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SOME NOTES ON A METHOD OF OBTAINING TIME AND LATITUDE.

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Some time ago the writer observed an article by W. E. Cooke, Government Astronomer of Western Australia, in the *Monthly Notices* of the Royal Astronomical Society, on a novel method of obtaining Time, Latitude, and Azimuth with an ordinary transit theodolite. The principle of the method was not original, being first advocated by Prof. Chandler, of Harvard University, but Mr. Cooke had adapted it to the ordinary field transit. He gave the results of some three or four nights' observing with an ordinary five-inch transit theodolite, and the writer was much struck by the remarkably close agreement of the different nights' results, made notes on the article, and determined to try the method when opportunity offered.

Principle of the Method.

If the telescope of any transit is clamped at an altitude equal to the observer's approximate latitude, and the instrument revolved in azimuth, the line or sight describes a small circle in the heavens, called by Prof. Chandler the Almucantur or co-latitude circle. The observation for time and latitude by this method consists in observing the time of transit of stars in crossing this circle.

Any ordinary field transit which has a level attached to the vernier arm of the vertical circle is suitable. Let this level be called the "latitude level," and the more sensitive it is the better. All instruments fitted this way have also two slow-motion screws to the telescope clamp. One, which we will call "X," moves telescope only, the other, "Y," moves telescope and bubble. A chronometer or good watch is also essential for this method.

The accuracy of the method is dependent upon the sensitiveness of the latitude level, the uniformity of rate of the chronometer or watch, and the personal equation of the observer, or, if the observer does not take the time himself, of the recorder also. On account of some stars crossing the horizontal wire very obliquely, this personal equation may be somewhat large.

OBSERVING LIST.

Let $\phi_0 =$ the observer's assumed latitude, always positive.

$\Delta\phi =$ the required correction to the assumed latitude.

$\alpha =$ the star's right ascension.

$\delta =$ the star's declination, positive when north.

$t =$ the star's hour angle, negative when east.

$\theta, \theta' =$ the sidereal time of transit across the co-latitude circle, east and west respectively.

$A =$ the star's azimuth (counting from the north) at transit across the co-latitude circle.

$T, T' =$ the observed time of transit over the horizontal wire reduced to sidereal time, east and west respectively.

$Tc, Tc' =$ the observed time of transit in sidereal time, corrected for level, watch rate, and diurnal aberration.

$\Delta Tc, \Delta Tc' =$ the corresponding watch error.

$d =$ the value of one division of the level in seconds of arc.

$R =$ the correction to watch for rate.

Then
$$\theta = \alpha + t \tag{1}$$
$$\cos t = \tan \phi_0 \cdot \tan (45^\circ - \tfrac{1}{2}\delta) \tag{2}$$
$$\cot \frac{A}{2} = \sin \phi_0 \cdot \tan t \tag{3}$$

The first step is to prepare an observing list. Decide between what hours the observation is to take place. Two series of stars are required for the observation.

(*a*) The time stars, or "Prime Vertical" stars as Mr. Cooke calls them, and these must cross the co-latitude circle within about 20° N. or S. of the prime vertical, i.e., their azimuth must lie between 70° and 110°, or 250° and 290°.

(*b*) The latitude stars, and the hour angle of these must not be greater than two and one-half hours, or their polar distance must be between, say, $180°-2\phi_0$ and $170°-2\phi_0$.

The Time Stars.—Starting with the two extreme values of A from (3) we find t must lie between two certain values. And from (2) we get values of δ for each of these two extreme values. From (1) we can now find the extremes of right ascension permissible for the two extremes of declination, both for east and for west stars.

The Latitude Stars. — We have δ and t at once from (b) and a is readily obtainable.

With these data, an inspection of the *British Nautical Almanac*, or *American Ephemeris* will give the stars. With any ordinary transit, stars up to the fourth magnitude should be observed without any difficulty, and in a dry atmosphere, or high altitude, the list may be extended to take stars up to the fifth magnitude.

