IMAGE EVALUATION
TEST TARGET (MT-3)

1.0 1.1 1.25 1.4 1.6 1.8 2.0 2.2 2.5 2.8 3.2 3.6 4.0 4.5 5.0 5.6 6.3 7.1 8.0

6"

Photographic Sciences Corporation

23 WEST MAIN STREET
WEBSTER, N.Y. 14580
(716) 872-4503

Technical and Bibliographic Notes/Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

- [ ] Coloured covers/ Couverture de couleur
- [ ] Covers damaged/ Couverture endommagée
- [ ] Covers restored and/or laminated/ Couverture restaurée et/ou pelliculée
- [ ] Cover title missing/ Le titre de couverture manque
- [ ] Coloured maps/ Cartes géographiques en couleur
- [ ] Coloured ink (i.e. other than blue or black)/ Encre de couleur (i.e. autre que bleue ou noire)
- [ ] Coloured plates and/or illustrations/ Planches et/ou illustrations en couleur
- [ ] Bound with other material/ Relié avec d'autres documents
- [ ] Tight binding may cause shadows or distortion along interior margin/ La reliure serrée peut causer de l'ombre ou de la distortion le long de la marge intérieure
- [ ] Blank leaves added during restoration may appear within the text. Whenever possible, these have been omitted from filming/ Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.
- [ ] Additional comments:/ Commentaires supplémentaires:

L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

- [ ] Coloured pages/ Pages de couleur
- [ ] Pages damaged/ Pages endommagées
- [ ] Pages restored and/or laminated/ Pages restaurées et/ou pelliculées
- [x] Pages discoloured, stained or foxed/ Pages décolorées, tachetées ou piquées
- [ ] Pages detached/ Pages détachées
- [x] Showthrough/ Transparence
- [ ] Quality of print varies/ Qualité inégale de l'impression
- [ ] Includes supplementary material/ Comprend du matériel supplémentaire
- [ ] Only edition available/ Seule édition disponible
- [ ] Pages wholly or partially obscured by errata slips, tissues, etc., have been refilmed to ensure the best possible image/ Les pages totalement ou partiellement obscurcies par un feuillet d'errata, une pelure, etc., ont été filmées à nouveau de façon à obtenir la meilleure image possible.

This item is filmed at the reduction ratio checked below/
Ce document est filmé au taux de réduction indiqué ci-dessous.

10X	12X	14X	16X	18X	20X	22X	24X	26X	28X	30X	32X
			✓								

The copy filmed here has been reproduced thanks to the generosity of:

National Library of Canada

The images appearing here are the best quality possible considering the condition and legibility of the original copy and in keeping with the filming contract specifications.

Original copies in printed paper covers are filmed beginning with the front cover and ending on the last page with a printed or illustrated impression, or the back cover when appropriate. All other original copies are filmed beginning on the first page with a printed or illustrated impression, and ending on the last page with a printed or illustrated impression.

The last recorded frame on each microfiche shall contain the symbol ―▶ (meaning "CONTINUED"), or the symbol ▼ (meaning "END"), whichever applies.

Maps, plates, charts, etc., may be filmed at different reduction ratios. Those too large to be entirely included in one exposure are filmed beginning in the upper left hand corner, left to right and top to bottom, as many frames as required. The following diagrams illustrate the method:

L'exemplaire filmé fut reproduit grâce à la générosité de:

Bibliothèque nationale du Canada

Les images suivantes ont été reproduites avec le plus grand soin, compte tenu de la condition et de la netteté de l'exemplaire filmé, et en conformité avec les conditions du contrat de filmage.

Les exemplaires originaux dont la couverture en papier est imprimée sont filmés en commençant par le premier plat et en terminant soit par la dernière page qui comporte une empreinte d'impression ou d'illustration, soit par le second plat, selon le cas. Tous les autres exemplaires originaux sont filmés en commençant par la première page qui comporte une empreinte d'impression ou d'illustration et en terminant par la dernière page qui comporte une telle empreinte.

Un des symboles suivants apparaîtra sur la dernière image de chaque microfiche, selon le cas: le symbole ―▶ signifie "A SUIVRE", le symbole ▼ signifie "FIN".

Les cartes, planches, tableaux, etc., peuvent être filmés à des taux de réduction différents. Lorsque le document est trop grand pour être reproduit en un seul cliché, il est filmé à partir de l'angle supérieur gauche, de gauche à droite, et de haut en bas, en prenant le nombre d'images nécessaire. Les diagrammes suivants illustrent la méthode.

1	2	3

1
2
3

1	2	3
4	5	6

NEW

A

S

Te

THIS

T

NEW EDITION. **REVISED AND ENLARGED.**

THE

NOVA SCOTIA

ARITHMETIC:

PREPARED AND DESIGNED FOR

SCHOOLS AND ACADEMIES,

FULLY EXPLAINING

THE PRINCIPLES OF THE SCIENCE.

BY

W. R. MULLHOLLAND,

Teacher of Mathematics, &c., in the Provincial Normal School, Truro, N. S.

THIS BOOK IS A COMPLETE SYSTEM OF ARITHMETIC. IT CONTAINS ALL THE NECESSARY RULES RELATING TO THE DECIMAL CURRENCY, WITH NUMEROUS EXAMPLES.

Authorized by the Council of Public Instruction for Nova Scotia.

A. & W. MACKINLAY,
PUBLISHERS,
HALIFAX, N. S.
1864.

PREFACE.

There are only two reasons that warrant, in our estimation, the publication of another Arithmetic. There is, first, its own intrinsic merit, which may arise either from the exposition of some new principle or principles, or from some new method of arrangement or illustration. Then, there is its suitableness to local circumstances, it may be, to some peculiarity in the educational or financial condition of those for whom it is mainly intended.

The title of our book sufficiently indicates the grounds of its appearance,— as to which of the above causes it owes its existence.

For a number of years, the educational authorities in the Province have been making an effort to introduce Arithmetic into the common schools at a much earlier period than heretofore. If some children are signalized for their observational powers, even from their youngest years, there are others equally so for their calculating or arithmetical powers; and hence it has been argued, and we think with great cogency, that it is as incumbent on the Teacher to use every legitimate means for the culture of the latter as it is for that of the former. But to teach arithmetic to the young when they enter school, in adaptation to, and for the development of, their intellectual endowment, requires special treatment, — even the teaching of their understanding through the medium of their perceptive faculties; in other words, arithmetic must be taught concretely, not abstractly. Hence the cause of the division of the following treatise into two parts — the first being intended for beginners of five or six years of age, and the second for those who, having gone through that course, have reached their eighth or ninth year.

Again, the value of mental arithmetic is now universally admitted, and is being introduced into all schools of note from the primary to the more advanced. It is of great practical utility to persons engaged in business, but it is of still greater benefit for disciplining the mind, exercising and unfolding at once the powers of attention, memory, reasoning and abstraction. It ought to precede the use of the slate, accompany it at every stage, and also succeed it. It holds a similar place to Arithmetic on the slate that Mental Composition does to that on paper. Hence, to save the expense of a separate treatise on Mental Arithmetic, special exercises in this department have been given throughout the work.

But the most important local circumstance that seems to demand a new Arithmetic, is the recent introduction of the decimal currency into our Province. When the Bill that legalized this currency had passed the Legislature, it was considered advisable in the Normal and Model Schools to work the arithmetical exercises both ways; first, according to the pounds, shillings, and pence system, and then the decimal, and thereby still to make use of the old arithmetical books. This mode of introducing the decimal currency into our schools is making but slow progress, and thus has rendered it expedient to try some other means more likely to effectuate the object; and this has been the issuing of the following work.

These are the principal causes that have led to the publication of the *Nova Scotia Arithmetic*. As to the plan pursued, or the way in which the work is done, it does not become us to speak. We may, however, say a word or two on a few of its more prominent features.

It will be seen that an unusually large space has been devoted to the discussion of what is generally termed the fundamental rules, viz.: Addition, Subtraction, Multiplication and Division. Under Multiplication, a considerable number of contracted rules has been given, which, it is hoped, will be found not only a pleasing feature of the work, but well adapted for mental culture.

Mental exercises follow the exposition of each rule. These will not only make the pupil familiar with the principle involved, but enable the teacher readily to see whether the pupil has thoroughly understood its application.

Decimals, to a limited extent, have been introduced immediately after the fundamental rules. There is no reason why a child may not work decimals up to a certain stage, as well as integral numbers. And the sooner he becomes acquainted with these the better, now that our currency is based on this form of notation.

The more important Tables of Weights and Measures, &c., are presented in a collected form, at the end of Part First. In Part Second, they are treated separately, and at considerable length, in their origin, history and application; and this on the principle that the pupil ought to be thoroughly drilled in one thing till he is completely master of it, before he proceeds to another. To ensure this thoroughness to a greater degree, miscellaneous exercises have been appended to every portion of the work.

It will be observed that Practice is taken up before Proportion, as it is more applicable to ordinary mercantile transactions.

In the discussion of Proportion, we have endeavored to show that a knowledge of Addition, Multiplication and Reduction is all that is necessary to produce efficiency in this department of arithmetic, and to point out, step by step, that Interest, Discount, Commission, Fellowship, &c., are mere applications of this rule.

It has been our aim so to explain the *rationale* of every rule that we have oftentimes left ourselves comparatively little space for examples. The intelligent Teacher can easily supply this deficiency.

PREFACE TO THE SECOND EDITION.

In issuing a new edition of the Nova Scotia Arithmetic the author and the publishers desire to tender their sincere thanks to the Press and the Teachers, generally, throughout the Province, for the favorable opinion they have formed of the book; and to express the hope that this edition will be found yet more worthy of their commendation.

The only exception, we believe, taken to this Arithmetic, has been *the fewness of the examples.* This objection is now, we trust, fully met.

Nearly 1000 new examples, arranged in consecutive order and rising, step by step, from the easy to the more difficult, are now appended.

Several errors, both in the figures and in the letter-press, unavoidably marred the first edition. These have all been carefully corrected. Every example that is given in the book has been worked and proved; and we cherish the hope that few or no mistakes will now be found to exist.

Notwithstanding the work has been much enlarged, and the quality of the paper and the ink greatly improved the publishers have decided to issue this edition at the same price as the former, hoping thereby to place it within the reach of all.

HINTS FOR THE TEACHING OF ARITHMETIC.

BY THE SUPERINTENDENT OF EDUCATION.

I. Arithmetic, like Penmanship, can only be effectively taught in accordance with what is usually designated the *Training System;* that is, the pupils must do the exercises themselves.

II. In the *first* part of the following *Arithmetic*, the pupils are supposed to perform the exercises, mentally, with the aid of visible objects; and, afterwards, with the aid of objects unseen, but with which they are perfectly familiar.

These exercises should be engaged in simultaneously, by all the children from 5 or 6 to 8 or 9 years of age, not longer than 5 or 8 minutes at a time, though oftentimes repeated throughout the day. To do full justice to this initiatory department, the teacher should possess a Black-board, an Arithmeticon, or Ball frame, a box of bricks, a good collection of the current coin of the country, of the weights and measures, &c.

III. In the various stages of slate arithmetic, the pupils must be thoroughly classified, if the emulative principle of their nature, through the sympathy of numbers, is to be operated upon and taken advantage of. In a miscellaneous school the arithmeticians may be conveniently divided into four groups, embracing:—1. The youngest children who are not yet using slates. 2. Those working the simple and compound fundamental rules, viz.: Addition, Subtraction, Multiplication and Division. 3. Those in Vulgar Fractions, Practice, Proportion, Interest, &c. 4. Those in Decimals and the higher rules. Each of these groups may be subdivided, corresponding to the exact attainments of the pupils, though one person could easily overlook the subdivisions of any one group.

IV. When any pupils evince general superior excellence in accuracy and expertness, they should at once be transferred to the higher groups, and others from the lower invited to take their place. This furnishes a much more powerful stimulant than the taking of places, and is preferable in a moral point of view.

V. An hour should be devoted to this branch every day; and the first half hour of every alternate day all the classes in group second should be collected into the gallery, or raised up benches, at one time, and thoroughly drilled in one or other of the simple rules; at another time, those in the compound rules; and, again, those in the more advanced rules—Practice, Proportion, Fractions, &c. These exercises should be conducted, at one time, with the mind alone, and at another, with the slate. These frequent revisals are of paramount importance. In fact, it is the want of being thoroughly grounded in the common rules, that accounts for so few persons being good arithmeticians; and so it is in other branches of education.

VI. In no other branch can Monitors, or Pupil teachers, be employed with equal benefit. The pupils, working exercises from the text-books or blackboard, can be arranged in the raised benches, and superintended by the Monitors, while the master retains the class-room or the platform for special instruction in the principles of the science.

VII. Before beginning the lesson, the teacher should see that the slates are all clean; and, in order to secure this, each slate should have a small piece of sponge attached to it. At the commencement of the lesson, a Monitor, appointed for the purpose, should pass along each bench, carrying a small vessel filled with clean water, into which each pupil dips his sponge. On no account should the pupils be allowed to drop saliva on their slates, or to rub them with their sleeve or any other part of their dress. When the slates are thus prepared, all the pupils should be required to show their slate pencils, and, at a given signal, the work should commence. These preliminary exercises contribute very materially to form and cherish habits of cleanliness and neatness.

PART I.

A few special explanations to the teacher in this department.

ART. 1. Children, at a very early period, obtain some idea of number, there being comparatively few of five years of age that cannot count ten or even twenty. As to the source or origin of such an idea, we do not here say anything. One thing, however, is certain, that the first conception of number in the mind of the young, is uniformly associated with the object or objects. They have evidently, at first, no idea of number in the abstract, but in the concrete. They understand perfectly the meaning of 5 apples or 2 marbles, whereas they know nothing of 5 or 2 in themselves. This is sufficient to furnish a Key to the teacher in his first lesson to young children, and points out very forcibly the absurdity of teaching the symbols first, and, thereafter, the things symbolized. This is reversing nature's method, and hence we need not wonder that the acquisition of multiplication and reduction tables has always been regarded as a work of such extreme difficulty, as to require some six months for its performance. If it be true that the early notions of children, in regard to number, are associated with objects, then it follows that we ought to introduce them to the knowledge of abstract numbers through the medium of objects which appeal to their senses. For this purpose, every common school, and especially every infant and primary school, should be furnished with a ball-frame, the decimal currency coins, as well as the sterling money, and a full set of weights and measures. The blackboard may be used with great effect, both along with, and after, the ball-frame, making strokes or dots, representative of units.

The second stage in this initiatory process is the numbering of objects with which the children are perfectly familiar, but which are not present. The third stage is the enumeration of units, tens, hundreds. Then the children will be prepared to take up the abstract numbers, not as unmeaning symbols, but as the signs of things — of realities. And if this course be pursued, and the children, as they advance, rendered thoroughly familiar with every principle of the science — so familiar that they are able to frame rules for themselves — Arithmetic would become not only one of the best cultivators of the human mind, but one of the most interesting and delightful studies.

FIRST LESSON IN ARITHMETIC.

COUNTING

Art. 2. Here every effort should be made to see that the young are carried on *from the known to the unknown.* Every child knows something, less or more, connected with the subject you wish to introduce to his notice, and this should be made the basis of instruction, and of all future progress. There is, for example, scarcely a child of five or six years old that cannot count ten, and counting should thus plainly form the first lesson in Arithmetic. Take a box of pencils, and, lifting the one after the other, make the scholars of the most initiatory class count; then take the frame and do the same with the balls. At the commencement, the lessons given to the very young should be short and frequent. After they have been well exercised, by a succession of lessons, in plain counting, you should proceed a step farther, and bring out the idea, that 2 are just two ones, and 3 three ones of the same sort only. To show this more clearly, take two different objects and count them alternately; such as slates and books, lead and slate pencils, boys and girls; and when you reach ten, ask if there are really ten slates, or ten books. This will lead them to think, and to see that though there are ten things or objects, there are only five slates and five pencils; and that though there are ten children or scholars, there are only five boys and five girls; and that the great lesson taught by all this, is, that in adding one unit to another, every succeeding one must be of the same nature. Another lesson on the same subject may consist in taking the scholars on from ten to twenty. Put the ten balls of the first line a little asunder; then take the second ten, one by one, and, as you count eleven, show that it is made up of ten and one; twelve, of ten and two; thirteen, of ten and three, and so on to twenty. And if you go farther, point out that a new name is given at the end of every ten. So much as to counting right on, as high as 50 or 100.

Another form of the same exercise is counting backwards, first from ten, then from twenty, &c. Try and vary every lesson, so as to impress the minds of the young with the idea that 20 is just twenty ones; and 100 a hundred ones; and the longer these simple points are dwelt upon, you are only weaving into their minds, more thoroughly, that which lies at the basis of the science of numbers.

ON SYMBOLS.

Art. 3. Here give a short oral lesson on signs. Take a signboard, such as the children may have often seen. It may be the signboard over a shoe-maker's door, with a boot or shoe painted upon it. From this, picture out in words the difference between the sign and the thing signified. Another illustration may be taken from letters being the signs of sounds, and words, of ideas. And so it is in figures. We have here also certain signs or symbols expressive of a certain number of units. It would be a very cumbersome, if not an impracticable operation, to write down three ones or six ones, in the shape of dots or strokes, if we wished to convey the idea of three or six; and, therefore, to simplify matters, symbols are employed. Here the teacher may write on the blackboard a series of ones, and place their corresponding figure underneath:

I	II	III	IIII	IIIII	IIIIII	IIIIIII	IIIIIIII	IIIIIIIII
1	2	3	4	5	6	7	8	9

IIIIIIIIII
10

These characters, nine of which are significant, and the cipher 0, are sometimes called digits, from the word that signifies fingers.

Here the teacher must explain everything connected with the different designations given to these numbers, such as integer, even, odd, simple, compound, abstract, prime, composite, &c. He should also show the Roman form of notation, as well as the one usually called the Arabic.

In part Second, these subjects are fully considered.

FUNDAMENTAL RULES.

ADDITION.

Art. 4. Here, by an oral lesson, picture out the idea, that the quantity of any thing can either be made larger or smaller, added to, or taken from, increased or diminished. For example, ask the scholars of this class to hold up their left hand, and then put to them the question: How many fingers have you upon it? *Five*, they will, with one voice, reply. Then tell them to hold up their right hand, and ask: How many fingers or digits have you upon it? The same answer. Tell them to put all the fingers of both hands together, and How many have you now? *Ten*, will be the immediate reply. Now what is this? It is adding. You have five fingers on one hand and five on the other, and by putting them both together, you have — *Ten;* and this is called the sum, and the act of bringing them together into one, is called — *Addition*. Does any of you know the symbol used to represent addition? No reply. Would you like to know it? *Yes*. It is this +; and the name given to it is plus. Can you point out the sign or symbol which expresses the sum of any two or more numbers? *No*. Well, then, I will tell you. It is =, $\underset{5}{|||||} + \underset{5}{|||||} = \underset{10}{||||||||||}$; that is, five plus five has for its sum ten, or is equal to ten, and hence it is sometimes called the sign of equality. Here give a great number of exercises, such as the following: (1.) $\underset{2}{||} + \underset{3}{|||} = \underset{5}{|||||}$; (2.) $\underset{2}{||} + \underset{4}{||||} = \underset{6}{||||||}$; (3.) $\underset{3}{|||} + \underset{4}{||||} = \underset{7}{|||||||}$, and so on.

When the children are perfectly familiar with these symbols, by having a great number of examples presented to them, the next step is to accustom them to add. For this purpose, very simple questions must at first be taken. Indeed, it would be desirable to begin with the constant addition of one. Thus;–1 ball and 1 ball are 2 balls, 2 balls and 1 ball are 3 balls, 3 balls and 1 ball are 4 balls, and so on. Vary the exercise by substitution of every object in the room, in place of balls: such as books, slates, pencils, panes of glass, seats, &c.; any thing to keep up the interest, and prevent the attention of the children from flagging. Then take the ball-frame and add 2. 2 and 2 balls are 4, and 2 balls are 6 balls, &c. Go on to 10. When the children are well acquainted with this form, take one more systematic: a regular addition table.

2 and			3 and			4 and			5 and		
2	are	4	2	are	5	2	are	6	2	are	7
3	—	5	3	—	6	3	—	7	3	—	8
4	—	6	4	—	7	4	—	8	4	—	9
5	—	7	5	—	8	5	—	9	5	—	10

and so on again to 10. After this, various interesting and amusing questions may be put. Take the following as a sample: (1.) William has 6 marbles in one pocket, and 5 in another; how many has he altogether? (2.) There are 7 girls in a class; if 3 more be added, how many will there then be? (3.) Henry got 9 pence, then 5 pence, and a penny; how much money had he? (4.) A man took 3 horses to be shod; one had to get all his feet shod, another had to get 3, and the last had only to get 1; how many shoes would the blacksmith require to furnish? (5.) How many feet have a horse, a cow, a sheep, a goose and a hen? Another sort of exercise may be resorted to on the ball-frame; and that is the breaking of a simple number down to its constituent parts. Thus laying off on one line 5 balls, and on another 3, the children should be asked, how many should be laid off, on a third line, to make up the same number as the two lines contain. Then they might be asked what other numbers would make up 8. All to be proved on the frame, that the children may learn to take nothing upon trust.

SUBTRACTION.

Art. 5. Here explain the term by a short oral lesson. Reversing the former, ask the children to hold up both hands. How many fingers have you on both? *Ten.* Take one hand down, and how many fingers do you hold up? *Five.* Well, then, do you now see that you can do something else besides adding to any quantity? *Yes, we can take from.* Do any of you know what that is called? *No.* It is called subtraction, and the symbol that represents it is just one line, — which is called minus. $\underset{5}{|||||} - \underset{2}{||} = \underset{3}{|||}$; $\underset{6}{||||||} - \underset{4}{||||} = \underset{2}{||}$. Give a large number of exercises on this plan, and afterwards with the ball-frame in the same systematic way as in Addition. Thus lay off 10 balls, and remove 1, and ask the children, 1 ball from 10 balls will leave how many balls? 9 *balls*, for 9 balls and 1 ball will give 10 balls. And so on, till the whole is gone over in the same way. The subtraction of the numbers successively from 10, as 9 from 10, 8 from 10, &c., and similar exercises, should follow. Vary the exercises, as much as possible, to keep up the interest. Take the following as a sample: (1.) John has eight pence in his pocket, and he gave James 5 pence for a knife; how much has John now? (2.) A window has 12 panes of glass; 5 of them were broken; how many were entire? (3.) A dozen birds sat on a tree; 7 flew away; how many remained? (4.) In two weeks a tradesman lost 4 days' work through ill health; what number of days did he work? (5.) A boy in a hay-field worked ten hours a day; how many in the 24 did he not work? &c. After some practice in these and similar exercises, double subtraction may be taken up. Thus, John had 6 marbles, and he gave 2 to James and 3 to Robert; how many had he left? A butcher had 12 sheep; he killed 3 of them and lost 4; how many had he left? It may be well now to join Addition and Subtraction, in every variety of way. Thus, John had 6 marbles; he won from Thomas 5 and from William 2; and afterwards lost 3 first game, and 5 second; how many had he then? Go on in this strain day after day, and week after week, the teacher keeping a note-book, and jotting down some new form every night, so as to go on steadily and progressively.

MULTIPLICATION.

Art. 6. For explanation of the term, give a short oral lesson. Ask the children how many rows of panes there are in one of the windows of the school-room? They all cry, *four.* How many panes are there in each row? *Three.* And how do you find how many there are altogether? The children, looking at the window, begin at once and count *one and one are two, and one is three*, and so on till they make 12. The teacher again asks, could you not shorten this? At once they say, *Three and three are six, and three are nine, and three are twelve.* Is there not a shorter way still? *Yes.* And they say 4 *times* 3 *are* 12. And what is the lesson we are taught by all this? That whenever a number exactly contains a certain number several times, it can be done in a short or abridged form. 12 contains 3 exactly 4 times. 12 is, in this case, said to be a multiple of 3; that is, it contains 3 four times, and the act of doing it is called Multiplication. Multiplication is, therefore, nothing but a short or an abridged or a contracted way of doing addition.

The sign of this is ×.

To familiarize children with the fundamental idea of Multiplication, the teacher should write down lines on the blackboard, as follows: and cause the children to repeat the results.

|| or two ones are 2, or 2 times 1 are 2
||| or three ones are 3, or 3 times 1 are 3
|||| or four ones are 4, or 4 times 1 are 4
||||| or five ones are 5, or 5 times 1 are 5
|||||| or six ones are 6, or 6 times 1 are 6

and so on, until the first line is familiar. Then the same exercise may be done with the ball-frame, or with objects: 4 boys have each a slate, how many have they all?

The second line of the Multiplication Table should be gone over in the same manner, either with the ball-frame or strokes on the blackboard, thus:

||, 2 ones are 2, or 2 times 1 are 2
|| ||, 2 twos are 4, or two times 2 are 4
|| || ||, 3 twos are 6, or 3 times 2 are 6
|| || || ||, 4 twos are 8, or 4 times 2 are 8
|| || || || ||, 5 twos are 10, or 5 times 2 are 10, &c.

|||, 3 ones are 3, or 3 times 1 are 3
||| |||, 2 threes are 6, or 2 times 3 are 6
||| ||| |||, 3 threes are 9, or 3 times 3 are 9
||| ||| ||| |||, 4 threes are 12, or 4 times 3 are 12
||| ||| ||| ||| |||, 5 threes are 15, or 5 times 3 are 15

and so on, till the whole of the Multiplication Table is accurately learned by means of visible objects, either on the blackboard or ball-frame, or with slate-pencils. It is not meant that the whole table should be taught at once. Numerous examples of a simple kind should be given at each step. Take the following as a specimen:

1. How many hands have 4 or 5 or 6 boys?
2. How many feet have 2 or 3 or 4 sheep?
3. How many legs have 6 or 7 or 8 cows?
4. How many half-pennies are there in 4 pence?
5. How many units are there in 3 tens?
6. Bought 3 eggs at 2 pence each; how much should I pay?
7. What should you pay the milk-man for 4 pints of milk, at 2 pence a pint?
8. Find the cost of 4 loaves, at 5 pence each.
9. If a man travels 3 miles in 1 hour, how far will he travel from 6 o'clock in morning till noon?
10. If one large ball is worth 5 little ones, how many little ones will 4 large ones be worth?

To resolve numbers into their factors, the simplest plan is to arrange the numbers in various forms. Thus, for example, to decompose 12, arrange the numbers as follows:

1st. { |||||| / |||||| } That is, $6 \times 2 = 2 \times 6 = 12$

2nd. { |||| / |||| / |||| } That is, $4 \times 3 = 3 \times 4 = 12$

The children should be made to arrange counters, marbles, bricks, or any such things in rectangles, like the foregoing, and from these to deduce their factors. Such exercises will be both instructive and amusing.

DIVISION.

ART. 7. A short oral lesson, to show why this term is used for another rule.

In the first reading class, how many girls are there? *Six.* Well, I wish to give them an equal share of 12 apples; how many have I to give each? *Two.* And why? Because twice 6 are 12, and by giving 2 to each, I divide equally the apples. This is done a great deal quicker than by subtracting 2 from 12, leaving 10, and 2 from 10, leaving 8, &c.; and from its showing how often one number is contained in another, it is called Division, and has for its sign $\div$. Division, then, is just a short or an abridged way of working Subtraction, in the same way as Multiplication is a short way of working Addition.

When the mind is thus prepared for understanding the nature of the work, the Multiplication Table should be referred to, and used as follows:

||, 2 ones are 2, the half of 2 is 1
|||, 3 ones are 3, the third of 3 is 1
|| ||, 2 twos are 4, the half of 4 is 2
||| ||| |||, 3 threes are 9, the third of 9 is 3
|||| |||| |||| ||||, 4 fours are 16, the fourth of 16 is 4

and so on with the whole table. Then give general exercises, such as the following:

1. How many apples at 1d. each, can I buy for 4d.?
2. How many at 2d. each for 6d., for 8d., for 10d.?

3. Four whips cost 8d.; what is the price of 1?
4. Three loaves cost 9d.; what is the price of 1?
5. Divide a shilling among 3 boys; how much has each?
6. Throw 18 marbles equally into 3 holes; how many will there be in each hole?
7. There are 36 boys in a class, seated equally in 4 forms; how many will be on each form?
8. A horse galloped 40 miles in 5 hours; what rate is that per hour?

NOTE. At the end of every new lesson, give a few exercises on the preceding one. When Subtraction is going on, combine it with Addition; when Multiplication, with Addition and Subtraction; and when Division, with Addition, Subtraction and Multiplication.

Here there should be given a large number of miscellaneous exercises on the fundamental rules.

FRACTIONAL NUMBERS.

ART. 8. Give an oral lesson, to show the real meaning of the word *Fractional.*

Children, do you see this apple? *Yes.* Is it a whole apple or part of one? *It is a whole one.* Could you give me another name for..*whole? Yes, entire.* Any other? *No answer.* Well, I will tell you. An Integer has just the same meaning. Well, this whole or integer apple I am going to cut into two equal parts. Done, and holding up the two parts. What do you call each of these? *The half.* Suppose I divide these halves again equally, how would the apple be divided? *Into four parts.* And can I go on dividing these as I will? *Yes.* And these parts, when applied to number, are called? *No answer.* Well, then, I will tell you. They are called Fractions, which just means broken. A fraction, then, is just a part of an integer or whole number. Come, now, and I will show you the symbols employed to represent these fractions or broken numbers. There is a half $\frac{1}{2}$, a third $\frac{1}{3}$, a fourth $\frac{1}{4}$, a fifth $\frac{1}{5}$, two thirds $\frac{2}{3}$, four fifths $\frac{4}{5}$.

Having thus fixed in the mind of the young the idea of a fractional number, it is very easy to go on, and, by dividing an apple into a great number of parts, to give more enlarged views on the same subject. Again, take an orange. If I wish to divide this orange equally among three boys, what must I do? How much must each get? Here is half an orange, and I divide it between two boys; How much of the half does each get? How much of the whole? Very likely this question would not be answered. An easy demonstration of it would consist in dividing a whole orange into two, then each half into two equal portions, and the children would at once see that one half of half an orange was exactly the same thing as one fourth of the whole orange.

The pupils having been thus rendered familiar with the elementary ideas of fractions, they should then be led on to the addition, subtraction and multiplication of fractional numbers, the teacher still availing himself of tangible, ocular demonstrations, or visible objects; and he need be at no loss for these.

Here follow a few exercises in the fundamental rules.

1. Two halves and one half are how many halves?
2. Three fourths and four fourths, how many fourths?
3. Four fifths and six fifths, how many fifths?
4. A boy has 3 half apples, and gives 1 of these halves to his neighbor; how many halves has he left? How many whole apples?
5. If I were to give one boy 6 fourths of an apple, and another boy 2 fourths, which would have more? how much more? how many apples more?

Note. All this is to be done with the apple, by submitting it to the children's observation.

6. Take 4 tenths from 6 tenths; 8 tenths from 10 tenths; 2 hundredths from 3 hundredths.
7. From 5 thirds take 2 thirds.
8. Let 5 boys hold up each 10 fingers. Now, if we take two tenths of these 50 tenths away, how many tenths will be left?

9. Six boys got half an apple each, how many halves had they all? How many wholes?
10. In three apples how many halves?
11. How many thirds are 4 times 2 thirds?
12. How many tenths do 4 times 3 tenths make?

The teacher can lay the whole school under contribution in these exercises.

APPLICATION OF NUMBER TO MONEY.

Art. 9. What is this (the teacher holding up a cent in his hand)? *A cent.* James, suppose I were to give you this cent, what would you do with it? *I would buy a pear.* Well, then, you go to Mrs. Thompson's, and you ask her to give you a pear for a cent; she gives you one, and you put down the cent on the counter. But, suppose you found the pear, the moment you took it into your hand, was rotten, what would you do? *I would say to Mrs. Thompson, this pear is rotten; give me another pear that is worth a cent*, or, *of the same value.*

When, then, you buy the pear, you give what you consider is of *equal value*, and what do you call the cent? *Money—a piece of money.* Any other name? *A coin.*

The cent, then, is of equal value, and you exchange it —*for the pear.* Could you get the pear for any thing else? *Yes:* if I were to give Mrs. Thompson a ball, which would be of a little more value than the pear. Would she like this as well? *No.* Why? *Because it is not nearly so convenient; and, very likely, she might not get any person to purchase the ball.*

Do you know any other kind of money? *Yes, a dime.* How many pears could you get for a dime? *Ten.* And why? *Because the dime is worth ten cents.*

What coin is this? *A quarter dollar.* And how many pears could you buy for it? *Twenty-five, because there are twenty-five cents in a quarter dollar.* What is this? *Half a dollar, or a Florin.* And how many pears could you purchase for it? and so on, till you go over all the decimal currency, holding up every coin before the children, and allowing them to touch it, if they will.

Take a farthing, a penny, a shilling and a pound, and go over them in the same way.

Now the children are prepared to make the application in accounts, like the following:

1. John gave 5 cents to James, and 10 cents to Andrew, and had four remaining; how many cents had he at first?
2. How many dimes are there in half a dollar? in a whole dollar? in two dollars?
3. If John has two pennies, how many farthings would you require to give him for them? How many half pence?
4. John has 4 pennies and Jane has 8; how many have they between them?
5. How many pence in one shilling? in two? in three?
6. How many shillings in one pound? in two? in three?
7. John goes to the grocers and buys 3 lbs. of sugar at 6d. a pound; how much should he bring home out of 2s.?
8. James bought a pair of shoes for a dollar and a half; a jacket for 2 dollars, and a cap for 75 cents; how much did he pay for all?

These may suffice as a specimen. The teacher can multiply these to any amount, and the more they are within the range of common life, the more clearly will the children see that Arithmetic has to do with every day transactions.

APPLICATION OF NUMBER TO MEASURES, WEIGHTS, &c.

Art. 10. The teacher, having beside him a few inch, foot and yard measures, will proceed to the first lesson, by giving to his pupils the names, if they do n't happen to know them, by comparing their lengths, laying them alongside each other, then drawing them on the blackboard, and requiring the pupils to do the same on their slates or blackboard. Having ascertained by actual measurement that there are 12 inches in a foot, and 3 feet in a yard, the various objects in the school may be taken and tested, the pupils being required to state what they believe to be the length of each object in rotation, and then to prove it by measuring it themselves. Similar exercises to the following may be given:

1. In 5 yds. how many feet? inches?
2. John is 5 feet 3 inches, and William is 4 feet 9 inches; what is the difference?
3. How many yards of cloth, at 2s. per yard, can be purchased for $ 3?
4. I bought 6 yards of cloth for 12 dollars; what was that for each yard?

The application of number to square measure and measure of capacity, may be taught exactly in the same way.

The application of Number to Weights may now be considered.

The teacher, being provided with a pair of scales, and the more common of the Avoirdupois, Troy and Apothecarie's Weights, will proceed to explain every thing connected with the scales, and then give the names of the different weights. Then he will take the ounce weight, and putting it into one of the scales, he will proceed to put the dram weights into the opposite scale, and the pupils will then see how many drams are in an ounce, which the teacher will mark on the blackboard, and the pupils on their slates. Again, putting the pound weight into one scale, and the ounce weights into the other, it will be seen how many ounces are in a pound. And so onward, through all the weights.

Exercises can easily be given, corresponding with those on the measures.

DECIMAL NOTATION.

Art. 11. The pupils are now in a state of preparedness for the use of the slate.

They ought to be kept in the foregoing preliminary exercises for two years at least, so that on the supposition that they entered school at six years of age, they are now about eight. The time they have spent at this initiatory work has not only had the effect of weaving into their mental frame-work a practical acquaintance with the elements of the science, but will prove of incalculable service in their whole subsequent career as arithmeticians; and even irrespective of any practical benefit, the development of mind, in adaptation to its germinative condition, which the great majority thus exercised have experienced, is far more than a compensation for all the time they have spent, and all the toil they have undergone.

The first thing to be done in slate arithmetic, is to give the pupils clear and correct notions of the decimal system of notation. As this system is entirely a conventional arrangement, the teacher cannot reason it out with the pupils as he would a theorem in Geometry. He must explain and illustrate, by many familiar examples, the principle on which this convention is based, showing that it is neither more nor less than the localizing of figures, by giving every figure placed on the left hand side of another, ten times its former value. Thus I place the figure 6 on the blackboard. It stands there in its own free, independent condition, as the representative of 6 units of any thing, and is therefore called an absolute figure. I place another 6 before it, or on its left hand side, 66 ; and according to the conventional arrangement, it is no longer a representative of 6 units, but of 6 tens ; and as it gets this property from the position it occupies, it is technically called its *local* value. And suppose I were to prefix another 6 to the two already on the blackboard, its value would be ten times greater than the former 6, that is, it would no longer represent tens, but hundreds, or ten tens. And so onwards.

This arrangement, so beautiful and so simple, was no doubt suggested to its inventor by the ten digits or fingers of the human hand. But be its direct origin what it may, to us it is perfectly manifest that the principle underlying the whole is that of Classification. So multitudinous and varied are the objects of nature, and so limited is our capacity, that to facilitate the memory of these objects, even in reference to the matter of number, some arrangement or classification was indispensable, in which classification we have the finest material or food provided for the exercise of our rational faculties.

This principle, we hold, lies at the very foundation of the decimal system of notation; at all events, we know of no more effectual way of rendering it plain and palpable to the understanding of the young, than by resorting to it.

Suppose that I had a very large box, say, of lead pencils, and that I wish to divide them; what am I to do? I place them before the children, and proceed to reduce them to something like a methodical arrangement, by tying them up in bundles of ten, until I have gone over them all, when I have, say, eight remaining. I place these eight aside, and write the figure 8 on the blackboard, telling the children that that figure represents the eight pencils I have set aside, or 8 ones.

I now tie every ten of these bundles together, and find, say, five remaining. Place these bundles of ten to the left of the eight ones, and write on the blackboard the figure 5 to the left of the 8, telling the children that that figure represents the five bundles of ten. Take the last made bundles, which contain each ten tens, or one hundred ones, and tie them also together by tens. Suppose I have two of these, with three of the bundles of one hundred remaining. I place aside to the left of the five bundles of ten, these three of one hundred, and write on the blackboard the figure 3 to the left of the 5, to represent three bundles of one hundred. Lastly, I place to the left of the three bundles of one hundred, the remaining two bundles, which contain each ten hundreds or one thousand, and write the figure 2 to the left of the 3, to represent the 2 thousands. I have now on the blackboard 2 3 5 8, which represents respectively two bundles of one thousand each, three bundles of one hundred, five bundles of ten, and eight ones.

Or we may take another way of representing the principle of decimal notation, by the drawing of lines on the blackboard.

I wish, for example, to write in figures fifteen. I draw fifteen lines on the board, placing five in one place and ten in another, because fifteen is thus compound. I put down 5, and place 1 as representative of ten before it, and I call this 15. And so on, up to 20, when the same process should be repeated. Two children might be requested to hold up each his ten fingers, and one three fingers. We have thus 2 tens and 3 ones; but 2 tens and 3 ones make 23. In this way the representation of numbers from 1 to 99 can be easily illustrated.

The same process must be again gone over, and the children taught that to put a figure two places to the left, increases its value one hundred times.

For some time, however, it will be desirable not to proceed beyond thousands.

Exercises in tens and units; hundreds, tens and units; thousands, hundreds, tens and units, should be given, so as to render the children perfectly at home in the decimal system of notation.

EXAMPLE 1. Add together 26 and 32.

Write on the blackboard

tens.	units.
2	6
3	2
5	8

Here we have 6 units and 2 units, which make 8 units, and 2 tens and 3 tens, which make 5 tens; and so we have 5 tens and 8 units, which make 58. And so on, to hundreds and thousands.

Ex. 2. Add 5 shillings and 4 pence, and 4 shillings and 3 pence.

Shillings.	Pence.
5	4
4	3
9	7

Here we have 4 pence and 3 pence, which make 7 pence, and 5 shillings and 4 shillings, which make 9 shillings.

EXERCISES.

1. A boy spent in the grocer's store 7s. 6d., and in the baker's 8s. 4d.; how much did he spend altogether?

2. John paid 3s. 8d. for a jacket, and 3s. 2d. for a vest; what did he pay for jacket and vest together?

3. James sold 6 lb. 8 oz. of sugar to one boy, and 8 lb. 3 oz. to another; how much did he sell altogether?

4. The length of this room is 30 ft. 6 in., and that of the class-room 15 ft. 4 in.; what is the length of the two together?

The exercises in Subtraction, Multiplication and Division, should be precisely of the same nature as the foregoing, and gone over in the same careful and accurate manner.

The more practical the questions, the more will they tend to show that Arithmetic is no abstract, barren science, but one of the utmost utility, and necessary for the most common transactions of life.

To give variety and consecutiveness to these exercises, the teacher ought to keep a memorandum-book, and jot down every night the general character of those that are to be given out the following day.

ARITHMETICAL TABLES.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

MONEY.

4 Farthings (*f*)	make	1 Penny (*d*)
12 Pence	- - - -	1 Shilling (*s*)
20 Shillings	- - - -	1 Pound or Sovereign (£)
5 Shillings	- - - -	1 Crown
21 Shillings	- - - -	1 Guinea
1 Farthing is	written	¼
2 Farthings are	- - -	½ Or 1 Half-penny
3 Farthings	- - - -	¾

AVOIRDUPOIS WEIGHT.	TROY WEIGHT.	APOTHECARIES' WEIGHT.
For all Common Goods.	For Gold, Silver and Jewelry.	For Drugs, and in Philosophical Experiments.
16 Drams make 1 Ounce (*oz.*)	24 Grains make 1 Pennyweight (*pwt*)	20 Grains make 1 Scruple
16 Ounces - - 1 Pound (*lb.*)	20 Pennyweights 1 Ounce	3 Scruples - - - - 1 Dram
14 Pounds - - 1 Stone (*st.*)	12 Ounces - - - 1 Pound	8 Drams - - - - - 1 Ounce
28 Pounds - - 1 Quarter (*qr.*)	7000 Grains - - 1 Pound *Av.*	12 Ounces - - - - - 1 Pound
4 Quarters(112 lbs.)1 Hundredweight(*cwt.*)		***The gr. oz. and lb. are the same as in Troy Weight.***
20 Hundredweight 1 Ton		

LONG MEASURE.

4 Inches	make	1 Hand
12 Inches	- - -	1 Foot (*ft.*)
3 Feet	- - - -	1 Yard (*yd.*)
6 Feet	- - - -	1 Fathom
5½ Yards	- - -	1 Rod, Pole or Perch
40 Poles (220 yds.)		1 Furlong
8 Furlongs (1760 yds.)		1 Mile
3 Miles	- - -	1 League
2¼ Inches	- - -	1 Nail } *Cloth M.*
4 Nails	- - -	1 Quarter } *Cloth M.*
4 Quarters	- -	1 Yard } *Cloth M.*
5 Quarters	- -	1 Ell } *Cloth M.*

SQUARE MEASURE.

144 Square Inches	make	1 Square Foot
9 Square Feet	- - -	1 Square Yard
30¼ Square Yards	- -	1 Sq. Rod, Pole or Perch (*P.*)
40 Perches	- - - -	1 Rood (*R.*)
4 Roods (4840 Sq. Yds)		1 Acre (*Ac.*)
640 Acres	- - - - -	1 Square Mile

SOLID MEASURE.

1728 Cubic Inches	make	1 Cubic Foot
27 Cubic Feet	- - -	1 Cubic Yard

MEASURE OF CAPACITY.

4 Gills or Noggins	make	1 Pint
2 Pints	- - -	1 Quart
2 Quarts	- - -	1 Pottle
4 Quarts	- - -	1 Gallon
2 Gallons	- - -	1 Peck
4 Pecks	- - -	1 Bushel
8 Bushels	- - -	1 Quarter
5 Quarters	- - -	1 Load
3 Bushels	- - -	1 Sack } *Coal Meas.*
12 Sacks	- - -	1 Chald } *Coal Meas.*
A Barrel of *Beer*	contains	36 Gallons
A Hogshead	- -	54 Gallons
A Hogshead of *Wine*	-	63 Gallons
A Pipe	- - - -	2 Hogsheads

TIME.

60 Seconds	make	1 Minute
60 Minutes	- -	1 Hour
24 Hours	- -	1 Day
7 Days	- -	1 Week
4 Weeks	- -	1 Lunar Month
365 Days	- -	1 Year

52 Weeks, or 13 Lunar Months, are sometimes reckoned as a year.

The Year is also divided into 12 Calendar Months, viz.:

January	31	May	31	September	30
February	28	June	30	October	31
March	31	July	31	November	30
April	30	August	31	December	31

February has 28 Days excepting in Leap Year, which takes place every Fourth Year, and then February has 29 Days. All the other Months contain 31 Days, excepting those named in the Rhyme,

Thirty Days have September,
April, June, and November.

PART II.

Article 1. Arithmetic may be considered either as a science, or as an art. As a science, it teaches the properties of numbers; as an art, it enables us to apply this knowledge to practical purposes; the former may be called theoretical, the latter practical arithmetic.

2. The grand elementary principle or idea in this science, is unity or a unit; and the only way of impressing it upon the mind is by presenting to the senses a single object, as one apple or one pear.

3. There are three signs by which this idea of *one* is expressed and communicated: the word *one*, the Roman character I, and the figure 1.

4. If one be added to one, the idea thus arising is different from the idea of one, and is complex. This new idea has also three signs, viz.: Two, II, and 2.

5. If we begin with the idea of the number one, and then add it to one, making two; and then add it to two, making three, and so on, it is plain we shall form a series of numbers, each of which will be greater by one than that which precedes it. Now, one, or unity, is the basis of this series of numbers, and each number may be expressed in the three ways before mentioned.

6. Number, then, is the name by which we signify how many objects or things are considered.

7. Numbers are considered either as abstract or concrete.

Abstract numbers are those which have no reference to any particular kind of unit; thus, five, as an abstract number, signifies five units only, without any regard to particular objects.

Concrete numbers are those which have reference to some particular kind of unit; thus, when we speak of five horses, seven cows, the numbers, five, seven, are said to be concrete numbers, having reference to the particular units, one horse, one cow, respectively.

8. All numbers in common arithmetic are expressed by means of the figure 0, commonly called zero or a cypher, which has no value in itself, and nine significant figures: 1, 2, 3, 4, 5, 6, 7, 8, 9, which denote respectively the numbers one, two, three, four, five, six, seven, eight, and nine. These ten figures are sometimes called DIGITS; but this name is often improperly limited to the nine significant figures above mentioned, which are then called nine digits.

This admirable method of writing all numbers by the use of the ten digits, is of great antiquity. It is sometimes called the Arabic method, on the supposition that it was invented by the Arabians. This, however, was not the case, as we have the highest authority for believing that the knowledge of this method was communicated to the Arabians by the Hindoos, and as it cannot be traced to any remoter period, the latter people are entitled to all the honor of its invention. But if the Arabians cannot claim its invention, they have the honor of introducing it into Europe. When they invaded Spain, about the beginning of the eighth century, it was in common use among them, and it is probable that a knowledge of it was soon afterwards communicated to the inhabitants of Spain, and gradually to those of the other European countries.

9. When any of these figures stands by itself, it expresses its simple or intrinsic value; thus 9 expresses nine abstract units, or nine particular things; but when it is followed by another figure, it then expresses ten times its simple value. Thus 94 expresses ten times nine units, together with four units more; when it is followed by two figures, it then expresses one hundred times its simple value; thus, 943 expresses one hundred times nine units, together with ten times four units, together with three units more; and so on, by a ten fold increase, for each additional figure that follows it.

The value which thus belongs to a figure, in consequence of its position or place, is called its LOCAL VALUE.

A system in which ten is the common ratio, is called *Decimal*. Our system is, therefore, a decimal one. If the common ratio were sixty, it would be a sexagesimal system; if twelve, a duodecimal. Both of these were formerly in use, and, as will be seen from the tables, still, to some extent, retained.

10. It appears, then, that in common Arithmetic, we proceed towards the left from units to tens of units; from tens of units to tens of tens of units, or hundreds of units; from hundreds of units to tens of hundreds of units, or thousands; from thousands of units to tens of thousands of units; from tens of thousands of units to tens of tens of thousands of units, that is, to hundreds of thousands of units; hence to tens of hundreds of thousands of units, or millions of units, and so on to billions, trillions, quadrillions, &c. Thus 10 represents one ten of units, together with no unit; or, as it is briefly read, ten. 11 represents one ten of units, together with one unit, and is briefly read, eleven. Similarly, 12, 13, 14, 15, 16, 17, 18, 19, respectively represent one ten of units, together with two, three, four, five units, &c.

The next ten numbers are expressed by 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, which respectively represent two tens of units, together with no, one, two, three, four, five, six, seven, eight, nine units; they are briefly read twenty, twenty-one, &c.

The next ten numbers are expressed by 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, which are respectively read thirty, thirty-one, thirty-two, &c.; we thus arrive at 40 (forty), 50 (fifty), 60 (sixty), 70 (seventy), 80 (eighty), 90 (ninety) — 99 being the largest number which can be expressed by two figures, since it represents nine tens of units, together with nine units. The next number to this is 100, which represents ten tens of units, or one hundred of units, together with no tens of units, together with no units; or is briefly read, one hundred.

By pursuing the same system in higher numbers, the figure occupying the fourth place from the right hand will represent so many tens of hundreds of units, or thousands of units; the figure in the fifth place will represent so many tens of thousands of units; and so on

5473 represents five thousands of units, together with four hundreds of units, together with seven tens of units, together with three units, or, as it is briefly read, five thousand four hundred and seventy-three.

NOTATION AND NUMERATION.

11. Notation is the art of expressing any number by figures which is already given in words. Numeration is the converse of Notation, being the art of expressing any number in words which is already given in figures.

Numeration Table.

	&c.		&c.
VI.	Quadrillions.	18	Hundreds of Quadrillions,
		17	Tens of Quadrillions,
		16	Quadrillions,
V.	Trillions	15	Hundreds of Trillions,
		14	Tens of Trillions,
		13	Trillions,
IV.	Billions	12	Hundreds of Billions,
		11	Tens of Billions,
		10	Billions,
III.	Millions	9	Hundreds of Millions,
		8	Tens of Millions,
		7	Millions,
II.	Thousands ..	6	Hundreds of Thousands,
		5	Tens of Thousands,
		4	Thousands,
I.	Units	3	Hundreds,
		2	Tens,
		1	Units,

The terms *hundred*, *thousand* and *million*, are primitive words, and bear no analogy to the numbers which they denote. The terms *billion*, *trillion*, *quadrillion*, &c., are formed from million and the Latin numerals, *bis*, *tres*, *quatuor*, &c. Thus, prefixing *bis* to *million*, by a slight contraction, for the sake of euphony, it becomes *billion*, &c.

12. *To express in words the numbers denoted by lines of figures.*

RULE. (1.) Commencing at the right hand side, divide the given figures into periods of three figures each, till not more than three remain. (2.) Then, commencing at the left hand, annex to the value expressed by the figures of each period, except that of the units, the name of the period, according to the numeration table.

Thus, the expression, 37053907, becomes, by division into periods, 37,053,907, and is read *thirty-seven millions, fifty-three thousand, nine hundred and seven*, the term *units* or *ones*, at the last, being omitted, as was explained in Art. 10.

NOTE.—Before the pupil be allowed to put the above rule into practice, he should be well exercised on the principles laid down in Art. 10. Thus, how many tens of units are there in 46, how many tens of tens of units, or hundreds, together with how many tens of units or tens, together with how many units or ones are there in 467; and so on to thousands, &c.

EXERCISES IN NUMERATION.

Write down in words, or name the numbers signified by the following exercises:

Ex. 1.	24	Ex. 4.	1000	Ex. 7.	1234
2.	124	5.	1200	8.	6789
3.	465	6.	1230	9.	36847

Ex. 10. 10076 Ex. 11. 876543219

13. *To express numbers by figures.*

RULE. Make a sufficient number of dots or cyphers, and divide them into periods of three each. Then, commencing at the left, place, in their proper position, beneath the dots or cyphers, the significant figures necessary for expressing the proposed number. If any places remain unoccupied, let them be filled with cyphers. Thus, the method of expressing the number two hundred and five millions, twenty thousand, seven hundred and nine, will be found in the following manner: 000, 000, 000
2 5   2   7 9

and hence, by filling the unoccupied places with cyphers, we get 205,020,709.

EXERCISES IN NOTATION.

Express the following in figures:

1. Fifty-four.
2. One thousand and twenty-four.
3. Sixty-four millions, three hundred and ten thousands, four hundred and six.

14. *Roman System of Notation.*

Our ordinary Numeral Characters have not been always, nor every where, used to express numbers. The letters of the alphabet, as being well known, naturally presented themselves for the purpose, and accordingly were very generally adopted. For example, by the Hebrews, Greeks, Romans, &c., each of course using their own Alphabet.

The pupil should be acquainted with the Roman Notation, on account of its being used to denote chapters, sections, and other divisions of books and discourses.

It will be found, by referring to the following table, that the Romans used very few characters — fewer, indeed, than we do, although our system is still more simple and effective, from our applying the principle of "position," unknown to them.

They expressed all numbers by the following symbols, or combinations of them: I, V, X, L, C, D, M.

The rules for Roman Notation are as follows:

By a *repetition* of a letter, the value denoted by the letter is represented as repeated: as II represents *two;* XX represents twenty.

The *annexing* a letter of a lower value to one of a higher, denotes the sum of both, or their joint value: as VI denote 6, XII denote 12.

The *prefixing* a letter of a lower value to one of a higher, denotes their difference: as IV denote 4; IX denote 9.

Every Ↄ *annexed* to IↃ, increases the value ten times. Thus, IↃↃ denote 5000.

Every C and Ↄ to the left and right of CIↃ, increases the value ten times. Thus, CCIↃↃ denote 10,000; CCCIↃↃↃ denote 100,000.

A line over *any* number increases its value a *thousand* fold. Thus, $\overline{X}$ denotes 10,000, $\overline{L}$ denotes 50,000.

In constructing their system, they evidently had a quinary in view, that is, one in which five would be the *common ratio;* for we find that they changed their character, not only at ten, ten times ten, &c., but also at five times five, &c.

As far as notation was concerned, what they adopted was neither a decimal nor a quinary system, nor even a combination of both. They appear to have supposed *two* primary groups or periods, one of five, the other of ten, and to have formed all the other groups or periods from these, by using ten as a *common ratio* of each resulting series.

They anticipated a change of character, one unit before it would naturally occur.

In this point of view, four is one unit before five; forty, one unit before fifty — ten being now the units under consideration; four hundred, one unit before five hundred — hundreds having become the units contemplated.

TABLE OF ROMAN NOTATION.

	Characters.	Nos. Expres'd.		Characters.	Nos. Expres'd
	I	1		XIX	19
	II	2		XX	20
	III	3		XXX, &c.	30, &c.
Anticipated change.	IIII or IV	4	Anticipated change.	XL	40
Change.	V	5	Change	L	50
	VI	6		LX, &c.	60, &c
	VII	7	Anticipated change	XC	90
	VIII	8	Change	C	100
Anticipated change.	IX	9		CC, &c.	200, &c.
Change.	X	10	Anticipated change.	CD	400
	XI	11	Change.	D or IↃ	500
	XII	12	Anticipated change.	CM	900
	XIII	13	Change.	M	1,000
	XIV	14		$\overline{V}$	5,000
	XV	15		$\overline{X}$	10,000
	XVI	16			
	XVII	17			
	XVIII	18			

It may be doubted whether any other ratio of increase would, on the whole, be more convenient, than that of the decimal system. If the ratio were *less*, it would require more places of figures to express larger numbers; if the ratio were *larger*, it would not indeed require so many figures, but the operations would manifestly be more difficult than at present, on account of the numbers in each order being larger. Besides, the decimal system is sufficiently comprehensive to express with all desirable facility, every conceivable number, the largest as well as the smallest; and yet so simple that a child may understand and apply it.

In a word, it is in every way adapted to the practical operations of business, as well as the most abstruse mathematical investigations. In whatever light, therefore, it is viewed, the *decimal* notation must be regarded as one of the most striking monuments of human ingenuity, and its beneficial influence on the progress of science and the arts, on commerce and civilization, must win for its unknown author the everlasting admiration and gratitude of mankind.

ARITHMETICAL SIGNS.

15. The sign $=$ (*equal*) indicates that the numbers between which it is placed, are *equal* to one another. Thus, 12 pence $=$ one shilling.

The sign $+$ (*plus*), placed between two numbers, indicates that they are to be *added together*. Thus, $5 + 7 = 12$.

The sign $-$ (*minus*), placed between two numbers, indicates that the *latter* is to be *taken* from the *former*. Thus, $12 - 7 = 5$.

The sign $\times$ (multiplied by), denotes that the number which precedes it is to be multiplied by that which follows it. Thus, $4 \times 6 = 24$.

The sign $\div$ (divided by), signifies that the number which precedes it is to be divided by that which follows it. Thus, $24 \div 6 = 4$. But division is more generally indicated by placing the dividend above a line, and the divisor under it. Thus, $\frac{24}{6} = 4$.

The signs :, : :, : indicate proportion. Thus, the expression 2 : 3 : : 6 : 9, denotes that 2 is to 3, as is 6 to 9.

The sign $\overline{\quad\quad}$ (*vinculum*), placed over numbers, and the sign ( ) or { } called *Brackets*, enclosing numbers within them, are used to denote that all numbers under the vinculum, or within the bracket, are equally affected by all numbers not under the vinculum, or within the brackets. Thus, $\overline{2 + 3}$, or $(2 + 3)$, or $\{ 2 + 3 \}$, each signify, that whatsoever is outside the vinculum or bracket which affects 2 in any way, must affect 3 in the same way, and conversely.

The *power* of a number, that is, the number of times that the number is to be multiplied into itself, is expressed by *indices* placed above the root. Thus, the expression, 4^2, 4^3, 4^4, signify the *second*, *third* and *fourth* powers of 4.

The sign $\sqrt{\ }$ signifies *square* root; as $\sqrt{16} = 4$, or $(16)^{\frac{1}{2}} = 4$.

$\sqrt[3]{\ }$, Sign of *Cube* root; as $\sqrt[3]{27} = 3$, or $(27)^{\frac{1}{3}} = 3$.

$\sqrt[4]{\ }$ Sign of the *Biquadrate* root; as $\sqrt[4]{1296} = 6$, or $(1296)^{\frac{1}{4}} = 6$.

The sign ∴ signifies therefore. The sign ∵ signifies because.

The pupil should be made PERFECTLY familiar with these symbols; or else so far from being, as they ought, a great advantage, they will serve only to embarrass him. There can be no doubt that the expressions of quantities by characters, and not by words written in full, tends to brevity and clearness. The same is equally true of the processes which are to be performed — the more concisely they are indicated the better.

ADDITION.

16. If numbers are changed by any arithmetical process, they are either increased or diminished; if increased, the effect belongs to *Addition;* if diminished, to *Subtraction.* Hence, all the rules of Arithmetic are ultimately resolvable into either of these, or combinations of both.

When any number of quantities, either *different* or *repetitions* of the same, are united together so as to form but one, we term the process simple "Addition." When the quantities to be added are the *same* (and we may have *as many* of them as we please), it is called "Multiplication." When they are not only the *same*, but their number is indicated by *one of them*, the process belongs to "Involution." That is, Addition restricts us neither as to the kind, nor the number, of the quantities to be added; Multiplication restricts us as to the kind, but not the number; Involution restricts both as to the kind and number. All, however, are really comprehended under the same rule — ADDITION.

There are two kinds of Addition — SIMPLE and COMPOUND.

Simple Addition is the process of uniting two or more abstract numbers; or denominate numbers containing but one denomination, *in such a way* that all the *units* which they contain may be expressed by a single number, called the *Sum*, or *sum total.*

Compound Addition is the uniting of two or more concrete numbers, of the same kind, but of different denominations of that kind; as years, months, and days, or gallons, quarts, and pints.

NOTE.—A truly philosophical arrangement of the simple or fundamental rules would require *Multiplication* to come after Addition, and *Division* after Subtraction. We have waived our preference, however, in deference to the universality of the old arrangement.

SIMPLE ADDITION.

17. RULE. Write down the given numbers under each other, so that units may come under units, tens under tens, hundreds under hundreds, and so on; then draw a straight line under the lowest line of the addenda.

Find the sum of the column of units; if it is under ten, write it down under the column of units, below the line just drawn; if it exceed ten, then write down the last figure of the sum under the column of units, and carry to the next column the remaining figure or figures; treat each succeeding column in the same way, and write down the full sum of the extreme left-hand column. The entire sum, so marked down, will be the sum total or amount of the separate numbers.

Ex. Add together 5469, 743, and 27. Proceeding by the Rule given above, we obtain—

$$\begin{array}{r} 5469 \\ 743 \\ 27 \\ \hline 6239 \end{array}$$

The reason for the rule will appear from the following considerations.

When we take the sum of 7 units, and 3 units, and 9 units, we get 10 units and 9 units, or 19 units; we therefore place the 9 units under the column of units, and carry on the 1 ten (10 *units*) to the next column, viz.: the column of tens.

Now, the sum of 1 ten, 2 tens, 4 tens, and 6 tens, is 10 tens and 3 tens, or 13 tens; we therefore place the 3 tens under the column of tens, and carry on the 1 hundred units (10 tens) to the next column, viz.: the column of hundreds.

Again, the sum of 1 hundred, and 7 hundreds, and 4 hundreds, is 10 hundreds and 2 hundreds, or 12 hundreds; we therefore place the 2 hundreds under the column of hundreds, and carry on the 1 thousand (10 hundreds) to the next column, viz.: those of thousands.

Again, the sum of 1 thousand, and 5 thousands, is 6 thousands; we therefore place the 6 under the column of thousands, and the entire sum is 6239.

18. The preceding example might have been worked thus, putting down at full length the local value of the figures:

$$\begin{aligned} \text{Thus, } 5469 &= 5000 + 400 + 60 + 9 \\ + 743 &= 700 + 40 + 3 \\ + 27 &= 20 + 7 \end{aligned}$$

Now, adding the columns, we get the sum—

$$= 5000 + 1100 + 120 + 19$$

$$= 5000 + \overline{1000 + 100} + \overline{100 + 20} + \overline{10 + 9}$$

(Since $1100 = 1000 + 100$, $120 = 100 + 20$, $19 = 10 + 9$)

$$= 6000 + 200 + 30 + 9$$

(Collecting the thousands together, the hundreds together, and so on), $= 6239$.

NOTE.—To the farmer and mechanic, the accountant as well as the mathematician, accuracy is indispensable; and as this can only be acquired by long experience, each sum should be proved, either by a direct method, by analysis, of the foregoing rule (we prefer the latter, as by it, these rules are rendered practical by their application), and no pupil should be satisfied with the correctness of the result, till so tested.

19, *First Method of Proof.* Begin at the top, and add the columns downward, in the same way as they were before added upward, and if the sums agree, the work is presumed to be right.

NOTE.—The reason of this proof is, that, by adding downward, the order of the figures is inverted; and therefore, any error made in the first addition, would probably be detected in the second. This method of proof is generally used in business.

Second Method of Proof. Separate the given numbers into two or more divisions. Find the sums of these divisions, severally, and add these partial sums together. If the last result be equal to that found by the commo method, the work is right.

NOTE.—This method of proof depends on the axiom, that the *whole* of a quantity is equal to the sum of all its parts.

Third Method of Proof. Commencing at the left hand, add the several columns *without carrying*, and set down the full sum of each column, with the units in their proper place, and the tens below the figure immediately to the left.

Add together these lines thus resulting, and if the last result agree with that obtained, by the common method, it may be concluded that both are right.

Thus, in the annexed example, the sum of the left hand column is 25, which is set down in full: the sum of the next column is 30; the cypher is set in its proper place, and 3 under the 5; and so with the rest. The sum of the two lines thus obtained, is equal to the sum found by the ordinary method.

This method of addition might be used instead of the common method; and, as it requires nothing to be carried, it may be employed with advantage when the calculator is liable to interruption.

```
 5946
 9738
 2697
 9868
-----
28249
-----
25029
 322
-----
28249
```

Fourth Method of Proof. Cast the 9s out of each of the given numbers separately, and place each excess at the right of the number. Then cast the 9s out of the sum of these excesses; also cast the 9s out of the amount; and if these *two* excesses are equal, the work may be supposed to be right

Thus, as in the example:

```
5946  excess of 9s     = 6
9738    "      "       = 0
2697    "      "       = 6
9868    "      "       = 4
-----                    -
28249 excess of 9s = 7 = 7
```

Note.—This mode of proof is based on a peculiar property of the number nine, which will be further illustrated under Multiplication.

Mental Exercises in Addition.

1. Add 17, 8, 11, 14, 15, 10, 6, 5, 9; how many?
2. There were 4 boxes of oranges. The first contained 527, the second 265, the third 69, and the fourth 72; how many oranges were there altogether?

Note.—It is not necessary to multiply examples here, as the teacher can supply them to suit the capacity of his pupils.

18. Exercises in Slate Arithmetic.

1. 13678632	2. 6584334156
46795320	1290887091
4679532	787692123
19848015	356796432
8309169	45699543
93310668	9065409345

3. Add together 7384, 326, 6780, and 57. Ans. 14547.

4. Add together 89, 4500, 423, 2024, 5408, 60546, and 9401. Ans. 82391.

5. 986759 + 4976346 + 29483 + 898647 + 3984753 + 6489778 + 57893 + 2468144 + 576989 + 498653. Ans. 20967445.

6. £7654 + £50121 + £100 + £76767 + £675. Ans. £135317.

7. $ 10600 + $ 7676 + $ 6760 + $ 90017. Ans. $ 115053.

8. A merchant owes to A £1500; to B £408; to C £1310; to D £50; and to F £1900; what is the amount of all his debts? Ans. £5168.

9. A merchant received the following sums: $200, $315, $317, $10, $172, $513, and $9: what is the sum of all? Ans. $1536.

10. A merchant bought 7 casks of merchandize. No. 1 weighed 310 lbs.; No. 2, 420 lbs.; No. 3, 388 lbs.; No. 4, 335 lbs.; No. 5, 400 lbs.; No. 6, 412 lbs.; No. 7, 429 lbs.; what is the weight of the whole? Ans. 2694 lbs.

11. A farmer sold 9 loads of hay. The first weighed 2065 lbs.; second, 1896 lbs.; third, 1467 lbs.; fourth, 2000 lbs.; fifth, 1867 lbs.; sixth, 2891 lbs.; seventh, 1872 lbs., eighth, 784 lbs., and the ninth, 1740 lbs.; what is the total weight? Ans. 16582 bls.

12. Add together the following numbers: Fifteen thousand, seven hundred and ninety-six; four hundred and nine; two hundred and thirty-four thousand and fifty; four millions, three thousand and seventy-six; four thousand and thirty-six; ten thousand, nine hundred and one. Ans. 4268268

Note.—The pupil should not be allowed to leave Addition until he can, with great rapidity, continually add any of the nine digits to a given quantity. Thus, beginning with 9, to add 6, he should say: 9, 15, 21, 27, 33, &c., without hesitation, or further mention of the numbers.

For example, he should not be allowed to proceed thus: 9 and 6 are 15; 15 and 6 are 21, &c.; nor even, 9 and 6 are 15; and 6 are 21, &c. He should be able, ultimately, to add the following, or any other:

5638	in this manner: 2, 8, 16 (the sum of the first col-
4756	umn, of which one is carried, and 6 to be set
9342	down); 5, 10—13; 4, 11—17; 10, 14—19.
19736	

SUBTRACTION.

20. Subtraction is the method of finding what number remains when a smaller number is taken from a greater number.

The number found by subtracting the smaller of two numbers from the greater, is called the *Remainder*.

There are two kinds of Subtraction, Simple and Compound, which differ from each other in precisely the same way in which Simple and Compound Addition differ from each other.

The sign — (minus) placed between two numbers, signifies that the second number is to be taken from the first number. (Art. 15.)

SIMPLE SUBTRACTION.

21. Rule. Place the less number under the greater, so that units come under units, tens under tens, hundreds under hundreds, and so on; then draw a straight line underneath.

Take, if possible, the number of units in each figure of the lower line from the number of units in each figure of the upper line, which stands immediately over it, and put the remainder below the line just drawn, units under units, tens under tens, hundreds under hundreds, and so on; but if the units in any figure of the lower line exceed the number of units in the figure above it, add ten to the upper figure, and then take the number of units in the lower figure from the number in the upper figure thus increased; put down the remainder as before, and then carry one to the next figure of the lower line. The entire difference or remainder, so marked down, will be the difference or remainder of the given numbers.

22. In order that the difference of two numbers may remain the same, if we increase one of the numbers we must increase the other by the same quantity: for example, five apples minus two apples equal three apples, or $5 - 2 = 3$, and increasing each of the numbers by 1, $6 - 3 = 3$.

In like manner, 5 tens — 3 tens = 2 tens, and increasing each number by 1 ten: 6 tens — 4 tens = 2 tens, the same as before.

This axiom will enable us to explain the rule of Subtraction.

23. Subtract 356 from 634.

Hundreds.	Tens.	Units.
6	3	4
3	5	6
2	7	8

Here, as we cannot take 6 units from 4 units, we borrow one of the tens from the 3 tens, and then 6 units from 14 units, and 8 remain: we have now 5 tens to take from 2 tens, (why not from 3 tens? because we took or borrowed one to put along with the 4 units) as this cannot be done, we take or borrow a hundred or 10 tens from the 6 hundreds, and then we have 5 tens from 12 tens (for 10 tens + 2 tens = 12 tens) and 7 tens remain; lastly, we have 3 hundreds from 5 hundreds, (why not 6 hundreds? because we took or borrowed one of them to put along with the two tens as above) and 2 hundreds remain.

In the course of the foregoing demonstration we had 5 tens to take from 12 tens, now the result will not be altered if we increase each of the numbers by 1 ten, that is, if we say, 6 tens from 13 tens; in like manner, in the place of saying 3 hundreds from 5 hundreds, we may say, without altering the result, 4 hundreds from 6 hundreds: thus establishing the common rule of Subtraction, when we "borrow" 1 from a figure in the top line, we must "carry" 1 to the next figure in the bottom line.

24. As a farther illustration of the above, let us take the following example. Subtract 4938 from 5123 according to the rule

```
5123
4938
----
 185
```

Here we cannot take 8 units from 3 units, we therefore add 10 units to the 3 units, which are thus increased to 13 units; and taking 8 units from 13 units we have 5 units left; we therefore place 5 under the column of units; but having added 1 ten units to the upper line or number, we must add the same number of units (1 ten units) to the lower number, so that the difference between the numbers may remain the same; and adding 1 ten units to the 3 ten units in the lower number, we obtain 4 tens or 40 instead of 3 tens or 30.

Again, we cannot take 4 tens from 2 tens; we therefore add 10 tens or 1 hundred to the 2 tens, which thus becomes 12 tens, or 120; and then taking 4 tens or 40 from 12 tens or 120, we have 8 tens or 80 remaining; we therefore place 8 under the column of tens: but having added 1 hundred to the upper number, we must add 1 hundred to the lower number, for the reason given in Art. 21; and adding 1 hundred to 9 hundreds in the lower number, we obtain 10 hundreds or 1000 instead of 900.

Again, we cannot take 10 hundreds or 1000 from 1 hundred and we therefore add 10 hundreds or 1 thousand to the 1 hundred, which thus becomes 11 hundreds or 1100; and taking 10 hundreds or 1000 from 11 hundreds or 1100, we have 1 hundred or 100 left; we therefore place 1 under the column of hundreds; but having added 10 hundreds or 1 thousand to the upper number, we must add 1 thousand to the lower number, for the reason given above; and adding 1 thousand to the 4 thousands in the lower number, we obtain 5 thousands or 5000; and 5 thousands or 5000 taken from 5000 leaves 0; therefore the whole difference or remainder is 185.

24. The above Example might have been worked thus, putting down at full length the local values of the figures:

$$5123 = 5000 + 100 + 20 + 3$$

$$= 4000 + 1000 + 100 + 10 + 10 + 3$$

(collecting the first 10 with the 100, and the second 10 with the 3.)

$$\begin{array}{rrrrr} & = 4000 + & 1000 + & 110 + & 13 \\ \text{and } 4938 = & 4000 + & 900 + & 30 + & 8 \\ \hline & & 100 + & 80 + & 5 = 185 \end{array}$$

The truth of all results in Subtraction may be proved by adding the less number to the difference or remainder; if the sum equals the larger number, the result obtained by subtraction may be presumed to be correct.

Second Method. Cast the 9s out of the larger number, and place the excess to the right. Next, cast the 9s out of the less number and place the exc ss to the right of this number also, then take the latter excess, if possible, from the former excess, and note the difference: if the latter excess cannot be taken from the former, add 9 to it, and then take the difference and set it down as before; lastly, cast the 9s out of remainder, and if this excess be equal to the difference of the above excesses the result obtained by subtraction may be considered right.

$$\begin{array}{lllll} \text{Thus, } 5123 & \text{excess of 9s} & = 2 & \text{adding } 9 = & 11 \\ \phantom{\text{Thus, }} 4938 & \text{" 9s} & = 6 & \text{and} & 6 \\ \hline \phantom{\text{Thus, }} 185 & \text{excess of 9s} & = 5 = & & 5 \end{array}$$

Mental Exercises in Subtraction.

1. A chest of oranges contained 703; but 285 were bad. How many good ones were there? *Ans.* 418.

2. A boy had a 103 marbles, and lost 86 of them. how many had he left? *Ans.* 17.

3. A man bought a horse for $186 and sold him again for $225. How much did he gain by the bargain. *Ans.* $39.

4. A farmer procured 2896 poles for fencing, he took 1689 of them to go round a pasture. How many has he left? *Ans* 1207.

5. A farmer raised 896 bushels of wheat, and sold 675 bushels of it; how many did he reserve for his own use? *Ans.* 221.

EXERCISES IN ADDITION AND SUBTRACTION.

1. 16 + 17 + 19 + 10 + 7 — 8 + 11 — 14 + 16 —20. *Ans.* 54.

2. A man owing $767, paid at one time $190, at another time $131, at another time $155; how much did he then owe? *Ans.* 291 dollars.

EXERCISES FOR THE SLATE.

1. From 45079 take 32048. *Ans.* 13031.
2. From 1896481035 take 736792632. *Ans.* 1159688403.
3. From 4673842150 take 2186367025. *Ans.* 2487475125.
4. 153425178—53845248. *Ans.* 99579930.
5. Required the difference between three, and three hundred thousand. *Ans.* 299997.
6. Mont Blanc, the highest mountain in Europe, is 15,680 feet high; and the height of Chimborazo, the highest in America, is 21,427 feet; how much is the latter higher than the former? *Ans.* 5747 feet.

MULTIPLICATION.

25. MULTIPLICATION is a short method of finding the sum of any given number repeated as often as there are units in another given number (Art. 16.) thus: when 3 is multiplied by 4, the number produced by the multiplication is the sum of 3 repeated four times, which sum is equal to 3 + 3 + 3 + 3 or 12.

The number to be repeated, or added to itself, is called the *Multiplicand.*

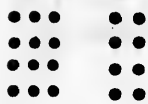
The number which shows how often the multiplicand is to be repeated or added to itself, is called the *Multiplier.*

The number found by the multiplication is called the *Product.*

The multiplicand and the multiplier are sometimes called '*Factors*' because they are factors or makers of the product.

26. The process of *Multiplication* is founded upon the axiom that any quantity taken a certain number of times is the same as the parts of that quantity taken the same number of times; for example, 3 times 27 = 3 times 20 + 3 times 7.

The truth of this axiom will be rendered more apparent by arranging counters, as in the following figure.



= 4 times 5 = 4 times 3 + 4 times 2.

In the same manner any other case may be illustrated.

27. Multiplication is of two kinds, SIMPLE and COMPOUND. It is termed Simple Multiplication, when the MULTIPLICAND is either an abstract number, or a concrete number of one denomination.

It is termed COMPOUND MULTIPLICATION when the multiplicand contains numbers of more than one denomination but all of the same kind.

28. One of the factors, namely the multiplier, must necessarily be an '*abstract number*,' since it would be absurd to speak of 6 shillings multiplied by 4 shillings. We can multiply 6 shillings by 4, i. e., we can find how many shillings there are in four times six shillings; but there is no meaning in 6 shillings multiplied by 4 shillings.

SIMPLE MULTIPLICATION.

29. RULE.—Place the multiplier under the multiplicand in such a way that units may be under units, tens under tens, and so on. Multiply each figure of the multiplicand, beginning with the units, by the figure in the units' place of the multiplier (by means of the table given for multiplication in the first part); set down and carry as in addition. Then multiply each figure of the multiplicand, beginning with the units, by the figure in the tens' place of the multiplier, placing the first figure so obtained under

the tens of the line above, the next figure under the hundreds, and so on. Proceed in the same way with each succeeding figure of the multiplier. Then add up all the results thus obtained, by the rule of Simple Addition.

NOTE.—If the multiplier does not exceed 12, the multiplication can and should be effected in one line.

Ex. Multiply 7654 by 397. Proceeding by the rule given above we obtain,

```
   7654
    397
-------
  53578
 68886
22962
-------
3038638
```

Here we place the 7 units under the 4 units, the 9 tens under the 5 tens, and so on. When 7654 is to be multiplied by 7, we first take 4 seven times which by the table gives 28, i. e. 8 units and 2 tens; we therefore put the 8 in the units' place and carry on the two tens: again 5 tens taken 7 times gives 35 tens, to which add 2 tens, and we obtain 37 tens, or 7 tens and three hundreds; we put down the 7 in the tens' place, and carry on the 3 hundreds: again, 6 hundreds are to be taken 7 times which gives 42 hundreds, to which add 3 hundreds and we obtain 45 hundreds, or 4 thousands and 5 hundreds; we put down the 5 in the hundreds' place, and carry on the 4 thousands: again, 7 thousands taken 7 times gives 49 thousands, to which we add the 4 thousands, thus obtaining 53 thousands which we write down.

Next, when we multiply 7654 by the 9, we in fact multiply it by 90; and 4 units taken 90 times give 360 units, or 3 hundreds, 6 tens and 0 units: therefore, omitting the cypher, we place the 6 under the tens place, and carry on the 3 to the next figure, and proceed with the operation as in the line above.

When we multiply 7654 by 3, we in fact multiply by 300; and 4 multiplied by 300 gives 1200, or 1 thousand 2 hundreds 0 tens, and 0 units: therefore omitting the cyphers, we place the first figure 2 under the hundreds' place, and proceed as before. Then adding up the three lines of figures as in simple addition, we obtain the product of 7654 by 397.

