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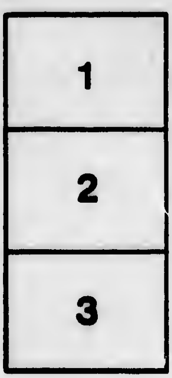
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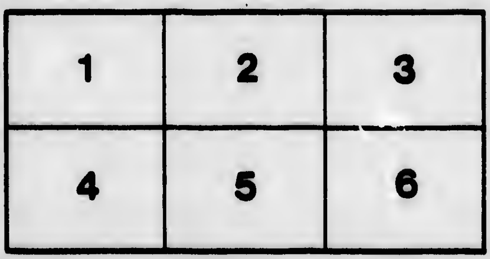
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[Haines' Arithmetic.]

THE

CANADIAN ARITHMETIC,

DESIGNED FOR

SCHOOLS AND ACADEMIES

IN

BRITISH AMERICA,

IN FOUR PARTS.

COMPILED FROM THE BEST AUTHORS,

BY A TEACHER.

—

PICTON, C. W.

PRINTED BY JOHN DOUGLASS,

1845.

PREFACE.

Among the many Arithmetics extant it is universally conceded that there is none that is in every respect adapted to the wants of Schools in this Province. There is scarcely a School in which there are not several different works on Arithmetic in use. This is very perplexing to the Teacher, as well as a positive disadvantage to the pupil. It would not only be of great advantage to the pupil, but a great saving of money to parents to have a uniformity of books in our Common Schools—for it frequently occurs, that when a pupil is removed from one School to another, a new set of books must be purchased, and the old ones laid aside as useless. Every school room should be furnished with a black board, about four by six feet in size, and all the pupils should be classed and exercised upon it daily, which cannot conveniently be done if each pupil uses a different author. Much time is wasted in many Schools by teaching individually instead of in classes.—While a class is reciting and exercising on the black board it affords an excellent opportunity for the Teacher to explain to the whole class at the same time any part that they do not understand. When a class is called up to recite, while one of them is solving the question given out by the Teacher, on the board, the rest of the class should solve the same question on their slates at the same time, and it should not be dismissed until each one of the class thoroughly understands it. When all the pupils in a School use the same author, and are classed and exercised in this way, it excites a spirit of emulation and interest, which is a powerful incentive to action. Many of the Arithmetics in use are American, and consequently are filled almost entirely with questions in Federal money, to the exclusion of the currency of this Province; others again are better calculated for those who are considerably advanced in the science, than to the capacities of the majority of pupils in our Common Schools. In many Arithmetics a mechanical method is presented of performing certain operations *according to rule*, without assigning any *reason* for such operations. Thus in Subtraction, the reason *why* one is carried, and ten borrowed; or in Multiplication, *why* the figures are placed in a certain method; or in Division, *why* Multiplication and Subtraction are performed, is never explained or illustrated. To the pupil they are a sort of cabalistical process, which he finds will *bring the right answer*, and this is all he can know from any thing he learns from the book.

Others again are too sparing of examples, especially in the simple rules; in consequence of which the pupil is hurried through them without understanding the first principles. Hence his ambition is fettered, and he can see no beauty in a science in which obscurity is behind him, and impenetrable darkness before him.

The compiler of the following work has enlarged upon and given numerous examples in the different rules, in proportion to their practical importance in the business transactions of life. At the commencement of each rule, one or more examples have been wrought at length, and the method of operating has been clearly and fully explained. No term is used until it has been defined—and the examples under the various rules are mostly of a practical nature. The arrangement of the rules is that which appeared to the compiler the most easy and natural. After Reduction and the Compound Rules, those which are of the most practical importance are first introduced. It is believed that this arrangement will be found of great advantage, especially to those who have not an opportunity to go through all the rules of arithmetic.

The intercourse and trade between this Province and the United States is so extensive, it is important that every one should understand the currency of that country—therefore a short article on Federal money has been introduced immediately after the compound rules. In the article on the subject of Proportion, the useless and perplexing distinction of *direct* and inverse proportion has been discarded, and a single rule, which is simple and general, has been adopted.

The solutions of some of the most difficult questions are intended to assist such as may study the science without the aid of a Teacher; and perhaps they may be of advantage to some teachers who have had but little experience in their profession.

To a business man, a practical and correct knowledge of Arithmetic is of the first importance; hence the public should receive with indulgence every attempt to improve this interesting and important department of instruction.

In a popular treatise on a subject which has engaged the attention of many Authors endued with talents of the highest order, much originality cannot be expected. This work, compiled from the best authors, is respectfully submitted to the inspection of the public, with the simple request, that they will examine critically and impartially before they condemn it.

Prince Edward Seminary, 7th mo. (July,) 1845.

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ADDRESS TO PUPILS.

My Young Friends,

This book is compiled expressly for your benefit, and is intended to assist you in acquiring a knowledge of one of the most useful, and if properly pursued, most interesting branches of science. Arithmetic is not pronounced a dry and difficult study by any one who pursues it understandingly; and that it may be perfectly understood by all who resolve to do so, there is no reason to doubt.

It is to be hoped that very few, if any of you, will be satisfied short of this; for it is not only a great waste of time and labor, but is also doubly perplexing to spend months, perhaps years in a dull monotonous drilling upon a subject, for want of a proper application, while the student who perseveringly removes each obstacle which he meets, will progress far more rapidly as well as pleasantly, and at the end will enjoy the gratification of having gained the object of his pursuit.

I trust, that in the following pages you will find no difficulty which may not by a little sober reflection be easily removed. An example in each case is worked out and explained, so that the most intricate may, by the application of your minds with a little assistance from a teacher, be rendered quite plain and easy, and you should make it a constant rule never to pass over any part without thoroughly understanding it. Although by this method, your progress may at first appear rather slow, your real advancement will be greatly increased.

The questions at the bottom of each page are designed to point out the portions to be committed to memory. The answers should be thoroughly learned, as you proceed in the examples. Take your books home and study these parts during the long winter evenings.

The study of mathematics, of which arithmetic is a branch, is peculiarly calculated to discipline and improve the intellectual faculties, and to fit for future usefulness and activity. This volume is intended as an introduction to that important and extensive field of science. Having gained a thorough acquaintance with arithmetic you will be prepared to advance with ease and alacrity and with invigorating powers in the path of mathematical knowledge. Be not discouraged then by apparent difficulties at the commencement. They may be, as they often have been, completely surmounted, and when once overcome they will, instead of perplexing, prove sources of amusement to your minds. A pupil engaged in this study has been aptly compared to an army marching through the country of an enemy. If any part is left unconquered, there will be difficulties both before and behind with but a poor prospect of success. I have read an anecdote of a lad who boasted that he had been through the arithmetic and could perform any sum in it. Some person gave him the following question: "How much will sixty pounds of beef amount to at three pence per pound, provided it is three fourths fat?" After pondering awhile, he gave it up in dispair saying, if it were not for the *fat* he thought he could do it. So much for going through the arithmetic without thinking.

You must not be afraid of *thinking*. It is the very thing that will strengthen and improve your minds, while at the same time it will make you the conquerors of the field before you. The only thing wanting with most young persons, is a *determination* to obtain knowledge. There is time enough wasted by nearly every one between the ages of fifteen and twenty, to acquire a good English education. Many of the most eminent men, whose names are justly honored, have attained the elevated stations which they occupy in society and in the world, by their own unaided and untiring exertions. Close application, joined with unconquerable perseverance, will overcome all obstacles; one after another will give way, until eventually, you will find yourselves standing upon the hill of science.

May you then my young friends, deeply possess your minds with an esteem for that which is truly worthy of your time and attention. Seek enjoyment in the study of truth in all its branches, in the undeviating practice of virtue, and in promoting the happiness of all around you. Virtue and intelligence will make you honored, and true piety will render you happy in every situation in life, and that you may become *deservedly* honored and *truly* happy, is the desire of

Your friend,

THE AUTHOR.

ARITHMETIC.

Arithmetic is the science of numbers. It contains the following five principal rules of operation, viz: Numeration, Addition, Subtraction, Multiplication and Division.

NUMERATION AND NOTATION.

Numbers are expressed by certain characters called figures. There are ten of these characters, viz: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0—the last of which is called a *cipher*, or naught. The nine others are called significant figures or digits.* They are also called Arabic characters, because they were first introduced into Europe by the Arabs.

A unit is a whole thing of any kind. Thus, if the number be eight feet, one foot is the unit; if it be four pounds, one pound is the unit, &c.

If the figure 1 stands alone, it represents one unit; figure 2, two units; figure 3, three units, and so on.

* From the Latin word digitus, a finger—their number being equal to that of the fingers on both our hands.

QUESTIONS.—What is Arithmetic? How many principal rules does it contain? What are they? How are numbers expressed? How many of these characters are there? What is the last of these characters called? What are the nine others called? From what is the word digit derived, and why are these characters so called? Why are they called Arabic characters? What is a unit? Give examples. What does the figure 1 represent when standing alone? Figure 2? Figure 3? 4? 5? &c.

If we wish to express a higher number than nine, we must *combine* these characters. For instance, if we wish to express the number ten, we must write a cipher on the right hand of 1, thus, 10; if one hundred, we must write two ciphers, thus, 100; if one thousand, three ciphers, &c. So we see that figures have a different value, depending upon the place they occupy. Take for example the number 100; when the figure 1 stands alone it represents but one unit—but remove it one place towards the left by putting a cipher on the right hand, and it becomes ten times as much, or ten units; remove it two places and it is again increased ten times, and represents one hundred. Hence it appears that the removal of a figure one place towards the left increases its value *ten times.*

1 is called a unit of the first order; 10 is called a unit of the second order, or order of tens; 100 is called a unit of the third order, or order of hundreds, &c. Thus we perceive that ten units of the first order make one unit of the second order, and ten units of the second make one of the third order, and ten units of the third order make one unit of the fourth order, or 1000.

Writing numbers by figures is called Notation. Numeration is the reading of numbers set down in figures.

QUESTIONS.—How can we express a higher number than nine? Give examples. What does the figure 1 represent when a cipher is placed on the right hand? Two ciphers? Three ciphers? How is the value of a figure affected by removing it one place towards the left hand? What is a unit of the first order? Second order? Third order? How many units of the first order make one of the second? How many of the second order make one of the third? How many of the third order make one of the fourth? What is Notation? What is Numeration?

NUMERATION TABLE.*

Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
		.			.			.			.			.			4
		.			.			.			.			.		2	5
		.			.			.			.			.	3	4	1
		.			.			.			.			5	7	0	1
		.			.			.			.		6	4	3	4	5
		.			.			.			.	1	3	2	1	9	8
		.			.			.			7	6	2	8	5	4	1
		.			.			.		4	3	1	2	3	4	7	9
		.			.			.	3	5	4	6	7	1	1	0	1
		.			.			1	4	3	7	6	7	0	5	4	3
		.			.		2	1	1	4	3	0	1	6	2	5	0
		.			.	4	3	4	5	6	7	8	4	0	1	0	0
		.			3	1	2	3	4	5	0	1	3	7	8	1	9
		.		2	1	5	7	0	1	0	1	2	0	1	0	1	0
		.	4	5	3	7	1	1	2	1	8	9	1	6	7	1	3
		5	1	2	1	3	4	1	6	7	9	1	2	4	8	9	1
	1	2	4	1	2	7	3	1	4	5	4	3	4	5	1	6	7
9	4	1	2	3	4	3	5	4	1	0	1	2	1	4	7	8	9

The words at the head of the above table, *units*, *tens*, *hundreds*, *&c.* are applicable to all numbers, and must be committed to memory by the pupil.

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* This table is formed according to the French method of numeration. The English method gives six places to thousands, &c.

QUESTIONS.—Repeat the numeration table. How may the reading of figures be facilitated?

In order to facilitate the reading of figures they are often separated into periods of three figures each, counting from the right hand towards the left. The first period is called the period of units —the second, thousands—the third, millions—the fourth, billions—the fifth, trillions—the sixth, quadrillions, &c.

Quad.	Tril.	Bil.	Mil.	Thous.	Units.
457,	341,	623,	589,	891,	213.

EXERCISES IN NOTATION.

Write four in figures. Write twenty one. Write seventy five. Write one hundred and one. Write six hundred and seventy nine. Write four thousand and twenty eight. Write nine thousand nine hundred & nine. Write ten thousand. Write one hundred thousand. Write one million. Write one hundred million. Write four billions.

Write two hundred and eighty five thousand seven hundred and nineteen.

Seven hundred and thirty six thousand one hundred and fifty six.

Seven million one hundred and sixty one thousand nine hundred and six.

Three million seven hundred thousand six hundred and seventy four.

Twenty seven million fifty four thousand three hundred and ninety six.

One hundred and eighty two million three hundred and seventy five thousand nine hundred and nine.

QUESTIONS.—What is the first period called? What is the second called? The third? The fourth? The fifth? The sixth?

ROMAN NUMERATION.

Before the introduction of the Arabic figures, a method of expressing numbers by Roman letters was employed. As this method is still in use, it is important that it should be learned. The letter I stands for one; V, for five; X, for ten; L, fifty; C, C, one hundred; D, five hundred; M, one thousand.

As often as a letter is repeated its *value* is repeated. When a *less* number is put before a greater, the less number is subtracted from the *greater*. But when the *less* number is put after the *greater*, *it is added.*

EXAMPLES.

In IV, the less number, I, is put before the greater number V, and is to be *subtracted*, making the number *four*.

In VI, the less number is put *after* the greater, and it is to be *added*, making the number *six*.

In XL, the ten is to be *subtracted* from fifty.

In LX, the ten is to be *added* to fifty.

ROMAN TABLE.

I - -	One	LX	Sixty
II - -	Two	LXX	Seventy
III - -	Three	LXXX	Eighty
IV - -	Four	XC	Ninety
V - -	Five	C	One hundred
VI - -	Six	CC	Two hundred
VII - -	Seven	CCC	Three hundred
VIII - -	Eight	CCCC	Four hundred
IX - -	Nine	D	Five hundred
X - -	Ten	DC	Six hundred
XX - -	Twenty	DCC	Seven hundred
XXX -	Thirty	DCCC	Eight hundred
XL - -	Forty	DCCCC	Nine hundred
L - -	Fifty	M	One thousand

A line drawn over a number increases it a thousand times, thus, $\overline{X}$ expresses ten thousand, and $\overline{XX}$ twenty thousand.

SIMPLE ADDITION.

If four apples are added to 5 apples, how many in all? Every one will answer, 9.

Here a single apple is the unit; and the number 9 contains as many units as the two numbers 4 and 5; and the operation by which this result is obtained is called addition. Hence, *addition is uniting several numbers in one.* The number which is obtained by uniting several numbers into one is called the *sum*, or *sum total.* In the above example, 9 is the sum of 4 and 5 added together.

THE SIGNS.

One straight line crossing another at right angles, thus $+$ is called plus, which signifies more. When placed between two numbers, it denotes that they are to be added together, thus $2+4+3$ denotes that 2, 4 and 3 are to be added together.

Two short parallel lines are called the sign of equality. Thus $2+3+4=9$ and $4+5+6=15$. When placed between two numbers it denotes that they are equal to each other.

NOTE.—Before adding large numbers the pupils should be able to add small numbers mentally, and not by counting their fingers, or something else, as many do. By being *thorough* at the commencement much time and labor will be saved.

The pupils should now be classed, and combinations like the following should be propounded until the nature of addition is well understood.

QUESTIONS.—What is addition? What is the number called which is obtained by uniting several numbers into one? What is the sum of 4 and 5? What is the sign of addition? What is it called? What does it signify? When placed between two numbers, what does it denote? Give an example. What is the sign of equality? When placed between two numbers what does it show?

1 and 0 are how many?
2 and 1———————?
3 and 2———————?
5 and 3———————?
8 and 4———————?
5 and 4———————?
6 and 3———————?
7 and 5———————?
8 and 6———————?
9 and 5———————?
10 and 6———————?
11 and 5———————?
12 and 6———————?

14 and 3 are how many?
15 and 4———————?
16 and 5———————?
17 and 6———————?
18 and 7———————?
19 and 8———————?
20 and 9———————?
9 and 4———————?
8 and 7———————?
7 and 9———————?
6 and 7———————?
5 and 8———————?
4 and 9———————?

4+9=how many?
8+7+2=how many?
5+4+3=how many?
6+5+4=how many?
2+0+4+6=how many?
9+3+2+1=how many?
10+4+6+2=how many?
11+3+1+0=how many?
12+5+6+4=how many?
13+6+5+4+2=how many?
14+7+6+5+4+1=how many?

RULE FOR ADDITION.

Write the numbers to be added—units under units, tens under tens, hundreds under hundreds, &c., and draw a line underneath.

Add each column separately, beginning with the right hand column. When the sum of any column is not more than nine, write it down under the column; but when it is more than nine, write only the right hand or unit figure under the column,

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QUESTIONS.—How do we write down the numbers for addition? Where do we begin to add? When the sum of any column is not more than 9 what do we do? When it exceeds 9 what do we do? What do we set down at the last column?

and carry the left hand figure or tens to the next column. Add each column in the same way and set down the entire sum of the last column.

EXAMPLES.

1. What is the sum of 4638 and 216, and 8329 and 1212.

	thou.	hun.	tens.	units.
	4	6	3	8
		2	1	6
	8	3	2	9
	1	2	1	2
Sum	1 4	3	9	5

In this example we first write the numbers under each other, units under units, tens under tens, &c. as directed in the rule. We then commence at the right hand column and add it up, and find the sum of the units to be 25, or 2 tens and 5 units, we write only the right hand figure or 5 units, and add the left hand figure or 2 tens to the line of tens.

In adding the tens we find the sum to be 9, we therefore write it down. Now there is none to carry.

In adding the hundreds the sum is 13. If we should write down 13, the 3 would stand under the column of hundreds, and the 1 under the column of thousands; therefore we write the 3 only, and add the one in with the thousands.

In adding up the last column we find it amounts to 14; we now set down the entire sum according to the rule.

```
(2) 22345
    -----
    67890
     8752
      340
      350
       78
    -----
Sum, 99755
    -----
    77410
    -----
Proof, 99755
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PROOF OF ADDITION.

Draw a line under the upper number—add the lower numbers together, and then add their sum to the upper. If the last sum is the same as the sum total first found, the work may be regarded as right.

QUESTION.—How do we prove addition?

(3)	(4)	(5)	(6)
22321	23432	110331	143450
41332	42212	224212	467089
12123	13124	103123	356748
13220	21101	220320	910310
88996	99869	657986	1877597

(7)	(8)	(9)	(10)
140670	23456	456780	541012
596704	54321	134108	134167
860342	12345	120212	34160
104539	67890	967342	5603
210110	30102	710011	414
121401	87549	81216	21

(11)	(12)	(13)	(14)
234561	1345601	5430161	1
123003	3413215	241678	54
456784	1014494	34124	671
341612	3742121	9671	3416
172310	34167	540	12467
416789	841	31	91511
432111	21	9	87894

PRACTICAL EXERCISES.

1. Find the sum of 4+8+6+10+12+16+18+20+81+13. Ans.

2. What is the sum of 100+101+141+106+91+ +10+4. Ans.

3. Five boys commenced playing marbles—Willet had 18, Levi 21, John 11, William 10, and Thomas 9; how many marbles had they all?

4. Charles bought apples as follows: at one time 5221, at another 7540; then 1368, then 5648, then 7300: how many did he buy in all? Ans. 27077.

5. A man had 241 sheep in one field, 104 in another, 91 in another, and 164 in another. How many in all?

6. The population of Montreal is 45,000, of Quebec 300,000, of Toronto 20,000, of Kingston 8,000; what is the entire population of the above named cities?

Ans. 103000.

7. From the creation of the world to the Deluge was 1656 years; thence to the building of Solomon's temple 1344 years; thence to the birth of Christ 1004 years. How old is the world the present year?

8. A drover paid 300 dollars for 200 sheep, 525 dollars for 250 sheep, and 1000 dollars for 504 sheep; how many did he buy, and what did the whole cost?

Ans. 954 sheep, and they cost 1825 dollars.

9. St. Paul's Cathedral in London cost 800,000 pounds sterling, the Royal Exchange 80,000 pounds, the Mansion House 40,000 pounds, Blackfriars bridge 152,840 pounds, Westminster bridge 389,000 pounds, and the Monument 13,000 pounds; what is the amount of these sums?

Ans. 1,474,840 pounds.

10. The British dominions are estimated in square miles as follows: England and Wales 57,812, Scotland 32,167, Ireland 31,874, Islands in the British Seas 332, Colonies and Dependencies in Europe [illegible]4, in Asia 1,204,664, in Africa 200,723, in North America 754,577, in the West Indies 77,552, in South America 52,400, and in Australasia 500,000; what is the whole amount?

Ans. 2,912,225.

11. The population of the British Empire and Colonies is as follows: England 14,995,000, Wales 911,000, Scotland 2,628,000, Ireland 8,466,000, North America 1,580,000, West Indies and South America 845,000, Africa 300,000, Asiatic 124,541,000; Ceylon, Chin-India &c. 1,400-000, Oceanica 575,000; query, the entire population?

Ans. 156,241,000.

12. The population of London is 1,500,000, Manchester 182,000, Liverpool 165,000, Birmingham 146,000, Leeds 123,000, Bristol 117,000, Plymouth 75,000, Nor-

wich 61,000, Sheffield 59,000, Hull 54,000, Nottingham 51,000, Portsmouth 50,000, Cambridge 21,000, Oxford 21,000, and York 25,000; how many inhabitants in all? Ans. 2,650,000.

13. According to the census of 1842, the population of Canada West was as follows: Eastern District 27618, Ottawa Dist. 7386, Johnstown Dist. 31839, Bathurst Dist. 21086, Dalhousie Dist. 15681, Prince Edward Dist. 14396, Midland Dist. 34438, Victoria Dist. 5214, Newcastle Dist. 30425, Colborne Dist. 13265, Home and Simcoe Dists. 83294, Niagara Dist. 34348, Gore Dist. 44232, Wellington Dist. 11418, Brock Dist. 17315, Talbot Dist. 10193, London Dist. 29657, Huron Dist. 6515, Western Dist. 21498. What was the entire population of Canada West at that time? Ans. 459,818.

SIMPLE SUBTRACTION.

1. If 4 apples are taken from 6 apples, how many will remain?

2. If 3 pence are taken from 8 pence, what will remain?

In the first example two apples will remain, and the two is called the difference between 4 apples and 6 apples. Hence *subtraction is taking a less number from a greater.* The greater of the two numbers is called the *minuend*, and the less the *subtrahend*, and the difference is called the *remainder.*

A short horizontal line (—), is the sign of subtraction; it is called minus, which is a Latin word, signifying less; it shows that the number after it is to be taken from the one before it. Thus, 9—4=5, and is read 9 minus 4 is equal to 5, or 9 less 4 equals 5.

Questions.—What is subtraction? What is the greater number called? What is the less number called? What is the difference called? What is the sign of subtraction? What is it called? What does the term signify? What does it show?

Subtraction is the reverse of addition, and may be proved by it, as a few examples will show.

3. A man bought 75 sheep, and sold 32 of them; how many had he left? 75—32=how many? Ans. 43.

4. A man sold 32 sheep and had 43 left; how many had he at first? 32+43=how many? Ans. 75.

48—24=24, 24+24=48, 9—4=5, 5+4=9.

NOTE.—The class should now be required to answer questions like the following, orally: thus 2 from 6 leaves how many?

1—0=how many?	7—5=how many?
5—2=how many?	9—6=how many?
8—4=how many?	13—9=how many?
9—3=how many?	11—5=how many?
9—5=how many?	15—7=how many?
7—4=how many?	17—8=how many?
10—4=how many?	19—10=how many?
12—7=how many?	19—3=how many?
14—8=how many?	18—7=how many?
16—10=how many?	21—10=how many?
18—12=how many?	22—12=how many?
20—13=how many?	25—13=how many?

RULE FOR SUBTRACTION.

Write down the numbers, the less under the greater, placing units under units, tens under tens, and draw a line beneath. Begin at the right hand or unit figure, and subtract each figure in the lower line from the one above it, and write the remainder directly below.

When the number in the upper line is smaller than the one under it, suppose 10 to be added to the upper figure, but in this case we must add 1 to the lower figure in the next column before subtracting. This is called borrowing 10.

QUESTIONS.—Of what is subtraction the reverse? How do we set down the numbers for subtraction? Where do we begin to subtract? How do we subtract? What do we do with the remainder? When the number in the upper line is smaller than the one below it what do we do? What do we add to the next lower figure? What is this called?

EXAMPLES.

	(1)	(2)
From	4567	3436 subtrahend.
Subtract	1314	2187 minuend.
	3243	1249 remainder.

In the first example, we begin at the right hand and subtract each figure in the lower line from the one above it, thus, 4 from 7 and 3 remains, &c. In the second example we meet with a difficulty, for we cannot take 7 units from 6, we obviate this difficulty by adding 10 to the minuend or 6, which makes 16, 7 from 16 and 9 remains. As 10 units have been added to the minuend, the same amount must be added to the subtrahend. The pupil will recollect that 10 units in the first place make one in the second place, we therefore add 1 to the 8 tens, making it 9 tens—we cannot subtract 9 tens from 3 tens, therefore we again add 10 to the minuend, which makes 13, 9 from 13 leaves 4, and so on through all the orders.

The above reasoning is predicated upon this principle, that if an equal amount be added to the minuend and the subtrahend, the remainder is unaltered.

EXAMPLE.

9—4=5. Now if we add 10 to 9 and also to 4 we will have 19—14=5 as before.

(3)

From	423652
Take	132941
Rem.	290711
Proof,	423652

PROOF.

Add the remainder and subtrahend together; if the work is right, the sum will be equal to the minuend.

Addition may be proved by subtracting continually from the amount the several numbers which were added to pro-

QUESTIONS.—If an equal amount be added to the minuend and the subtrahend, is the remainder affected? How do we prove subtraction? How may addition be proved by subtraction?

duce it, and if the work is right there will be no remainder.

ILLUSTRATION.—5, 4, 7=16: proof, 16—7=9, and 9—4=5, and 5—5=0.

	(4)		(5)
From	3621531	From	91041234
Take	1841675	Take	70134165
	1779856		20907069

(6)	(7)	(8)
345701341	12301014	841397
141870134	9109417	198745

(9)	(10)	(11)
7456789	1043456	10101010
946340	141897	1010101

12. From 41078912 take 19416781.
13. From 72416714 take 13741010.
14. From 91012412 take 9178743.

SIMPLE MULTIPLICATION.

1. A boy gives 8 apples to each of 3 companions; how many does he give to them all?

2. If 1 bushel of apples cost 9 pence, how many pence must I pay for 4 bushels?

3. If one orange costs 3 pence, what will 4 oranges cost?

The answers to the above questions may be obtained by addition; but the operation may be much facilitated by a rule called Multiplication.

In the first example the number 8 apples is repeated 3 times, we may therefore add 8 three times to itself, thus 8, 8, 8=24; or we may say 3 times 8=24, and so of the second and third examples.

Multiplication is a short method of repeating one number as many times as there are units in another.

The number to be repeated is called the *multiplicand.*

The number which shows how many times the multiplicand is to be repeated is called the *multiplier.*

The answer is called the *product,* because it is the sum produced by multiplication.

The multiplier and multiplicand taken together are called *factors.*

SIGN.—Two short lines crossing each other in the form of the letter X are the sign of multiplication; thus, $4\times2=8$, which means that two times 4 are equal to 8, or 4 times 2 are 8.

NOTE TO PUPILS.—I hope you will so far consult your own interest, as to commit the following table perfectly to memory before you attempt to proceed farther. No progress can be made without it. If you apply yourselves perseveringly to the task, you can soon accomplish it; and if you should proceed no farther, you will find it of great advantage to you through life.

QUESTIONS.—What is multiplication? What is the number called which is to be repeated? What is the multiplier? What is the answer called? Why? What are the multiplier and multiplicand taken together, called? In the first example, which is the multiplicand? The multiplier? The product?—What is the sign of Multiplication?

Multiplication Table.

2 times	0 are	0	4 times	0 are	0	6 times	0 are	0	8 times	0 are	0	10 times	0 are	0	12 times	0 are	0
2	1	2	4	1	4	6	1	6	8	1	8	10	1	10	12	1	12
2	2	4	4	2	8	6	2	12	8	2	16	10	2	20	12	2	24
2	3	6	4	3	12	6	3	18	8	3	24	10	3	30	12	3	36
2	4	8	4	4	16	6	4	24	8	4	32	10	4	40	12	4	48
2	5	10	4	5	20	6	5	30	8	5	40	10	5	50	12	5	60
2	6	12	4	6	24	6	6	36	8	6	48	10	6	60	12	6	72
2	7	14	4	7	28	6	7	42	8	7	56	10	7	70	12	7	84
2	8	16	4	8	32	6	8	48	8	8	64	10	8	80	12	8	96
2	9	18	4	9	36	6	9	54	8	9	72	10	9	90	12	9	108
2	10	20	4	10	40	6	10	60	8	10	80	10	10	100	12	10	120
2	11	22	4	11	44	6	11	66	8	11	88	10	11	110	12	11	132
2	12	24	4	12	48	6	12	72	8	12	96	10	12	120	12	12	144

3	0	0	5	0	0	7	0	0	9	0	0	11	0	0
3	1	3	5	1	5	7	1	7	9	1	9	11	1	11
3	2	6	5	2	10	7	2	14	9	2	18	11	2	22
3	3	9	5	3	15	7	3	21	9	3	27	11	3	33
3	4	12	5	4	20	7	4	28	9	4	36	11	4	44
3	5	15	5	5	25	7	5	35	9	5	45	11	5	55
3	6	18	5	6	30	7	6	42	9	6	54	11	6	66
3	7	21	5	7	35	7	7	49	9	7	63	11	7	77
3	8	24	5	8	40	7	8	56	9	8	72	11	8	88
3	9	27	5	9	45	7	9	63	9	9	81	11	9	99
3	10	30	5	10	50	7	10	70	9	10	90	11	10	110
3	11	33	5	11	55	7	11	77	9	11	99	11	11	121
3	12	36	5	12	60	7	12	84	9	12	108	11	12	132

RULE FOR MULTIPLICATION.

1. Set down the multiplier under the multiplicand, so that units shall fall under units, tens under tens, hundreds under hundreds, &c. and draw a line underneath.

When the multiplier does *not* exceed 12, begin at the right hand of the multiplicand, and multiply each figure contained in it by the multiplier, setting down and carrying as in addition.

2. When the multiplier exceeds 12, multiply by each figure of the multiplier separately; first by the units, then by the tens, then by the hundreds, &c., being careful always to place the first figure of each product directly under the figure by which you multiply.

Add up the several products, and their sum will be the product sought.

EXAMPLES.

1. Multiply 145 by 3; that is, find 3 times 145.

The answer to this example might be obtained by adding 145 together 3 times; thus, 145+145+145=435, or more readily by multiplying 145 by 3.

OPERATION.

Multiplicand	145
Multiplier	3
	——
Product	435

We first write the multiplier under the multiplicand, and then say 3 times 5 are 15, or 5 units and 1 ten—we write down the 5 units only, and reserve the ten to be added in the ten's place; we then say 3 times 4 are 12, and 1 more makes 13, that is 3 tens and one hundred. We now write down

QUESTIONS.—How do we set down numbers for multiplying? Where do we begin to multiply? How do we multiply? When the multiplier exceeds 12 how do we proceed? How do we place the products? What do we do with the several products? What is their sum?

the 3 tens in the tens place, and reserve the hundreds for the hundreds place, and then say 3 times 1 are 3 and 1 are 4 &c. which occupies the place of hundreds; so we find that 3 times 145 are 435, which is 145, 3 times repeated.

It is plain from the above example that *Multiplication is a short method of addition*, and that any product may be found by setting down the multiplicand as many times as there are units in the multiplier, and adding them together.

2. Multiply 365 by 246.

OPERATION.

Multiplicand	365
Multiplier	246
	2190
	1460
	730
Product	89790

In this example the multiplier consists of 3 figures; we first write it under the multiplicand, units under units, &c., as the rule directs—we then multiply the multiplicand by the 6 units, then by the 4 tens, then by the 2 hundreds, and place the product directly under the figure we multiply by; lastly we add the several products together, agreeably to the directions of the rule.

In multiplying by 6 units the product is	2190
In multiplying by 4 tens, or 40, the product is	14600
In multiplying by 2 hundreds, or 200, the product is	73000
Consequently the sum of these products must be the entire product of 365 multiplied by 246.	89790

Either of the factors may be used as the multiplier without altering the product. For example, 4×5 is the same as 5×4, and $8\times6=6\times8=48$.

PROOF.

Write the multiplicand in the place of the multiplier, and find the product as before: if the two products are alike, the work may be regarded as right.

QUESTIONS.—What may multiplication be considered? How may any product be found? May either of the factors be used as the multiplier without altering the product? Give examples. How do we prove multiplication?

(3)	(4)	(5)
4372543	427365	5729385
2	3	4
8745086		

(6)	(7)	(8)
4567145	3541678	13456789
5	6	7

(9)	(10)	(11)	(12)
3451248	2314521	314156	1213451
8	9	10	12

13.	Multiply	480	by	36	Product	17280
14.	"	1324	by	45	"	59580
15.	"	3648	by	72	"	265248
16.	"	3725436	by	43	"	160193748
17.	"	12765235	by	275	"	3510439625
18.	"	537467	by	367	"	197250389
19.	"	673526	by	2674	"	1801008524
20.	"	346726	by	3426	"	1187883276

CONTRACTIONS IN MULTIPLICATION.

I. When the number is 1, and any number of ciphers after it, as 10, 100, 1000, &c.

We have already learned that a cipher placed on the right of a number, changes the units place into tens, the tens into hundreds, &c. Hence,

When the multiplier is 10, 100, or 1 with any number of ciphers annexed, place as many ciphers to the multiplicand as there are ciphers in the multiplier, and the multiplicand so increased will be the product required.

Questions.—How is a number affected by placing a cipher on the right of it? When the multiplier is 10, 100, 1000, &c. how do we proceed?

EXAMPLES.

1. Multiply 341 by 10 Ans. 3410.
2. " 475 by 100. Ans. 47500.
3. " 3159 by 1000. Ans. 3159000.
4. " 2346 by 10000. Ans. 23460000.

II. When there are ciphers on the right hand of one or both of the factors.

RULE.

Neglect the ciphers and multiply by the significant figures only; then place as many ciphers to the right hand of the product, as there are in both of the factors.

EXAMPLES.

(1)	(2)	(3)
365	4567	46000
40	300	340
Ans. 14600	Ans. 1370100	184
		138
		Ans. 15640000

4. 76400×24 Ans. 1833600.
5. 7532000×580 Ans. 4368560000.
6. 21200×70 Ans. 1484000.
7. 4871000×270000 Ans. 1315170000000.
8. 7496430×695000 Ans. 5210018850000.

III. *When there are ciphers standing between significant figures of the multiplier they may be disregarded.*

EXAMPLE.

1. What is the product of 12318 multiplied by 7004?

QUESTIONS.—When there are ciphers on the right hand of one or both the factors, what is the method of proceeding? How many ciphers should be placed at the right hand of the product? When there are ciphers standing between significant figures of the multiplier how should we treat them.

$$\text{Operation.}\left\{\begin{array}{r} 12318 \\ 7004 \\ \hline 49272 \\ 86226 \\ \hline 86275272 \end{array}\right.$$

2. What is the product of 506 multiplied by 302 ?
Ans. 152812.

3. Multiply 154326 by 3007. Ans. 464058282.

IV. When the multiplier is a composite number.

A composite number is the product of two or more numbers, which are called the *components* or factors. Thus, $4\times3=12$. Here 12 is a composite number, and 3 and 4 are the factors, $8\times5=40$; 40 is also a composite number.

RULE.

When the multiplier is a composite number, multiply by each of the factors in succession, and the last product will be the entire product sought.

EXAMPLES.

1. Multiply 365 by 16.

The factors of 16 are 4 and 4, or 2 and 8, or they are 2 and 2 and 4; for $4\times4=16$ and $2\times8=16$; also, $2\times2\times4=16$.

	365	365	365
	4	8	2
	1460	2920	730
	4	2	2
Product	5840	5840	1460
			4
		Product	5840

QUESTIONS.—What is a composite number? Give an example. What are the factors? When the multiplier is a composite number how do we multiply? What will the last product be?

2. Multiply 5709 by $6\times8=48$ Ans. 274032.
3. Multiply 2573624 by $5\times3=15$ Ans. 38604360.
4. Multiply 8423675 by 64 Ans. 539115200.
5. Multiply 5246789 by 81 Ans. 424989909.
6. Multiply 4103413 by 100 Ans. 410341300.

PRACTICAL EXERCISES.

1. What number is that, the factors of which are 9, 5, 4 and 2?

$9\times5\times4\times2$ are how many?

2. A certain orchard consists of 126 rows, 109 trees in a row, and suppose it to have 1007 apples on each tree, how many trees and apples does the orchard contain?
Ans. 13734 trees, 13830138 apples.

3. There are 24 hours in a day, and seven days in a week, how many hours in a week. Ans. 168 hours.

4. Suppose a man were to travel 32 miles a day, how far would he travel in 365 days? Ans. 11680.

5. If 46 men can do a piece of work in 60 days, how many men will it take to do it in one day?
Ans. 2760 men.

6. What is that number, of which 9, 12, 14, and 21 are factors?
Ans. 31752.

7. There are 320 rods in a mile; how many rods in 340 miles?
Ans. 172800 rods.

8. What will 194 chests of tea cost, at 75 dollars a chest?
Ans. 14550.

9. A man sold a farm containing 495 acres, for 19 dollars an acre; what did the whole come to?
Ans. 9405 dollars.

10. Suppose a book to contain 470 pages, 45 lines on each page, and 50 letters in each line, how many letters in the book?
Ans. 1057500.

11. The earth is computed to move at the rate of 58000 miles per hour; how far does it move in a year consisting of 8766 hours?
Ans. 508428000 miles.

12. Two men depart from the same place and travel in opposite directions—one at the rate of 31 miles a day,

the other 45 miles a day; how far will they be apart at the end of 12 days? Ans. 912 miles.

13. The component parts or factors of a certain number are 4, 6, 8, 9, 12, and 14; what is the number?
Ans. 290304.

14. A dollar contains 5 shillings; how many shillings in 299 dollars? Ans. 1495 shillings.

15. One pound contains 4 dollars; how many dollars in 1540 pounds? Ans. 6160 dollars.

SIMPLE DIVISION.

John has 25 apples, and wishes to divide them equally among 8 boys.

Operation.	
	25
	8
1st remainder	17
	8
2nd remainder	9
	8
3rd remainder	1

In this example, giving each boy 1 would take 8 and leave 17; giving each 1 a second time would take 8 and leave 9, and giving each 1 would again take 8 and leave 1. 8 has been subtracted from 25 3 times; hence 8, is contained 3 times in 25 and there is 1 over.

By continued subtraction we can always find how many times one number is contained in another, and likewise what remains when it is not contained an exact number of times.

We can however arrive at the same result by a much shorter and more expeditious method called Division.

QUESTIONS.—When John divides 25 apples equally among 8 boys, how many does he give to each? How many times does 25 contain 8? How many remain? By continued subtraction what can we find? By what other method may we arrive at the same result?

SIMPLE DIVISION.

Division teaches the manner of finding how many times a less number is contained in a greater. It is also a *short* method of performing many subtractions of the same number.

The number to be divided is called the *Dividend.*

The number by which we divide is called the *Divisor.*

The number expressing how many times the dividend contains the divisor is called the *Quotient* or *answer.*

SIGNS.

There are three signs used to denote division. 1st. A short horizontal line between two dots, thus ÷ it shows that the number *preceding* it is to be divided by the one *following* it, thus 24÷4=6. 2nd. The dividend is written over the line, and the divisor under it, thus $\frac{24}{4}$=6. 3rd. Two curved lines drawn to the right and left of the dividend, with the divisor at the left, thus 4)24(.

NOTE.—This rule involves subtraction and multiplication, and to beginners is somewhat difficult; they should therefore be exercised in the division of small numbers mentally, until the process becomes familiar and easy.

2 in 2 how many times?	
2 in 4 " " ?	3 in 6 how many times?
2 in 6 " " ?	3 in 9 " " ?
2 in 8 " " ?	4 in 12 " " ?
2 in 10 " " ?	4 in 16 " " ?
2 in 12 " " ?	4 in 8 " " ?
2 in 14 " " ?	5 in 10 " " ?
2 in 16 " " ?	5 in 15 " " ?
2 in 18 " " ?	6 in 12 " " ?
	6 in 18 " " ?

QUESTIONS.—What does division teach? What is the number to be divided called? What is the number to divide by called? What is the answer called? How many signs are there in division? Make them.

7 in 14	how many	times?		11 in 22	how many	times	?
7 in 21	"	"	?	11 in 33	"	"	?
8 in 16	"	"	?	12 in 24	"	"	?
8 in 24	"	"	?	12 in 36	"	"	?
9 in 18	"	"	?	12 in 60	"	"	?
9 in 27	"	"	?	12 in 96	"	"	?
10 in 20	"	"	?	12 in 108	"	"	?
10 in 30	"	"	?	12 in 144	"	"	?
10 in 40	"	"	?	9 in 81	"	"	?
9 in 63	"	"	?	8 in 72	"	"	?

$28 \div 7$ = how many?	$\frac{49}{7}$ = how many?
42 " 6 = how many?	$\frac{66}{11}$ = how many?
54 " 9 = how many?	$\frac{84}{12}$ = how many?
32 " 4 = how many?	$\frac{108}{12}$ = how many?
33 " 11 = how many?	$\frac{72}{6}$ = how many?
64 " 8 = how many?	$\frac{110}{10}$ = how many?
81 " 9 = how many?	$\frac{144}{12}$ = how many?

RULE FOR DIVISION.

I. When the divisor *does not exceed* 12, set it down on the left of the dividend, draw a curved line between them, and a straight line under the dividend. Find how many times the divisor is contained in the left hand figure, or figures of the dividend, and write the figure expressing the number of times underneath. If there be a remainder over, conceive it to be prefixed to the next figure of the dividend, and divide the next figures as before, and so on through the dividend.

II. When the divisor *does exceed* 12, set down the divisor and dividend as above directed; draw a curved line to the right and also to the left hand of the dividend; then find how many times the divisor is contained in the fewest figures of the

---

Questions.—How do we set down the numbers for division? What do we do next? If there be a remainder what do we do with it? When the divisor exceeds 12 how do we proceed?

dividend that will contain it; place the result to the right hand of the dividend for the first quotient figure. Then multiply the divisor by this quotient figure—write the product under the first figure or figures of the dividend and subtract it from them; to the remainder bring down the next figure of the dividend—see how many times it contains the divisor—place the result in the quotient; then multiply and subtract as before, and so on until all the figures of the dividend shall have been brought down and divided.

EXAMPLES.

1. Divide 468 by 2.

Operation.
Divisor 2)468 Dividend.
—
234 Quotient.

In this example there are 4 hundreds, 6 tens and 8 units to be divided. We say, 2 in 4, 2 times, which being 2 hundreds, we write it under the 4 hundreds. Then we say 2 in 6, 3 times, which being tens, is written under the 6 tens; then we say 2 in 8, 4 times, which are 4 units, and are written under the 8 units.

2. Divide 568 by 4.

Operation.
4)568
—
142

Here we say 4 in 5, once and 1 over—we set the 1 under the 5 and join the 1 over with the 6, making 16; then say 4 in 16, 4 times, and 4 in 8, 2 times, &c.

3. Divide 1547 by 5.

Operation.
5)1547
—
309–2

In this example we say, 1 does not contain 5, but 15 contains 5, 3 times; we then say 5 in 4, 0 times, and set the 0 in the quotient; then say 5 in 47, 9 times,

QUESTIONS.—Where do we place the result? What is the next step? Where do we write the product, and what do we do with it? What do we do with the remainder, and how do we proceed to the end? If the divisor is contained in hundreds, of what order will the quotient be? If in tens? If in units?

and a remainder of 2, this 2 being a part of the dividend, should be divided by 5, but it cannot be, we therefore write the divisor 5 under the remainder 2, thus, $\frac{2}{5}$, which expresses that the 2 is to be divided by 5, and the quotient of 1547 divided by 5 is $309\frac{2}{5}$.

Hence when there is a remainder after division, it may be written after the quotient, with the divisor placed under it.

The foregoing method of performing division, when the divisor does not exceed 12, is called *Short Division.*

PROOF.

Multiply the quotient by the divisor, and to the product add the remainder, if any, and the sum will be equal to the dividend.

	(4)	(5)
	4)3695672	5)145678
Quot.	923918	29135—3
	4	5
Proof	3695672	145678

It appears from the above examples that division is proved by multiplication, and as the one is the opposite of the other, *multiplication may be proved by division.* When two numbers are multiplied together, the multiplicand and multiplier are both factors of the product. Hence it follows that, if the product be divided by one of the factors, the quotient will be the other factor. Wherefore, *if the product of two numbers be divided by the multiplicand, the quotient will be the multiplier; or if it be divided by the multiplier, the quotient will be the multiplicand.*

Illustration, $12\times8=96\div12=8$ and $96\div8=12$.

$11\times9=99\div9=11$ and $99\div11=9$.

QUESTIONS.—If the divisor is contained in hundreds, of what order will the quotient be? If in tens? If in units? If there be a remainder after division what may be done with it?

(6)	(7)	(8)
3)846208	5)620482	6)131519
$282069\frac{1}{3}$	$124096\frac{2}{5}$	$21919\frac{5}{6}$

(9)	(10)	(11)
9)1434567	7)7841034	8)13456789

(12)	(13)	(14)
10)3415670	11)3891416	12)27483416

15. Divide 640 pounds among 4 men.

$640 \div 4$, or $\frac{640}{4}=160$ pounds. Ans.

16. $\frac{510421}{7}=$how many?

17. $\frac{8941674}{8}=$how many?

18. $\frac{141304416}{9}=$how many?

When the divisor exceeds 12, as in the examples that follow, the operation is called long division.

1. Divide 840 by 24.

```
Operation.
          Dividend
Divisor 24)840(35Quotient
           72
           ---
           120
           120
           ---
```

In this example we set the divisor on the left hand of the dividend as in short division, and draw a curved line to the right of the dividend as the rule directs. We now wish to find how many times 840 contains 24. The fewest figures of the dividend that will contain the divisor is 84.—We conclude that 84 will contain 24, 3 times. We place the 3 in the quotient, and multiply the divisor by it, the

QUESTIONS.—When the divisor does not exceed 12 what is the operation called? How is division proved? How may multiplication be proved? In multiplication, if the product be divided by the *multiplicand* what will the quotient be? If the product be divided by the *multiplier* what will the quotient be? When the divisor exceeds 12, what is the operation called.

product is 72, which is placed under 84 and subtracted from it and 12 remains. We now bring down the next figure of the dividend, and conclude that 120 will contain 24 5 times; we therefore place 5 in the quotient and multiply as before. Thus we find 840 contains 24 35 times.

2. Divide 2756 by 26.

```
          Operation.
          Dividend.
Divisor 26)2756(106 Quotient.
           26
           ——
            156
            156
            ———
```

In this example we say 26 in 27 once, and place the 1 in the quotient, multiply 26 by 1, subtract and bring down the 5; we then say 26 in 15, 0 times and place the 0 in the quotient, we then bring down the 6, and find that the divisor is contained in 156, 6 times.

NOTE 1. If the remainder at any time be greater than the divisor, the *quotient figure must be increased.*

NOTE 2. If the product of the divisor and quotient figure exceed that part of the dividend directly above it, the *quotient figure must be diminished.*

3. Divide 30660 by 84.