It remains to calculate by (1), (2), and (3) the θ and A of each of the stars selected. Five-figure logarithms are sufficient.

The Observation.

1. Set up and level the instrument very carefully. The sun shade will be found a convenience in preventing the dew from gathering on the object glass. Set the telescope at an altitude equal to ϕ_0 and clamp firmly. Then by means of screw "Y" bring the latitude level to the middle of its run. Since this level is much more sensitive than the plate levels, it is as well now to revolve the instrument in azimuth over the parts of the circle which will be used for the observation and see if the level remains fairly steady in the middle. If not, relevel until it does, as it is desirable to make the level correction as small as possible. Shortly before the first star is expected, set for it in azimuth and clamp. If, when the star is seen in the field, it appears that it will not cross the horizontal wire at the computed time, as a matter of convenience, the altitude may be altered by screw "X" to make it do so, but after the first star, screw "X" must on no account be touched at all until the whole observation is complete. The reason for the star not crossing the horizontal wire at the computed time may be an inaccurate telescope setting or a wrong assumption of the watch error, but this will not affect the accuracy of the results. If the star appears to be moving in such a manner that it will not pass near the intersection of the wires, the azimuth slow-motion screw may be used at any time.
2. Observe the times of transit across the horizontal wire, moving the azimuth screw so that the star crosses near, but not quite at the centre.
3. Read the latitude level as quickly as possible, after each transit, being careful to keep the warm breath or heat from the reading lamp away from the level.

Computation.

By means of the bubble we have observed the transit of a number of stars over the horizontal wire at some one definite zenith distance, the accuracy of which depends upon the sensitiveness of the bubble.

From the prime vertical stars, we shall first find the quantity

z whereby the circle actually traced by the instrument differs from the true co-latitude circle, and secondly the watch error.

From the latitude stars combined with z just obtained we shall compute our error in assumed latitude:

1. Reduce the observed times of transit to sidereal times T.
2. The sidereal time of transit T must now be corrected for

(*a*) level,
(*b*) watch rate,
(*c*) diurnal aberration, if the observation is considered to have been taken with sufficient accuracy to warrant it.

Correction for Level. — The correction for level in seconds of time $= E\varepsilon$, (4)

$$\text{Where } E = \frac{1}{15 \,.\, \cos \phi_0 \,.\, \sin A} \tag{5}$$

$$\text{and } \varepsilon = \frac{w - e}{2} .d \text{ for all Time stars} \tag{6}$$

$$\varepsilon = \frac{n - s}{2} .d \text{ for Latitude stars east of south} \tag{6'}$$

$$\varepsilon = \frac{s - n}{2} .d \text{ for Latitude stars west of south} \tag{6''}$$

if the level divisions are numbered in both directions from the middle.

If the level divisions are numbered continuously from one end of the level to the other, with the zero at the eye end, the level readings must all be referred to some one reading c_0, the centre of the bubble, and the formula then becomes

$$\varepsilon = \left(c_0 - \frac{w + e}{2}\right) .d \text{ for } E \text{ Time stars} \tag{7}$$

$$\varepsilon = \left(\frac{w + e}{2} - c_0\right) .d \text{ for } W \text{ Time stars} \tag{7'}$$

$$\varepsilon = \left(c_0 - \frac{n + s}{2}\right) .d \text{ for Latitude stars east of south} \tag{7''}$$

$$\varepsilon = \left(\frac{n + s}{2} - c_0\right) .d \text{ for Latitude stars west of south} \tag{7'''}$$

Three-figure logarithms are sufficient.

Correction for Watch Rate.—If the watch rate is not zero the chronometer correction changes during the progress of the observation. To reduce each observed time of transit across the horizontal wire to what it would have been if the rate had been zero (and the correction equal to the correction which actually existed at the mean epoch of the set) apply the correction

$$R = (T - T_0) r_h \tag{8}$$

in which T_0 is the mean of the observed times T and r_h is the hourly rate of the watch, + when losing — if gaining $(T - T_0)$ being expressed in hours.