30. If the multiplier, or multiplicand, or both, end with cyphers, we may omit them in the working; taking

care to affix to the product as many cyphers as we have omitted from the end of the multiplier or multiplicand, or both.

Thus, if 263 be multiplied by 6200, and 570 be multiplied by 3200, we have

263	570
6200	3200
526	114
1578	171
1630600	1824000

The reason is clear; for, in the first case, when we multiply by 2, in fact we multiply by 200; and 3 multiplied by 200 gives 600. In the second case, the 7 multiplied by the 2 is the same as 70 multiplied by 200; and 70 multiplied by 200 gives 14000.

31. If the *Multiplier* contains any cypher in any other place, then, in multiplying by the different figures of the multiplier, we may pass over the cypher, taking care, however, when we multiply by the next figure, to place the first figure arising, from that multiplication, under the third figure of the line above, instead of the second figure. The reason of this is clear; for, if we were multiplying by 206, when we multiply by the 6, we take the multiplicand 6 times; when we multiply by the 2, we really take the multiplicand, not 20 times, but 200 times.

32. When two numbers are to be multiplied together, it is a matter of indifference, so far as the product is concerned, which of them be taken as the multiplicand or multiplier; in other words, the product of the first, multiplied by the second, will be the same as the product of the second multiplied by the first.

$$\text{Thus, } 2 \times 4 = 2 + 2 + 2 + 2 = 8$$
$$4 \times 2 = 4 + 4 = 8$$

Therefore the results are the same, that is, $2 \times 4 = 4 \times 2$.

33. We have hitherto confined our attention to products formed by the multiplication of two factors only. Products may arise from the multiplication of three or more

factors. This is termed *Continued Multiplication*. Thus, $2 \times 3 \times 4$, denotes the continued multiplication of the factors 2, 3, and 4; and means that 2 is first to be multiplied by 3, and the product thus obtained to be then multiplied by 4. The result of such a process would be 24, which is, therefore, the continued product of 2, 3, and 4. We may thus express it: $2 \times 3 \times 4 = 24$.

34. Methods of Proof.

First. Make the multiplicand the multiplier, and the multiplier the multiplicand; proceed as before, and if the results are the same, the work may be considered right.

Second. Begin at the left hand of the multiplicand, and add together its successive digits towards the right, till the sum obtained equals or exceeds the number nine. If it equals it, drop the nine, and begin to add again at this point, and proceed till you obtain a sum equal to, or greater than, nine. If it exceeds nine, drop the nine as before, and carry the excess to the next figure, and then continue the addition as before. Proceed in this way till you have added all the figures in the multiplicand, and rejected all the nines contained in it, and write the final excess at the right hand of the multiplicand.

Proceed in the same manner with the multiplier, and write the final excess under that of the multiplicand. Multiply these excesses together, and place the excess of nines in their product at the right.

Then proeeed to find the excess of nines in the product obtained by the original operation; and if the work is correct, the excess thus found will be equal to the excess contained in the product of the above excesses of the multiplicand and the multiplier.

Example.	*Multiplicand.*	12345 =	6 excess	
	Multiplier.	2231 =	8 excess	
		12345	48 = 3	Proof.
		37035		
		24690		
		24690		
		27541695 =	3	

Note.—This method of proof, though perhaps sufficiently sure for common purposes, is not always a test of the correctness of an operation. If two or more figures in the work should be transposed, or the value of one figure be just as much too great as another is too small, or if a nine be set down in place of a cypher, or the contrary, the excess of nines will be the same, and still the work may not be correct. Such a balance of errors will not, however, be likely to occur unless designedly effected.

Third. Commencing at the units of the multiplicand, add together the digits in the odd places, rejecting 11 as often as it occurs (the same as the 9 in the former), and reserve the result. Proceed in the same manner with the digits in the even places, and from the former result, increased if necessary by 11, take this result, and place the excess opposite the multiplicand. In a similar way, find the excesses in the multiplier and product; then multiply the excesses of the two factors together, and find, in the same manner, the excess of their product. If this excess, and the excess of the product of the two factors be the same, the work is generally correct. If they differ, it must be wrong.

Ex. *Multiplicand*, $84963 = 9 - 10 = 10$
Multiplier, $44085 = 9 - 1 = 8$

Product, $3745593855 = 1 - 9 = 3$, 80 excess $= 3$.

In the above example (the work of which, for brevity, is omitted), we say $3 + 9$ are 12, which exceeds 11 by 1; $1 + 8$ are 9; then $6 + 4$ are 10, which being taken from $20, = (9 + 11)$, the excess is 10. Then in the multiplier, the sum of $5 + 0 + 4$ are 9, from which 1, the excess of $8 + 4$ above 11, being taken, the excess is 8. In the product, $8 + 5$ are 13; 2, the excess, and 9 are 11, which is rejected; 5 and 7 are 12, the excess of which is 1; then taking the even numbers 5 and 3 and 5 are 13; the excess $2 + 4 + 3$ are 9; and 9 from 12 ($= 11 + 1$), and the excess is 3. The product of the first two excesses is 80, and the excess of 11s is 3, the same as the excess of the product of the factors.

Fourth. Divide the multiplicand by 11, and set the excess or remainder to the right. Do the same with the multiplier; then divide the product of these excesses by

11, and set down the remainder. Also divide the product of the factors by 11, and if the excess of 11s be the same as the excess of 11s in the product from the above excesses, the work is right. Thus, in the foregoing example, the excess of the multiplicand is 10, and the excess in the multiplier 8, which being multiplied by 10, gives 80, the excess of which is 3.

The excess of the product is also 3, which proves the work to be right.

Note.—This method is not applicable in this place, as the pupil is not supposed to be able to do Division.

The reasons for these last three methods of proof are given in Art. 93, props. 15 and 18.

Mental Exercises in Multiplication.

1. A ship sails 11 miles in an hour; how far will she sail in 7 hours? in 4 hours? in 8 hours?

2. At 3 shillings a pound, what will 12 lbs. of tea cost? 6 lbs.? 8 lbs.?

3. If the interest of 1 dollar is 6 cents for 1 year, what is the interest on 8 dollars for the same time? of 10 dollars?

4. If a box hold 36 apples, how many will 6 boxes of the same size hold? 8 boxes? 10 boxes?

5. If a man earn 4 dollars in one week, how many will he earn in 8 weeks?

Mental Exercises in Multiplication, Addition, and Subtraction combined.

1. 3 fives, and 6 fives, and 7 fives, are how many fives? are how many?

2. John has 5 pens; Robert has 5 times as many, and Charles 6 times as many as John; how many times 5 pens, and how many pens do they all possess?

3. Lucy has 10 cents; her father gave her 6 more, her brother 5, and she worked a purse which she sold for 37 cents; how many cents had she remaining after buying 3 books at 8 cents each?

4. 6 + 8 + 7 multiplied by 7, + how many are 160?

Exercises for the Slate.

1.	1816 × 10	3.	40376 × 400	5.	7854 × 16
2.	40376 × 40	4.	81967 × 13	6.	4079 × 12000

ANSWERS.

1.	18160	3.	16150400	5	125664
2.	1615040	4.	1065571	6.	48948000

		ANSWERS.
7.	148 × 53	7844
8.	958 × 34	32572
9.	31416 × 175	5497800
10.	15607 × 3094	48288058
11.	1368752 × 72	98550144
12.	1267887621 × 468	593371406628
13.	14638887425 × 3672	53753994624600

14. Find the continued product of 12 17, and 19.
Ans. 3876.

15. Find the continued product of 3781, 3782, and 3783.
Ans. 54095923986.

16. How many yards of linen are there in 759 pieces, each containing 25 yds.?
Ans. 18975.

17. It is found by microscopic observations, that in each square inch of the human skin there are about 1000 pores; and the surface of the body of a middle-sized man contains about 2304 inches, or 16 square feet. Required, the number of pores in the surface of such a body, 999 being supposed to be contained in each inch?
Ans. 2301696.

CONTRACTIONS IN MULTIPLICATION.

35. The general rule is adequate to the solution of all examples that occur in Multiplication. In many instances,

however, by the exercise of judgment in applying the preceding principles, the operation may be very much *abridged.*

36. The contractions under this rule, if perfectly familiar both in their extent and application, will enable the pupil to abridge very materially nearly one-half of the usual business processes in multiplication.

The abridged processes can be applied with a very great saving of time and labor, in computing interest and other usual counting-house calculations.

The contractions, which are limited in their application, if they have but little practical value, will serve as excellent exercises for mental training.

NOTE.—The following contractions may be studied by the pupils as soon as they are prepared to understand them. The principle of each contraction should be fully shown to the pupils.

37. Any number which may be produced by multiplying two or more numbers together, is called a *Composite number.*

Thus, 4, 15, 132 are composite numbers; for 4 = 2 × 2; 15 = 5 × 3; 132 = 4 × 3 × 11.

NOTE.—The *factors* which, being multiplied together, produce a composite number, are sometimes called *component parts* of the number. The process of finding the factors of which a given number is composed, is called *resolving the number into factors.*

The *factors* into which a number may be *resolved*, must not be confounded with the *parts* into which it may be separated. The former has reference to Multiplication, the latter to Addition. Thus 56 may be resolved into two *factors*, 8 and 7; it may be separated into two or more *parts*, 50 or 5 tens and 6, or 25 and 31.

38. *To Multiply by a Composite Number.* Resolve the multiplier into two or more factors, and proceed as in Continued Multiplication, (Art. 33.)

39. *To Multiply by* 5. Add a cypher, or rather conceive it to be added to the multiplicand, and take half the result. This is evidently the same as multiplying by 10, and taking half the product.

40. *To Multiply by* 15. Conceive a cypher to be added to the multiplicand, and to the result add half of itself

41. *To Multiply by* 25. Conceive two cyphers to be added to the multiplicand, and take one fourth of the result. In like manner, because 125 is one eighth of 1000. To multiply by 125, conceive three cyphers to be added to the multiplicand, and take one eighth of the result.

42. *To Multiply by* 75. Conceive two cyphers to be added to the multiplicand, and from the result take one fourth of itself.

43. *To Multiply by* 175. Divide 700 times the multiplicand by 4.

44. *To Multiply by* 9, 99, 999, *or any other number of* 9s. Annex as many cyphers to the multiplicand as there are 9s in the multiplier; from the result subtract the given multiplicand, and the remainder will be the answer required.

44. *To Multiply by* 11. Set down the unit figure of the multiplicand for the first figure of the product, then add the digit in the ten's place in the multiplicand to the units of the same for the second figure of the product; proceed in the same manner with the others. Thus—

$$\begin{array}{r} 186743 \times 11 \\ 11 \\ \hline 2054173 \end{array}$$

We first set down the 3, then $4 + 3 = 7$, in the place of tens; $7 + 4 = 11$, which gives 1 in the place of hundreds, and one to carry; $6 + 7 + 1 = 14$, setting down the 4 in the place of thousands, and carrying 1 to the next place; and so on, we obtain the correct result. This is simply multiplying by 10 first, and adding once the multiplicand at the same time.

45. *To Multiply by any Multiple of* 11. By the figure denoting how many times 11 are contained in the multiplier, multiply first the unit's figure of the multiplicand, then the sum of the units and tens, then the sum of the tens and hundreds, &c., and lastly the left hand figure.

Thus, $176 \times 55 = (5 \times 11)$

$$\begin{array}{r} 5 \\ \hline 9680 \end{array}$$

$5 \times 6 = 30$, then $(7 + 6)\ 5 + 3 = 68$, and $(1 + 7)\ 5 + 6 = 46$, and finally $5 \times 1 + 4 = 9$.

46. *To find the Product of any Two Numbers between 10 and 20.* Arrange in decimal order, first the right hand figure of the product of the units, then the right hand figure of the sum of the units, then the product of the tens.

NOTE.—In writing figures in the decimal order, whenever the product or sum contains more than one figure, the left hand figure must be added to the figure in the next place on the left.

Ex. $16 \times 17 = 272$. Proceeding by the rule given above, the right hand figure of the product of the units (6×7) is 2; the right hand figure of the sum of the units $(6 + 7)$ is 3, plus the tens of the product of the units (4) is 7; the product of the tens (1×1) plus 1, is $2 = 272$.

The reason of tne rule may be illustrated by writing at full length the local value of the figures

Thus, $16 = 10 + 6$
and $17 = 10 + 7$

We have first the product of the units, then	42
the 7 units by the 10, and the 6 units by	13
the 10, which is the same as $(7 + 6) \times 10$,	1
or 13 in the ten's place; and lastly we have	
the product of the 1 ten by 1 ten $= 100$, or	272
1 in the place of hundreds.	

NOTE.—All the following rules may be illustrated in nearly the same manner.

EXERCISES.

1.	16×12	6.	14×19	11.	19×19
2.	18×12	7.	15×13	12.	19×18
3.	19×16	8.	17×14	13.	19×17
4.	15×14	9.	19×13	14.	19×15
5.	18×13	0.	13×12	15.	17×15

17. In one foot tnere is 12 inches; how many inches are there in 13 feet? in 16 feet?

18. In one pound, Troy, there are 12 ounces; how many are there in 15 lbs.? in 16 lbs.?

19. In one pound, Avoirdupois, there are 16 ounces; how many are there in 17 pounds? how many in 13 pounds?

20. How many rods in a piece of land that is 18 rods long, and 17 rods wide?

47. *To find the Product of any Two Numbers, of two figures each, when the unit's figure in each is* 1. rrange in decimal order the unit figure (1), the sum of the tens, and the product of the tens. Ex. 61 × 71 = 4331. The unit figure is 1; the sum of the tens (6 + 7) = 13; the product of the tens (6 × 7), plus the right hand figure of the sum of the tens (1), is 43.

EXAMPLES.

1.	21 × 21	5.	61 × 21	9.	31 × 31
2.	31 × 21	6.	71 × 21	10.	41 × 31
3.	41 × 21	7.	81 × 21	11.	51 × 31
4.	51 × 21	8.	91 × 31	12.	61 × 31

13. In one guinea there are 21 shillings; how many are there in 61 guineas?

14. If I pay 31 cents for one pound of butter, how many cents will I have to pay for 51 pounds?

NOTE.—By the use of the extended Multiplication Table, we can use the above rule when there are three figures, as 121 x 171, or we may make use of the former rule to multiply 12 by 17. The same may be said of all the others.

48. *To find the Product of Two Numbers, when the sum of the units is ten, and the preceding figure or figures are alike in each.* Multiply the preceding figure or figures of one of the numbers by the preceding figure or figures of the other, increased by one, and prefix the product to the product of the units. When the product of the units is less than ten (10), a cypher must be written in the ten's place.

Ex. 21 × 29 = 609. The preceding figure of one number (2) by the same figure increased by one (2 + 1), is = 6; the product of the units (9 × 1), is 9; written with a cypher prefixed, 09.

Exercises.

1.	62 × 68	5.	141 × 149	9.	195 × 195
2.	63 × 67	6.	161 × 169	10.	194 × 196
3.	84 × 86	7.	95 × 96	11.	174 × 176
4.	91 × 99	8.	184 × 186	12.	33 × 37

13. If a car-wheel turn 188 times in a minute, how many times will it turn in 182 minutes?

49. *To find the Product of Two Numbers containing two figures each, when the unit figures are alike, and the sum of the tens is* 10. Prefix the product of the tens, plus the unit's figure of one of the numbers to the product of the units. Thus, 24 × 84 = 2016. When the product of the units is less than ten (10), a cypher must be written in the place of tens.

Ex. In the above, the product of the tens is 16, plus the unit figure (4), = 20; the product of the units, 4 × 4, = 16, which, with the 20 prefixed, = 2016

Exercises.

1.	22 × 82	5.	89 × 99	9.	28 × 88
2.	23 × 83	6.	99 × 19	10.	33 × 73
3.	47 × 67	7.	56 × 56	11.	163 × 43
4.	49 × 69	8.	76 × 36	12.	121 × 131

13. What will 46 bus. oats cost at 36 cents per bushel?

14. 93 sheep at 13 dollars each?

50. *To find the Product of Two Numbers when the sum of the units is* 10, *and the difference of the figures preceding the units' figures is* 1. Prefix the square of the figure or figures preceding the units of the larger number, less 1, to the difference between the square of the units of the larger number and 100.

Ex. 126 × 134 = 16884. The square of 13 (the figures preceding the units of the larger number), is 169; less 1, is 168. The square of 4 (the units of the larger number) is 16; 100 — 16 = 84; 168 prefixed to 84 is 16884.

Note.—The product of a number multiplied into itself is called the square of that number.

EXERCISES.

1.	21 × 39	5.	43 × 57	9.	54 × 66
2.	31 × 49	6.	51 × 69	10.	55 × 65
3.	27 × 33	7.	52 × 48	11.	101 × 119
4.	36 × 44	8.	53 × 67	12.	149 × 151

13. How many square feet are there in a lot of land that is 146 feet long, and 134 feet wide.

51. *To find the Product of Two Numbers containing two figures each, when the figures in the place of units are alike.* Arrange in decimal order, first the right hand figure of the product of the units; then the right hand figure of the product of the sum of the tens by the unit's figure of one of the numbers; then the product of the tens.

Ex. 27 × 47 = 1269. The product of the units (7 × 7) is 49; the right hand figure is 9. The sum of the tens (4 + 2), multiplied by the units (7), gives 42; plus 4 (from the 49) are 46; the right hand figure is 6. The product of the tens (4 × 2) is 8; plus the 4 (from the 46) are 12 = 1269.

EXERCISES.

1.	22 × 32	5.	26 × 46	9.	29 × 59
2.	22 × 72	6.	26 × 56	10.	29 × 69
3.	23 × 33	7.	28 × 48	11.	29 × 89
4.	24 × 44	8.	28 × 88	12.	29 × 99

13. In one day there are 24 hours; how many are there in 84 days?

14. A person bought 65 turkeys at 65 cents each; what did they cost him?

52. *To find the Product of Two Numbers containing two figures each when the figures in the place of tens are alike.* Arrange in decimal order the right hand figure of the

product of the units, the right hand figure of the product of the sum of the units by one of the tens, and then the product of the tens.

Ex. $46 \times 47 = 2162$. The product of the units (7×6) is 42; the right hand figure is 2. The sum of the units $(6 + 7)$, multiplied by one of the tens (4), gives 52; plus 4 (from the 42) are 56; the right hand figure is 6. The product of the tens (4×4) is 16; plus 5 (from the 56) are $21 = 2162$.

Exercises.

1. 37×37	4. 88×88	7. 72×78
2. 38×38	5. 99×99	8. 97×97
3. 43×47	6. 87×83	9. 106×107

53. *To find the Product of any two Mixed Numbers whose fractional parts are halves.* Prefix the product of the whole numbers, plus one half their sum, to the fraction $\frac{1}{4}$. Thus, $3\frac{1}{2} \times 3\frac{1}{2} = 12\frac{1}{4}$.

When the sum of the whole number is not an even number, add one half of the next lower number to the product, and prefix the sum to $\frac{3}{4}$. Thus, $2\frac{1}{2} \times 3\frac{1}{2} = 8\frac{3}{4}$.

Exercises.

1. $6\frac{1}{2} \times 8\frac{1}{2}$	5. $8\frac{1}{2} \times 12\frac{1}{2}$	9. $10\frac{1}{2} \times 10\frac{1}{2}$
2. $6\frac{1}{2} \times 10\frac{1}{2}$	6. $8\frac{1}{2} \times 11\frac{1}{2}$	10. $10\frac{1}{2} \times 11\frac{1}{2}$
3. $7\frac{1}{2} \times 8\frac{1}{2}$	7. $9\frac{1}{2} \times 10\frac{1}{2}$	11. $12\frac{1}{2} \times 12\frac{1}{2}$
4. $7\frac{1}{2} \times 11\frac{1}{2}$	8. $9\frac{1}{2} \times 11\frac{1}{2}$	12. $11\frac{1}{2} \times 12\frac{1}{2}$

13. If a man walk $3\frac{1}{2}$ miles in one hour, how many miles can he walk in $15\frac{1}{2}$ hours?

14. What will $16\frac{1}{2}$ lbs. of raisins cost at $12\frac{1}{2}$ cents a pound?

54. *To find the Product of Two Mixed Numbers, when the whole numbers are alike, and the sum of the fraction is 1.* Multiply one whole number by the other, increased by 1, and prefix the product to the product of the fractions. Thus, $6\frac{2}{3} \times 6\frac{3}{5} = 42\frac{6}{25}$.

Exercises.

1.	$4\frac{1}{4} \times 4\frac{3}{4}$	4.	$7\frac{1}{8} \times 7\frac{7}{8}$	7.	$8\frac{2}{13} \times 8\frac{11}{13}$
2.	$5\frac{2}{5} \times 5\frac{3}{5}$	5.	$7\frac{2}{10} \times 7\frac{8}{10}$	8.	$10\frac{6}{11} \times 10\frac{5}{11}$
3.	$5\frac{2}{7} \times 5\frac{5}{7}$	6.	$8\frac{1}{4} \times 8\frac{3}{4}$	9.	$10\frac{10}{13} \times 10\frac{3}{13}$

10. Coal from Pictou is sold for $8\frac{3}{4}$ dollars a chaldron; what is paid for $8\frac{1}{4}$ chaldrons?

55. *To find the Product of any Two Mixed Numbers, when the difference of the whole numbers is* 1, *and the sum of the fractional parts is* 1. Prefix the square of the larger number, less 1, to the difference of the square of the fraction of the larger number and 1. Thus, $5\frac{1}{6} \times 6\frac{5}{6} = 35\frac{11}{36}$.

Exercises.

1.	$2\frac{1}{3} \times 3\frac{2}{3}$	4.	$7\frac{3}{11} \times 8\frac{8}{11}$	7.	$15\frac{1}{8} \times 14\frac{7}{8}$
2.	$5\frac{5}{6} \times 6\frac{1}{6}$	5.	$8\frac{3}{5} \times 9\frac{2}{5}$	8.	$17\frac{1}{3} \times 16\frac{2}{3}$
3.	$7\frac{3}{4} \times 8\frac{1}{4}$	6.	$13\frac{1}{7} \times 14\frac{6}{7}$	9.	$19\frac{1}{3} \times 20\frac{2}{3}$

56. *To find the Product of any Two Numbers ending in* 5. Prefix the product of the figures preceding the 5 in each number, plus $\frac{1}{2}$ their sum to 25. Thus, $185 \times 45 = 8325$

When the sum of the preceding figures is an odd number, *add to the product* $\frac{1}{2}$ *of the next smaller number, and prefix the sum to* 75. Thus, $55 \times 25 = 1375$

Examples.

1.	25×25	4.	75×85	7.	145×175
2.	35×45	5.	95×85	8.	165×135
3.	85×45	6.	105×125	9.	125×145

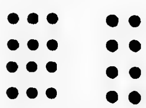
DIVISION.

57. Division is the method of finding how often one number, called the Divisor, is contained in another number, called the Dividend. The result is called the Quotient. The number which is sometimes left, after dividing, is called the Remainder, and is always of the same name as the dividend.

Division is the opposite of Multiplication, and as the latter is an extension of Addition, so, in like manner, Division may be regarded as an extension of Subtraction.

58. If any number be divided into two or more groups of units, then the collected units will contain the divisor as often as it is contained in the parts.

Thus, $20 = 12 + 8$; therefore $\frac{20}{4} = \frac{12}{4} + \frac{8}{4}$.



To illustrate this axiom, let 20 counters or dots be arranged, as in the annexed figure: here we first observe that 4 can be taken out of 20 5 times. In the group to the left there are 12 counters, out of which 4 can be taken 3 times, and in the group of the right there are 8 counters, out of which 4 can be taken 2 times. That is, 4 will be contained in 20 the same number of times that it is contained in 12, together with the number of times it is contained in 8. It is on this principle that we perform operations of Division.

59. Division is of two kinds—SIMPLE and COMPOUND. It is called Simple Division, when the dividend and divisor are, both of them, either abstract numbers, or concrete numbers, of one and the same denomination.

It is called Compound Division, when the dividend, or when both divisor and dividend, contain numbers of different denominations, but of one and the same kind.

60. In Division, if the dividend be a concrete number, the divisor may be either a concrete number or an abstract number, and the quotient will be an abstract number or a concrete number, according as the divisor is concrete or abstract.

For instance, 5 shillings taken 6 times, give 30 shillings; therefore, 30 shillings divided by 5 shillings, give the abstract number 6 as quotient; and 30 shillings divided by 6, give the concrete number 5 shillings as quotient.

SIMPLE DIVISION.

61. Simple Division is generally divided into two kinds —Short Division and Long Division.

When the process of dividing is carried on in the mind, and the quotient only is set down, the operation is called Short Division.

When the result of each step in the operation is written down, the process is called Long Division.

SHORT DIVISION.

62. **Rule.** Place the divisor and dividend thus:

Divisor,) Dividend,
Quotient.

By the Multiplication Table, find how often the divisor is contained in the first figure, or, if necessary, in the number expressed by the first two, or the first three figures of the dividend, and write down the figure denoting the number of times directly under the figure or figures divided. Find the product of this figure and the divisor, and take it from the number expressed by the figure or figures of the dividend formerly used. If there is a remainder after dividing any figure, prefix it mentally to the next figure of the dividend, and divide as before; and if the divisor is not contained in any figure of the dividend, place a cypher in the quotient, and prefix this figure to the next one of the dividend, as if it were a remainder. If there be a remainder at the conclusion, write it, with the divisor under it, at the end of the quotient.

In order to render the division complete, it is obvious that the *whole* of the dividend must be divided. But when there is a remainder after dividing the last figure of the dividend, it must of necessity be smaller than the divisor, and cannot be divided by it. We therefore represent the division by placing the remainder over the divisor, and annex it to the quotient.

The reason of this is clear; for, were it required to divide 7 apples among 3 boys, each boy would get 2 whole apples, and there would be 1 remaining. Now, each boy is as much entitled to a share of this 1 apple, namely, $\frac{1}{3}$, as he is to the 2 whole apples. We therefore write the quotient $2\frac{1}{3}$.

Ex. Let it be required to divide 1738 dollars among 7 persons. Putting, then, the divisor and dividend as directed, we say 7 is contained in 17 (or, for brevity, 7 in 17) twice and 3 over, and we write 2 under the 7 in the dividend; then the remainder 3 and the next figure 3 annexed to it, expresses 33—7 in 33 four times and 5 over; writing 4 under the 3, and to the remainder 5, annexing, mentally, the 8, we have 58: lastly, 7 in 58, 8 times and 2 over; we set 8 under the 8 in the dividend, and after it, the remainder 2, with the divisor 7 under it, to complete the quotient. Hence, the share of each would be 248 dollars, with a seventh part of 2 dollars that remain at the end of the operation.

$$\begin{array}{r} 7)1738 \\ \hline 248\frac{2}{7} \end{array}$$

The expression $\frac{2}{7}$, meaning, as we have seen, a seventh part of 2 dollars, is less than one dollar; and being, therefore, as it were, a broken quantity in comparison of a *whole* dollar, it is called a Fraction of a dollar; while the dollar, in reference to the fraction, is called the Integer.

As 2 dollars are twice as great as 1 dollar, a seventh part of 2 dollars is evidently 2 times or twice as great as a seventh part of 1 dollar. It is plain, therefore, that the expression $\frac{2}{7}$ is the same in value as two sevenths of 1 dollar; and hence it is read *two sevenths* of a dollar. In like manner, $\frac{5}{8}$ is *read* five eighths; $\frac{1}{6}$ one sixth, &c.

It may be shown, also, in a similar manner, that, if a shilling be the integer, the fraction $\frac{2}{7}$ means either two sevenths of one shilling, or one seventh of two shillings; while, in reference to a ton, $\frac{2}{7}$ means equally two sevenths of one ton, or one seventh of two tons.

In any fraction, expressed in the same manner that has been now explained, the upper number is called its Numerator, and the lower its Denominator. Thus, in $\frac{2}{3}$, 2 is the numerator, and 3 the denominator.

Note.—The theory and management of fractions form an important branch of Arithmetic, which will be discussed at due length, in a more advanced part of the work. With the few views and explanations given above, as well as those given in Art. 8, Part I., the pupil should be made thoroughly acquainted.

Mental Exercises in Division.

1. If one yard of cloth can be bought for 5 dollars, how many can be bought for 50 dollars? how many for 35 dollars?

2. How many eggs can be bought for 63 cents, at the rate of a dozen for 9 cents?

3. How long will it take a horse to travel 76 miles, if he travels 8 miles in each hour?

4. If one man can do a piece of work in 56 days, how many men will be required to do the same in 7 days?

5. I have 47 pounds of tea in a chest; how many packages of 8 pounds can I put up, and how many pounds will remain in the chest? If I make 6 packages out of the whole, how many pounds will be in each package?

Exercises for the Slate.

1. 470850 ÷ 3	4 3782047 ÷ 6	7. 74593822 ÷ 9
2. 1829765 ÷ 4	5. 7165537 ÷ 7	8. 53248675 ÷ 11
3. 4265983 ÷ 5	6. 27459332 ÷ 8	9. 49275189 ÷ 12

ANSWERS.

1. 156950	4. $630341\frac{1}{6}$	7. $8288202\frac{4}{9}$
2. $457441\frac{1}{4}$	5. $1023648\frac{1}{7}$	8. $4840788\frac{7}{11}$
3. $853196\frac{3}{5}$	6. $3432416\frac{4}{8}$	9. $4106265\frac{9}{12}$

LONG DIVISION.

63. Rule. Place the divisor and dividend thus:

Divisor,) Dividend, (Quotient.

Take off from the left hand of the dividend the least number of figures which make a number not less than the divisor; then find (by the Multiplication Table), how often the first figure on the left hand side of the divisor is contained in the first figure, or the first two figures on the left hand side of the dividend, and place the figure which denotes this number of times in the quotient; multiply the divisor by this figure, and bring down the product, and subtract it from the number which was taken off at the left of the dividend; then bring down the next figure of the dividend, and place it to the right of the remainder, and proceed as before. If the divisor be greater than this remainder, affix a cypher to the quotient, and bring down the next figure from the dividend to the right of the remainder, and proceed as before. Carry on this operation till all the figures of the dividend have been thus brought down, and the quotient, if there be no remainder, will be thus determined, or if there be a remainder, the quotient and the remainder will be thus determined.

NOTE.—If any product be greater than the number which stands above it, the last figure in the quotient must be changed for one of smaller value; but if any remainder be greater than the divisor, or equal to it, the last figure of the quotient must be changed for one of greater value.

Ex. Divide 2338268 by 6758.

Proceeding by the Rule given above, we obtain—

```
6758 ) 2338268 ( 346, Quotient.
       20274··
       -----
        31086
        27032
        -----
         40548
         40548
         -----
```

The reason for the Rule will appear from the following considerations.

The divisor represents six thousand, seven hundred and fifty-eight; the first five figures on the left hand side of the dividend represent two millions, three hundred and thirty-eight thousand, and two hundred.

Now, the divisor is contained in this 300 times; and $6758 \times 300 = 2027400$, or, omitting the two cyphers at the end for convenience in working, we properly place 4 under 2 in the line above; we subtract the product thus found, and we obtain a remainder of 3108, which represents three hundred and ten thousand, and eight hundred. Bring down the six, by the Rule, (it is proper, for preventing mistakes, to put a dot below each figure of the given dividend, when it is *brought down*); this 6 denotes 6 tens or 60, but the cypher is omitted for the reason above stated; the number now represents three hundred and ten thousand, eight hundred and sixty; 6758 is contained 40 times in this, and 6758 $\times$ 40 = 27032. We omit the cypher at the end as before, and subtract the 27032 from the 31086; and, after subtraction, the remainder is 4054, which represents forty thousand, five hundred and forty. Bring down the 8, by the Rule, and the number now represents forty thousand, five hundred and forty-eight; 6758 is contained 6 times exactly in this number.

Therefore, 346 is the quotient of 2338268 by 6758.

64. The above example, worked without omitting the cyphers, would have stood thus:

```
6758)2338268(300 + 40 + 6.
     2027400
     -------
      310868
      270320
     -------
       40548
       40548
     -------
```

Hence it appears that the divisor is subtracted from the dividend 300 times, and then 40 times from what remains, and then 6 times from what remains, and there being now no remainder, 6758 is contained exactly 346 times in 2338268 (Art. 58).

NOTE.—It will help us to understand how greatly division abbreviates subtraction, if we consider how long a process would be required to discover, by actually subtracting it, how often 6758 is contained in 2338268.

65. *If the divisor terminate with cyphers, the process can be abridged by the following method.*

RULE. Cut off the cyphers from the divisor, and as many figures from the right hand of the dividend as there are cyphers so cut off at the right hand of the divisor; then proceed with the remaining figures according to the Rule (Art. 63); and to the last remainder annex the figures cut off from the dividend for the total remainder

Ex. Divide 537523 by 3400.
Proceeding by Rule—

```
34,00)5375,23(158
      34··
      ----
      197
      170
      ----
       275
       272
      ----
         3
```

Therefore 3400 is contained in 537523, 158 times, with remainder 323.

The reason of the Rule will appear from the following considerations.

537523 is 5375 hundreds and 23, of which 537500 contains 3400 158 times, with a remainder 300 over; and as 23 does not contain 3400 at all, the quotient will evidently be 158, with remainder 300 + 23, or 323.

NOTE.—The same rule applies when the divisor and dividend both terminate with cyphers.

METHODS OF PROOF.

First. Find the product of the divisor and quotient, and add to it the remainder; if the sum be equal to the dividend, the work is correct.

Second. First cast the 9s out of the divisor and quotient, and multiply the excesses together; to the product add the excess of 9s in the remainder, if any after division; cast the 9s out of this sum, and set down the excess; finally cast the 9s out of the dividend, and if the *excess* is the *same* as that obtained from the divisor and *quotient*, the work may be considered right.

Third. Commencing at the units of the divisor, add together the digits in the odd places, rejecting 11 as often as possible, and reserve the result; proceed in the same manner with the digits in the even places, and from the former result, increased, if necessary, by 11, take this result, and place the excess below the divisor. In a similar way find the excesses of the dividend, quotient and remainder; then multiply the excess of the divisor by the excess of the quotient and add to the product the excess of the remainder, and if the excess of this sum be equal to the excess of the dividend, the work is right; if they differ, the work is wrong.

EXERCISES.

		ANSWERS.
1	764235 ÷ 51	14985
2.	557442 ÷ 31	17982
3.	945054 ÷ 43	21978
4.	3387612 ÷ 121	$27996\frac{96}{121}$
5.	51846734 ÷ 102	$508301\frac{23}{102}$
6.	980263711 ÷ 809	$1211698\frac{29}{809}$
7.	1700649160000 ÷ 759	$2240644479\frac{439}{759}$
8.	9302688 ÷ 14356	648
9.	75843639426 ÷ 8593	$8826211\frac{8303}{8593}$
10.	1111111111111 ÷ 854	$1301066874\frac{715}{854}$
11.	1000000000000000 ÷ 111	$9009009009009\frac{1}{111}$
12.	1000000000000000 ÷ 81	$12345679012345\frac{55}{81}$
13.	267817938473 ÷ 8760	$30572824\frac{233}{8760}$

DIVISION BY COMPOSITE NUMBERS.

66. A number which cannot be separated into factors, which are respectively greater than unity, is called a PRIME Number.

Thus, 3, 5, 7, 11, 13, are prime numbers.

A number which can be separated into factors respectively greater than unity, or which, in other words, is produced by multiplying together two or more numbers respectively greater than unity, is called a COMPOSITE Number. Thus, 4, which $= 2 \times 2$; 6, which $= 2 \times 3$; 8, which $= 2 \times 2 \times 2$, are composite numbers, because they are composed or consist of the product of two or more numbers, each of which is greater than unity.

Numbers which have no common factor greater than unity, are said to be PRIME to one another. Thus, the numbers 3, 5, 8, 11, are prime to each other.

NOTE.—The learner must be careful not to confound numbers which are *prime to each other* with *prime* numbers: for numbers that are prime to each other may themselves be *composite* numbers. Thus, 4 and 9 are prime to each other, while they are composite numbers.

67. When the divisor is a composite number, and made up of two or more factors, neither of which exceeds 12, the dividend may be divided by one of the factors in the way of SHORT DIVISION, and then the result (or quotient) by the other factor; and so on, till all the factors are employed. The last quotient will be the answer.

68. To find the *true* remainder.

If the divisor is resolved into but two factors, multiply the last remainder by the first divisor, and to the product add the first remainder, if any, and the result will be the true remainder.

When more than two factors are employed, multiply each remainder by all the preceding divisors, to the sum of their products add the first remainder, and the result will be the true remainder.

Or multiply the last remainder by the preceding divisor and add in the preceding remainder; then multiply this sum by the next preceding divisor, and to it add the next preceding remainder, and so on, till all the remainders have been added or taken in. The last result will be the true remainder.

Ex. Divide 507 by 64.

64 {	2	507			
	8	253	— 1 rem.		= 1
	4	31	— 5 rem.	Now 5 × 2	= 10
		7	— 3 rem.	And 3 × 8 × 2	= 48
					59 true rem.

Second method:

$$64\left\{\begin{array}{l|l} 2 & 507 \\ 8 & 253 - 1 \\ 4 & 31 - 5 \\ & 7 - 3 \end{array}\right. \qquad \left.\begin{array}{l} 3 \times 8 + 5 = 29 \end{array}\right\} 29 \times 2 + 1 = 59$$

The reason for the above Rules is manifest from the following considerations:

31 is 4 times 7, together with 3.
and 253 is 8 times 31, together with 5.
and 507 is 2 times 253, together with 1.
or is 2 times (8 times 31 + 5), together with 1.
or is 16 times (4 times 7 + 3) together with 10 + 1.
or is 64 times 7 + 48 + 10 + 1.
or is 64 times 7 + 59.

Exercises.

		Answers.
.	795456 ÷ 84	$9469\frac{62}{84}$
2.	41763481 ÷ 625	$66821\frac{356}{625}$
3.	317682 ÷ 18	17649
4.	25760 ÷ 56	460
5.	234765 ÷ 192	$1222\frac{141}{192}$

GENERAL PRINCIPLES IN DIVISION.

69. From the nature of Division, it is evident that the *value* of the *quotient* depends both on the *divisor* and the *dividend.*

70. If a given divisor is contained in a given dividend a certain number of times, the same divisor will obviously be contained,

In *double* that dividend, *twice* as many times.

In *three times* that dividend, *thrice* as many times, &c. Hence, *If the divisor remains the same, multiplying the dividend by any number, is in effect multiplying the quotient by that number.*

Thus, 6 is contained in 12, 2 times; in 2 times 12, or 24, 6 is contained 4 times (i. e. twice 2 times); in 3 times 12, or 36, 6 is contained 6 times (i. e. thrice 2 times), &c.

71. Again, if a given divisor is contained in a given dividend a certain number of times, the same divisor is contained,

In *half* that dividend, *half* as many times.

In a *third* that dividend, a *third* as many times, &c. Hence, *If the divisor remains the same, dividing the dividend by any number, is in effect dividing the quotient by that number.*

Thus, 8 is contained in 48, 6 times: in 48 ÷ 2, or 24 (half of 48), 8 is contained 3 times (i. e. half of 6 times); in 48 ÷ 3, or 16 (a third of 48), 8 is contained 2 times (i. e. a third of 6 times), &c.

72. If a given divisor is contained in a given dividend a certain number of times, then, in the same dividend,

Twice that divisor, only *half* as many times;

Three times that divisor, a *third* as many times, &c. Hence, *If the dividend remains the same, multiplying the divisor by any number, is in effect dividing the quotient by that number.*

Thus, 4 is contained in 24, 6 times; 2 times 4, or 8, is contained in 24, 3 times (i. e. half of 6 times); 3 times 4, or 12, is contained in 24, 2 times (i. e. a third of 6 times), &c.

73. If a given divisor is contained in a given dividend a certain number of times, then in the same dividend,

Half that divisor is contained *twice* as many times;

A *third* of that divisor, *three times* as many times, &c. Hence, *If the dividend remains the same, dividing the divisor by any number, is in effect multivlying the avotient by that number.*

Thus, 6 is contained in 36, 6 times; 6 ÷ 2, or 3 (half of 6), is contained in 36, 12 times (i. e. twice 6 times); 6 ÷ 3, or 2 (a third of 6). is contained in 36, 18 times (i e. thrice 6 times), &c.

74. From the preceding articles, it is evident that any given divisor is contained in any given dividend just as many times as *twice* that divisor is contained in *twice* that dividend; *three times* that divisor in *three times* that dividend, &c.

Conversely, any given divisor is contained in any given dividend just as many times as *half* that divisor is contained in *half* that dividend; a *third* of that divisor in a *third* of that dividend, &c. Hence,

75. *If the divisor and dividend are both multiplied, or both divided by the same number, the quotient will not be altered.*

Thus, 6 is contained in 12, 2 times;
2 times 6 is contained in 2 times 12, 2 times;
3 times 6 is contained in 3 times 12, 2 times, &c.
Again, 12 is contained in 48, 4 times;
$12 \div 2$ is contained in $48 \div 2$, 4 times;
$12 \div 3$ is contained in $48 \div 3$, 4 times, &c.

76. If the *sum* of two or more numbers is divided by any number, the quotient will be equal to the *sum* of the quotients which will arise from dividing the given numbers separately.

Thus, the sum of $12 + 18 = 30$; and $30 \div 6 = 5$.

Now, $12 \div 6 = 2$; and $18 \div 6 = 3$; but the sum of $3 + 2 = 5$.

Again, the sum of $32 + 24 + 40 = 96$; and $96 \div 8 = 12$.

Now, $32 \div 8 = 4$; $24 \div 8 = 3$; $40 \div 8 = 5$; but $4 + 3 + 5 = 12$.

77. If a given divisor is contained a certain number of times in a given dividend, then in the same dividend,

If the divisor be increased by *one half*, *one third*, *three fourths*, or any other fractional part of itself, the quotient will be the same fractional part of itself too small.

If the divisor be diminished by any fractional part of itself, the quotient will be the same fractional part of itself too great.

78. If the divisor remain the same, and the dividend be increased *one half*, the quotient will be too great by *one third* of itself; and if increased *one third*, it will be too great by *one fourth* of itself, &c.

If the dividend be diminished by *one third* of itself, the quotient will be too small by *one half* of itself; and if diminished by *one fourth*, too small by *one third* of itself, &c.

79. It is evident, from the foregoing principles, whatever operation is performed on the divisor, in order to find the true value of the quotient, the same must be performed on the dividend

MULTIPLICATION AND DIVISION BY FRACTIONAL NUMBERS.

80. In Arithmetical operations, it is often necessary to multiply or divide by numbers containing fractions; and, though the method of doing this will be fully given in the Articles on Multiplication and Division of Fractions, it may be proper here to point out the modes of proceeding in the simplest cases. Every such operation requires both multiplication and division; and hence it may be introduced with propriety here.

Ex $1234 \times 29\frac{1}{2}$

$$\begin{array}{r} 1234 \\ 29\frac{1}{2} \\ \hline 11106 \\ 2468 \\ 617 \\ \hline 36403 \end{array}$$

Here we multiply 1234 by 29 in the usual manner; but, before adding, we divide 1234 by 2, and, writing the quotient, 617, under the partial products, adding the three lines together, we find the required product is 36,403.

The answer would also be found by doubling $29\frac{1}{2}$, which gives 59; and then by multiplying 1234 by 59, and taking half the product.

Ex. 2. If the circumference of a carriage-wheel be $14\frac{2}{3}$ feet, how often will it turn round in going a mile, the mile containing 5280 feet?

$$\begin{array}{rl} 14\frac{2}{3}) & 5280(360 \\ 3 & 3 \\ \hline 44 & 15840 \\ & 132\cdot\cdot \\ \hline & 264 \\ & 264 \\ \hline & 0 \end{array}$$

In this example, the denominator is 3; we treble both divisor and dividend (Art. 78), thus obtaining 44 and 15840; and then by dividing the latter by the former, we get 360, the number of times required.

Exercises.	Answers.	Exercises.	Answers.
1. $13746 \times 2\frac{1}{2}$	34365	4. $41785 \div 2\frac{1}{2}$	16714
2. $477121 \times 1\frac{1}{2}$	$715681\frac{1}{2}$	5. $24579 \div 12\frac{4}{5}$	$1920\frac{15}{64}$
3. $4275 \times 4\frac{1}{4}$	$18168\frac{3}{4}$	6. $24248 \div 2\frac{3}{7}$	$9984\frac{8}{17}$

CONTRACTIONS IN DIVISION.

81. *To Divide by* 5. Double the dividend, and cut off the last figure of the result, the half of which figure will be the remainder. This is the same as doubling the dividend, and dividing by 10, (Art. 78).

82. *To Divide by* 15, 35, 45, or 55. Double the dividend, divide the result by 30, 70, 90, or 110, respectively, and for the true remainder take half the remainder thus found.

83. *To Divide by* 25. Multiply the dividend by 4, cut off two figures from the result, and take one fourth of the number expressed by them for the remainder.

84. *To Divide by* 125. Multiply the dividend by 8, cut off three figures from the result, and take one eighth of the number expressed by them for a remainder.

85. *To Divide by* 75. Multiply the dividend by 4, divide the product by 300, and for the true remainder take one fourth of the remainder thus found.

86. *To Divide by* 175, 225, *and* 275. Multiply the dividend by 4, divide the result in the first case by 700, in the second by 900, and in the third by 1100, and take one fourth of the remainder, in each case, for the true remainder.

EXERCISES.

		ANSWERS.
1.	317684 ÷ 5	$63536\frac{4}{5}$
2.	1623841617 ÷ 25	$64953664\frac{17}{25}$
3.	813764 ÷ 125	$6510\frac{14}{125}$
4.	12347681 ÷ 75	$164635\frac{56}{75}$
5.	247632 ÷ 175	$1415\frac{7}{175}$
6.	487695 ÷ 225	$2167\frac{120}{225}$
7.	8941364 ÷ 275	$32514\frac{14}{275}$
8.	12345678 ÷ 125	$98765\frac{53}{125}$

CANCELLATION.

87. We have seen that Division is finding a quotient, which, multiplied into the divisor, will produce the dividend (Art. 57). If, therefore, the dividend is resolved into two such factors that one of them is the divisor, the other factor will, of course, be the quotient. Suppose, for example, 42 is to be divided by 6. Now, the factors of 42 are 6 and 7, the first of which being the divisor, the other must be the quotient. Therefore, *cancelling a factor of any number, divides the number by that factor*. Hence,

88. When the dividend is the product of two factors, one of which is the same as the divisor,

Cancel the factor common to the dividend and divisor; the other factor of the dividend will be the answer.

Ex. Divide the product of 34 into 28 by 34.

Common method.

```
      34
      28
     ---
     272
     68
     ---
34)952(28
    68
    ---
    272
    272
    ---
```

By cancellation.

$$\frac{\not{3}\not{4})\not{3}\not{4} \times 28}{28}$$, Answer.

Cancelling the factor 34, which is common to both the divisor and dividend, we have 28 for the quotient, the same as before.

89. The method of contracting arithmetical operations, by rejecting equal factors, is called CANCELLATION.

90. When the divisor and dividend have common factors,

Cancel the factors common to both; then divide the product of those remaining in the dividend by the product of those remaining in the divisor.

Ex. $1260 \div 210$; $1260 = 15 \times 7 \times 12$, $210 = 5 \times 3 \times 7 \times 2$

$$\text{and } \not{5} \times \not{3} \times \not{7} \times \not{2})\overset{3}{\not{1}\not{5}} \times \not{7} \times 12$$

$$6$$

Here we say, first, that 5 is contained in 15, 3 times; again, 3 will cancel 3, 7 will cancel 7, and 2 into 12, 6 times.

NOTE.—The further *development* and *application* of the principles of this most useful Rule, may be seen in the Reduction of Compound Fractions to Simple ones; in Multiplication and Division of Fractions; in Simple and Compound Proportion.

ON THE CONSTRUCTION OF QUESTIONS.

91. When a class of beginners is being exercised in any or all of the fundamental Rules, it is often a great labor for the teacher to look over the work of each pupil; in such cases, the following elegant and really useful method of constructing questions is well deserving the

attention of the teacher. It will be observed that the questions may be formed as quickly as the figures can be written, and that the law of the figures, testing the accuracy of the answer, may be seen at a glance.

Ex.	167,832
	476,523
	18,981
	67,932
	731,268

Addition. Here, in each row, the corresponding figures on the left hand, added to those on the right, produce nines; and the same law is observed in the answer. The only precaution is simply that the sum of the last column should not exceed nine.

Ex. 1	78649,21350
	64327,35672
	14321,85678

Subtraction. Here the rows are formed in the same way as in Addition, and the figures in the answer follow the same law.

Ex. 2.	7467,1421
	3648,5240
	3818,6181

Instead of the figures in the question forming nines, any other number may be selected. In the above example, 8 is the number taken; but here, also, the figures in the answer follow the law of nines.

Multiplication. Ex. 1. 256,9,743 × 34 = 87371262.

Here the figures to the left and right of the nine, are formed in the same way as in addition, and the figures in the product follow the same law. When there are *three* figures in the multiplier, there must be *two* central nines, and so on.

The only precaution, is simply that the product by the last figure of the multiplier, when added to the other partial products, does not produce a number greater than 9, as was remarked in Addition, which may be easily obviated by taking low numbers for the highest digit's place in both multiplicand and multiplier.

Ex. 2. 136,8,752 × 18 = 2463,7536. Here there is a central 8, the other figures make up eights, the addition of the figures in the multiplier make up 9, and the answer is the law of Nines. When the addition of the figures in the multiplier makes nine or nines, the figures in the multiplicand may make up any particular number whatever.

Division. Ex. 1. 51)764,235(14985. Here the divisor is written at pleasure; the first three figures, in the dividend, are found by multiplying the divisor by 15, and taking 1 from the unit's figure. The remaining figures make up nines, as in Addition; the quotient figures follow the law of Nines.

Ex. 2. 62; 31)557442(17982. Here, to modify the form, we first write 62, and multiply by 9 (any other number will do), and thus form the dividend as in the last example; then take the half of 62, which gives 31, for the divisor. The figures in the quotient follow the law of Nines, as in the last example.

Ex. 3. 86; 43)945,054(21978.

Ex. 4. 484; 242)33876612(139986.

Ex. 5. 484: 121)33876612(279972.

PROPERTIES OF NUMBERS.

92. The progress, as well as the pleasure, of the student in Arithmetic, depends very much upon the accuracy of his knowledge of the terms which are employed in mathematical reasoning. Particular pains should therefore be taken to understand their true import.

Def. An integer signifies a *whole* number.

2. Whole numbers or integers are divided into *prime* and *composite* numbers.

3. A *Composite* number, we have seen, (Art. 66), is one which may be produced by multiplying two or more numbers together.

4. A *Prime* number is one which *cannot* be produced by multiplying any two or more numbers together; or which *cannot* be exactly divided by any *whole* number except a *unit* and *itself*.

5. An *Even* number is one which can be divided by 2, without a remainder, as 4, 6, 10, 12.

6. An *Odd* number is one which cannot be divided by 2 without a remainder, as 3, 7.

NOTE.—All even numbers, except 2, are *composite* numbers; an odd number is sometimes a *composite*, and sometimes a *prime* number.

7. One number is a *measure* of another, when the former is *contained* in the latter any number of times without a remainder. Thus, 3 is a measure of 15, &c.

8. One number is a *multiple* of another when the former can be *divided* by the latter without a remainder. Thus, 6 is a multiple of 3; 20 is a multiple of 5, &c.

NOTE.—A multiple is therefore a *composite* number, and the number thus contained in it, is always one of its factors.

9. The *Aliquot parts* of a number are the parts by which it can be *measured* or divided without a remainder. Thus, 5 and 7 are the aliquot parts of 35.

10. The *Reciprocal* of a number is the quotient arising from dividing a *unit* by that number. Thus, the reciprocal of 2 is $\frac{1}{2}$; of 3 is $\frac{1}{3}$, &c.

11. A perfect number is one which is equal to the sum of all its aliquot parts. Thus, $6 = 1 + 2 + 3$, the sum of its aliquot parts, and is therefore a perfect number.

All perfect numbers terminate with 6 or 28.

93. By the term *properties* of numbers, is meant those qualities or elements which are inherent and inseparable from them. Some of the more prominent are the following:

PROP. 1. The sum of any *two or more even* numbers is even number.

2. The difference of any *two even* numbers is an even number.

3. The sum or difference of *two odd* numbers is *even;* but the sum of *three odd* numbers is odd.

4. The sum of any *even* number of odd numbers is even, but the sum of any *odd* number of odd numbers is odd.

5. The sum, or difference, of an *even* and an *odd* number is an odd number.

6. The product of an *even* and an *odd* number, or of *two even* numbers, is even.

7. If an even number be divisible by an odd number, the *quotient* is an even number.

8. The product of any number of factors is *even*, if any one of them be even.

9. An odd number *cannot* be *divided* by an even number without a remainder.

10. The product of any *two* or *more odd* numbers is an odd number.

11. If an odd number divides an even number, it will also divide the *half* of it.

12. If an even number be divisible by an odd number, it will also be divisible by *double* that number.

13. Any number that *measures* two others, must likewise measure their *sum*, their *difference*, and their *product*.

14. A number that *measures* another, must also measure its *multiple* or its *product* by any *whole* number.

15. Any number expressed by the decimal notation, divided by 9, will leave the *same remainder* as the sum of its figures or digits divided by 9. Thus, take any number, 6357: now, separating it into its several parts, it becomes $6000 = 6 \times 1000 = 6 \times (999 + 1) = 6 \times 999 + 6$. In like manner, $300 = 3 \times 99 + 3$, and $50 = 5 \times 9 + 5$. Hence, $6357 = 6 \times 999 + 6 + 3 \times 99 + 3 + 5 \times 9 + 5 + 7$, or $6 \times 999 + 3 \times 99 + 5 \times 9 + 6 + 3 + 5 + 7$; and $6357 \div 9 = (6 \times 999 + 3 \times 99 + 5 \times 9 + 6 + 3 + 5 + 7) \div 9$. But $6 \times 999 + 3 \times 99 + 5 \times 9$ is evidently divisible by 9; therefore, if 6357 be divided by 9, it will leave the same remainder as $6 + 3 + 5 + 7 \div 9$. The same will be found true of any number whatever.

NOTE.—This property of the number 9 affords an ingenious method of proving the fundamental rules. The same property belongs to the number 3; but it belongs to no other figure.

The preceding is not a *necessary* but an *incidental* property of the number 9.

It arises from the law of increase in the decimal notation. If the *radix* of the system were 8, it would belong to 7; if the radix were 12, it would belong to 11: and, universally, it belongs to the number that is *one less* than the *radix* of the system of notation.

16. If the number 9 is multiplied by any *single* figure or digit, the *sum* of the figures composing the product will make 9.

17. If we take any two numbers whatever, then *one* of them or their *sum*, or their *difference*, is divisible by 3. Thus, take 11 and 17: though neither of the numbers themselves, nor their sum, is divisible by 3, yet their difference is, for it is 6.

18. Any number divided by 11, will leave the *same remainder* as the sum of its *alternate* digits in the *even* places, reckoning from the right, taken from the sum of its alternate digits in the *odd* places, increased by 11, if necessary

19. Every prime number, except 2, if increased or diminished by 1, is divisible by 4.

20. Every prime number, except 2 and 3, if increased or diminished by 1, is divisible by 6.

21. Every prime number, except 2 and 5, is contained, without a remainder, in the number expressed in the common notation, by as many 9s as there are units, less one, in the prime number itself. Thus, 3 is a measure of 99; 7 of 999,999.

22. Every prime number, except 2, 3 and 5, is a measure of the number expressed in common, by as many 1s as there are units, less one, in the prime number. Thus, 7 is a measure of 111,111; and 13 of 111,111,111,111.

23. All *prime* numbers, except 2 and 5, must terminate with 1, 3, 7 or 9; all the other numbers are *composite*.

94. To find the *prime* numbers in any series of numbers.

Write in their proper order all the *odd* numbers contained in the series. Then, reckoning from 3, place a point over every third number in the series; reckoning from 5, place a point over every fifth number; reckoning from 7, place a point over every seventh number and so on. The numbers remaining without points, together with the number 2, are the prime numbers.

Take the series of numbers up to 40. Thus: 1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39; then adding the number 2, the prime numbers are 1, 2, 3, 5, 7, 11, 13, &c.

GREATEST COMMON MEASURE.

95. A *Common Divisor* of two or more numbers, is a number which will divide each of them without a remainder. Thus, 2 is a common divisor of 6, 8, 12, 16, 18, &c.

96. The *Greatest Common Divisor*, or, as it is called, the *greatest common measure* of two or more numbers, is the *greatest* number which will divide them without a remainder Thus, 6 is the greatest common measure of 36, 12 18 and 24.

NOTE.—We may here remark, that the measure of two or more quantities can sometimes be found by inspection. The following facts may assist the learner in finding the common measure:

1. Any number ending in 0, or an even number, may be divided by 2.
2. Any number ending in 5 or 0, may be divided by 5.
3. Any number ending in 0, may be divided by 10.
4. When the two right hand figures are divisible by 4, the whole number may be divided by 4.
5. If the three right hand figures of any number are divisible by 8, the whole is divisible by 8.

To find the Greatest Common Measure of Two Numbers

97. RULE. Divide the greater number by the less; if there be a remainder, divide the first divisor by it. If there be still a remainder, divide the second divisor by this remainder, and so on, always dividing the last preceding divisor by the last remainder, till nothing remains. The last divisor will be the greatest common measure required.

Ex. Required, the greatest common measure of 475 and 589. Proceeding by the rule given above—

```
475)589(1
    475
    ---
    114)475(4
        456
        ---
         19)114(6
            114
            ---
              0
```

Therefore, 19 is the greatest common measure of 475 and 589.

Reason for the above process.

Any number which measures 589 and 475 also measures their difference, or 589 — 475, or 114 (Art. 93, prop. 13); also measures any multiple of 114, and therefore 4 × 114 or 456, (Art. 93, prop. 14).

And any number which measures 456 and 475, also measures their difference, or 475 — 456, or 19; and no number greater than 19 can measure the original numbers 589 and 475; for it has just been shown that any number which measures them must also measure 19.

Again, 19 itself will measure 589 and 475.
For 19 measures 114 (since 114 = 6 × 19).
Therefore, 19 measures 4 × 114, or 456, Art. 93, pro. 14).
Therefore, 19 measures 456 + 19, or 475, (Art. 93, pro. 13).
Therefore, 19 measures 475 + 114, or 589;
Therefore, since 19 measures them both, and no number greater than 19 can measure them both,
19 is their greatest common measure.

Exercises.

Find the greatest common measure of

1.	16 and 72,	Ans. 8.	6.	532 and 1274,	Ans. 14.
2.	55 and 121,	11.	7.	2145 and 3471,	39.
3.	272 and 425,	17.	8.	4872 and 81,	3.
4.	128 and 324,	4.	9.	126 and 162,	18.
5.	825 and 960,	15.	10.	176 and 1000,	8.

To find the Greatest Common Measure of Three or more Numbers.

98. RULE. Find the greatest common measure of the first two numbers; then the greatest common measure of the common measure so found and the third number; then that of the common measure last found and the fourth number, and so on. The last measure so found will be the greatest common measure required.

Ex. Find the greatest common measure of 16, 24 and 18. Proceeding by the Rule given above—

```
16)24(1
   16
   —
    8)16(2
      16
      —
```

Therefore 8 is the greatest common measure of 16 and 24.

Now find the greatest common measure of 8 and 18.

```
8)18(2
  16
  —
   2)8(4
     8
     —
```

Therefore, 2 is the greatest common measure required.

Reason of the above process.

It appears, from Art. 93, prop. 13, that every number which measures 16 and 24, measures 8 also;

Therefore, every number which measures 16, 24 and 18, measures 8 and 18;

Therefore, the greatest common measure of 16, 24 and 18, is the greatest common measure of 8 and 18.

But 2 is the greatest common measure of 8 and 18;

Therefore, 2 is the greatest common measure of 16, 24 and 18.

Exercises.

Find the greatest common measure of

1.	14, 18 and 24.	Ans. 2.
2.	13, 52, 416 and 78.	Ans. 13.
3.	805, 1311 and 1978.	Ans. 23.
4.	504, 5292 and 1520.	Ans. 4.

LEAST COMMON MULTIPLE.

99. One number is said to be a multiple of another, when the former can be divided by the latter without a remainder (Art. 92, def. 8). Hence,

100. A *Common Multiple* of two or more numbers, is a number which can be *divided* by *each* of them without a remainder. Thus, 144 is a common multiple of 3, 9, 18 and 24.

101. The *Continued Product* of two or more given numbers will always form a common multiple of those numbers. The same number may have an unlimited number of common multiples; for, multiplying their continued product by any number, will form a new common multiple (Art. 93, prop. 14).

102. The *Least Common Multiple* of two or more numbers, is the least number which will contain each of the given numbers an exact number of times without a remainder.

Thus, 12 is the least common multiple of 4 and 6, for it is the least number which can be exactly divided by them.

Note.—The least common multiple of two or more numbers, is evidently composed of all the prime factors of each of the given numbers repeated *once* and only *once*. For, if it did not contain all the prime factors of any one of the given numbers, it could not be divided by that number.

On the other hand, if any prime factor is employed *more times* than it is repeated as a factor in some one of the given numbers, then it would not be the *least* common multiple.

103. *To find the Least Common Multiple of Two Numbers.*

RULE. Divide their product by their greatest common measure. Or, divide one of them by their greatest common measure, and multiply the quotient by the other. The result of either method will be the least common multiple of the numbers.

Ex. Find the least common multiple of 18 and 30.

Proceeding by the Rule given above, the greatest common measure of 18 and 30 (Art. 98), is 6, and $18 \times 30 = 540$, and $540 \div 6 = 90$.

Therefore, 90 is the least common multiple of 18 and 30.

Reason of the above process.

$18 = 3 \times 6$, and $30 = 5 \times 6$.

Since 3 and 5 are prime factors, it is clear that 6 is the greatest common measure of 18 and 30; therefore, the least common multiple must contain 3, 6 and 5 as factors.

Now, every multiple of 18 must contain 3 and 6 as factors; and every multiple of 30 must contain 5 and 6 as factors (Art. 93, prop. 14).

Therefore, every number, which is a multiple of 18 and 30, must contain 3, 5 and 6 as factors; and the least number which contains them is $3 \times 5 \times 6$, or 90.

Now, $90 = (3 \times 6) \times (5 \times 6)$, divided by 6;
$= 18 \times 30$, divided by 6;
$= 18 \times 30$, divided by the greatest common measure of 18 and 30.

104. Hence, it appears that the least common multiple of two numbers which are prime to each other, or have no common measure but unity, is their product.

105. *To find the Least Common Multiple of Three or More Numbers.*

RULE. Find the least common multiple of the first two numbers; then the least common multiple of that multiple, and the third number, and so on. The last common multiple so found will be the least common multiple required.

Ex. Find the least common multiple of 9, 18 and 24.

Proceeding by the Rule given above,

Since 9 is the greatest common measure of 18 and 9, their least common multiple is clearly 18.

Now, find the least common multiple of 18 and 24. The greatest common measure of 18 and 24 is 6 (Art. 98).

Therefore, the least common multiple of 18 and 24 is equal to $(18 \times 24) \div 6$, or $432 \div 6 = 72$.

Therefore, 72 is the least common multiple required.

Reason for the above process.

Every multiple of 9 and 18 is a multiple of their least common multiple, 18; therefore, every multiple of 9, 18 and 24, is a multiple of 18 and 24; and, therefore, the least common multiple of 9, 18 and 24 is the least common multiple.

106. *When the Least Common Multiple of several Numbers is required, the most convenient practical method is that given by the following Rule.*

Rule. Arrange the numbers in a line from left to right, with a comma placed between every two.

Divide by the smallest number which will divide any two or more of them without a remainder, and set the quotients and the undivided numbers in a line below. Divide this line, and set down the results as before; thus continue the operation, till there are no two numbers which can be divided by any number greater than 1. The continued product of the divisors into the numbers in the last line will be the least common multiple required.

Note.—The least divisor of every number is a prime number. For every whole number is either prime or composite; hence, dividing by the *smallest* number which will divide two or more of the given numbers, is dividing them by a prime number.

The result will evidently be the same if, instead of dividing by the smallest number, we divide the given numbers by *any prime* number that will divide two or more of them without a remainder.

The preceding operation, it will be seen, resolves the given numbers into their prime factors, then multiplies all the different factors together, taking each factor as many times in the product, as are equal to the *greatest number* of times it is found in either of the given numbers.

Ex. Find the least common multiple of 6, 8 and 12.

OPERATION.

$6 = 2 \times 3$
$8 = 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$
and $2 \times 2 \times 2 \times 3 = 24$

By resolving the given numbers into their prime factors, it will be seen that 2 is found *once* in 6; *twice* in 12; and *three times* in 8. It must therefore be taken *three times* in the product. Again, 3 is a factor of 6 and 12, consequently must be taken only *once* in the product. Thus, $2 \times 2 \times 2 \times 3 = 24$, which is, therefore, the least common multiple.

Ex. Find the least common multiple of 12, 18 and 36.

Proceeding by the Rule given,

First operation.

2)	12, 18, 36
2)	6, 9, 18
3)	3, 9, 9
3)	1, 3, 3
	1, 1, 1

$2 \times 2 \times 3 \times 3 = 36$. Ans.

Second operation.

9)	12, 18, 36
2)	12, 2, 4
2)	6, 1, 2
	3, 1, 1

Now, $9 \times 2 \times 2 \times 3 = 108$.

Third operation.

12)	12, 18, 36
3)	1, 18, 3
	1, 6, 1

and $12 \times 3 \times 6 = 216$

EXPLANATION. In the first operation, we divide by the *smallest* numbers which will divide any two or more of the given numbers without a remainder, and the product of the divisors, &c., is 36, which is the answer required.

In the second and third operations, we divide by numbers that will divide two or more of the given numbers without a remainder, and in both cases obtain erroneous answers.

NOTE.—1. It will be seen from the second and third operations given above, that "dividing by any number which will divide two or more of the given numbers without a remainder," according to the rule given by some authors, does not always give the *least* common multiple.

2. The reason for dividing by the *smallest* number, is because the divisor may otherwise be a *composite* number, and have a factor common to some one of the quotients or undivided numbers in the last line; consequently, the continued product of them would be too large for the least

common multiple. Thus, in the second operation, the divisor 9 is a composite number, containing the factor 3 common to the 3 in the quotient; consequently, the product is *three times too large*. In the third operation, the divisor 12 is a composite number, and contains the factor 6 common to the 6 in the quotient; therefore, the product is *six times too large*.

107. The process of finding the least common multiple may often be shortened by canceling in any line any number which is exactly contained in any other number in the same line.

Ex. Find the least common multiple of 4, 6, 10, 8, 12 and 15.

2)	~~4~~,	~~6~~,	10,	8,	12,	15
2)			~~5~~,	4,	6,	15
				2,	~~3~~,	15

Since 4 and 6 will exactly divide 8 and 12, we cancel them. Again, since 5 in the second line will exactly divide 15 in the same line, we therefore cancel it, as also 3 in the third line, and proceed with the remaining numbers as before. Thus, $2 \times 2 \times 2 \times 15 = 120$. Ans.

Exercises.

Find the least common multiple of the following numbers.

1.	12, 8 and 9,	Ans. 72.
2.	6, 10 and 15,	30.
3.	27, 24 and 15,	1080.
4.	6, 15, 24 and 25,	600.
5.	15, 35, 63 and 72,	2520.
6.	54, 81, 63 and 14,	1134.
7.	1, 2, 3, 4, 5, 6, 7, 8 and 9	2520.
8.	7, 8, 9, 18, 24, 72 and 144.	1008.

DECIMALS.

108. In Division of Whole Numbers, we found that sometimes there was a remainder, and that this remainder was less than the divisor (Art. 62). We therefore placed it over the divisor, and annexed it to the quotient, as in

Ex. 1. 8) 27 ($3\frac{3}{8}$. Ans.
24
—
$\frac{3}{8}$

Now, if we have multiplied this remainder by any number, say 16, and divide the product by the divisor, (Ex 2), the quotient would be 16 times too large (Art. 69). We therefore divide this quotient figure by 16, or, which is the same thing, place 16 under it, and annex it to the quotient as before.

Ex. 2. 8) 27 ($3\frac{6}{16}$
24
—
3
16
—
8) 48 (6
48

Ex. 3. 8) 27 ($3 + \frac{1}{4} + \frac{3}{24}$
24
—
3
4
—
8) 12 (1
8
—
4
6
—
8) 24 (3
24
—

Again, if we have taken any other number, as in Ex. 3, there would still be a remainder, and if this remainder be multiplied by any other number, and so on, it will be found that the sum of these, along with the whole number, will be the same as in Ex. 1.

Now, if we had multiplied each remainder, as it arose, by 10, instead of a variable number, we would have obtained numbers, every successive figure of which would decrease ten-fold. Thus, taking the same numbers as in the last example.

```
8)27(3 + 3/10 + 7/100 + 5/1000
  24
  —
   3
  10
  —
 8)30(3
  24
  —
   6
  10
  —
 8)60(7
  56
  —
   4
  10
  —
 8)40(5
  40
  —
```

Here we multiply the first remainder by 10, and divide by 8, placing the fraction ($\frac{3}{10}$) in the quotient. Again, we multiply the next remainder (6) by 10, and place the fraction $\frac{7}{100}$—because we multiply by ten twice, or 100—in the quotient, &c. The quotient thus obtained is the same as that of Ex. 1.

Or, let it be required to divide 27 apples among 8 boys. Here we find that each boy would get 3 whole apples, leaving 3 apples yet to be divided among them. We therefore take these 3 apples and cut each of them into 10 equal pieces, or multiply by 10, which is the same, and proceed to divide these 30 pieces, or 30 tenths of an apple, as before, giving each boy 3 tenths, leaving 6 tenths; and as it is evident each boy cannot get one whole tenth of these 6 tenths, we again divide these 6 tenths into ten equal parts each, or 60 hundredths of an apple, and proceed as before, giving each boy 7 of these equal parts, or 7 one hundredths of an apple; in the same manner we divide the remaining 4 one hundredths, giving each boy 5 one thousandths part of an apple. Therefore, each boy's share will be 3 whole apples, together with 3 tenths, together with 7 one hundredths, together with 5 one thousandths of an apple, or, as it is briefly written, 3.375 apples; or, in other words, we divide each of the 3 remaining apples into 1000 equal parts, and give each boy 375 of these parts, or 375 thousandths of an apple.

109. Thus, we have a series of orders, by which we can represent whole numbers or integers and fractions, by means of figures in the unit's place, and on each side of it.

Numbers, which assume this order of decrease, are called Decimals, or *Decimal Fractions*, in contradistinction to *Vulgar Fractions*, which, as we have seen, are represented by a different notation.

110. The denominator of a decimal fraction is always 1, with as many cyphers annexed to it as there are figures in the given numerator.

111. The names of the different *orders of decimals*, or places below units, may be easily learned from the following

Decimal Table.

			&c.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
0	0	0	0	0	0	0	0	0	.	0	0	0	0	0	0	0	0	0
										1st place,	2d place,	3d place,	4th place,	5th place,	6th place,	7th place,	8th place,	9th place,

112. It will be seen from this table, that the *value* of each figure or digit in *decimals*, as well as in whole numbers, depends upon the *place* it occupies, reckoning from units. Thus, if the figure stands in the *first* place on the right of units, it expresses *tenths;* if in the *second* place, *hundredths*, &c.

113. Each removal of a decimal figure one place from the units toward the right, diminishes its value ten times.

Prefixing a cypher, therefore, to a decimal, diminishes its value ten times; for it removes the decimal one place farther from the units' place. Thus, $.4 = \frac{4}{10}$, but $.04 = \frac{4}{100}$; for the denominator to a decimal fraction is 1, with as many cyphers annexed to it as there are figures in the numerator, (Art. 110).

Annexing cyphers to decimals does not alter their value; for each significant figure continues to occupy the same place as before.

Thus, $.5 = \frac{5}{10}$; so $.50 = \frac{50}{100}$, or $\frac{5}{10}$, by dividing the numerator and denominator by 10.

It will be perceived that the effect of annexing and prefixing cyphers to decimals is exactly opposite to that which is produced by affixing and prefixing cyphers to integers.

114. *To read Decimal Fractions.*

Beginning at the left hand, read the figures as if they were whole numbers, and to the last one add the name of its order. Thus,

.7	is read	7 tenths.
.36	" "	36 hundredths.
.475	" "	475 thousandths.
.6342	" "	6342 ten-thousandth.

NOTE.—Sometimes we pronounce the word *decimal* when we come to the separatrix, or decimal point, and read the figures as if they were whole numbers, or simply repeat them one after another. Thus, 125.427 is read one hundred and twenty-five, *decimal* four hundred and twenty-seven; one hundred and twenty-five, *decimal* four, two, seven; or point four, two, seven.

EXERCISES IN NOTATION AND NUMERATION OF DECIMALS.

Write the fractional parts of the following numbers in decimals.

(1.)	$25\frac{7}{10}$	(4.)	$4\frac{7}{100}$	(7.)	$3\frac{3126847}{10000000}$
(2.)	$30\frac{62}{100}$	(5.)	$6\frac{13}{1000}$	(8.)	$8\frac{4}{1000000}$
(3.)	$72\frac{80}{100}$	(6.)	$7\frac{80}{1000}$	(9.)	$2\frac{81}{1000000}$

10. Write 9 tenths, 25 hundredths, 45 thousandths.

11. Write 71 thousandths, 7 millionths.

115. Decimals are *added*, *subtracted*, *multiplied* and *divided*, in the same manner as whole numbers.

Addition of Decimal Fractions.

116. Rule. Place the numbers under each other, units under units, tens under tens. &c.; one-tenths under one-tenths, one-hundredths under one-hundredths, &c. Add as in whole numbers, and place the decimal point in the sum under the decimal point above.

Ex. Add together 27.5037, .042, 432 and 2.1.

Proceeding by the Rule given above,

```
 27.5037
   .042
432.
  2.1
--------
461.6457
```

Or, filling up the vacant places by Art. 13.

```
 27.5037
   .0420
432.0000
  2.1000
--------
461.6457
```

Note.—The same method of explanation holds for the fundamental rules of decimals, as also the proof, which has been given at length in explaining the Rules for Simple Addition, Simple Subtraction, and the other fundamental rules in whole numbers.

Mental Exercises in Addition of Decimals.

1. What is the sum of .1 and .05?
2. What is the sum of .6 and .45?
3. What is the sum of 4.25 dollars and 92 cents?

Note.—It is unnecessary to multiply questions for mental exercise, under any of these rules, as any teacher can supply an unlimited quantity to suit the capacity of his pupils.

Exercises for the Slate.

1.) .234 + 14.3812 + .01 + 32.47 + .00075. Ans. 47.09595.

(2.) 232.15 + 3.225 + 21 + .0001 + 34.005 + .001304. Ans. 290.381404.

(3.) .08 + 165 + 1.327 + .0003 + 2760.1 + 9. Ans. 2935.5073.

(4.) 725.1201 + 34.00076 + .04 + 50.9 + 143.713. Ans. 953.77386.

(5.) 67.8125 + 27.105 + 17.5 + .000375 + 255. Ans. 367.417875.

(6.) 1.83 + 5.674 + .3125 + 18.3 + 100 + 38.62 + 4.3957 + .5. Ans. 169.6322.

SUBTRACTION OF DECIMALS.

117. RULE. Place the less number under the greater, units under units, tens under tens, &c.; one-tenths under one-tenths, &c. Suppose cyphers to be supplied, if necessary, in the upper line, to the right of the decimal; then proceed as in Simple Subtraction of Whole Numbers, and place the decimal point under the decimal point above.

Ex. Subtract 5.473 from 6.23.

Proceeding by the Rule given above,

```
 6.23
 5.473
 -----
  .757
```

EXERCISES IN SUBTRACTION.

(1.) 213.5 — 1.8125. Ans. 211.6875.
(2.) 603 — .6584003. Ans. 602.3415997.
(3.) .582 — .09647. Ans. .48553.
(4.) 3.468 — 1.2591. Ans. 2.2089.
(5.) 34.528 — 10.6347. Ans. 23.8933.

MULTIPLICATION OF DECIMALS.

118. RULE. Multiply, as in whole numbers, and point off as many figures from the right of the product for decimals, as there are decimal places in both the multiplicand and multiplier.

If the product does not contain so many figures as there are decimals in both factors, supply the deficiency by *prefixing* cyphers.

Ex. 1. Multiply 5.34 by .21.

Proceeding by the Rule given above,

```
 5.34
  .21
 ----
  534
 1068
 ----
11214
```

Now, the number of decimal places in the multiplicand + the number of those in the multiplier = 2 + 2 = 4.

Therefore, the product = 1.1214.

Ex. 2. 5.34 × .0021.

```
 5.34
.0021
 ----
  534
 1068
 ----
11214
```

Here we must have 6 places of decimals in the product; but there are only 5 figures, and therefore we must prefix one zero, or cypher, and place a point before it, thus, .011214.

Reason for the above process.

The reason for pointing off as many decimal places in the product as there are decimals in both factors, may be illustrated thus:

Suppose it is required to multiply .25 by .5. Supplying the denominators, .25 = $\frac{25}{100}$, and .5 = $\frac{5}{10}$ (Art. 110); now, multiplying the denominators together, and also the numerator, we obtain $\frac{25}{100} \times \frac{5}{10} = \frac{125}{1000}$, or .125 (Art. 112); that is, the product of .25 × .5 contains just as many decimals as the factors themselves. In like manner, it may be shown that the product of two or more decimal numbers must contain as many decimal figures as there are places of decimals in the given factors. So, multiplying by any number of tens, it is only necessary to remove the point one place toward the right for each ten.

EXERCISES IN MULTIPLICATION OF DECIMALS.

		Answers.			Answers.
1.	62.38 × 7.	436.66.	4.	.1 × .1 × .001.	.00001.
2.	3.81 × 41.7	158.877.	5.	417 × .417.	173.889.
3.	.31 × .32.	.0992.	6.	.417 × .417.	.173889.

