £500, including a brokerage of ⅛ %, was $2,443.05. What was the quotation?

<div align="center">SOLUTION.</div>

$$100\% + \tfrac{1}{8}\% = 100\tfrac{1}{8}\%.$$

$$100\tfrac{1}{8}\% \text{ of cost of bill } = \$2,443.05$$

$$\therefore \text{ Cost of bill } = \frac{2,443.05 \times 100}{100\tfrac{1}{8}} = \$2,440.$$

$$\therefore \text{ £500 are worth } \quad \$2,440$$

$$\text{£1 is worth } \quad \frac{2,440}{500} = \$4.88, \text{ course of exchange.}$$

EXAMPLE 2.—The cost of a bill of exchange on Hamburg for 4,400 marks, including brokerage of ¼ %, was $1,057.32. What was the course of exchange on Hamburg?

<div align="center">SOLUTION.</div>

$$100\% + \tfrac{1}{4}\% = 100\tfrac{1}{4}\%.$$

$$100\tfrac{1}{4}\% \text{ of cost of bill } = \$1,057.32$$

$$\therefore \text{ Cost of bill } = \frac{\$1,057.32 \times 100}{100\tfrac{1}{4}} = \$1,056.$$

$$\therefore \text{ 4,400 marks are worth } \$1,056$$

$$1 \text{ mark is worth } 24\text{c.}$$

$$24\text{c.} \times 4 = 96\text{c.} = \text{ course of exchange. Art. 478.}$$

<div align="center">EXERCISE 108.</div>

Find the course of exchange of a bill.

	Face		Cost		Brokerage	
1.	Face	£5,000,	Cost	$24,250.50,	Brokerage	⅛ %.
2.	"	£2,000,	"	$9,732.15,	"	⅜ %.
3.	"	3,200 marks,	"	$765.66,	"	½ %.
4.	"	800 "	"	$184.23,	"	¼ %.
5.	"	1,600 guilders,	"	$645.61,	"	½ %.
6.	"	3,600 "	"	$1,680.75,	"	⅛ %.
7.	"	1,854 francs,	"	$360.45,	"	⅛ %.
8.	"	366.20 "	"	$72.09,	"	¼ %.
9.	"	2,200 reichsmarks,	"	$528.66,	"	⅛ %.
10.	"	5,500 " "	"	$1,321.65,	"	⅛ %.

11. A draft on Dublin for £360 cost $1,736. What was the course of exchange?

12. The cost in currency, when gold was at 104¼, for a bill of exchange for 12,800 guilders on Amsterdam was $6,245.80, including ⅛% brokerage. What was the course of exchange?

13. I paid $5,817 for a bill of exchange for £1,200 on Liverpool. What was the course of exchange, exclusive of brokerage?

14. The cost, including ⅛% brokerage, for a draft on Antwerp for 833 francs was $161. What was the course of exchange?

15. A merchant paid $755 for a bill of exchange for 3,200 marks on Frankfort. What was the course of exchange, no charges for brokerage being made?

482. To find the Face of a Foreign Bill of Exchange.

EXAMPLE 1.—A bill of exchange on Manchester, England, cost $1194.94 when exchange was 4.88. What was the face of the bill?

SOLUTION.

$4.88 = cost of £1

$1 = " £$\frac{1}{4.88}$

$1194.94 = " £$\frac{1194.99}{4.88}$ = £244.875.

= £244 17s. 6d. Face of bill.

EXAMPLE 2.—The cost of a bill of exchange on Bremen was $570, when exchange was 95. What was the face of the bill?

SOLUTION.

$.95 = cost of 4 marks. (Art. 478).

$1 = " $\frac{4}{.95}$ "

$570 = " $\frac{4 \times 570}{.95}$ "

= 2,400 marks, Face of bill.

EXAMPLE 3.—The cost of a bill of exchan on Paris was $500, when exchange was at 5.18. What was the face of the bill?

SOLUTION.

$1 = cost of 5.18 francs.

$500 = " 5.18 × 500 "

= 2,590 francs, Face of bill.

EXERCISE 109.

1. A bill of exchange on Montreal, cost £125 in Birmingham, England, exchange being at 8% premium for sterling ; required the face of the bill?

2. Bought a bill of exchange on London, when exchange was 4.90 and gold 102½. I paid $37,668.75 in currency. What was the face of the bill?

3. An agent remitted to his principal a draft on Toronto from Amsterdam at ⅛% brokerage, exchange being at 40. The cost of the draft in Amsterdam, including brokerage, was 960 guilders. What was the face of the draft?

4. A broker invested $1,158 in Paris francs at par. How many francs did he purchase?

5. What will be the face of a bill on Hamburg, exchange being quoted at 94½ and the cost of the draft $756?

6. An agent in Boston, having $7,536.30 due his employer in England, is directed to remit by a bill on Liverpool. What is the face of the bill which he can purchase for this money, exchange being at 11% premium?

7. A merchant in Chatham has 9,087 guilders, 10 stivers, due him in Amsterdam, and requests the remittance by draft. What sum will he receive, exchange on Canada being in Amsterdam at 2½ guilders for $1? (1 guilder = 20 stivers.)

8. What is the face of a 3 days' draft on Brem , that was purchased in Hamilton for $3,261.60, exchange 94⅜?

9. A trader in London, Eng., wishes to invest £2,500 in merchandise in Lisbon. If he remits to his correspondent at Lisbon a bill purchased for this sum at the rate of 64½d. sterling, per milree. What sum in the currency of Portugal will the agent receive?

10. G. B. Smith & Co., Toronto, instructed their agent at Berlin to draw on them for a bill of goods of 4,500 marks, exchange at 97½, brokerage ¼%. What did they pay in Canadian money for the goods?

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FOREIGN CIRCUITOUS EXCHANGE.

483. Arbitration of Exchange is the process of finding the cost of exchange between two places, if remittance be made through one or more intermediate places.

NOTE.—1. When there is only one intermediate exchange, the process is called Simple Arbitration; when there are two or more intermediate exchanges, the process is called Compound Arbitration.

2. The object of arbitration is to ascertain the most advantageous route for making drafts or remittances.

484. There are always three methods of receiving money from a place, or of transmitting money to a place, by means of indirect exchange through one intervening place. Thus,

If A. is to receive money from C. through B., 1st. A. may draw on B., and B. draw on C.; 2nd. A. may draw on B., and C. remit to B.; 3rd. B. may draw on C., and remit to A.

If A. is to transmit to C. through B., 1st. A. may remit to B., and B. remit to C.; 2nd. A. may remit to B., and C. draw on B.; 3rd. B. may draw on A., and remit to C.

EXAMPLE 1.—A man in Toronto paid a demand bill in Paris of 5,400 francs, by remitting to Amsterdam at the rate of 21 cents for 10 stivers, and thence to Paris at the rate of 28 stivers for 3 francs. How much Canadian money was required?

SOLUTION.

$$28 \text{ stivers } = 3 \text{ francs} \qquad \therefore 2\tfrac{2}{3} \text{ stivers } = 1 \text{ franc.}$$
$$21 \text{ cents } = 10 \text{ stivers} \qquad \therefore 2\tfrac{1}{10} \text{ cents } = 1 \text{ stiver.}$$

$$5,400 \text{ francs} = \frac{5,400 \times 28}{3} \text{ stivers}$$

$$\frac{5,400 \times 28}{3} \text{ stivers} = \frac{5,400 \times 28 \times 21}{3 \times 10} \text{ cents}$$

$$= \$1,058.40. \text{ Ans.}$$

EXPLANATION.

To reduce francs to stivers, multiply by $\frac{2.4}{5}$, because there are $\frac{2.4}{5}$ times as many stivers as there are francs.

To reduce stivers to cents, multiply by $\frac{4.4}{8}$, because there are $\frac{4.4}{8}$ times as many cents as there are stivers.

EXAMPLE 2.—A Montreal merchant remits 55,880 florins to Amsterdam by way of London and Paris, at a time when the exchange of Montreal on London is $4.885 for £1, of London on Paris is 25.4 francs for £1, and of Paris on Amsterdam is 212 francs for 100 florins; $\frac{1}{8}$ per cent. brokerage being paid in London and in Paris. How many dollars will purchase the bill of exchange?

SOLUTION.

100 florins $=$ 212 francs $\quad \therefore \quad \left(\frac{212}{100} \times \frac{100\frac{1}{8}}{100}\right)$ francs $=$ 1 florin.

25.4 francs $=$ £1 $\qquad \therefore £\left(\frac{1}{25.4} \times \frac{100\frac{1}{8}}{100}\right) =$ 1 franc.

$\qquad\qquad\qquad\qquad\qquad$ $4.885 \qquad =$ £1.

55,880 florins $\times \qquad = \dfrac{55,880 \times 212 \times 801}{100 \times 800}$ francs.

$\dfrac{55,880 \times 212 \times 801}{100 \times 800}$ francs $\qquad = £\dfrac{55,880 \times 212 \times 801 \times 801}{100 \times 800 \times 25.4 \times 800}$

$£\dfrac{55,880 \times 212 \times 801 \times 801}{100 \times 800 \times 25.4 \times 800} \qquad = \$\dfrac{55,880 \times 212 \times 801 \times 801 \times 4.885}{100 \times 800 \times 25.4 \times 800}$

$\qquad\qquad\qquad\qquad\qquad\qquad = \$22,840.634 +$ Ans.

EXPLANATION.

To reduce florins to francs, multiply by $\dfrac{212}{100} \times \dfrac{100\frac{1}{8}}{100}$, because there are $\left(\dfrac{212}{100} \times \dfrac{100\frac{1}{8}}{100}\right)$ times as many francs as there are florins.

To reduce francs to £, multiply by $\left(\dfrac{1}{25.4} \times \dfrac{100\frac{1}{8}}{100}\right)$, because there are $\left(\dfrac{1}{25.4} \times \dfrac{100\frac{1}{8}}{100}\right)$ times as many £ as there are francs.

To reduce £ to $, multiply by 4.885, because there are 4.885 times as many $ as there are £.

EXAMPLE 3.—A banker in New York remits $3,000 to Liverpool, by arbitration, as follows: First to Paris at 5 francs 40 centimes per $1; thence to Hamburg at 185 francs per 100 marcs; thence to Amsterdam at 35 stivers per 2 marcs; thence to Liverpool at 220 stivers per £1 sterling. How much sterling money will he have in bank at Liverpool, and what will be his gain over direct exchange at 10 % premium?

SOLUTION.

220 stivers = £1 ∴ $£\frac{1}{220}$ = 1 stiver.

2 marcs = 35 stivers ∴ $\frac{35}{2}$ stivers = 1 marc.

185 francs = 100 marcs ∴ $\frac{100}{185}$ marcs = 1 franc.

5 francs 10 cent = $1 ∴ $\frac{510}{100}$ francs = $1.

$3,000 = ... $\frac{3,000 \times 510}{100}$ francs.

$\frac{3,000 \times 540}{100}$ francs = ... $\frac{3,000 \times 510 \times 100}{100 \times 185}$ marcs.

$\frac{3,000 \times 540 \times 100}{100 \times 185}$ marcs. = $\frac{3,000 \times 540 \times 100 \times 35}{100 \times 185 \times 2}$ stivers.

$\frac{3,000 \times 540 \times 100 \times 35}{100 \times 185 \times 2}$ stivers = $£\frac{3,000 \times 540 \times 100 \times 35}{100 \times 185 \times 2 \times 220}$

= £696 11s. 2d. Circuitous exchange

$(\frac{4}{7} \times \frac{1}{1+2}) = £1$

∴ $3,000 = $£\frac{1111}{1}$ \times \frac{1}{5} \times \frac{1}{1+2}$ = £613 12s. 9d. Direct exchange.

£82 18s. 5d. Gain. Ans.

EXERCISE 110.

1. When exchange at New York on Paris is 5 francs 16 centimes per $1, and at Paris on Hamburg $2\frac{1}{8}$ francs per marc banco, what will be the arbitrated price in New York of 7,680 marc bancos of Hamburg?

2. The exchange at Paris upon London is at the rate of 25 francs 70 centimes for £1 sterling, and the exchange at Vienna upon Paris is at the rate of $40\frac{1}{2}$ Austrian florins for 20 francs: find how many Austrian florins should be paid at Vienna for a £50 note.

3. An agent in Boston, having $7,536.80 due his employer in England, is directed to remit by a bill on Liverpool. What is the face of the bill which he can purchase for this money, exchange being at $\frac{1}{1}$ % premium?

4. Bills on Amsterdam, bought in London at 12 florins 15 cents per £1 sterling, are sold in Paris at 57½ florins for 120 francs. What is the course of exchange between London and Paris?

5. If at Philadelphia, exchange on Liverpool is at 9¼ % premium, and at Liverpool on Paris 26 francs 86 centimes per £1; what is the arbitrated course of exchange between Philadelphia and Paris, through Liverpool?

6. A resident at Naples having a bequest of $8,720 made him in Boston, orders the remittance to be made to his agent in London, who remits the proceeds to Naples, reserving his commission of ½ % on the draft sent. If exchange on London is 9% in Boston, and the rate between London and Naples is £1 for 5 scudi, how much does the man realize from his bequest?

7. A merchant of Toronto wishes to transmit 2,400 marcs banco to Hamburg. He finds exchange between Toronto and Hamburg to be 35 cents for 1 marc. The exchange between Toronto and London is $4.83 for £1; that between London and Paris is 26 francs for £1; and that of Paris on Hamburg is 47 francs for 25 marcs. By what way should the Toronto merchant remit?

8. A person in London owes another in St. Petersburg 920 roubles, which must be remitted through Paris. He pays the requisite sum to his broker, at a time when the exchange between London and Paris is 25.15 francs for £1, and between Paris and St. Petersburg 1.2 francs for 1 rouble. The remittance is delayed until the rates are 25.35 francs for £1 and 1.15 francs for 1 rouble. What does the broker gain or lose by the delay?

9. A merchant in New York wishes to pay £3,000 in London. Exchange on Lo   at par; on Paris, 5

francs 25 centimes per $1; and on Amsterdam, 40 cents to a guilder. The exchange between France and England at the same time 25 francs to £1, that of Amsterdam on England is 12½ guilders to £1. Which is the most advantageous, the direct exchange, or through Paris, or through Amsterdam?

10. When the co   f exchange between London and
Paris is 9½d. per f   £1 3.63 f   s are equivalent to 1
Prussian thaler, a.   5 thalers to 34 Austrian florins,
and 25 Austrian   to 12.6 Venetian ducats,—if a
London merchant   one in Venice 1,000 ducats, will
it be more advantag   remit by way of Paris, Berlin,
and Vienna, or direct   Venice, supposing a ducat to be
equivalent to 4s. 2d

RATIO.

——

485. Ratio is the relation between two mer ' ᵣrs of the same denomination, expressed by the quotient ог the first divided by the second.

Thus the ratio of 9 to 6 is (9 + 6); the ratio of 6 to 9 is (6 + 9).

486. The Sign of ratio is the colon (:).

The ratio of 9 to 6 is expressed 9 : 6, or 9 + 6, or as a fraction ⁹⁄₆.

487. The Terms of a ratio are the numbers compared.

488. The Antecedent is the first term, or the dividend, or, if expressed as a fraction, the numerator.

489. The Consequent is the second term, or the divisor, or, if expressed as a fraction, the denominator.

490. The two terms together form a Couplet.

491. A Direct Ratio is the quotient of the antecedent divided by the consequent

492. An Inverse Ratio or Reciprocal Ratio is the quotient of the consequent divided by the antecedent.

493. Ratios are compared by comparing the fractions by which they are represented.

494. Ratios are compounded by multiplying together the fractions by which they are represented, and expressing the resulting fraction as a ratio.

Thus the ratio compounded of 3 : 5 and 7 : 9, is $\frac{3}{5} \times \frac{7}{9} = \frac{21}{45} = 21 : 45$.

PROPORTION.

—

495. **Proportion** consists in the equality of two ratios.

For example, the ratio of 27 yds. to 9 yds. is $\frac{27}{9} = 3$; the ratio of $37\frac{1}{2}$ cts. to $12\frac{1}{2}$ cts. is $\frac{37\frac{1}{2}}{12\frac{1}{2}} = 3$ and, therefore, the ratio of 27 yds. to 9 yds. is equal to the ratio of $37\frac{1}{2}$ cts. to $12\frac{1}{2}$ cts., since each ratio is equal to 3.

This is expressed thus:—27 yds. : 9 yds. $=$ $37\frac{1}{2}$ cts. : $12\frac{1}{2}$ cts., or 27 yds. : 9 yds. :: $37\frac{1}{2}$ cts : $12\frac{1}{2}$ cts., the double colon (::) being used instead of the sign of equality ($=$), or it may be expressed $\frac{27 \text{ yds.}}{9 \text{ yds.}} = \frac{37\frac{1}{2} \text{ cts.}}{12\frac{1}{2} \text{ cts.}}$

496. The arithmetic test of proportion is, therefore, that the two fractions representing the ratios must be equal.

Since $\frac{6}{12} = \frac{4}{8}$ therefore 6 : 12 :: 4 : 8.

497. The two terms 6 and 8 are called the **extremes**. The two terms 12 and 4 are called the **means**.

6 is called the *first* proportional. 12 is called the *second* proportional, 4 is called the *third* proportional. and 8 is called the *fourth* proportional.

498. Where the two means are the same number, that number is said to be a **mean proportional** between the two extremes.

Thus, in the proportion 4 : 6 :: 6 : 9, 6 is the mean proportional between 4 and 9.

499. When two quantities are connected in such a way, that, when the first is increased any number of times, the second is increased the same number of times, they are said to be in **direct** proportion.

For example, if 1 lb. of sugar cost 8 cts.

    2 lbs. will cost 2 times 8 cts.

    8 "   "   3   "   8 "

    4 "   "   4   "   8 "

        etc., etc.

That is, if we increase the weight any number of times we increase the cost the same number of times. *i.e.*, the cost of the sugar is *directly* proportional to its weight and *vice versa.*

Hence, 1 lb. : 7 lbs. :: 8 cts. : 7 times 8 cts.

500. When two quantities are connected in such a way, that, when the first is increased any number of times, the second is decreased the same number of times, they are said to be in **inverse** proportion.

For example, if one man can do a piece of work in 12 days,

    2 men will do the work in 12 days ÷ 2

    8   "   "   12 days ÷ 3

    4   "   "   12 days ÷ 4

        etc., etc.

That is, if we *increase* the number of men any number of times, we *decrease* the time the same number of times, *i.e.*, the number of men required to do the work is inversely proportional to the number of days, and *vice versa.*

Hence, 1 man : 4 men :: ¹² days : 12 days.

501. The student will obtain from the foregoing illustrations the following principles.

1. The product of extremes is equal to the product of the means.

2. Hence, the product of the extremes, divided by either mean, will give the other mean.

3. The product of the means, divided by either extreme will give the other extreme.

SIMPLE PROPORTION.

502. A Simple Proportion is an expression of equality between two simple ratios.

EXAMPLE 1.—Find the term omitted in the following proportion 3 : 16 :: no. required : 48.

SOLUTION.

3 × 48 + 16 = 9, no. required.   Principle 2.

EXAMPLE 2.—If 5 lbs. of sugar cost 60 cts, find the cost of 11 lbs.

SOLUTION.

Here *more requires more*, (*i.e.*, more weight requires more cost) hence the cost is *directly* proportional to the weight.

∴ 5 lbs. : 11 lbs. :: 60 cts. : required cost.

∴ required cost $= \dfrac{11 \times 60}{5} =$ $1.32 Ans.   Principle 3.

EXAMPLE 3.—If 3 men can do a piece of work in 25 days, how long will it take 5 men to do the same work ?

SOLUTION.

Here *more requires less* (*i.e.*, more men require less time to do the same work) hence, the time is *inversely* proportional to the number of men.

∴ 3 men : 5 men :: time required for 5 men : 25 days (time required for 3 men).

∴ time required for 5 men $= \dfrac{3 \times 25}{5} = 15$ days Ans.   Principle 2.

or,

5 men : 3 men :: 25 days (time required for 3 men) : no. of days required,

.. no. days required $= \dfrac{3 \times 25}{5} = 15$ days.   Principle 3.

EXAMPLE 4.—If 6 men can do a piece of work in 12 days, in what time will 4 men do the same work?

SOLUTION.

Here *less requires more* (*i.e.*, less men require more time to do the same quantity of work), hence the time is *inversely* proportional to the number of men.

∴ 6 men : 4 men :: time required for 4 men : 12 (time required for 6 men),

time required for 4 men $= \dfrac{6 \times 12}{4} = 18$ days, Ans.   Principle 4.

Notes 1.—If the terms of any couplet are of different denominations, they must be reduced to the same denomination.

2. If the odd term is a compound number reduce it to its lowest unit.

3. If the divisor and dividend contain factors common to both, cancel them.

EXERCISE 111.

Find the term omitted, and represented by x, in each of the following proportions :

1. $8 : 52 = 20 : x$.
2. $12 : x = 1 : 144$.
3. $x : 20 :: 120 : 50$.
4. $\$80 : \$4 = x : 8$.
5. $2.5 : 62.5 :: 5 : x$.

6. $\$175.35 : \$x :: \frac{1}{4} : \frac{1}{4}$.
7. $4\frac{1}{2}$ yd. $: x$ yd. $:: \$9\frac{3}{4} : \$27\frac{1}{4}$.
8. $x : 9.01 = 16.05 : 5.35$.
9. $\dfrac{x}{9.01} = \dfrac{16.05}{5.35}$.
10. $\frac{1}{4}$ yd. $: x$ yd. $:: \$\frac{1}{4} : \$5$.

11. If 12 gallons of wine cost \$30, what will 68 gallons cost ?

12. If 9 bush. of wheat make 2 bbl. of flour, how many barrels of flour will 100 bush. make ?

13. If $6\frac{1}{2}$ bush. of oats cost \$3, what will $9\frac{1}{4}$ bush. cost ?

14. What will 87.5 yd. of cloth cost, if $1\frac{3}{4}$ yd. cost \$.42 ?

15. If by selling \$1,500 worth of dry goods I gain \$275.40, what amount must I sell to gain \$1,000 ?

16. What will $11\frac{1}{4}$ lb. of tea cost, if 3 lb. 12 oz. cost \$3.50 ?

17. If a speculator in grain gain \$26.32 by investing \$325, how much would he gain by investing \$2,275 ?

18. In canning 5 lb. of raspberries 3 lb. sugar are needed, how many pounds sugar for 38 lb. of berries ?

19. If with the money I have, I can buy 84 lb. of coffee at 25c a lb., how many pounds can I buy for the same money at 30c a lb ?

20. If wall paper be 20 inches wide, I shall need 7 rolls to paper a room. How many rolls will suffice if the paper be 24 inches wide? If 30 inches wide?

21. If $750 will yield $120 interest in a certain time, what interest will $600 yield in the same time?

22. A man, whose step measures $\frac{5}{4}$ yard, counts 1,200 steps from his house to his office. How many steps will his son have to take, whose step measures $\frac{1}{2}$ yd?

23. If each man on board ship consumes daily $1\frac{1}{4}$ lb. bread, their bread will last $5\frac{1}{5}$ months. How much will each man get per day if it is to last $6\frac{1}{2}$ months?

24. The rate of two pedestrians is as $5:4$. How many miles will the first travel in the same time in which the second travels $84\frac{1}{2}$ miles?

25. At the rate of $180 for $\frac{3}{10}$ acre, what will 5 acres cost?

26. The heat produced by a cubic yard of beech-wood is to that produced by a cu. yd. of pine as $9:7$. How many cu. yd. of beech-wood are needed to produce the heat of 50 cu. yd. of pine?

27. If $1\frac{3}{4}$ yards of velvet cost $5\frac{1}{2}$, what will 9 yd. cost?

28. A farmer sowed 3 bush. of buckwheat on $2\frac{1}{4}$ acres. How much would he need for a field containing $4\frac{1}{2}$ acres?

29. $\frac{4}{5}$ of a sum of money is $800, how much is $\frac{5}{6}$ of it?

COMPOUND PROPORTION.

503. A Compound Proportion is an expression of equality between two ratios, one or both of which are compound.

Thus $\left.\begin{array}{c}3 : 4\\ 6 : 9\end{array}\right\}$ :: 14 : 28 is a proportion composed of a compound and a simple ratio, and may be expressed, $3 \times 6 : 4 \times 9 :: 14 : 28$, equivalent to a simple proportion, 18 : 36 :: 14 : 28.

504. The terms of a proportion have not only the relations of magnitude, but also the relations of **cause** and **effect.**

505. Causes, in proportion, are considered as things that produce a certain result : as, men at work, money lent, horses, time, etc.

506. Effects are the result of causes : as, work done, interest drawn, cost, distance travelled.

507. Every problem in proportion may be considered as a comparison of two causes and two effects ; these causes and effects being themselves either simple or compound.

Thus if 4 tons of hay as a *cause*, will bring, when sold, $24 as an *effect*, 12 tons, when sold, as a *cause*, will bring $72 as an *effect*. Or, if 6 horses as a *cause*, draw 10 tons as an *effect*, 9 horses as a *cause*, will draw 15 tons as an *effect*.

508. *Since like causes produce like effects, the ratio of two like causes must equal the ratio of two like effects produced by these causes.*

Hence every question in proportion must give one of the following statements :

1st cause : 2nd cause :: 1st effect : 2nd effect.
or 1st effect : 2nd effect :: 1st cause : 2nd cause.

EXAMPLE 1.—If 4 horses consume 24 bushels of oats in 12 days, how many bushels will 20 horses eat in 16 days ?

SOLUTION.

1st cause : 2nd cause   ::   1st effect : 2nd effect.

$$\left. \begin{array}{l} \text{4 horses} \quad : \text{20 horses} \\ \text{12 days} \quad : \text{16 days} \end{array} \right\} :: \left\{ \text{24 bush.} : \text{No. bush. required.} \right.$$

∴ No. bush. required $= \dfrac{20 \times 16 \times 24}{4 \times 12} = 160$ bush. Ans. Prin. 3.

EXAMPLE 2.—If 2 workmen dig a ditch 24 yards long and 3 feet wide, and 2 feet deep, in 5 days, how long will it take 3 workmen to dig a ditch 30 yards long, 4 feet wide, and 3 feet deep?

SOLUTION.

1st cause   :   2nd cause   ∵   1st effect : 2nd effect.

$$\left. \begin{array}{ll} \text{2 workmen :} & \text{3 workmen} \\ \text{5 days} \quad : & \text{No days required} \end{array} \right\} :: \left\{ \begin{array}{ll} \text{24 yards} & : \quad \text{30 yards.} \\ \text{3 feet} & : \quad \text{4 feet.} \\ \text{2 feet} & : \quad \text{3 feet.} \end{array} \right.$$

Here one part of the means is missing, and it may be found by dividing the product of the extremes by the product of the given parts of the means.

Hence, required time $= \dfrac{2 \times 5 \times 30 \times 4 \times 3}{3 \times 24 \times 3 \times 2} = 8\frac{1}{3}$ days. Ans. Prin. 2.

EXERCISE 112.

Find the term omitted and represented by x in the following proportions.

1. $\left. \begin{array}{l} 8:9 \\ 3:4 \end{array} \right\} = 40 : x.$

2. $\left. \begin{array}{l} 480: \quad x \\ 30 : 15 \end{array} \right\} :: 84 : 21.$

3. $\left. \begin{array}{l} 6: \quad x \\ 14 : 12 \end{array} \right\} = \left\{ \begin{array}{l} 36 : 48, \\ 56 : 54. \end{array} \right.$

4. $\left. \begin{array}{l} 7 : 28 \\ 12 : \quad x \end{array} \right\} :: \left\{ \begin{array}{l} 60 : 80, \\ 8 : \quad 3, \\ 6 : \quad 8. \end{array} \right.$

5. Five clerks use 25 quires of paper in 8 days. At the same rate, how much paper will 6 clerks use in 10 days ?

6. Six lamps consume 2 gallons of petroleum in 8 days. How many lamps will consume 3 gallons in 12 days?

7. Two workmen dig a ditch of 24 yds. in length and 8 ft. in width in 5 days. How long will it take 3 workmen to dig a ditch 30 yds. long and 4 ft. wide?

8. Eight persons spend $296 on a journey of 7 days. How long will $300 last 7 persons at that rate?

9. If a block of marble 5 ft. long, 3 ft. wide, 2 ft. thick, weighs 4,850 lb., what will a block weigh measuring 7 ft. in length, 4 ft. in width, and 3 ft. in thickness?