```
   Operation.                 Proof.
 84)30660(365                  365  Quotient.
    252                         84  Divisor.
    ———                        ————
     546                       1460
     504                      2920
     ———                      —————
      420                     30660=the dividend.
      420
```

QUESTIONS.—If the remainder at any time be greater than the divisor, what must be done? If the product of the divisor and quotient figure exceed that part of the dividend directly above it, what must be done?

4. Divide 25648 by 21.

Operation.	Proof.
21)25648(1221	1221 Quotient.
21	21 Divisor.
46	1221
42	2442
44	7 remainder.
42	25648=the dividend.
28	
21	
7 rem.	

5. Divide 9420674 by 13. Ans. 724667— 3 rem.
6. Divide 3271916 by 17. Ans. 192465—11 rem.
7. Divide 9643007 by 23. Ans. 419261$\frac{4}{23}$.
8. Divide 6318 by 27. Ans. 234.
9. Divide 96414 by 34. Ans. 2835 $\frac{24}{34}$.
10. Divide 937387 by 54. Ans. 17359—1 rem.
11. Divide 147735 by 45. Ans. 3283.
12. Divide 145260 by 108. Ans. 1345.
13. Divide 24167 by 125, and prove the operation.
14. Divide 34108 by 87, " "
15. Divide 10416 by 140, " "
16. Divide 541678 by 341, " "
17. Divide 674160 by 410, " "
18. Divide 940161 by 365, " "
19. Divide 141041 by 1341, " "
20. Divide 8416759 by 2140. " "

CONTRACTIONS IN DIVISION.

I When the divisor is a composite number.

RULE.

When the divisor is a composite number, divide the dividend by one of the component parts, and the quotient arising from that division, by the other; the last quotient will be the answer.

EXAMPLES.

1. Divide 5430 apples equally among 15 boys.

Operation.
5)5430
———
3)1086 1st quo.
———
362 quo. sought.

In this example 5 and 3 are the component parts or factors of 15.—First divide the apples among 5 boys, and we find they will have 1086 apples a piece. Then let *each one* of these boys divide 1086 among 3 boys, and they will have 362, and the whole number of parts will be 15.

2. Divide 18576 by 48= 4×12. Ans. 387.
3. Divide 9576 by 72= 9× 8. Ans. 133.
4. Divide 19296 by 96=12× 8. Ans. 201.

To obtain the true remainder, when factors have been used as divisors, *multiply the last remainder by the first divisor, and to the product add the first remainder.*

1. Divide 4967 by 32=4×8. Ans. $155\frac{7}{32}$.

Operation,
4)4967
———
8)1241—3, 1st remainder.
———
155—1×4+3=7 the true remainder.

2. Divide 956789 by 7×8=56. Ans. $17085\frac{29}{56}$.
3. Divide 4870029 by 8×9=72. Ans. $67639\frac{21}{72}$.
4. Divide 674201 by 10×11=110. Ans. $6129\frac{11}{110}$.
5. Divide 445767 by 12×12=144. Ans. $3095\frac{87}{144}$.

QUESTIONS.—When the divisor is a composite number, how is the division performed? What will be the answer? How may the true remainder be obtained when factors have been used as divisors?

II. When the divisor is 10, 100, 1000, &c.

RULE.

Cut off as many figures from the right hand of the dividend, as there are ciphers in the divisor; the other figures of the dividend will be the *quotient*, and the figures cut off will be the remainder.

EXAMPLES.

1. Divide 14364 by 100. Ans. $143\frac{64}{100}$

Operation. In this example there are two 0s in the divisor; therefore we cut off two figures from the right hand of the dividend, and the quotient is 143, and the remainder 64.

1|00)143|64.

2. Divide 24367 by 10. Ans. $2436\frac{7}{10}$.
3. Divide 52164 by 100. Ans. $521\frac{64}{100}$.
4. Divide 1040241 by 1000. Ans. $1040\frac{241}{1000}$.

III. When there are ciphers on the right hand of the divisor.

RULE.

Cut off the ciphers and omit them in the operation, likewise cut off and omit the same number of figures from the right hand of the dividend. Annex to the remainder, if there be one, the figures cut off from the dividend: this will form the true remainder.

EXAMPLES.

1. How many times 900 are there in 741725.

Operation. We divide 7417 by 9: there remains 1, to which annex the 25, making the true remainder 125.

9|00)7417|25
824..125 rem.

2. How many times are 700 contained in 67389? Ans. $96\frac{189}{700}$.

QUESTIONS.—When the divisor is 10, 100, 1000, &c. what is the method of dividing? When there are ciphers on the right hand of the divisor, what is to be done? What, if there be a remainder?

3. How many times are 37000 contained in 8749632?
Ans. $236\frac{17632}{37000}$.

4. How many times are 6000 contained in 876000?
Ans. 146.

5. How many times are 400700 contained in 36599503?
Ans. $91\frac{135803}{400700}$.

PRACTICAL EXERCISES.

1. There are 7 days in a week: how many weeks in a year of 365 days? Ans. 52 weeks and 1 day over.

2. Divide 3125 pounds equally among 25 men.
Ans. 125.

3. Lake Ontario is 190 miles long: how many hours will it require for a boat to sail from one end to the other, if she sail 9 miles per hour? Ans. $21\frac{1}{9}$ hours.

4. How many days will a ship be in sailing from Quebec to Liverpool, allowing the distance to be 3000 miles, and the ship to sail 100 miles per day? Ans. 30 days.

5. If 6 bushels of apples make 1 barrel of cider, how many barrels will 1000 bushels make?
Ans. $166\frac{4}{6}$ barrels.

6. How many bags will it require to hold 1000 bushels of wheat, allowing each bag to hold 3 bushels?
Ans. 333 bags and 1 bushel over.

7. What number must be multiplied by 250 that the product may be 18750? Ans. 75.

8. If the divisor be 49 and the dividend 42581: what is the quotient? Ans. 869.

9. A man bought 241 acres of land, for which he paid 9468 dollars. Query, the price per acre?
Ans. $39\frac{69}{241}$ dollars.

10. A farmer raised 1500 bushels of wheat from 60 acres: what was that per acre. Ans. 25 bushels.

11. The number of letters in a volume being 2344125, of which 4605 were contained in a page. Required the number of pages. Ans. 525.

12. What will be the quotient of 974932, divided by 365? Ans. $2671\frac{17}{365}$.

13. Twenty persons dined together, their bill was 100 dollars. Query, the amount paid by each?

Ans. 5 dollars.

14. A gentleman has a garden walled in, containing 9625 yards; the breadth was 35 yards, what was the length? Ans. 275 yards.

15. What number added to the 43rd part of 4429 will raise it to 240? Ans. 137.

16. The quotient of a certain number is 1083, the divisor 28604, and the remainder 1788: what was the dividend? Ans. 30979920.

17. If the French army which invaded Russia in 1812, consumed 17534 barrels of flour, and 11698 barrels of meat in a week; what number of waggons, each carrying 16 barrels, would be required to carry the provisions consumed in a day? Ans. 261.

18. A square mile contains 640 acres of land, and a bushel of wheat is supposed to contain 491520 grains: how many grains of wheat would a square mile produce, each acre yielding 25 bushels? also, how many loaves of bread would this wheat make, allowing 30720 grains to a loaf? Ans. 7864320000 grains, and 256000 loaves.

FRACTIONS.

The unit 1 represents a whole or *entire* thing; as 1 orange, 1 yard of cloth, 1 pound of sugar.

When any unit, as an orange, or a pound, is divided into 2 equal parts, each part is called *one half* of the thing.

When divided into 3 equal parts, each part is called *one third*.

QUESTIONS.—What does the unit 1 represent? When it is divided into 2 equal parts, what is each part called? into 3 equal parts? into 4? into 10? into 15?

If divided into 4 equal parts, each part is called *one fourth.*

If divided into 10 equal parts, each part is called *one tenth.*

If into 15, each part is called *one fifteenth,* &c.

These parts, or broken numders are called *fractions,* and are written thus:

$\frac{1}{2}$	is read	one half.	$\frac{1}{7}$	is read	one seventh.
$\frac{1}{3}$	"	one third.	$\frac{1}{8}$	"	one eighth.
$\frac{1}{4}$	"	one fourth.	$\frac{1}{10}$	"	one tenth.
$\frac{1}{5}$	"	one fifth.	$\frac{1}{15}$	"	one fifteenth.
$\frac{1}{6}$	"	one sixth.	$\frac{1}{20}$	"	one twentieth, &c.

When we wish to express *more* than *one* of the equal parts of a thing, as *two thirds, three fourths,* &c. we write it thus:

$\frac{2}{3}$	is read	two thirds.	$\frac{7}{8}$	is read	seven eighths.
$\frac{3}{4}$	"	three fourths.	$\frac{9}{20}$	"	nine twentieths.

The number below the line is called the *denominator* because, it gives the fraction its denomination or name, and it shows *into how many equal parts the unit is divided.*

The number above the line is called the *numerator,* because it *numbers* the parts, or *shows how many of the parts are expressed by the fraction.*

In the fraction $\frac{4}{6}$, the *denominator* shows that a unit is divided into 6 equal parts, and the *numerator* shows that 4 of these parts are expressed in the fraction.

In the fraction $\frac{8}{10}$, the denominator shows that a unit is divided into 10 equal parts, and the numerator, that 8 of these parts are expressed, &c.

Questions.—What are these broken numbers called? In a written fraction what is the number below the line called? Why? What does it show? What is the number above the line called? Why? What does *it* show? In the fraction 4-6 which is the denominator? What does the denominator express? Which is the numerator? What does the numerator show?

The numerator and denominator taken together are called the *terms* of the fraction.

If a melon be divided into 8 equal parts and 4 of the parts be given to Charles and the other 4 parts be given to Henry, it is plain that each boy's share will be $\frac{4}{8}$, it is also plain that each boy has one half the melon. $\frac{4}{8}$ then is equal to $\frac{1}{2}$.

It will be seen that changing $\frac{4}{8}$ to $\frac{1}{2}$ does not alter its value. This operation is called reducing a fraction to its *lowest terms.*

A fraction is reduced to lower terms by dividing its numerator and denominator by any number which will divide them both without a remainder.

Thus $\frac{3}{9}$ may be divided by 3 without a remainder and $=\frac{1}{3}$, $\frac{5}{10}=\frac{1}{2}$, $\frac{3}{12}=\frac{1}{4}$, $\frac{2}{12}=\frac{1}{6}$, $\frac{2}{6}=\frac{1}{3}$, $\frac{8}{12}=\frac{2}{3}$, $\frac{2}{14}=\frac{1}{7}$, $\frac{5}{15}=\frac{1}{3}$.

It should be constantly borne in mind, that if the *numerator* and *denominator be divided by the same number, the value of the fraction remains the same.*

QUESTIONS.—What are the numerator and denominator taken together called? If a melon be divided into 8 equal parts, and 4 of them be given to Charles and the remaining 4 to Henry, what is each boy's share of the melon? What are 4-8 equal to? What is the difference in value between 4-8 and $\frac{1}{2}$? What is this operation of changing fractions called? How may a fraction be reduced to lower terms? If the numerator and denominator be divided by the same number what effect has it on the value of the fraction? If the numerator and denominator in the fraction 5-15 be both divided by 5, what will the expression be? Will its value be changed?

SUPPLEMENT TO MULTIPLICATION.

You have learned how to multiply by integers, or whole nnmbers. It is necessary that you now learn how to multiply by a mixed number, which is made up of a *whole* number and a *fraction*: thus, $2\frac{1}{2}$, $4\frac{1}{3}$, $6\frac{3}{4}$, $9\frac{2}{3}$, &c.

Multiplication has been defined to be the repeating of one number as many times as there are *units* in *another*. Hence multiplying by 1 is taking the multiplicand *once*, multiplying by 2 is taking it *twice*, multiplying by 3 is taking it *three times*, &c.

Multiplying by a broken number is taking a part of the multiplicand as many times as there are like parts of a unit in the multiplier.

Multiplying by $\frac{1}{2}$ is taking *one half* of the *multiplicand*.
Multiplying by $\frac{1}{4}$ is taking *one fourth* of it.
Multiplying by $\frac{3}{4}$ is taking *three fourths* of it, &c.

To multiply a whole number by a mixed number observe the following

RULE.

First multiply the multiplicand by the whole number of the multiplier; then multiply it by the numerator of the fraction, and divide that product by the denominator; add the two products together, and their sum will be the entire product of the mixed number.

QUESTIONS.—What is a mixed number? Give an example? How has multiplication been defined? What is multiplying by 1? by 2? by 3? What is multiplying by a broken number? Multiplying by $\frac{1}{2}$ is the same as what? by $\frac{1}{4}$? by $\frac{3}{4}$? Repeat the rule for multiplying by a mixed number?

EXAMPLES.

1. Multiply 48 by $\frac{1}{2}$.

Operation.

48
1
—
2)48
—
24 Product.

When the numerator is 1, as in this example, there is no necessity of multiplying by it, for it simply repeats the multiplicand.

2. Multiply 36 by $4\frac{3}{4}$.

Operation.

36	36
4	3
144	4)108
27	27 Product of $36\times\frac{3}{4}$
171 Product.	

In this example we first multiply 36 by 4, and the product is 144; we then set 36 down in another place, and multiply it by $\frac{3}{4}$, and find the product to be 27; the sum of these products are the entire product of 36 multiplied by $4\frac{3}{4}$.

3.	Multiply	39 by $\frac{1}{3}$	Ans. 13.
4.	———	96 by $\frac{1}{4}$	Ans. 24.
5.	———	150 by $\frac{1}{5}$	Ans. 30.
6.	———	69 by $2\frac{1}{3}$	Ans. 161.
7.	———	100 by $4\frac{1}{4}$	Ans. 425.
8.	———	48 by $2\frac{3}{4}$	Ans. 132.
9.	———	55 by $5\frac{1}{5}$	Ans. 286.
10.	———	346 by $2\frac{5}{7}$	Ans. $939\frac{1}{7}$.
11.	———	100 by $6\frac{2}{3}$	Ans. $666\frac{2}{3}$.
12.	———	240 by $18\frac{5}{9}$	Ans. $4453\frac{3}{9}$.

PRACTICAL EXERCISES.

1. There are $16\frac{1}{2}$ feet in a rod, and 320 rods in a mile: how many feet are there in a mile? Ans. 5280.

2. What is the cost of $8\frac{3}{4}$ tons of hay at 10 dollars per ton? Ans. $87\frac{1}{2}$ dollars.

3. If $4\frac{4}{5}$ bushels of wheat make 1 barrel of flour: how many bushels will it require to 250 barrels?

Ans. 1200 bushels.

4. There are $69\frac{1}{2}$ miles in one degree: how many miles in 360 degrees? Ans. 25028 miles.

5. What is the cost of 25 horses at $15\frac{1}{2}$ pounds each?

Ans. $387\frac{1}{2}$ pounds.

NOTE TO PUPILS—You have now been through the fundamental or *foundation* rules of *Arithmetic*, and I hope are able to solve all the questions thus far *understandingly*, and can also answer all the questions at the bottom of each page. A thorough knowledge of the foregoing rules will greatly facilitate your future progress. I will now present you with a few

PRACTICAL EXERCISES.

Involving the principles of the preceding rules.

1. $240+670+81+14+9+2=$ how many?

Ans. 1016.

2. $10460148-1341009=$ how many?

Ans. 9119139.

3. $340084\times4005=$ how many? Ans. 1362036420.

4. $2674236\div634=$ how many? Ans. $4218\frac{24}{634}$.

5. $\frac{148906}{3040}=$ how many? Ans. 49.

6. $25+18+10-13\times9\div3=$ how many? Ans. 120.

7. $100+75\times4-2\div4=$ how many? Ans. $174\frac{1}{2}$.

8. $12\times11+141-13\div10\times2=$ Ans. 52.

NOTE.—A line, or *vinculum*, drawn over several numbers, signifies, that the numbers under it, are to be taken as one whole number; thus $\overline{4+3}\times\overline{8-2}=42$.

9. $\overline{10-6+4}\times\overline{7-3+1}=$ how many? Ans. 40.

10. $\frac{\overline{5+6-4}\times\overline{9-6}}{9-5}=$ how many? Ans. $5\frac{1}{4}$.

QUESTION.—What does a line, or vinculum, drawn over several numbers signify?

11. From the sum of 100, 84, 75, 18 and 31 subtract the sum of 10, 9, 8 and 2. Ans. 279.

12. The sum of two numbers is 1045, one of the numbers is 109, what is the other? Ans. 936.

13. If the minuend be 1460 and the subtrahend 1390; what is the remainder?

14. If the minuend be 1460 and the remainder 70; what is the subtrahend?

15. If the subtrahend be 1390 and the remainder 70; what is the minuend?

16. What is the product of 537467 and 367?
Ans. 197250389.

17. If the divisor be 84 and the dividend 30660; what is the quotient? Ans. 365.

18. If the quotient be 365 and the divisor 84; what is the dividend?

19. The factors of a certain number are 86972 and 1208; what is the number? Ans. 105062176.

20. A farmer sells a horse for 18 pounds, 5 cows for 4 pounds a piece, 6 oxen for 5 pounds each, and 100 sheep at $1\frac{1}{2}$ pounds a head: How much did he receive for them all? Ans. 218 pounds.

21. A gentleman sells 150 bushels of wheat at $\frac{1}{4}$ (one fourth) of a pound per bushel, 25 tons of hay at 3 pounds per ton; and takes in part payment a wagon at 10 pounds, and a yoke of oxen worth 20 pounds, and the rest in cash: How much money did he receive? Ans. $82\frac{1}{2}$ pounds.

22. Sold a ship for 11516 pounds, and I owned $\frac{3}{4}$ (three fourths) of her; what was my part of the money?
Ans. 8637 pounds.

23. A gentleman left his son, sixteen thousand sixteen hundred and sixteen pounds, and his daughter one half as much: Query, the daughter's portion. Ans. 8808 pounds.

24. The circumference of the earth is 25000 miles: how long would it take a man to travel around it, supposing him to travel 42 miles a day? Ans. $595\frac{10}{42}$ days.

25. What will $12\frac{3}{4}$ yards of cloth come to at 6 dollars a yard? Ans. $76\frac{2}{4}$ dollars.

26. The Bible contains 31173 verses; how many verses must be read in a day, that it may be read through in 365 days? Ans. $85\frac{148}{365}$.

27. What is one-eighth of 59876? Ans. $7484\frac{1}{2}$.

28. What is four fifths of 64870? Ans. 51896.

29. Write down four thousand six hundred and seventeen; multiply it by 12, divide the product by 9, and add 365 to the quotient, then from that sum subtract five thousand five hundred and twenty-one, and the remainder will be 1000. Try it and see.

30. If a man's wages amount to 625 dollars in 52 weeks, how much may he spend a week and save 261 dollars? Ans. 7 dollars a week.

31. How many times can 24 be subtracted from 1416? Ans. 59.

32. Sold 5505 pounds of butter at 12 pence per pound, and took my pay in molasses at 36 pence a gallon; how many gallons did I receive? Ans. 1835 gallons.

33. How many barrels of flour, at 8 dollars per barrel will it take to buy 62 horses at 95 dollars each? Ans. $736\frac{2}{8}$.

34. How many men must be employed to do a piece of work in 1 day, that 11 men can perform in 18 days? Ans. 198 days.

35. Thomas and Joseph are studying arithmetic. Thomas is 322 examples in advance of Joseph, but Joseph performs 55 examples in a day, and Thomas 41. In how many days will Joseph overtake Thomas? Ans. 23 days.

36. A. B. and C. made up a purse of 500 pounds. A. put in 75 pounds, B. put in three times as much: How much did C. put in? Ans. 210 pounds.

37. What number must be subtracted from 294106 in order that the remainder shall be 14230? Ans. 279876.

38. In 30416 dollars; how many pounds. Ans. 7604.

39. In 8940 shillings; how many dollars. Ans. 1788.

40. A farmer purchased a farm, for which he paid 1850 dollars. He sold 50 acres for 60 dollars per acre,

and the remainder stood him in 50 dollars an acre: how much land did he purchase. Ans. 351 acres.

41. The mariner's compass was discovered in England in the year 1302: how long since that time?

42. A gentleman wishes to distribute 1200 apples among 5 boys: he gave the first boy one third of the whole: the second one fourth, the third one fifth the fourth one sixth; and the fifth the remainder: how many apples did each boy receive.

Ans.	1st boy	400,
	2nd "	300,
	2d "	240,
	4th "	200,
	5th "	60.

43. A speculator bought 536 acres of land at $10\frac{1}{2}$ dollars per acre: he sold A. 100 acres at 12 dollars an acre, B. 150 at 11 dollars per acre, C. 145 at 13 dollars per acre; he was obliged to sell the balance at 8 dollars per acre: did he gain or lose by the purchase, and how much? also, what was the average price that he obtained per acre.

Ans. he gained 235 dols.
Average price $10\frac{503}{536}$ dollars.

Note by the Printer.—The following exercises were inadvertently omitted by the compositor, and should have been inserted immediately after the examples in subtraction.

PRACTICAL EXERCISES IN SUBTRACTION.

1. From fifteen million take fifty six thousand, and what will remain. Ans. 14,944,000.

2. A man paid 150 dollars for a good horse and sold him for 175 dollars; how much did he gain? Ans. 25 dol.

3. What number is that which, taken from 5487, leaves 999? Ans. 4488.

4. Columbus discovered America in the year 1492, how many years since?

5. Queen Victoria was born in the year 1819 succeeded to the throne of England in 1837; what was her age at the latter period?

6. Subtract one from one million. Ans. 999999.

7. From one hundred million two hundred and forty seven thousand, take one million four hundred and nine. Ans. 99246591.

8. The number of inhabitants in the U. States, in 1830, was 12,840,540, in 1840 they amounted to 17,069,957. What was the increase in 10 years? Ans. 4,229,417.

9. The author of this book was born in the year 1814, What is his age the present year, 1845.

10. Canada was ceded to Britain, by the French, in 1763, how long since that time?

11. What number, together with these three, viz. 1301, 2561 and 3120, will make ten thousand. Ans. 3018.

12. What number must be added to 175 to make 1101. Ans. 926.

13. There are two numbers, whose difference is 10, 101; the greater number is 100,000: Query the less. Ans. 89,899.

14. A man dies worth 10,104 pounds, he leaves to his daughter 4,115 pounds, and the remainder to his son: what was the son's portion? Ans. 5989.

15. The sun is ninety five millions of miles from the earth, and the moon is two hundred and forty thousand miles, how much farther in the sun from us than the moon? Ans. 94,760,000.

16. The minuend being 135 and the subtrahend 100: what is the remainder? Ans. 35.

17. The subtrahend being 100 and the remainder 35; what is the minuend? Ans. 135.

18. The minuend is 135, and the remainder 35; what is the subtrahend. Ans. 100.

19. The sum of two numbers is 100, and one of the numbers is 35: what is the other? Ans. 65.

QUESTIONS.—When the minuend and subtrahend are given, how do we find the remainder? When the subtrahend and remainder are given, how do we find the minuend? When the sum of two numbers and one of the numbers are given, how do we find the other?

20. The greater of two numbers is 100 and their difference 35; what is the *less* number? Ans. 65.

21. The *less* of two numbers is 65, and their difference 35; what is the greater? Ans. 100.

QUESTIONS.—When we have the greater of two numbers and their difference, how do we find the less number? When we have the less of two numbers and their difference given, how do we find the greater?

END OF PART I.

PART II.

COMPOUND NUMBERS.

A number expressing things of the *same kind* is called a *simple number*. For example, 20 horses, 4 pounds, 8 bushels, are each of them *simple numbers*.

A *compound number*, expresses things of *different* kinds; thus 5 pounds, 8 shillings and 10 pence is a *compound number*, also, 1 year 2 months 2 weeks and 3 days, 2 bushels 3 pecks 4 quarts, are compound numbers. When numbers have different names as above they are mostly called *different denominations*.

TABLES OF COMPOUND NUMBERS.

PENCE TABLE.

d.	*s.*	*d.*	*s.*	*d.*
20=	1	8	2=	24
30=	2	6	3=	36
40=	3	4	4=	48
50=	4	2	5=	60
60=	5	0	6=	72
70=	5	10	7=	84
80=	6	8	8=	96
90=	7	6	9=	108
100=	8	4	10=	120
110=	9	2	11=	132
120=	10	0	12=	144

Questions.—What do simple numbers express? What do compound numbers express? Give an example of a simple number, of a compound number. What is understood by different denominations? Are 8 pounds and 4 pounds of the same denomination? Are 6 pounds and 4 shillings? Are several numbers of different denominations often connected together? Give an example?

ENGLISH MONEY.

The denominations of English Money, guineas, pounds, shillings, pence, and farthings.

4 farthings marked *far.*	make 1 penny marked	d.
12 pence	make 1 shilling ——	s.
20 shillings	make 1 pound ——	£.
21 shillings	make 1 guinea.	

NOTE.—Farthings are generally expressed in fractions of a penny. Thus, for 1 farthing we write $\frac{1}{4}$d. for 2 farthings $\frac{1}{2}$d. and for 3 farthings $\frac{3}{4}$d.

TROY WEIGHT.

Gold, silver, jewels and liquors, are weighed by this weight. Its denominations are pounds, ounces, pennyweights, and grains.

24 grains, (gr.)	make	1 pennyweight marked	pwt.
20 pennyweights	"	1 ounce, ——	oz.
12 ounces	"	1 pound, ——	lb.

APOTHECARIES' WEIGHT.

This weight is used by apothecaries and physicians in mixing their medicines. Its denominations are pounds, ounces, drachms, scruples, and grains.

20 grains, (grs.)	make	1 scruple.
3 scruples	"	1 drachm.
8 drachms	"	1 ounce.
12 ounces	"	1 pound.

QUESTIONS.—What are the denominations of English Money? Repeat the table. How are farthings generally expressed? How many pence in 6 farthings? In 8? In 10? In 12? How many shillings in 14 pence? In 16? In 18? In 23? In 27? In 32? In 44? How many pounds in 24 shillings? In 30 shillings? In 36? In 40? What articles are weighed by Troy Weight? What are its denominations? Repeat the table. What is the use of Apothecaries' Weight? What are its denominations? Recite the table.

AVOIRDUPOIS WEIGHT.

By this weight are weighed all things of a coarse drossy nature, as tea, sugar, grain, hay, tallow, leather, medicines, (in buying and selling) and all kinds of metals except gold and silver. The denominations are tons, hundreds, quarters, pounds, ounces, and drachms.

16 drachms (drs.)	make	1 ounce,	marked	oz.
16 ounces	"	1 pound	"	lb.
28 pounds	"	1 quarter	"	qr.
4 quarters	make	1 hundred weight,	marked	cwt.
20 hundred weight	make	1 ton,	"	T.

NOTE.—In this weight the words *gross* and *nett* are used. Gross is the weight of the goods, together with the box, cask, bag, bale, &c., which contains them. Nett is the weight of the goods only, or what remains after deducting from the gross weight, the weight of the box, bag, cask, &c. A hundred weight is 112 lbs., as appears from the table.

LONG MEASURE.

Long measure is used in measuring distances, or other things, where *length* is considered without regard to breadth. Its denominations are degrees, leagues, miles, furlongs, rods, yards, feet, inches, and barley corns.

3 barley corns, (*bar.*)	make	1 inch,	marked	in.
12 inches	"	1 foot,	"	ft.
3 feet	"	1 yard,	"	yd.
5½ yards or 16½ feet	"	1 rod, perch or pole,	"	rd.
40 rods	"	1 furlong,	"	fur.
8 furlongs or 320 rods	"	1 mile,	"	mi.
3 miles	"	1 league,	"	L.
60 geographical or 69½ statute miles,	"	1 degree,	"	deg. or °

QUESTIONS.—What are weighed by Avoirdupois Weight?—What are its denominations? Repeat the table. What is meant by gross weight? What by nett? What is a cwt.? When is long measure used? What are its denominations? Repeat the table.

360 degrees make a great circle, or circumference of the earth.

NOTE.—A hand is a measure of 4 inches, and is used to measure the height of horses. A fathom is 6 feet, and is mostly used to measure the depth of water.

CLOTH MEASURE.

By this measure all kinds of cloth, tapes, &c., are measured. Its denominations are ells French, ells English, ells Flemish, yards, quarters, nails and inches.

$2\frac{1}{4}$ inches (in.)	make	1 nail,	marked	na.
4 nails	"	1 quarter of a yard	"	qr.
4 quarters	"	1 yard,	"	yd.
3 quarters	"	1 Ell Flemish,	"	E. Fl.
5 quarters	"	1 Ell English,	"	E. E.
6 quarters	"	1 Ell French,	"	E. Fr.

LAND OR SQUARE MEASURE.

This measure is used in measuring land, or any thing in which *length* and *breadth* are *both* considered. The denominations are miles, acres, roods, perches, yards, feet and inches.

144 square inches (sq. in.)	make	1 square foot,	Sq. ft.
9 square feet	"	1 square yard,	Sq. yd.
$30\frac{1}{4}$ square yards	"	1 square pole,	P.
40 square poles	"	1 rood,	R.
4 roods, or 160 sq. rods	"	1 acre,	A.
640 acres	"	1 square mile,	M.

NOTE.—The Surveyor's or Gunter's chain is used in measuring land. It is 4 rods, or 66 feet in length, and is divided into 100 links.

QUESTIONS.—What is a hand in measure, and for what is it used? What is a fathom, and for what is it used? What are measured by Cloth Measure? What are its denominations? Repeat the table? For what is square measure used? What are the denominations. Repeat the table. For what is the Surveyor's or Gunter's chain used? What is its length? How is it divided?

SOLID OR CUBIC MEASURE.

Solid or cubic measure is used in measuring timber, stone, wood, earth, and such other things as have *length, breadth* and *thickness.* Its denominations are tons, cords, yards, feet and inches.

1728 solid inches (S. in.)	make	1 solid foot,	S. ft.
27 solid feet	"	1 solid yard,	S. yd.
40 feet of round timber	"	1 ton,	T.
50 feet of hewn timber	"	1 ton,	T.
128 solid feet	"	1 cord of wood,	C.

NOTE.—A pile of wood 8 feet long, 4 feet wide, and 4 feet high makes a cord. And what is called a cord foot of wood is 16 solid feet: that is 4 feet in length, 4 feet in width, and 1 foot in height, and 8 such feet, that is 8 *cord* feet make 1 cord.

WINE MEASURE.

Wine measure is used in measuring all liquors excepting beer and ale. Its denominations are tuns, pipes, hogsheads, barrels, gallons, quarts, pints, and gills.

4 gills (gi.)	make	1 pint,	marked	pt.
2 pints	"	1 quart,	"	qt.
4 quarts	"	1 gallon,	"	gal.
31½ gallons	"	1 barrel,	"	bar.
63 gallons	"	1 hogshead,	"	hhd.
2 hogsheads	"	1 pipe,	"	pi.
2 pipes or 4 hhds.	"	1 tun,	"	tun.

NOTE.—A gallon wine measure, contains 231 cubic inches.

QUESTIONS.—How is Solid or Cubic Measure used? What are its denominations? Recite the table. What is the length, breadth and height of a cord of wood? What is a cord foot? How many cord feet make a cord? What is the use of Wine Measure? What are its denominations? Repeat the table.—How many solid inches in a gallon, wine measure?

ALE OR BEER MEASURE.

This measure is used in measuring ale, beer and milk. Its denominations are hogsheads, barrels, gallons, quarts and pints.

2 pints (pt.)	make	1 quart,	marked	qt.
4 quarts	"	1 gallon,	"	gal.
36 gallons	"	1 barrel,	"	bar.
54 gallons	"	1 hogshead,	"	hhd.

NOTE.—A gallon, beer measure, contains 282 cubic inches.

DRY MEASURE.

Dry measure is used in measuring all dry articles, such as grain, fruits, roots, salt, coal, &c. Its denominations are chaldrons, quarters, bushels, pecks, quarts and pints.

2 pints (pt.)	make	1 quart,	marked	qt.
8 quarts	"	1 peck,	"	pk.
4 pecks	"	1 bushel,	"	bu.
8 bushels	"	1 quarter,	"	qr.
36 bushels	"	1 chaldron,	"	ch.

NOTE.—A gallon dry measure contains $268\frac{4}{5}$ cubic inches. A Winchester bushel is $18\frac{1}{2}$ inches in diameter, 8 inches deep, and contains $2150\frac{2}{5}$ cubic inches.

TIME.

Time is naturally divided into *days*, by the revolution of the earth upon its axis, once in 24 hours; and into *years*, by the revolution of the earth round the sun once in 365 days, 5 hours, 48 minutes, and 48 seconds; this period is called a *Solar*

QUESTIONS.—What is the use of Ale or Beer Measure? What are its denominations? Repeat the table. How many cubic inches in a gallon, beer measure? For what is Dry Measure used? What are its denominations? Repeat the table. How many inches, solid, are contained in a gallon, dry measure? How many inches in diameter is a Winchester bushel? How many inches deep? How many cubic inches does it contain?

year. In order to keep pace with the solar year in our reckoning, we make every fourth year to contain 366 days, and call it Leap year.

The denominations of time are years, months, weeks, days, hours, minutes and seconds.

60	seconds (sec.)	make	1 minute,	marked	m.
60	minutes	"	1 hour,	"	hr.
24	hours	"	1 day,	"	da.
7	days	"	1 week,	"	wk.
4	weeks	"	1 month,	"	mo.
13	months, 1 day, 6 hrs.	"	1 Julian year	"	yr.

The year is also divided into 12 calendar months, which contain an unequal number of days.

		Names.	No of days.
1	month,	January,	31
2	"	February,	28
3	"	March,	31
4	"	April,	30
5	"	May,	31
6	"	June,	30
7	"	July,	31
8	"	August,	31
9	"	September,	30
10	"	October,	31
11	"	November,	30
12	"	December,	31
		Total,	365

NOTE.—When any year can be divided by 4 without a remainder, it is called leap year, in which the 2d month (February) has 29 days.

QUESTIONS.—How is time divided into days? How into years? What is a solar year? Why is every fourth year called Leap year? What are the denominations of time? Repeat the table. How many calendar months in a year? Name them, and the number of days in each. How many days has February in the Leap year.

CIRCULAR MEASURE OR MOTION.

This measure is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies. Every circle is supposed to be divided into 360 equal parts, called degrees.

60 seconds (″)	make	1 minute,	marked ′
60 minutes	"	1 degree,	" °
30 degrees	"	1 sign,	" s.
12 signs, or 360 °	"	1 circle	" c.

TABLE OF PARTICULARS.

12 units	make	1 dozen.
12 dozen	"	1 gross.
12 gross or 144 dozen	"	1 great gross.
20 units	"	1 score.
24 sheets of paper	"	1 quire.
20 quires	"	1 ream.

BOOKS.

A sheet folded in	two	leaves is called	a folio.
"	four	"	a quarto, or 4 to.
"	eight	"	an octavo, or 8 vo.
"	twelve	"	a duodecimo, or 12 mo.
"	eighteen	"	an octodecimo, or 18 mo.

REDUCTION.

Reduction is changing the denomination, or name of a number without altering its *value*. The reducing of a denomination of *greater* value into a

---

QUESTIONS.—For what is Circular Measure used? How is every circle supposed to be divided? Recite the table. How many units make a dozen? How many dozens make a gross? A great gross? How many units make a score? How many sheets of paper make a quire? How many quires a ream.

When a sheet is folded into two leaves what is it called? In four leaves? In eight? In twelve? In eighteen?

denomination of *less* value, is called reduction descending, and is performed by *multiplication.*—The reducing of a denomination of *less* value into one of *greater* value is called reduction ascending, and is performed by *division.*

It is evident, if we wish to change 8 feet to inches, we must multiply by 12 because 12 inches make 1 foot thus, $8\times12=96$. If we wish to change 96 inches to feet it is also plain that we must divide by 12, thus $96\div12=8$ feet. Hence it follows that, reduction *descending* and *ascending* reciprocally prove each other.

RULE FOR REDUCTION.

I. To reduce from a higher to a lower denomination

Multiply the *greater* denomination by that number which is required of the *less*, to make *one* of the greater, adding to the product, as many of the *less* denomination, as are expressed in the given sum. And so on through all the denominations.

II. To reduce from a lower to a higher denomination. *Divide* the *less* denomination by that number which is required of the *less* to make *one* of the next greater; the quotient will be of the same name as the greater denomination, and the remainders will be of the same denomination with the dividend, and are to be put as a part of the answer.

QUESTIONS.—What is reduction? What is reduction descending? How is it performed? What is reduction ascending? How is it performed? How do we change feet to inches? Is this reduction ascending, or descending? How do we change inches to feet? Is this descending or ascending? How is reduction proved? How do we reduce from a higher to a lower denomination.

EXAMPLES IN ENGLISH MONEY.

1. In £2 9s. 6d. 3far. how many farthings?

```
      £   s.  d.  far.
      2   9   6   3
      20
      ----
      49 shillings.
      12
      ----
     594 pence.
       4
     ----
Ans. 2379 farthings.
```

In this example we first reduce the pounds to shillings by multiplying by 20, and add 9 shillings to the product —making 49 shillings. We then reduce the 49 shillings to pence, by multiplying by 12, and add the 6 pence to the product—-making 549 pence; we then reduce these pence to farthings, by multiplying by 4, and add the 3 farthings to the product, making 2379 farthings. This is called reduction descending, because we have changed a higher denomination to a lower.

2. Reduce 2379 farthing to pounds.

```
      4)2379
        ----
     12)594..3 far.
        ----
      20)49..6 pence.
        -----
        £2.. 9 shillings.
          £  s.  d. far.
Ans.      2  9   6  3
```

This example is the reverse of the first example.—We first reduce the farthings to pence by dividing by 4, and the quotient is 549 pence and 3 far. (or quarters marked qr.) over. We then reduce the pence to shillings by dividing by 12, and the quotient is 49 shillings and 6 pence over. We next reduce the 49 shillings to pounds, by dividing by 20, and find there is 2 pounds and 9 shillings over.

3. In 35 pounds how many shillings? Ans. 700s.

QUESTIONS.—How are pounds reduced to shillings? Shillings to pence? Pence to farthings?

How are farthings reduced to pence? Pence to shillings? shillings to pounds?

4. In £145 how many shillings? Ans. 2900s.
5. In £376 how many shillings and pence?
Ans. 7520s. 90240d.
6. Reduce £56 14s. 6d. to pence. Ans. 13614d.
7. In 165 pounds 13 shillings, how many farthings?
Ans. 159024qrs.
8. In 128s. how many pounds? Ans. £6 8s.
9. Reduce 1046 pence to pounds. Ans. £4 7s. 2d.
10. In 6169 pence, how many pounds?
Ans. £25 14s. 1d.
11. In 180960d. how many pounds. Ans. £754.
12. Reduce £967 14s. 7d. to pence. Result 232255d.
13. In 135764qrs. how many pounds?
Ans. £141 8s. 5d.
14. In 48 guineas how many pence? Ans. 12096d.
15. How many pence, shillings, and pounds, are there in 17280 farthings? Ans. 4320d. 360s. £18.
16. Reduce 369936 farthings to guineas.
Ans. 367 guineas.
17. Reduce 878 guineas to pence and these pence to pounds. Fact. 221256d. and £921 18s.
18. In 525 dollars at 5s. each how many pence?
Ans. 31500d.
19. In 126000 farthings how many dollars?
Ans. 525 dollars.

EXAMPLES IN TROY WEIGHT.

1. In 27lb. 10oz. 13dwts. of gold, how many grains?
Ans. 160632.
2. In 8lb. 0oz. 7dwt. 2grs. how many grains?
Ans. 46250.
3. Reduce 158262grs. to pounds.
Ans. 27lb. 5oz. 14dwt. 6grs.
4. Reduce 376457grs. to pounds.
Fact. 65lb. 4oz. 5dwt. 17grs.

QUESTIONS.—How are pounds reduced to ounces, in Troy Weight? Ounces to pennyweights? Pennyweights to grains? Grains to pennyweights? Pennyweights to ounces? Ounces to pounds?

5. In 375lb. 10oz. 16grs. how many grains?
Ans. 2164816grs.

6. In 167597dwts. how many pounds?
Ans. 698lb. 3oz. 17dwts.

7. In 25lb. 9oz. 10dwt. how many grains?
Ans. 148560grs.

8. Reduce 97645745grs. to pounds.
Result 16952lb. 4oz. 12dwt. 17grs.

EXAMPLES IN APOTHECARIES WEIGHT.

1. In 17lb. how many ounces, drachms, and scruples?
Ans. 204oz. 1632drs. 4886scru.

2. In 1332005 grains, how many pounds?
Ans. 231lb. 3oz. 5grs.

3. In 5lb. of drugs, how many parcels, each 16 drachms. Ans. 30 parcels.

4. Reduce 27lb. 3oz. 2drs. to grains.
Result 157080grs.

5. In 245 parcels of drugs each 10oz. 3drs. 2scru. how many pounds? Ans. 213lb. 6oz. 2drs. 1scru.

EXAMPLES IN AVOIRDUPOIS WEIGHT.

1. In 15 tons, how many hundred weight, quarters and pounds? Ans. 300cwt. 1200qrs. 33600lbs.

2. In 67200 lbs. how many tons? Ans. 30 tons.

3. In 9cwt. 5lbs. how many ounces?
Ans. 16208oz.

4. Reduce 20571005 drachms to tons.
Ans. 35T. 17cwt. 1qr. 23lbs. 7oz. 13dr.

5. One of the stones in the walls of Balbeck is said to have weighed 683T. 8cwt. Query, its weight in pounds?
Ans. 1530816lbs.

6. In 57T. 10cwt. 3qrs. 14lbs. how many drachms?
Ans. 32997888 drs.

7. Reduce 4768768 drachms to tons.
Ans. 8T. 6cwt. 1qr. 8lbs.

Questions.—In Apothecaries Weight how are pounds reduced to grains? Grains to pounds? In Avoirdupois weight, how are tons reduced to drachms? Drachms to tons?

8. A merchant would put 109cwt. 0qrs. 12lbs. of raisins into boxes, containing 26lbs. each; how many boxes will it require? Ans. 470 boxes.

EXAMPLES IN LONG MEASURE.

1. How many barley-corns will reach round the globe, it being 360 degrees? Ans. 4755801600 bar.

Operation.

2)360°
$69\frac{1}{2}$
———
180
3240
2160
———
25020 miles.
3020
———
500400
75060
———
8006400 rods.
$16\frac{1}{2}$
———
4003200
48038400
8006400
———
132105600 feet.
12
———
1585267200 inches.
3
———
4755801600 barleycorns.

In this example, we first multiply 360° by $69\frac{1}{2}$, the number of miles in a degree; then by 320, the number of rods in a mile: then by $16\frac{1}{2}$, the number of feet in a rod; then by 12 the number of inches in a foot, lastly by 3, the number of barleycorns in an inch,—which produces the answer.

2. Reduce 4755801600 barleycorns to degrees.

Ans. 360 degrees.

Operation.

3)4755801600

12)1585267200 in.

132105600 feet.
2

33)264211200$\frac{1}{2}$ feet.

320)8006400 rods.

25020 miles.
2

139)50040$\frac{1}{2}$ miles.

360 degrees.

This example is the reverse of the first example.—We first divide by 3, the no. of bar. in an inch; then by 12 the number of in. in a foot—and find the quotient to be 132105600 feet, which are to be reduced to rods.—We cannot easily divide by the mixed number $16\frac{1}{2}$ on account of the fraction $\frac{1}{2}$.—We overcome this difficulty, thus, $16\frac{1}{2}$ feet=33 half feet, and 132105600 feet = 264211200 half feet, which divided by 33 gives 8006400 rods; we reduce these rods to miles by dividing by 320, the number of rods in a mile, and to degrees by dividing by $69\frac{1}{2}$ miles=139 half miles, and 25020 miles=50040 half miles, which divided by 139 gives 360 degrees for the quotient.

Note.—When the divisor is a mixed number we may reduce it to halves, thirds, fourths, &c. and the *dividend* to the same; then divide, and the quotient will be the answer.

3. How many inches are in 273 miles?

Ans. 17297280in.

4. In 34594560in. how many miles? Ans. 546m.

5. Reduce 2m. 1fur. 8p. 3yds. 2in. to inches.

Ans. 136334 in.

6. Reduce 2280060 barley-corns to miles.

Ans. 11m. 7fur. 38p. 2yds. 2ft.

Questions.—In long measure, how are degrees reduced to barleycorns? Barleycorns to degrees?

7. Required the number of revolutions a wheel 18ft. 4in. circumference will make in running 150 miles.

Ans. 43200.

EXAMPLES IN CLOTH MEASURE.

1. In 15yds. 3qr. 1na. how many nails? Ans. 253na.
2. In 1012 nails of cloth, how many yards?

Ans. 63yds. 1qr.

3. Reduce 73 ells Flemish to quarters. Ans. 219qrs.
4. In 1752 nails, how many ells Flemish?

Ans. 146 ells.

5. How many ells English are in 1408 nails?

Ans. 70 E. E. 2qrs.

6. In 47656 nails how many ells English?

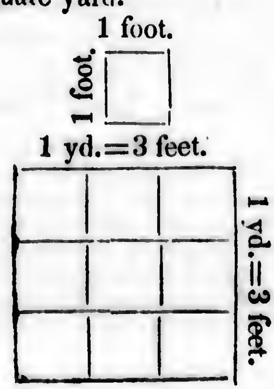
Ans. 2382 E. E. 4qrs.

7. Reduce 475 ells English to nails and these again to yards. Result 9500na. and 593yds. 3qrs.
8. In 4 bales of cloth, each 12 pieces, and each piece 24 ells English, how many yards and ells Flemish?

Ans. 1440yds. 1920 E. Fl.

EXAMPLES IN LAND OR SQUARE MEASURE.

By the table of long measure we learn that 3 feet in length make 1 yard, also by the table of square measure, that 9 square feet make 1 square yard, that is 3 feet in length and 3 feet in breadth. To make this plain let the small square represent 1 square foot. It will be perceived that the large square contains 9 such squares, or 1 square yard.



A square is the space included between 4 equal lines, drawn perpendicular to each other. Each line is called a side of the square. If each side be 1 foot, it is called a square foot, if 1 yard it is called a square yard: if 1 rod a square rod, &c.

QUESTIONS.—In cloth measure how do we reduce yds to inches? Ells English to inches? Ells Flemish to inches? Ells French to inches? Inches to yards, &c.?

NOTE.—When the length and breadth are given, we find its *square contents*, by multiplying the *length* by the *breadth*.

1. Required, the contents of a board 14 feet long and 2 feet wide. Ans. 28 feet

2. In 24 ac. 2 r. 26 p. how many perches?
Ans. 3946 perches.

3. Reduce 365 ac. 3 r. 13 p. to perches.
Ans. 58533 perches.

4. In 267464 perches, how many acres?
Ans. 1671ac. 2r. 24p.

5. If 2599200p. be divided into 25 equal tracts, how many acres will be in each? Ans. 649ac. 3r. 8p.

6. In 37456 square inches, how many square feet?
Ans. 260 sq. ft. 16 sq. in.

7. In 14972 square rods, how many acres?
Ans. 93a. 2r. 12p.

8. In 3674139p. how many square miles?
Ans. 35m. 563a. 1r. 19p.

EXAMPLES IN CUBIC MEASURE.

In long measure we consider length only: in square measure, length and breadth are both considered: cubic or solid measure has regard to *length*, *breadth* and *thickness*. A cube may be represented by a solid block having 6 equal sides. Suppose we have a number of blocks, containing a cubic foot each, if we lay 9 of these upon the floor, they will cover a square yard, and as each block contains a cubic foot, the solid contents of the square yard will be 9 cubic feet. Then if we

QUESTIONS.—How many feet in length make a yard? How many square feet make a square yard? What is a square? What is each line called? If each side be 1 foot, what is it called? If each side be 1 yard what is it called? When the length and breadth are given how do we find the contents of a square? How do we reduce miles to inches in Square Measure? Inches to miles?

cover this layer with another of blocks, containing also 9 solid feet, the pile will contain 18 solid feet; if we pile on yet another layer the pile will contain 27 solid feet, or one cubic yard, and as the pile is 3 feet in length, 3 feet in breadth, and 3 feet in depth, its solid contents are found by *multiplying* its *length* and *breadth*, and *depth together*.

1. How many cubic inches are there in a brick, that is 8 inches long, 4 inches wide, and 2 inches thick?

Ans. 64.

2. How many solid feet in a pile of wood 25 feet long, 4 feet wide, and 6 feet high? How many cords?

Ans. 600 feet and 4 cords and 88 feet.

3. In 14 tons of hewn timber, how many solid inches?

Ans. 1209600.

4. In 12 cords of wood, how many solid feet and inches? Ans. 1536 feet and 2654208in.

5. In 4608 solid feet of wood, how many cords?

Ans. 36.

EXAMPLES IN WINE MEASURE.

1. In 5 tuns 1 hogshead of wine, how many gallons?

Ans. 1323.

2. In 10584 quarts of wine, how many tuns?

Ans. 10 tuns, 2hhds.

3. In 24hhds. 18gals. 2qts. how many pints?

Ans. 12244.

4. In 1789 quarts of cider, how many barrels?

Ans. 14bbls. 25qts.

5. In 58 barrels of wine, how many gallons, quarts and pints? Ans. 1827gals. 7308qts. 14616pts.

QUESTIONS.—In long measure what do we consider? What in square measure? To what has cubic measure regard? How may a cube be represented? How many cubic feet in a cubic yard? What is its length? What is its breadth? What is its depth? How are its solid contents found? In cubic measure how are tons reduced to inches? How are cords reduced to inches? Inches to cords? To tons? How are tuns reduced to gills, in wine measure? Gills to tuns?

EXAMPLES IN ALE OR BEER MEASURE.

1. Reduce 47bar. 16gals. 4qts. to pints.
Ans. 13672pts.
2. In 64972 quarts of beer, how many barrels?
Ans. 451bar. 7gals.
3. In 27hhds. of beer, how many pints?
Ans. 11664.
4. Reduce 12528 pints to hogsheads. Ans. 29hhds.

EXAMPLES IN DRY MEASURE.

1. In 372 bushels, how many pints? Ans. 23808.
2. In 17408 pints, how many bushels? Ans. 272.
3. In 1597 quarts, how many bushels?
Ans. 49bush. 3pks. 5qts.
4. In 46321 pecks, how many chaldrons?
Ans. 321ch. 24bush. 1pk.

EXAMPLES IN TIME.

1. In 49 weeks, how many minutes? Ans. 493920.
2. In 24796800 seconds, how many weeks?
Ans. 41.
3. In 184009 minutes, how many days?
Ans. 127d. 18h. 49min.
4. In 214 days, 15 hours, 31 minutes and 25 seconds, how many seconds? Ans. 18545485.
5. In 126230400 seconds, how many years of 365 days? Ans. 4 years and 1 day.
6. How many hours in 4 years, allowing 365 days and 6 hours to the year? Ans. 35064.
7. In 12 years of 365 days, 23 hours, 57 minutes, 39 seconds each, how many seconds?
Ans. 379467108 sec.

EXAMPLES IN CIRCULAR MEASURE OR MOTION.

1. In 4s. 20° 15′ 34″ how many seconds?
Ans. 504934″.

QUESTIONS.—In beer measure how are hogsheads reduced to pints? Pints to hogsheads? In dry measure how do we reduce chaldrons to pints? Pints to chaldrons? How are years reduced to seconds? How are seconds reduced to years?

2. In 167573 seconds how many degrees?
Ans. 46° 32' 53''.

3. In 32295 minutes how many circles?
Ans. 1c. 5s. 28° 15'.

4. How many minutes and seconds in one complete revolution of a planet? Ans. 21600' 1296000''.

PRACTICAL EXERCISES.

1. In £25 14s. 1d. how many pence? Ans. 6169.

2. In 11520d. how many pounds? Ans. £48.

3. In 59lbs. 13pwt. 5grs. of gold, how many grains?
Ans. 340157.

4. In £85 8s. how many guineas?
Ans. 81 guineas, 7s.

5. How many cords are there in a pile of wood that is 36 feet long, 6 feet high and 4 feet wide?
Ans. 6 cords and 6 feet.

6. How many yards of carpeting, which is one yard wide, will be required to carpet a room 18 feet wide and 20 feet long? Ans. 40yds.

7. Reduce 346 ells Flemish to ells English.
Ans. $207\frac{3}{5}$ ells English.

8. A man would ship 720 bushels of corn in barrels which hold 3 bushels 3 pecks each; how many barrels must he get? Ans. 192.

9. How long will it take to count a million, at the rate of 50 a minute? Ans. 13d. 21h. 20m.

10. In 10 bales of cloth, each bale 12 pieces, and each piece 25 Flemish ells, how many yards?
Ans. 2250.

11. How many strokes does a regular clock strike in a year, striking once at one, twice at two, &c.
Ans. 56940.

12. In a cube, or square piece of wood 24 inches each way, how many small cubes of one inch each way can be sawed from it, allowing no waste in sawing?
Ans. 13824.

QUESTIONS.—In circular measure how is a circle reduced to seconds? Seconds to circles?

13. What number of bottles, containing a pint and a half each, can be filled from a barrel of cider?
Ans. 168.

14. In 3562 American dollars how many pounds sterling? the dollar being worth 4s. 7d. sterling.
Ans. £816 5s. 10d.

15. How many pints, quarts, and two quarts, each an equal number, may be filled from a pipe of wine?
Ans. 144.

16. In 4lb. 1oz. 1pwt. of silver, how many table-spoons, weighing 23pwt. each, and tea-spoons, weighing 4pwt. 6grs. each, can be made, and an equal number of each sort? Ans. 36.

17. How many steps of 2ft. 9in. each would a person take in walking from Kingston to Cobourg, allowing the distance to be 97 miles? Ans. 186240.

18. The sun is 95,000,000 of miles from the earth, and a cannon ball at its first discharge flies about a mile in $7\frac{1}{2}$ seconds; how long would it be at that rate in flying from here to the sun? Ans. 22yrs. 216d. 12h. 40m.

19. How often will a chariot wheel, 18 feet 4 inches in circumference, turn round in running 22 miles?
Ans. 6336.

20. How many bricks will it take to lay a floor 20 feet long and 18 feet wide, each brick being 8 inches square? Ans. 810 bricks.

21. If a field is 60 rods long, and 48 rods wide, how many acres does it contain? Ans. 18.

22. The forward wheels of a wagon are $14\frac{1}{2}$ feet in circumference, and the hind wheels 15 feet and 9 inches; how many more times will the forward wheels turn round than the hind wheels, in running from Boston to New York, it being 248 miles?
Ans. 7167 times, omitting remainders.

COMPOUND ADDITION.

Addition of compound numbers is collecting together two or more numbers of different denominations into one sum.

RULE.

Write the numbers so that those of the same denomination may stand directly under each other. Add the first column or denomination together as in simple numbers; then divide the sum by as many of the same denomination as make one of the next higher; set down the remainder under the column added, and carry the quotient to the next higher denomination, continuing the same to the last, which add, as in simple numbers.

Proof, the same as in addition of simple numbers.

EXAMPLES IN STERLING MONEY.

1. How many pounds, shillings, pence and farthings in £41 13s. 6d. 3qr. £48 11s. 4d. 3qr. £96 16s. 10d. 3qr.

Operation.

	£	s.	d.	qr.
	41	13	6	3
	48	11	4	3
	96	16	10	3
Sum	187	1	10	1

In the above example we first write the numbers of the same denomination under each other, as the rule directs. We then add up the column of farthings, and find it amounts to 9, but 9 farthings are equal to 2 pence and 1 farthing; we therefore write down the one farthing and carry the 2 pence to the column of pence.—In adding the column of pence we find the amount to be 22 pence, which is equal to 1 shilling, and 10 pence over; we write the 10 under the column of pence and carry the one to the column of shillings. We then find the sum of the shillings to be 41; that is 2 pounds, and 1 shilling over; carrying the 2 to the column of pounds we find the sum to be 187 pounds, 1s. 10d. 1qr.

Questions.—What is Compound Addition? Repeat the rule. How is Compound Addition proved?

NOTE.—In simple numbers we carry 1 for every 10; but in compound numbers we carry 1 for so many units of the lower denomination as make one of the next higher. For example, in passing from pence to shillings we carry 1 for every 12, and 1 for every 20 in passing from shillings to pounds, &c.

2. What is the sum of £16 18s. 9d. £14 13s. 8d. and £15 17s. 6d.?

Operation.

	£	s.	d.
	16	18	9
	14	13	8
	15	17	6
Sum,	47	9	11
	30	11	2
Proof,	47	9	11

(3)

	£	s.	d.
	173	13	5
	87	17	7¾
	75	18	7½
	25	17	8¼
	10	10	10½
Sum,	373	18	3

(4)

£	s.	d.
72	16	4¾
84	10	6½
12	12	9
28	8	1
41	19	3¼
50	1	8

(5)

£	s.	d.
31	11	11
7	0	8
19	18	9
41	16	0
59	0	6½
17	10	9

(6)

£	s.	d.
146	19	9½
310	17	10½
461	10	11¾
1	19	8¼
61	1	1
39	11	10¼

QUESTIONS.—In simple numbers when do we carry 1? In compound numbers when do we carry 1? In passing from pence to shillings what do we carry? In passing from shillings to pounds?

TROY WEIGHT.

Operation.
(1)

lb.	oz.	pwt.	gr.
11	8	18	19
114	9	6	16
223	10	17	18
Sum, 350	5	2	23

In adding up the column of grains, we find their sum to be 47=1pwt. 23gr. We set down the 23 and carry the 1pwt. to the column of pennyweights; we find their sum to be 42pwt. =2oz. 2pwt. Carrying 2 to the ounces, we find their sum to be 29oz.=2lb. 5oz. We then carry the 2 to the column of pounds, and find their sum to be 350.

(2)

lb.	oz.	pwt.	gr.
16	10	14	16
23	8	7	20
52	10	18	15
83	1	5	7
76	3	12	3
252	10	18	13

(3)

lb.	oz.	pwt.	gr.
273	1	15	14
342	9	3	5
657	10	13	21
724	9	10	17
652	7	18	23

APOTHECARIES WEIGHT.

(1)

lb.	oz.	dr.	scru.	gr.
24	7	2	1	16
17	11	7	2	19
36	6	5	0	7
15	9	7	1	13
9	3	4	1	9
16	10	3	2	17

(2)

lb.	oz.	dr.	scru.	gr.
12	11	6	1	15
4	9	7	0	12
9	10	1	2	16
4	8	1	2	19
9	0	0	1	10
4	9	2	1	6
46	1	4	1	18

AVOIRDUPOIS WEIGHT.

(1)

cwt.	qr.	lb.
2	3	27
1	1	17
4	2	26
6	1	13
3	3	15
6	2	16
26	0	2

(2)

lb.	oz.	dr.
24	13	14
17	12	11
26	12	15
16	8	7
24	10	12
11	12	12
122	7	7

(3)

T.	cwt.	qr.	lb.	oz.	dr.
91	17	2	24	13	14
19	9	0	17	10	12
14	13	2	4	9	11
47	11	3	19	14	5
69	0	1	0	0	12
77	19	3	27	15	11
320	12	2	11	1	1

4. A farmer sold 4 loads of hay, weighing as follows, viz: 12cwt. 3qrs. 16lbs., 13cwt. 2qrs. 12lbs. 13oz., 15cwt. 1qr. 21lbs. 11oz., 16cwt. 1qr. 24lbs. 8oz.; what was the weight of the whole in tons?

Ans. 2T. 18cwt. 1qr. 19lbs.

LONG MEASURE.

(1)

L.	mi.	fur.	rd.
16	2	7	39
327	1	2	20
87	0	1	15
1	1	1	1
432	2	4	35

(2)

yd.	ft.	in.	bar.
90	2	11	2
155	1	9	1
327	0	7	0
50	2	1	2
624	1	5	2

CLOTH MEASURE.

(1)

yds.	qrs.	na.
71	3	3
13	2	1
16	0	1
42	3	3
57	2	2
49	2	2
251	3	0

(2)

E.E.	qr.	na.
44	3	2
49	4	3
6	2	3
84	4	1
7	0	0
61	2	1
254	2	2

(3)

E.F.	qrs.	na.
84	2	1
7	1	3
76	0	2
52	2	3
53	2	2
9	2	3
285	0	2

LAND OR SQUARE MEASURE.

(1)

Sq. yd.	Sq. ft.	Sq. in.
97	4	104
22	3	27
105	8	2
37	7	127
263	5	116

(2)

M.	A.	R.	P.
1	700	3	37
6	375	2	25
7	450	1	31
11	30	0	25
27	277	0	38

SOLID OR CUBIC MEASURE.

(1)

T.	ft.
41	43
12	43
49	6
4	27
108	19

(2)

C.	ft.
3	122
4	114
7	83
10	127
27	62

(3)

feet.	inches.
13	1446
16	1726
3	866
14	284
48	866

WINE MEASURE.

(1)

gal.	qt.	pt.	gi.
39	3	1	3
17	2	1	2
24	3	0	1
19	1	1	2
8	0	0	3
40	2	1	1
150	2	1	0

(2)

hhd.	gal.	qt.	pt.
42	61	3	1
27	39	2	0
9	14	0	1
0	9	2	1
16	24	1	1
4	0	3	0
100	24	1	0

(3)

tun.	hhd.	gal.	qt.
34	2	34	2
19	1	59	1
28	2	2	1
19	0	32	2
37	3	11	1
0	1	9	0
139	3	22	3

DRY MEASURE.

(1)

ch.	bu.	pk.	qt.	pt.
27	25	3	7	1
59	21	2	6	3
2	1	2	7	1
5	9	1	8	2
94	22	3	7	1

(2)

ch.	bu.	pk.	qt.	pt.
141	36	3	7	2
21	32	2	4	1
85	9	1	0	3
10	4	4	1	3
259	12	0	0	1

TIME.

(1)

yr.	mo.	wk.	da.	hr.
4	11	3	6	20
3	10	2	5	21
5	8	1	4	19
101	9	3	7	23
55	8	4	6	17
172	2	1	4	4

(2)

wk.	da.	hr.	m.	sec.
8	8	14	55	57
10	7	23	57	49
20	6	14	42	10
6	5	23	19	59
2	2	20	45	48
50	4	1	41	34

CIRCULAR MOTION.

(1)

S.	°	′	″
3	29	17	14
1	6	10	17
4	18	17	11
6	14	18	10
16	8	2	52

(2)

S.	°	′	″
11	29	59	59
0	0	40	10
9	4	10	49
4	11	6	10
25	15	57	8

PRACTICAL EXERCISES.

1. A man bought a wagon for £18 16s. 8d., a plough for £2 10s., a span of horses for £55 10s. 6d.; what must he pay for the whole? Ans. £76 17s. 2d.

2. What is the sum of one hundred and five pounds, fourteen shillings, six pence, one farthing; eighty-four pounds, ten shillings, four pence, three farthings; and five hundred pounds, fifteen shillings, ten pence and three farthings? Ans. £691 0s. 9d. 3qr.

3. Bought a set of silver spoons weighing 1lb. 1oz. 9pwt. 17gr., a silver cup weighing 5oz. 10pwt. 14gr. and a silver tankard weighing 1lb. 11oz. 7gr.: what was the weight of the whole? Ans. 3lb. 6oz. 0pwt. 14gr.

4. The great bell at Moscow, the largest in the world, weighs 198tons, 2cwt. 1qr.: the bell at Oxford, the largest in England, weighs 7tons, 11cwt. 3qr. 4lb.; St. Paul's bell at London, 5tons, 2cwt. 1qr. 22lbs.; and the Tom of Lincoln, 4tons, 16cwt. 3qrs. 18lbs.: what is the sum of their weights? Ans. 215tons, 13cwt. 1qr. 16lbs.

5. A merchant bought 5 bales of cloth; the first contained 35yds. 2qr.; the second 42yds. 3qr.; the third 39yds. 3qr.; the fourth 45yds. 3qr., and the fifth 27yd. 2qrs.; how many yards did he buy? Ans. 191yds. 1qr.

6. If 1 cistern contains 25hhd. 27gal. 3qt.; a second 37hhd. 26gal. 2qt.; a third 35hhd. 54gal. 1qt.; and a 4th 45hhd. 15gal. 3qt.: what quantity will they jointly contain? Ans. 143hhd. 61gal. 1qt.

7. A man bought 3 piles of wood, which contain as follows, viz. 37 cords, 51 feet; 14 cords, 120 feet; 19 cords, 95; how many cords did he purchase? Ans. 72 cords 10 feet.

COMPOUND SUBTRACTION.

Compound subtraction like that of simple numbers, is taking a *less* number from a *greater*, but of different denominations. The principles involved are the same as those already explained in simple subtraction, except that it takes different numbers to make a unit in the next higher denomination as explained in compound addition.

RULE.

Write the less numbers under the greater, placing those of the same denomination directly under each other. Begin with the lowest denomination, and take successively the lower number in each denomination from the upper, and write the remainder underneath as in subtraction of simple numbers.

If the lower number be greater than the one above it, add as many units to the upper number, as make one of the next higher denomination, subtract the lower number therefrom, and add 1 to the next higher denomination in the subtrahend, and so on through all the denominations.

PROOF—as in simple subtraction.

EXAMPLES.

1. Subtract £29 19s. 8d. from £36 15s. 7d. Placing them according to the rule, they stand thus:

Operation.

£	s.	d.
36	15	7
29	19	8
6	15	11

In the above example 8d. cannot be taken from 7; we therefore add as many units of this denomination to 7, as are required to make a unit of the next higher, that is 12, which added to 7 make 19,

QUESTIONS.—What is Compound Subtraction? How does it differ from Simple Subtraction? Repeat the rule. If the lower number be greater than the one above it what is to be done?—How is Compound Subtraction proved?

subtract 8 from 19, and 11 remains. As 12 pence have been added to the minuend, an equal quantity must be added to the subtrahend, we therefore carry 1 shilling to the 19, which makes 20. As this cannot be subtracted from 15, we add to the 15 as many of this denomination as make one of the next higher; that is 20, which added to 15 makes 35. Subtract 20 from 35 and 15 remain. As 20s. have been added to the minuend, 1 pound must be added to the subtrahend of the next higher denomination, making it 30, which subtracted from 36 leaves 6.

(1)

	£	s.	d.	qr.
From	346	16	5	3
Take	128	17	4	2
Rem.	217	19	19	1
Proof,	346	16	5	3

(2)

	£	s.	d.	qr.
From	476	10	9	1
Take	277	17	7	1
Rem.	198	13	2	0

3. Borrowed £29 16s. and paid £16 11s. 9d.: how much remains due? Ans. £13 4s. 3d.

4. From £397 16s. 6d.; take £144 11s. 4d. 3qr. Ans. 253 5s. 1d. 1qr.

5. How much does £317 6s. exceed £178 18s. 5¼d.? Ans. £138 7s. 6¾d.

6. From eleven pounds take eleven pence. Ans. £10 19s. 1d.

7. From one hundred pounds, take four pence halfpenny. Ans. £99 19 7½d.

(8)

	lb.	oz.	pwt.	gr.
From	273	0	0	0
Take	98	10	18	21
Rem.	147	1	1	3
Proof,	273	0	0	0

(9)

lb.	oz.	dr.	scr.	gr.
40	7	3	2	7
24	4	5	2	8
16	2	5	2	19

(10)

T.	cwt.	qr.	lb.	oz.
7	14	1	3	6
2	6	3	4	11
5	7	1	26	11

(11)

Deg.	m.	fur.	pol.	yd.	ft.	in.	bc.
54	48	7	28	4	2	2	2
36	25	6	39	0	2	8	0
22	23	0	29	3	2	6	2

(12)

yd.	qr.	na.
47	3	2
27	2	3
20	0	3

(13)

A.	R.	P.
36	3	28
28	2	39
8	0	29

(14)

T.	ft.	in.
44	44	884
39	39	982
5	4	1630

(15)

T.	hhd.	gal.	qt.	pt.	gi.
88	3	55	3	0	0
48	0	62	2	0	2
40	2	56	0	1	2

(16)

ch.	bu.	pk.	qt.
47	34	3	6
20	35	0	7
26	35	2	7

(17)

yr.	mo.	wk.	da.	hr.	mi.	sec.
250	4	3	6	22	55	49
220	8	0	5	23	59	55
29	9	3	0	22	55	54

(18)

S.	°	′	″
9	23	45	54
3	7	40	56
6	15	44	58

PRACTICAL EXERCISES

IN COMPOUND ADDITION AND SUBTRACTION.

1. From one pound take one farthing.
Ans. 19s. 11d 3far.

2. What sum added to £17 11s. 8¼d. will make £100?

3. Lent a friend at one time £13 16s. 8d.; at another £35 10s. 6d.; he paid me at one time, £10 10s. 11d.; at another, £9 3s. 6d.; how much remains due?
Ans. £29 12s. 9d.

4. From 11 years, 6 months, 11 days, take 10 years, 6 months 29 days. Ans. 11mo. 12da.

5. A gentleman paid his 3 laborers as follows: to A. he gave £11 13s. 6d.; to B. 13s. 6d. more than to A.; and to C. he gave as much as A. and B. both; how much did the gentleman pay out? Ans. £48 1s.

6. A merchant bought 17cwt. 2qr. 14lb. of sugar, of which he sells 9cwt. 3qr. 25lb. how much of it remains unsold? Ans. 7cwt. 2qr. 17lb.

7. From a piece of cloth which contained 52yds. 2na. a tailor was ordered to take 3 suits, each 6yds. 2na.; how much remains of the piece? Ans. 32yds, 2qr. 2na.

8. The revolutionary war broke out between Great Britain and the Uunited States, April 19th, 1775, and a general peace took place January 20th, 1783; how long did the war continue? Ans. 7yr. 9mo. 1d.

9. A merchant buys two hogsheads of sugar, one weighing 8cwt. 3qr. 21lb., the other 9cwt. 2qr. 6lb.; he sells two barrels, one weighing 3cwt. 1qr. 21lb. 14oz., the other, 2cwt. 3qr. 15lb. 6oz.; how much remains on hand? Ans. 12cwt. 26lb. 12oz.

10. A boy sets out upon a journey, and has 200 miles to travel; the first day he travels 9 leagues, 2 miles, 7 furlongs, 30 rods; the second day 12 leagues, 1 mile, 1 furlong; the third day 14 leagues; the fourth day 15 leagues, 2 miles, 5 furlongs, 35 rods; how far had he then to travel? Ans. 14l. 1mi. 1fur. 15rd.

COMPOUND MULTIPLICATION.

To multiply a compound number by a simple one, is to repeat the compound number as many times as there are units in the multiplier.

I. When the multiplier does not exceed 12.

RULE.

Write down the compound number and set the multiplier under the lowest denomination. Multi-

---

QUESTIONS.—What is multiplying a compound number by a simple one? When the multiplier does not exceed 12 what is the rule?

ply the several denominations in succession by the multiplier; reduce the results to the next higher denomination, and set down the excess as in addition; proceed in the same way through all the denominations, and set down the entire product when you come to the last,

EXAMPLES.

1. Multiply £3 9s. 10d. by 4.

Operation.	£	s.	d.
	3	9	10
			4
Product,	13	19	4

Place the numbers as the rule directs, and then say 4 times 10d. are 40d.=3s. 4d.; we set down the 4d. in the lower line; then 4 times 9s. are 36s. and 3 shillings to carry make 39s.=£1, and 19s. over—set down the 19s.; then 4 times £3 are £12, and £1 to carry make £13, and the product of £3 9s. 10d. by 4=£13 19s. 4d.

	£	s.	d.	qr.	
2. Multiply	1	11	6	2	by 5
				5	
Product,	£7	17	8	2	

(3)

T.	cwt.	qr.	lb.	oz.
0	9	3	27	12
				7
3	9	3	26	4

4. What will be the cost of 5 yards of calico, at 3s. 9d. per yard? Ans. 18s. 9d.

5. A man bought 7 sheep at 9s. 6d. a head; what did they cost him? Ans. £3 6s. 6d.

6. What will be the cost of 9 hats at 9s. 9d. each? Ans. £4 7s. 9d.

7. Bought 9 pieces of shirting, each containing 28yds. 2qrs. 2na.; how many yards in all? Ans. 257 yds. 2qrs. 2na.

8. What is the cost of 12 bushels of wheat at 9s. 6d. per bushel? Ans. £5 14s.

9. What will 12 horses come to, at £29 16s. 8d. each? Ans. £358.

II. When the multiplier exceeds 12 and is a composite number.

RULE.

Multiply the compound number by one of the component parts or factors, and then that product by the other factor, and the last product will be the answer.

EXAMPLES.

1. What will 16 yards of broadcloth come to at £2 3s. 3d. per yard?

Operation.

	£2	3	3
			4
	8	13	0
			4
Ans.	34	12	0

In this example 4 and 4 are the factors of 16, because $4 \times 4 = 16$.

2. What will 20 barrels of flour come to, at £1 6s. 3d. each? Ans. £26 5s.

3. What must a man pay for 36 sheep, at 9s. 4d. each? Ans. £16 16s.

4. What is the weight of 48 pipes of wine, each weighing 18cwt. 2qrs. 14lbs. Ans. 894cwt. or 44T. 14cwt.

5. In 21 pieces of cloth, each containing 24yds. 2qrs. 3na.; how many yards? Ans. 518yds. 1qr. 3na.

6. How much water will be contained in 96hhds. each containing 62gal. 1qt. 1pt. 1gi. Ans. 55991gal.

7. What is the cost of 120 dozen of candles at 5s. 9d. per dozen? Ans. £34 10s.

III. When the multiplier exceeds 12, and is not a composite number.

QUESTION.—When the multiplier exceeds 12 and is a composite number what is the rule?

RULE.

Multiply the simple number by each of the denominations separately, and reduce each product to the highest denomination named. Then add the several products together, and their sum will be the answer sought.

EXAMPLES.

1. Multiply £5 3s. 8d. by 13.

Operation.

13	13
3s.	8d.
39s.=£1 19s.	104d.=8s. 8d.

13		
5£		
65£		
1	19s.	
	8	8d.
Ans. 67	7	8

In this example we first multiply the 13 by 8d., and the product is 104d.=8s.8d. then the 13 by 3s. and the product is 39s.=£1 19s.; then the 13 by £5, and the product is £65. We then add the several products together for the answer, as the rule directs.

2. What is the cost of 139 oxen at £6 8s. 9d. each.

Operation.

139×9d.=1251d.=£	5	4s.	3d.
139×8s.=1112s.=£	55	12s.	0
139×£6=£834=£	834	0	0
Ans. £	894	16	3

3. What is the worth of 47 pounds of butter at 9½d. per pound? Ans. £1 17s. 2½d.

4. What will 19 yards of cambric come to, at 11s. 6d. per yard? Ans. £10 18s. 6d.

5. What is the cost of 52 pounds of tea, at 5s. 9d. per pound? Ans. £14 19s.

Question.—When the multiplier exceeds 12, and is not a composite number, how do we proceed?

6. What is the weight of 17 hogsheads of sugar, each weighing 8cwt. 3qrs. 14lbs. Ans. 150cwt. 3qrs. 14lbs.

7. If one yard of cloth cost 7 shillings and 10 pence, what will 65 yards cost? Ans. £25 9s. 2d.

8. What is the cost of 46 bushels of wheat at 4s. 7¼d. per bushel? Ans. £10 11s. 9½d.

9. What is the cost of 117cwt. of raisins, at £1 2s. 3d. per cwt. Ans. £130 3s. 3d.

10. In 357 hogsheads of sugar, each 15cwt. 3qrs. 17lbs. how many cwt.? Ans. 5676cwt. 3qrs. 21lbs.

11. If a steamboat in crossing the Atlantic goes 211mi. 4fur. 32rd. a day, how far will she go in 15 days? Ans. 3174 miles.

COMPOUND DIVISION.

Compound Division is finding how often one number is contained in another of *different denominations*, and is the *reverse* of compound multiplication. When the price of one yard, one pound, &c. is given to find the price of a *quantity*, it is *done* by *multiplication;* but when the price of a quantity is given to obtain the price of *one article*, we *must divide* the *price* by the *quantity*, and the quotient will be the price of one pound, one yard, &c. &c.

RULE.

Place the divisor at the left hand of the dividend, and divide the left hand denomination, the same as a sum in simple division, and if any thing remains,

---

Questions.—What is compound division? Of what is it the reverse? When the price of one article is given to find the price of a quantity how is it done? When the price of a quantity is given to obtain the price of one article how is it performed? Repeat the rule. What will the quotient be, and in what denominations? How is multiplication proved? How is division proved?

reduce it by reduction, to the next lower denomination, and to the product add the next lower denomination of the dividend; then divide as before, and so continue to do, till the whole dividend be divided. The quotient figures will then be the answer in the same denominations as the dividend that produced them.

PROOF.—To prove multiplication we divide the product by one of the factors, and if the work be right the quotient will be the other factor. As division is the reverse of multiplication, to prove it we must multiply the quotient by the divisor, and if the product is equal to the dividend, the work is right.

EXAMPLES.

1. If 6 yards of cloth cost £7 14s. 4½d., what is the price per yard?

Operation.

	£	s.	d.	qr.	
6)	7	14	4	2	cost of 6yds.
	1	5	8	3	price of 1 yd.
				6	
	7	14	4	2	Proof.

In this example we say 6 is contained in £7 once, and £1 over; we write down the quotient, and reduce the remaining £1 to shillings, which with the 14s. added, make 34s.; we then say 6 in 34s. goes 5 times, and 4s. over; this 4s. reduced to pence=48d., which with the given pence, 4d. in the dividend, make 52d.; 6 in 52d. goes 8 times, and 4d. over; 4d.=16 qrs., which with the given quarters make 18, and 18 contains 6 three times, and it is evident that the several quotients £1 5s. 8d. 3qrs. are the true quotient arising from the division of the compound number by 6.

2. Bought 139 yards of cloth, for £461 11s. 11d.; what was that per yard?

Operation.