	Answers.
7. 71956 × .000025.	1.7989.
8. 7.6 × .071 × 2.1 × 29.	32.86164.
9. 1.05 × 1.05 × 1.05	1.157625.
10. 7.49 × 63.1.	472.619

Contractions in Multiplication of Decimals.

119. When the number of decimal places in the multiplier and multiplicand is large, the number of decimals in the product must also be large. But decimals below the fifth or sixth place express so small parts of a unit, that when obtained they are commonly rejected.

It is therefore desirable to avoid the unnecessary labor of obtaining those which are not to be used.

Ex. It is required to multiply 1.3569 by .36742, and retain only five places of decimals in the product.

```
   1.35691
    ·36742
   -------
 4 0707|3
  8141|46
   949|837
    54|2764
     2|71382
   ----|-----
 .49855|58722
```

It is evident, from the nature of decimal notation, that if we begin to multiply by the left hand figure of the multiplier first, instead of the right hand, and advance the partial product of each figure in the multiplier one place to the right instead of the left, the operation will correspond with the descending scale, and at the same time will give the true product. But since only five places of decimals are required, those on the right of the perpendicular are useless. Our present object is to show how the answer can be obtained without them.

```
1.3569'1
 .3674 2
 -------
 4070 7
  814 1
   95 0
    5 4
      3
 -------
 .49855
```

Beginning at the right hand, as before, we first multiply the fourth figure, or ten thousands' place of the multiplicand, by the tenths' or left hand figure of the multiplier (for 4 decimals in the multiplicand and 1 decimal place in the multiplier will give 5 places in the product, (Art. 118,) and place the first figure of the partial product under the figure multiplied. In obtaining the second partial product (i. e. multiplying by 6), it is plain we omit the *right hand* figure of the multiplicand, for if multiplied, its product will fall to the right of the perpendicular line, and therefore will not be used But if we multiply 9 into 6, the product will be 54 ; consequently, there would be 5 to carry to the next product. We therefore carry 5 to 36, which makes 41. Again, in the third partial product (i. e. in multiplying by 7), we may omit the *two* right hand figures of the multiplicand, for their product will fall to the right of the perpendicular line. But by recurring to the rejected figures, it will be seen that the product of 7 into 6 is 42, and 6 to carry makes 48 ; we therefore add 5 to the product of 7 into 5, because 48 is nearer 50 than 40 ; consequently, it is nearest the truth to carry 5 than 4. In the fourth partial product, we may omit the *three* right hand figures, and in the fifth or last, the *four* right hand figures.

Again, it may be seen from the last operation, that if we had placed the figure occupying the tenths' place in the multiplier under the fourth figure of the multiplicand, and written the other figures in reversed order, then multiplied as before, the result would have been the same. Thus,

```
1.35691
2 4763
-------
4 0707
  8141
   950
    54
     3
-------
.49855
```

we place 3 under 9, and multiply, and set down the right hand figure of the partial product, 7, and carry 2, &c. Again, we multiply 6 into 6, and to this product add the carrying figure arising from 6 into 9, viz., 5, and place the right hand figure of this product under the 7, &c.; proceeding in the same way with the other figures, only setting down the product of each figure of the multiplier into the one directly above it for the right hand figure of the partial product.

NOTE.—In finding what is to be carried for the rejected figures, it is generally sufficient to go one figure back, but in doubtful cases it may be well to go farther, but, even then, the last figure cannot be depended on. It is therefore better to work for one figure more than it is necessary to have true, and to reject it at the conclusion. And in lengthened computations, such as many of those in compound interest and annuities, it may be right to work for two or three additional figures. When the work is carried to one or two places more than required, it is not necessary to go back farther than one figure to obtain the carrying figure.

120. *To Multiply Decimals and retain only a given number of Decimal Figures in the Product.*

Rule. Count off, after the decimal point in the multiplicand (annexing cyphers if necessary), as many figures of decimals as requisite to have in the product. Below the last of these, write the *unit* figure of the multiplier, and write its other figures in reversed order. Then multiply by each figure of the multiplier, thus inverted, neglecting all the figures of the multiplicand to the right of that figure, except to find what is to be carried; and let all the partial products be so arranged that their right hand figures may stand in the same column.

Lastly, from the sum of these partial products, cut off the assigned number of decimal places.

In carrying from the rejected figures, we should take what is *nearest* the truth, whether it be too great or too small.

Ex. 1. Multiply 7.24651 by 16.3476, so that there may be only four places of decimals in the product.

```
 7.2465 1 0
  6743.6 1
 ----------
  72465 1 0
  43479 0 6
   2173 9 5
    289 8 6
     50 7 2
      4 3 4
 ----------
118.463 0[3
```

Here, the unit figure of the multiplier is written under the fifth figure of the multiplicand, because we carry it one place farther than necessary in the product; 1, the figure which precedes it (6) is put *after* it; 3, the figure which *follows* it, is written or set *before* it, &c. We then proceed as in the former examples, rejecting the first column, and pointing off four places of decimals: the number required.

Ex. 2. Multiply .681472 by .01286, so as to retain only 4 places of decimals.

```
.6814 7'2
6 8210.0
---------
    68 1
    13 6
     5 4
       3
---------
.0087[4
```

In this example, since the multiplier contains no integer, a cypher is placed below the fifth place (one more than required in the product to insure accuracy) of the multiplicand; and then the multiplier being written in reversed order, the work proceeds as in the last example.

Exercises

To be worked by the contracted method, and proved by the former method.

1. 1.23467 × 4.896 to 5 places.
2. 4.8367 × 12.63 to 3 places.
3. 17.674 × 3.298 to 4 places.
4. 1.65 × 1.65 to 6 places.
5. .0006 × 3.48 to 4 places.
6. 186.784 × .2986 to 4 places.
7. .00678 × .46743 to 6 places.
8. .863541 × .10983 to 5 places.
9. 6.74321 × .0006 to 8 places.
10. 4867.632 × 123.45 to 2 places.
11. 98.98 × 98.98 × 98.1 to 4 places.
12. .167 × 167.1 × 16.7848 to 6 places.

Note.—The same contractions may be used in Decimals as were used in Multiplication of Integers, always bearing in mind to count off from the product the same number of decimals as there are decimal places both in the multiplier and multiplicand.

Division of Decimals.

121. *First. When the number of Decimal Places in the Dividend exceeds the number of Decimal Places in the Divisor.*

Rule. Divide as in whole numbers, and mark off in the quotient a number of decimal places equal to the excess of the number of decimal places in the dividend over the number of decimal places in the divisor; if there are not figures sufficient, prefix cyphers as in Multiplication.

Ex. 1. Divide 1.1214 by 5.34.

Proceeding by the Rule given above,

```
5.34 ) 1.1214 (.21
       1 068
       -----
         534
         534
         ---
```

Now, the number of decimal places in the dividend — the number of decimal places in the divisor = 4 — 2 = 2.

Therefore, the quotient = .21.

Ex. 2. Divide .011214 by 5.34.

```
5.34 ) .011214 ( 21
        1068·
        ----
         534
         534
         ---
```

Now, the number of decimal places in the dividend — the number of decimal places in the divisor = 6 — 2 = 4; therefore, we prefix two cyphers, and the quotient = .0021.

Reason for the above process.

We have seen in Multiplication of Decimals that the product has as many decimal figures as the multiplier and multiplicand (Art. 118). Now, since the dividend is equal to the product of the divisor and quotient, it follows that the dividend must contain as many decimal places as the *divisor* and *quotient together;* consequently, the quotient will contain a number of decimal places equal to the number in the dividend less those in the divisor.

122. *Secondly. When the number of Decimal Places in the Dividend is less than the number of Decimal Places in the Divisor.*

Rule. Affix cyphers to the dividend until the number of decimal places in the dividend equals the number of decimal places in the divisor; the quotient up to this point of the division will be a whole number; if there be a remainder, and the division be carried on farther, the figures in the quotient after this point will be decimals.

Ex. 1. Divide 1121.4 by .534.

```
.534 ) 1121.400 ( 2100
       1068 ·
       ----
        534
        534
        ---
```

We have 3 places in the divisor and only one place in the dividend; we therefore affix two cyphers to the dividend.

Ex. 2. Divide 172.9 by .142 to 3 places of decimals.

Proceeding by the Rule given on foregoing page,

```
.142 ) 172.900,000 ( 1217.605
       142 ···  ···
       ---
        30 9
        28 4
        ----
         2 50
         1 42
         ----
         1 080
           994
         -----
            86 0
            85 2
            ----
              800
              710
              ---
               90
```

123. *Instead of the foregoing Rules, the following may be used with advantage, particularly with beginners.*

Rule. If the divisor and dividend do not contain the same number of decimal places, supply the deficiency by annexing cyphers. Then, rejecting separating points, divide as in whole numbers, and the quotient will be a whole number. If there be a remainder, after all the figures of the dividend have been used, cyphers may be annexed, till nothing remains, or till as many figures are found as may be judged necessary. The part of the quotient thus obtained, will be a decimal.

If, after rejecting the decimal points, the divisor be greater than the dividend, the quotient will contain no whole number.

Ex. Divide 1346.5 by 43.68.

Proceeding by the Rule given above,

4368) 134650 (30.826465, &c.

Here, by annexing a cypher to the dividend, and rejecting the point, we have 4368 for the divisor, and 134650 for the dividend. Hence, dividing in the common way, we find 30 for the integral part, and annexing cyphers to the remainders, and continuing the operation, we get .826465, &c. The answer, therefore, is 30.826465.

Reason for the above Rule and process.

The value of 1346.5 is not changed by the annexing of a cypher, (Art. 113); and the removal of the points merely multiplies each of the given numbers by 100. (See Art 75.) It is evident, therefore, that the value of the quotient will not be affected; since, while the dividend is multiplied by 100, the divisor is increased in the same ratio.

The reason of removing the points, is to make the divisor and dividend whole numbers, and thus render the operation, as much as possible, the same as in simple division.

Ex. 2. Divide .1342 by 67.1.

Here, by annexing three cyphers to the divisor, and rejecting the decimal points, we get for the divisor 671000, and for the dividend 1342. Then, the divisor being greater than the dividend, the quotient will contain no integral part; and the annexing of a cypher to the dividend gives one cypher for the quotient; the annexing of a second cypher to the dividend gives another cypher; but the annexing of a third gives 2. Hence, the quotient is .002.

The work is left for the learner to perform.

EXERCISES.

1. 10.836 ÷ 5.16. Ans. 2.1.
2. 34.96818 ÷ .381. Ans. 91.78.
3. .025075 ÷ 1.003. Ans. .025.
4. .02916 ÷ .0012. Ans. 24.3.
5. .00081 ÷ 27. Ans. .00003.
6. 1.77089 ÷ 4.735. Ans. 374.
7. 1 ÷ .1 ÷ .01 ÷ .0001. Ans. 10,100, 10000.

124. When the divisor consists of many figures, the work will be shortened, if, instead of annexing a cypher to each remainder, a figure be cut off from the divisor. In

this case, each product is to be increased by *carrying* from the product of the figure last cut off, and of the figure last placed in the quotient.

Ex. 1. Divide 2.3748 by 1.4736, so that the quotient may contain three places of decimals.

```
14736) 23748 (1.611
 ···   14736
       -----
        9012
        8842
        ----
         170
         147
         ---
          23
          15
          --
           8
```

In this example, the numbers being prepared according to the former rule, and the first figure of the quotient being found, instead of adding a cypher to the remainder, 9012, we omit the last figure of the divisor, to denote which a point is placed below it. Then 6, being put in the quotient, we multiply 6, the figure cut off, by it, and without setting anything down, we carry 4, because the product 36 is nearer 40 than 30. After that, 3 is cut off in like manner, and then 7.

The quotient is found to be 1.611, or more nearly 1.612. because the remainder 8 is rather more than the half of 14,

Miscellaneous Exercises on the foregoing Rules.

1. Find the sum and difference of 9090909 and 90909.
   Ans. 9181818; 9000000.

2. A person, whose age is 73, was 37 years old at the birth of his eldest son; what is the age of his son?
   Ans. 36.

3. Find the value of the following expression: $15 \times 37153 - (73474 - 67152) \div 4 + 40734 \times 2$.
   Ans. $356954\frac{1}{2}$

4. The annual deaths in a town being 1 in 45, and in the country 1 in 50; in how many years will the number of deaths out of 18675 persons living in the town, and 79250 persons living in the country, amount together to 10,000?
   Ans. 5 years.

5. Find the value of 494871 — 94853 + (45079 — 3177) — (54312 — 3987) — (1763 + 231) + 379 × 379.
Ans. 147802420.

6. What number divided by 528 will give 36 for the quotient, and leave 44 as a remainder? Ans. 19052.

7. The Iliad contains 15683 lines, and the Ænead contains 9892 lines; how many days will it take a boy to read through both of them, at the rate of eighty-five lines a day? Ans. 300 days, and 75 lines rem.

8. At a game of cricket, A, B and C together score 108 runs; B and C together score 90 runs, and A and C together score 51 runs; find the number of runs scored by each of them. Ans. A, 18; B, 57; C, 33.

9. The remainder of a division is 97, the quotient 665, and the divisor 91 more than the sum of both. What is the dividend? Ans. 567342.

10. The quotient arising from the division of 9281 by a certain number is 17, and the remainder is 373. Find the divisor. Ans. 524.

11. What number multiplied by 86 will give the same product as 163 by 430? Ans. 815.

12. Find the greatest number which can divide each of the two numbers, 849 and 1132; also, the least number which can be divided by each of them.
Ans. 283; 3396.

13. From 46 hundredths take 46 thousandths.
Ans. 0.414.

14. In one rod there are 16.5 feet; how many are there in 41.3 rods? Ans. 681.45 feet.

15. How many suits of clothes will 29.6 yards of cloth make, allowing 3.7 yards to a suit? Ans. 8 suits.

16. How many bales of cotton are there in 56343.75 pounds, allowing 375 pounds to a bale?
Ans. 150.25 bales.

REDUCTION, ADDITION, SUBTRACTION, &c., OF COMPOUND OR DENOMINATE NUMBERS.

125. Our operations hitherto have been carried on with regard only to abstract numbers, or concrete numbers of one denomination. It is evident that if concrete numbers were all of one denomination; if, for instance, shillings were the only units of money, yards of length, years of time, and so on, such numbers would be subject to the common rules for abstract numbers. Again, if the concrete numbers were different denominations, and these denominations differed from each other by 10, or multiples of 10, then all operations with such concrete numbers could be carried on by the rules which have been given for Decimals. But, generally, with concrete numbers, such a relation does not hold between the different denominations, and therefore it is necessary to commit to memory tables which connect the different units of money together, the different units of time together, the different units of lengths together, and so on.

We shall now put down some of the most useful of these tables, with a brief remark on each, and show the manner of applying them to the Reduction, Addition, Subtraction, Multiplication and Division of each denomination separately.

TABLE OF MONEY.

ENGLISH OR STERLING MONEY.

2	Farthings	make	1 Half-penny.
2	Half-pence	"	1 Penny.
12	Pence	"	1 Shilling.
20	Shillings	"	1 Pound.

Pounds, shillings, pence and farthing were formerly denoted by £, S, D, and q, respectively, these letters being the first letters of the Latin words *libra*, *solidus*, *denarius*, and *quadrans*, the Latin names of certain Roman coins or sums of money. £, S, D, are still the abbreviated forms for pounds, shillings, and pence, respectively; but $\frac{1}{4}$ annexed to pence, denotes 1 farthing; $\frac{1}{2}$ denotes a half-penny; $\frac{3}{4}$ denotes three farthings—showing that one farthing, two farthings, and three farthings, are respectively $\frac{1}{4}$th, $\frac{2}{4}$ths or $\frac{1}{2}$, and $\frac{3}{4}$ths of the concrete unit one penny.

The mark /. which is often placed between shillings and pence is a corruption of the long ſ.

The following coins are at present in common use in England:

COPPER COINS.

A Farthing, the coin of least value.
A Half-penny = 2 Farthings.
A Penny = 4 Farthings.

GOLD COINS.

A Half-Sovereign = 10 Shillings.
A Sovereign = 20 Shillings.

SILVER COINS.

Three-penny piece = 3 pence.
Four-penny piece = 4 pence.
A Shilling = 12 pence.
A Florin = 2 Shillings.
A Half-Crown = 2 Shillings and 6 pence.
A Crown = 5 Shillings.

Money, as expressed by means of these denominations, is commonly called *Sterling Money*, in order to distinguish it from *Stock*, &c., which is only *nominal*.

Though all commercial transactions are conducted by means of the money enumerated in the preceding table, there are coins or denominations frequently met with, and some of them more particularly in old documents, of which the most important, and their *value* in *current* money, is here annexed:

	£. S. D.
A Groat or Fourpenny,	0— 0—4
A Tester,	0— 0—6
A Seven-Shilling Piece,	0— 7—0
A Guinea,	1— 1—0
A Half-Guinea,	0—10—6
A Noble,	0— 6—8
An Angel,	0—10—0
A Mark or Merk,	0—13—4
A Carolus,	1— 3—0
A Jacobus,	1— 5—0
A Moidore,	1— 7—0
A Six-and-Thirty,	1—16—0

The office at which coin is made and stamped, so as to pass or become current for legal money, is called *the Mint*.

The standard of gold coin in England is 22 parts of *pure gold* and 2 *parts copper*, melted together.

From a pound, Troy, of standard gold, there are coined at the Mint $46\frac{29}{40}$ sovereigns, or £ 46. 14. 6., so that the weight of each is exactly 5 dwt. $3\frac{171}{623}$ grs., or nearly 123.-274 grs.; and the mint price of standard gold is £ 3. 17. $10\frac{1}{2}$ per ounce, (12 oz. Troy, = 1 pound Troy).

The standard of silver coin is 37 parts of *pure silver*, and 3 parts of *copper*. From a pound, Troy, of standard silver are coined 66 shillings.

Therefore, the mint price of silver is 5s. 6d. per ounce, standard.

In the copper coinage, 24 pence are coined from a pound, Avoirdupois, of copper. Therefore, 1 penny should weigh $\frac{1}{24}$th of a pound, Avoirdupois,

The *copper* coinage is not, according to the present law, a *legal tender* for more than 12d.; nor is the *silver* coinage for more than 40s.—the *gold* coinage being the standard of Britain.

REDUCTION.

126. **Reduction** is the method of expressing numbers of a superior denomination in units of a lower denomination, and conversely. Thus, £ 1 is the same value as 240 pence, and £ 21 as 5040 d., and conversely; and the process by which we ascertain this to be so, is termed *Reduction.*

First. To express a number of a higher denomination in units of a lower denomination.

Rule. Multiply the number of the highest denomination in the proposed quantity by the number of units of the next lower denomination contained in one unit of the highest, and to the product add the number of that lower denomination, if there be any in the proposed quantity; repeat this process for each succeeding denomination, till the required one is arrived at.

Ex. How many pence are there in £23, 15s. 0d.?

Proceeding by the Rule given on preceding page,

£23, 15s. 0d.
20

460 + 15, or 475s.
12

5700d., or £23. 15s. = 5700d.

Reason for the process.

There are 20 shillings in £1.

Therefore, there are (23 × 20) shillings, or 460s. in £23, and so there are 460s. + 15s., or 475s. in £23, 15s.

Again, since there are 12 pence in 1 shilling, therefore there are (475 × 12) d., or 5700d. in 475s.: i. e. in £23, 15s.

In practice, we do not affix the shillings to the product by means of the sign +, but merely add them as we proceed with the work.

NOTE.—It has been stated (Art. 28) that one of the factors of any product must be an abstract number; and as neither £23 nor 20s. are such, we should have reasoned thus: as there are 20s. in £1, therefore there are 23 times 20 shillings in £23; i. e. we should have multiplied 20s. by 23, instead of 23 by 20—for £23 x 20 equals £460—but for convenience we multiply £23 by 20, and call the product shillings (Art. 32), and so with the pence, &c.

Secondly. To express a number of inferior denomination in units of a higher denomination.

RULE. Divide the given number by the number of units which connect that denomination with the next higher, and the remainder, if any, will be the number of surplus units of the lower denomination. Carry on this process till you arrive at the denomination required.

Ex. How many pounds and shillings are there in 5700 pence?

Proceeding by the Rule given above,

$$\begin{array}{r} 12\,)\,5700 \\ \hline 2{,}0\,)\;\;47{,}5 \\ \hline \pounds 23,\ 15s.\ 0d. \end{array}$$

In dividing 475 by 20, we cut off the 0 and 5, by Art. 65.

Reason for the above process.

Since 12 pence = 1 shilling, therefore, in any given number of pence, for every 12 pence there is 1 shilling, so that in 5700d. or (475 × 12) d. there are 475 shillings.

Again, since 20s = £1, therefore, in any given number of shillings, for every 20 shillings there is £1.

Hence, in 475s. or (20 × 23 + 15) s., there are £23, and 15s. over.

NOTE.—Since each of the above Rules is the converse of each other, the accuracy of any result obtained by either of them may be tested by working the result back again by the other Rule.

REDUCTION OF STERLING MONEY.

MENTAL EXERCISES.

The pupil should be made well acquainted with the pence and shilling table

1. Change 102 pence to their value in shillings.
2. In 5s. 8d. how many pence are there?
3. What is the difference, in farthings, between 15 shillings and ½ of a guinea?
4. What is the difference, in pence, between ½ of a sovereign and a Mark?

EXERCISES FOR THE SLATE.

Reduce (verifying each result)

1. £57 to pence; and 613 guineas to farthings. Ans. 13680d.; 617904q.

2. £15, 12s, to pence; and 5000 guineas to pence.
Ans. 3744d.; 1260000d.

3. 8s. 4½d. to half pence; and £1, 0s. 3¾d. to farthings.
Ans. 201 half pence; 975q.

4. 738 half-crowns to farthings; 22½ guineas to sixpences.
Ans. 88560q.; 945 sixpences.

5. How many half-crowns, how many sixpences, and how many fourpences are there in 25 pounds?
Ans. 200 half-crowns, 1000 sixpences, 1500 fourpences.

6. In 351 seven-shilling pieces, how many half-guineas are there, and how many moidores?
Ans. 234 half-guineas; 91 moidores

COMPOUND ADDITION.

127. Compound Addition is the method of collecting several numbers of the same kind, but containing different denominations of that kind, into one sum.

Rule. Arrange the numbers so that those of the same denomination may be under each other, in the same column, and draw a line below them.

Add the numbers of the lowest denomination together, and find by reduction how many units of the next higher denomination are contained in the sum.

Set down the remainder, if any, under the column just added, and carry the quotient to the next column; proceed thus with all the columns.

Ex. Add together £2, 4s. 7½d., £3, 5s. 10¼d., £15, 15s. 0d., and £33, 12s. 11½d.

Proceeding by the Rule given above,

£	s.	d.
2	4	7½
3	5	10¼
15	15	0
33	12	11½
54	18	5¼

Reason for the foregoing process.

The sum of 2 farthings, 1 farthing and 2 farthings = 5 farthings, = 1 penny and 1 farthing. We therefore put down $\frac{1}{4}$, that is, 1 farthing, and carry 1 penny to the column of pence. Then,

$(1 + 11 + 10 + 7)$ pence = 29d. = $(12 \times 2 + 5)$ d. or 2 shillings and 5 pence. We therefore put down 5d., and carry on the 2 to the column of shillings.

Then, $(2 + 12 + 15 + 5 + 4)$ s. = 38s. = $(20 \times 1 + 18)$s. = £1 and 18s. We therefore put down 18s., and carry on the 1 pound to the column of pounds. Then,

$(1 + 33 + 15 + 3 + 2)$ pounds = £54.

Therefore, the result is £54, 18. $5\frac{1}{4}$.

ADDITION OF STERLING MONEY.

Mental Exercises.

1. A gentlemen paid £8 for a cow, £22, 10. for a horse, and £5, 6. 3. for a saddle; how much did he pay for all?

2. John had 10 pence, Hugh gave him 11 pence and James gave him 1 shilling and $10\frac{1}{2}$.; how much had he then?

3. A farmer has three cows worth £21, a calf worth £2, and three sheep worth £1, 18. 9.; how much are they all worth?

Exercises for the Slate.

1. Find the sum of £28, 14. $6\frac{3}{4}$., £27, 18. $4\frac{1}{2}$., £79, 12. 6., £19, 18. $10\frac{1}{2}$. and £85, 14. $3\frac{3}{4}$.
Ans. £241, 18. $7\frac{1}{2}$.

2. Find the sum of £678, 10. 2., £325, 6. 5., £487, 18. 9., £507, 0. 11. and £779, 10. 8.
Ans. £2778, 6. 11.

3. Find the sum of £306217, 13. $9\frac{3}{4}$., £55, 0. 9., £450812, 15. $2\frac{1}{2}$., £9837. 1. $5\frac{1}{2}$. and £2939, 3. $11\frac{3}{4}$.
Ans. £769861, 15. $2\frac{1}{2}$.

4. Find the sum of £485, 12. $7\frac{3}{4}$., £49, 16. $8\frac{1}{2}$., £186, 13. $11\frac{3}{4}$., £787, 10. $8\frac{1}{4}$., £239, 9. $9\frac{1}{2}$., £843, 11. $4\frac{1}{2}$., £374, 16. 7., £285, 4. $9\frac{3}{4}$ and £599, 19. 8.

Ans. £3852, 15. 10.

5. Add together £18, 14. $8\frac{1}{2}$., £12, 13. $9\frac{1}{2}$., £21, 12. 10., £32, 9. $10\frac{1}{2}$., £63, 13. $9\frac{1}{2}$., £16, 4. $8\frac{1}{2}$.. £35, 14. $9\frac{1}{2}$., £17, 16. $7\frac{1}{2}$., £23, 15. $9\frac{1}{2}$., £35, 17. $2\frac{1}{2}$., £8, 19. 8., £12, 10. $0\frac{1}{2}$. and £13, 8. $8\frac{1}{2}$. Ans. £313, 12. $6\frac{1}{2}$.

COMPOUND SUBTRACTION.

128. COMPOUND SUBTRACTION is the method of finding the difference between two numbers of the same kind, but containing different denominations of that kind.

RULE. Place the less number below the greater, so that the numbers of the same denomination may be under each other in the same column, and draw a line below them. Begin at the right hand, and subtract if possible each number of the lower line from that which stands above it, and set the remainder underneath. But when any number in the lower line is greater than the number above it, add to the upper one as many units of the same denomination as make one unit of the next higher denomination; subtract as before, and carry one to the number of the next higher denomination in the lower line. Proceed thus throughout the columns.

Ex. Subtract £88, 18. $8\frac{1}{2}$ from £146, 19. $6\frac{1}{4}$.

Proceeding by the Rule given above,

£	s.	d.
146	19	$6\frac{1}{4}$
88	18	$8\frac{1}{2}$
58	0	$9\frac{3}{4}$

Reason for the above process.

Since $\frac{1}{2}$d. is greater than a $\frac{1}{4}$d., we add 4 farthings or 1 penny, thus raising it to 5 farthings; and when 2 farthings are subtracted from 5 farthings, we have 3 farthings left.

We therefore place down $\frac{3}{4}$d., and in order to increase the lower number equally with the upper number, we add 1 penny to 8 pence.

Now, 9 pence cannot be taken from 6 pence. We therefore add 12 pence or 1 shilling to 6 pence, thus raising the latter to 18d.; we take the 9 pence from 18 pence, and put down the remainder 9d.; then adding 1s to 18s., the latter becomes 19s.; 19s. taken from 19s. leave no remainder. We then subtract £88 from £146, as though they were abstract numbers. It is manifest that in this process, whenever we add to the upper line, we also add a number of the same value to the lower line, so that the final difference is not altered (Art. 22).

SUBTRACTION OF STERLING MONEY.

Mental Exercises.

1. A man owing £19, 10. 0., paid all but 15 shillings; how much did he pay?

2. A merchant sold a piece of cloth for £10, 5. 0., which was £1, 2. 6. more than he paid for it; how much did it cost him?

3. A man bought 1 barrel of flour at £1, 17. 6., a barrel of corn meal at £1, 2. 6.; he sold both together for £2, 18. 9.; how much did he lose by the bargain?

4. A man having £20, gave £1 for a hat, £3, 15. for a coat, and £1, 5. for a pair of boots; how much had he left?

Exercises for the Slate.

1. From £19, 3. 10. take £8, 15. 3$\frac{3}{4}$.
   Ans. £10, 8. 6$\frac{1}{4}$.

2. From £575, 15. 1$\frac{1}{2}$. take £124, 13. 4.
   Ans. £451, 1. 9$\frac{1}{2}$.

3. From £192, 11. 4$\frac{1}{2}$. take £88, 16. 9$\frac{1}{2}$.
   Ans. £103, 14. 7.

4. What sum added to £947, 19. 7$\frac{3}{4}$. will make £1000?
   Ans. £52, 0. 4$\frac{1}{4}$.

5. A furnished house is worth £4759, 10. $9\frac{1}{4}$.; unfurnished, it is worth £1494, 11. $9\frac{3}{4}$. By how much does the value of the furniture exceed the value of the house?

Ans. £1770, 7. $1\frac{3}{4}$.

COMPOUND MULTIPLICATION.

129. Compound Multiplication is the method of finding the amount of any proposed compound number, that is, of any number composed of different denominations, but all of the same kind, when it is repeated a given number of times.

Rule. Place the multiplier under the lowest denomination of the multiplicand; multiply the number of the lowest denomination by the multiplier, and find the number of units of the next denomination contained in this first product. If there be a remainder, place it down, adding on the number of units just found to the second product; for this second product, multiply the number of the next denomination in the multiplicand by the multiplier, and after carrying on to it the above-mentioned number of units, proceed with the result as with the first product. Carry this operation through with all the different denominations of the multiplicand.

Ex. Multiply £56, 4. $6\frac{1}{2}$. by 5.

Proceeding by the Rule given above

£	s.	d.
56	4	$6\frac{1}{2}$
		5
281	2	$8\frac{1}{2}$

Reason for the above process.

$\frac{1}{2}$d. multiplied by 5 is the same as $(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2})$ d. $=$ 5 half pence $= 2\frac{1}{2}$d. We therefore put down $\frac{1}{2}$d., and carry on 2d. to the denomination of pence.

6d. multiplied by 5 = 30d.; therefore, $(6 \times 5 + 2) =$ 32d. $= (2 \times 12 + 8)$ d. $=$ 2s. $+$ 8d. We therefore put down 8d., and carry on 2s. to the denomination of shillings.

4s. multiplied by 5 = 20s.; therefore, (4 × 5 + 2) s. = 22s. = (20 + 2) s. = £1 + 2s. We therefore put down 2s., and carry on £1 to the denomination of pounds.

Now, by simple multiplication, £56 × 5 = £280; therefore, £(1 + 56 × 5) = £(1 + 280) = £281.

Therefore, the total amount is £281, 2. $8\frac{1}{2}$.

130. When the multiplier exceeds 12, it will be the easiest method to split the multiplier into factors or into factors and parts. Thus, 15 = 3 × 5; 17 = 3 × 5 + 2; 23 = 5 × 4 + 3, and so on.

Ex. Multiply £55, 12. $9\frac{1}{4}$ by 23

£	s.	d.	
55	14	$9\frac{1}{4}$	
		4	
222	11	1,	value of £55, 12. $9\frac{1}{4}$, multiplied by 4.
		5	
1112	15	5,	value of £222, 11. 1., multiplied by 5, or of £55, 12. $9\frac{1}{4}$., multiplied by (4 × 5 or 20).
166	18	$3\frac{3}{4}$,	value of £55, 12. $9\frac{1}{4}$., multiplied by 3.
1279	13	$8\frac{3}{4}$,	value of £55, 12. $9\frac{1}{4}$., multiplied by (20 + 3), or 23.

NOTE 1.—When the multiplicand contains farthings, if one of the factors of the multiplier be even, it will often be advantageous to use it first, as the farthings may disappear.

NOTE 2.—Should the multiplier consist of *many* factors, it will be found in that case convenient to reduce the multiplicand to the lowest denomination contained in it, then multiply this result by the multiplier, and then to reduce the result back again.

MULTIPLICATION OF STERLING MONEY.

MENTAL EXERCISES.

1. What will 17 sheep come to, at 15 shillings each?
2. What will 125 pairs of boots come to, at 25s. a pair?

3. What cost 36 yds. of cloth, at £2, 3. 9. a yard?

4. A and B engage to saw 60 cords of wood, at 2 shillings a cord; A saws 3 cords for every 2 cords sawed by B. How much will A receive more than B?

Exercises for the Slate.

1.	£1, 10. 8 × 90	Ans. £138, 0. 0.
2.	£2, 19. 6 × 121	Ans. £359, 19. 6.
3.	£1, 12. 5 × 75	Ans. £121, 11. 3.
4.	£2, 15. 2¼ × 196	Ans. £540, 16. 9.
5.	£3, 6. 5¼ × 3178	Ans. £10556, 18. 4½.
6.	£2, 6. 9½ × 938	Ans. £2194, 10. 7.

7. What cost 112 pounds of indigo, at 11s. 4½d. per pound? Ans. £63, 14. 0.

8. What is the amount of duty on 149 lbs. of West India coffee, at 7¾d. per pound? Ans. £4, 16. 2¾.

COMPOUND DIVISION.

131. **Compound Division** is the method of finding a compound number, that is, a number composed of several denominations, but all of the same kind, into as many equal parts as the divisor contains units; and also of finding how often one compound number is contained in another of the same kind.

When the divisor is an abstract number.

Rule. Place the numbers as in Simple Division; then find how often the divisor is contained in the highest denomination of the dividend; put this number down in the quotient. Multiply as in Simple Division, and subtract. If there be a remainder, reduce that remainder to the next inferior denomination, adding to it the number of that denomination in the dividend, and repeat the division. Carry on this process through the whole dividend.

Ex. Divide £199, 6. 8. by 130.

Proceeding by the Rule given on preceding page.

```
       £    s.  d.
130 ) 199,  6.  8. ( £1.
      130
      ---
       69
        20
      ----
130 ) 1386 ( 10s.
      130
      ----
        86
        12
      ----
130 ) 1040 ( 8d.
      1040
      ----
```

Therefore, the answer is £1, 10. 8.

Reason for the above process.

We first subtract £1, taken 130 times, from £199, 6. 8., and there remains £69, 6. 8.

Now, £69, 6. 8. = 1386s. 8d.; from this amount we subtract 10s., taken 130 times, and there remains 86s. 8d.

Again, 86s. 8d. = 1040d.; from this amount we subtract 8d., taken 130 times, and nothing remains.

Therefore, £1, 10. 8. is contained 130 times in £199 6. 8.

NOTE.—When the divisor is not greater than 12, the division can be easily performed in one line. Thus, for example, divide £8, 18. 6. by 12.

```
     £   s.   d.
12 ) 8   18   6
   -------------
     0   14   10½
```

Since we cannot divide 8 by 12, we reduce £8 to shillings, and adding in the term 18s., we have 178s. to divide by 12. We obtain 14s., with remainder 10s.; and since 10s. = 120d., therefore, adding in the term 6d., we have to divide 126d. by 12. We obtain 10d., with remainder 6d.; and since 6d. = 24q., we divide 24q. by 12, and thus obtain 2q., or $\frac{1}{2}$d.

DIVISION OF STERLING MONEY.

Mental Exercises.

1. How many lbs. of sugar, at 9d. per lb., may be bought for 117d.?
2. How much coffee, at 8d. per lb., may be bought for 4s. 8d.?
3. How much wheat, at 8s. per bushel, may be bought for £2, 16. 0.?
4. In 243 farthings, how many pence? how many shillings? how many pounds?
5. A goldsmith sold a tankard for £10, 8. 0., at the rate of 5s. 4d. per ounce. How much did it weigh?
6. In £84 how many shillings? In these shillings, how many guineas?
7. Paid £7, 8. 6. for fire-wood, at 9s. per cord; how many cords did I get?

Exercises for the Slate.

1.	£409, 6. 2. ÷ 8.	Ans. £51, 3. 3$\frac{1}{4}$.
2.	£386, 16. 5$\frac{1}{4}$. ÷ 11.	Ans. £35, 3. 3$\frac{3}{4}$.
3.	£12, 18. 4$\frac{1}{2}$. ÷ 39.	Ans. £0, 6. 7$\frac{1}{2}$.
4.	£130264, 9. 6. ÷ 9416.	Ans. £13, 16. 8$\frac{1}{4}$.
5.	£1746 ÷ 2737.	Ans. £0, 12. 9$\frac{279}{2737}$.

132. When the divisor is a composite number, it may sometimes be found convenient to break up the divisor into factors (Art. 67). Thus,

Ex. 1. Divide £37, 14. 0. by 24. 24 = 4 × 6.

		£	s.	d.
24 {	4)	37	14	0
	6)	9	8	6
		1	11	5

Ex. 2. Divide £131, 2. $8\frac{1}{2}$ by 48, and also by the factors 6 and 8, and show that the results coincide.

£ s. d.
48) 131 2 $8\frac{1}{2}$ (£2.
96
———
35
20
———
48) 702 (14s.
48
———
222
192
———
30
12
———
48) 368 (7d
336
———
32
4
———
48) 130 (2q., or $\frac{1}{2}$d.
96
———
34 farthings, or $8\frac{1}{2}$d.

Now, dividing by the factors 6 and 8, we get

£ s. d.
48 { 6) 131 2 $8\frac{1}{2}$
8) 21 17 $1\frac{1}{4}$ + 4
2 14 $7\frac{1}{2}$ + 5 } 34

Therefore, the answer is
£2, 14. $7\frac{1}{2}$., 34 rem.

Therefore, the answer is £2, 14. $7\frac{1}{2}$., 34 farthings remaining; or £2, 14. $7\frac{1}{2}$d. $\frac{34}{48}$.; or £2, 14. $7\frac{1}{2}$d. $\frac{17}{24}$q.

Note.—The true remainder in the second operation is found by Art. 68.

Division of Sterling Money.—*Continued.*

In the following examples, divide by the numbers themselves, and then by any factors composing them, and show th the results are the same.

1. £440, 16. $9\frac{1}{2}$. ÷ 15. Ans. £29, 7. $9\frac{1}{4}$. $\frac{11}{15}$q.
2. £123, 13. $0\frac{3}{4}$. ÷ 99. Ans. £1, 4. $11\frac{3}{4}$. $\frac{2}{33}$q.
3. £678, 19. $9\frac{3}{4}$. ÷ 32. Ans. £21, 4. $4\frac{1}{4}$. $\frac{23}{32}$q.
4. £113, 14. 9. ÷ 96. Ans. £1, 3. $8\frac{1}{4}$., 9d. rem.
6. £100, 0. 0. ÷ 121. Ans. £0, 16. $6\frac{1}{4}$., $11\frac{3}{4}$. rem.

133. If the Divisor be 10, 100, 1000, &c., the operation of Division is usually performed by pointing off as decimals, one, two, three, &c., figures accordingly, at the right hand of the dividend.

Ex 1. Divide £5362, 10. 0. by 100.

```
       Long Method.                              Usual Method.
         £     s.  d.                              £     s.  d.
100 ) 5362   10   0 ( £53.                       53,62  10   0
      500·                                          20
      ----                                       -----
       362                                       12,50
       300                                          12
       ---                                       -----
        62                                        6,00
        20
      ----
100 ) 1250 ( 12s.
      100·
      ----
       250
       200
       ---
        50
        12
       ---
100 )  600 ( 6d
       600
       ---
```

Therefore, the quotient is £53, 12. 6.

Reason for the preceding process.

$$\pounds 5362, 10s. \div 100 = \pounds \tfrac{5362}{100} + \tfrac{10}{100}s.$$
$$= \pounds 53.62 + \tfrac{10}{100}s. = \pounds 53 + \pounds \tfrac{62}{100} + \tfrac{10}{100}s.$$
$$= \pounds 53 + \tfrac{(62 \times 20)}{100}s. + \tfrac{10}{100}s.$$

$= \pounds 53 + \frac{(1240 + 10)}{100}s.$	$= \pounds 53, 12. + \frac{(50 \times 12)}{100}d.$
$= \pounds 53 + \frac{1250}{100}s.$	$= \pounds 53, 12. + \frac{600}{100}d.$
$= \pounds 53 + 12.50s.$	$= \pounds 53, 12. + 6.00d.$
$= \pounds 53, 12. + \frac{50}{100}s.$	$= \pounds 53, 12. 6.$

Ex 2. Divide £1668, 15. 0. by 1500.

$$1500 = 3 \times 5 \times 100.$$

First divide by the factors 3 and 5, and then by 100; it will be found best in all cases of this kind to do so.

```
              £       s.   d.
    ( 3 ) 1668     15    0
 15 {  ---------------------
    ( 5)  1,11      5    0
            20
          ----
          2,25
            12
          ----
          3,00
```

Therefore, the quotient is £1, 2. 3

EXERCISES.

1.	£396, 9. 2. ÷ 10.	Ans. £39, 12. 11.
2.	£1787, 10. 0. ÷ 100.	Ans. £17, 17. 6.
3.	£262, 10. 0. ÷ 2400.	Ans. £0, 2. $2\frac{1}{4}$.
4.	£26380, 4. 2. ÷ 25000.	Ans. £1, 1. $1\frac{1}{4}$.

134. *When the divisor and dividend are both compound numbers of the same kind.*

RULE. Reduce both numbers to the same denomination; divide as in Simple Division, and the result will be the answer required.

Ex. How often is £0, 5. $3\frac{3}{4}$. contained in £15, 18. 9.?

Proceeding by the Rule given on preceding page.

s. d.		£ s. d.
5 $3\frac{3}{4}$		15 18 9
12		20
63		318
4		12
255		3825
		4
	255)	15300 (60
		1530·
		0

Therefore, 60 is the answer.

Reason for the above process.

5s. $3\frac{3}{4}$d. = 255 farthings; £15, 18. 9. = 15300 farthings; and 255 farthings, subtracted 60 times from 15300 farthings, leave no remainder.

EXERCISES.

1. £2, 12. 3. ÷ 1s. $4\frac{1}{2}$d. Ans. 38.
2. £55, 18. $10\frac{1}{4}$ ÷ £2, 8. $7\frac{3}{4}$. Ans. 23.
3. £160, 4. $8\frac{1}{4}$ ÷ £1, 10. $6\frac{1}{4}$. Ans. 105.
4. £3824, 5, $11\frac{1}{4}$ ÷ £22, 19. $4\frac{1}{2}$. Ans. $166\frac{1}{2}$.

135. It was shown, Art. 80, how to multiply or divide by numbers containing fractions; and it may be proper here to give some examples and exercises of a similar kind in Compound Multiplication and Division.

Ex. 1. Required, the price of $22\frac{5}{8}$ cwt. of pork, at £1, 16. 3.

In this example, to multiply by $22\frac{5}{8}$, we first multiply by 22, in the way already explained. Then, for $\frac{5}{8}$, we multiply, in a separate place, £1, 16. 3. by 5, and divide the product by 8. The result is £1, 2. $7\frac{7}{8}$., which, being added to the product by 22, the sum is £41, 0. $1\frac{7}{8}$, the product required. The work for the fractional part is not performed here, as the learner can do that part himself.

£	s.	d.
1	16	3
		2
3	12	6
		11
39	17	6
1	2	$7\frac{7}{8}$
41	0	$1\frac{7}{8}$

The answer might also have been found by multiplying by 23, and taking $\frac{3}{8}$ of £1, 16. 3. from the product; or by means of aliquot parts (Art. 92, def. 9). Thus,

	£	s.	d.	
	1	16	3	
			2	
	3	12	6	
			11	
$\frac{4}{8} = \frac{1}{2}$)	39	17	6	
$\frac{1}{8} = \frac{1}{4}$)	0	18	$1\frac{1}{2}$	= price of $\frac{4}{8}$ cwt., at £1, 16. 3.
	0	4	$6\frac{3}{8}$	= " " $\frac{1}{8}$ " " £1, 16. 3.
	41	0	$1\frac{7}{8}$	

Since $\frac{4}{8}$ cwt. $= \frac{1}{2}$ cwt., the cost of $\frac{4}{8}$ cwt. will be half the cost of 1 cwt. We therefore add $\frac{1}{2}$ of £1, 16. 3.

Again, as $\frac{1}{8} = \frac{1}{4}$ of $\frac{4}{8}$, the cost of $\frac{1}{8}$ cwt. will be $\frac{1}{4}$ the cost of $\frac{1}{2}$ cwt. We therefore take $\frac{1}{4}$ of £0, 18. $1\frac{1}{2}$., and add it to the former result, which gives the same as before.

Ex. 2. If $125\frac{1}{2}$ gal. oil cost £28, 9. $11\frac{3}{4}$., what is the cost per gallon?

Here we multiply 125 by 2, the denominator of the fraction, and to the product of 250 we add 1, the numerator. We multiply the given dividend also by 2 (Art. 75). Then, dividing this product by 251, we find £0, 4. $6\frac{1}{2}$. for the required quotient.

```
          £    s.   d.
125½) 28    9   11¾
   2                  2
 ———  ————————————
 251   56 19   11½(£0, 4. 6½
          20
        ————
        1139
        1004
        ————
         135
          12
        ————
        1631
        1506
        ————
         125
           2
        ————
         251
         251
        ————
```

Exercises.

1. Multiply £13, 12. $11\frac{1}{4}$. by $2\frac{7}{8}$.
   Ans. £37. 18. $1\frac{3}{4} + \frac{2}{3}$q.
2. Multiply £20, 18. $2\frac{1}{2}$. by $12\frac{10}{12}$.
   Ans. £268, 7. $0\frac{1}{12}$.
3. Divide £50, 10. 7. by $\frac{1}{8}$. Ans. £404, 4. 8.
4. Divide £9, 9. $7\frac{5}{16}$. by 3s. 9d. Ans. $50\frac{81}{144}$.

CURRENCY OF NOVA SCOTIA.

136. By an Act passed in 1860, the Currency of Nova Scotia was changed from Pounds, Shillings, and Pence, to that of Dollars and Cents (See Statutes of 1860, Chap. 3, Sects. 1–3).

The denominators are Dollars and Cents, or Dollars, Dimes, Cents, and Mills.

TABLE OF DECIMAL CURRENCY.

10 Mills (*m.*)	Make 1 Cent.	Marked *ct.* or $\frac{1}{100}$.
10 Cents, 100 Mills	" 1 Dime.	" *d.*
10 Dimes, 100 cents, 1000 Mills,	Make 1 Dollar.	Marked $, or *doll.*

It is usual, in writing dollars and cents, to place the sign ($) of dollars in front of the sum, and a point (.) between the dollars and cents. Thus, fifty-six dollars, four dimes, six cents, and five mills, would be written $ 56.465, or $ 56.46½, and read 56 dollars and 46½ cents.

If the sum consists of dollars, and a number of cents less than ten, there must be a cypher between the dollars and the cents in place of dimes. Thus, 5 dollars and 4 cents must be written $5.04; this is evident from Art. 112, for 4 occupies the second place from the decimal point, it being hundredths of a dollar.

137. As the above currency is based on the principles of the *Decimal Notation*, it is evident that any operation, as Addition, Multiplication, &c., may be performed upon it in the same manner as upon decimals or decimal fractions.

NOTE 1.—The names of the coins or denominations less than a dollar, are significant of their value. The term *dime* is derived from the French *disme*, which signifies *ten*; the term *cent* and *mill* are from the Latin *centum* and *mille*, the former of which signifies a hundred, and the latter a thousand.

NOTE 2.—The sign $, which is prefixed to sums of money, is borrowed from the United States.

It is said to be a contraction of "U. S.," the initials of *United States*, which were originally prefixed to sums of money expressed in Federal currency. At length the two letters were moulded or merged into a single character, by dropping the curve of the U, and writing the S over it. Thus, the sum of seventy-six dollars, which was originally written U. S. 76 dollars, is now written $ 76.00.

138. Accounts are also kept in Pounds, Shillings, and Pence.

TABLE.

2 Half-pence	make	1 Penny.
12 Pence	"	1 Shilling.
20 Shillings	"	1 Pound.

Since the denominations in the above table are the same as those of Sterling, all operations under it are to be performed in the same manner.

As Nova Scotia has only issued coins of copper (pence and half-pence) and bronze (cents and half-cents), the gold and silver coins of Britain are used for those of higher value.

The Doubloon and Mexican Dollar are also legal tenders.

The following table will show the coins, &c., in circulation, with their value in dollars and cents; and in pounds, &c.

GOLD COINS.

1 Sovereign	= $ 5.00	= £1, 5. 0.
½ Sovereign	= $ 2.50	= £0, 12. 6.
1 Doubloon	= $ 16.00	= £4, 0. 0.

SILVER COINS.

1 Crown	= $ 1.25	= £0, 6. 3.
½ Crown	= $ 0.62½	= £0, 3. 1½.
1 Florin	= $ 0.50	= £0, 2. 6.
1 Shilling, Stg.	= $ 0.25	= £0, 1. 3.
6 Pence "	= $ 0.12½	= £0, 0. 7½.
4 " "	= $ 0.08	= £0, 0. 4.

The Notes for Twenty Shillings each, issued by the Treasury or Banks, are computed at $ 4.00.

NOTE.—The Treasury and Bank Notes, also the silver coins of New Brunswick and Canada, are computed, in mercantile transactions, at the same value as in those Provinces.

The letters Stg. are generally placed after British money to distinguish it from the currency of those countries using a like denomination.

To Reduce Pounds, Shillings and Pence to Dollars and Cents.

RULE. Multiply the pounds by 4 for dollars, the shillings by 20 for cents, the pence by 5, and divided by 3, for cents also

Ex. Reduce £4, 16. $4\frac{1}{2}$. to Dollars and Cents.

£	s.	d.	
4	16	$4\frac{1}{2}$	
4			
16.00			$16 \times 20 = 320$ cents, or \$3.20
3.20			
$0.07\frac{1}{2}$			
\$$19.27\frac{1}{2}$			$4\frac{1}{2}$d. $\times 5 = 22\frac{1}{2}$, which, divided by 3, $= 7\frac{1}{2}$.

When the shillings in the sum amount to, or exceed, 5, it will be found more convenient to add one dollar for each five shillings to the product of the pounds by 4, then multiply the remaining shillings, if any, by 20, for cents, &c. Thus,

£	s.	d.	
4	16	$4\frac{1}{2}$	Here, 16 shillings $= 5 + 5 + 5 + 1$ shilling. We therefore add 3 dollars to the product. We then multiply 1 shilling by 20, for cents, as before, &c.
4			
\$19.00			
20			
$7\frac{1}{2}$			
\$$19.27\frac{1}{2}$			

To Reduce Dollars and Cents to Pounds, &c.

Reckon every 4 dollars = £1.
" " 1 dollar = 5 Shillings
" " 20 cents = 1 Shilling.
" " 10 cents = 6 Pence.
" " 5 cents = 3 Pence.

And if there are any cents remaining, multiply these by 3. and divide by 5, for pence and fractions of pence also.

Ex. Reduce $165.72 to Pounds, &c

Proceeding by the above Rule,

$165 = £41 + 1 dollar	=	£41,	5.	0.
70 cents = 3 × 20 cents, + 10 cents	=		3	6.
2 cents multiplied by 3, and divided by 5,	=			$1\frac{1}{5}$.
		£41,	8.	$7\frac{1}{5}$.

MENTAL EXERCISES.

1. In 7 dollars and 56 cents, how many cents?
2. In 753 dolls., how many cents?
3. Reduce £1, 10. 6. to dollars, &c.
4. Reduce £4, 15. $7\frac{1}{2}$. to dollars and cents.
5. Reduce $$156.62\frac{1}{2}$ to pounds, &c.
6. What will 125 lbs. of tea cost, at 35 cents per lb.?
7. What cost 19 gallons of oil, at $1.28 a gallon?
8. What cost 124 bottles of wine, at $1.24 a bottle?
9. What cost 165 lbs. of sugar, at $0.125 a pound?
10. What cost 124 tons of hay, at $12.60 a ton?

The answer to be given in pounds, &c.

EXERCISES FOR THE SLATE.

1. What will be the expense of forming 146 miles of railway, at $15.75 per yard? Ans. $4,047,120.00.

2. A man bought 10 horses for £200, 15. 0.; how many dollars did each cost? Ans. $80.30.

3. I have a bank-note of £20, a note-of-hand for £6, 10. 0., and in several coins as follows: in copper, 45 half-pence; in silver, 36 six-penny pieces, sterling; 97 florins; in gold, 36 sovereigns, 21 half-sovereigns, and 6 doubloons. How much have I altogether in Dollars and Cents? Ans. $ 487.87½.

4. What is the cost of 196¾ acres of land, at $ 26.56 per acre? Ans. $ 5225.68.

5. At $ 19.45 per 1000, what cost 2680 feet of pine boards? Ans. $ 52.126

6. When ginger is sold at $ 16.55 per 100 lbs., what is it per lb.? $ 0.1655.

BILLS, ACCOUNTS, ETC.

7. Find the cost of the several articles, and the amount of the bill

Halifax, July 16th, 1863.

C. B. PICKMAN, Esq.

Bot of LEVERETTE, GRIGGS & Co.

163	lbs. Butter,	@	$0.14½	$
235	lbs. Coffee,	@	.08¼	
86	lbs. Chocolate,	@	.11	
685	lbs. Sugar,	@	.10½	
21	doz. Eggs,	@	.13	
860	lbs. Lard,	@	.09½	
				$ 208.838

(8.)

Pictou. Aug. 17th, 1862.

MESSRS. HENRY & BROTHERS,

To J. L. HOFFMAN & Co. *Dr.*

15260	lbs. Pork,	@	$0.05½	$
7265	lbs. Cheese,	@	0.08½	
11521	bush. Oats,	@	0.50	
1560	bbls. Flour,	@	6.12½	

Credit.

By	1150 lbs Soda,	$\$0.06\frac{1}{4}$	\$
"	8256 lbs. Sugar,	0.07	
"	6450 gal. Molasses,	$0.37\frac{1}{2}$	
"	Cash to balance account,		

What is the amount of Cash requisite to balance the account? Ans. \$13703.78

NOTE.—The pupils should be required to reduce the examples given under Sterling to Nova Scotia currency, and perform the same operations as required in the several questions, and verify the results.

139. It has been remarked (Art. 138, Note) that the currency of New Brunswick and Canada is taken at the same value as in the above-mentioned places; but Nova Scotia currency does not pass for the same amount in those Provinces as it does here. The following table will therefore be found useful to those having small transactions with the other British North American Provinces.

TABLE.

N. Scotia.	N. Brunswick.	Canada.	N. F. Land.	P. E. Island.
\$5.00	$= \$4.86\frac{2}{3}$	$= 4.86\frac{2}{3}$	= £1, 4. 0.	= £1, 10. 0.
\$0.25	$= \$0.25$, or $24\frac{1}{3}$c.	$= 0.25$, or $24\frac{1}{3}$.	= £0, 1. 2.	= £0, 1. 6.

NOTE.—When the amount is large, it comes under the rule of Exchange, which will be fully treated of in a more advanced portion of the work.

To reduce small amounts Nova Scotia Currency

To New Brunswick Currency,		deduct	$\frac{2}{75}$;
To Canada	"	deduct	$\frac{2}{75}$;
To P. E. Island	"	add	$\frac{1}{5}$;
To New Foundland	"	deduct	$\frac{1}{15}$.

TABLE OF WEIGHTS AND MEASURES.

140. Weights and Measures were invented 869 B. C.; fixed to a standard in England, A. D. 1257; regulated, 1492; equalized, 1826.

Agreeable to the Act of Uniformity, which took effect 1*st January*, 1826,

The term *Measure* may be distinguished into seven kinds, viz.: *Length, Surface, Volume, Capacity, Specific Gravity, Space, Time and Motion.*

The several denominations of these measures have reference to certain *Standards*, which are entirely arbitrary, and consequently vary among different nations. In England and her Colonies, the standard of

Length is a *Yard*,
Surface is a *Square Yard*, $\frac{1}{4840}$ *of an acre*,
Solidity is a *Cubic Yard*,
Capacity is a *Gallon*,
Weight is a *Pound*.

The *standards* of *angular measure* and of *time* are the same in all European and most other countries.

The *Imperial standard yard*, and the *Imperial standard pound, Troy*, of 1758 and 60, in the custody of the Clerk of the House of Commons, having been destroyed by the fire at the House of Parliament in 1834, Restored Standards of *Weights* and *Measures* have been legalized by 18 and 19 Vict., Cap. 72.

MEASURE OF LENGTH.

Table of Lineal Measure.

141. It is recorded that the various denominations of length were constructed from a *corn* of *barley*, 3 of which, taken from middle of the ear, and well dried, made an inch. Other terms were taken from the human body, such as the Digit or finger's breadth, Palm, Hand, Cubit or length of the arm from the elbow to the wrist; Fathom, from the extremity of one hand to that of the other, the arms oppositely extended.

It is stated that Henry I., in 1101, commanded that the ulna or ancient ell, which answers to the modern yard, should be made the length of his arm; and that the other measures of length were hence derived, whether *Lineal, Superficial* or *Solid.*

The *Restored Standard* of Lineal Measure, whose length is called a yard, is a solid square Bar, thirty-eight inches long, and one inch square, in transverse section, the Bar being of Bronze or Gun Metal, at the temperature of 62° of Fahrenheit's Thermometer, marked Copper, 16 oz.; Tin, 2½ oz.; Zinc, 1 oz.; and near to each end a Cylindrical Hole is sunk to the depth of half an inch—the distance between the centres of the two holes being 3 Feet or 36 Inches, or one *Imperial Standard Yard*.

The standard of *Square* and *Cubic* measures will therefore depend entirely upon it.

At present, we have no means of ascertaining why this particular length was originally fixed upon; but as it is most essential that it should always remain the same, it will be found convenient to refer it to something else, which we have no reason to suppose ever undergoes any change.

Now, the length of a *Pendulum*, vibrating *seconds*, or forming 86,400 oscilations in the interval between the sun's leaving the meridian of a place and returning to it again, is always the same at a *fixed* place and under the *same* circumstances; and if this length be divided into 391,392 equal parts, the yard is *defined* to be equivalent to 360,000 of these parts; also, conversely, since a yard is equal to 36 inches, it follows that the length of the seconds' pendulum, expressed in inches, is 39.1392.

The *Pendulum* referred to, is one vibrating seconds at *Greenwich* or in *London*, at the level of the sea, in a vacuum or non-resisting medium; and if the *standard yard* be at any time lost or destroyed, it is easy to have recourse to experiment for its recovery.

The *standard yard* being the general *unit* of lineal measure, it follows that all lengths less than a *yard* will be expressed by fractions; and it is on this account that a lineal *inch*, or *ten thousands* of the expressed portions of the *pendulum*, is conveniently adopted as a *unit* of lineal measure, when applied to small magnitudes.

Hence, also, by the same means, the *standard superficial* and *solid* measures will be accurately ascertained and kept correct.

In this measure, which is used to measure distances, lengths, breadths, heights, depths, and the like, of places or things,

12	Inches (in length)	make	1	Foot,	written	1 ft.
3	Feet	"	1	Yard,	"	1 yd.
6	Feet	"	1	Fathom	"	1 fath.
$5\frac{1}{2}$	Yards	"	1	Rod, Pole or Perch,	"	1 po.
40	Poles	"	1	Furlong,	"	1 fur.
8	Furlongs	"	1	Mile,	"	1 m.
3	Miles	"	1	League,	"	1 lea.
$69\frac{1}{2}$	Miles	"	1	Degree,	"	1 deg. or 1°

An inch is the smallest lineal measure to which a name is given; but subdivisions are used for many purposes. Among mechanics, the Inch is commonly divided into *eighths* and *sixteenths*. By the officers of the revenue, and by scientific persons, it is divided into *tenths*, *hundredths*, &c. The inch, three-fourths inch, half-inch and quarter-inch, divided into *twelfths*, are used by architects.

The following measurements may be added, as useful in certain cases:

4 Inches make 1 Hand (used in measuring horses).

22 Yards make 1 Chain, } Used in measuring land.
100 Links make 1 Chain, }

A Palm = 3 inches, a Span = 9 inches, a Cubit = 18 in.

A Pace = 5 feet, 1 Geographical Mile = $\frac{1}{60}$th of a degree.

CLOTH MEASURE.

142. This measure, which is a species of Long Measure, is used for all kinds of cloth, muslin, ribbon, &c.

The yard, in Cloth Measure, is the same as in Long Measure, but differs in its divisions and subdivisions.

$2\frac{1}{4}$	Inches	make	1	Nail.
4	Nails	"	1	Quarter, 1 qr.
4	Quarters	"	1	Yard, 1 yd.
5	Quarters	"	1	English Ell.
6	Quarters	"	1	French Ell.
3	Quarters	"	1	Flemish Ell.

Mental Exercises.

1 In 8 ft. 7 in. how many inches are there?

2 In 2 fath. 1 ft. 9 in. how many inches are there?

3. How many feet of line will reach the bottom in $9\frac{2}{3}$ fathoms of water, the deck of the vessel being 15 ft. above the water?

4. A school-room is 2 rods 15 feet in length, and $2\frac{1}{2}$ rods in breadth; how many inches does the length exceed the breadth?

5. James lives $\frac{1}{4}$ of a mile from the school-house; if he takes 2 steps every yard, how many steps will he take in going to school?

6. What is the use of Long Measure? What is the Standard, and how obtained?

7. If it requires $2\frac{1}{2}$ yds. of cloth to make a coat, how many nails of cloth will there be in 16 coats?

Exercises for the Slate

Reduce, verifying the result in each case, the following:

1. 3 m. 7 fur. 8 po. to yards; and 573 miles to inches.
Ans. 6864 yds.; 36305280 in.

2. 1364428 in. to leagues; and 74 m. 3 fur. 4 yds. to inches. Ans. 7 lea. 4 fur. 10 po. 5 yds. 2 ft. 4 in.
and 4712544 in.

3. 7 fur. 200 yas. to chains; and 6 cubits, 1 span to feet. Ans. 79 ch. 2 yds.; 9 ft. 9 in.

4. 1000000 inches to miles.
Ans. 15 m. 6 fur. 10 po. 2 yds. 2 ft. 4 in.

5. A person engages to walk 16 times, without stopping, between two places whose distance is 1 m. 3 fur. 14 po. 4 yds 2 ft. 3 in.; what will be the whole distance he will have to walk?
Ans. 22 m. 5 ft. 37 po. 4 yds. 1 ft. 6 in.

MEASURE OF SURFACE

Table of Square Measure.

143. The *Imperial square yard* contains 9 imperial square feet, and the *Imperial square foot* 144 square inches; the *circular foot* (that is, a circle whose diameter is 1 foot) contains 113,097 square inches; and the *square foot* contains 183,346 circular inches (that is, inches whose diameters are each 1 inch).

This measure is used to measure all kinds of superficies, such as land, paving, flooring, in fact everything in which length and breadth are to be taken into account.

A square is a figure which has four equal sides, each perpendicular to the adjacent ones.

A square inch is a square, each of whose sides is an inch in length; a square yard is a square, each of whose sides is a yard in length, &c.

The table of Square Measure is formed from that of Long Measure, by multiplying each lineal dimension by itself. Thus,

A Square Foot is $= 12 \times 12 = 144$ square inches.

144	Square Inches	make	1 Square Foot, 1 sq. ft. or 1 ft.
9	Square Foot	"	1 Square Yard, 1 sq. yd. or 1 yd.
30¼	Square Yards	"	1 Square Pole, 1 sq. po. or 1 po.
40	Square Poles	"	1 Square Rood, 1 ro.
4	Roods	"	1 Acre, 1 ac.
640	Acres	"	1 Square Mile.
30	Acres	"	1 Yard of Land.
100	Acres	"	1 Hide of Land.
40	Hides	"	1 Barony.

25000 Square Links = 1 Rood.
100000 Square Links = 1 Acre.
10 Square Chains = 1 Acre.

Note 1.—The chain, referred to in the above and preceding table, is called *Gunter's Chain;* it is 4 perches in length, and is divided into 100 equal parts, called *links*. Four perches being equivalent to 792 inches, it follows, from dividing by 100, that the length of a link is 7 92-100 inches.

Surveyors compute by Chains and Links, but exhibit the result in

NOTE 2.—In many parts of England, Ireland and Scotland, the old measure is still retained. The following table will therefore be found useful in some cases:

$6\frac{1}{4}$ Yards = 1 Perch, Cunningham measure.
6 Yards = 1 Perch, Woodland or Burleigh measure.
7 Yards = 1 Perch, Irish measure.
8 Yards = 1 Perch, Forest "

The roads, in Ireland, are measured by the Irish perch.

In some parts of Ireland, land is rented, bought and sold by the Irish acre, or, as it is sometimes called, *Irish plantation measure;* in some, by *Cunningham measure*, and in others, by *English statute measure.*

In *Irish measure*, 64 acres are equivalent to 49 acres in *Forest measure;* 625 to 784 in *Cunningham measure;* 36 to 49 in *Woodland* or *Burleigh measure;* 121 to 196 in *English statute measure;* and 1369 to 1764 *Scotch acres.*

These numbers may be found by multiplying each of the numbers expressing the length of the perches, in the different kinds of measure, by itself.

The Scotch chain of 74 feet, or 24 ells of 37 inches each, is partially used yet.

Ex. 1. Reduce 13 ac. 3 ro. 14 po. to square yards.

Proceeding by the Rule given (Art. 126),

ac.	ro.	po.
13	3	14
4		
55		
40		
2214		
$30\frac{1}{4}$		
66420		
$553\frac{1}{2}$		
$66973\frac{1}{2}$		

Ex. 2. Reduce 1896784 sq. yds. to acres.

Proceeding by Rule given (Art. 126).

```
                                 Square yards.
                                   1896784
                                         4
                                  --------
30¼ reduced to        ( 11 ) 7587136
      fourths, = 121 {        --------
                      ( 11 )  689739"7 )
                              ---------  } 73 quarters
                  4,0 )      6270,3"6  )    = 18¼ yds.
                             ---------
                    4 )      1567"23 per.
                             ------------
                              391ac. 3ro. 23per. 18¼yds.
```

Ex. 3. How many acres are there in a field which is 12 chains, 78 links long, and 4 chains, 72 links broad?

```
12 ch. 78 links = 1278 links.
 4 ch. 72 links =  472 links.
                  ----
                  2556
                 8946
                5112
                -------
                6,03216
                      4
                -------
                0,12864
                     40
                -------
                5,14560
                    30¼
                  -------
                  436800
                    3640
                  -------
                 4,40440
```

Ans. 6 ac. 0 ro. 5 po. 4⅖ yds.

Reason for the preceding process.

When links are multiplied by links, the product is called square links. Therefore, 1278 × 472 = 603216 square links, and as there are 100000 square links in an acre, we divide the former by the latter for acres; that is, we cut off five figures toward the right hand.

Again, a *Rood* being the next unit of measure, the remainder will contain 4 times as many roods as acres. We therefore multiply by 4, and divide as before. In the same manner, we multiply by 30¼ for yards, &c.

MENTAL EXERCISES.

1. How many square inches are there in 2 sq. ft. 12 sq. in.?

2. Change ⅜ of a sq. ft. to sq. in., and add 6 sq. in. to it.

3. What will it cost to dig a garden 900 feet long and 30 feet wide, @ 3 cents per sq. yard?

4. What cost 240 sq. rods of land, at $1000 an acre?

5. What is the use of Square Measure? By what measure would you ascertain the distance from your home to the school-room? Would you use the same kind to measure your slate?

6. How wide must a piece of land be that is 16 rods long, to make an acre?

7. Find the numbers for the foregoing table.

EXERCISES FOR THE SLATE.

Verify each of the following results:

1. Reduce 35 ac. 2 ro. to poles. Ans. 5680 po.
2. Reduce 3 ro. 37 po. 26 yds. to inches.
   Ans. 6188724.
3. Reduce 15 ac. 3 ro. to links. Ans. 1575000.
4. Reduce 93827 perches to acres.
   Ans. 586 ac. 1 ro. 27 po.

5. Find the sum of 25 ac. 2 ro. 16 po.; 30 ac. 2 ro. 25 po.; 26 ac. 2 ro. 35 po.; 63 ac. 1 ro. 31 po.; and 34 ac. 2 ro. 29 po. Ans. 181 ac. 0 ro. 16 po.

6. A man having 56 ac. 2 ro. 34 po., sold 48 ac. 3 ro. 38 per.; how much had he remaining?

Ans. 7 ac. 2 ro. 36 po.

7. Multiply 380 ac. 3 ro. 32 po. by 106.

Ans. 40380 ac. 2 ro. 32 po.

8. A person, at his death, bequeathed his farm, containing 1867 ac. 3 ro. 14 po. to his three sons, in equal shares; how much does each get?

Ans. 622 ac. 2 ro. 18 po.

ARTIFICERS' MEASURE.

144. Flooring, roofing, &c., are measured by the square of 100 feet.

Plastering, by the square yard. Plasterers, generally, deduct $\frac{1}{3}$ of all the openings from the total content, for the true content.

Bricklayers' work, by the pole of $16\frac{1}{2}$ feet, the square of which is $272\frac{1}{4}$ feet, though this is partly a cubic measure, as the brickwork is reckoned to be 14 inches, or $1\frac{1}{2}$ brick thick.

Masons calculate by the rood of 36 sq. yds. This is partly a cubic measure, also, as the work is reckoned to be 22 inches thick.

Ex. 1. Required, the number of square feet in a roof which is 46 feet, 10 inches long, and 21 feet, 3 inches of rafter.

46 ft. 10 in. = 562 inches.
21 ft. 3 in. = 255 inches.

144) 143310 sq. inches.

995 sq. ft. 30 inches.

Ex. 2. Find the expense of paving a floor, whose length is 33 ft. 2 in., and breadth 18 feet, @ 6s. per square yard.

```
33 ft. 2 in.  =   398 inches.
18 ft.        =   216
                 -----
      ( 12)85968
  144 {      -----
      ( 12)  7164
             -----
          9)  597
              ---
              66⅓ sq. yds., @ 6s. per sq. yd
            -------
          £3,  6.  4. = cost, @ 1s. per yd.
                   6
         -------
        £19, 18.  0. = cost, @ 6s. per yd.
```

It may be well to remark that each of the foregoing examples might have been worked in the fractional form, or by reducing the lower denominations into decimals of the highest involved.

There is, however, a method of working examples in square and cubic measure, without reducing the different denominations to the same denomination.

This method is styled CROSS MULTIPLICATION OR DUODECIMALS.

Duodecimals are calculations by feet, inches and parts, which decrease and increase by twelves. Hence, they take their name.

TABLE.

12 Thirds	make 1 Second,	Marked	1″
12 Seconds	" 1 Prime,	"	1′
12 Primes or Inches	" 1 Foot,	"	ft.

The inches or primes, seconds, &c., being distinguished by the signs ′, ″, ‴, ⁗, &c., the place of any product may be known by adding the signs of the factors together. Thus, 9 feet × 5″ = 45″ = 3′, 9″; 4′ × 5″ = 20‴ = 1″, 8‴.

To Multiply Feet and Inches, or Feet, Inches and Parts Duodecimally.

Rule. Place feet below feet, inches below inches, &c. First multiply the whole multiplicand by the feet in the multiplier, as in compound multiplication; then by the inches in the same manner, placing the product, however, one place farther to the right; then the seconds in the same manner, &c. The sum of these products will be the product of the two quantities.

Ex. 1. Multiply 4 ft. 6 in. by 2 ft. 3 in.

Proceeding by the Rule given above,

4 ft.	6 in.	
2 ft.	3 in.	
9	0′	
1	1′	6″
10 ft.	1′	6″, or 10 ft. 18 in.

Note.—In the above example of Cross Multiplication, we see t[illegible] mixed decimal and duodecimal scale of notation is employed, the fig[illegible] of the feet being expressed and multiplied in the ordinary way; whereas, in other places the number 12 is always used instead of 10. Cross Multiplication is not, therefore, properly termed Duodecimal Multiplication or Duodecimals, because, although the different denominations are connected with each other by the number 12, still the different digits of those denominations are connected with each other by the number 10.

After multiplying by the feet, it will often be found more convenient and expeditious to perform the rest of the operation by means of aliquot parts (Art. 92, def. 9).

Ex. What is the superficial content of a floor whose length is 40 feet 6 inches, and breadth 28 feet 9 inches?

	Ft.	′	″	
	40	6		
	28	9		
	14	0′		= 6′ × 28 = 14 ft.
	320			
	80			
6′ = ½ of 1 ft. =	20	3′		
3′ = ½ of 6′ =	10	1′	6″	
	1164	4′	6″	

or 1164 sq. ft. 54 sq. in.

Ft.	′	″
40	6	
28	9	
1134	0′	
30	4′	6″
1164	4′	6″

or 1164 ft. 54 sq. in.

Mental Exercises.

1. How many square inches in a board 15 inches wide and 11 feet long? How many sq. ft.?

2. What will it cost to plaster a room 9 feet long, 8 feet wide, and 9 feet high, at 15 cents per yard?

3. How many feet are there in a board 3 feet 2 inches long, and 23 inches wide?

Exercises for the Slate.

1. What is the superficial content of a plank 16 feet 8 inches long, and 15 inches broad?
Ans. 20 ft. 10′, or 120 in.

2. Find the content of a board 9 ft. 8 in. long, and 4 ft. 6½ in. broad. Ans. 43 ft. 10′, 10″, or 43 ft. 130 sq. inches.

3. How many square yards of paving in a court 38 ft. 7 in., by 20 ft. 9 in.? Ans. 88 yds. 8 ft. 7′, 3″

4. How many yards of flooring in a house of two stories, 40 ft. 9 in. long, by 32 ft. 0 in. within, deducting from both flats the well-hole of stair, 10 ft. by 6 ft. 6 in.?
Ans. 130 yds. 4 ft.

5. How many square feet of board will be in a log 28 feet 3 inches long, 16½ inches deep, cutting 10 boards?
Ans. 388 feet, 63 inches.

MEASURE OF SOLIDITY.

Table of Solid or Cubic Measure.

145. The *Imperial cubic* (or solid) *yard* contains 27 imperial cubic feet, and the *imperial cubic foot* contains 1728 cubic inches. The *cylindric foot* (that is, a cylinder 1 foot long and 1 foot in diameter) contains 1357.17 cubic inches. The *conical foot* (that is, a cone 1 foot in height, and 1 foot at the base) contains 452.39 cubic inches.

A *Cube* is a solid body, and contains length, breadth, and thickness, having six equal sides.

A *Cube Number* is produced by multiplying a number twice into itself. Hence, the following table is formed from the table of lineal measure by multiplying each lineal dimension by itself twice.

Cubic Measure is used in measuring solid bodies, or things which have *length*, *breadth*, and *thickness:* such as timber, stone, boxes of goods, the capacity of rooms, ships, &c.

1728 Cubic Inches	= 1 Cubic Foot.
27 Cubic Feet	= 1 Cubic Yard.
40 Cubic Feet of Rough, or 50 Cubic Feet of Hewn Timber	= 1 Ton.
42 Cubic Feet of Timber	= 1 Shipping Ton.
128 Cubic Feet	= 1 Cord of Wood.
5 Cubic Feet	= 1 Barrel bulk.

NOTE 1.—By a ton of round timber, is meant such a quantity of timber in its rough or natural state, as when hewn, will make 40 cubic feet, and is supposed to be equal in weight to 50 feet of hewn timber.

NOTE 2.—The truth of the foregoing Tables of Square and Cubic Measures, should be illustrated to the pupils by means of small blocks, of one inch each, and by diagrams.

Ex. What is the solid content of a block which is 10 feet 3 inches long, 2 feet 4 inches wide, and 3 feet 5 inches deep?

```
10 ft. 3 in. =  123 Inches.
 2 ft. 4 in. =   28
                ---
                984
               246
               ----
               3444 Sq. Inches.
 3 ft. 5 in. =   41 Inches.
               ----
               3444
              13776
             ------
             141204 Cubic Inches.
           = 81 cub. ft. 1236 cub. inches.
```

By Duodecimals.

Ft.	′		
10	3		
2	4		
20	6′		
3	5′	0″	
23	11′	0″	
	3	5′	
71	9′	0″	
9	11′	7″	0‴
81	8′	7″	0‴

or 81 cub. ft. 1236 cub. inches.

	Ft.	′	
	10	3	
	2	4	
	20	6′	
$4' = \frac{1}{3} =$	3	5	
	23	11′	
	3	5′	
	71	9′	
$4' = \frac{1}{3} =$	7	11′	8″
$1' = \frac{1}{4} =$	1	11′	11″
	81	8′	7″

Mental Exercises.

1. How many solid inches does a block 2 inches long, 2 inches wide, and 1 inch thick, contain? 2 inches thick? 4 inches thick?

2. How many cubic inches does a block that is 1 foot long, 1 inch thick, and 1 inch wide, contain? How many if 2 inches wide? 4 inches wide?

3. What is the cost, per foot, of wood, at $1.60 per cord, and what will a pile of such wood, 5 ft. 6 in. long, 4 ft. 2 in. high, and 4 feet deep, cost?

Exercises for the Slate.

1. How many cubic inches in 767 cubic feet?
Ans. 1325376.

2. Reduce 157248 cubic inches to yards?
Ans. 3 cub. yds. 10 cub. ft.

3. Find the sum of 3 c. yds. 23 c. ft. 171 c. in.; 17 c. yds. 17 c. ft. 31 c. in.; 28 c. yds. 26 c. ft. 1000 c. in., and 34 c. yds. 23 c. ft. 1101 c. in.
Ans. 85 c. yds. 9 c. ft. 575 c. in.

4. Required, the content of a log 28 ft. 8 in. long, 17 in. wide, and 24½ deep. Ans. 82 ft. 10 in. 11″, 8‴.

5. Find the content of a tank 12 ft. 6 in. by 11 ft. 4 in. at top, by 10 ft. 8 in. at bottom, and 6 ft. deep.

Ans. 764 ft. 72 in.

Note.—When the article to be measured tapers in either length, breadth, or depth, take half the sum of that at the two ends for a mean.

MEASURE OF CAPACITY.

Table of Liquid and Dry Measure.

146. The *Imperial Gallon* is the standard unit of the measure of capacity, and is defined to be 277.274 cubic inches, the lineal inch being that above mentioned. The gallon, and its multiples and parts, are used to measure both *liquids*, as water, spirits, &c.; and *dry goods*, as malt, corn, &c., and the system is therefore called the *Imperial Liquid and Dry Measure.*

						Cubic Inches
4 Gills	make	1 Pint,	marked	1	pt.	34.6592.
2 Pints	"	1 Quart,	"	1	qt.	69.3185.
4 Quarts	"	1 Gallon,	"	1	gal.	277.274.
2 Gallons	"	1 Peck,	"	1	pk.	554.548.
4 Pecks	"	1 Bushel,	"	1	bus.	2218.192.
8 Bushels	"	1 Quarter,	"	1	qar.	17745.536.
36 Bushels	"	1 Chaldron,	"	1	ch.	