10. Ten cwt. of merchandise cost $2½ freight for 245 miles. What will 5 cwt. cost for 210 miles?

11. If $700 at interest amounts to $770 in 15 months, what sum must be put at the same rate to amount to $845 in the same time?

12. From the milk of 20 cows, each giving 18 qts. daily, 16½ cheeses of 50 lb. each are made in 42 days. How many cows, giving but 16 qts. daily, will be needed to make 83 cheeses of 60 lb. each in 28 days?

13. Being asked to find the number of bricks in a wall 10 ft. high, 922 ft. long, and 16 in. thick, I found that a part of the wall, 4 ft. high, 4 ft. long, and 16 in. thick, contained 448 bricks. How many in the whole wall?

14. If $750 gain $202.50 in 4 years 6 months, what sum will gain $15..52 in 1 year 6 months?

15. If it require 1,200 yds. of cloth ⅘ wide to clothe 500 men, how many yards which is ⅞ wide will clothe 960 men?

16. If by travelling 6 hours a day at the rate of 4½ miles an hour, a man perform a journey of 540 miles in 20 days, in how many days, travelling 9 hours a day at the rate of 4⅔ miles an hour, will he travel 600 miles?

17. What sum of money will produce $300 in 8 months, if $800 produce $70 in 15 months ?

18. How many days will 21 men require to dig a ditch 80 ft. long, 3 ft. wide, and 8 ft. deep, if 7 men can dig a ditch 60 ft. long, 8 ft. wide, and 6 ft. deep, in 12 days ?

19. How many men will be required to dig a cellar 45 ft. long, 34.6 ft. wide, and 12.3 ft. deep, in 12 days of 8.2 hours each, if 6 men can dig a similar one 22.5 ft. long, 17.3 wide, and 10.25 ft. deep, in 3 days of 10.25 hours each ?

20. If a bin 8 ft. long, $1\frac{1}{2}$ ft. wide, and $2\frac{1}{2}$ ft. deep hold $67\frac{1}{2}$ bush., how deep must another bin be made, that is 18 f long and $3\frac{5}{8}$ ft. wide, to hold 450 bush. ?

21. How long should A. lend B. $1,175, to balance loans from B. to A. of $100 for 3 months, $400 for 2 months, and $600 for 6 months ? How much should A. lend B. for 10 months, to balance these loans ?

DISTRIBUTIVE PROPORTION.

509. Distributive or Partitive Proportion is the method of dividing a number, or quantity, into parts which are proportional to given numbers.

510. The principle of this rule can be applied to the solution of numerous questions of a practical nature, such as determining the profits and losses of partners in trade, apportioning shares of participators of prize money, finding the relative proportion of ingredients requisite to form a given quantity of a compound, apportioning taxes, school rates, averaging, etc.

EXAMPLE 1.—Divide $600 among A. B. C. and D., so that their shares may be in the proportion of 3, 4, 5 and 6.

SOLUTION 1.

$$3 + 4 + 5 + 6 = 18$$

$18 : 3 :: \$600 : $ A.'s share	$\therefore$ A.'s share $= \$100$
$18 : 4 :: \$600 : $ B.'s share	$\therefore$ B.'s share $= \$133\frac{1}{3}$
$18 : 5 :: \$600 : $ C.'s share	$\therefore$ C.'s share $= \$166\frac{2}{3}$
$18 : 6 :: \$600 : $ D.'s share	$\therefore$ D.'s share $= \$200.$

EXPLANATION.

Altogether there are 18 shares, of which A. gets 3, B. 4, C. 5, D. 6, and the problem then becomes: If 18 shares represent $600, what is represented by 3 shares? by 4 shares? by 5 shares? by 6 shares? These give rise to the preceding proportions.

SOLUTION 2.

A. 3 shares
B. 4   "
C. 5   "
D. 6   "
Total   18 shares.

18 shares $= \$600$

$\therefore$ 1 share $= \frac{600}{18}$

A. gets 3 shares $= \frac{600}{18} \times 3 = \$100$

B. gets 4 shares $= \frac{600}{18} \times 4 = \$133\frac{1}{3}$,
    etc

SOLUTION 3.

A. 8 shares   A. gets $\frac{8}{18}$ of the whole and $\therefore$ $\frac{8}{18}$ of $600 = $100

B. 4 "   B. gets $\frac{4}{18}$ of the whole and $\therefore$ $\frac{4}{18}$ of $600 = 133\frac{1}{3}$

C. 5 "   etc.

D. 6 "

Total   18 shares.

The student is recommended to use either the second or third method of solution

EXAMPLE 2 —Divide $2,000 among A., B., C., so that B. may have $300 more than A., and C. $200 more than B.

SOLUTION.

A.'s share = A.'s share

B.'s share = A.'s share + $300

C.'s share = A.'s share + $300 + $200

Total   = 3 times A.'s share + $800

$\therefore$ 3 times A.'s share + $800 = $2,000

$\therefore$ 3 " "   = $1,200

$\therefore$ A.'s share = $400

B.'s share = $400 + $300 = $700

C.'s share = $700 + $200 = $900.

EXERCISE 113.

1. Divide $60 into two parts proportional to 11 and 9.

2. Divide $2,500 into parts proportional to 2, 3, 7, 8.

3. Divide $8,470 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$.

4. Gunpowder is made of saltpetre, sulphur and charcoal in parts proportional to 75, 10 and 15 ; how many pounds of each are contained in 12 cwt. of gunpowder ?

5. The sides of a triangle are as 3, 4, 5, and the sum of the lengths of the sides is 480 yards : find the sides.

6. Divide $640 among A., B. and C., so that A. may have three times as much as B., and C. as much as A. and B. together.

7. Divide the number 582 into 4 such parts that the second may be twice the first, the third 21 more than the second, and the fourth 54 more than the first.

8. If C. has twice as much money as B., and if $12 be taken from A.'s money, it will be equal to $\frac{1}{2}$ of B.'s; how much has each, the sum of their moneys being $645?

9. A man left his property to be divided among his 3 sons in proportion to their ages, which are 21, 18, and 12 years. The share of the youngest is $1,440. · What was the value of the property?

10. A., B., C., and D. commenced business with a capital of $18,500; A. invested $800 less than B., and C. invested $1,000 more than A., and D. $900 less than C.; how much did each invest?

11. Divide 560 into parts, so that the second may be 4 times the first.

12. A force of police 1,921 strong is to be distributed among 4 towns in proportion to the number of inhabitants in each; the population being 4,150, 12,450, 24,900, and 29,050 respectively. Determine the number of men sent to each.

13. Divide 450 shares of stock among 3 persons, in proportion to the number of shares owned by each; A. holds 400, B. 200, and C. 300; how many shares will each receive?

14. A piece of land of 200 acres is to be divided among 4 persons, in proportion to their rentals from surrounding property. Supposing these rents to be £500, £350, £800, and £90, how many acres must be allotted to each?

15. If $\frac{2}{3}$ of A.'s money, and $\frac{3}{4}$ of B.'s equal $900, and $\frac{1}{2}$ of B.'s is twice $\frac{2}{3}$ of A.'s, what sum has each?

16. A father divided $18,500 among 3 children, so that the portion of the second was greater by one-half than that of the first, and $\frac{1}{2}$ the first was equal to $\frac{1}{3}$ of the third; what was the share of each?

PARTNERSHIP.

----

511. A Partnership is an association of two or more persons, who combine their capital, skill or labor, or all of them, for the purpose of carrying on some lawful business, and for participating in the profits or losses arising therefrom, according to the terms of their agreement.

512. The business association is called a *Firm, House,* or *Company :* and each individual of the association is called a *Partner.*

513. Partners may be classified as—

1. Active partners.
2. Silent or dormant partners.
3. Nominal partners.
4. Special partners.

514. An Active Partner is one who has an interest in the business, and is *known to the public* as a partner.

515. A Silent or Dormant Partner is one who has an interest in the business, but is unknown to the   ic as a partner.

516. A Nominal Partner is one who allows his name to be used for the benefit of the firm, without having any pecuniary interest in its business.

517. A Special Partner is one who is held liable for only a specified amount.

518. In an ordinary partnership, each member is liable to the full extent of his means for the liabilities of the firm; but in a joint stock company, each shareholder is liable only for the amount of his unpaid capital. This explains the meaning of the term "*Limited*," which is added to the names of companies, as for example, "The Canada Publishing Co." (Limited).

519. Capital is the money or property invested in the business.

520. The Resources or Assets of a firm consist of the property it owns and the debts due the firm.

521. The Liabilities of a firm embrace all the debts or obligations due by the firm to its creditors.

522. The Investment is the aggregate of the money or property jointly contributed by the partners.

523. The Net Capital is the excess of the Assets or Resources over all Liabilities.

524. The Net Insolvency is the amount which the liabilities exceed the resources.

525. The Net Investment of a firm is the difference between the total sum invested and the total withdrawals.

526. The Net Gain is the excess of the gains over the losses, during a certain time.

527. The Net Loss is the excess of the losses over the gains, during a certain time.

528. A Partnership Settlement is an adjustment of the partners' accounts setting forth the net investment, liabilities assumed, withdrawals, gains, losses, and showing his net capital or net insolvency at closing or settling the partnership's interests.

529. To divide the Gain or Loss, when each partner's capital has been employed for the same period of time.

EXAMPLE.—A. and B. formed a partnership; A. furnished $3,000, B. $5,000; they gained $2,000, and agreed to share the profit or loss in proportion to the capital of each; what was each partner's share?

SOLUTION.

A.'s capital = 3,000
B.'s " = 5,000
Total " = $8,000

∴ A. furnishes $\frac{3000}{8000}$ or $\frac{3}{8}$ of capital.
B " $\frac{5000}{8000}$ or $\frac{5}{8}$ "

∴ A.'s share of gain = $\frac{3}{8}$ of $2,000 = $750.
B.'s " = $\frac{5}{8}$ of $2,000 = $1,250.

or,

Total gain ($2,000) = $\frac{2000}{8000}$ or $\frac{1}{4}$ of capital = .25 of capital.
∴ A.'s share of gain = $3,000 × .25 = $750.
B.'s " = $5,000 × .25 = $1,250

EXERCISE 114.

1. A. and B. buy a store which rents for $950 a year; A. advanced $3,500, B. $4,800; how much rent should each receive?

2. A. and B. form a partnership, A. furnishing $2,200 and B. $2,500; they lose $800; what is each one's share of the loss?

3. A. put $7,500, and B. $6,000 into a land speculation: and A.'s share of the loss was $225; what was B.'s share?

4. Two men formed a partnership, the former furnishing 3 times as much capital as the latter; they gained $12,500; what was each one's share of the gain?

5. The net gains of A., B., and C. for a year are $12,800; A. furnishes $25,000, B. $18,000, and C. $15,000; how should the profit be divided?

6. X., Y. and Z. bought a ship on speculation ; X. put in $30,000, Y., $20,000, and Z., $15,000 : they sold it at a loss of $7,500, what was each man's share of the loss?

7. A., B., C. and D. form a partnership with a capital of $37,000; A. furnishing $10,000, B. 12,000, C. $5,000, and D. the remainder; they gain 15 % of the joint stock; what is each partner's share of the profit?

8. A., B. and C. entered into partnership; A. furnishing $\frac{1}{2}$, B. $\frac{1}{3}$, and C. the rest of the capital. On winding up the business, C.'s share of the profit was $4,518; what were the respective dividends of A. and B. ?

9. A. invested $12,000 and B. $8,000 in a business. A.'s share of the gain or loss is to be $\frac{2}{3}$ and B.'s $\frac{1}{3}$. At the close of the year their resources are $25,000 in goods and cash, and liabilities $15,000; what is the net capital, and what each partner's share of the gain or loss?

10. Four persons engage in the lumber trade, and invest jointly $22,500; at the expiration of a certain time, A.'s share of the gain is $2,000, B.'s $2,800.75, C.'s $1,685.25, and D.'s $1,014; how much capital did each put in?

11. Three persons enter into partnership for the manufacture of coal oil, with a joint capital of $18,840. A. puts in $3 as often as B. puts in $5, and as often as C. puts in $7. Their annual gain is equal to C.'s stock; how much is each partner's gain?

12. A., B. and C. are employed to do a piece of work for $26.45. A. and B. together are supposed to do $\frac{3}{4}$ of the work, A. and C. $\frac{9}{10}$, and B. and C. $\frac{13}{20}$, and are paid proportionally; how much must each receive?

13. Three men trade in company. A. furnishes $8,000, and B. $12,000. Their gain is $1,680, of which C.'s share is $800; required, C.'s stock, and A.'s and B.'s gain.

14. Six persons are to share among them $6,300; **A.** is to have $\frac{1}{4}$ of it, B. $\frac{1}{6}$, C. $\frac{2}{9}$, D. is to have as much as **A.** and C. together, and the remainder is to be divided between E. and F. in the ratio of 3 to 5. How much does each receive?

15. A., B. and C. form a company for the manufacture of woollen cloths. A. puts in $10,000, B. $12,800, and C. $3,200. C. is allowed $1,500 a year for personal attention to the business; their expenses for labor, clerk hire, and other incidentals for 1 year are $3,100, and their receipts during the same time are $9,400. What is A.'s, B.'s and C.'s income respectively from the business?

530. To divide the gain or loss according to the amount of capital invested and time it is employed.

EXAMPLE.—A., B. and C. are partners in a business; A. invested $3,000 for four years, B. invested $5,000 for three years, and C. invested $4,500 for two years. How should a gain of $18,000 be divided?

SOLUTION.

A.'s investment of $3,000 for 4 yrs. = an investment of $12,000 for 1 yr.

B.'s   " $5,000 for 3 yrs =   " $15,000   "

C.'s   " $4,500 for 2 yrs. =   " $ 9,000   "

                           Total investment = $36,000   "

A furnishes $\frac{12}{36}$ of investment $\therefore$ his gain = $\frac{12}{36}$ of $18,000 = $6,000.

B   " $\frac{15}{36}$ " $\therefore$ "   = $\frac{15}{36}$ of $18,000 = $7,500.

C.   " $\frac{9}{36}$ " $\therefore$ "   = $\frac{9}{36}$ of $18,000 = $4,500.

or,

Total gain ($18,000) = $\frac{18000}{36000}$ or $\frac{1}{2}$ of investment = .5 of investment.

$\therefore$ A.'s share of gain = $12,000 × .5 = $6,000.

       B.'   "   = $15,000 × .5 = $7,500.

       C.   "   = $ 9,000 × .5 = $4,500.

RULE.

Multiply each partner's capital by the time it is employed, consider these products as their respective capitals and proceed as in Art. 529.

EXERCISE 115.

1. A., B. and C. form a partnership; A. furnishing $3,000 for 9 months, B $3,400 for 10 months, and C. $2,800 for 15 months; they lose $3,200; what is each man's share of the loss?

2. January 1st, 1889, A., B. and C. form a partnership; A. puts in $8,000, but after six months withdrew $2,000; B. puts in $6,000, and adds $500 after 4 months; C. puts in $4,000 for the year; they gain $3,600; what is the share of each?

3. Three men hire a pasture for $175. A. put in 20 cows for 7 months, B. 120 sheep for 5 months, and C. 24 horses for 8 months; 5 sheep being considered equal to 1 cow, and 4 horses equal to 5 cows; how much should each pay?

4. A. and B. are partners, A. putting in $4,500 and B. $2,500; after 6 months they take in C., who furnished $10,000; their gain for the year was $5,000; what was the   of each?

5. ., .. and Z. formed a partnership; X. putting in $8,000 for 1 year, Y. $4,500 for 8 months, and Z. $5,000 for 6 months; they lost $4,000; what was each man's share of the loss?

6. A. and B. formed a partnership and divided the gain or loss in proportion to their average investments. A. put in $6,000 for 12 months, and afterwards $4,000 for 6 months. He withdrew $3,000 for 4 months, then $6,000 for 2 months, before the close of the partnership. B. put in $7,000 for 12 months, then 6,000 for 8 months. He withdrew $4,000 for 5 months, then $8,000 for 2 months. They gained $4,560; what was each partner's share?

7. A., B. and C. began business Jan. 1st, when A. put in $7,500, and July 1st he put in $2,500 more; B. put in

Jan. 1st $12 000, and May 1st withdrew $4,000; C. put in Jan. 1st $10,000, Aug. 1st he added $3,000, and Oct. 1st he withdrew $7,900. At the close of the year the profit was $8,500; how much ought each to have, the gains being divided according to their average investment?

8. Howard & Salter commenced business with a capital of which Howard furnished $2 to Salter's $1. At the end of 3 months, Howard withdrew half of his capital, and Salter increased his 25%. At the end of 9 months, they had $3,150 to divide. What was the share of each?

9. Mills, Ross and McAdams, having been in partnership for one year, under an agreement to divide the profit proportionally to their respective shares of capital, have made $2,403. On the first day of the year, each put in $10,000; but Ross in 4 months withdrew 20% of his share, and McAdams at the end of six months put in $2,000 more. Find each partner's share of the profit.

10. R. E. Walker and John Lawson engaged in a lumber business on January 1st, 1889. Mr. Walker invested $6,000, and Mr. Lawson invested $6,000. On March 1st, Mr. Lawson made an additional investment of $3,000, and Mr. Walker withdrew $1,500. July 1st, Mr. Walker invested $2,900, and Mr. Lawson withdrew $3,000. The profits for the year were $4,620. What was each partner's average investment and share of the profits, if the profits were divided in proportion to the capital invested and the time it was employed?

11. S. Morgan, J. R. Street and R. C. Cheswright formed a co-partnership, and invested respectively, $9,600, $8,400 and $7,200. At the end of four months, Mr. Morgan invested $2,000, Mr. Street $1,400, and Mr. Cheswright $800. The net profits for the year were $12,800. What was each partner's share, the gains and losses being divided in proportion to their average investments?

12. Three men take an interest in a coal mine. B. invests his capital for 4 months, and claims $\frac{1}{10}$ of the profits; C.'s capital is in 8 months; and D. invests $6,000 for 6 months, and claims $\frac{2}{5}$ of the profits; how much did B. and C. put in?

13. A. and B. are partners. A.'s capital is to B.'s as 5 to 8; at the end of 4 months A. withdraws $\frac{1}{2}$ of his capital, and B. $\frac{2}{3}$ of his; at the end of the year their whole gain is $4,000; how much belongs to each?

14. Three men engage in trade. A.'s money was in 10 months, for which he received $456 of the profits; B.'s was in 8 months, for which he received $342.20 of the profits; and C.'s was in 12 months, for which he received $750 of the profits. Their whole capital invested was $14,345; how much was the capital of each?

15. A., B. and C. engage in manufacturing shoes. A. puts in $1,920 for six months; B. a sum not specified for 12 months; and C. $1,280 for a time not specified. A. received $2,400 for his stock and profits, B. $4,800 for his, and C. $2,080 for his. Required, B.'s stock and C.'s time?

16. B. commenced business with a capital of $15,000. Three months afterward C. entered into partnership with him, and put in 125 acres of land. At the close of the year their profits were $4,500, of which C. was entitled to $1,800; what was the value of the land per acre?

17. B., C. and D. form a manufacturing company, with capitals of $15,800, $25,000, and $30,000 respectively. After 4 months, B. draws out $1,200, and in two months more he draws out $1,500 more, and four months afterwards puts in $1,000. C. draws out $2,000 at the end of 6 months, and $1,500 more 4 months afterwards, and a month later puts in $800. D. puts in $1,800 at the end of

7 months, and 8 months after draws out $5,000 If their gain at the end of 18 months be $15,000, how much should each receive ?

18. July 1st, 1886, A. and B. commenced business with a capital of $7,500, for which A. furnished ⅔ and B. the remainder ; May 1st, 1887, B. invested $1,500, and A. withdrew $600 ; Oct. 1st, 1887, they admitted C. as a partner, with an investment of $4,500 ; Jan. 1st, 1888, each partner invested $1,000, and on Jan. 1st, 1889, each partner withdrew $500. On closing business, Oct. 1st, 1889, it is found that a net loss of $3,000 has been sustained. Find each partner's proportion of the loss.

19. Gibson and Montague dissolved a three-years partnership Aug. 1st, 1888, having resources of $16,500, and liabilities of $2,150. At first Gibson invested $2,750, and Montague $2,500 ; at the end of the first year Gibson drew out $1,500, and Montague invested $3,000 ; six months later each invested $1,200. No interest account being kept, what has been the gain or loss, and the share of each partner, if apportioned according to average investments ?

20. Day, Scott and Carruthers, each invested $15,500 in a business that gave the firm a profit of $21,000 in one year. Nine months before dissolution, Day increased his investment $3,000, and Scott and Carruthers each withdrew $3,000 ; six months before dissolution, Scott invested $2,000, and Day and Carruthers each drew out $2,000 ; three months before dissolution, Carruthers invested $1,000, and Day and Scott each drew out $1,000. If no interest account was kept, and the gain be divided according to average investment, what is each partner's share ?

21. A. and B. formed a co-partnership for 3 years. A. investing $7,200, and B. investing $5,400. At the end of 6 months A. increased his investment by $1,500,

and B. withdrew $900 ; one year before the expiration of the partnership, each withdrew $1,000, and six months later each invested $500. The net loss was $2,400. How much should be sustained by each, if the partners receive credit for interest at the rate of 6 % on all investments, and are charged interest on all sums drawn out, and the loss be sustained in proportion to average investment ?

22. April 1st, 1884, Craig and Cowan commenced business as partners, Craig investing $8,000, and Cowan $6,000 ; six months later each increased his investment $1,500; and on Jan. 1st, 1885, Allan was admitted as a partner with an investment of $2,400. On Oct. 1st, 1885, each partner drew out $1,500 ; on Apr. 1st, 1886, Craig and Cowan each drew out $1,000, and Allan invested $6,000. On Jan. 1st, 1889, it was found that a net gain of $37,500 had been realized. What was the share of each, if by agreement Craig, at final settlement, was to be allowed $1,200 per year for keeping the books of the concern ?

531. To find the net gain or loss, the net resources or the liabilities of a partnership.

EXAMPLE 1.—A. and B. commenced business with a capital of $10,000 cash ; merchandise as per inventory, $5,000 ; bills payable, $1,500. At the end of the year they had cash $6,500 ; merchandise as per inventory, $5,400 ; bills receivable, $3,200 ; debts owed by firm, $650. What was the net gain or loss of the firm ?

SOLUTION.

ASSETS AT COMMENCEMENT.		ASSETS AT CLOSE.	
Cash	$10,000	Cash	$6,500
M'dse	5,000	M'dse.	5,400
Total Assets	$15,000	Bills receivable	3,200
Liabilities	1,500	Total Assets	$15,100
Net Capital	$13,500	Liabilities	650
		Net Capital	$14,450

Net gain = $14,450 − $13,500 = $1,950.

EXAMPLE 2.—A. and B. are partners, A. sharing ⅔ of the gain or loss and B. ⅓. A. invests $5,000, and B. $2,350. At the end of the year their resources and liabilities are as follows : merchandise on hand, as per inventory, $2,000; real estate, $7,000 ; cash on hand and in bank, $1,532; due on personal accounts, $1,640.25 ; bills receivable, $1,000 · bills payable, $800 ; owing by the firm to sundry persons. $4,471.69. What is the amount of net resources belonging to each partner ?

SOLUTION.

RESOURCES.

M'dse. on hand $2,000.00		
Real estate 7,000.00		
Cash on hand and in bank .. 1,532.00		
Personal account 1,640.25		
Bills receivable 1,000.00		
		$13,172.25

LIABILITIES.

Bills payable $800.00		
Personal accounts 4,471.69	$5.271.69
Present worth	$7,900.56
Less investments	7,350.00
Total net gain	$550.56

⅔ of $550.56 = $367.04, A.'s share of gain.

⅓ of $550.56 = $183.52, B.'s   "

A.'s investment = $5,000.00

A.'s gain .. .. = 367.04

A.'s present worth $5,367.04

B.'s investment = $2,350.00

B.'s gain .. .. = 183.52

B.'s present worth $2,533.52

Present worth as before $7,900.56

532. To find each partner's interest, when each partner is allowed to withdraw a certain sum, and when no interest account is kept.

EXAMPLE.— A. and B. are partners, each invested $6,000, and agreed to share the gains and losses equally. A. drew out $1,200 and B. $1,000. Required their gains at the end of the year, their books showing the following result:

RESOURCES.		LIABILITIES.	
Cash	$7,000	Debts firm owe as per ledger	$3,000
Mdse. per inventory ..	7,200	Bills payable	1,600
Bills receivable..	2,400	Total liabilities	$4,600
Debts due firm as per ledger	5,000		
Total resources	$21,600		

Net capital at closing, $21,600 – $4,600 = $17,000

A. invested	$6,000
A. withdrew	1,200
A.'s credit balance	$4,800
B. invested	$6,000
B. withdrew	1,000
B.'s credit balance	$5,000
	$9,800

Net gain of firm $7,200

A.'s ½ net gain = $3,600
B.'s ½   "   = $3,600

PROOF.

A. invested..	$6,000	B. invested	$6,000
A. withdrew	1,200	B. withdrew	1,000
	$4,800		$5,000
A.'s ½ net gain	3,600	B.'s ½ net gain	3,600
A.'s net capital at closing	$8,400	B.'s net capital at closing	$8,600

$8,400 + $8,600 = $17,000, firm's net capital.

533. To find each partner's interest, when one or more partners are allowed a fixed salary and no interest account is kept.

EXAMPLE.—A., B. and C. entered into partnership January 1st, 1899. A. invested $14,000, B. $14,000, and C. $28,000. A. to share ¼ of the gains and losses, B. ¼, and C. ½. A. was to receive a salary of $1,000 per year, B. $1,200, and C. $600 for services. A. drew out $1,300, B. $900, and C. $1,800. What was each partner's interest in the firm January 1st, 1890, when their resources were $108,000, and their liabilities $27,000?

SOLUTION.

Resources ..	$108,000
Liabilities	27,000
Firm's net capital	$81,000

A.'s investment	$14,000	
a. s salary	1,000	
	$15,000	
Less amount withdrawn	1,300	
A.'s credit balance		$13,700
B.'s investment	$14,000	
B.'s salary	1,200	
	$15,200	
Less amount withdrawn	900	
B.'s credit balance		$14,300
C.'s investment	$28,000	
C.'s salary	600	
	$28,600	
Less amount withdrawn	1,800	
	$26,800	$54,800
Firm's net gain		$26,200

A.'s credit bal. $13,700	B.'s credit bal. $14,300	C.'s credit bal. $26,800
A.'s ½ gain .. 6,550	B.'s ½ gain .. 6,550	C.'s ½ gain .. 13,100
A.'s net capital $20,250	B.'s net capital $20,850	C.'s net capital $39,900

PROOF.

A.'s net capital	$20,250
B.'s "	20,850
C.'s "	39,900
Firm's net capital	$81,000

534. To find each partner's interest at the end of the year or close of partnership when amounts withdrawn are averaged, and interest is charged and allowed.

EXAMPLE.—A. and B. entered into partnership January 1st, 1889, and agreed to share the gains or losses equally. A. invested $6,000, and B. $7,250; each partner was allowed 6% on his investment and was charged 6% for the sums withdrawn. A. drew as follows: March 1st, $300; July 9th, $250; September 10th, $200; December 18th, $150. B. drew, April 17th, $100; August 4th, $400; November 23rd, $250. What was each partner's interest in the business January 1st, 1890, their resources and liabilities being as follows:

RESOURCES.			LIABILITIES.		
Cash	$1,800		Personal debts firm owe ..	$5,750	
Personal debts due firm ..	8,000		Bills payable	250	
Bills receivable	700		Total liabilities	$6,000	
M'dse. as per inventory ..	13,000				
C. P. R. Railway Stock ..	3,000		Firm's net capital	$20,500	
Total resources	$26,500			$26,500	

SOLUTION.

A.'s amount withdrawn $900; average date July 7th. From July 7th to January 1st = 178 days.

B.'s amount withdrawn $750; average date August 27th. From August 27th to January 1st = 127 days.

A.'s investment	$6,000.00		
Less withdrawn	900.00	$5,100.00	
Int. on investment for 1 year ..	$360.00		
Less int. on $900 for 178 da. at 6%	26.33	333.67	
A.'s credit balance			$5,433.67
B.'s investment	$7,250.00		
Less withdrawn	750.00	$6,500.00	
Int. on investment for 1 year ..	$435.00		
Less int. on $750 for 127 da. at 6%	15.66	419.34	
B.'s credit balance			$6,919.34
Firm's net capital			$20,500.00
A.'s credit balance	$5,433.67		
B.'s "	6,919 34	$12,353.01	
Firm's net gain			$8,146.99

A.'s credit balance .. $5,433.67		B.'s credit balance.. .. $6,919.34	
A.'s ½ gain.. .. .. 4,073.49½		B.'s ½ gain 4,073.49½	
A.'s net capital $9,507 16½		B.'s net capital .. $10,992.83½	

Firm's net capital $20,500.