```
       £   s.  d.
139)461  11  11(3
    417
    ---
     44
     20
    ---
    891(6s.
    834
    ---
     57
     12
    ---
    695(5d.
    695
    ---
```

Here we say £461 contains the divisor 3 times, and £44 remain, which we reduce to shillings and add in the 11s., making 891s., in which the divisor is contained 6 times, and 57s. remain, which must be reduced to pence, and 11d. added, making 695d., which contains the divisor 6 times and no remainder.

NOTE.—When the divisor does not exceed 12 the work may be performed by short division—and if the divisor be a composite number, it may be performed as already explained in simple numbers.

3. Divide £28 2s. 4d. equally among 21 men.

Operation.

```
7)£28  2s. 4d.
  ----------
 3)4   0   4
  ----------
Ans. £1  6  9⅓
```

We divide by 7 and 3 because $7\times3=21$; hence each man receives £1 6s. $9\frac{1}{3}$d.

4. If £12 9s. 8d. be equally divided among 4 men, how much will each receive? Ans. £3 2s. 5d.

5. Three cows cost £22 3s. 9d.; what was the cost of each? Ans. £7 7s. 11d.

6. If 8 horses cost £185 17s. 6d., what was the cost of one horse? Ans. £23 4s. $8\frac{1}{4}$d.

7. A man paid £2 10s. for 15 bushels of corn; what did he pay per bushel? Ans. 3s. 4d.

8. Bought 36 bushels of apples for £2 14s.; what was that per bushel? Ans. 1s. 6d.

9. Sold 81 barrels of flour for £147 16s. 6d.; how much was that per barrel? Ans. £1 16s. 6d.

10. Bought 64 gallons of oil for £30 8s.; what did it cost per gallon? Ans. 9s. 6d.

11. Bought 144 reams of paper for £96; what did it cost per ream? Ans. 13s. 4d.

12. 176 men consumed in a week 13cwt. 3 qrs. 1lb. 6oz. of bread; how much did each man consume?

Ans. 8lb. 12oz. 2dr.

13. If 232 bushels 3 pecks 7 quarts of wheat be put into 105 bags, how much will each bag contain?

Ans. 2bu. 7qt.

14. Divide 18 gallons equally among 144 soldiers.

Ans. 1 pint each.

PRACTICAL EXERCISES

IN COMPOUND MULTIPLICATION AND DIVISION.

1. In 15 loads of hay, each weighing 1 ton 3cwt. 2qrs. how many tons? Ans. 17T. 12cwt. 2qrs.

2. What will 24 barrels of flour cost at £2 12s. 4d. per barrel? Ans. £62 16s.

3. The Prince of Wales receives a salary of £150,000 a year; how much is that per day?

Ans. £410 19s. 2d.

4. Bought a dozen silver spoons, which together weighed 3lb. 2oz. 13pwt. 12gr.; how much silver did each spoon contain? Ans. 3oz. 4pwt. 11gr.

5. Bought 17cwt. 3qrs. 19lbs. of sugar, and sold out one third of it; how much remains unsold?

Ans. 11cwt. 3qrs. 22lbs.

6. If 168 bushels 1 peck 6 quarts of wheat be put into 35 bags, how many bushels in each?

Ans. 4bu. 3pks. 2qts.

7. In 35 pieces of cloth, each measuring 27¾ yards, how many yards? Ans. 971yds. 1qr.

8. If a man's wages amount to £257 2s. 5d. in 12 months, what is that per month? Ans. £21 8s. 6¼d.

9. A privateer took a prize of £30,000, of which the owner took one third, and the officers one fourth; the remainder to be equally divided among 125 seamen; how much must each seaman receive? Ans. £100.

10. A certain gentleman lays up every year £294 12s. 6d., and spends daily £1 12s. 6d.; what is his income? Ans. £887 15s.

11. If 1cwt. of sugar cost £3 10s. what is it per pound? Ans. 7½d.

PRACTICAL EXERCISES

IN THE FOUR COMPOUND RULES.

1. Find the amount of forty pounds nine shillings, sixty-four pounds and nine pence, ninety-five pounds nineteen shillings, and seventeen shillings fourpence half-penny. Ans. £201 6s. 1½d.

2. Received of 4 men the following sums of money, viz: the first paid me £37 11s. 4d.; the second £25 16s. 7d.; the third, £19 14s. 6d., and the fourth as much as all the other three, lacking 19s. 6d. I demand the whole sum received. Ans. £165 5s. 4d.

3. Borrowed £100, and paid in part as follows: at one time £21 11s. 6d.; at another time £19 17s. 4½d.; at another time 10 dollars, at 6s. each, and at another time two guineas, at 28s. each, and two pistareens, at 14½d. each; how much remains due, or unpaid? Ans. £52 12s. 8½d.

4. How many days are there in 15 years of 365 days 5hrs. 48m. 51sec. each? Ans. 5478da. 15hr. 12m. 45sec.

5. A club in Quebec, consisting of 25 men, joined for a lottery ticket of £10 value, which came up a prize of

£4000. I wish to know what each man contributed, and what each man's share came to.

Ans. Each contributed 8s.; each share £160.

6. Three Merchants A. B. & C. have a ship in company. A. has $\frac{5}{8}$, B. $\frac{2}{8}$, and C. $\frac{1}{8}$, and they receive for freight £228 16s. 8d. It is required to divide it among the owners, according to their respective shares.

Ans. A's share £143 0s. 5d. B's £57 4s. 2d. and C's £28 12s. 1d.

7. A man paid £67 4s. for a pile of wood containing 64 cords; he sold 30 cords for £29 16s.; for how much must he sell the remainder per cord, so as not to lose?

Ans. £1 2s.

8. A gentleman purchased of a silversmith, 2 dozen silver spoons, each weighing 3oz. 4pwt. 1gr.; 2 dozen of tea spoons, each weighing 15pwt. 16gr.; 3 tankards, each weighing 22oz. 14pwt. He sold him old silver to the amount of 6lb. 10oz. 3pwt.; how much remained to be paid for? Ans. 6lb. 9oz. 12pwt.

9. A farmer has 6T. 8cwt. 2qrs. 14lbs. of hay to be removed in 6 equal loads; how much must be carried at each load? Ans. 1T. 1cwt. 1qr. 21lbs.

10. A merchant had £19118 to begin trade with; for 5 years together, he cleared £1086 a year; the next 4 years he cleared £2715 10s. 6d. a year; but the last 3 years he was in trade he had the misfortune to lose one year with another, £475 4s. 6d. a year; what was his real fortune at the end of 12 years? Ans. £33984 8s. 6d.

11. What will 1cwt. of cheese come to at $2\frac{1}{2}$d.; at $2\frac{3}{4}$d.; at 3d.; at 2d.; at $3\frac{1}{2}$d. per pound?

Answers in course: £1 3s. 4d.; £1 5s. 8d.; £1 8s.; 18s. 8d.; £1 12s. 8d.

12. Out of a pipe of wine a merchant draws 12 bottles, each containing 1 pint 3 gills; he then fills six 5 gallon demijohns: then he draws off 3 dozen bottles, each containing 1 quart 2 gills; how much remained in the cask?

Ans. 82gal. 1pt.

13. A privateer takes a prize worth £12465, of which the owner takes one half, the officers one fourth, and the remainder is equally divided among the sailors, who are 125 in number; how much is each sailor's part?

Ans. £24 18s. $7\frac{1}{5}$d.

14. A printer uses one sheet of paper for every 16 pages of an octavo book; how much paper will be necessary to print 500 copies of a book containing 336 pages, allowing 2 quires of waste paper in each ream.

Ans. 24 reams, 5 quires, 12 sheets.

BILLS OF PARCELS.

NOTE.—In keeping accounts, making out bills of parcels, we draw an oblique line and place the shillings on the left hand of it and the pence on the right, to designate the price of one article: thus, 2/6, 4/9, which are read 2s. 6d. and 4s. 9d.

Picton, October 14th, 1845.

1. Joseph Faithful,

Bought of James Trust,

8 yards of Calico at 1/3
$9\frac{1}{2}$ " " " 1/
10 " Cloth " 5/6
50 lbs. of Sugar, " $6\frac{1}{2}$d.
25 " of Rice " $3\frac{1}{2}$d.
30 " of Codfish " $4\frac{1}{2}$d.

Total Cost, £6 0s. $1\frac{1}{2}$d.

Kingston, Sept. 19th, 1845.

2. Otis Smith,

Bought of Richard Good,

15 yards of Satin, at 9/6
18 " of Silk, " 17/4
12 " of Fine Cloth, " 19/8,
16 " of Cambric, " 3/2,
13 " of Velvet, " 27/6,
23 " of Sheep's grey, at 6/3

Total Cost, £62 2s. 5d.

Quebec, June 18th, 1845.

3. Calvin Hawley,

Bought of John Punctual,

54 bushels	of Wheat,	at 4/9,	
75 "	of Oats,	" 1/8,	
25½ "	of Corn,	" 2/6,	
4½ tons	of Hay,	" 40/6,	
			£31 7s. 6d.

Montreal, July 24th, 1845.

4. Samuel Edwards,

Bought of Andrew White,

90 yards	of Broad Cloth,	at 8/4,	
100 "	"	" 10/6,	
112 "	Satinet,	" 3/7½,	
126 "	"	" 12/11¾,	
144 "	"	" 19/11,	
162 "	"	" 9/3,	
70 "	of Bombazine,	" 19/7½,	
198 "	of Italian Silk,	" 16/½,	
132 "	"	" 8/11,	
66 "	"	" 16/11½,	
			Ans. £752 14s. 1½d.

Toronto, Nov. 19th, 1845.

5. William Clark,

Bought of John Black, & Co.

92 yards of Satinet,	at	3/5¼,	
94 " of "	"	6/9¼,	
102 " of Durant,	"	1/8,	
104 Silk Vests,	"	6/7,	
106 Leghorn Hats,	"	11/9½,	
114 pieces Nankin,	"	8/3¾,	
116 pounds of Thread,	"	9/11¼,	
			Ans. £257 17s. 8d.

FEDERAL MONEY.

Federal Money is the national currency of the United States. Its denominations are,—the *mill*, the *cent*, the *dollar* and *eagle*.

TABLE OF FEDERAL MONEY.

10 mills are equal in value to 1 cent.
10 cents are equal to 1 dime.
10 dimes, or 100 cents, are equal to 1 dollar.
10 dollars are equal to one eagle.

In this table, 10 units of either denomination, make one unit of the next higher denomination, and this is the same way that simple numbers increase from the right to the left. Therefore, the denominations *of federal money may be added, subtracted, multiplied and divided by the same rules that have already been given for simple numbers.*

From the table it appears—First, *that cents may be changed into mills by multiplying by* 10, *or by annexing a cipher*. Thus, 5 cents=50 mills. *Second, that dollars may be changed into cents by multiplying by* 100, *or annexing two ciphers, and into mills by annexing three.* Thus, 8 dollars=800 cents, or 8000 mills.

This character, $, placed before a number shows the number to express dollars, thus $14, is 14 dollars. When dollars and cents are written in one sum, they are separated by a point, thus, $10.45: to be read 10 dollars and 45 cents. *Take notice, there must be two places of figures for cents:* therefore if the cents be less than 10, a *cipher*

QUESTIONS.—What is federal money? What are its denominations? Repeat the table. How many units of either denomination make one of the next higher? How may federal money be added? subtracted? multiplied? and divided? How may cents be changed to mills? Dollars into cents? How into mills? How many cents in 8 dollars? in 9? in 10? What character stands for dollars? When dollars and cents are written together, how are they separated? How many places must there be for cents?

must be placed on the left hand of the cents, thus, 84 dollars and 4 cents is written, $84.04.

Federal money is generally read in dollars and cents.—Cents are reduced to dollars by dividing by 100, or, which is the same thing, by cutting off the two right hand figures, mills to dollars by cutting off three right hand figures.

EXAMPLES.

1. How many cents in 89 dollars? Ans. 8900 cents.
2. How many cents in 468 dollars? Ans. 46800 cents.
3. How many cents in 48 and 19 cents? Ans. 4819 cents.
4. How many dollars in 8643 cents? Ans. $86.43.
5. How many dollars in 1903 cents? Ans. $19.03.
6. How many dollars in 6489 mills? Ans. $6.48.9.
7. What is the sum of $34.25, $18.04, $142, $176.81 and $0.58.

Operation.

34.25	In writing federal money for addition be careful to place dollars under dollars, cents under cents, &c. Then add the same as in simple numbers.
18.04	
142.00	
176.81	
.58	
$371.68	

8. Add together 36 dollars, 7 dollars and 45 cents, 86 cents, 130 dollars and 6 cents, and 340 dollars 1 cent. Ans. 514 dollars 38 cents.
9. Add together 46 dollars 9 cents, 100 dollars 7 cents, 99 dollars 75 cents, 451 dollars 99 cents, and 1 dollar 1 ct. Ans. 698 dollars 91 cents.
10. What is the expense of one quarter's schooling, allowing $19 for board, $9 for tuition, $3.75 for books, and 92 cents for stationary? Ans. $32.67.

QUESTIONS.—When the cents are less than 10 what is to be done? How is federal money generally read? How are cents reduced to dollars? How are mills reduced to dollars? How do we set down federal money for addition? How do we add federal money?

11. A man paid $75.41 for a horse, $54.04 for a yoke of oxen, $21 for a cow, $7.41 for 4 sheep, $1.50 each for two pigs, and $64 for a wagon; how much money did he pay out? Ans. $224.86.

12. From 319 dollars take 47 dollars and 56 cents.

Operation.
```
 319,00
  47,56
 ------
$271,44
```

When either of the sums in Federal money presented for subtraction has no cents expressed, the places of cents may be supplied by two ciphers; then proceed as in simple numbers.

13. Subtract 654 dollars from 783 dollars and 48 cents. Ans. $

14. Subtract $31,12 from $5390. Ans. $

15. Subtract 42 cents from $51. Ans. $

16. Subtract 7 cents from $1. Ans. $

17. Subtract 5 cents from $754. Ans. $

18. Subtract 4 cents from $4 Ans. $

19. A man's income is $3000 a year; he spends $187,50; how much does he save? Ans. $2812,50.

20. How much must be added to $40,17 to make $100. Ans. $59,83.

21. How much is 18 times 4 dollars 72 cents?

Operation.
```
  4,72
    18
 -----
  3776
  472
 -----
$84,96
```

$4,72 is the same as 472 cents; we therefore multiply it as 472 cents, and the product is 8496 cents. To change these cents to dollars we divide them by 100; this is done by pointing off two figures for a remainder.—The quotient, 84, is dollars, and the remainder, 96, is cents.

NOTE.—If we wish to obtain the value of several articles in Federal money, the *articles may be multiplied by the price of one, or the price by the articles*, and the *pro-*

QUESTIONS.—When either of the sums presented for subtraction has no cents expressed, what do we do? How do we then proceed? How do we obtain the value of several articles in federal money? In what denomination will the answer be?

duct will be the answer in the lowest denomination mentioned in the price.

22. What must be paid for 6 pounds of tea, at $1,20 per pound? Ans. $7,20.

23. At $1,05 per pound, what is the value of 5 chests of tea, each chest containing 64 pounds? Ans. $336.

24. What will 55 yards of cloth come to at 37 cents per yard? Ans. $20,35.

25. What will 300 bushels of wheat come to at $1,25 per bushel? Ans. $375.

26. What is the value of 9704 oranges, at 3½ cents each? Ans. $339,64.

27. What will be the cost of 47 barrels of apples at 1¾ dollars per barrel? Ans. $82,25.

28. If 637 dollars be divided equally among 24 men, what will each man receive?

Operation.

```
24)637(26 dollars.
   48
   ——
   157
   144
   ———
    1300(54 cts.
    120
    ———
     100
      96
     ———
Rem.  4 cents.
```

After dividing the dollars by the number of men, it appears from the quotient and remainder, that each man can have $26, and still 13 dollars will remain undivided. We change $13 to cents, by annexing two ciphers, and then divide the cents by the number of men, from which it appears each man will have 54 cts., and 4 cents will remain.

29. A man bought a piece of cloth containing 72yds. for $252: what did he pay per yard? Ans. $3,50.

30. A farmer purchased a farm containing 725 acres, for which he paid $18306.25: what did it cost him per acre? Ans. $25.25.

Question.—In dividing dollars, if a remainder occur, what is to be done in order to divide the remainder?

31. A farmer receives $840 for the wool of 1400 sheep: how much does each sheep produce him?
Ans. $0.60.

32. At $954 for 3816 yards of flannel, what is that a yard? Ans. $0.25.

33. Bought 72 pounds of raisins for $8, what was that a pound? Ans. $11\frac{1}{9}$ cents.

PRACTICAL EXERCISES IN FEDERAL MONEY.

1. Bought 1 barrel of flour at 6 dollars 75 cents, 10lbs. of coffee for 2 dollars 30 cents, 7 pounds of sugar for 92 cents, 1 pound of raisins for $12\frac{1}{2}$ cents, and 2 oranges for 6 cents; what was the whole amount? Ans. $\$10.15\frac{1}{2}$.

2. A man having $500 lost 83 cents: how much had he left? Ans. $499.17.

3. How much must be added to $16.82 to make $25?

4. If I pay 22 cents a gallon for 72 hogsheads of molasses, each hogshead containing 63 gallons, and then sell the whole for $936, how much do I lose? Ans. $61.92.

5. From 3 dollars take 7 cents. Ans. $2.93.

6. A man dies leaving an estate of $33000 to be equally divided among his four children after his wife shall have taken her third. What was the wife's portion, and what the part of each child?

Ans. { $11000 wife's part.
{ $ 5500 each child's part.

7. A person on settling with his butcher finds that he is charged with 126 pounds of beef at 9 cents per pound; 85 pounds of veal at 6 cents per pound: 6 pairs of fowls at 37 cents a pair: and three hams at $1.50 each: how much does he owe him? Ans. $23,16.

8. A farmer bargains with his tailor for a new coat every six months, a new vest every three months, and three pairs of pantaloons a year: the coats to cost $29.50 each, the vests $3 a piece, and the pantaloons $12 a pair: at the end of two years how much did he owe him. Ans. $214.

9. A farmer has six ten acre lots, in each of which he pastures 6 cows: each cow produces 112 pounds of but-

ter, for which he receives $18\frac{1}{2}$ cents per pound: the expences of each cow are 5 dollars and a half: how much does he make by his dairy? Ans. $547.92.

10. A man lets out 2000 sheep with the condition that he is to have three fourths of what they produce after deducting the expenses of shearing: they yield 4 pounds of wool a head, which is sold at $47\frac{1}{2}$ cents per lb. The expense of shearing is one tenth of the whole: what does the owner of the sheep receive? Ans. $25.65.

END OF PART II.

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ion
uce
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lb.
hat
65.

PART III.

SIMPLE INTEREST.

Interest is money paid for the use of money that has been borrowed. The *sum* of money lent is called the *principal.* The *sum paid* for the use of money is called *interest.* *Amount* is the *principal* and *interest* added together. *Per annum* signifies *by the year.* It is customary to pay a certain sum for every hundred pounds, dollars &c.— In this Province six pounds a year is paid for the use of every hundred, and in England five pounds for every hundred that is borrowed. These rates are established by law and are called *legal interest.*

Usury is taking more interest than the law allows.

The expressions six per cent, seven per cent, &c., signify that six or seven pounds or dollars are paid for every hundred borrowed. Per signifies for, and cent. is the abbreviation of centum, the Latin word for hundred. *Rate per cent.*, then signifies *rate by the hundred.*

In all notes on interest if no particular *rate per cent.* is mentioned, it is always understood to be *legal interest* that is promised. In this work 6 *per cent.* will be understood when no rate per cent. is mentioned.

Questions.—What is interest? What is the principal?—What is the sum called which is paid for the use of the principal? What is the amount? What is meant by per annum? What is legal interest? What is usury? What is the meaning of per cent.? When no rate per cent. is mentioned what interest is understood?

CASE I.

To find the interest of any given sum for one or more years.

RULE.

I. Multiply the principal by the rate per cent, and divide the product by 100 and the quotient will be the interest for one year.

II. When the number of years exceeds one, multiply the interest for one year by the number of years: the product will be the interest for that number of years.

EXAMPLES.

1. What is the interest of £150 10s. 6d. for one year, at 6 per cent?

Operation.

```
£150  10s.  6d.
              6
---------------
 £9|03   3   0
    20
---------------
 s.0|63
    12
---------------
 d.7|56
     4
---------------
qr.2|24 Rem.
```

We first multiply the principal by the rate per cent. and divide the product by 100, by cutting off the two right hand figures in the pounds—We then multiply the remaining right hand figures by the next inferior denomination, and divide by 100 as before, and so on through all the denominations.

Ans. £9 0s. 7d. 2qr.

2. What is the interest of £155 for one year?
Ans. £9 6s.

3. What is the interest of £236 10s. 4d. for a year at 5 per cent? Ans. £11 16s. 6d.

4. What is the interest of £2 12s. 9½d. for a year?
Ans. £0 3s. 2d.

Questions.—How do we find the interest on any given sum for one year? How do we find the interest for several years?

5. Required the interest of £200 10s. 4d. for 3 years at 7 per cent. Ans. £42 2s. 2d.

6. What is the interest of £150 8s. 3d. for 5 years at 6 per cent? Ans. £45 2s. 5½d.

7. Required the amount of £547 15s. at 5 per cent. per annum for 3 years. Ans. £629 18s. 3d.

8. What is the amount of £500 for 5 years at 5 per cent? Ans. £625.

9. What is the interest of £325 12s. 3d. for five years at 6 per cent? Ans. £97 13s. 8d.

10. Required the interest of £855 17s. 6d. for one year at 5¾ per cent per annum. Ans. £49 4s. 3d.

11. What is the interest of £246 18s. for five years at 4¼ per cent per annum? Ans. £52 9s. 3¼d.

CASE II.

To find the simple interest of any sum of money, for any number of years, and parts of a year.

I. Find the interest on the given sum for 1 year.

II. Multiply the interest of one year by the given number of years, and the product will be the answer for that time.

III. If there be parts of a year, as months and days, work for the months by the aliquot or even parts of a year, and for the days by the aliquot parts of a month, or for the *days* multiply the interest of one year by the number of days and divide the product by 365.

NOTE.—An exact or even part of a quantity is called an aliquot part, as ½, ¼, ⅙, &c. If we wish to find the interest of any sum for any part of a year it may be done by dividing the interest for a year by that part. For example, for six months, which is half a year, we divide the yearly interest by 2, for 1 month by 12, &c. and the same in regard to parts of a month. By a little exercise of the mind it is plain that one month is $\frac{1}{12}$ of a year; 2 months the ⅙ of a year; 3 months the ¼ of a year; 4 months the ⅓, and 6 months the ½ of a year, &c. And so of the days, allowing 30 days to the month; 1 day is $\frac{1}{30}$ of a month, 2 days $\frac{1}{15}$, 3 days $\frac{1}{10}$, 5 days $\frac{1}{6}$, 6 days $\frac{1}{5}$, 10 days $\frac{1}{3}$, and 15 days $\frac{1}{2}$ of a month, &c. It is sometimes more convenient to take parts of parts.

EXAMPLES.

1. What is the interest of £150 16s. 8d. for 4 years, 7 months and 20 days at 6 per cent?

Operation.		
£	s.	d.
150	16	8
		6
9|05	0	0
20		
1|00		

First find the interest for 1 year and then for the number of years. Then because 6 mo. are the one-half of a year, we divide the interest of one year by 2, and the quotient is the interest for 6 mo. We then divide the interest of 6 mo. by 6 for the interest of one month, and the interest of 1 mo. by 2 because 15 days are the $\frac{1}{2}$ of 1 mo. and the interest of 15 days by 3 because 5 days are the $\frac{1}{3}$ of 15 days. Wherefore the sum of these several numbers must be the interest for the whole time.

	£	s.	d.		
6 mo. = $\frac{1}{2}$ year.	£9	1s.		= Interest for 1 year.	
		4			
	36	4		= do.	for 4 do.
1 mo. is $\frac{1}{6}$ of 6 mo.	4	10	6	= do.	for 6 months.
15 da. is $\frac{1}{2}$ of 1 mo.	0	15	1	= do.	for 1 month.
5 da. is $\frac{1}{3}$ of 15 da.	0	7	6 2	= do.	for 15 days.
	0	2	6 $0\frac{2}{3}$	= do.	for 5 do.
	41	19	7 $2\frac{2}{3}$	Ans.	

2. What is the interest of £100 10s. for 11 months?

QUESTIONS.—How do we find the interest for months? How for days? By what other method may the interest for days be found?

Operation.

```
          £100  10s.
                 6
          ----------
6 mo.|½|  603  00 Int.for 1 yr
     | |  --------
4 mo.|⅓|  301  10 do. 6 mo.
1 mo.|¼|  201  00 do. 4 mo.
     | |   50  05 do. 1 mo.
          ----------
         £5,52  15
                20
          ----------
         s.10,55
                12
          ----------
           d.6,60
                 4
          ----------
           qr.2,40
          ----------
```

Ans. £5 10s. 6d. 2qrs.

We find the interest for 6 mo. by dividing the interest for 1 year by 2, then for 4 mo. by dividing the interest for 1 year by 3, and for 1 mo. by dividing the interest for 4 mo. by 4, then add the several sums together; 6 mo. 4 mo. and 1 mo.=11 months.

We may if we please add the several sums together before dividing by 100, as in the above example, which is frequently the better way.

3. What is the interest of £250 16s. 8d. for 28 days?

```
      £   s.
      15  1 Int. for 1 yr.
         28 numb. of d'ys
    ----------
365)421  8(£1
    365
   -----
     56
     20
   -----
```

First, find the interest for 1 year, multiply it by the no. of days, and divide the product by 365 as the rule directs.

(*Over.*)

QUESTIONS.—What part of a year is 1 month? 2 months?—3 months? 4 months? 6 months What part of a month is a day? 2 days? 3 days? 5 days? 6 days? 10 days? 15 days?

```
 1128(3s.
 1095
 ----
   33
   12
 ----
  396(1d.
  365
 ----
   31
```

Ans. £1 3s. 1d.

4. What is the interest of £57 17s. 8d. for three months? Ans. 17s. 4¼d.

5. Required the interest of £174 10s. 6d. for 3 years and 6 months? Ans. £36 13s.

6. Of £150 16s. 8d. for 4 years and 7 months? Ans. £41 9s. 7d.

7. Of £75 8s. 4d. for 5 years and 2 months? Ans. £23 7s. 7d.

8. What will £3000 amount to in 12 years and 10 months? Ans. £5310.

9. What is the interest of £257 5s. 1d. for 1 year and three quarters, at 4 per cent? Ans. £18 0s. 1¾d.

10. What will £279 13s. 8d. amount to in 3 years and a half at 5¼ per cent. per annum? Ans. £331 1s. 6d.

11. What is the interest of £71 3s. 11½d. for 1 year, 5 months and 25 days? Ans. £6 6s. 11.

12. What is the interest of £397 9s. 5d. for 2 years 3 months, at 3½ per cent? Ans. £31 6s.

13. Required the interest of £300 for 2 years, 7 mo. and 20 days? Ans. £47 10s.

14. What is the interest of £80 19s. 11d. for 3 years, 10 months and 10 days? Ans. £18. 15s. 3¼d.

15. What is the interest of £320 10s. 8d. for 2 years, 10 months and 20 days? Ans. £55 11s. 2d.

16. Required the interest of £500 for 3 years, 11 mo. and 11 days, at 3½ per cent.? Ans. £69 1s. 6¼d.

17. What is the interest of £100 for for 45 days? Ans. 14s. 9d. 2qr.

18. What is the interest of £25 18s. 9d. for 9 years 9 months and 9 days at 5 per cent? Ans. £12 13s. 6d. 1qr.

RATIO.

The word *ratio* means *relation*; and when it is asked what ratio one number has to another, it means *in what relation* does *one number stand to another?*

Thus, when we say the ratio of 1 to 2 is $\frac{1}{2}$, we mean that the relation in which 1 stands to 2 is that of *one-half* to a *whole.*

Again, the ratio of 3 to 4 is $\frac{3}{4}$, that is, 3 is $\frac{3}{4}$ of 4, or stands in the relation of $\frac{3}{4}$ to the 4. So also the ratio of 4 to 3 is $\frac{4}{3}$; for the 4 is 4 thirds of 3, and stands to it therefore in the relation of $\frac{4}{3}$.

The relation of 12 to 24 is $\frac{1}{2}$, and the relation of 24 to 12 is 2.

When therefore we find the ratio of one number to another, we find *what part of one number another is.* Then the ratio of 4 to 6 is $\frac{4}{6}$; that is, 4 is 4 sixth of 6. The ratio of one number to another

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QUESTIONS.—What is the meaning of the word ratio? When it is asked what ratio one number bears to another, what is meant? What is the ratio of one to 2? What is the relation of 12 to 24? Of 24 to 12? When we obtain the ratio of one number to another what do we find? What part of 6 is 4?

then may always be expressed by a fraction, in which the *first number* is put for *numerator*, and is called the *antecedent*, and the second number is put for *denominator*, and is called the *consequent*. Thus the ratio of 8 to 4 is $\frac{8}{4}$ or 2. That is, 8 is twice 4, or stands to 4 in the relation of a *duplicate* or *double*.

RULE OF THREE, OR PROPORTION.

When quantities have the *same ratio*, they are said to be *proportional* to each other. Thus the ratio of 2 to 4 is $\frac{1}{2}$, and the ratio of 4 to 8 is $\frac{1}{2}$; that is, 1 has the same relation to 2 that 4 has to 8; therefore these numbers are called *proportionals*. Again, 4 is the same *portion* or *part* of 8 that 10 is of 20; wherefore these numbers are called *proportionals*. A *proportion* then is a *combination* of *equal ratios*.

Points are used to indicate that there is a proportion between numbers. Thus, 4 : 8 : : 9 : 18 is read thus, 4 has the same ratio or relation to 8 that 9 has to 18. Or more briefly, 4 is to 8, as 9 to 18.

The fourth term of *every proportion* may be found by *multiplying* the *second* and *third terms together* and *dividing* their *product* by the *first term*. For example, if the first three terms of a proportion are 3, 9, 12, the fourth is 36, for $9 \times 12 \div 3 = 36$.

QUESTIONS.—How may the ratio of one number to another be expressed? Which number is put for numerator? What is put for denominator? What is this called? When are quantities said to be proportional to each other? Give examples. What are used to indicate that there is a proportion between numbers? Give an example on the slate. How is it read? How may the fourth term of any proportion be found? If the first three terms are 2, 4 and 6, what is the fourth?

The first and fourth *terms* of a proportion are called the two *extremes*, and the second and third *terms* are called the two *means*.

In *every* proportion the *product* of the two *extremes* is *equal* to the *product* of the two *means*.

Eor example, in the proportion 4 : 12 : : 8 : 24 $4\times24=8\times12=96$.

The Rule of Three takes its name from the circumstance that *three* numbers are *always given* to find a *fourth*, which shall bear the same proportion to one of the *given numbers* as exists between the *other two*.

To find the fourth term when three are given we have the following general

RULE.

I. Write that number for the *third term* which is of the same kind with the answer or number sought.

II. Then consider from the nature of the question, whether the answer required must be greater or less than this third term; if *greater*, write the *greater* of the other given numbers for the *second term*, and the *less* for the first; but if the required answer must be *less* than the third term, set down the less of the other two numbers for the second term, and the greater for the first.

III. If the first and second terms contain *different denominations* they must be reduced to the

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QUESTIONS.—What are the first and fourth terms of a proportion called? What are the second and third terms called? In every proportion what is the product of the two extremes equal to? Give an example. From what does the Rule of Three take its name? In stating a question what is the first thing to be done? What is the next? If the required answer must be greater than the third term how must we place the other two numbers? How if the answer is to be less than the third term?

same denomination, and the *third* to the *lowest* denomination mentioned in it.

Then multiply the *second* and *third terms* together, and divide the product by the *first term,* and the quotient will be the fourth term or *answer* sought of the *same denomination* as that to which the *third term* was reduced.

NOTE.—The same rule is applicable, whether the given quantities be integral, fractional or decimal.

PROOF.—Divide the product of the extremes by one of the mean terms, and if the work is right the quotient will be the other mean term.

EXAMPLES.

1. If 5 pounds of butter cost 75 pence; how much will 13 pounds cost?

Operation.