HEAPED MEASURE.

147. Potatoes, Turnips, Fruit, Lime, Coals, and a few other articles, are bought and sold by *heaped measure.*

				Cubic Inches
		1 Peck,	=	703.87148.
4 Pecks	=	1 Bushel,	=	2815.4871.
3 Bushels	=	1 Sack or Tub,	=	8446.45776.
12 Tubs	=	1 Chaldron,	=	101357.49309.

The diameter of the exterior brim of the bushel is to be 19½ inches, and the height of the heap at least 6 inches. The content of the heap is therefore 597.29518 cubic inches, which, added to 2218.192, the content of the bushel, gives 2815.4871 cubic inches for the content of the heaped bushel, and the contents of the other measures are in proportion.

The outside diameter of the measures less than a bushel, are as follows:

Half-bushel	=	$15\frac{1}{2}$ Inches.	Gallon	=	$9\frac{3}{4}$ Inches.
Peck	=	$12\frac{1}{4}$ Inches.	Half-gallon	=	$7\frac{3}{4}$ Inches.

Mental Exercises.

1. At 43 cents a peck, what cost 14 bus. 3 pks. of wheat?

2. At 3 cents per quart, what will 5 bus. 3 pks. 2 qts. of salt come to?

3. A man sold 63 gal. molasses @ $0.40 per gal., and received his pay in corn @ $0.84 per bus. How many bushels did he receive?

4. In 48 pecks how many pints?

5. By what measure would you buy potatoes? oats? oil? boards? ribbon? cloth?

Exercises for the Slate.

1. In 186040 pks. how many chaldrons?
Ans. 1291 ch. 34 bus.

2. In 365843 gills how many gallons?
Ans. 11432 gal. 2 pt. 3 gi.

3. Add together 39 gal. 3 qt. 1 pt.; 48 gal. 2 qt. 1 pt.; 56 gal. 1 pt.; 74 gal. 3 qt.; 84 gal. 6 qt. 1 pt.
Ans. 305 gal. 0 qt.

4. Find the difference between 23 chal. 5 bus. 2 pk. and 14 chal. 6 bus. 3 pk. Ans. 8 chal. 34 bus. 3 pk.

5. Multiply 57 gal. 3 qt. separately by 10 and 257.
Ans. 577 gal. 2 qt; 14841 gal. 3 qt.

APOTHECARIES' FLUID MEASURE.

60 Minims (♏)	= 1	Fluid Dram,	Marked	f℥.
8 Drams	= 1	" Ounce,	"	f℥.
16 Ounces	= 1	" Pint,	"	O.
8 Pints	= 1	" Gallon,	"	gall.

Note.—In some places a pint equals 20 ounces. A minim may be reckoned 2 drops, a dram about a tea-spoonful, and 1 ounce about 2 table-spoonfuls.

MEASURES OF WEIGHT.

Table of Troy Weight.

148. The origin of all *weight* in England was derived from a *grain* of wheat. *Vide* Statute of 51 Henry III.; 31 Edward I., and 12 Henry VII., which enacted, that 32 of them gathered from the middle of the ear, and well dried, were to make 1 pennyweight; 20 pennyweights 1 ounce; and 12 ounces 1 pound.

It was subsequently thought better to divide the *penny-weight* into 24 equal parts, called grains.

William the Conqueror introduced into England what is called Troy-weight, from *Troyes*, a town in the province of Champagne, in France, now in the department of Aube, where a large and celebrated fair was held.

It seems to have been brought hither from Egypt.

It has also been derived from *Troy-novant*, the monkish name for London.

This weight was formerly used for weighing articles of every kind. It is now only employed in weighing gold, silver, diamonds, and other articles of a costly nature; also in determining specific gravity.

The different units are grains (grs.), pennyweights (dwt.), ounces (oz.), and pounds (lbs.), and are connected thus:

24 Grains	make	1 Pennyweight,	1 dwt.
20 Pennyweights	"	1 Ounce,	1 oz.
12 Ounces	"	1 Pound,	1 lb.

NOTE 1.—It was called a *pennyweight* from its being the weight of the silver penny then in circulation.

The term ounce comes from the Latin word *uncia*, which signifies a twelfth part.

In the abbreviation, dwt., for pennyweight, d is from the Latin word *denarius*, a penny; wt, the first and last letters of the English word weight. Oz, is from the Spanish *onza*, an ounce.

NOTE 2.—Diamonds, and other precious stones, are weighed by "Carats," each carat weighing 3 17-101, or nearly 3 1-6 grains, Troy. The term carat, applied to gold, has a relative meaning only; any quantity of pure gold, or of gold alloyed with any other metal, being supposed to be divided into 24 equal parts (carats): if the gold be pure, it is said to be 24 carats fine; if 22 parts be pure gold, and 2 parts alloy, it is said to be 22 carats fine.

Standard gold is 22 carats fine; jewellers' gold is 18 carats fine. Thus we generally perceive "18" on the cases of gold watches. This indicates that they are "18 carats fine," the lowest degree of purity which is marked; but many articles are manufactured as low as 9 carats fine.

TABLE OF APOTHECARIES' WEIGHT.

149. Apothecaries' weight only differs from Troy weight in the subdivisions of the pound, which is the same in both. This table is only used in mixing medicines, as they buy and sell by Avoirdupois weight.

The different units are grains (grs.), scruples (℈), drams (ʒ), ounce (℥), pound (lb.), and these are connected thus:

20 Grains	make	1 Scruple,	1 sc.	or 1 ℈.
3 Scruples	"	1 Dram,	1 dr.	or 1 ʒ.
8 Drams	"	1 Ounce,	1 oz.	or 1 ℥.
12 Ounces	"	1 Pound,	1 lb.	or 1 ℔.

MENTAL EXERCISES IN TROY AND APOTHECARIES WEIGHT.

1. How many grains are there in 1 dwt.?
2. In 3 dwt. 6 gr. how many grains are there?
3. In 3 scruples how many grains are there?
4. How often is 36 drams contained in $\frac{3}{4}$ lb.?
5. 3 dr. 1℈ 20 gr. are how many times 22 grains?
6. $\frac{3}{4}$ of a pound + 2 oz. are equal to how many dwt.?
7. When gold is selling for $16\frac{3}{4}$ dollars an ounce, what is the value of a pound?

Exercises for the Slate.

1. Reduce 536 lb. to scruples. Ans. 154368 sc.

2. Reduce 17 lb. 2℥ 2℈ to grains. Ans. 98920 gr.

3. Reduce 29 lb. 7 oz. 3 dwt. to grains. Ans. 170472 gr.

4. Reduce 6237 drams (apoth.) to lbs. Ans. 64 lb. 11 oz. 5 dr.

5. A gentleman's silver plate weighs as follows, viz.: salvers, 12 lb. 3 oz. 20 gr.; tankards, 9 lb. 6 oz. 13 dwt. 17 gr.; spoons, 21 lb. 14 dwt. 6 gr.; knives and forks, 16 lb. 3 oz. 15 dwt. 21 gr.; and salts, 2 lb. 6 dwt. Required, the total weight of his plate. Ans. 61 lb. 2 oz. 10 dwt. 16 gr.

6. A person having a piece of pure gold weighing 13 oz. 5 dwt. 9 gr., offered to exchange it for a piece of plate weighing 12 oz. 3 dwt. 3 gr., provided he would get 84 cents per dwt. for the difference in weight. How much would he receive? Ans. $18.69.

AVOIRDUPOIS WEIGHT.

150. When William the Conqueror introduced Troy weight, the English were dissatisfied with it, because it did not weigh so much as the pound then in use. Hence arose the term *Avoir-du-poids*, which was a medium between the French and the ancient English weight.

Avoirdupois Weight was first made a legal tender in the reign of Henry VII., and its particular use was to weigh provisions and coarse heavy articles. Henry fixed the stone at 14 lbs., which has since been confirmed by Act of Parliament.

The *Imperial Pound, Avoirdupois*, which is the standard unit by means of which all heavy goods of large masses is weighed, is *defined* to be the weight of *one-tenth* part of an imperial *gallon*, or of 27.7274 cubic inches of distilled water, ascertained at a time when the barometer stands at 30 ins., and the height of *Fahrenheit's* thermometer is 62°; and this standard may consequently be verified or recovered at any time, when it may be necessary to appeal to experiment.

If the weight of a cubic inch of distilled water be divided into 505 equal parts, and each of such parts be defined to be a *half-grain*, it follows that 27.7274 cubic inches contain very nearly 7000 such grains; and it is hence declared by Act of Parliament that 7000 *grains* exactly, shall hereafter be considered as a pound, avoirdupois; and that 10 *grains* shall be equivalent to 1 *scruple;* and 3 *scruples* to 1 *dram*. But these latter denominations are seldom necessary, unless great nicety is required.

This weight receives its name from *avoirs*, the ancient name of *goods* and *chattels*, and *poids*, signifying *weight*, in the ordinary language of the country at the time of the *Normans*.

The *Restored Imperial Standard Pound*, *Avoirdupois*, is constructed of platinum, the form being that of a *cylinder*, nearly 1.35 inch in height, and 1.15 inch in diameter, marked P. S. 1844, 1 ℔.

The different units are drams (drs.), ounces (oz.), pounds (lb.), tons (ton), and they are connected thus:

				Grains, Troy.
16 Drams	make	1 Ounce,	1 oz. =	437.50
16 Ounces	"	1 Pound,	1 lb. =	7000
28 Pounds	"	1 Quarter,	1 qr. =	196000
4 Quarters	"	1 Hundredweight,	1 cwt. =	784000
20 Hundredweight	"	1 Ton,	1 ton =	15680000

In general, 1 Stone (1 st.) = 14 lbs. avoirdupois, but for butchers' meat or fish, 1 stone = 8 lbs.

In Ireland, in the sale of some articles, the stone is = 16 lbs. avoirdupois.

NEW SYSTEM OF WEIGHT.

151. By an Act of the Provincial Parliament, passed A. D. 1859, the hundredweight is to contain 100 lbs. avoirdupois, instead of 112 lbs., and the ton 2000 lbs. instead of 2400 lbs., as formerly.

The different units are the same as in the old system. Thus:

				Troy grains.
16 Drams	make	1 Ounce,	1 oz. =	437.5
16 Ounces	"	1 Pound,	1 lb. =	7000
25 Pounds	"	1 Quarter,	1 qr. =	175000
4 Quarters	"	1 Hundredweight,	1 cwt. =	700000
20 Hundredweight	"	1 Ton,	1 ton =	14000000

NOTE.—The old system of weight is called long, and the new system short weight.

Ex. 1. Reduce 23 cwt. 3 qr. 14 lb. long weight, to pounds.

Proceeding by the Rule given (Art. 126),

```
cwt. qr. lb.
 23   3   14
  4
 --
 95
 28
---
 774
190
----
2674 pounds.
```

SECOND METHOD.

```
cwt. qr. lb.
 23   3   14
------
2300 = 23 × 100 }
 276 = 23 × 12  } = 112 × 23.
  84 = 3 qr. × 28.
  14 = 14 lbs.
----
2674 pounds.
```

Ex. 2. Reduce 18967432 dr. to hundredweight, short weight.

Proceeding by Rule given (Art. 126),

```
        ( 4 ) 18967432
16 dr. {  ------------
        ( 4 )  4741858,0 )
          ------------   } 8 dr.
        ( 4 )  1185464,2 )
16 oz. {  ------------
        ( 4 )   296366,0 )
          ------------   } 8 oz
                740,91,2 )
```

In dividing by 100, we merely cut off the two right hand figures. Therefore, the answer is

740 cwt. 91 lb. 8 oz. 8 dr., or
740 cwt. 3 qr. 16 lb. 8 oz. 8 dr.

Mental Exercises.

1. How many oz. are there in 16 dr.? how many in 80 dr.?
2. In 3 oz. 3 dr. how many drams are there?
3. In 2 ton 12 cwt. 3 qr. how many qr.?
4. At 9 cents a pound, what cost 3 cwt. 2 qr. 16 lb. short weight, of sugar?
5. In 25 lb. how many grains?

Exercises for the Slate.

1. Reduce 6 cwt. 1 qr. 18 lb. long weight, to drams.
Ans. 183308.

2. In 30 ton 18 cwt. 2 qr. 20 lb. 12 oz. 15 dr. short weight, how many drams? Ans. 15,838,927.

3. A man bought 15 loads of hay, each weighing 1 ton 270⅓ lb.; what was the weight of the whole?
Ans. 17 ton 55 lb.

4. In 25 packages, each containing 20336 oz., how many pounds? and what will the whole cost at 22½ cents per lb.? Ans. $285.97½.

MEASURE OF SPACE.

Angular Measure or Division of the Circle.

152. 1 Second is written 1sec., or 1″.

60 Seconds	make	1 Minute,	1min.	or	1′.
60 Minutes	"	1 Degree,	1deg.	or	1°.
90 Degrees	"	1 Right Angle,	1rt. ang.	or	90°.

The circumference of every circle is considered to be divided into 360 equal parts, each of which is often called a degree, as it subtends an *angle* of 1° at the centre of the circle.

MEASURE OF TIME.

Table of Time.

153. 1 Second is written thus: 1″.

60 Seconds	make	1 Minute,	1′.
60 Minutes	"	1 Hour,	1hr.
24 Hours	"	1 Day,	1day.
7 Days	"	1 Week,	1wk.
28 Days	"	1 Lunar Month.	
28, 29, 30 or 31 Days	"	1 Calendar Month.	
12 Calendar Months	"	1 Year.	
365 Days	"	1 Common Year.	
366 Days	"	1 Leap Year.	

The number of days in each month is easily remembered by means of the following lines:

Thirty days hath September,
April, June, and November;
February hath twenty-eight alone,
And all the rest have thirty-one;
But leap-year, coming once in four,
February then has one day more.

A *day*, or rather a *mean solar day*, which is divided into 24 equal portions, called *mean solar hours*, is the standard unit for the measurement of time, and is the mean or average time which elapses between two successive transits of the sun across the meridian of any place.

The time between the sun's leaving a certain point in the *Ecliptic*, and its return to that point, consists of 365.242218 mean *solar days*, or 365 days, 5 hours, 48 minutes, $47\frac{1}{2}$ seconds, very nearly, and is called a *solar year*. Therefore, the *civil* or *common year*, which contains 365 days, is about $\frac{1}{4}$th of a day less than the *solar* year; and this error would of course, in time, be very considerable, and cause great confusion.

Julius Cæsar, in order to correct this error, enacted that every 4th year should consist of 366 days. This was called *Leap* or *Bissextile year*. In that year February had 29 days, the extra day being called the *Intercalary* day.

But the Solar year contains 365.242218 days, and the Julian year contains 365.25, or $365\frac{1}{4}$ days.

Now, $365.25 - 365.242218 = .007782$. Therefore, in one year, taken according to the Julian calculation, the sun would have returned to the same place in the ecliptic .007782 of a day before the end of the Julian year.

Therefore, in 400 years the sun would have to come to the same place in the ecliptic $.007782 \times 400$, or 3.1128 days before the end of the Julian year; and, in 1257 years, would have come to the same place $.007782 \times 1257$, or 9.7819, or about 10 days before the end of the Julian year.

Accordingly, the vernal equinox which, in the year 325, at the Council of Nice, fell on the 21st March, in the year 1582 (that is, 1257 years later), happened on the 11th of March. Therefore, Pope Gregory caused 10 days to be omitted in that year, making the 15th of October immediately succeed the 4th, so that, in the next year, the vernal equinox again fell on the 21st of March; and, to prevent the recurrence of the error, ordered that, for the future, in every 400 years, 3 of the leap years should be omitted, viz.: those which complete a century, the numbers expressing which century, are *not divisible* by 4. Thus, 1600 and 2000 are leap years, because 16 and 20 are exactly divisible by 4; but 1700, 1800, and 1900 are not leap years, because 17, 18, and 19 are not exactly divisible by 4.

This Gregorian Style, which is called the *New Style*, was adopted in England on the 2d of September, 1752, when the error amounted to 11 days.

The Julian calculation is called the *Old Style*. Thus, Old Christmas takes place 12 days after New Christmas.

In Russia, they still calculate according to the *Old Style;* but in the other countries of Europe, the New Style is used. Sir Harris Nicolas, in his Chronology, gives the dates at which the New Style was adopted in different countries.

Of course it was almost immediately adopted by most of the Roman Catholic Courts of Europe.

Mental Exercises.

1. How many seconds are there in 3 min. 48 sec.?
2. If a person works $\frac{3}{8}$ths of a day, at $12\frac{1}{2}$ cents an hour, how much should he receive?
3. In 37 days there are how many weeks?
4. If the sun travels round the earth in 24 hours, through how many degrees will he pass in 1 hour? in 4 minutes?
5. The longitude of Boston is 71° West, and that of Chicago $87\frac{1}{2}$ West; what is the difference of time? When it is 6 o'clock in the latter place, what time is it at the former?

Exercises for the Slate.

1. One year being equivalent to $365\frac{1}{4}$ days, find how many seconds there are in 27 years 245 days.
   Ans. 873223200.
2. From 9 o'clock, P. M., Aug. 5, 1852, to 6 o'clock, A. M., March 3, 1853, how many hours are there? and how many seconds? Ans. 5025 hr.; 18090000 sec.
3. How many days are there from the 12th of Aug. till the 24th of next April, in a common year? Ans. 255.
4. If the 8th of August be on Monday, on what day of the week will the 1st of November be?
   Ans. Tuesday.

5. If a leap year commence on Wednesday, what day of September will be the first Monday of that month? Ans. The 7th.

A Catalogue of Useful Things.

A Standard Gallon contains 10lb. avoirdupois, distilled water.		A Firkin of Butter,	56 lb.
A Barrel of Beer,	36 gal.	A Decker of Gloves,	1 doz. pr.
A Quintal of Fish,	112 lb.	A Bushel of Oats,	34 lb.
Do. in U. States,	100 lb.	A Gallon of Flour,	7 lb.
		A Barrel of Flour,	196 lb.

There are several measures mentioned in commerce, as Tierce, Hogshead, Puncheon, Pipe, Butt, and Tun; but these may be considered rather as the names of the casks in which commodities are imported, than as expressing any definite number of gallons. It is the practice to gauge all such vessels, and to charge them according to their actual contents.

Commercial Numbers.

12 Articles	make 1 Dozen.		4 Quarters	make	1 Hundred.
13 "	" 1 Long Dozen.		24 Sheets Paper	"	1 Quire.
12 Dozen	" 1 Gross.		20 Quires	"	1 Ream.
20 Articles	" 1 Score.		2 Reams	"	1 Bundle.
5 Score	" 1 Common Hundred.		10 Reams	"	1 Bale.
6 Score	" 1 Gt. Hundred.		5 Doz. Skins of Parchment	"	1 Roll.
80 Deals	" 1 Quarter.				

90 Words in Chancery, 8 Words in Exchequer, or 72 in Common Law, } 1 Folio.

Sizes of Books.

The terms *Folio*, *Quarto*, *Octavo*, &c., applied to books, denote the number of leaves into which a sheet of paper is folded.

A sheet folded into 2 leaves forms a Folio.
A sheet " " 4 leaves " a Quarto, or 4 to.
A sheet " " 8 leaves " a Octavo, or 8 vo.
A sheet " " 12 leaves " a Duodecimo, or 12mo.
A sheet " " 18 leaves " an 18mo.
A sheet " " 36 leaves " a 36mo.

Sizes of Drawing-Paper.

Wove Antique,	4ft.	4in.	×	2ft.	7in.
Double Elephant,	3	4	×	2	2
Atlas,	2	9	×	2	2
Columbies,	2	10	×	1	11
Elephant,	2	4	×	1	11
Imperial,	2	5	×	1	9
Super-Royal,	2	3	×	1	7
Royal,	2	0	×	1	7
Medium,	1	10	×	1	6
Demy,	1	8	×	1	3

WEIGHTS AND MEASURES OF THE UNITED STATES CURRENCY.

154. *Federal Money* is the currency of the *United States.* The denominations are *Eagles*, *Dollars*, *Dimes*, *Cents*, and *Mills.*

10 Mills (*m.*)	make	1 Cent,	marked	ct.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	doll. or $.
10 Dollars	"	1 Eagle,	"	E.

The National Coins of the United States are of three kinds, viz.: Gold, Silver, and Copper or Bronze.

The gold coins are the *eagle*, the *double-eagle*, *half-eagle*, *quarter-eagle*, and *gold dollar.*

The eagle contains 258 grains of *standard* gold; the half-eagle and quarter-eagle like proportions.

The silver coins are the *dollar*, *half-dollar*, *quarter-dollar*, the *dime*, *half-dime*, and *three-cent-piece.*

The dollar contains 412½ grains of *standard* silver; the others, like proportion.

The copper and bronze coins are the *cent* and *half-cent.*

The present *standard* for both *gold* and *silver coin* is 900 parts of pure metal and 100 parts of alloy. The alloy of gold coin is composed of silver and copper, the silver not to exceed the copper in weight.

The alloy of silver coin is pure copper.

Measure of Length, etc.

The standard measure of Length, Surface, and Solidity are the same as those used in the British dominions.

Liquid Measure.

The standard *Unit of Liquid Measure*, adopted by the *United States*, is the *Wine Gallon*, of 231 cubic inches (old wine gallon of England), which is equal to 58372.175 grains of distilled water, at the maximum density, weighed in air at 30 inches barometer, or 8.338 lbs. avoirdupois, very nearly.

Dry Measure.

The *Standard Unit of Dry Measure*, adopted by the *United States*, is the *Winchester Bushel*, which is equal to 77.627413 pounds, avoirdupois, of distilled water, at the maximum density, and contains 2150.4 cubic inches, or 7.75557 imperial gallons. 1 bushel, imperial, is equal to 1.032 bushel, U. S., nearly.

Note.—The standard bushel of the State of New York is the *imperial bushel* of 80 pounds.

Measures of Weight.

The standard measures of all weights in the United States are the same as those of England.

FRENCH MONEY, WEIGHTS, AND MEASURES.

155. The new System of Money, Weights, and Measures of France, was formed according to the *decimal* notation.

French Money.

The *Franc* is the *unit* money of the new system of French currency. It is a silver coin, consisting of $\frac{9}{10}$ pure silver, and $\frac{1}{10}$ alloy.

10 Centimes	make	1 Decime.
10 Decimes	"	1 Franc.
20 Francs	"	1 Louis.

A Franc passes current in Nova Scotia for $0.16½.

French Lineal Measure.

The *Standard unit* of French *lineal measure* is the *Metre*. Its length, according to the mean of several comparisons, is equal to 39.3809171 imperial inches.

10 Metres	make	1 Decametre,	= 32.817431	feet.
10 Decametres	"	1 Hectometre,	= 328.17431	"
10 Hectometres	"	1 Kilometre,	= 3281.7431	"
10 Kilometres	"	1 Myriametre,	= 32817.431	"

The *standard* by which the new *French measures* of length is determined, is the *quadrant* of a meridian of the earth, or the *terrestrial arc* from the equator to the pole, in the meridian of Paris. The *ten-millionth* part of this is called a *metre*, which is equal to 39.381 imperial inches, nearly.

The *metre* is subdivided into 10 decimetres; the *decimetre* into 10 centimetres; the *centimetre* into 10 *millimetres*.

French Square Measure.

The *unit* of *French Superficial Measure* is the *Are*, whose sides are each a decametre in length. Consequently, it contains 100 square metres, or 119.6648496 imperial square yards.

10 Ares	make	1 Decare,	= 1196.648496	sq. yds.
10 Decares	"	1 Hectare,	= 11966.48496	"
10 Hectares	"	1 Kilare,	= 119664.8496	"
10 Kilares	"	1 Myriare,	= 1196648.496	"

The *are* is subdivided in the same manner as the *metre*.

French Cubic Measure.

The *unit of French Cubic Measure* is the *Stere*, which is a cubic *metre*, and is equal to 61074.1564445 imperial cubic inches.

10 Decisteres	make	1 Stere,	= 35.24384	cubic feet.
10 Steres	"	1 Decastere	= 353.4384	"

French Liquid and Dry Measure.

The *unit of French Liquid* and *Dry Measure* is the *Litre*, which is a *cubic decimetre*, and is equal to 61.074154445 imperial cubic inches, or .88106 imperial quarts.

10 Litres	make	1 Decalitre,	= 2.20266	gall.
10 Decalitres	"	1 Hectolitre,	= 22.0266	"
10 Hectolitres	"	1 Kilolitre,	= 220.266	"

The *litre* is subdivided in the same manner as the *stere*.

French Weight.

The *unit* of *French Weights* is the weight of a cubic *centimetre* of distilled water, at the maximum density, and is called a *Gramme*. It is equal to 15.433159 Troy grains.

10 Grammes	make	1 Decagramme,	= 154.33159	grs.
10 Decagrammes	"	1 Hectogramme,	= 1543,3159	"
10 Hectogrammes	"	1 Kilogramme,	= 15433.159	"
10 Kilogrammes	"	1 Myriagramme,	= 154331.59	"

The *gramme* is divided into 10 decigrammes; the *decigramme* into 10 centigrammes; the *centigramme* into 10 milligrammes.

Note.—The denomination chiefly used in the making out invoices of goods sold by weight, and in business transactions, is the *Kilogramme*, which is equal to 2.21 lbs. avoirdupois, very nearly.

French Circular Measure.

The *circle* is divided into 400 parts, called *grades*, and the *quadrant* into 100 *grades*.

The grade is divided into 100 equal parts, and each of these parts is subdivided into 100 other equal parts, according to the *centesimal* scale. Hence,

The Seconde	= .00009	English	Degree.
The Minute	= .009	"	"
The Grade	= .9	"	"

NOTE.—The names of the denominations *larger* than the *unit* in the French concrete numbers, are formed by prefixing to the name of the unit the Greek words *deca*, *hecto*, *kilo*, and *myria*; those *less* than the *unit* are formed by prefixing to the name of the unit the Latin words *deci*, *centi*, and *milli*.

MISCELLANEOUS QUESTIONS AND EXAMPLES ON ARTS. 125–157.

1. Explain the meaning of the term 'Reduction.'
Reduce 537983 half-guineas to seven-shilling pieces.
Ans. 806974½.

2. Can concrete numbers, of the same or different kinds, be multiplied together? Give the reason.
What is the cost of school accommodation for 13750 children, @ £1, 18. 6½. each? Ans. £26497, 7. 11.

3. A person bought 1763 yds. of cloth, @ $1.10 per yard, and sold at 6s. 11d. per yd. What was his profit?
Ans. £124, 17. 7.

4. Divide £3, 13. 9. between two persons, so that one shall receive half as much as the other. Reduce the result to dollars and cents. Ans. $9.83⅓, and $4.91⅔.

5. A servant's wages are £10, 8. a year. How much ought he to receive in 7 weeks (supposing the year 52 weeks)? Ans. $5.60.

6. A factor bought 56 pieces of stuff for £1569, 17. 4. at 4s. 10d. a yard; how many yards were there in each piece? Ans. 116 yds.

7. A merchant bought 450 yards of cassimere at 8s. 6d. per yd.; while lying in his wareroom 14 yds. were rendered useless; he sold the remainder at $2.06⅔ per yard. What did he gain? Ans. $136.06⅔.

8. Two boys run a race of 1 mile; one of them gains 5 feet in every 110 yds. How far will the other be left behind at the end of the race? Ans. $26\frac{2}{3}$ yds.

9. A merchant buys 10 gallons of spirit, at 12s. a gallon; 15 gallons, at $ 2.90 a gallon; and 18 gallons, at 15s. 9d. a gallon. What will be the price of a gallon of the mixture, so that he may gain $ 9.10 on his outlay? Ans. $ 3.10.

10. An apothecary bought 100 lb. opium (Troy wt.), at 16s. 3d. a pound, and sold it at 19s. 0d. per pound, avoirdupois; did he gain or lose? Ans. Lost £ 3, 1. $6\frac{6}{7}$.

11. A gentleman sent a tankard to his silversmith, which weighed 100 oz. 16 dwt., and ordered him to make it into spoons, each weighing 2 oz. 16 dwt.; how many spoons did he receive? Ans. 36 spoons.

12. Divide £ 17, 3. 5. by £ 14, 3. 6. to 4 places of decimals. Can these sums be multiplied together? Ans. 1.2113.

13. A certain number of men, twice as many women, and three times as many boys, earned in 5 days £ 7, 15.; each man earned 30 cents, each woman 10 pence, and each boy $13\frac{1}{2}$ cents a day. How many were there of each? Ans. 6 men, 12 wo. 18 boys.

14. What is the expense of plastering the walls of a room, the perimeter of which is 58 ft. 4 in., and the walls 11 ft. high, deducting three doors, each 7 ft. 10 in. by 3 ft. 3 in., two window-openings each 10 ft. 6 in. by 6 ft., and a fire-place 6 ft. by 5 ft. 1 in., @ 15 cents per yard; likewise a cornice, at 5 cents per foot, allowing a foot in length at each corner? Ans. $ $9.92\frac{1}{2}$.

15. An apothecary bought 126 gallons of essence of lemon, @ $ $3.07\frac{1}{2}$ per quart, and sold it [illegible] ounce, fluid measure; how much did he gain? Ans. $ 466.20.

16. If 5000 people took in hand to count a trillion of dollars, and, beginning their work at the commencement of the year 1852, could each count on the average 100 dollars a minute (without intermission), when would they finish their task? Ans. 20th Oct., 1855.

17. I hire a house at £ 90 a year, which is assessed in the rate-book at $\frac{4}{5}$ths of its rent; I agree to pay the rates upon it, viz.: 3 poor's-rates, @ 9d., 10d., and 1s. 3d., respectively in the £; a church-rate of 8d. in the £; and a paving-rate of 1s. 7d. in the £. What is the whole annual cost of the house? Ans. £ 108, 6, 0.

RULES FOR MENTAL ARITHMETIC.

156. Mental calculation sharpens the intellect, quickens the parts, enlarges the views, strengthens the mental powers, stimulates emulation, makes dry calculations seem pleasing and inviting, and is of very great importance in domestic affairs, and in conducting the business of every household.

Our limits will not allow of giving rules for all the transactions which may occur in business, but the following, if taken with those already given under Multiplication, will be found to combine nearly all that is necessary.

(1.) *To find the amount of any number of yards, pounds, gallons, &c., at a given price per yard, &c.*

Rule. Find the amount at one penny, and multiply by the price in pence. Should a fraction occur in the price, add the same part of what it amounts to @ a penny, as the fraction is of a penny, namely: if $\frac{1}{4}$d. add one fourth, if $\frac{1}{2}$d. add one half, &c.

Ex. What will 163$\frac{1}{4}$ yards cost, at 6$\frac{1}{2}$d. per yard?

163$\frac{1}{4}$ pence =		13.	7$\frac{1}{4}$.
			6$\frac{1}{2}$.
	4.	1.	7$\frac{1}{2}$.
$\frac{1}{2}$ =		6.	9$\frac{5}{8}$.
	£4.	8.	5$\frac{1}{8}$.

Note.—If there are shillings in the price, find the amount for the shillings first, and that for the pence afterward, and add these two amounts for the whole.

(2.) *To find the price of any number of dozens.*

Rule. Regard the pence in the price as shillings, and multiply by the number of dozens.

Ex. Find the price of 72 (6 doz.). at $8\frac{3}{4}$d.

		s.	d.
$8\frac{3}{4}$d. regarded as shillings, =		8	9
			6
	£2	12	6

(3.) *When the number of articles does not consist of exact dozens.*

Rule. Find the price of the nearest number of dozens, by the last Rule, and increase or diminish it, as the case may require, by the price of the other articles.

Ex. Find the price of 69 articles, @ $8\frac{3}{4}$d.

		s.	d.
$8\frac{3}{4}$d. regarded as shillings, =		8	9
69 = 6 dozen, wanting 3. Multiplying by			6
	£2	12	6
Deducting the price of 3,		2	$2\frac{1}{4}$
	£2	10	$3\frac{3}{4}$

(4.) *To find the price of any number composed of sixteens.*

Rule. Reduce the price to farthings, multiply by the number of sixteens, and divide the product by 3, for shillings. If there is a remainder, after dividing by 3, reckon each one over as 4 pence.

Ex. Find the price of 16 articles, at $4\frac{3}{4}$d. each. Also, 64 articles, at $6\frac{3}{4}$d.

$4\frac{3}{4}$d. =	19	$6\frac{3}{4}$d. =	27
	1	64 = 4 × 16	4
3)	19	3)	108
	6s. 4d.		36s. or £1. 16. 0.

Reason for the preceding process.

16 articles, at ¼d., = 4d. Therefore, multiplying the farthings in the price by the number of sixteens in the whole number, the product will be so many four-penny-pieces; hence, dividing by 3, will give shillings.

(5.) *To find the price of* 48.

RULE. Reduce the price to farthings, and reckon the product as shillings.

Ex. Find the price of 48 articles, at 5¾d.

5¾d = 23 farthings;
Reckoned 23 shillings, = £1, 3. 0.

Reason for the above Rule.

48 farthings being equivalent to 1 shilling, therefore each farthing in the price will be equal to 1 shilling.

(6.) *To find the price of* 96.

RULE. Reduce the price to farthings, double the unit figure for shillings, and consider the other as pounds.

Ex. Find the price of 96 articles, at 5¾d.

5¾d. reduced to farthings, = 23
2
£2, 6. 0.

The reason for the above may be easily deduced from the preceding Rule.

(7.) *To find the price of* 112 *articles.*

RULE. Find the price of 16 by the former rule, and multiply by 7.

Ex. What cost 112 lb. (7 × 16) of butter, at 9¾d.

16 lb. @ 9¾d. = 13 0
7
£4, 11. 0.

(8.) *To find the price of* 100 *articles.*

RULE. Reduce the price to farthings, and take those farthings as pence, and their double as shillings.

Ex. Find the price of 100 lb. of nails, at 3$\frac{1}{4}$d. per lb.

3$\frac{1}{4}$d. = 13 farthings,	× 2 = 26;	as shillings,	=	£1, 6. 0.
and 13 "	× 1 = 13;	as pence,	=	1. 1.
				£1, 7. 1.

Reason for the above process.

100 articles, at $\frac{1}{4}$d. each, will amount to 2 shillings and 1 penny. Therefore, twice as many shillings, and once as many pence, as there are farthings in the price, will give the true result.

If there are shillings in the price, reckon each as £5.

(9.) *To find the price of* 120 *articles, at any given number of pence.* (2.) *At any given number of pence and farthings.*

RULE. (1.) Regard the pence in the given price as pounds, and divide by 2. (2.) Regard the farthings in the price as pounds, and divide by 8.

Ex. 1. What will 120 lb. tea cost, at 1s. 6d. per lb.

1s. 6d. = 18, which, divided by 2, = £9, 0. 0.

Ex. 2. Find the price of 120 lb. soda, at 4$\frac{1}{2}$d.

4$\frac{1}{2}$d. = 18 farthings, divided by 8, = £2, 5. 0.

(10.) *Find the price of any number of articles, at any given price, in cents or dollars and cents.*

RULE. Multiply the number of articles by the cents or dollars and cents in the price, and point off as in decimals.

NOTE.—As dollars and cents are in decimal notation, several contractions can be made in the operation, to which the attention of the learner should be directed by the teacher.

To Calculate Interest Mentally.

157. When money or property is hired or let out, a certain stipulated sum, or an equivalent, has to be paid by the borrower, for the loan of that property; this, in the case of money, is called Interest.

Interest, therefore, is the sum of money paid for the loan or use of some other sum of money, lent for a certain time, at a fixed rate; generally, at so much for each £100 or $100, for one year.

The money lent is called *The Principal.*

The interest of £100 or dollars, for a year, is called the *Rate per Cent.*

The principal, + the interest, is called the *Amount.*

Computations in Interest and Discount may sometimes be performed mentally with considerable facility, especially by means of contrivances which have been fallen upon to shorten the work in particular cases. The following are some of the more useful of these.

(1.) *To find the interest of any number of pounds, for a given number of months, @ 5 per cent.*

Rule. Take the pounds as pence, and multiply by the number of months.

Ex. Find the interest of £186, 10. for 5 months, at 5 per cent.

Proceeding by the above Rule,

£186, 10., taken as pence, = 186½ pence:
= £0, 15. 6½. Interest for 1 month.
5 months.

£3, 17. 8½.

Reason for the preceding process.

The interest of £100, at 5 per cent., for one year, is £5, and the interest of £1 would be $\frac{1}{100}$th of this, or 1 shilling, for one year, and therefore 1 penny for 1 month. Hence, £186, 10., for one month, would be 186½ pence, or 15s. 6½d., and for 5 months, 5 times as much, or £3, 17. 8½.

(2.) *To find the interest of any number of pounds, &c., for any given number of months, at 6 per cent. per annum.*

RULE. Multiply the principal by the months; increase the unit figure by a fifth of itself, to find the pence of the answer, and take the others as expressing shillings.

For 5s. add ¼d.	For 15s. add ¾d.
For 10s. add ½d.	To 16s. 8d. and above, add 1d.

Ex. Required, the interest of £16, 3. 4. for 7 months, at 6 per cent.

Proceeding by the above Rule,

£	s.	d.
16	3	4
		7
11,3	3	4

11,3 3 4 = £0, 11. $3\frac{3}{5}$.

So 3s. 4d. = $\frac{1}{6}$ of £1. Therefore, $\frac{1}{6}$ of $\frac{6}{5}$ or $\frac{6}{30}$ ... $\frac{1}{5}$.

£0, 11. $3\frac{4}{5}$.

Reason for the above process.

It is evident from the preceding rule, that the interest of £1, for 1 month, at 6 per cent., will be $1\frac{1}{5}$d., and as 200 times $1\frac{1}{5}$d. = £1, the product of the principal and months, divided by 200, will give pounds. We therefore cut off the unit figure, which is the same as dividing by 10, and taking the rest as shillings instead of pounds, divides by 20. (20 × 10 = 200).

The figure cut off is plainly tenths of shillings, and increasing of it by a fifth of itself, gives twelfths of a shilling or pence.

(3.) *To find the interest of any number of dollars and cents, for a given number of mouths, at 6 per cent. per annum.*

Rule. Multiply the principal by half the number of months, and consider the product as cents.

Ex. Required, the interest of \$126.15, for 8 months, at 6 per cent.

Proceeding by the Rule given above,

$\$126.15 \times 4 = 504.60$ cents, or $\$5.04\frac{6}{10}$.

(4.) *To find the interest of any number of pounds, &c., for a given number of days, at 5 per cent. per annum.*

Rule. Multiply the principal by one third the number of days, or multiply a third of one of them by the other; a tenth of the result is the answer in pence, nearly. To correct it, reject a penny for every 6 shillings, or $3\frac{1}{4}$d. for every pound contained in it. Should this correction be considerable, add a penny for every 6 shillings in it to the answer.

Ex. Required, the interest of £141, 10. for 27 days, at 5 per cent.

Proceeding by the Rule given above,

	£141,	10.	0.
			9
	1273,	10.	0.

Dividing by 10 $= 127\frac{3}{10}$, or $127\frac{1}{4}$d. $=$ 10s. $7\frac{1}{4}$.
Rejecting a penny for every 6 shillings, $1\frac{1}{2}$.

£0, 10. $5\frac{3}{4}$.

(5.) *To find the interest of any number of pounds. &c., for a given number of days, at 6 per cent. per annum.*

Rule. Divide the product of the principal and days by 100; take one third of the quotient for shillings, and one sixth of the remainder for farthings, and from the sum thus obtained, reject a penny for every six shillings contained in it. The remainder will be the interest required, nearly. If the interest is large, correct it as in 5 per cent.

Ex. Find the interest of £163, 5. 3. for 25 days, at 6 per cent.

Proceeding by the Rule given above,

	£163,	5.	3.	
			5	
	816,	6.	3.	
			5	
Dividing by 100,	40.81,	11.	3.	
$\frac{40}{3}$ =	£0,	13.	4.	
$\frac{81}{6}$ for farthings,		0.	$3\frac{1}{4}$.	
	£0,	13.	$7\frac{1}{4}$.	
Rejecting 1d. for 6s.,			$2\frac{1}{4}$.	
	£0.	13.	5.	Interest.

(6.) *To find the interest of any number of dollars and cents, for any given number of days, at* 6 *per cent. per annum.*

Rule. Multiply the principal by one sixth of the number of days, and consider the product as mills.

Ex. Required, the interest of $146.16, for 18 days, at 6 per cent.

Proceeding by the above Rule,

$146.16 × 3, = 438.46 mills, or $0.44 cents, nearly.

Note 1.—When the interest is large, or even amounts to dollars, deduct $1\frac{1}{2}$ cents from it, for the true interest.

Note 2.—The reason for the deduction in the preceding rules, is in consequence of reckoning the commercial year as 12 months, of 30 days each, or 360 instead of 365 days, which gives the interest one seventy-second part of itself more than it should be. This will be better understood when the learner has worked the Rule of Proportion. (See Note, Rule 3, Art. 221).

FRACTIONS.

VULGAR FRACTIONS.

158. If a quantity, considered as a whole, be divided into any number of equal parts, a FRACTION, in reference to that quantity, signifies one or more such parts; and the quantity divided is called, in respect to the fraction, the INTEGER or UNIT.

A fraction is expressed by two numbers, or TERMS, called the *numerator* and the *denominator*.

The DENOMINATOR is written below the numerator, and expresses the number of parts into which the integer is divided; and the NUMERATOR expresses the number of such parts denoted by the fraction.

Thus, $\frac{4}{5}$, which is read *four-fifths*, is a fraction, and signifies that a unit, a day, for instance, is divided into 5 equal parts, and that four of these parts are taken.

If the numerator of a fraction be taken as an integer, and be divided into as many equal parts as there are units in the denominator, the fraction may also be regarded as expressing *one* of these parts.

Thus, for instance, if 4 days be divided into 5 equal parts, the fraction $\frac{4}{5}$ expresses one of these parts, and is therefore the same as *one-fifth* of 4 days. Hence, $\frac{3}{4}$ of a pound means either three times one-fourth of a pound, or one-fourth of three pounds.

A PROPER FRACTION is that whose numerator is less than its denominator.

An IMPROPER FRACTION is that whose numerator is *not* less than its denominator.

Thus, $\frac{3}{8}$ and $\frac{1}{7}$ are proper fractions; and $\frac{8}{8}$, $\frac{5}{3}$ and $\frac{11}{6}$ are improper fractions.

A number consisting of a whole number, with a fraction annexed, as $6\frac{2}{3}$, is termed a MIXED NUMBER.

A Simple Fraction expresses one or more of the equal parts into which a unit is divided, without reference to any other fraction.

A Compound Fraction expresses one or more of the equal parts into which another fraction, or a mixed number, is divided.

Thus, $\frac{5}{6}$ is a simple fraction; but $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{12}$, and $\frac{1}{3}$ of $\frac{1}{8}$ of $1\frac{6}{7}$, are compound fractions. Compound fractions have the word *of* interposed between the simple fractions of which they are composed.

A Complex Fraction is that which has a fraction either in its numerator or denominator, or in each of them.

Thus, $\frac{5\frac{1}{4}}{7}$, $\frac{7}{3\frac{1}{8}}$ and $\frac{5\frac{1}{4}}{6\frac{2}{3}}$, are complex fractions.

When fractions are considered in the abstract, without reference to any particular integer, we say briefly four-sevenths, instead of four-sevenths of one, the former expression being *understood* to be equivalent to the latter.

We thus see that the two definitions of a fraction, given in the text, though they suggest to the mind ideas which are apparently different, are in effect the same. Agreeable to either, the lower number is very properly called the *denominator*, as it gives *name* (such as *fifths*, *sixths*, &c.,) to the parts into which the integer is divided; and the upper number is called the *numerator*, as it *numbers* the parts expressed by the fraction, or is the number of the integers, which are divided by the denominator.

According to these views, every fraction would have its numerator less than its denominator, and would consequently be less than its unit, as its name imports.

Such a view, however, is too limited to answer the general purposes of calculation; and it is often necessary to consider, as fractions, quantities which equal or exceed the integer, or which *indicate* the quotients of numbers divided not only by the greater, but also by equal or by smaller numbers, without the actual performance of the division. Quantities of the latter kind are therefore called *improper fractions;* and, for the sake of distinction, others, in reference to them, are called *proper fractions.* It is evident, from either view of the nature of a fraction, that it is less or greater than a unit, accordingly as its numerator is less or greater than its denominator, and that it is equal to a unit, if its numerator and denominator be equal.

In addition to the preceding, it may be proper to state, that fractions are of the same nature as the subordinate quotients in compound division.

Thus, if 27 cwt. be divided by 20, the quotient is 1 ton 7 cwt., or $1\frac{7}{20}$ ton, each of these expressing precisely the same quantity, since 1 cwt. is one twentieth, and, consequently, seven cwt. seven twentieths of a ton.

From these principles, and those laid down in Art. 75, we arrive at the following important conclusions:

159. *If the terms of any fraction be both multiplied, or both divided, by the same number, the resulting fraction is equivalent to the original one.*

Thus, if the numerator and denominator of the fraction $\frac{2}{7}$ be multiplied by 3, the fraction resulting will be $\frac{6}{21}$, which is of the same value as $\frac{2}{7}$.

Reason of the process.

In the fraction $\frac{2}{7}$, the unit is divided into 7 equal parts, and 2 of these parts are taken; in the fraction $\frac{6}{21}$, the unit is divided into 21 equal parts, and 6 of these parts are taken.

Now, there are 3 times as many parts taken in the second fraction as there are in the first; but 3 parts of the second fraction are only equal to 1 part in the first fraction. Therefore, the 6 parts taken in the second fraction equals the 2 parts taken in the first fraction; therefore, $\frac{2}{7} = \frac{6}{21}$.

160. Hence, it follows that a whole number may be converted into a fraction with any denominator, by multiplying the number by the required denominator for the numerator of the fraction, and placing the required de-

nominator underneath; for $6 = \frac{6}{1}$, and to convert it into a fraction, with the denominator 5, or any other number, we have

$$6 = \frac{6}{1} = \frac{6 \times 5}{1 \times 5} = \frac{30}{5}.$$

161. *Again, to multiply the numerator of a fraction by any numbers is the same in effect as dividing the denominator by it, and conversely.*

For, if the numerator of the fraction $\frac{6}{8}$ be multiplied by 4, the resulting fraction is $\frac{24}{8}$; and if the denominator be divided by 4, the resulting fraction is $\frac{6}{2}$.

Now, the fraction $\frac{24}{8}$ signifies that unity is divided into 8 equal parts, and that 24 such parts are taken; these are equivalent to 3 units. Also, $\frac{6}{2}$ signifies that unity is divided into 2 equal parts, and that 6 such parts are taken; these are equivalent to 3 units. Hence, $\frac{24}{8}$ and $\frac{6}{2}$ are equal.

Again, if we divide the numerator of the fraction $\frac{6}{8}$ by 2, the resulting fraction is $\frac{3}{8}$; and if we multiply the denominator by 2, the resulting fraction is $\frac{6}{16}$.

Now, $\frac{3}{8}$ signifies that unity is divided into 8 equal parts, and that 3 of such parts are taken; and $\frac{6}{16}$ signifies that the unit is divided into 16 equal parts, and that 6 of such parts are taken. But each part in $\frac{3}{8}$ is equal to 2 parts in $\frac{6}{16}$; therefore, $\frac{3}{8}$ is of the same value as $\frac{2 \times 3}{16}$ or $\frac{6}{16}$.

The same operations can be performed on fractional as on integral quantities.

Hence, the doctrine of Fractions comprises the *addition*, *subtraction*, *multiplication* and *division* of fractions.

Before entering on these operations, it is proper to show how such quantities may be modified, without changing their value, so as to fit them for the operations to be performed on them, and for the uses to which they may be applied.

This will consititute *Reduction of Fractions.*

Reduction of Vulgar Fractions.

162. When the numerator and denominator of a fraction are not prime to each other, they have a common factor greater than unity (Art. 92, def. 4). If we divide each of them by this, there results a fraction *equal* to the former, but of which the terms, that is, the numerator and denominator, are less, or *lower* than those of the original fraction; and it may be considered the same in lower *terms*.

When the numerator and denominator of a fraction are Prime to each other, that is, have no common factor greater than unity, it is clear that its terms cannot be made lower by division of this kind, and on this account the fraction is said to be in its Lowest Terms.

163. *To Reduce a Fraction to its lowest terms.*

Rule. Divide the numerator and denominator by their greatest common measure.

Ex. Reduce $\frac{6465}{7335}$ to its lowest terms.

The greatest common measure of 6465 and 7335 is 15 (Art. 97), and

$$15\left\{\begin{array}{r} 3\ )\ 6465 \\ \hline 5\ )\ 2155 \end{array}\right. \qquad\qquad 15\left\{\begin{array}{r} 3\ )\ 7335 \\ \hline 5\ )\ 2445 \end{array}\right.$$

$$\begin{array}{r} \hline 431 \end{array} \qquad\qquad\qquad\qquad \begin{array}{r} \hline 489 \end{array}$$

Therefore, the fraction, in its lowest terms, $= \frac{431}{489}$.

Reason for the above process.

If the numerator and denominator of a fraction be divided by the same number, the value of the fraction is not changed (Art. 159); and the greatest number which will divide the numerator and denominator is their greatest common measure.

Mental Exercises.

Reduce each of the following fractions to its lowest terms.

1.	$\frac{4}{8}$	5.	$\frac{21}{49}$	9.	$\frac{64}{96}$	13.	$\frac{24}{30}$
2.	$\frac{8}{10}$	6.	$\frac{11}{55}$	10.	$\frac{10}{15}$	14.	$\frac{48}{96}$
3.	$\frac{12}{18}$	7.	$\frac{26}{39}$	11.	$\frac{14}{21}$	15.	$\frac{31}{62}$
4.	$\frac{18}{36}$	8.	$\frac{56}{63}$	12.	$\frac{18}{27}$	16.	$\frac{24}{48}$

Exercises for the Slate.

1.	$\frac{246}{342}$	Ans.	$\frac{41}{57}$	4.	$\frac{81}{4872}$	Ans.	$\frac{27}{1624}$
2.	$\frac{625}{9000}$	"	$\frac{5}{72}$	5.	$\frac{10812}{22800}$	"	$\frac{901}{1900}$
3.	$\frac{4301}{95897}$	"	$\frac{253}{5641}$	6.	$\frac{128352}{238368}$	"	$\frac{7}{13}$

163. *To Reduce Fractions to equivalent ones, with a common denominator.*

Rule. Find the least common multiple of the denominators; this will be the common denominator. Then divide the common multiple so found by the denominator of each fraction, and multiply each quotient so found into the numerator of the fraction which belongs to it for the new numerator of that fraction.

Ex. Reduce $\frac{5}{12}$, $\frac{9}{16}$, $\frac{11}{24}$, $\frac{17}{33}$ into equivalent fractions, with a common denominator.

Proceeding by the Rule given above,

We find the least common multiple to be 528 (Art. 105).
Therefore, the fractions become respectively,

$$\frac{5}{12} \times \frac{44}{44} = \frac{220}{528} \text{ (since } \frac{528}{12} = 44),$$
$$\frac{9}{16} \times \frac{33}{33} = \frac{297}{528} \text{ (since } \frac{528}{16} = 33),$$
$$\frac{11}{24} \times \frac{22}{22} = \frac{242}{528} \text{ (since } \frac{528}{24} = 22),$$
$$\frac{17}{33} \times \frac{16}{16} = \frac{272}{528} \text{ (since } \frac{528}{33} = 16),$$

Or, the fractions with a common denominator, are

$$\frac{220}{528}, \frac{297}{528}, \frac{242}{528}, \frac{272}{528}.$$

Reason for the above process.

The least common multiple of the denominators of the given fractions will evidently contain the denominator of any one of the fractions an exact number of times. If both the numerator and denominator of that fraction be multiplied by that number, the value of the fraction will not be altered (Art. 159) ; and the denominator will then be equal to the least common multiple of all the denominators. If this be done with all the fractions, they will evidently be, in like manner, reduced to others of the same value, and having the least common multiple of all the denominators for the denominator of each fraction.

NOTE.—If the denominators have no common measure, we must then multiply each numerator into all the denominators, except its own, for a new numerator for each fraction, and all the denominators together for the common denominator.

EXERCISES FOR THE SLATE.

Reduce the following sets of questions to others having common denominators.

	EXERCISES.	ANSWERS.
1.	$\frac{2}{5}$, $\frac{2}{3}$, $\frac{5}{9}$ and $\frac{7}{10}$,	$\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{63}{90}$.
2.	$\frac{2}{5}$, $\frac{35}{80}$ and $\frac{14}{200}$,	$\frac{240}{400}$, $\frac{175}{400}$, $\frac{28}{400}$.
3.	$\frac{7}{9}$, $\frac{5}{11}$, $\frac{13}{18}$, $\frac{3}{22}$ and $\frac{1}{36}$,	$\frac{308}{396}$, $\frac{180}{396}$, $\frac{286}{396}$, $\frac{54}{396}$, $\frac{11}{396}$.
4.	$\frac{31}{60}$, $\frac{17}{90}$, $\frac{13}{25}$, $\frac{1}{105}$ and $\frac{5}{9}$,	$\frac{3255}{6300}$, $\frac{1190}{6300}$, $\frac{3276}{6300}$, $\frac{60}{6300}$, $\frac{3500}{6300}$.
5.	$\frac{9}{10}$, $\frac{9}{100}$, $\frac{9}{1000}$ and $\frac{9}{10000}$,	$\frac{9000}{10000}$, $\frac{900}{10000}$, $\frac{90}{10000}$, $\frac{9}{10000}$.
6.	$\frac{31}{54}$, $\frac{11}{28}$, $\frac{53}{63}$ and $\frac{3}{12}$,	$\frac{434}{756}$, $\frac{297}{756}$, $\frac{636}{756}$, $\frac{189}{756}$.
7.	$\frac{1}{3}$, $\frac{2}{9}$, $\frac{4}{27}$, $\frac{8}{81}$, $\frac{16}{243}$, $\frac{32}{729}$,	$\frac{243}{729}$, $\frac{162}{729}$, $\frac{108}{729}$, $\frac{72}{729}$, $\frac{48}{729}$, $\frac{32}{729}$.

164. *To Reduce an Improper Fraction to a whole or mixed number.*

RULE. Divide the numerator by the denominator; the quotient will be the whole number required. And if there be any remainder, write it over the given denominator for the fractional part of the required result.

Ex. Reduce $\frac{120}{8}$ and $\frac{56}{6}$ to whole or mixed numbers.

Proceeding by the Rule given above,

$\frac{120}{8}$) 120 —— 15. Ans.

$\frac{56}{6}$) 56 —— $9\frac{2}{6}$, or $9\frac{1}{3}$. Ans.

The *reason for this Rule*, and that of the next, are evident from the nature of Fractions.

Mental Exercises.

Reduce the following fractions to whole or mixed numbers.

1.	$\frac{25}{5}$	5.	$\frac{64}{7}$	9.	$\frac{99}{12}$
2.	$\frac{6}{3}$	6.	$\frac{48}{5}$	10.	$\frac{37}{5}$
3.	$\frac{27}{6}$	7.	$\frac{76}{8}$	11.	$\frac{69}{7}$
4.	$\frac{5}{2}$	8.	$\frac{87}{11}$	12.	$\frac{101}{11}$

Exercises for the Slate.

1.	$\frac{751}{24}$	Ans.	$31\frac{7}{24}$	5.	$\frac{231750}{153}$	Ans.	$1514\frac{108}{153}$
2.	$\frac{5801}{63}$	"	$92\frac{5}{63}$	6.	$\frac{14264}{239}$	"	$59\frac{163}{239}$
3.	$\frac{648}{108}$	"	6	7.	$\frac{95950}{999}$	"	$96\frac{46}{999}$
4.	$\frac{10000}{111}$	"	$90\frac{10}{111}$				

165. *To Reduce a Mixed Number to an Improper Fraction.*

Rule. Multiply the whole number by the denominator of the fractional part, and to the product add the numerator; the sum will be the required numerator, below which write the given denominator.

Ex. Reduce $7\frac{4}{11}$ to an improper fraction.

Proceeding by the Rule given above,

$$\begin{array}{l} 7\frac{4}{11} \\ 11 \\ \hline \frac{77+4}{11} = \frac{81}{11}.\ \text{Ans.} \end{array}$$

Mental Exercises.

1. In $16\frac{1}{2}$ how many halves?

2. A lady purchased $\frac{18}{8}$ of a yard of silk for a dress, what number of yards did she buy?

3. Reduce 6, 7, 8, 9, 12 and 14 to improper fractions, the teacher naming the denominator of the fraction to which he wishes the number reduced.

4. If an acre of land produces $27\frac{2}{3}$ bus. of corn, how many thirds of a bushel does it produce? What will it come to at 14 cents each third?

EXERCISES FOR THE SLATE.

1.	$3\frac{95}{224}$	Ans.	$\frac{767}{224}$	5.	$157\frac{122}{137}$	Ans.	$\frac{21631}{137}$
2.	$26\frac{201}{202}$	"	$\frac{5453}{202}$	6.	$17\frac{2605}{2859}$	"	$\frac{51208}{2859}$
3.	$164\frac{118}{441}$	"	$\frac{72442}{441}$	7.	$427\frac{5}{1107}$	"	$\frac{472694}{1107}$
4.	$106\frac{119}{851}$	"	$\frac{90325}{851}$	8.	$100\frac{413}{720}$	"	$\frac{72413}{720}$

166. *To Reduce a number, or a fraction, of any denomination, to a fraction of another denomination.*

RULE. "Reduce the given number, or fraction, and also the number or fraction to the fraction of which it is to be reduced, to their respective equivalent values in terms of some one and the same denomination; then the fraction of which the former is made the numerator, and the latter the denominator, will be the fraction required."

Ex. 1. Reduce 4s. 6d. to the fraction of £1.

Proceeding by the above Rule,

4s. 6d. = 54 pence
£1 = 240 pence.

Therefore, fraction required $= \frac{54}{240}$, or $\frac{9}{40}$.

Reason for the above process.

£1, or unity, is here divided into 240 equal parts; and 54 of such parts being taken, the part of unity, or £1, which they make up, is represented by $\frac{54}{240}$, or $\frac{9}{40}$.

Ex. 2. Reduce $\frac{5}{8}$ of 1 cwt. to the fraction of 27 lbs.

$\frac{5}{8}$ of 1 cwt. $=$ 112 times $\frac{5}{8}$ of 1 lb.

$$= \frac{5 \times 112}{8}$$

$$= \frac{5 \times 14}{1} \text{ (Art. 87.)}$$

27 lbs. $=$ 27 lbs.

Therefore, $\dfrac{\frac{5 \times 14}{1}}{27} = \frac{5 \times 14}{1} \times \frac{1}{27} = \frac{70}{27}$

Ex. 3. Reduce £$\frac{1}{35}$ to the fraction of a farthing.

$\frac{1}{35}$ of £1 $= (\frac{1}{35} \times 20 \times 12 \times 4)$ farthings, $= \frac{960}{35}$, $= \frac{192}{7}$ farthings, and 1 farthing $=$ 1 farthing, therefore the fraction required.

Exercises for the Slate.

1. Reduce 12s. 6d. to the fraction of 1 £. Ans. $\frac{5}{8}$.
2. Reduce £18, 7. 6. to the fraction of 2 £ " $\frac{147}{16}$.
3. Reduce 6s. 7$\frac{6}{7}$d. to the fraction of 7s. 9d. " $\frac{559}{651}$.
4. Reduce 1s. 2d. to the fraction of 27s. " $\frac{7}{162}$.
5. Reduce 2 ac. 1 ro. to the fraction of 9 ac. 2 ro. Ans. $\frac{9}{38}$.
6. Reduce 6 ft. 3$\frac{5}{8}$ in. to the fraction of 13 ft. 8$\frac{9}{10}$ in. Ans. $\frac{3025}{6596}$.

167. *To find the value of a Fraction in the denominations contained in the integer.*

Rule. Divide the numerator, considered as so many times the integer, by the denominator.

Ex. 1. Required, the value of £$\frac{5}{6}$.

Proceeding by the Rule given above,

The numerator considered as so many times the unit, $=$ £5, which, divided by 6, $=$

$$\begin{array}{r} 6)\,£5,\ 0.\ 0. \\ \hline £0,\ 16.\ 8. \end{array}$$

Ex. 2. Required, the value of $\frac{3}{8}$ of £5, 13. 9.

Here, the integer is £5, 13. 9.; then the numerator expressing three times this quantity, we multiply by 3, and divide by the denominator 8; the quotient, £2, 2. $7\frac{1}{8}$, is the required value.

$$\begin{array}{r} £5,\ 13.\ \ 9. \\ 3 \\ \hline 8\,)\ 17,\ \ 1.\ \ 3. \\ \hline £2,\ \ 2.\ \ 7\frac{1}{8}. \end{array}$$

Exercises.

1. Required, the value of £$\frac{3}{16}$. Ans. £0, 3. 9.

2. Required, the value of $\frac{6}{11}$ of £148, 5.
Ans. £80, 17. $3\frac{3}{11}$.

3. Required, the value of $\frac{1}{11}$ of 48 ac. 1 ro. 13 po.
Ans. 4 ac. 1 ro. 23 po.

4. Required, the value of $6\frac{26}{29}$ bus.
Ans. 6 bus. 3 pk. $1\frac{5}{29}$ gal.

Note.—The method of reducing Compound to Simple Fractions will be given in Multiplication of Fractions, and that of reducing Complex Fractions to Simple ones, in Division of Fractions.

Addition of Vulgar Fractions.

168. Rule. Reduce the given fractions to others having a common denominator, if they be not such already.

When they are in this state, add all the numerators, and below their sum write the common denominator; the result will be the sum of the fractions, which, if it be an improper fraction, must be reduced to a whole or mixed number.

If some of the given quantities be mixed numbers, the fractional parts are to be added by the preceding part of the Rule; and the whole numbers, with any *integral* part that may be obtained by summing of the fractions, are to be added by simple addition.

Ex. 1. Add $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{16}$.

Proceeding by the preceding Rule,

First finding the least common multiple of the denominators — 48 (Art. 105).

Therefore, the fractions become

$$\left.\begin{array}{r} \frac{3}{4} = 36 \\ \frac{5}{6} = 40 \\ \frac{7}{16} = 21 \end{array}\right\} 48, \text{ Common Denominator.}$$

$$\frac{97}{48} = 2\tfrac{1}{48}.$$

Ex. 2. Add together $14\frac{3}{8}$, $12\frac{4}{5}$ and $\frac{3}{20}$.

In this case, by reducing the given fractions to equivalent ones having a common denominator, we have

$$\left.\begin{array}{rr} 14\frac{3}{8} & = 15 \\ 12\frac{4}{5} & = 32 \\ \frac{3}{20} & = 6 \end{array}\right\} 40, \text{ C. D.}$$

$$27\tfrac{13}{40} \qquad \frac{53}{40} = 1\tfrac{13}{40}. \text{ Answer.}$$

Mental Exercises.

1. What is the sum of $4\frac{2}{7}$ and $5\frac{6}{7}$?
2. What is the sum of $9\frac{5}{8}$ and $8\frac{6}{8}$?
3. What is the sum of $10\frac{5}{9}$ and $8\frac{3}{18}$.
4. A farmer sold $7\frac{5}{8}$ tons of hay to one man, and $3\frac{3}{16}$ tons to another man; how much did he sell altogether?
5. A merchant bought $\frac{7}{9}$ of a ship, and afterwards $\frac{1}{18}$ more; what part of the ship had he?

Exercises for the Slate.

Find the sum of the following:

1. $\frac{10}{14}$, $\frac{2}{15}$ and $\frac{18}{70}$. Ans. $1\frac{11}{105}$.
2. $\frac{4}{5}$, $\frac{7}{10}$, $\frac{4}{7}$ and $\frac{2}{21}$. Ans. $2\frac{1}{6}$.
3. $\frac{1}{8}$, $\frac{7}{12}$, $\frac{5}{9}$ and $\frac{9}{20}$. Ans. $1\frac{257}{360}$.
4. $\frac{1}{5}$, $6\frac{4}{5}$ and $\frac{4}{21}$. Ans. $7\frac{4}{21}$.
5. $100\frac{2}{5}$, $64\frac{5}{9}$ and $420\frac{3}{5}$. Ans. $585\frac{5}{9}$.
6. $261\frac{1}{3}$, $174\frac{3}{4}$ and $8\frac{3}{4}$. Ans. $444\frac{5}{6}$.
7. $387\frac{1}{2}$, $285\ \frac{1}{4}$, $394\frac{1}{3}$ and $1481\frac{3}{5}$. Ans. $2548\frac{41}{60}$.

SUBTRACTION OF VULGAR FRACTIONS.

169. RULE. Set the less quantity below the greater, and prepare them both as in Addition of Fractions. Then, if possible, subtract the lower numerator from the upper; below the remainder write the common denominator; and if there be whole numbers, find their difference, as in Simple Subtraction.

But if the lower numerator exceed the upper, subtract it from the common denominator; to the remainder add the upper numerator; write the common denominator beneath the sum, and carry one to the whole number in the lower line.

Ex. 1. From $4\frac{3}{4}$ take $2\frac{3}{8}$.

Proceeding by the above Rule,

$$\left.\begin{array}{l} 4\frac{3}{4} = 6 \\ 2\frac{3}{8} = 3 \end{array}\right\} 8, \text{ Common Denominator.}$$

$$\begin{array}{ll} \overline{2\frac{3}{8}} & \overline{\frac{3}{8}} \end{array}$$

Ex. 2. From $6\frac{1}{18}$ take $3\frac{3}{4}$.

Proceeding by the above Rule,

$$\left.\begin{array}{l} 6\frac{1}{18} = 2 \\ 3\frac{3}{4} = 27 \end{array}\right\} 36, \text{ C. D.}$$

$$\begin{array}{ll} \overline{2\frac{11}{36}} & \overline{\frac{11}{36}}. \text{ Answer.} \end{array}$$

Reason for the above process.

In the second example, after reducing $\frac{1}{18}$ and $\frac{3}{4}$ to fractions, with a common denominator, viz.: $\frac{2}{36}$ and $\frac{27}{36}$, we find that we cannot take $\frac{27}{36}$ from $\frac{2}{36}$. We therefore add 1 to $\frac{2}{36}$ (or $\frac{36}{36}$ to $\frac{2}{36}$), and now take $\frac{27}{36}$ from $\frac{38}{36}$, which leaves a remainder of $\frac{11}{36}$.

Again, having added 1 to the upper number, we must add 1 to the lower number, so that the difference between the two numbers may not be altered; and adding 1 to 3, we obtain 4, which, taken from 6, leaves 2. Therefore, the difference or remainder is $2\frac{11}{36}$.

Note.—It is more convenient, in practice, to take the lower numerator from the denominator, as the Rule directs. The reason is the same.

Mental Exercises.

1. What is the difference between $\frac{48}{69}$ and $\frac{23}{69}$?
2. What is the difference between $2\frac{3}{4}$ and $1\frac{7}{8}$?
3. What is the difference between $\frac{3}{4} + \frac{3}{8}$ and $\frac{1}{2} + \frac{5}{6}$?
4. Ellen bought $14\frac{3}{4}$ yards of merino for a dress, and $2\frac{1}{12}$ yards of colored cambric for lining; how many yards were there in both? how many more in the dress than in the lining?

Exercises for the Slate.

Find the difference between—

1. $\frac{7}{12}$ and $\frac{8}{15}$.	Ans. $\frac{1}{20}$	5. $13\frac{5}{12}$ and $9\frac{7}{13}$.	Ans. $3\frac{137}{156}$
2. $\frac{17}{18}$ and $\frac{11}{12}$.	Ans. $\frac{1}{36}$	6. $6\frac{3}{8}$ and $4\frac{1}{2}$.	Ans. $1\frac{7}{8}$
3. $\frac{8}{15}$ and $\frac{9}{20}$.	Ans. $\frac{1}{12}$	7. $15\frac{32}{729}$ and $1\frac{16}{243}$.	" $13\frac{713}{729}$
4. $50\frac{1}{16}$ and $47\frac{1}{24}$.	Ans. $3\frac{1}{48}$	8. $46\frac{5}{8}$ and $15\frac{1}{6}$.	" $31\frac{11}{24}$

Multiplication of Vulgar Fractions.

170. **Rule.** If any of the quantities be mixed numbers, reduce them to improper fractions.

Find the product of the numerators for the numerator of the required result, and the product of the denominators for its denominator.

Compound Fractions are reduced to Simple ones in the same manner.

Ex. Multiply $\frac{7}{8}$ by $\frac{6}{17}$.

Proceeding by the above Rule,

$$\frac{7}{8} \times \frac{6}{17} = \frac{42}{136}.$$
$$\text{Reduced,} = \frac{21}{68}.$$

Reason for the above Rule.

If $\frac{7}{8}$ be multiplied by 6, the result is $\frac{42}{8}$. But this result must be 17 times too large, since, instead of multiplying by 6, we have only to multiply by $\frac{6}{17}$, which is 17 times smaller than 6, or, in other words, is one-seventeenth part of 6. Consequently, the product above, viz.: $\frac{42}{8}$, must be divided by 17, and $\frac{42}{8} \div 17, = \frac{42}{136}$, or $\frac{21}{68}$.

It has been shown that a fraction is reduced to its lowest terms by dividing its numerator and denominator by their greatest common measure, or, in other words, by the product of those factors which are common to both. Hence, in all cases of multiplication of fractions, it will be well to split up the numerators and denominators as much as possible into the factors which compose them; and then, after putting the several fractions under the form of one fraction, the sign of $\times$, placed between each of the factors in the numerator and denominator, to cancel those factors which are common to both, before carrying into effect the final multiplication.

Ex. 1. Multiply $\frac{5}{6}$ by $\frac{4}{5}$.

Product $= \frac{5}{6} \times \frac{4}{5}$, cancelling the 5 in each,

$= \frac{4}{6}$, or $\frac{2}{3}$.

Ex. 2. Multiply $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{8}{11}$ and $\frac{11}{12}$ together.

$$\text{Product} = \frac{3 \times 2 \times 5 \times 8 \times 11}{4 \times 3 \times 6 \times 11 \times 12}$$

$$\text{Splitting up,} = \frac{3 \times 2 \times 5 \times 2 \times 4 \times 11}{4 \times 3 \times 2 \times 3 \times 11 \times 2 \times 6}$$

Cancelling $\frac{5}{18}$. Answer.

NOTE.—Instead of splitting up the factors, it is often more convenient to divide by some number which is common to one of the factors in each, (numerator and denominator), placing the quotient above the factor divided, in the numerator, and below the factor divided in the denominator. Thus,

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{11}} \times \frac{\cancel{11}}{\underset{6}{\cancel{12}}}.$$ Cancelling 11 and 3 in each.

Next dividing 8 and 4 by 4. Again, dividing 2 and 6 by 2, and 2 and 12 by 2, we have 5 in the numerator and 3 and 6 in the denominator, which gives $\frac{5}{3 \times 6} = \frac{5}{18}$.

Mental Exercises.

1. What is the product of 9 multiplied by $\frac{5}{8}$? by $\frac{7}{8}$? by $\frac{9}{8}$?

2. What is the product of $12 \times \frac{3}{9}$? $12 \times \frac{5}{9}$? $12 \times \frac{7}{9}$? $12 \times \frac{10}{9}$?

3. A man who had found a purse containing 13 dollars, paid $\frac{3}{10}$ of the money for advertising it; how much did he pay for advertising it?

4. At 20 dollars a ton, what is the cost of $\frac{3}{8}$ of a ton of hay? what cost $\frac{9}{8}$ of a ton?

5. An inch is $\frac{1}{12}$ of $\frac{1}{3}$ of a yard; what fraction of a yard is 8 inches?

6. A grocer sells sperm oil for $1\frac{3}{25}$ dollars a gallon; what is the cost of $\frac{3}{4}$ of a gallon?

Exercises for the Slate.

1. $\frac{36}{55} \times \frac{22}{23}$. Ans. $\frac{72}{115}$.
2. $\frac{15}{79} \times \frac{19}{20}$. Ans. $\frac{57}{316}$.
3. $7\frac{1}{2} \times \frac{1}{5}$ of $\frac{4}{9}$. Ans. $\frac{2}{3}$.
4. 12 by $\frac{2}{3}$ of 5. Ans. 40.
5. $\frac{12}{13}$ of $3\frac{2}{3}$ by $1\frac{1}{33}$ of $\frac{39}{51}$ of $\frac{3}{8}$. Ans. 1.
6. $\frac{1}{2}$ of $\frac{2}{3}$ by $5\frac{3}{8}$ of 3. Ans. $5\frac{3}{8}$.
7. $\frac{147}{196} \times \frac{1764}{1491} \times \frac{355}{529} \times \frac{207}{507} \times 1\frac{110}{189}$. Ans. $\frac{5}{13}$.
8. $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$. Ans. $\frac{1}{6}$.
9. $5\frac{2}{19} \times 3\frac{1}{4}$ of $\frac{8}{117}$ of 34 by $\frac{3}{194}$ of $\frac{9}{68}$ of $1\frac{1}{3}$ of 19. Ans. 2.

Division of Vulgar Fractions.

171. Rule. Reduce mixed or whole numbers to improper fractions, and compound to simple ones, if any such be given. Then invert the divisor, i. e. take its numerator as a denominator, and its denominator as a numerator, and proceed as in multiplication.

A Complex Fraction is reduced to a Simple one by dividing its numerator by the denominator.

Ex. 1. Divide $\frac{5}{18}$ by $\frac{7}{12}$.

Proceeding by the Rule given above,

$$\frac{5}{18} \div \frac{7}{12} = \frac{5}{\underset{3}{\cancel{18}}} \times \frac{\overset{2}{\cancel{12}}}{7} = \frac{10}{21}$$

Reason for the preceding Rule.

If $\frac{5}{18}$ be divided by 7, the result is $\frac{5}{18 \times 7}$ or $\frac{5}{126}$, (Art. 161).

This result is 12 times too small, or, in other words, is only one-twelfth part of the required quotient, since, instead of dividing by 7, we have to divide by $\frac{7}{12}$, which is only one-twelfth part of 7; and the quotient of $\frac{5}{18}$, divided by $\frac{7}{12}$, must therefore be 12 times greater than if the divisor were 7.

Hence, the above result, $\frac{5}{126}$, must be multiplied by 12, in order to give the true quotient.

Therefore, the quotient $= \frac{5}{126} \times 12 = \frac{60}{126}$, or $\frac{10}{21}$.

Ex. 2. Divide $4\frac{3}{5}$ by $6\frac{3}{8}$, reducing the fractions.

$4\frac{3}{5} = \frac{23}{5}$.
$6\frac{3}{8} = \frac{51}{8}$.
$\frac{23}{5} \div \frac{51}{8}$.
$\frac{23}{5} \times \frac{8}{51}$.
$\frac{184}{255}$.

Ex. 3. Required the value of the complex fraction, $\dfrac{5\frac{1}{8}}{6\frac{3}{5}}$

Here $5\frac{1}{8} = \frac{41}{8}$, and $6\frac{3}{5} = \frac{33}{5}$.
Therefore, $\frac{41}{8} \div \frac{33}{5}$,
$= \frac{41}{8} \times \frac{5}{33}$,
$= \frac{205}{264}$.

172. *To Divide a Fraction by a whole number.*

Rule. Divide the numerator by the whole number when it can be done without a remainder; but when this cannot be done, multiply the denominator by the whole number, and reduce, if necessary.

Ex. What is the quotient of $\frac{15}{22}$ divided by 5.

First Method.	Second Method (Art. 161).
$\frac{15}{22} \div 5 = \frac{3}{22}$. Ans.	$\frac{15}{22} \div 5 = \frac{15}{110} = \frac{3}{22}$.

Mental Exercises.

1. What is the quotient of $\frac{18}{2} \div 6$?
2. What is the quotient of $\frac{16}{5} \div 8$?
3. If 10 yards of venetian stair-carpeting cost $9\frac{1}{8}$ dollars, what cost 1 yard? what is the cost of 6 yards?

4. If 6 yards of 3-ply carpeting cost $7\frac{6}{7}$ dollars, what is the cost of 1 yard? what is the cost of 21 yards?

5. In what two ways can we multiply a fraction by a whole number? In what two ways can we divide a fraction by a whole number?

6. A lady purchased 7 yards of linen for $6\frac{1}{8}$ dollars; what would 8 yards cost at the same rate?

Exercises for the Slate.

1. Divide $\frac{13}{20}$ by $\frac{3}{5}$. Ans. $1\frac{1}{12}$.
2. Divide $\frac{51}{64}$ by $3\frac{1}{6}$. Ans. $\frac{153}{608}$.
3. Divide $\frac{16}{45}$ by $\frac{14}{15}$. Ans. $\frac{8}{21}$.
4. Divide $2\frac{8}{21}$ by $4\frac{1}{6}$. Ans. $\frac{4}{7}$.
5. Divide $3\frac{3}{4}$ of $3\frac{1}{3}$ of $\frac{1}{2}$ by 75. Ans. $\frac{1}{12}$.
6. Divide $3\frac{1}{5}$ of $5\frac{5}{8}$ of $3\frac{5}{8}$ by $9\frac{3}{4}$ of $\frac{8}{33}$ of $7\frac{1}{3}$. Ans. $3\frac{9}{13}$.

7. A merchant wishes to lay out $657\frac{1}{2}$ dollars for wheat, which is worth $1\frac{1}{8}$ of a dollar a bushel; how many can he buy? Ans. $584\frac{4}{9}$ bus.

8. Paid $575\frac{3}{4}$ dollars for $96\frac{7}{8}$ yards of cloth; what was the cost per yard? Ans. $\$5\frac{731}{775}$.

Reduction of Decimal Fractions.

173. *Certain Vulgar Fractions can be expressed accurately as Decimals.*

Rule. Make the numerator of the given fraction the dividend; place a dot or decimal point after it, and affix cyphers for decimals. Divide by the denominator, as in Division of Decimals, and the quotient will be the decimal, or the whole number and decimals required.