EXERCISE 116.

1. At the expiration of a year from the commencement of their business, Baker, Morgan & Co., after taking an account of stock, find the amount of merchandise, as per inventory, to be $17,450; cash on hand, $10,250; debts due the firm, $11,300; amount of firm's indebtedness,

$15,500. Make out a statement, showing the resources and liabilities of the firm, with net capital and gain; and find each partner's share of the latter, the respective shares of capital being as follows: J. Baker, $8,000; S. Morgan, $5,000; and J. Murray, $3,000.

2. A. put $10,000 into a partnership and B. $5,000. They agreed to divide the gain or loss in proportion to their original investments, and to keep no interest account. During the year A. withdrew $800 and B. $500; what was the net capital of each at the close of the year, their resources being $25,800 and their liabilities $18,500? What per cent of their investment was the gain or loss?

3. Duff, Fry & Rowat became partners, each investing $15,000, and each to have one-third of the gains or sustain one-third of the losses. Duff withdrew $2,100 during the time of the partnership, Fry $1,800, and Rowat $2,000. At close of business their resources were: cash, $3,540; mdse., $14,785; notes, acceptances, and accounts receivable, $16,250; real estate, $28,500. They owed on their outstanding notes $8,125, and on sundry personal accounts, $1,950. Find the present worth of each partner at closing.

4. A., B., and C. formed a partnership; A. put in $5,000, B. $4,000, and C. $2,500. A. withdrew $1,000, B. $800, and C. $500. They agreed to share the gain or loss in proportion to their original investments, no interest account being kept. At the close, what was each partner's share of gain or loss, and the net capital of each, as shown by the following statement:

RESOURCES.		LIABILITIES.	
Cash in bank............	$3,475	Bills payable.............	$3,000
Mdse. per inventory......	5,150	Rent, etc.................	700
Bills receivable..........	4,225	Debts firm owe...........	2,300
Debts due firm..........	3,150	Total liabilities.........	$6,000
Total resources........	$16,00'		

5. At the time of closing business, the resources of a firm were: cash, $931.50; mdse., per inventory, $18,196.25; notes and accounts due it, $8,154; interest on same, $211.50; real estate, $11,150. The firm owed, on its notes, acceptances and bills outstanding, $7,142, and interest on the same, $348.50; and there was an unpaid mortgage on the real estate of $2,500, with interest accrued thereon of $88.50. If the invested capital was $22,500, what was the net solvency or net insolvency of the firm at closing, and how much has been the net gain or net loss?

6. The firm of A. & B. formed a partnership Jan. 1st for 1 year, investing $8,000 each. They were to have 6 % interest on their capital and be charged 6 % on sums withdrawn. The gains or losses were to be shared equally. April 4th, A. drew out $500, July 10th, $400, and Sept. 5th, $200. B. drew out May 6th, $700, Aug. 12th, $300, and Oct. 4th, $400. What was each partner's net capital on closing, the net gains being $3,850?

7. Johnston and Atkinson became partners April 1st, 1888, under an agreement that each should be allowed 6 % simple interest on all investments, and that, on final settlement, Johnston should be allowed 10 % of the net gains, before other division, for superintending the business, but that otherwise the gains and losses be divided in proportion to average investment. April 1st, 1888, Atkinson invested $18,000, and Johnston, $4,000; Jan. 1st, 1889, Atkinson withdrew $5,000, and Johnston invested $3,000; Aug. 1st, 1889, Atkinson withdrew $1,500; Dec. 1st, 1889, the partners agreed upon a dissolution of the partnership, having resources and liabilities as follows:

RESOURCES.		LIABILITIES.	
Cash on hand	$ 1,101 05	Bills payable	$6,520 00
Accounts receivable..	16,405 50	Outstanding accounts ..	1,246 50
Bills receivable..	2,550 00	Rent due	1,200 00
Interest......	287 41		
Mdse., as per inventory	9,716 55		

If, of the accounts receivable, only 80% prove to be good, what has been the net gain or loss? What has been the gain or loss of each partner? What is the firm's net insolvency at dissolution? What is the net insolvency of each?

8. A., B., and C., formed a co-partnership for 2 years, investing equal sums, with the agreement that each shall receive interest the rate of 6% on all sums invested, be charged interest at the same rate on all sums withdrawn, and the gains or losses shown on final settlement be apportioned according to average net investment. Three months after the formation of the partnership A. drew out $1,200, and six months later B. and C. each drew out $1,000, and A. invested $6,000; at the end of the first year each drew out $500. On closing the affairs of the firm, the following statement was made: net gain, $15,000; present worth, $75,000. What was the original investment of each? What was the present worth of each at the time of the dissolution? What was each partner's share of the gain?

9. A. and B. became partners for one year; A. investing ⅓ of the capital, and B. ⅔; the agreement being that the gains or losses shall be apportioned according to average net investment, and that each partner be allowed 6% interest per annum on all investments, and be charged interest at that rate on all sums withdrawn. At the end of the year the firm had as resources: mdse., per inventory, $21,460; real estate, $15,000; cash, $1,950; bills receivable, $13,146.50; interest accrued on the same, $519.25; accounts due it, $11,218.50; store furniture, $1,320; delivery wagons and horses, $2,100. The liabilities were: mortgage on real estate, $7,000; interest on same accrued, $210; notes outstanding, $26,950; interest accrued on same, $811.75. The firm owes H. W. Darling & Co., Toronto, $33,560. It is found that 33⅓% of the

accounts due the firm are uncollectable. If the firm's losses during the year have been $12,000, how much was invested by each partner? What is the present worth or net insolvency of the firm, and of each partner, at closing?

10. Sills and Jones became partners July 1st, 1886, under a 3-year's contract, which provided that Sills should have $1,500 each year for superintending sales, and that Jones should have $1,000 each year for keeping the books of the concern, and that these salaries should be adjusted at the end of each year, and before other apportionment of gains or losses was made. July 1st, 1886, each invested $12,500. Six months later, each increased his investment $5,000. July 1st, 1887, Sills drew out $3,600 and Jones drew out $3,000. Oct. 1st, 1837, Sills withdrew $1,000 and Jones invested $2,000. July 1st, 1888, each drew out $1,500. At the expiration of the time of the contract, the resources exceeded all liabilities by $47,280. What was the gain of each, and the present worth of each?

11. A. and B. commenced business as partners. A. invested $20,000, and B. $10,000, A. sharing ⅔ and B. ⅓ of the gains and losses. No interest account was kept. A. drew out $1,700, and B. $2,150. Their assets at the close of the year consisted of—cash, $4,200; bills receivable, $8,800; mdse., $26,000; and personal debts, $16,000. 10 % of the personal debts are considered bad. Their liabilities are—bills payable, $3,250; personal accounts, $11,250. If B. should retire from the firm, how much ought he to receive?

12. On January 1st, 1889, A. E. Brock, W. McMaster and H. Crawford entered into a co-partnership. Brock was to invest ⅘ of the capital and share ⅘ of the gains. McMaster was to invest ⅜ of the capital and share ⅜ of the gains, and Crawford was to invest ⅔ of the capital and share ⅔ of

the gains.   Interest at the rate of 10 % per annum was to
be allowed to each partner should he invest more than his
proportion ; and interest, at the same rate, was to be
charged each partner if he failed to invest his proportion.
A settlement was made at the end of the year, and the net
gain was $3,600.   Find Brock's and McMaster's net
interest, and Crawford's insolvency Jan. 1st, 1890, the
following being a statement of each partner's account.

Dr.	A. E. Brock.		Cr.
1889.—April 23, Drew out	$3,000	1889.—Jan. 1, Invested	$32,000
" June 16, "	1,600	" Mar. 18, "	4,800
" Aug. 17, "	1,800	" Oct. 20, "	6,000
" Total withdrawn	$6,400	" Total investment	$42,800

Dr.	W. McMaster.		Cr.
1889.—July 28, Drew out	$1,200	1889.—Jan. 1, Invested	$24,000
" Dec. 4, "	1,600	" " 21, "	3,600
" Total withdrawn	$2,800	" May 17, "	1,200
		" Total investment	$28,800

Dr.	H. Crawford.		Cr
1889.—Mar. 30, Drew out	$12,000	1889.—Jan. 1, Invested	$12,000
" Sept. 5, "	8,000	" Aug. 3, "	1,200
" Total withdrawn	$20,000	" Total invested	$13,200

BANKRUPTCY.

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535. Bankruptcy is the formal acknowledgement in accordance with the law, by a person or firm, of inability to pay indebtedness.

536. A Bankrupt is a person who is insolvent, or unable to pay his debts.

537. After the assets of a bankrupt have been applied to meet his liabilities, he still remains liable for them unless discharged, or unless a compromise has been effected with his creditors.

538. The **Assets** of a bankrupt are his entire property.

539. The **Liabilities** of a bankrupt are the debts and obligations due by him to his creditors.

540. The **Net Proceeds** are the assets less the expense of settlement. They are divided among the creditors according to their claims.

The claims of a certain class of creditors, as employees and others, are paid in full up to a certain amount. These are called "Preferred Creditors."

541. An **Assignee** is a person appointed in accordance with the law, to take charge of the bankrupt's property, to make collections of debts due the estate, and after deducting the expenses of the assignment, to pay such proportion of the debts due the creditors as the available assets will allow.

542. To find each creditor's dividend, the liabilities and net proceeds being given.

EXAMPLE.—A merchant failing in business gave the following statement of his assets and liabilities: Assets, cash, $5,474; real estate, $3,000; merchandise, $4,000; personal accounts, $1,900. Liabilities, bills payable, $2,400; due R. E. Walker & Co., $5,000; dne A. Boyle & Co, $17,500. The expenses of assignment were $430. How much did each creditor receive ?

SOLUTION.

ASSETS.		LIABILITIES.	
Cash	$5,474	Bills payable	$2,400
Real estate	3,000	R. E. Walker & Co. ..	5,000
Mdse.	4,000	A. Boyle & Co.	17,500
Personal acconnts.. ..	1,900	Total	$24,900
Total..	$14,374		
Less expenses	430		
Net assets..	$13,944		

$13,944 ÷ $24,900 = .56, or 56 %, rate on dollar.

$2,400 × .56 = $1,344 on bills payable.

$5,000 × .56 = $2,800 to R. E. Walker & Co.

$17,500 × .56 = $9,800 to A. Boyle & Co.

$24,900 $13,944

EXERCISE 117.

1. A bankrupt owes A. $6,500, B. $4,600, and D. $3,800; his assets are $5,950, and the expenses of settling $1,700; what per cent. and how much will each creditor receive ?

2. J. Gould & Co. failed with liabilities amounting to $300,000. The assets of the firm were $186,294. How much should each creditor receive on the dollar, and what sum was allowed J. P. Hume & Co., whose claim was $17,314, the expenses of settling being $6,294 ?

3. J. Wild & Co., went into bankruptcy, owing $48,500, and having $13,300 assets; the expense of settling was 5 % of the amount distributed to creditors. What per cent. and how much did a creditor receive on $8,350 ?

4. A grain firm failed with liabilities amounting to $24,500. The assets were: cash, $1,080; real estate, $8,250; notes on hand, $1,170. The expenses of settling were 2% of the assets. How much should W. H. Hull & Co. receive, whose claim against the firm was $6,308.50?

5. A manufacturer failed, owing A. $12,260, B. $13,850, and C. $14,560; his assets were $28,350, and the expenses of settling were $1,250. He owed $850 to employees who were to be paid in full; what per cent. and how much did the other creditors receive?

6. The real estate of a bankrupt firm was sold by an assignee for $24,000, goods in store for $12,244. There were collected on notes due the firm $4,214, and on personal accounts $5,346. The total liabilities of the firm were $54,067.50, and the expenses of settling $1,350. How much on the dollar can be paid, and what should Howard Bros. receive, whose claim is $12,430?

7. A. Reid's claim against a bankrupt firm was $7,200, and J. Taylor's 70 % of that of A. Reid's. After the expenses of the assignment were deducted from the assets, there remained $18,260. The total liabilities were $24,480. How much did A. Reid and J. Taylor respectively receive?

8. A firm failed with liabilities amounting to $26,125. The assets of the firm exclusive of real estate were $1,521.25. The assignee obtained for a warehouse and three building lots the sum of $15,675. The expenses for settling the bankruptcy was $237.50. W. Alexander's claim against the firm was $3,642; J. Moblo's, $3,191; R. A. Harrison's, $2,897; D. McGregor's, $2,383.50; W. Ayer's, $1,982. How much did each of these creditors receive?

ANNUITIES.

———

543. **An Annuity** is a specified sum of money paid annually, or at equal periods as, half-yearly, quarterly, etc., to continue a given number of years, for life, or for ever.

544. A **Certain Annuity** is one which begins and ends at a fixed time.

545. A **Perpetual Annuity** or **Perpetuity** is one which continues for ever.

546. A **Contingent Annuity** is one whose time of commencement or ending, or both, is uncertain, and which depends upon some unforeseen event, as the death of an individual, or his arrival at a certain age. Life insurance, pensions, dowers, leases, etc., belong to this class of incomes.

547. An **Annuity in Possession** or an **Immediate Annuity** is one that begins immediately.

548. A **Deferred Annuity** or an **Annuity in Reversion** is one that begins at some future time, it may be at some specified time, or at the occurrence of some event.

549. An **Annuity in Arrears** or **Forborne** is one on which the payments were not made when due.

550. The **Amount** or **Final Value** of an annuity is the sum to which all its payments with interest on each, will amount at its termination.

551. The Present value of an Annuity is the sum which at the given rate of interest, will amount to its final value.

NOTE 1.—The present value of a deferred annuity is that principal which will amount, at the time the reversion expires, to what will then be the present value of the annuity.

2. The present value of a perpetual annuity is the sum whose interest equals the annuity.

8. Annuities and their values are computed by simple interest or by compound interest.

552. To find the amount of an annuity at simple interest when the time and rate are given.

EXAMPLE.—What is the amount of $500 annuity for 5 years at 6 % simple interest?

SOLUTION.

ANNUITY.	INT.	AMT.
$500 +	$120 =	$620
500 +	90 =	590
500 +	60 =	560
500 +	30 =	530
500 +	0 =	500

Amount $2,800

EXPLANATION.

The interest on $500 for 1 year at 6 % = $30. The first annuity is not due until the end of the first year, and hence draws interest for only 4 years = $120. The second is not due until the end of the second year, and hence draws interest for only 8 years, etc.

553. To find the present worth of an annuity at simple interest.

EXAMPLE.—What is the present value of an annuity of $500 for 5 years, when money is worth 6 % simple interest?

SOLUTION.

By the preceding example the final value of the annuity is $2,800. The present worth of $2,800 due in 5 years at 6 % = $\frac{100}{130}$ of $2,800 = $2153.846.

EXERCISE 118.

1. What is the amount of an annuity of $150 for 8 years, when money is worth 6 % simple interest?

2. A man works for 1 year and 6 months at $20 per month, payable monthly; and these wages remain unpaid until the expiration of the whole time of service. How much is due to the workman, allowing simple interest at the rate of 6 % per annum ?

8. A father deposits $50 a year for his son, commencing on the son's 10th birthday. What amount will the son have on his 21st birthday, if the payments draw simple interest at the rate of 8 % per annum ?

4. A lady has $300 a year left to her for 10 years. What is its present cash value, at 7 % simple interest ?

5. What is the present worth of an annuity of $600 for 4 years, money being worth 6 % simple interest ?

6. How much will an annuity of $100 amount to in 8 years at 8 % simple interest ?

7. An annuity of $200 for 12 years is in reversion for 6 years. What is its present worth, simple interest at 6 % ?

ANNUITIES AT COMPOUND INTEREST.

554. The labor of computing the values of annuities at compound interest is greatly diminished by the use of the following tables. The tables are always used in practice.

Table 1.

Amount of $1 annuity at compound interest, from 1 year to 40, inclusive.

Yrs.	3 %.	3½ %.	4 %.	5 %.	6 %.	7 %.	Yrs.
1	1.000 000	1.000 000	1 000 000	1.000 000	1.000 000	1.000 000	1
2	2.030 000	2.035 000	2.040 000	2.050 000	2.000 000	2.070 000	2
3	3.090 900	3.106 225	3.121 600	3.152 500	3.183 600	3.214 900	3
4	4.183 627	4.214 943	4.216 464	4.310 125	4.374 616	4.439 943	4
5	5.309 136	5.362 466	5.416 323	5.525 631	5.637 093	5.750 739	5
6	6.468 410	6.550 152	6.632 975	6.801 913	6.975 319	7.153 291	6
7	7.662 462	7.779 408	7.898 294	8.142 008	8.393 838	8.654 021	7
8	8.892 336	9 051 687	9.214 226	9.549 109	9.897 468	10.259 803	8
9	10.159 106	10.368 196	10.582 795	11 026 564	11.491 316	11.977 989	9
10	11.463 879	11.731 393	12.006 107	12.577 893	13.180 795	13.816 448	10
11	12.807 796	13 141 992	13.486 351	14.206 787	14.971 643	15.783 599	11
12	14.192 030	14.601 962	15 025 805	15.917 127	16.869 941	17.888 451	12
13	15.617 790	16.113 030	16 626 838	17.712 983	18.882 138	20.140 643	13
14	17.086 324	17.676 986	18.291 911	19.598 632	21.015 066	22.550 488	14
15	18.598 914	19.295 681	20 023 588	21.578 56	070	25.129 022	15
16	20.156 881	20.971 030	21.824 531	23.657 492	25.670 528	27.888 054	16
17	21.761 588	22.705 016	23.697 512	25.840 366	28.212 880	30.840 217	17
18	23.414 435	24.499 691	25.645 413	28.132 385	30.905 653	33.999 033	18
19	25.116 868	26.357 180	27.671 229	30.539 004	33 759 992	37.378 905	19
20	26.870 374	28.279 682	29.778 079	33 065 954	36 785 591	40.995 492	20
21	28.676 486	30.269 471	31.969 202	35.719 252	39.992 727	44.865 177	21
22	30.536 780	32.328 902	34.247 970	38.505 214	43 392 290	49.005 739	22
23	32.452 854	34.460 414	36.617 889	41.430 475	46 995 828	53.436 141	23
24	34.426 470	36.666 528	39 082 604	44.501 999	50.815 577	58 176 671	24
25	36.459 264	38.949 857	41.645 908	47.727 099	54.864 512	63.249 030	25
26	38.553 042	41.313 102	44.311 745	51.113 454	59.156 383	68 676 470	26
27	40.709 634	42.759 060	47.084 214	54.669 126	63.705 766	74.483 823	27
28	42.930 923	46.290 627	49.967 583	58.402 583	68.528 112	80.697 691	28
29	45.218 850	48.910 799	52.966 286	62.322 713	73 639 798	87.316 529	29
30	47.575 416	51.622 677	56.084 938	66.438 848	79.058 186	94.460 786	30
31	50.002 678	54.429 471	59.328 335	70.760 790	84.801 677	102.073 041	31
32	52.502 759	57.334 502	62.701 469	75 298 829	90.899 778	110.218 154	32
33	55.077 841	60.341 210	66.209 527	80.063 771	97.343 165	118 933 425	33
34	57.730 177	63.453 152	69.857 909	85.066 959	104.183 755	128.258 765	34
35	60.462 082	66.674 013	73.652 225	90.320 307	111.434 786	138.236 878	35
36	63.271 944	70.007 603	77.598 314	95.836 323	119 120 867	148.913 460	36
37	66.174 223	73.457 869	81.702 246	101 628 139	127.268 119	160.337 400	37
38	69.159 449	77.028 895	85.970 336	107.709 546	135.904 206	172.561 020	38
39	72.234 233	80.724 906	90.409 150	114.095 023	145.058 458	185.640 292	39
40	75.401 260	84.550 278	95.025 516	120.799 774	154.761 966	199.635 112	40

TABLE 2.

555. Present worth of $1 annuity at compound interest, from 1 year to 40, inclusive.

Yrs	3 %	3½ %.	4 %.	5 %.	6 %.	7 %.	Yrs
1	0 970 871	0 966 184	0.961 538	0.952 341	0 943 396	0 934 579	1
2	1.913 470	1 899 694	1 886 095	1 859 410	1.833 393	1 808 017	2
3	2.828 611	2.801 637	2.775 091	2.723 248	2.673 012	2 624 314	3
4	3.717 098	3.673 079	3.629 895	3 545 951	3.465 106	3 387 249	4
5	4.579 707	4.515 052	4.451 822	4.329 477	4 212 364	4 100 195	5
6	5.417 191	5.329 553	5.242 137	5.075 692	4.917 324	4.766 537	6
7	6 230 283	6 114 544	6 002 455	5.786 373	5.582 381	5 389 286	7
8	7 019 693	6.873 956	6 732 745	6 463 213	6 209 711	5 971 276	8
9	7.786 109	7.607 687	7 435 332	7 107 822	6 801 692	6 515 228	9
10	8 530 203	8.316 605	8.110 896	7.721 735	7 360 087	7.023 577	10
11	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7 498 669	11
12	9.954 004	9.663 334	9 385 071	8 863 252	8.383 844	7.942 671	12
13	10 634 955	10.302 738	9.985 648	9 393 573	8.852 683	8.357 635	13
14	11 296 073	10.920 520	10.563 123	9.898 641	9.294 984	8.745 452	14
15	11.937 935	11.517 411	11.118 387	10.379 658	9 712 249	9 107 898	15
16	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632	16
17	13.166 118	12 651 321	12.165 669	11.274 066	10 477 260	9.763 206	17
18	13.753 513	13 189 682	12.659 297	11.689 587	10 827 603	10 059 070	18
19	14.323 799	13 709 837	13.133 939	12.085 321	11.158 116	10 335 578	19
20	14.877 475	14.212 403	13.590 326	12.462 210	11.469 421	10.593 997	20
21	15 415 024	14 697 974	14 029 160	12 821 153	11.764 077	10 835 527	21
22	15.936 917	15.167 125	14.451 115	13.163 003	12 041 582	11 061 241	22
23	16 443 608	15 620 410	14.856 842	13.488 571	12 303 379	1 272 187	23
24	16.935 542	16.058 363	15.246 963	13 798 642	12.550 358	11 469 334	24
25	17.413 148	16.481 515	15.622 080	14.093 945	12 783 356	11 653 583	25
26	17.876 842	16.890 352	15.982 769	14.375 185	13 003 166	11.825 779	26
27	18.327 031	17.285 365	16 329 586	14 643 034	13 210 534	11.986 709	27
28	18.764 108	17.667 019	16.663 663	14.898 127	13 406 164	12.137 111	28
29	19.188 455	18.035 767	16 983 715	15 141 074	13.590 721	12.277 674	29
30	19.600 441	18.392 045	17.292 033	15.372 451	13.764 831	12.409 041	30
31	20 000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814	31
32	20.338 766	19 068 865	17.873 552	15.802 677	14.084 043	12 646 555	32
33	20.765 792	19 390 208	18.147 646	16.002 549	14 230 230	12 753 790	33
34	21.131 837	19.700 684	18.411 198	16 192 204	14.368 141	12 854 009	34
35	21.487 220	20 000 661	18.664 613	16.374 194	14.498 246	12.947 672	35
36	21 832 252	20.290 494	18.908 282	16.546 852	14.620 987	13.035 208	36
37	22.167 235	20.570 525	19.142 579	16.711 287	14 736 780	13 117 017	37
38	22 492 462	20.841 067	19 367 864	16 867 893	14.816 019	13 193 473	38
39	22 808 215	21.102 500	19 584 485	17.017 041	14.949 075	13.264 928	39
40	23 114 772	21.355 072	19.792 774	17.159 066	15.046 297	13.331 709	40

556. To find the final value of an annuity by compound interest.

EXAMPLE 1.—What is the final value of an annuity of $500 for 6 years at 5%.

SOLUTION.

By Table 1 the final value of an annuity of

$1, at 5 % for 6 years = $6.801913.

∴ final value of an annuity of $500 = 6.801913 × 500 = $3400.9565.

NOTE.—When payments are made half-yearly, take from the table double the time, and $\frac{1}{2}$ the rate.

557. To find the present value of an Annuity.

EXAMPLE.—What is the present worth of an annuity of $500 for 5 years at 6 %.

SOLUTION.

By Table 2 the present worth of an annuity of $1 for 5 years at 6 % is $4.212364.

∴ The p. w. of an annuity of $500 = $4.212364 × 500 = $2106 182.

558. To find the present worth of an annuity in reversion.

EXAMPLE.—What is the present worth of an annuity in reversion of $500 at 6 %, which begins in 4 years, and then terminates after 6 years.

SOLUTION.

The p. w. of an annuity of

$1, at 6 % for 10 yrs. = $7.360087.
          "   4 yrs. = $3.465106.

∴ The present worth of an annuity of $1 to begin in 4 years, and then to continue 6 years = $3.894981, the difference

∴ p.w. of $500 = 3.894981 × 500 = $1947.4905.

559. To find the present worth of a perpetual annuity.

EXAMPLE 1.—A perpetual scholarship of $150 per year is established at Queen's University. What sum must be invested at 5% to yield this income.

SOLUTION.

5 % of the investment = $150.

∴ the investment = $\frac{150}{5}$ × 100 = $3,000 Ans.

EXAMPLE 2.—What is the present worth of a perpetual annuity of $300 in arrears for 20 years, allowing 5% compound interest.

SOLUTION.

There is now due the amount of $300 for 20 years at 5% compound interest, together with the present worth of the perpetual annuity of $300.

The p. w. of the perpetual annuity of

     $300, by Example 1 = $\frac{1}{5}$ × 100 = $6,000.

Amount of annuity of

     $1 for 20 years at 5 % = $33.065954.   (Table 1).

     $300   "   = $33.065954 × 300 = $9919.7862.

   ∴ total present worth = $9919 7862 + $6,000 = $15919.7862. **Ans.**

EXERCISE 119.

1. Money being worth 6 %, how much mus . be presented to a college, to insure $50 a year forever, for an annual prize ?

2. A person left $5,000 for the poor of his native town. How great was the perpetuity realized from it, at 6 % ?

3. What is the final value of an annual pension of $150 for 15 years at 4 % compound interest ?

4. A widow is entitled to $140 a year for 18 years, at 10 % semi-annual compound interest: what is its final value ?

5. An annuity of $350 was left to A., and one of $550 to B., by the same person ; both were to run 12 years. Allowing compound interest, at 6 %, by how much would the amount of A.'s exceed that of B.'s in the given time ?

6. How much will an annuity of $100 amount to, in 8 years, at 8 % simple interest ? How much at 6 % compound interest ?

7. A soldier 57 years old, having a pension of $80 a year, agreed to sell it for cash at 10 % less than its present value, compound interest being allowed at 7 %. How much should he receive, his expectation of life being 74 years of age ?

8. A lawyer collected for a client an annuity of $700, in arrears for 4 years, the legal rate of interest being 6 %. He charged 15 % on the amount collected. At this rate, how much greater would have been his fee had he been able to collect compound interest ?

9. A clerk saves from his salary $50 every year, and puts it in a savings bank which allows interest compounded annually at 6%. If he draws no checks on the bank, how much will he have there at the end of 10 years?

10. A person aged 54 has a life annuity of $400. What is its present value, allowing compound interest at 4%, his expectation of life being 19 years?

11. At the age of 20, and every year after, a young man places $200 at compound interest at 5%. How much will he have at the age of 30? At the age of 40?

12. How much a year must be invested for a boy 11 years old, that the sums thus invested, with compound interest at 5%, may make a total of $10,000 by the time he becomes of age?

13. What is the present worth of an annuity of $500 for 8 years, at 4% compound interest?

14. What is the present worth of an annuity of $3,000 for 20 years at 3% compound interest?

15. What is the present worth of an annuity in reversion of $1,000, at 6% compound interest, which begins in 3 years and then terminates in 8 years?

16. The reversion of a lease of $450 per year, at 5%, begins in 3 years and continues 16 years. What is its present worth?

17. A father bequeathed to his son, 11 years of age, a 5% annuity of $1,000, to begin in 3 years and continue 10 years. What would be the amount when the son was 21 years old? What is its present worth?

18. What is the present worth of a perpetual annuity of $250, in arrears for 10 years, allowing 8%, compound interest?

19. What is the present worth of a perpetuity of $500 in arrears for 30 years, allowing compound interest at 5%?

SINKING FUNDS.