```
As 5lb.: 13lb. : : 75d.
                   13
                  ---
                  225
                  75
                  ---
                5)975
                  ---
                  195 pence.
```

In this example it is plain that the answer must be money; we therefore write the 75 pence as the third term. It is also plain that the price of 13 pounds is greater than the price of 5 pounds, that is, the required answer is greater than the third term; we therefore set down the 13lbs. for the second term, and the 5lbs. for the first term; we then multiply the second and third terms together, and divide by the first, as the rule directs, and the quotient is the answer in the same denomination as the third term.

2. If a footman perform a journey in 21 days, when

QUESTIONS.—If the first and second terms contain different denominations what must be done? How must we reduce the third term? Which two do we multiply together? By what do we divide the product? What will the quotient be? Of what denomination?

the days are 15 hours long, in how many days of 9 hours can he perform the same journey?

Operation.	
hrs. hrs. days.	
9 : 15 :: 21	
15	
9)315	
Ans. 35 ds.	

Here the term similar to the required answer is 21 days, which is consequently the third term. Then, since the footman will not travel so far in a day of 9 hours as in one of 15 hours, the required answer must be greater than the third term; the 15hrs. must therefore be the second term and 9hrs. the first.

Operation of Proof.
$9\times35\div15=21$ or
$9\times35=315\div21=15$

In this example the terms 9, 15 and 21 are given, and the last term 35 is found. Then the product of the extremes 9, 35, is 315; this being divided by either of the means gives the other.

3. If 3cwt. of sugar cost £9 2s. 0d. what will 4cwt. 3qrs. 26lbs. cost at the same rate?

Operation.

cwt.		cwt.	qr.	lb.		£	s.	d.
3	:	4	3	26	::	9	2	0
4		4				20		
12		19				182		
28		28				12		
336lb.		558lb.				2184d.		
						558		

336)1218672(3627d.
12)3627
20)302 3d.
£15 2s. 3d. Ans.

QUESTION.—How is the Rule of Three proved?

First state the question as the rule directs. We then reduce the first and second terms to pounds, then the third to pence, which is the lowest denomination named in it. We then multiply the second and third terms together and divide by the first, and the quotient is pence, which we reduce to pounds, &c.

4. If a man's wages be £75 10s. for 12 months, what is that a month? Ans. £6 5s. 10d.

5. If one pair of gloves cost 4s. 6d. what will 19 dozen pairs cost? Ans. £51 6s.

6. At 10½d. per pound, what is the value of a firkin of butter weighing 56lbs.? Ans. £2 9s.

7. If a man spends 3s. 4d. per day, how much is that a year? Ans. £60 16s. 8d.

8. What is the value of 2cwt. of sugar at 5d. a pound? Ans. £4 13s. 4d.

9. If 240 sheep yield 660lbs. of wool, how many pounds will be obtained from 1200 sheep? Ans. 3300lbs.

10. If 4½ tons of hay will keep 3 cattle over the winter, how many tons will it take to keep 25 cattle the same time? Ans. 37½ tons.

11. Bought 8 chests of sugar, each 9cwt. 2qrs.; what do they come to at £2 5s. per cwt.? Ans. £171.

12. How much butter can you buy for £23 2s., at 9d. per pound? Ans. 5cwt. 2qrs.

13. If 30 barrels of flour will support 100 men for 40 days, how long would it subsist 25 men? Ans. 160 days.

14. A owes B £679 6s., but compounds with him by paying 3s. 4d. on the pound; how much does B receive for his debt? Ans. £113 4s. 4d.

15. A goldsmith sold a tankard for £8 12s. at 5s. 4d. per oz.; what was the weight of the tankard? Ans. 2lbs. 8oz. 5pwt.

16. If 2cwt. 3qrs. 21lbs. of sugar cost £6 1s. 8d. what cost 35¼cwt.? Ans. £73.

17. If a man spend 7d. per day for bitters, how much is that in a year? Ans. £10 12s. 11¼d.

18. If 90 bushels of oats will feed 40 horses for 6 days, how long will 450 bushels last them? Ans. 30 days.

19. If 5cwt. 3qrs. 14lbs. of sugar cost £6 1s. 8d., what will 35cwt. 28lbs. cost? Ans. £36 10s.

20. What is the cost of 3cwt. of coffee at 15d. per pound? Ans. £21.

21. If 3 quarters of a yard of velvet cost 7s. 3d. how many yards can be bought for £13 15s. 6d.
Ans. 28yds. 2qrs.

22. Sold a ship for £537, and I owned $\frac{3}{8}$ of her; what was my part of the money? Ans. £201 7s. 6d.

23. If a staff 5 feet long cast a shade on level ground 8 feet, what is the height of that steeple whose shade at the same time measures 181 feet? Ans. $113\frac{1}{8}$ feet.

24. Bought 50 pieces of kerseys, each 34 ells Flemish, at 8s. 4d. per ell English; what did the whole cost?
Ans. £425.

25. Bought 200 yards of cambric for £90, but being damaged, I am willing to lose £7 10s. by the sale of it; what must I demand per ell English? Ans. 10s. $3\frac{3}{4}$d.

26. If an ingot of gold weighing 9lb. 9oz. 12pwt. be worth £470 8s. what is that per grain? Ans. 2d.

27. Bought 4 bales of cloth, each containing 6 pieces, and each piece 27 yards at £16 4s. per piece; what is the value of the whole and the cost per yard?
Ans. £388 16s. at 12s. per yd.

28. What will be the cost of 72 yards of cloth at the rate of £5 12s. for 9 yards? Ans. £44 16s.

29. A person's annual income is £146; how much is that per day? Ans. 8s.

30. If 3 paces or common steps of a person be equal to 2 yards, how many yards will 160 paces make?
Ans. 106 yards 2ft.

31. How many yards of carpeting that is 3 feet wide, will cover a floor that is 27 feet long and 20 feet broad?
Ans. 60 yards.

32. What is the cost of 6 bushels of coal at the rate of £1 14s. 0d. the chaldron? Ans. 5s. 9d.

33. When hens are 9 shillings a dozen, what will be the price of 6 dozens of eggs at 2 pence for 3 eggs?
Ans. 4s.

34. If 6352 stones 3 feet long will complete a certain quantity of wall, how many stones of 2 feet long will raise the like quantity? Ans. 9528.

35. If a person can count 300 in two minutes, how many can he count in a day? Ans. 216000.

36. A garrison of 536 men have provisions for 365 days; how long will those provisions last if the garrison be increased to 1142 men? Ans. 174 days and $\frac{64}{1142}$.

37. What will be the tax upon £763 15s. at the rate of 3s. 6d. per pound sterling? Ans. £133 13s. 1½d.

38. A certain work can be raised in 12 days by working 4 hours each day; how long would it require to raise the work by working 6 hours per day? Ans. 8 days.

39. When oats are 2s. a bushel, and Indian corn 4s. a bushel, what will be the price of 37 bushels of provender at 3s. a bushel? Ans. £5 11s.

40. What quantity of corn can I buy for 90 guineas, at the rate of 6 shillings a bushel? Ans. 315 bushels.

41. What is the cost of 30 pieces of lead, each weighing 1cwt. 12lbs., at the rate of 16s. 4d. the cwt.
Ans. £27 2s. 6d.

42. What length must be cut off from a board that is 9 inches wide to make a square foot? Ans. 16 inches.

43. A and B depart from the same place and travel the same road; but A goes 5 days before B, at the rate of 15 miles a day; B follows at the rate of 20 miles a day; what distance must he travel to overtake A?
Ans. 300 miles.

44. A factor bought a certain quantity of broadcloth and drugget which together cost £81; the quantity of broadcloth was 50 yards, at 18s. per yd., and for every 5 yards of broadcloth he had 9 yards of drugget. I demand how many yards of drugget he had, and what it cost him per yard? Ans. 90yds. and 8s. per yd.

45. Suppose a gentleman's income to be 600 guineas

a year, and that he spends 25s. 6d. per day, one day with another, how much will he have at the end of the year? Ans. £164 12s. 6d.

46. The flash of a gun was observed 38 seconds before hearing the report; required the distance, sound being supposed to move at the rate of 1142 feet per second. Ans. 43396ft.

47. What is the weight of a pea to a steelyard which being suspended 39 inches from the centre of motion will equipoise 208lbs. suspended at the draught end 3 quarters of an inch? Ans. 4lbs.

48. There was a certain building raised in 8 months by 120 workmen; but the same being demolished, it is required to be re-built in 2 months; I demand how many men must be employed about it. Ans. 480 men.

49. A cistern containing 200 gallons is filled by a pipe which discharges 3 gallons in 5 minutes; but the cistern has a leak which empties 1 gallon in 5 minutes. Now if the water begins to run in when the cistern is empty how long will it be in filling? Ans. 8hrs. 20m.

50. A, leaves the city of New York, to go to Montreal, and travels at the rate of 35 miles a day; B, at the same time leaves Montreal to go to New York, and travels the same road, at the rate of 30 miles a day; how far from the city of New York will they meet, allowing the distance to be 390 miles? Ans. 210 miles.

51. As I was hunting on the forest grounds,
Up starts a hare, before my two grey-hounds;
The dogs, being light of foot, did fairly run,
Unto her fifteen rods, just twenty-one.
The distance that she started up before
Was fourscore and sixteen rods, just, and no more;
Now this I'd have you unto me declare,
How far they ran before they caught the hare?

Ans. 336 rods.

PRACTICE.

Practice is a short method of finding the answers to questions in the Rule of Three, when the first term is a unit or one.

It has acquired its name from its daily use among merchants and business men, it being an easy method of working, where the price of a unit is given to find the price of a quantity.

For example, if one yard of cloth cost ten shillings, what will 40 cost? This question may be easily solved by the rule called Practice.

If the cloth had been £1 per yard, the cost of 40 yards would have been £40; but since it is only a part of a pound per yard, the whole cost will be the same part of £40 that the cost of one yard is of one pound, that is $\frac{1}{2}$ of 40. Hence the cost is $\frac{1}{2}$ of £40, or £20.

One number is said to be an aliquot part of another when it forms an exact or even part of it. For example, 4d. is an aliquot or even part of a shilling. So is 5s. of one pound; it is one fourth part, being contained in 20s. 4 times.

TABLE OF ALIQUOT PARTS.

Parts of £1.		Parts of 1s.		Parts of 1 Ton.		Parts of a cwt.	
				cwt.		qr. lb.	
10s.	$=\frac{1}{2}$	6d.	$=\frac{1}{2}$	10	$=\frac{1}{2}$	2 or 56	$=\frac{1}{2}$
6s. 8d.	$=\frac{1}{3}$	4d.	$=\frac{1}{3}$	5	$=\frac{1}{4}$	1 or 28	$=\frac{1}{4}$
5s.	$=\frac{1}{4}$	3d.	$=\frac{1}{4}$	4	$=\frac{1}{5}$	14	$=\frac{1}{8}$
4s.	$=\frac{1}{5}$	2d.	$=\frac{1}{6}$	$2\frac{1}{2}$	$=\frac{1}{8}$	Parts of a qr.	
3s. 4d.	$=\frac{1}{6}$	$1\frac{1}{2}$d.	$=\frac{1}{8}$	2	$=\frac{1}{10}$	14lbs.	$=\frac{1}{2}$
2s. 6d.	$=\frac{1}{8}$	1d.	$=\frac{1}{12}$			7	$=\frac{1}{4}$
1s. 8d.	$=\frac{1}{12}$					4	$=\frac{1}{7}$
						$3\frac{1}{2}$	$=\frac{1}{8}$

Questions.—What is Practice? From what has it derived its name? When is one number said to be an aliquot part of another? Mention some of the aliquot parts of £1. Of a shilling. Of a ton. Of a cwt. Of a quarter, &c.

CASE I.

When the price of one yard, pound, &c. is *less* than a penny.

RULE.

Find the value of the given quantity at 1d. a yard, pound, &c. and divide it by that even part, and the quotient will be the answer in pence.—But if the price be not an even part of 1d., as 3qrs., take parts of parts, and add the results together for the answer in pence.

EXAMPLES.

1. What is the value of 4528 oranges at $\frac{1}{4}$d. each?

Operation.

|$\frac{1}{4}$|$\frac{1}{4}$|4528
——
1132d.=£4 14s. 4d.

It is evident that the price of 4528 oranges at 1d. is 4528d. It is also plain that at $\frac{1}{4}$d. the price would be $\frac{1}{4}$ as many pence as there are oranges; we therefore divide by 4 and the quotient is the answer in pence.

2. What is the value of 4528 eggs at $\frac{3}{4}$d. each?

Operation.

|$\frac{1}{2}$|$\frac{1}{2}$|4528
——
|$\frac{1}{4}$|$\frac{1}{2}$|2264 value at $\frac{1}{2}$d.
1132 value at $\frac{1}{4}$d.
——
3396 value at $\frac{3}{4}$d.
—— Ans. £14 3s.

$\frac{3}{4}$d.=3qr. the greatest even part of which is 2qr. or $\frac{1}{2}$d.; we therefore divide by 2, and the quotient is the price at $\frac{1}{2}$d., and because $\frac{1}{4}$d. is $\frac{1}{2}$ of $\frac{1}{2}$d. we divide this quotient by 2 for the price at $\frac{1}{4}$d.; we then add the quotients together and the sum is the price at $\frac{3}{4}$d.

3. What is the value of 5704 lemons at $\frac{1}{4}$d.?
Ans. £5 18s. 10d.

4. What is the value of 6813 do. at $\frac{1}{2}$d.?
Ans. £14 3s. $\frac{1}{2}$d.

QUESTIONS.—When the price of a yd. lb. &c. is an even part of a penny what is first to be done? What will the quotient be? If it be not an even part of 1d.? What part of $\frac{1}{2}$ is $\frac{1}{4}$? When the price of a unit is $\frac{3}{4}$ of 1d. how do we divide?

5. What is the value of 9424 do. at ¾d?

Ans. £29 9s.

6. What is the value of 1487 do. at ¾d.?

Ans. £4 12s. 11¾d.

CASE II.

When the price of one yard, pound, &c. is *less* than 1s.

RULE.

Find the value of the quantity at 1s. a yd. &c., then take such part or parts as the price is of 1s.; add the quotients together, and their sum will be the answer in shillings.

EXAMPLES.

1. What is the value of 3711lbs. of butter at 7¼d. per pound?

Operation.

6	½	3711		
1	$\frac{1}{6}$	1855	6	
½	½	309	3	
¼	½	154	7	2
		77	3	3
		2396s.	8d.	1qr.

£119 16s. 8d. 1qr.

It is plain that the price of 3711 lbs. at 1s. is 3711s. then because 6d. is ½ of 1s. we divide by 2 for the price at 6d., then by 6, because 1 is $\frac{1}{6}$ of 6d., then by 2 again, because 2qrs. is ½ of 1d.; again by 2, because 1qr. is ½ of 2qrs.; we then add their quotients as the rule directs.

2. What is the cost of 862 yards at 2d?

Ans. £7 3s. 8d.

3. " " 749 " 4d.?

Ans. £12 9s. 8d.

4. " " 113 " 6d.?

Ans. £2 16s. 6d.

5. " " 899 " 8d.?

Ans. £29 19s. 4d.

Questions.—When the price of a yard, pound, &c. is less than 1 shilling what is the first thing to be done? What part or parts of this do we take? What will the sum of the quotients be?

6. What is the cost of 2147 yards at 3½d.?
Ans. £31 6s. 2½d.

7. " " 2456 " 4¼d.?
Ans. £43 9s. 10d.

8. " " 3271 " 7d.?
Ans. £95 8s. 1d.

9. " " 2759 " 8½d.?
Ans. £97 14s. 3½d.

10. " " 5272 " 9d.?
Ans. £197 14s. 0d.

11. " " 3254 " 10½d.?
Ans. £142 7s. 3d.

12. " " 7972 " 11¾d.?
Ans. £390 5s. 11d.

CASE III.

When the price is any number of shillings under 20, or an even part of 1 pound.

RULE.

Multiply the quantity by the price for the answer in shillings, or

Find the value at £1 per yard, &c. and then take parts, or parts of parts as the case may require, and the quotient or sum of the quotients will be the answer in pounds.

EXAMPLES.

1. What will 129½ bushels of oats cost at 2s. 6d. per bushel?

			£	s.	
Operation	2s. 6.	⅛	129	10	value at £1 per bushel.
		Ans.	£16	3s. 9d.	value at 2s. 6d.

Because 2s. 6d. is ⅛ of a pound we divide the price at £1 by 8, aud it is evident that the quotient will be the price at 2s. 6d.

QUESTIONS.—When the price is any part of a pound how may the answer be found? How is it done otherwise? In what denomination will the quotient or quotients be?

2. What is the value of 527 yards at 4s.?
Ans. £105. 8s.
3. " " 3271 " 5s.?
Ans. £817 15s.
4. " " 2710 " 6s.?
Ans. £813 0s.
5. " " 191 " 8s.? 76 8s.
6. " " 600 " 13s.?
Ans. £390 0s.
7. " " 1075 " 16s.?
Ans. £860 0s.
8. " " 2150 " 19s.?
Ans. 2042 10s.
9. " " 543 " 6s. 8d.?
Ans. 181 0s.
10. " " 127 " 3s. 4d.?
Ans. £21 3s. 4d.
11. " " 461 " 1s. 8d.?
Ans. £38 8s. 4d.

CASE IV.

When the price is pounds, or pounds, shillings, pence, and quarters.

RULE.

Multiply the given quantity by the pounds, then work for the shillings by case 3d. for the pence by case 2nd, and for the quarters by case 1st, add the several quotients together and the sum will be the answer.

EXAMPLES.

1. What is the cost of 680 acres of land at £3 9s. 7½d. per acre?

QUESTIONS.—Repeat the rule for performing the operation when the price is in pounds, or pounds, shillings, pence, and quarters.

Operation.

5s.	$\frac{1}{4}$	680			
4s.	$\frac{1}{5}$	3			
		2040			value at £3 per acre.
		170			do at 5s. per do.
		136			do at 4s. per do.
6d.	$\frac{1}{8}$	17			do at 6d. per do.
1d.	$\frac{1}{6}$	2	16	8	do at 1d. per do.
2qr.	$\frac{1}{2}$	1	8	4	do at ½d. per do.
Ans.		£2367	5	0	do at £3 9s. 7½d.

We first multiply the price by the pounds, then it is evident that the price of 680 acres at £1 per acre would be £680, then as 5s. and 4s. are even parts of 20s. we take the $\frac{1}{4}$ and the $\frac{1}{5}$ of 680 for the price at 9s.—then because 4s.=48d. and 6d. is $\frac{1}{8}$ of 48d. we take $\frac{1}{8}$ of the price at 4s. for the price at 6d., then $\frac{1}{6}$ of that for the price at 1d., and $\frac{1}{2}$ of the price at 1d. for the price at 2qrs. and their sums is the answer sought.

2. What is the value of 124 acres at £3 5s. 6½d.?
Ans. £406 7s. 2d.

3. " " 47 " £3 3s. 4d.?
Ans. £148 16s. 8d.

4. " " 20 " £4 13s. 4d.?
Ans. £93 6s. 8d.

5. " " 71 " £6 13s. 4d.?
Ans. £473 6s. 8d.

6. " " 37 " £1 19s. 5¾d.?
Ans. £73 0s. 8¼d.

7. " " 2715 " £1 17s. 2½d.?
Ans. £5051 0s. 7½d.

8. " " 3210 " £1 18s. 6¾d.?
Ans. £6189 5s. 7½d.

CASE V.

When the price and quantity are of several denominations.

RULE.

Multiply the price by the integers, or whole numbers in the given quantity, and take parts for the rest from the price of an integer; which added together will be the answer.

EXAMPLES.

1. What cost 5 cwt. 3 qr. 14lbs of raisins at £2 11s. 8d.?

Operation.

		£	s.	d.	
2qr.	$\frac{1}{2}$	2	11	8	
				5	
		12	18	4	cost of 5 cwt.
		1	5	10	do. of 2 qr.
1qr.	$\frac{1}{2}$		12	11	do. of 1 qr.
14lb.	$\frac{1}{2}$		6	$5\frac{1}{2}$	do. of 14 lb.
Ans.		£15	3	$6\frac{1}{2}$	whole cost.

We first multiply the price by the 5 cwt. then because 2 qr. is $\frac{1}{2}$ cwt. we divide the price of cwt. by 2, and the quotient is the cost of 2 qr.; then as 1 qr. is $\frac{1}{2}$ of 2 qr. we take $\frac{1}{2}$ the price of 2 qr. for the price of 1 qr.; also, because 14 lb. is $\frac{1}{2}$ 1 qr. we take $\frac{1}{2}$ the price of one quarter, and the quotient is the price of 14 lbs.

2. What is the cost of 5cwt. 1qr. of sugar at £2 17s. per cwt.? Ans. £14 19s. 3d.

3. What is the cost of 14cwt. 3qr. 7lb. of beef at 13s. 8d. per cwt.? £10 2s. $5\frac{1}{4}$d.

4. At £1 4s. 9d. per cwt. what is the value of 17cwt. 1qr. 17lbs. cheese? Ans. £21 10s. 8d.

5. At £3 17s. 6d. per cwt. what is the value of 25 cwt. 2qr. 14lb. tobacco! Ans. £99 5s. $11\frac{1}{4}$d.

6. Bought 78cwt. 3qr. 12lb. of currants at £2 17s. 9d. per cwt.; what did I give for the whole? Ans. £227 14s.

Questions.—When the price and quantity are of several denominations, what is the rule?

PRACTICAL EXERCISES.

1. What is the cost of 650 pigeons at ¼d. each.?
Ans. 13s. 6½d.

2. What is the value of 245 ducks at ½d. each.?
Ans. 10s. 2½d.

3. Bought a box of oranges containing 525, at ¾d. each; what did they cost me.? Ans. £1 12s. 9¾d.

4. What is the value of 120 lb. of rice at 3d. per lb.?
Ans. £1 10s.

5. Bought 8012 lb. chalk at 2¾d. per lb.
Ans. £91 16s. 1d.

6. How much will 3906 lb. of beef come to at 7½d. per lb.? Ans. £122 1s. 3d.

7. What will 1847 yards of cloth come to at 5s. 8d. per yard? Ans. £523 6s. 4d.

8. If an ell of Holland cost 4s. 6d. what is the value of 5 pieces each 12 ells? Ans. £13 10s.

9. What is the value of 1234 yards of muslin at 1s. 11¾d. per yard? Ans. £122 2s. 3½d.

10. What cost 287 bushels of wheat at 17s. 6d. per bushel? Ans. £251 2s. 6d.

11. How much will 47 tons of hay amount to at £6 6s. 8d. per ton. Ans. £297 13s. 4d.

12. Sold 26 acres of land for £11 14s. per acre; what is the amount? Ans. £304 4s.

13. If 1 yard of cloth cost £1 19s. 4d. how much will 1677 yards come to. Ans. £3298 2s.

14. Sold 16cwt. 2qr. 17lb. of sugar at £2 15s. 11d. per cwt. what was its value? Ans. £46 11s. 1d.

15. Sold 56cwt. 1qr. 17lb. of sugar at £2 15s. 9d. the cwt.; what does it come to? Ans. £157 4s. 4¼d.

16. What will 51 acres of land be worth at £3 2s. 2d. per acre. Ans. £158 10s. 6d.

17. What will 4 E. E. 3qr. 2na. of broadcloth cost at £2 3s. 8d. per yard? Ans. £12 16s. 6½d.

COMMISSION AND BROKERAGE.

Commission is an allowance made to a *Factor* or *person* engaged in buying and selling goods for another. A *Factor* is an agent who transacts business for his employer.

Brokerage is an allowance made to dealers in money or stocks.

The *allowance* made is generally a certain per cent. or rate per hundred on the monies paid out or received, and the work is the same as casting the interest on the same sum for one year.

EXAMPLES.

1. If I employ a factor to sell goods for me to the value of £2575 17s. 6d.; what must I pay him at 4 per cent.?

Operation.

```
 £      s.  d.
 2575   17  6
             4
 ------------
 103,03 10  0
     20
 ------
   0,70
     12
 ------
   8,40
      4
 ------
   1,60       Ans. £103 0s. 8¼d.
```

Here the work is the same as simple interest, we multiply by the rate per cent, and divide by 100.

2. My correspondent sends me word that he has bought goods to the amount of £1286 on my account; what will his commission come to at 2½ per cent.? Ans. £32 3s.

QUESTIONS.—What is Commission? What is a factor? What is Brokerage? What is the allowance generally made? To what is the work similar?

3. A factor sells land to the amount of £25,500 and is to receive $2\frac{1}{2}$ per cent. commission; how much must he pay over to his principal? Ans. £24862 10s.

4. What is the commission on £3496 at 6 per cent.? Ans. £263 15s. $2\frac{1}{4}$d.

5. A gentleman sent a broker £3825 to be invested in stock, the broker is to receive 2 per cent. on the amount paid for the stock; what was the value of the stock purchased?

Operation.

```
   100
     2
  ----
  2,00
 100
 -----
102 : 100::3825
        3825
      ------
102)382500(3750 Ans.
```

As the broker is to receive 2 per cent., it follows that £102 of the money received by him, will purchase £100 of stock: therefore 100*l.* added to the commission is to 100, as the given sum to the stock which it will purchase.

PROOF.–Commission on 3750 at 2 per cent. is £75 and 3750 +75=3825.

6. A factor receives £708 15s. and is directed to purchase steel at £45 per ton: he is to receive 5 per cent. on the money paid: how much steel can he purchase? Ans. 15 tons.

7. A broker bought 200 shares of bank stock for A.—He paid £197 per share, and he is to receive one fourth per cent. on the money he received; how much must A. pay for the stock? Ans. £39498 10s.

8. A bank fails, and has in circulation bills to the amount of £267581. It can pay only $9\frac{1}{2}$ per cent.: how much money is there on hand? Ans. £25420 3s. $10\frac{3}{4}$d.

9. A merchant shipped to his agent in Montreal 1000 barrels of flour, which was sold at 20s. per barrel: what

did he obtain for the flour, and what commission did he pay at $1\frac{3}{4}$ per cent?

Ans. { He received £982 10s. for the flour.
And paid £17 10s. commission.

INSURANCE.

An *Insurance Company* is a body of men who in return for a certain compensation, promise to pay for the loss of property insured. The written engagement they give is called a *Policy*.

The sum paid by those who own the property, to the Company who insure it, is called *Premium*. It is reckoned at so much per cent. on the value of the property insured.

EXAMPLES.

1. What will be the premium for insuring a ship and cargo from Quebec to Amsterdam, valued at £37800, at $4\frac{1}{2}$ per cent.? Ans. £1701.

2. What would be the premium for the insurance of a house valued at £5500, against loss by fire for 1 year at $\frac{1}{2}$ per cent.? Ans. £27 10s.

3. What would be the premium for insuring a ship and cargo, valued at £37500 from Montreal to Liverpool, at $3\frac{1}{2}$ per cent.? Ans. £1312 10s.

4. What would be the insurance of a steamboat from Kingston to Toronto, valued at £14000, at $1\frac{1}{2}$ per cent? Also, at $\frac{3}{4}$ per cent? At $\frac{1}{2}$ per cent? At $\frac{1}{3}$ per cent? At $\frac{1}{4}$ per cent.?

Answers in course, £210, £105, £70, £46 13s. $3\frac{1}{2}$d. £35.

QUESTIONS.—What is an Insurance Company? What is the written engagement which they give called? What is the Premium? How is it reckoned?

5. What is the insurance on a store and goods, valued at £27000, at $2\frac{1}{4}$ per cent? At 2, at $1\frac{1}{2}$, at $\frac{3}{4}$, at $\frac{1}{2}$, at $\frac{1}{4}$, at $\frac{1}{5}$, and at $\frac{1}{6}$ per cent.?

Answers in course, £607 10s., £540, £405, £202 10s., £135, £67 10s., £54, £45.

DISCOUNT.

Discount is a deduction made from a debt, for paying it before it is due.

If, for example, I owe a man £300 two years hence, and am willing to pay him now, I ought to pay only *that* sum which, with its *interest*, would in two years amount to £300.

The sum which, in the time mentioned, would by the addition of its interest, amount to the sum which is due, is called the *present worth.*

The question then is in the above example, what sum together with its interest at a certain per cent. would in two years amount to £300? This is found by the following

RULE.

Find the amount of £100 or dollars, for the time and rate proposed in the question. Then, as the amount found is to the amount given, so is £100 or dollars to the principal or present worth required. The present worth, *subtracted* from the whole debt will leave the discount.

EXAMPLES.

1. A debt of £372 is due 4 years hence; what money paid down will discharge it, allowing 6 per cent. per annum discount?

Questions.—What is Discount? What is called the present worth of a note, or debt? Repeat the rule? When the present worth is found, how do we find the discount?

Operation.

```
 100
   6
 ---
 600
   4
 ----
2400
100
----
124 amount of £100 for the time.
     Then as 124 : 372 : : 100
                           100
                          -----
                   124)37200(300 present worth.
     372—300=72 discount.
```

It is evident that £100 is the present worth of £124 due 4 years hence, because £100 amounts to £124 for the time and rate given ; hence *L*124 bears the same relation to *L*372 as *L*100 to the discount on *L*372.

NOTE.—This method of computing discount is the correct one, but the mode generally adopted at the banks is to compute the interest on the whole note to be discounted in a manner which produces a small excess, and, deducting this interest, advance the remainder to the holders—thus virtually charging interest not only on the sum advanced, but on the part withheld. This when the sum is small is a trivial error, but in a large one the error is sometimes considerable. For example, the interest of *L*500 at 5 per cent. for 12 years, exceeds the discount of the same sum for the same time and at the same rate by *L*112 10s., a sum too great to lose.

2. What sum in ready money will discharge a debt of *L*925 due one year and 8 months hence at 6 per cent.?
Ans. *L*840 18s. 2d.

3. What is the present worth of *L*600 due 4 years hence, at 5 per cent.? Ans. *L*500.

4. What is the discount of *L*275 10s. for 10 months, at 6 per cent. per annum? Ans. *L*13 2s. 4½d.

5. What is the present worth and discount of *L*550 10s. for 9 months, at 5 per cent. per annum?

Ans. *L*530 12s. 0½d. the present worth, and *L*19 17s. 11¼d. discount.

6. Bought to the value of *L*35 8s. to be paid 8 months hence; what ready money will pay for them, at 6 per cent discount? Ans.

NOTE.—When payments are to be made at different times, *find the present value of the several sums separately, and their sum will be the present value of the note.*

7. What is the discount of *L*1500, one half payable in 6 months, and the other half at the expiration of a year, at 7 per cent. per annum? Ans. *L*74 8s. 6¾d.

8. What will be the present worth of *L*150, one third payable at 4 months, one third at 8 months, and one third at 12 months, at 5 per cent. discount?

Ans. *C*145 3s. 8½d.

9. A merchant owes *L*110, payable in 20 months, and *L*224 payable in 24 months; the first he pays in 5 months, and the other in 1 month after that, discounting at 8 per cent. per annum. I demand the sum he paid.

Ans. *L*300.

LOSS AND GAIN.

Loss and gain is a rule by which merchants and traders discover their profit or loss in buying and selling their goods. It also instructs them how much to increase or diminish the price of their goods so as to make or lose so much per cent.

Questions in this rule are worked by the Rule of Three.

QUESTIONS.—What is loss and gain? By what rule are questions in loss and gain worked?

EXAMPLES.

1. Bought a piece of cloth containing 75yds., at *L*1 5s. per yard, and sold it at *L*1 15s. per yard; how much was gained in the whole?

Operation.

*L*1 15s. price of 1yd.
1 5s. cost of 1 yd.

10s. profit on 1yd.

yd. yd. s.
1 : 75 : : 10 : *L*37 10s. Ans.

We first find the profit on a single yd., and then on the 75 yards.

2. Bought a piece of cloth containing 50 yards at 2s. 6d. per yard, what must it be sold for per yard to gain *L*1 0s. 10d.?

50 yards at 2s. 6d.=*L*6 5s.
Profit,=*L*1 0s. 10d.

It must sell for *L*7 5s. 10d.

50) 7 5 10(2s. 11d. Ans.

3. Bought 11cwt. of sugar at 6½d. per pound but could not sell it again for any more than *L*2 16s. per cwt.: did I gain or lose by my bargain? Ans. Lost, *L*2 11s. 4d.

4. Bought 44lb. at *L*6 12s. and sold it again for *L*8 10s. 6d.: what was the profit on each pound? Ans. 10½d.

5. Bought a hogshead of wine at *L*1 5s. per gallon, and sold it for *L*78: was there a loss or gain?

Ans. Loss of 15s.

II. To know what is gained or lost per cent.

RULE.

First find what the gain or loss is by subtraction, then, as the price it cost : is to the gain or loss : : so is £100 to the gain or loss per cent.

QUESTION.—What is the Rule to find what is gained or lost per cent.?

EXAMPLES.

1. A boy bought a knife for 2s. and sold it again for 2s. 8d.; what did he gain per cent. or in laying out *L*100?

It is plain that the boy gains 8d. in selling his knife; that is 2s. gained 8d. We then say, if 2s. gain 8d. what will £100 gain? because the gain on £100 must be in the same proportion as the gain on 2 shillings.

Operation.
2s. 8d.
2 0d.
——
8d. gain.
s. d. £
2 : 8 : : 100 : £33 6s. 8d. Ans.

2. Bought sugar at $8\frac{1}{4}$d. per lb. and sold it again at £4 17s. per cwt.; what did I gain per cent.?

Ans. £25 19s. $5\frac{3}{4}$d.

3. At $1\frac{1}{2}$d. profit on a shilling, how much per cent.?

Ans. £12 10s.

4. If I buy 12hhds. of wine for £204 and sell the same again at £14 17s. 6d. per hhd. do I gain or lose, and what per cent.? Ans. I lose $12\frac{1}{2}$ per cent.

5. At 5s. profit on a pound, how much per cent?

Ans. 25 per cent.

6. If by selling pepper at $10\frac{1}{2}$d. per pound there are 2d. lost on each: required the loss per cent. Ans. 16.

III. To know how a commodity must be sold to gain or lose so much per cent.

RULE.

As £100 : is to the price it cost : : so is £100 with the profit added, or loss subtracted, to the gaining or losing price.

EXAMPLES.

1. If I buy Irish linen at 2s. 3d. per yard, how must I sell it per yard to gain 25 per cent.?

As £100 : 2s. 3d. : : £125 to 2s. 9d. 3qrs. Ans.

QUESTION.—What is the rule for finding how a commodity must be sold, to gain or lose so much per cent.?

2. Bought cloth at 17s. 6d. per yard, which not proving so good as I expected, I am obliged to lose 15 per cent. by it; how must I sell it per yard? Ans. 14s. 10½d.

3. Bought a cow for £5; what must I sell her for, in order to gain 25 per cent.? Ans. £6 5s.

IV. When there is gain or loss per cent., to know what the commodity cost.

RULE.

As £100, with the gain per cent. added, or the loss per cent. subtracted, is to the price, so is £100 to the first cost.

EXAMPLES.

1. If a yard of cloth be sold at 14s. 7d. and there is gained £16 13s. 4d. per cent.; what did it cost per yard? Ans. 12s. 6d.

£100+£16 13s. 4d.=£116 13s. 4d. : 14s. 7d. :: £100 to 12s. 6d.

2. A farmer sold a horse for 25 pounds, and lost 15 per cent.; what did the horse cost him?

£100—15=£85 : 25 :: 100 to £29 8s. $2\frac{4}{17}$d. Ans. or cost.

3. If a parcel of cloth be sold for £560, and at 12 per cent. gain, what was the prime cost? Ans. £500.

4. If by selling cloth at 9s. per yard I gain 12½ per cent, what was the prime cost of a yard? Ans. 8s.

V. If by wares sold at a given rate, there is so much gained or lost per cent., to know what would be gained or lost per cent. if sold at another rate.

RULE.

As the first price is to £100 with the *profit* per cent. *added*, or the *loss* per cent. *subtracted*, so is the *other price*, to the gain or loss per cent. at the other price.

QUESTIONS.—When there is gain or loss per cent. how do we find the cost? If there is so much gained or lost per cent. by wares sold at a given rate, how do we find what would be gained or lost if sold at another rate?

N. B.—If the answer *exceed* 100, the excess is *gain* per cent., but if it be *less* than 100, the *deficiency* is *loss* per cent.

EXAMPLES.

1. If I sell cloth at 5s. per yard, and thereby gain 15 per cent. what shall I gain per cent. if I sell it at 6s. per yd.

As 5s. : £115 : : 6s. : £138.

Ans. gained 38 per cent.

2. If I sell a barrel of sugar for £8, and thereby lose 12 per cent., what shall I gain or lose per cent. if I sell 4 barrels of the same sugar for £36?

Ans. I lose 1 per cent.

3. A gentleman sold a silver watch for £17 1s. 5d. and by so doing lost 15 per cent. whereas he ought in trading to have cleared 20 per cent.; how much was it sold under its real value?

	£	£ s. d.		£	£ s. d.	
First as	85 :	17 1 5	: :	100 :	20 1 8	the prime cost.
Second as	100 :	20 1 8	: :	120 :	24 2 0	the real value.

Then £24 2s.—£17 1s. 5d. the answer.

EQUATION OF PAYMENTS.

Equation of payments is a method of finding the mean time of payment of several sums, due at different times, and at *such* a time that *neither* shall lose interest.

In how many months will £1 gain as much at interest as £8 will gain in 4 months. Now as the £1 is 8 times *less* than 8, it will require 8 times *more* time, or $8\times4=32$ months.

A man owes me £12 payable in 3 mo., £18 in 4 mo. and £20 in 9 mo. He wishes to pay the whole at once; in what time ought he to pay?

QUESTIONS.—If the answer exceed 100, what is the excess? If it be less than 100, what is the deficiency? What is Equation of Payments?

The interest of £12 for 3 mo.=int. of £1 for	36	mo.
do of £18 for 4 mo.=int. of £1 for	72	do
do of £20 for 9 mo.=int. of £1 for	180	do
£50	288	

Now it appears that it will be the same to him to have £1 for 36, for 72, and for 180 months, as it would to have the 12, the 18, and the 20 pounds for the number of months specified.

He might therefore keep £1 just 288 months, and it would be the same as keeping the £50 for the number of months specified. But as the whole sum of money lent was £50, he may keep this only one *fiftieth* of the time he might keep £1. Therefore if 288 months be divided by 50, the quotient will be the equated time of payment, which is $5\frac{38}{50}$ months.

RULE.

Multiply each payment by the time before it becomes due, and divide the *sum* of the products by the *sum* of the payments; the quotients will be the mean time.

EXAMPLES.

2. A owes B £600 : £200 is to be paid in 2 months, £200 in 4 months, and £200 in 6 months; what is the mean time for the payment of the whole?

Operation.

$200\times2=400$
$200\times4=800$
$200\times6=1200$

$2400\div600=4$mo. Ans.

We here multiply each sum by the time at which it becomes due, and divide the sum of the products by the sum of the payments.

3. A man owes me £300, to be paid as follows; $\frac{1}{3}$ in 3 months; $\frac{1}{4}$ in 4 months, and the rest in 6 months; what is the mean time for payment? Ans. $4\frac{1}{2}$ months.

4. A merchant has due him £300 to be paid in 60 days, £500 to be paid in 120 days, and 750 to be paid in

Repeat the Rule for Equation of Payments.

180 days; what is the equated time for the payment of the whole? Ans. $137\frac{13}{31}$ days.

5. A owes B £1200, $\frac{1}{2}$ is to be paid in 6 months, $\frac{1}{3}$ in 8 months, and the remainder in 10 months; what is the equated time for the payment of the whole?

Ans. $7\frac{1}{3}$ months.

FELLOWSHIP.

The *Rule of Fellowship* is a method of ascertaining the respective gains or losses of individuals engaged in joint trade.

The money, or value of the articles employed in trade is called the *Capital* or *Stock*. The gain or loss to be shared is called the *Dividend*.

It is plain that each man's gain or loss should be in proportion to his share of the Stock. Hence the following

RULE.

As the whole stock is to the whole gain or loss, so is each man's share to his share of the gain or loss.

PROOF.

Add all the separate profits or shares together; their sum should be equal to the whole profit or stock.

EXAMPLES.

1. A and B buy certain merchandise amounting to £160, of which A pays £90, and B £70; they gain by the purchase £32; what is each one's share of the profits?

£90+£70=£160, then £160 : 32 { 90 : £18 A's share.
70 : £14 B's share. }

2. Three merchants make a joint stock of £1200, of

QUESTIONS.—What is Fellowship? What is the Capital or Stock? What is the dividend? Repeat the rule. How is it proved?

which A put in £240, B £360, and C £600; and by trading they gain £325; what is each one's part of the gain?

Ans. A's part £65, B's £97 10s., C's £162 10s.

3. A bankrupt is indebted to A £211, to B £300, and o C £390, and his whole estate amounts only to £675 10s. which he gives up to those creditors; how much must each have in proportion to his debt?

Ans. A must have £158 0s. $3\frac{3}{4}$d., B £224 13s. $4\frac{1}{2}$d. and C £292 16s. $3\frac{3}{4}$d.

DOUBLE FELLOWSHIP.

When several persons who are joined together in trade employ their capital for different periods of time, the partnership is called *Double Fellowship.*

For example, suppose A puts £200 in trade for 4 years, B £300 for 3 years, and C £100 for 1 year; this would make a case of double Fellowship.

Now it is evident that there are two circumstances which should determine each one's share of the profit; first, the amount of capital he puts in; and secondly, the time which it is continued in the business. Wherefore each one's share should be proportional to the capital he puts in, multiplied by the time it is continued in trade. Hence the following

RULE.

Multiply each man's stock or share by the time it was continued in trade; then,

As the sum of the several products is to the whole gain or loss, so is each man's particular product to his particular share of the gain or loss.

Questions.—What is Double Fellowship? What two circumstances should determine each one's share of the profits? Repeat the rule.

EXAMPLES.

1. A and B enter into partnership. A puts in £840 for 4 months, and B puts in £650 for 6 months; they gain £300. What is each one's share of the profits?

A's stock £840×4=3360
B's stock £650×6=3900

£7260 : 300 :: { 3360 : £138 16s. 10d
3900 : £161 3s. 1d. }

2. A put in trade £50 for 4 months, and B £60 for 5 months; they gained £24. How is it to be divided between them? Ans. A's share £9 12s., B's £14 8s.

3. C and D hold a pasture together, for which they pay £54; C pastures 23 horses for 27 days, and D 21 horses for 39 days. How much of the rent ought each one to pay? Ans. C £23 5s. 9d.; D £30 14s. 3d.

4. A, B and C hold a pasture in common, for which they pay £19 per annum. A put in 8 oxen for 6 weeks, B 12 oxen for 8 weeks, and C 12 oxen for 12 weeks. What must each pay of the rent?

Ans. A must pay £3 3s. 4d.; B, £6 6s. 8d.; and C, £9 10s.

TARE AND TRET.

Tare and *Tret* are allowances made in selling goods by weight.

Draft is an allowance on the gross weight in favour of the buyer or importer. It is always deducted before the *Tare*.

Tare is an allowance made to the buyer for the weight of the hogshead, barrel or bag, &c., containing the commodity sold.

QUESTIONS.—What are Tare and Tret? What is Draft? What is Tare?

Gross weight is the whole weight of the goods, together with that of the hogshead, barrel, bag, &c., which contains them.

Suttle is what remains after a *part* of the allowances have been deducted from the gross weight.

Nett weight is what remains after all the deductions are made.

All the questions in this rule may be worked by the Rule of Three.

EXAMPLES.

1. What is the nett weight of 112cwt. 3qrs. 12lbs. of tobacco; tare on the whole, 6cwt. 3qrs. 20lbs.

	cwt.	qrs.	lbs.	
	112	3	12	gross weight,
	6	3	20	tare.
Ans.	105	3	20	nett weight.

2. If the tare be 4lbs. per cwt. what will the tare be on 6T. 2cwt. 3qr. 14lbs. Ans. 4cwt. 1qr. 15½lbs.

3. What is the nett weight of 3 casks of indigo, each weighing 4cwt. 2qrs. 14lbs. gross; tare on each cask 1cwt. 0qr. 12lbs.? Ans. 10cwt. 2qrs. 6lbs.

4. What is the nett weight of 20 hogsheads of sugar, weighing in all 246cwt. 3qrs. 7lbs. gross, tare 16lbs. per cwt.? Ans. 211cwt. 2qrs. 6lbs.

5. What is the nett weight of 132cwt. 1qr. 20lbs. gross, tare 14lbs. per cwt.? Ans. 108cwt.

6. At £7 5s. per cwt. nett, how much will 16hhds. of sugar come to, each weighing gross 8cwt. 3qrs. 7lbs., tare 12lbs. per cwt.? Ans. £912 14s. 5½d.

7. What is the nett weight of 18hhds. of tobacco, each weighing gross 8cwt. 3qrs. 14lbs., tare 16lbs. per cwt.? Ans. 6T. 16cwt. 3qrs. 20lbs.

8. At £1 5s. per cwt. nett, tare 4lbs. per cwt., what

QUESTIONS.—What is Gross weight? What is Suttle? What is Nett weight? How may questions in this rule be worked?

will be the cost of 4 hogsheads of sugar weighing in all 49cwt. 0qrs. 14lbs. gross? Ans. £59 4s. 3d.

9. What is the nett weight of 495cwt. 1qr. 2lbs. gross, tare 28lbs. per cwt., and tret 4lbs. for every 104lbs.?

Ans. 357cwt. 0qr. $18\frac{1}{2}$lbs.

VULGAR FRACTIONS.

1. If a unit be divided into two equal parts, what is each part called? How do we express one of the parts? How many halves are there in one thing?

2. If a unit be divided into three equal parts, what is each part called? How do we express one of the parts? How do we express two of them? Three of them? How many thirds in one thing?

3. If a unit be divided into four equal parts, what is each part called? How do we express one of the parts? Two of them? Three of them? Four of them? How many fourths or quarters in one thing? How much greater is a half than a quarter? What is the sum of one fourth and two fourths?

4. If a unit be divided into six equal parts, what is each part called? How do we express one sixth? How do we express two of the parts? Three of them? Six of them? How many sixths are there in a unit? How much greater is a third than a sixth?

5. If a unit be divided into ten equal parts, what is each part called? How many tenths in a whole thing? How many fifths in one thing? How much greater is a fifth than a tenth?

6. If a unit be divided into twelve equal parts, what is each part called? How is it expressed? How are five of the parts expressed? Six of them? Eight of them? Eleven of them? How many sixths in a unit? How many twelfths? How much greater is a sixth than a twelfth?

7. How many halves in two units? How many thirds in three units? How many fourths in three units? In four? In five? In six? How many tenths in two? In three? In four?

8. How many sevenths in a unit? How many fourteenths? How many fourteenths are equal to one seventh? How many are equal to three sevenths? To five sevenths? To seven sevenths?

9. What is one half of one half? One half of one third? What is the half of one fourth? Of one fifth? Of one sixth? Of one seventh?

10. What is the sum of one half and one half? Of one third and two thirds? Of one fourth and one fourth? Of one fourth and two fourths? Of one fourth and three fourths? Two fourths and two fourths?

11. What is the sum of one fifth and two fifths? What is their difference? What is the sum of two fifths and three fifths? What is their difference?

12. What is the difference between six eighths and three eighths? What is their sum? What is the sum of five eighths and two eighths? What is their difference? What is the sum of four eighths and four eighths? What is their difference?

13. What is the sum of four twelfths and eight twelfths? What is their difference? What is the difference between five twelfths and seven twelfths? What is their sum?

14. How many halves in one? How many thirds? Fourths? Fifths? Sixths? Sevenths? Eighths? Ninths? Tenths? Elevenths? Twelfths?

15. How many halves in two? How many fourths? How many eighths? How many elevenths? How many twelfths?

16. How many fourths in four? In five? In six? In seven? How many fifths in four? How many sevenths? How many tenths?

17. How many elevenths in three? In six how many? In nine? In ten?

18. How many whole units in two halves? In three halves? In four halves? In five halves? In six halves? In eight halves? In nine halves?

19. How many units in three thirds? In four thirds? In five thirds? In seven thirds? In nine thirds? In eleven thirds?

20. How many units in four fourths? In five fourths? In eight fourths? In nine fourths? In eleven fourths? In sixteen fourths?

21. How many units in ten tenths? In fifteen tenths? In twenty tenths? In twenty four tenths? In thirty three tenths?

22. How many eights is one fourth equal to? What is the sum of one fourth and one eighth? Two fourths and two eighths? One fourth and five eighths? Three fourths and two eighths?

23. How many tenths are two fifths equal to? What is the sum of one fifth and eight tenths? What is the sum of one sixth and one twelfth? Of one sixth and ten twelfths?

Before proceeding farther the pupil is requested to review carefully what is said of Fractions on the 44th, 45th and 46th pages.

A *fraction* is the *expression* of one or more parts of a unit.

There are five kinds of Vulgar Fractions, viz: *Proper*, *Improper*, *Simple*, *Compound* and *Mixed*.

A *Proper Fraction* is one in which the *numerator* is *less* than the *denominator*. The value of every proper fraction is *less* than 1, as the following:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{3}{4}, \frac{5}{6}, \frac{8}{10}.$$

An Improper Fraction is one in which the *numerator equals* or *exceeds* the *denominator*. They are called *improper* fractions because they are *equal to* or *exceed unity*. When the *numerator* is *equal* to the *denominator*, as $\frac{4}{4}$, the *value of the fraction is equal to* 1. If the numerator *exceed* the denominator as $\frac{5}{4}$, the value of the fraction is *greater than* 1, as the following:

$$\frac{3}{2},\ \frac{4}{3},\ \frac{6}{5},\ \frac{8}{7},\ \frac{9}{3},\ \frac{10}{8},\ \frac{12}{2},\ \frac{14}{7}.$$

A *Simple Fraction* is a single expression, as $\frac{3}{8}$. It may be either proper or improper. The following are simple fractions:

$$\frac{1}{4},\ \frac{2}{3},\ \frac{4}{5},\ \frac{3}{6},\ \frac{5}{3},\ \frac{7}{4},\ \frac{8}{6},\ \frac{9}{7}.$$

A *Compound Fraction* is a fraction of a fraction, or several fractions connected by the word *of*, as the following:

$\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{4}$ of $\frac{1}{6}$, $\frac{1}{3}$ of $\frac{1}{5}$ of 10, $\frac{1}{4}$ of $\frac{1}{3}$ of 12.

A *Mixed Number* is composed of a whole number and a fraction, as the following:

$$2\tfrac{1}{2},\ 4\tfrac{3}{4},\ 6\tfrac{2}{3},\ 9\tfrac{1}{7}.$$

A *Mixed Fraction* is one whose numerator or denominator is a mixed number, as $\dfrac{4\frac{1}{2}}{10}$.

A whole number may be expressed fractionally by writing 1 below it for a denominator.

Thus, 2 may be written $\frac{2}{1}$, and is read 2 ones.
4 " " $\frac{4}{1}$, " " 4 ones.
7 " " $\frac{7}{1}$, " " 7 ones.

But 2 ones are equal to 2, 4 ones are equal to 4, 7 ones to 7, &c. Therefore the value of a number is not altered by placing 1 under it for a denominator.

QUESTIONS.--What is a fraction? How many kinds of Vulgar Fractions are there? What are they? What is a Proper Fraction? Is its value greater or less than 1? Give an example of a proper fraction. What is an Improper Fraction? Why is it called improper? When is its value equal to 1? When is it greater than 1? Give an example of an improper fraction? What is a Simple Fraction? Give an example. What is a Compound Fraction? Give an example. What is a Mixed Number? Give an example. Is five-eighths a proper or improper fraction? What kind of a fraction is eight-fourths? What is its value? What kind of a fraction is nine-eighths? What is its value? What kind of a fraction is one-half of one-third? What kind of a fraction or number is 4 three-fourths? 7 one-seventh? 9 two-thirds?

You have learned (page 45) that the denominator shows into how many equal parts a unit is divided, and the numerator shows how many of the parts are expressed by the fraction.

You have also learned that the numerator and denominator taken together are called the terms of the fraction, and that *dividing both terms by the same number does not change the value of the fraction.*

REDUCTION OF VULGAR FRACTIONS.

CASE I.

To reduce a fraction to its lowest terms.

RULE.

Divide the numerator and denominator by any number which will divide them both without a remainder, and those quotients again in the same way until there is no number greater than 1 that will divide them both without a remainder.

EXAMPLES.

1. Reduce $\frac{6}{12}$ to its lowest terms.

Operation.

$$\frac{6)\,6}{6)\,12} = \frac{1}{2} \text{ Ans.}$$

Here it will be seen that the fraction is in the lowest terms, as no number greater than 1 will divide the numerator and denominator.—It will also be seen that its *terms only* are altered, *not* its *value.*

Questions.—How may a whole number be expressed fractionally? Does this alter its value? Give an example. What does the denominator of a fraction show? What does the numerator show? What are the numerator and denominator taken together called? If both terms be divided by the same number does it change the value of a fraction? Repeat the rule for reducing a fraction to its lowest terms.

2. Reduce $\frac{55}{100}$ to its lowest terms.

Operation. $5)\frac{55}{5)100}=\frac{11}{20}$. Ans.

3. Reduce $\frac{104}{312}$ to its lowest terms.

Operation. $\frac{2)104}{2)312}=\frac{2)52}{2)156}=\frac{2)26}{2)78}=\frac{13)13}{13)39}=\frac{1}{3}$ Ans.

4. Reduce $\frac{275}{440}$ to its lowest terms. Ans. $\frac{55}{88}$.

5. Reduce $\frac{1344}{1536}$ to its lowest terms. Ans. $\frac{7}{8}$.

6. Reduce $\frac{1049}{8392}$ to its lowest terms. Ans. $\frac{1}{8}$.

GREATEST COMMON DIVISOR.

There is another method of reducing a fraction to its lowest terms, which is often preferable to the above, viz: dividing the terms by their greatest common divisor. In the first example above, 6 is a common divisor of both terms of the fraction $\frac{6}{12}$: it is also their greatest common divisor.

Any number greater than 1 that will divide two or more numbers without a remainder is called their common divisor; and the greatest number that will so divide them is called their greatest common divisor.

The greatest common divisor of two numbers is found by the following

RULE.

Divide the greater number by the less, then the divisor by the remainder, and continue to divide the last divisor by the last remainder until nothing remains.

The last divisor will be the common divisor sought.

Questions.—What other method have we for reducing a fraction to its lowest terms? What is a common divisor? What is the *greatest* common divisor of any two numbers? Repeat the rule for finding the greatest common divisor of two numbers.

EXAMPLES.

1. Find the greatest common divisor of the two numbers 135 and 165.