Ex. 1. Convert $\frac{5}{8}$ into a decimal.

$$\begin{array}{r} 8\,)\,5.000 \\ \hline .625 \end{array}$$

There are three places of decimals in the dividend, and none in the divisor. Therefore, there are three places in the quotient.

In reducing any such fraction as $\frac{3}{50}$ or $\frac{3}{500}$ to a decimal, we may proceed in the same way as if we were reducing $\frac{3}{5}$, taking care, however, in the result, to move the decimal point one place further to the left for each cypher cut off.

Thus, $\frac{3}{5} = .6$, $\frac{3}{50} = .06$, $\frac{3}{500} = .006$; for, in fact, we divide by 5, and then by 10, 100, &c., according as the divisor is 50, 500, &c.

Ex. 2. Convert $\frac{3}{512}$ and $\frac{3}{51200}$ into decimals.

Now, $512 = 8 \times 64, = 8 \times 8 \times 8$.

$$512\left\{\begin{array}{l} 8)\,3.000 \\ \hline 8)\ \ .375000 \\ \hline 8)\ \ .046875000 \\ \hline \qquad .005859375. \end{array}\right.$$

Or, $\frac{3}{512}$ is equivalent to .005859375.
And $\frac{3}{51200}$ is equivalent to .00005859375.

Exercises.

Reduce the following fractional quantities to decimals.

1. $\frac{9}{25}$. Ans. .36.
2. $\frac{19}{20}$. Ans .95.
3. $\frac{66}{128}$. Ans. .515625.
4. $\frac{54}{125}$. Ans. .432.
5. $\frac{570}{200}$. Ans. 2.85.
6. $\frac{170}{125}$. Ans. 1.36.
7. $\frac{1}{160}$. Ans. .00625.
8. $6\frac{11}{64}$. Ans. 6.171875.
9. $\frac{57}{240}$. Ans. .2375.
10. $15\frac{588}{78125}$. Ans. 15.0075264.
11. $3\frac{4}{25} + \frac{33}{110} + 81\frac{37}{1000} + \frac{7\frac{3}{4}}{3\frac{7}{8}}$. Ans. 86.497.

NOTE. 10 is sometimes called the *first power of* 10.
10×10 is sometimes called the *second power of* 10.
$10 \times 10 \times 10$ is sometimes called the *third power of* 10, and so on, similarly of other numbers.

174. We have seen that, in order to convert a Vulgar Fraction into a Decimal, after affixing cyphers to the numerator, we have in fact to divide 10, or some multiple of 10 or of its powers, by the denominator of the fraction. Now, $10 = 2 \times 5$, and these are the only factors into which 10 can be broken up. Therefore, when the fraction is in its lowest terms, if the denominator be not composed solely of the factors 2 and 5, or one of them, or of powers of 2 and 5, or one of them, then the division of the numerator by the denominator will never terminate. Decimals of this kind, that is, which never terminate, are called indeterminate decimals; and they are also called CIRCULATING, REPEATING OR RECURRING DECIMALS, from the fact that when a decimal does not terminate, the same figures must come round again, or recur, or be repeated. For, since we always affix the same figure to the dividend, namely, a cypher, whenever any former remainder recurs, the quotient will also recur.

Now, when we divide by any number, the remainder must always be less than that number, and therefore some remainder must recur before we have obtained a number of remainders equal to the number of units in the divisor.

175. PURE CIRCULATING DECIMALS are those which recur from the beginning. Thus, .333 .. .., .272727 .. .., are pure circulating decimals.

MIXED CIRCULATING DECIMALS are those which do not begin to recur till after a certain number of figures.

Thus, .12888 .. .. .., .01263535 .. .. .., are mixed circulating decimals.

The circulating part, or the part which is repeated, is called the PERIOD OR REPETEND.

Pure and Mixed Circulating Decimals are generally written down only to the end of the first period, a trait or accent being placed over the first and last figures of that period.

Thus, .3′ represents the pure circulating decimal .333 .. ..
.3′6′ represents the pure circulating decimal .363636 ..
.6′39′ " " " " .639639 ..
.126′ " the mixed " " .12666 ..
.0126′23′ " " " " .012623623 ..

Reduce the following vulgar fractions and mixed numbers to circulating decimals.

	ANSWERS.		ANSWERS.
1. $\frac{5}{9}$.	.5′.	6. $\frac{3231}{3520}$.	.9178977′2′.
2. $\frac{17}{30}$.	.56′.	7. $7\frac{962}{3367}$.	7.2′85714′.
3. $\frac{368}{495}$.	.74′3′.	8. $\frac{17}{99000}$.	.0001′7′.
4. $\frac{16}{81}$.	1′97530864′.	9. $24\frac{83}{9768}$.	24.0084′97133′.
5. $15\frac{52}{333}$.	15.1′56′.	10. $17\frac{13}{700}$.	17.018′57142′.

176. *To convert Pure Circulating Decimals into their equivalent Vulgar Fractions.*

Rule. Make the period or repetend the numerator of the fraction, and for the denominator put down as many *nines* as there are figures in the period or repetend.

Exs. Reduce the following pure circulating decimals, .3′, .2′7′, .8′57142′, to their respective equivalent vulgar fractions.

Proceeding by the Rule given above,

$$.3' = \frac{3}{9} = \frac{1}{3}, \qquad .2'7' = \frac{27}{99} = \frac{3}{11}.$$

$$.8'57142' = \frac{857142}{999999},$$
$$= \frac{95237}{111111} = \frac{6 \times 15873}{7 \times 15873},$$
$$= \frac{6}{7}.$$

The truth of these results will appear from the following considerations.

Let the circulating decimal, .3333 .. .., be represented by the symbol x; then $x = .3333$ .. .. ..

Therefore, 10 times x = 10 times .3333 .. .. ..
= 3.333 .... (Art. 118).

Now, 10 times x, diminished by 1 times x, will leave 9 times x, and 3.333.... — .333.... = 3.333...

$$\begin{array}{r} 3.333\ldots \\ -\ .333\ldots \\ \hline 3 \end{array}$$

or 9 times $x = 3$

Therefore, 1 time x, that is, x or .333... $= \frac{3}{9} = \frac{1}{3}$.

Next, let the circulating decimal, .2727 .. ..., be represented by x. Then $x = .2727.....$

Here, since there are two figures in each period, we multiply by 100, and we have

100 times x = 100 times .2727....
= 27.2727 (Art. 118).

Therefore, 100 times x, diminished by 1 time x, will be equal to 27.27... — .2727...
or 99 times $x = 27$.

Therefore, x or .2727.... $= \frac{27}{99} = \frac{3}{11}$.

In like manner, 999999 times $x = 857142$,
$x = \frac{857142}{999999}$.

Note.—The object in each case is to multiply the recurring decimal by such a power of 10 as will bring out the period a whole number.

Otherwise.

To investigate this problem otherwise, let us recur to the origin of circulating decimals, or the manner of obtaining them. For example, $\frac{1}{9}$ = .1111, &c., or .1′. Therefore, the true value of .1111, &c., or .1′, must be $\frac{1}{9}$ from which it arose.

For the same reason, .2222, &c., or .2′, must be twice as much or $\frac{2}{9}$ (Art. 160 and 170);

.3333.... or .3′ $= \frac{3}{9}$; .4′ $= \frac{4}{9}$; .5′ $= \frac{5}{9}$, &c.

Again, $\frac{1}{99}$ = .010101...., or .0′1′; consequently, .0101...., or .0′1′ $= \frac{1}{99}$; .0202....., or .0′2′ $= \frac{2}{99}$; .0′3′ $= \frac{3}{99}$; .0′7′ $= \frac{7}{99}$, &c. So, also, $\frac{1}{999}$ = .001001001..., or .0′01′. Therefore, .001001...., or .0′01′ $= \frac{1}{999}$; .0′02′ $= \frac{2}{999}$, &c.

In like manner, $\frac{1}{7}$ = .1′42857′ (Art. 174); and .1′42857′

$= \frac{142857}{999999}$; for, multiplying the numerator and denominator of $\frac{1}{7}$ by 142857, we have $\frac{142857}{999999}$ (Art. 159). So, $\frac{2}{7}$ is twice as much as $\frac{1}{7}$; $\frac{3}{7}$ three times as much as $\frac{1}{7}$; $\frac{4}{7}$ four times as much, &c.

177. *To convert Mixed Circulating Decimals into their equivalent Vulgar Fractions.*

RULE. Subtract the figures which do not circulate from the figures taken to the end of the first period, as if both were whole numbers; make the result the numerator, and write down as many *nines* as there are figures in the circulating part, followed by as many *zeros* as there are figures in the non-circulating part, for the denominator.

EXS. Reduce the following mixed circulating decimals, .14′, .0138′, .24′18′, to their respective equivalent vulgar fractions.

Proceeding by the above Rule,

$$.14' = \frac{14-1}{90} = \frac{13}{90},$$

$$.0138' = \frac{138-13}{90} = \frac{125}{9000} = \frac{1}{72}, \text{ in its lowest terms.}$$

$$.24'18' = \frac{2418-2}{9990} = \frac{2416}{9990} = \frac{1208}{4995}.$$

The truth of these results will appear from the following considerations:

Taking the last as an example, and separating the mixed decimal into its terminate and periodical parts, we have .24′18′ $=$ .2 $+$ .04′18′. Now .2 $= \frac{2}{10}$ (Art. 110); and .04′18′ $= \frac{418}{9990}$; for, the pure period 4′18′ $= \frac{418}{999}$ (Art. 176); and since the mixed period, .04′18′, begins in hundredths' place, its value is evidently only $\frac{1}{10}$ as much; but $\frac{418}{999} \div 10, = \frac{418}{9990}$ (Art. 172). Therefore, .24′18′ $= \frac{2}{10} + \frac{418}{9990}$. Now, to perform the addition, we must reduce them to a common denominator, when they become (Art. 163),

$$\frac{2\times 999}{10\times 999}+\frac{418}{9990},\ \text{(and since } 999=1000-1),$$

$$=\frac{2\times(1000-1)}{9990}+\frac{418}{9990}=\frac{2\times 1000-2}{9990}+\frac{418}{9990},$$

$$=\frac{2000-2}{9990}+\frac{418}{9990}=\frac{2000+418-2}{9990},$$

$$=\frac{2418-2}{9990}=\frac{2416}{9990}=\frac{1208}{4995}.$$

178. Quantities composed of whole numbers, and either pure or mixed circulating decimals, may be reduced to their equivalent vulgar fractions in the same manner, taking care to use cyphers in the denominator for the non-circulating decimals only.

Ex. 1. Reduce 17.6′ to the form of a vulgar fraction.

Proceeding by the preceding Rule,

$$17.6'=\frac{176-17}{9}=\frac{159}{9}=\frac{53}{3}.$$

Ex. 2. Find the vulgar fraction equivalent to the quantities 17.63′, and 1′7.6′.

$$17.63'=\frac{1763-176}{90}=\frac{1587}{90}+\frac{529}{30}.$$

First Method.

$$1'7.6'=17.61'76'=\frac{176176-176}{9990}$$

$$=\frac{176000}{9990}=\frac{17600}{999}.$$

Second Method.

Affixing a cypher for each whole number, and forming the denominator as before directed, we have $17.6'=\frac{17600}{999}$.

Exercises.

For exercises to Articles 176, 177 and 178, verify the result of those in Article 175.

179. *To Reduce a Number or Fraction of any Denominator to the Decimal of another Denomination.*

RULE. Reduce the given number or fraction to a fraction of the proposed denomination (Art. 166), and then reduce this fraction to its equivalent decimal.

Ex. 1. Reduce $\frac{2}{5}$ of £1 to the decimal of 1 guinea.

$$\tfrac{2}{5} \text{ of } £1 = \tfrac{2 \times 20}{5}\text{s.} = 8\text{s.}$$

$$1 \text{ guinea} = 21\text{s.}$$

Therefore, the fraction required $= \frac{8}{21}$.

$$21 \begin{cases} 7 \,|\, 8.0. \\ 3 \,|\, 1.1'42857' \end{cases}$$

.3'80952', Decimal required.

Ex. 2. Reduce 13s. $6\frac{1}{4}$d. to the decimal of £1.

$$13\text{s. } 6\tfrac{1}{4}\text{d.} = \tfrac{649}{4}\text{d.}$$

$$£1 = \tfrac{960}{4}.$$

Therefore, the fraction $\dfrac{\frac{649}{4}}{\frac{960}{4}} = \frac{649}{960} = .6760416'$.

We may work such an example as the above more expeditiously, by first reducing $\frac{1}{4}$d. to the decimal of a penny, which decimal will be .25, and then reducing 6.25d. to the decimal of a shilling, by dividing by 12, which decimal will be .52083', and then reducing 13.52083's. to the decimal of £1, by dividing by 20, which process gives .6760416' as the required decimal of £1.

The mode of operation may be shown thus:

4	1.00
2	6.25
2,0	13.52083'
	.6760416'.

Ex. 3. Reduce 3 roods 11 po. to the decimal of an acre.

```
4,0 | 11.000
    |-------
  4 |  3.275
    ---------
       .81875, Decimal required.
```

Exercises.

Reduce

1. 6s. 4d. to the decimal of £1. Ans. .316′.
2. £3, 11. $9\frac{3}{4}$. to the decimal of £2, 10. Ans. 1.43625.
3. 2 oz. 13 dwt. to the decimal of 1 lb. Ans. .22083′.
4. $27\frac{1}{2}$ gals. to the decimal of $1\frac{1}{3}$ qts. Ans. 82.5.
5. 3 reams to the decimal of 19 sheets. Ans. 75.789.
6. $5\frac{3}{4}$ yds. to the decimal of 2 Fr. ells. Ans. 1.916′.

180, *To Reduce a Decimal of any Denomination to its proper value.*

Rule. Multiply the decimal by the number of units connecting the next lower denomination with the given one, and point off for decimals as many figures in the product, beginning from the right hand, as there are figures in the given decimal. The figures on the left of the decimal point will represent the whole numbers in the next denomination. Proceed in the same way with the decimal part for that denomination, and so on.

Ex. Find the value of .5375 of £1.

Proceeding by the Rule given above,

```
 £.5375
     20
 -------
 10.7500
     12
 -------
  9.0000
```

Therefore, the value of .5375 of £1 = 10s. 9d.

With respect to the *reason* of the process, it is only necessary to observe, that it is exactly the same as finding the value of £ $\frac{5375}{10000}$, by Art. 167, the pointing off of the decimals serving the purpose of dividing by the denominator.

Exercises.

Required, the value of the following decimals:

1.	.0675 cwt.	Ans. $7\frac{14}{25}$ lbs.
2.	.0625 of a guinea.	Ans. 1s. $3\frac{3}{4}$d.
3.	.875 of a league.	Ans. 2 m. 1100 yds.
4.	1.605 of £3, 2. 6.	Ans. £5, 0. $3\frac{3}{4}$.
5.	1.005 of 15 guineas.	Ans. £15, 16. $6\frac{3}{4}$. .6q.
6.	.4′ foot, *long measure.*	Ans. $5\frac{1}{3}$ inches.
7.	.063′ of 100 guineas.	Ans. £6, 13. 0.
8.	.2383′ of a degree.	Ans. 14′. 18″.

Addition and Subtraction of Circulating Decimals.

181. In arithmetical operations, where circulating decimals are concerned, and the result is only required to be true to a small number of decimal places, it will be sufficient to carry on the circulating part to two or three decimal places more than the number required, taking care that the last figure retained be increased by 1, if the succeeding figure be 5, or greater than 5; but when it is required to find the true sum, difference, &c., the periods must be rendered similar and coterminous, or reduced to their equivalent vulgar fractions before being operated upon.

182. *To make any number of Periodical Decimals similar and coterminous.*

Rule. Extend each of the circles as many places beyond the longest finite part as is denoted by the least common multiple of the *number* of places in the given circles.

Ex. Make .31′6′, .4′287′ and .5142′73′ similar and coterminous.

Here the longest finite part in these decimals is 514 in the last, and the least common multiple of 2, 4 and 3, the number of places in the circles, is 12. Therefore, extend each of the circles to 12 figures beyond the 4, when the same figures will again fall under one another, if repeated. Thus,

.31′6′	=	.31′6′	161616161616	1
.4′287′	=	.4′28	7′42874287428	7
.5142′73′	=	.514	2′73′273273273	2

183. When you add or subtract the right hand column, include the carriage that would arise by extending the circles still farther; or, in other words, to find the carrying figure, go towards the left hand as many places as the periods have been extended, to make them similar and coterminous. Thus, if they are extended 6 places go back 6; if 10, go back 10, &c.

Exercises.

1. Add 31.61′4′, 3.416′23′, 80.57′631′ and 6.3′.
Ans. 121.940′02861547416′.

2. Add 8.1576′4′, .5′1387625′, 8.13′042′ and .4.
Ans. 17.2019′4576208′.

3. From 21.63′ take $3.87\dot{6}\dot{1}$.
Ans. $17.75\dot{7}\dot{1}$.

4. From $.\dot{8}10\dot{2}$ take $.5\dot{3}7\dot{6}$.
Ans. $.2\dot{7}2643390465\dot{1}$.

Multiplication of Circulating Decimals.

184. *When the Multiplicand only is interminate.*

Rule. Place the multiplier as usual below the multiplicand, and when you multiply the right hand figure, include the carriage that would arise by extending the repeating figures, and before adding up the partial products, extend the circles or repeating figures as in Addition.

Exercises.

1. Multiply $1.3\dot{8}$ by 2.76. Ans. $3.8\dot{3}$.

2. Multiply $15.3\dot{9}\dot{6}$ by 7.89. Ans. $121.482\dot{0}\dot{9}$.

3. Multiply $20.\dot{4}38\dot{7}$ by 6.73. Ans. $137.55\dot{2}74\dot{6}$.

185. *When the Multiplier is interminate also.*

Rule. Reduce the multiplier to a vulgar fraction, and multiply by the numerator and divide by the denominator. Or, reduce both factors to vulgar fractions, and multiply.

When there is an integer, the decimal portion only may be reduced to a vulgar fraction: then multiply, as in Art. 80.

Exercises.

1. Multiply $4.10\dot{8}$ by $.3\dot{8}\dot{7}$. Ans. $1.593\dot{7}5084\dot{1}$.

2. Multiply $\dot{6}81.\dot{7}3\dot{8}$ by 2.363′. Ans. $1611.17\dot{5}8\dot{8}$.

3. Multiply $27.1\dot{8}2\dot{4}$ by $3.\dot{2}8571\dot{4}$. Ans. $89.3\dot{1}3871\dot{0}$.

Division of Circulating Decimals.

186. *When the Dividend only is interminate.*

Rule. Divide as in finite decimals, but annex the repeaters or circulating figures instead of cyphers.

187. *When the Divisor is interminate.*

Rule. Reduce the divisor to a vulgar fraction, then multiply by the denominator, and divide by the numerator.

Exercises.

Verify the results of those in Articles 184 and 185.

Miscellaneous Exercises on Articles 158–187.

1. A clerk spent $26\frac{3}{8}$ dollars for a coat, $9\frac{3}{4}$ dollars for pants, $6\frac{3}{4}$ dollars for a vest, $5\frac{1}{2}$ dollars for a hat, and $6\frac{1}{4}$ dollars for a pair of boots; how much did the suit cost him? Ans. \$$54\frac{5}{8}$.

2. What number added to $\frac{3}{4}$ of $(\frac{1}{3} + \frac{1}{5} - \frac{4}{15} + \frac{1}{9})$ makes $3\frac{1}{4}$? Ans. $2\frac{29}{30}$.

3. If I pay away $\frac{1}{3}$ of my money, then $\frac{1}{2}$ of what remains, and then $\frac{1}{4}$ of what still remains, what fraction of the whole will be left? Ans. $\frac{1}{4}$.

4. Find the sum of the greatest and least of these fractions: $\frac{3}{8}$, $\frac{5}{12}$, $\frac{4}{9}$ and $\frac{7}{20}$; the sum of the other two; and the difference of these sums. Ans. $\frac{1}{360}$.

5. A man has $\frac{3}{8}$ of an estate; he gives his son one-half of his share. What portion of the estate has he then left? Ans. $\frac{3}{16}$.

6. Multiply $3\frac{1}{3}$ by $3\frac{1}{10}$, and divide $\frac{20\frac{3}{4}}{3}$ by $\frac{41\frac{1}{2}}{4}$ and find the difference between the sum and difference of these results. Ans. $1\frac{1}{3}$.

7. Add together $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$; subtract the sum from 2; multiply the result by $\frac{2}{3}$ of $\frac{27}{40}$ of 8, and find what fraction this is of 99. Ans. $\frac{43}{1650}$.

8. Work each of the foregoing decimally, and thus verify the results.

PRACTICE.

188. **Practice** is a compendious mode of finding the value of any number of articles by means of **Aliquot** parts, when the price of an **unit** of any denomination is given.

Practice may be separated into two cases, *Simple* and *Compound*.

I.—Simple Practice.

In this case, the given number is expressed in the same denomination as the unit whose value is given; as, for instance, 21 oz. at 1s. 3d. per oz.; or 460 articles at £1, 2. 6. each.

(1.) *To find the price of a commodity, when the price of each article as well as the quantity, is of one denomination.*

Rule. Multiply the price of each article by the number of articles.

Note.—All the exercises under this article should be worked by dollars and cents also, and thus verify the results.

Ex. Required, the price of 368 cwt. @ £2 per cwt.

Here, the price of 368 cwt., at £1 per cwt., is evidently £368, and the price of the same at £2 per cwt., must obviously be twice as much.

368 cwt. @ £2 per cwt.
£368 = price, @ £1 per cwt.
2
£736, = price at £2 per cwt.

Exercises.

1. 344 @ £2. Ans. £683.	3. 161 @ £5. Ans. £805.
2. 421 @ £3. " £1268.	4. 121 @ £2. " £242.

(2.) *When the price is an aliquot part of a higher denomination.*

Rule. Take a like part of the number of articles, and the result will be the price in the higher denomination.

Ex. 1. What is the value of 127 lbs. of tea, at 2s. 6d. per lb.

127 lbs. @ 2s. 6d. per lb.
£127 = price, at £1 each.
2s. 6d. = $\frac{1}{8}$ of £1. £15, 17. 6. = price at 2s. 6d. each.

In this example, since 127 lbs., at £1 each, would cost £127, it is evident that, at 2s. 6d., the same number would cost one-eighth as much, 2s. 6d. being that part of £1. We therefore divide £127 by 8, and the quotient, £15, 17. 6., is the price required.

Ex. 2. Find the price of 163 articles, at 3d. each.

Here, 163, at 1 shilling, would cost 163 shillings; and the price of the same, at 3d., would evidently be one-fourth of that, or £2, 0. 9.

163 @ 3d. each.

163 = price, at 1s. each.

3d. = $\frac{1}{4}$ of 1s. = 40, 9. = price at 3d. each.

£2, 0. 9.

Exercises.

1.	346 at 10s.	Ans.	£173, 0. 0.
2.	327 at 6s. 8d.	"	£109, 0. 0.
3.	271 at 3s. 4d.	"	£ 45, 3. 4.
4.	141 at 6d.	"	£ 3,10. 6.
5.	146 at 2d.	"	£ 1, 4. 4.
6.	189 at 4d.	"	£ 3, 3. 0.

(3.) *To find the price of any number of articles, at 2 shillings each.*

Rule. Double the last figure of the number for shillings, and take the number expressed by the preceding figures as pounds.

Ex. What is the cost of 587 yards of muslin, at 2s. per yard?

587 yds. @ 2 shillings.
2

£58, 14.

The Reason of this Rule is, that 2 shillings are one-tenth of a pound, and the work is merely a contracted form of dividing by ten. Thus, in the above example, by dividing by 10, we would have £58, with the fraction £$\frac{7}{10}$, or, by doubling both the terms (Art. 159), £$\frac{14}{20}$, or 14 shillings.

(4.) *If the rate be an even number of shillings.*

RULE. Multiply by half the number of shillings, and in multiplying, double the last figure for shillings; the rest will be pounds.

Ex. Required, the price of 186 lbs. of indigo, at 8 shillings per lb.

186 lbs. at 8s.
4
£ 74, 8. 0.

The operation of this rule is evidently an extension of the last. In the annexed example, the price is $\frac{8}{20}$ or $\frac{4}{10}$ of a pound; and hence we multiply by 4, and the doubling of the last figure of the product for shillings is equivalent to a division by 10.

(5.) *When the number of shillings is odd.*

RULE. Find, by the last rule, the amount at one shilling less than the given rate, and for that shilling, take one-twentieth of the price at one pound, or add the aliquot part that it is of the even number of shillings in the price.

Ex. What is the cost of 787 pounds of tea, at 7 shillings per lb.?

	787	
	3	
	£236, 2. 0.	= price, at 6s. per lb.
1s. = £$\frac{1}{20}$	39, 7. 0.	= price, at 1s. per lb.
	£275, 9. 0.	7s.

In this example, the price is first found at 6s. per pound, by multiplying by 3, and doubling the last figure of the product for shillings; then, for the remaining shilling, a twentieth part of £787, the price at 20s. per lb., is taken, and the sum of both results is £275, 9. 0., the price at 7 shillings.

(6.) *When the number of articles is even and the price odd, half the number may be taken, multiply by the whole number of shillings and double the last figure for shillings.*

Ex. Required, the cost of 126 articles, at 11 shillings each?

```
2 ) 126
    ---
     63
     11 shillings.
  ---------
 £69, 6. 0.
```

EXERCISES.

1.	397 at 2s.	Ans.	£39, 14. 0.
2.	418 at 3s.	"	£62, 14. 0.
3.	763 at 6s.	"	£228, 18. 0.
4.	937 at 9s.	"	£421, 13. 0.
5.	675 at 13.	"	£438, 15. 0.
6.	319 at 16.	"	£255, 4. 0.
7.	868 at 17s.	"	£737, 16. 0.
8.	480 at 18s.	"	£432. 0. 0.

The preceding rules should be worked mentally as well as on the slate. They might have been given under the Article on Mental Calculation, but have been inserted here, instead, to render the present Article complete in itself.

(7.) *To find the price of any number of articles when the price is of more than one denomination.*

RULE. Multiply the number of articles by the integer of the price, and take parts of the integer for the lower denominations. The sum of the results thus obtained, will be the price required.

Ex. 1. Find the value of 484 things, at £2, 16. 8. each.

484, at £2, 16. 8.

£484, at £2, = price at £1 each.
2

£968, at £2, = price at £2 each.

10s. 0d. = $\frac{1}{2}$ of £1	242, 0. 0.	= price, at	10s. each.
6s. 8d. = $\frac{1}{3}$ of £1	161, 6. 8.	= price, at	6s. 8d. each.
	£1371, 6. 8.	= price, at	£2, 16. 8. each.

Ex. 2. What cost 643 yards of linen, at 3s. 9d. per yd.?

643 yds. at 3s. 9d. per yd.

£643, 0. 0. = cost of 643 yd. at £1 per yd.

2s. 6d. = $\frac{1}{8}$ of £1	80, 7. 6.	= cost of 643 yd. at 2s.6d. "
1s.3d. = $\frac{1}{2}$ of 2s.6d.	40, 3. 9.	= cost of 643 yd. at 1s.3d. "
	£120, 11. 3.	= cost of 643 yd. at 3s.9d. "

SECOND METHOD.

643 yds. at 3s. 9d. per yard.

643s. = value, at 1s. per yard.
3

6d. = $\frac{1}{2}$ of 1s.	1929s. 0d.	= value, at 3s. per yard.
3d. = $\frac{1}{2}$ of 6d.	321s. 6d.	= value, at 0s. 6d. per yard.
	160s. 9d.	= value, at 0s. 3d. per yard.
	2,0) 241,1. 3d.	3s. 9d.

£120, 11. 3. Ans. *as before.*

Ex. 3. Find cost of 1263 yards ribbon, at $6\frac{3}{4}$d. per yd.

1263 yds. @ $6\frac{3}{4}$d. per yd.

1263s. 0d. price, at 1s. 0d. per yd.

6d. = $\frac{1}{2}$ of 1s.	631s. 6d.	= price, at 0s. 6d. per yd.
$\frac{1}{2}$d. = $\frac{1}{12}$ of 6d.	52s. $7\frac{1}{2}$d.	= price, at 0s. $0\frac{1}{2}$d. per yd.
$\frac{1}{4}$d. = $\frac{1}{2}$ of $\frac{1}{2}$d.	26s. $3\frac{3}{4}$d.	= price, at 0s. $0\frac{1}{4}$d. per yd.
	2,0) 71,0s. $5\frac{1}{4}$d.	= price, at 0s. $6\frac{3}{4}$d. per yd.

£35, 10. $5\frac{1}{4}$. Answer.

SECOND METHOD.

1263 yds. at $6\frac{3}{4}$d. per yard.

6d. = $\frac{1}{2}$ of 1s.	631s. 6d. = cost, at 6d. per yard.
$\frac{3}{4}$d. = $\frac{1}{8}$ of 6d.	78s. $11\frac{1}{4}$d. = cost, at $\frac{3}{4}$d. per yard.
	2,0) 71,0s. $5\frac{1}{4}$d. = cost, at $6\frac{3}{4}$d. per yard.
	£35, 10. $5\frac{1}{4}$. Ans.

NOTE.—The student must use his own judgment in selecting the most convenient 'aliquot' parts, taking care that the sum of those taken make up the *given price of the unit*.

In taking aliquot parts, it sometimes shortens the work to take the same part twice, as the result may thus be copied without working for it again. Thus, 18s. 6d. may be divided into 10s., 4s., 4s. and 6d. Sometimes the price at a smaller rate may be found, and from it the price of a greater may be obtained by multiplication. Thus, 16s. 4d. may be divided into 2s., 14s. and 4d.; the price at 14s. being 7 times the price at 2s.

EXERCISES.

Find the value of

1. 454 things, @ 2s. 9d. each. Ans. £62, 8. 6.
2. 80 things, @ 4s. $4\frac{1}{2}$d. each. Ans. £17, 8. 4.
3. 898 things, @ 18s. $7\frac{3}{4}$d. each. Ans. £837, 3. $11\frac{1}{2}$.
4. 4681 things, @ $8\frac{3}{4}$d. each. Ans. £170, 13. $2\frac{3}{4}$.
5. 7382 things, @ £3, 15. $4\frac{1}{2}$ each. Ans. £27820, 18. 3.
6. 43.35 things, @ 8s. $11\frac{1}{4}$d. Ans. £19, 7. $5\frac{1}{4} + \frac{3}{20}$q.
7. 147.625 things, at 19s. $7\frac{1}{2}$d. Ans. £144, 14. $0\frac{3}{4}$. $+ \frac{1}{8}$q.

The price may often be determined very easily, by finding the amount at a rate higher than the given rate, and deducting from the amount the price at the difference between the given rate and the assumed one.

Ex. What cost 189 tons, at 17s. 6d. per ton?

	189, at 17s. 6d.
	189, 0. 0. = price, at £1 per ton.
2s. 6d. = $\frac{1}{8}$ of £1.	23, 12. 6. = price, at £0, 2. 6. per ton.
	£165, 7. 6. = price, at 0. 17. 6. per ton.

Exercises.

1.	358 articles, at 13s. 4d.	Ans. £238. 13. 4.
2.	599 articles, at 0s. $10\frac{1}{2}$d.	Ans. £26. 4. $1\frac{1}{2}$.
3.	276 articles, at £2, 15. 0.	Ans. £756, 5. 0.
4.	721 articles, at £0, 19. 0.	Ans. £684, 19. 0.

II.—Compound Practice.

In this case, the given number is not wholly expressed in the same denomination as the unit, whose value is given; as, for instance, $79\frac{3}{8}$ lbs. at 2s. 9d. per lb., 126 cwt. 3 qrs. 14 lbs. at £2, 9. 6. per cwt.

(8.) *When the quantity is not expressed by a whole number of one denomination.*

Rule. Compute the price of the integral part by some of the methods already given. Then find the price of the fractional parts, or lower denominations, from the given rate, by means of aliquot parts, or otherwise. The sum of all will be the whole price required. Or,

(9.) Find the price of the entire given quantity at £1 or $1 for each unit of the integral part, valuing the subordinate parts at the same rate. Then the operation will proceed in the same manner already explained, without farther work for the subordinate parts.

Ex. Find the cost of $167\frac{7}{8}$ cwt. at £2, 5. 6 per cwt.

	$167\frac{7}{8}$ cwt. at £2. 5. 6. per cwt.	
	167, 0. 0.	= price of 167, at £1 per cwt.
	2	
	334, 0. 0.	= price of 167, at £2 per cwt.
5s = $\frac{1}{4}$ of £1.	41,15. 0.	= price of 167, at 5s. per cwt.
6d. = $\frac{1}{10}$ of 5s.	4, 3. 6.	= price of 167, at 6d. per cwt.
$\frac{4}{8} = \frac{1}{2}$ of 1 cwt. (∴ $\frac{1}{2}$ of £2, 5. 6.)	1, 2. 9.	= price of $\frac{4}{8}$ cwt. at £2, 5. 6.
$\frac{2}{8} = \frac{1}{2}$ of $\frac{4}{8}$.	0,11. $4\frac{1}{2}$.	= price of $\frac{2}{8}$ cwt. at £2, 5. 6.
$\frac{1}{8} = \frac{1}{2}$ of $\frac{2}{8}$.	0. 5. $8\frac{3}{4}$.	= price of $\frac{1}{8}$ cwt. at £2, 5. 6.
	£381,18. $3\frac{3}{4}$.	

SECOND METHOD.

	167 $\frac{7}{8}$ cwt. at		£2, 5. 6. per cwt.	
	2			
		$\frac{4}{8}=\frac{1}{2}$ of 1 cwt.	1, 2. 9.	"
	334, 0. 0.	$\frac{2}{8}=\frac{1}{2}$ of $\frac{4}{8}$ "	0,11. 4$\frac{1}{2}$.	"
5d. = $\frac{1}{4}$ of £1	41,15. 0.	$\frac{1}{8}=\frac{1}{2}$ of $\frac{2}{8}$ "	0, 5. 8$\frac{1}{4}$.	"
6s. = $\frac{1}{10}$ of 5s.	4. 3. 6.			
			£1,19. 9$\frac{3}{4}$.	

£379,18. 6. price 167 cwt. at £2, 5. 6. "
1,19. 9$\frac{3}{4}$. price $\frac{7}{8}$ cwt. at 2, 5. 6. "
£381,18. 3$\frac{3}{4}$. price 167$\frac{7}{8}$ cwt. £2, 5. 6. "

THIRD METHOD.

167$\frac{7}{8}$ cwt. at £2, 5. 6. per cwt.

	167, 17. 6.	= value 167$\frac{7}{8}$ cwt. at £1 per cwt.					
	2						
	335, 15. 0.	= "	"	"	£2	"
5s. = $\frac{1}{4}$ of £1	41, 19. 4$\frac{1}{2}$.	= "	"	"	5s.	"
6d. = $\frac{1}{10}$ of 5s.	4, 3. 11$\frac{1}{4}$.	= "	"	"	6d.	"
	£381, 18. 3$\frac{3}{4}$.	= "	"		£2, 5. 6.	"

BY DOLLARS AND CENTS.

167$\frac{7}{8}$ cwt. at \$9.10 per cwt.

167.875 cwt. at \$9.10 per cwt.
9.10

1678750
1510875

\$1527.66250 = £381,18. 3$\frac{3}{4}$.

SECOND METHOD.

$167\frac{7}{8}$ cwt. at \$9.10 per cwt.

	$167\frac{7}{8}$
	63.70
	546.0
	910.
$\frac{4}{8} = \frac{1}{2}$ of 1 cwt. at	4.55
$\frac{2}{8} = \frac{1}{2}$ of $\frac{4}{8}$ cwt. at	2.275
$\frac{1}{8} = \frac{1}{2}$ of $\frac{2}{8}$ cwt. at	1.1375
	\$1527.6625=£381, 18. $3\frac{3}{4}$.

The examples under this and the following rules in this article should also be worked by Art. 133.

EXERCISES.

1. $165\frac{7}{8}$ at £2, 5. 6. Ans. £377, 7. $3\frac{3}{4}$.
2. $7538\frac{3}{4}$ at £0, 2. 4. Ans. £879, 10. 5.
3. $164\frac{7}{8}$ at \$9.10. Ans. $\$1500.36\frac{1}{4}$.
4. 239 lbs. 12 oz. at $\$2.16\frac{2}{3}$ Ans. $\$519.45\frac{5}{6}$.
5. $257\frac{1}{12}$ at £2, 9. 4. Ans. £636, 3. $10\frac{1}{2}$.

(10.) *In calculating the price of hundred, quarters, and pounds*, the price, at £1 per cwt. will be found by multiplying the pounds by $2\frac{1}{7}$, and considering the product as pence; and by multiplying the quarters by 5, and considering the product as shillings.

Ex. Required the price of 319 cwt. 3 qr. 16 lbs. at £2, 12. 6. per cwt.

	319 cwt. 3. 16.	at £2, 12. 6. per cwt.
	5. $2\frac{1}{7}$	
	319, 17. $10\frac{2}{7}$	= price at £1 per cwt.
	2	
	£639, 15. $8\frac{4}{7}$	= price at £2 per cwt.
10s. = $\frac{1}{2}$ of £1	159, 18. $11\frac{1}{7}$	= " 10s. "
2s. 6d. = $\frac{1}{4}$ of 10s.	39, 19. $8\frac{11}{14}$	= " 2s. 6d. "
	£839, 14. $4\frac{1}{2}$.	= " £2, 12. 6. "

Reason for the preceding process.

It is evident that as £1 per cwt. each quarter would cost 5 shillings, and each pound the twenty-eighth of this, or $2\frac{1}{7}$ pence,

Therefore, multiplying the pounds by $2\frac{1}{7}$, we say, one-seventh of 16 is $2\frac{2}{7}$, and twice 16 are 32, and $2\frac{2}{7}$ are $34\frac{2}{7}$ pence, or 2s. $10\frac{2}{7}$d., which is the value of the pounds at £1 per cwt. We then set down $10\frac{2}{7}$d. and carry 2 to 5 times 3, or 15 shillings, the price of 3 quarters; and we thus find the price of 3 qr. 16 lbs., at £1 per cwt., to be 17s. $10\frac{2}{7}$ pence; to which £319, the price of 319 cwt. at the same rate is prefixed. The rest of the operation proceeds in the usual manner.

The above might have been calculated by Rule 7.

Thus, 319 cwt. 3 qr. 16 lbs. at £2, 12. 6.
2

	£638, 0. 0.	=	price 319 cwt. at £2 per cwt.
10s. = $\frac{1}{4}$ of £2	159, 10. 0.	=	" " at £0, 10. 0. "
2s. 6d = $\frac{1}{4}$ of 10s	39, 17. 6.	=	" " at 0. 2. 6. "
2 qr. = $\frac{1}{2}$ 1 cwt.	1, 6. 3.	=	" 2 qr. at £2, 12. 6. "
1 qr. = $\frac{1}{2}$ 2 qr.	0, 13. $1\frac{1}{2}$.	=	" 1 qr. " " "
14 lb. = $\frac{1}{2}$ 1 qr.	0, 6. $6\frac{3}{4}$.	=	" 14 lb. " " "
2 lb. = $\frac{1}{7}$ 14 lb.	0, 0. $11\frac{1}{4}$.	=	" 2 lb. " " "
	£839, 14. $4\frac{1}{2}$.		

The same by Dollars and Cents.

319 cwt. 3 qr. 16 lbs at $10.50 per. cwt.

In some cases it will be found more convenient to multiply the price by the quantity. Thus,

	$10.50
	319. 3. 16.
	94.50
	105.0
	3150.
2 qr. = $\frac{1}{2}$ of 1 cwt.	5.25
1 qr. = $\frac{1}{2}$ of 2qr.	2.625
14 lb. = $\frac{1}{2}$ of 1 qr.	1.3125
2 lb. = $\frac{1}{7}$ of 14 lb.	.1875
	$3358.8750 = £839, 14. $4\frac{1}{2}$.

It will be found in nearly all the examples in practice, that dividing by the aliquot parts &c., gives origin to vulgar fractions, which, of necessity, must be added to find the true resuts. This may be obviated by proceeding on the same principle, but converting the fractions into decimals, which will be found to be sufficient to carry to two places each. Thus the work may be as follows,

319 cwt. 3 qrs. 16 lbs. at £2, 12. 6. per cwt.

	5 $2\frac{1}{7}$
	£319, 17. 10.29
	2
	639, 15. 8.58
10s.=$\frac{1}{2}$ of £1	159, 18. 11.14
2s. 6d.=$\frac{1}{4}$ 10s.	39, 19. 8.78
	£839, 14. 4.50 or £839, 14. $4\frac{1}{2}$.

Here, in taking $\frac{1}{7}$ of 16, we have 2 to carry, and 2 remaining, then conceiving a cipher annexed to this remainder, and dividing 20 by 7, we set down the quotient 2, and conceiving a cypher annexed to the remainder 6, we have contained most nearly 9 times in 60. We then proceed as before, and find the price at £1 per cwt. £319, 17. 10.29 nearly.

After this the work proceeds as before, only that in each line the pence and the decimal are multiplied and divided as if they were a single whole number, the point being retained. Thus in finding the price at 2s. 6d., after having found £39, 19, we have 2s. 11d., or 35d. remaining; we then divide 35.14 by 4, as if it were all one number, and find for the quotient 878 or with the point, 8.78.

Note.—In working by this method it is evident that each penny is supposed to be divided into 100 equal parts. Therefore 25d.=$\frac{1}{4}$, .50d=$\frac{1}{2}$, .75d.=$\frac{3}{4}$. *In valuing the decimal found in the answer, the pupil should consider to which of these it is nearest, and value it accordingly.*

EXERCISES.

	Cwt.	qrs.	lbs.			
1.	134.	1.	21.	at £0, 18. 4.	Ans.	£123, 4. 8¼.
2.	812.	3.	7.	at £6, 12. 8.	Ans.	£5391, 13. 1½.
3.	786.	2.	8.	at $3.75	Ans.	$2949.64⅙.
4.	179.	3.	25.	at $14.25	Ans.	$2564.61⅔.

(11.) *In computing the price of tons, hundreds, and quarters, at £1 per ton,* take the tons and hundreds as pounds and shillings, and multiply the quarters by 3 for pence.

Ex. Find the price of 163 tons 2 cwt. 3 qr. at £2, 16. 3. per ton.

163 tons 2 cwt. 3 qr. at £2, 16. 3. per ton.
3

	£163, 2. 9. =	price at £1 per ton.	
	2		
	326, 5. 6. =	"	£2 "
10s. = ½ of £1.	81, 11. 4.50 =	"	10. "
5s = ½ of 10s.	40, 15. 8.25 =	"	5. "
1s. 3d. = ¼ of 5s.	10, 3. 11.06 =	"	1. 3.
	£458, 16. 5¾.81		£2, 16. 3.

By Dollars and Cents.

163 tons 2 cwt. 3 qrs. at $11.25 per ton
$11.25
163 tons 2 cwt. 3 qr.

	33.75
	675.0
	1125.
2 cwt. = $\frac{1}{10}$ of 1 ton	1.125
2 qrs. = ¼ of 2 cwt.	.28125
1 qr. = ½ of 2 qr.	.140625
	1835.296875 = £458. 16. 5¾

Exercises.

1.	175 tons 18 cwt. 1 qr. at £38, 13.0.	per ton	£6799. 0. 4$\frac{1}{2}$
2.	219 tons 16 cwt. 3 qrs. at $45.50	"	10002.60\frac{5}{12}$
3.	2 tons 15 cwt. 2 qrs. at $7. 84$\frac{1}{2}$	"	$21.77 nearly
4.	163 tons 2 cwt. 1 qr. at £2, 19. 6.	"	£485, 5. 2$\frac{1}{4}$

(12.) *In computing the price of acres, roods and perches at £1 per acre*, multiply the perches by 1$\frac{1}{2}$ for pence, and the roods by 5 for shillings.

The reason for this rule and the following depend on the same principle as Rule 10.

Ex. Required the price of 127 acres 3 ro. 14 po. at £2, 11. 6. per acre.

	127 acres	3ro.	14 po.	at £2, 11. 6. per acre.
		5	1$\frac{1}{2}$	
	£127,	16.	9.	= value at £1 per acre.
			2	
	£255,	13.	6.	= value at £2 per acre
10s. = $\frac{1}{2}$ of £1.	63,	18.	4.50	= " " 10. "
1s. = $\frac{1}{10}$ 10s.	6,	7.	10.05	= " " 1. "
6d. = $\frac{1}{2}$ of 1s.	3,	3.	11.02	= " " 6. "
	£329,	3.	7$\frac{1}{2}$(57	= £2,11.6.

By Dollars and Cents.

127 acres 3 ro. 14 po. at $10.30 per acre.

```
 $10.30
   127
 ------
  7210
 2060
1030
 ------
```

	$1308.10	= price of 127 ac. at 10.30 per ac.
2 ro. = $\frac{1}{2}$ of 1 ac.	5.15	= price of 2 ro. at 10.30 per ac.
1 ro. = $\frac{1}{2}$ of 2 ro.	2.575	= " 1 ro. " "
10 po. = $\frac{1}{4}$ 1 ro.	.64375	= " 10 po. " "
4 po. = $\frac{1}{10}$ 1 ro.	.2575	= " 4 po. " "
	$1316.72625	= " 127 ac. 3 ro. 14 po.

or £329, 3 7$\frac{1}{2}$.

Exercises.

	Ac. ro. po.		
1.	23. 3. 5. at £2, 12. 6.	Ans.	£62, 8. 6$\frac{1}{4}\frac{9}{10}$
2.	225. 1. 19. at £0, 13. 2$\frac{1}{2}$ per ro.	Ans.	£595, 6. 11$\frac{3}{4}+\frac{3}{20}$ q.
3.	45. 2. 35. at £0, 16. 6.	Ans.	£37, 14. 4$\frac{1}{4}$.
4.	311. 2. 26. at $4.55	Ans.	1418.06\frac{1}{4}$

(13.) *In computation in Troy weight at* £1, *per oz.*, take the ounces as pounds, the penny-weights as shillings, and half the grains as pence.

Ex. Required the price of 16 oz. 3 dwt. 14 grs. Troy weight at 17s. 6d. per ounce.

	16 oz.	3 dwt.	14 grs.	at £0, 17. 6. per oz.		
			$\frac{1}{2}$			
	£16,	3.	7.	=price at	£1,	per oz
2s. 6d.=$\frac{1}{8}$ £1 de.	2,	0.	6$\frac{3}{8}$.	=price at	2. 6d.	"
	£14,	3.	1$\frac{5}{8}$.	= "	17. 6.	

By Dollars and Cents.

16 oz. 3 dwt. 14 grs. at $3.50 per oz.

	$3.50			
	16			
	$56.00	=price of	16 oz. at $3.50 per oz.	
2dwt.=$\frac{1}{10}$ 1oz.	.35	= "	2dwt.	"
1dwt.=$\frac{1}{2}$ 2 dwt.	.175	= "	1dwt.	"
12grs.=$\frac{1}{2}$ 1 dwt.	.0875	= "	12grs.	"
2grs.=$\frac{1}{6}$ 12 grs.	.014583′	= "	2grs.	"
	$56.627083′	= "	16 oz. 3 dwt. 14 grs.	

=£14, 3. 1$\frac{5}{8}$.

Exercises.

1.	15 oz. 6 dwt. 17 grs. at £0, 5. 10.	Ans.	£4, 9. 5$\frac{1}{2}$
2.	93 oz. 7 dwt. 15 grs. at £0, 10. 4.	Ans.	£48, 4. 11$\frac{1}{4}$
3.	263 oz. 16 dwt. 9 grs. at £0. 11. 3.	Ans.	£148. 7. 11$\frac{1}{2}$
4.	3 lbs. 11 oz. 12 dwt. at 1.16\frac{1}{2}$ per oz.	Ans.	55.45\frac{1}{8}$

(14.) *In computing the price of yards, quarters, and nails at £1 per yard*, take each quarter at 5 shillings, and each nail at 1s. 3d.

Ex. Find the cost of 183 yds. 3 qrs. 2 nls. at 15. 6. per yd.

183 yds. 3 qrs. 2nls. at £0, 15. 6d per yd.
5. 1s. 3d½.

	£183.	17.	6. =	price at £1 per yard.	
10s = ½ of £1	91,	18.	9. =	"	10. "
5s = ½ of 10s.	45,	19.	4.50 =	"	5. "
6d. = $\frac{1}{10}$ of 5s.	4,	11.	11.25 =	"	" 6d. "
	£142,	10	0¾.75 =		15. 6.

By Dollars and Cents.

183 yds. 3 qrs. 2 nls. at $3.10 per yd.
3.1

183
549

	$567.30=	cost of 183 yds. at $3.10 per yd.		
2 qrs.=½ of 1 yd.	1.55=	"	2qrs.	"
1 qr.=½ of 2 qrs.	0.775=	"	1 qr.	"
2 nls=½ of 1 qr.	.3875=	"	2 nls.	"
	$570.0125= £142, 10. 0¾.		183 yds. 3qrs. 2n.	

MISCELLANEOUS EXERCISES ON THE FOREGOING RULES.

1. 645 things at £0, 2. 6. each. Ans. £80, 12. 6.
2. 52 things at £0, 3. 9. each. Ans. £9, 15. 0.
3. 80 things at £0, 4. 4½ each. Ans. £17, 8. 4.
4. 454 things at $0.55 each. Ans. $249.70.
5. 51143 things at 19.55\frac{5}{12}$ each. Ans. 1000058.74\frac{7}{12}$.
6. 4013⅝ things at £2, 16. 6¼ each. Ans. £11342, 13. 5¼.

7. Find the value of 5 cwt. 2 qr. 14 lbs. at £2. 5. 6. per cwt. Ans. 12, 15. 11¼.

8. Find the value of 7 cwt. 1 qr. 15½ lbs. at 2£, 0. 7. per cwt. Ans. £14, 19, 10$\frac{33}{224}$.

9. Find the value of 9 yds. 2 ft. 10 in. at $1.12½ per yard. Ans. $11.18¾.

10. Find the value of 317 gal. 3 pts. at $2.10 per gal. Ans. $666.48¾.

11, What is the cost of 38 qr. 6 bus. 3 pk. at £1, 18. 10½ per quarter? Ans. £75, 10. 0½ + $\frac{7}{16}$.

First Principles.

189. Examples which are usually classed under particular Rules, such as the Rule of Proportion, &c., can nevertheless be readily solved independently by means of the foregoing principles which are therefore called First Principles.

To work a question by *first principles* is to employ only the elementary rules, Multiplication, Division, Reduction, &c., and thus show that no new rule is necessary to solve questions in Proportion, &c., whatever expediencies such rules may in some cases possess.

The following examples which are worked out, are intended to exemplify various methods of reasoning; the number of such examples must in this place be very limited, and therefore the student is strongly recommended to apply to all questions which are hereafter given under particular Rules, an independent method of solution, as well as the one denoted by the Rule to which they are respectively affixed.

Ex. 1. If 6 men can do a piece of work in 10 days, how many men would do the same in 15 days?

6 men can perform the whole work in 10 days,
∴ 60 men can do it in one day.

And 60 men performing it in 1 day, it will take $\frac{1}{15}$ as many to do it in 15 days.

$\therefore \frac{60}{15} = 4$, the number required.

Ex. 2. How many yards of cloth at 5s. 6d. per yard, should be given for 19 lbs of tea, 3s. 8d. per lb?

Change into the equivalent question: How many yards at 66d. should be given for 19 *yards* at 44d.?

44d. for each yard in 19 yards
amounts to 1d. for each yard in 19×44,
or to 66d. for each yard in $\frac{19\times44}{66}$ yds.
$= \frac{19\times2}{3} = 12\frac{2}{3}$ yards.

Ex. 3. Find the interest of £182, 10. for 20 days at 6 per cent.

£100 will yield for interest £6 in 365 days,
∴ £1 will give £$\frac{6}{36500}$ in 1 day
and in 20 days, 20 times as much.
∴ $\frac{6\times20}{36500} = £\frac{6}{1825}$ = Interest of £1 for 20 days,
and £182. 10. will give $182\frac{1}{2}$ times as much,
∴ £$\frac{6\times182\frac{1}{2}}{1825}$ = £0, 12. 0.

Ex. 4. After taking from my purse $\frac{1}{4}$ of my money, I find that $\frac{2}{3}$ of what is there left amounts to 7s. 6d.; what money have I in my purse at first?

Let Unity or 1, denote the sum in the purse at first. After taking away $\frac{1}{4}$, $\frac{3}{4}$ remains. Now by the question $\frac{2}{3}$ of $\frac{3}{4}$ of unity, or $\frac{2}{3}$ of $\frac{3}{4}$ of the sum in the purse at first = 7s. 6d.

or $\frac{1}{2}$ of the sum in the purse at first = 7s. 6d.
∴ sum in the purse at first = 15s.

Ex. 5. If a person, travelling $13\frac{3}{4}$ hours a day, perform a journey in $27\frac{5}{8}$ days, in what length of time will he perform the same if he travel $10\frac{2}{7}$ hours a day?

If he travel $13\frac{3}{4}$ hours a day, he does the journey in $27\frac{5}{8}$ days.

If he travel 1 hour a day, he does the journey in $(27\frac{5}{8}\times13\frac{3}{4}.)$

If he travel $10\frac{2}{7}$ hours a day, he does the journey in $\frac{(27\frac{5}{8}\times13\frac{3}{4}.)}{10\frac{2}{7}}$, which worked out gives $36\frac{2141}{2304}$ days.

Ex. 6. Gunpowder is composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts; find how much of each is required for 16 cwt.

The whole number of parts = (15+3+2, = 20 of every 20 parts.

$\frac{15}{20}$ or $\frac{3}{4}$ is nitre, $\frac{3}{20}$ is charcoal, $\frac{2}{20}$ or $\frac{1}{10}$ is sulphur.
$\therefore$ $\frac{3}{4}$ of 16 cwt. or 12 cwt. = quantity of nitre.
$\frac{3}{20}$ of 16 cwt. or $2\frac{2}{5}$ cwt. = quantity of charcoal.
$\frac{1}{10}$ of 16 cwt. or $1\frac{3}{5}$ cwt. = quantity of sulphur.

Ex. 7. A can do a piece of work in 5 days, B can do it in 6 days, and C can do it in 7 days; in what time will A, B, and C, all working together finish the work?

Representing the work by Unity, or 1.

In one day A does $\frac{1}{5}$ part of the work.
In one day B does $\frac{1}{6}$ part of the work.
In one day C does $\frac{1}{7}$ part of the work.

$\therefore$ In one day A, B, and C, do $(\frac{1}{5}+\frac{1}{6}+\frac{1}{7})$, or $\frac{107}{210}$ part;
$= 1 \div \frac{107}{210}$ days $= 1\frac{103}{107}$ days.

RATIO.

190. We may compare one number with another, or ascertain the relation which one bears to the other in respect to magnitude, in two different ways.

First. By considering how much one is greater or less than the other.

Second. By considering what multiple, part, or parts one is of the other, that is, how many times or parts of a time, or both, one number is contained in the other.

191. The relation of one number to another in respect of magnitude is called RATIO; and when the relation is considered in the first of the above methods, that is, when it is estimated by the difference between the two numbers, it is called ARITHMETICAL RATIO; but when it is considered according to the second method, it is called GEOMETRI-

CAL RATIO. Thus, for instance, the arithmetical ratio of the numbers 5 and 6 is one; while their geometrical is $\frac{5}{6}$.

192. As the term *Arithmetical Ratio* is merely a substitute for the word *difference*, the term difference in the succeeding pages, is used in its stead; and when the word *Ratio* is simply used, it signifies that which is denoted by the quotient of the one divided by the other, that is, by the fraction which the first number is of the second.

193. The ratio of one number to another is often denoted by placing a colon between them.

Thus the ratio of 7 to 13 is denoted by 7 : 13. As we have shown that the ratio of one number to another may be expressed by the fraction in which the former is the numerator and the latter the denominator, we see that 7 : 13 $= \frac{7}{13}$.

The two numbers which form a ratio are called its *terms;* the *first* number, or the number compared, being called the first term, or THE ANTECEDENT, and the *second* number or that with which the former is compared, the second term, or THE CONSEQUENT, of the ratio.

194. If the two numbers to be compared together be concrete, they must be of the *same kind.*

We cannot compare together 7 days and 13 miles in respect of magnitude; but we can compare 7 days and 13 days; and it is clear that 7 days will have the same relation to 13 days in respect of magnitude, which the number 7 has to the number 13, so that the ratio of 7 days to 13 days, will be the same as the ratio of the abstract number 7 to the abstract number 13, and may be expressed by the fraction $\frac{7}{13}$. If however the concrete, though of the same kind, be not in the same denomination of that kind, it will be convenient to reduce them to one and the same denomination, in order to find the ratio. Thus, if one of the numbers be 7 days and the other 13 hours, the ratio of the former to the latter will not be that of 7 to 13, but that of 7 *days* to 13 hours, that is 168 hours to 13 hours, which will clearly be the same as that of the abstract number 168 to the abstract number 13, and so will be expressed not by $\frac{7}{13}$, but by $\frac{168}{13}$. We see, then, that 7 days : 13 hours is the same as 168 : 13, and that each is $= \frac{168}{13}$. Thus it is plain that when the numbers are concrete, we may reduce them to one and the same denomination, and then in considering their ratio, treat them as abstract numbers.

195. A *Direct* ratio is that which arises from the dividing the antecedent by the consequent. (Art. 192.)

196. An *inverse* or *reciprocal* ratio, is the ratio of the *reciprocals* of two numbers (Art. 92, Def. 10). Thus the direct ratio of the numbers 9 and 3, is 9 to 3, or $\frac{9}{3}$: the reciprocal ratio is $\frac{1}{9} : \frac{1}{3}$ or $\frac{1}{9} : \frac{1}{3} = \frac{3}{9}$ (Art. 171), that is, the consequent 3, is divided by the antecedent 9.

An inverse or reciprocal ratio is expressed by inverting the fraction which expresses the direct ratio; or when the notation is by points, by inverting the order of the terms. Thus 8 is 4 invertedly, as 4 to 8.

197. A *simple* ratio is a ratio which has but *one antecedent* and *one consequent*, and may be either direct or inverse; as 9 : 3, or $\frac{1}{9} : \frac{1}{3}$.

198. *A compound* ratio is the ratio of the *Products* of the corresponding terms of two or more simple ratios. Thus,

The simple ratio of 9 ; 3 is 3
And simple ratio of 8 : 4 is 2

The ratio compounded of these is, 72 : 12 = 6.

From the principles of fractions already established, we may therefore deduce the following truths respecting ratios.

199. *To multiply or divide both the antecedent and consequent of a ratio by the same number does not alter the ratio:* for, multiplying or dividing both the numerator and denominator of a fraction by the same number does not alter its value. (Art. 159).

Thus the ratio of 12 : 4 is 3
The ratio of 12 × 2 : 4 × 2 is 3
And the ratio of 12 ÷ 2 : 4 ÷ 2 is 3

200. *If to or from the terms of a ratio, two other numbers having the same ratio be added or subtracted, the sums or remainders will also have the same ratio.* Thus the ratio of 12 : 3 is the same as that of 20 : 5. And the *ratio* of the sum of the *antecedents* 12 + 20 to the *sum* of the *consequents* 3 + 5, is the same. That is, 20 + 12 or 3 + 5, or $\frac{20+12}{3+5} = \frac{32}{8} = 4$.

So also the *ratio* of the *difference* of the *antecedents*, to the *difference of the consequents*, is the same.

That is 20 — 12 : 5 — 3 is the same as 12 : 3.

for $\frac{20-12}{5-3} = \frac{8}{2} = 4$.

PROPORTION.

201. Proportion is the equality of two ratios; so that when the ratio of one number to a second is equal to the ratio of a third number to a fourth, proportion is said to exist among the numbers, and the numbers are called PROPORTIONALS. Thus, the ratio of 8 to 9 is equal to that of 24 to 27, for the former ratio is $\frac{8}{9}$, and the latter $\frac{24}{27}$, which is also equal to $\frac{8}{9}$. The ratios being equal, proportion exists among the numbers 8, 9, 24, 27; and those numbers are proportionals.

NOTE.—The expression *equality of ratios* is not strictly accurate. The Ratio of 3 to 9, viz., 1-3, is not properly said to be equal to that of 5 to 15, viz., 1-3; the two are *identical*. It would be more accurate to say Proportion is the equivalence of two expressions denoting the same ratio.

202. When proportion exists among four numbers, that is, when the ratio of the first to the second is equal to that of the third to the fourth, this proportion or equality is often denoted by writing down the two ratios in the manner mentioned in (Art. 193) in one line, and placing a

double colon (: :) between them. Thus, the existence of proportion among the numbers 6, 8. 24, 32, is indicated as follows: 6 : 8 : : 24 : 32, which is commonly read, "Six are to eight as twenty-four is to thirty-two," or, "as six to eight, so is twenty-four to thirty-two."

203. In order to form a proportion four numbers are required. It may indeed happen that the second and third are the same, in which particular case it might be said that only three numbers are required. Thus, 9 : 6 : : 6 : 4; but even in such a case it is better to consider the second and third as distinct numbers, and to regard the proportion as consisting of four numbers, of which indeed two are equal. The four numbers required to form a proportion are called its *Terms*

204. The *first* and *last* terms are called the *extremes;* the other two, the *means*.

205. *Direct* proportion is an equality between two *direct* ratios. Thus, 12 : 4 : : 9 : 3 is a direct proportion.

206. *Inverse* or *reciprocal* proportion is an equality between a *direct* and a *reciprocal* ratio. Thus, 8 : 4 : : $\frac{1}{3}$: $\frac{1}{6}$; or 8 is to 4 reciprocally, as 3 is to 6.

207. If four numbers are proportional, the product of the extremes is equal to the product of the means.

208. It has been stated that proportion is the equality of two ratios, and we have explained that the two numbers constituting a ratio must either be both abstract, or (if concrete) both of the same kind. In a proportion, if one of the ratios be formed by two abstract numbers, the other may arise from two concrete numbers. For it has been explained (Art. 194) that if a ratio consist of two concrete numbers, we may reduce them both to the same denomination, and then treat the resulting numbers as abstract, the ratio of those abstract numbers being the same as that of the two concrete numbers from which they have arisen. For the same reason, one of two ratios constituting a proportion may be formed from concrete numbers of one kind, while the other is formed from concrete numbers of a dif-

ferent kind; for 7 days : 13 days : : 7 miles : 13 miles, each ratio being in fact that of 7 to 13. Indeed it appears by (Art. 194) that the ratio of two concrete numbers may always be expressed by a ratio of two abstract numbers. If both or either of the ratios in a proportion be formed from concrete numbers, we may thus replace each such ratio by one arising from abstract numbers, and in this way every term of the proportion will become an abstract number; so that, notwithstanding the remark in (Art. 28), any one of the terms may then be multiplied or divided by the other.

209. If only three of the numbers in a proportion be given, we can by means of them find the fourth, and the method or Rule by which it may be found is one of very great importance in Arithmetic. Almost all questions which arise in the common concerns of life, so far as they require calculation by numbers, might be brought within the scope of the Rule of Proportion, which enables us to find the fourth term in a proportion, and which, on account of its great use and extensive application, is often called the Golden Rule.

210. The Rule of Proportion, then, is a method by which we are enabled, from three numbers which are given, to find a fourth which shall bear the same ratio to the third as the second to the first, that is, shall be the same multiple, part or parts of the third, as the second is of the first; or, in other words, it is a Rule by which, when three terms of a proportion are given, we can find or determine the fourth

Proportion in Arithmetic is usually divided into Simple and Compound.

Simple Proportion.

211. Simple proportion is an equality of two *simple* ratios. It may be either *direct* or *inverse* (Arts. 197, 205, 206.)

Rule. Leaving out of consideration superfluous quantities, find, out of the three quantities given, that which is of the same kind as the fourth or required quantity; or that which is distinguished from the other terms by the nature of the question; place this quantity as the third term of the proportion.

Now consider whether from the nature of the question, the fourth term will be greater or less than the third; if it be greater, then put the larger of the other two quantities in the second term, and the smaller in the first term; but if less, put the smaller in the second term and the larger in the first term.

When necessary, reduce the first and second terms to the same name, and the third to the lowest name mentioned in it. Multiply the second and third terms together, and divide by the first, treating all three as abstract numbers. The quotient or answer will be of the same name as the third term, or of the name to which it shall have been reduced.

Ex. If 12 yards of cloth cost £15, what would 8 yds. cost at the same rate?

Proceeding by the Rule we observe that the £15 is of the same kind as the required term, viz., money; we make that the third term of the proportion; and since the required sum (cost of 8 yards) must necessarily be less than £15, (the cost of 12 yards,) we make 8 the second term, and 12 the first term.

And by the rule must now treat the numbers as abstract, multiply the second and third together, and divide by the first. Thus,

```
        yds.  yds.   £.
     As 12  :  8  :: 15
                      8
                    ---
                 12)120
                    ---
                    £10
```

The reason for the process may be shown as follows:

The cost of 12 yards is £15.

∴ the cost of 1 yard is $\pounds\frac{15}{12}$

∴ the cost of 8 yards is $\pounds\frac{15}{12}\times 8=\pounds 10$.

NOTE. The operation may also be explained in the following manner: since 12 yards cost £15, 8 times 12 yards, or 96 yards, would cost 8 times £15, or £120; then, dividing by 12, we find that a twelfth part of 96 yards would cost a twelfth part £120; that is, 8 yards would cost £10.

212. The process denoted by the foregoing Rule may often be much abbreviated by dividing the first and second, or the first and third terms, (but never the second and third,) by any number which will divide each of them without a remainder, and using the quotients instead of the numbers themselves. (Art. 87—90.)

Ex. If 9 cwt. of sugar cost £21, what would 12 cwt. cost at the same rate?

Stating the question according to the Rule given,

as ~~9~~ :	~~12~~ : :	£~~21~~
~~3~~	4	7
1		4
		£28. Answer.

Here, as 3 is a common factor of 9 and 21, we divide these numbers by it and set down the quotients 3 and 4. Again, as 3 is a common factor of the numbers 3 and 12, we divide these numbers by 3, and set down the quotients 1 and 4. Then multiplying 4 and 7 together we obtain the answer, viz., £28.

213. Operations in the rule of proportion may often be abbreviated by the method of aliquot parts, whether the first term is a unit or not.

Ex. If 2 cwt. 3 qr. 16 lbs. of sugar cost £6, how much can be purchased for £16. 11. 6.?

	£. £. s. d.	cwt. qrs. lbs.
	As 6 : 16, 11. 6 : :	2, 3, 16.
		4
		11, 2, 8
		4
		46, 1, 4,
10s. = ½ of 1£		1, 1, 22,
1s. = $\frac{1}{10}$ of 10s.		0, 0, 16.20
6d. = ½ of 1s.		0, 0, 8.10
		6)47, 3. 22.30
		7, 3. 27.05—or,
		7 cwt. 3 qr. 27$\frac{1}{20}$ lbs.

Note. The student should prove the following exercises by making the answer found one of the terms of a new question, as well as by other methods.

Exercises.

1. If 27 yards cost £11, how much will £33 buy?
   Ans. 81 yds.

2. If 156 yards cost $700, how many will purchase 39 yards? Ans. $175.

3. If $204 pay for 10 cwt. 2 qr. 14 lbs. of sugar, what would 4 cwt. 1 qr. 14 lbs. cost at the same rate?
   Ans. $84.

4. How many yards at 8s. 6d. per yard, must be given for 66 yds. at 5s. 9d. per yard? Ans. 44 yd. 2 qr. 2$\frac{6}{17}$.

5 If the rent of 5 acres be £4, 13s. 4d., what would be the rent of 75a. 2r. 10 p. at the same rate?
   Ans. £70, 10s. 6d.

6. A person travelling at the rate of 20 miles a day, performs a journey in 18 days; at what rate must he travel per day that he may return in 16 days? Ans. 22½.

7. Find the value of 23 yds. 1ft. of cloth, supposing 4 yds. 31 in. of the same quality to cost \$15. Ans. \$72.

8. If a property worth £185, 10s. pay a tax of \$21.64$\frac{1}{6}$, what ought a property worth 1000 guineas sterling, to pay? Ans. \$153.12$\frac{1}{2}$.

9. If 26 yds. of cloth cost 48s., what must it be sold at per foot, to gain 4s. on the purchase? Ans. 8d.

10. If the prime cost of 247 cwt. 3 qrs. 14 lbs. of sugar amount to £457, 14s. 4$\frac{1}{2}$d. and the charges, freight, duty, &c., to £6, 3s. 9d. what does it stand per cwt.?
Ans. \$7.48$\frac{3}{5}$.

11. If 3$\frac{5}{9}$ oz. avoir. cost \$1.40, what will 30$\frac{3}{7}$ lbs. cost?
Ans. \$191.70.

12. How many yards of drugget an ell wide will cover 40 yds. of carpet $\frac{3}{4}$ yd. wide? Ans. 24 yds.

13. A servant enters on a situation at 12 o'clock at noon on Jan. 1, 1863, at a yearly salary of 35 guineas sterling, he leaves at noon on the 27th of May following; how many dollars should he receive for his services?
Ans. \$73.50.

14. A person, after paying 7d. in the £ for poor's-rate on his income, has £1632, 18s. 10d. remaining; what had he at first? Ans. £1682.

15. The circumference of a circle is to its diameter as 3.1416 : 1; find (in feet and inches) the circumference of a circle whose diameter is 22.5 feet.
Ans. 70 ft. 8.232 in.

16. A watch is 10 minutes too fast at 12 o'clock (noon) on Monday and it gains 3′ 10″ a day; what will be the time by the watch at a quarter past 10 o'clock A. M. on the following Saturday? Ans. 10h. 40′ 36$\frac{7}{48}$″.

17. How many yards of carpet $\frac{3}{4}$ yd. wide will cover a room whose width is 16 feet, and length 27.5 feet?
Ans. 65$\frac{5}{27}$ yds.

18. A regiment of 1000 men are to have new coats; each coat is to contain 2$\frac{1}{2}$ yds of cloth 1$\frac{1}{4}$ yd. wide; and it is to be lined with shalloon of $\frac{3}{4}$ yd. wide; how many yards of shalloon will be required? Ans. 4166$\frac{2}{3}$.

19. If two numbers are as 8 to 12, and the less is 320, what is the greater? Ans. 480.

20. If an ounce of gold be worth £4.189583, what is the value of .36822916 lbs.? Ans. £18, 10. 3. nearly.

21. If 1000 men have provisions for 85 days, and if, after 17 days 150 of the men go away; find how long the remaining provisions will serve the number left.

Ans. 80 days.

22. If 4 horses and 6 cows together find sufficient grass on a certain field; and 7 cows eat as much as 9 horses; what must be the size of a field relatively to the former, which will support 18 horses and 9 cows? Ans. 207 : 82.

23. What must be the breadth of a piece of ground whose length is $40\frac{1}{2}$ yds, in order that it may be twice as great as another piece of ground whose length is $14\frac{5}{8}$ yds., and whose breadth is $13\frac{2}{13}$ yds.? Ans. $9\frac{1}{2}$ yards.

Compound Proportion.

214. There are many questions which are of the same nature with those belonging to simple proportion, but which, if worked out by means of that Rule as before given, would require two or more distinct applications of it. Every such question, in fact, may be considered to contain two or more distinct questions belonging to the Rule of simple proportion, and when each of these questions have been worked out by means of the Rule, the answer obtained for the last of them will be the answer to the original question.

215. The following example may serve to illustrate the preceding remarks. If the carriage of 15 cwt. for 17 miles cost $17, what would the carriage of 21 cwt. for 16 miles cost at the same rate?

This question, though of a like nature with those which engaged our attention under the last Rule, is nevertheless of a more complicated description; for we observe that instead of three quantities here are five, every one of which must necessarily have a bearing on the answer, so that none of them can be superfluous.

If, however, the question be divided into two distinct questions, each of these, when the superfluous terms are rejected, will be found to comprise only three given terms of a proportion, from which three terms the fourth is to be ascertained: so that in this way we obtain the correct answer by applying the Rule of proportion twice over.

The first question may be this: If the carriage of 15 cwt. for 17 miles cost $17, what would the carriage of 21 cwt. cost? In this question 17 miles would have no effect upon the answer, because any other number of miles, or the words, a certain distance, might have been used instead of 17 miles. This number may therefore be neglected as superfluous. Solving the question by the Rule of simple proportion, we find the answer will be $23.80.

The second question may be this: If the carriage of 21 cwt. for 17 miles cost $23.80, what will the carriage of 21 cwt. for 16 miles cost?

In this question, for reasons similar to those given above, the 21 will be a superfluous quantity. Applying the Rule of simple proportion to the question we find the answer to be $22.40.

From the connection of these two questions with that originally proposed we observe that the answer thus obtained through the means of two distinct applications of the Rule of simple proportion, must be the answer to the original question.

216. Compound Proportion is a shorter or more compendious method of working such questions as would require two or more applications of the Rule of Simple Proportion.

217. For the sake of convenience, we may divide each question into two parts, the *supposition* and *demand;* the former being the part which expresses the conditions of the question, and the latter the part which mentions the thing demanded or sought. In the above question, the words, "*if the carriage of* 15 *cwt., for* 17 *miles cost* $17, form the supposition; and the words, *what would the carriage of* 21 *cwt., for* 16 *miles cost at the* same rate? form the demand. Adopting this distinction, the following rule

will be found applicable for working out questions in Compound Proportion.

218. Rule.—Take from the supposition that quantity which corresponds with the quantity sought in the demand; and write it down as the third term. Then take one of the other quantities in the supposition and the corresponding quantity in the demand, and consider them with reference to the third term *only* (regarding each other quantity in the supposition and its corresponding quantity in the demand as being equal to each other); when the two quantities are so considered, if from the nature of the case, the fourth term would be greater than the third, then, as in the Rule of Simple Proportion, put the larger of the two quantities in the second term, and the smaller in the first term; but if less, put the smaller in the second term, and the larger in the first term.

Again, take another of the quantities given in the supposition. and the corresponding quantity in the demand; and retaining the same third term, proceed in the same way to make one of those quantities a first term and the other a second term.

If there be other quantities in the supposition and demand, proceed in like manner with them.

In each of these statings reduce the first and second terms to the same denomination. Let the common third term be also reduced to a single denomination if it be not already in that state. The terms may then be treated as abstract numbers.

Multiply all the first terms together for a final first term, and all the second terms for a final second term, and retain the former third term. In this final stating multiply the second and third terms together and divide by the first. The quotient will be the answer to the question in the denomination to which the third term was reduced.

219. As a contraction in the use of this rule, divide an antecedent, and either the last term or any consequent, by any number that will divide them without remainders, and employ the quotients instead of those terms; or if an antecedent and any consequent, or an antecedent and the last term be the same, reject them. (Art. 87—90.)

Ex. If 7 horses be kept 20 days for \$56, how many will be kept 7 days for \$112?

The 7 horses in the supposition correspond to the required quantity (number of horses) in the demand. Make this the third term. Then taking 20 days in the supposition, and the 7 days in the demand, and considering them with reference to our third term, we observe that if the number of days be diminished, the number of horses which can be kept in them for a given sum of money will be increased, and thus a fourth term will be greater than the third; we therefore place the 7 days in the first term, and the 20 days in the second. Again, taking the \$56 in the supposition, and the \$112 in the demand, and considering them with reference to the third term, we observe that if the sum be increased the number of horses which can be kept by it in a given time will be increased; so that here also a fourth term will be greater than the third; we therefore place the \$56 in the first term and the \$112 in the second term. We thus obtain the following statement:—

7 horses 20 days, \$56	}	Supposition.
? 7 days, \$112.	}	Demand.

As 7 days : 20 days } :: 7 horses,
\$56 : \$112 }

$$56 \times 7 \quad 20 \times 112 \ :: \ 7$$
$$= 392 : 2360 :: 7$$
$$= 2240 \times 7 \div 392 = 40 \text{ horses.}$$

By Art. 219.

5
As ~~7~~ : ~~20~~ } :: ~~7~~
~~14~~ ~~56~~ : ~~112~~ }

8 × 5 = 40 horses, as before.

Here, the 7 in first term cancels the 7 in the third term. Again dividing 56 in the first term and 20 in the second term by 4, we set down the quotients 14 and 5. For similar reasons we omit 14 and write 8 instead of 112. We then multiply 5 and 8 together and find the answer as before.

The student should work each of the following questions by simple statements, and thus verify the results.

Exercises.

1. If 7 men can reap 6 acres in 12 hours, how many men will reap 15 acres in 14 hours? Ans. 15 men.

2. If a family of 8 persons expend £200 in 9 months; how much will serve a family of 18 persons for 12 mos.? Ans. £600.

3. If 120 bushels of oats serve 14 horses 56 days; how many days will 94 bushels serve 6 horses? Ans. $102\frac{16}{45}$ days.

4. If 180 men, in 6 days, of 10 hours each, can dig a trench 200 yards long, 3 wide, and 2 deep; in how many days of 8 hours long, will 100 men dig a trench of 360 yards long, 4 wide, and 3 deep? Ans. $48\frac{3}{5}$ days.

5. If the rent of a farm of 17 ac. 3 ro. 2 po. be £39, 4. 7., what would be the rent of another farm, containing 26 ac. 2 ro. 23 po., if 6 acres of the former be worth 7 acres of the latter? Ans. $201.75.

6. If 1500 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of 25 sheets, of the same size as the former? Ans. 500 reams.

7. A pit 24 feet deep, 14 sq. feet, horizontal section, cost $12 to dig out; how deep will a pit be of horizontal section, 7 ft. by 9 ft., which cost $18? Ans. 8 ft.

8. If 1 man and 2 women do a piece of work in 10 days, find in how long a time 2 men and 1 woman will do a piece of work 4 times as great, the rates of working of a man and woman being as 3 to 2. Ans. 35 days.

INTEREST.

220. Some information regarding Interest has been already given in (Art. 157.) It is repeated here, however in substance, to render the present article complete in itself.

INTEREST is the sum of money paid for the loan or use of some other sum of money, lent for a certain time at a fixed rate; generally at so much for each £100 or $100 for one year.

The money lent is called the PRINCIPAL.

The interest of £100 or $100 for a year is called THE RATE PER CENT.

The Principal + the interest is called THE AMOUNT.

Interest is divided into Simple and Compound. When interest is reckoned only on the original principal, it is called SIMPLE INTEREST.