560. Sinking Funds are sums of money set apart at regular intervals for the payment of indebtedness.

561. Sinking Fund Bonds are securities issued by corporations, based on the pledge of a special income, which is funded for their redemption.

562. To find what sum must be set apart annually, as a sinking fund, to pay a debt in a given time.

EXAMPLE.—The Town of Woodstock borrowed $20,000 to build a High School, and agreed to pay 5 % compound interest. What sum must be set apart annually, as a sinking fund, to pay the debt in 12 years?

SOLUTION.

Amount of

$1 at 5 % compound interest for 12 years = $1.795856.

∴ $20,000 = $1.795856 × 20,000 = $35,917.12.

Amount of annual payment of $1 for 12 years at 5% = $15.917127. Art. 554.

∴. amount necessary to pay a debt of

$35,917.12 will require $35,917.12 ÷ 15.917127 = $2,256.58.   Ans.

RULE.

Divide the amount of the debt at its maturity at compound interest, by the amount of an annuity of $1 for the given time and rate, and the quotient will be the sinking fund required.

563. To find the number of years required to pay a given debt, by a given annual sinking fund.

EXAMPLE.—The Town of Port Hope built a Court House at a cost of $15,000, and raised $1,300 a year to pay for it. Allowing 6 % compound interest, how many years will it require to cancel the debt?

SOLUTION.

A sinking fund of $1,800 has a present worth of $15,000 for a certain time at 6%.

∴ A sinking fund of $1 has a present worth of $\frac{15000}{1300}$ = $11.538461, for the required time at 6%.

Looking in Table 2, Art. 555, in the column 6%, we find the nearest number less than 11.538461, to be 11.469421, the present worth of $1 annuity for 20 years.

20 years is therefore the number of whole years required.

Again :

The amount of the debt $15,000 at 6% compound interest

for 20 years   ..   .. = $48,107 40

The amount of a sinking fund $1,300 at 6% compound

interest   ..   .. = 47,821.27

Balance due at end of 20 years   = $286.13

RULE.

Divide the debt by the given sinking fund, and the quotient will be the present worth of $1 annuity for the given time.

Look for this number in Table 2, Art. 555, in the column denoting the given rate, and opposite in the column of time will be found the number of whole years.

NOTES 1.—If the exact number is not found in the column, take the years standing opposite the next smaller number.

2. To ascertain the balance due at the end of the number of whole years, find the difference between the amount of the debt, at the given rate, for the time taken, and the amount of the sinking fund for the same time and rate.

EXERCISE 120.

1. If a railroad company sets apart an annual sinking fund of $20,000, and loans it at 5% compound interest. What will be its amount in 12 years?

2. What will be the amount in 15 years of a sinking fund of $12,000, yielding 4% compound interest?

3. What sum must be set apart annually to rebuild a bridge costing $80,000, estimated to last 17 years, allowing 5% compound interest?

4. A railroad company bought $100,000 worth of rolling stock, payable in 5 years with 6 % compound interest ; what sum must be set apart annually as a sinking fund to discharge the debt ?

5. A man buys a farm for $5,000, and agrees to pay for it in six equal annual instalments. What is the amount of each payment, money being worth 5 % compound interest ?

6. A railroad company issued sinking fund bonds at 6 % for $200,000, payable in 10 years. If at compound interest, what sum must be set apart annually to meet interest and principal when due ?

7. What would be the amount in 10 years, at 6 % simple interest ?

8. If the funded securities were drawing an annual income of 4 % compound interest, by how much would the amount necessary to meet principal and interest at 6 % be reduced ?

9. With the above reduction, what sum would be needed annually as a sinking fund to pay the amount when due at 4 %.

10. A man buys a farm for $6,000, and agrees to pay $700 each year until paid, allowing 6 % compound interest, both on the debt and on the payments. How many number of whole years will he be in paying for the farm ? What is the balance then due ?

11. A village built a school-house costing $12,000, and raised $1,700 a year to pay for it ; allowing 6 % compound interest. How many whole years will it require to cancel the debt ? What will be the balance then due ?

GROUND RENTS.

564. Ground Rents is a term applied to leases of building lots, the rent of which is considered equal to the interest on the valuation of the land. The payment is generally secured by a claim on the building erected on the land occupied.

565. When the party who rents the ground has the privilege of purchasing it, the Ground Rent is said to be **redeemable**; otherwise, it is **irredeemable**. The rentor of the land usually erects buildings thereon in his own right and pays a specified sum quarterly, semi-annually, or yearly, for the use of the ground. In some cities the issue of irredeemable ground rents is prohibited.

566. Building lots are sometimes sold at so much per foot frontage ground rent. Thus, a lot valued at $4,000, with a frontage of 20 feet, drawing interest at 8 %, is said to be worth $16 per foot. The interest on $4,000 for 1 year at 8 % is $320, which, being divided by 20, the number of feet on the front, gives $16 as the price.

When a 6 % ground rent yields the owner $180 per year, the value of the ground is estimated at $3,000, since $180 is the interest on $3,000 for 1 year at 6 %.

EXERCISE 121.

1. What is the capitalized value of ground, which at 5 % ground rent, yields the owner $600 per year?

2. What will be received as ground rent for a ⸱⸱ ⸱ valued at $5,000, leased at a ground rent of 8 % ?

8. What is the ground rent price per foot frontage of a lot 80 feet front, valued at $12,000 and paying a ground rent of 7 %.

4. If $192 be received yearly from a ground rent bought for $8,840, what is the rate per cent. ground rent ?

5. I bought three lots, each 25 feet front and 140 feet in depth, at $50 per foot frontage, and leased them at 4½ % ground rent. What income do I receive from my investment ?

6. A real estate owner sold a ground rent of $75 at 6 %. What did he receive for it ?

7. The annual income received on a 6 % ground rent was $540. If the ground rent be sold at its value and the proceeds applied to the purchase of a mining stock at $50 per share, how many shares can be bought ?

8. Find the present worth of a ground rent of 8 % on a lot valued at $4,500, to commence in 3 years and to then continue 15 years, if money be worth 5 % co⸱ ⸱ ound interest.

LIFE INSURANCE.

567. **Life Insurance** is a contract by which a company (the insurer), in consideration of certain payments, agrees to pay to the heirs of a person, when he dies, or to himself, if living at a specified age, a certain sum of money.

568. The principal kinds of policies issued by Life Insurance Companies are the following : Ordinary Life, Limited Payment Life, Endowment, and Annuity.

569. An **Ordinary Life Policy** is one on which a certain premium is to be paid every year until the death of the insured, when the policy becomes payable to the persons named in the policy as the beneficiaries.

570. A **Limited Payment Life Policy** is one on which the premium is paid annually for a certain number of years, fixed upon at the time of insuring, or until the death of the insured, should that occur prior to the end of the selected period. The policy is payable on the death of the insured.

571. An **Endowment Policy** is one which is payable to the person insured, if he survives a certain number of years, or to his heirs, if he should die before the expiration of such period, in consideration of certain regular payment from the person insured.

572. An **Annuity Policy** is one which secures to the holder the payment of a certain sum of money every year during his life-time. It is secured by a single payment.

573. A **Non-Forfeiting Policy** is one which does not become void on account of non-payment of premium.

574. The **Surrender Value** of a policy is the amount of cash which the Company will pay the holder on the surrender of the policy. It is the legal reserve less a certain per cent. for expenses.

575. The Reserve of Life Insurance Policies is the present value of the amount to be paid at death less the present value of all the net premiums to be paid in the future.

576. The Reserve Fund of a Life Insurance Company is that sum on hand which invested at a given rate of interest together with future premiums on existing policies, should be sufficient to meet all obligations as they become due. It is the sum of the separate reserves of the several policies outstanding.

577. The Premium is the sum paid for the insurance of a person's life. It is paid annually, semi-annually, or quarterly.

578. The Premium consists of three elements: 1st. The Reserve, or that portion of each premium which must be kept and improved by interest, to pay the policy at its certain maturity.

2nd. An estimated amount for each man's share of the annual losses of the company.

3rd. Loading, or a certain per cent. to be added to the net premium to cover the general expenses of the business, and to provide against unusual contingencies.

579. The Sum Insured is the sum which is payable by the company upon the conditions mentioned in the contract.

580. Tables of Mortality are tables showing the average rate of deaths in every ten thousand persons.

581. Expectation is the average number of years which a person of a certain age is expected to live, based on a Table of Mortality.

582. The Rates of premium for Life Insurance, as fixed by different companies, are based on the probabilities of life, determined by a table of mortality, and the probable rates of interest which money will bear, and a loading or margin for expenses.

583. Expectation of Life.

The following table shows the number living, the number dying, and the expectation or duration of life of each individual, calculated from the Combined Experience Mortality Table :

Age.	Living.	Dying.	Expectation.	Age.	Living.	Dying.	Expectation.
10	100000	676	48.36	55	63469	1375	16.86
11	99324	674	47.68	56	62094	1436	16.22
12	98650	672	47.01	57	60658	1497	15.59
13	97978	671	46.33	58	59161	1561	14.97
14	97307	671	45.64	59	57600	1627	14.37
15	96636	671	44.96	60	55973	1698	13.77
16	95965	672	44.27	61	54275	1770	13.18
17	95293	673	43.58	62	52505	1844	12.61
18	94620	675	42.88	63	50661	1917	12.05
19	93945	677	42.19	64	48744	1990	11.51
20	93268	680	41.49	65	46754	2061	10.97
21	92558	683	40.79	66	44693	2128	10.46
22	91905	686	40.09	67	42565	2191	9.96
23	91219	690	39.39	68	40374	2246	9.47
24	90529	694	88.68	69	38128	2291	9.00
25	89835	698	37.98	70	35837	2327	8.54
26	89137	703	87.27	71	33510	2351	8.10
27	88434	708	86.56	72	31159	2362	7.67
28	87726	714	85.86	73	28797	2358	7.26
29	87012	720	85 15	74	26439	2339	6.86
30	86292	727	34 48	75	24100	2303	6.48
31	85565	734	33.72	76	21797	2249	6.11
32	84831	742	83.01	77	19548	2179	5.76
33	84089	750	82.30	78	17369	2092	5.42
34	83339	758	31.58	79	15277	1987	5.09
35	82581	767	30.87	80	13290	1866	4.78
36	81814	776	30.15	81	11424	1730	4.48
37	81038	785	29.44	82	9694	1582	4.18
38	80253	795	28.72	83	8112	1427	3.90
39	79458	805	28.00	84	6685	1268	8.63
40	78653	815	27.28	85	5417	1111	8.86
41	77838	826	26.56	86	4306	958	8.10
42	77012	839	25.84	87	3348	811	2.84
43	76173	857	25.12	88	2537	673	2.59
44	75316	881	24.40	89	1864	545	2.35
45	74435	909	28.69	90	1319	427	2.11
46	73526	944	22.97	91	892	322	1.89
47	72582	981	22.27	92	570	231	1.67
48	71601	1021	21.56	93	339	155	1.47
49	70580	1063	20.87	94	184	95	1.28
50	69517	1108	20.18	95	89	52	1.12
51	68409	1156	19.50	96	87	24	0.99
52	67253	1207	18.82	97	13	9	0.89
53	66046	1261	18.16	98	4	3	0.75
54	64785	1316	17.50	99	1	1	0.50

584.   Table of Rates.

RATES FOR WHOLE LIFE INSURANCE.

PREMIUMS TO INSURE $1,000 PAYABLE AT DEATH, WITH PROFITS.

Age.	Annual Premiums.	Single Premiums.	Annual Premiums for 5 Years.	Annual Premiums for 10 Years.	Annual Premiums for 15 Years.	Annual Premiums for 20 Years.	Age.
20	17.80	265.17	60.22	35.03	26.95	23.10	20
21	18.20	270.07	61.34	35.69	27.46	23.59	21
22	18.62	275.11	62.50	36.38	28.00	24.05	22
23	19.06	280.38	63.71	37.09	28.55	24.54	23
24	19.51	285.79	64.95	37.82	29.13	25.04	24
25	19.99	291.39	66.24	38.58	29.72	25.55	25
26	20.49	297.17	67.57	89.37	30.34	26.09	26
27	21.01	303.15	69.94	40.18	30.97	26.65	27
28	21.56	309.32	70.36	41.02	31.64	27.23	28
29	22.13	315.70	71.83	41.90	32.32	27.83	29
30	22.73	322.28	73.35	42.80	33.03	28.45	30
31	23.36	329.08	74.92	43.73	33.76	29.10	31
32	24.02	336.10	76.55	44.70	84.52	29.78	32
33	24.71	343.33	78.22	45.70	35.31	30.48	33
34	25.44	350.81	79.95	46.73	36.13	31.21	34
35	26 21	358.53	81.74	47.80	86.98	31.97	35
36	27.01	366.50	83.59	48.90	37.87	32.77	36
37	27.86	374.73	85.50	50.05	38.79	33.60	37
38	28.76	383.23	87.48	51.24	39.75	34.47	38
39	29.71	392.02	89.53	52.48	40.76	35 39	39
40	30.71	401.10	91.67	53.77	41 81	36.35	40
41	31.78	410.49	93.84	55.12	42.92	37.37	41
42	32.91	420.19	96.13	56.53	44.08	38.45	42
43	34.11	430.22	98.50	58.01	45.30	39.58	43
44	35.39	440.54	100.96	59.55	46.59	40.78	44
45	36.74	451.13	103.51	61.15	47.93	42.04	45
46	38.17	461.96	106.13	62.82	49.33	43.37	46
47	39 67	472.99	108.81	64.53	50.79	44.76	47
48	41.26	484.23	111.57	66.31	52.32	46.22	48
49	42.93	495.66	114.39	68.04	53.90	47.75	49
50	44.70	507.27	117.28	70.05	55.56	49.37	50
51	46.56	519.06	120.24	72.01	57 30	51.07	51
52	48.53	531.01	123.28	74.05	59.11	52.86	52
53	50.61	543.10	126.38	76.16	61.00	54.75	53
54	52.81	555 33	129.55	78.33	63.00	56.75	54
55	55.14	567.70	132.79	80.61	65.09	58.86	55
56	57.61	580.17	136.11	82.97	67.29	61.11	56
57	60.22	592.74	139.51	85.43	69.61	63.49	57
58	63.00	605.41	143.00	88.00	72.07	66.03	58
59	65.94	618.17	146.58	90.69	74.68	68.74	59
60	69.07	630.98	150.26	93.51	77.44	71.63	60

585.   RATES FOR ENDOWMENT INSURANCE.

ANNUAL PREMIUMS TO INSURE $1,000, PAYABLE AT DEATH OR THE EXPIRATION OF THE FOLLOWING TERMS, WITH PROFITS.

Age.	10 Years.	15 Years.	20 Years.	25 Years.	30 Years.	35 Years.	Age
20	95.33	61.63	45.02	35.31	29.10	24.94	20
21	95.39	61 70	45 10	35.41	29.22	25.10	21
22	95.45	61.77	45.19	35.52	29 36	25.27	22
23	95.51	61.85	45.29	35.63	29.50	25.46	23
24	95.58	61 94	45.39	35.76	29.66	25.66	24
25	95.65	62.03	45.50	35.89	29.83	25.87	25
26	95.73	62.12	45.61	36.04	30.02	26.11	26
27	95.81	62.22	45.74	36.19	30.22	26.36	27
28	95.89	62.33	45 87	36.36	30 44	26.64	28
29	95.98	62.44	46.01	36.54	30.67	26.94	29
30	96.08	62.55	46.16	36.74	30.93	27.27	30
31	96.18	62.68	46 32	36.96	31.21	27.63	31
32	96 28	62 81	46.50	37.20	31.52	28.01	32
33	96 39	62.95	46.69	37.46	31.86	28.44	33
34	96 50	63.11	46.90	37.74	32.23	28.90	34
35	96 63	63 28	47.14	38 06	32.63	29.40	35
36	96.76	63.46	47.40	38.40	33.07	....	36
37	96.90	63.67	47.69	38.78	33.56	....	37
38	97.05	63.90	48.01	39.20	34.09	....	38
39	97.23	64.16	48.37	39.67	34.68	....	39
40	97.43	64.46	48 77	40.19	35.33	....	40
41	97.66	64.80	49.22	40.77	....	....	41
42	97.94	65.18	49.72	41.41	....	....	42
43	98.25	65.61	50 28	42.12	....	....	43
44	98 62	66.10	50.91	42.91	....	....	44
45	99.02	66.63	51.60	43.77	....	....	45
46	99.47	67.23	52.36	....	....	....	46
47	99.96	67.87	53.18	....	....	....	47
48	100.50	68.58	54.09	....	....	....	48
49	101.08	69.35	55.07	....	....	....	49
50	101.72	70.19	56.15	....	....	....	50
51	102.41	71.12	....	....	....	....	51
52	103.17	72.12	....	....	....	....	52
53	104.00	73.22	....	....	....	....	53
54	104.90	74.42	....	....	....	....	54
55	105.89	75.74	....	....	....	....	55
56	106.97	....	....	....	....	....	56
57	108.16	....	....	....	....	....	57
58	109.47	....	....	....	....	....	58
59	110.91	....	....	....	....	....	59
60	112.50	....	....	....	....	....	60

EXERCISE 122.

1. Find the amount of premium for an ordinary life policy of $4,000, issued to a person 40 years of age. (Art. 583.)

2. Find the annual premium for a 10-payment life policy of $5,000, issued to a person 35 years of age. (Art. 585.)

8. When 40 years of age, a person took out a 20-year endowment policy of $10,000. He survived the endowment period. How much less did he receive than he paid as premiums, not reckoning interest?

4. The annual premium, without profits, on a life policy of $5,000 at the age of 35 is $111. How much would be necessary to invest at 6 % interest to secure the payment of the annual premium?

5 Mr. A., age 30, insures his life for $10,000, ordinary life plan, with profits. How much must he place in trust so that the interest at 5 % will be sufficient to pay the premiums on the policy?

6. A single premium for an insurance of $1,000, without profits, for a person 32 years of age, is $300. What would be the excess of the insurance over the amount produced by placing the money at compound interest at 4 %, supposing the insured to live 20 years?

7. Mr. A., aged 36, insured his life for $5,000, and paid an annual premium of $135; supposing he died at the age of 68, how much did the premiums he paid exceed the face of his policy, money being worth 6 % compound interest?

8. Mr. A., at the age of 35, takes out a 20-year endowment policy for $3,000 and pays an annual premium of $141. By what amount will the premiums exceed the face of the policy at the end of the endowment period, money being worth 5 % compound interest?

MISCELLANEOUS.

EXERCISE 123.

I.

1. Which is the better investment, a $3,000 7 % bond, or a house which rents for $240 a year, taxes being $30.50, and annual repairs $40 ?

2. A person exchanges 250 shares of 6 % stock, at 70, for stock bearing 8 %, at 120 ; what is the difference in his income ?

8. A gentleman has been receiving 12 % on his capital in Canada. He goes to England to reside, and invests it in the 8 per cents. at $94\frac{3}{8}$, and his income in England is £2,400. What was his income in Canada, the £ being equal to 4.86\frac{2}{3}$?

4. Find the alteration in income occasioned by shifting £3,200 stock from the 8 per cents. at $86\frac{3}{8}$, to 4 per cent. stock at $114\frac{7}{8}$: the brokerage being $\frac{1}{8}$ %.

5. Suppose a railroad stock, actually worth $100 a share, to be " watered " by the issue of a stock dividend of 20 % to the stockholders, what would the watered stock be worth ?

6. A person bought stock at $95\frac{1}{4}$, and after receiving the half yearly dividend at the rate of 7 % per annum, sold out at $92\frac{3}{4}$ and made a profit of $37.50. How much stock did he buy ?

7. At what price must U. S. $4\frac{1}{2}$'s be bought, to yield the interest on the investment that 5 % bonds will at 110 ?

What amount of the latter bonds (par value) must be sold at 108, leaving brokerage out of account, that with the proceeds a sufficient amount of $4\frac{1}{2}$'s may be bought, at par, to yield a semi-annual income of $364.50 ?

8. A person invests the proceeds of a note for $9,607.50, due 18 months hence, discounted (true discount) at $4\frac{1}{2}$ %, in 6 % stock at 91, brokerage $\frac{1}{4}$ %. Find his net annual income from this investment after deducting an income tax of $2\frac{1}{2}$ %.

9. The present income of a railway company would justify a dividend of $3\frac{3}{4}$ %, if there were no preference shares; but as $1,200,000 of the stock consists of such shares, which are guaranteed 5 % per annum, the ordinary shareholders receive only 3 %. What is the whole amount of stock ?

10. A gentleman has $25,000 of Bank of Commerce stock which pays a dividend of 8 %. When money is worth 7 % he sells out, and invests in Bank of Toronto stock at 205, which pays a dividend of 12 %. What difference in his income after allowing his agent $\frac{1}{4}$ % commission for each transaction ?

11. A man invests $19,450 in Bank of Montreal stock at 194, and $19,850 in Bank of Toronto stock at 198, paying his broker in each case $\frac{1}{2}$ % on the amount of stock purchased. If the former pays a half-yearly dividend of $6\frac{1}{2}$ %, and the latter a half-yearly dividend of $6\frac{1}{4}$ %, find his total income for the half-year.

12. A man invested a certain sum in Bank of Commerce stock, which is at 120, and pays $4\frac{3}{4}$ % half-yearly dividends; and $62\frac{1}{2}$ per cent. more than that sum in Dominion Bank stock, which is at 130, and pays $4\frac{1}{2}$ % half-yearly dividends; his income from both investments is $222.50. Find the amount of money invested in each kind of stock.

II.

1. Jan. 1st, 1889, three persons began business. A. put in $1,200, B. put in $500, and May 1st $800 more, C. put in $700, and July 1st $400 more. At the end of the year the profits were $875. How shall it be divided ?

2. A. B. and C. commence business; A. puts in 250 firkins of butter, B. puts in $2,500, and C. $4,100. Their profits amounted to $2,210, of which A. took $560. How much was his butter a pound, and to how much were B. and C. entitled?

8. A building worth $28,500 is insured in the Ætna for $3,200, in the Western for $4,200, and in the Mutual for $6,500. It having been partially destroyed, the damage is set at $10,500. What should each company pay?

4. A. had $8,800 at interest for 60 days; B. had $4,100 at interest for 45 days; and C. had $4,950 at interest for 70 days. They received $162 interest money. What did each get, and what was the rate per cent?

5. A. and B. formed a partnership Jan. 1st, 1889. A. put in $6,000, and at the end of 3 months $900 more, and at the end of 10 months drew out $800; B. put in $9,000, and 8 months after $1,500 more, and drew out $500 Dec. 1st. At the end of the year the net profits were $8,900. Find the share of each.

6. Two persons commence trade with the same amount of money. The first man spends 48 % of his money yearly, and the second spends a sum equal to 25 % of what both had at first. At the end of the year they both together had $8,468. How much had each at the end of the year?

7. A. commenced business with a capital of $10,000, on the 1st of January, 1889; on the 1st of May, B. entered into partnership with him, and put in 1,500 barrels of flour. On the first of January, 1890 their profits were $5,100, of which B. was entitled to $2,100. What was the value of the flour per barrel?

8. Three persons formed a partnership, with a capital of $4,600. The first man's stock was in trade 8 months and

gained $752; the second man's stock was in trade 12 months and gained $600; and the third man had his stock in 16 months and gained $640. What was each man's stock?

9. Three men engaged in the manufacture of pails; A. put in $2,550 for 8 months; B., a sum not specified for 12 months; C., $1,080 for a time not specified. A. received for his stock and profit $3,400; B., $1,200 for his; C., $1,485 for his. Required, B.'s stock and C.'s time.

10. On the 1st of January, 1889, James Wilson opened a hardware store with a stock of $17,200; on the 1st of April, Joseph Brooks entered into partnership with him, and advanced $12,000; on the 1st of July, Abraham Miller put in goods to the amount of $16,000; on the 1st of January, 1890, when the balance sheet was exhibited, there appeared a net profit of $8,060. To how much was each partner entitled?

11. A., B. and C. engaged in business. A. puts in $400 at first, and $400 more at the end of 6 months; B. puts in $900 at first, and withdraws one-third of his capital at the end of 6 months; C. puts in $200 at the end of every 6 months. At the end of two years they have gained $6,700. What share of the profits should C. receive in addition to 25 % of the total profit for managing the business?

12. A., B. and C. formed a partnership for 2 years; A. put in $10,000, B. $5,000, and C. $2,500; it was agreed that C. should receive $1,500 a year for superintending the business. A. drew out $1,000 at the end of each quarter for one year, and at the end of 13 months put in $15,000 more; B. withdrew $600 at the end of each quarter. At the time of settlement the net gain was $22,500. Required each one's share.

III.

1. A draft on Winnipeg bought at $\frac{3}{4}$% premium for $12,000, was sent to an agent to pay for cotton purchased at $2\frac{1}{2}$% commission; what was the value of the cotton?

2. A commission merchant in Peterborough wishes to remit to his employer in Belleville $512.36 by draft at 60 days; what is the face of the draft that he can purchase with this sum, exchange being at $2\frac{1}{4}$% discount, interest 7%?

3. Shipped to Liverpool, 2,000 barrels of flour, which cost in Montreal $4.50 per barrel; it was sold at £1 18s. 6d. per barrel, when the premium was $8\frac{1}{2}$%; how much was the gain?

4. A grain dealer bought 10,000 bushels of corn, at $88\frac{3}{8}$ cts. a bushel. He sent it to London, where it brought 28s. 9d. a quarter, when the premium was $9\frac{1}{2}$%; the cost of transportation was $12\frac{1}{2}$ cts. per bushel; how much was gained?

5. A person in Barrie received £1,000 sterling, from England, when the premium was 9%. He put it out at interest for 9 months, 18 days at 6% per annum; to how much did it amount?

6. A merchant sent his agent in London 425 bales of cotton weighing 356 lbs. apiece, which cost him $9\frac{1}{4}$ cents a lb.; the agent paid $\frac{3}{8}$d. a lb. for freight, £43 for cartage, sold it at 8d. a lb., and charged $2\frac{1}{2}$% commission. If the merchant sells a bill of exchange for the amount, at $10\frac{1}{2}$%, will he make or lose by the operation. How much?

7. Received from my correspondent in New York $6,150 U. S. currency, with instructions to deduct my commission at 2½ %, and invest the remainder in Canadian Tweeds worth $1.03½ per yard. How many yards should I send him, gold being quoted at 115 ?

8. An importer bought 1,565 yards of silk, at 5s. 6d. per yard ; paid £7 12s. for freight, 25 % duties, and remitted a bill on London at 9¼ % premium ; how must he sell it per yard on 6 months, in order to make 12½ %, allowing 7 % interest ?

9. Exchange between Paris and Amsterdam being at the rate of 2 francs 20 centimes to the guilder, that between London and Paris at the rate of 25 francs 80 centimes to the £, and that from New York on London at 9½ % premium, what will be the cost of a remittance for 1,000 guilders from New York to Amsterdam by bills of exchange through London and Paris ?

10. A merchant in Toronto wishes to pay £3,000 in London. Exchange on London is 9½ % premium ; on Paris, 5 francs 25 centimes per $1 ; and on Amsterdam, 40 cents to a guilder. The exchange between France and England at the same time is 25 francs to £1, and that of Amsterdam on England 12½ guilders to £1. Which is the most advantageous, the direct exchange, or through Paris, or through Amsterdam ?

11. A Hamilton merchant, owing 2,400 florins in Amsterdam, can buy exchange on that city for 41¼. Is it better for him to do so, or to remit to London, and thence to Amsterdam,—exchange on London being 4.87 in Hamilton, exchange on Amsterdam being 12 florins to the pound sterling in London, and brokerage for purchasing the exchange in London being ½ of 1 % ?

12. A banker in Toronto remits $10,000 to Liverpool as follows : First to Paris, at 5 francs 40 centimes per $1 ; thence to Hamburg, at 185 francs per 100 marcs ; thence to Amsterdam, at 17½ stivers per marc ; thence to Liverpool, at 220 stivers per £ sterling ; how much sterling money will he have in bank at Liverpool, and what will be his gain over direct exchange at 10 % premium ?

IV.

1. Allowing 6 % compound interest on an annuity of $200 which is in arrears 20 years, what is its present amount ?

2. What is the present worth of an annuity of $500 for 7 years, at 6 % compound interest ?

3. Find the annuity whose amount for 25 years is $16,459.35, allowing compound interest at 6 %.

4. The present worth of an annuity to be continued 10 years at 6 %, compound interest, compounded annually, is $7,360.08. What is the annuity ?

5. A man bought a farm for $4,500, and agreed to pay principal and interest in 4 equal annual instalments ; how much was the annual payment, interest being 6 % ?