```
        Operation.                  Proof.
        135)165(1                15)135(9
            135                      135
            ---                      ---
         30)135(4                15)165(11
            120                      165
            ---
Greatest common div. 15)30(2
                        30
```

Let us try if the less number 135 is the greatest common divisor. It will exactly divide itself, but will not divide 165 without a remainder; we divide it therefore, by this remainder, and find still a remainder of 15. We divide the last divisor by this remainder, and nothing is left. Therefore 15 is the greatest common divisor of 135 and 165.

2. What is the greatest common divisor of 323 and 475? Ans. 17.

3. Required the greatest common divisor of 2310 and 4626? Ans. 6.

4. What is the greatest common divisor of 1092 and 1428? Ans. 84.

5. What is the greatest common divisor of 1197 and 805? Ans. 7.

NOTE.—To find the greatest common divisor of more than two numbers; find the common measure of two of them as above, then find the greatest divisor of this common measure and a third given number; and so proceed to the last.

The pupil may now reduce the following fractions to their lowest terms by dividing both terms by their greatest common divisor.

EXAMPLES.

1. Reduce $\frac{70}{175}$ to its lowest terms.

Operation.

$$70)175(2 \\ 140 \\ \overline{\quad 35)70(2} \\ 70$$

$$\frac{35)\ 70}{35)\ 175}=\frac{2}{5}, \text{Ans.}$$

We first find the greatest common divisor of 70 and 175 to be 35; then reduce $\frac{70}{175}$ to its lowest terms by dividing at once by this number.

2. Reduce $\frac{63}{81}$ to its lowest terms by the last method. Ans. $\frac{7}{9}$.

3. Reduce $\frac{315}{405}$ to its lowest terms by the last method. Ans. $\frac{7}{9}$.

4. Reduce $\frac{792}{1386}$ to its lowest terms by both methods. Ans. $\frac{4}{7}$.

5. Reduce $\frac{384}{1152}$ to its lowest terms by both methods. Ans. $\frac{1}{3}$.

6. Reduce $\frac{1428}{2856}$ to its lowest terms by both methods. Ans. $\frac{1}{2}$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction, to this product add the numerator, and place their sum over the given denominator.

EXAMPLES.

1. Reduce $12\frac{3}{8}$ to its equivalent improper fraction.

$12\times8=96+3=99$. Ans. $\frac{99}{8}$.

1 whole thing is equal to 8 eighths—hence, 12 whole things equal 96 eighths.

It is plain that multiplying 12 by the denominator 8, makes it 96 eighths. Then 3 eigths added make $\frac{99}{8}$.

2. Reduce $45\frac{1}{6}$ to an improper fraction? Ans. $\frac{271}{6}$.

3. How many 24ths in $365\frac{6}{24}$? Ans. $\frac{8766}{24}$.

4. Reduce $192\frac{45}{60}$ to its equivalnt improper fraction? Ans. $\frac{11565}{60}$.

5. Reduce $240\frac{67}{138}$ to its equivalent improper fraction. Ans. $\frac{33187}{138}$.

6. Reduce $876\frac{250}{300}$ to its equivalent improper fraction. Ans. $\frac{263050}{300}$.

CASE III.

To reduce an improper fraction to its equivalent whole or mixed number.

RULE.

Divide the numerator by the denominator, the quotient will be the whole number; and if there be a remainder place it over the given denominator.

EXAMPLES.

1. Reduce $\frac{48}{7}$ to its equivalent whole or mixed number.

Operation.
7)48
—
$6\frac{6}{7}$ Ans.

2. Reduce $\frac{84}{7}$ to a whole number. Ans. $84 \div 7 = 12$.

From the above examples we may perceive the truth of the following principle, viz:—*The value of every fraction is equal to the quotient arising from dividing the numerator by the denominator.*

3. Reduce $\frac{148}{12}$ to a whole or mixed number. Ans. $12\frac{4}{12}$.

4. In $\frac{51}{9}$ of bushels, how many bushels? Ans. $5\frac{6}{9}$.

5. If I give $\frac{1}{4}$ of an orange to each of 12 children, how many oranges do I give? Ans. 3.

QUESTIONS.—How many eighths in 12 whole numbers? How many eighths in 12 and three-eighths? Repeat the rule for reducing a mixed number to its equivalent improper fraction?—How many whole numbers in 48 sevenths? and how many sevenths over? Repeat the rule for reducing an improper fraction to its equivalent whole or mixed number.

6. Reduce $\frac{327}{125}$ to its whole or mixed number?
Ans. $2\frac{77}{125}$.

7. Reduce $\frac{3672}{153}$ and $\frac{50287}{6041}$ to their equivalent whole or mixed numbers. Ans. 24 and $8\frac{1959}{6041}$.

8. Reduce $\frac{36}{20}$, $\frac{760}{40}$, $\frac{875}{150}$ and $\frac{3465}{450}$ to whole or mixed numbers. Ans. $1\frac{4}{5}$, 19, $5\frac{5}{6}$ and $7\frac{7}{10}$.

CASE IV.

To reduce a whole number to an equivalent fraction having a given denominator.

RULE.

Multiply the whole number by the given denominator, and set the product over the said denominator.

EXAMPLES.

1. Reduce 8 to a fraction whose denominator shall be 5. Here $8\times5=40$; therefore $\frac{40}{5}$ is the required fraction, for $40\div5=8$, according to case III.

2. Reduce 18 to a fraction whose denominator shall be 8. Ans. $\frac{144}{8}$.

3. Reduce 125 to a fraction whose denominator shall be 15. Ans. $\frac{1875}{15}$.

4. Reduce 135 to a fraction whose denominator shall be 175. Ans. $\frac{23625}{175}$.

CASE V.

To reduce a compound fraction to its equivalent simple one.

RULE.

I. Reduce all mixed numbers to their equivalent improper fractions by case II.

QUESTIONS.—To what is the value of every fraction equal?—Repeat the rule for reducing a whole number to an equivalent fraction having a given denominator. How many whole numbers in 63 sevenths? In 96 eighths? In 100 tenths? How many 8ths in 7 units? How many 11ths in 6? How many 9ths in 7? If the denominator be 5 what fraction do we form of 9? Of 11? Of 12?

II. Then multiply all the numerators together for a numerator, and all the denominators together for a denominator; their products will form the fraction required.

EXAMPLES.

1. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ to a simple fraction. Ans. $\frac{1}{8}$.

$\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$.

If $\frac{1}{4}$ of 1 is $\frac{1}{4}$, then $\frac{1}{4}$ of $\frac{1}{2}$ must be half as much, or $\frac{1}{8}$ of 1.

2. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{7}$ to a simple fraction. Ans. $\frac{5}{42}$.

3. Reduce $\frac{5}{3}$ of $\frac{3}{6}$ of $\frac{6}{7}$ to a simple fraction.

Here $\frac{5}{3}\times\frac{3}{6}\times\frac{6}{7}=\frac{90}{126}=\frac{10}{14}=\frac{5}{7}$ by reducing the fraction to its lowest terms, as shewn in case I.

Or, by cancelling or drawing a perpendicular line after the 3's and 6's in the numerator and denominator, thus, $\frac{5}{3|}\times\frac{3|}{6|}\times\frac{6|}{7}=\frac{5}{7}$.

By cancelling the 3's we only divide both terms by 3; and in cancelling the 6's we divide by 6. *Hence the value of the fraction is not affected by thus cancelling like figures*, which should always be done when the numerator of one is like the denominator of another.

4. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ to a simple fraction.

Here $\frac{6}{8}\times\frac{8}{9}\times\frac{9}{10}=\frac{432}{720}=\frac{48}{80}=\frac{6}{10}=\frac{3}{5}$. Ans.

Or, $\frac{6}{8|}\times\frac{8|}{9|}\times\frac{9|}{10}=\frac{6}{10}=\frac{3}{5}$ Ans.

5. Reduce $\frac{3}{4}$ of $\frac{9}{10}$ of $\frac{15}{16}$ to a simple fraction. Ans. $\frac{81}{128}$.

6. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a single fraction.

Ans. $\frac{624}{306}=2\frac{2}{51}$.

7. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{12}{13}$ of $\frac{8}{12}$ of $\frac{4}{5}$ to a single fraction.

Ans. $\frac{16}{75}$.

8. Required the value of $\frac{1}{2}$ $\frac{12}{13}$ of $33\frac{1}{3}$ in a single fraction. Ans. $15\frac{5}{13}$.

9. How many apples are $\frac{6}{7}$ of $\frac{5}{6}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of $\frac{7}{8}$ of 40 apples? Ans. 20.

QUESTIONS.—How do we reduce a compound fraction to a simple one? When there are like figures in the numerator and denominator, what do we do with them? Does this alter the value of the fraction? What is half of one-third? What is one-third of one-fifth? What is one-third of three-twelfths? two-thirds of six-ninths? three-fourths of eight-elevenths? four-fifths of five-twelfths?

CASE VI.

To reduce fractions of different denominators to equivalent fractions having a common denominator.

RULE.

I. Reduce compound fractions to simple ones, and whole or mixed numbers to improper fractions.

II. Then multiply each numerator by all the denominators except its own, for the new numerators, and all the denominators together for a common denominator; the common denominator placed under each of the new numerators will form the several fractions sought.

EXAMPLES.

1. Reduce $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{4}{7}$ to a common denominator.

$1\times3\times7=21$	the new numerator of the		1st.
$1\times4\times7=28$	"	"	2nd.
$4\times4\times3=48$	"	"	3d.
$4\times3\times7=84$	the common denominator.		

Therefore $\frac{21}{84}$, $\frac{28}{84}$ and $\frac{48}{84}$ are the equivalent fractions.

It is plain that this reduction does not alter the value of the fractions, for the numerator and denominator of each are multiplied by the same number, and by reducing each to its lowest terms, we should have again the original fractions. For $\frac{21}{84}=\frac{1}{4}$, $\frac{28}{84}=\frac{1}{3}$, and $\frac{48}{84}=\frac{4}{7}$.

Hence we have the following general principle:

If the numerator and denominator be both multiplied by the same number the value of the fraction will remain unchanged.

2, Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ to a common denominator.

Ans. $\frac{32}{64}$, $\frac{48}{64}$ and $\frac{56}{64}$.

Questions.—In reducing fractions to a common denominator, what is the first thing to be done? What is the second? Does this reduction alter the value of the several fractions? Why not? How may we change them back to the original fractions? What general principle have we under this rule?

3. Reduce $\frac{5}{6}$, $2\frac{3}{5}$ and 4 to a common denominator.

Ans. $\frac{25}{30}$, $\frac{78}{30}$ and $\frac{120}{30}$.

4. Reduce $\frac{7}{8}$, $\frac{2}{3}$, $\frac{6}{7}$ and $\frac{1}{5}$ to a common denominator.

Ans. $\frac{735}{840}$, $\frac{560}{840}$, $\frac{720}{840}$ and $\frac{168}{840}$.

5. Reduce $\frac{1}{5}$, $\frac{5}{7}$, $\frac{9}{10}$ and $\frac{1}{2}$ to a common denominator.

Ans. $\frac{28}{140}$, $\frac{100}{140}$, $\frac{126}{140}$ and $\frac{70}{140}$.

6. Reduce $7\frac{1}{2}$, $\frac{31}{18}$, $6\frac{1}{4}$ to [illegible] common denominator.

Ans. $\frac{1080}{144}$, $\frac{24}{14}$ [illegible] $\frac{900}{144}$.

NOTE.—To reduce fractions to their *least common denominator*, it is necessary first to learn the method of finding the

LEAST COMMON MULTIPLE

Of two or more numbers. A number is said to be a *common multiple* of other numbers, when it can be divided by each of them without a remainder.

Thus 8 is a common multiple of 2 and 4, because it may be exactly divided by each of them. Also 12 is a common multiple of 2, 3, 4 and 6. The *least common multiple* of two or more numbers is the *least* number which they will separately divide without a remainder.

For example, 16 is a common multiple of 2 and 4, but it is not the *least* common multiple, because 8 is also exactly divisible by 2 and 4; and as it is the least number that may be so divided by those numbers, it is their least common multiple.

The least common multiple of several numbers may be found by the following

RULE.

I. Place the numbers in a line, and divide by

QUESTIONS.—What is a common multiple? Give an example. What is the least common multiple of several numbers? What is the least common multiple of 2 and 4? What is the first step in finding the least common multiple of two or more numbers? What is the second? What is the least common multiple of 4, 6 and 12?

any number that will divide two or more of them without a remainder, and place the quotients and undivided numbers in a line below.

II. Divide these numbers in the same way, and so continue, until no number greater than 1 will exactly divide any two of them. The numbers in the lower line and the divisors multiplied together, will produce the least common multiple.

EXAMPLES.

1. Find the least common multiple of 4, 6 and 12.

Operation.

2)4 . . . 6 . . . 12
―――――――――
3)2 . . . 3 . . . 6
―――――――――
2)2 . . . 1 . . . 2
―――――――――
1 . . . 1 . . . 1
―――――――――
Ans. $2 \times 3 \times 2 = 12$

We first divide by 2, which we find will divide 4, 6 and 12 without a remainder. We then divide this line by 3, which is a common divisor of 3 and 6; and as 2 cannot be divided by it, we bring it down to the lower line. We then find the numbers in this line divisible by 2, except the 1, which we place below, and find this last line to consist of 1's. As multiplying by the 1's would not alter the result, we leave them and multiply the divisors together, $2 \times 3 \times 2 = 12$, the least common multiple of 4, 6 and 12.

2. Find the least common multiple of 3, 7 and 9.

Operation.

3)3 . . . 7 . . . 9
―――――――――
1 . . . 7 . . . 3
―――――――――
Ans. $3 \times 7 \times 3 = 63$

Here, as there is no common divisor between any two numbers in the last line, we multiply them together, and also by the divisor 3, and find the least common multiple to be 63.

3. Find the least common multiple of 12, 16, 20 and 30? Ans. 240.

4. Required the least common multiple of 21 and 49? Ans. 149.

5. Required the least common multiple of 4, 14, 28 and 98? Ans.

6. Find the least common multiple of 25, 35, 60 and 72? Ans. 12600.

7. What is the least common multiple of 11, 17, 19, 21 and 7? Ans. 74613.

8. What is the least number that can be divided by the nine digits separately, without a remainder? Ans. 2520.

To reduce fractions to their *least common denominator*, observe the following

RULE.

I. Find the least common multiple of the several denominators as shown above, and it will be the least common denominator.

II. Divide the common multiple by the denominator of each fraction, and by each of these quotients multiply its respective numerator; the products will be the numerators of the required fractions, ander which write the least common denominator.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{3}{8}$ and $\frac{5}{6}$ to their least common denominator.

2)2 . . . 8 . . . 6

1 . . . 4 . . . 3 and $3\times4\times2=24$, the least common denominator.

$24\div2=12\times1=12$, 1st numerator.
$34\div8=3\times3=9$, 2nd numerator.
$24\div6=4\times5=20$, 3rd numerator.

Ans. $\frac{12}{24}$, $\frac{9}{24}$ and $\frac{20}{24}$.

These fractions may be reduced back to their former terms, thus: $\frac{12}{24}=\frac{1}{2}$, $\frac{9}{24}=\frac{3}{8}$, and $\frac{20}{24}=\frac{5}{6}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to fractions having the least common denominator. Ans. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$ and $\frac{10}{12}$.

Questions.—In reducing fractions to their least common denominator, what is the first step? What is the second? Does this reduction alter their values? May they be reduced back to their original terms?

3. Reduce $\frac{4}{5}$, $\frac{8}{9}$ and $\frac{3}{15}$ to their least common denominator. Ans. $\frac{36}{45}$, $\frac{40}{45}$ and $\frac{9}{45}$.

4. What is the least common denominator of $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$ and $\frac{7}{10}$. Ans. $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$ and $\frac{63}{90}$.

5. Find the least common denominator of $\frac{3}{15}$, $\frac{4}{24}$ and $\frac{8}{9}$. Ans. $\frac{72}{360}$, $\frac{60}{360}$ and $\frac{320}{360}$.

6. Reduce $\frac{5}{7}$, $\frac{9}{14}$, $\frac{5}{21}$ and $\frac{8}{42}$ to their least common denominator. Ans. $\frac{30}{42}$, $\frac{27}{42}$, $\frac{10}{42}$ and $\frac{8}{42}$.

7. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$ and $\frac{17}{24}$ to equivalent fractions having the least common denominator.

Ans. $\frac{16}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$, $\frac{33}{48}$ and $\frac{34}{48}$.

CASE VII.

To reduce a mixed fraction to a simple one.

RULE.

Multiply the numerator and denominator of the given fraction by the denominator of the fraction annexed, to the product of the numerator adding the numerator of the annexed fraction, and the products will be the terms of the fraction required.

NOTE.—In the application of this rule it should be considered that a fraction multiplied by a whole number equal to its denominator, produces a whole number equal to its numerator.

EXAMPLES.

1. Reduce $\frac{42\frac{7}{8}}{49}$ to a simple fraction.

$$\left.\begin{array}{ll} 42\tfrac{7}{8}\times 8+7=343 & \text{numerator.} \\ 49\times 8 \quad =392 & \text{denominator.} \end{array}\right\} = \frac{7}{8} \text{ Ans.}$$

2. Reduce $\frac{34\frac{1}{2}}{46}$ to a simple fraction. Ans. $\frac{3}{4}$.

QUESTIONS.—How may a fraction, whose numerator or denominator is a mixed number, be reduced to a simple fraction? What does a fraction, multiplied by a whole number equal to its denominator, produce?

3. Reduce $\dfrac{34}{45\frac{1}{3}}$ to a simple fraction. Ans. $\frac{3}{4}$.

4. Reduce $\dfrac{73}{131\frac{2}{5}}$ to a simple fraction. Ans. $\frac{5}{9}$.

The following cases relate to fractions of different denominations. The next two cases are the reverse of each other.

CASE VIII.

To reduce a fraction from a lower to a higher denomination.

RULE.

I. Consider how many units of the given denomination make one of the next *higher*, and place 1 *over* that number forming a second fraction.

II. Proceed in the same manner from the second denomination to the third, and so on to the denomination desired.

III. Connect the several fractions thus formed by the word *of*, making a compound fraction, then reduce the compound fraction to a simple one by Case V.

EXAMPLES.

1. Reduce $\frac{5}{6}$ of a penny to the fraction of £1.

$\frac{5}{6}$ of $\frac{1}{12}$ of $\frac{1}{20} = \frac{5}{1440} = \frac{1}{288}$ of £1, Ans.

Here the given fraction is $\frac{5}{6}$ of a penny; but one penny is $\frac{1}{12}$ of a shilling, and one shilling is $\frac{1}{20}$ of a pound; hence $\frac{5}{6}$ of a penny is equal to $\frac{5}{6}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a £$=\frac{1}{288}$. Therefore the reason of the rule is evident.

2. Reduce $\frac{3}{8}$ of a barleycorn to the fraction of a yard.

Ans. $\frac{3}{864}$ of a yd.

Operation. $\frac{3}{8}$ of $\frac{1}{3}$ of $\frac{1}{12}$ of $\frac{1}{3} = \frac{3}{864}$.

QUESTIONS.—To reduce a fraction from a lower to a higher denomination, what is the first step? What is the second? The third? Are $\frac{1}{4}$ of a £ and of $\frac{1}{4}$ of a £ the same, or different denominations? One-fifth of a £ and $\frac{1}{5}$ of a shilling? $\frac{1}{3}$ of a day and $\frac{1}{4}$ of an hour? One-seventh of a week and $\frac{1}{4}$ of a month? $\frac{1}{3}$ of a toot and $\frac{2}{5}$ of a rod? $\frac{1}{4}$ of an inch and two-fifths a yard?

Three barleycorns make an inch, we therefore first place 1 over 3; as 12 inches make a foot, we next place 1 over 12, and as 3 feet make a yard, we place 1 over 3.

3. Reduce $\frac{1}{2}$ of a farthing to the fraction of a shilling. Ans. $\frac{1}{96}$.

4. Reduce $\frac{8}{9}$ of an ounce Troy, to the fraction of a pound. Ans. $\frac{2}{27}$.

5. Reduce $\frac{6}{7}$ of a pound Avoirdupois, to the fraction of a cwt. Ans. $\frac{3}{392}$.

6. Reduce $\frac{1}{3}$ of a farthing to the fraction of a £. Ans. $\frac{1}{2880}$ £.

7. Reduce $\frac{1}{2}$ of an ounce to the fraction of a ton. Ans. $\frac{1}{71680}$ T.

8. Reduce $\frac{187}{127}$ of a minute to the fraction of a day. Ans. $\frac{187}{182880}$ day.

CASE IX.

To reduce a fraction from a higher to a lower denomination.

RULE.

I. Consider how many units of the next *lower* denomination make one unit of the given denomination, and place 1 *under* the number forming a second fraction.

II. Proceed in the same manner with the denomination still lower, and so on to the denomination desired.

III. Connect the several fractions thus formed, making a compound fraction, which reduce to a simple one by Case V.

EXAMPLES.

1. Reduce $\frac{1}{7}$ of a £ to the fraction of a penny. Ans. $\frac{240}{7}$d.

Operation. $\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}$d.

QUESTIONS.—In reducing fractions from a higher to a lower denomination what is the first step? What the second? Third?

Here $\frac{1}{7}$ of a £ is $\frac{1}{7}$ of 20 shillings; but 1 shilling is equal to 12 pence, hence $\frac{1}{7}$ of a £ $=\frac{1}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{7}$. The reason of the rule is therefore apparent.

2. Reduce $\frac{4}{7}$ cwt. to the fraction of a pound.
Ans. $\frac{448}{7}$lb.

3. Reduce $\frac{2}{27}$ of a pound Troy, to the fraction of an ounce.
Ans. $\frac{8}{9}$oz.

4. Reduce $\frac{7}{85}$ of a week to the fraction of a day.
Ans. $\frac{49}{85}$ day.

5. Reduce $\frac{3}{504}$ of a hogshead to the fraction of a gallon.
Ans. $\frac{3}{8}$ gal.

6. Reduce $\frac{5}{8}$ of a tun to the fraction of a gill.
Ans. $\frac{40320}{8}$ gill.

7. Reduce $\frac{1}{1584}$ of a day to the fraction of a minute.
Ans. $\frac{10}{11}$ min.

8. Reduce $\frac{7}{5760}$ of a furlong to the fraction of a foot.
Ans. $\frac{77}{96}$ ft.

CASE X.

To find the value of a fraction in whole numbers of a lower denomination.

RULE.

I. Reduce the numerator to the next lower denomination and divide the result by the denominator.

II. Reduce the remainder, if there be one, to the denomination still less, and divide again by the denominator, and so proceed to the lowest denomination. The several quotients placed in order, will be the value of the fraction in the different denominations.

QUESTIONS.—To find the value of a fraction in the lower denominations of a whole number what is the first thing to be done? What is the next? How do we reduce an integer to a fraction of a given denomination? To find the value of a fraction in the lower denominations of a whole number what is the first thing to be done?

EXAMPLES.

1. What is the value of $\frac{2}{3}$ of a £.

Operation.

```
  2
 20
 ——
3)40
 ——
 13s . . 1 Remainder.
        12
        ——
      3)12
        ——
        4d.
       ————
Ans. 13s. 4d.
```

We first reduce the numerator from the denomination of pounds to that of shillings. Dividing by the denominator gives 13s. and 1 over. Reducing this to pence and dividing as before gives 4d.

2. What is the value of $\frac{3}{10}$ of a day? Ans. 7h. 12m.
3. Find the value of $\frac{7}{8}$ of an acre. Ans. 3R. 20P.
4. Find the value of $\frac{5}{16}$ of a cwt. Ans. 1qr. 7lbs.
5. What is the value of $\frac{5}{6}$ of a hogshead? Ans. 52gals. 2qts.
6. What is the distance of $\frac{9}{10}$ of a mile? Ans. 7fur. 8p.
7. Reduce $\frac{1}{2}\frac{9}{0}$ of an ell English to its proper value. Ans. 1yd. 0qr. 3na.
8. Reduce $\frac{4}{7}$ of a mile to its proper quantity. Ans. 4fur. 22rds. 4yds. 2ft. 1in. $2\frac{1}{7}$bc.

CASE XI.

To reduce an integer to a fraction of a given denomination.

RULE.

Reduce the number to the lowest denomination mentioned in it; then if the reduction is to be

Question.—How do we reduce an integer to a fraction of a given denomination?

made still lower, proceed as in Case IX, but if to a higher denomination, proceed as in Case VIII.

EXAMPLES.

1. Reduce 4d. 2qrs. to the fraction of a shilling.

Ans. $\frac{3}{8}$.

Operation.

4d. 2qrs. = 18qrs.

Then 18 of $\frac{1}{4}$ of $\frac{1}{12}=\frac{18}{48}=\frac{3}{8}$ qr.

We first reduce the given number to the lowest denomination mentioned in it, viz: qrs. Then as the reduction is to be made to a higher denomination, we reduce as in Case VIII.

2. Reduce 2 feet 2 inches to the fraction of a yard.

Ans. $\frac{13}{18}$ yd.

3. What part of a hogshead is 3qts. 1pt.?

Ans. $\frac{1}{72}$ hhd.

4. Reduce 13 hours 30min. to the fraction of a day.

Ans. $\frac{9}{16}$ day.

5. Reduce 3cwt. 2qrs. 14lbs. to the fraction of a ton.

Ans. $\frac{29}{160}$ ton.

6. What part of a mile is 6ft. 7in.? Ans. $\frac{79}{63360}$.

7. What part of a mile is 1 inch? Ans. $\frac{1}{63360}$.

ADDITION OF VULGAR FRACTIONS.

Addition of fractions teaches how to express the value of several fractions by a single fraction.

It is plain that fractions cannot be added so long as they have different units; for, $\frac{1}{2}$ of a £ and $\frac{1}{2}$ of a shilling neither make £1 nor 1 shilling.

Neither can we add parts of the same unit unless they are like parts, for $\frac{1}{3}$ of a £ and $\frac{1}{4}$ of a £ neither make $\frac{2}{3}$ of a £ nor $\frac{2}{4}$ of a £. But $\frac{1}{3}$ of a £ and $\frac{1}{3}$ of a £ may be added, and make $\frac{2}{3}$ of a £.

QUESTIONS.—What does addition of fractions teach? Before fractions can be added what two things must be done?

We see therefore that before fractions can be added they must be first reduced to the same denomination; and secondly, to a common denominator.

CASE I.

When the fractions are of the same denomination and have a common denominator.

RULE.

Add the numerators together, and place their sum over the common denominator; then reduce the fraction to its lowest terms, or to its equivalent mixed number.

EXAMPLES.

1. Add $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{5}{4}$ together. Ans. $\frac{11}{4}=2\frac{3}{4}$.

Operation. $1+2+3+5=11$. Hence $\frac{11}{4}$=their sum.

It is plain that as all the parts are fourths their true sum will be expressed by the number of fourths; that is 11 fourths, which equal $2\frac{3}{4}$.

2. Add $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{4}{5}$ of a £ together. Ans. £$\frac{7}{5}=1\frac{2}{5}$.
3. What is the sum of $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{7}{8}$? Ans. $\frac{16}{8}=2$.
4. What is the sum of $\frac{2}{15}$, $\frac{4}{15}$, $\frac{7}{15}$ and $\frac{11}{15}$? Ans. $1\frac{9}{15}$.
5. Add together, $\frac{8}{17}$, $\frac{11}{17}$, $\frac{9}{17}$, $\frac{14}{17}$, $\frac{21}{17}$ and $\frac{35}{17}$.

Result, $\frac{98}{17}=5\frac{13}{17}$.

CASE II.

When the fractions are of the same denomination, but have different denominators.

RULE.

Reduce mixed numbers to improper fractions, by Case II, page 148; compound fractions to simple ones by Case V, page 150; and all the frac-

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QUESTIONS.—When fractions are of the same denomination and have a common denominator, how do we find their sum? What is the sum of 2 thirds, 4 thirds and 1 third? Of 1 fourth, 2 fourths and 6 fourths? When fractions have different denominators how do we add them? How do we reduce fractions to a common denominator? How may 1 fourth and 1 half be added?

tions to a common denominator, by Case VI, page 152. Then add them as in the last article.

EXAMPLES.

1. Add $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{2}{5}$ together. Ans. $\frac{142}{30}=4\frac{11}{15}$.

Operation.
$6\times3\times5=90$, 1st numerator.
$4\times2\times5=40$, 2nd numerator.
$2\times3\times2=12$, 3rd numerator.
$2\times3\times5=30$, the denominator.

After reducing to a common denominator, the new fractions are $\frac{90}{30}$, $\frac{40}{30}$, $\frac{12}{30}=\frac{142}{30}$, which, reduced to its lowest terms, becomes $4\frac{11}{15}$.

2. Add $\frac{6}{9}$, $\frac{5}{8}$ and $\frac{3}{4}$ together. Ans. $2\frac{2}{48}$.

3. Add $4\frac{3}{8}$ and $9\frac{5}{8}$ together. Ans. 14.

4. Find the lowest common denominator, and add $\frac{6}{12}$, $\frac{3}{5}$, $\frac{4}{8}$ and $\frac{6}{30}$. Ans. $1\frac{96}{120}$.

Note.—When there are mixed numbers, instead of reducing them to improper fractions, it is better to add the whole numbers and fractional parts separately, and then add their sums.

5. Add $19\frac{1}{7}$, $6\frac{2}{3}$, and $4\frac{4}{5}$ together. Ans. $30\frac{64}{105}$.

Operation. Operat'n. fract'l p'ts.
$19+6+4=29$ whole numbers. $\frac{1}{7}+\frac{2}{3}+\frac{4}{5}=\frac{169}{105}=1\frac{64}{105}$.
Then $29+1\frac{64}{105}=30\frac{64}{105}$, the sum.

6. Add $12\frac{1}{2}$, $3\frac{2}{3}$, and $4\frac{3}{4}$ together. Ans. $20\frac{11}{12}$.

7. Add $\frac{9}{11}$, $\frac{5}{7}$, $4\frac{3}{4}$, and $\frac{3}{4}$ of $\frac{5}{9}$ together. Ans. $6\frac{328}{462}$.

8. Add together $\frac{1}{3}$ of 95, and $\frac{7}{8}$ of 14. Ans. $43\frac{11}{12}$.

9. Add $3\frac{1}{4}$, $6\frac{5}{7}$, $8\frac{9}{15}$, and $65\frac{9}{8}$. Ans. $84\frac{579}{840}$.

10. Add $\frac{3}{8}$ of $\frac{5}{6}$ of $1\frac{3}{5}$ apples, $\frac{2}{7}$ of $\frac{1}{2}$ of $2\frac{1}{2}$ apples, $\frac{5}{9}$ of $\frac{3}{5}$ of $7\frac{1}{2}$ apples, and $\frac{2}{3}$ of $\frac{11}{12}$ of $3\frac{3}{5}$ apples together.

Result, $5\frac{39}{70}$.

CASE III.

When fractions are of different denominations.

Questions.—When there are mixed numbers to be added, what is the best method? When the fractions are of different denominations what is first to be done? What is next? What part of a pound is $\frac{1}{2}$ of an ounce Troy weight? Then what is the sum of three-twenty-fourths of a pound and half an ounce?

RULE.

Reduce the fractions to the same denomination. Then reduce them to a common denominator and add as in Case I.

EXAMPLES.

1. Add $\frac{3}{8}$ of a pound Troy, to $\frac{5}{6}$ of an ounce. Ans. $5\frac{2}{6}$oz.

Operation.

$\frac{3}{8}$ of $\frac{12}{1}=\frac{36}{8}$

Then $\frac{36}{8}+\frac{5}{6}=\frac{216}{48}+\frac{40}{48}=\frac{256}{48}$

$\frac{256}{48}=5\frac{2}{6}$ ounces.

In this example we first reduce $\frac{3}{8}$ of a pound to the fraction of an ounce, and find it is $\frac{36}{8}$ of an oz. Then $\frac{36}{8}$ and $\frac{5}{6}$ reduced to a common denominator are $\frac{216}{48}$ and $\frac{40}{48}$, which added together, make $\frac{256}{48}$, and reduced to a mixed number equal $5\frac{2}{6}$ ounces.

Or the $\frac{5}{6}$ of an ounce might have been reduced to the fraction of a pound, thus, $\frac{5}{6}$ of $\frac{1}{12}=\frac{5}{72}+\frac{3}{8}=\frac{27}{72}=\frac{32}{72}$ of a pound, which being reduced by Case X, equals $5\frac{2}{6}$ oz.

2. Add $\frac{2}{5}$ of a day to $\frac{5}{6}$ of an hour. Result, 10hrs. 26min.

3. Add $\frac{7}{9}$ of a ton, to $\frac{9}{10}$ of a cwt. Result, 16cwt. 1qr. $23\frac{1}{45}$ lbs.

4. Add $\frac{5}{9}$ of a week, $\frac{7}{10}$ of a day, and $\frac{6}{7}$ of an hour together. Result, 4 days, 14hrs. $59\frac{3}{7}$ min.

5. What is the sum of $\frac{2}{7}$ of £15, £$3\frac{3}{7}$, $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of a £, and $\frac{2}{8}$ of $\frac{3}{7}$ of a shilling? Ans. £7 17s. $5\frac{1}{7}$d.

6. Required the sum of $\frac{3}{4}$ of $\frac{5}{6}$ of $3\frac{1}{3}$ tons, $\frac{2}{3}$ of $\frac{9}{10}$ of $2\frac{1}{2}$ tons, and $\frac{5}{6}$ of $\frac{7}{10}$ of $5\frac{3}{4}$ cwt. Ans. 3T. 17cwt. $11\frac{2}{3}$lb.

NOTE.—The value of each fraction may be found separately, and their several values then added.

7. Add $\frac{3}{5}$ of a year, $\frac{1}{3}$ of a week, and $\frac{1}{8}$ of a day,

$\frac{3}{5}$ of a year $=\frac{3}{5}$ of $\frac{365}{1}$ days $=$ 219 days,
$\frac{1}{3}$ of a week $=\frac{1}{3}$ of 7 days $=$ 2 days, 8hrs.
$\frac{1}{8}$ of a day $=$ 3hrs.

Ans. 221 days, 11hrs.

8. Add $\frac{3}{4}$ of a cwt., $\frac{42}{2}$ of a lb. 13oz. and $\frac{1}{2}$ of a cwt. 6lbs. together. Ans. 1cwt. 1qr. 27lbs. 13oz.

9. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{13}{4}$ of a minute together. Ans. 2 days, 2hrs. 30' 45''.

10. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile together. Ans. 1540yds. 2ft. 9in.

SUBTRACTION OF VULGAR FRACTIONS.

We have seen that fractions cannot be added until they are reduced to the same denomination and to a common denominator. The same is necessary before they can be subtracted.

Subtraction of Vulgar Fractions teaches how to take a less fraction from a greater.

CASE I.

When the fractions are of the same denomination, and have a common denominator.

RULE.

Subtract the less numerator from the greater and place their difference over the common denominator.

EXAMPLES.

1. What is the difference between $\frac{7}{9}$ and $\frac{4}{9}$?
Here $7-4=3$, hence, $\frac{3}{9}=$the difference. Ans. $\frac{3}{9}$.
2. Subtract $\frac{15}{28}$ from $\frac{24}{28}$. Ans. $\frac{9}{28}$.
3. From $\frac{230}{375}$ take $\frac{145}{375}$. Ans. $\frac{85}{375}$.
4. From $\frac{4978}{9765}$ take $\frac{1697}{9765}$. Ans. $\frac{3281}{9765}$.
5. From $\frac{18345}{425}$ take $\frac{11234}{425}$. Ans. $\frac{7111}{425}$.

Questions.—Can one-third of an hour be subtracted from two-thirds of a day without reduction? Can one-fourth of a day be subtracted from one-sixth of a day? Before subtracting fractions what reductions are necessary? What does subtraction of fractions teach?

CASE II.

When fractions are of the same denomination, but have different denominators.

RULE.

Reduce mixed numbers to improper fractions, compound fractions to simple ones, and all the fractions to a common denominator; then subtract as in case I.

EXAMPLES.

1. What is the difference between $\frac{4}{5}$ and $\frac{3}{4}$? Ans. $\frac{1}{20}$.
$\frac{4}{5}=\frac{16}{20}$ and $\frac{3}{4}=\frac{15}{20}$; therefore $\frac{16}{20}-\frac{15}{20}=\frac{1}{20}$ difference.
2. What is the difference between $\frac{9}{14}$ and $\frac{17}{27}$? Ans. $\frac{5}{378}$.
3. From $14\frac{1}{4}$ take $\frac{2}{3}$ of 19 — Ans. $1\frac{7}{12}$.
4. From $\frac{37}{80}$ take $\frac{111}{240}$. — Remainder 0.
5. From 14 take $\frac{13}{15}$. — Rem. $13\frac{2}{15}$.
6. From $\frac{3}{4}$ take $\frac{2}{3}$ of $\frac{7}{8}$. — Ans. $\frac{1}{6}$.
7. What is the difference between $\frac{2}{3}$ of $\frac{4}{5}$ of 20, and $\frac{1}{4}$ of $\frac{2}{6}$ of $12\frac{1}{2}$? — Ans. $8\frac{1}{6}$.
8. What is the difference between £$2\frac{1}{2}$ and £$\frac{3}{15}$? — Ans. £2 6s.
9. From $\frac{1}{7}$ of $\frac{3}{6}$ of 7, take $\frac{3}{8}$ of $\frac{5}{4}$. — Ans. $\frac{1}{32}$.
10. From $37\frac{11}{15}$, take $3\frac{5}{7}$ of $\frac{1}{3}$. — Ans. $36\frac{52}{105}$.

CASE III.

When the fractions are of different denominations.

RULE.

Reduce the fractions to the same denomination; then to a common denominator, and add as in case I.

Questions.—When fractions are of the same denomination, and have a common denominator, how do we subtract them? When they have different denominators what must be done?—When the fractions are of different denominations what is the rule?

EXAMPLES.

1. What is the difference between $\frac{1}{2}$ of a £ and $\frac{1}{3}$ of a shilling? Ans. 9s. 8d.

$\frac{1}{3}$ of a shilling $=\frac{1}{3}$ of $\frac{1}{20}=\frac{1}{60}$ of a £.

$\frac{1}{2}$ of a £ $=\frac{30}{60}-\frac{1}{60}=\frac{29}{60}$ of a £ $=$ 9s. 8d. difference.

2. From $\frac{3}{5}$ of an oz. take $\frac{7}{8}$ of a pwt.

Ans. 11pwt. 3grs.

3. Subtract $\frac{7}{12}$ of a lb. from $\frac{1}{2}$ of a cwt.

Ans. 1qr. 27lbs. 6oz. 10$\frac{8}{12}$dr.

4. From $3\frac{2}{3}$ weeks take $\frac{1}{7}$ of a day, and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of an hour. Ans. 3w. 4da. 12hr. 19m. $17\frac{1}{7}$sec.

5. From $1\frac{3}{4}$ of a lb. troy wt. take $\frac{1}{6}$ of an ounce.

Ans. 1lb. 8oz. 16pwt. 16gr.

6. What is the difference between $\frac{4}{15}$ of a hogshead and $\frac{6}{19}$ of a quart? Ans. 16gals. 2qts. 1pt. $\frac{37}{95}$gills.

7. What is the difference between $\frac{5}{9}$ of $\frac{8}{15}$ of $3\frac{1}{4}$lbs. Troy wt. and $\frac{7}{16}$ of $\frac{4}{9}$ of $3\frac{3}{8}$lbs? Ans. 3oz. 13pwt. $14\frac{2}{3}$grs.

8. From the sum of $\frac{5}{9}$ of 7 miles, $\frac{2}{3}$ of $\frac{7}{8}$ of $2\frac{1}{4}$ miles, and $\frac{5}{6}$ of $\frac{3}{10}$ of $3\frac{1}{2}$ miles, take the sum of $\frac{5}{7}$ of $\frac{3}{5}$ of $1\frac{3}{4}$ miles, $\frac{8}{9}$ of $\frac{9}{10}$ of $2\frac{1}{5}$ miles, and $\frac{7}{11}$ of $\frac{5}{7}$ of $2\frac{3}{4}$ miles.

Result, 2m. 2fur. $21\frac{1}{4}\frac{1}{5}$rods.

MULTIPLICATION OF VULGAR FRACTIONS.

If 1 apple cost $\frac{1}{3}$ of a penny what will 2 apples cost? 3 apples? 5 apples? 7 apples? 8 apples? 9 apples?

Multiply the fraction $\frac{4}{12}$ by 4.

$\frac{4}{12}\times4=\frac{16}{12}=\frac{4}{3}=1\frac{1}{3}$ Ans.

Or by dividing the denominator by 4 we have $\frac{4}{12}\times4=$

$4)\frac{4}{12}=\frac{4}{3}=1\frac{1}{3}$ Ans.

Multiplying a fraction by a whole number is *increasing* the *value* of the *fraction* as many times as there are *units* in the *multiplier*. This we have seen in the above example, may be done, by either *multiplying* the *numerator*, or *dividing* the *denominator*.

Thus $\frac{3}{8}\times 8=\frac{24}{8}=3$.

Or $8)\frac{3}{8}=\frac{3}{1}=3$.

Hence the following general principle:

If the denominator remains unchanged, multiplying the numerator of a fraction by any number is multiplying the fraction by that number.

Or, *If the numerator remains unchanged, dividing the denominator by any number, is multiplying the fraction by that number.*

For, the *less* the *denominator* the *greater* is the *size* of the parts into which a unit is divided, as $\frac{1}{3}$ is more than $\frac{1}{4}$; and the *greater* the *numerator*, the greater the *number* of parts expressed by the fraction, as $\frac{3}{4}$ is greater than $\frac{1}{4}$.

Hence,

CASE I.

To multiply a fraction by a whole number, we have the following

RULE.

Multiply the numerator, or divide the denominator by the whole number.

EXAMPLES.

1. Multiply $\frac{37}{144}$ by 12. Ans. $3\frac{1}{12}$.

Here, $12)\frac{37}{144}=\frac{37}{12}=3\frac{1}{12}$ Or $\frac{37\times 12}{144}=\frac{444}{144}=3\frac{1}{12}$.

2. Multiply $\frac{47}{49}$ by 7. Ans. $6\frac{5}{7}$.

QUESTIONS.—What is multiplying a fraction by a whole number? Repeat the principle stated above.

3. Multiply $\frac{175}{47}$ by 9. Ans. $\frac{1575}{49}$.
4. Multiply $\frac{127}{15}$ by 5. Ans. $42\frac{1}{3}$.
5. Multiply $\frac{369}{145}$ by 49. Ans. $124\frac{101}{145}$.

CASE II.

To multiply one fraction by another.

You have already learned that multiplying by a fraction is taking a part of the multiplicand as many times as there are like parts of a unit in the multiplier.

For example to multiply 8 by $\frac{3}{4}$ is to take $\frac{3}{4}$ of 8 which is 6. Hence, when the multiplier is less than 1, we do not take the whole of the multiplicand, but only such a part of it as the fraction is of unity. Thus, if the multiplier is one half of unity, the product will be one half of the multiplicand; if the multiplier be $\frac{1}{4}$ of unity, the product will be $\frac{1}{4}$ of the multiplicand.

Hence, to multiply by a *proper fraction* does not imply increase, as in multiplication of whole numbers.

For example. Multiply $\frac{3}{4}$ by $\frac{2}{3}$.

Here $\frac{3}{4}$ is to be taken $\frac{2}{3}$ times, that is $\frac{3}{4}$ is to be multiplied by 2 and the product divided by 3. This result is obtained by multiplying the numerator by the numerator and the denominator by the denominator.

For the numerator $3\times2=6$, and the denominator $4\times3=12$, thus, $\frac{6}{12}$, and as twelfths are three times less than fourths, it follows that the fraction has been divided by 3 as well as multiplied by 2.

Hence we have the following

QUESTIONS.—Why does dividing the denominator increase the value of a fraction? Why does multiplying the numerator increase the fraction? How do we multiply a fraction by a whole number? What is multiplying by a fraction? When the multiplier is less than 1 what part of the multiplicand do we take? If the multiplier is $\frac{1}{2}$ what part of the multiplicand will the product be? If it is $\frac{1}{4}$? Does multiplying by a proper fraction imply increase? Does multiplying the denominator increase or diminish the value of the fraction? Why?

RULE.

Reduce mixed numbers to improper fractions, and compound fractions to simple ones; then multiply the numerators together for a numerator, and the denominators together for a denominator.

EXAMPLES.

1. Multiply $\frac{1}{5}$ by $\frac{3}{7}$. Ans. $\frac{3}{35}$.

Here $\frac{1}{5 \times 7} = \frac{1}{35}$; $\frac{1 \times 3}{35} = \frac{3}{35}$. It will be seen that $\frac{1}{35}$ is only $\frac{1}{7}$ of $\frac{1}{5}$, and $\frac{1}{5}$ multiplied by $\frac{3}{7}$ is not the whole of one fifth, but only 3 7ths of it.

2. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. Ans. $\frac{21}{24} = \frac{7}{8}$.

3. Multiply $\frac{1}{2}$ of $\frac{2}{3}$, by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$. Ans. $\frac{5}{36}$.

This may be somewhat shortened by cancelling; thus, $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3} \times \frac{2}{3}$ of $\frac{3}{4} = \frac{2}{4}$ of $\frac{5}{6} = \frac{10}{72} = \frac{5}{36}$. Ans.

4. Multiply $\frac{1}{9}$ of $\frac{7}{8}$ of $\frac{11}{12}$, by $\frac{6}{7}$ of $5\frac{3}{7}$. Result, $\frac{209}{504}$.

5. Required the product of 6 by $\frac{2}{3}$ of 5. Ans. 20.

6. What is the product of $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$? Ans. $\frac{23}{84}$.

7. Required the product of $7\frac{5}{9}$, $2\frac{1}{4}$, $3\frac{1}{2}$, and $\frac{6}{7}$ of $\frac{13}{17}$? Ans. 39.

8. What is the product of 5, $\frac{2}{3}$, $\frac{2}{7}$ of $\frac{3}{5}$ and $4\frac{1}{6}$? Ans. $2\frac{8}{21}$.

9. Required the product of $4\frac{1}{2}$, $\frac{3}{4}$ of $\frac{1}{7}$ and $18\frac{4}{5}$? Ans. $9\frac{9}{140}$.

10. Required the product of 14, $\frac{5}{6}$, $\frac{4}{5}$ of 9 and $6\frac{3}{7}$? Ans. 540.

11. What is the continued product of $6\frac{1}{3}$, $2\frac{2}{3}$, $\frac{4}{7}$ of $\frac{3\frac{1}{2}}{24}$ and $\frac{3}{9\frac{4}{5}}$ Ans. $\frac{5}{22}$.

DIVISION OF VULGAR FRACTIONS.

Suppose there are $\frac{16}{4}$ of apples which we wish to divide equally among 4 children; we should take the parts ex-

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QUESTIONS.—Repeat the rule for multiplying one fraction by another. How many ways are there of dividing a fraction by a whole number? What are they? Is there any difference in the results?

pressed by the numerator 16, and divide them again into four portions, this would be dividing the numerator by 4. If we wished to take one half the parts we should take one half of 16, this would be dividing the numerator by 2, and the fraction would be $\frac{8}{4}$.

Again,

If we have $\frac{3}{4}$ of an apple and wish to divide it among 4 children, we must divide the parts again, in order to share it equally; let each fourth be divided into 3 equal parts, each part will be $\frac{1}{12}$ of an apple and each one's share of the whole will be $\frac{3}{12}$ of an apple.

From these examples it appears that there are two ways of dividing a fraction by a whole number, viz. To divide the numerator, or, if this cannot conveniently be done, To multiply the denominator.

$\frac{3)3}{6}=\frac{1}{6}$. Or $\frac{3}{6\times3}=\frac{3}{18}=\frac{1}{6}$ Here it is plain that whether the numerator is divided by 3, or the denominator multiplied by 3, the result is the same.

From what has been shown we have the following *general principle*, viz.

If the denominator remain unchanged, dividing the numerator by any number, is dividing the fraction by that number.

Or, *If the numerator remain unchanged, multiplying the denominator by any number, is dividing the fraction by that number.*

CASE I.

To divide a fraction by a whole number.

RULE.

Divide the numerator, or multiply the denominator, by the whole number.

QUESTIONS.—How does multiplying the denominator divide or diminish the value of the fraction? Which is the greater, three-fourths or three-twelfths? What general principle is stated above? Repeat the rule for dividing a fraction by a whole number.

EXAMPLES.

1. Divide $\frac{3}{4}$ by 2. Ans. $\frac{3}{8}$.
2. Divide $\frac{4}{3}$ by 2. Ans. $\frac{2}{3}$.
3. Divide $\frac{28}{15}$ by 2, by 7 and by 14. Ans. $\frac{4}{15}$, $\frac{4}{15}$, $\frac{2}{15}$.
4. Divide $\frac{18}{37}$ by 9. Ans. $\frac{18}{333}=\frac{2}{37}$.
5. Divide $\frac{405}{19}$ by 15. Ans. $\frac{405}{285}=\frac{27}{19}$.
6. Divide $\frac{2755}{37558}$ by 19. Ans $\frac{145}{37558}$.
7. Divide $\frac{379}{1267}$ by 15. Ans. $\frac{379}{19005}$.

CASE II.

To divide one fraction by another.

EXAMPLES.

1. Divide $\frac{8}{20}$ by $\frac{4}{5}$.

1st Operation.

$$\frac{4}{5}=\frac{4\times1}{5}$$

$$\frac{8}{20}\div4=\frac{8}{80}$$

$$\frac{8\times5}{80}=\frac{40}{80}=\frac{1}{2}$$

If the divisor were 4 the quotient would be $\frac{8}{80}$. But since the divisor is only $\frac{1}{5}$ of 4, the true quotient must be 5 times $\frac{8}{80}$, for the fifth of a number will be contained in the dividend 5 times more than the number itself. In this operation we have actually multiplied the numerator of the dividend by 5 and the denominator by 4; that is, we have *inverted the terms of the divisor and multiplied the fractions together.*

2nd Operation.

$$\frac{8}{20}\div\frac{4}{5}=\frac{4)\ 8}{5)20}=\frac{2}{4}=\frac{1}{2}$$

Since multiplying the denominator by 4 is the same as dividing the numerator, and multiplying the numerator by 5 the same as dividing the denominator, we may, if we please, divide 8 by 4 and 20 by 5.

QUESTIONS.—How do we divide one fraction by another? Is multiplying the denominator the same as dividing the numerator? Is multiplying the numerator the same as dividing the denominator? If then we divide the numerator by the numerator and the denominator by the denominator, or if we invert the divisor and multiply the fractions together, will the result be the same?

Hence for division of one fraction by another, we have the following

RULE.

Prepare the fractions as in multiplication; then divide the numerator by the numerator, and the denominator by the denominator, if they will exactly divide. If they will not, then invert the terms of the divisor and proceed as in multiplication.

EXAMPLES.

1. Divide $\frac{15}{48}$ by $\frac{5}{6}$.

1st Operation.

$\frac{5)15}{6)48}=\frac{3}{8}$ Ans.

Here we divide the numerator by the numerator, and the denominator by the denominator.

2nd Operation.

$\frac{15}{48}\div\frac{5}{6}$ is equal to $\frac{15}{48}\times\frac{6}{5}=\frac{90}{240}=\frac{3}{8}$ Ans.

Here we invert the terms of the divisor, and multiply the fractions together.

2. Divide $\frac{1}{2}$ by $\frac{1}{2}$. Quotient, 1.
3. Divide $\frac{1}{2}$ by $\frac{1}{4}$. Quot. 2.
4. Divide $\frac{3}{4}$ by $\frac{1}{4}$. Quot. 3.
5. Divide $2\frac{1}{4}$ by $1\frac{1}{2}$. Quot. $1\frac{1}{2}$.
6. Divide $10\frac{3}{8}$ by $2\frac{1}{8}$. Quot. $4\frac{15}{17}$.
7. Divide $16\frac{1}{2}$ of $\frac{1}{3}$ by $4\frac{1}{7}$. Ans. $1\frac{19}{58}$.
8. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{2}{3}$.
9. Divide 5 by $\frac{7}{10}$. Ans. $7\frac{1}{7}$.
10. Divide $371\frac{1}{2}$ by $1\frac{1}{414}$. Ans. $370\frac{251}{415}$.
11. Divide $\frac{5}{6}$ of 50 by $4\frac{1}{3}$. Ans. $9\frac{8}{13}$.
12. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$. Ans. $9\frac{1}{2}$.
13. Divide $28\frac{3}{9}$ by $2\frac{2}{3}$. Ans. $7\frac{29}{33}$.
14. Divide £259 equally among 15 persons, and what is the share of each? Ans. £$17\frac{71}{240}$.
15. Divide $\frac{8}{9}$ of $\frac{5}{7}$ of $4\frac{3}{4}$, by $\frac{2}{3}$ of $\frac{4}{5}$ of $2\frac{5}{7}$. Ans. $2\frac{1}{12}$.

GENERAL REVIEW.

1. *A fraction is the expression of one or more parts of a unit.*

2. The *denominator* of a fraction shows into how many equal parts a unit is divided, and the *numerator* shows how many of the parts are taken in the fraction.

3. The *value* of every fraction is equal to the quotient of the *numerator* divided by the *denominator*.

4. When the *numerator* is less than the *denominator*, the value of the fraction is less than 1.

5. When the *numerator* is equal to the *denominator*, the value of the fraction is equal to 1.

6. When the *numerator* is greater than the *denominator*, the *value* of the fraction is greater than 1.

7. *If the denominator remains unchanged, multiplying the numerator by any number is multiplying the fraction by that number, and dividing the numerator is dividing the fraction.*

8. *If the numerator remains unchanged, multiplying the denominator by any number is dividing the fraction by that number,* and *dividing the denominator is multiplying the fraction.*

9. Hence it follows, that *dividing the numerator* by any number, has the same effect on the value of the fraction, as *multiplying the denominator*, and *multiplying the numerator* has the same effect as *dividing the denominator*.

10. It is also evident that if the *numerator and denominator be both multiplied*, or *both divided by the same number, the value of the fraction will remain the same.*

EXERCISES IN THE FOUR PRECEDING RULES.

1. What is the sum of $26\frac{2}{3}$, $18\frac{7}{8}$, $19\frac{3}{5}$, $13\frac{1}{4}$ and $\frac{117}{8}$?

Ans. $93\frac{1}{60}$.

2. Bought $\frac{1}{2}$ of $3\frac{1}{2}$ of 5cwt. of sugar at one time; at

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QUESTIONS.—What is a fraction? What does the denominator show? What does the numerator show? To what is the value of every fraction equal? When the numerator is less than the denominator what is the value of the fraction? When the numerator is equal to the denominator? When the numerator is greater than the denominator? Repeat the 7th proposition.

another, $\frac{1}{3}$ of $5\frac{1}{8}$ of 6cwt.; at another, $\frac{1}{5}$ of $\frac{6}{7}$ of 8cwt.; how much did I buy? Ans. $20\frac{331}{420}$.

3. What is the value of $\frac{4}{7}$ of a ton, and $\frac{9}{10}$ of a cwt.?
Ans. 12cwt. 1qr. 8lbs. $12\frac{8}{10}$ oz.

4. Bought 3 pieces of cloth; the first contained $\frac{1}{2}$ of 3 of $\frac{6}{2}$ of $\frac{2}{6}$ yards; the second $\frac{1}{4}$ of $\frac{4}{5}$ of 5; and the third $\frac{1}{6}$ of $\frac{3}{2}$ of $\frac{1}{3}$; what did they all contain?
Ans. 2yds. 2qrs. $1\frac{1}{3}$ na.

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1. From $\frac{3}{8}$ of an ounce take $\frac{7}{8}$ of a pwt.
Ans. 6pwt. 15gr.

2. Take $\frac{1}{7}$ of a day and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of an hour from $3\frac{2}{3}$ weeks. Ans. 3wk. 4da. 12hr. 19m. $17\frac{1}{7}$ sec.

3. From $1\frac{1}{2}$ of a £, take $\frac{3}{4}$ of a shilling.
Ans. £1 9s. 3d.

4. One man bought $\frac{1}{9}$ of $\frac{4\frac{1}{2}}{4\frac{1}{2}}$ cwt. of iron, another $\frac{1}{9}$ of $\frac{5\frac{1}{2}}{5\frac{1}{2}}$ cwt.; how much did one buy more than the other?
Ans. $3\frac{1201}{1995}$ drachms.

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1. Multiply $\frac{4}{5}$ of a bushel by $\frac{2}{3}$ of 7. Ans. $3\frac{11}{15}$ bu.

2. If I own $\frac{6}{7}$ of a ship, and sell $\frac{1}{3}$ of $\frac{1}{9}$ of my share, what part is it of the whole? Ans. $\frac{2}{63}$.

3. How many miles are $\frac{9}{10}$ of 7 miles, multiplied by $\frac{11}{15}$ of $87\frac{3}{11}$? Ans. $403\frac{1}{5}$ miles.

4. What will be the cost of $17\frac{1}{2}$ yards of cambric at $2\frac{1}{2}$ shillings per yard? Ans. £2 3s. 9d.

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1. If $\frac{4}{7}$ of a yard of cloth cost 3s. what is the price per yard? Ans. $5\frac{1}{4}$s.

2. Paid $666\frac{2}{3}$ pence for marbles at 6d. a piece; how many did I buy? Ans. $111\frac{1}{9}$.

3. In $8\frac{1}{2}$ weeks a family consumes $165\frac{3}{9}$ lbs. of butter; how much do they consume a week? Ans. $19\frac{68}{153}$ lbs.

4. If 50 bushels of wheat cost £$17\frac{3}{5}$, what is it per bushel? Ans. 7s. 0d. $1\frac{23}{25}$qrs.

Repeat the 8th. The 9th. The 10th.

DECIMAL FRACTIONS.

The division of the unit into tenths, hundredths, thousandths, &c. forms a system of numbers called *Decimal Fractions*, (from the Latin word *decem*, which signifies *ten*,) because they *increase* and *decrease* in a *tenfold* proportion, in the same manner as whole numbers.

The denominator of a decimal fraction is never written; the numerator is written with a point prefixed to it, and the denominator is understood to be 1, with as many ciphers annexed as there are figures in the numerator.—Thus: ,5 tenths is the same as $\frac{5}{10}$, and ,75 hundredths is the same as $\frac{75}{100}$, and ,316 thousandths is the same as $\frac{316}{1000}$, &c.

When a whole number and a decimal are written together, the decimal point is placed between them. Thus: 24,6 is $24\frac{6}{10}$; 5,71 is $5\frac{71}{100}$; 48,364 is $48\frac{364}{1000}$.

Decimals *decrease* in a tenfold proportion, counting from the left to the right. Thus: 5 is only one tenth the value it would express in the place of units, by taking away the decimal point; and ,05 is only one tenth as much as ,5. So it is plain that they diminish in a tenfold proportion as they recede from the place of units.

Ciphers placed on the right hand of decimal figures do not alter the value of the decimal, because the figures still remain unchanged in their distance from the unit's place. For instance; ,5, ,50, and ,500 are of equal value—they are each equal to *five-tenths*. But every cipher that is placed on the left of a decimal renders its value *ten times smaller*, by removing the figures one place further from the units place. Thus: if we prefix one cipher to ,5 it becomes ,05 hundredths; if we prefix two ciphers, it becomes ,005 thousandths, &c.

QUESTIONS.—How are decimal fractions formed? What is the meaning of *decem?* Why are they called decimals? How are decimal fractions expressed? Give an example. When a whole number and a decimal are written together, where is the decimal point placed? Give an example. What is the value of 5, written as a decimal compared with its value in the unit's place? How do decimals decrease?

DECIMAL NUMERATION TABLE.

	Tenths.	Hundredths.	Thousandths.	Tens of thousandths.	Hundreds of thousandths.	Millionths.	Tens of millionths.	
,	4							is read 4 tenths.
,	6	4						" 64 hundredths.
,	0	6	4					" 64 thousandths.
,	6	7	5	4				" 6754 ten thousandths.
,	0	1	2	3	4			" 1234 hundred thousandths.
,	0	0	7	6	5	4		" 7654 millionths.
,	0	0	4	3	6	0	4	" 43604 ten millionths.

Decimal fractions are numerated from the left hand to the right, beginning with the tenths, hundredths, &c., as in the above table.

EXERCISES.

Write upon the black board or slate, fifteen, and three-tenths. Eighteen, and seventy-five hundredths. Five, and five thousandths. One, and one millionth. Five, and five tenths. Seventy-five, and nine-tenths.

ADDITION OF DECIMALS.

RULE.

Write the numbers under each other, tenths under tenths, hundredths under hundredths, &c., then add as in whole numbers, setting the decimals in the sum directly under those in the numbers to be added.

QUESTIONS.—Do ciphers placed on the right hand of decimals affect their value? Why not? What is the value of ,5 written as a decimal? Of ,50? Of ,500? To what is each one equal?

EXAMPLES.

1. What is the sum of 37,04, 704,3 and ,0376?

Operation.

37,04	In this example we place those of the same value under each other, tenths under tenths, &c., then add as in whole numbers.
704,3	
,0376	
Ans. 741,3776	

2. Add 4,035, 763,196, 445,3741 and 91,3754 together. Ans. 1303,9805.

3. Add 72,5+32,071+2,1574+371,4+2,75. Ans. 480,8784.

4. Add ,7509+,0074+,69+,8408+,6109. Ans. 2,9.

5. To 9,999999 add one millionth part of a unit, and the sum will be 10.

6. What is the sum of one tenth, one hundredth, and one thousandth. Ans. ,111.

7. What is the sum of 4, and 6 ten thousandths? Ans. 4,0006.

8. Find the sum of Twenty-five hundredths. Three hundred and sixty-five thousandths. Six-tenths and nine millionths. Ans. 1,215009.

SUBTRACTION OF DECIMALS.

RULE.

Place the numbers according to their value, then subtract as in whole numbers, and point off the decimals as in Addition.

EXAMPLES.

1. From 837,642 take 579,358. Ans. 258,284.

QUESTIONS.—How does every cipher placed on the left of a decimal affect its value? Why so? Give examples. How are decimal fractions numerated? Repeat the rule for addition of decimals. What is the rule for subtraction of decimals?

	(2)	(3)	(4)
From	27,15	14,674	719,10009
Take	1,51679	5,91	7,121
Differ.	25,63321		
Proof,	27,15000		

5. From 480 take 245,0075. Ans. 234,9925.
6. From 236 take ,549. Ans. 235,451.
7. From ,145 take ,09684. Ans. ,04816.
8. From one take one millionth. Ans. ,999999.
9. From one hundred take one tenth. Ans. 99,9.
10. From nine-tenths take 75 hundredths. Ans. ,15.

MULTIPLICATION OF DECIMALS.

RULE.

Multiply as in simple numbers, and point off in the product from the right hand as many figures for decimals as are equal to the number of decimals in the multiplicand and multiplier; and if there be not so many in the product, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Multiply 3,024 by 2,23.

Operation.
3,024
2,23
6,74352 Product.

In this example there are three decimal figures in the multiplicand, and two in the multiplier, making five in both; we therefore point off five in the product, as the rule directs.

(2)		(3)	
Multiply	365,491	Multiply	496,0135
by	,001	by	1,496
Ans.	,365491	Ans.	742,0361960

QUESTIONS.—How do we multiply decimals? How many figures should be pointed off in the product? If there be not so many what must be done?

4. Multiply 25,238 by 12,17. Ans. 307,14646.
5. Multiply 2461 by ,0529. Ans. 130,1869.
6. Multiply 7853 by 3,5. Ans. 27485,5.
7. Multiply ,007853 by ,035. Ans. ,000274855.
8. Multiply ,004 by ,004. Ans. ,000016.
9. What is the product of five-tenths by five-tenths? Ans. ,25.

10. What is the product of five-tenths by five thousandths? Ans. ,0025.

11. Multiply one hundred and forty-seven millionths, by one millionth. Ans. ,000000000147.

To multiply by 10, 100, 1000, &c., remove the separating point so many places to the right hand as the multiplier has ciphers. For example; ,425 multiplied by 10 makes 4,25, and ,425×100=42,5, and ,425×1000=425.

DIVISION OF DECIMALS.

RULE.

Divide as in simple numbers; and in the quotient point off from the right hand so many places for decimals as the decimal places in the dividend exceed those in the divisor. That is, make the decimal places in the divisor and quotient counted together, equal to the decimal places in the dividend; and if there are not so many, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Divide 1,38483 by 6021.

QUESTIONS.—How do we multiply by 10, 100, 1000, &c.? Give examples. How are decimals divided? How many places should be pointed off for decimals?

Operation.
60,21)1,38483(23
12042
———
18063
18063
———
Ans. ,023.

There are 5 decimal places in the dividend, and 2 in the divisor; there must therefore be 3 places in the quotient. Hence, one 0 must be prefixed to the 23, and the decimal point placed before it.

2. Divide 2,3421 by 2,11. Ans. 1,11.
3. Divide 12,82561 by 3,01. Ans. 4,261.
4. Divide 77,4114 by 9,51. Ans. 8,14.
5. Divide 206,79 by 2,46. Ans. 84,06.
6. Divide 5,8674 by 127. Ans. ,0462.
7. Divide 2033,100 by ,324. Ans. 6275.
8. Divide 8,2470 by ,002. Ans. 4123,5.

To divide a decimal number by 10, 100, 1000, &c. remove the decimal point as many places to the left as there are ciphers in the divisor; and if there be not so many, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Divide 687 by 10. Ans. 68,7.
2. Divide 489 by 100. Ans. 4,89.
3. Divide 1678 by 1000. Ans. 1,678.
4. Divide 1895 by 10000. Ans. ,1895.

When there are more decimal places in the divisor than in the dividend, annex as many ciphers to the dividend as are necessary to make its decimal places equal to those of the divisor; *all the figures of the quotient will then be whole numbers.*

QUESTIONS.—How many decimal places should there be in the divisor and quotient counted together? What must be done if there are not so many? If there are more decimals in the divisor than in the dividend what should be done? What will all the quotient figures then be?

EXAMPLES.

1. Divide 4397,4 by 3,49. Ans. 1260.
2. Divide 2194,02194 by ,100001. Ans. 21940.
3. Divide 9811,0047 by ,325947. Ans. 30100.
4. Divide ,1 by ,0001. Ans. 1000.
5. Divide 10 by ,1. Ans. 100.

After bringing down all the figures in the dividend, if there be a remainder, we may annex ciphers, and carry on the quotient to any degree of exactness.

EXAMPLES.

1. Divide 37,4 by 4,5. Ans. 8,3111.+
2. Divide 4,18 by ,1812. Ans. ,23068.+
3. Divide 586,4 by 375. Ans. 1,563.+
4. Divide 94,0369 by 81,032. Ans. 1,160.+

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to its equivalent decimal.

RULE.

I. Annex one or more ciphers to the numerator, and then divide by the denominator.

II. If there is a remainder, annex a cipher or ciphers, and divide again, and so continue until the quotient is sufficiently exact, and there must be as many places pointed off in the quotient for decimals as there were ciphers used; if there be not so many supply the deficiency with ciphers.

EXAMPLES.

1. Reduce $\frac{7}{12}$ to a decimal.

Operation.

12)70000

———

Ans. ,5833+

By annexing four ciphers, we obtain four decimal figures. We might if we choose, annex more ciphers and carry the decimal lower.

QUESTIONS.—If there be a remainder after bringing down all the figures in the dividend, how do we proceed? How is a vulgar fraction reduced to its equivalent decimal? If there be a remainder what must be done with it? How many decimal places must be pointed off in the quotient?

2. Reduce $\frac{3}{4}$ and $\frac{1}{4}$ to decimals. Ans. ,75 & ,25.
3. Reduce $\frac{1}{5}$ to a decimal. Ans. ,2.
4. Reduce $\frac{1}{2}$ to a decimal. Ans. ,5.
5. Reduce $\frac{5}{8}$ to a decimal. Ans. ,625.
6. Reduce $\frac{1}{8}$ to a decimal. Ans. ,125.
7. Reduce $\frac{11}{16}$ to a decimal. Ans. ,6875.
8. Reduce $\frac{3}{32}$ to a decimal. Ans. ,09375.
9. Reduce $\frac{1}{3}$ to a decimal. Ans. ,333333+.
10. Reduce $\frac{1}{27}$ to a decimal. Ans. ,037037+

CASE II.

To reduce quantities of several denominations to a decimal.

RULE.

Write down the given numbers, from the least to the greatest, in a perpendicular column, then divide each denomination by such a number as will reduce it to the next higher denomination; in each place annexing the quotient to the right hand of the next superior denomination, and the last quotient will be the decimal required.

EXAMPLES.

1. Reduce 12s. 6d. 3qr. to the decimal of a pound.

Operation.

4)3,

12)6,75

20)12,5625

,628125 Ans.

We first place the numbers as the rule directs, and reduce 3 farthings to the decimal of a penny by dividing by 4, and place the quotient ,75 to the right of 6d. We next divide by 12, giving ,5625, which is the decimal of a shilling, this we annex to the pounds, and then divide by 20 and the work is done.

2. Reduce 15s. 7d. 2qr. to the decimal of a pound. Ans. ,78125=15s. 7d. 2qr.

QUESTION.—How are quantities of several denominations reduced to a decimal?

3. Reduce 9d. 3qr. to the decimal of a shilling.
Ans. ,8125.

4. Reduce 10s. 6d. to the decimal of a pound.
Ans. ,525.

5. Reduce £19 17s. 3¼d. to the decimal of a pound.
Ans. £19 ,863+

6. Reduce 7½d. to the denomination of shillings.
Ans. ,625s.

7. Reduce 12s. to the decimal of a pound.
Ans. ,6.

8. What is the decimal expression of £4 19s. 6½d.?
Ans. £4 ,97708+

9. Bring £34 16s. 7¾d. into a decimal expression.
Ans. £34,8322916+

10. Reduce 3qr. 2na. to the decimal of a yard.
Ans. ,875.

11. Reduce 1 gallon to the decimal of a hogshead.
Ans. ,015873.

12. Reduce 7 oz. 19pwt. to the decimal of a lb. Troy.
Ans. ,6625.

13. Reduce 3qrs. 21lbs. Avoirdupois, to the decimal of a cwt.
Ans. ,9375.

14. Reduce 2 roods 16 perches to the decimal of an acre.
Ans. ,6.

15. Reduce 2 feet 6 inches to the decimal of a yard.
Ans. ,833333+

16. Reduce 5fur. 16p. to the decimal of a mile.
Ans. ,675.

17. Reduce 4½ calendar months to the decimal of a year.
Ans. ,375.

18. Reduce 109 days 12 hours to the decimal of a year.
Ans. ,3.

19. Reduce 3qr. 12lbs. 5oz. 1,92 dr. to the decimal of a cwt.
Ans. ,86.

20. Reduce 3 pecks 6 quarts 1 pint to decimals of a bushel.
Ans. ,953125 bu.

21. Reduce 5cwt. 3qr. 16lb. to decimals of a ton.
Ans. ,2946428+ton.

CASE III.

To reduce a decimal fraction to its value.

RULE.

I. Multiply the decimal by that number which it takes of the next lower denomination to make one of this higher, and point off so many places for a remainder, to the right hand, as there are places in the given decimal.

II. Multiply the remainder by the next inferior denomination, and cut off a remainder as before, and so on through all the parts of the integer, and the several denominations standing on the left hand make the answer.

EXAMPLES.

1. What is the value of ,832296 of a pound Sterling?

Operation.

,832296 20 ——— s.16,645920 12 ——— d.7,751040 4 ——— far.3,004160 ———	We first multiply the decimal by 20, which brings it to shillings, and after cutting off from the right as many places for decimals as in the given number, we have 16s. and the decimal ,645920 over. This we reduce to pence by multiplying by 12, and then reduce to farthings by multiplying by 4.

Ans. 16s. 7d. 3far.

2. What is the value of ,5724 of a £?

Ans. 11s. 5d. 1,5 qr.

3. What is the value of ,85251 of a £?

Ans. 17s. 0d. 2,4qr.

4. What is the value of ,040625 of a pound Sterling?

Ans. $9\frac{3}{4}$d.

QUESTIONS.—To reduce a decimal fraction to its value, what is the first step? What is the second?

5. What is the value of £1,88? Ans. £1 17s. 7d.+
6. What is the value of £,3375? Ans. 6s. 9d.
7. What is the value of ,875cwt.? Ans. 3qrs. 14lbs.
8. Reduce ,67457lbs. Avoirdupois to its proper value. Ans. 10oz. 12,68992dr.
9. What is the value of ,617 of a cwt.? Ans. 2qr. 13lbs. 1oz. 10,6dr.
10. Find the value of ,76442 of a pound Troy. Ans. 9oz. 3pwt. 11grs.
11. What is the value of ,875 of a yd.? Ans. 3qr. 2na.
12. What is the value of ,875 of a hhd. of wine? Ans. 55gal. 1pt.
13. Find the proper quantity of ,089 of a mile. Ans. 28po. 2yd. 1ft. 11,04 in.
14. Find the proper quantity of ,9075 of an acre. Ans. 3roods. 25,2po.
15. What is the value of ,569 of a year of 365 days? Ans. 207da. 16h. 26m. 24sec.
16. What is the value of ,712 of a furlong? Ans. 28po. 2yds. 1ft. 11,04in.
17. What is the proper quantity of 142465 of a year? Ans. 51,9998725 days.
18. What is the difference between ,82 of a day and ,32 of an hour? Ans. 19h. 21m. 36sec.

RULE OF THREE, OR PROPORTION.

A correct knowledge of the Rule of Three is of the utmost importance, it being applicable in almost all arithmetical operations. I will now show you how the work may, in particular cases, be considerably shortened.

I. When the second or third term is a multiple or an aliquot part of the first; divide the second or third term by the first.

QUESTION.—What is the first method proposed for shortening operations in the Rule of Three?

1. If 6 yards of cloth cost £2 3s. 4d. what will 36 yards cost?

Opeartion.