When the interest at the end of the first period, instead of being paid by the borrower, is retained by him and added on as principal to the former principal, interest being calculated on the new principal for the next period, and this again, instead of being paid, is retained and added on to the last principal for a new principal, and so on; it is called COMPOUND INTEREST.

NOTE.—The rate of interest has varied much at different periods, and in different countries, but it has been generally observed to diminish as commerce extends. In Italy, about the beginning of the thirteenth century, it varied from 20 to 30 per cent. per annum; and in the Netherlands, it was fixed by Charles V. in 1560, at 12 per cent. By an act of the 37th year of Henry VIII., interest in England was not to exceed 10 per cent. By the 21st of James I., it was reduced to 8 per cent. Soon after the Restoration, it was reduced still farther, to 6 per cent.; and in the 12th of Anne, to 5 per cent., the present rate. The legal rate of interest in Nova Scotia is at present 6 per cent.

SIMPLE INTEREST.

221. In interest five quantities are concerned, the principal, the rate, the time, the interest, and the amount, and any three of these except the principal, interest and amount being given the rest may be found. Hence com-

putations in interest admit of several problems all of which depend upon either Simple or Compound Proportion or both; for the interest of any sum for any time is directly proportional to the principal sum, and also to the time of continuance, and conversely.

(1.) *To find the interest of a given sum for a year, at a given rate per cent. per annum.*

RULE.—Multiply the principal by the rate, and divide the product by 100. (Art. 133.) or, As 100 : rate : : principal : Its interest for one year.

Ex. 1. What is the interest of £168, 16. 3. for 1 year at 6 per cent.?

£. s. D.	Dollars.
168, 16. 3.	675.25
6	6.
10,12, 17. 6.	40,51.50.
20	
2,57	$40.51½.
12	
6,90	
4	
3,60	
£10, 2. 6¾.	

The *reason of the above operation* is quite evident, as it is nothing more than this: As the principal £100 or $100 is to its interest £6, or $6, so is the principal £168, 16. 3. or $675.25 to its interest; and it is evident that as often as the one principal contains its interest, so often will the other contain its interest: that is, by the nature of proportion, the interest will be proportional to the principal.

Ex. 2. What is the interest of £127, 13. 4½. for 1 year at 5⅞ per cent.?

	£. s. d.		Dollars.
	127, 13. 4½		510.675
	5⅞		5⅞
	638, 6. 10½		2553.375
$\frac{4}{8} = \frac{1}{2}$ of 1 unit	63, 16. 8¼	$\frac{4}{8} = \frac{1}{2}$ of 1	255.337
$\frac{2}{8} = \frac{1}{2}$ of $\frac{4}{8}$	31, 18. 4	$\frac{2}{8} = \frac{1}{2}$ of $\frac{4}{8}$	127.668
$\frac{1}{8} = \frac{1}{2}$ of $\frac{2}{8}$	15, 19. 2	$\frac{1}{8} = \frac{1}{2}$ of $\frac{2}{8}$	63.884
	7,50, 1. 0¾.		30,00.264
	20		
	10,01		$30.00.
	£7, 10. 0.		

NOTE.—The remainders after pence may be rejected as they do not effect the result. The same may be observed with regard to the decimal form.

For mental exercises to this and succeeding Rules see (Art. 157).

EXERCISES.

Find the interest of the following sums, for 1 year at the given rates per cent. per annum.

1. £376, 12. 8. at 4 per cent. Ans. £15, 1. 3¾.
2. £774, 13. 3. at 5 per cent. Ans. £38, 14. 7¾.
3. £637, 11. 0. at 5⅚ per cent. Ans. £37, 3. 9¾.
4. $2687.90 at 4¼ per cent. Ans. $114.23⅕.
5. $69.40 at 3½ per cent. Ans. $2.42 11/12.
6. $189.63½ at 6¾ per cent. Ans. $12.80.

(2.) *To find the interest of a given principal for years and months.*

RULE.—Find the interest for 1 year by the last Rule, multiply the number of years and take parts of a year for the months. Or

Multiply the principal by the rate; then by the number of years, taking aliquot parts for the months; and, last of all, dividing by 100.

Ex. Required the interest of £99, 2. $4\frac{1}{2}$. for 2 years, 9 months, at 4 per cent. per annum.

FIRST METHOD.

	£99, 2. $4\frac{1}{2}$.	
	4	
	3,96, 9. 6	
	20	
	19,29	
	12	
	3,54	
	£3, 19. 3.54	= Ins. for 1 year
	2	at 4 per cent.
	7, 18. 7.08	= Int. for 2 years.
6 mos. = $\frac{1}{2}$ of 1 year	1, 19. 7.77	= Int. for 6 months.
3 mos. = $\frac{1}{2}$ of 6 mos.	0, 19. 9.88	= Int. for 3 months.
	£10, 18. 0.73	= Int. for 2 y. 9 mo.

SECOND METHOD.

	£99, 2. $4\frac{1}{2}$.
	4
	396, 9. 6.
	2
	792, 19. 0.
6 = $\frac{1}{2}$	198, 4. 9.
3 = $\frac{1}{2}$	99, 2. $4\frac{1}{2}$.
	10,90, 6. $1\frac{1}{2}$.
	20
	18,06
	12
	0,73
	£10, 18. 0.73.

Or thus

£99, 2. 4½. for 2 years and 9 months at 4 per cent. is equivalent to £99. 2. 4½. for 1 year at 11 per cent. Therefore

```
£99. 2. 4½.
        11
-----------
10,90, 6. 1½.
        20
-----------
   18,06
      12
-----------
    0,73
```

£10, 18. 0.73 as before.

The reason for the above methods is evident from (Art. 213).

Exercises.

Find the interests of the following principals, for the given times, and at the given rates per cent., per annum.

1. £355, 15. 0. for 4 y., at 4 per cent. Ans. £56, 18. 4¾.
2. £319, 0. 6. for 5¾ y., at 3¾ per cent. Ans. £68, 15. 9½.
3. £712, 6. 0. for 8 mo., at 7½ per cent. Ans. £35, 12. 3½.
4. £25, 0. 0. for 1 y. 9 mo., at 5 per cent. Ans. £2, 3. 9.
5. $738.00 for 1 y. 2 mo., at 7 per cent. Ans. $60.27.
6. $894.00 for 1 y. 8 mo., at 6 per cent. Ans. $89.40.
7. $65256 for 4 mo., at 7 per cent. Ans. $1522.64.

(3.) *To find the interest of a given principal for years, months and days.*

Rule.—Find the interest for the years and months by the last Rule, then take such a fractional part of one month's interest, as is denoted by the given number of days.

Note 1.—In calculating interest, a month, whether it contains 30 or 31 days, or even but 28 or 29, as in the case of February, is assumed to be *one-twelfth* of a year. Again 30 days are commonly considered a month; consequently the interest for 1 day, or any number of days under 30, is so many *thirtieths* of a month's interest.

This practice seems to have been originally adopted on account of its convenience. Though not strictly accurate, it is sanctioned by general usage.

Allowing 30 days to a month, and 12 months to a year, a year would contain only 360 days, which in point of fact $\frac{5}{365}$, or $\frac{1}{73}$ less than an ordinary year.

Hence to find the interest for any number of days with *entire accuracy*, by means of aliquot parts, we must take so many 365ths of 1 year's interest, as is denoted by the given number of days; or find the interest for the days as above, from this subtract $\frac{1}{73}$ of itself and the remainder will be the exact interest. (See Note 2, Art. 157, Rule 6.)

NOTE 2.—If the interest has to be calculated from one given day to another, as for instance from the 30th of January to the 7th of February, the 30th of January must be left out in the calculation, and the 7th of February must be taken into account, for the borrower will not have had the use of the money for one day till the 31st of January.

Ex. Required the interest of $148 for 8 mo. 12 d., at 6 per cent. per annum.

	$148		£37
	6 per ct.		6
	888		222
6 mo. = $\frac{1}{2}$ of 1 y.	444.00	6 mo. = $\frac{1}{2}$ of 1 y.	111
2 mo. = $\frac{1}{3}$ of 6 mo.	148.00	2 mo. = $\frac{1}{3}$ of 6 mo.	37, 0. 0.
10 d. = $\frac{1}{6}$ of 2 mo.	24.666	10 d. = $\frac{1}{6}$ of 2 mo.	6, 3. 4.
2 d = $\frac{1}{5}$ of 10 d.	4.933	2 d. = $\frac{1}{5}$ of 10 d.	1, 4. 8.
	621.599		1,55, 8. 0.
			20
	6.21\frac{6}{10}$.		11,08
			12
			0,96
			£1, 11. 1. nearly.

Find the interest of the following sums for the given time, and at the given rates per cent., per annum.

1. £127, 13, 4. for 6 mo. 10 d., at 4 per cent.
Ans. £2, 13. 10¾.

2. £362, 11. 0. for 9 mo. 20 d., at 4¾ per cent.
Ans. 13, 17. 5½.

3. $287.50 for 1 y. 3 mo. 9 d., at 6 per cent.
Ans. $22.00 nearly.

4. $189 for 11 mo. 20 d., at 5¼ per cent.
Ans. $9.65.

5. $1763.16½ for 8 mo. 15 d., at 4⅛ per cent.
Ans. $51.51 $\frac{7}{10}$.

(4.) *To find the interest accurately of any sum for any time, and at any rate per cent., per annum.*

Rule.—As £100 or $100 is to the rate per cent. and as 1 year in the same name as the given time is to the given time, so is the principal to the interest on that principal for the given time.

Ex. Required the interest of £456, 10. 0. for 31 days at 5 per cent.

```
       20                      £   s
    As 100 : 5  }          : : 456 10.
       365 : 31 }               31
    ───────────                ────
      7300 : 31                 456
                               1368
10s. = ½ of £1 ∴ ½ of 31 as £. 's.  15.10
                               ────────  Interest.
                        7300)14151.10(£1, 18. 9¼.
                              7300
                              ────
                              6851
                                20
                             ──────
                             137030
                              7300
                             ──────
                              64030
                              58400
                             ──────
                               5630
                                 12
                             ──────
                              67560
                              65700
                             ──────
                               1860
                                  4
                              ─────
                               7440
                               7300
                              ─────
                                140
```

In the above example, if we had adhered to the Rule given for Compound Proportion, we should have placed the principal in the second term instead of the rate, but the present mode which, for brevity, is that generally adopted in calculating interest, as it saves the trouble of reducing the £100 to the lowest denomination mentioned in the principal.

If the principal consists of Dollars and Cents the Rule in Compound Proportion will be equally short.

EXERCISES.

Find the interest of	Answers.
1. £690, 10. 6. for 85 days, at 4 per cent.	£6, 8. $7\frac{1}{2}$.
2. £573, 8. 3. for 73 days, at $3\frac{1}{2}$ per cent.	£4, 0. $3\frac{1}{4}$.
3. £684, 7. 6. for 56 days, at $3\frac{1}{2}$ per cent.	£3, 13. 6.
4. \$1719.46$\frac{2}{3}$ for 86 days, at 4 per cent.	\$16.20$\frac{1}{2}$.
5. \$3311.50 for 292 days, at $2\frac{1}{2}$ per cent.	\$66.23.

Calculate the amount of £643, 16. 6.

6. From Feb. 20 till July 18th, at $4\frac{1}{2}$ per cent. Ans. £655, 11. $5\frac{1}{4}$.

7. From May 26th till Nov. 26th, at $3\frac{1}{2}$ per cent. Ans. £655, 3. $8\frac{1}{4}$.

8. From June 3d till May 3d, at 5 per cent. Ans. £673, 5. $7\frac{1}{2}$.

9. Required the interest of \$63 from March 17th, 1840, till January 26th, 1842, at 6 per cent. per annum. Ans. \$7.04$\frac{1}{6}$.

10. What is the interest of \$213.33$\frac{1}{3}$ from June 14, 1841, till Sept. 22, 1843, at $4\frac{1}{2}$ per cent.? Ans. \$21.82$\frac{1}{12}$.

11. What is the interest of £52, 10. for 1 year and 2 months, at 6 per cent., per annum. Ans. £3, 13. 6.

(5.) *To find the interest of a given sum for any number of days.*

RULE.—Multiply the principal by twice the rate, and the product by the days, and divide the result by 73,000.

The division by 73000 may be performed by the following Rule: Below the dividend write one-third of itself, one-tenth of that third, and one-tenth of that tenth, rejecting shillings and remainders; then add the four lines together, divide the sum by 100.000 (or cut off five figures toward the right), and reject a farthing for every £10 or a mill for every \$10 in the result.

The tenths will be obtained by setting the figures one place to the right hand, and rejecting the last of them, or the remainders may be treated as in dollars and cents.

Ex. Find the interest of £372, 10. 10. for 309 days, at $4\frac{1}{2}$ per cent. per annum.

£372, 10. 10.
9

£3352, 17. 6. or
£3353, 0. 0. nearly.
309

30177
10059

1036077
$\frac{1}{3}$ 345359
$\frac{1}{10}$ 34536
$\frac{1}{10}$ 3453

14,19425
20

3,88500
12

10,62000
4

2,48000
£14, 3. $10\frac{1}{2}$.
Correction $\frac{1}{4}$

£14, 3. $10\frac{1}{4}$. Ans.

The reason for the above process will be evident from the operation by Compound Proportion.

If, instead of £100 and the rate per cent., their doubles be employed (Art. 199). Thus we have in this exercise

```
As £200     :  £9.      }
   365 days :  309 days, }  £372, 10. 10.

      | 73000      309 × 9 × 372, 10. 10.
  1/3 | 24333'                        9
 1/10 |  2433'              ------------
 1/10 |   243'              £3352. 17. 6. or
      ------
   100010 or                £3353, 0. 0.
                                 309
   100000                  | 1036077
                       1/3 |  345359
                      1/10 |   34536
                      1/10 |    3453
                           ---------
                           14)19425 or
                          £14, 3. 10½.
```

Now as 73000 becomes the division and the continued product of the time, double the rate and principal, viz., £1036077 becomes the dividend, it is evident, from (Art. 79) that if we increase the dividend by $\frac{1}{3}$ of itself we must increase the divisor also by $\frac{1}{3}$ of itself; again when we increase the dividend by $\frac{1}{10}$ of this third and also $\frac{1}{10}$ of that tenth, we must of necessity increase the divisor in the same proportion.

We have now 100010 for the divisor and 1419425 for the dividend, rejecting the 10 and dividing by 100,000 or cutting off 5 figures, &c., we obtain £14, 3. 10½. for the quotient. Now we divide by 100,000 instead of 100010, therefore the quotient will be too great by the same fractional part of itself as the 10 is of 100010 (Art. 77). Hence the correction.

(6.) *To find the interest of a given principal for any number of days at 4 per cent. per annum.*

Rule.—Multiply the principal by the days: to the product add one-tenth of itself: from the sum take four times the same product wanting the last three figures: divide what remains by 10,000 (or cut off 4 figures); the quotient will be the answer nearly. When the interest is large reject a farthing for each £10 or 1 mill for each $10 contained in it.

For other rates than 4 per cent. increase or diminish the product of the principal and days, by the method of aliquot parts, and then proceed by the rule.

Ex. Required the interest of $35942.80 for 12 days, at 4 per cent., per annum.

Here the product of the principal and days is 431313.69 and the tenth of this (found by setting each figure one place nearer the right hand side) being added to it the sum is 474444.96. After this we multiply 43131 by 4 and increase the product by 1 (carried for 4 times 3, the first of the figures cut off). The result 1724.25, is then subtracted, and the remainder 472720.71, divided by 10,000 in the way pointed out in the last example. The quotient is $47.272 from which subtract 1 mill for each $10, viz., 4, the remainder $47.26¾ is the interest required.

```
$35942.80
       12
---------
431313.60
 43131.36
---------
474444.96
  1724.25
---------
47,2720.71

$47.272
      4 Correction.
-------
$47.268.
```

The reasons for the above process are similar to those given for Rule 5. It may also be reasoned otherwise.

The exercises under Rule 4 may be used for Rules 5 and 6.

The following rules serve for the resolution of the remaining cases of interest. As these cases are of minor importance the rules are given without illustration. They are easily proved by the principles of proportion.

(7.) *To find what principal, in a given time, would produce a given interest, at a given rate per cent. per annum.*

RULE.

As the rate : £100 or $ } :: the int. : the principal.
The given time : 1 y. }

Exer. 1. What sum will produce for interest £56, 14. in $2\frac{1}{4}$ years at $4\frac{1}{2}$ per cent. Ans. £560, 0. 0.

2. What principal at 5 per cent. per annum, will bring a yearly income of $1365? Ans. $27300.09.

(8.) *To find the time in which, at a given rate per cent. per annum, a given principal would produce a given interest.*

RULE.

As the principal : £100 or $100 } :: 1 year : time reqd.
The rate : the interest }

Exer. 1. In what time will £560 amount to £616, 14. at $4\frac{1}{2}$ per cent. per annum? Ans. $2\frac{1}{4}$ years.

2. How long must $8000 be lent at simple interest, at $3\frac{1}{2}$ per cent. per annum, to amount to $9120?
Ans. 4 years.

(9.) *To find at what rate per cent. a given principal would gain a given interest in a given time.*

RULE.

As the given time : 1 year } :: interest : rate.
the principal : £100 or $ }

Exer. 1. At what rate will £157, 15. 4. amount to £295, 16. 3. in 25 years at simple interest?
Ans. $3\frac{1}{2}$ per cent.

2. If $1 amount to $1.13 cents $7\frac{1}{2}$ mills in $3\frac{1}{4}$ years, at simple interest, at what rate per cent. per annum, must it have been lent? Ans. 4.2307 per cent.

(10.) *To find what principal, in a given time would increase to a given amount, at a given rate per cent. per annum.*

RULE.—To the product of the time and rate, add the product of £100 or $100 and 1 year, in the same name as the given time. Then as the sum is to the above mentioned product of £100 or $100 and 1 year, so is the amount to the principal.

Exer. 1. What sum of money will amount to £256, 10. in 4 years, at $3\frac{1}{2}$ per cent., simple interest?

Ans. £225, 0. 0.

2. What sum must be lent, at simple interest, at 4 per cent., per annum, that the amount, at the end of 2 years 10 months, may be $2511.70? Ans. 2256.01\frac{3}{5}$.

COMMISSION, INSURANCE, BROKERAGE, &c.

222. Commission is *the per cent.* or *sum charged* by agents for their services in buying and selling goods, or transacting other business.

Note.—An agent who buys and sells goods for another is called a *Commission Merchant, a Factor, or Correspondent.*

Brokerage is of the same nature as Commission, but has relation to money transactions, rather than dealings in goods or merchandise.

Insurance is a contract, by which one party on being paid a certain sum or *Premium* by another party on property which is subject to risk, undertakes, in case of loss, to make good to the owner the value of that property.

Note.—The written *instrument* or *contract* is called the *Policy.* When duty is charged on the Policy it is calculated on even hundreds. Thus $650 would be $700.

(1.) *To Compute the Commission, Brokerage, Insurance, or any other allowance on a given sum, at a given rate per cent.*

Rule.—Multiply the sum by the rate per cent., and divide the product by 100; or as £100 or dollars are to the rate per cent., so is the given sum to the required allowance.

Ex. Required the premium of insurance on £512, 9. 4. at £6, 16. 6. per cent.

	£512, 9. 4.		$2049.866′
	6		6.82\frac{1}{2}$
	£3074, 16. 0.		12299.199
12s. 0. = $\frac{1}{10}$ of £6	307, 9. 7$\frac{1}{4}$.	50ct.=$\frac{1}{2}$	1024.933′
4s. 0. = $\frac{1}{3}$ of 12s.	102, 9. 10$\frac{1}{2}$.	20ct.=$\frac{1}{5}$	409.973′
0s. 6. = $\frac{1}{8}$ of 4s.	12, 16. 2$\frac{3}{4}$.	12$\frac{1}{2}$ct.=$\frac{1}{8}$	256 233′
	100) 3497, 11. 8$\frac{1}{2}$.		139,90.339
Ans.	£34, 19. 6$\frac{1}{4}$.	Ans.	$139,90$\frac{1}{4}$.

(2.) *To find how much must be insured on property worth a given sum, so that, in case of loss, both the value of the property and the premium may be repaid.*

Rule.—Subtract the rate from £100 or Dollars. As the remainder is to £100 or dollars, so is the value of the property to the sum to be insured.

Ex. How much must be insured at 8$\frac{1}{2}$ per cent. on goods worth £600, that in case of loss not only the value of the goods, but also the premium of insurance, may be paid?

Here, as £100 — 8$\frac{1}{2}$, or £91$\frac{1}{2}$: £100 : : £600 : : £655. 14.9. The accuracy of this operation is proved by finding the premium on £655.14.9 at 8$\frac{1}{2}$ per cent. This is found to be £55.14.9. Hence, in case of the property being lost, the owner will receive, not only £600, the value of the goods, but also £55.14.9 the premium; and he will therefore sustain no loss whatever.

The reason will appear manifest from considering that, in receiving £100, which has been insured at 8$\frac{1}{2}$ per cent, the owner would receive but £91$\frac{1}{2}$ in lieu of the goods, 8$\frac{1}{2}$ having been paid for the insurance.

Exercises.

1. What is the commission on £942, 16. 3. at 4$\frac{1}{2}$ per cent.? Ans. £42, 8. 6$\frac{1}{4}$.

2. Required the brokerage on £946, 18. 10. at 5s. 6d. per cent. Ans. £2, 12. 1.

3. Find the commission on \$2278.95 at $7\frac{1}{2}$ per cent. Ans. \$$170.92\frac{1}{2}$.

4. What sum must be insured at £1, 10. per cent. on goods worth £1200, so that in case of loss, both the value of the goods and the premium may be repaid? Ans. £1218, 5. 6., nearly.

5. At \$$2.27\frac{1}{2}$ per cent., what will be the cost of insuring goods worth \$6240, so that in case of loss, the owner may be entitled to the value of the goods, and the premium? Ans. \$$145.26\frac{1}{4}$.

Application of Interest.

223. In the application of interest to business transactions the following particulars deserve attention.

(1.) A *promissory* note is a writing which contains a promise of the payment of money or other property to another, at or before a time specified in consideration of value received by the promiser or *maker* of the note.

The words "value received" were at one time supposed to be an essential part of a bill or note, but as a valuable consideration is always presumed until the contrary is proved, they are not at all necessary, though usually inserted.

(2.) The person who signs a note is called the *maker*, *drawer*, or *giver* of the note. The person to whom a note is made payable, is called the *payee;* the person who has legal possession of a note is called the *holder* of it.

(3.) A note which is made payable "to order" "or bearer," is said to be negotiable; that is, the holder may *sell* or *transfer* it to whom he pleases, and it can be collected by any one who has lawful possession of it. Notes without these words are not negotiable. (See Nos. 1 and 2.)

(4.) If the holder of a negotiable note which is made payable *to order* wishes to sell or transfer it, the law requires him to *endorse* it or write his name on the back of it. The person to whom it is transferred, or the holder of it, is then empowered to collect it of the drawer; if the drawer is *unable* or *refuses* to pay it, then the endorser is responsible for its payment. (See No. 1.)

(5.) When a note is made payable to the *bearer*, the holder can sell or transfer it without endorsing it, or incurring the liability for its payment. Bank notes are of this description. (See No 2.)

(6.) When a note is made payable to any particular person without the words *order* or *bearer*, it is *not negotiable;* for it cannot be collected or sued except in the name of the person to whom it is made payable. (See No. 3.)

(7.) A note should always specify the time at which it is to be paid; but if no time is mentioned, the presumption is that it is intended to be paid on *demand*, and the giver must pay it when demanded.

(8.) According to custom, a note or draft is not presented for collection until *three* days after the time specified for its payment. These three days are called *days of grace*. Interest is therefore reckoned for *three* days more than the time specified in the note. When the last day of grace comes on Sunday, or a national holiday, it is customary to pay a note on the day previous.

(9.) If a note is not paid at *maturity* or the *time specified*, it is necessary for the holder to *notify the endorser* of the fact in a legal manner, as soon as circumstances will admit; otherwise the responsibility of the endorser ceases.

(10.) Notes do not draw interest unless they contain the words "with interest." But if a note is not paid when it becomes due, it then draws legal interest till paid, though no mention is made of interest.

(11.) Notes which contain the words "*with interest*," though the *rate* is not mentioned, are entitled to the legal rate established by the Kingdom, State or Province in which the note is made.

In writing notes, therefore, it is unnecessary to specify the rate unless by agreement it is to be less than the legal rate.

(12.) When *two or more* persons jointly and severally give their note, it may be collected of either of them. (See No. 4.)

(13.) The *sum* for which a note is given, is called the *principal*, or *face of the note*, and should always be written out in words.

(*No.* 1.)

$450.

HALIFAX, April 16th, 1863.

Sixty days after date, I promise to pay George Kinman, or order, Four Hundred and Fifty Dollars, with interest, value received.

JOHN JOHNSON.

(*No.* 2.)

$630.

BOSTON, May 23d, 1863.

Thirty days after date, for value received, I promise to pay John Holmes, or bearer, Six Hundred and Thirty Dollars with interest.

JAMES GOODYEAR.

(*No.* 3.)

$850.

WINDSOR, March 16th, 1863.

Four months after date, I promise to pay Horace Williams, Eight Hundred and Fifty Dollars, with interest, value received.

SANDFORD ATWATER.

(*No.* 4.)

$1000.

TRURO, April 14th, 1863.

For value received, we jointly and severally promise to pay to the order of John K. Blair, One Thousand Dollars, in one year from date, with interest.

ROBERT L. ARCHIBALD,
WILLIAM SIMMONDS.

DISCOUNT.

224. DISCOUNT is the *abatement* or *deduction* made for the payment of money before it is due.

For example, if I owe a man $100, payable in one year without interest, the *present worth* of the note is less than $100; for if $100 were put at interest for 1 year at 6 per cent., it would amount to $106; at 7 per cent., to $107, &c. In consideration, therefore, of the *present payment* of the note, justice requires that he should make some *abatement* from it. This abatement is called *discount*.

(1.) *To find the present worth of a bill or debt.*

RULE. Find the interest of the debt or note, at the given rate, and for the given time; consider this interest as discount, and subtract it from the debt or note to find the present worth.

Ex. Required the present worth of a note for £416, 3. 4. drawn March 1st at 7 months, discounted June 9th, at 4 per cent.

By counting forward 7 months from the 1st of March, and adding three days of grace, we find this bill to be due on the 4th of October. The number of days from the 9th of June till this date, is 117; (Art. 221, Rule 3, Note 2,) and the interest of £416, 3. 4. for 117 days at 4 per cent. per annum, is found, by any of the methods formerly explained, to be £5, 6. 8$\frac{3}{4}$. and consequently the present worth is £410, 16. 7$\frac{1}{4}$.

NOTE 1.—If a bill without the days of grace, should appear to be due on the 31st of any month having only 30 days, the last day of that month, and not the first day of the next, is considered as the day on which the bill is due. Thus a bill drawn on the 31st of October, at 4 months, would be really due, adding in the days of grace, on the 3d of March.

NOTE 2.—When Merchants, Tradesmen, &c., make a discount on any of their book accounts, it is calculated without reference to time.

Exercises.

Find the present worth of the following bills at the given rates per cent. per annum.

1. £607, 3. 4. drawn May 22, at 5 mos. Discounted July 10th at $5\frac{1}{2}$ per cent. Ans. £597, 7. $6\frac{1}{2}$.

2. £486, 18. 8. drawn March 25, at 10 months. Discounted June 19th, at 5 per cent. Ans. £472, 1. 2.

3. $1152.50 drawn Dec. 8th, at 6 mos. Discounted March 25th at 6 per cent. Ans. $1137.72.

4. $4000 drawn Feb. 16th, at 11 mos. Discounted Sept. 12th at $5\frac{1}{2}$ per cent. Ans. $3922.25.

5. What is the discount on £549, for 32 days, at 5 per cent. per annum. Ans. £2, 8. $1\frac{1}{2}$.

6. Five volumes of a work can be bought for a certain sum, payable at the end of a year: and six volumes of the same work can be bought for the same sum ready money; what is the rate of discount? Ans. 20 per cent.

7. A tradesman marks his goods with two prices, one for ready money, and the other for one year's credit, allowing discount at 5 per cent.; if the credit price be marked $9.80, what ought to be the cash price? Ans. £ 2, 6, 8.

Note.—This last exercise is to be calculated by the rule for finding the true discount. (See next Art.)

225. The rule which has been given for the calculation of discount, is that which is always adopted in actual practice. It is founded, however, on a principal radically false; and always *gives the discount too large, and consequently the present worth too small, by the interest of the true discount.* This will appear manifest, if we consider, that *the true present worth of any debt is such a sum as would if lent at interest at the assigned rate, amount to that debt at the time at which it would have been due:* and consequently, the discount, or difference between the present worth and the debt, should be, not the interest of the debt, but the interest of the present worth; and, therefore, the interest of the debt will exceed the true discount, that is, the interest of the present worth, by the interest of that discount.

(2.) *To find the true present worth of a bill or debt.*

Rule.—State, as the amount of £100 or dollars, for the given time, and at the proposed rate, is to £100 or dollars, so is the debt to its true present worth; and the present worth being subtracted from the debt, the remainder is the discount.

Ex. Find the true present worth of £463, for 7 months, at 5 per cent., per annum.

```
   1 £        £           £
As 100     : 5       }  : : 100
   12 mo. : 7 mo.    }       1
   ---------------
   12      : 35  : : 1
            12)35
            ---------
              £2, 18. 4. interest of £100 for 7 mos.
```

∴ £100 + £2, 18. 4. = £102, 18. 4. = Amount of £100 for the given time.

```
                 £    s.   d.     £      £
Again,   As   102,  18.   4.  :  100 : : 463
                20                20
              ----              ----
              2058              2000
                12                12
              -----             -----
             24700             24000
                                 463      Present worth.
                               -------
          24700)11112000(£449, 17. 6½.
```

The *reason for this rule* will be evident from the consideration, that £100 or dollars is the present worth of its amount regarded as a debt; and, consequently, the analogy given above will become simply this: as the amount of £100, considered as a debt is to £100, the present work of that debt, so is any other debt to its present worth.

It is obvious also, that, for the first two terms of the analogy, we might use the amount of *any sum whatever*, and that sum itself; but it is generally more simple and easy to employ £100 or dollars and its amount.

By comparing the above result with that found by the common method we see that the error amounts to 7s. $7\frac{1}{2}$.

Where the principal is given in pounds, &c., it will be found more convenient to add the interest of £100 for the given time to £100 in the form of a fraction. Thus

$$\text{As } \begin{array}{r} \not{1}\not{0}\not{0} : 5 \\ 12 : 7 \end{array} \Big\} :: 100$$

$$\overline{12 \quad 35} = £\tfrac{35}{12} \text{ interest of } £100.$$

Again, $\quad$ As £$100\frac{35}{12}$: £100 : : £463 : £449, 17. $6\frac{1}{2}$.

Both of these are, in reality, the same as Rule (10) in interest.

Stocks.

226. If the 3 per cent. Consols be quoted in the money-market at $96\frac{5}{8}$, the meaning of this is, that for £96, 12. 6. of money a person can purchase £100 stock, for which he will receive an acknowledgment which will entitle him to half-yearly dividends from Government at the rate of 3 per cent., per annum on the stock held by him.

Similarly, if the shares in any trading company, which were originally fixed at a given amount, say, $100 each be advertised in the share market at 86, the meaning of this is, that for $86 of money one share can be obtained, and the holder of such share will receive dividends at the end of each half-year upon the $100 share according to the state of the finances of the company.

Stock may therefore be defined to be the *capital* of moneyed institutions, as incorporated Banks, Railroad, Insurance Companies, and Manufactories, &c.; or to be the money borrowed by our or any Government at so much per cent., to defray the expenses of the nation.

The amount of debt owing by the Government is called the National Debt, or the Funds. The Funds represent the credit of the country, which is bound to pay whatever debts are contracted by its Government. The Government, however, reserves to itself the option of paying off the principal at any future time whatever; pledging itself nevertheless, to pay interest on it regularly at fixed periods, in the meantime.

If money would always bring the same amount of interest, the average price of £100 or $100 stock would be always the same (viz., £100, or $100 the price first given for it)—we say *average* price, because even then the price would evidently be somewhat less immediately *after* the payment of a dividend, than it would be immediately *before* it: but not only does this cause affect the price of Stocks, but the continual fluctuations in the value of money, arising from commercial or political changes or expectations abroad and at home, are constantly disturbing it, even two or three times in the *same* day according to the news which reach us. The price of stock will *rise* and *fall* according as it seems most likely that money would fetch elsewhere a *higher* or a *lower* rate of interest; i. e. would be more *scarce*, and in *demand*, as in prospect of war, or of active speculation, or be lying *upon hand* and *plentiful*, as when trade is looking dull, and there are no means of employing capital.

Thus if at a time A wished to sell his stock, money was elsewhere making 5 per cent., it is plain that no one would give him $100 for the right to receive only 4, but since $80 of common money would now bring $4 interest, he would be able to sell his $100 stock for $80, and the 4 per cents. would be said to be selling at 80.

When the price of £100 or $100 stock is £100 or $100 in money, the stock is said to be at *par*.

When the price is more than the original price it is said to be at a *premium*, and when less at a *discount*.

All examples in stocks depend on the principles of proportion, those of most frequent occurrence will now be explained.

Ex. 1. Required, the sum which will purchase £1500 in the 3 per cents. at 82.

In this case £100 stock is worth £82 in money.

∵ As £100 : £1500 : : £82 money : Money required
whence required sum of money = £1230.

Ex. 2. What amount of stock in the $3\frac{1}{2}$ per cents. at 90 will \$4050 purchase?

In this case \$90 money will purchase \$100 stock.

$\therefore$ As \$90 : \$4050 : : \$100 stock : requd. amt. of stock
whence required amount of stock = \$4500.

Ex. 3. If I buy £1520 3 per cent. consols at $93\frac{1}{4}$, and pay $\frac{1}{8}$ per cent. for brokerage, what does it cost me?

Every £100 stock costs me (£$93\frac{1}{4}$ + £$\frac{1}{8}$) or £$93\frac{3}{8}$.

$\therefore$ As £100 stock : £1520 stock : : $93\frac{3}{8}$: requ. s. of money
whence required sum of money = £1419, 6s.

Ex. 4, What sum shall I receive for £1920, 13. 4. in the $3\frac{1}{2}$ per cents. at $98\frac{7}{8}$, brokerage being £$\frac{1}{8}$ per cent?

£100 stock realises (£$98\frac{7}{8}$ — $\frac{1}{8}$) = £$98\frac{3}{4}$.

$\therefore$ As £100 stock : £$1920\frac{2}{3}$ stock : : £$98\frac{3}{4}$: required sum
whence required sum = £1896, 13. 2.

Ex. 5. If I invest \$7927.50 in the 3 per cents. at $94\frac{3}{8}$, what annual income shall I receive from the investment?

For every \$$94\frac{3}{8}$ I get \$100 stock, and the interest on \$100 stock is \$3, therefore for every \$$94\frac{3}{8}$ of money I get \$3 interest.

$\therefore$ As $94\frac{3}{8}$: 100 : : \$3 : required income
whence required income = \$252.

NOTE 1.—If it be required to find the income arising from a certain quantity of stock, it is merely a question of simple interest.

Note 2.—It may be noticed in the above examples, that when the question was simply to find the amount of stock, or money realized by sale of stock, the 3, 4, or other rates per cent. never entered into the *statement;* and when the question was simply to find the income arising from any sum invested in the funds, then the £100 or \$100 never entered into the *statement.*

Ex. 6. Which is the best investment, \$4000 in the 3 per cents. at $89\frac{1}{2}$, or the $3\frac{1}{2}$ per cents. at $98\frac{1}{2}$.

In the first case every \$$89\frac{1}{2}$ gives \$3 interest.

$\therefore$ every \$1 of money gives $\$\frac{3}{89\frac{1}{2}}$ or $\$\frac{6}{179}$ interest.

In the second case every \$$98\frac{1}{2}$ gives \$$3\frac{1}{2}$ interest.

$\therefore$ every \$1 of money gives $\$\frac{3\frac{1}{2}}{98\frac{1}{2}}$ or $\$\frac{7}{197}$ interest.

and comparing the fractions $\frac{6}{179}$ and $\frac{7}{197}$,

$$\frac{1182}{35263} \text{ and } \frac{1253}{35263},$$

the *second* fraction is greater than the *first*, and therefore the second investment is the best.

Ex. 7. How much stock can be purchased by the transfer of £2000 stock from the 3 per cents. at 90 to the $3\frac{1}{2}$ per cents. at 96, and what change will be effected in income by it?

It is evident that we can purchase less for a certain sum at 96 than if it were only 90, so that we must state as follows:

As 96 : 90 : : £2000 stock : required amount of stock whence required amount of stock = £1875.

Income in first case = £60

Income in second case = £65, 12. 6.

Therefore income increased by £ 5, 12. 6.

NOTE. The last question might have been worked thus: first sell out the stock at 90, and then invest the proceeds in $3\frac{1}{2}$ per cents. at 96.

Ex. 8. A person purchases \$4000 3 per cent. consols at $97\frac{1}{8}$, and sells out again when they have sunk to $83\frac{1}{2}$; how much does he lose by the transaction?

He loses on every \$100 stock (\$$97\frac{1}{8}$ — $83\frac{1}{2}$), or \$$13\frac{5}{8}$.

∴ his total loss = (\$$13\frac{5}{8}$ × 40) = \$ 545.00.

EXERCISES.

1. The 4 per cents. being at $82\frac{1}{8}$, what must be given for £1000 stock, and what sum would be gained by selling out again at $86\frac{1}{4}$? Ans. £821, 5. 0; £41, 5. 0.

2. If I lay out \$12,000 in the 3 per cents. when they are at $84\frac{3}{8}$, what income should I thence derive? Ans. \$$426.66\frac{2}{3}$.

3. What is the cost of £850 Bank stock at $90\frac{5}{8}$, $\frac{1}{8}$ per cent. being paid for brokerage; and what sum would be lost by selling out at $89\frac{1}{2}$? Ans. £771, 7, 6.; £10, 12. 6.

4. A person transfers \$4000 stock from the 4 per cents. at 90, to the 3 per cents. at 72; find the alteration in his income. Ans. \$10.00.

5. A person having £4236 sterling invested in the 3 per cents, sells out at 75 and reinvests the proceeds in this country at 6 per cent. per annum, allowing exchange to be at par, what is the difference of his income.

Ans. An increase of $317.70 Nova Scotia currency.

PROFIT AND LOSS.

227. All questions in Arithmetic which relate to gain or loss in mercantile transactions, fall under the head of PROFIT AND LOSS.

Examples in profit and loss are worked by the principles of proportion.

The method of working the first four examples is so obvious as not to require a formal rule.

Ex. If 112 lbs. of rice be bought for £1, 15. 0. and sold at 4½d. per lb., what is the gain?

112 at 4½ per lb.	=	£2, 2. 0.
Prime cost	=	£1, 15. 0.
Gain		£0, 7. 0. Ans.

EXERCISES.

1. If a piece of linen, containing 25½ yds., cost £3, 8. 3., what is gained by selling it at 3s. 9½. per yard?

Ans. £1, 8. 5¼.

2. If a pipe of wine, containing 138 gals., cost $455, what is gained by selling it at $3.70 per gallon?

Ans. $55.60.

3. If 113 pounds of tea be sold at 2s. 9d. per lb., what is the gain, the first cost being $48.02½?

Ans. $14.12½

4. If a roll of tobacco, containing 25lbs., be bought at 3s. 6d. per lb.; what is gained by selling it at 3¼d. per oz., a pound and a half being lost by weighing it out in small quantities, and by drying? Ans. £0, 14. 4.

228. *From the prime cost and the selling price, to find the gain or loss per cent.*

RULE. As the prime cost is to the gain or loss on that cost, so are £100 or $100 to the gain or loss per cent.

Ex. If tea be bought at 2s. 3d. per lb,, and sold at 3s. 1½d. per lb., what is gained per cent?

Here, 3s. 1½d. — 2s. 3d. = 10½d. gain on 2s. 3.
Then, As 2s. 3d. : 10½ : : £100 regarded as a first cost : £38, 17. 9¼, the gain on 100 pounds

By Dollars and Cents.

3s. 1½d. = 62½ cents = selling price,
2s. 3 d. = 45 cents = prime cost,

17½ = gain on 45 cents.

∴ As 45 cents : 17½ cents : : $100 : $38.88⅞.
∴ the gain is 38⅞ per cent.

229. *To find how a commodity must be sold to gain or lose a certain rate per cent.*

RULE. As £100 or dollars are to the gain or loss per cent., so is the prime cost to the gain or loss on that cost; and from this and the prime cost, the selling price will be found by addition or subtraction.

Ex. How must nutmegs, which cost $1.20 per pound, be sold to gain 16 per cent.?

As $100 (regarded as the first cost) : $16 (gain on $100) : : $1.20 (first cost of 1 lb.) : $0.192, the gain per lb., which being added to $1.20, the amount is 1.39\frac{2}{10}$.

NOTE. It should be particularly remarked that *by the gain or loss per cent. is to be understood the sum that would be gained or lost at the given prices*, not on a hundred pounds or a hundred dollars' worth sold, but *on a hundred pounds or dollars laid out in prime cost, and in charges, if there be any.*

EXERCISES.

1. If paper which cost 19s. 2d. per ream, be sold for 17s. 11d. per ream, how much is lost per cent.?
Ans. £6, $10.5\frac{5}{23}$.

2. How must linen which cost 3s. $1\frac{1}{2}$d. per yard, be sold to gain 16 per cent.? Ans. $\$0.72\frac{1}{2}$.

3. If Broadcloth cost $2.35 per yard, how much is gained per cent. by selling one part at $2.80 per yard, and how much by selling another at $2.90?
Ans. $19\frac{7}{47}$ and $23\frac{19}{47}$ per cent.

4. Isinglass, which cost 8s. 6d. per lb., is sold at a loss of 11 per cent.: at what rate is it sold, and how much is lost on the sale of 17 cwt. 1 q. long wt. at the same rate?
Ans. 7s. $6\frac{39}{50}$d.; £90, 6. $5\frac{1}{25}$.

5. Bought 2048 yards of cambric at 3s. $2\frac{1}{2}$d. per yard, and sold the whole for $1443.95. Required, the whole gain and the gain per cent. Ans. $\$129.81\frac{2}{3}$; $\$39.51\frac{1}{4}$.

6. If soap cost £3, 5. per cent., how must it be sold to gain 10 per cent. and how to gain 20 and 30 per cent.?
Ans. £3, 11. 6.; £3, 18.; £4, 4. 6.

230. *From the gain or loss per cent, and the selling price, to find the first cost.*

RULE. As £100 or $100 together with the gain per cent., or diminished by the loss per cent, are to £100 or $100, so is the selling price to the prime cost.

Ex. 1. What was the first cost of flax seed, which being sold at $14.10 per hogshead, the seller gained 13 per cent?

As $\$100 + \$13 : \$100 :: \$14.10 : \$12.47\frac{2}{3}$. In this example it is evident, that what cost $100 is sold for $113, and the analogy used above is no more than this: as the selling price $113 is to its first cost $100, so is the selling price $14.10 to the corresponding first cost $\$12.47\frac{2}{3}$.

Ex. 2. If 15 per cent. be gained by selling sugar at £2, 10. per cwt., how much is gained by selling it at £2, 5. 6. per cent.?

As £2, 10.: £2, 5. 6.:: £115: £104, 13. and £104, 13 — £100 = £4, 13. Ans.

This question might have been worked by finding the first cost, and thence the selling price, as in the preceding examples.

EXERCISES.

1. If 11 per cent. be lost by selling 128 yards of broadcloth for £98, 18. 8., what was the first cost per yard? Ans. £0, 17. 4½.

2. If a book be sold for $19.95, and 17 per cent. be gained, what was the first cost? Ans. $17.05.

3. What was the first cost of tar, which being sold at £0, 17. 4. per barrel, the merchant loses 9 per cent., and how much is lost on the sale of 63 barrels?

Ans. 19s. 0$\frac{4}{7}$d. and £5, 8. 0.

4. If by selling muslin at $0.62½ per yard 2 per cent. be lost, how must it be sold to gain 25 per cent.?

Ans. $0.80 nearly.

DIVISION INTO PROPORTIONAL PARTS AND RECIPROCAL PROPORTION.

231. *To divide a given number into parts which shall be proportional to certain other given numbers.*

Rule. State thus: As the sum of the given parts : any one of them : : the entire quantity to be divided : the corresponding part of it.

When all the parts except one, have been determined, that one may be found by adding the rest together and taking the sum from the number to be divided. It is better however, to find them all by proportion, as the operation can then be tested by adding them together.

Ex. 1. Divide a purse containing 16s. 8d. among 3 boys according to their ages: A is 10, B is 12, and C is 14. Required, their shares?

A = 10 years of age.
B = 12 ,, ,,
C = 14 ,, ,,

As 36 : 10 : : 16s. 8d. : 4s. 7$\frac{5}{9}$. = A's share.
As 36 : 12 : : 16s. 8d. : 5s. 6$\frac{2}{3}$. = B's ,,
As 36 : 14 : : 16s. 8d. : 6s. 5$\frac{7}{9}$. = C's ,,

16s. 8d. proof.

Ex. 2. Divide \$1000 among 4 persons A, B, C, and D, in the proportions of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

$$\left.\begin{array}{l}\frac{1}{2} = 30 \\ \frac{1}{3} = 20 \\ \frac{1}{4} = 15 \\ \frac{1}{5} = 12\end{array}\right\} 60 \text{ L. C. M.}$$

As 77 : 30 : : \$1000 : \$389.61 A's share.

The other shares may be found in the same mannner.

232. *To divide a given quantity into parts which shall be in inverse or reciprocal proportion to the given numbers.*

Rule. Invert the terms on which the inverse proportion depends, proceed with these fractions as in the former rule.

When other conditions in *direct* proportion are given, multiply each of these by the inverted term, and proceed with these fractions as before.

Ex. A gentleman left \$4000 to his three children to be divided in inverse proportion to their ages, so that the younger may receive the greater share; their ages are 10, 12, and 15 years, what must be the share of each child?

$$\left.\begin{array}{l}\frac{1}{10} = 6 \\ \frac{1}{12} = 5 \\ \frac{1}{15} = 4\end{array}\right\} 60 \text{ L. C. M.}$$

As 15 : 6 : : \$4000 : \$1600 = share of youngest.
As 15 : 5 : : \$4000 : \$1333.33$\frac{1}{3}$ = ,, second.
As 15 : 4 : : \$4000 : \$1066.66$\frac{2}{3}$ = ,, eldest.

\$4000.00 proof.

Exercises.

1. Divide 398 into three parts, which shall be to one another as the numbers 5, 7, and 11.

Ans. $86\frac{12}{23}$, $121\frac{3}{23}$, and $190\frac{8}{23}$.

2. Divide 80 Miles into 4 parts, proportional to the numbers 10, 9, 8, and 7.

Ans. $23\frac{9}{17}$, $21\frac{3}{17}$, $18\frac{14}{17}$, and $16\frac{8}{17}$.

3. How much copper and how much tin will be required to make a cannon weighing 16c. 1q. 20 lbs.; gun-metal being composed of 100 parts of copper and 11 of tin?

Ans. 14c. 3q. $5\frac{73}{111}$lbs, ; 1c. 2q. $14\frac{38}{111}$.

4. 76 parts of nitre, 14 of charcoal, and 10 of sulphur, compose gun-powder; how much of these ingredients will form 4650 lbs. of powder.

Ans. 3534lbs., 651lbs., 465lbs.

5. Divide 1065 into parts which shall be to each other in the ratio of 3, 5, 7; and also into parts which shall be in the ratio of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$.

Ans. 213, 355, 497; 525, 315, 225.

6. The standard gold coin of England and her colonies is made of gold 22 carats fine, and a pound Troy of this metal yields $46\frac{29}{40}$ sovereigns; what weight of pure gold is there in 100 sovereigns?

Ans. 1lb. 11oz. 10 dwt. $20\frac{100}{623}$ grs.

7. Divide $1200 among three persons, so that the first shall have twice as much as the second, and the third twice as much as the other two together.

Ans. $\$266.66\frac{2}{3}$, $\$133.33\frac{1}{3}$, $800.

8. British silver coin consists of 37 parts of pure silver and three of copper; how much of each does the florin contain, each pound, Troy weight, being coined into 66 shillings? Ans. 8 dwt. $9\frac{9}{11}$ qrs. and $16\frac{4}{11}$ grs.

9. A common consisting of 200 acres, is to be divided among three adjoining proprietors in direct proportion to the extent of their estates, but in inverse proportion to their value; A possesses 200 acres, valued at £60 per acre, B 300 acres, valued at £80 per acre, and C 600 acres at £40 per acre. Required, their shares of the common?

Ans. A 30ac. 0ro. $30\frac{10}{53}$po., B 33ac. 3ro. $33\frac{51}{53}$po., C. 135ac. 3ro. $15\frac{44}{53}$po.

FELLOWSHIP OR PARTNERSHIP.

232. FELLOWSHIP OR PARTNERSHIP is a method by which the respective gains or losses of partners in any mercantile transactions are determined.

Fellowship is divided into SIMPLE and COMPOUND FELLOWSHIP: in the former, the sums of money put in by the several partners continue in business for the same time; in the latter, for different periods of time.

SIMPLE FELLOWSHIP.

233. *When the shares are in proportion to the stock or claims.*

RULE.—State thus:—As the whole stock : the whole gain or loss : : the stock of any partner : his gain or loss.

This rule is merely a particular application of the one given in Art. 10, and therefore requires no separate illustration.

NOTE.—The estate of a Bankrupt may be divided among his creditors by the same method.

234. *When interest is allowed upon the stocks and the partners have either equal or proportional shares.*

RULE.—Find the interest of the several stocks, subtract the amount from the gain, and divide the remainder equally among the partners, or according to their claims, as in the former rule.

EXERCISES.

1. A's and B's stocks are \$1500 and \$1700 respectively Required the share of each in a gain of \$960.

Ans. A's share, \$450; B's, \$510.

2. A's stock £1750, B's £1250; whole gain £565, 12. 0. Required each person's share.

A's, £329, 18. 8.; B's, £235, 13. 4.

3. C's stock, £475, 15. 8., D's, £346, 12. 4., E's, £396, 17. 6.; whole gain, £279. 4. 10.

Ans. C's gain, £108, 19. $3\frac{1}{2}$., D's, £79, 7. $7\frac{1}{4}$., and E's, £90, 17. $10\frac{1}{4}$.

4. Three merchants enter into partnership; A advanced $2720, B $2320, and C $1200; their net gain at the end of the year is $2080. How much will each partner draw, allowing 5 per cent. for stock, and dividing the remainder equally among the partners?

Ans. A, 725.33\frac{1}{3}$.; B, 705.33\frac{1}{3}$.; C, 649.33\frac{1}{3}$.

5. Two merchants join stocks; W advances £2400, and M £3000; the interest on each man's stock is to be at 4 per cent.; and W, for superintending the warehouse, is to have three shares, while M, acting as traveller, is to have five. Required each man's share of the first year's gain, which was £850? Ans. W, £333, 15.; M, £516, 5.

6. R and I are in partnership; R possesses $4000 of the stock and I $4800, for which they receive 4 per cent.; they admit C, their foreman into the firm, and, having no stock, he is only allowed one-fourth share of the profits, the remainder being equally divided between R and I. Required each man's share of $5800,00, the first year's gain!

Ans. R, $2203.00: I, $2235.00; C, $1362.00.

COMPOUND FELLOWSHIP.

235. Rule.—Reduce all the times into the same denomination, and multiply each man's stock by the time of its continuance, and then state thus:

As the sum of all the products : each particular product : : the whole quantity to be divided : the corresponding share.

Ex. A and B enter into partnership; A puts in $2400 for 13 months, and B $3200 for 10 months. Required the share of each in a gain of $2600.

A's stock, $2400 × 13 = 31200.
B's $3200 × 10 = 32000.

63200. Sum.

As 63200 : 31200 : : $2600 : 1283.54\frac{2}{5}$, A's share.
As 63200 : 32000 : : 2600 : 1316.45\frac{3}{5}$, B's share.

$2600.00. Proof.

The reason for the above process is evident from the CONSIDERATION, that a stock of \$2400 for 13 months, would be equivalent to 13 times \$2400 for 1 month: and one of \$3200 for 10 months, to 10 times \$3200 for 1 month. Hence if these increased stocks be employed, it is evident, that since the times are then to be regarded as equal, the work will proceed in the same manner as in simple fellowship.

EXERCISES.

1. A's stock \$1120 for 5 months, B's, \$1066.66⅔ for 6 months; whole gain, \$1326.50.
   Ans. A's gain \$619.03⅓; B's, \$707.46⅔.

2. A's stock £170 for 8 months, B's, £280 for 6 months; whole gain, £250.
   Ans. A's gain £111, 16. 10¼; B's, £138, 3. 1¾.

3. A and B enter into partnership; A contributes £3000 for 9 months, and B £2400 for 6 months; they gain £1150. Find each man's share of the gain.
   Ans. A's share £750, and B's £400.

4. There were at a feast 20 men, 30 women, and 15 servants; for every 10 shillings that a man paid a woman paid 6 shillings, and a servant 2 shillings; the bill amounted to £41. How much did each man, woman, and servant pay?
   Ans. Each man £1; each woman 12s.; and each servant 4s.

ALLIGATION.

236. The process of finding the mean rate or price of a mixture compounded of several ingredients, or the quantity of ingredients of different rates necessary to make a mixture of a required rate, or quality, is called ALLIGATION.

ALLIGATION is usually divided into two kinds, *Medial* and *Alternate*.

Alligation Medial.

237. Alligation Medial is the process of finding the mean price of two or more ingredients, or articles, of different values.

From the quantity and rate of several ingredients to be mixed, to find the mean rate.

Rule.—Multiply each quantity by its rate, and divide the sum of the products by the sum of the quantities.

Ex. A farmer mixed 20 bushels of wheat at 5s. per bushel, and 50 bushels at 3s. per bushel, with 10 bushels at 2s. per bushel. What is a bushel of this mixture worth?

$$\begin{array}{rcr} 20 \times 5 & = & 100 \\ 50 \times 3 & = & 150 \\ 10 \times 2 & = & 20 \\ \hline 80 & & 80)270 \\ & & \overline{3 \text{ s. } 4\tfrac{1}{2}\text{d.}} \text{ Ans.} \end{array}$$

Exercises.

1. A grocer mixes 10 lbs. of tea at 4s., 12 lbs. at 4s. 9d., 18 lbs. at 5s. 2d., and 20 lbs. at 5s. 6d. What is a pound of this mixture worth? Ans. 5s.

2. A goldsmith melted together 8 oz. of gold, 16 carats fine, 10 oz. of 18 carats fine, 18 oz. of 20 carats fine, and 4 oz. of alloy. Of what fineness was the mass?
Ans. $16\frac{7}{10}$ carats.

3. A vinter mingles 15 gallons of canary, at $1.60 per gallon, with 20 gallons at $1.50 per gallon, 10 gallons of sherry, at $2.15 per gallon, and 24 gallons of white wine, at $0.80 per gallon. What is the worth of a gallon of this mixture? Ans. 1.37\frac{1}{4}$ nearly.

Alligation Alternate.

238. Alligation Alternate is the process of finding what quantity of any given number of ingredients, whose prices are given will form a mixture of a *given mean price*.

Alligation Alternate embraces three varieties of examples, which are sometimes called, *Alternate*, *Partial*, and *Total.*

1. *To find the quantity of each ingredient, when its price and that of the required mixture are given.*

RULE.—Let the rates of the ingredients, all in the same denomination, be written in a line; and let the mean rate in the same denomination be written above them. Take one rate which is greater, and one which is less, than the mean rate, and write the difference between each of them and the mean rate below the other.

Proceed thus with the rates two by two, if there be more than two, till one or more differences stand below each. Then, if only one difference stand below any rate, it will be the quantity required at that rate; but if there be more than one, their sum will be the required quantity.

NOTE.—The connecting or *linking* of the rates with crooked or curved lines, in the use of this rule, is attended with no advantage, and has an awkward appearance. Should that method be preferred, however, it can present no difficulty, as each rate less than the mean rate is to be connected with one greater, and each greater with one less, and the differences are to be set below the rate to which the line directs.

EX. A man mixed four kinds of oil, worth 8s., 9s., 11s., and 12s. per gal.; the mixture was worth 10s. per gal. Required, the quantity of each.

10

8	9	11	12
1	2	2	1

Here the mean rate, 10 shillings, is set above the other rates. Then the difference between 12 and the mean is set below 9, and the difference between 9 and the mean is set below 12. Again, the difference between 11 and the mean is set below 8, and the difference between 8 and the mean is set below 11. Hence we find, that for 1 gallon at 8s. we must take 2 at 9s., 2 at 11s., and 1 at 12s.

With respect to *the reason* of the operation, it is obvious from the previous article, that 1 gallon at 8s., and 2 gallons at 11s. each, would make a mixture worth 10s. per gallon; and likewise that a mixture of 1 gallon at 9s., and 2 at 9s., would be worth the same per gallon; and it is evident, that both mixtures, taken together, must make a mixture of the same value still, per gallon: and in the same way every operation in this rule may be explained.

Note.—It is manifest that other answers may be obtained by connecting the prices in a different manner. It is also manifest, if we multiply or divide the answers already obtained by any number, the results will fulfil the conditions of the question; consequently the number of answers is unlimited.

(2.) *When the quantity of one of the ingredients and the mean price are given.*

Rule.—Find the difference between the price of each ingredient and the mean price, as before. Then,

As the difference of that ingredient whose quantity is given: the rest of the differences severally:: the quantity given: the quantity required of each ingredient.

Ex. How many pounds of sugar at 10, and 15 cents a pound, must be mixed with 20 lbs. at 9 cents, so that the mixture may be worth 12 cents a pound?

	12	
9	10	15
3	3	3
		2
3	3	5

As 3 : 3 : : 20 : 20 lbs. at 10 cents a pound.
As 3 : 5 : : 20 : $33\frac{1}{3}$ lbs. at 15 cents a pound.

It appears, therefore, that 20 lbs. at 10 cents, $33\frac{1}{3}$ lbs. at 15 cents, and 20 lbs. at 9 cents, will compose a quantity worth 12 cents per pound, at an average.

This question belongs to what is usually called *Alligation partial.*

(3.) *When the quantity to be mixed and the mean price of the required mixture are given.*

Rule.—Find the difference between the price of each ingredient and the mean price of the required mixture, as before. Then, As the sum of the differences : each particular difference :: the whole quantity to be mixed : required of each ingredient.

Ex. A grocer has raisins worth 8, 10, and 16 cents a pound: how many of each kind may be taken to form a mixture of 112 lbs. worth 12 cents a pound?

	12		
8	10	16	
4	4	4	
		2	
4	4	6	or, dividing by 2,
2	2	3	

As 7 : 2 : . 112 : 32 lbs. at 8 cents a pound.
As 7 : 3 : : 112 : 48 lbs. at 16 cents a pound.

It appears, therefore, that 32 lbs. at 8 cents, the same quantity at 10 cents, and 48 lbs. at 16 cents, will form a compound of 112 lbs. worth 12 cents per lb. This question belongs to what is usually called *Alligation total.*

NOTE.—*To prove questions in this rule*, find the value of all the ingredients at their given prices; if this be equal to the value of the whole mixture at the given price, the work is right.

EXERCISES.

1. A goldsmith has gold of 18, 20, 22, and 24 carats fine: how much of each may be taken to form a mixture 21 carats fine?

Ans. 3 of 18 car., 1 of 20, 1 of 22, and 3 of 24 car. fine.

2. How much wool at 20, 30, and 54 cents a pound must be mixed with 95 lbs. at 50 cents, to form a mixture worth 40 cents a pound?

Ans. 133 lbs. at 20 cents, 95 lbs. at 30 cents, and 190 lbs. at 54 cents.

3. How much water must be added to a cask of spirits containing 84 gallons, worth 13s. 6d. per gallon, to reduce the value to 11s. $4\frac{1}{2}$d. per gallon? Ans. $15\frac{9}{13}$ gal.

4. A grocer having four sorts of tea, at 50 cents, 60 cents, 80 cents, and 90 cents per lb., wishes to form a mix-

ture of 87 lbs., worth 70 cents per lb. What quantity must there be of each sort?

Ans. 14½ lbs. at 50 cents, 29 lbs. at 60 cents, 29 lbs. at 80 cents, and 14½ at 90 cents.

CONJOINED PROPORTION.

239. When each *antecedent* of a compound ratio is *equal* in value to its *consequent*, the proportion is called CONJOINED PROPORTION.

Conjoined Proportion is often called the *chain rule*. It is chiefly used in comparing coins, weights and measures of two countries, through the medium of those of other countries, and in the higher operations of exchange. The *odd term* is called the *demand*.

RULE.—Taking the terms in pairs, place the first term on the left hand for the antecedent, and its equal on the right for the consequent, and so on. Then if the answer is to be of the same kind as the first term, place the odd term under the antecedents; but if not, place it under the consequents.

Cancel the factors common to both sides, and if the odd term falls under the consequents, divide the product of the factors remaining on the right by the product of those on the left, and the quotient will be the answer; but if the odd term falls under the antecedents, divide the product of the factors remaining on the left by the product of those on the right, and the quotient will be the answer.

NOTE.—In arranging the terms, it should be observed that the *first antecedent* and the *last consequent* will always be of the *same kind*.

All the quantities of the same kind must be reduced to the same denomination, if they be not so already.

Ex 1. If 20 lbs. Nova Scotia make 12 lbs. in Spain; and 15 lbs. in Spain 20 lbs. in Denmark; 40 lbs. in Denmark 60 lbs in Russia: how many pounds in Russia are equal to 100 lbs. Nova Scotia?

Proceeding by the Rule given above.

20 lb. N. S. = 3, 12 lb. Spain.
15 lb. Spain = 20 lb. Den.
10 40 lb. Den. = 4, 60 lb. Russ.
10, 100

$10 \times 4 \times 3 = 120$ lbs.

Ex. 2. If $18 U. S. are worth 8 ducats at Frankfort; 12 ducats at Frankfort 9 pistoles at Geneva; and 50 pistoles at Geneva, 24 rupees at Bombay how many rupees at Bombay are equal to $100 U States?

Proceeding by the Rule given above.

2, 18	8, 4
12	9
50	24, 2
	100, 2

$2 \times 2 \times 4 = 16$ rupees.

NOTE.—Questions in this rule may be proved by reversing the operation, taking the consequents for the antecedents, and the answer for the odd term, or by as many simple statements as the question requires.

EXERCISES.

1. If 10 yds. at New York make 9 yds. at Athens; and 90 yds. at Athens, 112 yds. at Canton; how many yds. at Canton are equal to 50 yds. at New York?
Ans. 56 yds. at Canton.

2. If 50 yds. of cloth in Boston are worth 45 bbls. of flour in Philadelphia; and 90 bbls. of flour in Philadelphia 127 bales of cotton in New Orleans; how many bales of cotton at New Orleans are worth 100 yds. of cloth in Boston? Ans. 127 bales N. O.

3. If 140 braces at Venice are equal to 156 bonnets at Leghorn; and 7 bonnets at Leghorn equal to 4 yds. of cloth at Halifax; how many braces at Venice are equal to 16 yds. of cloth at Halifax? Ans. $25\frac{5}{39}$.

EXCHANGE.

240. *Exchange*, in commerce, signifies the receiving or paying of money in one place, for an equal sum in another, by *draft or bill of exchange.*

A *Bill of Exchange* is a written order, addressed to a person, directing him to pay, at a specified time, a certain sum of money to another person, or to his order.

The person who *signs* the bill is called the *drawer* or *maker;* the person in whose favor it is drawn, the *buyer* or *remitter;* the person on whom it is drawn, the *drawee;* and after he has accepted it, the *accepter;* the person to whom the money is directed to be paid, the *payee;* and the person who has legal possession of it, the *holder*.

On the reception of a Bill of Exchange, it should be immediately presented to the *drawee* for his *acceptance.*

241. The *acceptance* of a bill, or draft, is a promise to pay it at *maturity* or the *specified time.* The common method of accepting a bill, is for the drawee to write his name under the word *accepted*, across the bill, either on its face or back. The drawee is not responsible for its payment, until he has accepted it.

If the *payee* wishes to *sell* or *transfer* a bill of exchange, it is necessary for him to *endorse* it, or write his name on the back of it.

If the endorser directs the bill to be paid to a particular person, it is called a *Special endorsement* and the person named, is called the *endorser.* If the endorser simply writes his name on the back of the bill, the endorsement is said to be *blank.* When the endorsement is *blank*, or when a bill is drawn payable to the *bearer*, it may be transferred from one to another at pleasure, and the drawee is bound to pay it to the holder at maturity. If the drawee, or accepter of a bill, fail to pay it, the endorsers are responsible for it.

242. When *acceptance* or *payment* of a bill is refused, the holder should duly notify the endorsers and drawer of the fact by a legal *protest*, otherwise they will not be responsible for its payment.

A *protest* is a formal declaration in writing, made by a civil officer, termed a *Notary Public*, at the request of the holder of a bill, for its *non-acceptance*, or *non-payment*.

The *time specified* for the payment of a bill is a matter of agreement between the parties at the time it is negotiated. Some are payable at *sight*, others in a certain *number of days* or *months* after sight, or after date.

243. Bills of exchange are usually divided into *inland* and *foreign bills*. When the *drawer* and *drawee* both reside in the same country they are termed *inland bills* or *drafts;* when they reside in different countries, *foreign bills*. In negotiating foreign bills, it is customary to draw *three* of the *same date* and *amount*, which are called the *First*, *Second* and *Third* of *Exchange;* and collectively, a *Set of Exchange*. These are sometimes sent by different conveyances, and when the *first* that arrives, is accepted, or paid, the *others* become *void*. The object of this arrangement is to avoid delays, which might arise from accidents, miscarriage, &c.

FORM OF A FOREIGN BILL OF EXCHANGE.

No. ——
Exchange for HALIFAX, 4th May, 1863.
£1000, 0. 0. Stg.

Sixty days after sight of this First Exchange, second and third of the same tenor and date unpaid, pay to the order of Charles Ringland & Co., One Thousand Pounds Sterling, value received, which place to the account of

JOHN L. NEWCOMB.

To
Messrs. JOHN FELTHAM & Co.,
Bankers,
London.

FORM ON AN INLAND BILL OR DRAFT.

HALIFAX, May 4th, 1863.

$100

Thirty days after date, pay to the order of Messrs. Newman & Co., One Hundred Dollars, value received, and charge the same to

MACEY & BROWN.

To

Messrs. BLAIR & BROWN,
Windsor.

244. The term *par* of *exchange*, denotes the *standard* by which the comparative worth of the money of different countries is estimated. It is either *intrinsic* or *commercial*.

The intrinsic par is the real value of the money of different countries, determined by the *weight* and *purity* of their coin.

The commercial par is a nominal value, fixed by law or commercial usage, by which the worth of the money of different countries is estimated.

The intrinsic par remains the same, so long as the *standard coins* of each country are of the same *metals*, and of the same *weight* and *purity;* but in case the standard coins are of *different* metals, the intrinsic par must vary, as the comparative values of the metals vary.

The commercial par is conventional, and may at any time be changed by law or custom.

245. By the term, *course of exchange*, is meant the *current price* which is paid in one place for bills of a given amount drawn on another place.

The course of exchange is seldom *stationary* or at par. It varies according to the circumstances of trade. When the balance of trade is against a country, that is, when the exports are *less* than the imports, bills on the foreign country will be *above par*, for the reason that there will be a greater demand for them to pay the balance due abroad. On the other hand, when the balance of trade is in favor of a country, foreign bills will be *below par*, for the reason that few will be required.

It should be remarked that the course of exchange can never exceed very much the *intrinsic par value;* for it is plain that *coin* or *bullion*, instead of bills, will be remitted, whenever the course of exchange is such that the expense of insuring and transporting it from the debtor to the creditor country, is less than the *premium* for bills, and the exchange will soon sink to par

246. All the calculations in exchange may be performed by the rule of proportion: and the operations may often be abbreviated by the method of aliquot parts.

General Rule.—Place, as the second term in the analogy, that sum whose value is to be found in the money of another country; make that term of the rate which is of the same kind with the second term, the first term of the analogy, and the remaining term of the rate, the third term: then work the analogy in the usual way.

English and American Exchange.

247. Formerly £90 sterling were worth £100 British North American Currency, this is called the *old par value;* now, however, £90 are worth more, and as the rates of exchange on Great Britain are reckoned, in Nova Scotia, New Brunswick, Canada, and the U. States, at a certain per cent. on the old commercial par instead of the new, the rate of exchange must reach a nominal premium before it is *at par*, according to the new *standard.*

This nominal premium is just the difference per cent. between the old and new par value.

In Nova Scotia, the new par value is $12\frac{1}{2}$ per cent. greater than the old. Hence, when bills of exchange on England are selling at $12\frac{1}{2}$ per cent. premium, they are said to be at par.

In New Brunswick and Canada, it is $9\frac{1}{2}$ per cent. more than the old par value; consequently when exchange is $9\frac{1}{2}$ per cent. premium, it is said to be at par in those places.

In the U. States, according to the *old par*, the value of a pound sterling is \$$4.44\frac{4}{9}$, as fixed by act of Congress in 1799. According to the *new par* it is \$$4.84\frac{4}{9}$. Hence, 9 per cent. premium, added to \$4.44$\frac{4}{9}$, will give the new par value of £1 sterling.

248. *To find the value of any sum stg. in Nova Scotia, New Brunswick, Canada, and U. States currency.*

Rule.—State thus :—As 90 : 100 + the rate of premium :: \$4, the result being multiplied by the number of pounds sterling will give the equivalent number of dollars. Or,

To \$4.44$\frac{4}{9}$ add the premium, at the given rate, the sum being multiplied by the number of pounds sterling, will give the equivalent number of dollars.

Note.—Exchange which is variously quoted in the newspapers as 112$\frac{1}{2}$, 113, 170, &c., means 12$\frac{1}{2}$, 13, 70, &c., per cent. premium on the old par value.

Ex. A merchant in Halifax negotiated a bill on Liverpool, G. B., for £1000 stg., at 13$\frac{1}{2}$ per cent. premium: what did he pay for it?

FIRST METHOD.

As 90 : 113$\frac{1}{2}$: : \$4
2 2
180 227
45

And $\frac{227}{45}$ × £1000 = \$5044.44$\frac{4}{9}$ N. S. Currency.

SECOND METHOD.

\$4.444′	\$4.444′
.60	13$\frac{1}{2}$ per cent.
\$5.044′	57777′
1000	2222′
\$5044.44$\frac{4}{9}$ U. S. Currency.	.59999′ or .60

We might have used $4\frac{4}{9}$ and $113\frac{1}{2}$ instead of \$4.444′ and $13\frac{1}{2}$ per cent. thus:—

$$113\frac{1}{2} = 113.5 \text{ per cent.}$$
$$4\frac{4}{9}$$
$$454.0$$
$$\frac{4}{9} \text{ of } 1 = \frac{1}{9} \text{ of } 4 = \quad 50.444'$$
$$5.04\,444 \times 1000 = \$5044.44\tfrac{4}{9}.$$

THIRD METHOD.

£1000 × \$$4.44\frac{4}{9}$ = \$4444.444′ at old par value.
Then \$4444.444′ × .135 = 599.999 the premium.

\$5044.444′

249. *To find the value in sterling money of any sum : currency.*

RULE.—State thus:—As 100 + the rate per cent. premium : 90 :: *sum :* currency, (which, if in dollars, must be divided by 4) : the equivalent in pounds and decimal of a pound sterling. Or,

Multiply the amount currency by 100 and divide the product of $4\frac{4}{9}$ and the rate per cent. of exchange, the quotient will be the equivalent in pounds and decimal of a pound sterling.

Ex. A merchant paid \$1650 for a bill of exchange on London, at $13\frac{1}{2}$ per cent. premium: for what amount sterling was the bill drawn?

FIRST METHOD.

As $113\frac{1}{2}$: 90 : : \$1650 : 1308.37
and 1308.37 ÷ 4 = £327.092 stg.
= £327, 1. 10. stg.

SECOND METHOD.

\$1650
100

$113\frac{1}{2} \times 4\frac{4}{9} = 504\frac{4}{9}$) 165000 (£327, 1. 10. stg.

Exchange between England and Newfoundland.

250. In Newfoundland accounts are kept in pounds, shillings, and pence.

Rates of exchange are reckoned at a certain rate per cent. on the sterling. Trade between Newfoundland and England is in a state of equilibrium when exchange is 120 per cent., or 20 per cent. premium, which, if added to the sterling, will give the equivalent number of pounds, &c., Newfoundland currency.

Ex. 1. What will a bill cost on London, for £3486, 10. at $21\frac{1}{2}$ per cent. premium?

	£3486, 10. 0.
$20 = \frac{1}{5}$ of 100	697, 6. 0.
$1 = \frac{1}{20}$ of 20	34, 17. $3\frac{1}{2}$.
$\frac{1}{2} = \frac{1}{2}$ of 1	17, 8. $7\frac{3}{4}$.
	£4236, 1. $11\frac{1}{4}$. Currency.

Ex. 2. When exchange on England is at $21\frac{1}{2}$ per cent. premium, how much sterling can be obtained for £4236, 1. $11\frac{1}{4}$. currency?

As $121\frac{1}{2}$: 100 : : £4236, 1. $11\frac{1}{4}$.

£4236, 1. $11\frac{1}{4}$., or £4236, 2. 0. nearly

200

847200

2s. $= \frac{1}{10}$ 20

243)847220(£3486, 10. 0. Sterling.

Exercises.

1. A merchant in Paris draws a bill of 1500 francs upon a merchant in London for goods supplied. What sterling money will the latter have to pay, exchange being 24.25 francs for £1 sterling? Ans. £61, 17. $1\frac{35}{97}$.

2. What is the course of exchange between London and Lisbon when 594 milrees, 480 rees, are received for £158, 16. 9.? (1 milree = 1000 reers,) Ans. 64.124d. or 5s. $4\frac{1}{8}$d. nearly.

3. A merchant negotiated a bill on Glasgow for £1267, 10. at $12\frac{1}{2}$ per cent. premium. What did he pay for it?
Ans. $6337.50.

4. Merchant in Liverpool, G. B., draws a bill of £1020, 0. 0. upon a merchant in Halifax for goods supplied. What amount of currency must the latter pay, exchange being at 15 per cent? Ans. $\$5213.33\frac{1}{3}$.

5. A merchant in London draws a bill of $1867 upon Boston. What amount sterling will he have to pay, exchange between Boston and London being 150 per cent.?
Ans. £ 280, 1, 0.

Arbitration of Exchange.

251. When the course of exchange between the first place and the second, the second and third, the third and fourth, &c., of any number of places, are given; the method of finding the course of exchange between the first place and the last, corresponding to these courses, or of valuing any sum of the money of the first place in that of the last, through the medium of the others, is called Arbitration of Exchange.

As the actual course of exchange between the first place and the last, is almost always, from various circumstances, different from the arbitrated course, this method is of use in enabling a merchant, in one place, to discover whether he should draw and remit directly between his own place and another, or circuitously, through other places.

When there is but *one* intervening country, the operation is termed *Simple Arbitration*, when *more than one*, it is termed Compound Arbitration.

Problems in Arbitration of exchange may be solved by conjoined proportion, or by one or more analogies in simple proportion.

Ex. 1. What is the value of $1 N. S. Currency in New Brunswick, exchange between England and Nova Scotia at $13\frac{1}{2}$ per cent. and between England and New Brunswick $9\frac{1}{2}$ per cent?

Proceeding by the Rule given for conjoined proportion.

$$\begin{array}{rcl} 112\frac{1}{2} & = & \not{9}\not{0} \\ \not{9}\not{0} & = & 109\frac{1}{2} \\ & & \$1 \\ \hline 225 & & 219 \end{array}$$

$$\tfrac{219}{225} = \$0.97\tfrac{1}{3} \text{ N. B. Currency.}$$

Hence the rate of exchange between Nova Scotia and New Brunswick is $2\frac{2}{3}$ per cent. discount.

Ex. 2. What is the value of a franc in N. S. Currency, exchange between England and Nova Scotia at $13\frac{1}{2}$ per cent., and between England and France at 24 francs, 87 centimes, per pound sterling?

$$\begin{array}{lcl} \$113\tfrac{1}{2} \text{ N. S.} & = & £22, 10. \text{ or } \$90 \text{ sterling.} \\ £1 \text{ stg.} & = & 24 \text{ francs, } 87 \text{ centimes.} \\ 1 \text{ franc.} & & \end{array}$$

Reducing the quantities of the same kind to the same denomination.

$$\begin{array}{rrcl} & 113\frac{1}{2} & = & \not{5}\not{5}\ 11 \\ & 2 & = & 2487 \\ 20 & \not{1}\not{0}\not{0} & & \\ \hline & 4540 & & 27357 \end{array}$$

$$\tfrac{4540}{27357} = \$0.16\tfrac{1}{2}$$

Exercises.

1. A trader in London owes a debt of 508 pistoles to one in Cadiz: is it more advantageous to him to remit directly to Cadiz, or circuitously through France. The exchange being £1 = 25. 4 francs, 19 francs = 1 Spanish pistole, 4 Spanish pistoles = £3. Ans. Through France.

2. If the exchange of New York on London is 8 per cent. prem., and that of Amsterdam on London is 12 florins for £1, what is the arbitrated course of exchange between New York and Amsterdam; that is, how many florins are equal to $1 U. S. Ans. $1 = 2½ florins.

INVOLUTION.

252. A power of any number is the product obtained by the continual multiplication of that number, taken a certain number of times as factor.

A number, in relation to any power of it, is called the root of that power.

When the proposed number is used *twice* as factor, the product is called the SECOND POWER, or the SQUARE, of that number; when *three times*, the THIRD POWER, or CUBE; when *four times*, the FOURTH POWER, &c.

Powers are often denoted by writing after the proposed number, a little higher, the number which shows how often the proposed number is repeated as factor. This number is called the INDEX, or the Exponent, of the Power.

Thus 5×5, or 25, is the second power, or square, of 5, and may be written 5^2, where 2 is the index; while $7 \times 7 \times 7 \times 7$, or 2401, is the fourth power of 7, and may be written 7^4, where 4 is the index, &c. Also, 5 is the second or square root of 25, and 7 is the fourth root of 2401.