6. A man bought a piece of property for $10,000, and agreed to pay principal and interest in 3 equal annual instalments. How much was the annual payment, interest being 7 % ?

7. A father bequeathed his son, 11 years of age, a 6 % annuity of $2,500, to begin in 3 years and continue 10 years ; what would be the amount when the son was 21 years old ?

8. A man took out a life policy for $3,000, at the rate of $21.50 per $1,000. What sum must he deposit in a savings bank, the compound interest of which, at 5 %, payable semi-annually, shall discharge his annual premium ?

9. A man died leaving $5,000 to be divided between his three sons, aged 13, 15, and 16 years respectively, in such a proportion that the share of each being put at simple interest at 6 %, should amount to the same sum when they should arrive at the age of 21. How much was each one's share ?

10. A man paid annually $10 for tobacco from the age of 14 until he was 50, when he died, he left $1,000 for his heirs. What sum might he have left them had he dispensed with tobacco, and loaned the money thus saved at the end of each year at 6 % compound interest ?

11. A mortgage of $1,000, repayable in 5 years at $200 a year with interest at 6 % on the unpaid principal, is sold; what is its value allowing the purchaser 8 % for his money ?

12. A mortgage on a farm is payable in four equal annual instalments of $1,000 each. When the first instalment falls due the mortgagor offers in part payment $2,000 in 6 % municipal debentures upon which interest is due, and which mature in one year. What balance in cash should the mortgagor demand in exchange for the mortgage, money being worth 10 % ?

POWERS AND ROOTS.

586. A **Power** of a number is the number itself, or the product of equal factors, each of which is that number.

Thus, 8 is a power of 2, since $8 = 2 \times 2 \times 2$.

587. The **First Power** is the number itself.

588. The **Second Power** is the product of a number taken *twice* as a factor, and is called a Square.

Thus, 16 is the *square* of 4, since $16 = 4 \times 4$.

589. The **Third Power** is the product of a number taken *three* times as a factor, and is called a Cube.

Thus, 125 is the cube of 5, since $125 = 5 \times 5 \times 5$.

590. A **Root** is one of the equal factors of a number.

NOTE.—Roots are named from the number of equal factors they contain.

591. The **Square Root** is one of the *two equal factors* of a number.

Thus, 7 is the square root of 49, since $49 = 7 \times 7$.

592. The **Cube Root** is one of the *three equal factors* of a number.

Thus, 7 is the cube root of 343, since $343 = 7 \times 7 \times 7$.

593. The **Radical Sign** is the character $\sqrt{}$, which, placed before a number, indicates that its root is to be found.

594. The **Index** of the root is the figure placed above the radical sign to denote what root is to be taken. When no index is written, the index 2 is always understood.

NOTE.—The names of the roots are derived from the corresponding powers, and are denoted by the indices of the radical sign.

Thus $\sqrt{9}$ denotes the square root of 9, the $\sqrt[3]{9}$ denotes the cube root of 9, etc.

595. A **Perfect Square** is one whose exact square root can be found; as 9, 16, 36, etc.

596. A **Perfect Cube** is one whose exact cube root can be found; as 27, 64, 216, etc.

SQUARE ROOT.

597. Extracting the Square Root of a number is the process of finding one of the two equal factors of a number.

Note.—The student should memorize the squares of the first nine digits.

The squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, are respectively 1, 4, 9, 16, 25, 36, 49, 64, 81.

598. To extract the square root of a number.

EXAMPLE 1.—Extract the square root of 5,625.

PROCESS.

$$
\begin{array}{r l}
& 56 \mid 25\ (75 \\
& \underline{49} \\
145 & \overline{725} \\
& \underline{725}
\end{array}
$$

EXPLANATION OF THE METHOD.

Separate the given number into periods of two figures each, beginning at the units' figure.

Find the greatest square in the first period (56), which is 49, and place it under 56, also write the root of 49, which is 7, as the first figure in the required root.

Subtract 49 from 56, and to the remainder (7) affix the next period (25), giving 725 for a dividend. At the left of the dividend (725), write twice the root already found (7), which gives 14.

Divide 72 by 14, which gives a quotient (5).

Affix 5 to 14, giving 145, also place 5, as the second figure of the root.

Multiply 145 by 5, giving 725, which subtracted from the dividend (725), leaves no remainder.

75 is the required root.

EXAMPLE 2.—Extract the square root of 6,838,225.

PROCESS.

$$
\begin{array}{r l}
& 6 \mid 83 \mid 82 \mid 25\ (2615 \\
& \underline{4} \\
46 & \overline{283} \\
& \underline{276} \\
521 & \overline{782} \\
& \underline{521} \\
5225 & \overline{26125} \\
& \underline{26125}
\end{array}
$$

Explanation of the Method.

Separate the given number into periods of two figures each, commencing at the units' figure.

Find the greatest square in the first period (6), which is 4, and place it under 6. also write the root of 4, which is 2, as the first figure of the required root.

Subtract 4 from 6, and to the remainder (2) **affix** the next period (83), giving 283 as the dividend.

At the left of the dividend (283), write twice the root already found (2), which gives 4.

Divide 28 by 4, which gives 7 as a quotient.

Affix 7 to 4, giving 47, also place 7 as the second figure of the root, and multiply 47 by 7, which gives 329, a number **greater than the dividend** (283), showing that 7 is too large a number.

We next try 6 as the second figure of the root.

Affix 6 to 4, giving 46; and place 6 as the second figure of the root.

Multiply 46 by 6, giving 276, which subtracted from the dividend 283, **leaves** a remainder 7, to which affix the next period (82), giving **as the** next dividend 782.

Multiply the part of the root already found (26) **by** 2, obtaining 52, which place to the left of the dividend 782.

Divide 78 by 52, which gives a quotient of 1.

Affix 1 to 52, giving 521, also place 1 as the third figure of the root.

Multiply 521 by 1 and subtract from the dividend 782, after which proceed as before.

Notes 1.—If there is a *remainder* after the root of the last period is found, annex *periods of ciphers*, and proceed as before. The figures of the root thus obtained will be *decimals*.

2. If the trial divisor is not contained in the dividend, *annex a cipher* both to the root and to the divisor, and bring down the next period.

3. It sometimes happens that the remainder is *larger* than the divisor; but it does not necessarily follow that the *figure* in the root is too *small.*

599. To extract the square root of a decimal.

RULE.

Begin at the units' place, and proceed towards the left and right, to separate into periods of two figures each, then extract the root as in whole numbers.

Notes 1 —The *left hand* period in *whole numbers* may have but *one* figure; but in *decimals*, each period must have *two* figures. Hence, if the number of decimals is odd, a *cipher* must be annexed to complete the period

2. It must be kept in mind that no period should contain an integer and decimal, and that, if there is an odd number of decimal places in the given number, the last period must be completed by annexing a cipher.

600. To extract the square root of a fraction.

RULE.

Reduce the fraction to its simplest form and find the square root of each term separately.

NOTES 1.—If the denominator of the given fraction, when reduced, is an imperfect square, reduce the fraction to a *decimal*, and proceed as above.

2. Mixed numbers should be reduced to improper fractions, or the fractional part to a decimal.

EXERCISE 123.

Find the square root of—

1.	36864.	5.	244036.	9.	579121.	13.	966289.
2.	81225.	6.	258064.	10.	734449.	14.	1081600.
3.	168921.	7.	396900.	11.	820836.	15.	1177225.
4.	212521.	8.	499849.	12.	850625.	16.	1234321.

Find one of the two equal factors of—

17.	6838225.	20.	296356225.	23.	44502241.
18.	9048064.	21.	3196944.	24.	61685316.
19.	6885376.	22.	19228225.	25.	179586801.

Extract the square root of—

26.	.0961.	30.	28867.	34.	8819.24	38.	5416.96.
27.	15.21.	31.	33489.	35.	1.338649.	39.	50.1264.
28.	22.09.	32.	4.2849.	36.	226.8036.	40.	.00720801.
29.	.0004.	33.	17.3056.	37.	.00001024.	41.	290.225296.

Extract the square root of—

42.	5.	46.	2.	50.	$20\frac{1}{4}$.	54.	$8\frac{3}{4}$.
43.	.5.	47.	.06.	51.	$153\frac{1}{4}$.	55.	$35\frac{1}{4}$.
44.	.05.	48.	26.	52.	$1\frac{44}{169}$.	56.	$27\frac{3}{4}$.
45.	.005.	49.	.02.	53.	23.1.	57.	$36\frac{1}{4}$.

Find the square root of—

58.	$\frac{3}{4}$.	61.	$\frac{841}{961}$.	64.	$\frac{4}{25}$.	67.	$\frac{9}{4}$.
59.	$\frac{1}{2}$.	62.	$\frac{1156}{2525}$.	65.	$17\frac{1}{4}$.	68.	$5\frac{4}{9}$.
60.	$\frac{3}{8}$.	63.	$\frac{84681}{114025}$.	66.	$11\frac{37}{49}$.	69.	$38\frac{11}{49}$.

CUBE ROOT.

601. Extracting the Cube Root of a number is the process of finding one of the three equal factors of the number.

NOTE.—The student should memorize the cubes of the first nine digits

The cubes of 1, 2, 3, 4, 5, 6, 7, 8, 9 are respectively 1, 8, 27, 64, 125, 216 343, 512, 729.

602. To find the cube root of a number.

EXAMPLE.—Find the cube root of 32768.

PROCESS.

I.	II.	III.	
	8		32 | 768
			27
9	2	2700	5768
		184	
		2884	5768

32 is the cube root.

EXPLANATION OF THE METHOD.

First separate the given number into periods of three figures each, beginning at the units' figure.

Then take the nearest perfect cube not greater than 32, which is 27, and set down its cube root, which is 3 in column II., in line with 32768.

Then subtract 27 from 32, and to the remainder (5) annex the next period (768), giving 5768.

Next place 3 times the first figure (3) of the root, already found, which is (3 × 3) 9 in column I., and 3 times the square of the root (3) already found, which gives (3 × 3 × 3) 27, with two ciphers annexed to it, in column III., each opposite 5768.

Divide 5768 by 2700, which gives a quotient of 2.

Place 2 in column II., opposite 9.

Read 9-2 as one number 92, multiply this by 2, and place the product 184 under 2700, add and multiply their sum, 2884 by 2, and place their product 5768 under 5768, and subtract. As there is no remainder 32768 is a perfect cube

The figures in column II. taken in order give the cube root 32.

EXAMPLE 2.—Extract the cube root of 122615327232.

PROCESS.

I.	II.	III.		
	4			122 | 615 | 327 | 232
				64
12	9	4800	T.D.	58615
		1161)		
		5961 }	C D.	53649
		81)		4966327
147	6	720300	T.D.	4374936
		8856)		
		729156 }	C.D.	4374936
		36)		591391232
1488	8	73804800	T.D.	
		119104		
		73923904	C.D.	591391232

EXPLANATION OF THE METHOD.

Separate the given number into periods of three figures, each beginning at the units' figure.

Then as in Example 1, take the nearest perfect cube not greater than 122, which is 64, and set down its cube root which is 4 in column II., in line with the given number.

Subtract 64 from 122, and to the remainder (58) annex the next period (615), giving 58615.

Next place 3 times 4 (the first figure of the root), that is 12 in column I; and 3 times 4 × 4 (the square of 4), which equals 48 in column III; each in line with 58615, and annex two ciphers to 48 giving 4800.

Divide 58615 by 4800, and a quotient 12 is obtained.

Now 9 is the largest number we can have as a figure of the root, and we therefore use 9, placing it in column II. opposite 12.

Read 12 9 as one number 129. Multiply 129 by 9, and place the product 1161 under 4800, to which it is then added, giving as a result 5961.

Multiply 5961 by 9, and place the product 53649 under 58615 and subtract, and to the remainder 4966 annex the next period 327.

Next place the square of 9, which is 81, under 5961, add the three numbers connected by the bracket, and to their sum 7203 annex two ciphers.

Then place 3 times 49 (the part of the root already found), which is 147, in column I., in the position indicated in the solution.

Divide 4966327 by 720300, and a quotient 6 is obtained. Place 6 in column II. opposite 147.

Read 147--6 as one number 1476. Multiply 1476 by 6, and add the product 8856 to 720300. Multiply their sum 729156 by 6, and place their product 4374936 in the position given in the solution, etc.

The attention of the student is directed first to the method of obtaining the numbers in column I. from those in column II; $12 = 4 \times 3$; $147 = 49 \times 3$; $1488 = 496 \times 3$; etc.

Second, to the formation of the following numbers:

$1161 = 129 \times 9$; $8856 = 1476 \times 6$; $119104 = 14888 \times 8$.

Third, to the formation of trial divisors marked T.D.

Fourth, to the formation of complete divisors marked C.D.

NOTES 1.—If there is a *remainder* after the root of the last period is found, annex periods of ciphers, and proceed as before. The root figures thus obtained will be *decimals*.

2. If a trial divisor is *not contained* in the dividend, put a *cipher* in the root, *two ciphers* on the right of the divisor, and bring down the next period.

3. If the product of the divisor completed into the figure last placed in the root *exceeds* the dividend, the root figure is too large. Sometimes the remainder is *larger* than the divisor completed; but it does not necessarily follow that the root figure is *too small*.

603. To extract the cube root of a decimal.

RULE.

Begin at the units' place, and proceed both toward the left and right to separate into periods of three figures each, then extract the root as in whole numbers.

NOTE.—The *left hand* period in *whole numbers* may have but *one* or *two* figures, but in *decimals* each period must have *three* figures. Hence, ciphers must be annexed to the right of the decimal to complete the periods, when necessary.

604. To extract the cube root of a fraction.

RULE.

Reduce the fraction to its lowest terms, then extract the root of its numerator and denominator.

NOTES 1.—When the denominator is not a *perfect cube*, the fraction should be reduced to a decimal, and the root of the decimal be found as above.

2. A *mixed* number should be reduced to an improper fraction.

EXERCISE 124.

Find the cube root of—

1.	6859.	4.	2406104.	7.	49027896.
2.	12167.	5.	3869893.	8.	66430125.
8.	27000.	6.	5545233.	9.	929714176.

Extract the cube root of—

10	1412467848.	12.	3341362375.	14.	8616805375.
11.	1865409391.	13.	2857243059.	15.	4065356786.

Find the cube root of—

16.	830.584.	18.	1.092727.	20.	.000175616.
17.	.970299.	19.	.002197.	21.	.007645373.

Find the cube root of the following numbers carrying incomplete roots to three or five decimal places, as may be required :

22.	1.	24.	.01.	26.	.001.	28.	$\frac{3}{4}$.	80.	$\frac{1}{3}$.
23.	2.	25.	.02.	27.	.002.	29.	$\frac{2}{3}$.	81.	$\frac{1}{4}$.

MICROCOPY RESOLUTION TEST CHART

(ANSI and ISO TEST CHART No. 2)

1.0

1.1

1.25

1.4

1.6

1.8

2.0

2.2

2.5

2.8

3.2

3.6

4.0

4.5

5.0

5.6

6.3

7.1

8.0

PRACTICAL MENSURATION.

———

605. Mensuration treats of the measurement of lines, surfaces and solids.

606. Lines are measured by expressing their length in inches, feet, yards, etc. (Linear Measure), or in links, chains, etc. (Surveyors' Measure.)

607. A Surface is that which has length and breadth only.

608. Surfaces are measured by expressing the number of times they contain the units of surface measure, *i.e.*, the sq. inch, sq. yard, etc. (Square Measure), or the sq. link, sq. chain (Surveyors' Square Measure).

609. If a straight edge laid anywhere upon a surface touches at every point, the surface is a **plane surface.**

610. A **Polygon** is a plane surface bounded by straight lines.

611. The **Area** of a plane surface is the space enclosed by the lines which bound it

612. A polygon takes its name from the number of sides which bound it, thus :

Triangle.   Quadrilateral.   Pentagon.   Hexagon.   Heptagon.   Octagon.

QUADRILATERALS.

613. A Right Angle is an angle formed by two lines perpendicular to each other.

614. Parallel Lines are lines in the same plane, which being produced both ways never meet, and which are therefore the same distance apart throughout their entire length.

615. Quadrilaterals are of three kinds, as follows:

Parallelogram. Trapezoid. Trapezium.

616. A Parallelogram has its *opposite sides* parallel; a Trapezoid has only *two sides* parallel; a Trapezium has has *no two sides* parallel.

617. Parallelograms are of four kinds, as follows:

Square. Rectangle. Rhomboid. Rhombus.

618. A Square has all its sides equal and all its angles right angles; a Rectangle has its opposite sides equal, and all its angles right angles; a Rhomboid has its opposite sides equal, and none of its angles right angles; a Rhombus has all its sides equal and none of its angles right angles.

619. The **Altitude** of a parallelogram or trapezoid is the perpendicular distance between the parallel sides.

620. The **Diagonal** of a quadrilateral is a straight line joining two opposite corners.

621. To find the area of a rectangle or square.

EXAMPLE 1.—Find the area of the rectangle whose sides are 8 inches and 5 inches in length.

SOLUTION.

5 sq. in. × 3 = 15 sq in.   Ans.
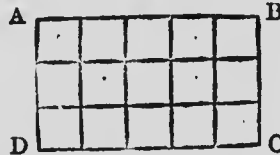
EXPLANATION.

In the figure A B C D, let A B be 5 inches, and A D be 3 inches. Let A B be divided into 5 equal parts, each 1 inch in length, and let A D be divided into 3 equal divisions each, 1 inch in length. Draw through these divisions the lines represented in the figure. The whole figure will then be divided into squares, each of whose sides is 1 inch in length, and hence each square is a *square inch.* In each horizontal row there are 5 square inches, and in the three horizontal rows there will be 3 times 5 square inches, or 15 square inches, and hence the solution, 5 sq. in. × 3 = 15 sq. in.

EXAMPLE 2.—Find the area of a square whose side is 8 inches.

SOLUTION.

8 sq. in × 8 = 64 sq. in.   Ans.

EXPLANATION.

Same as Example 1.

RULE.

Multiply the length by the breadth and the result will be the area.

NOTES 1.—The student will observe that the rule is only a shortened form of expressing the longer rule. Multiply the measure of the length expressed in units of square measure by the measure of the breadth.

2. All the following rules will be expressed in a shortened form.

The converse of the preceding rule must be true:

If the area of a rectangle be divided by a side, the quotient will be the other side, or if the square root of the area of a square be extracted, the result will be the length of a side.

622. To find the area of a rhomboid or rhombus, the length of a pair of opposite sides and the perpendicular distance between them being given.

EXAMPLE.—Find the area of a rhomboid, one pair of whose opposite sides are 10 feet in length, and the distance between them 6 feet.

SOLUTION.	EXPLANATION.

10 sq. ft. × 6 = 60 sq. ft. Ans.   It is proved in Euclid, Book I, proposition 35, that the area of a parallelogram is equal to the area of a rectangle on the same base, and of the same altitude, and hence the solution given.

RULE.

Multiply the length of one of the parallel side. by the perpendicular distance between them.

623. To find the area of a trapezoid, the lengths of the parallel sides and the perpendicular distance between them being given.

EXAMPLE.—Find the area of a trapezoid, the lengths of the parallel sides being 6 feet and 10 feet, and the perpendicular distance between them 5 feet.

SOLUTION.

(6 ft. + 10 ft.) ÷ 2 = 8 ft.   8 sq. ft. × 5 = 40 sq. ft.   Ans.

RULE.

Multiply one-half the sum of the parallel sides by the perpendicular distance between them.

TRIANGLES.

624. A Triangle is the space enclosed by three straight lines.

625. Triangles are named according to their sides, and also according to their angles, as follows:
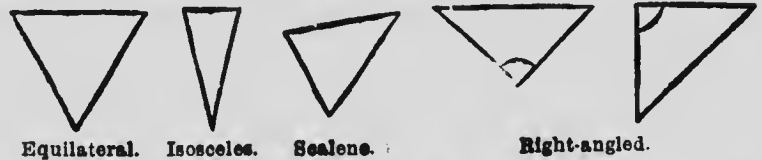
Equilateral.   Isosceles.   Scalene.   Right-angled.

626. An Equilateral Triangle has its three sides equal.

627. An Isosceles Triangle has only two sides equal.

628. A Scalene Triangle has all of its sides unequal.

629. A Right Angled Triangle has one of its angles a right angle.

630. The Base of a triangle is any side of a triangle upon which a perpendicular is let fall from the opposite angle.

631. The Altitude of a triangle is the length of the perpendicular let fall from an angle on the opposite side or the opposite side produced.

NOTE.—Dotted lines represent the altitude.
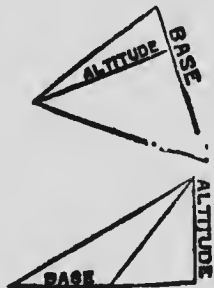
632. To find the area of a triangle.

EXAMPLE 1.—Find the area of a triangle whose base is 16 feet, and whose altitude is 9 feet.

SOLUTION.

$$(16 \text{ sq. ft} \div 2) \times 9 = 72 \text{ sq. ft.}$$

EXPLANATION.

It is proved in Euclid, Book I, proposition 41, that the area of a triangle is half the area of a parallelogram on the same base and of the same altitude, hence the solution given.

RULE.

Multiply one-half the base by the altitude.

The following rule is also necessary when three sides are given.

RULE

From half the sum of the sides subtract each side separately ; then multiply half the sum and the three remainders together, and extract the square root of the product.

EXAMPLE 2.—What is the area of a triangle whose sides are 12 feet, 16 feet, and 18 feet ?

SOLUTION.

$(12 + 16 + 18) + 2 = 23$

$23 - 18 = 5$ $23 \times 5 \times 7 \times 11 = 8,855.$

$23 - 16 = 7$

$23 - 12 = 11$ $\sqrt{8858} = 94.1$ sq. ft.   Ans.

633. *It is proved in Euclid, Book I, proposition 47, that in any right angled triangle the area of the square described on the side opposite the right angle, is equal to the sum of the areas of the squares described on the sides containing the right angle.*

In the accompanying figure, if A B C be a triangle having a right angle at C, the area of the square described on A B is equal to the sum of the areas of the squares described on A C and B C.
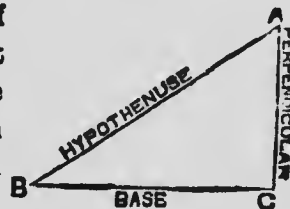
A B, the side opposite the right angle, is called the hypothenuse ; B C the bass ; and A C the perpendicular.

Hence, the square on the hypothenuse = square on the base + the square on the perpendicular.

EXAMPLE 1.—If the base of a right angled triangle be 8 feet, and the perpendicular be 6 feet, what is the length of the hypothenuse?

SOLUTION.

In the preceding figure,

$$\text{sq. on A B} = 8 \times 8 + 6 \times 6$$
$$= 100 \text{ sq. ft.}$$
$$\therefore \text{ A B} = \sqrt{100} = 10 \text{ ft. Ans.}$$

EXAMPLE 2.—The hypothenuse of a right angled triangle is 35 feet and the perpendicular is 28 feet, find the base.

SOLUTION.

$$35 \times 35 = \text{sq. on the base} + 28 \times 28$$
$$\therefore \text{ sq. on the base} = 35 \times 35 - 28 \times 28 = 441$$
$$\therefore \text{ the base} = \sqrt{441} = 21 \text{ ft. Ans.}$$

634. To find the area of a trapezium.

A trapezium may be divided into two triangles by joining two opposite corners, and hence it is only necessary to find the areas of the two triangles and to take their sum.

EXAMPLE.—Find the area of a trapezium whose sides are 10 feet, 11 feet, 12 feet, and 15 feet, the length of the line joining opposite corners being 13 feet.

SOLUTION.
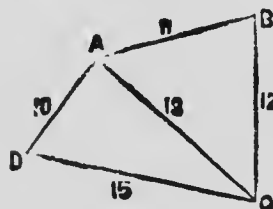
$$\text{Area A B C} = \sqrt{18 \times 7 \times 6 \times 5} = 61.48.$$
$$\text{(Art. 632.)}$$
$$\text{`` A C D} = \sqrt{19 \times 9 \times 6 \times 4} = 64.06.$$
$$\text{(Art. 632.)}$$
$$\therefore \text{Area A B C D} = 61.48 + 64.06 =$$
$$125.54 \text{ sq. ft. Ans.}$$

POLYGONS.

━━━ To find the area of a regular polygon containing more than four sides.

RULE.

Multiply the perimeter (sum of all the sides) of the base by one-half the perpendicular distance from the centre to one of the sides.

EXAMPLE.—What is the area of a hexagon, side 8 feet, the perpendicular distance from the centre to one of the sides being 6.928 + feet.

SOLUTION.

Perimeter = 8 ft. × 6 = 48 ft

Area = 48 sq ft. × $\frac{6.928}{2}$ = 166.272 + sq. ft.

The area of an ''lateral triangle equals the square of a side multiplied by .433, and the area of a hexagon, which is made up of 6 equilateral triangles, is therefore 6 × .433 times the square of a side.

THE CIRCLE.

636. A **Circle** is a plane figure bounded by a curve line called the **circumference,** every point of which is equally distant from a point called the **centre.**

637. The **Diameter** of a circle is a line drawn through the centre, and terminated at both ends by the circumference.

638. A **Radius** is a straight line drawn from the centre to the circumference and is equal to half the diameter.

Note.—From the definition of a circle, it follows that all the *radii* are *equal* ; also, that all *diameters* are *equal.*

639. Principles. ` *The circumference = the diameter 3.1416 nearly.*

2. Therefore the diameter = the circumference + 3.1416 nearly.

3. The area of a circle = the square of the radius × 3.1416 nearly.

4. The area of a circle = the circumference × half the radius.

5. Therefore the radius of a circle = sq. root of (the area + 3.1416) nearly.

Note.—The fraction 3⅟ is commonly used in place of the decimal 3.1416, and is near enough for common practical operations, and will be used in this work.

EXAMPLE 1. What is the circumference of a circle whose radius is 7 feet?

SOLUTION.

$$7 \text{ ft.} \times 2 = 1\text{. ft. diameter,}$$
$$14 \text{ ft.} \times 3\tfrac{1}{7} = 44 \text{ ft.} \quad \text{Ans. (Prin. 1.)}$$

EXAMPLE 2. The circumference of a circle is 176 feet. What is the diameter?

SOLUTION.

$$176 \div 3\tfrac{1}{7} = 56 \text{ ft.} \quad \text{Ans. (Prin. 2.)}$$

EXAMPLE 3. What is the area of a circle whose diameter is 14 feet?

SOLUTION 1.

$$14 \text{ ft.} \div 2 = 7 \text{ ft. the radius,}$$
$$7 \times 7 \times 3\tfrac{1}{7} = 154 \text{ sq. ft.} \quad \text{Ans. (Prin. 3.)}$$

SOLUTION 2.

$$14 \text{ ft.} \times 3\tfrac{1}{7} = 44 \text{ ft. the circumference. (Prin. 1.)}$$
$$14 \text{ ft.} \div 2 = 7 \text{ ft. the radius.}$$
$$44 \times \tfrac{7}{2} = 154 \text{ sq. t.} \quad \text{Ans. (Prin. 4.)}$$

EXAMPLE 4. The area of a circle is 616 square feet. Find the radius, diameter, and circumference.

SOLUTION.

$$\text{Radius} = \sqrt{616 \div 3\tfrac{1}{7}} = 14 \text{ ft.} \quad \text{(Prin. 5.)}$$
$$14 \text{ ft.} \times 2 = 28 \text{ ft. the diameter.}$$
$$28 \text{ ft.} \times 3\tfrac{1}{7} = 88 \text{ ft. the circumference. (Prin. 1.)}$$

MISCELLANEOUS.

EXERCISE 125.

1. How may acres in a piece of woodland 220 yards in length and 40 rods in width?

2. How many square miles in a township 5 miles and 40 chains square?

8. How many square feet in a floor 20 feet long and 5 yards wide?

4. Find the surface of a pane of glass measuring $37\frac{1}{2}$ inches long and 28 inches wide.

5. How many square yards in the four walls of a room 15 ft. 6 in. high and 80 feet in compass?

6. A rectangular pavement, 50 ft. 9 in. long and 12 ft. 6 in. wide, was laid with a central line of stone 5 feet wide at $.75 a running foot; the sides were flanked with brick at 50 cents per square yard. What did the paving cost?

7. How many square feet in a surface 24 feet long 20 feet wide? How many in another surface of half these dimensions?

8. Two fields contain 10 acres each; one is in the form of a square, the other is 4 times as long as it is wide. What would be the difference in expense of fencing them at $2.25 per rod?