Correction for Diurnal Aberration.—This correction is very small, and unless the observation is taken in conjunction with a chronometer and chronograph had better be neglected. The formula is:

The correction for diurnal aberration, $k = 0\overset{s}{.}0207 \sin \varphi_0$ (9)
This correction is to be subtracted from the observed time of transit.

3. Separate the Time stars from the Latitude stars, and arrange each in pairs east and west, pairing according to their respective hour angles t.

4. Compute rigorously and tabulate for each star, the value of θ by (1) and (2), using seven-figure logarithms and taking angles out to seconds of arc.

5. Compute also and tabulate $Z = \dfrac{1}{\sin t \,.\, \cos \delta}$ (10)

+ for west and — for east stars

$$L = \frac{1}{15 \,.\, \sin 2\varphi_0} \,.\, \cot. t \qquad (11)$$

same sign as Z.

Four-figure logarithms are sufficient in both cases.

6. Form the quantity $(\theta - Tc)$ for each star. If the watch is fast this quantity will probably be negative, and if slow, positive.

7. Distinguish all western observations with an accent, thus $(\theta' - Tc')$, Z', L'.

Computation for Time.

8. Tabulate for each pair of Time stars

$$z = \frac{(\theta' - Tc') - (\theta - Tc)}{Z' - Z} \qquad (12)$$

Be careful about the sign of z, as the correction to the latitude depends upon this.

9. The watch error ΔTc as given by each pair

$$= \frac{\{(\theta' - Tc') - Z'z\} + \{(\theta - Tc) - Zz\}}{2} \qquad (13)$$

ΔTc_0 is the mean of the ΔTc for all the pairs.

Computation for Latitude.

10. Find the mean z_0 of all the z's in the Time stars.

11. If Z_0, Z'_0, L_0, L'_0, represent the mean of all the Z's, Z''s, L's, L''s in the Time stars.

Then
$$\left.\begin{aligned} Z_1 &= \frac{Z'_0 - Z_0}{2} \\ L_1 &= \frac{L'_0 - L_0}{2} \end{aligned}\right\} \text{ for Time stars} \qquad (14)$$

12. Compute and tabulate for each pair of Latitude stars

$$a = (Z' - Z)\, z_{\circ} \qquad (15)$$

$$b = (\theta' - Tc') - (\theta - Tc) \qquad (16)$$

$$c = (L^{\varphi'} - L^{\varphi}) - \frac{L_1}{Z_1} \cdot (Z^{\varphi'} - Z^{\varphi}) \qquad (17)$$

13. The correction to latitude or $\Delta\varphi = \frac{b - a}{c}$ (18)

and will be in seconds of arc.

Results.

The writer had an opportunity to try this method in 1906, when engaged in making a survey of the harbour of Prince Rupert under instructions from W. J. Stewart, Esq., Chief Hydrographer, Department of Marine and Fisheries. Directions were given to observe for latitude at some suitable spot and tie on to the triangulation. The Dept. provided an eight-inch Troughton & Simms alt-azimuth with micrometer microscopes for both circles reading to ten seconds, and by estimation to say 2.5 seconds. The telescope was of about thirteen inches focal length, with an inch and three-quarter objective. The diaphragm had five vertical and three horizontal spider webs. One division of the latitude level (0.05 of an inch) was equal to 5 seconds. The level was not a chambered one, and was mounted as in the ordinary field transit, in Plaster of Paris. The striding level was similar to the latitude level. A sidereal watch was also provided.

There were, therefore, three methods available for latitude determination, namely, the usual method of transits of north and south stars across the meridian, Bessel's prime vertical, and Mr. Cooke's method, and the writer determined to try this last one.