The method of finding any assigned power of a given number, or as it is also expressed, the method of *raising* a number to any proposed power, is called *Involution*.

253. *To find any assigned power of a given number; to raise a given number to any proposed power.*

RULE.—Find the continual product of the given number repeated as a factor, as often as there are units in the index of the proposed power.

The process may often be abbreviated by multiplying together powers already found. In this case, the index of the power thus found is equal to the *sum* of the indices of the powers multiplied together.

When the given number is either wholly or partly a decimal, the operation may often be much abbreviated, by the rule for contracting the multiplication of decimals given in Art. 119.

Ex. 1. Required the fifth power of 23.

Here, by multiplying 23 by itself, we find 529 for the second power of 23. By multiplying this by 23, we get 12167 for the third power. By proceeding in like manner, we find the fourth power to be 279841, and the fifth to be 6436343.

Multiply	23	= 1st power.
	23	
Multiply	539	= 2d "
	23	
Multiply	12167	= 3d "
	23	
Multiply	279841	= 4th "
	23	
Ans.	6436343	= 5th "

The answer might also have been found by multiplying the second power, 529, by itself, and the product of 279841, which is the fourth power, by 23. The same result would also be obtained by multiplying the third power by the second.

Ex. 2. Required the fifth power of $\frac{3}{8}$. The fifth power of 3 is 243, while that of 8 is 32768. The answer, therefore, is $\frac{243}{32768}$. The *reason* of this is evident from the method of multiplying fractions.

Ex. 3. What is the third power of $1\frac{1}{4}$? This, by reduction to an improper fraction, becomes $\frac{5}{4}$, and by involving the numerator and denominator each to the third power, we find the answer $\frac{125}{64}$, or $1\frac{61}{64}$.

Each of the last examples might have been worked by reducing the fractions to decimals, and then working by the general rule.

Ex. 4. Required the sixth power of 1.12 true to 5 places of decimals.

By raising this to the third power, in the way already shown, we find 1.404928, and this being multiplied by itself as in the margin, we find for the sixth power 1.973822.

1.404928	= 3d power.
829404.1	
1404928	
561971	
5620	
1264	
28	
11	
1.973822	= 6th "

EXERCISES.

Involve the following numbers to the powers denoted by their respective indices.

1.	2880^2	Ans.	8294400	5.	$(\frac{2}{3})^6$	Ans.	$\frac{64}{729}$
2.	135^3	"	2460375	6.	$(2\frac{3}{4})^5$	"	$157\frac{283}{1024}$
3.	9^9	"	387420489	7.	$(3\frac{4}{7})^4$	"	$161\frac{1325}{2401}$
4.	86^5	"	4704270176	8.	1.05^{31}	"	4.538039

EVOLUTION.

254. EVOLUTION is the method of finding, or, as it is usually termed, extracting, an assigned root of a given number.

The INDEX of a root is a fraction whose denomination denotes the order of the root, and whose numerator is unity.

The root of a number is also expressed by prefixing to the number the sign $\sqrt{}$, with the number above it, which denotes the order of the root. In case of the square, or second root, however, the nnmber 2 is omitted.

Thus the fourth root of 16 is denoted by $16^{\frac{1}{4}}$ or $\sqrt[4]{16}$; and means a number whose fourth power is 16.

NOTE.—The sign $\sqrt{}$, called the radical sign, is the letter *r*, the initial of the Latin word *radix*, *a root*, changed in form by rapidity in writing it, and by its appropriation to a particular use.

SQUARE ROOT.

255. The SQUARE ROOT of a given number is a number, which, when multiplied by itself, will produce the given number.

256. The number of figures in the integral part of the Square Root of any whole number may readily be known from the following considerations:

The square root of 1 is 1
100 is 10
10000 is 100
1000000 is 1000
&c. is &c.

Hence it follows that the square root of any number between 1 and 100 must lie between 1 and 10, that is, will have one figure in its integral part; of any number between 100 and 10000, must lie between 10 and 100, that is, will have two figures in its integral part; of any number between 10000 and 1000000, must lie between 100 and 1000; that is, must have three figures in its integral part; and so on. Wherefore, if a point be placed over the units' figure of the number, and thence over every second figure to the left of that place, the points will show the number of figures in the integral part of the root. Thus the square root of 99 consists, so far as it is integral, of *one* figure; that of 198 of *two* figures; that of 176432 of *three* figures; and so on.

257. *To extract the square root of a given number.*

RULE.—Place a point or dot over the units' place of the given number, and thence over every second figure to the left of that place, thus dividing the whole into several periods.

Find the greatest number whose square is contained in the first period at the left; this is the first figure of the root, which place in the form of a quotient to the right of the given number. Subtract its square from the first period, and to the remainder bring down the second period. Divide the number thus formed, omitting the last figure, by twice the part of the root already obtained, and annex the result to the root and also to the divisor. Then multiply the divisor, as it now stands, by the part of the root last obtained, and subtract the product from the number formed, as above mentioned, by the first remainder and the second period. If there be more periods to be brought down, the operation must be repeated; and if any remain, proceed in the same manner to find decimals, adding, to find each figure, two cyphers, or if the given number end in an interminate decimal, the two figures that would next arise from its continuation.

Note.—If there be not whole numbers, or integral part in the given number, we must, in pointing, begin with the second figure from that which would be the units' place, if there were a whole number, and mark successively over every second figure to the right.

Ex. 1. Find the square root of 529.

```
                 5̇29̇(23
                 4
                 ———
{2 × 2 = 4} 43  |129
                |229
                 ———
```

After pointing, according to Rule, we take the first period, or 5, and find the greatest number whose square is contained in it. Since the square of 2 is 4, and that of 3 is 9, it is clear that 2 is the greatest number whose square is contained in 5; therefore place 2 in the form of a quotient to the right of the given number. Square this number, and put down the square under the 5; subtract it from the 5, and to the remainder 1 affix the next period 29, thus forming the number 129. Take 2 × 2, or 4, for a divisor: divide the 129, omitting the last figure, that is, divide the 12 by the 4, and we obtain 3. Annex the 3 to the 2 before obtained and to the divisor 4; then multiply the 43 by the 3 we obtain 129, which being subtracted from the 129 before formed, leaves no remainder; therefore 23 is the square root of 529.

Reason for the above process.

The *principle* on which the above process depends, is, that *the square of the sum of two numbers is equal to the squares of the numbers together with twice their product.* Thus 529 = 500 × 29, and the greatest square in 500 is 400, the root of which is 20, with a remainder of 100; consequently, the first part of the root must be 20, and the true remainder 100 × 29, or 129. And, since there are three figures in the given number, there must be two figures in the root; (Art. 259;) but the square of the sum of two numbers, is equal to the *square* of the first part added to *twice* the product of the two parts and the *square* of the last part; it follows, therefore, that the remainder, 129, must be twice the product of 20 into the part of the root still to found, together with the square of that part. Now, dividing 129 by 40, the double of 20, the quotient is 3, which being added to 40 makes 43; finally, multiplying 43 by 3, the product is 129, which is manifestly twice the product of 20 into 3, together with the square of 3.

In the same manner the operation may be proved in every case.

Ex. 2. Find the square root of 74684164.

74684164 (8642
64

{ $2 \times 8 = 16$ } 166 | 1068
996

{ $2 \times 86 = 172$ } 172 | 7241
6896

{ $2 \times 864 = 1728$ } 17282 | 34564
34564

Instead of doubling the part of the root obtained for a trial divisor, we may add the figure of the root last found to the former divisor, for a new trial divisor: thus

8) 74684164 (8642
8 64

166) 1068
6 996

1724) 7241
4 6896

17282) 34564
34564

258. *To extract the square root of a vulgar fraction.*

Rule.—Reduce it to its simplest form; if it be not so already, and extract the roots of both terms, if they be complete powers; otherwise, find their product, extract its square root, and divide the result by the denominator.

The root may also be found by reducing the fraction to a decimal, and taking the root of the result.

Exercises.

Find the square roots of

1. 2601	Ans.	51	10. $\frac{3}{8}$	Ans.	.61237243
2. 5329	"	73	11. $13\frac{1}{5}$	"	3.63318042
3. 784	"	28	12. 1728	"	41.5692193
4. 566.44	"	23.8	13. $1\frac{3}{8}$	"	1.1726039
5. 7.3441	"	2.71	14. $1\frac{3}{80}$	"	1.01857743
6. .81796	"	.9044	15. 33	"	5.7445626
7. 1169.64	"	34.2	16. $1\frac{13}{36}$	"	1.16′ or $1\frac{1}{6}$
8. .07	"	.26457	17. $6\frac{1}{7}$	"	2.4784789
9. .006	"	.0774597	18. $794\frac{1}{5}$	"	28.1815542

CUBE ROOT.

259. The Cube Root of a given number is a number which, when multiplied into itself, and the result again multiplied by it, will produce the given number. Thus 6 is the cube root of 216; for $6 \times 6 \times 6$ is $= 216$.

260. *To extract the cube root of a given number.*

Rule.—Place in succession, and at moderate intervals, two cyphers and the given number, as the commencements of three columns. Beginning at the unit figure of the given number, point off as many periods as possible, of three figures each. For the first figure of the root, take the root of the greatest cube contained in the left-hand period. Place the figure in the first column; and, having added it to what stands above it, multiply the sum by the same figure, writing the product in the second column. Add, in like manner, in the second column, and multiply the sum by the same figure; set the product in the third column, and subtract it from what stands above it. Perform a process exactly similar in the first and second columns; and, after that, add the figure found for the root, to what stands in the first column. Annex one cypher in the first column, and two in the second; and in the third the next period of the given number; or if there be no figures remaining, annex three cyphers. To find the next figure of the root, divide the number in the third column, by the one in the second. Place this figure in the first column, and proceed in the same manner. Then annex cyphers, &c., and continue the process till nothing remains, or till the root is carried out as far as may be considered necessary.

Care must be taken to insert the decimal point in the root, when the figures in the integral part of the given number have been all employed.

Ex. Extract the cube root of 926859375.

0	0	926859375 (975
9	81	729
9	81	197859
9	162	183673
18	24300	14186375
9	1939	14186375
270	26239	
7	1988	
277	2822700	
7	14575	
284	2837275	
7		
2910		
5		
2915		

Here, the greatest cube contained in the first period, 962, is 729: the root of which is 9, the first figure of the required root. This is placed under the first cypher; and, going through the form adding these, we get 9, the product of which by 9 is set in the second column. Then, by addition, we have 81, the product of which by 9 is 729. This is set under the first period, 926, and subtracted from it; and to the remainder, 197, the second period, 859, is annexed. Then commencing at the first column, we add 9; and multiply the sum, 18, by 9, we set the product, 162, in the second column, and adding it to the number above it, we get 243. We next add 9 in the first column; and annexing one cypher in that column, and two in the second, we finish all that is preparatory to the finding of the second figure of the root.

To find that figure, we divide the number in the third column by the one in the second. The quotient would appear to be 8; this, however, would be found on trial to be too large, and we therefore take 7, which answers. We add this to the first column, and multiply the sum, 277, by 7, setting the product in the second column. Then, by adding, we get 26239, the product of which by 7 is put in the third column. By taking this from the number above it, we find for remainder 14186, to which the third period, 375, is annexed. We then add 7 in the first column, and multiplying the sum by 7, we obtain 1988, which is added to the number in the second column. The operation preparatory to the finding of the third figure of the root, is completed by adding 7 in the first column, and annexing one cypher in it, and two in the second.

To find the third figure, we divide, as before, the number in the third column by the one in the second. We thus obtain 5, which is added to the first column, and the sum, 2915, being multiplied by 5, and the product being added to the number in the second column, the sum, 2837275, is multiplied by 5; and the product being exactly equal to the number in the third column, there is no remainder, and the work terminates, the root being 975.

261. *To extract the cube root of a vulgar fraction.*

Rule.—If, when the fraction is in its lowest terms, the numerator and denominator be exact cubes, extract their roots for the numerator and denominator of the answer. If only the denominator be an exact cube, the answer will be obtained by finding the cube root of the numerator, and dividing it by that of the denominator. If the denominator be not a cube number, multiply the numerator by the square of the denominator; take the cube root of the product, and divide it by the denominator.

In every case, the vulgar fraction may be reduced to a decimal, the root of which taken by the former rule, will be the required root.

The cube root of a mixed number is generally best found, by reducing the fractional part to a decimal, by annexing this decimal to the integral part, and then extracting the root by the general rule.

Exercises.

Find the cube roots of the following numbers.

1. 123	Ans.	4.973190	6. 1234567	Ans.	107.276572
2. 517	"	8.025957	7. 44.6	"	3.546323
3. 900	"	9.654894	8. $\frac{4}{15}$	"	.6436595897
4. 123456789	"	497.933859	9. $\frac{8}{9}$	"	.9614997135
5. 12345678	"	231.120418	10. 376	"	7.217652

COMPOUND INTEREST.

262. The method that naturally presents itself for finding the amount of a sum at compound interest, is to find its amount at simple interest at the end of the first year; then to take this amount as a new principal, and find its amount in like manner, which would be the amount at compound interest at the end of the second year, and the principal for the third year: the amount of which must be found in like manner. Continuing the process, we should thus find the amount at the end of the proposed time. This will be illustrated in the following example.

Ex. 1. Required the amount of £2500 at the end of 4 years at 6 per cent. per annum, compound interest.

Here, the amount for one year is £2650; the amount of which for one year also is £2809, the amount at compound interest for 2 years. The amount of this again, for 1 year, or the amount of the given sum at the end of the third year, is £2977, 10. $9\frac{1}{2}$.; the amount of which for another year is £3156, 3. 10., the amount of £2500 for four years.

When the time is short, this method may be practised without much trouble: but when it is long the labor becomes very great. In this case the following method should be employed.

263. *To find the amount, or the interest, of any sum, at compound interest for a given time, and at a given rate.*

Rule.—Find the *amount* of £1 or $1 for the time of the first payment; raise this amount to such a power as is denoted by the number of payments, and multiply this power by the principal for the amount, from which subtract the principal for the interest.

Ex. 1. What is the amount of £162, 10. for 6 years, payable half-yearly, at 5 per cent. per annum, compound interest?

$$\text{As } \left.\begin{matrix} £100 : £5 \\ 12 : 6 \end{matrix}\right\} :: £1 : £.025. \quad \text{Interest for 6 months.}$$

And $£1 + £.025 = £1.025$, amount of £1 for do.

$£1.025^{12} = £1.3448888.$

$£1.3448888 \times £162, 10. 0. = £218.54443.$

£218, 10. 10. amt. of £162, 10. for 6 years, &c.

The above might have been more easily found thus:

The amount of £100 for 6 months, at 5 per cent., is £102, 10., or £102.5; and consequently that of £1 for the same time is £1.025.

Reason for the above process will appear from the following considerations.

Since the amount of £1 for a year, or any given time, will evidently be a hundredth part of the amount of £100 for the same time: and as £1 is to its amount for a year, or any given time, so is any other principal to its amount for the same time. Hence, to take a particular instance, the amount of £1 for a year at 5 per cent. will be £1.05: and by the nature of compound interest, this will be the principal for the second year. Then, as the principal, £1 : £1.05, its amount : : the principal, £1.05 : $£1.05^2$, the amount at the end of the second year, and the principal for the third. Again, as £1 : £1.05, its amount : : the principal, $£1.05^2$: $£1.05^3$, the amount at the end of the third year, and the principal for the fourth. It will thus appear, that the amount of £1 for any number of years, will be equal to £1.05 raised to the power denoted by the number of years. The amount of £1 being thus determined, it is plain, that the amount of any other principal will be had by multiplying the amount of £1 by that principal, since the amount will evidently be proportional to the principal.

Ex. 2. Required the amount of $780 for 12 years at 5 per cent., per annum, compound interest.

The amount of $1 for a year is the hundredth part of the amount of $100, or $1.05, the twelfth power of which is $1.795856. This being multiplied by $780, the result is 1400.76\frac{7}{10}$, the amount.

Exercises.

Find the amounts of the following sums, at the given rates per cent., per annum:

1.	£251, 16. 6.	for 9 yrs,	at 5	per ct.	&c.	Ans.	£390, 13. 3
2.	£212, 0. 0.	" 15	" 4	"	"	"	£381, 16. 0
3.	$500.00	" 15	" 6	"	"	"	$1198.28
4.	$960.00	" 10	" 7	"	"	"	$1888.46½
5.	£151, 12. 1¼	" 12	" 4¾	"	"	"	£264, 11. 8¼
6.	£1000, 0. 0.	" 22	" 6	"	"	"	£3603, 10. 8½

264. *To find the principal, which, at a given rate, and in a given time, would amount to a given sum.*

Rule.—Divide the given sum by the amount of £1 or $1 for the given time.

Ex. What sum must be lent out at compound interest, at 5 per cent. per annum, so as to amount to £3000, 6. 8.

at the end of 21 years? Here the amount of £1 for 21 years being 2.785962, we have for answer £3000.3′ ÷ 2.785962 = £1076.9469 = £1076, 18. 11¼.

For exercises to this rule, prove those of the former.

MISCELLANEOUS EXERCISES.

1. The sum of two numbers is 980, and their difference 62: what are the numbers? Ans. 459 and 521.

2. What number multiplied by $28\frac{5}{7}$, will produce 145? Ans. $5\frac{10}{201}$.

3. If an army of 24000 men have 520000 lbs. of bread, how long will it last them, allowing each man $1\frac{1}{12}$lb. per day? Ans. 20 days.

4. For what sum must a note be drawn, payable in 4 months, the proceeds of which shall be $1800, discounted at a bank at 7 per cent.? Ans. $1843.003.

5. A merchant bought 500 yds. of cloth for $1800: how must he retail it by the yard to gain 25 per cent.? Ans. $4.50.

6. A man bought 640 lbs. of beef for £1250, and sold it at a loss of 12 per cent.: how much did he get a barrel? Ans. £1, 14. $4\frac{1}{2}$.

7. How many dollars, each weighing $412\frac{1}{2}$ grs., can be made from 16 lbs. 5 oz. of silver? Ans. $229\frac{13}{55}$.

8. How many revolutions will the hind wheel of a carriage 5 ft. 6 in. in circumference, make in 2 miles 4 furlongs? Ans. 2400.

9. What cost $15\frac{2}{5}$ lbs. of cheese, at $8\frac{5}{8}$ per hundred pounds? Ans. $1.328.

10. How many yards of carpeting $\frac{3}{4}$ yd. wide will it take to cover a floor 18 ft. long and 15 ft. wide? Ans. 40 yds.

11. If a man travelling 14 hours per day, perform half his journey in 9 days, how long will it take him to go the other half travelling 10 hours a day? Ans. $12\frac{3}{5}$ days.

12. If 150 apples cost 9s. $4\frac{1}{2}$d., how many of them must be sold at the rate of 8 for $6\frac{1}{2}$d., and how many at the rate of 3 for $2\frac{1}{2}$d., that the gain on the whole may be 10 per cent.? Ans. 90 at 3 for $2\frac{1}{2}$d., and 60 at 8 for $6\frac{1}{2}$d.

13. A merchant engages a clerk at the rate of £20 for the first year, £25 for the second, £30 for the third, &c., thus augmenting his salary by £5 each year. How long must the clerk retain his situation so as to receive on the whole as much as he would have received had his salary been fixed at £52, 10. per annum? Ans. 14 years.

14. A son asked his father's age, the father replied: "Your age is twelve years; to which if five-eighths of both our ages be added, the sum will be equal to mine." What was the father's age? Ans. 52 years.

15. If one person lies in bed 9 hours per day, and another 6 hours, how much time will the one gain over the other in 20 years? Ans. 2 y. $182\frac{1}{2}$ d.

16. A man and his wife can drink a barrel of beer in 30 days, and the man alone can drink it in 40 days: how long will it last the wife? Ans. 120 days.

17. A teacher being asked how many scholars he had, replied, $\frac{1}{4}$ study Arithmetic, $\frac{1}{5}$ study Latin, $\frac{3}{10}$ study Algebra, $\frac{1}{20}$ study Geometry, and 24 study French. How many scholars had he? Ans. 120.

18. If A could reap a field in 13 days, and B in 16 days, in what time would both together reap it?

Ans. In $7\frac{5}{29}$ days.

19. Suppose 17 gallons of spirits, at 10s. 6d. per gallon, to be mixed with 7 gallons at a different price. What was the price of the latter per gallon, if 20 per cent. be gained by selling the mixture at 13s. per gallon? Ans. 11s. $7\frac{5}{7}$d.

19. If gold be beaten out so thin that an oz. avoirdupois will cover 20 square yards, how many leaves of this thickness will make an inch thick, the weight of a cubic foot of gold being 10 cwt. 95 lbs.? Ans. 291600 leaves.

20. A man owes a debt, to be paid in four equal instalments at the end of 4, 9, 12, and 20 months respectively; and he finds, that discount being allowed, according to the true method, at 5 per cent., per annum, \$3000 paid at present will discharge the whole debt. How much did he owe? Ans. \$3136$\frac{3812776}{5284036}$.

21. The liabilities of a bankrupt are \$63240, and his assets \$12648: what per cent. can he pay?

Ans. 20 per cent.

22. Four men, A, B, C, and D, spent £255, and agreed that A should pay $\frac{3}{8}$; B $\frac{1}{4}$: C $\frac{1}{2}$; and D $\frac{3}{4}$. How much must each pay? Ans. A £51; B £34; C £68; D £102.

23. A man wished to tie his horse by a rope so that he could feed on just an acre of ground. How long must the rope be? Ans. 7.13645 r.

24. How many bushels will a cubical bin contain whose side is 9 feet? Ans. 585.80357 bush.

25. In how many years will the error of the Julian calendar involve the loss of a day? Ans. 128$\frac{4}{7}$ yrs.

26. A contractor sends in a tender of £5000 for a certain work; a second sends in a tender of £4850, but stipulates to be paid £500 every three months; find the difference of the tenders, supposing the work in both cases to be finished in two years, and money to be worth 4 per cent. simple interest. Ans. Diff. = £10.

27. Exchange between London and Paris is 25.5 francs per pound sterling; between Paris and Amsterdam is 117 francs for 55 florins; between Amsterdam and Hamburg is 11 florins for 13 marks; what is the exchange between London and Hamburg? Ans. 14$\frac{1}{6}$ marks.

28. What must A bequeath to B so that B may receive £1000, after a legacy duty of 10 per cent. has been deducted? Ans. £1111, 2. 2$\frac{2}{3}$.

29. A railway train travels 27 miles per hour, including stoppages, and 30 miles per hour when it does not stop; in what distance will it lose 20′ by stopping?

Ans. 90 miles.

30. The expenses of constructing a railway is £2,000,000, of which $\frac{1}{4}$th part was borrowed on mortgage at 5 per cent. and the remaining $\frac{3}{4}$ths was held in shares; what must be the average weekly receipts so as to pay the shareholders 6 per cent., the expenses of working the railroad being 45 per cent. of the gross receipts?

Ans. £4020, 19. 6$\frac{3}{4}$.

31. How much sugar at 6d. and 8d. per lb. must be mixed with 12 lbs. at 7d., 16 lbs. at 9$\frac{1}{4}$d., and 20 lbs. at 10d., that the mixture may sell at 8$\frac{1}{2}$d.?

Ans. 8 lbs. of each.

32. A cistern can be filled by a pipe in 20 minutes, and emptied by another in 25 minutes; in what time may the cistern be filled, when empty, by opening both pipes at the same time? Ans. I hour 40 min.

33. A market-woman bought 120 apples at 2 a penny, and 120 more at 3 a penny, but not liking her bargain, she offered them to her neighbor at 5 for 2 pence, being what they cost her, she said. Did she gain or lose by this transaction, and how much? Ans. Lost 4 pence.

34. Sold cloth at 15s. 4d. per yard by which I lost 8 per cent., whereas I ought to have gained 15 per cent.; how much was it sold at per yard below value?

Ans. 3s. 10d.

35. In what time will £2500 double itself at 4 per cent. simple interest? Ans. 25 years.

36. Of 138,918 persons, 30.66 per cent. can read and write; 58.89 per cent. can do neither; and the rest can only read; find the numbers in each class.

Ans. 42592.2588 can read and write; 81808.8102 can do neither; 14516.931 can read only.

37. Find the value of 57 kilogr. 8 decagr. 4 grains of any article which cost £17, 11. 4$\frac{4}{5}$ per kilogramme. Express the result in dollars and cents. Ans. \$4011.86$\frac{1}{4}$.

38. Find the squares of 1039681 and 328776; and divide the greater result by the less, to the first significant figure in the decimal places. Ans. 10.000000000009.

39. How much in the 3 per cents. at 96 must be sold out to pay a bill of £1654, 9 months before it becomes due, true discount being allowed at 4½ per cent. per annum?

Ans. £1666, 13. 4.

40. How much ought the price of the three per cent. consols to sink below par, in order that a broker may be enabled to obtain four per cent. on money.

Ans. 25 per cent.

41. The cost of carpeting a room twice as long as it is broad at 5s. per square yard amounted to £6, 2. 6.; and the painting of the walls at 9d. per square yard amounted to £2, 12. 6. Find the height of the room. Ans. 10 ft.

ADDITION.

(PAGES 40—45.)

Ex. 1. Find the sum of 4738685,237869513,14879434-3978, 865, 4647,250,68539582,78602045,370489000,70555-91284,276,9123456789,5000.

Ans. 165733641864.

2. Add together 629405,7629,31000401,263012,13005-12,390217,13268.

Ans. 33604444.

3. What is the sum of 6457,29301,82406,7589,63489, 101364,46745?

Ans. 337351.

4. Add together 432678902,310046734,2167005,327861, 293000428.

Ans. 1038220930.

5. Add together 493742,56710607,23461,400072,681-1004,8999003,26501.

Ans. 73464390.

6. Add the following amounts, $2425, $3282, $2793, $2354, $4262, $9158, $2653, $3424, $1266, $8742, $2126, $5387.

Ans. $47872.

7. Required the amount of the following: $46519, $32-271, $17436, $81587, $28333, $52745, $23052, $20158, $71-232, $39467, $18643, $42027, $73235, $24103.

Ans. $570808.

8. Required the amount of the following: £421536, £310101, £797019, £233680, £124402, £255353, £8520-57, £618041, £100266, £971134, £536920, £703352, £420-503, £312675.

Ans. £6657039.

9. 607253 + 232012 + 211849 + 380436 + 578551 + 231-349 + 145763 + 605037 + 760155 + 357676 + 544844 + 276-232 + 803383 + 725918.

Ans. 6460458.

10. Add together one thousand, four hundred and eighty-three; seven hundred and ninety-six; thirty-nine; forty thousand, seven hundred and forty-four; five thousand, eight hundred and sixty; fifty thousand and seven.

Ans. 98929.

11. Add together the following numbers: twenty-two millions, six hundred thousand, five hundred and three; five hundred and sixty-three millions, seventy-six thousand and thirty-four; one hundred and eleven millions, six hundred and fifty thousand and fifty; three hundred and twenty-six millions, seven thousand, nine hundred and ninety one; one billion, seven hundred and ten millions, one thousand, seven hundred and ten; one billion, three hundred thousand and five. Ans. 3733636293.

SUBTRACTION.

(PAGES 45—49.)

Ex. 1. From 9876102 take 1050671. Ans. 8825431

2. From 20030000 take 72534. Ans. 19957466

3. From 34200591 take 8888888. Ans. 25311703

4. From 95246300 take 9438675. Ans. 85807625

5. From 96531768 take 873625. Ans. 95658143

6. From 6764+3764 take 6500+2430. Ans. 1598.

7. From 6008+9270 take 5136 — 2352. Ans. 12494.

8. From 9687—3401 take 3021+1754. Ans. 1511.

9. A gentleman worth $163,250 bequeathed $15200 to each of his two sons, $16500 to his daughter, and to his wife as much as his three children, and the remainder to a hospital: how much did his wife receive, and how much the hospital?

Wife $46900.
Hospital $69450.

10. A speculator gained $3560, and afterwards lost $2500; at another time he gained $6283, and then lost $3450; how much more did he gain than lose? Ans. $3893.

11. Subtract five hundred and eighty-four thou-

sand and seventy-six, from fifteen millions, one hundred thousand and three.

Ans. 14515927.

12. A man bought a house for MDCCCC XXXVII dollars, and sold it for DCXVIIII dollars less than he gave: how much did he sell it for?

Ans. $1318.

MULTIPLICATION.

PAGES 49 — 64.

Ex. 1. Multiply 61835720 by 1320. Ans. 81623150400.

2. Multiply 67243 by 99999. Ans. 6724232757.

3. Multiply 31890420 by 85672.

Ans. 2732116062240.

4. Multiply 2364793 by 8485672.

Ans. 20066857745896.

5. Multiply 1256702 by 999999.

Ans. 1256700743298.

6. Multiply 73084163 by 7584.

Ans. 554270292192.

7. Multiply 79548050 by 97280.

Ans. 7738434304000.

8. Multiply eight hundred and seventy-seven millions, five hundred and ten thousand, eight hundred and sixty-four, by five hundred and forty-five thousand, three hundred and fifty-seven. Ans. 478556692258448.

9. Find the continued product of 6565, 6786 and 9898. Ans. 440956790820.

10. A convoy of army bread, consisting of 896 wagons, and each wagon containing 18600 loaves, having been intercepted by the enemy, what is the number of loaves lost? Ans. 16665600.

DIVISION.

(PAGES 64 — 77.)

Ex. 1. Divide 72091365 by 5201. Ans. $13861\frac{304}{5201}$.

2. Divide 4637064283 by 57606. Ans. $80496\frac{11707}{57606}$.

3. Divide 353008972662 by 5406. Ans. 65299477.

4. Divide 67157148372 by 90009.
Ans. $746115\frac{83337}{90009}$.

5. Divide 908070605040 by 654321.
Ans. $1387805\frac{649635}{654321}$.

6. Divide 65358547823 by 2789.
Ans. $23434402\frac{645}{2789}$.

7. Divide 3968901531620 by 687637943.
Ans. $5771\frac{542962567}{687637943}$.

DIVISION BY COMPOSITE NUMBERS.

Ex. 1. Divide 41763481 by 625. Ans. $66821\frac{356}{625}$.
2. Divide 4276318 by 450. Ans. $9502\frac{209}{225}$.
3. Divide 618374121 by 125. Ans. $4946992\frac{121}{125}$.
4. Divide 126789432 by 9072. Ans. $13975\frac{8232}{9072}$.
5. Divide 9060708093 by 24192. Ans. $374533\frac{5757}{24192}$.

MULTIPLICATION AND DIVISION OF FRACTIONAL NUMBERS.

(PAGES 77—78.)

Ex. 1. Multiply 27346 by $2\frac{1}{2}$. Ans. 68365.
2. Multiply 477121 by $1\frac{1}{2}$. Ans. $715681\frac{1}{2}$.
3. Multiply 567893 by $3\frac{11}{12}$. Ans. $2224247\frac{7}{12}$.
4. Multiply 5691036 by $7\frac{4}{5}$. Ans. $44390080\frac{4}{5}$.
5. Divide 25974 by $17\frac{1}{2}$. Ans. $1484\frac{8}{35}$.
6. Divide 367849 by $12\frac{1}{3}$. Ans. $29825\frac{22}{37}$.
7. Divide 2906709 by $121\frac{1}{9}$. Ans. $24000\frac{381}{1090}$.
8. Divide 4632987 by $133\frac{2}{3}$. Ans. $34660\frac{301}{401}$.

GREATEST COMMON MEASURE.

(PAGES 86—89.)

Find the greatest Common Measure of

1. 105 and 165. Ans. 15.
2. 285 and 465. Ans. 15.
3. 1879 and 2425. Ans. 1.
4. 108, 126 and 162. Ans. 18.
5. 140,210, and 315. Ans. 35.
6. 285714 and 999999. Ans. 142857.
7. 324, 612 and 1032. Ans. 12.
8. 16, 24, 48 and 74. Ans. 2.
9. 837, 1134 and 1347. Ans. 3.

10. 28, 84, 154 and 343. Ans. 7.
11. 396, 5184 and 6914. Ans. 2.
12. 56, 84, 140 and 168. Ans. 28.

LEAST COMMON MULTIPLE.

(PAGES 89 — 93.)

Find the least Common Multiple of the following numbers.

Ex. 1. 8, 16, 18 and 24. Ans. 144.
2. 9, 15, 12, 6 and 5. Ans. 180.
3. 5, 10, 8, 18 and 15. Ans. 360.
4. 36, 25, 60, 72 and 35. Ans. 12600.
5. 27, 54, 81, 14 and 63. Ans. 1134.
6. 9, 12, 72, 36 and 144. Ans. 144.
7. 8, 12, 20, 24 and 25. Ans. 600.
8. 63, 12, 84 and 7. Ans. 252.
9. 72, 120, 180, 24 and 36. Ans. 360.
10. 256, 512 and 1728. Ans. 13824.
11. 375, 850 and 3400. Ans. 51000.
12. 225, 255, 289, 1023 and 4095. Ans. 2017790775

DECIMALS.

ADDITION OF DECIMALS.

(PAGES 98 — 99.)

Ex. 1. Add together 467.3004; 28.78249; 1.29468; and 3.78241. Ans. 501.15998.

2. Add together 21.6434; 800.7; 29.461; 1.7506 and 3.45. Ans. 857.005.

3. Add together 45.001; 163.4234; 20.3045; 924.00369 and 62. 1346. Ans. 1214.86719.

4. Add together 293.0072; 89.00301; 29.84567; 924.00369 and 72.39602. Ans. 1408.25559.

5. Add together 1.721341; 8.620047; 51.720345; 2.684 and 62.304607. Ans. 127.05034.

6. 1.293062 + 3.00042 + 9.7003146 + 3.600426 + 7.0040031 + 8.7200489. Ans. 33.3182746.

7. 394.61 + 81.928 + 3624.8103 + 640.203 + 6291.302 + 721,004 + 3920.304. Ans. 15674.1613.

8. Add together 25 hundredths, 8 tenths, 65 thousandths, 16 hundredths 142 thousandths, and

89 hundredths. Ans. 1.807.

9. Add together 9 tenths, 92 hundredths, 162 thousandths, 489 thousandths, and 92 millionths. Ans. 2.471092.

10. Add together 95 thousandths, 61 millionths, 6 tenths, 11 hundredths, and 265 hundred-thousandths. Ans. 0.807711.

11. Add together 96 hundred-thousandths, 92 millionths, 25 hundredths, 45 thousandths, and seven tenths. Ans. 0.996052.

12. Add together five hundred and nine hundredths; three hundred and seventy-five; twenty thousand and eighty-four, and seventy-eight hundred thousandths; eleven millions, two thousand, and two hundred and nine millionths; eleven hundred-millionths; one billion, and one billionth.
Ans. 1011022464.090989111.

SUBTRACTION OF DECIMALS.

Page 99.

Ex. 1. From .0516 take .0094187. Ans. .0421813.

2. From 17.5 take 13.0046. Ans. 4.4954.

3. From 456.0546 take 364.3123. Ans. 91.7423.

4. Take 57.704 from 713.00683. Ans. 655.30283.

5. From 1460.39 take 32.756218.
Ans. 1427.633782.

6. Take 9.163 from 81.6823401. Ans. 72.5193401.

7. What is the difference between 25 and .25?
Ans. 24.75.

8. What is the difference between 3.29 and .999?
Ans. 2.291.

9. What is the difference between 10 and .00000001? Ans 9.99999999.

10. From one thousandth take one millionth.
Ans. 0.000999.

11. From one billionth take one trillionth.
Ans. 0.000000000999.

12. From 8436 hundred-millionths take 426 ten-billionths. Ans. 0.0000843174.

MULTIPLICATION OF DECIMALS.

(PAGES 99 — 104.)

To be proved, true to 6 places of decimals, by Rule Art. 120, page 103 —

Ex. 1. Multiply 40 . 4369 by 1 .2904.
Ans. 52 . 17977576.

2. Multiply 100 . 0008 by . 000306.
Ans. 0 . 0306002448.

3. Multiply 75 . 35060 by 62 . 3906.
Ans. 4701 . 169144360.

4. Multiply 31, 50301 by 17 . 0352.
Ans. 536 . 660075952

5. Multiply 0 . 000713 by 2 . 30561.
Ans. 0 . 00164389993.

6. Multiply 42 . 10062 by 3 . 821013.
Ans. 160 . 86701632806.

7. Multiply 0 . 834567834 by . 00000008.
Ans. 0 . 00000006676542672.

8. Multiply 840003 . 1709 by 112 . 10371.
Ans. 94167471 . 869654039.

9. Multiply 65 ten thousandths by 1000. Ans. 6 . 5.

10. Multiply 248 thousandths by 10,000.
Ans. 2480.

11. Multiply 2564 . 21035 by 4 . 300506.
Ans. 11027 . 40199543710.

12. Multiply 446 . 3214032 by 10 000.
Ans. 4463214 . 032.

DIVISION OF DECIMALS.

(PAGES 104 — 108.)

Ex. 1. Divide 123 . 70536 by 54 . 25. Ans. 2 . 2802+.

2. Divide 4 . 00334 by 6 . 31. Ans. 0 . 6344+.

3. Divide 73 . 8243 by . 061. Ans. 1210 . 2344+.

4. Divide 0 . 00033 by . 011. Ans. 0 . 03.

5. Divide 684234 . 6 by 2682. Ans. 255 . 1210+.

6. Divide 46 . 634205 by 4807 . 65. Ans. 0 . 0097.

7. Divide 176432 . 76 by . 01257.
Ans. 14036019 . 0930.

8. Divide 1 . 157625 by 1 . 05×1 . 05. Ans. 1 . 05

9. Divide 32 . 86164 by 7 . 6×0 . 071. Ans. 60 . 9.
10. Dvide 8444 . 443752 by 6 . 84. Ans. 1234 . 5678.
11. Divide 5 . by 1024. Ans. 0 . 0048828125.
12. Divide 134642 . 156 by 1622 . 2. Ans. 82 . 9997+.

MISCELLANEOUS EXERCISES ON THE FOREGOING RULES.

Ex. 1. The sum of two numbers is 980, and their difference 62 : what are the numbers ? Ans.459 and 521.

2. The product of two numbers is 4410, and one is 63 : what is the other ? Ans. 70.

3. What number multiplied by $28\frac{5}{7}$ will produce 145 ? Ans. $5\frac{10}{201}$.

4. What number multiplied by $6\frac{1}{2}$, will produce the product of $7\frac{1}{4}$ multiplied by $5\frac{1}{2}$? Ans. $6\frac{7}{52}$.

5. If an army of 24000 men have 520000 lbs. of bread, how long will it last them, allowing each man $1\frac{1}{12}$ lbs. per day ? Ans. 20 days.

6. The age of the father and son together is 60 years and if 18 be taken from the father's age and added to the son's their ages will be equal : what is the age of each ? Ans. Father 48, son 12.

7. From a certain sum 152 persons took $17 each, and there remained $13 : what was the sum ? Ans. $2597.

8. If you multiply a certain number by 7 you will augment it 1548 ; required the number. Ans. 258.

9. Two lines are 41 . 06328 and . 0438 of an inch long respectively. How many lines as long as the latter can be cut off from the former ? What will be the length of the remaining line. Ans. 937 lines.
Ans. . 02268 in.length of remaining line.

REDUCTION.

REDUCTION OF STERLING MONEY.

(PAGE 114)

Ex. 1. In 4927 farthings how many pounds, shillings and pence ? Ans. £5. 2. $7\frac{3}{4}$

2. In 189345 pence how many florins ?
Ans. $7889\frac{1}{8}$ florins.

3. Reduce 2673 half guineas to farthings, and $22\frac{1}{2}$ guineas to sixpences. Ans. 1347192 far.
945 sixpences.

4. How many half crowns, how many sixpences and how many fourpences are there in £263 . 10 . 0.
Ans. 2108 half crowns.
10540 sixpences.
15810 fourpences.

5. In 1896784 farthings how many pounds, shillings and pence? Ans. £1975. 16. 4.

6. Reduce 128000 farthings to pounds, &c.
Ans, £133. 6. 8.

7. Reduce £3769. 16. 0 to shillings. Ans. 75396.

8. In 15624 half pence how many guineas?
Ans. 31 guineas.

9. How many farthings are there in 36 guineas.
Ans. 36288.

10. Reduce 1738 half crowns to farthings.
Ans. 208560.

CURRENCY OF NOVA SCOTIA, &c.

(PAGES 180—183.)

Ex. 1. Reduce $127\frac{1}{2}$ florins to pence. Ans. 3825.

2. Reduce £186. 14. $6\frac{1}{2}$ to farthings. Ans. 179258.

3. In 486 doubloons how many pence? Ans. 466560

4. In £1867. 13. $7\frac{1}{2}$ how many dollars and cents?
Ans. $\$7470.72\frac{1}{2}$.

5. Reduce 18967 florins to dollars. Ans. $9483.50.

6. Reduce $\$1789.33\frac{1}{3}$ to £. S. D. Ans. £447. 6. 8.

7. Reduce £176. 13. 9. P. E. Island currency to Nova Scotian currency. $\$588 \cdot 95\frac{5}{8}$.

8. Reduce £16.13.$4\frac{1}{2}$ N. F. Land currency to Nova Scotian currency at par? Ans, $69,453.

9. Reduce $1896.30 Nova Scotia currency to New Brunswick currency. Ans. $\$1845.73\frac{1}{3}$.

10. Reduce £196. 15. 6 Sterling to N. Scotia currency at par. Ans. $\$983.87\frac{1}{2}$.

11. How many £. S. D. are there in 415739 farthings. Ans. £433. 1. $2\frac{3}{4}$.

12. Reduce 150233 Mills to Dollars.
Ans. $\$150.23\frac{3}{10}$.

13. Reduce \$4896.62½ N. S. currency to Canada currency, also to P. E. Island currency.
Canada; \$4766.04$\frac{8}{10}$.
P. E. I. £1468.19.9.

14. How many farthings are thero in 5 half sovereigns, 5 half-crowns, 5 sixpences, and 5 half-pence sterling? Ans. 3130 Stg.
3912½ N. S.

REDUCTION.

MEASURE OF LENGTH.

(PAGES 136—139.)

Ex. 1. Reduce 290375 feet to miles,
Ans. 54 m. 7 l. 38 p. 2 y. 2 ft.

2. In 1081080 inches, how many miles, &c.,
Ans. 17 miles 20 rod.

3. How many nails in 160 yards?
Ans. 2560 na,

4. Reduce 223267 nails to French ells,
Ans. 9302 e. 4 q. 3 n.

5. How many inches in 1000 English ells?
Ans. 45000.

6. How many feet are in the circumference of the earth, supposing it to be 25020 miles? Ans. 132105600.

7. Reduce 326 miles, 27 per., 2 yd., 1 ft.. 6 in., to yards. Ans. 573911.

8. In 4 miles, 3 fur., 21 po., 3 yd., 0 ft., 6 in.. how many feet? Ans. 23456.

9. Reduce 7 fur., 200 yards to chains,
Ans. 79 chains 2 yds.

10. In 46785 chains, how many Irish miles?
Ans. 459 miles, 3 fur., 38 po., 4 y.

11. Reduce 126 miles, 3 fur., 6 in., Irish measure to inches. Ans. 10190880 inches.

12. Reduce 1000007 inches to miles,
Ans. 15 m., 6 fur., 10 po., 2 y., 2 f., 11 in.

MEASURE OF SURFACE.

(PAGES 140—144.)

Ex. 1. Reduce 25363896 sqr. feet to acres, &c..
Ans. 582 a., 1 r., 3 po., 269½ sq. feet.

2. Reduce 892 590 sq. rods to square inches.
Ans. 32640858360.

3. Reduce 1728 sq. rods, 23 yards, 5 feet, to feet.
Ans. 470660.

4. In 1239493$\frac{3}{8}$ yards, how many acres, &c.?
Ans. 256 ac. 15 per.

5. In 22 acres, 3 rods, 33 poles, 2$\frac{3}{4}$ yards, how many yards? Ans. 111111.

6. Reduce 15 ac., 3 ro. to links. Ans. 1575000.

7. Reduce 312 ac. 2 ro. to poles. Ans. 50000.

8. Reduce 189678 links to acres, &c.
Ans. 1 ac. 3 ro. 23 po. 14$\frac{3}{4}$ yards nearly.

9. In 49 ac. 28 po. 10 yds. 8 ft. 112 inches, how many square inches? Ans. 308471296.

10. Reduce 308471296 inches to acres, &c.
Ans. 49 ac. 28 po. 10 yds. 8 ft. 112 in.

11. A piece of land is 12964 perches long and 1$\frac{1}{2}$ perches wide, how many acres, &c. are there in it?
Ans. 121 ac, 2 ro. 6 po.

MEASURE OF SOLIDITY.

(PAGES 147 — 150.)

Ex. 1. In 150 cubic feet, how many inches?
Ans. 259200.

2. In 97 yards, 15 feet, how many cubic inches?
Ans. 4551552.

3. In 49 cords, 23 feet, how many cubic inches?
Ans. 10877760.

4. Reduce 84673 cubic inches to feet.
Ans. 49 feet 1 inch.

5. In 39216 cubic feet, how many cords?
Ans. 306$\frac{3}{8}$.

6. In 65 tons of round timber, how many cubic inches? Ans. 4492800.

7. In 4562100 cubic inches, how many tons of hewn timber? Ans. 52 tons, 40 feet, 180 inches.

8. Reduce 88459776 cubic inches, to yards.
Ans. 1896.

9. Reduce 157248 cubic in. to yards.
Ans. 3 cubic yards, 10 feet.

10. There are three piles of wood, the first con-

tains 1220 feet, the second 45986 feet and the third 3290 feet, how many cords are in all? Ans. 394½.

ARTIFICERS, WORK AND DUODECIMAL MULTIPLICATION.

(PAGES 144—147.)

Ex. 1. How many square feet are there in a piece of marble 15 feet, 7 in. long, and 1 ft, 10 in, wide,

Ans. 28 feet, $6'$. $10''$, or 28 feet 82 inches.

2. How many cubic feet in a stick of timber 15 ft. 3 in. long, 2 ft. 4 in. wide, and 1 ft. 8 in. thick?

Ans. 59 ft. $3'$. $8''$, or 59 ft. 528 in.

3. How many cubic feet in a block of granite 18 ft. 5 in. long, 4 ft. 2 in. wide, and 3 ft. 6 in. thick?

Ans. 268 feet, $6'$, $11''$, or 268 feet, 996 in.

4. How many square feet in a stock of 10 boards, 15 ft. 8 in. long, and 1 ft., 6 in. wide? Ans. 235 feet.

5. How many square feet in a stock of 15 boards, 20 feet 3 inches long, and 2 feet 5 inches wide?

Ans. 734 feet, $0'$. $9''$, or 734 feet, 9 inches.

6. How many squares of flooring in a house of 3 stories, 40 ft. 6 in. long, and 35 ft. 9 in. wide, allowing stair-way in all the flats, 9 ft. by 6 ft. 8 in. and places for 2 chimneys, 2 ft. 6 in. by 4 ft. Ans. 41 squares, 14 ft. 126 in.

7. Multiply 16 ft. $3'$, $4''$ by 6 ft. $5'$, $8''$, $10'''$.

Ans. 105 ft. $5'$. $4''$, $5'''$, $5''''$, $4'''''$.

8. Multiply 20 ft., $4'$, $8''$, $5'''$, by 7 ft., $6'$, $9''$, $4'''$,

Ans. 154 ft., $3'$, $1''$, $5'''$, $4''''$, $6'''''$, $8''''''$.

9. How many cords of wood in a pile 50 ft., 6 in. long, 8 ft, 3 inches wide, and 7 ft. 4 in. high?

Ans. 23 c., 111 ft., 36 in.

10. If a cistern is 30 ft., 10 in. long, 12 ft. 3 in. wide, and 10 feet 2 in. deep, how many cubic feet, &c. will it contain? Ans. 3840 ft., $0'$, $5''$ or 3840 ft. 5 in.

11. What will it cost to plaster a room 20 ft. 6 in. long, 18 ft. wide, and 10 ft. high at 12½ cents per square yard? Contents, Ans. 126⅝.

Cost $15 . 819+.

12. The soil taken from a certain mound covered an acre of land to the depth of 2 feet; how many cubic yards of earth were there? Ans. 3226⅔.

MEASURE OF CAPACITY.

(PAGES 150—151.)

Ex. 1. In 15 bushels, 1 peck, how many quarts? Ans. 488.

2. In 763 bus. 3 pks. how many qts? Ans. 24440.

3. In 56 quarters, 5 bushels, how many pints? Ans. 28992 pints.

REDUCTION.

Ex. 4. Reduce 45672 quarts to bushels. &c. Ans. 1427 bushels, 1 peck.

5. Reduce 260200 pints to quarts. Ans. 130100.

6. In 3674 gals. how many cubic inches? Ans. 1018704·676.

7. How many cubic feet in the hold of a ship, which contains 1000 bushels imperial? Ans. 1283 ft. 1168 inches.

8. How many bushels of coals will a box hold which is 8 ft. long, 4 ft. wide, and 4 ft. 1½ inch deep? Ans. 81.015 nearly.

9. In 186043 pecks how many chaldrons? Ans. 1291 ch. 34 bushs. 3 pecks.

10. Reduce 365843 gills to gallons. Ans. 11432 gals. 1 qt, 3 gills.

MEASURE OF WEIGHT.

TROY WEIGHT.

(PAGE 152)

Ex. 1. Reduce 29 lbs., 7 oz., 3 dwts. to grains. Ans. 170472.

2. Reduce 37 lbs. 6 oz. to dwts. Ans. 9000.

3. Reduce 175 lbs., 4 oz., 5 dwts., 7 grs. to grs. Ans. 1010047.

4. Reduce 12256 grains to pounds, &c. Ans. 2 lb. 1 oz. 10 dwts. 16 grs.

5. In 42672 dwts. how many pounds, &c. Ans. 177 lbs. 9 oz. 12 dwts.

6. In 167040 grains, how many pounds? Ans. 29 lbs.

7. Reduce 11 oz. 12 dwt. 12 grs. to grains. Ans. 5580.

8. How many grains are there in 18 lbs. 3 oz. 16 dwt.; and what will the cost amount to at $3\frac{1}{2}$ cents per grain? Ans. $3692 64.

9. How many papers of $4\frac{1}{2}$ grains each can be made out of 1 lb., 5 oz., 15 dwt., 12 grs.? Ans. 1896.

10. In 700 lbs. troy of silver, how many pounds avoirdupois? Ans. 576 lbs. avoir.

11. In 840 lbs., 6 oz., 10 dwt.; how many pounds, &c., avoirdupois? Ans. 691 lbs., 10 oz., 5 $\frac{43}{175}$ drms.

12. A merchant bought 1500 lbs. of lead, troy weight, and sold it by avoirdupois weight; how many pounds did he lose?

Ans. $265\frac{5}{7}$ lbs. troy, or 218 $\frac{158}{245}$ lbs. avoir.

APOTHECARIES WEIGHT.

(PAGE 153.)

Ex. 1. In 130 pounds, how many scruples?
Ans. 37440.

2. In 6237 drams how many pounds?
Ans. 24 lbs. 5 oz., 13 drs.

3. Reduce 25463 scruples to ounces, &c.,
Ans. 1060 ℥, 7 ʒ, 2 ℈.

4. Reduce 11 lbs., 10 ℥, 6 ʒ, 1 ℈, to grains,
Ans. 68540.

5. Reduce 17 lbs., 2 ℥, 2 ℈, to grains, Ans. 98920.

6. In 34678 grs. how many ounces?
Ans. 72 ℥, 1 ʒ, 2 ℈, 18 grs.

7. What is the weight of 186 powders, each made up of three ingredients, $1\frac{1}{2}$, $2\frac{1}{4}$ an $1\frac{1}{4}$ grs. respectively?
Ans. 1℥, 7ʒ, 1℈, 10 grains.

AVOIRDUPOIS WEIGHT.

(PAGES 155—157.)

Ex. 1. In 16256 ounces, how many hundred weights, old or long weight? Ans. 9 cwt., 0 qrs., 8 lbs.
New. 10 cwt., 16 lbs.

2. Reduce 1876338 drams to hundred weights, &c., of both kinds. Ans., old, 65 cwt., 1 qr., 21 lbs. 7 oz. 2 drs.
New, 73 cwt., 29 lbs., 7 oz., 2 drs.

3. Reduce 18967 lbs. to tons.
Ans., old, 8 tons, 9 cwt., 1 qrs., 11 lbs.
New, 9 tons, 9 cwt., 67 lbs.

4. Reduce 137 tons old weight to pounds,
Ans. 306880.

5. Three loads of hay weighed 1896, 4327 and 1234 pounds respectively, how many tons are there in all?
Ans. old 3 tons, 6 cwt., 2 qrs., 9 lbs.
Ans. new 3 tons, 14 cwt., 2 qrs., 7 lbs.

6. A person purchased 16 cwt., 3 qrs., 15 lbs., new weight, of pork at 5½ cents per pound, what did it cost him? Ans. $92 . 95.

7. In 48 pounds avoirdupois, how many pounds troy? Ans. 58⅓ lbs.

8. A druggist bought 1260 lbs. of alum avoirdupois, and retailed it by troy weight; how many pounds did he sell more than he bought? Ans. 271 lbs. 3 oz.

9. Reduce 1867 lbs., 14 oz. 13 drs. avoir. to pounds, &c, troy.
Ans. 2270 lbs., 0 oz., 11 dwt., 16½ grs., nearly.

10. In 111111 lbs. avoirdupois, how many pounds troy? Ans. 135030 lbs., 420 grs.

11. Reduce 186321 drs. to pounds.
Ans. 727 lbs., 13 oz., 1 dr.

MEASURE OF TIME, &c.

(PAGES 158 — 160.)

Ex. 1. Reduce 25 days, 6 hours to minutes.
Ans. 36360.

2. Reduce 365 days, 6 hours to seconds.
Ans. 31557600.

3. In 847125 min., how many weeks, &c.?
Ans. 84 wks. 6 ho., 45′.

4. Reduce 5623480 seconds to days,
Ans. 65 days, 2 h., 4 m., 40 s.

5. How many seconds in a solar year?
Ans. 31556927½.

6. How many seconds in 30 years, allowing 365 days, 6 hours to a year? Ans. 946728000.

7. How many years of Sabbaths are there in 70 years? Ans. 10 years.

8. In 110 degrees, 20 minutes, how many seconds?
Ans. 397200″.

9. In 11 signs, 45 degrees, how many seconds? Ans. 1350000″.

10. Reduce 7654314 seconds to degrees, Ans. 2126°, 11′, 54″.

11. In 124678 seconds French, how many French grades? How many degrees? Ans., grades 12 ge.,46′, 78″.
Ans. degrees, 11°, 13′, 15 . 672″.

12. How many seconds are there between the first of August at 12 o'clock noon and 4 o'clock, P. M., on the 16th of November? Ans. 9259200″.

COMPOUND ADDITION.

Ex. 1. Add together £12. 13. 9½, £156. 13. 4¼, £12. 0.0, £18. 2. 6¾, £146, £1276. 19. 11¾, £15.10, £12789. 16. 10½, and £0. 18. 4¾. Ans. £14428. 14. 11½.

2. Find the sum of £478. 9. 11, £147. 13. 0½, £111.-11. 11½, £112. 11. 11¼, £167. 19. 9½. £13. 13. 6¾, £127.19.-9½ and £100. 10. 11. Ans. £1260. 10. 11.

3. Add £16. 8. 9. £8, 5. 6, £25. 6. 8 together. Ans. £50. 0. 11.

4. Find the sum of £68. 17. 10¾, £10. 9. 6, £43·-10. 11½ and £65. 14. 8¼. Ans. £188. 13. 0½.

5. Find the sum of $189.62½, $196.78, $12.96½ $134.72½, $14.00½, $17.08¼, $190.90 and $365.71½. Ans. $1121.79¾.

6. Find the sum of $126.33½, £14.3.9, $20.10, $180.15, £13.13.9¼, , £127.18.7½ and $200.15.
Ans. in $1149.96 5/12. Ans. in £287.9.9 17/20.

7. $78.62+$33.90+$120.16¼+£19.19.10½+£14.-13.6¾+$122.82+$0.05+£14.6.7½.
Ans. in $551.56½. Ans. in £137.17.9 9/10.

8. Add together 126 miles, 3 fur., 14 po., 5 yds., 3 m., 4 f., 13 po., 126 m., 7 fur., 18 per., 3 yds., 163 m, 7 fur., 14 per., 1367 m., 3 fur., 10 per., 1 yd. and 176348 yards. Ans. 1888 m., 3 f., 13 po., 5 yds.

9. Add 126 m., 3 fur., 14 per., 16 m., 3 f. 10 po., 14 m. 7 fur., 16 po., 5 yds., 126 m., 3 f., 7 per. 3 yds., 18 m. 2 f., 16 per., and 183467 ft. together.
Ans. 337 m. 1 fur. 23 po., 3y., 2 ft.

10. Add together 5 leagues, 2 m., 4 fur. 7 po., 4 y.; 18 lea., 2 m., 3 fur., 21 per,, 5 yds.; 85 lea., 6 fur., 10 po., 2 yds, Ans. 109 lea., 2 m., 6 fur.

11. Add together 19 po., 12 ft, 8 in.; 64 po., 13 ft,, 8 in.; 28 po., 10 ft., 5 in.; 60 po., 9 ft., 11 in.

Ans. 4 fur., 13 po., 13 ft., 3 in.

12. Add together 19 yds,, 3 qrs., 3 na.,; 21 yds., 2 qrs,, 1 na.; 42 yds., 1 qr., 2 na.; 30 yds., 3 qrs., 2 na.

Ans. 114 yds., 3 qrs.

13. Add together 65 yds., 3 qrs., 1 na.; 81 yds., 2 qrs., 2 na.; 100 yds., 3 qrs., 1 na.: 95 yds., 1 qr., 1 na.; 15 yds. 3 na.; 28 yds., 2 qrs. Ans. 387 yds., 1 qr

14. Add together 12 ac., 3 ro., 30 po.; 16 ac. 29 po.; 19 ac., 1 ro., 19 po.; 9 ac., 2 ro., 3 po.; 17 ac., 3 ro.; 10 ac., 21 po.; 18 ac, 1 ro., 14 po. Ans. 114 ac., 0 ro., 36 po.

15. Find the sum of 13 ac., 3 ro, 27 po.; 45 ac., 1 ro., 27 po.; 63 ac., 2 ro., 17 po.; 26 ac., 2 ro.. 26 po,; 16 ac., 3 ro, 34 po.; 21 ac, 8 po.; 55 ac., 2 ro., 31 po.; 37 ac., 2 ro., 18 po.; 44 ac. 2 ro., 20 po,; 57 ac., 0 ro., 19 po.; 61 ac. 3 ro,, 18 po.; 39 ac. 2 ro.; and 5 ac,, 1 ro., 30 po. Ans. 489 ac. 1 ro., 35 po.

16. Find the sum of 25 ac., 2 ro., 16 po.; 30 ac,, 2 ro., 25 po.; 26 ac. 2 ro., 35 po.; 53 ac. 1 ro., 31 po.; 34 ac., 2 ro., 29 po.; 5 ac., 2 ro., 15 po.; 25¼ yds., 101 sq. in.; 9 ac., 1 ro. 35 po., 12½ yds,, 87 in.; 42 ac. 3 ro., 24 po., 23¾ yds., 57 in.; 12 ac., 2 ro. 5 po., 13¾ yds., 23 in.; 17 ac., 0 ro., 24 po., 30 yds., 113 In.

Ans. 268 ac., 3 ro., 2 po., 14 yds., 7 ft., 21 in.

17. One room in a house contains 15 sq. yds., 5 ft., 7 in. of plastering; another 10 yds., 7 ft., 30 in.; another 9 yds., 6 ft., 25 in.; another 7 yds., 5 ft. 63 in.; how much plastering is there in all of them?

Ans. 43 yds. 5 ft., 125 in.

18. Find the sum of 3 c. yds. 23 c, ft. 171 c, in.; 71 c. yd., 17 c. ft. 31 c. in.; 28 c. y., 26 c. ft. 1000 c. in. and 34 c. yd. 23 c. ft., 1101 c. in.

Ans. 85 c. yds. 9 c. ft. 575 c. in.

19. Add together 196 sq. ft. 11′, 11″, 2‴, 27 ft., 6′, 10″; 136 ft., 4′, 11‴ and 1 ft., 3′, 10″.

Ans. 362 ft., 2′. 8″, 1‴, or 362 ft., 385 in.

20. Add together 127 gals., 3 qts., 1 pt.; 127 gal., 1 qt.; 44 gals., 3 qts., 1 pt.; 93 gals., 1 qt.; 73 gals. 3 qts., 1 pt. and 126 gals., 3 qts., 1 pt. Ans. 594 gals.

21. A merchant bought a cask of oil, containing 73 gals., 3 qts.; another 60 gals., 2 qts.; another 40 gals.,

1 qt.; another 65 gals., 2 qts.: how much oil did he buy? Ans. 240 gals.

22. Add together 21 lbs., 7 oz., 12 dwt. 10 grs.; 28 lbs. 5 oz., 8 dwt. 7 grs.; 7 lbs., 6 dwt., 15 grs.; 41 lbs., 6 oz., 20 grs.; and 9 lbs., 7 grs.
Ans. 107 lbs., 7 oz., 8 dwt., 11 grs.

23. What is the sum of 16 lbs., 6 oz., 6 dwts. 19 grs.; 100 lbs., 8 oz., 16 dwt.; 97 lbs., 2 oz., 10 grs.; 115 lbs., 9 grs? Ans. 330 lbs., 2 oz., 3 dwt., 5 grs.

24. Find the sum of 7 qrs., 6 bush., 1 pk., 3 qts.; 27 qrs., 6 bush., 6 qts.; 34 qrs., 1 bush., 6 qts.; 65 qrs., 6 bush., 3 qts. Ans. 135 qrs., 3 bush., 3 pks., 2 qts.

25. In 4 fluid pints, 14 f ℥, 6 f ʒ. 36 m.; 19 pts. 11 f ℥, 1 ʒ, 56 m.; and 3 pts., 15 f ℥, 16 m. how many pints? Ans. 28, O. 9 f. ℥. 0 ʒ. 48 m.

26. Add together 3 lbs., 0 ℥., 7 ʒ.; 1℈.; 13 lbs., 11 ℥, 7 ʒ, 2 ℈, 19 grs.; 14 lbs., 10℥, 7 ʒ, 2℈, 18 grs., and 6℥, 2℈, 17 grs. Ans. 32 lb., 6℥, 14 grs.

27. Add together 3 drs., 2 scr., 19 grs.; 2 drs., 2 scr., 11 grs.; 7 drs., 17 grs.; 6 drs., 1 scr., 9 grs.; and 5 drs., 1 scr., 13 grs. Ans. 3 oz., 2 drs., 9 grs.

28. Add together 19 cwt., 3 qrs., 14 lbs.; 16 cwt., 3 qrs., 11 lbs.; 27 cwt., 3 qrs., 27 lbs.; 18 cwt., 2 qrs., 11 lbs.; and 196 cwt., 1 qr., 1 lb., old weight.
Ans. Old, 279 cwt., 2 qrs., 8 lbs. New, 313 cwt., 12 lbs.

29. Find the aggregate of 126 cwt., 1 qr., 11 lbs.; 313 cwt., 24 lbs.; 18 cwt., 1 qr., 21 lbs.; and 11 cwt., 2 qrs., 24 lbs. New weight.
Ans. New, 469 cwt., 3 qrs., 5 lbs.
Old, 419 cwt., 1 qr., 24 lbs.

30. Find the sum of 11 cwt., 2 qrs., 13 lbs.; 12 cwt., 2 qrs., 8 lbs., 14 oz.; 33 cwt., 2 qrs., 11 lbs., 14 oz.; 9 cwt., 2 qrs., 13 oz.; 126 cwt., 1 qr., 27 lbs., 13 oz.; 17 lbs., 8 oz. and 1267 oz., old weight.
Ans. Old, 194 cwt., 2 qrs. 19 lbs., 1 oz.
New, 218 cwt., 3 lbs., 1 oz.

31. What is the sum of 10 wks., 5 d., 12 h. 40′, 21 wks., 3 d., 9 h., 15′; 40 wks., 4 d., 17 h., 32′, and 42 wks., 1 d.? Ans. 115 wks. 15 h., 27 m.

32. Add together 7 years, 28 wks., 3 sec.; 26 y., 5

w., 5 d.; 58 y., 6 d., 23 h., 59 sec.; 43 w., 23 h., 50 m., 12 sec., and 124 y., 14 w., 19 h. 37 sec.

Ans. 216 years, 39 w., 6 d., 17 h., 51 m., 51 sec.

33. When B. was born, A.'s age was 2 y., 3 mo., 3 w., 2 d., 10 h., 11 m.; when C. was born, B.'s age was 3 y., 2 mo., 3 w., 6 d., 11 h., 39 m., 44 sec.; when D. was born, C.'s age was 11 y., 11 m, 3 w, 6 d., 21 h., 16 m., 49 sec. What was A.'s age when D.'s age was 14 y., 11 m., 3 w., 1 hour?

Ans. 32 y., 6 m., 2 w., 1 d., 20 h., 7 m., 33 sec.

COMPOUND SUBTRACTION.

Ex. 1. From £196.3.9½ take £127.13.10.

Ans. £68.9.11½.

2. From £147.13.6¾ take £27.14.7¼.

Ans. £119.18.11½. $519,79⅙.

3. From £160½.6⅓ s. 3¾ d, take £100⅕. 8 s.

Ans. £60.4.7¾. $240.92+$\frac{1}{12}$.

4. What sum added to £189.13.7¾ will make £1000.10?

Ans. 3243.27\frac{7}{12}$.

5. From 69 miles, 3 f. 14 po. take 57 m., 2 f. 8 per.

Ans. 12. m. 1 f. 6 per.

6. From 36 lea. 1 m. 3 fur. 4 po. 3 y. take 26 lea. 2 m. 1 f. 2 yds. Ans. 9 lea., 2 m., 2 f., 4 po., 1 yd.

7. Subtract 29 yds., 2 qrs., 3 na., from 85 yds., 1 qr., 2 na. Ans. 55 yds., 2 qr., 3 n.

8. Subtract 55 yds., 2 qr., 1 n. from 100 yds.

Ans. 44 yds., 1 qr., 3 n.

9. A. gives to B. 16 ac., 3 ro., 14 po. off a farm of 126 ac., 2 ro., 19 po.; how much has he left?

Ans. 109 ac., 3 ro., 5 po.

10. A person has a farm of 93 ac., 2 ro., 10 po., 4 yd., how much must he purchase to increase it to 100 ac, 3 ro?

Ans. 7 ac., 29 po., 26¼ yds.

11. A person had a farm of 186 ac., 3 ro., 10 po., and he sold off it, at different times, 20 ac., 2 ro.; 21 ac., 3 ro.; 14 po., and 14 ac., 2 ro., 39 po., how much had he left?

Ans. 129 ac., 2 ro., 37 po.

12. A person having 167 cords, 93 ft., 16 in. of wood, sells 136 cords, 118½ feet; what quantity has he remaining? Ans. 30 cords, 102 ft., 890 inches.

13. From 196 cubic feet, 144 inches, take 186 cubic feet, 1701 in. Ans. 9 ft., 171 cu. in.

14. From 163 sq. ft., 6'. 7" take 27 ft. 3'. and 11", and reduce the remainder to sq. in., Ans. 19616 in.

15. From 36 chaldrons, 13 bush., 1 qt., take 27 chs., 9 bush., 3 gals., 3 qts. Ans. 9 ch., 3 bush., 4 gals, 2 qts.

16. From 85 bush., 2 pks., 4 qts., take 49 bush., 3 pks., 6 qts. Ans. 35 bush., 2 pks., 6 qts.

17. From 16 lbs., 11 oz., 19 dwt., take 11 lbs., 3 oz., 15 dwts., 23 grs. Ans. 5 lbs., 8 oz., 3 dwts., 1 gr.

18. From 18 lb., 3 ℥, 7 ʒ, 2 ℈ take 17 lbs., 11 ℥, 3 ʒ, 2 ℈. 19 grs. Ans. 4 ℥. 3 ʒ. 2℈. 1 gr.

19. From 18 tons, 16 cwt., 3 qrs., 11 lbs., take 14 tons, 13 cwt., 2 qrs., 27 lbs., old weight.

Ans. Old wt., 4 tons., 3 cwt., 12 lbs.
New wt., 4 tons, 13 cwt., 8 lbs.

20. From 14 lbs., 3 oz., 15 drs. take 12 lbs. 8 oz., 14 drs. Ans. 1 lb., 11 oz., 1 dr.

21. Subtract 13 lbs., 4 oz., 15 drs. from 126 lbs., 1 oz., 14 drs. Ans. 112 lbs., 12 oz., 15 drs.

22. From 160 years, 11 mo., 2 wks., 5 d., 16 h., 30', 40", take 106 yrs., 8 mos., 3 wks., 6 d., 13 h., 45', 34"
Ans. 54 y., 2 mo., 2 wk., 6 d., 2 h., 45'., 6".

23. The latitude of a certain place is 40°, 16', 10", and that of another 50°, 27', 35": required the difference. Ans. 10°, 11', 25".

24. The latitude of St. Peter's, at Rome, is 41°, 53', 54" north, and that of St. Paul's, at London, is 51°, 30', 49", north. Find the difference. Ans. 9°, 36', 55".

COMPOUND MULTIPLICATION.

Ex. 1. What cost 7 acres of land, at £35. 6 s. 7 d, per acre? Ans. £247, 6.1. \$989.21$\frac{2}{3}$.

2. What cost 18 barrels of flour, at £1, 6, 8$\frac{1}{2}$. per bbl.? Ans. £24, 0, 9.

3. Multiply £583. 0, 10 separately, by 13 and 16, and reduce the sum of both results to \$. Ans. \$67632.83$\frac{1}{3}$.

4. Multiply £2579, 0, 0$\frac{3}{4}$ separately, by 147, 155, 474 and 2331,

Ans. £379113, 9, 2$\frac{1}{4}$. Ans. £1222447, 9, 7$\frac{1}{2}$.
£399745, 9, 8$\frac{1}{4}$. £6011656, 5, 8$\frac{1}{4}$.

5. Find the price of 1897$\frac{1}{2}$ lbs. of tea, at 75 cts,

per pound. Ans. $1423.12½.

6. What is the cost of 183 gals. oil at 62½ cts. per gallon? Ans. $114.37½.

7. What does 127½ cords of wood cost, at $1.37½? Ans. $175.31¼. £43.16.6¾.

8. Multiply 175 miles, 7 fur. 18 po. by 84. Ans. 14778 m., 1 f., 32 po.

9. Multiply 40 lea., 2 m., 5 fur. 15 po. by 50. Ans. 2044 l., 1 m., 4 fur. 30 po.

10. Multiply 70 yds., 2 ft., 10 in. separately by 7 and 29. Ans. 2 fur., 10 po., 1 yd., 1 ft., 10 in. 1 m., 1 fur., 14 po., 1 ft., 2 in.

11. Multiply 67 yds., 1 qr., 2 na., separately by 9 and 53. Ans. 606 yds., 1 qr., 2 na. 3570 yds., 3 qrs., 2 na.

12. Multiply 310 ac., 3 ro., 3 po., by 81. Ans. 25172 ac., 1 ro., 3 po.

13. Multiply 380 ac., 3 ro., 32 po., separately by 12 and 106. Ans. 4571 ac., 1 ro., 24 po. and 40380 ac., 2 ro., 32 po.

14. Multiply 146 sq. ft., 14 in. separately, by 12 and 28. Ans. 1753 ft., 24 in. 4090 ft. 104 in.

15. Multiply 36 sq. yds., 3 ft., 16 in. separately, by 14 and 34. Ans. 508 yds., 7 ft., 80 in. 1235 yds., 6 ft., 112 in.

16. Multiply 126 cu. ft., 133 in., by 14 and 56 separately. Ans. 1765 ft., 134 in. 261 yds., 13 ft., 536 in.

17. A bushel contains 2218.192 cub. in., what is the contents of 196 bush.? Ans 251 ft., 1037 . 632 in.

18. Multiply 14 ft., 3′, 6″ separately, by 14 and 35, Ans. 200 ft., 12 in. 500 ft., 30 in.

19. Multiply 126 ft., 3′, 6″ separately, by 182 and 400. Ans. 22985 ft., 12 in. 50516 ft., 8′.

20. If a ship sails 3°, 24′, 10″ per day, how far will she sail in 60 days? Ans. 204°, 10′, 0″.

21. If 1 ac. of land produce 45 bush., 26 qts., how much will 100 ac. produce? Ans. 4581 bush., 8 qts.

22. If one barrel of flour requires 4 bush., 3 pks., 5 qts., of wheat, how much will 500 barrels require? Ans. 2453 bush., 4 qts.

23. Multiply 16 lbs., 3 oz., 5 dwts., 9 grs. by 26. Ans. 423 lbs., 19 dwts., 18 grs.

24. Multiply 5 ℥, 7ʒ, 2℈, 15 grs. separately, by 16

and 48. Ans. 95 ℥. 6 ʒ. 2 ℈, and 287 ℥. 4 ʒ.

25. Multiply 3 tons. 27 lbs., 13 oz., old wt., by 11.
Ans. Old, 33 tons, 2 cwt., 2 qrs., 25 lbs., 15 oz.
New 37 tons, 2 cwt., 1 qr., 15 oz.

26. Multiply 16 cwt., 3 qrs., 24 lbs., new wt. by 76.
Ans. New, 64 tons, 11 cwt., 24 lbs.
Old, 57 tons, 12 cwt., 3 qrs., 16 lbs.

27. A person purchased 50 lambs at 15s., 6d., each; 40 sheep at £1, 5, 6 each; 30 cows at £12, 10, 6 for three, and 28 horses at £25 sterling each; the expenses for getting them home amounts to 3 s. 6 d. per head. What is the whole cost? Ans. $4463.60.

COMPOUND DIVISION.

Ex. 1. Divide £254, 7 s. 6 d. by 111. Ans. £2. 5. 10.

2. Divide £160.4.8$\frac{1}{4}$ by 105. Ans. $6. 10.$\frac{5}{12}$.

3. Divide £346, 9, 9$\frac{1}{2}$ by 155. Ans. £2. 4. 8$\frac{1}{2}$,

4. Divide £111. 2. 6 by 180. Ans. £0. 12. 4, rem. $\frac{2}{3}$.

5. Divide £2045. 16. 5$\frac{1}{4}$ by 4083.
Ans. £0. 10. 0$\frac{1}{4}$, rem. $\frac{1}{4}$ $\frac{1}{2}$

6. Three persons purchased a ship at $111267.36. the first takes 2 shares, the second 5 shares, and the third 7 shares, how much do they severally pay?
Ans. 15895.33\frac{5}{7}$; 39738.34\frac{2}{7}$; $55633.68.

7. Divide 75 ac., 3 ro., 39 po. by 26.
Ans. 2 ac., 3 ro., 27 po., 19 yds., 7 ft., 1$\frac{5}{13}$ in.

8. Divide 13 ac., 1 ro. by 147.
Ans. 14 po., 12 yds., 6 ft., 119$\frac{1}{49}$ in.

9. Divide 91 yds., 2 qrs., 1 na. by 903. Ans. 1 $\frac{562}{903}$ na.

10. A tailor put 216 yds., 3 qrs. of cloth into 20 cloaks; how much did each cloak contain?
Ans. 10 yds., 3 qrs., 1$\frac{2}{5}$. na.

11. A person travelled 1000 miles in 12 days: at what rate did he travel per day?
Ans. 83 m., 2 fur., 26 po., 11 ft.

12. Divide 164 cords, 30 ft., by 17 in.
Ans. 9 cords, 84 ft., 1016$\frac{8}{17}$ in.

13. Divide 410 cords, 10 ft., 21 in. by 61.
Ans. 6 cords, 92 ft., 850$\frac{11}{61}$ in.

14. A farmer raised 1000 bush., 3 pks. 16 qts. of wheat on 40 acres; how much was that per acre.
Ans. 25 bush., 1$\frac{1}{8}$ pts

15. Divide 77 ac., 1 ro. 33 po., by 51.
Ans. 1 ac., 2 ro., 3 po.

16. Divide 206 mo., 4 da. by 26.
Ans. 7 mo., 3 wks., 5 days.

17. Divide 75 cwt., 2 qrs., 10 lbs. by 35.
Ans. 2 cwt., 16 lbs.

18. Divide 312 lbs., 9 oz., 18 dwt., by 43.
Ans. 7 lbs., 3 oz., 6 dwt.

19. Divide 17 lbs., 9 oz., 2 grs. by 7.
Ans. 2 lbs., 6 oz., 8 dwt., 14 grs.

20. Divide 17 lbs., 5 oz., 2 drs., 1 scr., 4 grs. by 12.
Ans. 1 lb., 5 oz., 3 drs., 1 scr., 12 grs.

21. Divide 178 cwt., 3 qrs., 14 lbs. old weight, by 53.
Ans. 3 cwt., 1 qr., 14 lbs.

22. Divide 365 days, 10 h., 40′ by 15.
Ans. 24 d., 8 h., 42′, 40″.

MULTIPLICATION AND DIVISION OF COMPOUND NUMBERS, BY NUMBERS CONTAINING FRACTIONS.

Ex. 1. Multiply £163, 13 s., $6\frac{1}{2}$ d. by $4\frac{1}{12}$.
Ans. £804, 14, $10\frac{23}{24}$.

2. Multiply \$1896.$13\frac{1}{3}$ by $8\frac{1}{9}$. Ans. \$15379.$74\frac{22}{27}$.

3. Divide £1876, 14, $3\frac{1}{2}$ by $4\frac{7}{16}$ Ans. £422, 18, $5\frac{13}{71}$.

4. Divide 127 yds., 3 qrs., 2 nails by $14\frac{1}{2}$.
Ans. 8 yds., 3 qrs., $1\frac{3}{29}$ nails.

5. Divide 196 cwt., 3 qrs., 14 lbs., new wt., by $14\frac{1}{8}$.
Ans. 13 cwt., 3 qrs., $18\frac{103}{113}$.

6. Multiply 144 ft., 2′, 11″ by $16\frac{1}{8}$.
Ans. 2325 ft., $132\frac{3}{8}$ in.

7. Divide 178 cubic feet, 3′, 4″, 10‴ by $14\frac{1}{9}$.
Ans. 12 feet, $1096\frac{2}{127}$.

8. Divide 1234 gals., 3 pts. by $111\frac{1}{10}$.
Ans. 11 gals. 0 qts., $0\frac{491}{1111}$.

MISCELLANEOUS QUESTIONS ON ART'S 125—157.