9. If the fence were built $4\frac{1}{2}$ feet high, of boards 8 inches wide, the lower one raised 2 inches above the ground, and a space of 3 inches between the boards, how many square feet of boards would be required for both fields?

10. How many more for one than for the other?

11 A piece of land containing 2 acres is 5 times as long as it is broad. What is its length and breadth?

12. How many bricks 8 inches long and 4 inches wide will pave a yard that is 100 feet by 50?

13. What will it cost to pave a roadway 80 feet long and 15 feet wide, at $1.50 per square yard?

14. I have a box without a lid; it is 5 feet long, 4 feet wide, and 3 feet deep, interior dimensions. How many square feet of zinc will it take to line the bottom and sides of the box?

15. Find the area of a rhomboid whose length is 1 yd. 2 ft. 6 in., and whose width is 2 ft. 8 in.

16. The base of a rhombus is 10 ft. 6 in., and its altitude 8 feet. What is its area?

17. How many acres in a piece of land in the form of a rhomboid, the base being 8.75 ch. and altitude 6 ch.?

18. A man bought a farm 198 rods long and 150 rods wide, and agreed to give $32 an acre. What did the farm cost?

19. A certain rectangular piece of land measures 1,000 links by 100. How many acres does it contain?

20. How many square feet in a board 16 feet long 18 inches wide at one end and 25 inches wide at the other end?

21. Required the area of a trapezoid whose parallel sides are 178 and 146 feet, and the altitude 69 feet.

22. One side of a quadrilateral field measures 38 rods; the side opposite and parallel to it measures 26 rods, and the distance between the two sides is 10 rods. Find the area.

23. The parallel sides of a trapezoid measure respectively 8½ feet and 6 inches; the perpendicular distance between them is 2 feet. What is the area?

24. Find the area of a trapezium whose diagonal is 168, and one perpendicular 42, the other 56.

25. Find the area of a trapezium whose diagonal is 35 ft. 6 in., and the perpendiculars to this diagonal 9 feet and 12½ feet.

26. How many acres in a quadrilateral field whose diagonal is 30 rods, and the perpendiculars to this diagonal 20.458 and 50.832 rods. ?

27. What is the base of a triangle whose area is 156 square feet, and its altitude 12 feet ?

28. What is the base of a triangle whose area is 144 acres and its altitude 60 rods ?

29. Find the base of a triangle whose area is 5,280 square yards, and altitude 240 yards.

30. What is the area of a triangle whose three sides are 18, 14, and 15 feet ?

31. What is the area in acres of a triangular field whose three sides measure respectively 47, 58, and 69 rods ?

32. What is the area of a triangle whose base is 24 feet and altitude 16 feet ?

33. The base of a triangle is 28 inches and the altitude 16 inches; what is the area ?

34. A board 16 feet long is 22 inches wide at one end, and tapers to a point; what is the value at $4\frac{1}{2}$ cents a square foot?

35. Find the area of a triangle whose base is 12 ft. 6 in. and altitude 6 ft. 9 in.

36. Whose base is 25.01 chains and altitude 18.14 chains.

37. What is the cost of a triangular piece of land whose base is 15.48 ch. and altitude 9.67 ch. at $60 an acre?

38. At $.40 a square yard, find the cost of paving a triangular court, its base being 105 feet, and its altitude 21 yards?

39. Find the area of a circular pond, its circumference being 200 chains.

40. The distance around a circular park is $1\frac{1}{2}$ miles. How many acres does it contain?

41. How much land in a circular garden that requires 84 rods of fencing to inclose it?

42. Find the difference in cost at $87\frac{1}{2}$ cts. per rod between fencing a square field of 10 acres and a rectangular field 32 rods wide of the same area.

43. Draw a square containing 81 square inches; inscribe a circle in this square. What is the superficies of this circle in square inches?

44. A cow is tethered to a post driven in the centre of a lot 100 feet square; the tether is just long enough for her to reach the fence. How much of the surface of the field is she unable to crop?

45. If the diameter of an iron column is 3 ft. 5 in., what is the circumference ? If the girth of a tree is 5 ft. 9 in., what must be its diameter ?

46. If the equatorial diameter of the earth is 7,925 miles, how long in miles and rods is the equator ?

47. The distance from the centre of the hub of a wheel to the outer edge of the felly is 15 inches. How long must the tire be ?

48. If the length of an oar from the thole-pin to the end of the blade is 5 feet, how many feet would the end of the blade travel in the water during 6,000 strokes, each describing an arc of 60° ? (60° = $\frac{1}{6}$ of the circumference.)

49. If the circumference of a circular pond is 628 318 rods, what part of a mile must I row to pass from shore to shore across the centre of the pond ?

50. If a horse is tethered to the middle post of a fence, from which he can graze out into the field in a curved line 78.539314 feet long, how long is the tether ?

51. What will be the circumference of the largest circle that can be drawn on a sheet of paper 12 inches wide and 18 inches long ?

SOLIDS.

640. A **Solid** is that which has length, breadth, and thickness.

641. A **Prism** is a solid whose bases are similar, equal, and parallel polygons, and whose sides are parallelograms.

642. Prisms take their **names** from the forms of their bases, as *triangular, rectangular, pentagonal, hexagonal*, etc.
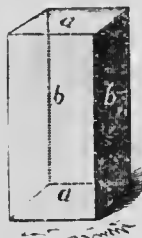
643. A **Cube** is a rectangular prism whose faces are all equal squares.

644. A **Cylinder** is a circular body of uniform diameter whose ends are equal and parallel circles.

645. The **Altitude** of a prism or cylinder is the perpendicular distance between its bases.

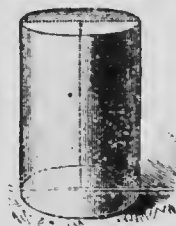Triangular prism.   Rectangular prism.   Pentagonal prism.   Hexagonal prism
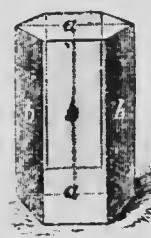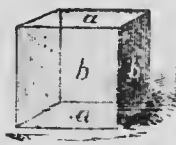
Cube.   Cylinder.

646. To find the convex surface of a prism or cylinder

Suppose a block of the shape of one of the preceding prisms to have been fitted with a piece of paper so as to exactly cover its convex surface. Now if the paper be unrolled it will be found to be the shape of a rectangle, one side being equal to the height, and the other side equal to the perimeter of the base. Hence, the following rule.

RULE.

1. Multiply the perimeter (sum of all the sides) of th' base by the altitude.

2. To find the entire surface, add the areas of the bases to the convex surface.

EXAMPLE 1. Find the convex surface and also the entire surface of a rectangular prism whose ends are 5 inches by 7 inches, and whose altitude is 12 inches.

SOLUTION

$$\text{Perimeter of the base} = (5 + 7 + 5 + 7) \text{ in.} = 24 \text{ in.}$$
$$\text{Altitude} = 12 \text{ in.}$$
$$\therefore \text{Convex surface} = 24 \text{ sq. in.} \times 12 = 288 \text{ sq. in.}$$
$$\text{Again, area of base} = 7 \text{ sq. in.} \times 5 = 35 \text{ sq. in.}$$
$$\therefore \text{Entire surface} = 55 \text{ sq. in.} + 35 \text{ sq. in.} + 288 \text{ sq. in.} = 358 \text{ sq. in}$$

EXAMPLE 2. Find entire surface of a cylinder the diameter of whose base is 14 inches, and whose altitude is 20 inches.

SOLUTION.

$$\text{Perimeter of base} = 14 \times 3\tfrac{1}{7} = 44 \text{ in.}$$
$$\therefore \text{Convex surface} = 44 \text{ sq. in} \times 20 = 880 \text{ sq. in.}$$
$$\text{Again, area of base} = 7 \times 7 \times 3\tfrac{1}{7} = 154 \text{ sq. in.}$$
$$\therefore \text{Entire surface} = (154 + 154 + 880) \text{ sq. in.} = 1188 \text{ sq. in.}$$

647. To find the volume of a prism or cylinder.

RULE.

Multiply the area of the base by the altitude.

EXAMPLE. 1. Find the volume of a rectangular prism whose base is 4 inches by 6 inches, and altitude 10 inches.

SOLUTION.

$$\text{Area of base} = 6 \text{ sq. in.} \times 4 = 24 \text{ sq. in.}$$
$$\text{Volume} = 24 \text{ cub. in.} \times 10 = 240 \text{ cub. in.}$$

The base can be divided into 24 squares each side of which is 1 inch. If a piece of the prism 1 inch in thickness be cut off by a plane parallel to the base it can be divided in 24 small blocks, corresponding to the 24 squares into which the base can be divided, each of these small blocks will therefore be 1 inch long, 1 inch wide, and 1 inch in thickness. Hence the part cut off will contain 24 cubic inches, 10 such pieces can be cut off the whole block, and the whole block therefore contains

$$24 \text{ cub. in. } \times 10 = 240 \text{ cub. in.}$$

EXAMPLE. 2. What is the volume of a triangular prism whose base is an equilateral triangle each side 8 inches, and whose altitude is 12 inches?

SOLUTION.

Area of base $= \sqrt{12 \times 4 \times 4 \times 4} = 27.712 +$ sq. in.
Volume $= 27.712$ cub. in. $\times 12 = 332.544$ cub. in.

EXAMPLE. 3. Find the volume of a cylinder, the diameter of whose base is 14 inches and altitude 20 inches.

SOLUTION.

Area of base $= 7 \times 7 \times 3\frac{1}{7} = 154$ sq. in.
Volume $= 154$ cub. in $\times 20 = 3080$ cub. in.

648. A **Pyramid** is a solid whose base is a polygon and whose sides terminate in a point called the vertex.

649. A **Cone** is a solid which has a circle for its base, and terminates in a point called the vertex.

650. The **Altitude** of a pyramid or cone is the perpendicular distance from the base to the vertex.

651. The **Slant** height of a pyramid is the distance from the vertex to the middle point of any side of the base.

652. A **Frustrum** of a pyramid or cone is the part which is left after the top is cut off by a plane parallel to the base.

653. The **Altitude** of a frustrum is the perpendicular distance between its ends.

654. The **Slant Height** of a frustrum of a pyramid is the distance between the middle points of two parallel sides of one of its faces.
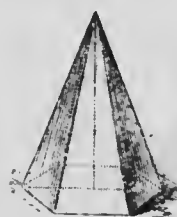
Pyramid.   Cone.   Frustrum of a pyramid.   Frustrum of a cone.

655. To find the convex surface of a pyramid or cone.

RULE.

1. Multiply the perimeter by one-half the slant height.

2. To find the entire surface add the area of the base to the area of the convex surface.

EXAMPLE 1. - Find the entire surface of a pyramid whose base is a square side 6 inches, and whose slant height is 10 inches.

SOLUTION.

Perimeter of base   = 16 in.
Convex surface   = 16 sq. in. $\times$ $\frac{10}{2}$ = 80 sq. in.
Area of base   = 4 sq. in. $\times$ 4 = 16 sq. in.
$\therefore$ Entire surface   = (80 + 16) sq. in. = 96 sq. in.

EXAMPLE 2.—Find entire surface of a cone, the diameter of the base ing 14 inches, and slant height 30 inches.

SOLUTION.

Perimeter of base   = 14 in. $\times$ $3\frac{1}{7}$ = 44 in.
Convex surface   = 44 sq. in $\times$ $\frac{30}{2}$ = 660 sq. in.
Area of base   = 7 $\times$ 7 $\times$ $3\frac{1}{7}$ = 154 sq. in.
Entire surface   = (660 + 154) sq. in. = 814 sq. in.

656. To find convex surface of a frustrum of a cone or pyramid.

RULE.

1. Multiply one-half the sum of the perimeters of the ends by the slant height.

2. To find the entire surface, add the areas of the ends to the area of the convex surface.

EXAMPLE.—Find entire surface of the frustrum of a cone, the diameters of whose ends are 7 inches and 14 inches, and whose slant height is 20 inches.

SOLUTION.

Perimeter of ends = 7 in. × $3\frac{1}{7}$ = 22 in., and 14 in. × $3\frac{1}{7}$ = 44 in.

Convex surface $= \left(\dfrac{44 + 22}{2}\right)$ sq. in. × 20 = 660 sq. in.

Area of smaller end = 7 × 7 × $3\frac{1}{7}$ = $38\frac{1}{2}$ sq. in.

Area of larger end = 7 × 7 × $3\frac{1}{7}$ = 154 sq. in.

Entire surface = (660 + $38\frac{1}{2}$ + 154) sq. in. = $852\frac{1}{2}$ sq. in.

657. To find the volume of a cone or pyramid.

RULE.

Multiply area of the base by one-third the altitude.

EXAMPLE.—Find volume of a cone, whose base is 14 inches in diameter, and whose altitude is 24 inches.

SOLUTION.

7 × 7 × $3\frac{1}{7}$ (area of base) × 8 = 1232 cub. in.

658. To find the volume of the frustrum of a cone or pyramid.

RULE.

$(A + a + \sqrt{A \times a}) \times h \times \frac{1}{3}$, *where* '*A*' *stands for the area of the larger end,* '*a*' *for the area of the smaller end, and* '*h*' *for the perpendicular height.*

EXAMPLE.—Find the volume of the frustrum of a cone, whose end diameters are 7 f   and 14 feet, and whose altitude is 12 feet.

SOLUTION.

Area of smaller end = $\frac{7}{2}$ × $\frac{7}{2}$ × $3\frac{1}{7}$ = $38\frac{1}{2}$ sq. ft.

Area of larger end = 7 × 7 × $3\frac{1}{7}$ = 154 sq. ft.

Volume = (154 + $38\frac{1}{2}$ + $\sqrt{154 + 38\frac{1}{2}}$) × 12 × $\frac{1}{3}$ = 1078 cub. ft.

659. A **Sphere or Globe** is a solid terminated by a curve surface, every part of which is equally distant from a point within, called the centre.

660. The **Diameter** of a sphere is a straight line drawn through its centre and terminated at both ends by the surface.

661. A Hemisphere is one-half a sphere.

662. The Radius of a sphere is a straight line drawn from its centre to any point in its surface.

663. To find the surface of a sphere.

RULE.

Multiply the square of the diameter by 3⅐.

EXAMPLE.—What is the surface of a sphere whose diameter is feet ?

SOLUTION.

$$7 \times 7 \times 3⅐ = 154 \text{ sq. ft. Ans.}$$

664. To find the volume of a sphere.

RULE.

Multiply the cube of the diameter by 3⅐, and divide the result by 6.

EXAMPLE.—What is the volume of a sphere whose diameter is 7 feet ?

SOLUTION.

$$7 \times 7 \times 7 \text{ (cube of the diameter)} \times 3⅐ \times ⅙ = 179⅔ \text{ cub. ft.}$$

CISTERNS AND BINS.

665. To find the number of gallons in a cistern.

RULE.

Find the volume in cubic inches and divide the result by 231.

NOTE.—There are 231 cubic inches in one gallon.

EXAMPLE.—Find the number of gallons in a rectangular cistern, 8 feet by 6 feet, and 3 feet deep.

SOLUTION.

Volume $= (8 \times 6 \times 3)$ cub. ft. $= (8 \times 6 \times 3) \times 1728$ cub. in.

$\therefore$ No gallons $= 8 \times 6 \times 3 \times 1,728 \div 231 = 1,077\frac{1}{3}$ gal.

666. To find the number of bushels of wheat in a bin or pile.

RULE.

Find the volume in cubic inches and divide the result by 2150.42.

NOTE.—There are 2150.42 cubic inches in one bushel.

EXAMPLE.—How many bushels of grain in a bin 4 feet by 6 feet, and 8 feet deep?

SOLUTION.

Volume $= 4 \times 6 \times 8 \times 1,728$ cub. in.

$\therefore$ No. bushels $= 4 \times 6 \times 4 \times 1,728 \div 2150.42 = 53$ bush. nearly

GAUGING OF CASKS.

667. Gauging is the process of finding the capacity or volume of casks and other vessels.

NOTE.—A cask is equivalent to a cylinder, having the same length and a diameter equal to the *mean diameter* of the cask.

668. To find the mean diameter of a cask (nearly).

RULE.

Add to the head diameter $\frac{2}{3}$. *or, if the staves are but little curved,* $\frac{1}{2}$ *of the difference between the head and bung diameters.*

669. To find the volume of the cask in gallons.

RULE.

Multiply the square of the mean diameter by the length (both in inches), and this product by .0034.

EXAMPLE.—How many gallons in a cask whose head diameter is 24 inches, bung diameter 30 inches, and length 34 inches?

SOLUTION.

Mean diameter $= \{24 + (30 - 24) \times \frac{2}{3}\} = 28$ in.
Capacity $\qquad = 28 \times 28 \times 34 \times .0034 = 90.63$ gal.

EXERCISE 126.

1. What is the solidity of a triangular prism whose length is 12 feet, and one of the equal sides of one of its equilateral ends is 3 feet ?

2. How many gallons of water would a cylindrical boiler contain if 25 inches high and 12 inches in diameter ?

3. Find the cubic inches in the largest cone that can be cut from a cylinder 2 ft. 6 in. high and 14 inches in diameter.

4. A sphere 8 inches in diameter is placed in a cubical box whose interior dimensions are 8 inches. How much vacant space is left?

5. I have a cylindrical tank which contains 160 gallons, it is 6 ft. 5 in. in diameter. How deep is it?

6. How many square feet of canvas will be required to cover a cylinder 16½ feet in circumference and 25 feet long?

7. How many square inches of surface in a stove pipe 22 inches in circumference and 12 feet long?

8. What is the convex surface of a log 25 feet in circumference and 18 feet long?

9. What is the convex surface of a cylinder 3 feet long and 1½ feet in diameter? What is its entire surface?

10. What are the contents of a log 15 feet long and 2 feet in diameter?

11. The standard liquid gallon is 231 cubic inches; how many gallons in a can 22 inches in diameter and 3 feet high?

12. How many cubic feet in a triangular prism, the area of whose base is 920 square feet and height 20 feet?

13. What are the contents of a quadrangular prism whose length is 25 centimeters, and the base a rectangle 3 by 5 centimeters?

14. What is the lateral surface of a regular pyramid whose slant height is 15 feet, and whose base is 30 feet square?

15. What is the surface of a pyramid whose base is an equilateral triangle measuring 4 feet on each side, and slant height 16 feet?

16. What is the convex surface of a cone, the diameter of whose base is 7 feet and its slant height 12 feet?

17. What is the entire surface of a triangular pyramid whose slant height is 25 feet, and each side of the base 10 feet?

18. What is the entire surface of a right cone, the diameter of the base and the slant height being each 40 feet?

19. Find the cubic feet in a log 30 feet long and 2 feet in diameter at the larger and 1 ft. 10 in. at the smaller end.

20. Find the cubic contents of a pyramid, base 300 feet square, and altitude 80 feet.

21. How many cubic feet in a circular mound 48 feet high, and having a diameter of 86 feet at the top, and a circumference of 471.24 feet at the bottom?

22. How many cubic miles in the earth, supposing it to be a perfect sphere 8,000 miles in diameter?

23. How many barrels of oil in a tank 60 feet in diameter if the oil is 5 feet deep? (40 gal. to the barrel.)

24. A monument in the form of a square pyramid, is 2 ft. 10 in. square at base, and 11 feet high; at 175 pounds to a cubic foot what is its weight?

25. What are the contents of a round log whose length is 20 feet, diameter of larger end 12 inches, and smaller end 6 inches?

26. The altitude of a frustrum of a pyramid is 27 feet, the ends are 4 feet and 3 feet square; what is its solidity?

27. What are the contents of a pyramid whose base is 144 square feet, and its altitude 33 feet?

28. Find the solidity of a sphere whose diameter is 12 inches.

29. What are the contents of a cone the area of whose base is 1,865 sq. feet, and its altitude 36 feet ?

30. Find the convex surface of a frustrum of a cone whose slant height is 15 feet, the circumference of the lower base 30 feet, and of the upper base 16 feet.

31. ..nat will it cost to gild a ball 12 inches in diameter, at 10 cents a square inch ?

32. The standard bushel of the United States is 18½ inches in diameter and 8 inches deep; how many cubic inches does it contain ?

33. How many square yards in the convex surface of a frustrum of a pyramid, whose bases are heptagons, each side of the lower base being 8 feet, and of the upper base 4 feet, and the slant height 55 feet ?

34. Find the contents in gallons of a cask whose length is 54 inches, its bung diameter 42, and head diameter 86 inches.

35. Required the contents in gallons of a rectangular istern 4½ feet long, 3¼ feet wide, and 6 feet deep.

36. What are the contents in gallons of a cask 36 inches long, its head diameter 26 inches, and bung diameter 32 inches ?

37. How many gallons in a cask whose head diameter is 24 inches, bung diameter 30 inches, and its length 34 inches ?

38. What is the volume of a cask whose length is 40 inches the diameters 21 and 30 in. respectively ?

39. How many gallons in a cask of slight curvature, 3 ft. 6 in. long, the head diameter being 26 inches, the bung diameter 31 inches ?

MEASUREMENT OF CARPETING.

670. Carpet is sold by the linear yard, and is of various widths. The more common widths are 27 inches and 36 inches.

671. In determining the number of yards of carpet that will be required to cover a room, it is first necessary to decide whether the strips of carpeting shall run lengthwise of the room or crosswise. Economy in matching usually decides this.

672. In determining the length of each strip of carpet, allowance must be made for waste in matching.

673. To find the number of yards of carpeting required for a room of given dimensions.

EXAMPLE 1.— How many yards of carpet 27 inches wide will be required for a rectangular room 21 feet long and 18 feet wide, if the strips run lengthwise and no waste in matching ?

SOLUTION.

18 ft. = 216 in.
216 ÷ 27 = 8, No. strips of carpet.
1 strip is 21 ft. or 7 yds. long.
8 strips are, 7 yds × 8 = 56 yds. Ans.

EXAMPLE 2.—How many yards of carpet 36 inches wide will be required for a rectangular room 20 feet 6 inches long, and 16 feet 9 inches wide, if the strips run crosswise, and 4 inches per strip be allowed for matching ?

SOLUTION.

16 ft. 9 in. = 201 in.
201 in. ÷ 36 in. = 5 times and 21 in. remaining.
∴ It will take 6 strips of carpet.
Length of each strip = 20 ft. 6 in. + 4 in. = 20 ft. 10 in.
1 strip is 20 ft. 10 in. long.
∴ 6 strips are, 20 ft. 10 in. × 6 = 125 ft. or $41\frac{2}{3}$ yds. Ans.

EXERCISE 127.

1. A rectangular room 26 ft. 8 in. long, and 16 ft. 6 in wide, is to be covered with carpet 1 yard wide. Which way of the room should the strips run that there may be the least turned under or cut off from one side of a breadth ?

2. In No. 1, if the strips were 16 ft. 6 in. long, how many strips would be required ?

8. In No. 1, if the strips were 26 ft. 3 in. long, how many would be required.

4. In No. 1, if the strips were 16 ft. 6 in. long, and there was no waste in matching, how many yards would it take ?

5. In No. 1, if the strips were 26 ft. 3 in. long, and there were no waste in matching, how many yards would it take ?

6. How many yards of carpeting 27 inches wide will be required for a room 17 ft. 6 in. by 15 ft. 5 in., if the strips run crosswise, and 7 inches be wasted in matching each strip ?

7. A room is 15 feet by 17 ft. 6 in., and the carpet is $\frac{3}{4}$ of a yard wide. What must be the length of the strips to have the least waste ? How many strips will be required ?

8. In No. 7, how many yards of carpet would be required if there were a waste of 8 inches in matching each strip, except the first ? Why should there be no waste in the first strip ?

9. Find the cost of carpeting a room 22 ft. 8 in. by 13 ft. 4 in. if the carpeting be 27 inches wide, and cost $1.80 per yard, there being a waste of 8 inches per strip in matching, the strips running lengthwise.

10. A parlor 20 feet by 17 feet is carpeted with a carpet 1 yard wide, at $1.20 per yard, surrounded with a carpet border 1 foot wide, at 75 cents a yard. Find the total cost.

11. Find the cost of carpeting a room 28 ft. 10 in. long, by 17 ft. 8 in. wide, with carpet ¾ of a yard wide, at $1.80 per yard, if the strips run lengthwise of the room, and 9 inches per strip be wasted in matching.

12. Find the cost of the carpet for a stair of 17-12 inch steps, each rising 8 inches, at 90 cents a yard.

18. Find the cost of the stair carpet at $1.20 a yard, for a flight of stairs of 22 steps, 11 inches wide, with 7 inches rise, allowing 1 yard extra at the top.

14. Find the cost of covering the floor of a hall 24 feet long by 8 feet wide, with oil-cloth 4 feet wide, no waste in matching.

MEASUREMENT OF WALL PAPER.

674. Wall paper is sold by the roll, any part of a roll being counted as a whole roll.

675. Canadian and American wall papers are 18 inches wide, and have 8 yards in a roll. For convenience wall paper is done up in double rolls of 16 yards.

676. In estimating the number of rolls necessary for a certain room, paper-hangers ascertain the height of the room and its perimeter, making an allowance in the perimeter of 3 feet for each door or window.

677. The exact cost of papering a room can be ascertained only by taking account of the number of rolls of paper actually used in doing the work.

678. To find the number of rolls of paper required for a room.

EXAMPLE 1.—How many rolls of wall paper will be required for the walls of a rectangular room 20 feet by 16 feet, with a 12 foot ceiling, there being one door 3 feet 8 inches wide, and 2 windows each 4 feet 2 inches wide?

SOLUTION.

Perimeter of room is   (20 ft. + 16 ft.) × 2   = 72 ft.
Width of door,   3 ft. 8 in.
Width of 2 windows (4 ft. 2 in.) × 2 = 8 ft. 4 in.   12 ft.
Perimeter after deducting width of door and windows = 60 ft.
60 ft. = 720 inches.
720 in. ÷ 18 in. (width of paper) = 40, number of strips.
1 strip is 12 ft. long.
∴ 40 strips are 480 ft. or 160 yds. long.
160 yards ÷ 8 yds. (No. yds. in a roll) = 20, No. of rolls. Ans.

EXAMPLE 2.—Find the cost of the wall paper at 80 cents a roll and bordering at 7 cents a yard for a room 18 feet 9 inches long by 16 feet 5 inches wide, with the ceiling 10 feet 9 inches above the base boards, allowing for 2 doors each 3 feet 8 inches wide, and 3 windows each 3 feet 6 inches wide, also an allowance of 9 inches on each strip for matching. (In reckoning the cost of the bordering no allowance is made for the doors and windows.)

SOLUTION.

Perimeter of room is (18 ft. 9 in. + 16 ft. 5 in.) × 2 = 70 ft. 4 in.

Width of doors   (3 ft. 8 in.) × 2 = 7 ft. 4 in.

Width of windows (3 ft. 6 in.) × 3 = 10 ft. 6 in. 17 ft. 10 in.

Perimeter of room after deducting width of doors and windows = 52 ft. 6 in.

52 ft. 6 in. = 630 in.

630 in. ÷ 18 in. = 35, No. of strips.

To allow for matching, the paper will cut into strips of

(10 ft. 9 in. + 9 in.) = 11 ft. 6 in. in length.

One roll will practically cut into 2 strips.

∴ No. of rolls = 35 ÷ 2 = 17½

∴ It will take 18 rolls

1 roll is worth   80 cents

∴ 18 rolls are worth 80 cents × 18 = $14.40, Cost of wall paper.

70 ft. 4 in. = 24 yds. nearly

1 yard is worth 7 cents

∴ 24 yds. are worth 7 cents × 24 = $1.68, Cost of border.

$16.08. Total cost.

EXERCISE 128.

1. How many strips of paper will go around a room 18 feet by 24 feet ?

2. How many strips of paper are required for a room 30 feet by 24, if there are 4 windows and 2 doors ? (Art. 676.)

3. How many rolls will paper a ceiling 24 feet by 18 feet ?

4. How many double rolls are required for a hall 21 feet long and 13 feet high, with a cornice 1 foot deep ?

5. Find the cost of the paper for a room 36 feet by 24 feet and 11 feet high, with a cornice 1 foot deep, and a wainscoting 2 feet deep, at 50 cents per   uble roll.

6. How many double rolls of wall paper will be required for a room 18 ft. 6 in. by 15 ft. 4 in., the ceiling 8 feet above the base-boards, allowance being made for 1 door 3 ft. 8 in. wide and 2 windows each 4 feet wide ?