```
yd.   yd.      £ s. d.
 6| : 3|6| :: 2  3  4
                    6
             --------
            13  0  0 Ans.
```

We state the question as in page 111, then because the first term is an aliquot part of the second, we divide the second by the first, and multiply the third term by the quotient 6, and the result is the same as though the second and third term had been multiplied, and the product divided by the first term.

2. If £6 buy 24 yards of cloth, how many yards may be bought for £11?

```
 £    £    yds.
 6| : 11 :: 2|4|
              4×11=44 yds. Ans.
```

II. When the first is a multiple of either the second or third, divide the first by the second or third.

1. If 12 yards of cloth cost £18 what will 4 yards cost?

```
 yd.   yd.    £
1|2| : 4| :: 18
 3               Then 18÷3=6 Ans.
```

2. If 36 yards cost £2 3s. 4d. what will 6 yards cost?

```
 yd.   yd.    £ s. d.
3|6| : 6| :: 2  3  4
             --------
             £0 7 2⅔ Ans.
```

III. When the first and either of the other given terms have a common measure, divide them by it and use the quotients instead of the given numbers.

QUESTIONS.—What is the second method mentioned? What is the third?

1. If 36 yards cost £3 2s. 6d. what will 24 yards cost?

	yds.		yds.		£	s.	d.	
Operation.	12)36	:	24	::	3	2	6	
	3		2				2	
					3)6	5	0	
					£2	1	8	Ans.

2. If 3 barrels of flour cost 12 dollars what will 16 barrels cost?

	bbl.		bbl.		$	
3)	3	:	16	::	12	$
	1				$4 \times 16 = 64$	Ans.

3. If 25 yards of cloth cost £2 3s. 4d. what will 5 yards cost? Ans. 8s. 8d.

4. If 12 hats cost 60 dollars, how much will 40 cost? Ans. 200 dollars.

5. If 30 barrels of flour will subsist 100 men for 40 days, how long will it subsist 25? Ans. 160.

6. If 120 sheep yield 360 lbs. of wool, how many pounds will be obtained from 600? Ans. 1800.

7. If a man travel 210 miles in 6 days, how far will he travel in 40 days? Ans. 1400 miles.

RULE OF THREE BY ANALYSIS.

The solution of questions by analysis consists in finding the ratio of two of the given terms, and multiplying this ratio by the other term.

The ratio of two of the terms will generally express the value or cost of a single thing.

EXAMPLES.

1. If 3 barrels of flour cost 24 dollars, what will seven barrels cost.

Questions.—In what does the solution of questions by analysis consist? What will the ratio of two of the terms generally express?

By dividing the 24 dollars by 3 we get the cost of 1 bbl. For, if 24 dollars will buy 3 barrels, it is plain that $\frac{1}{3}$ of it will buy 1 barrel. This, multiplied by 7, gives 56 dollars, the cost of 7 barrels.

2. If a family of ten persons spend 3 bushels of malt in a month, how many bushels will serve them when there are 30 in the family?

If 10 persons spend 3 bushels, it is plain that 1 person in the same time, would spend $\frac{1}{10}$ of 3 bushels, that is $\frac{3}{10}$ of a bushel: and 30 persons would spend 30 times as much, that is, $\frac{90}{10}=9$ bushels, Ans.

All the questions in Proportion may be solved on general principles as above, without the formality of a statement.

3. If a field will feed 6 cows 91 days, how long will it feed 21 cows. Ans. 26 days.

4. If I walk 84 miles in three days, how far should I walk at the same rate in 9? Ans. 252.

5. If 2 lbs. of sugar cost 25 cents, what will 100 lbs. of coffee cost, if 8 lbs. of sugar are worth 5 lbs. of coffee.

Ans. 20 dollars.

QUESTIONS INVOLVING FRACTIONS.

RULE.

State the question according to the directions given in page 111. Then having reduced when necessary, the similar terms to the same denomination, and mixed numbers to improper fractions, invert the first term, and multiply this term thus inverted and the other two continually together, for the answer sought, which will be of the same denomination as the third term.

EXAMPLES.

1. If $\frac{5}{8}$ of a yard cost $\frac{3}{7}$ of a pound, what will $\frac{9}{15}$ of an ell English cost? $\frac{5}{8}$yd. $=\frac{5}{8}$ of $\frac{4}{1}$ of $\frac{1}{5}=\frac{20}{40}=\frac{1}{2}$ ell Eng.

QUESTIONS.—May all questions in proportion be solved without a statement? Repeat the rule for operations in which there are fractions. Why is the first term to be inverted?

Ell. Ell. £

As $\frac{1}{2}$: $\frac{9}{15}$ :: $\frac{3}{7}$ First term inverted $\frac{2}{1}\times\frac{9}{15}\times\frac{3}{7}=\frac{45}{105}$£$=$ 10s. 3d. $1\frac{5}{7}$qr. Ans.

2. If $\frac{3}{5}$ of a yard cost $\frac{7}{8}$ of a pound, what will $40\frac{3}{4}$ yds. come to? Ans. £59 8s. $6\frac{1}{2}$d.

3. If $\frac{5}{7}$oz. cost £$\frac{11}{12}$, what will 1oz. cost?

Ans. £1 5s. 8.

4. A person having $\frac{3}{5}$ of a vessel, sells $\frac{2}{3}$ of his share for £312; what is the whole vessel worth? Ans. £780.

5. A merchant bought $5\frac{1}{2}$ pieces of cloth, each containing $24\frac{1}{3}$ yards at 9s. $\frac{1}{2}$d. per yard; what did the whole amount to? Ans. £60 10s. 0d. $3\frac{3}{8}$qr.

6. What is the value of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of a pound, at the rate of $\frac{9}{10}$ of a £ for $\frac{8}{9}$ of a pound? Ans. £$\frac{27}{64}$.

7. If $\frac{7}{8}$ of a ship be worth $\frac{2}{9}$ of her cargo, valued at £8000; what is the whole ship and cargo worth?

Ans. £10031 14s. $11\frac{1}{21}$d.

8. If $\frac{11}{13}$lbs. of sugar cost $\frac{7}{15}$ of a shilling, what will $\frac{32}{43}$ of a pound cost?

Let us solve this by analysis. First, divide $\frac{7}{15}$ of a shilling by $\frac{11}{13}$ of a lb. and the quotient is $\frac{91}{165}$s. which is the price of one pound; then, $\frac{32}{43}$ of $\frac{91}{165}$s.$=\frac{2912}{7095}$s., the cost of $\frac{32}{43}$ of a pound. $\frac{2912}{7095}$s.$=$4d. $3\frac{4971}{7095}$qr. Ans.

9. If 7 pounds of sugar cost $\frac{3}{4}$ of a dollar, what will 12 pounds cost?

It is plain that if 7lb. cost $\frac{3}{4}$ of a dollar, 1lb. will cost $\frac{1}{7}$ of $\frac{3}{4}=\frac{3}{28}$ of a dollar, it is also evident, that 12 pounds will cost 12 times $\frac{3}{28}$ of a dollar$=\frac{3}{28}\times12=\frac{36}{28}=1\frac{2}{7}$ dol. Ans.

10. How many pieces of merchandise, at $20\frac{1}{8}$s. a pc. must be given for 240 pieces, at 12s. a piece?

Ans. 149 $\frac{11}{161}$.

11. If 16 men finish a piece of work in $28\frac{1}{3}$ days, how long will it take 12 men to do the same work?

Ans. $37\frac{7}{9}$ days.

12. If $\frac{1}{3}$lb. less by $\frac{1}{6}$, costs $13\frac{1}{5}$d., what costs 14lbs. less by $\frac{1}{5}$ of 2lbs.? Ans. £4 9s. $9\frac{3}{25}$d.

13. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of 7 dollars for 5

yards, and sold them out at the rate of 11 dollars for 7 yds. and gained 200 dollars by the bargain; how many bales were there? Ans. 9 bales.

First find what he gained in selling 1 yard; then it will be easily ascertained how many yards he must sell to gain $200.

COMPOUND PROPORTION.

Compound Proportion, is a method of performing such operations as require two or more statings. It is commonly called Double Rule of Three, because its operations can be performed by two statings in the Rule of Three.

RULE.

Make that number which is of the same kind with the required answer the *third term*. Then with this third term and each pair of similar terms complete a stating in the single rule as already taught; then, having reduced the similar terms to like denominations, and the third to its lowest given denomination, multiply the numbers in the second and third places continually together for a dividend, and those in the first for a divisor; divide the former by the latter, and the quotient will be the required answer, of the same denomination as the third term.

NOTE.—The principles of contraction given in single proportion, are equally applicable in this rule.

EXAMPLES.

1. If 225 bushels of oats be eaten in 30 days, by 20 horses, how many bushels will suffice for 50 horses 16 days?

QUESTIONS.—What is compound proportion? Why is it called double rule of three? Repeat the rule for compound proportion.

Operation.

```
horses.
20 : 50 ⎫ bushels.
days.   ⎬ : : 225
30 : 16 ⎪       800
——— ——— ⎭
6|00  800     ———————
           6)1800|00
             ————
         Ans. 300
```

In this example it is plain that the term similar to the number sought is 225 bushels; which is therefore set in the third place. Then putting the question on the number of horses, the fourth term must be more than the third, for it is evident that 50 horses will require more in a given time than 20 horses, hence 50 must be the second, and 20 the first term. Again, putting the question on the number of days, since a given number of horses will eat less in 16 days than in 30; the fourth term here is less than the third; whence 16 is the second term and 30 the first.

2. If 20 horses in 30 days eat 225 bushels of oats, in how many days will 50 horses eat 300 bushels?

Operation.

```
    horses.        ⎫
 5|0| : 2|0|       ⎪   days.
  5|     2         ⎬ : : 3,0
   bushels.        ⎪
2|2|5| : 3|0|0|    ⎪      6|
   3|     4        ⎭      2
                          4
                          —
                          8
                          2
                          —
                  Ans. 16da.
```

We first state this question as the rule directs. Then because 10 is an aliquot part of 50 and 20, we divide them by 10 and cancel them. For the same reason we divide 225 and 300 by 75, and cancel these numbers. We then divide the third term, 30 by 5, and cancel it, and the 6 by 3, and cancel the 6 and 3. Then as the numbers are all cancelled in the first term, we multiply those in the second and third together, and the product is the answer.

3. If a family of 8 persons expend 360 dollars in 9 months, how much will serve a family of 18 persons 12 months? Ans. $1080.

4. If 3 men in 4 days eat 15lbs. of bread, how much will suffice 5 men for 12 days? Ans. 75lbs.

5. If 20cwt. be carried 50 miles for $15, how much

will 45cwt. cost to be conveyed 80 miles? Ans. $54.

6. If 12 men in 6 days mow 80 acres, in how many days will 25 men mow 250 acres? Ans. 9 days.

7. If 5 men make 300 pairs of shoes in 40 days, how many men will make 900 pairs in 60 days?

Ans. 10 men.

8. If 144 men can build a wall 32 feet high in 8 days, in how many days can 63 men build a wall 28 feet high, of the same length? Ans. 16 days.

9. If a footman, when the days are 14 hours long, can travel 276 miles in 16 days; in how many days can he travel 828 miles, when the days are but 12 hours long?

Ans. 56 days.

10. If the wages of 6 men for 14 days be 84 dollars, what will be the wages of 9 men for 11 days?

Let us work this question by analysis. First, $84 ÷ 14 = 6 dollars, what 6 men earn in 1 day; then $6 ÷ 6 men = 1 dollar, what 1 man earns in 1 day. Then $1 × 9 × 11 = 99 dollars, what 9 men will earn in 11 days.

Ans. $99.

11. If 56lbs of bread be sufficient for 7 men 14 days, how much bread will serve 21 men 3 days?

If 7 men consume 56lbs. of bread, one man in the same time would consume $\frac{1}{7}$ of 56lbs. $= \frac{56}{7}$lbs.; and if he consumes $\frac{56}{7}$lbs. in 14 days, he would consume $\frac{1}{14}$ of $\frac{56}{7} = \frac{56}{98}$ lbs. in 1 day. 21 men would consume 21 times as much as one man; that is 21 times $\frac{56}{98} = \frac{1176}{98}$lbs. in 1 day, and in 3 days they would consume 3 times as much; that is, $\frac{3528}{98} =$ 36lbs. Ans.

Or place the numbers that occupy the third and second places above a line, and the first terms below it, and cancel them, thus,

$$\frac{\overset{4}{5|6|} \times 2\overset{3}{|1|} \times 3}{1|4| \times 7|} = 36 \text{ Ans. as before.}$$

12. If 9 students spend £$10\frac{7}{8}$ in 18 days, how much will 20 students spend in 30 days?

£$10\frac{7}{8} \div 9 =$ £$\frac{87}{81}$ = what 1 student spends in 18 days, and £$\frac{97}{81}$: 18 $=\frac{97}{1458}=$ what 1 student spends in 1 day. Then $\frac{97}{1458}\times20=$ £$\frac{1940}{1458}=$ what 20 students spend in 1 day, and £$\frac{1940}{1458}\times30=$ what 20 students spend in 30 days $=$ £39 18s. $4\frac{20}{81}$. Ans.

13. If 3 men receive £$8\frac{9}{10}$ for $19\frac{1}{2}$ days' work, how much must 20 men receive for $100\frac{1}{4}$ days?

$$\frac{89\times \overset{2}{2|}\times 2|0|\times 401}{1|0|\times 3\times \underset{2}{4|}}=\frac{35689}{117}=£305\ 0s.\ 8\tfrac{8}{39}\ \text{Ans.}$$

14. If a barrel of beer last 7 persons 12 days, how much will be drank by 42 persons in a year?

Ans. 182 bar. 18gal.

15. If 32 men build a wall 36 feet long, 8 feet high, and 4 feet wide, in 4 days, in what time will 48 men build a wall 864 feet long, 6 feet high and 3 feet wide?

Ans. 36 days.

16. Supposing 3 compositors to set $15\frac{1}{2}$ pages in $2\frac{7}{9}$ hours, how many will be required to set $69\frac{3}{4}$ pages in $6\frac{1}{4}$ hours? Ans. 6.

17. If 12 oxen eat $7\frac{3}{5}$ acres of pasture in $5\frac{1}{2}$ weeks, how many oxen will be required to eat $13\frac{1}{3}$ acres in $10\frac{15}{19}$ weeks? Ans. 11 oxen.

18. If 16 compositors set 150 pages of types, each page containing 48 lines, and each line 50 letters, in 3 days of 10 hours each; how many compositors will be required to set 500 pages of 72 lines each, and 45 letters in a line, in 6 days of 8 hours each? Ans. 45.

SIMPLE INTEREST BY DECIMALS.

In calculating interest, the *rate per cent.* is a certain number of *hundredths* of the sum lent. Thus, if 1 per cent. is paid for £100, it is $\frac{1}{100}=,01$ part of the sum lent. If 6 per cent. is paid, it is the $\frac{6}{100}=,06$ part of the sum lent.

QUESTIONS.—In calculating interest, how is the rate per cent. to the sum lent? If 1 per cent is paid, what part of £100 is it?

For this reason all calculations in interest are properly sums in decimal multiplication.

The *rate per cent.* may always be written as a *decimal fraction* of the order of *hundredths.* Thus, 1 per cent. may be written ,01; 2 per cent. ,02; 3 per cent. 0,3; 4 per cent. ,04; 5 per cent. ,05; 6 per cent. ,06, and so on.

RULE.

Reduce the shillings and pence to the decimal of a pound—see page 183. Then find the interest by the rule on page 104: after which reduce the decimal part of the answer to shillings and pence, —see page 185.

EXAMPLES.

1. What is the interest at 6 per cent. of £27 15s. 9d. for 2 years?

Operation.

```
£27 15s. 9d.=£27,7857
                    6
             --------
             1,667250
                    2
             --------
            £3,334500
                   20
             --------
           s.6,690000
                   12
             --------
           d.8,280000
                    4
             --------
          qr.1,120000
             --------
      Ans. £3 6s. 8¼d.+
```

We first find the interest for one year; then multiply by 2, which gives the interest for two years. We then reduce to pounds, shillings and pence.

QUESTIONS.—If 6 per cent. is paid, what part of £100 is it? What are all calculations in interest? How may the rate per cent. always be written? Repeat the rule.

2. What is the interest of £121 8s. 6d. for 4½ years, at 6 per cent. per annum? Ans. £32 15s. 8d. 1,36qr.

3. What is the interest on £67 19s. 6d. at 6 per cent. for 3 years, 8 months, 16 days? Ans. £15 2s. 8½d.

4. What is the interest on £127 15s. 4d. at 6 per cent. for 3 years and 3 months? Ans. £24 18s. 3¼d.+

5. What is the interest of £107 16s. 10d. at 6 per cent, for 3 years, 6 months and 6 days? Ans.

6. What will £279 13s. 8d. amount to in 3 years and a half, at 5¼ per cent. per annum? Ans. £331 1s. 6d.+

7. What is the interest of £514 10s. 2d. for 3 years and a half, at 4 per cent.? Ans. £72 0s. 7¼d.+

8. What is the interest on £255 10s. 8d. at 6 per cent. per annum, for 6 years and 6 months?

Ans. £99 13s. 1¾d.

9. What is the interest on £53 18s. 5d. at 6 per cent. for 7 years and 12 days? Ans. £22 15s. 1d.+

Find the interest on the following note.

£127 10s. Toronto, Jan. 1st, 1845.

For value received, I promise to pay on the 10th day of June next, to T. Lawrence or order, the sum of one hundred and twenty-seven pounds and ten shillings, with interest from date, at 7 per cent. S. S. SMITH.

Ans. £3 19s. 2¼d.

To find what remains due at the end of a given time, on notes, bonds, mortgages, &c. when payments are made at different times.

There are various methods of casting interest when several payments are made, no two of which produce exactly the same result. The following is as simple and comprehensive as any with which I am acquainted.

RULE.

Multiply the principal by the time it bears interest, before any part of the debt is discharged,

---

Repeat the rule for casting interest on notes &c. when partial payments are made at different times.

and from the given principal deduct the first payment; multiplying the remainder by the time between the first and second payments; from the remainder deduct the second payment, and multiply the remainder by the time between the second and third payments, and so proceed through all the payments. Then add all the products together, and find the interest on the sum for 1 year, 1 month, or 1 day, according as the times of payment are, years, months, or days, and this interest added to the last remainder will be the sum due at the end of the given time.

EXAMPLES.

£600 Kingston, May 13th, 1845.

One year from date, for value received, I promise to pay Wm. Howland or bearer six hundred pounds currency, with interest at 5 per cent. per annum. H. GOOD.

Now the maker of this note, H. Good, pays the 9th of July £200, and the 17th of September £150; how much principal and interest is he to pay at the end of the year?

Operation.

£600×57 =34200
200
400×70 =28000
150
———
250×238=59500
————
121700
5
————
365)6085,00
————
£16 13s. $5\frac{7}{73}$d.
£250
16 13 $5\frac{7}{73}$
————
266 13 $5\frac{7}{73}$ Ans.

In this example we first multiply the principal by the time that expires before the first payment is made, which is 57 days; then deduct the payment £200 from the principal, and multiply the remainder by the time between the first and second payment, which is 70 days, then deduct the second payment £150 and multiply the remainder by the time between the second payment and the time of settlement, which is 238 days.—Then add the several products together and find the interest on the sum for 1 day at 5 per cent., by the rules previously given. The interest is £16 13s. $5\frac{7}{73}$d., which being added to £250, the part of the principal remaining unpaid, will give the sum yet due, which is £266 13s. $5\frac{7}{73}$d. Ans.

By a little reflection this method is evident, because the several principals multiplied by their respective times, will produce sums which will bring in as much interest in 1 month or 1 day as the several principals would in their respective times.

£300. Toronto, June 14th, 1845.

Due J. Robson or bearer, for value received, three hundred pounds currency. J. THOMAS.

Three months after date £60 was paid and endorsed on this note; four months after that £100, and five months after that £75; how much is due on the note at the end of 18 months? Ans. £79 15.

A merchant borrows £250 for 2 years at 8 per cent. and agrees to pay as fast as he can; now at the expiration of 9 months he paid £80, and 6 months after that £70, leaving the remainder the full term of two years. How much principal and interest has the merchant then to pay?

Ans. £127 16s.

A gives to B. on interest on first of November, 1844, £6000 at $4\frac{1}{2}$ per cent. B. is to pay him at the expiration of 2 years, having liberty to pay before that time as much of the principal as he pleases.

Now B. pays,	The 16th Dec. 1844,	£900
	The 11th of March, 1845,	1260
	The 30th ditto,	600
	The 17th August,	800
	The 12th of February, 1846,	1048

How much principal and interest is B. to pay on the 1st November, 1846. Ans. £1642 9s. $2\frac{1}{2}$d.+

VARIOUS EXERCISES IN INTEREST.

I. To find the principal, when the time, rate and amount are known.

If in 1 yr. 4. mo. the interest and principal on a sum at 6 per cent. amount to £61,02, what is the principal?

We first find what will be the amount of a £, with its interest, for the given time. This amounts to £1,08 now, as every £ in the original sum gained ,08 of a £ interest, there were as many pounds as there are £1,08 in £61,02.

Ans. £56 10s.

RULE.

Find the amount of £1 for the given time, and divide the sum given by this amount.

EXAMPLES.

What principal at 8 per cent. will amount to £85,12 in 1 year 6 months? Ans. £76.

II. To find the principal, when the time, rate and interest are known.

What sum at interest at 6 per cent. will gain £10,5 in 1yr. 4mo.?

One pound put at interest for that time, would gain £,08, and therefore it requires as many pounds as there are £,08 in £10,5. Ans. £131 5s.

RULE.

Find interest of £1 for the given rate and time. Divide the given interest by this and the quotient is the principal.

EXAMPLES.

1. A man paid £4,52 interest at the rate of 6 per cent. at the end of 1 year 4 months; what was the principal.

Ans. £56 10s.

2. A man received £20 for interest on a certain note at the end of 1 year at the rate of 6 per cent.; what was the principal? Ans. £333,3333.

QUESTION.—How do we find the principal, when the time, rate, and amount are known? How do we find the principal when the time, rate, and interest are known?

III. To find the rate when the principal, interest and time are known.

If £3,78 is paid for the use of £54, 1 year and 6 mo. what is the rate per cent.?

If this sum were at interest at 1 per cent. it is plain that it would produce £,54.

As many times, therefore, as £,54 is contained in £3,78, so much more than one per cent. is the rate.

RULE.

Find the interest on the given sum at 1 per cent. for the given time, by which divide the given interest: the quotient will be the rate at which interest was paid.

EXAMPLES.

1. If £2,34 is paid for the use of £468 for 1 month, what is the rate per cent.? Ans. 6 per cent.

2. At £46 16s. for the use of £520 for 2 years, what is it per cent.? Ans. 4½ per cent.

IV. The principal, rate per cent. and interest being given to find the time.

What is the time required to gain £3,78 on £36, at 7 per cent.?

The interest on £36, 1 year at 7 per cent. is £2,52; hence £3,78 ÷ £2,52 = 1,5 years, the time required.

RULE.

Find the interest for 1 year on the principal given, at the given rate, by which divide the given interest; the quotient will be the time required in years and decimal parts of a year.

EXAMPLES.

1. Paid £20 for the use of £600 at 8 per cent.; what was the time? Ans. 5 mo. nearly.

2. Paid £28,242 for the use of £217 5s. at 4 per cent. what was the time. Ans. 3 yrs. 3 mo.

QUESTIONS.—How do we find the rate, when the principal interest and time are known? How is the time found, when the principal, rate per cent. and interest are known?

EXCHANGE.

Exchange is a rule by which the money of one state or country is reduced to that of another.

Par is equality in value; but the course of exchange is frequently above or below par.

Agio, is a term used to signify the difference, in some countries, between bank and current money.

Whenever, in the commercial transactions between distant places, the debts and credits are nearly equal, bills of exchange may be considered as negotiable according to the intrinsic value of the nominal amount; exchanges are then said to be at par. But when the debts become greater in amount than the credits, exchanges must rise or sell above par.

CURRENCY TABLE.

	£	s.	d.
British Sovereign,	1	4	4
Pound Sterling,	1	4	4
United States Eagle, coined before 1834,	2	13	4
do. do. do. between 1834 and 1841,	2	10	0
United States and Mexican Dollar,	0	5	1
Half Dollars of the above,	0	2	6½
Quarter do. do.	0	1	3
Eighth do. do.	0	0	7½
Six-pence do.	0	0	3½
French five frank pieces,	0	4	8
British Crown,	0	6	1
British half do.	0	3	0½
British Shilling,	0	1	2⅗
British Six-pence,	0	0	7 3/10

Questions.—What is Exchange? What is Par? What is Agio?

CANADIAN CURRENCY.

To change Canadian Currency to Federal Money, or Federal Money to Canadian Currency.

1 dollar in this currency is 5s.=60d., and 60d.=100 cents Federal money, or 3d.=5 cents; hence cents may be changed to pence, or pence to cents by the Rule of Three.

EXAMPLES.

1. In 40 cents how many pence?
   As 5 cents : 3d. :: 40 cents : 24d. Ans.
2. In 95 cents how many pence? Ans. 57d.
3. In 18d. how many cents? Ans. 30 cents.
4. In 54d. how many cents? Ans. 90 cents.
5. Reduce 9s. 3d. to Federal money. Ans. $1,85.
6. Reduce $2,50 to currency. Ans. 12s. 6d.
7. Reduce £25 10s. 6d. to Federal money.

1 pound=4 dollars; a dollar therefore is ¼ of a pound; hence there will be as many dollars as there are quarters in the sum. Therefore,

Reduce the sum to the decimal of a pound and multiply it by 4.

Operation. { £25 10s. 6d.=£25,525.
25,525×4=$102,10 Ans. }

8. Change £69 15s. 5d. to Federal money.
   Ans. $297,083+
9. Reduce £6 11s. 6¼d. to Federal money.
   Ans. $27,304.
10. Reduce £95 19s. 11¼d. to Federal money.
    Ans. $383,9875.
11. Reduce $102,85 to pounds.

Operation.
4)102,85
————
25,7125
Ans. £25 14s. 3d.

Here we divide by 4, because 1 dollar is ¼ of a pound, and the quotient is pounds and decimals of a pound.

QUESTIONS.—In 60d. how many cents? In 3d. how many? How do we reduce pence to cents? Cents to pence? How do we change pounds, shillings and pence to Federal money? How Federal money to pounds, shillings, &c.

12. Reduce $500,80 to Currency. Ans. £125 4s.
13. Change $118,25 to Currency. Ans. £29 11s. 3d.
14. Change $250 to Currency. Ans. £62 10s.

To change Sterling money to Currency, or Currency money to Sterling—work by the Rule of Three.

EXAMPLES.

1. Reduce £150 10s. English money, to Canadian Currency.

As £1 : £1 4 4 : : £150 10 to £183 2 2 Ans.

2. Reduce £183 2s. 2d. Currency to Sterling money.

As £1 4 4 : £1 : : £183 2 2 to £150 10s. Ans.

The pupil will perceive that these two examples reciprocally prove each other, and as £1 Sterling is equal to £1 4s. 4d. Currency, the operation is evident.

3. Reduce £500 Sterling to Currency.

Ans. £608 6s. 8d.

4. Reduce £125 Currency to Sterling money.

Ans. £102 14s. 9½

5. Reduce £250 10s. Sterling money to Canadian Currency. Ans. £304 15s. 6d.

6. A person in England deposits the sum of £50 Sterling in the Bank of British North America, London, and receives a draft on the branch of the Bank at Toronto, Canada, for the amount, which he sends to his correspondent in Canada. How much must the latter receive in Canadian Currency, exchange being at par—how much, if at 5 pr cent. premium?

Ans. { £60 16s. 8d. at par.
{ £63 17s. 6d. at 5 per cent.

7. A person in Kingston being desirous to remit to London £60 Canadian Currency—for what amount British must the bill be, exchange being at par.

Ans. £49 6s. 3½d.

QUESTIONS.—How do we reduce Sterling money to Currency? How Currency to Sterling?

END OF PART III.

PART IV.

INVOLUTION.

When a number is multiplied into itself, it is said to be *involved*, and the process is called *involution*.

Hence, *Involution teaches the method of finding the powers of numbers.*

The *product* which is obtained by multiplying a number into itself, is called a *Power*.

Thus, when 2 is multiplied into itself *once*, it is 4, and this is called the *second power* of 2. If it is multiplied into itself twice, ($2\times2\times2=8$) the answer is 8, and this is called the third power.

The number which is involved is called the root, or *first power*. Thus, 2 is the root of its second power 4, and also the root of its third power 8.

A *power* is *named* or *numbered* according to the number of times its root is used as a factor.

Thus the number 4 is called the *second* power of its root 2, because the root is twice used as a factor; thus $2\times2=4$. The number 8 is called the *third power* of its root 2, because the root is used three times as a factor, thus $2\times2\times2=8$.

The method of expressing a power is by writing its root and then placing a small figure above it, to show the number of times that the root is used as a factor. Thus the second power of 2 is 4, but instead of writing the product 4, we write it thus 2^2.

QUESTIONS.—What is Involution? What does Involution teach? What is a power? What is the second power of 2? The third? What is the root or first power? Give an example? How is a power named or numbered? Give examples? What is the method of expressing a power? Express the second power of 2? The third? The fourth? What is the Index or Exponent? For what is it used? Repeat an example.

The third power of 2 is written thus 2^3. The fourth power of 2 is 16, and is written thus 2^4.

The small figure that indicates the *number of times that the root is used as a factor* is called the *Index*, or *exponent.*

The index is used to denote the power to which the root is to be raised, thus 5^4 denotes that 5 is to be involved to the fourth power.

The second power is called the *square*. The third power is called the *cube*,—the fourth power is called the *Biquadrate*,—the fifth power is called the *sursolid*, and the sixth power is called the *square-cubed*.

RULE OF INVOLUTION.

Multiply the number continually by itself as many times less 1 as there are units in the exponent: the last product will be the power sought.

EXAMPLES.

1. What is the cube of 5? Ans. $5\times5\times5=125$.
2. What is the cube of 7? Ans. 343.
3. What is the fourth power of 4? Ans. 256.
4. What is the square of 14? Ans. 196.
5. What is the 5th power of 2? Ans. 32.
6. What is the 7th power of 2? Ans. 128.

A fraction is *involved by involving both numerator and denominator*.

7. What is the square of $\frac{1}{2}$? Ans. $\frac{1}{4}$.
8. What is the cube of $\frac{2}{3}$? Ans. $\frac{8}{27}$.
9. What is the fourth power of $\frac{3}{5}$? Ans. $\frac{81}{625}$.
10. What is the square of $5\frac{1}{2}$? Ans. $30\frac{1}{4}$.
11. What is the square of $30\frac{1}{4}$? Ans. $915\frac{1}{16}$.
12. Perform the involution of 8^5. Ans. 32768.
13. Involve $\frac{4}{39}$, $\frac{11}{12}$, and $\frac{8}{9}$ to the third power each. Ans. $\frac{64}{59319}$; $\frac{1331}{1728}$; $\frac{512}{729}$.
14. Involve 211^3. Ans. 9393931.
15. Raise 25 to the fourth power. Ans. 390625.
16. Find the sixth power of 1,2. Ans. 2,985984.

QUESTIONS.—What is the second power of a number called? The third? The fourth? The fifth? The sixth? Repeat the rule of Involution. How is a fraction involved?

EVOLUTION.

Evolution is the process of *finding the root of any number;* that is of finding that number which multiplied into itself, will produce the given number.

The *Square Root* is a number which being *squared* will produce the given number.

It is expressed either by this sign $\sqrt{}$ placed before a number, thus $\sqrt{4}$, or by a fraction $\frac{1}{2}$ placed above a number, thus, $4^{\frac{1}{2}}$.

The *Cube Root* or *Third Root* is a number which being *cubed*, or multiplied by itself *twice*, will produce the given number. It is expressed thus, $\sqrt[3]{}$; or thus $27^{\frac{1}{3}}$.

All the other roots are expressed in the same manner. The *Fourth Root* has this sign $\sqrt[4]{}$ put before a number, or else $\frac{1}{4}$ placed above it. There are some numbers whose roots cannot be precisely obtained, but by means of *decimals*, we can *approximate* to the number which is the root.

Numbers whose roots can be exactly obtained are called *rational numbers*.

Numbers whose precise roots cannot be obtained are called *surd numbers*.

When the root of several numbers united by the sign $+$ or — is indicated, a *vinculum*, or line is drawn from the sign of the root over the numbers. Thus, the square root of 59—10 is written $\sqrt{59-10}$.

The root of a rational number, is a *rational root*, and the root of a surd number, is a *surd root*.

QUESTIONS.—What is Evolution? What is the Square Root? Show how the Square Root is expressed. What is the Cube Root? Express the fourth root, &c. Can the roots of all numbers be exactly obtained? How may we approximate to the number which is the root? What are rational numbers?

When we express the root of several numbers united by the signs $+$ or —, how do we indicate that the numbers are all affected by the sign of the root? Express the Square Root of 48+6—4. What is a rational root?—A surd root?

EXTRACTION OF THE SQUARE ROOT.

Extracting the square root is finding a number, which multiplied into itself, will produce the given number; or it is finding the length of *one side* of a certain quantity, when that quantity is placed in an exact square. Thus, $\sqrt{4\times4}=\sqrt{16}=4$, and $\sqrt{49}=7$ for $7\times7=49$.

RULE FOR EXTRACTING THE SQUARE ROOT.

I. Point off the given number, into periods of two figures each, beginning at the right hand.

II. Find the greatest square in the first left hand period, and subtract it from that period. Place the root of this square in the quotient. To the remainder bring down the next period for a dividend.

III. Double the root already found and place it on the left for a divisor. Seek how many times the divisor is contained in the dividend, exclusive of the right hand figure, and place the figure in the root and also at the right hand of the divisor.

IV. Multiply the divisor, thus increased, by the last figure of the root, and subtract the product from the dividend. To the remainder bring down the next period, for a new dividend. But if the product should exceed the dividend, diminish the last figure of the root.

V. Double the root already found for a new divisor, and proceed as before, until all the periods are brought down.

EXAMPLES.

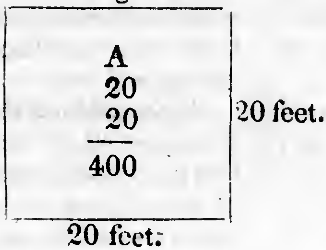
1. What is one side of a square room, which contains 784 square feet?

QUESTIONS.—What is the extraction of the Square Root?—What is the first step in extracting the Square Root of numbers? What is the second? What the third? The fourth? The fifth? Repeat the entire rule?

Operation.

7'84(2
4
———
384

Fig. 1.



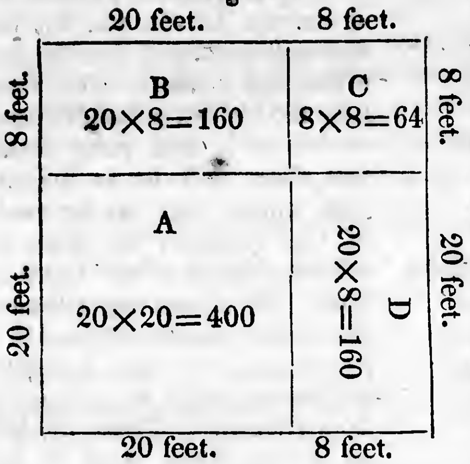
We first point off the number as the rule directs, and find the root will consist of two figures, a *ten* and a *unit*. We then take the highest period 7 (hundreds) and find how many feet there will be in the largest square that can be made of this quantity, the sides of which must be of the order of *tens*. No square larger than 4 (hundreds) can be obtained in 7 (hundreds), the sides of which will be each 20 feet, because $20 \times 20 = 400$. These 20 feet or 2 tens being sides of the square are placed in the quotient as the first figure of the root.

This Square may be represented by Fig. 1.

We now take out the 400 from 700, and 300 square feet remain. These are added to the next period 84, making 384, which are to be arranged around the square A in such a way as not to destroy its *square* form; consequently the additions must be made on *two* sides.

To ascertain the *breadth* of these additions, the 384 must be divided by the length of the two sides $20 \times 20 = 40$, and as the root already found is *one* side, we *double* this root for a divisor, making 4 tens or 40, for as 40 feet is the length of these sides, there will be as many feet in breadth as there are forties in 384. The quotient arising from the division is 8, which is the *breadth* of the addition to be made, and which is placed in the quotient, after the 4 tens.

Fig. 2.



It will be seen by Fig. 2, that to complete the square, the corner C. must be filled by a small square, the sides of which are each equal to the *width* of B and C, that is 8 feet.

Adding this to the 4 tens, or 40, we find that the whole *length* of the addition to be made around the sqr. A, is 48 feet, instead of 40. This multiplied by its breadth, 8 feet, the quotient figure, gives the *contents* of the whole addition, 384 feet.—

```
     7'84(28 root.
     4
     ---
48)  384
     384
```

As there is no remainder, the work is done, and 28 feet is the side of the given square. It may be proved by involution, thus, 28 ×28=784, or by adding together the several parts of the figure thus,

A	contains	400	square	feet.
B	"	160	"	"
D	"	160	"	"
C	"	64	"	"
		784		

If in any case, there is a *remainder*, after the last period is brought down, it may be reduced to a decimal fraction by annexing two ciphers for a new period, and the same

QUESTIONS.—If there is a remainder after the last period is brought down how may we proceed? If the dividend be too small to contain the divisor what must be done? How many figures will there always be in the root?

process continued. If at any time the dividend be too small to contain the divisor a cipher must be placed in the root, and another period brought down.

The pupil will find by trial, that the root *always contains just half as many, or one figure more than half as many figures as are in the given quantity.* Hence, the propriety of pointing off into periods of two figures each, and there will always be as many figures in the roots as there are periods.

2. What is the square root of 998001?