The approximate latitude was obtained by a couple of north and south stars in the usual way. The observation covered a period of nine hours. The method of taking the time of transit was the crudest. The "tip" at the instant of transit was given to the recorder, who read the face of the watch by eye. In this way the errors of judgment of two men instead of one enter into the result. Transits were observed across the three horizontal wires, and the mean of the three taken as the observed time of transit T. Ten pairs of Time stars and fifteen pairs of Latitude stars were taken. The probable error of the mean of the Latitude pairs was ± 0".34.

Later, an effort was made to verify the accuracy of the result by Bessel's prime vertical method. But fine nights that season were few and far between, and on both nights the work of verifying was stopped by clouds.

A determination at Port Simpson, in 1895, by the U. S. Coast and Geodetic Survey, the result of 32 pairs of stars with one of their large zenith telescopes, and reduced through the triangulation of Chatham Sound, was 0".26 greater than the writer's value.

Remarks.

The writer does not believe that with one night's observing he could have obtained as good results with the same instrument by any other method, nor with any other instrument equally portable, which had stood an equal amount of rough usage and knocking about. On the other hand, the calculations, though quite simple, are certainly found long and tiresome. A little good judgment on the part of the computer will materially reduce the time and labour of the calculations. As in nearly all calculations, a ruled form is of great assistance, saving time and mistakes.

A copy (reduced) of the form used is appended. The corresponding calculations for the different stars should be carried along together. All calculations may be made with the slide rule except in the case of finding the hour angle t by (2), where seven-figure logarithms must be used. The writer used logarithms throughout, and checked everything, except the above-mentioned seven-figure logarithmic calculation, with the slide rule.

Mr. Cooke, in his paper, does not apply any level correction with the smaller instruments, but by means of screw "Y" brings the latitude bubble to exactly the same position just prior to each observed transit.

The writer is of the opinion that very fair results might be obtained with the ordinary transit which has the level attached to the telescope, by levelling the instrument very carefully at the beginning of the observation with this level, and handling the instrument very lightly throughout the observation. To test this, the observation mentioned was calculated without applying any correction for level or watch rate. The result differed by 1".3 from the true observed latitude. The writer is therefore probably justified in considering that, using this method, an ordinary six-inch railway transit in good condition would give results about as good as the above 1".3, and that by no other method with such an instrument could an equally good determination be obtained.

For all reconnaissance work, this method might no doubt be used to advantage. It entails a good deal more work than the usual *Polaris* latitude used by so many on railway reconnaisance, but the writer thinks the results would justify the extra work incurred. The observation is quite simple, and the computation can be left to some convenient time.

Mr. Cooke also recommends the same method for azimuth determinations, and gives formulae for the same, but as it was thought that for simplicity and accuracy the method of circum-polars can hardly be excelled, his method was not tried.

LATITUDE OBSERVATION RECORD

PRIME VERTICAL STARS

Place .... Instrument ....

Date .... One division of level ....

Observer .... Computer ....

Star	E or W	Wire 1	Wire 2	Wire 3	T	Level E & S	Level W & N	Level Mean	E	Level Corr. E_t	R	Tc	θ	$\theta - Tc$	$\theta' - T'c$	Z −	Z' +	L −	L' +	Z_θ & Z'_θ	Z_θ Z'_θ $(\theta - Tc)$ $(\theta' - T'c)$	ΔTc	Remarks

LATITUDE OBSERVATION RECORD

LATITUDE STARS

Place .... Instrument ....

Date .... One division of level ....

Observer .... Computer ....

Star	E or W	Wire 1	Wire 2	Wire 3	T	Level E & S	Level W & N	Level Mean	E	Level Corr. E_t	R	Tc	θ	$\theta - Tc$	$\theta' - T'c$	$Z' + Z$ −	Z'	Z	$L' + L$ −	L'	L	a	b	$b\ a$	c	$\Delta\phi$	v	v^2