Ex. 1. How long will a person be in saving £150 sterling, if he save 2s. 6d., N. S. currency per week?
Ans. 28 yrs., 44 wks.

2. How many dozen of tea-spoons, each spoon weighing 1 oz., 3 dwt., can be made out of 25 lbs., 10 oz., 10 dwt. of silver? Ans. $22\frac{1}{2}$ doz.

3. If a pile of wood is 140 ft. long, and 3 ft., 6 in. wide, how high must it be to contain 18 cords?

Ans. 4 feet, 8$\frac{104}{245}$ in.

4. If 9 cows eat 21 tons, 7 cwt., 2 qrs., 12 lbs., new weight, of hay in a year, how much will 2 cows eat in the same time? Ans. 4 tons, 15 cwt., 0 qrs., 2$\frac{2}{3}$ lbs.

5. If a gentleman's annual income be £1000, and his daily expenses £1, 17, 3$\frac{1}{2}$, how much does he save in 9 years? Ans. £2874, 16, 10$\frac{1}{2}$.

6. If a newspaper has a circulation of 1260 daily, and be sold at 2$\frac{1}{4}$ d. each copy, what amount in dollars an cents will be realized by its sale for one year (313 days)?

Ans. $14789.25.

7. If I purchase 20 pieces of cloth, each piece 20 ells, for $2.50 per ell, what is the value of the whole in pounds? Ans. £250.

8. How many yards of cloth may be bought for £21, 11, 1$\frac{1}{2}$, when 3$\frac{1}{2}$ yds. cost $10.85?

Ans. 27 yds., 3 qrs., 1$\frac{1}{31}$ nls.

9. The difference between two numbers is 12, and their quotient is 4; what are the numbers? Ans. 4 and 16.

10. If 24 lbs of raisins cost 6 s. 6 d., what will 326$\frac{1}{4}$ lbs. cost at the same rate? Ans. £4, 8, 4 $\frac{5}{16}$.

11. If 2$\frac{1}{2}$ oz., of silver cost $2.50, what is the price of 14 ingots, each weighing 7 lbs., 5 oz., 10 dwts.?

Ans. $1253.00.

12. How many gallons will a box hold, which is 3 feet 6 inches long, 2 ft., 6 in. wide, and 4 ft., 6 in. high?

Ans. 245 gals., 1 qt., 1 pt., $\frac{15563}{138637}$.

13. How many feet of boards one inch thick, can be cut from a log which is 22 ft., 6 in. long, 14$\frac{3}{4}$ in. deep on the one side, and 17$\frac{1}{2}$ in. on the other, allowing $\frac{1}{4}$ inch waste for each cut? Ans. Cutting boards 17$\frac{1}{2}$ wide, 393$\frac{3}{4}$ feet.
" " " 14$\frac{3}{4}$ do. 387 ft., 2′,3″.

FRACTIONS.

REDUCTION OF VULGAR FRACTIONS.

(ART. 163. PAGE 180.)

Reduce the following fractions to their lowest terms:

Ex. 1.	$\frac{299}{713}$,	Ans.	$\frac{13}{31}$.	Ex. 4.	$\frac{252}{324}$,	Ans.	$\frac{7}{9}$.
2.	$\frac{522}{1044}$,	"	$\frac{1}{2}$.	5.	$\frac{374}{561}$,	"	$\frac{2}{3}$.
3.	$\frac{360}{468}$,	"	$\frac{10}{13}$.	6.	$\frac{1293}{1724}$,	"	$\frac{3}{4}$.

Ex. 7. $\frac{1457}{5921}$.	Ans. $\frac{47}{191}$,	Ex. 11. $\frac{1760}{1903}$.	Ans. $\frac{169}{173}$.
8. $\frac{8465}{7335}$,	" $\frac{431}{489}$.	12. $\frac{1664}{2483}$.	" $\frac{128}{191}$.
9. $\frac{4201}{11132}$,	" $\frac{41}{92}$.	13. $\frac{144000}{160016}$,	" $\frac{9000}{10001}$.
10. $\frac{396}{432}$,	" $\frac{11}{12}$.	14. $\frac{1136}{2201}$,	" $\frac{16}{31}$.

Reduce the following fractions to others having a common denominator.

Ex. 1. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$, Ans. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.
2. $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$ and $\frac{9}{10}$, $\frac{2160}{2520}$, $\frac{2205}{2520}$, $\frac{2240}{2520}$, $\frac{2268}{2520}$.
3. $\frac{11}{12}$, $\frac{12}{13}$, $\frac{13}{14}$ and $\frac{14}{15}$, $\frac{5005}{5460}$, $\frac{5040}{5460}$, $\frac{5070}{5460}$, $\frac{5096}{5460}$.
4. $\frac{2}{9}$, $\frac{5}{12}$, $\frac{17}{48}$, $\frac{37}{72}$, $\frac{32}{144}$, $\frac{60}{144}$, $\frac{51}{144}$, $\frac{74}{144}$.
5. $\frac{4}{9}$, $\frac{7}{23}$, $\frac{8}{30}$ and $\frac{9}{22}$, $\frac{10120}{22770}$, $\frac{6930}{22770}$, $\frac{6072}{22770}$, $\frac{9315}{22770}$.
6. $\frac{31}{45}$, $\frac{17}{60}$, $\frac{14}{35}$ and $\frac{24}{70}$. $\frac{868}{1260}$, $\frac{357}{1260}$, $\frac{504}{1260}$, $\frac{432}{1260}$.
7. $\frac{81}{90}$, $\frac{65}{180}$, $\frac{49}{60}$ and $\frac{95}{270}$. $\frac{486}{540}$, $\frac{195}{540}$, $\frac{441}{540}$, $\frac{190}{540}$.

(ARTICLE 164. PAGE 182.)

Reduce the following fractions to whole or mixed numbers.

Ex. 1. $\frac{11}{9}$	Ans. $1\frac{2}{9}$.	7. $\frac{51}{21}$	Ans. $2\frac{3}{7}$.
2. $\frac{22}{13}$	" $1\frac{9}{13}$.	8. $\frac{76}{8}$	" $9\frac{1}{2}$.
3. $\frac{161}{32}$	" $5\frac{1}{32}$	9. $\frac{196}{7}$	" 28.
4. $\frac{196}{11}$	" $17\frac{9}{11}$.	10 $\frac{5932}{121}$	" $49\frac{3}{121}$.
5. $\frac{1001}{100}$	" $10\frac{1}{100}$	11 $\frac{19876}{321}$	" $61\frac{295}{321}$.
6. $\frac{5432}{196}$	" $27\frac{5}{7}$	12. $\frac{111111}{1111}$	" $100\frac{1}{101}$.

(ARTICLE 165. PAGE 183.)

Reduce the following whole and mixed numbers to improper fractions.

Ex. 1. Reduce 8, 9, 10, 11 and 12 to fractions whose denominator shall be 21.

Ans. $\frac{168}{21}$, $\frac{189}{21}$, $\frac{210}{21}$, $\frac{231}{21}$ and $\frac{252}{21}$.

2. $7\frac{7}{11}$	Ans. $\frac{84}{11}$.	7. $789\frac{13}{17}$	Ans. $\frac{13426}{17}$.
3. $8\frac{21}{32}$	" $\frac{277}{32}$.	8. $296\frac{1}{101}$.	" $\frac{29897}{101}$.
4. $10\frac{11}{12}$	" $\frac{131}{12}$.	9. $496\frac{3}{22}$	" $\frac{10915}{22}$.
5. $161\frac{1}{3}$	" $\frac{484}{3}$.	10. $18801\frac{1}{6}$	" $\frac{112807}{6}$.
6. $129\frac{11}{19}$	$\frac{2462}{19}$.	11. $37892\frac{111}{10101}$,	" $\frac{41719203}{10101}$.

12. Reduce 61, 32, 41 and 72 to sixths.

$\frac{366}{6}$, $\frac{192}{6}$, $\frac{246}{6}$ and $\frac{432}{6}$.

13. Reduce 72 to sevenths, 93 to eighths, and 121 to sixteenths.

Ans. $\frac{504}{7}$, $\frac{744}{8}$, and $\frac{1936}{16}$.

(ARTICLE 166. PAGE 184.)

Ex. 1. Reduce £1. 13. $6\frac{1}{2}$ to the fraction of £2. Ans. $\frac{201}{240}$.
2. Reduce £3. 6. 7 to the fraction of $19.20 " $\frac{738}{1152}$.
3. Reduce 16 lbs., 3 oz. to fraction of cwt. " $\frac{259}{1792}$.

4. Reduce $\$1.27\frac{1}{2}$ to the fraction of £1. Ans. $\frac{51}{160}$.
5. Reduce 11 lbs. 3 oz. troy to fraction of 3 lbs. 2 oz. avoirdupois. Ans. $\frac{2592}{875}$ or $2\frac{842}{875}$.
6. Reduce 1 gal. 3 pts. to fraction of 2 bus. Ans. $\frac{11}{128}$.

Art. 167. Page 185.

Ex. 1. What is the value of $\frac{1}{8}$ of £19. 10. 6? Ans. £2. 8. $9\frac{3}{4}$.
2. What is the value of $\frac{2}{3}$ of 1 ton? (old weight.) Ans. 13 cwt. 1 qr. $9\frac{1}{3}$ lbs.
3. Find the value of $\frac{9}{10}$ of 2 cords of wood. Ans. 1 cord, $102\frac{2}{5}$ ft.
4. Find the value of $\frac{12}{17}$ of 3 ac. 2 ro. 14 po. Ans. 2 ac. 2 ro. $5\frac{3}{17}$ po.
5. Required the value of $7\frac{11}{12}$ chal. Ans. 7 chal. 33 bus.
6. Find the value of $\frac{9}{11}$ of $\$166.33\frac{1}{2}$. Ans. $136.09\frac{5}{22}$.
7. What is the value of $\frac{9}{11}$ of a year? Ans. 298 days, 15 hours, 16 min. $21\frac{9}{11}$ sec.
8. What is the value of $\frac{9}{11}$ of a mile? Ans. 6 fur. 21 po. 4 yd. 1 ft. 6 in.

ADDITION OF VULGAR FRACTIONS.

Art. 168. Page 186.

Find the sum of the following.

Ex. 1. 320, and $\frac{7}{15}$. Ans. $320\frac{7}{15}$.
2. $635\frac{15}{16}$, $427\frac{12}{48}$, $1625\frac{7}{8}$. Ans. $2689\frac{1}{16}$.
3. $21\frac{1}{2}$, $35\frac{1}{8}$, $\frac{53}{16}$, and $\frac{7}{13}$. Ans. $61\frac{5}{8}$.
4. $326\frac{1}{112}$, $98\frac{1}{12}$, $186\frac{11}{126}$, $14\frac{1}{2}$, and $\frac{1}{2}$. Ans. $575\frac{181}{1008}$.
5. $268\frac{1}{2}$, $16\frac{1}{2}$, $189\frac{1}{12}$, $14\frac{3}{28}$, and $1\frac{1}{3}$. Ans. $489\frac{11}{21}$.
6. $189\frac{1}{12}$, $18\frac{1}{3}$, $146\frac{1}{7}$, $\frac{175}{35}$, and $\frac{1}{10}$. Ans. $853\frac{29}{420}$.
7. $11\frac{1}{2}$, 14, $16\frac{1}{3}$, $18\frac{1}{8}$, and 196. $255\frac{23}{24}$.
8. $61\frac{3}{7}$, $803\frac{5}{28}$, and $4\frac{1}{2}$. Ans. $868\frac{27}{28}$.
9. $418\frac{4}{9}$, $643\frac{3}{4}$, $518\frac{1}{8}$. Ans. $1580\frac{23}{72}$.

SUBTRACTION OF VULGAR FRACTIONS.

Art. 169. Page 188.

Find the difference between —

1. $\frac{18}{27}$ and $\frac{3}{9}$. Ans. $\frac{3}{9}$.
2. $\frac{63}{74}$ and $\frac{26}{43}$. " $\frac{785}{3182}$.
3. $\frac{48}{55}$ and $\frac{42}{67}$. " $\frac{906}{3685}$.
4. $\frac{10}{18}$ and $\frac{7}{40}$. " $\frac{137}{360}$.
5. $69\frac{1}{9}$ and $29\frac{1}{3}$. " $39\frac{7}{9}$.
6. $16\frac{1}{15}$ and $9\frac{5}{7}$. " $6\frac{37}{105}$.
7. £$\frac{3}{11}$ and 1 s. $1\frac{2}{7}$. Ans. 2s. $6\frac{10}{77}$.
8. $\frac{5}{9}$ yd. and $1\frac{6}{7}$ nls. Ans. 1 qr. $3\frac{2}{63}$ nls.
9. $146\frac{1}{6}$ and $145\frac{1}{12}$. Ans $1\frac{1}{12}$.
10. $196\frac{11}{101}$ and $192\frac{11}{1010}$. Ans. $4\frac{99}{1010}$.
11. 147 and $136\frac{11}{136}$. Ans. $10\frac{125}{136}$.
12. $183\frac{1}{3}$ and $126\frac{11}{12}$. Ans. $56\frac{5}{12}$.

MULTIPLICATION OF VULGAR FRACTIONS.

ART. 170. PAGE 189.

Mu tiply

1. $\frac{18}{19}$ by $\frac{14}{15}$.	Ans. $\frac{84}{95}$.	5. $11\frac{1}{9}$ of $1\frac{1}{8}$ of $\frac{3}{4}$.	Ans. $9\frac{3}{8}$.
2. $\frac{16}{17}$ by $\frac{11}{12}$.	" $\frac{44}{51}$.	6. $\frac{10}{21}$ by $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{5}$.	" $\frac{10}{49}$.
3. $16\frac{1}{2}$ by $13\frac{1}{3}$.	" 220.	7. $\frac{9}{10}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$.	" $\frac{3}{8}$.
4. $4\frac{1}{2}$ by $16\frac{1}{8}$.	" $72\frac{9}{16}$.	8. $\frac{3}{4}$ of 9 by $\frac{7}{8}$.	" $5\frac{29}{32}$.

9. What is the product of $(8\frac{1}{2} + 3\frac{1}{3} - 2\frac{1}{6}) \times (9\frac{1}{12} - 8\frac{11}{12}) \times (3\frac{1}{8}$ of $\frac{5}{6}$ of $\frac{9}{10} - \frac{8}{11})$? Ans. $2\frac{3829}{6336}$.

DIVISION OF FRACTIONS.

ART. 171. PAGE 191.

Ex. 1. Divide $\frac{33}{42}$ by $\frac{42}{99}$. Ans. $1\frac{1503}{1764}$.
2. Divide $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{3}{8}$ by $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$. " $\frac{1}{8}$.
3. Divide $16\frac{100}{151}$ by $\frac{48}{49}$. " $17\frac{17}{1812}$.
4. Divide $18\frac{4}{9}$ by 11. " $1\frac{67}{99}$.
5. Divide $15\frac{3}{4}$ by $\frac{9}{10}$ of $\frac{3}{4}$. " $23\frac{1}{3}$.
6. Divide $\frac{25}{42}$ of 7 by $\frac{50}{63}$ of 42. " $\frac{1}{8}$.
7. Divide $\frac{45}{28}$ of $2\frac{1}{15}$ of $\frac{9}{12}$ of $\frac{10}{13}$ by $\frac{8}{25}$ of $\frac{20}{36}$ of $\frac{3}{4}$ of 5. " $3\frac{57}{65}$.
8. Divide $\frac{3}{\frac{4}{5}}$ by $\frac{\frac{3}{4}}{5}$. " 25.
9. Divide $\frac{27}{\frac{5}{19}}$ by $2\frac{1}{13}$ of $\frac{6}{23}$. " $248\frac{3}{70}$.
10. Divide $\frac{\frac{4}{5}}{\frac{3}{7}}$ by $\frac{\frac{3}{4}}{1\frac{3}{5}}$. " $3\frac{221}{225}$.

REDUCTION OF DECIMAL FRACTIONS.

ART. 173. PAGE 193.

Reduce the following fractions to decimals :

1 $\frac{23}{40}$.	Ans. .575.	7. $\frac{9}{16}$.	Ans. .5625.
2. $\frac{82}{125}$.	" .656.	8. $\frac{43}{80}$.	" .5375.
3. $\frac{51}{250}$.	" .204.	9. $\frac{371}{2240}$.	" .165625.
4. $\frac{1001}{10000}$.	" .1001.	10. $\frac{37}{320}$.	" .115625.
5. $\frac{461}{2500}$.	" .1844.	11. $\frac{3}{32}$.	" .09375.
6. $\frac{76}{625}$.	" .1216.	12. $\frac{5}{1024}$.	Ans. .0048828125.

ART. 175. PAGE 195.

Reduce the following to circulating decimals :

1. $\frac{4}{909}$.	Ans. $.\dot{0}04\dot{4}$.	7. $\frac{160}{27}$.	Ans. $5.\dot{9}2\dot{5}$.
2. $\frac{4}{9009}$.	" $.\dot{0}0044\dot{4}$.	8. $2\frac{1}{54}$.	" $2.0\dot{1}8\dot{5}$.
3. $\frac{3}{286}$.	" $.0\dot{1}0489\dot{5}$.	9. $\frac{49}{75}$.	" $.65\dot{3}$.
4. $\frac{574}{999}$.	" $.\dot{5}7\dot{4}$.	10. $17\frac{1874}{2475}$.	" $17.75\dot{7}\dot{1}$.
5. $\frac{82674}{99900}$.	" $.82\dot{7}5\dot{6}$.	11. $\frac{2904528}{4999500}$.	" $.580\dot{9}62\dot{6}$.
6. $\frac{3}{14}$.	" $.2\dot{1}4285\dot{7}$.	12. $121\frac{47727}{99000}$.	Ans. $121.482\dot{0}\dot{9}$

13. $81\frac{204}{495} + 8\frac{20721}{49950} + 80\frac{28818}{49995} + 6\frac{1}{3}$. Ans. $121.94\dot{0}0286154741\dot{6}$

Art. 176. Page 196.

Reduce the following decimals to vulgar fractions :

1. $.\dot{4}$.	Ans. $\frac{4}{9}$.	5. $.\dot{1}65\dot{3}$.	Ans. $\frac{551}{3333}$.
2. $.\dot{3}3\dot{6}$.	" $\frac{336}{999}$.	6. $.\dot{1}82\dot{7}$.	" $\frac{203}{1111}$.
3. $.\dot{4}59\dot{8}$.	" $\frac{4598}{9999}$.	7. $.\dot{4}56\dot{3}$.	" $\frac{507}{1111}$.
4. $.\dot{5}\dot{3}$.	" $\frac{53}{99}$.	8. $.\dot{6}6\dot{6}$.	" $\frac{2}{3}$.

Arts. 177 and 178. Pages 198 and 199.

1. $.13\dot{8}$.	Ans. $\frac{5}{36}$.	5. $3.\dot{6}$.	Ans. $3\frac{2}{3}$.
2. $.5\dot{9}2\dot{5}$.	" $\frac{16}{27}$.	6. $14.06\dot{9}\dot{4}$.	" $14\frac{172}{2475}$.
3. $.02\dot{2}\dot{7}$.	" $\frac{1}{44}$.	7. $15.069\dot{4}$.	" $15\frac{5}{72}$.
4. $.47\dot{4}\dot{5}$.	" $\frac{261}{550}$.	8. $123.\dot{6}549\dot{1}$.	Ans. $123\frac{65368}{99999}$.

Art. 179. Page 200.

1. Reduce 4s. 6d. to the decimal of £1. Ans. .225.
2. " 14 oz. 3 dwt. 19 gr. to the decimal of 1 lb. " 1.192882.
3. " 3 cwt. 2 qr. 11 lb. (old weight) to the decimal of 1 ton. Ans. .1798.
4. " 3 qrs. 1 nl. to the decimal of 1 yd. " .8125.
5. " 3 fur. 14 per. 5 yd. 6 in. to the dec. of 1 mile. " .405902.
6. " 16 hrs. 13 min. 14 sec. to the decimal of 2 days. " .337928.
7. Express £4. 13. 6. in £.'s and decimal of £. " 4.675.
8. " 5 miles, 3 fur. 14 per. in the dec. of a mile. " 5.41875.
9. " 44 ac. 3 ro. 14 po. to the decimal of an acre. " 44.8375.
10. " 19 gal. 3 qt. 1 pt. to the decimal of a gallon. " 19.875.
11. " 127 oz. 3 dr. 2 scr. 3 gr. to the decimal of a pound. Ans. $10.6220486\dot{1}$.
12. " 10 feet, 10′, 10″ to the decimal of a foot. " 10.9027.

Art. 180. Page 201.

Required the value of the following decimals :

1. .06789 of a ton (new weight). Ans. 1 cwt. 1 qr. $10\frac{39}{50}$ lb.
2. .06789 of a ton (old weight). " 1 cwt. 1 qr. $12\frac{46}{625}$.
3. $.32\dot{9}\dot{8}$ of a mile. Ans. 2 fur. 25 po. 3 yd. $0\frac{2789}{9000}$ ft.
4. $.\dot{6}78\dot{3}$ of a league. Ans. 2 lea. 0 fur. 11 po. $1\frac{8875}{9990}$. yd.
5. $.8763\dot{4}5\dot{2}$ of a cord. Ans. 112 ft. $297\frac{12643}{23125}$ in.
6. $.32\dot{9}\dot{8}$ of £3. 2. 6. " £1. 0. $7\frac{14}{33}$.
7. $.\dot{4}8\dot{9}$ of \$4.25. " \$2.08$\frac{41}{198}$.
8. $.32\dot{6}78\dot{9}$ of 12 feet, 10 in. " 4 ft. $2\frac{162789}{499950}$. in.
9. $.187\dot{5}4\dot{3}$ of a pound, avoirdupois. " 3 oz. $0\frac{464}{41625}$ drs.
10. $.187\dot{5}4\dot{3}$ of 1 pound, troy. Ans. 2 oz. 5 dwt. $0\frac{1082}{4125}$. gr.

ART. 181. PAGE 202.

Make the following similar and coterminous ::

(1.) 1.36789. Ans. 1.3678967896789678967896789.
4.89678. " 4.89678967896789678.
16.87329876. " 16.87329876876876876.
.18645. " .18645645645645645.
.000023. " .00002323232323232.

(2.) 486.7834. Ans. 486.7834.
298.6743. " 298.6743434.
273.1236. " 273.1236666.
1.4863. " 1.4863486.
101.89635. " 101.8963535.

ADDITION OF CIRCULATING DECIMALS.

ART 183. PAGE 203.

1. Add 24.132, 2.23, 85.24, and 67.6. Ans. 179.2745563.
2. Add 328.126, 81.23, 5.624, 61.6. Ans. 476.65028119.
3. Add 42.1673, 18.96326, 45.98763. Ans. 107.118225128245881918765.
4. Add 3.267, 3.846589452. Ans. 7.11385671992621253316179.

SUBTRACTION OF CIRCULATING DECIMALS.

1. From 1.3967 take .98764. Ans. .4091480319203.
2. From 14.6732 take 11.1987654. Ans. 3.47446682.
3. From 32.9874678 take 11.12343. Ans. 21.864033612472335344415555.

MULTIPLICATION OF CIRCULATING DECIMALS.

ARTS. 184 AND 185. PAGES 203 AND 204.

1. Multiply 7.6849 by 4.17. Ans. 32.046387.
2. " 20.4387 by 6.73. " 137.552746.
3. " 11.681096 by .07. " .8176.
4. " 1.4763 by 5. " 7.3816.
5. " 4.678 by 3.13. " 14.6598.
6. " 8.965 by 333.3. " 2988.518.
7. " 8574.3 by 87.5. Ans. 750780.518.
8. " 7.72 by .297. Ans. 2.29950617283.

DIVISION OF CIRCULATING DECIMALS.

ARTS. 186 AND 187. PAGE 204.

1. Divide 137.552746 by 6.73. Ans. 20.4387.
2. " 32.046387 by 4.17. " 7.6849.
3. " 121.48209 by 7.89. " 15.396.

4. Divide 15.78698 by 419. Ans. 3.767.
5. " 2.297 by .297. Ans. 7.72.
6. " 1.593750841 by .387. Ans. 4.108.
7. " 1611.17588 by 2.363. Ans. 681.738.
8. " 89.3138710 by 3.285714. Ans. 27.1824.

MISCELLANEOUS EXERCISES.

(ON ARTICLES 158—187.)

1. From the sum of $\frac{3}{4}$, $7\frac{4}{9}$ and 3. take the sum of $2\frac{1}{2}$ and $1\frac{1}{12}$, and divide the remainder by $\frac{11}{36}$. Ans. $24\frac{10}{11}$.

2. Whether is the sum of $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$, greater or less than $\frac{3}{4}$ of the sum of $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$ and $\frac{1}{9}$?

Ans. The latter is greater by $\frac{41}{840}$.

3. Divide the product of $3\frac{3}{7}$ and $3\frac{5}{12}$, successively, by their sum and difference. Ans. $1\frac{409}{575}$, and 984.

4. Bought $98\frac{5}{8}$ yds. of cloth, at £1.16 per yd., and sold it at £$1\frac{3}{8}$ per yd., how much did I gain on the whole?

Ans. £20. 10. $11\frac{1}{4}$.

5. From $\frac{3}{8}$ of $1\frac{1}{2}$ ton take $\frac{1}{6}$ of $\frac{3}{5}$ cwt. Ans. $11\frac{3}{20}$. cwt.

6. Divide $1968.30 among 4 men, 5 women and 9 children, giving each woman $\frac{3}{4}$ of a man's share, and each child $\frac{1}{2}$ of a woman's share. Ans. 4 men 707.7032.

5 women, 663.4715.

9 children, 597.1248.

7. A, B and C rent a pasture for £40, A, puts in 8 cattle, B. 9 and C. 11; how much should each pay for his share.

Ans. A. £11. 8. $6\frac{6}{7}$. B. £12. 17. $1\frac{5}{7}$. C. £15. 14. $3\frac{3}{7}$.

8. If a spout runs $8\frac{5}{8}$ gals. in 4 minutes, how long will it take to run 1 gallon? Ans. $\frac{32}{69}$ minutes.

9. A weaver weaves $9\frac{2}{5}$ yds. of cloth in $2\frac{5}{6}$ days; how many yards does he weave per day? Ans. $3\frac{27}{85}$ yds.

10. A tradesman can finish a piece of work in $\frac{2}{7}$ of a day, another can do the same in $\frac{5}{7}$ of a day. What time will they take to do it together; what part of the work will each do, and what part of the price will each get supposing the whole cost to be $96.36.

Ans. In $\frac{10}{49}$ of a day, first will do $\frac{5}{7}$, second will do $\frac{2}{7}$. First will receive 68.82\frac{6}{7}$; second will receive 27.53\frac{1}{7}$.

11. Divide $2250 between three persons so that the second may receive $\frac{3}{4}$ of the first one's share, and the

third $\frac{2}{7}$ of what the two first have together.

Ans. \$1000.00; \$750.00; \$500.00.

12. A watch is now regular, gets out of order and gains 15′ $\frac{1}{8}$ seconds each day; in how many days will it show the right time? Ans. 140$\frac{20}{41}$ days.

13. With 19$\frac{1}{3}$ yds. of silk $\frac{3}{4}$ yds. wide I can line a vestment; how many yards $\frac{4}{5}$ wide will do the same?

Ans. 18$\frac{1}{8}$ yds.

14. Reduce $\frac{5}{33}$, $\frac{7}{51}$, $\frac{8}{11}$ and $\frac{9}{13}$ to decimals and find the continued product of the three first by the last.

Ans.

PRACTICE.

RULE 1. ART. 188. PAGE 205.

1.	126 at £3.	Ans. £378.	5.	1234 at £6.	Ans. £7404.
2.	129 at \$2.	" \$258.	6.	987 at \$3.	" \$2961.
3.	4987 at \$4.	" \$19848.	7.	1783 at £11.	" £19613.
4.	1783 at £5.	" \$35660.	8.	2367 at \$22.	" \$52074.

(RULE 2. PAGE 206.)

1.	1278 at 5 s.	Ans.	£319, 10. 0.
2.	1987 at 6 s. 8 d.	"	£662, 6. 8.
3.	1968 at \$0.50.	"	\$984. 00.
4.	11267 at \$0.33$\frac{1}{3}$.	"	\$3755. 66$\frac{2}{3}$
5.	4878 at \$1.50.	"	\$7317. 00.
6.	9876 at 4 d.	"	£164. 12. 0.

RULES 3, 4, 5 and 6. PAGES 207, 8, 9.

1.	496 at 2 s.	Ans.	£49. 12. 0.
2.	1893 at 2 s.	"	£189. 6. 0.
3.	1763 at 3 s.	"	£264. 9. 0.
4.	1789 at 4 s.	"	£357. 16. 0.
5.	17832 at 9 s.	"	£8024. 8. 0.
6.	17698 at 13 s.	"	£11503. 14. 0.
7.	4299 at 15 s.	"	£3224. 5. 9.
8.	1496 at 16 s.	"	£1196. 16. 0.
9.	11111 at 17 s.	"	£9444. 7. 0.
10.	12121 at 18 s.	"	£10908. 18. 0.
11.	16789 at 19 s.	"	£15949. 11. 0.
12.	14678 at 14 s.	"	£10274. 12. 0.

(RULE 7. PAGE 209.)

1.	487 at 2 s. 7 $\frac{1}{2}$ d.	Ans.	£63. 18. 4$\frac{1}{2}$.
2	1489 at 2 s. 10$\frac{1}{2}$ d.	"	£214. 0. 10$\frac{1}{2}$.
3.	1678 at £2. 14. 6.	"	£4572. 11. 0.
4.	1798 at £3. 16. 3.	"	£6854. 17. 6.

5. 14987 at £2. 14. $3\frac{1}{2}$. Ans. £40683. 9. $2\frac{1}{2}$.
6. 19876 at $\$2\frac{7}{8}$. " \$57143. 50.
7. 1467 at $\$3\frac{5}{8}$. " \$5317. $87\frac{1}{2}$.
8. 1985 at £0. 13. 6. " £1339. 17. 6.
9. 7896 at £2. 19. 6. " £23490. 12. 0.
10. 11112 at £0. 18. 9. " £10417. 10. 0.
11. 7894 at £14. 6. 8. " £113147. 6. 8.
12. 2178 at £15. 15. 5. " £34348. 17. 6.

(RULE 8 & 9. PAGE 212.)

1. $123\frac{7}{8}$ at \$9. $33\frac{1}{3}$. Ans. \$1156. $16\frac{2}{3}$.
2. $1176\frac{1}{8}$ at £2. 14. 6. " £3204. 18. $9\frac{3}{4}$.
3. $1987\frac{1}{12}$ at £1. 14. 6. " £3429. 3. $1\frac{1}{2}$.
4. 126 lbs., 14 oz. at 4 s. 6 d. " £28. 10. 11.
5. 1789 lbs., 9 oz. troy at £1. 6. 8. " £2386. 6. 8.
6. $7896\frac{1}{16}$ at £2. 19. 6. " £23492. 12. $10\frac{3}{4}$.
7. $198\frac{4}{15}$ at \$12. $92\frac{1}{2}$. " \$2571. $21\frac{1}{3}$.
8. $146\frac{5}{16}$ at £3. 4. $6\frac{1}{2}$. " £474. 3. $7\frac{3}{32}$.

(RULE 10. PAGE 214.)

1. 163 cwt., 2 qrs. 14 lbs., at £2. 9. 6. Ans. £404. 19. $5\frac{1}{4}$.
2. 189 cwt., 3 qrs., 15 lbs. at \$44. $62\frac{1}{2}$. " \$8473.572.
3. 178 cwt., 3 qrs., 14 lbs. (new wt.) at £2. 3. 9.
Ans. £391. 6. $5\frac{1}{4}$.
4. 11 cwt., 2 qrs., 11 lbs. (new wt.) at£14. 3. $6\frac{1}{2}$.
Ans. £164. 11. $11\frac{1}{40}$.
5. 166 cwt., 3 qrs., 15 lbs. (new wt.) £2. 13. 9.
Ans. £448. 10. $10\frac{1}{2}$.
6. 16 cwt., 2 qrs. 24 lbs. (old wt.) at £1. 19. 6.
Ans. \$132.042.
7. 3 cwt., 2 qrs., 14 lbs. (old wt.) at \$14.25.
Ans. £12. 19. $6\frac{3}{8}$.
8. 19 cwt., 2 qrs., 15 lbs. (new wt.) at £2. 15. $4\frac{1}{2}$.
Ans.\$217. 62.

(RULE 11. PAGE 217.)

1. 127 tons, 13 cwt., 2 qrs. at £11. 10. 5.
Ans. £1470. 18. $5\frac{3}{8}$.
2. 17 tons, 15 cwt., 3 qrs. at \$15. 55. Ans. \$276. 595.
3. 11 tons, 14 cwt., 2 qrs. at £14. 14. $4\frac{1}{2}$. Ans. £172. 11. $6\frac{1}{20}$.
4. 17 tons, 2 cwt., 1 qr. at £4. 13. 6. Ans. £80. 0. 0. $\frac{9}{40}$.
5. 15 tons, 13 cwt., 1 qr. at \$19. $33\frac{1}{3}$. " \$302. 80.
6. 166 tons, 14 cwt. 3 qrs. at £17. 6. 3. " £2887. 1. $6\frac{3}{16}$.
7 11 tons, 3 qrs. at 126 francs, " \$229. 47.

8. 19 tons, 2 qrs. at $163. 17½. Ans. £776. 2. 0½.

(RULE 12. PAGE 218.)

	Ac.	ro.	po.		
1.	196	3	14	at $126.13.	Ans. $24827.1138.
2.	111	1	29	at £1. 13. 6.	Ans. £186. 12. 11¼.
3.	112	3	39½	at $14.16⅛.	Ans. $1600. 177.
4.	163	3	28	at £0. 16. 3½.	" £133. 10. 7¼.
5.	798	3	14	at £19. 6. 8.	Ans. £15444. 3. 10.

6. 11 1/16 sq. miles of land at £4. 5. 6 pr. ac. £30267. 0. 0.
7. 12 1/16 miles of land at $196.10 per ac. Ans. $236545.62½.
8. 72 1/12 sq. miles of land at $12. 13⅓ pr. ac. $559751. 11⅓.

(RULE 13. PAGE 219.)

1. 19 oz., 3 dwt., 19 grs. at £0. 16. 4½. Ans. £15. 14. 2½.
2. 126 oz., 14 dwt., 20 grs. at $19. 61½. Ans. $2486. 037.
3. 196 oz., 3 dwt., 5½ grs., at £1. 10. 7½. " £300. 7. 5⅓.
4. 72 lbs., 3 oz., 5 dwt., 9 grs. at £0. 17. 6 per oz. Ans. £758. 17. 2½.
5. 21 lbs., 15 dwt., 19 grs. at $6. 30¼ per ounce. Ans. $1593.206.
6. 11 oz., 9 dwts., 13 grs. at £0. 16. 4½. Ans. £9. 7. 11¾.
7. 14 oz., 5 dwts., 16 grs. at $19. 16. Ans. $273. 67.
8. 127 oz., 15 dwts. at £16. 1567. Ans. £2064. 0. 4½.
9. 177 oz., 13 dwts., 9 grs, at £14.1875. Ans. £2520.13.6.
10. 123 oz., 5 grs. at 4s. 6½ d. Ans. £27. 18. 8.
11. 312 oz., 19 dwts., 22 grs. at £18 1/12. Ans. £5920.16.9.
12. 463 oz., 15 dwts., 15 gr., at £19⅞. Ans. $9217. 13. 0½.

(RULE 14. PAGE 220.)

1. 196 yds. 3 qrs., 2 nls. at £16. 3′. Ans. £3215. 12. 6.
2. 148 yds., 2 qrs. at £1.56′. Ans. £232. 13. 0.
3. 172 yds., 3 qrs., 2 nls. at $11. 55. Ans. $1996. 706.
4. 12 yds., 2 qrs., 1 nl. at £1. 13. 3½. Ans. £20. 18. 2½.
5. 12 yds., 1 qr., 2 nls. at £0, 17. 9½. Ans. £11. 0. 2¼.
6. 127 yds., 1 nl. at £0. 14. 7⅛. Ans. £92. 14. 3¾.
7. 1234 yds., 3 qrs., 3 nls. at $1. 52½. Ans. $1883. 28.
8. 1189 yds., 2 qrs., 3 nls. at 19 s. 7¼. Ans. £1166. 2. 9¾.
9. 4123 yds., 1 qr., 1 nl. at $1. 75. Ans. $7215. 79½.
10. 397 yds., 3 qrs., 3 nls. at £7⅛. Ans. £2835. 5. 1⅛.
11. 123 yds., 3 qrs., 1 nl. at £0. 18 9/22. Ans. £113. 19. 3 2/8⅞.
12. 15 yds., 2 qrs., 3 nls. at £0. 6. 3⅜. Ans. £4. 18. 6¼.

SIMPLE PROPORTION.

(ARTICLES 212—213. PAGE 228.)

1. If 367 cwt. 3 qrs. 14 lbs., new weight, of flour cost \$1296.37, what cost 2 tons, 3 cwt., 3 qrs., old weight, at the same rate? Ans. \$172.66$\frac{2}{3}$.

2. What cost 197 bush. of wheat, if 372 bushels of the same quality cost £139. 10. 0? Ans. \$295.50.

3. If 2$\frac{1}{2}$ bush. of beans cost \$7.25, how many bushels can be bought for £18. 13. 6$\frac{1}{2}$ sterling, at par? Ans. 32$\frac{1}{5}$ bush.

4. If 196 bush. potatoes cost £14. 14. 0, in P. E. Island, how many bushels can be purchased in the same place for \$1896.75, N. S. currency? Ans. 7587 bushels.

5. If 160 bbls. of flour can be bought for \$640, how many can be purchased for \$2240.00. Ans. 560 bbls.

6. If a person walk 396 miles in 14 days of 12 hours each, in how many days of nine hours each, can he walk the same distance? Ans. 18$\frac{2}{3}$ days.

7. A hare pursued by a dog, was 96 yds. before him at starting. The dog ran 7 yds. while the hare ran 5. How far did the dog run before overtaking the hare? Ans. 336 yds.

8. If A can cut 3 cords of wood in 14$\frac{1}{2}$ hours, and B can cut 4 cords in 16$\frac{1}{2}$ hours, how many cords can they both cut in 2 days of 10 hours each? Ans. 9$\frac{371}{3641}$ cords.

9. If a board be 11 inches wide, what must be its length to contain 72 sq. ft. Ans. 78$\frac{6}{11}$.

10. At what time between 5 and 6 o'clock, are the hands of a watch together? Ans. 27m., 16$\frac{4}{11}$ sec. past 5 o'clock.

11. If one man can do a piece of work in 14 days. and another person the same piece in 16 dys. what part of it can both do in 4 days? Ans. $\frac{15}{28}$.

12. A town lot 375 feet, 6 in., by 75 ft, 6 in., cost \$472.50; what will be the cost of a similar piece 278 feet, 9 inches by 151 feet. Ans. \$700.84$\frac{1}{2}$.

13. A can do a piece of work in 5 days and B in 6 days, working 11 hours a day; find in what time A and B can do it together, working 10 hours a day. Ans. 3 days.

COMPOUND PROPORTION.

(ARTICLE 214—219. PAGE 233.)

1. If 360 sheep cost \$980, what cost 127 oxen: 9

sheep costing as much as 1 ox? Ans. $3111\frac{1}{2}$.

2. What cost 196 lbs. tea, if 397 lbs. cost £22. 16. 3 sterling: 9 lbs. of the former being equal in value to $7\frac{1}{2}$ of the latter? Ans. $45.36, N. S. cur.

3. If 196 men can reap 567 ac. Irish, in 18 days of 11 hours each, in how many days of 8 hours each, can 127 women reap 200 ac. English; 6 men being able to reap as much as 7 women? Ans. 14 days, 7 h., 56 min.

4. If 126 men dig a trench 263 yds. long, 3 feet deep, and $2\frac{1}{2}$ feet wide in 14 days of 9 hours each; what length of trench which is 11 feet wide, and $3\frac{1}{2}$ feet deep can 15 men dig, working 10 days of 10 hours each?

Ans. 4 yds. 2 feet, $6\frac{1}{4}$ in.

5. If 136 bush. of oats serve 16 horses for 7 weeks, how many bushels will serve 144 horses for 76 days?

Ans. $1898\frac{22}{49}$.

6. If 3 men and 2 boys can reap 12 ac., 3 ro. in 14 days of 10 hours each, how many acres can 14 men and 5 boys reap in 10 days of 8 hours each: the working rates of the men and boys being as 3 to 5.

Ans. 29 ac., 1 ro., 38 $\frac{18}{49}$ po.

7. If $127\frac{1}{2}$ yds. of cloth 18 in. wide cost £228. 16. 8, what will $118\frac{1}{2}$ yds. of yard wide cloth cost at the same rate? Ans. £424. 9. $3\frac{1}{2}$.

8. If 15,000 copies of a book of 110 sheets require 166 reams of paper, how much paper will be required for 5000 copies of a book of 250 sheets of the same size as the former? Ans. $125\frac{25}{33}$.

9. If a person is able to perform a journey of 146.812 miles in $4\frac{1}{12}$ days when the day is 11.06 hours long; how many days will he be in travelling 1055. 6′ miles when the days are 8.5 hours long? Ans. 38.20441 days.

10. If 16 men in a tour of 6 months duration spend £900, how much will it cost 6 men and 3 boys for 2 months; each boy spending $\frac{3}{8}$ as much as a man?

Ans. £133. 11. $10\frac{1}{2}$.

11. If 365 oxen and 120 sheep eat 764 tons, new wt., of hay in 7 months, how many tons, old weight, will be required for 33 oxen and 325 sheep for 8 months at the same rate; 15 sheep eating as much as 2 oxen?

Ans. 156 tons, 3 cwt., 3 qrs., 8 lbs.

12. If 126 men dig a trench 14 yds. long, 13 ft. wide

and 12 ft. deep in 14 days of 8 hours each, in how many days of 10 hours each will 96 men dig a trench 11 yds. long, 10 ft. wide, and 9 ft. deep, supposing that the relative difficulties are as 3 to 5?

Ans. $11\frac{11}{104}$ days.

13. If 163 men dig a trench 110 yds. long, 3 ft. wide, and 8 ft. deep in 6 days of 11 hours each, another trench is dug by half the number of men in 12 days of 7 hours each: how many gallons of water will the latter hold? Ans. 81409.796 gals.

INTEREST.

RULE 1. ARTICLES 220 AND 221. PAGE 238.

Find the interests of the following sums, for 1 year, at the given rates per cent. per annum:

1.	£193 10 6 at $6\frac{1}{4}$ per cent.	Ans. £12 1 $10\frac{3}{4}$
2.	£111 11 11 at $4\frac{1}{8}$ " "	Ans. £4 12 $0\frac{3}{4}$
3.	\$1278.$66\frac{1}{2}$ at $3\frac{1}{2}$ " "	Ans. \$44.$75\frac{1}{3}$
4.	\$1126.72 at 4 " "	Ans. \$45.$06\frac{22}{25}$
5.	£16 16 3 at 4 " "	Ans. £0 13 $5\frac{1}{4}$
6.	£18 17 $6\frac{1}{2}$ at 8 " "	Ans. £1 9 $4\frac{3}{4}$
7.	\$136.763 at $4\frac{1}{6}$ " "	Ans. \$5.698
8.	\$1123.45 at $7\frac{1}{5}$ " "	Ans. \$80.$888\frac{1}{2}$

RULE 2. PAGE 240.

Find the interests of the following sums of money for the given times, and at the given rates per cent. per annum:

1.	£173 10 6 for 4 y. at $4\frac{1}{2}$ per cent.	Ans. £31 4 $8\frac{1}{4}$
2.	£396 10 11 for $2\frac{1}{2}$ y. at 5 "	" £49 11 $4\frac{1}{4}$
3.	\$196.$33\frac{1}{2}$ for 4 y., 7 mo. at 6 per cent.	" \$53.992.
4.	\$1183.505 for 2 y., 9 mo. at $7\frac{1}{2}$ "	" \$244.097
5.	£126 13 $7\frac{1}{2}$ for 2 y., 1 mo. at 5 "	" £13 3 11
6.	£1269 14 6 for 1 y., 8 mo. at $7\frac{1}{4}$ "	" £153 8 6
7.	\$$293\frac{1}{2}$ for 2 y. 7mo. at $4\frac{1}{8}$ "	" \$31.276
8.	\$296.73 for 1 y., 11 mo. at 5 "	" \$28.436
9.	£147 13 6 for 7 mo. at 4 "	" £3 8 $10\frac{3}{4}$
10.	\$1289.55 for 4 mo. at $5\frac{1}{2}$ "	" \$23.64.

RULE 3. PAGE 242.

Find the interests of the following sums of money for the given times, and at the given rates per cent. per annum:

1.	£168 13 6 for 7 mo., 6 days, at $5\frac{1}{4}$ per cent.	Ans. £5 6 3.
2.	£1695 14 9 for 2 y., 5 mo., 8 d., at 4 per cent.	Ans. £164 9 $8\frac{3}{4}$
3.	\$123.46 for 1 y., 2 mo., 12 d. at $3\frac{1}{2}$ per cent.	" \$5.$18\frac{1}{2}$
4.	\$1836.$14\frac{1}{2}$ for 3 y., 120 d. at $4\frac{1}{4}$ per cent.	" \$260.12
5.	£197 14 $3\frac{1}{4}$ for 1 y., 160 d. at 5 per cent.	" £14 5 6
6.	\$1367.$92\frac{1}{2}$ for 130 d. at $4\frac{1}{8}$ per cent.	" \$25.078.
7.	£467 19 $10\frac{1}{2}$ for 4 mo., 11 d. at $4\frac{1}{2}$ per cent.	" £7 13 $3\frac{1}{4}$
8.	\$1111.$11\frac{1}{2}$ for 3 mo., 15 d. at $2\frac{1}{8}$ per cent.	" \$6.886

Rules 4 and 5. Pages 244–248.

Find the interest of

1.	\$987.27 for 89 days at 6 per cent.	Ans. \$14.44½
2.	£137 13 6 for 127 days at 5 per cent.	" £2 7 10¾
3.	£123 14 6½ for 189 days at 4½ per cent.	" £2 17 7¾
4.	\$1234.66 for 111 days at 2¼ per cent.	" \$8.76
5.	£999 19 9½ for 186 days at 4 per cent.	" £20 7 8
6.	\$1483.055 for 86 days at 2½ per cent.	" \$8.73½

Calculate the amount of £146 13 9¼

7.	From 1st May, 1864, till 14th July, 1865, at 4½ per ct.	" £154 12 11
8.	From 22d July, 1863, till 15th Nov., 1864, at 5 per ct.	" £156 7 1
9.	Required the interest on £1987 14 3 from 2d May till 18th October at 7 per cent.	" £64 8 5¾
10.	What is the interest of \$1768.443 from 18th March, 1862, till 17th May, 1864, at 4¾ per cent?	" \$182.04
11.	Find the interest on \$4987.72 from 1st Nov., 1848, till 2d January, 1864, at 3½ per cent.	" \$2795.08½

Rule 6. Page 248.

Find the interest of the following principals, for the given times at 4 per cent. per annum:

1.	£488 13 6 for 89 days.	Ans. £4 9 11¾
2.	\$1144.13½ for 126 days	" \$15.80
3.	\$1001.15 for 1 year, 101 days	" \$51.18¼
4.	£1685 15 6 for 2 years, 11 days.	" £182 16 11
5.	£487 13 2 for 2 years, 16 days.	" £39 17 4
6.	\$489.19 for 1876 days.	" \$100.58
7.	£156 14 for 123 days.	" £2 2 3

Rule 7. Page 249.

1.	What sum will produce for interest £4 5 0 in 1 year at 5 per cent?	" £85 0 0
2.	What principal at 3½ per cent. will produce a yearly income of £6 18 10?	" £198 6 8
3.	What principal lent out for 3 years, 8 mo., at 3½ per cent. will produce for interest £48 2 6?	" £375
4.	What sum of money lent out at 4¾ per cent. will produce for interest \$111.804 2 y., 5 mo.?	" \$978.97 $\frac{21}{40}$
5.	How many guineas must be lent for 117 days at 3½ per cent. so as to receive for interest £0 2 4 $\frac{497}{1825}$.	" 10 guineas.
6.	What sum lent on the 16th March, 1850, till 23d Jan., 1851, at 3⅛ per cent. will produce for interest \$4.50 $\frac{15}{73}$	" \$168.00
7.	What sum must be lent from Mar. 14th till June 8th, at 6 per cent. to produce for interest £3 9 10?	" £247 0 0

Rule 8. Page 250.

1.	In what time will \$4000 amount to \$4340 at 4¼ per cent. per annum?	" 2 years.
2.	In what time will £2833 6 8 amount to £3215 16 8 at 3 per cent.?	" 4½ years.

3. In what time will \$1786.00 amount to \$2076.22½ at 5 per cent.? Ans. 3 yrs. 3 mos.

4. How long must £243 10 be lent at simple interest, at 4¾ per cent., to amount to £271 9 0$\frac{17}{40}$? Ans. 2 yrs. 5 mos.

5. If \$706.26⅔ be lent out on the 17th day of March, at 5½ per cent.; on what day was it paid when the amount was \$723.40? Ans. Aug. 25th.

6. £70 6 0 was lent on June 9th, at 4 per cent., simple interest, and when paid amounted to £70 13 8¾; on what day was it paid? Ans. Dec. 6th.

RULE 9. PAGE 250.

1. At what rate will £236 6 8 produce for interest £17 14 6 in 2½ years? Ans. 3 per cent.

2. If \$3746.67 amount to \$4629.47½ in 4⅙ years, at what rate was it lent? Ans. 4⅚ per cent.

3. At what rate per cent., simple interest, will \$160 double itself in 25 yrs? Ans. 4 per cent.

4. At what rate will \$1900 amount to \$2185 in 3 yrs? " 5 per cent.

5. If £225 amount to £256 10 in 4 years, at what rate was it lent? " 3½ per cent.

6. £593 12 6 was lent 12th May, and the interest due 29th October was £11 1 2¼; required the rate pr ct. " 4 per cent.

RULE 10. PAGE 250.

1. What sum will amount to \$1488.00 at 6 per cent. in 4 years? " \$1200

2. What sum will amount to \$921.05 in 18 months at 6 per cent? " \$845.00

3. What sum must be lent at simple interest at 7 per ct. for 4 yrs and 5 mo., to amount to £571 8 0¼ " £436 9 4

4. What sum will amount to £739 1 8$\frac{61}{100}$ in 1 year, 11 mo., at 4⅛ per cent.? " £684 18 8

5. What sum of money lent out for 56 days at 3½ per cent. will amount to £688 1 0 " £684 7 6

6. What sum of money lent out for 4½ years at 2½ per cent. will amount to £804 15 1⅕ " £723 7 6

7. What sum lent out for 292 days at 2½ per cent. will amount to £844 8 7⅘ " £827 17 6

8. What sum lent June 8th, at 6½ per cent., will amount to \$506.35, Nov. 1st? " \$493.518

9. What sum lent March 3d, at 5 per cent., will amount to \$3653.55⅝ October 28th. " \$3537.73⅛

COMMISSION, INSURANCE, BROKERAGE, ETC.

ART. 222. PAGE 251. RULES 1 AND 2.

1. What is the commission on \$486.27 at 3½ per cent.? Ans. \$17.02

2. What is the commission on £86 5 at 7⅞ per cent.? " £6 15 10

3. Find the commission on £774 11 3 at 5 per cent. " £38 14 6¾

4. Find the commission on \$1987.37½ at 4⅛ per ct. Ans. \$81.98 nearly.

5. What is the brokerage on £196 17 6 at 2s. 6d. per ct. Ans. £0 4 10$\frac{1}{16}$

6. What must be the sum insured at \$5.75 per cent. on goods worth \$7754.50, so that, in case of loss, both the value of the goods and the premium may be repaid? " \$8227.58⅛

7. At $9.10 per cent., what will it cost to insure goods worth $6240 so that, in case of loss, the owner may be entitled to the value of the goods and the premium? $145.26¼.

8. What must be the sum insured at 4½ per cent. on goods worth £1910, so that, in case of loss, the worth of the goods and the premium may be recovered? Ans. £2000.

9. At 7½ per cent., what will be the cost of insuring goods or property worth 500 guineas, so that in case of loss, the worth of the property and the premium of insurance may be repaid? Ans. £42 11 $4\frac{8}{87}$.

10. Find the brokerage on $1121.33⅓ at $0.82½ per cent.
Ans. $9.25.

11. Find the expense of insuring $3200 on goods at $4.72½ per cent., policy-duty $0.25, and allowing ½ per cent. for commission.
Ans. $175.20.

12. Find the cost of insuring goods worth $1963 at 3½ per cent., policy duty $0.33⅓ per cent., commission ½ per cent. Ans. $85.18⅔.

13. Calculate the expense of insuring $1920 at 4 per cent., and policy duty $0.25. Ans. $81.80

DISCOUNT. — Rule 1. Page 256.

Find the present worth of the following bills at the given rates per cent. per annum.

		Drawn.		Discounted.	Answers.
1.	£388 2 6,	Dec. 8, at	6 mos.,	Mar. 25, at 6 per ct.	£383 2 11¾.
2.	£649 13 4,	Nov. 9, "	9 "	Ap. 19, " 5½ " "	£638 8 2.
3.	$2274.55,	Apr. 27, "	7 "	June 8, " 5 " "	$2218.46⅔.
4.	$66.00,	Sept. 25, "	5 "	Nov. 30, " 6¼ " "	$64.98⅓.
5.	£112 19 6,	Aug. 12, "	7 "	Dec. 22, " 5¼ " "	£111 12 6½.
6.	£467 13 6½	May 14, "	5 "	Aug. 14, " 6⅛ " "	£462 13 1½.
7.	$874.33⅓	Aug. 14, "	4 "	Oct. 3, " 4 " "	$867.14 7/12.
8.	$1790.50	June 23, "	6 "	July 8, " 5¾ " "	$1742.26⅔.
9.	$90.00	Mar. 31, "	7 "	May 8, " 6½ " "	$87.13⅓.
10.	$3582.50	Jan. 5, "	11 "	" 9, " 5 " "	$3477.92½.
11.	£146 7 11	Apr. 1, "	8 "	Oct. 3, " 4⅛ " "	£145 7 5.

12. 12 volumes of a work can be bought for a certain sum payable in 6 months: and 13 volumes of the same work can be purchased for the same sum, ready money; what is the rate of discount?
Ans. 16⅔ per cent.

13. For what sum must a note be drawn, at three months, so that the present worth, if discounted at the Bank at 6 per cent., may be $489.00? Ans. $496.59.

14. For what sum must a note be drawn for 2 months, which, when discounted at the Bank at 6 per cent., will liquidate a debt of $189,157? Ans. $191,136.

Rule 2. Page 258.

Find the true present worth of the several amounts in the preceding Rule:

TRUE PRESENT WORTHS.

1. £383 4 2¾	3. $2219.81⅜	6. £462 14 1¾.	9. $87.22
2. £638 12 0	4. $65.00	7. $867.20½	10. £145 7 5½.
	5. £111 12 10½	8. $1743.53½	

STOCKS.—Art. 226. Page 259.

What is the yearly income arising from the investment of

1. £2850 in the 3 per cents at 75. Ans. £114 0 0.
2. \$7129.00 in the $3\frac{1}{2}$ per cents at 85. Ans. \$293,54$\frac{14}{17}$.
3. \$2016.00 in the 4 per cents at 96. Ans. \$84.00.
4. £3500 in the 3 per cent. consols, at $94\frac{1}{8}$, brokerage $\frac{1}{8}$ per cent. Ans. £111.405.
5. \$4896.96 in the 4 per cents, at $87\frac{1}{2}$. Ans. \$223.86$\frac{1}{8}$.

Find the value of the following, $\frac{1}{8}$ per cent. for brokerage being allowed:

6. £183 16 3 in the 3 per cent. consols, at 78. Ans. £143 12 $2\frac{3}{4}$.
7. \$4832.96 in the 4 per cents, at 75. Ans. \$3618.68.
8. £3416 0 0, 3 per cent. stock, at 89. Ans. £3035 19 $4\frac{3}{4}$.
9. £1000 0 0, 4 per cent. stock, at $97\frac{5}{8}$. Ans. £972 10 0.
10. \$136,789.72, bank stock, at $188\frac{7}{8}$ per cent. Ans. \$258,532.57$\frac{8}{100}$.
11. What sum of money must be invested in the 3 per cents., at 85, to give an income of £160 0 0? Ans. £4533 6 8.
12. How much money must a broker invest in the funds when consols are selling at 90, so as to produce the same income as if he had invested £1100 when consols were at 99? Ans. £1000 0 0.
13. A person invests \$90,000 in the $3\frac{1}{2}$ per cents, at 85, and sells out at $82\frac{1}{2}$, how much does he lose by the transaction? Ans. \$2647.06 nearly.

PROFIT AND LOSS.—Art. 227. Page 263.

1. If 169 yds. of cloth be bought at \$1.25 per yd. and sold at \$1.37$\frac{1}{2}$ per yd., what is gained? Ans. \$21.12$\frac{1}{2}$.
2. Bought 186 lbs. of tea for \$116.25, and sold it 8s. $9\frac{1}{2}$d. per lb., what is the gain? Ans. £6 4 0.
3. A person purchased 400 yds. of silk at 7s. 6d. per yd., and sells 300 yds. at 8s. 6d., and the rest, which is damaged, at 25 cents per yd., how much did he lose? Ans. \$65.00.
4. Bought goods at £4 10 0 per cwt., old weight, and sold them at $16\frac{1}{2}$ cents per lb., how much was gained on the sale of 10 cwt. 2 qr. 14 lbs. new weight? Ans. \$1.90.
5. If 84 gals. of wine cost \$449, what is gained by selling it at \$6.30 per gallon? Ans. \$80.20.

Art. 228. Page 264.

1. Bought goods for \$127.50, and sold them for \$137.50, what was the gain per cent.? Ans. \$7$\frac{43}{51}$.
2. Bought cloth at \$2.10 per yd., and sold it at £0 10 0, what was lost per cent.? Ans. $4\frac{16}{21}$.
3. Bought 1987 lbs. of tea at 50 cts., sold 1560 lbs. at $53\frac{1}{2}$ cts. per lb., and the remainder at 55 cts., what was gained per cent.? Ans. $7\frac{1281}{1987}$.
4. Bought a quantity of cotton at 12 cts. per yd. and sold it at $12\frac{1}{2}$ cts., what was gained per cent.? Ans. $4\frac{1}{6}$.
5. A stock broker invested \$75,000 in the 4 per cents, which he sold for \$77,225, what per cent. did he make? Ans. $2\frac{29}{30}$.

Art. 229. Page 264.

1. If molasses cost $34\frac{1}{2}$ cts. per gal., how must it be sold to gain 10 per cent.? Ans. $37\frac{19}{20}$.

2. Bought a stock of goods for $5245; for how much must they be sold to gain 13 per cent.? Ans. $5926.85.

3. Bought oatmeal 15s. per cwt.; how must it be sold to gain 25 per cent.? Ans. £0 18 9.

4. Bought tea at 83⅓ cts. per lb., which, getting damaged, I propose to sell it at a loss of 5 per cent.; required the price. Ans. $0.79⅙.

5. A person having bought goods for £60 10, sells half of them at a gain of 10 per cent.; for how much must he sell the remainder so as to gain 30 per cent. on the whole? Ans. £45 9 6.

6. A merchant having bought a lot of goods for $489.60, sells one fourth at a loss of 5 per cent.; by what increase per cent. must he raise that selling price in order that by selling the remainder at the increased rate he may gain 5½ per cent. on the whole transaction? Ans. $11\frac{158}{2907}$.

Art. 230. Page 265.

1. If I sell coffee at 40 cts. per lb. and gain $17\frac{8}{41}$ per cent., what was the prime cost? Ans. $0.34⅛.

2. By selling an article at $1.00, a person lost 5 per cent.; what was the first cost, and what must he sell it at to gain 4½ per cent.?
Ans. Prime cost, 1.05\frac{5}{19}$; selling price, $1.10.

3. A merchant sold a lot of cloths for $7265, which was 15 per cent. more than cost; how much did they cost? Ans. $6317.391.

4. An importer sold a library for £769 10, which was 12½ per cent. advance on the cost; what was that cost? Ans. £684 0 0.

5. A merchant paid 50 per cent. for importing goods which he sells for $300, thus gaining 25 per cent. on the whole cost; what was the prime cost? Ans. $160.

6. Bought 120 gals. of spirits, into which I put 10 gals. of water, and sold the mixture at $1.80 per gal., realizing a profit of 30 per cent.; what was the prime cost? Ans. $1.50.

DIVISION INTO PROPORTIONAL PARTS AND RECIPROCAL PROPORTION. — Arts. 231–2. Page 266.

1. Divide $1896.35 among 3 men, according to their ages. A. is 40, B. is 50, and C. is 35. Required their shares.
Ans. A.'s, 433.45\frac{1}{7}$; B.'s, 541.81\frac{3}{7}$; C.'s, 921.08\frac{3}{7}$.

2. Three men, in company, gain $1566. A.'s stock is $1600; B.'s, $2400; and C.'s $3200. What is each man's share?
Ans. A.'s, $348; B.'s, $522; C.'s, $696.

3. Divide $1896.32½ among three persons. A., B., and C. in the proportion of $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$.
Ans. A.'s share, 1083.61\frac{3}{7}$; B.'s, 541.80$\frac{5}{7}$; C.'s, 270.90\frac{5}{14}$.

4. A person bequeathed £1890 10 6 to his three children to be divided in inverse proportion to their ages. Their ages are 18, 16, and 13½ yrs. What must be the share of each child?
Ans. £546 13 1$\frac{78}{83}$, £614 19 9$\frac{51}{83}$, £728 17 6$\frac{42}{83}$.

5. Three persons make a joint stock. A. takes 8 shares; B., 11; and C., 12. They lose $216. Required how much each man must advance.
Ans. A., 55.74\frac{6}{31}$; B., 76.64\frac{16}{31}$; C., 83.61\frac{9}{31}$.

6. Gunpowder being composed of 15 parts nitre, 3 parts charcoal, and 2 parts sulphur, find how much of each is required for 1792 lbs.

Ans. Nitre, 1344 lbs. ; charcoal, $268\frac{4}{5}$ lbs. ; sulphur, $179\frac{1}{5}$ lbs.

7. Divide $1860 between A., B., and C., so that as often as A. gets $5 B. shall get $4, and as often as B. gets $3 C. shall get $1.

Ans. A.'s share, $900 ; B.'s, $720 ; C.'s, $240.

8. If a cistern, when in good condition, can be filled by a spout in 2 hours, how long would it take if the cistern has a leak which would empty it in 10 hours? Ans. $2\frac{1}{2}$ hours.

9. Four men agree to make up $1100 among them. The second gives one half of what the first gave, the third three fourths of what the second gave, and the fourth two fifths of the amount paid by the first. What did each man give?

Ans. 1st, $$483.51\frac{59}{91}$; 2d, $$241.75\frac{75}{91}$; 3d, $$181.31\frac{79}{91}$; 4th, $$193.40\frac{60}{91}$.

FELLOWSHIP. — Arts. 232–234. Page 269.

1. Two merchants, A. and B., form a joint capital. A puts in $960, and B. $1440. They gain $320. How ought the gain to be divided between them? Ans. A.'s share, $128 ; B.'s, $192.

2. A. B. and C. form a partnership. A. puts in $1200 of the capital, B. $1600, and C. $2000. They gained $960. What was each man's share of the gain? Ans. A.'s share, $240 ; B.'s, $320 ; C.'s, $400.

3. Three persons enter into partnership. A. advances $6000, B. $4500, and C. $9000. Their gain, at the end of 8 months, is $2560. How much will each partner draw, allowing 6 per cent. on the capital advanced and dividing the balance equally among them?

Ans. A., $$883.33\frac{1}{3}$; B., $$778.33\frac{1}{3}$; C., $$958.33\frac{1}{3}$.

4. Two merchants join stocks. A. advances £980 ; B., £1100. The interest on each man's stock is to be 5 per cent. ; and A. is to have 3 shares for superintending the sales ; while B., acting as traveller, is to have 5 shares. Required each man's share of the 1 year's gain which is £950. Ans. A.'s, £366 5 0 ; B.'s, £583 15 0.

5. A bankrupt owes three creditors, A., B., and C., $1895, $2875, and $400 respectively. His property is worth $4090. What ought they each to receive? Ans. A., $1499.14 ; B., $2274.42 ; C., $316.44.

Art. 235. Page 270.

1. The firm of C., B., and E. lost $4500. C. had $3200 employed for 6 months ; B., $2400 for 7 months ; and E., $1800 for 9 months. What was each partner's loss?

Ans. C.'s, $1655.172 ; B.'s, $1448.276 ; E.'s, 1396.552.

2. A., B., and C. hired a pasture for $60. A. puts in 15 oxen for 20 days ; B., 17 oxen for $16\frac{1}{2}$ days ; and C., 22 oxen for 10 days. What rent ought each man to pay?

Ans. A., $22.486 ; B., $21.024 ; C., $16.490.

3. In a certain adventure A. put $12,000 for 4 months, then adding $8000 he continues the whole for 2 months longer ; B. put in $25,000, and after 3 months took out $10,000, and continued the rest for 3 months longer ; C. put in $35,000 for 2 months, then withdrawing $\frac{2}{7}$ of his stock, continued the remainder 4 months longer. They gained $15,000. What was the share of each?

Ans. A.'s share, $3492.06 ; B.'s, $4761.91 ; C.'s, $6746.03.

4. A gentleman left £2988 to his two sons, aged 12 and 15 years respectively, to be divided in inverse proportion to their ages; the share of each to be invested at 5 per cent. per annum, also the interest arising therefrom, except £30 per annum each for their education, till the younger be 21 years old. When the younger was 16 years old, their uncle died, bequeathing them £11,000 to be divided between them in direct proportion to the amount each then had. What was the share of each, and how much does each now possess?

Ans. Younger, £6157 19 2¾; amount, £8046 7 11½.
Elder, £4842 0 9¼; amount, £6326 18 6½.

ALLIGATION. — ART. 237. PAGE 272.

1. If 16 bushels of oats, at 2s. per bush., 10 bush. of corn, at 3s. 3d. per bush., and 12 bush. of barley, at 3s. 9d. per bush., were mixed together, what would be the price of the mixture? Ans. 2s. 10$\frac{11}{19}$d.

2. A grocer mixes 10 lbs. of tea, at 2s. per lb., 20 lbs., at 2s. 3d. per lb., 30 lbs., at 2s. 6d. per lb. What would the mixture be worth?

Ans. 2s. 4d.

3. A silversmith meted together 4 lbs. of silver 16 carats fine, 14 lbs., 18 carats fine, 20 lbs., 17 carats fine, and 2 lbs. of alloy. Of what fineness was the mass? Ans. 16$\frac{3}{5}$.

4. A grocer mixes 14 lbs. of tea, at 40 cts., 20 lbs., at 50 cts., 20 lbs., at 30 cts. What is a lb. of this mixture worth? Ans. $0.40.

ART. 238. PAGE 272. RULE 1ST.

1. A person mixes four kinds of sugar worth 6 cts., 10 cts., 11 cts., and 12½ cts. per lb. The mixture was worth 10½ cts. per lb. Required the proportion of each.

Ans. 1 lb., 6 cts.; 4 lbs., at 10 cts.; 9 lbs., at 11 cts.; 1 lb., 12½ cts.

2. A man mixes four kinds of oil worth 8s., 9s., 11s., and 12s. per gal. The mixture was worth 10s. per gal. Required the quantity of each.

Ans. 2 gals., at 8s.; 1 gal., 9s.; 1 gal., 11s.; 2 gals., at 12s.

3. How many lbs. of sugar, worth 9d., 10d., and 10½d., must be taken to make a mixture worth 9¾d. per lb.?

Ans. 4 lbs., at 9d.; 3 lbs., at 10d.; and 3 lbs., at 10½d.

4. In what proportions must three kinds of tea, which cost 40 cts., 45 cts., and 50 cts. respectively, be mixed so that the mixture may be sold at 47½ cts. per lb.?

Ans. 5 lbs., at 40 cts.; 10 lbs., at 45 cts.; and 25 lbs., at 50 cts

RULE 2. PAGE 274.

1. How many lbs. of sugar, at 12½ cts. and 15 cts. per lb., must be mixed with 42 lbs. at 10 cts., so that the mixture may be worth 12 cts. per lb.? Ans. 24 lbs., at 12½ cts.; and 24 lbs., at 15 cts.

2. How much gold, 16, 18, and 22 carats fine, must be mixed with 10 oz., 24 carats fine, that the mixture may be 20 carats fine?

Ans. 10 oz. 16 car. fine, 5 oz. 18 car. fine, 5 oz. 22 car. fine.

3. How much sugar, at 6d., 5$\frac{3}{8}$d., and 4$\frac{1}{8}$d. per lb., must be mixed with 30 lbs. at 3$\frac{5}{8}$d. per lb., that the compound may be worth 4$\frac{5}{8}$d. per lb.?

Ans. 29 lbs., at 6d.; 2 lbs., at 5$\frac{3}{8}$d.

4. How much tea, worth 6s. per lb., must be mixed with 12 lbs., at 3s. 8d., so that the mixture may be worth 4s. 4d. per lb.? Ans. 4⅘ lbs.

RULE 3. PAGE 274.

1. It is required to make 27 lbs. of a mixture worth 4s. 4d. per lb. It is to contain ingredients at 3s. 6d. and 5s. per lb. How much of each must be taken? Ans. 15 lbs. at 5s. and 12 lbs. at 3s. 6d.

2. How much sugar, worth 10 cts., 12 cts., and 15 cts. per lb., must be mixed together to form 128 lbs., worth 12½ cts. per lb.?
Ans. 40 lbs., at 10 cts.; 40 lbs., at 12 cts.; and 48 lbs., at 15 cts.

3. How must rice, which cost 3, 4, and 5 cents per lb., be mixed to be worth 4½ cts., and how much of each sort will be required to form 1236 lbs. of the mixture?
Ans. 309 lbs., at 3 cts.; 309 lbs., at 4 cts.; and 618 lbs., at 5 cts.

4. A merchant bought 280 pairs of braces and gloves for $79.33⅓, — the braces at 20 cts., and the gloves at 25 cts. per pair. How many pairs of each were there? Ans. 200 prs. gloves and 80 prs. braces.

CONJOINED PROPORTION. — ART. 239. PAGE 276.

1. If £1 stg. is worth $5 in Nova Scotia, and $4.86⅔ in New Brunswick, what is $1.00 N. S. worth in the latter place? Ans. $0.97⅓

2. If $162½ United States is worth £22. 10. 0. stg, and 5 shillings stg. worth $1.25 N. S, what is $1.00 N. S. worth in U. S.? Ans. $1.44⁴⁄₉.

3. If £1 stg. is worth £1. 4. 0. Newfoundland, and 5 shillings stg. worth $1.25 N. S., what is $1.00 N. S. worth in N. F. Ans. £0. 4. 9⅗d.

4. If 40 lbs. of sugar are worth 30 lbs. of butter, and 16 lbs. of butter worth 25⅗ lbs. of rice, and 20 lbs. of rice worth 12½ doz. eggs, how many dozens of eggs must be given for 130 lbs. sugar? Ans. 97½ doz.

5. If 100 lbs. U. States make 95 lbs. Italian, and 19 lbs. Italian, 25 lbs. in Persia, how many lbs. in U. S. are equal to 50 lbs. in Persia?
Ans. 40.

EXCHANGE. — ARTS. 240–246. PAGES 278–281.

1. What is the value of £105, at 25 francs 15 centimes per £?
Ans. 2640 fr. 75 cent.

2. What sum should be placed to my credit at St. John, N. B., when I remit $486.25 N. Scotia currency at par? Ans. $473.38⅓.

3. How much should I receive in P. E. Island for $1986.16 N. Scotia, at par? Ans. £595. 16. 11⅓.

4. What is the value of $1234.90 N. Scotia in Newfoundland at par?
Ans. £371. 7. 6¼.

ART. 248. PAGE 282.

1. A merchant in Truro negotiated a bill on London for £129. 16. stg., at 13½ per cent. premium; what did it cost? Ans. $653.32½.

2. What will a bill on Dublin for £1000 10. stg. cost in St. John, N. B., at 14 per cent. premium? Ans. $5069.20.

3. A merchant in Boston wishes to purchase a bill on London for £127 stg.; how much will he have to pay, exchange being quoted at 172¼?
Ans. $972.25⅛.

4. What will a bill for £1867 16 3 sterling cost in New York, exchange at 172 per cent? Ans. $10456.16½.

5. If gold is quoted in New York at 60 per cent premium, what is the rate of exchange between that place and London, and what is the value of £180 stg. Ans. Rate of exchange, 74⅔ per cent. prem. Value, $1395.20.

ART. 249. PAGE 283.

1. A merchant in Boston bought a bill of exchange on Liverpool, G. B., for $1987, at 65 per cent. prem.; for what amount was the bill drawn? Ans. £270 19 1.

2. A commission merchant in Halifax sold goods for a London firm to the amount of $1052.63; for what amount sterling should a bill be drawn, allowing himself 5 per cent. commission on the sales, exchange being 13½ per cent. prem.? Ans. £208 13 5½.

3. What is the value in Dublin of $1896.25 U. States currency, exchanged at 160 per cent.? Ans. £266 13 2½.

4. London remits to Quebec £4590 0 0 stg.; what must be received for it, exchange at 13 per cent.? Ans. $23052.00.

ART. 250. PAGE 284.

1. What will a bill for £189 stg. cost at 22¼ per cent. premium? Ans. £231 1 0⅗.

2. When exchange on England is quoted at 120½ per cent., what is the value of a bill for £489 10 stg.? Ans. £589 16 11¼.

3. A merchant in Newfoundland wishes to get a bill on Liverpool, G. B., for £189 16 8. currency; for what amount sterling is the bill drawn, exchange at 21¾ per cent.? Ans. £155 18 0¾.

ART. 251. PAGE 285.

1. If exchange between London and Hamburg is £1 for 13 marks 14 schillings, and between Hamburg and Paris 100 marks for 186 francs 4 centimes, how many francs in Paris through Hamburg is worth £1 in London, one mark being equal to 16 schillings? Ans. 25 francs 82 cent.

2. If exchange between London and Paris is 28 francs per pound, stg., and between Paris and Boston 18 cents per franc, what is the rate of exchange between Boston and London through Paris? Ans. $5.04 per £1 stg.

INVOLUTION.—ART. 253. PAGE 287.

Involve the following numbers to the powers denoted by their respective indices:

1. 36^2. Ans. 1296.
2. 148^3. " 3241792.
3. 296^4. " 7676563456.
4. $.0067^4$. Ans. .0000000020151121.
5. $.1\dot{6}^4$. Ans. .0007716+.
6. $.\dot{3}^8$. " .00015241+.
7. $(\frac{81}{144})^2$. Ans. $\frac{6561}{20736}$.
8. $(\frac{5}{9})^8$. " $\frac{690625}{43046721}$.
9. $1.\dot{9}^3$. " 8.
10. 26.1^4. Ans. 46407.0641.
11. 11.1^5. " 168505.8155.
12. $.10^8$. .00000001

EVOLUTION. — Art. 255. Page 289.

Find the square root of the following:

1. 87.	Ans. 9.327+.		7. $\frac{25}{49}$.	Ans. $\frac{5}{7}$.
2. 4761.	" 69.		8. $\frac{121}{256}$.	" $\frac{11}{16}$.
3. 7056.	" 84.		9. $17\frac{3}{8}$.	Ans. 4.1683+.
4. 9801.	" 99.		10. $\frac{5}{8}$.	" 79056+.
5. 152399025.	" 12345.		11. $34967\frac{2}{11}$.	" 186.9951+.
6. 10342656.	" 3216.		12. $207\frac{25}{36}$.	" 14.4116+.

Art. 260. Page 293.

Required the cube root of the following numbers:

1. 91125.	Ans. 45.	7. 20.570824.	Ans. 2.74.
2. 140608.	" 52.	8. .241804367.	" 0.623.
3. 571787.	" 83.	9. $\frac{125}{343}$.	" $\frac{5}{7}$.
4. 2515456.	" 136.	10. $44\frac{3}{5}$.	Ans. 3.5463+.
5. 10218313.	" 217.	11. $49\frac{8}{27}$.	" $3\frac{2}{3}$.
6. 11543.176.	" 22.6.	12. $\frac{4}{15}$.	" .643659+.

COMPOUND INTEREST. — Arts. 262–264. Page 296.

Find the amounts of the following sums, at the given rates per cent. per annum:

1.	$960	for 10 years,	at 7 per cent.	Ans.	$1888.464.
2.	$1000	" 9 "	at 5 " "	"	$1551.328.
3.	$1460	" 12 "	at 4 " "	"	$2337.506.
4.	$5000	" 20 "	at 6 " "	"	$16035.675.
5.	£198 10	" 30 "	at 5 " "	"	£857 18 1¼.
6.	£136 15	" 25 "	at 3 " "	"	£286 6 5¾.
7.	$10000	" 40 "	at 7 " "	"	$149744.
8.	£1674	" 10 "	at 4 " "	"	£2477 18 6¾.
9.	£1000	" 4 "	at 5 " "	"	£1215 10 1¼.
10.	£198 16 3	" 11 "	at 6 " "	"	£375 12 1¼.

Art. 264. Page 298.

1. What sum of money, lent out at compound interest, at 6 per cent., will amount to $35948.37 in 15 years? Ans. $15000.00.

2. A person wishes his son, who is three years old, to have $11782.80 when he comes to the age of 21 years; how much must he deposit in the bank to amount to that sum, at 5 per cent. compound interest? Ans. $4896.00.

TABLE OF CONTENTS.

TABLE OF CONTENTS.

TABLE OF CONTENTS.

TABLE OF CONTENTS.