```
               99'80'01)999 root.
               81
               ----
           189)1880
Operation.     1701
               ----
          1989)17901
               17901
               -----
```

3. Find the square root of 676. Ans. 26.
4. Find the square root of 625. Ans. 25.
5. What is the square root of 487204? Ans. 698.
6. " " 638401? Ans. 779.
7. " " 556516? Ans. 746.
8. " " 488601? Ans. 699.
9. " " 32761? Ans. 181.
10. " " 69? Ans. 8,3066+
11. " " 83? Ans. 9,1104+
12. " " 299? Ans. 17,2916+
13. " " 892? Ans. 29,8663+
14. " " 9712,693809? Ans. 98,553.

It was shewn in the article on Involution, that a fraction is involved by involving both numerator and denominator. Hence, to find the root of a fraction, *extract the root of the numerator and denominator.* If this cannot be done the fraction may be reduced to a decimal, and its root extracted.

QUESTIONS.—How may the root of a fraction be found? If this cannot be done, how may we proceed?

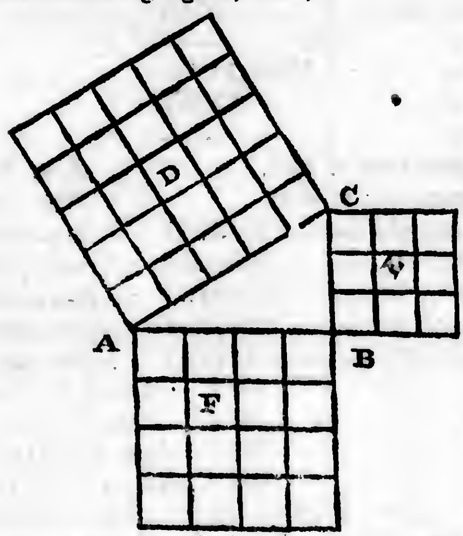
PRACTICAL EXERCISES.

1. If a square field contains 2025 square rods, how many rods does it measure on each side? Ans. 45 rods.

2. How many trees in each row of a square orchard containing 5625 trees? Ans. 75.

3. A square pavement contains 20736 square stones, all of the same size, what number is contained in one of its sides? Ans. 144.

NOTE.—The square of the hypothenuse, or longest side of a right angled triangle, is equal to the sum of the squares of the other two sides: and consequently the difference of the square of the longest, and either of the other sides, is the square of the remaining one. This is made plain by the following figure, thus,



If A B C be a right angled triangle, right angle at B, then will the square D described on A C be equal to the sum of the squares E&F on the sides A B and B C. And the difference between the square D and the square E is equal to the square F, also the difference between the squares D and F is equal to the square E. The truth of which may be seen by counting the small squares contained in E D and F.

This is called the carpenter's theorem, and is used for finding the length of braces, rafters, &c.

QUESTIONS.—To what is the square of the longest side of a right angled triangle equal? What is the difference of the square of the longest and either of the other sides?

4. A ladder being placed at the distance of 12 feet from the bottom of an upright wall, is found to reach a window 16 feet from the ground: what is its length?

Ans. 20 feet.

5. A line of 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad; the height of the wall is required?

Ans. 26,24+feet.

6. A ladder 50 feet long, being placed in the street, will reach a window 40 feet high, on one side and, without moving the foot, will reach one 30 feet high on the other: how wide was the street? Ans. 70 feet.

7. The side of a square meadow is 325 yards, what is the length of the diagonal or line drawn from corner to corner? Ans. 459,61942.

8. If a house is 50 feet wide, and the upright which supports the ridge poll is 12 feet high, what will be the length of the rafters? Ans. 27,7 feet+.

9. There is a square field containing 90 acres; how many rods in length is each side of the field? and how many rods apart are the opposite corners?

Answers, 120 rods; and 169,7+rods.

EXTRACTION OF THE CUBE ROOT.

A *Cube* is a solid body, having six equal sides, each of which is an exact square. Thus a solid which is 1 foot long, 1 foot high, and 1 foot wide, is a *cubic foot:* and a solid whose length, breadth and thickness are each one yard, is called a *cubic yard.*

The root of a cube is always the *length* of one of its sides; for as the *length, breadth* and *thickness* of such a body are the same, the length of one side, raised to the third power, will show the *contents* of the whole.

Extracting the Cube Root of any quantity, therefore is finding a number, which multiplied into itself *twice* will produce that quantity; or it is finding the length of one side of a given quantity, when that quantity is placed in an exact cube.

RULE FOR EXTACTING THE CUBE ROOT.

I. Point off the given number into periods of three figures each, beginning at the right hand.

II. Find the greatest cube in the left hand period, and put its root in the quotient.

III. Subtract the cube thus found from the said period, and to the remainder bring down the next period, and call this the *dividend*.

IV. Square the root already found, and multiply it by 300 for a divisor.

V. Find how many times the dividend contains the divisor, and place the result in the root; then multiply the divisor by this quotient figure, and place the product under the dividend.

VI. Multiply the square of this quotient figure by the former *figure* or *figures* of the root, and this product by 30, and place the product under the last. Finally, cube this quotient figure, and place its cube under the other products, and add these three results together for a subtrahend.

VII. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before; and so on until the whole is finished.

NOTE.—If the divisor is not contained in the dividend, a cipher must be placed in the root, and the next period brought down for a dividend.

The same rule must be observed for continuing the operation, and pointing off for decimals, as in the square root.

QUESTIONS.—What is a cube? What is a cubic foot? What is a cubic yard? What is the root of a cube? Why? What is extracting the cube root of any quantity? What is the first step in extraction of the cube root? What is the second? The third? the fourth? The fifth? The sixth? The seventh? Why do we point off the number into periods of three figures each? How many figures will the root always contain?

The pupil will perceive that the number which we call *the divisor*, when multiplied by the last quotient figure, does not produce so large a number as the real subtrahend; hence, the figure in the root must frequently be less than the quotient figure.

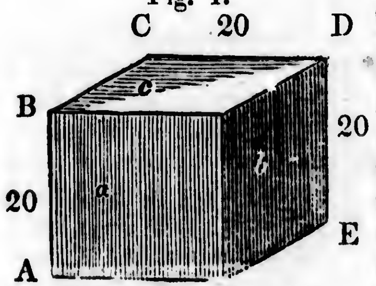
1. What is the cube root of 13824?

We first ascertain the number of figures of which the root will consist, by pointing off the number into periods of three figures each. We do this because the cube of a *unit figure will not give a higher order than hundreds*, for the cube of 9 is 729, and as 9 is the greatest unit figure, the cube of a unit will not consist of more than three figures. Also the cube of 10 is 1,000 and the cube of 9 tens for 90 is 729,000, hence the cube of tens will not give a lower order than thousands, nor a higher order than hundreds of thousands, which consists of six figures; therefore we point off into periods of three figures each, and the root always contains as many figures as there are periods.

Operation.

13824(2
8
5824

Fig. I.



20
20
400
20
8000 Solid feet.

We find the root will consist of two figures, a *ten* and a *unit*. We now seek for the first figure, or tens of the root, which must be extracted from the left hand period, 13 thousands. The greatest cube in 13 thousands we find *by trial*, to be 8 thousands, the root of which is 2 tens; we therefore place 2 tens in the root. The root it will be recollected, is one side of a cube. Let us then form a cube, (Fig. I.) each side of which shall be supposed 20 feet, expressed by the root now obtained. The contents of this cube are $20\times20\times20=8000$ solid feet which are now disposed of, and which consequently are to be deducted from the whole number of feet, 13824. 8000 taken from 13824 leave 5824 feet. This deduction is most readily performed by subtracting the cubic number 8, or the cube of 2, (the figure of the root already found,) from the period 13 thousands, and bringing down the next period by the side of the remainder, making 5824, as before.

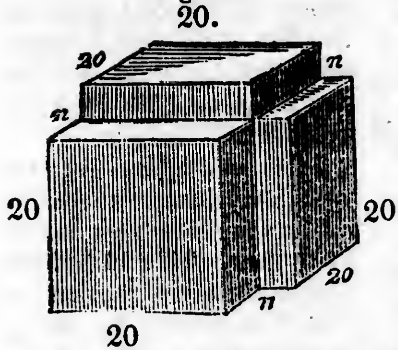
The cube A D is now to be enlarged by the addition of 5824 solid feet, and in order to preserve the cubic form of the block, the addition must be made on one half of its sides, that is on 3 sides, *a*, *b* & *c*. Now, if the 5824 solid feet be divided by the square contents of these 3 equal sides, that is, by 3 times $(20\times20=400)=1200$, the quotient will be the thickness of the addition made to each of the sides *a*, *b*, *c*. But the root, 2 tens, already found, is the length of *one* of these sides; we therefore square the root, 2 tens, $=20\times20=400$, for the *square contents* of *one* side, and multiply the product by 3, the *number* of sides, $400\times3=1200$; or, which is the same in effect, and more convenient in practice, we may square the 2 tens, and multiply the product by 300, thus, $2\times2=4$, and $4\times300=1200$, for the divisor, as before.

Operation continued.

```
             13824(24 root.
              8
            -----
divisor, 1200)5824 dividend.
            -----
             4800
              960
               64
            -----
             5824 subtrahend.
            -----
             0000
```

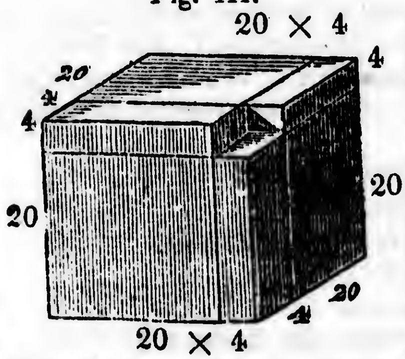
The divisor 1200, is contained in the dividend 4 times: consequently, 4 feet is the thickness of the addition made to each of the three sides, *a*, *b*, *c*, and $4\times1200=4800$ is the solid feet contained in these additions; but, if we examine Fig. II. we shall perceive that this addition to the 3 sides does not complete the cube; for

Fig. II.



there are deficiencies in the 3 corners, *n*, *n*, *n*. Now the *length* of each of these *deficiencies* is the same as the *length of each side*, that is 2 tens=20, and their *width* and *thickness* are each equal to the *last* quotient figure 4; their contents, therefore, or the number of feet required to *fill* these deficiencies, will be found by multiplying the *square* of the last quotient figure $4^2=16$, by the length of *all* the deficiencies, that is, by 3 times the length of *each* side, which is expressed by the former quotient figure, 2 tens. 3 times 2 tens are 6 tens=60; or, what is the same in effect, and more convenient in practice, we may multiply the quotient figure 2 tens, by 30, thus, $2\times30=60$, as before; then $60\times16=960$, contents of the three deficiencies *n*, *n*, *n*.

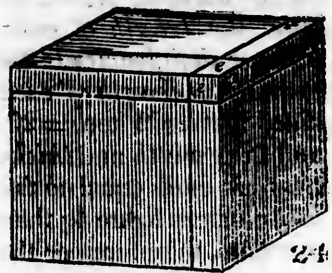
Fig. III.



By examining Fig. III. we perceive there is still a deficiency in the corner where the last blocks meet —this deficiency is a cube, each side of which is equal to the last quotient figure, 4. The cube of 4, therefore, $(4\times4\times4=64)$ will be the solid contents of this corner, which in Fig. IV. is seen filled.—Now the sum of these several additions, viz. $4800\times960\times64=5824$, will make the subtrahend, which, subtracted from the dividend, leaves no remainder, and the work is done.

Fig. IV.

24 feet.



24

24 feet.

Fig. IV. shows the pile which 13824 solid blocks of one foot each would make, when laid together, and the root 24, shows the length of one side of the pile.

The correctness of the work may be ascertained by cubing the side now found, 24^3, thus, $24\times24\times24=13824$, the given number; or it may be proved by adding together the contents of all the several parts, thus,

Feet.
8000 = contents of Fig. I.
4800 = addition of the sides *a*, *b*, and *c*, Fig. I.
960 = addition to fill the deficiencies *n*, *n*, *n*, Fig. II.
64 = addition to fill the corner *e*, *e*, *e*, Fig IV.
———
13842 = contents of the whole pile, Fig. IV. 24 feet on each side.

Note.—The best method of illustrating the extraction of the cube root, is by means of small wooden blocks, one in the form of a cube, to represent Fig. I. and 7 smaller ones to represent the deficiencies. The form and explanation of the figures will suggest the method of making them.

2. What is the cube root of 34645976?

Operation.

34645976(326 Root.
27
———
$3^2\times300=2700$) 7645 first dividend.
———
5400
$2^2\times3\times30=$ 360
$2^3=$ 8
——— *Carried up.*

5768 first subtrahend.

$32^2 \times 300 = 307200$)1877976 second dividend.

1843200
$6^2 \times 32 \times 30 =$ 34560
216

1877976 second subtrahend.

0000000

3. What is the cube root of 729? Ans. 9.
4. What is the cube root of 12167? Ans. 23.
5. What is the cube root of 4826809? Ans. 169.
6. What is the cube root of 9129329? Ans. 209.
7. What is the cube root of 15625000? Ans. 250.
8. What is the cube root of 997002999? Ans. 999.
9. What is the cube root of 469097433? Ans. 777.
10. What is the cube root of 445943744? Ans. 764.
11. What is the cube root of 41421,736? Ans. 34,6.
12. What is the cube root of 84,604519? Ans. 4,39.
13. What is the cube root of 49? Ans. 3,659+
14. What is the cube root of ,981? Ans. 9,936+

The cube root of a *fraction* is obtained by extracting the root of the numerator and denominator, but if this cannot be done, it may be changed to a decimal, and the root extracted.

15. What is the cube root of $\frac{8}{27}$? Ans. $\frac{2}{3}$. Of $\frac{125}{343}$? Ans. $\frac{5}{7}$. Of $\frac{1}{125}$? Ans. $\frac{1}{5}$. Of $\frac{27}{1331}$? Ans. $\frac{3}{11}$. Of $\frac{13824}{29791}$? Ans. $\frac{24}{31}$. Of $\frac{7301384}{26730899}$? Ans. $\frac{194}{299}$.

APPLICATION.

1. The content of a cubical piece of timber 103823 solid inches; how many inches is it each way? Ans. 47.

2. What is the side of a cubical mound, equal to one 288 feet long, 216 feet broad, and 48 feet high?

Ans. 144 feet.

QUESTION.—How is the cube root of a fraction obtained?

3. The statute bushel contains 2150,4252 cubic or solid inches; what must be the side of a cubic box that will contain the same quantity? Ans. 12,907 inches.

4. A stone of a cubic form contains 474552 solid inches; what is the superficial content of one of its sides? Ans. 6084.

MENSURATION.

Mensuration signifies to measure, hence measuring surfaces is called mensuration of *superfices;* and measuring of solids is called mensuration of *solids.*

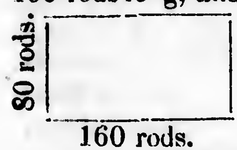
To find the area of a square or of a rectangle.

RULE.

Multiply the length by the breadth, and the product will be the area, or superficial content.

EXAMPLES.

1. How many acres are contained in a piece of land 160 rods long, and 80 rods wide?



A rectangle is a four sided figure like a square, in which the sides are perpendicular to each other, but the adjacent sides are not equal.

$160 \times 80 = 12800 \div 160 = 80$ acres.

2. How many acres are there in a lot of land 320 rods long, and 160 rods wide? Ans. 320 acres.

3. What is the area of a square field of which the sides are each 33,08 chains? Ans. 109A. 1R. 28P.+

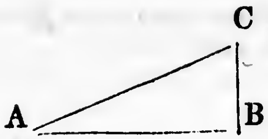
4. What is the content of a square piece of land of which the sides are 25 rods each? Ans. 3A. 3R. 25P.

To find the area of a triangle.

RULE.

Multiply the base by the perpendicular height and *one half* of the product will be the area.

QUESTION.—What is Mensuration? How do we find the area of a square or rectangle? How do we find the area of a triangle?



A triangle is a figure bounded by three straight lines. Thus, A B C is a triangle. A B is the base, B C the perpendicular, and A C the hypothenuse.

EXAMPLES.

1. The base of a triangle is 40 yards and the perpendicular 20 yards; what is the area? Ans. 400 square yards.

2. In a triangular field the base is 40 chains and the perpendicular 15 chains: how much does it contain?

Ans. 30 acres.

3. How many acres are contained in a triangle whose base is 320 rods, and perpendicular 40 rods? Ans. 40 acres.

To find the area of a triangle having the three sides given.

RULE.

I. From half the sum of the three sides subtract each side severally.

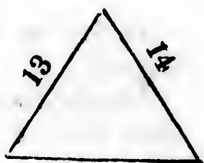
II. Multiply the half sum and the three remainders continually together, and the square root of the product will be the area required.

EXAMPLES.

1. Required the area of an oblique triangle, the three sides of which are 13, 14, and 15 rods.

13	21	21	21	21	½ the sum.
14	13	14	15	6	
15	—	—	—	—	
—	8	7	6	126	
2) 42 sum.				7	
—				—	
21 half the sum.				882	
				8	
				—	

$\sqrt{}$70·56(84 rods.



QUESTION.—What is the rule for finding the area of a triangle having the three sides given?

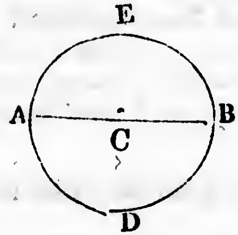
2. Required the area of an oblique triangle, the three sides of which are 80,120, and 160 rods.

Ans. 29 acres 7 poles.

To find the circumference of a circle when the diameter is given.

RULE.

Multiply the diameter by 3,1416 and the product will be the circumference.



A circle is the portion of a plane bounded by a curved line every part of which is equally distant from a certain point within, called the centre. The curved line AEBD is called the *circumference;* the point C the centre; the line AB passing through the centre a *diameter*, and CB the radius. The circumference AEBD is 3,1416 times greater than the diameter AB. Hence, if the diameter is 1, the circumference will be 3,1416. Wherefore if the diameter is known, the circumference is found by multiplying 3,1416 by the diameter.

EXAMPLES.

1. The diameter of a circle is 93 feet; what is the circumference? Ans. 292,1688 feet.

2. The diameter of a circle is 20 rods, what is the circumference? Ans. 62,832 rods.

To find the diameter of a circle when the circumference is given

RULE.

Divide the circumference by 3,1416 and the quotient will be the diameter,

EXAMPLES.

1. What is the diameter of a circle whose circumference is 78,54 feet? Ans. 25 feet.

QUESTIONS.—How do we find the circumference of a circle when the diameter is given? How do we find the diameter of a circle when the circumference is given?

2. What is the diameter of a circle whose circumference is 11752,1944 rods? Ans. 37,09 rods.

To find the area of a circle when the diameter is known.

RULE.

Square the diameter, and then multiply by the decimal ,7854.

EXAMPLES.

1. What is the area of a circle whose diameter is 5 feet? Ans. 19,6350.

2. What is the area of a circle whose diameter is 8,75 feet? Ans. 60,1322 sq. feet.

3. How many acres are there in a circle of one mile diameter? Ans. 502A. 2R. 24P.+

To find the surface of a sphere or ball.

RULE.

Multiply the square of the diameter by the decimal 3,1416.

EXAMPLES.

1. What is the surface or area of a sphere whose diameter is 12 feet? Ans. 452,3904 sq. ft.

2. What is the surface of a globe whose diameter is 7? Ans. 153,9384.

3. Required the area of the surface of the earth, its mean diameter being 7918,7 miles?
Ans. 196996571,722104 sq. miles.

To find the solidity of a sphere or globe.

RULE.

Multiply the cube of the diameter by the decimal ,5236 and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a sphere whose diameter is 17 inches.

$17 \times 17 \times 17 \times ,5236 = 2572,4468$ solid inches. Ans.

QUESTIONS.—How is the area of a circle found, when the diameter is known? How do we find the surface of a sphere?—How do we find the solidity of a sphere?

2. What is the solidity of a globe whose diameter is 12 inches? Ans. 904,7808 solid inches.

To find the convex surface of a cylinder.

RULE.

Multiply the circumference of its base by the altitude, and the product will be the answer.

EXAMPLES.

1. What is the convex surface of a cylinder, the diameter of whose base is 20 and altitude 50?

Operation.

```
 3,1416
     20
 ------
62,8320
     50
 ------
```

Ans. 3141,6000

We first multiply the diameter by by 3,1416 which gives the circumference of the base. Then multiplying by the altitude, we obtain the convex surface.

2. Required the surface of a cylinder, the diameter of whose base is 20 and the altitude 20? Ans. 1256,64.

TIMBER MEASURE.

I. To find the solid contents of a square or four sided stick of timber of equal bigness from end to end.

RULE.

Multiply the breadth by the depth, and then multiply the product by the length, and the result will be the solid contents.

EXAMPLES.

1. A squared piece of timber 15 inches broad and 15 inches deep, and 18 feet long; how many solid feet does it contain? Ans. 28,152.

QUESTIONS.—How may the convex surface of a cylinder be found? How is the solid contents of a four sided stick of timber found, the size being equal from end to end?

2. What is the solid contents of a piece of timber whose breadth is 16 inches, depth 12 inches, and length 12 feet?
Ans. 16 feet.

3. How many solid feet in a stick of ship timber, whose breadth is 3 feet, and depth $2\frac{1}{2}$ feet, and length 45 feet?
Ans. 337,5 feet.

II. To find the solid contents of a round stick of timber of equal bigness from end to end.

RULE.

Multiply the area of one end by the length, and the product will be the solid contents.

EXAMPLES.

1. What is the solid contents of a round stick of timber of equal bigness from end, whose diameter is 28 inches and length 25 feet?
Ans. $106\frac{136}{144}$ feet.

2. Required the solid contents of a round stick of timber, whose diameter is 18 inches and length 50 feet?
Ans. 88,3575 solid feet.

III. To find the solid contents of a tapering stick of timber, whether square or round, when one end is a point.

RULE.

Multiply the area of the big end by one third of its length, and the product will be the answer.

EXAMPLES.

1. What is the contents of a tapering square stick of timber 24 feet 9 inches long, 16 inches square at one end, and a point at the other?
Ans. $14\frac{96}{144}$ feet.

2. What is the contents of a tapering round stick of timber 30 feet long, 18 inches diameter at one end, and a point at the other?
Ans. 17+feet.

QUESTIONS.—How do we find the solid contents of a round stick of timber of equal size from end to end?

How do we find the contents of a tapering stick of timber when one end is a point?

IV. To find the solidity of a square piece of timber which tapers regularly but does not come to a point.

RULE.

1st. Add together the breadths at the two ends and also the depths.

2nd. Multiply these sums together, and to the result add the products of the depth and breadth at each end.

3d. Multiply the last result by the length, and take one sixth of the product, which will be the solidity.

EXAMPLES.

1. How many cubic feet in a piece of timber, the breadth and depth of the large end being 14 inches and 12 inches: and of the smaller, 6 and 4 inches, and the length $30\frac{1}{2}$ feet.

14	12	$16\times20=320$
6	4	$14\times12=168$
—	—	$6\times 4=\ 24$
20	16	——
		512 square inches.

But 512 sq. in. $=\frac{32}{9}$ sq. ft.

Then, $\frac{32}{9}\times30\frac{1}{2}\times\frac{1}{6}=18\frac{2}{27}$, solid feet.

2. What is the number of cubic feet in a stick of hewn timber, whose ends are 30 inches by 27 and 24 inches by 18, and the length 24 feet. Ans. 102 solid feet.

3. How many cubic feet in a stick of timber whose larger end is 25 feet by 20, the smaller 15 feet by 10, and the length 12 feet? Ans. 3700 solid feet.

V. To find the solid contents of a tapering round stick of timber, when the small end is not a point.

QUESTIONS.—To find the solidity of a square stick of timber which tapers regularly but does not come to a point, what is the first thing to be done? What is the second step? The third?

RULE.

Multiply each diameter by itself separately; multiply one diameter by the other: add these three products together, multiply their sum by the length, annex two ciphers to the product, and divide by 382; the quotient will be the solid contents.

EXAMPLES.

1. What is the solid contents of a round stick of timber whose diameter at the big end is 12 inches, at the small end 9 inches, and length 30 feet? Ans. 18$\frac{23}{144}$ feet.

2. What is the solid contents of a round block of marble, whose diameter at the big end is 23 inches, and small end 15 inches, and length 34 feet 8 inches?

Ans. 68 feet+.

VI. To find how many solid feet a round stick of timber of equal thickness from end to end, will contain when hewn square.

RULE.

Take one half of the diameter in inches and square it, this square being doubled and multiplied by the length gives the content in inches.

EXAMPLES.

1. If the diameter of a round stick of timber be 18 inches, and its length 30 feet, how many solid feet will it contain when hewn square? Ans. 33$\frac{108}{144}$ feet.

2. If a round stick of timber 28 feet long and 22 inches diameter, was hewn square, how many solid feet will it contain. Ans. 47$\frac{8}{144}$ feet.

VII. To find how many square edged boards of a given thickness can be sawn from a log of a given diameter.

Questions.—How do we find the solidity of a tapering round stick of timber, when the small end is not a point? How may we find what will be the content of a round stick of timber of equal size from end to end, when hewn square?

RULE.

1st. Find the solid contents of the log when made square by the last rule.

2nd. Then say, as the thickness of the board, including the saw carf, is to the solid feet, so is 12 inches to the number of feet of boards.

EXAMPLES.

1. How many feet of square edged boards, $1\frac{1}{2}$ inches thick, including the saw gap, can be sawn from a log 16 feet long and 18 inches diameter? Ans. 144.

2. How many square feet of boards, $1\frac{3}{4}$ inches thick, including the saw gap, may be sawn from a log 28 feet long and 24 inches diameter? Ans. 384.

GAUGING.

Gauging is finding the contents of any box, tub, cask or other vessel.

EXAMPLE.

1. There is a cask whose head diameter is 25 inches, bung diameter 31 inches, and whose length is 36 inches; how many wine gallons does it contain? Also, how many beer gallons?

The mean diameter of a cask is found by adding to the head diameter two thirds of the difference between the bung and head diameters, or if the staves are not much curved, by adding three fifths. This reduces the cask to a cylinder.

Then, to find the solidity, we multiply the square of the mean diameter by the decimal ,7854 and the product by the length; this will give the solid contents in cubic inches, which divided by 231 (the cubic inches in a gallon wine

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QUESTIONS.—What is the rule for finding how many square edged boards of a given thickness may be sawn from a log of a given diameter? What is gauging? How is the mean diameter of a cask found?

measure) will give the content in wine gallons, and divided by 282 (the cubic inches in a gallon beer measure) will give the content in ale or beer gallons.

In this process we see that the square of the mean diameter will be multiplied ,7854, and divided for wine gal. by 231. Hence we may contract the operation by only multiplying their quotient ($\frac{7854}{231}$=,0034;) that is, ,0034 (or by ,34, pointing off 4 figures from the product for decimals.) For the same reason we may, for *beer* gallons, multiply by ($\frac{7854}{282}$=,0028, nearly,) ,0028. Hence, the following

RULE.

Multiply the square of the mean diameter by the length; then multiply this product by 34 for *wine*, or 28 for *beer*, and pointing off 4 decimals, the product will be the content in gallons and decimals of a gallon.

In the above example, the bung diameter is 31in.—25 inches the head diameter=6in. difference, and $\frac{2}{3}$ of 6=4 inches; 25in.+4in.=29in. mean diameter.

Then 29^2=481, and 481×36in.=30276.

Then { 3276×24=1029384. Ans. 102,9384 wine gal.
{ 3276×28=847728. Ans. 847728 beer gal.

2. How many wine gallons in a cask whose bung diameter is 36 inches, head diameter 30 inches, and length 50 inches. Ans. 196,52gal.

3. How many wine gallons, and how many beer gallons, in a cask whose length is 36 inches, bung diameter 35in., and head diameter 30in.?

Ans. { 136 wine gal.
{ 112 beer gal.

QUESTIONS.—How do we find the solidity of a cask? What will the product give? How are the cubic inches reduced to wine gallons? How to beer gallons? How may the operation be contracted? Repeat the rule.

DUODECIMALS.

Duodecimal is derived from the Latin word *duodecim*, signifying *twelve.*

They are fractions of *a foot*, which is supposed to be divided into *twelve* equal parts called *primes*, marked thus, ($'$). Each prime is supposed to be subdivided into 12 equal parts called *seconds*, marked thus, ($''$). Each second is also supposed to be divided into 12 equal parts called *thirds*, marked thus, ($'''$), and so on to any extent.

It thus appears that $1'$ an inch or prime is $\frac{1}{12}$ of a foot. $1''$ a second is $\frac{1}{12}$ of $\frac{1}{12}$, or $\frac{1}{144}$ of a foot. $1'''$ a third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, or $\frac{1}{1728}$ of a foot, &c. Whenever therefore any number of *seconds* (as $4''$), are mentioned, it is to be understood as so many $\frac{1}{144}$ of a foot, and so of the *thirds*, *fourths*, &c.

Duodecimals are *added* and *subtracted* like *other compound numbers*, 12 of a less order making 1 of the next higher, thus,

$12''''$ fourths make 1 third $1'''$.
$12'''$ thirds make 1 second $1''$.
$12''$ seconds make 1 prime or inch $1'$.
$12'$ inches or primes make 1 foot.

These marks are called " "' are called *indices*.

Duodecimals are chiefly used in measuring *surfaces* and *solids*.

MULTIPLICATION OF DUODECIMALS.

RULE.

Set the multiplier in such order that the feet thereof may stand under the lowest denomination of the multiplicand. Then multiply and carry one for every 12 from one denomination to another,

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QUESTIONS.—From what is duodecimal derived? What are duodecimals? How are feet supposed to be divided? How are these parts again divided? How are duodecimals added and subtracted? How many of one order make one of the next higher? What are the distinguishing marks called?

and set down the result, so that the lowest denomination in the product may stand directly under the number we multiply by. Then add as in compound addition.

EXAMPLES.

1. A load of bark measures 12ft. 3 in. long, 4ft. 2in. high, and 3ft. 6in. wide. Required the number of solid feet therein.

Operation.

```
  12  3′
      4  2′
  --------
  49  0
   2  0′  6″
  ---------
  51  0′  6″
          3  6′
  -------------
 153  1   6
  25  6   3   0
 --------------
Ans. 178 7′ 9″ 0‴
```

We first place the numbers as the rule directs. We then multiply the lowest denomination in the multiplicand by the highest in the multiplier, which is 4 feet; then by 2in. and place the product directly under the multiplier, and find the product to be 51 feet 0in. and 6 seconds, which being multiplied by 3ft. 6in. gives the solid content of the load.

In multiplication of duodecimals it will in all cases be found to hold true, that the *product of any two denominations will always be of the denomination denoted by the sum of their indices*; thus, the sum of the indices of 8′×4′ is ″ hence the product is 32″; and thus *primes* multiplied by *primes* will produce *seconds*: *primes* multiplied by seconds produce thirds; *fourths* multiplied by *fifths* produce *ninths*, &c.

2. How many square feet in a board 16 feet 9 inches long, and 2 feet 3 inches wide? Ans. 37ft. 8in. 3″.

3. How many solid feet in a block 15ft. 8in. long, 1ft. 5in. wide, and 1ft. 4in. thick. Ans. 29ft. 7′ 1″ 4‴.

QUESTIONS.—How are duodecimals chiefly used? How do we set down the numbers for multiplying? How do we multiply, carry, and set down the result? In multiplication of duodecimals what will be found to hold true? Repeat some examples.

4. A board measures 28ft. 10in. long, and 3ft. 5in. wide; required its contents. Ans. 98ft. 6in. 2''.

5. In a pile of wood 176ft. in length, 3ft. 9' wide, and 4ft. 3' high, how many cords?

Ans. 21 cords, and $7\frac{5}{16}$ cord ft. over.

6. How many square feet in a pavement 371ft. 2' 6'' in length, and 181ft. 1' 9'' in breadth?

Ans. 67242ft. 10' 1'' 4''' 6''''.

7. What is the price of a marble slab, whose length is 5ft. 7in. and the breadth 1 foot 10in. at 6s. per foot?

Ans. £3 1s. 5d.

8. What will the paving of a court yard come to at 3s. 2d. per yard, if the length be 27 feet 10 inches, and the breadth 14 feet 9 inches? Ans. £7 4s. 5d.

POSITION.

Position is a rule for finding, an unknown number, by means of supposed numbers. It is of two kinds, *Single* and *Double*.

These rules are generally given by writers on Arithmetic; yet most of the questions that are usually solved by them may be easily worked on general principles, and all of them admit of very simple solutions, by Algebra.

SINGLE POSITION.

Single Position includes the solution of questions by one supposed number.

RULE.

Suppose any number, and perform on it the operation indicated in the question; then

As the result of the operation,
Is to the number given;
So is the supposed number,
To the number sought.

QUESTIONS.—What is Position? How many kinds are there? What does single Position include? What is the Rule?

Proof.—Perform, on the number found, the operation indicated in the question, if the work be right the result will be the same as the given number.

EXAMPLES.

1. A School Master being asked how many pupils he had, answered. If I had as many more as I now have, half as many, one-third, and one-fourth as many, I should then have 148. How many scholars had he?

Operation.

Suppose he had 12 pupils.	
as many $=12$	Here the supposed number is 12, then as many more, the $\frac{1}{2}$, the $\frac{1}{3}$, and the $\frac{1}{4}$ of 12 added together make 37, then according to the rule we say as 37 : 148 : : 12 to 48 the answer.
$\frac{1}{2}$ as many $=6$	
$\frac{1}{3}$ as many $=4$	
$\frac{1}{4}$ as many $=3$	
37	

As 37 : 148 : : 12 : 48 Ans.

2. What number is that, which being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, the sum shall be 125? Ans. 60.

3. A. B. C. buy a quantity of cloth for £340; of which A. pays three times more than B. and B. four times more than C; what did each pay?
Ans. A. paid £240, B. £80, and C. £20.

4. A person bought a chaise, horse and harness, for £60; the horse came to twice the price of the harness, and the chaise to twice the price of the horse and harness. What did he give for each? Ans. harness, £6 13s. 4d.; horse, £13 6s. 8d.; chaise, £40.

5. A gentleman being asked how much money he had on hand, said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ of his money amounted to £114. What amount of money had he? Ans. £120.

6. A Gentleman being asked his age, said, if $\frac{3}{5}$ of the years I have lived, be multiplied by 7, and $\frac{2}{3}$ of them be added to the product, the sum will be 219; what was his age? Ans. 45 years.

Question.—How is Position proved?

DOUBLE POSITION.

Double Position is used for solving such questions as require two supposed numbers.

RULE.

Suppose any two convenient numbers, and proceed with each of them separately according to the conditions of the question: then mark the errors with the signs +, or — according as the result of the operation, *exceeds* or *falls short* of the given number in the question. Then multiply the first supposed number by the second error, and the second supposed number by the first error, and divide the sum of the products by the sum of the errors, if they are differently marked; or the difference of the products by the difference of the errors, if they are marked alike, and the quotient will be the answer.—PROOF, as in Single Position.

EXAMPLES.

Divide £1000, so that B. may have twice as much as A., wanting £80, and C. three times as much, wanting £150; what was the share of each?

Operation.

First, suppose A. had £200
then B. had 320
and C. had 450
970
Supposed number too small, hence the error is also. 1000 —30 error.

2nd, suppose A. had £180
then B. had 280
and C. had 390
850
Supposed number too small, hence the errors are alike. 1000 –150 error.

We now multiply the first supposed number 200 by the last error 150=30000, also the last supposed number 180 by the first error 30=5400. Then as the errors are marked alike, we divide the difference of the products 30000—-5400 =2460 by the difference of the errors 150 —30=120, and the quotient is £205 A's

QUESTION.—For what is double position used? Repeat the rule.

share, then 205×2—80=330 B's share and 205×3—150=C's share.

2. A. B. and C. built a house which cost £500, of which A. paid a certain sum; B. paid £10 more than A. and C. paid as much as A. and B. both; how much did each man pay?

Ans. A. paid £120, B. £130, and C. £250.

3. What number is that, which being multiplied by 3, the product increased by 4, and the sum divided by 8, the quotient will be 32? Ans. 84.

4. If my horse cost six times as much as it did, more £4, the sum would be £100; what was the price of the horse? Ans. £16.

5. Two men, A. and B. lay out equal sums of money in trade; A. gains £126, and B. loses £87, and A's money is now double to B's; what did each lay out?

Ans. £300.

6. A laborer was hired for 60 days, upon this condition, that for every day he wrought he should receive 4s. and for every day he was idle should forfeit 2s; at the expiration of the time he received £7 10s.; how many days did he work, and how many was he idle?

Ans. He worked 45 days, was idle 15 days.

7. There was a fish caught below Kingston, whose head was 9 inches long; his tail was as long as his head and half his body, and the length of his body was equal to that of his head and tail: what was his whole length?

Ans. 6 feet.

8. A person has two horses, and a saddle worth £50; now, if the saddle be put on the back of the first horse, it will make his value double that of the second; but if it be put on the back of the second, it will make his value triple that of the first; what is the value of each horse?

Ans. one £30, the other £40.

9. Two gentlemen, A. and B. have both the same income; A. saves $\frac{1}{5}$ of his, but B. by expending £50 a year

more than A. at the end of 4 years finds himself £100 in debt; what does each receive and spend per annum?

Ans. Each receives £125 per annum. A. spends £100, and B. £150 a year.

ANNUITIES.

An annuity is a sum of money, payable every year, for a certain length of time, or forever.

An annuity, in the proper sense of the word, is a sum paid *annually*; yet payments made at different periods are called annuities.

Pensions, rents, salaries &c. belong to annuities. When annuities are not paid at the time they become due, they are said to be in *arrears*.

The sum of all the annuities in arrears, with the interest on each for the time they have remained due, is called the *amount*.

When an annuity is to continue forever, its present worth is a sum, whose yearly interest equals the annuity.

I. To find the amount of an annuity at simple interest.

RULE.

First, find the interest of the given annuity for one year, and then for 2, 3, 4, and so on, up to the given number of years, less 1. Then multiply the annuity by the given number of years, and add the product to the whole interest, and the sum will be the amount sought.

EXAMPLES.

1. What is the amount of an annuity of £100 for four years, computing interest at 7 per cent.?

QUESTIONS.—What is an annuity? When are annuities said to be in arrears? What is the amount? When an annuity is to continue forever, what is its present worth? How do we find the amount of an annuity at simple interest?

The interest of £100, at 7 per cent. for 1 year, is £7.
for 2 years, 14.
for 3 years, 21.
Four year's annuity, at £100 per year is £100×4=400.

Ans. £442

2. If an annuity of £70 be forborne 5 years, what will be due for the principal and interest at the end of said term, simple interest being computed at 5 per cent. per annum? Ans. £385 0s.

3. A house being let upon a lease of 7 years, at £400 per annum, and the rent being in arrear for the whole term, I demand the sum due at the end of the term, simple interest being allowed at £6 per cent. per annum. Ans. £3304.

II. To find the present worth of an annuity at simple interest.

RULE.

First, find the present worth of each year by itself, discounting from the time it becomes due; then the sum of all these will be the present worth.

NOTE.—This rule depends on the principles of discount, see page 129.

EXAMPLES.

1. What is the present worth of £400 per annum, to continue 4 years, at 6 per cent. per annum?

106	: 100 : : 400 :	377,35849=	pres't worth of	1st. y'r.
112		357,14285=	"	2d. y'r.
118		338,98305=	"	3d. y'r.
124		322,58064=	"	4th. y'r.

£1396,06503

2. How much present money is equivalent to an annuity of £100, to continue 3 years; rebate being made at 6 per cent.? Ans. £268,37.

QUESTIONS.—How is the present worth of an annuity at simple interest found? On what principles does this rule depend?

3. What is £80 yearly rent to continue 5 years worth in ready money at 6 per cent.? Ans. £340 15s. 4d.

ALLIGATION.

The rule of Alligation teaches how to compound or mix together several simples of different qualities or prices, so that the composition may be of some intermediate quality or price.

It consists of two kinds, *Alligation Medial*, and *Alligation Alternate.*

ALLIGATION MEDIAL

Teaches how to obtain the value, or *mean price* of a mixture, when the quantities and prices of the several articles are given.

RULE.

As the *whole mixture* is to the *whole value*, so is any part of the composition to its mean price.

EXAMPLES.

1. A merchant mixes 20 bushels of wheat at 10s. per bushel, with 36 bushels of corn at 6s. per bushel, and 40 bushels of oats at 4s. per bushel; what is a bushel of the mixture worth?

Operation.

$20 \times 10 = 200$s.	We first find the whole mixture to be
$36 \times 6 = 216$	96bu., and the whole value to be 576s.
$40 \times 4 = 160$	then according to the rule we say, as
— ——	96 : 576 : : 1 bushel to the worth of one
96bu. 576s.	bushel of the mixture.

As 96 : 576 : : 1 : 6s. Ans.

2. A grocer mixes 60lbs. of sugar at 8d. per pound with 20lbs. worth 12d. per pound; what is the value of one pound of the mixture? Ans. 9d.

Questions.—What does Alligation teach? Of what kinds does it consist? What does Alligation Medial teach? Repeat the rule.

3. A tobacconist mixed 36lbs. of tobacco, at 1s. 6d. per pound, 12lbs. at 2s. per pound, with 12lbs. at 1s. 10d. per pound; what is a pound of this mixture worth?

Ans. 1s. 8d.

4. A grocer mixed 2cwt. of sugar at 56s. per cwt. and 1cwt. at 43s. per cwt. and 2 at 50s. per cwt. together; I demand the price of 3cwt. of this mixture? Ans. £7 13s.

5. A refiner melted together 8oz. of gold, of 22 carats fine, 10oz. of 20 carats fine, 12oz. of 16 carats fine, 8oz. of 18 carats fine; what was the value of the composition?

Ans. $18\frac{14}{19}$ carats fine.

ALLIGATION ALTERNATE

Is the method of finding what quantity of each of the ingredients whose rates are given, will compose a mixture of a given rate; so that it is the reverse of Alligation Medial, and may be proved by it. *This is called Alligation Alternate, because the same question frequently admits of different answers.*

CASE I.

When the prices of the several simples are given, to find how much of each at their respective rates must be taken to make a compound or mixture, at any proposed price.

RULE.

I. Reduce the mean price and the prices of each separate article to the same denomination.

II. Connect with a line each price that is less than the mean price, with one or more that is greater; and each price greater than the mean price, with one or more that is less.

III. Write the difference between the mean price and the price of each separate article opposite the price with which it is connected; then the

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QUESTIONS.—What is Alligation Alternate? How may it be proved? Why is it called Alligation Alternate? How do we find the proportional parts to be taken when the prices of the several simples are given?

sum of the differences standing against any price will express the relative quantity to be taken of that price.

EXAMPLES.

A merchant would mix wines worth 16s. 18s. and 22s. per gallon, in such a way that the mixture be worth 20s. per gallon; how much must be taken of each sort?

Operation.