7. If a roll of paper cuts into two strips, and 10 strips be allowed for doors and windows, find the cost of papering a room 24 ft. 8 in. long by 16 feet wide with paper at 45 cents a roll and bordering at 7 cents a yard, the hanging of the paper costing 15 cents a roll.

8. Find the cost of paper for a hall 72 feet by 44 feet, 14 feet high, below the cornice, allowing for 8 windows each 4 ft. 2 in. wide and 2 doors each 3 ft. 8 in. wide, the paper costing 45 cents per double roll.

9. With paper at 1.   per roll, and border at 3 cents a yard, what is the cost of paper and border for a room 24 feet by 20 feet and 12½ feet high, with cornice 6 inches deep, there being 5 openings of an average width of 3 feet ?

10. If the paper-hanger charges $3, and the paper costs 80 cents a double roll and the border 4 cents a yard, find the cost of papering a room 18 ft. 9 in. long, 16 ft. 8 in. wide, with a ceiling 13 ft. 6 in. high, allowing for two doors, each 3 ft. 9 in. wide, and 3 windows, each 4 ft. 2 in. wide ; also for a base-board 18 inches deep.

MEASUREMENT OF SAW-LOGS.

679. TABLE OF LUMBER AND LOG MEASUREMENT.

Showing net proceeds (fractions of feet omitted) of logs in 1 inch boards, deducting saw kerf and slabs. The length will be found in the left hand column, and the diameter in inches on the head of the other columns.

Length of log in Feet	Diam. 10	Diam. 11	Diam. 12	Diam. 13	Diam. 14	Diam. 15	Diam. 16	Diam. 17	Diam. 18	Diam. 19	Diam. 20
10	23	31	40	50	62	75	90	105	122	140	160
11	25	34	44	55	69	83	99	116	135	154	176
12	27	37	48	61	75	91	108	126	147	169	192
13	29	40	52	66	81	98	117	137	159	183	208
14	32	43	56	71	88	106	126	148	171	197	224
15	34	46	60	76	94	113	135	158	184	211	240
16	36	49	64	81	100	121	144	169	196	225	256
17	38	52	68	86	106	128	153	179	208	239	272
18	41	55	72	91	112	136	162	190	220	253	288
19	43	58	76	96	119	143	171	201	232	267	304
20	46	61	80	101	125	151	180	211	244	280	320
21	48	64	84	106	131	158	189	222	257	293	336
22	50	67	88	111	137	166	198	232	269	280	352
23	52	70	92	116	144	174	207	243	281	309	368
24	54	74	96	122	150	181	216	254	294	323	384
25	56	77	100	127	156	189	225	264	308	338	400
26	59	80	104	132	163	196	234	274	318	351	416
27	61	83	108	137	169	204	243	285	330	366	432
28	63	85	112	142	175	212	252	296	342	380	448
29	65	89	116	147	182	219	261	306	355	394	464
30	68	92	120	152	188	226	270	316	363	408	480
31	70	95	124	157	193	234	279	327	380	422	496
32	72	98	128	162	200	242	288	338	392	436	512
33	74	101	132	167	206	249	297	348	404	450	528
34	77	104	136	172	212	256	306	358	416	478	544
35	79	107	140	177	219	265	315	369	428	492	560
36	81	110	144	182	224	272	324	380	440	508	576

Length of log in feet	Diam. 21	Diam. 22	Diam. 23	Diam. 24	Diam. 25	Diam. 26	Diam. 27	Diam. 28	Diam. 29	Diam. 30	Diam. 31	Diam. 32	Diam. 33	Diam. 34	Diam. 35	Diam. 36
10......	180	202	225	250	276	302	330	360	391	422	456	490	526	562	601	640
11......	198	223	248	275	303	333	363	396	430	465	502	539	578	619	661	704
12......	217	243	271	300	331	363	397	432	469	507	547	588	631	675	721	768
13......	235	263	293	325	359	393	430	468	508	549	592	637	683	731	781	832
14......	253	283	313	350	386	424	463	504	547	591	638	686	736	787	841	896
15......	271	303	336	375	413	454	496	540	586	633	683	735	788	844	901	960
16......	289	324	359	400	441	484	529	576	625	676	729	784	841	900	961	1024
17......	307	344	383	425	468	514	562	612	664	718	774	833	893	956	1021	1088
18......	325	364	406	450	496	544	595	648	703	760	820	882	946	1012	1081	1152
19......	343	384	429	475	523	575	628	684	742	803	865	931	999	1069	1141	1216
20......	361	404	452	500	550	605	661	720	782	845	911	980	1051	1125	1201	1280
21......	379	425	473	525	579	635	694	756	820	887	957	1029	1104	1181	1261	1344
22......	397	445	496	550	606	665	727	792	860	929	1002	1078	1156	1238	1321	1408
23......	415	465	519	575	632	695	760	828	898	972	1048	1127	1209	1294	1381	1472
24......	433	486	541	600	660	726	793	864	937	1014	1093	1176	1261	1350	1441	1536
25......	451	506	562	625	689	756	826	900	977	1056	1139	1225	1314	1406	1501	1600
26......	470	526	586	650	716	786	860	936	1016	1098	1185	1274	1366	1462	1562	1664
27......	488	546	606	675	744	817	893	972	1055	1141	1230	1323	1419	1518	1622	1728
28......	506	566	626	700	772	847	926	1008	1094	1183	1276	1372	1472	1575	1682	1792
29......	524	586	649	725	799	877	959	1044	1133	1225	1321	1421	1524	1631	1742	1856
30......	542	606	672	750	826	907	992	1080	1172	1267	1367	1470	1577	1688	1802	1920
31......	560	627	695	775	854	938	1025	1116	1211	1310	1412	1519	1629	1744	1862	1984
32......	578	648	718	800	882	968	1058	1152	1250	1352	1458	1568	1682	1800	1922	2048
33......	596	668	742	825	910	998	1091	1188	1289	1394	1504	1617	1735	1856	1982	2112
34......	614	688	766	850	937	1028	1124	1224	1328	1436	1548	1666	1787	1912	2042	2176
35......	632	708	789	875	965	1058	1157	1260	1367	1479	1594	1715	1840	1968	2102	2240
36......	650	728	812	900	992	1089	1190	1296	1406	1521	1640	1764	1892	2025	2162	2304

680. In some parts of Canada saw-logs are bought and sold by the *Standard*, in other parts with reference to the number of feet of inch lumber which they will produce.

681. A Standard Log is 12 feet long and 21 inches in diameter, and will produce 1,085 feet of inch lumber.

682. The measurement of a log is always taken at the small end and between the bark.

683. To find the number of standards in a given number of saw-logs.

EXAMPLE 1.—How many standards are there in 4 saw-logs, each 12 feet long, the diameters of which are 16 inches, 20 inches, 22 inches, and 25 inches respectively?

SOLUTION.

$$16^2 = 256$$
$$20^2 = 400$$
$$22^2 = 484$$
$$25^2 = \underline{625}$$
$$\text{Sum} = 1,765$$

$$1,765 \div 21_2 = 1,765 \div 441 = 4. \quad \text{No. standard. Ans.}$$

EXAMPLE 2.—How many standards are there in 5 logs, each 16 feet long, the diameters of which are 18, 20, 21, 24, and 30 inches respectively?

SOLUTION.

$$18^2 = 324$$
$$20^2 = 400$$
$$21^2 = 441$$
$$24^2 = 576$$
$$30^2 = \underline{900}$$
$$\text{Sum} = 2,641$$

$$2,641 \div 441 = 6 \text{ nearly.} \quad \text{No. of standards 12 feet long.}$$
$$16 = 1\tfrac{1}{3} \text{ times } 12$$
$$\therefore \text{ No. of standards} = 6 \times 1\tfrac{1}{3} = 8. \quad \text{Ans.}$$

EXERCISE 129.

1. How many standards are there in 6 saw-logs, each 12 feet long, the diameters of which are 12, 16, 20, 25, 26 and 28 inches respectively?

2. How many standards are there in 5 logs, each 18 feet long, the diameters of which are 14, 20, 22, 24 and 30 inches respectively?

8. What is the side of the largest square piece of timber which can be sawn from a log, the diameter of which is 28 inches?

4. From the Table, Art. 679, find cut the quantity of inch lumber that can be sawn from the following:

3 logs 10 feet long, diameters 15, 20 and 32 inches respectively.
2 " 14 "   "   18 and 24   "   "
4 " 16 "   "   16, 20, 22 and 80   "   "
2 " 18 "   "   20 and 26   "   "

5. A man wishes a piece of timber 18 inches square, what is the diameter of the smallest log from which it may be sawn?

MEASUREMENT OF LUMBER.

684. Lumber, as the term is used here, includes all kinds of sawed boards, plank, scantling, joists, etc.

685. A foot of lumber, or a board foot, is the unit of measurement. It is 1 foot long, 1 foot wide, and 1 inch thick.

686. The term scantling is given to lumber 8 or 4 inches wide, and from 2 to 4 inches thick.

Joist is usually from 2 to 4 inches thick, and from 6 to 16 inches wide.

Lumber heavier than joist or scantling is called timber. A broad piece of lumber thicker than a board,—usually from $1\frac{1}{2}$ to 4 inches thick, is called a plank.

687. All lumber less than one inch in thickness is considered inch lumber in measuring.

688. In measuring the width of a board a fraction greater than a half inch is called a half, and if less than a half it is rejected. Thus a board $5\frac{7}{8}$ inches wide would be considered 6 inches wide. a board $9\frac{3}{8}$ inches wide would be considered 9 inches wide.

689. The price of lumber is usually quoted at a certain rate per thousand feet, board measure.

690. To find the number of board feet or feet of lumber in a board, plank, joist, etc.

EXAMPLE 1.—Find the number of feet of lumber in a board 14 feet long, 12 inches wide, and 1 inch thick.

SOLUTION.

$(14 \times 12 \times 1) \div 12 = 14$ feet. Ans.

Example 2.—Find the number of feet of lumber in a plank 16 feet long, 14 inches wide, and 3 inches thick.

Solution.

$(16 \times 14 \times 3) + 12 = 56$ feet. **Ans.**

RULE.

Multiply the length in feet by the width and thickness in inches, and divide the product by 12, and the result will be the number of board feet of lumber.

EXERCISE 130.

1. Find the number of feet of lumber in 24 boards 14 feet long and 10 inches wide.

2. Find the cost of fifty 2-inch plank 16 feet long and 10 inches wide at $18 per thousand.

8. How many square feet are there in the surface of a board 16 feet by 9 inches ?

4. How many feet of lumber are there in a board 12 feet long, 6 inches wide and 1 inch thick ?

5. How many feet of lumber are there in the following bill ?—24 joists 16 feet by 10 inches, 2 inches thick ; 210 pieces of siding, 12 feet long, 4 inches wide, $\frac{3}{4}$ inch thick ; 14 beams 20 feet long, and 9 inches square ; 16 scantling, 2 inches by 4 inches, 16 feet long.

6. How many feet of lumber in a 140 pieces of siding, each 12 feet long, 6 inches wide, and $\frac{1}{2}$ inch thick ?

7. How much lumber is there in eighty 2 x 4 scantling 14 feet long ?

8. Find the cost of 2,250 feet of lumber at $20 per thousand.

9. Find the cost of 1½ inch flooring required to lay a floor 42 feet by 24 feet at $24 per thousand.

10. Find the cost of flooring a bridge 320 yards long by 20 feet wide with 3 inch oak planks, at $22 per thousand.

11. If 2 x 4 studs are used, and they are placed 16 inches apart, from centre to centre, how many feet of lumber are there in the studding of a wall 20 feet long and 12 feet high?

12. How many 12 foot boards 6 inches wide are required to put a wainscoting 3 feet high around a kitchen 12 feet by 16 feet, allowing for 2 doors, each 3½ feet wide?

13. Find the cost of the lumber for two floors of a house 24 feet long and 18 feet wide, if the lower floor is 1½ inches thick, and the upper floor 1 inch, at $20 a thousand.

14. A barn is 64 feet long and 40 feet wide, and 20 feet high to the eaves; the gables are 8 feet high, and the rafters 22 feet, 6 inches long. Find the number of feet of inch boards necessary to inclose the two sides, allowing for two doors 12 feet by 16 feet.

15. In No. 5, find the number of feet of lumber in the ends and gables.

16. In No. 5, find the number of feet of lumber required to sheet the roof.

17. In No. 5, find the cost of the lumber for the doors at $20 a thousand.

18. In No. 5, find the cost of the 2 inch plank needed for the floor at $24 a thousand.

19. If 4 x 5 rafters are used, and they are placed 30 inches apart, from centre to centre, how many feet of lumber are there in the 20 foot rafters of a double roof 40 feet long?

20. Find the price of the following bill of lumber at $24 per thousand :—

    120 2-inch plank 10 inches wide, 14 feet long.
    125 boards 10 inches wide, 16 feet long.
    80 2 x 4-inch scantling, 14 feet long.
    50 3 x 4-inch   "   1.   "
    120 3 x 10-inch joist, 16 feet long.

21. How many feet of lumber are there in the 2 x 4-inch studs of a partition wall 32 feet long and 14 feet high ?

Nⲟᴛᴇ.—The studs of partition walls are usually placed 16 inches apart from centre to centre.

22. How many 12-foot strips 2½ inches wide will lay a walk 4 feet wide and 80 yards long, allowing half an inch between the strips ?

23. If lumber 10 inches wide is used in sheeting the roof in No. 19, and the boards are placed two inches apart, allowing for a projection of one foot at each end, how many feet of lumber will be required ?

24. How many feet of lumber are there in the 12-inch base board of a square 10 acre field ?

25. Find the cost of the lumber for the dressed door facings of 18 doors, each 7 feet high and 2 feet 8 inches wide, the facings being 6 inches wide, at $30 per thousand feet.

MEASUREMENT OF SHINGLING.

691. Shingles are sold by the bunch, each bunch contains a quarter thousand. A bunch of shingles is 20 inches wide, and has 25 courses on each side. Dealers will not sell a part of a bunch.

692. Ordinary shingles have an average width of 4 inches, and are generally laid 4 inches to the weather.

693. Allowing for waste, 1000 shingles will cover a surface of 100 square feet (a square of shingling), 4 inches to the weather; laid 4½ inches to the weather, 900 shingles are required.

EXERCISE 131.

1. How many shingles are there in 24 bunches?

2. How many bunches are there in 15½ thousand?

3. How many thousand are there in 48 bunches?

4. Laid 4 inches to the weather, how many square inches are covered by the exposed part of one shingle?

5. How many shingles are required for a roof having a surface of 2,400 square feet?

6. How many bunches of shingles will shingle a roof 82 feet by 24 feet?

7. How many shingles are required for a double roof 36 feet long, with 20-foot rafters?

8. Find the cost of laying a double roof 48 feet long, rafters 24 feet long, with shingles 4 inches to the weather at $3.20 per thousand.

9. Find the cost of shingles for a double roof 36 feet long, rafters 21 feet long, at 60 cents a bunch, if the shingles are laid 4½ inches to the weather.

10. At $3.60 per thousand, find the cost of the shingles for a roof of a building 60 feet long, 40 feet wide, having a gable 12 feet high, and the rafters having an 18-inch heel.

FENCING.

EXERCISE 132.

1. How many fence posts are required for a fence 80 rods long, if the posts are placed 8 feet apart?

2. How many posts are required for a fence around a field 40 rods square, if they are placed 8 feet apart?

8. How many posts are required for a square 10-acre field, if they are placed 8 feet apart?

4. Find the cost of the posts for a fence around a garden plot 250 yards by 220 yards, if the posts are placed 6 feet apart and cost 10 cents each.

5. In No. 4, how many 2 x 4 scantling, 12 feet long will be required for the 2 stringers of the fence?

6. In No. 3, find the cost of 2 x 4 scantling, 16 feet long, that will be required for the 2 stringers of the fence, if the lumber is worth $18 per thousand.

7. How many feet of lumber are required for a 10-inch base board around the field in No. 2?

8. How many 2-inch pickets are required for a fence 40 rods long, if the pickets are placed 2 inches apart?

9. How many 2½-inch pickets, placed 2 inches apart, are required for a fence around a garden 200 yards by 150 yards?

10. How much lumber is there in a common board fence 40 rods long, consisting of 5 rounds of 6-inch boards?

11. What will it cost to fence 5 miles of railway, both sides, with 6 rounds of 6-inch boards, at $12 per thousand feet ?

12. What will it cost at $10 per thousand to fence a field 40 rods by 60 rods with 1 round of 12-inch boards, and 5 of 6-inch boards ?

13. What will be the cost per mile to fence a railway with 5 strands of barbed wire, which weighs 1 lb. per rod, at 8 cents a pound ?

14. Find the cost of a quarter mile of fence with the posts 8 feet apart, a 12-inch base, a 2 x 4 rail at top, and 4 strands of barbed wire ; the posts cost 10 cents each, the lumber $12 per thousand, and the wire at 7 cents a pound. (A pound stretches 16½ feet.)

MEASUREMENT OF PAINTING, KALSO-MINING AND PAVING.

694. The unit of measurement of painting, kalsomining, and paving is the square yard.

EXERCISE 133.

1. How many square yards of painting are there in a floor 30 feet by 28 feet ?

2. Find the cost of kalsomining the ceiling of a hall 64 feet long and 36 feet wide, at 20 cents a square yard.

3. What will it cost to paint a close board fence 6 feet high around a lot 36 yards long by 24 yards wide ?

4. What will it cost to paint a house 36 feet by 30 feet, which has an average height of 18 feet, at 18 cents a square yard ?

5. What will it cost to kalsomine a room 20 feet by 18 feet and 10 feet high, at 7 cents a square yard ?

6. Find the cost of painting a double roof 44 feet long by 24 feet, at 12 cents a square yard.

7. What will it cost to tuckpoint the front of a brick house 36 feet long and 22 feet high, allowing for half the openings which form one quarter of the surface, $1.25 per square yard ?

8. Find the cost of paving a street half a mile long and 60 feet wide, at 30 cents a square yard.

9. Find the cost of paving a street one-eighth of a mile long and 1½ chains wide, at 25 cents per square yard.

10. A circular plot of ground, 4 chains in diameter, has a walk 8 feet wide, formed around the outer edge. Find the cost of gravelling the walk, at 15 cents a square yard.

MEASUREMENT OF LATHING AND PLASTERING.

695. Laths are sold by the bunch.   There are 50 laths in a bunch, each lath being 4 feet long and $1\frac{1}{2}$ inches wide. They are usually laid about three-eights of an inch apart.

696. Allowing for waste, contractors reckon that a bunch of laths will cover 3 square yards of surface.

697. Lathing and plastering are estimated by the square yard.   Only one-half the surface of openings is allowed.

698. To find the cost of lathing and plastering a room of given dimensions.

EXAMPLE.—A rectangular room 24 feet by 18 ft. 9 in., and 10 ft. 10 in. high.   The base board is 10 inches high; there are two doors 8 feet by 4 ft. 3 in. each, and three windows 6 ft. 4 in. by 4 feet each.   Find the cost of lathing and plastering the walls and ceiling at 30 cents a square yard.

SOLUTION.

Perimeter of room   = (24 ft. + 18 ft. 9 in.) × 2 = 85 ft. 6 in.
Height of walls above base board – 10 ft. 10 in. – 10 in. = 10 ft.
Area of walls = 85 ft. 6 in. × 10 ft. = ... 855 sq. ft.
Area of ceiling = 24 ft. × 18 ft. 9 in. = ... 450 sq. ft.
                    Total gross area = ... 1,305 sq. ft.
Area of 2 doors = (8 ft. × 4 ft. 3 in.) × 2 = 68 sq. ft.
Area of 3 windows = (6 ft. 4 in. × 4 ft.) × 3 = 76 sq. ft.
        Total area of doors and windows = 144 sq. ft.
Half of 144 sq. ft. is allowed   = ... 72 sq. ft.
Net area to be lathed and plastered = ... 1,233 sq. ft.
            1,233 sq. ft. = 137 sq. yds.
            1 sq. yd. is worth 30 cents.
            137 sq. yds. are worth 30 cents × 137 = $41.10.   Ans.

EXERCISE 134.

1. Including one of the spaces between the laths, how many square inches does one lath cover?

2. How many square feet will a bunch of laths cover?

3. How many bunches of laths will be required for a wall 36 feet long and 12 feet high?

4. How many bunches of laths will be required for the ceiling of a room 32 feet by 28 feet?

5. How many bunches of laths are required for the walls and ceiling of a room 15 feet by 18 feet, and 9 feet high?

6. How many bunches of laths are required for a hall 84 feet long, 52 feet wide, and 21 feet high, allowing for 4 doors and 10 windows, each having an average surface of 82 square feet. Art. 696.

7. At 80 cents a bunch, find the cost of the laths for a room 20 feet by 24 feet and 15 feet high, there being 8 windows and 2 doors, each 8 feet by 4 feet.

8. At 25 cents a bunch, find the cost of the laths for a room 24 feet by 16 feet and 10 feet high, allowing for a door 8 feet by 3 ft. 6 in., and a window 7 feet by 4 feet.

9. How many square yards of plastering are there in the ceiling of a room 60 feet by 32 feet?

10. How many square yards of plastering are there in the walls and ceiling of a room 36 feet by 24 feet and 12 feet high?

11. Allowing for an 18-inch base-board, find the number of yards of plastering in a room 36 feet by 30 feet and 14 feet high.

12. Find the cost of plastering the ceiling of a room 36 feet by 32 feet, at 9 cents per square yard.

13. Find the cost of plastering the walls and ceiling of a room 18 feet by 24 feet, 12 feet high, at $12\frac{1}{2}$ cents a square yard.

14. At 15 cents a square yard, find the cost of plastering the walls and ceiling of a room 21 feet long, 14 feet wide, and 12 feet high, with 4 openings, each 8 feet by 4 feet.

15. At $12\frac{1}{2}$ cents a square yard, find the cost of plastering a room 20 feet by 16 feet and 12 feet high, with an 18-inch base, and having 4 openings, averaging 32 square feet each.

16. Find the cost of lathing and plastering a room 16 feet by 18 feet and 12 feet high, with laths at 30 cents a bunch, and plastering at 15 cents a square yard.

17. Find the cost of cementing a circular cistern 8 feet in diameter and 9 feet high, at 8 cents per square foot.

MEASUREMENT OF STONE-WORK.

699. A cord of stone is of the same size as a cord of wood. In estimating stone-work no smaller part than quarter-cords is allowed.

700. A cord of stone will make about 100 cubic feet of wall.

701. In estimating the cost of mason-work, it is customary to take the outside measurement of the wall, and make no allowance for openings, except they are large.

702. It takes about three bushels of lime and a cubic yard of sand to lay a cord of stone.

703. Stone-work is usually estimated by the perch.

704. A perch of stone-work is 1 rod long, $1\frac{1}{2}$ feet thick, and 1 foot high. It contains $24\frac{3}{4}$ cubic feet.

EXERCISE 135.

1. How many cubic feet of stone are there in a pile 38 feet long, 6 feet wide, and 4 feet high?

2. How many cubic feet of stone are there in wagon-box 9 feet long, $3\frac{1}{2}$ feet wide, and $1\frac{1}{2}$ feet high? What part of a cord does it contain?

3. How many cords of stone are there in a pile 20 feet long, 8 feet wide, and 3 feet high?

4. In No. 3, how many cubic feet of wall will the stone build?

5. How many cords of stone will build a wall 200 feet long, 6 feet high, and 3 feet thick?

6. How many cords of stone will build a wall 60 yards long, 6 feet high, and 18 inches thick? How many perch of stone-work in the wall?

7. Find the cost of the stone in a wall 42 feet long, 8 feet high, 18 inches thick, at $6 per cord.

8. How many cords of stone are required for a cellar 36 feet long, 30 feet wide, if the wall be built 8 feet high, and two feet thick?   Find the cost of the mason work at 50 cents a perch.

9. How many cords of stone are required for the foundation of a bank barn 60 feet long, by 36 feet wide, if the foundation wall be 7 feet high and 3 feet thick?   Find the cost of building the foundation at 60 cents a perch.

10. At 60 cents per perch, what is the cost of the stone-work for the basement of a house which has an outside perimeter of 160 feet, the wall being 8 feet high and 20 inches thick?

11. How much lime and sand will be required for the mortar of an 18-inch wall 8 feet high, under a house 40 feet by 30 feet?

12. In No. 9, find the cost of the material at $6 per cord for the stone, 30 cents a bushel for the lime, and $1.20 per cubic yard for the sand.

13. A stone house is 36 feet by 24 feet; the cellar walls are 9 feet high and 3 feet thick; the walls of the ground floor are 12 feet high and 2 feet thick; the walls of the second floor are 8 feet high and 18 inches thick; the gable walls are 7 feet high and 12 inches thick; find—

1st. Number of perches of mason work in the building, and cost of labour at $1.10 a perch.

2nd. Cost of the stone at $5 a cord.

3rd. Cost of the lime at 35 cents a bushel.

4th. Cost of the sand at $1.10 per cubic yard.

MEASUREMENT OF BRICK-WORK.

705. Bricks vary so much in size and style, that to give the exact dimensions of the different styles is impracticable. Ordinary bricks are 8 inches long, 4 inches wide, and 2½ inches thick.

706. It is sufficiently accurate, in making an estimate of the number of brick needed for a certain work, to reckon 20 bricks to the cubic foot *laid dry.*

707. In *half-brick* walls, such as in veneering wooden houses, each brick, with the mortar required to lay it, has an external surface of 8½ × 3, or for about every 25 square inches of surface.

708. In *single-brick* walls, each brick, with the mortar required to lay it, has an external surface of 4½ × 3, or one brick is required for about every 13 square inches of surface.

709. In a *brick-and-a-half* wall, a brick is required for about every 8⅔ square inches.

710. In *double-brick* walls, a brick is required for about every 6½ square inches of surface.

711. In estimating material, corners are measured once, and allowance is made for doors and windows

In estimating labor, the corners are measured twice, that is, the outside measurement is taken, and allowance is usually made for one-half the openings.

EXERCISE 136.

1. A pile of ordinary bricks is 8 feet 6 inches high, 14 feet long, and 15 feet wide. What is the pile worth at $8 per thousand?

2. How many bricks are there in a wall 36 feet long, 12 feet high, and half a brick thick?

3. How many bricks are required to veneer the front of a house 18 feet wide and 25 feet high?

4. How many bricks are required for a single brick partition between two houses, 40 feet deep and 24 feet high?

5. How many bricks are required to build a house 30 feet by 24 feet, and 18 feet high, with single brick walls?

6. How many bricks are required for a double brick wall of a basement, 48 feet by 32 feet, and 10 feet high?

7. What will it cost to lay the brick of a house 40 feet by 32 feet, and 21 feet high, with a flat roof and double walls, at $2.75 per thousand?

8. Find the cost of the brick in the wall around a garden, 400 feet by 200 feet, 6 feet high, and a brick and a half thick at $7 per thousand.

9. At $8 per thousand, find the cost of the brick in the front walls of a terrace block, 120 feet long and 22 feet high. There are 6 doors each 8 feet by $3\frac{1}{2}$, and 20 windows, each 8 feet by 4 feet, the wall being a brick and a half thick.

10. How many bricks will be required for a house 40 feet by 30 feet; the basement walls are 8 feet high and 2 brick thick, one door 4 feet by 6 feet; the ground floor is 11 feet between the floors, and the walls a brick and a half thick, 2 doors and 4 windows, each 8 feet by $3\frac{1}{2}$ feet; the second floor is 10 feet high between the floors, and the the walls one brick thick, 6 windows, each 8 feet by $3\frac{1}{2}$ feet; the gables are 10 feet high and half a brick thick.

THE METRIC SYSTEM OF MEASUREMENT.

"The real beginning of exact knowledge, or science, lies in measuring, and the faithful observer of nature is always occupied in measuring."—*Kart.*
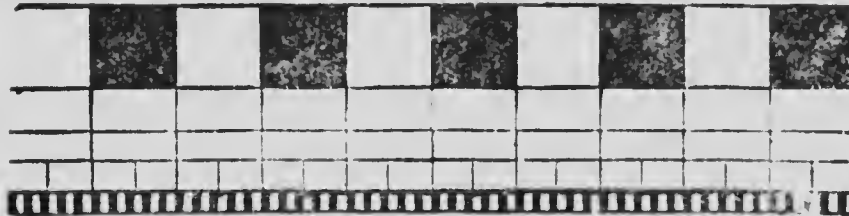
This engraving is a *Decimetre* (exact size), or the tenth part of a *METRE*. The large white-and-black squares at top show its division into 10 *Centimetres*, each of which is sub-divided into 10 *Millimetres*, as shown at bottom by the small white-and-black strips.   A complete *Metre* can be easily constructed from this illustration.