```
   ( 16 ——  2 at 16s.
20 { 18 ——  2 at 18s.
   ( 22 ——  4×2=6 at 22s.
```

In this example 20s. is the mean price; then as 16s. is less, and 22s. greater than the mean price, we connect 16 and 22; then as 18s. is less than 20s. it must also be connected with 22s.; then as the difference between 16s. and 20s. is 4s., and between 18s. and 20s. is 2s. the 4 and 2 must both be placed opposite 22, because 16 and 18 are both connected with that number, also because 22 is connected to 16 and 18 the difference between 20 and 22 must be placed opposite those two numbers. So that 2 gal. at 16s. 2 at 18s. and 6 at 22s. will make a mixture worth 20s. per gallon, or any other quantities bearing the proportion of 2, 2, and 6.

2. What proportions of tea at 8s. at 9s. at 11s. and at 12s. must be taken to make a mixture worth 10s. per pound. Ans. 2lbs. at 8 & 12s. and 1lb. at 9 and 11s.

3. A goldsmith has gold of 16, of 18, of 23 and of 24 carats fine; what part must be taken of each, so that the mixture shall be 21 carats fine?

Ans. 3 of 16, 2 of 18, 3 of 23, and 5 of 24.

4. What portions of wine at 14s., at 24s., at 21s., and at 10s. per gallon, must be mixed together so that the mixture shall be worth 18s. per gallon? Ans. 6 gal. at 10s., 3 at 14s. 4 at 21s. and 8gal. at 24s.

CASE II.

When one of the ingredients is limited to a certain quantity, to find the several quantities of the rest, in proportion to the given quantity.

RULE.

I. Find the proportional quantities of the simples as in Case I.

II. Then say, as the number opposite the simple whose quantity is given, is to the given quantity, so is either proportional quantity to the part of its simple to be taken.

EXAMPLES.

1. How much wine at 5s., at 5s. 6d., and 6s. per gal. must be mixed with 4 gallons at 4s. per gallon, so that the mixture shall be worth 5s. 4d. per gallon?

Operation.

64	48	8	. . Simple whose quantity is known.
	60	2	Proportional quantities.
	66	4	
	72	16	

Then 8 : 4 :: 2 : 1
8 : 4 :: 4 : 2
8 : 4 :: 16 : 8

Ans. 1 gal. 5s., 2 at 5s. 6d., and 8 at 6s.

2. How much water must be mixed with 100 gallons of wine worth 7s. 6d. per gallon to reduce it to 6s. 3d. per gallon? Ans. 20 gallons.

3. A farmer made a mixture of wheat at 4s. per bush., rye at 3s., barley at 2s., with 12 bushels of oats at 18d. per bushel: how much is taken of each sort when the mixture is worth 3s. 6d.

Ans. { 96 bu. of wheat; 12 bu. of rye; 12 bu. of barley; and 12 of oats.

4. With 95 gallons of wine at 8s. per gal. I mixed other wine at 6s. 8d. per gal. and some water: then I found it stood me in 6s. 4d. per gallon. I demand how much wine and how much water I took.

Ans. 95 gal. wine at 6s. 8d. and 30 gal. water.

QUESTIONS.—When one of the ingredients is limited to a certain quantity, how do we find the proportional quantities of the rest?

CASE III.

When the whole composition is limited to a ceartain quantity.

RULE.

I. Find the proportional quantities as in Case I.

II. Then say, as the sum of the proportional quantities is to the given quantity, so is each proportional quantity to the part to be taken of each.

EXAMPLES.

1. A grocer has four sorts of sugar worth 12d., 10d., 6d., and 4d., per pound; he would make a mixture of 144 lbs. worth 8d. per lb.: what quantity must be taken of each sort?

Operation.

8	4	4	Then 12 : 144 :: 4 : 48 lb. at 4d.	Answers.
	6	2	12 : 144 :: 2 : 24 " " 6d.	
	10	2	12 : 144 :: 2 : 24 " " 10d.	
	12	4	12 : 144 :: 4 : 48 " " 12d.	
		12 Sum of the proportional parts.		

2. A grocer had four sorts of tea at 1s., 3s., 6s., and 10s. per lb., the worst would not sell, and the best was too dear; he therefore mixed 120 lbs. and so much of each sort as to sell it at 4s. per lb.; how much of each kind did he take? Ans. 60lbs. at 1s., 20lbs. at 3s., 10lbs. at 6s. and 30 lbs. at 10s.

3. How much water at 0 per gallon, must be mixed with wine at 18s. per gallon, so as to fill a vessel of 100 gallons, which may be afforded at 12s. per gallon?

Ans. $33\frac{1}{3}$ gal. water, and $66\frac{2}{3}$ gal. wine.

4. A goldsmith has two sorts of silver bullion, one of 10 oz. and the other of 5 oz. fine, and has a mind to mix a pound of it so that it shall be 8 oz. fine; how much of each sort must he take?

Ans. $4\frac{4}{5}$ of 5 oz. fine, and $7\frac{1}{5}$ 10 oz. fine.

QUESTION.—When the whole composition is limited to a certain quantity, what is the rule?

ARITHMETICAL PROGRESSION, OR EQUI-DIFFERENT SERIES.

Any rank or series of numbers, consisting of more than two terms, which increases or decreases by a common difference, is called an *Arithmetical Series* or *Progression*.

When the series *increases*, that is, when it is formed by the constant *addition* of the common difference, it is called an ascending series, thus,

1, 3, 5, 7, 9, 11, 13, &c.

Here it will be seen that the series is formed of a continual addition of 2 to each succeeding figure. When the series decreases, that is, when it is formed by the constant *subtraction* of the common difference, it is called a descending series, thus,

14, 12, 10, 8, 6, 4, 2, &c.

Here the series is formed by a continual *subtraction* of 2 from each preceding figure.

The figures that make up the series are called the *terms* of the series. The *first* and *last* terms are called the *extremes*, and the other terms the *means*.

From the above, it may be seen that any term in a series may be found by continued addition or subtraction, but in a long series this process would be tedious. A much more expeditious method may be found.

The ages of six persons are in arithmetical progression. The youngest is 8 years old, and the common difference is 3; what is the age of the eldest? In other words, what is the last term of an arithmetical series, whose first term is 8, the number of terms 6, and the common difference 3?

8, 11, 14, 17, 20, 23.

Examining this series, we find that the common difference, 3, is added 5 times, that is, *one less* than the number of terms, and the last term 23, is larger than the first

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QUESTIONS.—What is an arithmetical series, or progression? When is it called an ascending series? When is it called a descending series? What are the terms of the series? What are the *extremes?* What are the *means?* How may any term in a series be found?

term, by 5 times the addition of the common difference, three; Hence,

The age of the elder person is $5\times3+8=23$ Ans.

Therefore, when the first term, the number of terms, and the common difference, are given, to find the last term we have the following

RULE.

Multiply the common difference into the number of terms, less 1, and add the product to the first term.

EXAMPLES.

1. The first term is 3, the common difference 2, and the number of terms 19: what is the last term?

Operation.

18 number of terms less 1.
 2 common difference.
—
36
 3 1st term.
—
Ans. 39 last term.

2. If the first term be 4, the common difference 3, and the number of terms 100, what is the last term? Ans. 301.

3. John owes Samuel a certain sum, to be paid in arithmetical progression; the first payment is 6d., the number of payments 52, and the common difference of the payments is 12d. What is the last payment? Ans. £2 11s. 6d.

4. A man put out £100, at 7 per cent. simple interest, which amounted to £107 in a year, £114 in 2 years, and so on in arithmetical progression, with the common difference of £7: what was the amount due at the expiration of 50 years? Ans. £450.

5. A man bought 100 yards of cloth in arithmetical progression: for the first yard he gave 4 pence and for the last 301 pence, what was the common increase on the price of each yard?

QUESTION.—When the first term, the number of terms, and the common difference are given, how do we find the last term?

As he bought 100 yards, and at an increased price upon every yard, it is evident that this increase was made 99 times, or *once less*, than the number of terms in the series. Hence the price of the last yard was greater than the first, by the addition of 99 times the regular increase. Therefore if the first price be subtracted from the last, and the remainder be divided by the number of additions 99, the quotient will be the common increase; $301-4=297 \div 99=3$, the common difference. Hence, when the extremes and number of terms are given, to find the common difference, we have the following

RULE.

Subtract the less extreme from the greater, and divide the remainder by the number of terms less 1, and the quotient will be the common difference.

EXAMPLES.

1. The extremes are 4 and 104, and the number of terms 26; what is the common difference?

$104-4 \div \overline{26-1}=4$ Ans.

2. A man has 8 sons, the youngest is 4 years old and the eldest 32, their ages increase in arithmetical progression: what is the common difference of their ages? Ans. 4 years.

3. A man is to travel from Kingston to a certain place in 12 days: to go three miles the first day, increasing every day by the same number of miles, so that the last day's journey may be 58 miles: required the daily increase.

Ans. 5 miles.

Suppose we are required to find the sum of all the terms in a series whose first term is 2, the number of terms 10, and the common difference 2.

2,	4,	6,	8,	10,	12,	14,	16,	18,	20.
20,	18,	16,	14,	12,	10,	8,	6,	4,	2.
22,	22,	22,	22,	22,	22,	22,	22,	22,	22.

The first row of figures above, represents the given se-

QUESTION.—When the extremes and number of terms are given how do we find the common difference?

ries. The second, the same series with the order inverted, and the third, the sums of the additions of the corresponding terms in the two series. By examining these series we shall find that the sums of the corresponding terms are the same, and that each of them is equal to the *sum* of the extremes, viz. 22. Now, as there are 10 of these pairs in the two series, the sum of the terms in *both,* must be $22\times10=220$.

But it is evident, that the sum of the terms in *one* series can be only *half* as great as the sum of both, therefore, if we divide 220 by two, we shall find the sum of the terms in one series, which was the thing required. $220\div2=110$, the sum of the series. Hence,

When the extremes and number of terms are given, to find the sum of the terms, we have the following

RULE.

Multiply the sum of the extremes by the number of terms, and divide the product by 2.

EXAMPLES.

1. The extremes are 2 and 100, and the number of terms 22: what is the sum of the series? Operation.

2 first term.
100 last term.

102 sum of extremes.
22 number of terms.

2)2244=1122 Ans.

2. How many strokes does the hammer of a clock strike in 12 hours? Ans. 78.

3. A man bought 19 yards of linen in arithmetical progression; for the first yard he gave 1s. and for the last yard £1 17s.; what did the whole come to? Ans. £18 1s.

4. There are in a certain triangular field, 41 rows of corn; the first row, being in one corner, is a single hill,

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QUESTION.—When the extremes and number of terms are given, how do we find the sum of the terms?

and the last row, on the opposite side, contains 81 hills; how many hills of corn in the field? Ans. 1681.

5. Suppose 144 oranges were laid 2 yards distant from each other, in a right line, and a basket placed two yards distant from the first orange, what length of ground will that boy travel over, who gathers them up singly, returning with them one by one to the basket?

Ans. 23 miles 5 fur. 180 yards.

GEOMETRICAL PROGRESSION.

Any series of numbers, consisting of more than two terms, which increases by a common multiplier, or decreases by a common divisor, is called a *Geometrical series.* Thus, the series 2, 4, 8, 16, 32, &c., consists of terms, each of which is *twice* the preceding, and this is an *increasing* or *ascending* Geometrical series. The series 32, 16, 8, 4, 2, consists of numbers, each of which is *one half* the preceding, and this is a *decreasing* or *descending* Geometrical series. The common multiplier or divisor is called the *Ratio,* and the numbers which form the series are called *Terms.* In *Arithmetical* and in *Geometrical* progression, if any three of the five following terms be given; the other two may be found.

1st. The first term. 2nd. The last term. 3d. The number of terms. 4th. The common difference. 5th. The sum of all the terms.

A man bought a piece of cloth containing 12 yards, the first yard cost 3d., the second 6, the third 12, and so on doubling the price to the last, what cost the last yard?

$3\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2=3\times2^{11}=$ 6144 Ans.

In examining the above process, it will be seen that the price of the second yard is found by multiplying the first

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QUESTIONS.—What is a Geometrical series? Give an example of an *ascending* Geometrical series? Describe a *descending* Geometrical series. What is the Ratio? What are the terms? What are the five terms, any three of which being given, the other two may be found.

payment into the ratio 2 once; the price of the *third* yard by multiplying by 2 *twice*, &c., and that the ratio 2 is used as a factor *eleven* times, or *once less* than the number of terms. The last term then, is the *eleventh* power of the ratio 2, multiplied by the first term 3. Hence, *when the first term, ratio, and number of terms being given, to find the last term, we have the following*

RULE.

Raise the ratio to a power whose exponent is one less than the number of terms, and then multiply the power by the first term, the product will be the last term.

EXAMPLES.

1. The first term is 3 and the ratio 2; what is the 6th term?

$$2\times2\times2\times2\times2=2^5=32$$

3 first term.

Ans. 96

2. A man purchased 12 pears: he was to pay 1 farthing for the first, 2 farthings for the second, and so on, doubling each time: what did he pay for the last?

Ans. £2 2s. 8d.

3. A gentleman dying, left 9 sons, and bequeathed his estate in the following manner: to his executors £50: his youngest son to have twice as much as the executors, and each son to have double the amount of the son next younger: what was the eldest son's portion? Ans. £25600.

4. A man plants 4 kernels of corn, which at harvest produce 32 kernels; these he plants the second year; now suppposing the annual increase to continue 8 fold, what would be the produce of the 16th year, allowing 1000 kernels to a pint? Ans. 2199023255,552 bushels.

The most obvious method of obtaining the *sum of the terms* in a geometrical series, might be by *addition*, but this is not the most expeditious, as will be seen.

Question.—When the first term, ratio, and number of terms are given, how do we find the last term?

A man bought 5 yards of cloth, giving 2 pence for the first, 6 pence for the second, and so on in a threefold ratio; what did the whole cost him?

2, 6, 18, 54, 162,
6, 18, 54, 162, 486.

The first of the above lines represents the original series. The second, that series, multiplied by the ratio 3.

By examining these series, it will be seen that their terms are all alike, excepting two: the *first* term of the first series, and the *last* of the second series. If now we subtract the first series from the last, we have for a remainder 486—2=484 as all the intermediate terms vanish in the subtraction.

Now the last series is *three* times the first, (for it was made by multiplying the first series by 3,) and as we have already subtracted *once* the first, the remainder must of course be *twice* the first.

Therefore if we divide 484 by 2, we shall obtain the sum of the first series. 484÷2=242 Ans.

As in the preceding process, all the preceding terms vanish in the subtraction, excepting the first and last, it will be seen, that the result would have been the same, if the last term only had been multiplied, and the first subtracted from the product.

Hence, *when the extremes and ratio are given, the sum of all the terms may be found by the following*

RULE.

Multiply the greater term by the ratio, from the product subtract the least term, and divide the remainder by the ratio less 1.

EXAMPLES.

1. The first term is 3, the ratio 2, and last term 192; what is the sum of the series?

Ans. $192\times2-3=381\div\overline{2-1}=381$.

QUESTIONS.—When the exrremes and ratio are given, how may the sum of all the terms be found? If the last term be not given in the question what must be done?

NOTE.—If the last term be not given in the question; first find it by the rule in the last article, then proceed as above.

2. A gentleman whose daughter was married on New Year's day, gave her one shilling towards her portion, and was to double it on the first day of every month during the year; what was her portion? Ans. £204 15s.

3. What is the sum of the series 16, 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, and so on to an infinite extent? Ans. $21\frac{1}{3}$.

Here it is evident the last term is 0, or indefinitely near to nothing; the extremes therefore are 16 and 0, and the ratio 4.

4. What debt can be discharged in a year, by paying 1 farthing the first month, 10 farthings the second, and so on, each month in a ten fold proportion?

Ans. £115740740 14s. $9\frac{3}{4}$d.

5. A man bought a horse, and by agreement was to give a farthing for the first nail, 2 for the second, 4 for the third, &c. There were 4 shoes, and 8 nails in each shoe; what did the horse come to at that rate?

Ans. £4473924 5s. $3\frac{3}{4}$d.

PERMUTATION.

Permutation is the method of finding how many changes may be made in the order in which things succeed each other.

What number of permutations may be made on the letters A and B? They may be written A B. or B A.

What number on the letters A B C?

Placing A first, A B C, or A C B,

Placing B first, B A C, or B C A,

Placing C first, C A B, or C B A.

From these examples it will be seen, that of two things, there may be two changes, $1\times2=2$, and of 3 things there may be 6 changes, $1\times2\times3=6$.

QUESTIONS.—What is Permutation? How may we find what number of changes or permutations may be made on any given number of things?

Hence, to find the number of different changes, or permutations of which any number of different things are capable,

Find the continued product of the natural series of numbers, from 1 to the given number.

EXAMPLES.

1. Four gentlemen agreed to remain together, as long as they could arrange themselves differently at dinner; how many days did they remain? Ans. 24.

2. How many variations may there be in the position of the nine digits? Ans. 362880.

3. Seven gentlemen met at an inn, and were so well pleased with their host, and with each other, that they agreed to tarry so long as they, together with their host, could sit every day in a different position at dinner; how long must they have staid at said inn to have fulfilled their agreement? Ans. $110\frac{170}{365}$ years.

MISCELLANLOUS EXERCISES.

1. $\overline{7+4-\overline{2+3}+40}\times5$ = how many? Ans. 230.

2. $\dfrac{\overline{3+6-2}\times\overline{4-2}}{2\times2}$ = how many? Ans. $3\frac{1}{2}$.

3. What number is that which being divided by 19, the quotient will be 72? Ans. 1368.

4. What number is that which being multiplied by 15 the product will be $\frac{3}{4}$? Ans. $\frac{1}{20}$.

5. What is the difference between six dozen dozen, and half a dozen dozen? Ans. 792.

6. What is the difference between thrice five and thirty, and thrice thirty five? Ans. 60.

7. What fraction is that, to which if you add $\frac{2}{5}$, the sum will be $\frac{5}{6}$? Ans. $\frac{13}{30}$.

8. What number is that which being divided by $\frac{3}{4}$, the quotient will be 21? Ans. $15\frac{3}{4}$.

9. What number taken from the square of 54 will leave 19 times 46? Ans. 2042.

10. If I buy 1000 Ells Flemish of linen for £90, what must it be sold for per Ell English, to make £10 by the purchase? Ans. 3s. 4d.

11. A snail in getting up a pole 20 feet high, was observed to climb up 8 feet every day, but to descend 4 feet every night; in what time did he reach the top of the pole? Ans. 4 days.

12. What number added to the 43rd part of 4429, will make the sum 240? Ans. 137.

13. How many bushels of potatoes, at 1s. 6d. per bushel, must be given for 32 bushels of barley, at 2s. 6d. per bushel? Ans. $53\frac{1}{3}$ bu.

14. A boy bought a number of apples; he gave away 10 of them to his companions, and afterwards bought 34 more, and divided one half of what he then had among four companions, who received 8 apples each; how many apples did the boy first buy? Ans. 40.

15. A man married at the age of 23; he lived with his wife 14 years; she then died, leaving him a daughter 12 years of age; 8 years after, the daughter was married to a man 5 years older than herself, who was 40 years of age when the father died; how old was the father at his death? Ans. 60 years.

16. There is a room 18 feet in length, 16 feet in width, and 8 feet in height; how many rolls of paper, 2 feet wide, and containing 11 yards in each roll, will it take to cover the walls? Ans. $8\frac{16}{66}$ rolls.

17. How many steps of 30 inches each must a man take in travelling $54\frac{1}{2}$ miles? Ans. 115104 steps.

18. How much time would a person redeem in 40 years, by rising each morning half an hour earlier than he now does? Ans. $304\frac{1}{6}$ days.

19. There is a house, the roof of which is $44\frac{1}{2}$ feet in length, and 20 feet in width on each of the two sides; if 3 shingles in width cover one foot in length, how many shingles will it take to lay one course on this roof? if 3

courses make one foot, how many courses will there be on one side of the roof? how many shingles will it take to cover one side, also to cover both sides? Ans. 16020 shingles.

20. Said John to Dick, my purse and money are worth £9 2s., but the money is 25 times as much as the purse; I demand how much money was in it? Ans. £8 15s.

21. The third part of an army was killed, the fourth part taken prisoners, and 1000 fled; how many were in this army, how many killed, and how many captives?

Ans. 2400 in the army, 800 killed, and 600 taken prisoners.

22. If 3 men can do a piece of work in $4\frac{1}{2}$ hours, in how many hours will 10 men do the same work?

Ans. $1\frac{7}{20}$ hours.

23. Jacob, by contract was to serve Laban for his two daughters, 14 years; and when he had accomplished 11 years, 11 months, 11 weeks, 11 days, 11 hours, 11 minutes, how long had he to serve?

Ans. 1yr. 11mo. 3w. 2da. 12h. 49min.

24. A man had two silver cups of unequal weight, having one cover to both, weighing 5oz.; now if the cover is put on the less cup, it will be double the weight of the greater cup, and put on the greater cup it will be three times as heavy as the less cup; what is the weight of each cup?

Ans. 3oz. less; 4oz. greater.

25. A man and his wife can drink out a cask of beer in 12 days, but when the man was from home it lasted the woman 30 days; how many days would the man be in drinking it alone? Ans. 20 days.

26. The great bell of Moscow, in Russia, the largest in the world, is 67 feet in circumference, 19 feet high, and weighs about 448000 pounds; how many teams would it require to move this bell, if each team draw 30cwt.?

Ans. $133\frac{1}{3}$ teams.

27. From each of 16 pieces of gold a person filed the worth of 2s. 6d., and then offered them in payment for their original value, and the fraud being detected, and the pieces weighed, they were found to be worth in the whole, no

more than 8 guineas; what was the original value of each piece? Ans. 13s.

28. Two men carry a kettle weighing 200 pounds; the kettle is suspended on a pole, the bale being 2ft. 6in. from the hands of one; and 3ft. 4in. from the hands of the other; how many pounds does each bear?

Ans. { $114\frac{2}{7}$ pounds.
{ $85\frac{5}{7}$ pounds.

29. A person bought 160 oranges at 2 for a penny, and 180 more at 3 for a penny; after which he sold them out at the rate of 5 for 2 pence; did he make or lose, and how much? Ans. He lost 4 pence.

30. If a person step 70 paces per minute, and 28in. each pace; how fast is that per hour?

Ans. $1\frac{113}{132}$ miles.

31. A wall of 700 yards in length was to be built in 29 days. Twelve men were employed on it for 11 days, and only completed 220 yards; how many men must be added to complete the wall in the required time?

Ans. 4 men.

32. There is a stone which measures 4 feet 6 inches long, 2 feet 9 inches broad, and 5 feet 4 inches deep; how many solid feet does it contain? Ans. 66ft.

33. What is the product of 2s. 6d. multiplied by 2s. 6d.? Ans. 6s. 3d.

34. ——— I sum up half mankind,
And add two thirds of the remaining half,
And find the total of their hopes and fears,
Dreams, empty dreams. COWPER.

What part of mankind are mere dreamers, according to this author? Ans. $\frac{5}{6}$.

35. Wh.. time, between 4 and 5 o'clock, are the hour and minute hands of a watch exactly together?

Ans. $21\frac{9}{11}$ min. past 4.

36. A. can mow an acre of grass in $7\frac{1}{2}$ hours, B. in $8\frac{1}{2}$ hours, in what time will they jointly perform it?

Ans. 4 hours.

37. A captain, mate, and twenty seamen, took a prize worth £3501, of which the captain takes 11 shares, and the mate 5 shares; the remainder of the prize is equally divided among the sailors; how much did each man receive?

Ans. { The captain, £1069 15s.
The mate, 486 5s.
Each sailor, 97 5s. }

38. Divide the number 360 into 3 parts, which shall be to each other as 2, 3 and 4. Ans. 80, 120 and 160.

39. Two merchants have gained £450, of which A is to have three times as much as B; how much is each to have? Ans. A. £337 10s. and B. £112 10s.

40. A. and B. traded together, and gained £100; A. put in £640.; B. put in so much that he must receive £60 of the gain; I demand B's stock. Ans. £960.

41. The wall that separates China from Tartary was built 2000 years ago; it crosses the largest rivers and mountains, and is 1200 miles in length, 30 feet high, and 24ft. broad; how many cubic feet does it contain?

Ans. 4561920000.

42. The surface of a middle sized man is 16 square feet, and the skin is said to be perforated by a thousand holes in the space of a square inch. How many pores does the human body contain, according to this calculation?

Ans. 2304000 pores.

43. Divide 97deg. 55mi. 7fur. 35po. 4ft. 2in. 1b. by 6.

Ans. 16deg. 20min. 7fur. 12po. 8ft. 11in. $1\frac{1}{6}$ b. c.

44. If a herring and a half cost a penny and a half, how many can be bought for eleven pence?

45. The entire amount of specie throughout the world is estimated at one billion nine hundred millions of dollars; how long would it take to count this sum at the rate of 50 a minute?

Ans. 72 years, 108 days, 21 hours and 20min.

46. One end of a certain pile of wood is perpendicular to the horizon, the other is in the form of an inclined plane; the length of the pile at the bottom is 64 feet, length at the top 50 feet, height 12 feet, length of the wood 5 feet; re-

quired the number of cords it contains? Ans. $26\frac{23}{32}$.

47. A may-pole, whose top was broken off by a blast of wind, struck the ground at 15 feet distant from the bottom of the pole; what was the height of the whole maypole, supposing the length of the broken piece to be 39ft.? Ans. 75 feet.

48. In the midst of a meadow well stored with grass,
I took just an acre to tether my ass;
How long must the cord be, that feeding all round,
He may 'nt graze less or more than an acre of ground?
Ans. 39,25+ yards.

49. If a quantity of provisions serves 1500 men 12 weeks, at the rate of 20 ounces a day for each man; how many men will the same provisions maintain for 20 weeks, at the rate of 8oz. a day for each man? Ans. 2250 men.

50. A younger brother received $8400, which was just $\frac{7}{9}$ of his elder brother's fortune; what was the father worth? Ans. $19200.

51. If 20 men can perform a piece of work in 12 days, how many men will accomplish three times as much in one-fifth of the time? Ans. 300.

52. Suppose that I have [illegible] of a ship worth $1200; what part have I left after selling $\frac{3}{[illegible]}$ of $\frac{4}{5}$ of my share, and what is it worth? Ans. [illegible] left, worth $986,66+

53. What number is that which being multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of $1\frac{1}{2}$, the product will be 1? Ans. $1\frac{1}{4}$.

54. My horse and saddle together are worth $132, and my horse is worth 10 times as much as the saddle: what is the value of the horse? Ans. $120.

55. A farmer being asked how many sheep he had, answered, that he had them in five fields; in the first he had $\frac{1}{4}$ of his flock, in the second $\frac{1}{6}$, in the third $\frac{1}{3}$, in the fourth $\frac{1}{12}$, and in the fifth 450; how many had he?
Ans. 1200.

56. Sound travels about 1142 feet in a second. Now if the flash of a cannon be seen at the moment it is fired, and the report heard 45 seconds after, what distance would the observer be from the gun? Ans. 9mi. 5fur. 34rd.+

57. In a certain orchard, $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ of them bear peaches, $\frac{1}{6}$ of them plums, 120 of them cherries, and 80 of them pears; how many trees are there in the orchard? Ans. 240.

58. A circular fish-pond is 865 feet in diameter; what is its circumference, and what is its area?

Ans. { Circumference 2717,484ft.
Area 587655,915ft. square.

59. A well is to be stoned, of which the diameter is 6 feet 6 inches; the thickness of the wall is to be 1 foot 6 inches, leaving the diameter of the well within the stones 3 feet 6 inches. If the well is 40 feet deep how many feet of stone will be required? Ans. 942,48 feet.

60. A ship has a leak by which it would fill and sink in 15 hours, but by means of a pump it could be emptied, if full, in 16 hours. Now if the pump is worked from the time the leak begins, how long before the ship will sink?

Ans. 240 hours.

61. How many planks 15ft. long and 15in. wide, will floor a barn which is $60\frac{1}{2}$ feet long and $33\frac{1}{2}$ wide?

Ans. $108\frac{7}{75}$.

62. A person dying worth $5460, left a wife and two children, a son and daughter, absent in a foreign country. He directed that if his son returned, the mother should have one third of the estate, and the son the remainder; but if the daughter returned, she should have one third, and the mother the remainder. Now, it so happened that they both returned; how must the estate be divided to fulfil the father's intentions?

Ans. Daughter $780, Son $3120, Mother $1560.

63. A cistern containing 60 gallons of water has three unequal cocks for discharging it; the largest will empty it in 1 hour, the second in 2, and the third in 3 hours; in what time will the cistern be emptied if they all run together?

Ans. $32\frac{8}{11}$ min.

64. A. can do a piece of work alone in 10 days, and B. in 13 days; in what time can they do it if they work together? Ans. $5\frac{15}{23}$ days.

65. The accounts of a certain school are as follows, viz: $\frac{1}{16}$ of the boys learn geometry, $\frac{3}{8}$ learn grammar, $\frac{3}{10}$ learn arithmetic, $\frac{3}{20}$ learn to write, and 9 learn to read; what is the number in each branch?

Ans. { 5 learn geometry, 30 grammar, 24 arithmetic, 12 writing, and 9 reading.

66. A stationer sold quills at 11s. a thousand, by which he cleared $\frac{3}{8}$ of his money; but they growing scarce he raised the price to 13s. 6d. a thousand; what did he clear at the last price, on each £100 laid out?

Ans. £96 7s. $3\frac{3}{11}$d.

67. A water tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes; the tap discharges at a medium 40 gallons in 31 minutes. Now, supposing these to be left open, and the water to be turned on at 2 o'clock in the morning; a servant at 5 shuts the tap, and wishes to know in what time the tub will be filled in case the water continues to flow.

Ans. The tub will be full at 3 min. $48\frac{114}{217}$ sec. after 6.

68. Take $\frac{1}{2}$ a square foot from $\frac{4}{11}$ of an acre.

Ans. 1R. 18P. 5yd. 4ft.

69. Two men and a boy were engaged to do a piece of work; one of the men could do it in 5 days, the other in 8 days, and the boy in 10 days; how long would it take the three together to do it? Ans. $2\frac{6}{17}$ days.

70. After laying out $\frac{1}{4}$ of my money, and $\frac{1}{5}$ of the remainder, I had 72 guineas left; how much had I at first?

Ans. 120 guineas.

71. Two persons, A. and B., are on the opposite sides of a wood which is 536 yards in circumference; they begin to travel in the same direction at the same moment.—A. goes at the rate of 11 yards per minute, and B. at the rate of 34 yards in 3 minutes. The question is, how many times the quicker one must go round the wood before he overtakes the slower? Ans. 17 times.

72. If a person take snuff once in 10 minutes, and spend $\frac{3}{4}$ of a minute in the process of snuffing, adjusting the box, and blowing the nose, how many hours will be thus

spent in 7 years, allowing $13\frac{4}{5}$ hours to a snuff-taking day, and $365\frac{1}{4}$ days to the year?

Ans. { $2646\frac{189}{500}$ hours, or the wakeful hours of more than 5 months.

73. A father dying bequeaths to one of his sons $\frac{2}{7}$ of his estate, and to another $\frac{12}{35}$ of the remainder; upon a division the latter bequest is found to be \$200 less than the former. Query, the legacy of each?

Ans. The former \$1400, the latter \$1200.

74. If A. can mow an acre of grass in $5\frac{2}{3}$ hours, and B. can mow $1\frac{3}{4}$ acres in $9\frac{1}{3}$ hours, in what time can they jointly cut $8\frac{1}{4}$ acres? Ans. $22\frac{2}{3}$ hours.

75. How many pounds of quick lime must be thrown into a well containing $64\frac{1}{2}$ cubic feet of carbonic acid gas, to render it respirable; a cubic foot of that gas containing $\frac{5}{43}$ of a pound of carbon, and 1lb. of lime being capable of absorbing $\frac{43}{57}$ of a pound of carbon? Ans. $9\frac{81}{86}$ pounds.

76. If a horse can draw $6\frac{1}{2}$ tons on a railroad which ascends $25\frac{3}{4}$ feet in a mile, what weight would be drawn if the ascent was only $15\frac{3}{8}$ft. in a mi. the force required to move a given wt. being as the ascent per mile? Ans. $10\frac{109}{123}$ tons.

77. A person in health has about 75 pulsations, or beats of the artery in a minute. Now, a gun being fired on one side of a river, an observer directly opposite counts nine pulsations at his wrist between seeing the flash and hearing the report; what was the breadth of the river?

Ans. 1mi. 4fur. 100yds. 2ft.

78. A hare starts 40 yards before a greyhound, and is not perceived by him till she has been up 40 seconds; she runs at the rate of ten miles an hour, and the dog, on view, makes after her at the rate of 18 miles an hour; how long will the course hold, and what space will be run over from the spot where the dog started?

Ans. $60\frac{5}{22}$ sec., and 530yds. space.

79. If to my age there added be,
One-half, one-third, and three times three,
Six score and ten the sum will be;
What is my age, pray show it me? Ans. 66yrs.

80. A gentleman divided his fortune among his three sons, giving A. £9 as often as B. £5, and to C. but £3 as often as B £7, and yet C's dividend was £2584; what was the amount of the whole estate?

Ans. £19466 2s. 8d.

81. The yearly interest of Mary's money, at 6 per cent. exceeds $\frac{1}{20}$ of the principal by £100, and she does not intend to marry any man who is not scholar enough to tell her fortune; pray what is it? Ans. £10000.

82. A. and B. can do a piece of work in 4 days, and B. and C. in 6 days, and A. and C. in 5 days; in what time can they all do it together? Ans. $3\frac{9}{37}$ days.

83. A. and B. can do a piece of work in 5 days; A. can do it in 7 days; in how many days can B. do it?

Ans. $17\frac{1}{2}$ days.

84. A man died, leaving £1000 between his two sons, one 14, and the other 18 years of age, in such proportion that the share of each, being put to interest at 6 per cent., should amount to the same sum when they should arrive at the age of 21; what did each receive?

Ans. The elder £546,153+; the younger £453,846+

85. A. B. and C. would divide £100 between them, so as that B. may have £3 more than A., and C. £4 more than B.; how much must each man have?

Ans. A. £30, B. £33, and C. £37.

86. A. and B. undertake a piece of work for $54, on which A. employed 3 hands 5 days, and B. employed 7 hands 3 days; what part of the work was done by A., what part by B., and what was each one's share of the money? Ans. A., $\frac{5}{12}$; B., $\frac{7}{12}$; A's money, $22,50; B's, $31,50.

87. A. B. and C. traded in company; A. put in $500, B. $350, and C. 120 yards of cloth; they gained $332,50, of which C's share was $120; what was the value of C's cloth per yard, and what was A. and B's share of the gain?

Ans. { C's cloth per yard, $4.
A's share of the gain, $125.
B's do. do. $87,50. }

88. There are 3 horses, belonging to 3 men, employed to draw a load of goods from Kingston to Toronto, for £26,45. A. and B's horses together are supposed to do $\frac{3}{4}$ of the work; A. and C's $\frac{9}{10}$; B. and C's $\frac{13}{20}$; they are to be paid proportionately; what is each one's share of the money?

Ans. { A's £11,5 ($=\frac{10}{23}$)
B's £5,75 ($=\frac{5}{23}$)
C's £9,2 ($=\frac{8}{23}$) }

89. A gentleman left his son a fortune, $\frac{5}{16}$ of which he spent in 3 months; $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only £537 left; what was the sum bequeathed him by his father?

Ans. £2082 18s. 2$\frac{2}{11}$d.

90. There is a square field, each side of which is 50 rods; what is the distance between opposite corners?

Ans. 70,71+

91. What is the area of a square field, of which the opposite corners are 70,71 rods apart? and what is the length of each side? Ans. to last, 50 rods, nearly.

92. A trader being embarrassed, owes $3400, which the creditor requires to be immediately paid. He has goods which he can sell at auction for cash at 15 per cent. below the just value; he cannot borrow money without allowing a premium of 9 per cent., and paying interest at 6 per cent. per annum on the whole. Now, admitting he can sell his goods for their value within a year, which will be more elligible, to send them to auction, or to borrow money on these conditions, to satisfy his creditor?

Ans. To borrow, by $71,64.

93. Two travellers, A. and B., at the distance of 59 miles from each other, set out in order to meet. A. begins his journey 1 hour before B., and travels 7 miles in 2 hours; B. proceeds 8 miles in 3 hours; how far will they travel respectively before they meet? Ans. A. 35mi. B. 24.

94. A hare starts up 50 of its own leaps before a grey hound, and takes 4 leaps while the hound takes 3; but the hound goes as far at 2 leaps as the hare does at 3; how

many leaps will the hound take to catch the hare? Ans. 300.

95. A. B. and C. can complete a piece of work in 15 days; A. can do it in 30 days, and B. in 40; in what time can C. perform it? Ans. 120 days.

96. A servant, having eloped from his master, travels 14 hours a day, at the rate of $3\frac{1}{2}$ miles an hour; at the end of two days a courier is sent in pursuit, who travels 9 hours in the day, at the rate of 7 miles an hour; in what time will he overtake him? Ans. 7 days.

97. A Greek epitaph, designed for the tomb of Diophantus, is said to have stated that he passed $\frac{1}{6}$ of his life in childhood, $\frac{1}{12}$ in adolescence; that after $\frac{1}{7}$ and 5 years more had been passed in a married state, he had a son who lived to $\frac{1}{2}$ his own age, and whom he survived 4 years. What then was the age of Diophantus? Ans. 84 yrs.

98. Three masons, A. B. and C. are to build a wall. Now A. and B. can build it in 12 days; A. and C. in 15; B. and C. in 20. In what time can they jointly effect it? and how long will they severally require?

Ans. Jointly, 10 days, A. 20, B. 30, C. 60.

99. A gentleman meeting, by accident, with a dangerous wound, sends for three physicians, promising to divide 100 dollars among them, in the ratio of the reciprocals of the number of minutes which shall elapse before they attend. The first arrives in 25 minutes, the second in 30, and the third in 35. Query, their respective shares?

Ans. The 1st, \$39,25; 2nd, \$32,71; 3rd, \$28,04.

100. A. B. and C. form a joint stock of £4392, by which they gain £234. A's money is in trade 4 months, B's 5 months, and C's 13 months; and their shares of the gain are as the numbers 2, 3, and 4, respectively. Required the gain and stock of each?

Ans. { A's gain £52, stock £1560.
B's gain £78, stock £1872.
C's gain £104, stock £960.

SOLUTIONS IN MISCELLANEOUS EXERCISES.

Ex. 14. Take the last number of apples, 8, and reverse the process, thus, $8\times4=32\times2=64-34=30+10=40$. Ans.

Ex. 15. $23+14=37$, age of the father when his wife died; $37-12=25$, his age when his daughter was born; and $25-5=20$, his age when his daughter's husband was born; then, $20+40=60$ years old at his death, Ans.

Ex. 20. $1+25=26$; then 26 : 25 :: £9 2s. : £8 15s. Ans.

Ex. 21. This and similar questions are usually worked by Position, but they may be easily solved on general principles. Thus, $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ of the army; therefore, 1000 is $\frac{5}{12}$ of the whole number of men; and if 5 twelfths be 1000, 12 twelfths is 2400 = the whole army; and $\frac{1}{3}$ of 2400 = 800 killed; and $\frac{1}{4}$ of 2400 = 600 taken prisoners.

Ex. 22. As 10 men :: 3 men : $4\frac{1}{2}$ hours to $1\frac{7}{20}$ Ans.

Ex. 23.

		yrs.	m.	w.	da.	h.	min.
He was to serve		14	0	0	0	0	0
of which he accomplished	yrs. 11, m. 11, w. 11, da. 11, h. 11, min. 11 =	12	1	0	4	11	11
	Ans.	1	11	3	2	12	49

Ex. 24. Suppose the weight of the less cup = 1oz., then $1+5=6$oz. = double of the greater cup; and $\frac{6}{2}+5=3+5=8$oz., which should have been 3oz. (8oz.—3oz.) = 5oz. = first error. Again, suppose the weight of the less cup = 2oz. then $2+5=7$ = double the greater; and $3\frac{1}{2}+5=8\frac{1}{2}$oz., which should have been 6oz. Then the second error is $2\frac{1}{2}$; here the errors are alike; then by the rule $(5\times2-1\times2\frac{1}{2})\div2\frac{1}{2}=(20-5)\div5=15\div5=3$oz. = the weight of the less cup; and $3+5\div2=8\div2=4$oz. = the weight of the greater cup.

Ex. 25. $\frac{1}{12}-\frac{1}{30}=\frac{1}{20}$ = the part the man drank in a day. Therefore 20 days are the number required.

Ex. 27. Since the reduced value of the 16 pieces is £8 8s. and the part taken from them is 16×2s. 6d. = £2, the original value was £10 8s.=208s. Consequently, 208÷16=13s. the original value of each.

Ex. 28. 3 feet 4 inches=40 inches, and 2 feet 6in. =30 inches; then as they would carry parts inversely as the distance, 40+30=70in. : 30in. : : 200lbs. : 85$\frac{5}{7}$lbs. Ans.; and 70in. : 40in. : : 200lbs. : 114$\frac{2}{7}$lbs. Ans.

Ex. 31. 12×11=132 days' work built 22 yards, and 480 yards remain; then 220 : 480 : : 132 : 288. But it is to be built in 18 days : hence 288÷18=16 men; that is it requires 16 men, or 4 must be added.

Ex. 35. The minute hand passes the hour hand 11 times in 12 hours, the fourth time between 4 and 5. Therefore as 11 : 4 : : 12 hours : 4 hours 21$\frac{9}{11}$ min.

Ex. 36. A. can mow an acre in $\frac{22}{3}$ hrs. or $\frac{3}{22}$ acres in 1 hour; B. can mow $\frac{5}{44}$ acre in 1 hour. Therefore $\frac{3}{22}$+$\frac{5}{44}$=$\frac{11}{44}$=number mowed by both in 1 hour. Whence, $\frac{44}{11}$hrs.=4 hours, the time required.

Ex. 37. 11+5+20=36 : 11 : : £3501 : £1069 15s. =Captain's share; and 36 : 5 : : £3501 : £486 5s.=the mate's share. Then, £1069 15s.+£486 5s.=£1556 and £3501—£1556=£1945÷20 sailors=£97 5s. each sailor's share.

Ex. 39. 1+3=4 : 3 : : £450 : £337 10s. A's share; then 4 : 1 : : £450 : £112 10s. B's share.

Ex. 40. £100—60=40 gain; then A's £40 : gain £60 : : £640 stock : £960, Ans.

Ex. 48. To find the area of a circle when the diameter is given we square the diameter and multiply the product by the decimal ,7854; hence, when the area is given to find the diameter we must reverse the process; 160 rods×30$\frac{1}{4}$ yards=4840 yrds; 4840÷,7854=6162,4649; then, $\sqrt{61624649}$=78,50, and 78,50÷2=39,25 Ans.

Ex. 49. 7×20=140oz. used by 1 man in a week, 140×12×1500=2520000 total ounces used; 7×8=56, what 1 man would use in 1 week by the second allowance : 56×20=1120, what he would use in 20 weeks. Then, 2520000÷1120=2250, Ans.

Ex. 50. As $\frac{7}{9}$: 1 : : \$8400 : \$10800, elder brother's part; then, $8400+10800=\$19200$, Ans.

Ex. 51. Three times as much in the same will require $20\times3=60$ men; and in $\frac{1}{5}$ of the time $60\times5=300$ Ans.

Ex. 52. $\frac{2}{5}$ of $\frac{4}{9}$ of $\frac{3}{16}=\frac{1}{30}$, then $\frac{3}{16}-\frac{1}{30}=\frac{37}{240}$. As $\frac{3}{16}:\frac{37}{240}$: : \$1200 : \$986,66, Ans.

Ex. 54. The saddle is 1 part, and horse 10 parts; hence, $1+10=11$ parts; then, $132\div11=12$, worth of the saddle; and $10\times12=120$ worth of the horse.

Ex. 55. $\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{12}=\frac{5}{8}$. Then $1-\frac{5}{8}=\frac{3}{8}$ the sheep in the fifth lot. Hence, $\frac{3}{8}=450$: or $\frac{1}{8}=150$ and the whole or $\frac{8}{8}=1200$ Ans.

Ex. 57. $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$, consequently $\frac{1}{12}$ remains, which is equal to $120+80=200$. Hence, $200\times12=2400$ Ans.

Ex. 59. 6ft. 6in.$=6,5\times6,5\times,7854=33,183150$ outer area. Then $3,5\times3,5\times,7854=9,621150$ inner area. Then $33,183150-9,621150=23,562\times40=942,480$.

Ex. 60. It will fill $\frac{1}{15}$ in an hour; they pump out $\frac{1}{16}$, hence the water gains $\frac{1}{15}-\frac{1}{16}=\frac{1}{240}$ of the ship per hour; hence it will fill in 240 hours.

Ex. 62. The mother was to have twice as much as the daughter and half as much as the son. Hence, the daughter 1 part, the mother 2, and son $4=7$ parts in all, then $5460\div7=\$780$ daughter's part, $780\times2=\$1560$ mother's, and $1560\times2=\$3120$ son's part.

Ex. 63. The first will empty in 1min. $\frac{1}{60}$ of it; second $\frac{1}{120}$ of it; third $\frac{1}{180}$ of it, together $\frac{1}{60}+\frac{1}{120}+\frac{1}{180}=\frac{11}{360}$ in 1 min. Then $\frac{11}{360}$: 1 : : 1min. : $32\frac{8}{11}$min. Ans.

Ex. 64. A. does in 1day $\frac{1}{10}$ of the work; B. $\frac{1}{13}$; together $\frac{1}{10}+\frac{1}{13}=\frac{23}{130}$; then $\frac{23}{130}$: 1 : : 1day : $5\frac{15}{23}$days Ans.

Ex. 65. $\frac{1}{16}+\frac{3}{8}+\frac{3}{10}+\frac{3}{20}$ will be equal to all the school except the 9 who read. Of the denominators of these fractions 80 is the least common multiple; hence $\frac{5}{80}+\frac{30}{80}+\frac{24}{80}+\frac{12}{80}=\frac{71}{80}$; and $1\frac{71}{80}=\frac{9}{80}$ the residue of the school, which is$=9$; then if $\frac{9}{80}$ of a school is equal to 9, how many in the school? It is plain there are 80; then $\frac{1}{16}$ of $80=5$; $\frac{3}{8}$ of $80=30$; $\frac{3}{10}$ of $80=24$; $\frac{3}{20}$ of $80=12$.

Ex. 66. 11s.$\times\frac{3}{8}$=what he first cleared on each thousand; hence, they cost him $\frac{5}{8}$ of 11s.=6s. 10$\frac{1}{2}$d. He afterwards sold for 13s. 6d.; then 13s. 6d.—6s. 10$\frac{1}{2}$d.=6s. 7$\frac{1}{2}$d. what he cleared per thousand by the latter price.—Then, as 6s. 10$\frac{1}{2}$d. : : 6s. 7$\frac{1}{2}$d. : £100 : £96 7s. 3$\frac{3}{11}$d. Ans.

Ex. 67. In 1min. the pipe brings $\frac{14}{9}$ gal. The tap discharges in the same time $\frac{40}{31}$ gal.; hence it fills in 1 min. $\frac{14}{9}-\frac{40}{31}=\frac{74}{279}$gal. It so runs for 3hrs.=180min.; hence, it fills in that time $\frac{74}{279}\times180=\frac{1480}{31}$gal. It has then 147 $-\frac{1480}{31}=\frac{3077}{31}$gal.; then, as 14 : $\frac{3077}{31}$: : 9min. : $\frac{27693}{434}$ min=63min. 48$\frac{114}{217}$sec., that is, after 5 o'clock it fills in 63 min. 48$\frac{114}{117}$sec.; that is at 6 o'clock 3 min. 48$\frac{114}{217}$sec.

Ex. 69. All can do in 1 day, $\frac{1}{5}+\frac{1}{8}+\frac{1}{10}=\frac{17}{40}$. Then, $\frac{17}{40}$: 1 : : 1da. : 2$\frac{6}{17}$ day, Ans.

Ex. 70. $1-\frac{1}{4}=\frac{3}{4}$; $\frac{1}{5}$ of $\frac{3}{4}=\frac{3}{20}$; then, $\frac{1}{4}+\frac{1}{20}=\frac{8}{20}$, what was spent, and $1-\frac{8}{20}=\frac{12}{20}$, what was left; then, as $\frac{12}{20}$: $\frac{20}{20}$: : 72 : 120 Ans.

Ex. 71. A. goes 11 yards per minute, or 33 yards in 3min, while B. goes 34 yards; hence B. gains 1 yd. in 3 min.; and to gain $\frac{536}{2}$=268yd. he must travel 268$\times$3= 804 minutes. Then, 3 : 804 : : 34 : 9112 distance travelled; then, 9112÷536=17 times round.

Ex. 72. 13$\frac{4}{5}$hr.=$\frac{69}{5}$hr.; 365$\frac{1}{4}$da.=$\frac{1461}{4}$; then $\frac{69}{5}\times\frac{1461}{4}\times\frac{7}{1}=\frac{705663}{20}$ hours. As $\frac{1}{6}$: $\frac{705663}{20}$: : $\frac{3}{4}$ min. : $\frac{6350967}{40}$=2646$\frac{189}{800}$hr.

Ex 73. $\frac{7}{7}-\frac{2}{7}=\frac{5}{7}$; then $\frac{12}{35}$ of $\frac{5}{7}=\frac{12}{49}$; also, $\frac{2}{7}-\frac{12}{49}=\frac{2}{49}$. Then, as $\frac{2}{49}$: $\frac{2}{7}$: : 200 : 1400—200=1200.

Ex. 74. As 5$\frac{2}{3}$hr. : 9$\frac{1}{3}$hr. : : 1A. : $\frac{28}{17}$A. quantity A. would mow in 9$\frac{1}{3}$hr.: then $\frac{28}{17}+\frac{7}{4}=\frac{231}{68}$ acres, quantity A. and B. together would mow in 9$\frac{1}{3}$hours. As $\frac{231}{68}$: $\frac{33}{4}$: : $\frac{28}{3}$: $\frac{68}{3}$=22$\frac{2}{3}$ hours Ans.

Ex. 75. $\frac{129}{2}\times\frac{5}{43}=\frac{15}{2}$; then, as $\frac{43}{57}$: $\frac{15}{2}$: : $\frac{1}{1}$: 9$\frac{81}{86}$lb.

Ex. 76. As 15$\frac{3}{8}$ft. : 25$\frac{3}{4}$ft. : : 6$\frac{1}{2}$ tons : 10$\frac{109}{123}$ Ans.

Ex. 77. The velocity of sound is 1142ft. per second, the distance passed over in 1min. would be 1142$\times$60= 68520 feet. But in this time the pulse beats 75 times. Hence, 75 puls. : 9 puls. : : 68520ft. : 8222ft.=1mi. 4fur. 100yds. 2ft.

Ex. 78. Because 40 seconds is just $\frac{1}{90}$ of an hour, and the hare runs 17600 yards per hour. She must run $17600 \div 90 = 195\frac{5}{9}$yds. in 40sec., and would be $195\frac{5}{9} + 40 = 235\frac{5}{9}$yds. before the dog at his starting; but as the dog gained 8 miles in 18, he would gain 8 yards in 18, or 4 in 9; hence, $4 : 9 :: 235\frac{5}{9} : 530$yds. run by the dog. And $1760 \times 18 = 31680 : 530 :: 60$min. : 1min. $0\frac{5}{22}$sec.

Ex. 79. The meaning of this question is, that the number 9 added to once his age, together with $\frac{1}{2}$ and $\frac{1}{3}$ of his age, the sum shall be 130; or since the sum of the parts 1 and $\frac{1}{2}$ and $\frac{1}{3}$ is $\frac{11}{6}$, that of his age is $130 - 9 = 121$; then $11 : 6 :: 121 : 66$, Ans. Or by Position.

Ex. 80. As $7 : 5 :: 3 : 2\frac{1}{7}$, and $9 + 5 + 2\frac{1}{7} = 16\frac{1}{7}$; then, as $2\frac{1}{7} : 16\frac{1}{7} ::$ £2584 : £19466 2s. 8d. Ans.

Ex. 81. Suppose 200; then $200 \times 6 \div 100 = 12$ interest; then $\frac{200}{20} = 10$, and $12 - 10 = 2$; then $100 - 2 = 98$, error. Again, suppose 300; then $300 \times 6 \div 100 = 18$ interest; then $\frac{300}{20} = 15$, and $18 - 15 = 3$; then $100 - 3 = 97$, error. Then $97 \times 200 = 19400$; and $98 \times 300 = 29400$; then $29400 - 19400 = 10000$ pounds, Ans.

Ex. 82. A. and B. can do $\frac{1}{4}$, B. and C. $\frac{1}{6}$, and A. and C. $\frac{1}{5}$ per day; then $\frac{1}{4} + \frac{1}{6} + \frac{1}{5} = \frac{37}{60}$, which $\div 2$ (because each man, by the conditions, is taken twice) $= \frac{37}{120}$ what all would do in 1 day; then $1 \div \frac{37}{120} = 3\frac{9}{37}$ days, Ans.

Ex. 83. A. and B. together can do $\frac{1}{5}$, and A. can do $\frac{1}{7}$ alone; then $\frac{1}{7} - \frac{1}{5} = \frac{2}{35}$ what B. can do in a day; then $1 \div \frac{2}{35} = 17\frac{1}{2}$ days, Ans.

Ex. 84. Amount of £1 for (21—18) 3yrs. = £1,18.)
Amount of £1 for (21—14) 7yrs. = £1,42.)
= £2,60.

Then (as they will receive inversely as the time.)
As £2,60 : £1,42 : : £1000 : £546,153 Ans.
As £2,60 : £1,18 : : £1000 : £453,846, Ans.

Ex. 85. B. has 3, and C. 7 more than A.; $7 + 3 = 10$, to be taken from $100 = 90 \div 3 = 30$ A's; then, $30 + 3 = 33$. B's, and $30 + 7 = 37$ C's share, Ans.

Ex. 86. 3 men 5 days = 15 men 1 day, and 7 men 3 days = 21 men 1 day; $15 + 21 = 36$ days; $\frac{15}{36} = \frac{5}{12}$, A.;

$\frac{21}{36}=\frac{7}{12}$, B; then $\$54\times\frac{5}{12}=\$22,50$, A's money, and $\$54\times\frac{7}{12}=\$31,50$, B's money, Ans.

Ex. 87. First, to find the gain of A. and B.; C's gain being \$120, \$332,50—120=212,50, the gain of A. and B. together; then, \$850 : \$500 : : \$212,50 : \$125, A's; and \$850 : \$350 : : \$212,50 : \$87,50, B's. To find the price of C's cloth 500+350=850; then as 212,50 : 120 : : 850 to C's stock, \$480÷120=\$4 per yard.

Ex. 88. $\frac{3}{4}+\frac{9}{10}+\frac{13}{20}=\frac{46}{20}\div2$, (as each man's horses are taken twice in the question)$=\frac{23}{20}$; then, $\frac{23}{20}-\frac{3}{4}$ (A. and B.)$=\frac{8}{20}$, C's; $\frac{23}{20}-\frac{9}{10}$ (A's and C's)$=\frac{5}{20}$, B's; and $\frac{23}{20}-\frac{13}{20}$ (B's and C's)$=\frac{10}{20}$, A's; then A. will have $\frac{10}{23}$ of \$26,45=\$11,50; B. will have $\frac{5}{23}$=\$5,75, and C. $\frac{8}{23}$ =\$9,20.

Ex. 89. $\frac{3}{4}$ of $\frac{5}{6}=\frac{5}{8}$; then, £537 is $\frac{8}{8}-\frac{5}{8}=\frac{3}{8}$, and £537$\div\frac{3}{8}$=£1432, the sum he had after he had spent $\frac{5}{16}$ of his fortune, and consequently this must be $\frac{11}{16}$ of what he had at first; then, £1432$\div\frac{11}{16}$=£2082 18s. $2\frac{2}{11}$d. Ans.

Ex. 92. As the goods, when sold at auction, sell at 15 per cent. below their just value, a quantity worth \$100 must be sold for \$85. Therefore, as \$85 : \$3400 : : \$100 : \$4000, the value of the goods which must be sold at auction to pay the debt. Debt \$3400, premium at 9 per cent. \$306; amount of debt and premium \$3706. Interest on this sum for 1 year \$222,36; amount \$3928,36, which is less than \$4000 by \$71,64, Ans.

Ex. 93. A. travels $\frac{7}{2}$ miles, and B. $\frac{8}{3}$ miles in 1 hour; $\frac{7}{3}+\frac{8}{3}=\frac{37}{6}$ miles distance travelled by both in 1 hour; 59 $-\frac{7}{2}=\frac{111}{2}$ mi.=the distance they are apart when B. sets out. $\frac{111}{2}\times\frac{6}{37}=9$, number of hours B. was travelling; then $\frac{7}{2}\times\frac{10}{1}=35$ miles A. travelled; $\frac{8}{3}\times9=24$ miles B. travelled.

Ex. 94. As 2 : 3 : : 3 : $4\frac{1}{2}$=number of the hare's leaps, which are equal 3 of the hound. But the hare takes but 4 leaps while the hound takes 3. Therefore, the hound by taking 3 leaps gains $\frac{1}{2}$ leap on the hare; he must consequently take 300 to gain 50.

Ex. 95. $\frac{1}{15}-\frac{1}{30}-\frac{1}{40}=\frac{1}{120}$=part C. does in 1 day; he will therefore do the whole in 120 days.

Ex. 97. $\frac{1}{6}+\frac{1}{12}+\frac{1}{7}+\frac{1}{2}=\frac{75}{84}$; $\frac{84}{84}-\frac{75}{84}=\frac{9}{84}$=part of his life which the 5+4 years composed; therefore, $\frac{9}{84}$: 9 : : 1 : 84 years, Ans.

Ex. 98. The part done in a day by A. and B. is $\frac{1}{12}$, by A. and C. $\frac{1}{15}$; and by B. and C. $\frac{1}{20}$. The sum of these fractions, $\frac{1}{5}$, including the part of each twice, is evidently twice the part performed in a day by A. B. and C. together. They therefore perform $\frac{1}{10}$ part of the work in a day, or the whole in 10 days. Again, $\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$ part A. can perform in a day; $\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$ part B. does in a day; $\frac{1}{10}-\frac{1}{12}=\frac{1}{60}$ part C. does in a day.

Ex. 99. The reciprocal of a number is the number inverted; hence, $\frac{1}{25}+\frac{1}{30}+\frac{1}{35}=\frac{107}{1050}$; then, as $\frac{107}{1050}:\frac{1}{25}$: : \$100 : \$39,25 and as $\frac{107}{1050}:\frac{1}{30}$: : \$100 : \$32,71, also, $\frac{107}{1050}:\frac{1}{35}$: : \$100 : \$28,04. Ans.

Ex. 100. 2+3+4=9, $\frac{2}{9}$ of $\frac{234}{1}$=£52 A's gain; $\frac{3}{9}$ of $\frac{234}{1}$=£78 B's gain; $\frac{4}{9}$ of $\frac{234}{1}$=£104 C's gain. Now $\frac{52}{4}$=13; $\frac{78}{5}$=15,6; $\frac{104}{13}$=8: and 13+15,6+8=36,6.

$$\text{As } 36{,}6 : \left\{\begin{array}{c}13\\15{,}6\\8\end{array}\right\} : : \begin{array}{c}£\\4392\end{array} : \left\{\begin{array}{l}£1560 \text{ A's stock.}\\£1872 \text{ B's do.}\\£\ 960 \text{ C's do.}\end{array}\right.$$

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PART IV.