712. The Metric System (pronounced Met -ric) is a system of weights and measures expressed in the **decimal scale.**   It is now legal in nearly all civilized countries.   It was legaliz   in Canada by Act of Dominion Parliament in 1886 (chap. 101, sec. 21), and all contracts based upon it are now enforcible at law.   It was legalized in the United States in 1866, and copies of the standard metre furnished to all the States.   This system of measurement is used in all countries for scientific purposes on account of its exactness, and in many countries it is used for ordinary purposes.   Since 1840 the metric measures have been the only ones in common use in France.

713. The Standard Metre, which is the basis of the Metric System of Measurement, is a bar of platinum 39.37 inches long   This length was chosen because it was supposed to be one ten-millionth (10,000,000 or .0000001) of a quarter of the earth's circumference measured by a line passing through Paris, France, from the equator to the pole.   The

original bar, or metre, was made by Borda in 1795 at Paris, where it is carefully preserved, accurate copies being furnished to the governments of all civilized nations. Its length being nearly 3 ft. $3\frac{3}{8}$ in., **the metre** may be remembered as **the rule of the three threes.**

714. The Standards used in a general scheme of measurement are called **Units.** Thus, the Metre in France forms the foundation and starting-point of every measure in existence.

715. All the Units of measures are derived in a simple manner from the Metre. Thus:

The **Metre** is the unit of **Length.** It is a bar 39.37 inches long.

The **Ar (or Are)** is the unit of **Land Measure.** It is a square whose side is 10 metres. **1 Ar** = 119.6 sq. yds.

The **Litre** (Lee-ter) is the unit of **Capacity.** It is a cubic decimeter; that is, a cube whose edge is a decimetre long. A **Litre** = 1.76 pint.

The **Gram** is the unit of **Weight.** It is the weight of a cubic centimetre of water.

As the terms used in the Metric System are derived from the Greek, Latin and French languages, we have thought it best to give them English spellings, dropping the final "me" in "*gramme,*" etc.

716. The Metre is sub-divided always into tenths, hundredths, thousandths, &c., or decimal parts, thus:

Decimetre (dm) Latin **decem,** ten = $\frac{1}{10}$ or .1 metre (m).
Centimetre (cm)   "   **centum,** hundred = $\frac{1}{100}$ or .01 metre.
Millimetre (mm)   "   **mille,** thousand = $\frac{1}{1000}$ or .001   "

The names of these lower denominations are formed by prefixing *Latin* numerals (deci, centi, milli,) and writing the abbreviations (dm, cm, mm,) *without* Capital letters. All the compound names are accented on the first syllable thus, *mil'limetre.*

Therefore 1 metre = 10 decimetres = 100 centimetres = 1000 mm.
                1 decimetre = 10 centimetres = 100 mm.
                      1 centimetre =   10 mm.

7. **Multiples of the Metre** are as follows :

Decametre (Dm) Greek Deka, ten = 10 metres.
Hectometre (Hm) " Hekaton, hundred = 100 metres.
Kilometre (Km) " Kilioi, thousand = 1000 metres.
Myriametre (Mm) " Myria, ten thousand = 10,000 metres.
Megametre (Mgm) " Mega, million = 1,000,000 metres.

The names of these higher denominations are formed by prefixing *Greek* numerals (deka, hekto, kilo, myria, mega) and writing the abbreviations (Dm, Hm, Km, Mm, Mgm) *with* Capital letters.

718. A person who wished to buy 125 metres of cloth would not ask for " 1 hectometre, 2 decametres, 5 metres," any more than a Boston merchant would tell a person who owes him $25.96 that his bill is 2 eagles, 5 dollars, 9 dimes, 6 cents.

719. **Comparative Lengths** are as follows :

	Inches.	Feet.	Yards.
1 Metre =	39.37079	3.2808992	1.0936331
1 Decimetre =	3.93708	.3280809	.1093633
1 Centimetre =	.39371	.032809	.0109363
1 Millimetre =	.03937	.0032809	.0010936

720. The **Metre**, like the yard-stick, is used in measuring cloth and short distances ; the **Kilometre** is used in measuring long distances.

721. Since, in the Metric System, 10, 100, 1000, etc., units of a lower denomination make a unit of a higher denomination, it follows that any one of the metric measures may be expressed in terms of another measure by simply moving the decimal point to the right or left.

1. *A number is reduced to a LOWER denomination by removing the decimal points as many places to the RIGHT as there are ciphers in the multiplier.*

2. *A number is reduced to a HIGHER denomination by removing the decimal point as many places to the LEFT as there are ciphers in the divisor.*

Thus 12,465,687^{mm} may be written as **Kilo**-metres by observing that **Milli**-metres are changed to metres by mov-

ing the point **three** places to the left, and metres are changed to Kilo-metres by carrying the point **three** places further, making in all six places.

Therefore $12,465,687^{\overset{\cdot\cdot}{mm}} = 12.465687^{km}$

RULE.—First count the number of places needed to convert the given measures into terms of the principal unit; then the number needed to convert the principal into the required units.

Before adding or subtracting, the quantities must be written in the **same unit of measure.**

722.   MEASURES OF LENGTH.

10 millimetres,	marked mm.	are	1 centimetre,	marked	cm.
10 centimetres,	" cm.	"	1 decimetre,	"	dm.
10 decimetres,	" dm.	"	1 **metre,**	"	m.
10 metres,	" m.	"	1 dekametre,	"	Dm.
10 dekametres,	" Dm.	"	1 hektometre,	"	Hm.
10 hektometres,	" Hm.	"	1 Kilometre,	"	Km.
10 Kilometres,	" Km.	"	1 Myriametre,	"	Mm.

723. To Reduce 3.825 m. to cm.

SOLUTION.—To reduce metres to centimetres, multiply by 100. Write 3825, and place the decimal point between 2 and 5, two orders farther to the right than it is in 3.825. **Ans.** 382.5 cm.

724. To Reduce 1025.5 m. to Km.

SOLUTION.—To reduce metres to kilometres, divide by 1000. Write 10255, and place the decimal point between 1 and 0, three orders farther to the left than it is in 1025.5. **Ans.** 1.0255 Km.

725. To Reduce 2.15 Dm. to centimetres.

SOLUTION.—To reduce dekametres to centimetres, multiply $10 \times 100 = 1000$. Write 215 and annex a cipher. **Ans.** 2150 cm.

LAND OR SQUARE MEASURE.

726. The **Are** is the unit of Land measure (or Area). It is legal at 119.6 sq. yds. The **Are** is the principal unit of

surfaee of small plots of land. The area of a farm is expressed in **Hektars**; of a eountry in **square Kilometres**.

TABLE.

100 centiares, marked ca., are 1 **Are**, marked **a.**
100 ares " a., " 1 hektar " Ha.

727. An **Are** is 10υ square metres, marked m². The **Hektar** is nearly 2½ aeres (2.47).

728. For measuring other surfaces, squares of the metre and its sub-divisions are used.

1. Reduce 397.8 a. to hektars. A.—3.978 Ha.
2. " 3.8 a. to square metres. A.—380 m².

MEASURES OF CAPACITY.

729. The **Litre** is the unit of capaeity. It is legal at 1.0567 quarts, Liquid measure.

TABLE.

10 centilitres, marked cl., are 1 decilitre, marked dl.
10 decilitres, " dl., " 1 **litre**, " l.
10 litres, " l., " 1 dekalitre " Dl.
10 dekalitres, " Dl., " 1 hektolitre " Hl.

730. The measures commonly used are the **litre** and the **hektolitre**. The litre is very nearly a quart; it is used in measuring milk, wine, etc., in moderate quantities. The hektolitre is about 2 bu. 3½ pk.; it is used in measuring grain, fruit, roots, etc. in large quantities.

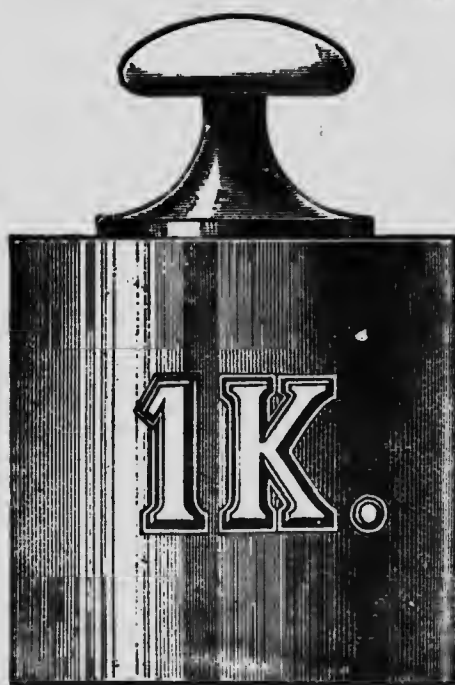**731.** For measuring wood the **Stere** is used; it is a eubic metre (= 35.316 cub. ft.)

MEASURES OF WEIGHT.

732. The **Gram** is the unit of Weight; it is legal at 15.432 grains Troy.

733.   Table.

10 milligrams,	marked	mg.,	are	1 centigram,	marked	cg.
10 centigrams,	"	cg.,	"	1 decigram	"	dg.
10 decigrams	"	dg.,	"	1 **gram**,	"	g.
10 grams,	"	g.	"	1 dekagram,	"	Dg.
10 dekagrams,	"	Dg.,	"	1 hektogram,	"	Hg.
10 hektograms,	"	Hg.,	"	1 kilogram,	"	Kg.
10 kilograms,	"	Kg.,	"	1 myriagram,	"	Mg.
10 myriagrams,	"	Mg.,	"	1 quintal,	"	Q.

10 quintals or 1000 kilograms **are** 1 Metric ton, marked M.T.

734. The weights commonly used are the **Gram, Kilogram**, and **Metric ton.** The **Gram** is used in mixing medicines, in weighing the precious metals, and in all cases where great exactness is required. The **Kilogram,** (commonly called the "Kilo"), is the usual weight for Groceries and coarse articles generally ; it is very nearly $2\frac{1}{5}$ lbs. Avoir. The **metric ton** is used for weighing hay and other heavy articles; it is about 204 lbs. more than our ton.

1 Kilogram=1000 grams, (exact size), commonly called the "Kilo."

735. Legal and Approximate Values are as follows:

Denomination.	Legal Value.	Approximate Value.
Metre.....................39.37 inches.............3 ft. $3\frac{3}{8}$ inches.
Centimetre............   .39371   "   ....$\frac{3}{8}$ inch.
Kilometre.............   .62137 mile......................$\frac{5}{8}$ mile.
Square Metre ....1.196 sq. yards...... ....$10\frac{3}{4}$ sq. feet.

Legal and Approximate Values (continued).

Denomination.	Legal Value.	Approximate Value.
Are....................119.6 sq yards.............4 sq. rods.		
Hektar...................2.471 acres.................2½ acres.		
Cubic Metre.............1.3′8 cub. yds.............35⅓ cub. feet.		
Stere.................. .2759 cord............. ....¼ cord.		
Litre...................1.0567 quarts......... { 1¹⁄₂₀ liquid quart. / 1⁶₀ dry quart.		
Hektolitre .... 2.8375 bushels.............2 bush. 3½ pk.		
Gram..................15.432 gr. Troy..............15½ grains.		
Kilogram .. ..2.2046 lb. Avoir.............2⅕ pounds.		
Metric Ton (or tonneau)..2204.6 lb. " ....1 T. 204 lbs.		
Stere....................0.2759 cord.... ....¼ cord.		

736. The legal value is used in solving the following examples.

737.   MISCELLANEOUS EXAMPLES.

1. How many yards, feet, etc., in 4 M.?

OPERATION.

SOLUTION. — In one metre there are 39.87 in.; in 4 metres there are 4 times 39.87 in., which are 157.48 in.; 157.48 in. reduced to integers cf higher denominations are 4 yds. 1 ft. 1.48 in.

```
    39.37
       4
12)157.48
3)13 ft.  1.48 in.
  4 yds.   1 ft.
```

2. What is the value of 36 lbs. in kilograms?

SOLUTION.—In one kilogram there are 2.2046 lbs.; in 36 lbs. there are as many kilograms as 2.2046 are contained times in 36, which are 16.329 +.

OPERATION.

```
2.2046)36.0000(16.329 +
       22 046
       13 9540
       13 2276
          72640
          66138
          65020
          44092
         209280
         198414
```

3. What is the value of 20 Km.?

12.4274 miles.

4. How many hektars in 160 acres?

64.75 + Ha.

5. What is the value of 49 m. ?   9 rd. 4 yd. 3.13 in.

6. How many hektolitres in 42 bu. ?   14.8 + Hl.

7. How many square yards in a roll of paper 9 m. long and 5 m. wide?   5.382 sq. yd.

8. The five-cent piece weighs 5 grams; how much will 100 such pieces weigh ?   .5 Kg.

9. Ten litres of a certain liquid weigh 92 Kg. ; what is the weight of a decilitre ?   .92 Kg.

10. One hektogram of goods costs $5.35; what costs one kilogram ? $53.50

11. A piece of money weighs 10 g.; how many such pieces in a bag weighing 1 Kg. ?   100

12 A hektolitre of wheat costs $6.25 ; what is the price of a dekalitre ? $.625

13. A hektolitre of wine costs $25.10 ; what is the price of a litre ? $.251

14. A kilogram of wool costs $1.875 ; what is the cost of 100 kilograms ? $187.50

15. A litre of wine weighs 880 g. ; what is the weight of a hektolitre?   88 Kg.

16. Add 45 kilograms, 4 hektograms, 5 dekagrams ; 35 kilograms, 8 dekagrams, 7 grams ; and 45 hektograms, 4 grams.   85.041 Kg.

17. A wine merchant sold 1270 litres, 487 litres, 1563 litres, 1000 litres, and 2345 litres ; how many hektolitres did he sell ?   66.65 Hl.

18. A vase, weighing 24.67 hektolitres, contains 18.79 hektolitres of liquid ; what is the weight of the empty vase ?   5.88 Hl.

19. From a barrel containing 147 litres of wine, 42.75 litres leaked out ; how much remained ?   104.25 l.

20. How much will 135.60 m. of cloth cost at $1.16 a metre ? $157.296

21. A grocer bought 3845 Kg. of sugar at 19 cents a kilogram; how much did it cost? $730.55

22. Bought 25 hogsheads of wine, of 225 litres each, at the rate of $.156 a litre; how much did it cost? $877.50

23. What is the cost of 21 pieces of cloth of 42 m. each, at $5.69 a metre? $5018.58

24. I have an article that sells for 26 cents a pound; how much is it worth a kilogram? $.573 +

25. A man bought 25 lbs. of tea at $1.80 a pound; he exchanged it for five times its weight in coffee, which he sold at $.86 a kilogram; did he gain or lose by the bargain, and how much? $.376 +

26. How many metres of carpeting, .75 m. wide, will cover a floor 8 m. long and 5 m. wide?   53 33 + m.

27. I paid $13 for a barrel of vinegar containing 140 l.; I lost 22 l. by leakage, and sold the remainder at 20 cents a litre; how much did I gain? $10.60

INSTITUTE OF CHARTERED ACCOUNTANTS.

ORGANIZATION.

738. This Institute, which received its charter from the Ontario. Legislature in 1883, comprises in its membership the leading Accountants of Canada. The chief aim of the Institute is to raise the standard of accountancy ; and in order to increase the knowledge, skill and proficiency of its members, it is empowered to establish classes, lectures and examinations ; to prescribe tests of competency ; to grant diplomas entitling members to use the distinguishing letters F.C.A. (Fellow of the Chartered Accountants) ; and to affiliate with any other similar bodies for mutual benefit

AFFILIATION.

739. Business Colleges and other Educational institutions having a department devoted to the study of Accounts may become affiliated with the Institute, and may conduct the Intermediate Examinations in connection therewith, on terms fixed from time to time by the Council.

740. Students-at-Accounts, of the age of 16 years or over, are admitted to registration under two classes : (1) Primary Students and (2) Intermediate Students or Book-keepers. Such Students are entitled to attend the meetings of the Institute and take part in discussion of papers. Students may form an Association for the better advancement of their studies and professional knowledge, and for making recommendations to the Council affecting their joint interests.

741. The **Primary Examination** required of students on entrance comprises Business Composition and Correspondence, Spelling and Punctuation, Arithmetic, Penmanship, Elementary Book-keeping, Common Latin Terms and Roots, British and Canadian History, Geography, Stenography (the last optional). This examination may be conducted in any affiliated institution, or the Council may waive this examination on students showing that they have passed one equivalent, or have had practical experience at accounts which may be deemed equivalent. The object of the Primary Examination is to reasonably ensure that future candidates for membership shall be men of good general education, the Council holding the view that the comparatively slow progress made hitherto, towards obtaining recognition from the public of the claims of accountancy to be considered as a profession, has been due in no small measure to the superficial character of the education deemed to be necessary to fit a man for intelligently undertaking the duties of an accountant, or even of a book-keeper (understood in the sense of one versed in one branch only of accountancy). While it may be true that every accountant will find his own level, on the ground of natural ability alone, it is equally certain that the accountant who has had the initial advantage of a good general education, supplemented by a judicious course of special training for his calling or profession, will out-distance the accountant who has not had these advantages, everything else being equal.

742. The **Intermediate Examination** is open to any one who has registered as a Student-at-Accounts, 19 years of age or over, after one year from passing the Primary or equivalent Examination. The Intermediate Examination comprises Mercantile Arithmetic, Negotiable Instruments, Book-keeping, Auditing, Shareholders' and Partners' Accounts, Insolvency. This examination may be held in affiliated institutions. Every person passing the Intermediate Examination is entitled to a Certificate to that effect, and setting forth in suitable terms his attainments as a book-keeper. The Intermediate Examinations are intended to afford to students who desire to take up accountancy as a profession, an opportunity to test their general progress in professional knowledge, to enable the Council to form an estimate of their capabilities, and to advise upon and direct, so far as may be, their course of preparation for the Final Examination, which qualifies for admission to membership as an Associate. There is the further intention to provide recognition of the attainments of those candidates who do not purpose attempting the Final Examinations, but desire to have the Certificate of the Institute of competency to undertake the duties of a book-keeper. The scope of the Intermediate Examinations, therefore, will, generally speaking, be limited to a thorough comprehension of the duties of one required to undertake the duties of chief book-keeper in a first-class business.

743. **Final Examinations.** Any person who has passed the Intermediate may apply for membership in the Institute, and if of the age of 21 or over, the Council will set a Final Examination comprising Book-keeping, Auditing, Insolvency, Joint Stock Companies, Mercantile Law, Partnerships and Executorships. This Final Examination shall be held in Toronto, and any who pass, upon being admitted to the Institute by ballot shall receive a Certificate of membership, and right to use the appellation "Chartered Accountant," and to be styled "Associate."

"F. C. A"

744. A Chartered Accountant who has been in continuous practice as such for three years after admission as a member may be admitted a "Fellow of the Chartered Accountants" upon passing the tests, viz.: (1) Known standing and reputation as a Public Accountant, and (2) a thesis upon some subject to be approved by the Council. Upon passing these tests a "Diploma of Fellowship" is issued to the candidate, giving him the right to use the letters "F.C.A"

745. Every Commercial Student should aim to secure membership in this Institute of Chartered Accountants, and to pass through the various grades above outlined till the goal is reached—the high honors and privileges of a "Fellow of the Chartered Accountants," upon whom the stamp of this honorable Institute is placed in the letters "F.C.A." In order to help our readers to reach this end, the above information is given and the following Examination Papers are quoted.

MERCANTILE ARITHMETIC.

Problems set for Candidates in Intermediate Examination, Institute of Chartered Accountants, May, 1897.

1. A nail manufacturer has 3 grades of nails which he wants to net him per keg, $2.75, $2.80, $2.85.   He desires to make a list of prices to sell at 50%, 10%, 5% discount to net the above prices. Give the list prices and show how it is worked out.

2. A Trustee invests $4,000 in Ontario Bank stock at 80, paying 6%, and $1,000 in Dominion Bank stock at 200 paying 10%.   After two years he sells the former at 86 and the latter at 180.   What rate of interest has he received during the period of investment and how has the value of the capital changed ?

3. Convert £855 5s. 10d. into currency, exchange being 9.78.

4. Convert $750 into Francs, Sterling exchange being at $9\frac{1}{4}$, $25\frac{1}{4}$ Francs representing £1 Sterling.

5. Find the equated time of paying the balance of the following account on basis of 360 days to the year.   Interest 6%.

1896				1896			
Jan. 5	oods	4/m,	$175	Feb. 9	By Cash,	$100
Jan. 25	"	2/m,	75	Mar. 2	By Cash,	50
Mar. 1	"	1/m,	125				
Mar. 14	"	Net,	50	Apr. 3	By Cash,	60
Apr. 9	"	3/m,	200	May 7	By Cash,	200
May 7	"	2/m,	100		Balance,	315
			$725				$725

May 7th, Balance, $315

Adjust the interest and state what amount is due in Cash May 7th.

6. A merchant has a line of tweeds which he is selling in 50 yd. ends, for $75 per end, a profit of 25% on cost.   His clerk, in order to make quick cash sales, sells for 15% cash discount.   What advance over cost did he net ?

7. A note of $500, dated April 1st, 1895, payable July 1st (without grace) with interest at 6% was discounted May 1st at 8%. Find the proceeds. Interest on basis of 360 days to the year.

8. A Board of School Trustees desire to issue Debentures to the amount of $2,500. Interest 5% payable annually 1st January each year, the whole amount with interest to be paid in five equal annual payments. Divide the amount into five debentures, one to mature each year.

Find the face amount of each debenture numbering them 1, 2, 3, 4, 5, and the amount of coupons due each year.

9. A merchant has 6 chests (of 30 lbs. each) of Tea at the following prices :—

         1 at 80c. per lb.
         1 " 75c.   "
         1 " 50c.   "
         1 " 60c.   "
         1 " 25c.   "
         1 " 20c.   "

He desires to make 1 chest of a blend containing all these grades to sell at $1.00 per lb. which will give him an advance over costs of 100%. Find how many pounds of each he must use.

10. If the profits are divided in proportion to the capital invested and the time it was employed, at the end of a year what would be each partner's average investment and share of the profits from the following accounts. Net profit $500.

	Jno. Roberts			Harry Jones	
Dr.	Cr.		Dr.	Cr	
Apr. 1, $2,000	1 Jan. $4,000		May 1, $300	Jan. 1, $2,000	
	1 Aug. 3,000			Sept. 1, 1,000	

Problems set for Candidates in Intermediate Examination, Institute of Chartered Accountants, Nov. 1895.

MERCANTILE ARITHMETIC.

1. A merchant buys a sort of wine at $2 per gallon, and another at $1.50 per gallon. At what price must he sell a blend of 7 parts of the former and 3 parts of the latter to realize 20 per cent. profit?

2. You manage an estate, and receive as your remuneration 5 per cent. of the net amount paid to the beneficiaries. Taxes, repairs and sundry expenses in a given year are $540. Your commissions amount to $350. Find the gross revenue of the estate.

3. Find the present value of $3,250 due 3 years and 6 months hence at 4 per cent. per annum. Show working.

4. Average the following account :

Jan. 20.—Merchandise, 30 days.........			$150 00
27.—	"	4 months.......	100 00
Feb. 15.—	"	net	150 00
			$400 00
	Cr.		
Feb. 10.—Cash. ....,................			75 00
	Balance.............		$325 00

5. A certain stock pays a semi-annual dividend of $3\frac{1}{2}$ per cent. What is it worth to an investor who wants a return of $4\frac{1}{2}$ per cent. per annum upon his investment ?

6. Convert $,1000 into sterling at ten and one-half per cent.

7. Find the cost of papering a room 30 x 22 feet, and 12 feet high, with paper 18 inches broad, costing eighty cents per roll of 12 yards, deducting 20 yards of paper for window and door spaces.

8. A merchant imports as follows :

       850 yards sheeting at 5 cents ;
      1,400 yards flannel at 13 cents.

The duty on sheeting is 20 per cent. ad val., and 5 cents per lb. (9 yards to 2 lbs.) ; the duty on flannel is 30 per cent. (4 yards to the lb.) Packages are charged at $4. Freight $6.50. Cartage $1. Find the cost per yard of each laid down in his warehouse.

9. An insolvent estate realized, after payment of expenses, $1,840.72. The claims to rank are as follows : A, $3,400.60 ; B, $1,347.85 ; C, $890.96 ; D, $870.42 ; E, $391.80 ; F, $102 ; G, $84.58. Prepare a dividend sheet showing the rate per cent. and the amount coming to each.

10. You are being charged interest monthly at 7 per cent. per annum on an overdraft at your bankers. They offer to discount your bills at three months at $6\frac{1}{2}$ per cent. per annum. Which is the more profitable transaction, and by how much ?
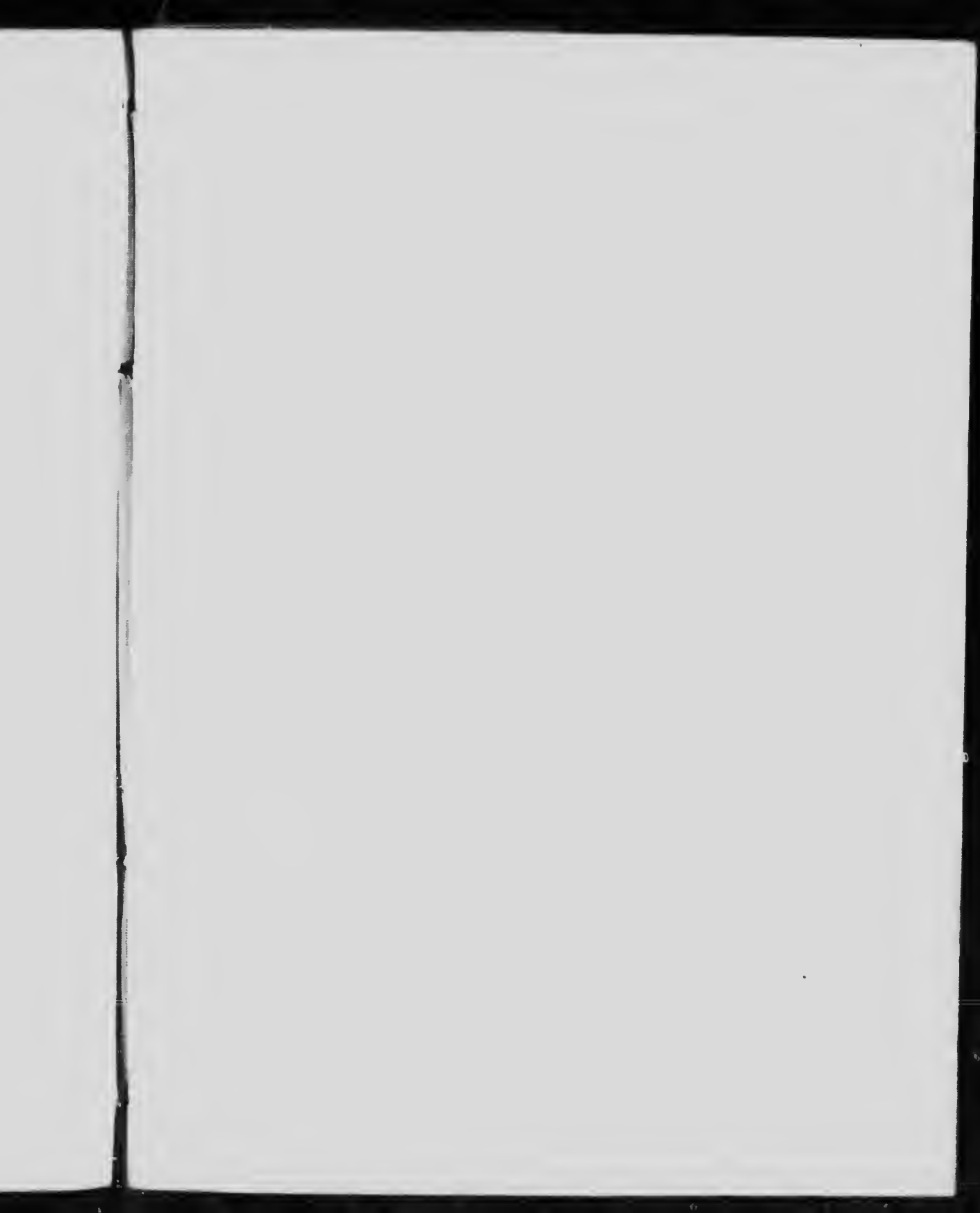
FICHE 6 NOT REQUIRED

FICHE 7 NOT REQUIRED